Title: Precision gravity from Effective Field Theory.

Date: Nov 04, 2008 02:00 PM

URL: http://pirsa.org/08110024

Abstract: TBA

Precision Gravity from EFT

based on (w/ I. Rothstein):
hep-th/0409156
hep-th/0511133
hep-th/060216, hep-th/0605238
(review: hep-ph/0701129)
+ in progress with A. Ross, I . Rothstein

In this talk, I'll consider astrophysical black holes (and other compact objects) interacting with long wavelength gravitational fields (e.g . field produced by other BH in orbit).

Such systems have a natural formulation in terms of effective field theories (EFTs). They are also relevant for experiments in gravitational wave detection, e.g. LIGO and LISA.

Main Motivation

An interesting class of signals for gravitational wave detectors (eg, LIGO/LISA) consists of radiation induced inspiral of compact binary systems (BH or NS constituents). Why?

1. Such systems are strong emitters of gravitational radiation:

$$h \sim \frac{1}{R} G_N E_{int} \sim 10^{-19} \frac{1}{R(\mathrm{Mpc})} \frac{m}{m_\odot}$$

giving values in the LIGO range $h = \Delta L/L \sim 10^{-21} - 10^{-22}$
for, eg, solar mass NS/NS at $R \sim 3000\mathrm{Mpc}$

2. Many expected inspiral events per year for upgraded LIGO:

	NS/NS	NS/BH	BH/BH in field	BH/BH in clusters
$\mathcal{R}_{\rm gal}$, yr^{-1}	10^{-6}–5×10^{-4}	$\lesssim 10^{-7}$–10^{-4}	$\lesssim 10^{-7}$–10^{-5}	$\sim 10^{-6}$–10^{-5}
$D_{\rm I}$	20 Mpc	43 Mpc	100	100
$\mathcal{R}_{\rm I}$, yr^{-1}	$3 \times 10^{-4} - 0.3$	$\lesssim 4 \times 10^{-4} - 0.6$	$\lesssim 4 \times 10^{-3} - 0.6$	$\sim 0.04 - 0.6$
$D_{\rm WB}$	300 Mpc	650 Mpc	$z = 0.4$	$z = 0.4$
$\mathcal{R}_{\rm WB}$, yr^{-1}	$1 - 800$	$\lesssim 1 - 1500$	$\lesssim 30 - 4000$	$\sim 300 - 4000$

3. Signal is long duration: For binary in close but non-relativistic orbit, can use virial thm. to get estimate of orbital dynamics:

$$v^2 \sim \frac{G_M m}{r} \equiv \frac{r_s}{2r}$$

e.g LIGO can detect signals in a frequency band $10 \text{ Hz} < \nu < 1 \text{ kHz}$. This correspond to orbital parameters

$$r(10 \text{ Hz}) \sim 300 \text{ km} \left(\frac{m}{m_\odot}\right)^{1/3} \rightarrow r(1 \text{ kHz}) \sim 14 \text{ km} \left(\frac{m}{m_\odot}\right)^{1/3}$$

(for comparison, $r_s \sim 1 \text{ km}$ for $m \sim m_\odot$)

$$v(10 \text{ Hz}) \sim 0.06 \left(\frac{m}{m_\odot}\right)^{1/3} \rightarrow v(1 \text{ kHz}) \sim 0.3 \left(\frac{m}{m_\odot}\right)^{1/3}$$

note: $m_{NS} \sim m_\odot, m_{BH} \sim 10 m_\odot$).

In this regime, the energy loss of the binary to GW's is approx. given by the quadrupole rad. formula:

$$\frac{d}{dt}\left(-\frac{1}{2}mv^2\right) = -\frac{32}{5}G_N^{-1}v^{10}$$

Solving this gives:

$$\Delta t \sim \frac{5}{512}\left(\frac{1}{v_i^8} - \frac{1}{v_f^8}\right) \sim 5 \text{ min.} \left(\frac{m}{m_\odot}\right)^{-8/3}$$

for the duration of the inspiral event in the LIGO band, and

$$N \sim \int_{t_i}^{t_f} \omega(t)dt = \frac{1}{32}\left(\frac{1}{v_i^5} - \frac{1}{v_f^5}\right) \sim 4 \times 10^4 \left(\frac{m}{m_\odot}\right)^{-5/3} \text{ radians}$$

for the number of orbital cycles spent in the detector band.

Long integration times $\Longrightarrow$ waveform is highly sensitive to small relativistic effects. LIGO is sensitive to at least $(v/c)^6$ corrections beyond Newtonian gravity for binary dynamics (more for BH/BH). (Cutler et. al. astro-ph/9208005)

Thus, by comparing LIGO measurement and theoretical prediction for waveforms, one can hope to obtain:

1. Accurate measurements of masses, spins, distances for compact binaries out to $R \sim 1000\text{Mpc}$.

2. Stringent tests of (classical) General Relativity.

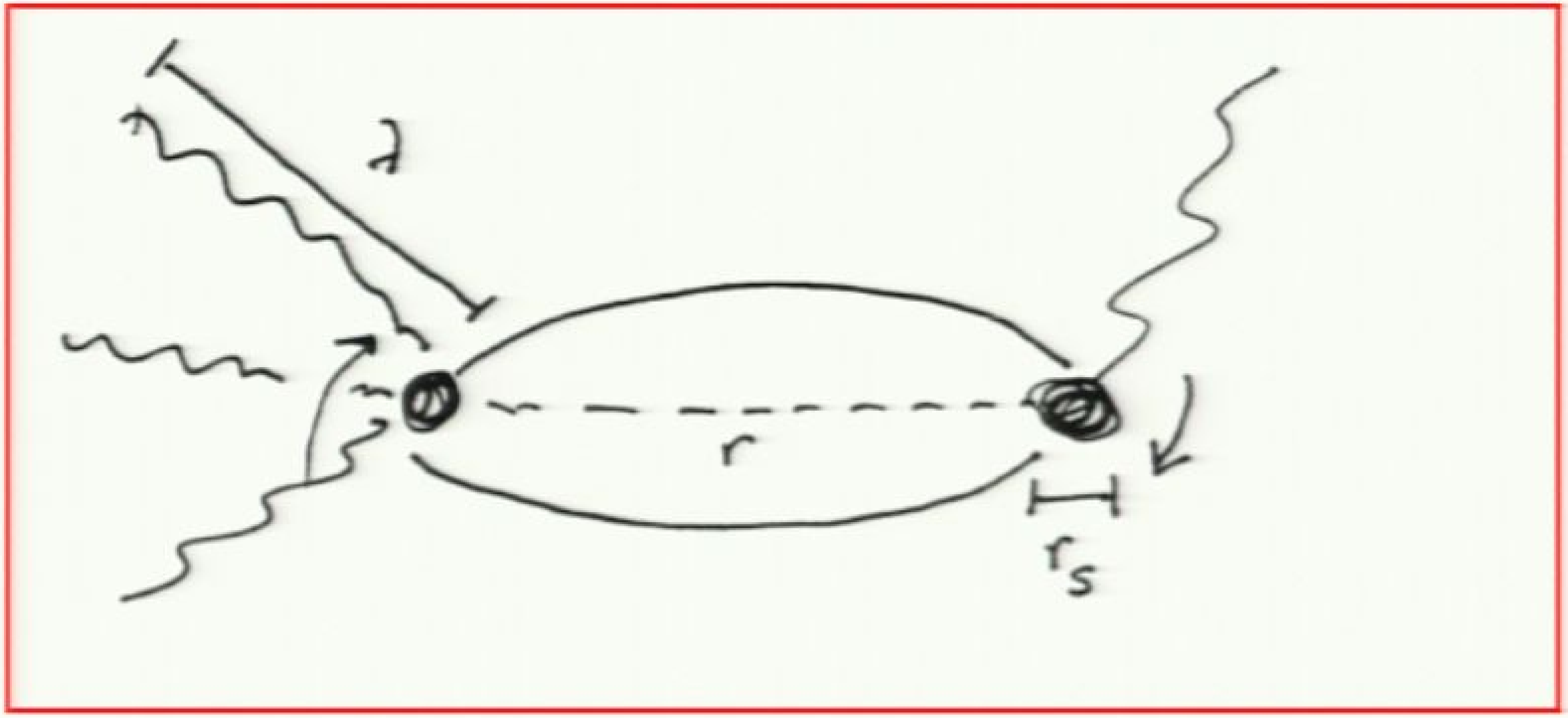
3. Structure of black holes or neutron stars? (eg., dynamics of BH horizons?)

(note: For LISA, binary inspirals follow exactly the same dynamics. LISA band is $10^{-5}\ \text{Hz} < \nu < 1\ \text{Hz}$ so typical sources have much larger mass. For LISA, one has:

BH/BH: $m_{BH} \sim 10^{5-8} m_{\odot}$

BH/NS: $m_{BH} \sim 10^{5-7} m_{\odot}$)

Non-relativistic binary problem is also interesting from the point of view of field theory, as it is a problem with many different length scales. E.g, for binary BH:



$r_s =$ Black hole radius $\quad r =$ Orbital radius

$\lambda =$ Wavelength of grav. radiation

cales are correlated: $\quad r \sim r_s/v^2$ (virial thm.) $\quad \lambda \sim r/v$

$v \ll 1$ Typical three-velocity

Because scales are correlated, a single expansion parameter controls qualitatively different physical effects....

This has been a source of problems for traditional "Post-Newtonian" methods of computing binary inspiral observables over the last ~30 years (Damour-Blanchet et. al; Will et. al). The expansion is plagued with e.g.:

Ultraviolet divergences, due to the use of singular (delta fn.) pt particle sources in $T_{\mu\nu}$

"Ambiguities" in the calculation of certain terms in the GW observables at order v^6 in the expansion (Blanchet, Damour et. al).

The reason why this is difficult to decide in traditional PN approaches is that there is no manifest separation of scales.

This motivates an alternative, "Wilsonian" approach, that treats each physical scale independently. In such an approach:

1. Finite size effects are parametrized as Wilson coefficients in an pt particle theory.

2. These Wilson coefficients can be obtained by a matching calculation done at $\mu \sim r_s^{-1}$ in the one-body sector, where the dynamics simplifies.

3. Short distance (UV) divergences can be handled in the usual way and absorbed into finite size coefficients.

4. Logs of scale ratios $= \ln v$ can be understood by RG methods.

Binary stars as an EFT problem

inary problem involves a hierarchy of scales, $r_s \ll r \ll r/v$

implify by integrating out one at a time

Extended objects (BH/NS) + GR (the "full theory")

match ——— $\frac{1}{\mu} = r_s$

Relativistic point particle + GR

↓

2-body NR problem

match ——— $\frac{1}{\mu} = r$

Composite object + radiation gravitons (NRGR)

↓ non-trivial RG running

——— $\frac{1}{\mu} = \frac{r}{v}$

The EFTs:

t each stage in the calculation, the relevant dof's are worldline localized
+1) dim d.o.f's with local $SO(3)$ indices, coupled to gravity.

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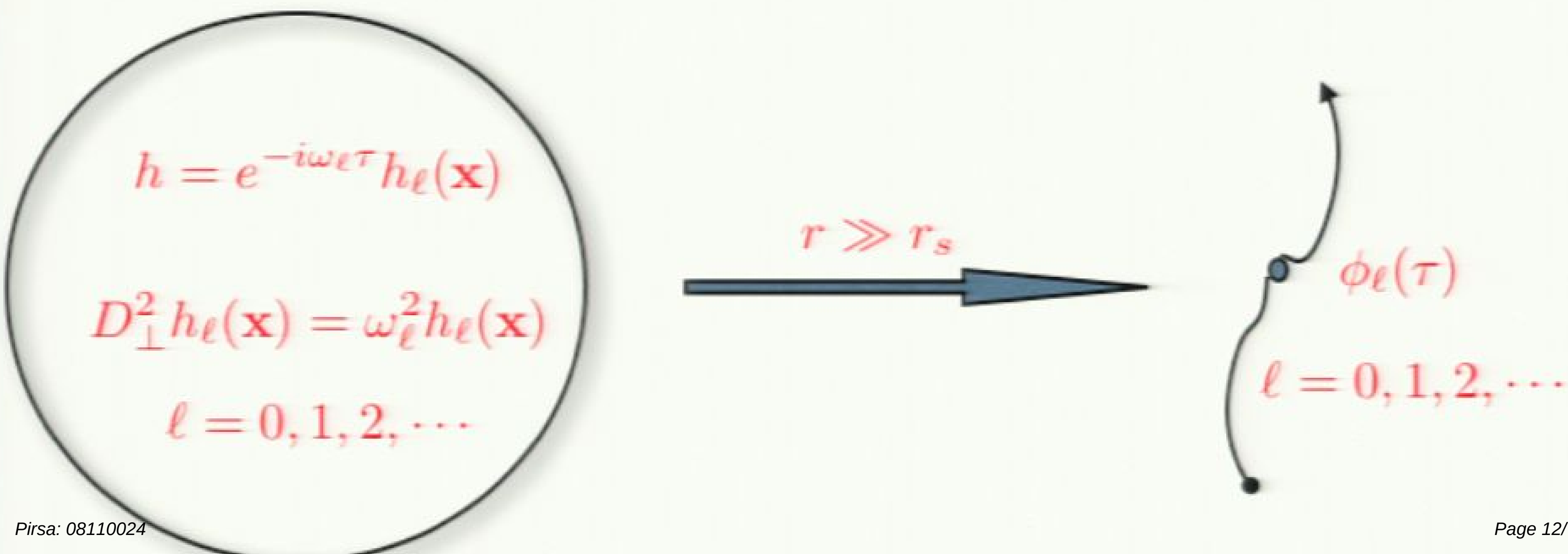

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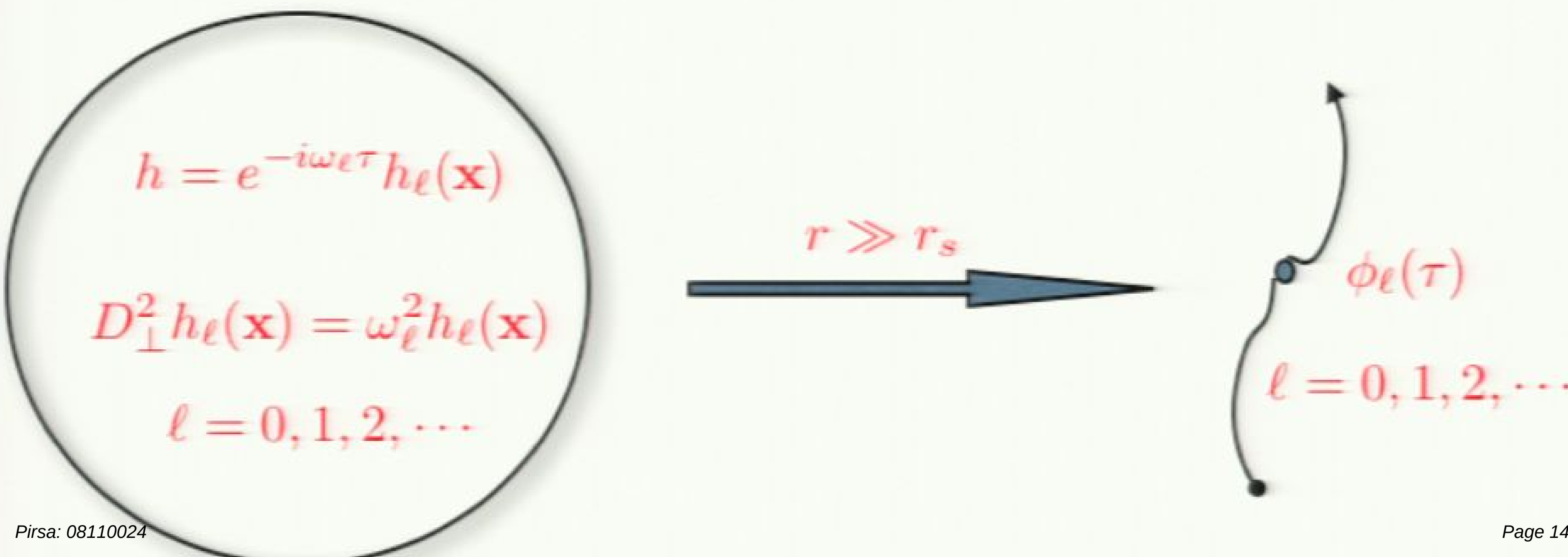

eg, for a Schwarschild black hole, the spectrum contains an infinite tower of modes. In this case there are some zero modes:

Mode	Freq.	J^P
$m(\tau)$	0	0
$x^\mu(\tau)$	0	1^+
$\omega_{ij}(\tau)$ (spin)	0	1^-

here are also massive "states":

n	$\ell = 2$		$\ell = 3$		$\ell = 4$	
0	0.37367	-0.08896 i	0.59944	-0.09270 i	0.80918	-0.09416 i
1	0.34671	-0.27391 i	0.58264	-0.28130 i	0.79663	-0.28443 i
2	0.30105	-0.47828 i	0.55168	-0.47909 i	0.77271	-0.47991 i
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Table 1: *The first four QNM frequencies (ωM) of the Schwarzschild black hole for $\ell = 2, 3$, and 4 [135]. The frequencies are given in geometrical units and for conversion into kHz one should multiply by $2\pi(5142Hz) \times (M_\odot/M)$.*

from Kokkotas and Schmidt, gr-qc/9909058).

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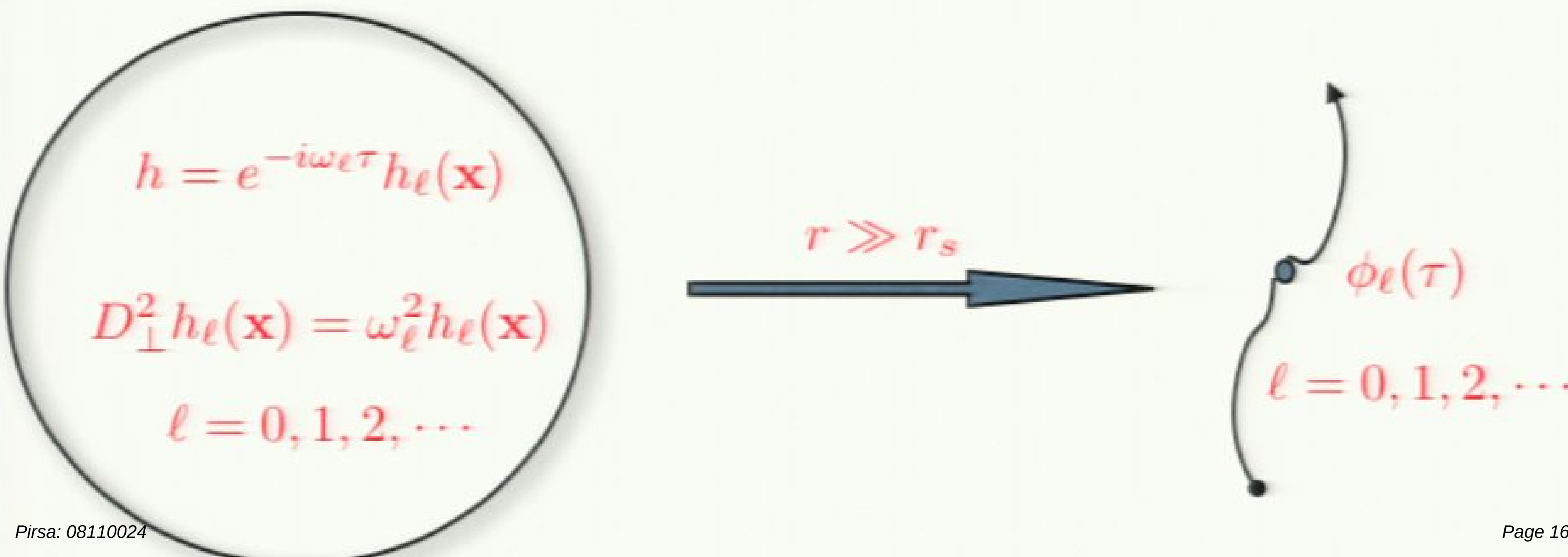

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$$S = S_{EH} + S_{pp}$$

$$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R(x) \qquad (m_{Pl}^2 = 1/(32\pi G_N))$$

$$S_{pp} = -m \int d\tau + c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu} + \cdots$$

noring spin dofs (see R. Porto, gr-qc/0511061; Porto+Rothstein gr-qc/0604099),
is is the most general action consistent with symmetries of the Schwarzschild
H. Here

$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$ =proper time along worldline

$E_{\mu\nu}$ = "electric parity" components of Weyl tensor $\sim C_{0i0j}$

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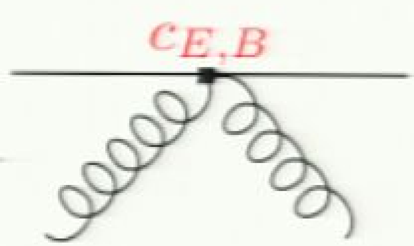
$$S_{pp} = \cdots + c_E \int d\tau E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau B_{\mu\nu}B^{\mu\nu}$$

the effective Lagrangian cause deviations from pure geodesic motion. They
e therefore associated with tidal = finite size effects. By including all such ops.
e are systematically parametrizing finite size effects within the pt particle EFT.

How are the coefs $c_{E,B}$ calculated? Through matching: Compare amplitudes
n full BH theory vs. pt. particle EFT.

ook at the elastic process $g + \mathrm{BH} \to g + \mathrm{BH}$. In the EFT, the amplitude has a
erm

$i\mathcal{A}_{EFT} = \cdots +$  $\sim \cdots + \frac{ic_{E,B}}{m_{Pl}^2}\omega^4 + \cdots$

o that the cross section has a term

$$\sigma_{EFT}(\omega) \sim \cdots + \frac{c_{E,B}^2}{m^4}\omega^8 \cdots$$

n the full BH theory, the only scale is the Schwarzschild radius, so

$$\sigma_{BH}(\omega) = r_s^2 f(r_s\omega)$$

vhere $f(r_s\omega)$ can be expanded in powers of $r_s\omega$ (possibly times logs) for $r_s\omega \ll 1$

n the limit $r_s\omega \ll 1$, the full BH result should agree with the pt. particle EFT calculation. Thus the matching condition is

$$\sigma_{BH}(\omega) \sim \cdots + r_s^{10}\omega^8 + \cdots = \sigma_{EFT}(\omega) \sim \cdots + \frac{c_{E,B}^2}{m_{Pl}^4}\omega^8$$

nd therefore $$c_{E,B} \sim m_{Pl}^2 r_s^5$$

ight expect this effect to be at most $\left(\frac{r_s}{r}\right)^5 \sim v^{10}$. This is a lower bound n finite size corrections

Large enhancement possible for NS, $R_{NS} \sim 10 \times r_s$. See Flannagan + Hinderer arXiv:0709.1915)

Calculating Observables:

After integrating out the internal size scale r_s, all the long distance two-body physics is calculable from

$$S_{pp} = -\sum_a m_a \int d\tau_a + c_E^{(a)} \int d\tau_a E_{\mu\nu}^2 + c_B^{(a)} \int d\tau_a B_{\mu\nu}^2 + \cdots$$

Interferometric detectors (LIGO/LISA) are most sensitive to the phase $\phi(t)$ of gravitational wave signals. Theoretically this can be calculated from

$E(v) =$ grav. binding energy, as a function of orbital velocity.

$\mathcal{F}(v) =$ radiated power, as a function of orbital velocity.

Setting $\frac{dE}{dt} = -\mathcal{F}$ the GW phase measured by LIGO/LISA can be calculated. E.g., for extreme mass ratio binary

$$\phi(\nu) = \phi_0 + \frac{1}{2r_s} \int_{\nu(v)}^{v(\nu_0)} dv' v'^3 \frac{dE(v')/dv'}{\mathcal{F}(v')}$$

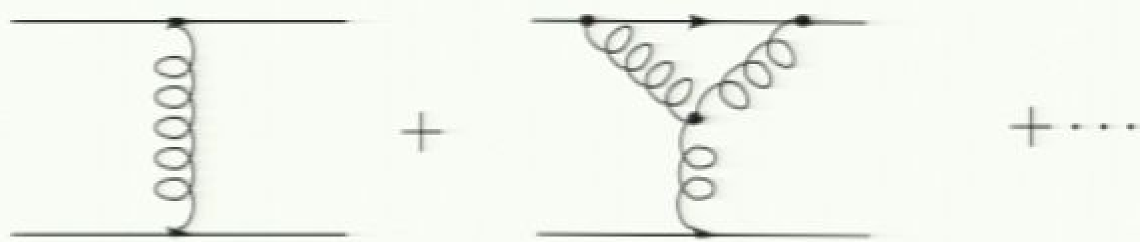
n (classical) field theory, these quantities can be calculated perturbatively rom a gravitational "Wilson loop" observable. Hold $x_a^\mu(\tau)$ fixed, and ompute

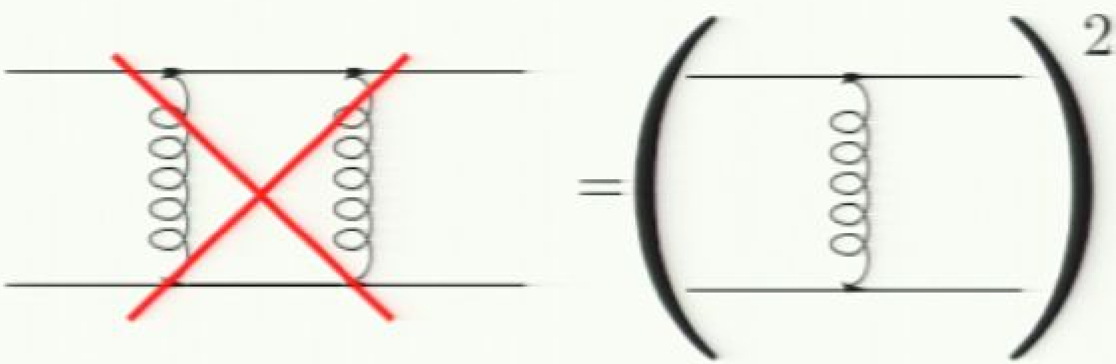
$$\exp[iS_{eff}(x_a)] = \int \mathcal{D}h_{\mu\nu} \exp[iS_{EH} + iS_{pp}].$$

vhere I expanded $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Diagrammatically:

$iS_{eff}(x_a) =$ diagrams that remain connected if particle lines are removed

$iS_{eff}(x_a) =$





(no propagators for particle lines)

(Donoghue, gr-qc/9512024)

$S_{eff}(x_a)$ generates all relevant observables:

$\mathrm{Re}\, S_{eff}[x_a] \Rightarrow$ generates classical e.om.'s for x_a^μ

nd for a fixed $\{x_a^\mu\}$ config., over a large time $T \to \infty$

$$\frac{1}{T}\mathrm{Im} S_{eff}[x_a] = \frac{1}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega},$$

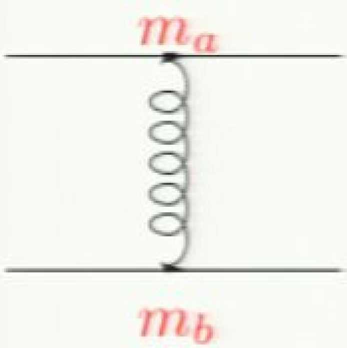
vhere $d\Gamma$ is the differential rate for graviton emission from binary system.
Classical power spectrum can be obtained from this:

$$dP = E d\Gamma,$$

ith E — tot energy radiated in gravitons

It is OK to set up the perturbative expansion in this way. But it is not optimal for the NR two-body problem:

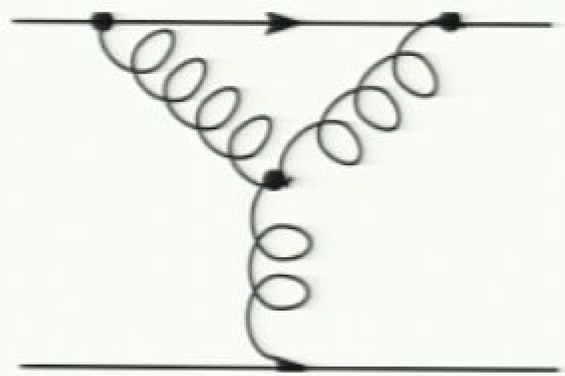
Feynman rules mix different orders in the velocity expansion.
eg, the one-graviton term in $S_{eff}[x_a]$



$$= \sum_{a,b} \frac{m_a m_b}{16 m_{Pl}^2} \int d\tau_a d\tau_b \left[1 - 2(\dot{x}_a \cdot \dot{x}_b)^2\right] D_F(x_a - x_b)$$

expanding $D_F(x)$, $d\tau = dx^0 \sqrt{1 - \mathbf{v}^2}$ generates an infinite number of terms with different velocity scaling. Not useful for calculating observables in the NR limit.

The reason for this is that the pt. particle + gravity theory used to generate the Feynman rules contains scales that have not been properly disentangled. Eg, look at



$$\sim v^?$$

Hard to tell what order in v this first comes in. This is because momentum integrals in this diagram contain all scales in the binary problem

$$r_s \ll r \lll \lambda$$

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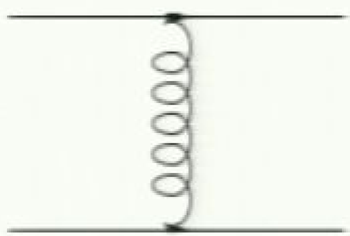
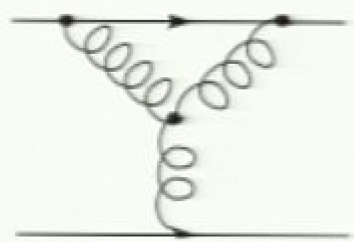
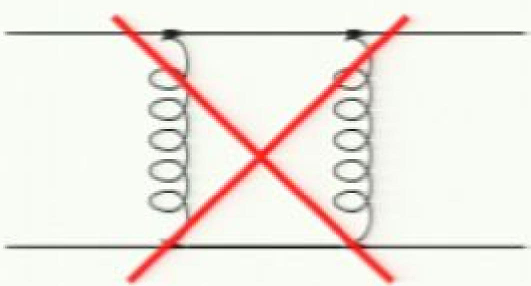
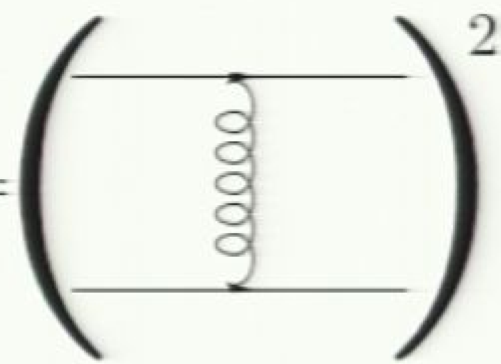
$iS_{eff}(x_a) =$  $+$  $+\cdots$

 $=$ 

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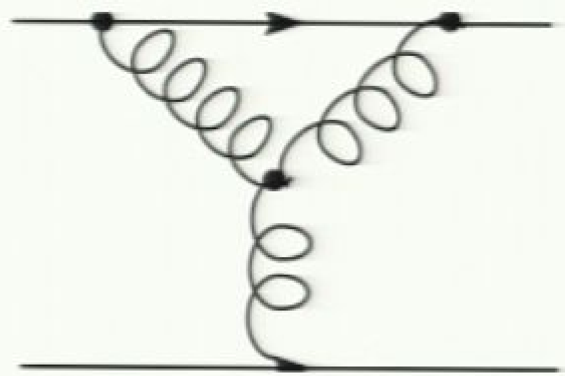

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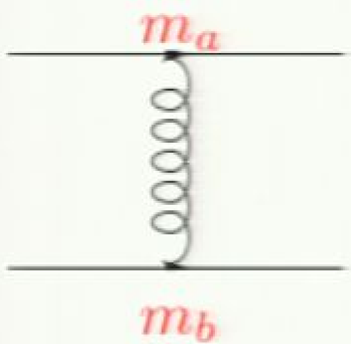
It is OK to set up the perturbative expansion in this way. But it is not optimal for the NR two-body problem:

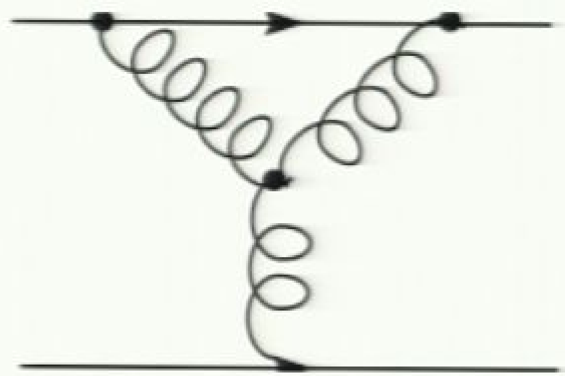
Feynman rules mix different orders in the velocity expansion.
eg, the one-graviton term in $S_{eff}[x_a]$



$$= \sum_{a,b} \frac{m_a m_b}{16 m_{Pl}^2} \int d\tau_a d\tau_b \left[1 - 2(\dot{x}_a \cdot \dot{x}_b)^2\right] D_F(x_a - x_b)$$

expanding $D_F(x)$, $d\tau = dx^0 \sqrt{1-\mathbf{v}^2}$ generates an infinite number of terms with different velocity scaling. Not useful for calculating observables in the NR limit.

The reason for this is that the pt. particle + gravity theory used to generate the Feynman rules contains scales that have not been properly disentangled. Eg, look at



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NR two-body kinematics

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oint particle momenta: $(E \sim mv^2, \mathbf{p} \sim mv)$ with velocity v elated to m, r through Virial theorem:

$$v^2 \sim \frac{m}{m_{Pl}^2 r}$$

wo types of graviton modes in a typical Feynman diagram:

Potential: $(k^0 \sim v/r, \mathbf{k} \sim 1/r)$ mediate binding forces, generate orbits. Never on-shell, so must integrate out.

Radiation: $(k^0 \sim v/r, \mathbf{k} \sim v/r)$ These are the modes that propagate to the detector. They generate cuts in diagrams at k^0 ~orbital freq.

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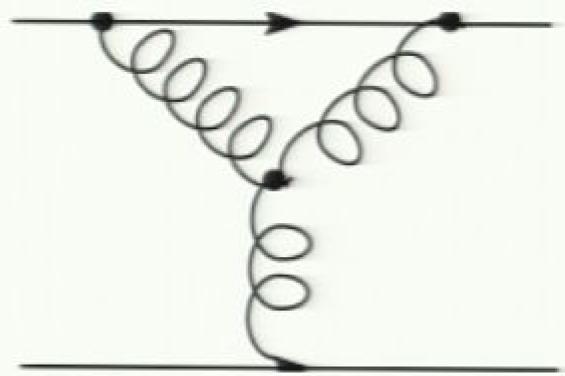

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So we have to integrate out the short distance potential modes from the theory.

To accomplish this, decompose

$$h_{\mu\nu}(x) = \bar{h}_{\mu\nu}(x) + H_{\mu\nu}(x),$$

The potential graviton modes are encoded in $H_{\mu\nu}$, with momentum components

$$\partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu} \quad \partial_0 H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu},$$

It is convenient to make the explicit the difference between ∂_i and ∂_0 , so take FT

$$H_{\mu\nu}(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} H_{\mathbf{k}\mu\nu}(x^0)$$

The radiation modes are encoded in $\bar{h}_{\mu\nu}$, viewed as long wavelength background field:

$$\partial_\alpha \bar{h}_{\mu\nu} \sim \frac{v}{r} \bar{h}_{\mu\nu}.$$

Now we can integrate out the potential modes. Formally,

$$\exp[iS_{NRGR}[x_a, \bar{h}]] = \int \mathcal{D}H_{\mu\nu} \exp[iS[\bar{h}+H, x_a] + iS_{GF}],$$

where we keep the diffeomorphism invariance with respect to the background field $\bar{h}_{\mu\nu}$:

$$S_{GF} = m_{Pl}^2 \int d^4x \sqrt{\bar{g}} \Gamma_\mu \Gamma^\mu,$$

vith $\Gamma_\mu = D_\alpha H^\alpha_\mu - \frac{1}{2} D_\mu H^\alpha_\alpha$. (ignore ghosts). The potential propagator is

$$\langle H_{\mathbf{k}\mu\nu}(x^0) H_{\mathbf{q}\alpha\beta}(0) \rangle = -(2\pi)^3 \delta^3(\mathbf{k}+\mathbf{q}) \frac{i}{\mathbf{k}^2} \delta(x^0) P_{\mu\nu;\alpha\beta}$$

ut $x^0 \sim r/v, \mathbf{k} \sim 1/r$

➡ $H_{\mathbf{k}\mu\nu}/m_{Pl} \sim v^2/\sqrt{L}$ $\quad (L = mvr \gg 1)$

In addition, to integrating out potentials, must perform multipole expansion of radiation field

$$\bar{h}_{\mu\nu}(x^0, \mathbf{x}) = \bar{h}_{\mu\nu}(x^0, \mathbf{X}) + \delta\mathbf{x}_i \partial_i \bar{h}_{\mu\nu}(x^0, \mathbf{X}) + \frac{1}{2}\delta\mathbf{x}_i \delta\mathbf{x}_j \partial_i \partial_j \bar{h}_{\mu\nu}(x^0, \mathbf{X}) + \cdots,$$

in both its couplings to (1) point particles (2) potential modes (thru graviton self interactions). Doing so yields Feynman rules compatible with small velocity limit. Can work out the scaling of any vertex from the rules

$$x^\mu \sim \frac{r}{v} \quad \mathbf{k} \sim \frac{1}{r} \qquad \frac{m^2}{m_{Pl}^2} \sim vL \qquad L = mvr$$

(virial theorem)

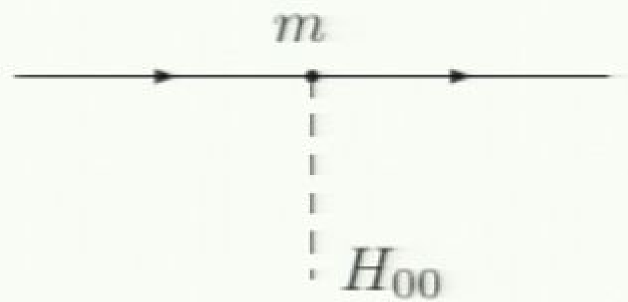
$$H_{\mathbf{k}\mu\nu}/m_{PL} \sim v^2/\sqrt{L} \qquad h_{\mu\nu}/m_{Pl} \sim v^{5/2}/\sqrt{L}$$

Any Feynman diagram scales as $L^n v^k$

w/ $n \leq 1, k \geq 0$ i.e., L=loop counting factor

Examples:

t. particle Newton potential
nteraction:

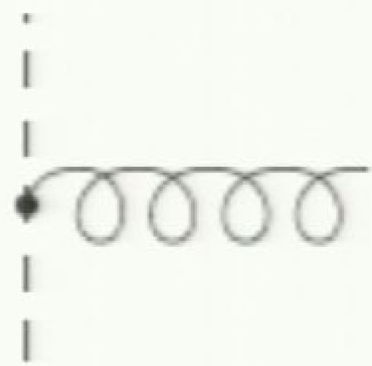


$$\sim \sqrt{L}$$

otential 3-graviton vertex:

$$\sim \frac{v^2}{\sqrt{L}}$$

adiation-potential interaction:



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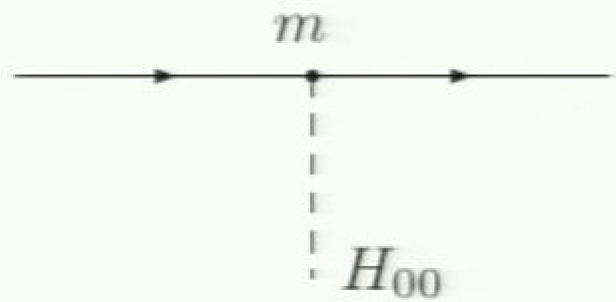

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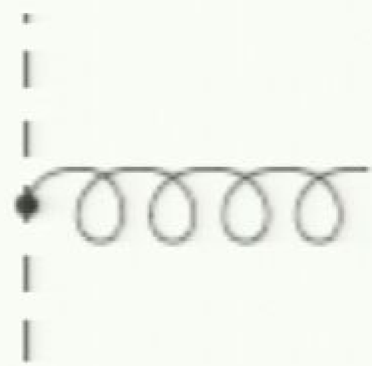


$\sim \frac{v^{5/2}}{\sqrt{L}}$

What is the effective theory below the scale $\mu = 1/r$?

Integrate out potential modes to get effective Lagrangian for radiation modes:

$$\Gamma[\bar{h}] = \Gamma_0 + \Gamma_1 + \cdots$$

$\mathcal{O}(\bar{h}^0)$ → Γ_0, $\mathcal{O}(\bar{h}^1)$ → Γ_1

$\Gamma_0 = \int dt L[\mathbf{x}_a] =$ many-particle Lagrangian, Feynman graphs w/ no ext. $\bar{h}_{\mu\nu}$

$$\Gamma_1 = -\frac{1}{2m_{Pl}} \int d^4x T^{\mu\nu}(x) \bar{h}_{\mu\nu}(x)$$

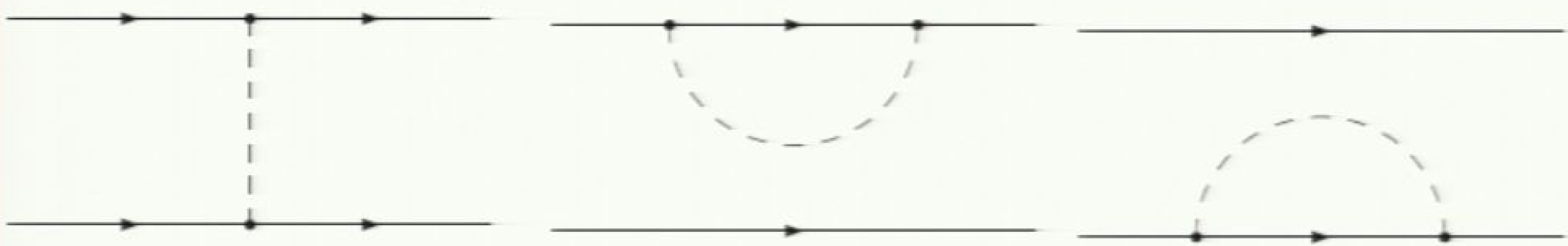
from graphs with one ext. $\bar{h}_{\mu\nu}$

$T^{\mu\nu}(x) =$ grav. energy-mom. "pseudo-tensor"

$\partial_\mu T^{\mu\nu} = 0$ (Ward id.)

Zero graviton sector

Integrate out $H_{\mu\nu}$ at lowest order. Graphs with no external $\bar{h}_{\mu\nu}$



(a) (b) (c)

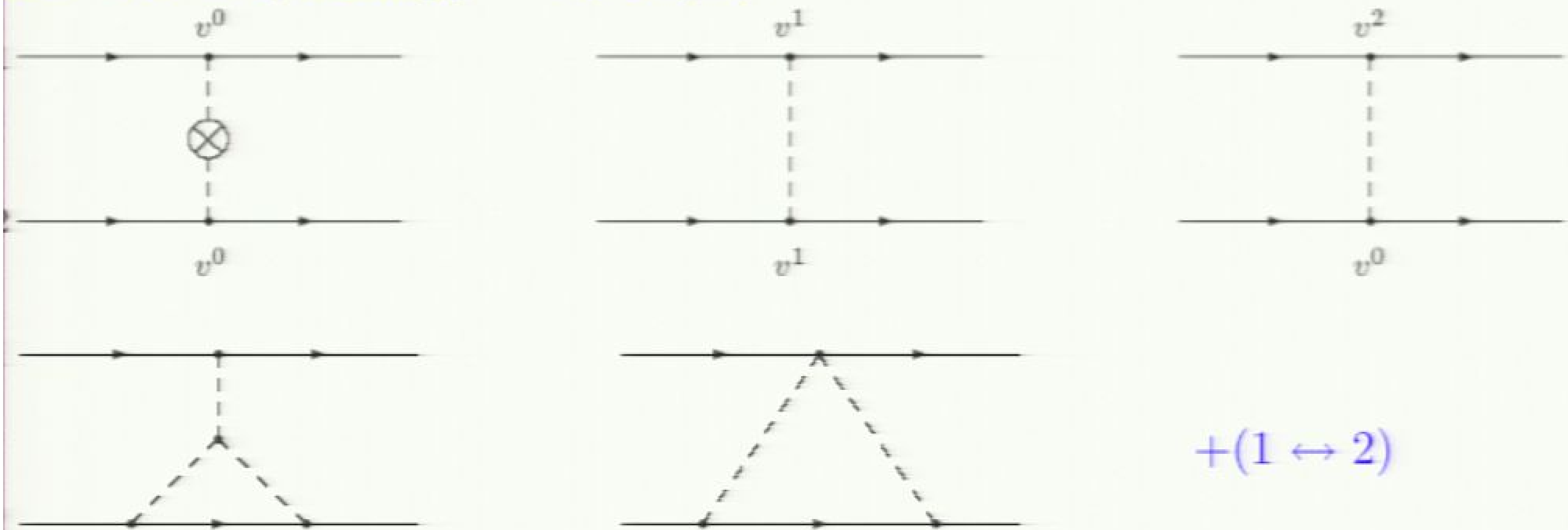
b),(c): Calculating in dim. reg. $\sim \int \frac{d^{3-\epsilon}\mathbf{k}}{(2\pi)^{3-\epsilon}} \frac{1}{\mathbf{k}^2} = 0$

(a) $\left(-\frac{im_1}{2m_{Pl}}\right)\left(-\frac{im_2}{2m_{Pl}}\right)\int dt_1 dt_2 \langle H_{00}(x_1) H_{00}(x_2)\rangle = \frac{im_1 m_2}{32\pi m_{Pl}^2}\int dt \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|}$

So at this order

$$S_{eff}[x_a] = \int dt \left[\frac{1}{2}\sum_a m_a \mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}\right] + \cdots$$

ith $G_N \equiv 1/32\pi m_{Pl}^2$ At $\mathcal{O}(v^2)$



$+(1 \leftrightarrow 2)$

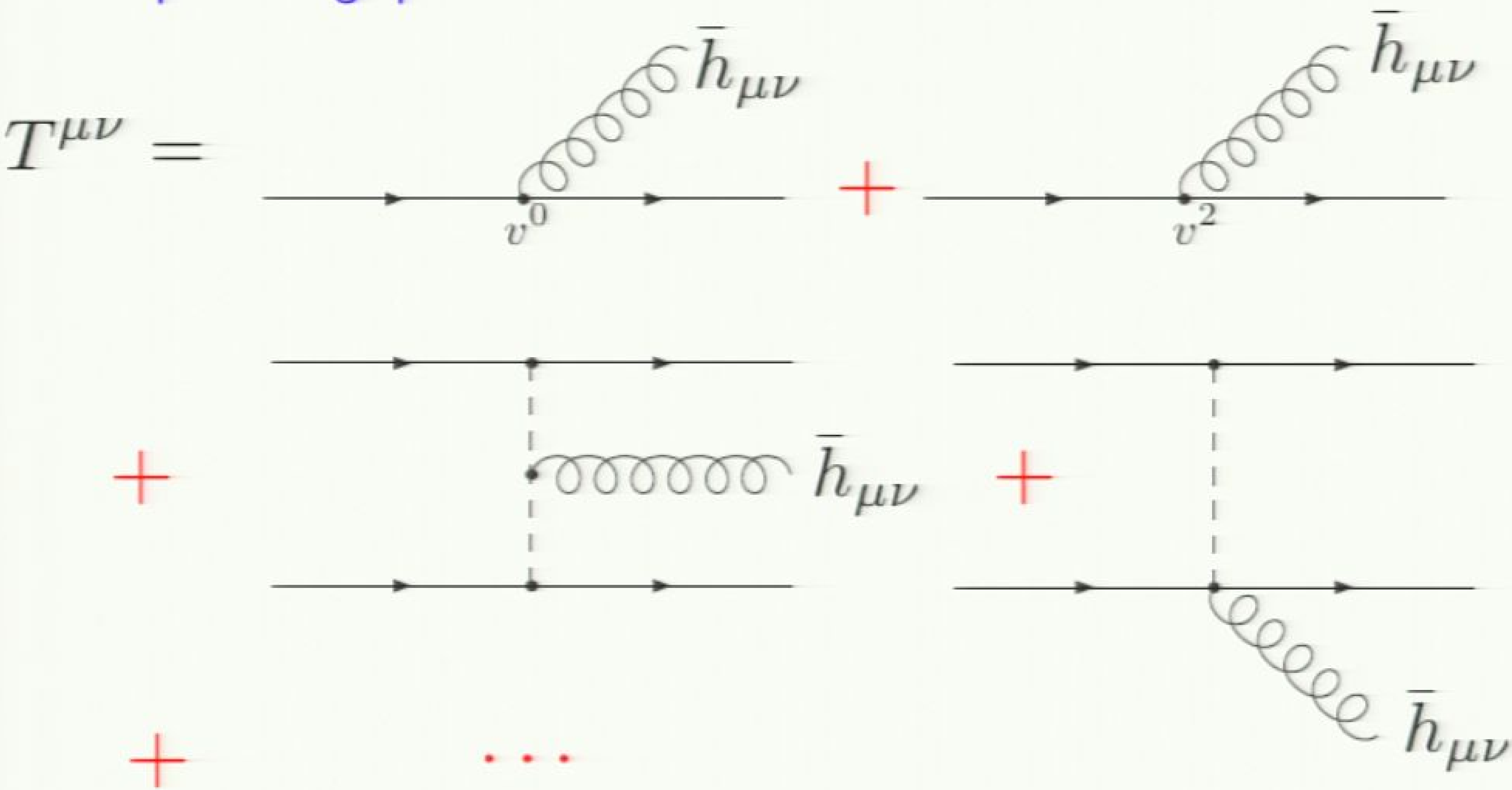
Adding these terms plus relativistic corrections to kinetic terms:

$$L_{EIH} = \frac{1}{8}\sum_a m_a \mathbf{v}_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7\mathbf{v}_1\cdot\mathbf{v}_2 - (\mathbf{v}_1\cdot\mathbf{n})(\mathbf{v}_2\cdot\mathbf{n})\right] - \frac{G_N^2 m_1 m_2(m_1+m_2)}{2\,|\mathbf{x}_1 - \mathbf{x}_2|^2}$$

which was first calculated in 1938 by Einstein, Infeld and Hoffmann.

One graviton sector

Must compute the graphs



(1st graph=LO. Last three graphs are NLO).

$T^{\mu\nu}$ encodes all moments. After multipole expanding

$$\bar{h}_{\mu\nu}(x^0,\mathbf{x}) = \bar{h}_{\mu\nu}(x^0,\mathbf{X}) + \delta\mathbf{x}_i\partial_i\bar{h}_{\mu\nu}(x^0,\mathbf{X}) + \frac{1}{2}\delta\mathbf{x}_i\delta\mathbf{x}_j\partial_i\partial_j\bar{h}_{\mu\nu}(x^0,\mathbf{X}) + \cdots,$$

btain the gauge invariant couplings

tot. energy → $m\bar{h}_{00}$; tot. mom. → $P_i\bar{h}_{0i}$

$$\Gamma_{1\partial^0} = -\frac{1}{2m_{Pl}}\int dt\,[m\bar{h}_{00} + 2P_i\bar{h}_{0i}]$$

CM → $X^i\partial_i\bar{h}_{00}$; ang. mom. → $L^{ij}\omega_{ij}$

$$\Gamma[\bar{h}]_{1\partial^1} = -\frac{1}{2m_{Pl}}\int dx^0\,[X^i\partial_i\bar{h}_{00}(x^0,0) + L^{ij}\omega_{ij}],$$

E-quad → E^{ij}; B-quad → $B^{i,jk}$; E-octo → E^{ijk}

$$\Gamma[\bar{h}]_{1\partial^2} = \frac{1}{2m_{Pl}}\int dx^0\left[E^{ij}R_{0i0j} + \frac{4}{3}B^{i,jk}R_{0jik} + \frac{1}{3}E^{ijk}\partial_k R_{0i0j}\right]$$

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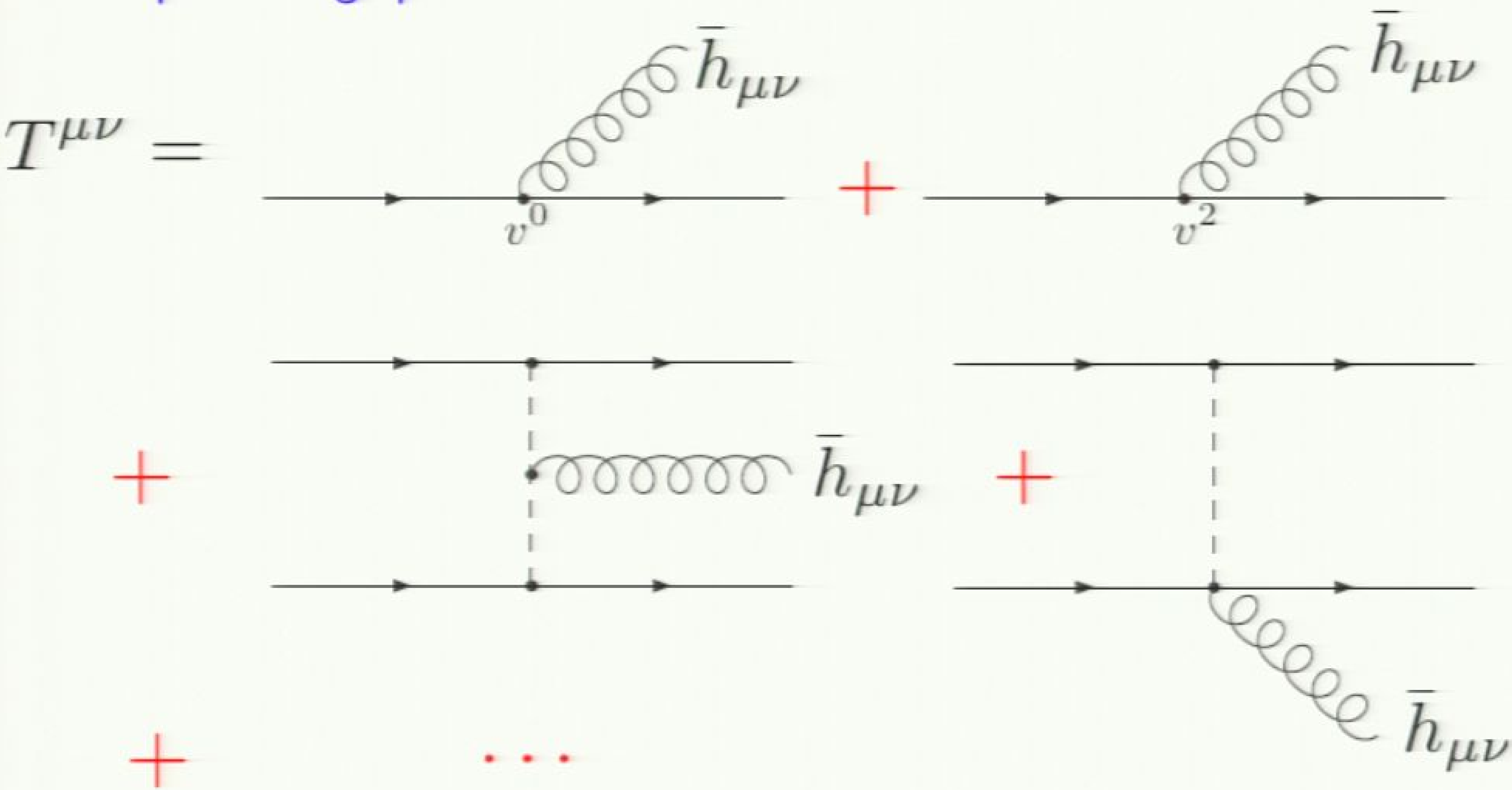

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/ e.g.

$$m = \int d^3\mathbf{x} T^{00}(x^0, \mathbf{x}) = \sum_a m_a \left[1 + \frac{1}{2}\mathbf{v}_a^2 - \frac{1}{2}\sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right] + \mathcal{O}(v^4)$$

$$\dot{m} = 0$$

$$X^i = \int d^3\mathbf{x}\, x^i T^{00}(x^0, \mathbf{x}) = \sum_a m_a \mathbf{x}_a^i \left[1 + \frac{1}{2}\mathbf{v}_a^2 - \frac{1}{2}\sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right] + \mathcal{O}(v^4$$

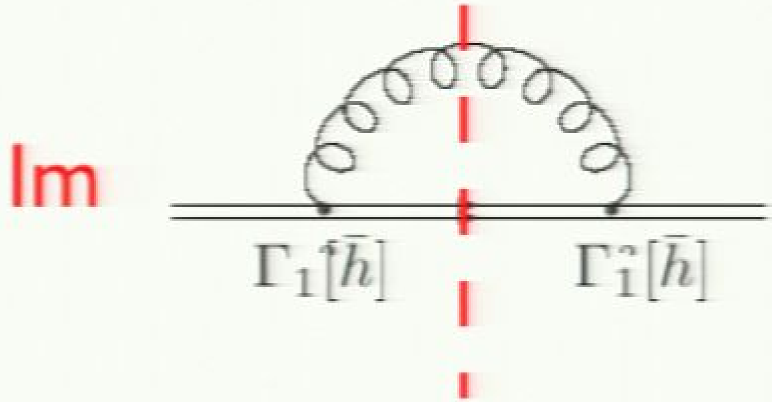
$$\dot{X}^i = P^i/m$$

$$^{ij} \quad = \quad \int d^3\mathbf{x} \left[T^{00} + T^{aa} + \frac{11}{42}\mathbf{x}^2 \ddot{T}^{00} - \frac{4}{3}\dot{T}^{0k} x^k\right] \left[x^i x^j\right]^{TF} + \mathcal{O}(v^4)$$

$$= \sum_a m_a \mathbf{x}_a^i \mathbf{x}_a^j \left[1 + \frac{3}{2}\mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right] + \frac{11}{42}\sum_a m_a \frac{d^2}{dt^2}(\mathbf{x_a}^2 \mathbf{x}_a^i \mathbf{x}_a^j)$$

$$-\frac{4}{3}\sum_a m_a \frac{d}{dt}(\mathbf{x_a} \cdot \mathbf{v}_a \mathbf{x}_a^i \mathbf{x}_a^j) - \text{traces} + \mathcal{O}(v^4)$$

an now use these moments to compute observables, e.g. radiated power to ILO. Use optical theorem to get



$$\frac{E}{lt} = \frac{G_N}{5}\langle\left(\frac{d^3}{dt^3}E^{ij}(t)\right)^2\rangle + \frac{16G_N}{45}\langle\left(\frac{d^3}{dt^3}B^{ij}(t)\right)^2\rangle + \frac{G_N}{189}\langle\left(\frac{d^4}{dt^4}E^{ijk}(t)\right)^2\rangle$$
$$+\cdots$$

g, binary system in circular orbit

$$\frac{dE}{dt} = \frac{32}{G_N}\left(\frac{\mu}{M}\right)^2 v^{10}\left[1 - \frac{1247}{336}v^2 + \cdots\right]$$

Radiative (long dist.) corrections:

Can use EFT to compute radiative corrections. Everything is calculable in terms of the radiation Lagrangian obtained by matching

$$S[\bar{h}] = S_{EH}[\bar{h}] + S_{GF} + \Gamma[\bar{h}]$$

$$\Gamma[\bar{h}] \supset \frac{1}{2m_{Pl}} \int dx^0 \left[E^{ij} R_{0i0j} + \frac{4}{3} B^{i,jk} R_{0jik} + \frac{1}{3} E^{ijk} \partial_k R_{0i0j} \right] + \cdots$$

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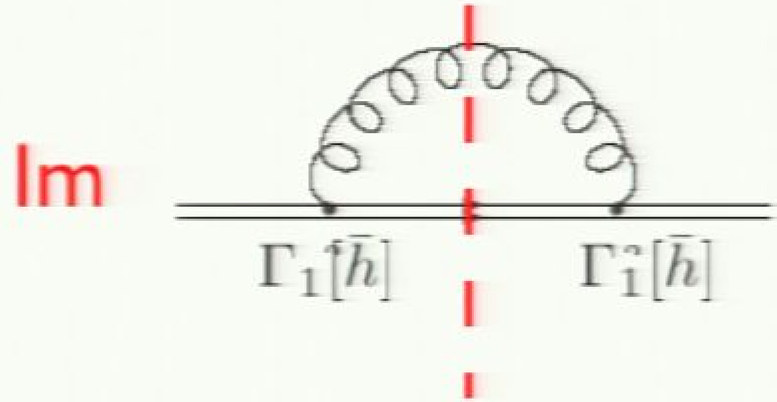

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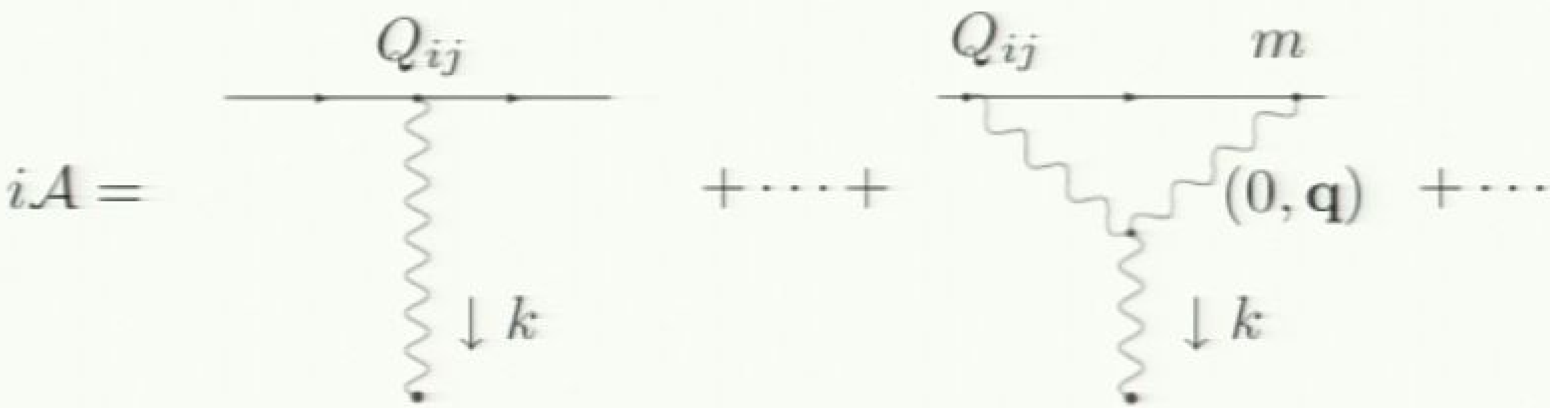
xample: the "tail effect". Look at the amplitude for rad. graviton mission from two-body system



'hysically: Outgoing graviton can interact with Newtonian $1/r$ potential of two-body source. Leading effect is from interference. Explicitly

$$iA = \frac{i\mathbf{k}^2}{4m_{Pl}}\epsilon^*_{ij}(k)Q_{ij}(-|\mathbf{k}|)\left[1 - \frac{m}{2m^2_{Pl}}\mathbf{k}^2\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\mathbf{q}^2}\frac{1}{\mathbf{k}^2 - (\mathbf{k}+\mathbf{q})^2 + i\epsilon}\right]$$

dropped pure Im terms that do not give rise to interference)

Note that the integral

$$I = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\mathbf{q}^2}\frac{1}{\mathbf{k}^2-(\mathbf{k}+\mathbf{q})^2+i\epsilon} = \frac{i}{16\pi|\mathbf{k}|}\left[\frac{1}{\epsilon_{IR}}-\gamma-\ln\left(\frac{-\mathbf{k}^2-i\epsilon}{\pi\mu^2}\right)\right]$$

is log divergent in the IR as $\mathbf{q}\to 0$

$$I \sim \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\mathbf{q}^2(\mathbf{k}\cdot\mathbf{q})} \qquad \text{for} \qquad \mathbf{q}\to 0$$

Physically this effect is easy to understand. For $\mathbf{q}\to 0$, intermediate rad. graviton is on shell. The IR singularity is then entirely analogous to the IR divergences found in QM scattering off a Coulomb $1/r$ potential. In fact, the singularity drops from the interference term with the LO amp:

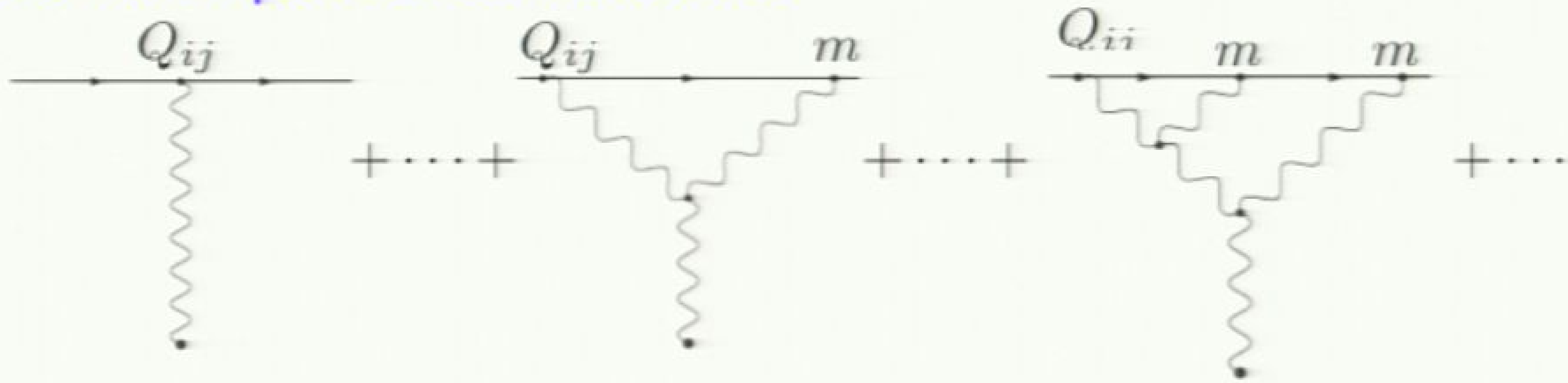
$$|\mathcal{A}|^2 = |\mathcal{A}_{LO}|^2\left[1-(2\pi)G_N m|\mathbf{k}|+\cdots\right]$$

For circular orbits, this leads to:

$$\frac{\dot{E}_{tail}}{\dot{E}_{LO}} = -4\pi v^3$$

which is non-analytic in the expansion parameter = v^2. Note that this a LARGE correction due to factor of 4π.

t is amusing to look at further interactions between the outgoing graviton and the Newtonian potential of the source:

$iA =$ 

one finds the structure (keep most IR singular terms in each graph):

$$iA = iA_{LO}\left[1 - \frac{m}{2m_{Pl}^2}\mathbf{k}^2 I(\mathbf{k}) + \frac{1}{2!}\left(-\frac{m}{2m_{Pl}^2}\mathbf{k}^2 I(\mathbf{k})\right)^2 + \cdots\right]$$

By analogy with Coulomb corrections to scattering, expect the series of tail corrections to exponentiates into a Coulomb phase (Weinberg, 1965). One finds:

$$iA = iA_{LO}\exp\left[-\frac{m}{2m_{Pl}^2}\mathbf{k}^2 I(\mathbf{k})\right]$$

$\mathrm{Im} I =$ Coulomb phase. $\mathrm{Re} I =$ "Sudakov" his can be shown by brute force calculation. Exponentiation also xpected on general grounds, based on the analogy with Coulomb cattering:

1. Exponentiation is necessary in order to ensure all orders cancellation of $1/\epsilon_{IR}$ pole terms.

2. Summation is equivalent to using the graviton propagator in Newton background.

his leads to a prescription for (partially) resumming the PN expansion:

$$Q_{i_1\cdots i_\ell}(|\mathbf{k}|) \rightarrow Q^{\text{“dressed”}}_{i_1\cdots i_\ell}(|\mathbf{k}|) \equiv Q_{i_1\cdots i_\ell}(|\mathbf{k}|) \exp\left[-\frac{m}{2m_{Pl}^2}\mathbf{k}^2 I(\mathbf{k})\right]$$

(“factorization theorem”)

his prescription leads to a systematic procedure for removing IR

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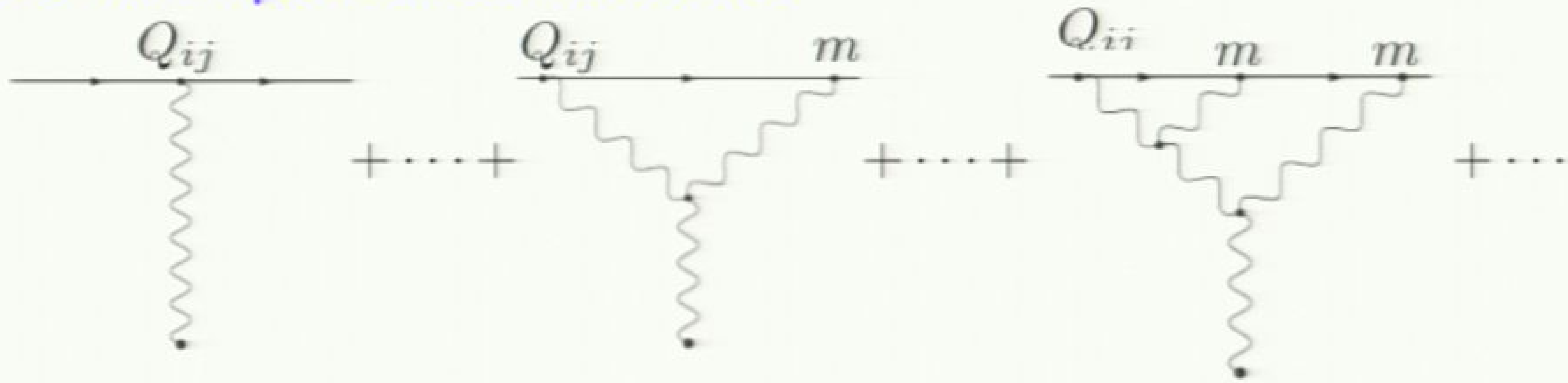
$iA =$ 

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xpected on general grounds, based on the analogy with Coulomb
cattering:

1. Exponentiation is necessary in order to ensure all orders cancellation of $1/\epsilon_{IR}$ pole terms.

2. Summation is equivalent to using the graviton propagator in Newton background.

'his leads to a prescription for (partially) resumming the PN expansion:

$$Q_{i_1\cdots i_\ell}(|\mathbf{k}|) \to Q_{i_1\cdots i_\ell}^{\text{“dressed”}}(|\mathbf{k}|) \equiv Q_{i_1\cdots i_\ell}(|\mathbf{k}|)\exp\left[-\frac{m}{2m_{Pl}^2}\mathbf{k}^2 I(\mathbf{k})\right]$$

(“factorization theorem”)

'his prescription leads to a systematic procedure for removing IR
livergences due to tail effects

ls a test of this prescription, can calculate radiation at order v^5 in terms f the moments at order v^2 previously calculated. Plug into

$$\frac{dE}{dt} = \frac{G_N}{T}\int_0^\infty \frac{d\omega}{2\pi}\omega^6\left[\frac{1}{5}|E^{ij}(\omega)|^2 + \frac{16}{45}|B^{ij}(\omega)|^2 + \frac{4}{756}|E^{ijk}(\omega)|^2 + \cdots\right]$$

naking the replacements

$$E_{ij}(\omega) \to E_{ij}(\omega)\exp\left[-\frac{m}{2m_{Pl}}\omega^2 I(\omega)\right]$$

and also for E_{ijk}, B_{ij} gives after expanding, e.g for circular orbit binary

$$\left.\frac{dE}{dt}\right._{v^5} = -\frac{8191\pi}{672}v^5\left.\frac{dE}{dt}\right._{LO}$$

n agreement w/ known results (Blanchet, 1996)

It would be interesting to see the full consequences of this "improvement" of perturbation theory for GW phenomenology.

Can get a rough estimate by looking at binary system in circular orbit. The resummed power is

$$\frac{\dot{E}_{\rm resum}}{\dot{E}_{LO}} = e^{-4\pi v^3}$$

Recall that, for e.g. LIGO, expansion parameter is:

$$v(10\ \mathrm{Hz}) \sim 0.06 \left(\frac{m}{m_\odot}\right)^{1/3} \rightarrow v(1\ \mathrm{kHz}) \sim 0.3 \left(\frac{m}{m_\odot}\right)^{1/3}$$

So for a BH with $m_{BH} \sim 10 m_\odot$ correction due to resummation can be large since:

$$0.03 < 4\pi v^3 < 3.4$$

however, at the upper bound, expansion in v probably not very reliable.

ALSO: Need to make sure that this expansion is systematic (i.e. are we really summing largest effect at each order?). This is work in progress.

Finite Size Effects:

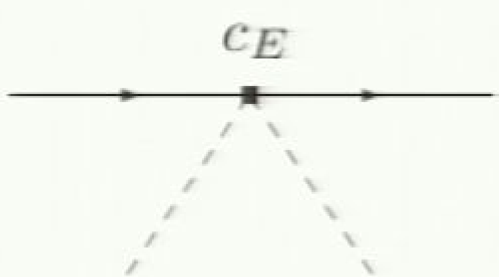
Recall that in the pt. particle EFT the structure of the binary constituents (BH or NS) is encoded in the "tidal" operators

$$S_{pp} = \cdots + c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu}$$

w/

$$c_{E,B} \sim m_{Pl}^2 r_s^5$$

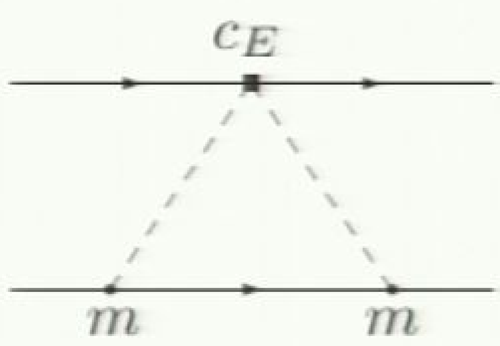
iven the scaling of the coefficients + power counting rules in the $v \ll 1$ mit, it is easy to determine the order at which finite size effects come in. or example:



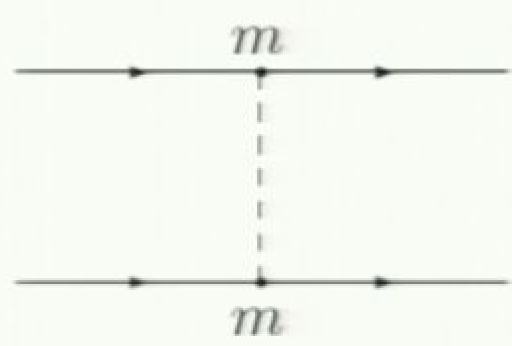
$$\sim \qquad v^{10}$$

ote: $(r_s/r)^5 \sim v^{10}$

Leading tidal effect is due to potential graviton exchange:



$$\sim L v^{10} \sim$$



$$\times v^{10}$$

I.e., tidal effects are suppressed by v^{10} (!) relative to Newtonian gravity, so are completely irrelvant for LIGO. However, expect an enhancement for less compact objects.

$$c_{E,B} \sim m_{Pl}^2 R_{NS}^5 \qquad \frac{R_{NS}}{r_s} \sim 10$$

So finite size effect is more like $\sim 10^5 \times v^{10}$

could make a significant difference in the "endpoint" region $v \to 0.1$

Detectable by LIGO? See Flannagan + Hinderer, arXiv:0709.1915

There are other internal structure effects at lower order in the expansion:

BH horizon absorption: Dissipative effects can be treated within the pt particle EFT (see WG+Rothstein hep-th/0511133). For non-spinning BH's this is a v^8 effect, so too small for LIGO (LISA?). Absorption is enhanced by spin to v^5 (Porto, arXiv:0710.5150)

Spin: Classically this is by definition a finite size effect. Effects of spin on two-body potentials and eqns. of motion at v^6

$V_{3PN} \sim \mathbf{S}_1 \cdot \mathbf{S}_2$ Porto+Rothstein, gr-qc/0604009
arXiv:0804.0260

$V_{3PN} \sim \mathbf{S}_1^2$ Porto+Rothstein, arXiv:0804.0260

these are new results.

Dont know how to explicitly include the dof's that capture BH absorption. However, by $SO(3)$ symmetry of Schwarzschild soln, must couple to gravitons through gauge invariant term

$$S = -\int d\tau Q^E_{ab} E^{ab} - \int d\tau Q^B_{ab} B^{ab} + \cdots ,$$

$E_{ab}, B_{ab} =$ "electric" and "magnetic" components of the Riemann tensor (classified according to parity).

$Q^E_{ab}, Q^B_{ab} =$ "electric" and "magnetic" quadrupole moment operators (composite operators built out of unknown worldline theory).

All the dynamics can be calculated if we know the correlation functions

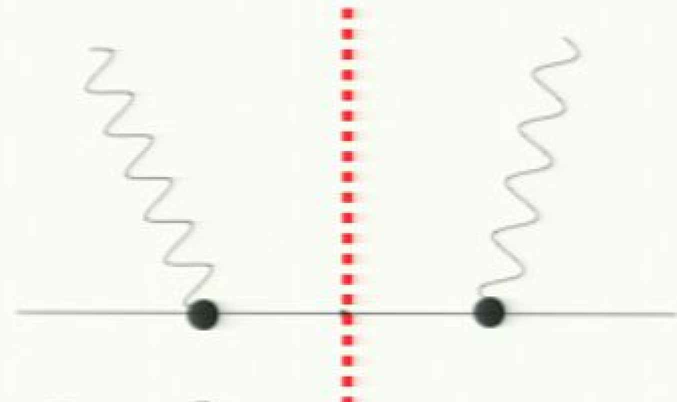
$$\langle Q^{E,B} \cdots Q^{E,B} \rangle$$

At leading order need only two-pt fns.   Obtain by matching to

ull theory: Solve Schrodinger eqn. for $h_{\mu\nu}$ in Schwarzschild background,
et

$$\sigma_{abs,p}(\omega) = \frac{1}{45} 4\pi r_s^6 \omega^4,$$

FT: Flat space calculation



$$\sigma_{abs,p}(\omega) = \frac{\omega^3}{2m_{Pl}^2} \mathrm{Im} F(\omega).$$

v/

$$\int dx^0 e^{-i\omega x^0} \langle T Q_{ab}^E(0) Q_{cd}^E(x^0) \rangle = -\frac{i}{2} \left[\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \frac{2}{3}\delta_{ab}\delta_{cd} \right] F(\omega),$$

$$\mathrm{Im} F(\omega) = 16 G_N^5 m^6 \omega / 45$$

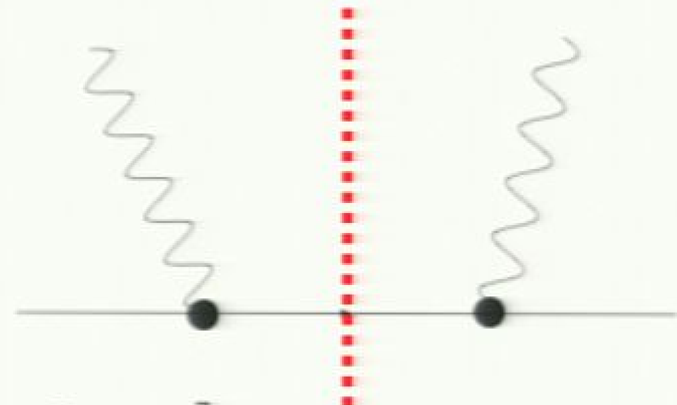
analogous to AdS/CFT calculations of brane absorption).

At leading order need only two-pt fns. Obtain by matching to

Full theory: Solve Schrodinger eqn. for $h_{\mu\nu}$ in Schwarzschild background, get

$$\sigma_{abs,p}(\omega) = \frac{1}{45} 4\pi r_s^6 \omega^4,$$

EFT: Flat space calculation



$$\sigma_{abs,p}(\omega) = \frac{\omega^3}{2m_{Pl}^2} \mathrm{Im} F(\omega).$$

w/

$$\int dx^0 e^{-i\omega x^0} \langle T Q^E_{ab}(0) Q^E_{cd}(x^0)\rangle = -\frac{i}{2}\left[\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \frac{2}{3}\delta_{ab}\delta_{cd}\right] F(\omega),$$

$$\mathrm{Im} F(\omega) = 16 G_N^5 m^6 \omega / 45$$

(analogous to AdS/CFT calculations of brane absorption).

Can also use the same methods to obtain dissipative corrections in stars made of ordinary matter (e.g. neutron star):

$$\frac{dP_{abs}}{d\omega} = -\frac{1}{T}\frac{G_N}{64\pi^2}\sum_{a\neq b}\frac{\sigma^{(b)}_{abs}(\omega)}{\omega^2} m_a^2 |q^{(a)}_{ij}(\omega)|^2,$$

where $\sigma_{abs}(\omega)$ is the body's low energy graviton absorption cross section.

Would expect that $\sigma_{abs}(\omega) < \sigma^{BH}_{abs}(\omega)$ on general grounds, however.

Conclusions

EFT methods are a useful way of organizing the calculation of gravitational wave processes.

1. Systematically parametrize and power count finite size/short distance effects.

2. Calculations can be done unambiguously in a pt. particle limit, where they simplify.

3. Separation of scales disentangles UV and IR physics. Can exploit this to sum/remove IR and UV singularities.

ere, presented the non-relativistic expansion, but the methods work in other nematic limits of interest to LIGO/LISA, with a suitably modified power ounting scheme (eg, $\lambda = m/M \ll 1$,....)