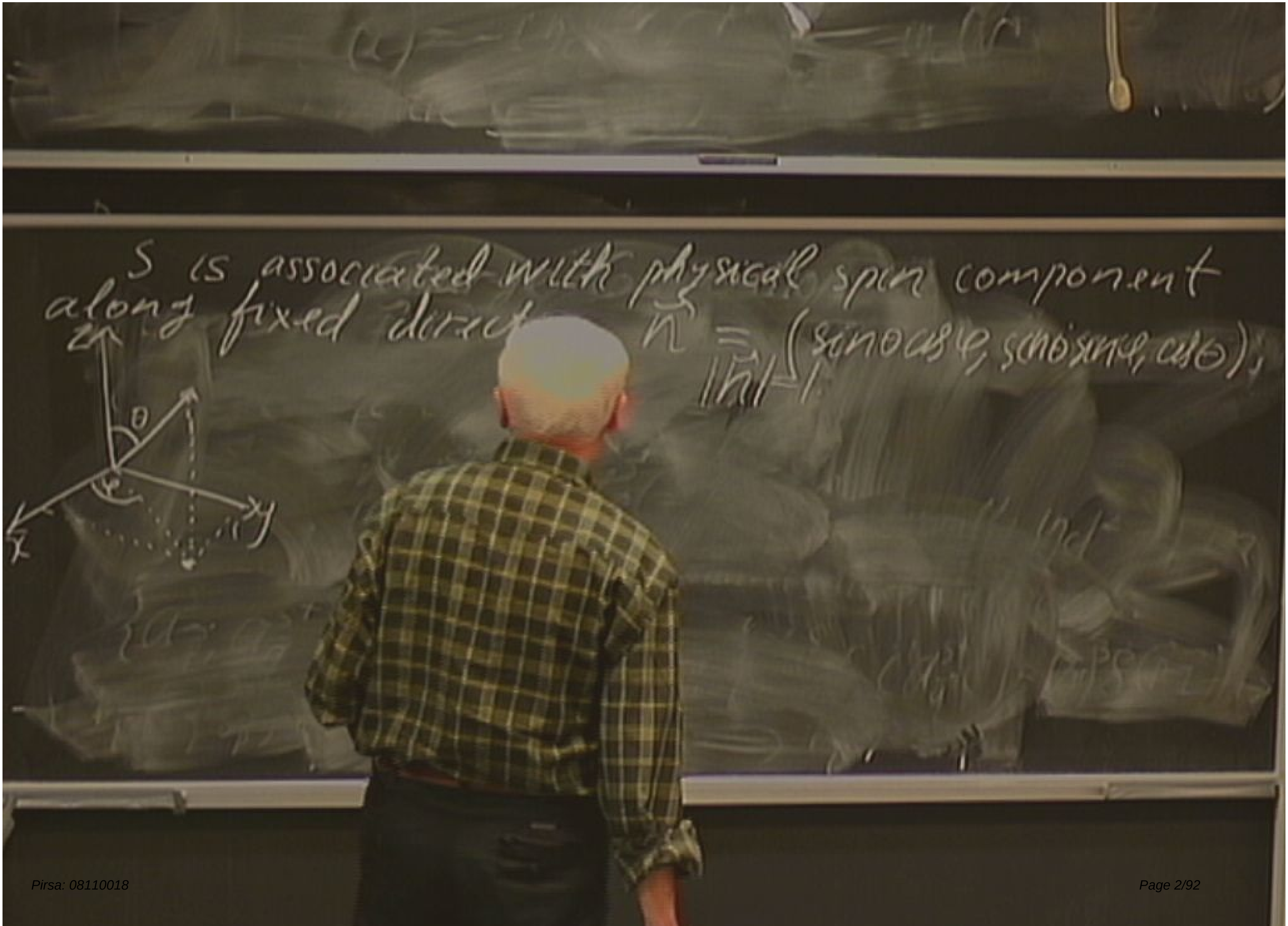
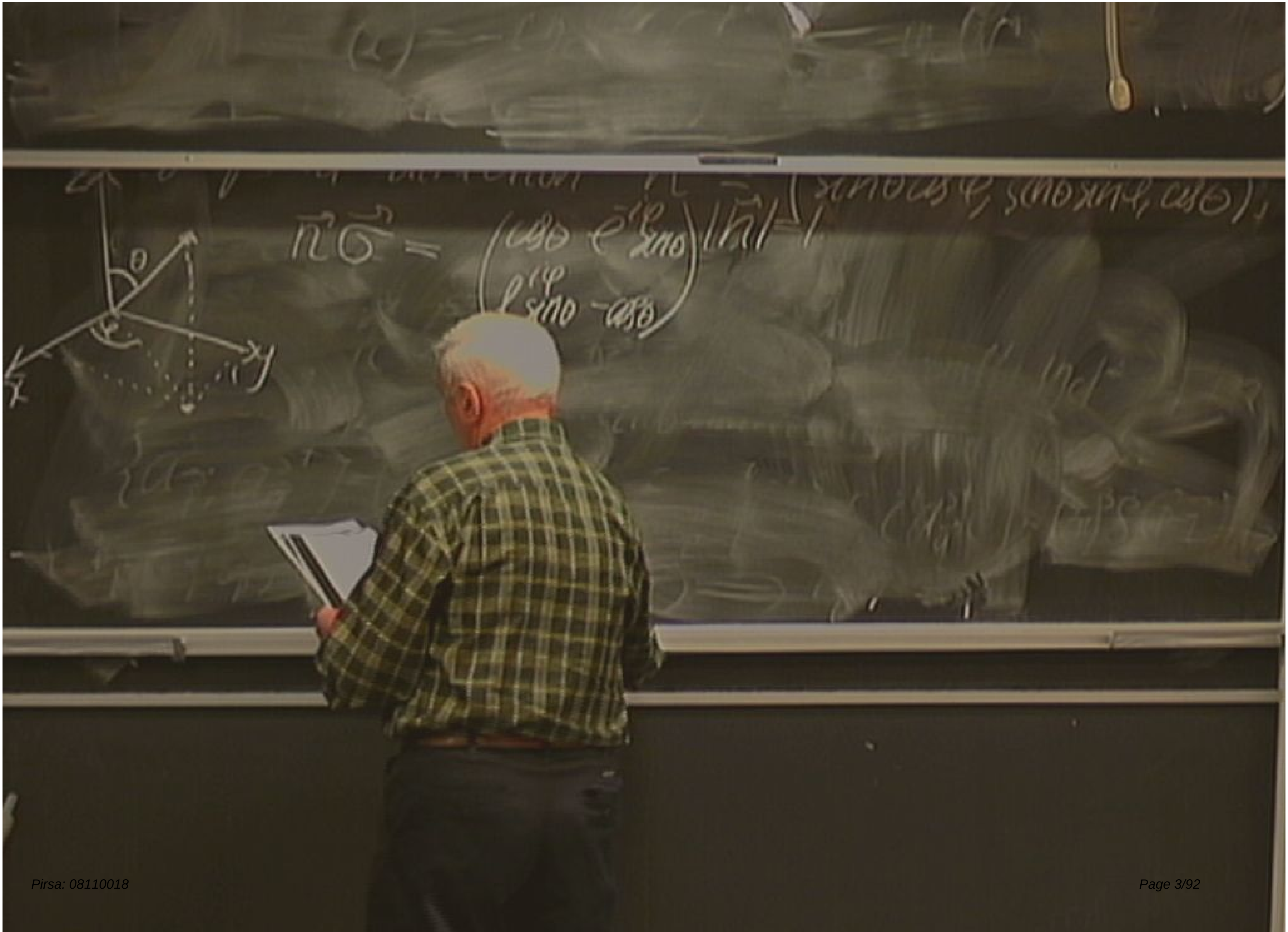
Title: Quantum Field Theory 1 - Lecture 11B

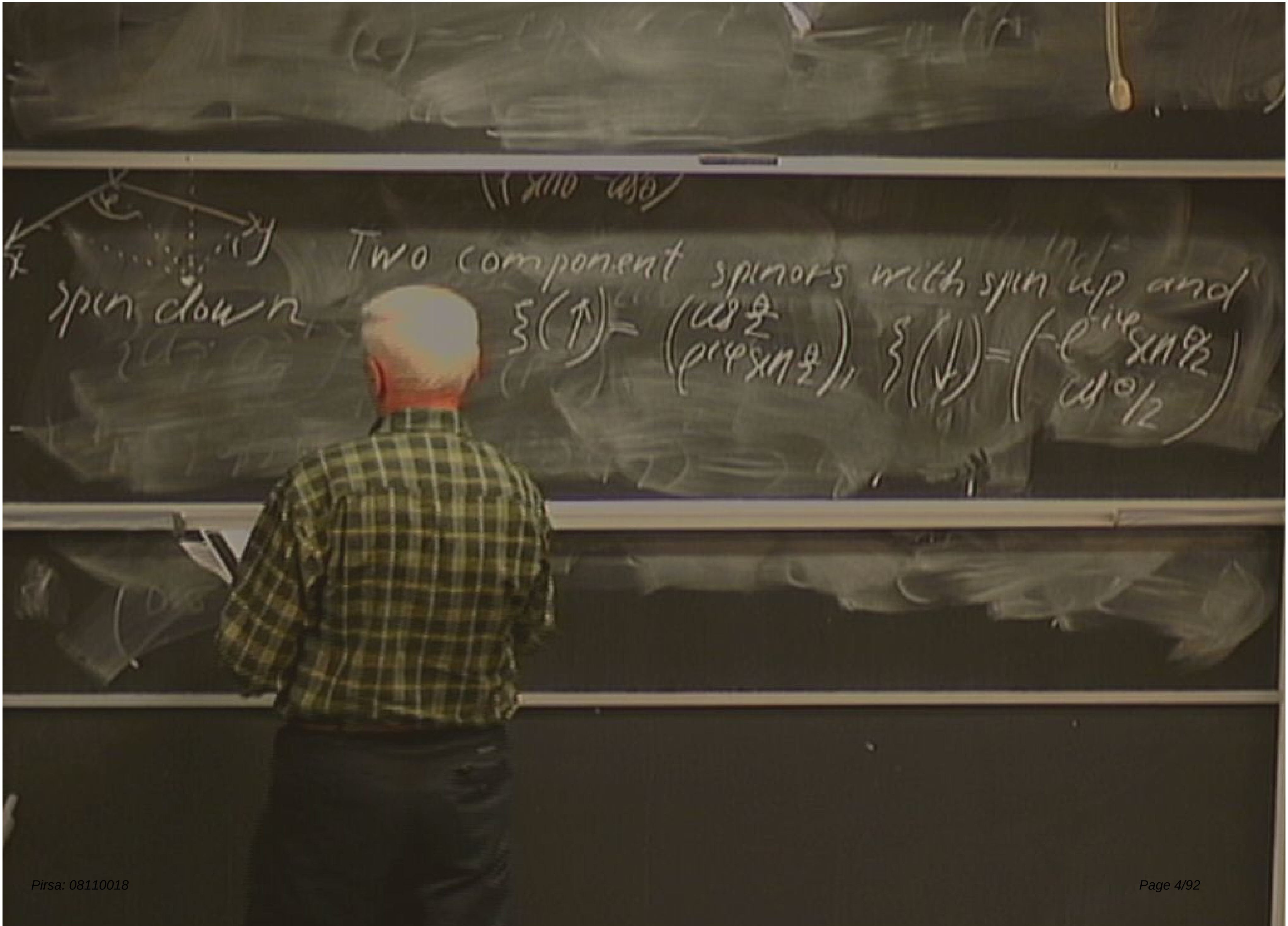
Date: Nov 19, 2008 03:30 PM

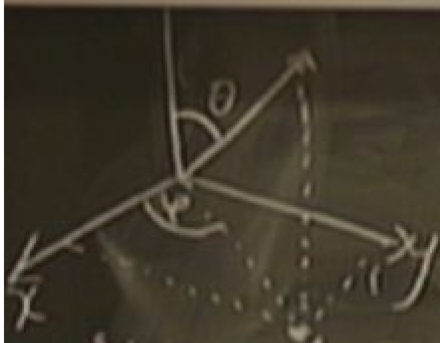
URL: http://pirsa.org/08110018

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario


S is associated with physical spin component
along fixed direct... n = (sinθcosφ, sinθsinφ, cosθ);
|n| = 1
z
θ
φ
x
y




Two component spinors with spin up and
spin down
$\xi(\uparrow) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}, \quad \xi(\downarrow) = \begin{pmatrix} -e^{-i\varphi}\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$

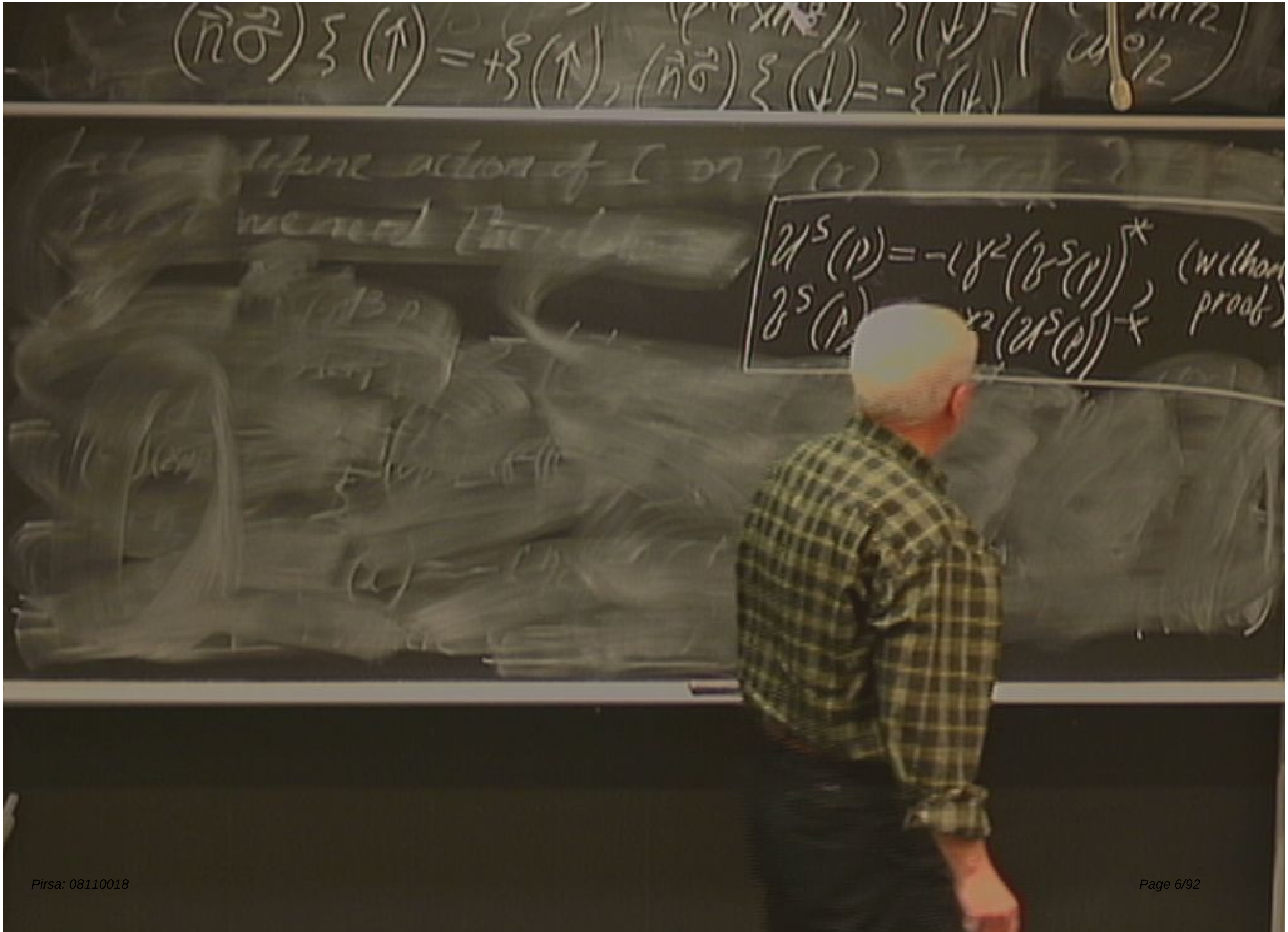


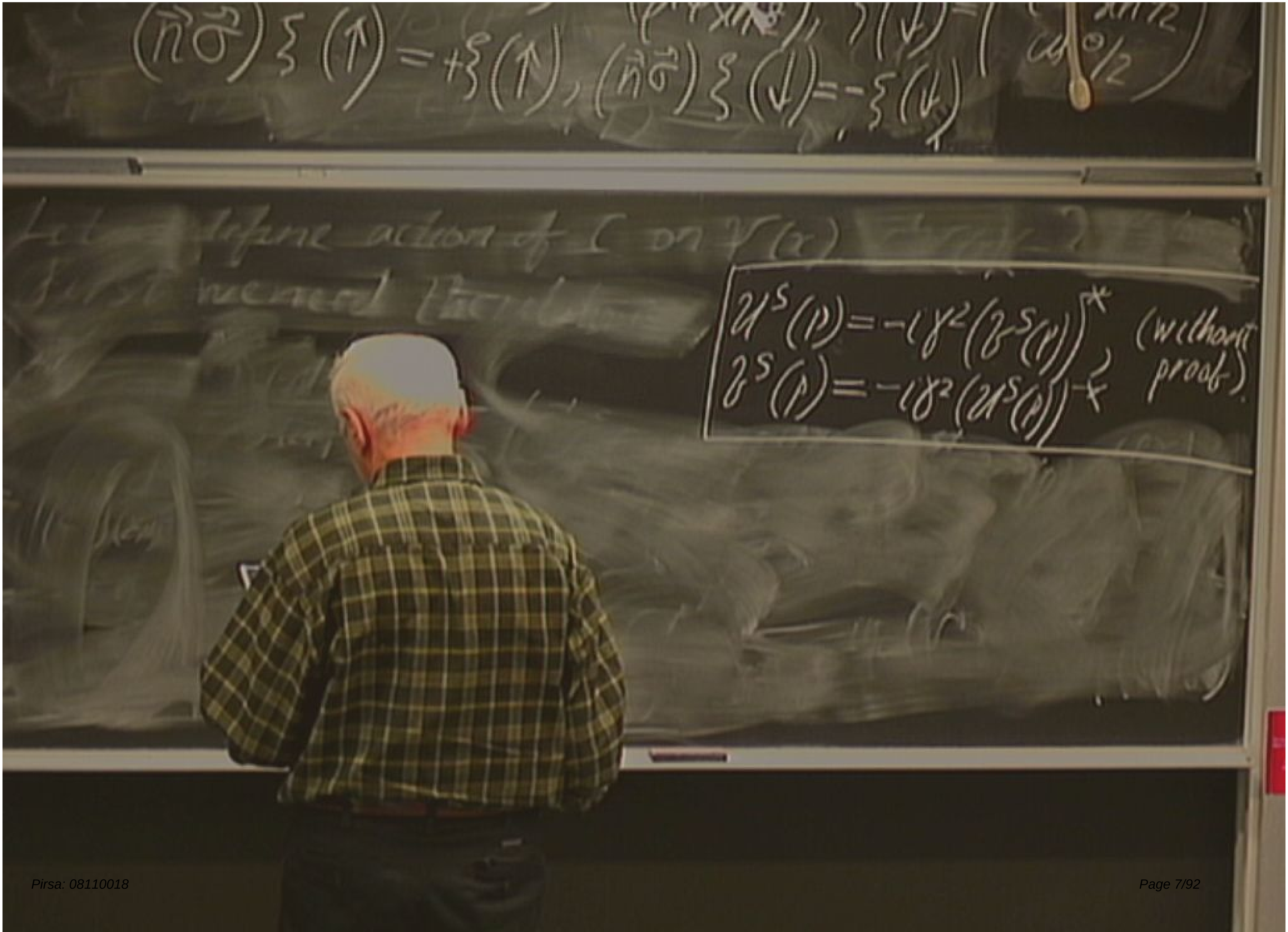
$$\vec{n}\vec{\sigma} = \begin{pmatrix} \cos\theta & e^{-i\varphi}\sin\theta \\ e^{i\varphi}\sin\theta & -\cos\theta \end{pmatrix}, \quad |\vec{n}|=1.$$

Two component spinors with spin up and spin down are:

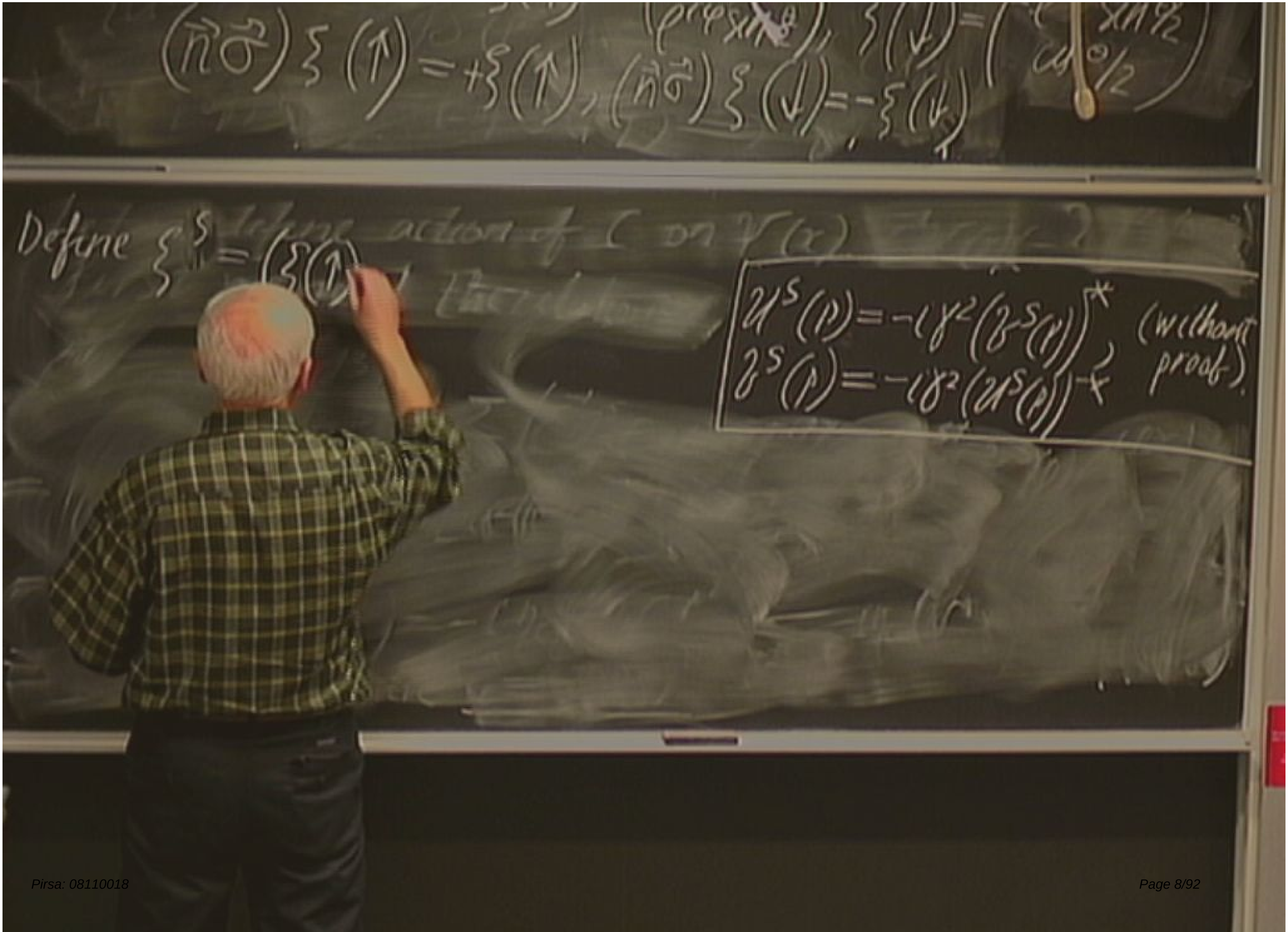
$$\xi(\uparrow) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}, \quad \xi(\downarrow) = \begin{pmatrix} -e^{-i\varphi}\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$$

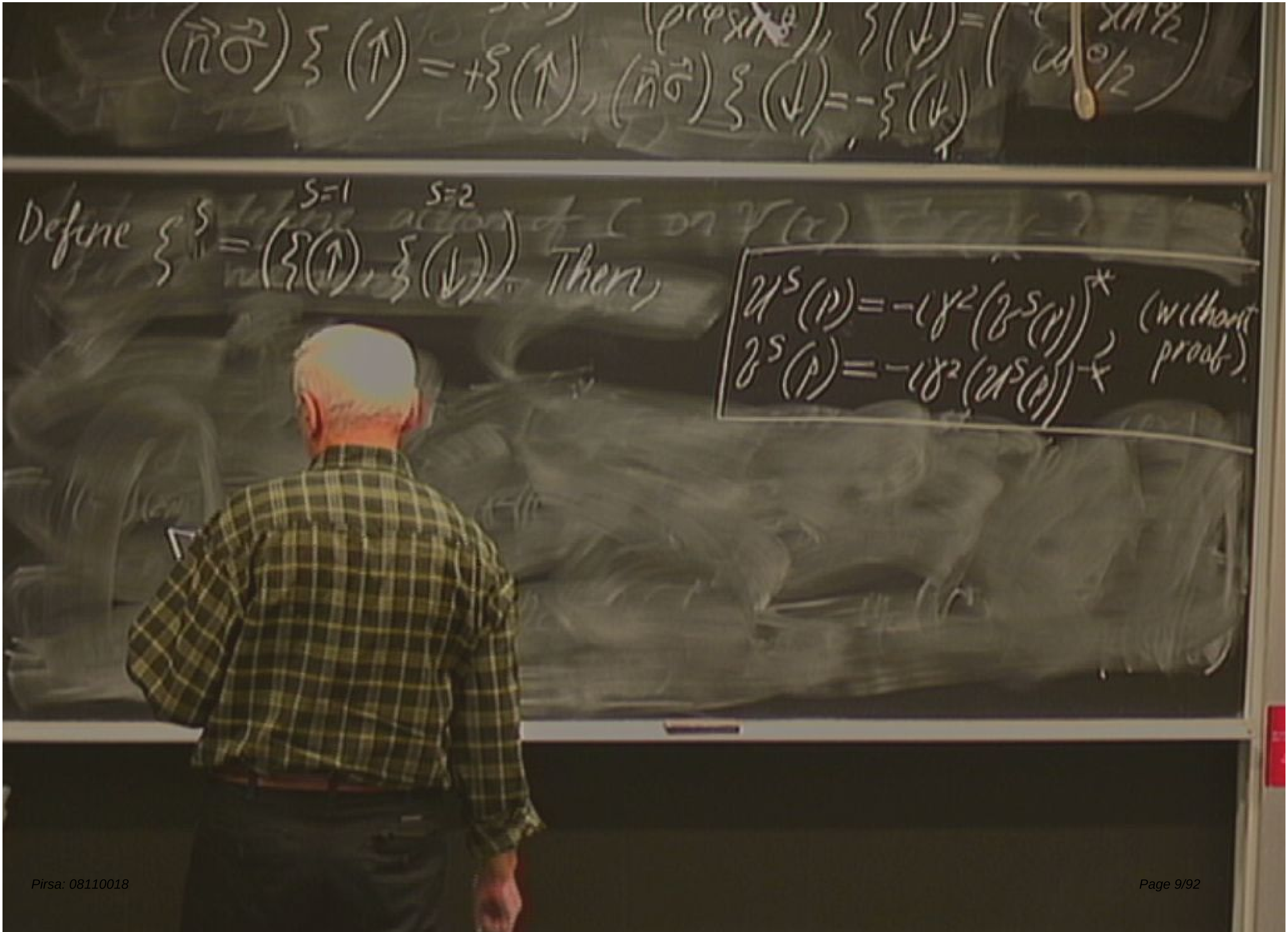
$$(\vec{n}\vec{\sigma})\,\xi(\uparrow) = +\xi(\uparrow), \quad (\vec{n}\vec{\sigma})\,\xi(\downarrow) = -\xi(\downarrow)$$

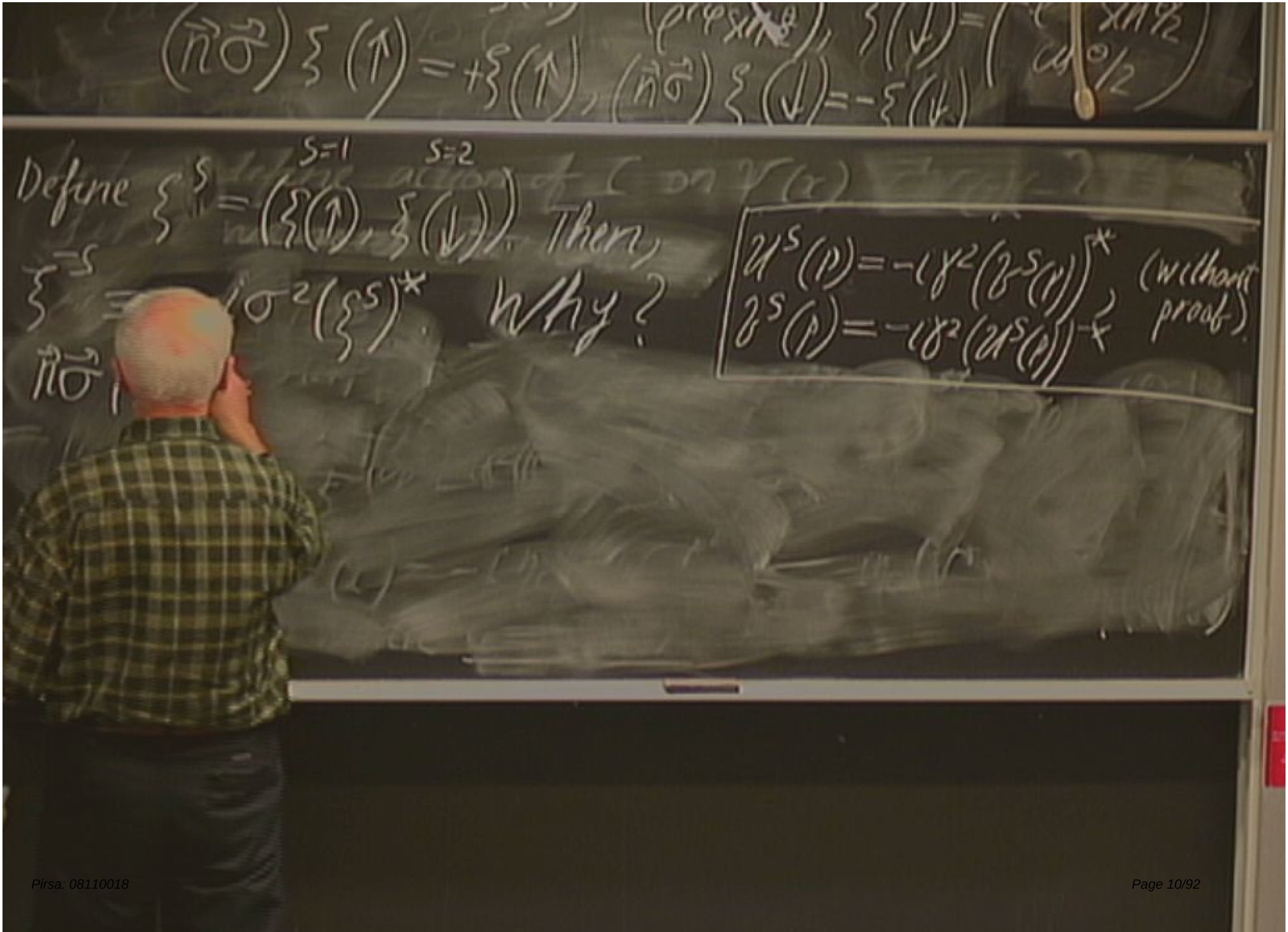


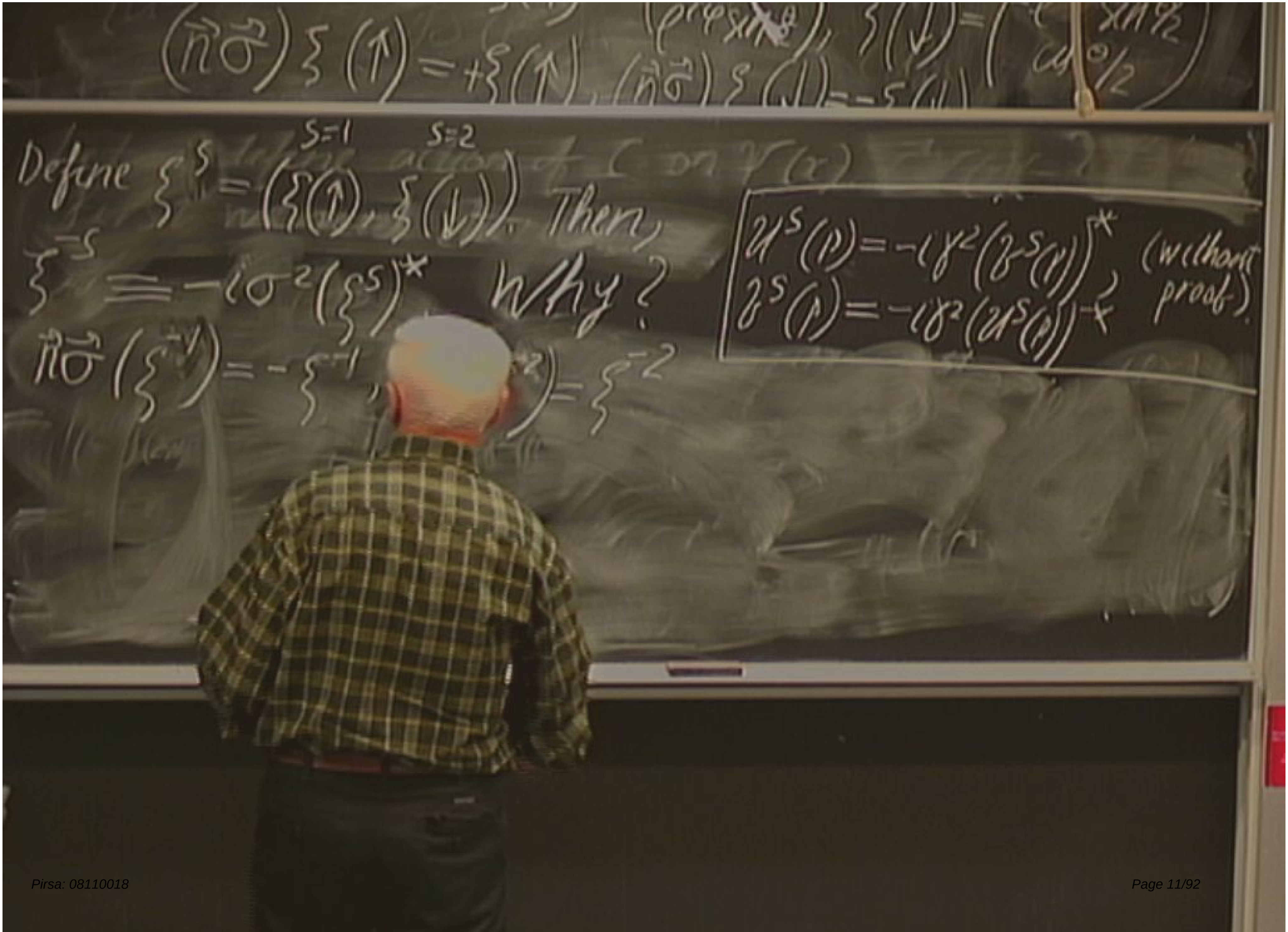


$(\vec{n}\vec{\sigma})\xi(\uparrow) = +\xi(\uparrow), (\vec{n}\vec{\sigma})\xi(\downarrow) = -\xi(\downarrow)$
$\mathcal{U}^S(p) = -i\gamma^2(\mathcal{V}^S(p))^*$
$\mathcal{V}^S(p) = -i\gamma^2(\mathcal{U}^S(p))^*$
(without proof)

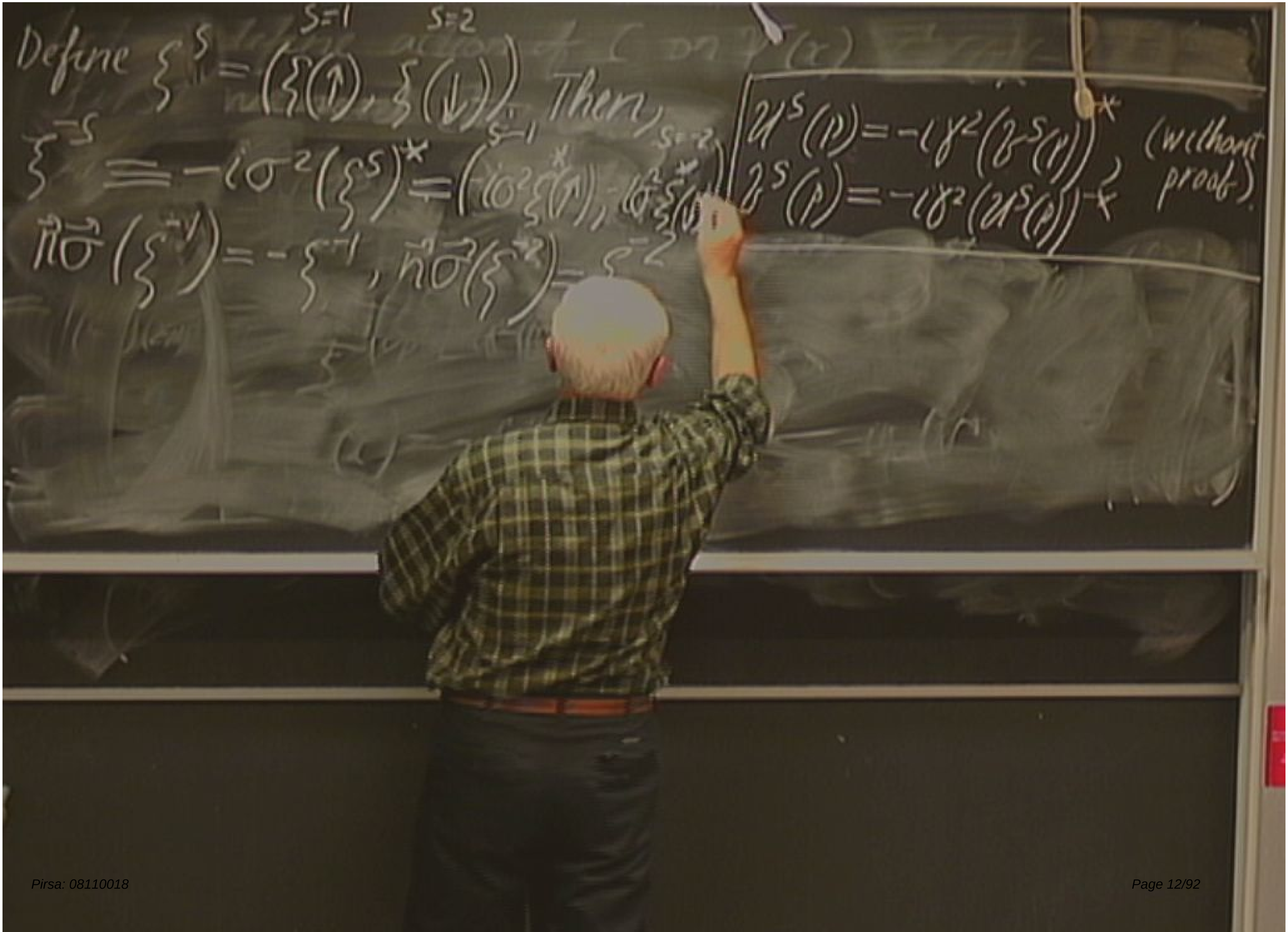

(n⃗σ⃗) ξ(↑) = +ξ(↑), (n⃗σ⃗) ξ(↓) = −ξ(↓)
Define ξ^s = (ξ(↑)
u^s(p) = −iγ²(v^s(p))* (without proof).
v^s(p) = −iγ²(u^s(p))*

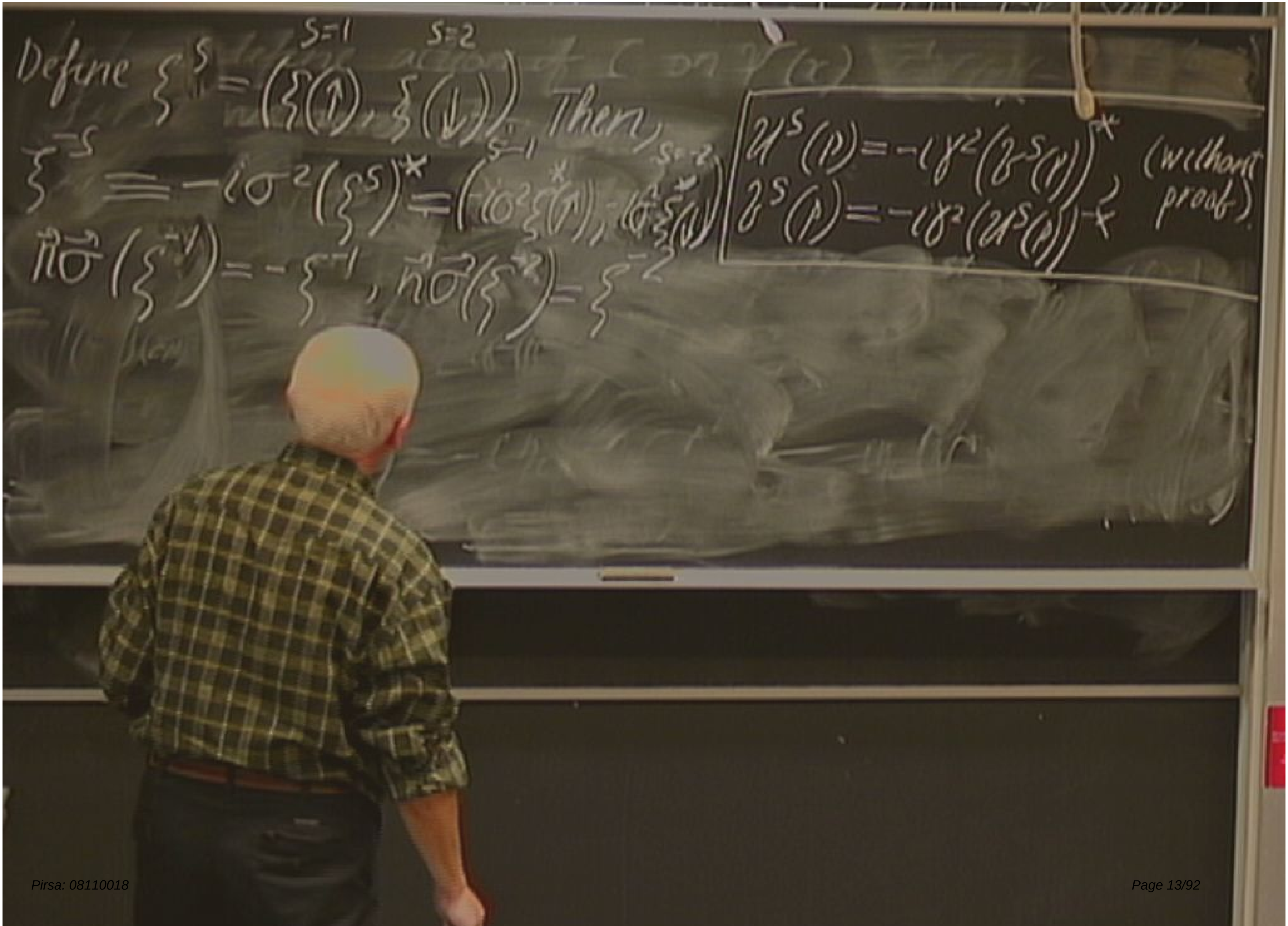

(n⃗σ⃗) ξ(↑) = +ξ(↑), (n⃗σ⃗) ξ(↓) = −ξ(↓)
s=1 s=2
Define ξ^s = (ξ(↑), ξ(↓)). Then,
u^s(p) = −iγ²(v^s(p))*
v^s(p) = −iγ²(u^s(p))*
(without proof)

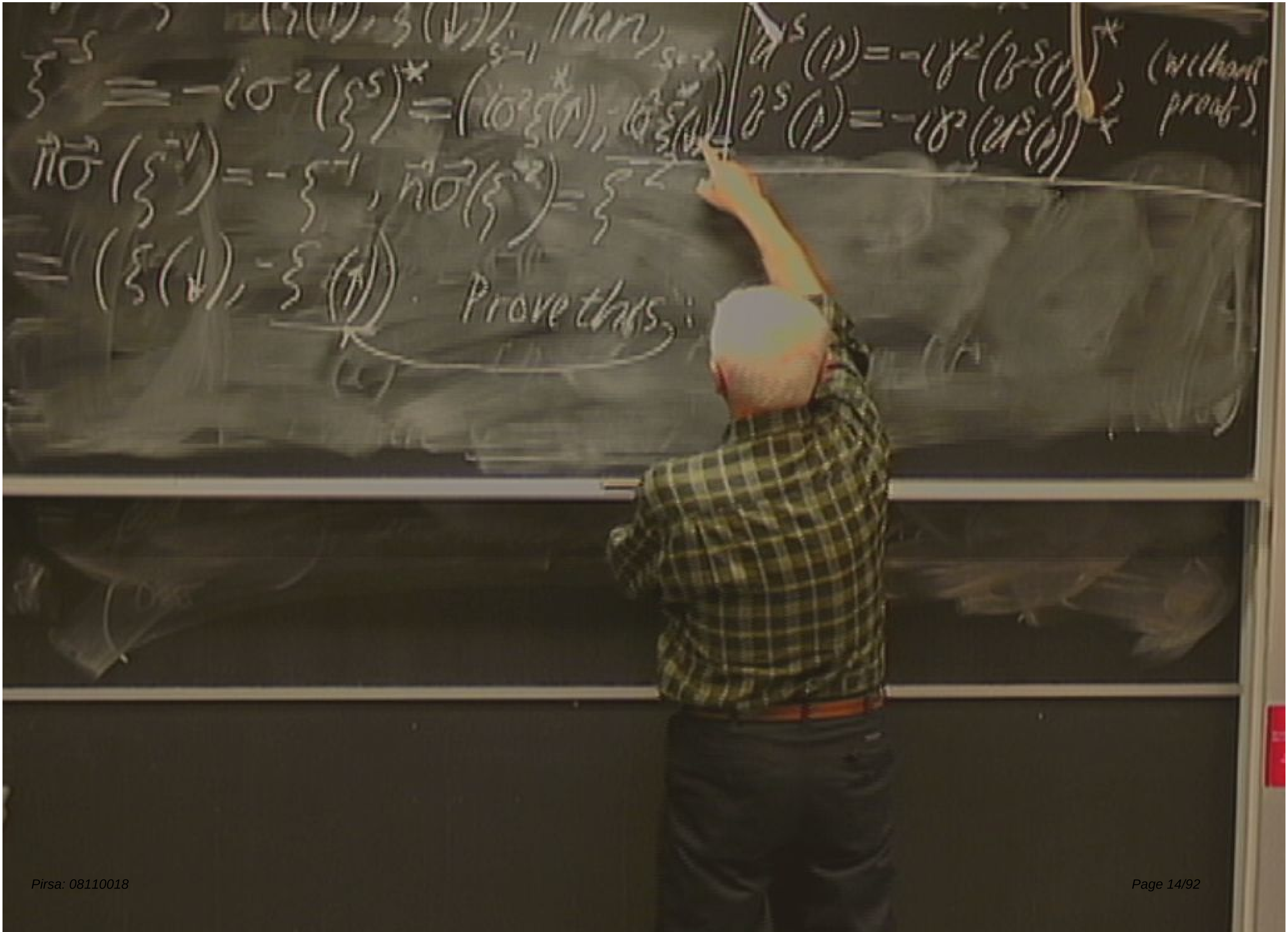

s=1
s=2
Define
Then,
Why?
(without
proof)

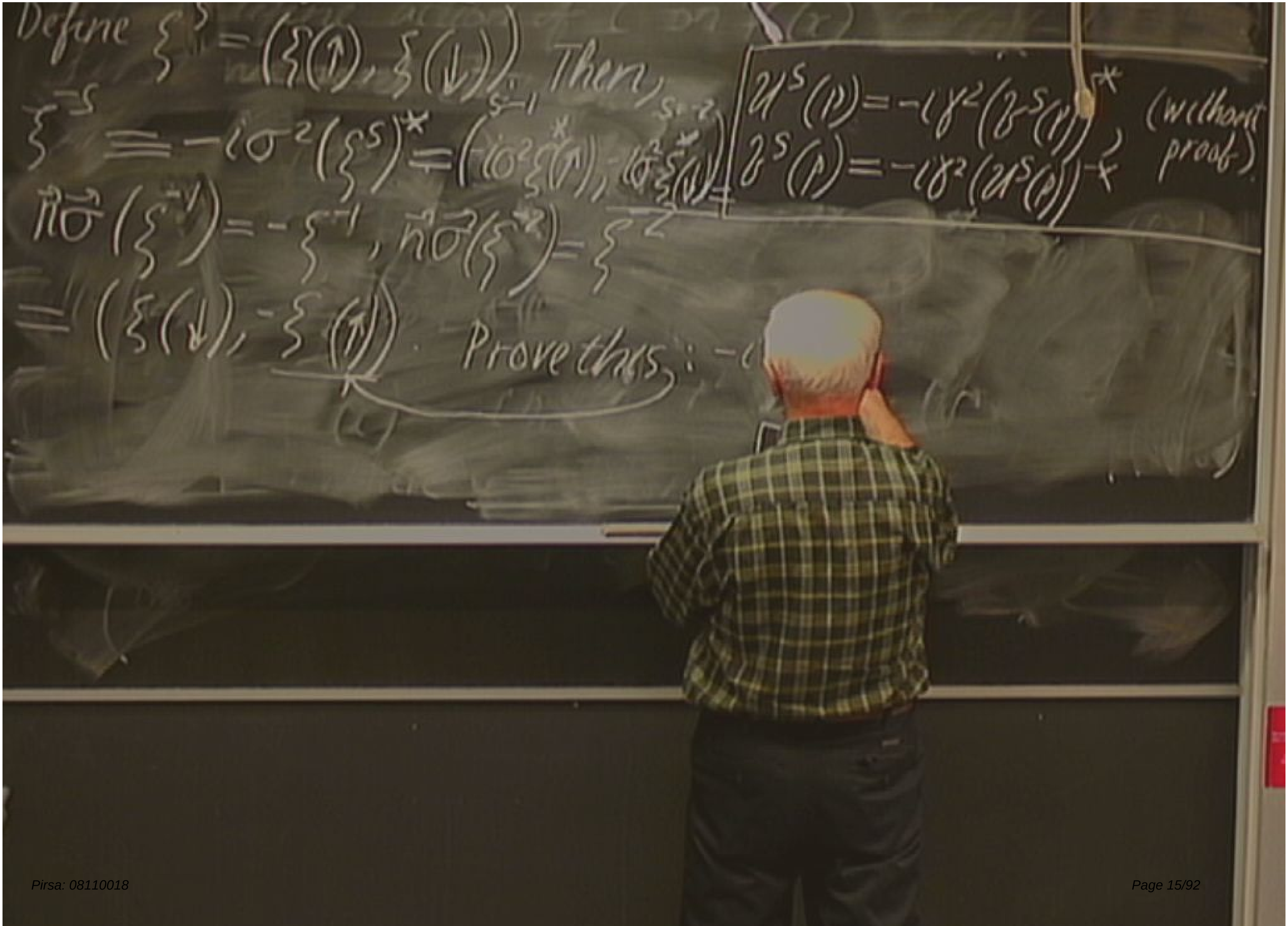


s=1 s=2
Define ξ^s = (ξ(↑), ξ(↓)). Then,
ξ^{-s} = -iσ²(ξ^s)* Why?
u^s(p) = -iγ²(v^s(p))*
v^s(p) = -iγ²(u^s(p))*
(without proof).

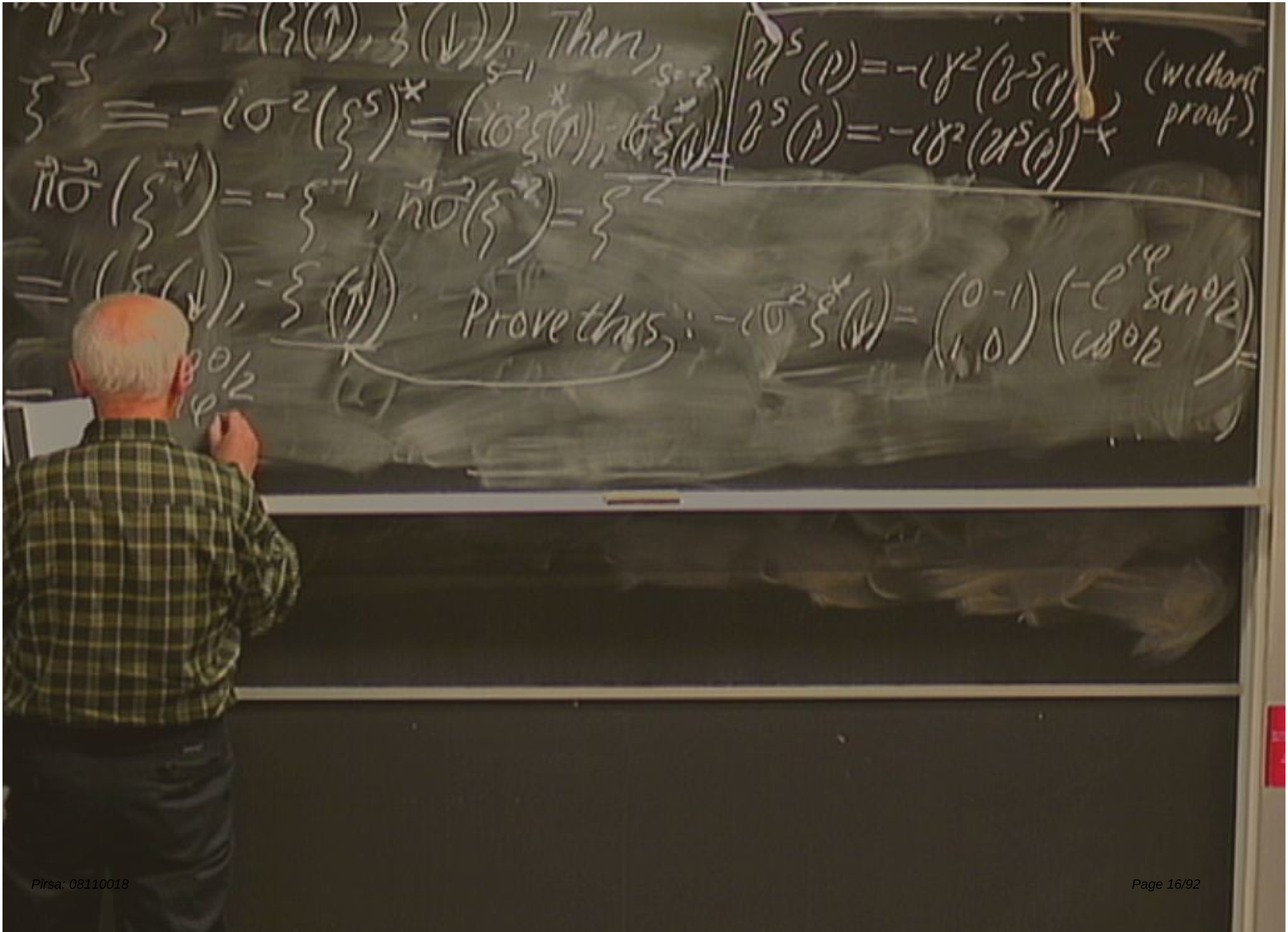

Define
Then,
(without proof).

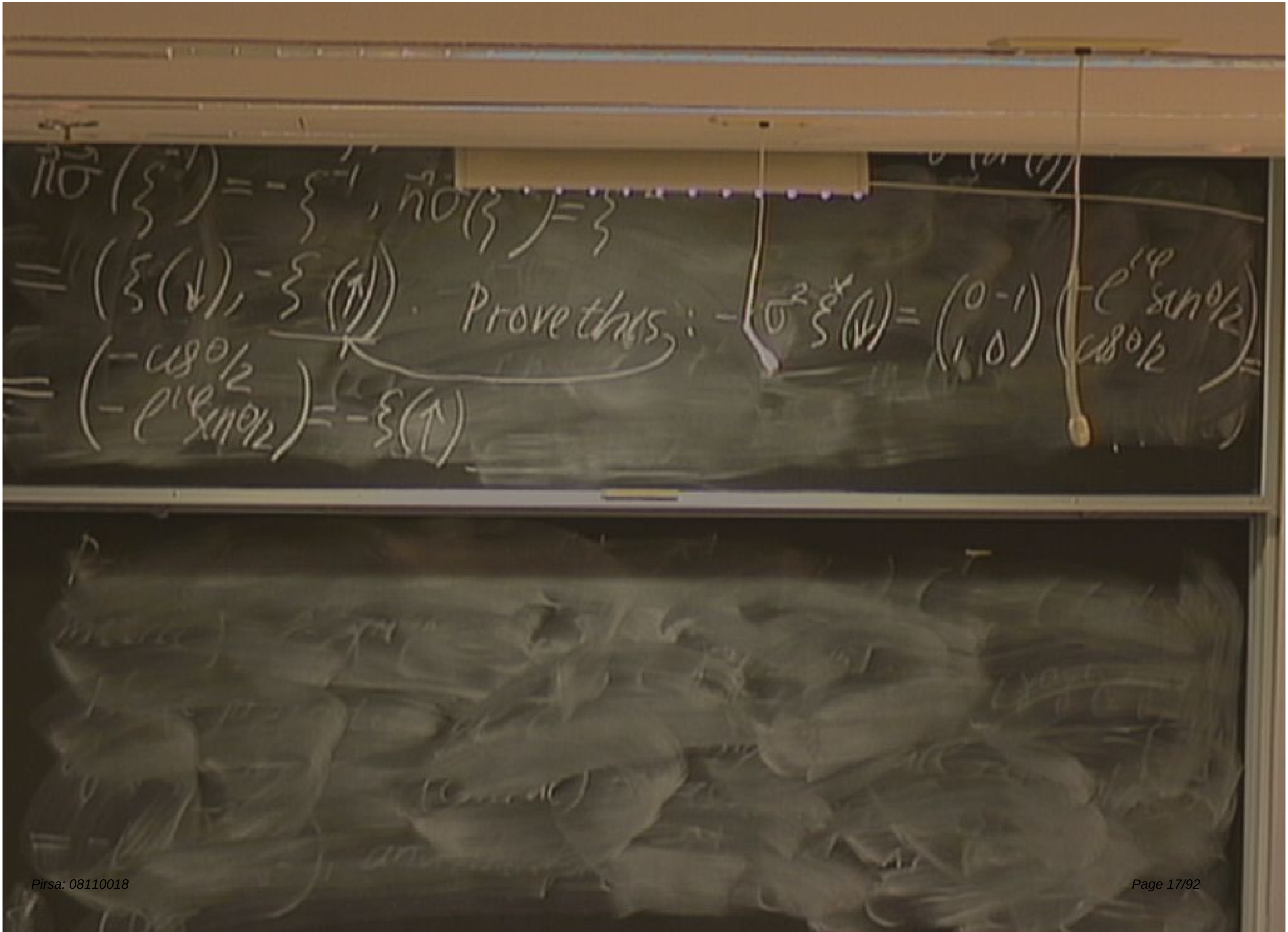

s=1
s=2
Define
Then
(without proof)

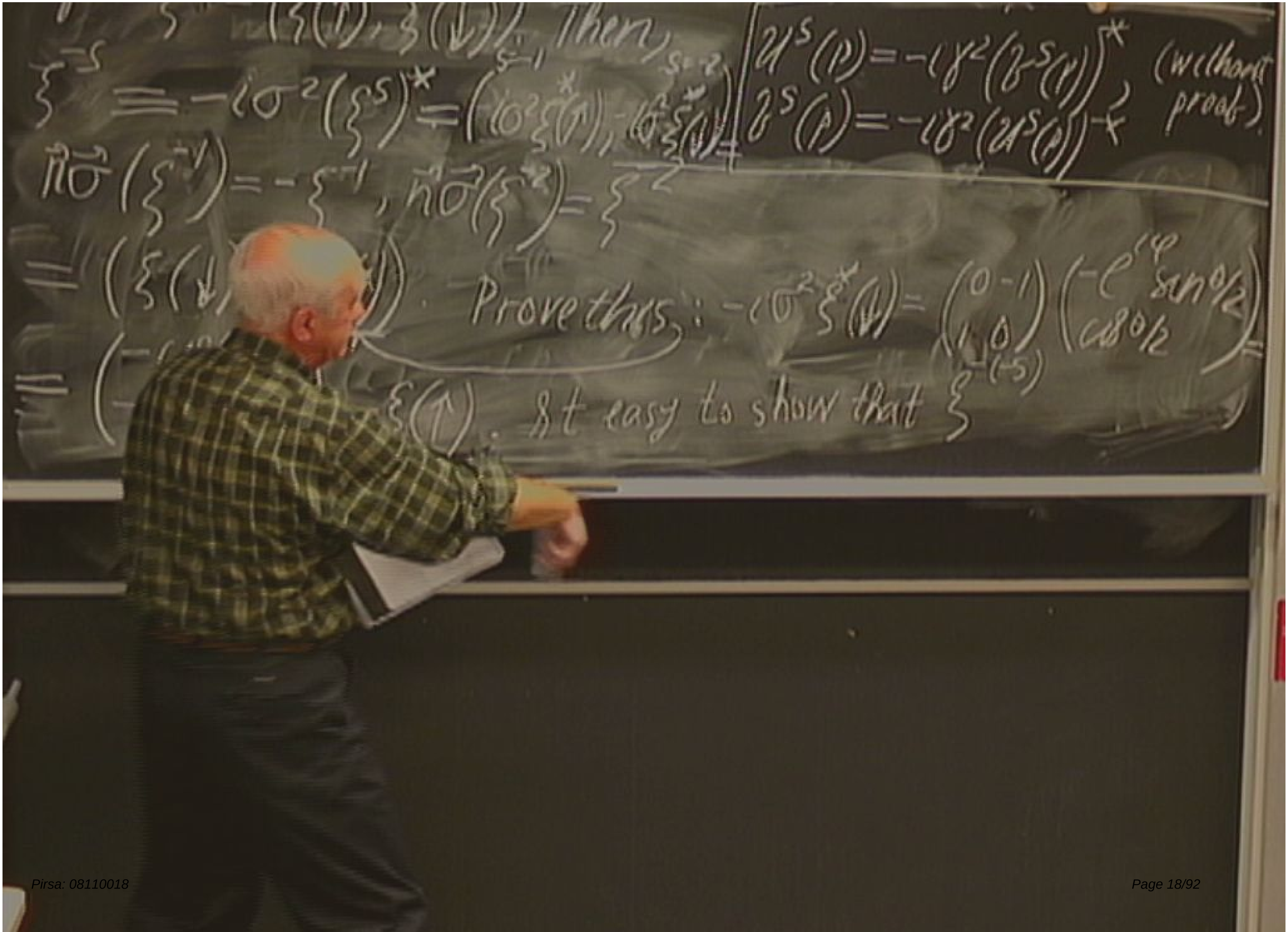

(without proof)
Prove this:

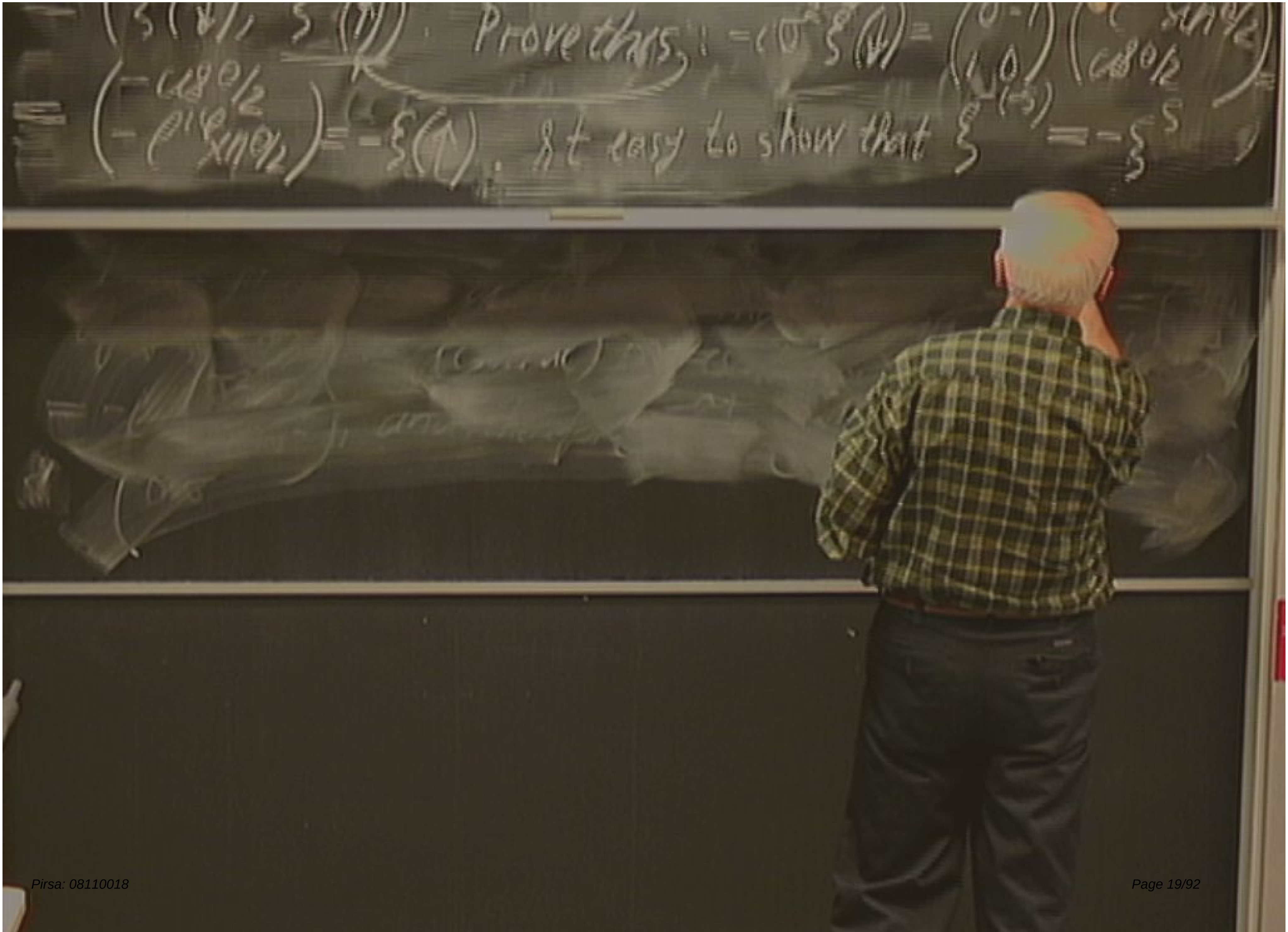


Define $\xi^s = (\xi(\uparrow), \xi(\downarrow))$. Then,
$\xi^{-s} = -i\sigma^2(\xi^s)^* = (-i\sigma^2\xi^*(\uparrow), -i\sigma^2\xi^*(\downarrow))$
$= (\xi(\downarrow), -\xi(\uparrow))$
Prove this:
$u^s(p) = -i\gamma^2(v^s(p))^*$
$v^s(p) = -i\gamma^2(u^s(p))^*$
(without proof)


Then,
(without proof).
Prove this:


Prove this:


Then,
(without proof)
Prove this:
It easy to show that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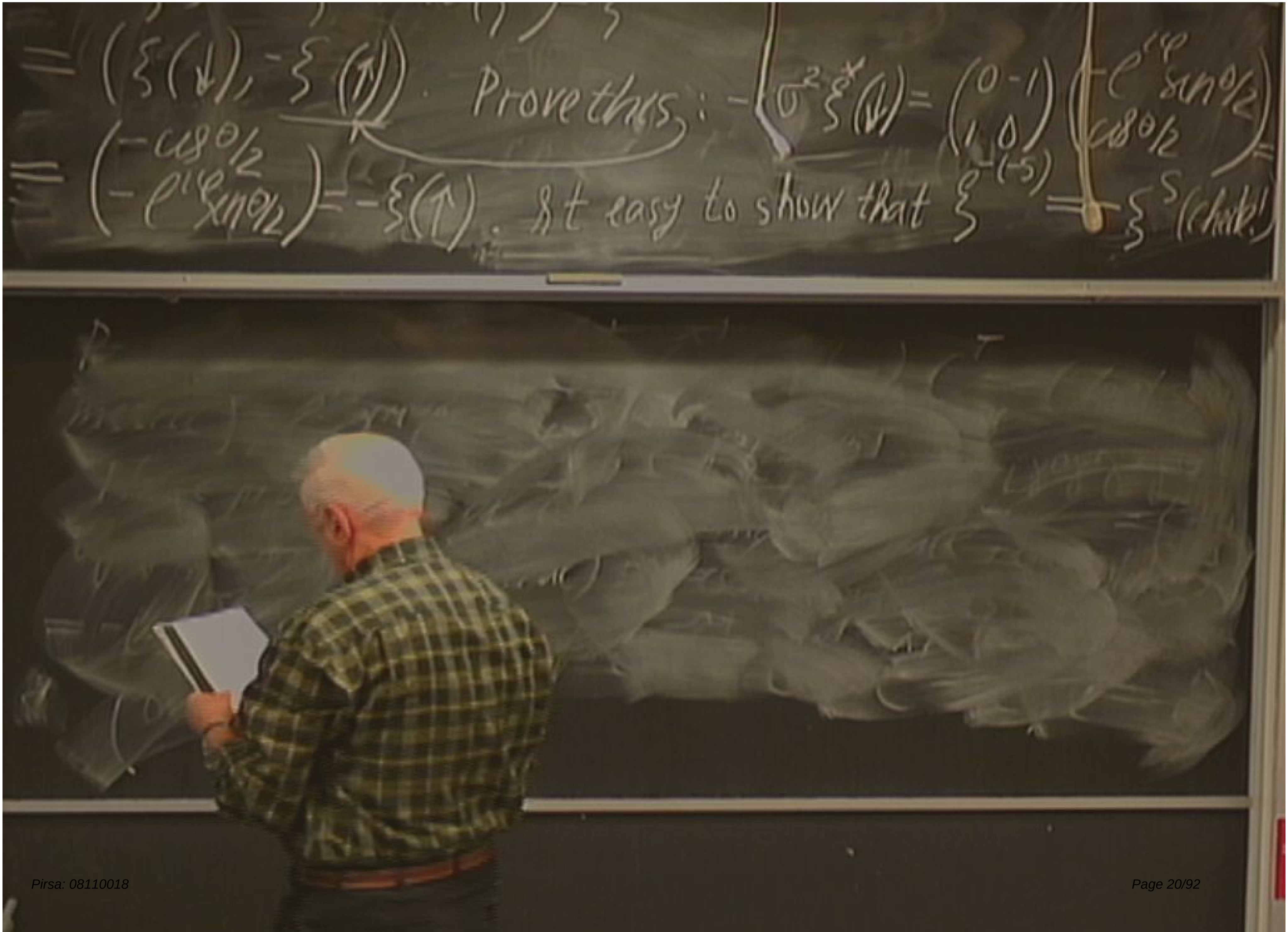
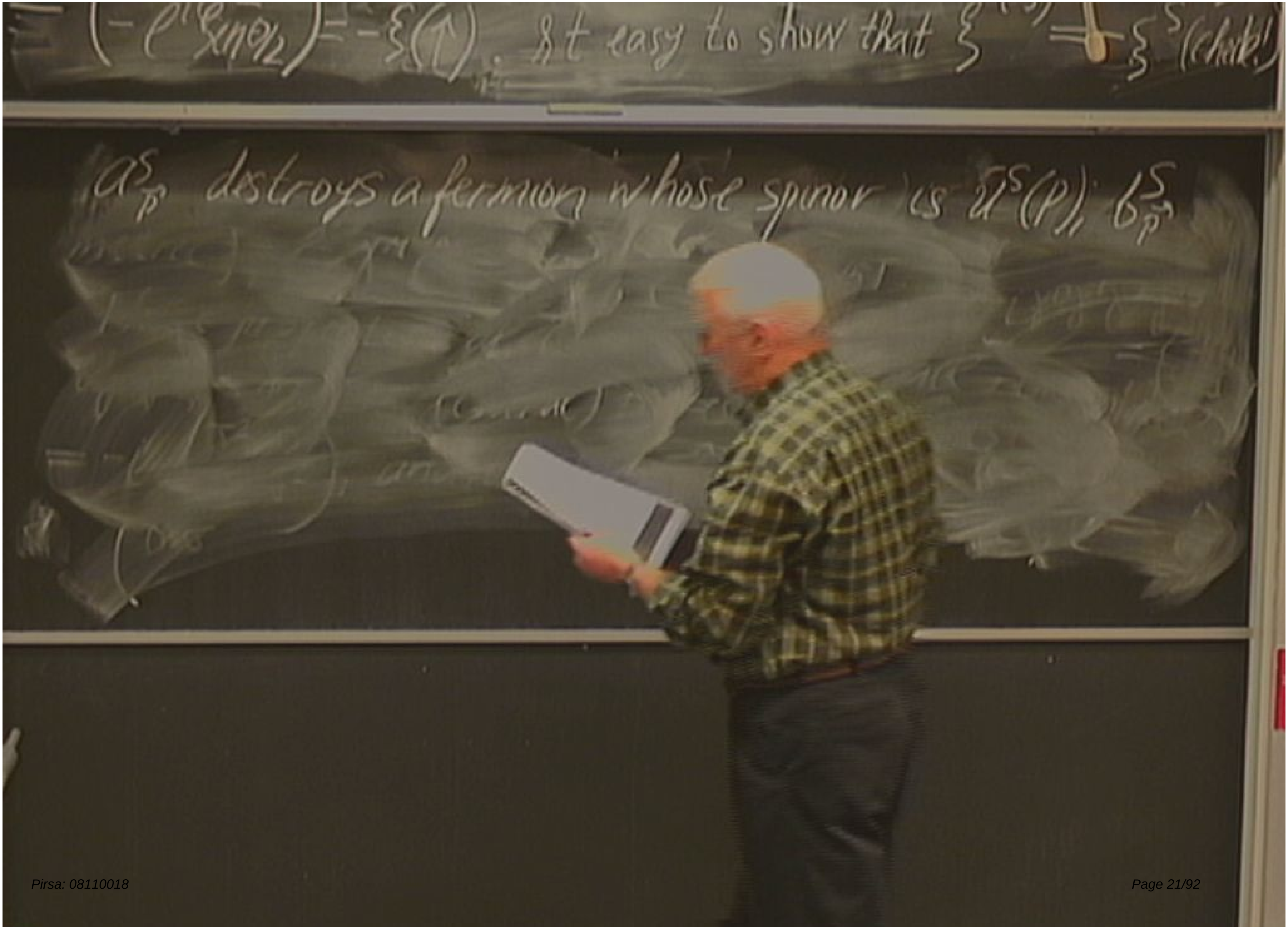
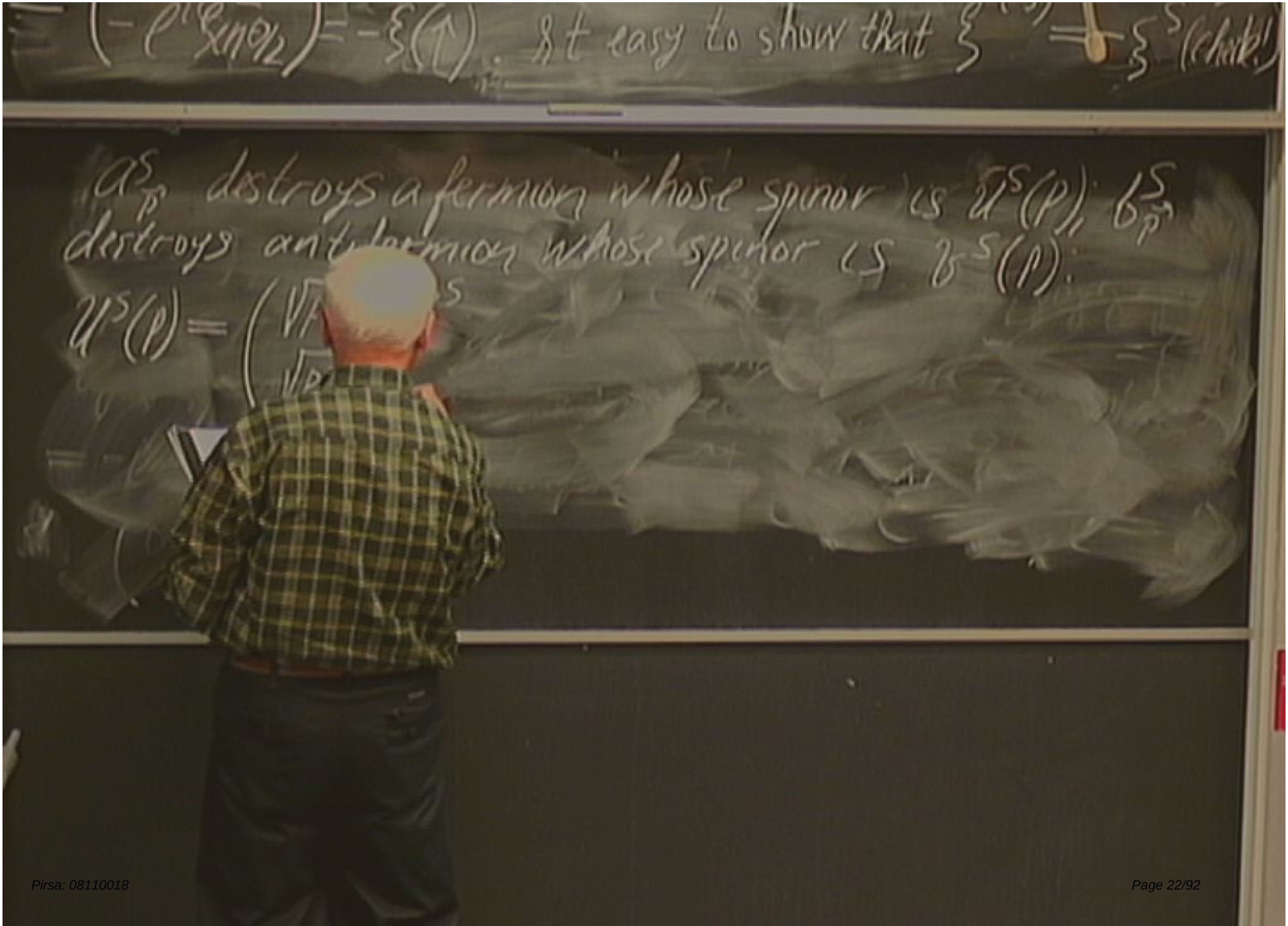

Prove this:
It easy to show that
(check!)


It easy to show that
(check!)
destroys a fermion whose spinor is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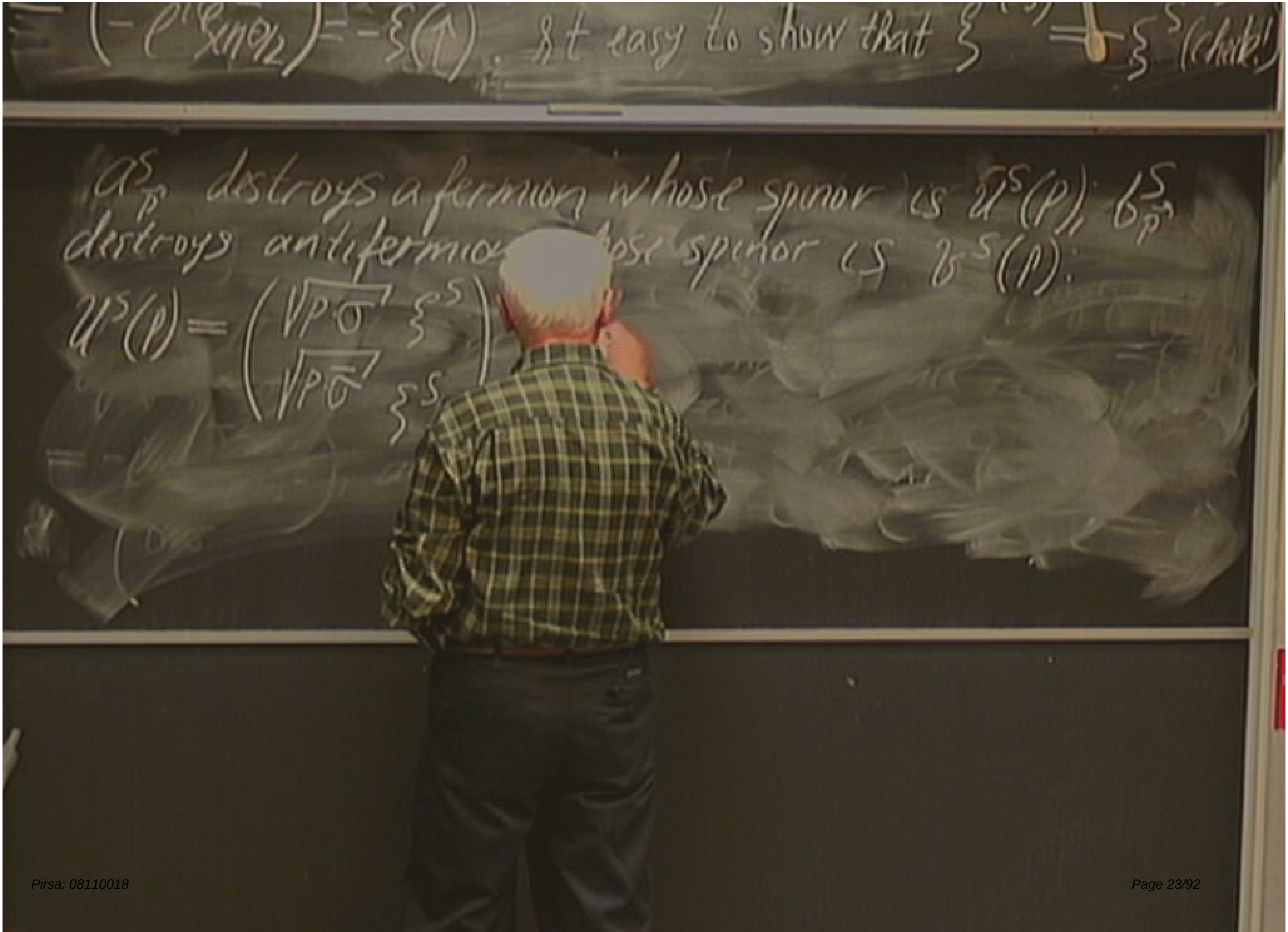
It easy to show that
(check!)
a destroys a fermion whose spinor is u(p), b
destroys antifermion whose spinor is v(p).

$= \begin{pmatrix} -e^{i\varphi} \times \ldots \theta/2 \end{pmatrix} = -\xi(\uparrow)$. It easy to show that $\xi^{-s} = \ldots \xi^s$ (check!)

$a^s_{\vec p}$ destroys a fermion whose spinor is $u^s(p)$, $b^s_{\vec p}$ destroys antifermion whose spinor is $v^s(p)$.

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar\sigma}\,\xi^s \end{pmatrix}, \quad v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^{-s} \\ -\sqrt{p\cdot\bar\sigma}\,\xi^{-s} \end{pmatrix}$$

$\left(-e^{i\varphi} \ldots \pi\theta/2\right) = -\xi(\uparrow)$. It easy to show that $\xi^{-s} = \xi^{s}$ (check!)

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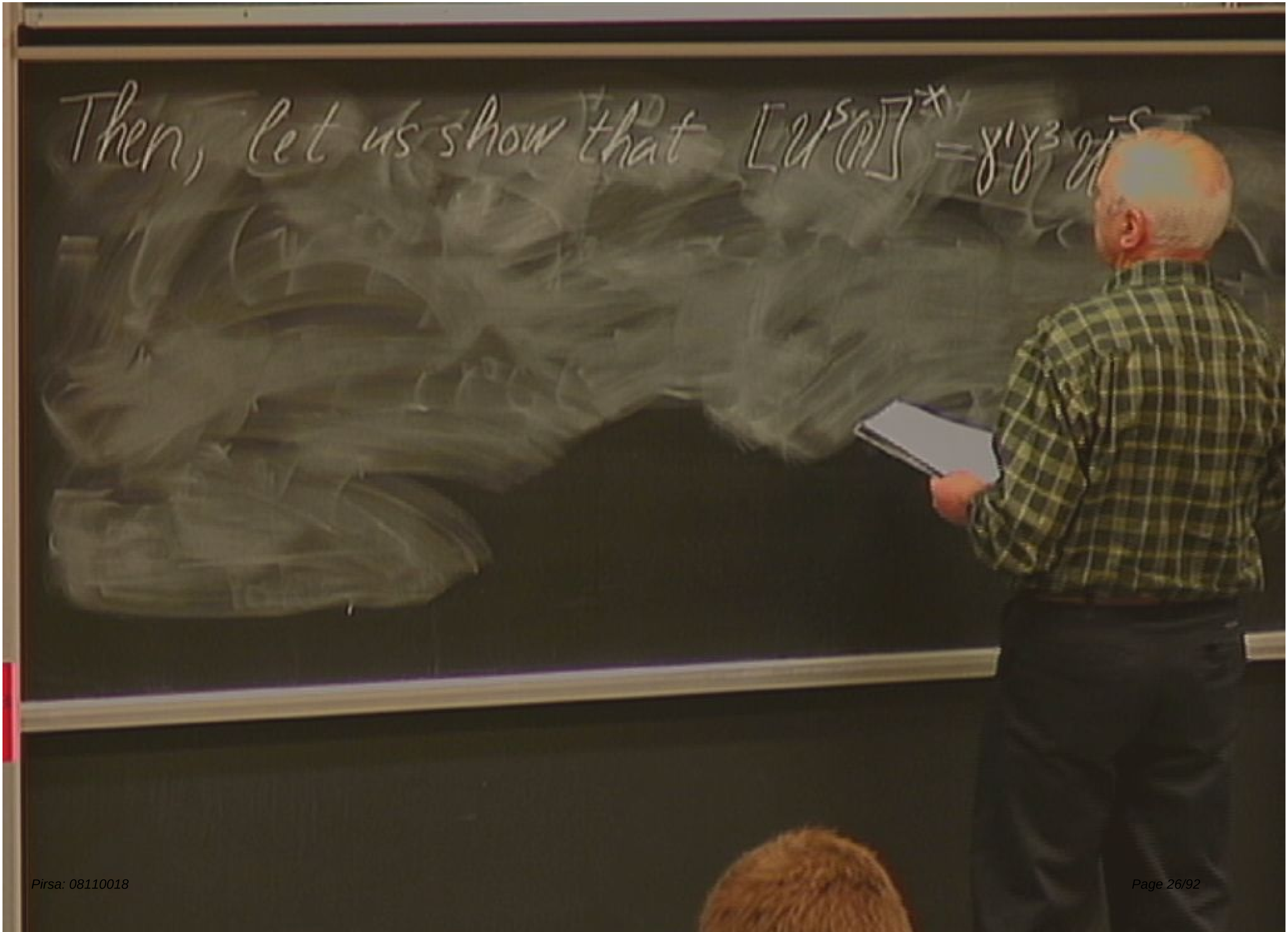
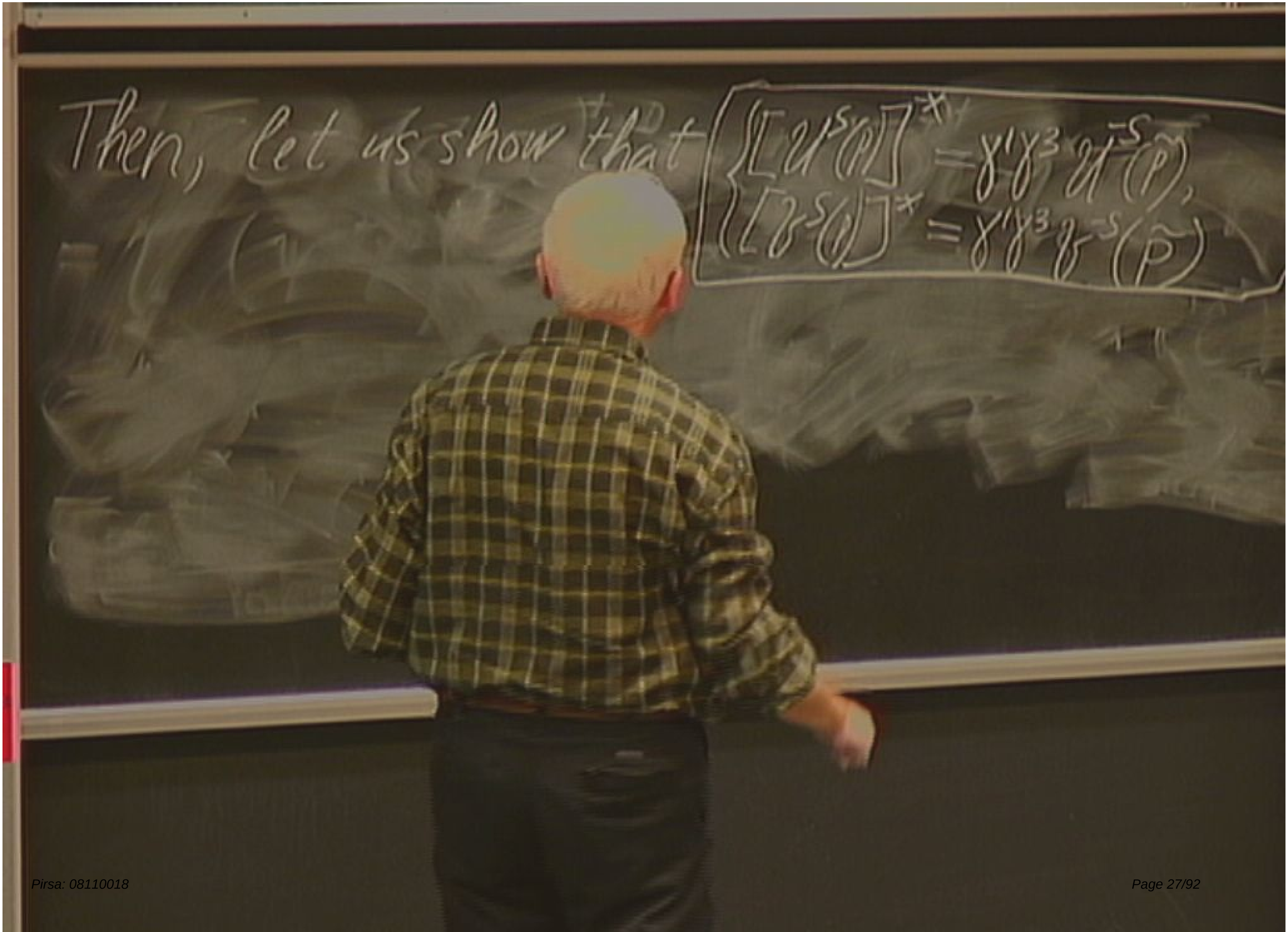
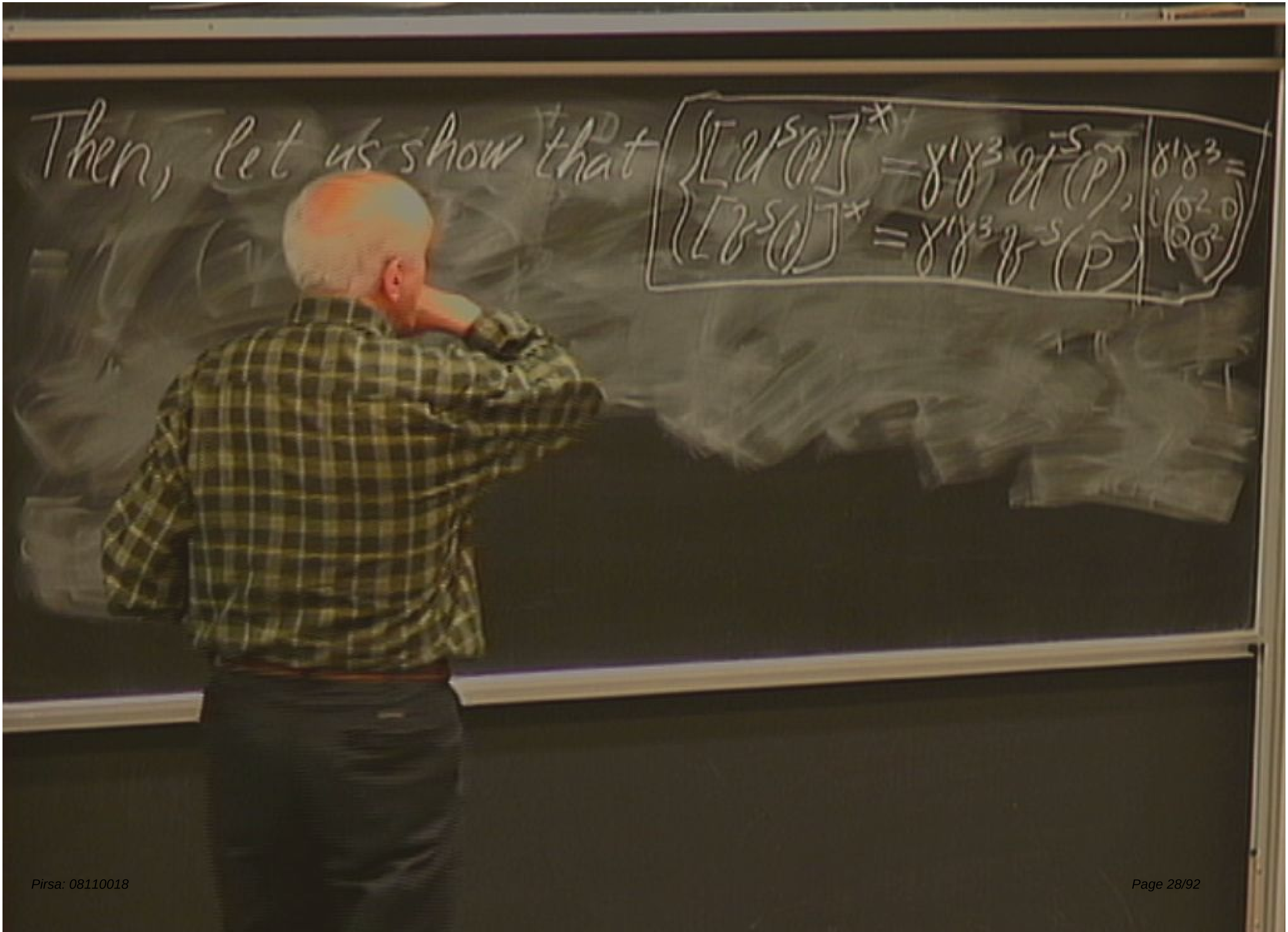
$$u^{s}(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^{s} \\ \sqrt{p\cdot\bar\sigma}\,\xi^{s} \end{pmatrix}, \quad v^{s}(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^{-s} \\ -\sqrt{p\cdot\bar\sigma}\,\xi^{-s} \end{pmatrix}$$

Without, use $\sqrt{\tilde p\cdot\sigma}\,\sigma^{2} = \sigma^{2}\sqrt{p\cdot\sigma^{*}}$, $\tilde p = (p^{0}, -\vec p)$


Then, let us show that $[u^s(p)]^* = \gamma^1\gamma^3 u^s$


Then, let us show that
[u^s(p)]* = γ'γ³ u^-s(p̃),
[v^s(p)]* = γ'γ³ v^-s(p̃)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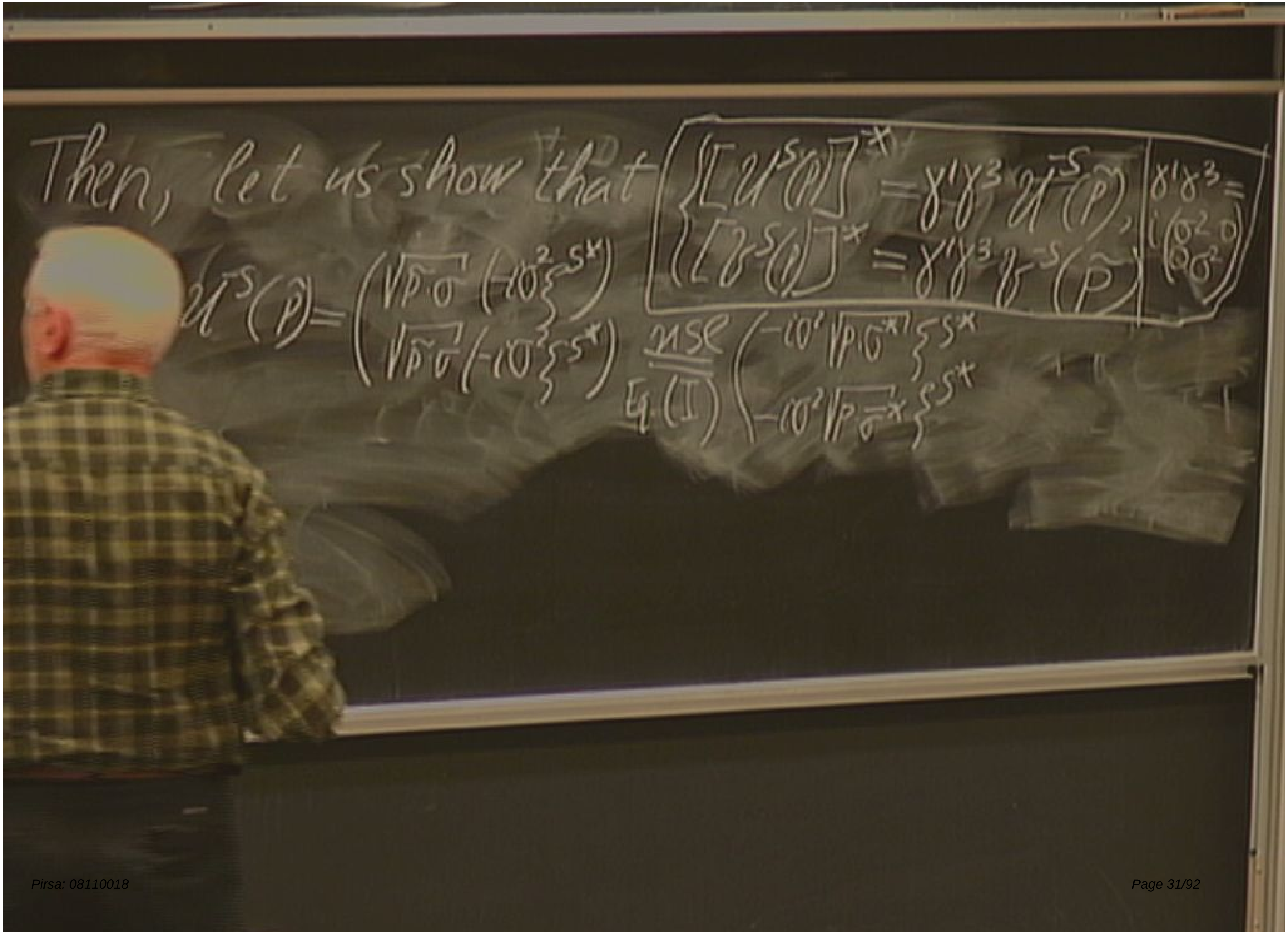
$$\begin{cases} [u^s(p)]^* = \gamma^1\gamma^3 u^{-s}(\tilde{p}), \\ [v^s(p)]^* = \gamma^1\gamma^3 v^{-s}(\tilde{p}) \end{cases} \qquad \gamma^1\gamma^3 = i\begin{pmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{pmatrix}$$

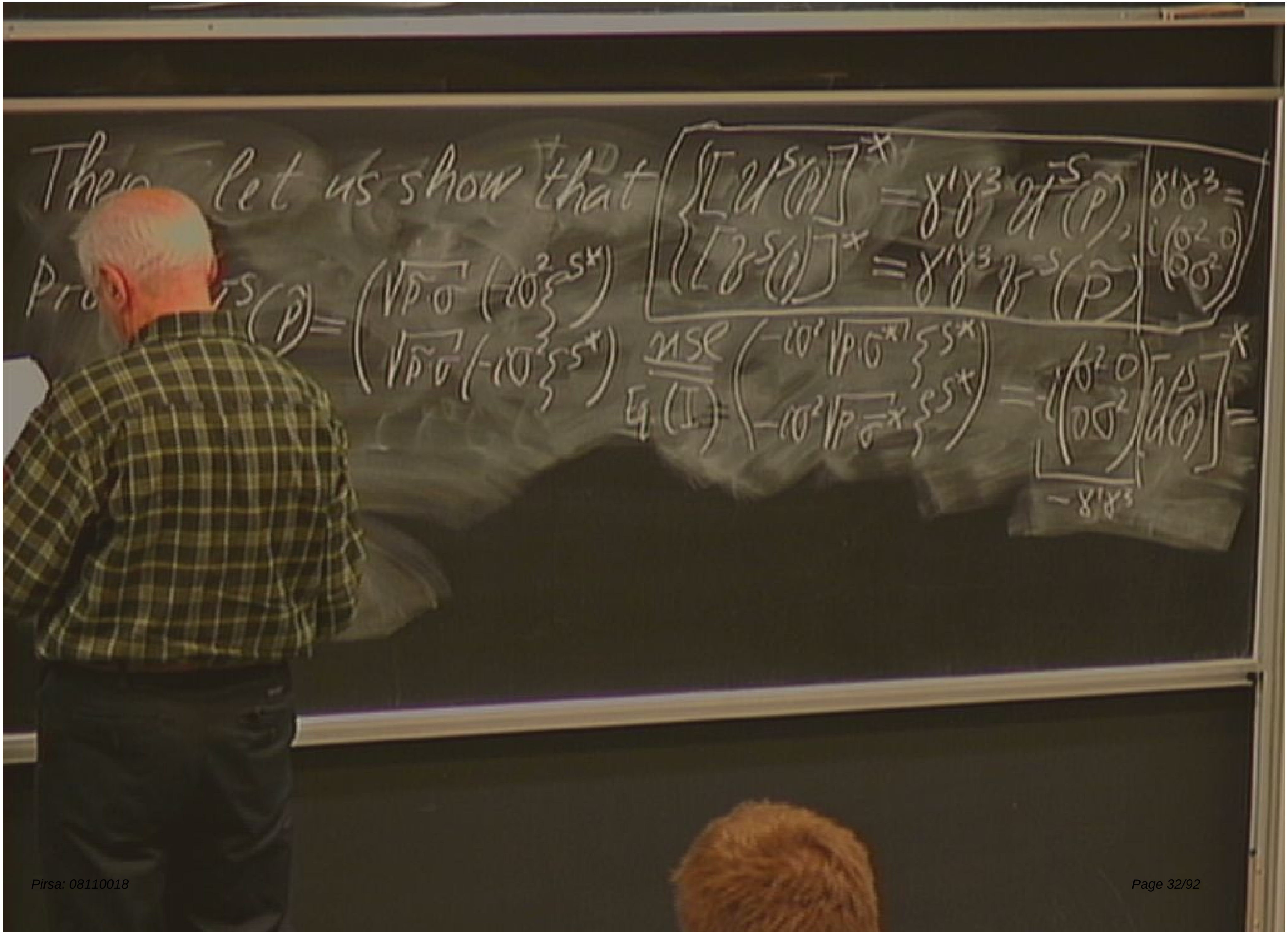
Proof: $u^{-s}(\tilde{p}) = \begin{pmatrix} \sqrt{\tilde{p}\cdot\sigma}\,(-i\sigma^2\xi^{s*}) \\ \sqrt{\tilde{p}\cdot\bar{\sigma}} \end{pmatrix}$

Then, let us show that

$$\left\{\begin{aligned} \left[u^s(p)\right]^* &= \gamma^1\gamma^3\, u^{-s}(\tilde{p}), \\ \left[v^s(p)\right]^* &= \gamma^1\gamma^3\, v^{-s}(\tilde{p}) \end{aligned}\right. \qquad \gamma^1\gamma^3 = i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$$

Proof: $u^{-s}(\tilde{p}) = \begin{pmatrix} \sqrt{\tilde{p}\cdot\sigma}\,(-i\sigma^2\xi^{s*}) \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}\,(-i\sigma^2\xi^{s*}) \end{pmatrix} = ($


Then, let us show that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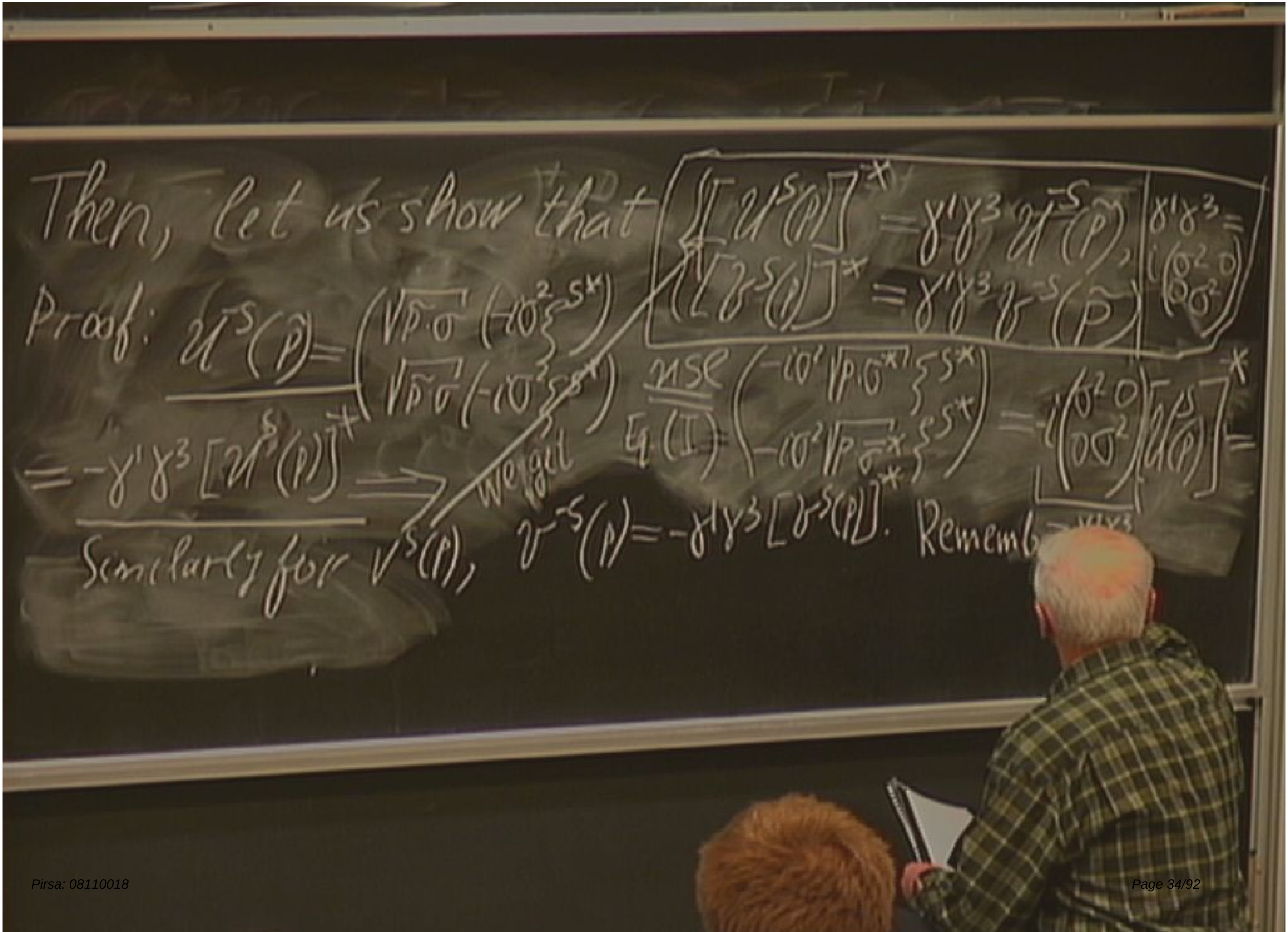
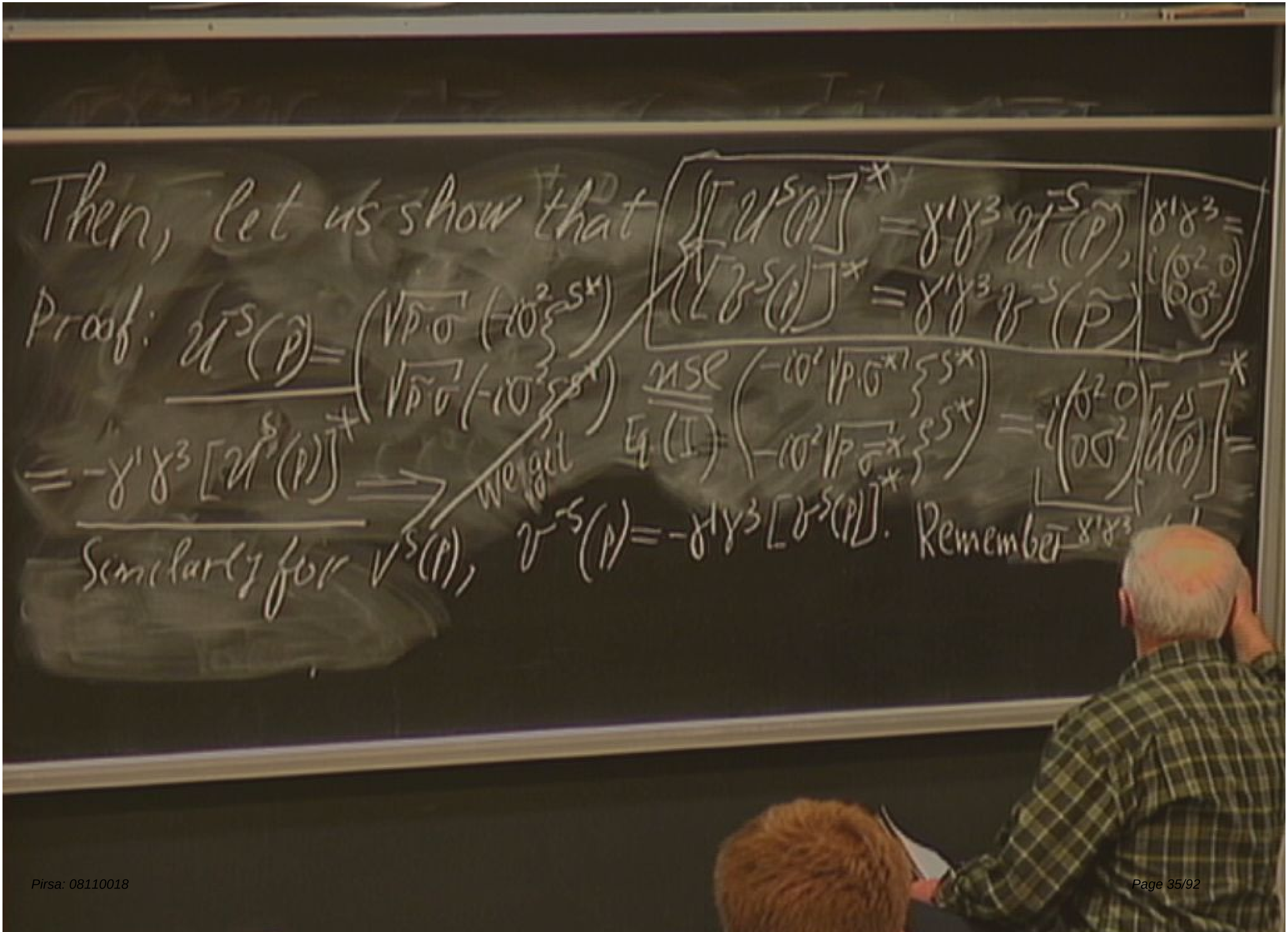
Then, let us show that

$$\begin{cases}[u^s(p)]^* = \gamma^1\gamma^3 u^{-s}(\tilde{p}), \\ [v^s(p)]^* = \gamma^1\gamma^3 v^{-s}(\tilde{p})\end{cases} \qquad \gamma^1\gamma^3 = i\begin{pmatrix}\sigma^2 & 0 \\ 0 & \sigma^2\end{pmatrix}$$

Proof: $$u^{-s}(\tilde{p}) = \begin{pmatrix}\sqrt{\tilde{p}\cdot\sigma}\,(-i\sigma^2\xi^{s*}) \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}\,(-i\sigma^2\xi^{s*})\end{pmatrix} \underset{\text{Eq.(I)}}{\overset{\text{use}}{=}} \begin{pmatrix}-i\sigma^2\sqrt{p\cdot\sigma^*}\,\xi^{s*} \\ -i\sigma^2\sqrt{p\cdot\bar{\sigma}^*}\,\xi^{s*}\end{pmatrix} = \underbrace{-i\begin{pmatrix}\sigma^2 & 0 \\ 0 & \sigma^2\end{pmatrix}}_{-\gamma^1\gamma^3}[u^s(p)]^* =$$

$$= -\gamma^1\gamma^3[u^s(p)]^*$$


Then, let us show that
Proof:
we get
Similarly for


Then, let us show that [u^s(p)]* = γ1γ3 u^-s(p̃), γ1γ3 = i(σ2 0; 0 σ2)
[v^s(p)]* = γ1γ3 v^-s(p̃)
Proof: u^-s(p̃) = (√p·σ (-iσ2 ξ^s*) ; √p̄·σ̄ (-iσ2 ξ^s*))
use Eq.(I) = (-iσ2 √p·σ* ξ^s* ; -iσ2 √p̄·σ̄* ξ^s*) = -i(σ2 0; 0 σ2)[u^s(p)]* =
= -γ1γ3 [u^s(p)]* ⟹ we get
Similarly for v^s(p), v^-s(p̃) = -γ1γ3 [v^s(p)]*. Remember γ1γ3

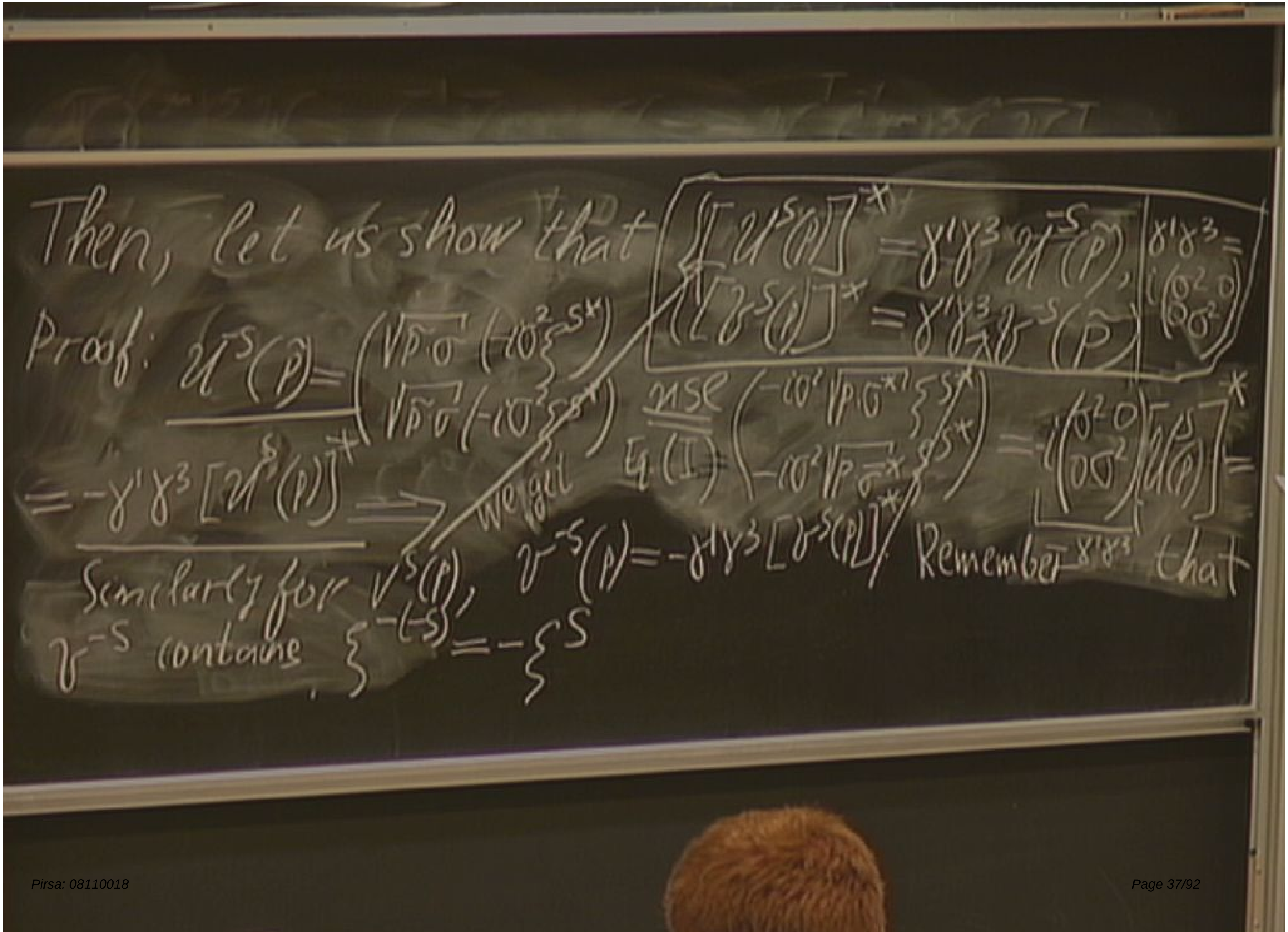
Then, let us show that $[u^s(p)]^* = \gamma^1\gamma^3 u^{-s}(\tilde{p})$, $[v^s(p)]^* = \gamma^1\gamma^3 v^{-s}(\tilde{p})$, $\gamma^1\gamma^3 = i\begin{pmatrix}\sigma^2 & 0 \\ 0 & \sigma^2\end{pmatrix}$

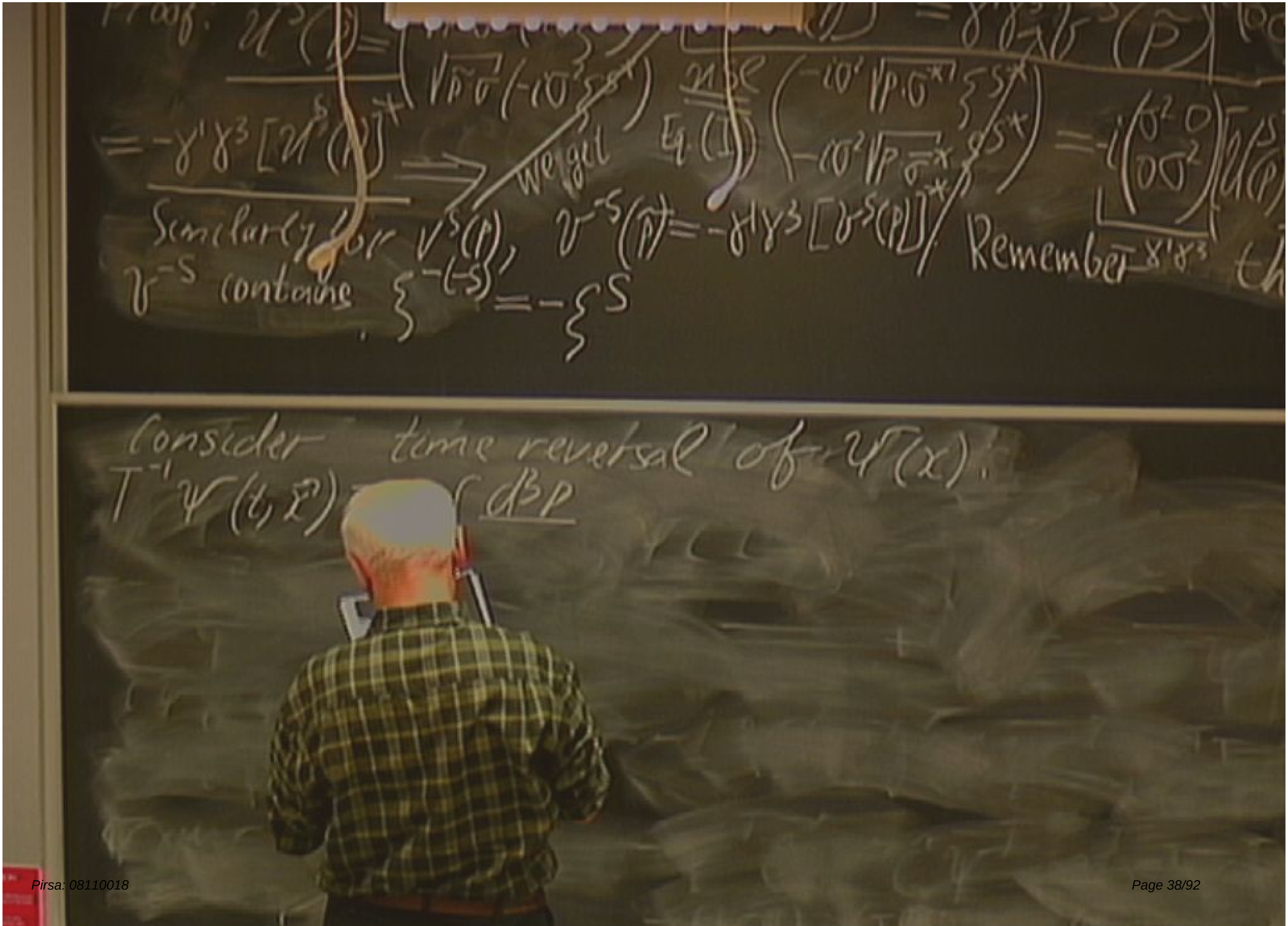
Proof: $u^{-s}(\tilde{p}) = \begin{pmatrix}\sqrt{\tilde{p}\cdot\sigma}(-i\sigma^2\xi^{s*}) \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}(-i\sigma^2\xi^{s*})\end{pmatrix} \underset{\text{Eq. (I)}}{\overset{\text{use}}{=}} \begin{pmatrix}-i\sigma^2\sqrt{p\cdot\sigma^*}\xi^{s*} \\ -i\sigma^2\sqrt{p\cdot\bar{\sigma}^*}\xi^{s*}\end{pmatrix} = -i\begin{pmatrix}\sigma^2 & 0 \\ 0 & \sigma^2\end{pmatrix}[u^s(p)]^* =$

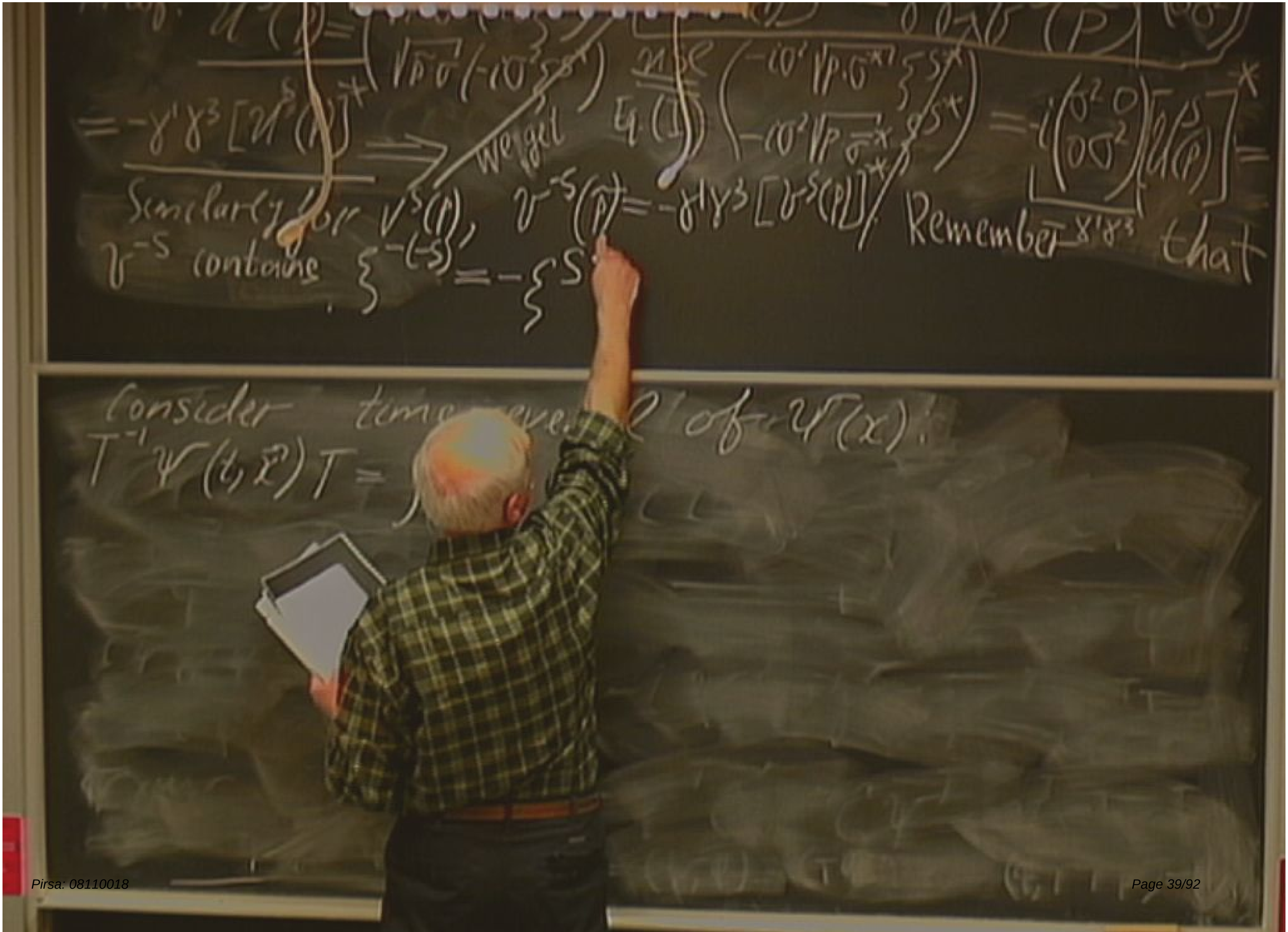
$= -\gamma^1\gamma^3[u^s(p)]^*$ $\Rightarrow$ we get

Similarly for $v^s(p)$, $v^{-s}(\tilde{p}) = -\gamma^1\gamma^3[v^s(p)]^*$. Remember that

v^{-s} contains $\xi^{-(-s)} = -\xi^s$


Then, let us show that $\{[u^s(p)]^* = \gamma^1\gamma^3 u^{-s}(\tilde{p})$, $[v^s(p)]^* = \gamma^1\gamma^3 v^{-s}(\tilde{p})$
$\gamma^1\gamma^3 = i\begin{pmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{pmatrix}$
Proof: $u^{-s}(\tilde{p}) = \begin{pmatrix}\sqrt{p\cdot\sigma}\,(-i\sigma^2\xi^{s*})\\ \sqrt{p\cdot\bar{\sigma}}\,(-i\sigma^2\xi^{s*})\end{pmatrix}$ use Eq.(I) $\begin{pmatrix}-i\sigma^2\sqrt{p\cdot\sigma^*}\,\xi^{s*}\\ -i\sigma^2\sqrt{p\cdot\bar{\sigma}^*}\,\xi^{s*}\end{pmatrix} = -i\begin{pmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{pmatrix}[u^s(p)]^* =$
$= -\gamma^1\gamma^3[u^s(p)]^*$ ⟹ we get
Similarly for $v^s(p)$, $v^{-s}(p) = -\gamma^1\gamma^3[v^s(p)]^*$. Remember $\gamma^1\gamma^3$ that
v^{-s} contains $\xi^{-(-s)} = -\xi^s$


Proof:
we get
Similarly for
Remember
contains
Consider time reversal of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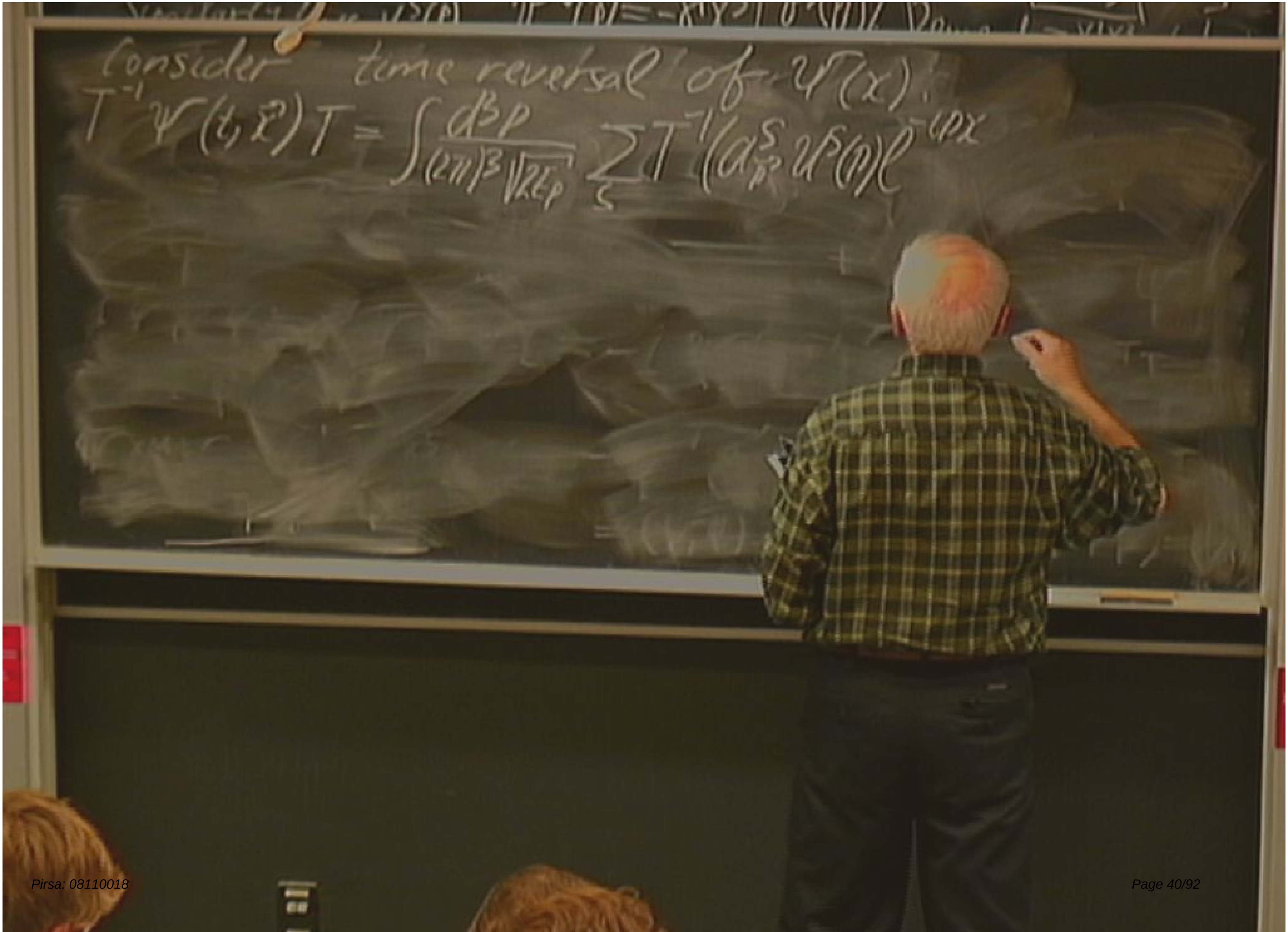
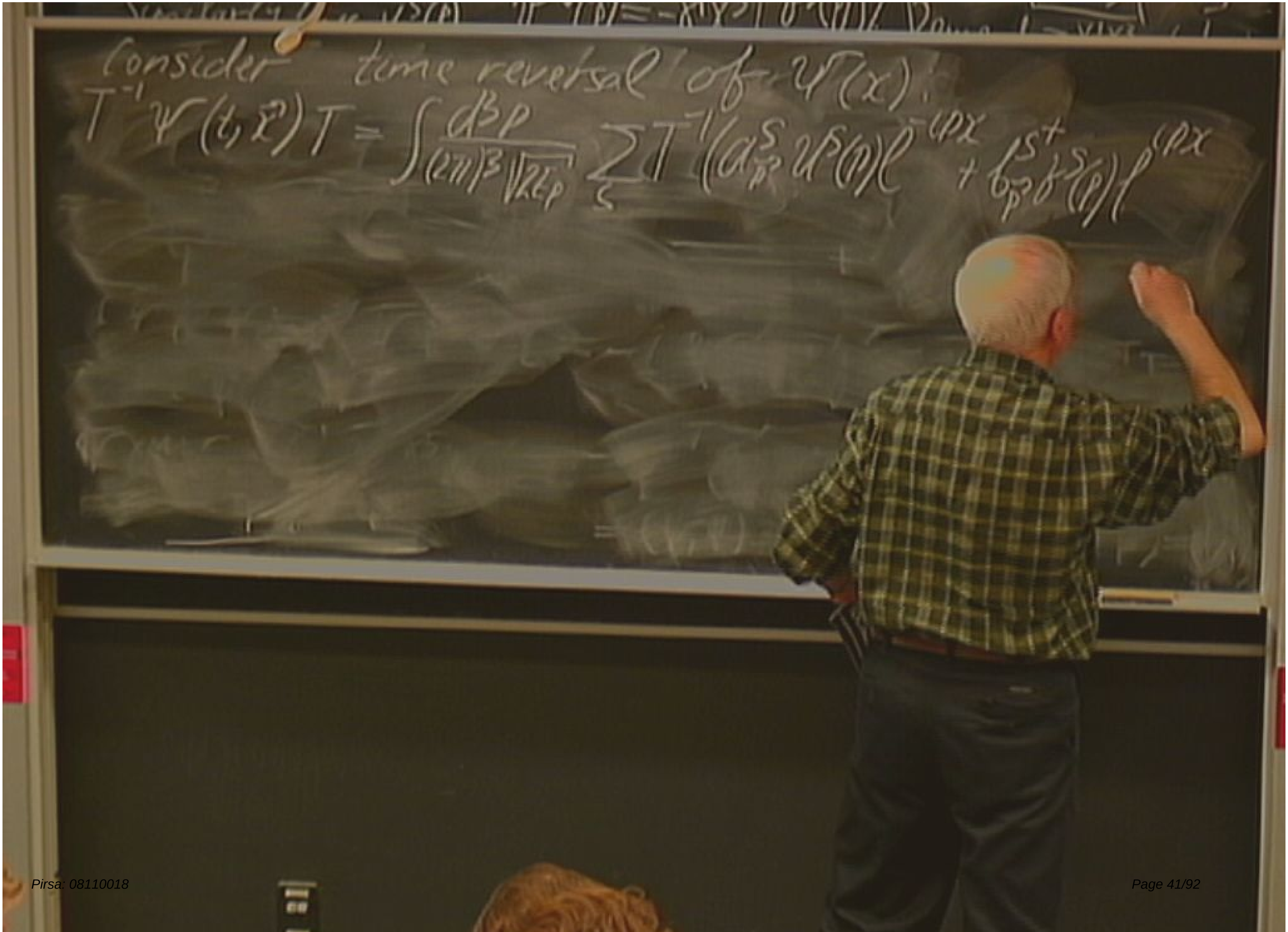
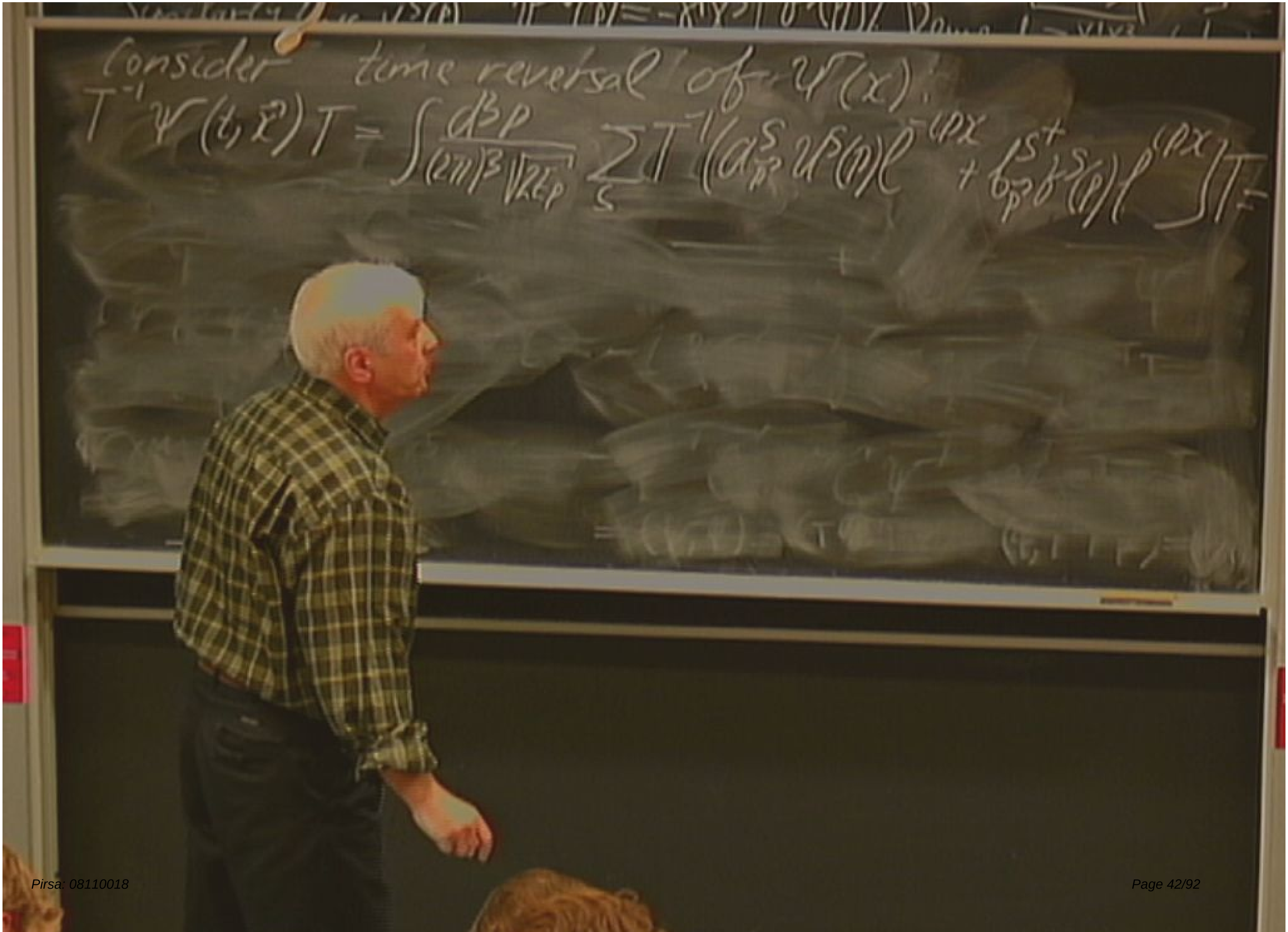

Consider time reversal of Ψ(x):


Consider time reversal of ψ(x):
T⁻¹ ψ(t, x⃗) T = ∫ d³p / ((2π)³ √(2E_p)) Σ_s T⁻¹ (a_p^s u^s(p) e^{-ipx} + b_p^{s†} v^s(p) e^{ipx}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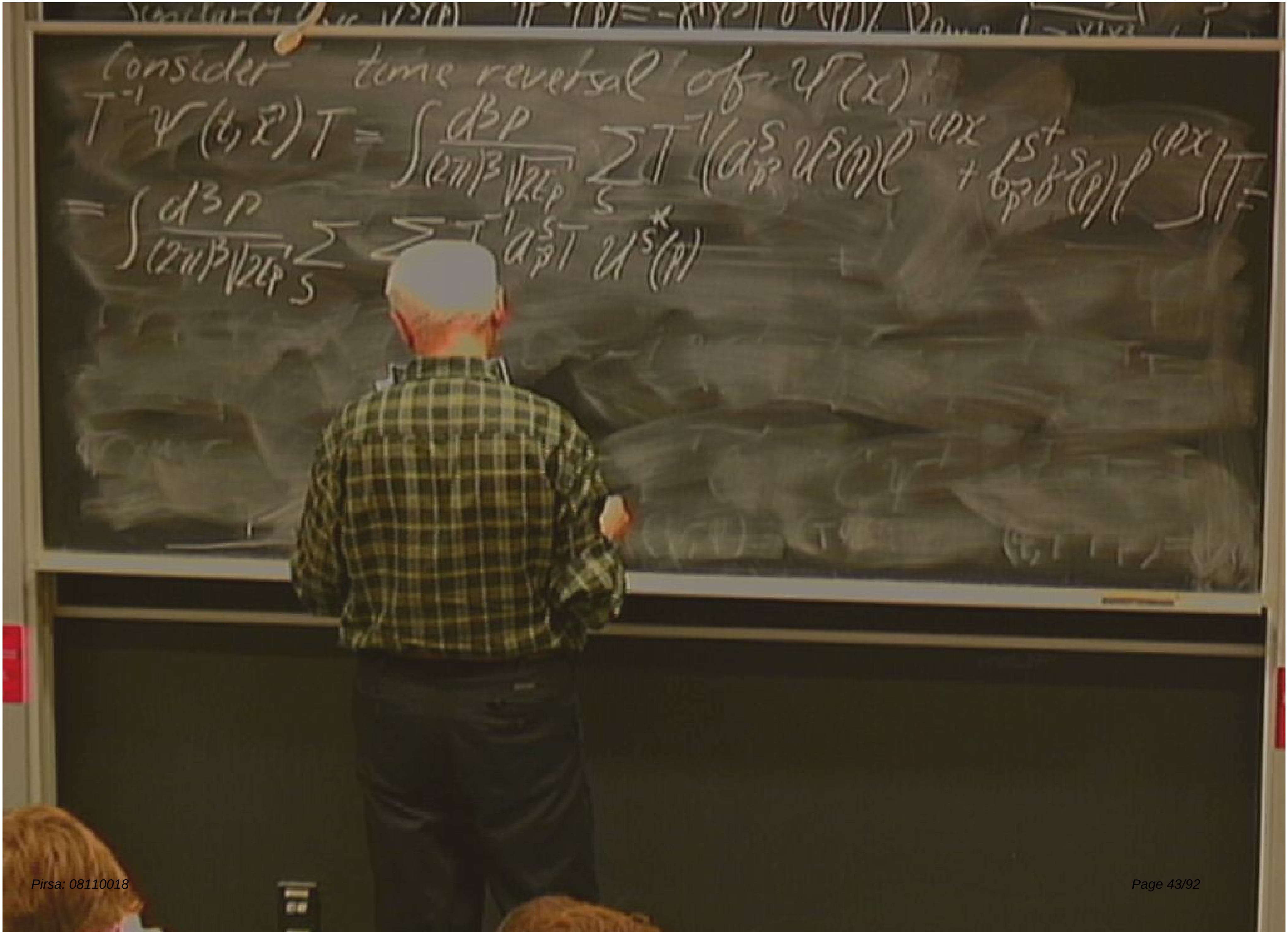
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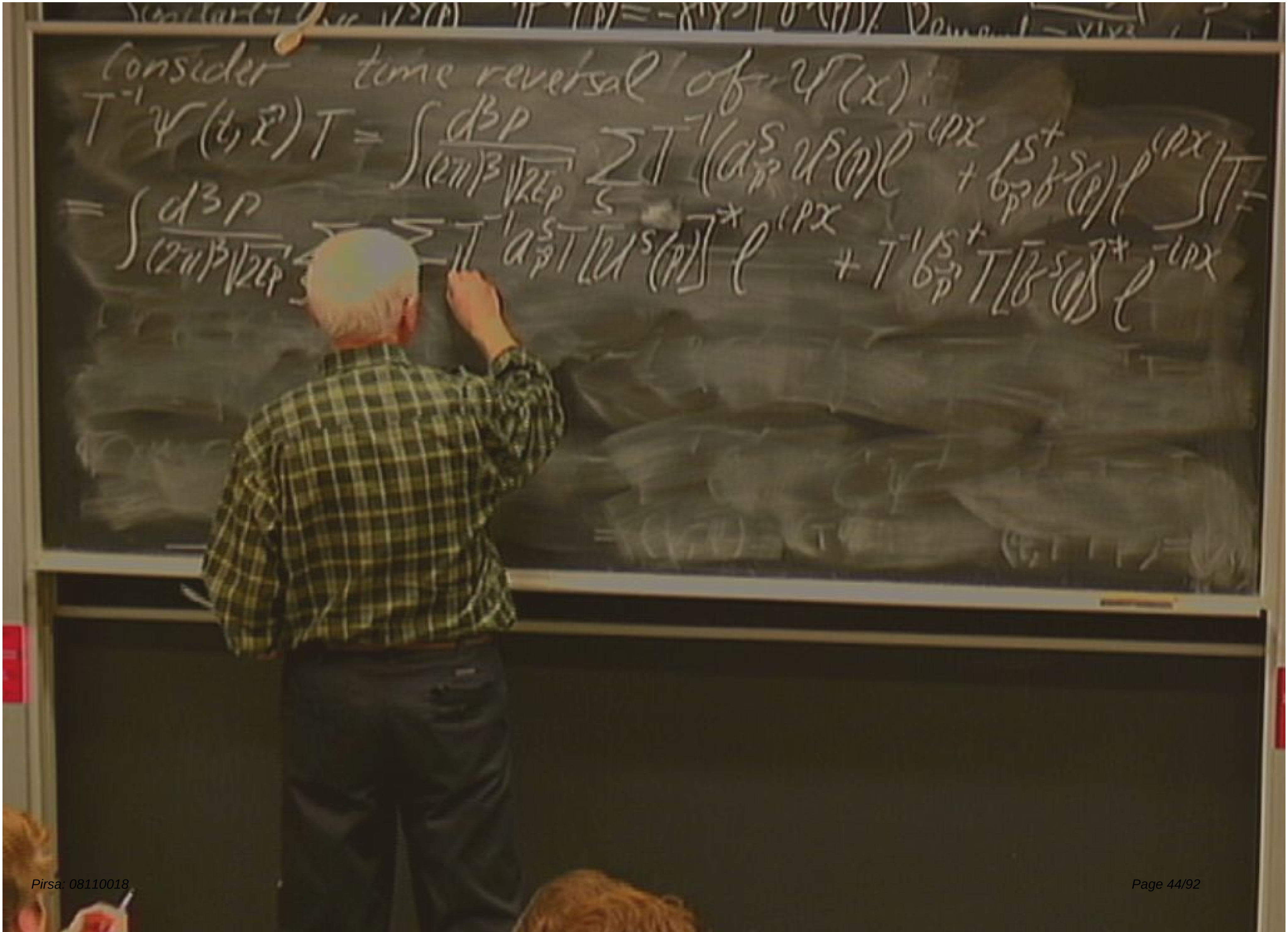
T⁻¹ψ(t, x⃗)T = ∫ d³p / ((2π)³ √(2E_p)) Σ_s T⁻¹(a_p^s u^s(p) e^{-ipx} + b_p^{s†} v^s(p) e^{ipx})T =
= ∫ d³p / ((2π)³ √(2E_p)) Σ_s T⁻¹ a_p^s T u^{s*}(p)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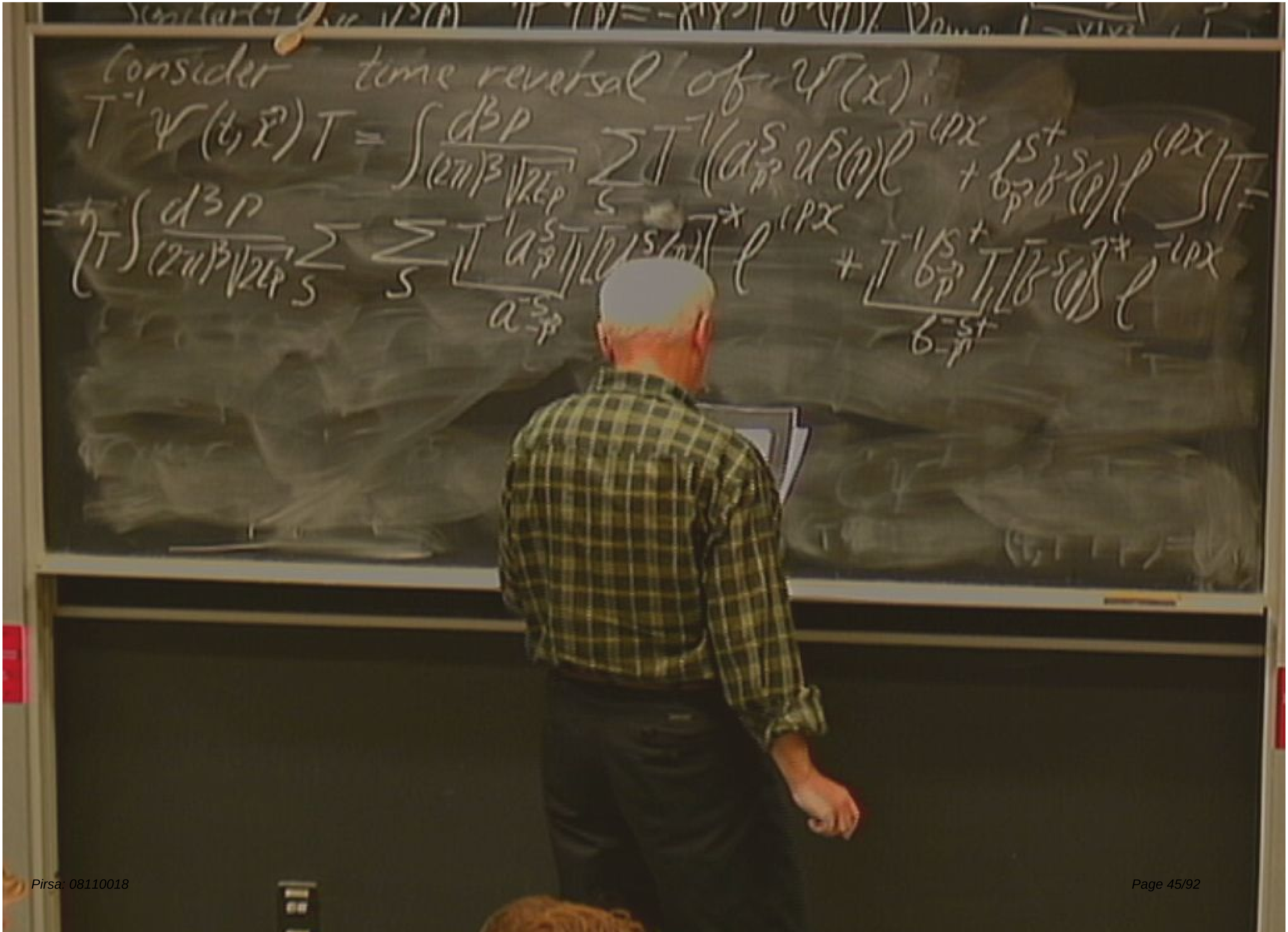
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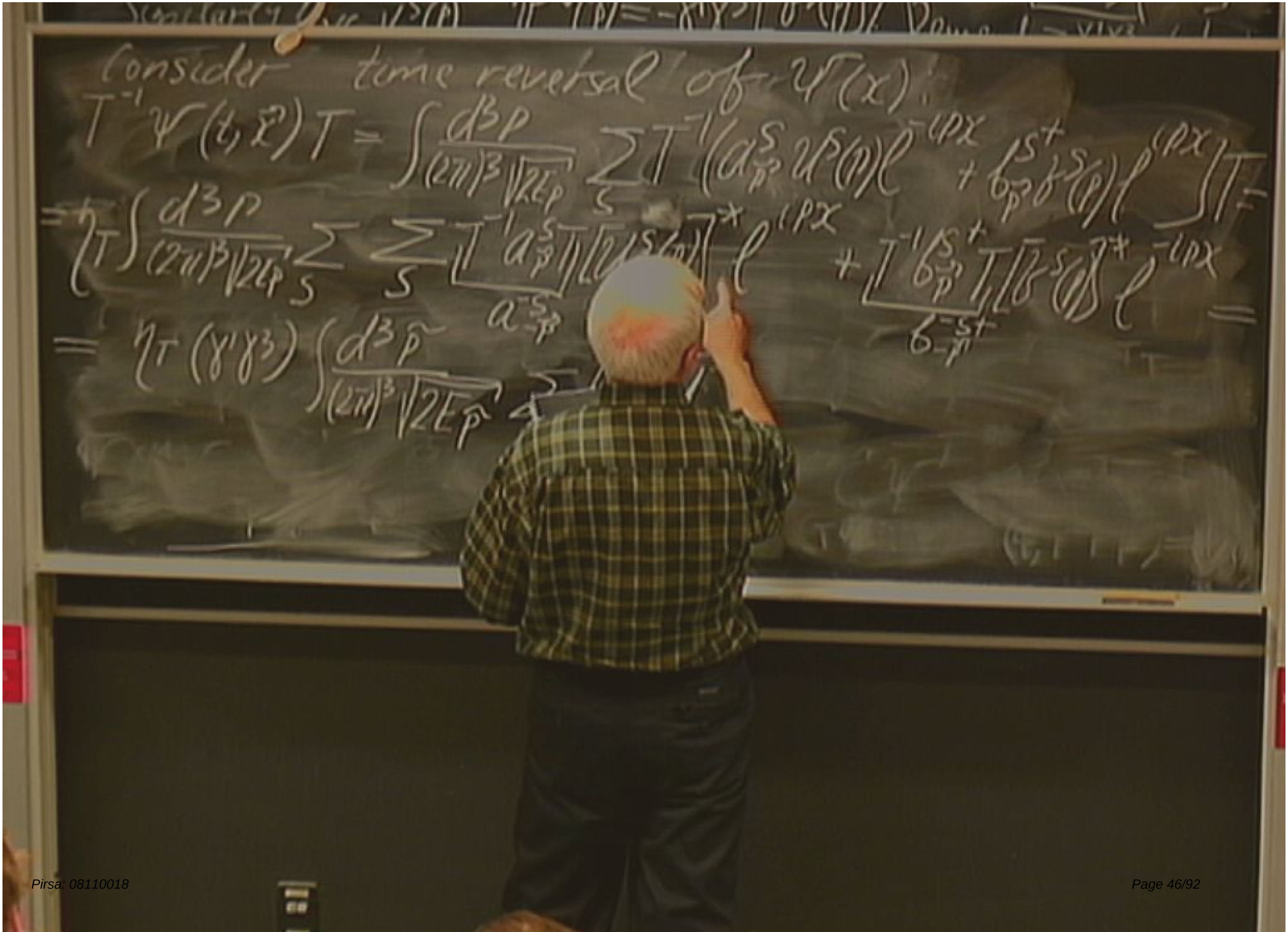
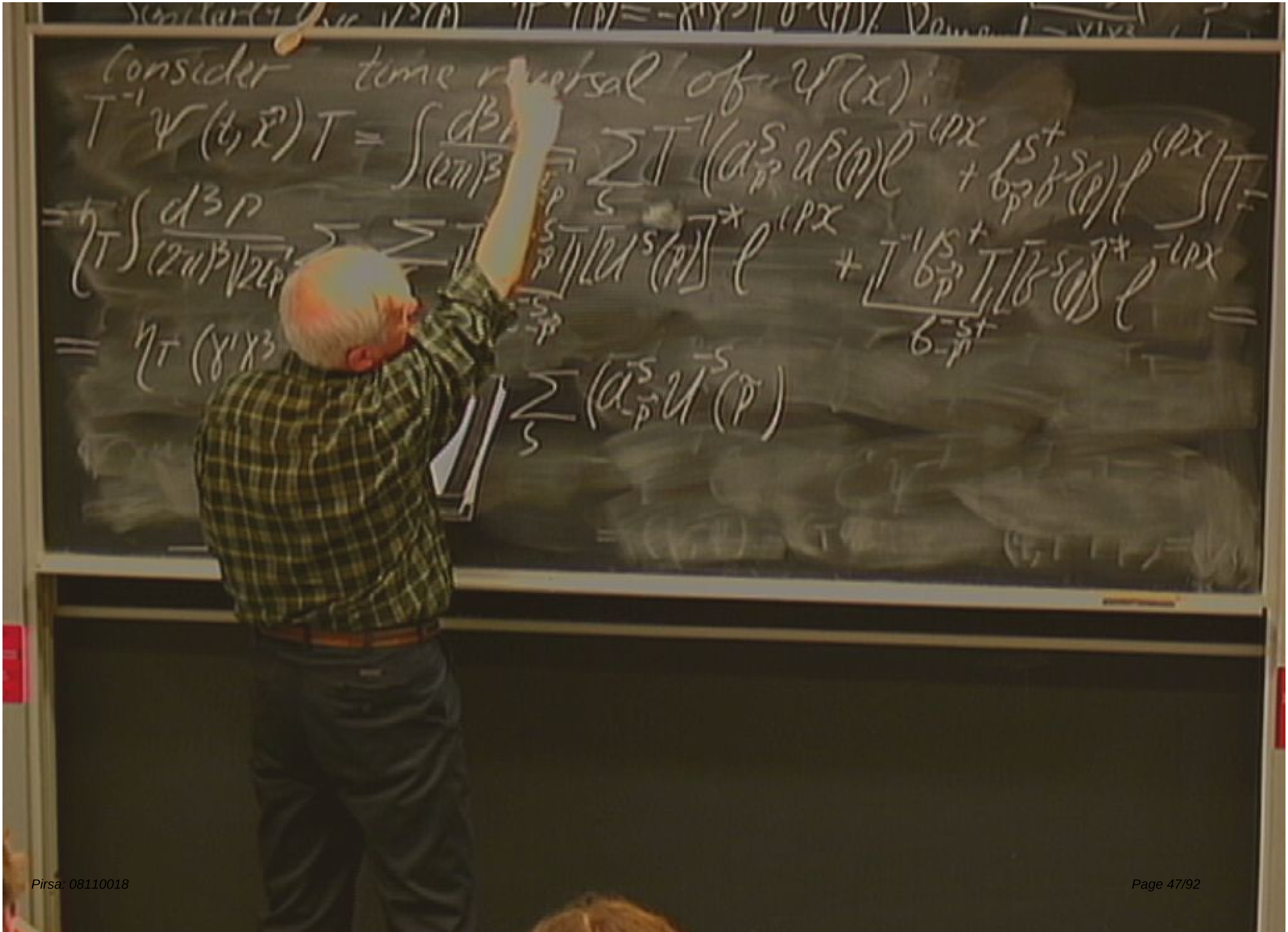
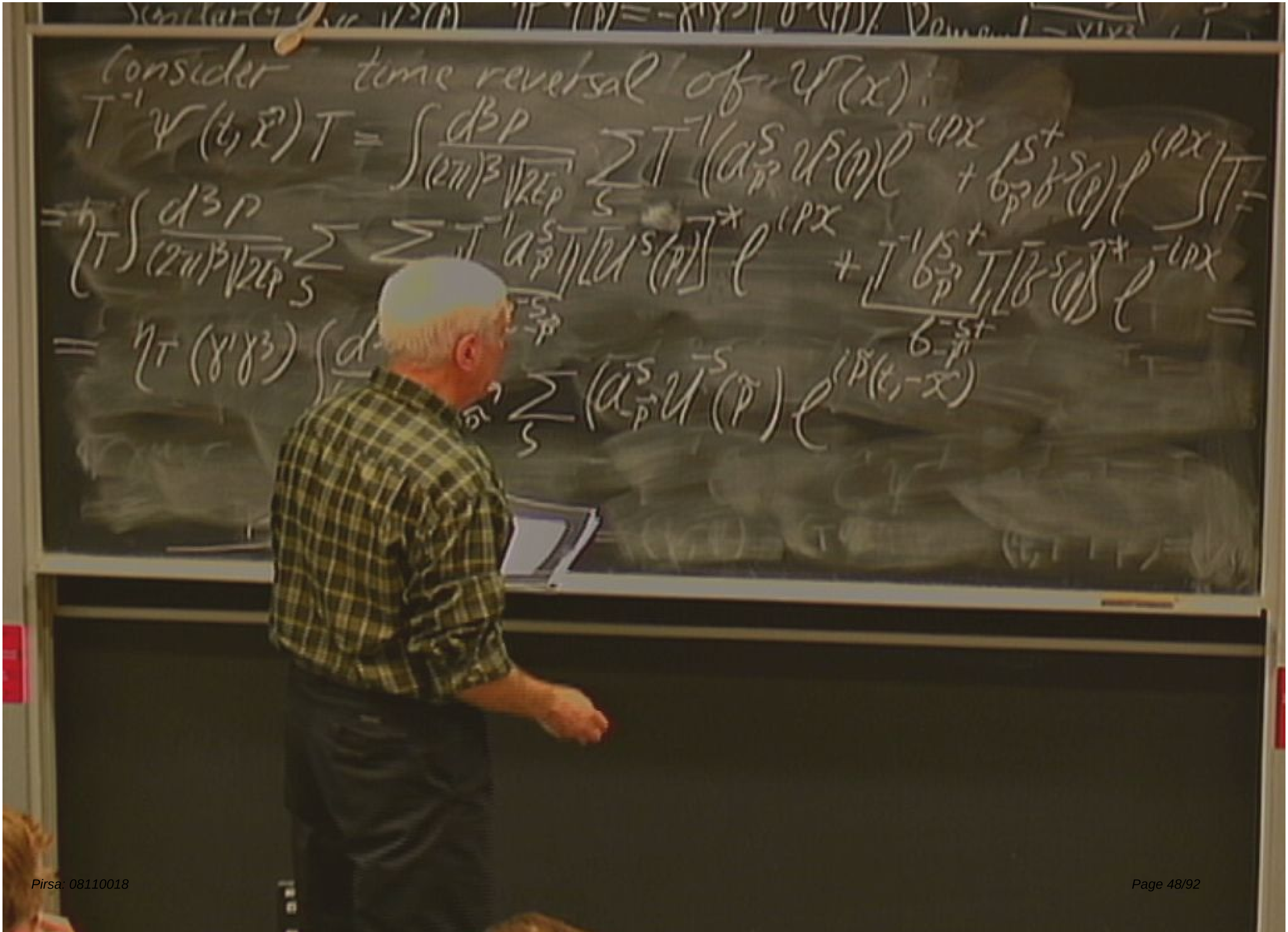
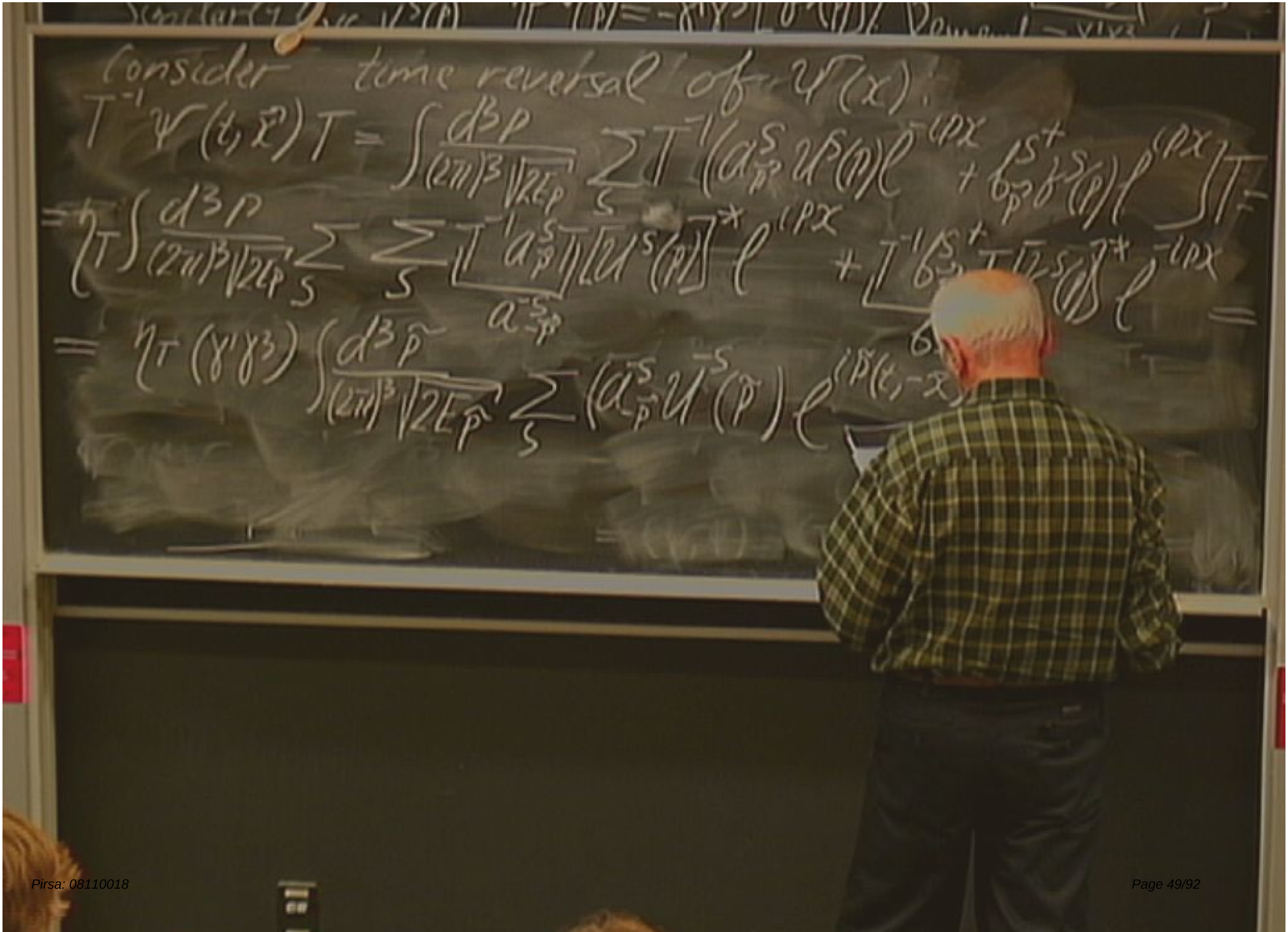
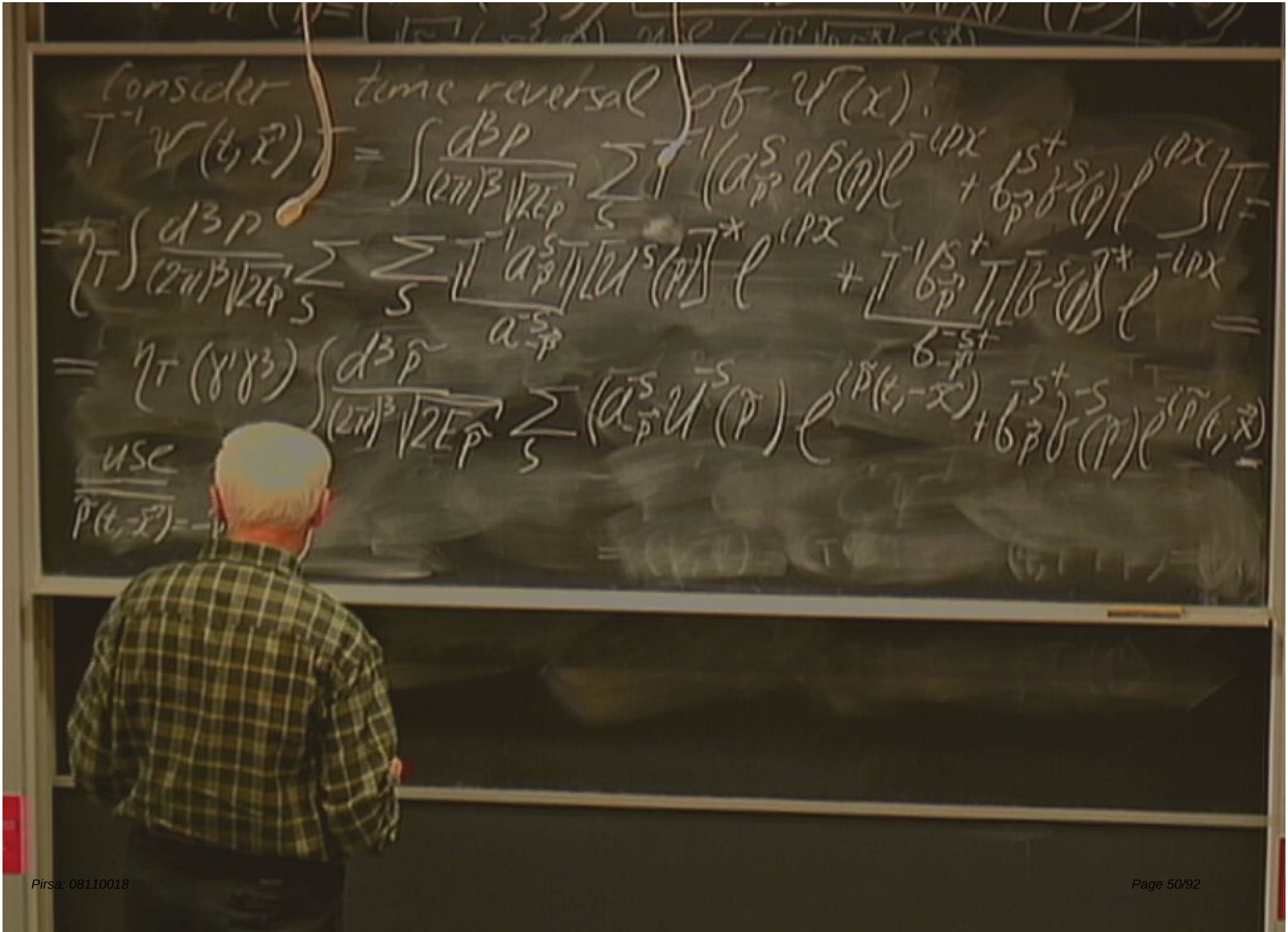

Consider time reversal of Ψ(x):


Consider


Consider time reversal of ψ(x):


Consider time reversal of ψ(x):


Consider time reversal of ψ(x):
use

Consider time reversal of $\psi(x)$:

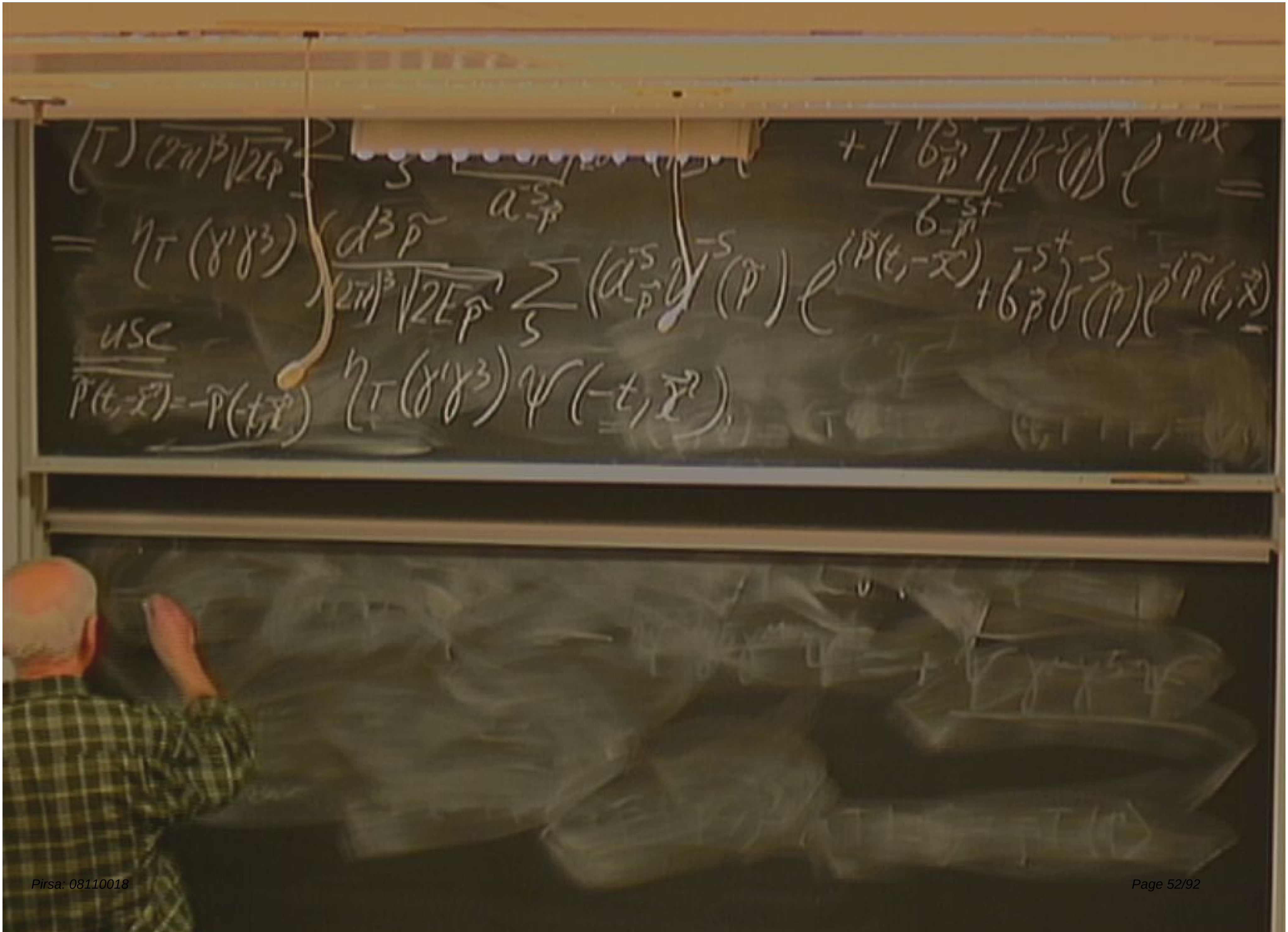
$$T^{-1}\psi(t,\vec{x})T = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s T^{-1}\left(a^s_{\vec{p}}\, u^s(p)\, e^{-ipx} + b^{s+}_{\vec{p}}\, v^s(p)\, e^{ipx}\right)T =$$

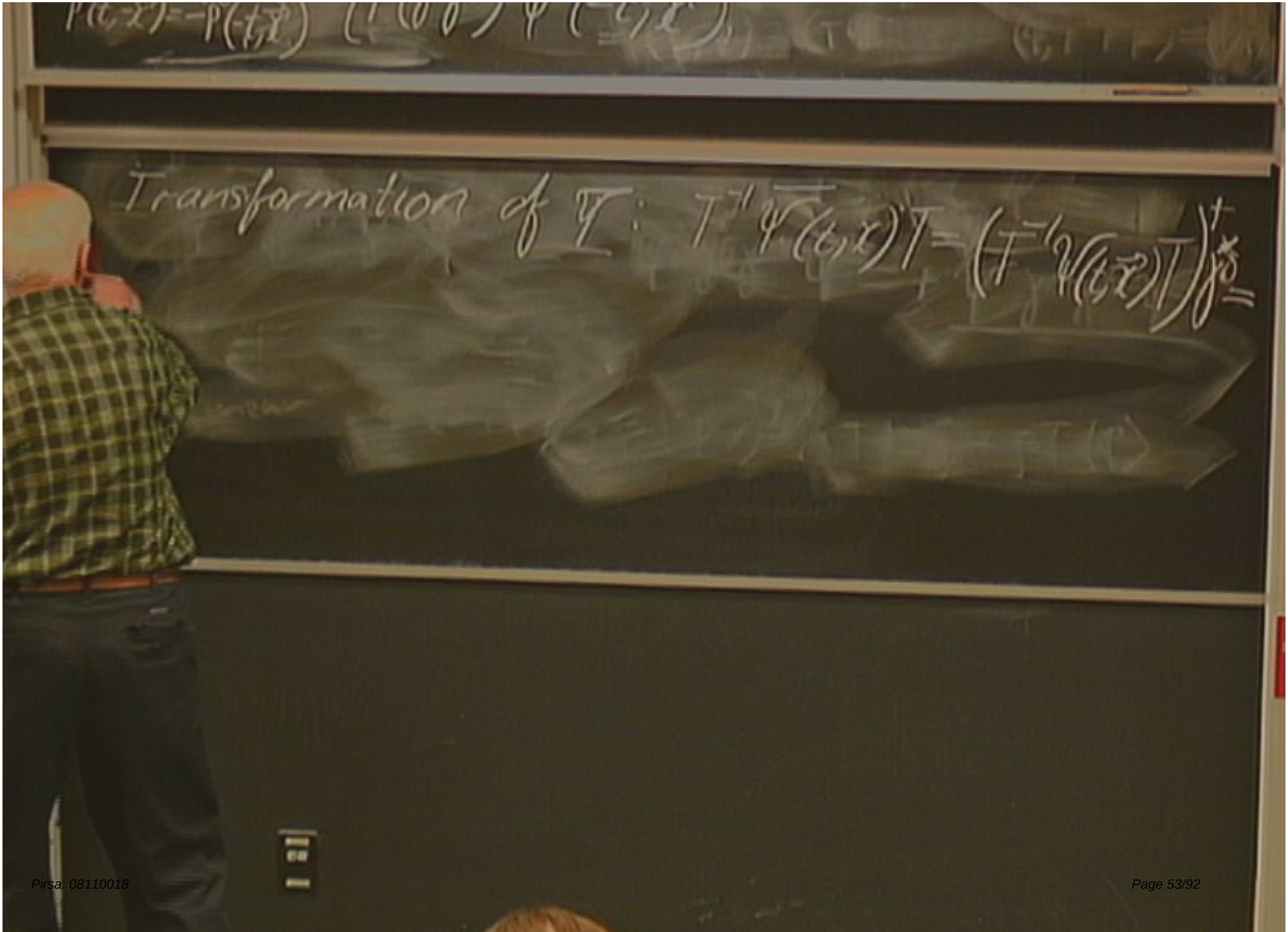
$$= \eta_T \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \sum_s \underbrace{T^{-1}a^s_{\vec{p}}T}_{a^{-s}_{-\vec{p}}}\,[u^s(p)]^*\, e^{ipx} + \underbrace{T^{-1}b^{s+}_{\vec{p}}T}_{b^{-s+}_{-\vec{p}}}\,[v^s(p)]^*\, e^{-ipx} =$$

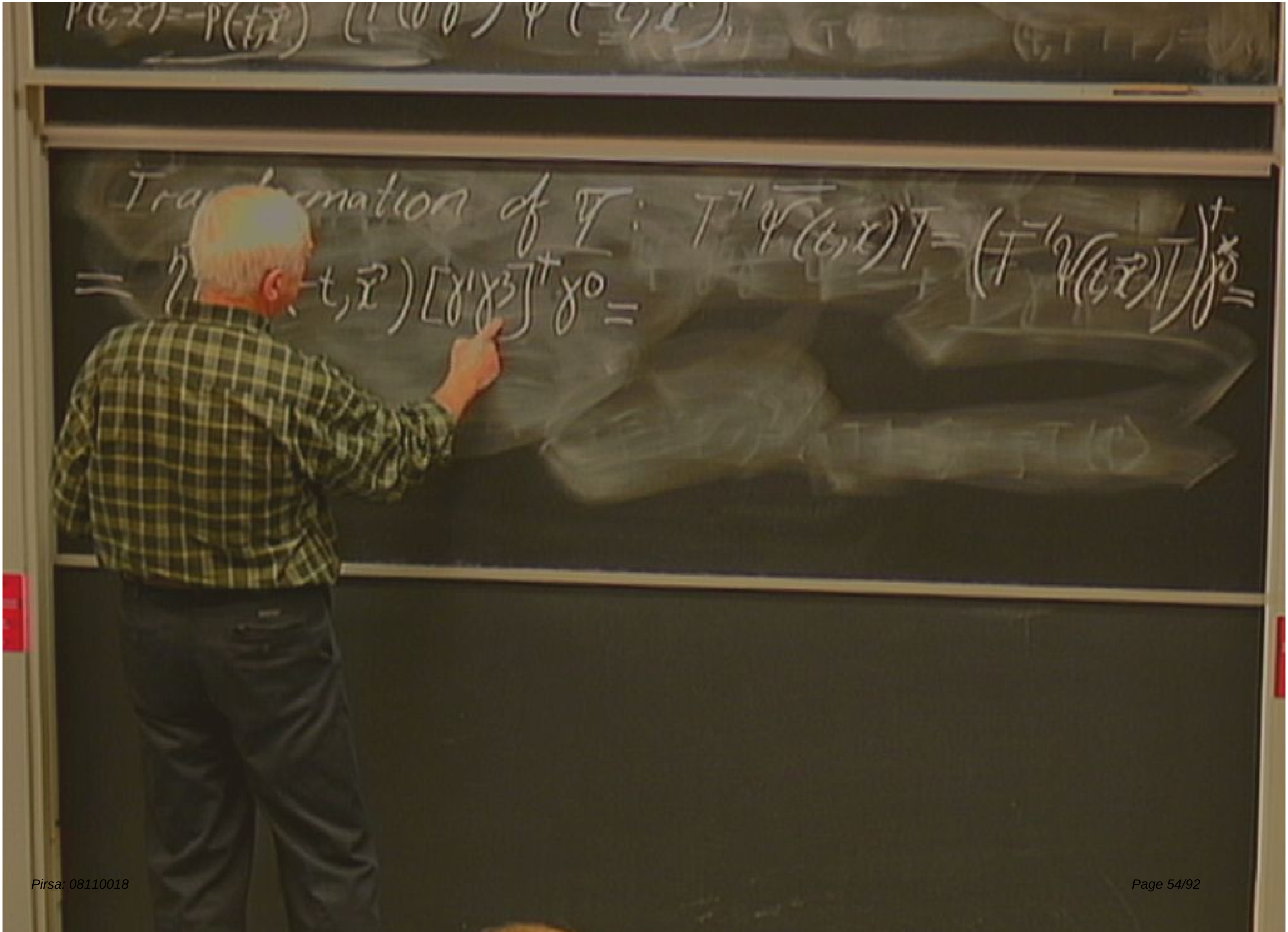
$$= \eta_T(\gamma^1\gamma^3)\int \frac{d^3\tilde{p}}{(2\pi)^3\sqrt{2E_{\tilde{p}}}} \sum_s \left(a^{-s}_{\tilde{p}}\, u^{-s}(\tilde{p})\, e^{i\tilde{p}(t,-\vec{x})} + b^{-s+}_{\tilde{p}}\, v^{-s}(\tilde{p})\, e^{-i\tilde{p}(t,\vec{x})}\right) =$$

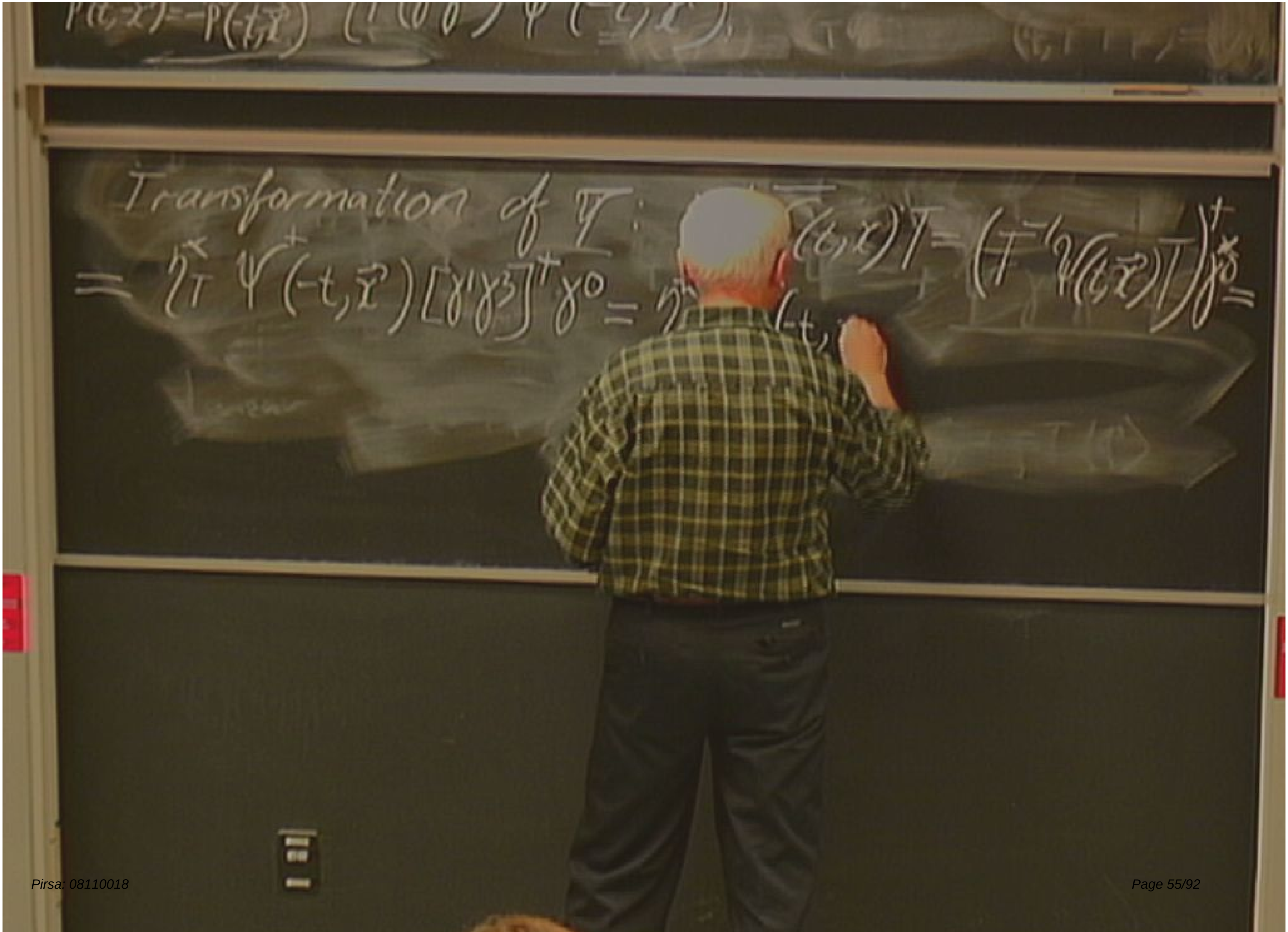
use $\tilde{p}(t,-\vec{x}) = -\tilde{p}(-t,\vec{x})$

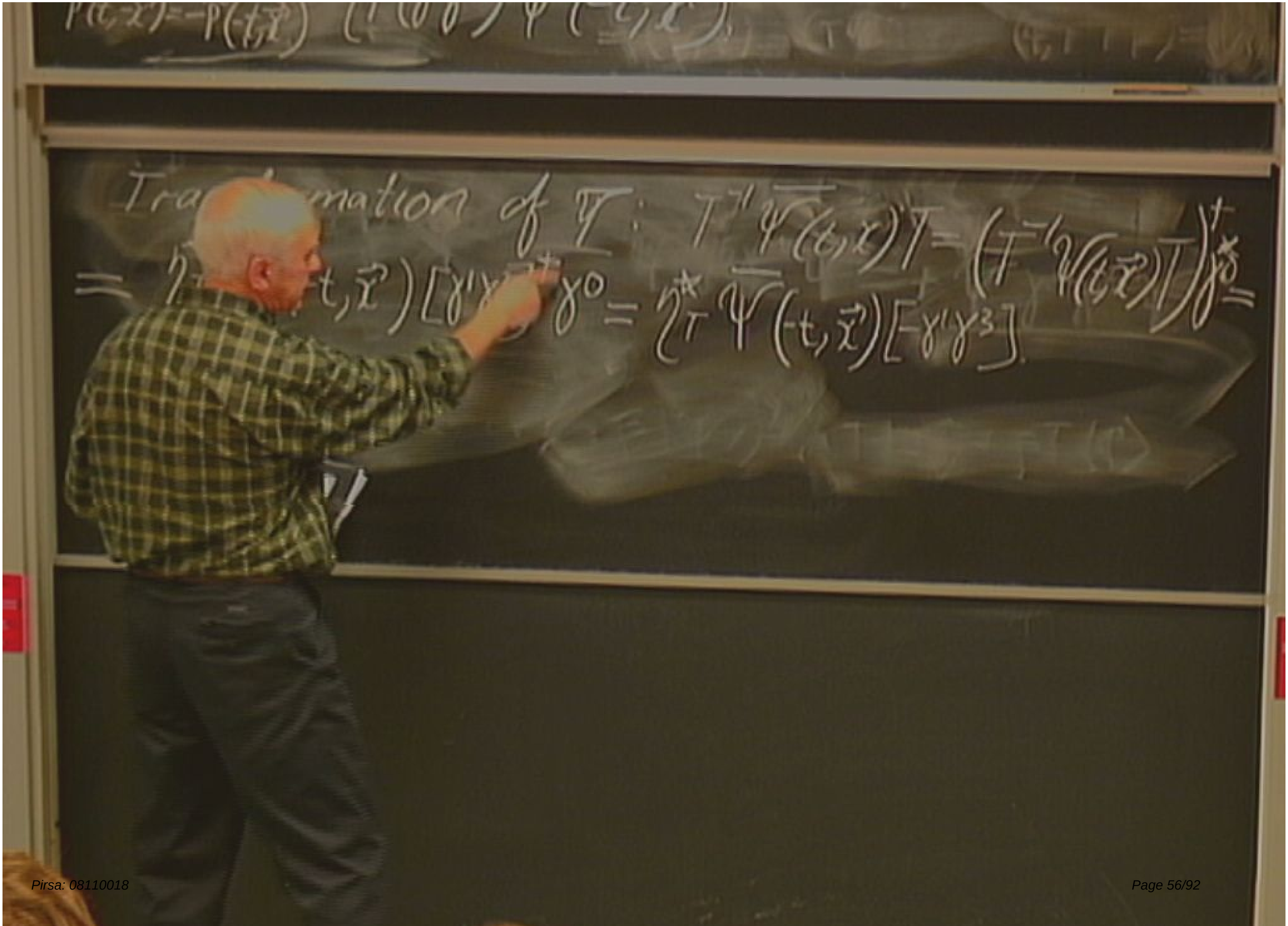
$$\eta_T(\gamma^1\gamma^3)\psi(-t,\vec{x})$$

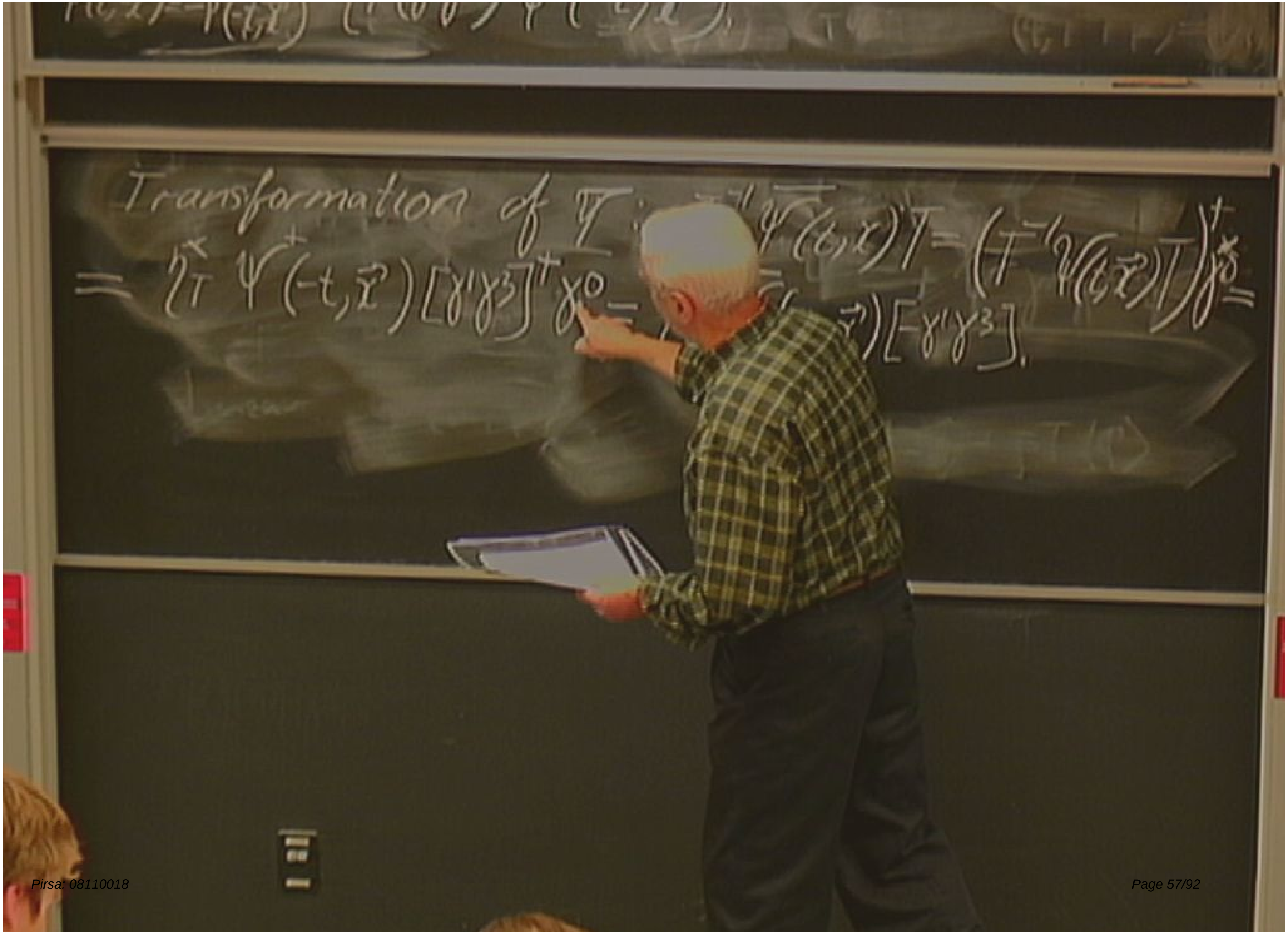


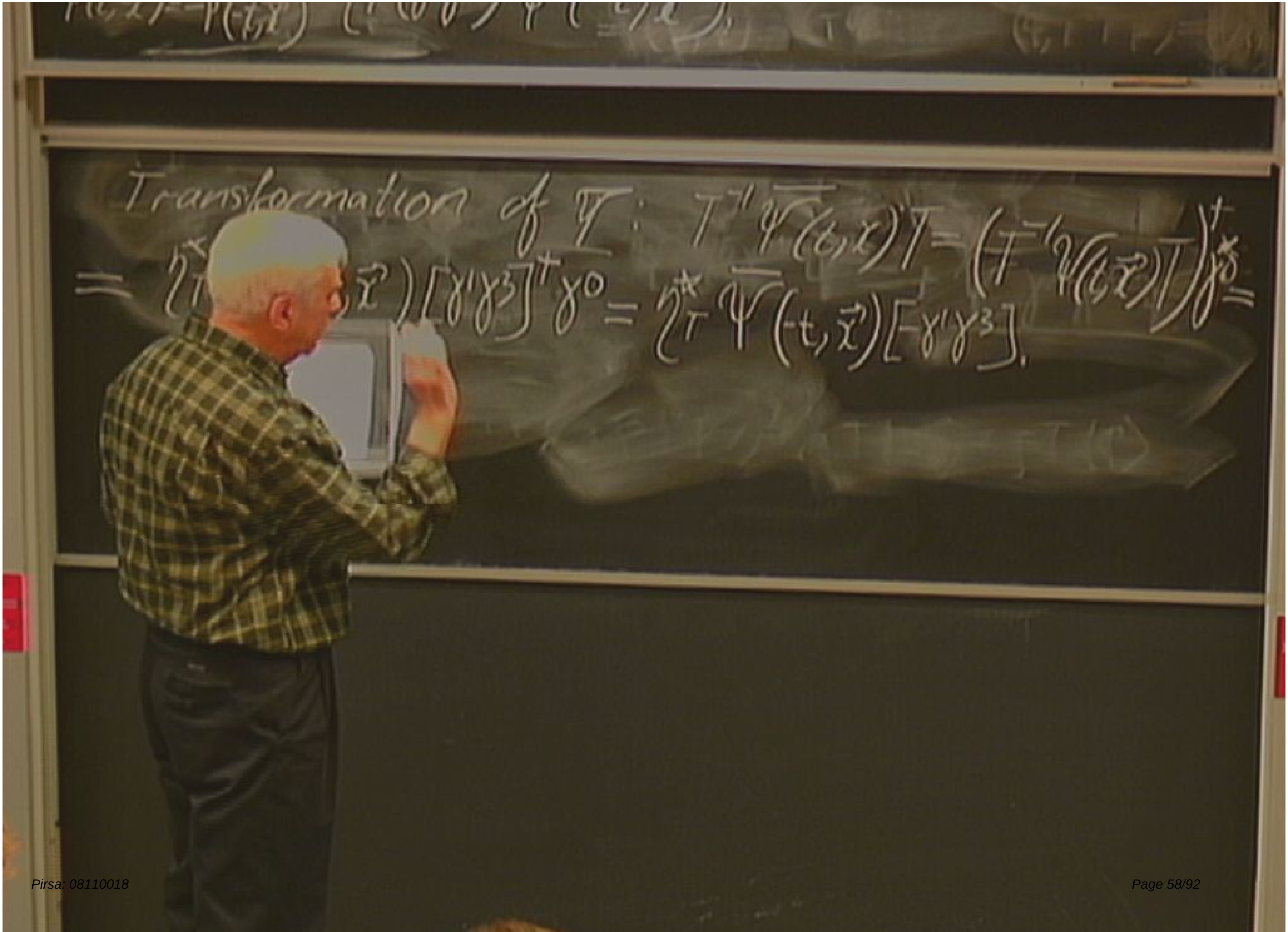

Transformation of $\bar{\Psi}$: $T^{-1}\bar{\Psi}(t,\vec{x})T = (T^{-1}\Psi(t,\vec{x})T)^{\dagger}\gamma^0 =$

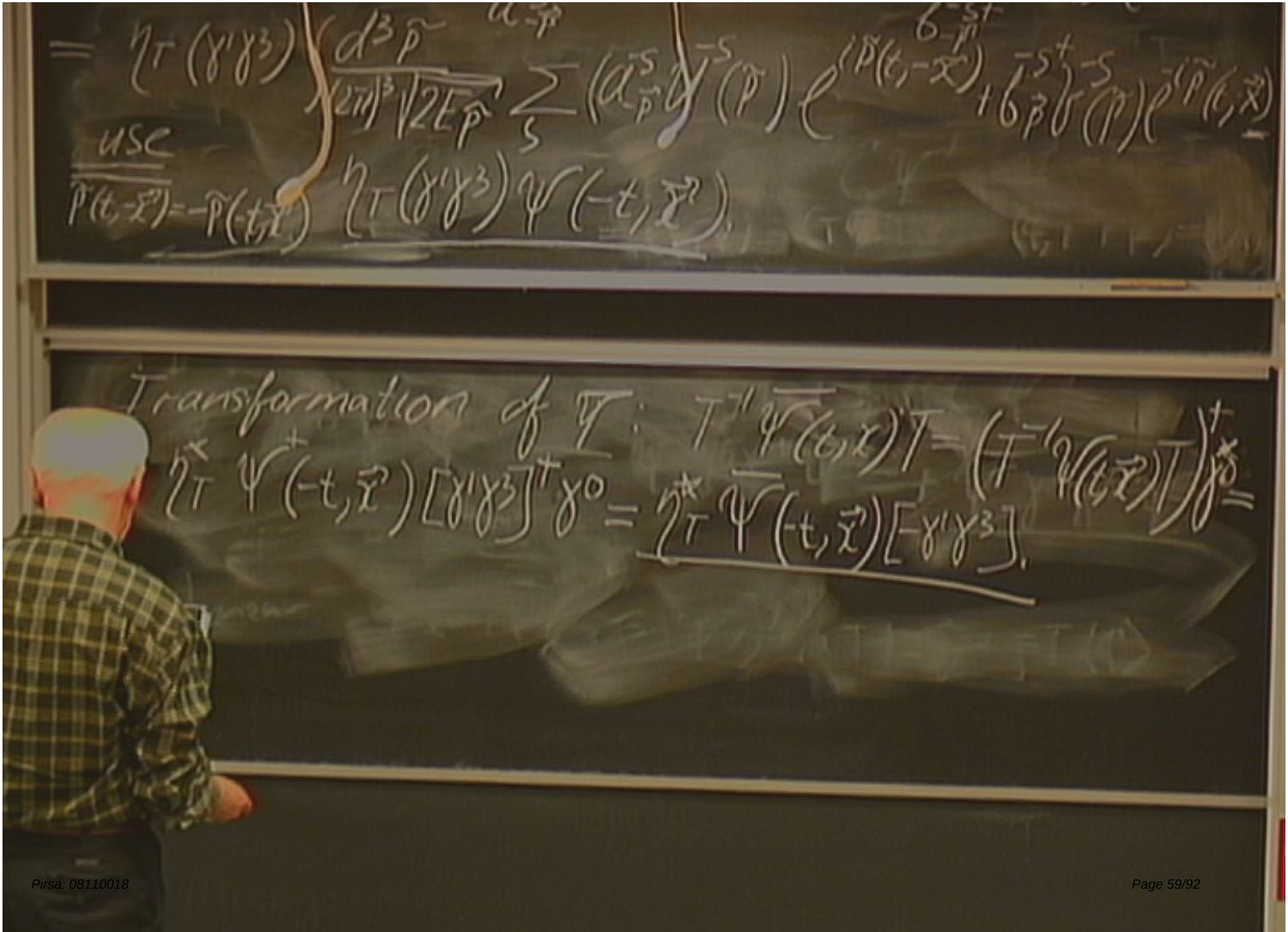



Transformation of Ψ̄ :




Transformation of Ψ̄ :


Transformation of


= η_T (γ¹γ³) ∫ d³p̃ / ((2π)³ √(2E_p̃)) Σ_s (...) e^{ip̃(t,-x⃗)} + ... e^{-ip̃(t,x⃗)} =
usc
p̃(t,-x⃗) = -p̃(-t,x⃗)
η_T (γ¹γ³) ψ(-t,x⃗).
Transformation of ψ̄: T⁻¹ψ̄(t,x)T = (T⁻¹ψ(t,x⃗)T)† γ⁰ =
η*_T ψ†(-t,x⃗) [γ¹γ³]† γ⁰ = η*_T ψ̄(-t,x⃗) [-γ¹γ³].

$$= \eta_T(\gamma^1\gamma^3)\int \frac{d^3\tilde{p}}{(2\pi)^3\sqrt{2E_{\tilde{p}}}}\sum_s \left(a^{-s}_{-\vec{p}}\, u^{-s}(\tilde{p})\, e^{i\tilde{p}(t,-\vec{x})} + b^{-s\dagger}_{\vec{p}}\, v^{-s}(\tilde{p})\, e^{-i\tilde{p}(t,\vec{x})}\right) =$$

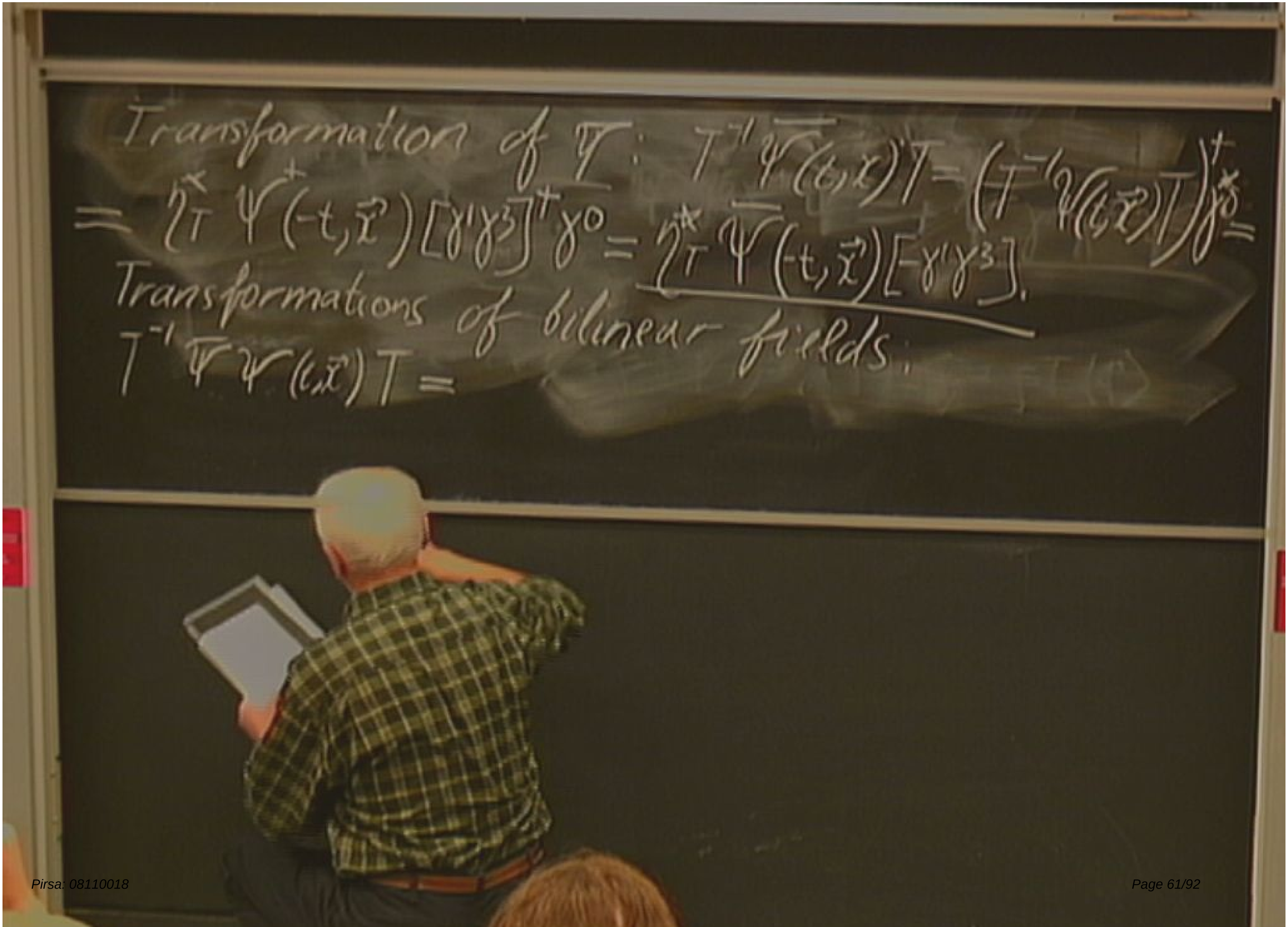
use $\tilde{p}(t,-\vec{x}) = -\tilde{p}(-t,\vec{x})$

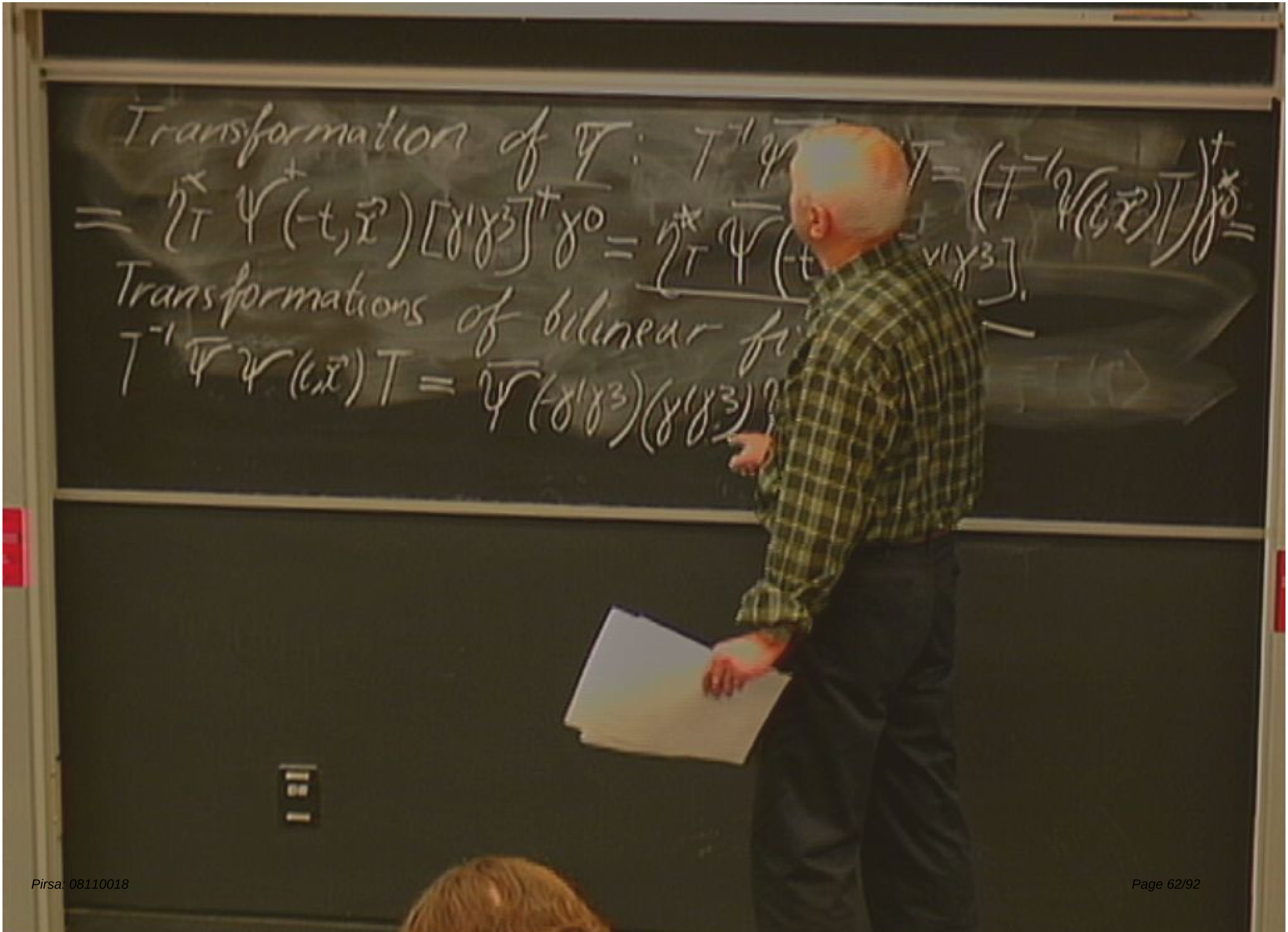
$$\eta_T(\gamma^1\gamma^3)\,\Psi(-t,\vec{x}).$$

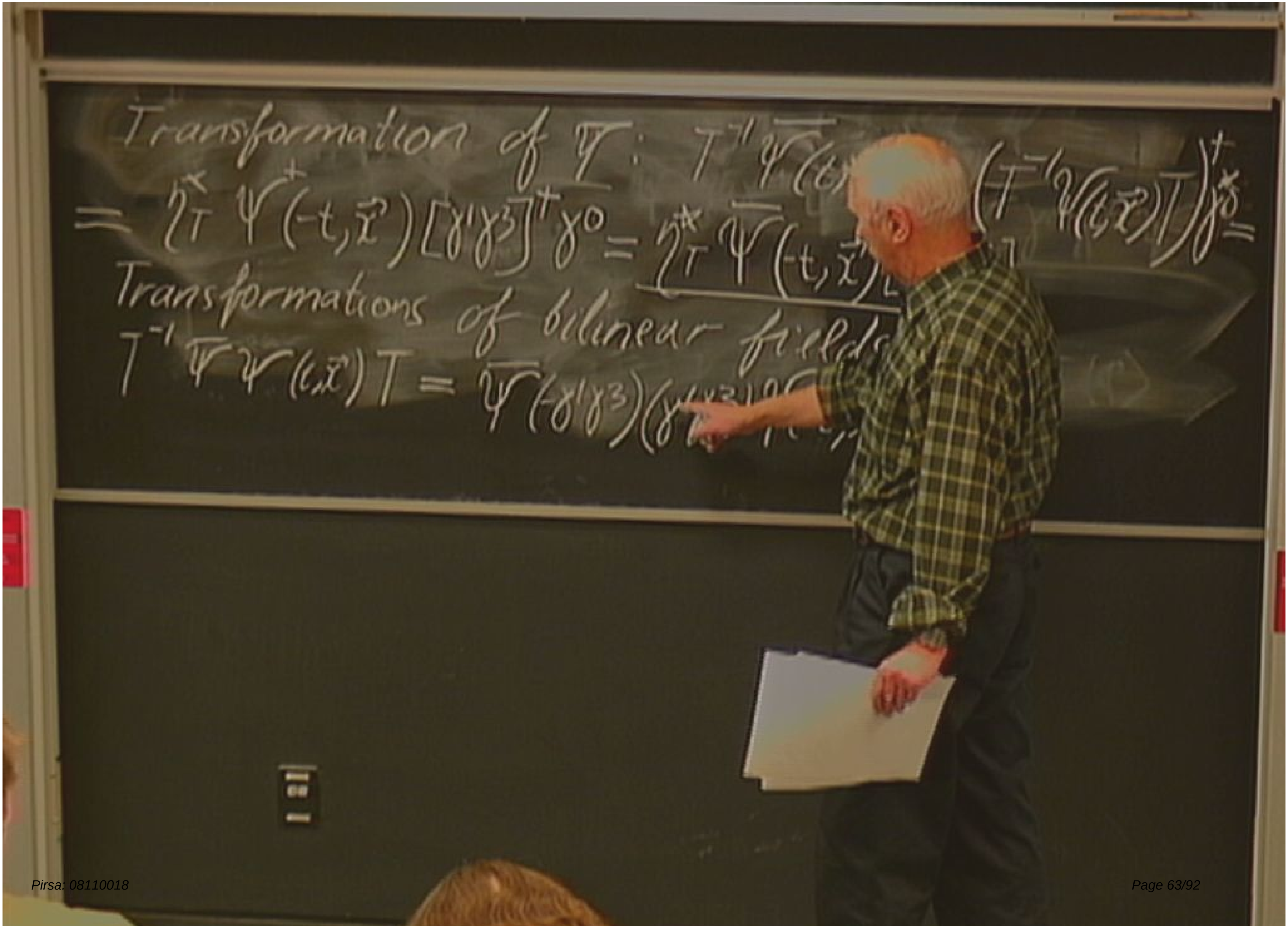
Transformation of $\bar{\Psi}$: $T^{-1}\bar{\Psi}(t,\vec{x})T = \left(T^{-1}\Psi(t,\vec{x})T\right)^{\dagger}\gamma^0 =$

$$= \eta_T^{*}\,\Psi^{\dagger}(-t,\vec{x})\,[\gamma^1\gamma^3]^{\dagger}\gamma^0 = \eta_T^{*}\,\bar{\Psi}(-t,\vec{x})\,[-\gamma^1\gamma^3].$$

T


Transformation of $\bar{\Psi}$: $T^{-1}\bar{\Psi}(t,\vec{x})T = (T^{-1}\Psi(t,\vec{x})T)^{\dagger}\gamma^0 =$
$= \eta_T^{*}\Psi^{\dagger}(-t,\vec{x})[\gamma^1\gamma^3]^{\dagger}\gamma^0 = \eta_T^{*}\bar{\Psi}(-t,\vec{x})[-\gamma^1\gamma^3].$
Transformations of bilinear fields:
$T^{-1}\bar{\Psi}\Psi(t,\vec{x})T =$


Transformation of $\bar{\Psi}$: $T^{-1}\bar{\psi}$
$= \eta_T^* \psi^\dagger(-t,\vec{x})[\gamma^1\gamma^3]^\dagger\gamma^0 = \eta_T^* \bar{\psi}(-t$
Transformations of bilinear
$T^{-1}\bar{\psi}\psi(t,\vec{x})T = \bar{\psi}(-\gamma^1\gamma^3)(\gamma^1\gamma^3)$

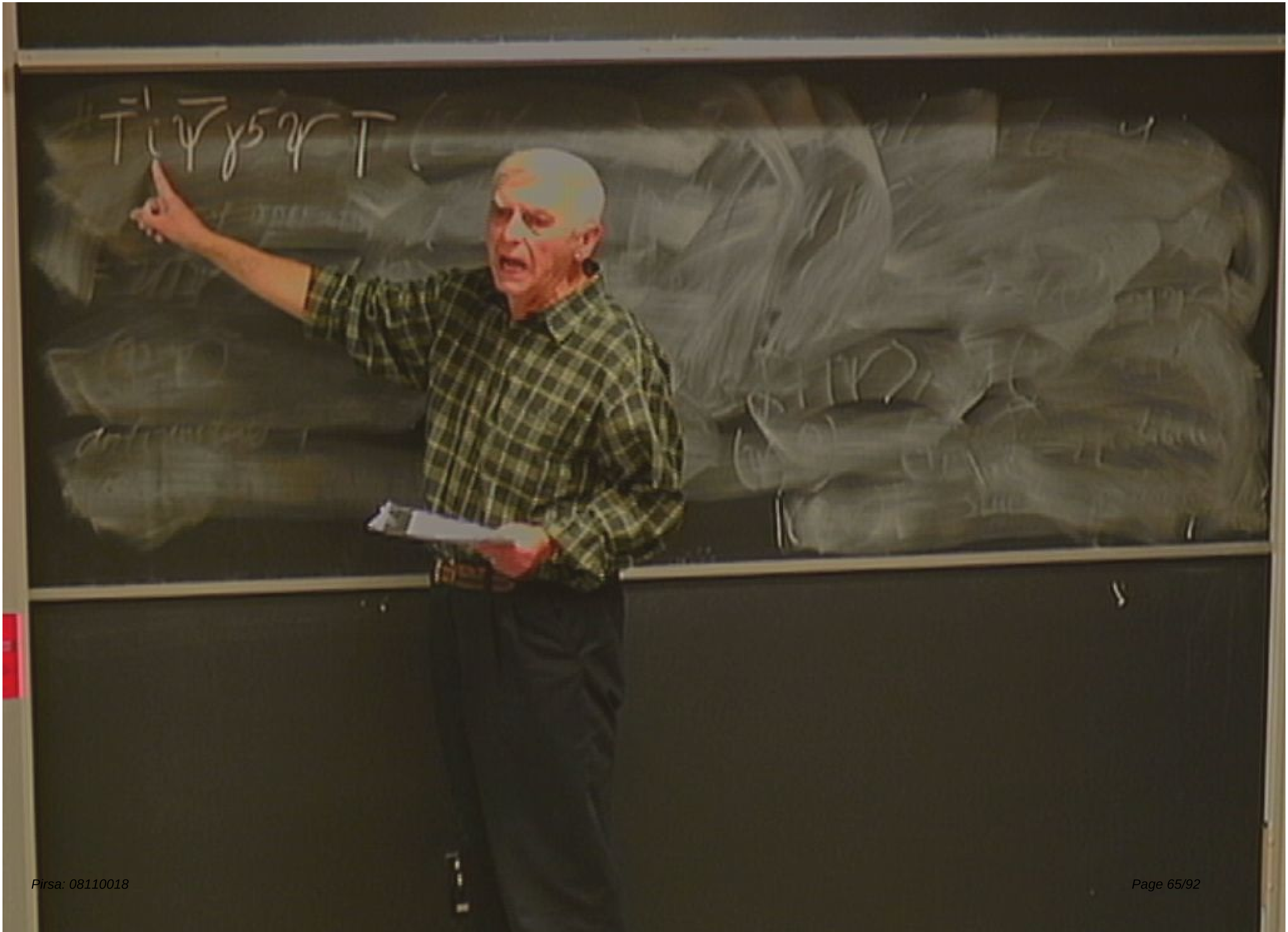

Transformation of Ψ̄:
Transformations of bilinear fields

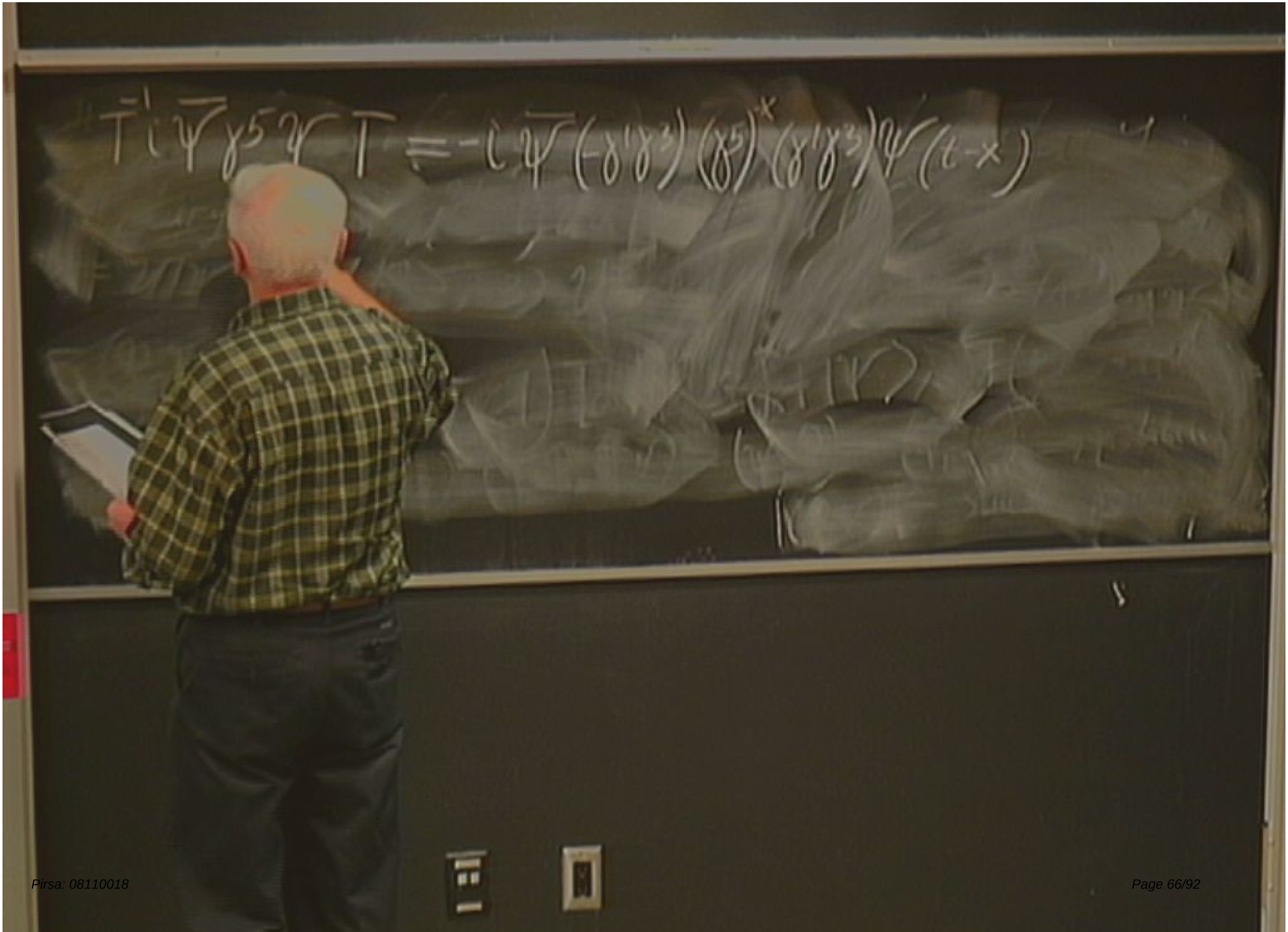
Transformation of $\overline{\Psi}$: $T^{-1}\overline{\Psi}(t,\vec{x})T = (T^{-1}\Psi(t,\vec{x})T)^{\dagger}\gamma^0 =$

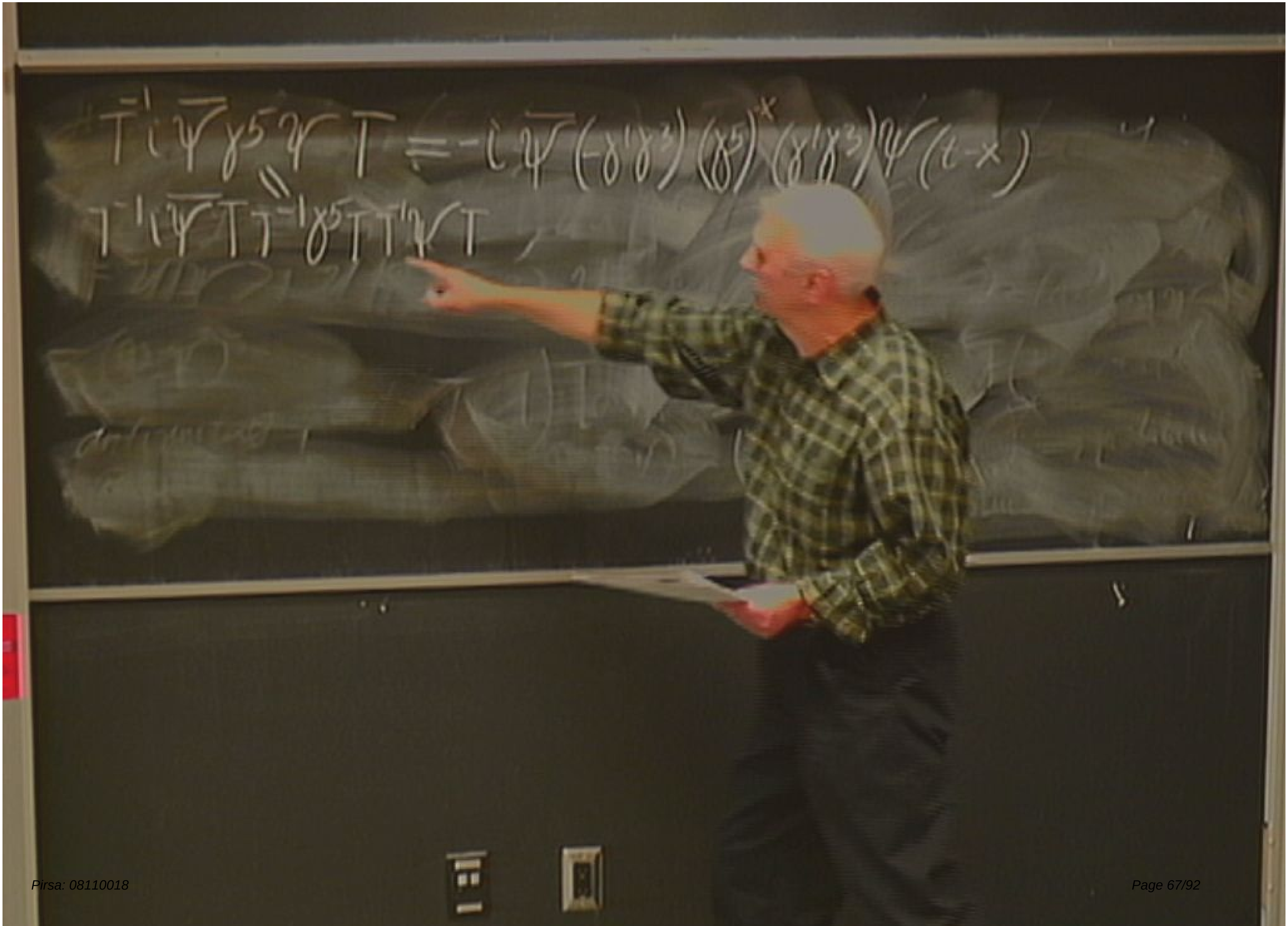
$= \eta_T^* \Psi^{\dagger}(-t,\vec{x})[\gamma^1\gamma^3]^{\dagger}\gamma^0 = \eta_T^* \overline{\Psi}(-t,\vec{x})[-\gamma^1\gamma^3]$.

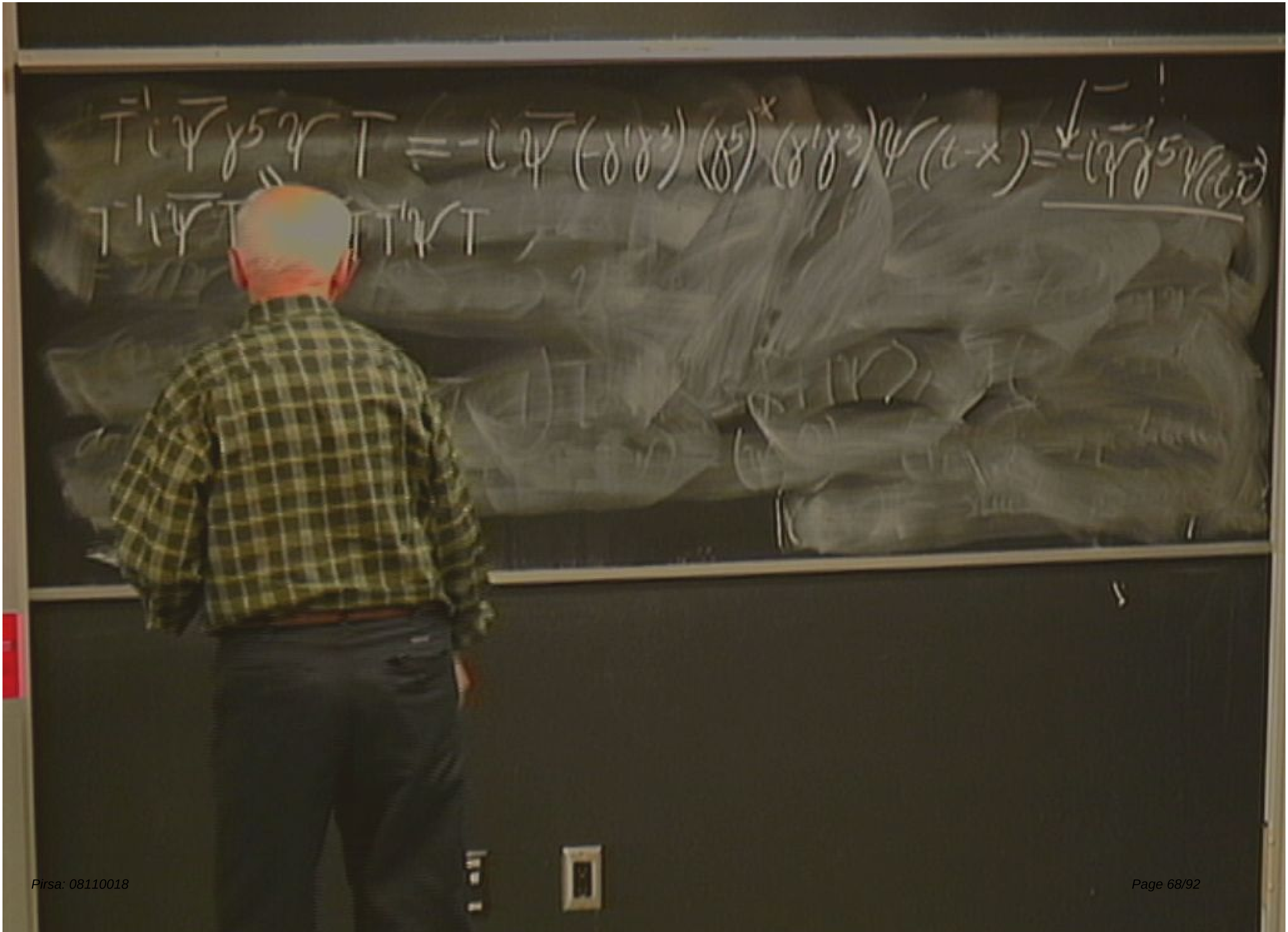
Transformations of bilinear fields:

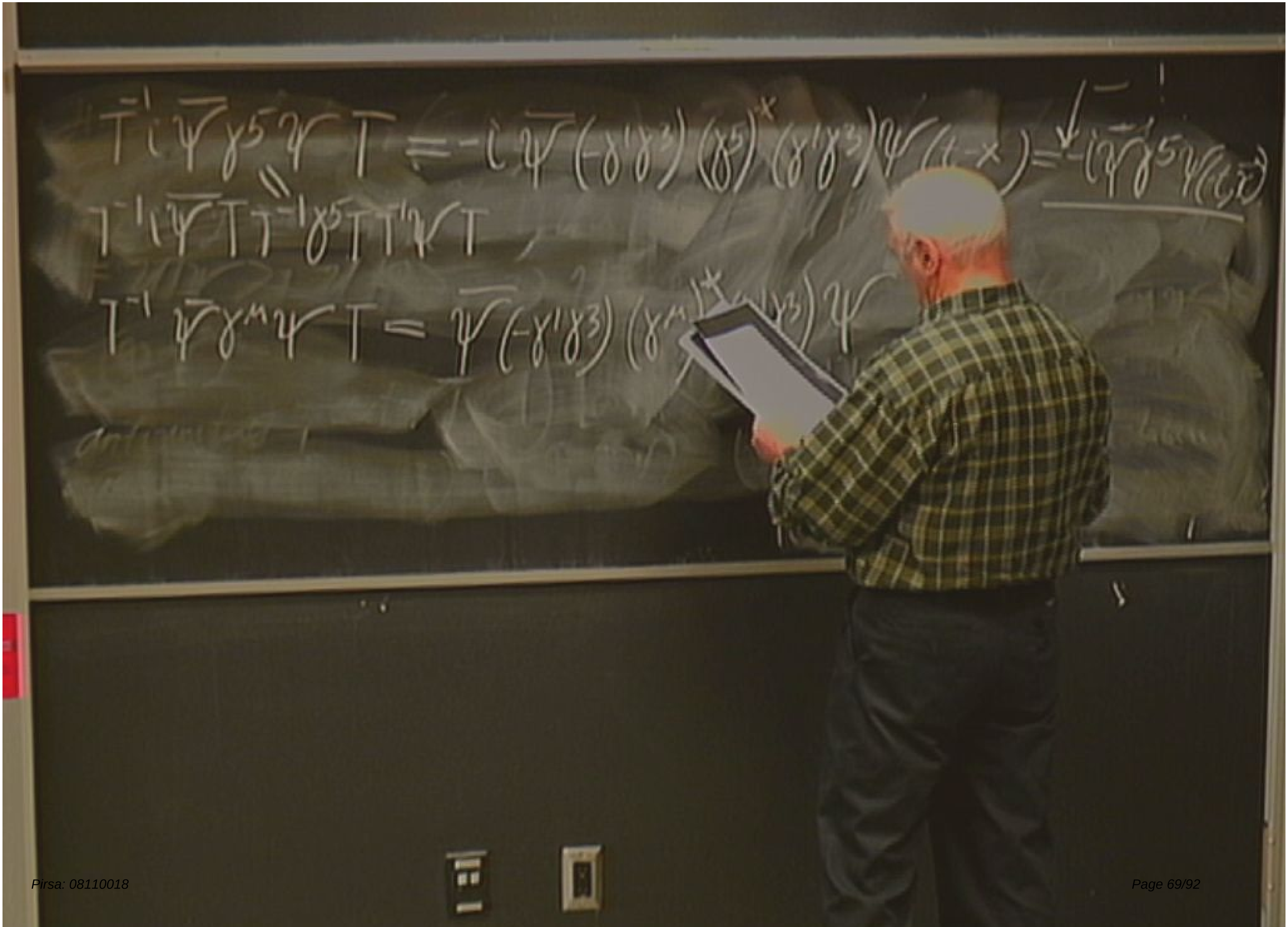
$T^{-1}\overline{\Psi}\Psi(t,\vec{x})T = \overline{\Psi}(-\gamma^1\gamma^3)(\gamma^1\gamma^3)\Psi(-t,\vec{x}) = \overline{\Psi}\Psi(-t,\vec{x})$,

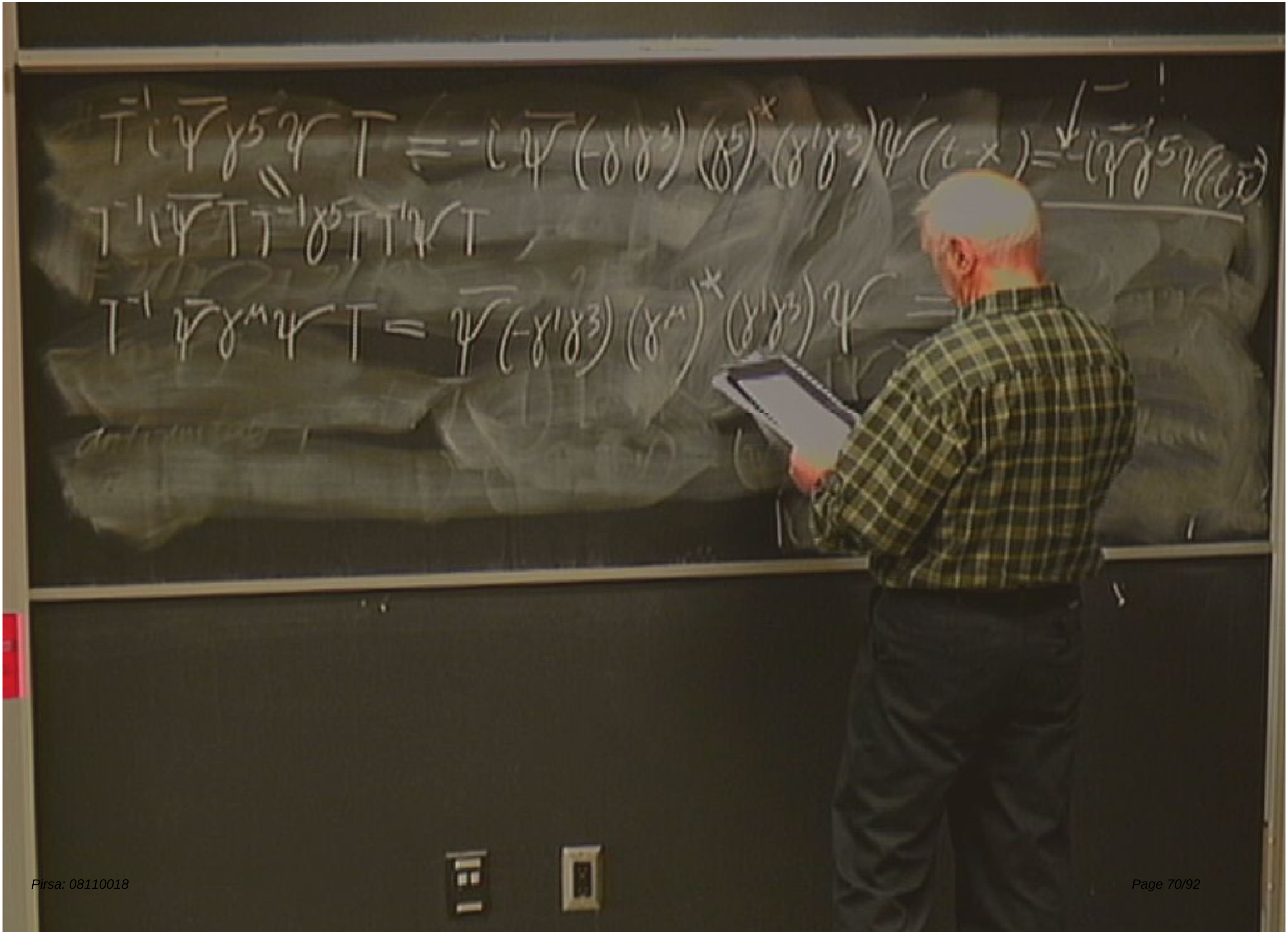

$T^{-1} i \bar{\psi} \gamma^5 \psi T$

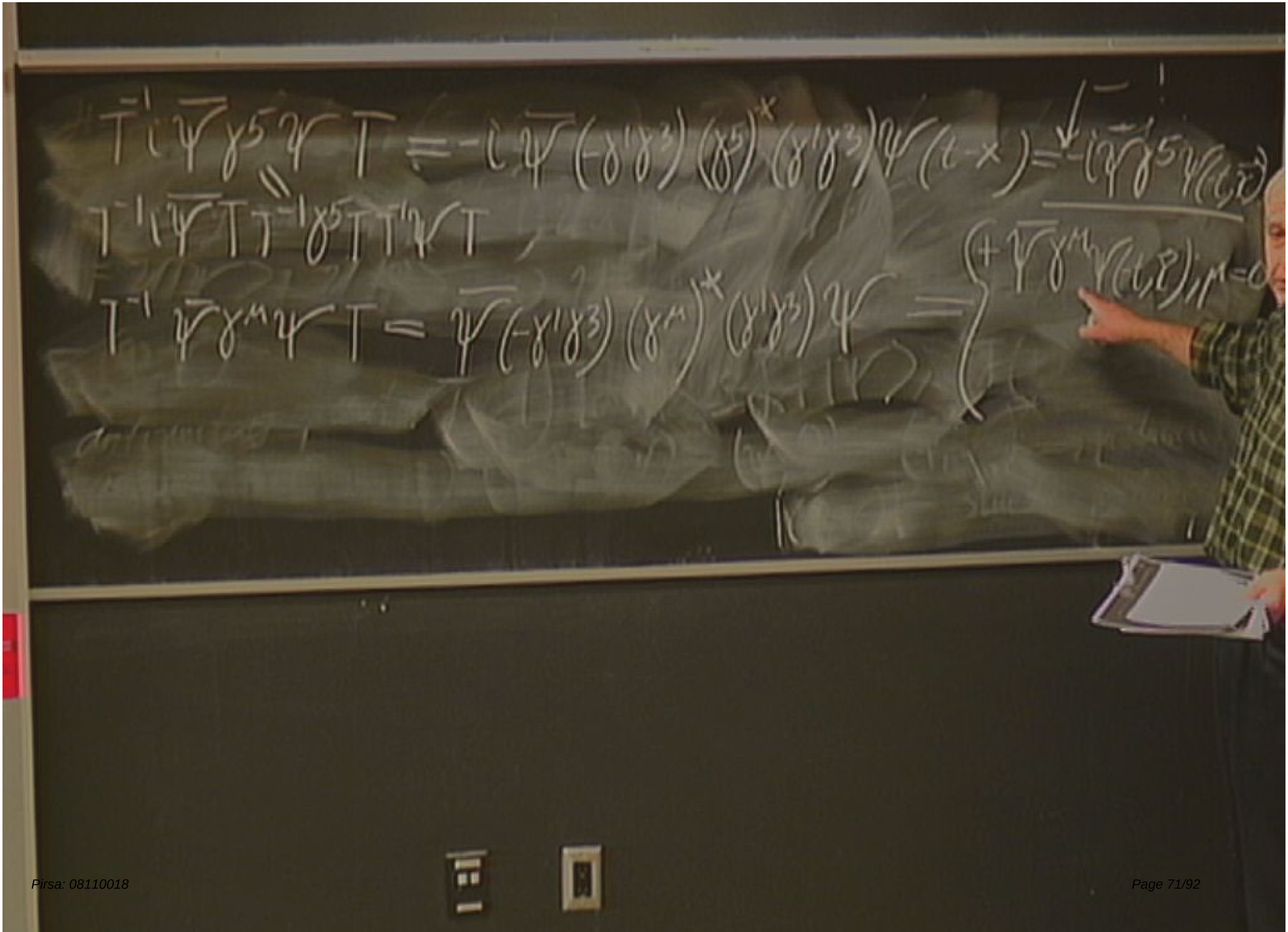

T⁻¹ i ψ̄ γ⁵ ψ T = −i ψ̄ (−γ¹γ³) (γ⁵)* (γ¹γ³) ψ (t, −x)

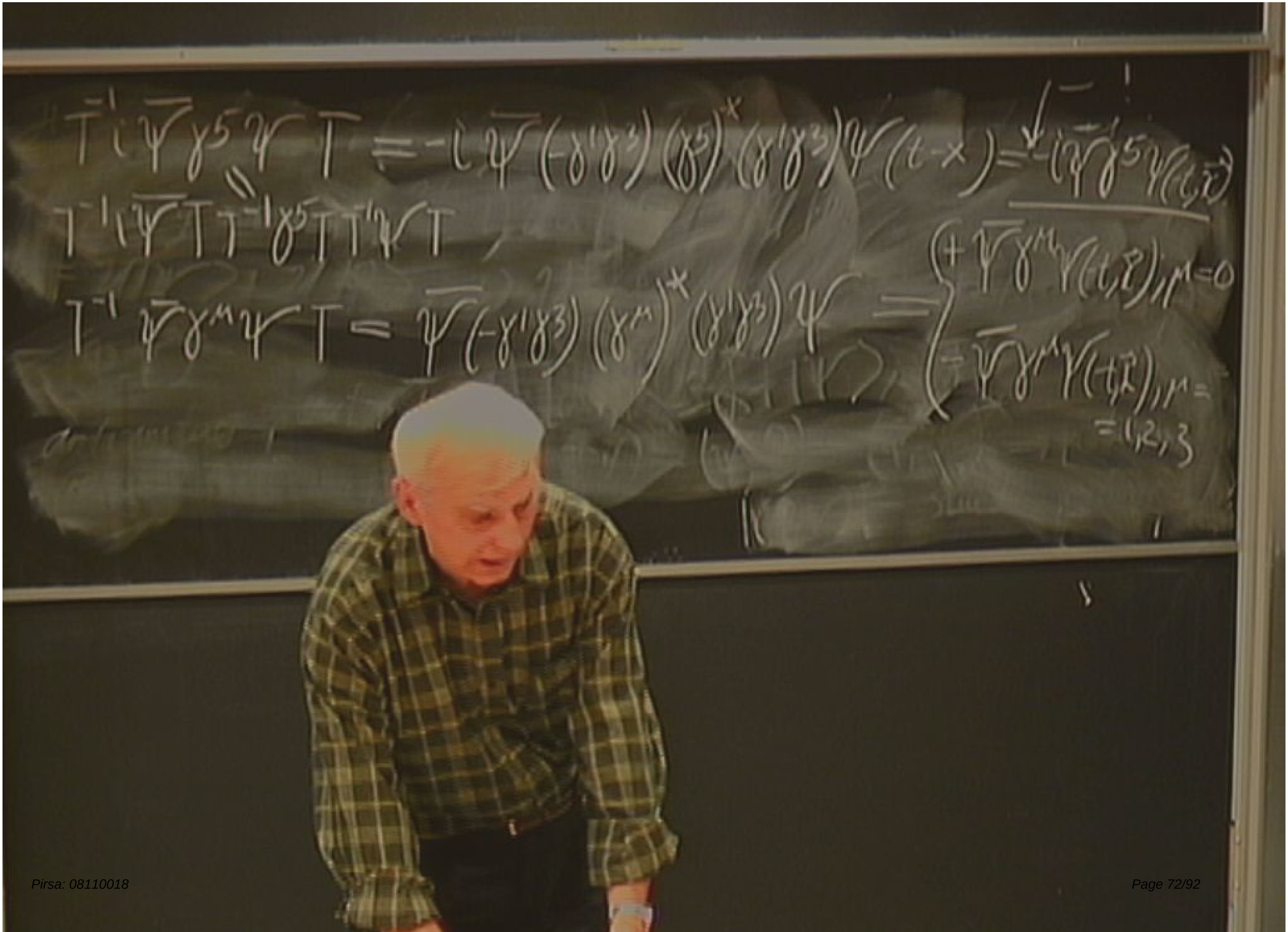

T⁻¹ i ψ̄ γ⁵ ψ T = −i ψ̄ (−γ¹γ³) (γ⁵)* (γ¹γ³) ψ (t, −x)
T⁻¹ i ψ̄ T T⁻¹ γ⁵ T T⁻¹ ψ T

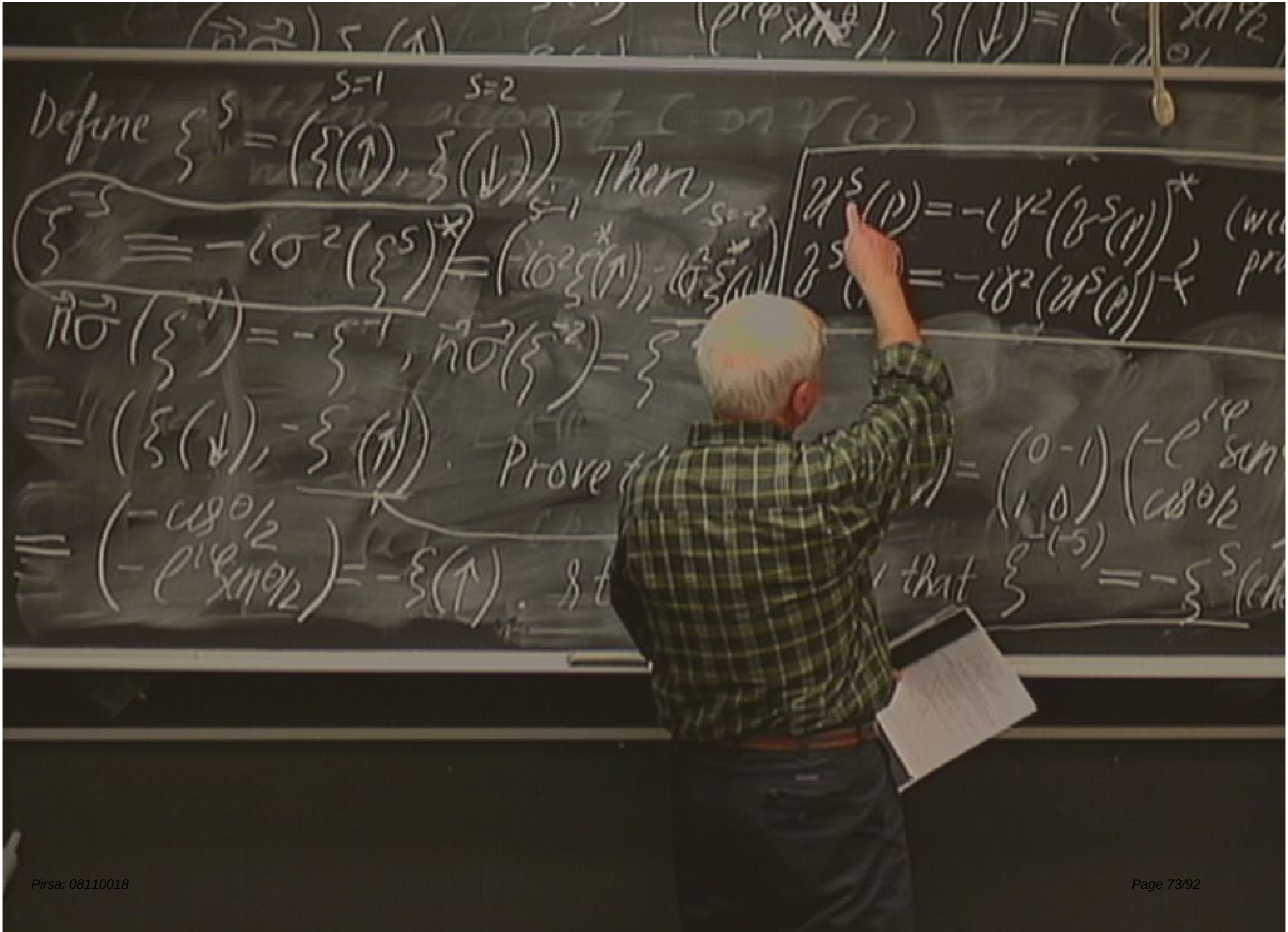


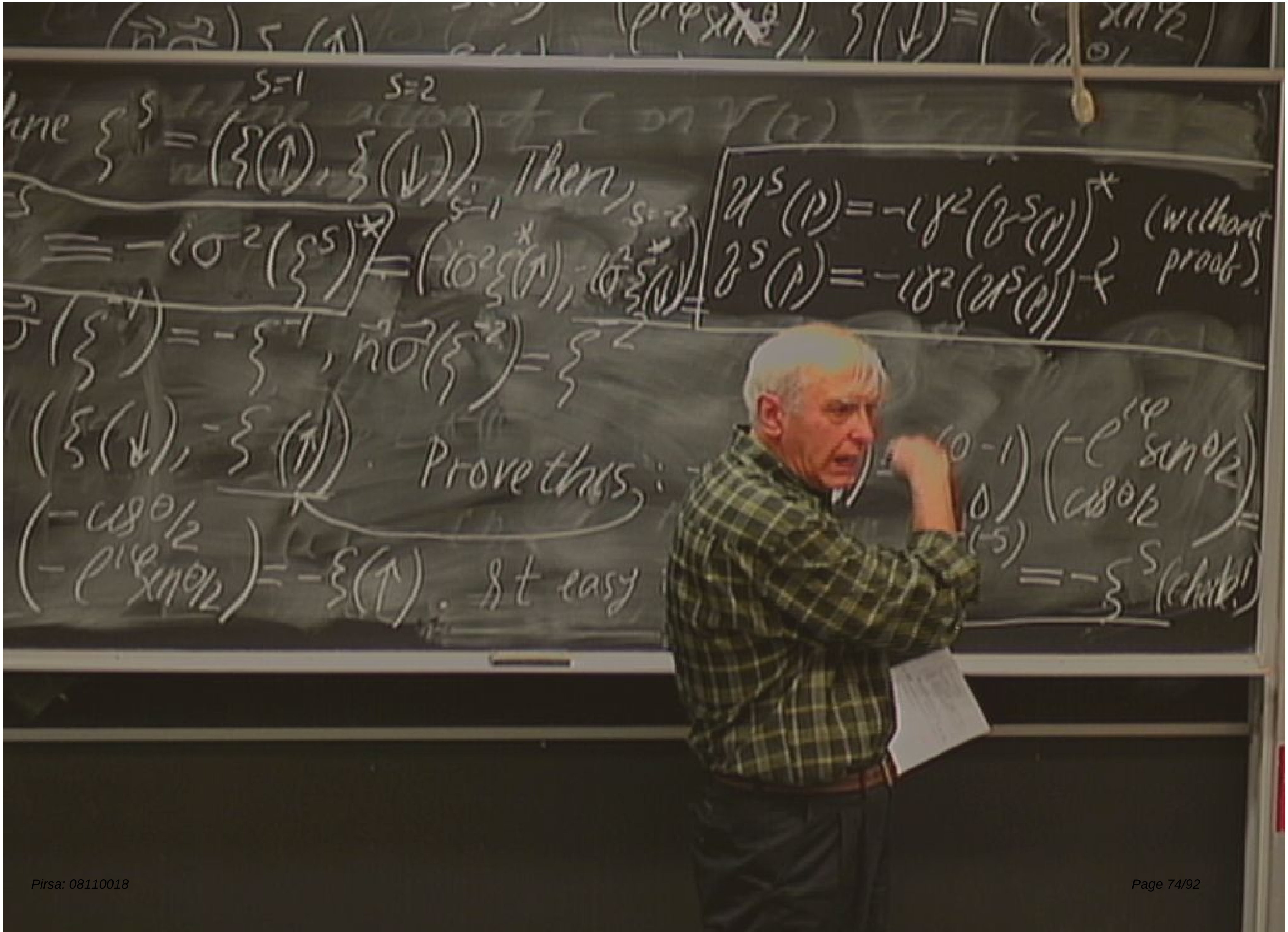

$T^{-1} i \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (-\gamma^1\gamma^3)(\gamma^5)^* (\gamma^1\gamma^3) \psi(t,-x) = -i \bar{\psi} \gamma^5 \psi(t,\vec{x})$
$T^{-1} i \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$
$T^{-1} \bar{\psi} \gamma^\mu \psi T = \bar{\psi} (-\gamma^1\gamma^3)(\gamma^\mu)^* (\gamma^1\gamma^3) \psi$


T⁻¹ i ψ̄ γ⁵ ψ T = −i ψ̄ (−γ¹γ³) (γ⁵)* (γ¹γ³) ψ(t, x) = −i ψ̄ γ⁵ ψ(−t, x)
T⁻¹ i ψ̄ T T⁻¹ γ⁵ T T⁻¹ ψ T
T⁻¹ ψ̄ γ^μ ψ T = ψ̄ (−γ¹γ³) (γ^μ)* (γ¹γ³) ψ =

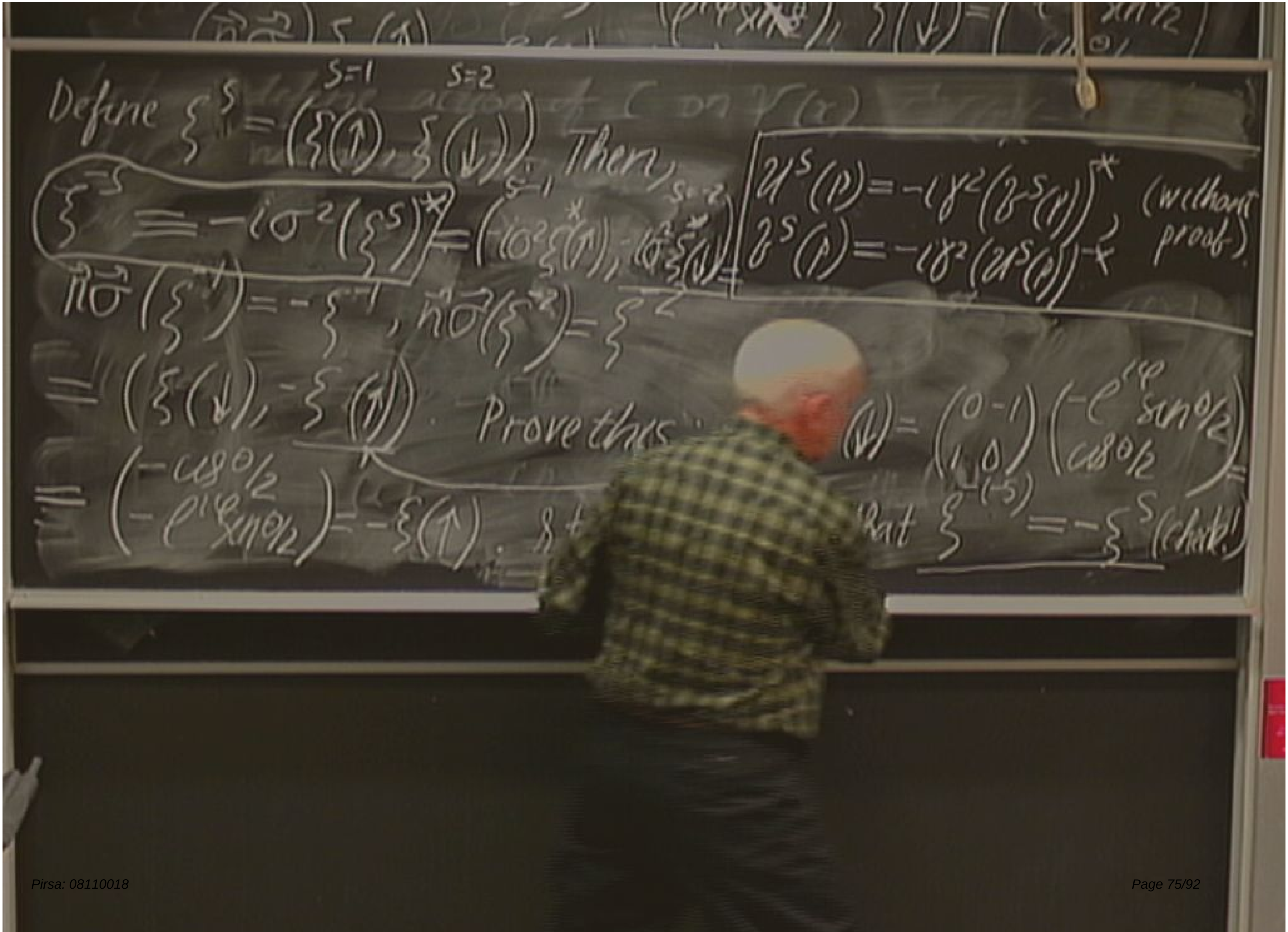

$T^{-1} i \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (-\gamma^1\gamma^3)(\gamma^5)^* (\gamma^1\gamma^3) \psi(t, -x) = -i \bar{\psi} \gamma^5 \psi(-t, \vec{x})$
$T^{-1} i \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$
$T^{-1} \bar{\psi} \gamma^\mu \psi T = \bar{\psi} (-\gamma^1\gamma^3)(\gamma^\mu)^* (\gamma^1\gamma^3) \psi = \{ + \bar{\psi} \gamma^\mu \psi(-t, \vec{x}), \mu = 0$




S=1
S=2
Define
Then,
Prove
that



Then
(without proof)
Prove this


s=1 s=2
Define ξ^s = (ξ(↑), ξ(↓)). Then,
ξ^{-s} = −iσ²(ξ^s)*
u^s(p) = −iγ²(v^s(p))*
v^s(p) = −iγ²(u^s(p))*
(without proof).
Prove this
(check!)

Define $\xi^s = \left(\underset{s=1}{\xi(\uparrow)}, \underset{s=2}{\xi(\downarrow)}\right)$. Then,

$$\tilde{\xi}^s = -i\sigma^2(\xi^s)^* = \left(\underset{s=1}{-i\sigma^2\xi^*(\uparrow)}, \underset{s=2}{-i\sigma^2\xi^*(\downarrow)}\right)$$

$$u^s(p) = -i\gamma^2\left(v^s(p)\right)^*$$
$$v^s(p) = -i\gamma^2\left(u^s(p)\right)^*$$
(without proof).

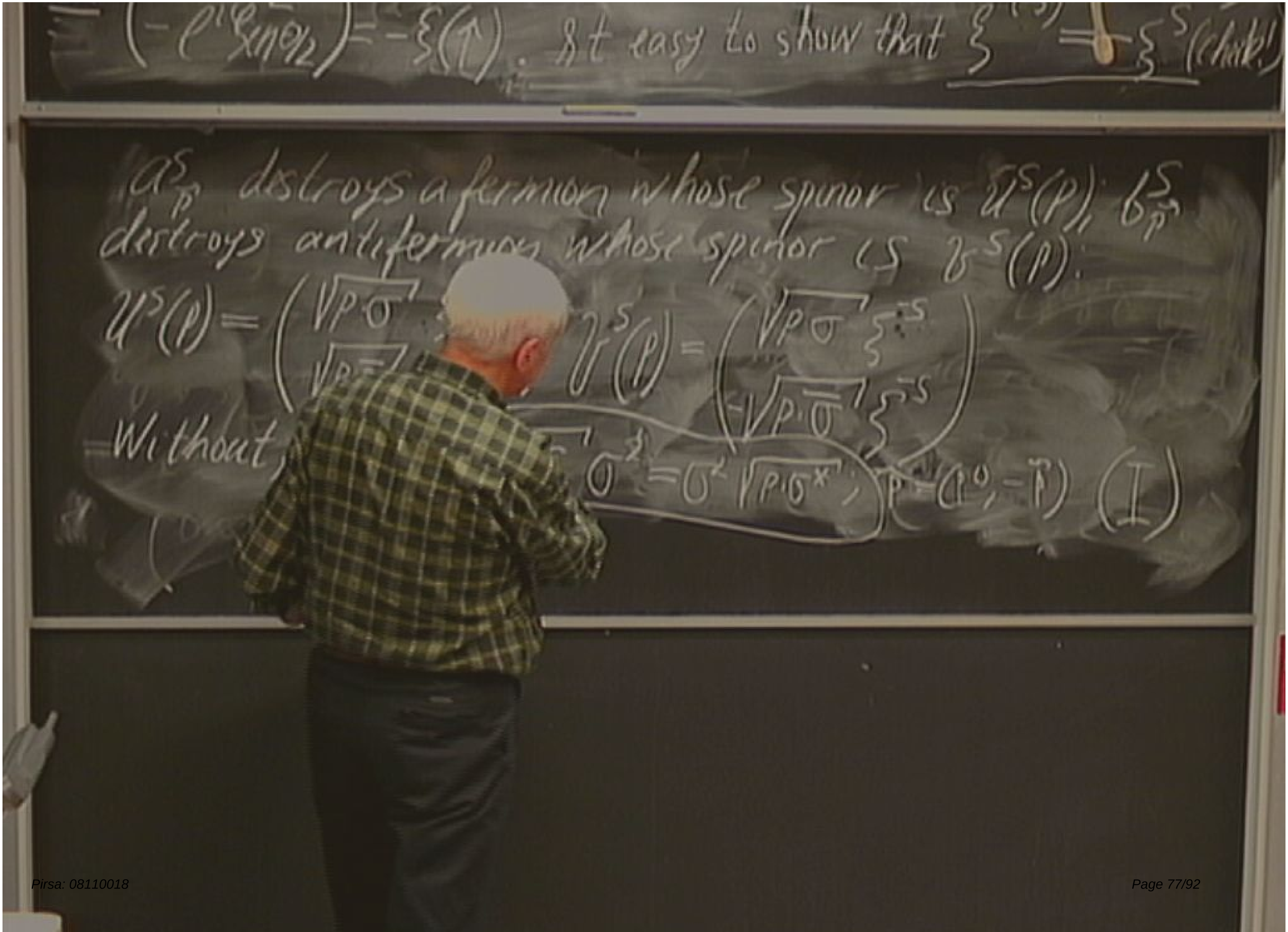
$$\hat{n}\cdot\vec{\sigma}\left(\tilde{\xi}^1\right) = -\tilde{\xi}^1, \quad \hat{n}\cdot\vec{\sigma}\left(\tilde{\xi}^2\right) = \tilde{\xi}^2$$

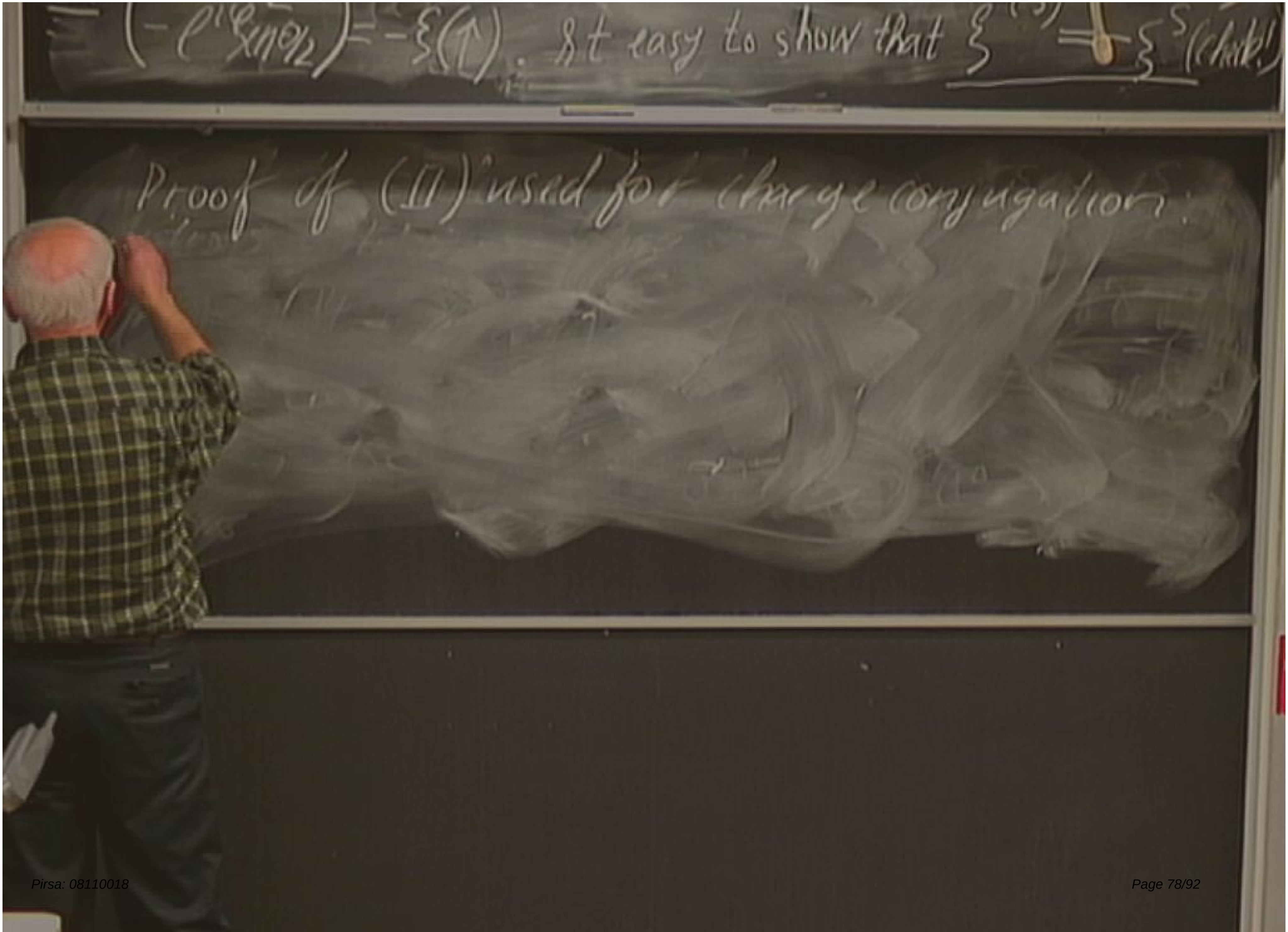
$$= \left(\xi(\downarrow), -\xi(\uparrow)\right).$$

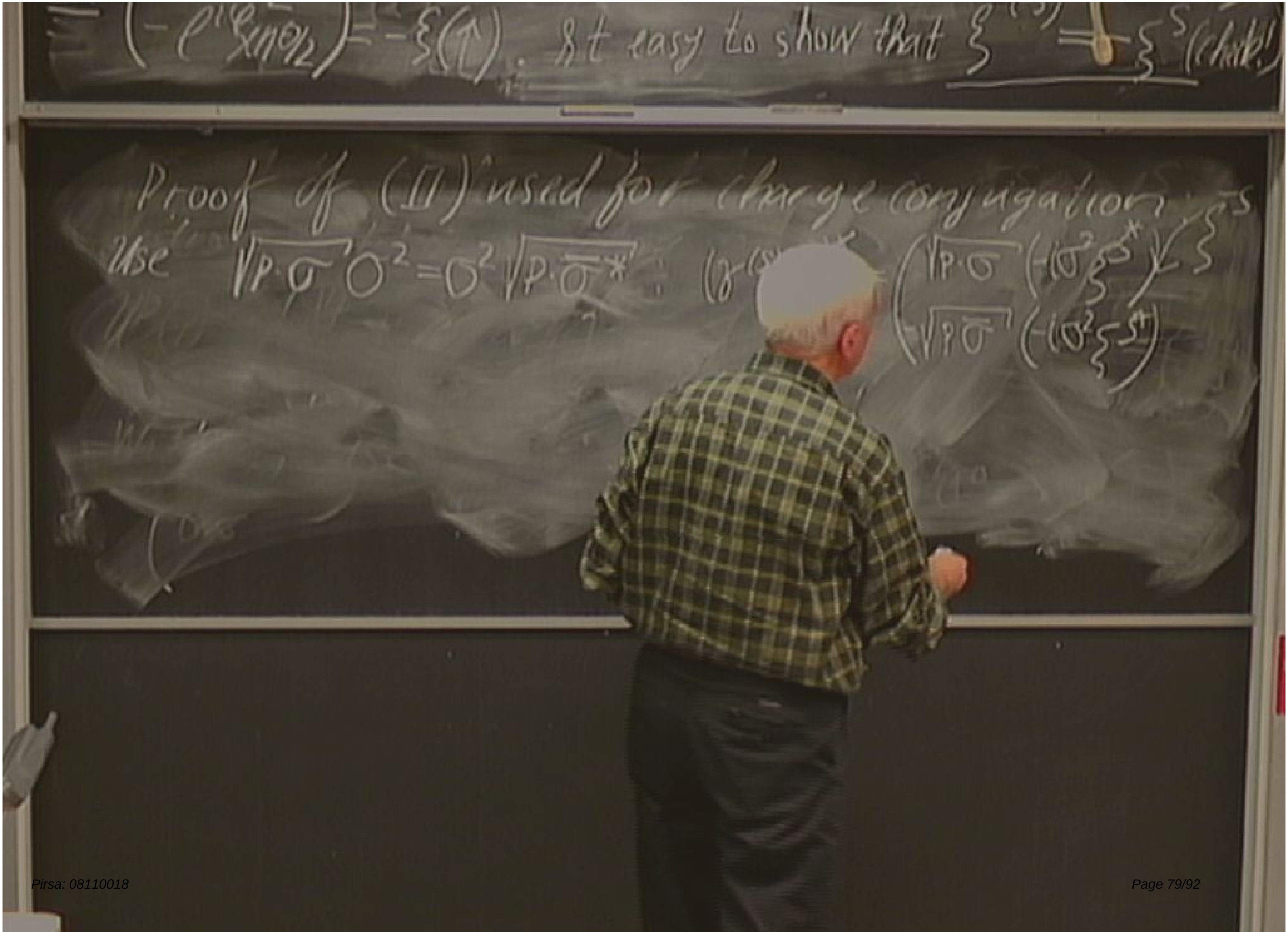
Prove this: $-i\sigma^2\xi^*(\downarrow) = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-e^{i\varphi}\sin\theta/2\\ \cos\theta/2\end{pmatrix}$

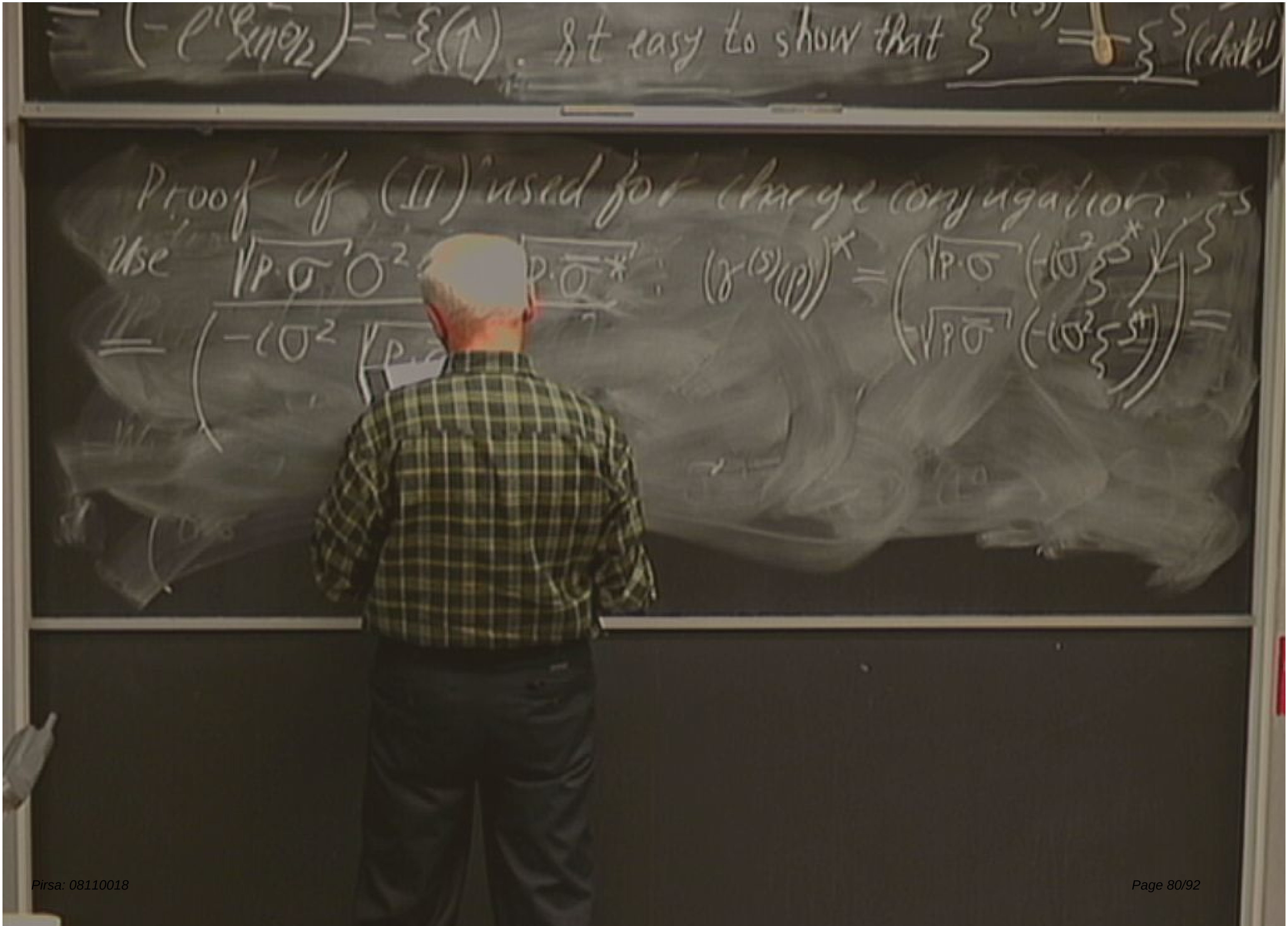
$$= \begin{pmatrix}-\cos\theta/2\\ -e^{i\varphi}\sin\theta/2\end{pmatrix} = -\xi(\uparrow).$$

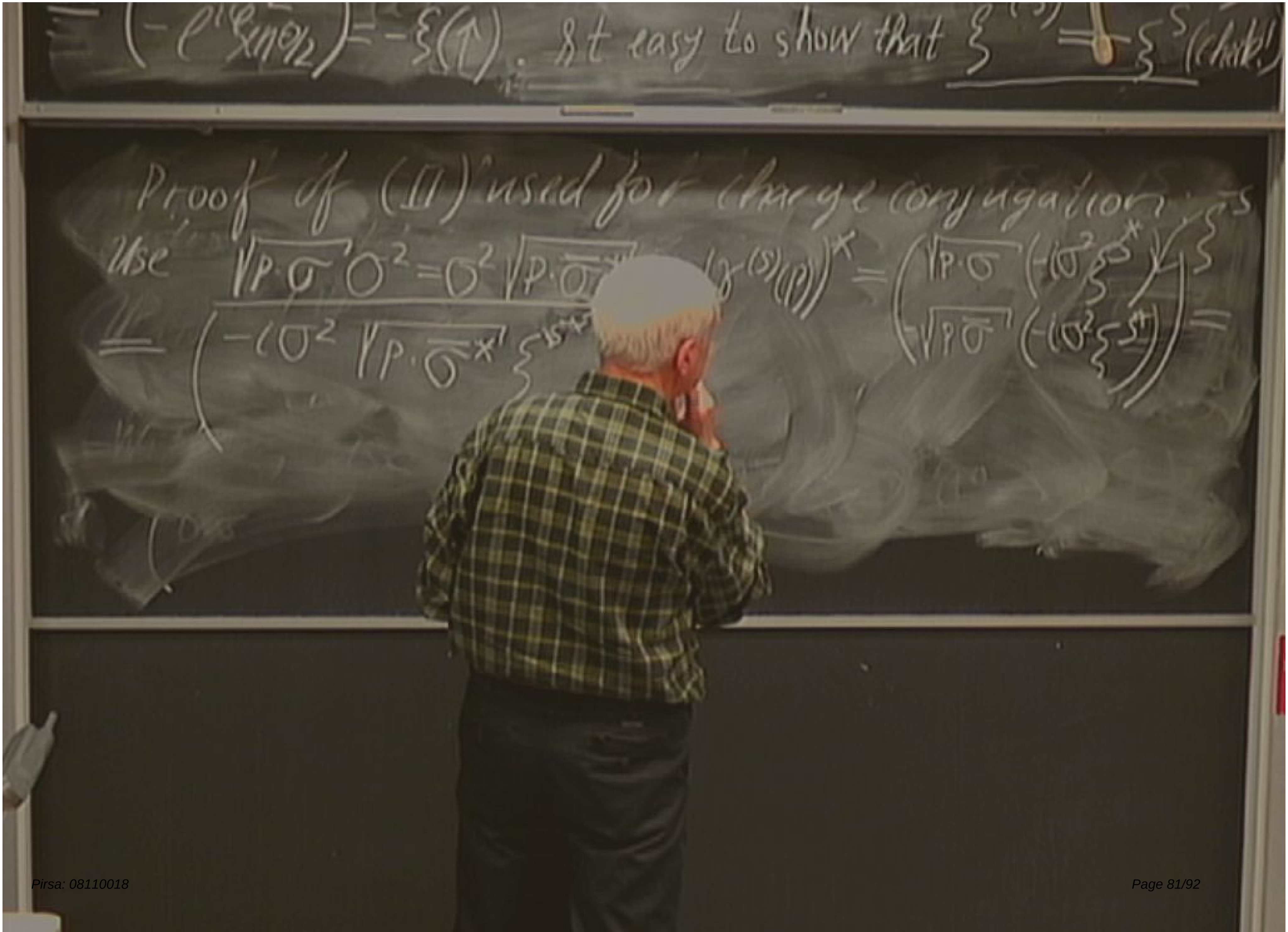
It easy to show that $\tilde{\xi}^{(-s)} = -\xi^s$ (check!)

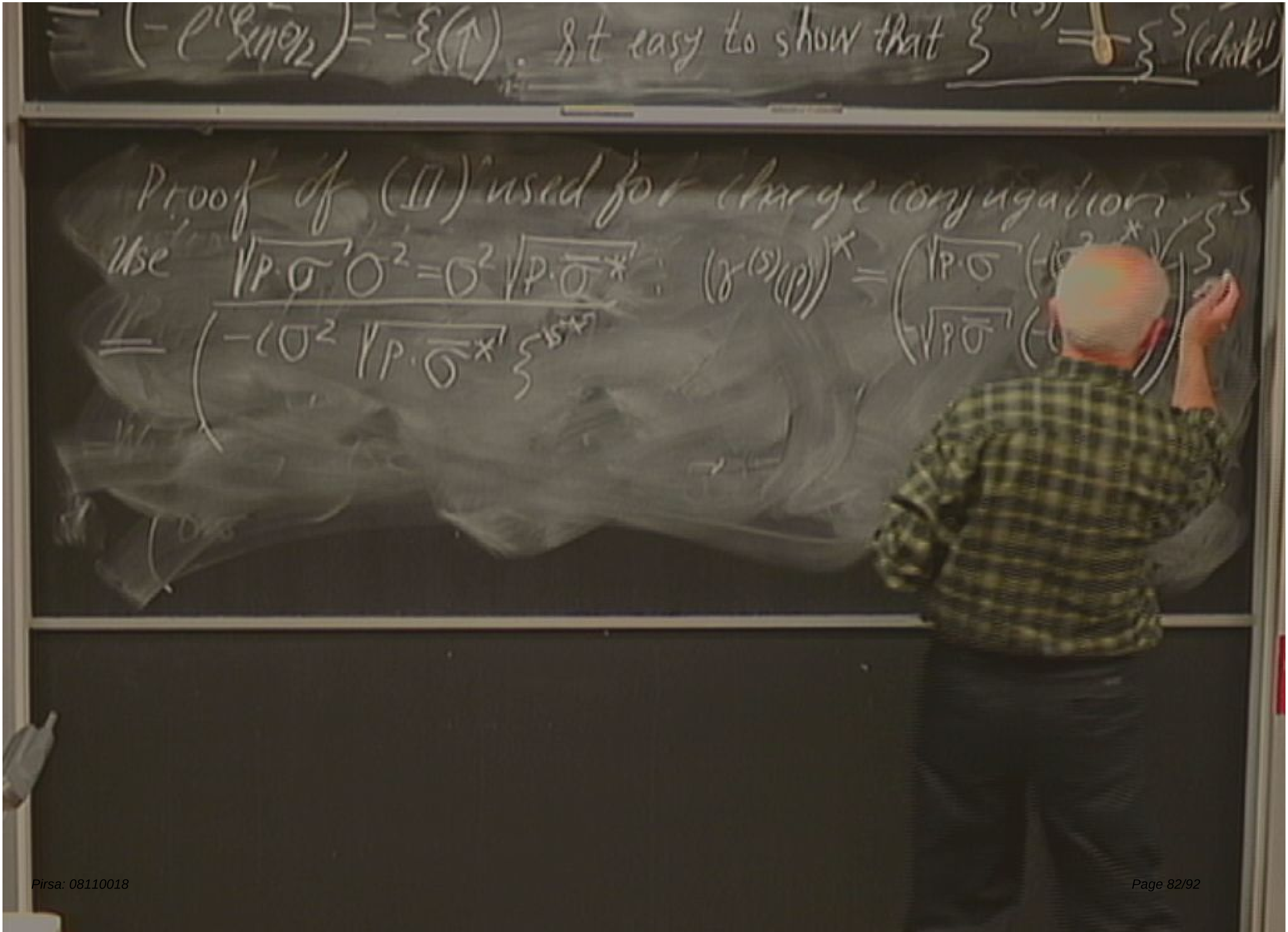

It easy to show that
(check!)
destroys a fermion whose spinor is
destroys antifermion whose spinor is
Without

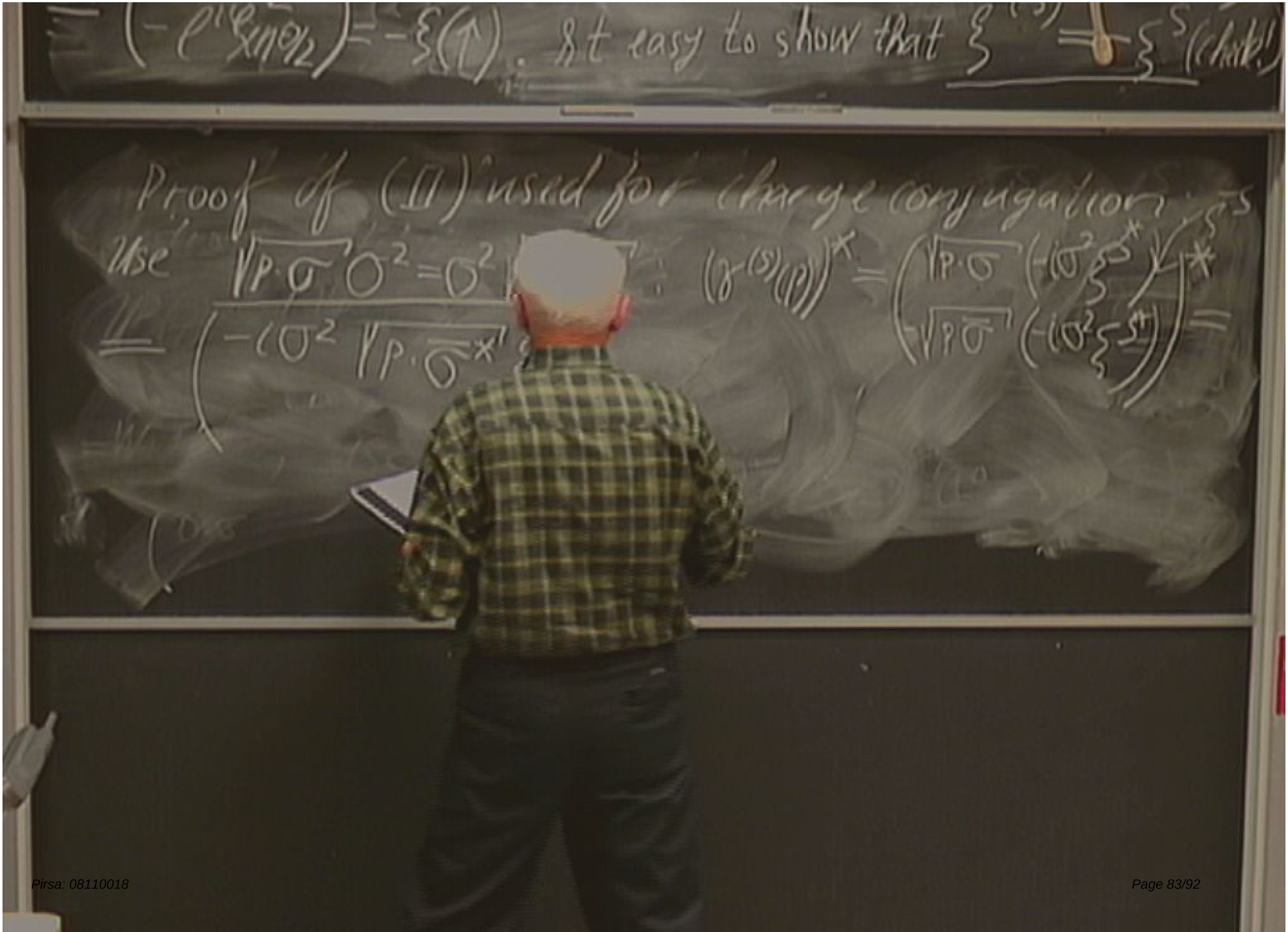

It easy to show that
(check!)
Proof of (II) used for charge conjugation:

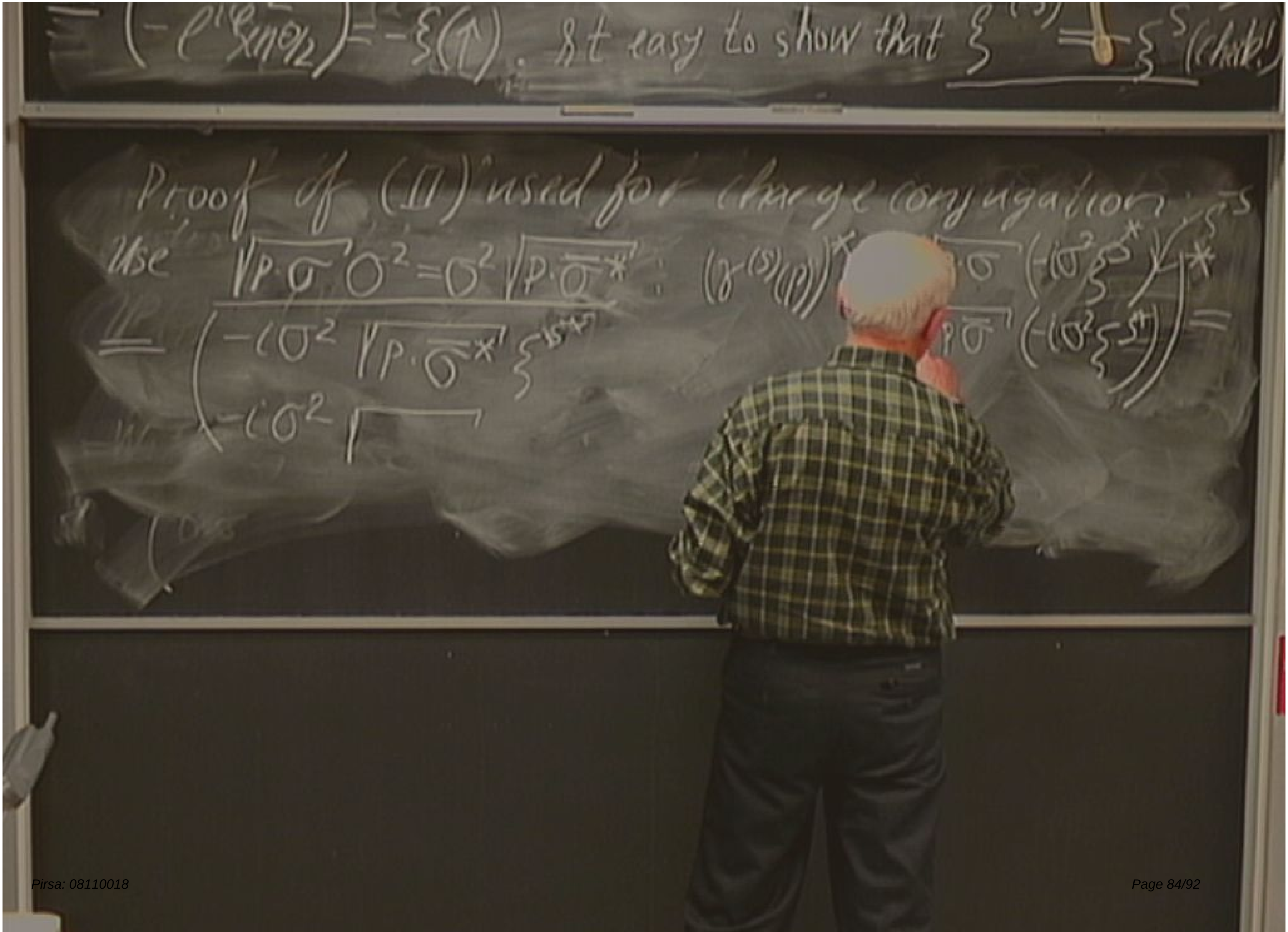

It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use

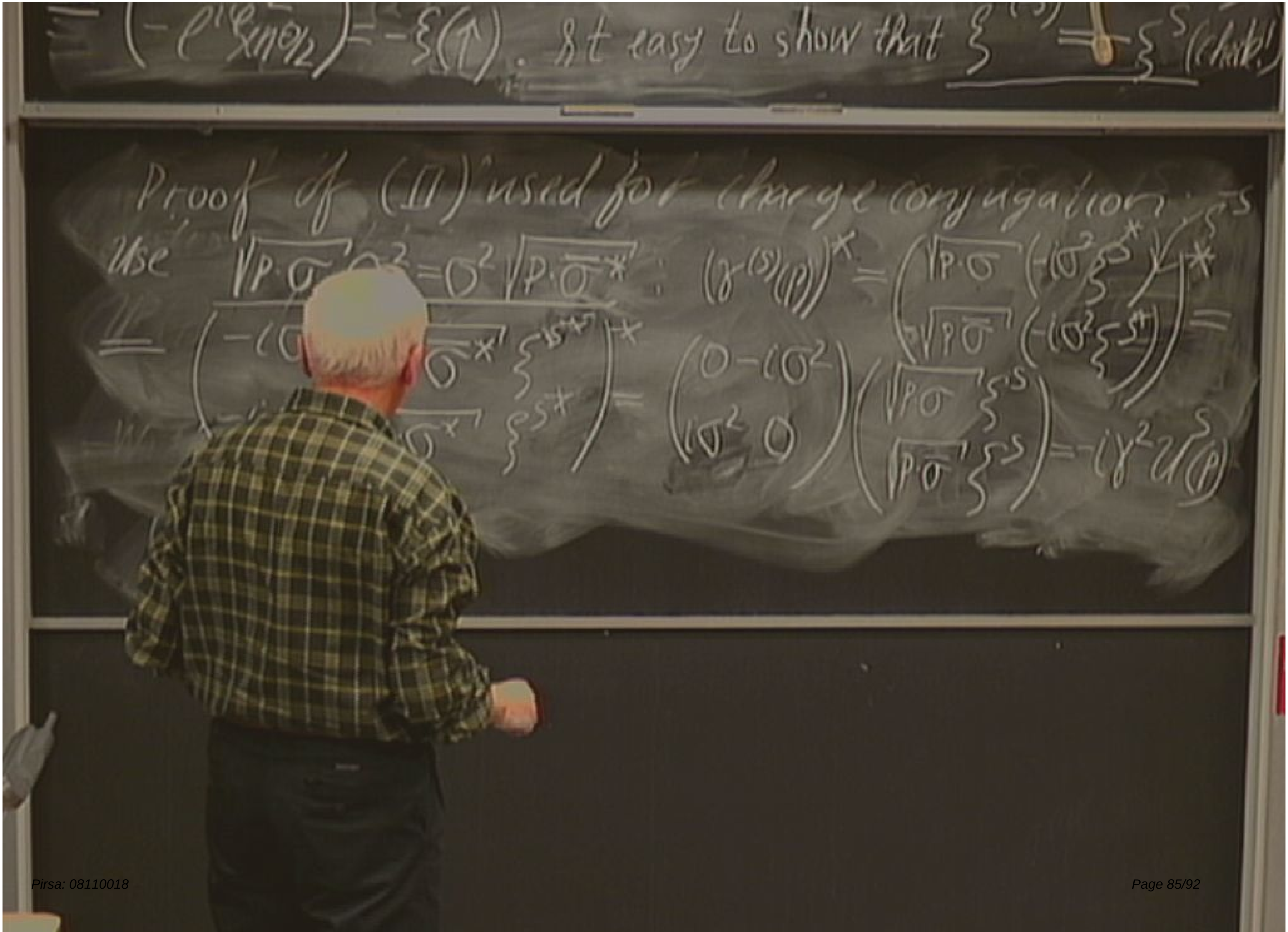

It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use

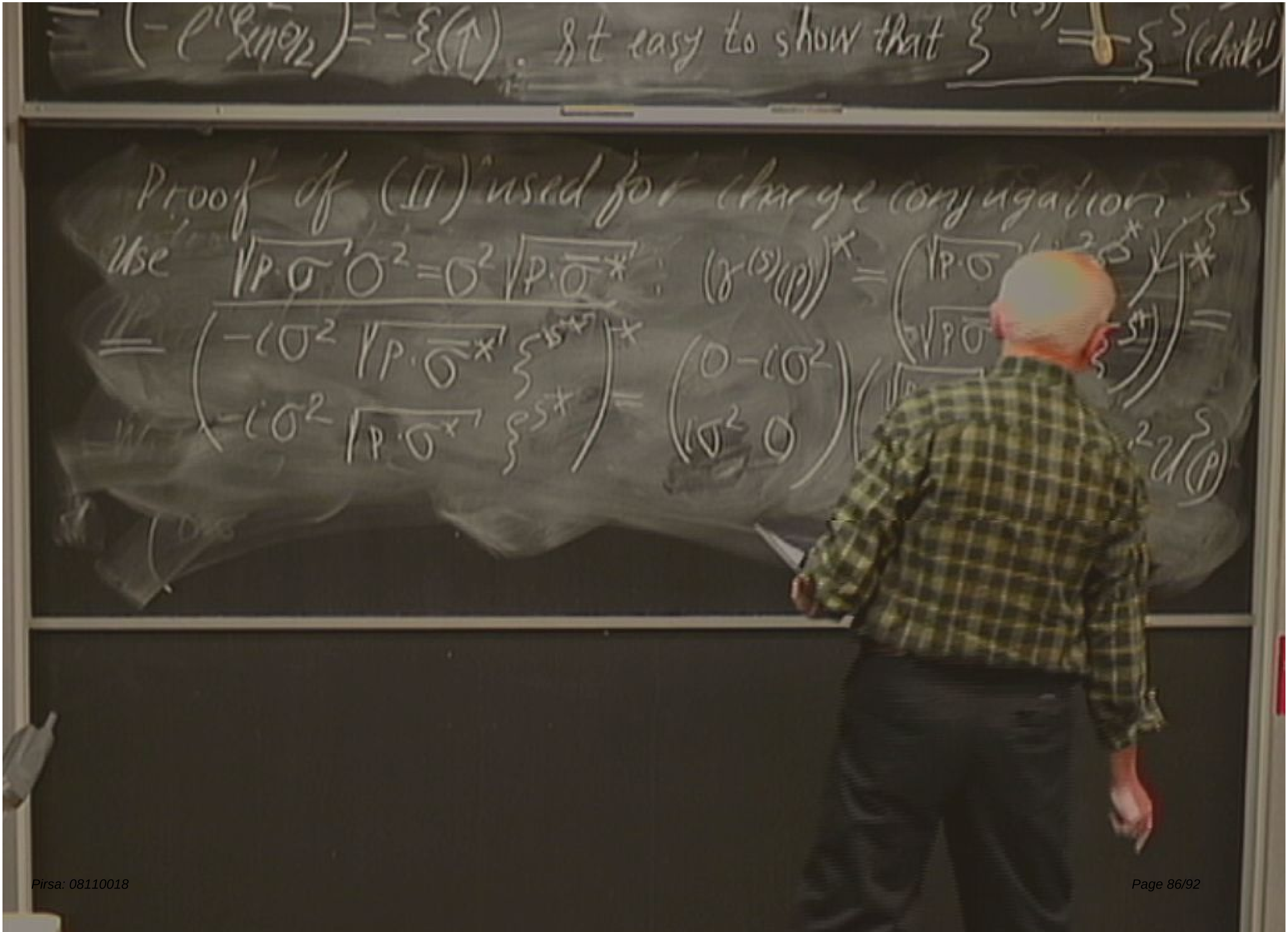

It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use

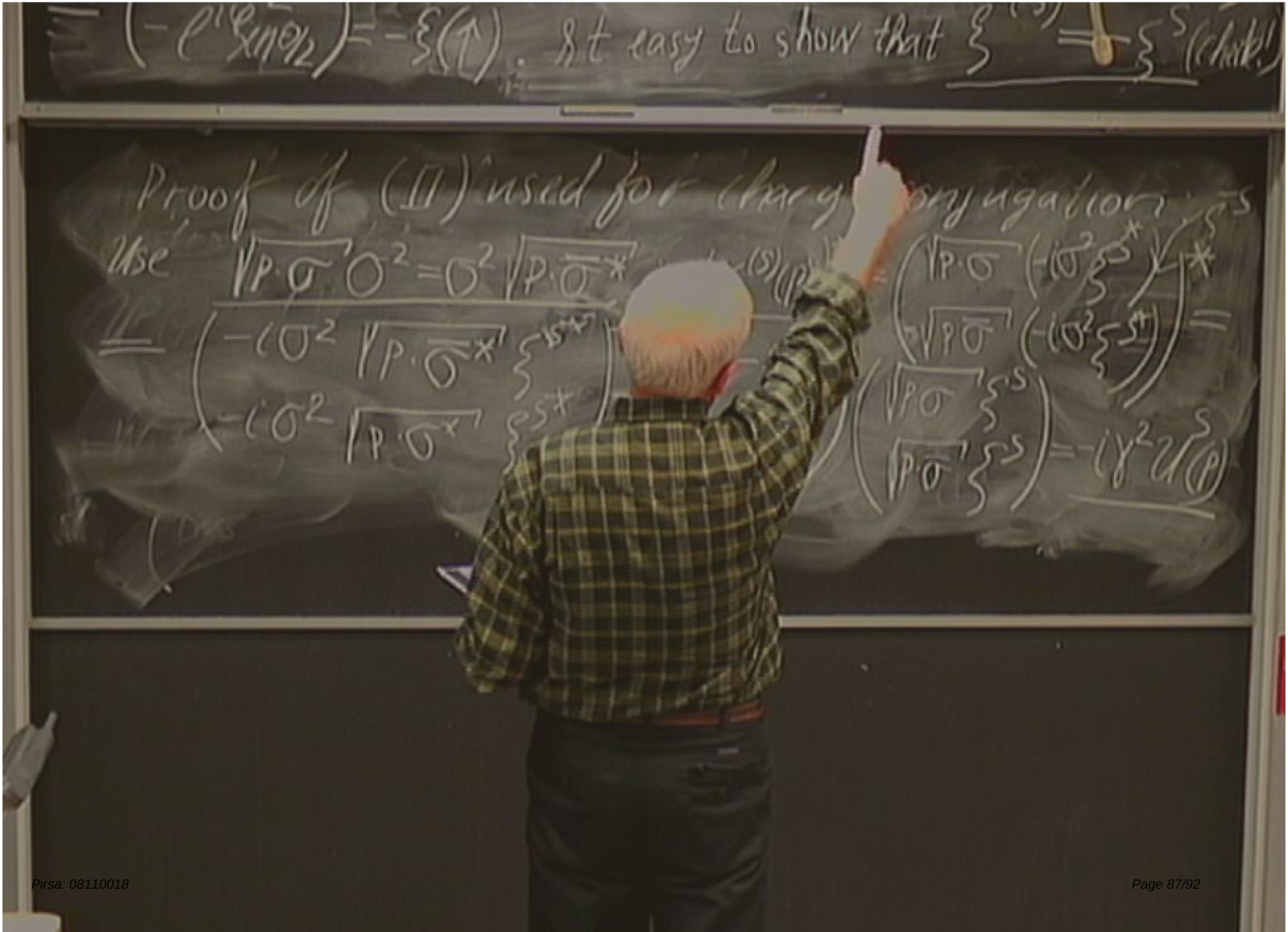

It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use

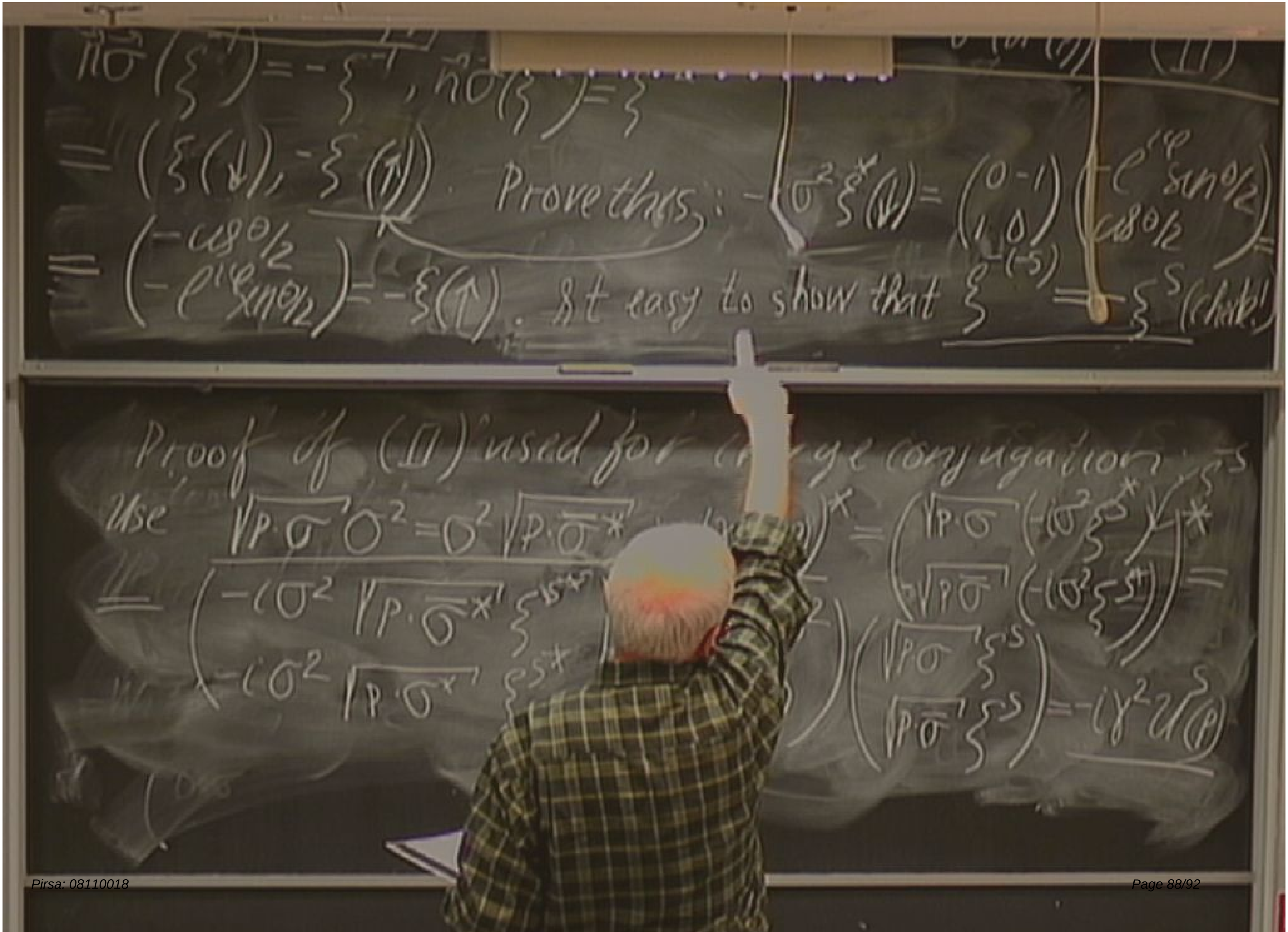

It easy to show that
(chalk!)
Proof of (II) used for charge conjugation
Use

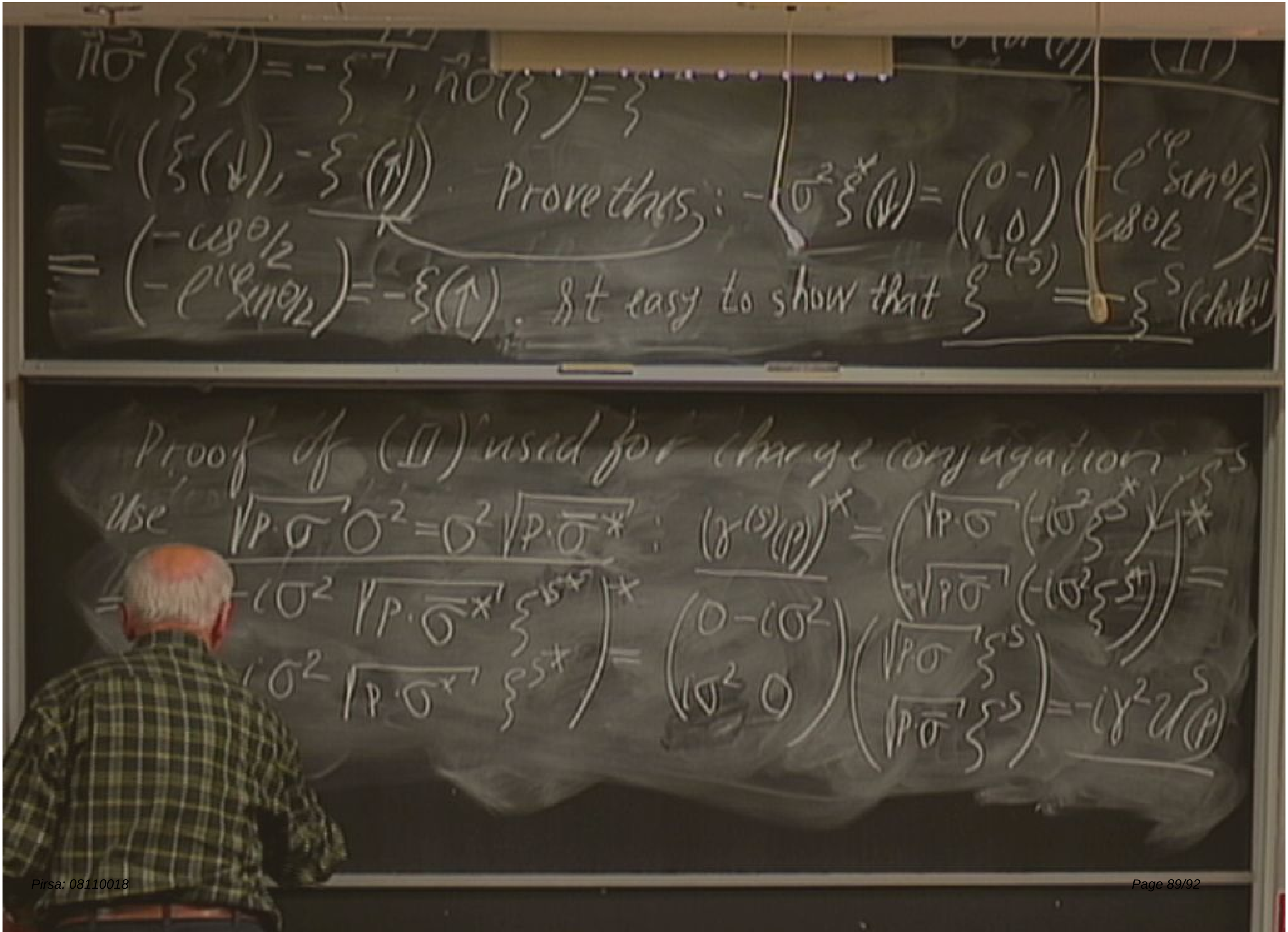

It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use

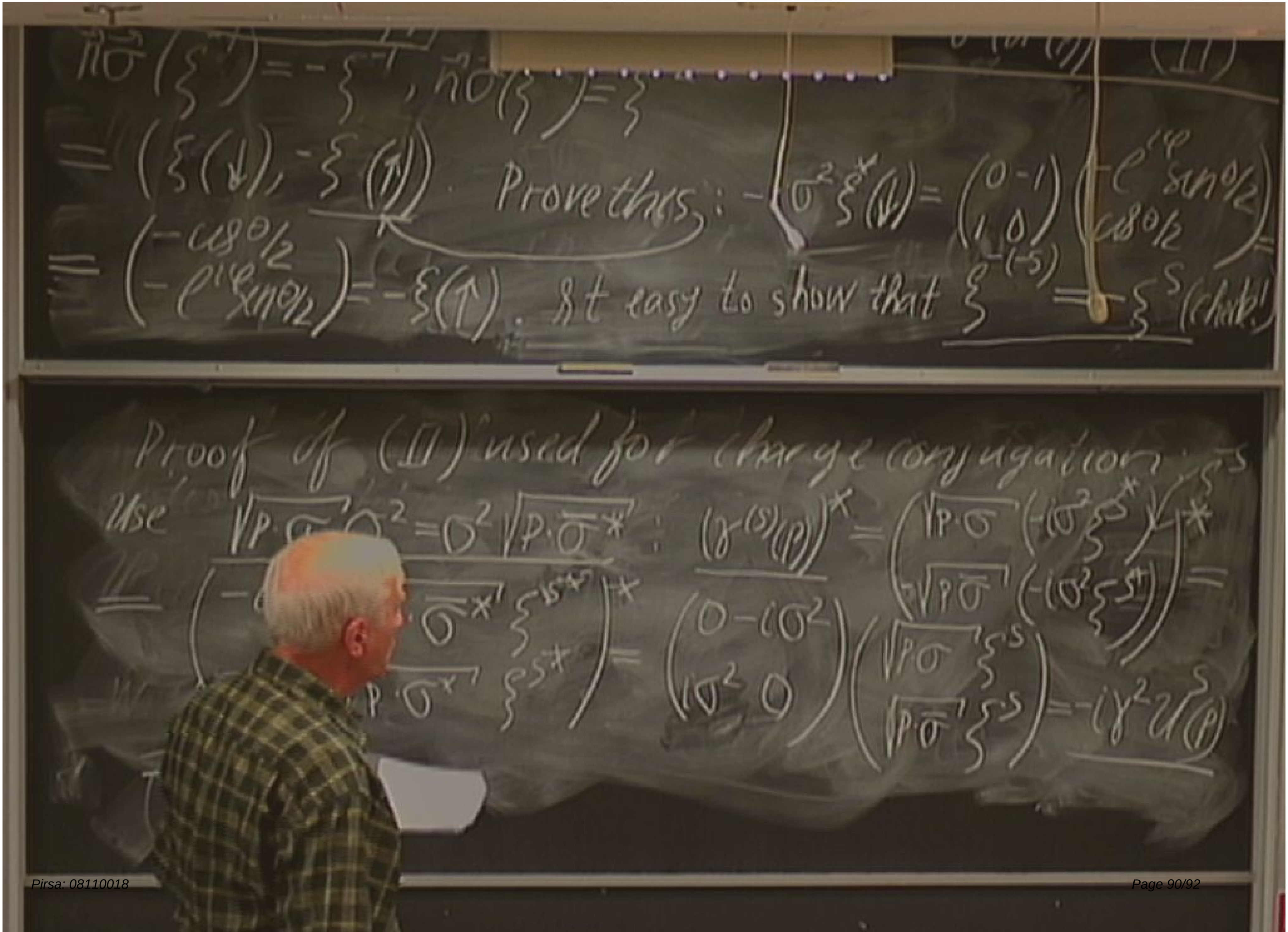

easy to show that
(check!)
Proof of (II) used for charge conjugation
Use


It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use


It easy to show that
Proof of (II) used for charge conjugation
Use


Prove this:
It easy to show that
Proof of (II) used for charge conjugation
Use


Prove this:
It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use


Prove this:
It easy to show that
(check!)
Proof of (II) used for charge conjugation
Use

$\hat{n}\vec{\sigma}(\xi^{-s}) = -\xi^{-s}$, $\hat{n}\vec{\sigma}(\xi^{s}) = \xi^{s}$

$= (\xi(\downarrow), -\xi(\uparrow))$. Prove this: $-i\sigma^2\xi^*(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -e^{i\varphi}\sin\theta/2 \\ \cos\theta/2 \end{pmatrix} =$

$= \begin{pmatrix} -\cos\theta/2 \\ -e^{i\varphi}\sin\theta/2 \end{pmatrix} = -\xi(\uparrow)$. It easy to show that $\xi^{-(-s)} = \xi^{s}$ (check!)

Proof of (II) used for charge conjugation:

Use $\sqrt{p\cdot\sigma}\,\sigma^2 = \sigma^2\sqrt{p\cdot\bar{\sigma}^*}$: $(\gamma^{(s)}(p))^* = \begin{pmatrix} \sqrt{p\cdot\sigma}\,(-i\sigma^2\xi^{s*}) \\ \sqrt{p\cdot\bar{\sigma}}\,(-i\sigma^2\xi^{s*}) \end{pmatrix}^* =$

$= \begin{pmatrix} -i\sigma^2\sqrt{p\cdot\bar{\sigma}^*}\,\xi^{s*} \\ -i\sigma^2\sqrt{p\cdot\sigma^*}\,\xi^{s*} \end{pmatrix}^* = \begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^{s} \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^{s} \end{pmatrix} = -i\gamma^2 u(p)$

$\Rightarrow$ (II)

$\hat{n}\cdot\vec{\sigma}\,(\xi^{-s}) = -\xi^{-s}$, $\hat{n}\cdot\vec{\sigma}\,(\xi^{s}) = \xi^{s}$

$= (\xi(\downarrow), -\xi(\uparrow))$. Prove this: $-i\sigma^2\xi^*(\downarrow) = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-e^{i\varphi}\sin\theta/2\\ \cos\theta/2\end{pmatrix}^* =$

$= \begin{pmatrix}-\cos\theta/2\\ -e^{i\varphi}\sin\theta/2\end{pmatrix} = -\xi(\uparrow)$. It's easy to show that $\xi^{-(-s)} = -\xi^{s}$ (check!)

Proof of (II) used for charge conjugation: ξ^{s}

Use $\sqrt{p\cdot\sigma}\,\sigma^2 = \sigma^2\sqrt{p\cdot\bar{\sigma}^*}$: $\left(v^{(s)}(p)\right)^* = \begin{pmatrix}\sqrt{p\cdot\sigma}\,(-i\sigma^2\xi^{s*})\\ -\sqrt{p\cdot\bar{\sigma}}\,(-i\sigma^2\xi^{s*})\end{pmatrix}^* =$

$= \begin{pmatrix}-i\sigma^2\sqrt{p\cdot\bar{\sigma}^*}\,\xi^{s*}\\ -i\sigma^2\sqrt{p\cdot\sigma^*}\,\xi^{s*}\end{pmatrix}^* = \begin{pmatrix}0 & -i\sigma^2\\ i\sigma^2 & 0\end{pmatrix}\begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^{s}\\ \sqrt{p\cdot\bar{\sigma}}\,\xi^{s}\end{pmatrix} = -i\gamma^2 u^{s}(p)$

$\Rightarrow$ (III)