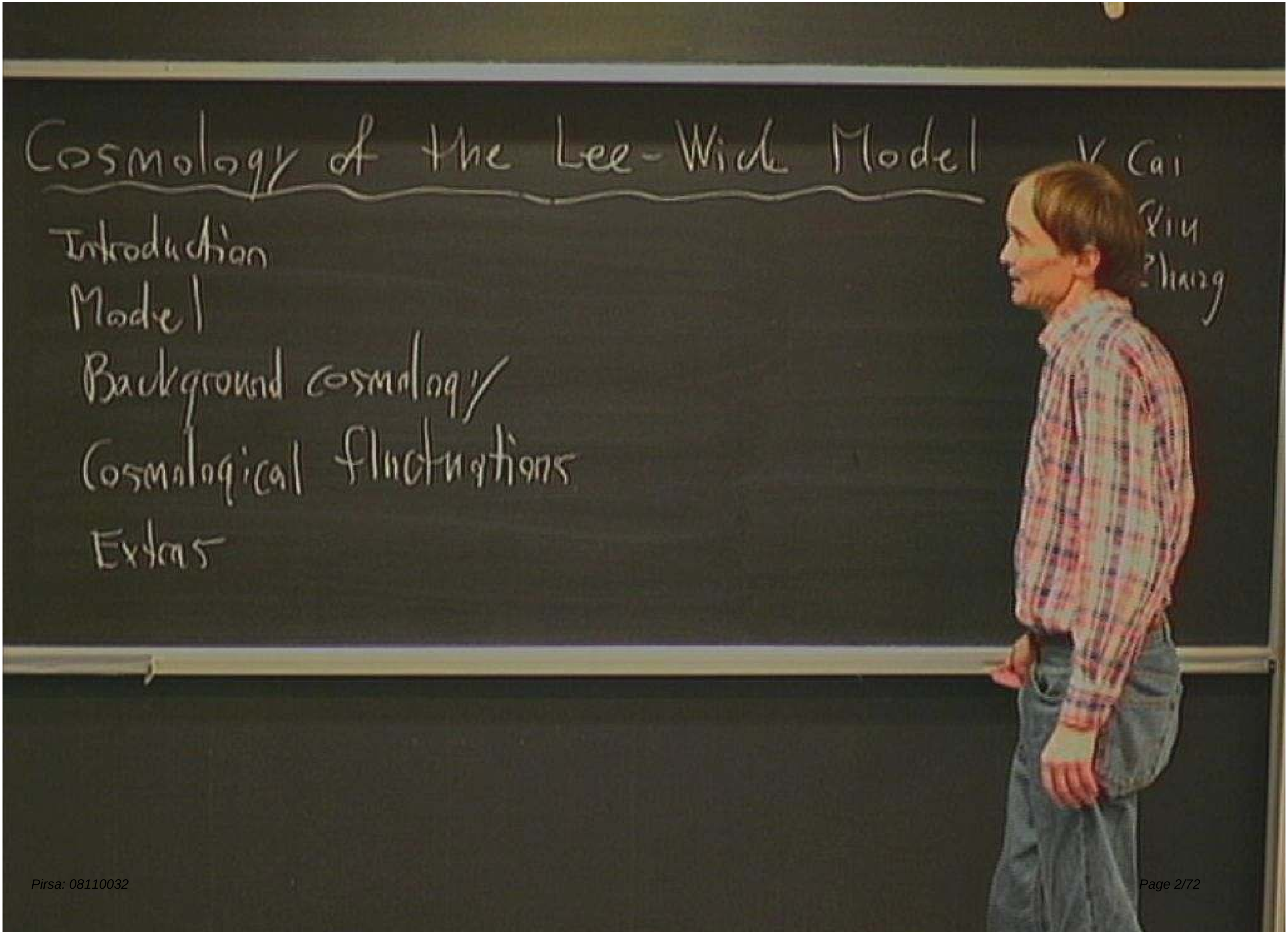
Title: Cosmology of the Lee-Wick Model

Date: Nov 11, 2008 02:00 PM

URL: http://pirsa.org/08110032

Abstract: The Lee-Wick model has recently been put forwards as an alternative to supersymmetry for solving the hierarchy problem of particle physics. I will show that, modulo important consistency questions, coupling the Lee-Wick model to cosmology leads to a bouncing universe cosmology with a scale-invariant spectrum of cosmological fluctuations emerging from quantum vacuum fluctuations in the contracting phase.


Cosmology of the Lee-Wick Model
Y. Cai
Introduction
Model
Background cosmology
Cosmological fluctuations
Extras

Cosmology of the Lee-Wick Model

Y. Cai
T. Qiu
X. Zhang

Introduction

Model

Background cosmology

Cosmological fluctuations

Extras

Cosmology of the Lee-Wick Model

Y. Cai
T. Qiu
X. Zhang

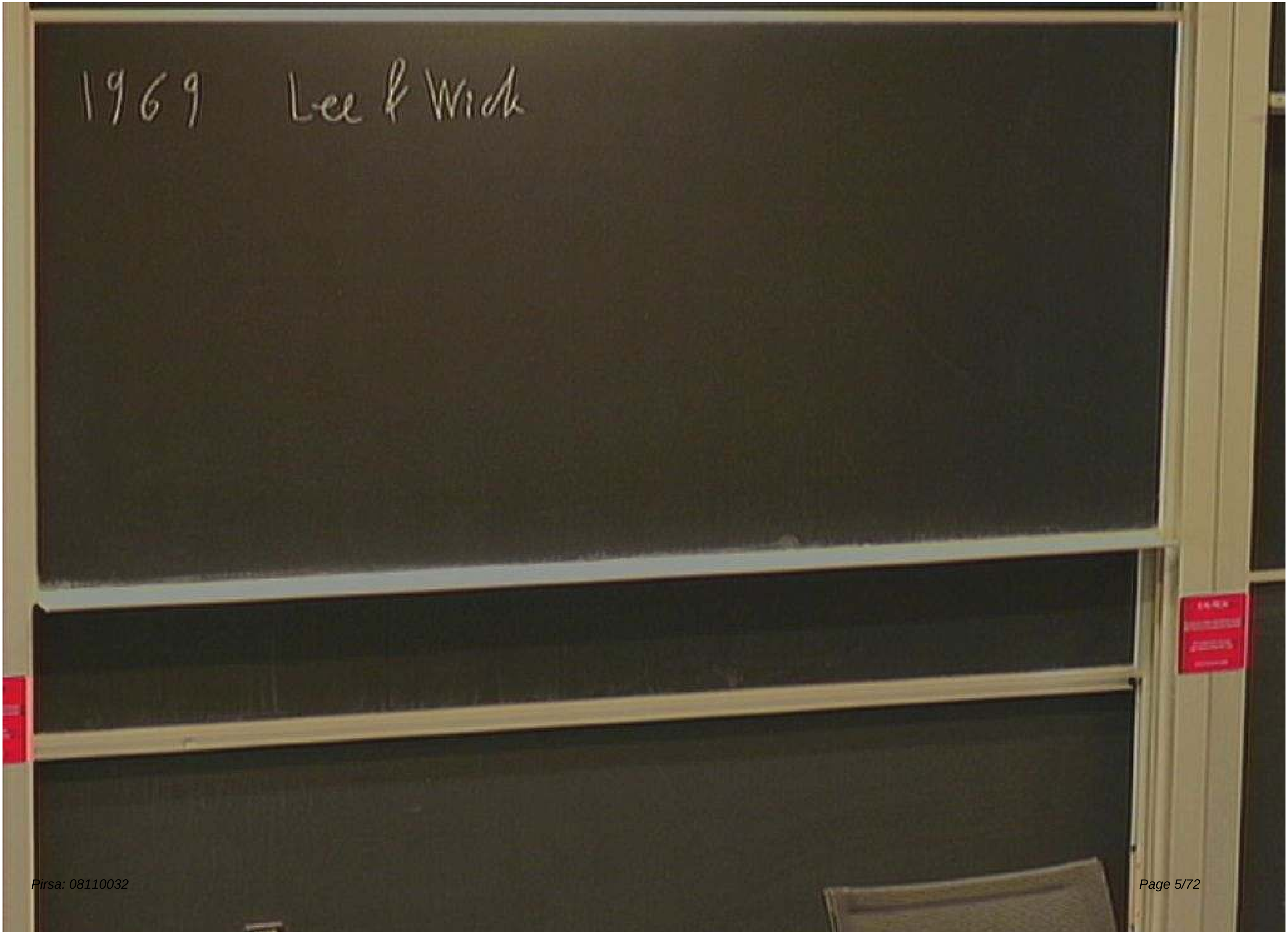
Introduction

Model

Background cosmology

Cosmological fluctuations

Extras


1969 Lee & Wick

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr.

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase

→ scale-inv. sp. of cosm. fluctr

Higgs sector of Lee-Wick SM

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$

non singular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$

$\longrightarrow$ scale-inv. sp. of conf. fluct.

Higgs sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2 - \lambda(\phi-\tilde{\phi})^4$$

$\rightarrow$ scale-inv. sp. of conn. fluct.

"Higgs" sector of Lee-Wick SM

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2 - \lambda(\phi-\tilde{\phi})^4$$

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2 - \lambda(\phi-\tilde{\phi})^4$$

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

nonsingular cosmology

vacuum fluctr. in contracting phase

$\longrightarrow$ scale-inv. sp. of cosm. fluctr.

ϕ original Higgs

$\tilde{\phi}$ Lee-Wick partner

$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} + \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\tilde{\phi}^2$$

$$- \lambda(\phi - \tilde{\phi})^4$$

ghost!

1969 Lee & Wick

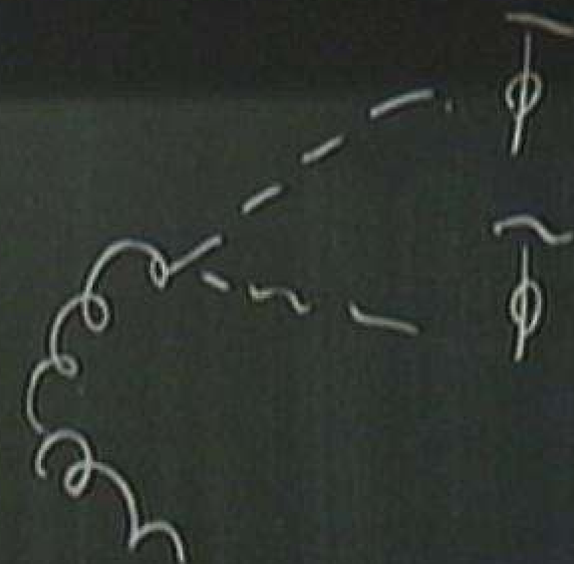
2007 Grinstein, O'Donnell & Wise

non singular cosmology

vacuum fluctr. in contracting phase

Lorentz invariance

stability



$$\mathcal{L}_{LW} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\ldots$$

$$S_{tot} = S_g^{\text{ghost}} + \int d^4x\sqrt{-g}\,\mathcal{L}_{LW} \quad \frac{\lambda}{4}(\phi-\tilde{\phi})^4$$

1969 Lee & Wick

2007 Grinstein, O'Donnell & Wise

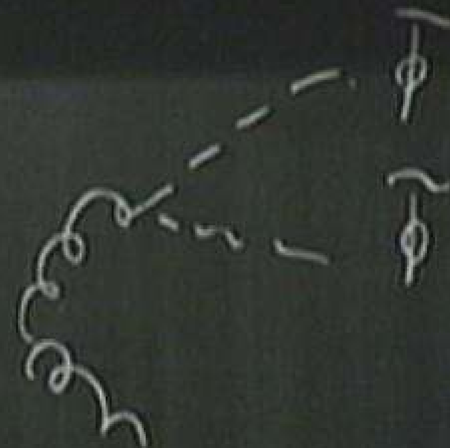
nonsingular cosmology

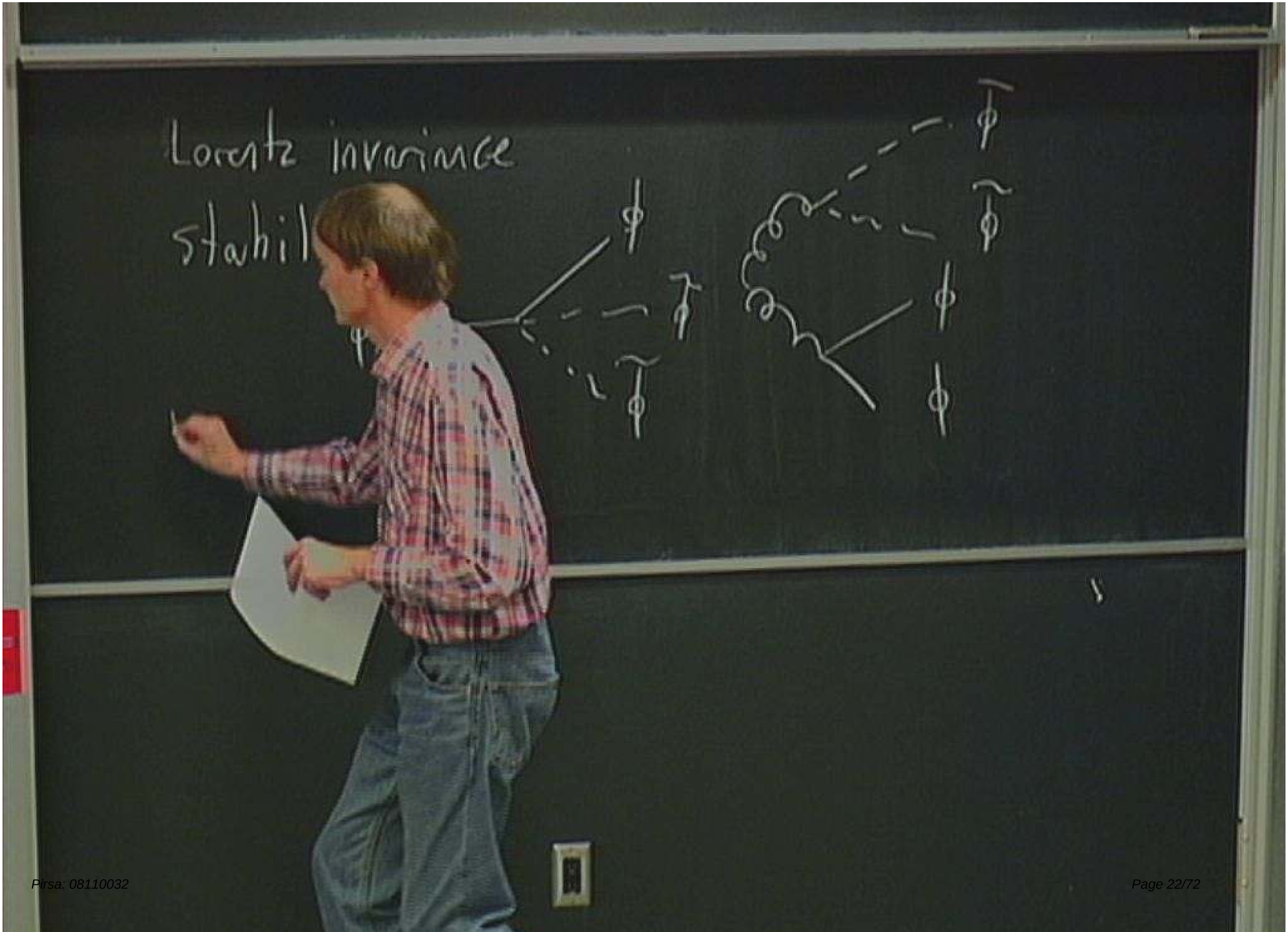
vacuum fluctr. in contracting phase

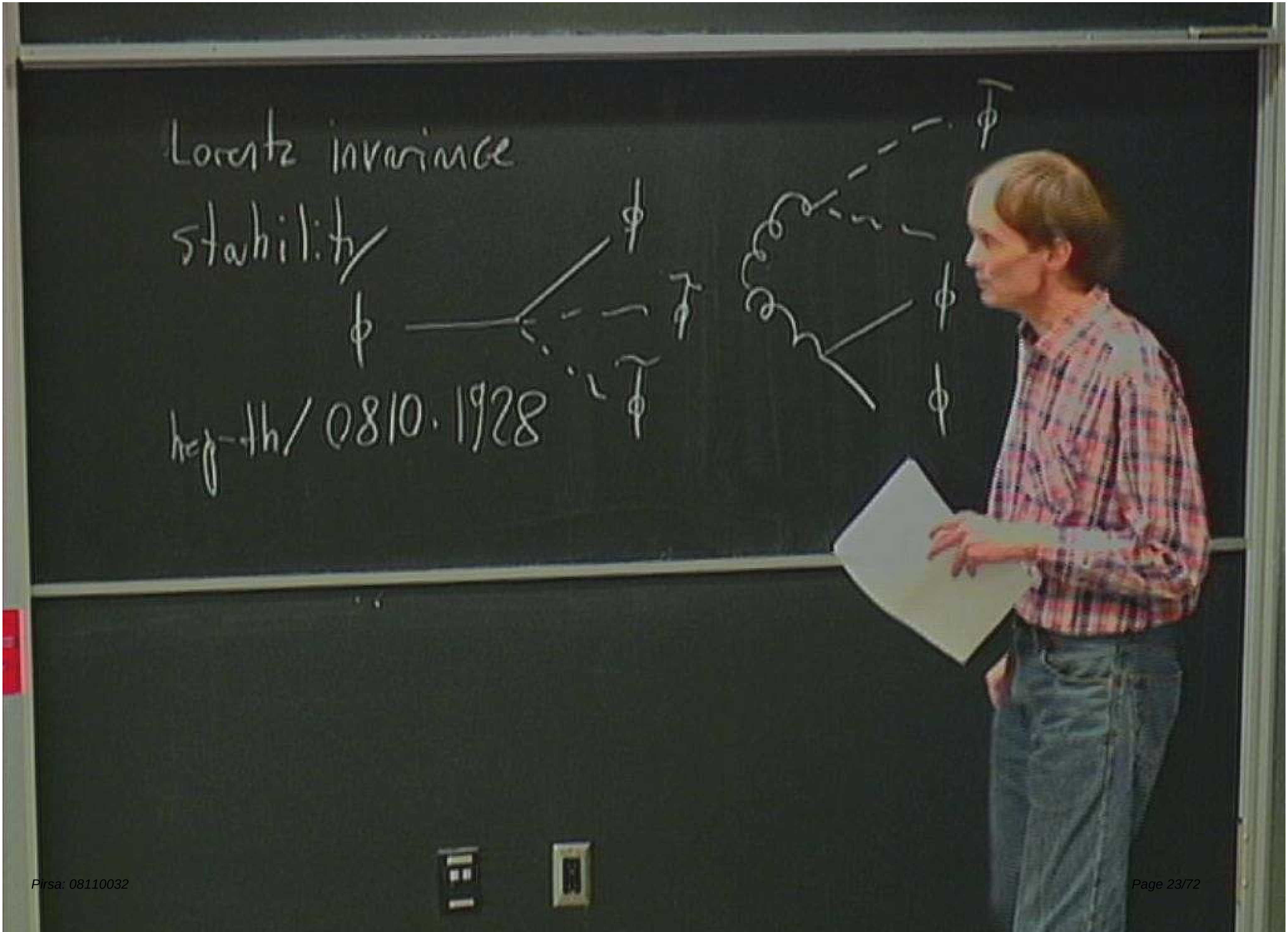
$\longrightarrow$ scale-inv. sp. of cosm. fluctr.

Lorentz invariance

stability

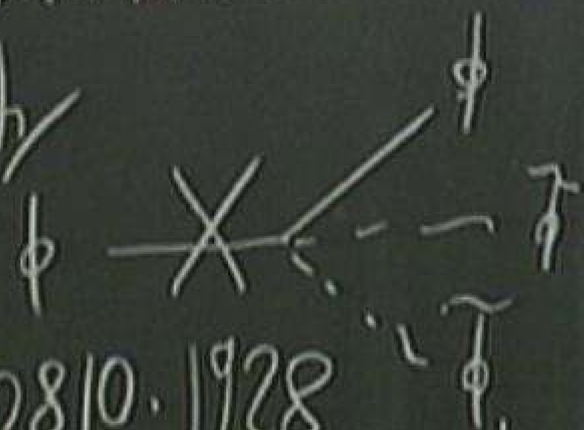


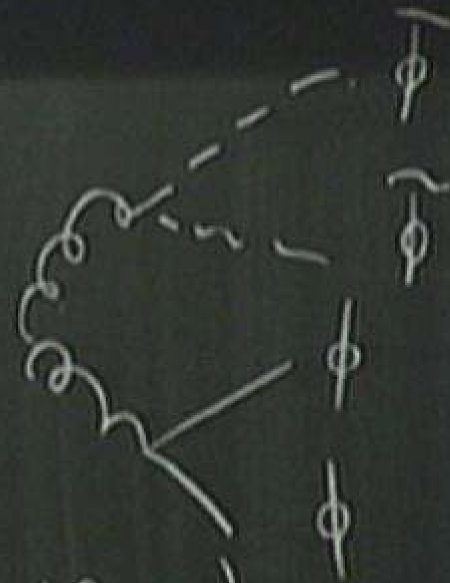

Lorentz invariance
stabil


Lorentz invariance
stability
hep-th/0810.1928

Lorentz invariance

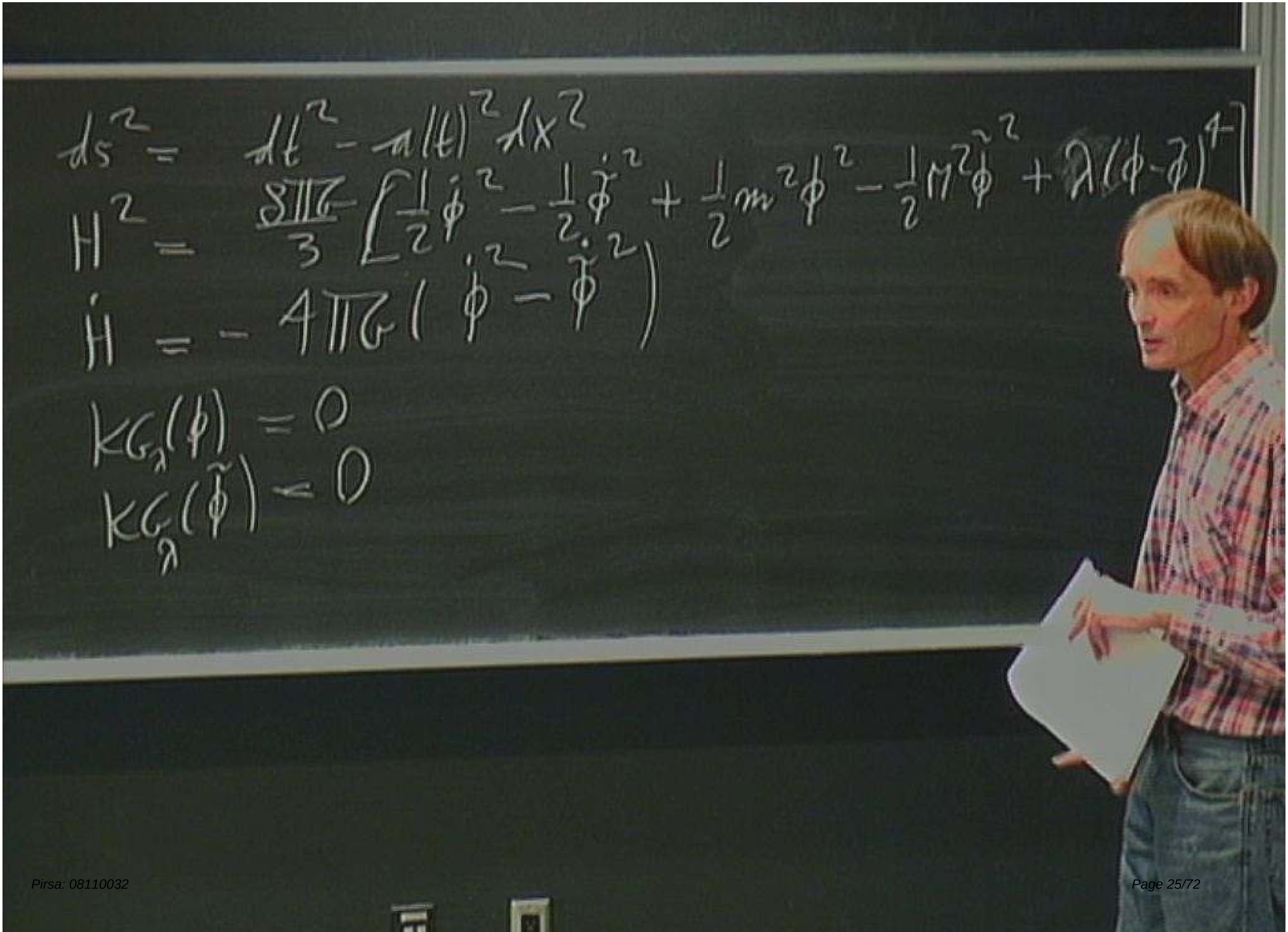
stability

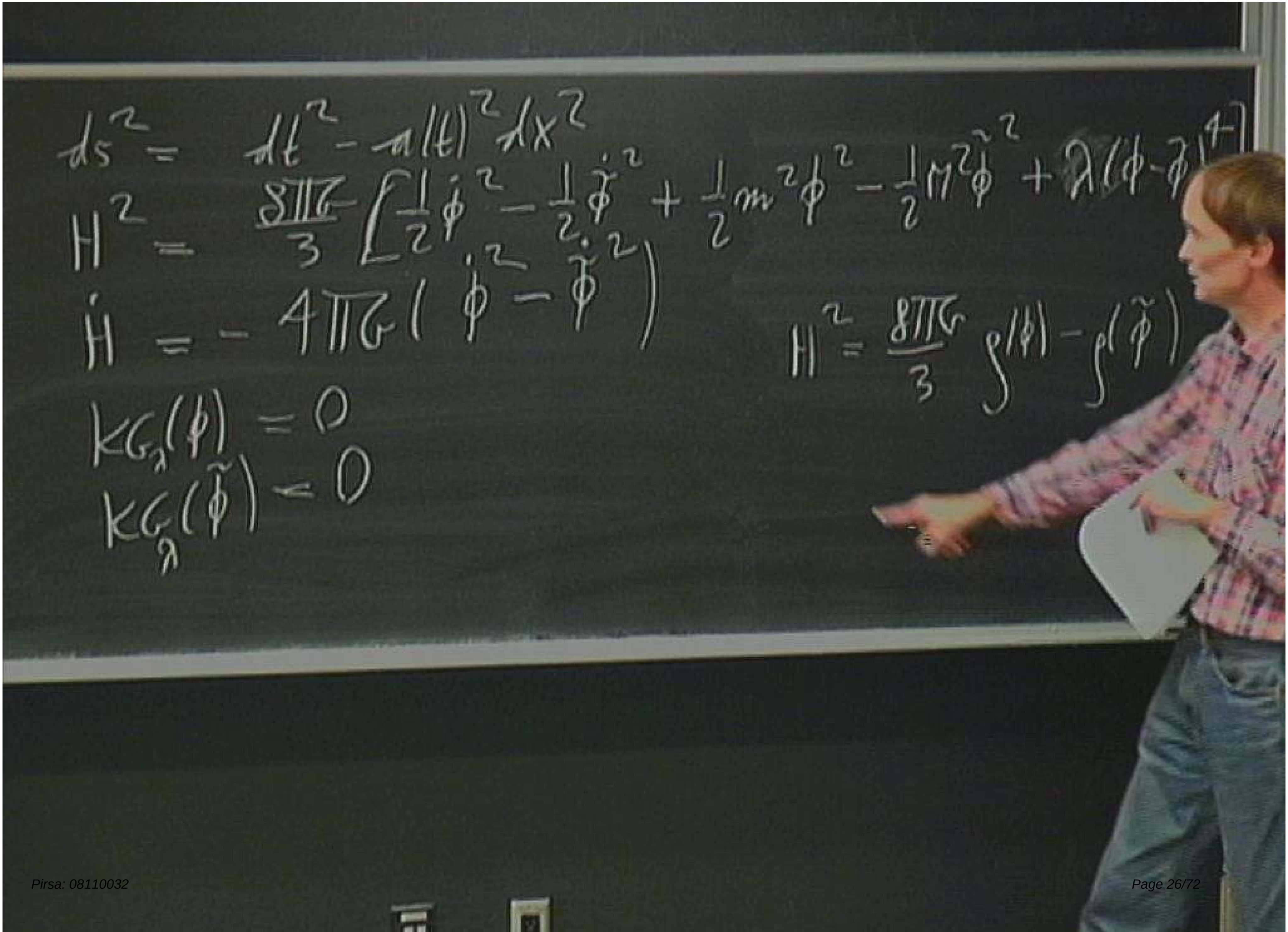




hep-th/0810.1928

A. van Tonder (Brown)


$ds^2 = dt^2 - a(t)^2 dx^2$
$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 - \frac{1}{2}\dot{\tilde\phi}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde\phi^2 + \lambda(\phi-\tilde\phi)^4\right]$
$\dot H = -4\pi G(\dot\phi^2 - \dot{\tilde\phi}^2)$
$KG_\lambda(\phi) = 0$
$KG_\lambda(\tilde\phi) = 0$


$ds^2 = dt^2 - a(t)^2 dx^2$
$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 - \frac{1}{2}\dot{\tilde\phi}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde\phi^2 + \lambda(\phi-\tilde\phi)^4\right]$
$\dot H = -4\pi G(\dot\phi^2 - \dot{\tilde\phi}^2)$
$KG_\lambda(\phi) = 0$
$KG_\lambda(\tilde\phi) = 0$
$H^2 = \frac{8\pi G}{3}\rho(\phi) - \rho(\tilde\phi))$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2 + \lambda(\phi - \tilde{\phi})^4\right]$$

$$\dot{H} = -4\pi G\left(\dot{\phi}^2 - \dot{\tilde{\phi}}^2\right)$$

$$H^2 = \frac{8\pi G}{3}\left[\rho(\phi) - \rho(\tilde{\phi})\right]$$

$$KG_\lambda(\phi) = 0$$

$$KG_\lambda(\tilde{\phi}) = 0$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

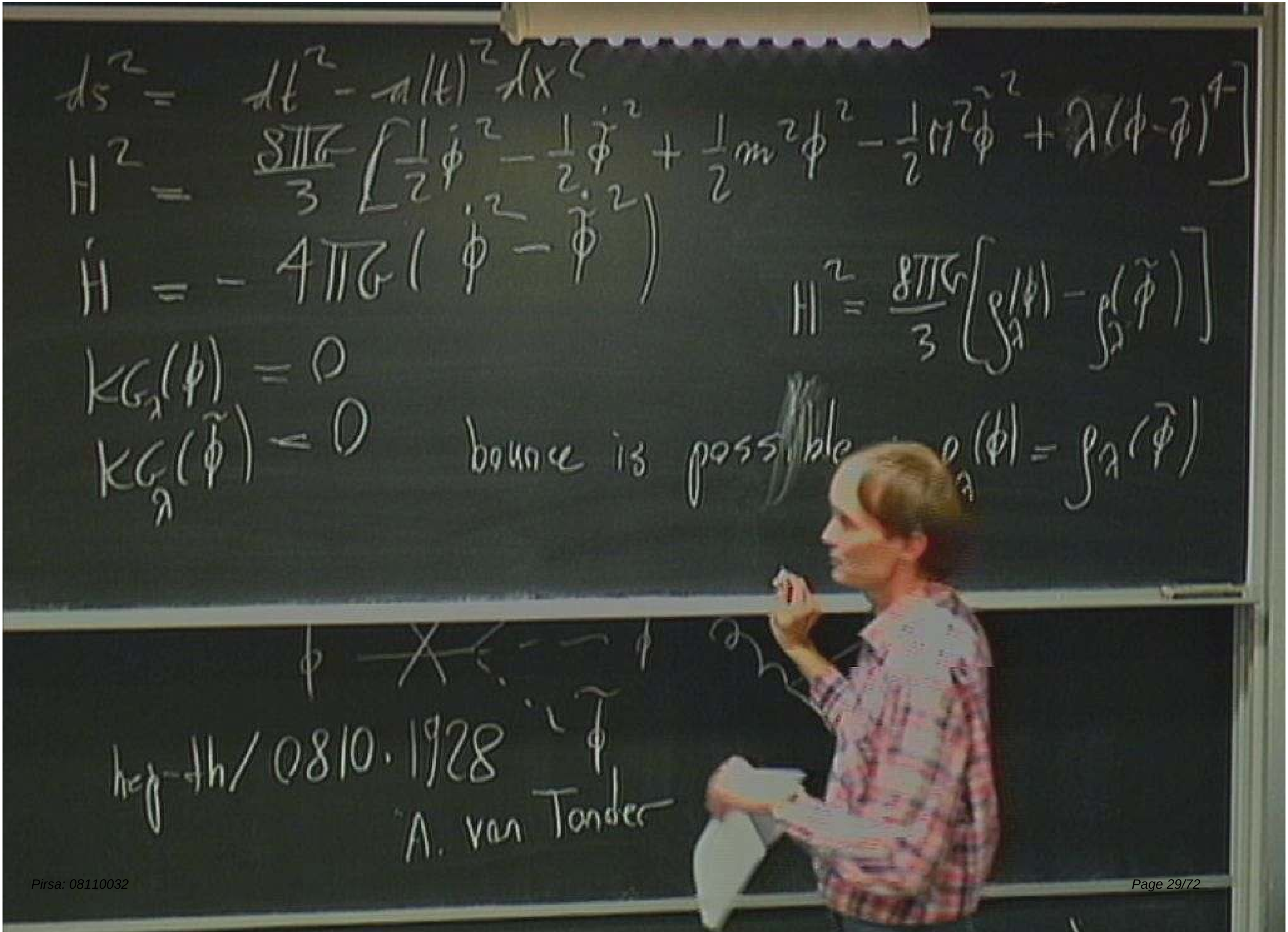
$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2 + \lambda(\phi-\tilde{\phi})^4\right]$$

$$\dot{H} = -4\pi G\left(\dot{\phi}^2 - \dot{\tilde{\phi}}^2\right)$$

$$KG_\lambda(\phi) = 0$$

$$KG_\lambda(\tilde{\phi}) = 0$$

$$H^2 = \frac{8\pi G}{3}\left[\rho_\lambda(\phi) - \rho(\tilde{\phi})\right]$$


$ds^2 = dt^2 - a(t)^2 dx^2$
$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 - \frac{1}{2}\dot{\tilde\phi}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m^2\tilde\phi^2 + \lambda(\phi-\tilde\phi)^4\right]$
$\dot H = -4\pi G(\dot\phi^2 - \dot{\tilde\phi}^2)$
$H^2 = \frac{8\pi G}{3}\left[\rho_\lambda(\phi) - \rho_\lambda(\tilde\phi)\right]$
$KG_\lambda(\phi) = 0$
$KG_\lambda(\tilde\phi) = 0$
bounce is possible $\rho_\lambda(\phi) = \rho_\lambda(\tilde\phi)$
ϕ
ϕ
$\tilde\phi$
hep-th/0810.1928
A. van Tonder

$$ds^2 = dt^2 - a(t)^2 dx\,dx$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m^2\tilde{\phi}^2 + \lambda(\phi-\tilde{\phi})^4\right]$$

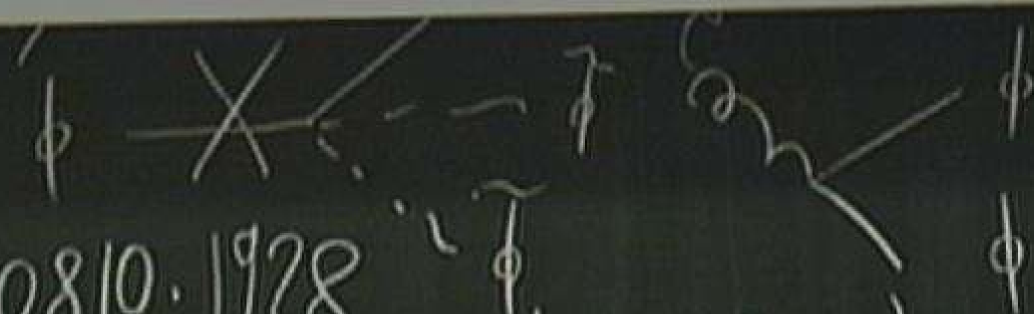
$$\dot{H} = -4\pi G\left(\dot{\phi}^2 - \dot{\tilde{\phi}}^2\right)$$

$$H^2 = \frac{8\pi G}{3}\left[\rho_\lambda(\phi) - \rho_\lambda(\tilde{\phi})\right]$$

$$KG_\lambda(\phi) = 0$$

$$KG_\lambda(\tilde{\phi}) = 0$$

bounce is possible : $\rho_\lambda(\phi) = \rho_\lambda(\tilde{\phi})$



hep-th/0810.1928

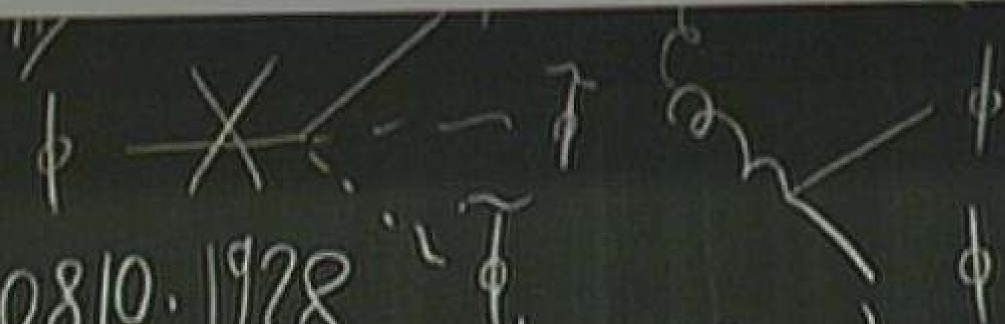
A. van Tonder (Brown)

$\dot{H} = -4\pi G(\phi - \phi)$

$H^2 = \frac{8\pi G}{3}\left[\rho(\phi) - \rho(\tilde{\phi}) + \tilde{\lambda}(\phi - \tilde{\phi})^4\right]$

$KG_\lambda(\phi) = 0$

$KG_\lambda(\tilde{\phi}) = 0$ bounce is possible : $\rho(\tilde{\phi}) = \rho(\phi) + \lambda(\phi - \tilde{\phi})^4$



hep-th/0810.1928

A. van Tonder (Brown)

$$S_{tot} = S_g + \int d^4x \sqrt{-g}\, h_{LW}$$

IC: matter-dom. contraction

$$S_{tot} = S_g + \int d^4x \sqrt{-g}\; h_{LW}$$

IC: matter-dom. contraction

stage 1: $\phi(t)$, $\tilde{\phi}(t)$ oscillating

$S_{tot} = S_g + \int d^4x \sqrt{-g}\, h_{LW}$

IC: matter-dom. contraction

stage 1 $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t)$

$\rho(\tilde{\phi}) \ll \rho(\phi)$

$S_{tot} = S_g + \int d^4x \sqrt{-g}\; h_{LW}$

IC: matter dom. contraction

stage 1. $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi)\, F^{-1}$

IC: matter-dom. contraction

stage 1. $\phi(t)$, $\tilde\phi(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde\phi) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

IC: matter-dom. contraction

stage 1: $\phi(t)$, $\tilde\phi(t)$ oscillating

$\mathcal{A}(t), \tilde{\mathcal{A}}(t) \sim a(t)^{-3/2}$

$\rho(\tilde\phi) = \rho(\phi) F^{-1}$

stage 2: $\mathcal{A} = \frac{1}{(12\pi)^{1/2}} m_{\chi}$ slow rolling

$\tilde{\mathcal{A}}(t) \sim a(t)^{-3/2}$

IC: matter-dom. contraction

stage 1. $\phi(t)$, $\tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$

IC: matter-dom. contraction

stage 1: $\phi(t), \tilde\phi(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde\phi) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation

IC: matter dom. contraction

stage 1: $\phi(t)$, $\tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H$

IC: matter dom. contraction

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) \cong \rho(\phi) \mathcal{F}^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t H \sim \ln \mathcal{F}$

IC: matter dom. contraction → bounce stage 3

stage 1: $\phi(t)$, $\tilde{\phi}(t)$ oscillating || stage 4

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln F$

IC: matter-dom. contraction $\to$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho'(\tilde{\phi}) \equiv \rho(\phi) F^{-1}$

stage 4 $= (\text{stage } 2)^{-1}$

" 5 $= (\text{stage } 1)^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln F$

IC: matter dom. contraction $\rightarrow$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

$\rho(\tilde{\phi}) = \rho(\phi) F^{-1}$

stage 4 $= (\text{stage } 2)^{-1}$

" 5 $= (\text{stage } 1)^{-1}$

stage 2: $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling

$\tilde{A}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln F$

IC: matter-dom. contraction $\rightarrow$ bounce stage 3

stage 1: $\phi(t), \tilde{\phi}(t)$ oscillating

$\mathcal{A}(t), \tilde{\mathcal{A}}(t) \sim a(t)^{-3/2}$

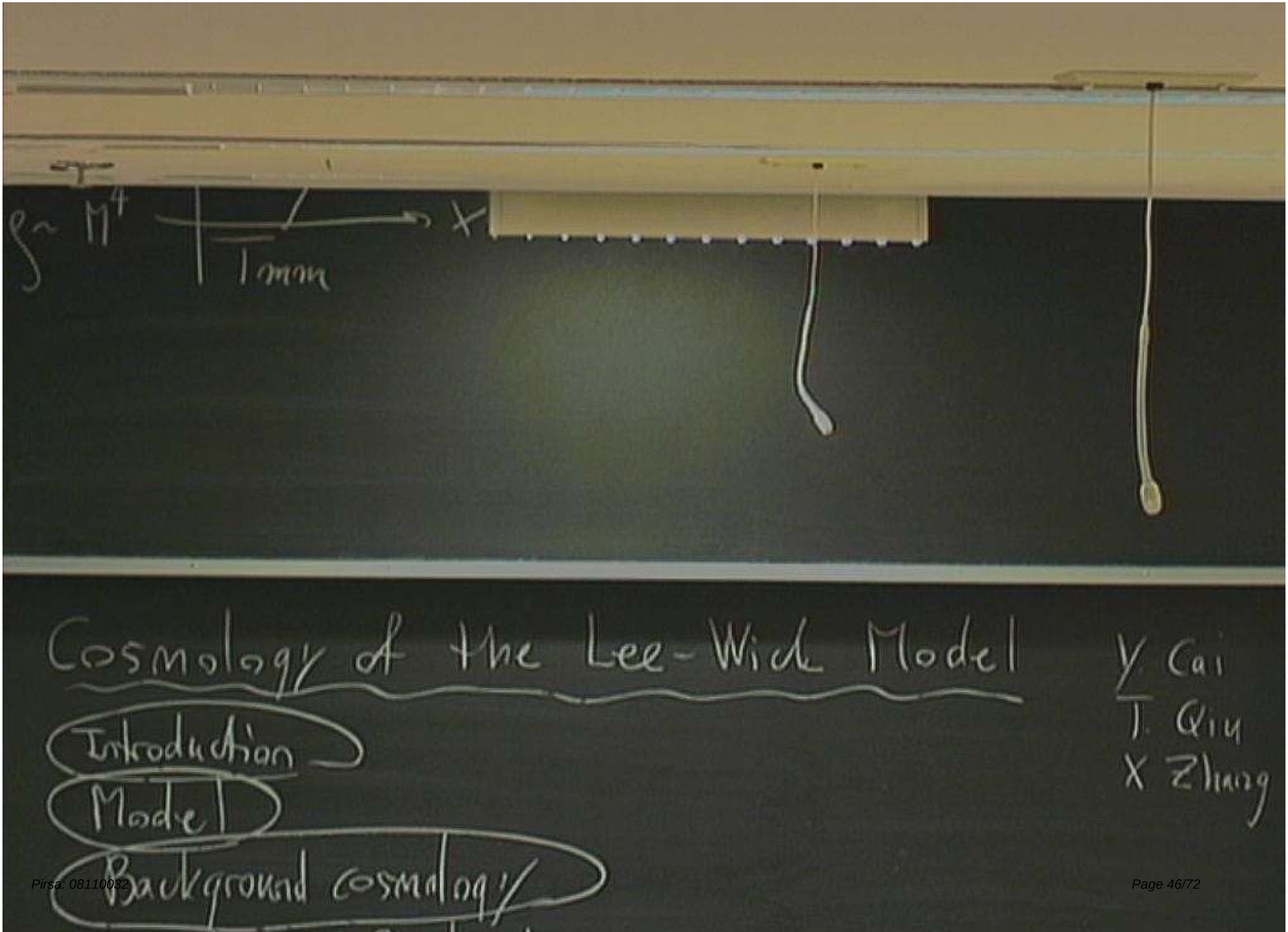
$\rho(\tilde{\phi}) = \rho(\phi)\, F^{-1}$

stage 4 $= (\text{stage } 2)^{-1}$

" 5 $= (\text{stage } 1)^{-1}$

stage 2: $\mathcal{A} = \frac{1}{(12\pi)^{1/2}} m_\chi$ slow rolling

$\tilde{\mathcal{A}}(t) \sim a(t)^{-3/2}$ deflation $\Delta t\, H \sim \ln F$


1mm
Cosmology of the Lee-Wick Model
Y. Cai
T. Qiu
X. Zhang
Introduction
Model
Background cosmology

Cosmology of the Lee-Wick Model

Y. Cai
T. Qiu
X. Zhang

Introduction

Model

Background cosmology

Cosmological fluctuations

Extras

$$ds^2 = dt^2 - a(t)^2 dx^2 \qquad \rho = 0 \quad \rho < 0$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 - \frac{1}{2}\dot{\tilde\phi}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m^2\tilde\phi^2 + \lambda(\phi - \tilde\phi)^4\right]$$

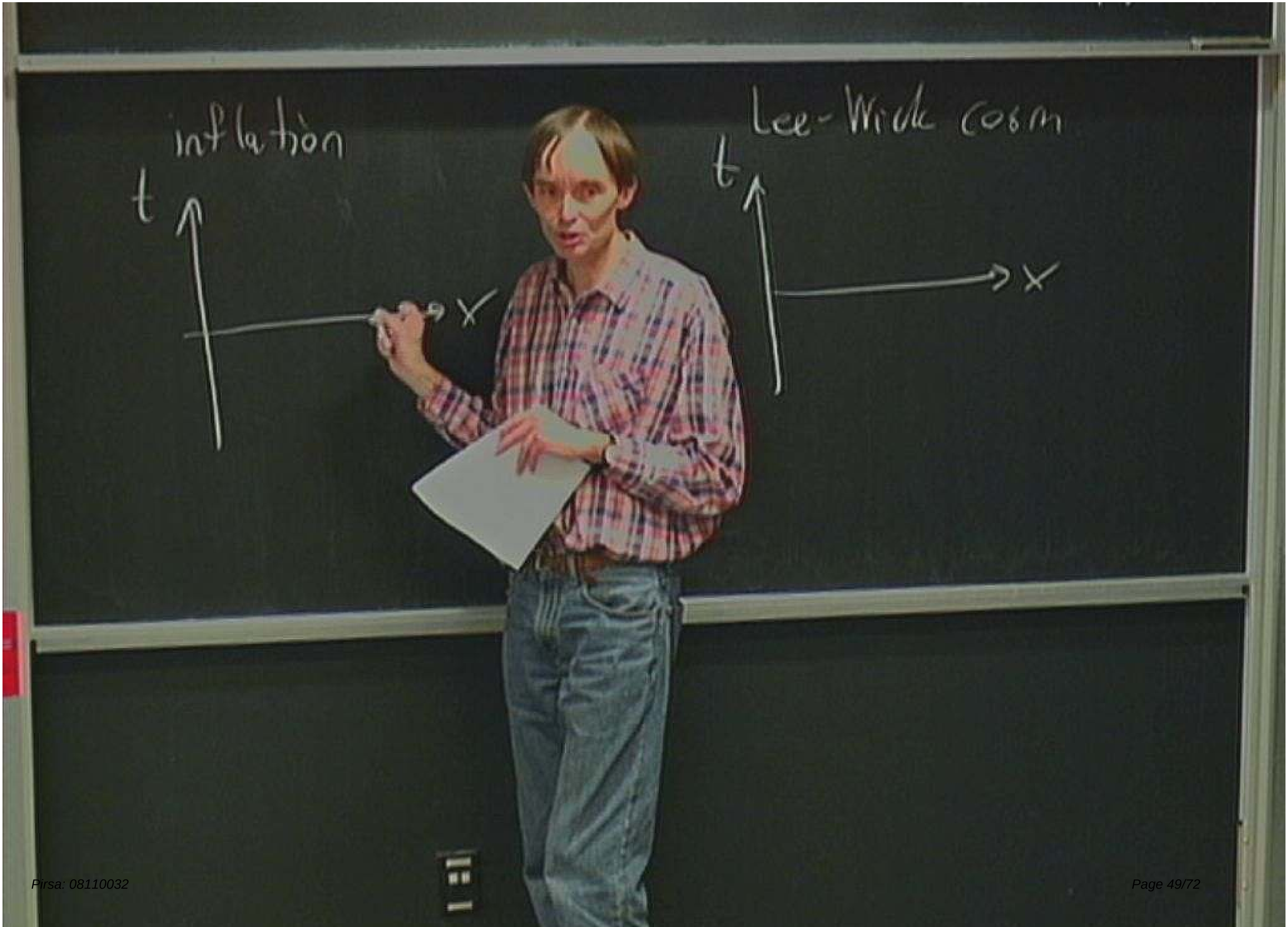
$$\dot H = -4\pi G\left(\dot\phi^2 - \dot{\tilde\phi}^2\right)$$

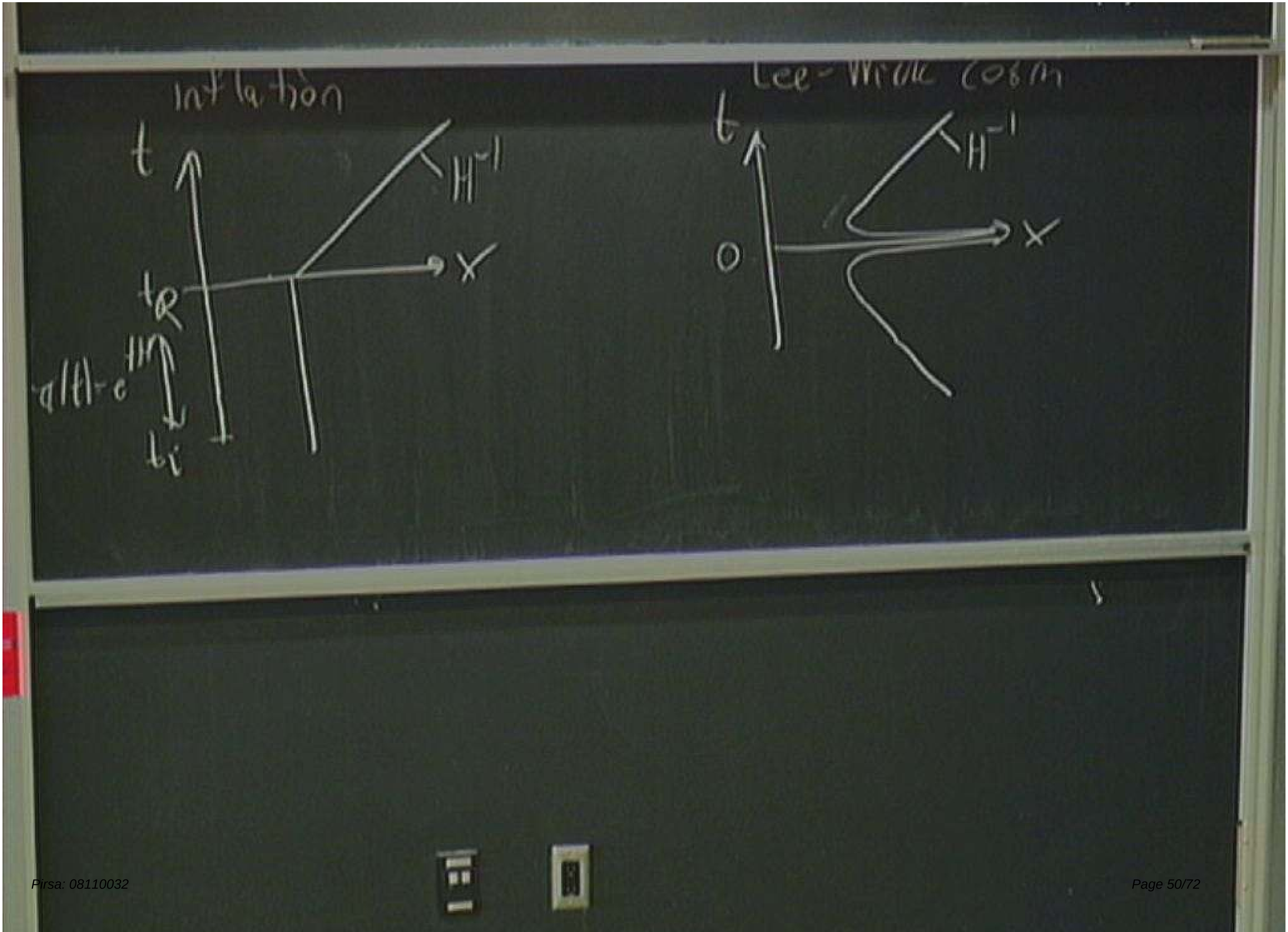
$$H^2 = \frac{8\pi G}{3}\left[\rho(\phi) - \rho(\tilde\phi) + \lambda(\phi - \tilde\phi)^4\right]$$

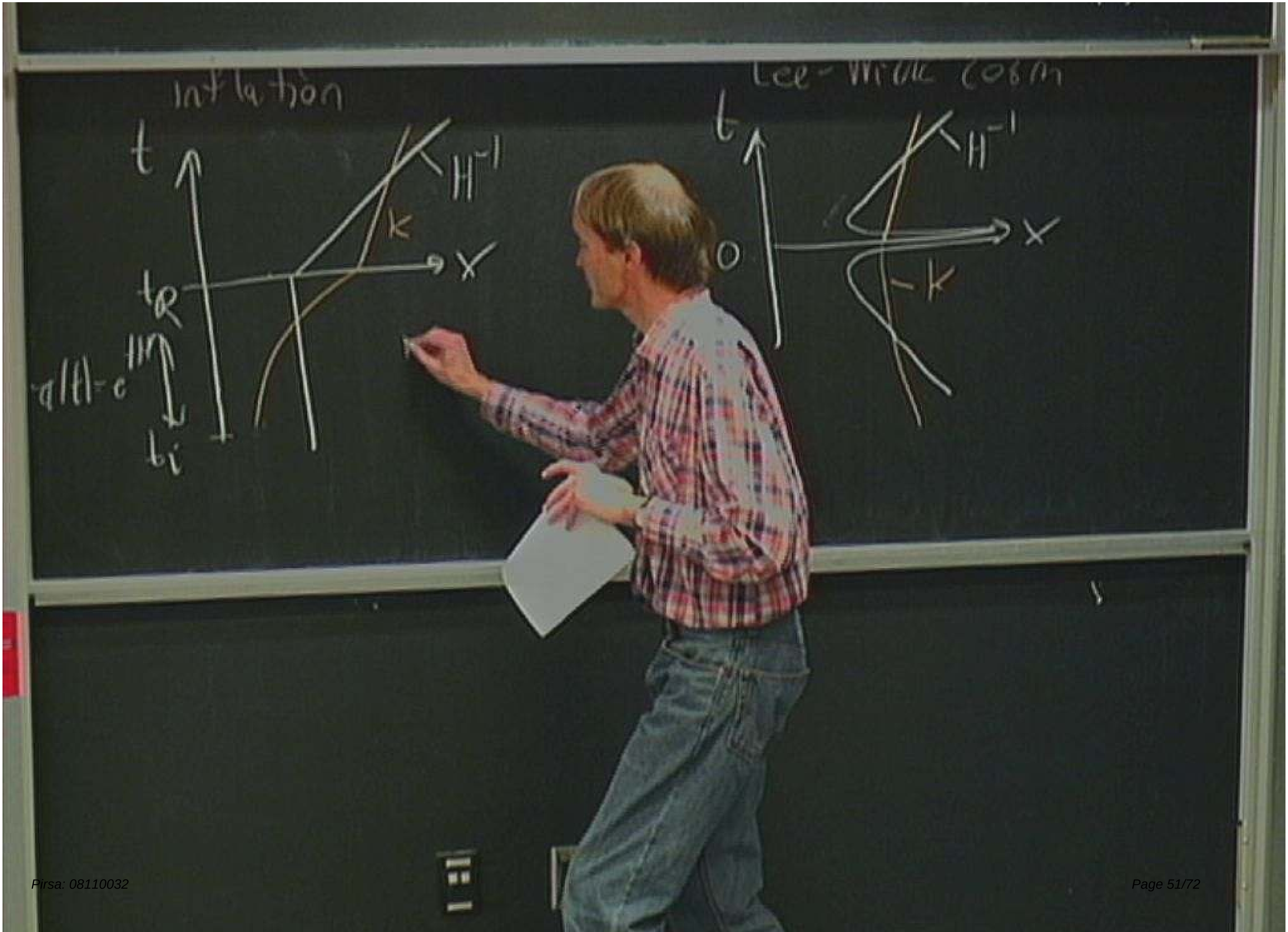
$$KG_\lambda(\phi) = 0$$

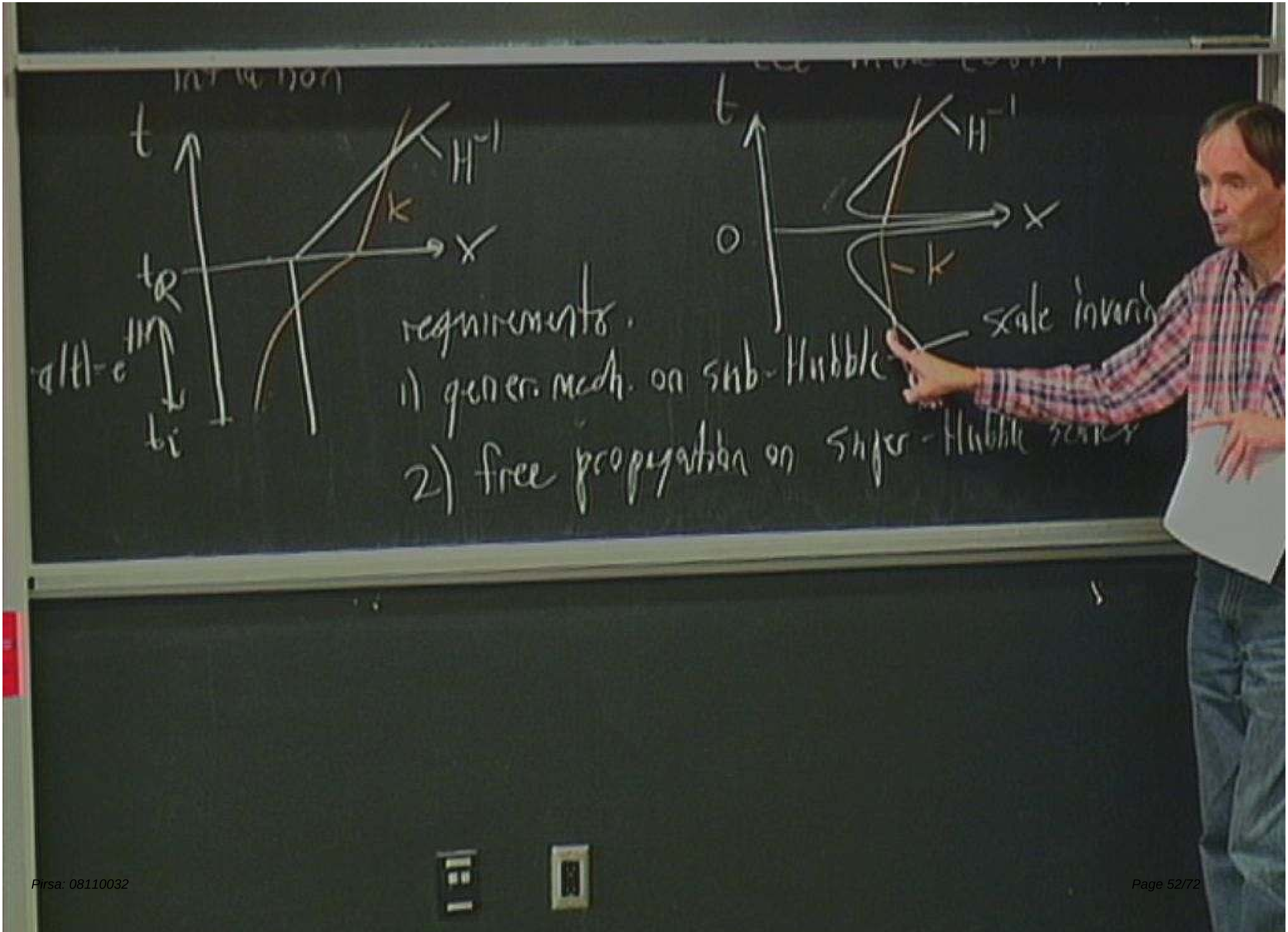
$$KG_\lambda(\tilde\phi) = 0$$

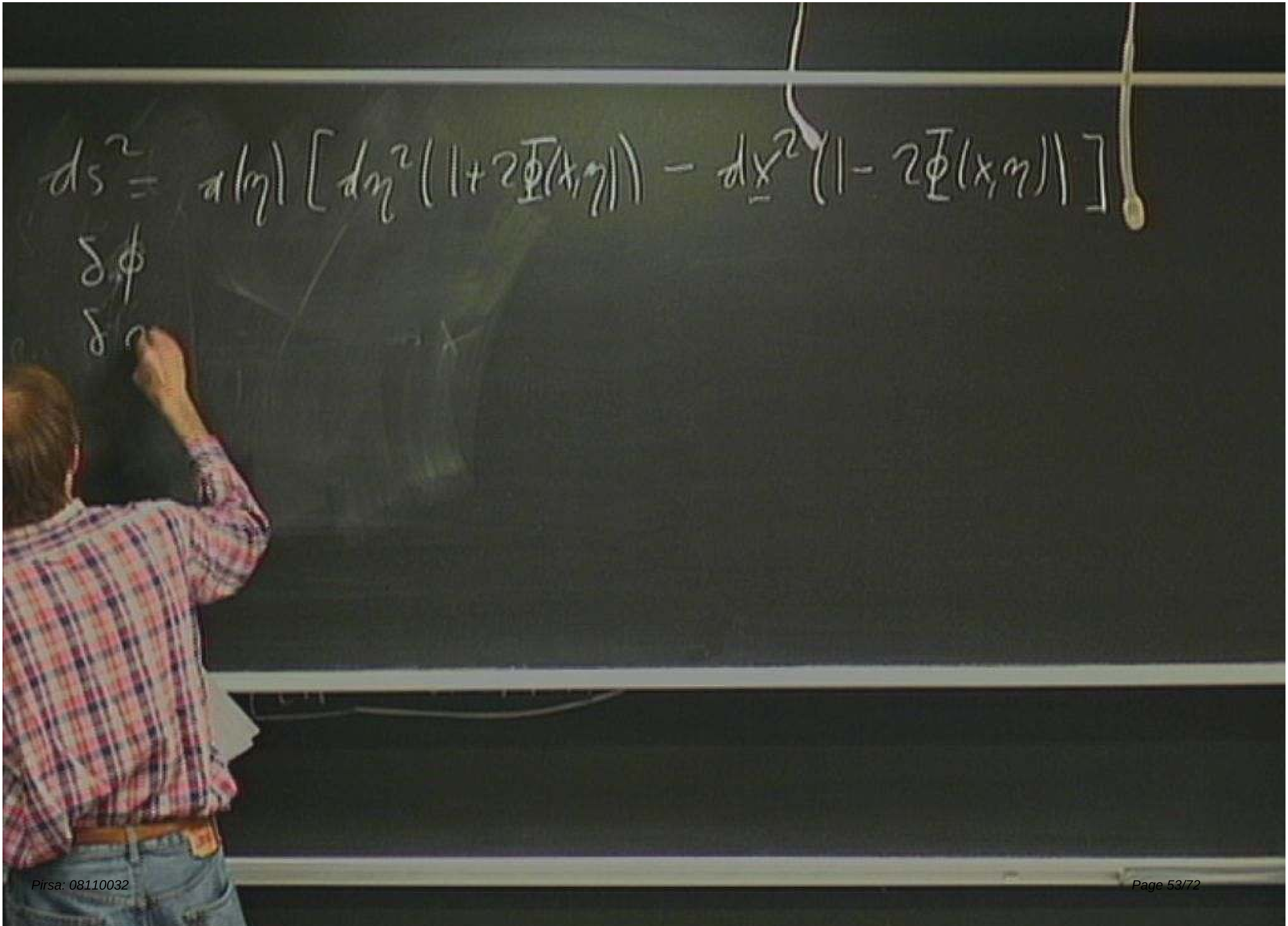
bounce is possible : $\rho(\tilde\phi) = \rho(\phi) + \lambda(\phi - \tilde\phi)^4$


inflation
t
x
Lee-Wick cosm
t
x


inflation
t
x
H^{-1}
t_R
$a(t) \sim e^{Ht}$
t_i
Lee-Wick cosm
t
0
x
H^{-1}


inflation
t
H^{-1}
k
x
t_R
t_i
Lee-Wick cosm
t
H^{-1}
x
0
-k


t
H^{-1}
k
x
t_R
$a(t) \sim e^{Ht}$
t_i
requirements:
1) gener. mech. on sub-Hubble
2) free propagation on super-Hubble scales
O
scale invari


$ds^2 = a(\eta)\left[d\eta^2(1+2\Phi(x,\eta)) - d\underline{x}^2(1-2\Phi(x,\eta))\right]$
$\delta\phi$
δ

$S_{tot} = S_g + \int d^4x \sqrt{-g}\ \mathcal{L}(\tilde{\phi})$

IC: matter dom. contraction → bounce stage 3

stage 1. $\phi(t), \tilde{\phi}(t)$ oscillating

$A(t), \tilde{A}(t) \sim a(t)^{-3/2}$

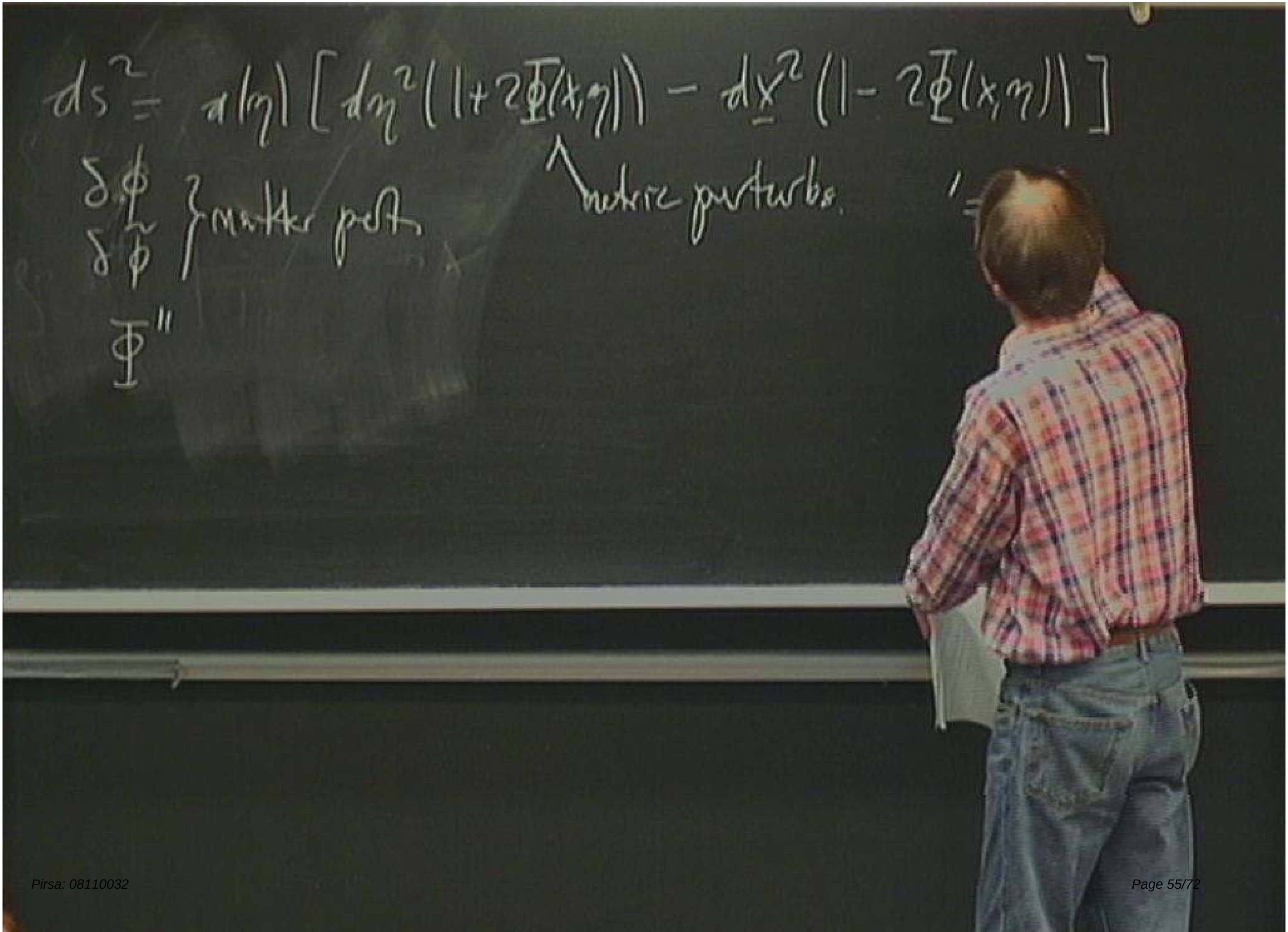
$\rho(\tilde{\phi}) \equiv \rho(\phi)\, F^{-1}$

stage 4 $= (\text{stage } 2)^{-1}$

" $5 = (\text{stage } 1)^{-1}$

stage 2. $A = \frac{1}{(12\pi)^{1/2}} m_{pl}$ slow rolling deflation $\Delta t\, H \sim \ln F$

$\tilde{A}(t) \sim a(t)^{-3/2}$


$ds^2 = a(\eta)\left[d\eta^2(1+2\Phi(x,\eta)) - d\underline{x}^2(1-2\Phi(x,\eta))\right]$
metric perturbs.
$\delta\phi$, $\delta\tilde{\phi}$ matter pert.
Φ''

$$ds^2 = a(\eta)\left[d\eta^2\left(1+2\Phi(x,\eta)\right) - d\underline{x}^2\left(1-2\Phi(x,\eta)\right)\right]$$

$\delta\phi$, $\delta\phi$ } matter pert.

^ metric perturbs.

$$' = \frac{\partial}{\partial\eta}$$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)$$

$$ds^2 = a(\eta)\left[d\eta^2\left(1+2\Phi(x,\eta)\right) - dx^2\left(1-2\Phi(x,\eta)\right)\right]$$

$\left.\begin{matrix}\delta\phi \\ \delta\tilde{\phi}\end{matrix}\right\}$ matter pert. metric perturbs. $' = \frac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta\tilde{\phi})$$

Model

Background cosmology

Cosmological fluctuations

Extras

$$ds^2 = a(\eta)\left[d\eta^2(1+2\Phi(x,\eta)) - dx^2(1-2\Phi(x,\eta))\right]$$

metric perturbs.

$\delta\phi$, $\delta\tilde{\phi}$ } matter pert.

$' = \frac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta\tilde{\phi})$$

Model

Background cosmology

Cosmological fluctuations

Extras

$$ds^2 = a(\eta)\left[d\eta^2(1+2\Phi(x,\eta)) - dx^2(1-2\Phi(x,\eta))\right]$$

$\delta\phi$, $\delta\dot\phi$ } matter pert.

metric perturbs.

$' = \frac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\left[\phi'\delta\phi + O(\delta\phi^2)\right]$$

Model

Background cosmology

Cosmological fluctuations

Extras

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi$$
$$+ O(\delta$$

$$f = z^{-1} v \qquad v'' + \left(k^2 - \frac{z''}{z}\right)v = 0 \qquad \Phi \sim \left(\frac{v}{z}\right)'$$

Background Cosmology

Cosmological fluctuations

Extras

$$ds^2 = a^2(\eta)\left[d\eta^2\left(1+2\Phi(x,\eta)\right) - dx^2\left(1-2\Phi(x,\eta)\right)\right]$$

$\left.\begin{matrix}\delta\phi \\ \delta\tilde{\phi}\end{matrix}\right\}$ matter pert. metric perturbs. $' = \frac{\partial}{\partial\eta}$

$$\Phi'' + 6\mathcal{H}\Phi' + 2(\mathcal{H}' + 2\mathcal{H}^2)\Phi + k^2\Phi = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\phi'\delta\phi + O(\delta\phi^2)$$

$$\delta\phi = z^{-1}v \qquad v'' + \left(k^2 - \frac{z''}{z}\right)v = 0 \qquad \Phi \sim \left(\frac{v}{z}\right)'$$

$$z = \frac{a\phi'}{\mathcal{H}}$$

Extras

What was known (vacuum IC)

i) expanding universe inflation ← s.i. spectrum of Φ

" of ζ

ii) contracting universe Ekpyrotic ← s.i. spectrum of Φ

not s.i. spectrum of ζ

$S_{tot} = S_g + \int d^4x \sqrt{-g} \ldots$

What was known (vacuum IC)

i) expanding universe   inflation ← s.i. spectrum of Φ
" of ζ

ii) contracting universe   Ekpyrotic ← s.i. spectrum of Φ
not s.i. spectrum of ζ

iii) " " matter dominated ← s.i. spectrum of ζ
k^{-4} " of Φ

$S_{tot} = S_g + \int \ldots$

What was known (vacuum IC)

i) expanding universe inflation ← s.i. spectrum of Φ

" of ζ

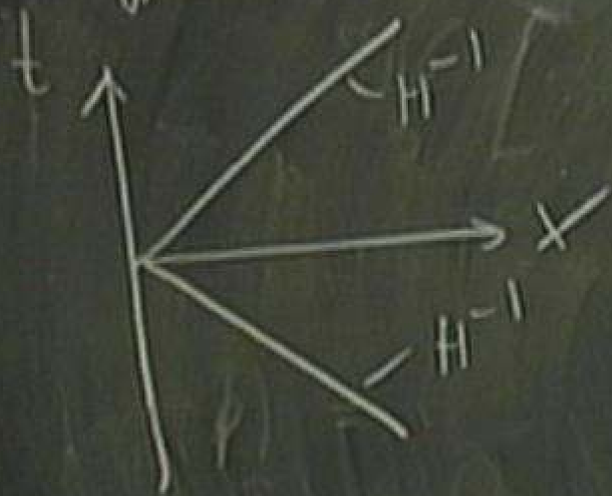
ii) contracting universe Ekpyrotic ← s.i. spectrum of Φ

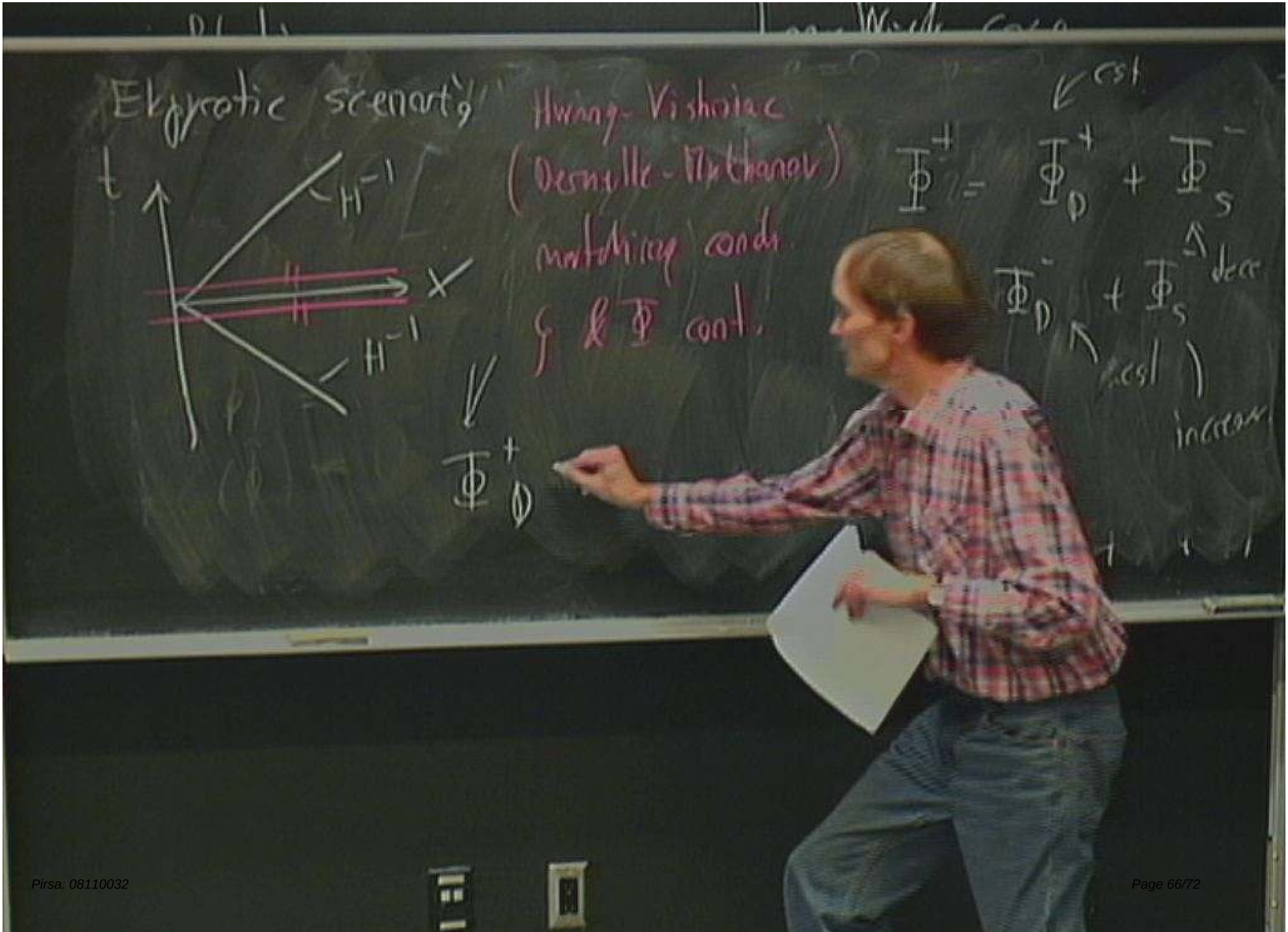
not s.i. spectrum of ζ

iii) " " matter dominated ← s.i. spectrum of ζ

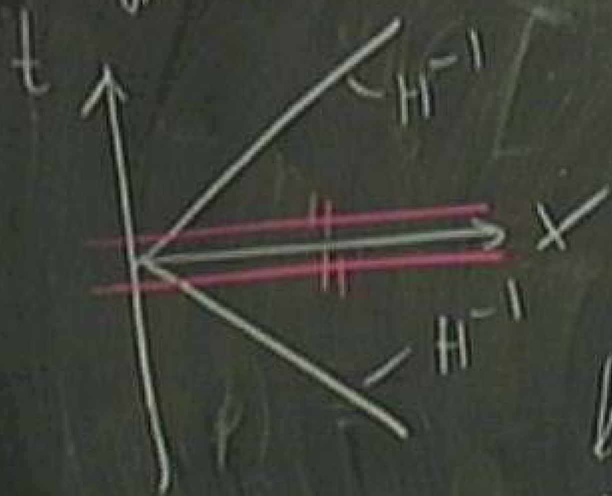
F. Finelli & R.B. 2001 k^{+4} " of Φ

Ekpyrotic scenario




Ekpyrotic scenario
Hwang-Vishniac
(Deruelle-Mukhanov)
matching cond
ζ & Φ cont.

Ekpyrotic scenario



Hwang-Vishniac

(Deruelle-Mukhanov)

matching cond.

ζ & Φ cont.

csl

$$\Phi^+ = \Phi^+_D + \Phi^-_S$$

decr

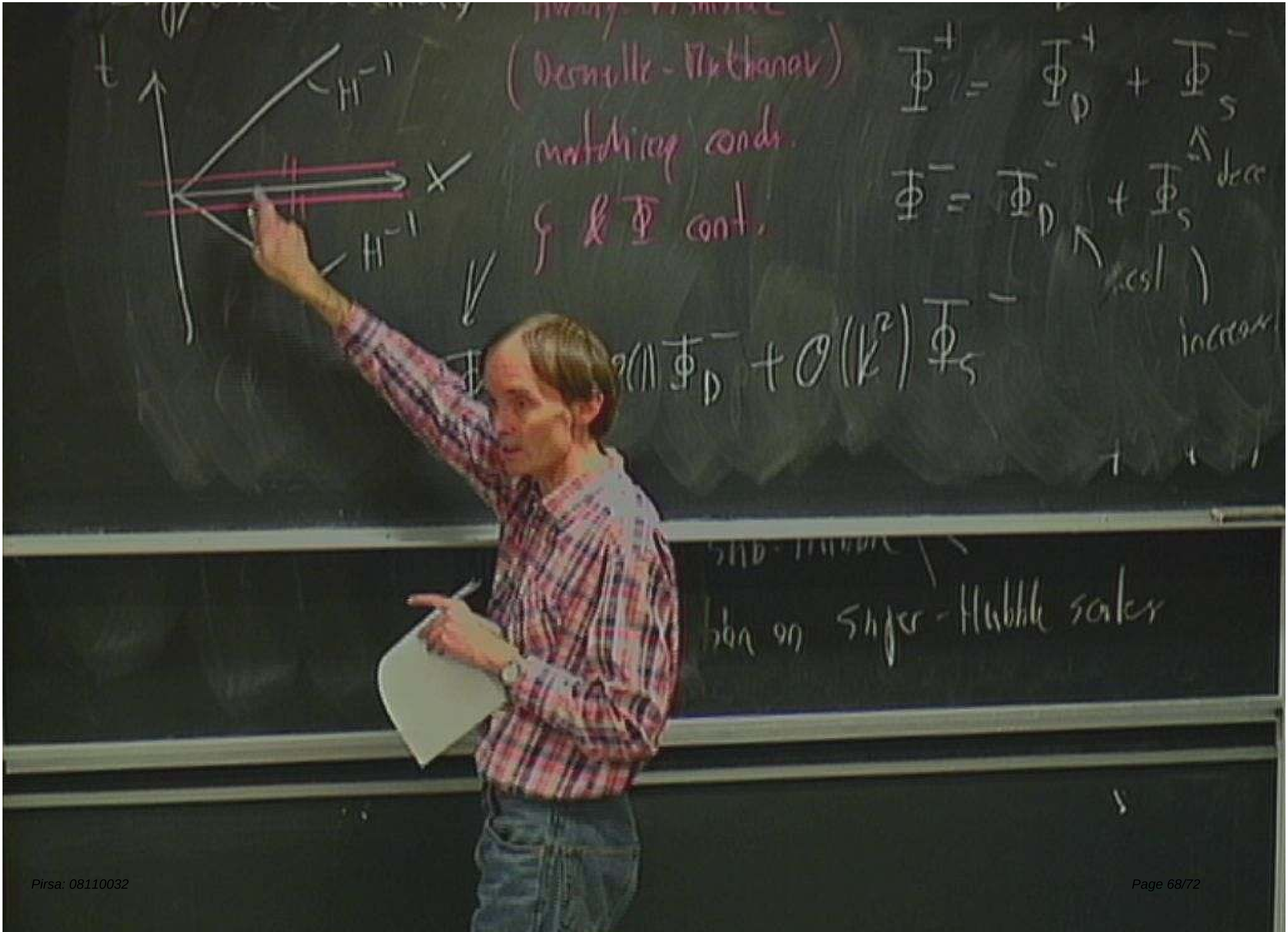
$$\Phi^- = \Phi^-_D + \Phi^-_S$$

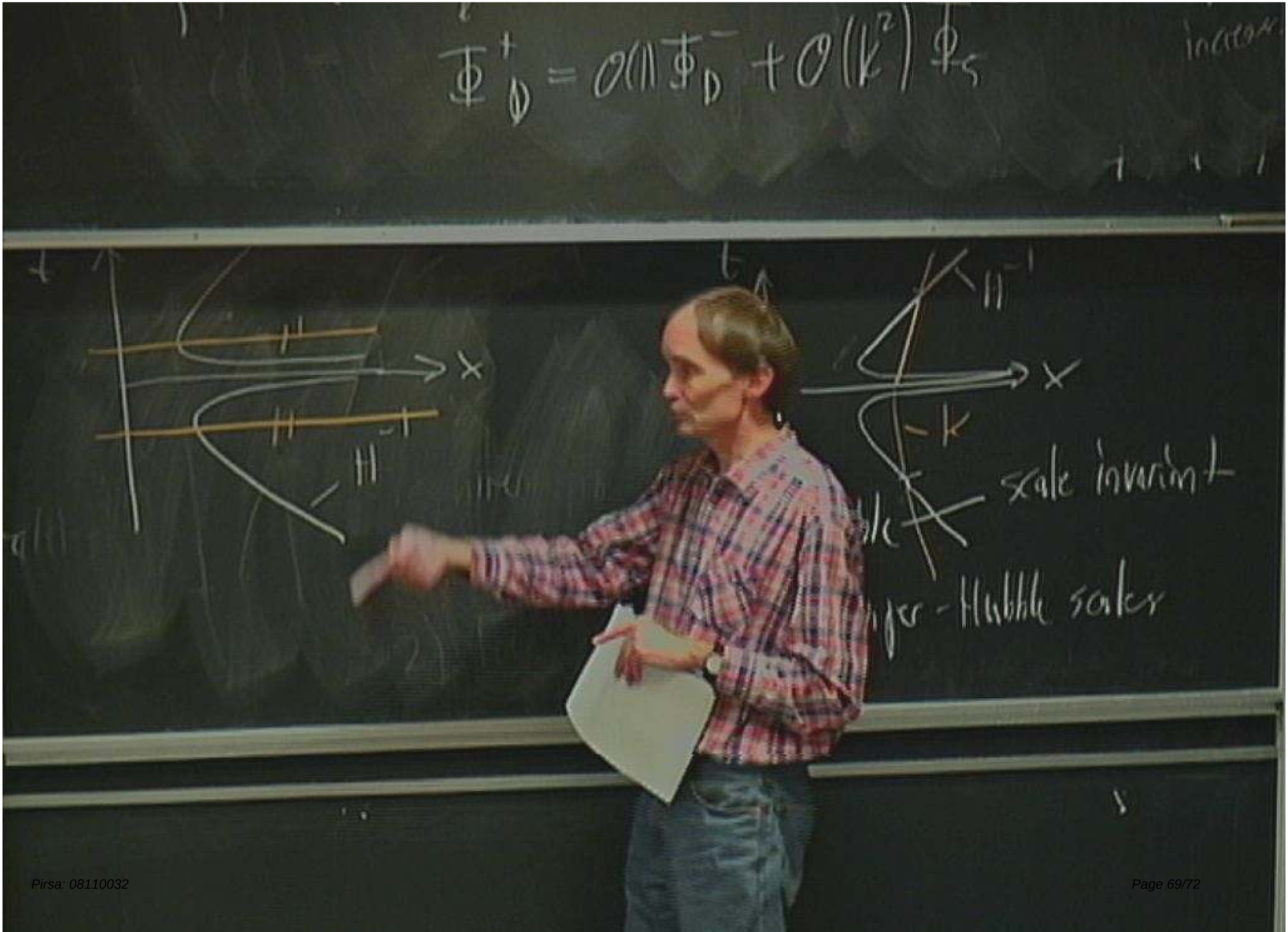
csl

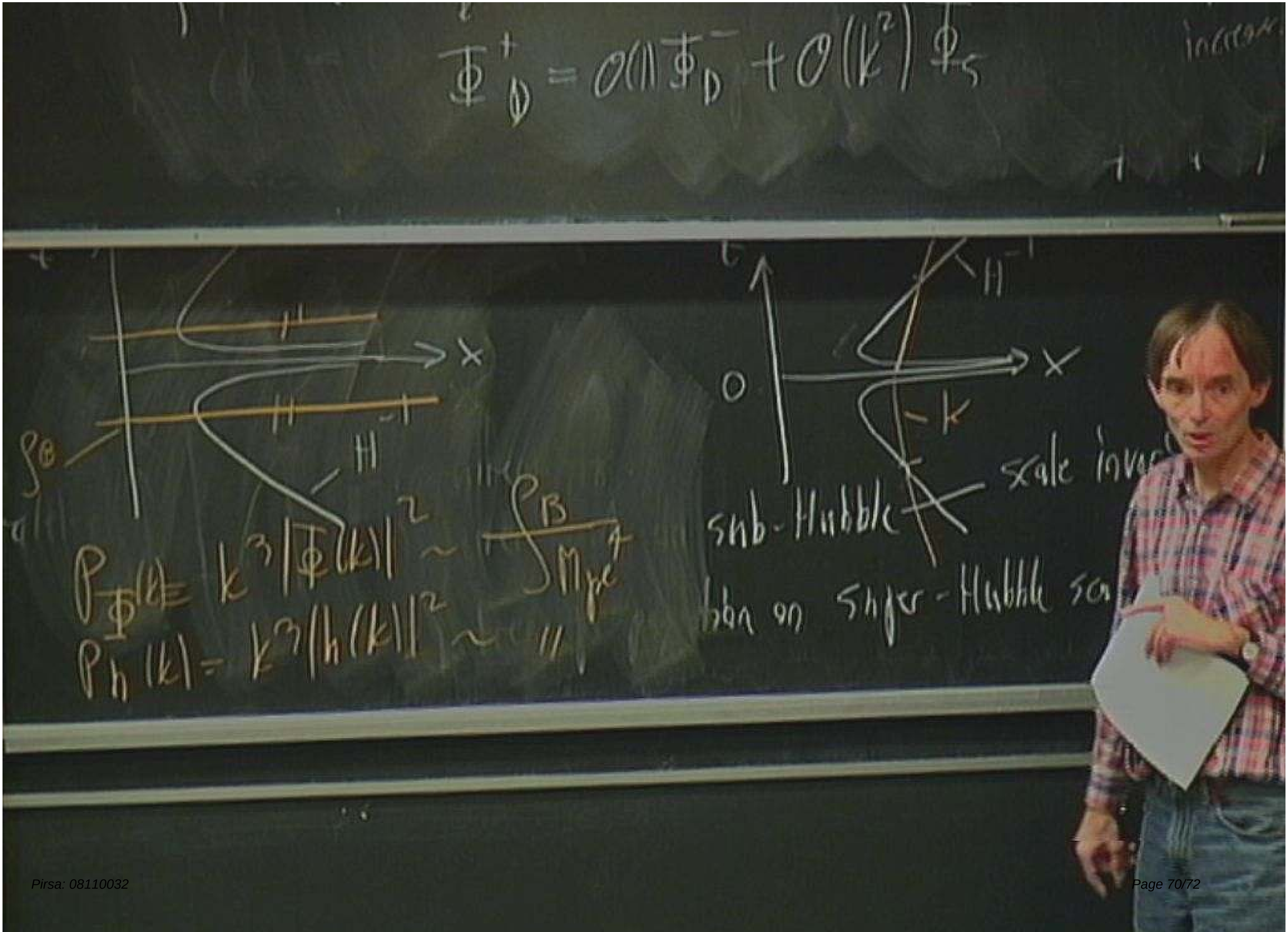
incr

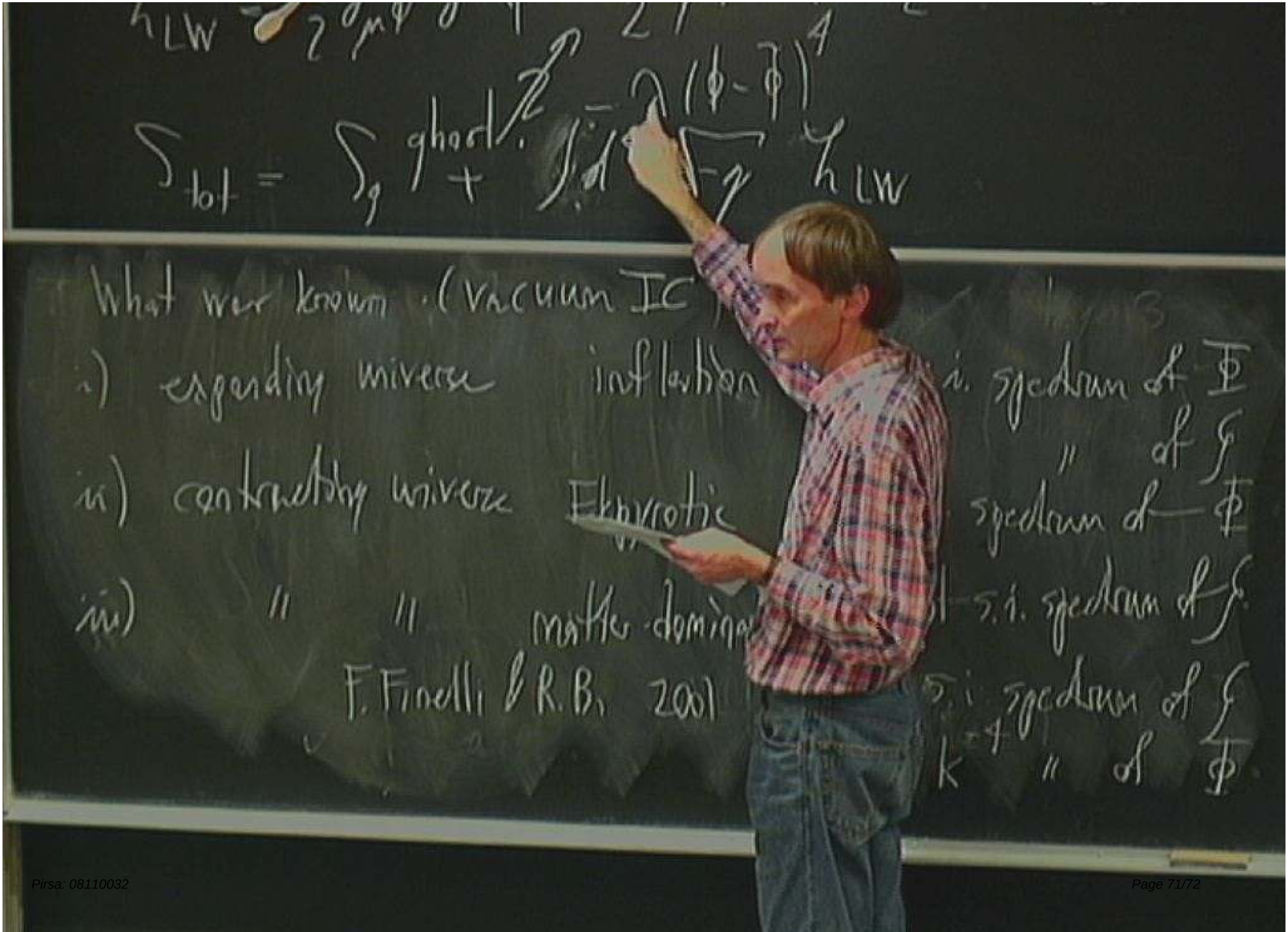
$$\Downarrow$$

$$\Phi^+_D = \mathcal{O}(1)\,\Phi^-_D + \mathcal{O}(k^2)\,\Phi^-_S$$

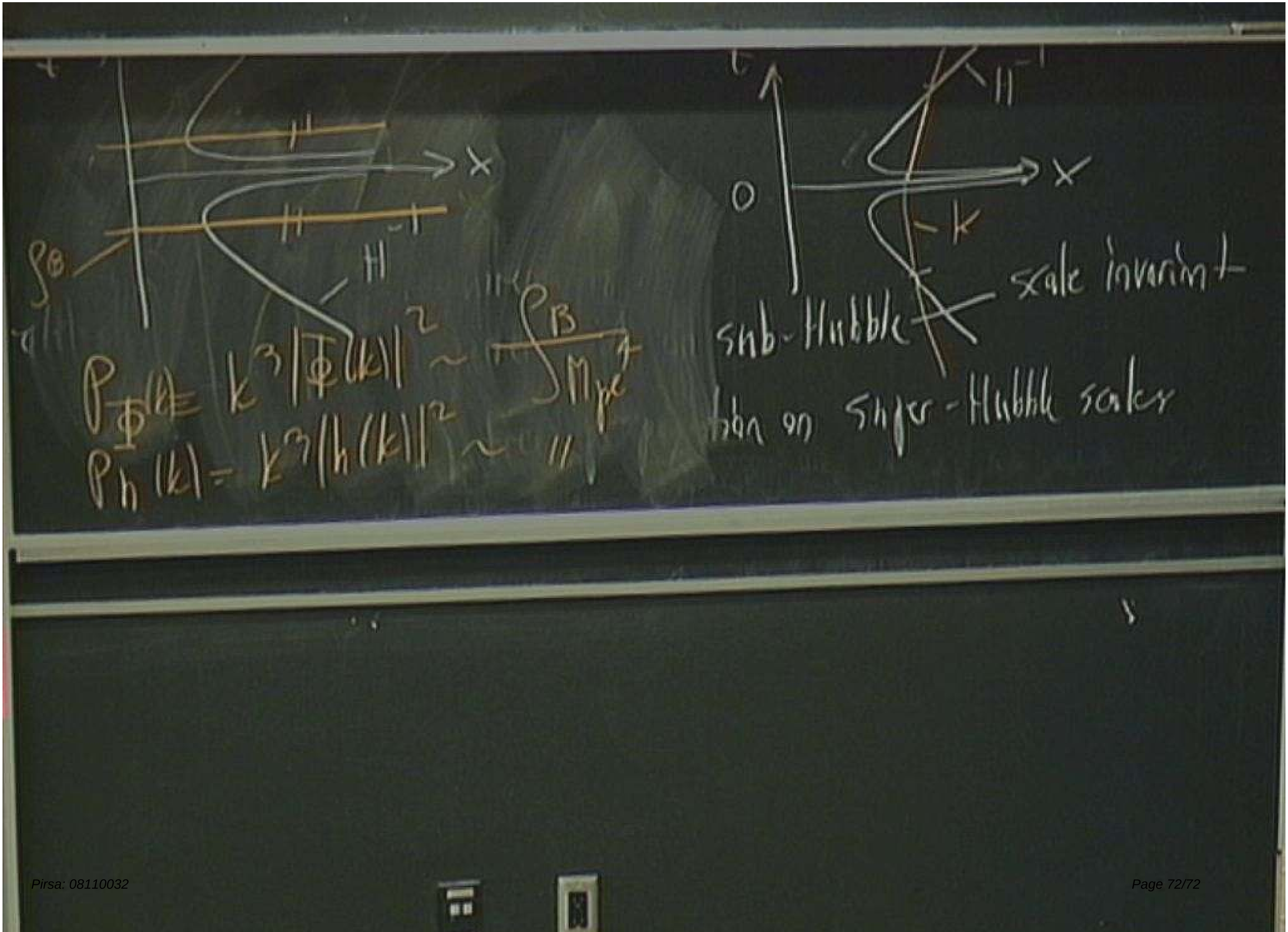

(Deruelle - Mukhanov)
matching cond.
super-Hubble scales


scale invariant


$\Phi_D^+ = O(1)\,\Phi_D^- + O(k^2)\,\Phi_S$
ρ_B
H
H^{-1}
x
t
0
$\sim k$
scale inv
sub-Hubble
$P_\Phi(k) = k^3|\Phi(k)|^2 \sim \frac{\rho_B}{M_{pl}^4}$
$P_h(k) = k^3|h(k)|^2 \sim$ "
super-Hubble



What was known (vacuum IC)
i) expanding universe inflation
s.i. spectrum of Φ
ii) contracting universe Ekpyrotic
spectrum of Φ
iii) " " matter-dominated
s.i. spectrum of ζ
F. Finelli & R.B. 2001
s.i. spectrum of ζ



scale invariant
sub-Hubble
on super-Hubble scales