Title: Hybrid quantum information processing and communication

Date: Nov 26, 2008 04:00 PM

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Abstract: With the aim of proposing feasible, quantum optical realizations of quantum information protocols and minimizing the resource costs in such implementations, we will discuss various, so-called hybrid approaches. These include, for instance, schemes based upon both discrete and continuous quantum variables.

Hybrid Quantum Information Processing and Communication

Optical Quantum Information Theory
Emmy Noether Junior Research Group
Max Planck Research Group
Institute of Optics, Information and Photonics
University of Erlangen/Nürnberg, Germany

Peter van Loock

Hybrid
Quantum Information Processing and Communication

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January 2009:
Max Planck Institute for the Science of Light

“Hybrid”

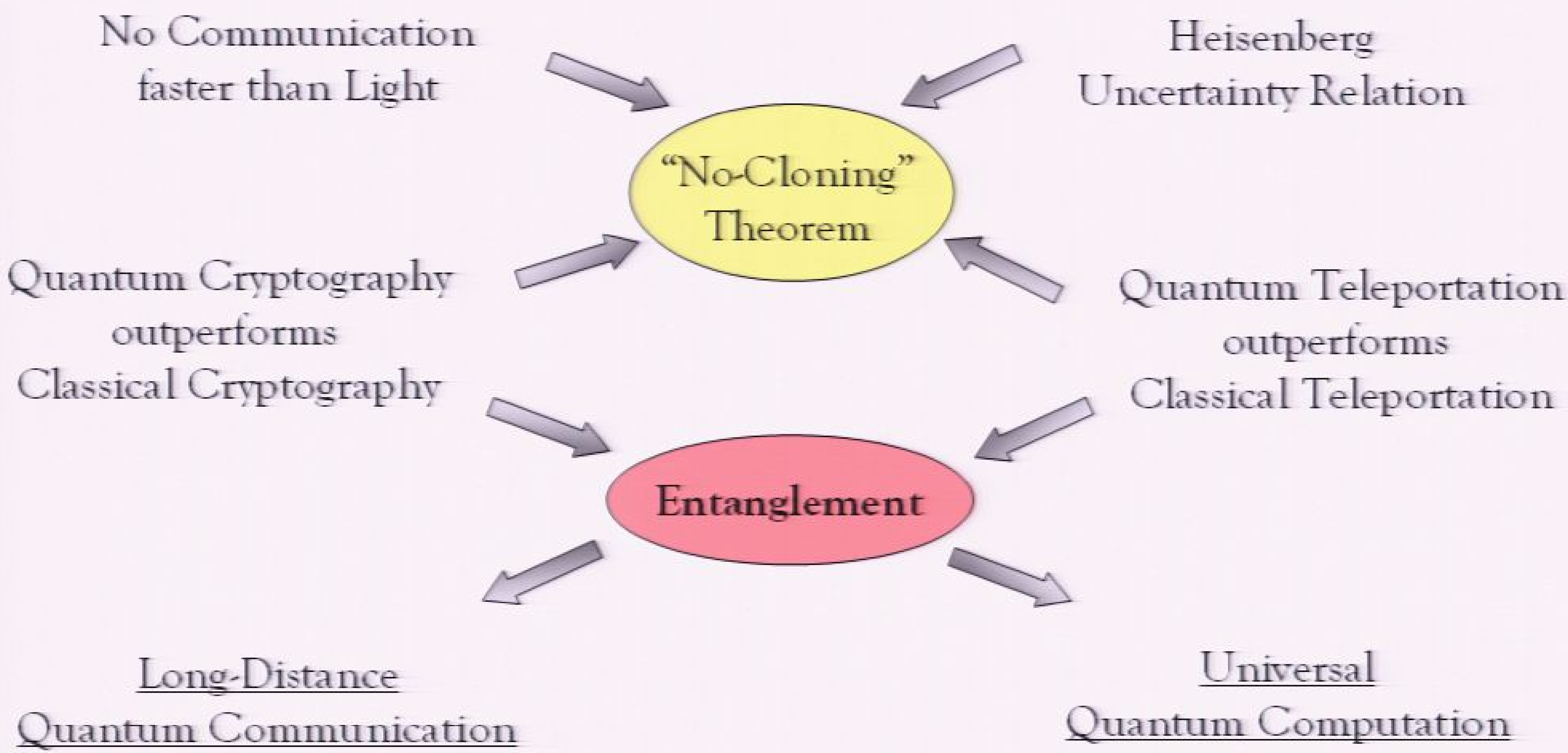
something heterogeneous in origin or composition
(in biology, crosses between different species);

something that has two different types of components
performing essentially the same function;

“Hybrid Computer”

a computer system consisting of a combination of analog and digital computer systems;

Quantum Information



Quantum Information

Entanglement

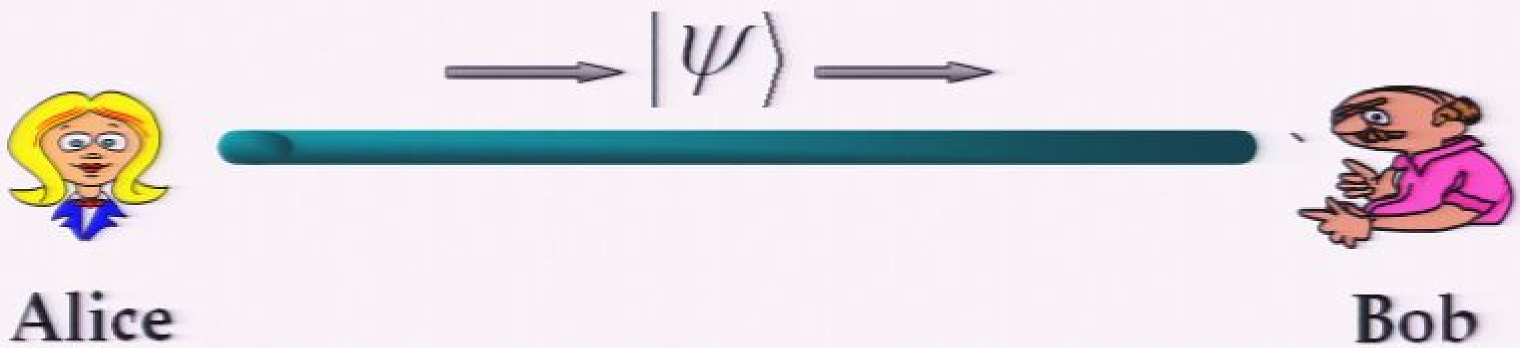
$$\int dx \left| x, x-c \right\rangle$$

$$\left| \Psi^+ \right\rangle = \left(\left|01\right\rangle + \left|10\right\rangle \right) / \sqrt{2}$$

Continuous Variables

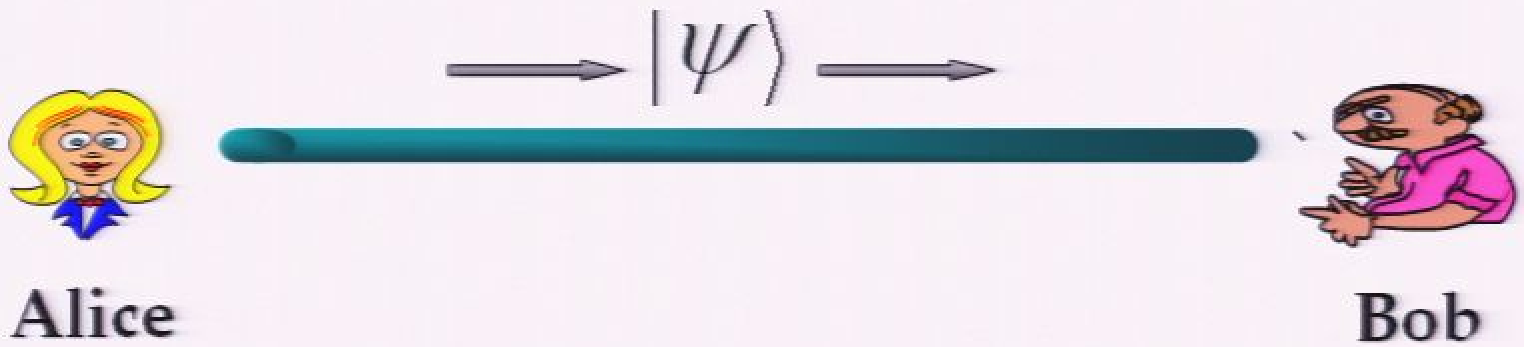
Discrete Variables

Quantum Communication



Quantum Communication

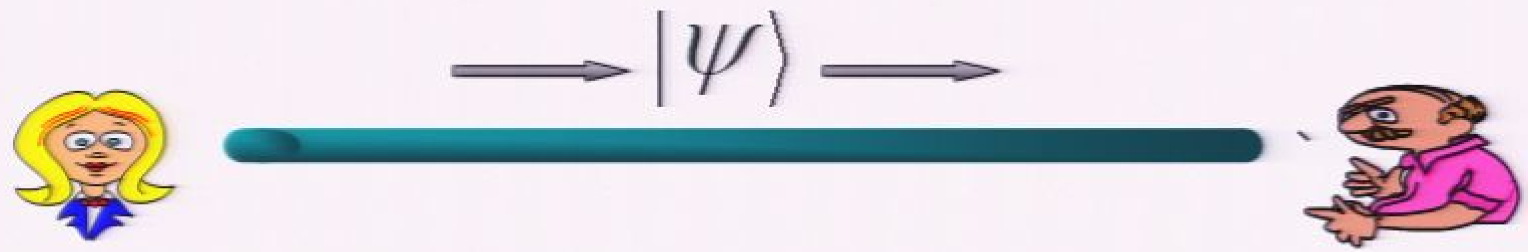
Quantum Communication



larger distances (1000 km) ?

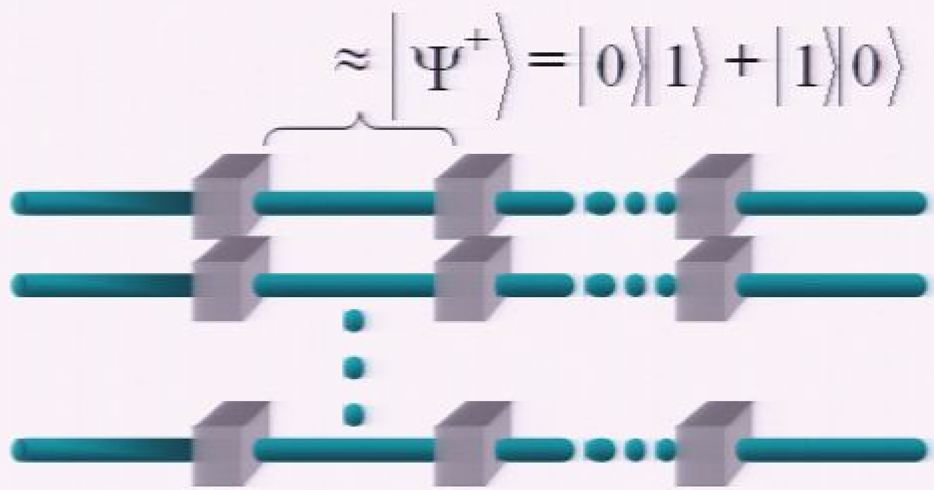
Entanglement: a universal resource

Quantum communication



larger distances (1000 km) ?

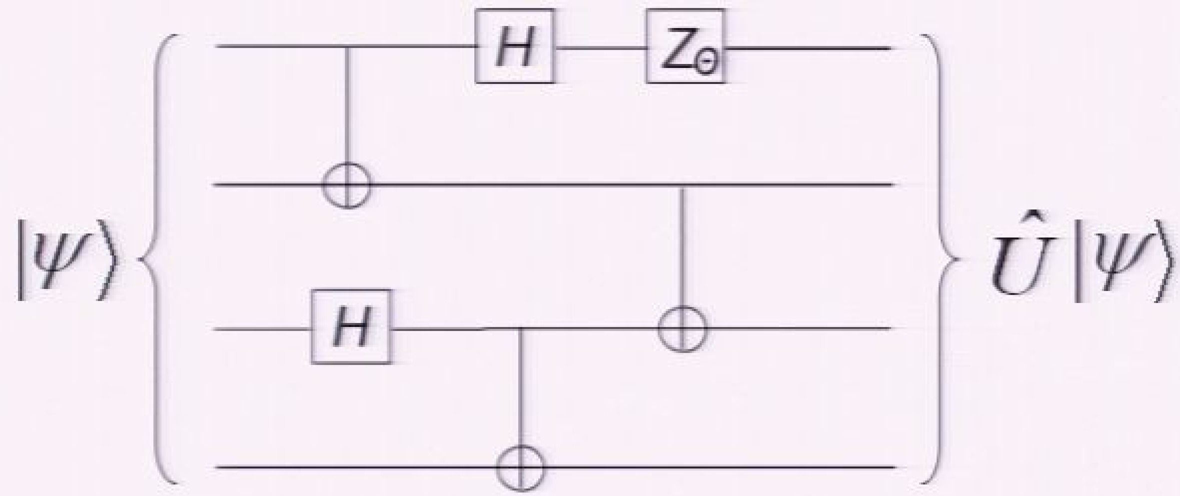
"Quantum Repeater"



H.-J. Briegel *et al.*, PRL **81**, 5932 (1998)

Quantum Computation

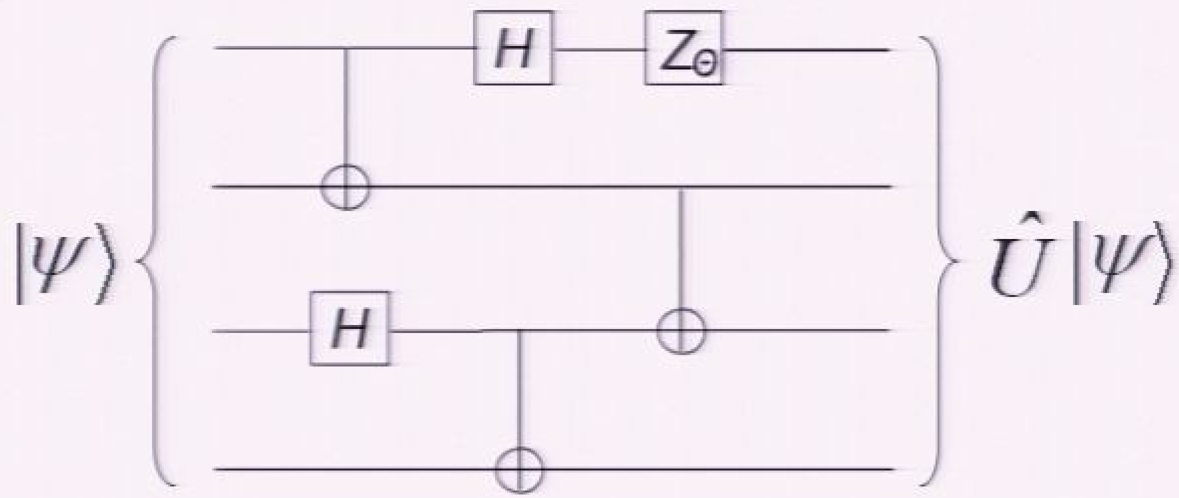
"Circuit Model"



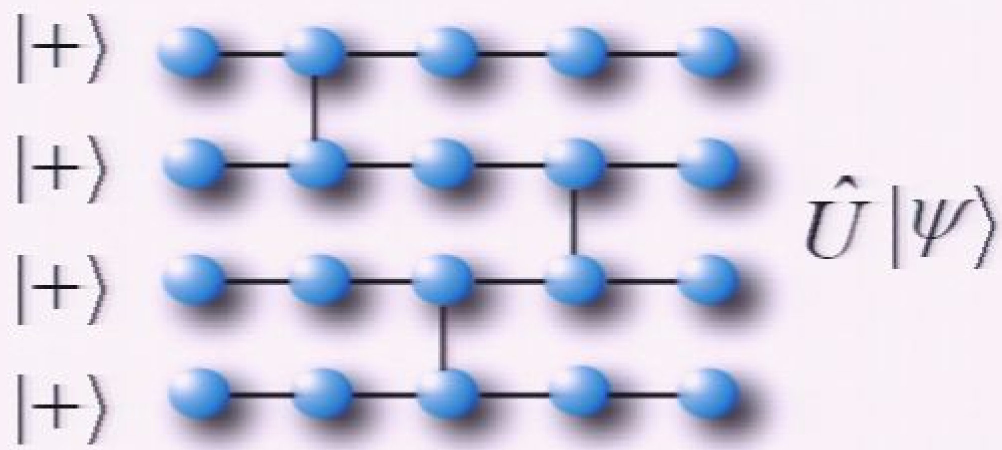
Entanglement: a universal resource

Universal Quantum Computation

"Circuit Model"



"One-Way Quantum Computation"



R. Raussendorf and H.-J. Briegel, PRL **86**, 5188 (2001)

Quantum Optics

Why Optics?

- ✓ light is a convenient medium for quantum communication
- ✓ light is useful for efficient, small-scale quantum logic / computation

Quantum Optics

... describe electromagnetic field by a discrete set of "mode variables" rather than the whole continuum of frequencies

$$\hat{\mathbf{E}}(\mathbf{r},t)=i\sum_k\left(\frac{\hbar\omega_k}{2\varepsilon_0}\right)^{1/2}\left(\hat{a}_k\,\mathbf{u}_k(\mathbf{r})\,e^{-i\omega_k t}-\hat{a}_k^{\dagger}\,\mathbf{u}_k^{*}(\mathbf{r})\,e^{+i\omega_k t}\right)$$

$$\hat{\mathbf{B}}(\mathbf{r},t)=i\sum_k\left(\frac{\hbar}{2\omega_k\varepsilon_0}\right)^{1/2}\left(\hat{a}_k\mathbf{k}\times\mathbf{u}_k(\mathbf{r})\,e^{-i\omega_k t}-\hat{a}_k^{\dagger}\mathbf{k}\times\mathbf{u}_k^{*}(\mathbf{r})\,e^{+i\omega_k t}\right)$$

$$[\hat{a}_k,\hat{a}_l^{\dagger}]=\delta_{kl}$$

Quantum Optics

$$\hat{H} = \frac{1}{2}\int d^3r \left(\varepsilon_0 \hat{E}^2 + \mu_0^{-1} \hat{B}^2 \right)$$

$$= \frac{1}{2}\sum_k \hbar\omega_k (\hat{a}_k^\dagger \hat{a}_k + \hat{a}_k \hat{a}_k^\dagger)$$

$$= \sum_k \hbar\omega_k (\underbrace{\hat{a}_k^\dagger \hat{a}_k} + 1/2)$$

photon number in mode k,
excitation number of
quantum harmonic oscillator

$$[\hat{a}_k , \hat{a}_l^\dagger] = \delta_{kl}$$

Quantum Optics

discrete number/Fock states:

$$\hat{a}_k | n_k \rangle = \sqrt{n_k} | n_k - 1 \rangle$$

$$\hat{n}_k | n_k \rangle = n_k | n_k \rangle$$

$$\hat{a}_k^\dagger | n_k \rangle = \sqrt{n_k + 1} | n_k + 1 \rangle$$

$$\hat{a}_k |0\rangle = 0$$ vacuum state

Fock basis:

$$\sum_{n_k=0}^{\infty} |n_k\rangle\langle n_k| = I_k \qquad \langle n_k | m_k \rangle = \delta_{n_k m_k}$$

convenient for few-photon states; hard to create or detect for large numbers

Quantum Optics

$$\hat{H}_k = \hbar\omega_k(\hat{a}_k^\dagger \hat{a}_k + 1/2)$$

$$= (\hat{p}_k^2 + \omega_k^2 \hat{x}_k^2)/2$$

unit mass

momentum and position of harmonic oscillator

$$\hat{x}_k = \sqrt{\hbar/2\omega_k}\,(\hat{a}_k + \hat{a}_k^\dagger), \quad \hat{p}_k = -i\sqrt{\hbar\omega_k/2}\,(\hat{a}_k - \hat{a}_k^\dagger)$$

$$[\hat{a}_k, \hat{a}_l^\dagger] = \delta_{kl} \qquad [\hat{x}_k, \hat{p}_l] = i\hbar\,\delta_{kl}$$

Quantum Optics

$$\hat{x}_k \rightarrow \sqrt{\omega_k/2\hbar}\ \hat{x}_k = \mathrm{Re}\,\hat{a}_k\,,\ \hat{p}_k \rightarrow \sqrt{1/2\hbar\omega_k}\ \hat{p}_k = \mathrm{Im}\,\hat{a}_k$$

continuous position/momentum states:

$$\hat{x}_k \left| x_k \right\rangle = x_k \left| x_k \right\rangle,\ \ \hat{p}_k \left| p_k \right\rangle = p_k \left| p_k \right\rangle$$

$$\left\langle 0 \left| \hat{x}_k^2 \right| 0 \right\rangle = 1/4\,,\ \ \left\langle 0 \left| \hat{p}_k^2 \right| 0 \right\rangle = 1/4$$ vacuum state

position/momentum basis:

$$\langle x | x' \rangle = \delta(x - x')\,,\ \langle p | p' \rangle = \delta(p - p')$$

$$\int dx\, |x\rangle\langle x| = I\,,\ \int dp\, |p\rangle\langle p| = I$$

convenient for Gaussian states; easy to create or detect even for large numbers

Quantum Optics

"continuum" of quadrature amplitudes:

$$\begin{pmatrix} \hat{x}_k^{(\theta)} \\ \hat{p}_k^{(\theta)} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{x}_k \\ \hat{p}_k \end{pmatrix}$$

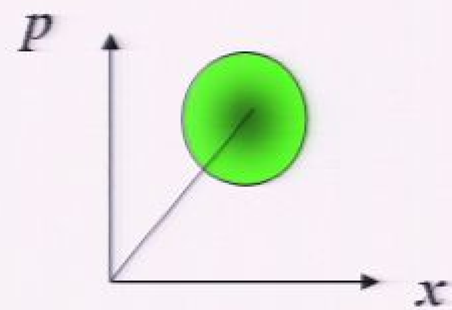
$$\hat{x}_k^{(\theta)} = (\hat{a}_k e^{-i\theta} + \hat{a}_k^{\dagger} e^{i\theta})/2$$
$$\hat{p}_k^{(\theta)} = (\hat{a}_k e^{-i\theta} - \hat{a}_k^{\dagger} e^{i\theta})/2i = \hat{x}_k^{(\theta+\pi/2)}$$

$$[\hat{x}_k^{(\theta)}, \hat{p}_l^{(\theta)}] = (i/2)\,\delta_{kl} \qquad \hat{x}_k = \hat{x}_k^{(\theta=0)},\ \hat{p}_k = \hat{x}_k^{(\theta=\pi/2)}$$

Quantum Optics

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

$$|\alpha\rangle = \hat{D}(\alpha)\,|0\rangle$$

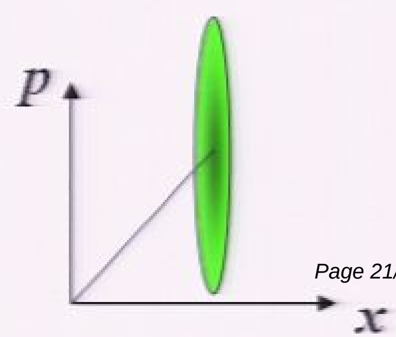


with $$\hat{D}(\alpha) = \exp(\alpha\,\hat{a}^{\dagger} - \alpha^{*}\,\hat{a})$$

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Squeezed States:

$$\hat{x} = e^{-r}\,\hat{x}^{(0)}, \quad \hat{p} = e^{+r}\,\hat{p}^{(0)}$$



Quantum Optics

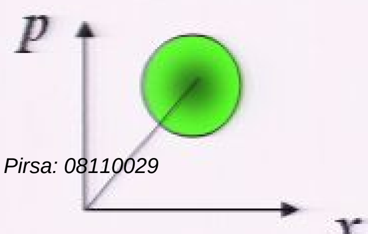
Wigner function:

$$W(x,p) = \frac{2}{\pi}\int dy\, e^{4iyp}\langle x-y|\hat{\rho}|x+y\rangle$$

Gaussian States:

$$W(\xi) = \frac{1}{(2\pi)\sqrt{\det V}}\exp(-\xi V^{-1}\xi^{T}/2)$$

2nd moment correlation matrix



$$W(x,p) = \frac{2}{\pi}\exp[-2(x-x_0)^2 - 2(p-p_0)^2]$$

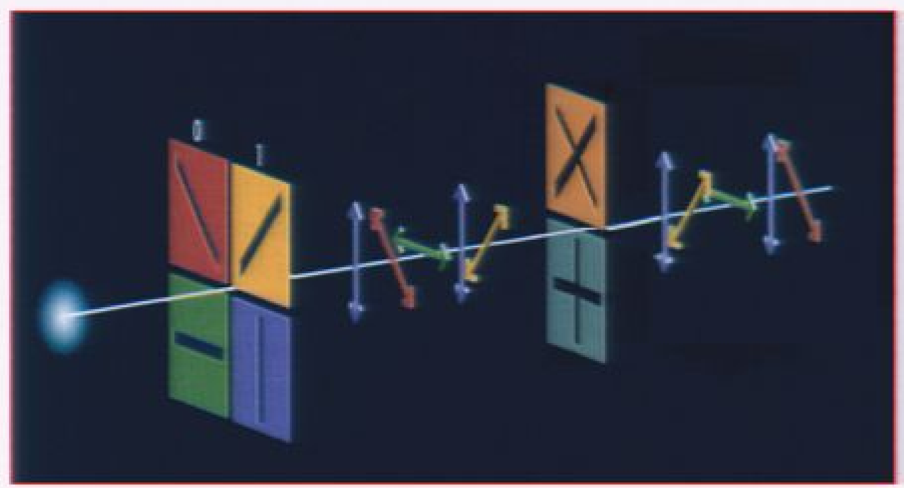
Discrete Variables (DV)

occupation number qubit … $\alpha\ |0\rangle + \beta\ |1\rangle$ (**one** mode)

dual-rail qubit … $\alpha\ |10\rangle + \beta\ |01\rangle$ (**two** modes)

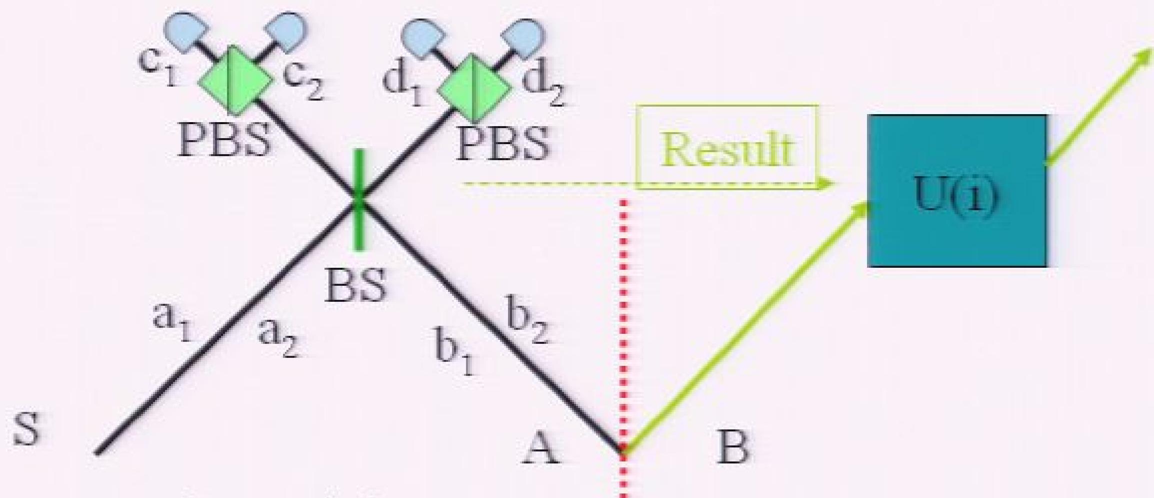
e.g., polarization of a single photon …

$$\alpha\ |\leftrightarrow\rangle + \beta\ |\updownarrow\rangle$$



Quantum Teleportation (DV)

Innsbruck Experiment, 1997 (Zeilinger)



$$\begin{pmatrix} c_1 \\ d_1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} c_2 \\ d_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

$\cos\alpha|\leftrightarrow\rangle_S + e^{i\varphi}\sin\alpha|\updownarrow\rangle_S$

$\frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_A|\updownarrow\rangle_B + |\leftrightarrow\rangle_A|\leftrightarrow\rangle_B\right)$

Analysis:

$$\frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_S|\leftrightarrow\rangle_A + |\leftrightarrow\rangle_S|\updownarrow\rangle_A\right) = \frac{1}{\sqrt{2}}\left(a_1^+ b_2^+ + a_2^+ b_1^+\right)|0\rangle \to \frac{1}{\sqrt{2}}\left(c_1^+ c_2^+ - d_1^+ d_2^+\right)|0\rangle$$

$$\frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_S|\leftrightarrow\rangle_A - |\leftrightarrow\rangle_S|\updownarrow\rangle_A\right) = \frac{1}{\sqrt{2}}\left(a_1^+ b_2^+ - a_2^+ b_1^+\right)|0\rangle \to \frac{1}{\sqrt{2}}\left(d_1^+ c_2^+ - c_1^+ d_2^+\right)|0\rangle$$

distinct click pattern (two separate clicks)

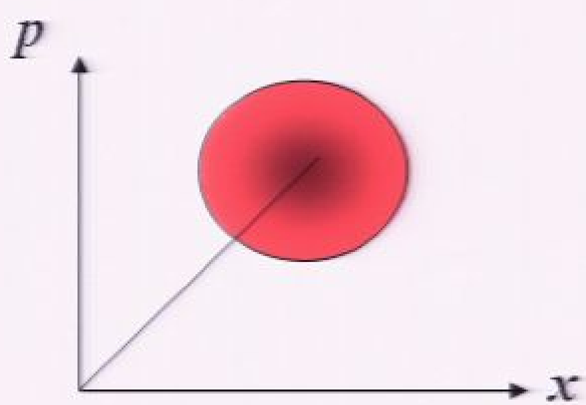
$$\frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_S|\updownarrow\rangle_A \pm |\leftrightarrow\rangle_S|\leftrightarrow\rangle_A\right) = \frac{1}{\sqrt{2}}\left(a_1^+ b_1^+ \pm a_2^+ b_2^+\right)|0\rangle \to \frac{1}{2\sqrt{2}}\left\{(c_1^+)^2 - (d_1^+)^2 \pm \left[(c_2^+)^2 - (d_2^+)^2\right]\right\}|0\rangle$$

undistinguishable click pattern ➔ 50% success rate

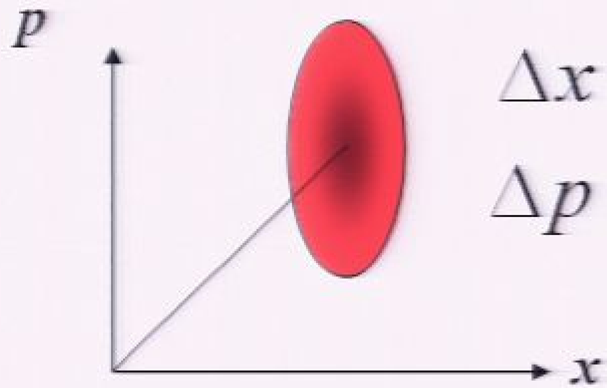
Continuous Variables (CV)

$$\hat{a} = \hat{x} + i\hat{p}, \quad [\hat{x}, \hat{p}] = i/2, \quad \Delta x \Delta p \geq 1/16$$

coherent state



squeezed state

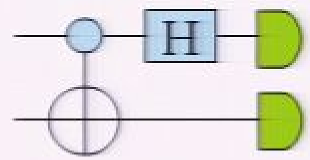


$$\Delta x = \lambda \, \Delta x_{vacuum}$$

$$\Delta p = (1/\lambda) \, \Delta p_{vacuum}$$

Quantum Teleportation (CV)

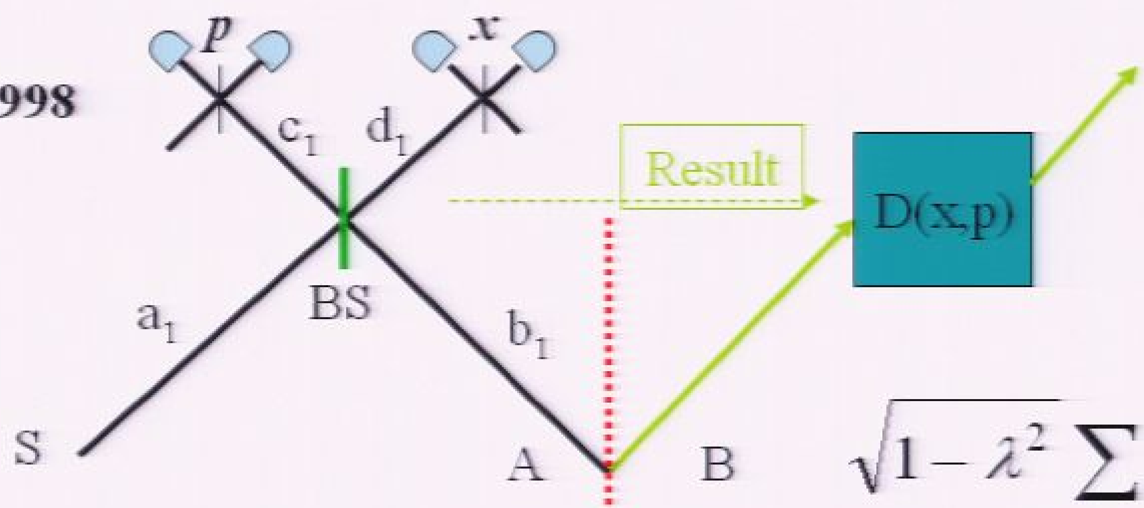
Hadamard ➔ Fourier transform:



$$F\,|p\rangle_{\text{momentum}} = \frac{1}{\sqrt{\pi}}\int dy\; e^{2ipy}\,|y\rangle_{\text{momentum}} = |x = p\rangle_{\text{position}}$$

CNOT ➔ beam splitter: $|x\rangle|y\rangle \rightarrow \left|\frac{1}{\sqrt{2}}(x+y)\right\rangle\left|\frac{1}{\sqrt{2}}(x-y)\right\rangle$

Caltech Experiment, 1998 (Kimble)

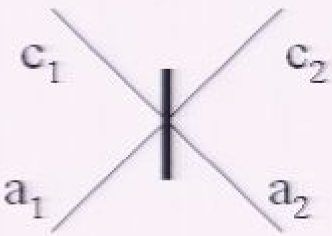


e.g. $|\alpha\rangle_S$

$$\sqrt{1-\lambda^2}\sum_n \lambda^n\,|n,n\rangle_{AB}$$

Linear Optics

beam splitter



$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} r & t \\ t & -r \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$|r|^2 + |t|^2 = 1$$

$$r t^* - r^* t = 0$$

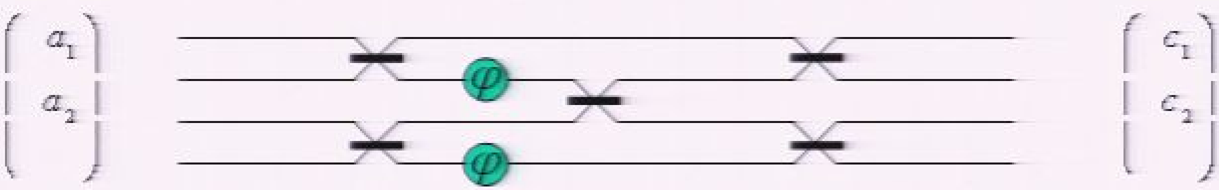
phase shifter

$$c = e^{i\varphi} a$$

$$\vec{c} = U \vec{a}$$

U unitary

linear network

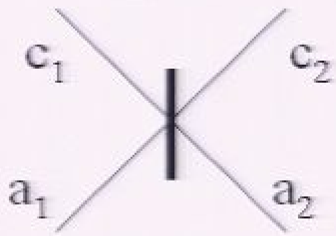


Any U can be realized ... M. Reck *et al.*, PRL **73**, 58 (1994)

Plus: photon counting or homodyne detection ...

Linear Optics

beam splitter



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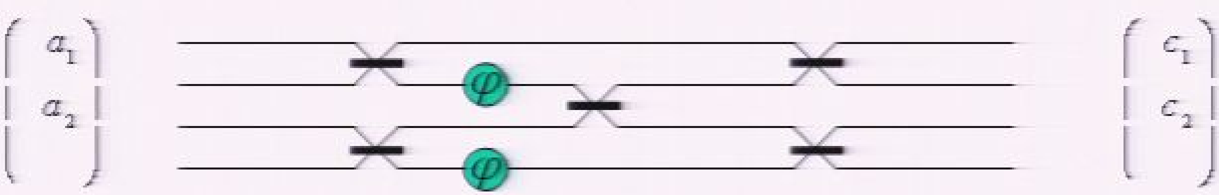
phase shifter

$$c = e^{i\varphi} a$$

any quadratic interaction including squeezing:

$$\vec{c} = A\vec{a} + B\vec{a}^\dagger + \vec{\gamma}$$

linear network



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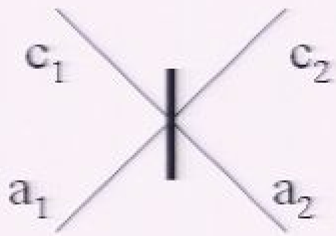

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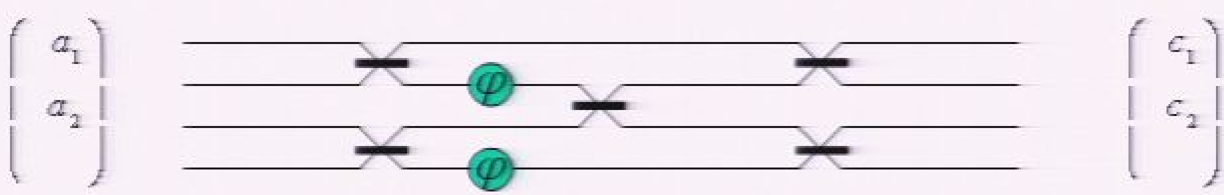
"LUBO", Gaussian transformations

linear network



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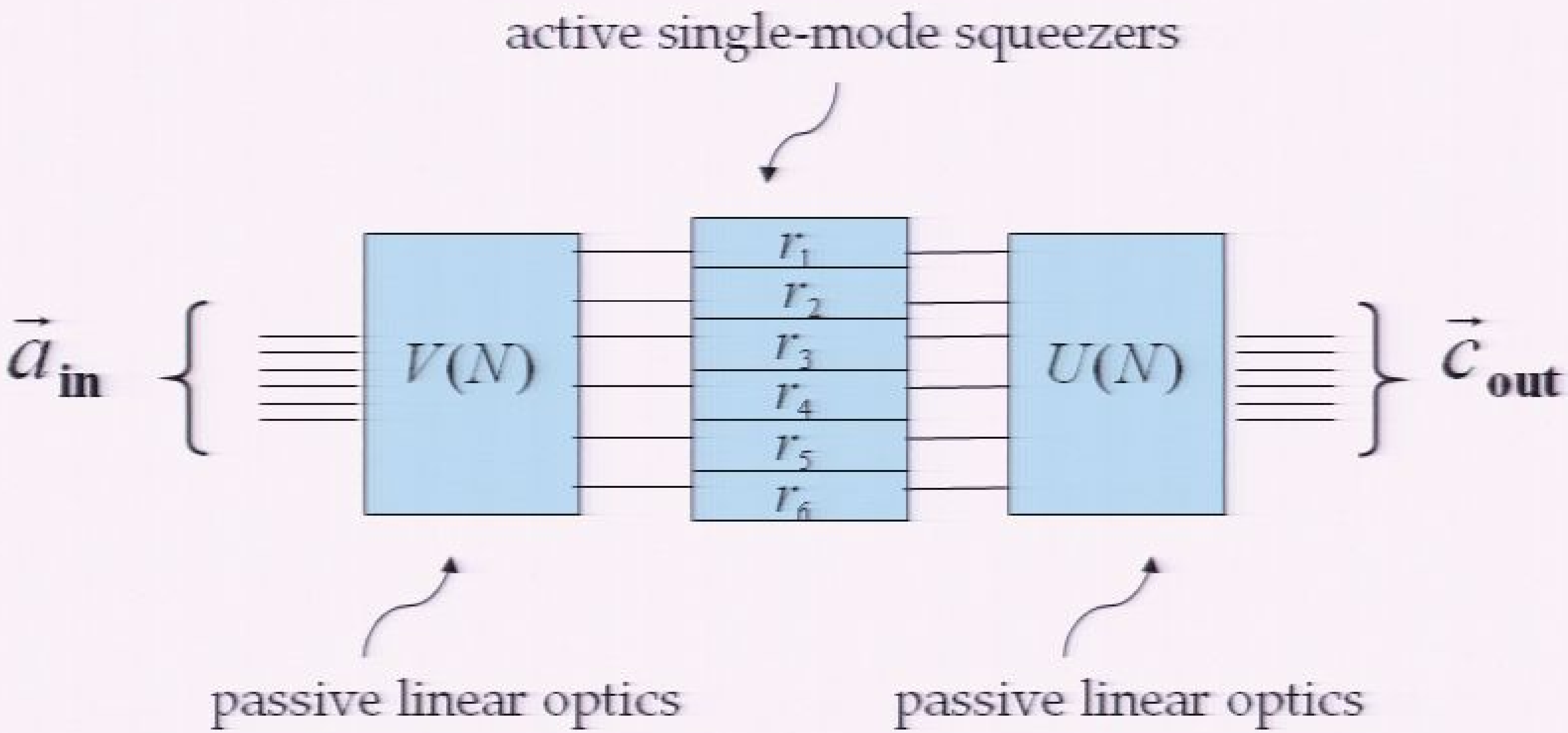
Plus: photon counting or homodyne detection ...

Linear Optics

any multi-mode Gaussian ("LUBO") transformation can be decomposed via SVD into



S. L. Braunstein, PRA **71**, 055801 (2005)

Nonlinear Optics (DV)

Cross Kerr ...

$$\hat{U} = e^{i\pi\, \hat{a}^\dagger \hat{a}\, \hat{b}^\dagger \hat{b}}$$

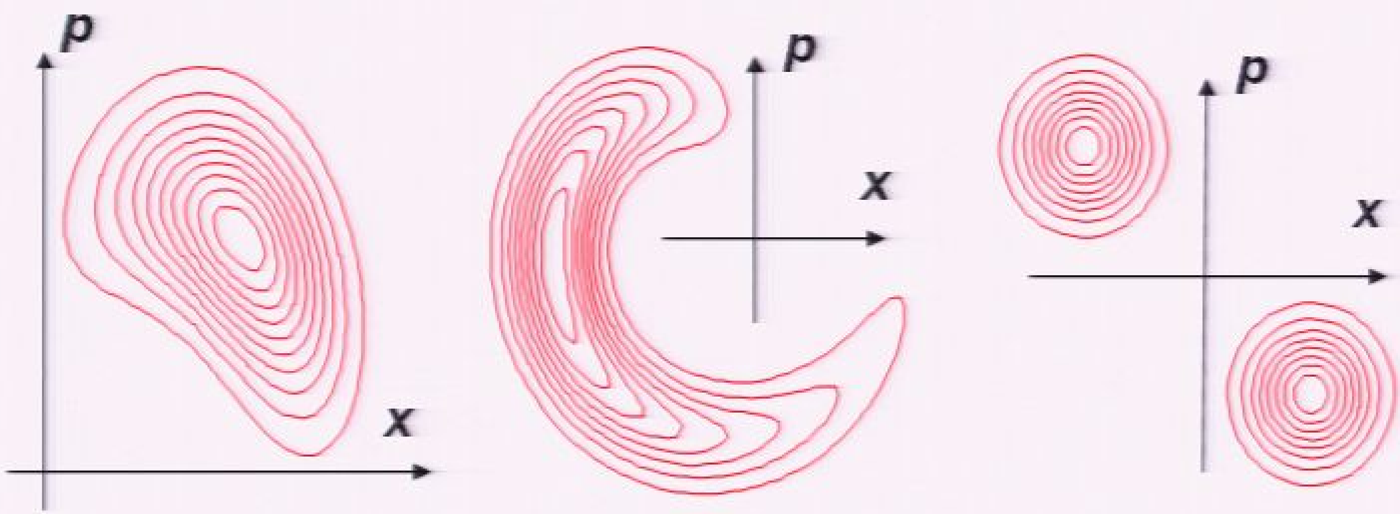
$$|00\rangle \xrightarrow{\hat{U}} |00\rangle,\quad |01\rangle \xrightarrow{\hat{U}} |01\rangle,\quad |10\rangle \xrightarrow{\hat{U}} |10\rangle,\quad |11\rangle \xrightarrow{\hat{U}} -|11\rangle$$

controlled sign gate

...hard to obtain for single-photon states

Nonlinear Optics (CV)

non-Gaussian states ...



„Cat States“

...hard to obtain for multi-photon states

Optical Quantum Information Processing

	unitary transformation	interaction (Hamiltonian)	input-output relation of mode operators
Gaussian	displacement in phase space	linear	"scalar"
Gaussian	beam splitter	quadratic	linear
Gaussian	squeezing		
	non-Gaussian (cubic phase gate, Kerr effect)	cubic or higher	nonlinear

linear, nonlinear optical interactions

Why Hybrid?

Discrete Variables

linear optics / linear transformations are not sufficient;

NOGO for universal quantum computing, NOGO for measurements, ...

Continuous Variables

Gaussian quantum computer can always be efficiently simulated
by a classical computer;

NOGO for universal quantum computing, NOGO for entanglement distillation and for quantum error correction, ...

Why Hybrid?

Discrete Variables $\alpha\,|\leftrightarrow\rangle + \beta\,|\updownarrow\rangle$

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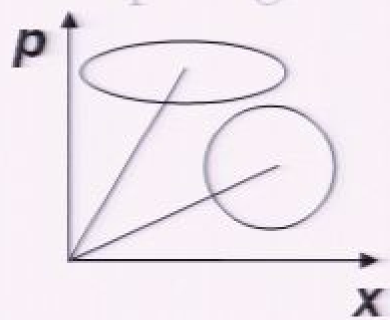

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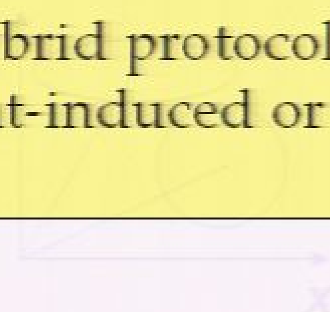
NOG



possible solution: hybrid protocols,
measurement-induced or weak nonlinearities

Continuous Variables

Gaussian quantum computer can always be efficiently simulated by a classical computer;

NOGO for universal quantum computing, NOGO for entanglement distillation and for quantum error correction, ...

Hybrid Approaches

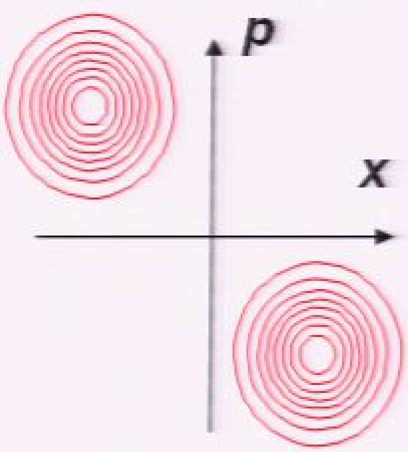
Hybrid...

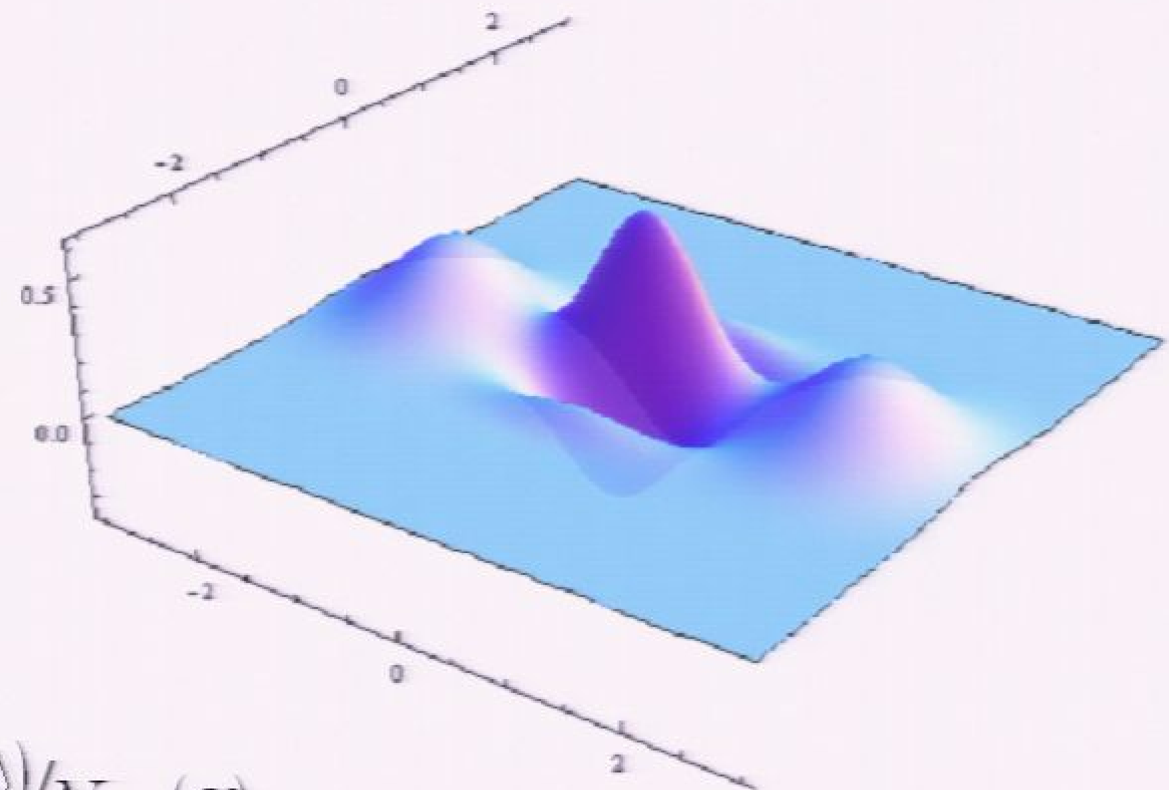
...combine different encodings, operations, and protocols

...minimize resource costs for implementations

Hybrid Approaches

- ✓ coherent-state-based qubits ("cat logic")





$$\left(\mu|\alpha\rangle + \nu\,|-\alpha\rangle\right)/N_{\mu\nu}(\alpha)$$

A.P. Lund *et al.*, PRL **100**, 030503 (2008) and refs. therein

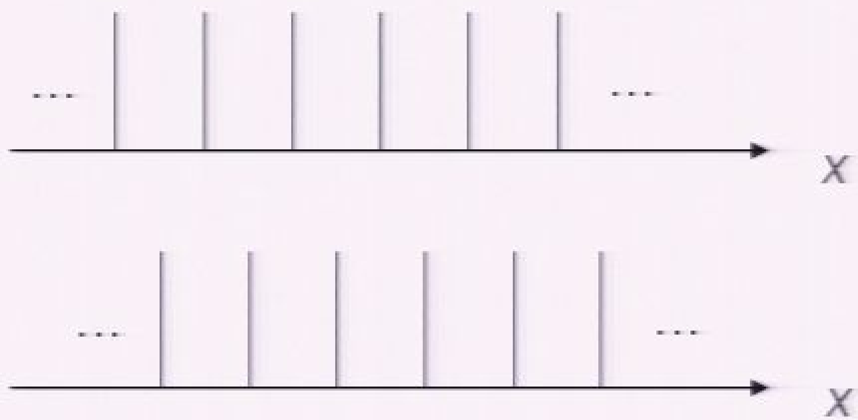
Hybrid Approaches

✓ coherent-state-based qubits ("cat logic")

✓ hybrid quantum error correction… encode discrete qubit into oscillator

$$|\bar{0}\rangle \qquad \sum_{s=-\infty}^{+\infty} |x = \Delta s\rangle$$

$$|\bar{1}\rangle \qquad \sum_{s=-\infty}^{+\infty} |x = \Delta s + \Delta/2\rangle$$



… useful for small, diffusive shifts
rather than stochastic, large errors

D. Gottesman *et al.*, PRA **64**, 012310 (2001)

Hybrid Approaches

- ✓ coherent-state-based qubits ("cat logic")
- ✓ hybrid quantum error correction... protect cv state against discrete errors

$$W_{out}(x,p) = (1-p)\,W_{in}(x,p) + p\,W_{error}(x,p)$$

for example, „erasure channel"

PvL, qph-archive/0811.3616 (2008)

Gaussian QECC not useful to protect Gaussian states against Gaussian errors

Niset *et al.*, qph-archive/0811.3128 (2008)

Shor QECC

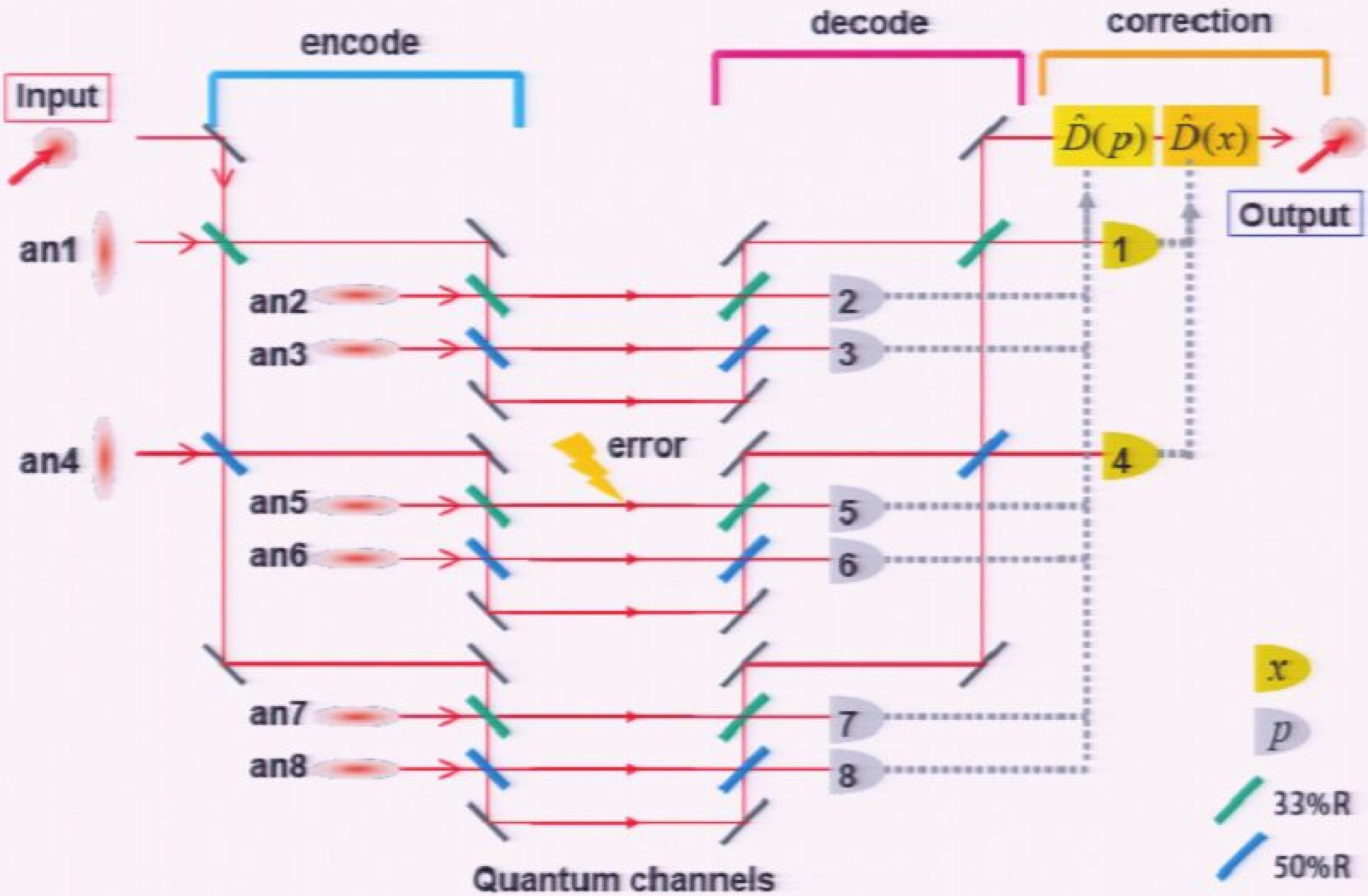
Shor code for qubits: $$\alpha\left|+++\right\rangle+\beta\left|---\right\rangle$$

with $$\left|\pm\right\rangle=\left(\left|000\right\rangle\pm\left|111\right\rangle\right)/\sqrt{2}$$

Shor-type code for CV: $$\int dp\,\psi(p)\left|p,p,p\right\rangle$$

with $$\left|p\right\rangle\propto\int dx e^{2ixp}\left|x,x,x\right\rangle$$

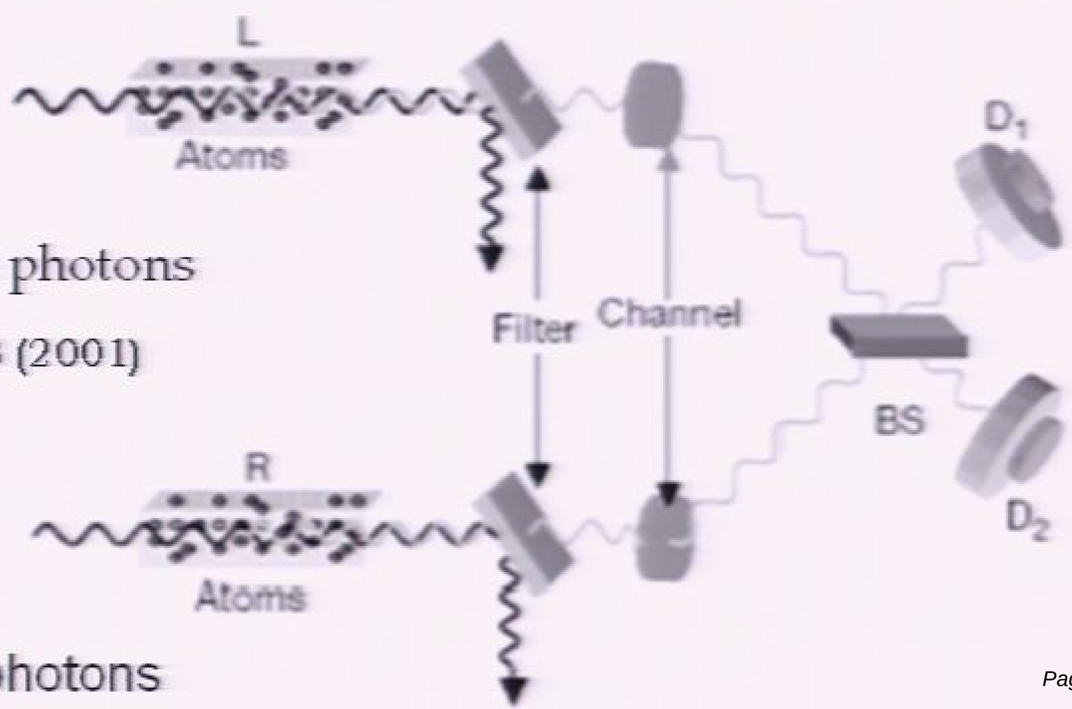
CV Quantum Error Correction



Aoki *et al.*, qph-archive/0811.3734 (2008)

Hybrid Approaches

- ✓ coherent-state-based qubits ("cat logic")
- ✓ hybrid quantum error correction... protect cv state against discrete errors
- ✓ light-matter interfaces for quantum communication



fully DV... many atoms/single photons

L. M. Duan *et al.*, Nature **414**, 413 (2001)

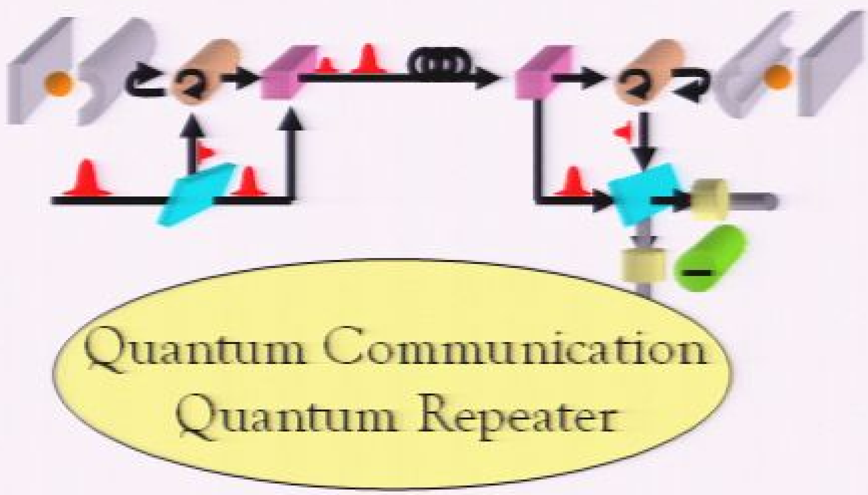
fully CV... many atoms/many photons

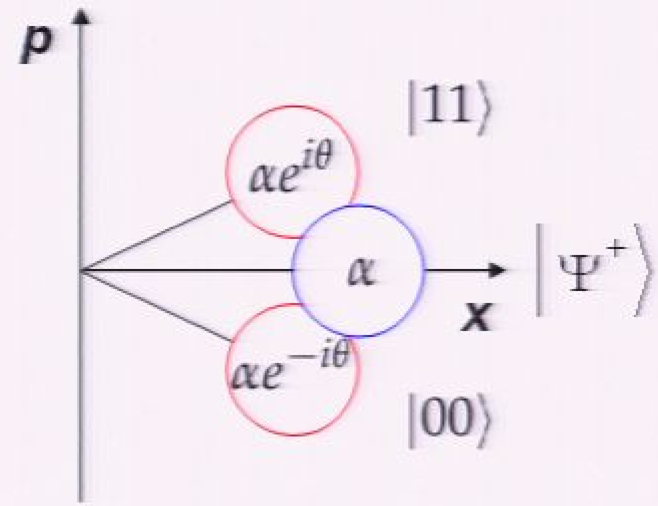
J. F. Sherson *et al.*, Nature **443**, 557 (2006)

Hybrid Approaches

- ✓ coherent-state-based qubits ("cat logic")
- ✓ hybrid quantum error correction... protect cv state against discrete errors
- ✓ light-matter interfaces for quantum communication
- ✓ hybrid quantum communication... continuous qubus for discrete qubits
- ✓ hybrid quantum computation... continuous qubus for discrete qubits
- ✓ hybrid one-way computer... combine discrete/continuous measurements
- ✓ hybrid quantum computation... combine circuit/cluster approaches
- ✓ hybrid quantum communication... combine teleportation/direct transfer
- ✓ ...

Hybrid Quantum Communication



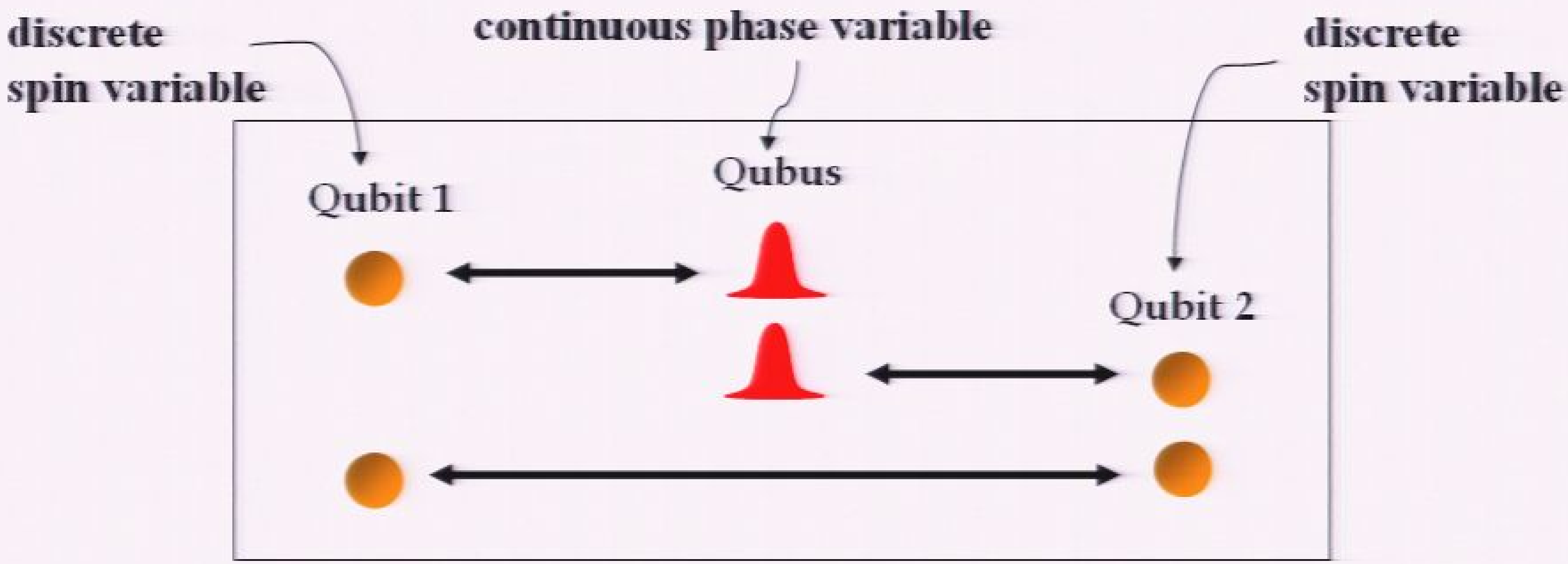


PvL *et al.*, Phys. Rev. Lett. **96**, 240501 (2006)

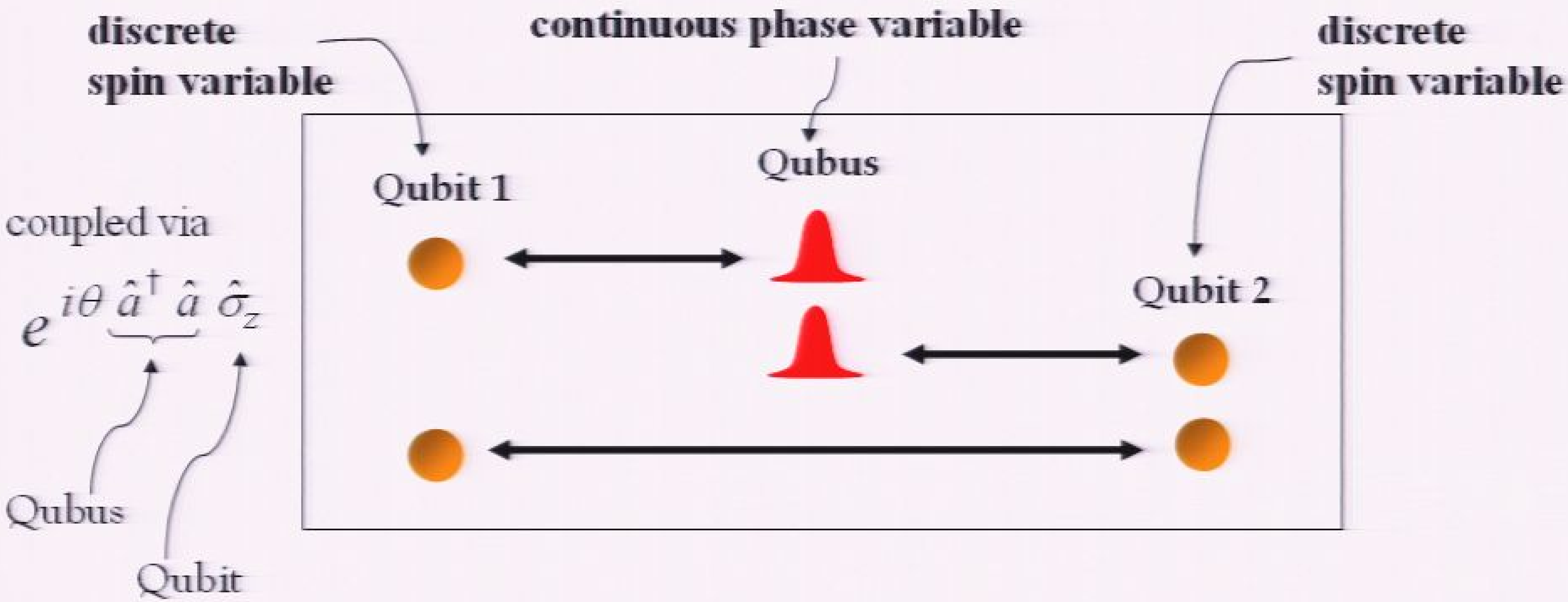
T. D. Ladd, PvL *et al.*, New J. Phys. **8**, 164 (2006)

…in collaboration with T.D. Ladd, W.J. Munro, K. Nemoto, Y. Yamamoto…

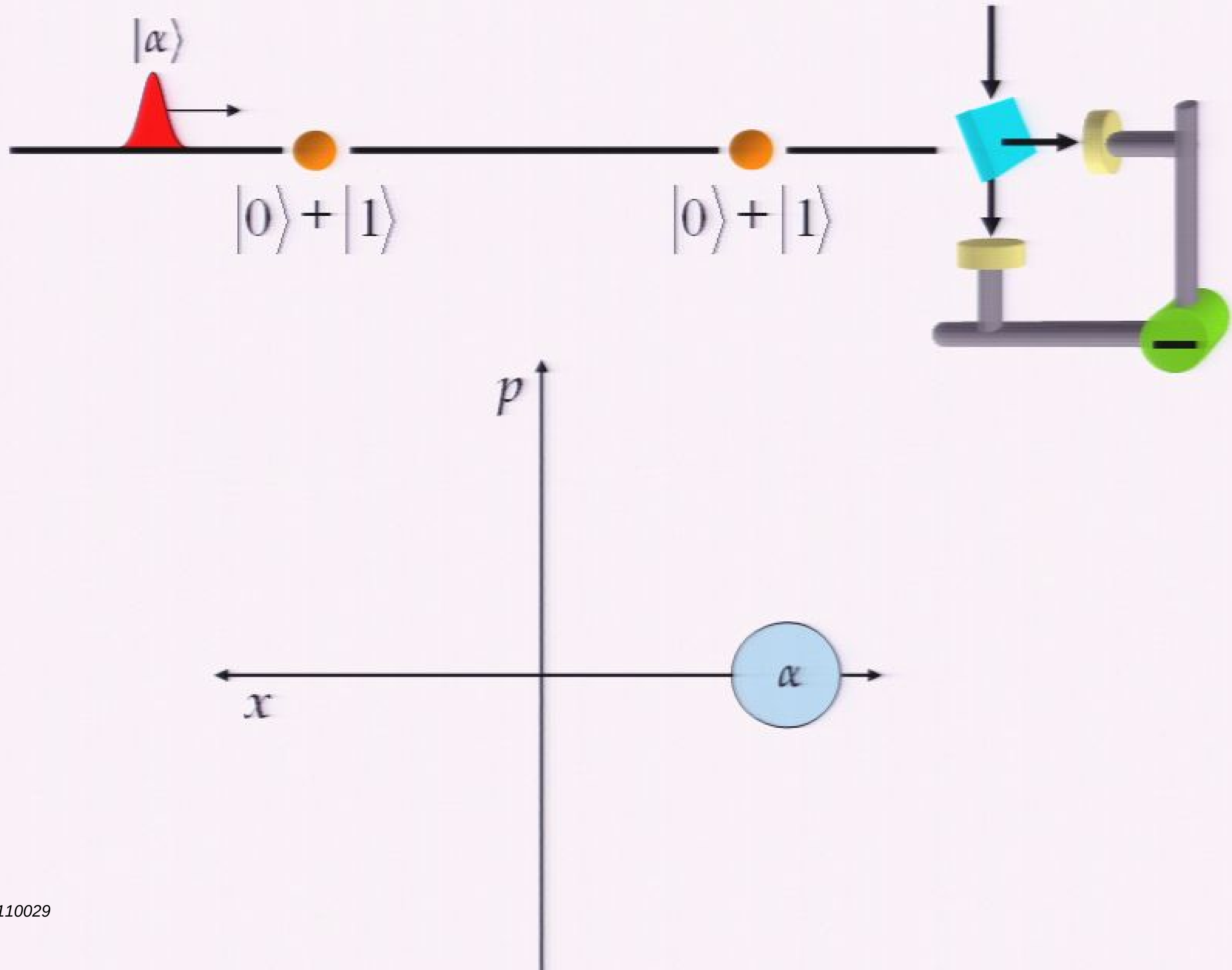
Hybrid Quantum Repeater



Hybrid Quantum Repeater



Entanglement Distribution



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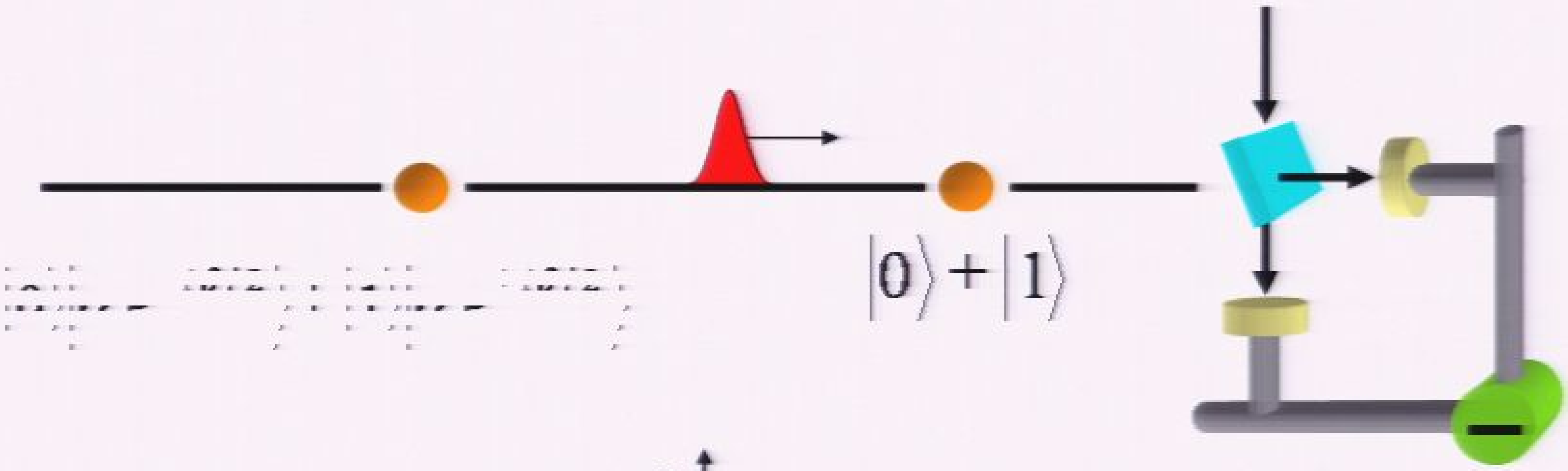

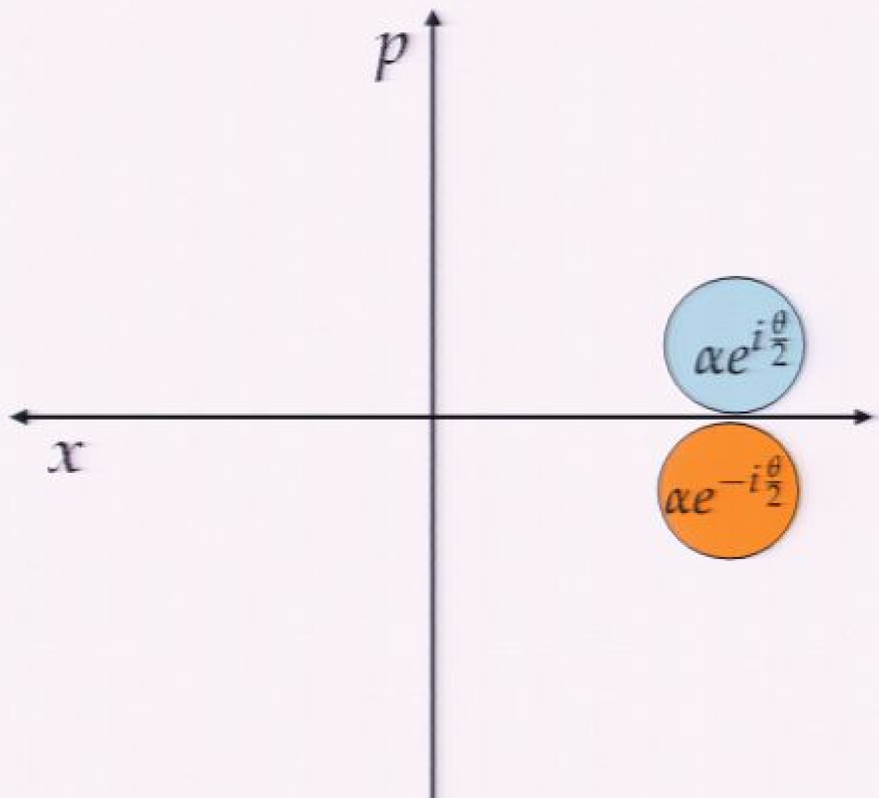

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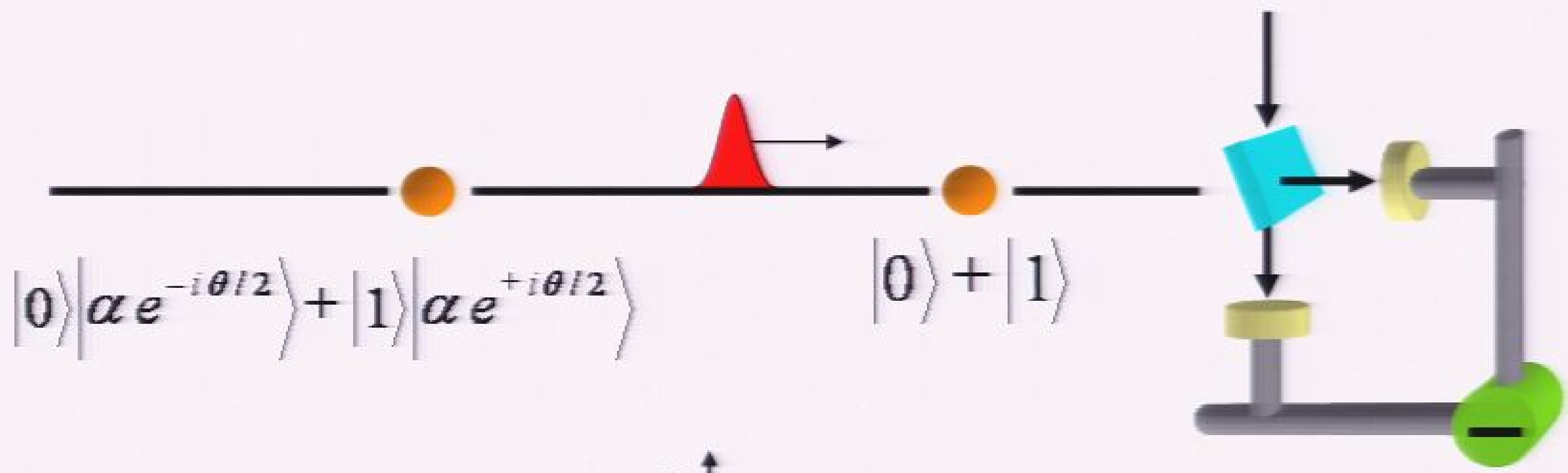

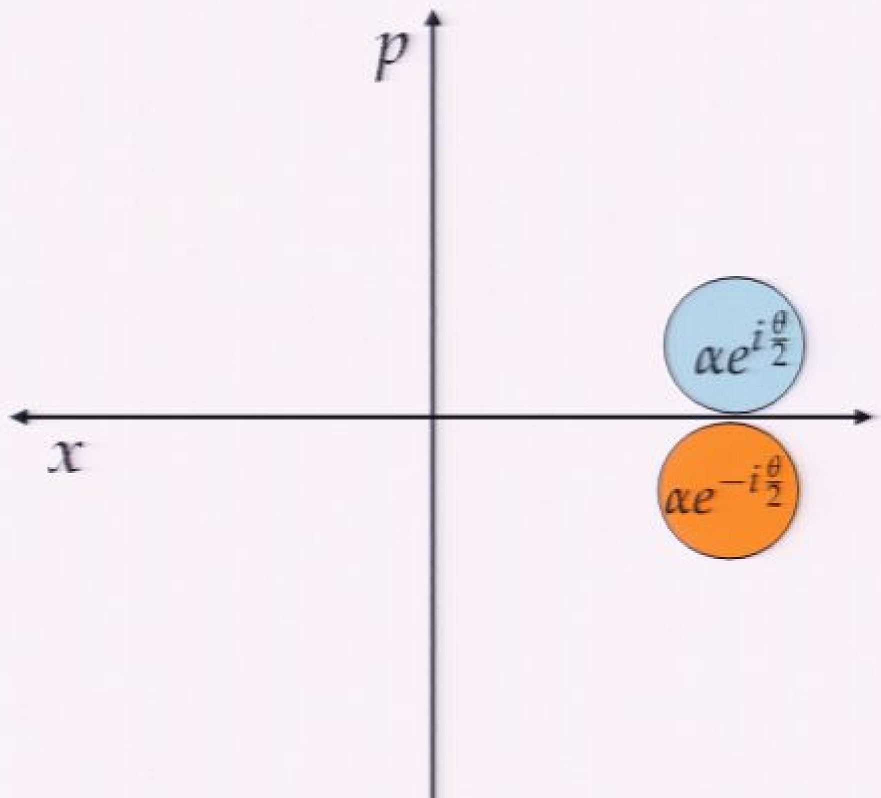

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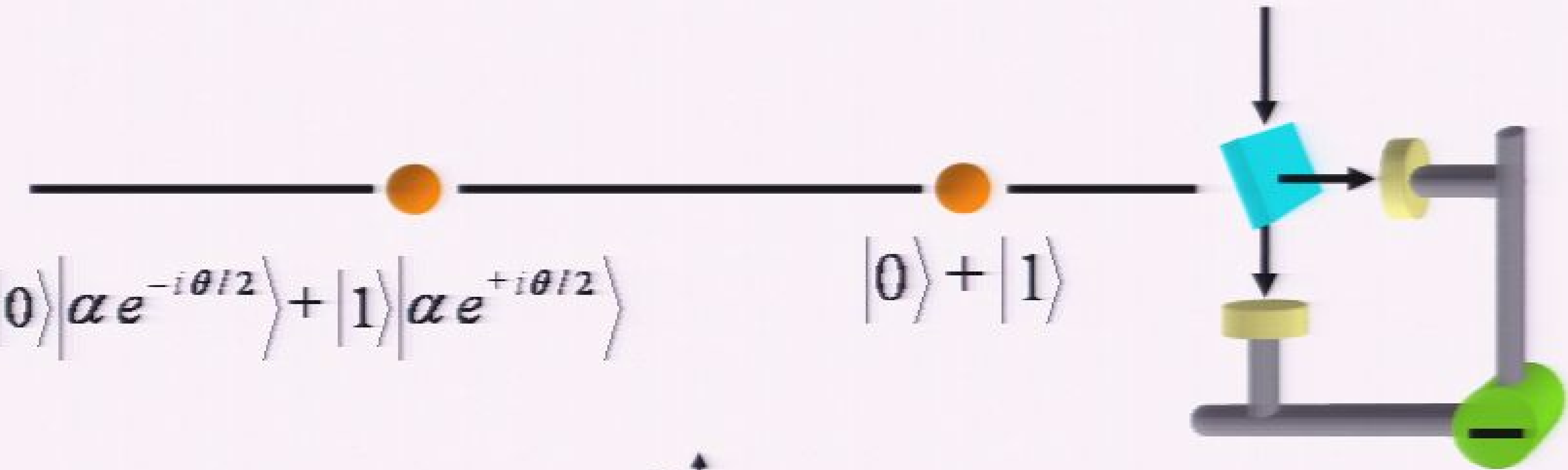
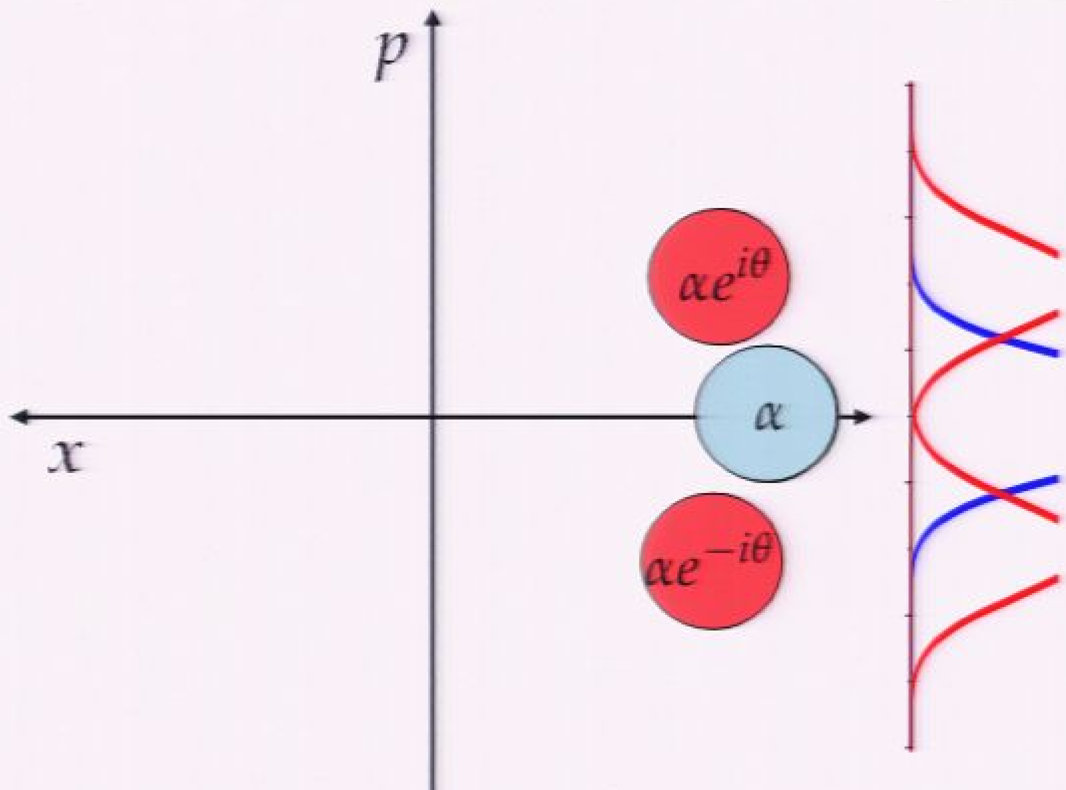


$$|0\rangle|\alpha e^{-i\theta/2}\rangle + |1\rangle|\alpha e^{+i\theta/2}\rangle \qquad |0\rangle + |1\rangle$$



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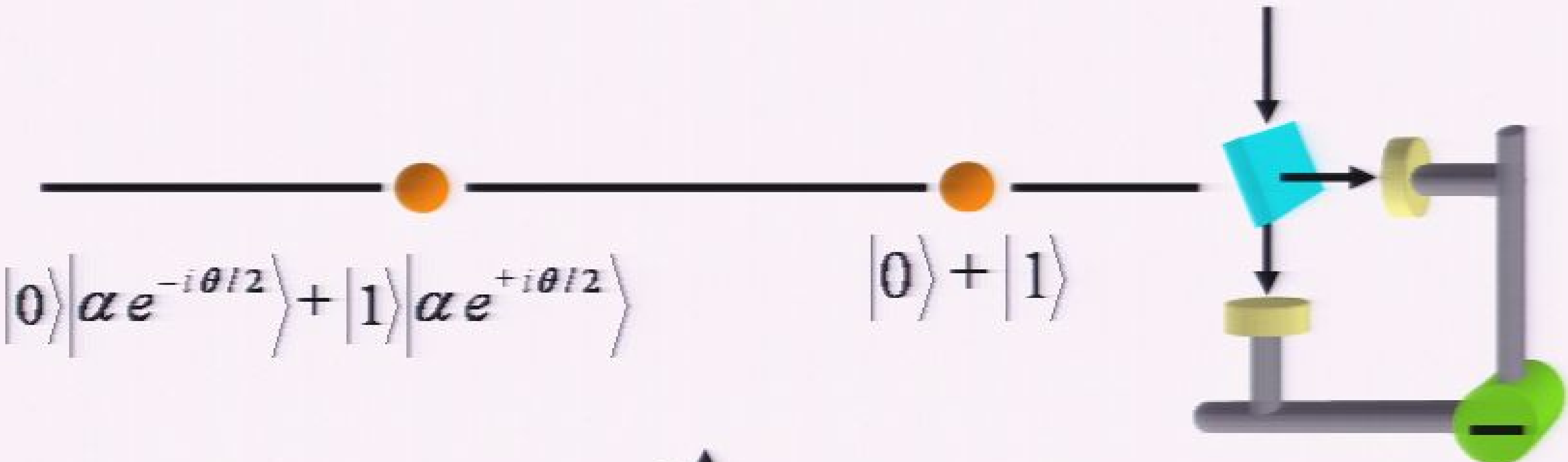

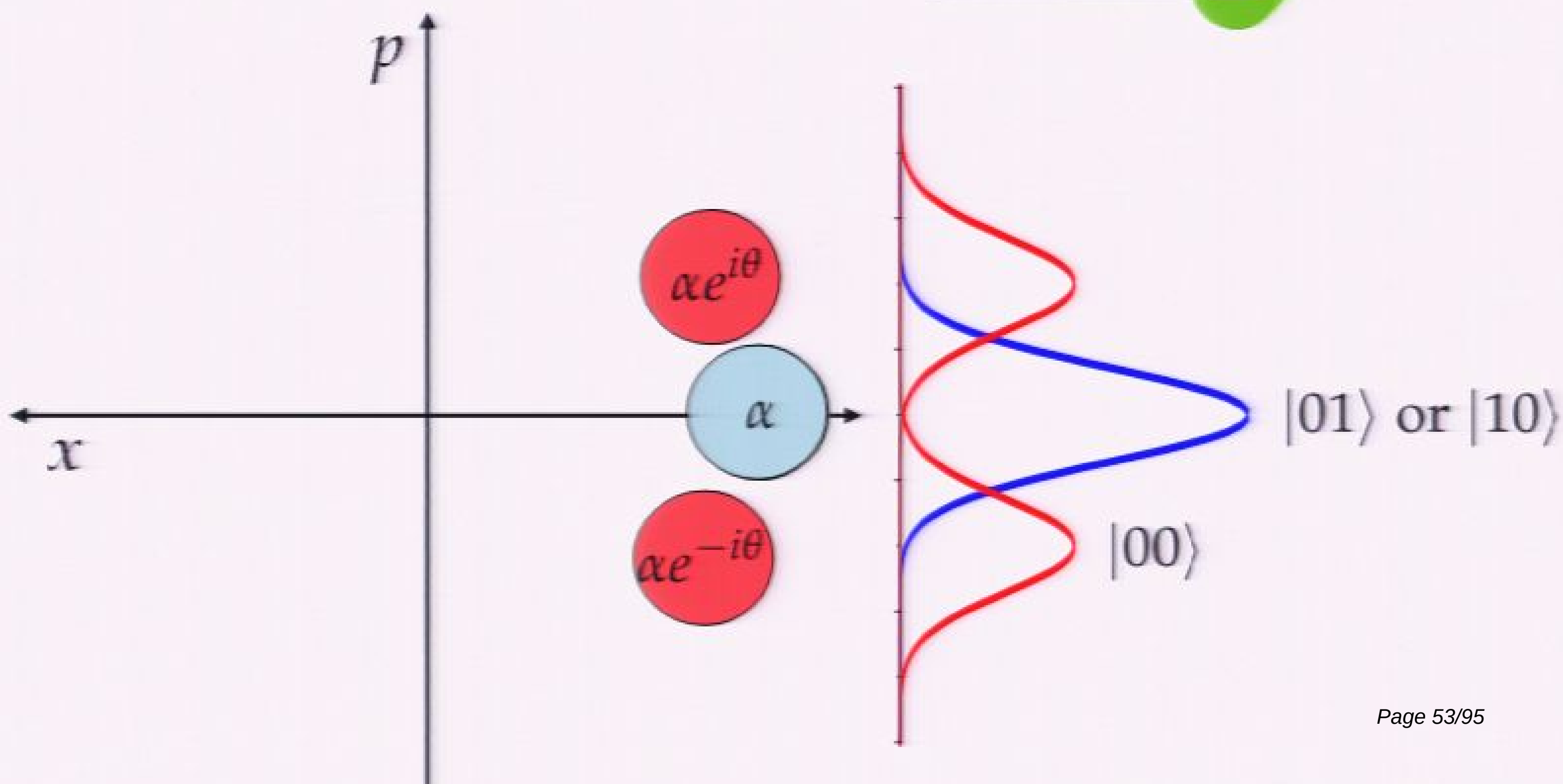

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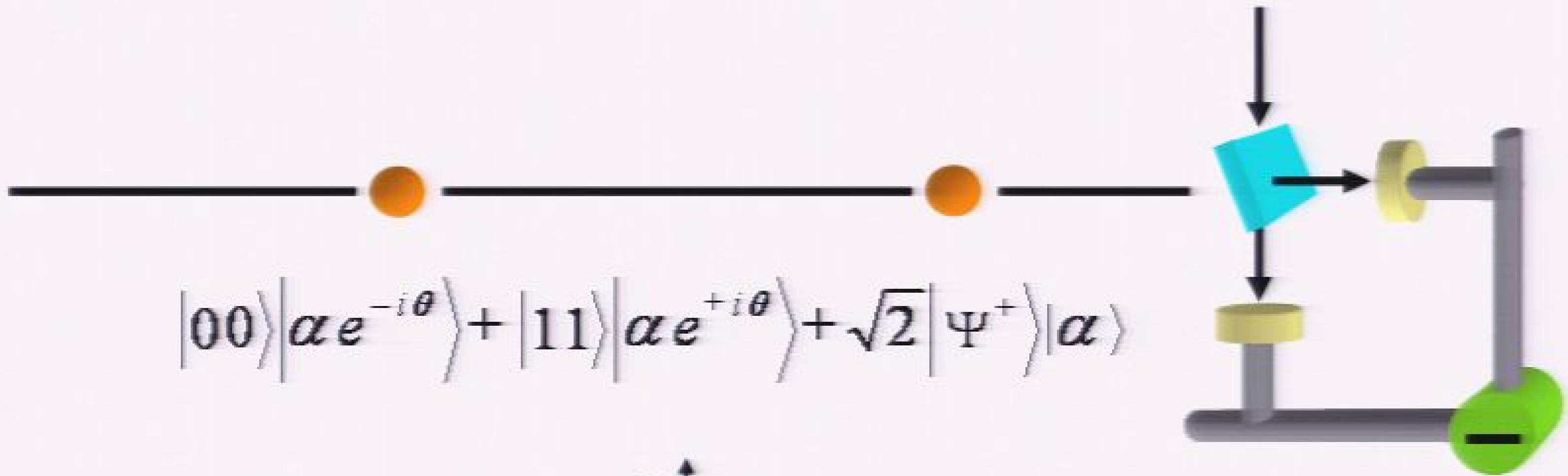
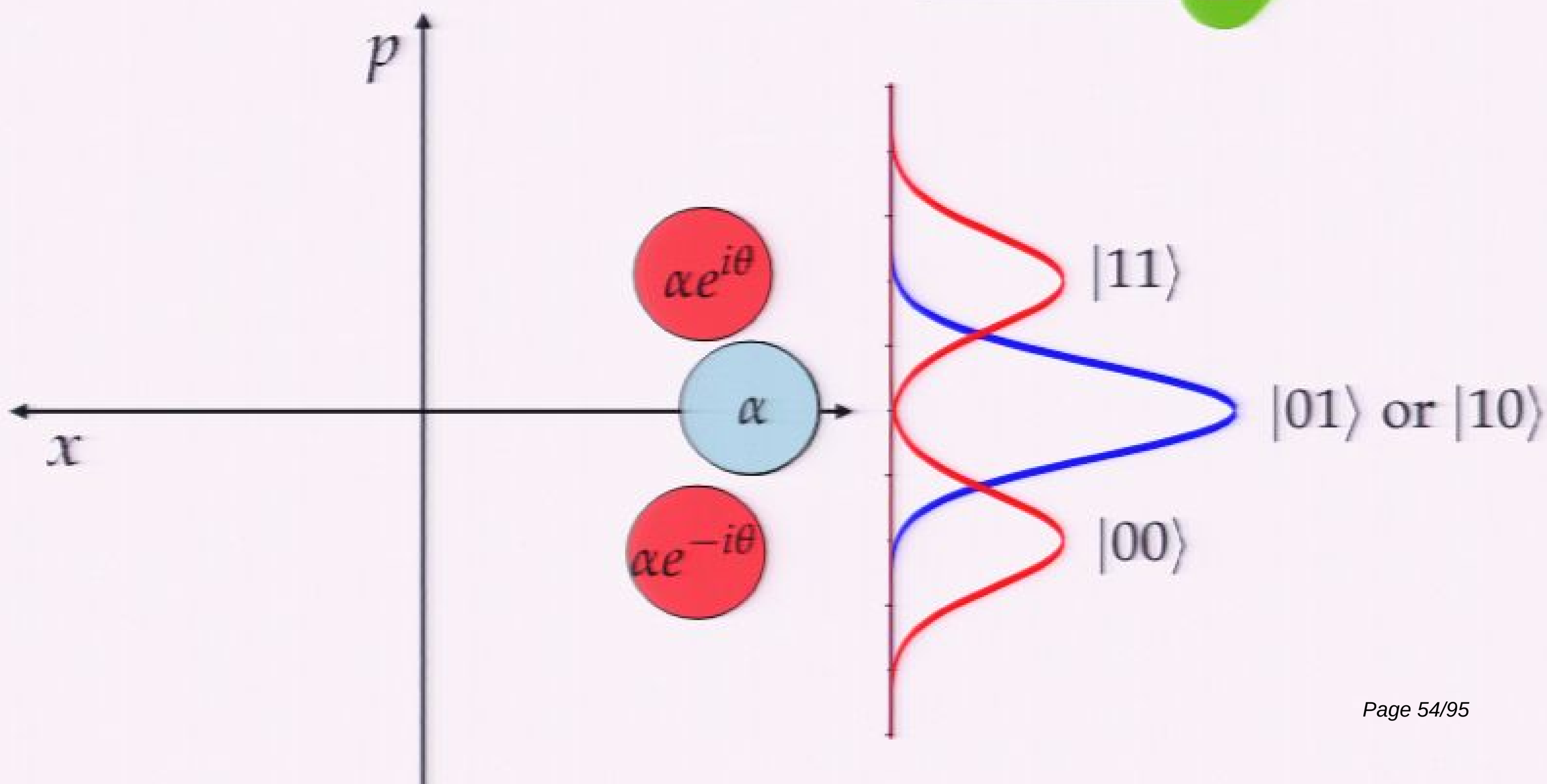


$$|00\rangle|\alpha e^{-i\theta}\rangle + |11\rangle|\alpha e^{+i\theta}\rangle + \sqrt{2}|\Psi^{+}\rangle|\alpha\rangle$$



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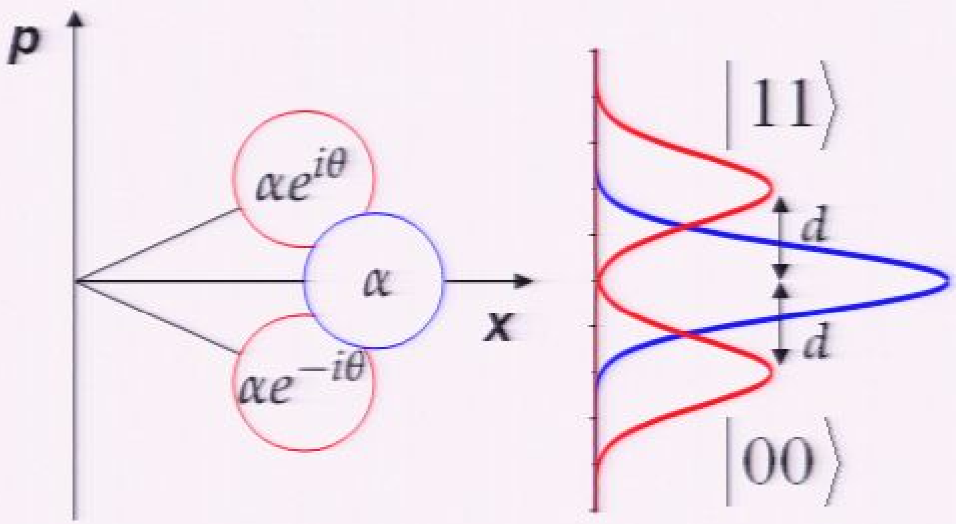
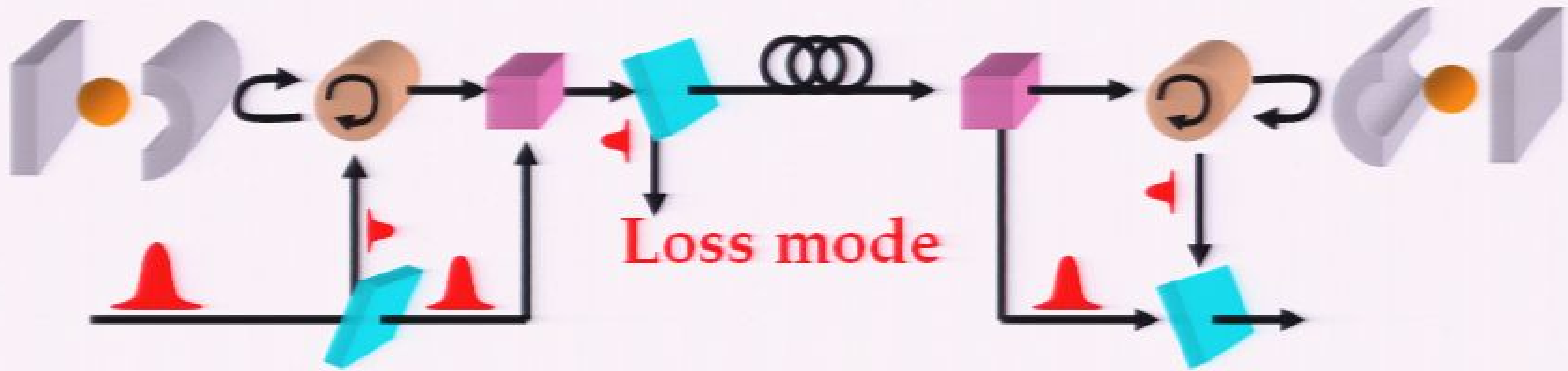


$$|\Psi^+\rangle = |01\rangle + |10\rangle$$

$$d \approx \alpha\theta$$

e.g., homodyne detection
and postselection

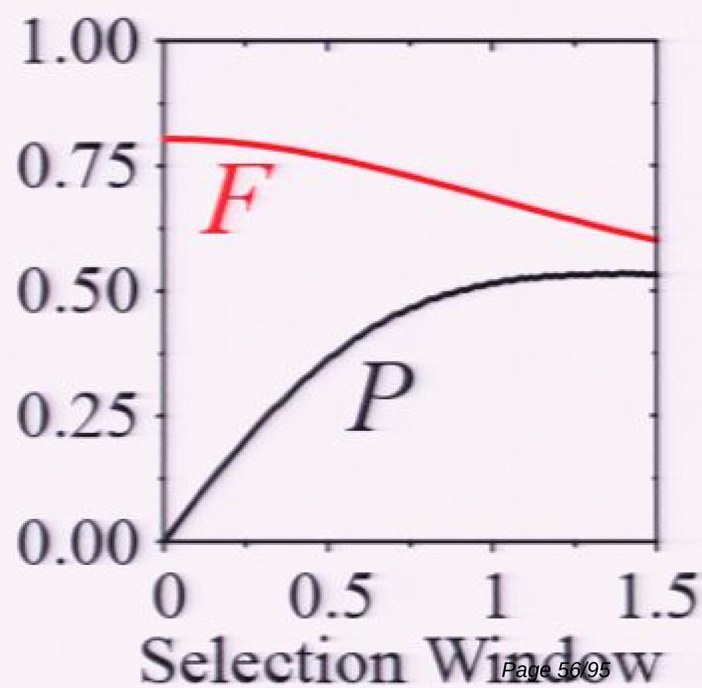
Entanglement Distribution



Ideal scheme:
make d arbitrarily large

Realistic scheme:
trade-off between distinguishability
and decoherence

$$d \sim 1, \text{ e.g. } \theta \approx 0.01$$
$$\alpha^2 \approx 10000$$
(probe photons)



Entanglement Distribution

… consider schemes without intrinsic errors

… generate mixtures of only two Bell states

$$F\,|\Phi^+\rangle\langle\Phi^+| + (1-F)\,|\Psi^+\rangle\langle\Psi^+|$$

$$|\Phi^+\rangle = \frac{1}{2}\left[|\sqrt{\eta}\,\alpha\rangle\left(|00\rangle + |11\rangle\right) + |\sqrt{\eta}\,\alpha\, e^{i\theta}\rangle|10\rangle + |\sqrt{\eta}\,\alpha\, e^{-i\theta}\rangle|01\rangle\right]$$

$$|\Psi^+\rangle = \frac{1}{2}\left[|\sqrt{\eta}\,\alpha\rangle\left(|00\rangle - |11\rangle\right) - |\sqrt{\eta}\,\alpha\, e^{i\theta}\rangle|10\rangle + |\sqrt{\eta}\,\alpha\, e^{-i\theta}\rangle|01\rangle\right]$$

Unambiguous State Discrimination

"+"	$\left\| \sqrt{2}\alpha \right\rangle$	$\left\| 0 \right\rangle$
"−"	$\left\| 0 \right\rangle$	$\left\| -\sqrt{2}\alpha \right\rangle$

$\left| \pm\alpha \right\rangle$ $\qquad$ $\left| A=\alpha \right\rangle$

Unambiguous State Discrimination

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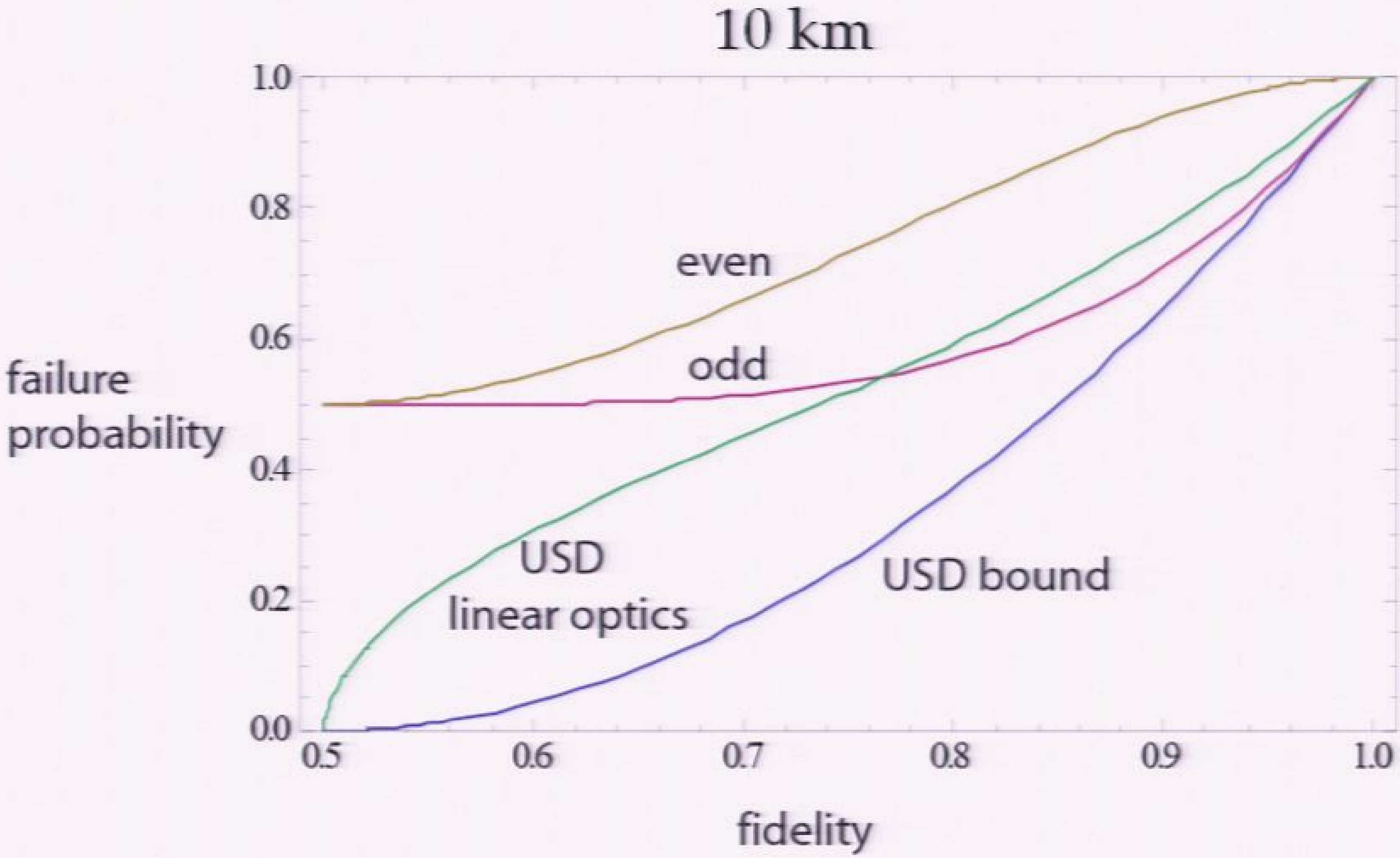
$\left|\pm\alpha\right\rangle$ $\left|A=\alpha\right\rangle$

$$\left\langle\alpha\middle|-\alpha\right\rangle = e^{-2\alpha^2}$$

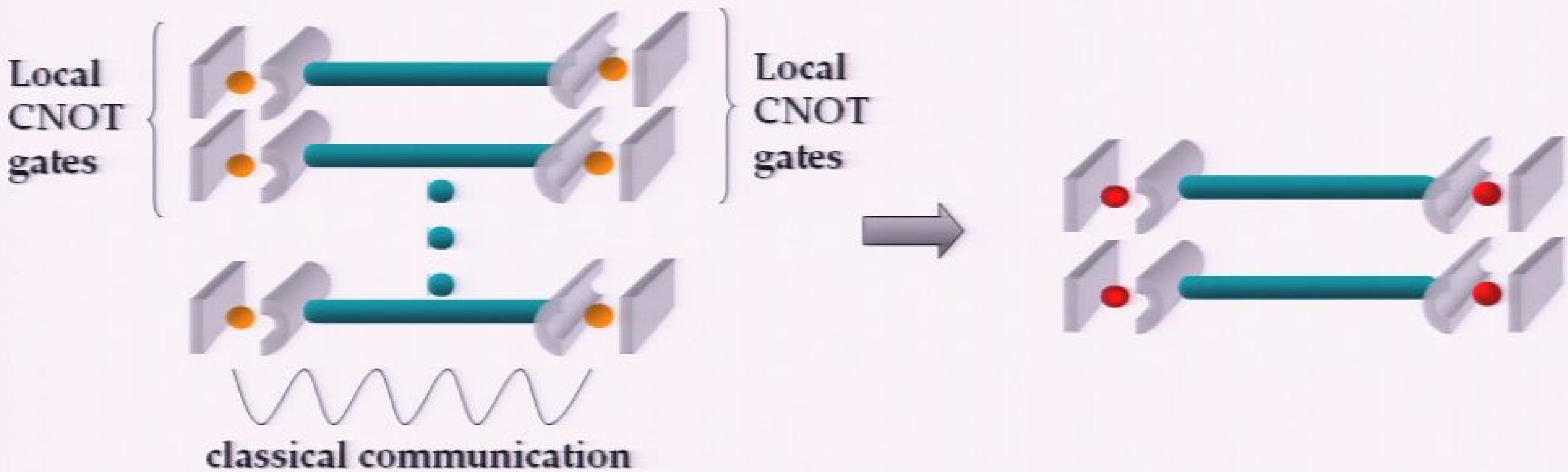
$$\left|\psi^{inconcl}\right\rangle = e^{-\alpha^2}\left|00\right\rangle$$

Unambiguous State Discrimination



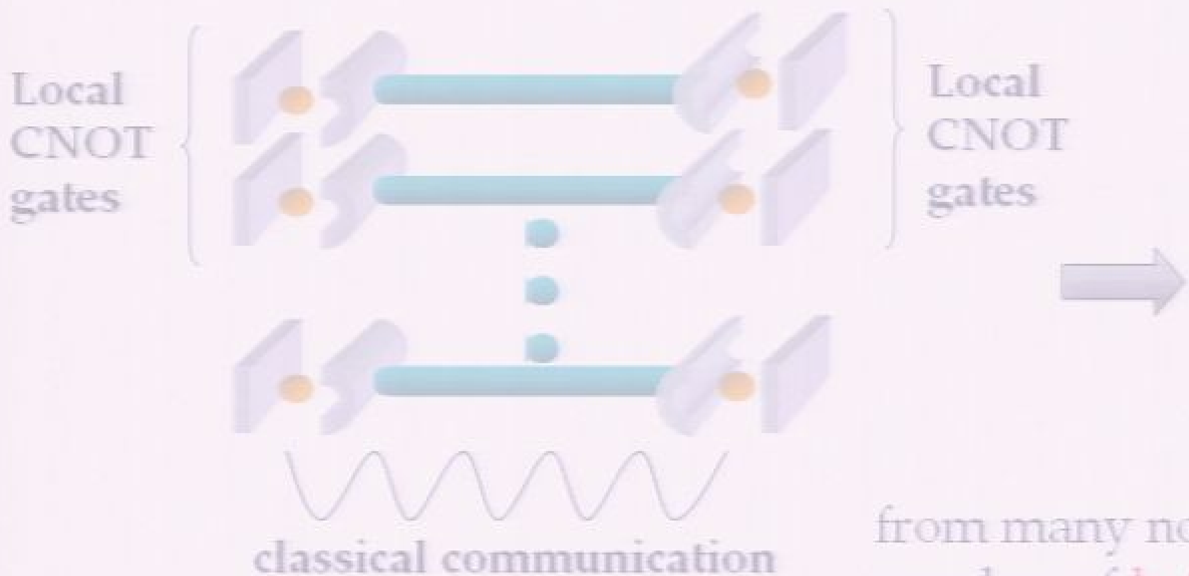
PvL *et al.*, Phys. Rev. A., accepted

Entanglement Purification and Swapping



from many noisy entangled pairs a smaller number of better entangled pairs can be distilled

Entanglement Purification and Swapping



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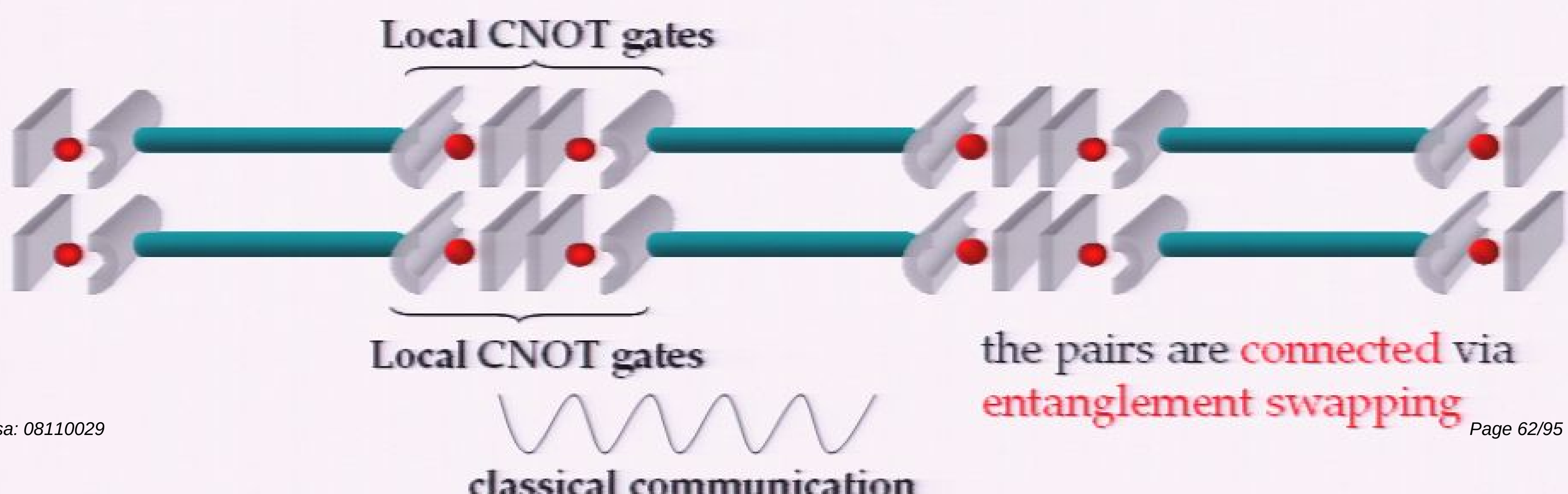


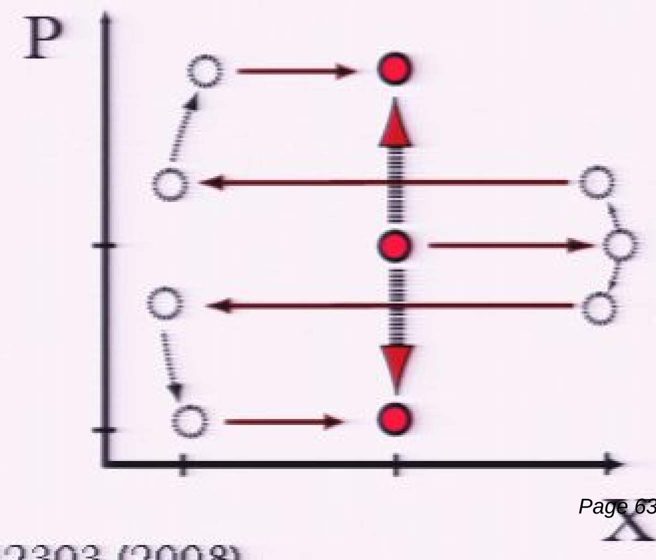
the pairs are connected via entanglement swapping

Deterministic C-NOT Gate

... simulate conditional displacements
via sequence of unconditional displacements
and conditional rotations ...

$$D(\boldsymbol{\beta}\cos\boldsymbol{\theta})\, e^{-i\hat{a}^{\dagger}\hat{a}\,\theta\,\hat{\sigma}_z}\, D(-2\boldsymbol{\beta})\, e^{i\hat{a}^{\dagger}\hat{a}\,\theta\,\hat{\sigma}_z}\, D(\boldsymbol{\beta}\cos\boldsymbol{\theta})$$

$$= e^{2i\,\beta\sin\theta\,(\hat{a}^{\dagger} + \hat{a})\,\sigma_z}$$



...works in any regime,
weak or strong coupling!

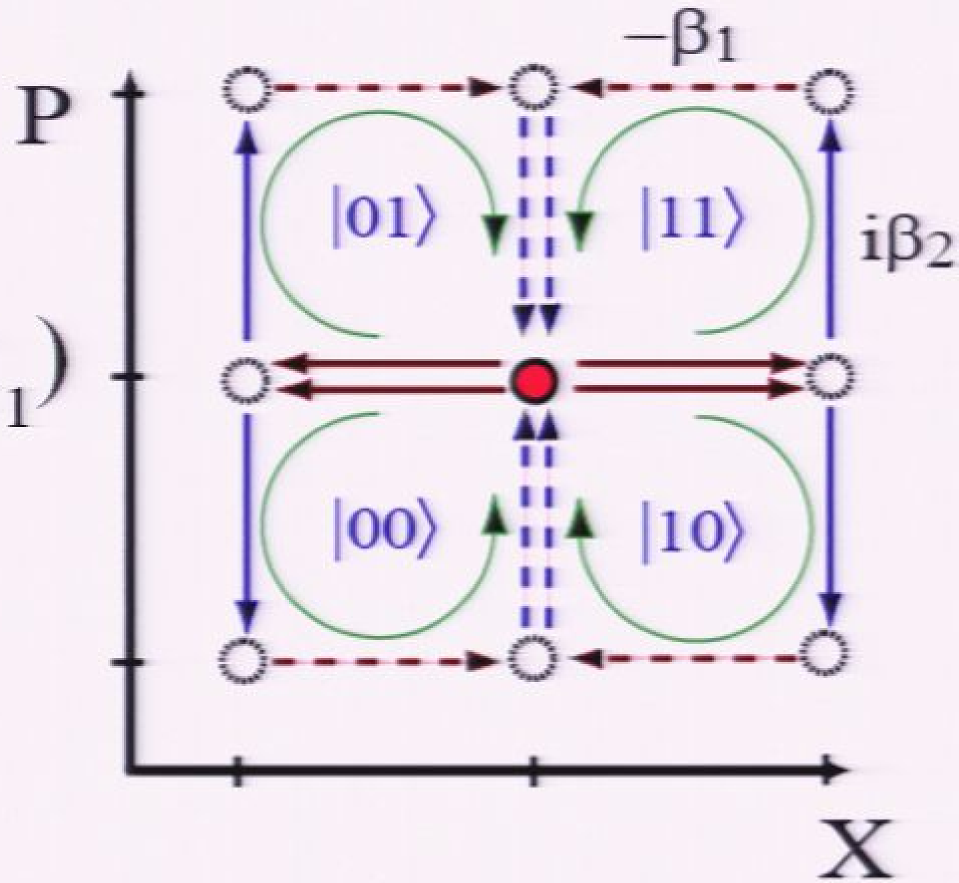
PvL *et al.*, Phys. Rev. A. **78**, 022303 (2008)

Deterministic C-NOT Gate

$$D(i\boldsymbol{\beta}_2\boldsymbol{\sigma}_{z2})\,D(\boldsymbol{\beta}_1\boldsymbol{\sigma}_{z1})$$
$$\times\, D(-i\boldsymbol{\beta}_2\boldsymbol{\sigma}_{z2})\,D(-\boldsymbol{\beta}_1\boldsymbol{\sigma}_{z1})$$

$$= e^{2i\,\beta_1\,\beta_2\,\boldsymbol{\sigma}_{z1}\boldsymbol{\sigma}_{z2}}$$

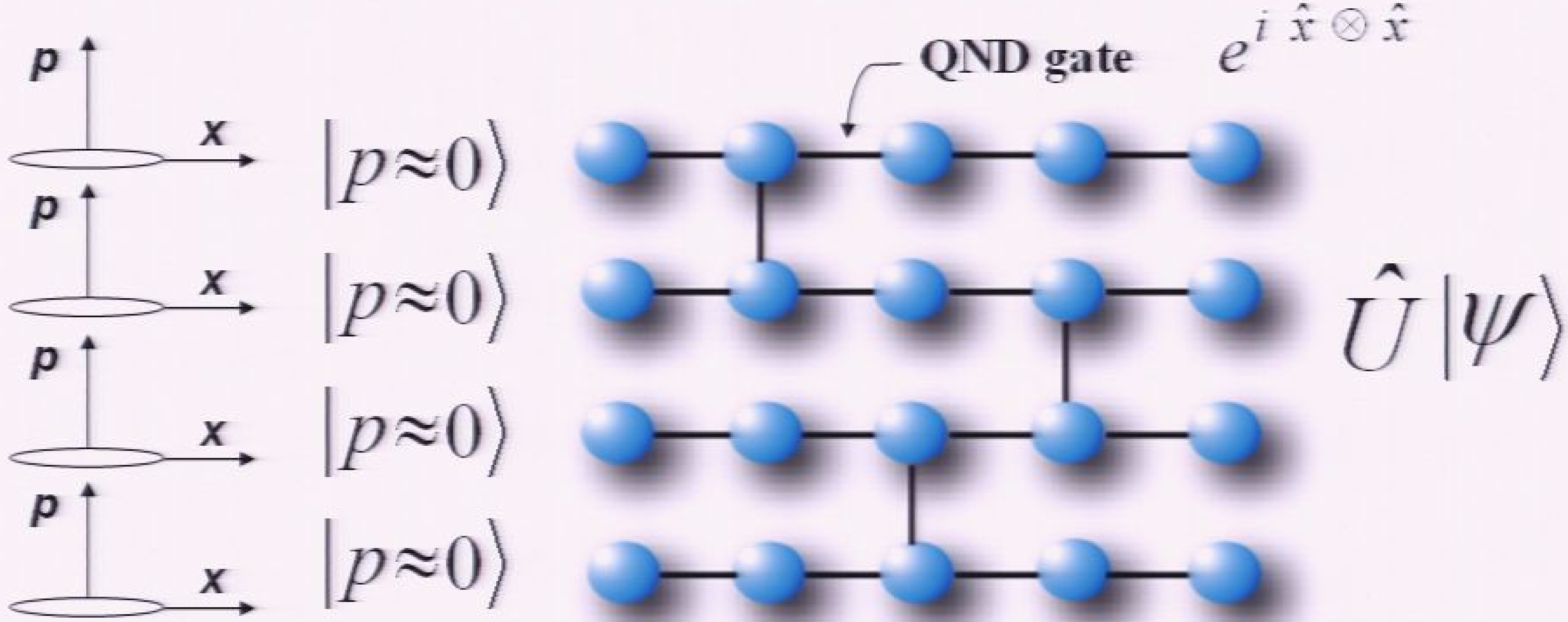
for $\beta_1\beta_2 = \frac{\pi}{8}$



this corresponds to a CNOT gate
up to a global phase and local rotations

Hybrid One-Way Quantum Computer

... universal quantum computing with Gaussian cluster states, via continuous and discrete measurements



...in collaboration with N. Menicucci, Mile Gu, C. Weedbrook, T.C. Ralph, M. Nielsen...

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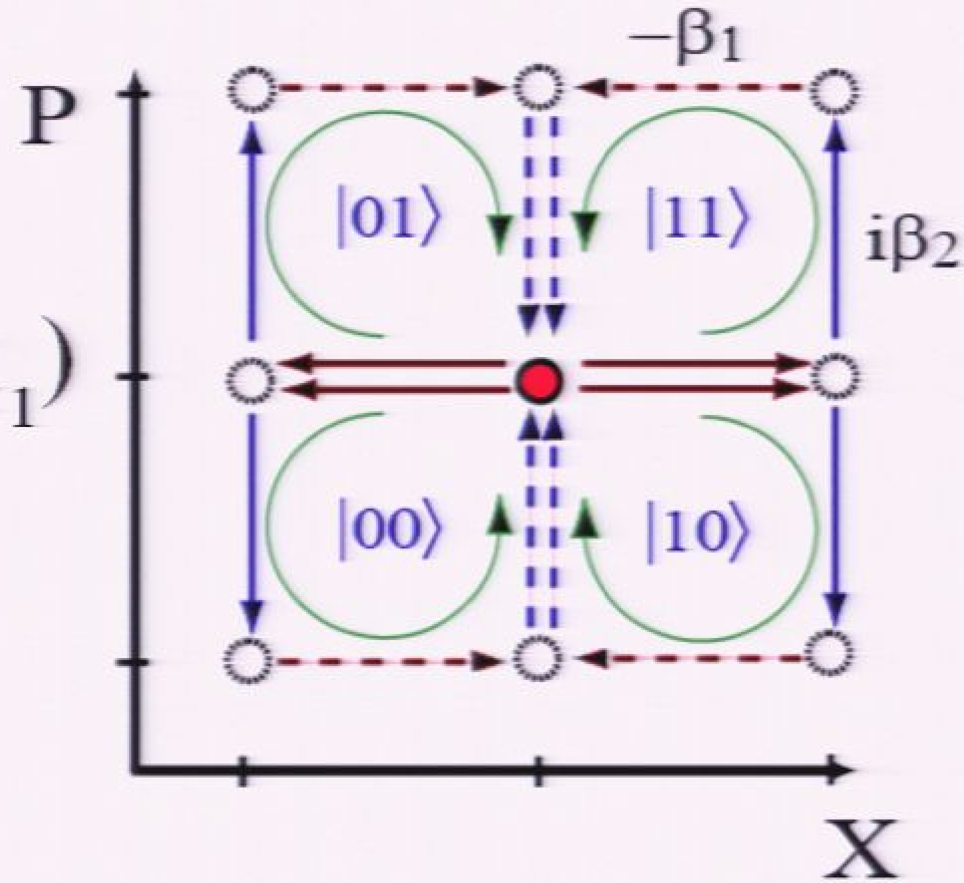

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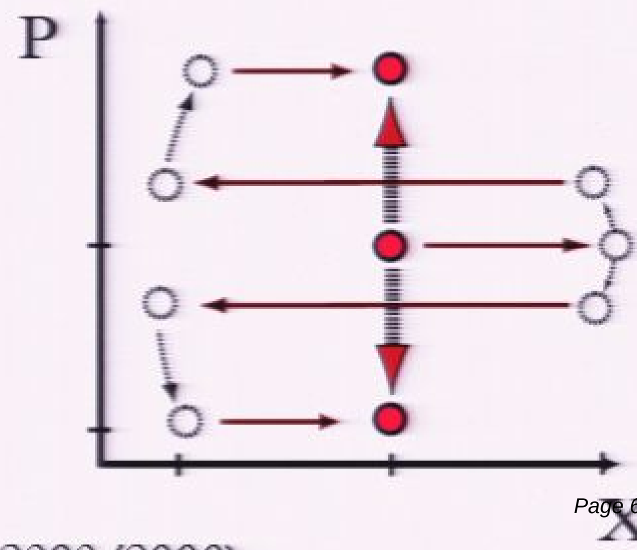
and conditional rotations ...

$$D(\beta\cos\theta)\,e^{-i\hat{a}^{\dagger}\hat{a}\theta\hat{\sigma}_z}\,D(-2\beta)\,e^{i\hat{a}^{\dagger}\hat{a}\theta\hat{\sigma}_z}\,D(\beta\cos\theta)$$

$$= e^{2i\beta\sin\theta\,(\hat{a}^{\dagger}+\hat{a})\,\sigma_z}$$



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PvL *et al.*, Phys. Rev. A. **78**, 022303 (2008)

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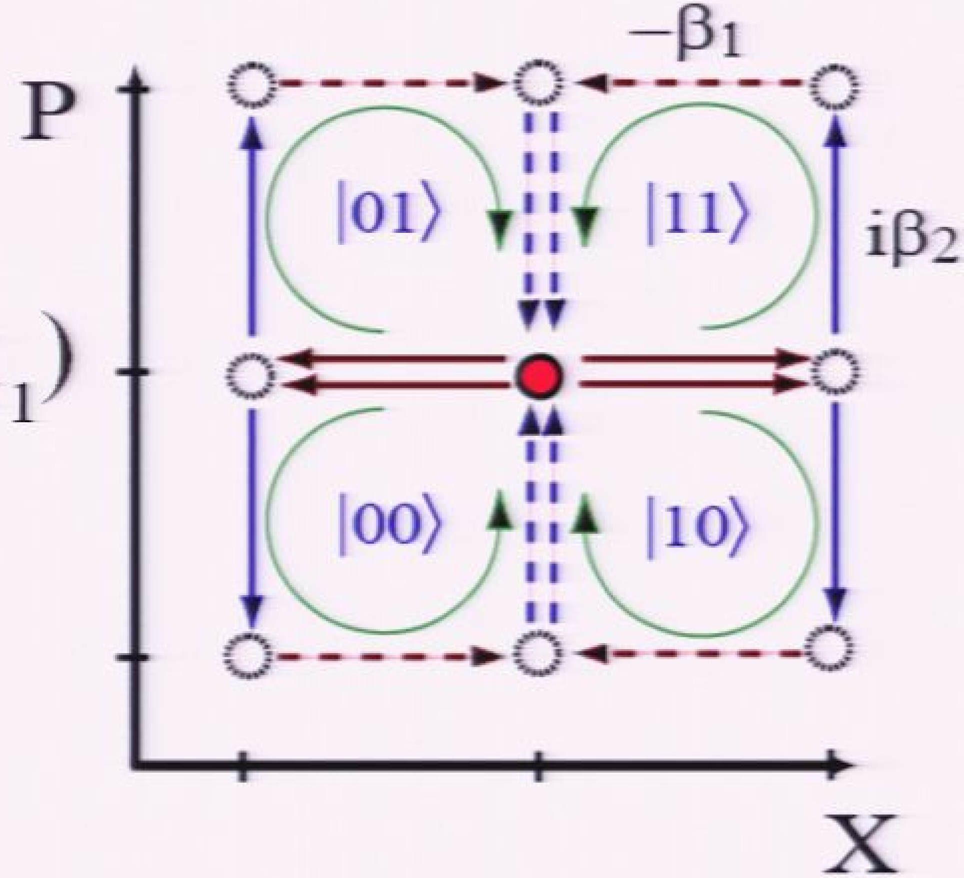
$$\times D(-i\boldsymbol{\beta}_2\boldsymbol{\sigma}_{z2})\,D(-\boldsymbol{\beta}_1\boldsymbol{\sigma}_{z1})$$

$$= e^{2i\,\beta_1\,\beta_2\boldsymbol{\sigma}_{z1}\boldsymbol{\sigma}_{z2}}$$



for $\beta_1\beta_2 = \frac{\pi}{8}$

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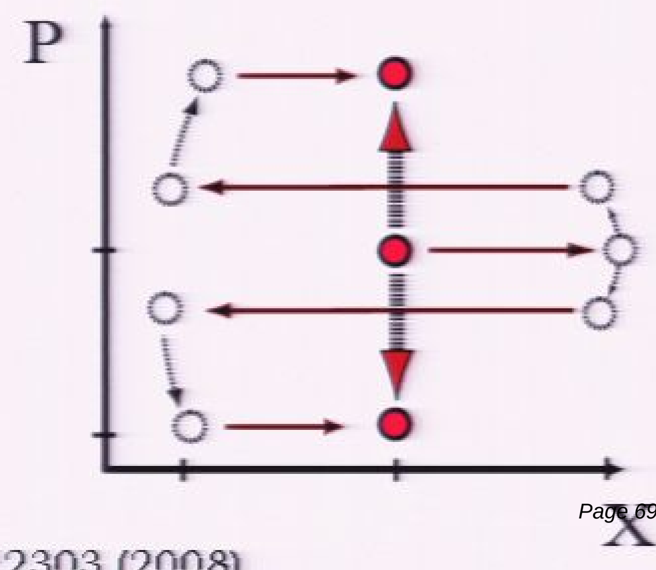

PvL *et al.*, Phys. Rev. A. **78**, 022303 (2008)

Hybrid One-Way Quantum Computer

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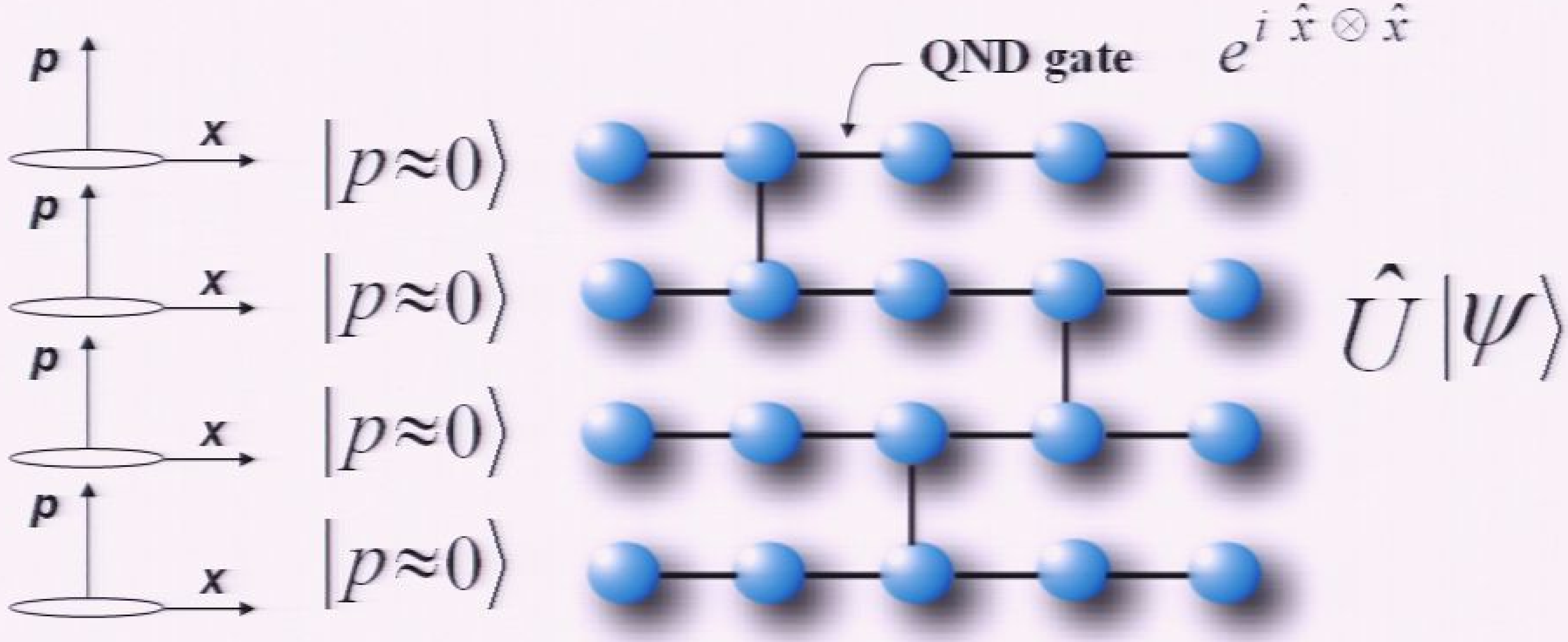

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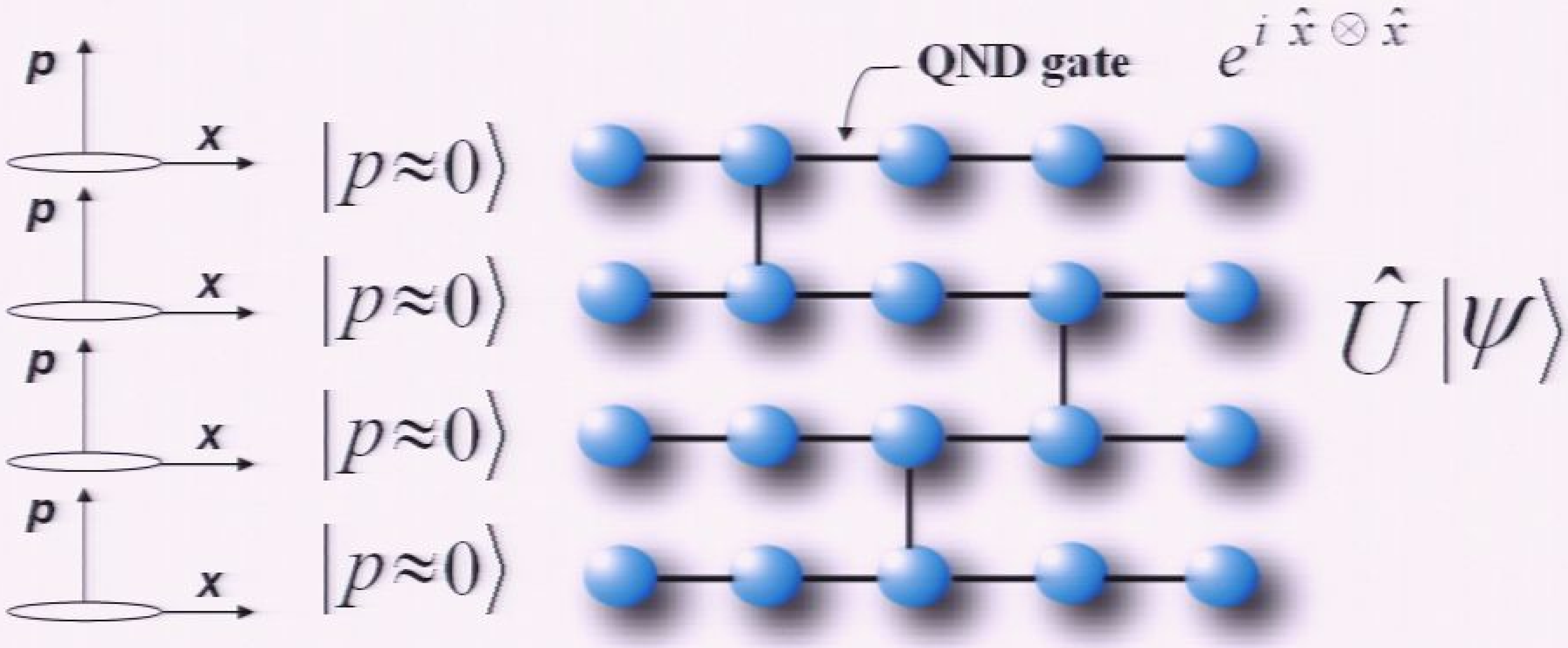
Hybrid One-Way Quantum Computer



- N.C. Menicucci, PvL, M. Gu, C. Weedbrook,T.C. Ralph, M.A. Nielsen, PRL **97**,110501 (2006)
- PvL, J. Opt. Soc. Am. B **24**, 340 (2007)
- efficient generation: PvL, C. Weedbrook, M. Gu, PRA **76**, 032321 (2007)

- compact generation: N.C. Menicucci, PRL **101**, 130501 (2008), etc.

Cluster Computation

<u>Essence of Cluster Computation:</u>

cluster state is independent of computation;

universality and "computational freedom" through choice of measurement bases

Quantum Computation via Continuous Variables

$$X(s) \equiv e^{-2is\hat{p}}, \quad Z(t) \equiv e^{2it\hat{x}}$$ (WH operators)

$$X(s)\left|p\right\rangle = e^{-2isp}\left|p\right\rangle, \quad Z(t)\left|p\right\rangle = \left|p+t\right\rangle$$

$$Z(t)\left|x\right\rangle = e^{2itx}\left|x\right\rangle, \quad X(s)\left|x\right\rangle = \left|x+s\right\rangle$$

(position/computational basis)

Quantum Computation via Continuous Variables

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$$X(s)|p\rangle = e^{-2isp}|p\rangle, \quad Z(t)|p\rangle = |p+t\rangle$$

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(position/computational basis)

$$D \equiv e^{i f(\hat{x})}$$

Quantum Computation via Continuous Variables

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(position/computational basis)

$$D \equiv e^{i f(\hat{x})}$$

$$F\left|x\right\rangle = \left|p=x\right\rangle, \text{etc.}, \qquad F\,\hat{x}\,F^{\dagger} = -\hat{p}, \quad F\,\hat{p}\,F^{\dagger} = \hat{x}$$

Quantum Computation via Continuous Variables

$$C_Z = e^{2i\,\hat{x}\otimes\hat{x}}$$

(controlled Z gate)

(QND gate)

Quantum Computation via Continuous Variables

$$C_Z = e^{2i\,\hat{x}\otimes\hat{x}}$$ (controlled Z gate)

(QND gate)

$$\left\{ F,\ Z(t),\ e^{i\kappa\hat{x}^2},\ C_Z,\quad e^{i\lambda\hat{x}^3};\ \ t,\kappa,\lambda\in\mathbf{R} \right\}$$

Clifford set = Gaussian

"cubic phase gate"

(universal set)

Quantum Computation via Continuous Variables

Clifford group:

$$U X U^{\dagger} = X'$$

qubits:

$$U \sigma U^{\dagger} = \sigma'$$

Quantum Computation via Continuous Variables

- any Clifford computation can be efficiently simulated on a classical computer

{S.D. Bartlett *et al.*, PRL **88**, 097904 (2002)}

- a single-mode non-Clifford (non-Gaussian) transformation is necessary and sufficient for universal quantum computation

{S. Lloyd and S.L. Braunstein, PRL **82**, 1784-1787 (1999)}

Elementary Teleportation Circuits

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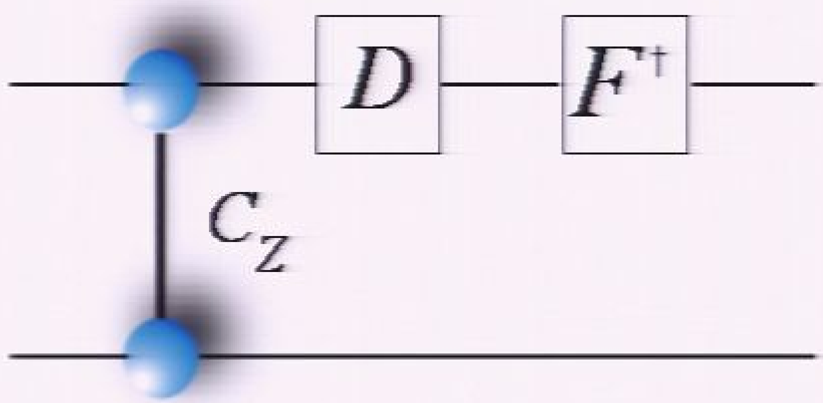
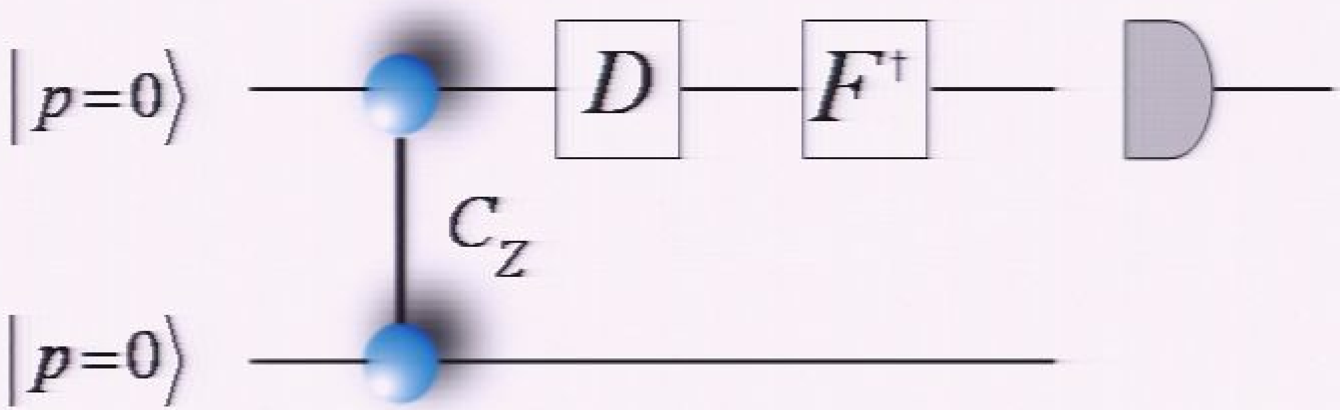
Elementary Teleportation Circuits



Elementary Teleportation Circuits



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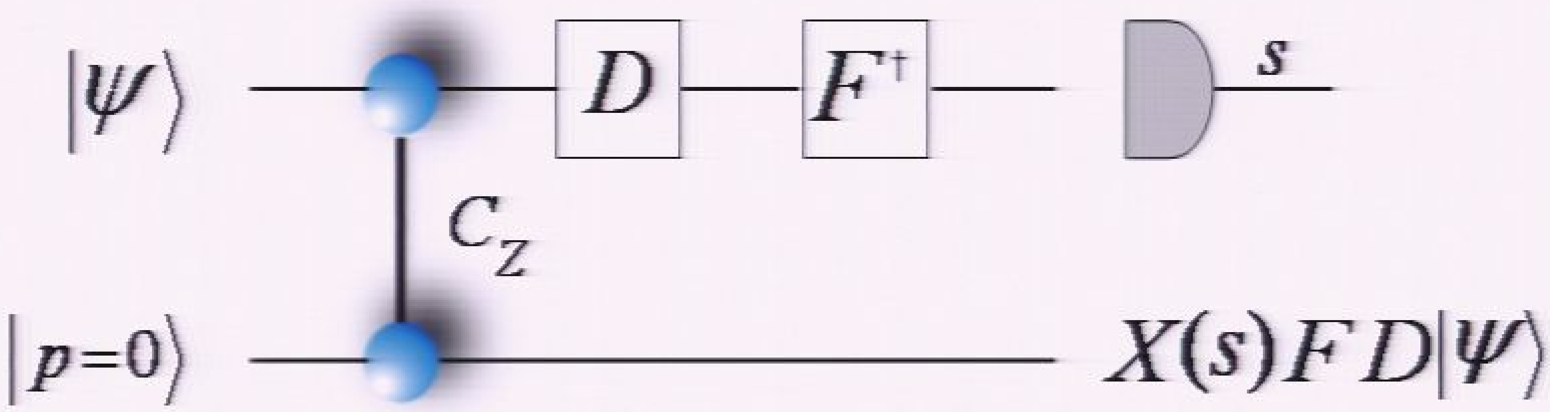

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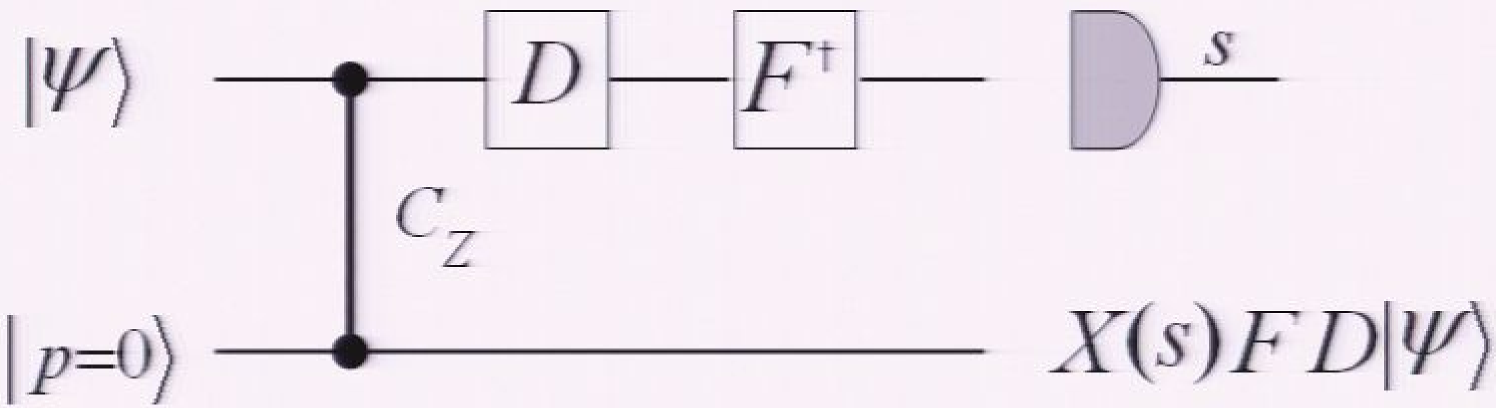

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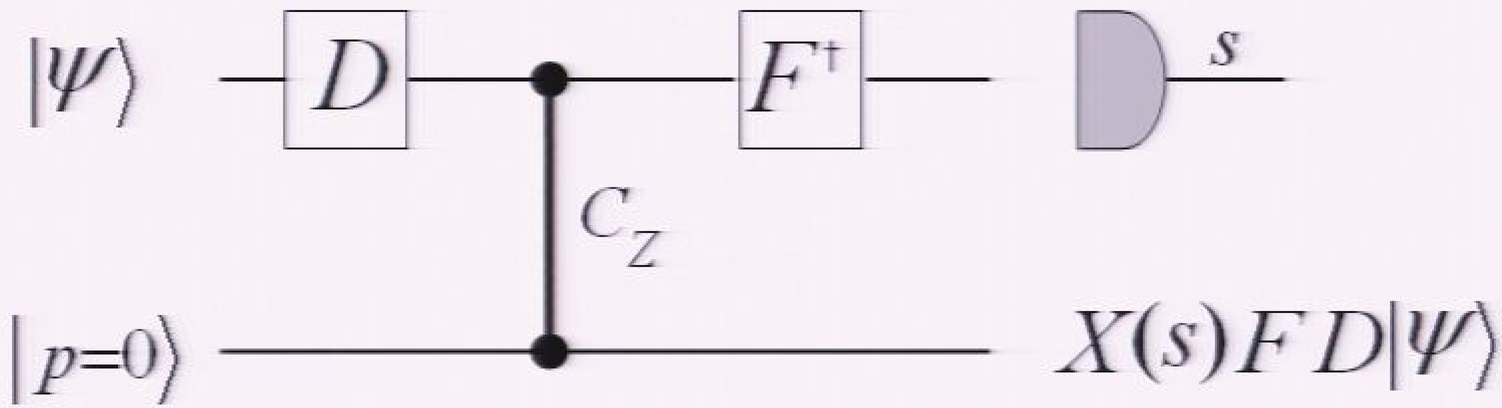
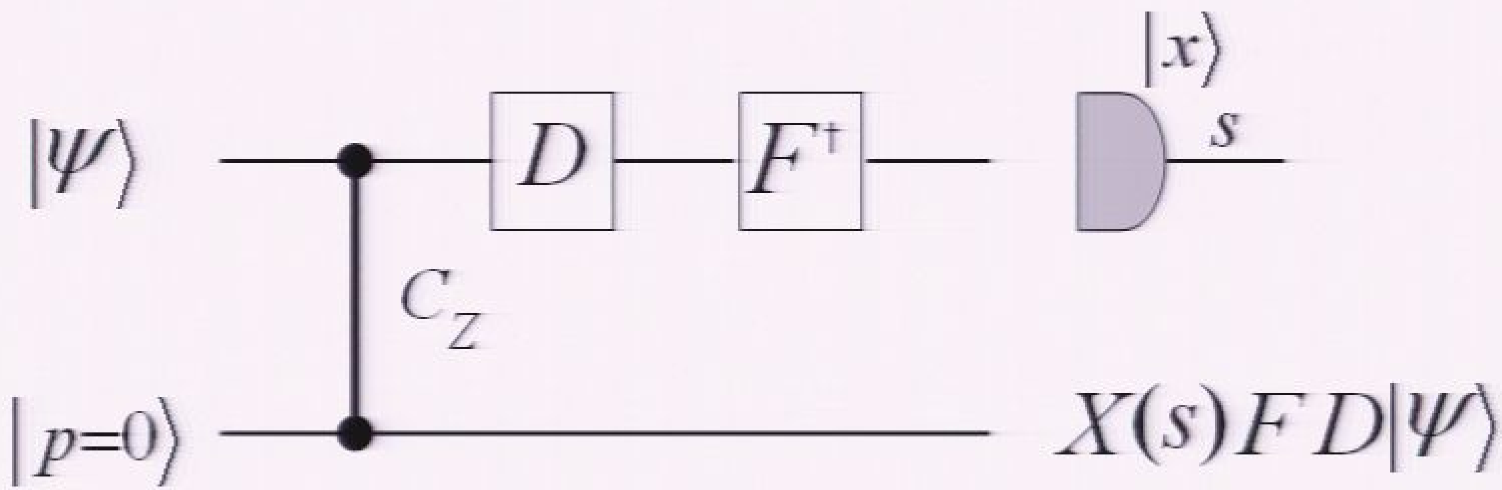


Elementary Teleportation Circuits



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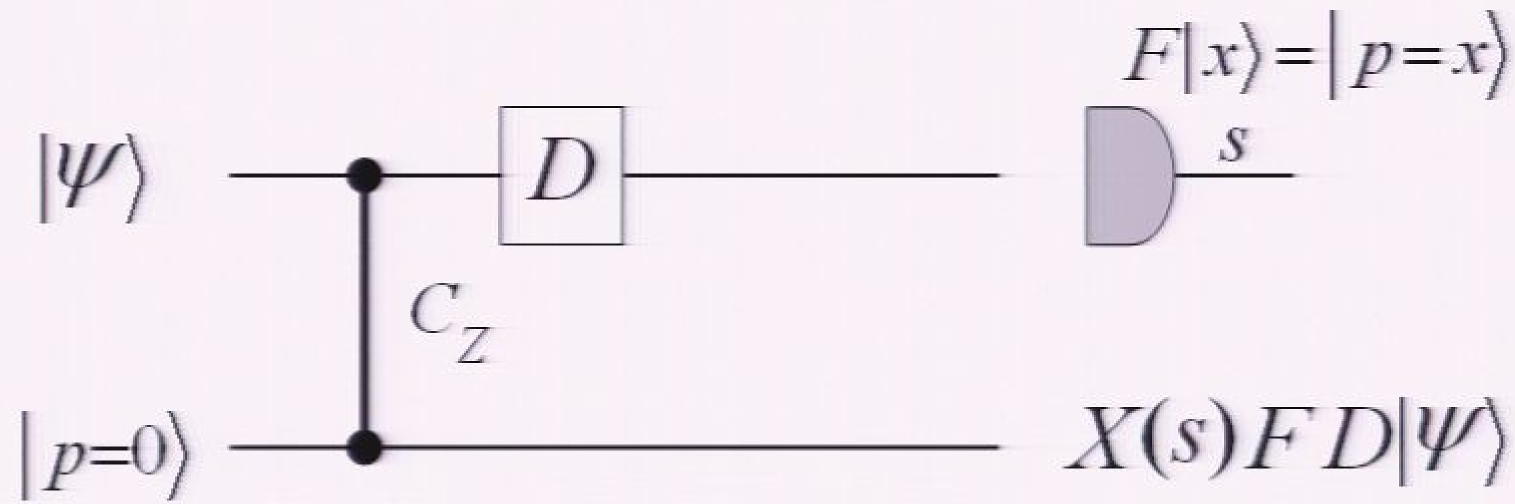

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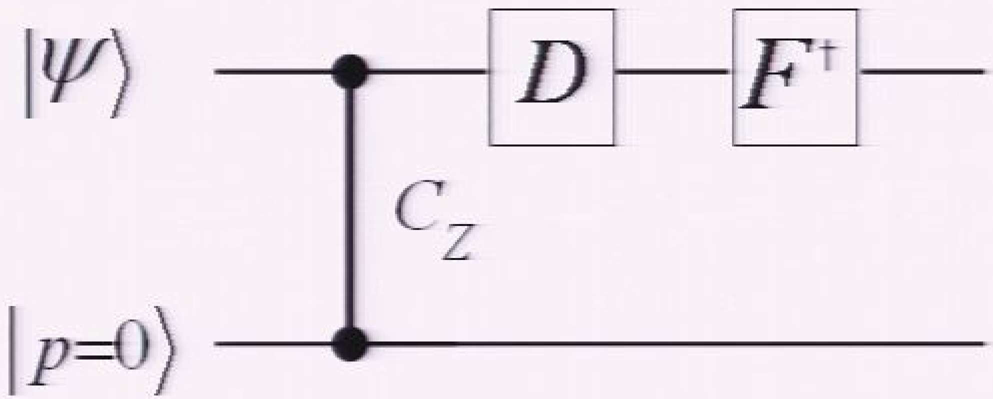

Elementary Teleportation Circuits

Unambiguous State Discrimination

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$$\left|\pm\alpha\right\rangle \qquad \left|A=\alpha\right\rangle$$

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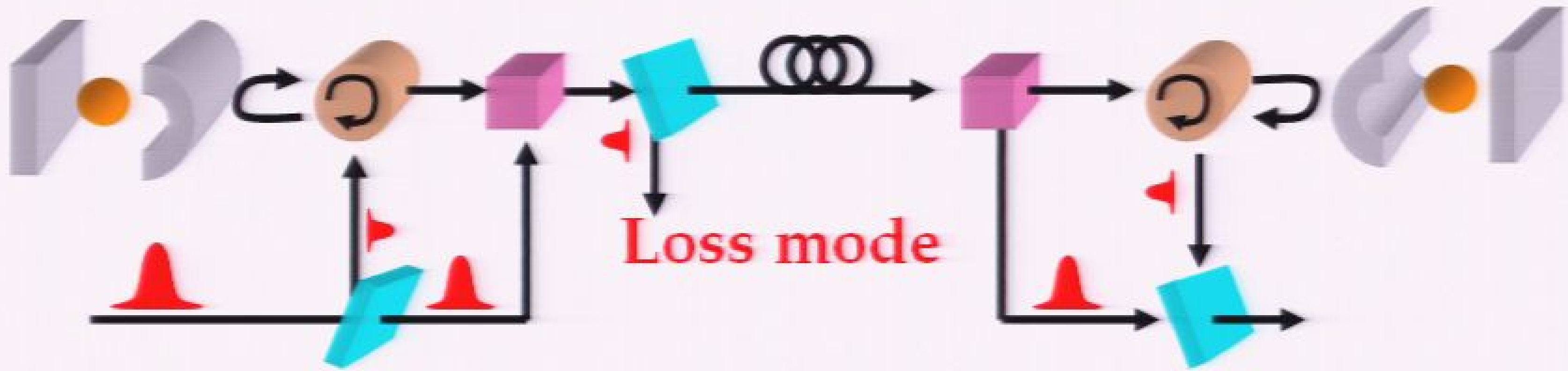
$$|\Psi^+\rangle = \frac{1}{2}\Big[\,|\sqrt{\eta}\,\alpha\rangle\big(|00\rangle - |11\rangle\big) - |\sqrt{\eta}\,\alpha\, e^{i\theta}\rangle|10\rangle + |\sqrt{\eta}\,\alpha\, e^{-i\theta}\rangle|01\rangle\Big]$$

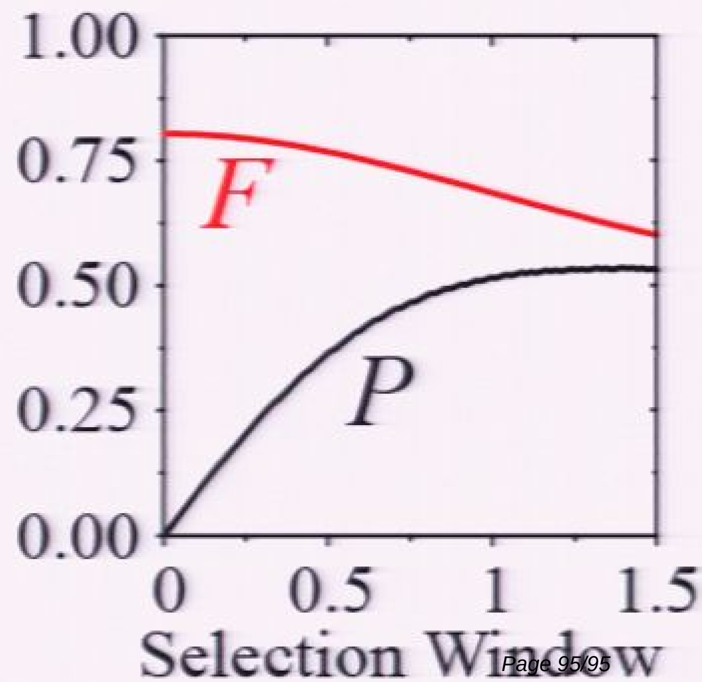
Entanglement Distribution



Ideal scheme:
make d arbitrarily large

Realistic scheme:
trade-off between distinguishability
and decoherence

$$d \sim 1, \text{ e.g. } \theta \approx 0.01$$
$$\alpha^2 \approx 10000$$
(probe photons)



(10km, 0.17dB/km)