Title: Non-conformal holography

Date: Nov 18, 2008 11:00 AM

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Abstract: We discuss holography for geometries that are asymptotic to non-conformal brane backgrounds. The near-horizon limit of all non-conformal branes, including D-branes and the fundamental string but excluding five-branes, is conformal to $AdS_{p+2} imes S^{8-p}$ with a linear dilaton. They exhibit a generalized conformal structure, both on the QFT and on the gravitational side. We develop holographic renormalization for all these cases and discuss a number of applications. We compute the holographic 2-point functions of the stress energy tensor and gluon operator and show they satisfy the expected Ward identities and the constraints of generalized conformal structure. The holographic results are also manifestly compatible with the M-theory uplift, with the asymptotic solutions, counterterms, one and two point functions etc. of the IIA F1 and D4 appropriately descending from those of M2 and M5 branes, respectively.

Introduction

In this talk I will discuss how to set up precision holography for backgrounds asymptotic to the near-horizon limit of the non-conformal branes.

- Shortly after the AdS/CFT conjecture, holographic dualities were also conjectured for the non-conformal branes [Itzhaki et al (1998)].
- Such dualities underpin interesting holographic models, for example the Witten-Sakai-Sugimoto model for QCD.
- However the detailed holographic dictionary was not set up for these dualities before our work.

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References

This talk will be based mostly on

- Ingmar Kanitscheider, Kostas Skenderis and Marika Taylor
  Precision holography for non-conformal branes
  0807.3324
- Related work appeared in Wiseman and Withers, 0807.0755.

Outline

1. Non-conformal branes
2. Generalized conformal structure
3. Non-conformal holography
4. Applications

Non-conformal branes

Recall that supergravity solutions for fundamental strings, Dp-branes and NS5-branes can all be written in the form

$$
\begin{aligned}
ds^2 &= H^a(H^{-1}ds^2(E^{p,1}) + ds^2(E^{9-p})) \\
e^{\phi} &= g_s H^b; \qquad A_{0\cdots p} = H^{-1} - 1
\end{aligned}
$$

for appropriate choices of the numbers (a, b) and where H is a single-centered harmonic function

$$H = (1 + Q/r^{7-p}).$$

with $Q \sim g_s N l_s^{7-p}$ for Dp-branes etc.

Field theory limit

- The field theory or decoupling limits of the Dp-brane backgrounds is:

$$g_s \to 0; \qquad \alpha' \to 0; \qquad U = r/\alpha' = fixed; \qquad g_d^2 N = fixed,$$

with U an energy scale.

- Here g_d^2 is the dimensionful Yang-Mills coupling, related to the string coupling via

$$g_d^2 \sim g_s \alpha'^{(p-3)/2}.$$

- Note that decoupling requires $N \to \infty$ when $p > 3$.

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Near-horizon limit

- In the decoupling limit the solutions become:

$$ds^2 = \left(\frac{Q}{r^{(7-p)}}\right)^{a-1} ds^2(E^{p,1}) + \left(\frac{Q}{r^{(7-p)}}\right)^{a} (dr^2 + r^2 d\Omega^2_{8-p})$$

$$e^{\phi} = g_s \frac{Q^b}{r^{b(7-p)}}$$

- The conjectured duality is then between string theory in this background and the non-conformal $(p+1)$-dimensional theories arising as low energy limits of the worldvolume theories.
- The metric is conformal to $AdS_{p+2} \times S^{8-p}$, except when $p = 5$ in which case the near-horizon geometry is conformal to $Mink_7 \times S^3$.

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Validity of approximations

- The dilaton in the case of Dp-branes can be expressed as

$$e^{\phi} = \frac{1}{N}(g_{eff}^{2}(u))^{\frac{7-p}{2(5-p)}}$$

where $g_{eff}^2(u) = g_d^2 N u^{p-3}$ and $u \sim E$ is the holographic radial direction (E is the energy scale of the dual theory).

- We consider $N \to \infty$ with $g_{eff}^2(u)$ fixed. In this limit the dilaton is small in all cases.
- $g_{eff}^2 \ll 1$: the SYM perturbative description is valid.
- $g_{eff}^2 \gg 1$: the supergravity description is valid.
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Remarks

- We initially include in the discussion fundamental strings and D6-branes.
- Whilst in both cases there is no decoupling, the conformally $AdS_{p+2} \times S^{8-p}$ geometries do solve the equations of motion - the question of holography in such a background is well posed.
- For fundamental strings, we will find that precision holography can be setup: the structure is inherited from the M2-brane and D1 case, in type IIA and IIB respectively.
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The dual frame [Boonstra, Skenderis, Townsend (1998)]

In the conformally AdS cases it useful to introduce a dual frame [Duff, Gibbons, Townsend (1994)] such that

$$ds^2_{dual} = (Ne^{\phi})^c ds^2,$$

with $c = -2/(7-p)$ for Dp branes and $c = -2/3$ for F1, and the action becomes:

$$S = \frac{N^2}{(\alpha')^4} \int d^{10}x \sqrt{-g} N^{\gamma} e^{\gamma\phi} [R + \beta(\partial\phi)^2 - \frac{1}{2N^2(8-p)!} |F_{8-p}|^2]$$

The field equations in this frame admit an $AdS_{p+2} \times S^{8-p}$ solution with linear dilaton.

Consistent truncation

- Reducing on the sphere and (consistently) truncating to the graviton and dilaton gives the $(p+2)$-dimensional action

$$S = L \int d^{d+1}x \sqrt{-g} e^{\gamma\phi} [R + \beta(\partial\phi)^2 + C].$$

where the constants (β, γ, C, L) depend on the case.

- The truncation corresponds to restricting to just the stress energy tensor and the gluon operator in the field theory; having dealt with this sector, it is straightforward to extend to other supergravity fields, or equivalently chiral operators.

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Linear dilaton AdS background

- The lower dimensional equations of motion admit an AdS_{d+1} solution with linear dilaton:

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\begin{aligned}
ds^2 &= \frac{d\rho^2}{4\rho^2} + \frac{dx_i dx^i}{\rho}; \\
e^{\phi} &= \rho^{\alpha},
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- AdS isometries are broken only by the non-trivial dilaton.
- Thus the background admits a generalized conformal "symmetry": the AdS isometries, which map to conformal transformations of the boundary theory, are restored if the string coupling g_s is transformed as a background field of appropriate weight. [Jevicki, Yoneya et al (1998)]

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Generalized conformal structure

- The generalized conformal structure underlies non-conformal brane dualities.
- Whilst it is less restrictive than full conformal invariance, nonetheless it implies properties such as Ward identities for correlation functions.

Generalized conformal structure in dual theory

- The (bosonic part of the) worldvolume theory of multiple Dp-branes is

$$S = -\int d^{p+1}x \mathrm{Tr}\left(-\frac{1}{4g_d^2}F_{ij}F^{ij} + \frac{1}{2}XD^2X + \frac{1}{4}g_d^2[X,X]^2\right)$$

The Yang-Mills coupling g_d^2 has (length) dimension $(p-3)$.

- This action can be coupled to a background metric $g_{(0)ij}$ in a Weyl invariant manner:

$$S[g_{(0)ij}(x),\Phi_{(0)}(x)] = -\int d^{p+1}x\sqrt{g_{(0)}}\left(-\Phi_{(0)}\frac{1}{4}\mathrm{Tr}F_{ij}F^{ij}\right.$$
$$\left.+\frac{1}{2}\mathrm{Tr}\left(X(D^2-\frac{(d-2)}{4(d-1)}R)X\right)+\frac{1}{4\Phi_{(0)}}\mathrm{Tr}[X,X]^2\right).$$

This reduces to the previous action when $g_{(0)ij} = \delta_{ij}, \Phi_{(0)} = 1/g_d^2$.

- Weyl transformations act on sources and fields, as usual

$$g_{(0)} \to e^{2\sigma}g_{(0)}; \quad \Phi_{(0)} \to e^{(4-d)\sigma}\Phi_{(0)}, \quad X \to e^{(1-\frac{d}{2})\sigma}X, \quad A_i \to A_i.$$

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The Yang-Mills coupling g_d^2 has (length) dimension $(p-3)$.

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- Weyl transformations act on sources and fields, as usual

$$g_{(0)} \to e^{2\sigma}g_{(0)}; \quad \Phi_{(0)} \to e^{(4-d)\sigma}\Phi_{(0)}, \quad X \to e^{(1-\frac{d}{2})\sigma}X, \quad A_i \to A_i.$$

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Ward identities

- The dilatation Ward identity is:

$$\langle T_i^i \rangle + (p-3)\Phi_{(0)}\langle \mathcal{O} \rangle = 0.$$

- The diffeomorphism Ward identity is:

$$\nabla_i \langle T^{ij} \rangle + \langle \mathcal{O} \rangle \partial_i \Phi_{(0)} = 0.$$

- These identities hold in the presence of sources, and thus can be functionally differentiated to give relations between correlation functions e.g.

$$\langle T_i^i(x)\mathcal{O}(0)\rangle + (p-3)\frac{1}{g_d^2}\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = 0.$$

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Anomalies

- Next one can ask about anomalies in these identities ...
- In a conformal field theory, anomalies in the trace of the stress energy tensor are classified by appropriate conformal invariants, e.g. in $d = 2$

$$\langle T_i^i \rangle = cR$$

  with c the central charge and R the Ricci scalar.
- Here one expects anomalies in the Ward identities to be determined by appropriate generalized conformal invariants.
- For most of the cases of interest here (D0, D1, D2,..) there are no anomalies in the Ward identities: this will come out of the holographic computation, and can presumably be derived directly from the field theory.

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D4-brane anomaly

- On the other hand, for the D4-brane theory there are possible anomaly terms, even though the field theory is in five dimensions:

$$\langle T^i_i \rangle + \Phi_{(0)} \langle \mathcal{O} \rangle = \mathcal{A} = \sum_a c_a \mathcal{A}_a$$

where the invariants $\mathcal{A}_a$ involve both the curvature of $g_{(0)}$ and $\Phi_{(0)}$.

- This is what one expects from the dimensional reduction of the conformal M5-brane theory, where the anomaly is

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Other implications of the generalized conformal structure

The generalized conformal structure leads to a number of other restrictions:

- The entropy at finite temperature T_H behaves as

$$S = \tilde{c}(g_{eff}^2(T_H), N, \cdots) V_p T_H^p.$$

  where $g_{eff}^2(T_H) = g_d^2 N T_H^{p-3}$ is the effective dimensionless coupling constant and $\tilde{c}$ is a generic function of the dimensionless parameters.

- The form of the 2-point function of scalar operators $\mathcal{O}$ of dimension Δ is

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Remarks

- The source $\Phi_{(0)}$ can also be thought as a conformal compensator: a theory which is not invariant under Weyl transformations, can be made Weyl invariant by introducing a new field φ which transforms linearly under Weyl transformations, $\delta_W \varphi = \sigma$, and appropriately multiplying each term with the new field.
- In flat space such theory will be conformally invariant provided the compensator transforms appropriately.
- A related example is the recent discussion of the Lorentzian BLG M2-brane theories [Bandes et al, Gomis et al]: The scalar field X^+ whose expectation value is g^2_{YM} can be thought of as the conformal compensator for the SYM theory.
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Outline

1. Non-conformal branes
2. Generalized conformal structure
3. Non-conformal holography
4. Applications

Generalities

- We now proceed to discuss setting up holography, i.e. develop holographic renormalization for the non-conformal backgrounds.
- But what do we actually mean with this?

Generalities

Recall that in holography:

1. Bulk fields Φ^I are in 1-1 correspondence with boundary gauge invariant operators $\mathcal{O}^I$.
2. The boundary fields $\phi^I_{(0)}$ parametrizing the boundary conditions of bulk fields Φ^I are identified with the sources of dual operators.
3. Within the low energy approximation, the bulk on-shell action $S_{onshell}[\phi^I_{(0)}]$ as a function of sources is identified with the generating functional of connected correlators of the boundary QFT.

Generalities: what is holographic renormalization?

So one has to:

1. Specify what are the appropriate boundary conditions.
   - The (non-linear) bulk field equations must admit boundary conditions specified by arbitrary functions, since QFT sources are unconstrained: we need to be able to functionally differentiate w.r.t them.
   - This is a non-trivial requirement. For example, it is not known whether such boundary conditions exist for asymptotically flat gravity. The straightforward generalization of the AdS case leads to constrained boundary conditions [de Haro, Solodukhin, KS (2000)].
2. Show that the bulk on-shell action can be rendered finite for arbitrary boundary conditions $\phi^I_{(0)}(x)$.
   - The objective is to show that $S_{onshell}[\phi^I_{(0)}(x)]$ exists as a generating functional of correlators, not just to show that one can extract a finite value for the action evaluated on a specific background.

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Generalities: general requirements

The process of holographic renormalization should

- depend only on the asymptotic form of bulk solutions. This is the bulk counterpart of the fact in QFT renormalization of UV divergences does not dependent on the IF region.
- be valid for all solutions of the bulk field equations with the prescribed boundary data.

Asymptotically locally $AdS \times X$ spacetimes

- In the case of asymptotically locally $AdS \times X$ spacetimes, with X a compact manifold, this problem has been solved in complete generality.
- Given any theory that admits such solutions, there is an algorithmic algebraic procedure that produces counterterms etc. and leads to **holographic 1-point functions in the presence of sources.**

— The objective of this work is to extend holographically renormalization for spacetimes with non-conformal asymptotics.

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Holographic Renormalization for Asymptotically locally AdS spacetimes

The original form of the method involved the following steps: [Henningson, Skenderis (1998), de Haro, Solodukhin, Skenderis (2000)]

1. Obtain the most general asymptotic solution of the bulk fields equations with the appropriate Dirichlet boundary conditions. E.g. for the metric

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j$$

$$g_{ij}(x,\rho) = g_{(0)}(x) + \rho g_{(2)}(x) + \cdots + \rho^{d/2}(g_{(d)ij} + \log \rho h_{(d)ij}) + \cdots$$

2. Evaluate the regulated on-shell action on the asymptotic solutions to obtain the most general divergences that can appear.
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4. Vary $S_{ren}[\phi_{(0)}]$ w.r.t. $\phi_{(0)}$ to obtain the holographic 1-point function in the presence of sources. This is given in terms of certain asymptotic coefficients. For example for the metric,

$$\langle T_{ij} \rangle = \frac{2}{\sqrt{g_{(0)}}} \frac{\delta S_{ren}}{\delta g_{(0)}^{ij}} = \frac{d}{16\pi G} [g_{(d)ij} + X_{ij}^{(d)}(g_{(0)})].$$

where $X_{ij}^{(d)}(g_{(0)})$ are local (non-linear) functions of $g_{(0)}$.

5. Higher point functions can be obtained by further functional differentiation. E.g.

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Holographic Renormalization for Asymptotically locally AdS spacetimes

The original form of the method involved the following steps: [Henningson, Skenderis (1998), de Haro, Solodukhin, Skenderis (2000)]

1. Obtain the most general asymptotic solution of the bulk fields equations with the appropriate Dirichlet boundary conditions. E.g. for the metric

$$
\begin{aligned}
ds^2 &= \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \\
g_{ij}(x,\rho) &= g_{(0)}(x) + \rho g_{(2)}(x) + \cdots + \rho^{d/2}(g_{(d)ij} + \log \rho h_{(d)ij}) + \cdots
\end{aligned}
$$

2. Evaluate the regulated on-shell action on the asymptotic solutions to obtain the most general divergences that can appear.
3. Show that these can be removed by covariant, local boundary counterterms. This results in the renormalized on-shell action, S_{ren}.

Holographic Renormalization for Asymptotically locally AdS spacetimes

4. Vary $S_{ren}[\phi_{(0)}]$ w.r.t. $\phi_{(0)}$ to obtain the holographic 1-point function in the presence of sources. This is given in terms of certain asymptotic coefficients. For example for the metric,

$$\langle T_{ij} \rangle = \frac{2}{\sqrt{g_{(0)}}} \frac{\delta S_{ren}}{\delta g_{(0)}^{ij}} = \frac{d}{16\pi G} [g_{(d)ij} + X_{ij}^{(d)}(g_{(0)})].$$

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Radial Hamiltonian formalism [Papadimitriou, KS (2005)]

- This form of the method is conceptually simple, but computationally inefficient as it does not exploit the underlying conformal structure.
- For most explicit computations, it is better to use the radial Hamiltonian formalism, a Hamiltonian formulation in which the radius plays the role of time.
- One relates the regularized holographic 1-point of an operator $\mathcal{O}_\Phi$ to the radial canonical momentum π_Φ of the corresponding bulk field Φ.

$$\delta S = \int dr\left(\frac{\partial L}{\partial \Phi} - \partial_r \frac{\partial L}{\partial(\partial_r \Phi)}\right)\delta\Phi + \left[\frac{\partial L}{\partial(\partial_r\Phi)}\delta\Phi\right]_r, \qquad L \equiv \int d^d x \sqrt{G}\mathcal{L}$$

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Radial Hamiltonian formalism: renormalization

One still has to renormalize ....

- A fundamental property of asymptotically locally AdS spacetimes is that scale transformations are part of the asymptotic symmetries and therefore every covariant quantity can be decomposed into a sum of terms each having a definite scaling.
- Thus the canonical momenta of a field dual to a dimension k operator are asymptotically expanded as

$$\pi^k = \pi^k_{(d-k)} + \cdots + \pi^k_{(k)} + \tilde{\pi}^k_{(k)} \log \rho + \cdots$$

with each coefficient $\pi^k_{(n)}$ having weight n.

- Each coefficient can be expressed (non-linearly) in terms of the asymptotic expansions, but the holographic 1-point functions are naturally expressed in terms of the coefficients of π^k, as these transform correctly. E.g.

$$\langle T_{ij} \rangle = \pi_{(d)ij}.$$

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Exploiting this approach has many advantages:

- One can bypass the on-shell action (and computation of counterterms) to compute renormalized correlators, as these can be obtained from the canonical momenta.
- Ward identities follow almost immediately.
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Asymptotics of non-conformal branes

- We now return to the case of interest. We define a linear dilaton asymptotically locally AdS structure as follows:
- We consider $(d+1)$-dimensional backgrounds which admit the following expansion near the conformal boundary at $\rho = 0$:

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{g_{ij}(x,\rho)dx^i dx^j}{\rho},$$

$$\phi(x,\rho) = \alpha \log \rho + \frac{\kappa(x,\rho)}{\gamma},$$

where $g(x,\rho)$ and $\kappa(x,\rho)$ are regular near $\rho = 0$:

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Most general asymptotic solution

The structure of the asymptotic solution depends on whether the parameter $\sigma = (d - 2\alpha\gamma)/2$ is (a) non-integral or (b) an integer (cf AdS case, where $\alpha = 0$).

(a) D0, D1, D2, F1: the structure is like that of odd d AdS_{d+1}.

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Non-integral cases: D0, D1, D2, F1

- All coefficients $(g_{(2n)}, \kappa_{(2n)})$ for $n < \sigma$ are locally determined via algebraic equations in terms of the non-normalizable terms $(g_{(0)}, \kappa_{(0)})$.
- $(g_{(2\sigma)}, \kappa_{(2\sigma)})$ are undetermined, apart from constraints on their trace and divergence.
- One can see that the undetermined terms occur at these powers on dimensional grounds: they should determine the vev of the dimension d stress energy tensor, but the overall normalization of the bulk action has dimension $2\alpha\gamma$. (Recall that $\sigma = (d - 2\alpha\gamma)/2$).

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Non-integral cases: D0, D1, D2, F1

- Holographic renormalization proceeds as in AdS: remove divergence with covariant, local counterterms, and obtain renormalized vevs from the renormalized onshell action.
- This results in simple formulae for the vevs in terms of the undetermined coefficients:

$$\langle T_{ij} \rangle = 2\sigma L e^{\kappa_{(0)}} g_{(2\sigma)ij},$$

with the trace and divergence constraints implying the expected Ward identities:

$$\langle T_i^i \rangle + (p-3)\Phi_{(0)} \langle \mathcal{O} \rangle = \nabla^i \langle T_{ij} \rangle + \partial_j \Phi_{(0)} \langle \mathcal{O} \rangle = 0.$$

The formulae are exactly analogous to those in even-dimensional AdS.

Integral case: D4 branes

- All coefficients $(g_{(2n)}, \kappa_{(2n)})$ for $n < \sigma$ are locally determined via algebraic equations in terms of $(g_{(0)}, \kappa_{(0)})$.
- One must include log terms $\rho^{\sigma} \ln(\rho)$ with coefficients $(h_{(2\sigma)}, \tilde{\kappa}_{(2\sigma)})$ determined in in terms of $(g_{(0)}, \kappa_{(0)})$.
- $(g_{(2\sigma)}, \kappa_{(2\sigma)})$ are undetermined, up to trace and divergence constraints.

Thus the structure for D4-branes indeed mirrors odd-dimensional AdS.

Holographic vevs for the D4 brane

- The expressions for the vevs in this case involve non-linear terms, i.e.

$$\langle T_{ij} \rangle = 2\sigma L e^{\kappa_{(0)}} g_{(2\sigma)ij} + \mathcal{T}_{ij}[g_{(0)}, \kappa_{(0)}],$$

  with $\mathcal{T}$ a known (complicated!) functional of the non-normalizable modes.
- The dilatation identity is in this case anomalous, namely

$$\langle T_i^i \rangle + \Phi_{(0)} \langle \mathcal{O} \rangle = \mathcal{A}[g_{(0)}, \kappa_{(0)}].$$

- The anomaly descends directly from the holographic Weyl anomaly of the M5-brane theory.

Uplift to M-theory

- The IIA F1 and D4 solutions lift to M2 and M5 solutions of M-theory.
- Reducing AdS_4/AdS_7 (in Fefferman-Graham coordinates) over the M-theory circle y:

$$ds^2_{d+1} = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}(g_{ij}dx^i dx^j + e^{2\kappa}dy^2).$$

leads to the F1/D4 solution in the dual-frame:

$$\begin{aligned} ds^2_d &= \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}g_{ij}dx^i dx^j \\ e^{4\phi/3} &= \frac{1}{\rho}e^{2\kappa}, \end{aligned}$$

- This extends to the general case: all holographic results for spacetimes with the F1/D4 asymptotics are in agreement with the reduction over the M-theory circle of the holographic results for asymptotically AdS_4 and AdS_7 specetimes, respectively.

Holographic vevs for the D4 brane

- The expressions for the vevs in this case involve non-linear terms, i.e.

$$\langle T_{ij} \rangle = 2\sigma L e^{\kappa_{(0)}} g_{(2\sigma)ij} + \mathcal{T}_{ij}[g_{(0)}, \kappa_{(0)}],$$

  with $\mathcal{T}$ a known (complicated!) functional of the non-normalizable modes.
- The dilatation identity is in this case anomalous, namely

$$\langle T_i^i \rangle + \Phi_{(0)} \langle \mathcal{O} \rangle = \mathcal{A}[g_{(0)}, \kappa_{(0)}].$$

- The anomaly descends directly from the holographic Weyl anomaly of the M5-brane theory.

Uplift to M-theory

- The IIA F1 and D4 solutions lift to M2 and M5 solutions of M-theory.
- Reducing AdS_4/AdS_7 (in Fefferman-Graham coordinates) over the M-theory circle y:

$$ds^2_{d+1} = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}(g_{ij}dx^i dx^j + e^{2\kappa}dy^2).$$

leads to the F1/D4 solution in the dual-frame:

$$\begin{aligned} ds^2_d &= \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}g_{ij}dx^i dx^j \\ e^{4\phi/3} &= \frac{1}{\rho}e^{2\kappa}, \end{aligned}$$

- This extends to the general case: all holographic results for spacetimes with the F1/D4 asymptotics are in agreement with the reduction over the M-theory circle of the holographic results for asymptotically AdS_4 and AdS_7 specetimes, respectively.

Extensions to include other operators

- We discussed renormalization for the truncation to the graviton and dilaton, or equivalently the stress energy tensor and gluon operator in the field theory.
- It is straightforward to generalize to linear dilaton asymptotically AdS solutions of gauged supergravities, and to the ten-dimensional linear dilaton conformally $AdS_{p+2} \times S^{8-p}$ backgrounds.
- The former allows us to include the R-currents and other low dimension scalar operators in the boundary theory; the latter involves retaining all operators dual to the Kaluza-Klein fields, and is the generalization of Kaluza-Klein holography [Skenderis, Taylor (2006)] to this case.

Counterterms, branes and the variational problem

- For fundamental strings (and branes) in AdS one can obtain a finite action by
  - (a) holographic renormalization by boundary counterterms [Graham-Witten (1999), Karch, O'Bannon, Skenderis (2005)]
  - (b) by adding boundary terms such that the variational problem is well-defined. [Drukker, Gross, Ooguri (1999)]
- These procedures are however equivalent: the counterterms in general are needed not just to renormalize the on-shell action, but to render the (appropriate) variational problem well-posed [Papadimitriou, Skenderis (2005)].
- Finding the counterterms for the brane actions is easier (no dynamical gravity) and can be done for all cases of interest, such as the D8-branes in Witten-Sakai-Sugimoto.

Outline

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2. Generalized conformal structure
3. Non-conformal holography
4. Applications

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Holographic data

From a given holographic supergravity background, we typically want to extract:

1. The on-shell action $\leftrightarrow$ Free energy (and hence phase structure);
2. The expectation values of gauge invariant operators, characterizing the vacuum structure;
3. Higher correlation functions.
4. Action, spectrum and correlation functions of probe strings and branes in such backgrounds corresponding to Wilson loops, flavors in the quenched approximation $N_f \ll N_c$ etc etc

Example 1: Action and vevs for non-extremal branes

- Consider non-extremal D1-branes, for which the 10d metric is:

$$ds^2 = H^{-1/2}(-f dt^2 + dx^2) + H^{1/2}(\frac{dr^2}{f} + r^2 d\Omega_7^2);$$

with appropriate dilaton and three form, where

$$H = 1 + \frac{\mu^6 \sinh^2 \alpha}{r^6}; \qquad f = (1 - \frac{\mu^6}{r^6}); \qquad Q \equiv r_o^6 = \mu^6 \sinh \alpha \cosh \alpha.$$

The decoupled and reduced solution is asymptotically AdS_3 with linear dilaton:

$$ds^2 = \frac{d\rho^2}{4\rho^2 f} + \frac{1}{\rho}(-f dt^2 + dx^2); \qquad e^{-4\phi/3} = \frac{1}{\rho}.$$

Example 1: Action and vevs for non-extremal branes

- Using the holographic formulae, we can immediately extract the vevs:

$$\langle T_{tt} \rangle = 16L\frac{\mu^6}{r_o^9}; \qquad \langle T_{yy} \rangle = 8L\frac{\mu^6}{r_o^9}; \qquad \langle \mathcal{O} \rangle = -4L\frac{\mu^6}{r_o^9},$$

which satisfy the dilatation Ward identity: this is what lattice techniques should reproduce.

- The renormalized on-shell (Euclidean) action I_E is given by

$$I_E = -2\pi\beta_H R_x L\frac{8\mu^6}{r_o^9},$$

whilst the entropy is given by

$$S = \frac{2^4\pi^{5/2}}{3^3}\frac{N^2}{g_{eff}(T_H)}(V_1 T_H).$$

This is indeed of the form dictated from the generalized conformal structure.

Example 2: the Witten-(Sakai-Sugimoto) model

- The Witten model is based on D4-branes wrapping a circle τ with antiperiodic fermionic boundary conditions. At low energies (below the KK scale of circle) the resulting non-susy theory is effectively four-dimensional QCD.
- The corresponding holographic 4-brane background is:

$$
\begin{aligned}
ds^2_{st} &= \left(\frac{U}{R}\right)^{3/2}[\eta_{ab}dx^a dx^b + f(U)d\tau^2] + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)\alpha'^2, \\
e^{\phi} &= g_s\left(\frac{U}{R}\right)^{3/4}, \qquad F_4 = 3N\alpha'^{3/2}d\Omega_4, \\
R^3 &= \frac{g^2_{YM}N}{\alpha'^2} = \frac{g_s N}{\alpha'^{3/2}}, \qquad f(U) = 1 - \frac{U^3_{KK}}{U^3},
\end{aligned}
$$

which reduces to an asymptotically AdS_6 linear dilaton background.

- Sakai-Sugimoto added D8-branes wrapping $E^{3,1} \times S^4$ with the embedding parameterized by a curve $\tau(U)$ to model chiral flavors in the effective four-dimensional gauge theory.

Example 2: the Witten model

- The holographic vevs in this case are:

$$\langle T_{\alpha\beta} \rangle = \langle \mathcal{O} \rangle \eta_{\alpha\beta}; \qquad \langle T_{\tau\tau} \rangle = -5 \langle \mathcal{O} \rangle; \qquad \langle \mathcal{O} \rangle = \frac{2^5 \pi^2}{3^7} N \frac{\lambda_4^2}{L_\tau^4}.$$

  Here L_τ is the radius of the circle and λ_4 is the four-dimensional 't Hooft coupling.
- In comparing to lattice QCD, it is natural to match the condensate values, and thus fix L_τ.
- Note that the dilatation Ward identity implies a Ward identity in the effective four-dimensional theory, and identifies the beta function (which is constant cf QCD!)

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Example 3: correlation functions

- To compute correlation functions, one needs to find regular solutions of the (diagonalised) fluctuation equations.
- Substituting into the holographic one point functions, and then functionally differentiating gives the correlation functions.
- We diagonalised the fluctuation equations for general RG flows, and solved the equations analytically for the non-conformal brane backgrounds themselves, giving results consistent with generalized conformal invariance. For example,

$$\langle T_i^i(x) T_j^j(0)\rangle \sim N^2 \mathcal{R} \frac{(g_{eff}^2(x))^{(p-3)/(5-p)}}{x^{2d}}.$$

where $g_{eff}^2(x) = g_d^2 N |x|^{3-p}$.

- For finite temperature, solitonic,... backgrounds, one can now proceed to solve the diagonalised fluctuation equations (numerically) to extract two point functions → transport properties.

Concluding remarks

- Holography can increasingly be promoted to a very precise framework that can be used to compute field theory properties from geometry and vice versa.
- In particular, we have discussed how to decode the hologram for non-conformal brane backgrounds.
- These tools can be used to extract detailed information from phenomenological holographic models, quantified how good/bad they are, and moreover play a key role in understanding how holography works in general.
- The generalized conformal structure which underlies these theories should be fully exploited in developing these dualities further.