

Title: Quantum Field Theory 1 - Lecture 12B

Date: Nov 26, 2008 03:30 PM

URL: <http://pirsa.org/08110019>

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

Same contribution for $\frac{i}{2} \chi^\dagger \not{\partial} \chi \xrightarrow[\text{answer}]{\text{final}} i \chi^\dagger \not{\partial} \chi$; O.E.D.
 Same conclusion is valid for $\frac{i}{2} \bar{\psi}_M (i \not{\partial}) \psi_M$.

(-)-operation

$$C = \eta_c \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \hat{=} -i \gamma_2^T \gamma_0^T$$

$$C^{-1} = \begin{bmatrix} \gamma_0^T \\ \gamma_1^T \\ \gamma_2^T \\ \gamma_3^T \end{bmatrix} = \psi_M \quad (II)$$

for ψ_M coincide from the invariance of charge parity.

η_c ? $\eta_c =$...
 permutation

Same contribution for $\frac{i}{2} \chi^\dagger \not{\partial} \chi \xrightarrow[\text{answer}]{\text{final}} i \chi^\dagger \not{\partial} \chi$; O. E. D.

Same conclusion is valid for $\frac{i}{2} \bar{\psi}_M (\not{\partial}) \psi_M$.

C-conjugation $c^{-1} \psi_M c = \eta_C \bar{\psi}_M$

$$c^{-1} \psi_M c = \eta_C \hat{C} \bar{\psi}_M^T, \quad \hat{C} = -i \gamma^2 \gamma^0 T$$

Let us show that $\hat{C} \bar{\psi}_M^T = \psi_M$ (II)

Fermions and antifermion for ψ_M coincide
What is η_C ? $\eta_C = \pm 1$ ← it follows from the invariance of
anticommutation relations — charge parity

At any fixed time $t=t_0$, we can write $\Psi(t_0, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^\dagger e^{-i\vec{p}\cdot\vec{x}})$. [It is important that in interacting theory, $[\Psi(t_0, \vec{x}), \Pi(t_0, \vec{y})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.]

The Heisenberg operator $\Psi(t, \vec{x})$ is

$$\Psi(t, \vec{x}) = e^{iH(t-t_0)} \Psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$



theory, $[\Psi(t_0, \vec{x}), \Pi(t_0, \vec{x})] = i\delta^3(\vec{x} - \vec{y})$ is still valid. ^{interacting}

The Heisenberg operator $\Psi(t, \vec{x})$ is

$$\Psi(t, \vec{x}) = e^{iH(t-t_0)} \Psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

[The following section of the chalkboard is heavily scribbled out with white chalk, obscuring any text that was written there.]

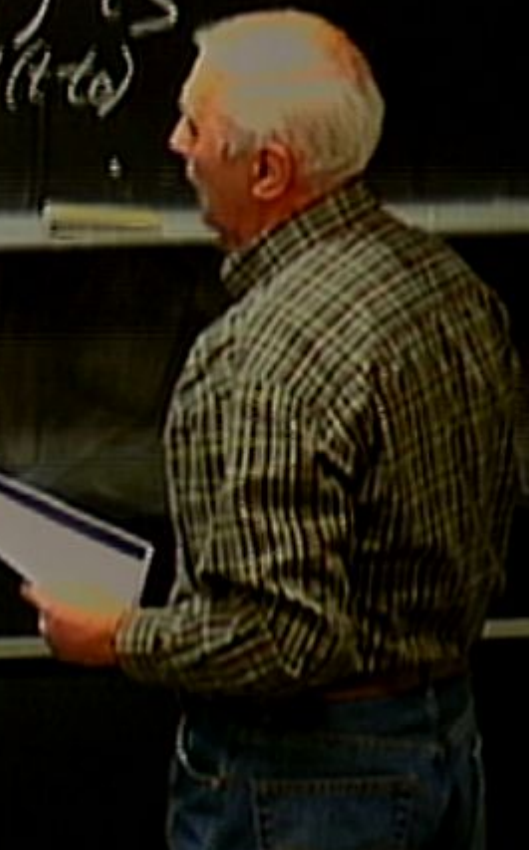
Higher orders in λ ?

Heisenberg field $\psi(x) = e^{iHt} \psi(\vec{x}) e^{-iHt}$ ← Schrodinger operator

At any fixed time $t=t_0$, we can write $\psi(t_0, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_p e^{i\vec{p}\cdot\vec{x}} + a_p^\dagger e^{-i\vec{p}\cdot\vec{x}})$. [It is important that in interacting theory, $[\psi(t_0, \vec{x}), \pi(t_0, \vec{y})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.]

The Heisenberg operator $\psi(t, \vec{x})$ is

$$\psi(t, \vec{x}) = e^{iH(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$



Idea: Introduce new, interaction picture,



Idea: Introduce new, interaction picture, field

ψ_I



Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x})$$



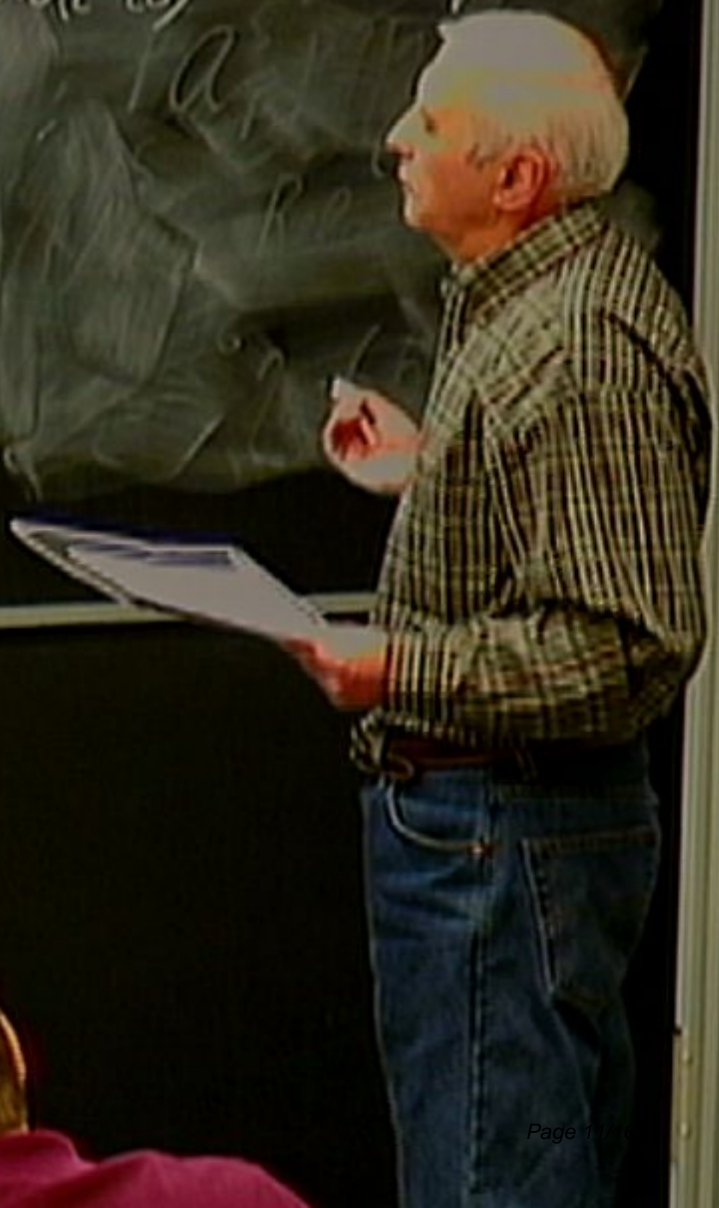
Idea: Introduce new, interaction picture, field:

$$\psi_I(t, \vec{x}) \equiv \mathcal{L}$$



Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$



$(a_{\vec{p}} + a_{\vec{p}}^\dagger)$ [... out in interacting theory, $[\psi(t_0, \vec{x}), \pi(t_0, \vec{x})] = i\delta^3(\vec{x} - \vec{y})$ is still valid.]

The Heisenberg operator $\psi(t, \vec{x})$ is

$$\psi(t, \vec{x}) = e^{iH(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

Idea: Introduce new, interaction picture, field:

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

$(a_{\vec{p}} + a_{\vec{p}}^\dagger)$ [Interaction term out in interacting theory, $[\psi(t_0, \vec{x}), \pi(t_0, \vec{x})] = i\delta^3(\vec{x} - \vec{y})$ is still valid.]

The Heisenberg operator $\psi(t, \vec{x})$ is

$$\psi(t, \vec{x}) = e^{iH(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

$(a_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^\dagger e^{i\vec{p}\cdot\vec{x}})$ [... out in interacting theory, $[\psi(t_0, \vec{x}), \pi(t_0, \vec{x})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.]

The Heisenberg operator $\psi(t, \vec{x})$ is

$$\psi(t, \vec{x}) = e^{iH(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i p \cdot x})$$

Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x}) / \sqrt{p^2 + m^2}$$

theory, $[\psi(t_0, \vec{x}), \pi(t_0, \vec{x})] = i\delta^3(\vec{x}-\vec{y})$ is still valid.

The Heisenberg operator $\psi(t, \vec{x})$ is

$$\psi(t, \vec{x}) = e^{iH(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

Idea: Introduce new, interaction picture, field:

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_p e^{-ipx} + a_p^\dagger e^{ipx})$$

Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)} \quad a \Rightarrow$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x}) \frac{1}{\sqrt{2\sqrt{p^2 + m^2}}}$$

Idea: Introduce new, interaction picture, field

$$\psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)} \Rightarrow$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-i(p \cdot x)} + a_{\vec{p}}^\dagger e^{i(p \cdot x)} \right) \Big|_{x^0 = t - t_0}$$

The Heisenberg operator $\Psi(t, \vec{x})$ is

$$\Psi(t, \vec{x}) = e^{iH(t-t_0)} \Psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

Idea: Introduce new, interaction picture, field:

$$\Psi_I(t, \vec{x}) \equiv e^{iH_0(t-t_0)} \Psi(t, \vec{x}) e^{-iH_0(t-t_0)} \Rightarrow$$

$$\Psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right) \Big|_{x^0 = t - t_0}$$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv e^{iH_0(t-t_0)} \underline{\psi(t_0, \vec{x})} e^{-iH_0(t-t_0)} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_p e^{-iEt + i\vec{p}\cdot\vec{x}} + b_p^\dagger e^{-iEt - i\vec{p}\cdot\vec{x}})$$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right) \Big|_{\substack{p^0 = \sqrt{\vec{p}^2 + m^2} \\ x^0 = t - t_0}}$$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x})$$

$$\psi(t, \vec{x}) = e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_p e^{-i(p \cdot x)} + b_p^\dagger e^{i(p \cdot x)} \right) e^{i(p_0 - t + t_0)}$$

$$\psi(t, \vec{x}) = e^{iH_A(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}$$

$$p_0 = \sqrt{\vec{p}^2 + m^2}$$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right) \Big|_{x^0 = t - t_0}$$

$$\psi(t, \vec{x}) = e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)} \quad \text{errors through } \psi(t, \vec{x})$$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right) \Big|_{\substack{p^0 = \sqrt{\vec{p}^2 + m^2} \\ x^0 = t - t_0}}$$

express
 $\psi(t, \vec{x})$ through
 $\psi_I(t, \vec{x})$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right) \Big|_{p^0 = \sqrt{\vec{p}^2 + m^2}}$$

$$\begin{aligned} \psi(t, \vec{x}) &= e^{iH_0(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)} \\ &= e^{iH(t-t_0)} e^{-iH_0(t-t_0)} \psi_I(t, \vec{x}) e^{iH_0(t-t_0)} \end{aligned}$$

express $\psi(t, \vec{x})$ through $\psi_I(t, \vec{x})$

Idea: Introduce new, interaction picture, field

$$\underline{\psi_I(t, \vec{x})} \equiv \underline{\psi(t_0, \vec{x})} e^{-iH_0(t-t_0)} \implies$$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_p e^{-i p \cdot x} + a_p^\dagger e^{i p \cdot x})$$

$$\psi(t, \vec{x}) = e^{iH(t-t_0)} \psi(t_0, \vec{x}) e^{-iH(t-t_0)} \text{ errors}$$

$$= e^{iH(t-t_0)} \underbrace{e^{-iH_0(t-t_0)} \psi_I(t, \vec{x}) e^{iH_0(t-t_0)}}_{\psi(t_0, \vec{x})} e^{-i(H-H_0)(t-t_0)} \psi(t_0, \vec{x})$$

Idea: Introduce new, interaction picture, field

$$\underline{\Psi_I(t, \vec{x})} \equiv \underline{\Psi(t_0, \vec{x})} e^{-iH_0(t-t_0)}$$

$$\Psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x})$$

$$\Psi(t, \vec{x}) = e^{iH_0(t-t_0)} \Psi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

$$= e^{iH(t-t_0)} \underbrace{e^{-iH_0(t-t_0)} \Psi_I(t, \vec{x}) e^{iH_0(t-t_0)}}_{\Psi(t_0, \vec{x})} e^{-iH(t-t_0)}$$

Idea: Introduce new, interaction picture, field

$$\underline{\Psi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \Psi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\Psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \left(a_{\vec{p}} e^{-iE_p t + i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^\dagger e^{iE_p t - i\vec{p}\cdot\vec{x}} \right) \frac{1}{\sqrt{E_p + m^2}}$$

$$\Psi(t, \vec{x}) = e^{iH_0(t-t_0)} \Psi(t_0, \vec{x}) e^{-iH(t-t_0)} \quad \text{errors} \quad \boxed{x^0 = t - t_0}$$

$$= \underbrace{e^{iH(t-t_0)} e^{-iH_0(t-t_0)} \Psi_I(t, \vec{x}) e^{iH_0(t-t_0)}}_{\Psi(t_0, \vec{x})} e^{-i(H-H_0)(t-t_0)} \Psi_I(t, \vec{x}) = \Psi_I(t, \vec{x})$$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$\alpha = \frac{1}{m} \frac{d\mathbf{p}}{dt}$

Mass term $m \bar{\psi} \psi$ in (t, \vec{x}) (χ)

$$\equiv U^\dagger(t, t_0) \left(\psi(t, \vec{x}) U(t, t_0), U(t, t_0) \right) =$$

$m \bar{\psi} \psi + m \bar{\psi} \psi$, we know that

Then the combination

$$\psi \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \psi$$

(compare with)

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$\alpha = \gamma^0 \gamma^i$

Mass term $m \bar{\psi} \psi$ to (t, \vec{x}) (λ)

$$\equiv U^\dagger(t, t_0) \left(\psi_I(t, \vec{x}) U(t, t_0), U(t, t_0) \right) = \begin{pmatrix} e^{iH_0(t-t_0)} & \\ & e^{-iH(t-t_0)} \end{pmatrix}$$

$$m \bar{\psi} \psi + m \bar{\psi} \psi$$

we know that

then the combination

$$\psi_L = \psi \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$\alpha = \gamma^0 \gamma^i$

Mass term $m \bar{\psi} \psi$ to (t, \vec{x}) (λ)

$$\equiv U^\dagger(t, t_0) U(t, \vec{x}) U(t, t_0) U(t, t_0) = \left\{ \begin{matrix} iH_0(t-t_0) \\ -iH(t-t_0) \end{matrix} \right\}$$

we know that

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term $m \bar{\psi} \psi$ by (set $\psi = \begin{pmatrix} \chi \\ \lambda \end{pmatrix}$)

U $U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0), U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right)$ is called interaction picture evolution



fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term $m \bar{\psi} \psi$... (λ)

$$\equiv U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0), \quad U(t, t_0) = e^{-iH_0(t-t_0)}$$

called interaction picture evolution operator

[Faded handwritten notes and equations on the chalkboard]



fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Mass term $m \bar{\psi} \psi$ in (set ψ) (λ)

$U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0), U(t, t_0) = \mathcal{P} \exp \left(\int_{t_0}^t iH_0(t-t_0) - iH(t-t_0) dt \right)$ is called interaction picture evolution operator. It



fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Mass term $m \bar{\psi} \psi$

in (set ψ) (λ)

$\equiv U^\dagger(t, t_0) (\psi(t, \vec{x})) U(t, t_0), U(t, t_0) = \mathcal{P} \exp \left(\int_{t_0}^t iH(t-t_0) dt \right)$ is called interaction structure evolution operator. It satisfies a relatively simple equation.

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term = $m \bar{\psi} \psi$... (λ)

$\equiv U^\dagger(t, t_0) \psi(t_0) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right)$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) =$$

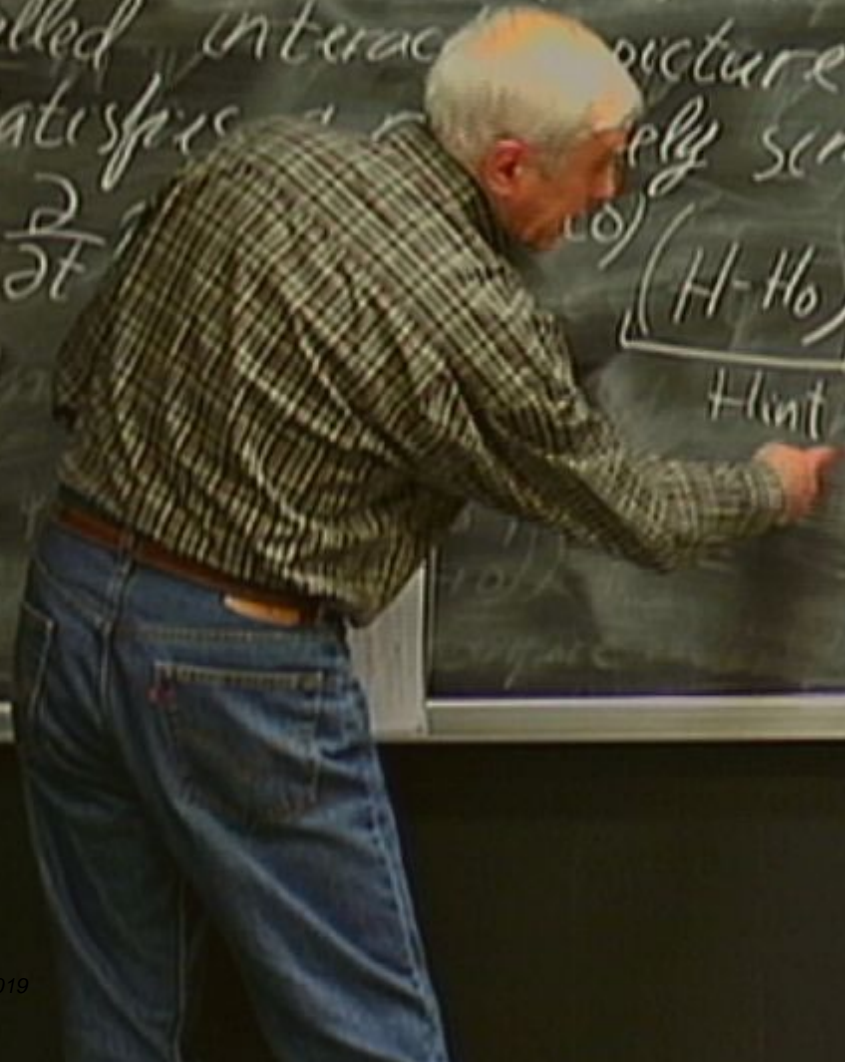
fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{\hbar^2} \psi^\dagger \psi$ $m(\psi^\dagger \psi)$ (λ)

$\equiv U^\dagger(t, t_0) \psi_I(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp\left(-i \int_{t_0}^t H(t') dt'\right)$ is called interaction picture evolution operator. It satisfies a very simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = (H - H_0) U(t, t_0)$$



fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{\hbar} \bar{\psi} \psi$

$\equiv U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \left(e^{iH_0(t-t_0)} e^{-iH(t-t_0)} \right)$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = \mathcal{P} \left(e^{iH_0(t-t_0)} (H - H_0) e^{-iH(t-t_0)} \right)$$

Hint

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{\hbar} \bar{\psi} \psi$ $m(\psi^\dagger \psi)$ (λ)

$\equiv U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right)$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right) (H - H_0) \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right) = \mathcal{P} \exp \left(-i \int_{t_0}^t H_{int}(t') dt' \right)$$

recombination
 $\psi = \psi \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
 (compact notation)

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi} \psi$ $m(\psi^\dagger \psi)$ (λ)

$\equiv U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0) = \left(e^{iH_0(t-t_0)} e^{-iH(t-t_0)} \right)$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = e^{iH_0(t-t_0)} (H - H_0) e^{-iH_0(t-t_0)} U(t, t_0)$$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $= \frac{m}{2} \bar{\psi} \psi = m (\psi^\dagger \psi)$

$$\equiv U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0), \quad U(t, t_0) = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$$

called interaction picture evolution operator
satisfies a relatively simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = e^{iH_0(t-t_0)} (H - H_0) e^{-iH(t-t_0)} = e^{iH_0(t-t_0)} H_{int} e^{-iH(t-t_0)}$$



fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi} \psi = -m (\psi^\dagger \cdot \psi)_{\alpha\beta} (\lambda)$

$\equiv U^\dagger(t, t_0) U(t, t_0) U(t, t_0)$; $U(t, t_0) = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$ is

called interaction picture evolution operator. It satisfies a relatively simple equation:

$$i \frac{d}{dt} U(t, t_0) = -i H_{int}(t) U(t, t_0)$$



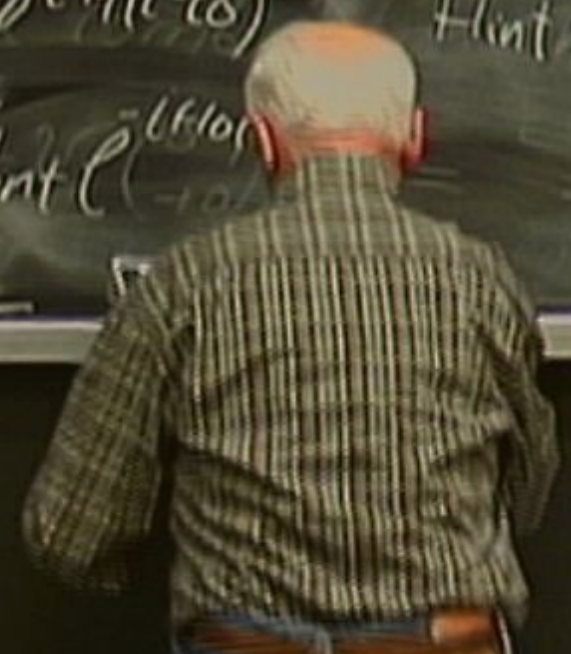
fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi} \psi = -m (\psi^\dagger, \psi^T) \psi (\chi)$

$\equiv U^\dagger(t, t_0) \psi(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t_0)} \right\}$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = \mathcal{P} \left\{ e^{iH_0(t-t_0)} (H - H_0) e^{-iH(t-t_0)} \right\} = \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_{int} e^{-iH_0(t-t_0)} \right\}$$



fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi}_m \psi_m = -\frac{m}{2} (\psi^+ \cdot \psi^T)_{10} (\lambda)$

$\equiv U^\dagger(t, t_0) U(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(\int_{t_0}^t (H_0(t') - iH(t')) dt' \right)$ is

called interaction picture evolution operator. It satisfies a very simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = (H - H_0) U(t, t_0) \quad \Rightarrow \quad \frac{\partial}{\partial t} U(t, t_0) = -i(H - H_0) U(t, t_0)$$

$\Rightarrow U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t (H - H_0) dt' \right)$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi}_m \psi_m = -\frac{m}{2} (\psi^+ \cdot \psi^T)_{10} (\lambda)$

$\equiv U^\dagger(t, t_0) \mathcal{U}_I(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(\int_{t_0}^t (H_0(t') - iH(t')) dt' \right)$ is

called interaction picture evolution operator. It satisfies a relatively simple equation:

$$i \frac{d}{dt} U(t, t_0) = (H - H_0) U(t, t_0)$$

$$U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t (H - H_0) dt' \right)$$

$$U(t, t_0) = \int d^3x \frac{\lambda}{4i} \mathcal{U}_I$$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi}_m \psi_m = -\frac{m}{2} (\psi^+ \cdot \psi^T)_{10} (\lambda)$

$\equiv U^\dagger(t, t_0) U_I(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(i \int_{t_0}^t H_0(t') dt' - i \int_{t_0}^t H_I(t') dt' \right)$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = \mathcal{P} \exp \left(i \int_{t_0}^t H_0(t') dt' \right) (H - H_0) \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right) = \mathcal{P} \exp \left(i \int_{t_0}^t H_0(t') dt' \right) H_{int} \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right)$$

$$= \int d^3x \frac{\lambda}{4!} \phi^4$$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Mass term: $-\frac{m}{2} \bar{\psi}_m \psi_m = -\frac{m}{2} (\psi^+ \cdot \psi^T)_{10} (\lambda)$

$\equiv U^\dagger(t, t_0) U_I(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t_0)} \right\}$ is called interaction picture evolution operator. It satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = \mathcal{P} \left\{ e^{iH_0(t-t_0)} (H - H_0) e^{-iH(t-t_0)} \right\} = \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_{int} e^{-iH_0(t-t_0)} \right\}$$

$$\mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t_0)} \right\} = H_{int}$$

$$\mathcal{P} \left\{ e^{iH_0(t-t_0)} H_{int} e^{-iH_0(t-t_0)} \right\} = \int d^3x \frac{\lambda}{4!} \phi^4$$

Mass term: $-\frac{m}{2} \bar{\psi}_M \psi_M = -\frac{m}{2} (\psi^\dagger - i \gamma^5 \psi) \psi$

$\equiv U^\dagger(t, t_0) \psi(t_0) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H(t') dt' \right)$ is called interaction picture evolution operator. It satisfies a simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = -i H_I(t) U(t, t_0)$$

$\mathcal{P} \exp \left(-i \int_{t_0}^t H_I(t') dt' \right)$

$$U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H_I(t') dt' \right)$$

$$H_I \equiv H_I(t)$$

Mass term: $-\frac{m}{2} \bar{\psi}_M \psi_M = -\frac{m}{2} (\bar{\chi}^+ - i \bar{\chi}^T \sigma_2) \psi_0 (\chi)$

$\equiv U^\dagger(t, t_0) H_I(t, \vec{x}) U(t, t_0)$, $U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H_0(t') dt' - i \int_{t_0}^t H_I(t') dt' \right)$ is called interaction picture evolution operator. It satisfies a relatively simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = (H - H_0) U(t, t_0) \Rightarrow \frac{\partial}{\partial t} U(t, t_0) = -i H_{int}(t) U(t, t_0)$$

$$U(t, t_0) = \mathcal{P} \exp \left(-i \int_{t_0}^t H_{int}(t') dt' \right)$$

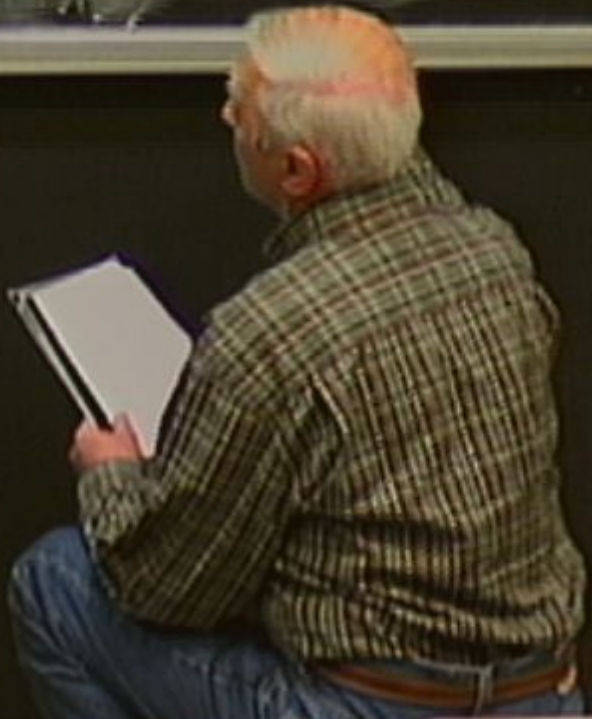
$$= \int \mathcal{D}\vec{x} \frac{1}{4!} \mathcal{L} \equiv H_I(t)$$

$U(t, t_0) = U_0(t, t_0) U_I(t, t_0)$
 called interaction picture-evolution operator. It
 satisfies a relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = U_0^{-1}(t, t_0) (H - H_0) U_0(t, t_0) U_I(t, t_0)$$

$$U_0^{-1}(t, t_0) (H - H_0) U_0(t, t_0) = H_I(t)$$

$$i \frac{\partial}{\partial t} U_I(t, t_0) = H_I(t) U_I(t, t_0)$$

$$U_I(t, t_0) = \mathcal{T} \exp \left(-i \int_{t_0}^t H_I(t') dt' \right) \equiv H_I(t)$$


called interaction picture evolution operator. It satisfies a relatively simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = \left(e^{iH_0(t-t_0)} (H - H_0) e^{-iH_0(t-t_0)} \right) U(t, t_0}$$

• $e^{iH_0(t-t_0)} e^{-iH(t-t_0)} = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$

$e^{iH_0(t-t_0)} H e^{-iH_0(t-t_0)} = \int dx \psi^\dagger \psi$

$\Rightarrow i \frac{\partial}{\partial t} U(t, t_0) = H_I(t) U(t, t_0)$

called interaction picture evolution operator. It satisfies a relatively simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = U(t, t_0) (H - H_0) U^\dagger(t, t_0)$$

$$U^\dagger(t, t_0) U(t, t_0) = I$$

$$U(t, t_0) U^\dagger(t, t_0) = I$$

$$U(t, t_0) \Rightarrow i \frac{\partial}{\partial t} U(t, t_0) = H_I(t) U(t, t_0)$$

$$U(t, t_0)|_{t=t_0} = I$$



called interaction picture evolution operator. It satisfies a relatively simple equation:

$$i \frac{\partial}{\partial t} U(t, t_0) = \left(e^{iH_0(t-t_0)} \right) \left[(H - H_0) \right] \left(e^{-iH_0(t-t_0)} \right) = \left(e^{iH_0(t-t_0)} \right) \left[H_{int}(t) \right] \left(e^{-iH_0(t-t_0)} \right)$$

$$\bullet \left(e^{iH_0(t-t_0)} \right) \left(e^{-iH_0(t-t_0)} \right) = H_{int} H_I(t) U(t, t_0) \Rightarrow i \frac{\partial}{\partial t} U(t, t_0) = H_I(t) U(t, t_0)$$

$$\left(e^{iH_0(t-t_0)} \right) H_{int} \left(e^{-iH_0(t-t_0)} \right) = \int d^3x \frac{\vec{\psi}^\dagger}{4!} \psi^4 \equiv H_I(t) \quad \left(U(t, t_0) \Big|_{t=t_0} = 1 \right)$$

fermions in superconductivity

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= -\frac{m}{2} (\chi^+, -i\chi^T \sigma^2) \begin{pmatrix} i\sigma^2 \chi \\ \chi \end{pmatrix} = -\frac{im}{2} (\chi^+ \sigma^2 \chi - \chi^T \sigma^2 \chi) =$$

$$= \frac{im}{2} (\chi^T \sigma^2 \chi - \chi^+ \sigma^2 \chi^*), \text{ Q.E.D.}$$

kinetic term:

$$\frac{c}{2} \psi_M^+ \partial_0 \psi_M = \frac{c}{2} \chi^+ \partial_0 \chi + \frac{c}{2} \chi^T \partial_0 \chi^*$$

use: $\frac{c}{2} \chi^T \partial_0 \chi^* = \frac{c}{2} \partial_0 (\chi^T \chi^*) - \frac{c}{2} \chi^T \partial_0 \chi^*$

Hint $\mathcal{L}(\dot{\chi}, \chi, t) = \int d^3x \frac{1}{4!} \mathcal{L}_I \equiv H_I(t)$ $(\mathcal{L}(t, \dot{\chi}, \chi))|_{t=t_0}$

Mass term: $-\frac{m}{2} \bar{\psi}_M \psi_M = -\frac{m}{2} (\chi^\dagger - i \chi^T \sigma_2) \gamma_0 (\chi)$

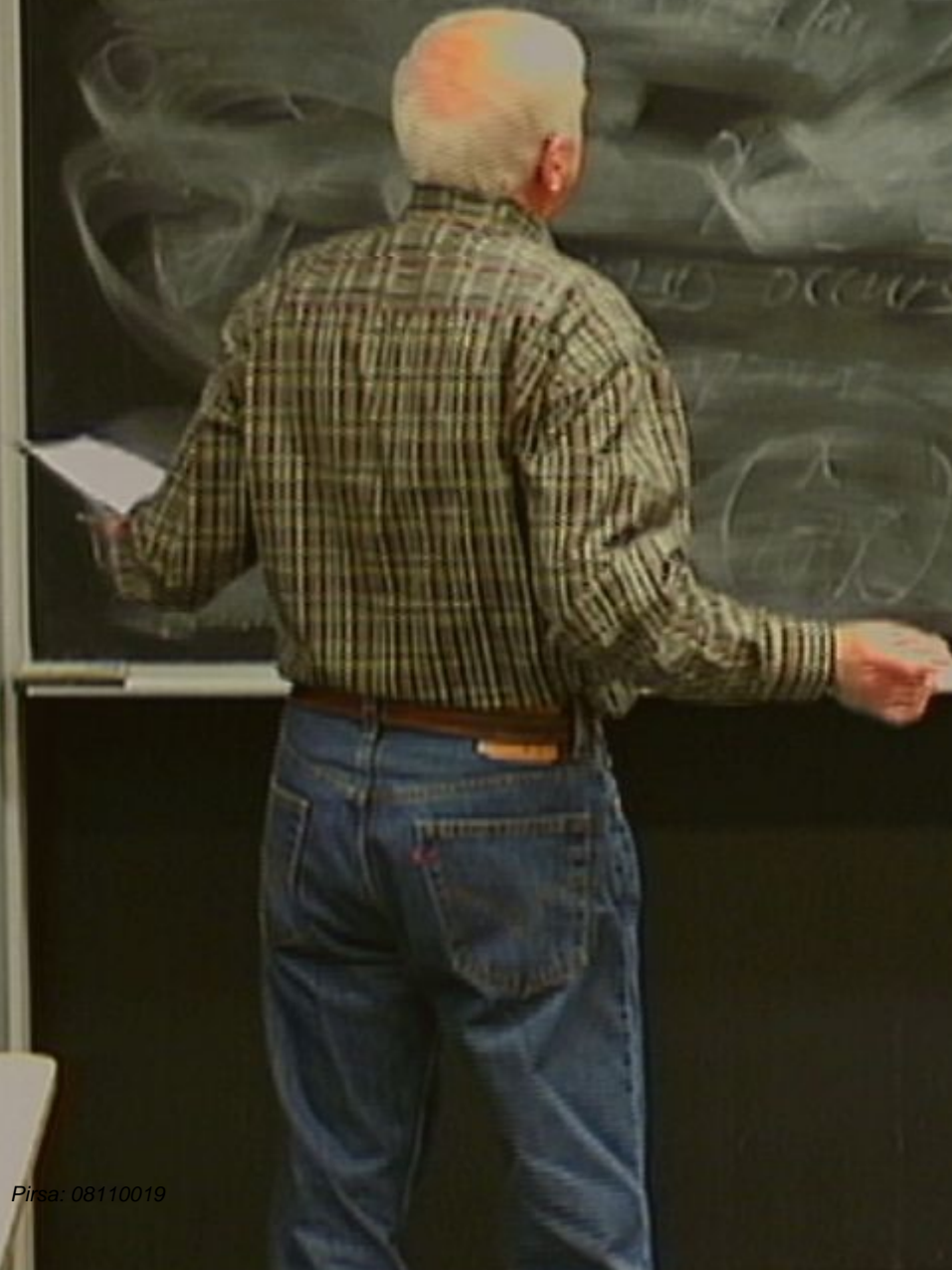
~~Equation~~ $\chi: \alpha = \chi^\dagger (\sigma^0 \partial_\mu \chi) - \frac{1}{2} (\chi^\dagger \sigma^2 \chi - \chi^T \sigma^2 \chi)$

$= \chi_1 \chi_2 - \chi_2 \chi_1 = \chi_1 \chi_2 + \chi_1 \chi_2 = 2 \chi_1 \chi_2$

mass occurs in the BCS theory of superconductivity
 two component formalism.

$\psi_M \Rightarrow \begin{pmatrix} \chi \\ i \sigma^2 \chi^* \end{pmatrix}$. Let us show that $\alpha = \frac{1}{2} \bar{\psi}_M (\gamma^\mu \partial_\mu - m) \psi_M$
 $\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

Guess: $U(t, t_0) \sim \exp(i H_I(t))$



Guess: $U(t, t_0) \sim \exp(-iH_I(t))$

The solution: $U(t, t_0) = 1 - i \int_{t_0}^t H_I(t') dt' + \dots$

Guess: $u(t, t_0) \sim \exp(-i H_I(t))$

The solution: $u(t, t_0) = 1 - i \int_{t_0}^t H_I(t_1) u(t_1, t_0)^2 dt_1$

Lyapunov exponent occurs in
 For component for



Guess: $U(t, t_0) \sim \exp(i H_I(t))$

The solution: $U(t, t_0) = 1 - i \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 + \dots$

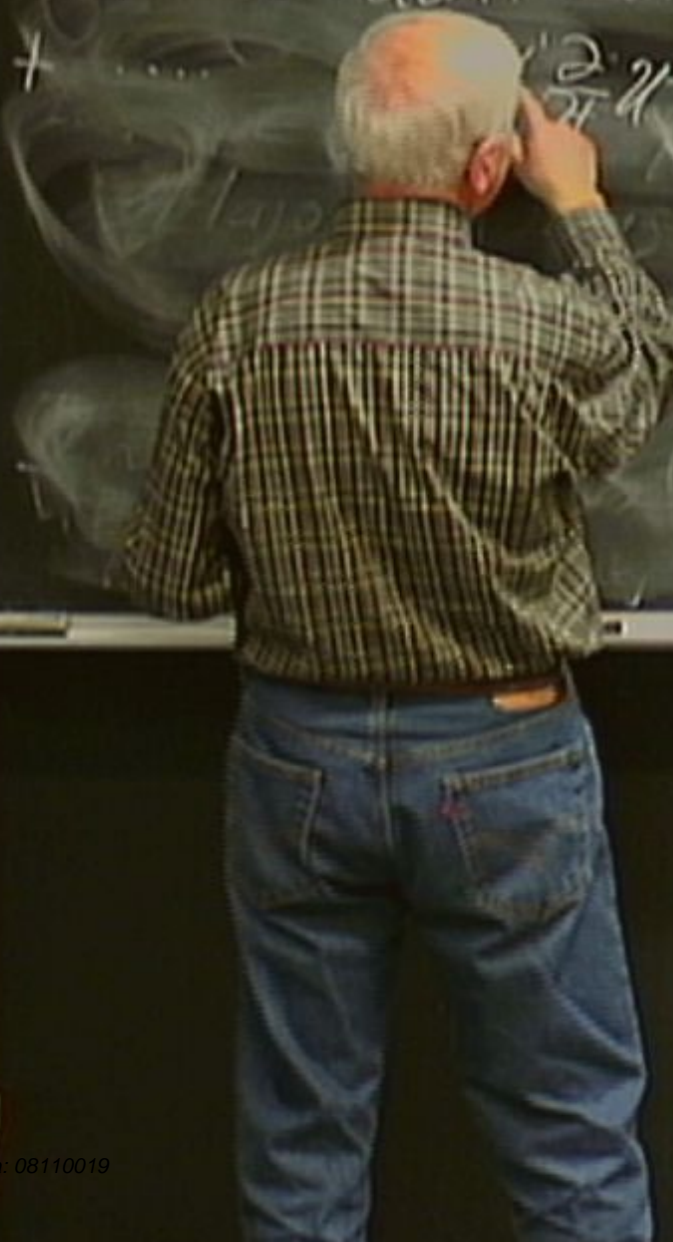
Majorana also occurs

For $\alpha = \pm 1/2$

Formal $\alpha = 1/2$

Guess: $U(t, t_0) \sim \exp(-i H_I(t))$

The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 + \dots$



$$\begin{aligned}
 & \int_{t_0}^t e^{iH_0(t-t_0)} \left[\frac{d}{dt} e^{-iH(t-t_0)} \right] dt = \int_{t_0}^t e^{iH_0(t-t_0)} \left[-iH e^{-iH(t-t_0)} \right] dt \\
 & = -i \int_{t_0}^t e^{i(H_0-H)(t-t_0)} dt = -i \int_{t_0}^t e^{-i(H-H_0)(t-t_0)} dt \\
 & = -i \int_{t_0}^t e^{-iH_I(t-t_0)} dt = -i \int_{t_0}^t e^{-iH_I(t-t_0)} dt = -i \int_{t_0}^t e^{-iH_I(t-t_0)} dt \\
 & = -i \int_{t_0}^t e^{-iH_I(t-t_0)} dt = -i \int_{t_0}^t e^{-iH_I(t-t_0)} dt = -i \int_{t_0}^t e^{-iH_I(t-t_0)} dt
 \end{aligned}$$

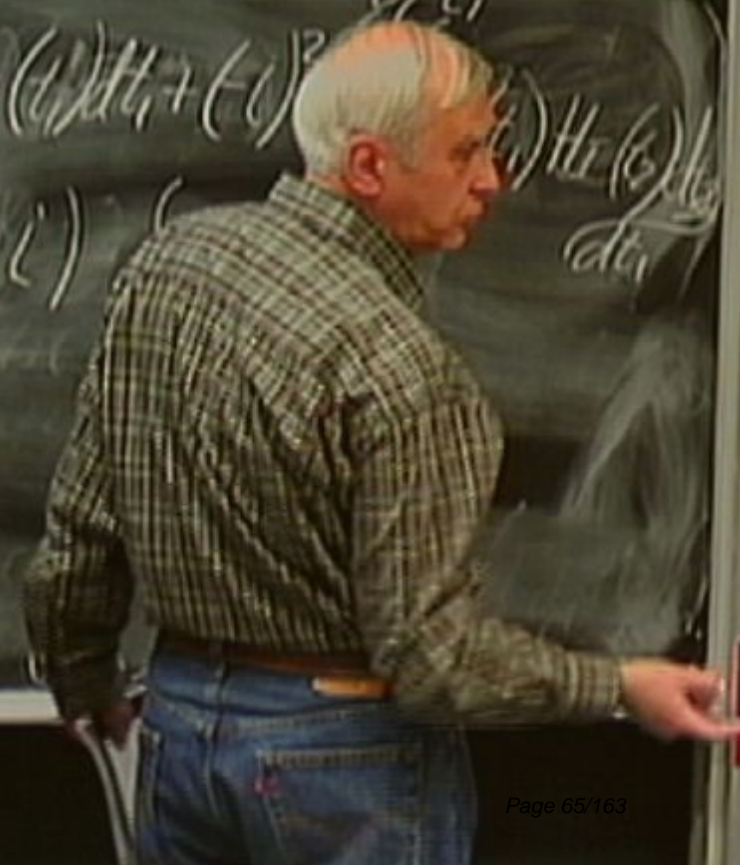
Guess: $U(t, t_0) \sim \exp(-iH_I(t))$
 The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t') dt' + (-i)^2 \int_{t_0}^t \int_{t_0}^{t'} H_I(t'') dt'' dt' + \dots$
 $i \frac{\partial U}{\partial t} = 0 + H_I(t)$

$$\begin{aligned}
 & e^{iH_0(t-t_0)} e^{-iH(t-t_0)} = \text{Hint } H_I(t) U(t, t_0) \Rightarrow \frac{d}{dt} U(t, t_0) = -i H_I(t) U(t, t_0) \\
 & e^{iH_0(t-t_0)} \text{Hint } e^{-iH(t-t_0)} = \int_{t_0}^t dt' \frac{d}{dt'} U(t, t') = H_I(t) U(t, t_0)
 \end{aligned}$$

Guess: $U(t, t_0) \sim \exp(-iH_I(t))$

The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t') dt' + (-i)^2 \int_{t_0}^t \int_{t_0}^{t'} H_I(t'') H_I(t') dt'' dt' + \dots$

$$i \frac{\partial}{\partial t} U = 0 + H_I(t) + (-i) \int_{t_0}^t H_I(t') \frac{d}{dt} dt'$$

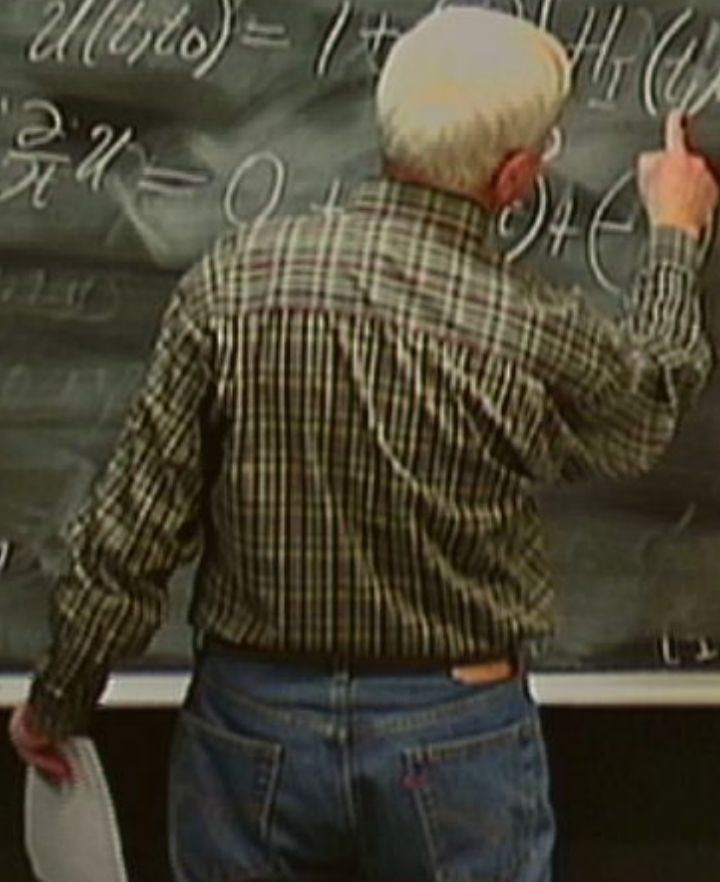


$$\begin{aligned}
 & e^{iH_0(t-t_0)} e^{-iH(t-t_0)} = \text{Hint } H_I(t) U(t, t_0) \Rightarrow \frac{d}{dt} U(t, t_0) = -H_I(t) U(t, t_0) \\
 & e^{iH_0(t-t_0)} \text{Hint } e^{-iH(t-t_0)} = \left(\frac{d}{dt} U \right) U^{-1} = H_I(t) U(t, t_0)
 \end{aligned}$$

Guess: $U(t, t_0) \sim \exp(-iH_I(t))$

The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 + \dots$

$$i \frac{\partial}{\partial t} U = 0 + (-i) H_I(t) \int_{t_0}^t H_I(t_2) dt_2$$



The solution, $u(t, x) = 1 + (-L) \int_{t_0}^t H_I(t_2) dt_2 + (-L)^2 \int_{t_0}^t \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$
 $(\frac{\partial}{\partial t} u) = 0 + H_I(t) + (-L) H_I(t) \int_{t_0}^t H_I(t_2) dt_2$

$2 (\lambda^0 \lambda - \chi^0 \sigma^2 \chi^*)$
 kinetic term.
 $\frac{1}{2} \psi_M^+ \partial_0 \psi_M = \frac{1}{2} \chi^+ \partial_0 \chi^*$
 use: $\frac{1}{2} \chi^T \partial_0 \chi^* = \frac{1}{2} \partial_0 \chi^T \chi^* = \frac{1}{2} \chi^+ \partial_0 \chi^*$

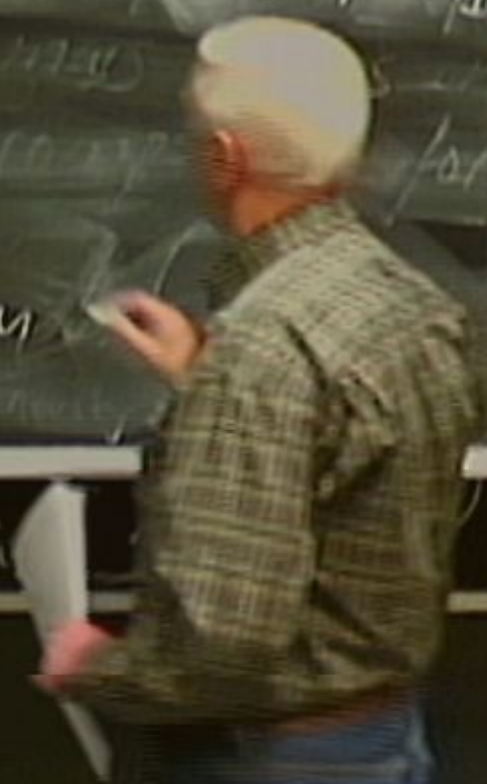
$$e^{iH_0(t-t_0)} e^{-iH(t-t_0)} = \text{Hint } H_I(t) U(t, t_0) \Rightarrow U(t, t_0) = e^{-iH(t-t_0)} e^{iH_0(t-t_0)} U(t, t_0)$$

Guess: $U(t, t_0) \sim \exp(-iH_I(t))$

The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$

$$i \frac{\partial}{\partial t} U = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^t H_I(t_2) dt_2$$

Because



Guess: $u(t, t_0) \sim \exp(-i H_I(t))$

The solution: $u(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 + \dots$

$i \frac{\partial}{\partial t} u = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^t H_I(t_2) dt_2$

Because of $t > t_1 > t_2$ order, one can write

$\int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 H_I(t_1) H_I(t_2)$

Rline

$$\frac{1}{2} \partial_0 \chi + \frac{1}{2} \chi^T \partial_0 \chi^* =$$

$$u \& \quad \frac{1}{2} \partial_0 (\chi^T \chi^*) - \frac{1}{2} (\partial_0 \chi^T) \chi^* = \frac{1}{2} \chi^T \partial_0 \chi$$

Guess: $u(t, t_0) \sim \exp(-L H_I(t))$

The solution: $u(t, t_0) = 1 + (-L) \int_{t_0}^t H_I(t_1) dt_1 + (-L)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$

$i \frac{\partial \psi}{\partial t} = 0 + H_I(t) + (-L) H_I(t) \int_{t_0}^t H_I(t_2) dt_2$

Because of $t > t_1 > t_2$ order, one can write

$\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^t \int_{t_0}^t dt_1 dt_2 H_I(t_1) H_I(t_2)$

Kinetic term:

$\frac{1}{2} \psi_M^+ \partial_0 \psi_M = \frac{1}{2} \chi^+ \partial_0 \chi$

Use: $\frac{1}{2} \chi^T \partial_0 \chi^* = \frac{1}{2} \partial_0 (\chi^* \chi) = \frac{1}{2} \chi^+ \partial_0 \chi$

$$i\hbar \frac{\partial \psi}{\partial t} = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^{t_1} H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$$

The solution:
$$U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$$

Because of $t > t_1 > t_2$ order, one can write

$$\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 T \{ H_I(t_1) H_I(t_2) \}$$

Kinetic term

$$\frac{1}{2} \psi_M^+ \partial_0 \psi_M = \frac{1}{2} \chi^T \partial_0 \chi^* = \frac{1}{2} (\partial_0 \chi^T) \chi^* = \frac{1}{2} \chi^+ \partial_0 \chi$$

Because of $t_2 > t_1 > t_3$ order, one

$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 T \{ H_I(t_1) H_I(t_2) \}$$

Mass term: $= \frac{m}{2} \psi_M \psi_M = \frac{m}{2} (\lambda^T, -i \lambda^T \sigma^2) \begin{pmatrix} i \sigma^2 \chi \\ \chi \end{pmatrix}$

$$= \frac{m}{2} (\lambda^T, -i \lambda^T \sigma^2) \begin{pmatrix} i \sigma^2 \chi \\ \chi \end{pmatrix} = \frac{im}{2} (\lambda^T \sigma^2 \chi - \chi^+ \sigma^2 \chi^+)$$

kinetic term:

$$\frac{1}{2} \psi_M^+ \partial_0 \psi_M = \frac{1}{2} \chi^+ \partial_0 \chi + \frac{1}{2} \chi^T \partial_0 \chi$$

use: $\frac{1}{2} \chi^T \partial_0 \chi^+ = \frac{1}{2} \partial_0 (\chi^T \chi^+) = \frac{1}{2} \chi^+ \partial_0 \chi$

Because of $t > t_1 > t_2$ order, one can write

$$\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \{ H_I(t_1) H_I(t_2) \}$$

Mass term: $-\frac{m}{2} \bar{\Psi}_M \Psi_M = -\frac{m}{2} (\lambda^+ - i \lambda^T \sigma^2) \gamma^0 \begin{pmatrix} \lambda \\ \chi^* \end{pmatrix}$

$$= -\frac{m}{2} (\lambda^+ - i \lambda^T \sigma^2) \begin{pmatrix} i \sigma^2 \chi^* \\ \chi \end{pmatrix} = -\frac{im}{2} (\lambda^+ \sigma^2 \chi^* - \lambda^T \sigma^2 \chi)$$

Q.E.D.

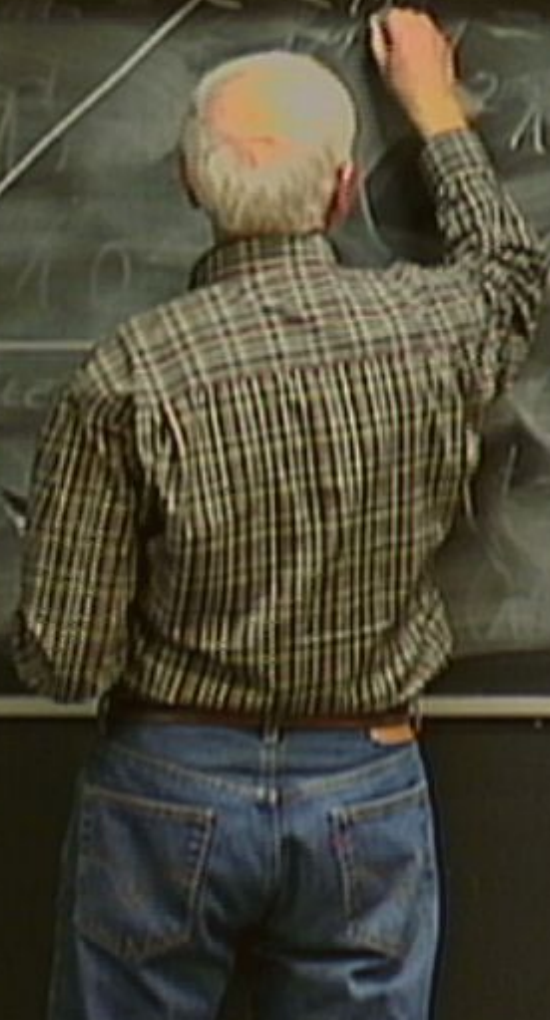
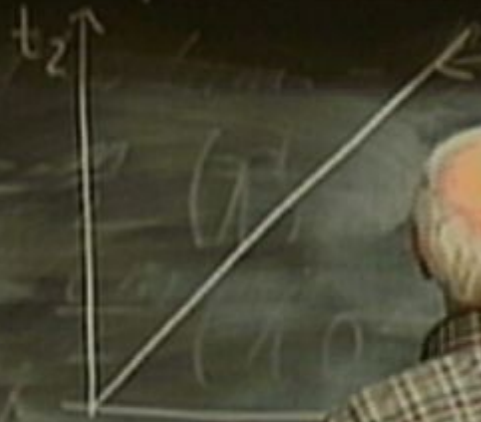
kinetic term.

$$\frac{i}{2} \bar{\Psi}_M \partial_0 \Psi_M = \frac{i}{2} \lambda^+ \partial_0 \lambda + \frac{i}{2} \lambda^T \partial_0 \chi^*$$

use: $\frac{i}{2} \lambda^T \partial_0 \chi^* = \frac{i}{2} \partial_0 (\lambda^T \chi^*) - \frac{i}{2} (\partial_0 \lambda^T) \chi^*$

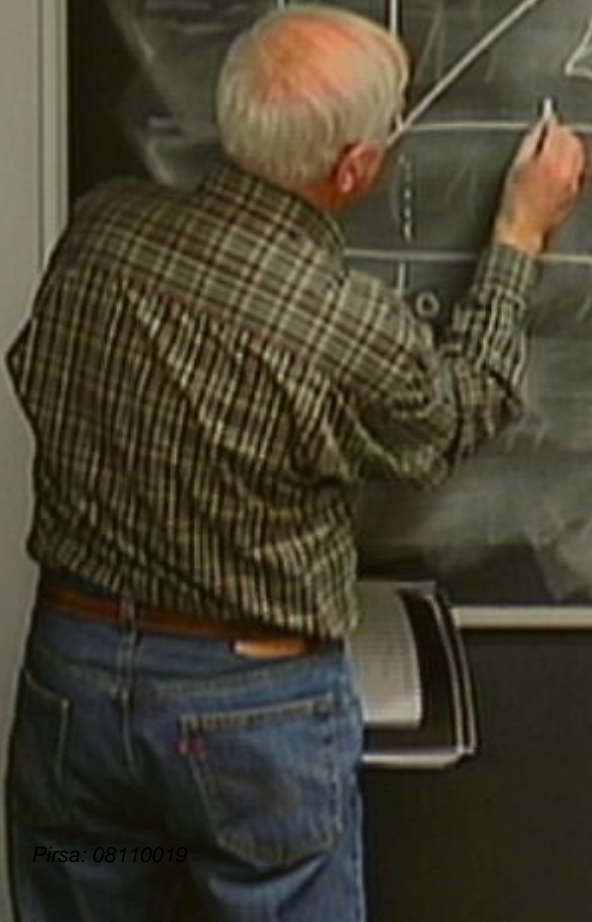
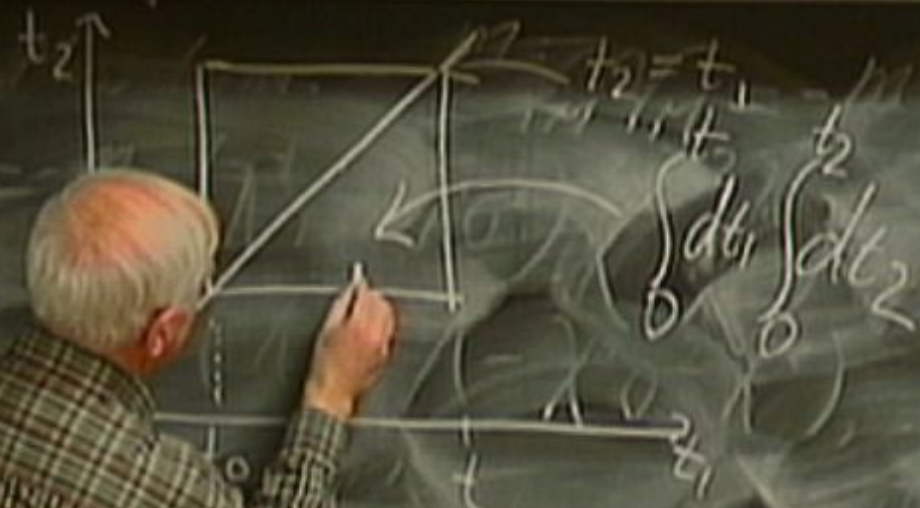
$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write



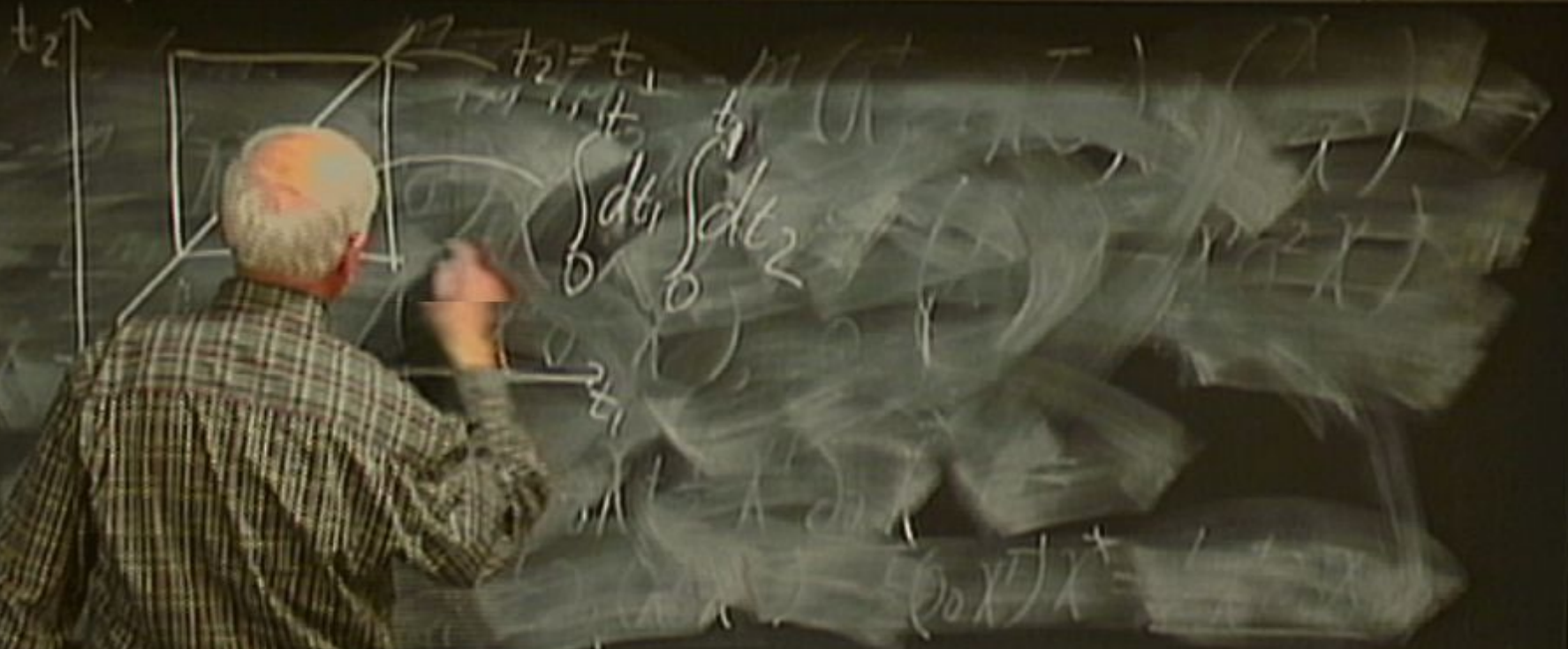
$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 \left[H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right]$$

to can write



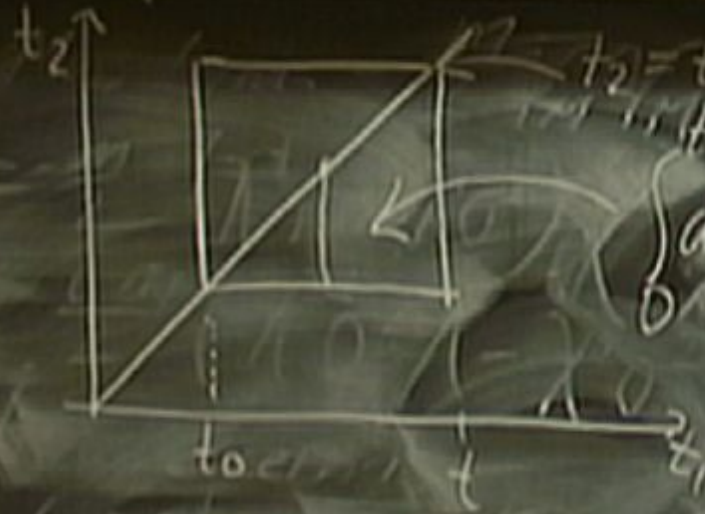
$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_2} \left[H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right] dt_1 dt_2$$

to can write



$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_2} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write

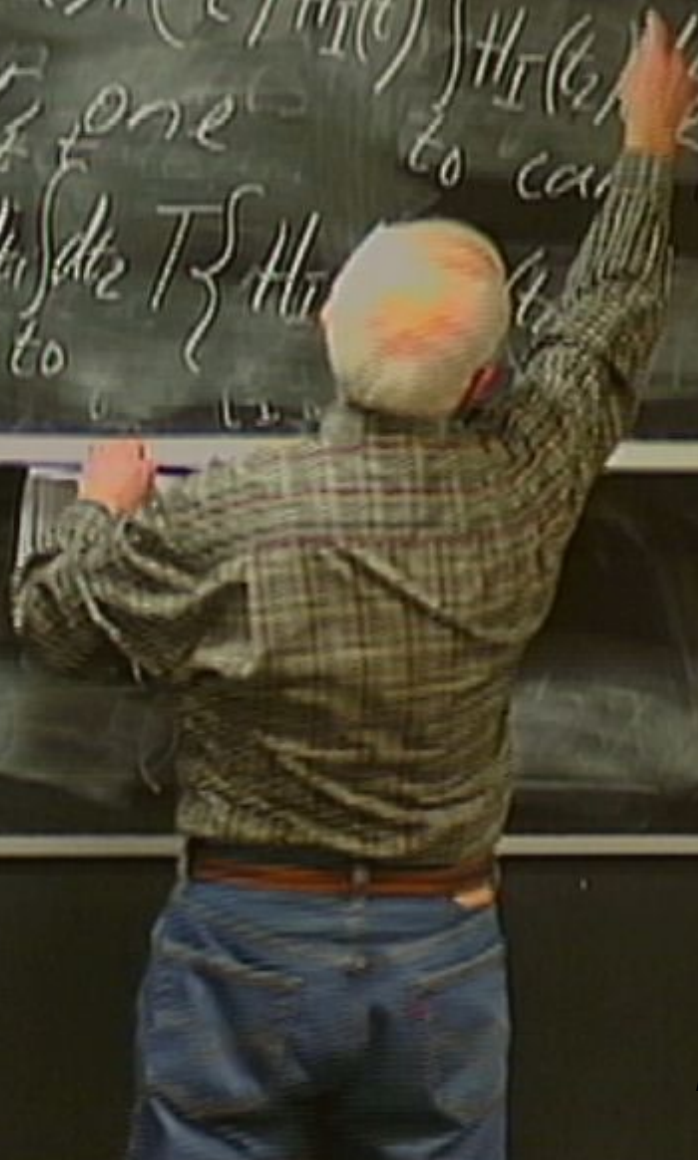


The solution. $u(t, t_0) = 1 + (t-t_0) \int_{t_0}^t H_I(t_1) dt_1 + (t-t_0)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_2) dt_2 dt_1 + \dots$

$i \frac{\partial u}{\partial t} = 0 + H_I(t) + (t-t_0) H_I(t) \int_{t_0}^t H_I(t_1) dt_1 + \dots$

Because of $t > t_1 > t_2$ order, one can write

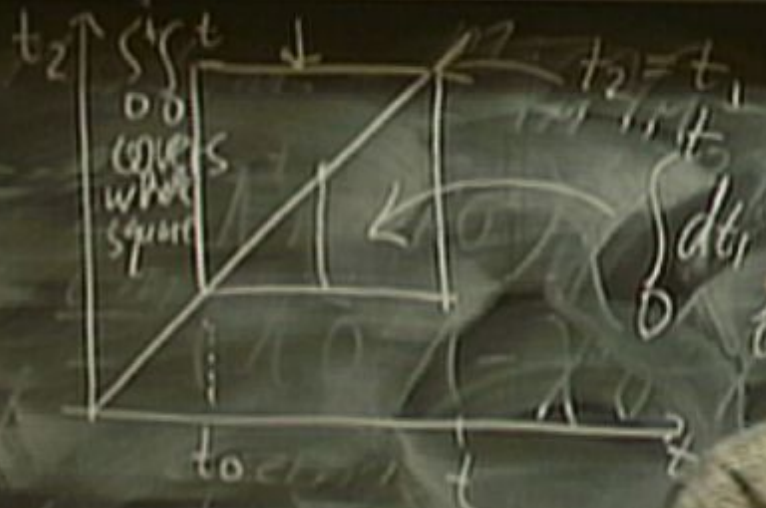
$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^t T \{ H_I(t_1) H_I(t_2) \} dt_1 dt_2$



$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 \left[H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right]$$

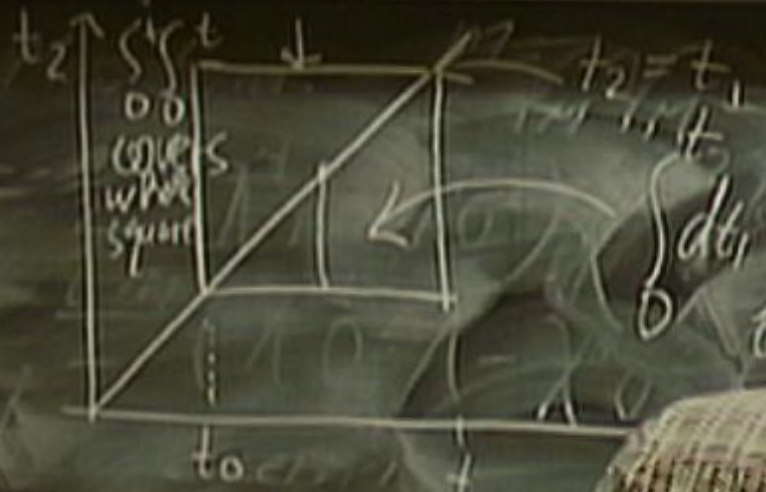
to to

to can write



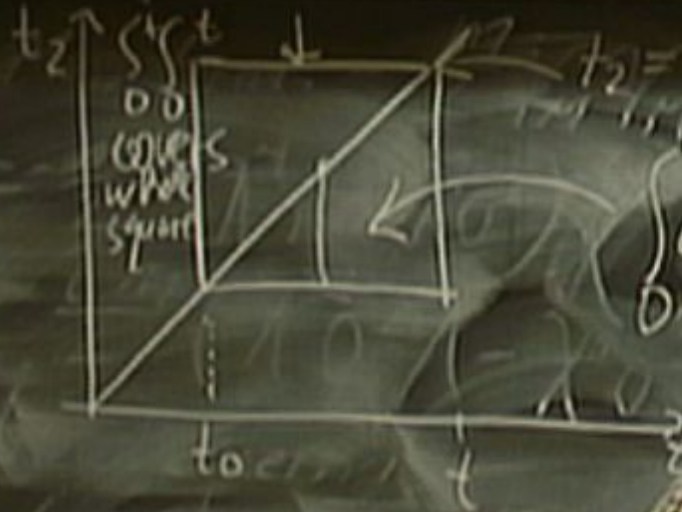
$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$$

to can write



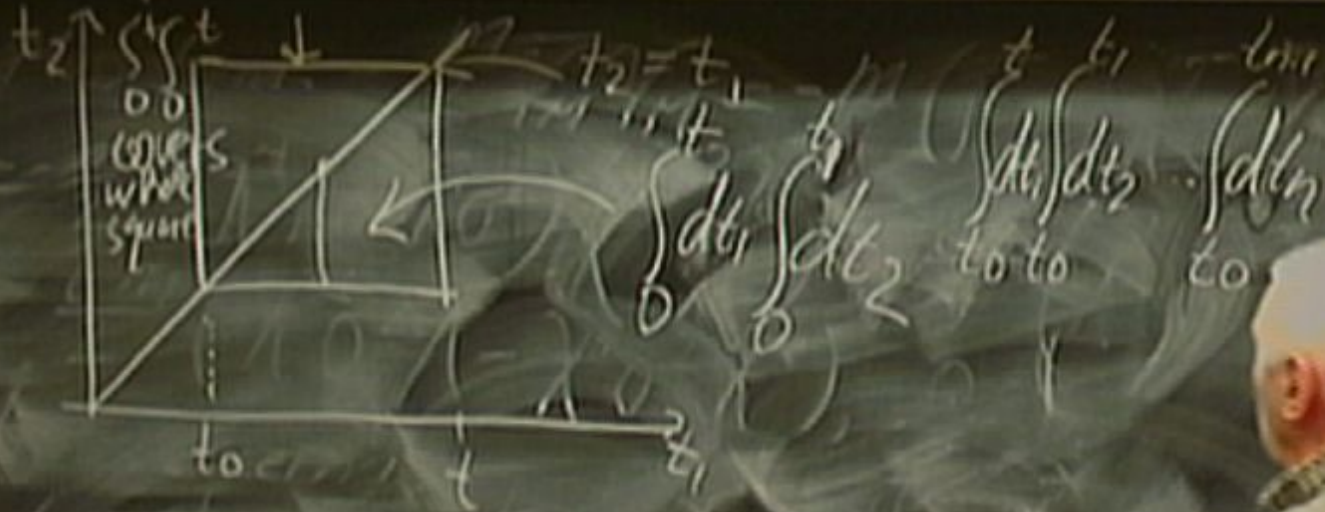
$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$$

to can write



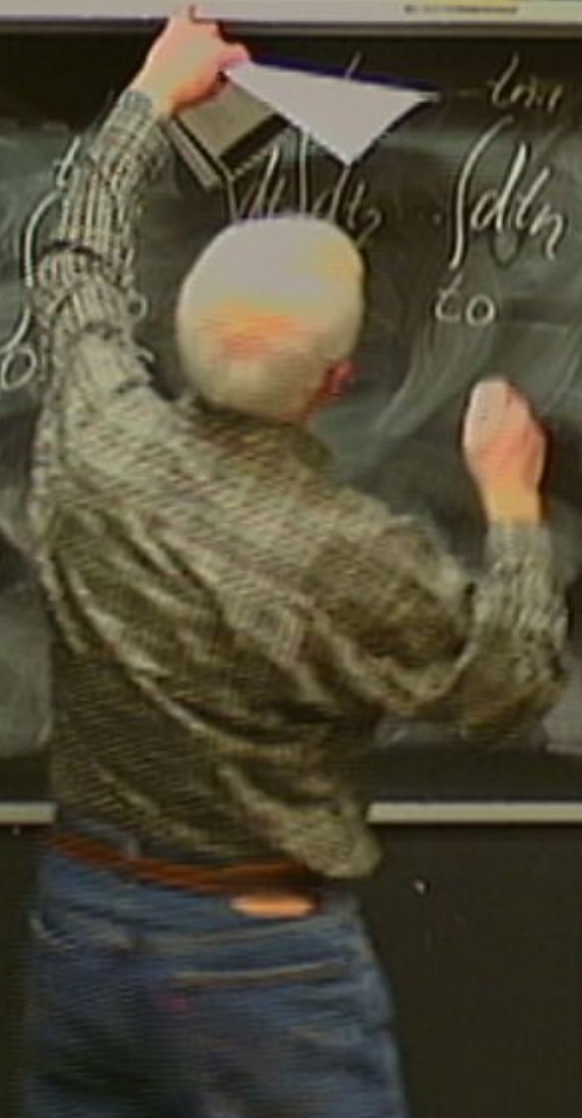
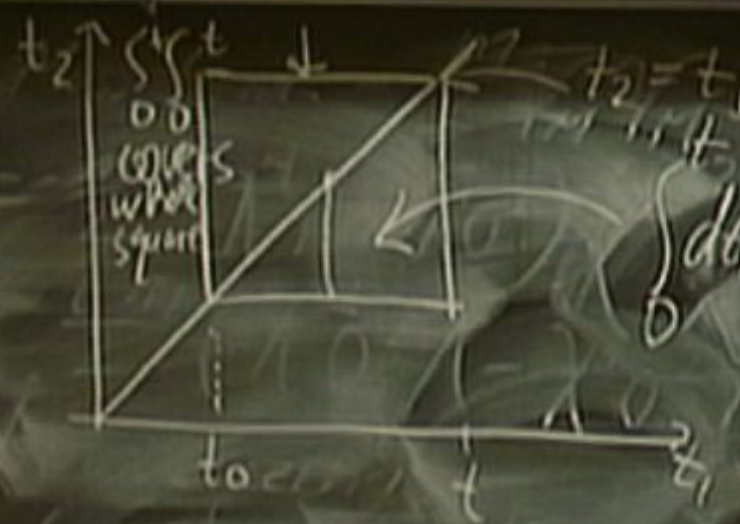
$$\int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write



$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$$

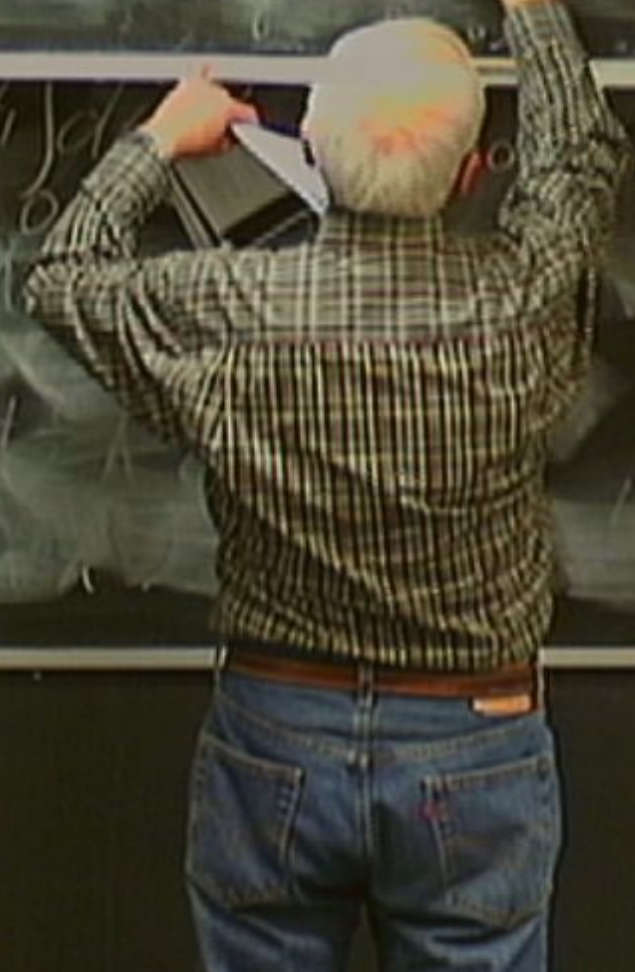
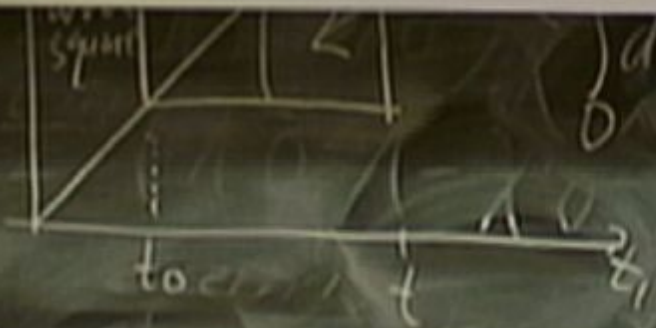
to can write



$\pi^k = 0 + H_I(t) + (-1)^k H_I(t)$

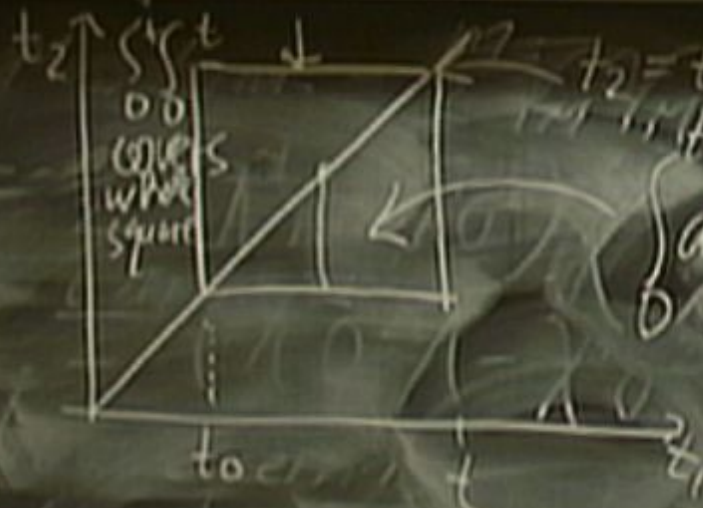
Because of $t > t_1 > t_2$ order, one $\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$ to can write

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)] dt_1 dt_2$$



$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_2} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write

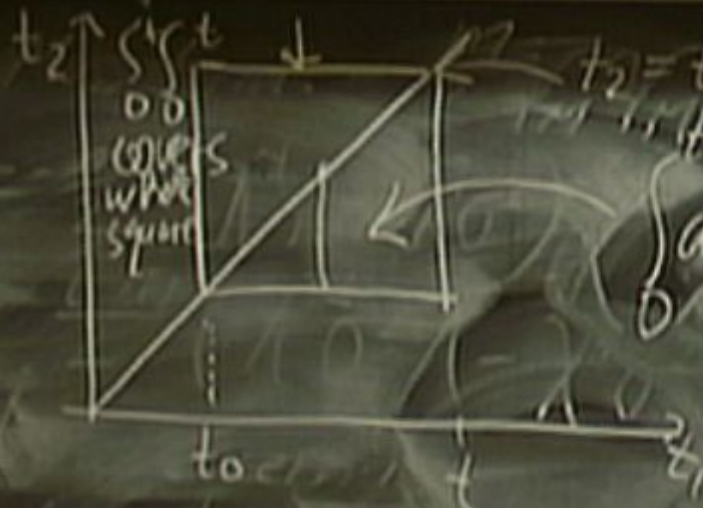


$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 + \int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_2) H_I(t_1) dt_1 dt_2$$

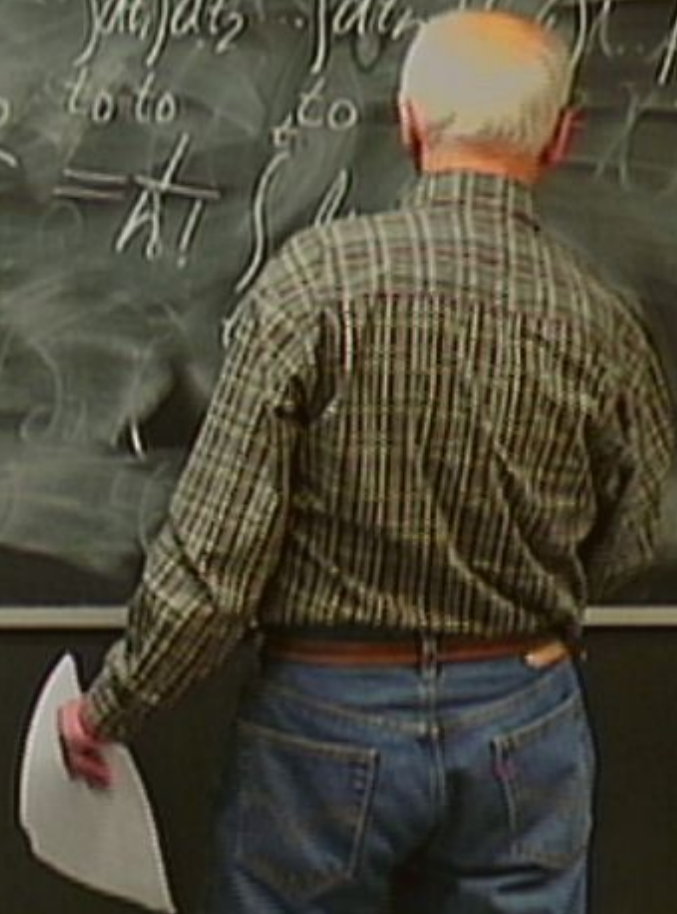


$$\int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write

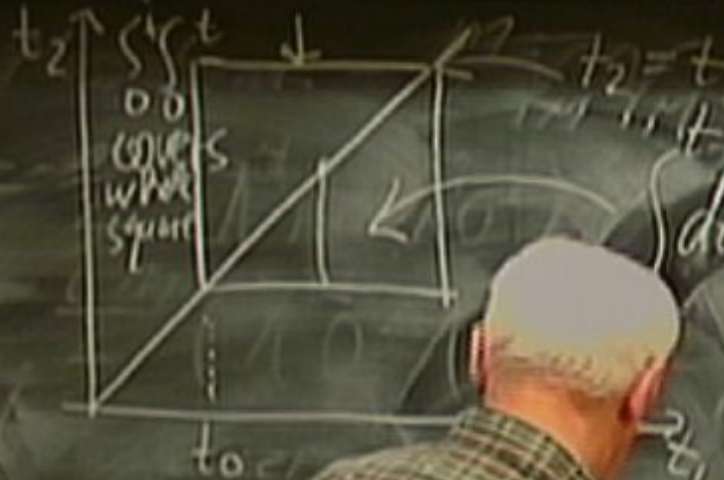


$$\int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_2) H_I(t_1) dt_1 dt_2$$



$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$$

to can write



$$\int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \int_{t_0}^{t_1} dt_1 H_I(t_1) \int_{t_0}^{t_1} dt_2 H_I(t_2) = \left(\int_{t_0}^{t_1} dt_1 H_I(t_1) \right)^2$$

to to

$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} dt_1 dt_2 \left\{ H_I(t_1) H_I(t_2) \right\}$$

to can write

$\equiv U^\dagger(t_1, t_0) U(t_1, t_0) U(t_1, t_0) = U(t_1, t_0) = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$ is called interaction picture evolution operator. It satisfies relatively simple equation.

$$i \frac{\partial}{\partial t} U(t, t_0) = (H - H_0) U(t, t_0)$$

$$U(t, t_0) = e^{iH_0(t-t_0)} U(t, t_0) e^{-iH_0(t-t_0)}$$

$$U(t, t_0) = e^{iH_0(t-t_0)} U(t, t_0) e^{-iH_0(t-t_0)}$$

$$U(t, t_0) = e^{iH_0(t-t_0)} U(t, t_0) e^{-iH_0(t-t_0)}$$

$$U(t, t_0) = e^{iH_0(t-t_0)} U(t, t_0) e^{-iH_0(t-t_0)}$$

Hint $H_I(t) \chi(t, t_0) \rightarrow \dots$

Guess: $U(t, t_0) \sim \exp(-i H_I(t))$

The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + \dots$

Because $t > t_1 > t_2$ order, one can write

$\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^t \int_{t_0}^t dt_1 dt_2 [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)]$



$$\begin{aligned}
 & \int_{t_0}^{t_1} \int_{t_0}^{t_1} \dots \\
 & \text{Hint } \int_{t_0}^{t_1} \int_{t_0}^{t_1} \dots = \int_{t_0}^{t_1} \int_{t_0}^{t_1} \dots \\
 & \text{Hint } \int_{t_0}^{t_1} \int_{t_0}^{t_1} \dots = \int_{t_0}^{t_1} \int_{t_0}^{t_1} \dots
 \end{aligned}$$

Guess

$$\begin{aligned}
 & u(t, t_0) \sim \exp(-i H_I(t)) \\
 & \text{The } u(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t') dt' + \dots
 \end{aligned}$$

But

$$\begin{aligned}
 & \text{for } t_1 > t_2 \text{ order, one} \\
 & \dots
 \end{aligned}$$

Solve

$$\begin{aligned}
 & \dots \\
 & \dots
 \end{aligned}$$

$$\begin{aligned}
 & \text{Hint } \mathcal{L}^{-1}\{H_I(t)\delta(t-t_0)\} = H_I(t) \delta(t-t_0) \Rightarrow \mathcal{L}\{H_I(t)\delta(t-t_0)\} = H_I(t_0) \\
 & \text{Hint } \mathcal{L}^{-1}\left\{\int_{t_0}^t \frac{d\tau}{\tau} \mathcal{L}' \equiv H_I(t)\right\} = \int_{t_0}^t \frac{d\tau}{\tau} \mathcal{L}' \equiv H_I(t)
 \end{aligned}$$

Given $u(t, t_0) \sim \exp(-L H_I(t))$

The solution $u(t, t_0) = 1 + (-L) \int_{t_0}^t H_I(\tau) d\tau + (-L)^2 \int_{t_0}^t \int_{t_0}^{\tau} H_I(\tau_1) H_I(\tau_2) d\tau_1 d\tau_2 + \dots$

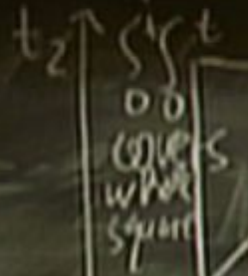
$\frac{\partial u}{\partial t} = 0 + H_I(t) + (-L) H_I(t) \int_{t_0}^t H_I(\tau) d\tau + \dots$

For $t_1 > t_2$ order, one can write

$$u(t_1, t_2) = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(\tau_1) H_I(\tau_2) d\tau_1 d\tau_2$$

$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$$

to can write

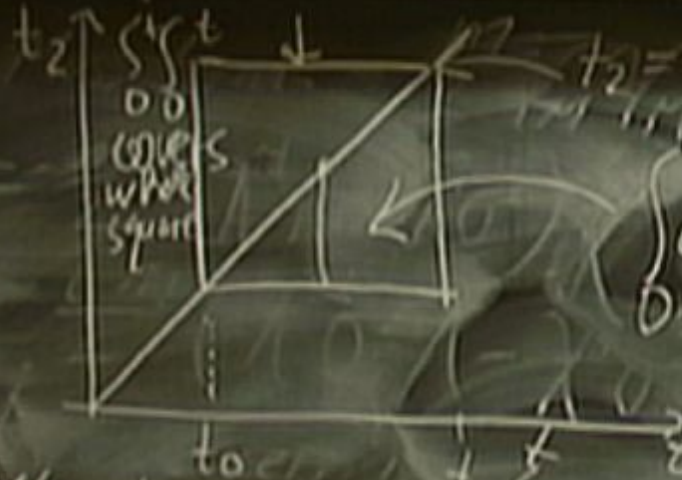


$$\int_0^t \int_0^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_0^t \int_0^t H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_0^t dt_1 \int_0^t dt_2 H_I(t_1) H_I(t_2)$$

$$u(t) = H(t) + \frac{(-1)^2}{2!} \int_0^t \int_0^t H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$$

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$$

to can write



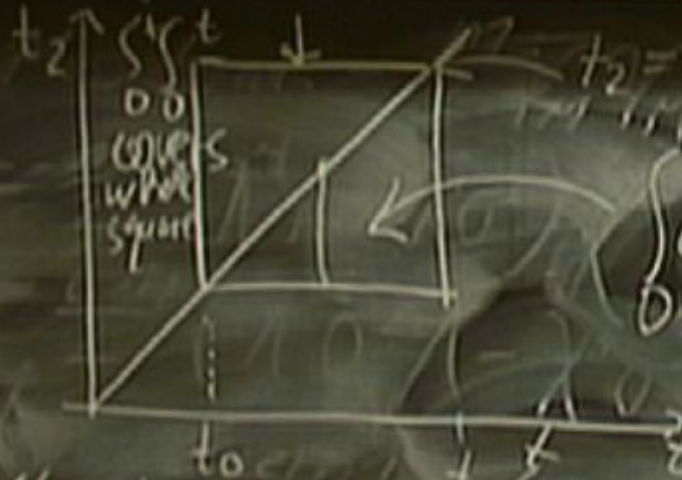
$$\int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 = \int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 \frac{d}{dt} (H_I(t_1) + H_I(t_2))$$

$$u(t, t_0) = 1 + (-1) \int_{t_0}^t H_I(t_1) dt_1 + \frac{(-1)^2}{2!} \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$$



$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write



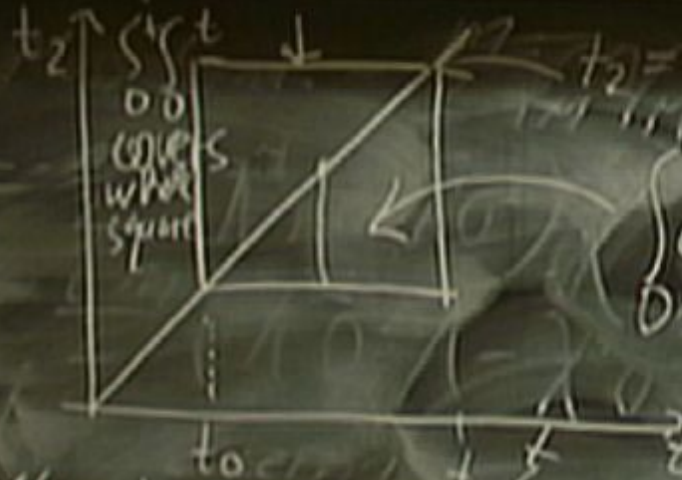
$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} \left(H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right) dt_1 dt_2$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + \frac{(-i)^2}{2!} \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 + \dots$$

$$= T \left\{ \exp \left[-i \int_{t_0}^t H_I(t) dt \right] \right\}$$

$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 \left[H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right]$$

to can write



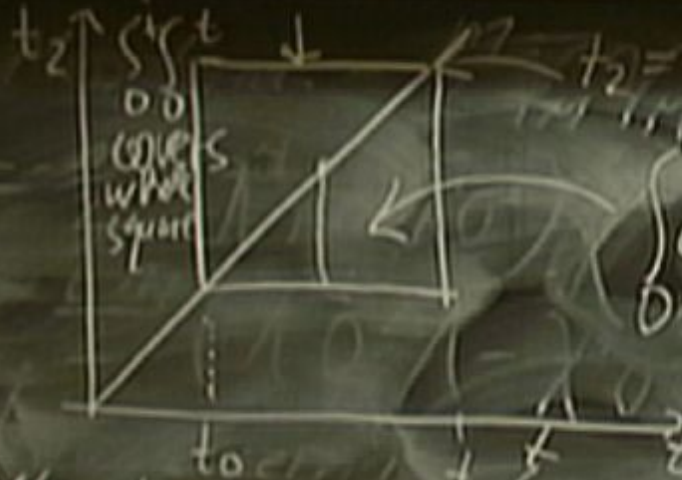
$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 H_I(t_1) H_I(t_2) = \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) + \int_{t_0}^{t_1} dt_1 \int_{t_1}^{t_2} dt_2 H_I(t_1) H_I(t_2)$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^t dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) + \dots$$

$$= T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}$$

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} T \{ H_I(t_1) H_I(t_2) \} dt_1 dt_2$$

to can write



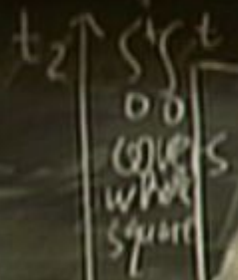
$$\int_0^t \int_0^{t_1} dt_1 dt_2 \int_0^{t_1} dt_3 H_I(t_1) H_I(t_2) H_I(t_3) = \frac{1}{3!} \int dt_1 dt_2 dt_3 T \{ H_I(t_1) H_I(t_2) H_I(t_3) \}$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^t dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 T \{ H_I(t_1) H_I(t_2) \} + \dots$$

$$= T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\} \rho(t_0)$$

$$\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write

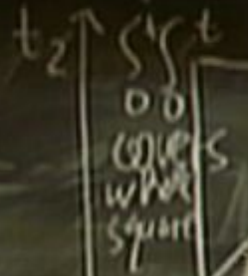


$$\int_0^{t_1} \int_0^{t_2} dt_1 dt_2 = \int_0^{t_1} \int_0^{t_1} dt_1 dt_2 = \frac{1}{2} \int_0^{t_1} \int_0^{t_1} dt_1 dt_2$$

$$= \frac{1}{2} \int_0^{t_1} \left[\int_0^{t_1} dt_2 \right] dt_1 = \frac{1}{2} \int_0^{t_1} dt_1 \left[\int_0^{t_1} dt_2 \right] = \frac{1}{2} \int_0^{t_1} dt_1 \left[\int_0^{t_1} dt_2 \right]$$

$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 \left[H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right]$$

to can write

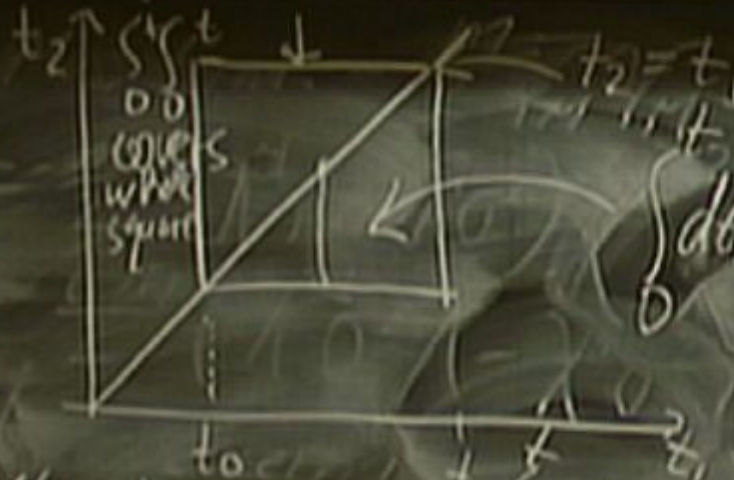


$$\int_0^t dt_1 \int_0^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_0^t dt_1 \int_0^t dt_2 [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)]$$

$$U(t) = 1 - i \int_0^t dt' H_I(t') - \frac{1}{2} \int_0^t dt_1 \int_0^{t_1} dt_2 [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)] + \dots$$

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write



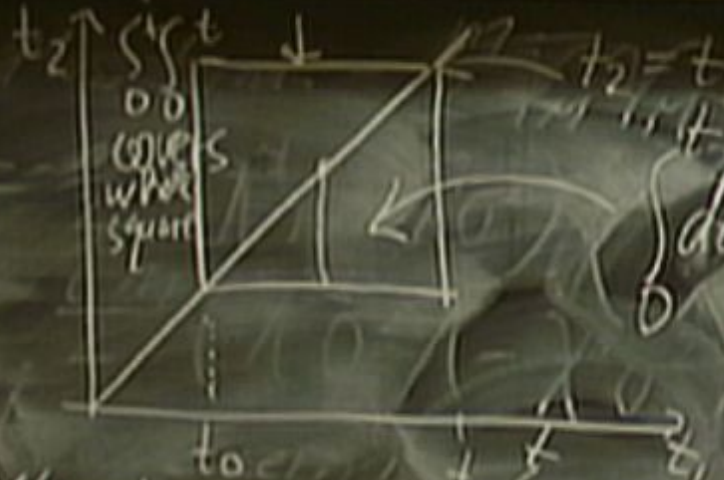
$$\int_0^t \int_0^{t_1} dt_1 dt_2 \int_0^{t_1} dt_3 H_I(t_1) H_I(t_2) H_I(t_3) = \frac{1}{6} \int_0^t \int_0^{t_1} \int_0^{t_1} dt_1 dt_2 dt_3 \left(H_I(t_1) H_I(t_2) H_I(t_3) + \dots \right)$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^t dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 T(H_I(t_1) H_I(t_2)) + \dots$$

$$= T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\} \rho(t_0)$$

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} \left\{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right\} dt_1 dt_2$$

to can write



$$\int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 = \int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 + \int_{t_0}^t \int_{t_0}^{t_1} dt_2 dt_1 = 2 \int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2$$

$$= \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 \left(H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \right)$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^t dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^t \int_{t_0}^{t_1} dt_1 dt_2 T(H_I(t_1) H_I(t_2)) + \dots$$

$$= T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}$$

Guess: $U(t, t_0) \sim \exp(-i H_I(t))$

The solution: $U(t, t_0) = 1 + \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 + \dots$

$$i \frac{\partial U}{\partial t} = 0$$

Because of $t > t_0$

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 = \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1$$

to can write $\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1$

$$\begin{aligned}
 & \int_{t_0}^t H_I(t-t_0) \dot{H}_I(t-t_0) dt \\
 & = \int_{t_0}^t \frac{d}{dt} \left[\frac{1}{2} H_I^2(t-t_0) \right] dt \\
 & = \frac{1}{2} H_I^2(t-t_0) \Big|_{t_0}^t \\
 & = \frac{1}{2} H_I^2(t-t_0) - \frac{1}{2} H_I^2(t_0-t_0) \\
 & = \frac{1}{2} H_I^2(t-t_0)
 \end{aligned}$$

Guess: $U(t) \sim \exp(-iH_I(t-t_0))$
 The solution is $U(t) = \exp(-iH_I(t-t_0))$
 Because of order one we can write

$$\int_{t_0}^t dt_2 \int_{t_0}^{t_2} dt_1 H_I(t_1) H_I(t_2)$$

$$\frac{\partial}{\partial t} u(t, t_0) = \dots$$

$$e^{iH_0(t-t_0)} \left[\frac{\partial}{\partial t} u(t, t_0) \right] e^{-iH_0(t-t_0)} = \dots$$

$$= \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \dots \equiv H_I(t)$$

$$\Rightarrow \frac{\partial}{\partial t} u(t, t_0) = H_I(t) u(t, t_0)$$

Guess: $u(t, t_0) \sim \exp(iH_I(t))$

The solution $u(t, t_0) = \dots$

Because of $\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 H_I(t_1) H_I(t_2)$

order one $\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 T \{ H_I(t_1) H_I(t_2) \}$

can write

$$i \frac{\partial}{\partial t} U(t, t_0) = (H(t) - H_0) U(t, t_0)$$

$$e^{i H_0 (t-t_0)} e^{-i H(t-t_0)} = \int_{t_0}^t dt_1 H_I(t_1) U(t, t_0) \Rightarrow \frac{\partial}{\partial t} U(t, t_0) = H_I(t) U(t, t_0)$$

$$e^{i H_0 (t-t_0)} H_I(t) e^{-i H(t-t_0)} = \int_{t_0}^t dt_1 \frac{\partial}{\partial t_1} U_I \equiv H_I(t)$$

Guess: $U(t, t_0) \sim \exp(-i H_I(t))$

The solution: $U(t, t_0) = (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1$

$i \frac{\partial}{\partial t} U = (-i) H_I(t) + (-i)^2 H_I(t) \int_{t_0}^t H_I(t_2) dt_2$

Because of $t_1 > t_2$ one can write

$$\int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_1 H_I(t_1) H_I(t_2) = T \int_{t_0}^{t_1} H_I(t_1) H_I(t_2)$$

$$i \frac{\partial}{\partial t} U(t, t_0) = (H(t) - H_0) U(t, t_0)$$

$$U(t, t_0) = \left[\exp\left(-i \int_{t_0}^t (H(t') - H_0) dt'\right) \right]$$

$$U(t, t_0) = \exp\left(-i \int_{t_0}^t H_I(t') dt'\right) \equiv H_I(t)$$

Guess: $U(t, t_0) \sim \exp(-i H_I(t))$

The solution: $U(t, t_0) = 1 + (-i) \int_{t_0}^t H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 + \dots$

$$i \frac{\partial}{\partial t} U = 0 + H_I(t) + (-i) H_I(t) \int_{t_0}^t H_I(t_2) dt_2$$

Because of $t > t_1 > t_2$ order, one can write

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_2 dt_1 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^t T \{ H_I(t_1) H_I(t_2) \} dt_1 dt_2$$

$i\frac{\partial}{\partial t}\psi = 0 + H_I(t) + (-i)H_I(t)$

Because of $t > t_1 > t_2$ order, one can write

$$\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)]$$

$\int_{t_0}^t dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)] + \dots$

$= \int_{t_0}^t dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 [H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1)] + \dots$

Because of $t > t_1 > t_2$ order one $\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2$ to can write

$$\int_{t_0}^t \int_{t_0}^{t_1} H_I(t_1) H_I(t_2) dt_1 dt_2 = \frac{1}{2} \int_{t_0}^t \int_{t_0}^{t_1} \{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \} dt_1 dt_2$$

$\psi(t, \vec{x}) = \psi(t, t_0) + \int_{t_0}^t \mathcal{L}(t, \vec{x}) dt$

$$\psi(t, t_0) = \psi(t_0, t_0) + \int_{t_0}^t \mathcal{L}(t, \vec{x}) dt = \psi(t_0, t_0) + \int_{t_0}^t \left[H_I(t) + \frac{(\hbar)^2}{2i} \int_{t_0}^t \int_{t_0}^{t_1} \{ H_I(t_1) H_I(t_2) + H_I(t_2) H_I(t_1) \} dt_1 dt_2 \right] dt$$



to to $\int_{t_0}^{t_1} \int_{t_0}^{t_2} H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_2} T \{ H_I(t_1) H_I(t_2) \}$ to can write

$\psi(t, \vec{x}) = U(t, t_0) \psi(t_0, \vec{x}) U(t_0, t_0)$

oo covers whole square

$\int_0^{t_1} \int_0^{t_2} dt_1 dt_2 = \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_2} dt_1 dt_2 \left[\int_{t_0}^{t_1} dt_3 H_I(t_3) + \frac{1}{2!} \int_{t_0}^{t_1} \int_{t_0}^{t_2} dt_1 dt_2 T \{ H_I(t_1) H_I(t_2) \} + \dots \right]$

$U(t, t_0) = T \left\{ \int_{t_0}^t dt' H_I(t') \right\}$

theory, $|\Omega\rangle$ is the ground state in φ^4 theory (in free theory, the ground state is $|0\rangle$)

Zero approximation: $\langle 0|T(\varphi(x)\varphi(y))|0\rangle_{free} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon}$

Idea: Introduce new, interaction picture, field:

$$\underline{\varphi_I(t, \vec{x})} \equiv \underline{e^{iH_0(t-t_0)} \varphi(t_0, \vec{x}) e^{-iH_0(t-t_0)}} \implies$$

$$\varphi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} + a_{-\vec{p}}^\dagger e^{i\vec{p}\cdot\vec{x}})$$

Higher orders in λ ? for $e^{iHt} \varphi(\vec{x}) e^{-iHt}$ ← Schrodinger operator
 Heisenberg field. $\varphi(x) = e^{iHt} \varphi(\vec{x}) e^{-iHt}$ ← ?

At any fixed time $t=t_0$, we can write $\varphi(t_0, \vec{x}) =$

$$(a_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} + a_{-\vec{p}}^\dagger e^{i\vec{p}\cdot\vec{x}}) \quad [\text{It is important that}]$$

theory, $|\Omega\rangle$ is the ground state in φ^4 theory (in free theory, the ground state is $|0\rangle$)

Zero approximation: $\langle 0 | T(\varphi(x)\varphi(y)) | 0 \rangle_{\text{free}} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon}$

Idea: Introduce new, interaction picture, field

$$\underline{\varphi_I(t, \vec{x})} \equiv e^{iH_0(t-t_0)} \varphi(t_0, \vec{x}) e^{-iH_0(t-t_0)} \Rightarrow$$

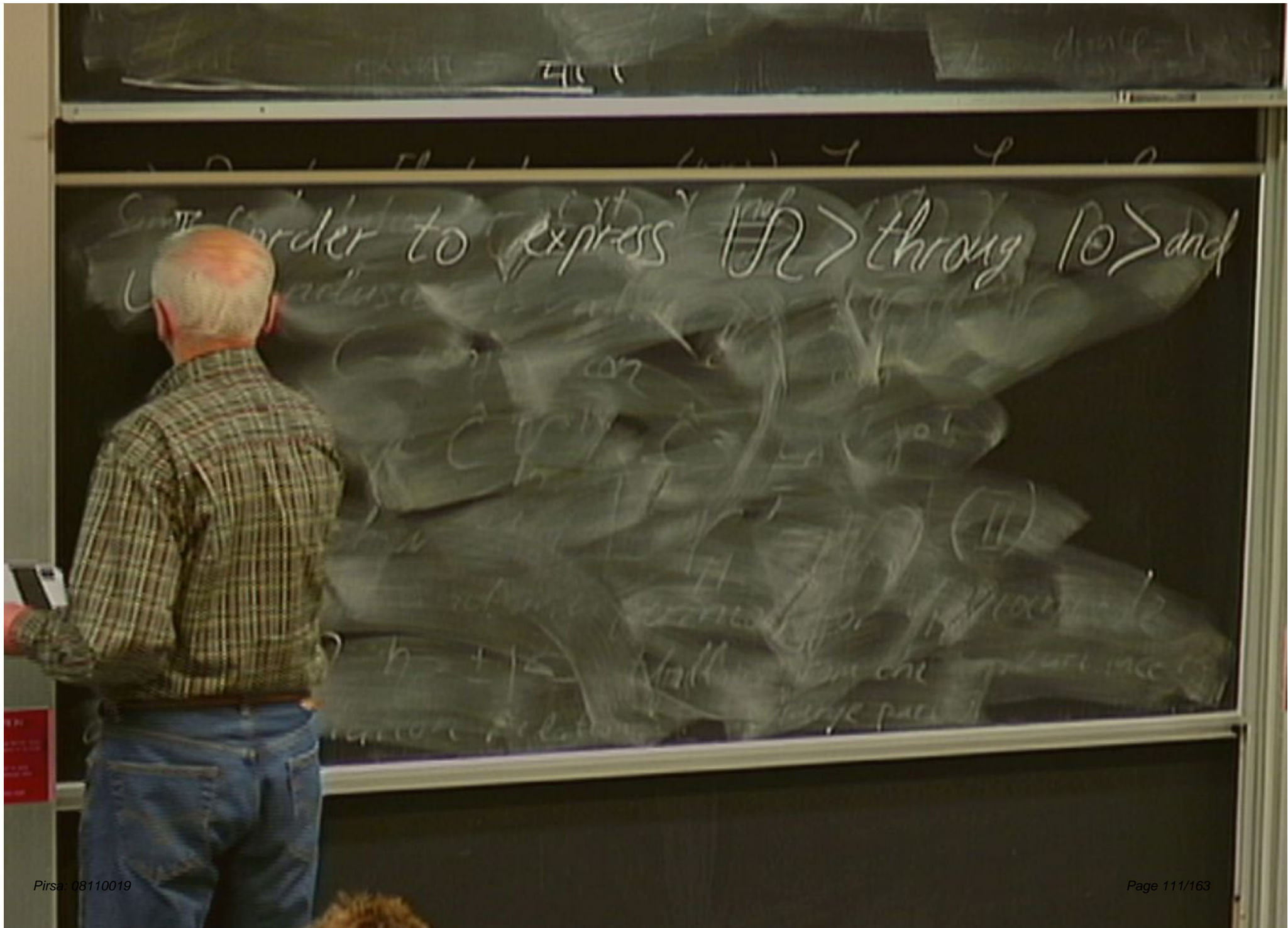
$$\varphi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\vec{p}} e^{-i(E_p t - \vec{p} \cdot \vec{x})} + a_{\vec{p}}^\dagger e^{i(E_p t - \vec{p} \cdot \vec{x})})$$

$$\varphi(t, \vec{x}) = e^{iH(t-t_0)} \varphi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

$$= e^{iH(t-t_0)} e^{-iH_0(t-t_0)} \varphi_I(t, \vec{x}) e^{-iH_0(t-t_0)} e^{-i(H-H_0)(t-t_0)}$$

Higher orders in λ
Husenberg field. $\varphi(x)$

inger operator
Ht
← ?



In order to express $\frac{1}{10}$ through 10 and $\psi(x)$, introduce $u(t, t')$



411

$\dim(\mathcal{C}) = 1$

2) $\mathcal{C} = \{ \text{Plücker coordinates} \}$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T$

Let's show \dots (II)
 \dots
 \dots
 \dots

41

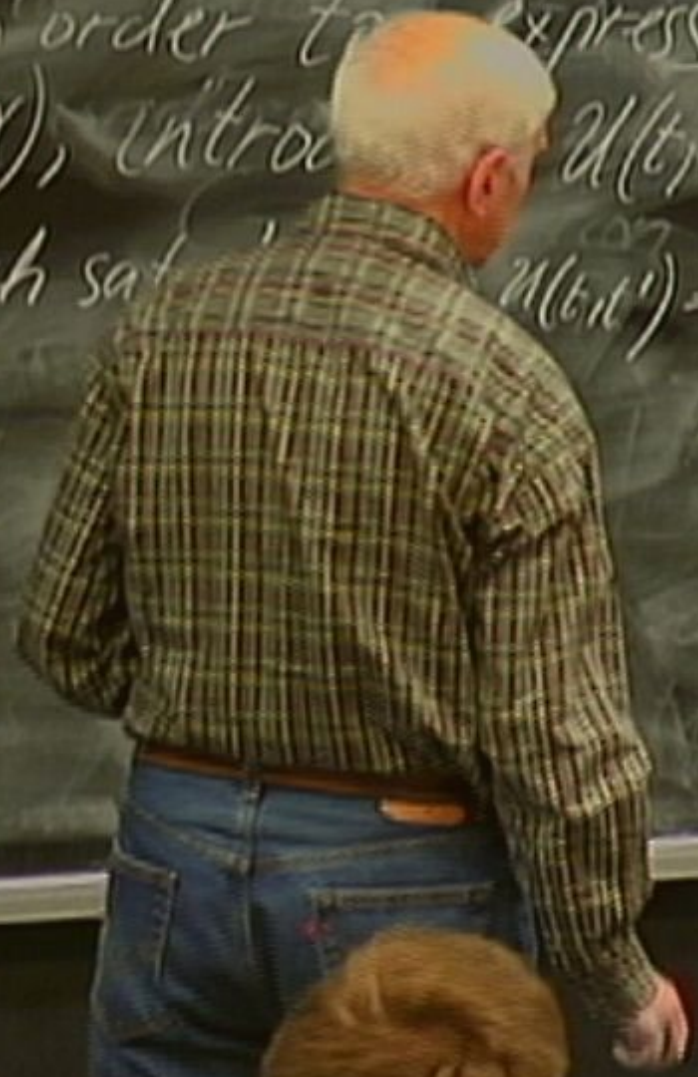
2) Quantum Electrodynamics (QED) $\gamma - \gamma - e$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$

Let $\psi_I(x)$ be a free field...
...for $t > t'$...
...the interaction...
...the propagator...

2) Quantum Electrodynamics (QED). $L_{int} = \mathcal{I} + \mathcal{I}'$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$,
 which satisfies $i \partial_t U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$



41

In order to express $|R\rangle$ through $|0\rangle$ and $G(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \partial_t U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$.



$H_I(t)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$,
 which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$



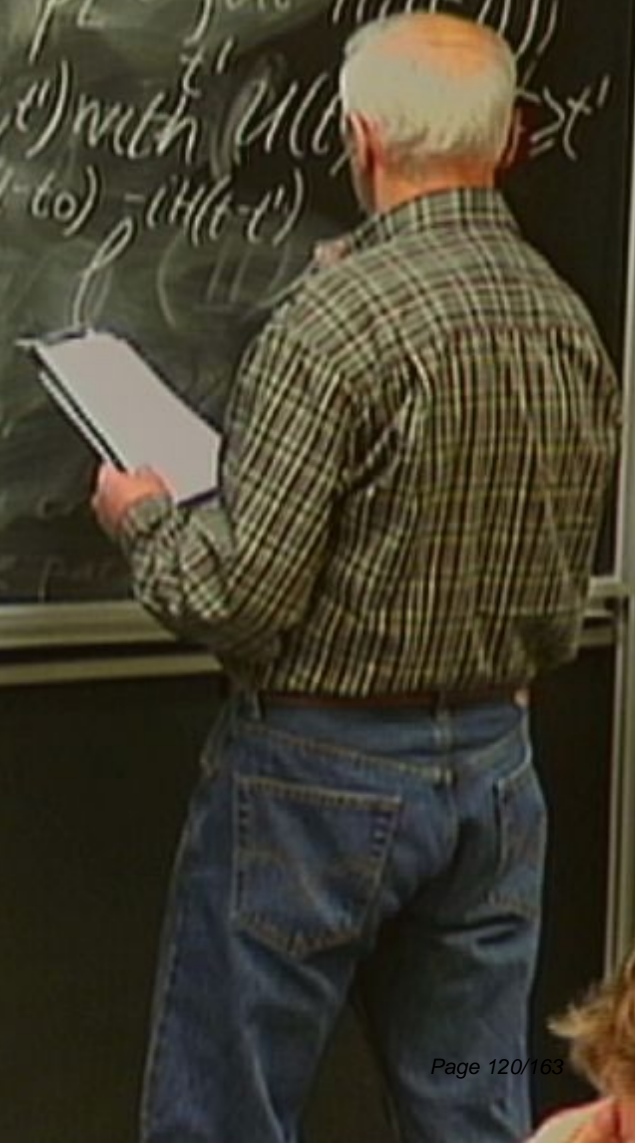
411

$H_I(t)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\}$,
which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$

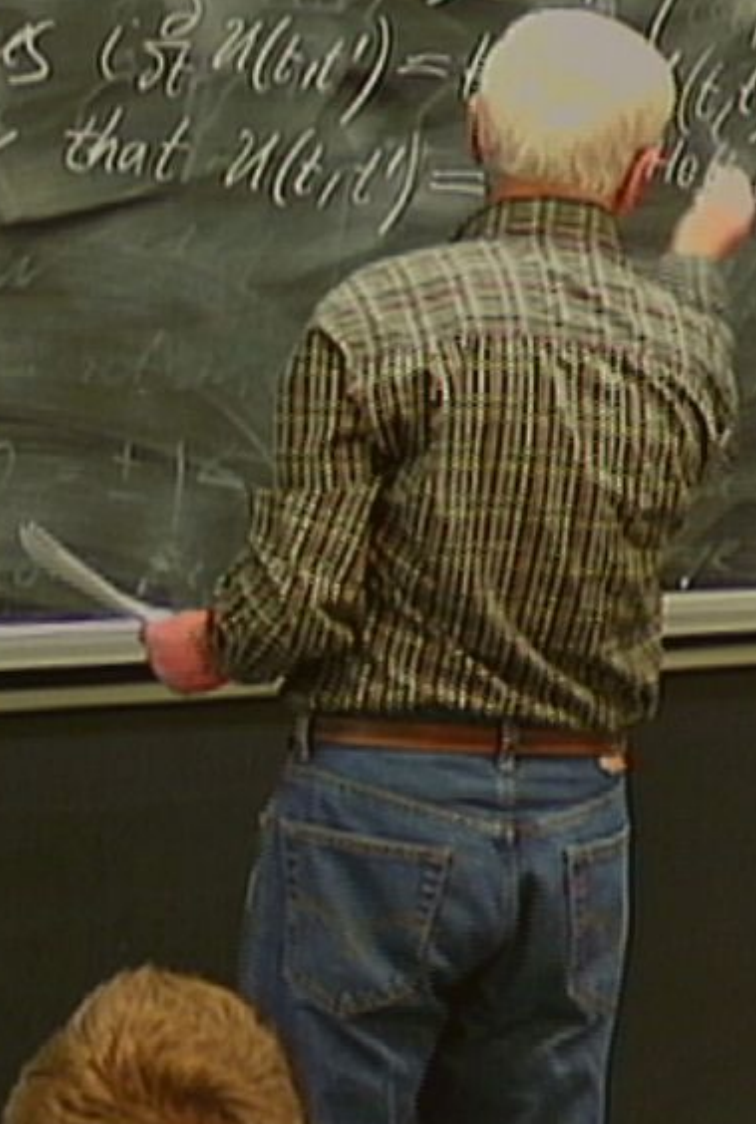


411

$H_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$,
 which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$
 Let us show that $U(t, t') = \left\{ \begin{matrix} -i H_0(t-t_0) & -i H(t-t') \end{matrix} \right\}$



$H_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$,
 which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$
 Let us show that $U(t, t') = \mathcal{P} \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right]$



411

$\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\}$,
 which satisfies $i \partial_t U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$
 Let us show that $U(t, t') = \mathcal{P} \exp \left[-i \int_{t'}^t H(t'') dt'' \right]$
 (check: $i \frac{\partial U(t, t')}{\partial t} = H(t) U(t, t')$)



$\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\}$,
 which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$.
 Let us show that $U(t, t') = \mathcal{P} \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right]$.
 (check: $i \frac{\partial}{\partial t} U(t, t') = - \left[H_I(t) U(t, t') - U(t, t') H_I(t) \right]$)



$\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\}$,
 which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$

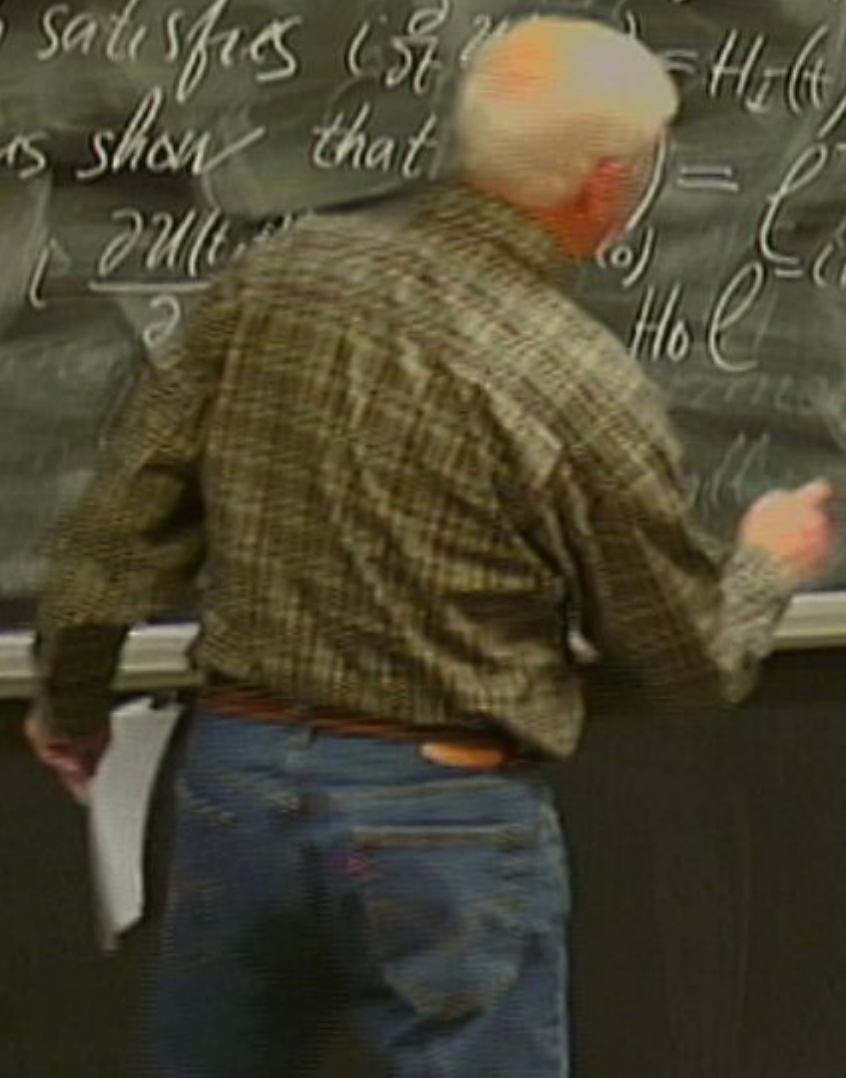
Let us show that $U(t, t') = \exp \left\{ -i H_0(t-t') - i H(t-t') - i H_0(t-t') \right\}$
 (check: $i \frac{\partial}{\partial t} U(t, t') = - \left\{ H_0 - i H(t-t') - H_0(t-t') \right\} U(t, t')$

~~...
 ...
 ...~~

411

through t_0 and $\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$

Let us show that $\frac{\partial}{\partial t} U(t, t') = -i H_I(t) U(t, t')$
check: $i \frac{\partial}{\partial t} U(t, t') = i \frac{\partial}{\partial t} T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\} = T \left\{ \frac{\partial}{\partial t} \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$
 $= T \left\{ -i H_I(t) \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\} = -i H_I(t) U(t, t')$



41

through t_0 and $\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$

show that $U(t, t') = \begin{cases} -iH_0(t-t_0) & -iH(t-t') & -iH_0(t'-t_0) \\ \dots & \dots & \dots \end{cases}$

$$\frac{\partial U(t, t')}{\partial t} = - \begin{pmatrix} -iH_0(t-t_0) & -iH(t-t') & -iH_0(t'-t_0) \\ \dots & \dots & \dots \end{pmatrix} + \dots$$



411

In order to express $|E\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$.

Let us show that $U(t, t') = e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' (H - H_0)}$

check $i \frac{\partial}{\partial t} U(t, t') = - \left(H_0 e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' (H - H_0)} + e^{-iH_0(t-t')} (H - H_0) e^{-i \int_{t'}^t dt'' (H - H_0)} \right)$



2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L + L_{int}$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$.

Let us show that $U(t, t') = e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' (H - H_0)}$
 check $i \frac{\partial}{\partial t} U(t, t') = -i H_0 e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' (H - H_0)} + e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' (H - H_0)} (H - H_0)$
 $= -i H_0 U(t, t') + U(t, t') (H - H_0) = -i (H - H_0) U(t, t')$

2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L_{EM} + L_{int}$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $(\partial_t U(t, t')) = H_I(t) U(t, t')$ with $U(t', t') = 1$ $t \geq t'$

Let us show that $U(t, t') = e^{-iH_0(t-t')} e^{-i\int_{t'}^t dt'' (H - H_0)}$
 (check $\frac{\partial U(t, t')}{\partial t} = -iH(t) U(t, t')$)
 $\frac{\partial}{\partial t} e^{-iH_0(t-t')} e^{-i\int_{t'}^t dt'' (H - H_0)} = -iH_0 e^{-iH_0(t-t')} e^{-i\int_{t'}^t dt'' (H - H_0)} - e^{-iH_0(t-t')} (H - H_0) e^{-i\int_{t'}^t dt'' (H - H_0)}$



2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L_{EM} + L_{int}$

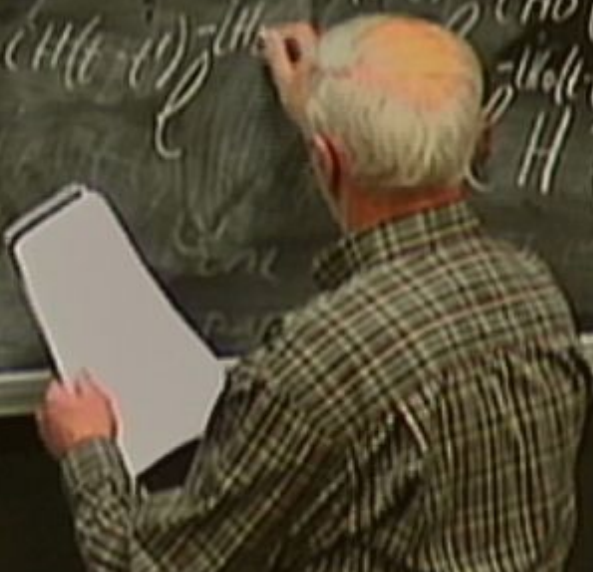
In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t', t') = 1$ $t \geq t'$

Let us show that $U(t, t') = e^{-iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 (check $i \frac{\partial}{\partial t} U(t, t') = -iH_0 e^{-iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)} - e^{-iH_0(t-t')} e^{-iH(t-t')} (-iH_0) e^{-iH_0(t'-t)}$
 $= -iH_0 e^{-iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)} + e^{-iH_0(t-t')} e^{-iH(t-t')} iH_0 e^{-iH_0(t'-t)}$
 $= -i(H - H_0) e^{-iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$)

2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L_{EM} + L_{int}$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$ $t \geq t'$

Let us show that $U(t, t') = \mathcal{P} \exp \left[-i \int_{t'}^t H_0(t'') dt'' - i \int_{t'}^t H_I(t'') dt'' \right]$
 (check $i \frac{\partial}{\partial t} U(t, t') = - \left(H_0(t) + H_I(t) \right) U(t, t')$)
 $\mathcal{P} \exp \left[-i \int_{t'}^t H_0(t'') dt'' \right] = \exp \left[-i H_0(t-t')$



2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L_{EM} + L_{int}$

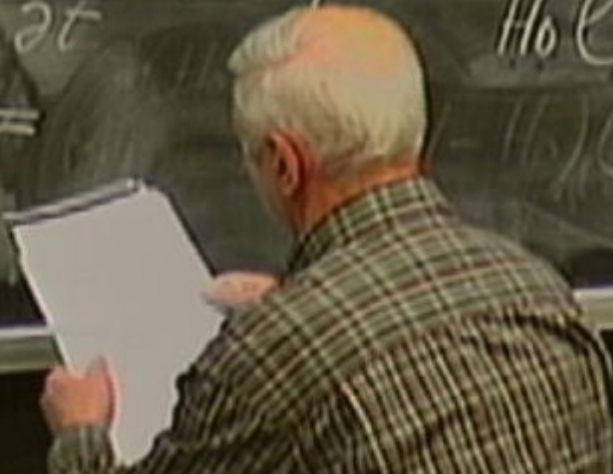
In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$

Let us show that $U(t, t') = \mathcal{P} \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right]$
check $i \frac{\partial U(t, t')}{\partial t} = -H_I(t) U(t, t')$
 $i \frac{\partial}{\partial t} \mathcal{P} \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] = -H_I(t) \mathcal{P} \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right]$

2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L_{EM} + L_{int}$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$

Let us show that $U(t, t') = \mathcal{P} \exp \left[-i \int_{t'}^t H_0(t'') dt'' - i \int_{t'}^t H_I(t'') dt'' \right]$
 (check $i \frac{\partial}{\partial t} U(t, t') = - \mathcal{P} \exp \left[-i \int_{t'}^t H_0(t'') dt'' - i \int_{t'}^t H_I(t'') dt'' \right] H_0(t) - \mathcal{P} \exp \left[-i \int_{t'}^t H_0(t'') dt'' - i \int_{t'}^t H_I(t'') dt'' \right] H_I(t)$)



2) Quantum Electrodynamics (QED). $L_{QED} = L_{Dirac} + L_{Maxwell} + L_{int} = \bar{\psi}(\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4} F_{\mu\nu}^2 - \bar{\psi} \gamma^\mu \psi A_\mu$

In order to express $|\Omega\rangle$ through $|0\rangle$ and $\psi(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[i \int_{t'}^t dt'' H_I(t'') \right] \right\}$, which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$

Let us show that $U(t, t') = e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' H_1(t'')} e^{-iH_0(t-t')}$
 (check $i \frac{\partial}{\partial t} U(t, t') = - \left[H_0(t) e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' H_1(t'')} e^{-iH_0(t-t')} + e^{-iH_0(t-t')} \frac{\partial}{\partial t} e^{-i \int_{t'}^t dt'' H_1(t'')} e^{-iH_0(t-t')} \right]$)
 $\cdot \left[H_0(t) e^{-iH_0(t-t')} e^{-i \int_{t'}^t dt'' H_1(t'')} e^{-iH_0(t-t')} + e^{-iH_0(t-t')} \frac{\partial}{\partial t} e^{-i \int_{t'}^t dt'' H_1(t'')} e^{-iH_0(t-t')} \right]$



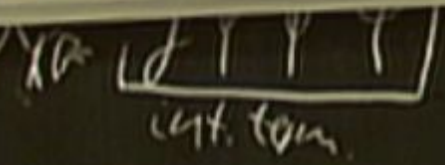
411

which satisfies $(\partial_t U(t,t')) = H(t)U(t,t')$ with $U(t,t') = 1$ $t=t'$

Let us show that $U(t,t') = e^{-iH_0(t-t)} e^{-iH(t-t)}$

check $\frac{\partial U(t,t')}{\partial t} = -iH(t)U(t,t')$

dim $\psi = 3/2$



411

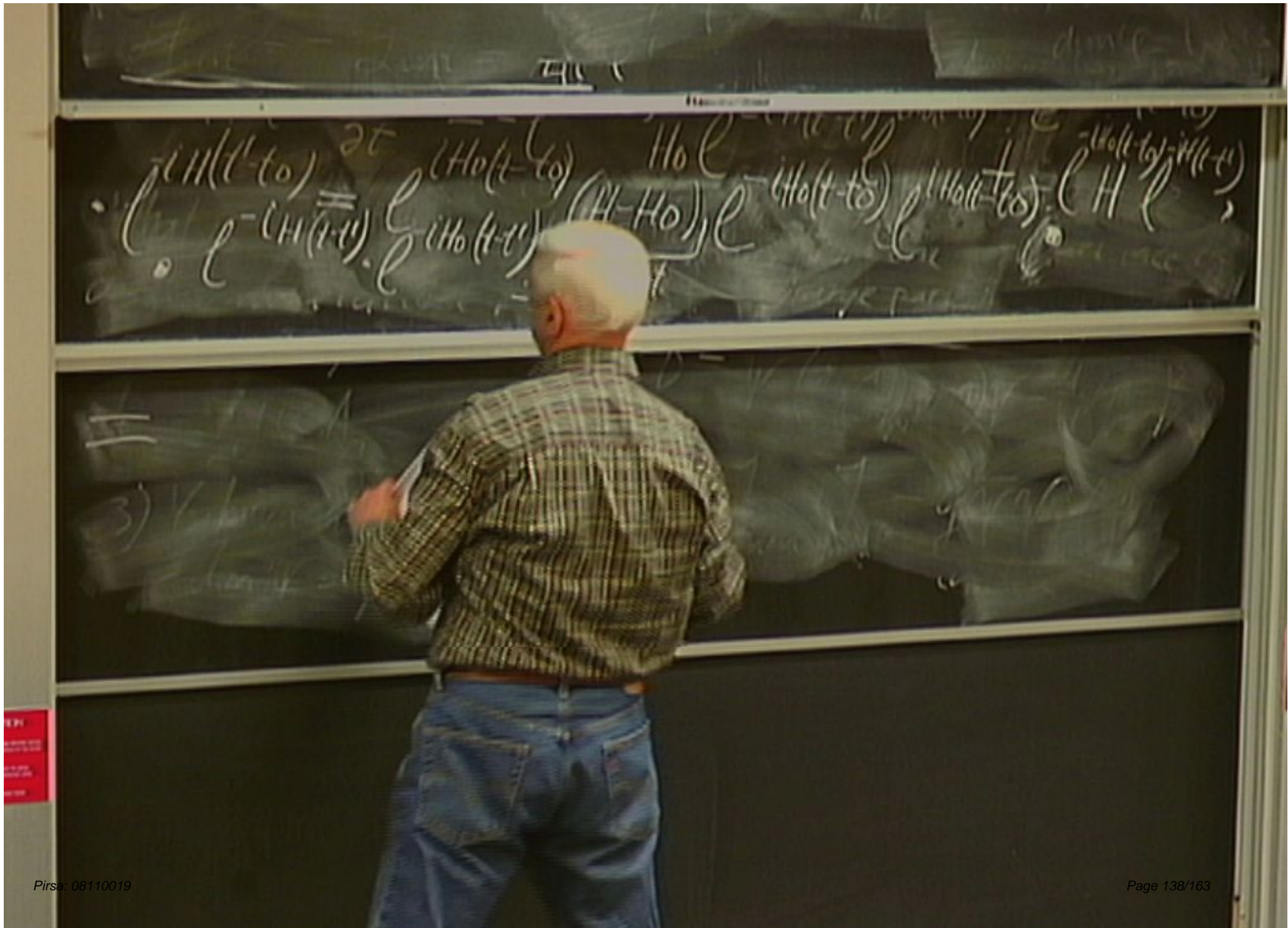
$$\begin{aligned}
 & \psi(t-t_0) = e^{-iH(t-t_0)} \psi(t_0) \\
 & \psi(t-t_0) = e^{-i(H_0(t-t_0))} e^{-i(H-H_0)(t-t_0)} \psi(t_0) \\
 & \text{Hint}
 \end{aligned}$$

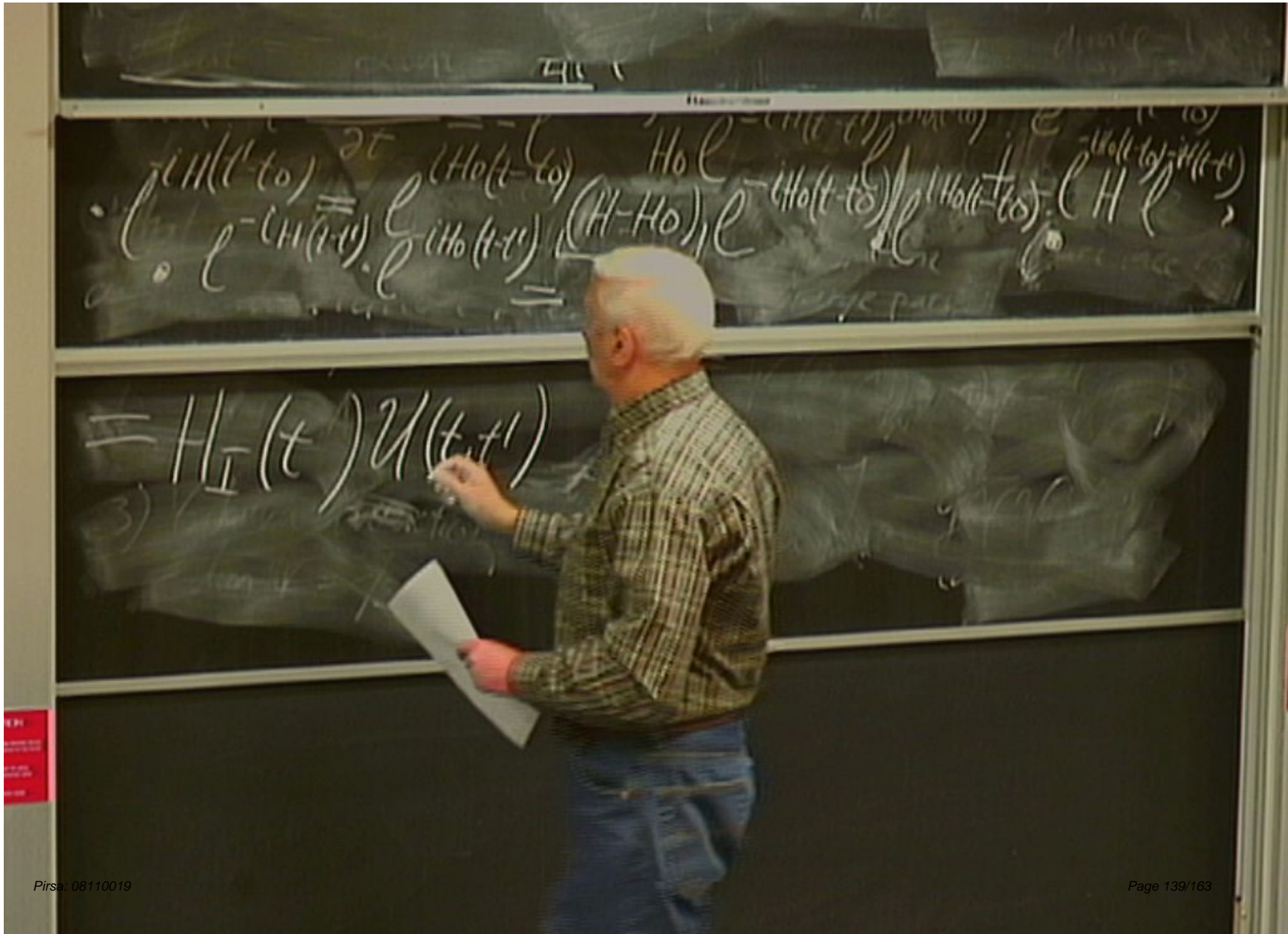
$$D_n = \partial_n + i e A_n$$

3) Yukawa interaction
 dim $\psi = 3/2$

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi$$

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Yukawa}} = -g \bar{\psi} \psi \phi$$





$$\begin{aligned}
 & e^{-iH(t-t_0)} = e^{-iH_0(t-t_0)} e^{-i(H-H_0)(t-t_0)} \\
 & e^{-iH(t-t_0)} = e^{-iH_0(t-t_0)} \left(1 - i(H-H_0)(t-t_0) + \dots \right)
 \end{aligned}$$

$$= U_I(t, t_0) U(t, t_0)$$

411

done

through t_0 and t_1
 $H_I(x)$, introduce $U(t, t') \equiv T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$,
 which satisfies $i \frac{\partial}{\partial t} U(t, t') = H_I(t) U(t, t')$ with $U(t, t) = 1$

Let us show that $U(t, t') = U(t, t_0) U(t_0, t')$
 (check $i \frac{\partial}{\partial t} U(t, t') = - \left(\begin{matrix} i H(t-t_0) \\ i H_0(t-t_0) \end{matrix} \right) U(t, t')$)

$$i \frac{\partial}{\partial t} U(t, t') = - \left(\begin{matrix} i H(t-t_0) \\ i H_0(t-t_0) \end{matrix} \right) U(t, t')$$

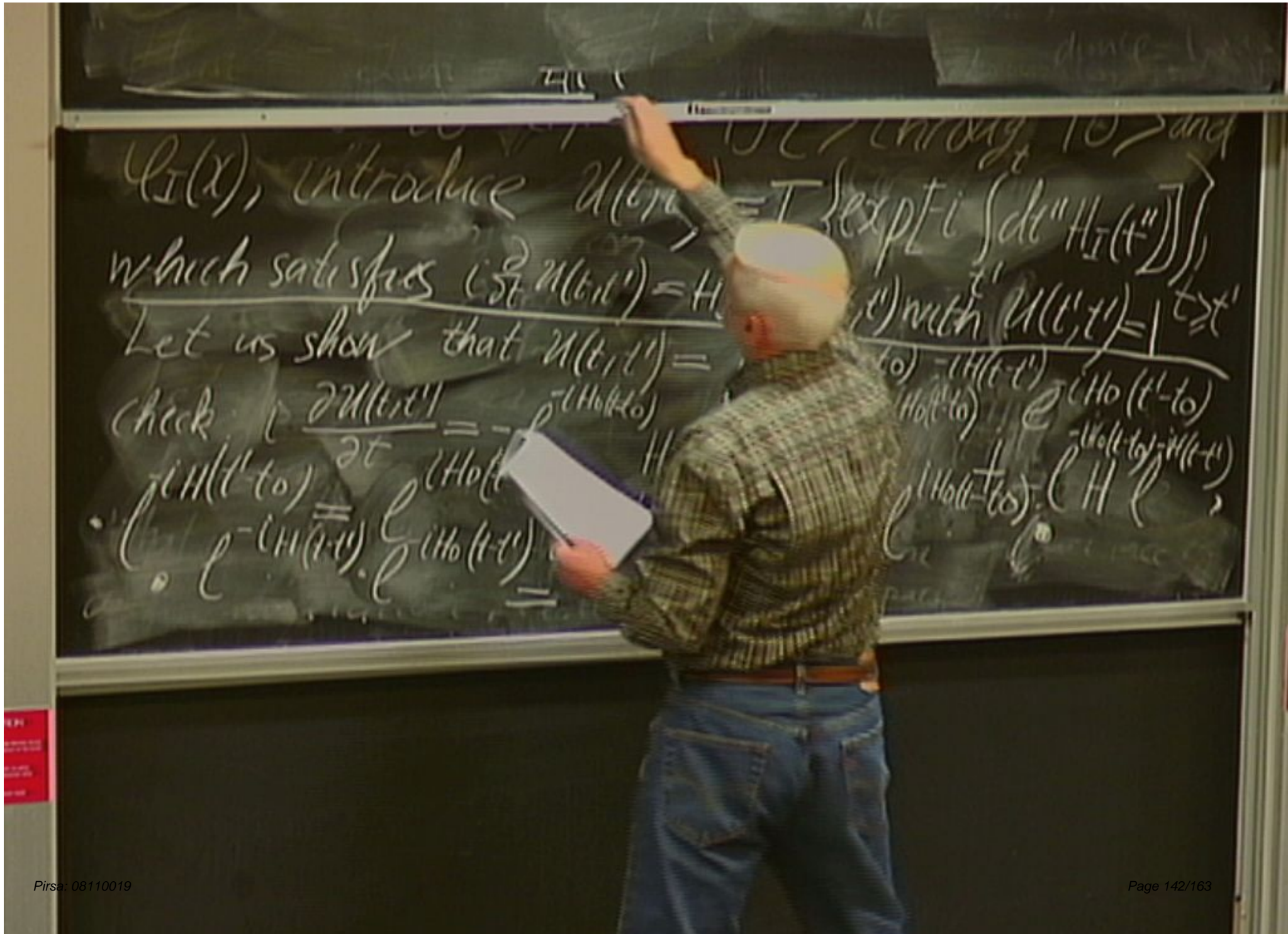
$$= - \left(\begin{matrix} i H(t-t_0) \\ i H_0(t-t_0) \end{matrix} \right) U(t, t_0) U(t_0, t')$$

$$= - \left(\begin{matrix} i H(t-t_0) \\ i H_0(t-t_0) \end{matrix} \right) U(t, t_0) U(t_0, t')$$

411

through t_0 and $\psi(x)$, introduce $u(t')$ $\equiv T \left\{ \exp \left[-i \int_{t_0}^{t'} dt'' H_I(t'') \right] \right\}$,
which satisfies $\partial_t u(t, t') = H_I(t) u(t, t')$ with $u(t, t') = 1$ $t = t'$

Let us show $u(t, t') = \left[\frac{H_0(t-t_0) - iH(t-t')}{H_0(t-t_0) - iH(t-t')} \right] e^{iH_0(t-t_0) - iH(t-t')}$
check $\partial_t u$
 $\partial_t \left[\frac{H_0(t-t_0) - iH(t-t')}{H_0(t-t_0) - iH(t-t')} \right] e^{iH_0(t-t_0) - iH(t-t')}$
 $\frac{H_0 - iH}{H_0 - iH} e^{iH_0(t-t_0) - iH(t-t')}$
Hint



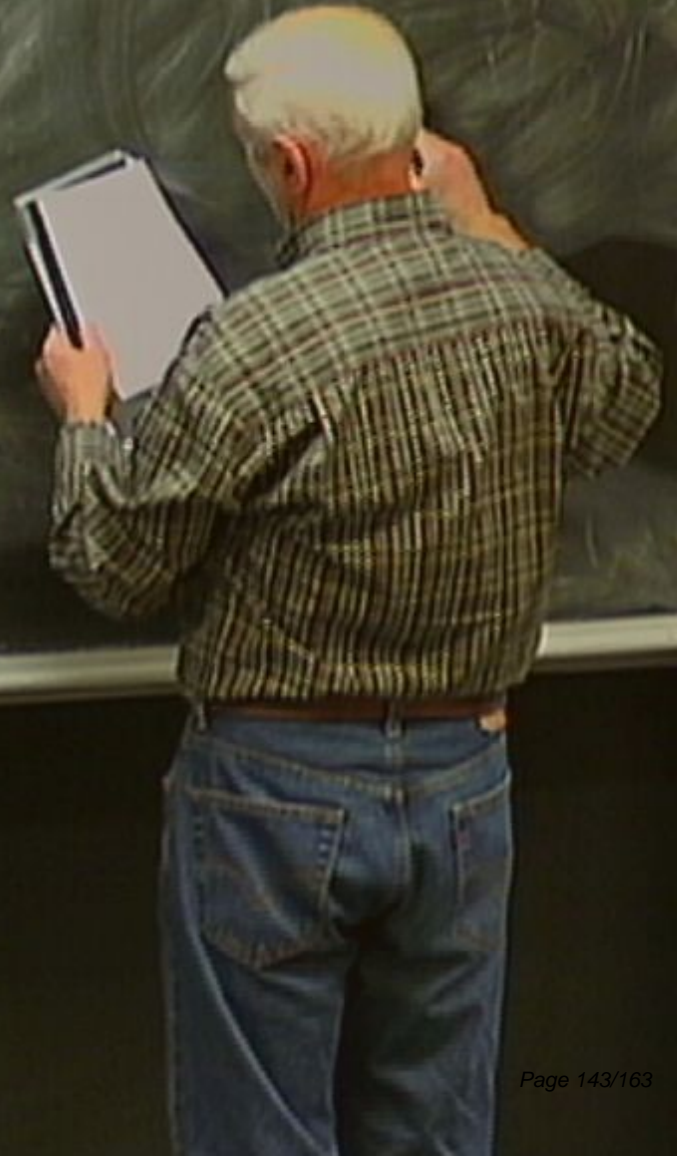
$U(t, t')$, introduce $U(t, t') = T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$,
which satisfies $(\partial_t U(t, t')) = -i H(t) U(t, t')$ with $U(t, t') = 1$ for $t = t'$

Let us show that $U(t, t') =$

check $\frac{\partial U(t, t')}{\partial t} = -i H(t) U(t, t')$

$$U(t, t') = \int_{t'}^t dt'' e^{-i \int_{t'}^{t''} H(t''') dt'''} e^{-i \int_{t''}^t H(t''') dt'''} = e^{-i \int_{t'}^t H(t'') dt''}$$

Let us show that $U(t, t') = e^{iH_0(t-t')}$



Let us show that $U(t, t') = \mathcal{P} \exp(iH_0(t-t') - iH(t-t'))$



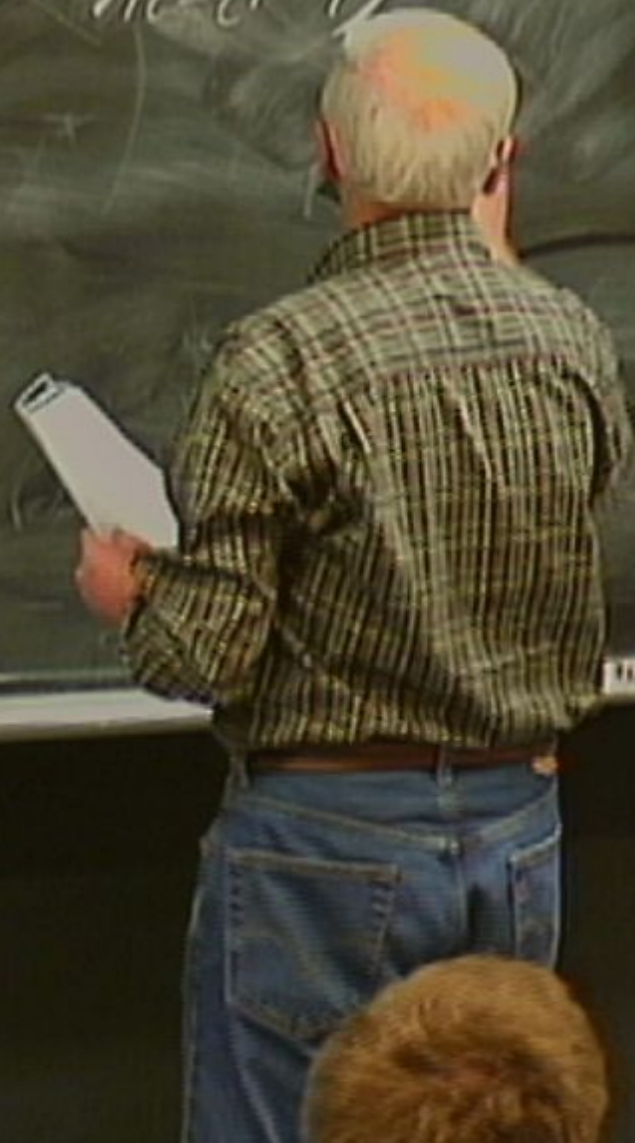
Let us show that $U(t, t') = \mathcal{P} \exp \left[i \int_{t'}^t H_0(t) dt - i H(t-t') - i H_0(t-t') \right]$

[The rest of the chalkboard contains heavily scribbled-out text, likely representing a derivation or proof.]

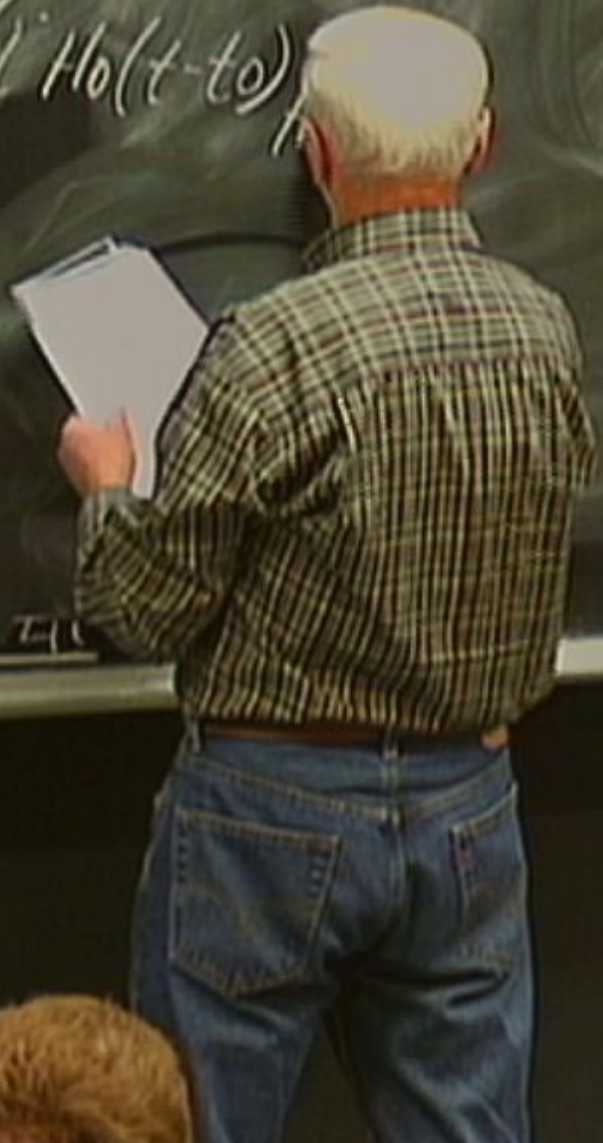


Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t-t')}$
(note $U(t, t')|_{t=t'} = 1$)

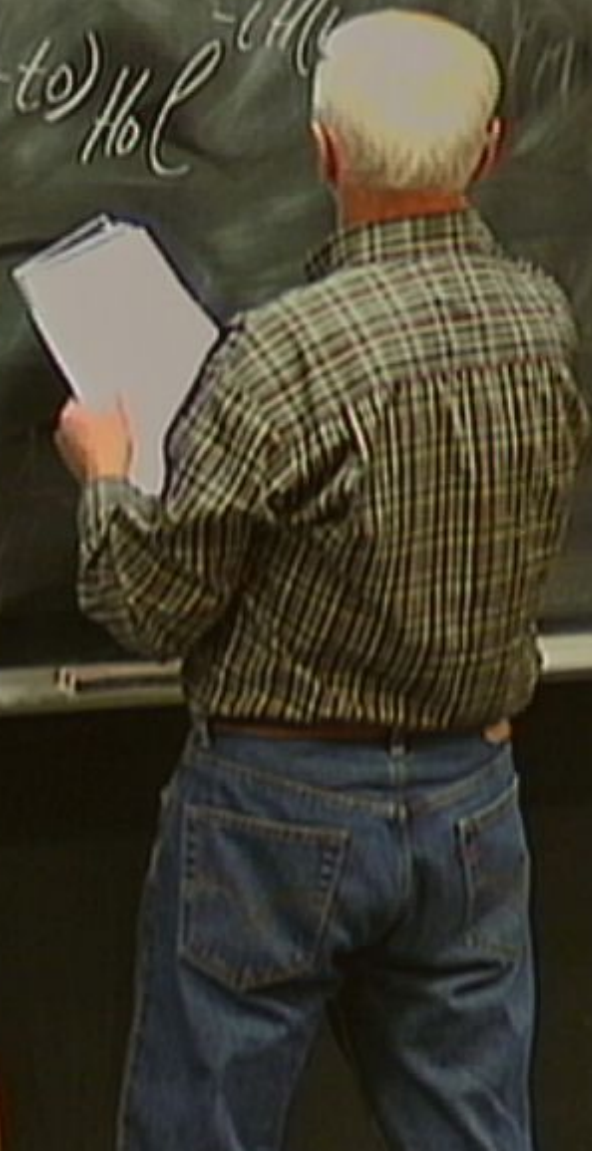
Let us show that $u(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t-t')}$
(note $u(t, t')|_{t=t'} = 1$)



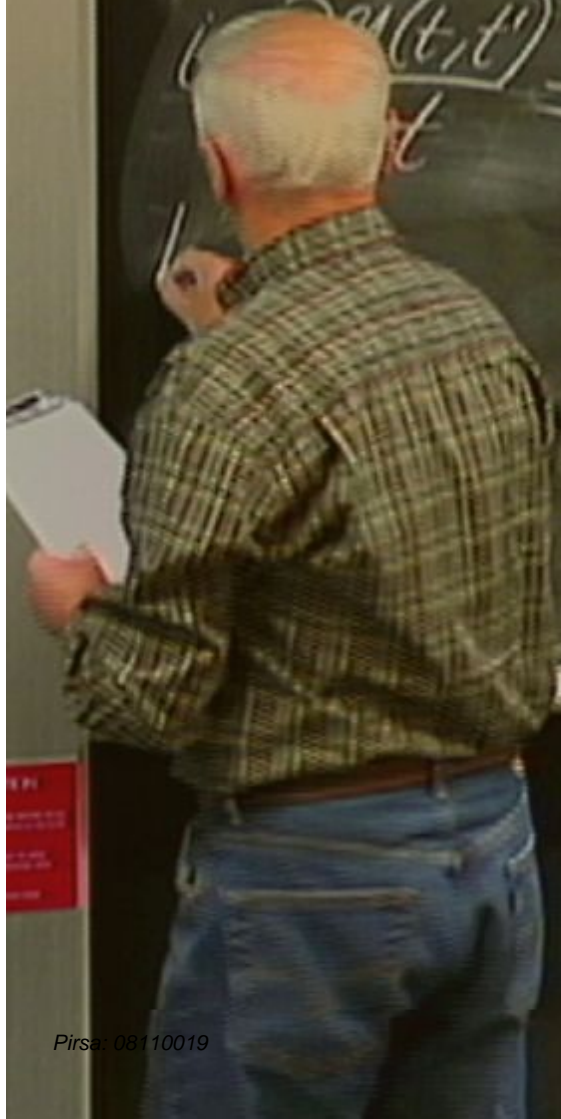
Let us show that $u(t, t') = e^{iH_0(t-t') - iH(t-t') - iH_0(t-t')}$
 (note $u(t, t')|_{t=t'} = 1$)
 $i \frac{\partial u(t, t')}{\partial t} = -e^{iH_0(t-t')}$



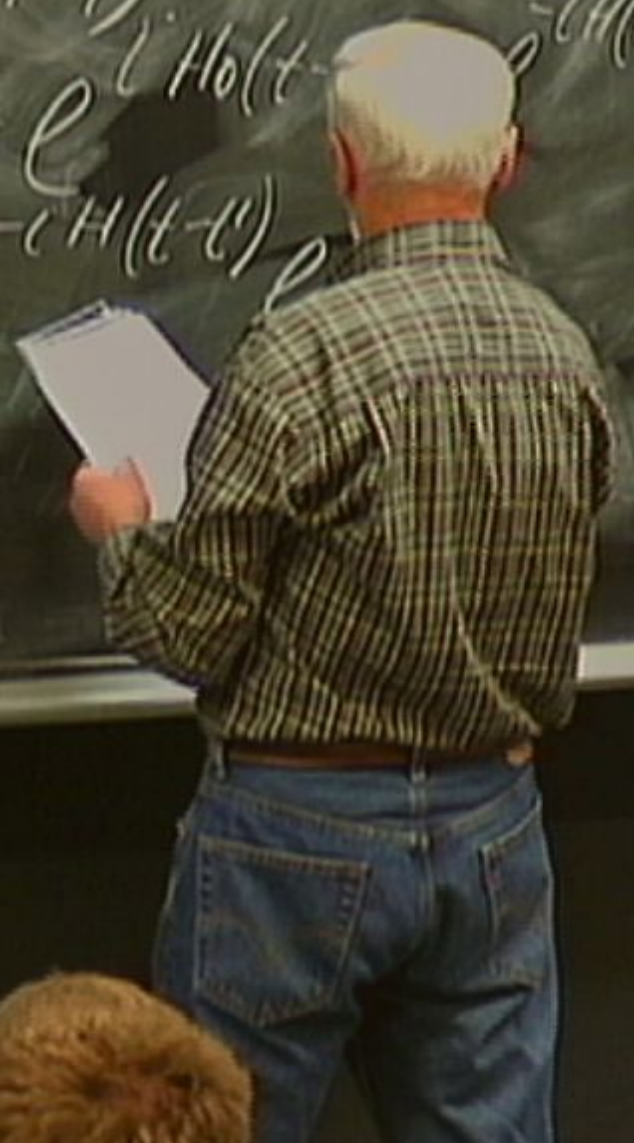
Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t-t')}$
 (note $U(t, t')|_{t=t'} = 1$)
 $i \frac{\partial U(t, t')}{\partial t} = -e^{iH_0(t-t')} H_0 e^{-iH(t-t')} e^{-iH_0(t-t')}$



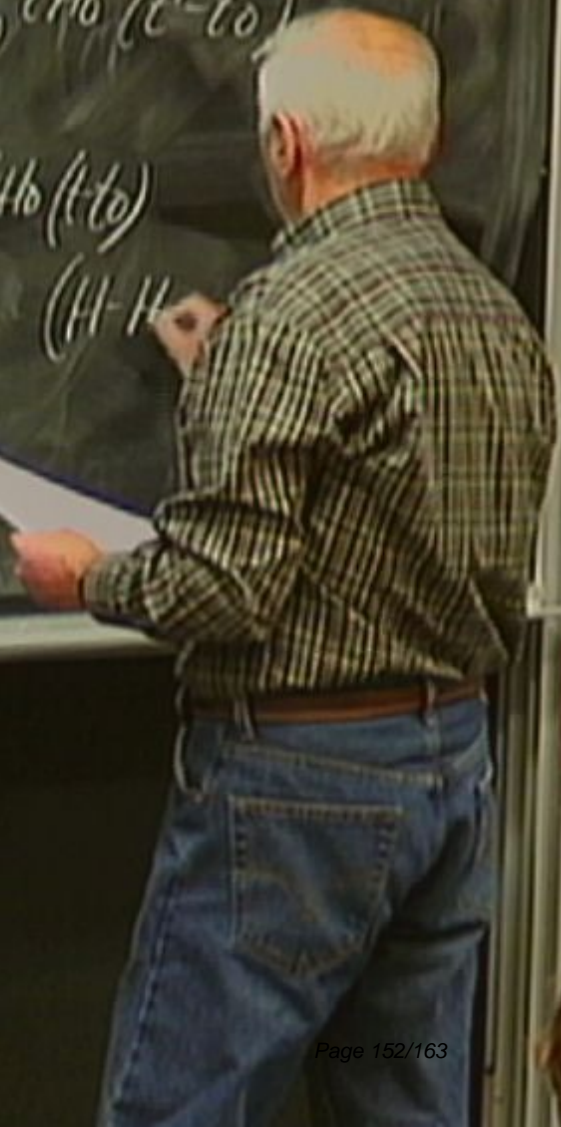
Let us show that $u(t, t') = \frac{1}{2} \left(e^{iH_0(t-t')} + e^{-iH_0(t-t')} \right)$
 (note $u(t, t')|_{t=t'} = 1$)
 $\frac{\partial u(t, t')}{\partial t} = -e^{iH_0(t-t')} + e^{-iH_0(t-t')}$



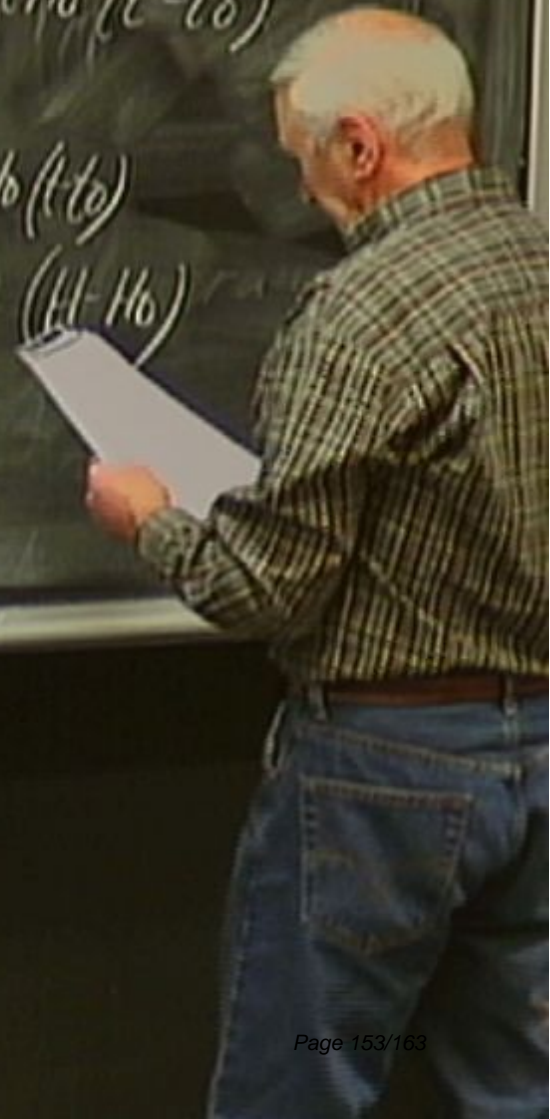
Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 (note $U(t, t')|_{t=t'} = 1$)
 $i \frac{\partial U(t, t')}{\partial t} = -e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 $+ e^{iH_0(t-t')} H e^{-iH(t-t')} e^{-iH_0(t'-t)}$



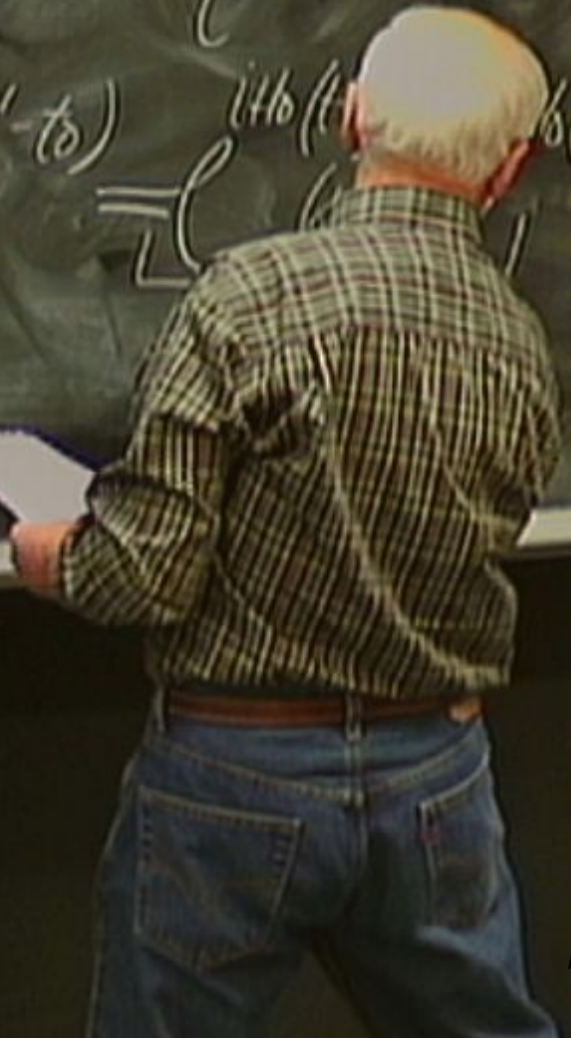
Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 (note $U(t, t')|_{t=t'} = 1$)
 $i \frac{\partial U(t, t')}{\partial t} = -e^{iH_0(t-t')} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 $+ e^{iH(t-t')} H e^{-iH(t-t')} e^{-iH_0(t'-t)} = e^{iH_0(t-t')} (H - H_0) e^{-iH_0(t'-t)}$



Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 (note $U(t, t')|_{t=t'} = 1$)
 $i \frac{\partial U(t, t')}{\partial t} = -e^{iH_0(t-t')} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 $+ e^{iH(t-t')} H e^{-iH(t-t')} e^{-iH_0(t'-t)} = e^{iH_0(t-t')} (H - H_0) e^{-iH_0(t'-t)}$



Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 (note $U(t, t')|_{t=t'} = 1$)
 $i \frac{\partial U(t, t')}{\partial t} = -e^{iH_0(t-t')} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t)} +$
 $+ e^{iH(t-t')} H e^{-iH(t-t')} e^{-iH_0(t'-t)} = e^{iH_0(t-t')} (H_0 - H) e^{-iH_0(t'-t)}$



Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = - e^{iH_0(t-t')} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t)} + e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)} + e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)}$$

$$= e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t)} \frac{(H - H_0) e^{iH_0(t-t')}}{H_0} e^{-iH_0(t-t')}$$

Let us show that $U(t, t') = e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = i H_0(t-t_0) e^{iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} + e^{iH_0(t-t')} i(H-H_0) e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$$

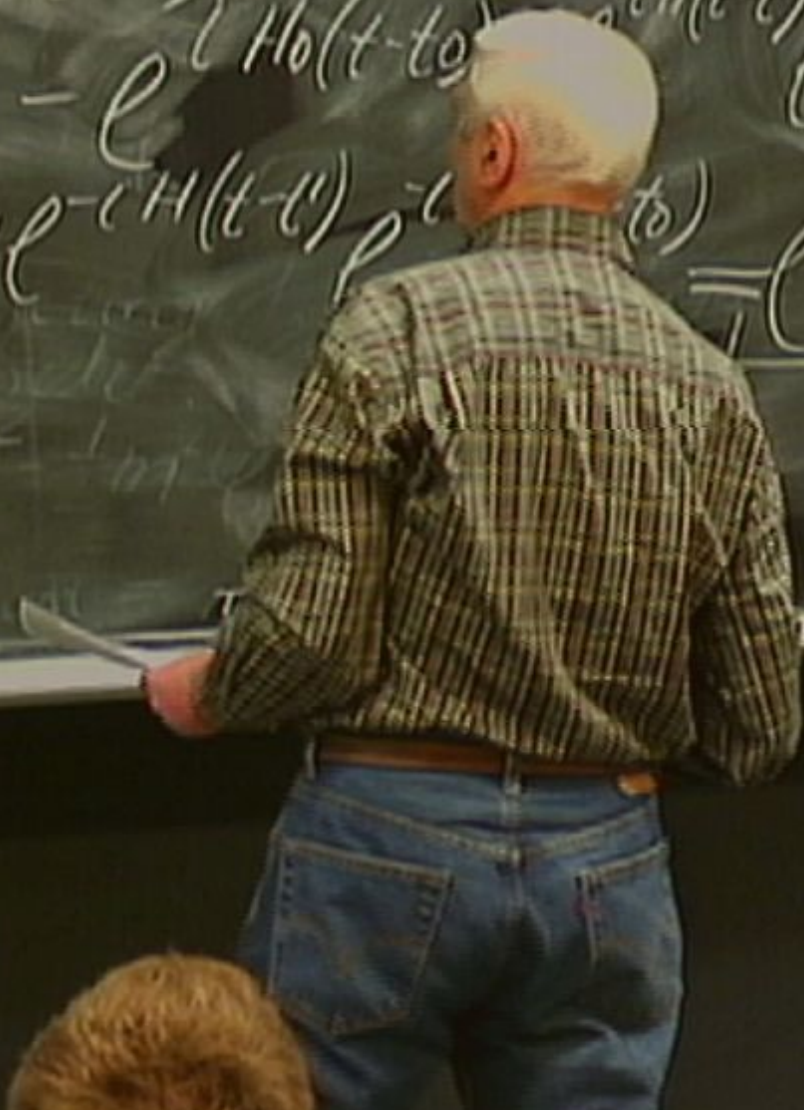
$$= e^{iH_0(t-t')} i(H-H_0) e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$$



Let us show that $U(t, t') = \mathcal{P} \left\{ e^{-iH_0(t-t')} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = - \mathcal{P} \left\{ e^{-iH_0(t-t')} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} + \mathcal{P} \left\{ e^{-iH_0(t-t')} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$$

$$= \mathcal{P} \left\{ e^{-iH_0(t-t')} (H - H_0) e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$$



Let us show that $U(t, t') = \mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = - \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} + \mathcal{P} \left\{ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$$

$$= \mathcal{P} \left\{ \frac{(H - H_0) e^{-iH(t-t')} e^{-iH_0(t'-t_0)}}{H_0} \right\}$$

$$= \mathcal{P} \left\{ \frac{(H - H_0)}{H_0} \right\} U(t, t')$$

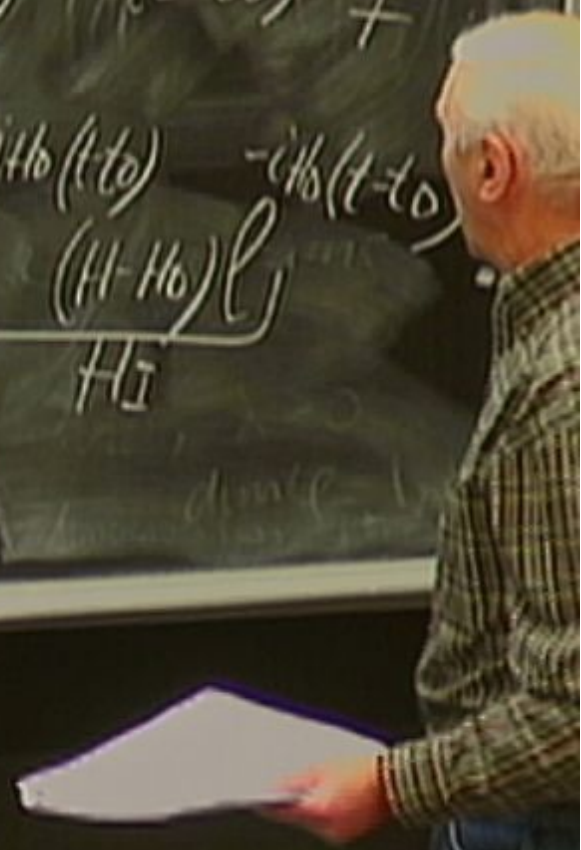


Let us show that $U(t, t') = \mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = - \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} +$$

$$+ \mathcal{P} \left\{ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} = \mathcal{P} \left\{ \frac{iH_0(t-t_0) - iH_0(t-t_0)}{i} \right\}$$

$$\underbrace{\mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}}_{U(t, t')} \Rightarrow$$



Let us show that $U(t, t') = \mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = - \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} +$$

$$+ \mathcal{P} \left\{ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} = \mathcal{P} \left\{ (H - H_0) e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$$

$$\underbrace{\mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}}_{U(t, t')} \Rightarrow i \frac{\partial U}{\partial t} = (H - H_0) U$$

Let us show that $U(t, t') = \mathcal{P} \left\{ e^{-iH_0(t-t')} e^{-i(H-H_0)(t-t')} \right\}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = \mathcal{P} \left\{ iH_0(t-t') e^{-iH_0(t-t')} e^{-iH_0(t'-t_0)} + \right.$$

$$\left. + \mathcal{P} \left\{ e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} \right\} = \mathcal{P} \left\{ (H-H_0) \right\}$$

$$\Rightarrow i \frac{\partial U}{\partial t} = H_I(t) U(t, t')$$



Let us show that $U(t, t') = \mathcal{P} \left\{ e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$
 (note $U(t, t')|_{t=t'} = 1$)

$$i \frac{\partial U(t, t')}{\partial t} = - \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} + \right.$$

$$+ \mathcal{P} \left\{ e^{iH(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} - \mathcal{P} \left\{ e^{iH_0(t-t_0)} H_0 e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$$

$$= \mathcal{P} \left\{ (H - H_0) e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\}$$

$$= \mathcal{P} \left\{ H_I e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right\} \Rightarrow i \frac{\partial U}{\partial t} = H_I(t) U(t, t')$$

$$\left[e^{iH_0(t-t_0)} \cdot e^{-iH(t-t')} \cdot e^{-iH_0(t'-t_0)} \right] \Rightarrow \left(\frac{\partial}{\partial t} = H_I(t) \right) U(t, t')$$

which satisfies $(\frac{\partial}{\partial t} U(t, t')) = H_I(t) U(t, t')$ with $U(t, t) = 1$

Let us show that $U(t, t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$

check $(\frac{\partial}{\partial t} U(t, t')) = -i \left[H_0 e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} - e^{iH_0(t-t_0)} H e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \right]$

$= e^{iH_0(t-t_0)} e^{-iH_0(t-t')} (H - H_0) e^{-iH_0(t'-t_0)}$

$= H_I(t) U(t, t')$