Title: Cosmology of the Lee-Wick Model

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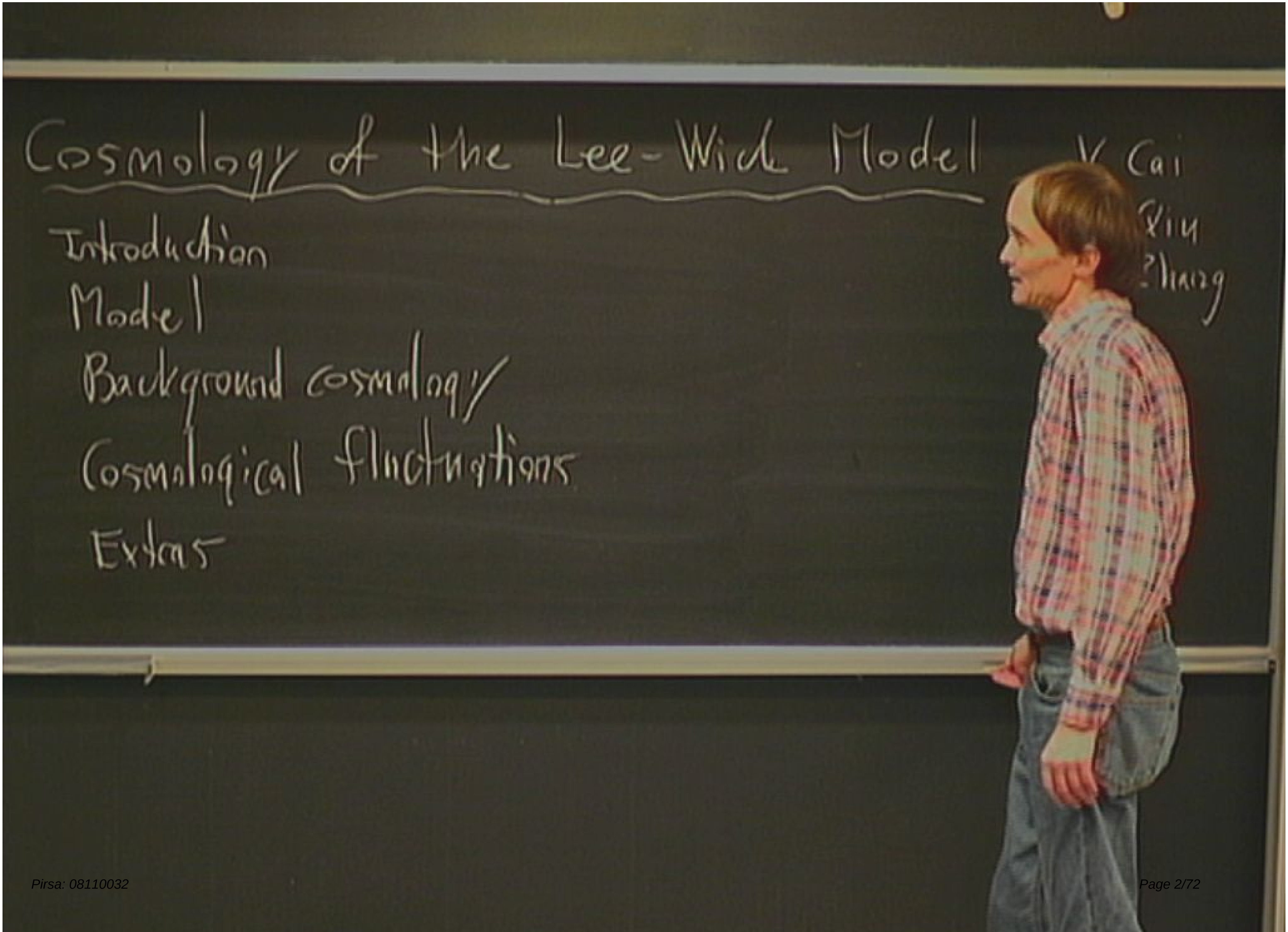
URL: http://pirsa.org/08110032

Abstract: The Lee-Wick model has recently been put forwards as an alternative to supersymmetry for solving the hierarchy problem of particle physics. I will show that, modulo important consistency questions, coupling the Lee-Wick model to cosmology leads to a bouncing universe cosmology with a scale-invariant spectrum of cosmological fluctuations emerging from quantum vacuum fluctuations in the contracting phase.


Cosmology of the Lee-Wick Model
Cai
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Background cosmology
Cosmological fluctuations
Extras

Cosmology of the Lee-Wick Model

Y. Cai
T. Qiu
X. Zhang

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1969 Lee & Wick

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

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1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr.

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr

Higgs sector of Lee-Wick SM

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$

Higgs sector of Lee-Wick SM

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non singular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr

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$\longrightarrow$ scale-inv. sp. of conf. fluct.

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2 - \lambda(\phi-\tilde{\phi})^4$$

→ scale-inv. sp. of conf. flucts.

"Higgs" sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$
$$-\lambda(\phi-\tilde{\phi})^4$$

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$
$$- \lambda(\phi - \tilde{\phi})^4$$

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr.

ϕ original Higgs

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$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2 - \lambda(\phi-\tilde{\phi})^4$$

ghost!

1969 Lee & Wick

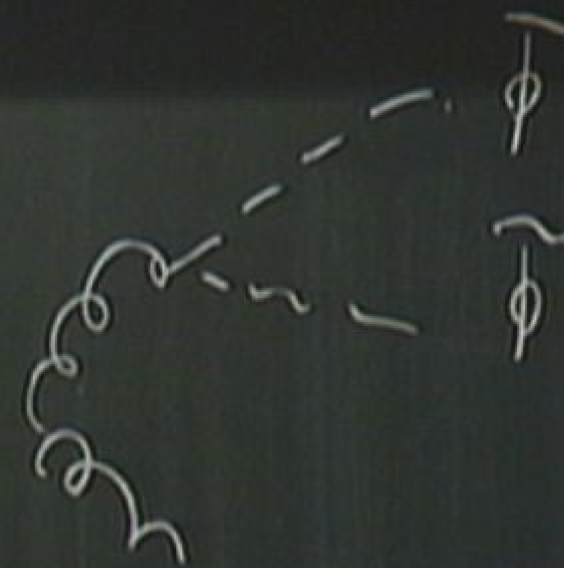
2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase ... fluctr.

Lorentz invariance

stability



$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\ldots$$

$$S_{tot} = S_g\ \text{ghost}\ + \int d^4x\sqrt{-g}\ \mathcal{L}_{LW} \qquad \cancel{\mathcal{L}} - \lambda(\phi-\tilde{\phi})^4$$

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

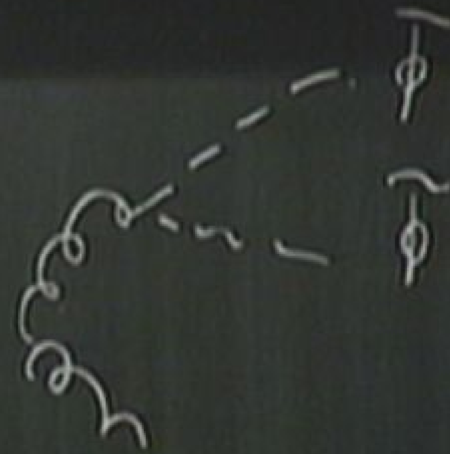
nonsingular cosmology

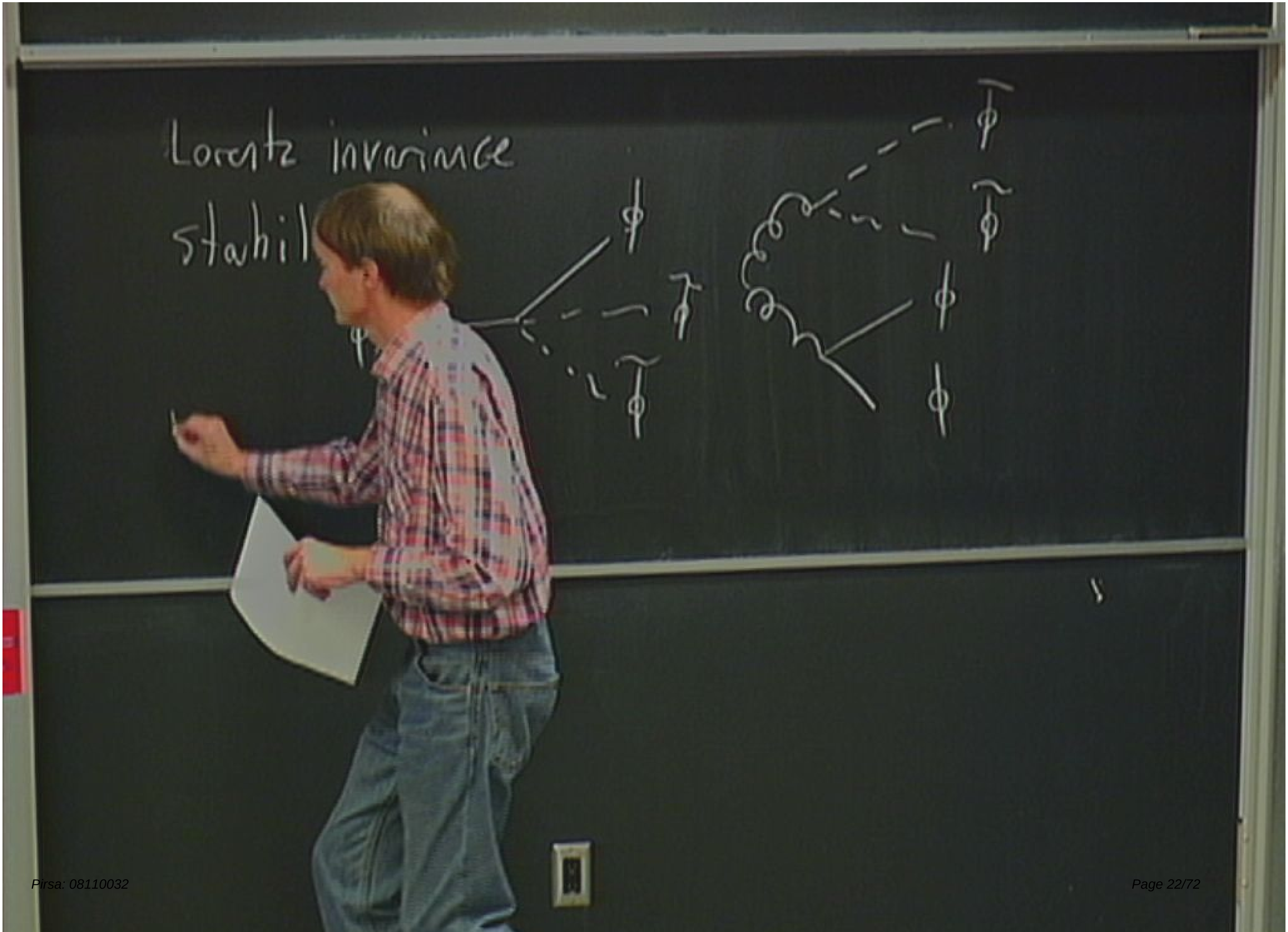
vacuum fluctr. in contracting phase

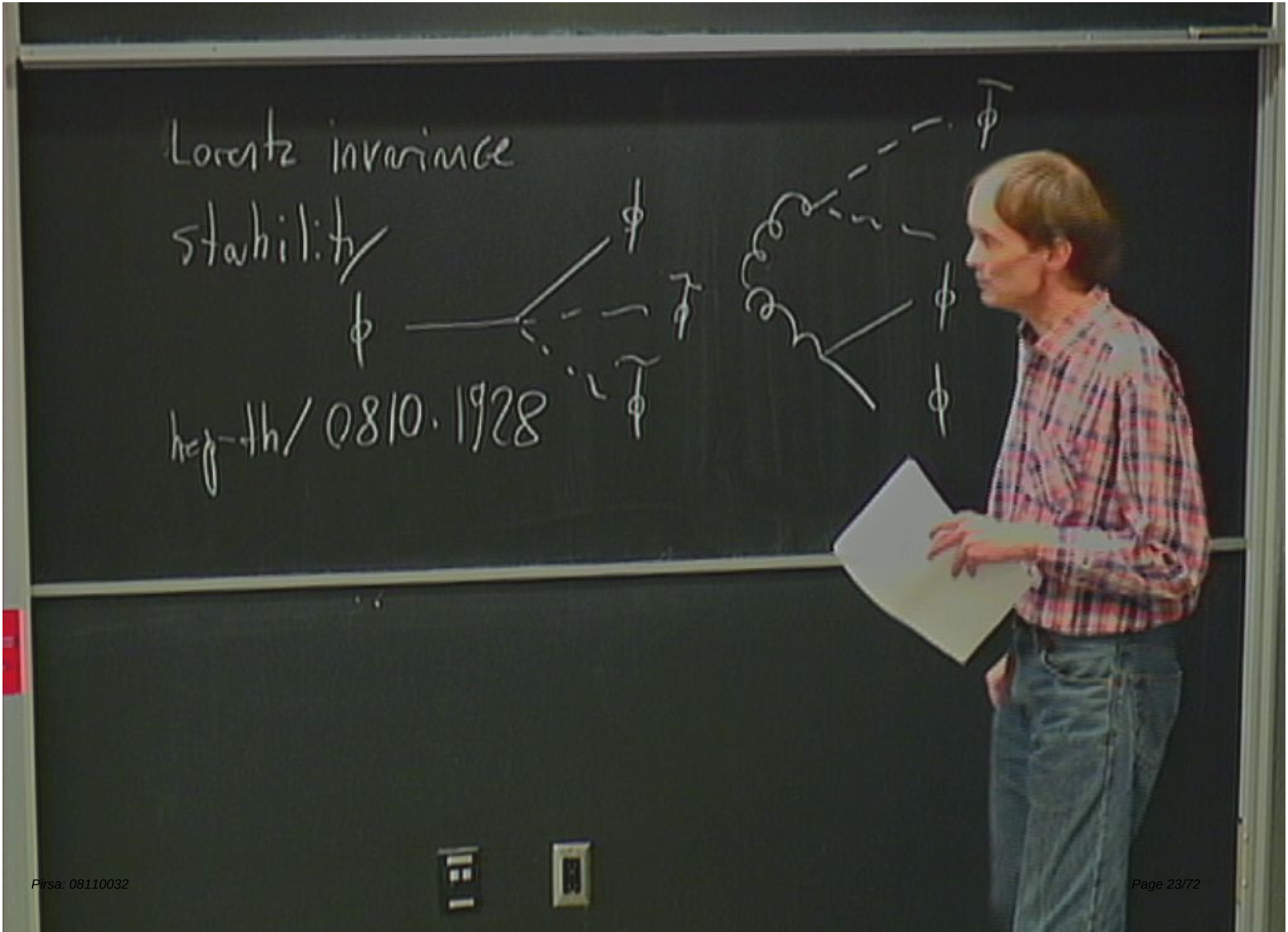
$\rightarrow$ scale-inv. sp. of cosm. fluctr.

Lorentz invariance

stability ✓

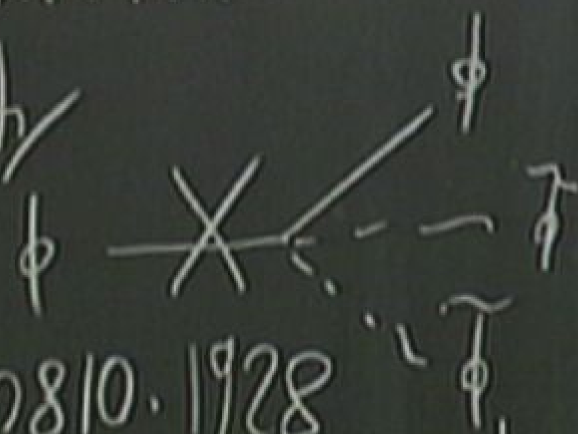


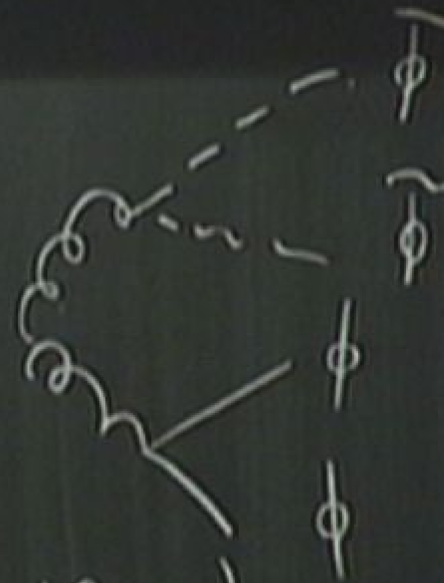

Lorentz invariance
stabil


Lorentz invariance
stability
hep-th/0810.1928

Lorentz invariance

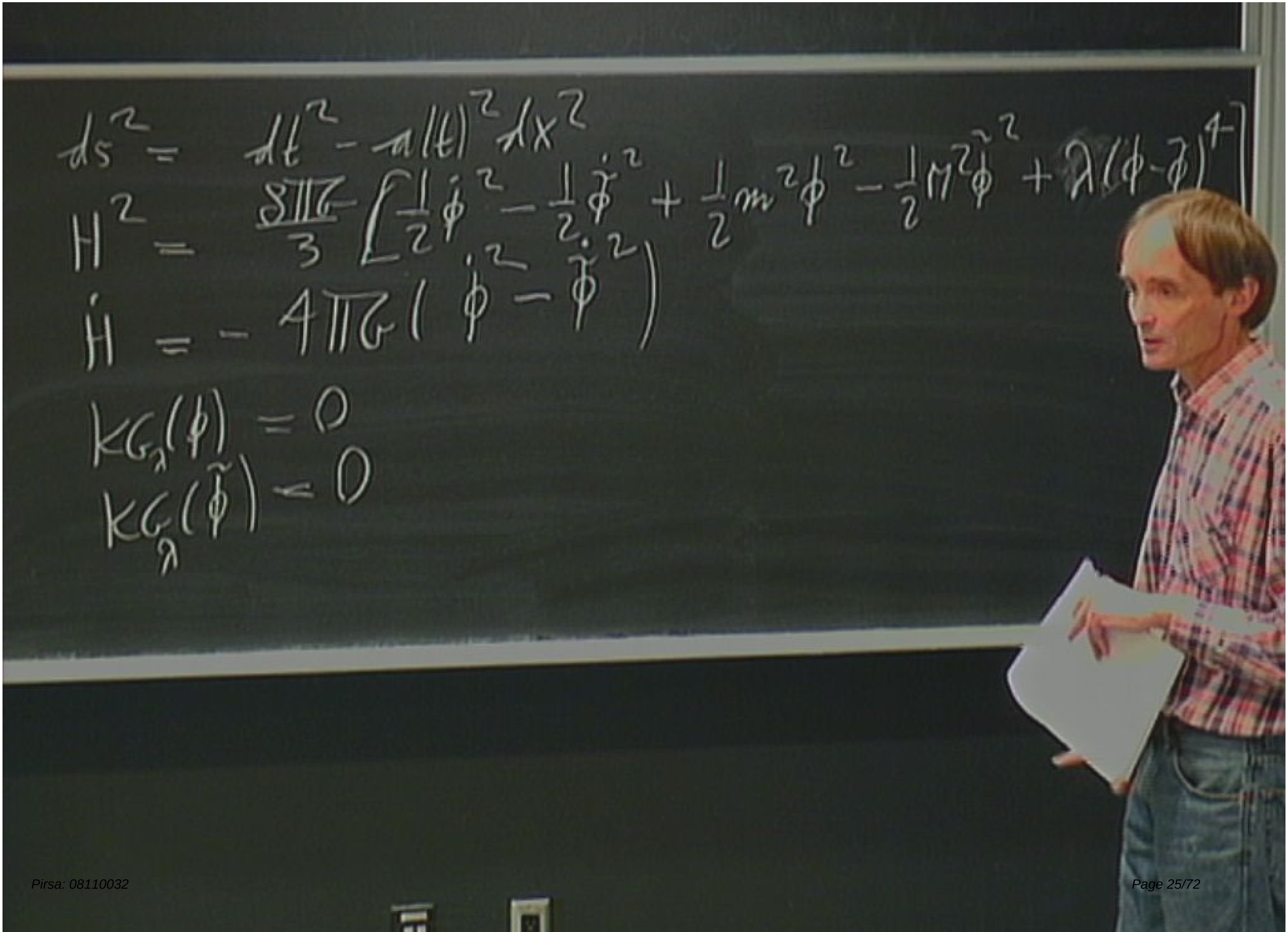
~~stability~~

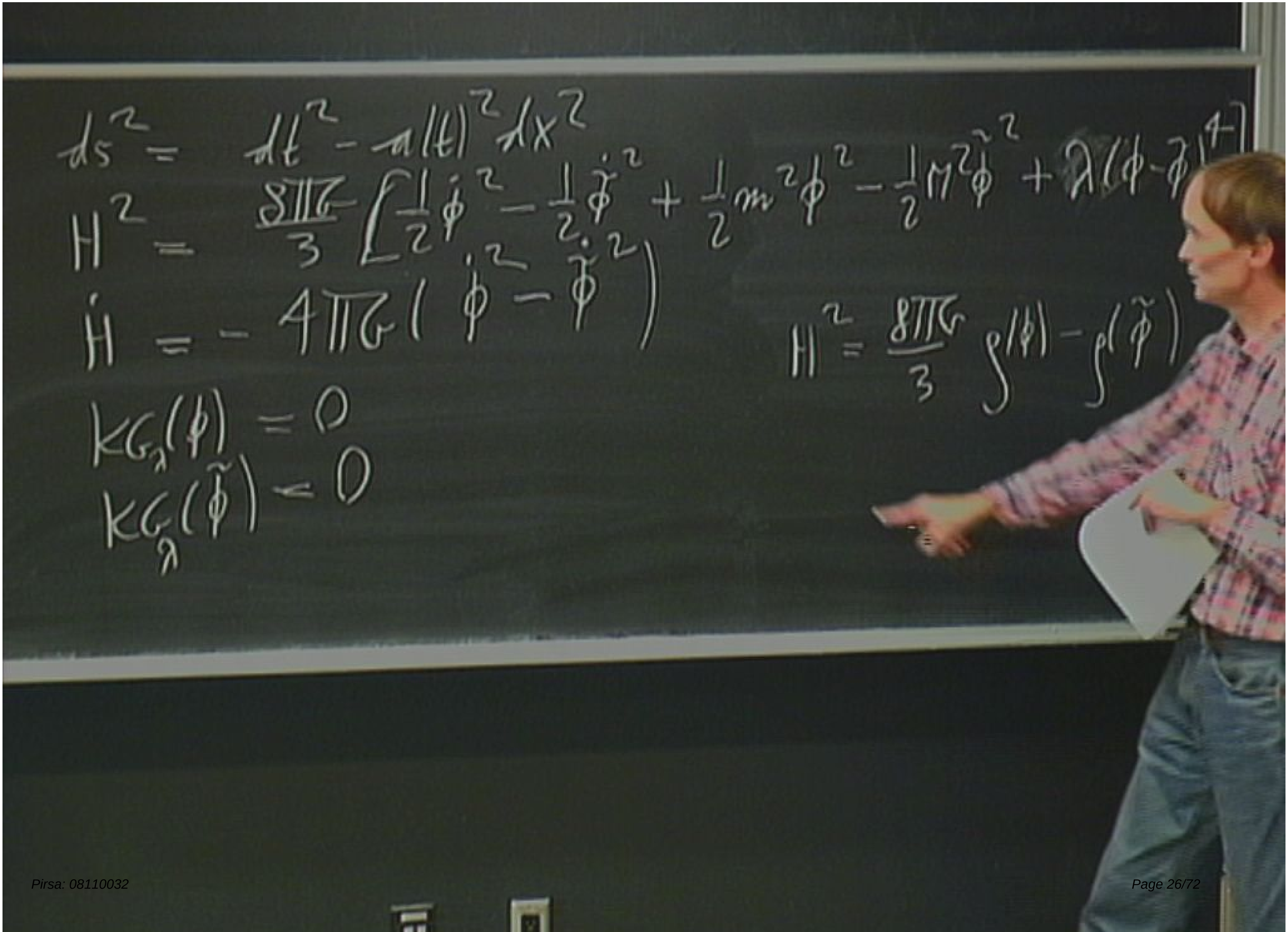




hep-th/0810.1928

A. van Tonder (Brown)


$ds^2 = dt^2 - a(t)^2 dx^2$
$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2 + \lambda(\phi - \tilde{\phi})^4\right]$
$\dot{H} = -4\pi G(\dot{\phi}^2 - \dot{\tilde{\phi}}^2)$
$KG_\lambda(\phi) = 0$
$KG_\lambda(\tilde{\phi}) = 0$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$\dot H = -4\pi G(\dot\phi^2 - \dot{\tilde\phi}^2)$
$H^2 = \frac{8\pi G}{3}\left(\rho(\phi) - \rho(\tilde\phi)\right)$
$KG_\lambda(\phi) = 0$
$KG_\lambda(\tilde\phi) = 0$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 - \frac{1}{2}\dot{\tilde\phi}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde\phi^2 + \lambda(\phi-\tilde\phi)^4\right]$$

$$\dot H = -4\pi G\left(\dot\phi^2 - \dot{\tilde\phi}^2\right)$$

$$H^2 = \frac{8\pi G}{3}\left[\rho(\phi) - \rho(\tilde\phi)\right]$$

$$KG_\lambda(\phi) = 0$$

$$KG_\lambda(\tilde\phi) = 0$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

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$$KG_\lambda(\phi) = 0$$

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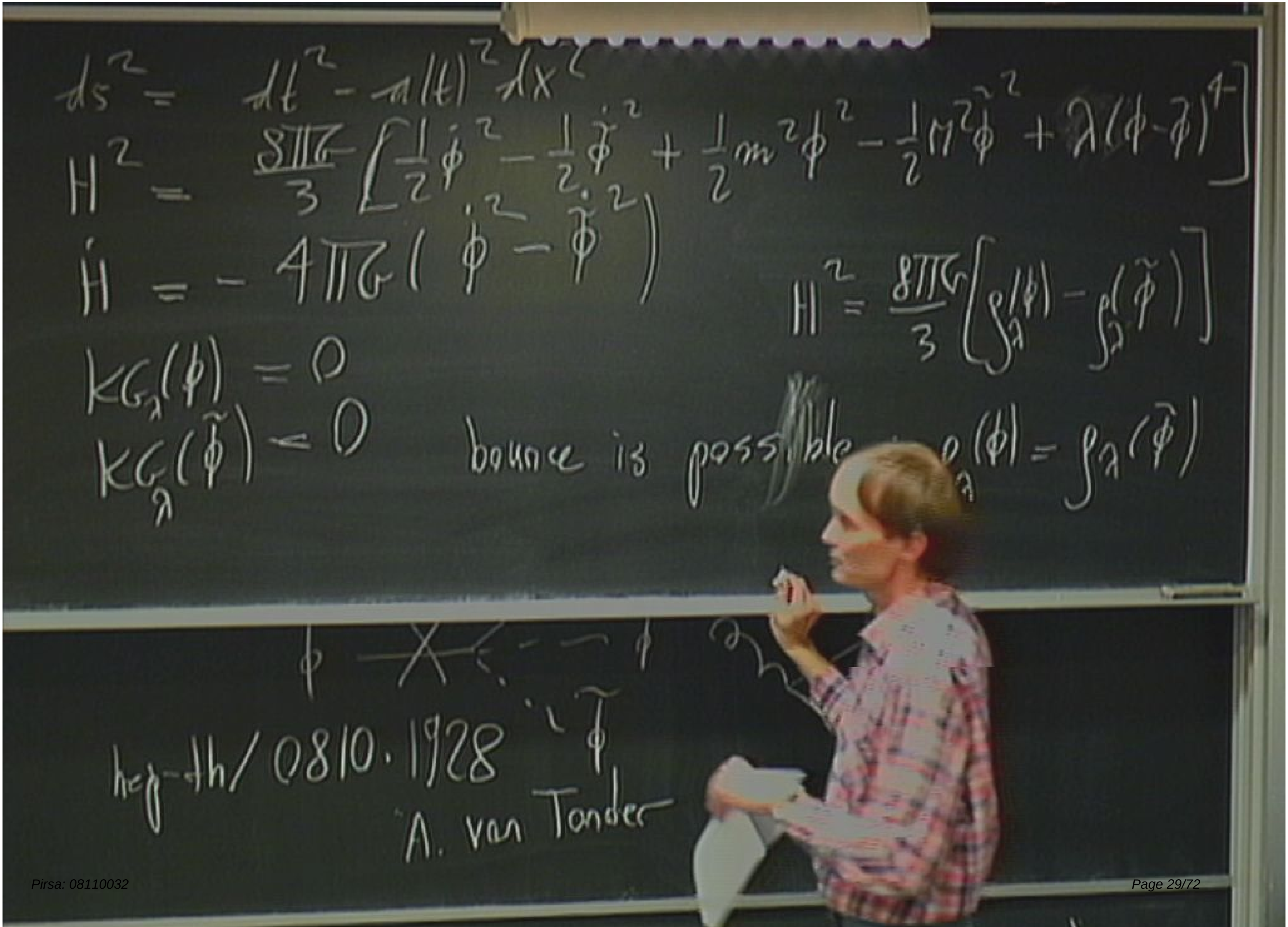
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$KG_\lambda(\phi) = 0$
$KG_\lambda(\tilde\phi) = 0$
bounce is possible $\rho_\lambda(\phi) = \rho_\lambda(\tilde\phi)$
hep-th/0810.1928
A. van Tonder

$$ds^2 = dt^2 - a(t)^2 dx\,dx$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2 + \lambda(\phi-\tilde{\phi})^4\right]$$

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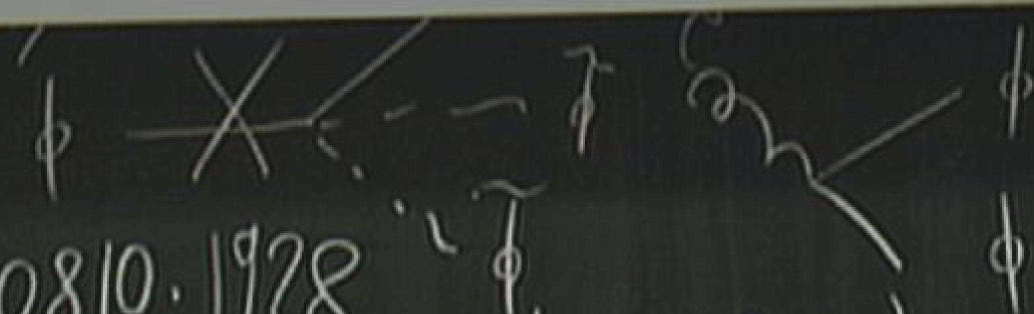
bounce is possible : $\rho_\lambda(\phi) = \rho_\lambda(\tilde{\phi})$



hep-th/0810.1928

A. van Tonder (Brown)

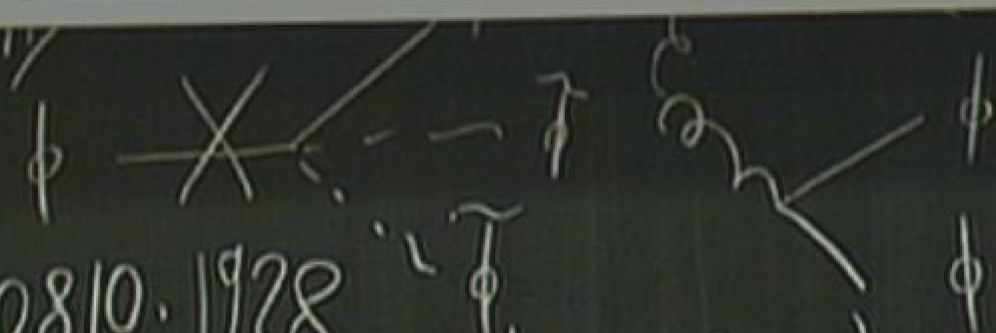
$$\dot{H} = -4\pi G(\phi - \phi)$$

$$H^2 = \frac{8\pi G}{3}\left[\rho(\phi) - \rho(\tilde\phi) + \tilde\lambda(\phi-\tilde\phi)^4\right]$$

$$KG_\lambda(\phi) = 0$$

$$KG_\lambda(\tilde\phi) < 0$$

bounce is possible : $\rho(\tilde\phi) = \rho(\phi) + \lambda(\phi-\tilde\phi)^4$



hep-th/0810.1928

A. van Tonder (Brown)

$$S_{tot} = S_g + \int d^4x \sqrt{-g}\, h_{LW}$$

IC: matter-dom. contraction

$S_{tot} = S_g + \int d^4x \sqrt{-g}\, \mathcal{L}_{LW}$

IC: matter-dom. contraction

stage 1: $\phi(t)$, $\tilde{\phi}(t)$ oscillating

$$S_{tot} = S_g + \int d^4x \sqrt{-g}\; h_{LW}$$

I.C.: matter-dom. contraction

stage 1: $\phi(t)$, $\tilde{\phi}(t)$ oscillating

$A(t)$, $\tilde{A}(t)$

$\rho(\tilde{\phi}) \ll \rho(\phi)$

$S_{tot} = S_g + \int d^4x \sqrt{-g}\, h_{LW}$

IC: matter dom. contraction

stage 1. $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

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stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

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$\tilde{A}(t) \sim a(t)^{-3/2}$

IC: matter-dom. contraction

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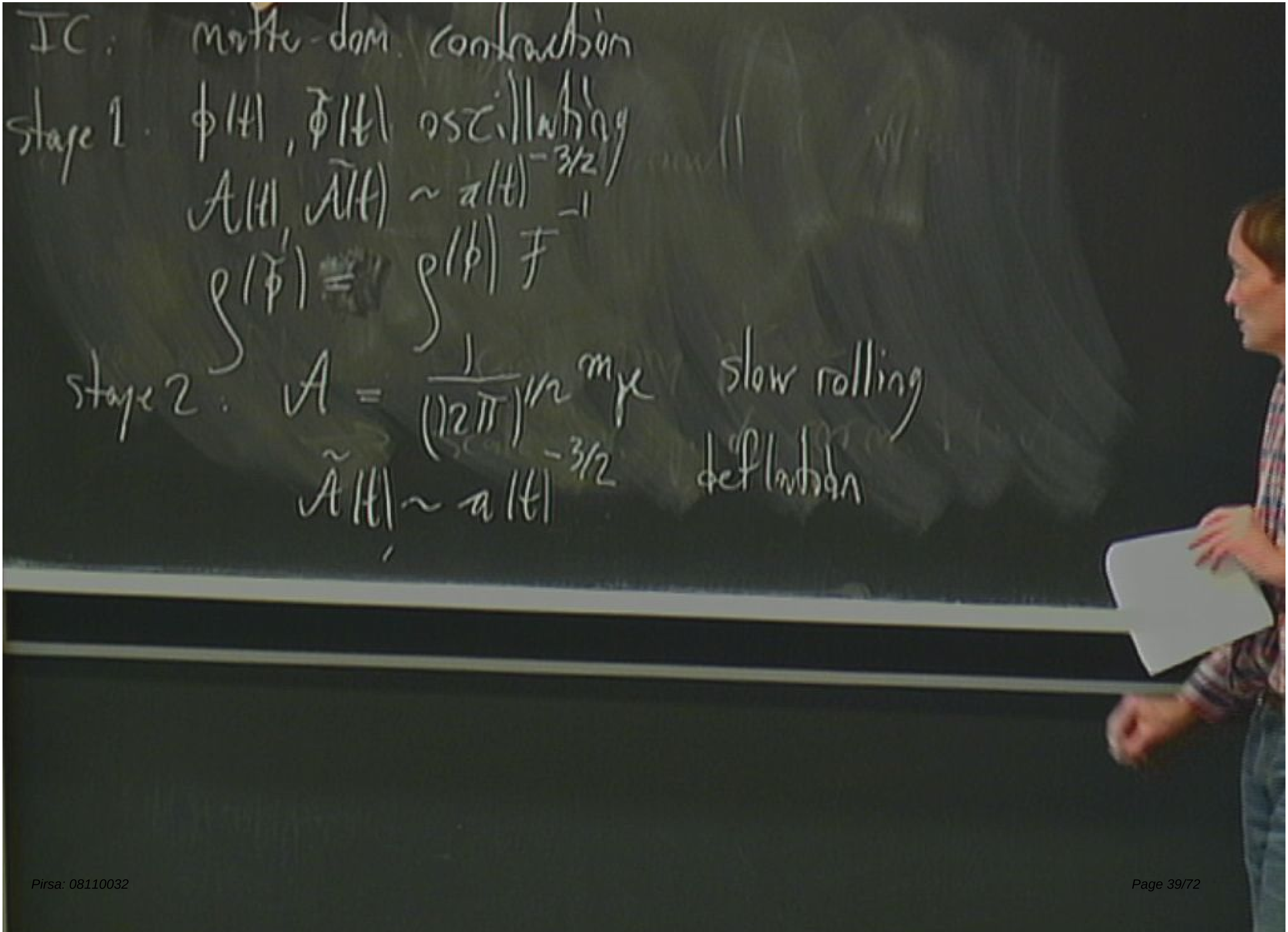
IC: matter-dom. contraction
stage 1: $\phi(t)$, $\tilde{\phi}(t)$ oscillating
$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$
$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$
stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling
$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation

IC: matter-dom. contraction

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$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H$

IC: matter-dom. contraction

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) \cong \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln F$

IC: matter-dom. contraction $\rightarrow$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating $\quad\Vert$ stage 4

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\quad \Delta t\, H \sim \ln F$

IC: matter-dom. contraction $\to$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) \cong \rho(\phi) F^{-1}$

stage 4 $= (\text{stage } 2)^{-1}$

" 5 $= (\text{stage } 1)^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln F$

IC: matter-dom. contraction $\to$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating || stage 4 $= (\text{stage } 2)^{-1}$

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$ " 5 $= (\text{stage } 1)^{-1}$

$\rho(\tilde{\phi}) = \rho(\phi) \mathcal{F}^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln \mathcal{F}$

IC: matter-dom. contraction $\to$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating $\quad$ stage 4 $= (\text{stage } 2)^{-1}$

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$ $\quad$ " 5 $= (\text{stage } 1)^{-1}$

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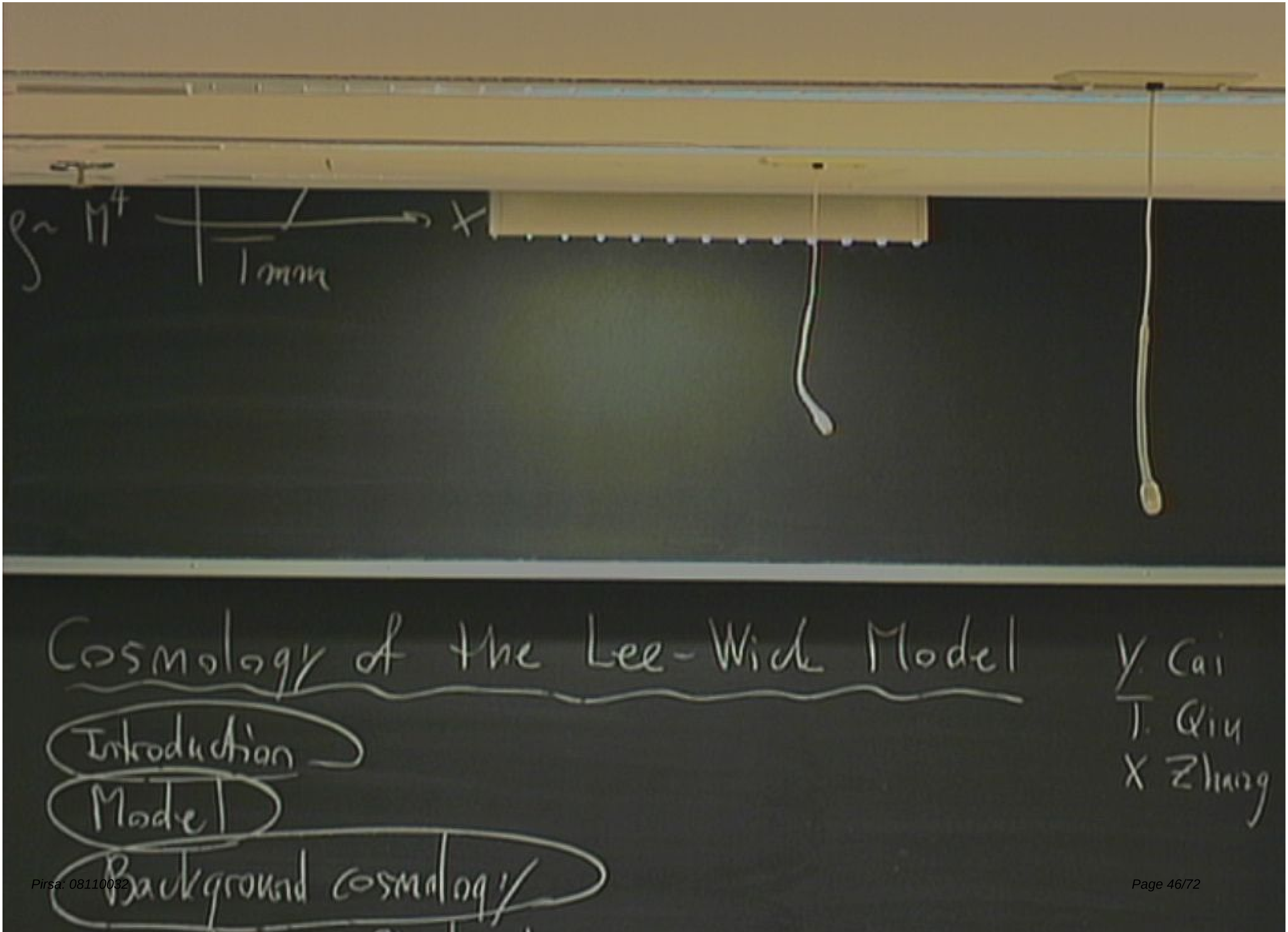
stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln \mathcal{F}$


ρ ~ M^4
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$$ds^2 = dt^2 - a(t)^2 dx^2 \qquad \rho = 0 \quad \tilde{\rho} < 0$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2 + \lambda(\phi-\tilde{\phi})^4\right]$$

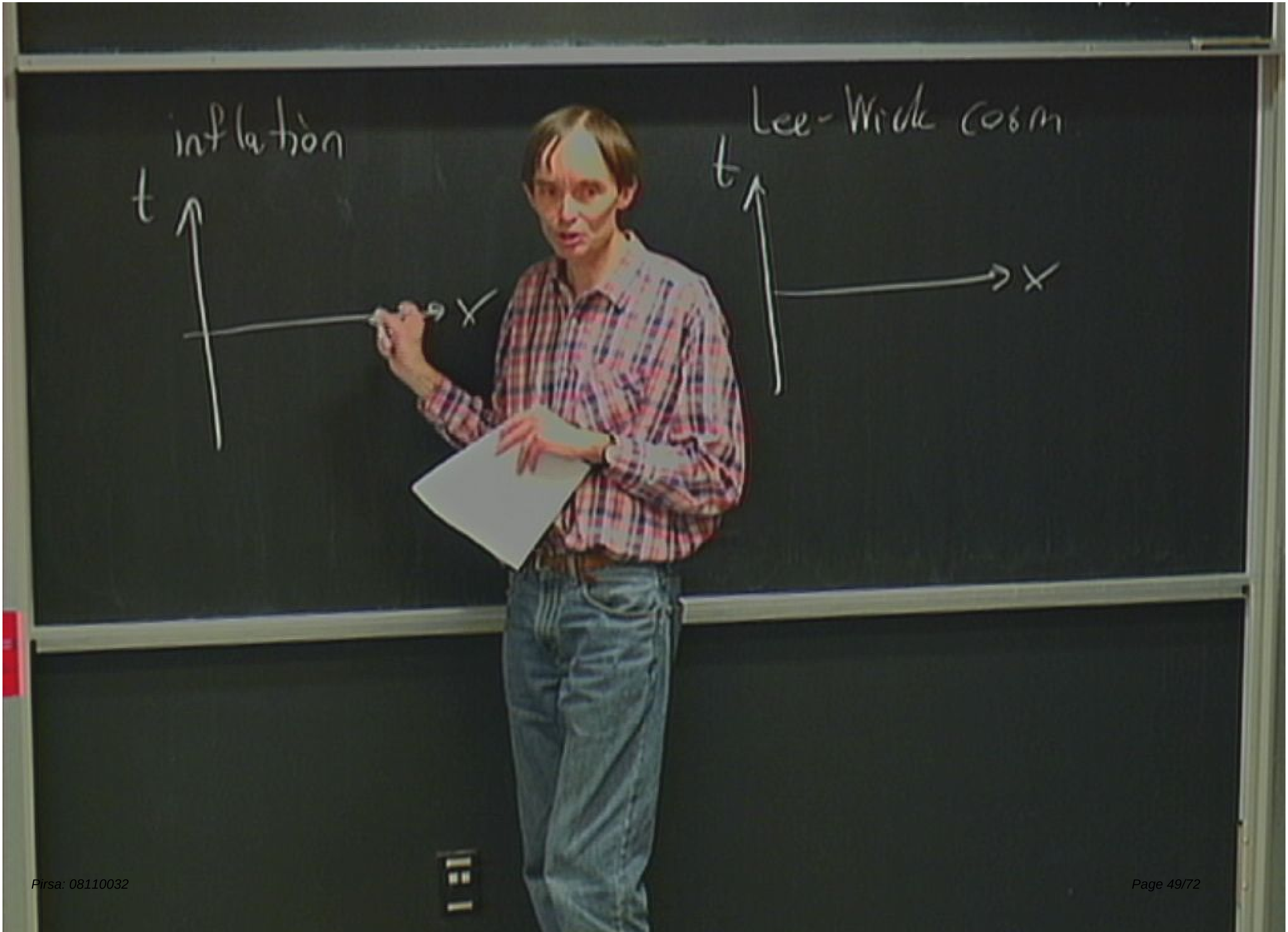
$$\dot{H} = -4\pi G(\dot{\phi}^2 - \dot{\tilde{\phi}}^2)$$

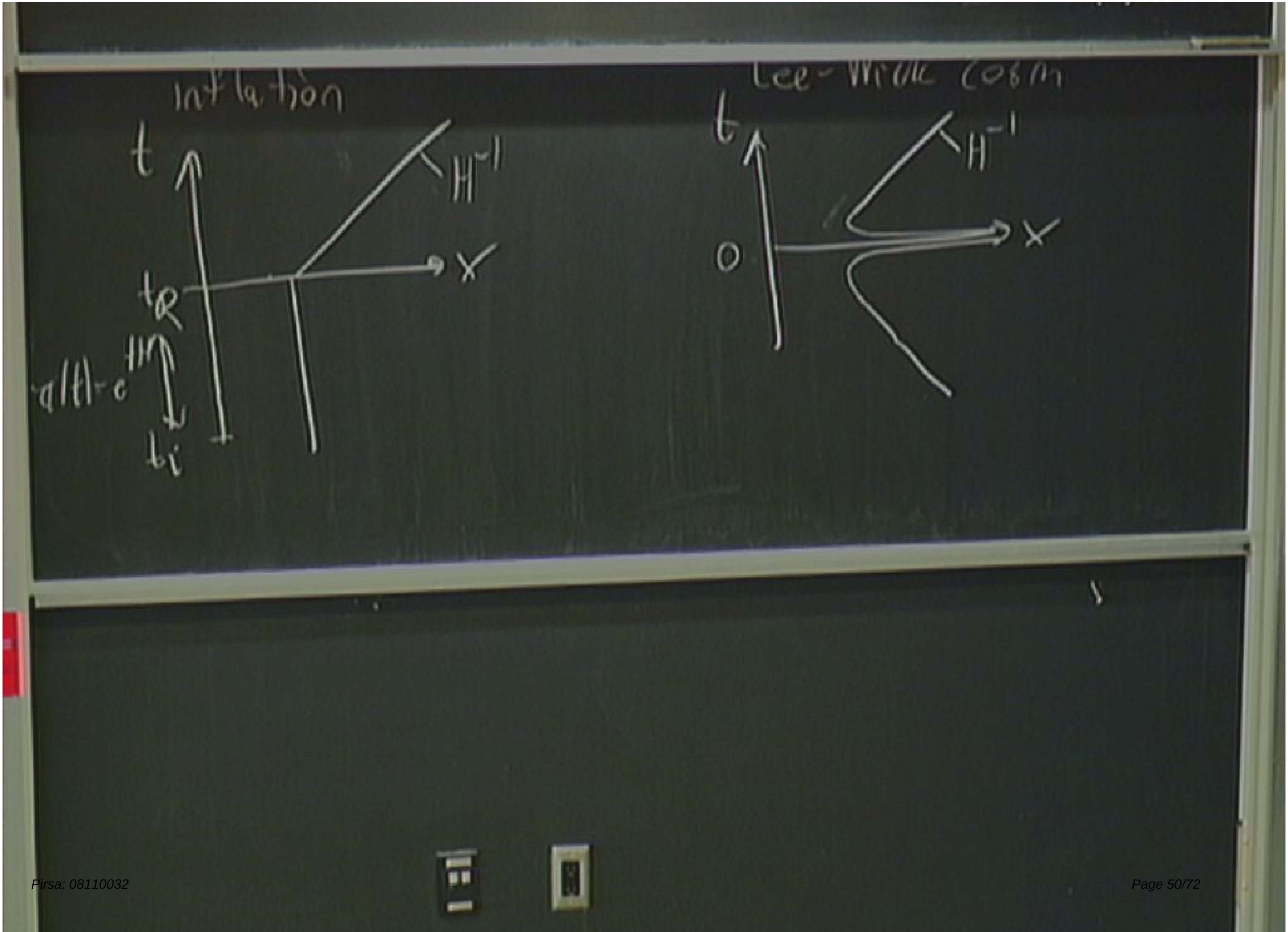
$$H^2 = \frac{8\pi G}{3}\left[\rho(\phi) - \rho(\tilde{\phi}) + \lambda(\phi-\tilde{\phi})^4\right]$$

$$KG_\lambda(\phi) = 0$$

$$KG_\lambda(\tilde{\phi}) = 0$$

bounce is possible : $\rho(\tilde{\phi}) = \rho(\phi) + \lambda(\phi-\tilde{\phi})^4$


inflation
t
x
Lee-Wick cosm
t
x


inflation
t
H^{-1}
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Lee-Wick cosm
t
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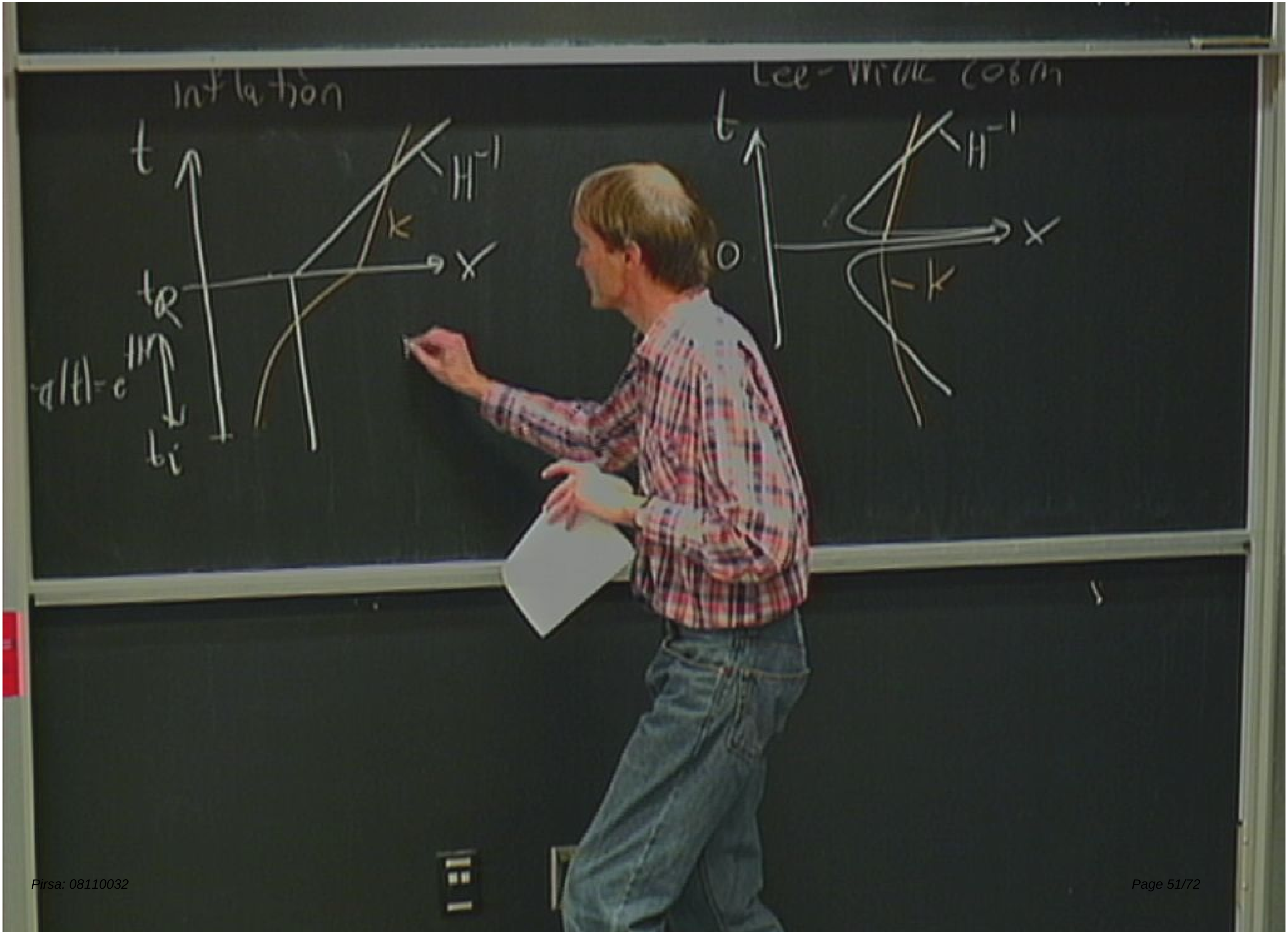
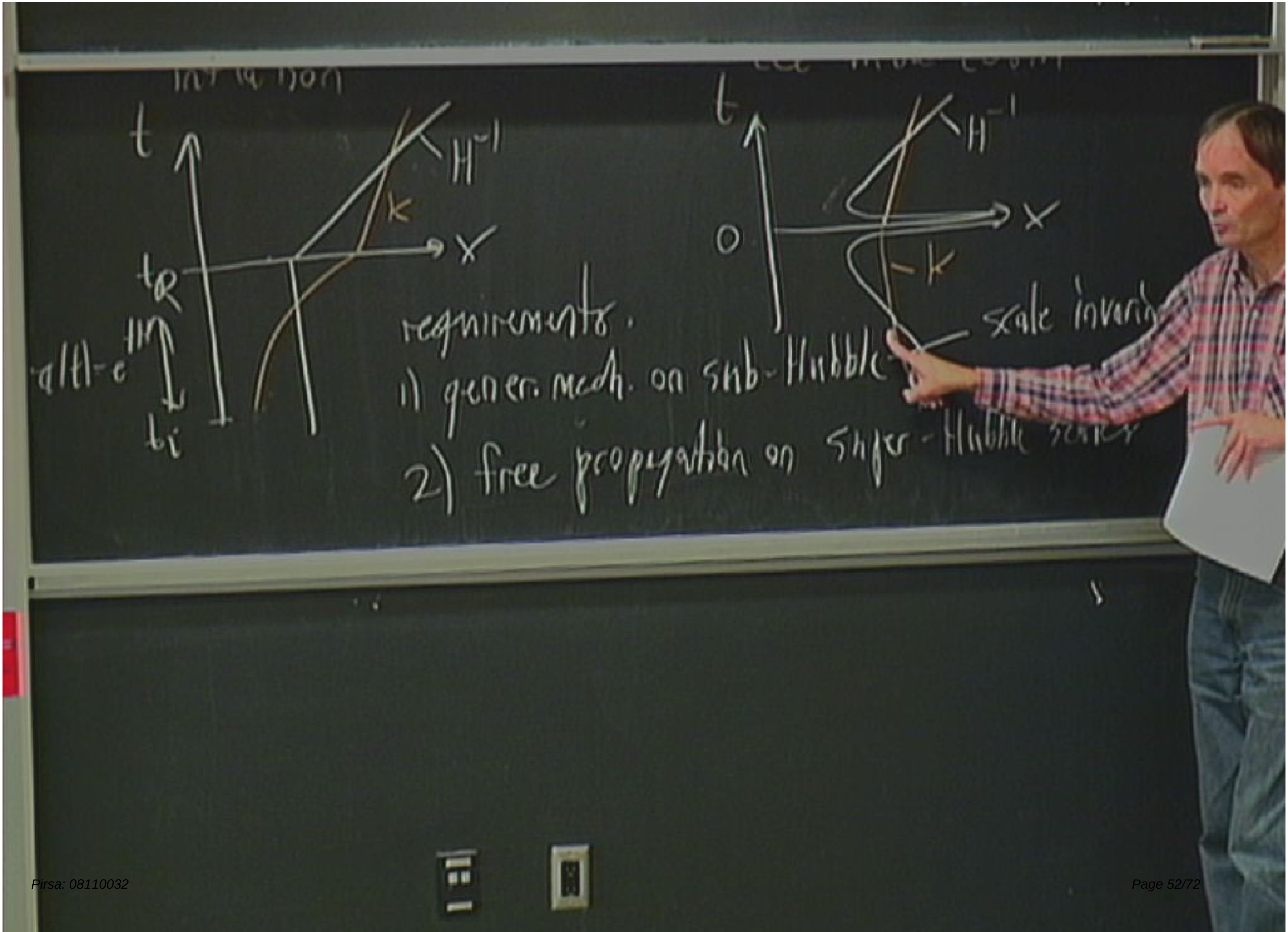

inflation
t
H^{-1}
k
x
t_R
$a(t)=e^{Ht}$
t_i
t
H^{-1}
x
0
-k


inflation
t
H^-1
k
x
t_R
t_i
0
-k
scale invari
requirements:
1) gener. mech. on sub-Hubble
2) free propagation on super-Hubble scales

$$ds^2 = a(\eta)\left[d\eta^2\left(1+2\Phi(\underline{x},\eta)\right) - d\underline{x}^2\left(1-2\Phi(\underline{x},\eta)\right)\right]$$

$\delta\phi$

δ

$S_{tot} = S_g + \int d^4x \sqrt{-g}\, \mathcal{L}_{LW}$

IC: matter dom. contraction → bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

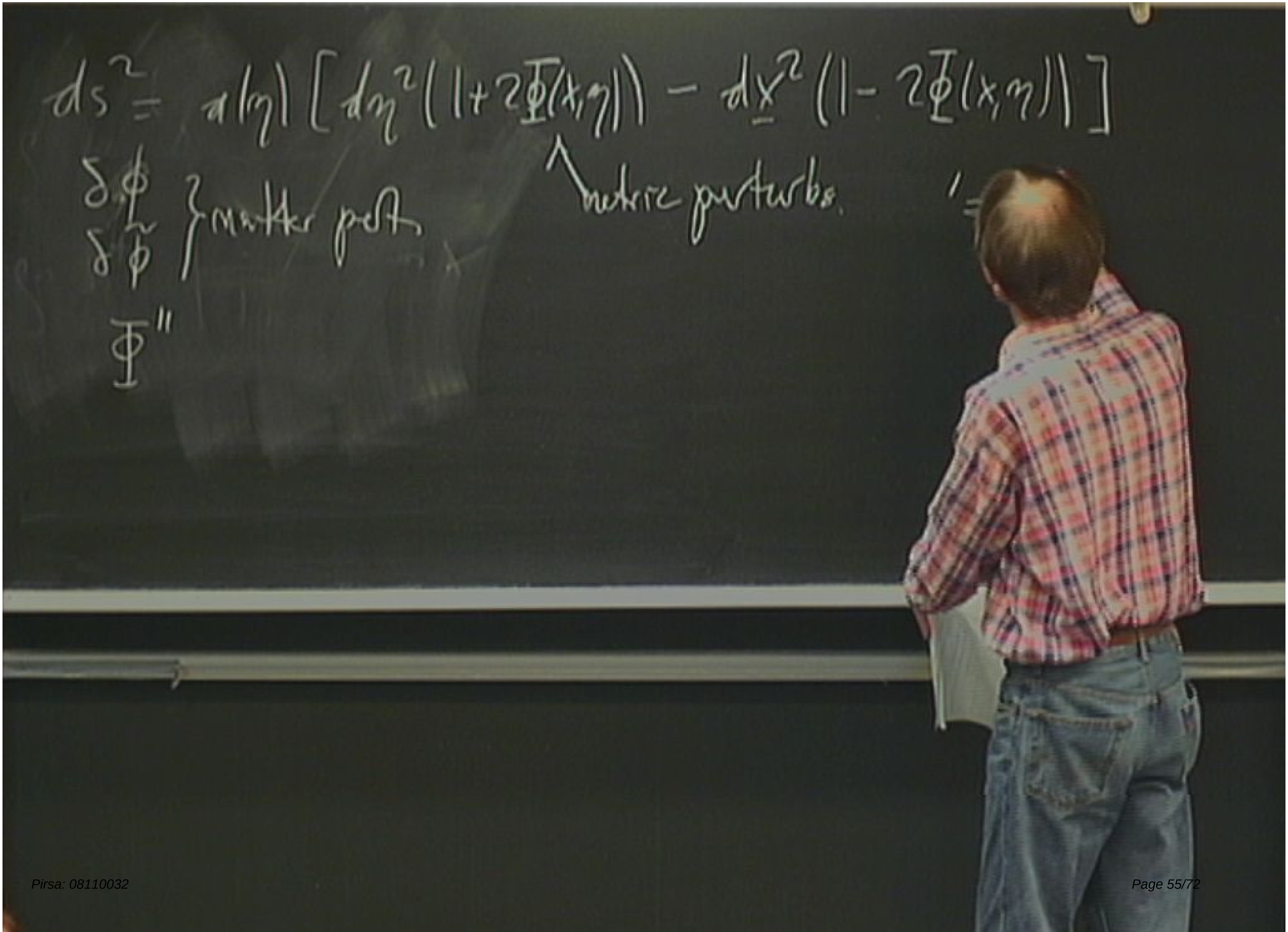
$\rho(\tilde{\phi}) = \rho(\phi)\, F^{-1}$

stage 4 = (stage 2)$^{-1}$

" 5 = (stage 1)$^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling deflation $\Delta t\, H \sim \ln F$

$\tilde{A}(t) \sim a(t)^{-3/2}$



$ds^2 = a(\eta)\left[d\eta^2\left(1+2\Phi(x,\eta)\right) - d\underline{x}^2\left(1-2\Phi(x,\eta)\right)\right]$
metric perturbs.
$\left.\begin{array}{l}\delta\phi\\ \delta\tilde{\phi}\end{array}\right\}$ matter pert.
Φ''

$$ds^2 = a(\eta)\left[d\eta^2\left(1+2\Phi(x,\eta)\right) - d\underline{x}^2\left(1-2\Phi(x,\eta)\right)\right]$$

metric perturbs.

$\left.\begin{array}{l}\delta\phi \\ \delta\tilde{\phi}\end{array}\right\}$ matter perts.

$' = \dfrac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)$$

$$ds^2 = a(\eta)\left[d\eta^2\left(1+2\Phi(x,\eta)\right) - d\underline{x}^2\left(1-2\Phi(x,\eta)\right)\right]$$

$\delta\phi$, $\delta\phi$ } matter perts

metric perturbs.

$$' = \frac{\partial}{\partial \eta}$$

$$\Phi'' + 6\mathcal{H}\Phi' + 2\left(\mathcal{H}' + 2\mathcal{H}^2\right)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta\phi^2)$$

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$\delta\phi$, $\delta\tilde{\phi}$ } matter pert.

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$$ds^2 = a(\eta)\left[d\eta^2(1+2\Phi(x,\eta)) - dx^2(1-2\Phi(x,\eta))\right]$$

$\delta\phi$, $\delta\phi$ } matter pert.

metric perturbs.

$' = \frac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta\phi^2)$$

Model

Background cosmology

Cosmological fluctuations

Extras

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta^2)$$

$$f = z^{-1}v \qquad v'' + \left(k^2 - \frac{z''}{z}\right)v = 0 \qquad \Phi \sim \left(\frac{v}{z}\right)'$$

Background Cosmology

Cosmological fluctuations

Extras

$$ds^2 = a^2(\eta)\left[d\eta^2(1+2\Phi(x,\eta)) - dx^2(1-2\Phi(x,\eta))\right]$$

$\delta\phi$, $\delta\tilde{\phi}$ } matter pert. — metric perturbs. $\quad ' = \frac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta\phi^2)$$

$$f = z^{-1}v \qquad v'' + \left(k^2 - \frac{z''}{z}\right)v = 0 \qquad \Phi \sim \left(\frac{v}{z}\right)'$$

$$z = \frac{a\phi'}{\mathcal{H}}$$

Extras

What was known (vacuum IC)

i) expanding universe   inflation ← s.i. spectrum of Φ

" of ζ

ii) contracting universe   Ekpyrotic ← s.i. spectrum of Φ

not s.i. spectrum of ζ

$S_{tot} = S_g + \int \sqrt{g} \ldots$

What we known (vacuum IC)

i) expanding universe   inflation $\leftarrow$ s.i. spectrum of Φ

" of ζ

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iii) " "   matter dominated $\leftarrow$ s.i. spectrum of ζ

k^{-4} " of Φ

What was known (vacuum IC)

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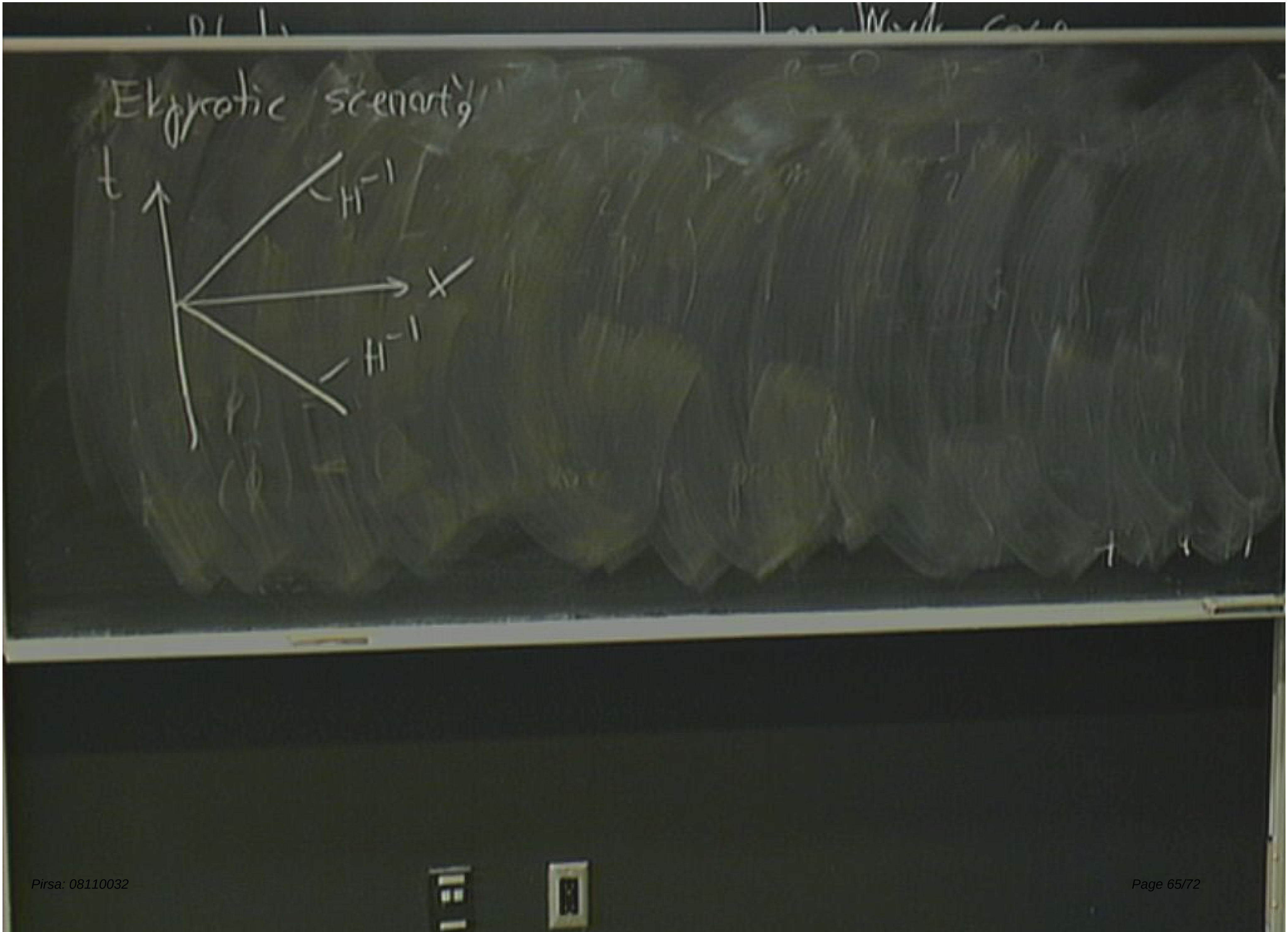
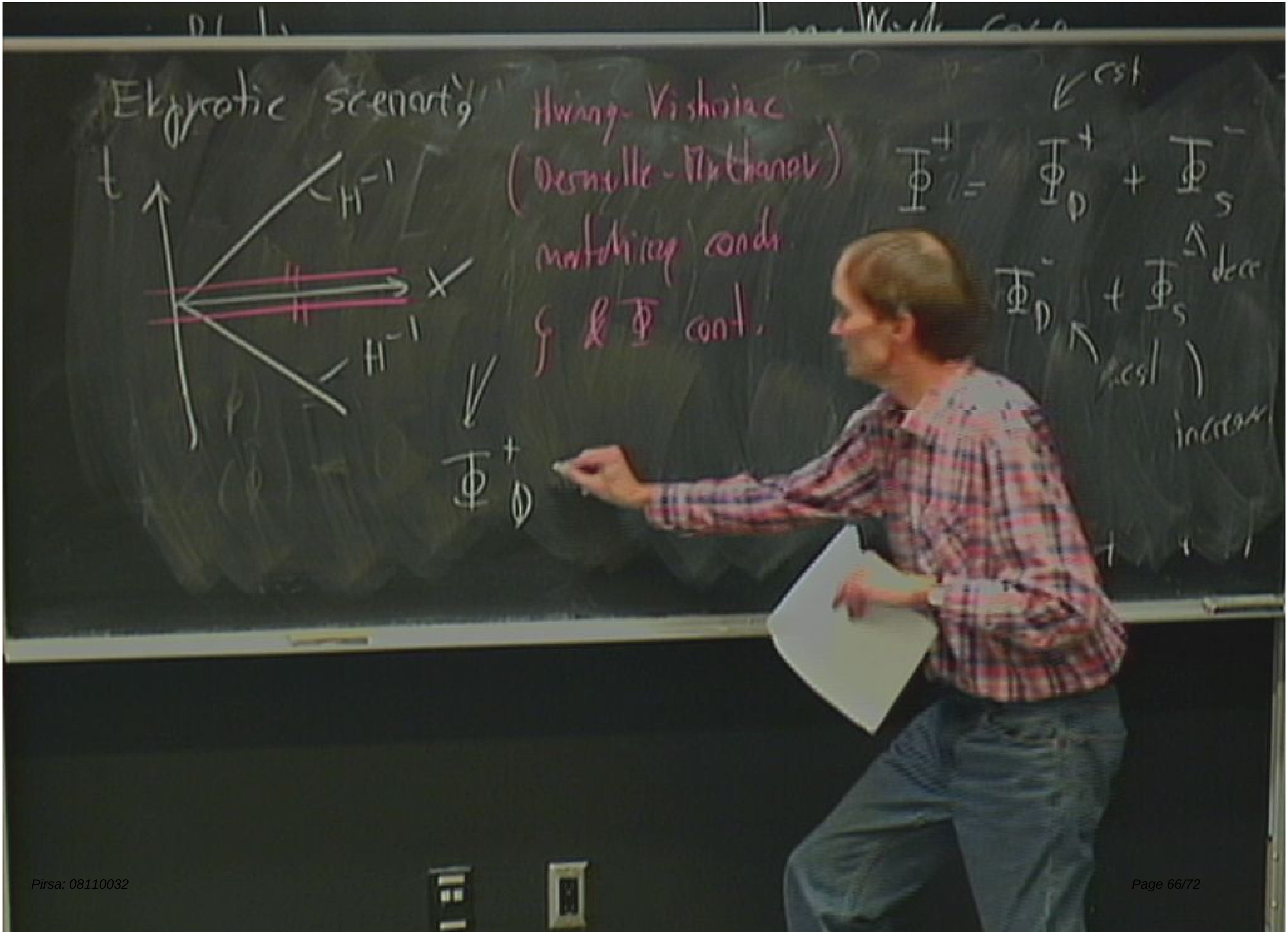
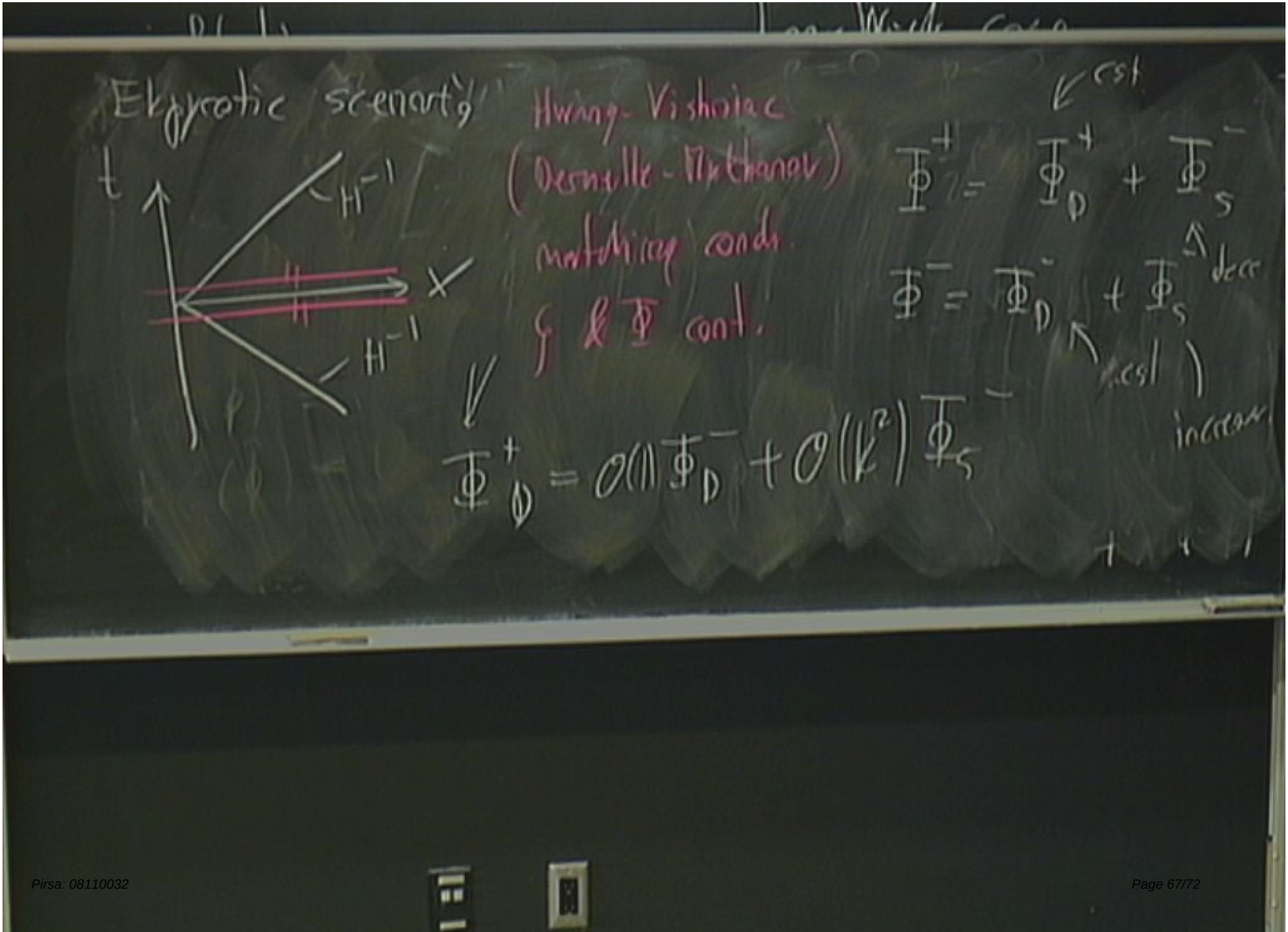
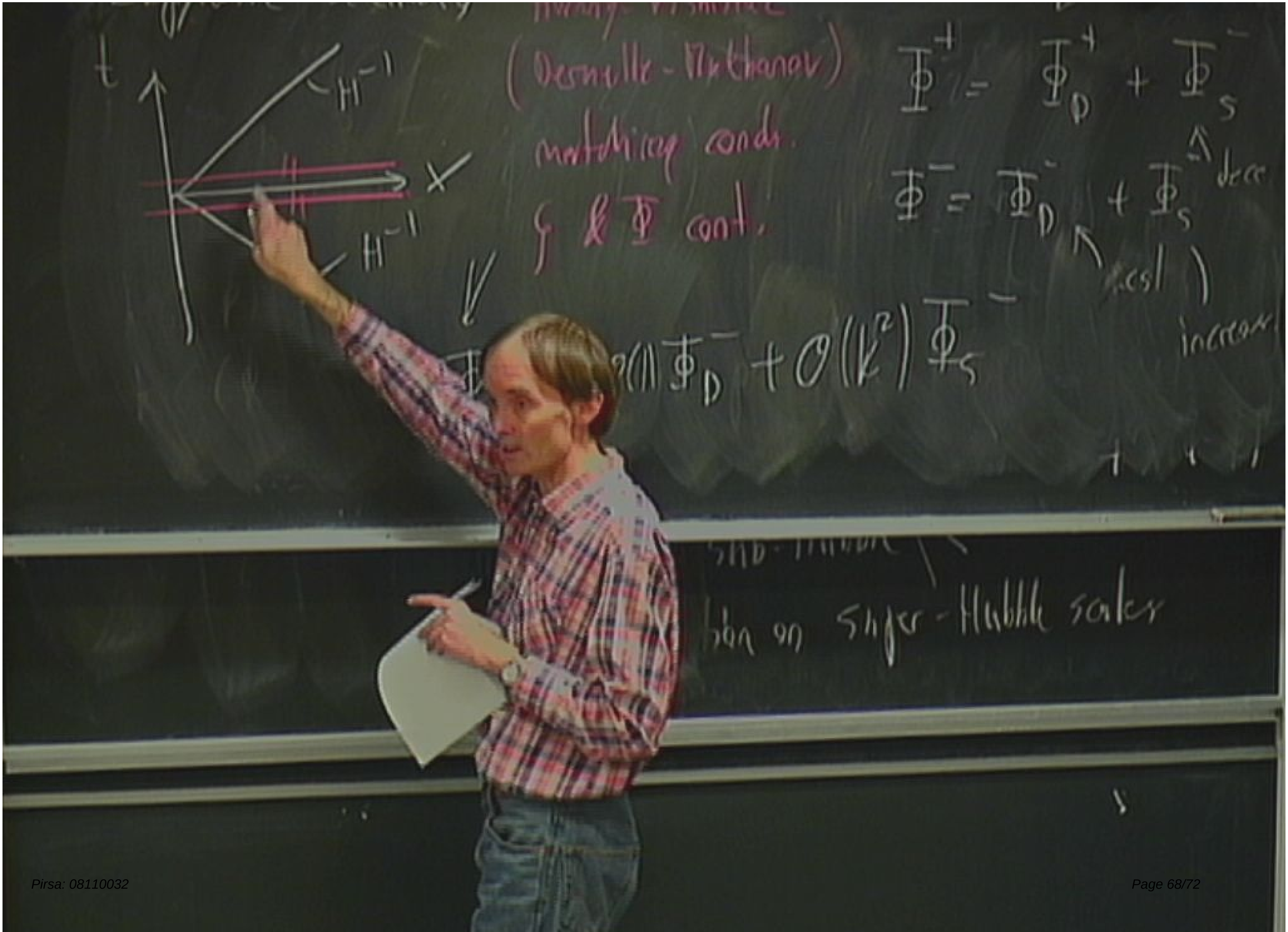
k^{-4} " of Φ

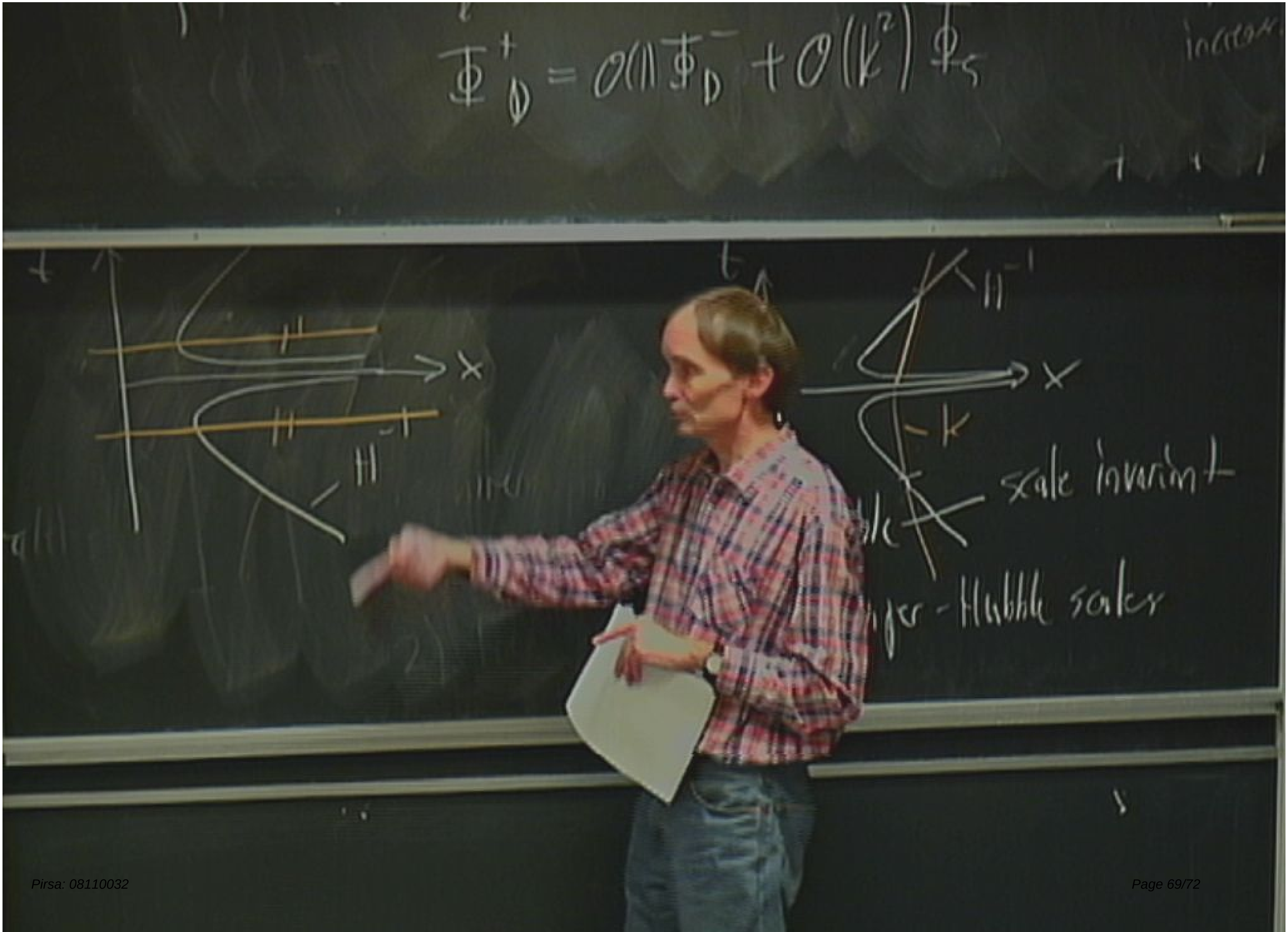
F. Finelli & R.B. 2001

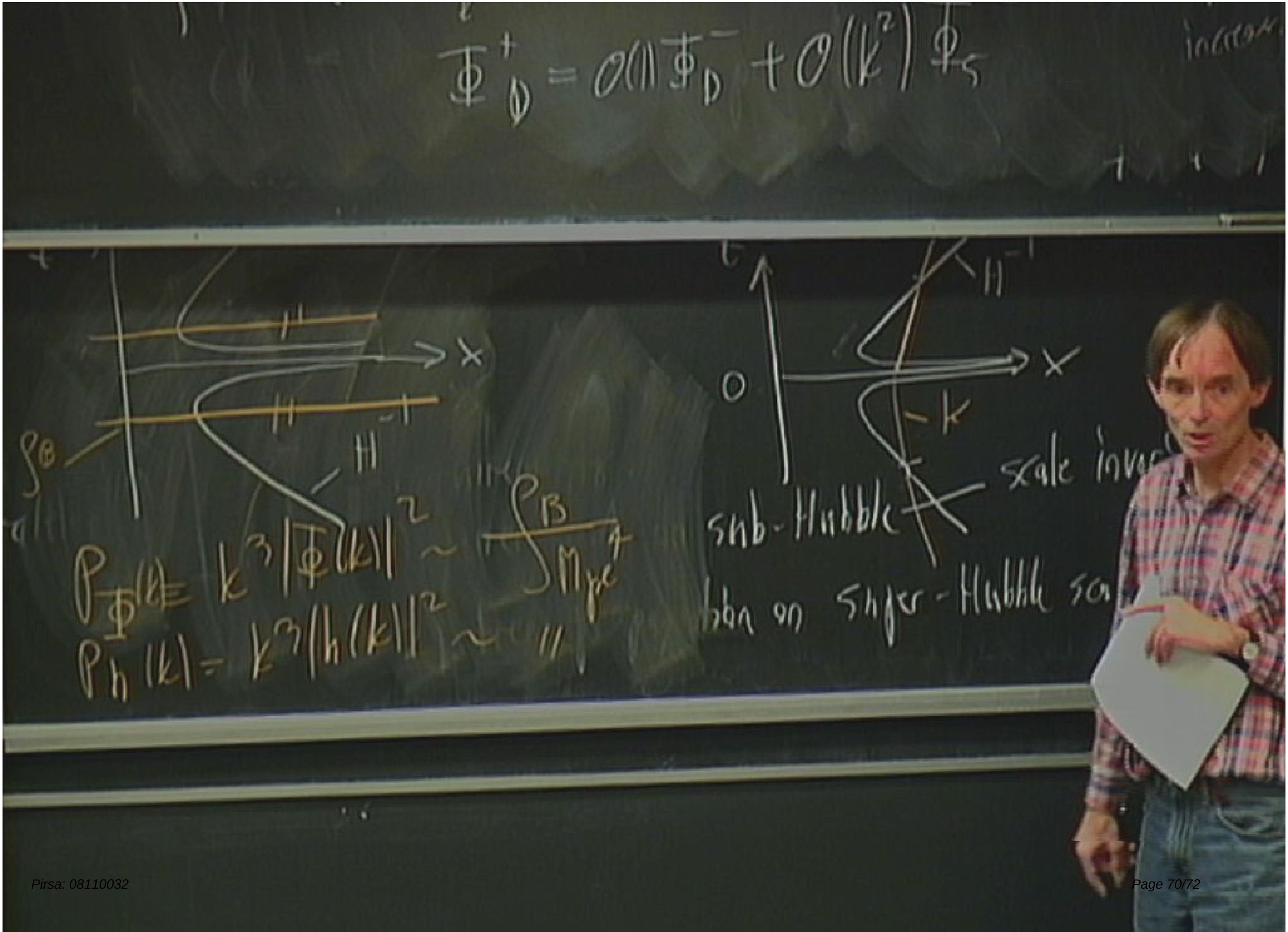

Ekpyrotic scenario
t
x
H^{-1}
H^{-1}

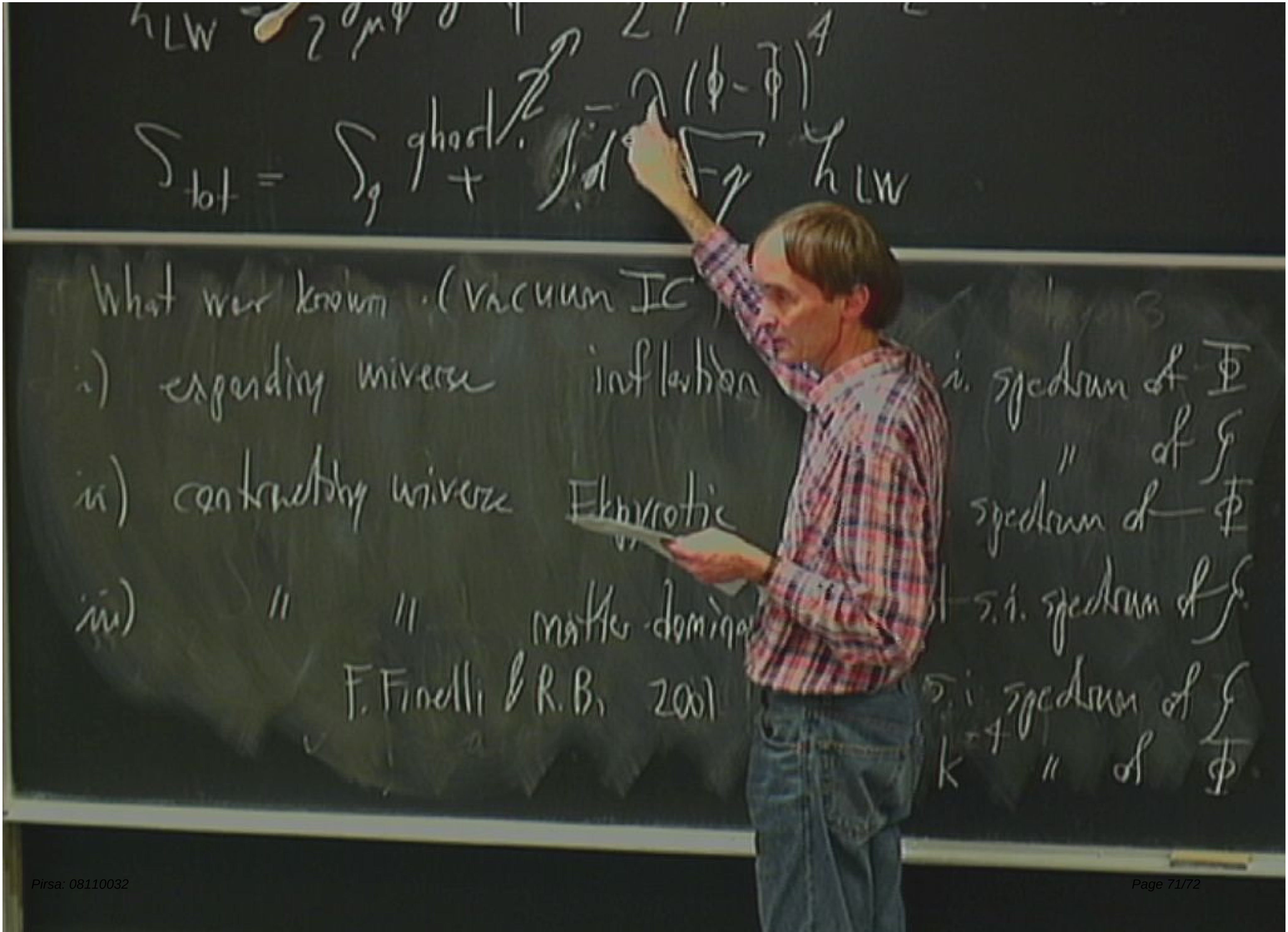

Ekpyrotic scenario
Hwang-Vishniac
(Deruelle-Mukhanov)
matching cond.
& cont.
t
x
csl
decr
incr


Ekpyrotic scenario
Hwang-Vishniac
(Deruelle-Mukhanov)
matching cond.
ζ & Φ cont.
t
x
H^{-1}
H^{-1}
$\Phi^{+} = \Phi_D^{+} + \Phi_S^{-}$
csl
decr
$\Phi^{-} = \Phi_D^{-} + \Phi_S^{-}$
csl
increas
$\Phi_D^{+} = O(1)\,\Phi_D^{-} + O(k^2)\,\Phi_S^{-}$


matching conds.
super-Hubble scales


Φ_D^+ = O(1) Φ_D^- + O(k^2) Φ_ς
t
x
H^-1
k
scale invariant
Hubble scales


sub-Hubble
scale inv
super-Hubble
x
0
t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spectrum of Φ
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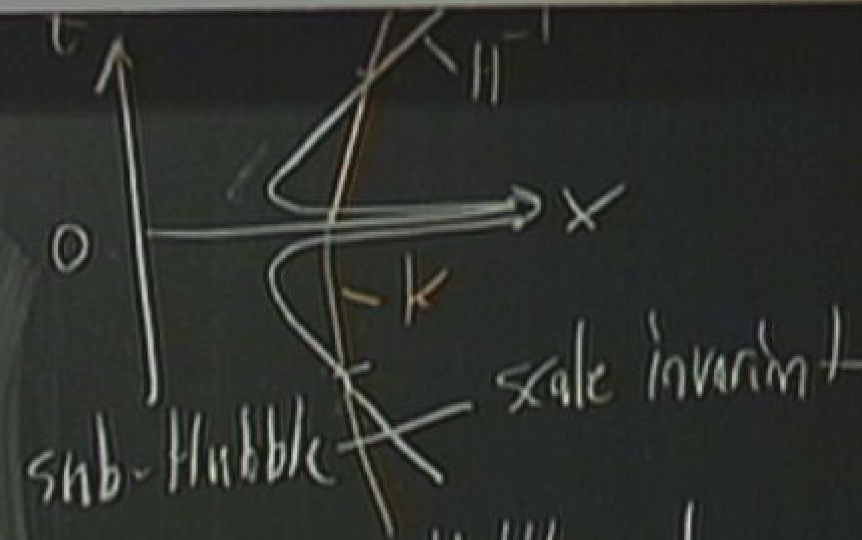
F. Finelli & R.B. 2001

$$P_\Phi(k) = k^3 |\Phi(k)|^2 \sim \frac{\rho_B}{M_{pl}^4}$$

$$P_h(k) = k^3 |h(k)|^2 \sim \quad ''$$



blue on super-Hubble scales