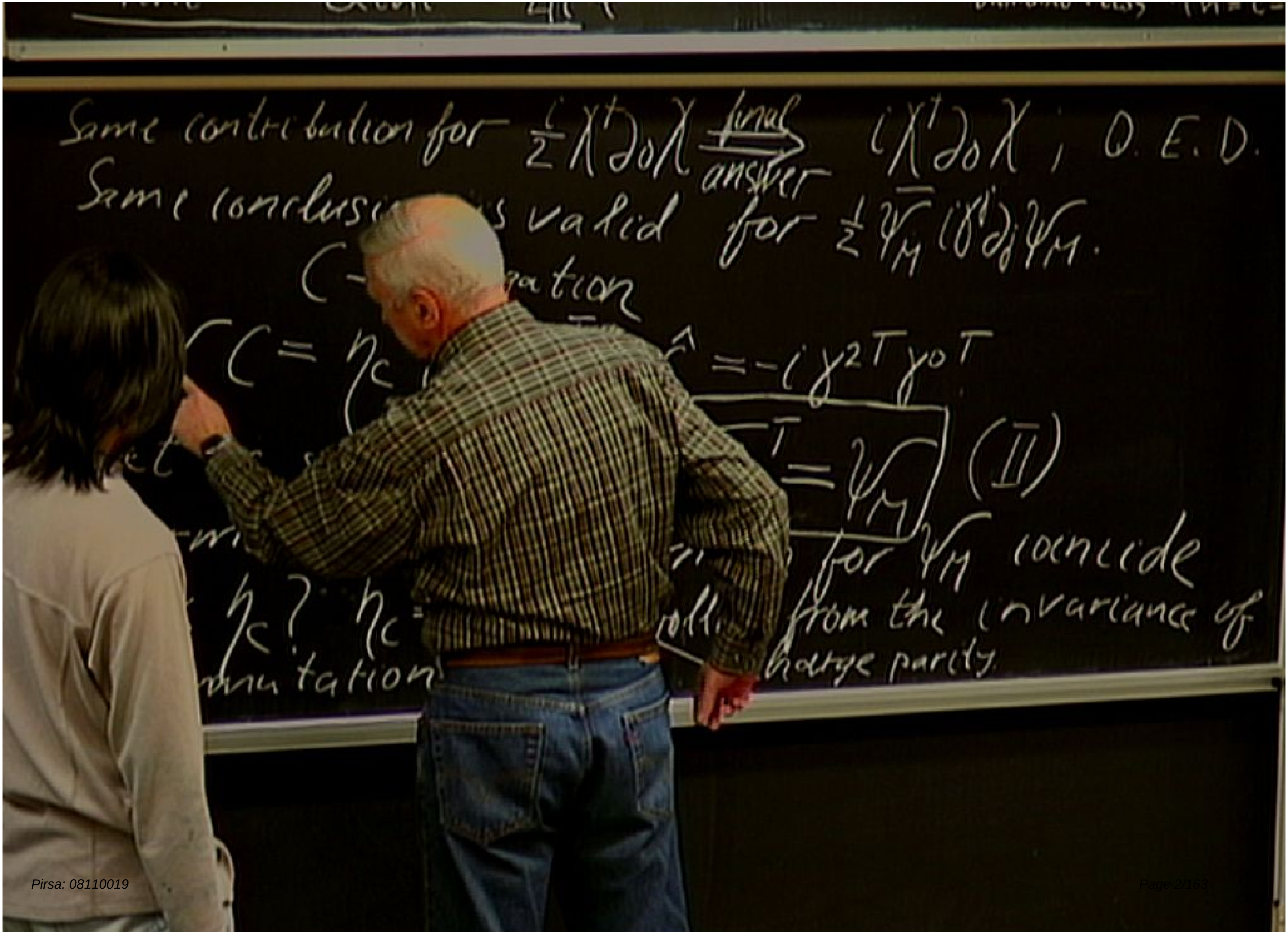
Title: Quantum Field Theory 1 - Lecture 12B

Date: Nov 26, 2008 03:30 PM

URL: http://pirsa.org/08110019

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario


Same contribution for $\frac{i}{2}\chi^T\partial_0\chi \xrightarrow[\text{answer}]{\text{final}} i\chi^\dagger\partial_0\chi$; Q. E. D.
Same conclusion valid for $\frac{1}{2}\bar{\psi}_M i\gamma^j\partial_j\psi_M$.
ation
$\hat{C} = -i\gamma^{2T}\gamma^{0T}$
$= \psi_M$ (II)
for ψ_M coincide
from the invariance of
tation
charge parity.

Same contribution for $\frac{i}{2}\chi^\dagger\partial_0\chi \xrightarrow[\text{answer}]{\text{final}} i\chi^\dagger\partial_0\chi$; Q.E.D.

Same conclusion is valid for $\frac{i}{2}\bar{\psi}_M i\gamma^j\partial_j\psi_M$.

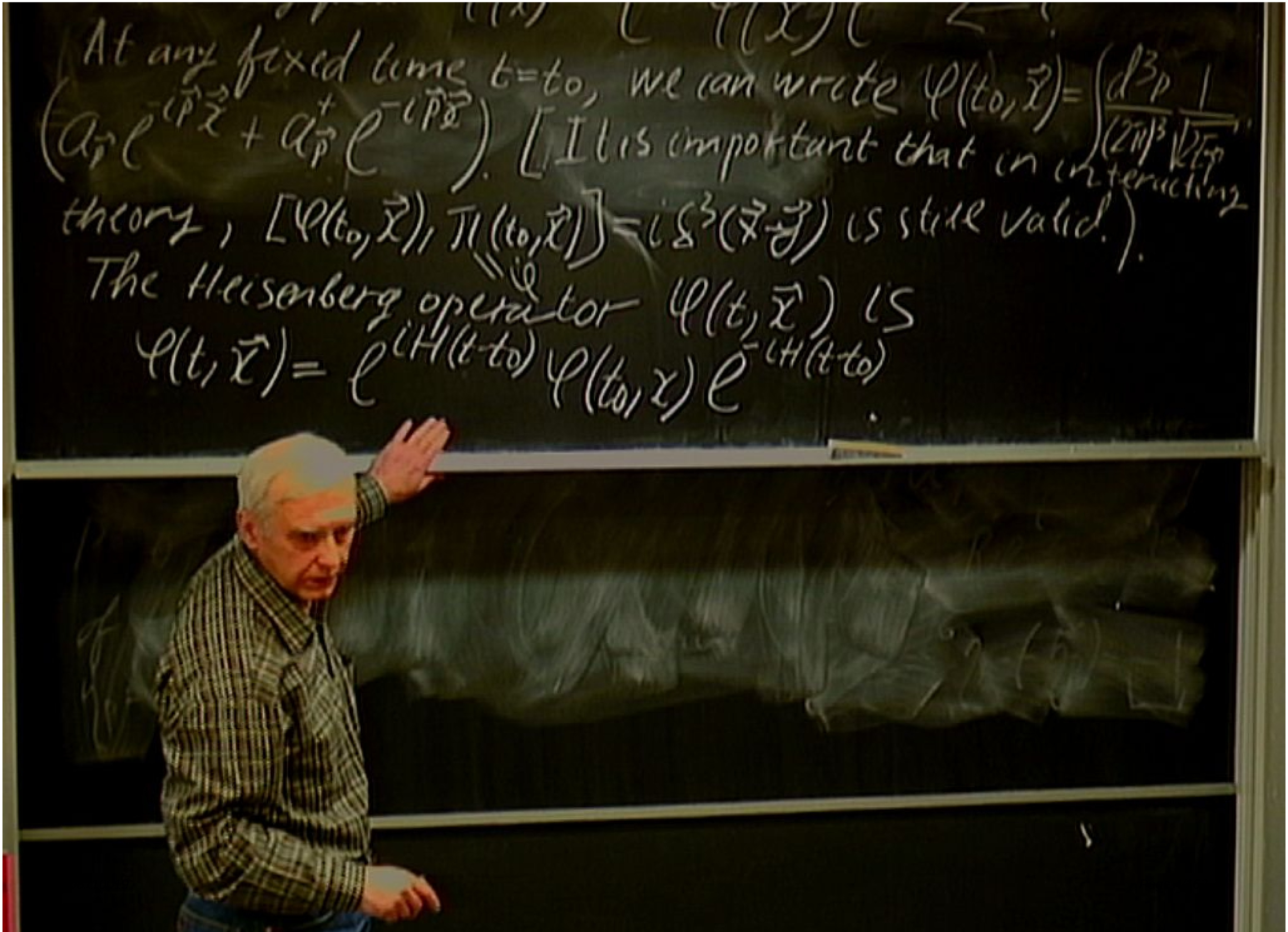
C - conjugation $C^{-1}\psi_M C = \eta_C \hat{C}\bar{\psi}_M^T$

$C^{-1}\psi_M C = \eta_C \hat{C}\bar{\psi}_M^T$; $\hat{C} = -i\gamma^{2T}\gamma^{0T}$

Let us show that $\boxed{\hat{C}\bar{\psi}_M^T = \psi_M}$ (II)

Fermions and antifermion for ψ_M coincide

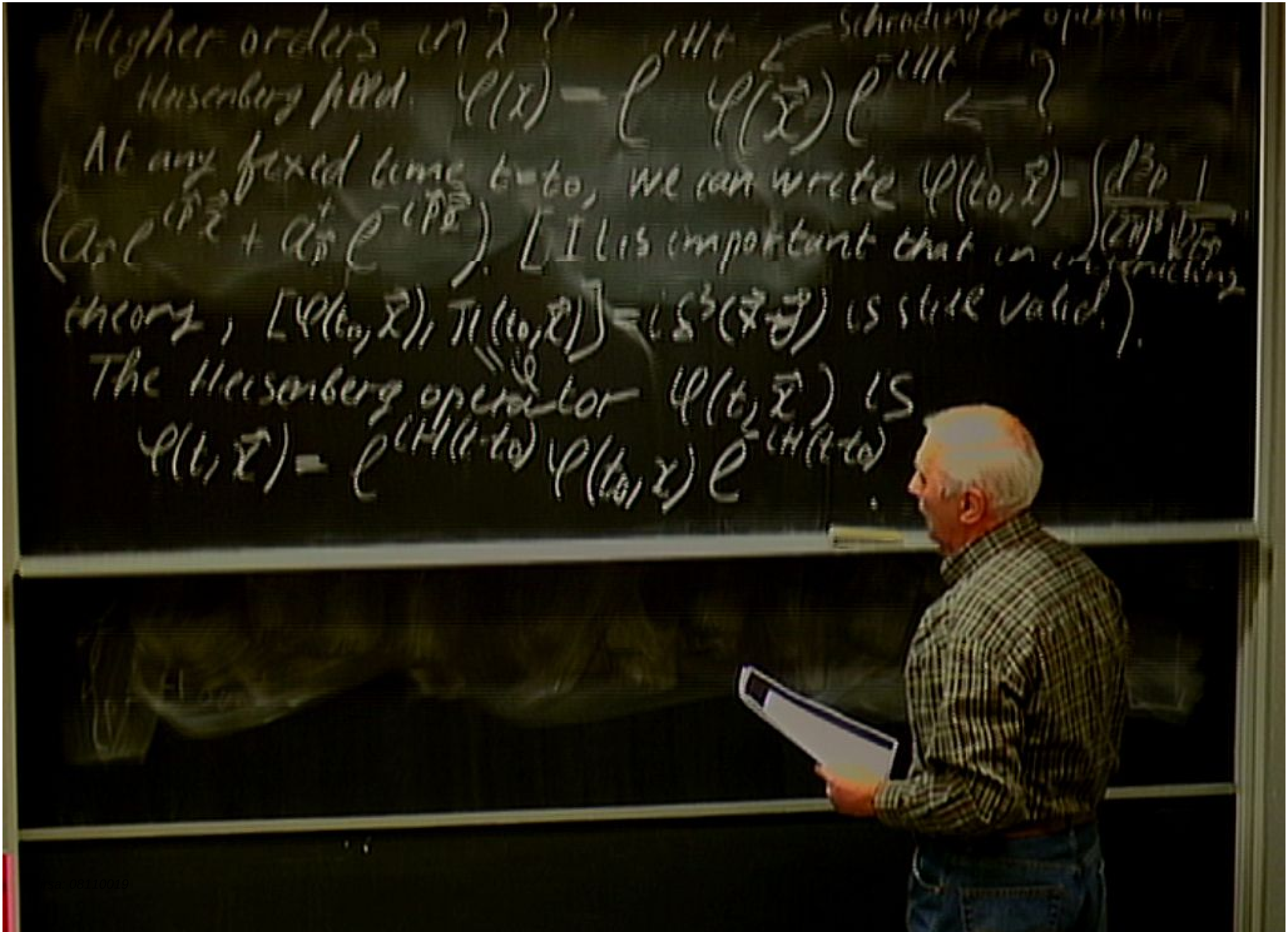
What is η_C? $\eta_C = \pm 1$ ← it follows from the invariance of anticommutation relations ~~charge parity~~

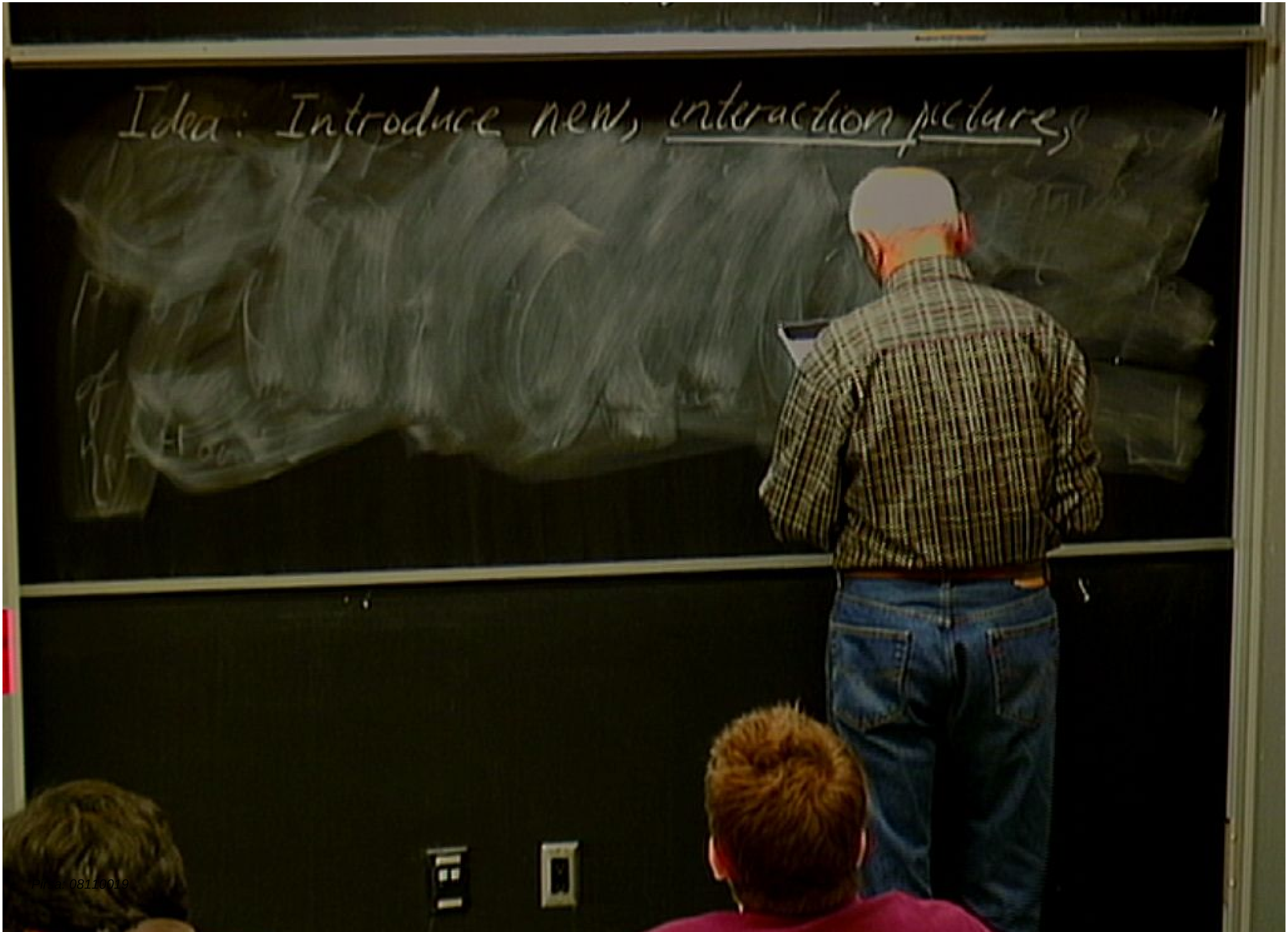

At any fixed time $t=t_0$, we can write $\varphi(t_0,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}}\, e^{i\vec{p}\vec{x}} + a_{\vec{p}}^{+}\, e^{-i\vec{p}\vec{x}} \right)$.
[It is important that in interacting theory, $[\varphi(t_0,\vec{x}), \pi(t_0,\vec{y})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.].
$\pi = \dot{\varphi}$
The Heisenberg operator $\varphi(t,\vec{x})$ is
$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\, \varphi(t_0,x)\, e^{-iH(t-t_0)}$

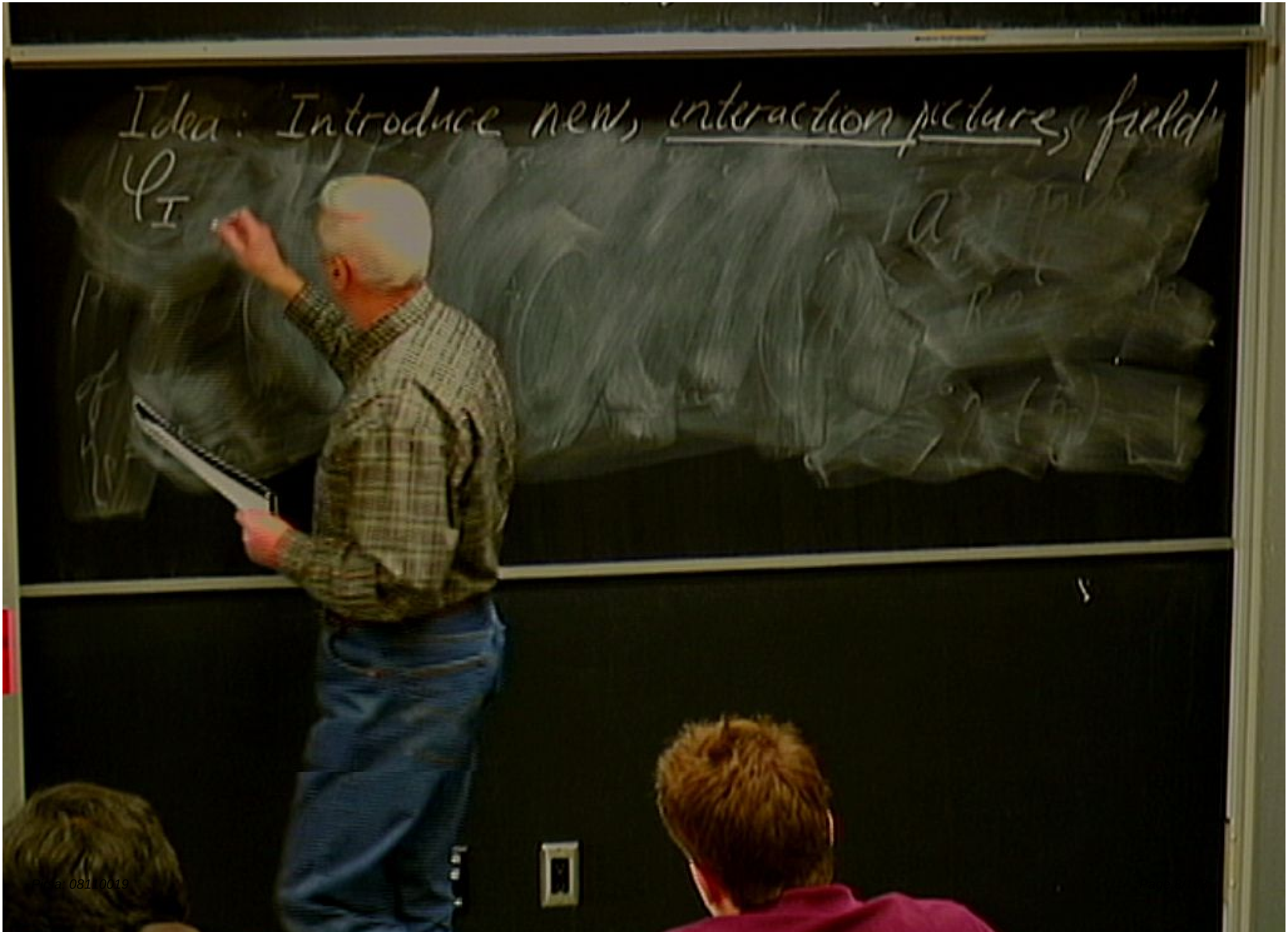
(... interacting theory, $[\varphi(t_0,\vec{x}), \pi(t_0,\vec{y})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.)

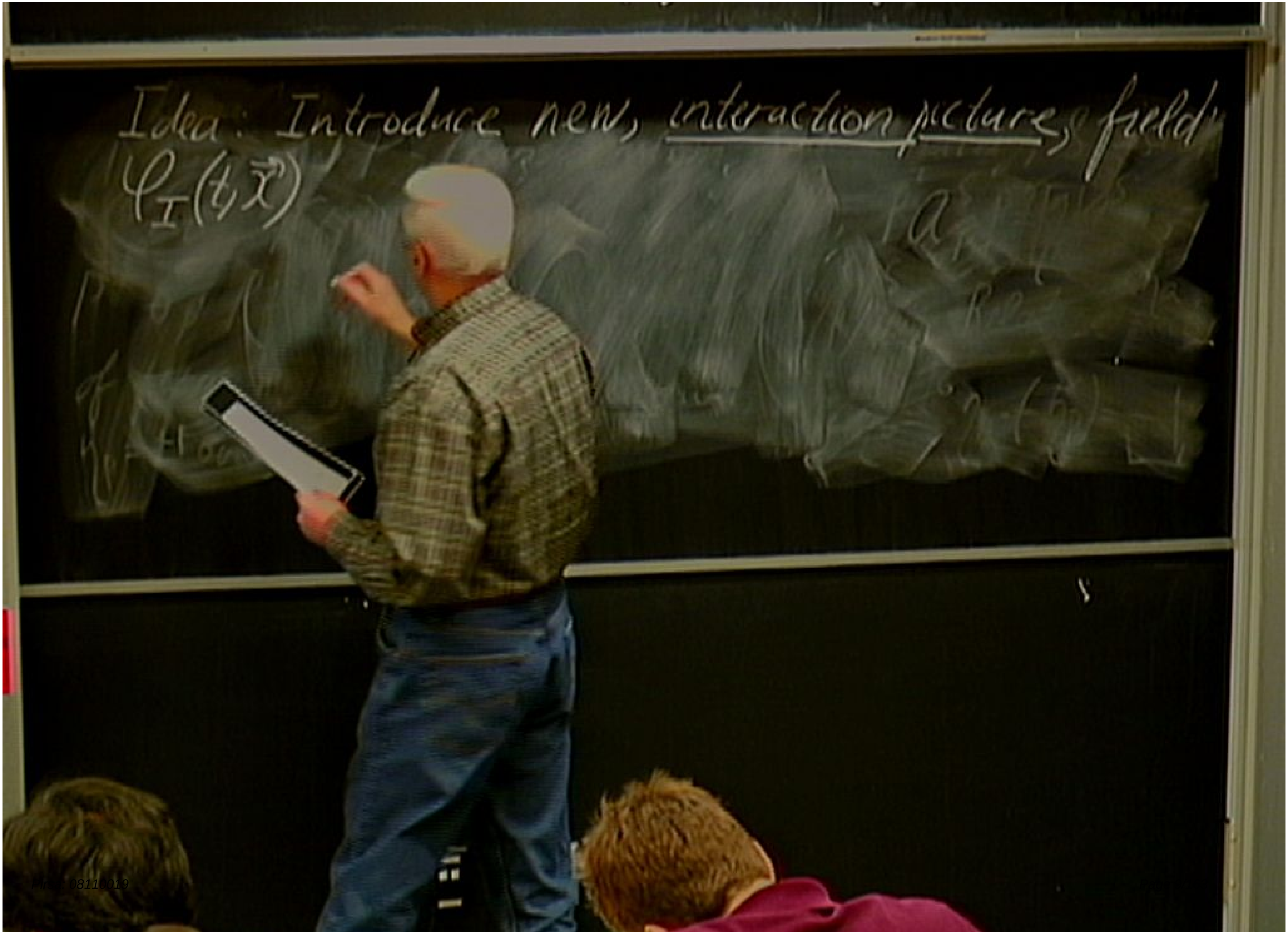
The Heisenberg operator $\varphi(t,\vec{x})$ is

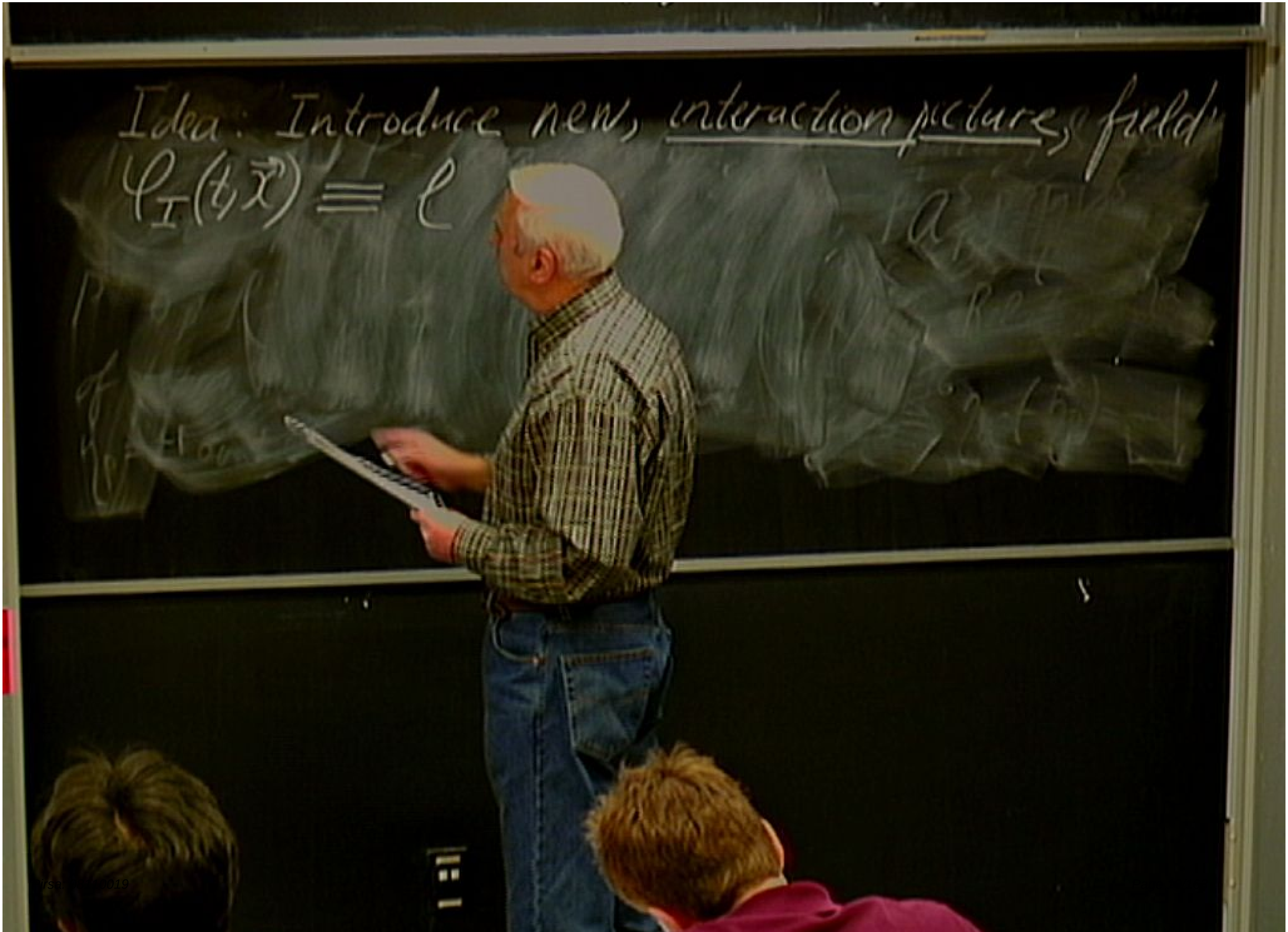
$$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\varphi(t_0,\vec{x})e^{-iH(t-t_0)}.$$

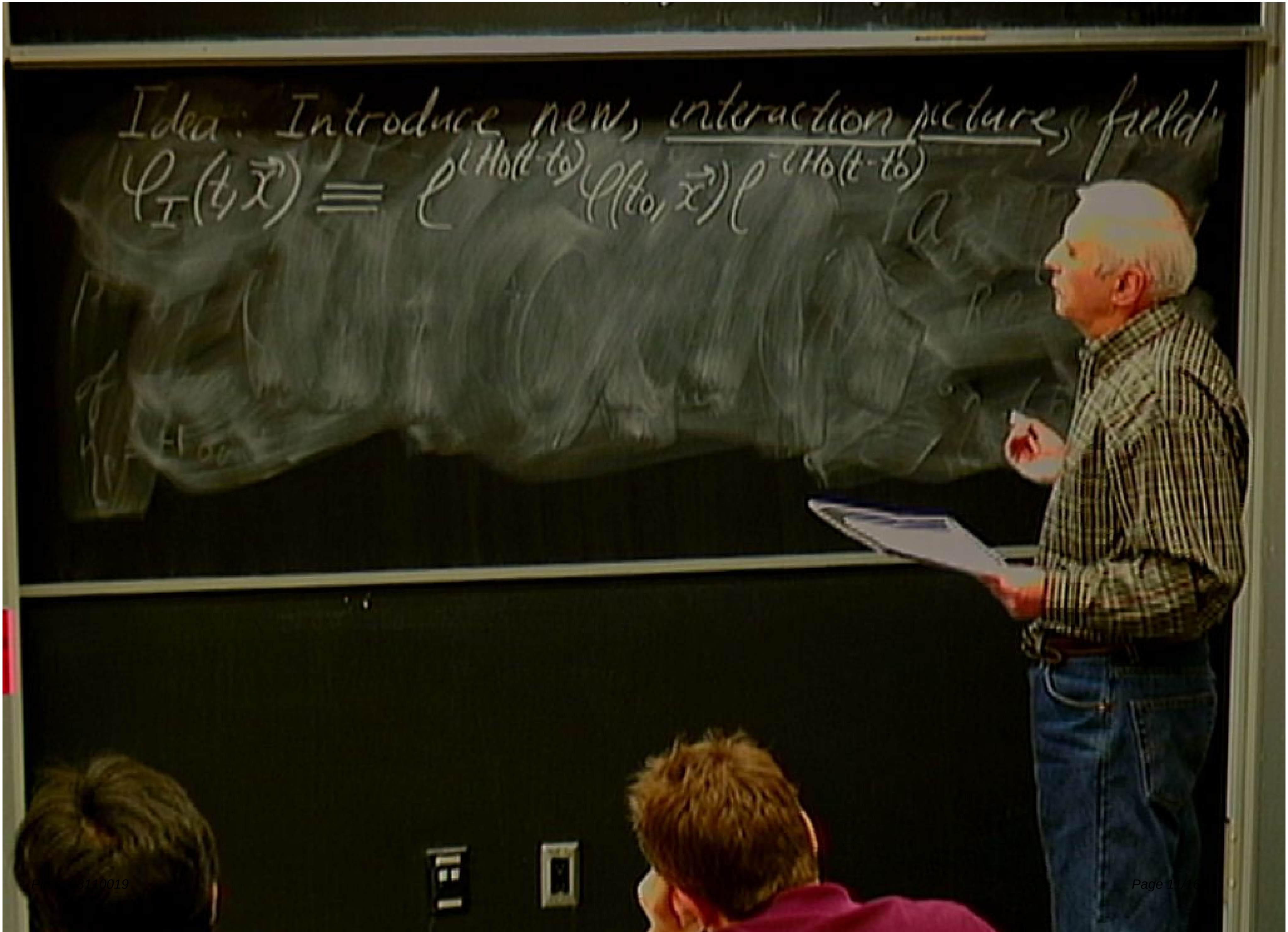

Higher orders in λ ?
Schrodinger operator
Heisenberg field φ(x) = e^{iHt} φ(x⃗) e^{-iHt} ?
At any fixed time t=t0, we can write φ(t0, x⃗) = ∫ d³p/(2π)³ 1/√(2Ep)
(ap e^{ip⃗x⃗} + ap† e^{-ip⃗x⃗}). [It is important that in interacting
theory, [φ(t0, x⃗), π(t0, y⃗)] = iδ³(x⃗-y⃗) is still valid.]
The Heisenberg operator φ(t, x⃗) is
φ(t, x⃗) = e^{iH(t-t0)} φ(t0, x⃗) e^{-iH(t-t0)}.

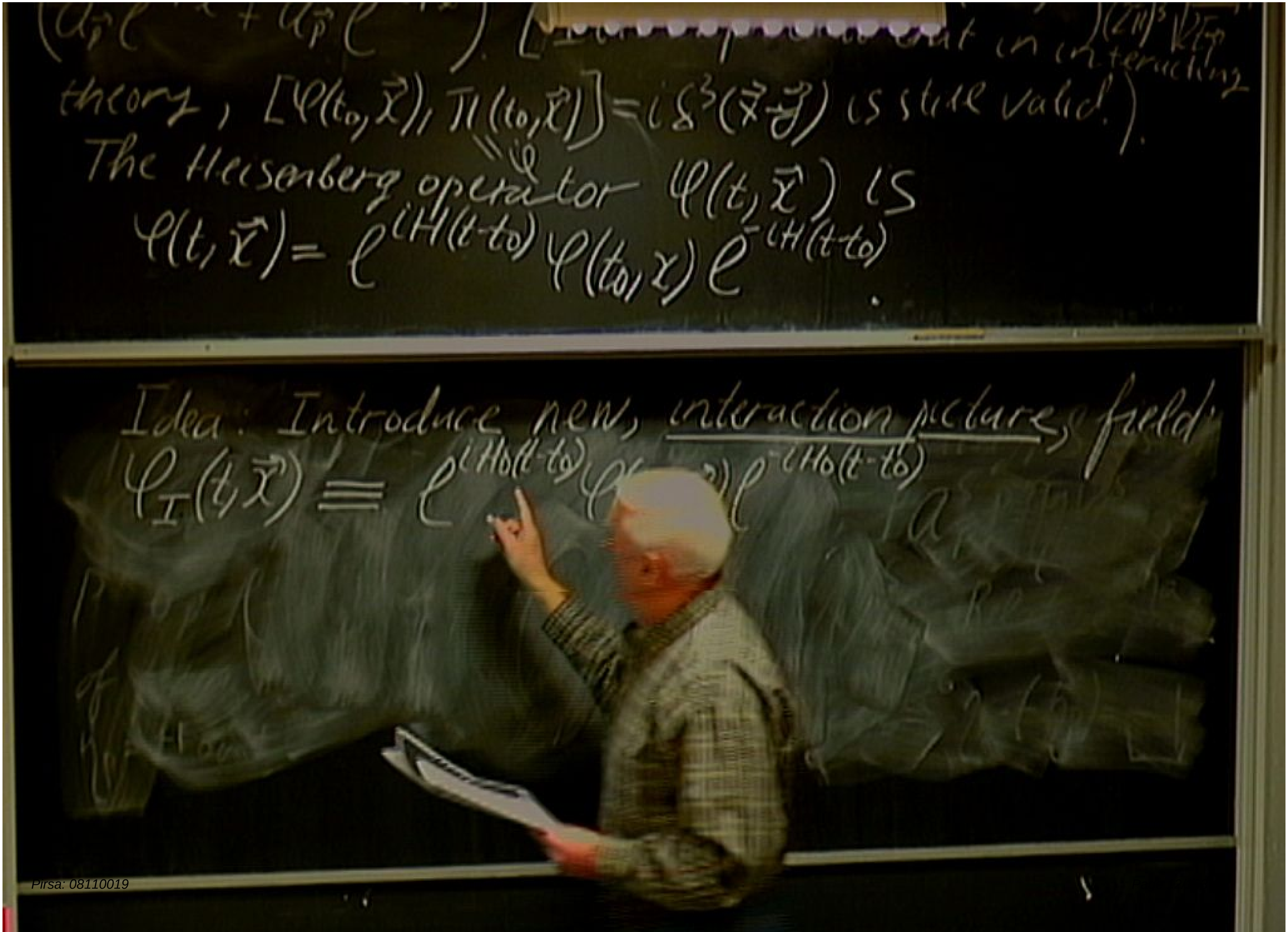

Idea: Introduce new, interaction picture,


Idea: Introduce new, interaction picture, field
φ_I


Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x})$


Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e$


Idea: Introduce new, interaction picture, field:
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)}$



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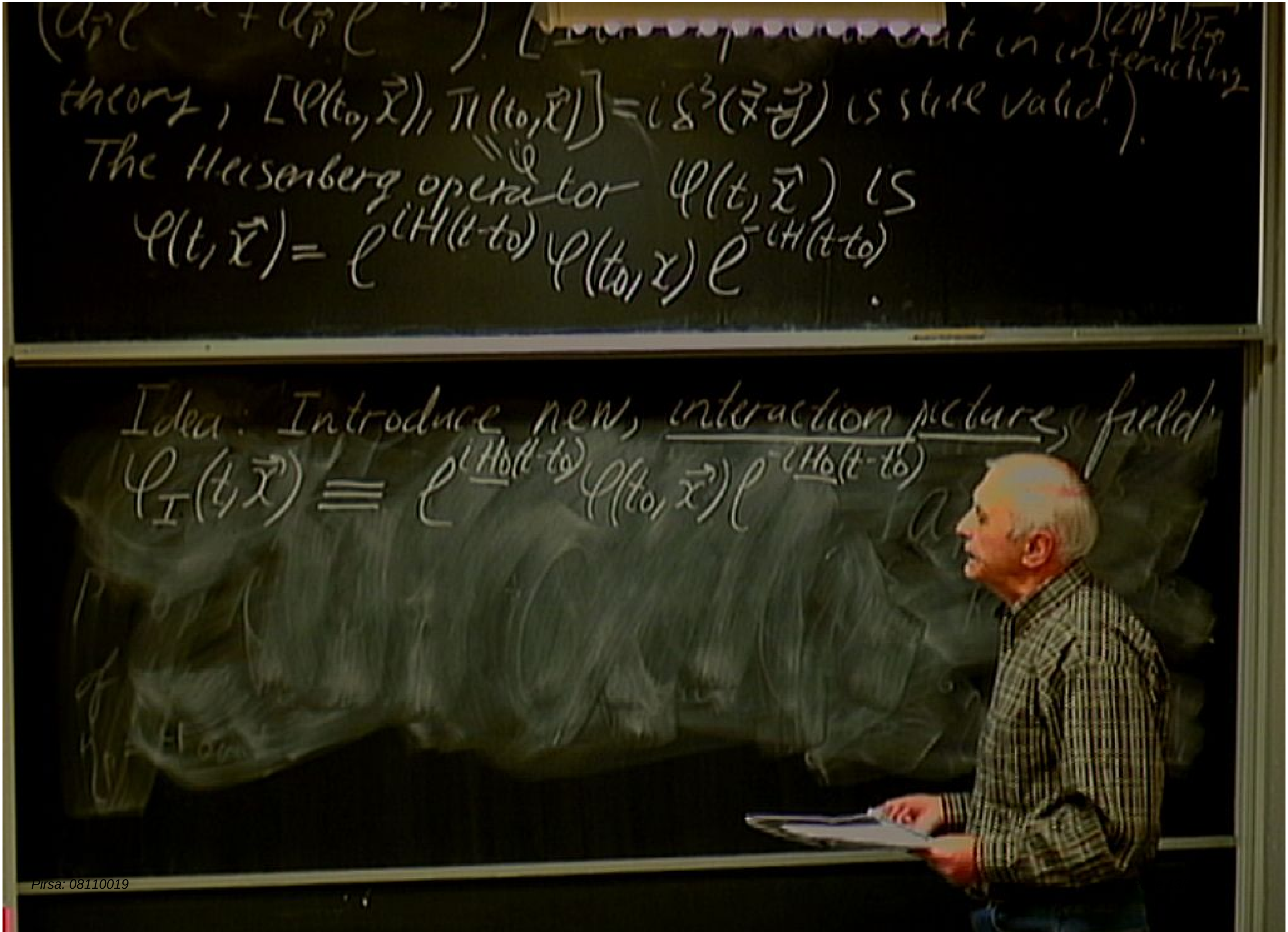
theory, $[\varphi(t_0,\vec{x}), \pi(t_0,\vec{y})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.).
The Heisenberg operator $\varphi(t,\vec{x})$ is
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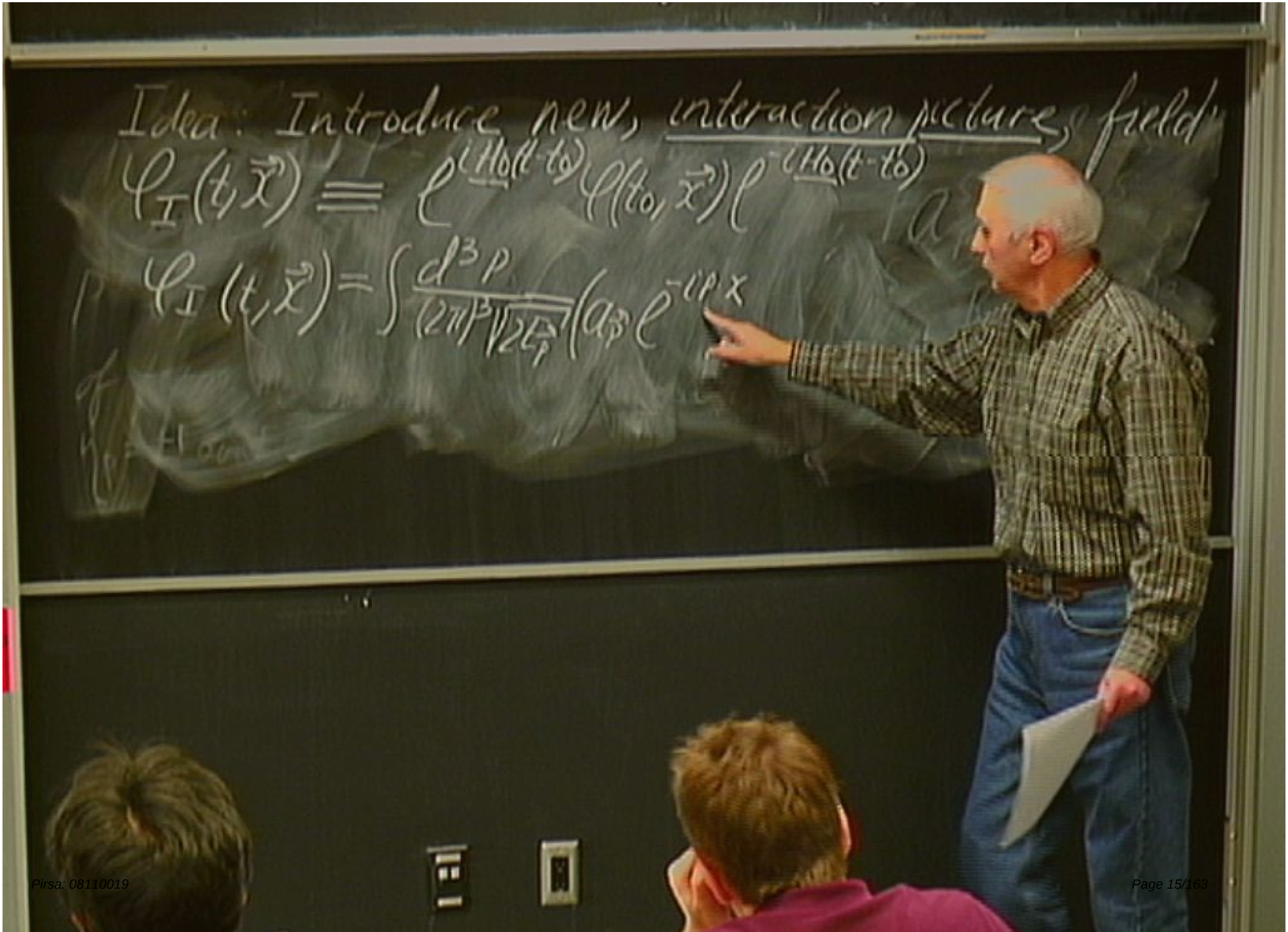
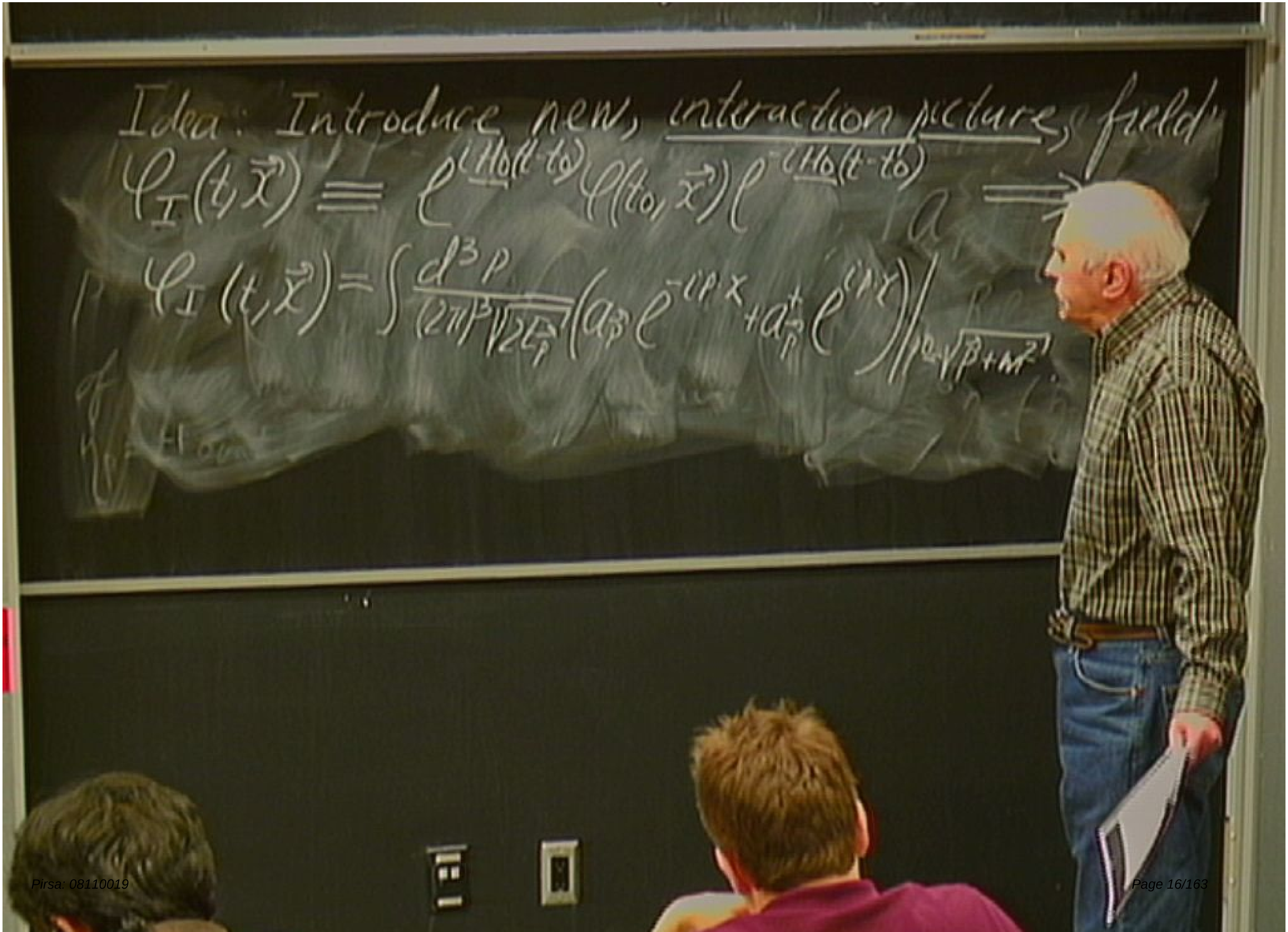

Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)}$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}(a_p e^{-ip\cdot x}$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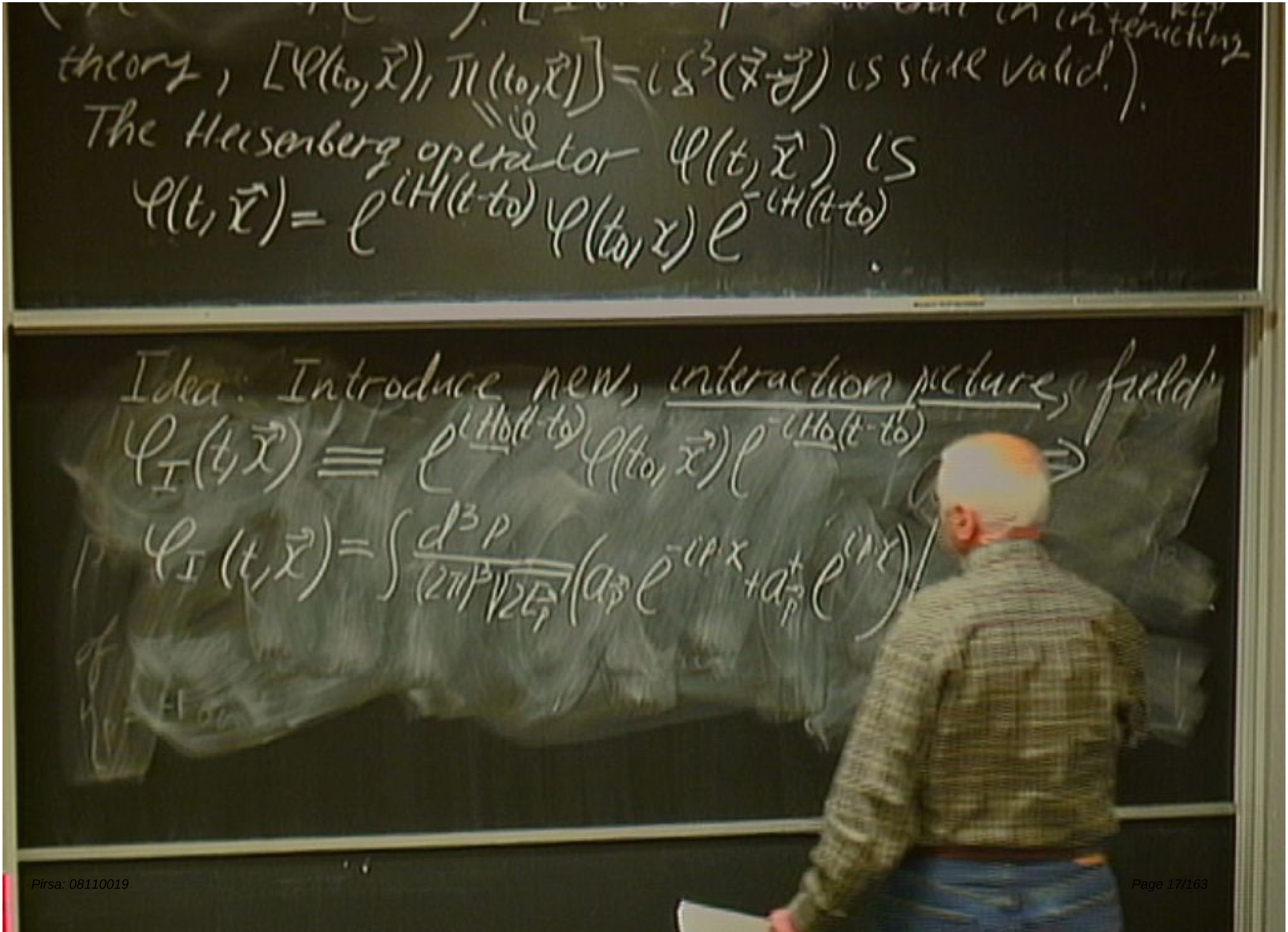
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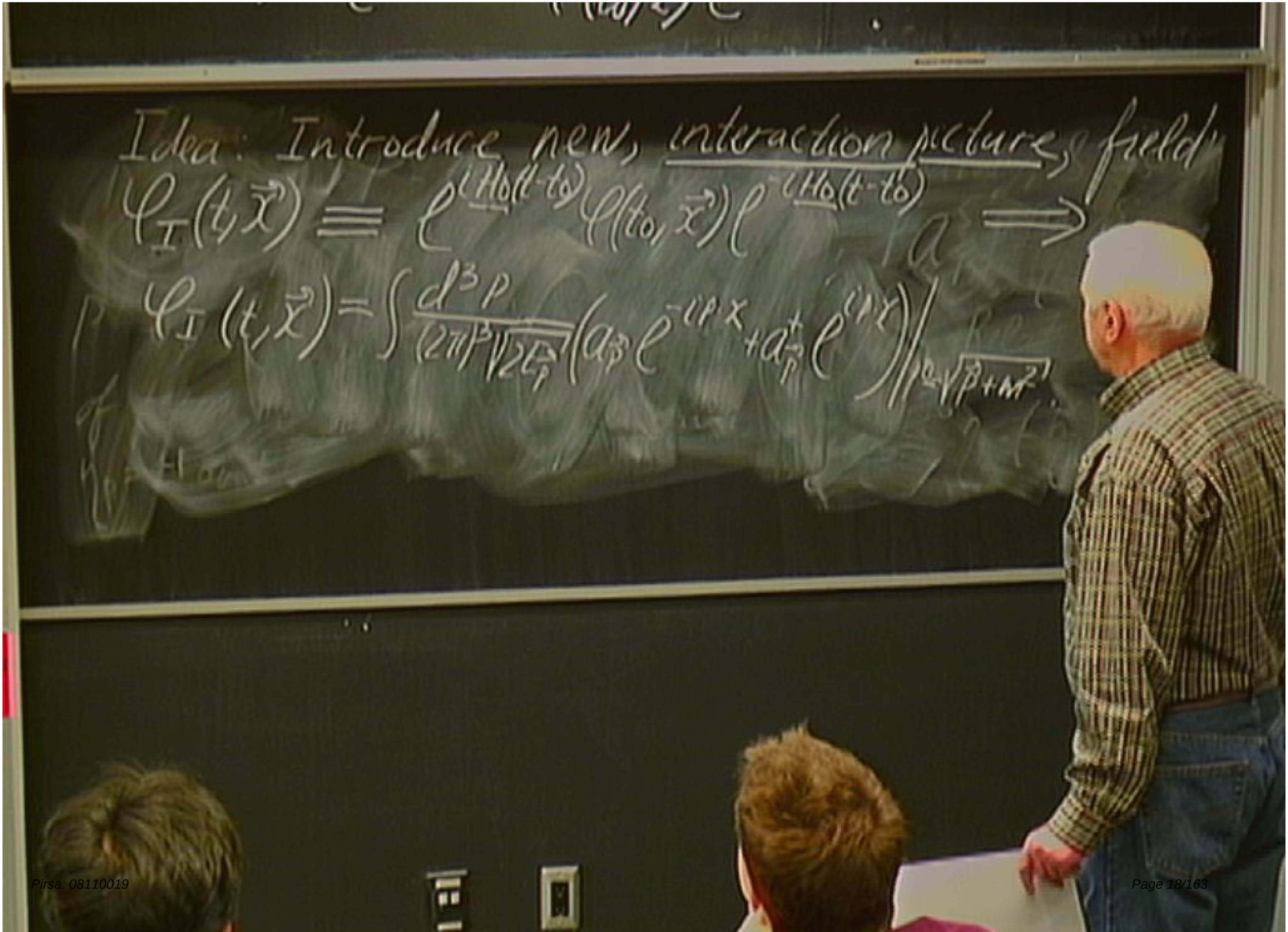
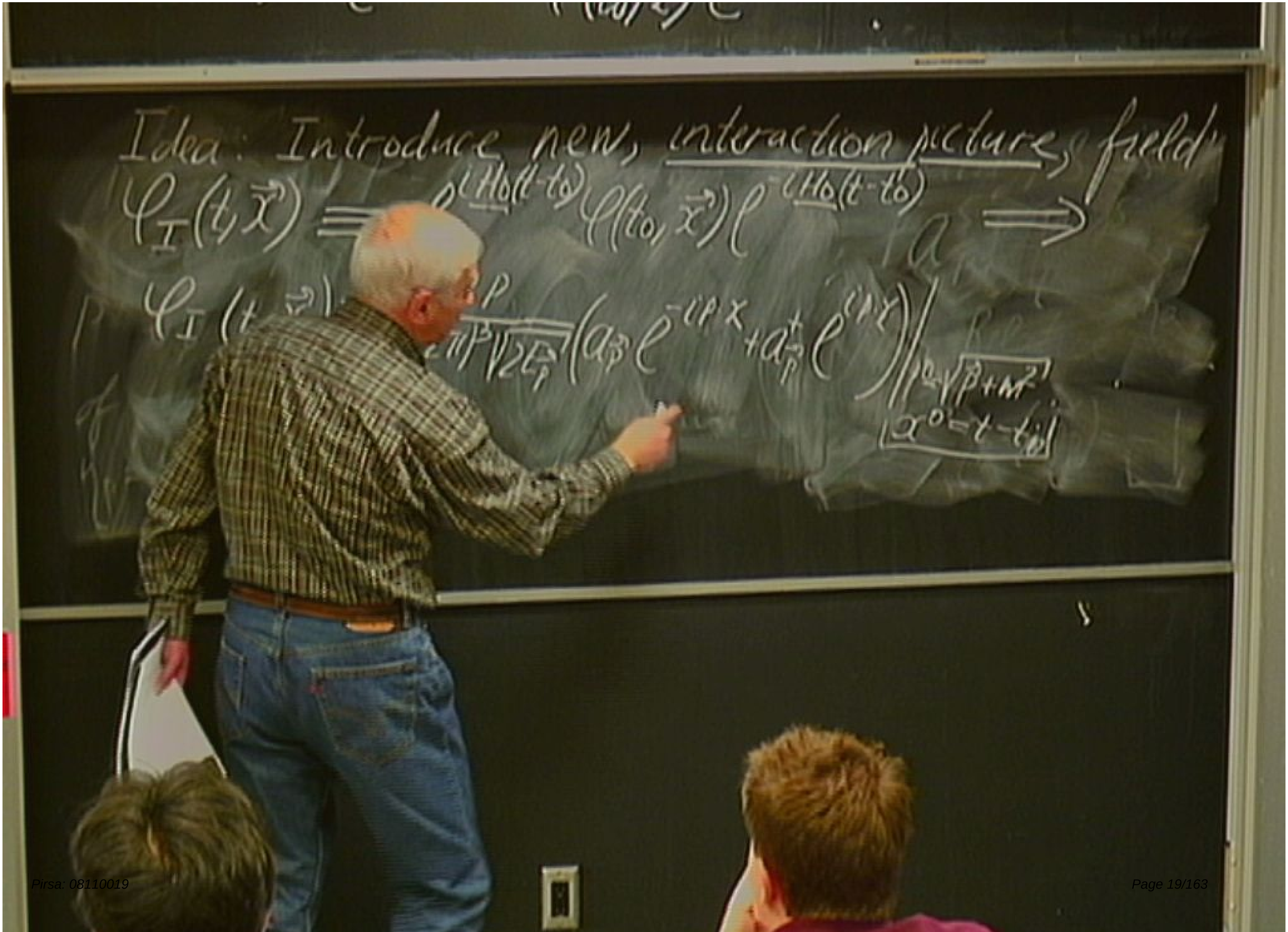
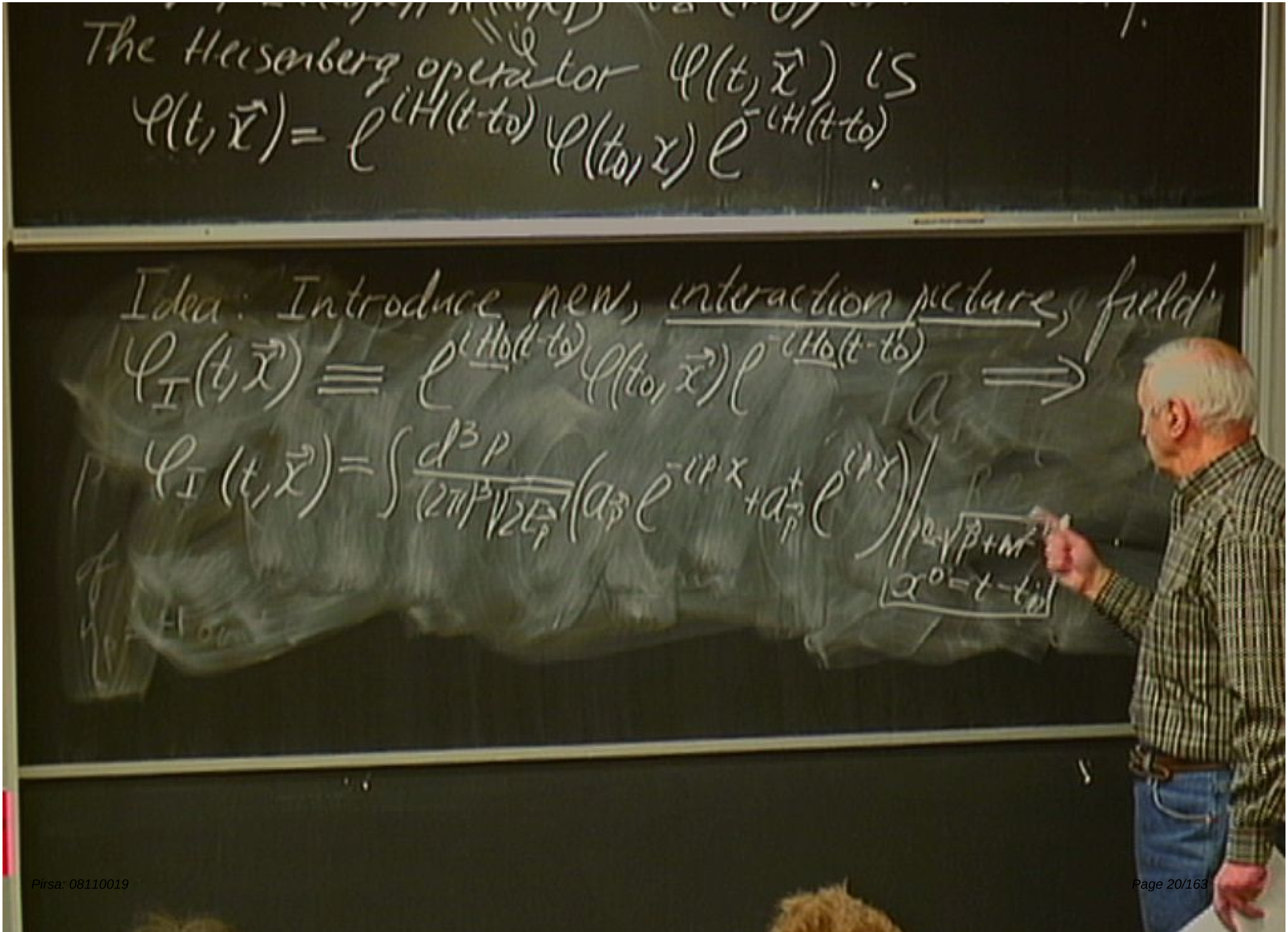

Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)} \Rightarrow$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\left(a_{\vec{p}}e^{-ip\cdot x} + a^\dagger_{\vec{p}}e^{ip\cdot x}\right)\Big|_{p^0=\sqrt{\vec{p}^2+m^2}}$


Idea: Introduce new, interaction picture, field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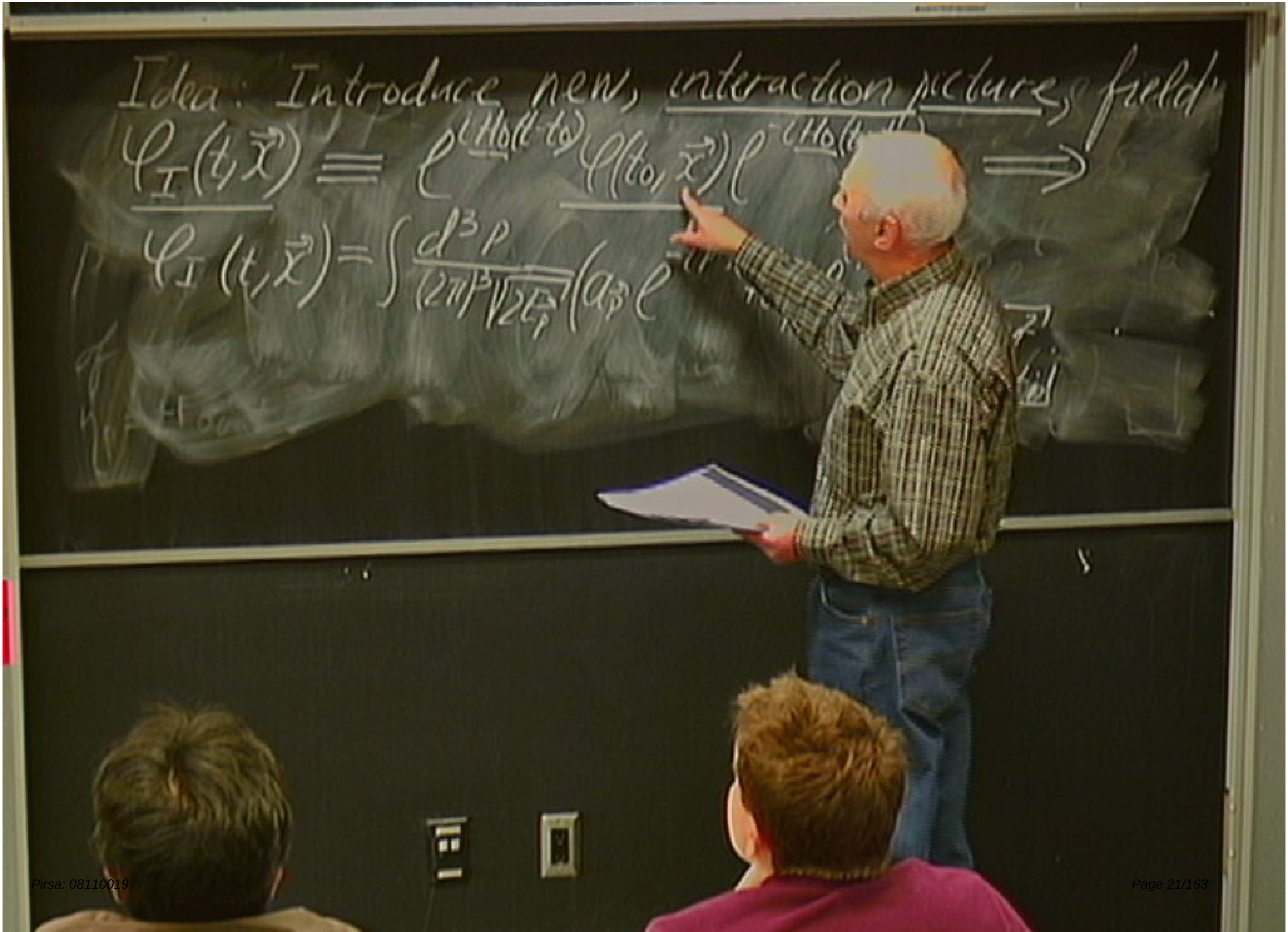
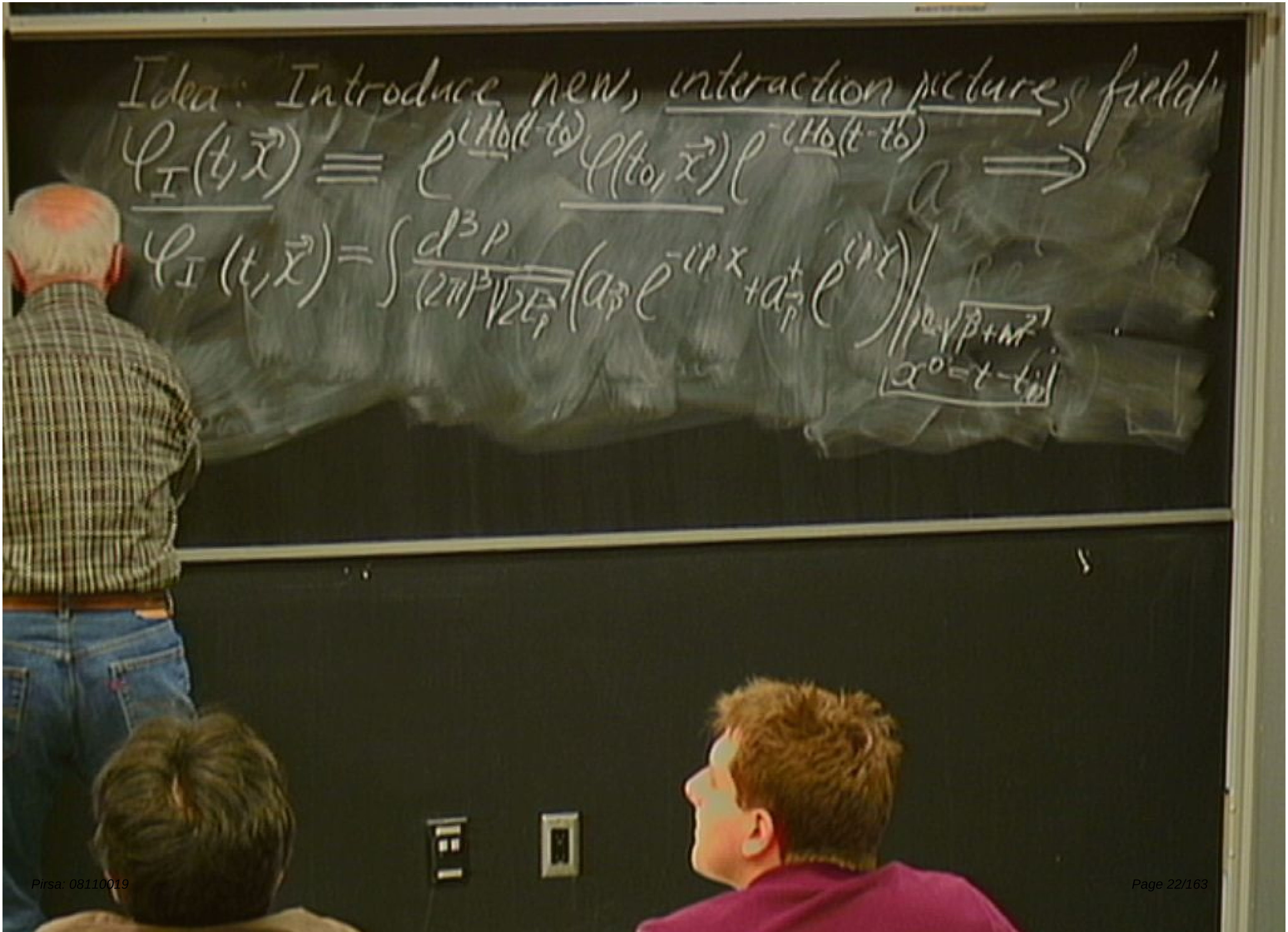

Idea: Introduce new, interaction picture, field


Idea: Introduce new, interaction picture field
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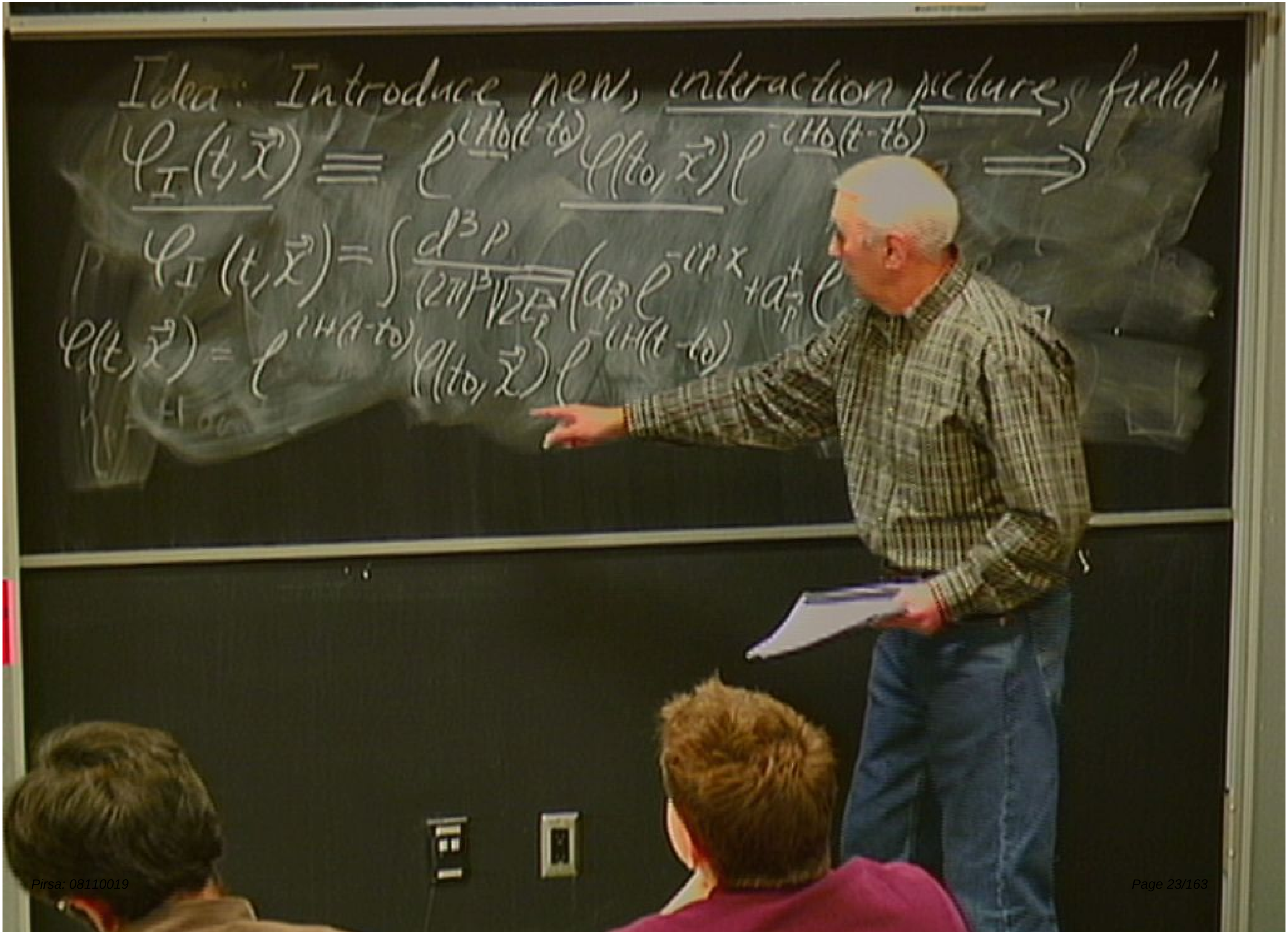
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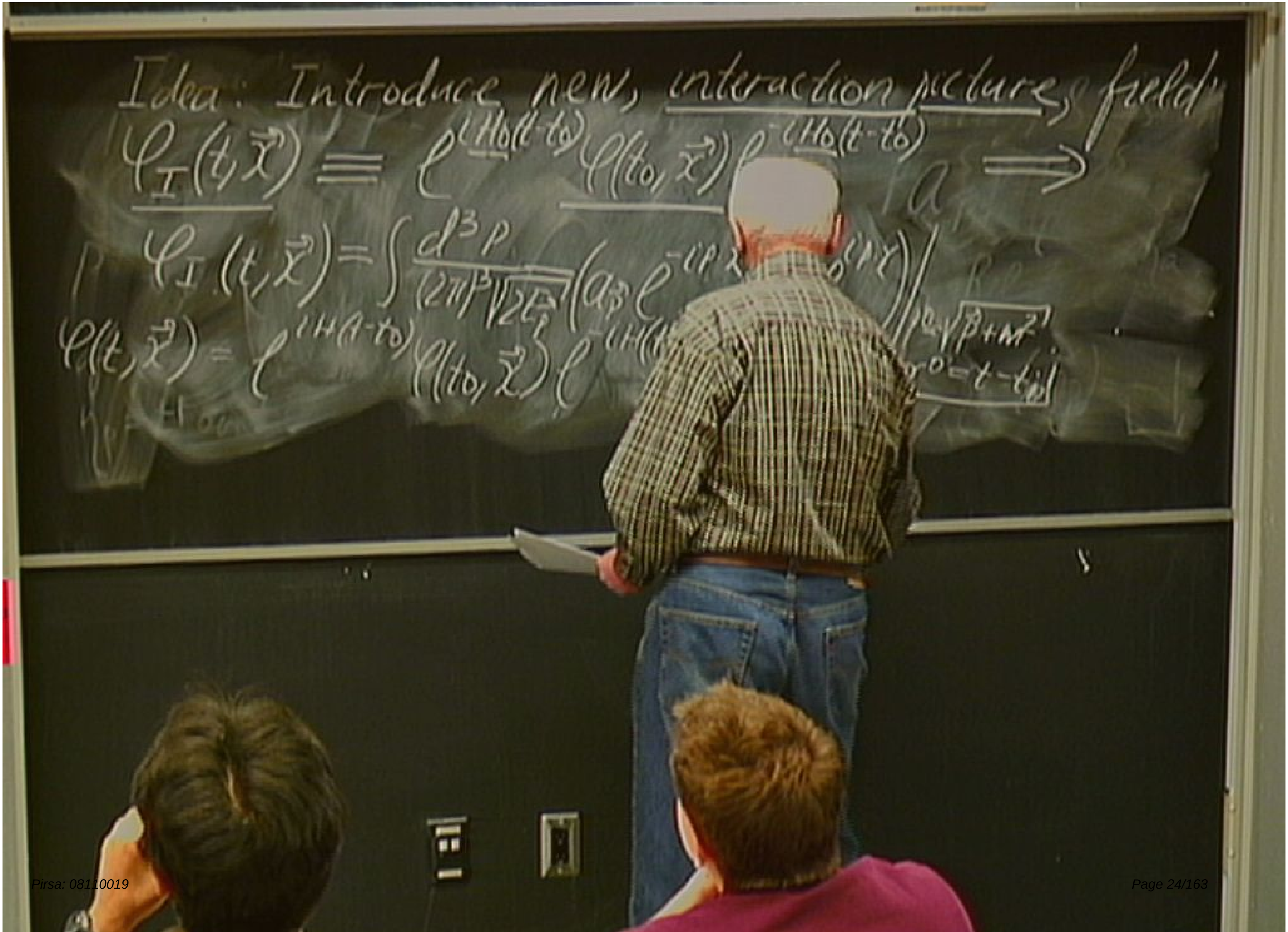
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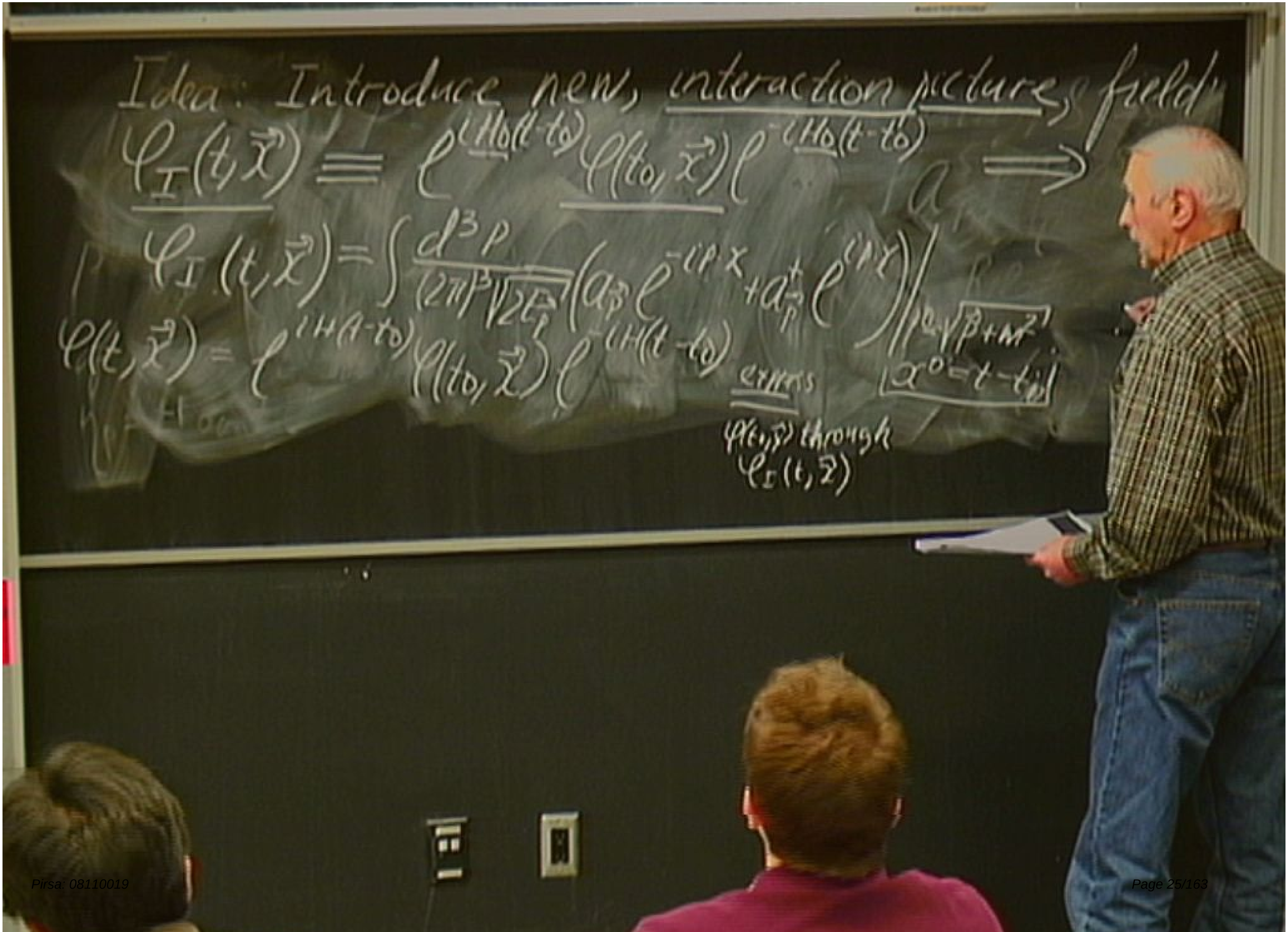
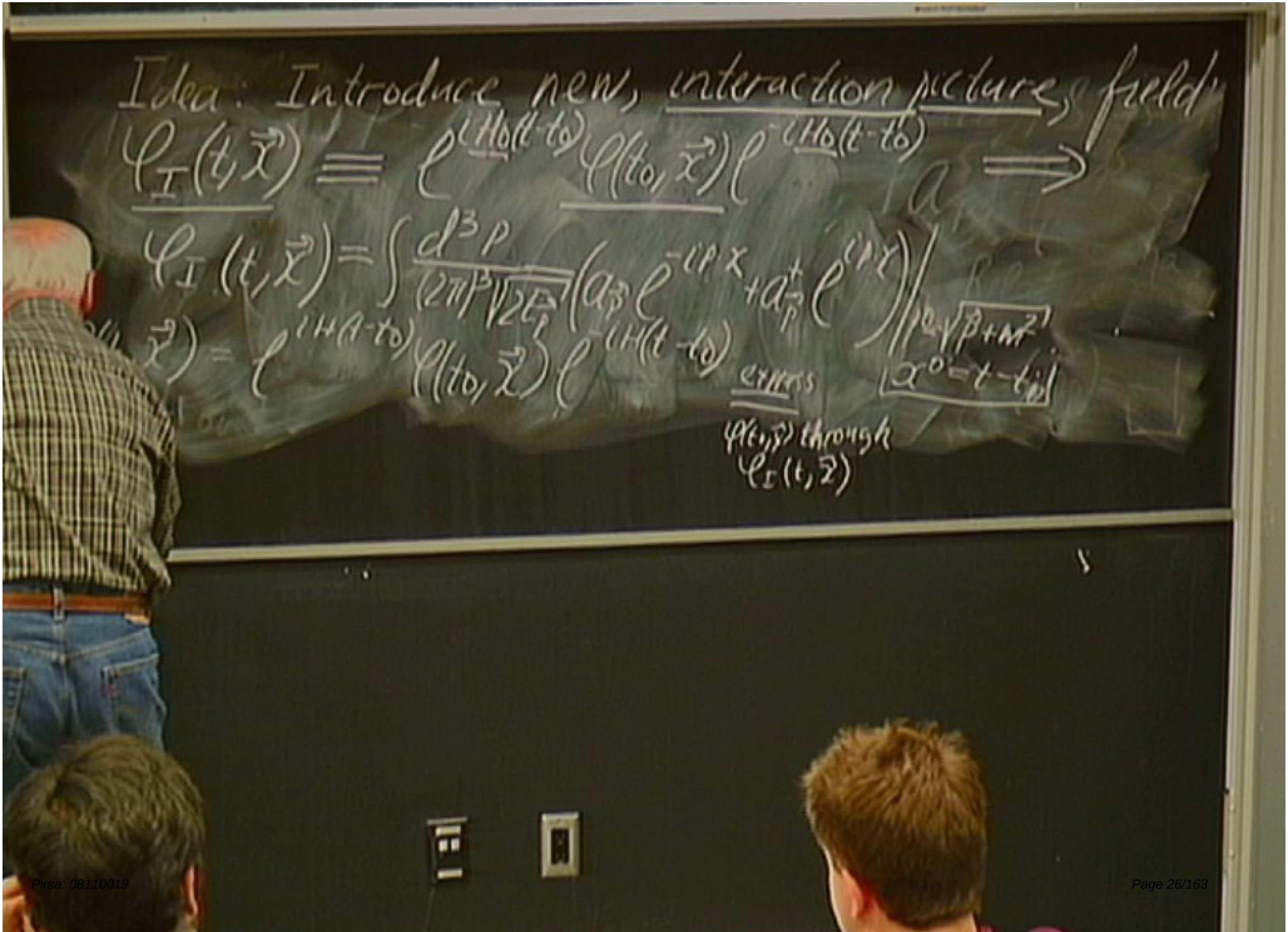

Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)} \Rightarrow$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\left(a_{\vec{p}}e^{-ip\cdot x}+a^\dagger_{\vec{p}}e^{ip\cdot x}\right)$
$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\varphi(t_0,\vec{x})e^{-iH(t-t_0)}$
$\varphi(t,\vec{x})$ through
$\varphi_I(t,\vec{x})$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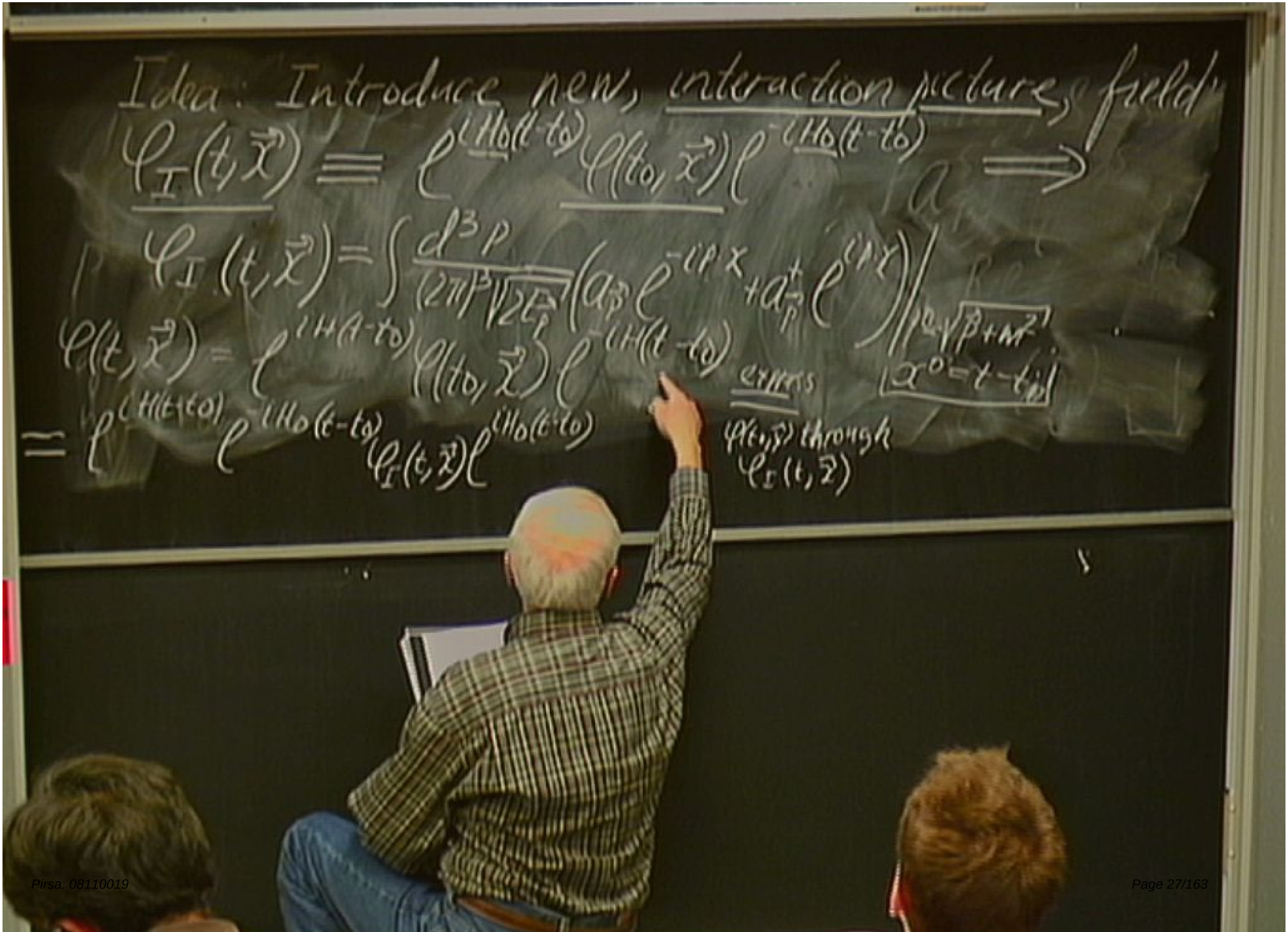
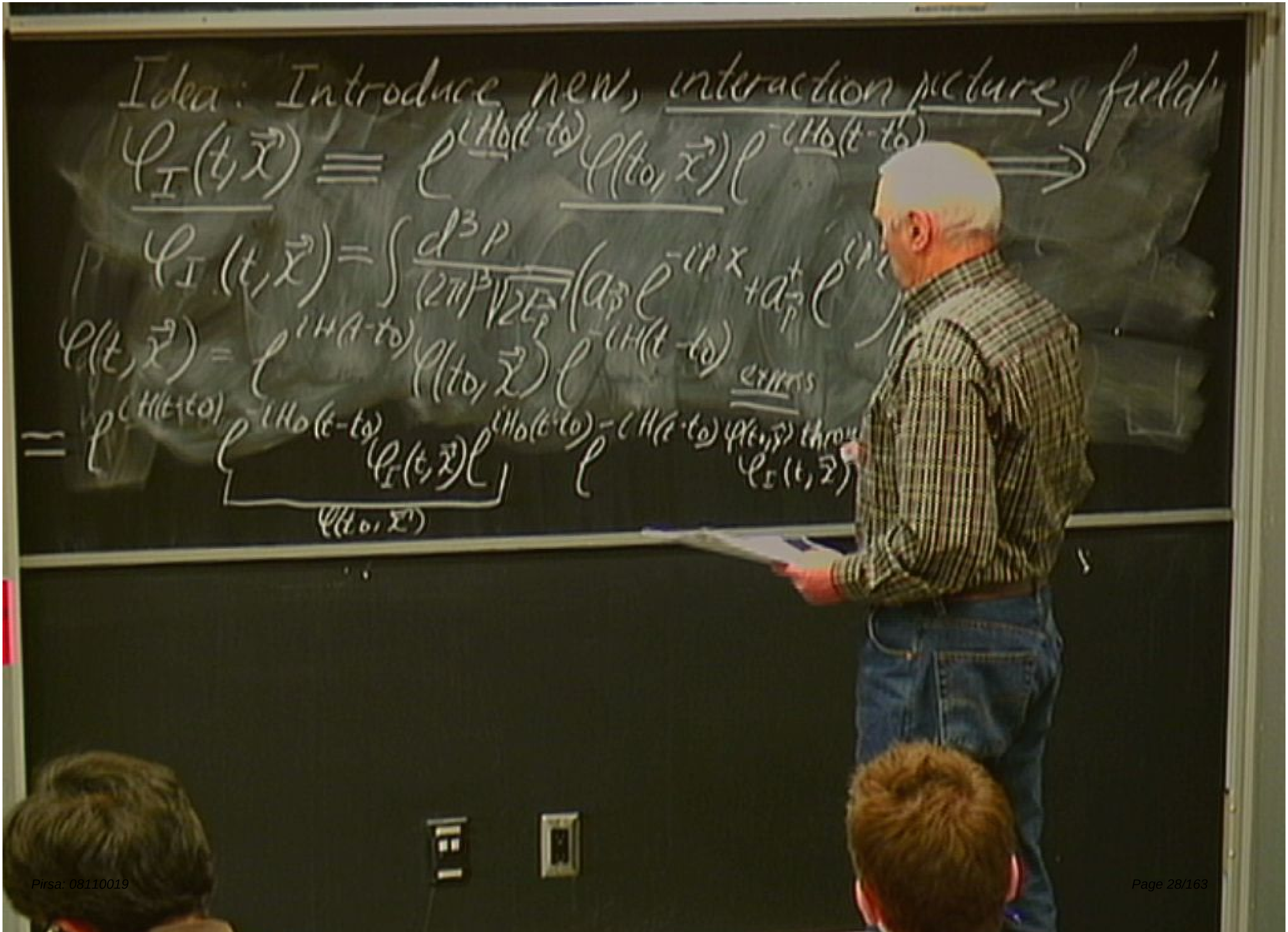
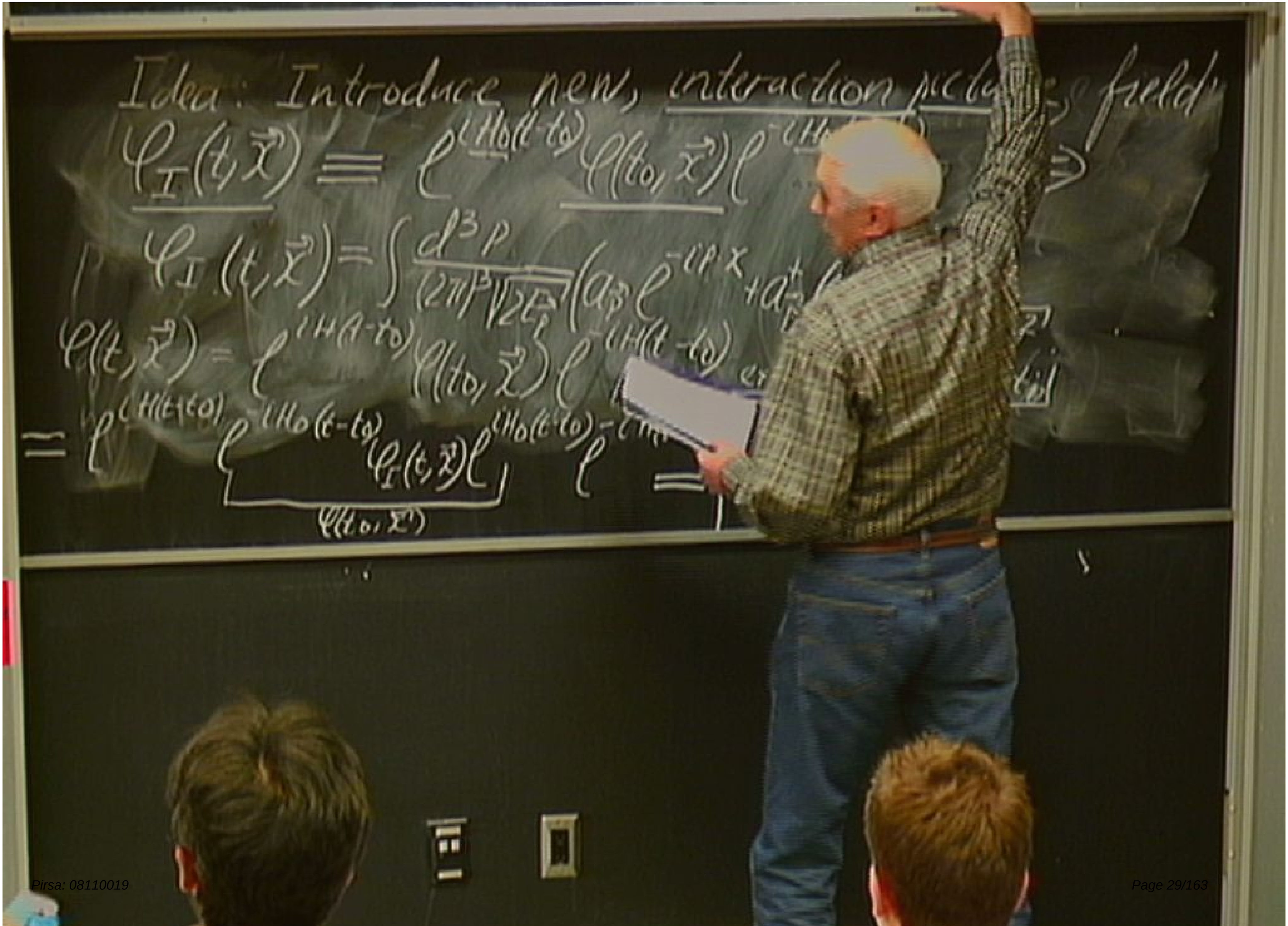
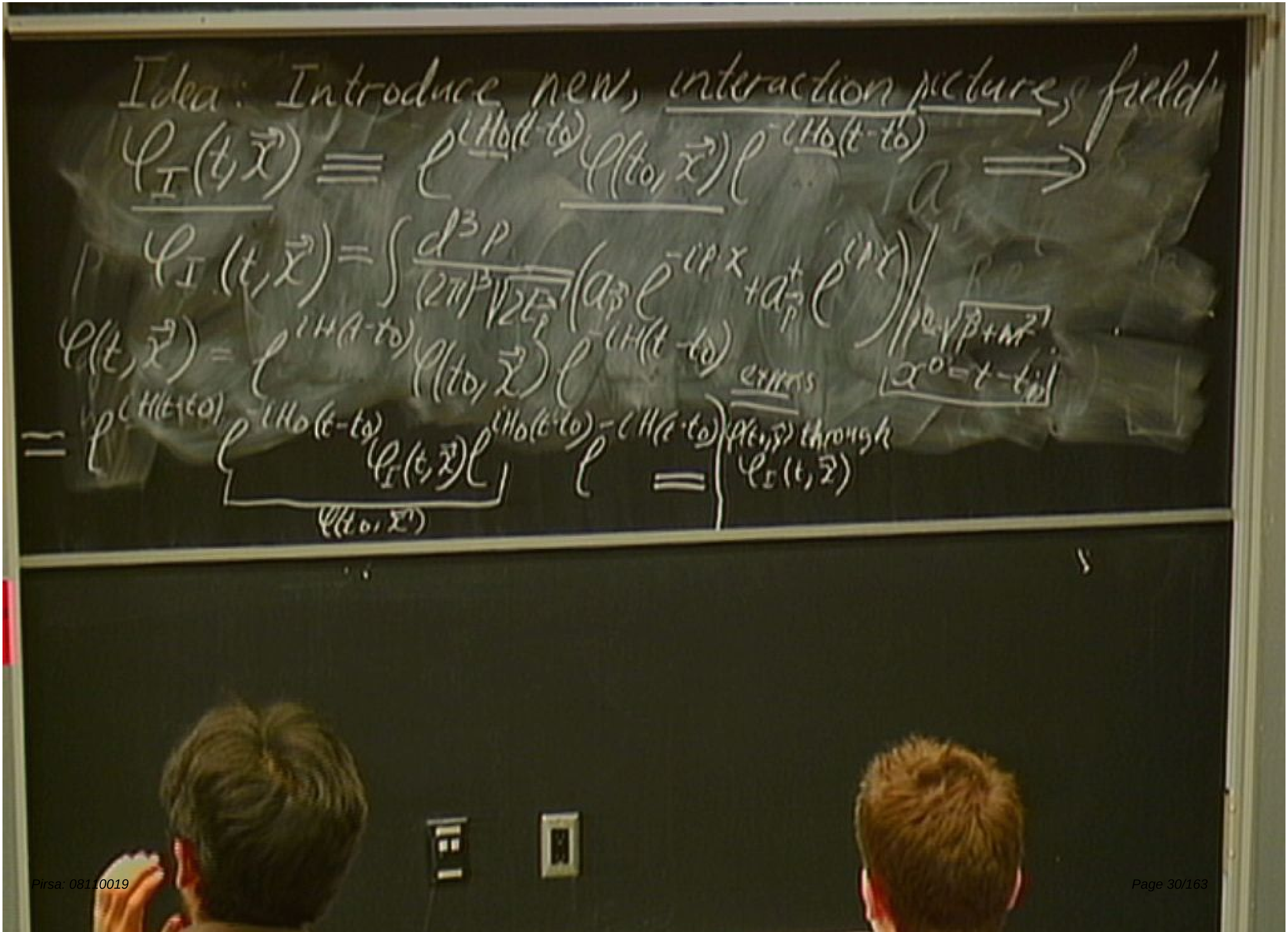
$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\varphi(t_0,\vec{x})e^{-iH(t-t_0)}$
express
$\varphi(t,\vec{x})$ through
$\varphi_I(t,\vec{x})$

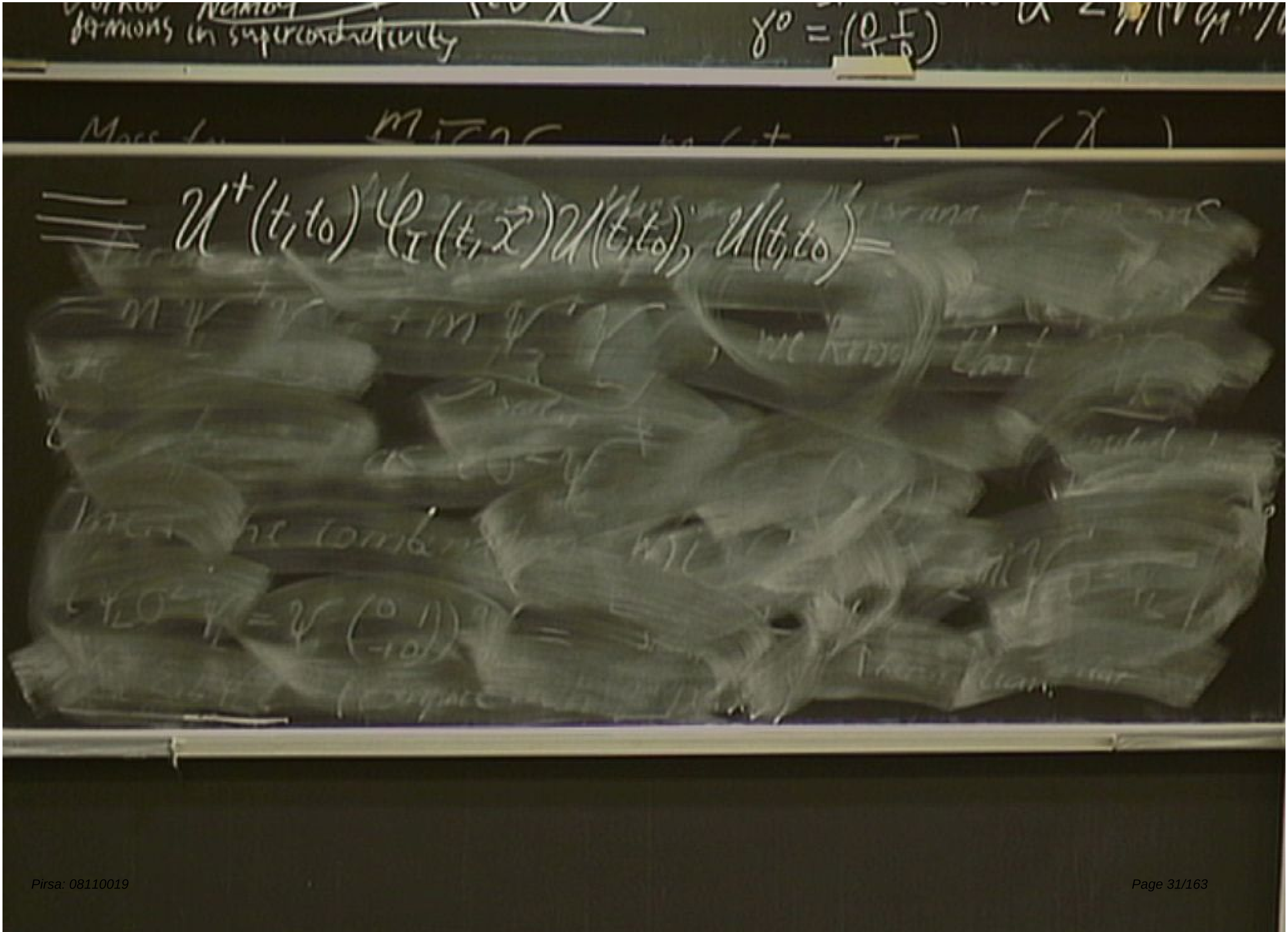


Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)}$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\left(a_{\vec p}e^{-ip\cdot x} + a^\dagger_{\vec p}e^{ip\cdot x}\right)\Big|_{p^0=\sqrt{\vec p^2+m^2},\ x^0=t-t_0}$
$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\varphi(t_0,\vec{x})e^{-iH(t-t_0)}$
$= e^{iH(t-t_0)}e^{-iH_0(t-t_0)}\varphi_I(t,\vec{x})e^{iH_0(t-t_0)}$
express
$\varphi(t,\vec x)$ through
$\varphi_I(t,\vec x)$


Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)} \Longrightarrow$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}(a_{\vec{p}}e^{-ip\cdot x} + a^\dagger_{\vec{p}}e^{ip\cdot x})$
$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\varphi(t_0,\vec{x})e^{-iH(t-t_0)}$
express
$= e^{iH(t-t_0)}e^{-iH_0(t-t_0)}\varphi_I(t,\vec{x})e^{iH_0(t-t_0)}e^{-iH(t-t_0)}$
$\varphi(t_0,\vec{x})$
$\varphi(t,\vec{x})$ throu
$\varphi_I(t,\vec{x})$


Idea: Introduce new, interaction
field

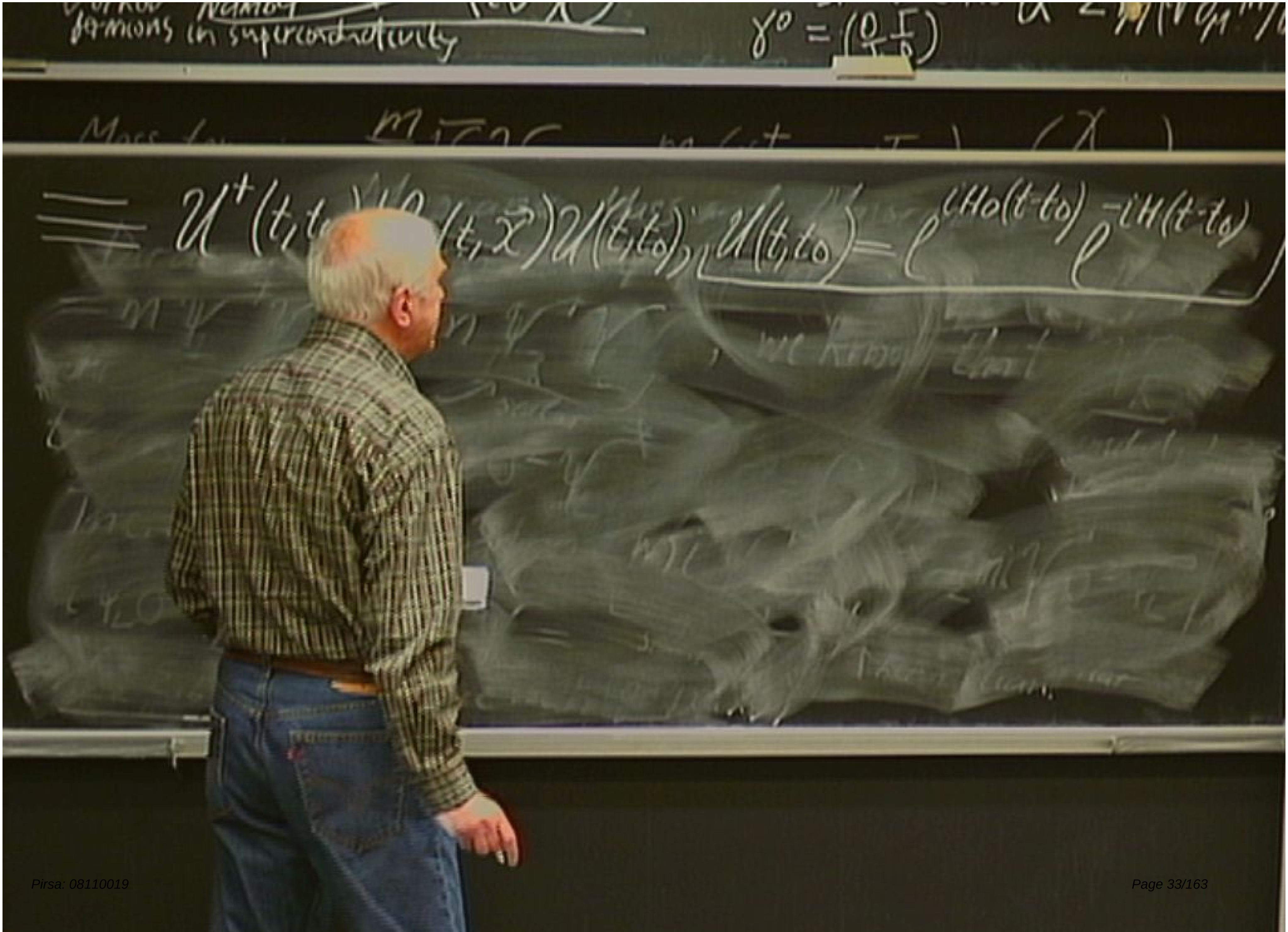

Idea: Introduce new, interaction picture, field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)}\varphi(t_0,\vec{x})e^{-iH_0(t-t_0)} \Longrightarrow a$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\left(a_{\vec{p}}e^{-ip\cdot x} + a^\dagger_{\vec{p}}e^{ip\cdot x}\right)\Big|_{\boxed{p^0=\sqrt{\vec{p}^2+m^2}},\ \boxed{x^0=t-t_0}}$
$\varphi(t,\vec{x}) = e^{iH(t-t_0)}\varphi(t_0,\vec{x})e^{-iH(t-t_0)}$
express $\varphi(t,\vec{x})$ through $\varphi_I(t,\vec{x})$
$= e^{iH(t-t_0)}\underbrace{e^{-iH_0(t-t_0)}\varphi_I(t,\vec{x})e^{iH_0(t-t_0)}}_{\varphi(t_0,\vec{x})}e^{-iH(t-t_0)} =$

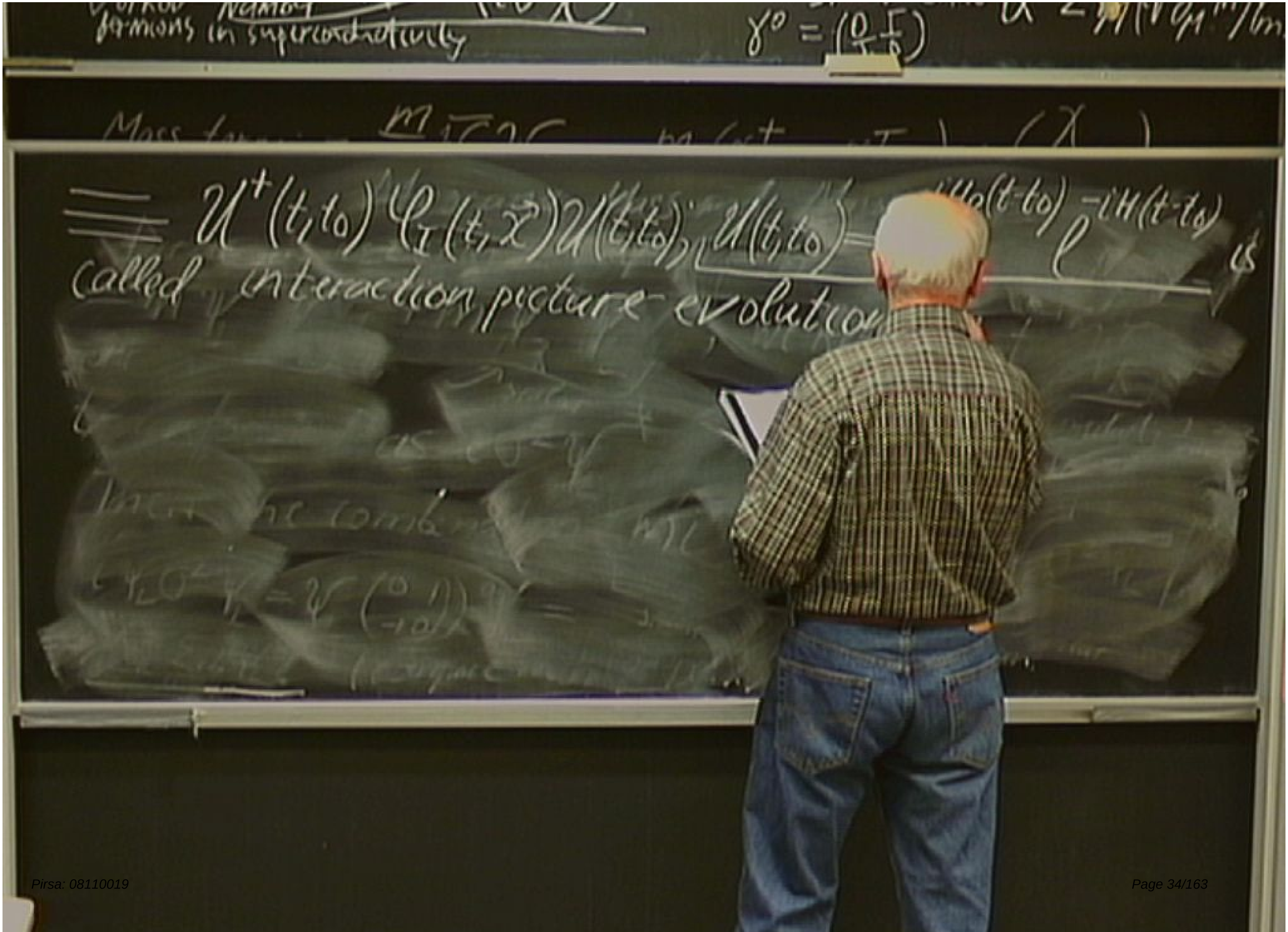

fermions in superconductivity
$\equiv U^+(t,t_0)\,\varphi_I(t,\vec{x})\,U(t,t_0)$; $U(t,t_0) =$

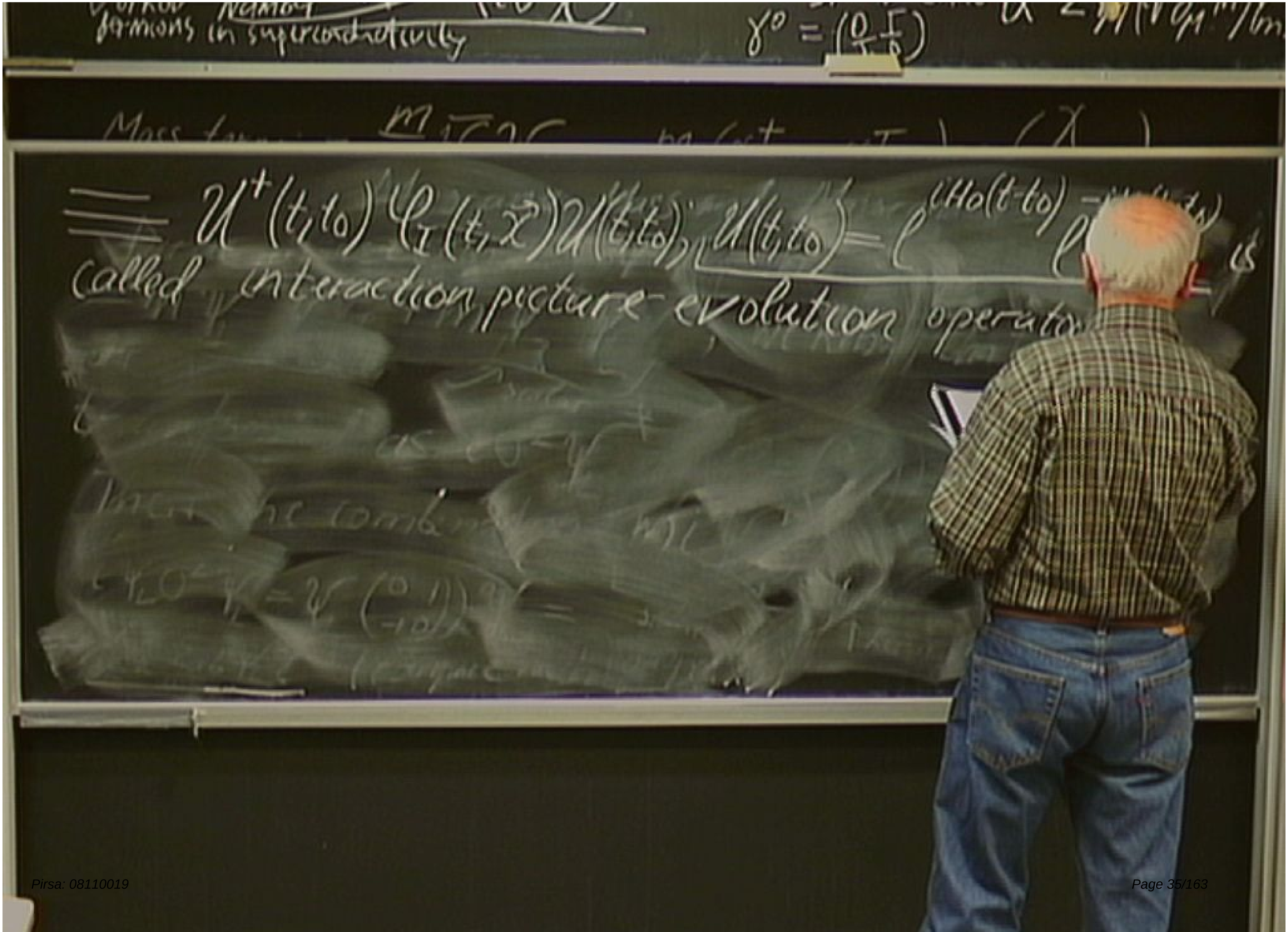
fermions in superconductivity

$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

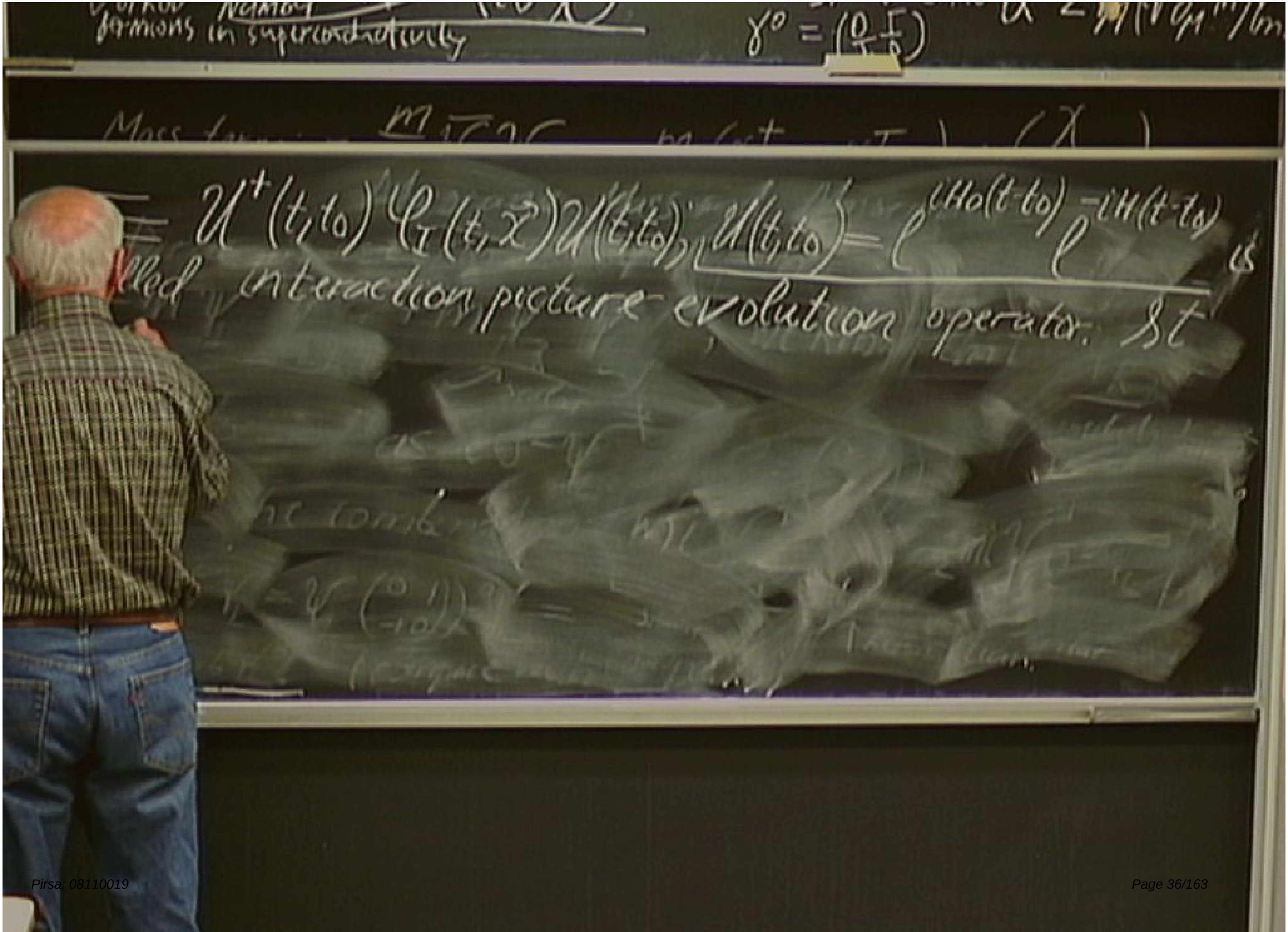
$$\equiv \mathcal{U}^\dagger(t,t_0)\,\psi_I(t,\vec{x})\,\mathcal{U}(t,t_0);\quad \mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$$


fermions in superconductivity
U(t,t0)=e^{iH0(t-t0)}e^{-iH(t-t0)}


fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
$\equiv \mathcal{U}^\dagger(t,t_0)\,\psi_I(t,\vec{x})\,\mathcal{U}(t,t_0);\ \mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$ is
called interaction picture evolution



fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
$\equiv \mathcal{U}^{+}(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0)$, $\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$ is
called interaction picture evolution operator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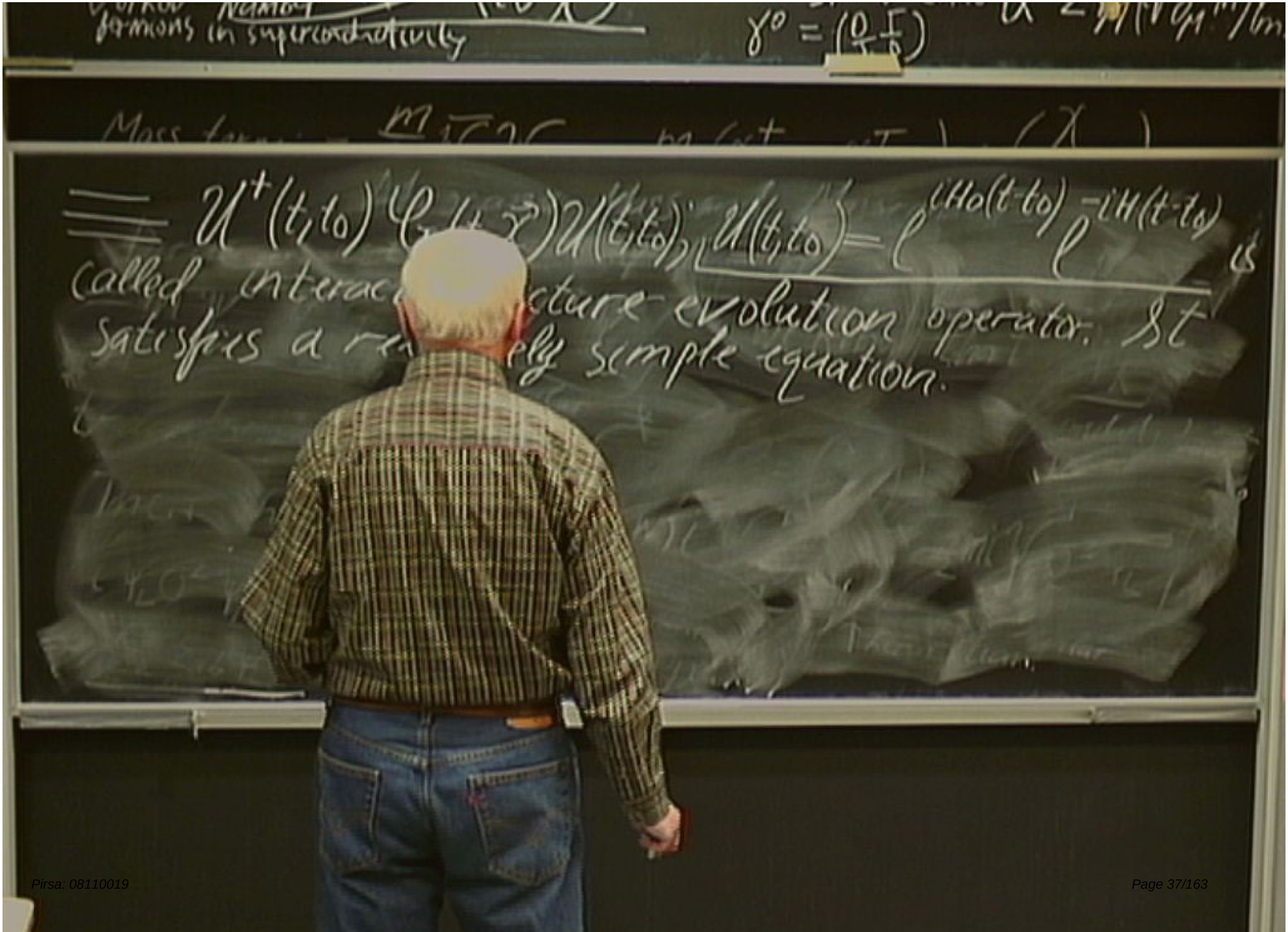
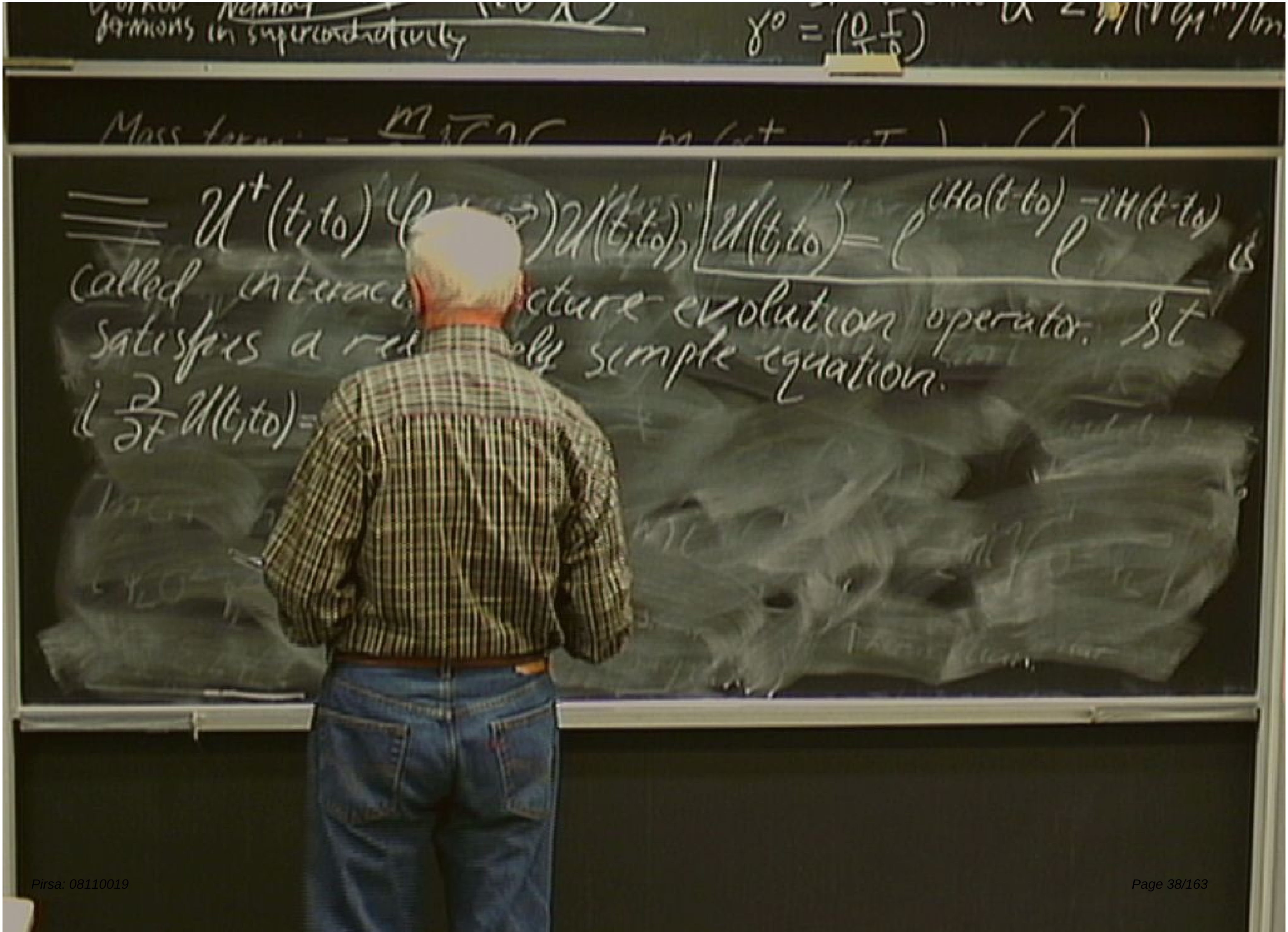
$= \mathcal{U}^\dagger(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0);\ \mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$ is
interaction picture evolution operator.


fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
$\equiv U^\dagger(t,t_0)\,\varphi(t,\vec{x})\,U(t,t_0)$; $U(t,t_0) = e^{iH_0(t-t_0)}e^{-iH(t-t_0)}$ is
called interac cture evolution operator. It
satisfies a re ly simple equation.


fermions in superconductivity
$U^+(t,t_0)$
$U(t,t_0) = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$ is
called interact
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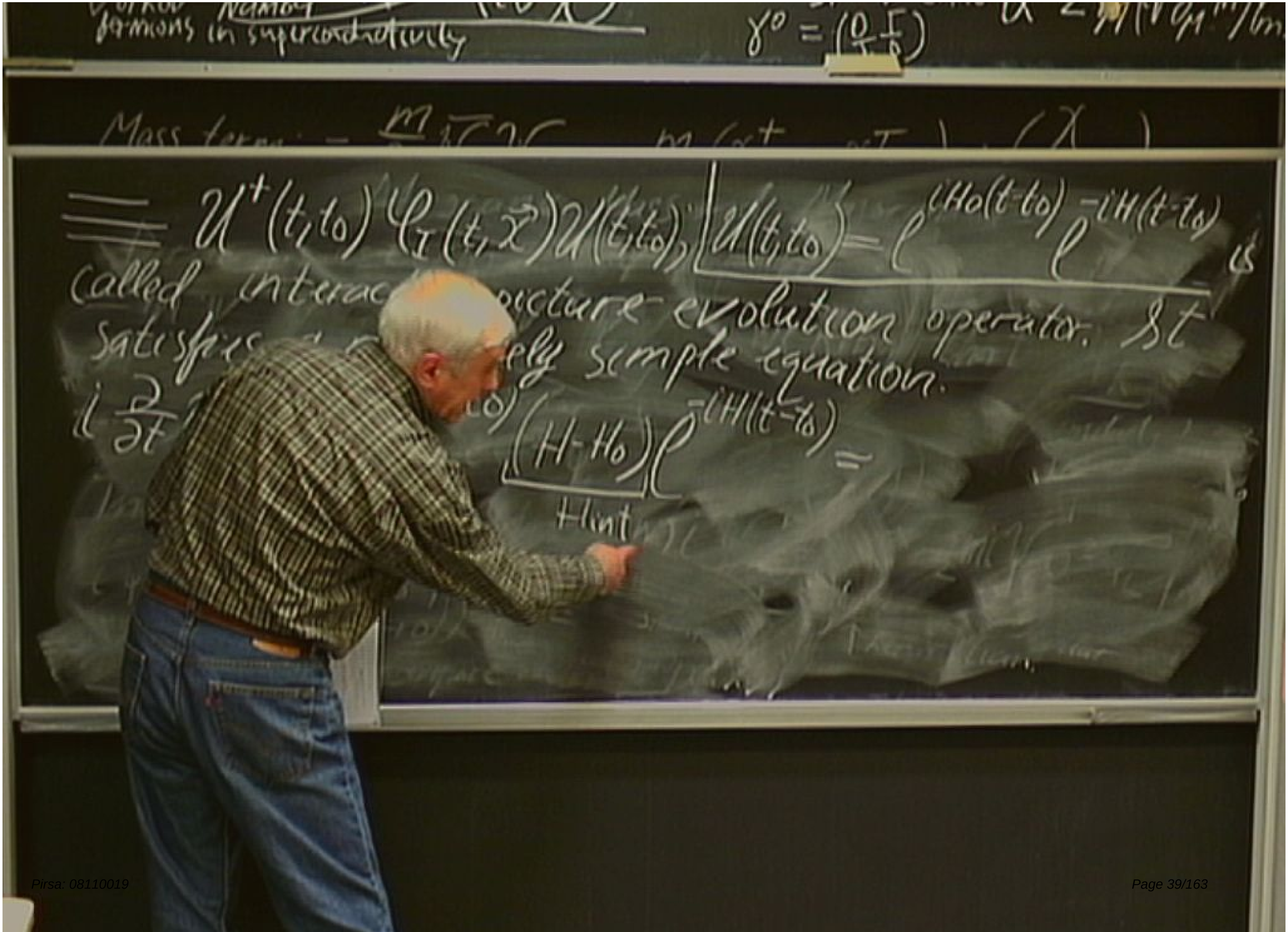
satisfies a rel
ly simple equation.
$i\frac{\partial}{\partial t}U(t,t_0)=$



fermions in superconductivity
$\mathcal{U}^+(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0)$, $\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$ is
called interac picture evolution operator. It
satisfies a ely simple equation.
$(H-H_0)\,e^{-iH(t-t_0)} =$
H_{int}

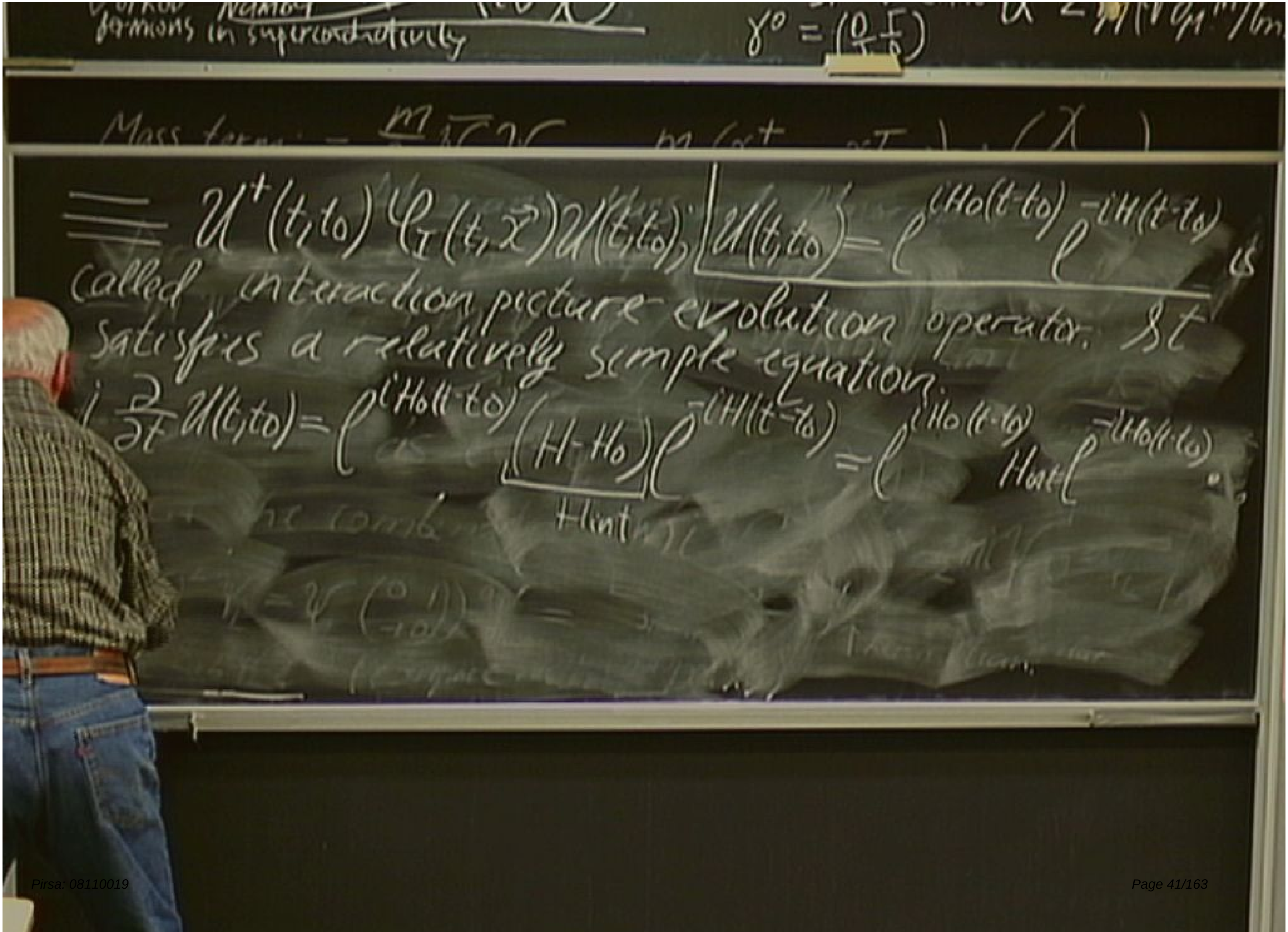
fermions in superconductivity

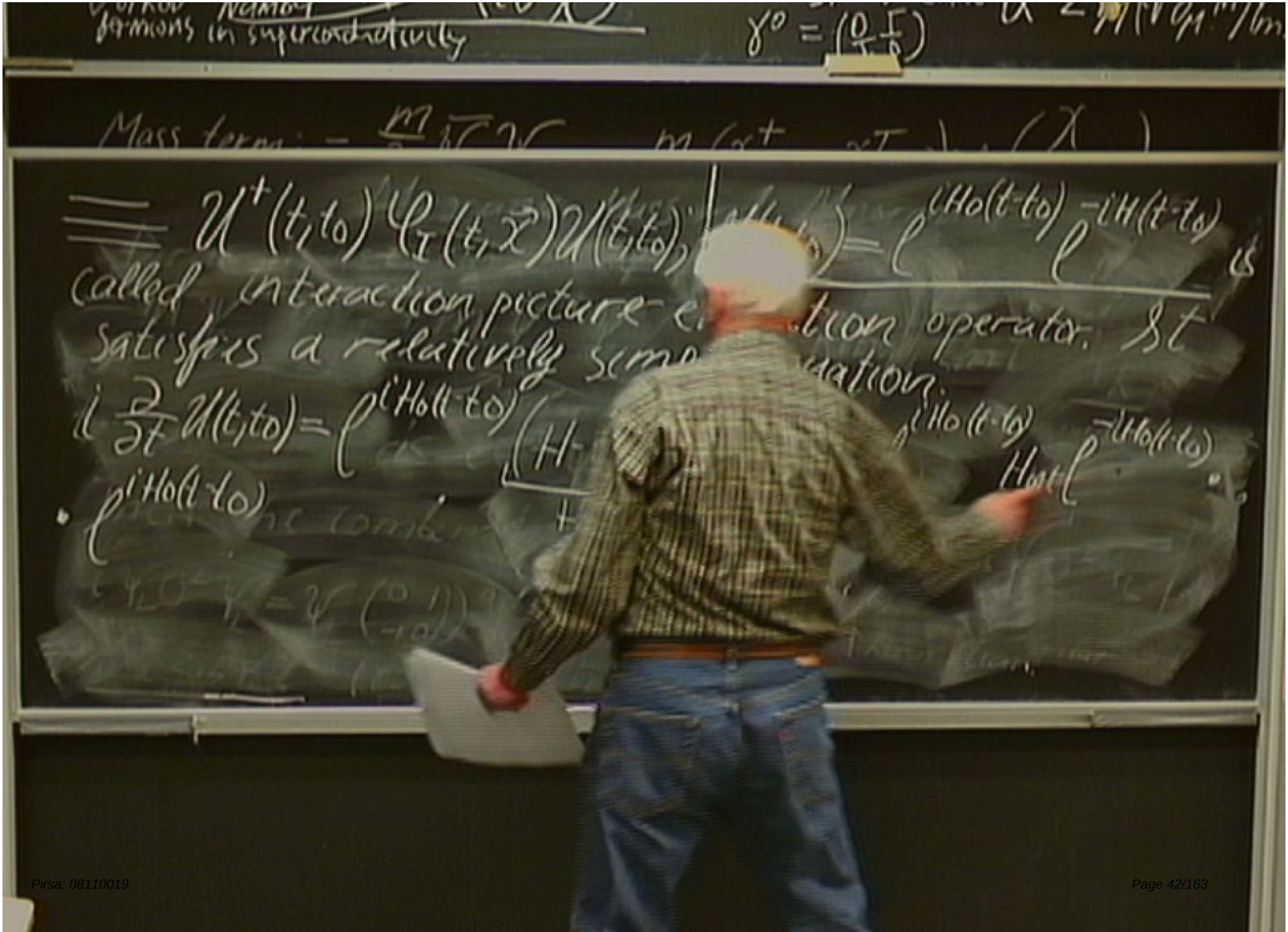
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

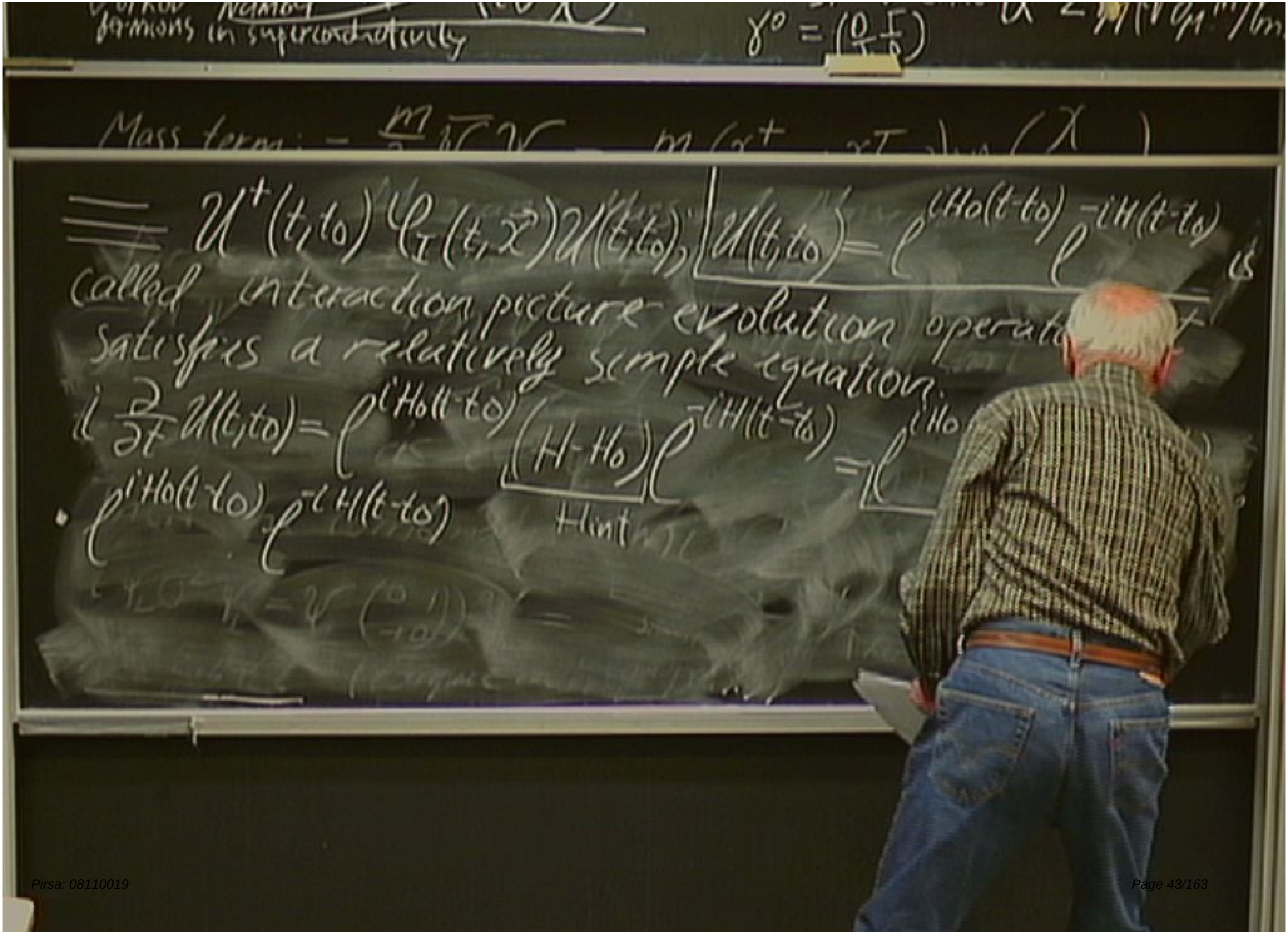
$$\equiv \mathcal{U}^{+}(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0), \quad \boxed{\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}}$$

is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,\underbrace{(H-H_0)}_{H_{int}}\,e^{-iH(t-t_0)} =$$


fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
$\equiv \mathcal{U}^\dagger(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0)$; $\boxed{\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}}$ is
called interaction picture evolution operator. It
satisfies a relatively simple equation:
$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}}e^{-iH(t-t_0)} = e^{iH_0(t-t_0)}\,H_{int}\,e^{-iH_0(t-t_0)}$.


fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
$= U^\dagger(t,t_0)\,\psi_I(t,\vec{x})\,U(t,t_0)$
$= e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$ is
called interaction picture
operator. It
satisfies a relatively
$i\frac{\partial}{\partial t}U(t,t_0) = e^{iH_0(t-t_0)}$


fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
Mass term:
$\equiv U^\dagger(t,t_0)\,\varphi_I(t,\vec{x})\,U(t,t_0)$, $\boxed{U(t,t_0) = e^{iH_0(t-t_0)}e^{-iH(t-t_0)}}$ is
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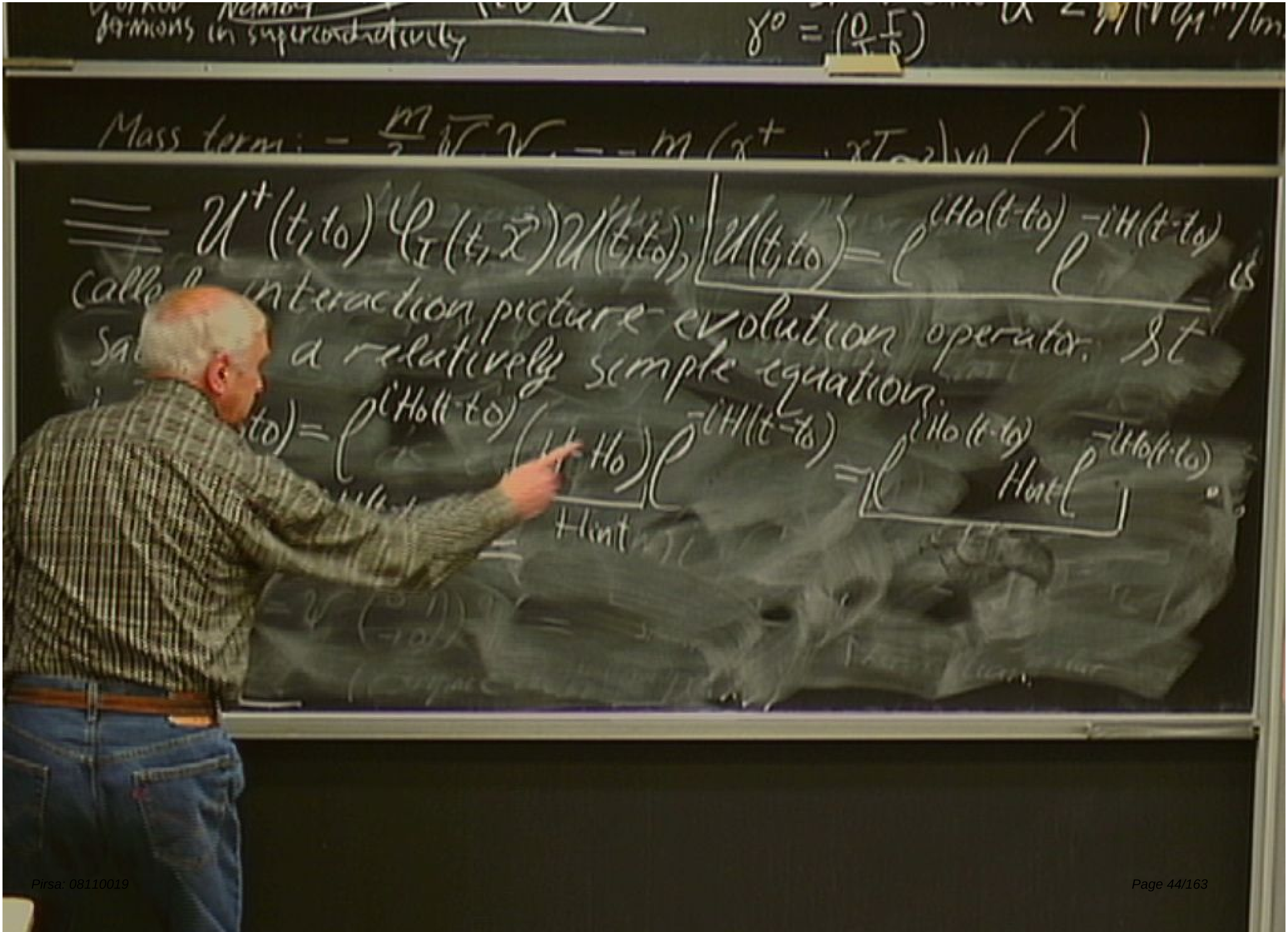
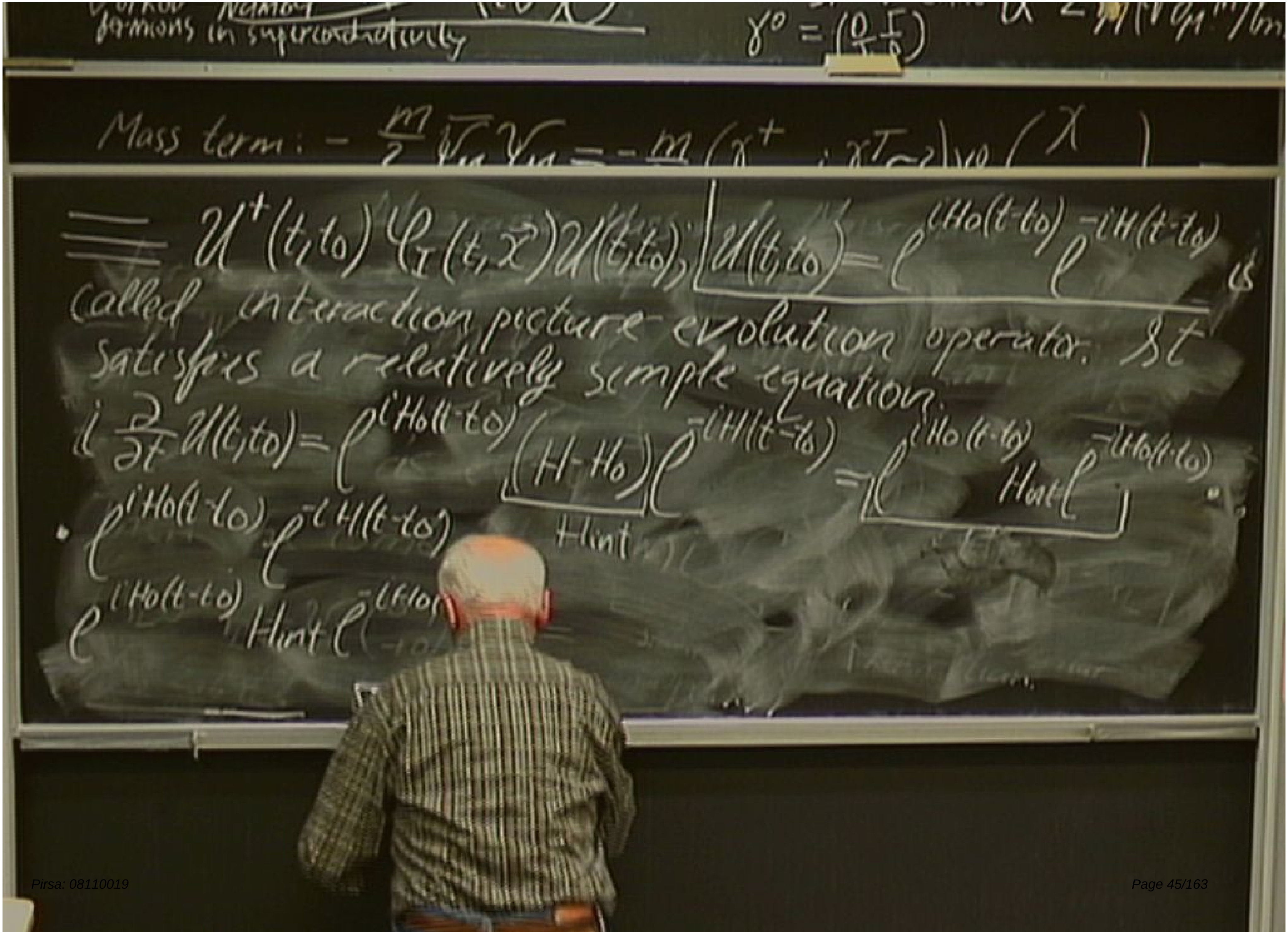
satisfies a relatively simple equation
$i\frac{\partial}{\partial t}U(t,t_0) = e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}}e^{-iH(t-t_0)} = e^{iH_0}$
$e^{iH_0(t-t_0)}e^{-iH(t-t_0)}$


fermions in superconductivity
Mass term:
called interaction picture evolution operator. It
a relatively simple equation.
Hint


fermions in superconductivity
$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
Mass term: $-\frac{m}{2}\bar{\psi}_M \psi_M = -m(\chi^+ \ldots$
$\equiv \mathcal{U}^+(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0)$, $\boxed{\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}}$ is
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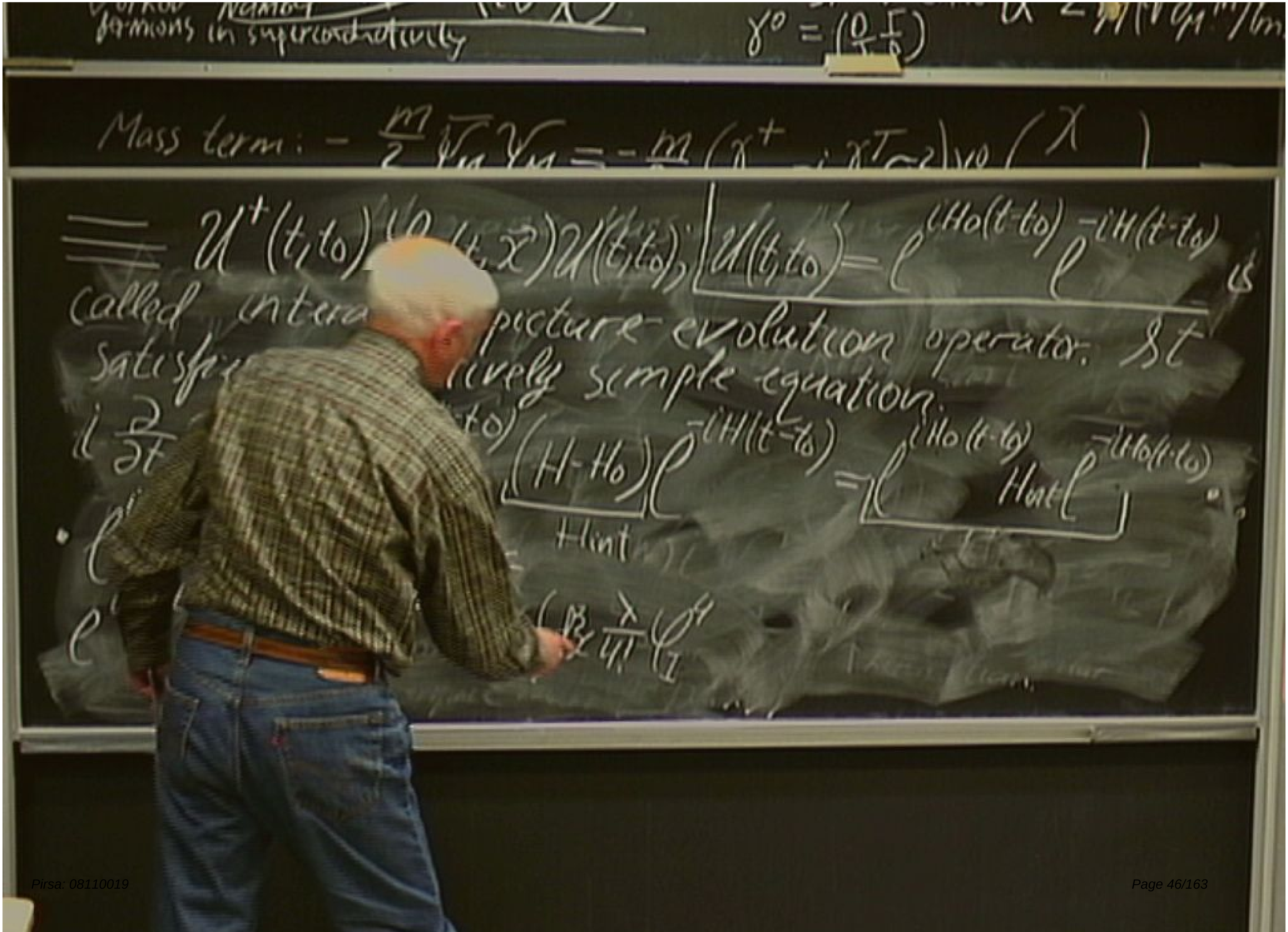
$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}} e^{-iH(t-t_0)} = \boxed{e^{iH_0(t-t_0)}\, H_{int}\, e^{-iH_0(t-t_0)}}$
$e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}$
$e^{iH_0(t-t_0)}\, H_{int}\, e^{-iH_0 \ldots}$


fermions in superconductivity
Mass term:
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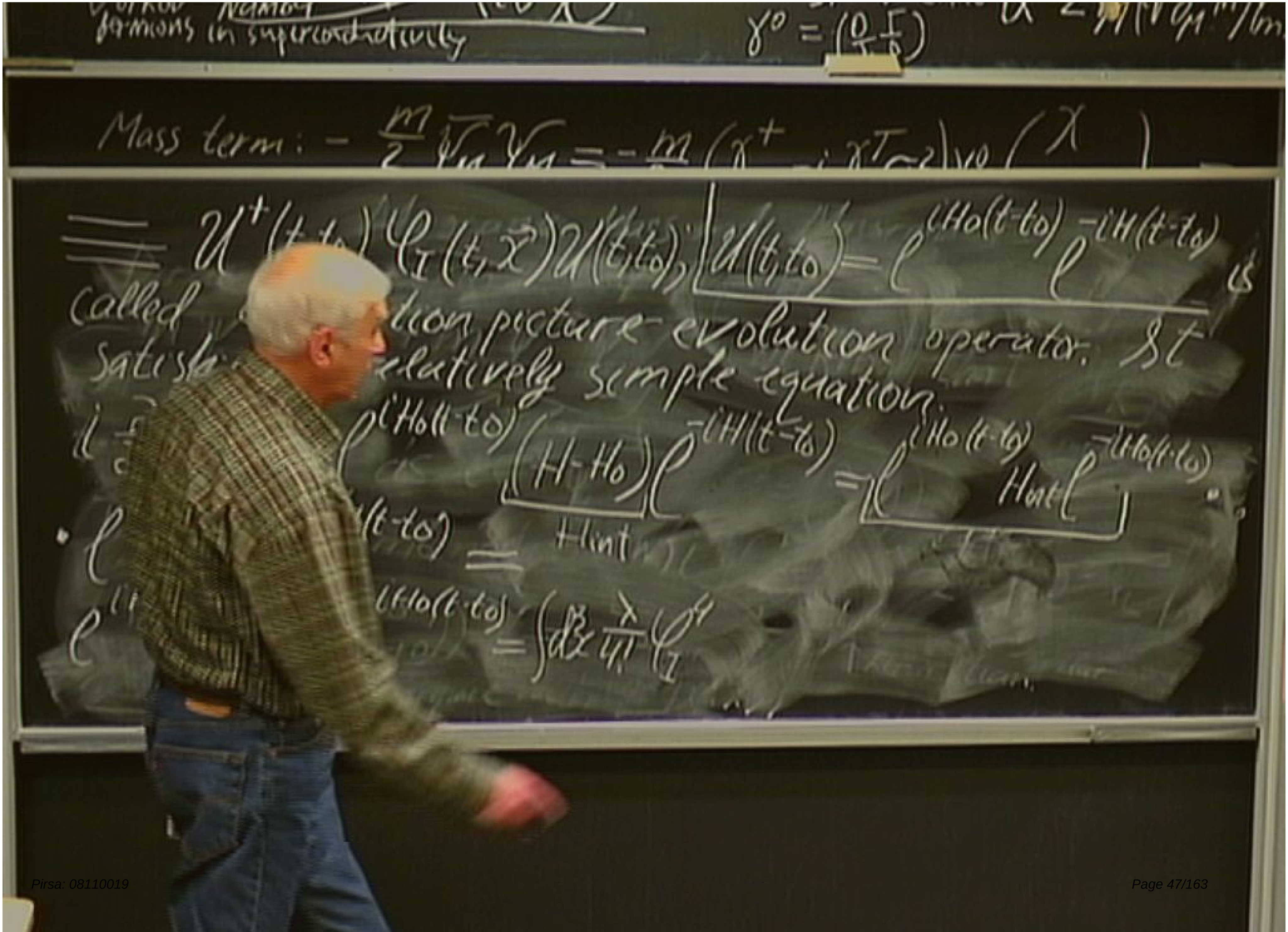
Satisf ively simple equation.
Hint



fermions in superconductivity
Mass term:
U(t,t0) =
called
tion picture evolution operator. It
relatively simple equation.
Hint

fermions in superconductivity $\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

Mass term: $-\frac{m}{2}\bar{\psi}_M \psi_M = -\frac{m}{2}(\chi^+ \; -i\chi^T\sigma_2)\gamma^0 \begin{pmatrix} \chi \\ \; \end{pmatrix}$

$\equiv \mathcal{U}^+(t,t_0)\varphi_I(t,\vec{x})\mathcal{U}(t,t_0)$, $\boxed{\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}}$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}}e^{-iH(t-t_0)} = \boxed{e^{iH_0(t-t_0)} H_{int} e^{-iH_0 \ldots}}$$

$$e^{iH_0(t-t_0)} e^{-iH(t-t_0)} =$$

$$e^{iH_0(t-t_0)} H_{int} e^{-iH_0(t-t_0)} = \int d^3x \frac{\lambda}{4!}\varphi_I^4$$

fermions in superconductivity $\qquad \gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

Mass term: $-\frac{m}{2}\bar{\psi}_M \psi_M = -\frac{m}{2}(\chi^+ \; -i\chi^T\sigma^2)\gamma^0 \begin{pmatrix} \chi \\ \ \end{pmatrix}$

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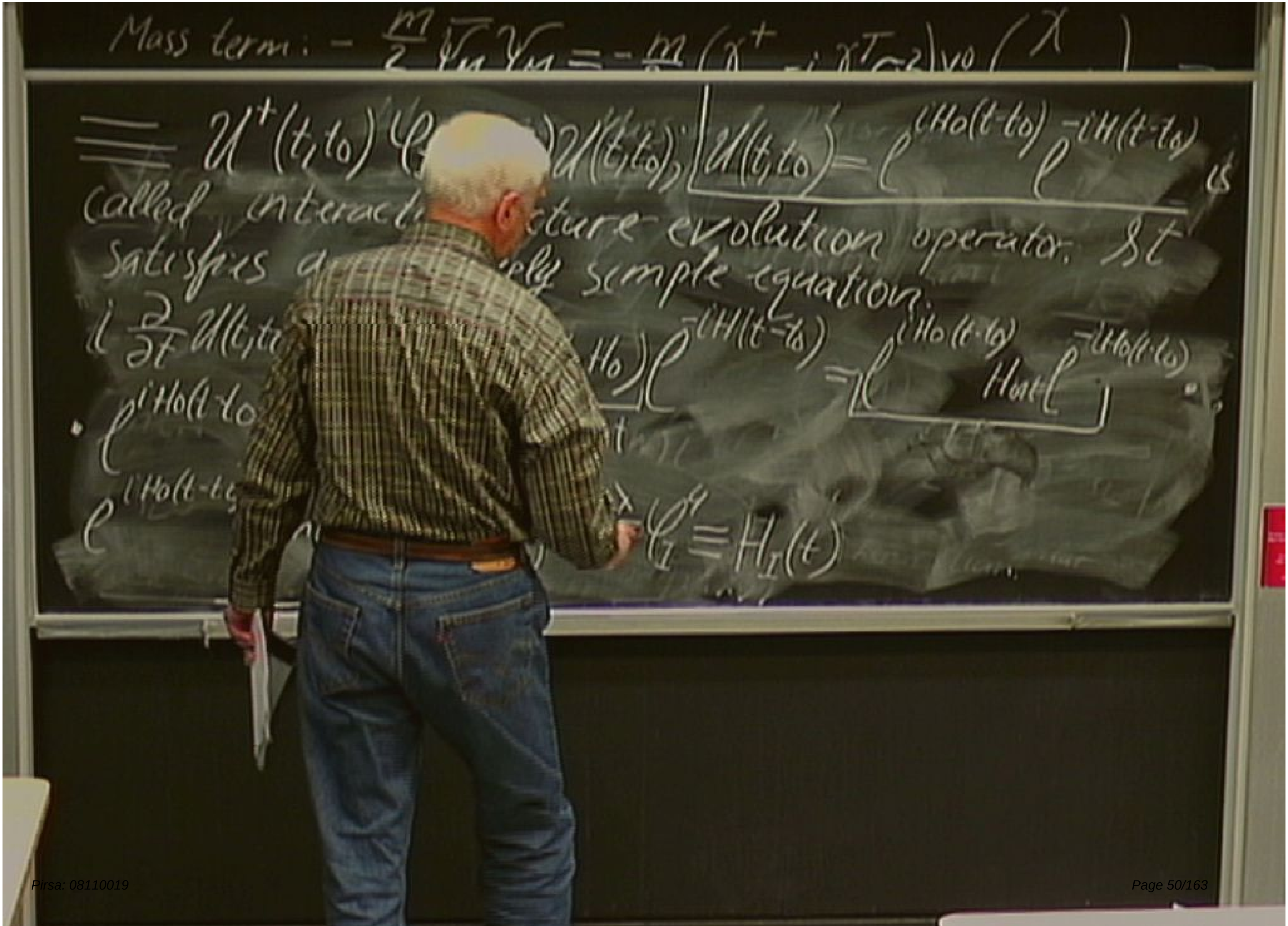
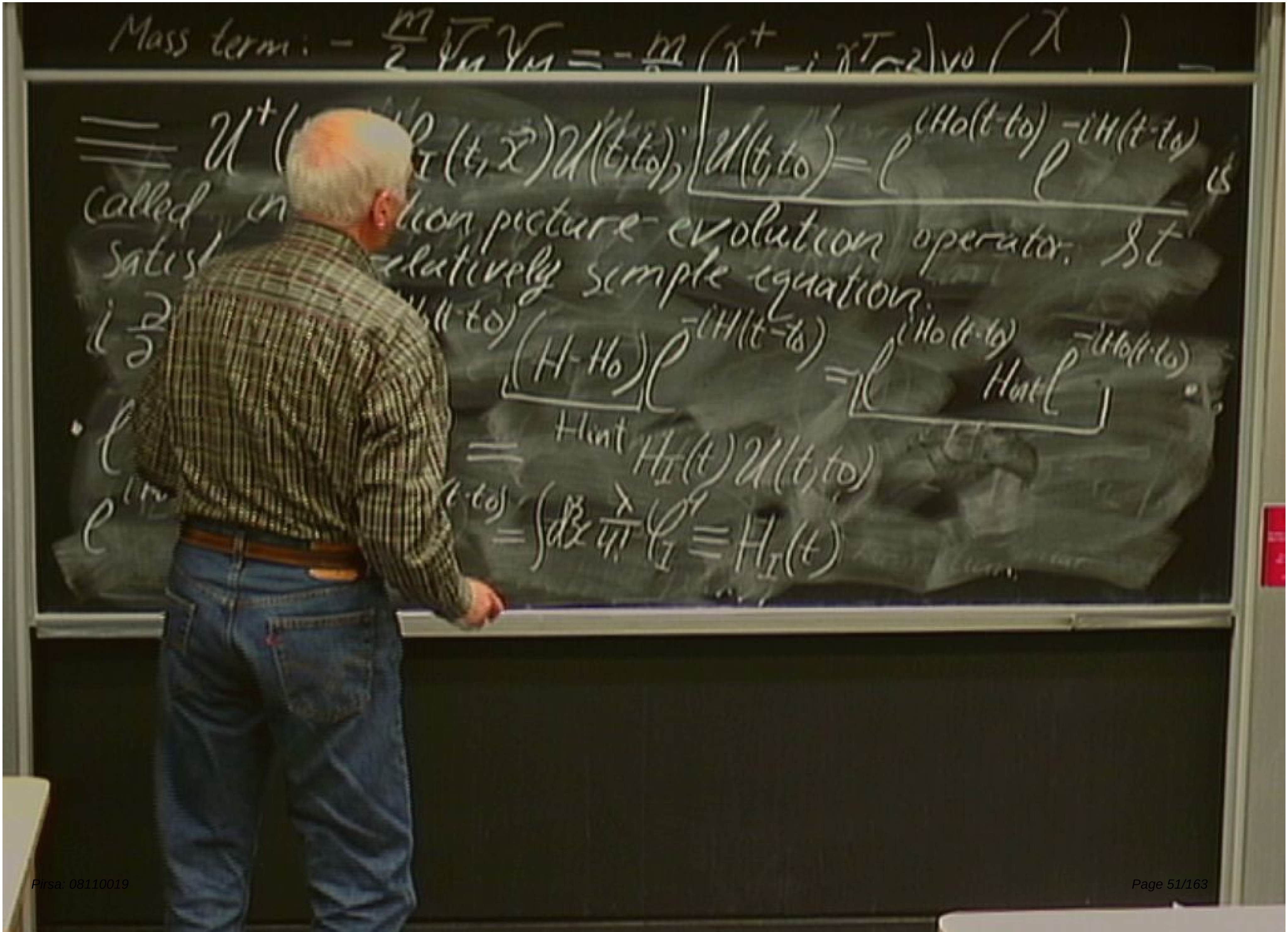
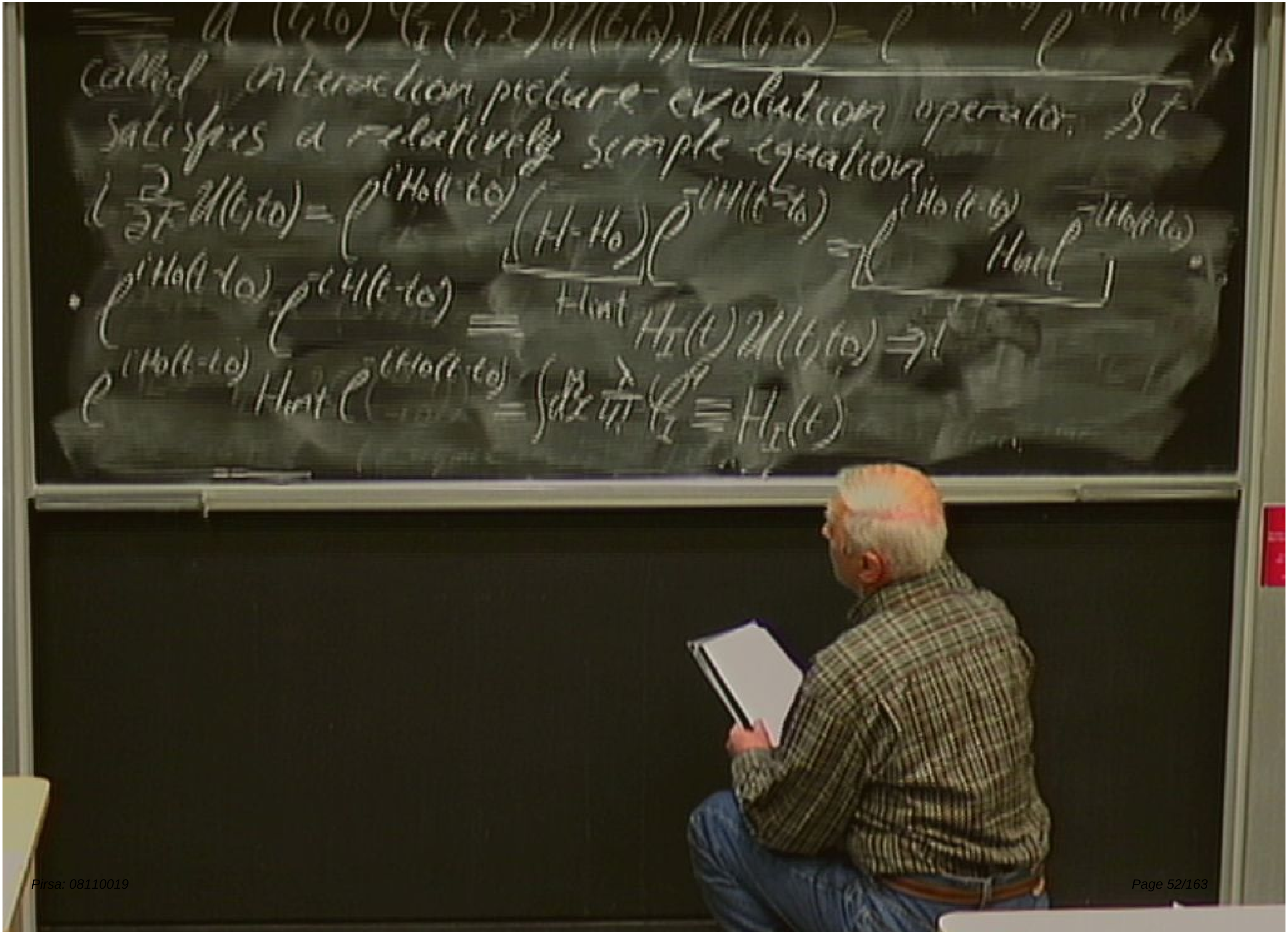
$$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}}e^{-iH(t-t_0)} = \boxed{e^{iH_0(t-t_0)}\,H_{int}\,e^{-iH_0(t-t_0)}}\cdot e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$$

$$e^{iH_0(t-t_0)}\,H_{int}\,e^{-iH_0(t-t_0)} = \int d^3x\,\frac{\lambda}{4!}\varphi_I^4$$


Mass term:
called
evolution operator. It
simple equation.
$= H_I(t)$


Mass term:
is
called
picture evolution operator. It
relatively simple equation.
Hint


is
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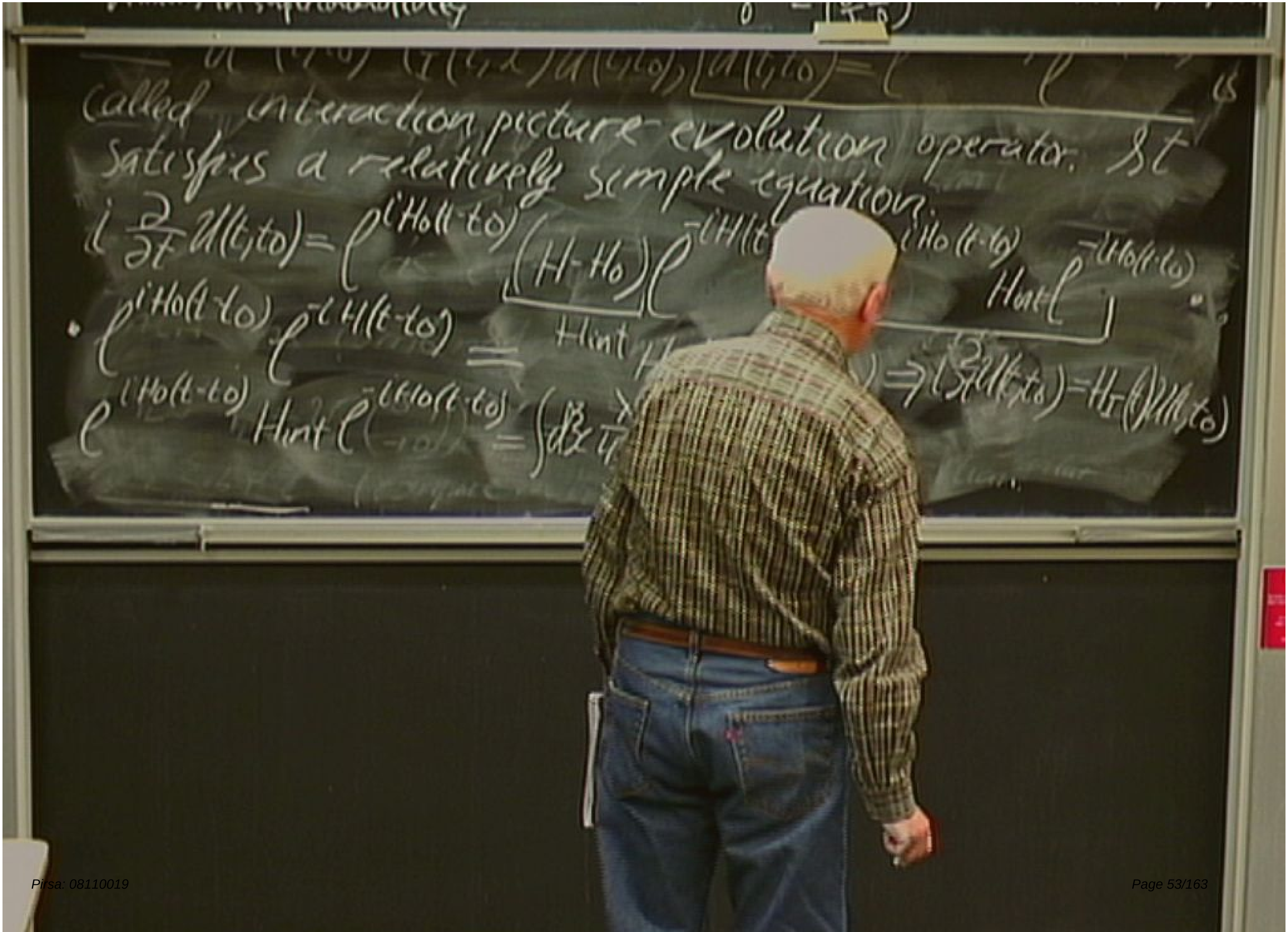
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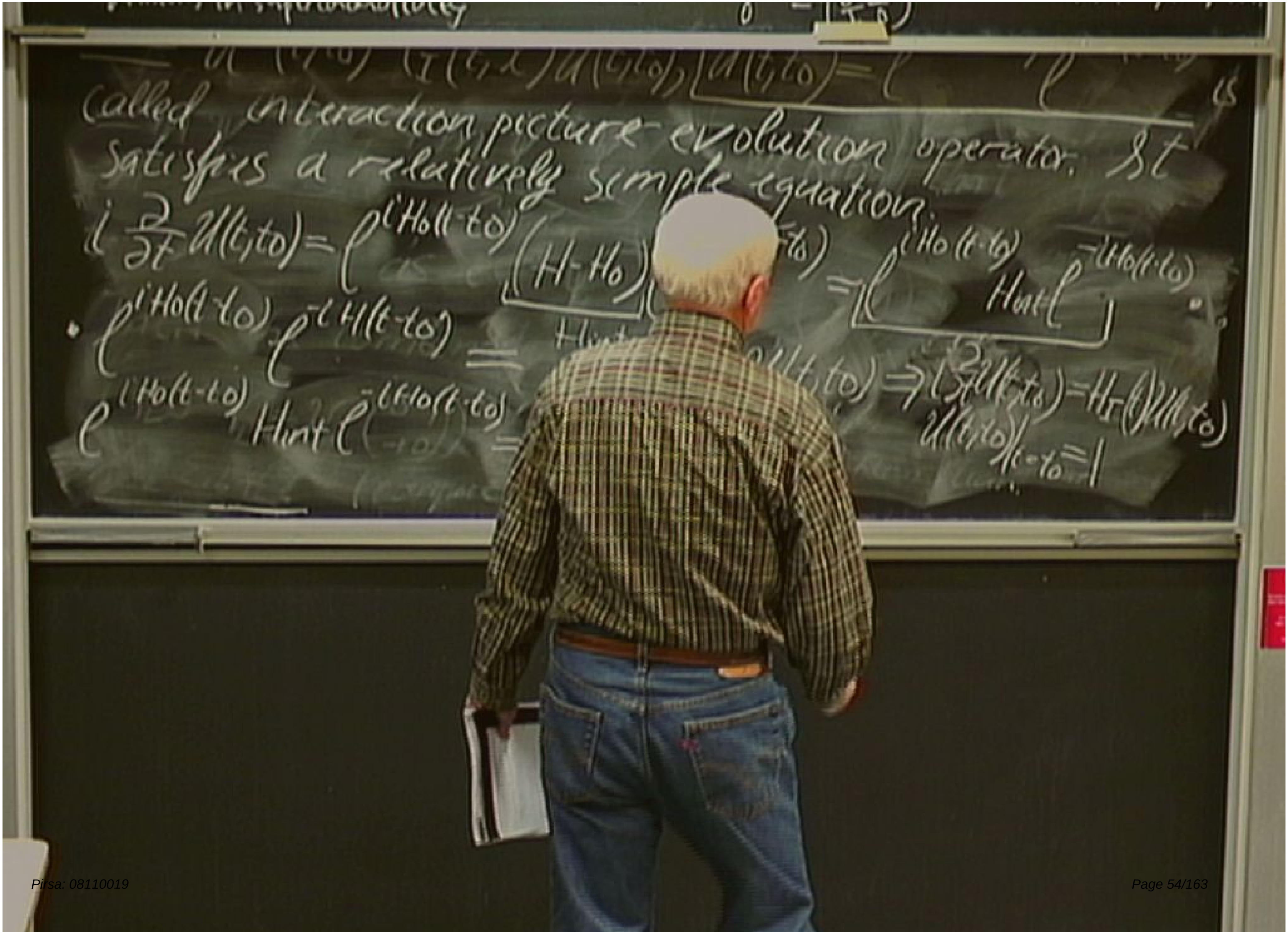
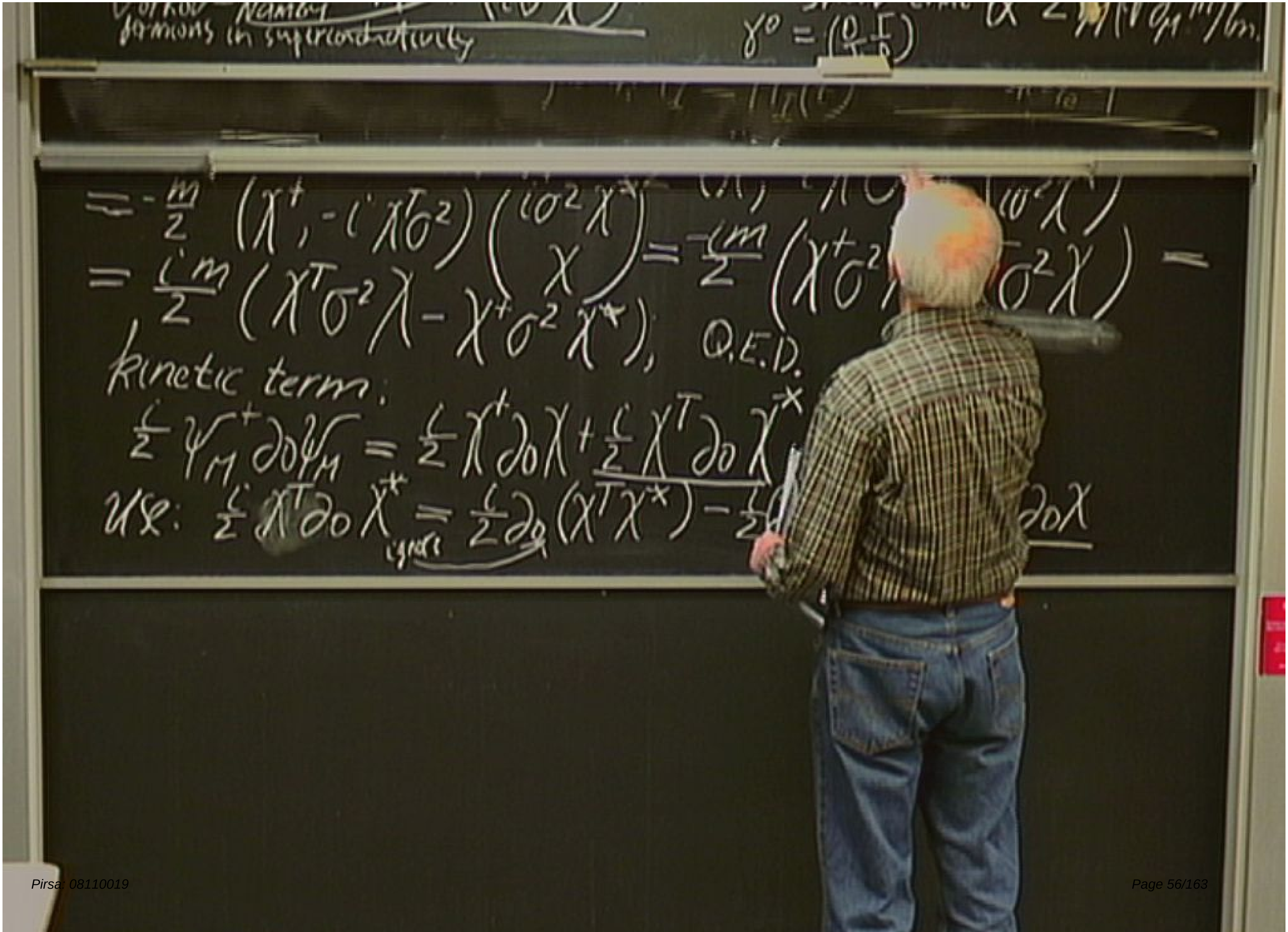

called interaction picture evolution operator. It
satisfies a relatively simple equation.

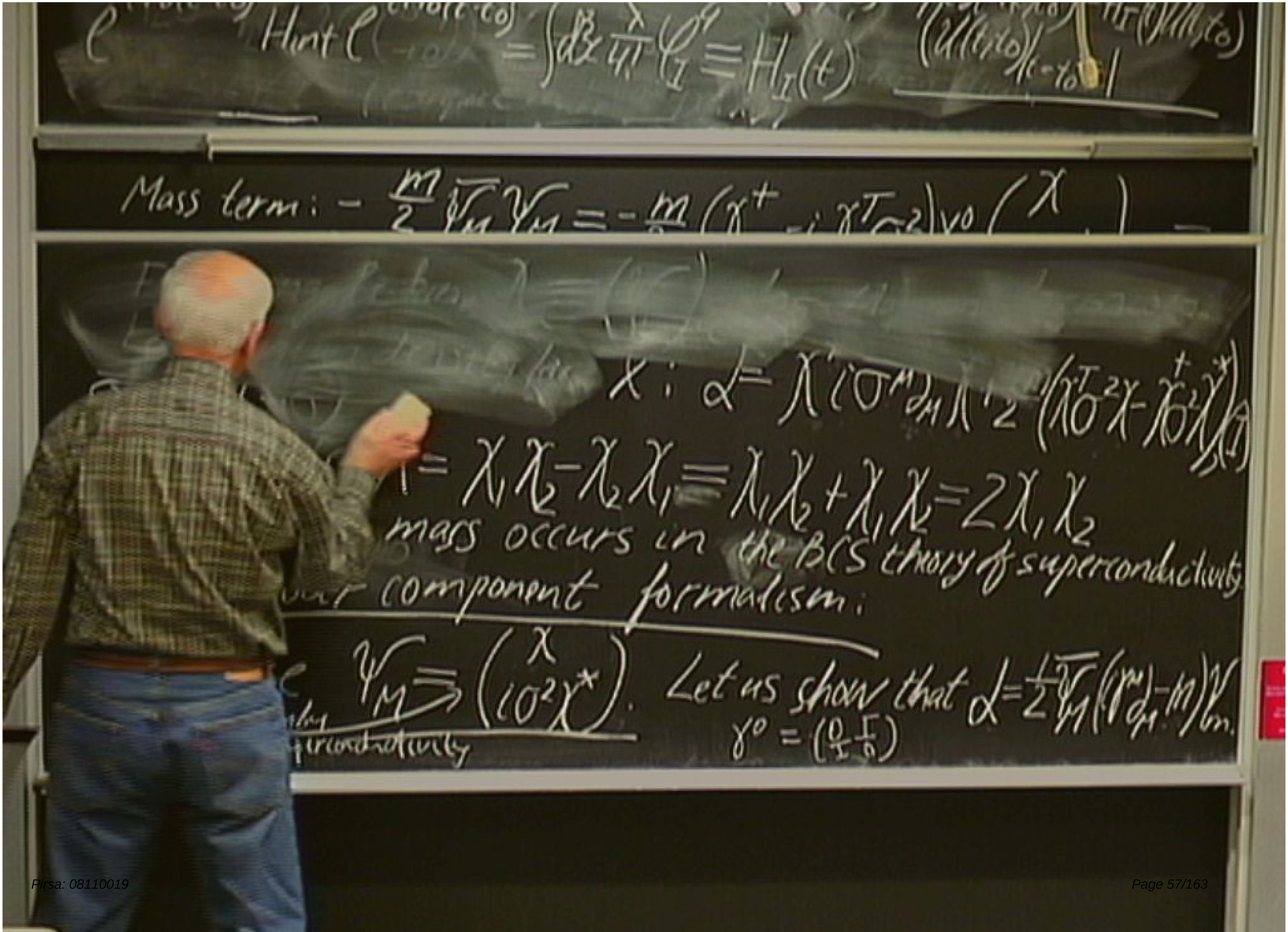
called interaction picture evolution operator. St satisfies a relatively simple equation

$$i\frac{\partial}{\partial t}U(t,t_0)=e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}}e^{-iH(t-t_0)}=\underbrace{e^{iH_0(t-t_0)}H_{int}e^{-iH_0(t-t_0)}}\,e^{iH_0(t-t_0)}e^{-iH(t-t_0)}$$

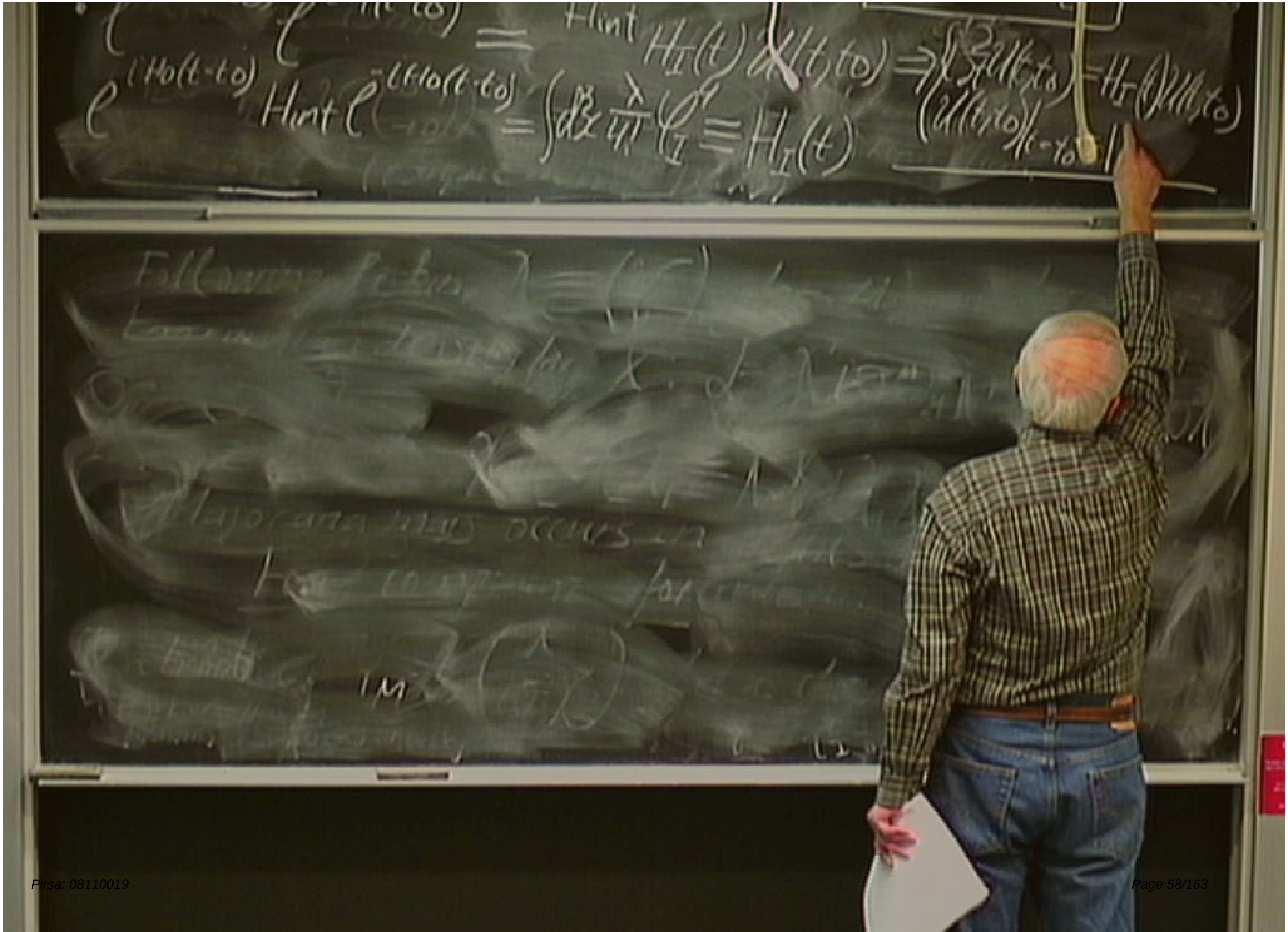
$$= H_I(t)\,U(t,t_0)\Rightarrow\left\{\begin{array}{l} i\frac{\partial}{\partial t}U(t,t_0)=H_I(t)U(t,t_0) \\ U(t,t_0)|_{t=t_0}=1\end{array}\right.$$

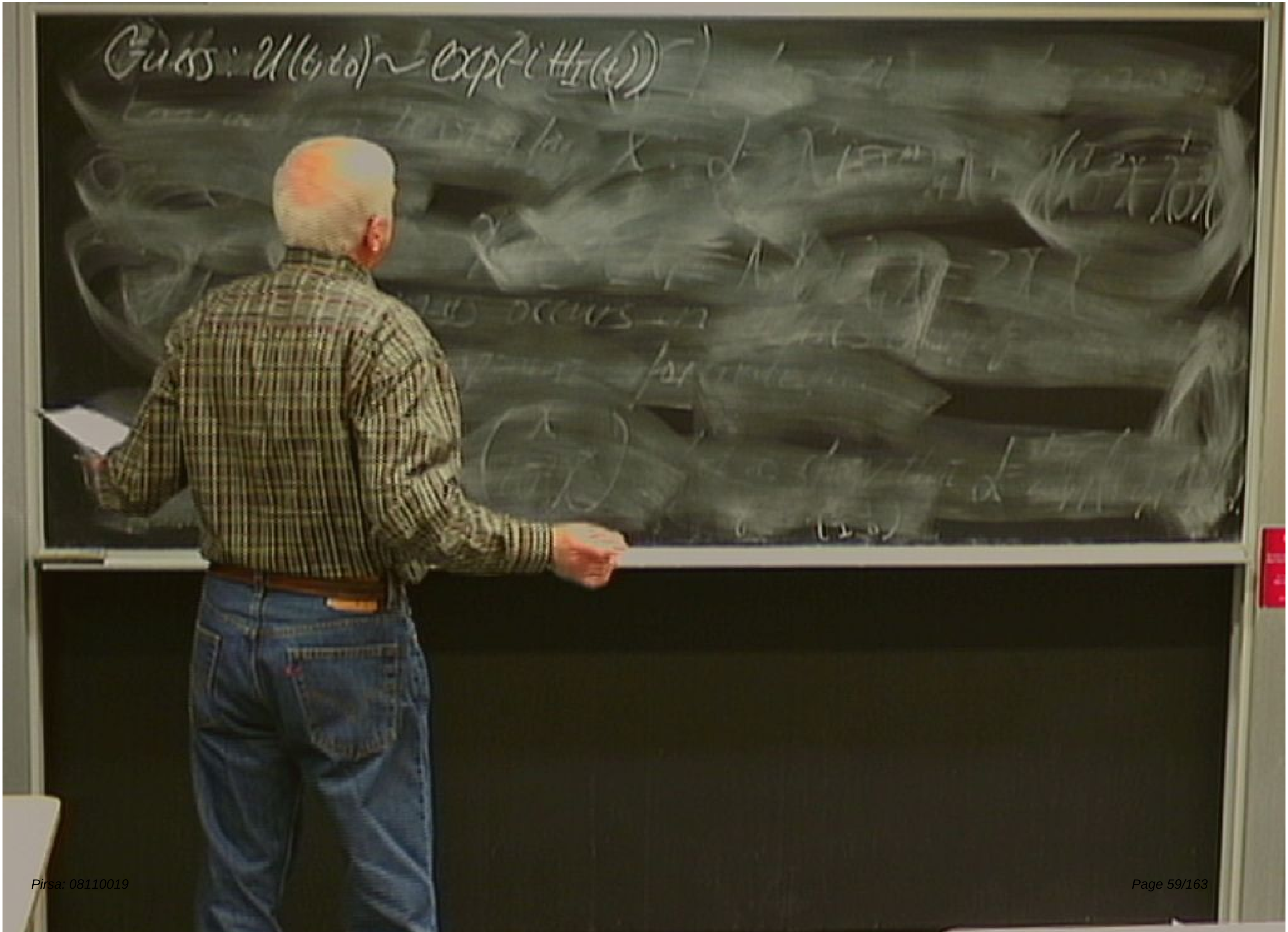
$$e^{iH_0(t-t_0)}H_{int}e^{-iH_0(t-t_0)}=\int d^3x\,\frac{\lambda}{4!}\phi_I^4\equiv H_I(t)$$

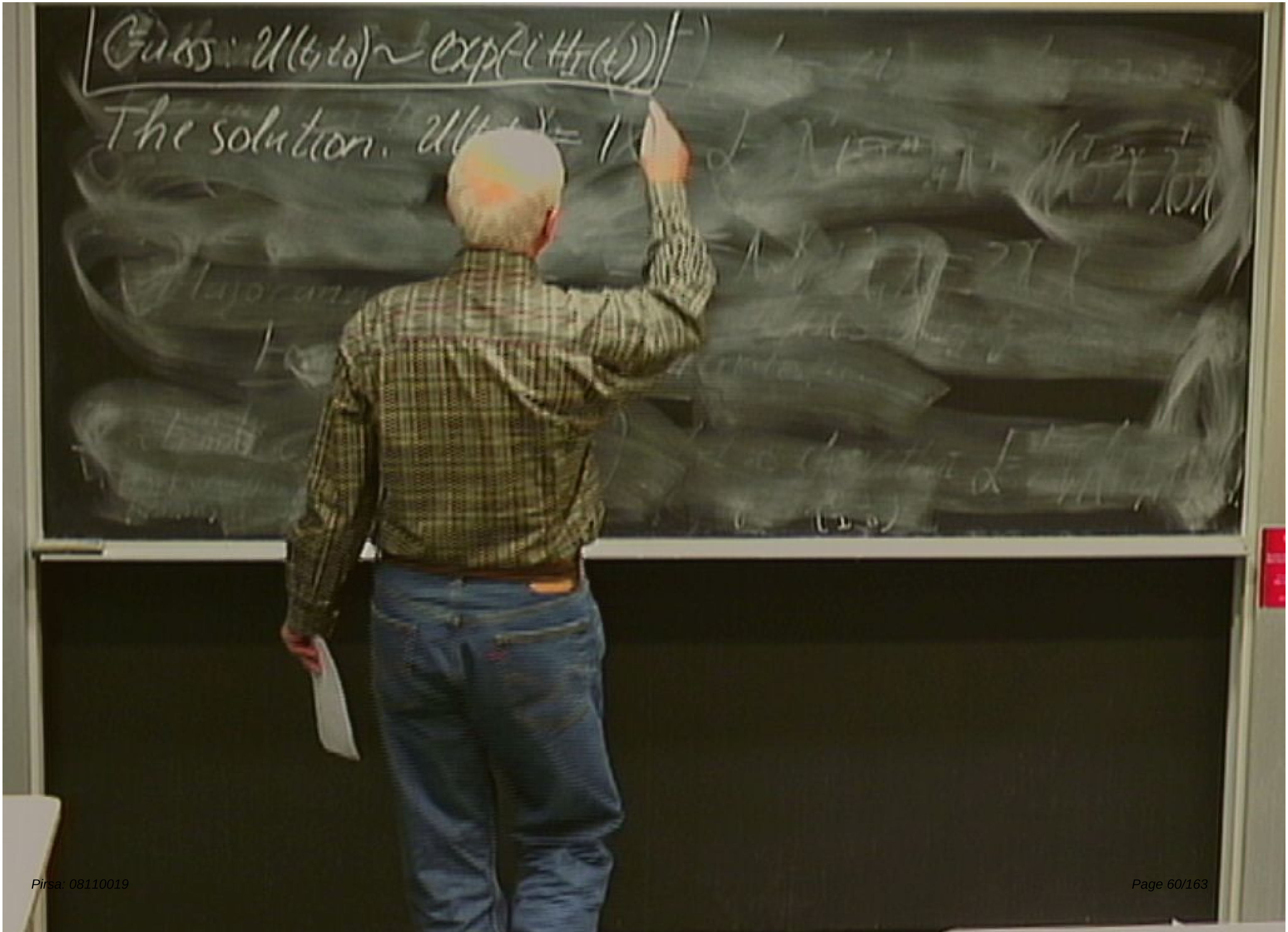

fermions in superconductivity
Q.E.D.
kinetic term:
use:



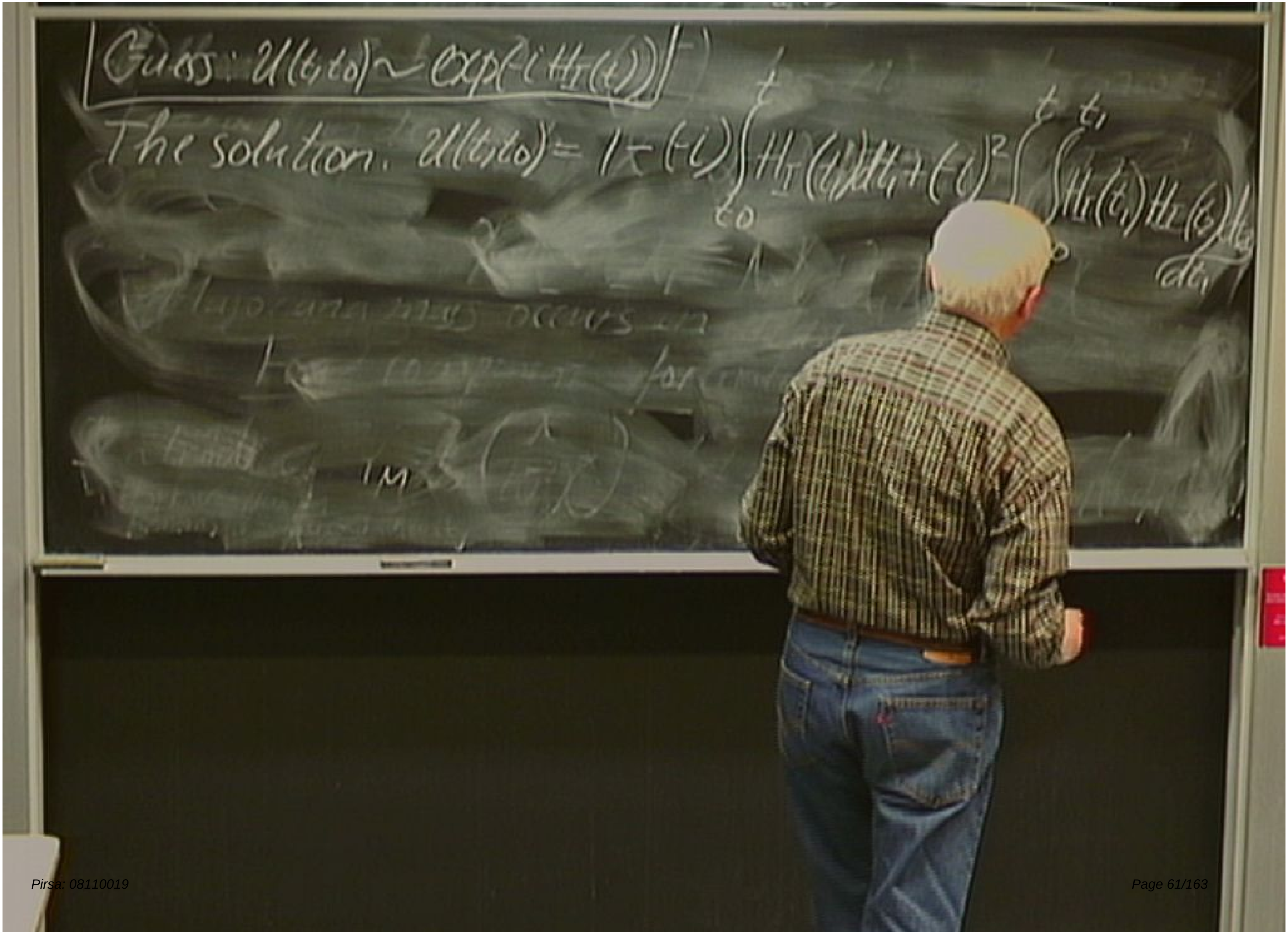
H_{int}
$= \int d^3x \frac{\lambda}{4!} \varphi_I^4 \equiv H_I(t)$
Mass term: $-\frac{m}{2}\bar{\psi}_M\psi_M = -\frac{m}{2}(\chi^\dagger - i\chi^T\sigma^2)\gamma^0 \Big(\chi$
χ: $\mathcal{L} = \chi^\dagger i\bar{\sigma}^\mu\partial_\mu\chi$
$= \chi_1\chi_2 - \chi_2\chi_1 = \chi_1\chi_2 + \chi_1\chi_2 = 2\chi_1\chi_2$
mass occurs in the BCS theory of superconductivity
component formalism:
$\psi_M \equiv \begin{pmatrix}\chi \\ i\sigma^2\chi^*\end{pmatrix}$. Let us show that $\mathcal{L} = \frac{1}{2}\bar{\psi}_M(i\gamma^\mu\partial_\mu - m)\psi_M$.
$\gamma^0 = \begin{pmatrix}0 & I \\ I & 0\end{pmatrix}$

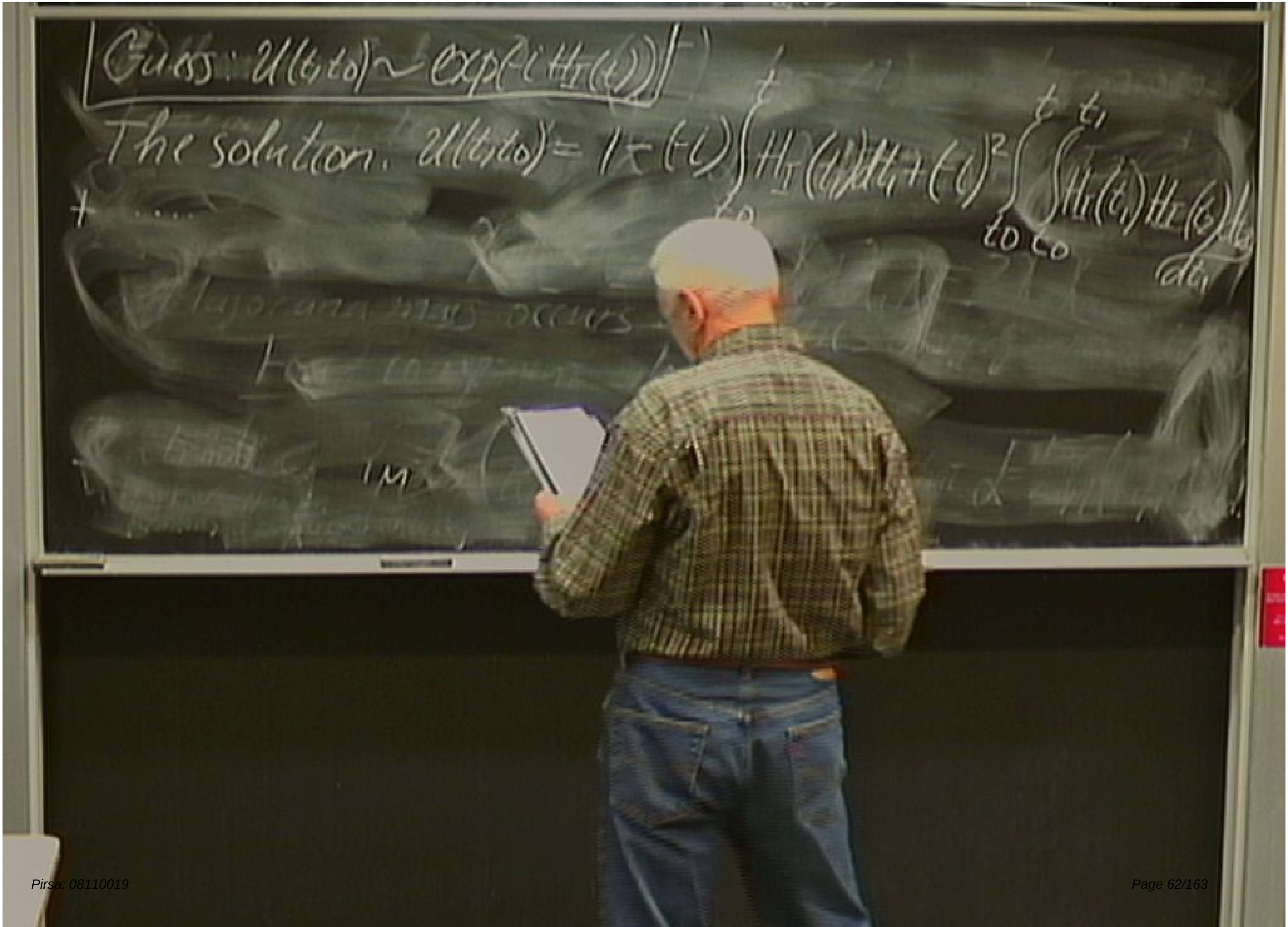


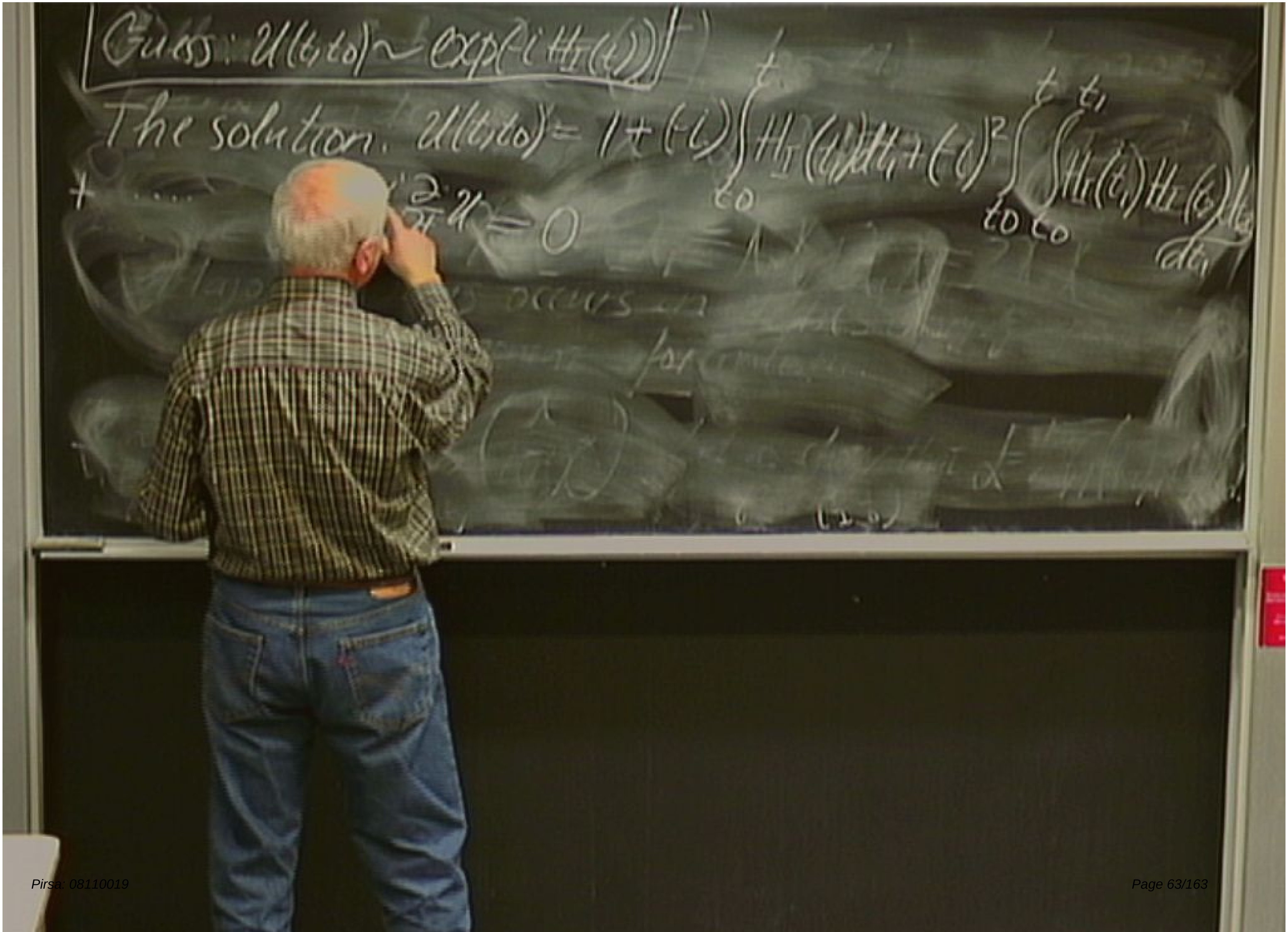

Guess: $U(t,t_0) \sim \exp(-i\, H_I(t))$
occurs in

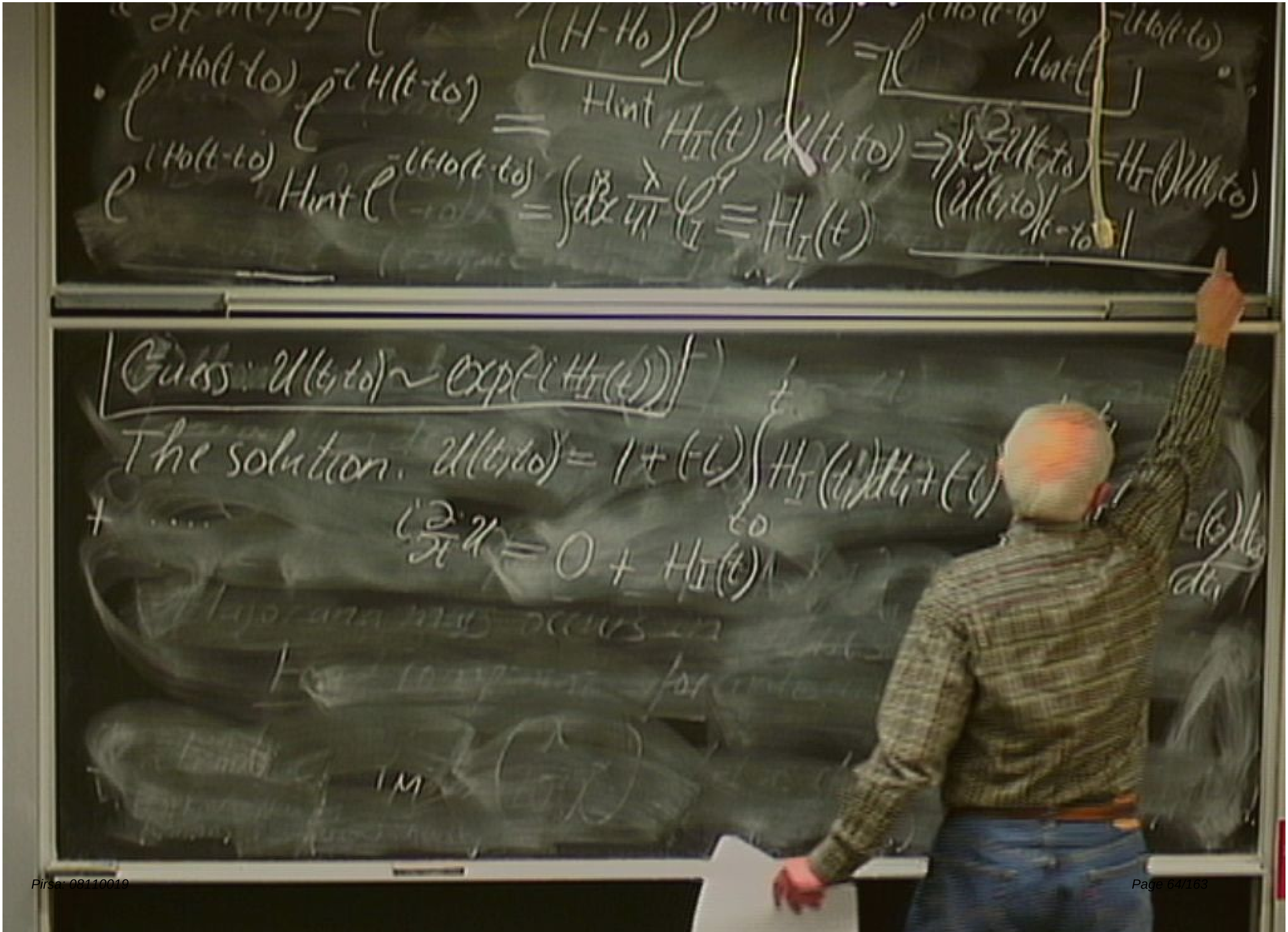


Guess: $U(t,t_0) \sim \exp(-i\,t\,H_I(t))$
The solution:

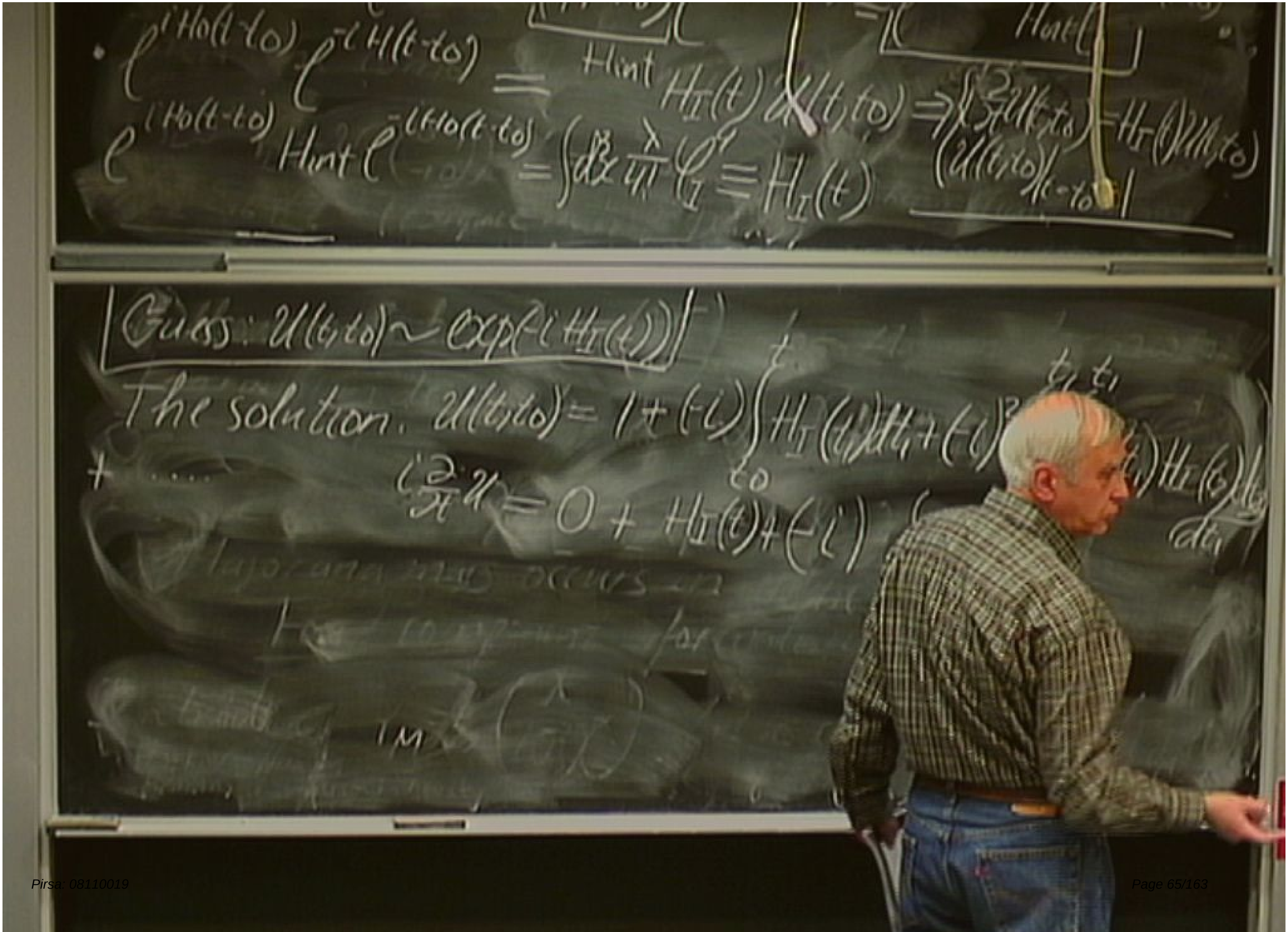

Guess: $U(t,t_0) \sim \exp(-i H_I(t))$
The solution: $U(t,t_0) = 1 + (-i)\int_{t_0}^{t} H_I(t_1)dt_1 + (-i)^2 \int_{t_0}^{t}\int^{t_1} H_I(t_1)H_I(t_2)dt_2\,dt_1$

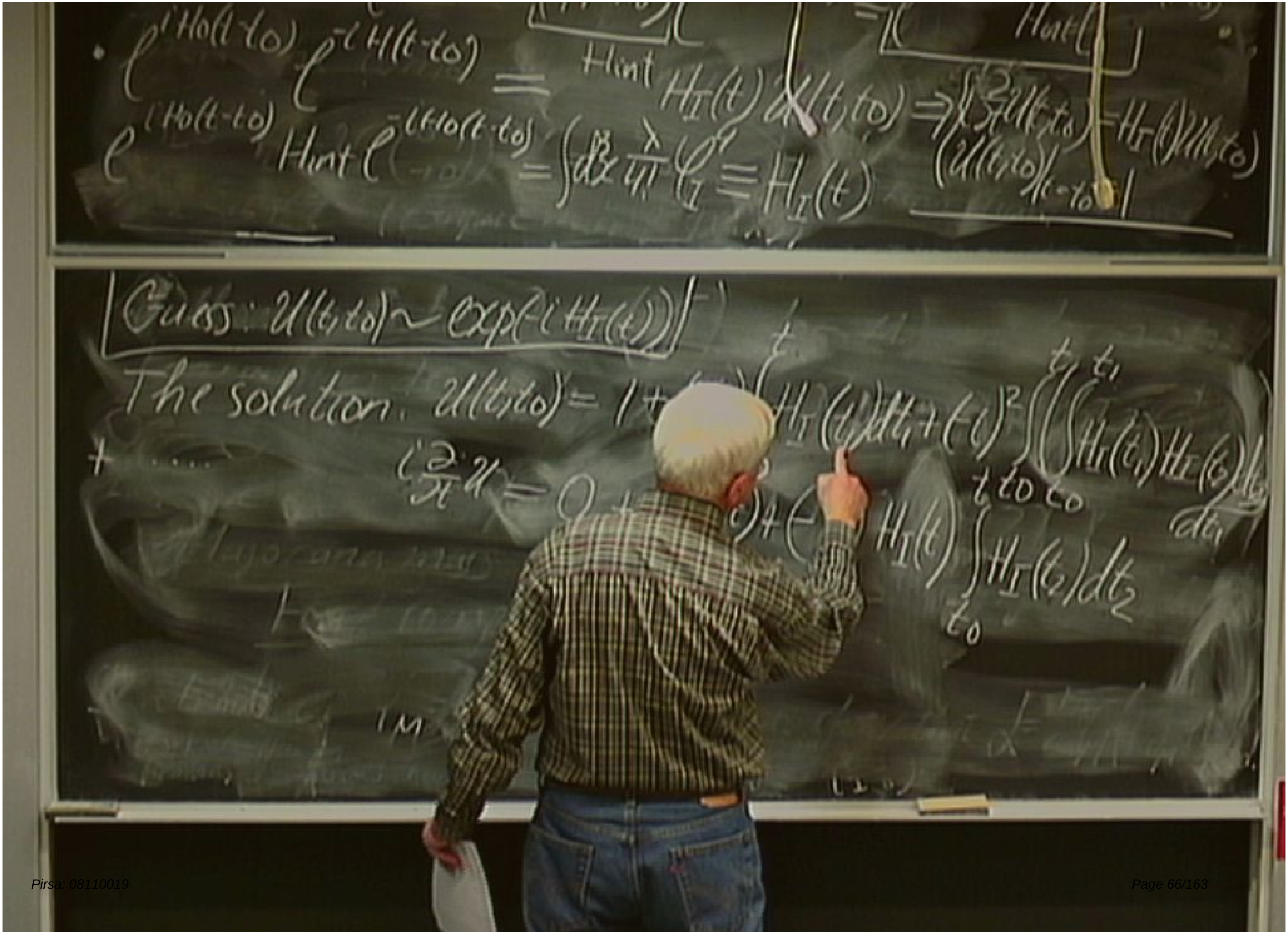

Guess: U(t,t0) ~ exp(-i HI(t))
The solution: U(t,t0) = 1 + (-i) ∫ HI(t1)dt1 + (-i)^2 ∫∫ HI(t1) HI(t2) dt2 dt1
+ ....


Guess: U(t,t0) ~ exp(-i H_I(t))
The solution: U(t,t0) = 1 + (-i) ∫ H_I(t1) dt1 + (-i)^2 ∫∫ H_I(t1) H_I(t2) dt2 dt1
+ ......
= 0
occurs in
for

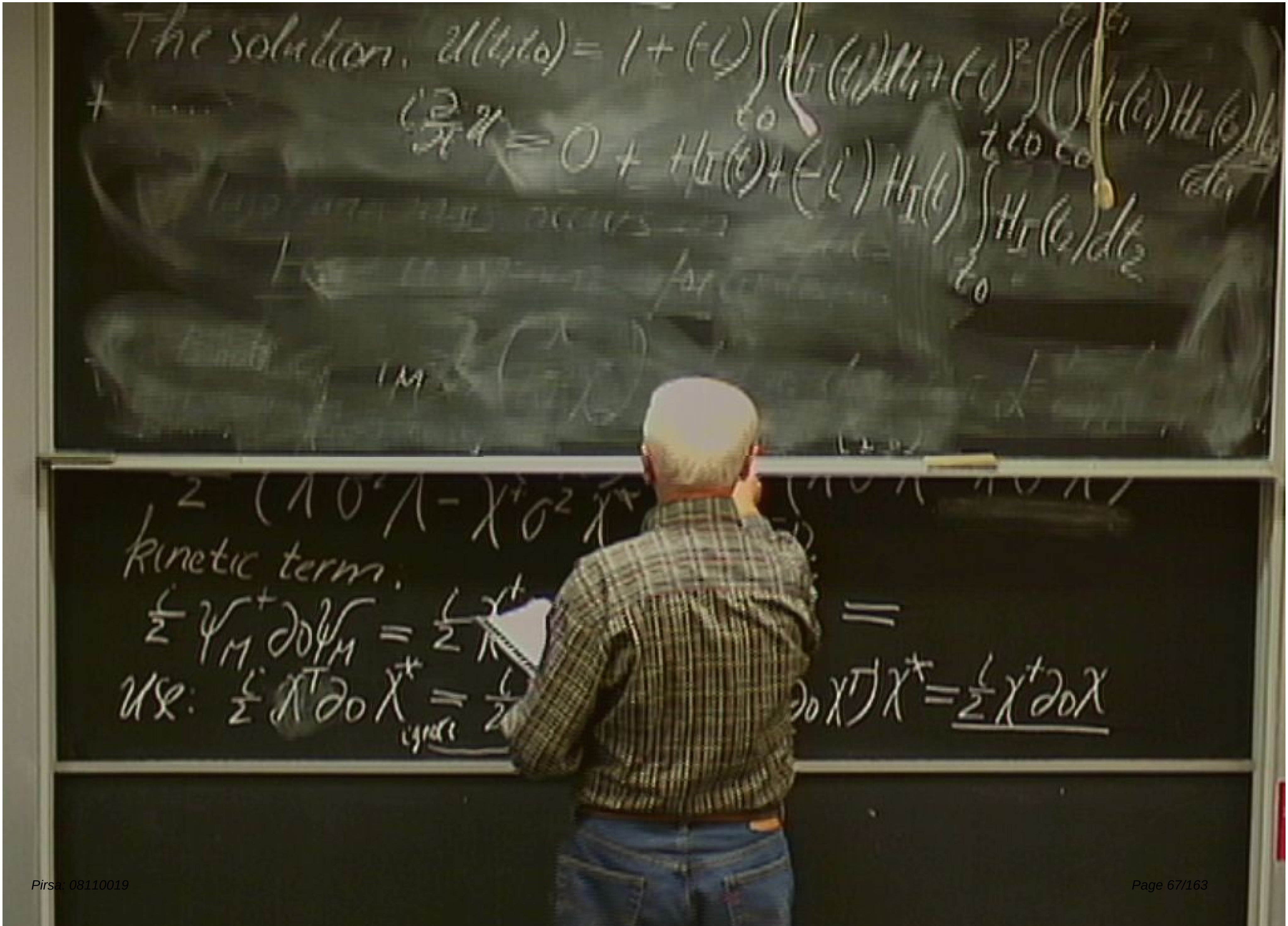


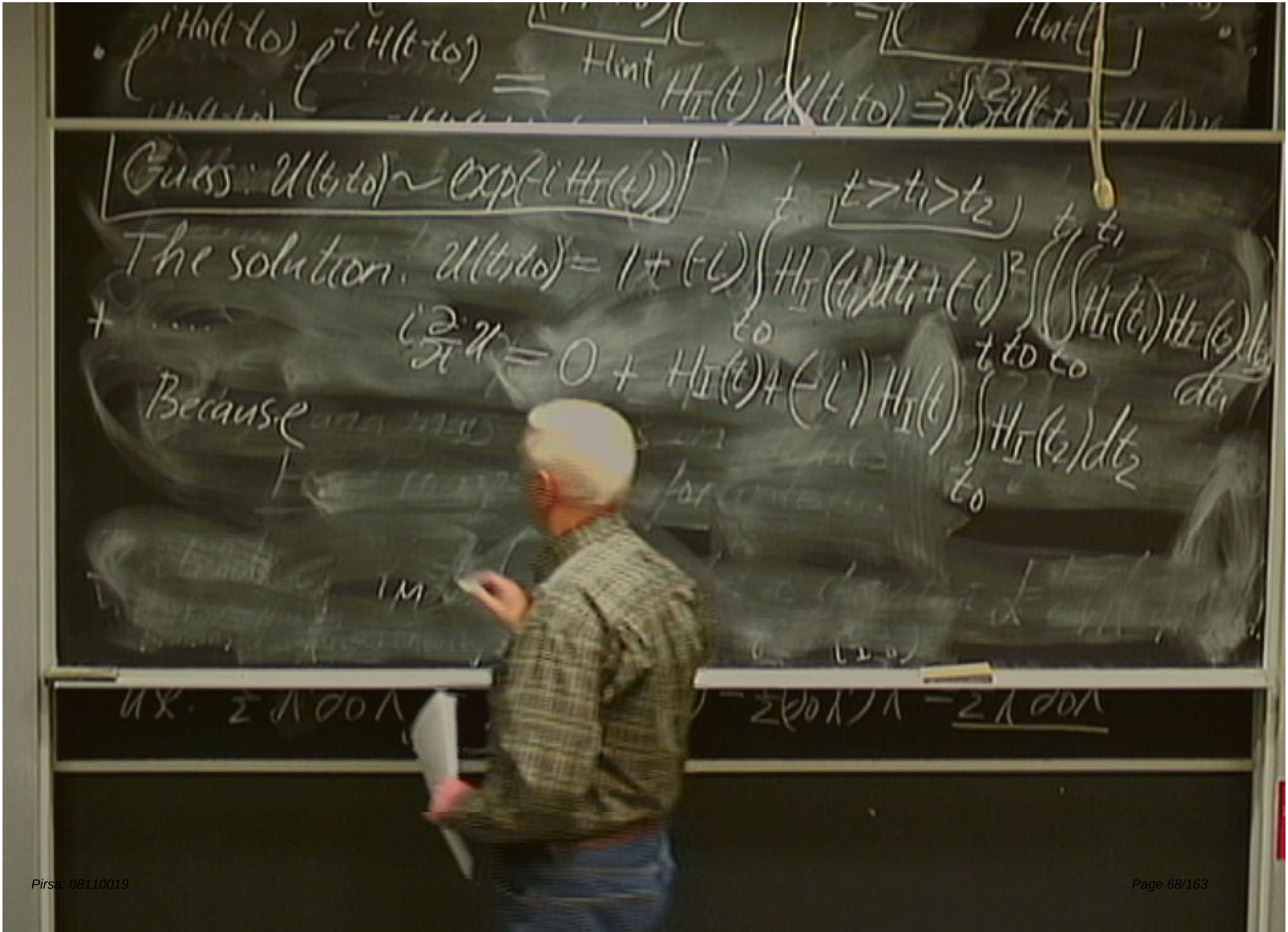
$e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$
Hint
$H_I(t)\, U(t,t_0) \Rightarrow i\frac{\partial}{\partial t}U(t,t_0) = H_I(t)\,U(t,t_0)$
$e^{iH_0(t-t_0)}\, H_{int}\, e^{-iH_0(t-t_0)} = \int d^3x \frac{\lambda}{4!}\varphi_I^4 \equiv H_I(t)$
$U(t,t_0)|_{t=t_0} = 1$
Guess: $U(t,t_0) \sim \exp(-iH_I(t))$
The solution. $U(t,t_0) = 1 + (-i)\int_{t_0}^{t} H_I(t_1)\,dt_1 + (-i)$
$+ \ldots$
$i\frac{\partial}{\partial t}U = 0 + H_I(t)$


Hint
Guess: U(t,t0) ~ exp(-i HI(t))
The solution. U(t,t0) = 1 + (-i)

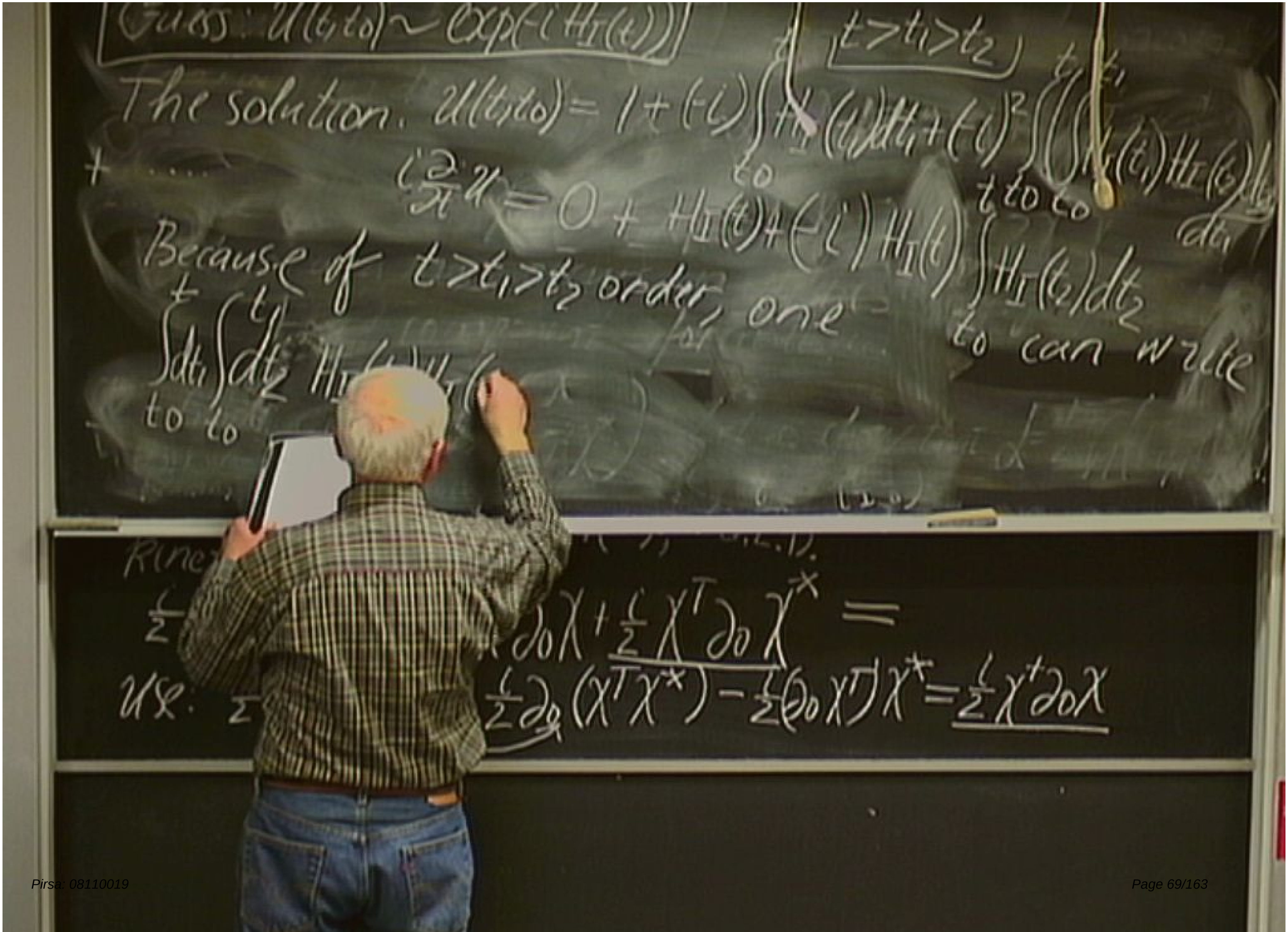


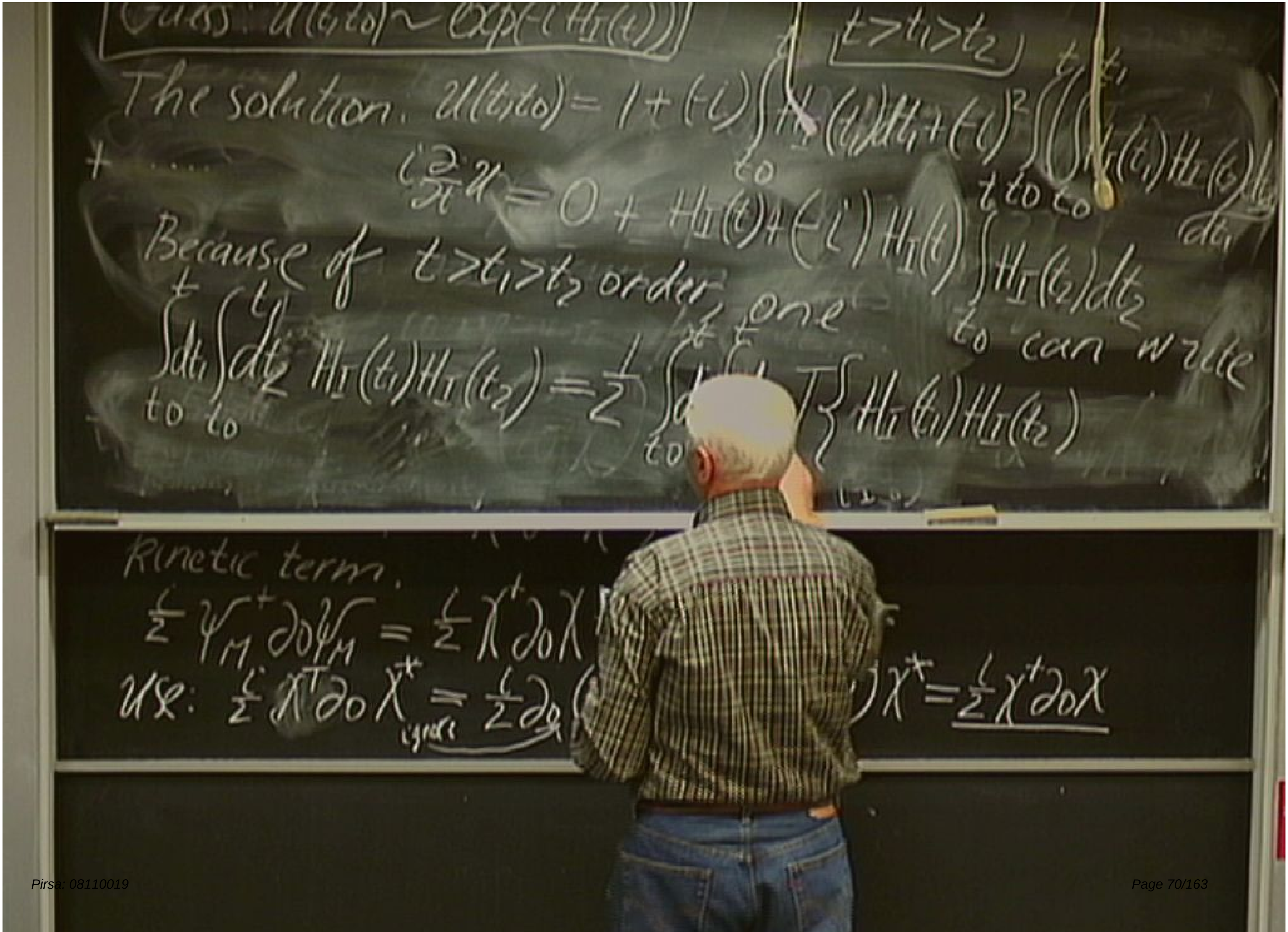
Guess: U(t,t0) ~ exp(-i HI(t))
The solution: U(t,t0) = 1 +

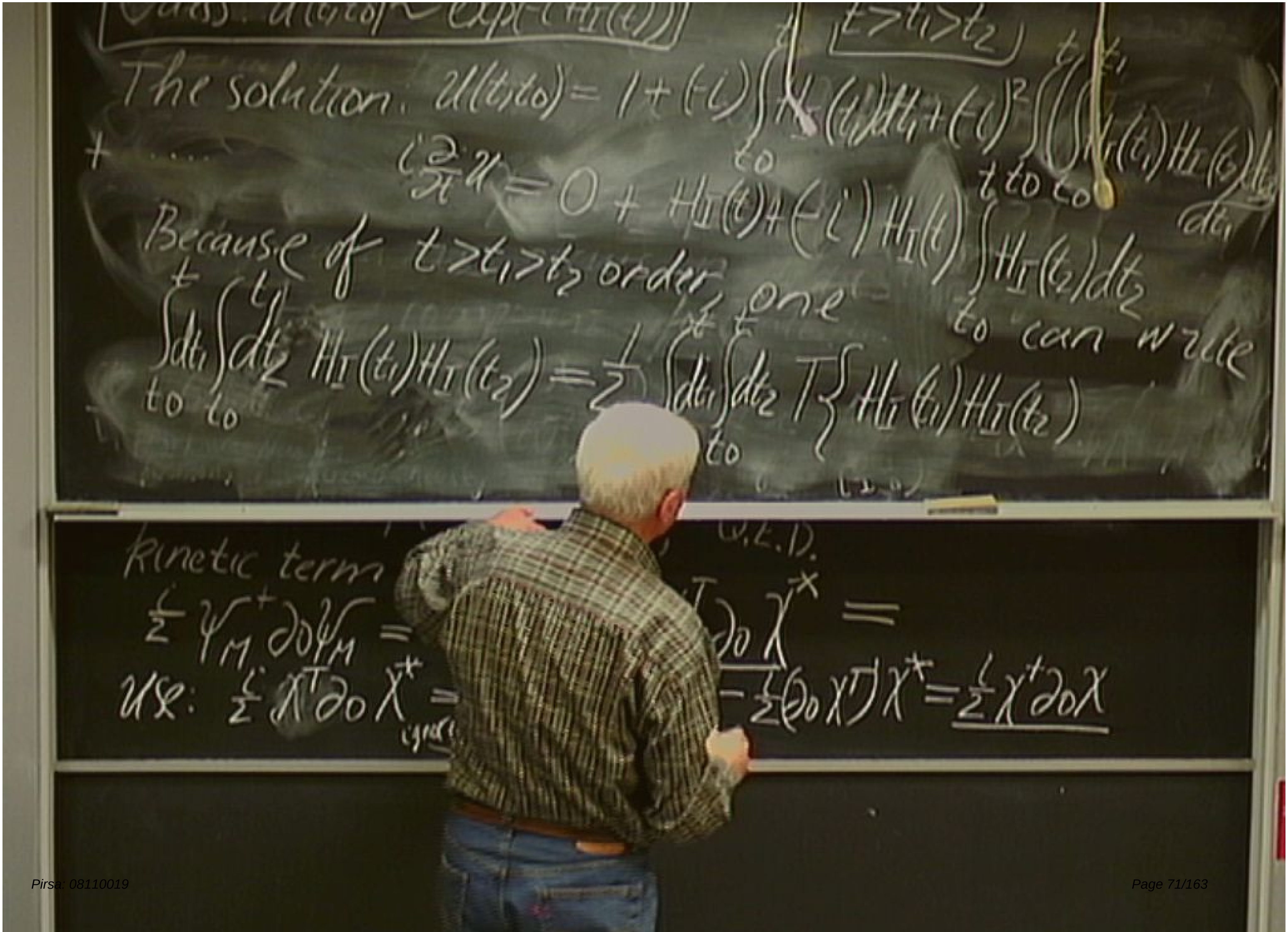

The solution.
kinetic term.

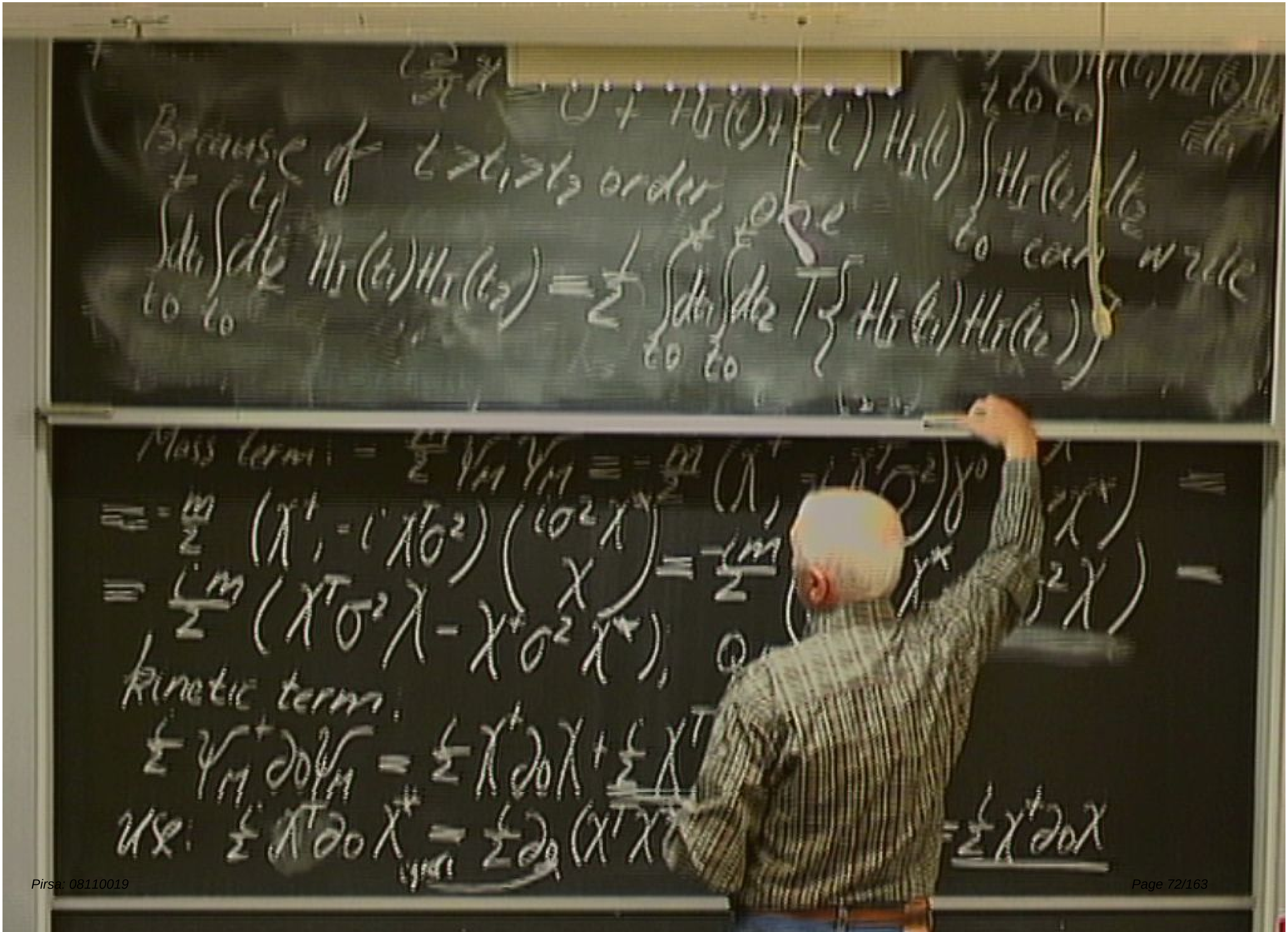


Hint
Guess: U(t,t0) ~ exp(-i HI(t))
t > t1 > t2
The solution.
Because


Guess: U(t,t0) ~ exp(-i H_I(t))
t > t1 > t2
The solution. U(t,t0) = 1 + (-i) ∫ H_I(t1) dt1 + (-i)^2 ∫∫ H_I(t1) H_I(t2) dt2 dt1
Because of t > t1 > t2 order, one can write


Guess: $U(t,t_0) \sim \exp(-i\, H_I(t))$
$t > t_1 > t_2$
The solution: $U(t,t_0) = 1 + (-i)\int_{t_0}^{t} H_I(t_1)\,dt_1 + (-i)^2 \int_{t_0}^{t}\int_{t_0}^{t_1} H_I(t_1) H_I(t_2)\,dt_2\,dt_1$
$+ \dots$
$i\frac{\partial}{\partial t} U = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^{t} H_I(t_2)\,dt_2$
Because of $t > t_1 > t_2$ order one can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t}\int_{t_0}^{t} T\{H_I(t_1) H_I(t_2))$
Kinetic term.
$\frac{i}{2} \psi_M^{+} \partial_0 \psi_M = \frac{i}{2} \chi^{+} \partial_0 \chi$
use: $\frac{i}{2} \chi^{T} \partial_0 \chi^{*} = \frac{i}{2} \partial_0 (\dots) \chi^{*} = \frac{i}{2} \chi^{+} \partial_0 \chi$
ignore


$t > t_1 > t_2$
The solution: $U(t,t_0) = 1 + (-i)\int_{t_0}^{t} H_I(t_1)dt_1 + (-i)^2 \int_{t_0}^{t}\int_{t_0}^{t_1} H_I(t_1)H_I(t_2)dt_2 dt_1$
$+ \ldots$
$i\frac{\partial}{\partial t}U = 0 + H_I(t) + (-i)H_I(t)\int_{t_0}^{t} H_I(t_2)dt_2$
Because of $t > t_1 > t_2$ order, one can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1)H_I(t_2)\}$
Q.E.D.
kinetic term


Because of $t>t_1>t_2$ order one can write
Mass term:
kinetic term:
use:

Because of $t>t_1>t_2$ order one can write

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1)H_I(t_2)\}$$

Mass term: $-\frac{m}{2}\bar{\psi}_M\psi_M = -\frac{m}{2}(\chi^+, -i\chi^T\sigma^2)\gamma^0\begin{pmatrix}\chi \\ i\sigma^2\chi^*\end{pmatrix} =$

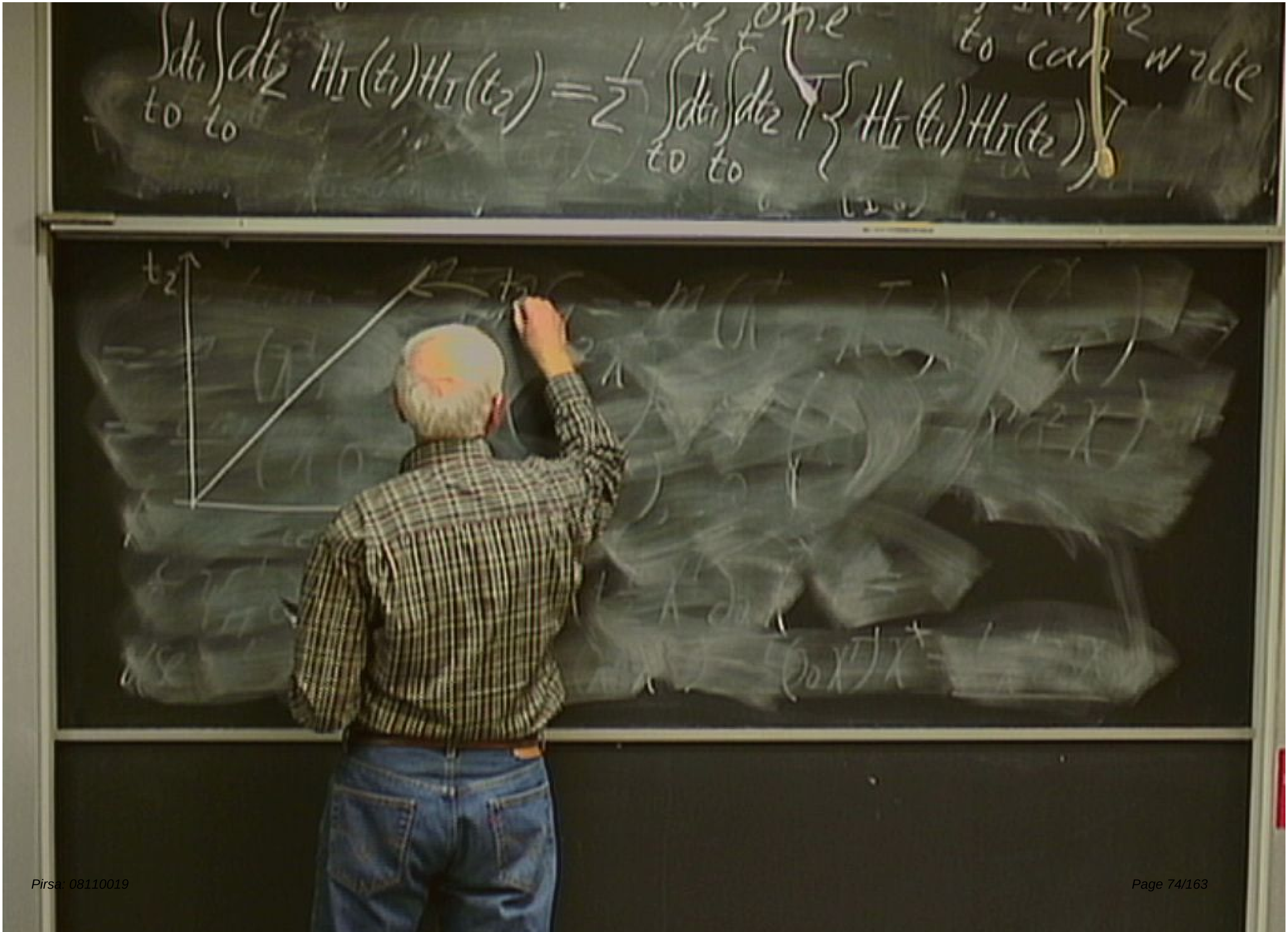
$= -\frac{m}{2}(\chi^+, -i\chi^T\sigma^2)\begin{pmatrix}i\sigma^2\chi^* \\ \chi\end{pmatrix} = -\frac{im}{2}(\chi^+\sigma^2\chi^* - \ldots\chi)$

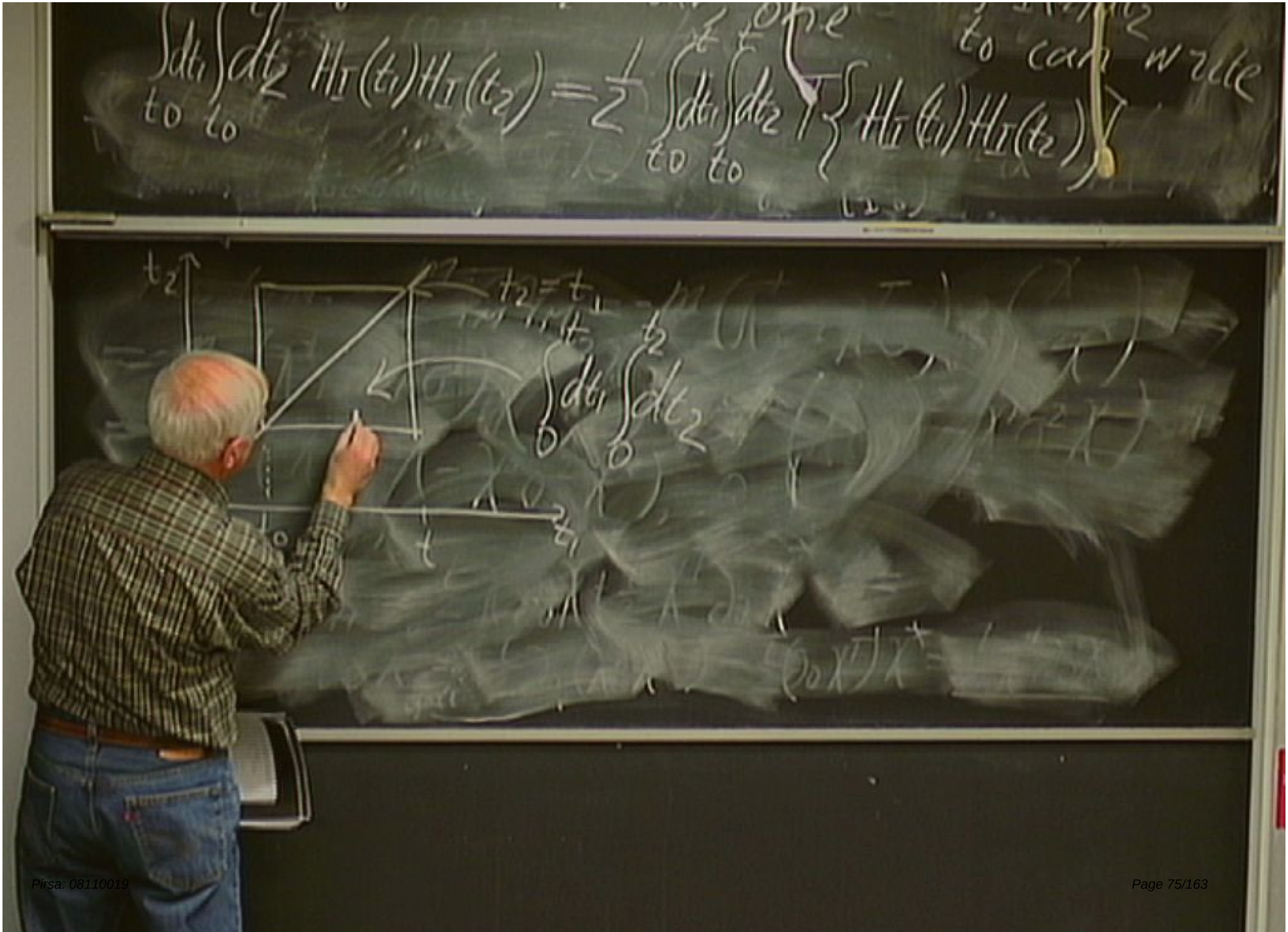
$= \frac{im}{2}(\chi^T\sigma^2\chi - \chi^+\sigma^2\chi^*)$, Q.E.D.

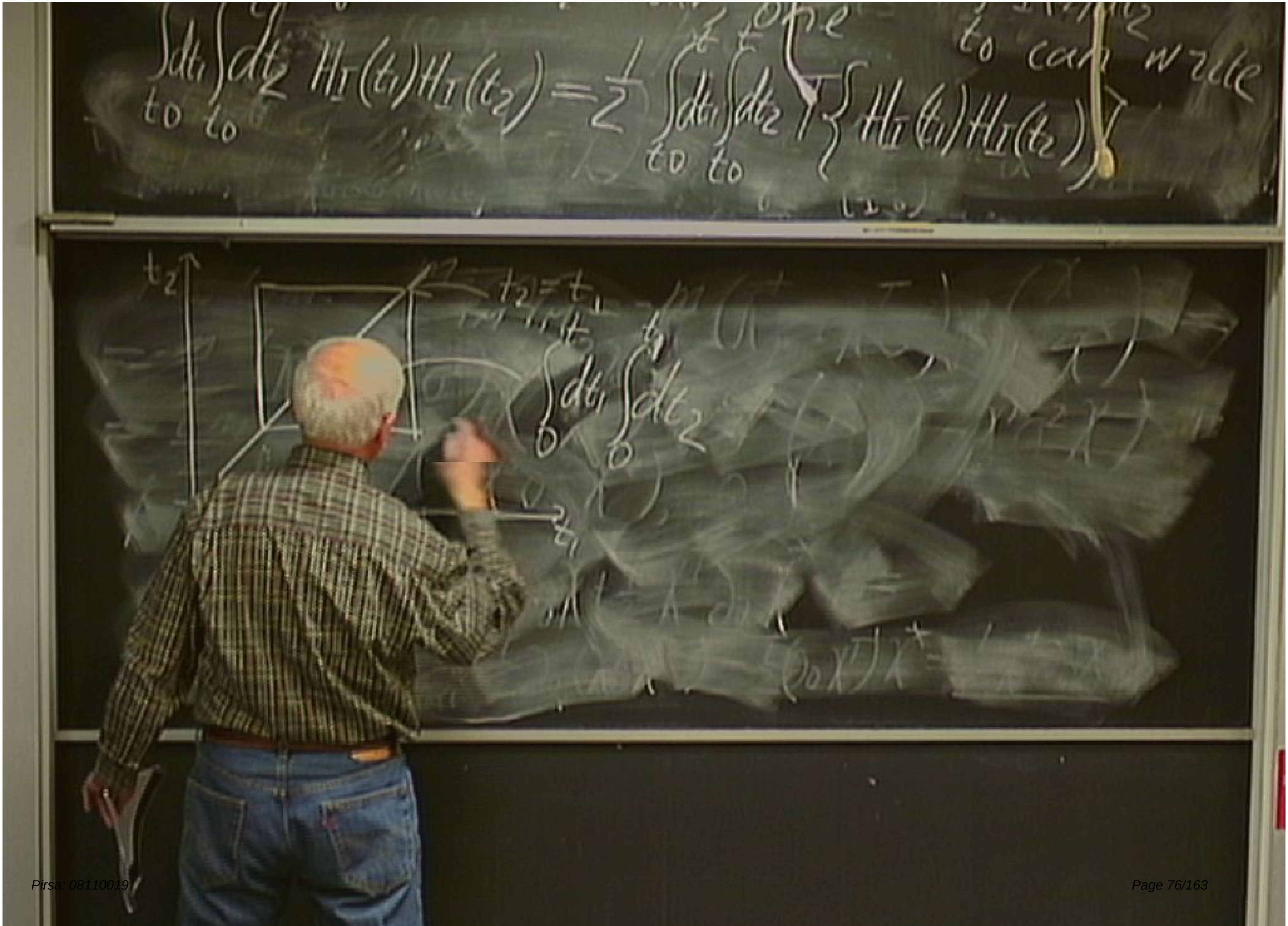
kinetic term:

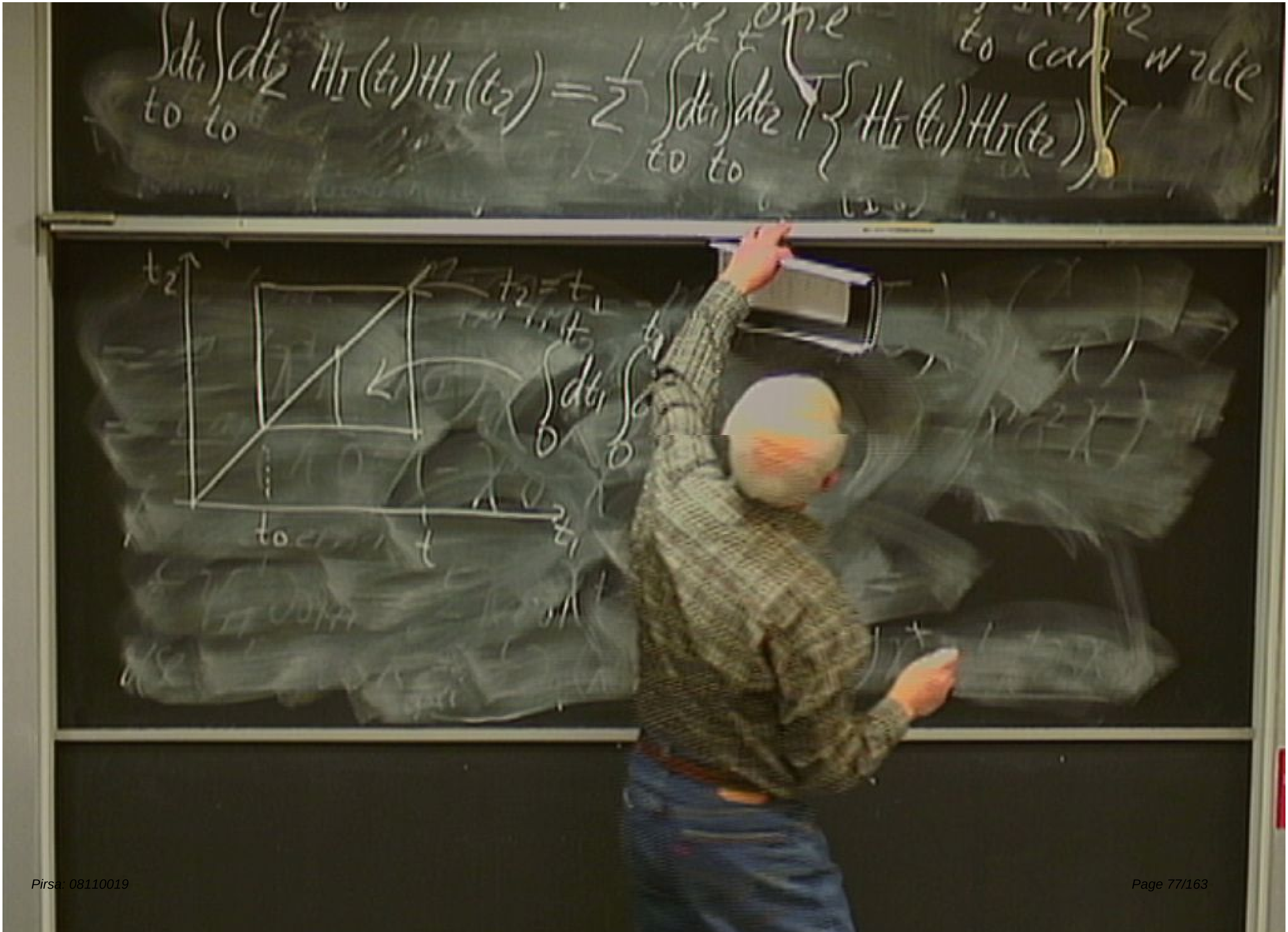
$\frac{i}{2}\psi_M^+\partial_0\psi_M = \frac{i}{2}\chi^+\partial_0\chi + \frac{i}{2}\chi^T\partial_0\chi^*$

use: $\frac{i}{2}\chi^T\partial_0\chi^* = \frac{i}{2}\partial_0(\chi^T\chi^*) - \frac{i}{2}(\ldots$ ignore

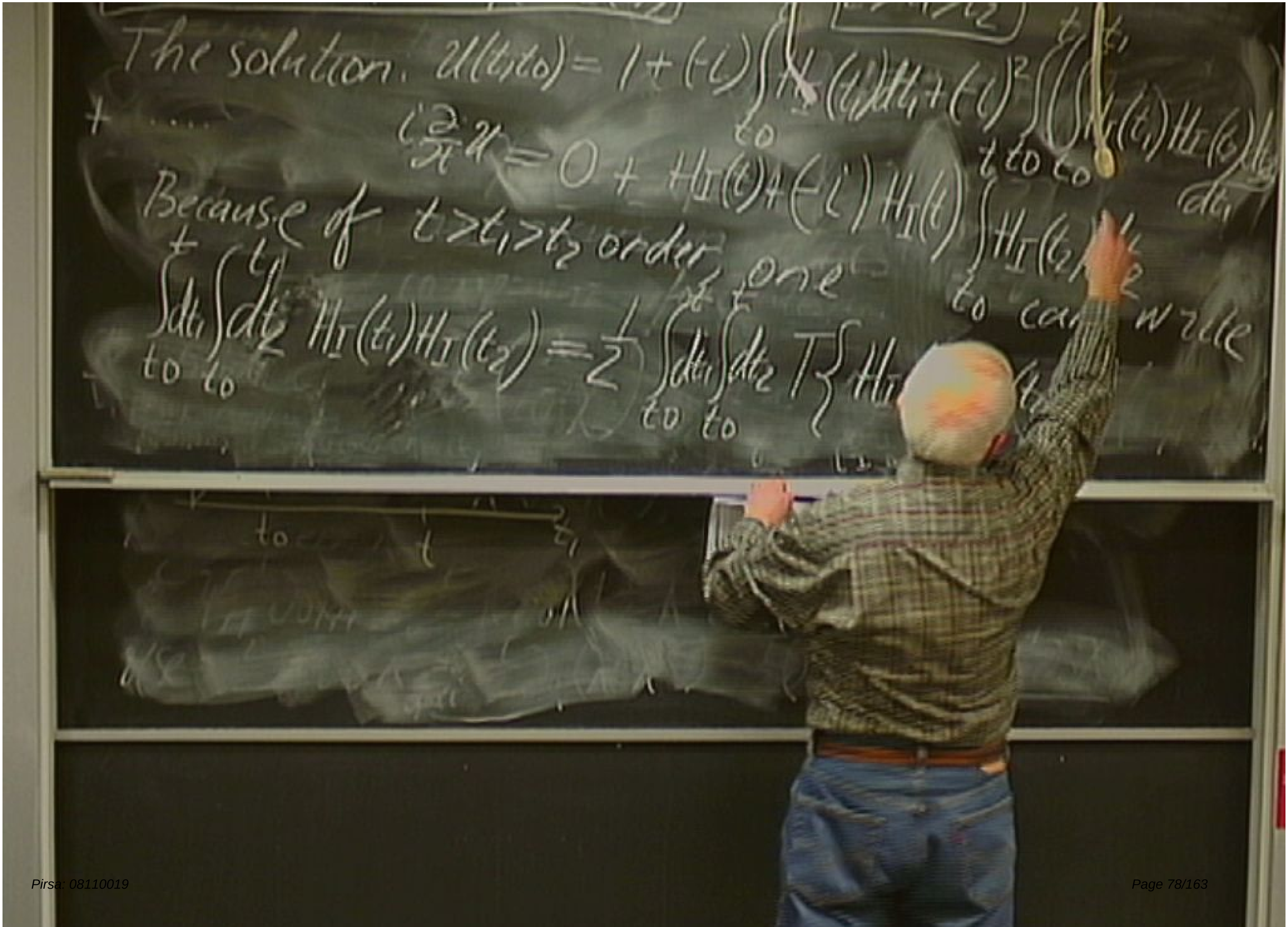

t_0 can write
$\int_{t_0} dt_1 \int_{t_0} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1) H_I(t_2)\}$
t_2

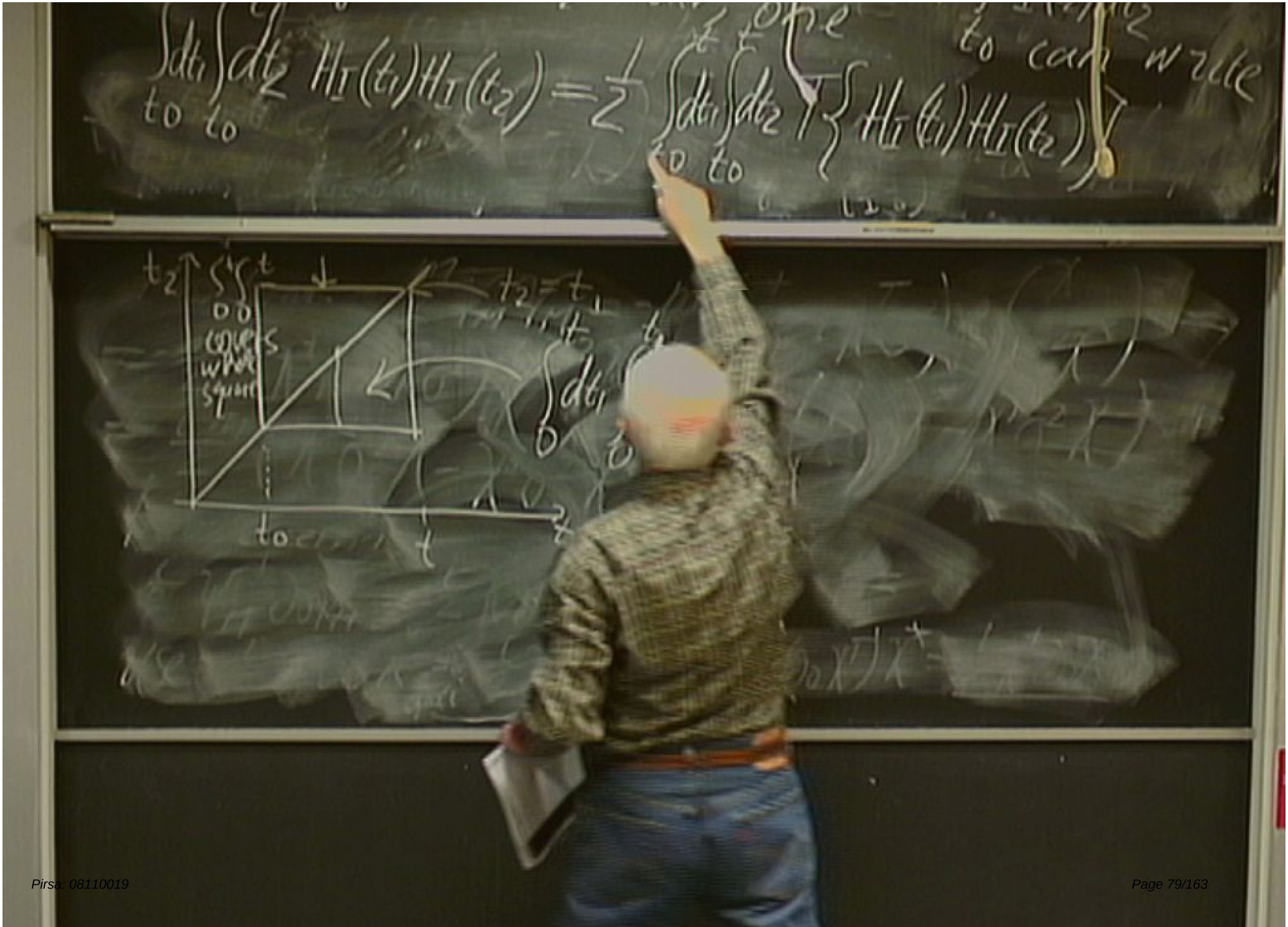

can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{ H_I(t_1) H_I(t_2) \}$
$t_2 = t_1$
$\int_0^{t} dt_1 \int_0^{t_2} dt_2$

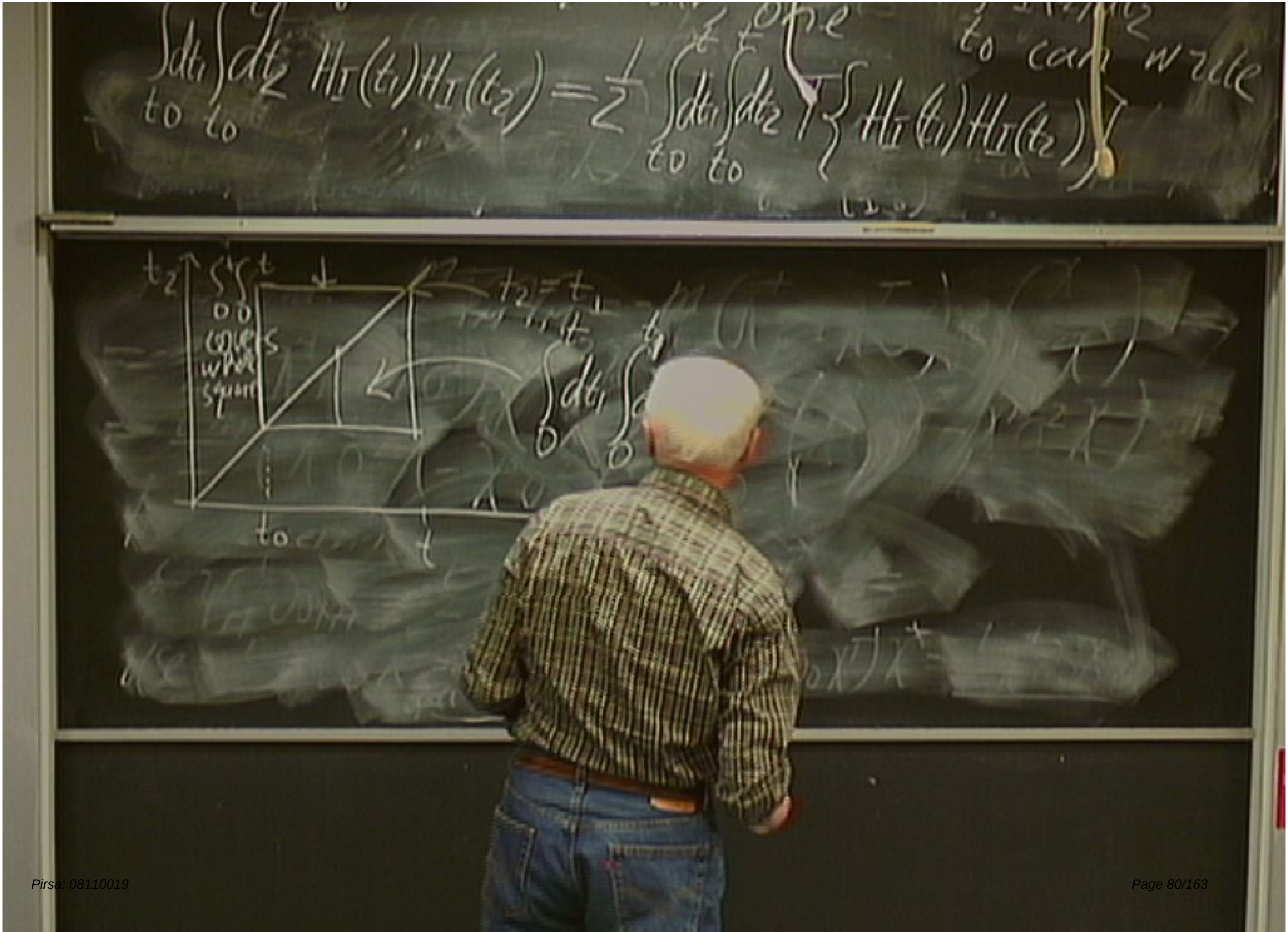

one
t_0 can write
$\int_{t_0} dt_1 \int_{t_0} dt_2 \, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \, T\{H_I(t_1) H_I(t_2)\}$
$t_2 = t_1$



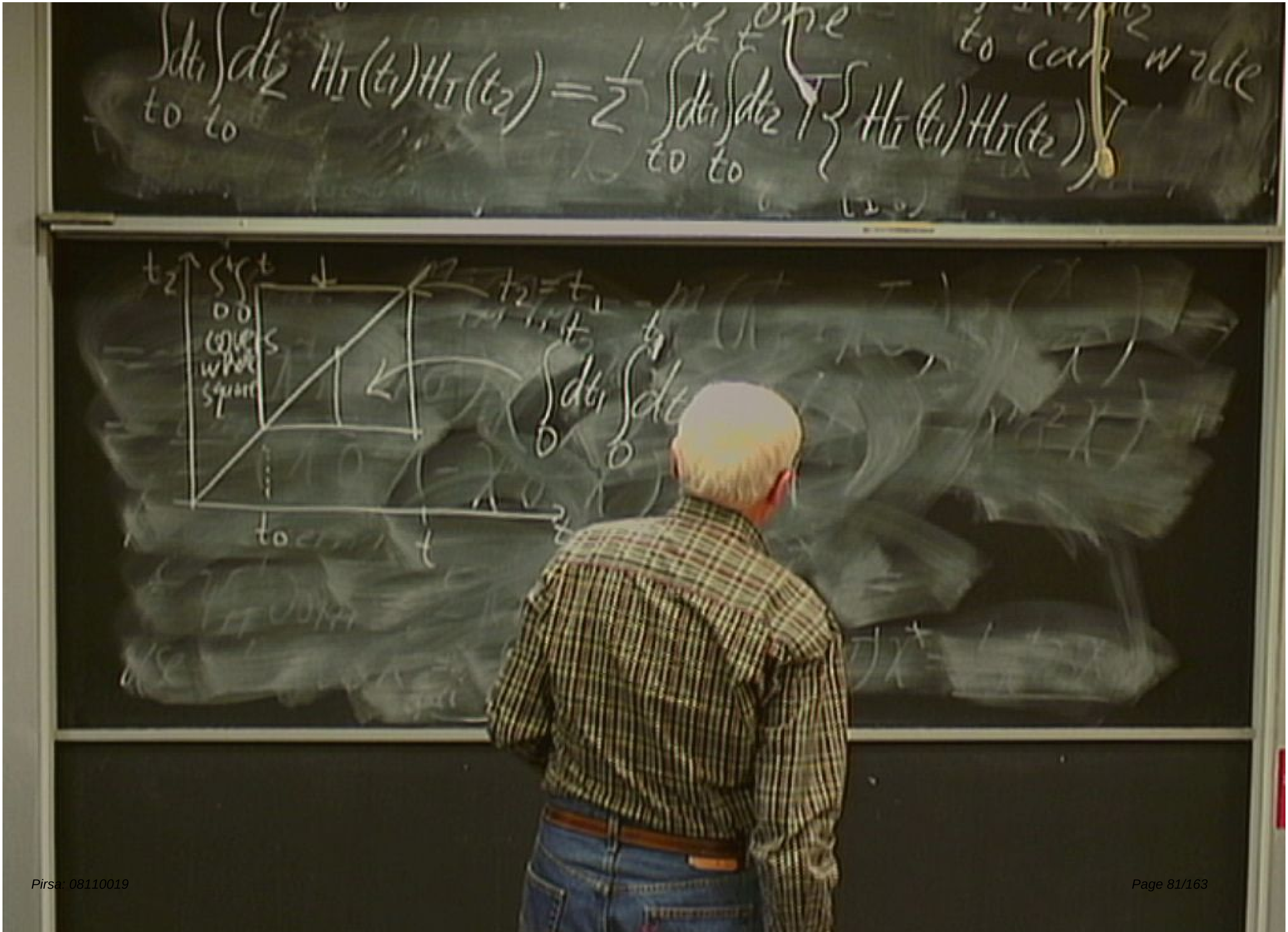
t₀ can write
∫dt₁∫dt₂ H_I(t₁)H_I(t₂) = 1/2 ∫dt₁∫dt₂ T{H_I(t₁)H_I(t₂)}
t₀ t₀ t₀ t₀
t₂
t₂=t₁
t₀
t
t₁


The solution: $\mathcal{U}(t,t_0) = 1 + (-i)\int_{t_0} H_I(t_1)dt_1 + (-i)^2 \int_{t_0}^{t}\int_{t_0}^{t_1} H_I(t_1) H_I(t_2)\, dt_2\, dt_1$
$+ \ldots$
$i\frac{\partial}{\partial t}\mathcal{U} = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^{t} H_I(t_2) dt_2$
Because of $t > t_1 > t_2$ order, one can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I$

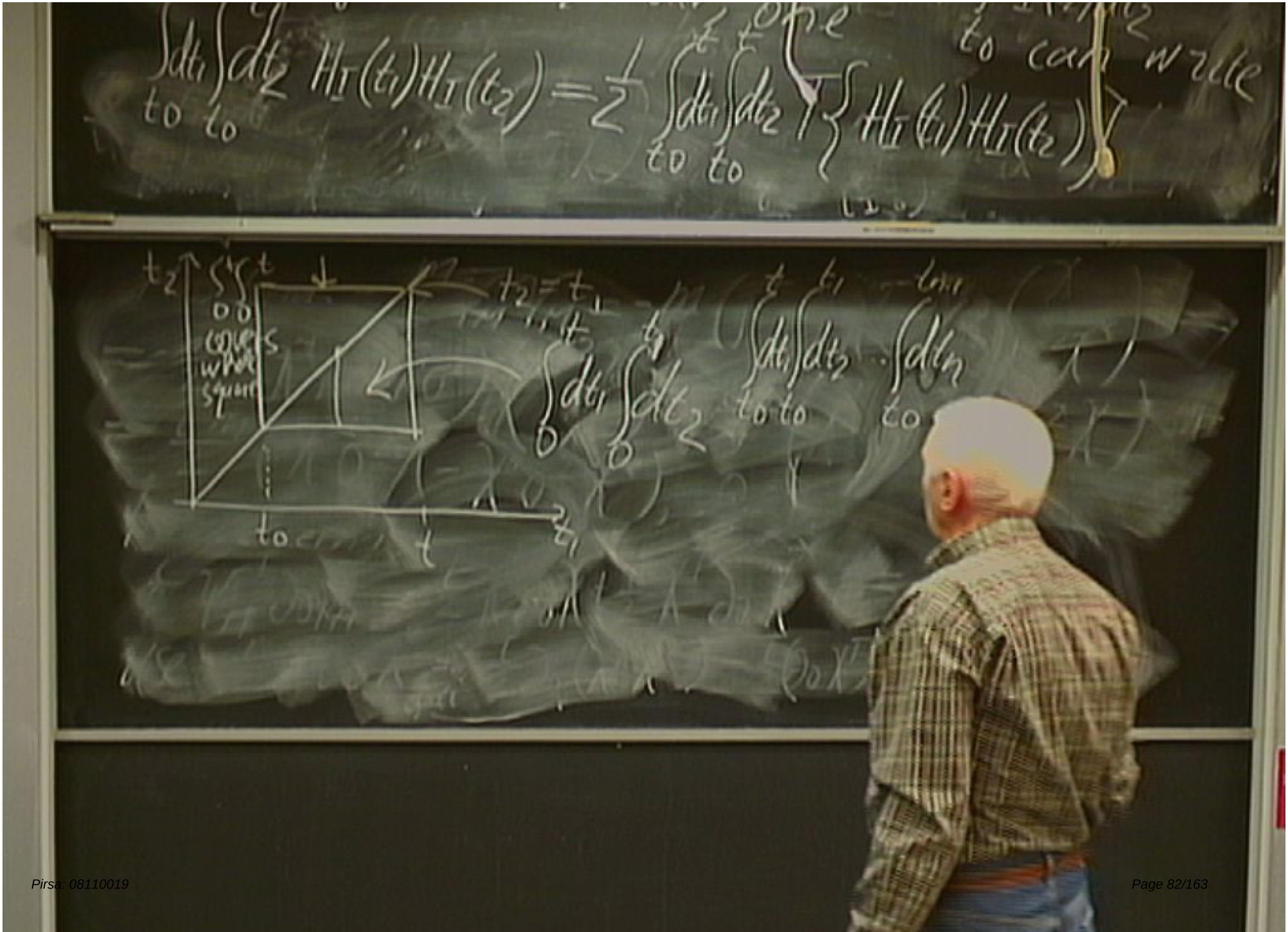

∫dt₁∫dt₂ H_I(t₁)H_I(t₂) = 1/2 ∫dt₁∫dt₂ T{H_I(t₁)H_I(t₂)}
to can write
covers whole square
t₂=t₁

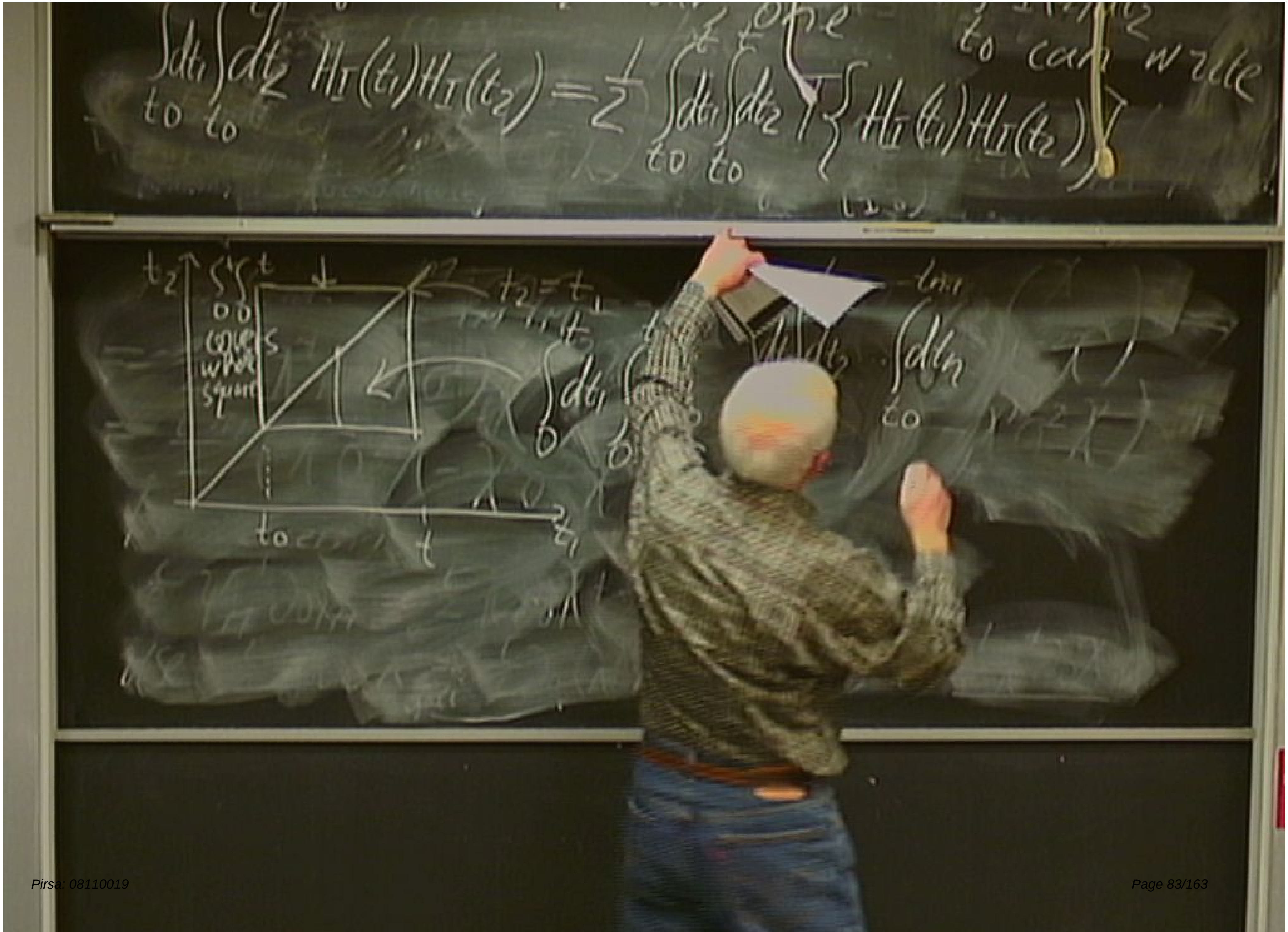


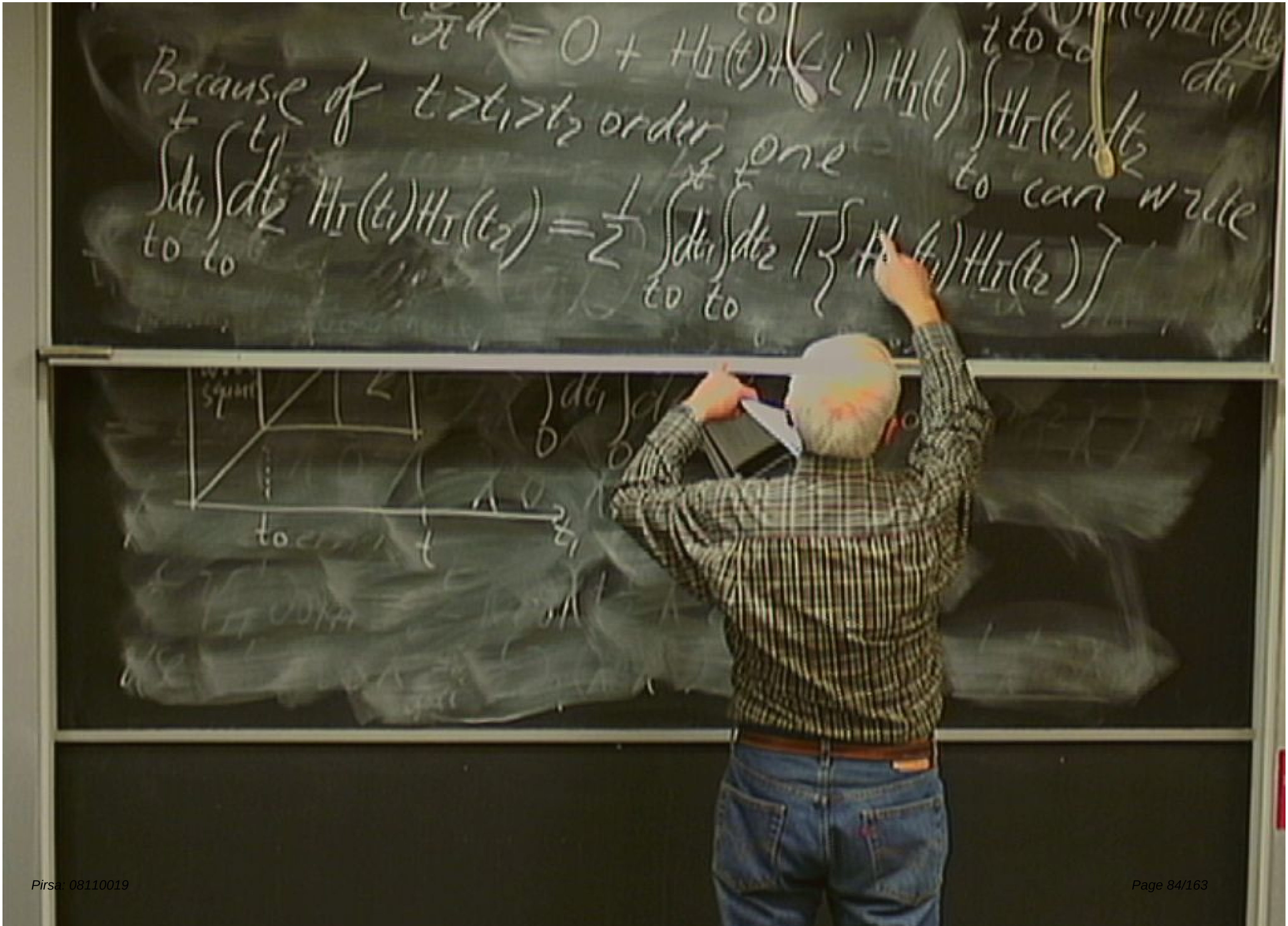
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1) H_I(t_2)\}$
can write
$t_2 = t_1$
covers whole square
t_0
t

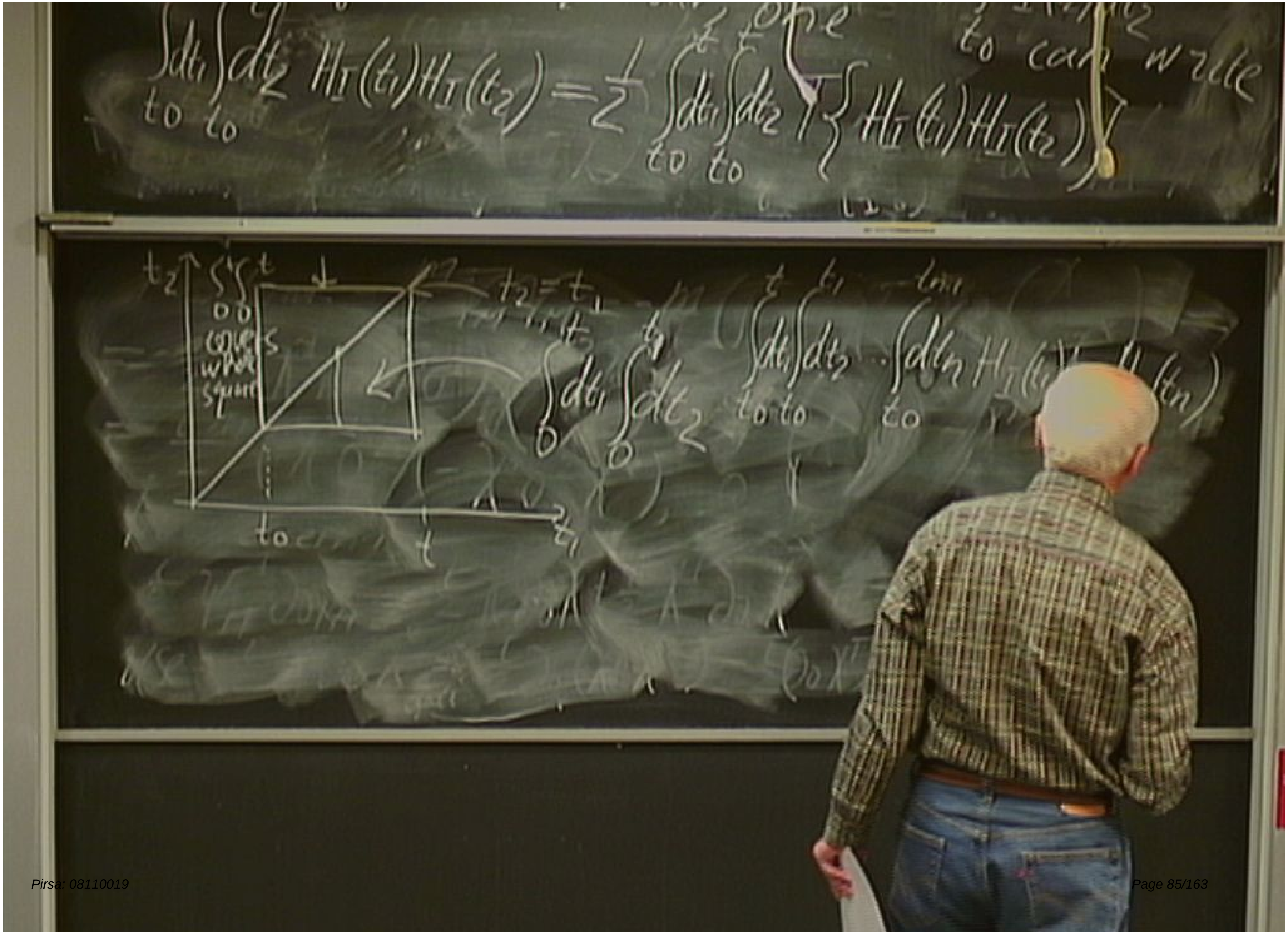


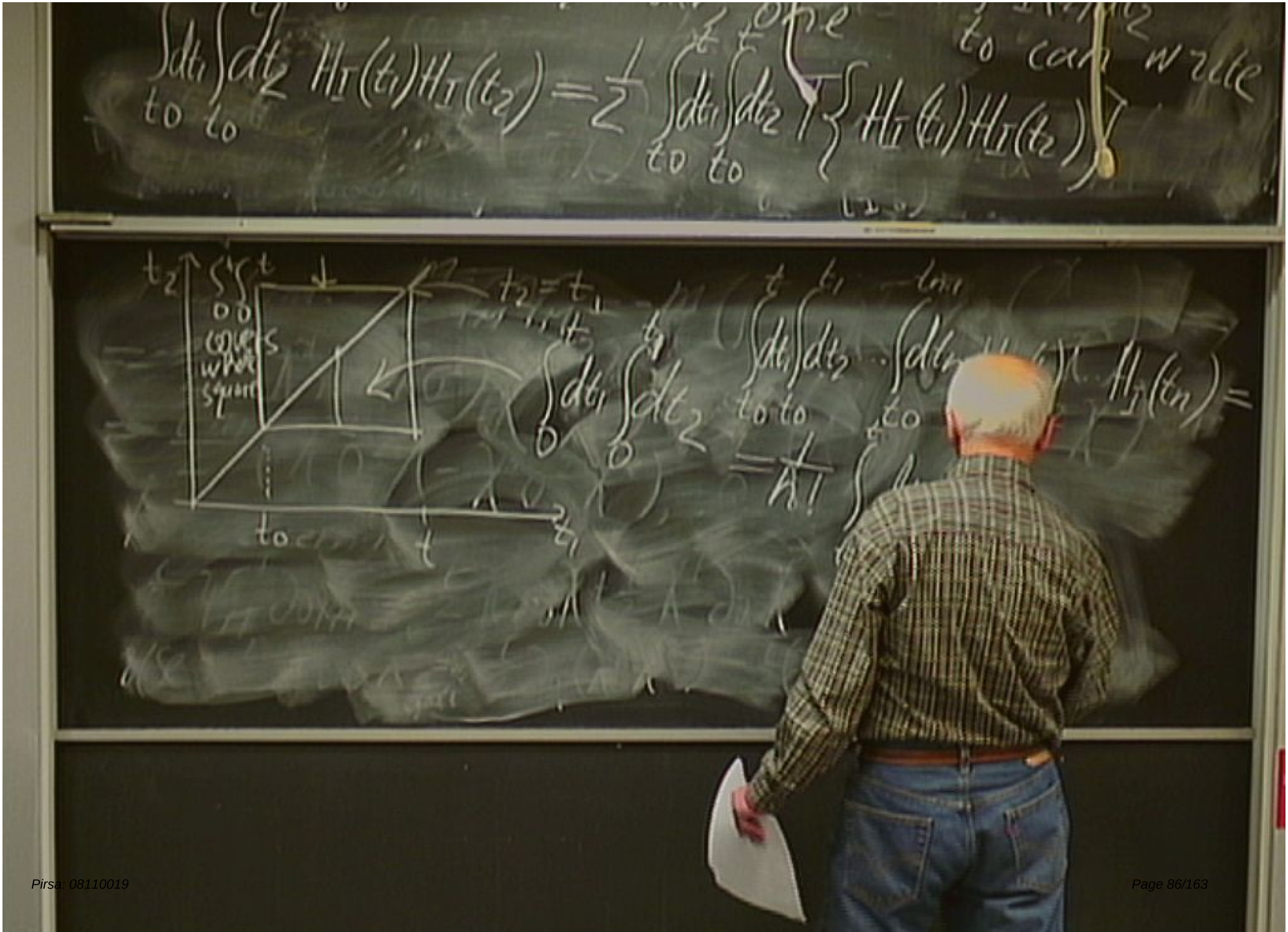
one
t_0 can write
$\int_{t_0} dt_1 \int_{t_0} dt_2 \, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0} dt_1 \int_{t_0} dt_2 \, T\{ H_I(t_1) H_I(t_2) \}$
t_2
covers whole square
$t_2 = t_1$
t_0
t


one
can write
covers whole square
$t_2 = t_1$
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1) H_I(t_2)\}$
$\int dt_1 \int dt_2 \cdots \int dt_n$

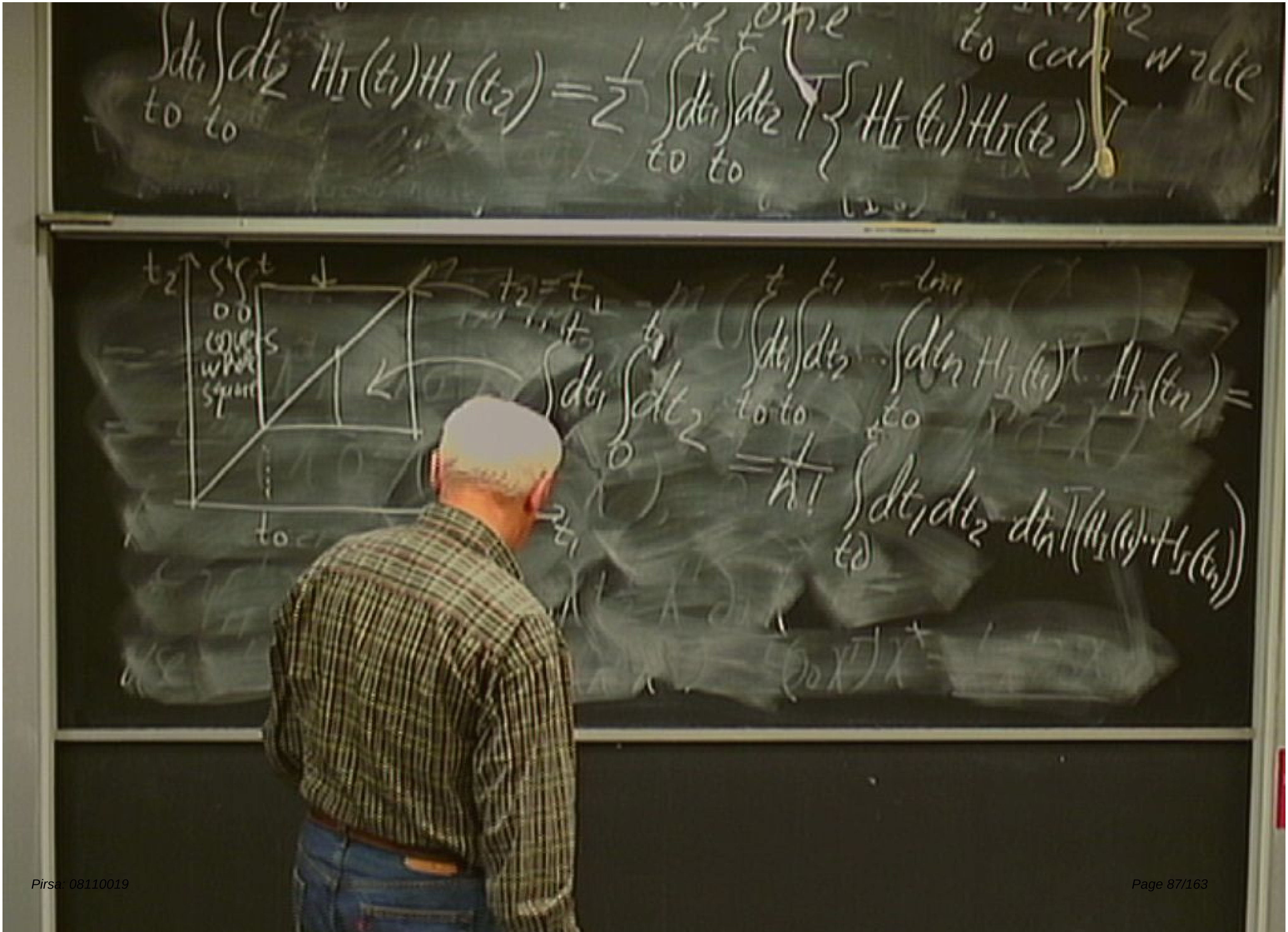

t0 can write
covers whole square
t2 = t1

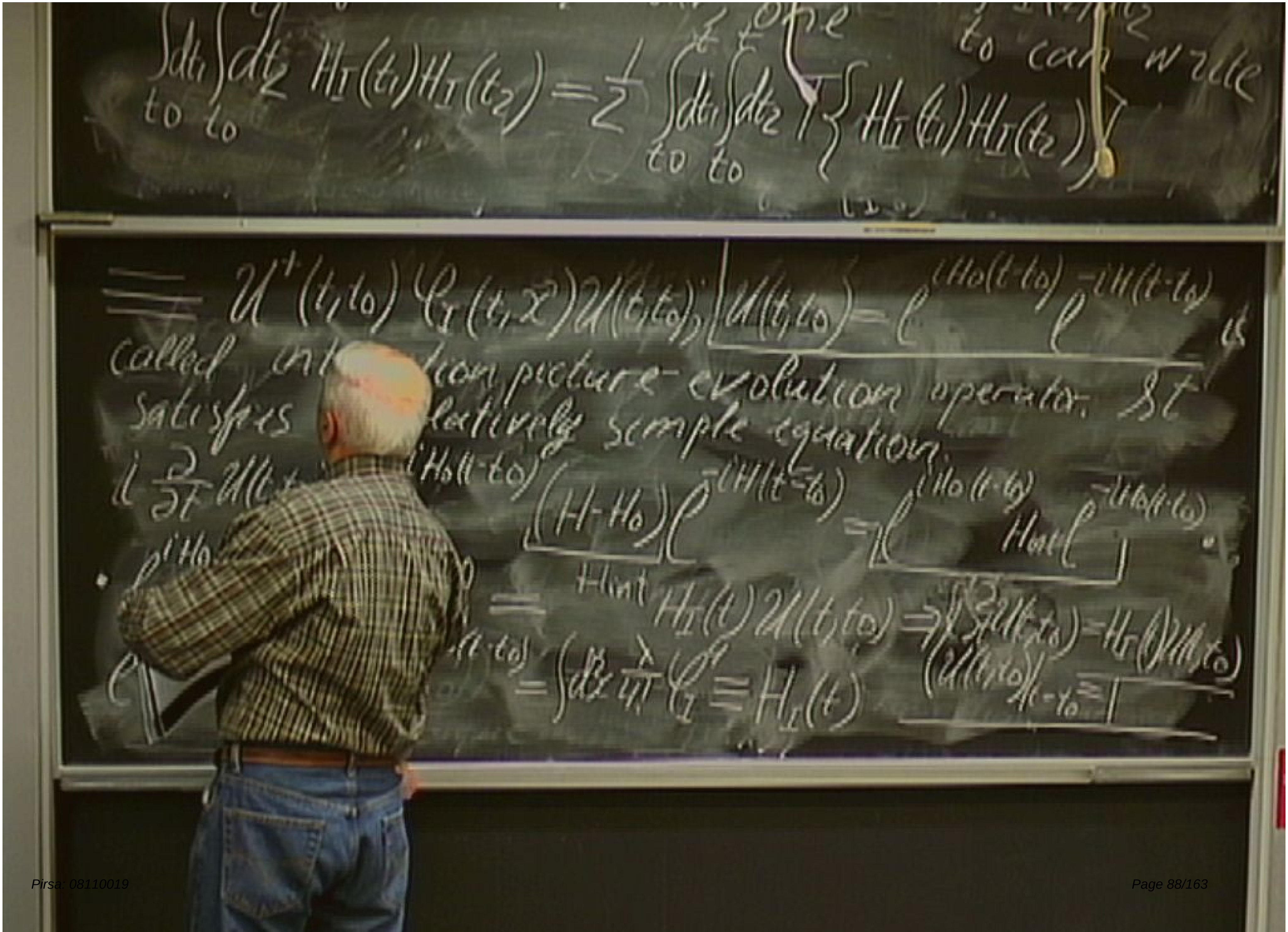

$\frac{\partial}{\partial t}U = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^{t} H_I(t_2)\,dt_2$
Because of $t > t_1 > t_2$ order, one can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1)H_I(t_2)\}$

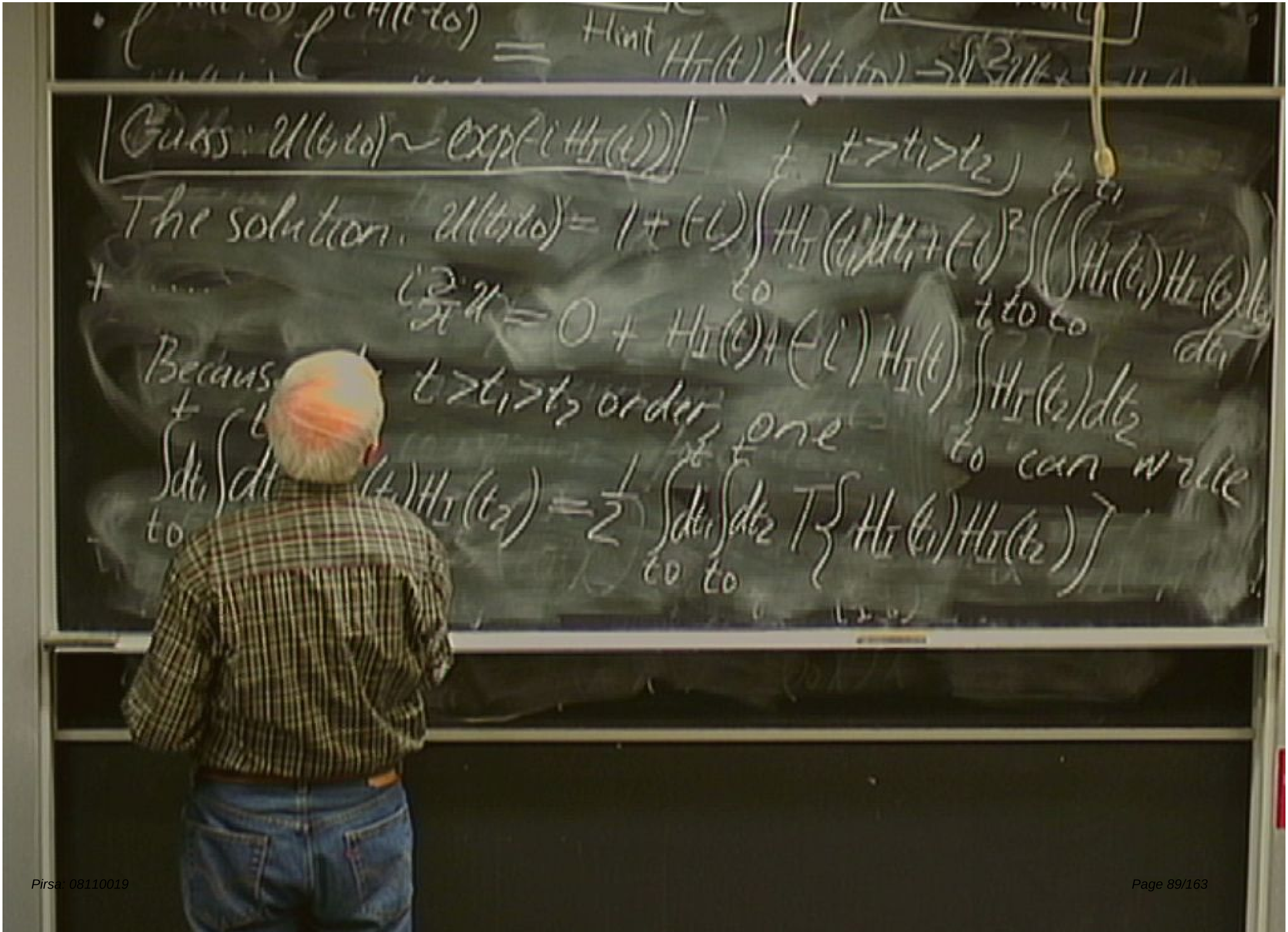

one
to can write
t2 = t1
covers whole square
to
t
t1

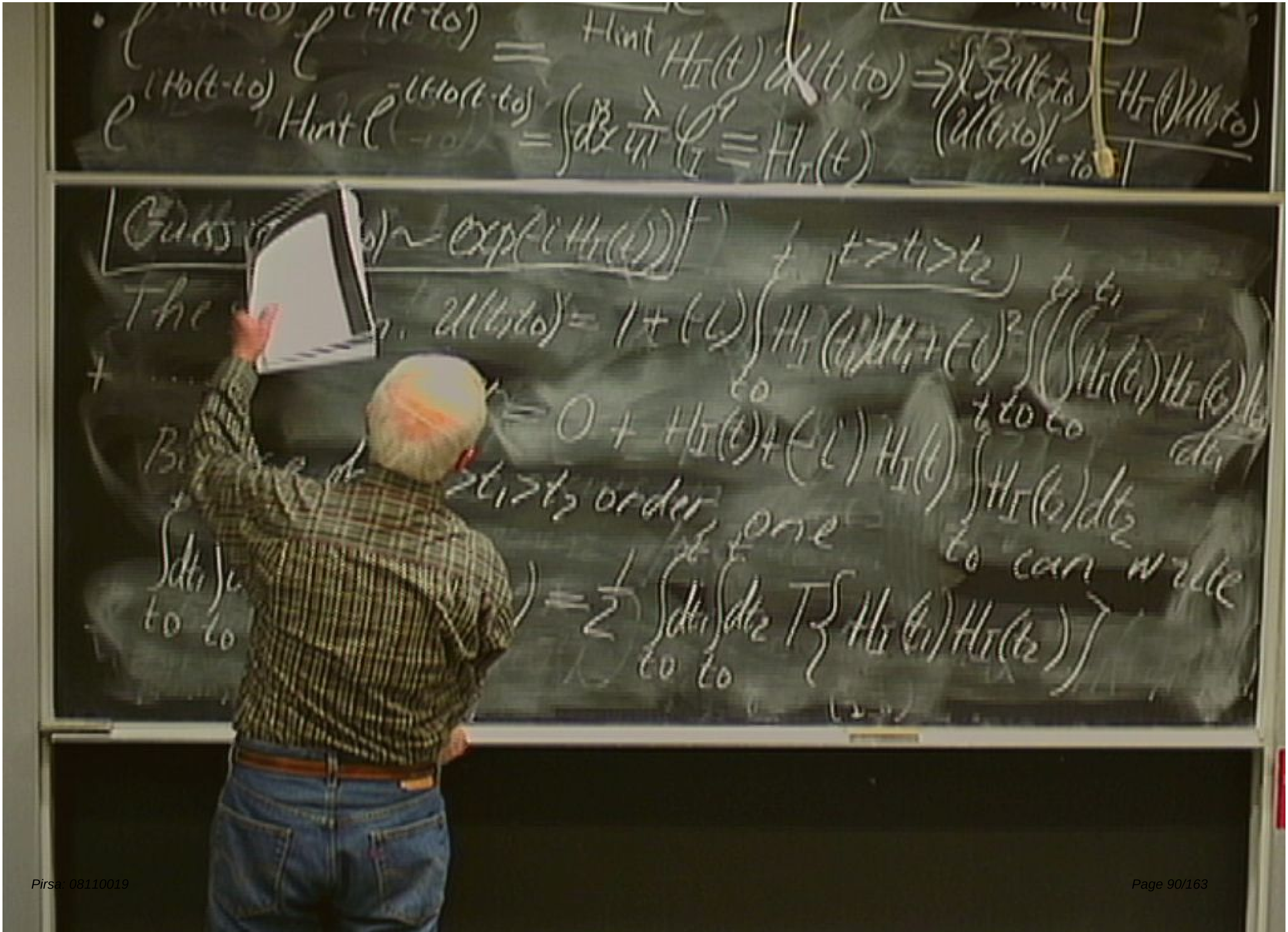


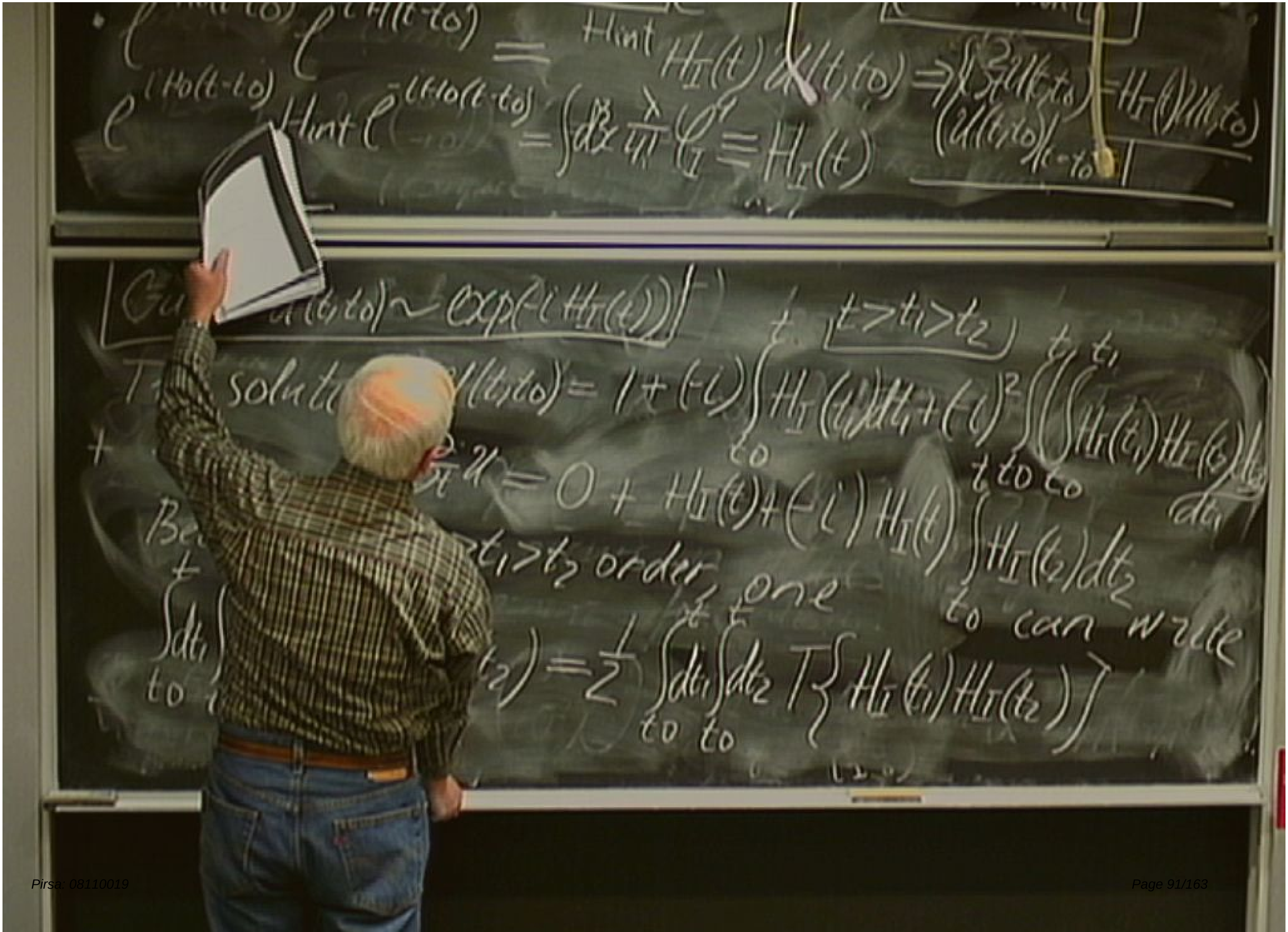
one
t_0 can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \, T\{H_I(t_1) H_I(t_2)\}$
covers whole square
$t_2 = t_1$
$= \frac{1}{n!}$

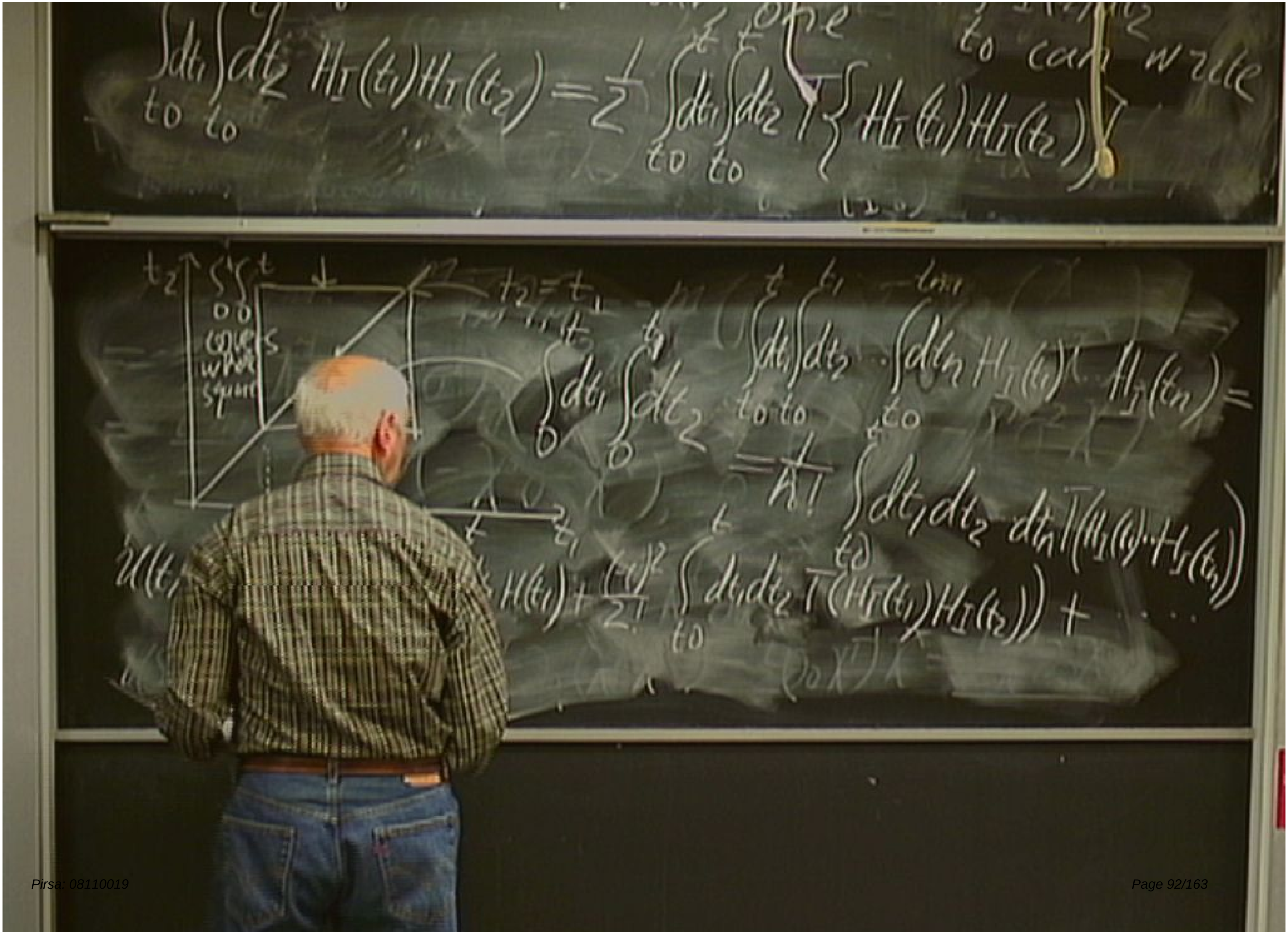

one
can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1) H_I(t_2)\}$
$t_2 = t_1$
covers whole square
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1) \cdots H_I(t_n) =$
$= \frac{1}{n!} \int_{t_0}^{t} dt_1\, dt_2 \cdots dt_n\, T(H_I(t_1) \cdots H_I(t_n))$

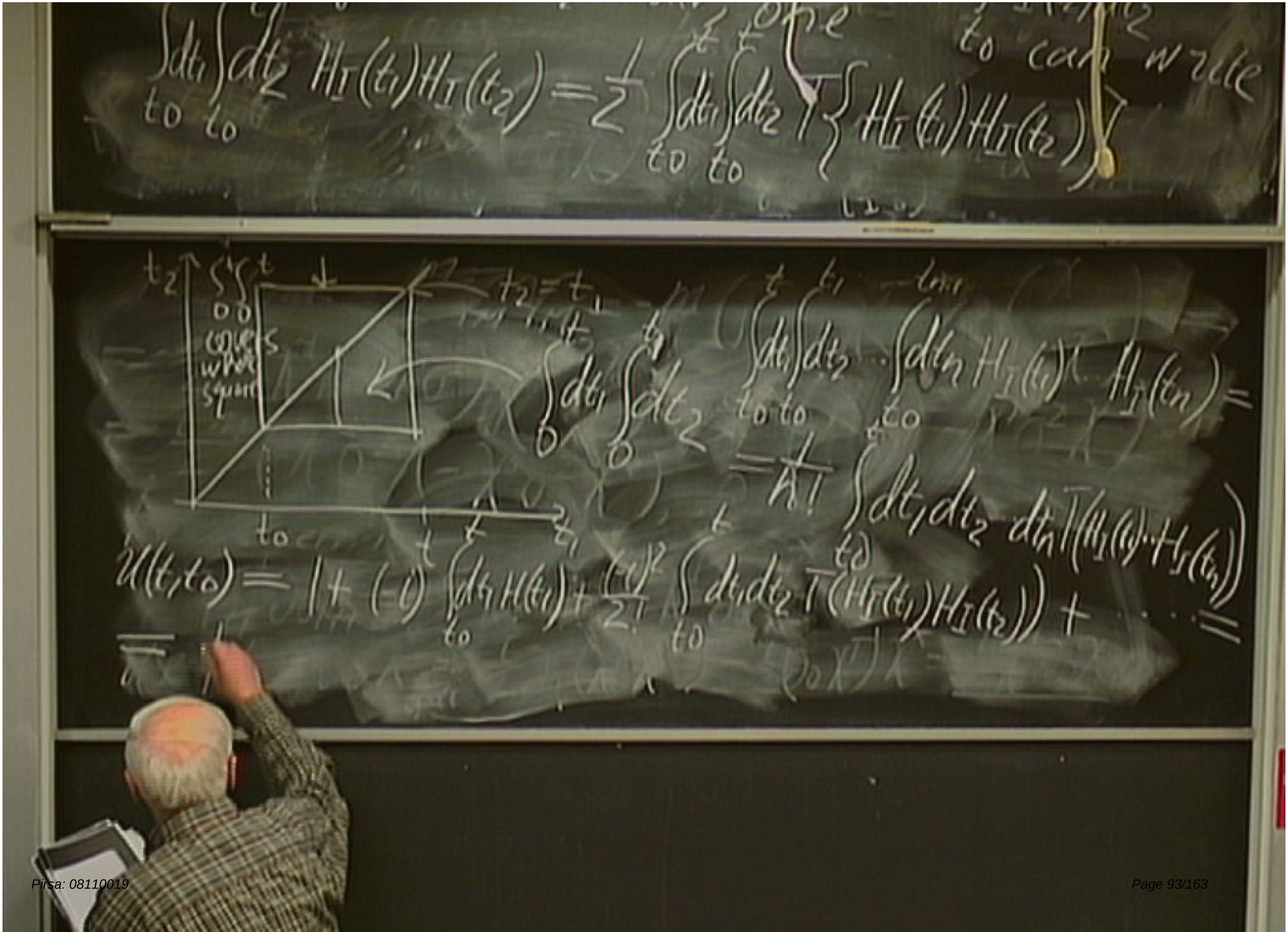

t0 can write
called
picture evolution operator. It
satisfies
latively simple equation:
Hint
$H_I(t)\,U(t,t_0)$
$= H_I(t)$

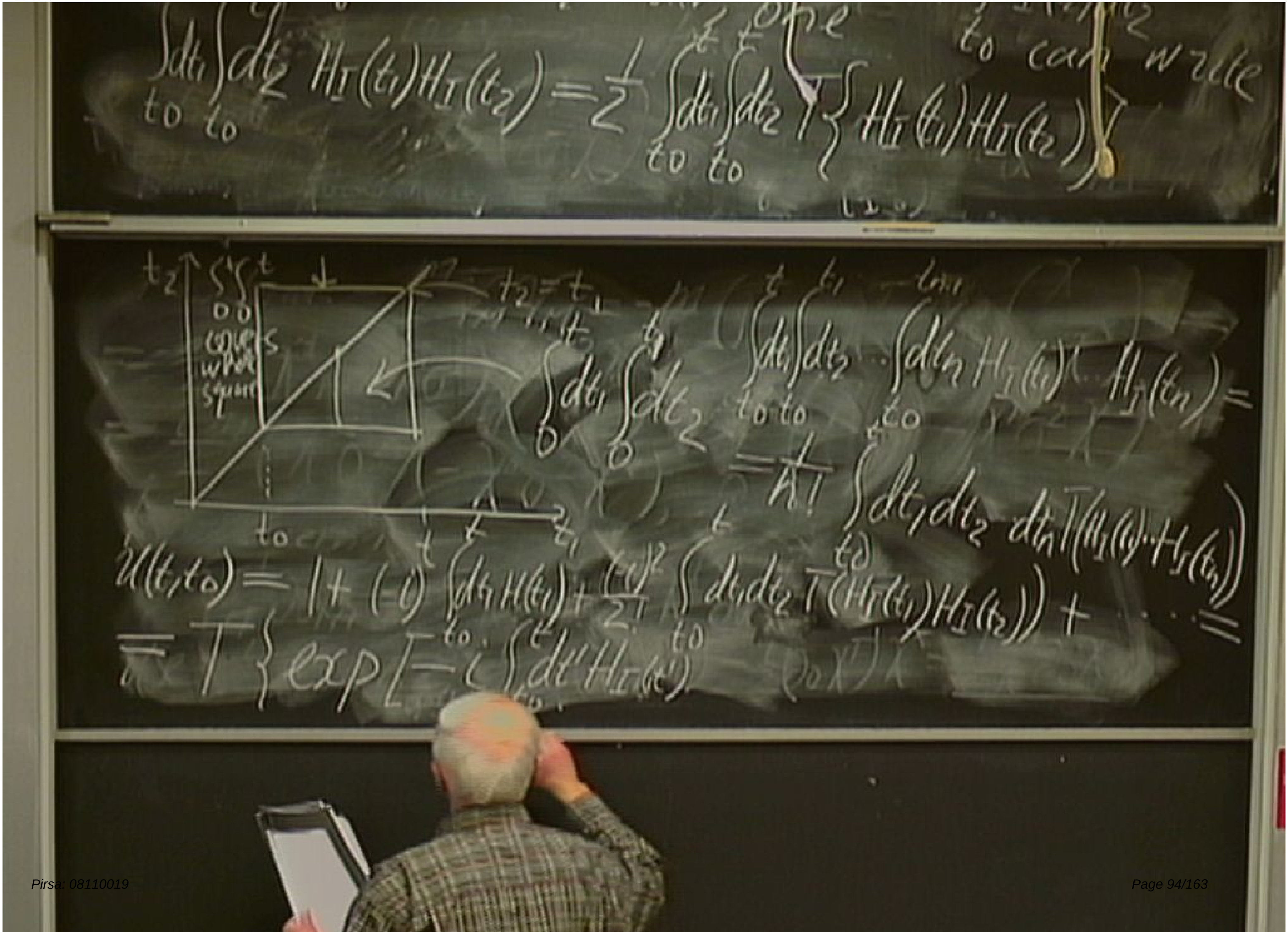

Hint
Guess: U(t,t0) ~ exp(-i HI(t))
t > t1 > t2
The solution: U(t,t0) = 1 + (-i) ∫ HI(t1) dt1 + (-i)² ∫ ∫ HI(t1) HI(t2) dt2 dt1
+ .....
i ∂/∂t U = 0 + HI(t) + (-i) HI(t) ∫ HI(t2) dt2
Because t > t1 > t2 order, one can write
∫ dt1 ∫ dt2 HI(t1) HI(t2) = 1/2 ∫ dt1 ∫ dt2 T{HI(t1) HI(t2)}


Hint
t>t1>t2
order
one
can write

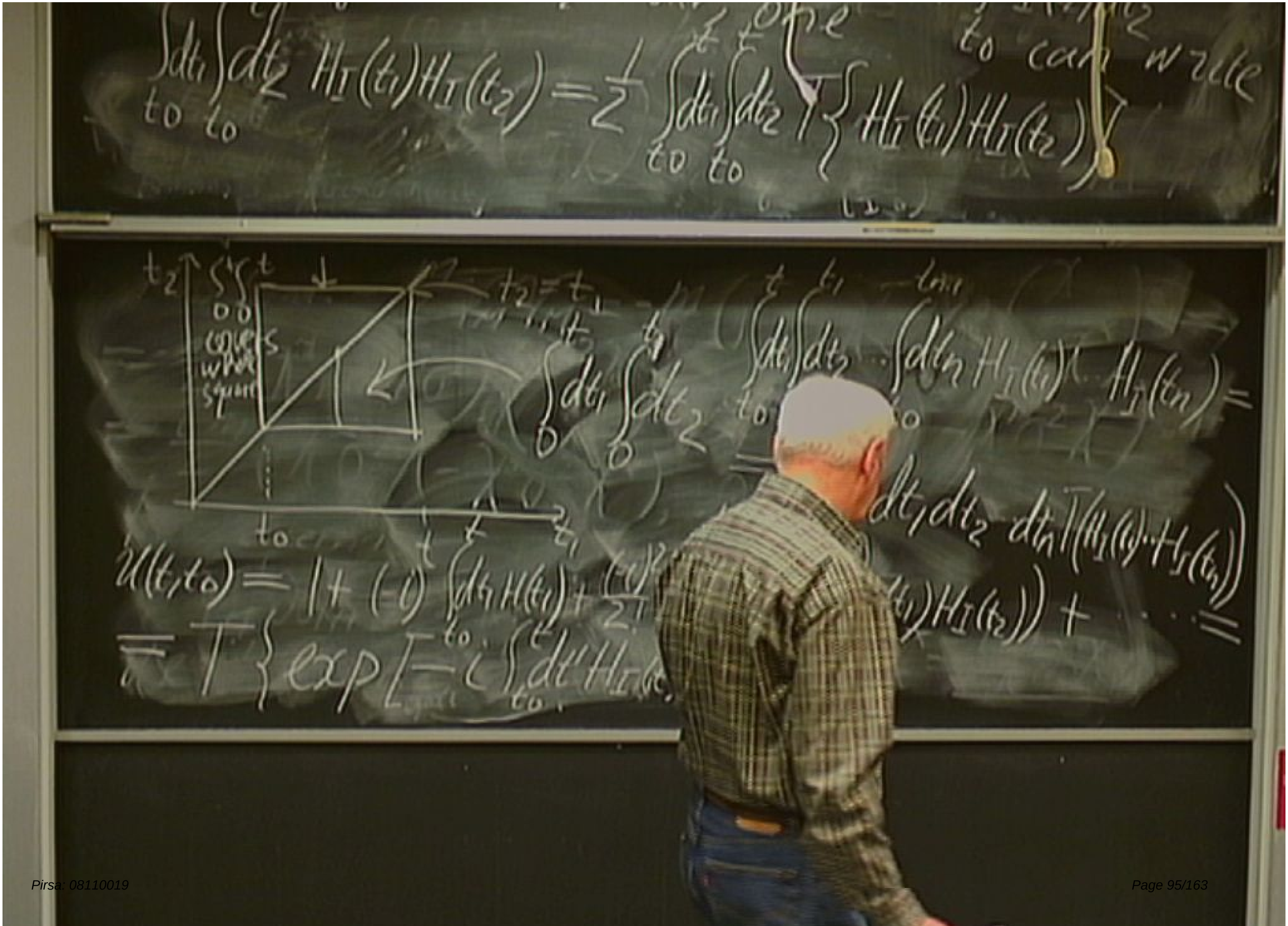

Hint
t > t1 > t2
t1 > t2 order, one
can write


one
can write


one
t_0 can write
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1)H_I(t_2)\}$
t_2
t_0
t
t_1
$t_2 = t_1$
covers whole square
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1)\cdots H_I(t_n) = \frac{1}{n!}\int_{t_0}^{t} dt_1 dt_2 \cdots dt_n\, T(H_I(t_1)\cdots H_I(t_n))$
$U(t,t_0) = 1 + (-i)\int_{t_0}^{t} dt_1 H(t_1) + \frac{(-i)^2}{2!}\int_{t_0}^{t} dt_1 dt_2\, T(H_I(t_1)H_I(t_2)) + \cdots =$



$\int_{t_0} dt_1 \int_{t_0} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1) H_I(t_2)\}$
one
t_0 can write
$t_2 = t_1$
$\int_0^t \int_0^t$ covers whole square
t_2
t_0
t
t_1
$\int_0^{t} dt_1 \int_0^{t_1} dt_2$
$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1) \cdots H_I(t_n) = \frac{1}{n!} \int_{t_0}^{t} dt_1 dt_2 \cdots dt_n\, T(H_I(t_1) \cdots H_I(t_n))$
$U(t,t_0) = 1 + (-i) \int_{t_0}^{t} dt_1 H(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^{t} dt_1 dt_2\, T(H_I(t_1) H_I(t_2)) + \cdots =$
$= T\{\exp[-i \int_{t_0}^{t} dt' H_I(t')$


one
can write
covers whole square

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \, T\{H_I(t_1) H_I(t_2)\}$$

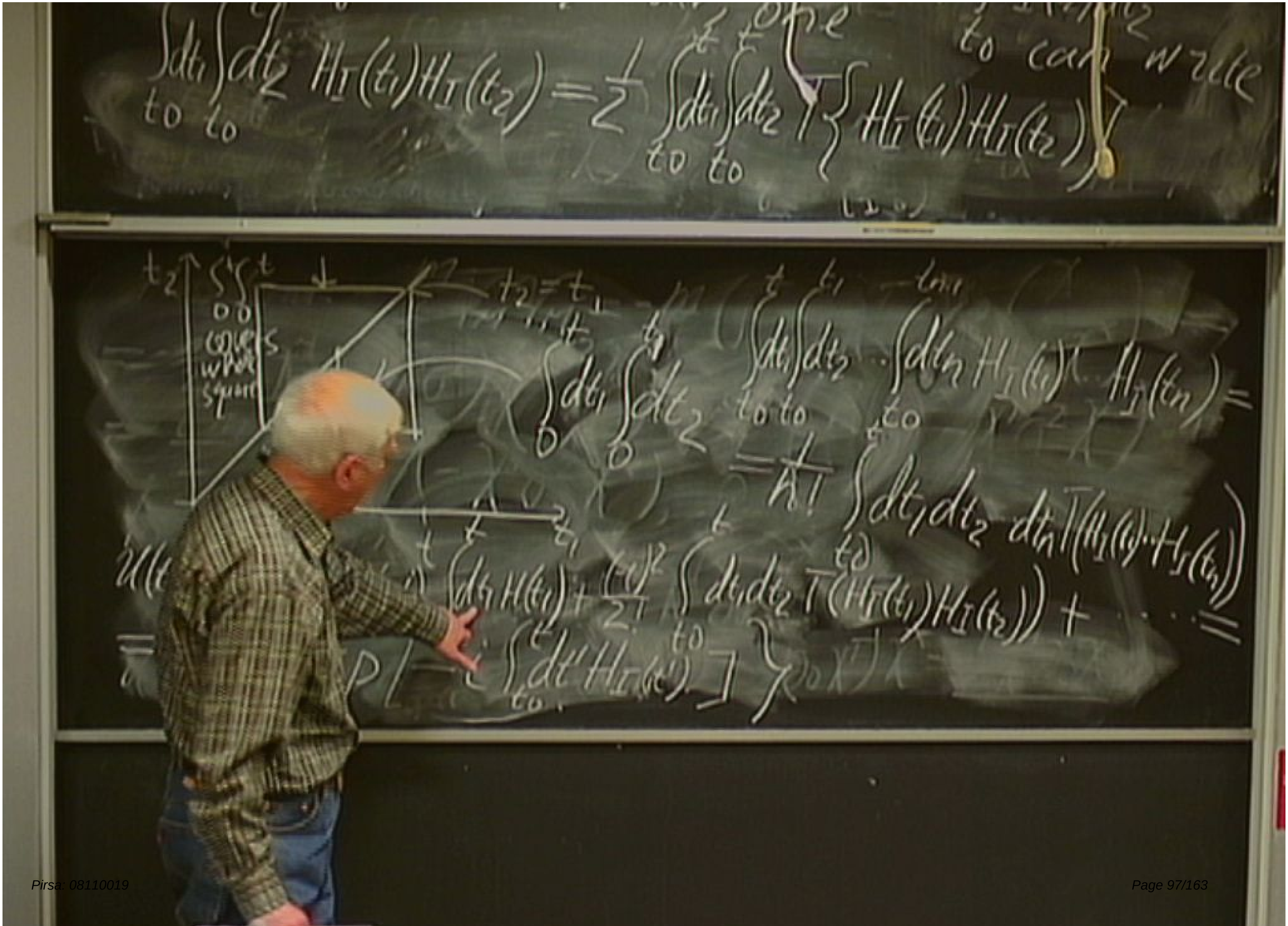
one ... t_0 can write

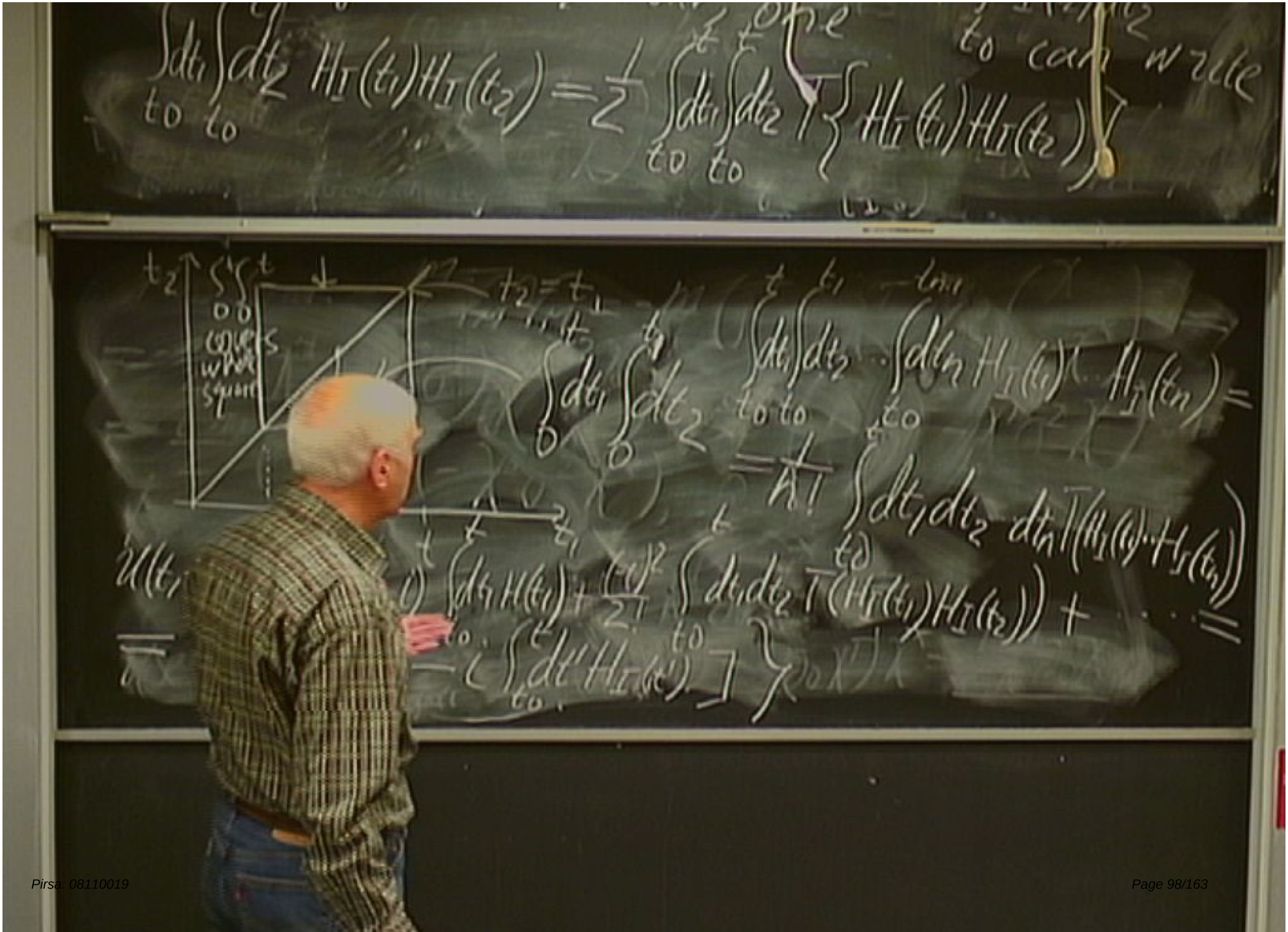
$$\int_0^{t} dt_1 \int_0^{t_1} dt_2$$

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n \, H_I(t_1) \cdots H_I(t_n) = \frac{1}{n!} \int_{t_0}^{t} dt_1 dt_2 \cdots dt_n \, T(H_I(t_1) \cdots$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^{t} dt_1 H(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^{t} dt_1 dt_2 \, T(H_I(t_1) H_I(t_2)) +$$

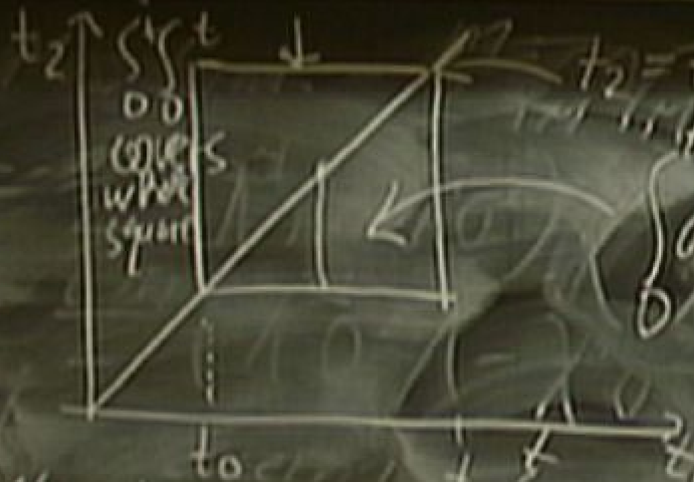
$$= T\left\{ \exp\left[-i \int_{t_0}^{t} dt' H_I(t') \right] \right\}$$


one
can write
covers whole square


one
can write
covers whole square
$t_2 = t_1$

one t_0 can write

$$\int_{t_0} dt_1 \int_{t_0} dt_2\, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1) H_I(t_2)\}$$



$$\int_0^{t} dt_1 \int_0^{t_1} dt_2$$

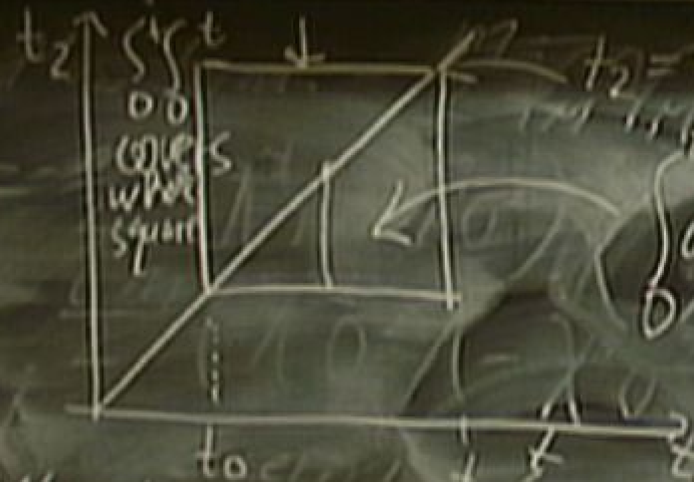
$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1) \cdots H_I(t_n) = \frac{1}{n!} \int_{t_0}^{t} dt_1 dt_2 \cdots dt_n\, T(H_I(t_1) \cdots H_I(t_n))$$

$$U(t,t_0) = 1 + (-i) \int_{t_0}^{t} dt_1 H(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^{t} dt_1 dt_2\, T(H_I(t_1) H_I(t_2)) + \cdots$$

$$= T\left\{ \exp\left[-i \int_{t_0}^{t} dt' H_I(t') \right] \right\}$$

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1)H_I(t_2)\}$$

one can write

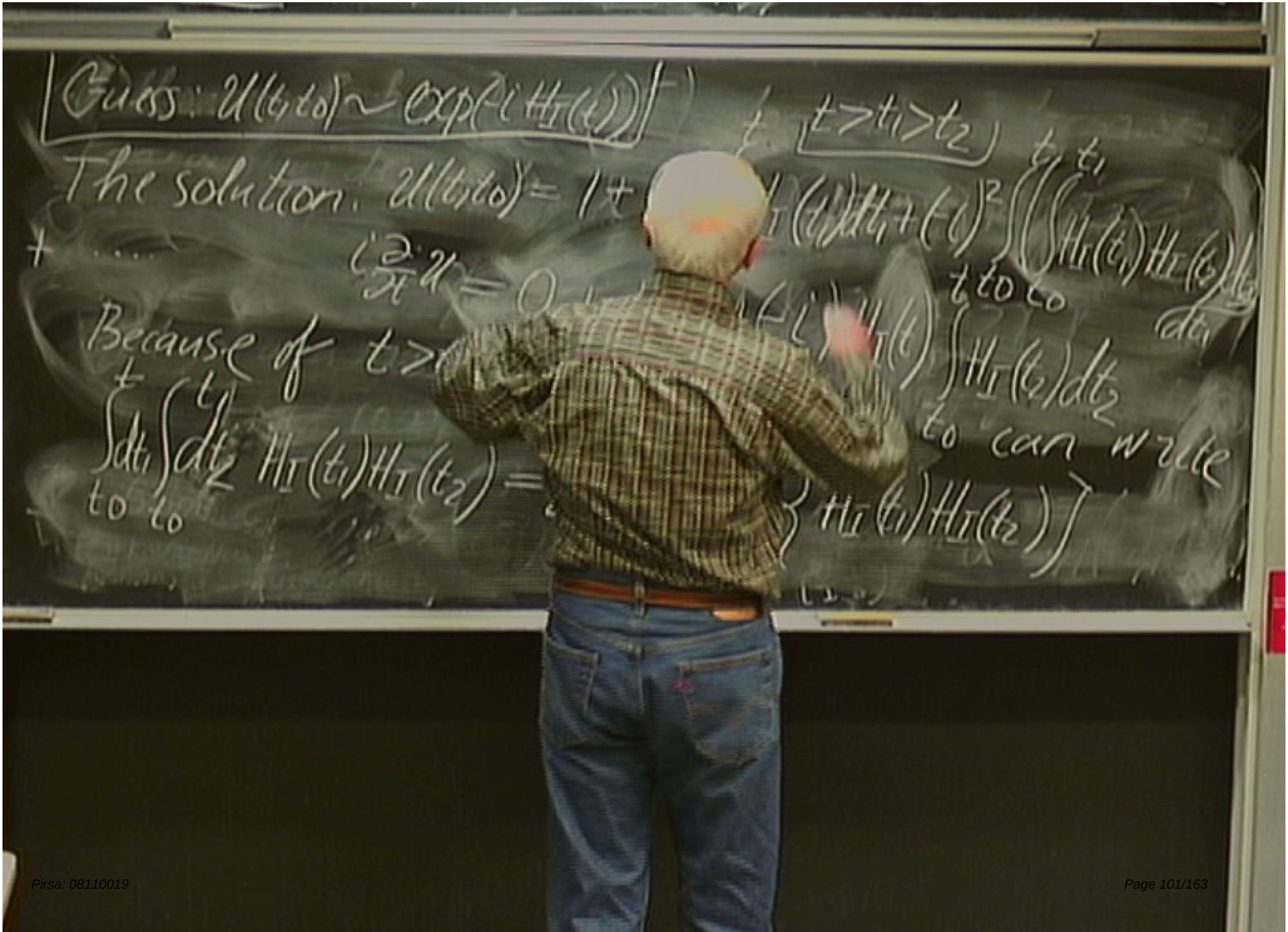


$$\int_0^t dt_1 \int_0^t dt_2$$

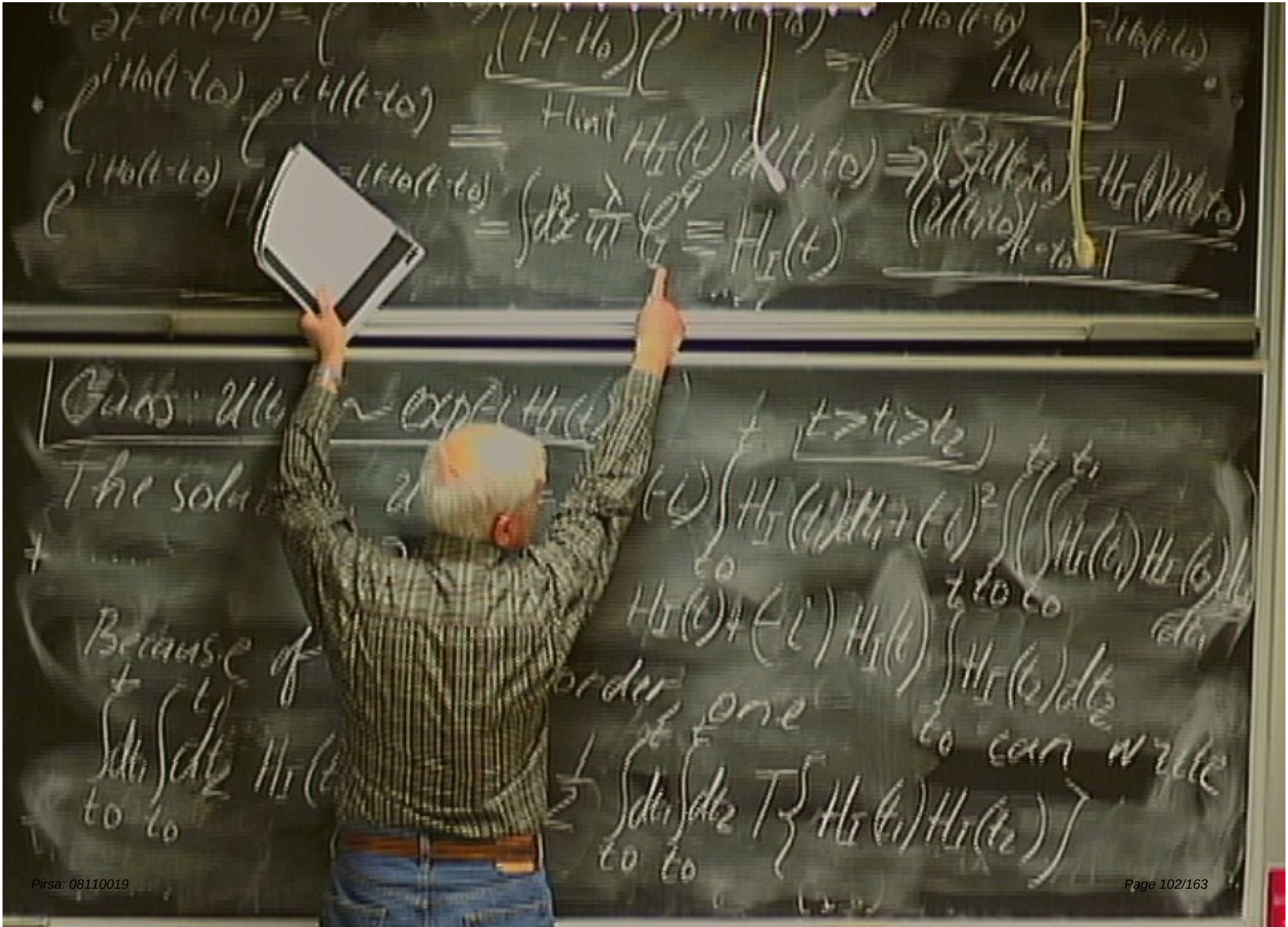
$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1)\cdots H_I(t_n) = \frac{1}{n!}\int_{t_0}^{t} dt_1\, dt_2 \cdots dt_n\, T(H_I(t_1)\cdots H_I(t_n))$$

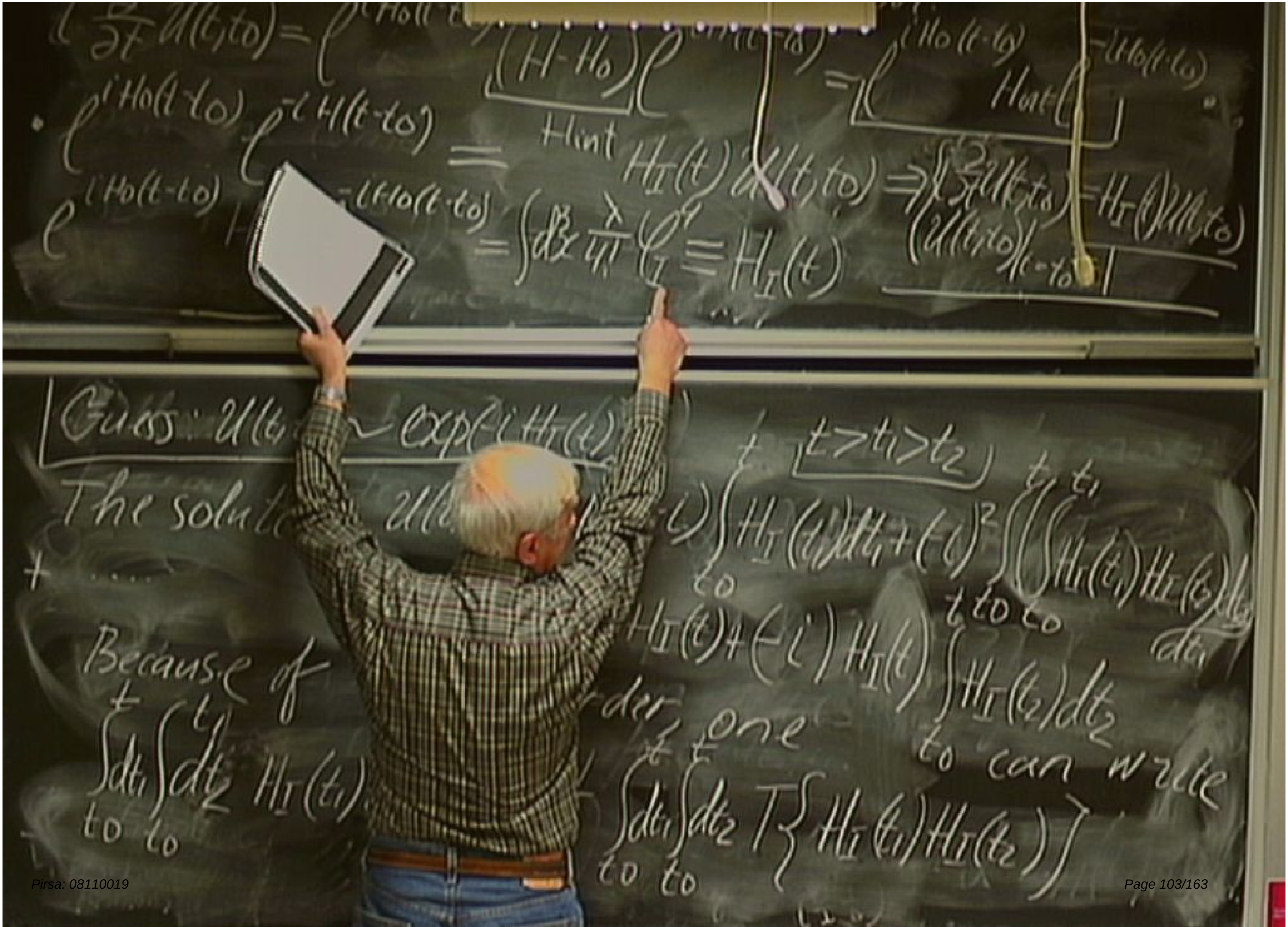
$$U(t,t_0) = 1 + (-i)\int_{t_0}^{t} dt_1 H(t_1) + \frac{(-i)^2}{2!}\int_{t_0}^{t} dt_1\, dt_2\, T(H_I(t_1)H_I(t_2)) + \cdots =$$

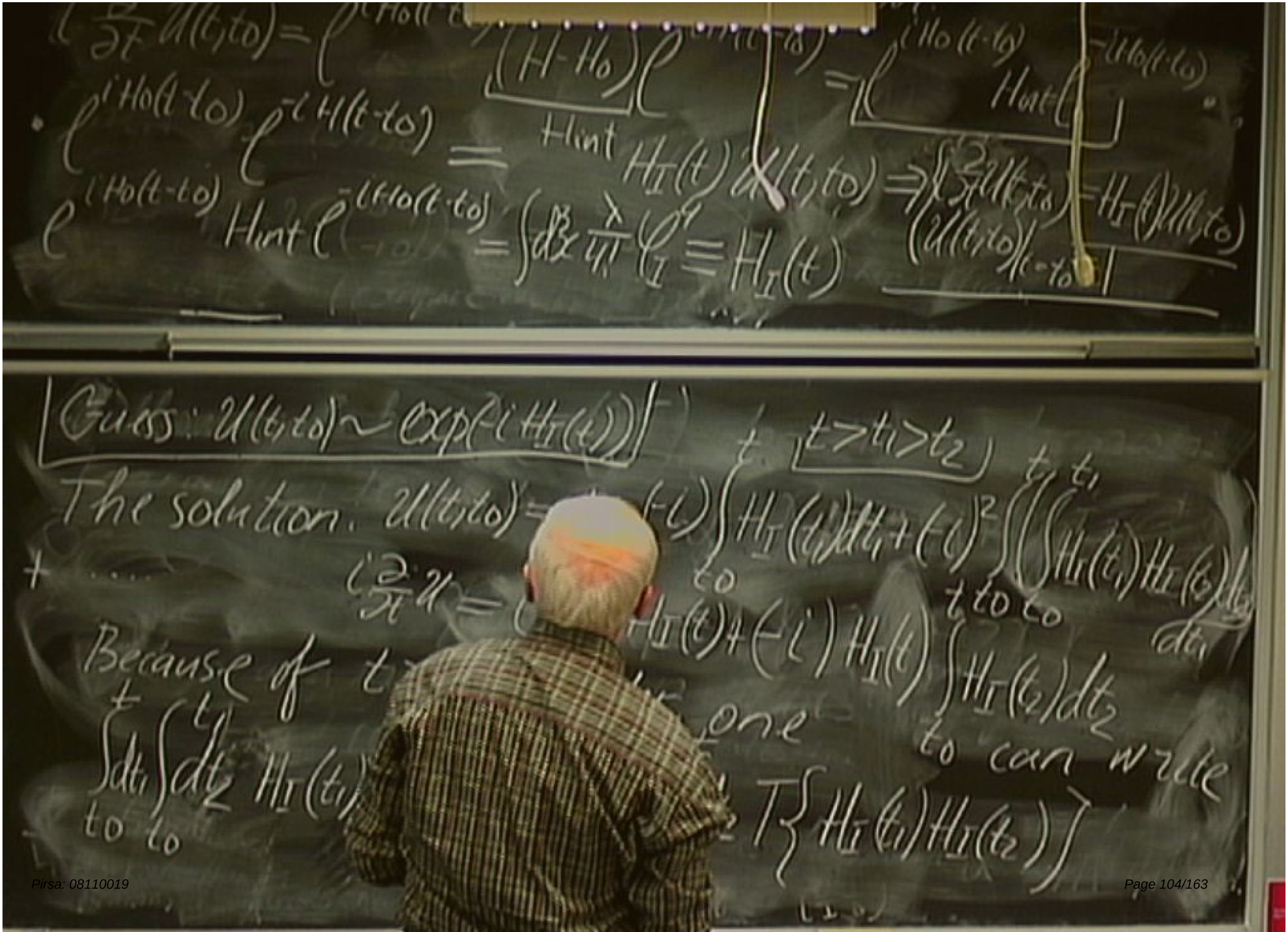
$$= T\left\{\exp\left[-i\int_{t_0}^{t} dt'\, H_I(t')\right]\right\}$$



Guess: U(t,t0) ~ exp(-i HI(t))
t > t1 > t2
The solution: U(t,t0) = 1 +
t to t0
Because of t >
to t0
t0 can write


Because of
order
one
can write





Hint
Guess: U(t,t0) ~ exp(-i HI(t))
t > t1 > t2
The solution. U(t,t0) =
Because of
one
t0 can write

$$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}\underbrace{(H-H_0)}_{H_{int}}e^{-iH(t-t_0)} = \underbrace{e^{iH_0(t-t_0)}H_{int}e^{-iH_0(t-t_0)}}$$

$$e^{iH_0(t-t_0)}e^{-iH(t-t_0)} \equiv H_I(t)\,\mathcal{U}(t,t_0) \Rightarrow \boxed{i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = H_I(t)\mathcal{U}(t,t_0)}$$

$$e^{iH_0(t-t_0)}H_{int}e^{-iH_0(t-t_0)} = \int d^3x \frac{\lambda}{4!}\phi_I^4 \equiv H_I(t) \qquad \boxed{(\mathcal{U}(t,t_0))_{t=t_0}}$$

$$\boxed{\text{Guess: } \mathcal{U}(t,t_0) \sim \exp(-iH_I(t))}$$

$$\boxed{t > t_1 > t_2}$$

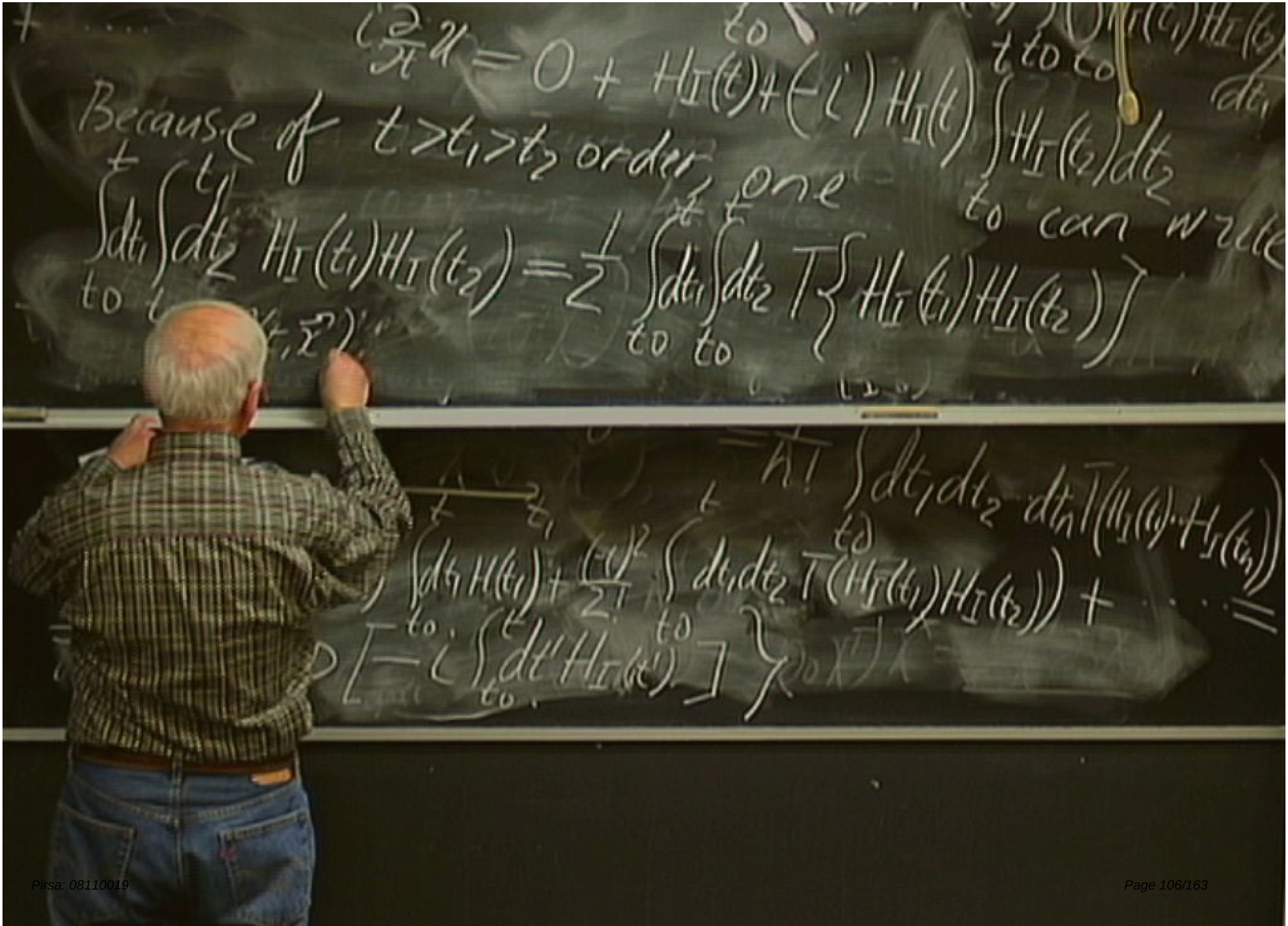
The solution: $\mathcal{U}(t,t_0) = 1 + (-i)\int_{t_0}^{t} H_I(t_1)dt_1 + (-i)^2 \int_{t_0}^{t}\left(\int_{t_0}^{t_1} H_I(t_1)H_I(t_2)dt_2\right)dt_1$

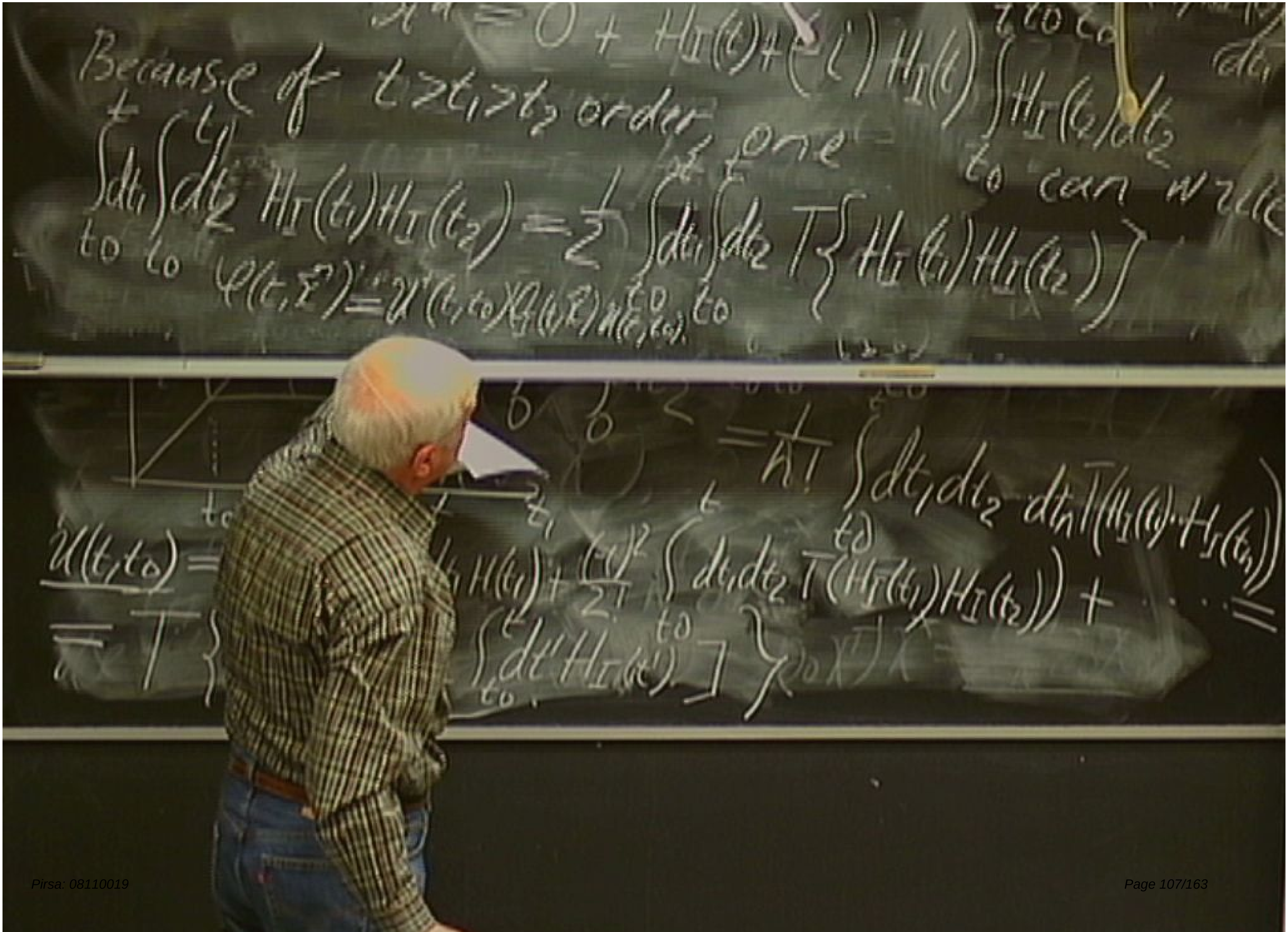
$+ \ldots$

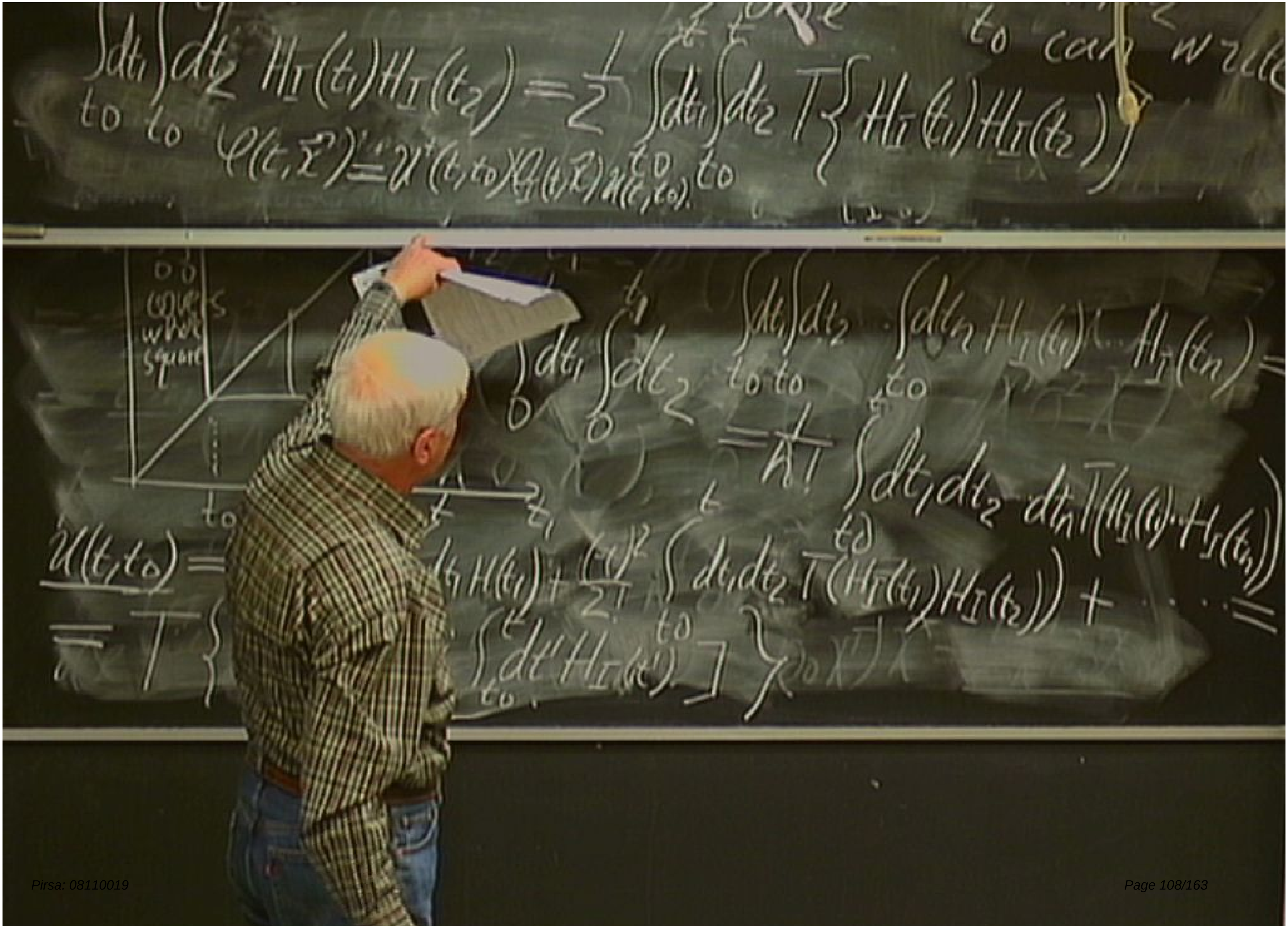
$$i\frac{\partial}{\partial t}\mathcal{U} = 0 + H_I(t) + (-i)\,H_I(t)\int_{t_0}^{t} H_I(t_2)dt_2$$

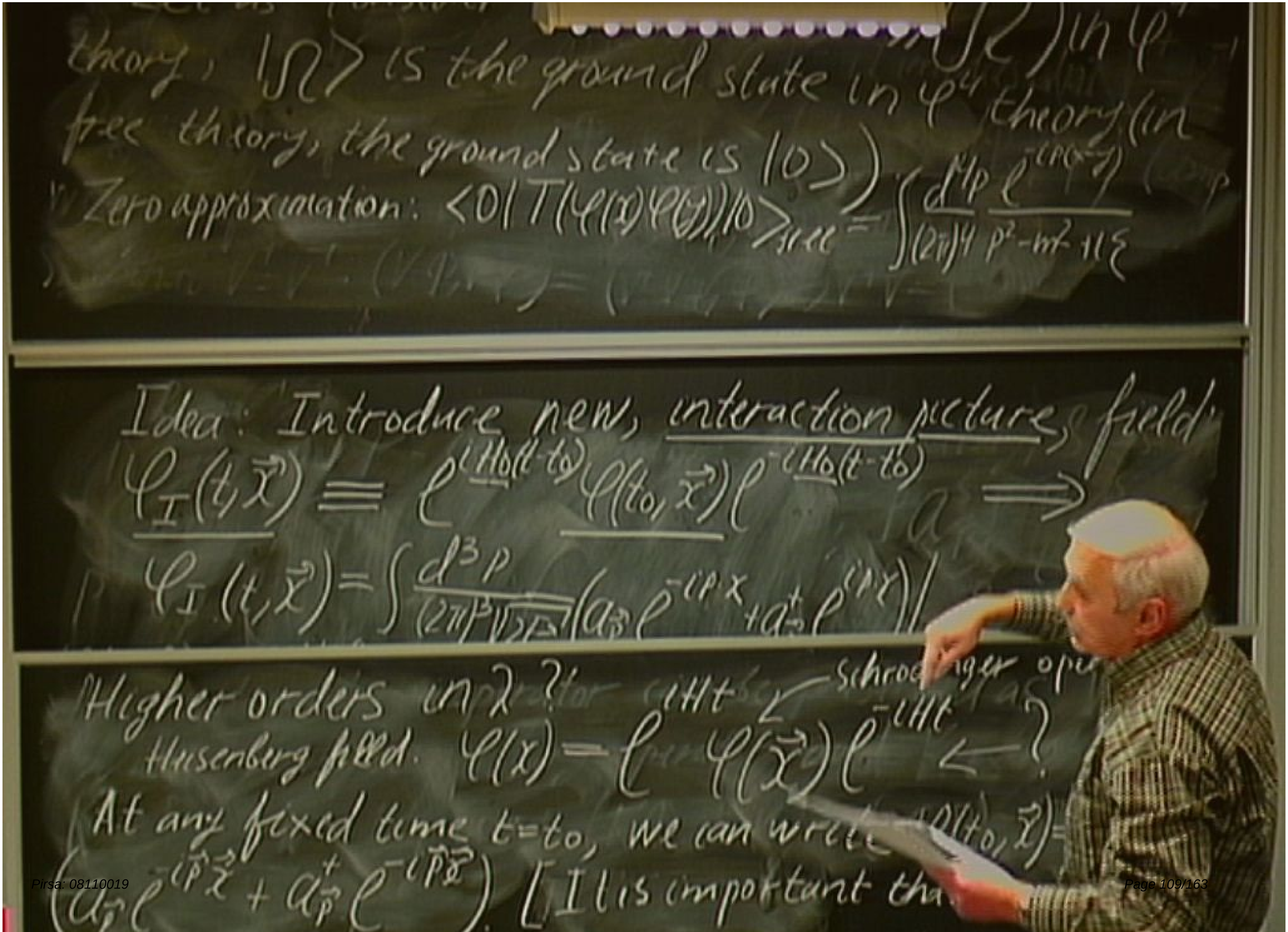
Because of $t > t_1 > t_2$ order, one can write

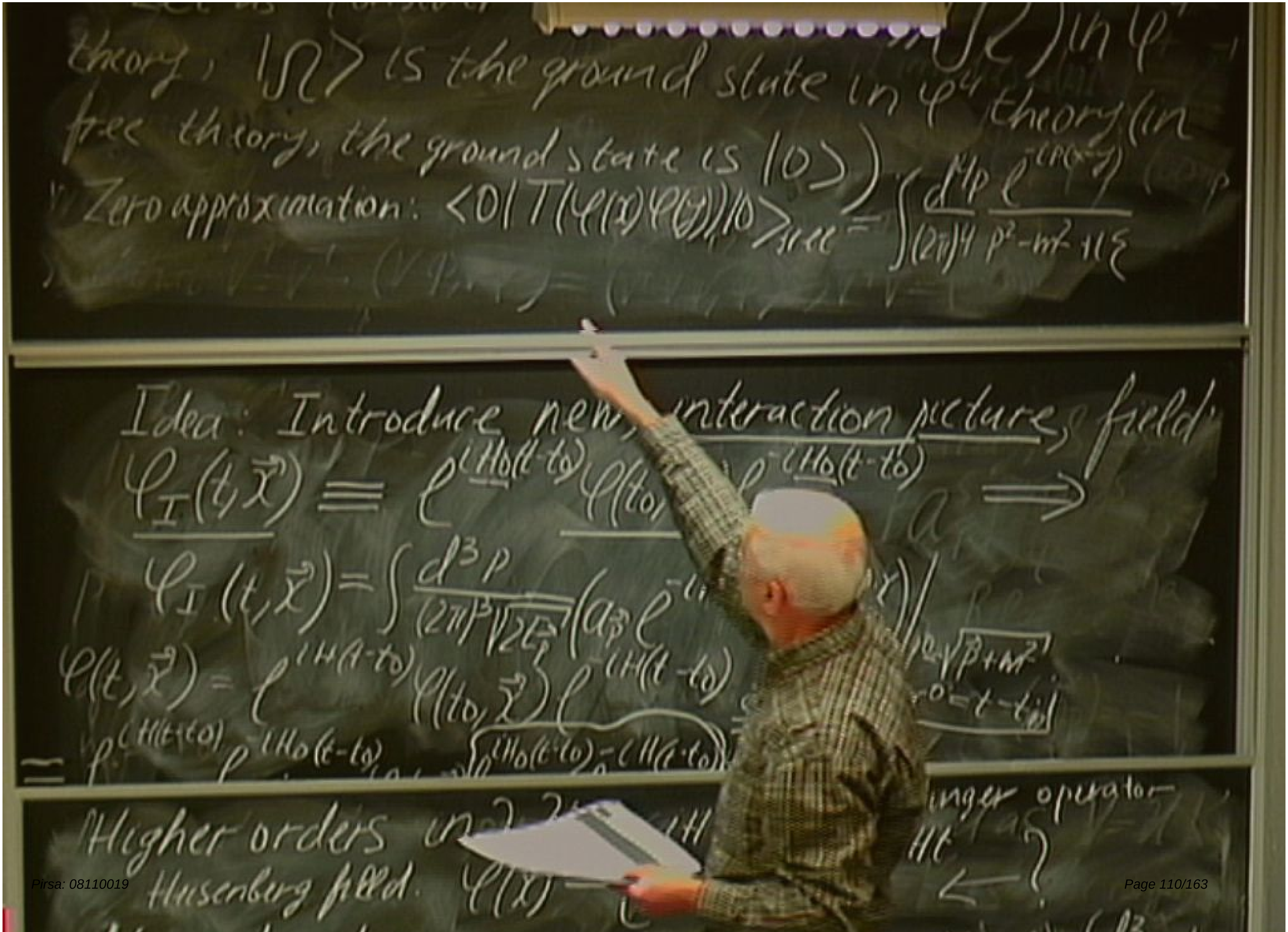
$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, T\{H_I(t_1)H_I(t_2)\}$$


Because of t>t1>t2 order one can write

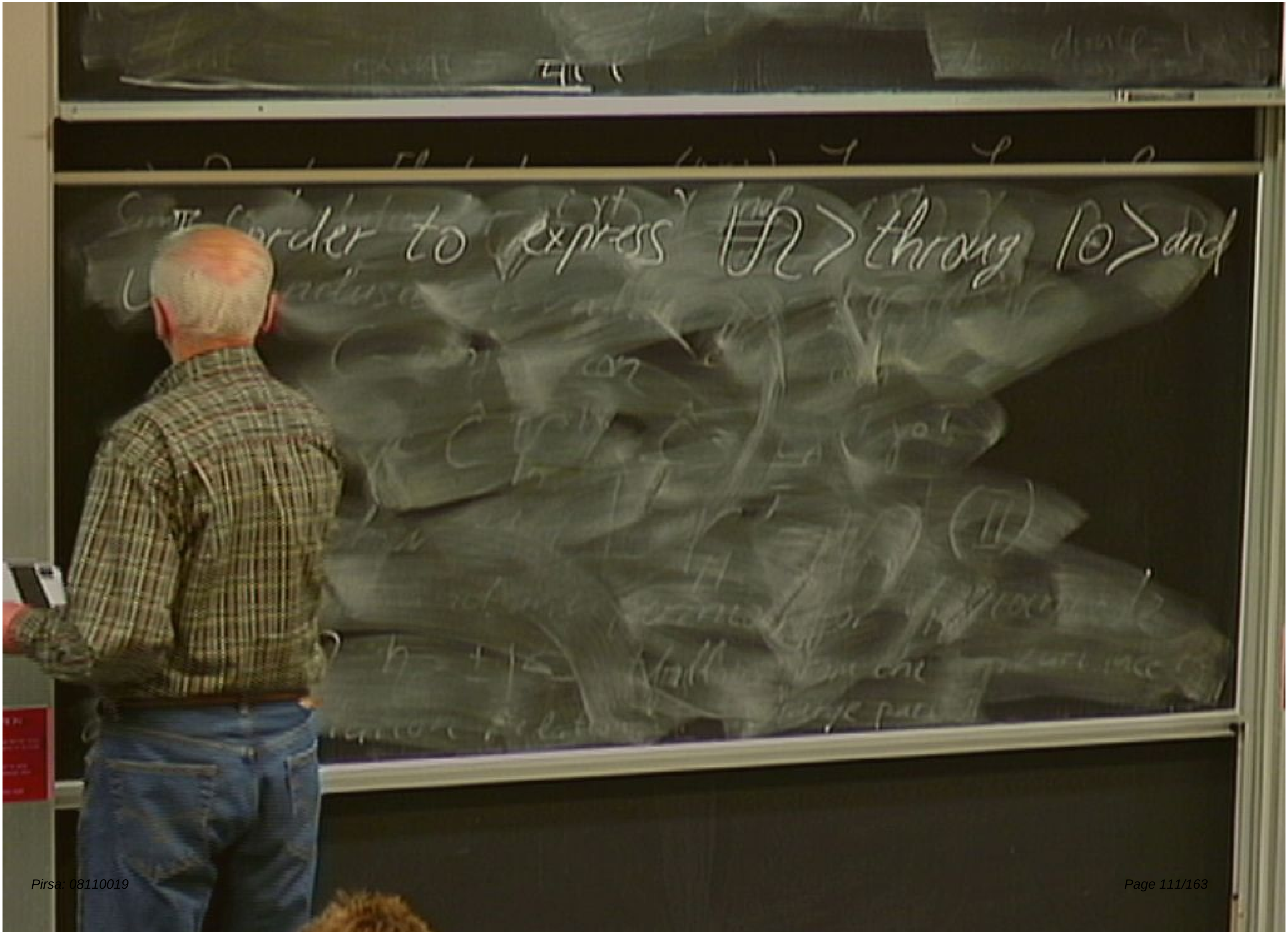

Because of t > t1 > t2 order one can write
∫dt1 ∫dt2 HI(t1)HI(t2) = 1/2 ∫dt1 ∫dt2 T{HI(t1)HI(t2)}
φ(t, x) = U†(t, t0)φ(t0, x)U(t, t0)
= 1/n! ∫dt1 dt2 … dtn T(HI(t1)…HI(tn))
U(t, t0) =
+ (−i)²/2! ∫dt1 dt2 T(HI(t1)HI(t2)) + …
= T{ ∫dt′ HI(t′)] }

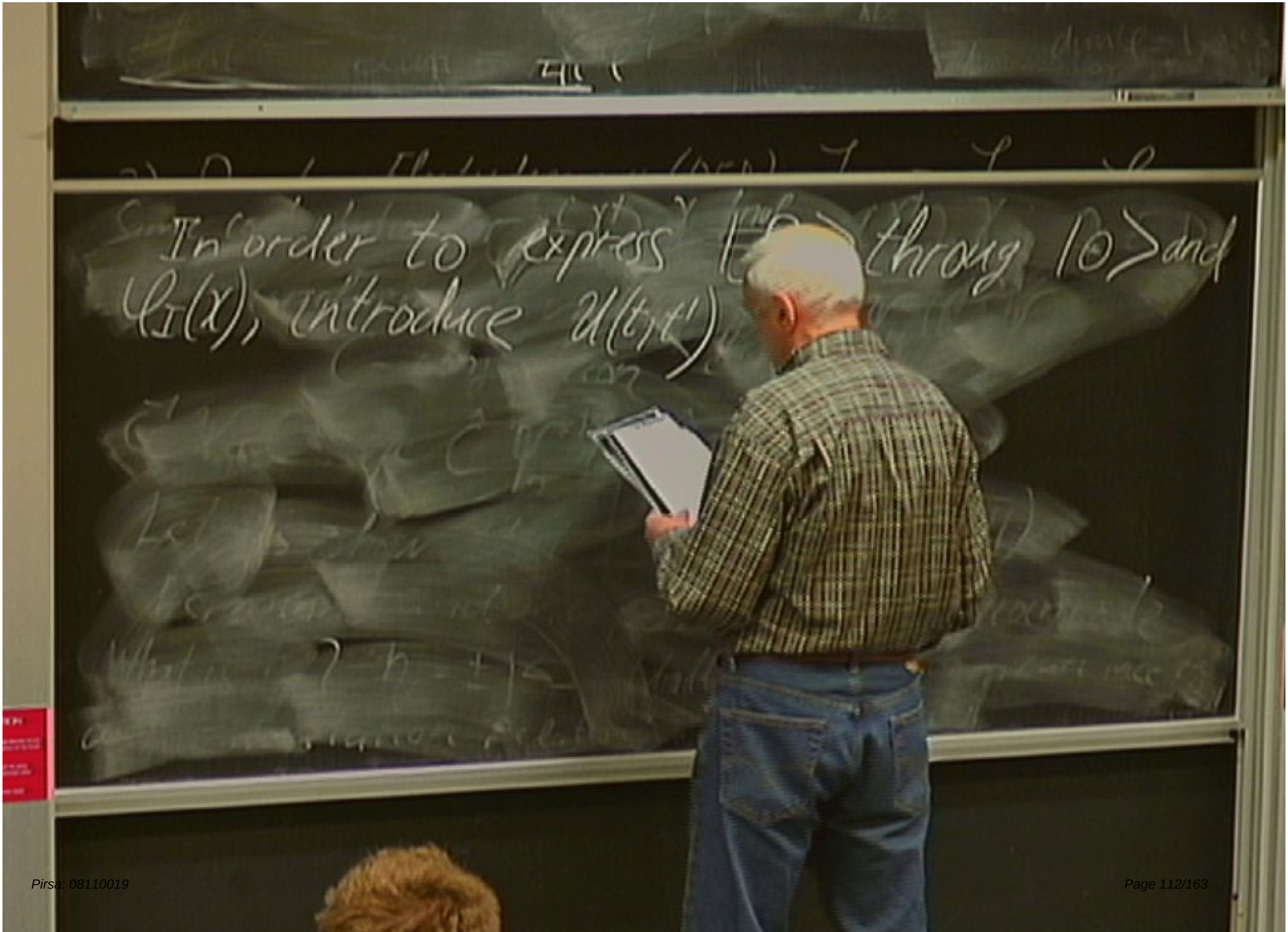

covers whole square


theory; |Ω> is the ground state in φ^4 theory (in
free theory, the ground state is |0>)
Zero approximation: <0|T(φ(x)φ(y))|0>_free = ∫ d^4p/(2π)^4 e^{-ip(x-y)}/(p^2-m^2+iε)
Idea: Introduce new, interaction picture, field:
φ_I(t,x) ≡ e^{iH_0(t-t_0)} φ(t_0,x) e^{-iH_0(t-t_0)} ⟹
φ_I(t,x) = ∫ d^3p/((2π)^3 √(2E_p)) (a_p e^{-ipx} + a_p^† e^{ipx})
Higher orders in λ?
Schroedinger op
Heisenberg field. φ(x) = e^{iHt} φ(x) e^{-iHt}
At any fixed time t=t_0, we can write φ(t_0,x) =
(a_p e^{ipx} + a_p^† e^{-ipx}). [It is important tha

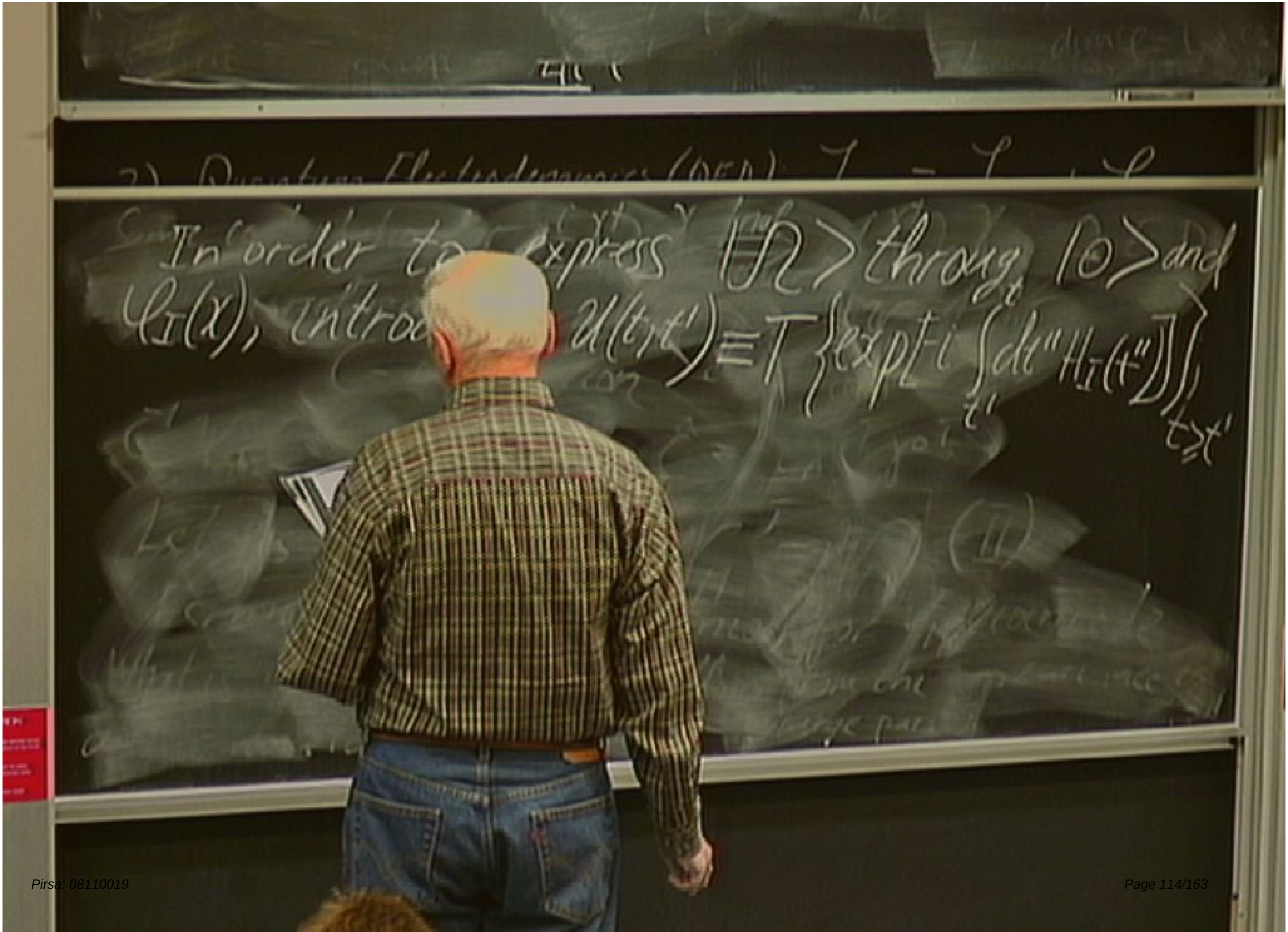


theory; $|\Omega\rangle$ is the ground state in φ^4 theory (in
free theory, the ground state is $|0\rangle$)
Zero approximation: $\langle 0|T(\varphi(x)\varphi(y))|0\rangle_{free} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2-m^2+i\epsilon}$
Idea: Introduce new interaction picture field
$\varphi_I(t,\vec{x}) \equiv e^{iH_0(t-t_0)} \varphi(t_0) e^{-iH_0(t-t_0)}$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} (a_p e^{-i
$\varphi(t,\vec{x}) = e^{iH(t-t_0)} \varphi(t_0,\vec{x}) e^{-iH(t-t_0)}$
Higher orders
Heisenberg field

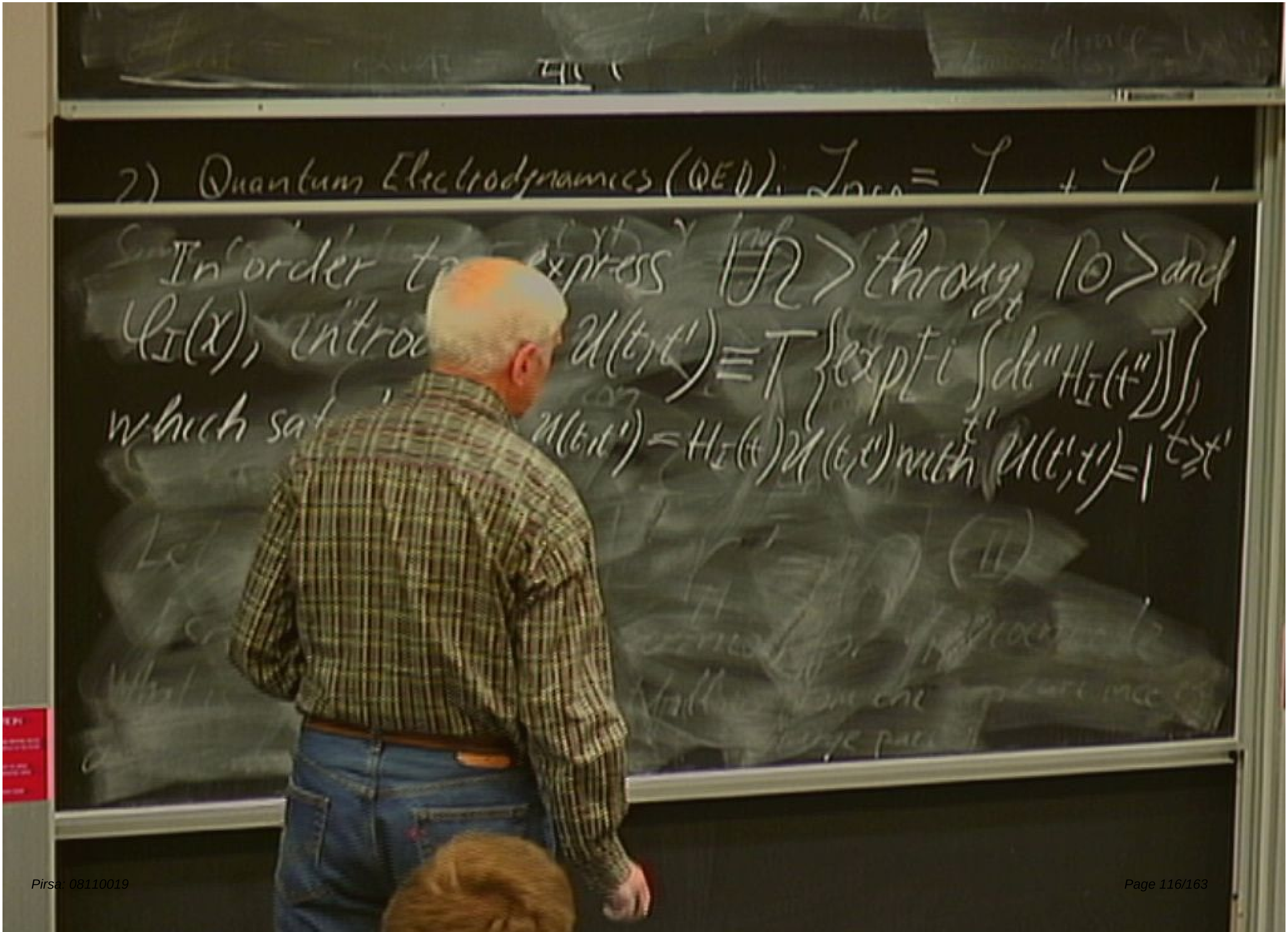

order to express |Ω> throug |0> and

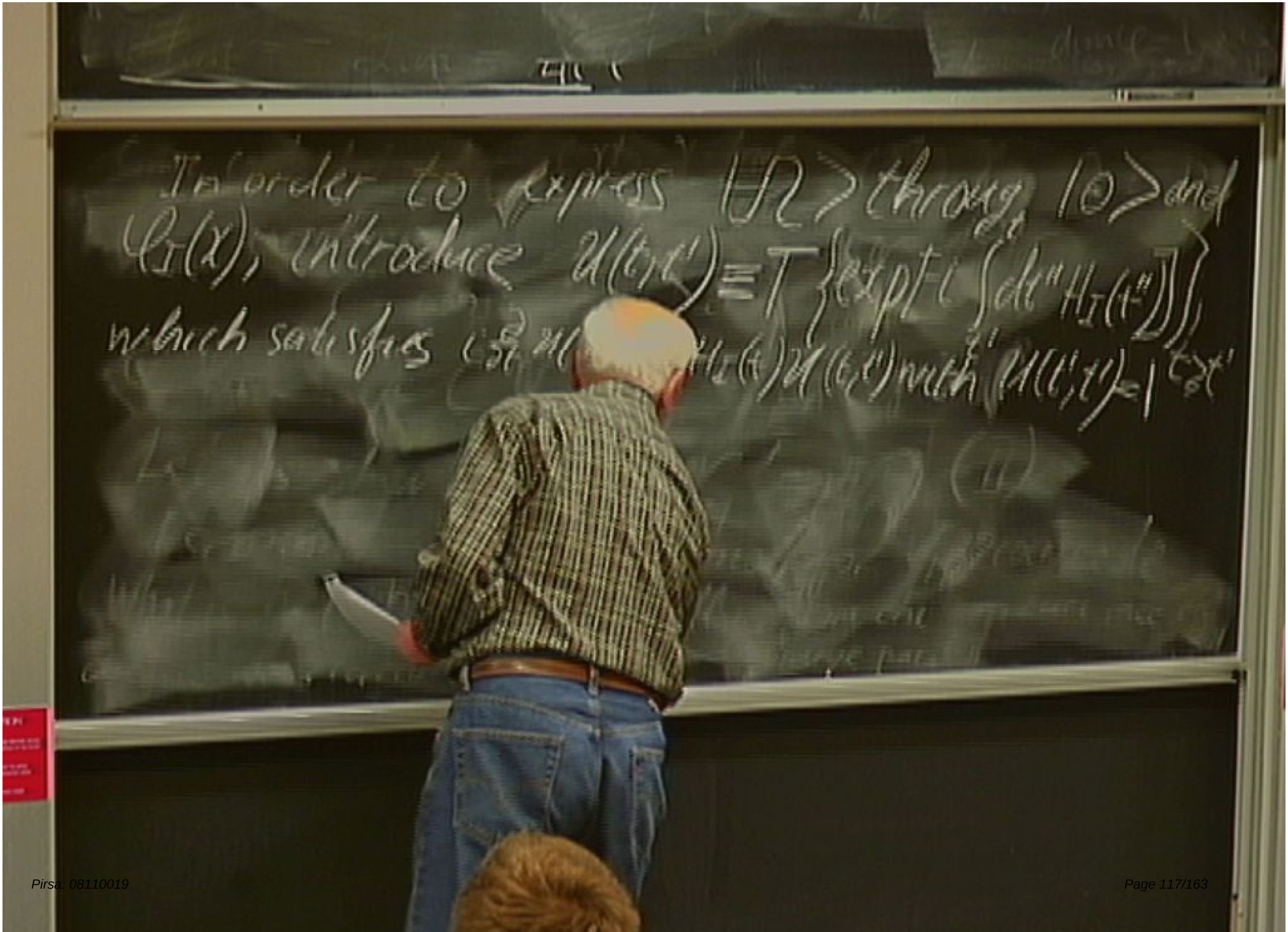

In order to express
throug |0> and
φ_I(x), introduce U(t,t')

In order to express $|\Omega\rangle$ throug $|0\rangle$ and $\varphi_I(x)$, introduce $\mathcal{U}(t,t') \equiv T$


In order to express |Ω> through |0> and
φ_I(x), introd
U(t,t') ≡ T{exp[-i ∫_{t'}^{t} dt'' H_I(t'')]}, t>t'

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\varphi_I(x)$, introduce $U(t,t') \equiv T\left\{\exp\left[-i\int_{t'}^{t} dt'' H_I(t'')\right]\right\}, \quad t>t'$


2) Quantum Electrodynamics (QED):
In order to express |Ω> through |0> and
φ_I(x), introd U(t,t') ≡ T{exp[-i ∫dt'' H_I(t'')]},
which sa U(t,t') = H_I(t) U(t,t') with U(t',t') = 1 t>t'


In order to express |Ψ(t)> throug |0> and
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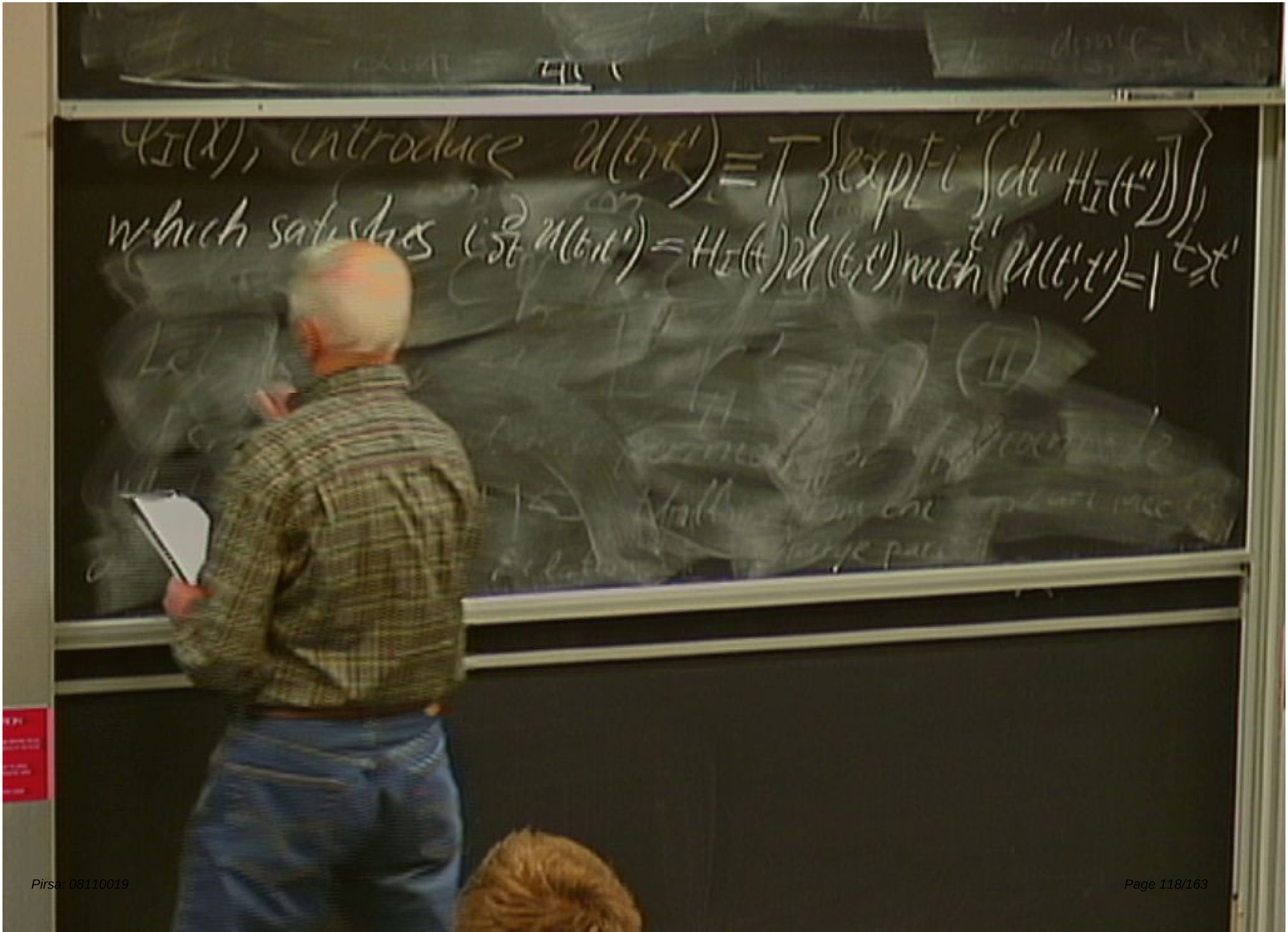
which satisfies
H_I(t)U(t,t') with U(t,t')=1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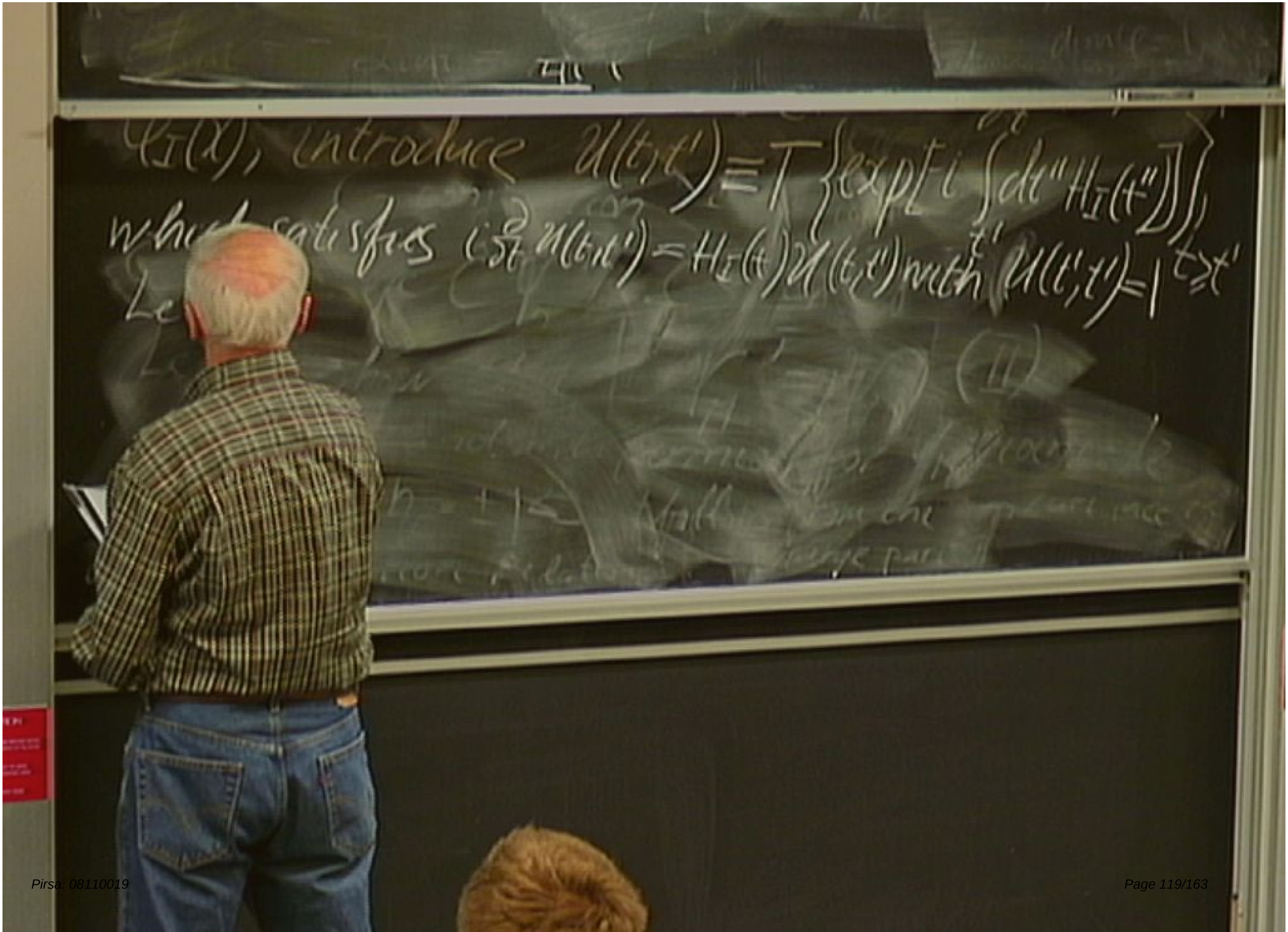
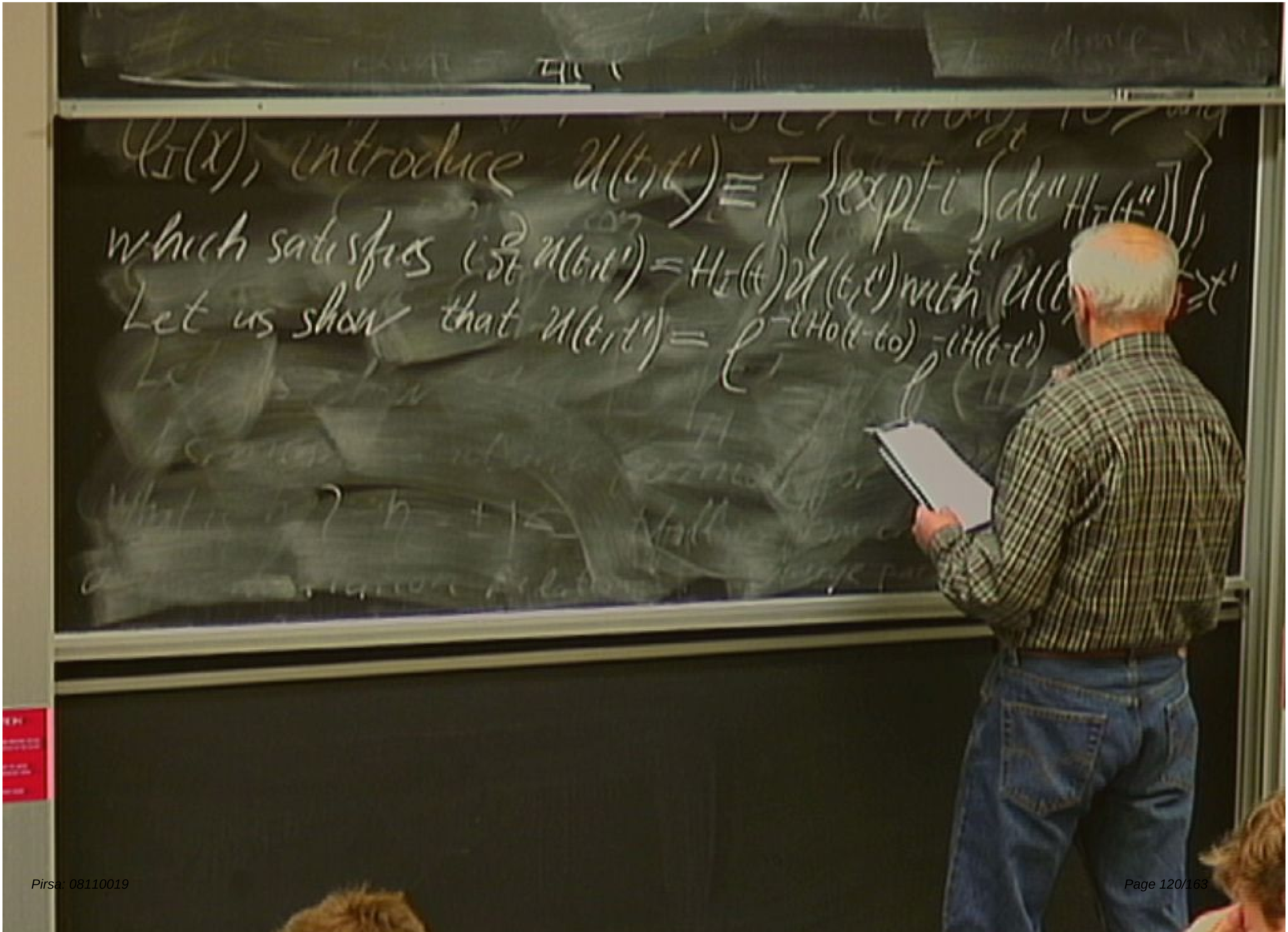
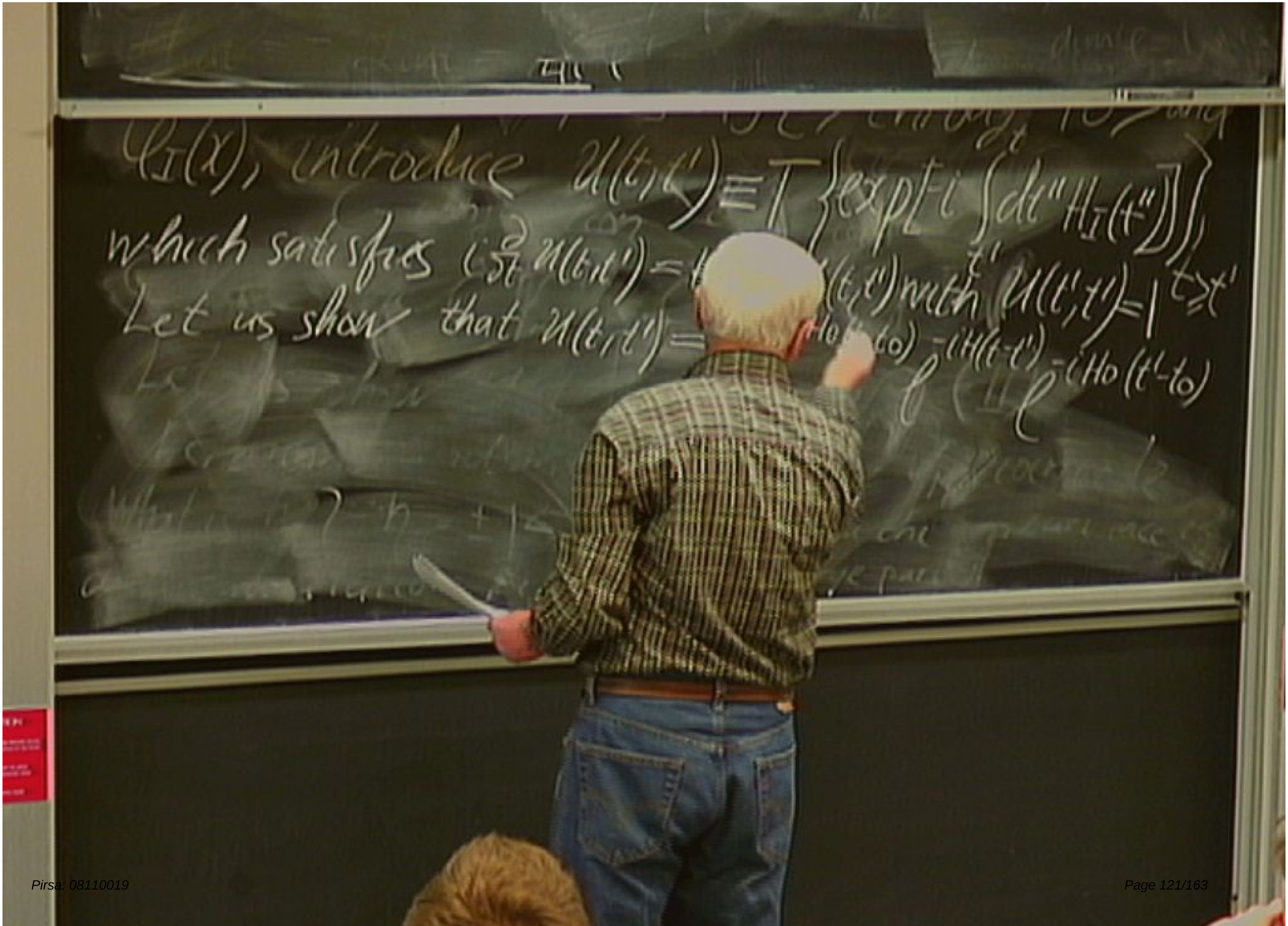

$\psi_I(x)$, introduce $U(t,t') \equiv T\{\exp[i\int_{t'}^{t} dt'' H_I(t'')]\}$,
which satisfies $i\frac{\partial}{\partial t} U(t,t') = H_I(t) U(t,t')$ with $U(t',t') = 1$ $t \geq t'$


φ_I(x), introduce U(t,t') ≡ T{exp[-i ∫ dt'' H_I(t'')]}
which satisfies i ∂/∂t U(t,t') = H_I(t) U(t,t') with U(t,
Let us show that U(t,t') = e^{-iH_0(t-t_0)} e^{-iH(t-t')}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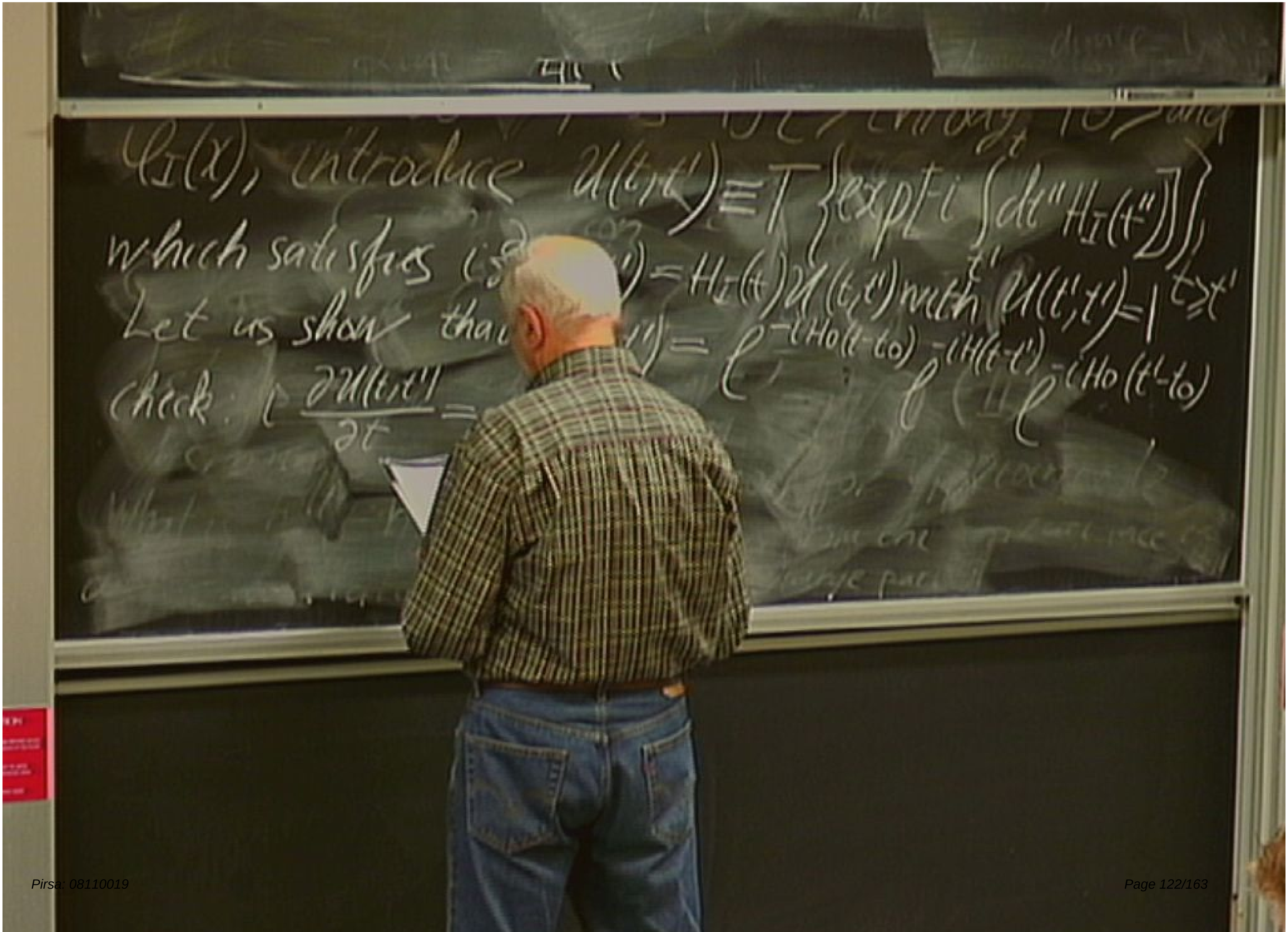
Let us show that U(t,t') =
H0(t-t0) e^{-iH(t-t')} e^{-iH0(t'-t0)}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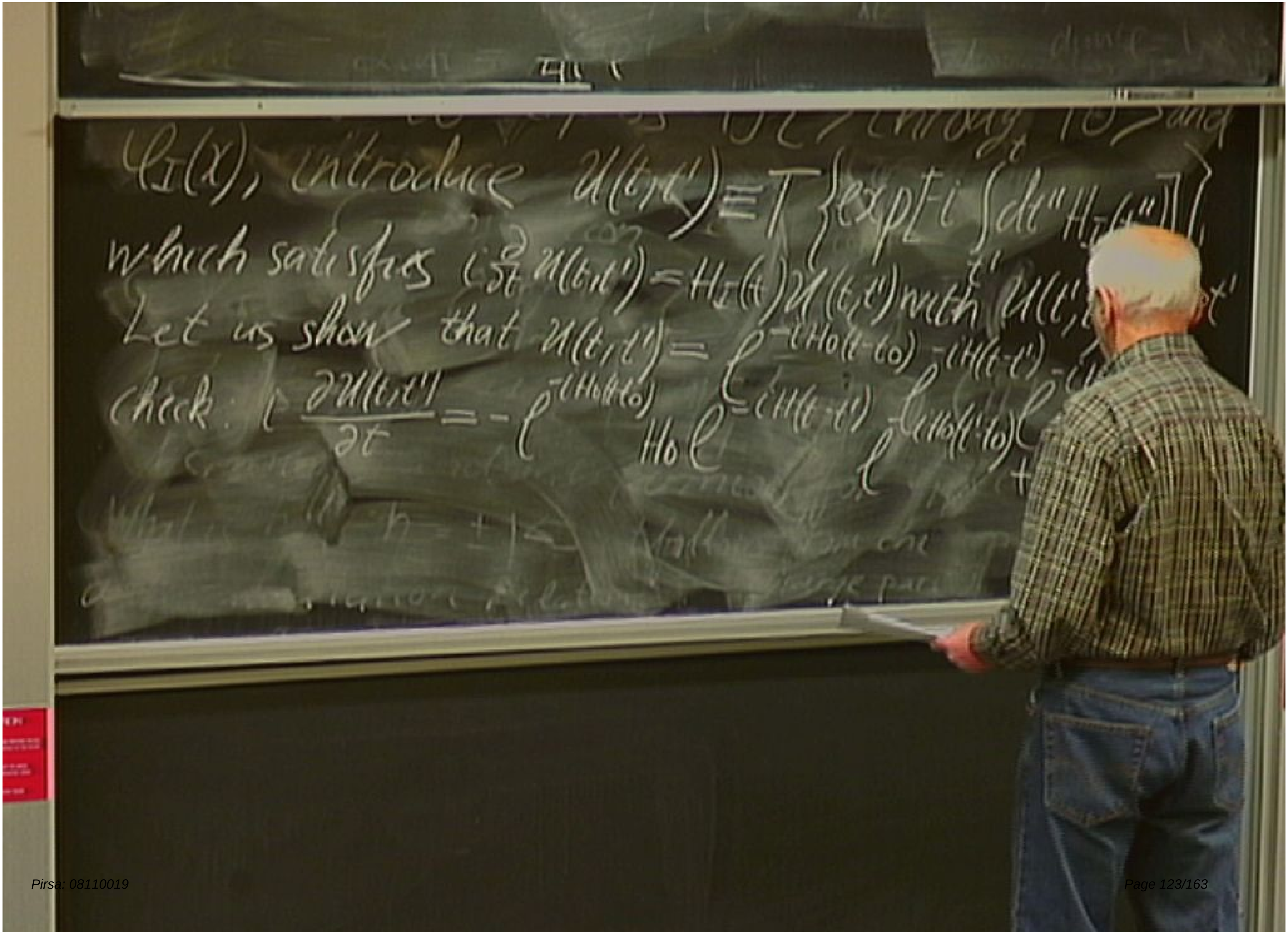
which satisfies (i∂ … t') = H_I(t) U(t,t') with U(t',t') = 1   t>t'
Let us show that … t') = e^{-iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}
check: i ∂U(t,t')/∂t =



$\varphi_I(x)$, introduce $U(t,t') \equiv T\{\exp[-i\int dt'' H_I(t'')]\}$
which satisfies $i\frac{\partial}{\partial t}U(t,t') = H_I(t)U(t,t')$ with $U(t',$
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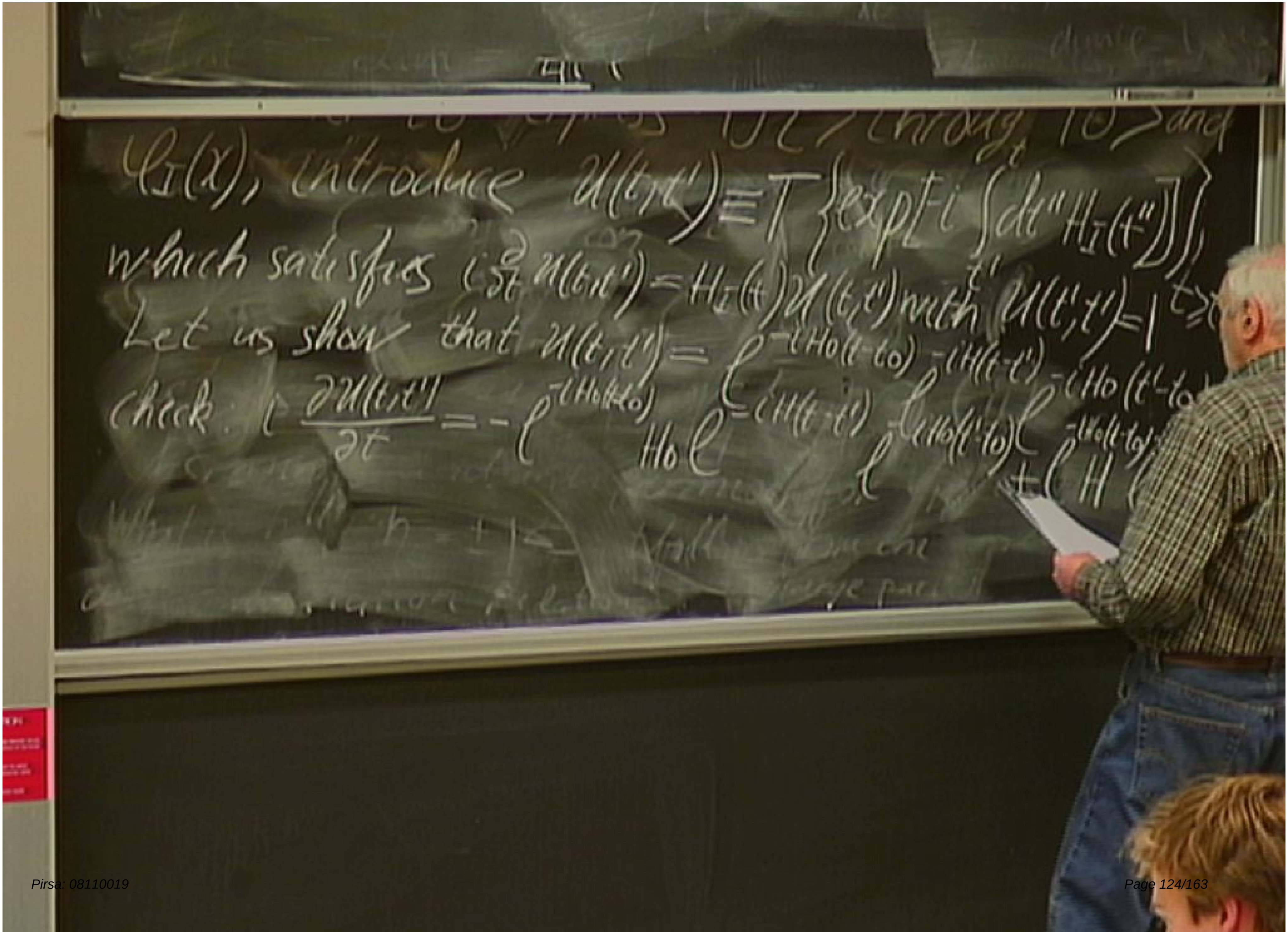

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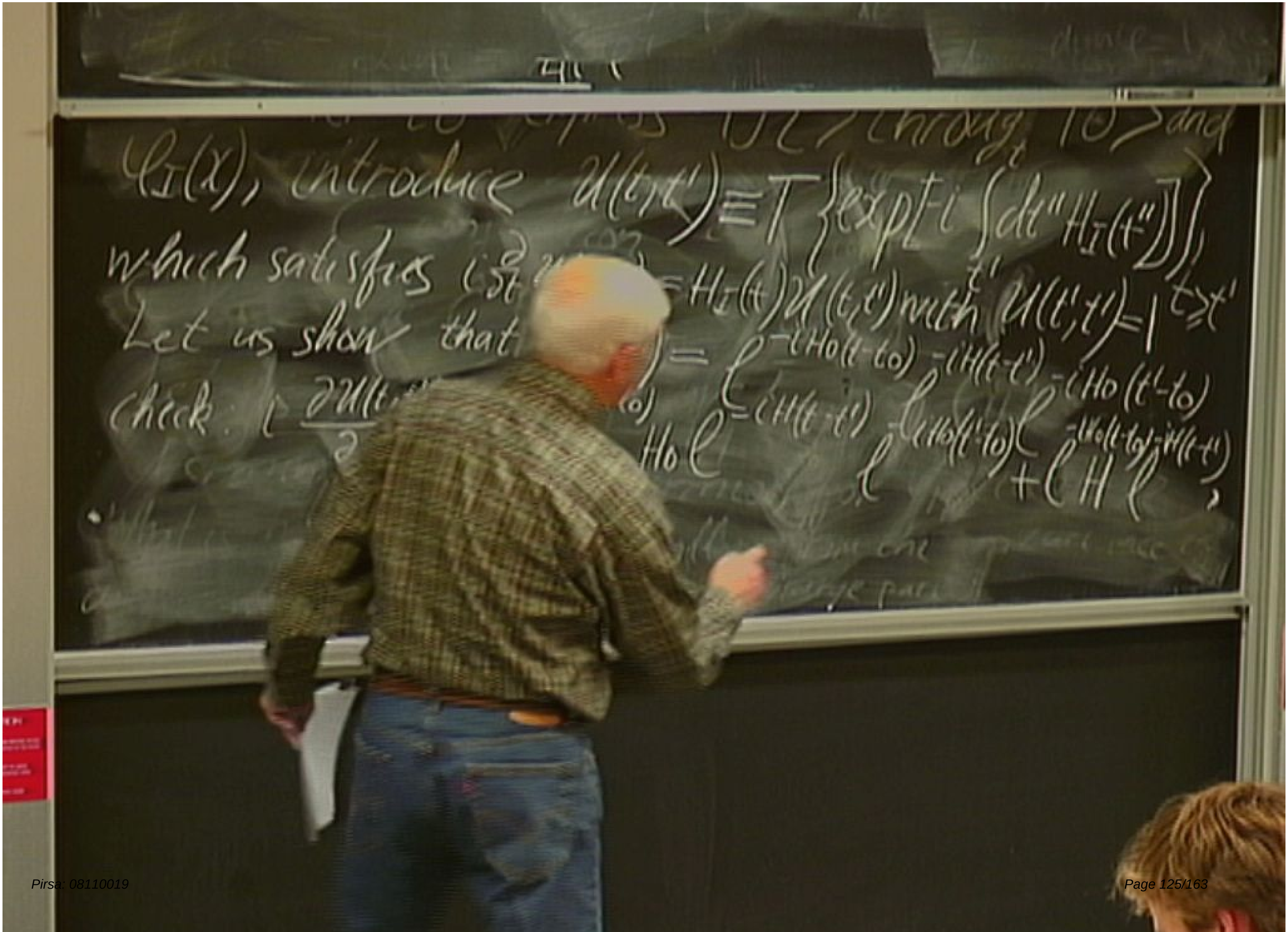

ψI(x), introduce U(t,t') ≡ T{exp[-i ∫ dt'' HI(t'')]},
which satisfies = HI(t) U(t,t') with U(t',t') = 1 t ≥ t'
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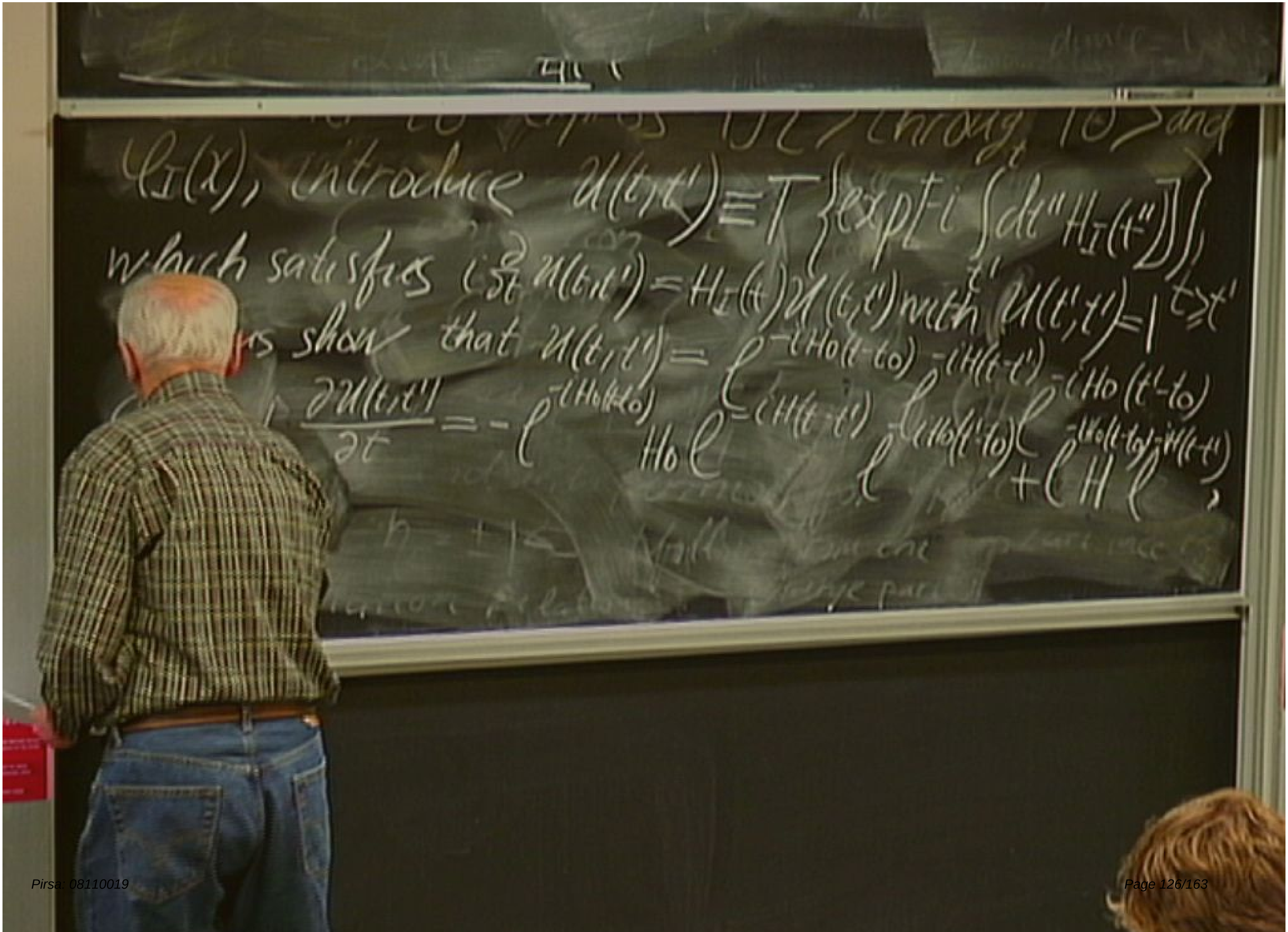
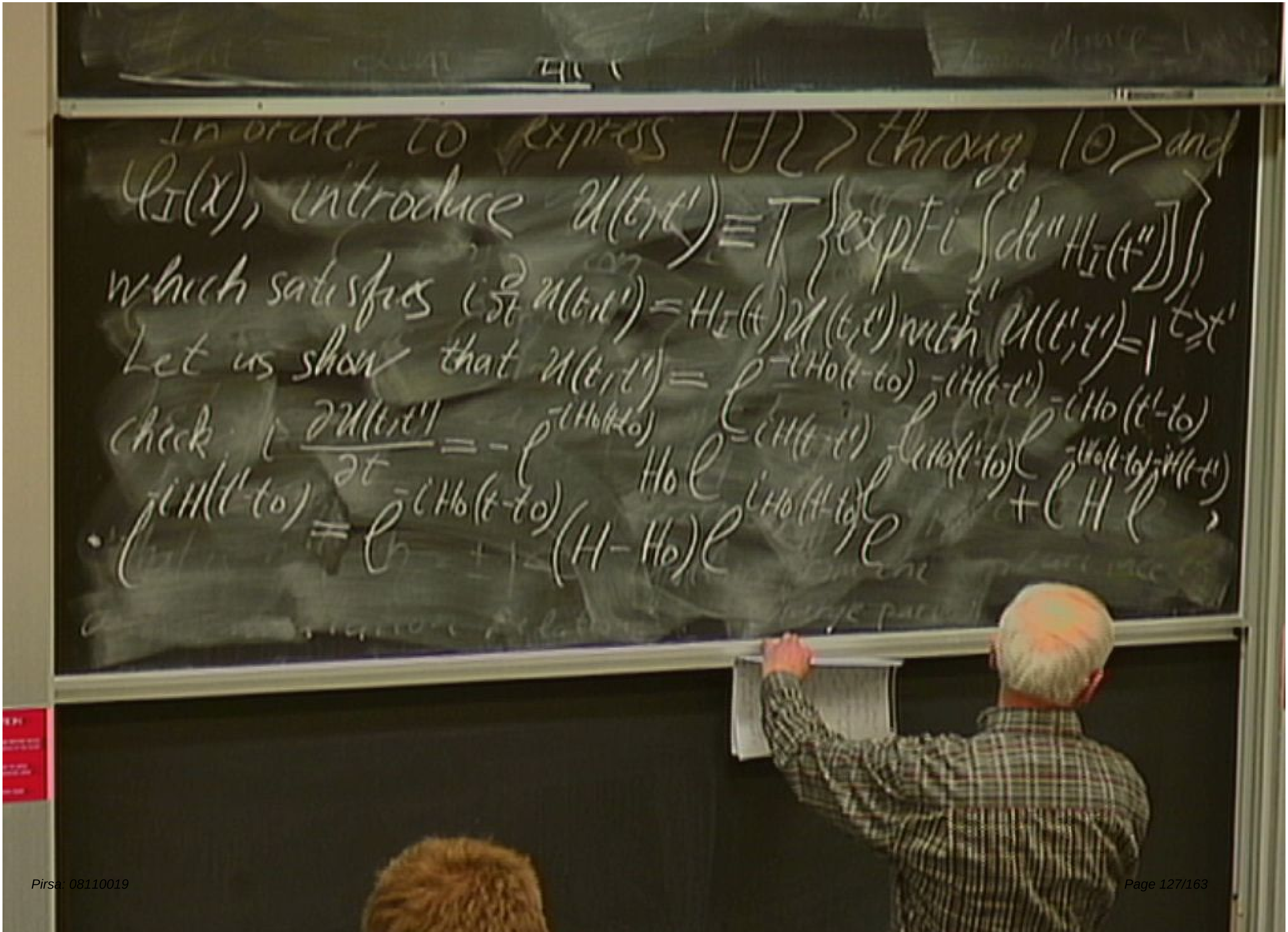
check:



ψI(x), introduce U(t,t') ≡ T{exp[-i ∫ dt'' HI(t'')]},
which satisfies i ∂/∂t U(t,t') = HI(t) U(t,t') with U(t',t') = 1
t>t'
that U(t,t') = e^{-iHo(t-to)} e^{-iH(t-t')} e^{-iHo(t'-to)}


In order to express $|\psi(t)\rangle$ through $|0\rangle$ and
$\varphi_I(x)$, introduce $U(t,t') \equiv T\{\exp[-i\int_{t'}^{t} dt'' H_I(t'')]\}$,
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Let us show that $U(t,t') = e^{-iH_0(t-t_0)}e^{-iH(t-t')}e^{-iH_0(t'-t_0)}$
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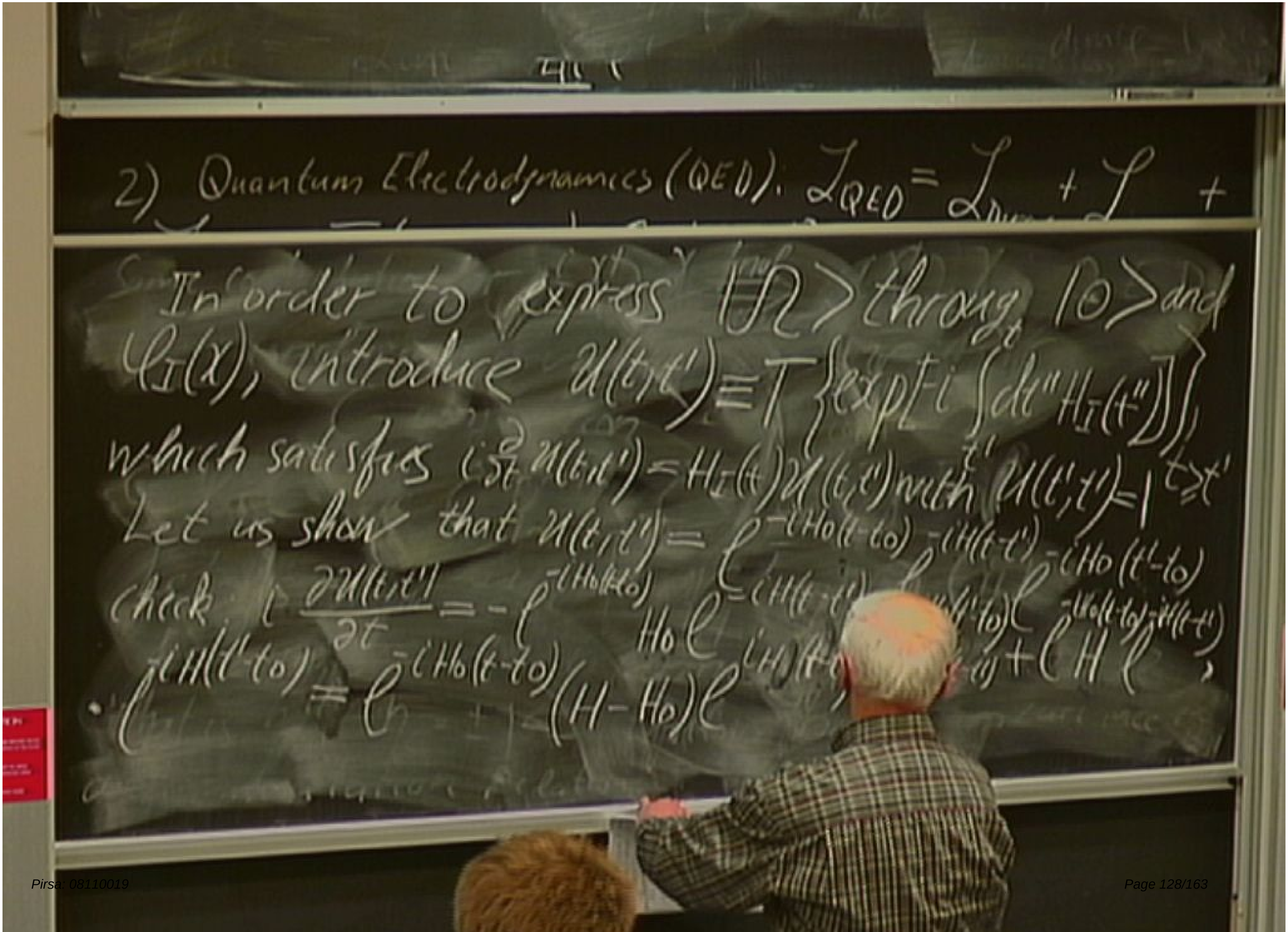
$= e^{-iH_0(t-t_0)}(H-H_0)e^{iH_0(t-t_0)}e$


2) Quantum Electrodynamics (QED): $\mathcal{L}_{QED} = \mathcal{L} + \mathcal{L} +$
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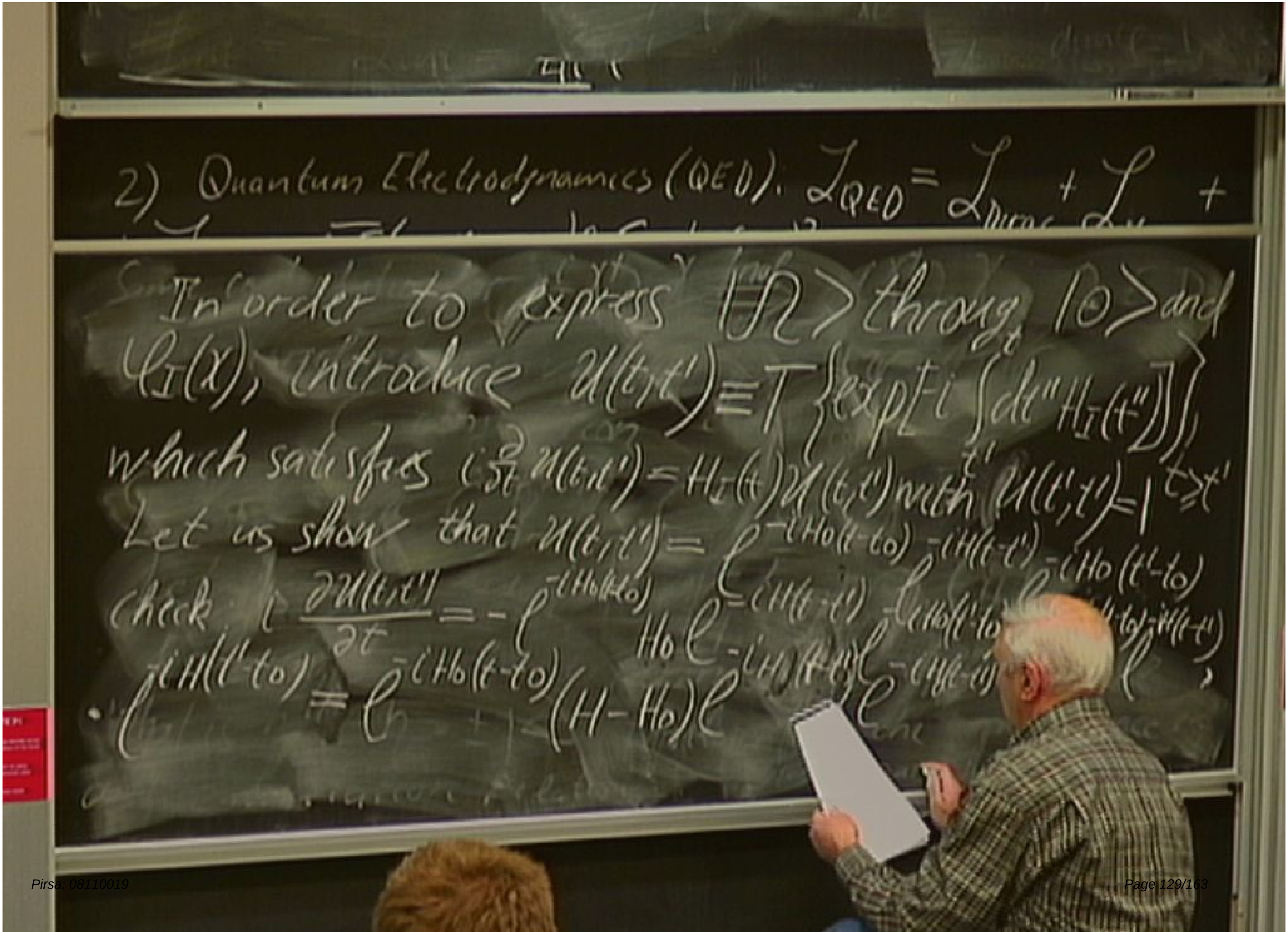
$e^{-iH(t'-t_0)} = e^{-iH_0(t-t_0)}(H-H_0)e$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In order to express $|\Omega\rangle$ through $|0\rangle$ and
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Let us show that $U(t,t') = e^{-iH_0(t-t_0)}\, e^{-iH(t-t')}\, e^{-iH_0(t'-t_0)}$
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In order to express $|\Omega\rangle$ through $|0\rangle$ and $\varphi_I(x)$, introduce $U(t,t') \equiv T\left\{\exp\left[-i\int_{t'}^{t} dt'' H_I(t'')\right]\right\}$,

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Let us show that $U(t,t') = e^{-iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$

check: $i\frac{\partial U(t,t')}{\partial t} = -e^{-iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} + e^{-iH_0(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$

$= e^{-iH_0(t-t_0)} (H - H_0) e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$

2) Quantum Electrodynamics (QED). $\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{M} +$

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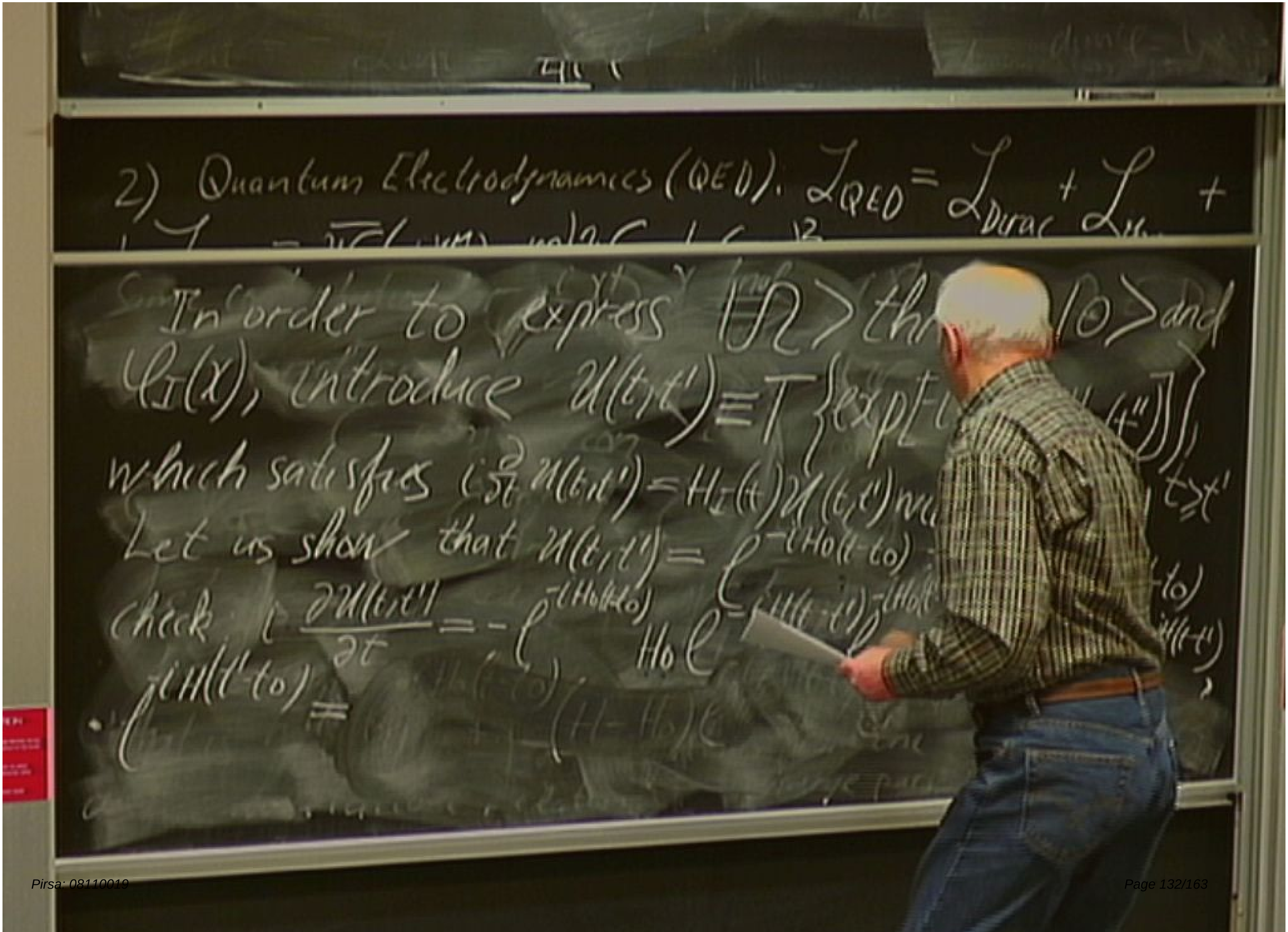
check $i\dfrac{\partial U(t,t')}{\partial t} = -e^{-iH_0(t-t_0)}H_0\,e^{-iH(t-t')}\,e^{-iH_0(t'-t_0)} + e^{-iH_0(t-t_0)-iH(t-t')}\,H\,e^{iH(t'-t_0)}$

$= e^{iH_0(t-t_0)}(H-H_0)e$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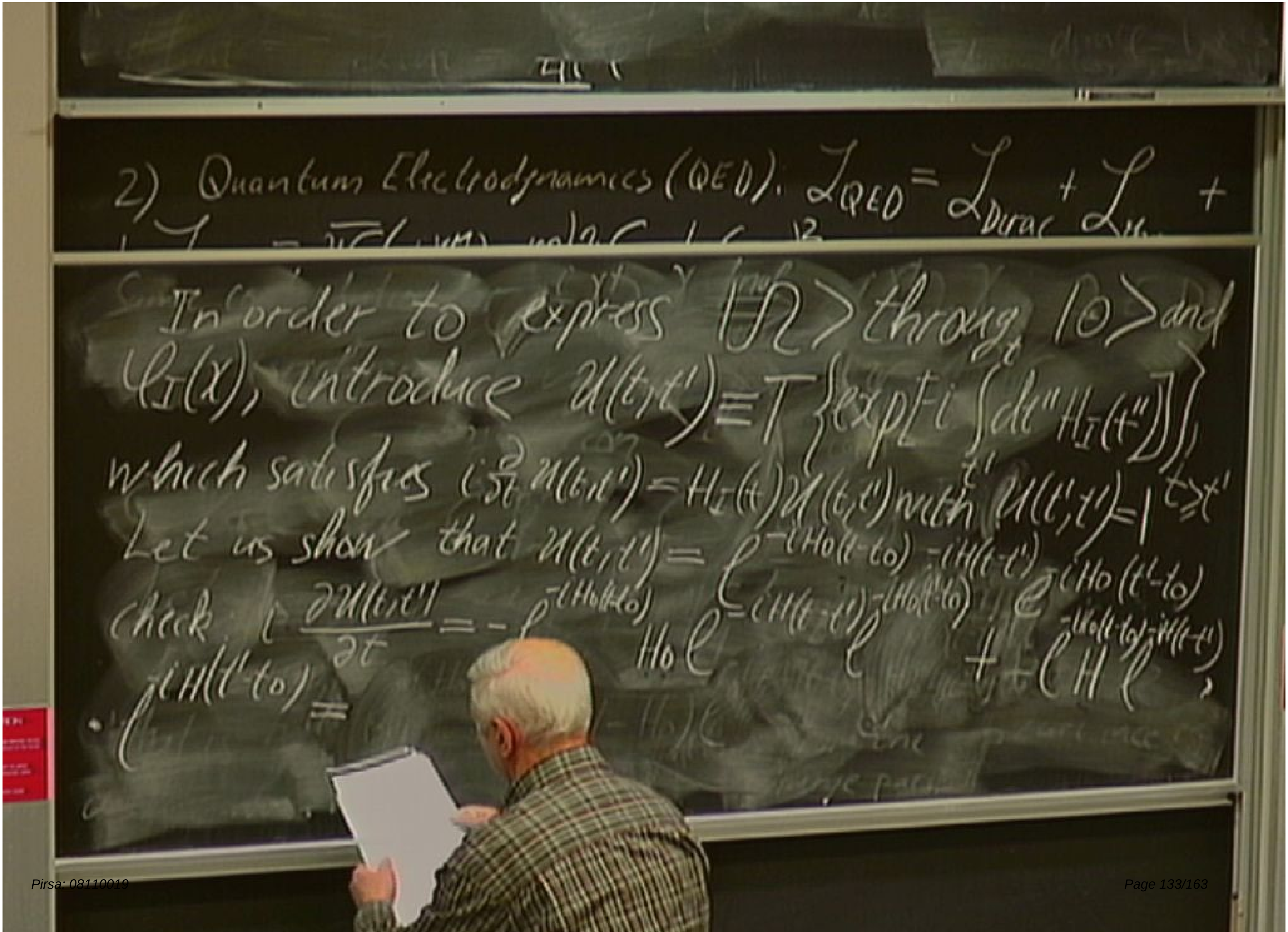
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check: $i\frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)}H_0e^{-iH(t-t')}e^{-iH_0(t'-t_0)} + e^{iH_0(t-t_0)}He^{-iH(t-t')}e^{-iH_0(t'-t_0)} =$

2) Quantum Electrodynamics (QED): $\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{Maxwell} +$

$+ \mathcal{L}_{int} = \bar{\psi}(i\gamma^{\mu} \ldots - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 \ldots$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\varphi_I(x)$, introduce $U(t,t') \equiv T\{\exp[-i\int_{t'}^{t} dt'' H_I(t'')]\}$,

which satisfies $i\frac{\partial}{\partial t}U(t,t') = H_I(t)U(t,t')$ with $U(t',t')=1$ $\quad t>t'$

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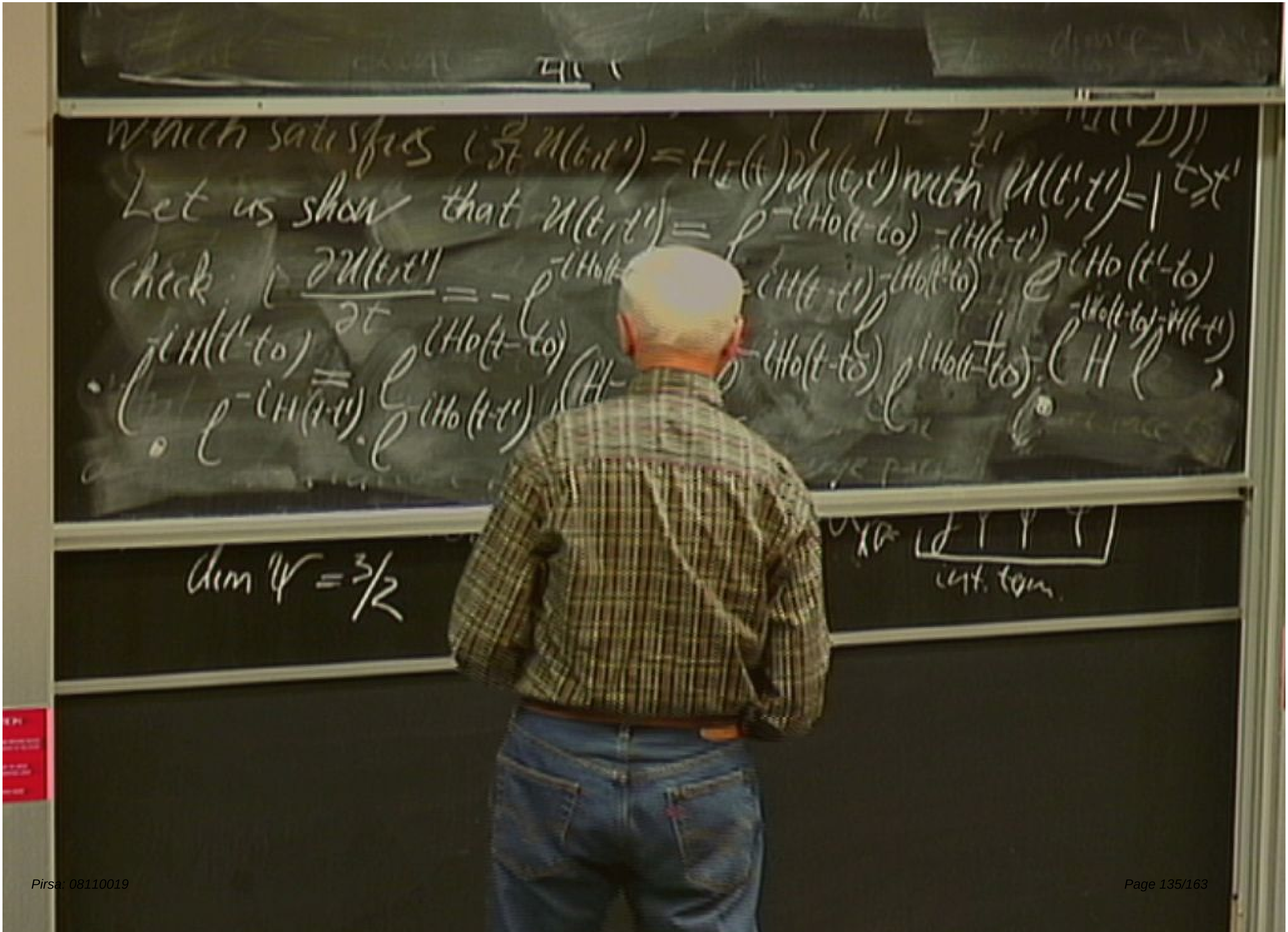
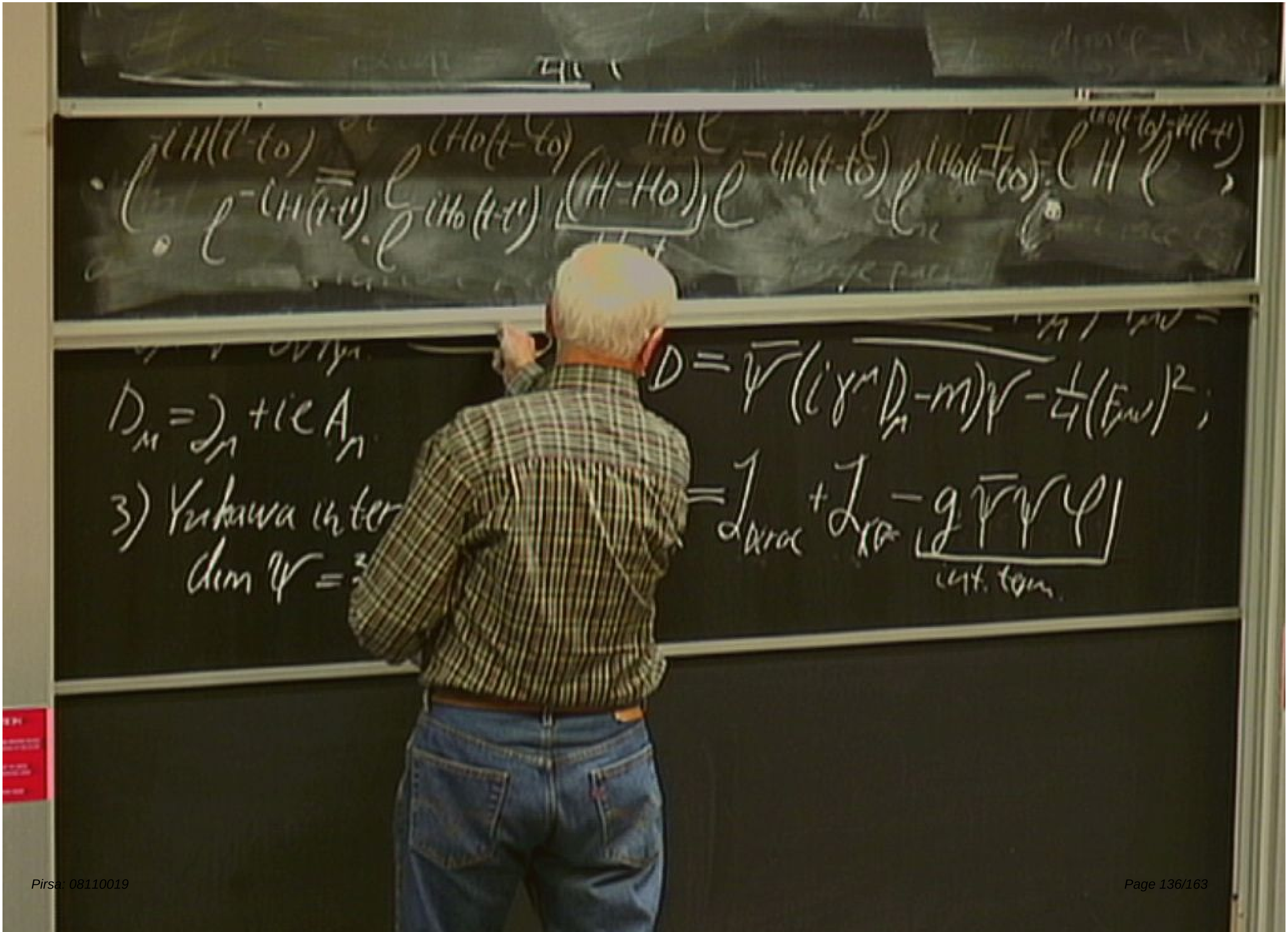
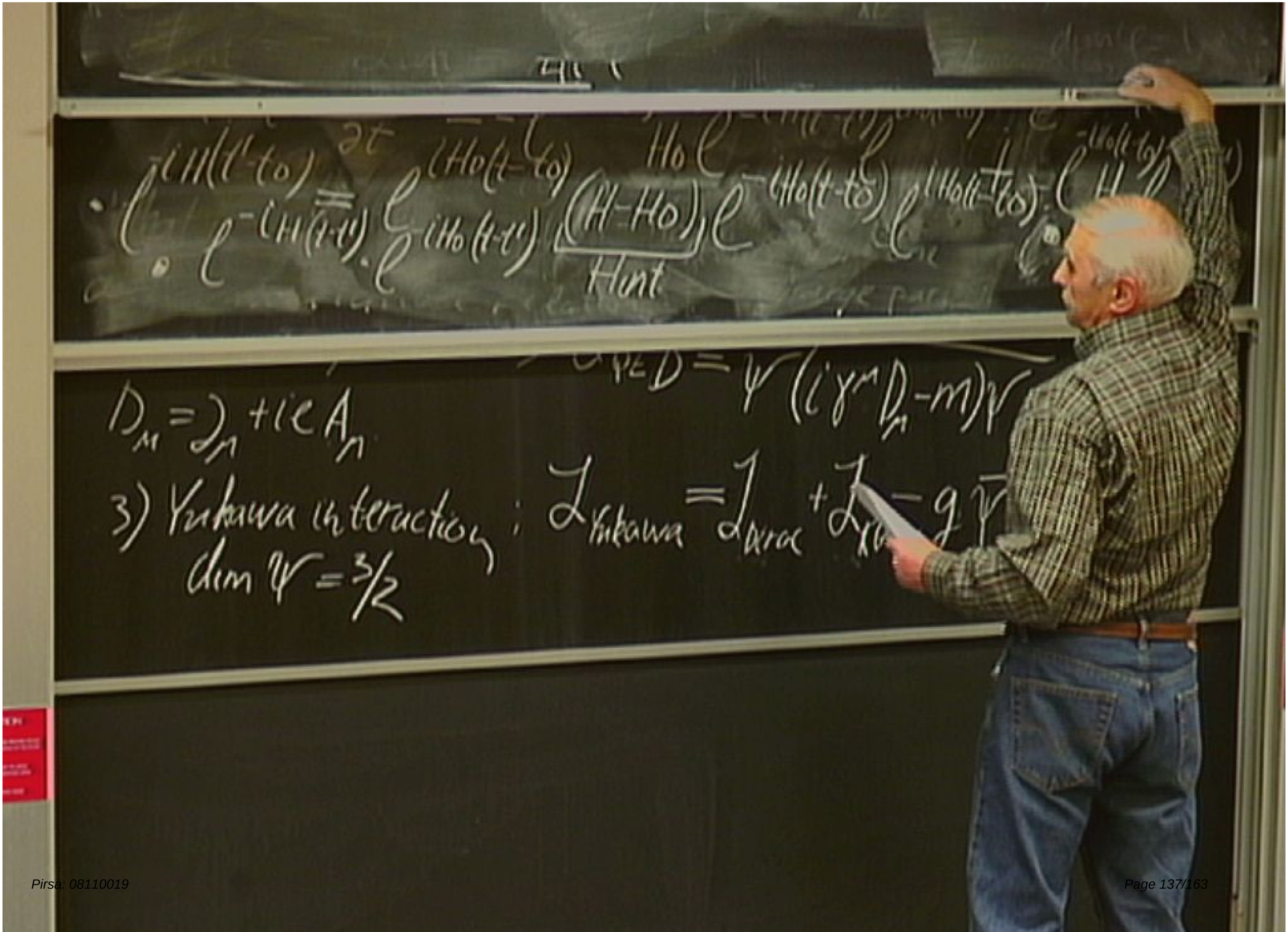
$e^{-iH(t'-t_0)} = e^{iH_0(t-t_0)} \underbrace{(H-H_0)}_{H_{int}} e^{\ldots}$

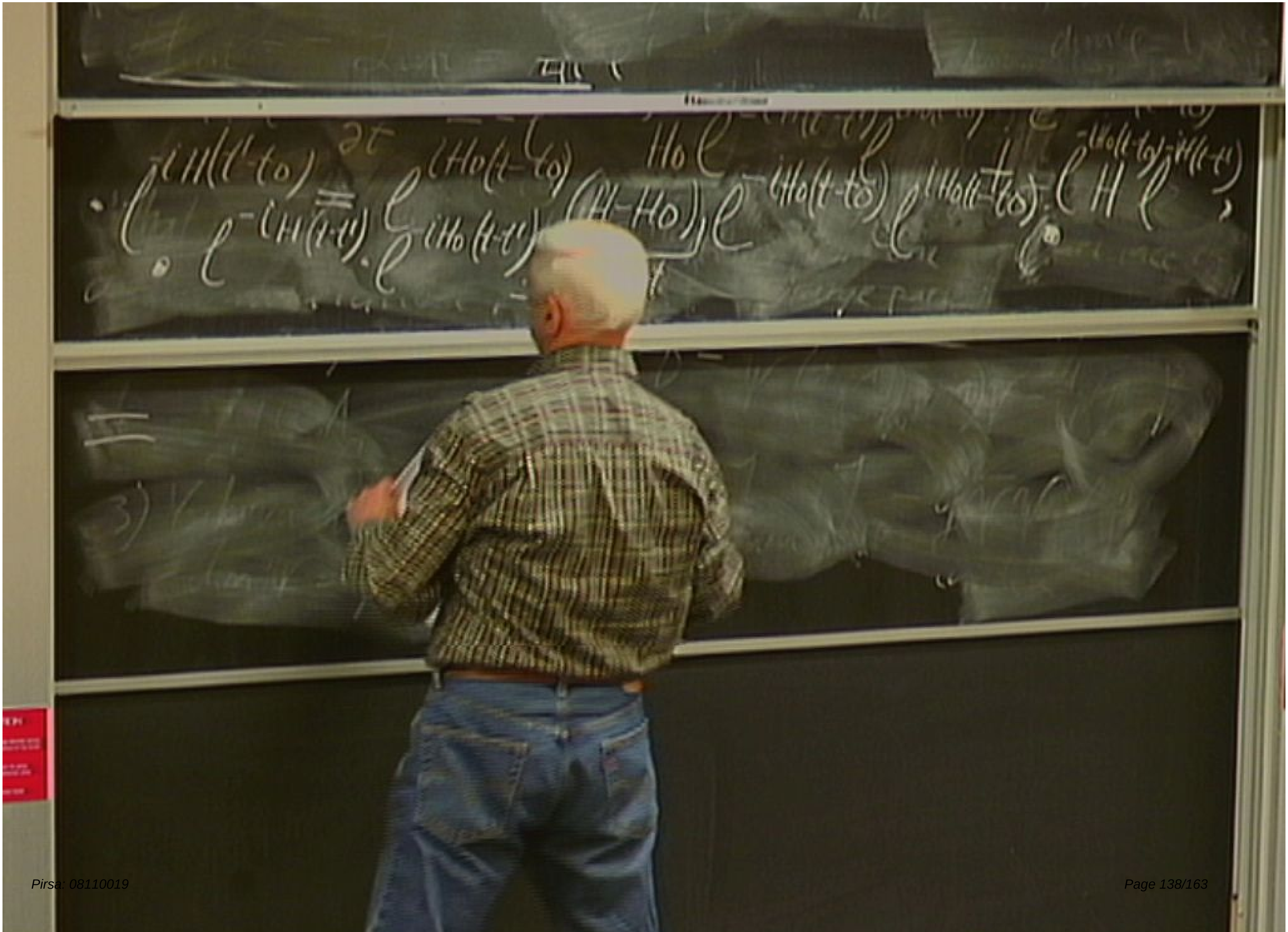


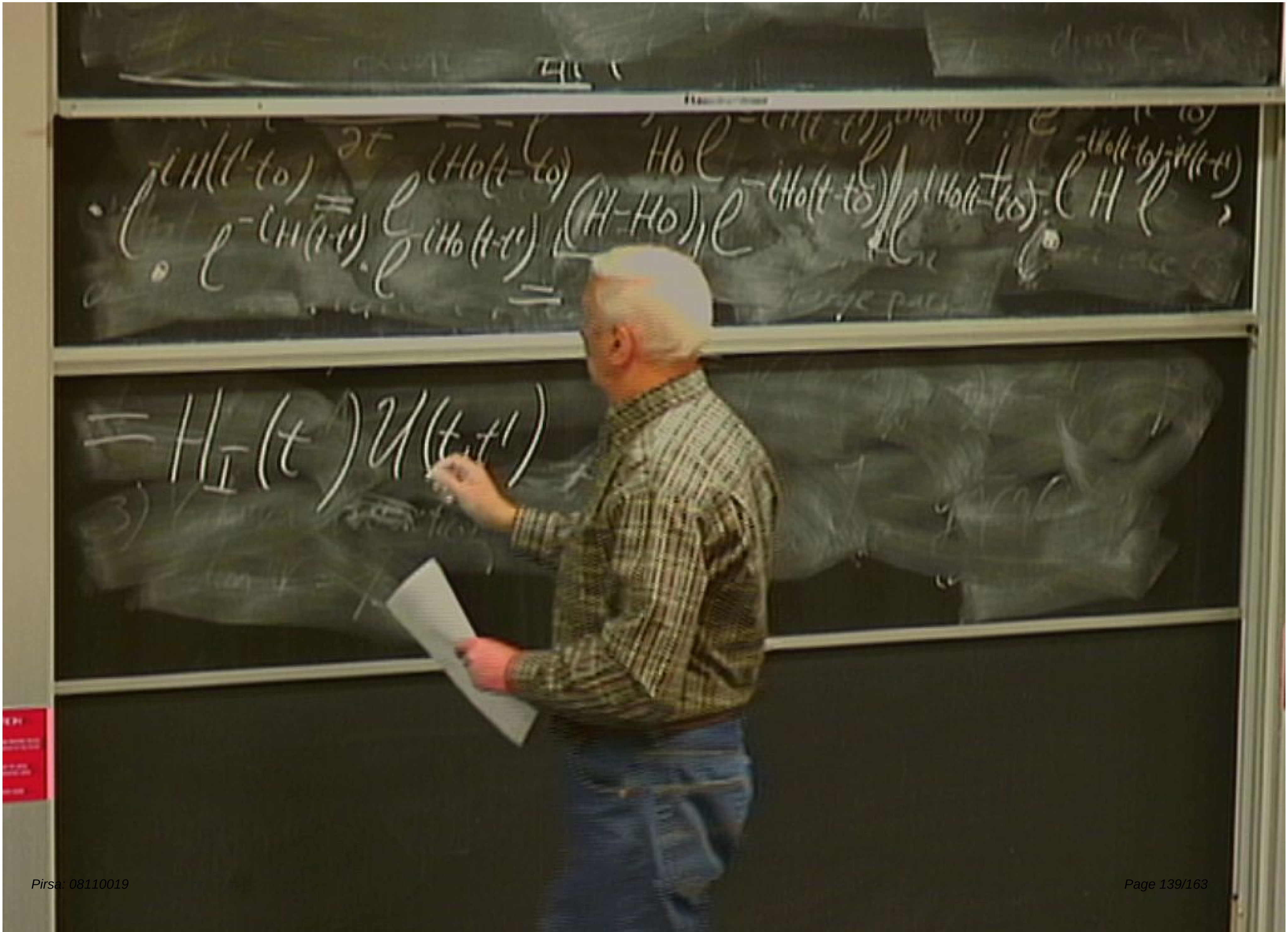
which satisfies $i\frac{\partial}{\partial t}U(t,t') = H_I(t)U(t,t')$ with $U(t',t') = 1$ $t>t'$
Let us show that $U(t,t') = e^{-iH_0(t-t_0)} e^{-iH(t-t')}$
check
$\dim \Psi = 3/2$
int. term

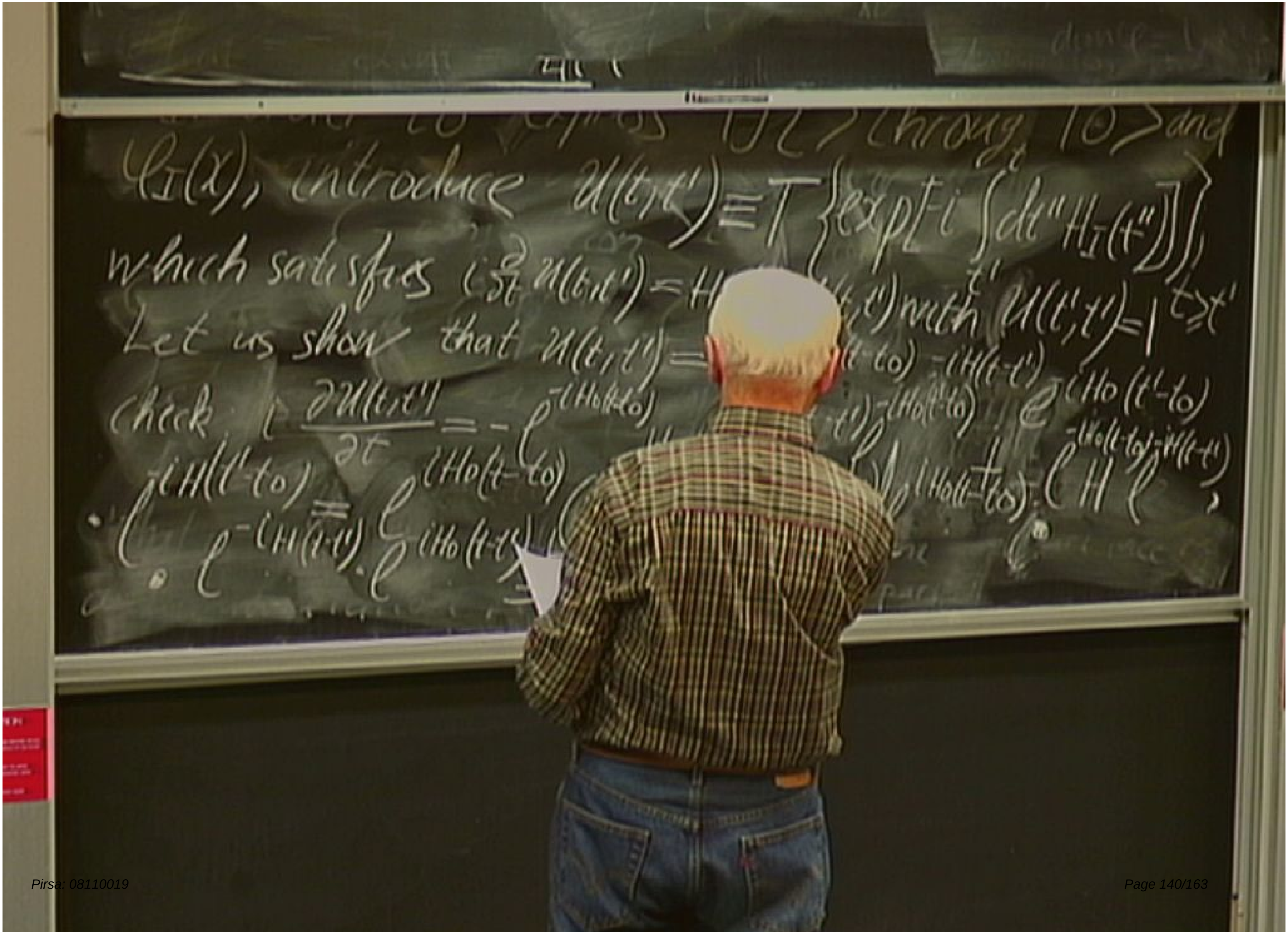


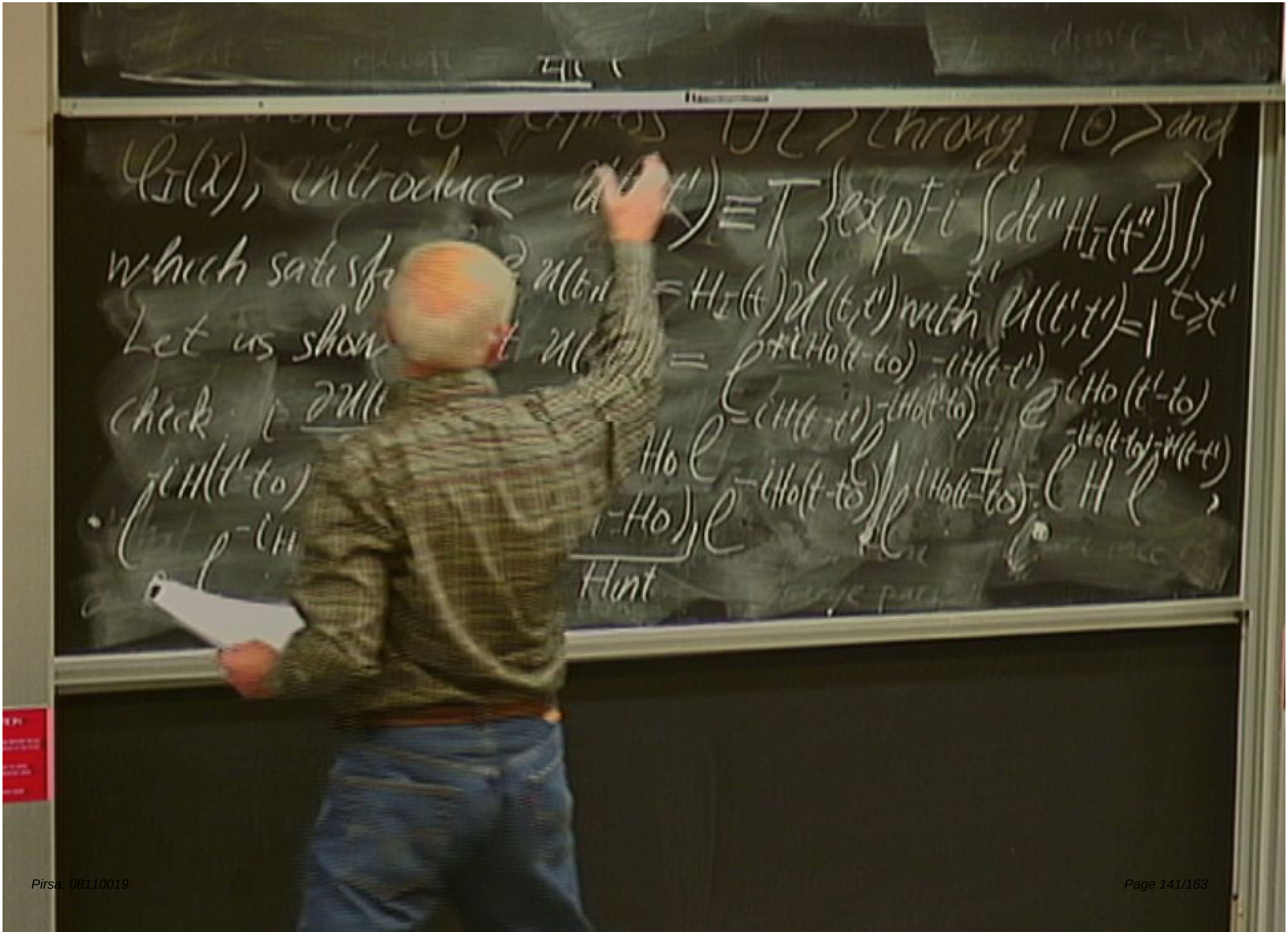
D_μ = ∂_μ + ie A_μ
3) Yukawa inter
dim Ψ = 3
int. term


$D_\mu = \partial_\mu + ieA_\mu$
3) Yukawa interaction ; $\mathcal{L}_{Yukawa} = \mathcal{L}_{Dirac} + \mathcal{L}_{KG} - g\bar{\psi}$
$\dim \psi = 3/2$
Hint




$= H_I(t)\, U(t,t')$


introduce
which satisfies
Let us show that
check


introduce
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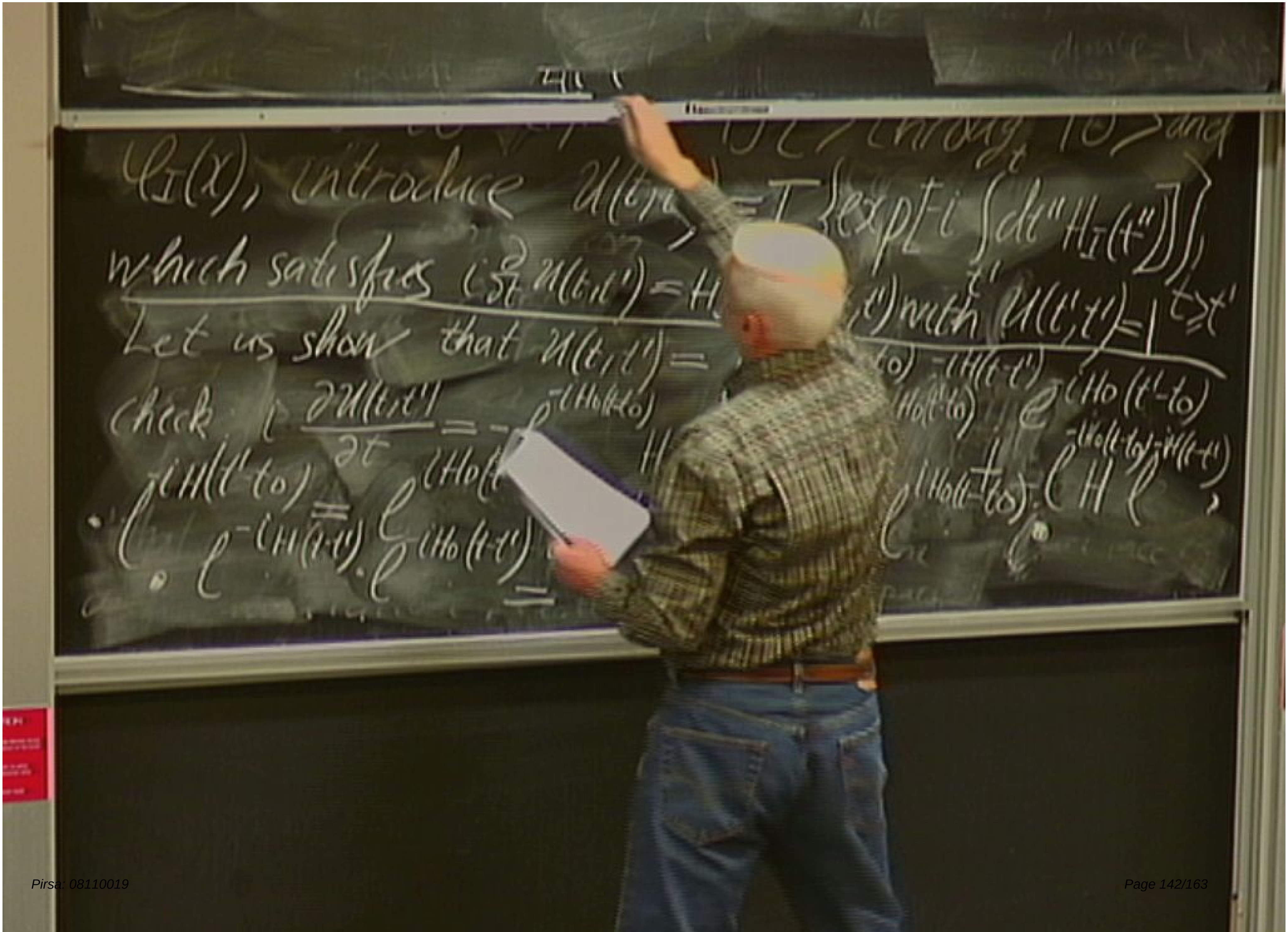
Hint



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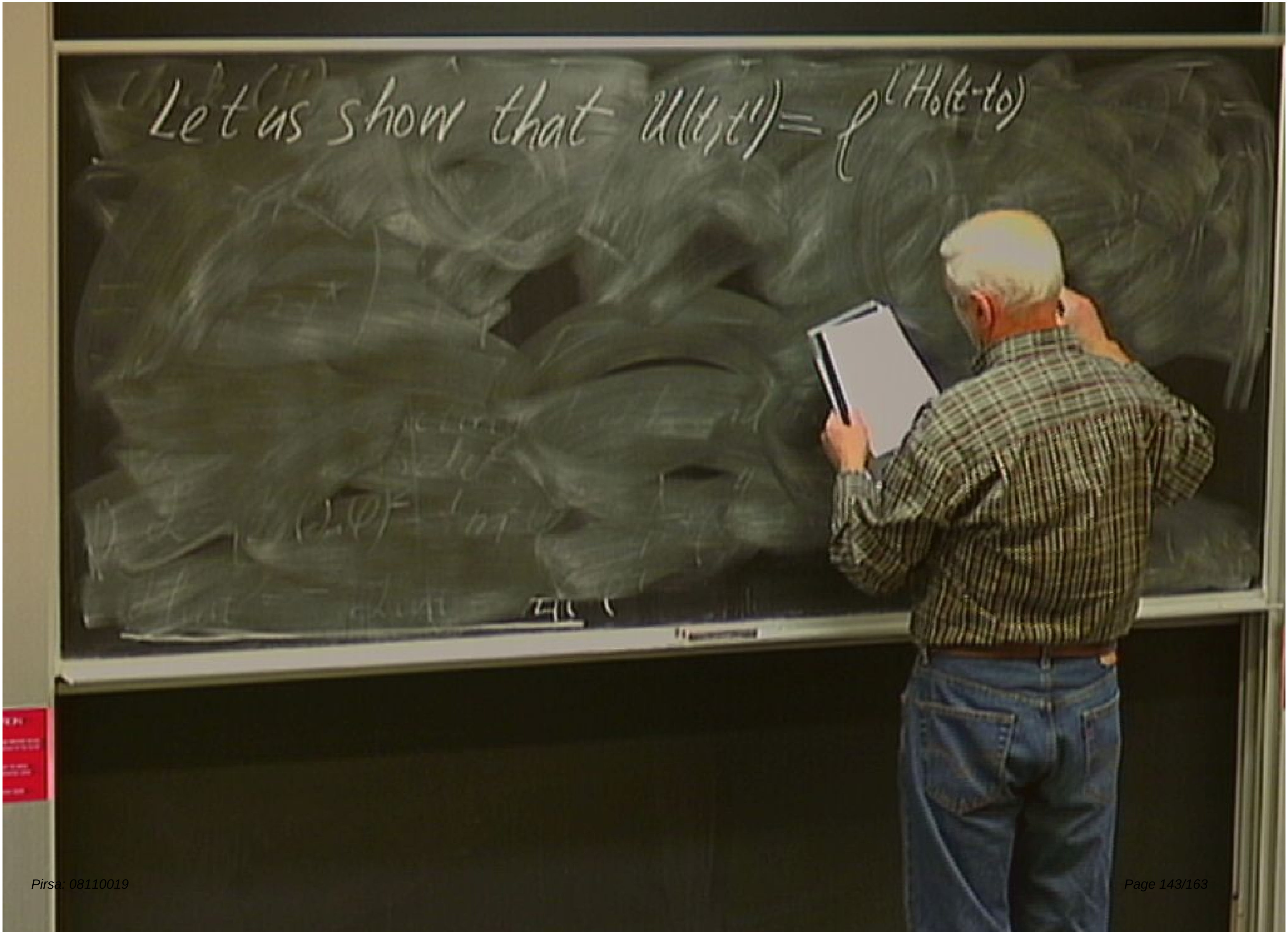
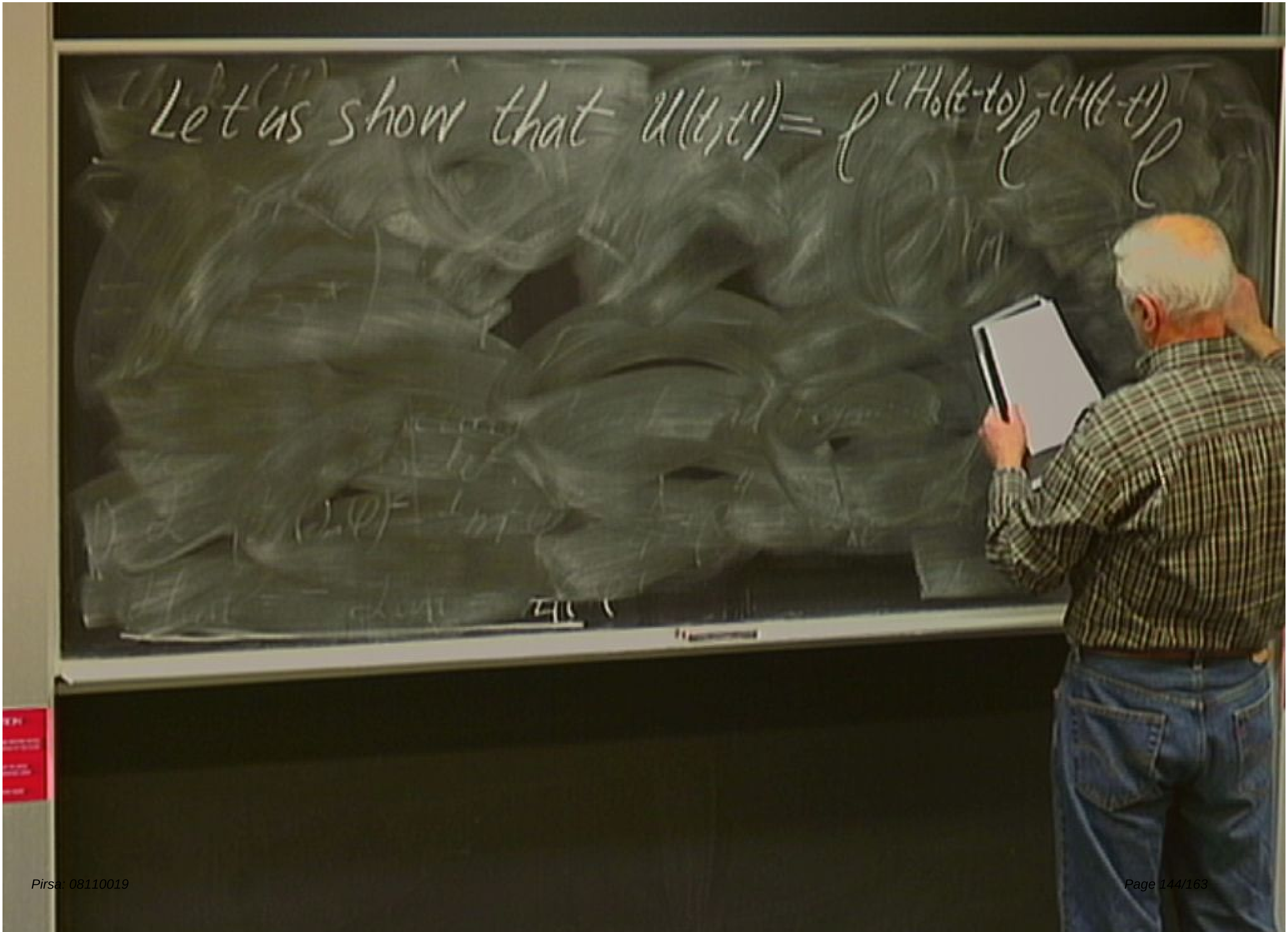
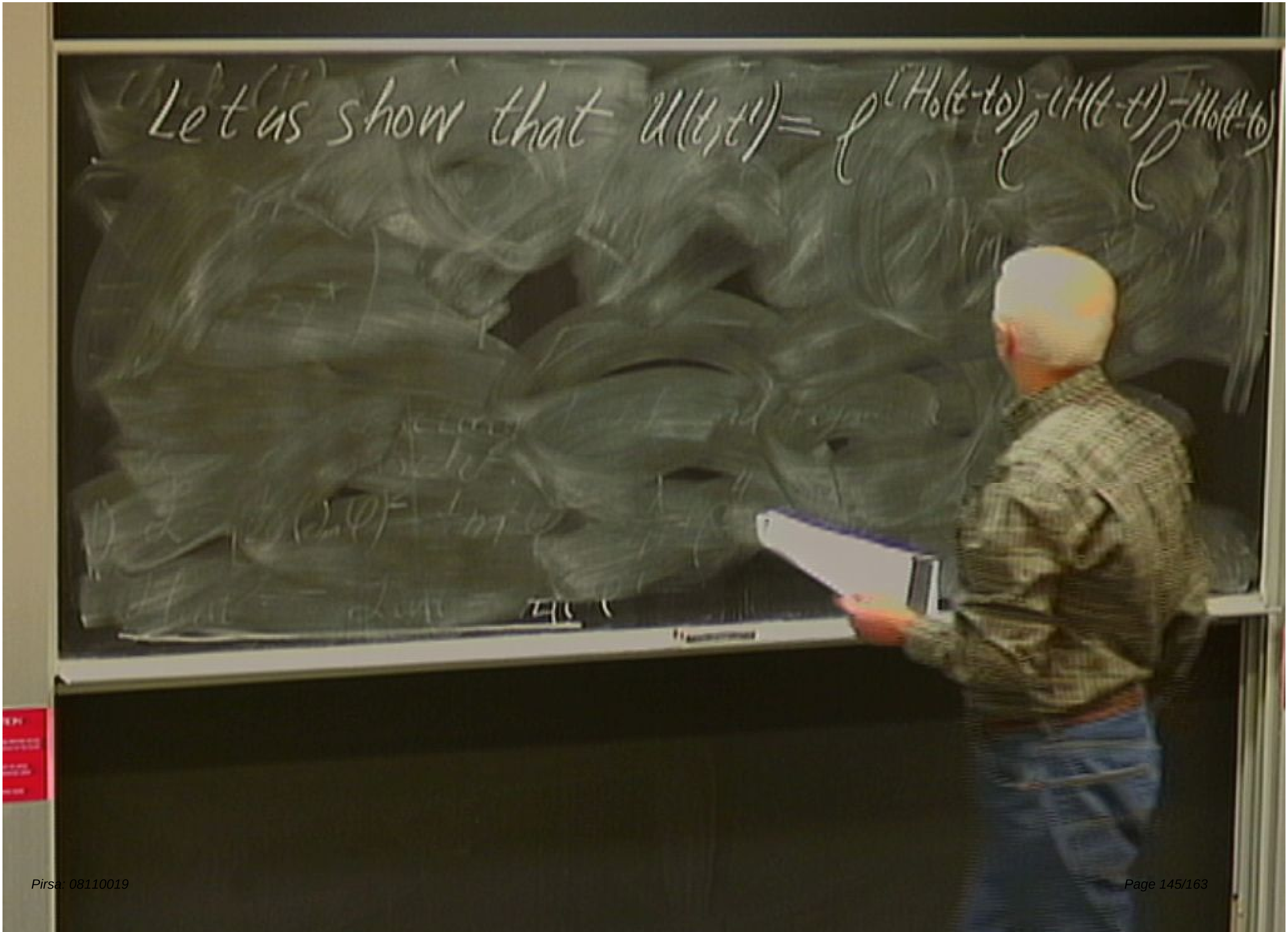
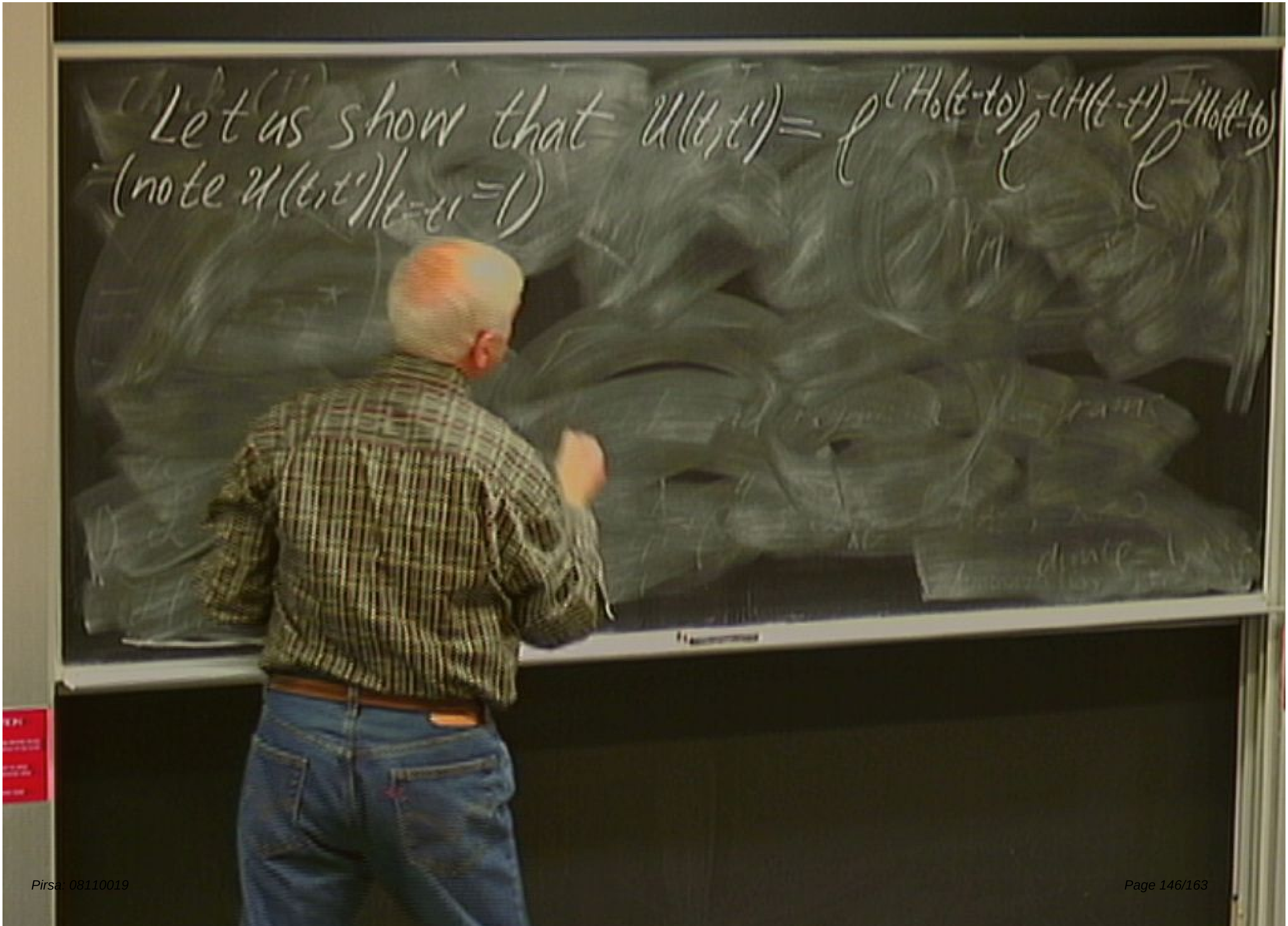
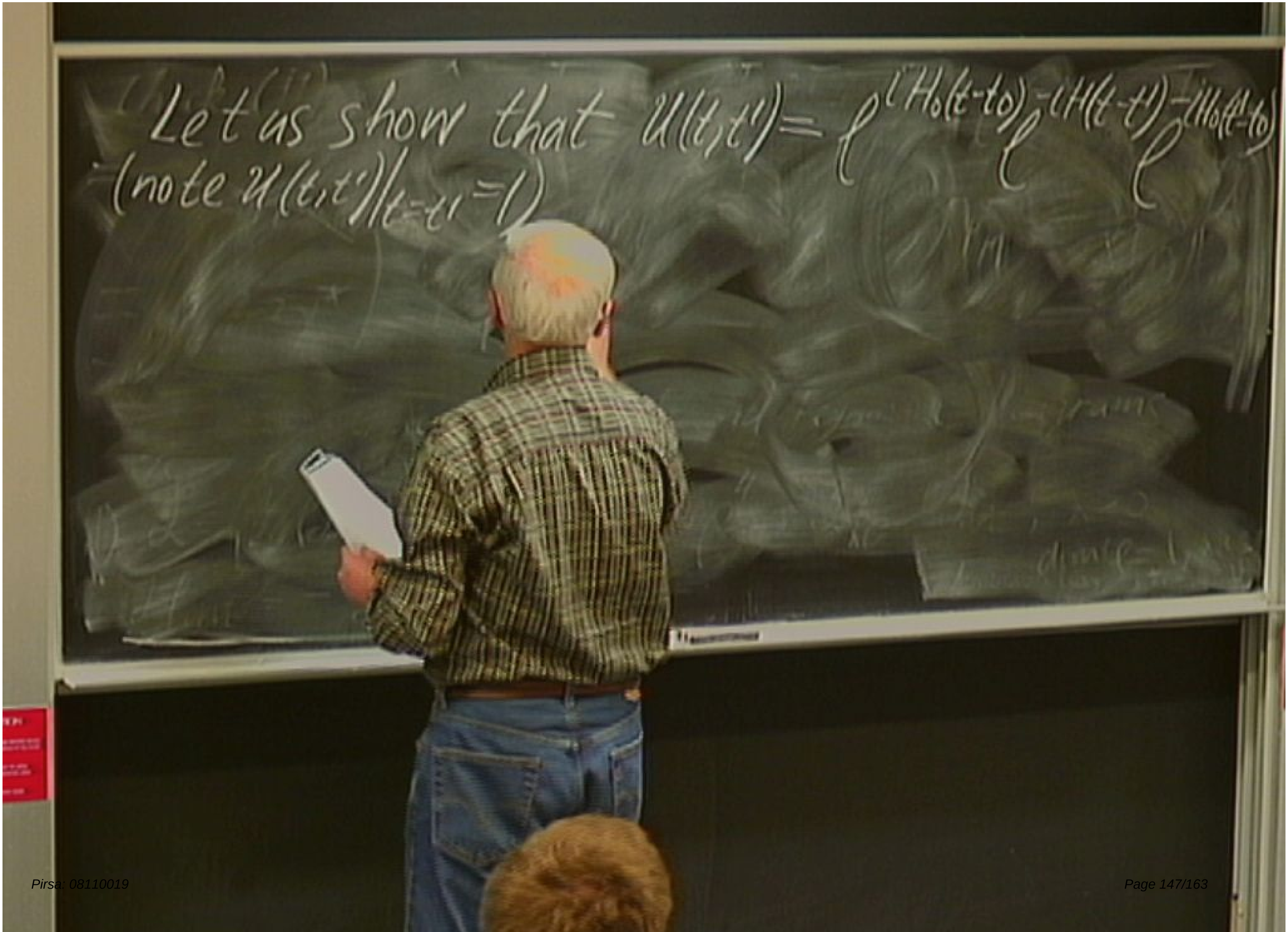
check


Let us show that $U(t,t') = e^{iH_0(t-t_0)}$


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e$


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
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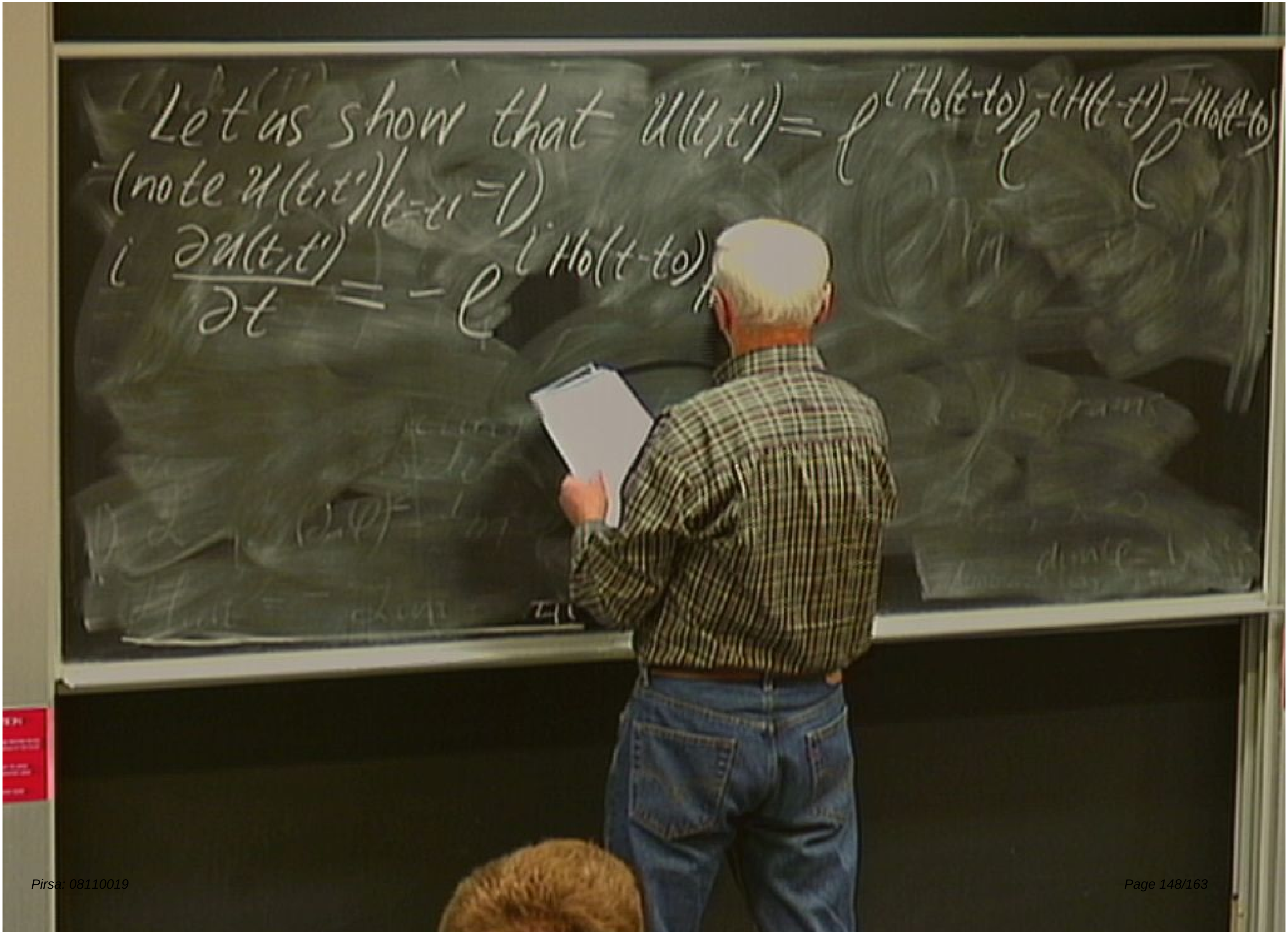
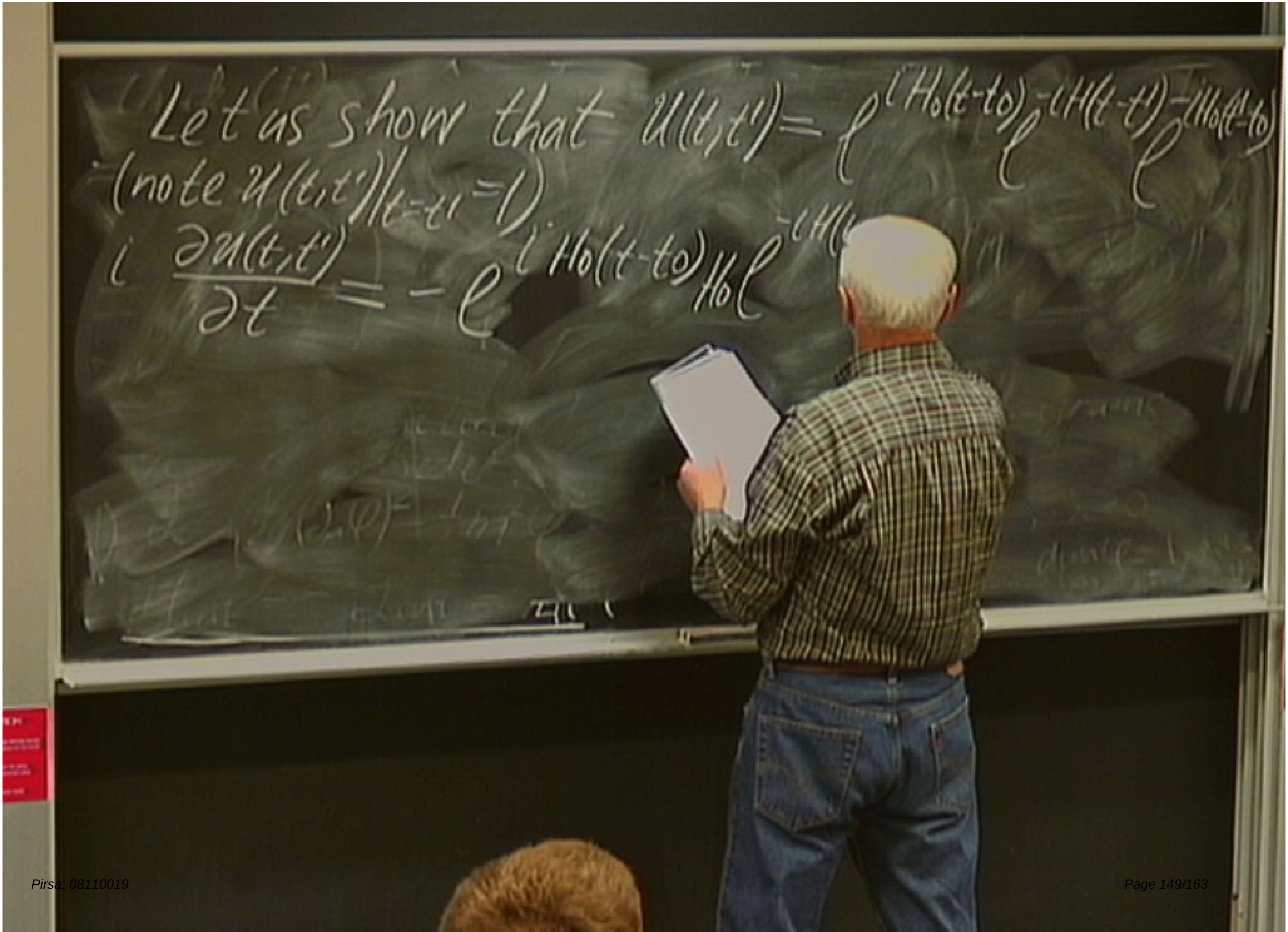
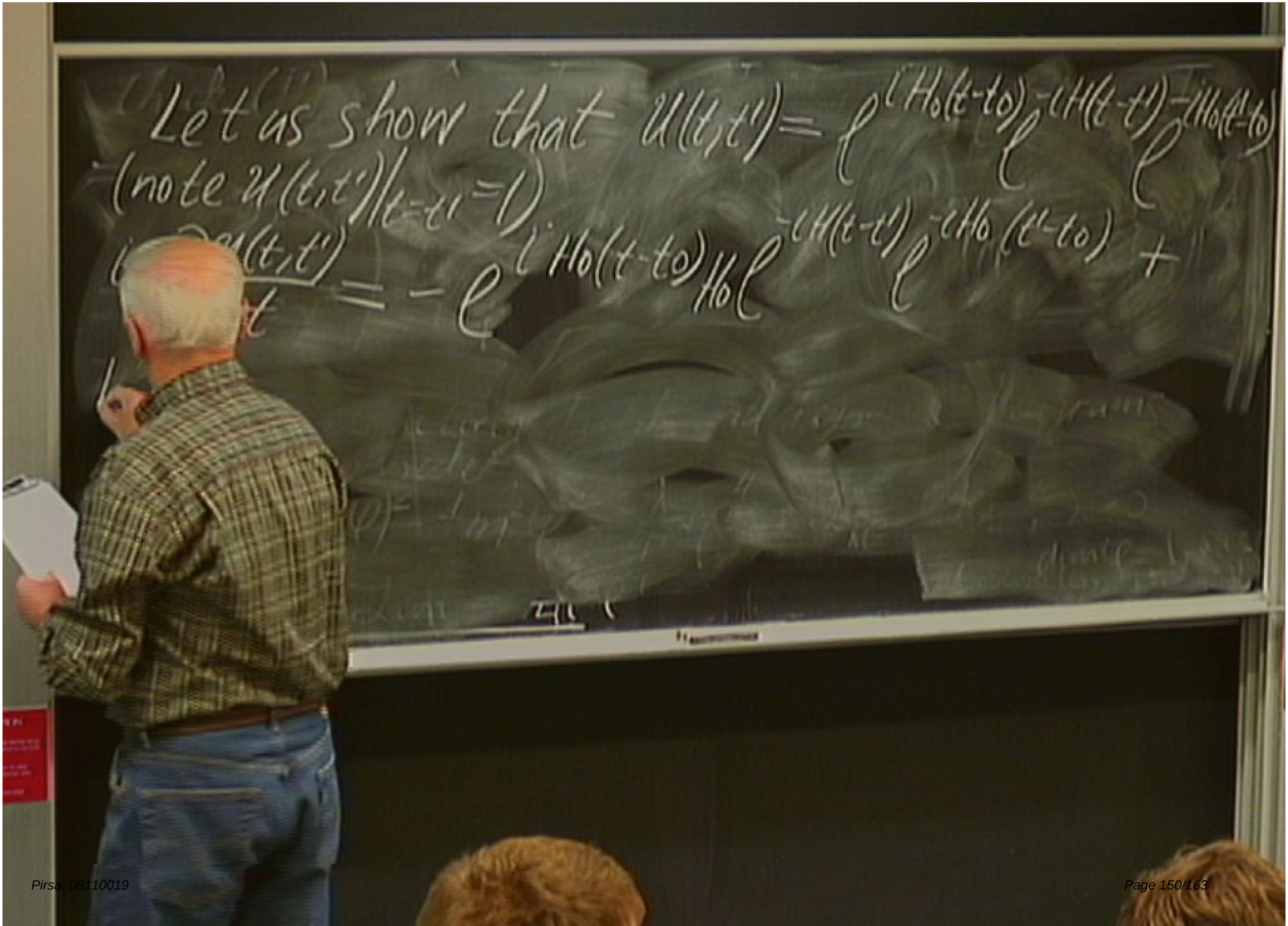
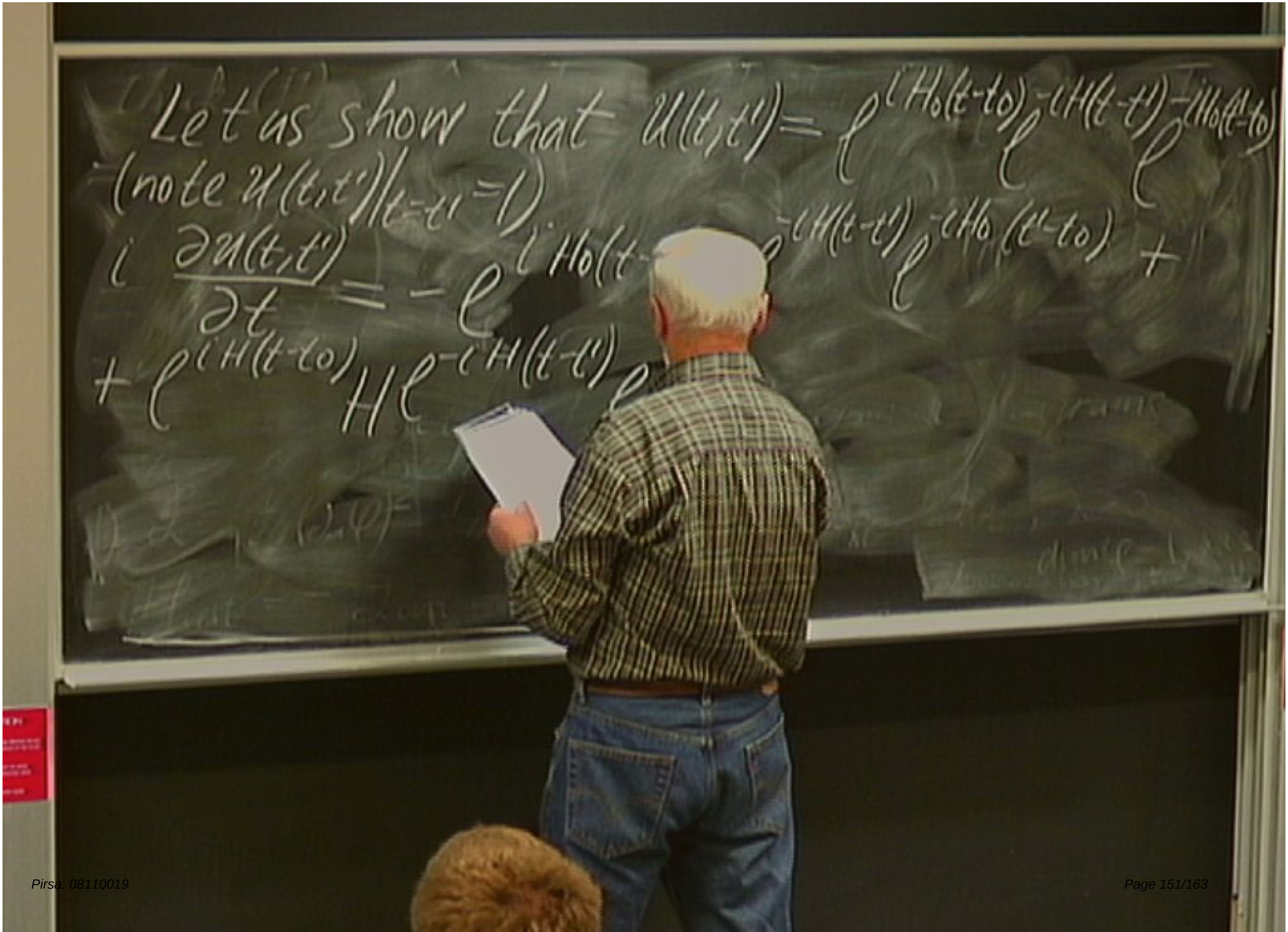
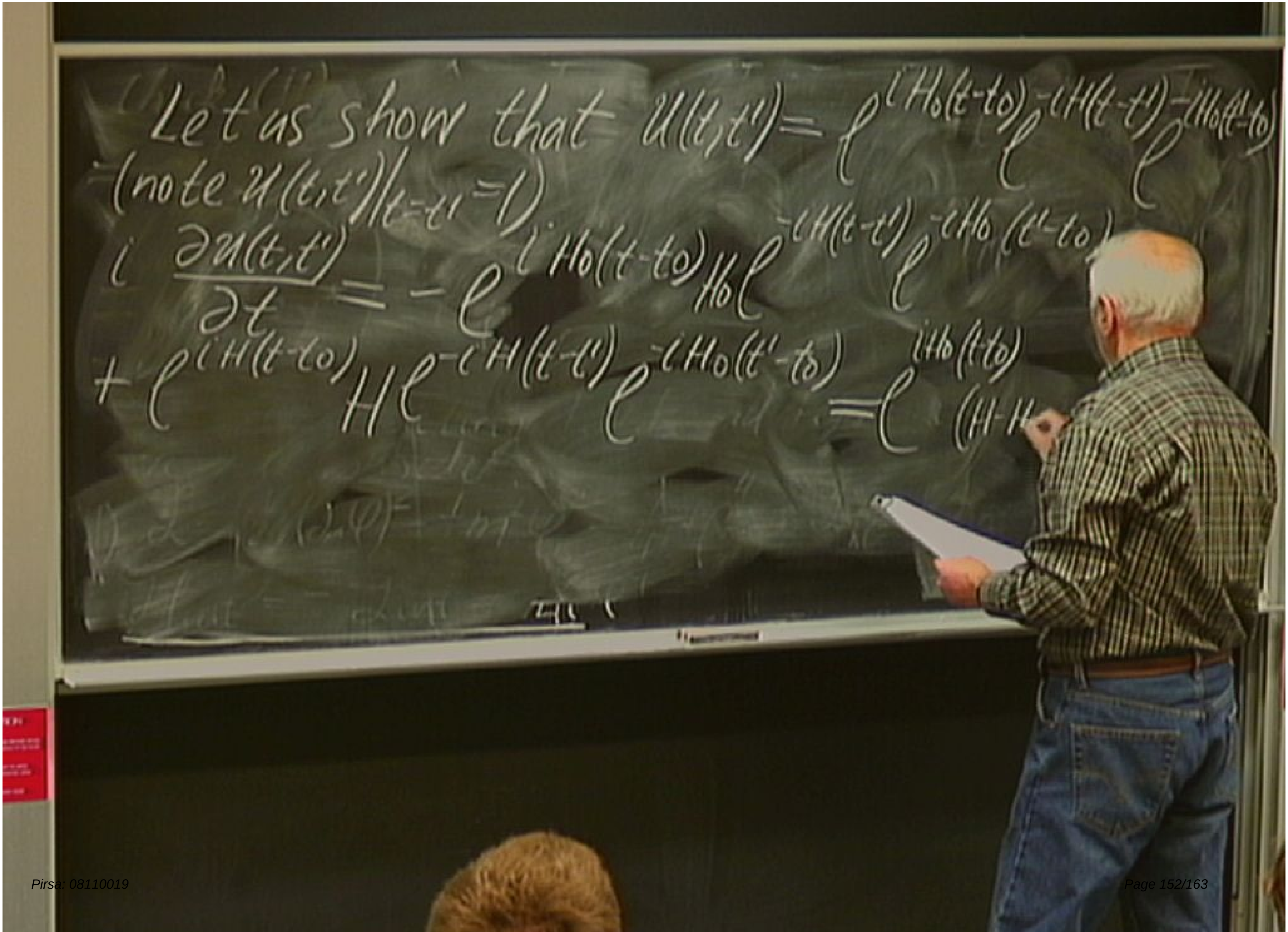
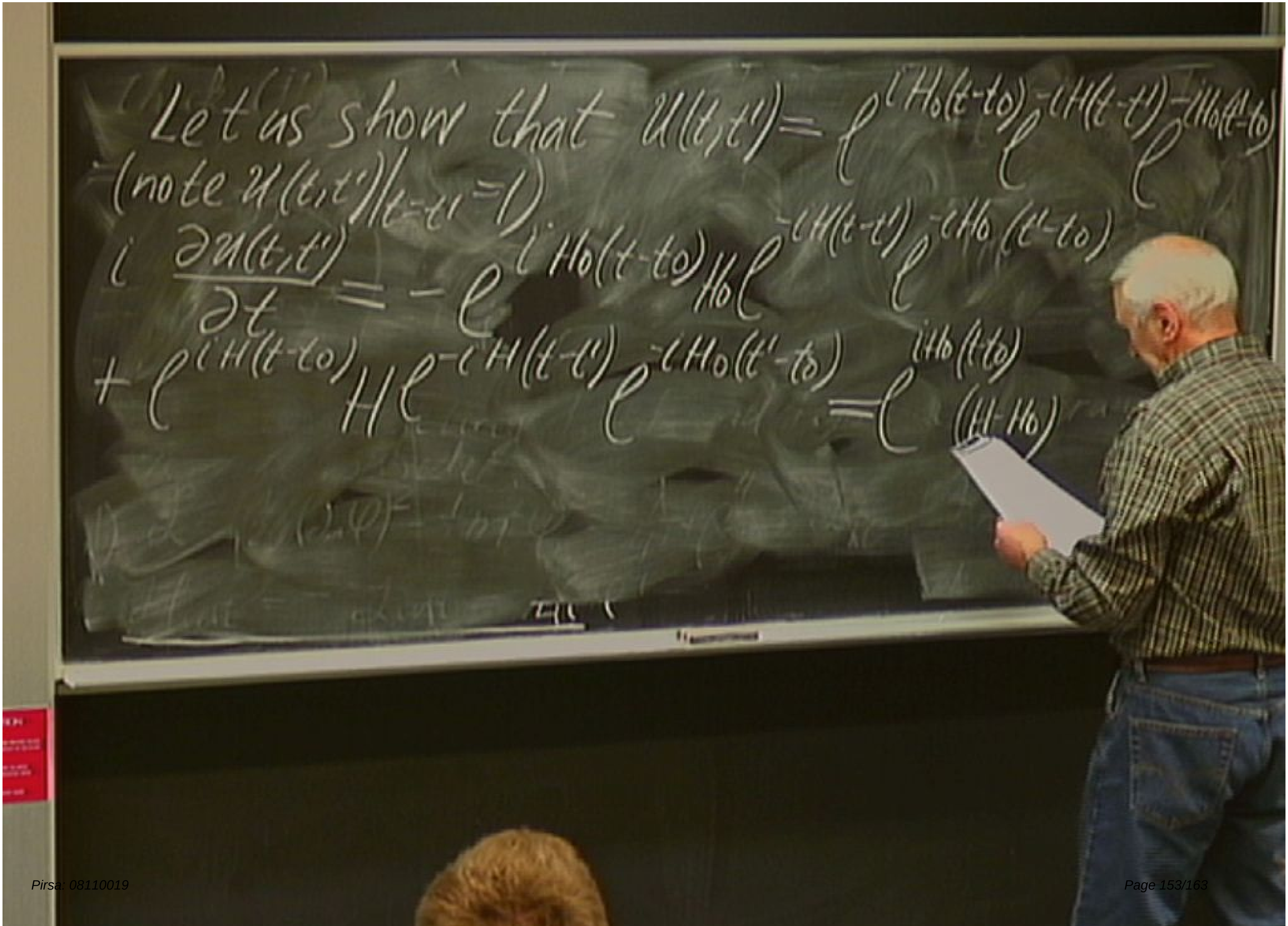

Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)}$

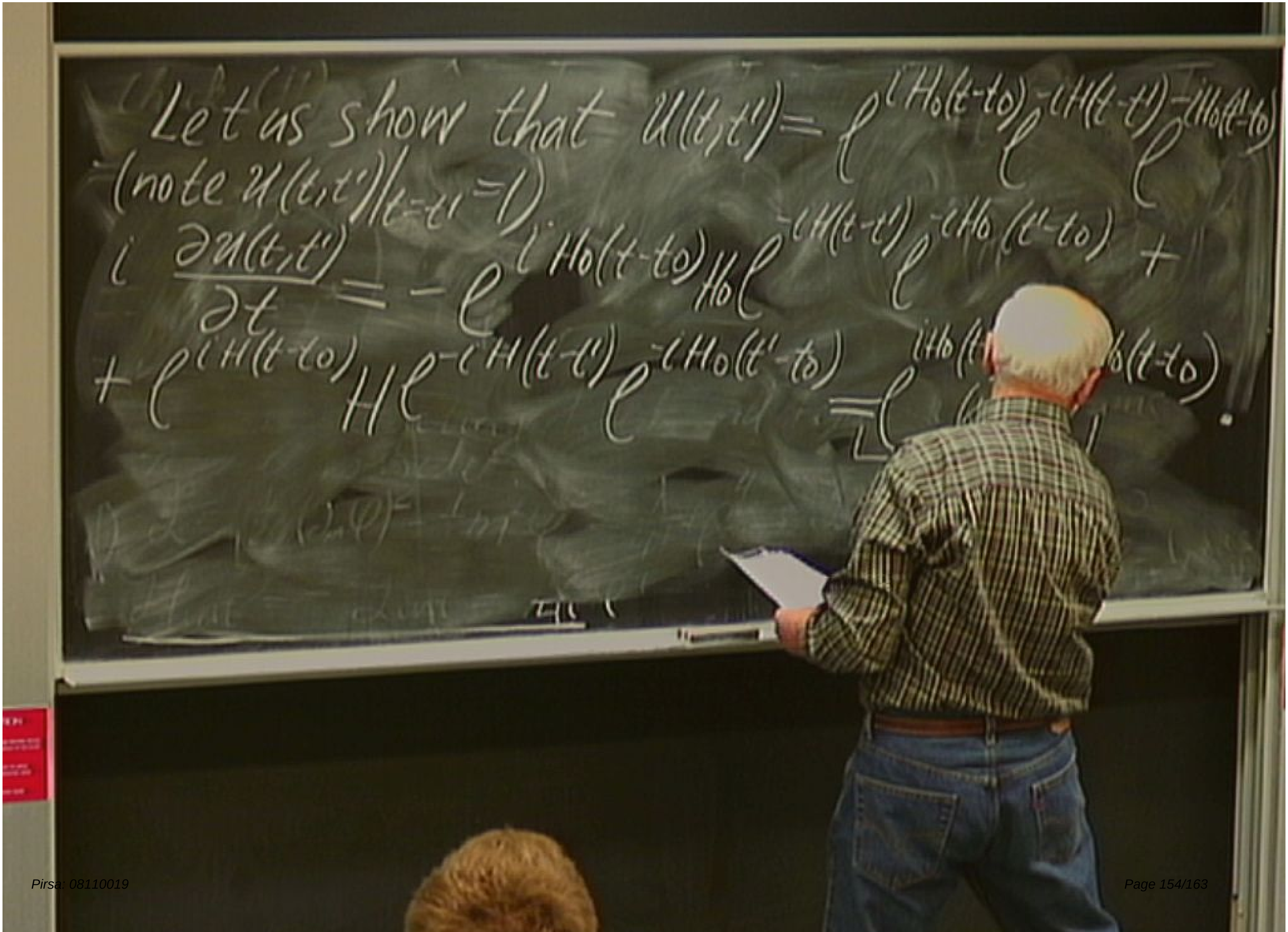

Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)} H_0 e^{-iH(}$

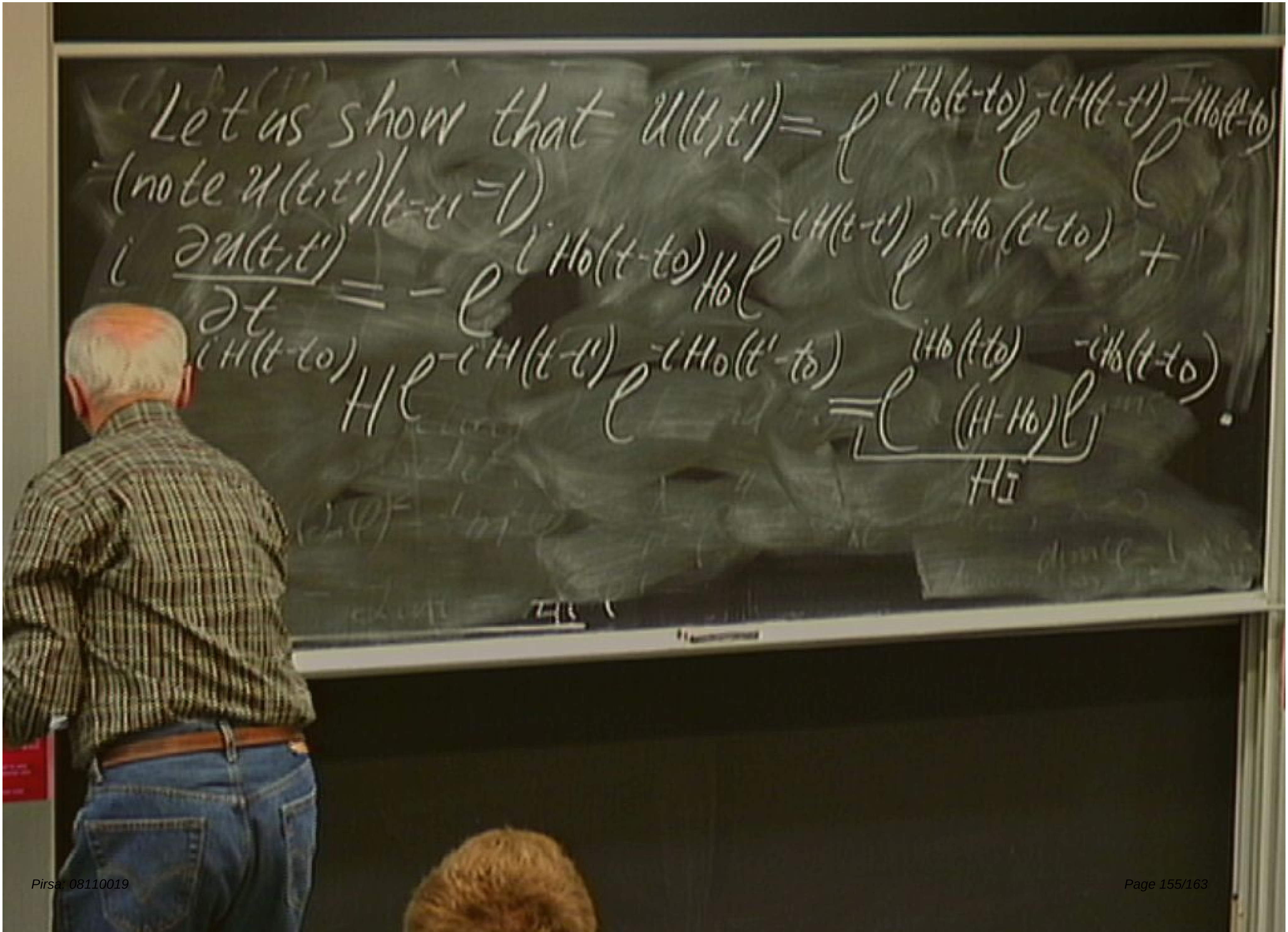

Let us show that U(t,t')= e^{iH0(t-t0)} e^{-iH(t-t')} e^{-iH0(t'-t0)}
(note U(t,t')|t=t' = 1)
i ∂U(t,t')/∂t = - e^{iH0(t-t0)} H0 e^{-iH(t-t')} e^{-iH0(t'-t0)} +

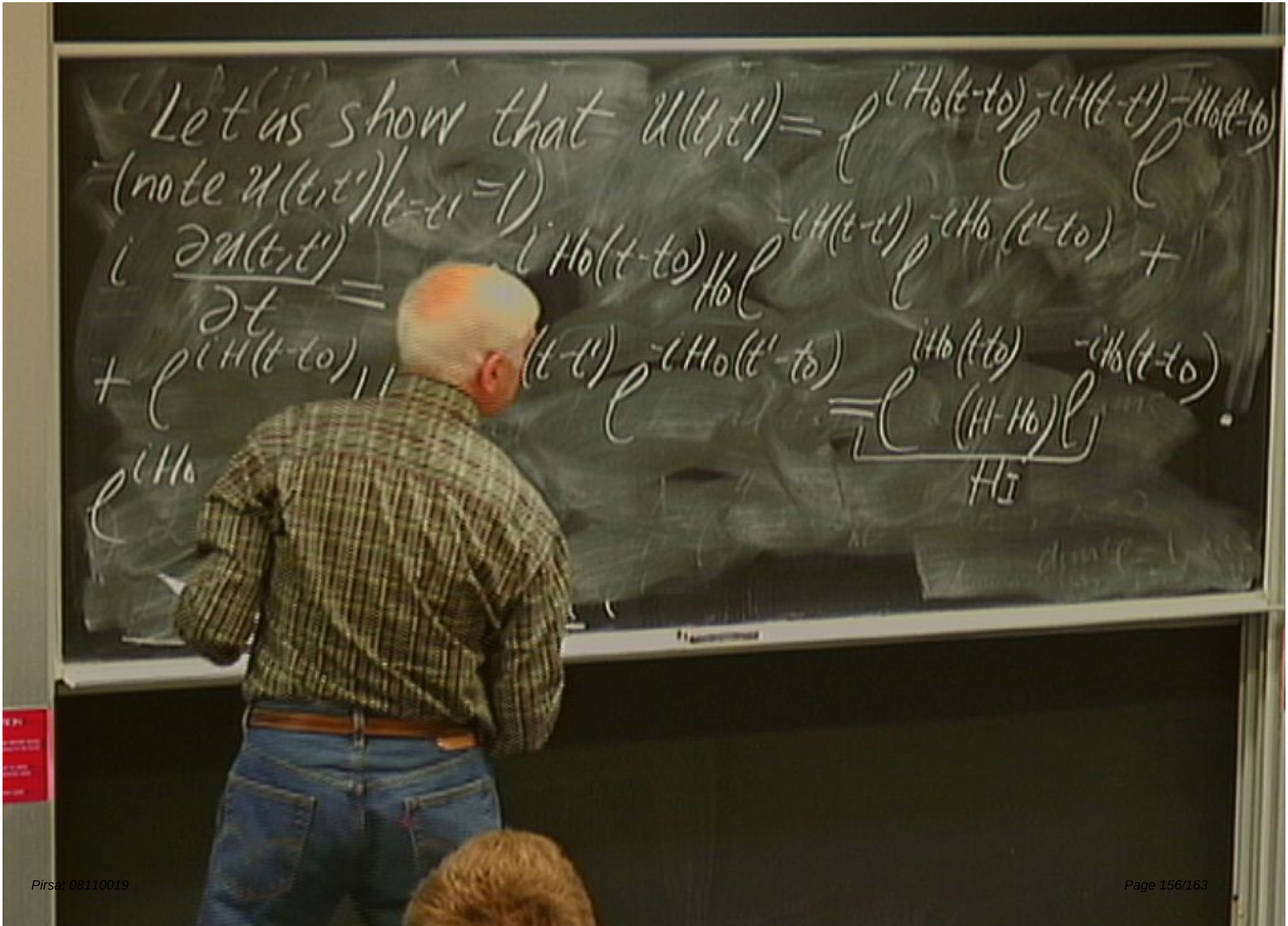

Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-} \quad e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$+ e^{iH(t-t_0)} H e^{-iH(t-t')} e$

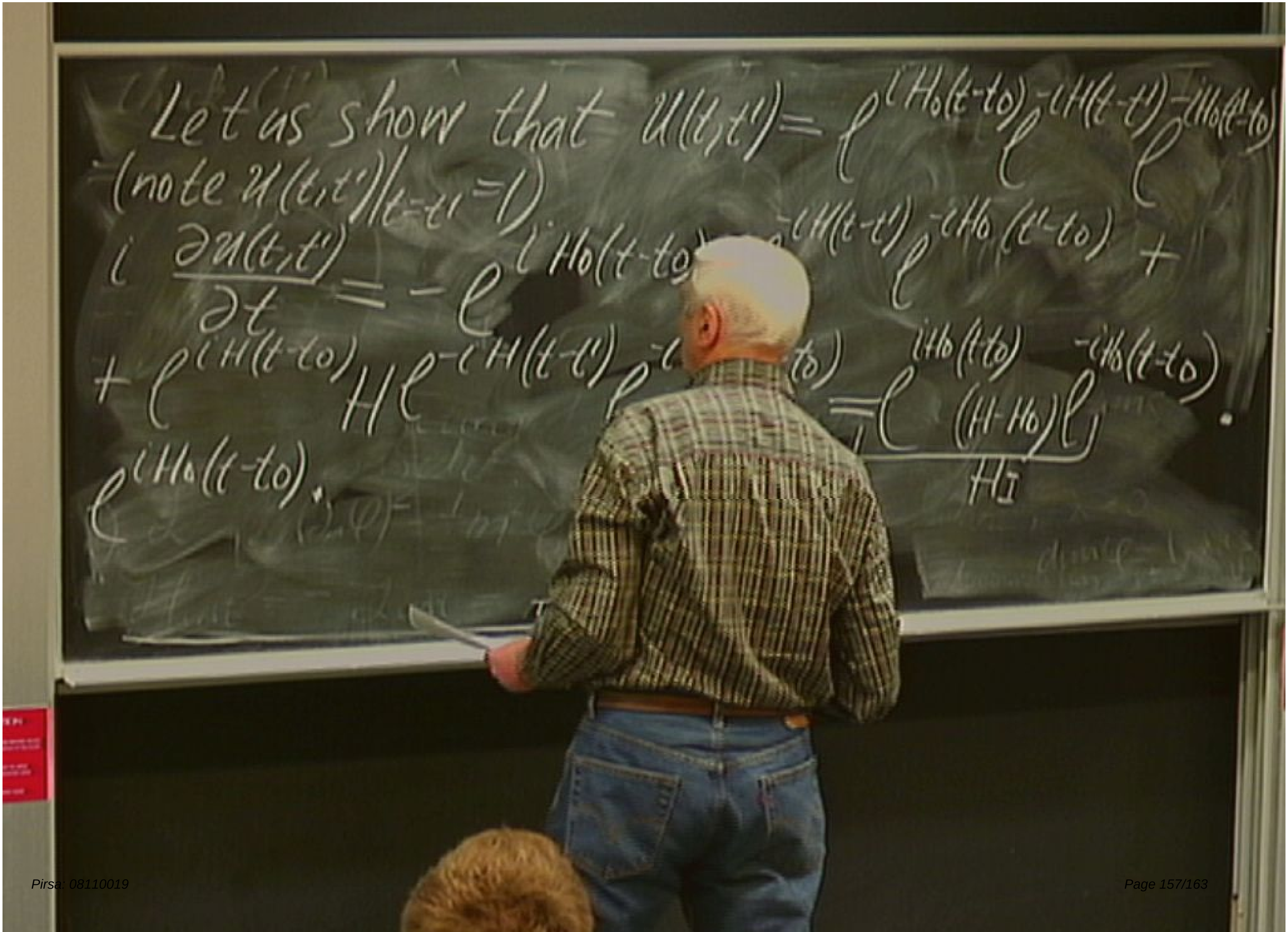

Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = - e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
$+ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} = e^{iH_0(t-t_0)} (H-H$

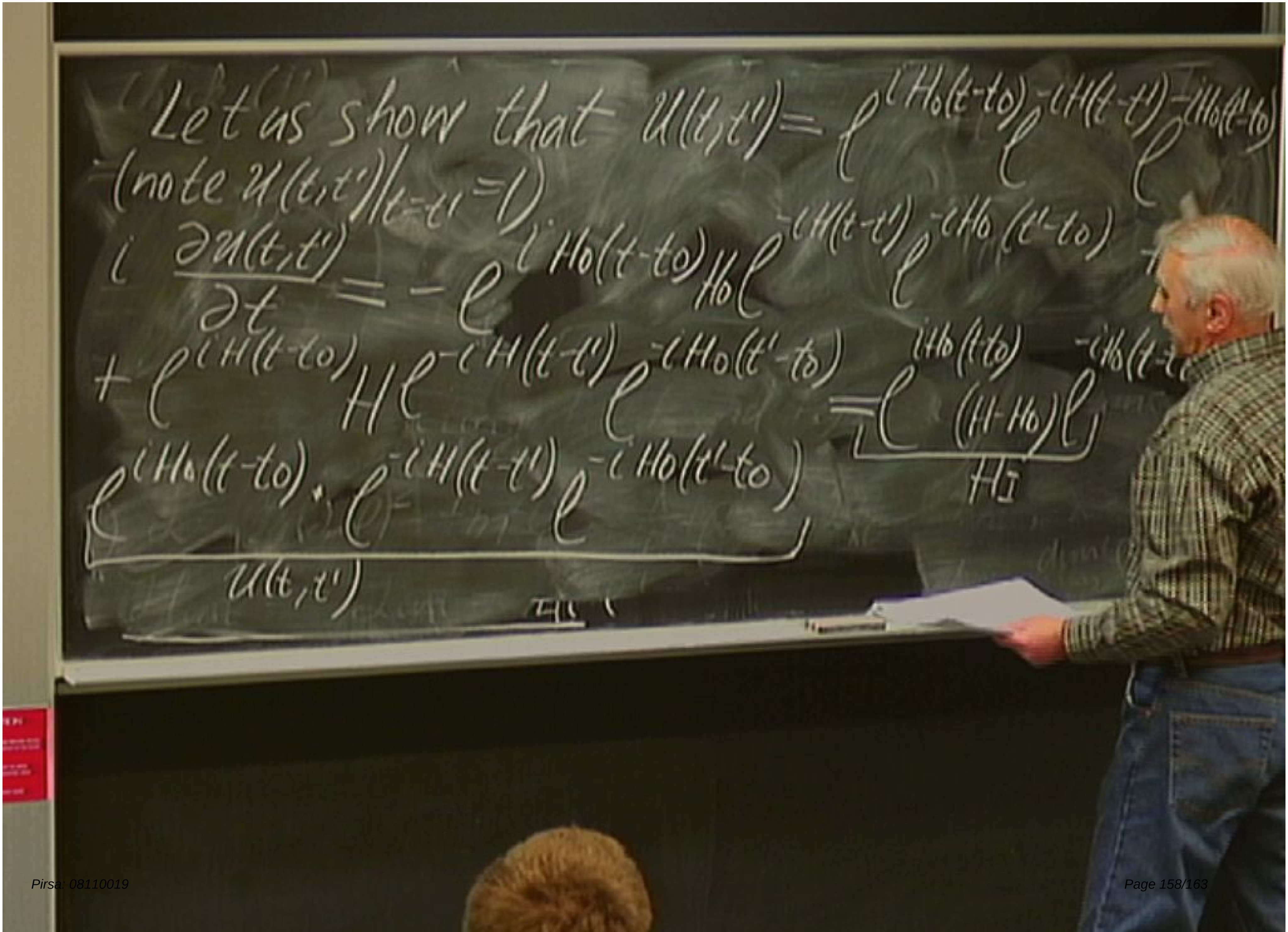

Let us show that U(t,t') = e^{iH₀(t-t₀)} e^{-iH(t-t')} e^{-iH₀(t'-t₀)}
(note U(t,t')|_{t=t'} = 1)
i ∂U(t,t')/∂t = - e^{iH₀(t-t₀)} H₀ e^{-iH(t-t')} e^{-iH₀(t'-t₀)}
+ e^{iH(t-t₀)} H e^{-iH(t-t')} e^{-iH₀(t'-t₀)} = e^{iH₀(t-t₀)} (H-H₀)


Let us show that U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}
(note U(t,t')|_{t=t'} = 1)
i ∂U(t,t')/∂t = − e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +
+ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)}

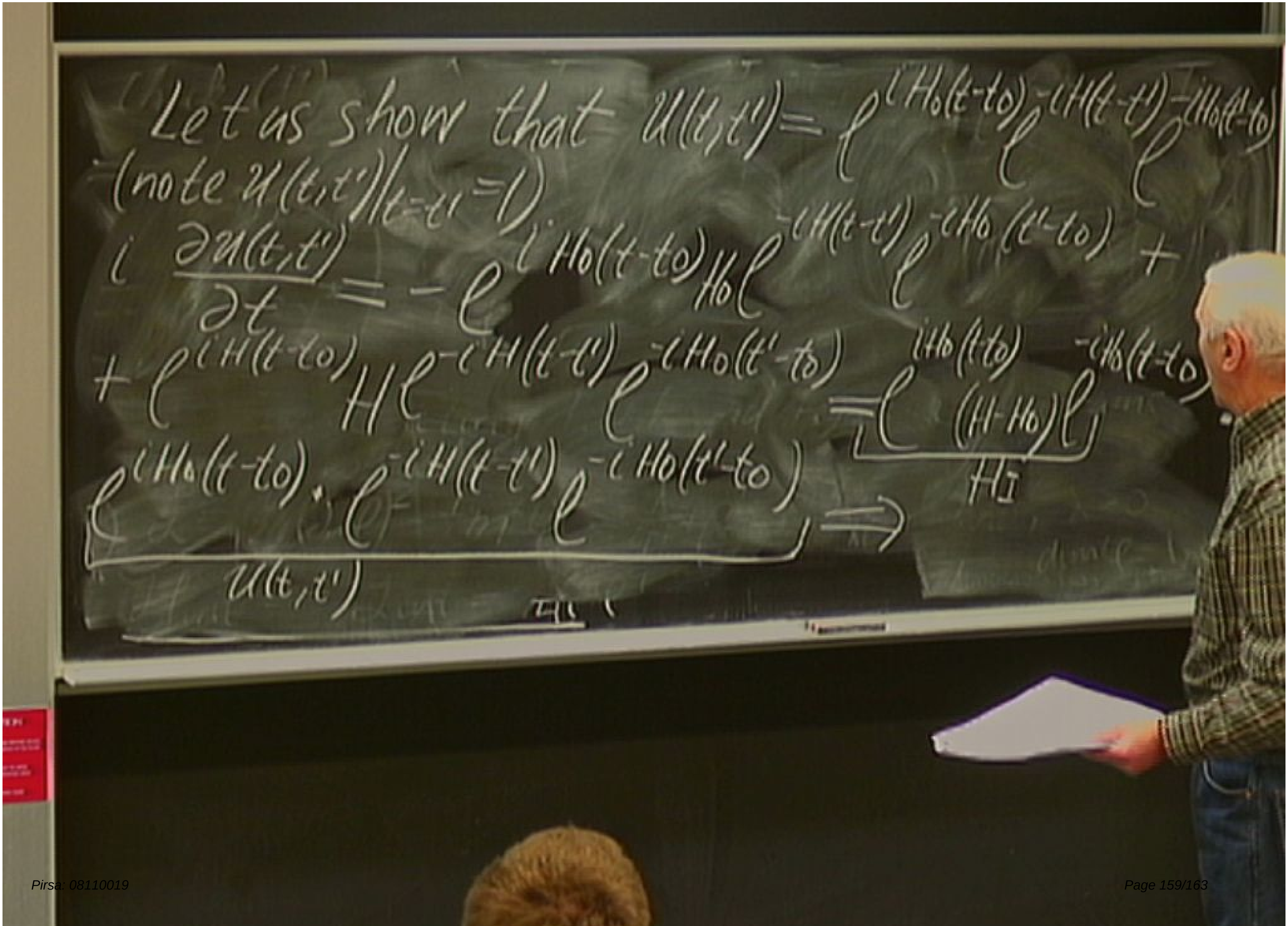

Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} = \underbrace{e^{iH_0(t-t_0)}(H-H_0)e^{-iH_0(t-t_0)}}_{H_I}$


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$+ e^{iH(t-t_0)}$
$e^{-iH_0(t'-t_0)}$
$= e^{iH_0(t-t_0)} \underbrace{(H-H_0)}_{H_I} e^{-iH_0(t-t_0)}$
e^{iH_0}

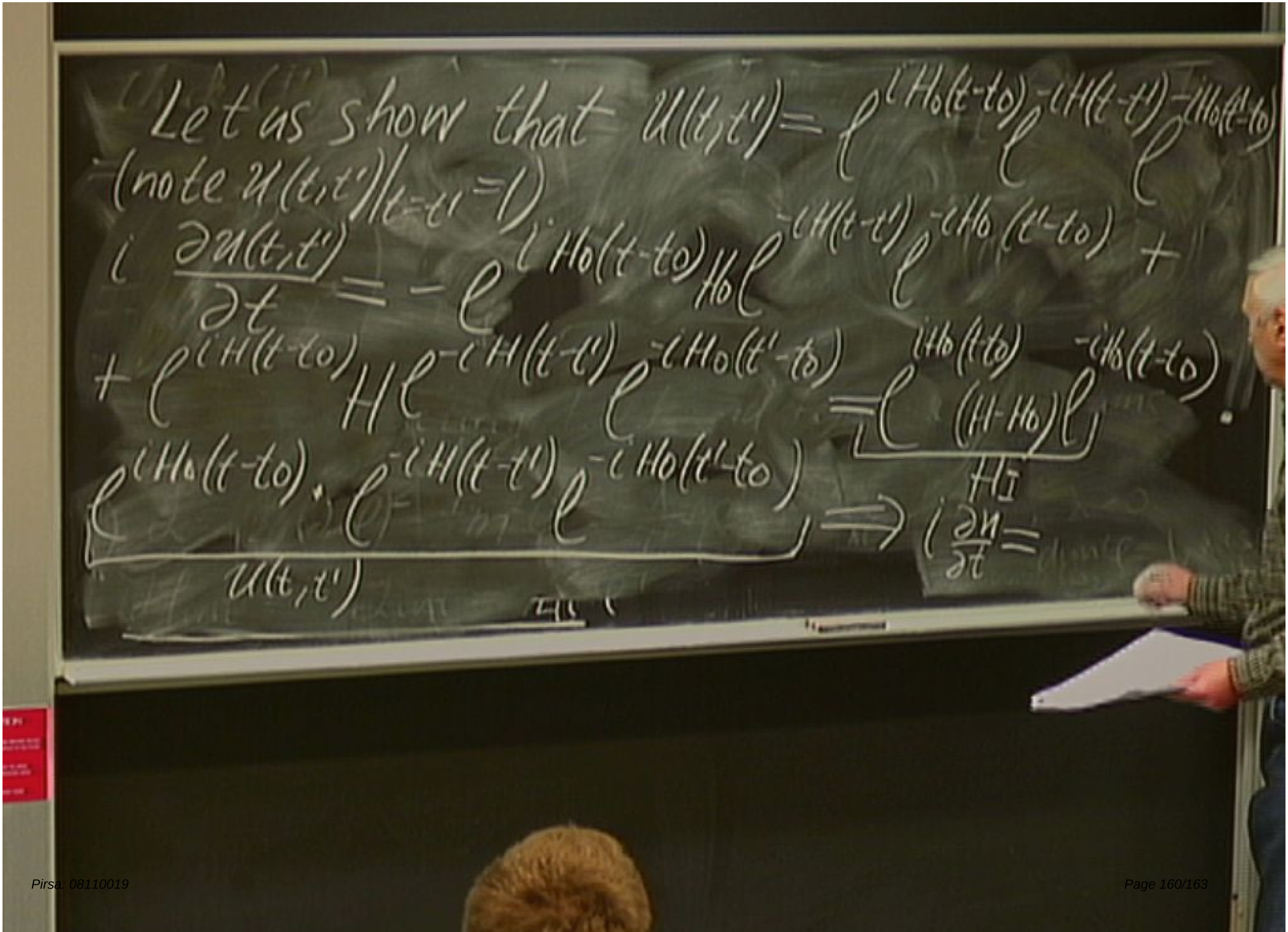

Let us show that U(t,t') = e^{iH0(t-t0)} e^{-iH(t-t')} e^{-iH0(t'-t0)}
(note U(t,t')|t=t' = 1)
i ∂U(t,t')/∂t = - e^{iH0(t-t0)} e^{-iH(t-t')} e^{-iH0(t'-t0)} +
+ e^{iH(t-t0)} H e^{-iH(t-t')} e
= e^{iH0(t-t0)} (H-H0) e^{-iH0(t-t0)}.
HI
e^{iH0(t-t0)}.

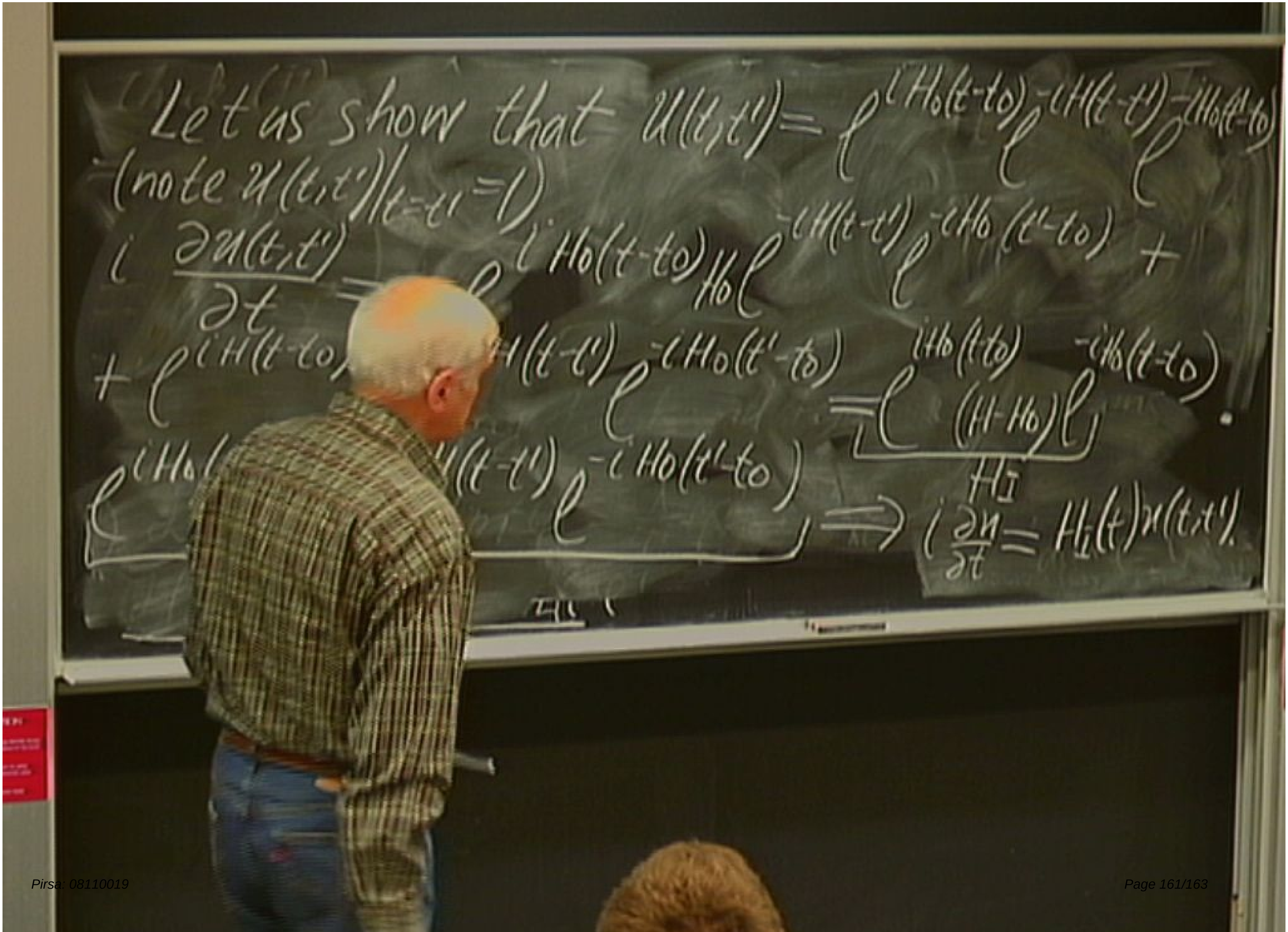


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$+ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
$= \underbrace{e^{iH_0(t-t_0)} (H-H_0) e^{-iH_0(t-t_0)}}_{H_I}$
$\underbrace{e^{iH_0(t-t_0)} \cdot e^{-iH(t-t')} e^{-iH_0(t'-t_0)}}_{U(t,t')}$



Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$+ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} = \underbrace{e^{iH_0(t-t_0)} (H-H_0) e^{-iH_0(t-t_0)}}_{H_I}$
$\underbrace{e^{iH_0(t-t_0)} \cdot e^{-iH(t-t')} e^{-iH_0(t'-t_0)}}_{U(t,t')} \Longrightarrow$


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} = -e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$+ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} = \underbrace{e^{iH_0(t-t_0)} (H-H_0) e^{-iH_0(t-t_0)}}_{H_I}$
$\underbrace{e^{iH_0(t-t_0)} \cdot e^{-iH(t-t')} e^{-iH_0(t'-t_0)}}_{U(t,t')} \Rightarrow i \frac{\partial U}{\partial t} =$


Let us show that $U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
(note $U(t,t')|_{t=t'} = 1$)
$i \frac{\partial U(t,t')}{\partial t} =$
$e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$
$e^{iH_0(t-t_0)}$
$e^{-iH_0(t'-t_0)}$
$= \underbrace{e^{iH_0(t-t_0)} (H-H_0) e^{-iH_0(t-t_0)}}_{H_I}$
$e^{-iH(t-t')}$
$e^{-iH_0(t'-t_0)}$
$\Longrightarrow i \frac{\partial U}{\partial t} = H_I(t) U(t,t')$.

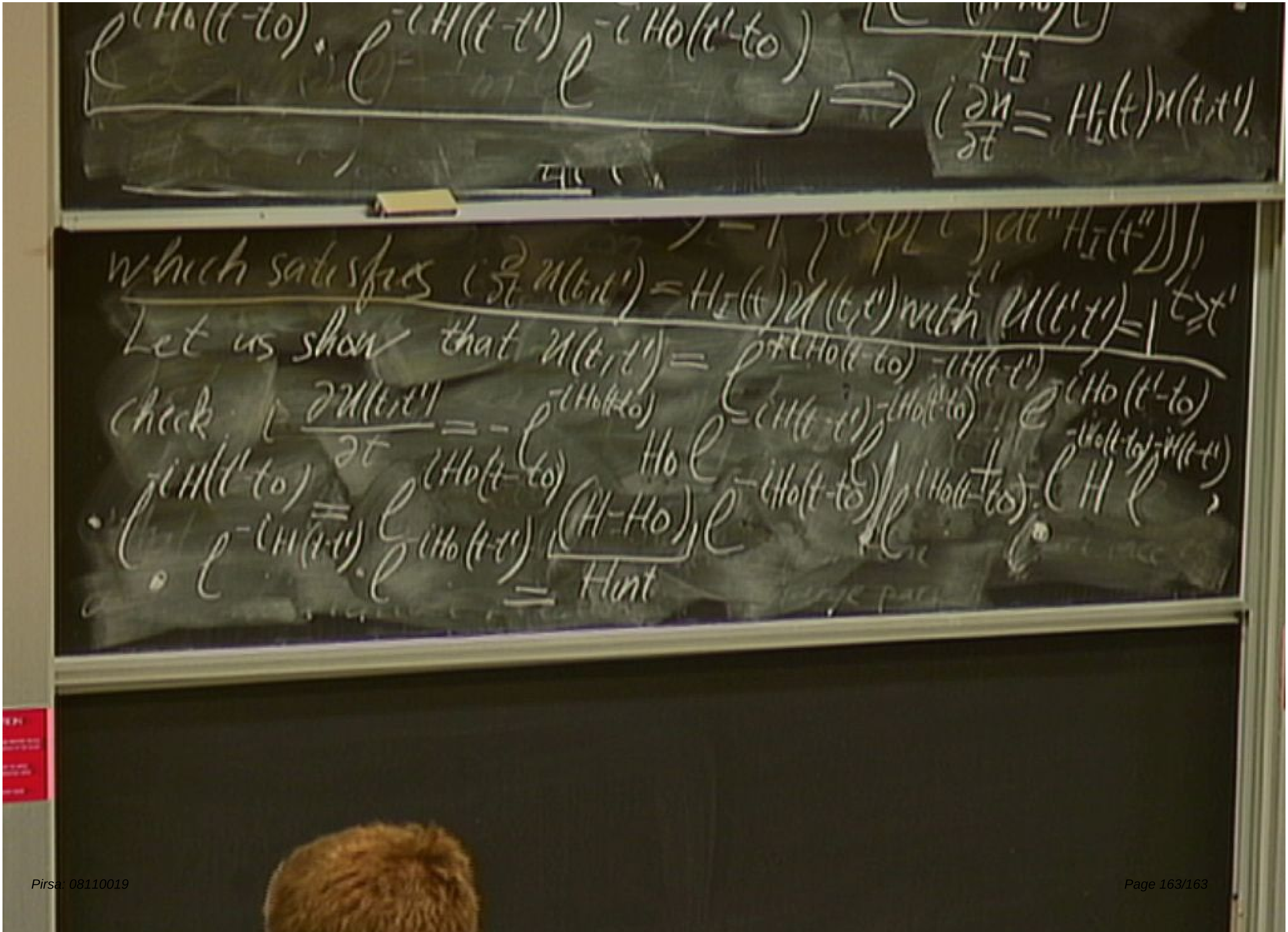
Let us show that $\mathcal{U}(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$

(note $\mathcal{U}(t,t')|_{t=t'} = 1$)

$$i\frac{\partial \mathcal{U}(t,t')}{\partial t} = -e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} +$$

$$+ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} = \underbrace{e^{iH_0(t-t_0)} (H-H_0) e^{-iH_0(t-t_0)}}_{H_I} \cdot$$

$$\underbrace{e^{iH_0(t-t_0)} \cdot e^{-iH(t-t')} e^{-iH_0(t'-t_0)}}_{\mathcal{U}(t,t')} \Longrightarrow i\frac{\partial \mathcal{U}}{\partial t} = H_I(t)\,\mathcal{U}(t,t').$$


which satisfies i ∂/∂t U(t,t') = H_I(t) U(t,t') with U(t',t') = 1   t>t'
Let us show that U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}
check
Hint