Title: Explaining Regularities: The Need for Singular Behaviour

Date: Nov 21, 2008 11:00 AM

URL: http://pirsa.org/08110027

Abstract:

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The Need for Singular Behaviour

Robert W. Batterman

## Far End Control

Explaining Regularities: The Need for Singularities Behaviour



Robert W. Batterman

University of Western Ontario Department of Philosophy



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## District

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Asymptotic Expansions

Asymptotic Representations and the World

the Philosophy of

Mathematics

- Aim is to discuss reasons for taking mathematically characterized idealizations very seriously in explaining physical phenomena.
- Suggest that the physical world, itself, requires or demands a certain type of mathematical representation and that, as a result, one should not be surprised that mathematical asymptotic explanations play an essential, ineliminable role in providing physical understanding.

Close

















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- Aim is to discuss reasons for taking mathematically characterized idealizations very seriously in explaining physical phenomena.
- Suggest that the physical world, itself, requires or demands a certain type of mathematical representation and that, as a result, one should not be surprised that mathematical asymptotic explanations play an essential, ineliminable role in providing physical understanding.
- The discussion relates to issues concerning the applicability of mathematics in the natural sciences (Wigner's Problem).

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Outline



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#### Slogan:

"Singularity is almost invariably a clue." —Sherlock Holmes in The Boscome Valley Mystery

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## Structure



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- Discuss the role of limits and asymptotics on the nontraditional approach.

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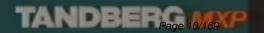












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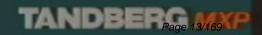












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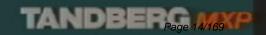












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- Discuss the role of asymptotic representations of various functions
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Discuss two approaches to idealization in physics.

- Traditional (Galilean)
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- Discuss the role of limits and asymptotics on the nontraditional approach.
- Discuss in detail a particular example—the understanding of features of the rainbow.
- Discuss the role of asymptotic representations of various functions.
- Offer a few remarks about idealizations, asymptotics, the world, and the role of mathematics in physics.

### Traditional Viewpoint



Almost all applications of mathematics to physical problems involve idealized models of some kind or other.

 One should try to find the most accurate and detailed mathematical representation of the problem at hand.

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- The aim is to try to effect a kind of convergence between model and reality. Ultimately, the goal is a complete (or true) description of the phenomenon of interest.

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- The aim is to try to effect a kind of convergence between model and reality. Ultimately, the goal is a complete (or true) description of the phenomenon of interest.
- On this view, a model is better the more details of the real phenomenon it is actually able to represent mathematically. In effect, the idealizations are introduced only to be removed through further work on the details.

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#### Idealization



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 Procedures/Recipe to gain insight, given a representative equation—the means for simplification.

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#### Idealization



 Procedures/Recipe to gain insight, given a representative equation—the means for simplification.

1 Nondimensionalize the equation—allows one to compare parameters as to their relative "size" (importance?).

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# Far End Control Idealization



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Remarks on the Philosophy of  Procedures/Recipe to gain insight, given a representative equation—the means for simplification.

1 Nondimensionalize the equation—allows one to compare parameters as to their relative "size" (importance?).

2 Take limits to "reduce" the equation—"small" nondimensional parameters approach zero or "large" nondimensional parameters approach infinity.

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#### Idealization



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- Procedures/Recipe to gain insight, given a representative equation—the means for simplification.
  - 1 Nondimensionalize the equation—allows one to compare parameters as to their relative "size" (importance?).
  - 2 Take limits to "reduce" the equation—"small" nondimensional parameters approach zero or "large" nondimensional parameters approach infinity.
- Prandtl (1948): "When the complete mathematical problem looks hopeless, it is recommended to enquire what happens when one essential parameter of the problem reaches the limit zero."

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#### Idealization



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1 Nondimensionalize the equation—allows one to compare parameters as to their relative "size" (importance?).

2 Take limits to "reduce" the equation—"small" nondimensional parameters approach zero or "large" nondimensional parameters approach infinity.

 Prandtl (1948): "When the complete mathematical problem looks hopeless, it is recommended to enquire what happens when one essential parameter of the problem reaches the limit zero."

 This is simplification by limiting idealizations. Not solely a pragmatic exercise to yield exactly solvable equations.

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### Minimal Models



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 Recall the traditional view aims, ultimately, to "de-idealize," to add more details to effect a convergence to a complete accurate description.

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#### Minimal Models



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#### Nontraditional View:

- Recall the traditional view aims, ultimately, to "de-idealize," to add more details to effect a convergence to a complete accurate description.
- Nontraditional view holds that a good model does not let these details get in the way.

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#### Nontraditional View:

- Recall the traditional view aims, ultimately, to "de-idealize," to add more details to effect a convergence to a complete accurate description.
- Nontraditional view holds that a good model does not let these details get in the way.
- Aims to find a "minimal model"—a model that "most economically caricatures the essential physics." (Goldenfeld)

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#### Minimal Models



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#### Nontraditional View:

- Recall the traditional view aims, ultimately, to "de-idealize," to add more details to effect a convergence to a complete accurate description.
- Nontraditional view holds that a good model does not let these details get in the way.
- Aims to find a "minimal model"—a model that "most economically caricatures the essential physics." (Goldenfeld)
- Holds that adding details to "improve" the model is self-defeating—such improvements are illusory.

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## Modeling Shocks



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 Region of dense molecular population separating two regions of lower molecular density. Momentum exchange across region as if there were a semipermeable membrane.

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Dense region is a shock. How can it be modeled?

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### Modeling Shocks



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 Region of dense molecular population separating two regions of lower molecular density. Momentum exchange across region as if there were a semipermeable membrane.

- Dense region is a shock. How can it be modeled?
  - Computationally track the molecules using the equations of molecular dynamics vs. . . .

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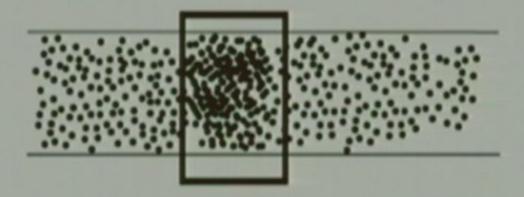
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## Modeling Shocks





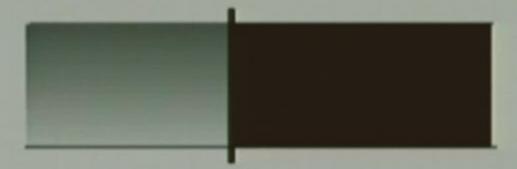


Figure: Modeling Shocks

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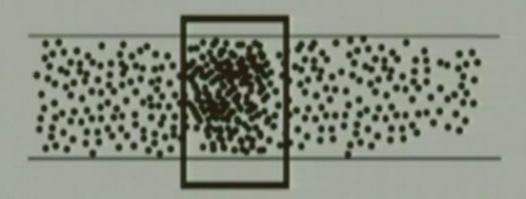
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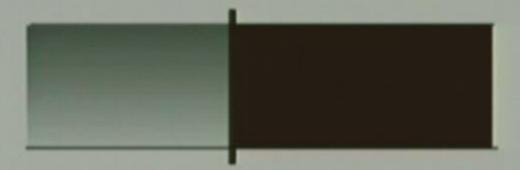


Figure: Modeling Shocks

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### **Modeling Shocks**



#### Continuum Idealization:

 The limit shrinks the shock region onto a two dimensional boundary.

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# Modeling Shocks



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#### Continuum Idealization:

- The limit shrinks the shock region onto a two dimensional boundary.
- On either side of the boundary the fluid's behavior is governed by the relevant partial differential equations of fluid dynamics.

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#### Continuum Idealization:

- The limit shrinks the shock region onto a two dimensional boundary.
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- Behavior at the boundary is not law governed—no differential equation holds; instead we have algebraic "jump conditions" —singular behavior across the boundary.

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# **Modeling Shocks**



#### Continuum Idealization:

- The limit shrinks the shock region onto a two dimensional boundary.
- On either side of the boundary the fluid's behavior is governed by the relevant partial differential equations of fluid dynamics.
- Behavior at the boundary is not law governed—no differential equation holds; instead we have algebraic "jump conditions"—singular behavior across the boundary.
- Standard view: Such boundaries are relatively unimportant to the physics—they are not law-governed. Covering law accounts of explanation relegate boundary conditions (and initial conditions) to secondary status in explanation.

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#### Continuum Idealization:

- Nonstandard View:
  - The boundary is the most important feature needed to understand the behavior of interest—the shock.

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# **Modeling Shocks**



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#### Continuum Idealization:

- Nonstandard View:
  - The boundary is the most important feature needed to understand the behavior of interest—the shock.
  - The shock/boundary "still dominates the overall behavior through the way in which it constrains the manner in which the 'law governed regions' piece together." (Mark Wilson)

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# Rays and the Rainbow Caustic





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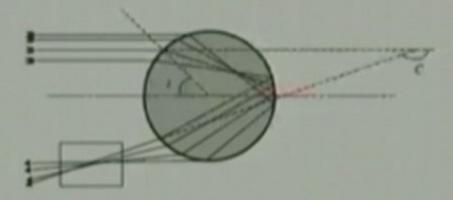


Figure: Raindrop and Light Rays

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# Modeling Shocks



#### Continuum Idealization:

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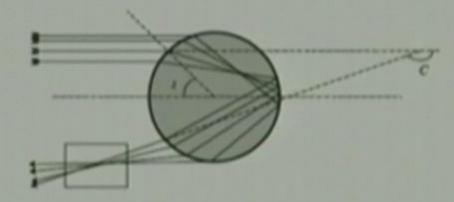


Figure: Raindrop and Light Rays

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#### The Need for Singular Behaviour

Rays and the Rainbow Caustic



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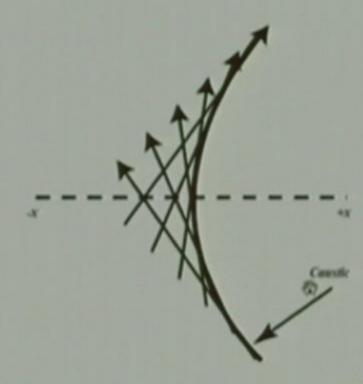


Figure: Fold Caustic

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# Rays and the Rainbow Caustic



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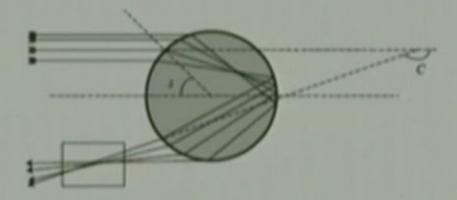


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# Rays and the Rainbow Caustic



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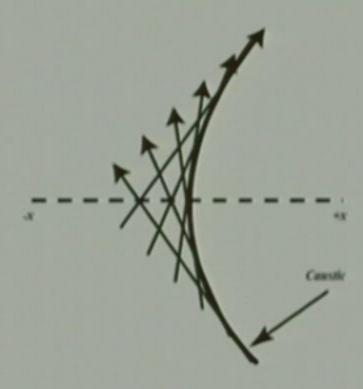


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# Rays and the Rainbow Caustic



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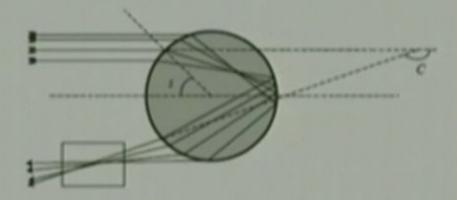


Figure: Raindrop and Light Rays

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# Rays and the Rainbow Caustic



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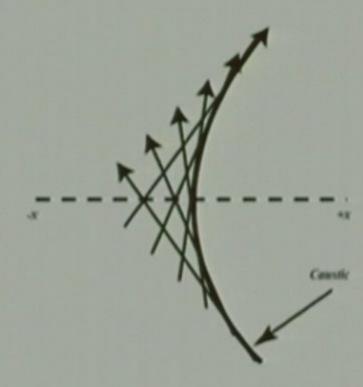


Figure: Fold Caustic

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# Ray Theory or Geometrical Optics



 The ray theory is capable of locating (only approximately) the primary bow of a rainbow—the caustic surface.

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### Far End Control

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- The ray theory fails to adequately account for the intensities that one observes. In fact, on the ray theory a caustic is a line on which the intensity of light is strictly speaking infinite.

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- The ray theory fails to adequately account for the intensities that one observes. In fact, on the ray theory a caustic is a line on which the intensity of light is strictly speaking infinite.
- This represents a kind of discontinuity or singularity in the theory. It is the λ → 0 limit of the wave theory and is analogous to the continuum limit yielding a singular shock.

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- This represents a kind of discontinuity or singularity in the theory. It is the  $\lambda \to 0$  limit of the wave theory and is analogous to the continuum limit yielding a singular shock.
- The ray theory is incapable of describing the so-called supernumerary bows—bows resulting from wave interference—that are sometimes seen as faint arcs of alternating pink and green on the lit side of the main rainbow arc.

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# Ray Theory or Geometrical Optics





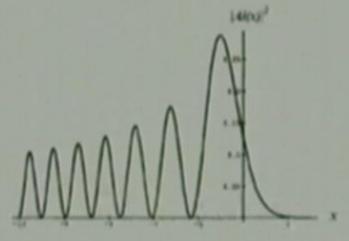


Figure: Supernumerary Bows

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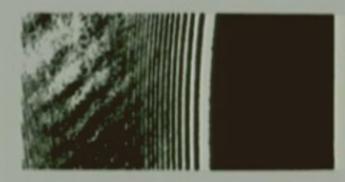
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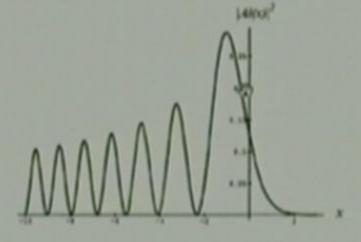


Figure: Supernumerary Bows

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# Ray Theory or Geometrical Optics



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So why do we need the ray theory at all?

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### Far End Control

# Ray Theory or Geometrical Optics



So why do we need the ray theory at all?

 Because the stability (under perturbation) of the caustics (of the ray-theoretic focal structures) plays an essential role in our understanding of the wave phenomena.

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# Airy Equation



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In 1838 George Biddell Airy derived a definite integral from the undulatory or wave theory of light from which one can determine the variation in intensity of light near a caustic of geometrical or ray optics. The term "caustic" literally means "burning surface" and in nature they are extremely bright lines and surfaces caused by the natural focusing of light. They are, in a very obvious sense "dominant" features of light phenomena.

Airy Equation



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Airy recognized that the singularity at the caustic was an artifact of the ray theory, and by properly incorporating effects of diffraction using the wave theory, he was able to derive the definite integral of equation (1).

$$Ai(x) \equiv \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt. \tag{1}$$

# Airy Equation



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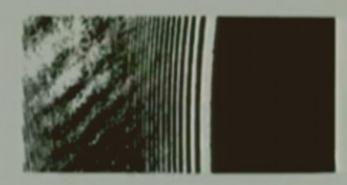
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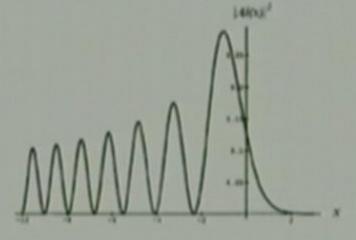


Figure: Airy Function and Supernumerary Bows

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#### The Need for Singular

# Airy's Convergent Expansion



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Using a convergent series representation of this function, Airy was able to compute values for Ai(x) for x between -5.6 to +5.6, where x is the distance along a normal to the caustic, negative values on the lit side, positive values in the shadow. Airy's series converges for all values of x and, as a result of this convergence, we may take it to be an exact representation of the Airy function. Unfortunately, this range of values allows one to locate only the first two dark bands on the lit side of the caustic. Because of the slow convergence of the series Airy was unable to extend his calculations beyond this limited range.

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# Airy Equation





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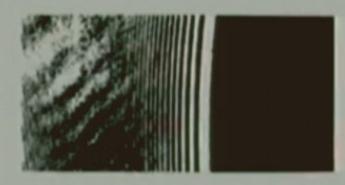
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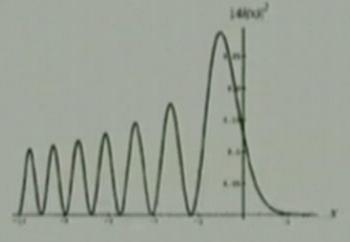


Figure: Airy Function and Supernumerary Bows

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# Divergent Asymptotic Expansions



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In 1850, in the first of several discussions having their genesis in Airy's caustic paper, Stokes noted that despite the fact that Airy's series converges, "... when [x] is at all large the calculation becomes exceeding laborious." [Stokes(1966b), p. 329] He was motivated, therefore, to express the Airy integral as a solution of a differential equation in a form that would exhibit "what terms are of most importance when x is large ...." [Stokes(1966b), p. 331] Stokes was aimed for physical understanding—a desire to display in the most perspicuous fashion those structures or features that dominate the phenomenon of interest; namely, the rainbow.

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# Divergent Asymptotic Expansions



 "After many trials I at last succeeded in putting Mr Airy's integral under a form from which its numerical value can be calculated with extreme facility when [x] [the distance from the geometrical caustic] is large, whether positive or negative, or even moderately large." [Stokes(1966b), p.330]

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#### The Need for Singular Behaviour

# Divergent Asymptotic Expansions



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 "Moreover the form of the expression points out, without any numerical calculation the law of the progress of the function when [x] is large." [Stokes(1966b), pp. 330]

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# Divergent Asymptotic Expansions



Had Stokes had large supercomputers at his disposal, there still would be compelling reasons for engaging in this asymptotic investigation—namely, that such investigations can often highlight important mathematical structures that are hidden in (or obscured by) the exact, convergent expansion.

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Stokes on the Airy Integral

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# Divergent Asymptotic Expansions



Stokes derived his asymptotic (divergent) expansion by finding the differential equation for which the Airy integral (1) was a solution and by using what is now known as the WKB method. He examined the general case where the argument can be complex and arrived at the following differential equation:

$$\frac{d^2u}{dz^2} - zu = 0. (2)$$

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# Divergent Asymptotic Expansions



Stokes reasoned as follows (and, at base, this is the type of asymptotic reasoning behind the WKB method). Let us focus on large values of |z| which reflects our interest in being able to describe the locations and intensities of the bows relatively far from the caustic. If we increase |z| by a small increment  $\delta z$ , the proportionate increase of |z| will be small. That is, for large |z| we can effectively regard z as a constant. If we make this assumption, then equation (2) has the approximate solution:

$$u(z) \equiv Ai(z) \approx Ae^{-\frac{2}{3}z^{\frac{3}{2}}} + Be^{\frac{2}{3}z^{\frac{3}{2}}}.$$
 (3)

where A and B are arbitrary constants.

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# Divergent Asymptotic Expansions



"The approximate integral [(3)] points out the existence of circular functions ... in the true integral." In fact, the approximate 'solution' (3) shows that when z is real and positive, the behavior of the solution to equation (2) can be exponential in character and when z is real and negative, its behavior will be oscillatory or trigonometric in character.

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# Divergent Asymptotic Expansions



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Airy vs. Stokes

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Stokes on the Airy Integral

Airy's convergent series:

$$Ai(z) = A \left\{ 1 + \frac{9z + 3}{2 \cdot 3} + \frac{9^2 z^6}{2 \cdot 3 \cdot 5 \cdot 6} + \frac{9^3 z^9}{2 \cdot 3 \cdot 5 \cdot 6 \cdot 8 \cdot 9} + B \left\{ z + \frac{9z^4}{3 \cdot 4} + \frac{9^2 z^7}{3 \cdot 4 \cdot 6 \cdot 7} + \frac{9^3 z^{10}}{3 \cdot 4 \cdot 6 \cdot 7 \cdot 9 \cdot 10} + \frac{9^3 z^{1$$

Stokes' divergent series:

$$Ai(z) \equiv Cz^{-\frac{1}{4}}e^{-2z^{\frac{3}{2}}}\left\{1 - \frac{1\cdot 5}{1\cdot 144z^{\frac{3}{2}}} + \frac{1\cdot 5\cdot 7\cdot 11}{1\cdot 2\cdot 144^{2}z^{3}} - \cdots \right.$$

$$+ Dz^{-\frac{1}{4}}e^{2z^{\frac{3}{2}}}\left\{1 + \frac{1\cdot 5}{1\cdot 144z^{\frac{3}{2}}} + \frac{1\cdot 5\cdot 7\cdot 11}{1\cdot 2\cdot 144^{2}z^{3}} + \cdots\right\}$$

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# Airy vs. Stokes



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 Stokes' "solution" and, in particular, its leading terms, explicitly exhibits the oscillatory and exponential character of the "true integral." This provides a crucial component of our understanding of the physical phenomenon of interest. Our understanding of the patterns present in the rainbow is provided by the relatively transparent mathematical representation of these dominant characteristics.

 Airy's convergent series provides an 'exact' solution to the equation for all values of |z|, but virtually no information about the dominant physical features of the phenomenon is conveyed by the analytical form of the terms of the series.

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# Airy vs. Stokes



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# Asymptotic Representations



#### Standard view:

 Asymptotic representations of various functions are particularly useful.

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# Asymptotic Representations



#### Standard view:

- Asymptotic representations of various functions are particularly useful.
- They provide quite accurate numerical values even when one considers very few terms in the series. For example, most applications of the WKB method retain only the first term in the asymptotic expansion.

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- However, because the late terms of such series typically diverge, they have historically been taken to be inherently vague and without any coherent meaning.

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# Asymptotic Representations



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- However, because the late terms of such series typically diverge, they have historically been taken to be inherently vague and without any coherent meaning.
- Contemporary understanding of asymptotic expansions rejects this skeptical assessment of the meaningfulness of asymptotic expansions, and, in fact, one can see, already in Stokes' own work, the seeds of this modern point of view.

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Asymptotic Expansions

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# Asymptotic Representations



- Stokes noted that "[a] semi-convergent [divergent] series (considered numerically, and apart from its analytical form) defines a function only subject to a certain amount of vagueness . . . . " [Stokes(1966a), p. 285]
- Stokes' parenthetical remark here seems to say that while there is numerical vagueness in asymptotic expansions, there may be exact formal information captured in the full asymptotic expansion.

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The Need for

R. B. Dingle (1973)

Singular Behaviour

Robert W. Batterman

Asymptotic Expansions

"[A]symptotic expansions are normal, immediately comprehensible, functions of their variables in so far as functional form is concerned. . . . A complete asymptotic expansion of a function f(x) may therefore be defined as an expansion containing an asymptotic series which formally exactly obeys-throughout a certain phase sector-all those relations satisfied by f(x) which do not involve any numerical value of x other than on the infinite circle  $|x| \to \infty$ : for instance,

- 1) Functional form as  $|x| \to \infty$ , i.e. boundary conditions on f and its derivatives at infinity.
- ② Differential, difference and integral equations.
- Relations involving other parameters incorporated, such as recurrence relations between orders." [Dingle(1973), pp. 19 - 20

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R. B. Dingle



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 In maintaining that a complete asymptotic expansion for a given function f(z) is not inherently vague or meaningless because of the divergence of the asymptotic series in the expansion, Dingle commits himself to providing an interpretation of the late terms of the series.

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R. B. Dingle



- In maintaining that a complete asymptotic expansion for a given function f(z) is not inherently vague or meaningless because of the divergence of the asymptotic series in the expansion, Dingle commits himself to providing an interpretation of the late terms of the series.
- To put this another way, Dingle is committed to interpreting the infinities arise when one takes the singularities in the theory (the places of breakdown) seriously.

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 In maintaining that a complete asymptotic expansion for a given function f(z) is not inherently vague or meaningless because of the divergence of the asymptotic series in the expansion, Dingle commits himself to providing an interpretation of the late terms of the series.

- To put this another way, Dingle is committed to interpreting the infinities arise when one takes the singularities in the theory (the places of breakdown) seriously.
- This interpretation is based on a theorem of Darboux from 1878.

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Asymptotic Expansions

# Asymptotic Interpretation



 Darboux's Theorem entails that the late terms in a convergent Taylor expansion depend only on the behavior of the function in the immediate neighborhood of the singularity closest to the point of expansion.

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# Asymptotic Interpretation



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# Asymptotic Interpretation



- Darboux's Theorem entails that the late terms in a convergent Taylor expansion depend only on the behavior of the function in the immediate neighborhood of the singularity closest to the point of expansion.
- Dingle: This extends to asymptotic (that is, divergent) series as well.

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# Asymptotic Interpretation



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- Darboux's Theorem entails that the late terms in a convergent Taylor expansion depend only on the behavior of the function in the immediate neighborhood of the singularity closest to the point of expansion.
- Dingle: This extends to asymptotic (that is, divergent) series as well.
- Almost without exception, the late terms of divergent series will have the same form.

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# Asymptotic Interpretation



The late terms  $(n \gg 1)$ 

of any asymptotic power series will transpire to be expressible in a standard limiting form  $(n + constant)!/(variable)^n$ , the accuracy of this limiting representation increasing with n . . . . This conclusion . . . is critically important in two ways: first, because it provides a valuable lead on how asymptotic power series and expansions containing them might best be defined; and second, because is shows that substantially a single theory of interpretation will apply equally to late terms of all such asymptotic series. [Dingle(1973), p. 4 with a slight change in notation]

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# Asymptotic Interpretation



 Almost does not matter what the nature of the singularity of the function is—whether it is a pole, a branch point, etc.

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# Asymptotic Interpretation



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- Almost does not matter how the asymptotic expansion is derived—whether from an integral representation, from a differential equation, etc.
- In virtually all cases, the late terms in the asymptotic expansions will have one of four basic forms that he calls "terminants."

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- In virtually all cases, the late terms in the asymptotic expansions will have one of four basic forms that he calls "terminants."
- A vast range of functions are such that through asymptotic analysis they can be transformed so as to exhibit a common, universal pattern:
   Function = first n terms of asymptotic series + n<sup>th</sup> × terminant. [Dingle(1973), p. 411]

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# Asymptotic Interpretation: Lessons



 Asymptotics provides complete information about the function, and hence, about the phenomenon it represents.

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## Asymptotic Interpretation: Lessons



- Asymptotics provides complete information about the function, and hence, about the phenomenon it represents.
- Asymptotic expansions often result from a focus on a singularity in a given theory or at the boundary between two theories.

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# Asymptotic Interpretation: Lessons



- Asymptotics provides complete information about the function, and hence, about the phenomenon it represents.
- Asymptotic expansions often result from a focus on a singularity in a given theory or at the boundary between two theories.
- The singularities very often are associated with the essential or dominant physical features—features that are in some sense generic and not specific to any one single instance: the shock boundaries, the stable focal surfaces of light.

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## Asymptotic Interpretation: Lessons



- Asymptotics provides complete information about the function, and hence, about the phenomenon it represents.
- Asymptotic expansions often result from a focus on a singularity in a given theory or at the boundary between two theories.
- The singularities very often are associated with the essential or dominant physical features—features that are in some sense generic and not specific to any one single instance: the shock boundaries, the stable focal surfaces of light.
- Thus, even though there may not be real physical singularities, the mathematical singularities reflect essential features of the situation.

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# Asymptotic Representations and Idealizations



 Most applications of mathematics to the description and explanation of physical phenomena do involve simplifications of a sort.

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# Asymptotic Representations and Idealizations



- Most applications of mathematics to the description and explanation of physical phenomena do involve simplifications of a sort.
- Most applications involve idealizations in which certain parameters are treated asymptotically.

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- Most applications of mathematics to the description and explanation of physical phenomena do involve simplifications of a sort.
- Most applications involve idealizations in which certain parameters are treated asymptotically.
- Such asymptotic limiting idealizations are essential for obtaining genuine scientific explanation and understanding.

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# Asymptotic Representations and Idealizations



 Many (if not most) investigations of physical phenomena focus on patterns of physical behavior—those aspects of a system that replay themselves in varying circumstances. This reflects a kind of stability under various changes of the phenomenon of interest. (Details surrounding distinct instances don't matter.)

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# Asymptotic Representations and Idealizations



- Many (if not most) investigations of physical phenomena focus on patterns of physical behavior—those aspects of a system that replay themselves in varying circumstances. This reflects a kind of stability under various changes of the phenomenon of interest. (Details surrounding distinct instances don't matter.)
- Asymptotic limits are most effective for examining what goes on at places where the 'laws' break down-that is, at places of singularities in the governing equations of the phenomena.

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# Asymptotic Representations and Idealizations



- Many (if not most) investigations of physical phenomena focus on patterns of physical behavior—those aspects of a system that replay themselves in varying circumstances. This reflects a kind of stability under various changes of the phenomenon of interest. (Details surrounding distinct instances don't matter.)
- Asymptotic limits are most effective for examining what goes on at places where the 'laws' break down-that is, at places of singularities in the governing equations of the phenomena.
- These 'physical' singularities and their 'effects'—how they dominate the observed phenomena—are themselves best investigated through asymptotic representations of the solutions to the relevant governing equations. Stokes and the rainbow.

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# Asymptotic Representations and Idealizations



 Interplay between the 'physical' singularities and the mathematical singularities.

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- Interplay between the 'physical' singularities and the mathematical singularities.
  - The ray-theoretic singularities need to be dealt with by the wave theory—accomplished by Airy in deriving his integral equation.

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## Asymptotic Representations and Idealizations



- Interplay between the 'physical' singularities and the mathematical singularities.
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# Asymptotic Representations and Idealizations



- Interplay between the 'physical' singularities and the mathematical singularities.
  - The ray-theoretic singularities need to be dealt with by the wave theory—accomplished by Airy in deriving his integral equation.
  - However, the ray-theoretic singularity dominates the physical phenomenon.
- The mathematical singularities in the asymptotic equations constrain the structure of the solutions to the governing equations.

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Alfred Seeger (2000) in the introduction to Special Functions: A Unified Theory Based on Singularities:

"In mapping a complex physical situation onto manageable mathematics, location and character of the singularities reflect the essentials of the situation, whereas the parameters not directly associated with the singularities usually carry incidental information, e.g. on the physical properties of the specific material under consideration. Recognizing this led to a new appreciation of the importance of asymptotic expansions and of the Stokes' phenomenon. On the mathematical side, it is the singularities of the differential equations resulting from the mapping that determine the character of the solutions."

[Slavyanov and Lay(2000), p. vii]

## Conclusions



Physical systems are often extremely complex.

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- Physical systems are often extremely complex.
- This complexity together with the existence of regular, stable features in the world demands that we idealize in a certain way.

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#### Conclusions



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Remarks on the Philosophy of Physical systems are often extremely complex.

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## Conclusions



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- Such idealizations focus on the "essentials of the situation."
- In turn, our mathematical attention is focused on certain singularities in the relevant mathematical equations which require asymptotic investigations.

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### Conclusions



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- Physical systems are often extremely complex.
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- Such idealizations focus on the "essentials of the situation."
- In turn, our mathematical attention is focused on certain singularities in the relevant mathematical equations which require asymptotic investigations.
- Interplay between physics and mathematics mediated by singularities.

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#### Explaining Regularities

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# Wigner's Problem



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The "unreasonable effectiveness of mathematics in the natural sciences."

 The "appropriateness of the language of mathematics for the formulation of the laws of physics" is a "miracle"—"a wonderful gift which we neither understand nor deserve." [Wigner(1967), p. 237]

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# Wigner's Problem



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Remarks on the Philosophy of Mathematics The "unreasonable effectiveness of mathematics in the natural sciences."

- The "appropriateness of the language of mathematics for the formulation of the laws of physics" is a "miracle"—"a wonderful gift which we neither understand nor deserve." [Wigner(1967), p. 237]
- Suggestion: The study of the asymptotics of various functions and the reasons for their usefulness in mathematical physics may very well help to dispel the appearance of the miraculous (and for some, the divine).

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# The Applicability of Mathematics



- Mark Steiner in, The Applicability of Mathematics as a Philosophical Problem, argues that contemporary mathematical physicists employ various (analogical) strategies in forming and discovering new theories about unobservable aspects of the world. He argues that these analogical strategies are fully anthropocentric—that is, that they depend for their success, upon humans having a special place in the world.
- He aims to maintain the miraculous nature of the applicability of mathematics to the world by demonstrating repeatedly that physicists employ analogies in discovery that are tied primarily to the formalism of existing theories and that cannot in any way be taken to be physically motivated.

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# The Applicability of Mathematics



 The discussion here provides some reasons to think that the world actually dictates that we employ asymptotic representations to best understand what we observe.

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# The Applicability of Mathematics



- The discussion here provides some reasons to think that the world actually dictates that we employ asymptotic representations to best understand what we observe.
  - The very nature of many phenomena that are investigated—their stability under changes in detail, their association with singularities or places of breakdown of various theories—constrains the nature of those investigations.

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## The Applicability of Mathematics



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  - The very nature of many phenomena that are investigated—their stability under changes in detail, their association with singularities or places of breakdown of various theories—constrains the nature of those investigations.
  - The dominant and real features of phenomena require that we employ limiting idealizations in forming the mathematical equations with which we may represent the phenomena.

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# The Applicability of Mathematics



I hope that the discussion here leads us to question the anthropocentric role of the mathematician's appreciation for beauty (or formal analogy) as an important criterion for what arguably should be paradigm examples of mathematics' applicability to the world; namely, the extraordinary effectiveness of mathematical asymptotics in describing and explaining the physical world.

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