

Title: Quantum Field Theory 1 - Lecture 10B

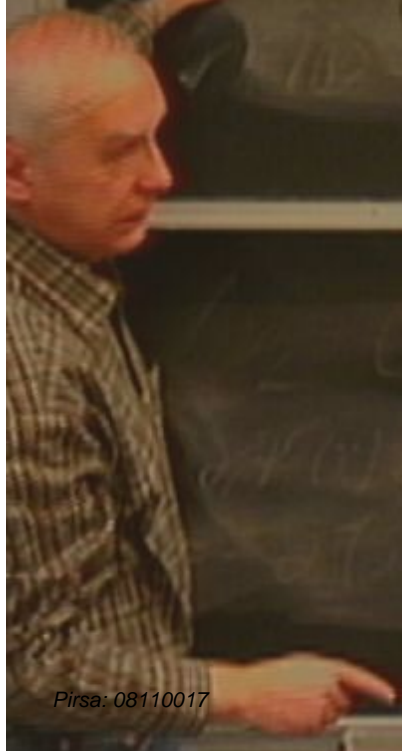
Date: Nov 12, 2008 03:30 PM

URL: <http://pirsa.org/08110017>

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

$$J_z^{orb} = J_z = i\vec{r} \times \nabla \psi = -i\vec{r} \times \nabla \psi = -i\vec{r} \times (\hat{x}\partial_y - \hat{y}\partial_x + \hat{z}\partial_z)\psi$$

$$\vec{J} = \frac{1}{\hbar} \left[\underbrace{\vec{r} \times (-i\vec{\nabla})}_{\text{orbital angular momentum}} + \frac{\hbar}{2} \vec{\sigma} \right] \psi = \psi \left(\vec{r} \times (-i\vec{\nabla}) + \frac{\hbar}{2} \vec{\sigma} \right)$$



P, C, T Transformations

P, C, T Transformations
Parity Transformation (P)
 $x = (t, \vec{x})$

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x})$$

$$\psi'(x) = P^{-1} \psi(x) P$$

Parity transformation (P)

$$P: x = (t, \vec{x}) \xrightarrow{P} \vec{x} = (t, -\vec{x})$$

$$\psi'(x) = P^{-1} \psi(x) P = \psi(P^{-1}x)$$

Parity transformation (P)

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x})$$

$$\psi'(x) = P^{-1} \psi(x) \quad \Lambda_P \psi(P^{-1}x)$$

Lorentz transformation (P)

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x})$$

$$\psi'(x) = P^{-1} \psi(Px) = \Lambda_P \psi(Px)$$

Parity transformation (P)

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x}), \quad \tilde{P} = P$$

$$\psi'(x) = P^{-1} \psi(x) P = \Lambda_P \psi(Px)$$

Parity transformation (P)

$$P: x = (t, \vec{x}) \xrightarrow{P} \vec{x}' = (t, -\vec{x}), \quad P^{-1} = P$$

$$\psi'(x) = P^{-1} \psi(x) P = \psi(Px)$$

Parity transformation (P)

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x}), \quad P^{-1} = P, \quad P^2 = I$$

$$\psi'(x) = P^{-1} \psi(x) \quad \Psi(P^{-1}x) = A_P \Psi(Px)$$

$$x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x}), \quad P^{-1} = P, \quad P^2 = I$$

$$\psi^{-1}(x) = P^{-1} \psi(Px) = \Lambda_P \psi(Px) = \Lambda_P \psi(x) = \Lambda_{P^{-1}}(\tilde{x})$$

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x}), \quad P^{-1} = P, \quad P^2 = I$$

$$\psi(\tilde{x}) = P^{-1} \psi(x) P = \Lambda_P \psi(P^{-1}x) = \Lambda_P \psi(Px) = \Lambda_P \psi(x)$$

$$x \rightarrow \tilde{x}, \quad (P^2)^{-1} \psi(x) P^2 = \Lambda_P^2 \psi(x)$$

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x}), \quad P^{-1} = P, \quad P^2 = I$$

$$\psi^{-1}(x) \xrightarrow{P^{-1}} \psi(x) P = \Lambda_P \Psi(P^{-1}x) = \Lambda_P \Psi(Px) = \Lambda_P \Psi(x) = \Lambda_P \Psi(\tilde{x})$$

$$P^2: \psi(x) \xrightarrow{P^2} \psi(x) P^2 = \Lambda_P^2 \Psi(x) = \Lambda_P \Psi(x)$$

$$P: x = (t, \vec{x}) \xrightarrow{P} \tilde{x} = (t, -\vec{x}), \quad P^{-1} = P,$$

$$\psi'(x) = P^{-1} \psi(x) P = \Lambda_P \Psi(P^{-1}x) = \Lambda_P \Psi(Px)$$

$$P^2: x \rightarrow x, \quad (P^2)^{-1} \psi(x) P^2 = \Lambda_P^2 \Psi(x) = \pm \psi(x)$$

$$\psi'(x) = P^{-1} \psi(x) P = \Lambda_P \psi(P^{-1}x) = \Lambda_P \psi(Px) = \Lambda_P \psi(x)$$

$$P^2: x \rightarrow x, (P^2)^{-1} \psi(x) P^2 = \Lambda_{P^2} \psi(x) = \pm \psi(x), \Lambda_{P^2} = \pm 1$$

$$P^2: x \rightarrow x; (P^2)\psi(x) = \Lambda_P^2 \psi(x) = \pm \psi(x), \Lambda_P = \pm 1$$



P, C, T Transformations

Parity Transformation (P)

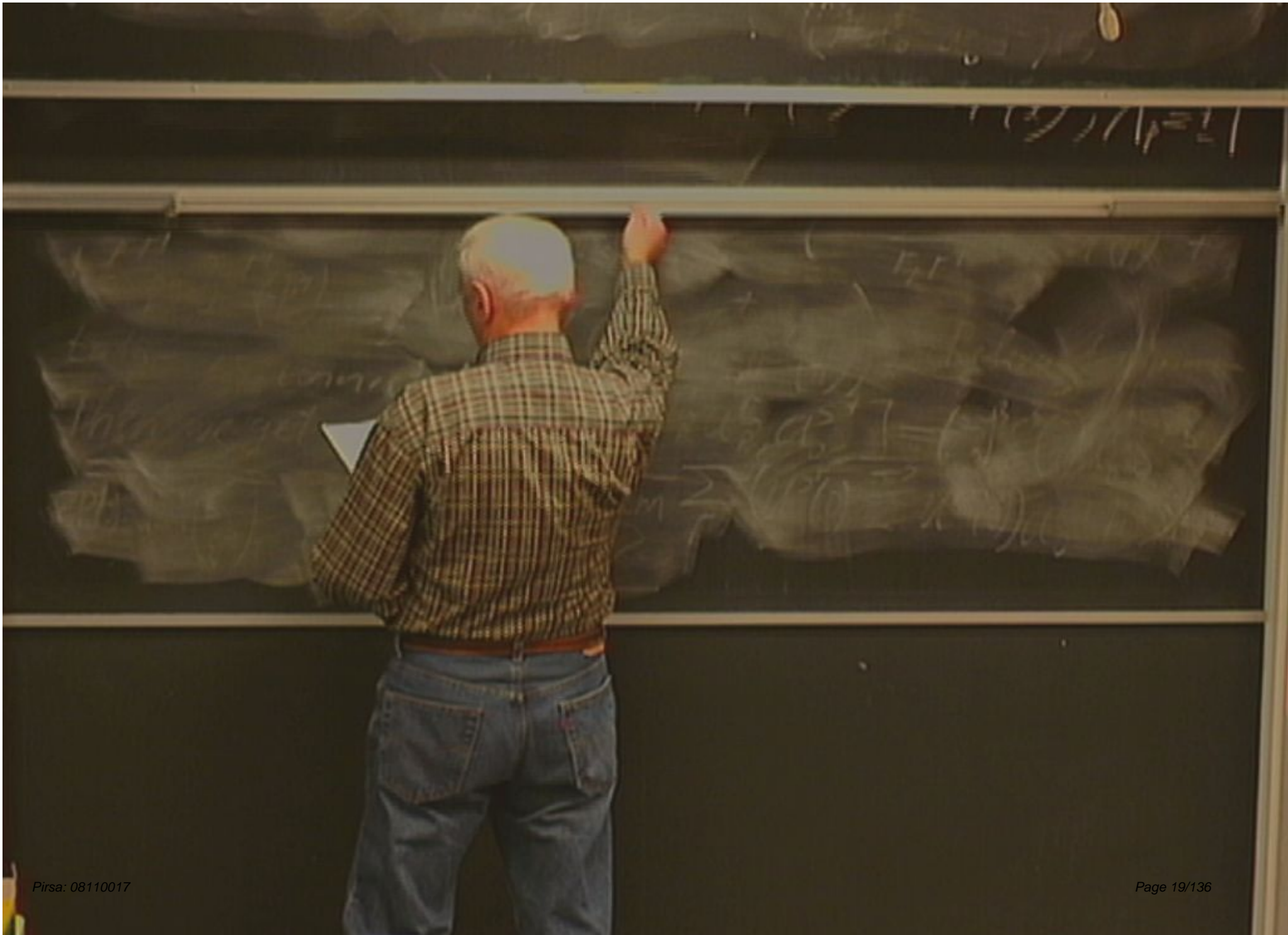
$$P: x = (t, \vec{x}) \xrightarrow{P} (t, -\vec{x}), \quad P^{-1} = P, \quad P^2 = I$$

$$\psi'(x) = P^{-1} \psi(x)$$

$$\psi(Px) = \Lambda_P \psi(x) = \Lambda_P \psi(\vec{x})$$

$$P^2: x \rightarrow x$$

$$P^2 \psi(x) = \pm \psi(x), \quad \Lambda_P^2 = \pm 1$$



$$J_z^{02} = J_z = i \vec{\psi} \gamma^0 \nabla \psi = -i \bar{\psi} \gamma^0 (x \partial_y - y \partial_x + \frac{1}{2} \Sigma^3) \psi$$

$$\vec{J} = \bar{\psi} \gamma^0 \left(\underbrace{\vec{x} \times (-i \vec{\nabla})}_{\text{orbital angular momentum}} + \frac{1}{2} \vec{\Sigma} \right) \psi = \psi^\dagger (\vec{x} \times (-i \vec{\nabla}) + \frac{1}{2} \vec{\Sigma}) \psi$$

To find Λ_p , use invariance of Dirac equation
under P .

$$(i\gamma^\mu \partial_\mu - \Lambda_P \Psi(x)) = \pm \Psi(x) \quad \Lambda_P^2 = \pm 1$$

To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\Psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \Psi(x) =$$



To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\tilde{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$



To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\tilde{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma} \implies$

$$\implies i\gamma^\mu \partial_\mu$$

To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\bar{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$ \Rightarrow

$$\Rightarrow i\gamma^\mu \partial_\mu \Lambda_P \psi(\bar{x}) =$$



To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\tilde{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma} \Rightarrow$

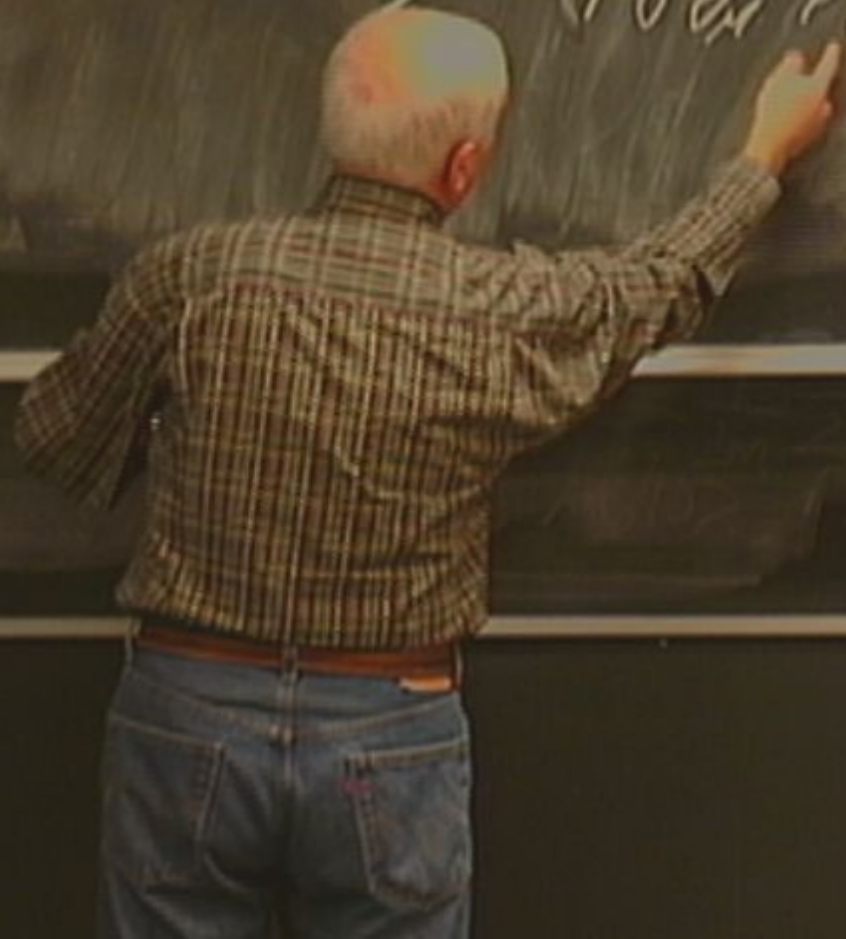
$$\Rightarrow i\gamma^\mu \partial_\mu \Lambda_P \psi(\tilde{x}) = \Lambda_P$$

To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\bar{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma_0 \Lambda_P = \gamma_0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$

$$\Rightarrow (i\gamma^\mu \partial_\mu \Lambda_P \psi(\bar{x})) = \Lambda_P (i\gamma^\mu \partial_\mu \psi(\bar{x}))$$



To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\bar{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma_0 \Lambda_P = \gamma_0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$

$$\Rightarrow i\gamma^\mu \partial_\mu \Lambda_P \psi(\bar{x}) = \Lambda_P i\gamma^\mu \partial_\mu \psi(\bar{x})$$



To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi'(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\bar{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$ \Rightarrow

$$\Rightarrow i\gamma^\mu \partial_\mu \Lambda_P \psi(\bar{x}) = \Lambda_P (i\gamma^\mu \partial_\mu \psi(\bar{x}))$$

To find Λ_P , use invariance of Dirac equation under P .

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\bar{x}) =$$

Take such a Λ_P that $\Lambda_P^{-1} \gamma_0 \Lambda_P = \gamma_0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$

$$\Rightarrow (i\gamma^\mu \partial_\mu \Lambda_P \psi(\bar{x})) = \Lambda_P (i\gamma^\mu \partial_\mu \psi(\bar{x})) \Rightarrow \Lambda (i\gamma^\mu \partial_\mu - m)\psi(\bar{x}) = 0$$



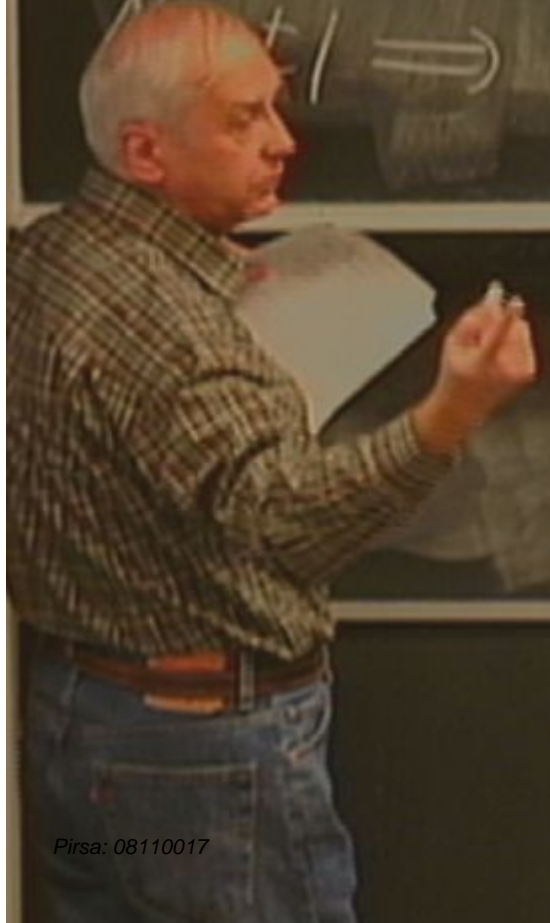
to find Λ_P , use invariance of Dirac equation under P . $(i\gamma^\mu \partial_\mu - m)\psi(\vec{x}) = (i\gamma^\mu \partial_\mu - m)\Lambda_P \psi(\vec{x}) = 0$

Take such a Λ_P that $\Lambda_P^{-1} \gamma_0 \Lambda_P = \gamma_0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma} \Rightarrow$

$\Rightarrow i\gamma^\mu \partial_\mu \Lambda_P \psi(\vec{x}) = \Lambda_P i\gamma^\mu \partial_\mu \psi(\vec{x}) \Rightarrow \Lambda_P (i\gamma^\mu \partial_\mu - m)\psi(\vec{x}) = 0$

$\Lambda_P = \eta_P \gamma_0$ satisfies this constraint.

$P^2 = 1 \Rightarrow$



To find Λ_p , use invariance of Dirac equation under P . $(i\gamma^\mu \partial_\mu - m)\psi(\vec{x}) = (i\gamma^\mu \partial_\mu - m)\Lambda_p \psi(\vec{x}) =$
 Take such a Λ_p that $\Lambda_p^{-1} \gamma_0 \Lambda_p = \gamma_0$, $\Lambda_p^{-1} \vec{\gamma} \Lambda_p = -\vec{\gamma} \implies$
 $\implies (i\gamma^\mu \partial_\mu \Lambda_p \psi(\vec{x})) = \Lambda_p (i\gamma^\mu \partial_\mu \psi(\vec{x})) \implies \Lambda_p (i\gamma^\mu \partial_\mu - m)\psi(\vec{x}) = 0$
 $\Lambda_p = \eta_p \gamma_0$ satisfies this constraint.
 $\Lambda_p^2 = \pm 1 \implies \eta_p^2 = \pm 1 \implies \eta_p = \pm 1 \text{ or } \pm i$

... such a Λ_P that $\Lambda_P \gamma^0 \Lambda_P^{-1} = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma} \Rightarrow$
 $\Rightarrow (\gamma^0 \vec{\gamma} \cdot \vec{x} \Lambda_P \psi(\vec{x})) = \Lambda_P (\gamma^0 \vec{\gamma} \cdot \vec{x} \psi(\vec{x})) \Rightarrow \Lambda_P (\gamma^0 \vec{\gamma} \cdot \vec{x} - m) \psi(\vec{x}) = 0$
 $\Lambda_P = \eta_P \gamma^0$ satisfies this constraint.
 $\Lambda_P^2 = \pm 1 \Rightarrow \eta_P^2 = \pm 1 \Rightarrow \eta_P = \pm 1$ or $\pm i$. $\psi_x = \eta_P \gamma^0 \psi(t, -\vec{x})$

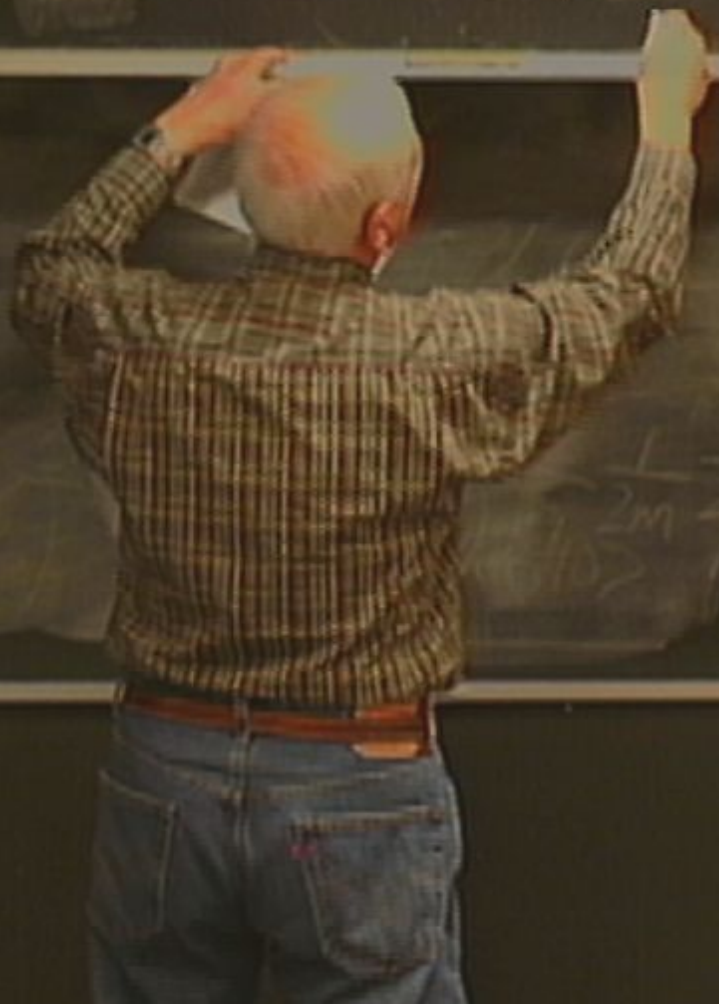


$\Rightarrow (\gamma^m \partial_m \Lambda_P \psi(\vec{x}) = \Lambda_P (\gamma^m \partial_m \psi(\vec{x}) - m \psi(\vec{x})) = 0$

$\Lambda_P = \eta_P \gamma^0$ satisfies this constraint.

$\Lambda_P^2 = \pm 1 \Rightarrow \eta_P^2 = \pm 1 \Rightarrow \eta_P = \pm 1 \text{ or } \pm i$.

$\psi_{\vec{x}} = \eta_P \gamma^0 \psi(t, -\vec{x})$



$\psi = \eta_p \gamma^0$ satisfies this constraint.
 $\eta_p^2 = \pm 1 \implies \eta_p = \pm 1 \implies \boxed{\eta_p = \pm 1 \text{ or } \pm i}$. $\boxed{\psi_x = \eta_p \gamma^0 \psi(1, -\vec{x})}$



$$\eta_P = \pm 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i} \cdot \boxed{\psi_x' = \eta_P \gamma^0 \psi(t, -\vec{x})}$$

$P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\vec{x})$; $\eta_P = \pm 1, \pm i$. What are transformations of a_p^s, b_p^s under P ?

$$\eta_P = \pm 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i} \quad \boxed{\psi'_x = \eta_P \delta^0 \psi(l, -\vec{x})}$$

$$\psi'(x) = P^{-1} \psi(x) P = \eta_P \delta^0 \psi(\vec{x}); \quad \eta_P = \pm 1, \pm i \quad (\text{I}) \quad \text{What are trans-}$$

formations of a_p^s, b_p^s under P ?

From (I)

$$\sum_{s \in \mathbb{Z}_2} (\bar{P} a_p^s P U(P) e^{-i p x} + P b_p^s P e^{i p x}) =$$

$$|\eta_P| = \pm 1 \implies \eta_P = \pm 1 \implies \boxed{\eta_P = \pm 1 \text{ or } \pm i} \quad \boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\vec{x})$; $\eta_P = \pm 1, \pm i$ (I) What are transformation
 of a_p^s, b_p^s under P ? $\mathcal{U}^S(P)$

From (I), $\int \frac{d^3x}{(2\pi)^3} \sum_s \left(\bar{P} a_p^s P \mathcal{U}^S(P) e^{-i p x} + P b_p^s P \mathcal{U}^S(P) e^{i p x} \right) =$

$$= \eta_P \int \frac{d^3x}{(2\pi)^3} \sum_s \left(\bar{P} a_p^s P \mathcal{U}^S(P) e^{-i p x} + P b_p^s P \mathcal{U}^S(P) e^{i p x} \right) =$$

$$|\eta_P| = \pm 1 \implies \eta_P = \pm 1 \implies \boxed{\eta_P = \pm 1 \text{ or } \pm i} \quad \boxed{\psi'_x = \eta_P \delta^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_P \delta^0 \psi(\vec{x})$; $\eta_P = \pm 1, \pm i$ (I) What are transformation of $a_{\vec{p}}^s, b_{\vec{p}}^s$ under P ? $\mathcal{U}^S(P)$

From (I), $\int \frac{d^3 p}{(2\pi)^3} \left(P^{-1} a_{\vec{p}}^s P U(P) e^{-i p x} + P b_{\vec{p}}^s P e^{i p x} \right) =$

$$= \eta_P \int \frac{d^3 p}{(2\pi)^3}$$

$$|\eta_P| = 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i} \quad \boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What is the transformation of $a_{\vec{p}}^s, b_{\vec{p}}^s$ under P ?

From (I),

$$\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (\tilde{P} a_{\vec{p}}^s P U(P) e^{-i p x} + \tilde{P} b_{\vec{p}}^s P V(P) e^{i p x})$$

$$= \eta_P \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s \gamma^0 U(P) e^{-i p \tilde{x}} + \tilde{P} b_{\vec{p}}^s V(P) e^{i p \tilde{x}})$$

$$|\eta_P| = 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i} \quad \boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What are transformation of $a_{\vec{p}}^s, b_{\vec{p}}^s$ under P ? $\mathcal{U}^S(P)$

From (I),

$$\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s \left(\tilde{P} a_{\vec{p}}^s P \mathcal{U}^S(P) e^{-i p x} + \tilde{P} b_{\vec{p}}^s P \mathcal{U}^S(P) e^{i p x} \right) =$$

$$= \eta_P \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s \left(a_{\vec{p}}^s \gamma^0 \mathcal{U}^S(P) e^{-i p \tilde{x}} + \tilde{P} b_{\vec{p}}^s P \gamma^0 \mathcal{U}^S(P) e^{i p \tilde{x}} \right)$$

$$|\eta_P| = 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i} \quad \boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What are transformation of $a_{\vec{p}}^s, b_{\vec{p}}^s$ under P ? $\mathcal{U}^S(P)$

From (I),
$$\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (P^{-1} a_{\vec{p}}^s P \mathcal{U}^S(P) e^{-i p x} + P^{-1} b_{\vec{p}}^s P \mathcal{U}^S(P) e^{i p x}) =$$

$$= \eta_P \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s \gamma^0 \mathcal{U}^S(P) e^{-i p \tilde{x}} + P^{-1} b_{\vec{p}}^s P \gamma^0 \mathcal{U}^S(P) e^{i p \tilde{x}})$$

change



[Faded handwritten text on a chalkboard, including the word 'change' and various mathematical or technical notations.]

change

$$P \rightarrow \tilde{P} = (P^0, -\vec{P}) \quad \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}}$$

T change

$$P \rightarrow \tilde{P} = (P^0, -\vec{P})$$

$$\tilde{P} \cdot \tilde{\chi} = P \cdot \chi$$

$$\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \chi_{021}^s(\tilde{p}))$$



T change

$$P \rightarrow \tilde{P} = (P^0, -\vec{P})$$

$$\tilde{P} \cdot \tilde{\chi} = P \cdot \chi$$

$$\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \chi_{0215}(\tilde{p})) e^{-i\tilde{p} \cdot \tilde{\chi}}$$

spin is one bit

+ it orders

$$|p| = 1 \Rightarrow \eta_p = \pm 1 \Rightarrow \boxed{\eta_p = \pm 1 \text{ or } \pm i} \quad \boxed{\psi_x = \eta_p \gamma^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_p \gamma^0 \psi(\tilde{x})$; $\eta_p = \pm 1, \pm i$ (I) What are the transformation of a_p^s, b_p^s under P ?

From (I),

$$\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(\psi(p) e^{-ipx} + P b_p^s P^{-1} e^{ipx} \right) = \eta_p \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(\psi(\tilde{p}) e^{-i\tilde{p}\tilde{x}} + \eta_p \gamma^0 b_{\tilde{p}}^s \gamma^0 e^{i\tilde{p}\tilde{x}} \right)$$

$$|p| = -1 \Rightarrow \eta_p = \pm 1 \Rightarrow \boxed{\eta_p = \pm 1 \text{ or } \pm i} \quad \boxed{\psi_x = \eta_p \gamma^0 \psi(t, -\vec{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_p \gamma^0 \psi(\tilde{x})$; $\eta_p = \pm 1, \pm i$ (I) What are the transformation of a_p^s, b_p^s under P ? $\mathcal{U}(P)$

From (I),
$$\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(\bar{P} a_p^s P \mathcal{U}(P) e^{-i p x} + P b_p^s P \mathcal{U}(P) e^{i p x} \right) =$$

$$= \eta_p \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(a_p^s \gamma^0 \mathcal{U}(P) e^{-i p \tilde{x}} + b_p^s \gamma^0 \mathcal{U}(P) e^{i p \tilde{x}} \right)$$

T change

$(\beta \tilde{p})$

$$P \rightarrow \tilde{P} = (P_0, -\tilde{P})$$

$$\sum_s (a_{\tilde{p}}^s \gamma_{0123}(p)) e^{-i\tilde{p}x} + b_{\tilde{p}}^{st} \gamma_{0123}(p) e^{i\tilde{p}x} =$$

spin is one half

change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \gamma^0 u^s(\tilde{p}))$ $\frac{1}{2} \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (b_{\tilde{p}}^{s\dagger} \gamma^0 v^s(\tilde{p}))$

$p \rightarrow \tilde{p} = (p^0, -\vec{p})$

$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p$

$\hbar/2 \rightarrow$ spin is one half

change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^{s\dagger})^\dagger \dots$

$P \rightarrow \tilde{P} = (P^0, -\vec{P})$

$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p$

$\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \dots$

$-i \tilde{p} \cdot \tilde{x} + b_{\tilde{p}}^{s\dagger} \gamma^0 \gamma^s(p) l^{\tilde{p} \cdot \tilde{x}} =$



change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \gamma^0 u^s(\tilde{p})) e^{-i(\tilde{p} \cdot \tilde{x})} \stackrel{st}{=} \int \frac{d^3 p}{(2\pi)^3 \sqrt{E_p}} \sum_s (a_p^s \gamma^0 u^s(p)) e^{-i(p \cdot x)}$

$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p, \quad \gamma^0 u^s(\tilde{p}) = \gamma^0 u^s(p)$

change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \gamma^0 u(\tilde{p})) e^{-i\tilde{p}\tilde{x}} + \dots =$

$p \rightarrow \tilde{p} = (p^0, -\vec{p})$

$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \sqrt{E_{\tilde{p}}} = \dots$

change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \gamma^0 u^s(\tilde{p})) e^{-i(\tilde{p}\tilde{x} - \tilde{p}^0 \tilde{t})} = \int \frac{d^3 p}{(2\pi)^3 \sqrt{E_p}} \sum_s (a_p^s \gamma^0 u^s(p)) e^{-i(p\cdot x - p^0 t)}$

$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tilde{p}^0 = \sqrt{|\tilde{p}|^2 + m^2}$ $\tilde{x} = \begin{pmatrix} \vec{x} \\ t \end{pmatrix}$ $\tilde{p} = \begin{pmatrix} \vec{p} \\ p^0 \end{pmatrix}$

$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 u^s(\tilde{p}) = \sqrt{\frac{E_p + m}{2m}} \begin{pmatrix} 1 \\ \frac{\vec{p} \cdot \vec{\sigma}}{E_p + m} \end{pmatrix} = \sqrt{\frac{E_p + m}{2m}} \begin{pmatrix} 1 \\ \frac{\vec{p} \cdot \vec{\sigma}}{E_p + m} \end{pmatrix}$

change

$$(P^0, -\vec{P}) \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \gamma^0 u^s(\tilde{p}) e^{-i\tilde{p}\cdot x} + b_{\tilde{p}}^{s\dagger} \gamma^0 v^s(\tilde{p}) e^{i\tilde{p}\cdot x}) =$$
$$\vec{x} = P \cdot x, \quad d^3 \tilde{p} = d^3 P; \quad \gamma^0 u^s(\tilde{P}) = \gamma^0 \begin{pmatrix} \sqrt{P \cdot \vec{\sigma}} \\ \sqrt{P \cdot \vec{\sigma}} \end{pmatrix} =$$

$\gamma^0 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ why \Rightarrow spin is one half

change $\int \frac{d^3\tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}}$ $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\hbar/2 \rightarrow$ spin is one half

$$P \rightarrow \tilde{P} = (P^0, -\vec{P}) \quad \int \left(a_{\tilde{p}}^s \gamma^0 u^s(\tilde{p}) e^{-i\tilde{p}\tilde{x}} + b_{\tilde{p}}^{s\dagger} \gamma^0 v^s(\tilde{p}) e^{i\tilde{p}\tilde{x}} \right) =$$

$$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad \gamma^0 u^s(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} =$$

change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (a_{\tilde{p}}^s \gamma^0 u^s(\tilde{p}) e^{-i\tilde{p}\tilde{x}} + b_{\tilde{p}}^{s\dagger} \gamma^0 v^s(\tilde{p}) e^{i\tilde{p}\tilde{x}})$

$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\hbar/2 \rightarrow$ spin is one half

$\tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 u^s(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix}$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

T-change $(d^3\tilde{p})$ $\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & \sigma^0 \end{pmatrix}$ \rightarrow spin is one half

$$P \rightarrow \tilde{P} = (P^0, -\vec{P})$$

$$= \sum_{\vec{p}, \vec{p}'} \left(a_{\vec{p}}^s \gamma^0 u^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} + b_{\vec{p}}^{st} \gamma^0 v^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}} \right)$$

$$d^3\tilde{p} = d^3P; \quad \gamma^0 u^s(\tilde{P}) = \gamma^0 \begin{pmatrix} \sqrt{P^0} \chi^s \\ \sqrt{P^0} \chi^s \end{pmatrix}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

T. change

$$P \rightarrow \tilde{P} = (P^0, -\vec{P}) \quad \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_{\lambda} (a_{\tilde{p}\lambda}^\dagger \gamma^0 \gamma^{\lambda} \tilde{p}) e^{-i\tilde{p}\tilde{x}} + b_{\tilde{p}\lambda}^{st} \gamma^0 \gamma^{\lambda} (\tilde{p}) e^{i\tilde{p}\tilde{x}} =$$

$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ \rightarrow spin is one half

$$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p$$

$$= \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \bar{\sigma}} \\ \sqrt{p \cdot \sigma} \end{pmatrix}$$

$$(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} =$$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

change $\int \frac{d^3\tilde{p}}{(2\pi)^3} \dots$ $\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & \sigma^0 \end{pmatrix}$ \rightarrow spin is one half

$$P \rightarrow \tilde{P} = (P^0, -\vec{P}) \quad \int \frac{d^3\tilde{p}}{(2\pi)^3} \left(a_{\vec{p}}^s \gamma^0 u^s(\vec{p}) e^{-i\tilde{p}\tilde{x}} + b_{\vec{p}}^{st} \gamma^0 v^s(\vec{p}) e^{i\tilde{p}\tilde{x}} \right) =$$

$$\tilde{P} \cdot \tilde{x} = P \cdot x = dP; \quad \gamma^0 u^s(\tilde{P}) = \gamma^0 \begin{pmatrix} \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \bar{\sigma}} \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \bar{\sigma}} \end{pmatrix} = u^s(P)$$

$$\sigma = (\sigma^0, \vec{\sigma}), \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$P = (P, -\vec{P}) \quad (2.11) \quad \frac{1}{2} (\not{P} + \not{\sigma} \cdot \vec{P}) \quad + \not{P} \not{\sigma} \cdot \vec{P}$$

$$\begin{aligned} \bar{P} \cdot \vec{X} &= P \cdot X, \quad d^3 \vec{P} = d^3 P, \quad \gamma^0 \gamma^i (\vec{P}) = \gamma^0 \begin{pmatrix} \sqrt{P \cdot \vec{\sigma}} \\ \sqrt{P \cdot \vec{\sigma}} \end{pmatrix} \\ &= \begin{pmatrix} \sqrt{P \cdot \vec{\sigma}} \\ \sqrt{P \cdot \vec{\sigma}} \end{pmatrix} = \begin{pmatrix} \sqrt{P \cdot \vec{\sigma}} \\ \sqrt{P \cdot \vec{\sigma}} \end{pmatrix} = \gamma^0 \begin{pmatrix} \sqrt{P \cdot \vec{\sigma}} \\ \sqrt{P \cdot \vec{\sigma}} \end{pmatrix} \end{aligned}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned} \tilde{p} \cdot \tilde{x} &= p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \mathcal{U}^S(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \vec{\sigma}} \\ \sqrt{\tilde{p} \cdot \bar{\sigma}} \end{pmatrix} = \\ &= \begin{pmatrix} \sqrt{\tilde{p} \cdot \vec{\sigma}} \\ \sqrt{\tilde{p} \cdot \bar{\sigma}} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \vec{\sigma}} \\ \sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = \mathcal{U}^S(p) \end{aligned}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned} \tilde{p} \cdot \tilde{\chi} &= p \cdot \chi, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \mathcal{U}^s(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \\ &= \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \bar{\sigma}} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^s(p) \end{aligned}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

change $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (\dots) \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$P \rightarrow \tilde{P} = (P^0, -\vec{P}) \quad \int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{E_{\tilde{p}}}} \sum_s (\dots) \gamma^0 = \int \frac{d^3 p}{(2\pi)^3 \sqrt{E_p}} \sum_s (\dots) \gamma^0 =$$

$$\tilde{P} \cdot \tilde{\chi} = P \cdot \chi, \quad d^3 \tilde{p} = \frac{d^3 p}{\sqrt{E_p}}$$

$$= \left(\frac{\sqrt{\tilde{P} \cdot \bar{\sigma}}}{\sqrt{\tilde{P} \cdot \sigma}} \right) = \left(\frac{\sqrt{P \cdot \sigma}}{\sqrt{P \cdot \bar{\sigma}}} \right) = \gamma^0 \begin{pmatrix} \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \bar{\sigma}} \end{pmatrix} = \gamma^0 \begin{pmatrix} \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \bar{\sigma}} \end{pmatrix} =$$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned}
 & p \cdot \tilde{\chi} = p \cdot \chi, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \mathcal{U}^S(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \\
 & = \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \bar{\sigma}} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) =
 \end{aligned}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned}
 & p \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \mathcal{U}^S(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = \\
 & \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) = \gamma^0 \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \bar{\sigma}} \end{pmatrix} =
 \end{aligned}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \quad \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned}
 & p \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \mathcal{U}^S(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} \\
 & = \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \bar{\sigma}} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) = \gamma^0 \begin{pmatrix} \sqrt{p \cdot \bar{\sigma}} \\ \sqrt{p \cdot \sigma} \end{pmatrix} \\
 & = \begin{pmatrix} \sqrt{p \cdot \bar{\sigma}} \\ -\sqrt{p \cdot \sigma} \end{pmatrix}
 \end{aligned}$$



$$\sigma = (\sigma^0, \vec{\sigma}), \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned}
 & p \cdot x = \tilde{p} \cdot x, \quad d^3 \tilde{p} = d^3 p, \quad \gamma^0 \mathcal{U}(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \\
 & = \begin{pmatrix} \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}(p), \quad \gamma^0 \mathcal{U}^s(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \\ -\sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \\
 & \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix}
 \end{aligned}$$

$$\sigma = (\sigma^0, \vec{\sigma}), \bar{\sigma} = (\sigma^0, -\vec{\sigma})$$

$$\begin{aligned}
 p \cdot x &= \tilde{p} \cdot x, \quad d^3 \tilde{p} = d^3 p, \quad \gamma^0 \mathcal{U}^S(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \\
 &= \begin{pmatrix} \sqrt{\tilde{p} \cdot \sigma} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) = \gamma^0 \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \\
 &= \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix} = -\mathcal{U}^S(p)
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{pmatrix} \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(P), \quad \gamma^0 \gamma^S(P) = \gamma^0 \\
 &= \begin{pmatrix} -\sqrt{P \cdot \sigma} \\ \sqrt{P \cdot \sigma} \end{pmatrix} \Rightarrow \begin{pmatrix} \sqrt{P \cdot \sigma} \\ -\sqrt{P \cdot \sigma} \end{pmatrix} = -\mathcal{U}^S(P)
 \end{aligned}$$

$\int d^3p$
 functions in Dirac theory
 $\frac{dP}{dt}$

$$= \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \gamma^5(p), \quad \gamma^0 \gamma^5(p) = \gamma^0$$

$$= \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix} \Rightarrow \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix} = -\gamma^5(p)$$

$$= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E}}$$

$$\begin{aligned}
 &= \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) = \gamma^0 \\
 &= \begin{pmatrix} -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} \Rightarrow \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = -\mathcal{U}^S(p)
 \end{aligned}$$

$$= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(a_{-\vec{p}}^S \mathcal{U}^S(p) e^{-i p \cdot x} + b_{-\vec{p}}^{S+} \mathcal{V}^S(p) e^{i p \cdot x} \right)$$

$$\begin{aligned}
 &= \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) = \gamma^0 \\
 &= \begin{pmatrix} -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} \Rightarrow \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \end{pmatrix} = -\mathcal{U}^S(p)
 \end{aligned}$$

$$= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(a_{-\vec{p}}^S \mathcal{U}^S(p) e^{-i p \cdot x} + b_{-\vec{p}}^{S\dagger} \mathcal{V}^S(p) e^{i p \cdot x} \right)$$

$$\begin{aligned}
 &= \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = \mathcal{U}^S(p), \quad \gamma^0 \mathcal{U}^S(p) = \gamma^0 \\
 &= \begin{pmatrix} -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} \Rightarrow \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \sigma} \end{pmatrix} = -\mathcal{U}^S(p)
 \end{aligned}$$

$$= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(\underline{a_{-\vec{p}}^S} \mathcal{U}^S(p) e^{-i p \cdot x} + \underline{b_{-\vec{p}}^{S\dagger}} \mathcal{V}^S(p) e^{i p \cdot x} \right)$$

$(\gamma^{\mu\nu}) \Lambda_P \psi(x) = \Lambda_P (\gamma^{\mu\nu})^x \psi(x) \Rightarrow \Lambda_P (\gamma^{\mu\nu})^x = \Lambda_P \gamma^{\mu\nu} \Lambda_P^{-1}$
 $\Lambda_P = \eta_P \gamma^0$ satisfies this constraint
 $\eta_P = \pm 1 \Rightarrow \eta_P^2 = 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i}$

$\psi(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(x)$; $\eta_P = \pm 1, \pm i$ (I) What are the
 transformation of a_p^s, b_p^s and P ? $\psi^s(p)$
 om (I). $\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s (\bar{P} a_p^s + P b_p^s) \psi^s(p)$
 $= \eta_P \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s (a_p^s \psi^s(p) + b_p^s \psi^s(p))$

$\psi(x) = \eta_P (\gamma^0)^{-1} \psi(\tilde{x}) \Rightarrow \psi(x) = \eta_P (\gamma^0)^{-1} \psi(\tilde{x})$
 $\psi_P = \eta_P \gamma^0$ satisfies this constraint.
 $\eta_P^2 = \pm 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i}$. $\boxed{\psi_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$

$\psi(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What are the transformation of $a_{\vec{p}}^s, b_{\vec{p}}^s$ under P ? $\mathcal{U}^S(P)$
 from (I). $\int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s \left(\bar{P} a_{\vec{p}}^s P \mathcal{U}^S(P) e^{-i p x} + \bar{P} b_{\vec{p}}^s P \mathcal{U}^S(P) e^{i p x} \right) =$
 $= \eta_P \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s \left(a_{\vec{p}}^s \gamma^0 \mathcal{U}^S(P) e^{-i p \tilde{x}} + b_{\vec{p}}^s \gamma^0 \mathcal{U}^S(P) e^{i p \tilde{x}} \right)$

$$= \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_S \left(\frac{a_{-\vec{p}}^S u^S(\vec{p}) e^{-i p x} + b_{-\vec{p}}^{S+} v^S(\vec{p}) e^{i p x} \right) \Rightarrow$$

$$\vec{p} = -\vec{p} \Rightarrow a_{-\vec{p}}^S$$

$$= \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_S \left(\frac{a_{-\vec{p}}^S u^S(\vec{p}) e^{-i p x} + b_{-\vec{p}}^{S\dagger} v^S(\vec{p}) e^{i p x}} \right) \Rightarrow$$

$$P = a_{-\vec{p}}^S, \quad P^\dagger = b_{\vec{p}}^{S\dagger} P = -b_{-\vec{p}}^{S\dagger}$$

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_S \left(\frac{a_{-\vec{p}}^S u^S(\vec{p}) e^{-i p x} + b_{-\vec{p}}^{S\dagger} v^S(\vec{p}) e^{i p x}} \right) \Rightarrow$$

$$\vec{p}^T a_{\vec{p}}^S P = a_{-\vec{p}}^S, \quad \vec{p}^T b_{\vec{p}}^{S\dagger} P = -b_{-\vec{p}}^{S\dagger}$$

orb. moment
 $J^y = x^z p^x - x^x p^z$

orb. moment

$$J^i - x^i p^j - x^j p^i \rightarrow J^{ij}$$

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(a_{-\vec{p}}^s u^s(\vec{p}) e^{-i p x} + b_{-\vec{p}}^{s\dagger} v^s(\vec{p}) e^{i p x} \right) \Rightarrow$$

$$P a_{\vec{p}}^s P = a_{-\vec{p}}^s, \quad P b_{\vec{p}}^{s\dagger} P = - b_{-\vec{p}}^{s\dagger}$$

Relative parity of fermions and antifermions $(S) = -1$

$$\Lambda_P^2 = \pm 1 \Rightarrow \eta_P^2 = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i}, \quad \boxed{\psi'_x = \eta_P \delta^0 \psi(t, -\tilde{x})}$$

$\psi'(x) = P^{-1} \psi(x) P = \eta_P \delta^0 \psi(\tilde{x}), \quad \eta_P = \pm 1, \pm i$ (I) What are the transformation of a_P^s, b_P^s under P ? $\mathcal{U}^s(P)$

From (I), $\int \frac{d^3P}{(2\pi)^3 V} \left(\overline{P a_P^s} P \mathcal{U}^s(P) e^{-iPx} + P b_P^s P \mathcal{U}^s(P) e^{iPx} \right) =$

$$= \eta_P \int \frac{d^3P}{(2\pi)^3 V} \left(\delta^0 \overline{a_P^s} \mathcal{U}^s(P) e^{-iP\tilde{x}} + \delta^0 b_P^s \mathcal{U}^s(P) e^{iP\tilde{x}} \right)$$

Relative parity of fermions and antifermions $(S) = -1$
Transformations of fermion field bilinears.

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(a_{-\vec{p}}^s u^s(\vec{p}) e^{-ipx} + b_{-\vec{p}}^{s+} v^s(\vec{p}) e^{ipx} \right) \Rightarrow$$

$$P a_{\vec{p}}^s P = a_{-\vec{p}}^s, \quad P b_{\vec{p}}^{s+} P = -b_{-\vec{p}}^{s+}$$

Relative parity of fermions and antifermions (S = -1)

Transformations of fermion field bilinears.

$$\bar{\psi}\psi, \quad \bar{\psi}\gamma_5\psi, \quad \bar{\psi}\gamma_\mu\psi$$

$$P a_{\vec{p}}^s P = a_{-\vec{p}}^s, \quad P b_{\vec{p}}^{s\dagger} P = -b_{-\vec{p}}^{s\dagger}$$

Relative parity of fermions and antifermions $(S = -1)$

Transformations of fermion field bilinears

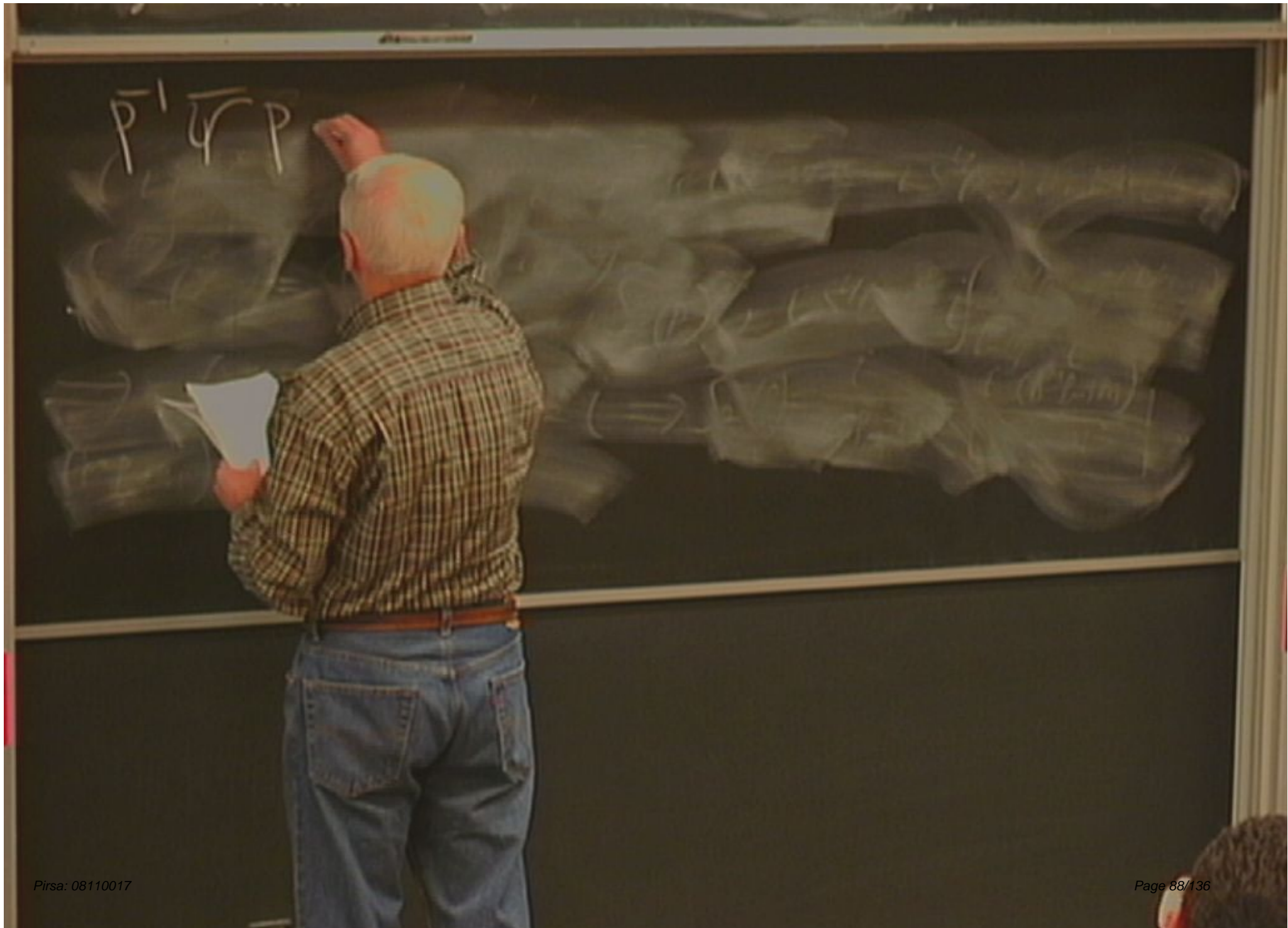
$$\bar{\psi}\psi, \quad \bar{\psi}\gamma_5\psi, \quad \bar{\psi}\gamma_\mu\psi, \quad \bar{\psi}\gamma_\mu\gamma_5\psi$$

$$U_{\vec{p}} P = a_{-\vec{p}}, \quad P^\dagger b_{\vec{p}} P = b_{-\vec{p}}$$

Relative parity of fermions and antifermions is -1

Transformations of fermion field bilinears

$\bar{\psi}\psi, \bar{\psi}\gamma_5\psi, \bar{\psi}\gamma^\mu\psi, \bar{\psi}\gamma^\mu\gamma_5\psi$
 They are hermitian.



$$\bar{P}^{-1} \bar{\psi} P = \bar{P}^{-1} \psi^{\dagger}(t, x) \gamma^0 P =$$

$$\bar{P}' \bar{\Psi} P = \bar{P}' \Psi^+(t, x) \gamma^0 P = \bar{P}' \underbrace{\Psi^+}_P \gamma^0 = (\bar{P}' \Psi P)^+ \gamma^0 =$$

$$\begin{aligned}
 \bar{P}^{-1} \bar{\psi} P &= \bar{P}^{-1} \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{P}^{-1} \psi^\dagger P \gamma^0 = (\bar{P}^{-1} \psi P)^\dagger \gamma^0 \\
 &= (\psi^\dagger(t, -\vec{x}))^\dagger \gamma^0 = \bar{\psi}^\dagger P \psi(t, -\vec{x}) \gamma^0
 \end{aligned}$$

$$\begin{aligned} \bar{p}' \bar{\psi} P &= \bar{p}' \psi^+(t, \vec{x}) \gamma^0 P = \bar{p}' \psi^+ P \gamma^0 = (\bar{p}' \psi P)^+ \gamma^0 \\ &= (\eta_P \gamma^0 \psi(t, -\vec{x}))^+ \gamma^0 = \bar{\eta}_P^* \bar{\psi}(t, -\vec{x}) \gamma^0 \end{aligned}$$

$$\begin{aligned}
 \bar{P}^{-1} \bar{\psi} P &= \bar{P}^{-1} \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{P}^{-1} \psi^\dagger P \gamma^0 = (\bar{P}^{-1} \psi P)^\dagger \gamma^0 = \\
 &= (\eta_P \gamma^0 \psi(t, -\vec{x}))^\dagger \gamma^0 = \underline{\underline{\bar{\eta}_P^* \bar{\psi}(t, -\vec{x})}} \gamma^0
 \end{aligned}$$

$$\begin{aligned}
 \bar{P}^{-1} \bar{\psi} P &= \bar{P}^{-1} \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{P}^{-1} \psi^\dagger P \gamma^0 = (\bar{P}^{-1} \psi P)^\dagger \gamma^0 = \\
 &= (\eta_P \gamma^0 \psi(t, -\vec{x}))^\dagger \gamma^0 = \bar{\eta}_P^* \bar{\psi}(t, -\vec{x}) \\
 \bar{P}^{-1} \bar{\psi} \psi P &= \bar{P}^{-1} \bar{\psi} P P^{-1} \psi P =
 \end{aligned}$$

$$\begin{aligned} \bar{\psi} P &= \bar{\psi}^{\dagger} \psi(t, \vec{x}) \gamma^0 P = \bar{\psi}^{\dagger} \psi P \gamma^0 = (\bar{\psi}^{\dagger} \psi P)^{\dagger} \gamma^0 \\ &= (\gamma^0 P \psi(t, -\vec{x}))^{\dagger} \gamma^0 = \bar{\psi}^{\dagger} \psi(t, -\vec{x}) \gamma^0 \end{aligned}$$

$$\bar{\psi} \psi P = \bar{\psi} P \psi P = |\psi_P|^2$$

$$\begin{aligned} \bar{P}^{-1} \bar{\psi} P &= \bar{P}^{-1} \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{P}^{-1} \psi^\dagger P \gamma^0 = (\bar{P}^{-1} \psi P)^\dagger \gamma^0 \\ &= (\eta_P \gamma^0 \psi(t, -\vec{x}))^\dagger \gamma^0 = \underline{\eta_P^* \bar{\psi}(t, -\vec{x})} \gamma^0 \end{aligned}$$

$$\bar{P}^{-1} \bar{\psi} \psi P = \bar{P}^{-1} \bar{\psi} P \bar{P}^{-1} \psi P = |\eta_P|^2 \bar{\psi}(t, -\vec{x}) \gamma^0$$

$$\begin{aligned}
 \bar{P}^{-1} \bar{\psi} P &= \bar{P}^{-1} \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{P}^{-1} \psi^\dagger P \gamma^0 = (\bar{P}^{-1} \psi P)^\dagger \gamma^0 = \\
 &= (\eta_P \gamma^0 \psi(t, -\vec{x}))^\dagger \gamma^0 = \bar{\eta}_P^* \bar{\psi}(t, -\vec{x}) \gamma^0 \\
 \bar{P}^{-1} \bar{\psi} \psi P &= \bar{P}^{-1} \bar{\psi} P \bar{P}^{-1} \psi P = |\eta_P|^2 \bar{\psi}(t, -\vec{x}) \gamma^0 \gamma^0 \psi(t, \vec{x}) =
 \end{aligned}$$

$$\begin{aligned}
 \frac{\bar{p}' \bar{\psi} P}{h_p} &= \bar{p}' \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{p}' \psi^\dagger P \gamma^0 = (\bar{p}' \psi P)^\dagger \gamma^0 = \\
 &= (\bar{p}' \psi(t, -\vec{x}))^\dagger \gamma^0 = \frac{\bar{p}' \psi(t, -\vec{x})}{h_p} \gamma^0 \\
 \bar{p}' P &= \bar{p}' \bar{\psi} P \bar{p}' \psi P = h_p^2 \bar{\psi}(t, -\vec{x}) \gamma^0 \gamma^0 \psi(t, \vec{x}) = \\
 &= \bar{p}' \psi(t, -\vec{x}) \quad (\text{scalar})
 \end{aligned}$$

$$\begin{aligned}
 \frac{\bar{\psi} \psi}{P} &= \bar{\psi} \psi^\dagger(t, \vec{x}) \gamma^0 P = \bar{\psi} \psi^\dagger P \gamma^0 = (\bar{\psi} \psi P)^\dagger \gamma^0 = \\
 &= (\eta_P \gamma^0 \psi(t, -\vec{x}))^\dagger \gamma^0 = \frac{\eta_P^* \bar{\psi}(t, -\vec{x})}{P} \gamma^0 \\
 \bar{\psi} \psi P &= \bar{\psi} \psi P \bar{\psi} \psi P = |\eta_P|^2 \bar{\psi}(t, -\vec{x}) \gamma^0 \gamma^0 \psi(t, \vec{x}) = \\
 &= \bar{\psi}(t, -\vec{x}) \psi(t, \vec{x}) \quad (\text{scalar})
 \end{aligned}$$

$$i\bar{p}'\psi\gamma^5\psi p = i\bar{p}'\psi p\gamma^5\bar{p}'\psi p =$$

$$i \bar{p}' \psi \gamma^5 \psi p = i \bar{p}' \psi p \gamma^5 \bar{p}' \psi p = m_p |p|^2 \bar{\psi}(t, \mathbf{x})$$

$$i\bar{p}'\psi\gamma^5\psi\rho = i\bar{p}'\psi\rho\gamma^5 p'\psi\rho = m_p|\psi(t,\vec{x})\gamma^0\gamma^5\gamma^0\psi(t,\vec{x})$$

$$i\bar{P}'\psi\gamma^5\psi P = i\bar{P}'\psi P\gamma^5\bar{P}'\psi P = |m_P|^2\bar{\psi}(t,\vec{x})\gamma^0\gamma^5\gamma^0\psi(t,\vec{x})$$

$$\gamma^0 = -\gamma^0\gamma^5$$

$$i \bar{P}' \bar{\psi} \gamma^5 \psi P = i \bar{P}' \bar{\psi} P \gamma^5 P' \psi P = m_P |z|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x})$$

$$\overline{\gamma^5 \gamma^0} = -\gamma^0 \gamma^5 \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x})$$

$$i \bar{P}' \bar{\psi} \gamma^5 \psi P = i \bar{P}' \bar{\psi} P \gamma^5 P' \psi P = |n_p|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x})$$

$$\overline{\overline{\gamma^5 \gamma^0 = -\gamma^0 \gamma^5}} \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudo-scalar}).$$

$$i \bar{P}' \bar{\psi} \gamma^5 \psi P = i \bar{P}' \bar{\psi} P \gamma^5 P' \psi P = |n_p|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x})$$

$$\overline{\overline{\gamma^5 \gamma^0 = -\gamma^0 \gamma^5}} - \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudo-scalar}).$$

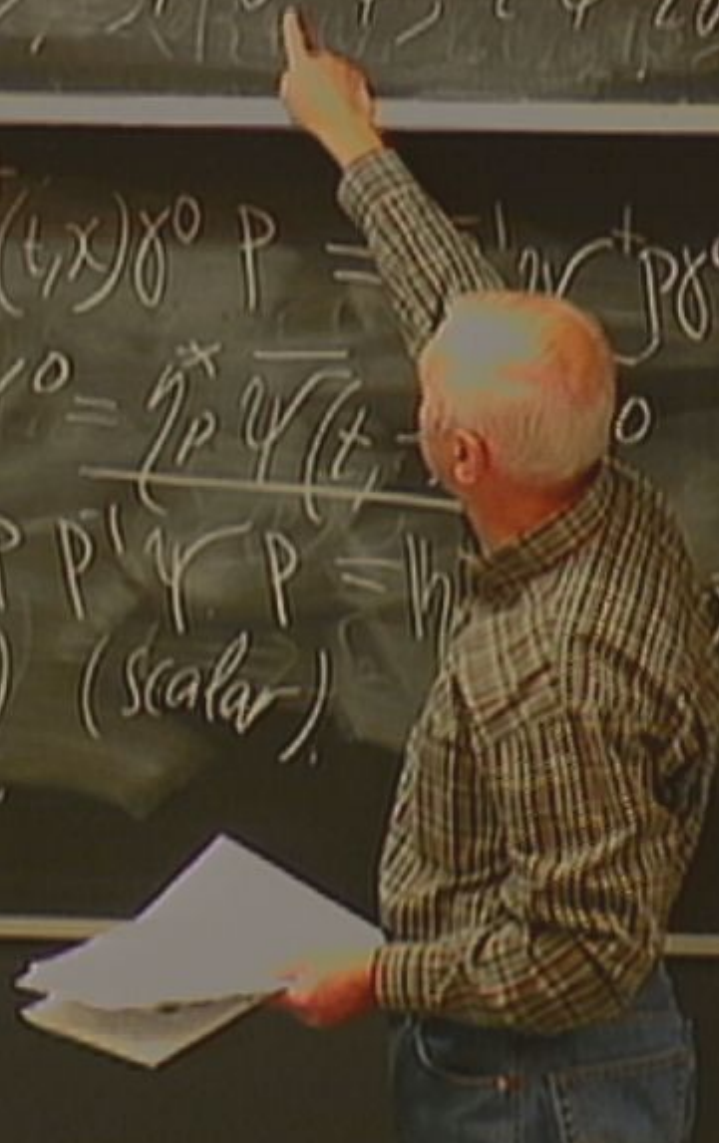
tive parity of fermions and antifermions (S = -1)
 transformations of fermion field bilinears
 $\bar{\psi}, \bar{\psi} \gamma^5 \psi, \bar{\psi} \gamma^{\mu} \psi, \bar{\psi} [\gamma^{\mu}, \gamma^{\nu}] \psi, \bar{\psi} \gamma^{\mu} \gamma^5 \psi$
 are hermitian.

$$\bar{\psi} P = \bar{\psi}' \psi^{\dagger}(t, \vec{x}) \gamma^0 P = \bar{\psi}'^{\dagger} P \gamma^0 = (\bar{\psi}' P)^{\dagger} \gamma^0 =$$

$$(\bar{\psi} \gamma^0 \psi(t, -\vec{x}))^{\dagger} \gamma^0 = \bar{\psi}'^{\dagger} \psi(t, -\vec{x})$$

$$\bar{\psi} \psi P = \bar{\psi}' P P' \psi P = \bar{\psi}'(t, -\vec{x}) \gamma^0 \gamma^0 \psi(t, \vec{x}) =$$

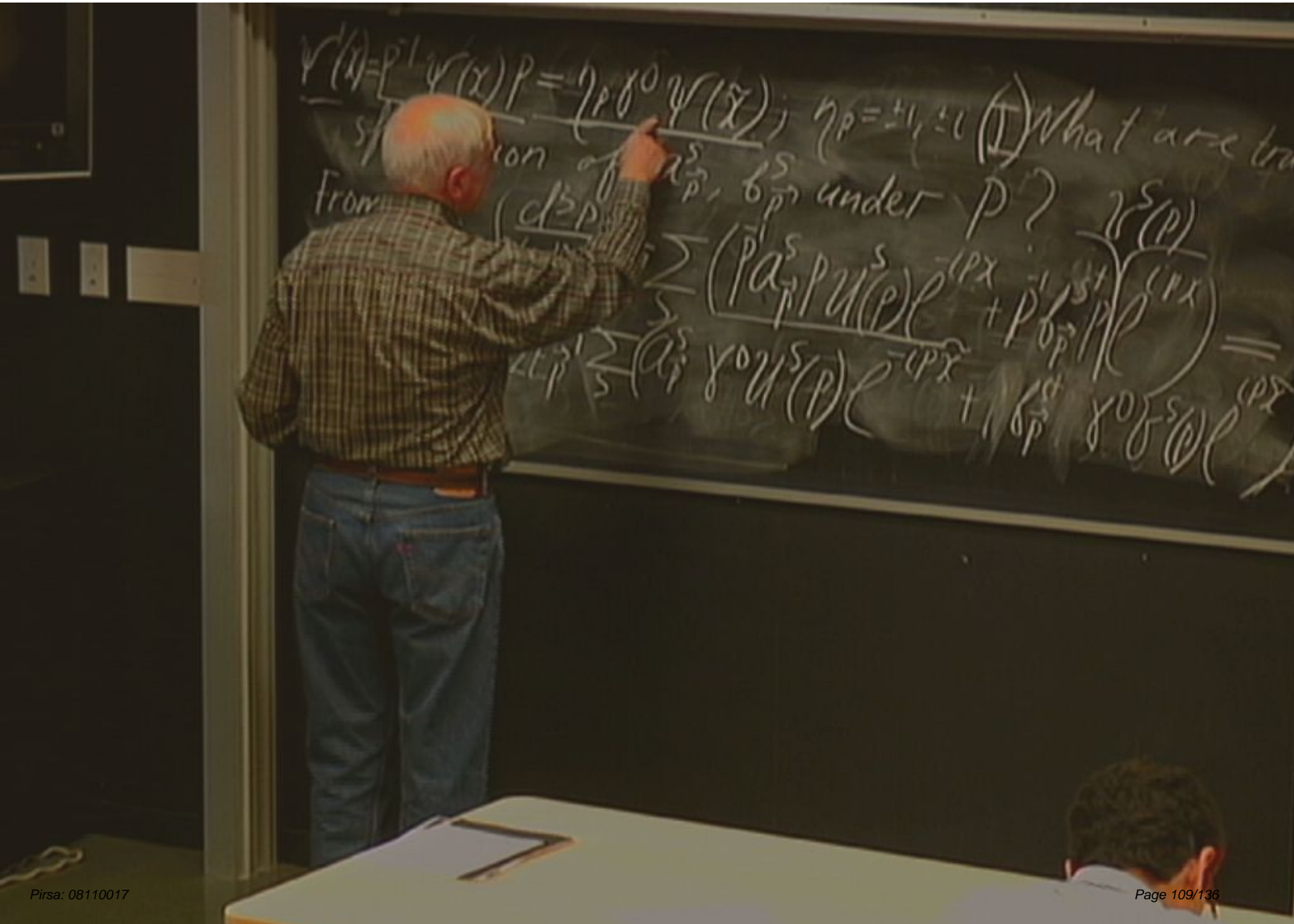
$$\bar{\psi}'(t, -\vec{x}) \psi(t, \vec{x}) \text{ (scalar)}$$



$$i \bar{P}' \bar{\psi} \gamma^5 \psi P = i \bar{P}' \bar{\psi} P \gamma^5 P' \psi P = |m_P|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x})$$

$$\overline{\gamma^5 \gamma^0} = -\gamma^0 \gamma^5 \quad \bar{\psi}(t, \vec{x}) \gamma^0 \psi(t, \vec{x}) \quad (\text{pseudo-scalar})$$

$$\bar{P}' \bar{\psi} \gamma^m \psi P =$$



$\psi'(x) = P^{-1} \psi(x) P = \eta_P \delta^0 \psi(\tilde{x}); \eta_P = \pm 1, \pm i$ (I) What are the
 From the transformation of $a_{p'}^s, b_{p'}^s$ under P ? $\gamma^s(P)$

$$\sum_{s'} (P^{-1} a_{p'}^s P U^s(P) e^{-i p x} + P b_{p'}^s P e^{i p x}) = \sum_{s'} (a_{p'}^s \gamma^0 U^s(P) e^{-i p x} + b_{p'}^s \gamma^0 \delta^s(P) e^{i p x})$$

$$i \bar{P}' \bar{\psi} \gamma^5 \psi P = i \bar{P}' \bar{\psi} P \gamma^5 P' \psi P = |m_P|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \psi(t, \vec{x})$$

$$\overline{\gamma^5 \gamma^0} = -\gamma^0 \gamma^5 \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudoscalar})$$

$$P' \bar{\psi} \gamma^m \psi P =$$

$$i P' \psi \gamma^5 \psi P = i \bar{P}' \bar{\psi} P \gamma^5 P' \psi P = |m_p|^2 \bar{\psi}(t, \vec{x}) \delta^0 \gamma^5 \delta^0 \psi(t, \vec{x})$$

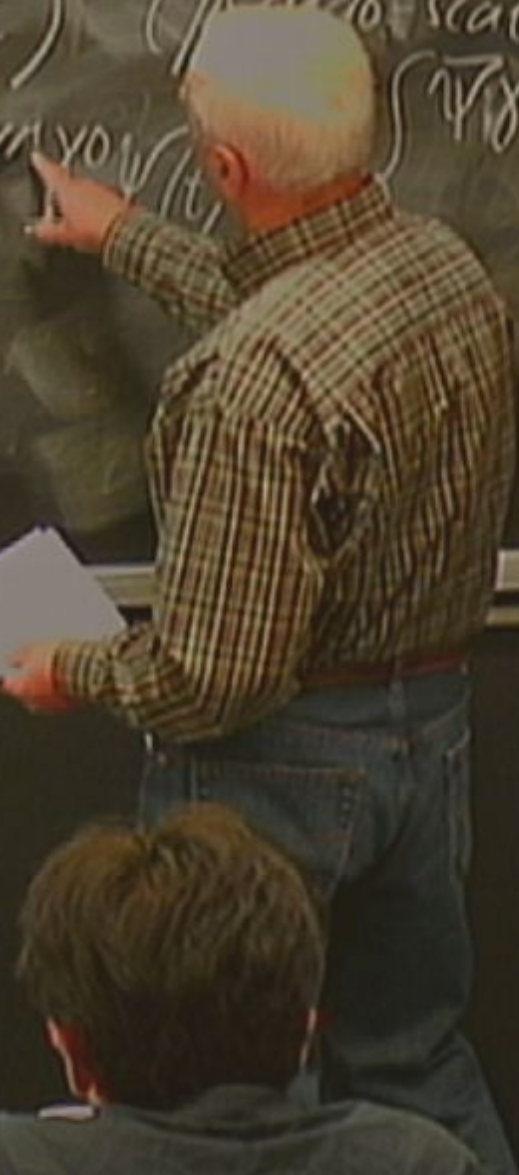
$$\overline{\delta^5 \delta^0 = -\delta^0 \delta^5} \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudoscalar})$$

$$P' \bar{\psi} \gamma^m \psi P = \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^m \gamma^0 \psi(t, \vec{x}) =$$

$$\begin{aligned}
 (1) \quad \bar{\psi} \gamma^0 \psi &= \bar{\psi} P \gamma^0 P \psi = |\eta_P|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x}) \\
 \overline{\gamma^5 \gamma^0} &= -\gamma^0 \gamma^5 \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudo-scalar}) \\
 \bar{\psi} \gamma^m \psi &= \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^m \gamma^0 \psi(t, \vec{x}) = \begin{cases} \bar{\psi} \gamma^0 \psi(t, \vec{x}) & \text{for } m=0 \\ \bar{\psi} \gamma^i \psi(t, \vec{x}) & \text{for } m=i \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad \bar{\psi} \gamma^0 \psi &= \bar{\psi} \psi \gamma^0 \psi = |\psi|^2 \psi(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x}) \\
 \overline{\gamma^5 \gamma^0} &= -\gamma^0 \gamma^5 \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudo}) \\
 \bar{\psi} \gamma^m \psi &= \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^m \gamma^0 \psi(t, \vec{x}) = \left. \begin{aligned} & \bar{\psi}(t, \vec{x}) \gamma^m \psi(t, \vec{x}) \text{ for } m=0 \\ & \bar{\psi}(t, \vec{x}) \gamma^m \psi(t, \vec{x}) \text{ for } m=1,2,3 \end{aligned} \right\}
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad \bar{\psi} \gamma^\mu \psi &= \bar{\psi} P \gamma^\mu P \psi = |\eta_P|^2 \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t, \vec{x}) \\
 \overline{\gamma^5 \gamma^0} &= -\gamma^0 \gamma^5 \quad \bar{\psi}(t, \vec{x}) \gamma^5 \psi(t, \vec{x}) \quad (\text{pseudo-scalar}) \\
 \bar{\psi} \gamma^{\mu\nu} \psi &= \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^\mu \gamma^0 \gamma^\nu \psi(t, \vec{x}) \quad \int \bar{\psi} \gamma^0 \psi(t, \vec{x}) \text{ for } \mu=0
 \end{aligned}$$



$$\begin{aligned}
 & \overline{\overline{\gamma^5 \gamma^0}} = -\gamma^0 \gamma^5 \quad \overline{\Psi}(t, \vec{x}) \gamma^5 \Psi(t, \vec{x}) \quad (\text{pseudo-scalar}) \\
 & \overline{P^{-1} \Psi} \gamma^m \Psi = \overline{\Psi}(t, \vec{x}) \gamma^0 \gamma^m \gamma^0 \Psi(t, \vec{x}) = \begin{cases} \overline{\Psi} \gamma^0 \Psi(t, \vec{x}) & \text{for } m=0 \\ -\overline{\Psi} \gamma^m \Psi(t, \vec{x}) & \text{for } m=1,2,3 \end{cases}
 \end{aligned}$$

$$\overline{\overline{\gamma^5 \gamma^0}} = -\gamma^0 \gamma^5 \quad \overline{\Psi(t, \vec{x})} \gamma^5 \Psi(t, \vec{x}) \quad (\text{pseudo-scalar})$$

$$P^{-1} \overline{\Psi} \gamma^m \Psi P = \overline{\Psi(t, \vec{x})} \gamma^0 \gamma^m \gamma^0 \Psi(t, \vec{x}) = \begin{cases} \overline{\Psi} \gamma^0 \Psi(t, \vec{x}) & \text{for } m=0 \\ -\overline{\Psi} \gamma^m \Psi(t, \vec{x}) & \text{for } m=1,2,3 \end{cases}$$

$$\begin{aligned}
 \overline{\gamma^5 \gamma^0} &= -\gamma^0 \gamma^5 \\
 \overline{\psi(t, \vec{x})} \gamma^5 \psi(t, \vec{x}) & \text{ (pseudo-scalar) } \quad \text{(scalar)} \\
 \overline{\psi} \gamma^m \psi &= \overline{\psi(t, \vec{x})} \gamma^0 \gamma^m \gamma^0 \psi(t, \vec{x}) = \begin{cases} \overline{\psi} \gamma^0 \psi(t, \vec{x}) \text{ for } m=0 \\ -\overline{\psi} \gamma^m \psi(t, \vec{x}) \text{ for } m=1,2,3 \end{cases} \\
 & \text{(vector)}
 \end{aligned}$$

$$\begin{aligned}
 \rho^{-1} \bar{\psi} \gamma^{\mu} \psi \rho = \bar{\psi}(t, -\vec{x}) \gamma_0 \gamma^{\mu} \psi(t, -\vec{x}) &= \begin{cases} \psi^0 \psi(t, -\vec{x}) & \text{for } \mu=0 \\ -\bar{\psi} \gamma^{\mu} \psi(t; \vec{x}) & \text{for } \mu=1,2,3 \end{cases} \\
 \rho^{-1} \bar{\psi} \gamma^{\mu} \gamma_5 \psi \rho = \bar{\psi}(t, -\vec{x}) \gamma_0 \gamma^{\mu} \gamma_5 \psi(t, -\vec{x}) &= \begin{cases} \text{vector} & \text{for } \mu=1,2,3 \end{cases}
 \end{aligned}$$



$$\begin{aligned}
 \bar{p} \gamma^{\mu} \psi &= \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^{\mu} \psi(t, \vec{x}) \\
 \bar{p} \gamma^{\mu} \psi &= \bar{\psi}(t, \vec{x}) \gamma^0 \gamma^{\mu} \psi(t, \vec{x})
 \end{aligned}$$



$$\begin{aligned}
 \rho' \psi \gamma^m \psi \rho = \bar{\psi}(t, \vec{x}) \gamma_0 \gamma^m \psi(t, \vec{x}) &= \begin{cases} \psi \gamma^0 \psi(t, \vec{x}) & \text{for } m=0 \\ -\bar{\psi} \gamma^m \psi(t, \vec{x}) & \text{for } m=1,2,3. \end{cases} \\
 \bar{\rho}' \bar{\psi} \gamma^m \psi \rho = \bar{\psi}(t, \vec{x}) \gamma_0 \gamma^m \psi(t, \vec{x}) &= \begin{cases} \bar{\psi} \gamma^0 \psi(t, \vec{x}) & \text{for } m=0 \\ \psi \gamma^m \psi(t, \vec{x}) & \text{for } m=1,2,3. \end{cases}
 \end{aligned}$$



$$\begin{aligned}
 & \int \psi^\dagger \psi = \int \psi^\dagger(x, t) \psi(x, t) = \int \psi^\dagger(x, t) \left[\sum_{\mu=1,2,3} \left(\psi_{\mu}(x, t) + \psi_{\mu}^{\dagger}(x, t) \right) \right] \\
 & \int \psi^\dagger \psi = \int \psi^\dagger(x, t) \psi(x, t) = \int \psi^\dagger(x, t) \left[\sum_{\mu=1,2,3} \left(\psi_{\mu}(x, t) + \psi_{\mu}^{\dagger}(x, t) \right) \right] \\
 & \int \psi^\dagger \psi = \int \psi^\dagger(x, t) \psi(x, t) = \int \psi^\dagger(x, t) \left[\sum_{\mu=1,2,3} \left(\psi_{\mu}(x, t) + \psi_{\mu}^{\dagger}(x, t) \right) \right]
 \end{aligned}$$



$$\begin{aligned}
 \bar{\psi} \gamma^\mu \psi &= \bar{\psi}(t, \vec{x}) \gamma^\mu \psi(t, \vec{x}) \\
 &= \begin{cases} \bar{\psi}(t, \vec{x}) \gamma^0 \psi(t, \vec{x}) & (\text{scalar}) \\ \bar{\psi}(t, \vec{x}) \gamma^i \psi(t, \vec{x}) & (\text{vector}) \end{cases} \quad \text{for } \mu=0,1,2,3 \\
 &= \begin{cases} \bar{\psi}(t, \vec{x}) \gamma^0 \psi(t, \vec{x}) & (\text{scalar}) \\ \bar{\psi}(t, \vec{x}) \gamma^i \psi(t, \vec{x}) & (\text{vector}) \end{cases} \quad \text{for } \mu=1,2,3
 \end{aligned}$$

A person in a plaid shirt is holding a stack of papers on the right side of the chalkboard.

relative parity of fermions and antifermions (S = -1)
 Transformations of fermion field bilinears
 $\bar{\psi}\psi, \bar{\psi}\gamma^5\psi, \bar{\psi}\gamma^\mu\psi, \bar{\psi}[\gamma^\mu, \gamma^\nu]\psi, \bar{\psi}\gamma^{\mu\nu}\psi$
 are hermitian.

$$\begin{aligned} \overline{P^{-1}\psi P} &= \overline{P^{-1}\psi^\dagger(t, \vec{x})\gamma^0 P} = \overline{P^{-1}\psi^\dagger P}\gamma^0 = (\overline{P^{-1}\psi P})^\dagger\gamma^0 \\ &= (\eta_P\gamma^0\psi(t, -\vec{x}))^\dagger\gamma^0 = \eta_P^*\overline{\psi(t, -\vec{x})}\gamma^0 \\ \overline{P^{-1}\bar{\psi}\psi P} &= \overline{P^{-1}\bar{\psi} P P^{-1}\psi P} = \eta_P^2\overline{\psi(t, -\vec{x})}\gamma^0\gamma^0\psi(t, \vec{x}) \\ &= \overline{\psi(t, -\vec{x})}\psi(t, \vec{x}) \quad (\text{scalar}) \end{aligned}$$

$$\begin{aligned}
 \vec{p}^i \vec{p}^j \vec{p}^k \vec{p}^l &= dA(\vec{r}, \vec{p}) = dA(\vec{r}, \vec{p}) = dA(\vec{r}, \vec{p}) \\
 &= \begin{cases} \text{(vector)} \\ \text{(pseudovector)} \end{cases} = \begin{cases} -\vec{\nabla} \cdot \vec{p} & \text{for } \mu=0 \\ +\vec{\nabla} \times \vec{p} & \text{for } \mu=1,2,3 \end{cases}
 \end{aligned}$$



$\gamma^5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3$

$\bar{\psi}(t, -\vec{x}) \gamma^5 \psi(t, \vec{x})$ (pseudoscalar)

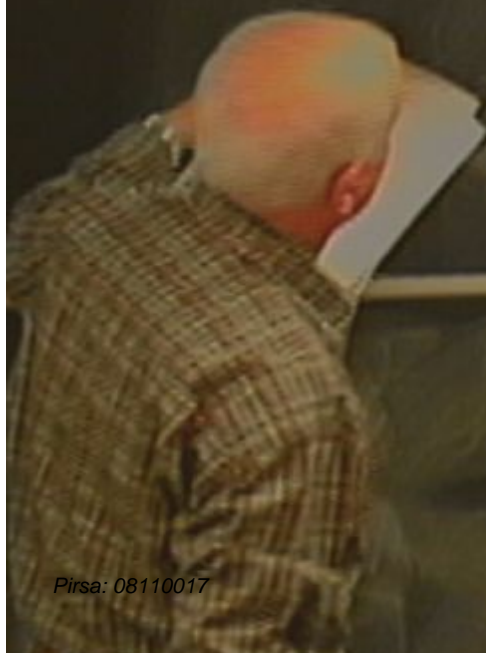
$\bar{\psi}(t, -\vec{x}) \gamma^0 \gamma^m \psi(t, \vec{x}) = \begin{cases} \bar{\psi} \gamma^0 \psi(t, \vec{x}) & \text{for } m=0 \\ -\bar{\psi} \gamma^m \psi(t, \vec{x}) & \text{for } m=1,2,3 \end{cases}$

$\bar{\psi}(t, -\vec{x}) \gamma^0 \gamma^m \gamma^5 \psi(t, \vec{x}) = \begin{cases} \bar{\psi} \gamma^0 \gamma^m \psi(t, \vec{x}) & \text{for } m=0 \\ -\bar{\psi} \gamma^m \gamma^5 \psi(t, \vec{x}) & \text{for } m=1,2,3 \end{cases}$

$\bar{\psi}(t, -\vec{x}) \gamma^m \psi(t, \vec{x}) = i(-1)^m (-1)^{\nu} \bar{\psi}(t, \vec{x}) \gamma^m \psi(t, \vec{x})$

where $(-1)^m = 1$ for $m=0$ and -1 for $m=1,2,3$.

(faded handwritten notes on the lower part of the chalkboard)



where $(-1)^m = 1$ for $m=0$ and -1 for $m=1, 2, 3, \dots$

Can we define P transformation for Majorana (L or R) fermions? The answer is "no"

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".
Write ψ_L and ψ_R in four-component form

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

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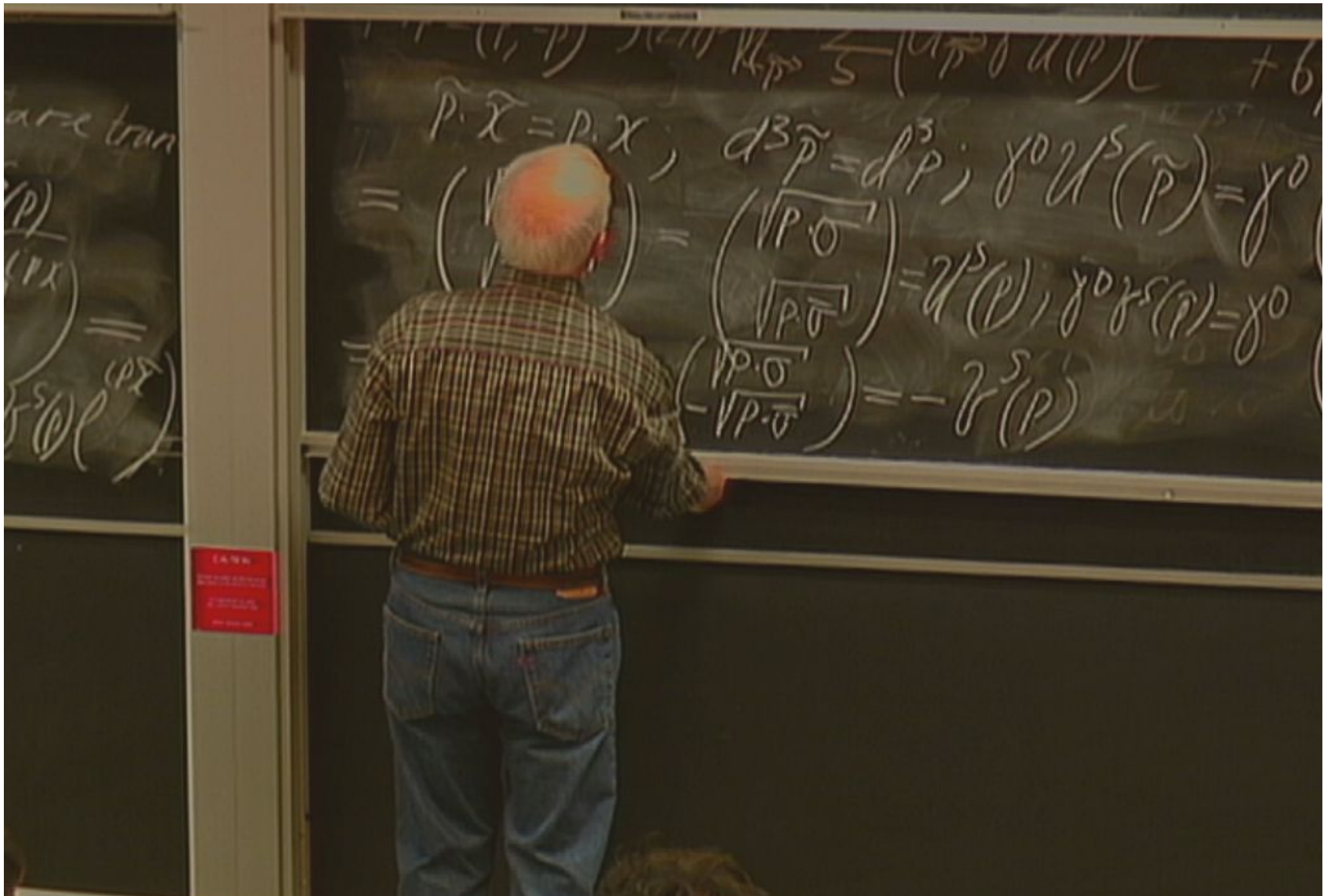
$$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

Write ψ_L and ψ_R in four-component form:
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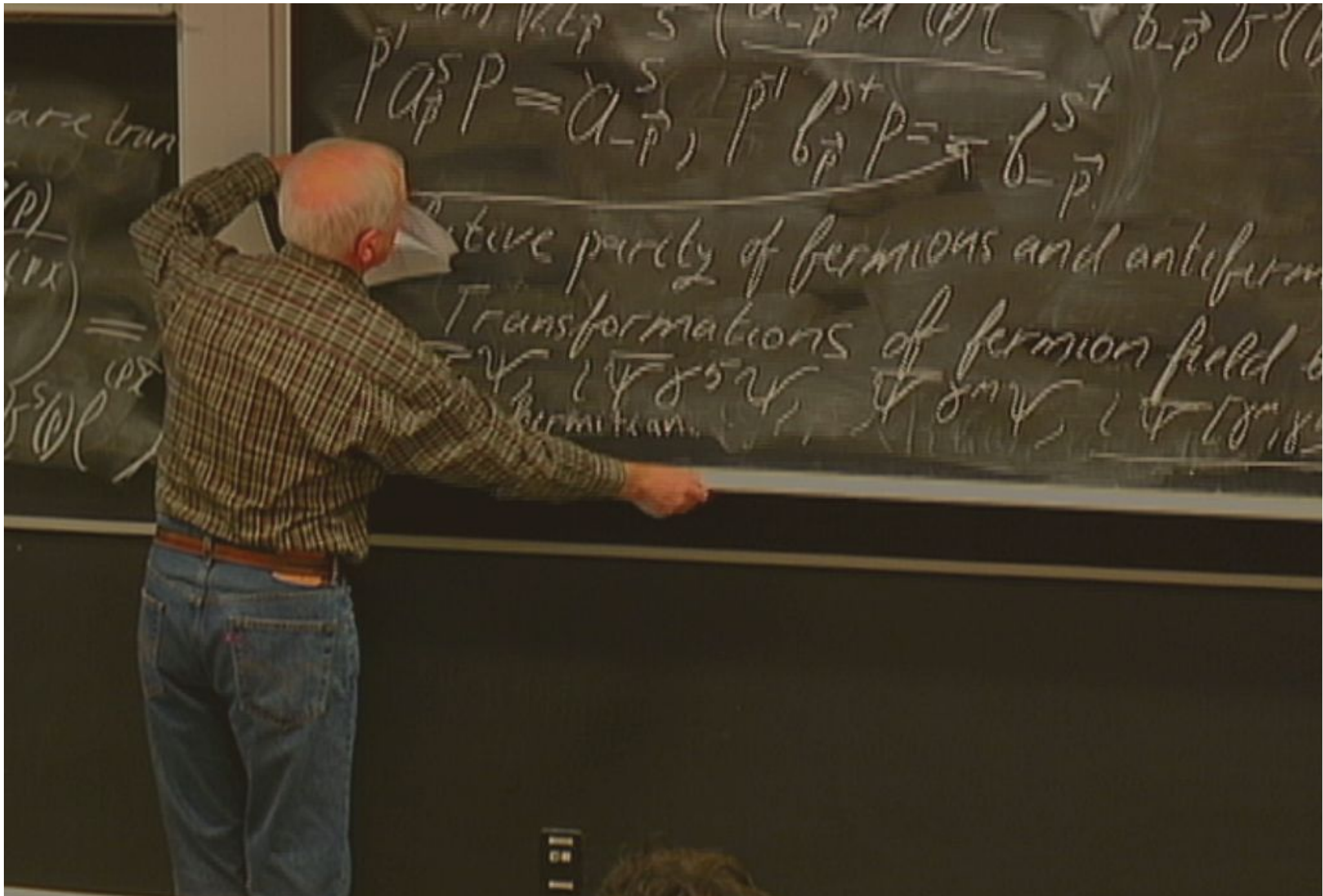
Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

Write ψ_L and ψ_R in four-component form:
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 (p)
 (px)

$$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3 \tilde{p} = d^3 p; \quad \gamma^0 \mathcal{U}^s(\tilde{p}) = \gamma^0 \mathcal{U}^s(p), \quad \gamma^0 \mathcal{V}^s(\tilde{p}) = \gamma^0 \mathcal{V}^s(p)$$
$$\begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = \mathcal{U}^s(p), \quad \begin{pmatrix} \sqrt{p \cdot \sigma} \\ -\sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = -\mathcal{V}^s(p)$$



$$P a_{\vec{p}}^s P = a_{-\vec{p}}^s, \quad P b_{\vec{p}}^{s\dagger} P = -b_{-\vec{p}}^{s\dagger}$$

Transformation of fermion field

$$\psi \rightarrow \gamma^0 \psi, \quad \bar{\psi} \rightarrow \bar{\psi} \gamma^0$$

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

Write ψ_L and ψ_R in four-component form:

$$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}. \quad \text{P-transformation:}$$

$$\bar{\psi}^{(L)} \psi^{(L)} P =$$

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$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$ Write ψ_L and ψ_R in four-component form:

$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$, $\psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$ P-transformation:

$\bar{P}^{-1} \psi^{(L)} P = \gamma^0 \psi^{(L)}(t, -\vec{x})$

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$$\bar{\psi}^{(L)} \psi^{(L)} = \bar{\psi}^0 \psi^L(t, \vec{x}) = \bar{\psi}^0 \begin{pmatrix} 0 \\ \psi_L \end{pmatrix} \leftarrow \text{right-handed fermion}$$

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$$\bar{\psi}^{(R)} \psi^{(R)} P = \eta_P \begin{pmatrix} \psi_R \\ 0 \end{pmatrix} \leftarrow \text{left-handed fermion}$$