

Title: Hybrid quantum information processing and communication

Date: Nov 26, 2008 04:00 PM

URL: <http://pirsa.org/08110029>

Abstract: With the aim of proposing feasible, quantum optical realizations of quantum information protocols and minimizing the resource costs in such implementations, we will discuss various, so-called hybrid approaches. These include, for instance, schemes based upon both discrete and continuous quantum variables.

Hybrid Quantum Information Processing and Communication

Optical Quantum Information Theory
Emmy Noether Junior Research Group
Max Planck Research Group
Institute of Optics, Information and Photonics
University of Erlangen/Nürnberg, Germany

Peter van Loock

Hybrid Quantum Information Processing and Communication

Optical Quantum Information Theory
Emmy Noether Junior Research Group
Max Planck Research Group
Institute of Optics, Information and Photonics
University of Erlangen/Nürnberg, Germany

Peter van Loock

Hybrid Quantum Information Processing and Communication

*Optical Quantum Information Theory
Emmy Noether Junior Research Group*

*Max Planck Research Group
Institute of Optics, Information and Photonics
University of Erlangen/Nürnberg, Germany*

Peter van Loock

January 2009:

Max Planck Institute for the Science of Light

“Hybrid”

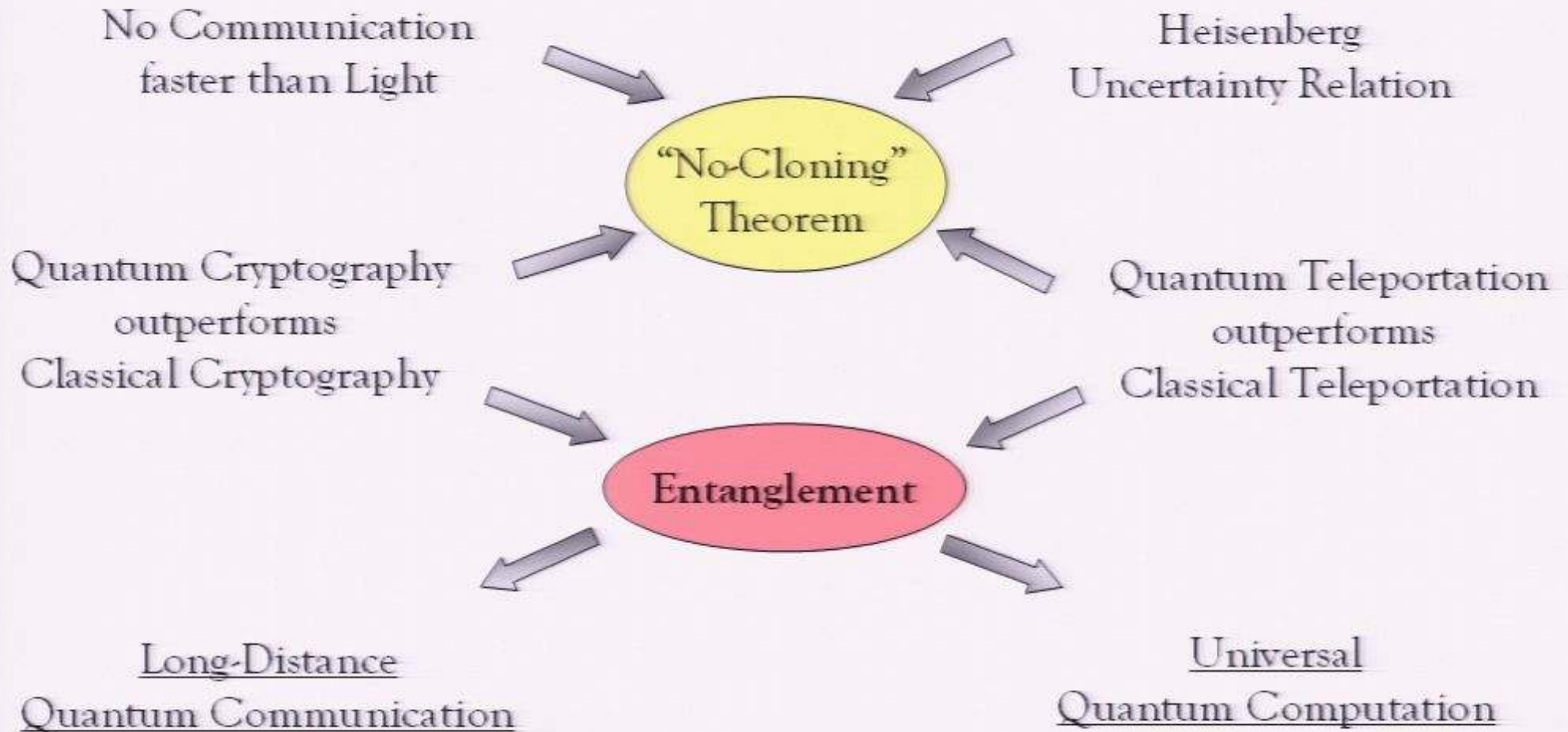
something **heterogeneous** in origin or composition
(in biology, crosses between different species);

something that has **two** different types of components
performing essentially the same function;

“Hybrid Computer”

a computer system consisting of a **combination** of **analog** and **digital**
computer systems;

Quantum Information



Quantum Information

Entanglement

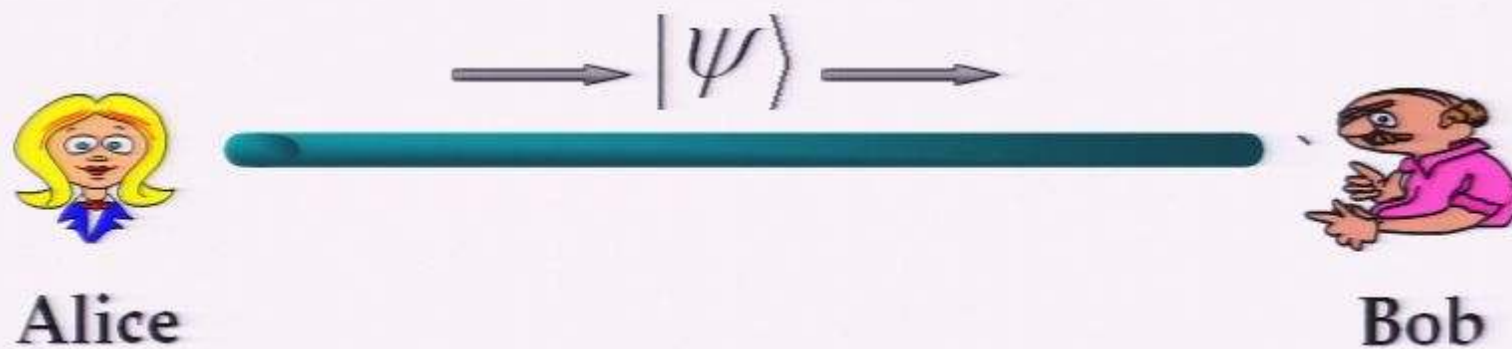
$$\int dx |x, x - c\rangle$$

Continuous Variables

$$|\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$$

Discrete Variables

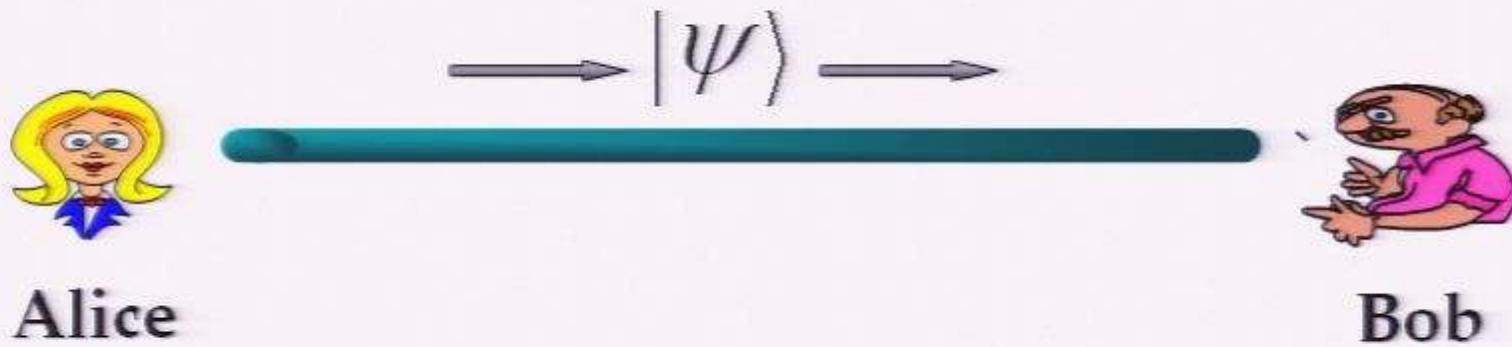
Quantum Communication



Quantum Communication



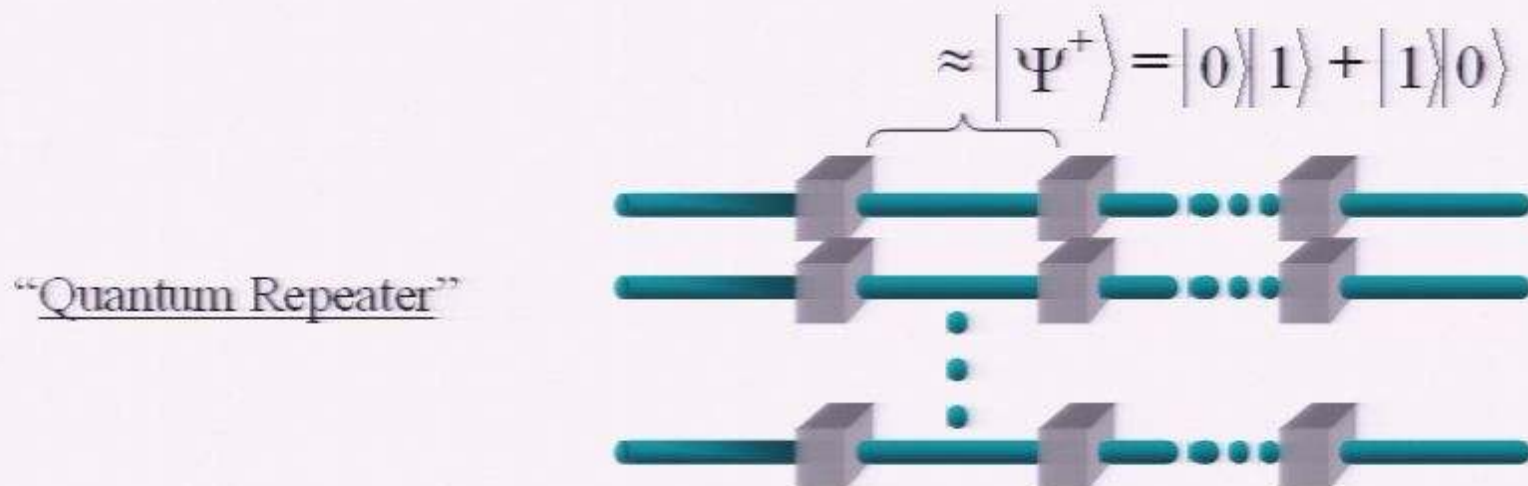
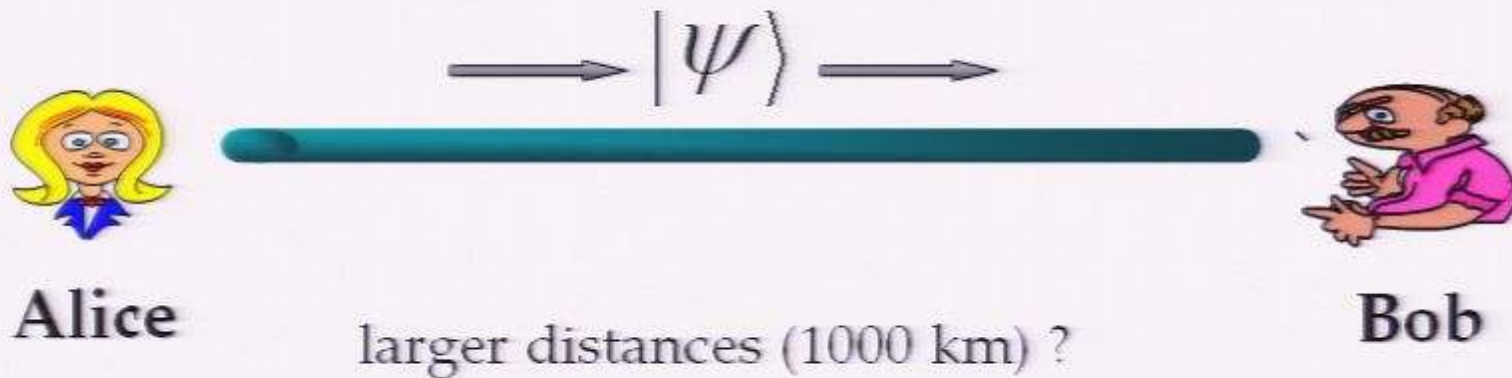
Quantum Communication



larger distances (1000 km) ?

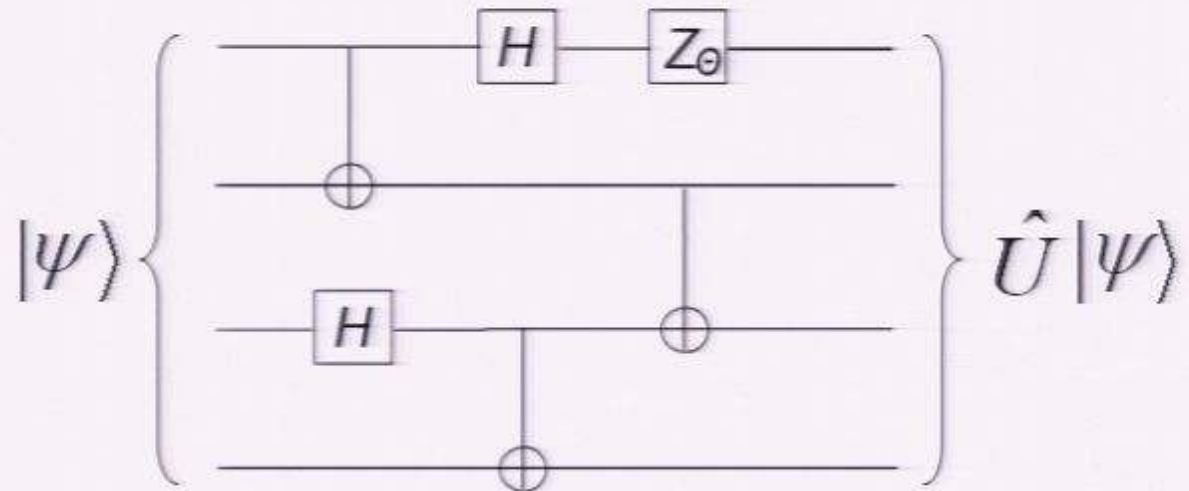
Entanglement: a universal resource

Quantum communication



Quantum Computation

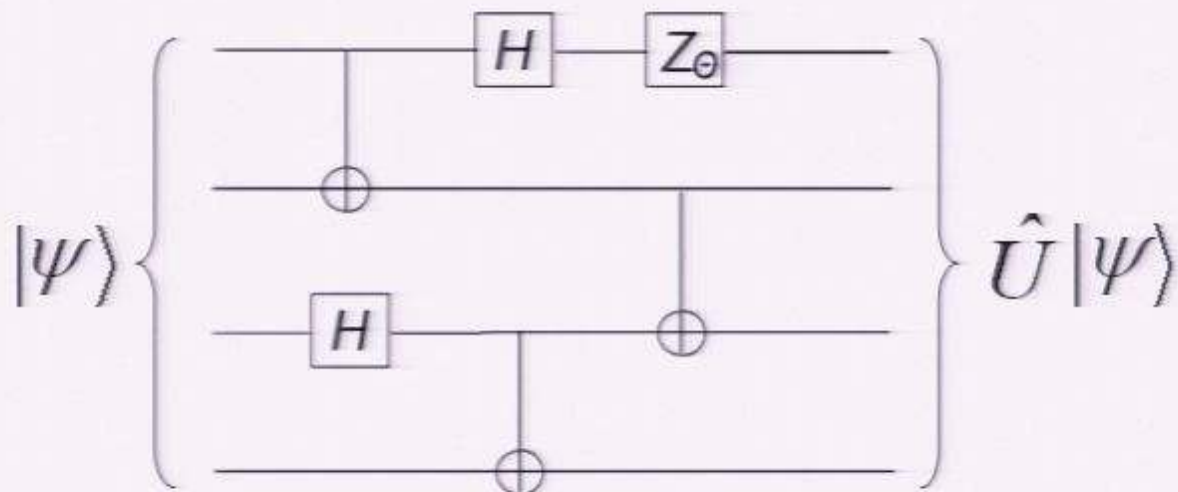
“Circuit Model”



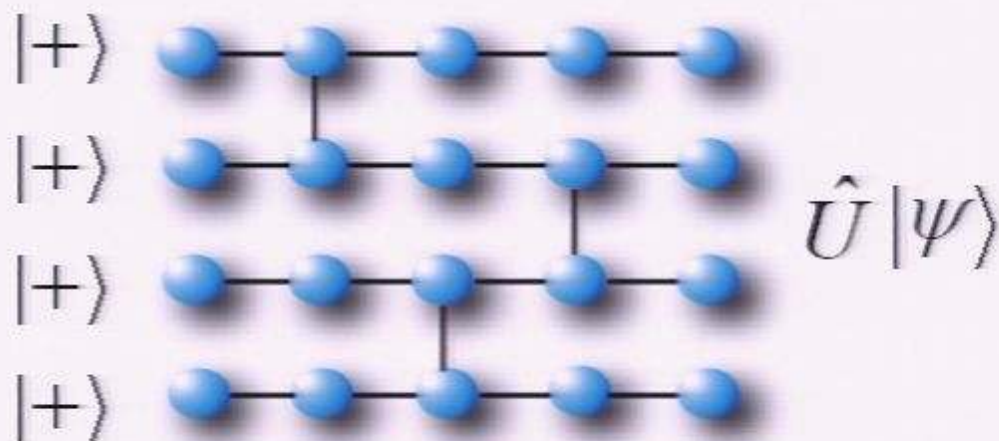
Entanglement: a universal resource

Universal Quantum Computation

“Circuit Model”



“One-Way Quantum Computation”



Quantum Optics

Why Optics?

- ✓ light is a convenient medium for quantum communication
- ✓ light is useful for efficient, small-scale quantum logic / computation

Quantum Optics

... describe electromagnetic field by a **discrete set** of “mode variables” rather than the whole continuum of frequencies

$$\hat{\mathbf{E}}(\mathbf{r}, t) = i \sum_{\mathbf{k}} \left(\frac{\hbar \omega_{\mathbf{k}}}{2\epsilon_0} \right)^{1/2} \left(\hat{a}_{\mathbf{k}} \mathbf{u}_{\mathbf{k}}(\mathbf{r}) e^{-i\omega_{\mathbf{k}} t} - \hat{a}_{\mathbf{k}}^\dagger \mathbf{u}_{\mathbf{k}}^*(\mathbf{r}) e^{+i\omega_{\mathbf{k}} t} \right)$$

$$\hat{\mathbf{B}}(\mathbf{r}, t) = i \sum_{\mathbf{k}} \left(\frac{\hbar}{2\omega_{\mathbf{k}} \epsilon_0} \right)^{1/2} \left(\hat{a}_{\mathbf{k}} \mathbf{k} \times \mathbf{u}_{\mathbf{k}}(\mathbf{r}) e^{-i\omega_{\mathbf{k}} t} - \hat{a}_{\mathbf{k}}^\dagger \mathbf{k} \times \mathbf{u}_{\mathbf{k}}^*(\mathbf{r}) e^{+i\omega_{\mathbf{k}} t} \right)$$

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{l}}^\dagger] = \delta_{\mathbf{k}\mathbf{l}}$$

Quantum Optics

$$\hat{H} = \frac{1}{2} \int d^3 r \left(\epsilon_0 \hat{E}^2 + \mu_0^{-1} \hat{B}^2 \right)$$

$$= \frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \left(\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger \right)$$

$$= \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \left(\underbrace{\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}}_{\text{photon number in mode } \mathbf{k}, \text{ excitation number of quantum harmonic oscillator}} + 1/2 \right)$$

*photon number in mode \mathbf{k} ,
excitation number of
quantum harmonic oscillator*

$$\boxed{[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{l}}^\dagger] = \delta_{\mathbf{k}, \mathbf{l}}}$$

Quantum Optics

discrete number/Fock states:

$$\hat{a}_k |n_k\rangle = \sqrt{n_k} |n_k - 1\rangle$$

$$\hat{n}_k |n_k\rangle = n_k |n_k\rangle$$

$$\hat{a}_k^\dagger |n_k\rangle = \sqrt{n_k + 1} |n_k + 1\rangle$$

$$\hat{a}_k |0\rangle = 0 \quad \text{vacuum state}$$

Fock basis:

$$\sum_{n_k=0}^{\infty} |n_k\rangle \langle n_k| = I_k$$

$$\langle n_k | m_k \rangle = \delta_{n_k m_k}$$

Quantum Optics

$$\hat{H}_k = \hbar\omega_k (\hat{a}_k^\dagger \hat{a}_k + 1/2)$$

$$= \left(\hat{p}_k^2 + \omega_k^2 \hat{x}_k^2 \right) / 2$$

unit mass *momentum and position of harmonic oscillator*

$$\hat{x}_k = \sqrt{\hbar/2\omega_k} (\hat{a}_k + \hat{a}_k^\dagger), \quad \hat{p}_k = -i\sqrt{\hbar\omega_k/2} (\hat{a}_k - \hat{a}_k^\dagger)$$

$$[\hat{a}_k, \hat{a}_l^\dagger] = \delta_{kl}$$

$$[\hat{x}_k, \hat{p}_l] = i\hbar\delta_{kl}$$

Quantum Optics

$$\hat{x}_k \rightarrow \sqrt{\omega_k/2\hbar} \hat{x}_k = \text{Re } \hat{a}_k, \quad \hat{p}_k \rightarrow \sqrt{1/2\hbar\omega_k} \hat{p}_k = \text{Im } \hat{a}_k$$

continuous position/momentum states:

$$\hat{x}_k |x_k\rangle = x_k |x_k\rangle, \quad \hat{p}_k |p_k\rangle = p_k |p_k\rangle$$

$$\langle 0 | \hat{x}_k^2 | 0 \rangle = 1/4, \quad \langle 0 | \hat{p}_k^2 | 0 \rangle = 1/4 \quad \text{vacuum state}$$

position/momentum basis: $\langle x | x' \rangle = \delta(x - x')$, $\langle p | p' \rangle = \delta(p - p')$

$$\int dx |x\rangle \langle x| = I, \quad \int dp |p\rangle \langle p| = I$$

Quantum Optics

“continuum” of quadrature amplitudes:

$$\begin{pmatrix} \hat{x}_k^{(\theta)} \\ \hat{p}_k^{(\theta)} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{x}_k \\ \hat{p}_k \end{pmatrix}$$

$$\hat{x}_k^{(\theta)} = (\hat{a}_k e^{-i\theta} + \hat{a}_k^\dagger e^{i\theta})/2$$

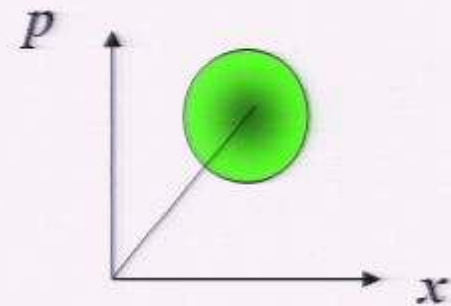
$$\hat{p}_k^{(\theta)} = (\hat{a}_k e^{-i\theta} - \hat{a}_k^\dagger e^{i\theta})/2i = \hat{x}_k^{(\theta + \pi/2)}$$

$$[\hat{x}_k^{(\theta)}, \hat{p}_l^{(\theta)}] = (i/2) \delta_{kl} \quad \hat{x}_k = \hat{x}_k^{(\theta=0)}, \quad \hat{p}_k = \hat{x}_k^{(\theta=\pi/2)}$$

Quantum Optics

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle$$

$$|\alpha\rangle = \hat{D}(\alpha) |0\rangle$$

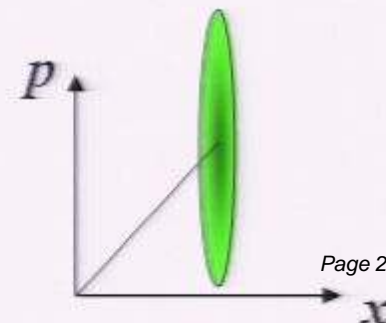


with $\hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$

$$|\alpha\rangle = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Squeezed States:

$$\hat{x} = e^{-r} \hat{x}^{(0)}, \quad \hat{p} = e^{+r} \hat{p}^{(0)}$$



Quantum Optics

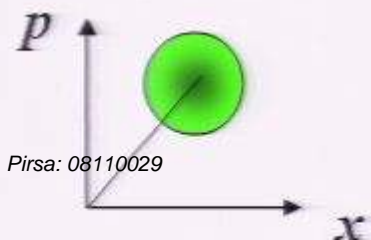
Wigner function:

$$W(x, p) = \frac{2}{\pi} \int dy e^{4iyp} \langle x-y | \hat{\rho} | x+y \rangle$$

Gaussian States:

$$W(\xi) = \frac{1}{(2\pi) \sqrt{\det V}} \exp(-\xi V^{-1} \xi^T / 2)$$

2nd moment correlation matrix



$$W(x, p) = \frac{2}{\pi} \exp[-2(x-x_0)^2 - 2(p-p_0)^2]$$

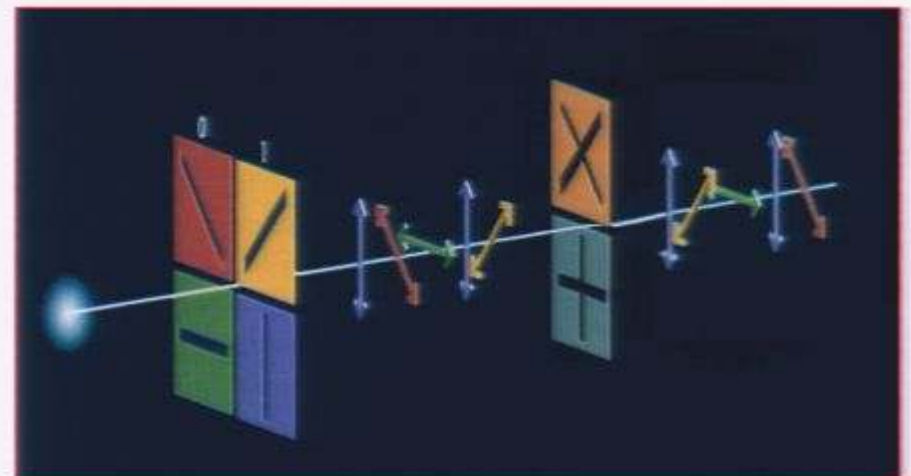
Discrete Variables (DV)

occupation number qubit ... $\alpha |0\rangle + \beta |1\rangle$ (one mode)

dual-rail qubit ... $\alpha |10\rangle + \beta |01\rangle$ (two modes)

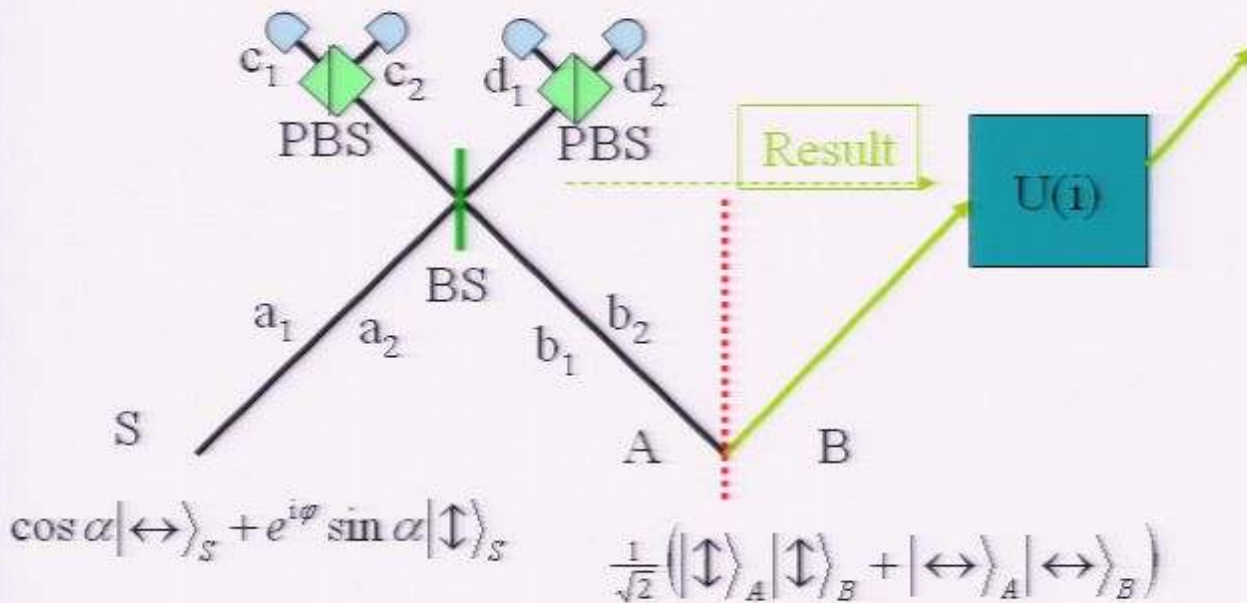
e.g., polarization of a single photon ...

$$\alpha |\leftrightarrow\rangle + \beta |\updownarrow\rangle$$



Quantum Teleportation (DV)

Innsbruck Experiment, 1997 (Zeilinger)



$$\begin{pmatrix} c_1 \\ d_1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} c_2 \\ d_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

Analysis:

$$\left. \begin{aligned} \frac{1}{\sqrt{2}} (|\updown\rangle_S |\leftrightarrow\rangle_A + |\leftrightarrow\rangle_S |\updown\rangle_A) &= \frac{1}{\sqrt{2}} (a_1^+ b_2^+ + a_2^+ b_1^+) |0\rangle \rightarrow \frac{1}{\sqrt{2}} (c_1^+ c_2^+ - d_1^+ d_2^+) |0\rangle \\ \frac{1}{\sqrt{2}} (|\updown\rangle_S |\leftrightarrow\rangle_A - |\leftrightarrow\rangle_S |\updown\rangle_A) &= \frac{1}{\sqrt{2}} (a_1^+ b_2^+ - a_2^+ b_1^+) |0\rangle \rightarrow \frac{1}{\sqrt{2}} (d_1^+ c_2^+ - c_1^+ d_2^+) |0\rangle \end{aligned} \right\} \begin{array}{l} \text{distinct click pattern} \\ \text{(two separate clicks)} \end{array}$$

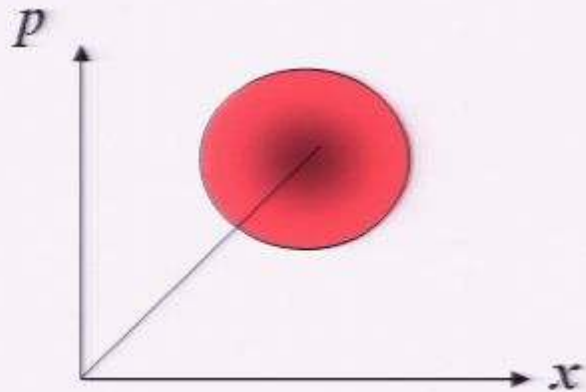
$$\frac{1}{\sqrt{2}} (|\updown\rangle_S |\updown\rangle_A \pm |\leftrightarrow\rangle_S |\leftrightarrow\rangle_A) = \frac{1}{\sqrt{2}} (a_1^+ b_1^+ \pm a_2^+ b_2^+) |0\rangle \rightarrow \frac{1}{2\sqrt{2}} \left\{ (c_1^+)^2 - (d_1^+)^2 \pm [(c_2^+)^2 - (d_2^+)^2] \right\} |0\rangle$$

undistinguishable click pattern \rightarrow 50% success rate

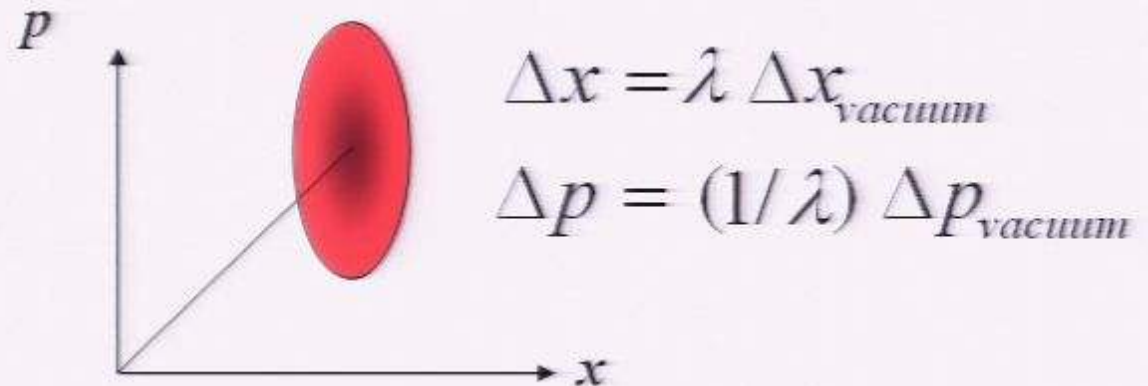
Continuous Variables (CV)

$$\hat{a} = \hat{x} + i\hat{p}, \quad [\hat{x}, \hat{p}] = i/2, \quad \Delta x \Delta p \geq 1/16$$

coherent state

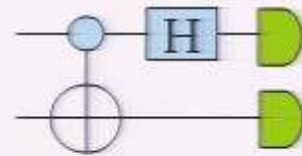


squeezed state



Quantum Teleportation (CV)

Hadamard \rightarrow Fourier transform:

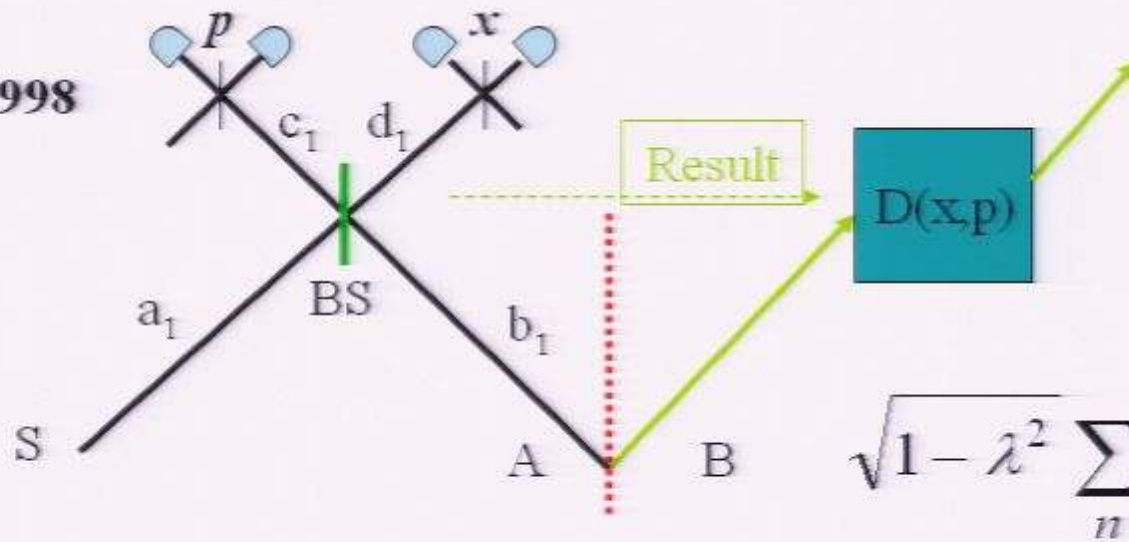


$$F |p\rangle_{\text{momentum}} = \frac{1}{\sqrt{\pi}} \int dy e^{2ipy} |y\rangle_{\text{momentum}} = |x=p\rangle_{\text{position}}$$

CNOT \rightarrow beam splitter:

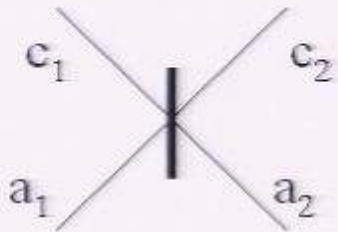
$$|x\rangle |y\rangle \rightarrow \left| \frac{1}{\sqrt{2}}(x+y) \right\rangle \left| \frac{1}{\sqrt{2}}(x-y) \right\rangle$$

Caltech Experiment, 1998
(Kimble)



Linear Optics

beam splitter



$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} r & t \\ t & -r \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{aligned} |r|^2 + |t|^2 &= 1 \\ rt^* - r^*t &= 0 \end{aligned}$$

phase shifter

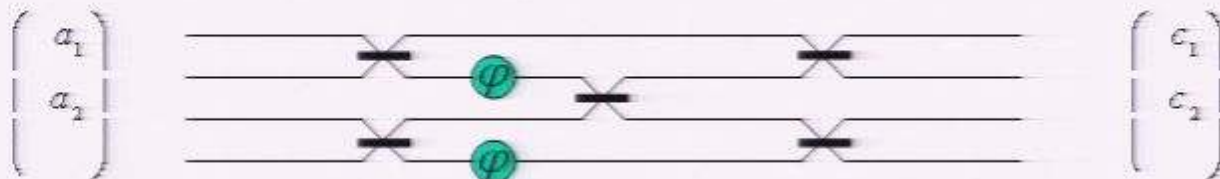


$$c = e^{i\varphi} a$$

$$\vec{c} = U \vec{a}$$

U unitary

linear network



Any U can be realized ... M. Reck *et al.*, PRL **73**, 58 (1994)

Linear Optics

beam splitter



$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} r & t \\ t & -r \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{aligned} |r|^2 + |t|^2 &= 1 \\ r t^* - r^* t &= 0 \end{aligned}$$

phase shifter

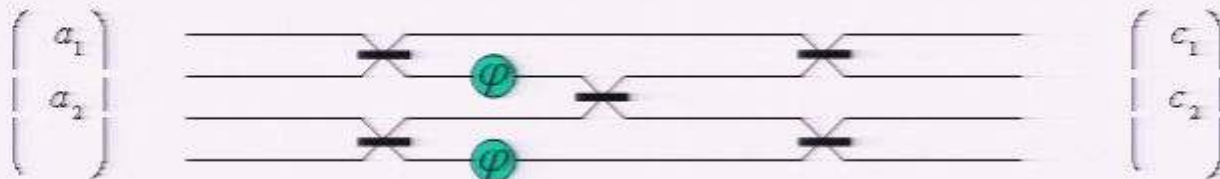


$$c = e^{i\varphi} a$$

**any quadratic interaction
including squeezing:**

$$\vec{c} = A\vec{a} + B\vec{a}^\dagger + \vec{\gamma}$$

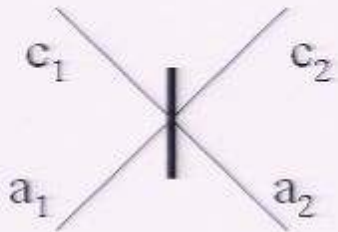
linear network



Any U can be realized ... M. Reck *et al.*, PRL **73**, 58 (1994)

Linear Optics

beam splitter



$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} r & t \\ t & -r \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{aligned} |r|^2 + |t|^2 &= 1 \\ r t^* - r^* t &= 0 \end{aligned}$$

phase shifter

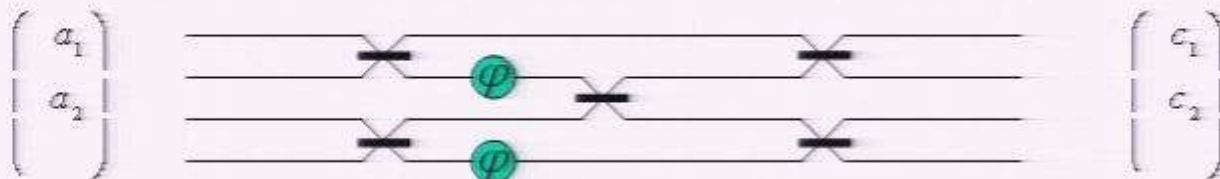


$$c = e^{i\varphi} a$$

any quadratic interaction
including squeezing:

$$\vec{c} = A\vec{a} + B\vec{a}^\dagger + \vec{\gamma}$$

linear network



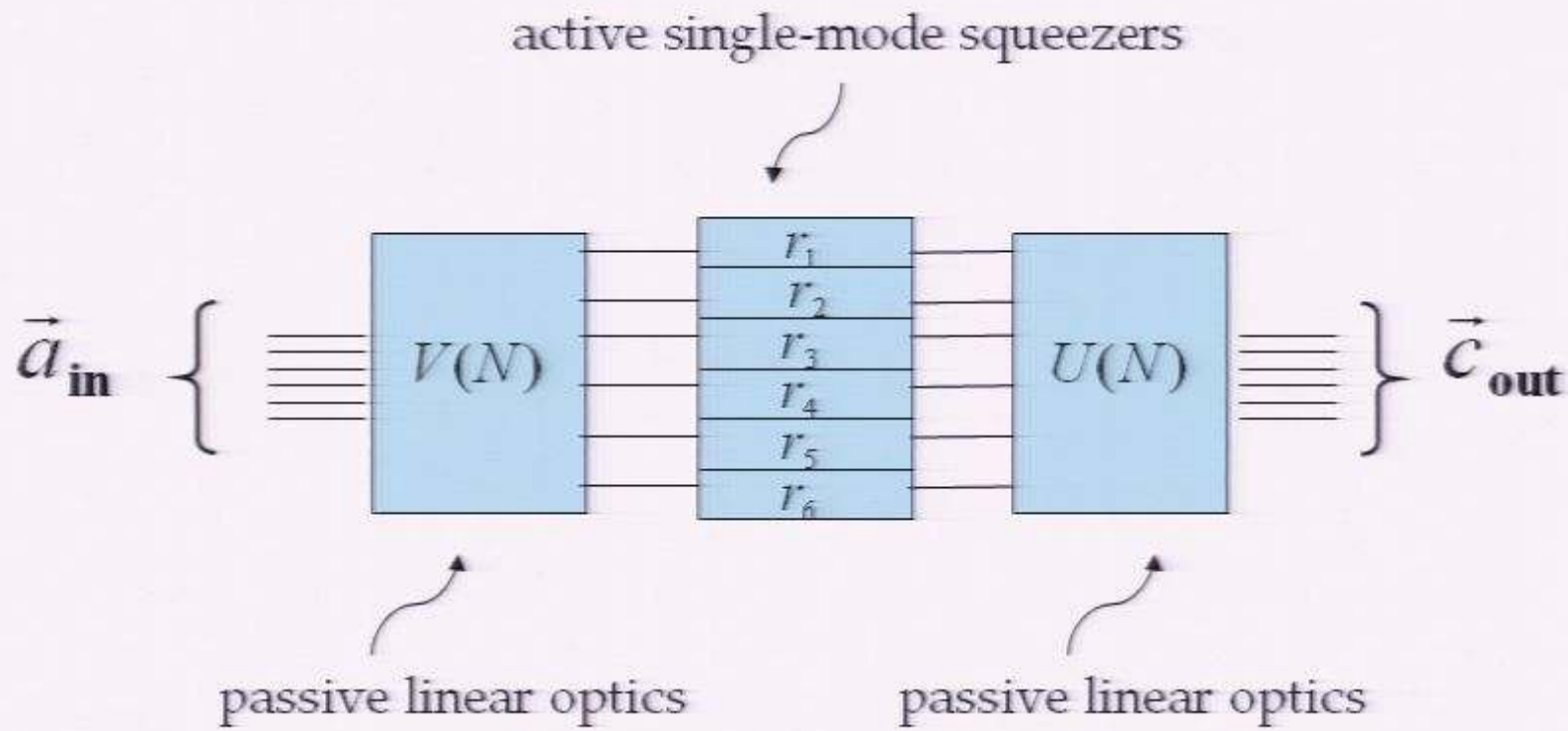
“LUBO”,
Gaussian

transformations

Any U can be realized... M. Reck *et al.*, PRL **73**, 58 (1994)

Linear Optics

any multi-mode Gaussian (“LUBO”) transformation can be decomposed via SVD into



Nonlinear Optics (DV)

Cross Kerr ...

$$\hat{U} = e^{i\pi \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b}}$$

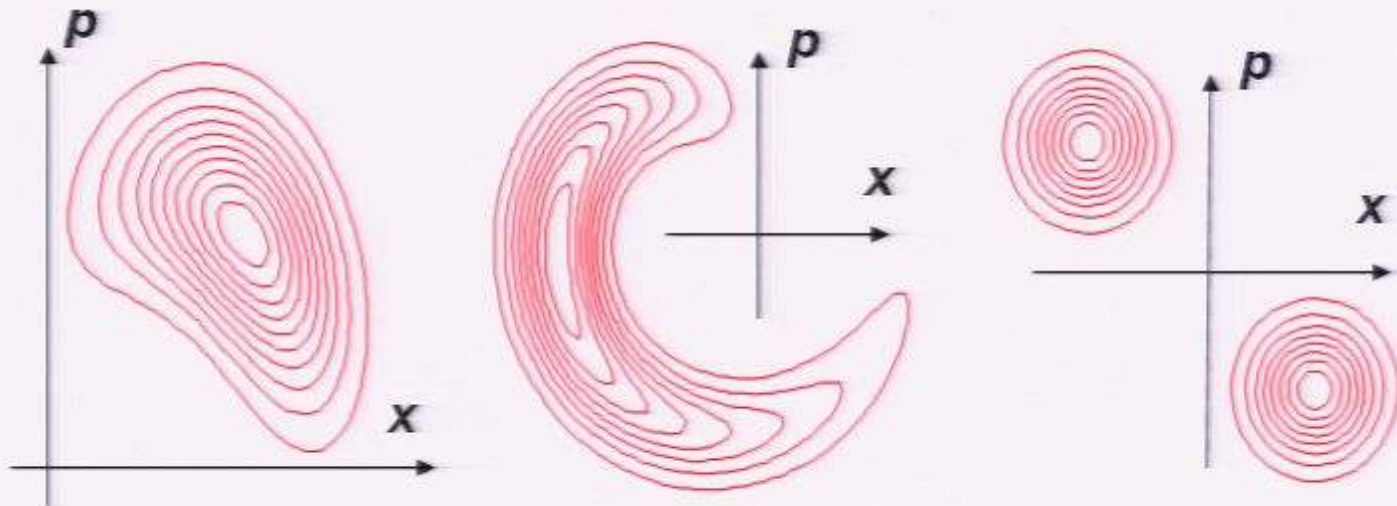
$$|00\rangle \xrightarrow{\hat{U}} |00\rangle, \quad |01\rangle \xrightarrow{\hat{U}} |01\rangle, \quad |10\rangle \xrightarrow{\hat{U}} |10\rangle, \quad |11\rangle \xrightarrow{\hat{U}} -|11\rangle$$

controlled sign gate

...hard to obtain for **single-photon states**

Nonlinear Optics (CV)

non-Gaussian states ...



„Cat States“

...hard to obtain for multi-photon states

Optical Quantum Information Processing

	unitary transformation	interaction (Hamiltonian)	input-output relation of mode operators
Gaussian	displacement in phase space	linear	“scalar”
	beam splitter	quadratic	linear
	squeezing		
	non-Gaussian (cubic phase gate, Kerr effect)	cubic or higher	nonlinear

linear, nonlinear optical interactions

Why Hybrid?

Discrete Variables

linear optics / linear transformations are not sufficient;

NOGO for universal quantum computing, **NOGO** for measurements, ...

Continuous Variables

Gaussian quantum computer can always be efficiently simulated by a classical computer;

NOGO for universal quantum computing, **NOGO** for entanglement distillation and for quantum error correction, ...

Why Hybrid?

Discrete Variables

$$\alpha |\leftrightarrow\rangle + \beta |\updownarrow\rangle$$

linear optics / linear transformations are not sufficient;

NOGO for universal quantum computing, **NOGO** for measurements, ...

Continuous Variables

Gaussian quantum computer can always be efficiently simulated
by a classical computer;

NOGO for universal quantum computing, **NOGO** for entanglement distillation
and for quantum error correction, ...

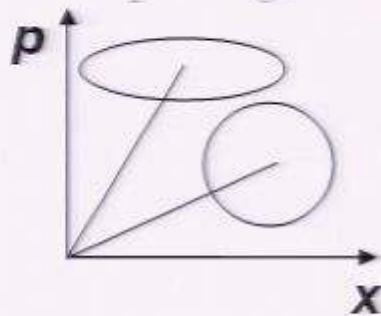
Why Hybrid?

Discrete Variables

$$\alpha |\leftrightarrow\rangle + \beta |\updownarrow\rangle$$

linear optics / linear transformations are not sufficient;

NOGO for universal quantum computing, **NOGO** for measurements, ...



Continuous Variables

Gaussian quantum computer can always be efficiently simulated by a classical computer;

NOGO for universal quantum computing, **NOGO** for entanglement distillation and for quantum error correction, ...

Why Hybrid?

Discrete Variables

linear optics / linear transformations are not sufficient;

NOGO

possible solution: hybrid protocols,
measurement-induced or weak nonlinearities

Continuous Variables

Gaussian quantum computer can always be efficiently simulated
by a classical computer;

NOGO for universal quantum computing, NOGO for entanglement distillation
and for quantum error correction, ...

Hybrid Approaches

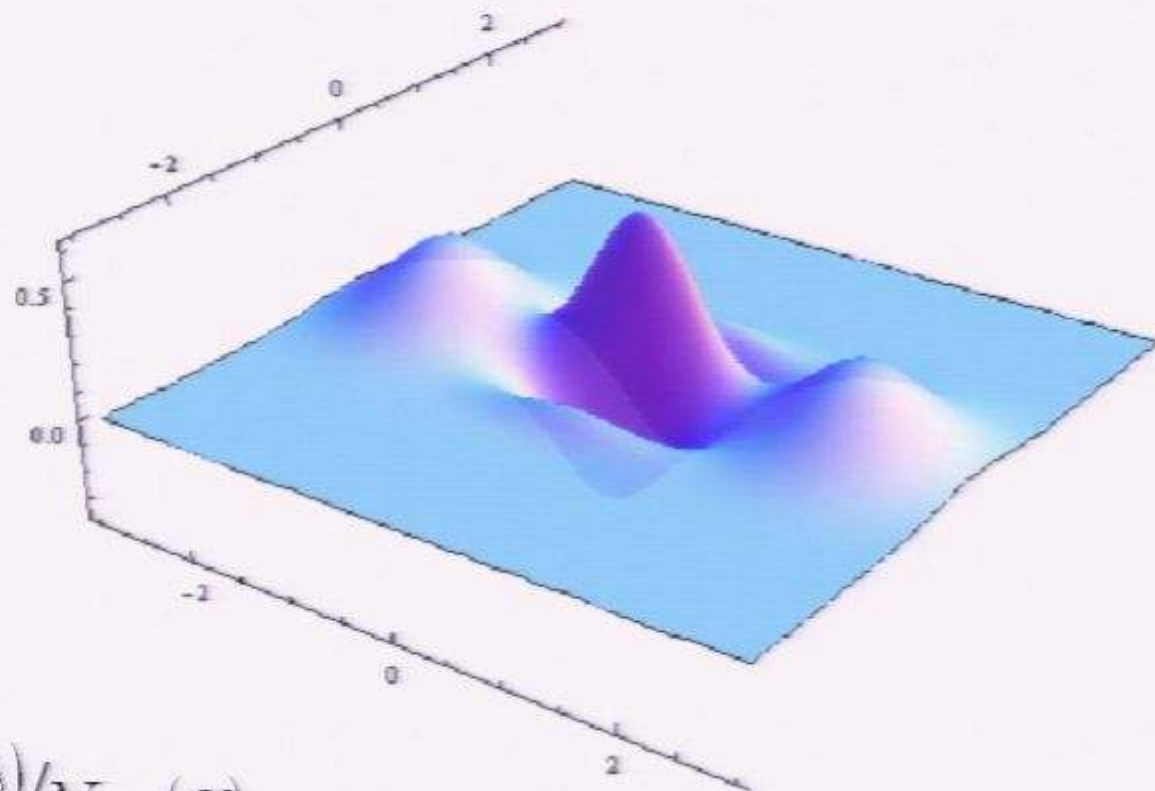
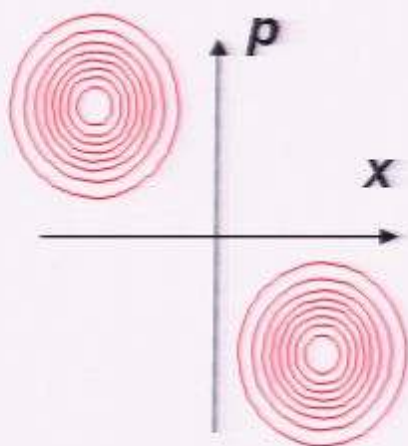
Hybrid...

... combine **different** encodings, operations, and protocols

... minimize **resource costs** for implementations

Hybrid Approaches

- ✓ coherent-state-based qubits (“cat logic”)



$$(\mu|\alpha\rangle + \nu|-\alpha\rangle)/N_{\mu\nu}(\alpha)$$

Hybrid Approaches

- ✓ coherent-state-based qubits (“cat logic”)
- ✓ hybrid quantum error correction... encode discrete qubit into oscillator

$$\begin{array}{l} |\bar{0}\rangle \quad \dots \left| \begin{array}{c} | \\ | \\ | \\ | \\ | \\ | \end{array} \right. \dots \quad \xrightarrow{x} \quad \sum_{s=-\infty}^{+\infty} |x = \Delta s\rangle \\ \\ |\bar{1}\rangle \quad \dots \left| \begin{array}{c} | \\ | \\ | \\ | \\ | \\ | \end{array} \right. \dots \quad \xrightarrow{x} \quad \sum_{s=-\infty}^{+\infty} |x = \Delta s + \Delta/2\rangle \end{array}$$

... useful for **small, diffusive** shifts
rather than stochastic, large errors

Hybrid Approaches

- ✓ coherent-state-based qubits (“cat logic”)
- ✓ hybrid quantum error correction... protect **cv** state against **discrete** errors

$$W_{out}(x, p) = (1-p) W_{in}(x, p) + p W_{error}(x, p)$$

for example, „**erasure channel**“

PvL, [qph-archive/0811.3616](#) (2008)

Gaussian QECC not useful to protect **Gaussian** states against **Gaussian** errors

Shor QECC

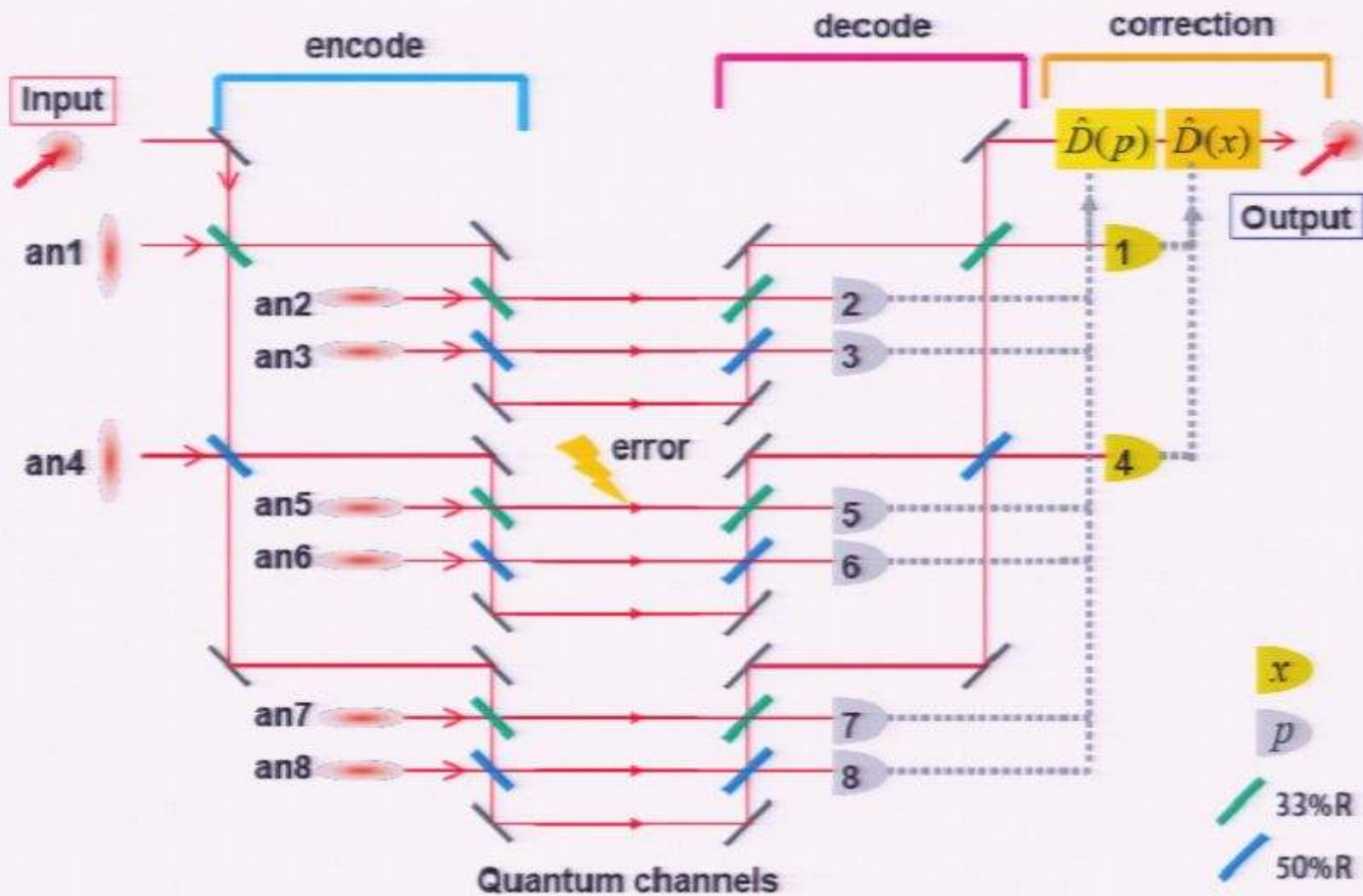
Shor code for **qubits**: $\alpha|++++\rangle + \beta|----\rangle$

with $|\pm\rangle = (|000\rangle \pm |111\rangle) / \sqrt{2}$

Shor-type code for **CV**: $\int dp \psi(p) |p, p, p\rangle$

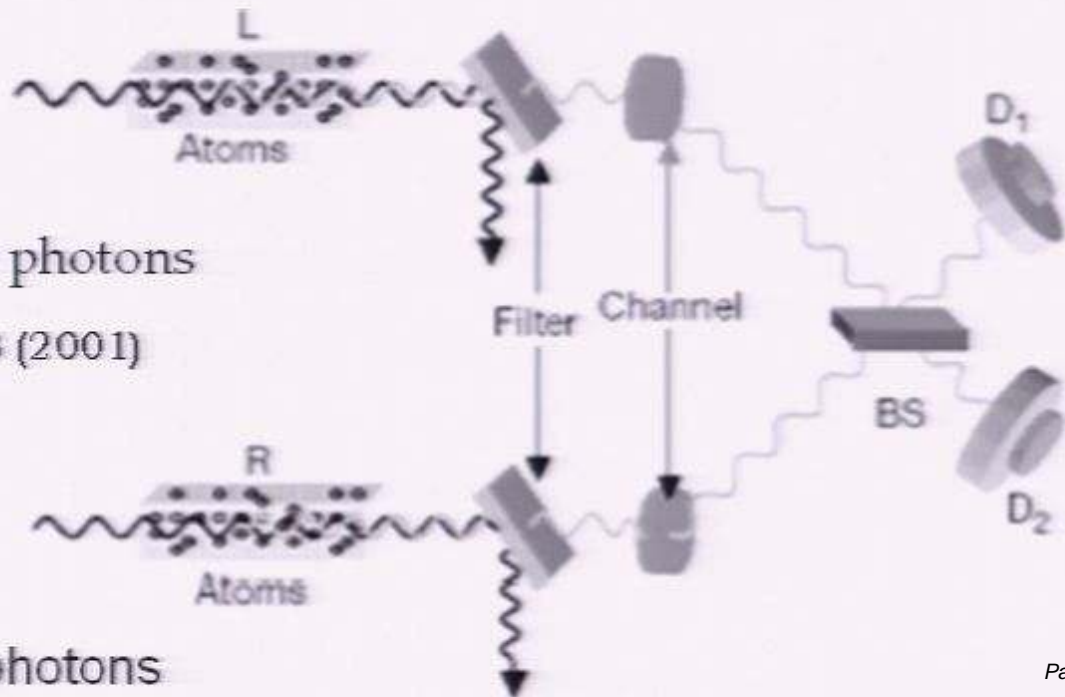
with $|p\rangle \propto \int dx e^{2ixp} |x, x, x\rangle$

CV Quantum Error Correction



Hybrid Approaches

- ✓ coherent-state-based qubits ("cat logic")
- ✓ hybrid quantum error correction... protect cv state against discrete errors
- ✓ light-matter interfaces for quantum communication



fully DV... many atoms/single photons

L. M. Duan *et al.*, Nature **414**, 413 (2001)

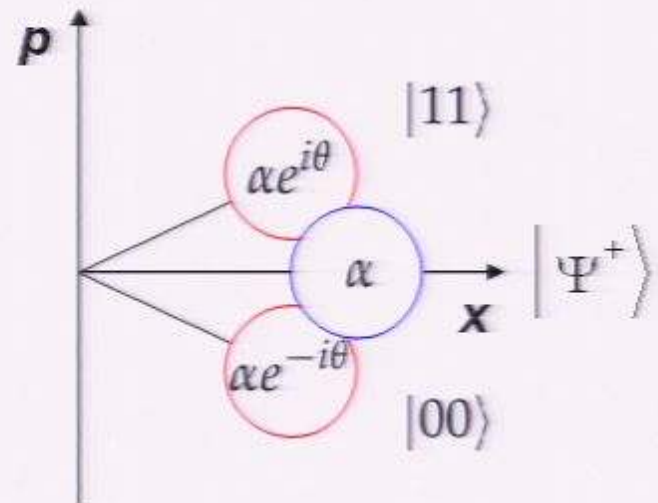
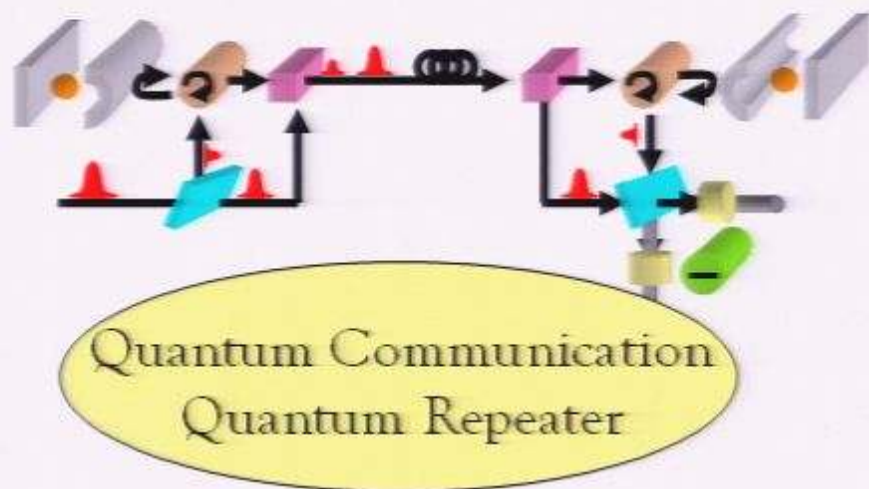
fully CV... many atoms/many photons

J. F. Sherson *et al.*, Nature **443**, 557 (2006)

Hybrid Approaches

- ✓ coherent-state-based qubits (“cat logic”)
- ✓ hybrid quantum error correction... protect cv state against discrete errors
- ✓ light-matter interfaces for quantum communication
- ✓ hybrid quantum communication... continuous qubus for discrete qubits
- ✓ hybrid quantum computation... continuous qubus for discrete qubits
- ✓ hybrid one-way computer... combine discrete/continuous measurements
- ✓ hybrid quantum computation... combine circuit/cluster approaches
- ✓ hybrid quantum communication... combine teleportation/direct transfer

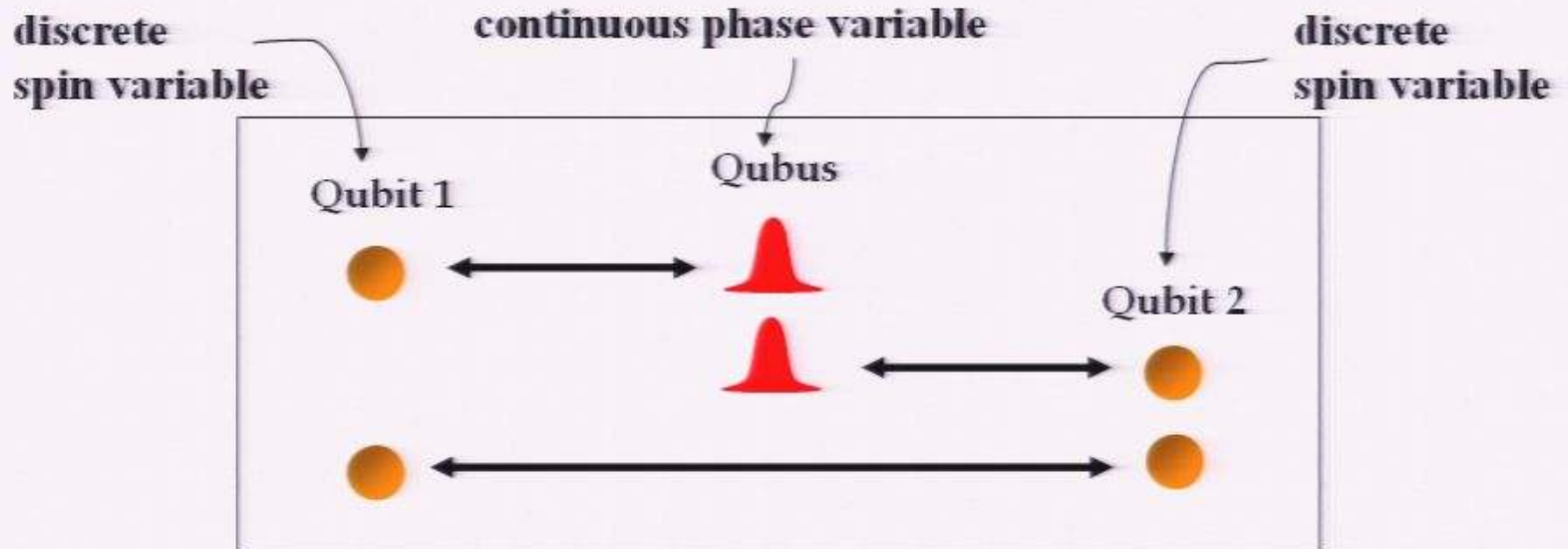
Hybrid Quantum Communication



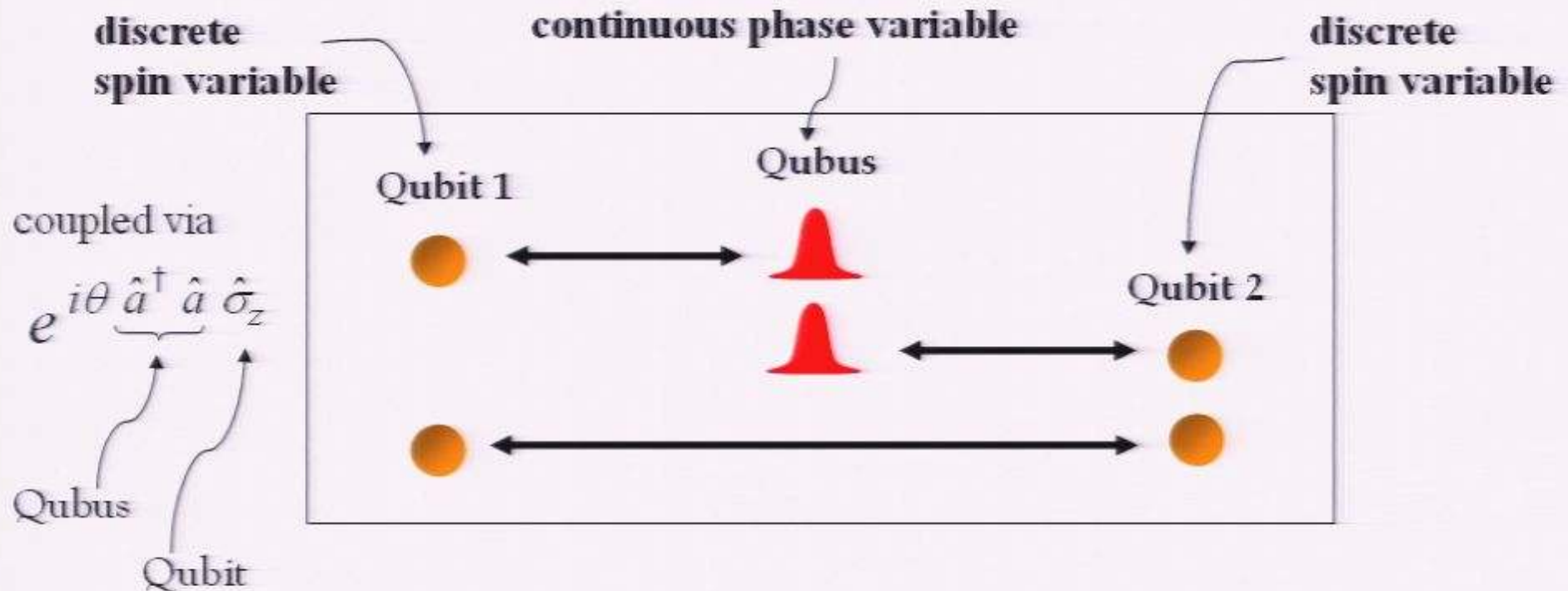
PvL *et al.*, Phys. Rev. Lett. **96**, 240501 (2006)

T. D. Ladd, PvL *et al.*, New J. Phys. **8**, 164 (2006)

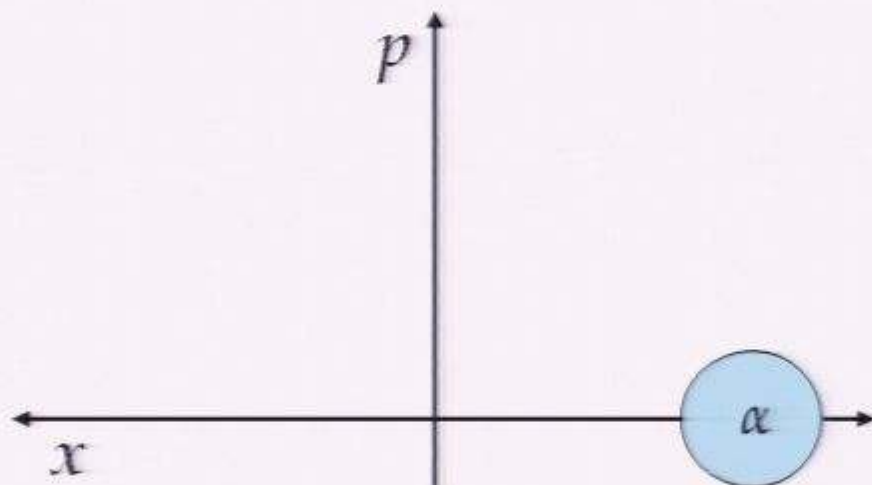
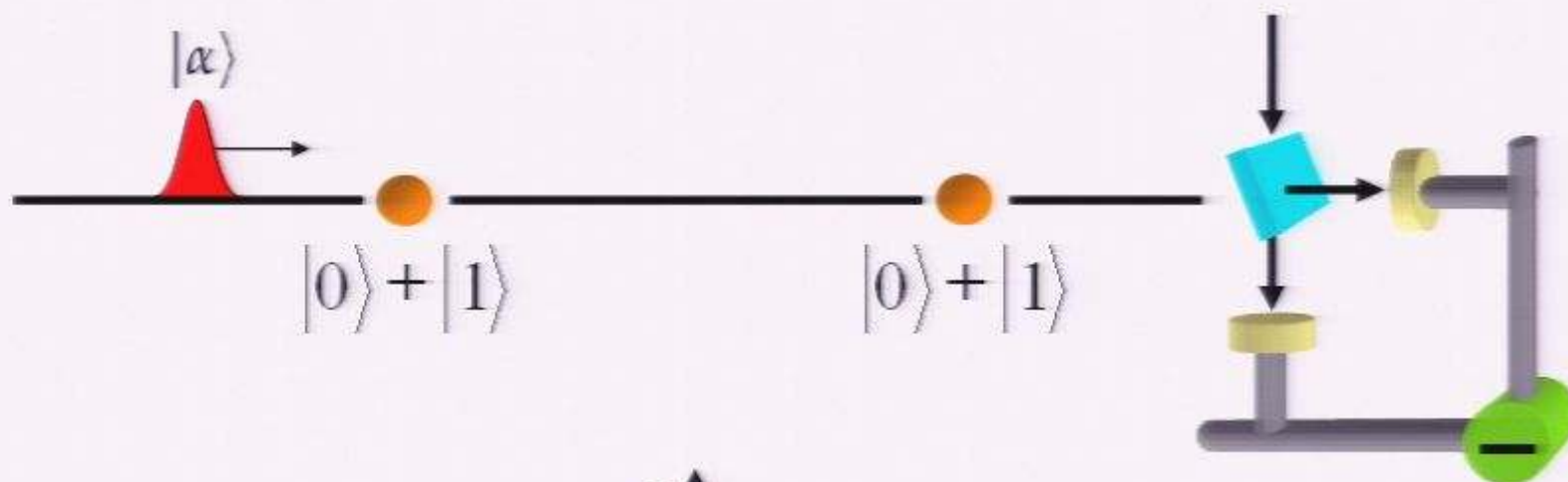
Hybrid Quantum Repeater



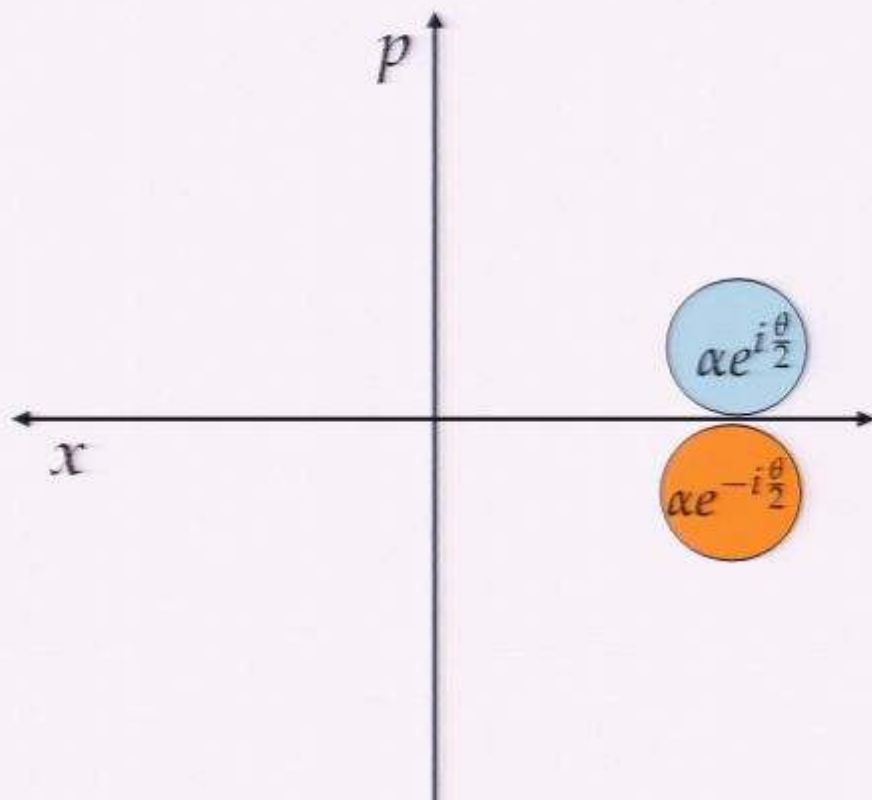
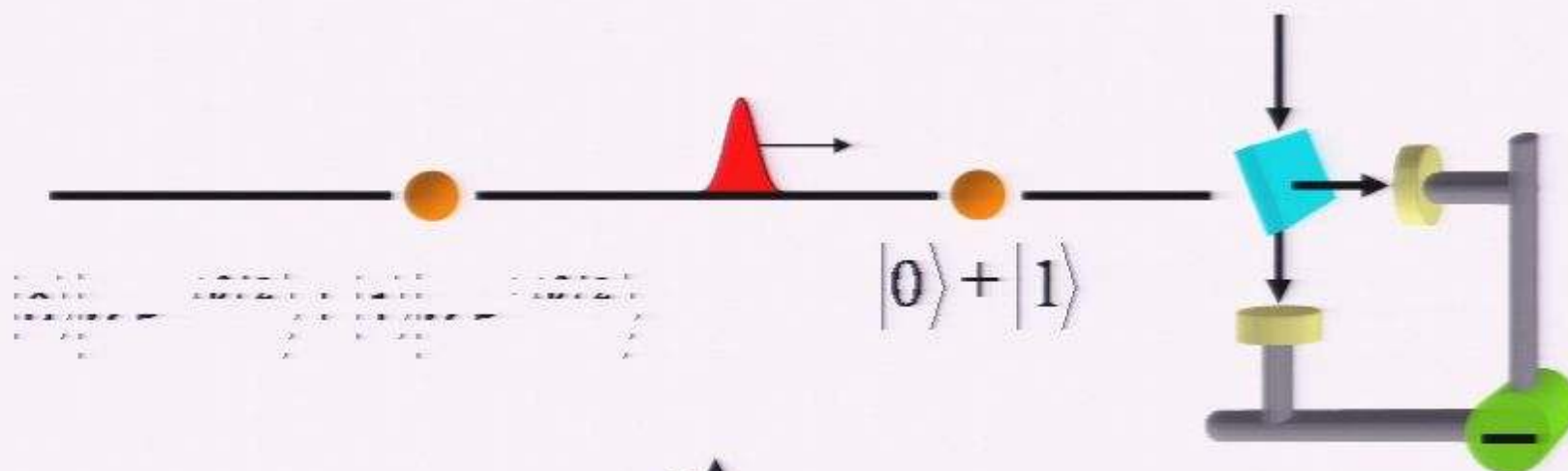
Hybrid Quantum Repeater



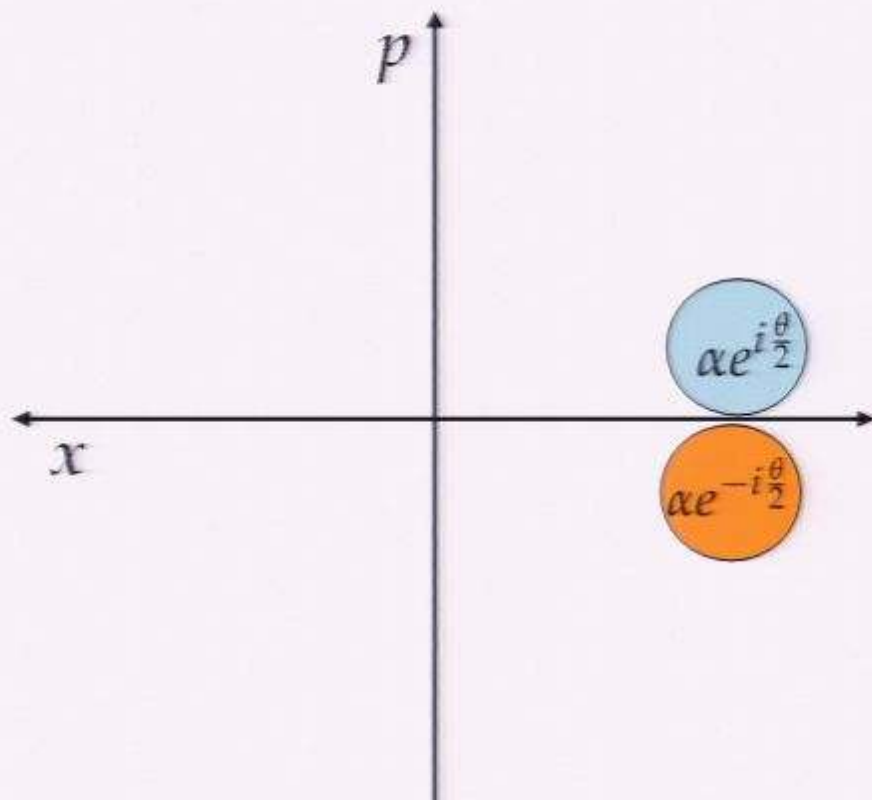
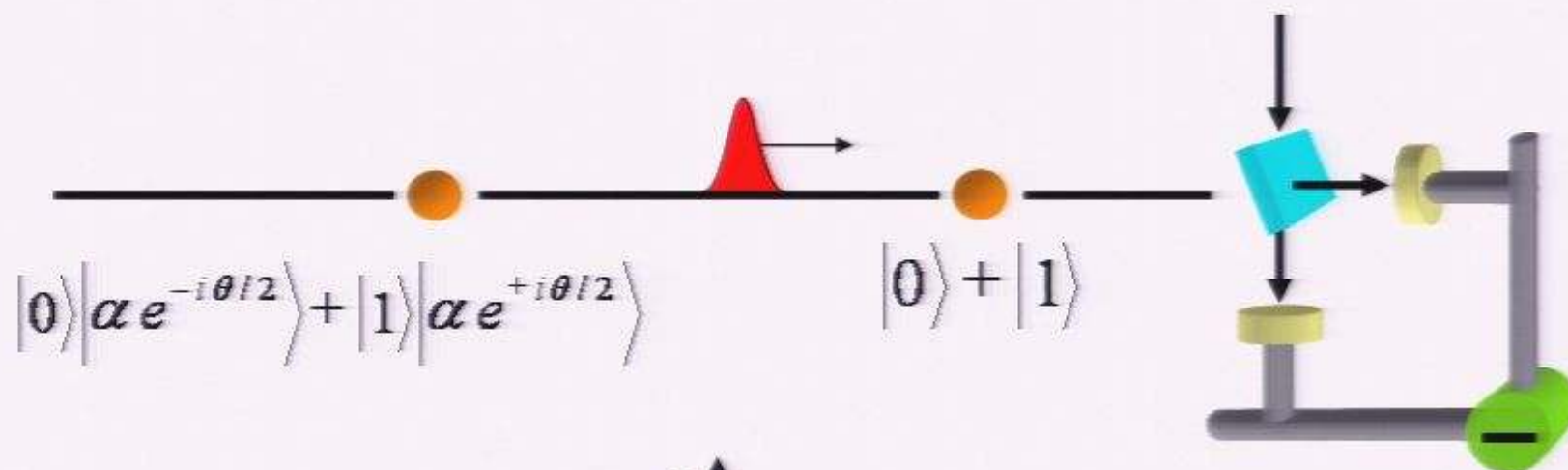
Entanglement Distribution



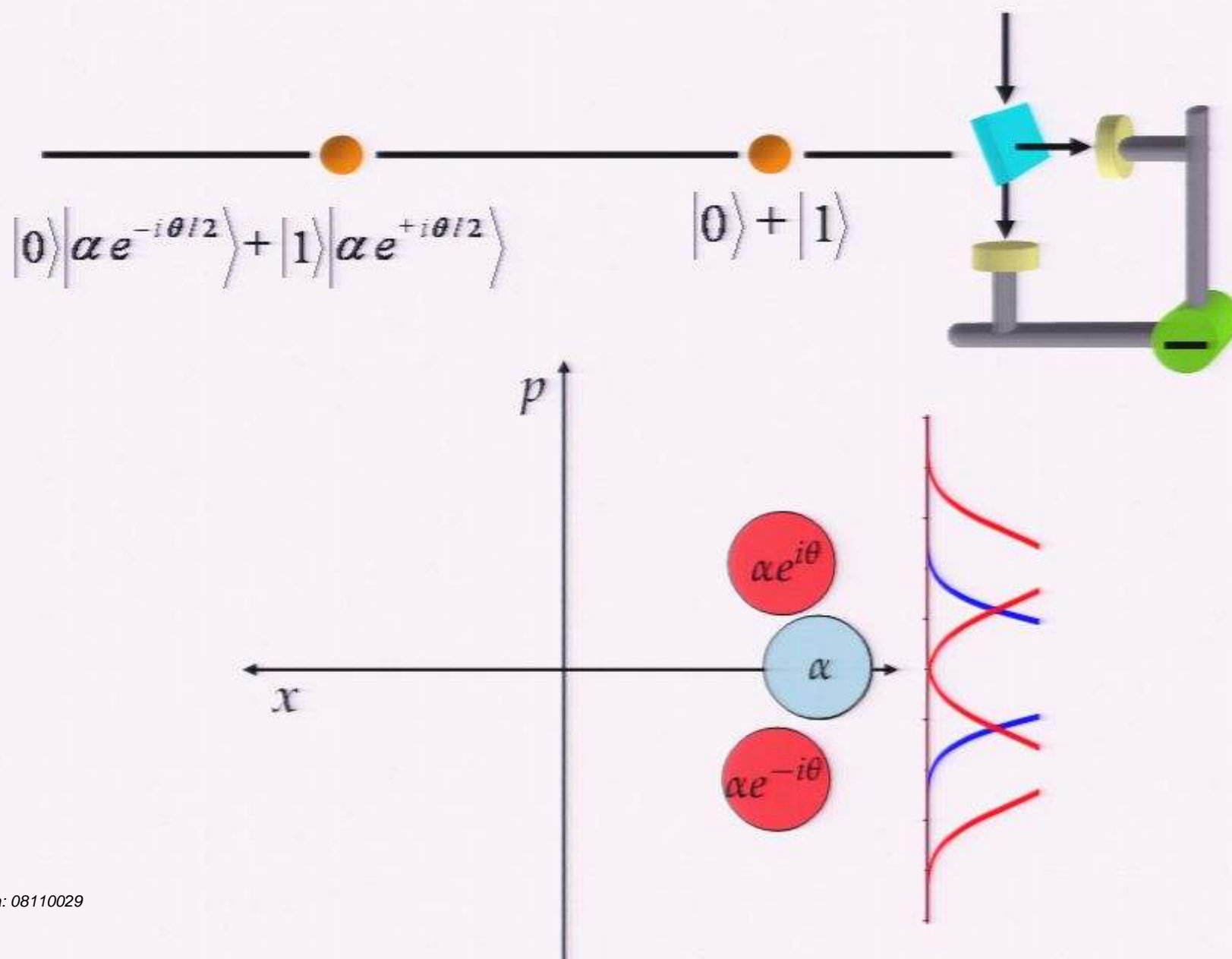
Entanglement Distribution



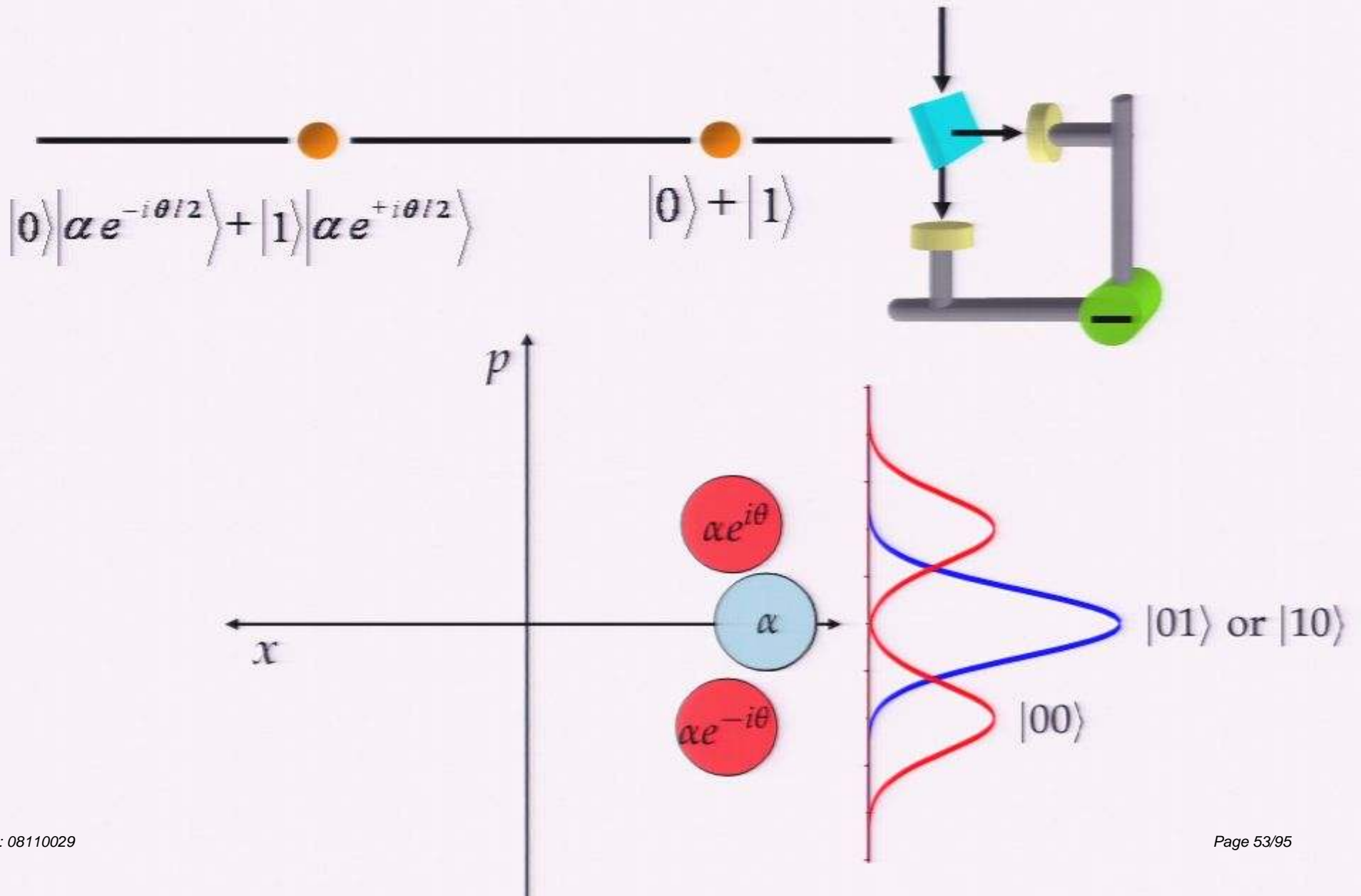
Entanglement Distribution



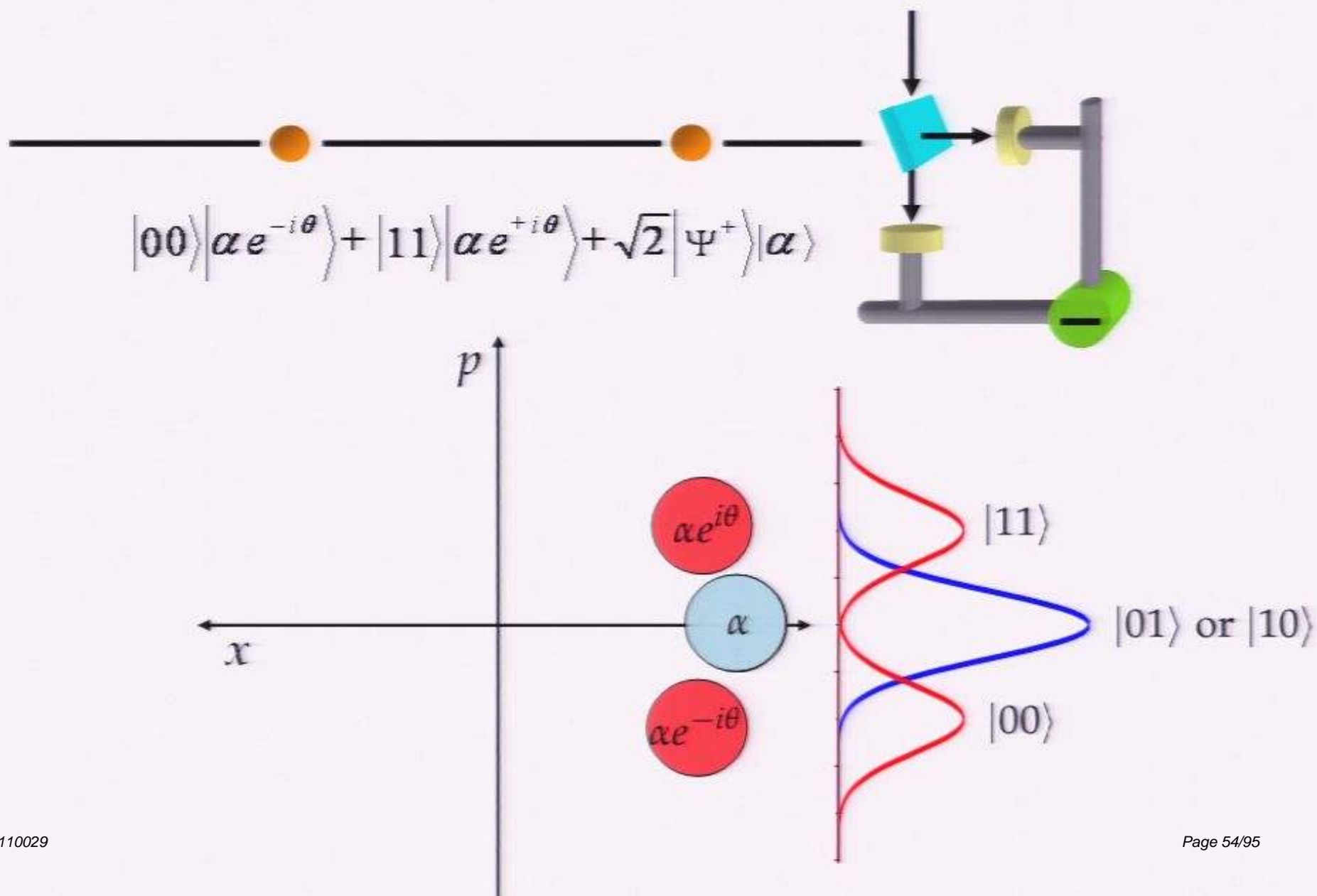
Entanglement Distribution



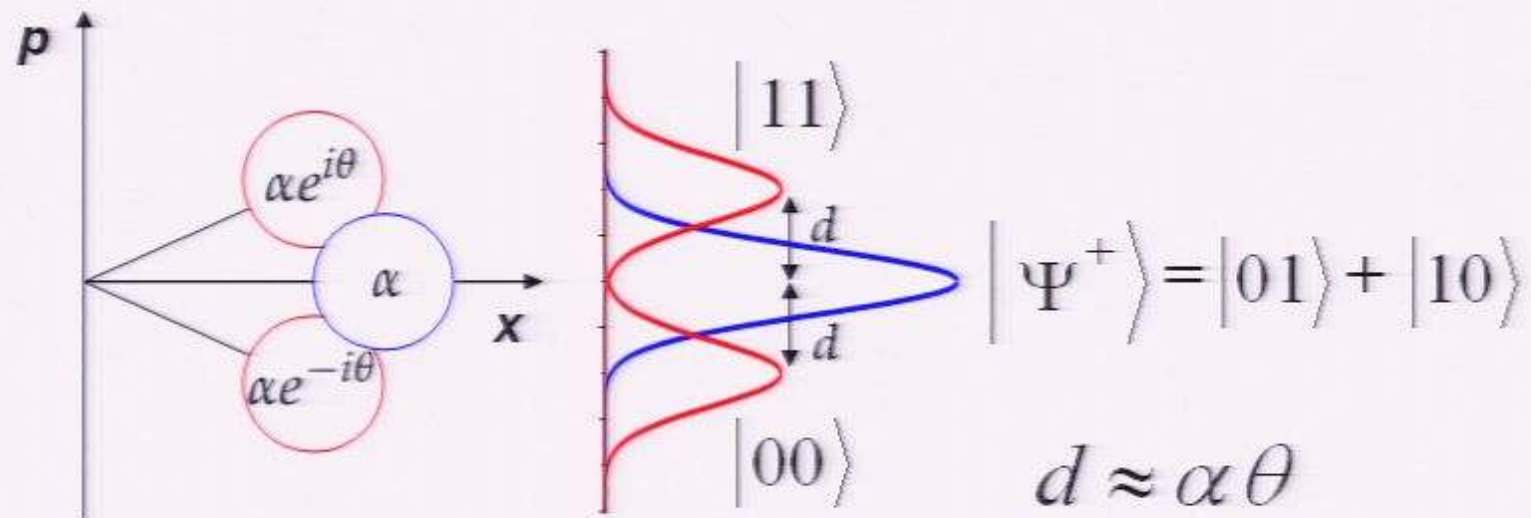
Entanglement Distribution



Entanglement Distribution

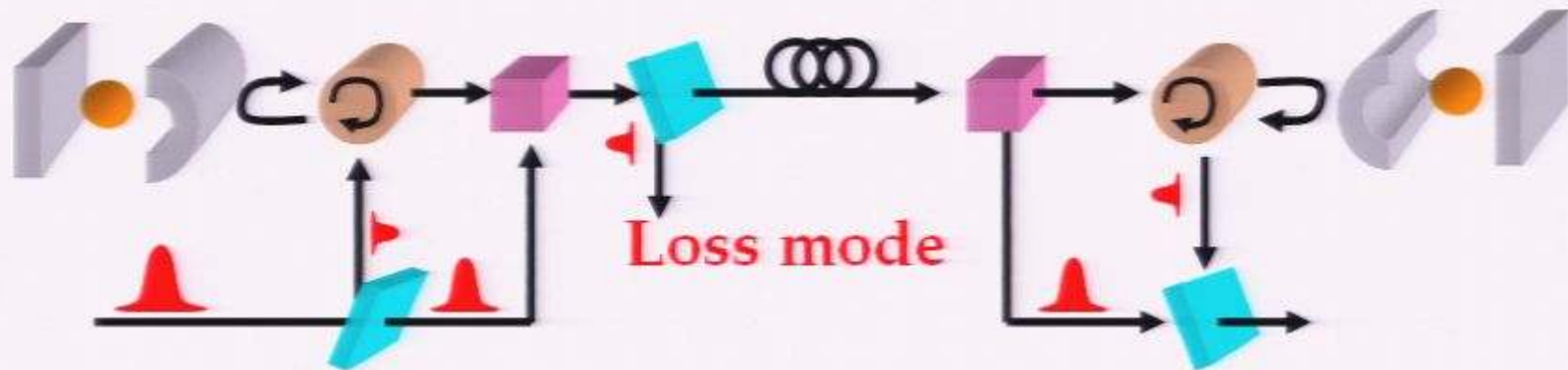


Entanglement Distribution



e.g., homodyne detection
and [postselection](#)

Entanglement Distribution



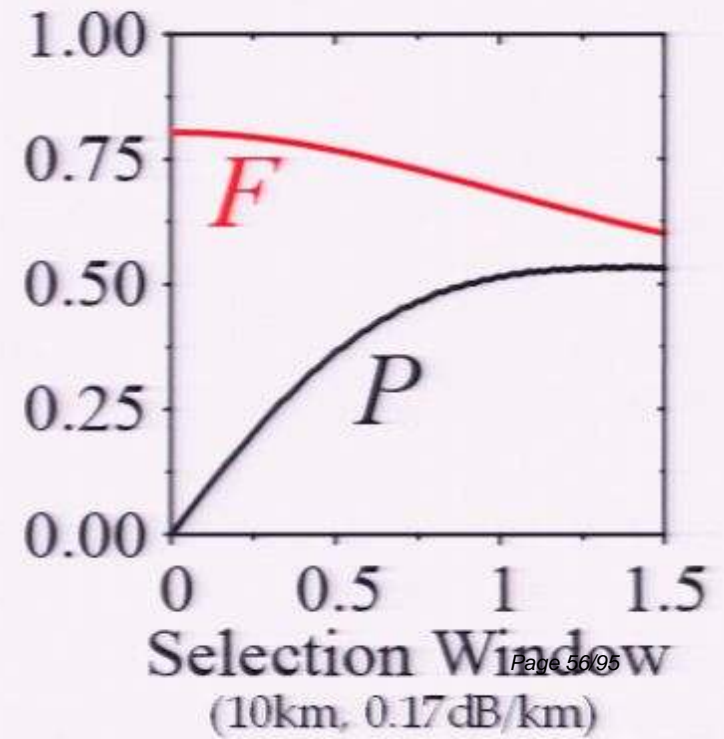
Ideal scheme:
make d arbitrarily large

Realistic scheme:
trade-off between distinguishability
and decoherence

$$d \sim 1, \text{ e.g. } \theta \approx 0.01$$

$$\alpha^2 \approx 10000$$

(probe photons)



Entanglement Distribution

... consider schemes **without** intrinsic errors

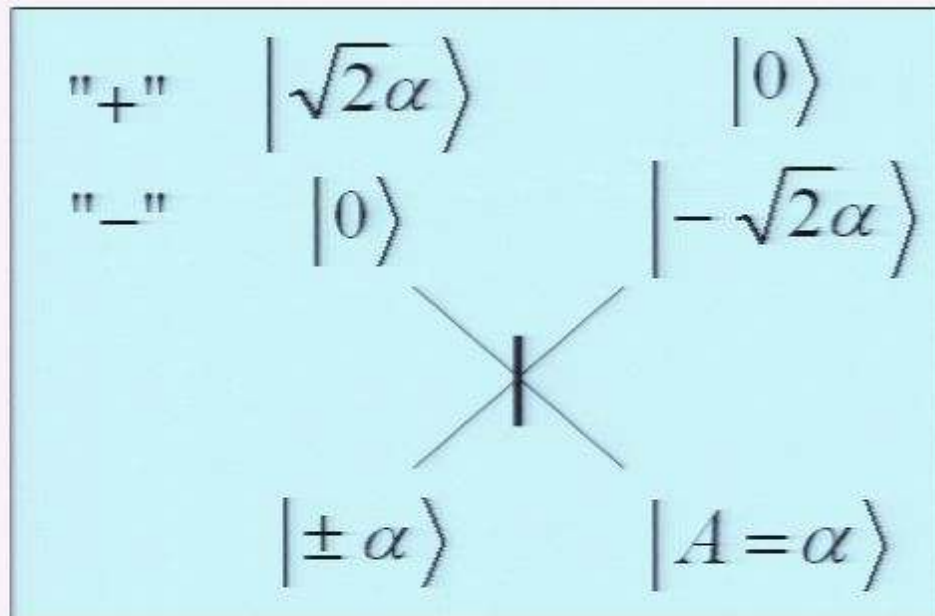
... generate mixtures of only **two** Bell states

$$F |\Phi^+\rangle\langle\Phi^+| + (1-F) |\Psi^+\rangle\langle\Psi^+|$$

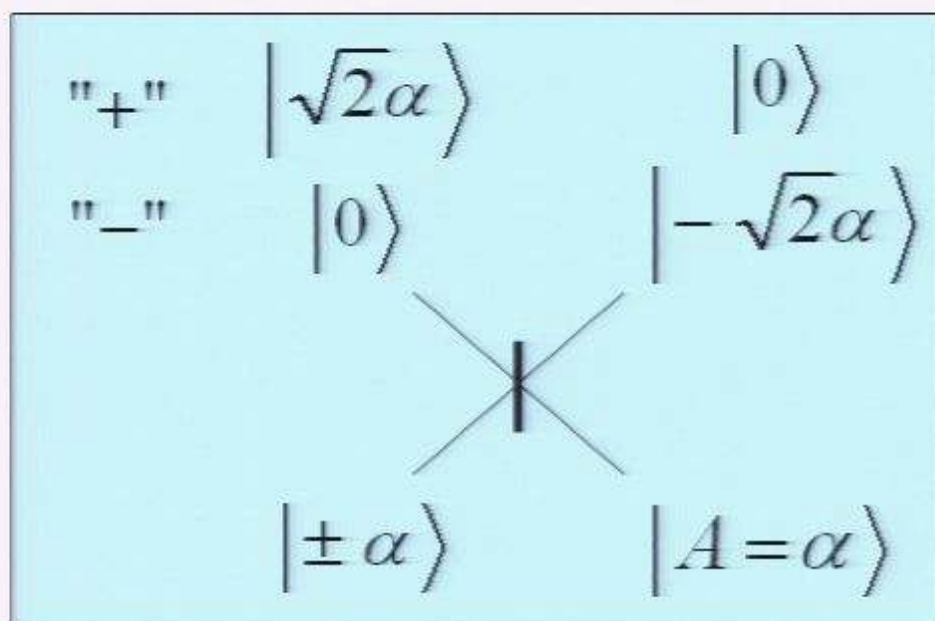
$$|\Phi^+\rangle = \frac{1}{2} \left[|\sqrt{\eta}\alpha\rangle (|00\rangle + |11\rangle) + |\sqrt{\eta}\alpha e^{i\theta}\rangle |10\rangle + |\sqrt{\eta}\alpha e^{-i\theta}\rangle |01\rangle \right]$$

$$|\Psi^+\rangle = \frac{1}{2} \left[|\sqrt{\eta}\alpha\rangle (|00\rangle - |11\rangle) - |\sqrt{\eta}\alpha e^{i\theta}\rangle |10\rangle + |\sqrt{\eta}\alpha e^{-i\theta}\rangle |01\rangle \right]$$

Unambiguous State Discrimination



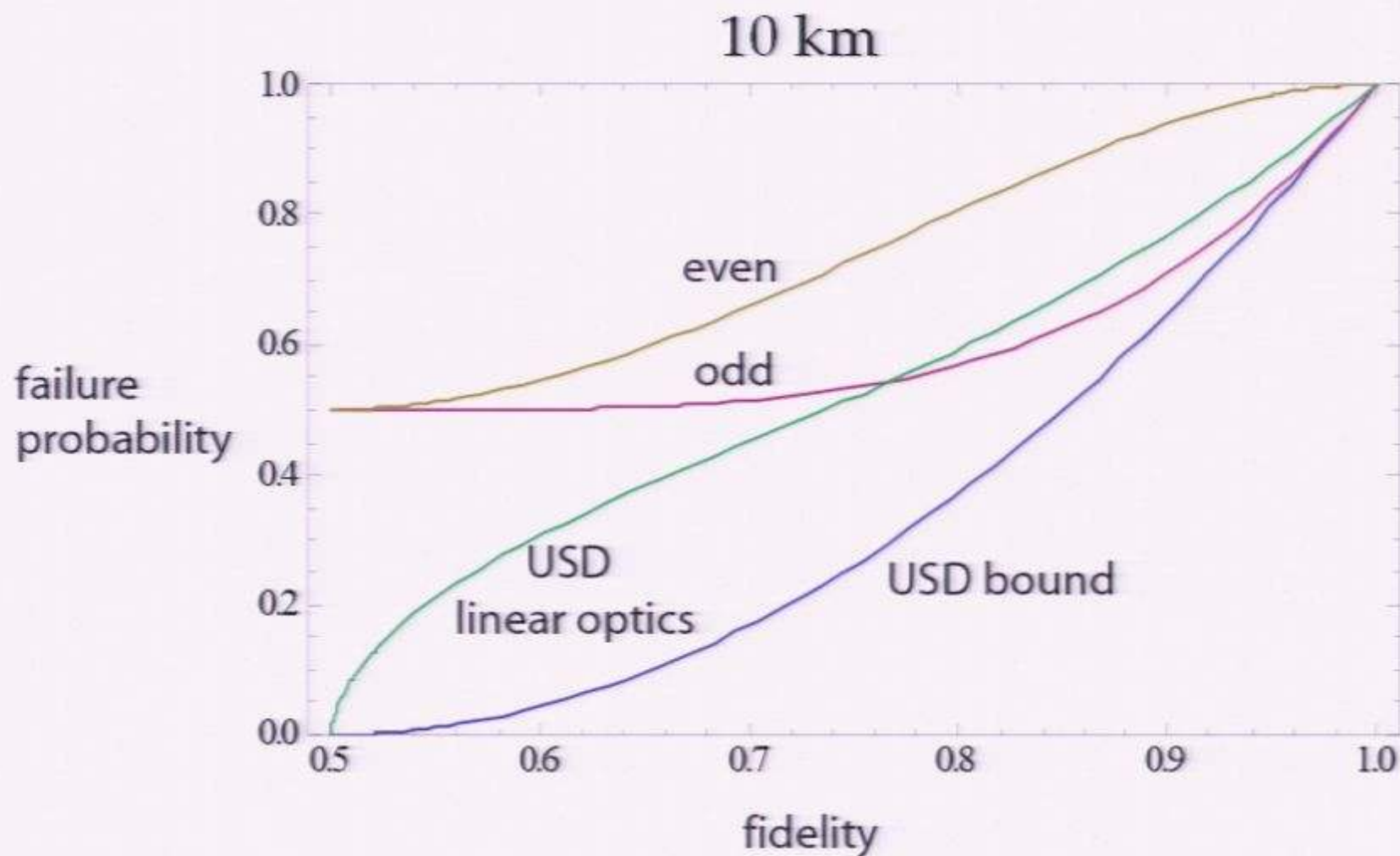
Unambiguous State Discrimination



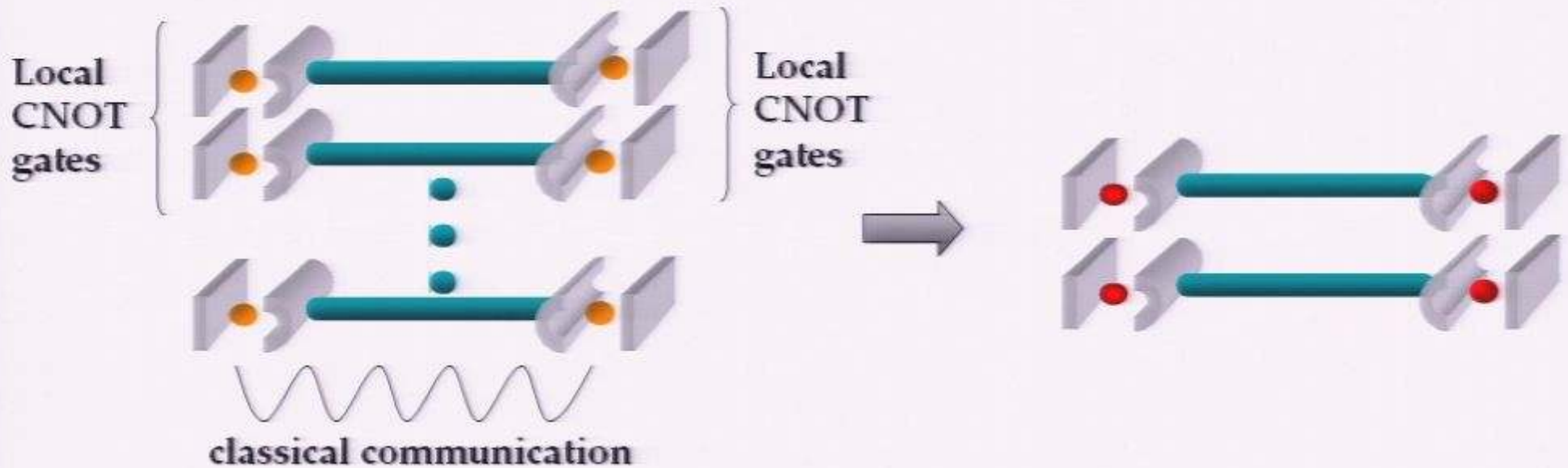
$$\langle \alpha | -\alpha \rangle = e^{-2\alpha^2}$$

$$|\psi^{inconcl}\rangle = e^{-\alpha^2} |00\rangle$$

Unambiguous State Discrimination

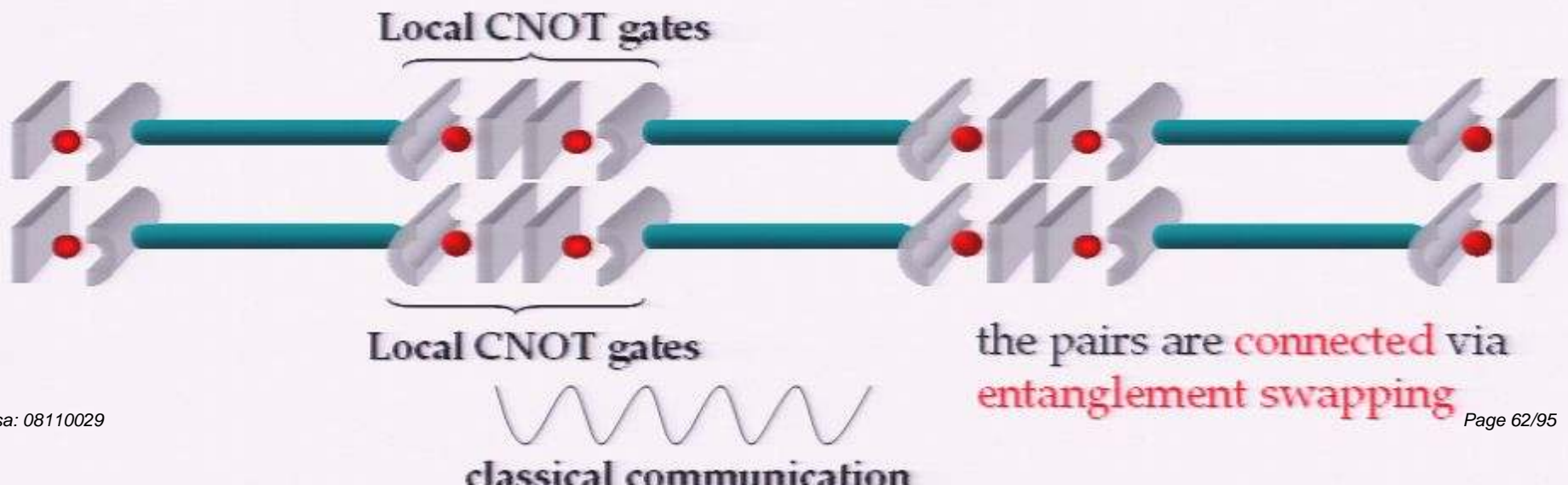
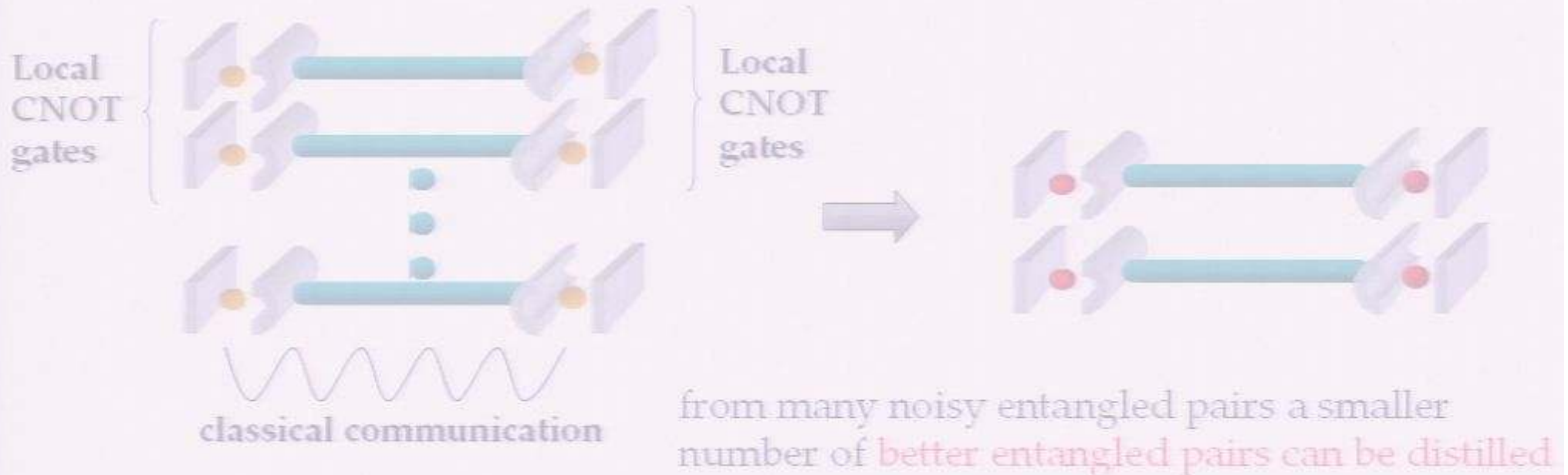


Entanglement Purification and Swapping



from many noisy entangled pairs a smaller number of **better entangled pairs can be distilled**

Entanglement Purification and Swapping



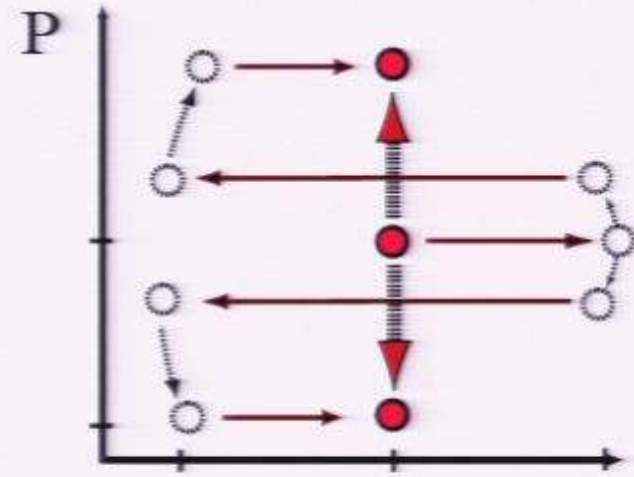
Deterministic C-NOT Gate

... simulate conditional displacements
via sequence of **unconditional displacements**
and **conditional rotations** ...

$$D(\beta \cos \theta) e^{-i \hat{a}^\dagger \hat{a} \theta \hat{\sigma}_z} D(-2\beta) e^{i \hat{a}^\dagger \hat{a} \theta \hat{\sigma}_z} D(\beta \cos \theta)$$

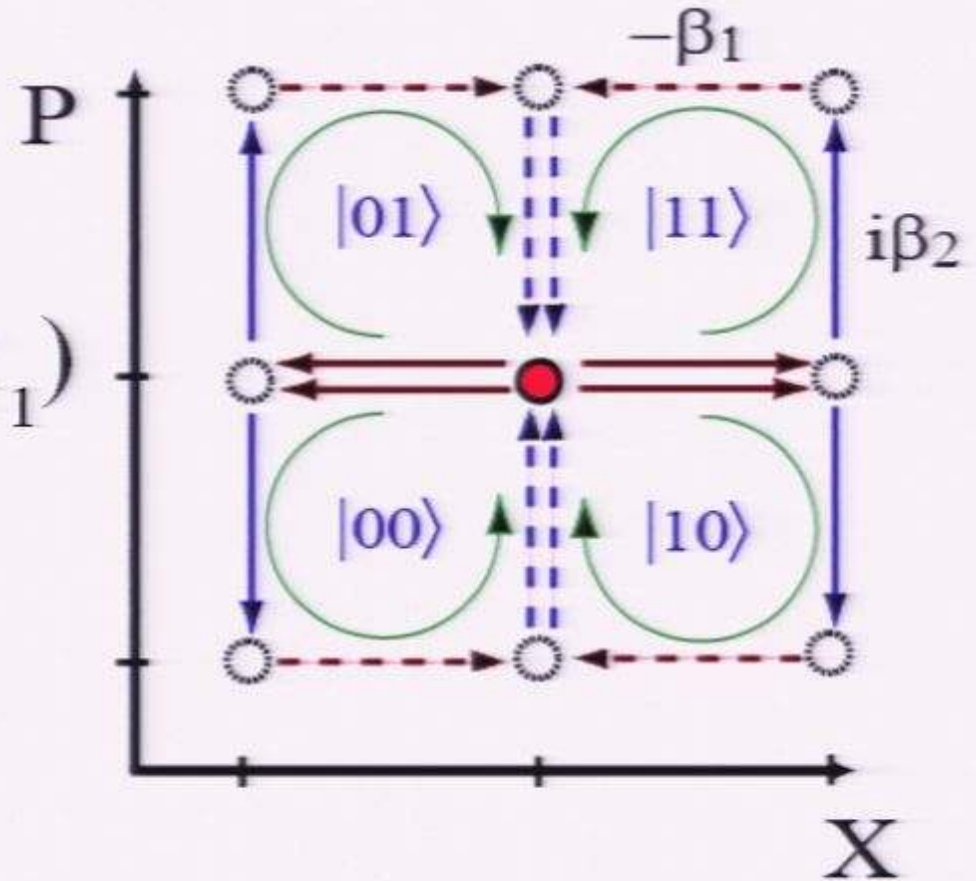
$$= e^{2i \beta \sin \theta (\hat{a}^\dagger + \hat{a}) \sigma_z}$$

...works in **any** regime,
weak or strong coupling!



Deterministic C-NOT Gate

$$\begin{aligned}
 & D(i\beta_2 \sigma_{z_2}) D(\beta_1 \sigma_{z_1}) \\
 & \times D(-i\beta_2 \sigma_{z_2}) D(-\beta_1 \sigma_{z_1}) \\
 & = e^{2i\beta_1\beta_2 \sigma_{z_1} \sigma_{z_2}}
 \end{aligned}$$

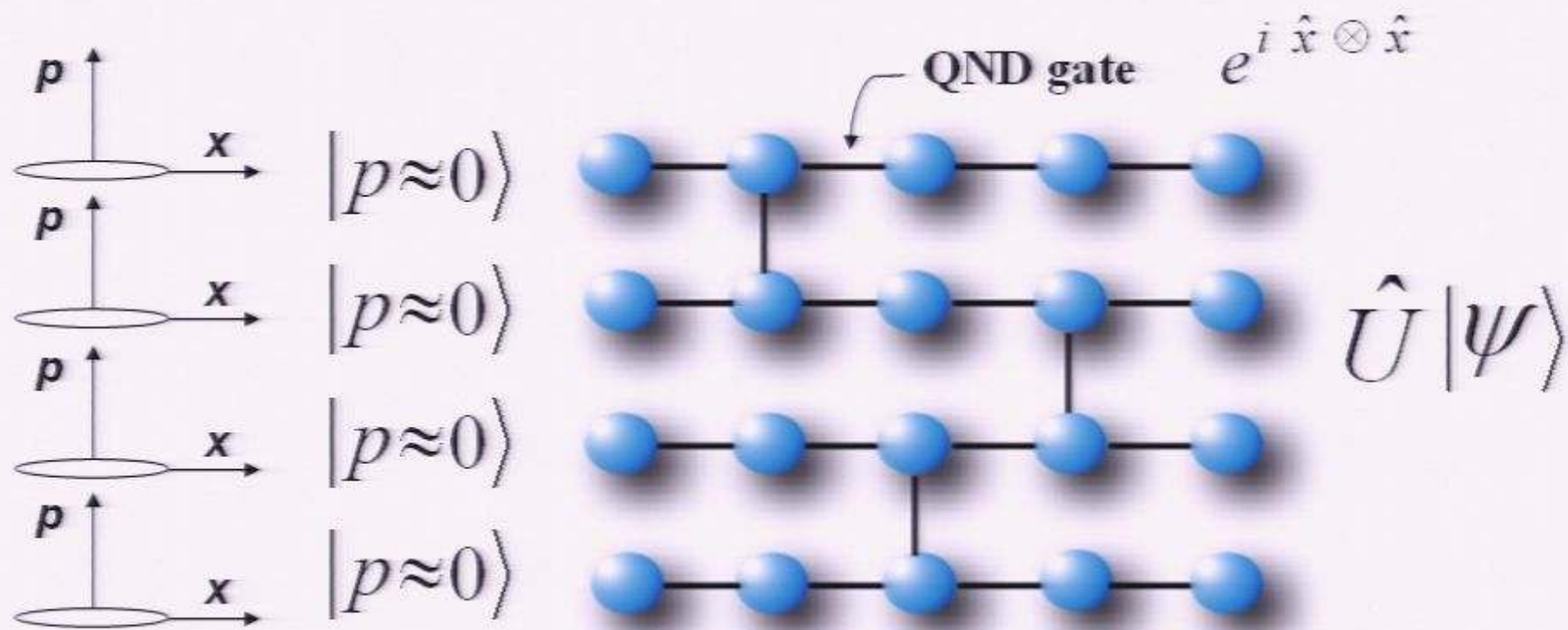


for $\beta_1\beta_2 = \frac{\pi}{8}$

this corresponds to a **CNOT gate**
up to a global phase and local rotations

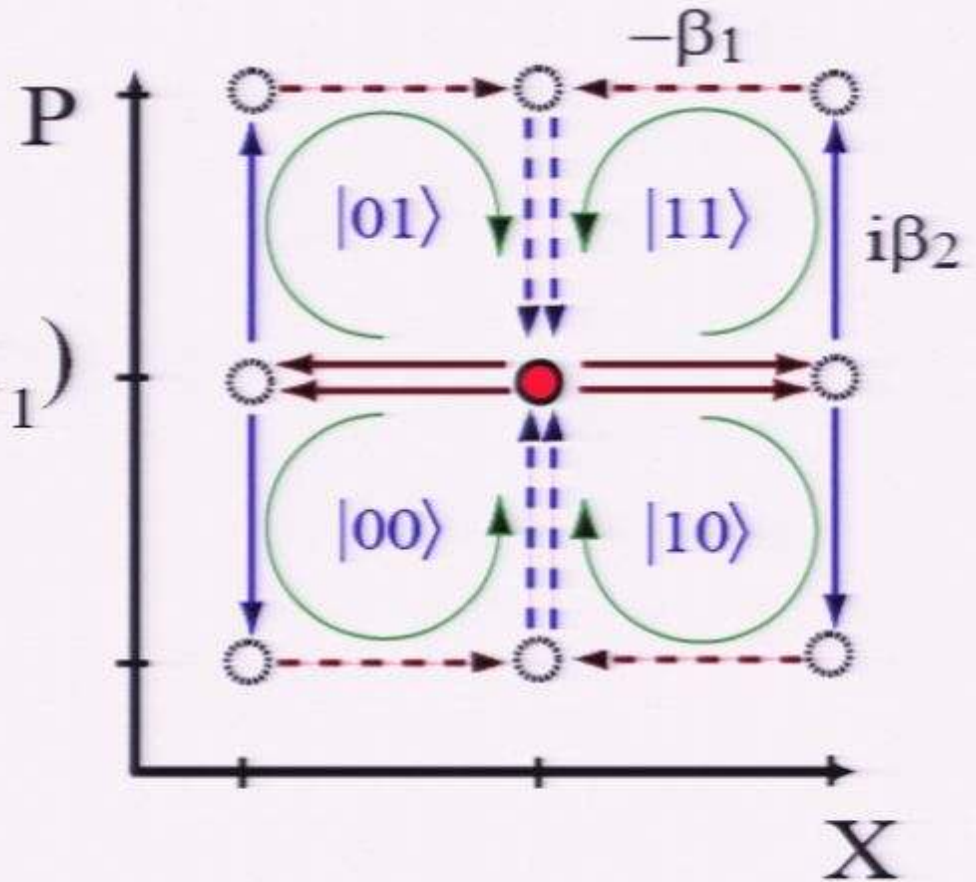
Hybrid One-Way Quantum Computer

... universal quantum computing with Gaussian cluster states, via **continuous** and **discrete** measurements



Deterministic C-NOT Gate

$$\begin{aligned}
 & D(i\beta_2 \sigma_{z_2}) D(\beta_1 \sigma_{z_1}) \\
 & \times D(-i\beta_2 \sigma_{z_2}) D(-\beta_1 \sigma_{z_1}) \\
 & = e^{2i\beta_1\beta_2 \sigma_{z_1} \sigma_{z_2}}
 \end{aligned}$$



for $\beta_1\beta_2 = \frac{\pi}{8}$

this corresponds to a **CNOT gate**
up to a global phase and local rotations

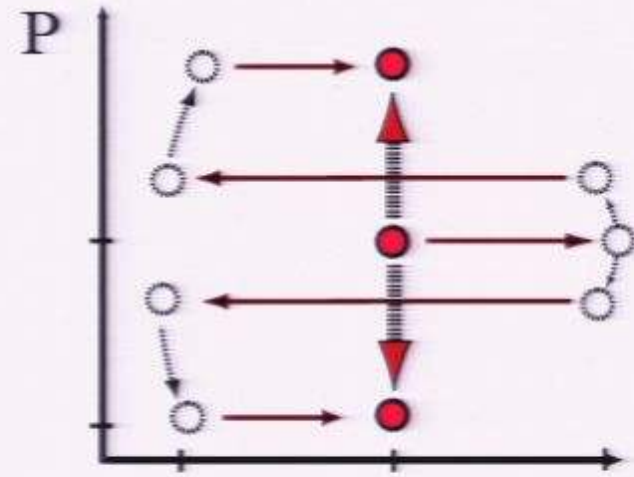
Deterministic C-NOT Gate

... simulate conditional displacements
via sequence of **unconditional displacements**
and **conditional rotations** ...

$$D(\beta \cos \theta) e^{-i \hat{a}^\dagger \hat{a} \theta \hat{\sigma}_z} D(-2\beta) e^{i \hat{a}^\dagger \hat{a} \theta \hat{\sigma}_z} D(\beta \cos \theta)$$

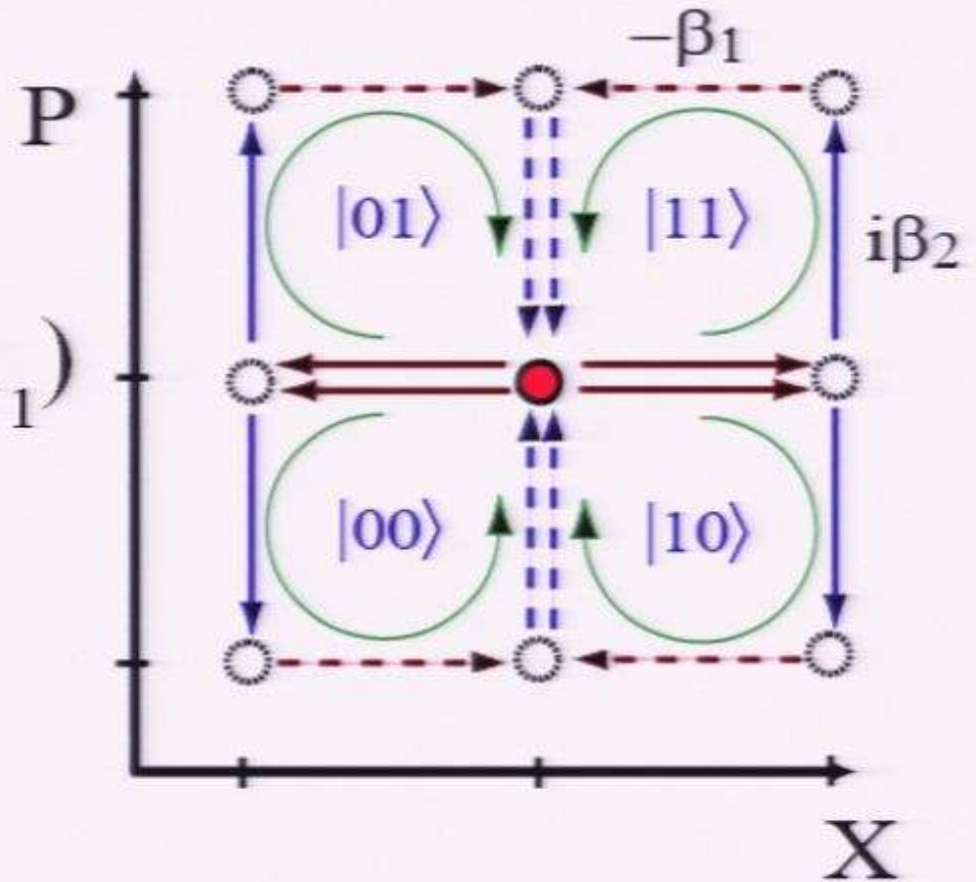
$$= e^{2i \beta \sin \theta (\hat{a}^\dagger + \hat{a}) \sigma_z}$$

...works in **any** regime,
weak or strong coupling!



Deterministic C-NOT Gate

$$\begin{aligned}
 & D(i\beta_2 \sigma_{z_2}) D(\beta_1 \sigma_{z_1}) \\
 & \times D(-i\beta_2 \sigma_{z_2}) D(-\beta_1 \sigma_{z_1}) \\
 & = e^{2i\beta_1\beta_2 \sigma_{z_1} \sigma_{z_2}}
 \end{aligned}$$



for $\beta_1\beta_2 = \frac{\pi}{8}$

this corresponds to a **CNOT gate**
up to a global phase and local rotations

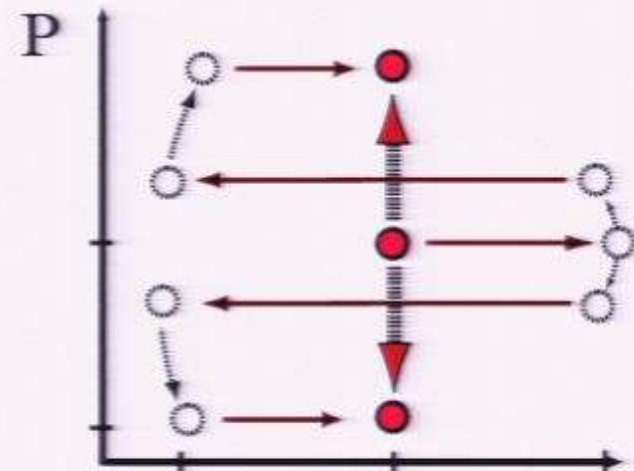
Deterministic C-NOT Gate

... simulate conditional displacements
via sequence of **unconditional displacements**
and **conditional rotations** ...

$$D(\beta \cos \theta) e^{-i \hat{a}^\dagger \hat{a} \theta \hat{\sigma}_z} D(-2\beta) e^{i \hat{a}^\dagger \hat{a} \theta \hat{\sigma}_z} D(\beta \cos \theta)$$

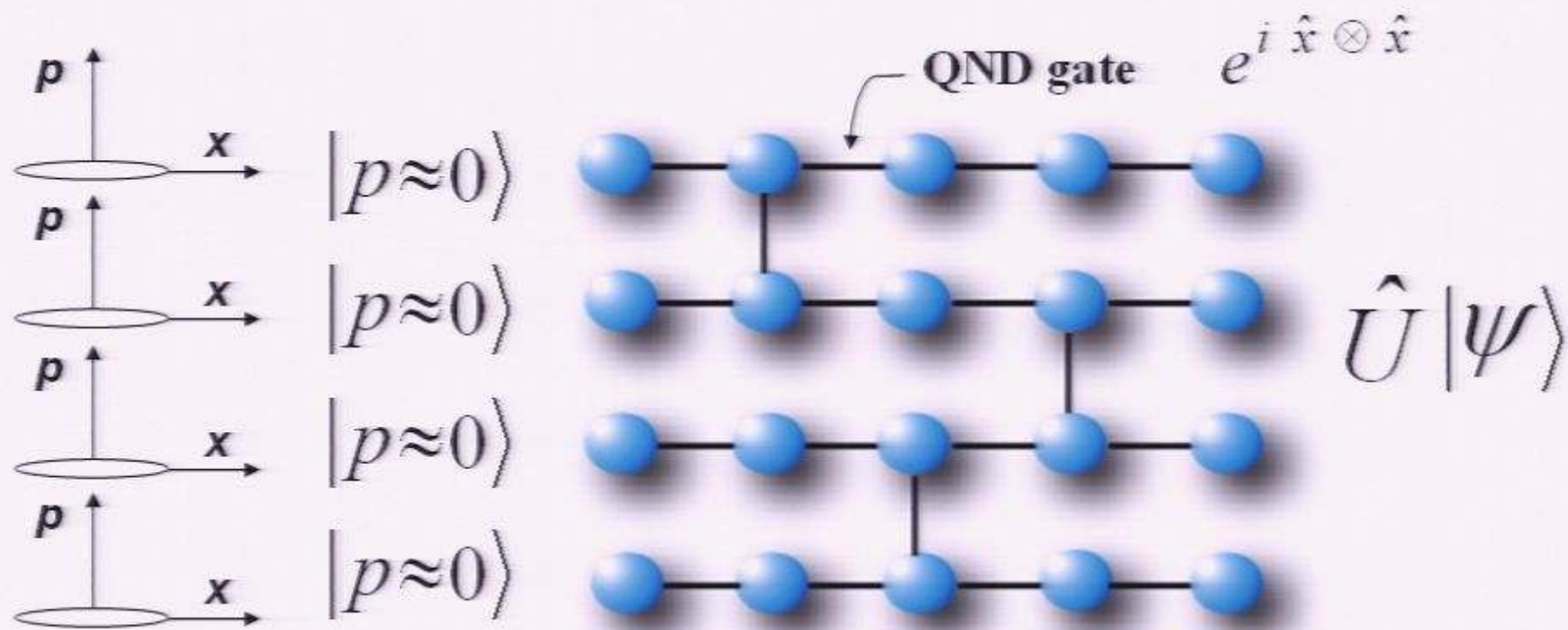
$$= e^{2i \beta \sin \theta (\hat{a}^\dagger + \hat{a}) \sigma_z}$$

...works in **any** regime,
weak or strong coupling!

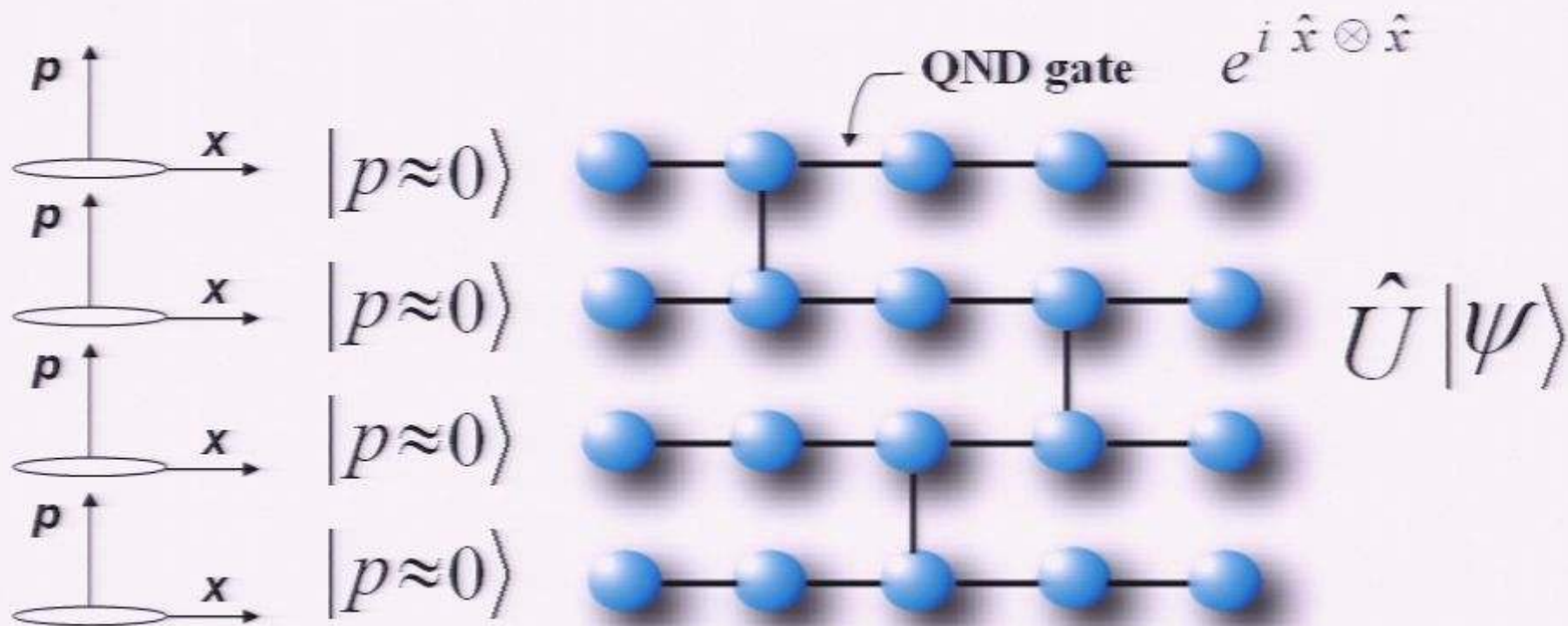


Hybrid One-Way Quantum Computer

... universal quantum computing with Gaussian cluster states, via **continuous** and **discrete** measurements



Hybrid One-Way Quantum Computer



- N.C. Menicucci, PVL, M. Gu, C. Weedbrook, T.C. Ralph, M.A. Nielsen, PRL **97**, 110501 (2006)
- PVL, J. Opt. Soc. Am. B **24**, 340 (2007)
- **efficient** generation: PVL, C. Weedbrook, M. Gu, PRA **76**, 032321 (2007)

- **compact** generation: N.C. Menicucci, PRL **101**, 130501 (2008), etc.

Cluster Computation

Essence of Cluster Computation:

cluster state is independent of computation;

universality and “computational freedom”
through choice of measurement bases

Quantum Computation via Continuous Variables

$$X(s) \equiv e^{-2is\hat{p}}, \quad Z(t) \equiv e^{2it\hat{x}} \quad (\text{WH operators})$$

$$X(s)|p\rangle = e^{-2isp}|p\rangle, \quad Z(t)|p\rangle = |p+t\rangle$$

$$Z(t)|x\rangle = e^{2itx}|x\rangle, \quad X(s)|x\rangle = |x+s\rangle$$

(position/computational basis)

Quantum Computation via Continuous Variables

$$X(s) \equiv e^{-2is\hat{p}}, \quad Z(t) \equiv e^{2it\hat{x}} \quad (\text{WH operators})$$

$$X(s)|p\rangle = e^{-2isp}|p\rangle, \quad Z(t)|p\rangle = |p+t\rangle$$

$$Z(t)|x\rangle = e^{2itx}|x\rangle, \quad X(s)|x\rangle = |x+s\rangle$$

(position/computational basis)

$$D \equiv e^{if(\hat{x})}$$

Quantum Computation via Continuous Variables

$$X(s) \equiv e^{-2is\hat{p}}, \quad Z(t) \equiv e^{2it\hat{x}} \quad (\text{WH operators})$$

$$X(s)|p\rangle = e^{-2isp}|p\rangle, \quad Z(t)|p\rangle = |p+t\rangle$$

$$Z(t)|x\rangle = e^{2itx}|x\rangle, \quad X(s)|x\rangle = |x+s\rangle$$

(position/computational basis)

$$D \equiv e^{if(\hat{x})}$$

$$F|x\rangle = |p=x\rangle, \text{ etc.}, \quad F\hat{x}F^\dagger = -\hat{p}, \quad F\hat{p}F^\dagger = \hat{x}$$

Quantum Computation via Continuous Variables

$$C_Z = e^{2i \hat{x} \otimes \hat{x}} \quad \begin{array}{l} \text{(controlled Z gate)} \\ \text{(QND gate)} \end{array}$$

Quantum Computation via Continuous Variables

$$C_Z = e^{2i \hat{x} \otimes \hat{x}} \quad \begin{array}{l} \text{(controlled Z gate)} \\ \text{(QND gate)} \end{array}$$

$$\left\{ F, Z(t), e^{i \kappa \hat{x}^2}, C_Z, e^{i \lambda \hat{x}^3} ; t, \kappa, \lambda \in \mathbf{R} \right\}$$

Clifford set = Gaussian

“cubic phase gate”

(universal set)

Quantum Computation via Continuous Variables

Clifford group:

$$U X U^\dagger = X'$$

qubits: $U \sigma U^\dagger = \sigma'$

Quantum Computation via Continuous Variables

- any Clifford computation can be efficiently simulated on a classical computer

{S.D. Bartlett *et al.*, PRL 88, 097904 (2002)}

- a single-mode non-Clifford (non-Gaussian) transformation is necessary and sufficient for universal quantum computation

{S. Lloyd and S.L. Braunstein, PRL 82, 1784-1787 (1999)}

Elementary Teleportation Circuits

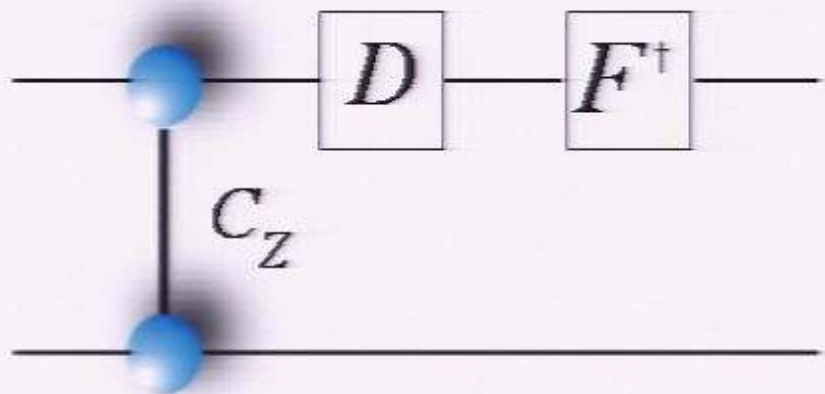
Elementary Teleportation Circuits



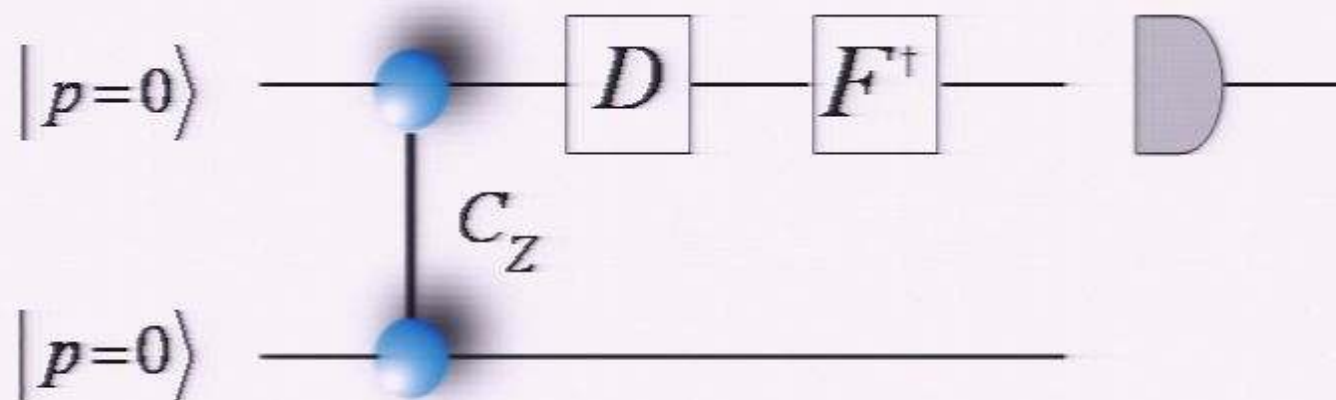
Elementary Teleportation Circuits



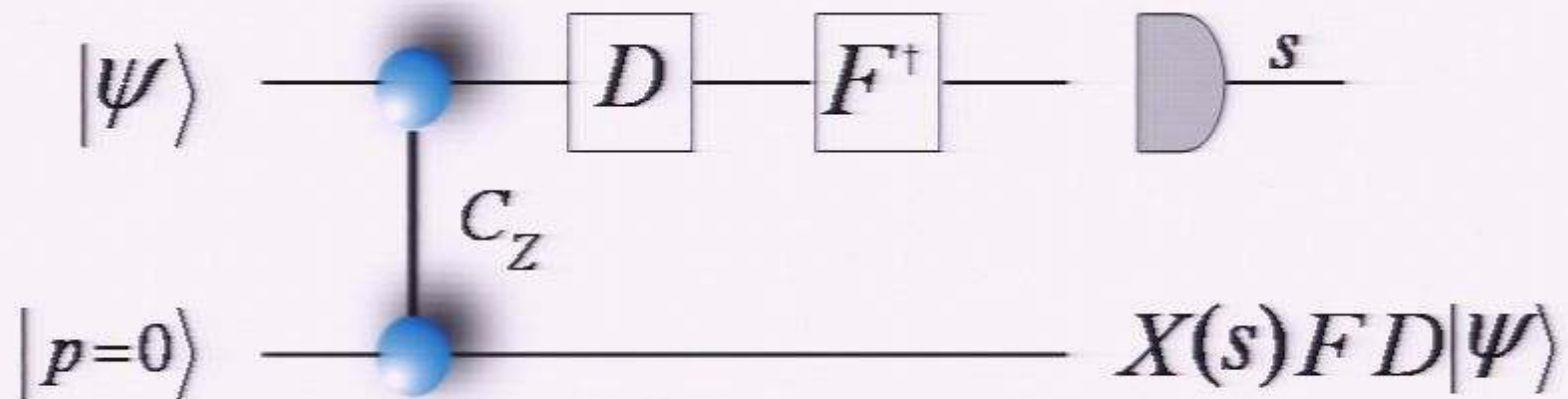
Elementary Teleportation Circuits



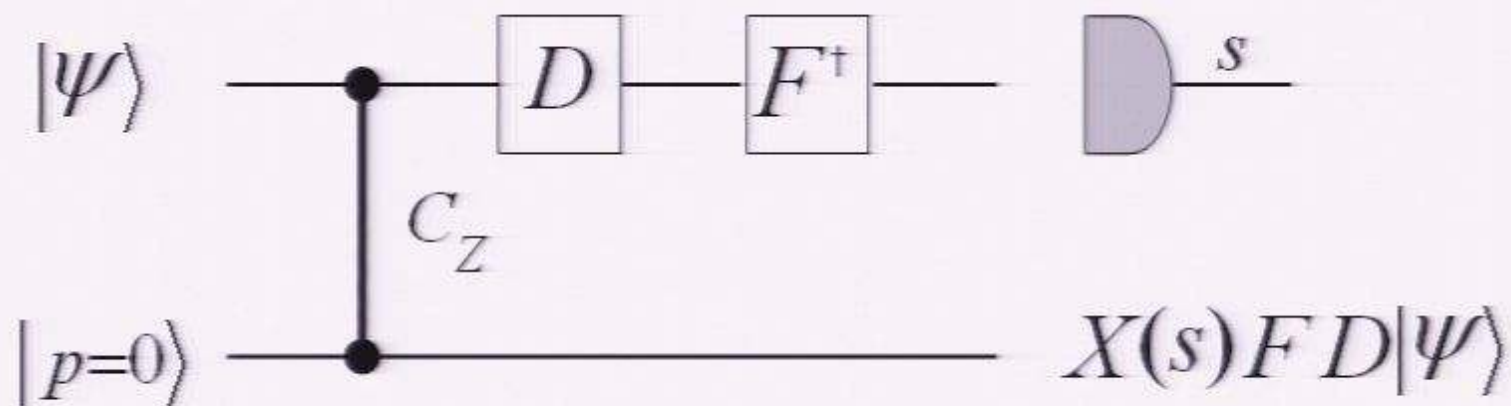
Elementary Teleportation Circuits



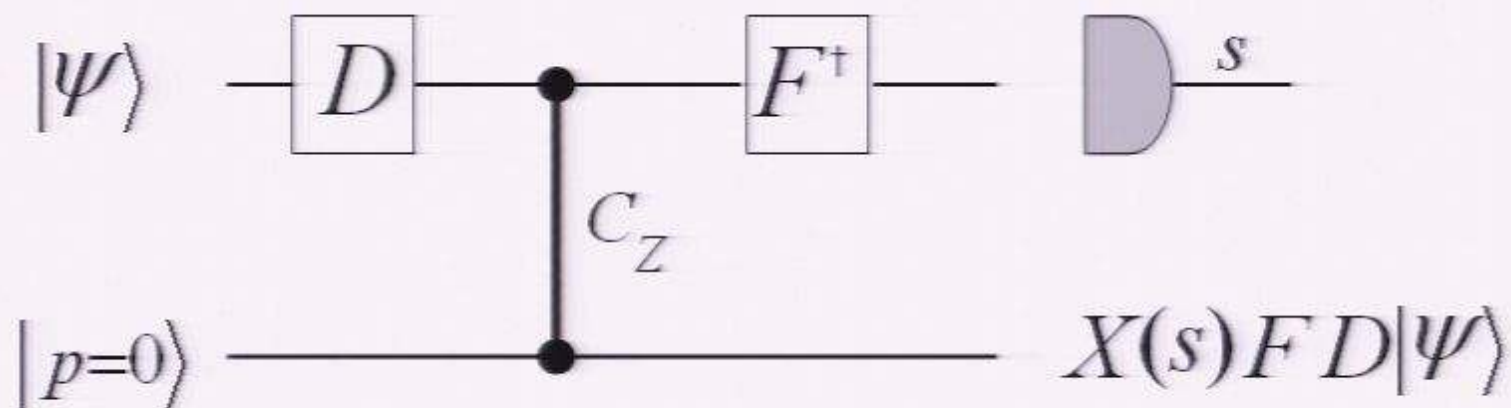
Elementary Teleportation Circuits



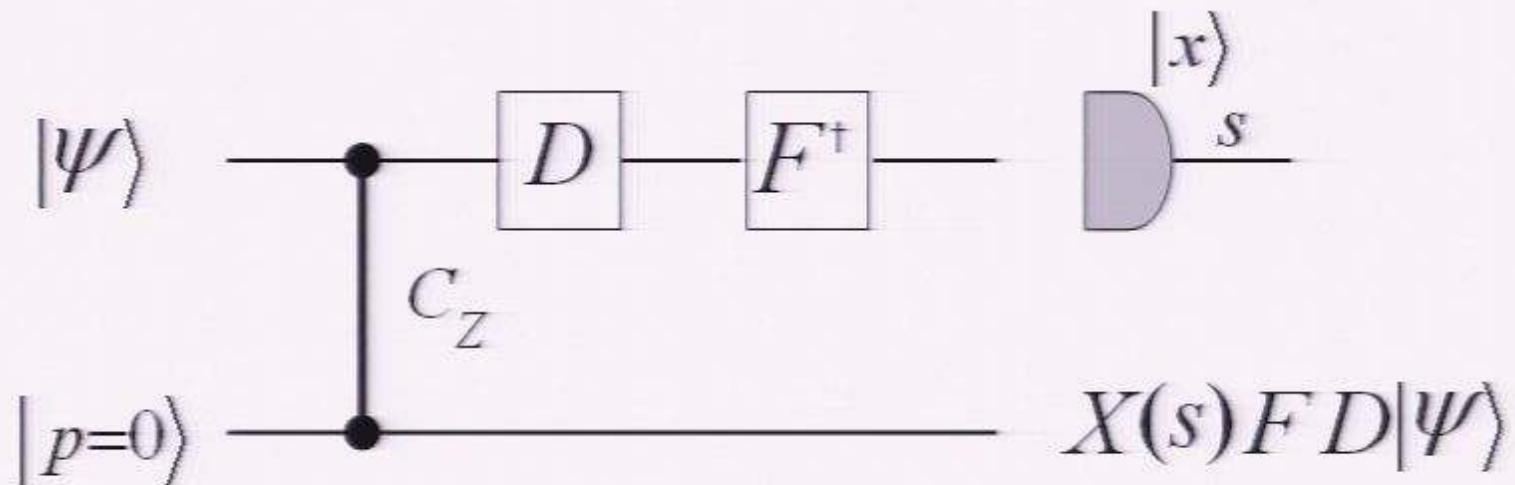
Elementary Teleportation Circuits



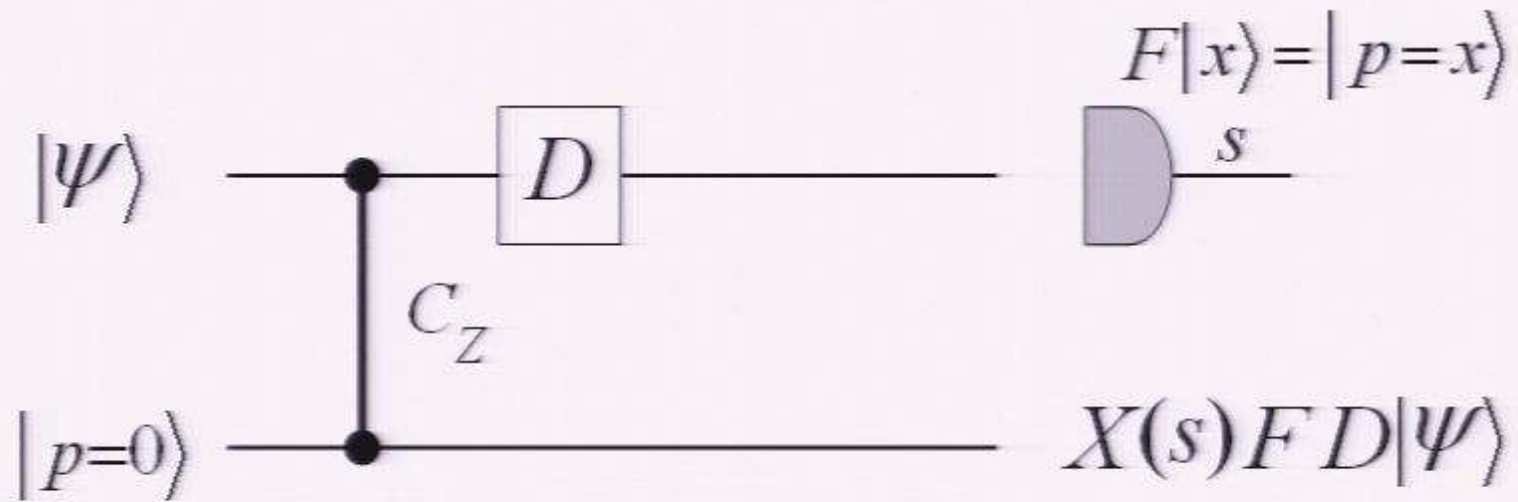
Elementary Teleportation Circuits



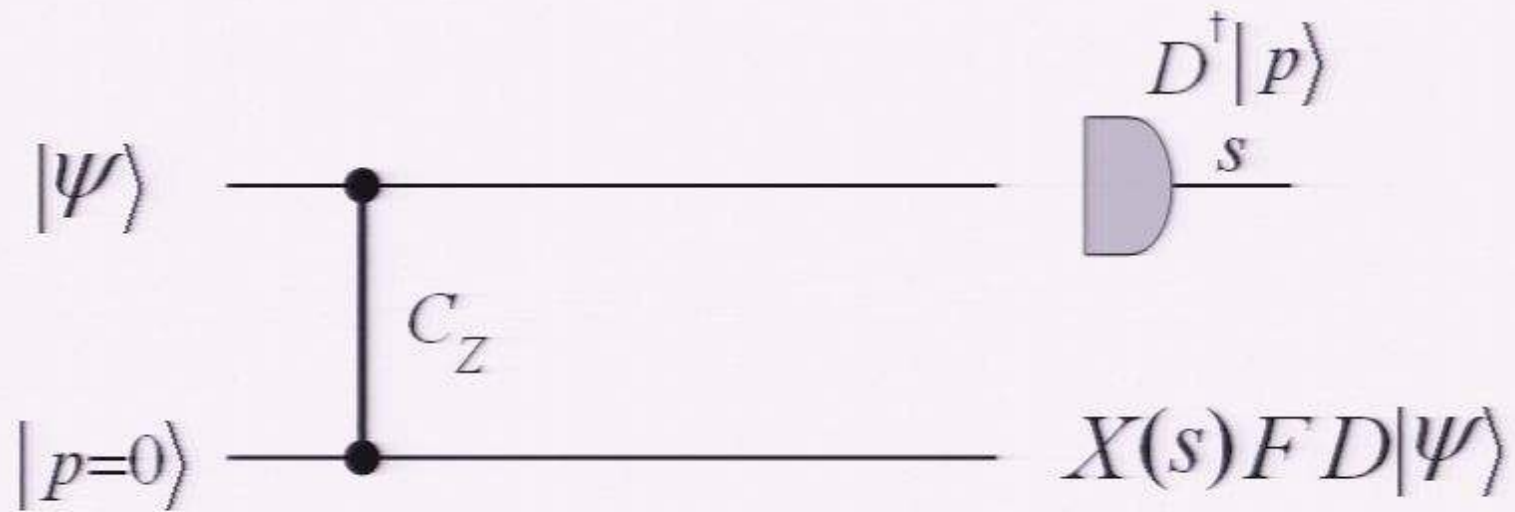
Elementary Teleportation Circuits



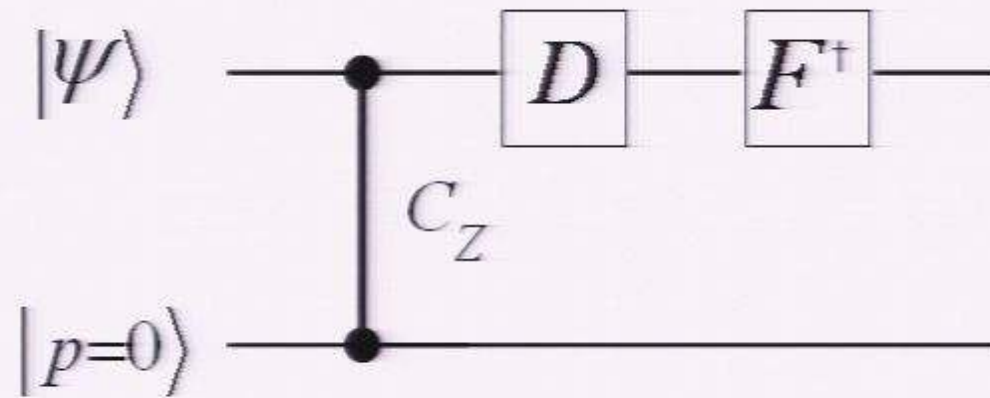
Elementary Teleportation Circuits



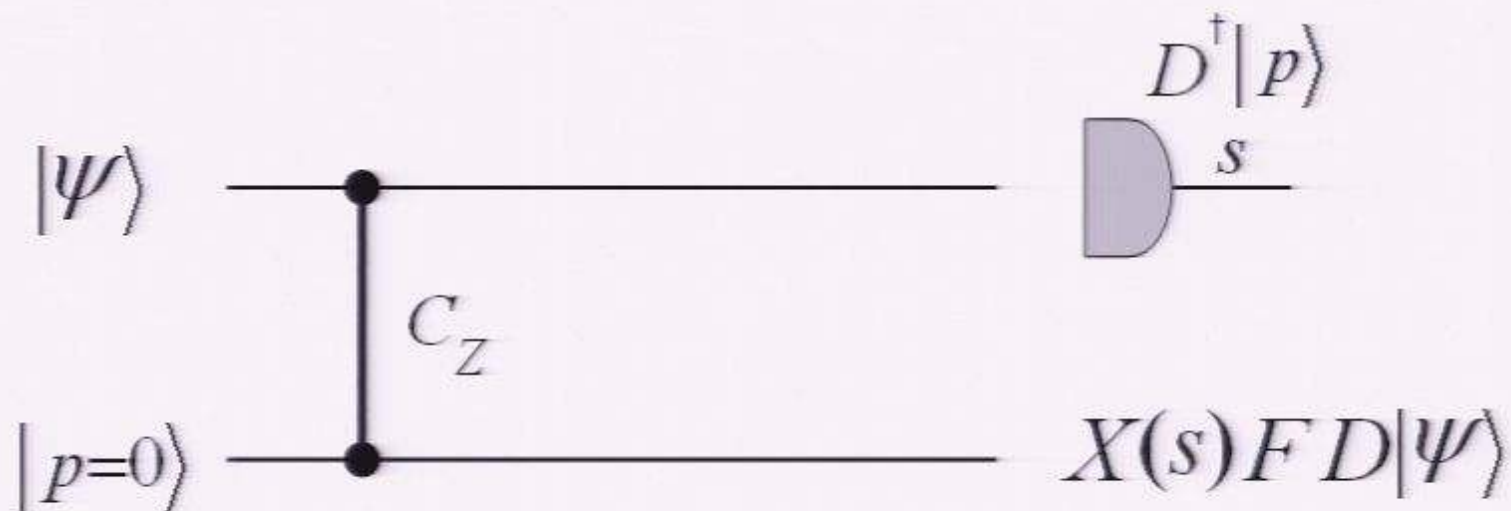
Elementary Teleportation Circuits



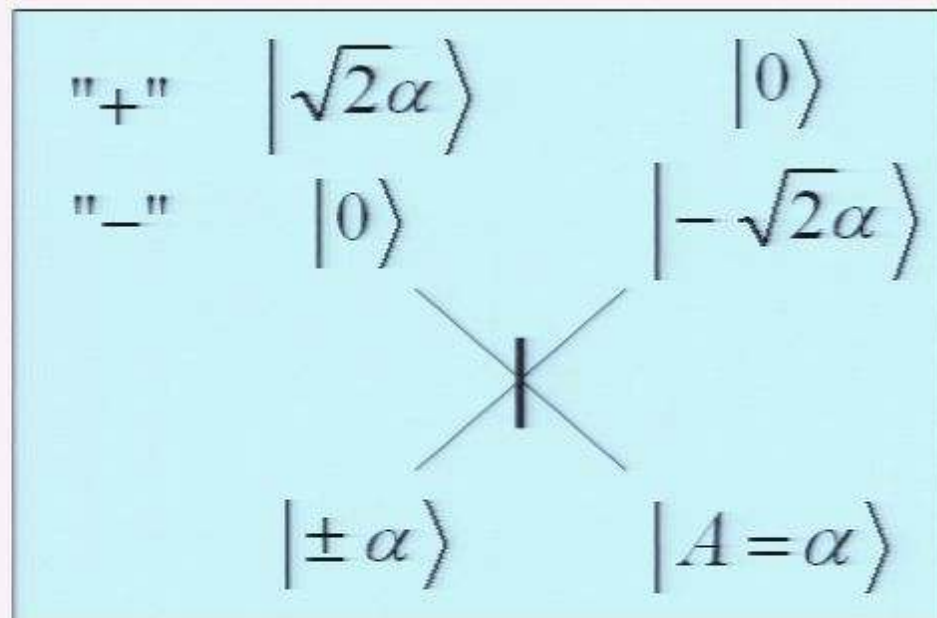
Elementary Teleportation Circuits



Elementary Teleportation Circuits



Unambiguous State Discrimination



Entanglement Distribution

... consider schemes **without** intrinsic errors

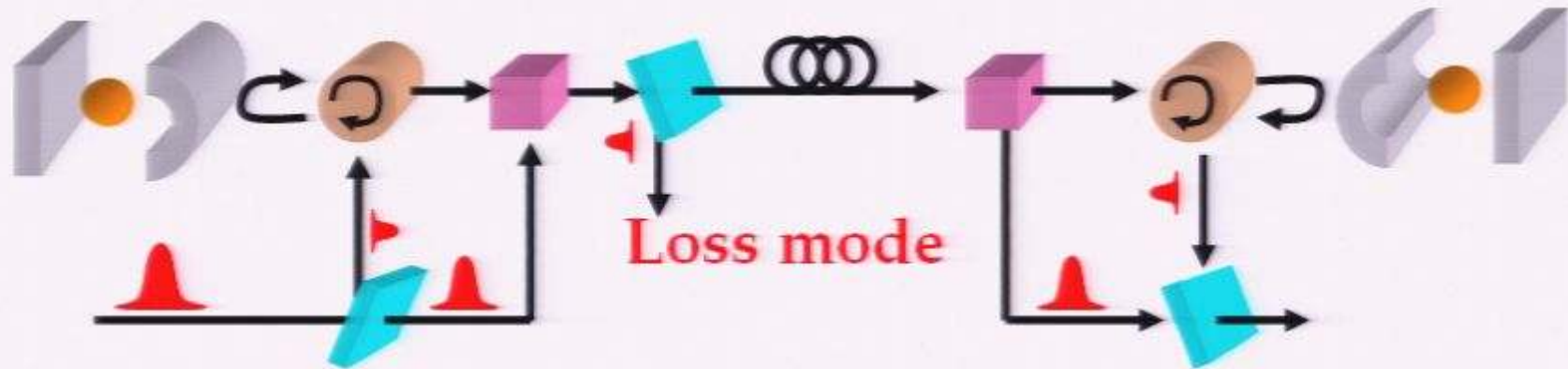
... generate mixtures of only **two** Bell states

$$F |\Phi^+\rangle\langle\Phi^+| + (1-F) |\Psi^+\rangle\langle\Psi^+|$$

$$|\Phi^+\rangle = \frac{1}{2} \left[|\sqrt{\eta}\alpha\rangle (|00\rangle + |11\rangle) + |\sqrt{\eta}\alpha e^{i\theta}\rangle |10\rangle + |\sqrt{\eta}\alpha e^{-i\theta}\rangle |01\rangle \right]$$

$$|\Psi^+\rangle = \frac{1}{2} \left[|\sqrt{\eta}\alpha\rangle (|00\rangle - |11\rangle) - |\sqrt{\eta}\alpha e^{i\theta}\rangle |10\rangle + |\sqrt{\eta}\alpha e^{-i\theta}\rangle |01\rangle \right]$$

Entanglement Distribution



Ideal scheme:
make d arbitrarily large

Realistic scheme:
trade-off between distinguishability
and decoherence

$$d \sim 1, \text{ e.g. } \theta \approx 0.01$$

$$\alpha^2 \approx 10000$$

(probe photons)

