Title: Finite States in Four Dimensional Quantized Gravity

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Abstract: The semiclassical-quantum correspondence (SQC) is a new principle which has enabled the explicit solution of the quantum constraints of GR in the full theory in the Ashtekar variables for gravity coupled to matter. The solutions, which constitute the physical space of states implementing the quantum dynamics of GR in the Dirac procedure, include a special class of states known as the generalized Kodama states (GKod). The GKodS can be seen as an analogue of the pure Kodama state (Kod) when quantum gravity (QGRA) is coupled to matter fields quantized on the same footing. The criterion for finiteness stems from a precise cancellation of the ultraviolet singularities stemming from the quantum Hamiltonian constraint, allowing for an exact solution. This signifies the following developments for 4D QGRA: (i) Equivalence among the Dirac, reduced phase, geometric and path integration approaches to quantization for GKods; (ii) A generalization of topological field theory to include matter fields via the instanton representation of GKod; (iii) A possible mechanism to establish 4D QGRA, via tree networks, as a renormalizable theory (iv) A direct link from QGRA to Minkoswki spacetime physics, which would enable tests of 4D QGRA without the necessity to access the Planck scale (v) A third-quantized analogy to second quantized spin network states implementing the quantum dynamics of GR. The aforementioned algorithm is designed to construct explicit solutions to the constraints of the full theory by inspection, while implementing any desired â€˜boundaryâ€™ conditions on the states necessary to reduce to the appropriate semiclassical limit. Conversely, the finite states of 4D QGRA can place severe restrictions on phenomena occurring in the weak gravitational limit below the Planck scale. While we demonstrate this for the GKodS in this talk, the procedure can be applied to obtain a family of states labeled by two arbitrary functions of position, which possess the requisite Hilbert space structure in the limit where the matter fields are turned off. Remaining areas of research in progress include the illumination of the Hilbert space structure of the GKodS, analysis of various models for which the SQC can produce tractable solutions, in the full theory and in minisuperspace, and the addressal of any issues of interest regarding the mathematical rigor of the states.

Finite States in Four-dimensional Quantized Gravity.

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Implications of a finite state of QGRA

- Formal equivalence of Dirac, RPS, path integration and geometric quantization procedures
  - Would imply convergence of the path integral for GR
  - Hence the effective action for gravity would be finite
- No need to access the Planck scale to test for or measure quantum gravitational effects
  - QGRA would place constraints on phenomena below the Planck scale consistent with a finite state, which cannot be deduced based on classical GR or QFT alone
  - This would imply a verifiable semiclassical limit
- No more cosmological constant problem
  - Λ is a fundamental expansion parameter of the theory
  - Nonperturbative QGRA would explain why Λ is so small

The pure Kodama state

$$\Psi_{Kod}[A] = \exp\left[-6(G\Lambda\hbar)^{-1} I_{CS}[A]\right]$$

- Chern-Simons functional of a self-dual SU(2) connection in the Ashtekar variables $A_i^a(\boldsymbol{x}, t), \widetilde{\sigma}_a^i(\boldsymbol{x}, t)$
- Exact solution to the quantum constraints of vacuum GR with cosmological term, discovered by Hideo Kodama: (Hideo Kodama: Phys. Rev. D42 (1990) 2548)
- Has a well-defined semiclassical limit below the Planck scale of quantum DeSitter spacetime (Lee Smolin: hep-th/0207079)
- Candidate for the ground state wavefunction of the universe
- Open issues with the Kodama state
  - Normalizability and Unitarity (Ed Witten objections by analogy to Yang-Mills theory: gr-qc/0306083)

Why is $\Psi_{Kod}[A]$ special?

- Only known nontrivial exact solution to the constraints of quantum GR in the **full** theory
- Both a semiclassical and **exact** quantum state
- Resembles TQFT, which is renormalizable/finite

$$\Psi_{Kod}[A] = \exp\left[-6(\hbar G\Lambda)^{-1}\int_{\partial M}\mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)\right]$$

$$= \exp\left[-6(\hbar G\Lambda)^{-1}\int_{M}\mathrm{tr}F\wedge F\right] \sim \exp\left[\int_{M}\mathrm{tr}\left(\Sigma\wedge F + \frac{\hbar G\Lambda}{24}\Sigma\wedge\Sigma\right)\right]$$

- But if our universe also includes quantized matter fields coupled to quantized gravity:
- How can $\Psi_{Kod}[A]$ be generalized accordingly?

Generalized Kodama states $\Psi_{GKod}[A_i^a(x), \psi^A(x)]$

- Is there an analogous wavefunction of the universe that
  - Exactly solves the quantum constraints of 4D general relativity, for the full theory, coupled to matter fields quantized on the same footing?
  - Bears an analogous topological representation in the presence of matter fields as the pure Kodama state does to 4D TQFT devoid of matter?
  - Is both a semiclassical and an exact quantum state to all orders?
  - Has a well-defined semiclassical limit below the Planck scale (e.g. QFT of matter fields on Minkowski spacetime, or whatever could be consistent with accelerator experiments or observation)?
- This leads to the principle of the Semiclassical/Quantum correspondence (SQC), which requires a consistent and exhaustive application of the CCR in the functional Schrodinger representation as a necessary condition for a finite state of full quantum gravity (Eyo Ita: Class. Quantum Grav. (2008) 125001 and 125002, and articles on gr-qc archive)

General procedure

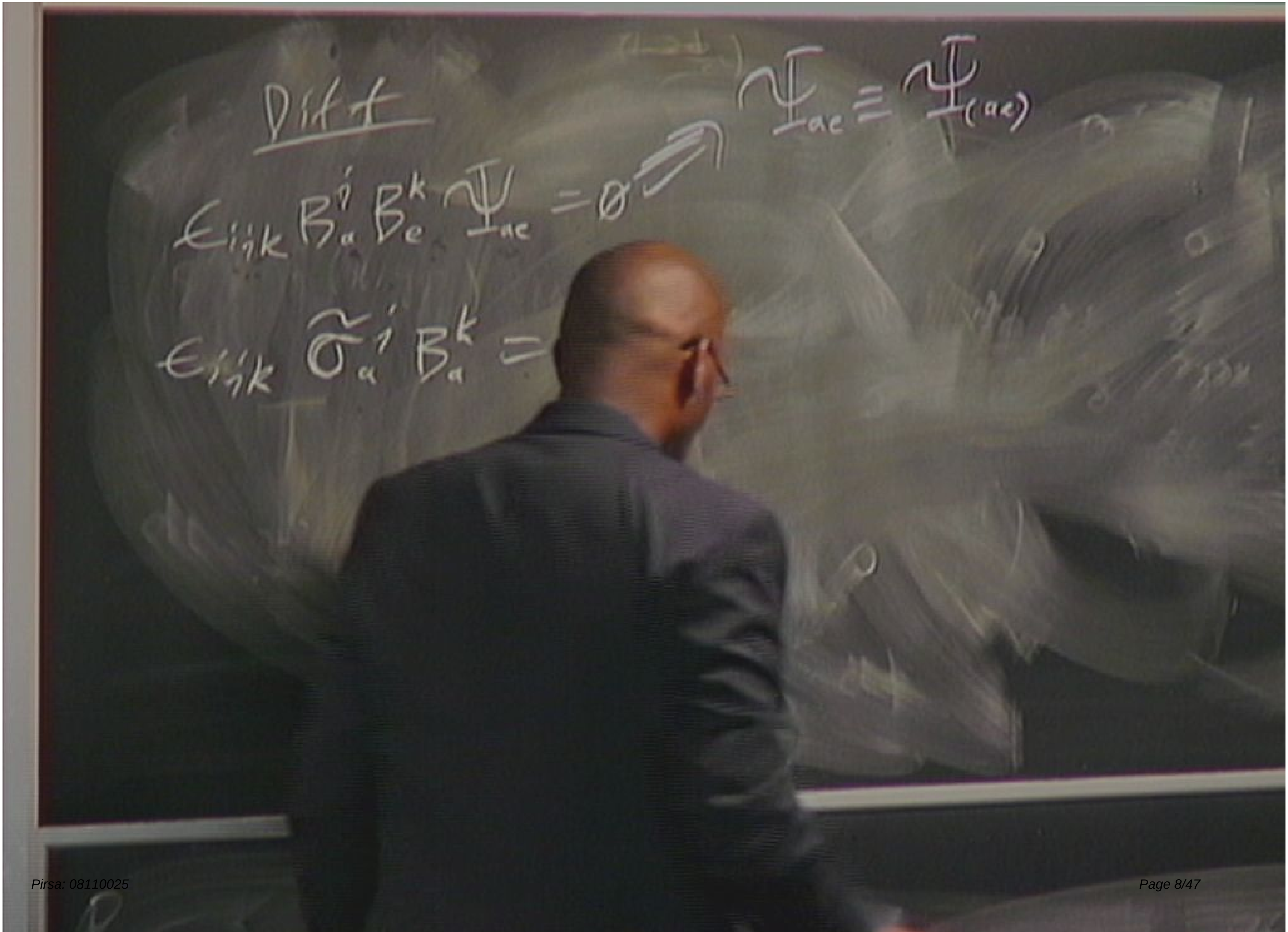
- CDJ Ansatz (Capovilla/Dell/Jacobson):
  - Solves classical constraints of GR with matter fields (Thiemann: Class. Quantum Grav. 10 (1993)1907-1921)

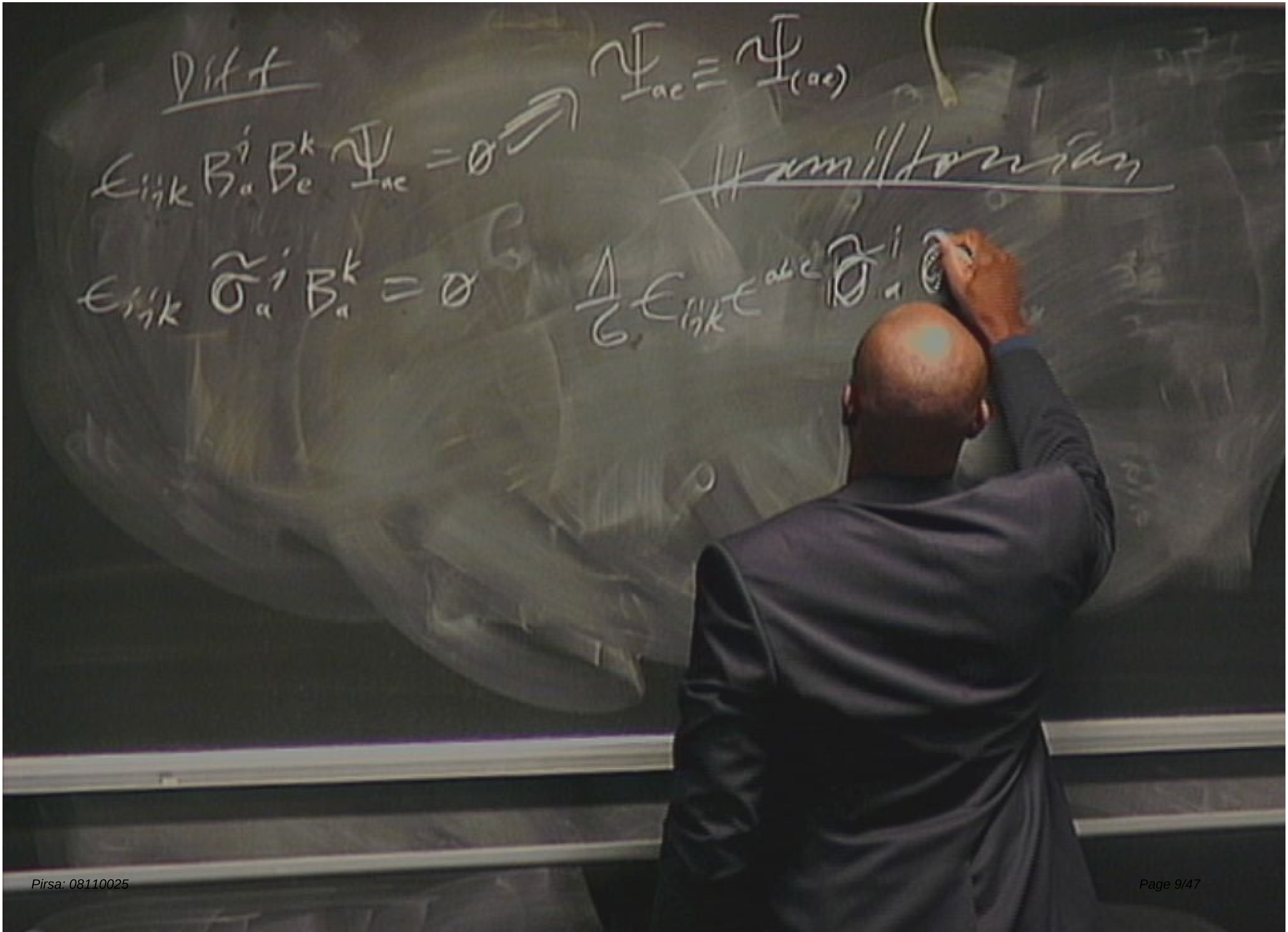
$$\widetilde{\sigma}^i_a(x) = \Psi_{ae}[A^a_i(x), \psi^A(x)]B^i_e(x)$$

- But we want the quantum constraints to be solved
  - Quantize the CDJ matrix subject to the SQC
  - Expand quantum statistical fluctuations about Ψ_{Kod}

$$\Psi_{ae} = -\frac{6}{\Lambda}\Big(\delta_{ae} + \frac{\Lambda}{6}\epsilon_{ae}\Big)$$

  - Use the D.O.F. of the CDJ matrix to eliminate all matter-induced UV singularities, not as a Lagrange multiplier


Diff
$\Psi_{ae} \equiv \Psi_{(ae)}$
$\epsilon_{ijk} B^j_a B^k_e \Psi_{ae} = 0$
$\epsilon_{ijk} \tilde{\sigma}^j_a B^k_a =$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$\epsilon_{ijk} B^j_a B^k_e \Psi_{ae} = 0$
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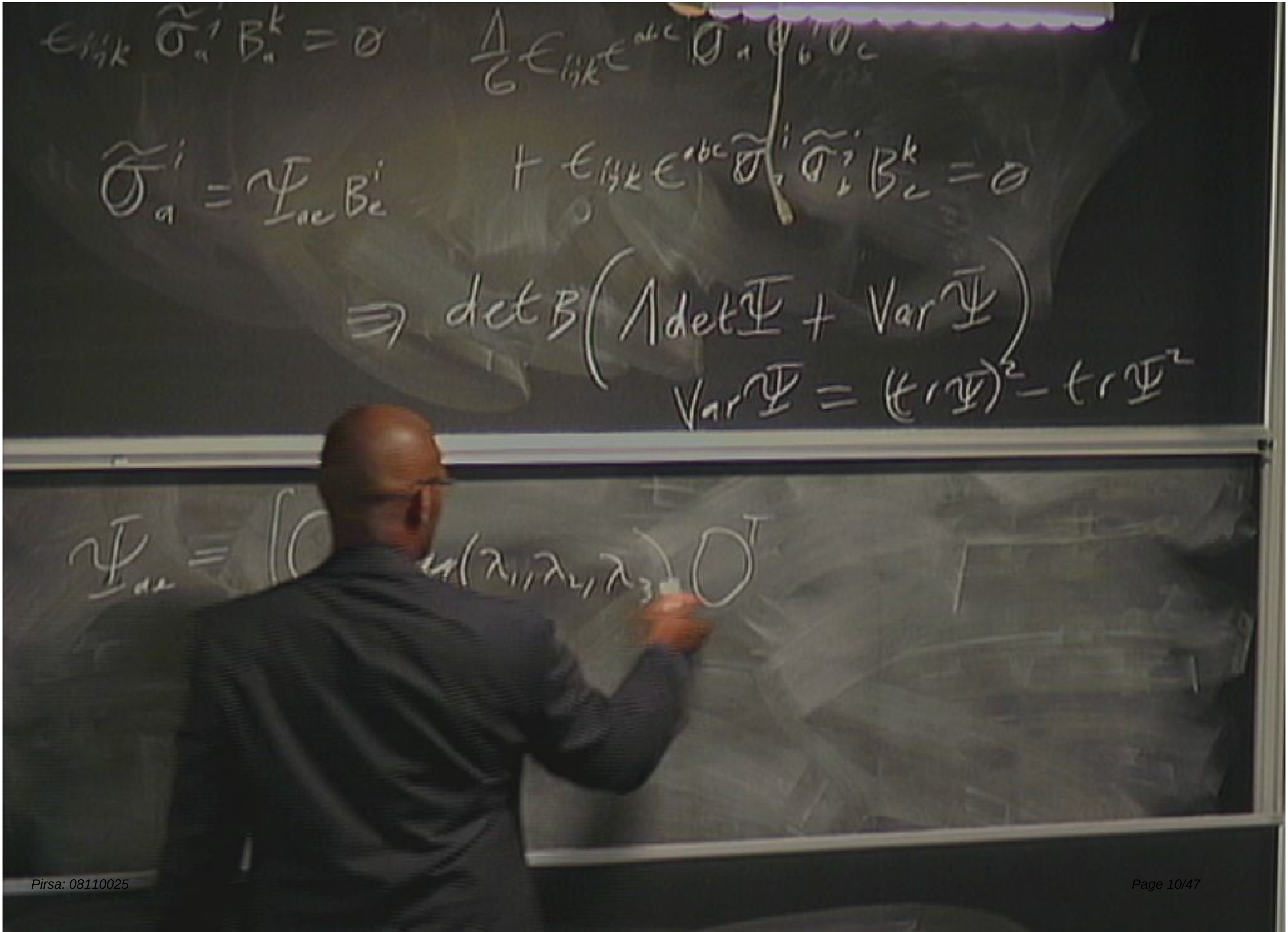
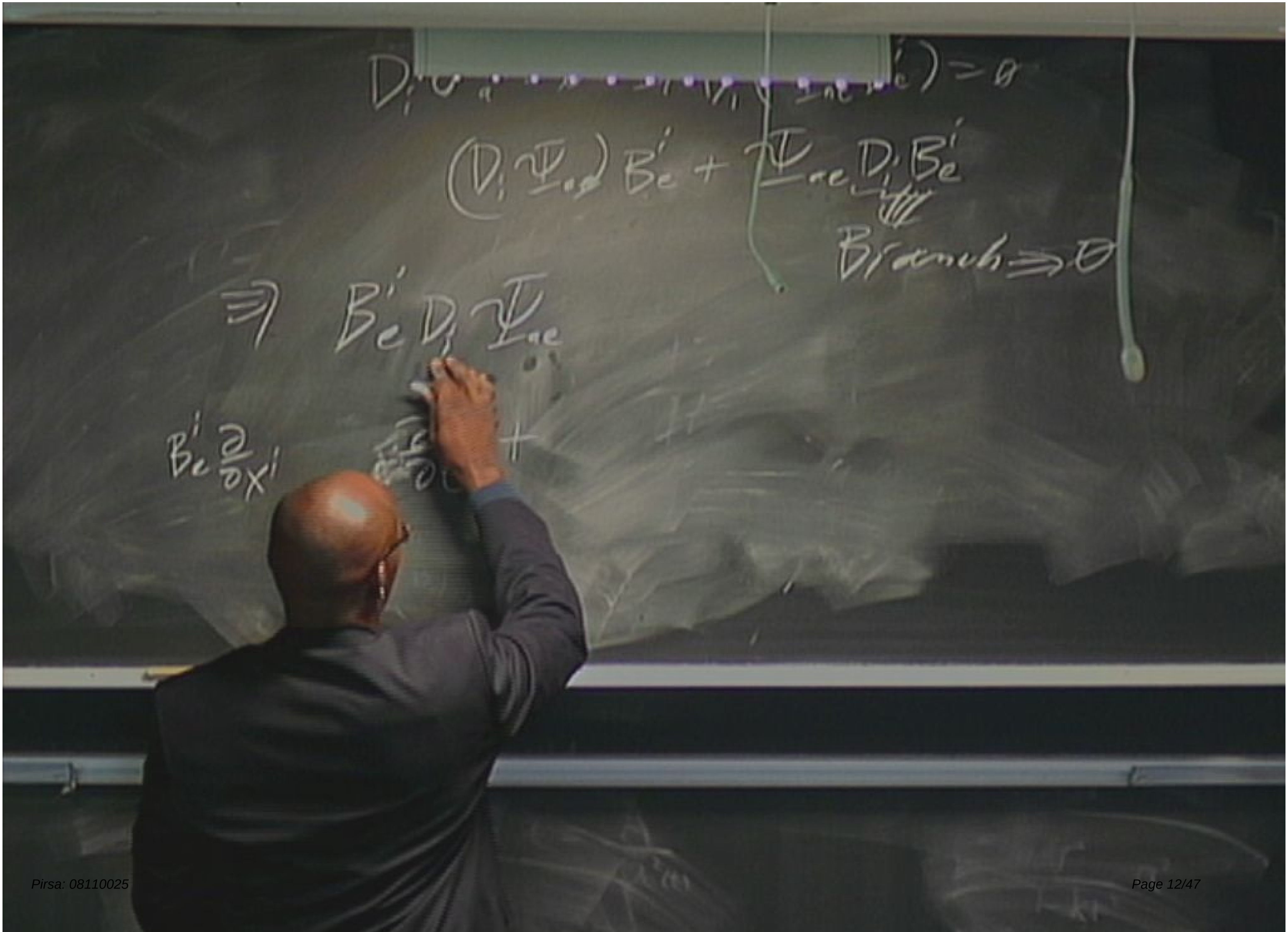
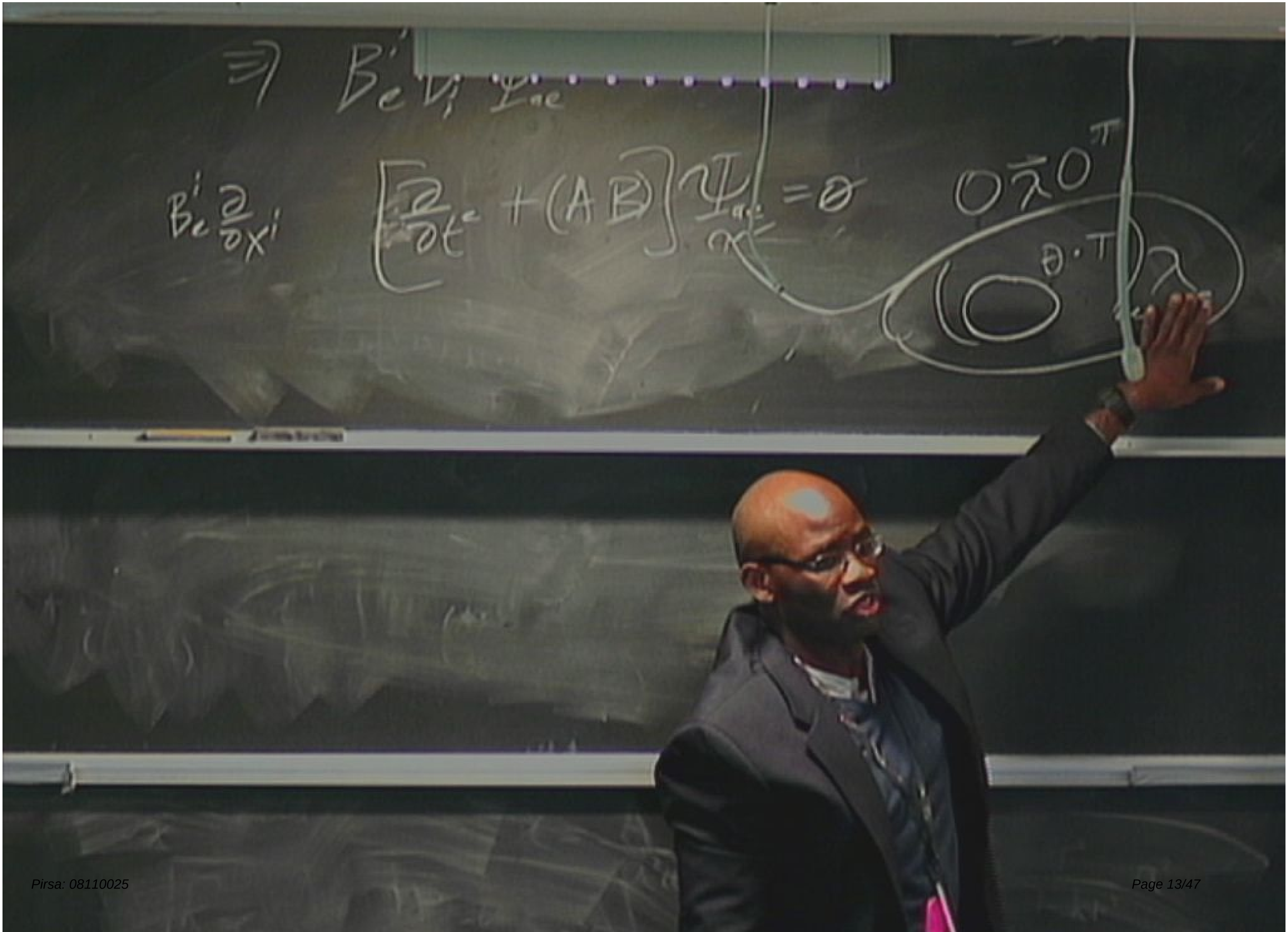
Hamiltonian

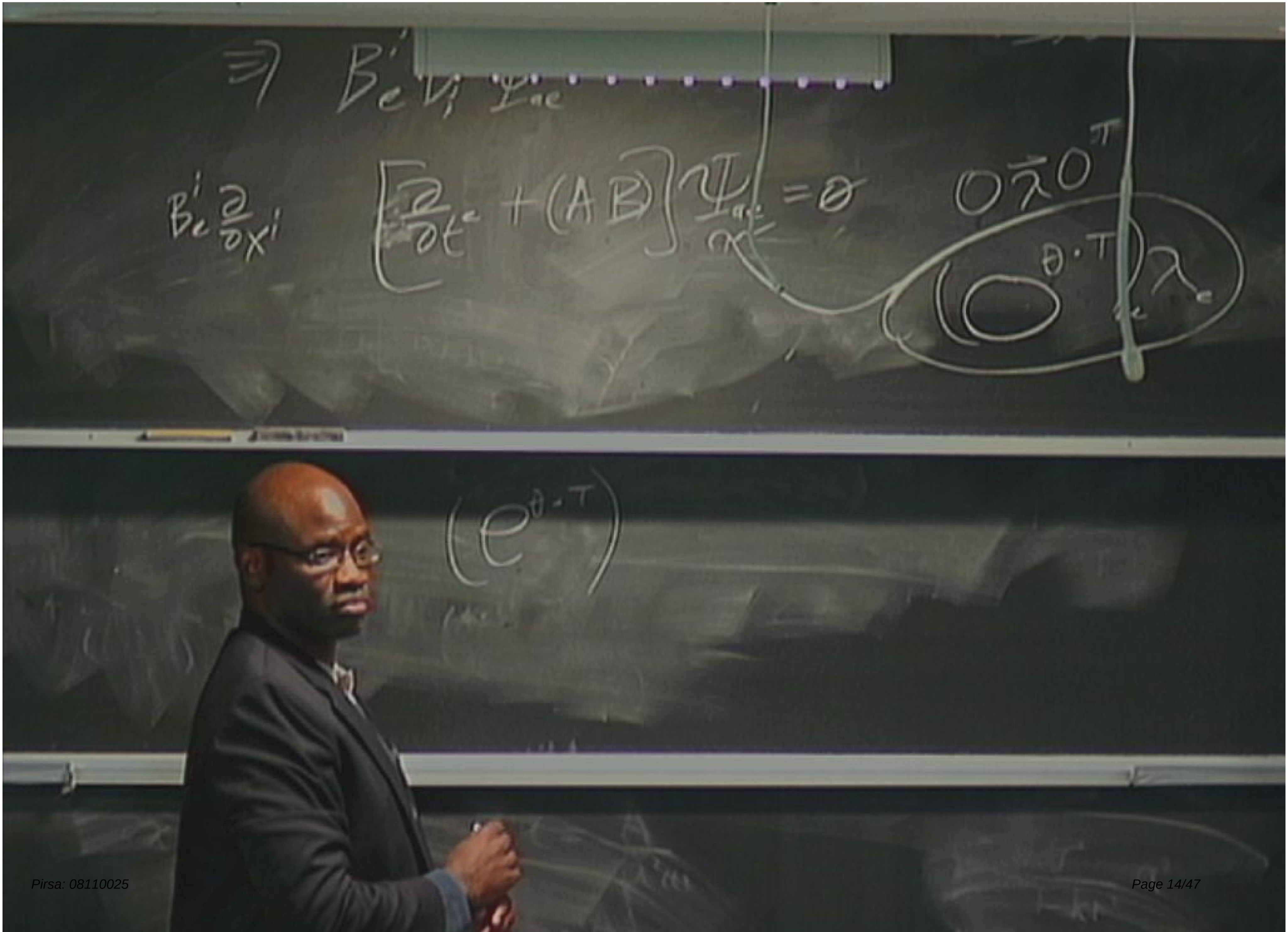

$\epsilon_{ijk}\tilde{\sigma}^j_a B^k_a = 0$
$\frac{\Lambda}{6}\epsilon_{ijk}\epsilon^{abc}\tilde{\sigma}^i_a\tilde{\sigma}^j_b\tilde{\sigma}^k_c$
$\tilde{\sigma}^i_a = \Psi_{ae}B^i_e$
$+\epsilon_{ijk}\epsilon^{abc}\tilde{\sigma}^i_a\tilde{\sigma}^j_b B^k_c = 0$
$\Rightarrow \det B(\Lambda \det\Psi + \mathrm{Var}\,\Psi)$
$\mathrm{Var}\,\Psi = (\mathrm{tr}\,\Psi)^2 - \mathrm{tr}\,\Psi^2$
$\Psi_{ae} =$

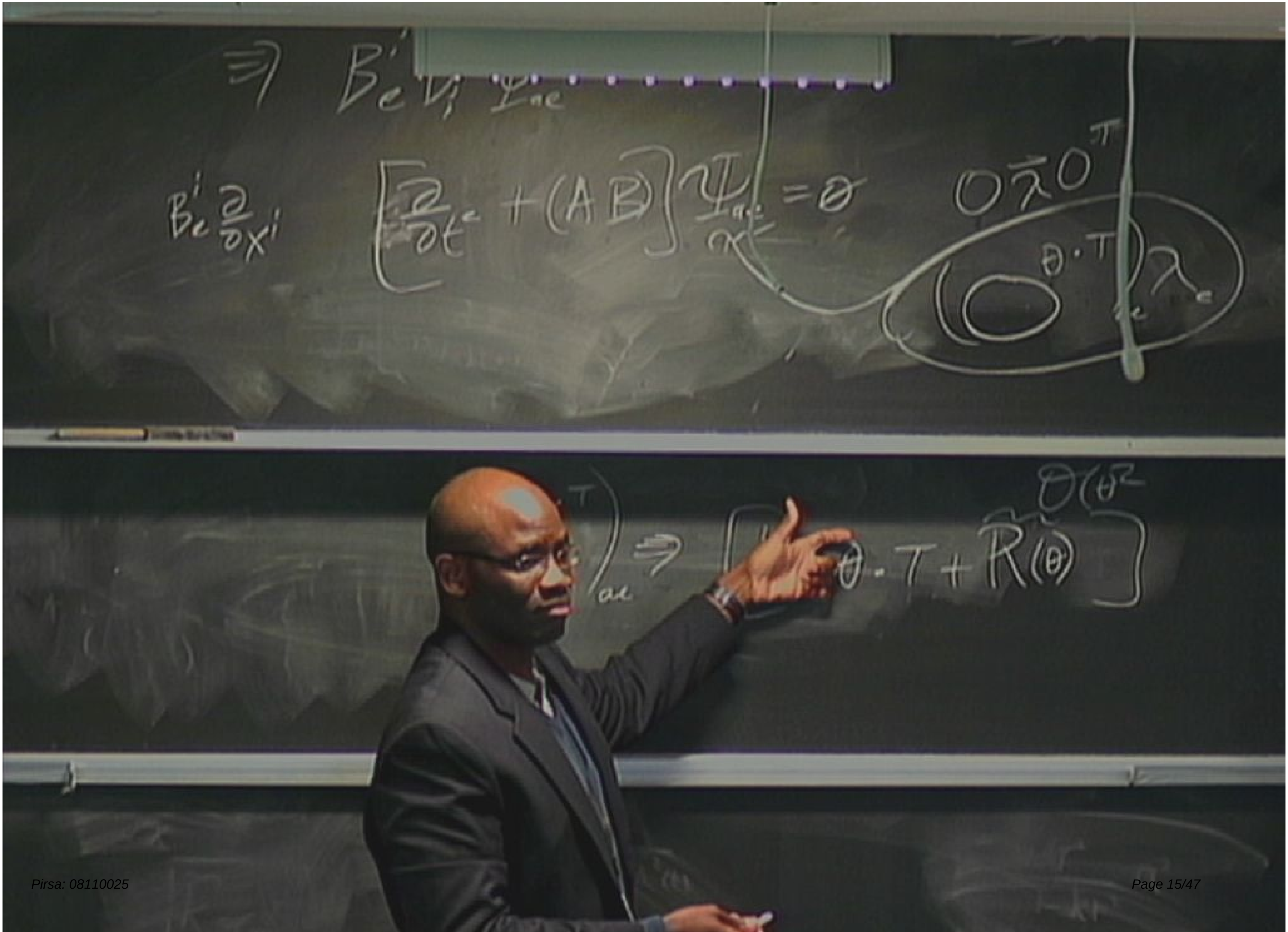
$$\Phi_{ac} = \left[O \, \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3) \, O^T \right]_{ac}$$

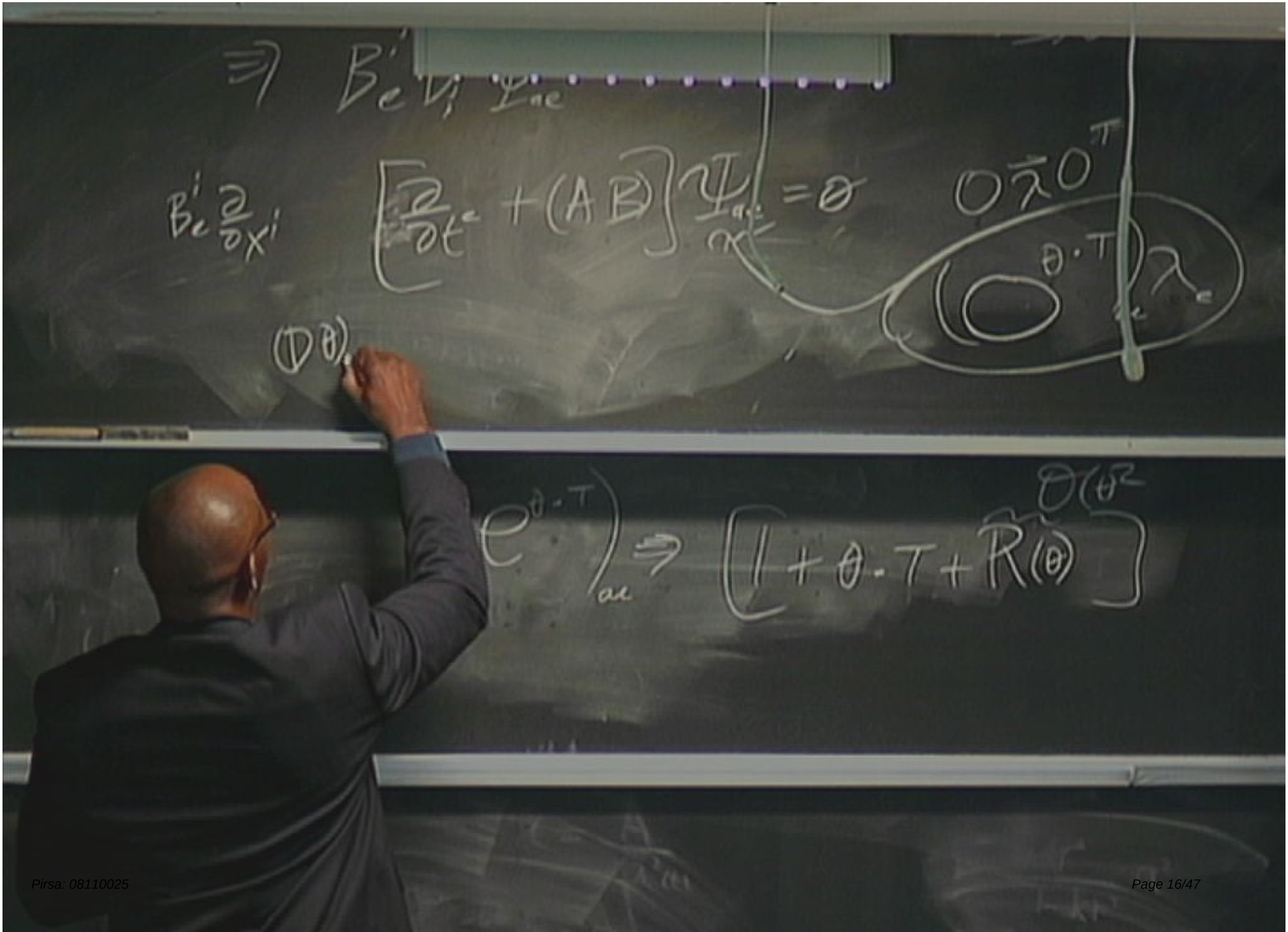
$$O_{ac} = \left(e^{\theta \cdot T} \right)_{ac}$$

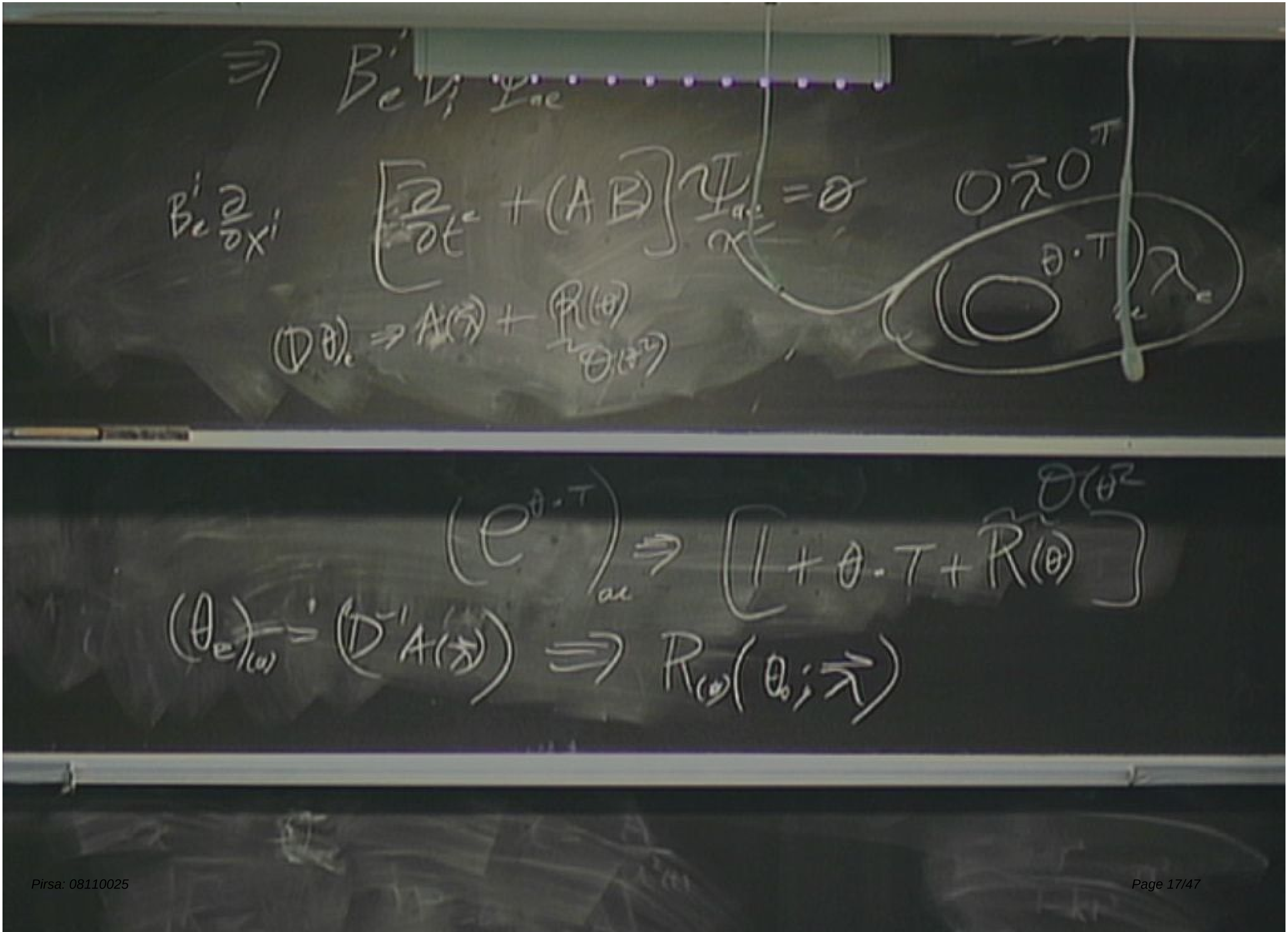












$$\theta_{n+1} = D^{-1} A(\vec{x}) + R(\vec{x}, \vec{\theta}_{(n)})$$

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$$\vec{\theta} = \lim_{n \to \infty} \vec{\theta}_n$$

$$\theta_{n+1} = D^{-1} A(\vec{\lambda}) + R(\vec{\lambda}, \vec{\theta}_{(n)})$$

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$$\tilde{\sigma}^{i}_{\theta} = \Psi_{ne} B^{i}_{e}$$

$$(\lambda_1, \lambda_2)$$

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Kinematic constraints of 4D QGRA

- Diffeomorphism constraint:

$$\epsilon_{aef}\epsilon_{ef}(x) - G\frac{B^i_a(x)}{\det B(x)}\pi_A(x)(D_i\psi(x))^A = 0$$

- Gauss' law constraint:

$$\left(\delta^{fg}B^i_a(x)\frac{\partial}{\partial x^i} + C^{fg}_a(x)\right)\epsilon_{fg}(x) + G\pi_A(x)(T_a)^A_B\psi^B(x) = 0$$

- Mixed partials condition (new, due to SQC):

$$\pi_A(x) - \frac{i}{G}\int_\Gamma B^i_e(x)\delta A^a_i(x)\frac{\partial\epsilon_{ae}}{\partial\psi^A} = f_A(\psi^B(x))$$

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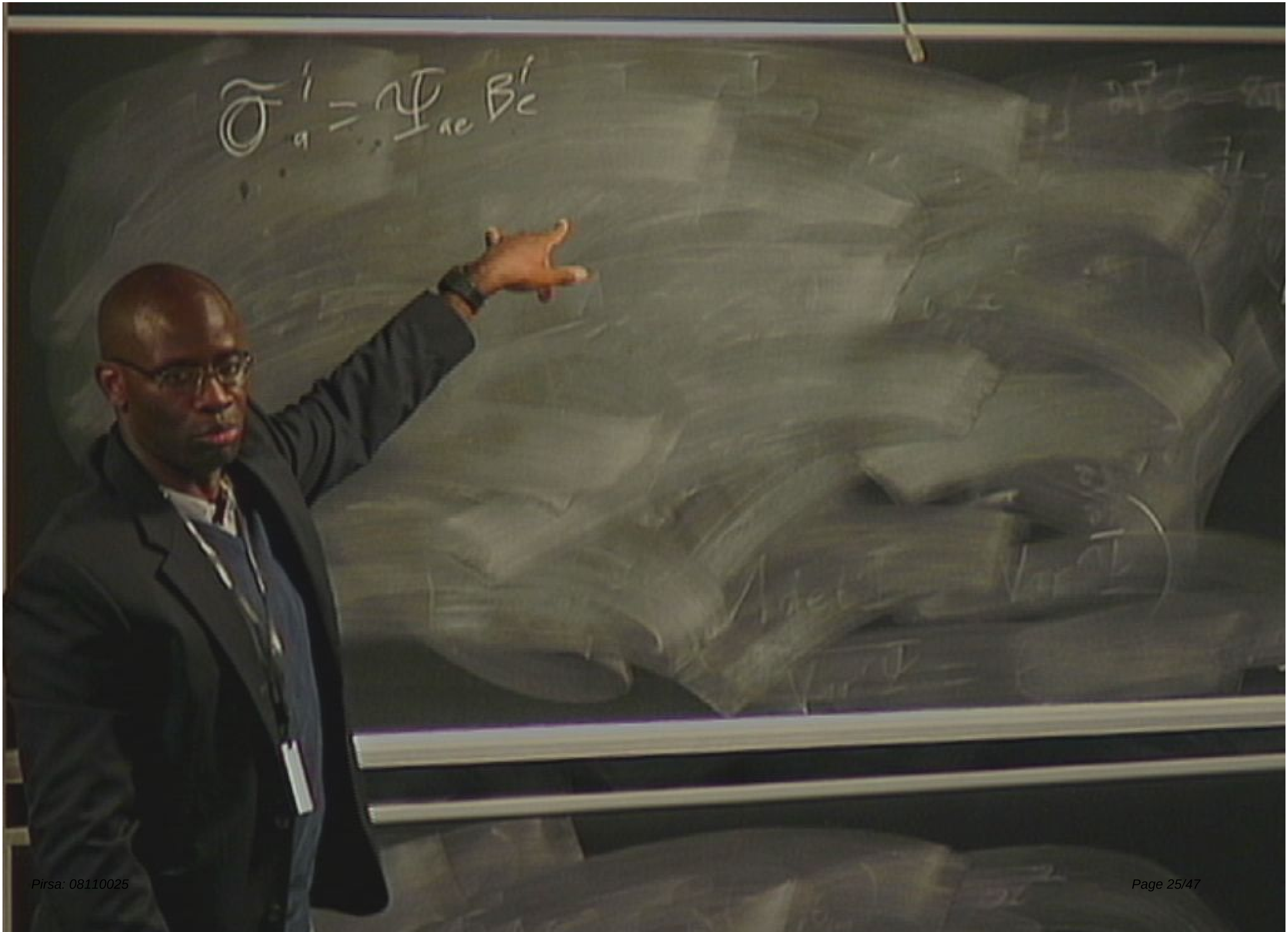
Quantum Hamiltonian constraint

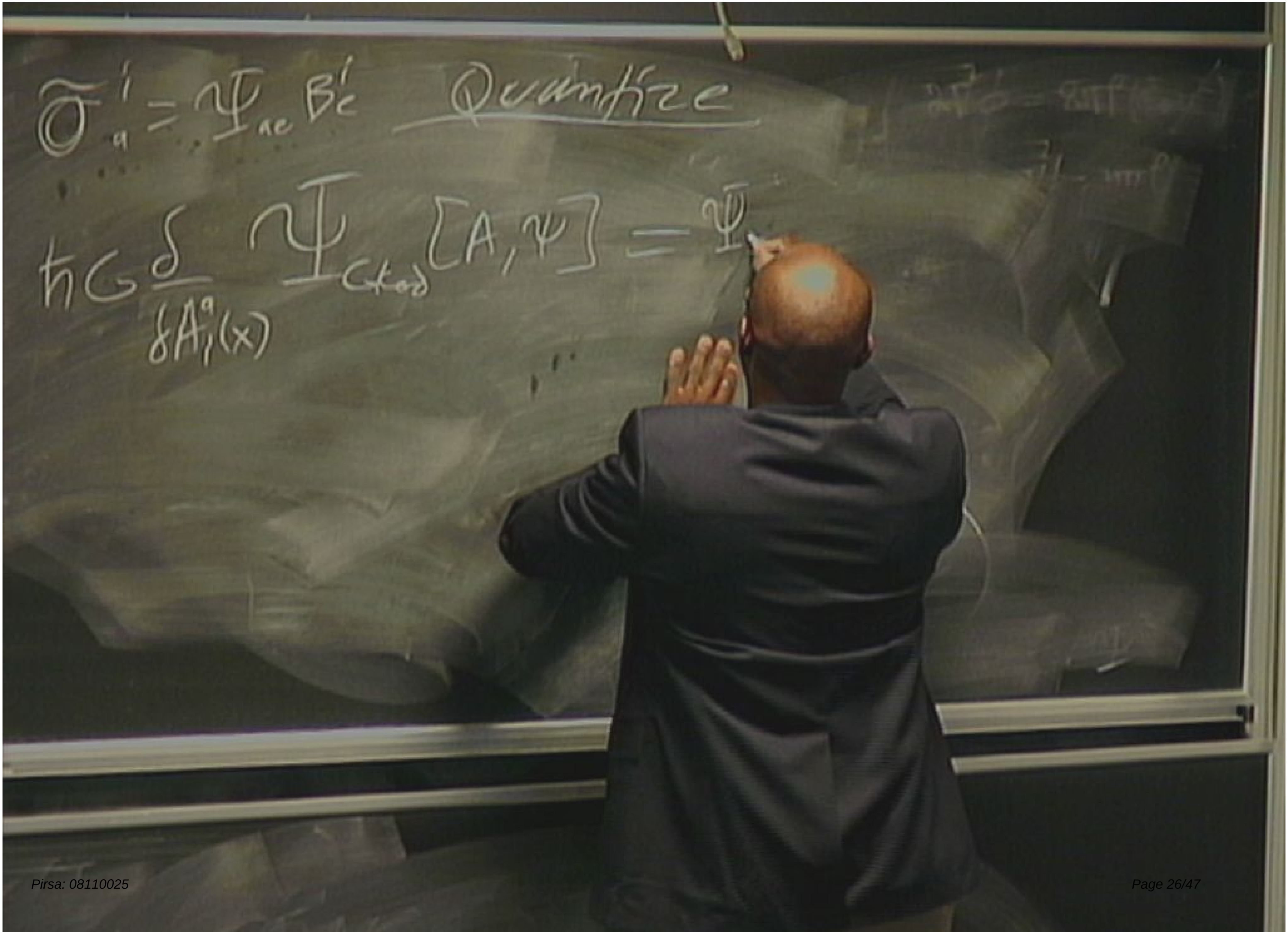
- SQC broken due to matter fields: Dilemma
  - (i) No physical states if constraints not satisfied
  - (ii) Problem of time in QGRA if constraints are satisfied

$$\hat{H}\Psi_{GKod}[A^a_i,\psi^A] = \Big[\frac{\Lambda}{6}\hbar^3 G^3 \epsilon^{abc}\epsilon_{ijk}\frac{\delta}{\delta A^a_i(x)}\frac{\delta}{\delta A^b_j(x)}\frac{\delta}{\delta A^c_k(x)}$$

$$\hbar^2 G^2 \epsilon^{abc}\epsilon_{ijk}\frac{\delta}{\delta A^b_j(x)}\frac{\delta}{\delta A^c_k(x)}B^i_a(x) + \hat{\Omega}[\psi^A,\delta/\delta\psi^A,\delta/\delta A^a_i]\Big]\Psi_{GKod}[A^a_i,\psi^A]$$

$$= \big(q_0(x) + \hbar G\delta^{(3)}(0)q_1(x) + (\hbar G\delta^{(3)}(0))^2 q_2(x)\big)\big|\Psi_{GKod}[A^a_i,\psi^A] = 0$$


$\tilde{\sigma}^i_a = \Psi_{ae} B^i_c$


Quantize

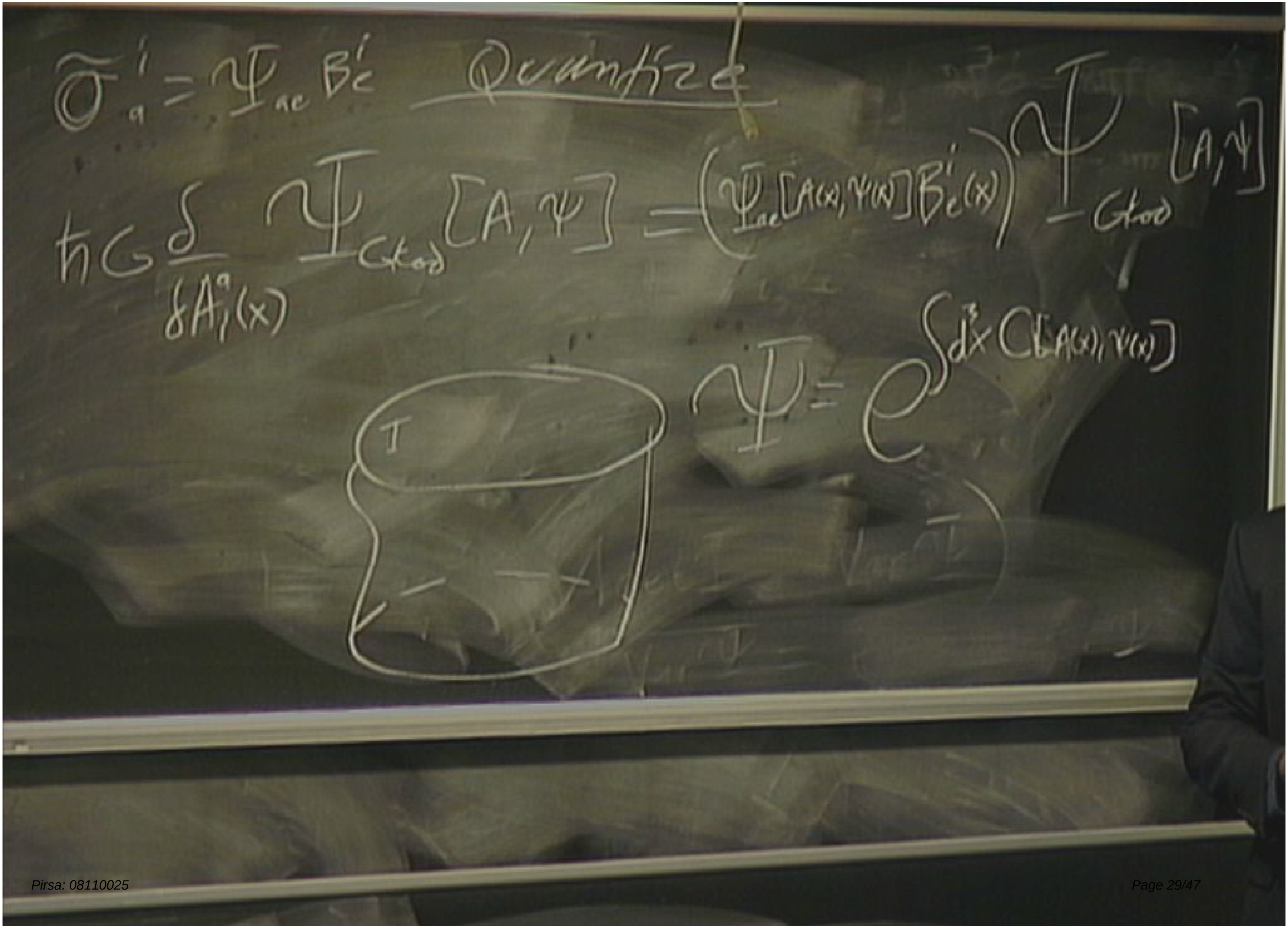
$$\tilde{\sigma}^i_a = \Psi_{ac} B^i_c$$

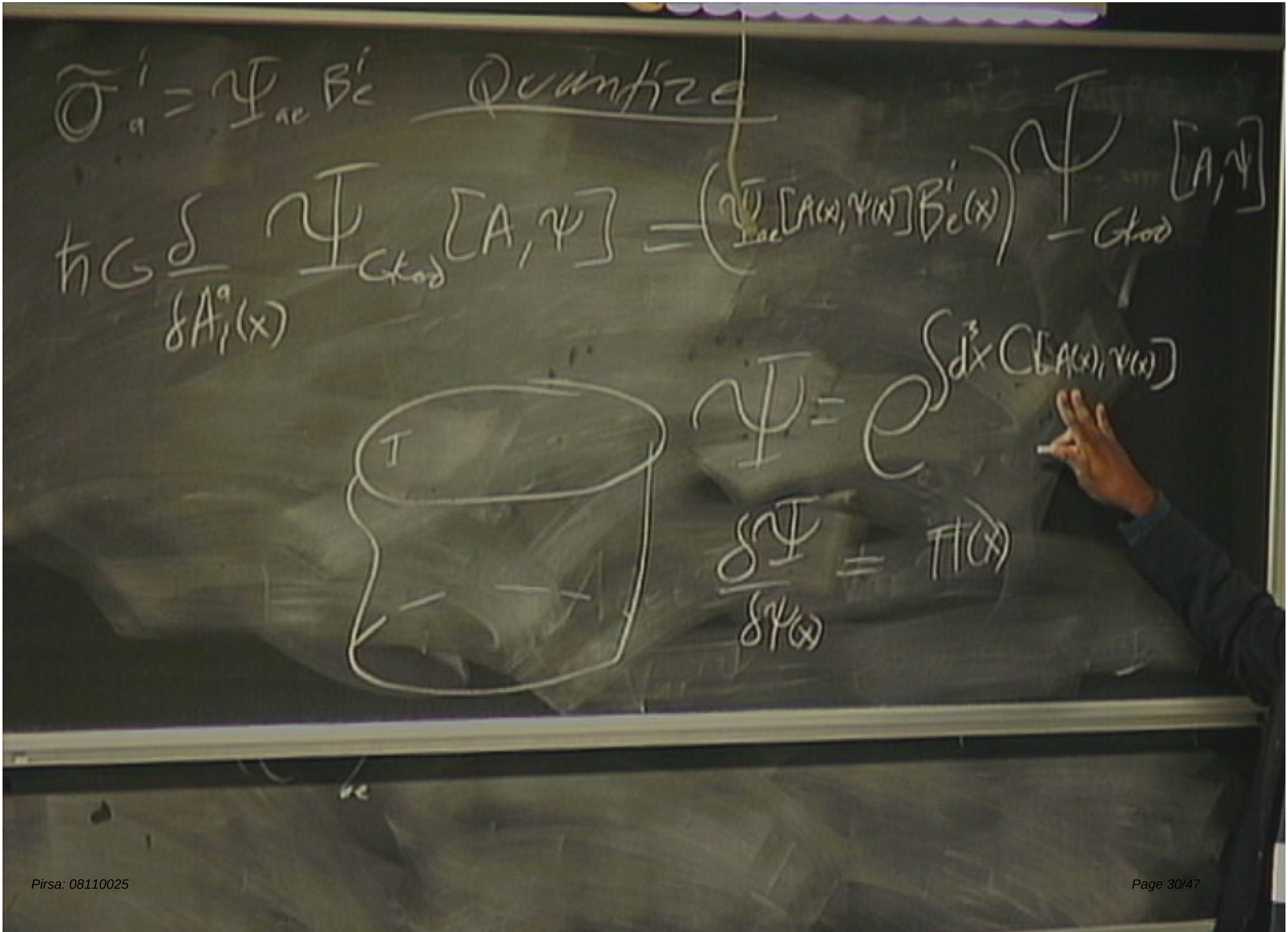
Quantize

$$\hbar G \frac{\delta}{\delta A^a_i(x)} \Psi_{CKod}[A,\psi] = \left(\Psi_{ac}[A(x),\psi(x)] B^i_c(x)\right) \Psi_{CKod}[A,\psi]$$

$$\tilde{\sigma}^i_a = \Psi_{ae} B^i_e \qquad \underline{\text{Quantized}}$$

$$\hbar G \frac{\delta}{\delta A^a_i(x)} \tilde{\Psi}_{Kod}[A,\psi] = \left(\Psi_{ae}[A(x),\psi(x)] B^i_e(x)\right) \tilde{\Psi}_{Kod}[A,\psi]$$


$\tilde{\sigma}^i_a = \Psi_{ac} B^i_c$
Quantize
$\hbar G \frac{\delta}{\delta A^a_i(x)} \Psi_{Chod}[A,\psi] = \left(\Psi_{ac}[A(x),\psi(x)] B^i_c(x)\right) \Psi_{Chod}[A,\psi]$
$\Psi = e^{\int d^3x\, C[A(x),\psi(x)]}$
T


Quantize

$\tilde{\sigma}^i_a = \Psi_{ae} B^i_e$ <u>Quantize</u>

$$\hbar G \frac{\delta}{\delta A^a_i(x)} \tilde{\Psi}_{CKod}[A,\psi] = -\left(\Psi_{ae}[A(x),\psi(x)] B^i_e(x)\right) \tilde{\Psi}_{CKod}[A,\psi]$$

$$\tilde{\Psi} = e^{\int d^3x\, C[A(x),\psi(x)]}$$

$$\frac{\delta \tilde{\Psi}}{\delta \psi(x)} = \pi(x) \tilde{\Psi}$$

$\tilde{\sigma}^i_a = \Psi_{ac} B^i_c$ Quantize

$$\hbar G \frac{\delta}{\delta A^a_i(x)} \Psi_{Chod}[A,\psi] = \left(\Psi_{ac}[A(x),\psi(x)] B^i_c(x)\right) \Psi_{Chod}[A,\psi]$$

$$\Psi = e^{\int d^3x\, C[A(x),\psi(x)]}$$

$$\frac{\delta \Psi}{\delta \psi(x)} = \pi(x)\, \Psi \qquad e^I$$

$\tilde{\sigma}^i_a = \Psi_{ac} B^i_c$ <u>Quantize</u>

$$\hbar G \frac{\delta}{\delta A^a_i(x)} \Psi_{CK\&D}[A,\psi] = -\left(\Psi_{ac}[A(x),\psi(x)]\, B^i_c(x)\right) \Psi_{CK\&D}[A,\psi]$$

$$\Psi = e^{\int d^3x\, C[A(x),\psi(x)]}$$

$$\Psi \quad e^{I}$$

$$\frac{\delta \Psi}{\delta \psi(x)} = \Pi(x)\, \Psi$$

$$\left(\frac{\delta I}{\delta A(x)}\right) e^{I}$$

Quantum Hamiltonian constraint

- SQC broken due to matter fields: Dilemma
  - (i) No physical states if constraints not satisfied
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$$\hat{H}\Psi_{GKod}[A^a_i,\psi^A] = \Big[\frac{\Lambda}{6}\hbar^3 G^3 \epsilon^{abc}\epsilon_{ijk}\frac{\delta}{\delta A^a_i(x)}\frac{\delta}{\delta A^b_j(x)}\frac{\delta}{\delta A^c_k(x)}$$

$$-\hbar^2 G^2 \epsilon^{abc}\epsilon_{ijk}\frac{\delta}{\delta A^b_j(x)}\frac{\delta}{\delta A^c_k(x)}B^i_a(x) + \hat{\Omega}\big[\psi^A,\delta/\delta\psi^A,\delta/\delta A^a_i\big]\Big]\Psi_{GKod}[A^a_i,\psi^A]$$

$$= \big(q_0(x) + \hbar G\delta^{(3)}(0)q_1(x) + (\hbar G\delta^{(3)}(0))^2 q_2(x)\big)\big|\Psi_{GKod}[A^a_i,\psi^A] = 0$$

Standard form for large class of models coupled to gravity: Regularization-independent at canonical level

- 9+N system for CDJ deviation matrix

$$O^{ae}_{gh}\epsilon_{ae} + \Lambda\Sigma^{aebf}_{gh}\epsilon_{ae}\epsilon_{bf} + \Lambda^2 E^{abcdef}_{gh}\epsilon_{ad}\epsilon_{be}\epsilon_{cf} = GQ_{gh}$$

- Identical structure to EOM of Yang Mills theory, which is renormalizable and finite

$$(\delta^\gamma_\nu\partial^2 - \partial^\gamma\partial_\nu)A^a_\nu + g\big(f^{ade}(\partial^\mu A^d_\mu)A^e_\nu - f^{abc}A^c_\mu\partial_\nu A^{b\mu}\big)$$
$$+g^2 f^{abc} f^{bde} A^c_\mu A^{d\mu} A^e_\nu - J^a_\nu = 0$$

Standard form for large class of models coupled to gravity: Regularization-independent at canonical level

- 9+N system for CDJ deviation matrix

$$O_{gh}^{ae}\epsilon_{ae} + \Lambda\Sigma_{gh}^{aebf}\epsilon_{ae}\epsilon_{bf} + \Lambda^2 E_{gh}^{abcdef}\epsilon_{ad}\epsilon_{be}\epsilon_{cf} = GQ_{gh}$$

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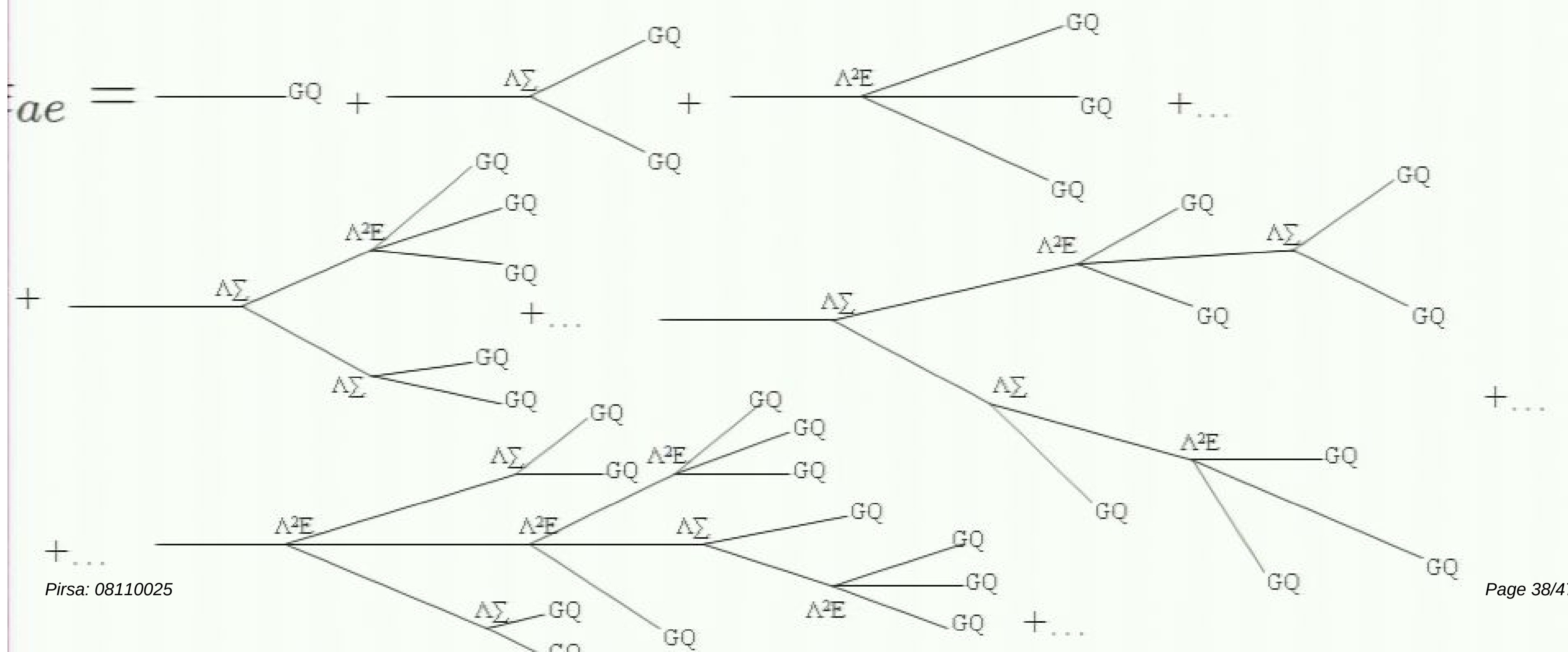
$$(\delta_{\nu}^{\gamma}\partial^2 - \partial^{\gamma}\partial_{\nu})A_{\nu}^{a} + g\big(f^{ade}(\partial^{\mu}A_{\mu}^{d})A_{\nu}^{e} - f^{abc}A_{\mu}^{c}\partial_{\nu}A^{b\mu}\big) + g^2 f^{abc}f^{bde}A_{\mu}^{c}A^{d\mu}A_{\nu}^{e} - J_{\nu}^{a} = 0$$

General solution by Feynman-like diagrammatic tree networks

- Third-quantized generalization of spin network state solving the quantum constraints of GR



Wavefunction of the Universe

- Perturbative expansion in dimensionless coupling constant GΛ or in GΛ' = GΛ+G²V(ψ)

$$\Psi_{ae} = -\Big[\frac{6}{\Lambda}\delta_{ae} + \sum_{n=0}^{\infty}(G\Lambda')^n\Big(\prod_i \Sigma^{a_ie_ib_if_i}_{g_ih_i}\prod_j E^{g_jh_j}_{a_je_jb_jf_jc_jd_j}\prod_k O^{c_kd_k}_{a_kb_k}Q_{c_kd_k}[f]\Big)_{ae}$$

$$\Psi_{GKod}[A^a_i,\psi^A] = \exp\Big[\int_M d^4x\big((\hbar G)^{-1}\Psi_{ae}B^i_e\dot{A}^a_i + \frac{i}{\hbar}f_A\dot{\psi}^A\big)\Big]$$

- Choose f to reproduce SR for matter below the Planck scale: sets boundary condition on QGRA

Representations of Ψ_{GKod}

- Boundary term on final spatial hypersurface: Problem of time has disappeared

$$\mathrm{xp}\Big[(\hbar G)^{-1}\int_{\partial M} d^3x \int_{\Gamma_A} \Psi_{ae} B^i_e \delta A^a_i\Big]\exp\Big[\frac{i}{\hbar}\int_{\partial M} d^3x \int_{\Gamma_\psi} f_A \delta\psi^A\Big]$$

- Instanton representation: TQFT plus matter

$$\exp\Big[(\hbar G)^{-1}\int_M \Psi_{ae} F^a \wedge F^e\Big]\exp\Big[\frac{i}{\hbar}\int_M d^4x f_A (D_0\psi)^A\Big]$$

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In progress research

- Formalizing Hilbert space structure of Ψ_{GKod}
- Generalization to include gravitational DOF (done)
- Develop interpretation as a third-quantized theory
- Semigroup interpretation of the Hamiltonian constraint functional Green's functions
- Yang-Mills theory requires special treatment since non-polynomial in the basic variables
- Creation of a library of states for different models in the full theory: Ultimate goal is the Standard Model

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$$ \Psi_{GKod}[A^a_i,\psi^A] = \exp\Big[\int_M d^4x\big((\hbar G)^{-1}\Psi_{ae}B^i_e\dot{A}^a_i + \frac{i}{\hbar}f_A\dot{\psi}^A\big)\Big] $$

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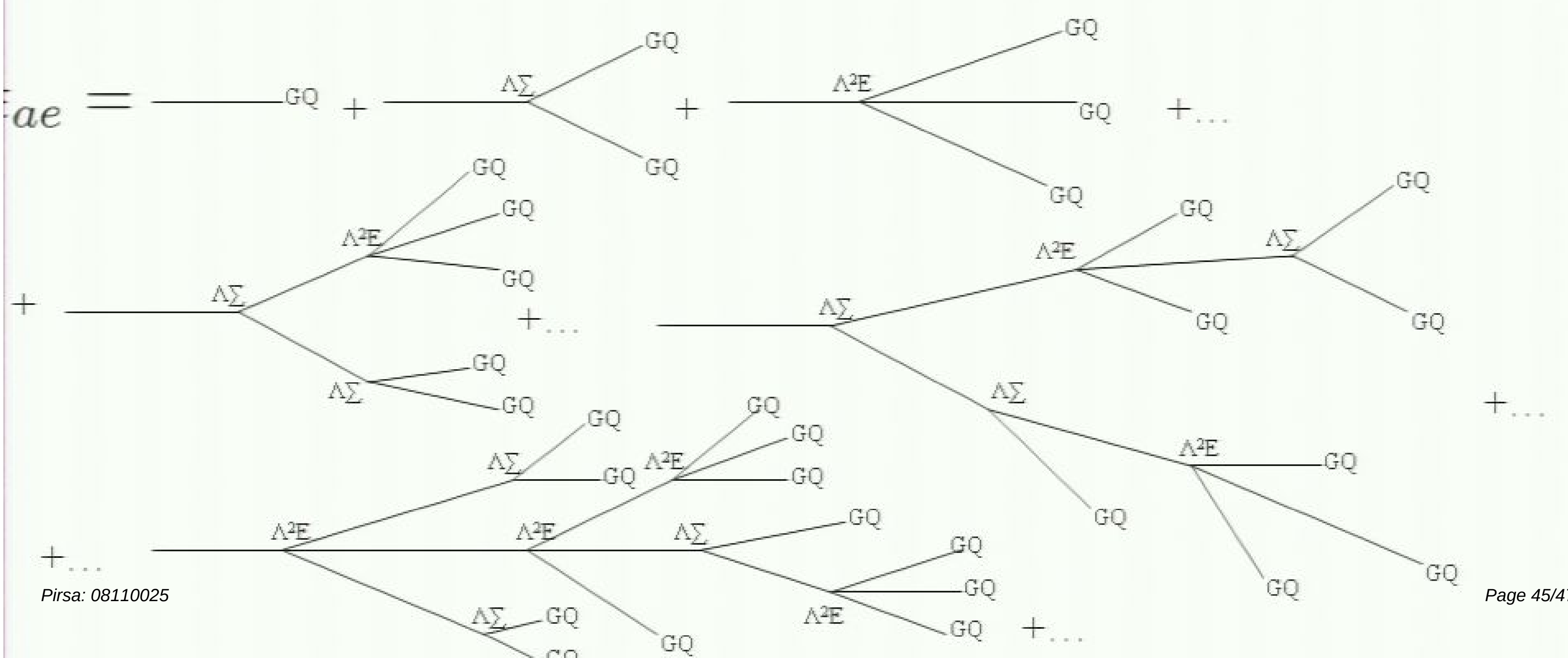


Solution: Set $q_0 = q_1 = q_2 = 0$

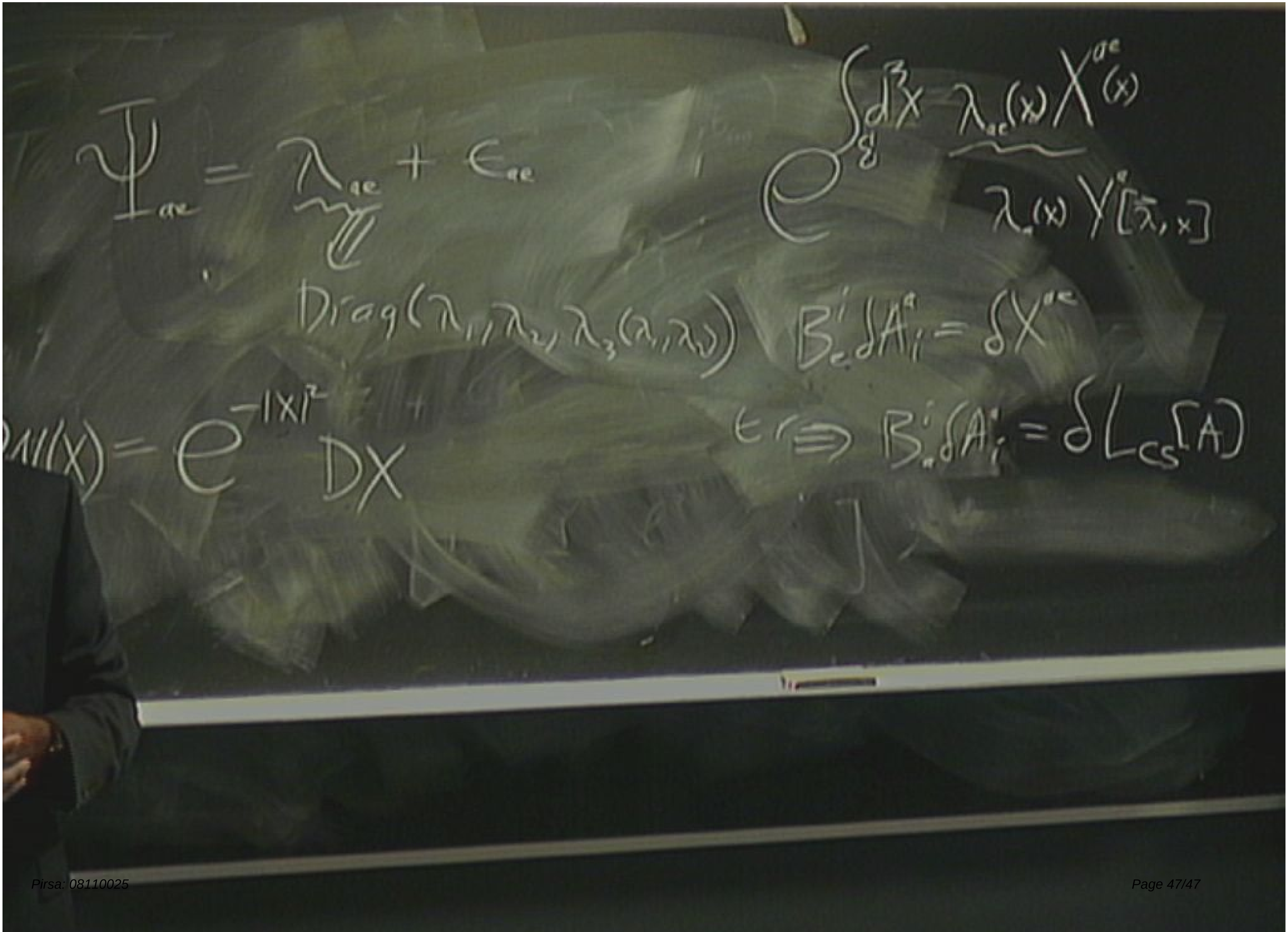
$$\Big(\delta_{ae} - \frac{G}{2}\frac{B^i_a B^j_e}{\det B}\partial_i\phi\partial_j\phi\Big)\epsilon_{ae} + \frac{\Lambda}{6}\Big(\delta_{ae}\delta_{bf} - \delta_{af}\delta_{be} - \frac{G}{4}\frac{B^i_e B^j_f}{\det B}\partial_i\phi\partial_j\phi\Big)\epsilon_{ae}\epsilon_{bf}$$

$$+\frac{\Lambda^2}{72}\epsilon_{abc}\epsilon_{def}\epsilon_{ad}\epsilon_{be}\epsilon_{cf} - \frac{G\Lambda}{12\det B}\Big(\frac{\pi^2}{2} + \frac{1}{2}B^i_a B^j_a \partial_i\phi\partial_j\phi\Big) = 0;$$

$$\det B\big(\delta_{bf}(B^{-1})^a_i - (B^{-1})^b_i\delta_{af} + \frac{G}{4}\delta_{ab}\tau_{if}\big)\frac{\partial}{\partial A^a_i} + 2\delta_{bf}\mathrm{tr}C\Big]\epsilon_{bf} - \frac{i}{4\det B}\frac{\partial\pi}{\partial\phi}$$

$$+\frac{\Lambda}{8}\Big(\epsilon^{abd}\epsilon^{fec}(B^{-1})^c_i\frac{\partial}{\partial A^d_i} + 4(\delta^a_e C^b_f - \delta^b_e C^a_f)\Big)\epsilon_{ae}\epsilon_{bf} = 0;$$

$$\Big[\epsilon_{ijk}\epsilon^{abc}B^k_e\frac{\partial}{\partial A^a_i}\frac{\partial}{\partial A^b_j} + 4\big(\delta^{ce}A^a_k - \delta^{ae}A^c_k\big)\frac{\partial}{\partial A^a_k} + 12\delta^{ce}\Big]\epsilon_{ce} = 0;$$


Ψ_ae = λ_ae + ε_ae
Diag(λ1, λ2, λ3(λ1, λ2))
e^{∫_Σ d³x λ_ae(x) X^ae(x)}
λ_e(x) Y^e[λ, x]
B^i_e δA^e_i = δX^ae
⇒ B^i_e δA^e_i = δL_CS[A]
(X) = e^{-|X|²} DX