

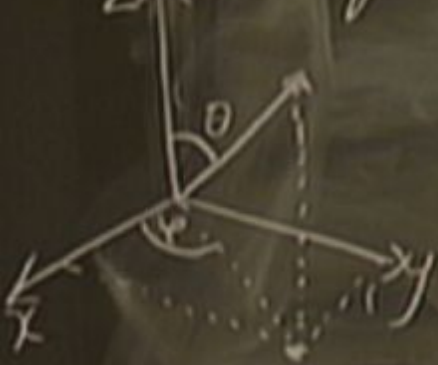
Title: Quantum Field Theory 1 - Lecture 11B

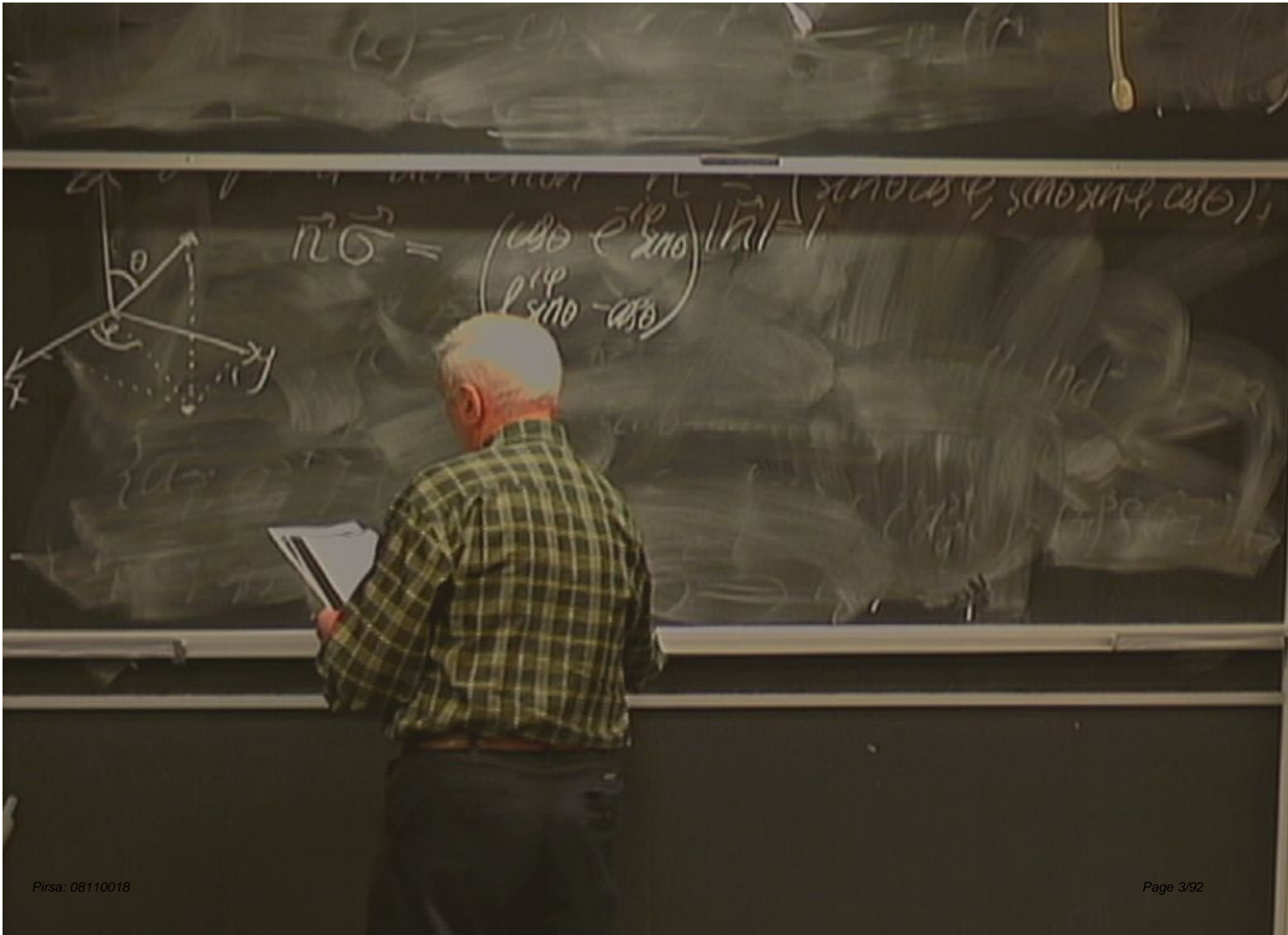
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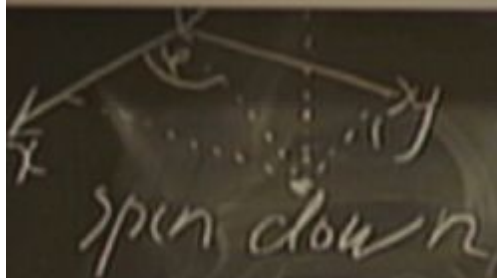
URL: <http://pirsa.org/08110018>

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

$S$  is associated with physical spin component along fixed direction  $\vec{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ ,  $|\vec{n}|=1$ .



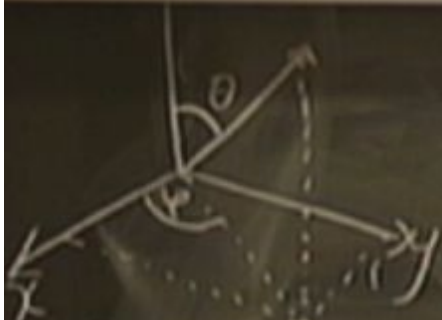




Two component spinors with spin up and

$$\chi(\uparrow) = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \end{pmatrix}, \quad \chi(\downarrow) = \begin{pmatrix} -e^{-i\phi} \sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} \end{pmatrix}$$





$$\vec{n} \cdot \vec{\sigma} = \begin{pmatrix} \cos\theta & e^{i\phi} \sin\theta \\ e^{-i\phi} \sin\theta & -\cos\theta \end{pmatrix} \quad |\vec{n}|=1$$

Two component spinors with spin up and spin down are:

$$\chi(\uparrow) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi} \sin\frac{\theta}{2} \end{pmatrix}, \quad \chi(\downarrow) = \begin{pmatrix} -e^{-i\phi} \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

$$(\vec{n} \cdot \vec{\sigma}) \chi(\uparrow) = +\chi(\uparrow), \quad (\vec{n} \cdot \vec{\sigma}) \chi(\downarrow) = -\chi(\downarrow)$$

$$(\vec{n} \cdot \vec{\sigma}) \begin{pmatrix} \uparrow \end{pmatrix} = +\begin{pmatrix} \uparrow \end{pmatrix}, \quad (\vec{n} \cdot \vec{\sigma}) \begin{pmatrix} \downarrow \end{pmatrix} = -\begin{pmatrix} \downarrow \end{pmatrix}$$

Let us define action of  $L$  on  $\psi(r)$   
 First moment theorem

$$\mathcal{L}^S(\rho) = -L^2 (\psi^S(\rho))^* \quad (\text{without proof})$$

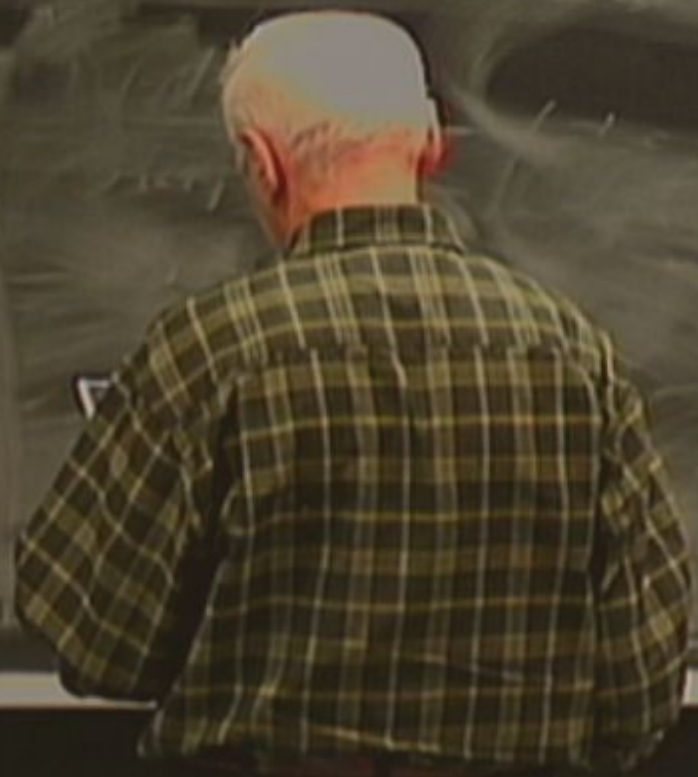
$$\psi^S(\rho) = \frac{1}{r^2} (\mathcal{L}^S(\rho))^*$$



$$(\vec{n} \cdot \vec{\sigma}) \xi(\uparrow) = +\xi(\uparrow), \quad (\vec{n} \cdot \vec{\sigma}) \xi(\downarrow) = -\xi(\downarrow)$$

Let us define action of  $L$  on  $\psi(x)$   
 First moment calculation

$$\begin{aligned} \mathcal{U}^S(p) &= -i\gamma^2 (\mathcal{Z}^S(p))^* && \text{(without proof)} \\ \mathcal{Z}^S(p) &= -i\gamma^2 (\mathcal{U}^S(p)) \end{aligned}$$



$$(\vec{n} \cdot \vec{\sigma}) \xi(\uparrow) = +\xi(\uparrow), \quad (\vec{n} \cdot \vec{\sigma}) \xi(\downarrow) = -\xi(\downarrow)$$

Define  $\xi^s = \xi(\uparrow)$  where action of  $C$  on  $\psi(x)$

$$\begin{aligned} \mathcal{U}^s(p) &= -i\gamma^2 (\mathcal{Z}^s(p))^* \quad (\text{without proof}) \\ \mathcal{Z}^s(p) &= -i\gamma^2 (\mathcal{U}^s(p)) \end{aligned}$$



$$(\vec{n} \cdot \vec{\sigma}) \xi(\uparrow) = +\xi(\uparrow), \quad (\vec{n} \cdot \vec{\sigma}) \xi(\downarrow) = -\xi(\downarrow)$$

Define  $\xi^s = \begin{pmatrix} \xi(\uparrow) \\ \xi(\downarrow) \end{pmatrix}$  <sup>s=1</sup> <sup>s=2</sup> There,

$$\begin{aligned} \mathcal{U}^s(p) &= -i\gamma^2 (\mathcal{Z}^s(p))^\dagger \quad (\text{without proof}) \\ \mathcal{Z}^s(p) &= -i\gamma^2 (\mathcal{U}^s(p))^\dagger \end{aligned}$$

$$(\vec{n} \cdot \vec{\sigma}) \xi(\uparrow) = +\xi(\uparrow), \quad (\vec{n} \cdot \vec{\sigma}) \xi(\downarrow) = -\xi(\downarrow)$$

Define  $\xi^s = (\xi(\uparrow), \xi(\downarrow))$  for  $s=1, 2$ . There,  $\xi^{-s} = i\sigma^2 (\xi^s)^*$ . Why?

$$\begin{aligned} \mathcal{U}^s(p) &= -i\gamma^2 (\mathcal{Z}^s(p))^* \\ \mathcal{Z}^s(p) &= -i\gamma^2 (\mathcal{U}^s(p))^* \end{aligned} \quad (\text{without proof})$$



$$(\vec{n} \cdot \vec{\sigma}) \xi(\uparrow) = +\xi(\uparrow), \quad (\vec{n} \cdot \vec{\sigma}) \xi(\downarrow) = -\xi(\downarrow)$$

Define  $\xi^s = (\xi(\uparrow), \xi(\downarrow))$  for  $s=1, 2$ . Then,

$$\xi^{-s} \equiv -i\sigma^2 (\xi^s)^* \quad \text{Why?}$$

$$\vec{n} \cdot \vec{\sigma} (\xi^{-s}) = -\xi^{-s}, \quad (\xi^{-s})^* = \xi^{-2s}$$

$$\begin{aligned} \chi^s(\rho) &= -i\gamma^2 (\xi^s(\rho))^* \\ \xi^s(\rho) &= -i\gamma^2 (\chi^s(\rho))^* \end{aligned} \quad (\text{without proof})$$



Define  $\xi^s = (\xi^s(\uparrow), \xi^s(\downarrow))$  <sup>S=1</sup> <sup>S=2</sup> action of  $C$  on  $\mathcal{P}(x)$ . Then,  $\xi^{-s} \equiv -i\sigma^2 (\xi^s)^* = (-i\sigma^2 \xi^s(\uparrow)^*, -i\sigma^2 \xi^s(\downarrow)^*)$ .  $\vec{n}\sigma(\xi^{-1}) = -\xi^{-1}$ ,  $\vec{n}\sigma(\xi^{-2}) = \xi^{-2}$ .

$$\begin{cases} \mathcal{U}^s(\rho) = -i\gamma^2 (\mathcal{U}^s(\rho))^* \\ \mathcal{V}^s(\rho) = -i\gamma^2 (\mathcal{V}^s(\rho))^* \end{cases} \quad (\text{without proof})$$

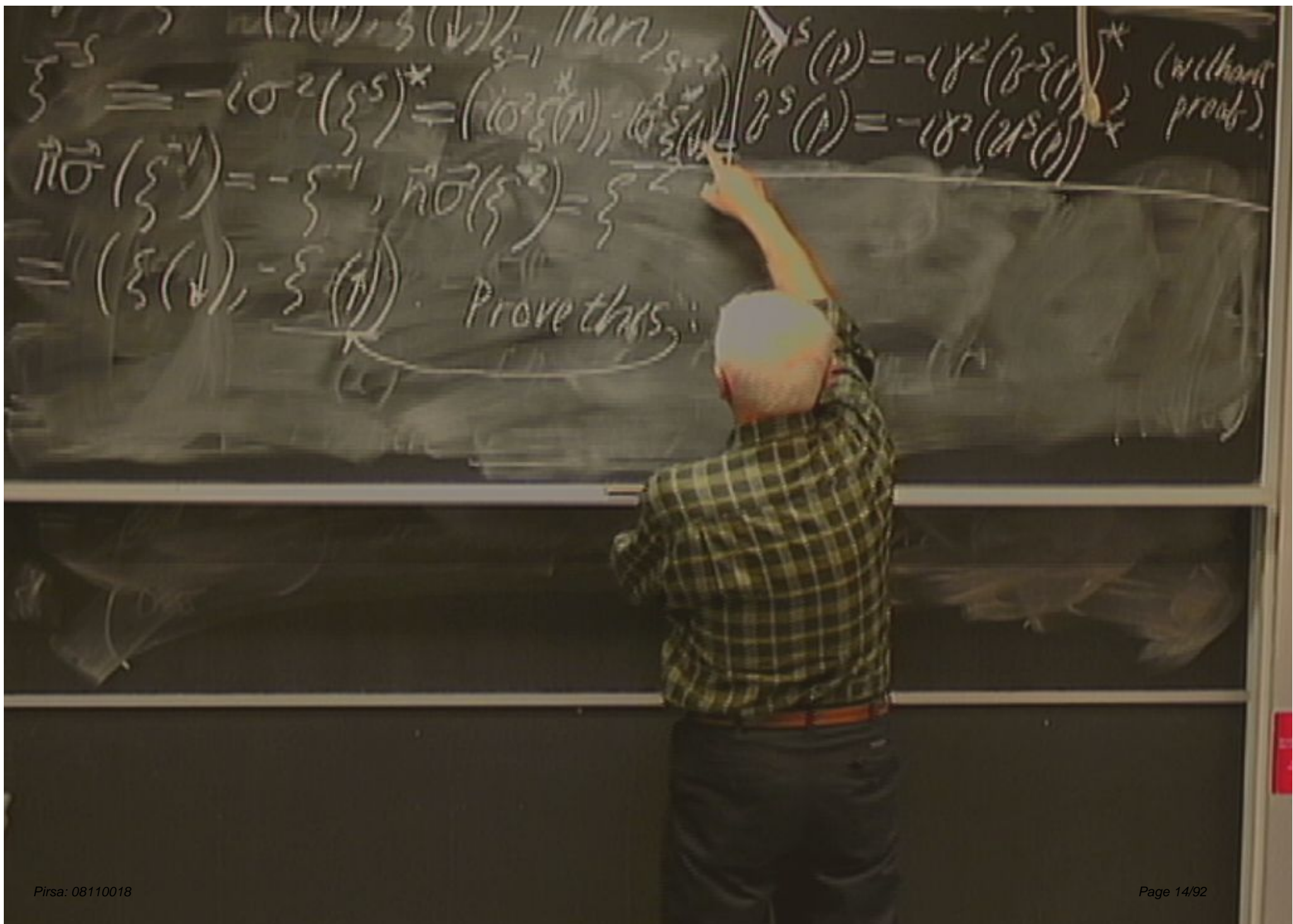
Define  $\xi^s = (\xi^s(\uparrow), \xi^s(\downarrow))$  for  $s=1, 2$ . Then,

$$\xi^{-s} \equiv -i\sigma^2 (\xi^s)^* = (-i\sigma^2 \xi^s(\uparrow)^*, -i\sigma^2 \xi^s(\downarrow)^*)$$

$$\vec{n}_\sigma(\xi^{-1}) = -\xi^{-1}, \quad \vec{n}_\sigma(\xi^{-2}) = \xi^{-2}$$

$$\begin{aligned} \mathcal{U}^s(\rho) &= -i\gamma^2 (\mathcal{U}^s(\rho))^* \\ \mathcal{V}^s(\rho) &= -i\gamma^2 (\mathcal{V}^s(\rho))^* \end{aligned} \quad (\text{without proof})$$





$\xi^{-s} = -i\sigma^2(\xi^s)^* = (i\sigma^2\xi(\uparrow), -i\sigma^2\xi(\downarrow))^*$

$\vec{\pi}_0(\xi^{-1}) = -\xi^{-1}, \vec{\pi}_0(\xi^{*2}) = \xi^2$

$= (\xi(\downarrow), -\xi(\uparrow))$

There,  $s-1$

$\vec{\pi}_0^s(P) = -i\gamma^2(\xi^s(P))^*$

$\vec{\pi}_0^s(P) = -i\gamma^2(\xi^s(P))^*$

(without proofs)

Prove this:



Define  $\xi^s = (\xi(\uparrow), \xi(\downarrow))$ . Then,

$$\xi^{-s} = -i\sigma^2 (\xi^s)^* = (-i\sigma^2 \xi^*(\uparrow), -i\sigma^2 \xi^*(\downarrow))$$

$$\vec{n}_\sigma(\xi^{-1}) = -\xi^{-1}, \vec{n}_\sigma(\xi^{s+2}) = \xi^{s+2}$$

$$= (\xi(\downarrow), -\xi(\uparrow))$$

Prove this:

$$\begin{aligned} \mathcal{U}^s(\rho) &= -i\gamma^2 (\mathcal{U}^s(\rho))^* \\ \mathcal{V}^s(\rho) &= -i\gamma^2 (\mathcal{V}^s(\rho))^* \end{aligned} \quad (\text{without proof})$$



$\xi^s = (\xi(\uparrow), \xi(\downarrow))_{s-1}$ . Then,  $s=2$

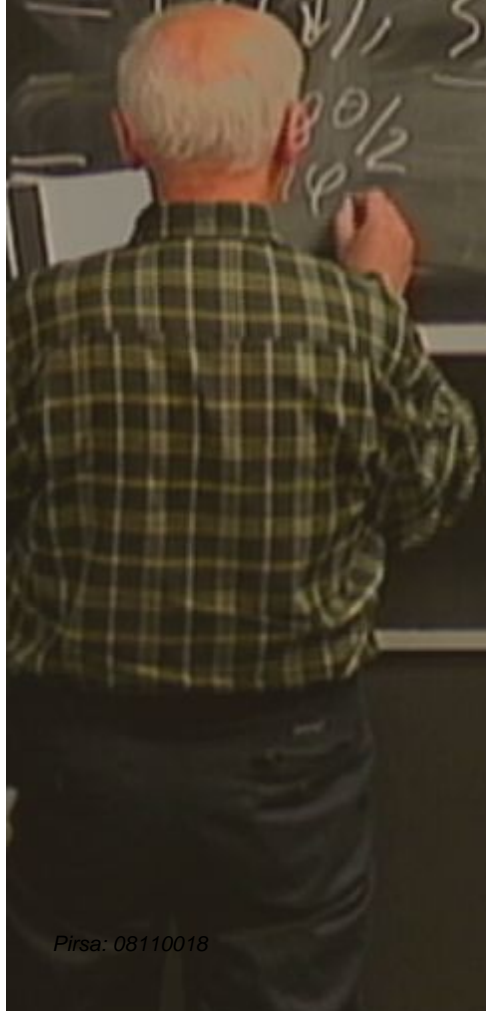
$$\xi^s \equiv -i\sigma^2 (\xi^s)^* = (-i\sigma^2 \xi^s(\uparrow), i\sigma^2 \xi^s(\downarrow))$$

$$\vec{n}\sigma(\xi^s) = -\xi^s, \vec{n}\sigma(\xi^{s+2}) = \xi^{s+2}$$

$$= (\xi(\downarrow), -\xi(\uparrow))$$

Prove this:  $-i\sigma^2 \xi^s(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -e^{i\varphi} \sin \theta/2 \\ \cos \theta/2 \end{pmatrix}$

$\chi^s(\uparrow) = -i\gamma^2 (\chi^s(\uparrow))^*$  (without proof)  
 $\chi^s(\downarrow) = -i\gamma^2 (\chi^s(\downarrow))^*$





$$\vec{n}_0(\xi^{-1}) = -\xi^{-1}, \vec{n}_0(\xi) = \xi$$

$$= \begin{pmatrix} \xi(\downarrow) \\ \xi(\uparrow) \end{pmatrix} \quad \text{Prove this: } -\sqrt{\sigma^2 \xi^x(\downarrow)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi} \sin \theta/2 \\ e^{i\varphi} \cos \theta/2 \end{pmatrix}$$

$$= \begin{pmatrix} -\cos \theta/2 \\ e^{i\varphi} \sin \theta/2 \end{pmatrix} = -\xi(\uparrow)$$



$\xi^s = -i\sigma^2 (\xi^s)^* = (i\sigma^2 \xi^s(\uparrow), -i\sigma^2 \xi^s(\downarrow))$  then,  $s=2$

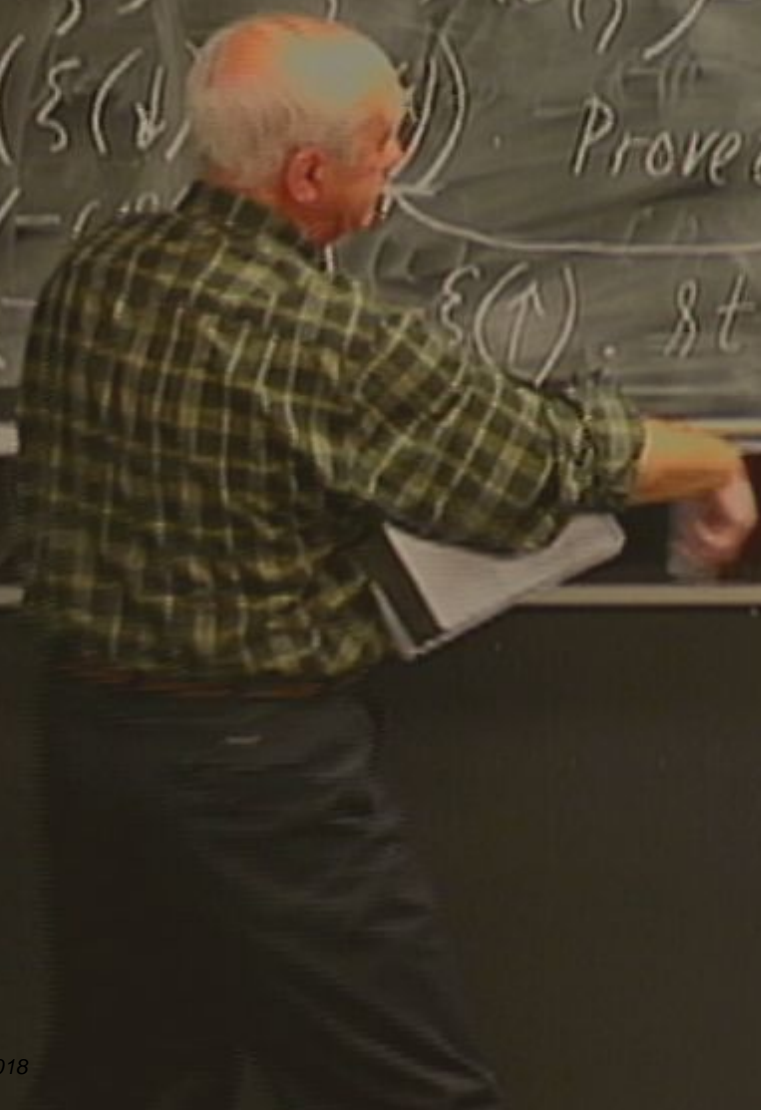
$\vec{n}_\sigma(\xi^{\uparrow}) = -\xi^{\uparrow}, \vec{n}_\sigma(\xi^{\downarrow}) = \xi^{\downarrow}$

$\vec{n}_\sigma(\xi^{\uparrow}) = -\xi^{\uparrow}, \vec{n}_\sigma(\xi^{\downarrow}) = \xi^{\downarrow}$

Prove this:  $-i\sigma^2 \xi^s(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -e^{i\varphi} \sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$

It easy to show that  $\xi^s$

$\chi^s(\uparrow) = -i\gamma^2 (\chi^s(\uparrow))^*$   
 $\chi^s(\downarrow) = -i\gamma^2 (\chi^s(\downarrow))^*$  (without proof)



$\begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} = -\xi(\theta)$ . Prove this:  $-\cos(\theta/2) = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix} = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$ . It is easy to show that  $\xi(\theta) = -\xi(\theta)$ .

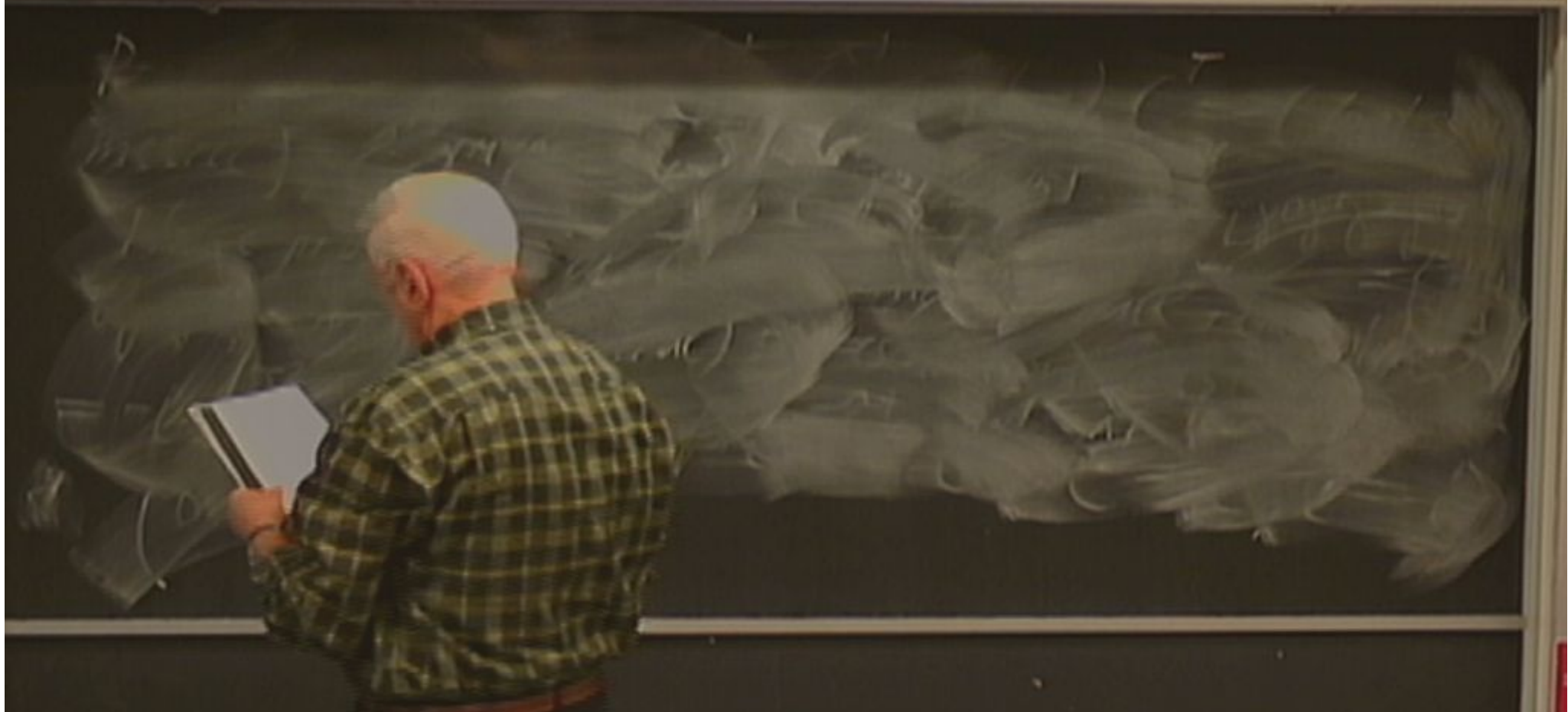
~~(Circuit) ...  
 ...  
 ...~~





$$= \begin{pmatrix} \xi(\downarrow) \\ -\xi(\uparrow) \end{pmatrix} \quad \text{Prove this: } -\sqrt{\sigma^2} \xi(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\phi} \sin \theta/2 \\ \cos \theta/2 \end{pmatrix}$$

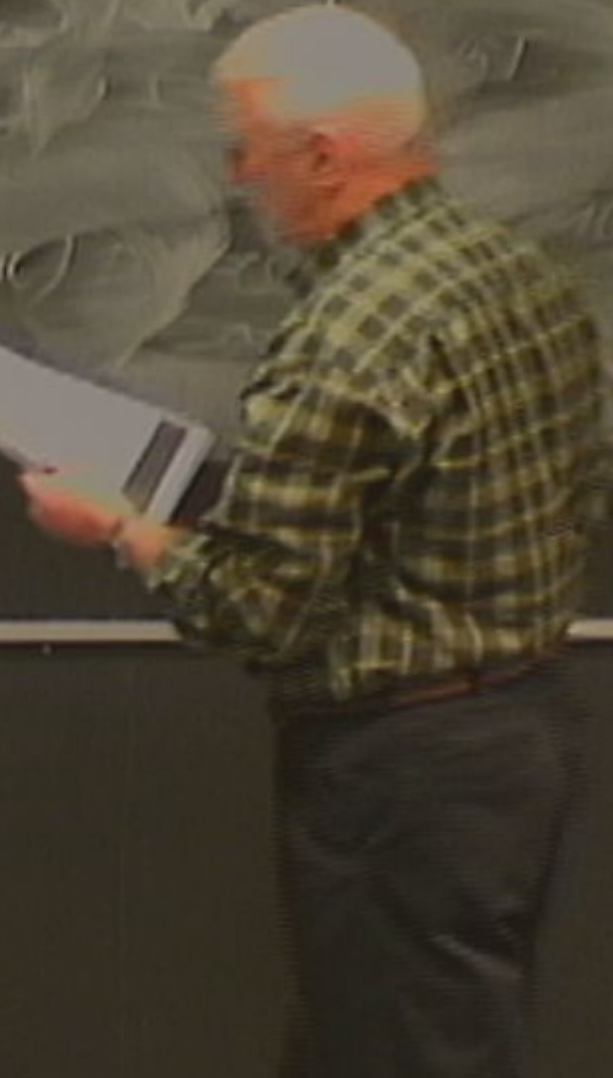
$$= \begin{pmatrix} -\cos \theta/2 \\ -e^{i\phi} \sin \theta/2 \end{pmatrix} = -\xi(\uparrow) \quad \text{It easy to show that } \xi(\uparrow) = \begin{pmatrix} \cos \theta/2 \\ e^{i\phi} \sin \theta/2 \end{pmatrix} \quad \text{(check!)}$$





$(-\rho^{11} \gamma_{11} \gamma_2) = -\xi(\uparrow)$ . It's easy to show that  $\xi^{\uparrow} = \xi^{\downarrow}$  (check!)

$a_{\vec{p}}$  destroys a fermion whose spinor is  $u^s(p)$ ,  $b_{\vec{p}}$  (annihilates)  $v^s(p)$



$(-\rho^{11} \chi_{110/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(s)} = \xi^s$  (check!)

$a_{\vec{p}}^s$  destroys a fermion whose spinor is  $u^s(p)$ ,  $b_{\vec{p}}^s$  destroys antifermion whose spinor is  $v^s(p)$ .

$$u^s(p) = \begin{pmatrix} \sqrt{E+m} \\ \sqrt{E-m} \end{pmatrix}$$



$(-\rho^{11} \chi_{110/2}) = -\xi(\uparrow)$ . It's easy to show that  $\xi^{(1)} = \xi^S$  (check!)

$a_{\vec{p}}$  destroys a fermion whose spinor is  $u^S(p)$ ,  $b_{\vec{p}}$  destroys antifermion whose spinor is  $v^S(p)$ .

$$u^S(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^S \\ \sqrt{p \cdot \bar{\sigma}} \xi^S \end{pmatrix}$$



$(-\rho^{11} \xi_{110/2}) = -\xi(\uparrow)$ . It's easy to show that  $\xi^{(1)} = \xi^s$  (check!)

$a_{\vec{p}}$  destroys a fermion whose spinor is  $u^s(p)$ ,  $b_{\vec{p}}$  destroys antifermion whose spinor is  $v^s(p)$ .

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix}, \quad v^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ -\sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix}$$

$(-\rho^{11} \xi_{110/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(1)} = \xi^s$  (check!)

$a_{\vec{p}}$  destroys a fermion whose spinor is  $u^s(p)$ ,  $b_{\vec{p}}$  destroys antifermion whose spinor is  $v^s(p)$ .

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix}, \quad v^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^{-s} \\ -\sqrt{p \cdot \bar{\sigma}} \xi^{-s} \end{pmatrix}$$

Without, use  $\sqrt{p \cdot \sigma} \sigma^3 = \sigma^2 \sqrt{p \cdot \sigma^*}$ ,  $\vec{p} = (p^0, -\vec{p})$

Then, let us show that  $[U^S(\rho)]^* = \gamma^1 \gamma^3 \gamma^5$



Then, let us show that  $\begin{cases} [U^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 (P) \\ [V^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 (P) \end{cases}$

Then, let us show that

$$\begin{cases} [U^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 (P) \\ [V^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 (P) \end{cases} \quad \gamma^1 \gamma^3 = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Then, let us show that

Proof: 
$$U^S(P) = \begin{pmatrix} \sqrt{P} \cdot 0 & (10^2 \cdot S^*) \\ \sqrt{P} \cdot 0 \end{pmatrix}$$

$$\left\{ \begin{array}{l} [U^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 (P) \\ [V^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 (P) \end{array} \right. \quad \left. \begin{array}{l} \gamma^1 \gamma^3 = \\ \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \right.$$



Then, let us show that

Proof:  $U^S(P) = \begin{pmatrix} \sqrt{p_0} (10^2 \xi^{5^*}) \\ \sqrt{p_0} (-10^2 \xi^{5^*}) \end{pmatrix} =$

$$\left\{ \begin{array}{l} [U^S(P)]^* = \gamma^1 \gamma^3 U^{-S}(\tilde{P}), \\ [V^S(P)]^* = \gamma^1 \gamma^3 V^{-S}(\tilde{P}) \end{array} \right\} \gamma^1 \gamma^3 = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Then, let us show that

$$u^S(p) = \begin{pmatrix} \sqrt{p\sigma} (-10^2 \xi^{S^*}) \\ \sqrt{p\sigma} (-10^2 \xi^{S^*}) \end{pmatrix}$$

Eq. (II)

$$\begin{pmatrix} -10^2 \sqrt{p\sigma} \xi^{S^*} \\ -10^2 \sqrt{p\sigma} \xi^{S^*} \end{pmatrix}$$

$$\left\{ \begin{aligned} [u^S(p)]^* &= \gamma^1 \gamma^3 u^S(p), & \gamma^1 \gamma^3 &= \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ [v^S(p)]^* &= \gamma^1 \gamma^3 v^S(p) \end{aligned} \right.$$



Then let us show that

Procs (P)

$$= \begin{pmatrix} \sqrt{P\sigma} (10^2 \xi^{S^*}) \\ \sqrt{P\sigma} (-10^2 \xi^{S^*}) \end{pmatrix}$$

$$\begin{cases} [U^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 \tilde{U}^S(P), & \gamma^1 \gamma^3 = i \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ [Z^S(P)]^* = \gamma^1 \gamma^3 \gamma^5 \tilde{Z}^S(P) \end{cases}$$

$$\text{Eq. (II)} \begin{pmatrix} -10^2 \sqrt{P\sigma} \xi^{S^*} \\ -10^2 \sqrt{P\sigma} \xi^{S^*} \end{pmatrix} = -i \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{P\sigma} \xi^{S^*} \\ \sqrt{P\sigma} \xi^{S^*} \end{pmatrix} = -\gamma^1 \gamma^3$$



Then, let us show that

$$\begin{aligned}
 \text{Proof: } \mathcal{U}^S(\tilde{p}) &= \begin{pmatrix} \sqrt{p\sigma} (-10^2 \xi^{S^*}) \\ \sqrt{p\sigma} (-10^2 \xi^{S^*}) \end{pmatrix} \\
 &= \underline{-\gamma' \gamma^3 [\mathcal{U}^S(\tilde{p})]^*} \\
 \text{Eq. (II)} &= \frac{\text{MSE}}{\begin{pmatrix} -10^1 \sqrt{p\sigma} \xi^{S^*} \\ -10^2 \sqrt{p\sigma} \xi^{S^*} \end{pmatrix}} = \frac{\begin{pmatrix} 10^2 0 \\ 0 0^2 \end{pmatrix} \mathcal{U}(\tilde{p})}{- \gamma' \gamma^3}
 \end{aligned}$$

Then, let us show that

$$\begin{bmatrix} [U^S(P)]^* \\ [V^S(P)]^* \end{bmatrix} = \gamma' \gamma^3 \begin{bmatrix} U^{-S}(P) \\ V^{-S}(P) \end{bmatrix}, \quad \gamma' \gamma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Proof:  $U^S(P) = \begin{pmatrix} \sqrt{P} \sigma^{-1} (-i \sigma^2 S^*) \\ \sqrt{P} \sigma^{-1} (-i \sigma^3 S^*) \end{pmatrix}$

$= -\gamma' \gamma^3 [U^S(P)]^* \Rightarrow$

MSE Eq. (II)  $\begin{pmatrix} -i \sigma^1 \sqrt{P} \sigma^{-1} S^* \\ -i \sigma^2 \sqrt{P} \sigma^{-1} S^* \end{pmatrix} = -i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} [U^S(P)]^*$

Similarly for  $V^S(P)$ ,  $V^{-S}(P) = -\gamma' \gamma^3 [V^S(P)]^*$ . Remember  $\gamma' \gamma^3$



Then, let us show that

Proof:  $u^s(p) = \begin{pmatrix} \sqrt{p} \sigma (-i \omega^2 s^x) \\ \sqrt{p} \sigma (-i \omega^2 s^x) \end{pmatrix}$

$= -\gamma' \gamma^3 [u^s(p)]^*$

Similarly for  $v^s(p)$ ,

$$\begin{aligned} [u^s(p)]^* &= \gamma' \gamma^3 u^{-s}(\tilde{p}), & \gamma' \gamma^3 &= i \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \\ [v^s(p)]^* &= \gamma' \gamma^3 v^{-s}(\tilde{p}) \end{aligned}$$

MSE Eq. (II)  $\begin{pmatrix} -i \omega^2 \sqrt{p} \sigma^* \} s^x \\ -i \omega^2 \sqrt{p} \sigma^* \} s^x \end{pmatrix} = -i \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} [u^s(p)]^*$

$v^{-s}(p) = -\gamma' \gamma^3 [v^s(p)]^*$ . Remember  $\gamma' \gamma^3$



Then, let us show that

$$\begin{aligned}
 \text{Proof: } \mathcal{U}^S(P) &= \begin{pmatrix} \sqrt{P} \sigma^{-1} (-i\sigma^2 S^*) \\ \sqrt{P} \sigma^{-1} (-i\sigma^2 S^*) \end{pmatrix} \\
 &= -\gamma' \gamma^3 [\mathcal{U}^S(P)]^* \Rightarrow \text{we get } \mathcal{V}^{-S}(P) = -\gamma' \gamma^3 [\mathcal{V}^{-S}(P)]^* \\
 &\text{Similarly for } \mathcal{V}^S(P), \mathcal{V}^{-S}(P) \text{ contains } \begin{pmatrix} -iS \\ -iS \end{pmatrix} = -\begin{pmatrix} S \\ S \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 [\mathcal{U}^S(P)]^* &= \gamma' \gamma^3 \mathcal{U}^{-S}(P) \\
 [\mathcal{V}^{-S}(P)]^* &= \gamma' \gamma^3 \mathcal{V}^S(P)
 \end{aligned}$$

$$\text{Eq (I)} \begin{pmatrix} -i\sigma^1 \sqrt{P} \sigma^{-1} S^* \\ -i\sigma^2 \sqrt{P} \sigma^{-1} S^* \end{pmatrix} = -i \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} \mathcal{U}^S(P)$$

Then, let us show that

$$\begin{aligned}
 \text{Proof: } \mathcal{U}^s(P) &= \begin{pmatrix} \sqrt{p\sigma} \{ -i\omega^2 s^* \} \\ \sqrt{p\sigma} \{ -i\omega^2 s^* \} \end{pmatrix} \\
 &= -\gamma' \gamma^3 [\mathcal{U}^s(P)]^* \xrightarrow{\text{weigel}} \begin{pmatrix} -i\omega^2 \sqrt{p\sigma}^* \{ s^* \} \\ -i\omega^2 \sqrt{p\sigma}^* \{ s^* \} \end{pmatrix} \xrightarrow{\text{Eq (I)}} \begin{pmatrix} i\omega^2 \sqrt{p\sigma} \{ s \} \\ i\omega^2 \sqrt{p\sigma} \{ s \} \end{pmatrix} = -i \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{pmatrix} \mathcal{U}^s(P) = \\
 & \text{Similarly for } \mathcal{V}^s(P), \mathcal{V}^{-s}(P) = -\gamma' \gamma^3 [\mathcal{V}^s(P)]^* \text{ Remember } \gamma' \gamma^3 \text{ that} \\
 & \mathcal{V}^{-s} \text{ contains } \{ -s \} = -\{ s \}
 \end{aligned}$$

$$\begin{aligned}
 [\mathcal{U}^s(P)]^* &= \gamma' \gamma^3 \mathcal{U}^{-s}(P) \\
 [\mathcal{V}^s(P)]^* &= \gamma' \gamma^3 \mathcal{V}^{-s}(P)
 \end{aligned}$$



Proof.  $\mathcal{U}^s(\rho) = \frac{1}{\sqrt{p_0}} (-i\sigma^3 \xi^s)$   $\mathcal{U}^s(\rho) = -\gamma^1 \gamma^3 \mathcal{U}^s(\rho)$

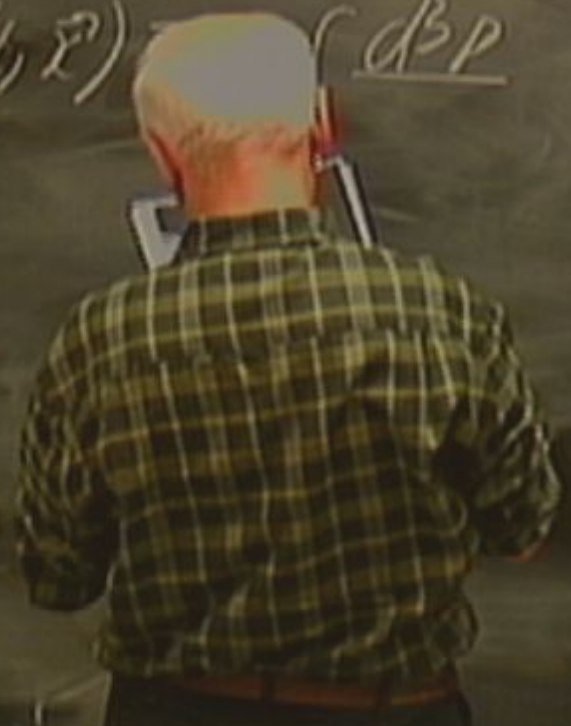
$= -\gamma^1 \gamma^3 [\mathcal{U}^s(\rho)]^* \Rightarrow$  weigel Eq. (I)  $\begin{pmatrix} -i\sigma^1 \sqrt{p_0} \xi^s \\ -i\sigma^2 \sqrt{p_0} \xi^s \end{pmatrix} = -i \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} \mathcal{U}^s(\rho)$

Similarly for  $\mathcal{U}^{-s}(\rho)$ ,  $\mathcal{U}^{-s}(\rho) = -\gamma^1 \gamma^3 [\mathcal{U}^{-s}(\rho)]^*$  Remember  $\gamma^1 \gamma^3$  th

$\mathcal{U}^{-s}$  contains  $\xi^{-s} = -\xi^s$

Consider time reversal of  $\psi(x)$ .

$T^{-1} \psi(t, \vec{x}) = \dots$  DBP





$$\begin{aligned}
 &= -\gamma^1 \gamma^3 [\psi^s(P)]^* \Rightarrow \text{weigel} \\
 & \text{Similarly for } \psi^s(P), \psi^{-s}(P) = -\gamma^1 \gamma^3 [\psi^s(P)]^* \text{ Remember } \gamma^1 \gamma^3 \text{ that} \\
 & \psi^{-s} \text{ contains } \xi^{-(-s)} = -\xi^s
 \end{aligned}$$

Consider time reversal of  $\psi(x)$ !

$$T^{-1} \psi(t, \vec{x}) T =$$

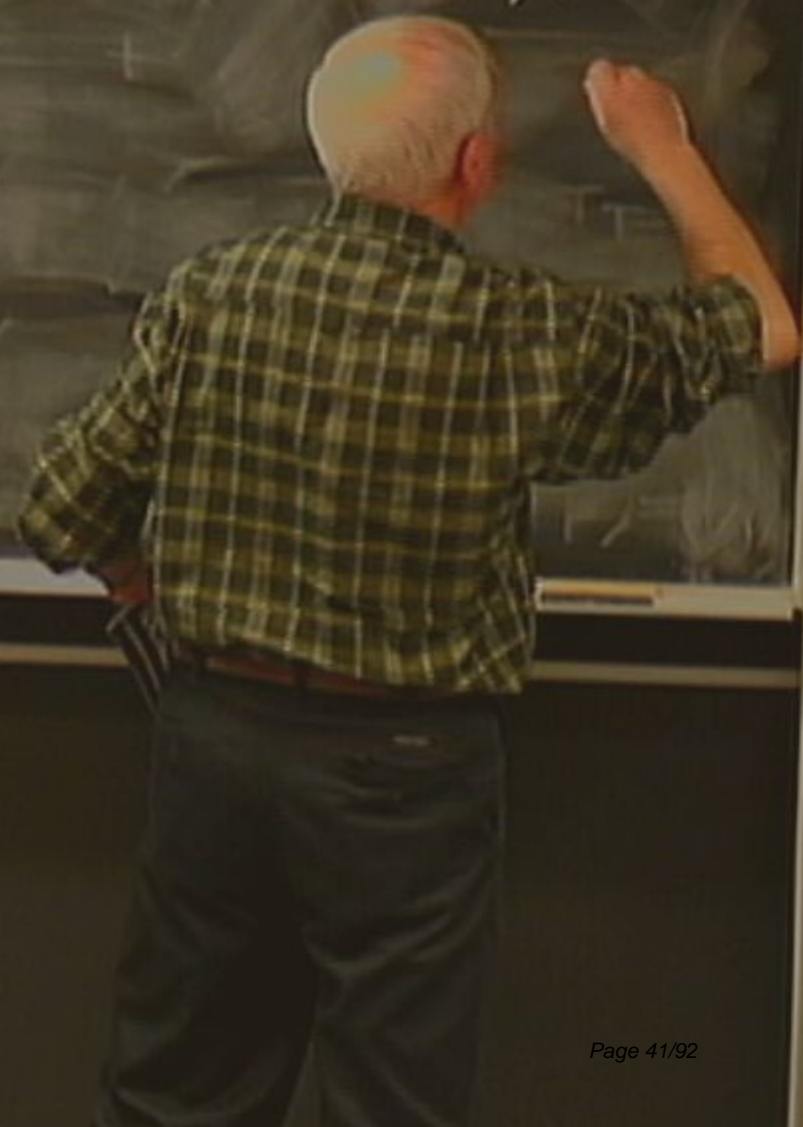
consider time reversal of  $\psi(x)$ .

$$T^{-1} \psi(t, \mathbf{x}) T = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_{\xi} T^{-1} (a_{\mathbf{p}, \xi} u^{\xi}(\mathbf{p}) e^{-i p x})$$



Consider time reversal of  $\psi(x)$ :

$$T^{-1} \psi(t, \mathbf{x}) T = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_{\mathbf{p}}^s u^s(\mathbf{p}) e^{-i p x} + b_{\mathbf{p}}^{s\dagger} v^s(\mathbf{p}) e^{i p x})$$





Consider time reversal of  $\psi(x)$ :

$$T^{-1} \psi(t, \vec{x}) T = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_{\vec{p}}^s u^s(p) e^{-ipx} + b_{\vec{p}}^{s\dagger} v^s(p) e^{ipx}) T$$


Consider time reversal of  $\psi(x)$ :

$$T^{-1} \psi(t, \vec{x}) T = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_p^s u^s(p) e^{-ipx} + b_p^{s\dagger} v^s(p) e^{ipx}) T$$

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \sum_{s'} T^{-1} a_p^{s'} T u^{s'}(p)$$





Consider time reversal of  $\psi(x)$ :

$$T^{-1} \psi(t, \vec{x}) T = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_p^s u^s(p) e^{-ipx} + b_p^{s\dagger} v^s(p) e^{ipx}) T$$

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} a_p^s T [u^s(p)]^* e^{ipx} + T^{-1} b_p^{s\dagger} T [v^s(p)]^* e^{-ipx}$$



Consider time reversal of  $\psi(x)$ :

$$T^{-1} \psi(t, \vec{x}) T = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S T^{-1} (a_{\vec{p}}^S u^S(p) e^{-i p x} + b_{\vec{p}}^{S\dagger} v^S(p) e^{i p x}) T$$

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \sum_{S'} \underbrace{T^{-1} a_{\vec{p}}^S T}_{a_{-\vec{p}}^{S'}} u^{S'}(p) e^{-i p x} + \underbrace{T^{-1} b_{\vec{p}}^{S\dagger} T}_{b_{-\vec{p}}^{-S\dagger}} v^S(p) e^{i p x}$$





Consider time reversal of  $\psi(x)$ :

$$\begin{aligned}
 T^{-1} \psi(t, \vec{x}) T &= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_p^s u^s(p) e^{-ipx} + b_p^{s\dagger} v^s(p) e^{ipx}) T \\
 &= \eta_T \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} \sum_s \left[ T^{-1} a_p^s T \right]^* e^{ipx} + \left[ T^{-1} b_p^{s\dagger} T \right]^* e^{-ipx} \\
 &= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} \sum_s \left[ a_{\tilde{p}}^s u^s(\tilde{p}) e^{i\tilde{p}x} + b_{\tilde{p}}^{s\dagger} v^s(\tilde{p}) e^{-i\tilde{p}x} \right]
 \end{aligned}$$



Similarity transformation  $T^{-1} \psi = -\gamma \gamma^T \psi$ . Volume  $V = \sqrt{1-v^2/c^2}$

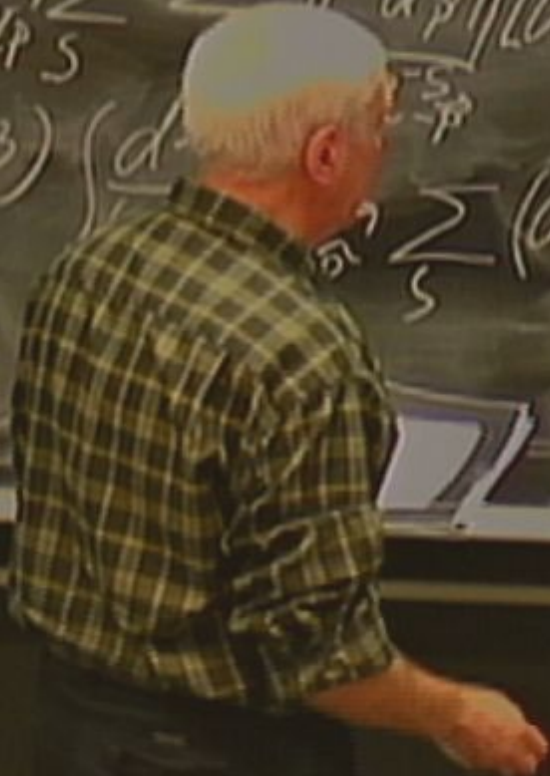
Consider time reversal of  $\psi(x)$ :

$$\begin{aligned}
 T^{-1} \psi(t, \vec{x}) T &= \int \frac{d^3 p}{(2\pi)^3} \sum_S T^{-1} (a_{\vec{p}}^S u^S(\vec{p}) e^{-i p x} + b_{\vec{p}}^{S+} v^S(\vec{p}) e^{i p x}) T \\
 &= \eta_T \int \frac{d^3 p}{(2\pi)^3} \sum_S T^{-1} \left[ \frac{1}{\sqrt{2E_{\vec{p}}}} \left( a_{\vec{p}}^S u^S(\vec{p}) e^{-i p x} + b_{\vec{p}}^{S+} v^S(\vec{p}) e^{i p x} \right) \right] T \\
 &= \eta_T (\gamma^T \gamma^3) \sum_S (a_{-\vec{p}}^S u^S(\vec{p}))
 \end{aligned}$$



Consider time reversal of  $\psi(x)$ :

$$\begin{aligned}
 T^{-1} \psi(t, \vec{x}) T &= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_{\vec{p}}^s u^s(\vec{p}) e^{-i p x} + b_{\vec{p}}^{s\dagger} v^s(\vec{p}) e^{i p x}) T \\
 &= \eta_T \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \sum_{s'} T^{-1} a_{\vec{p}}^s T^{-1} u^{s'}(\vec{p}) e^{i p x} + T^{-1} b_{\vec{p}}^{s\dagger} T^{-1} v^s(\vec{p}) e^{-i p x} \\
 &= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s (a_{-\vec{p}}^s u^s(\vec{p}) e^{i p(t, -\vec{x})} + b_{-\vec{p}}^{s\dagger} v^s(\vec{p}) e^{-i p(t, -\vec{x})})
 \end{aligned}$$



Consider time reversal of  $\psi(x)$ .

$$\begin{aligned}
 T^{-1} \psi(t, \vec{x}) T &= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_p^s u^s(p) e^{-ipx} + b_p^{s\dagger} v^s(p) e^{ipx}) T \\
 &= \eta_T \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \sum_s \left[ T^{-1} a_p^s T \right]^* u^s(p) e^{ipx} + \left[ T^{-1} b_p^{s\dagger} T \right]^* v^s(p) e^{-ipx} \\
 &= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} \sum_s (a_{-\tilde{p}}^s u^s(\tilde{p}) e^{i\tilde{p}(t, -\vec{x})} + b_{-\tilde{p}}^{s\dagger} v^s(\tilde{p}) e^{-i\tilde{p}(t, -\vec{x})})
 \end{aligned}$$





Consider time reversal of  $\psi(x)$ .

$$T^{-1} \psi(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_{\vec{p}}^s u^s(\vec{p}) e^{-ipx} + b_{\vec{p}}^{s\dagger} v^s(\vec{p}) e^{ipx}) T^{-1}$$

$$= \eta_T \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \sum_{s'} [T^{-1} a_{\vec{p}}^s T^{-1}]^* [u^{s'}(\vec{p})]^* e^{ipx} + [T^{-1} b_{\vec{p}}^{s\dagger} T^{-1}] [v^{s'}(\vec{p})]^* e^{-ipx}$$

$$= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} \sum_s (\tilde{a}_{\tilde{p}}^s u^s(\tilde{p}) e^{i\tilde{p}(t, -\vec{x})} + \tilde{b}_{\tilde{p}}^{s\dagger} v^s(\tilde{p}) e^{-i\tilde{p}(t, -\vec{x})})$$

USE  
 $\tilde{p}(t, -\vec{x}) = -$

Consider time reversal of  $\psi(x)$ .

$$\begin{aligned}
 T^{-1} \psi(t, \vec{x}) &= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s T^{-1} (a_{\vec{p}}^s u^s(\vec{p}) e^{-ipx} + b_{\vec{p}}^{s\dagger} v^s(\vec{p}) e^{ipx}) \\
 &\stackrel{[T]}{=} \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \sum_{s'} \underbrace{T^{-1} a_{\vec{p}}^s}_{a_{-\vec{p}}^{s'}} \underbrace{[u^s(\vec{p})]^*}_{u^{-s}(\vec{p})} e^{ipx} + \underbrace{T^{-1} b_{\vec{p}}^{s\dagger}}_{b_{-\vec{p}}^{-s\dagger}} \underbrace{[v^s(\vec{p})]^*}_{v^{-s}(\vec{p})} e^{-ipx} \\
 &= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 \vec{p}}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (a_{-\vec{p}}^s u^{-s}(\vec{p}) e^{i\vec{p}(t, -\vec{x})} + b_{\vec{p}}^{s\dagger} v^{-s}(\vec{p}) e^{-i\vec{p}(t, \vec{x})}) \\
 &\stackrel{usc}{=} \overline{\psi(t, -\vec{x})} = -\overline{\psi(-t, \vec{x})} \quad \eta_T (\gamma^0 \gamma^3) \psi(-t, \vec{x})
 \end{aligned}$$



$$\begin{aligned}
 & (T) \frac{(2\pi)^3 \sqrt{2E_{\vec{p}}}}{s} \frac{1}{a_{-\vec{p}}^{-s}} + \frac{1}{b_{\vec{p}}^{-s}} \frac{T}{1/b^s} \ell^{up} = \\
 & = \eta_T (\gamma^1 \gamma^3) \frac{d^3 \vec{p}}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (a_{-\vec{p}}^{-s} \psi^{-s}(\vec{p}) e^{i\vec{p}(t, -\vec{x})} + b_{\vec{p}}^{-s} \psi^{-s}(\vec{p}) e^{-i\vec{p}(t, \vec{x})}) \\
 & \quad \text{use} \\
 & \quad \bar{\psi}(t, -\vec{x}) = -\bar{\psi}(-t, \vec{x}) \quad \eta_T (\gamma^1 \gamma^3) \psi(-t, \vec{x})
 \end{aligned}$$

$$P(t, x) = -P(-t, x) \quad (1 \ 0 \ 0) \quad \Psi(-t, x)$$

Transformation of  $\Psi$ :  $T^{-1} \Psi(t, x) T = (T^{-1} \Psi(t, x) T)^{\dagger} \delta_0 =$



$$P(t, \vec{x}) = -P(-t, \vec{x}) \quad (1 \ 0 \ 0) \quad \Psi(-t, \vec{x})$$

Transformation of  $\Psi$ :  $T^{-1} \Psi(t, \vec{x}) T = (T^{-1} \Psi(t, \vec{x}) T)^\dagger \delta^0 =$   
 $= \Psi(-t, \vec{x}) [\gamma^0 \gamma^3]^\dagger \delta^0 =$

$$P(t, \vec{x}) = -P(-t, \vec{x}) \quad (1 \ 0 \ 0) \quad \Psi(-t, \vec{x})$$

Transformation of  $\Psi$ :

$$= \gamma_T^x \Psi^+(-t, \vec{x}) [\gamma^0 \gamma^3]^T \gamma^0 = \gamma_T^x \Psi^+(t, \vec{x}) = (\gamma_T^{-1} \Psi(t, \vec{x}))^T \gamma^0 =$$



$$P(t, \vec{x}) = -P(-t, \vec{x}) \quad (1 \ 0 \ 0) \quad \Psi(-t, \vec{x})$$

Transformation of  $\Psi$ :  $T^{-1} \Psi(t, \vec{x}) T = (T^{-1} \Psi(t, \vec{x}) T)^\dagger \gamma^0 =$   
 $= \gamma^0 \Psi(-t, \vec{x}) [\gamma^0 \gamma^i \gamma^0]^\dagger \gamma^0 = \gamma^0 \Psi(-t, \vec{x}) [\gamma^0 \gamma^i \gamma^0]$

Transformation of  $\Psi$ :  $\Psi(t, \vec{x})^\dagger = (F^{-1} \Psi(t, \vec{x})^\dagger)^\dagger =$   
 $= \tilde{\gamma}_T^\dagger \Psi(-t, \vec{x}) [\gamma^0 \gamma^3]^\dagger \gamma^0 = \tilde{\gamma}_T^\dagger \Psi(-t, \vec{x}) [\gamma^0 \gamma^3].$



Transformation of  $\Psi$ :  $T^{-1} \bar{\Psi}(t, \vec{x}) T = (T^{-1} \Psi(t, \vec{x}) T)^\dagger =$   
 $= \tilde{\Psi}^\dagger(t, \vec{x}) [\gamma^0 \gamma^3]^\dagger \gamma^0 = \tilde{\Psi}^\dagger(-t, \vec{x}) [\gamma^0 \gamma^3].$

$$= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 \vec{p}}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s \psi^s(\vec{p}) e^{i\vec{p}(t, -\vec{x})} + b_{\vec{p}}^{s\dagger} \psi^s(\vec{p}) e^{-i\vec{p}(t, \vec{x})})$$

USE

$$\overline{\psi(t, -\vec{x})} = -\overline{\psi(t, \vec{x})} \quad \eta_T (\gamma^0 \gamma^3) \psi(-t, \vec{x})$$

Transformation of  $\overline{\psi}$ :  $T^{-1} \overline{\psi}(t, \vec{x}) T = (\overline{T^{-1} \psi(t, \vec{x}) T})^\dagger =$

$$\eta_T^* \overline{\psi}(-t, \vec{x}) [\gamma^0 \gamma^3]^\dagger \gamma^0 = \eta_T^* \overline{\psi}(-t, \vec{x}) [-\gamma^0 \gamma^3]$$



$$= \eta_T (\gamma^0 \gamma^3) \int \frac{d^3 \vec{p}}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s \psi^s(\vec{p}) e^{i\vec{p}(t, -\vec{x})} + b_{\vec{p}}^{s\dagger} \psi^s(\vec{p}) e^{-i\vec{p}(t, \vec{x})})$$

USE

$$\overline{\psi(t, -\vec{x})} = -\overline{\psi(t, \vec{x})} \quad \eta_T (\gamma^0 \gamma^3) \psi(-t, \vec{x})$$

Transformation of  $\overline{\psi}$ :  $T^{-1} \overline{\psi}(t, \vec{x}) T = (T^{-1} \psi(t, \vec{x}) T)^\dagger \gamma_0 =$

$$= \eta_T^* \psi^\dagger(-t, \vec{x}) [\gamma^0 \gamma^3]^\dagger \gamma_0 = \eta_T^* \overline{\psi}(-t, \vec{x}) [-\gamma^0 \gamma^3]$$

Transformation of  $\bar{\Psi}$ :  $T^{-1} \bar{\Psi}(t, \vec{x}) T = (T^{-1} \Psi(t, \vec{x}) T)^{\dagger} =$   
 $= \tilde{\chi}_T^{\dagger} \Psi^{\dagger}(-t, \vec{x}') [\gamma^0 \gamma^3]^{\dagger} \gamma^0 = \tilde{\chi}_T^{\dagger} \bar{\Psi}(-t, \vec{x}') [-\gamma^0 \gamma^3].$

Transformations of bilinear fields:

$$T^{-1} \bar{\Psi} \Psi(t, \vec{x}) T =$$



Transformation of  $\Psi$ :  $T^{-1} \Psi T = (T^{-1} \Psi (t, \vec{x}) T) \delta_0^+$

$$= \tilde{\eta}_T^* \Psi(-t, \vec{x}) [\gamma^0 \gamma^3]^+ \delta_0^+ = \tilde{\eta}_T^* \bar{\Psi}(-t, \vec{x}) [\gamma^0 \gamma^3]$$

Transformations of bilinear forms

$$T^{-1} \bar{\Psi} \Psi (t, \vec{x}) T = \bar{\Psi} (\gamma^0 \gamma^3) (\gamma^0 \gamma^3) \Psi$$

Transformation of  $\bar{\Psi}$ :  $T^{-1} \bar{\Psi}(t, \vec{x}) = \bar{\Psi}^*(-t, \vec{x}') [\gamma^0 \gamma^3]^\dagger \gamma^0 = \bar{\Psi}^*(-t, \vec{x}')$

Transformations of bilinear fields

$$T^{-1} \bar{\Psi} \Psi(t, \vec{x}) T = \bar{\Psi}(-t, \vec{x}') \Psi(-t, \vec{x}')$$

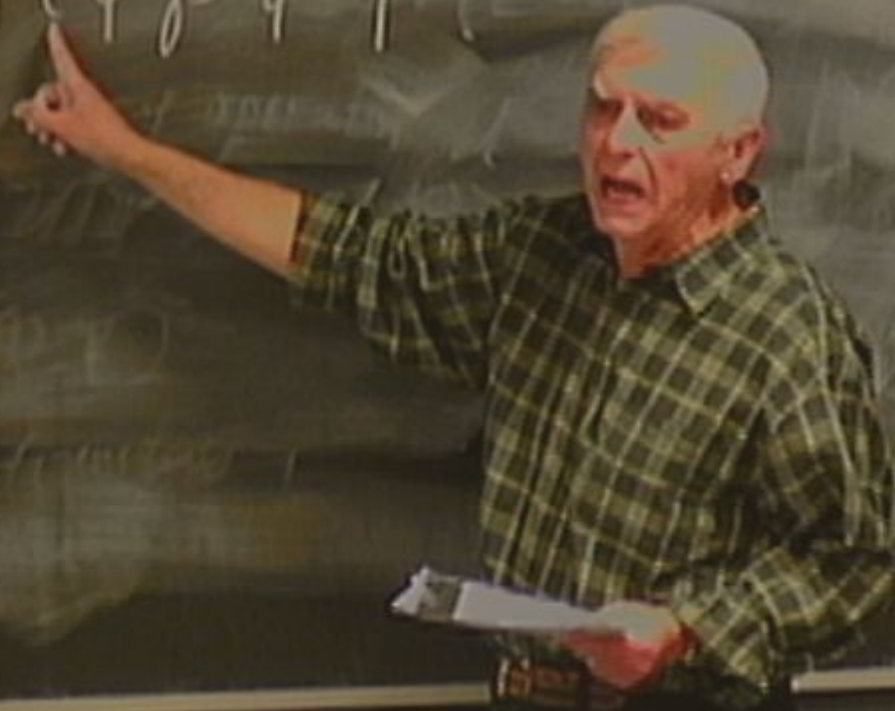


Transformation of  $\bar{\psi}$ :  $T^{-1} \bar{\psi}(t, \vec{x}) T = (T^{-1} \psi(t, \vec{x}) T)^{\dagger} \gamma_0$   
 $= \tilde{\gamma}_T^{\dagger} \psi^{\dagger}(-t, \vec{x}') [\gamma_1 \gamma_3]^{\dagger} \gamma_0 = \tilde{\gamma}_T^{\dagger} \bar{\psi}(-t, \vec{x}') [-\gamma_1 \gamma_3]$

Transformations of bilinear fields:

$$T^{-1} \bar{\psi} \psi(t, \vec{x}) T = \bar{\psi}(-\gamma_1 \gamma_3) (\gamma_1 \gamma_3) \psi(-t, \vec{x}') = \bar{\psi} \psi(-t, \vec{x}')$$

$T^1 \bar{\Psi} \gamma^5 \Psi T$

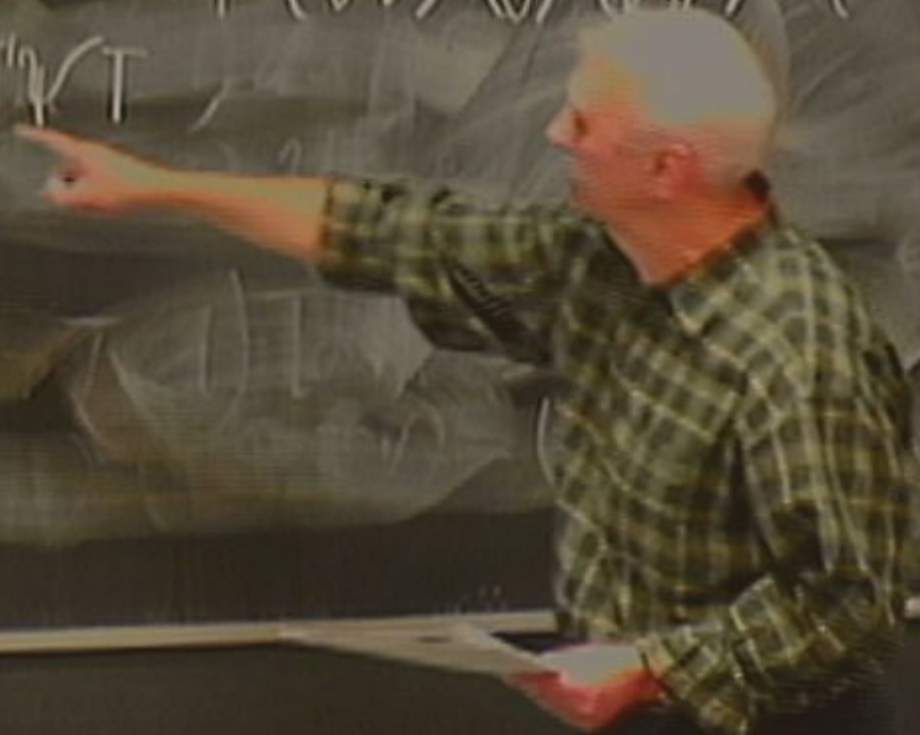




$$\bar{T} \psi \gamma^5 \psi T = -i \bar{\psi} (\gamma^0 \gamma^3) (\gamma^5)^* (\gamma^0 \gamma^3) \psi (t-x)$$

$$T^{-1} i \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (\gamma^0 \gamma^3) (\gamma^5)^* (\gamma^0 \gamma^3) \psi (t-x)$$

$$T^{-1} i \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$$





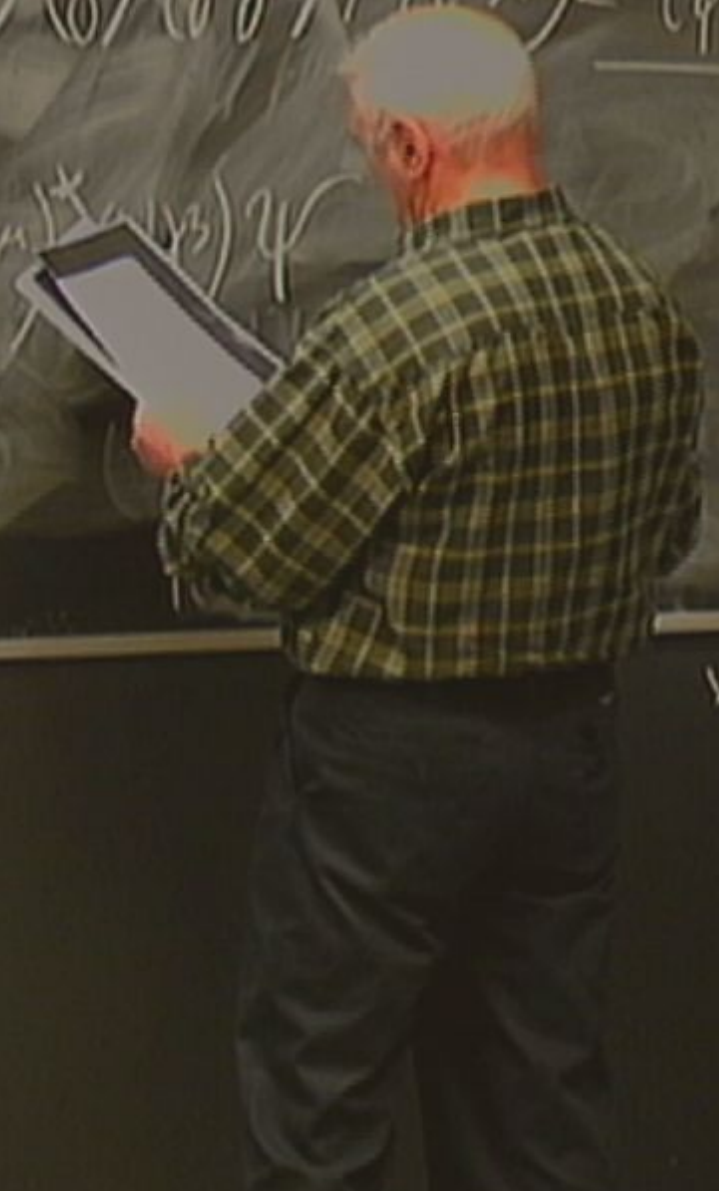
$$T^{-1} i \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (\gamma^1 \gamma^3) (\gamma^5)^* (\gamma^1 \gamma^3) \psi (t-x) = \sqrt{-1} \bar{\psi} \gamma^5 \psi (t-x)$$

$$T^{-1} i \bar{\psi} T = T^{-1} T i \psi T$$

$$T^{-1} \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^5)^* (\gamma^1 \gamma^3) \psi (t-x) = \sqrt{-1} \bar{\psi} \gamma^5 \psi (t-x)$$

$$T^{-1} \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$$

$$T^{-1} \bar{\psi} \gamma^{\mu} \psi T = \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^{\mu})^* (\gamma^1 \gamma^3) \psi$$

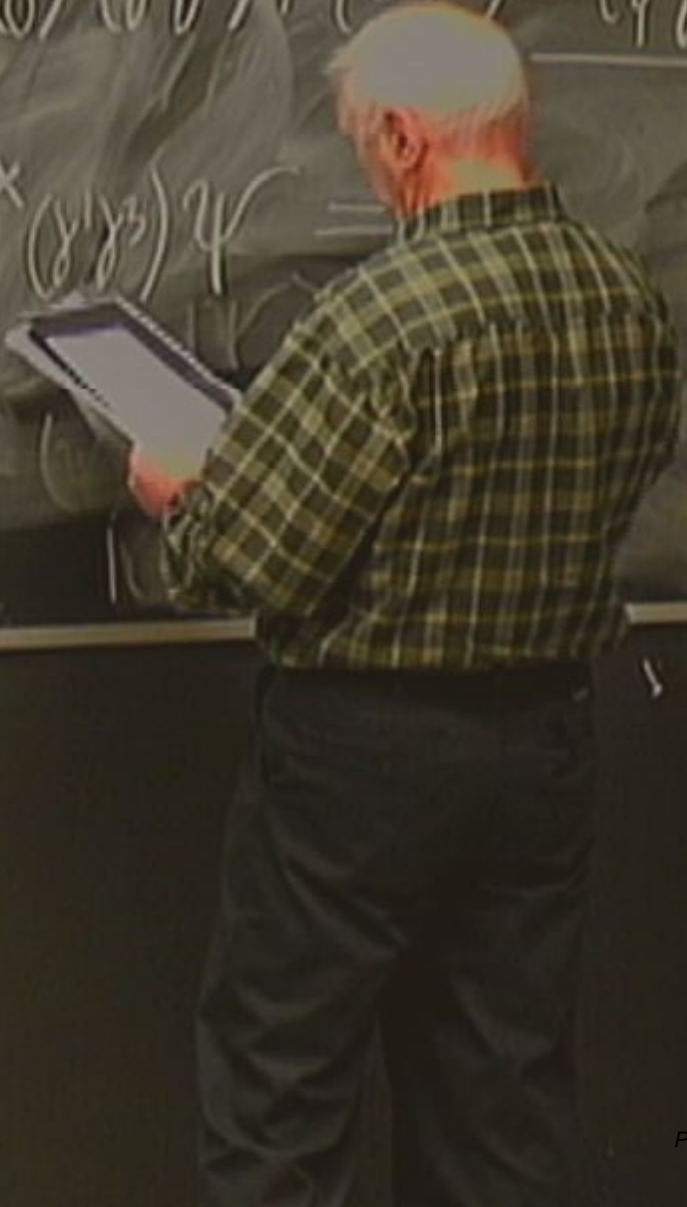




$$T^{-1} \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^5)^* (\gamma^1 \gamma^3) \psi (t-x) = \sqrt{-1} \bar{\psi} \gamma^5 \psi (t-x)$$

$$T^{-1} \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$$

$$T^{-1} \bar{\psi} \gamma^{\mu} \psi T = \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^{\mu})^* (\gamma^1 \gamma^3) \psi$$



$$T^{-1} \bar{\psi} \gamma^5 \psi T = -\bar{\psi} (-\gamma^1 \gamma^3) (\gamma^5)^* (\gamma^1 \gamma^3) \psi (t-x) = \sqrt{-1} \bar{\psi} \gamma^5 \psi (t, \vec{x})$$

$$T^{-1} \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$$

$$T^{-1} \bar{\psi} \gamma^{\mu} \psi T = \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^{\mu})^* (\gamma^1 \gamma^3) \psi = \left. \begin{aligned} &+ \bar{\psi} \gamma^{\mu} \psi (t, \vec{x}), \mu=0 \\ &- \bar{\psi} \gamma^{\mu} \psi (t, \vec{x}), \mu=1,2,3 \end{aligned} \right\}$$

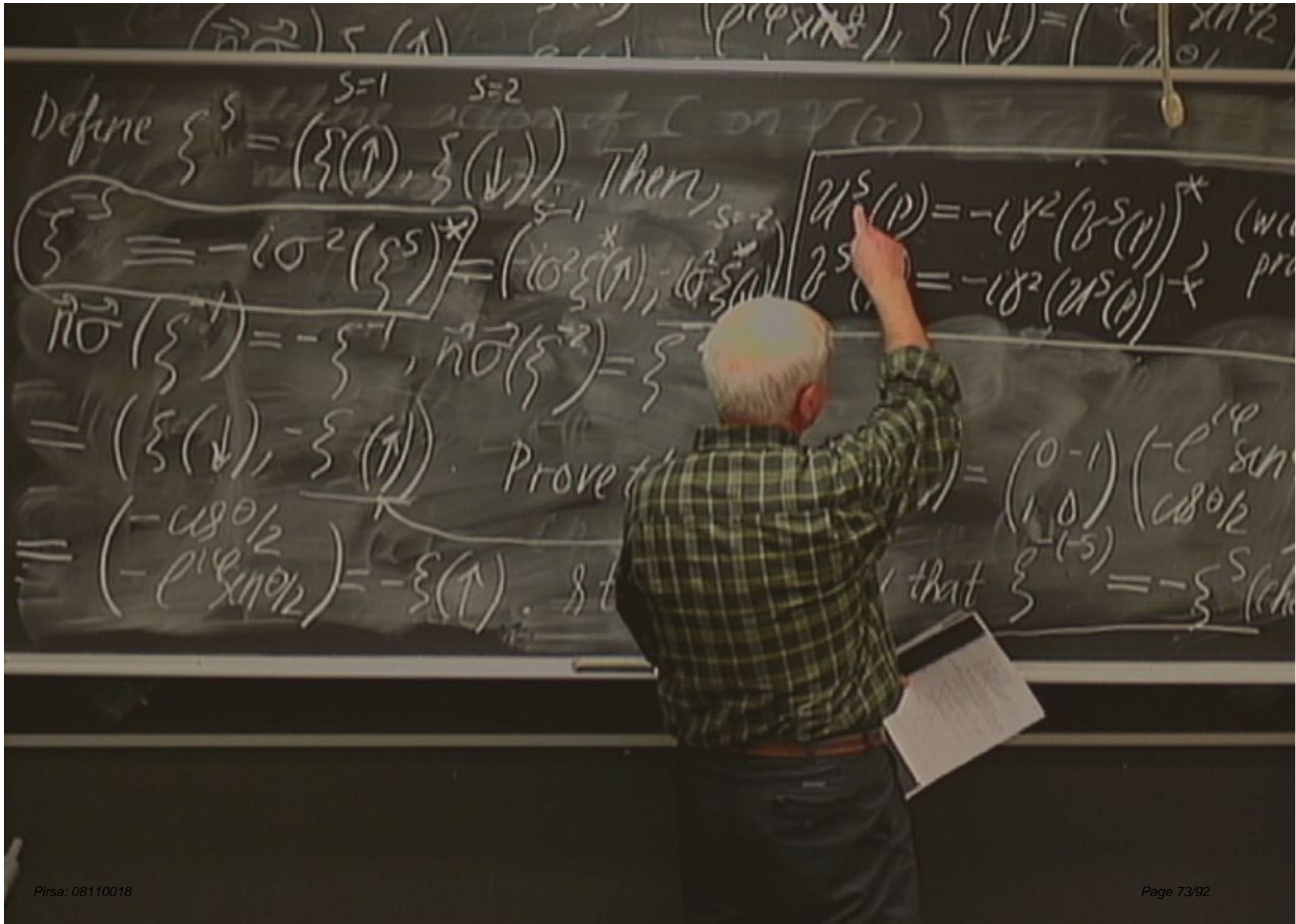


$$T^{-1} \bar{\psi} \gamma^5 \psi T = -i \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^5)^* (\gamma^1 \gamma^3) \psi (t-x) = \sqrt{-1} \bar{\psi} \gamma^5 \psi (t, \vec{x})$$

$$T^{-1} \bar{\psi} T T^{-1} \gamma^5 T T^{-1} \psi T$$

$$T^{-1} \bar{\psi} \gamma^{\mu} \psi T = \bar{\psi} (-\gamma^1 \gamma^3) (\gamma^{\mu})^* (\gamma^1 \gamma^3) \psi = \begin{cases} + \bar{\psi} \gamma^{\mu} \psi (t, \vec{x}), & \mu=0 \\ - \bar{\psi} \gamma^{\mu} \psi (t, \vec{x}), & \mu=1,2,3 \end{cases}$$





Define  $\xi^s = (\xi^{\uparrow}, \xi^{\downarrow})$  <sup>s=1</sup> <sup>s=2</sup>. Then,

$\xi^{-s} = -i\sigma^2 (\xi^s)^*$  <sup>s=1</sup> <sup>s=2</sup>

$\vec{n}\sigma (\xi^s) = -\xi^{-s}, \vec{n}\sigma (\xi^{-s}) = \xi^s$

$\vec{n}\sigma (\xi^{\uparrow}) = -\xi^{\downarrow}, \vec{n}\sigma (\xi^{\downarrow}) = \xi^{\uparrow}$

$\vec{n}\sigma (\xi^{\uparrow}) = -\xi^{\downarrow}, \vec{n}\sigma (\xi^{\downarrow}) = \xi^{\uparrow}$

Prove that  $\xi^{-s} = -i\sigma^2 (\xi^s)^*$

that  $\xi^{-s} = -i\sigma^2 (\xi^s)^*$

$$\begin{aligned} \mathcal{U}^s(p) &= -i\gamma^2 (\mathcal{U}^s(p))^* \\ \mathcal{V}^s(p) &= -i\gamma^2 (\mathcal{V}^s(p))^* \end{aligned}$$



$\vec{n} \cdot \vec{\sigma} = \begin{pmatrix} \cos \theta & e^{-i\phi} \sin \theta \\ e^{i\phi} \sin \theta & -\cos \theta \end{pmatrix}$

line  $\xi^s = (\xi^s(\uparrow), \xi^s(\downarrow))$ . Then,

$\xi^s = -i\sigma^2 (\xi^s)^*$

$\xi^s(\uparrow) = -\xi^s(\downarrow)^*$ ,  $\vec{n} \cdot \vec{\sigma} (\xi^s)^* = \xi^s$

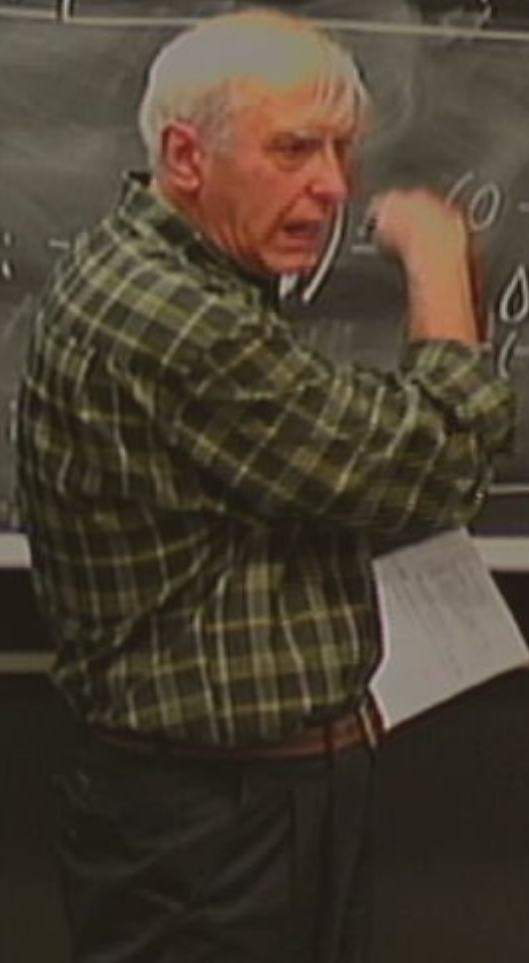
$(\xi^s(\downarrow), -\xi^s(\uparrow))$ . Prove this:

$\begin{pmatrix} -\cos \theta/2 \\ e^{i\phi} \sin \theta/2 \end{pmatrix} = -\xi^s(\uparrow)$ . It's easy

$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -e^{i\phi} \sin \theta/2 \\ \cos \theta/2 \end{pmatrix} = -\xi^s(\downarrow)$  (check!)

$\mathcal{U}^s(\rho) = -i\sigma^2 (\mathcal{U}^s(\rho))^*$  (without proof)

$\mathcal{U}^s(\rho) = -i\sigma^2 (\mathcal{U}^s(\rho))^*$



Define  $\xi^s = (\xi^{\uparrow}, \xi^{\downarrow})$  for  $s=1, 2$ . Then,

$$\xi^{-s} = -i\sigma^2 (\xi^s)^* = (-i\sigma^2 \xi^{\uparrow*}, -i\sigma^2 \xi^{\downarrow*})$$

$$\begin{aligned} \mathcal{U}^s(\rho) &= -i\gamma^2 (\mathcal{U}^s(\rho))^* \quad (\text{without proof}) \\ \mathcal{U}^s(\rho) &= -i\gamma^2 (\mathcal{U}^s(\rho))^* \end{aligned}$$

$$\vec{n}\sigma(\xi^{\uparrow}) = -\xi^{\downarrow}, \quad \vec{n}\sigma(\xi^{\downarrow}) = \xi^{\uparrow}$$

$$= (\xi^{\downarrow}, -\xi^{\uparrow}) \quad \text{Prove this}$$

$$= \begin{pmatrix} -\cos\theta/2 \\ -e^{i\phi} \sin\theta/2 \end{pmatrix} = -\xi^{\uparrow} \quad \text{etc}$$

$$\mathbb{U} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -e^{i\phi} \sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$$

$$\text{But } \xi^{(-s)} = -\xi^s \quad (\text{check!})$$



Define  $\xi^s = (\xi^s(\uparrow), \xi^s(\downarrow))$ . There,  $\xi^s = -i\sigma^2 (\xi^s)^*$ .  
 $\xi^s(\uparrow) = -i\sigma^2 \xi^s(\downarrow)^*$ ,  $\xi^s(\downarrow) = -i\sigma^2 \xi^s(\uparrow)^*$  (without proof)

$\vec{n}\sigma(\xi^s) = -\xi^{s-1}$ ,  $\vec{n}\sigma(\xi^{s+1}) = \xi^s$   
 $= (\xi^s(\downarrow), -\xi^s(\uparrow))$ . Prove this:  $-i\sigma^2 \xi^s(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -e^{i\varphi} \sin\theta/2 \\ \cos\theta/2 \end{pmatrix} = \begin{pmatrix} \cos\theta/2 \\ e^{i\varphi} \sin\theta/2 \end{pmatrix} = -\xi^s(\uparrow)$ . It easy to show that  $\xi^{s+1} = -\xi^s$  (check!)

$(-\ell^i \chi_{1/2}) = -\xi(\uparrow)$ . It's easy to show that  $\xi^{(+)}$   ~~$\neq$~~   $\xi^s$  (check!)

$a_{\vec{p}}$  destroys a fermion whose spinor is  $u^s(p)$ ,  $b_{\vec{p}}$  destroys antifermion whose spinor is  $v^s(p)$ .

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix} \quad v^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^{-s} \\ -\sqrt{p \cdot \bar{\sigma}} \xi^{-s} \end{pmatrix}$$

Without

$$\sigma^2 = \sigma^2 \sqrt{p \cdot \sigma^x}, \quad p = (p^0, -\vec{p}) \quad (I)$$



$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(1)} = \xi^S$  (check!)

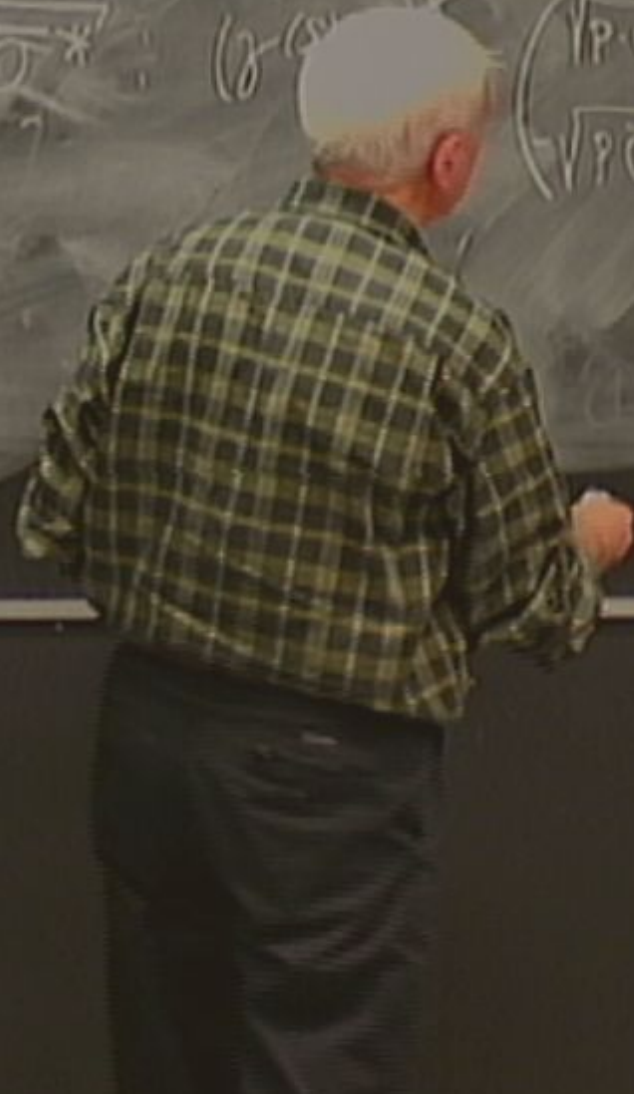
Proof of (II) used for charge conjugation:



$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(s)} = \xi^s$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$  :  $(\sigma^2)^{-1} \begin{pmatrix} \sqrt{p \cdot \sigma} & (-i\sigma^2 \xi^*) \\ \sqrt{p \cdot \sigma} & (-i\sigma^2 \xi) \end{pmatrix} \xi^s$





$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(s)} = \xi^s$  (check!)

Proof of (II) used for charge conjugation

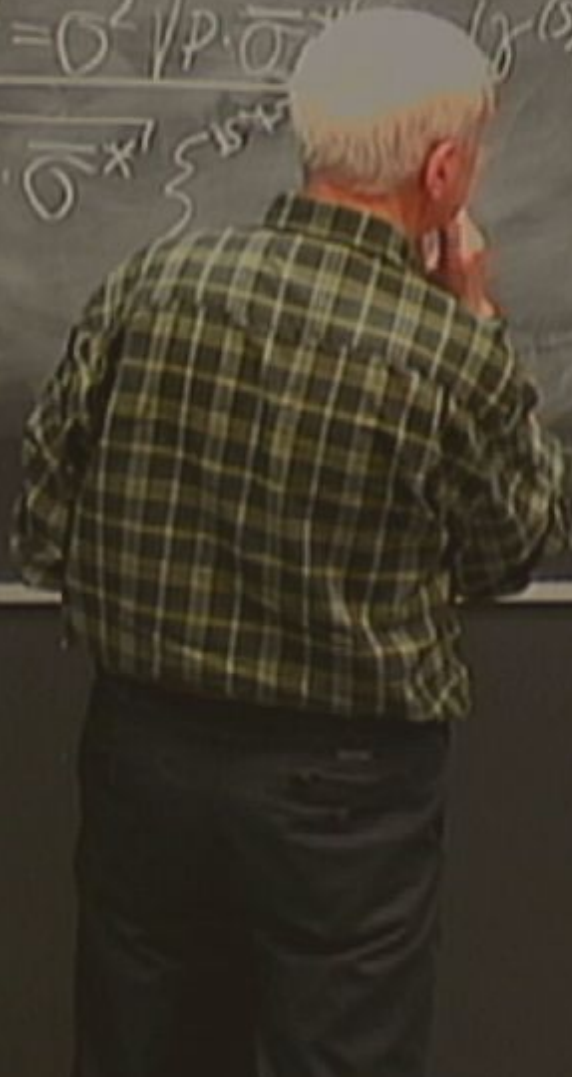
Use  $\frac{\sqrt{p \cdot \sigma} \cdot 0^2}{-i \sigma^2} \frac{p \cdot \sigma^*}{\sqrt{p \cdot \sigma}}$  :  $(\xi^{(s)}(p))^* = \begin{pmatrix} \sqrt{p \cdot \sigma} (-i \sigma^2 \xi^s)^* \\ \sqrt{p \cdot \sigma} (-i \sigma^2 \xi^s) \end{pmatrix} =$

$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(+)}$  ~~is~~  $\xi^s$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}$   $(\chi^{(s)}(p))^* = \begin{pmatrix} \sqrt{p \cdot \sigma} (-\sigma^2 \xi^s) \\ \sqrt{p \cdot \sigma} (-\sigma^2 \xi^{s*}) \end{pmatrix}$

$\underline{\underline{\begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma} \xi^{s*} \\ \sqrt{p \cdot \sigma} \xi^s \end{pmatrix}}}$



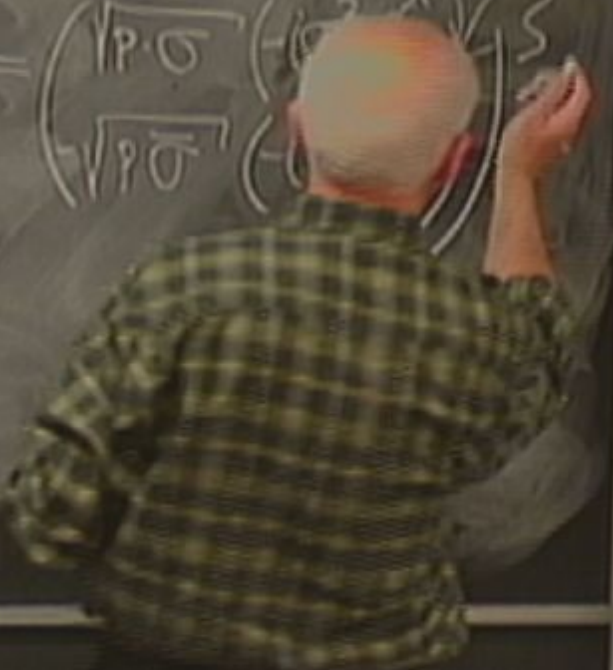


$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(s)} = \xi^s$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$  :  $(\xi^{(s)}(p))^* = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^{(s)*} \\ \sqrt{p \cdot \sigma} \xi^{(s)*} \end{pmatrix}$

$\underline{\underline{\begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{(s)*} \\ \xi^{(s)*} \end{pmatrix}}}$



$(-e^{i\pi/2}) = -i$ . It's easy to show that  $\xi^{(1)} = \xi^{(2)}$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}$  :  $(\psi^{(1)}(p))^* = \left( \begin{array}{c} \sqrt{p \cdot \sigma} (-i \sigma_2 \xi^*) \\ \sqrt{p \cdot \sigma} (-i \sigma_2 \xi^*) \end{array} \right)^*$

$\left( \begin{array}{c} -i \sigma_2 \sqrt{p \cdot \sigma} \xi \\ -i \sigma_2 \sqrt{p \cdot \sigma} \xi \end{array} \right)$





$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It's easy to show that  $\xi^{(1)} = \xi^5$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$  :  $(\sigma^{(1)}(p))^* = \sigma^2 (-\sigma^2 \xi^*)^*$

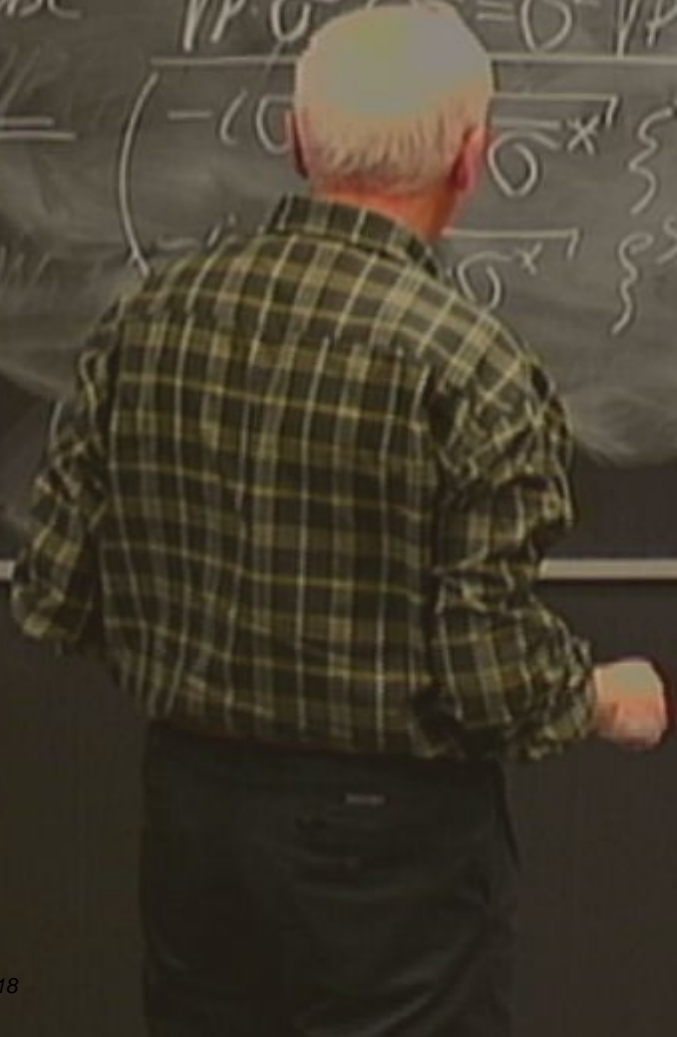
$$\begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{5*} \\ -\sigma^2 \sqrt{p \cdot \sigma} \xi^* \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^* \\ \sigma^2 \sqrt{p \cdot \sigma}^* \xi^{5*} \end{pmatrix}$$

$(-e^{i\pi/2}) = -\xi(\uparrow)$ . It easy to show that  $\xi^{(\downarrow)} = \xi^s$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$  :  $(\gamma^{(5)}(p))^* = \left( \begin{matrix} \sqrt{p \cdot \sigma} & (-i\sigma^2 \xi^s) \\ \sqrt{p \cdot \sigma} & (-i\sigma^2 \xi^s) \end{matrix} \right)^*$

$\left( \begin{matrix} -i\sigma^2 & \sqrt{p \cdot \sigma}^* \\ \sqrt{p \cdot \sigma} & 0 \end{matrix} \right)^* = \begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{p \cdot \sigma} & \xi^s \\ \sqrt{p \cdot \sigma} & \xi^s \end{pmatrix} = -i\gamma^2 U(p)$





$(-e^{i\pi/2}) = -i$ . It's easy to show that  $\xi^{(1)} = \xi^{(2)}$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$ :  $(\gamma^{(3)}(p))^* = \left( \begin{array}{c} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{array} \right)^*$

$$\begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{(1)*} \\ -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{(2)*} \end{pmatrix} = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = 2\sqrt{p \cdot \sigma} \xi^{(1)}$$

$(-e^{i\pi/2}) = -i$ . It's easy to show that  $\xi^{(1)} = \xi^S$  (check!)

Proof of (II) used for charge conjugation

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$

$$\begin{pmatrix} -i \sigma^2 \sqrt{p \cdot \sigma}^* \xi^{S*} \\ -i \sigma^2 \sqrt{p \cdot \sigma}^* \xi^{S*} \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{p \cdot \sigma} (-i \sigma^2 \xi^{S*}) \\ \sqrt{p \cdot \sigma} (-i \sigma^2 \xi^{S*}) \end{pmatrix}^* = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^S \\ \sqrt{p \cdot \sigma} \xi^S \end{pmatrix} = -i \gamma^2 U(p)$$



$$\begin{aligned}
\vec{n} \cdot \vec{\sigma} (\xi^\uparrow) &= -\xi^\uparrow, \quad \vec{n} \cdot \vec{\sigma} (\xi^\downarrow) = \xi^\downarrow \\
&= \begin{pmatrix} \xi^\downarrow \\ -\xi^\uparrow \end{pmatrix} \text{ Prove this: } -\sigma^2 \xi^\uparrow = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 \end{pmatrix} \\
&= \begin{pmatrix} -\cos \theta/2 \\ -\sin \theta/2 \end{pmatrix} = -\xi^\uparrow. \text{ It is easy to show that } \xi^\downarrow = \xi^\uparrow \text{ (check!)}
\end{aligned}$$

Proof of (II) used for charge conjugation:

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$

$$\begin{aligned}
&= \begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{s*} \\ -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{s*} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} (-\sigma^2 \xi^{s*}) \\ \sqrt{p \cdot \sigma} (-\sigma^2 \xi^{s*}) \end{pmatrix} \\
&= \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \sigma} \xi^s \end{pmatrix} = -\gamma^2 U(P)
\end{aligned}$$

$$\begin{aligned}
\vec{n} \cdot \vec{\sigma} (\xi^\uparrow) &= -\xi^\uparrow, \quad \vec{n} \cdot \vec{\sigma} (\xi^\downarrow) = \xi^\downarrow \\
&= (\xi^\downarrow, -\xi^\uparrow) \quad \text{Prove this: } -\left( \sigma^2 \xi^\downarrow \right) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\pi/2} \\ e^{i\pi/2} \end{pmatrix} \\
&= \begin{pmatrix} -e^{i\pi/2} \\ -e^{i\pi/2} \end{pmatrix} = -\xi^\uparrow. \quad \text{It's easy to show that } \xi^\downarrow = \xi^\uparrow \text{ (check!)}
\end{aligned}$$

Proof of (II) used for charge conjugation:

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$ :

$$\begin{aligned}
&= \frac{\begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{s*} \\ \sigma^2 \sqrt{p \cdot \sigma} \xi^s \end{pmatrix}^*}{\begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}} \begin{pmatrix} \sqrt{p \cdot \sigma} (-\sigma^2 \xi^{s*}) \\ \sqrt{p \cdot \sigma} \xi^s \end{pmatrix}^* \\
&= \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \sigma} \xi^s \end{pmatrix} = -\gamma^2 u(p)
\end{aligned}$$



$$\begin{aligned}
\vec{\pi} \cdot \vec{\sigma} (\xi^\dagger) &= -\xi^\dagger, \quad \vec{\pi} \cdot \vec{\sigma} (\xi) = \xi \\
&= \begin{pmatrix} \xi(\downarrow), -\xi(\uparrow) \end{pmatrix} \text{ Prove this: } -\begin{pmatrix} \sigma^2 \xi^\dagger(\downarrow) \\ \xi^\dagger(\uparrow) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\pi/2} \xi^\dagger(\downarrow) \\ e^{i\pi/2} \xi^\dagger(\uparrow) \end{pmatrix} \\
&= \begin{pmatrix} -e^{i\pi/2} \xi^\dagger(\downarrow) \\ e^{i\pi/2} \xi^\dagger(\uparrow) \end{pmatrix} = -\xi^\dagger(\uparrow). \text{ It's easy to show that } \xi^\dagger = \xi^S \text{ (check!)}
\end{aligned}$$

Proof of (II) used for charge conjugation:

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$ :

$$\begin{aligned}
\psi^{(S)}(p)^\dagger &= \left( \sqrt{p \cdot \sigma} \left( -\sigma^2 \xi^S \right) \right)^\dagger \\
&= \begin{pmatrix} -\sqrt{p \cdot \sigma}^* \xi^{S*} \\ \sqrt{p \cdot \sigma} \xi^S \end{pmatrix}^\dagger = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^S \\ \sqrt{p \cdot \sigma} \xi^S \end{pmatrix} = -\psi^{(A)}(p)
\end{aligned}$$

$$\begin{aligned}
\pi \vec{\sigma} (\xi^\dagger) &= -\xi^\dagger, \quad \pi \vec{\sigma} (\xi) = \xi \\
&= (\xi(\downarrow), -\xi(\uparrow)) \quad \text{Prove this: } -\sigma^2 \xi^\dagger(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi} \sin \theta/2 \\ e^{i\varphi} \cos \theta/2 \end{pmatrix} \\
&= \begin{pmatrix} -\cos \theta/2 \\ -e^{i\varphi} \sin \theta/2 \end{pmatrix} = -\xi(\uparrow). \quad \text{It is easy to show that } \xi = \xi^S \text{ (check!)}
\end{aligned}$$

Proof of (II) used for charge conjugation:

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$ :  $(\chi^{(S)}(p))^\dagger = \left( \sqrt{p \cdot \sigma} (-\sigma^2 \xi^S) \right)^\dagger$

$$\begin{aligned}
&\equiv \left( \begin{matrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{S*} \\ -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{S*} \end{matrix} \right)^\dagger = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^S \\ \sqrt{p \cdot \sigma} \xi^S \end{pmatrix} = -i \gamma^2 u(p) \\
&\Rightarrow \text{(II)}
\end{aligned}$$



$$\begin{aligned} \vec{n} \cdot \vec{\sigma} (\xi^\dagger) &= -\xi^\dagger, \quad \vec{n} \cdot \vec{\sigma} (\xi) = \xi \\ &= \begin{pmatrix} \xi(\downarrow), -\xi(\uparrow) \end{pmatrix} \text{ Prove this: } -\sqrt{\sigma^2} \xi^\dagger(\downarrow) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\phi} \sin \theta/2 \\ e^{i\phi} \cos \theta/2 \end{pmatrix} \\ &= \begin{pmatrix} -e^{i\phi} \cos \theta/2 \\ e^{i\phi} \sin \theta/2 \end{pmatrix} = -\xi(\uparrow). \text{ It is easy to show that } \xi^\dagger(\downarrow) = \xi^\dagger(\uparrow) \text{ (check!)} \end{aligned}$$

Proof of (II) used for charge conjugation:

Use  $\sqrt{p \cdot \sigma} \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^*$ :  $(\chi^{(S)}(p))^\dagger = \left( \sqrt{p \cdot \sigma} (-\sigma^2 \xi^{S\dagger}) \right)^\dagger$

$$\begin{aligned} &= \begin{pmatrix} -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{S\dagger} \\ -\sigma^2 \sqrt{p \cdot \sigma}^* \xi^{S\dagger} \end{pmatrix}^\dagger = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^S \\ \sqrt{p \cdot \sigma} \xi^S \end{pmatrix} = -i \gamma^2 u(p) \end{aligned}$$

$\Rightarrow$  (II)