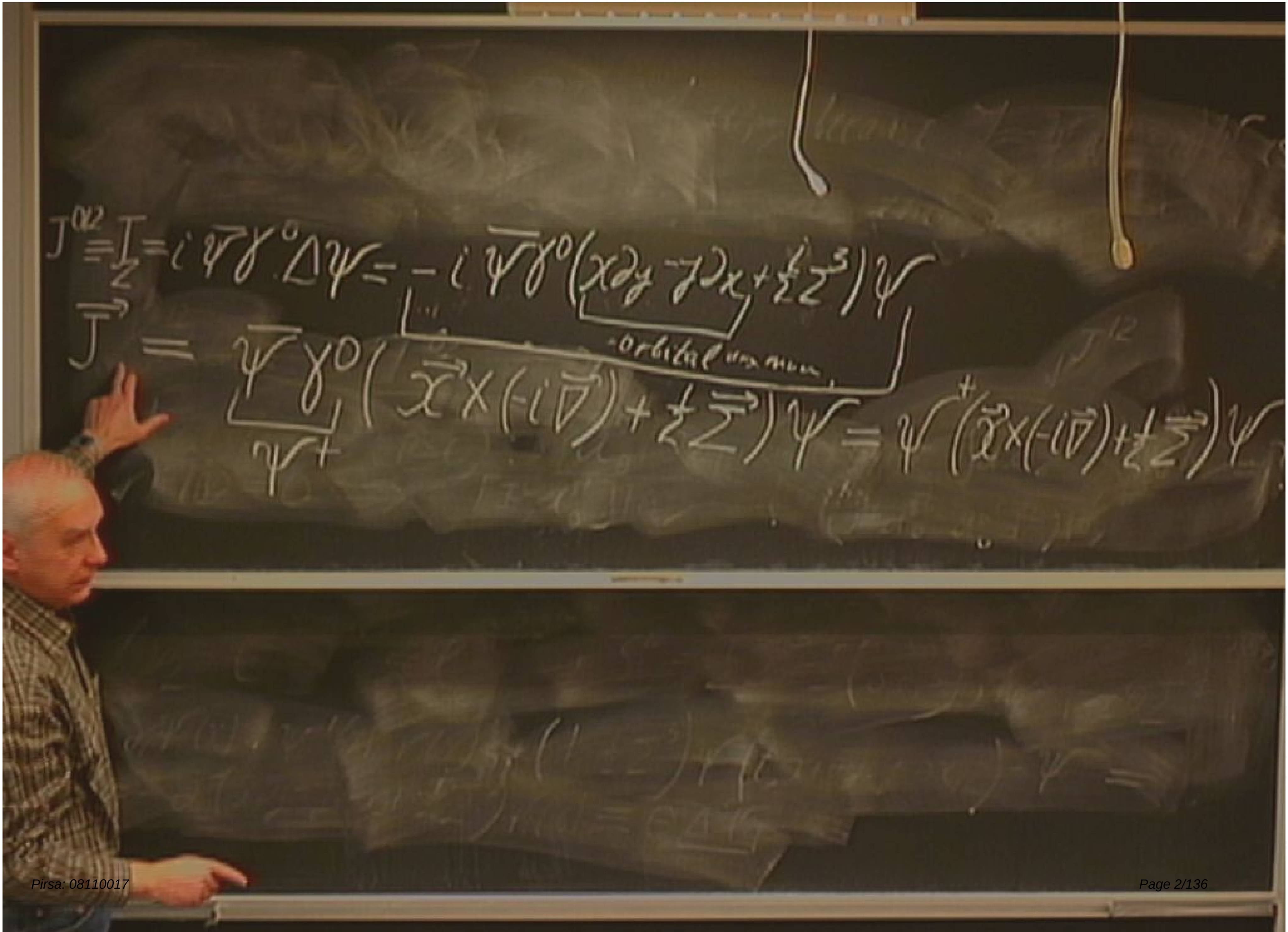
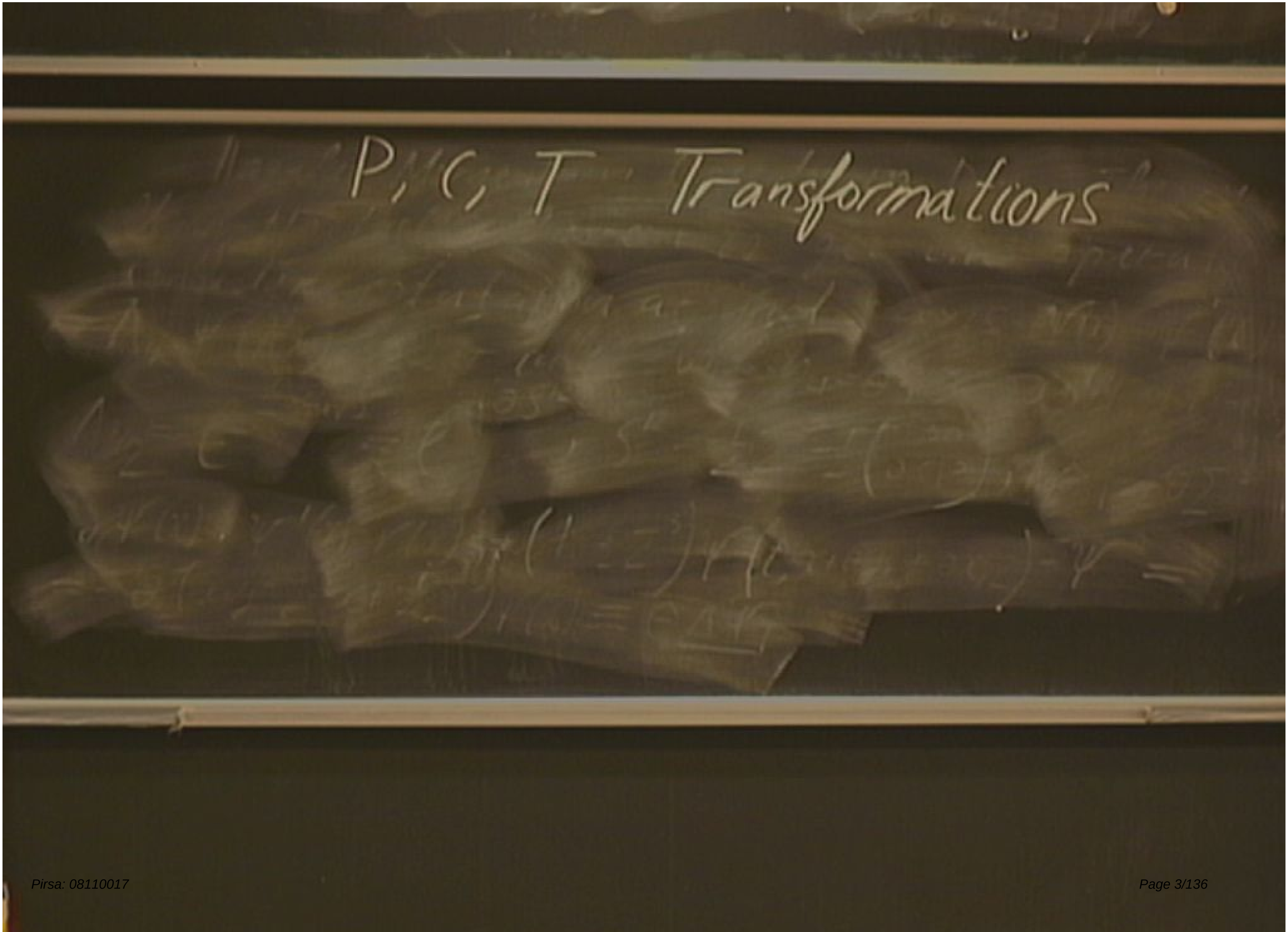
Title: Quantum Field Theory 1 - Lecture 10B

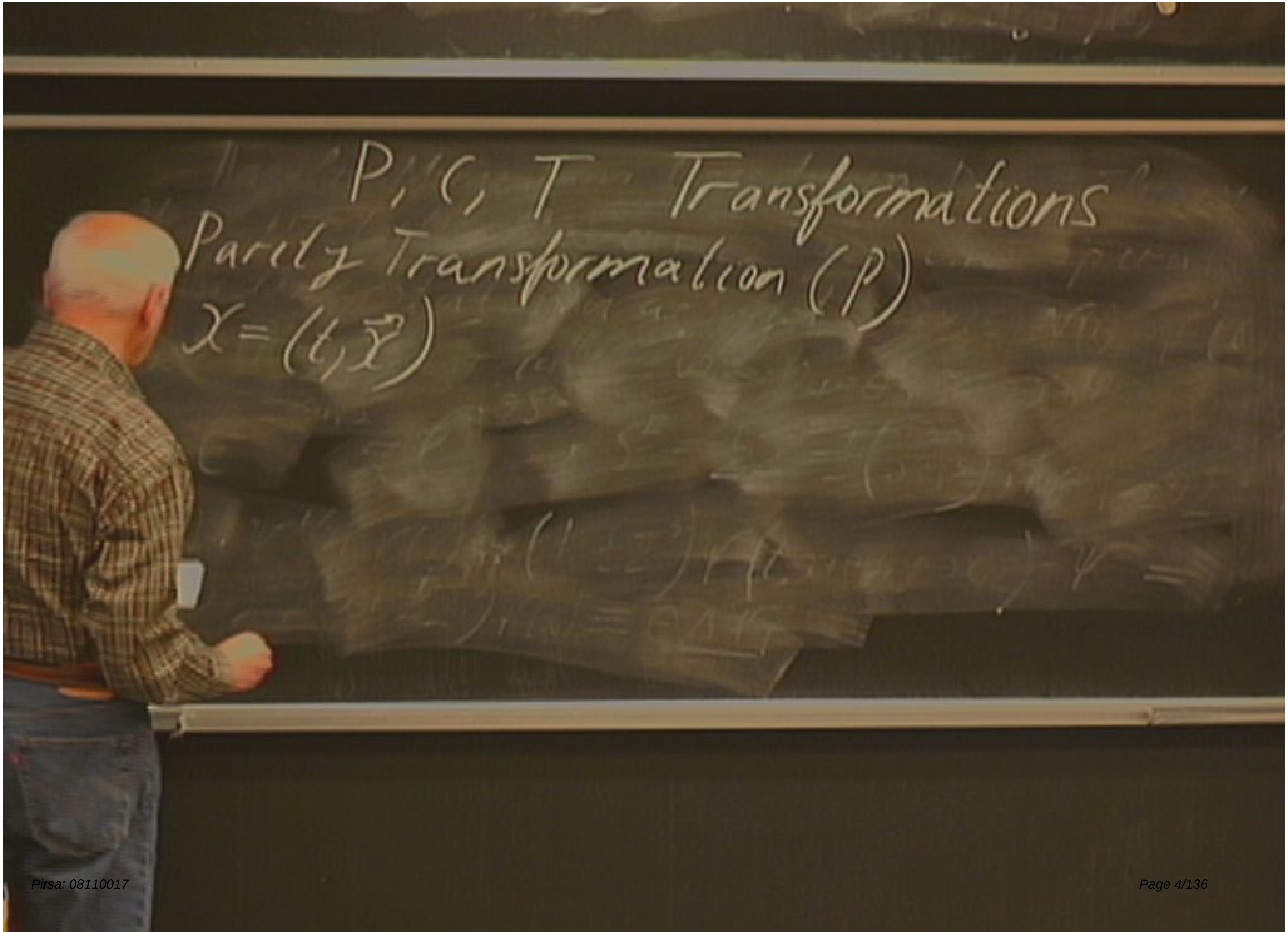
Date: Nov 12, 2008 03:30 PM

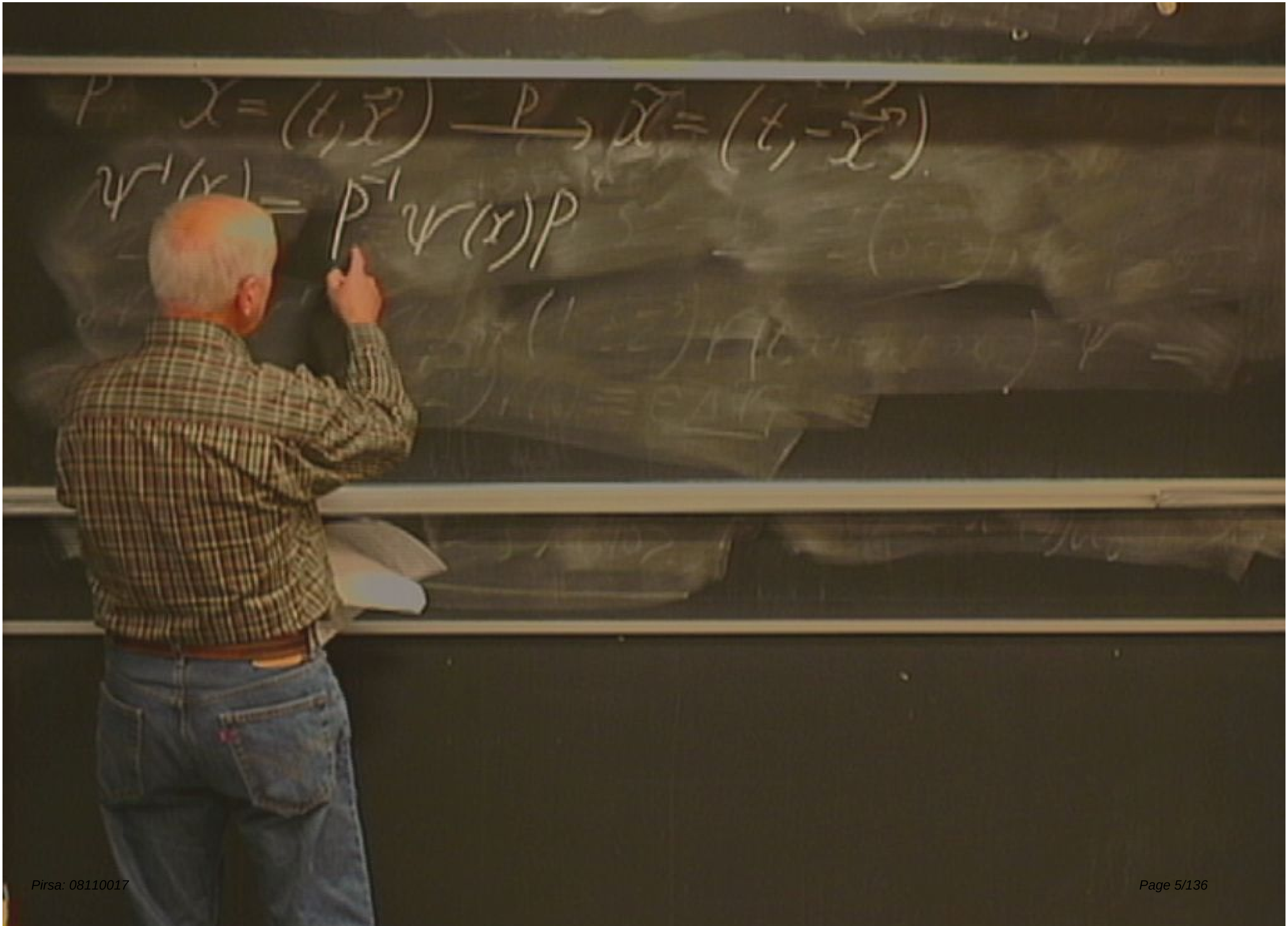
URL: http://pirsa.org/08110017

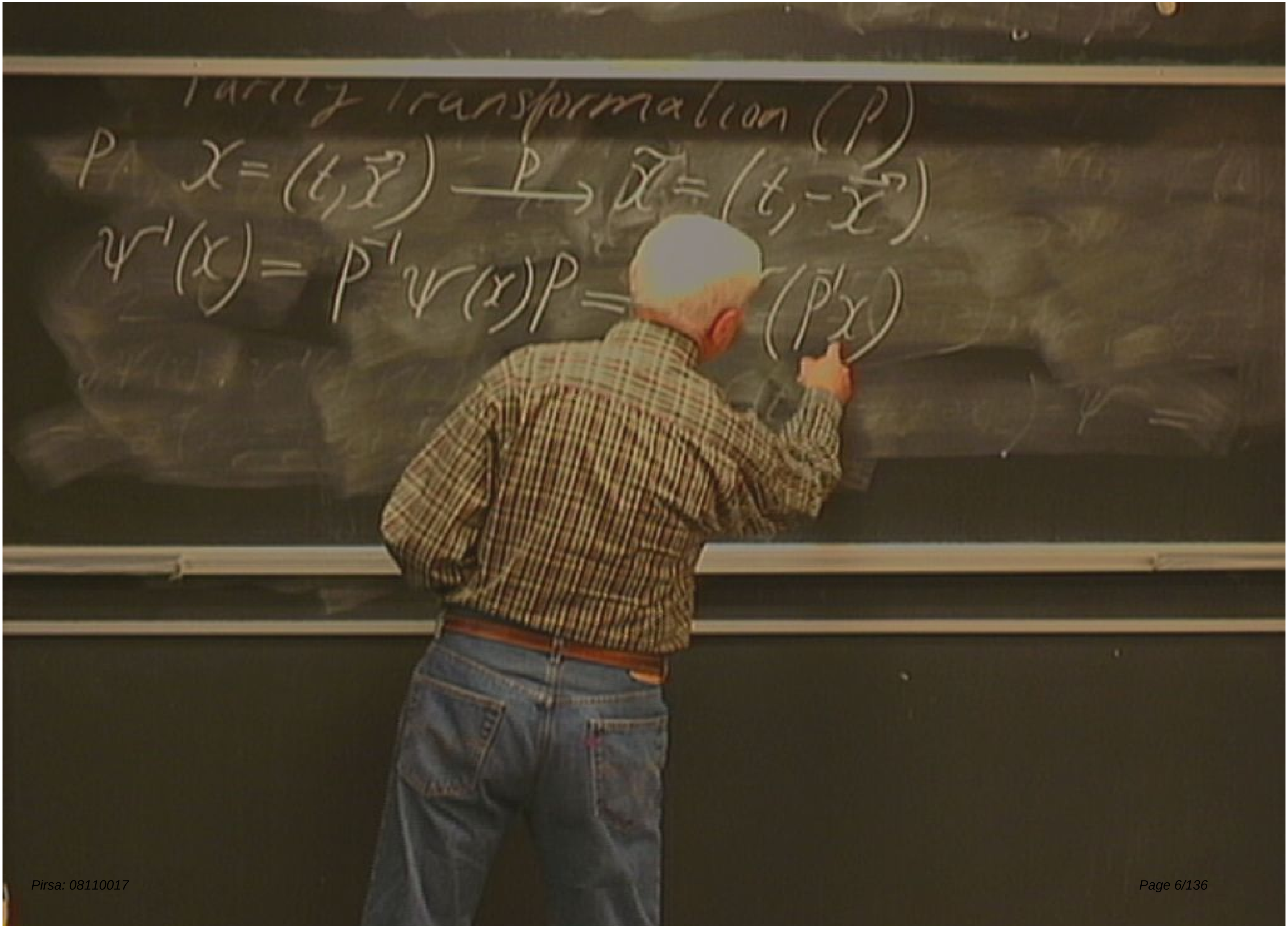
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

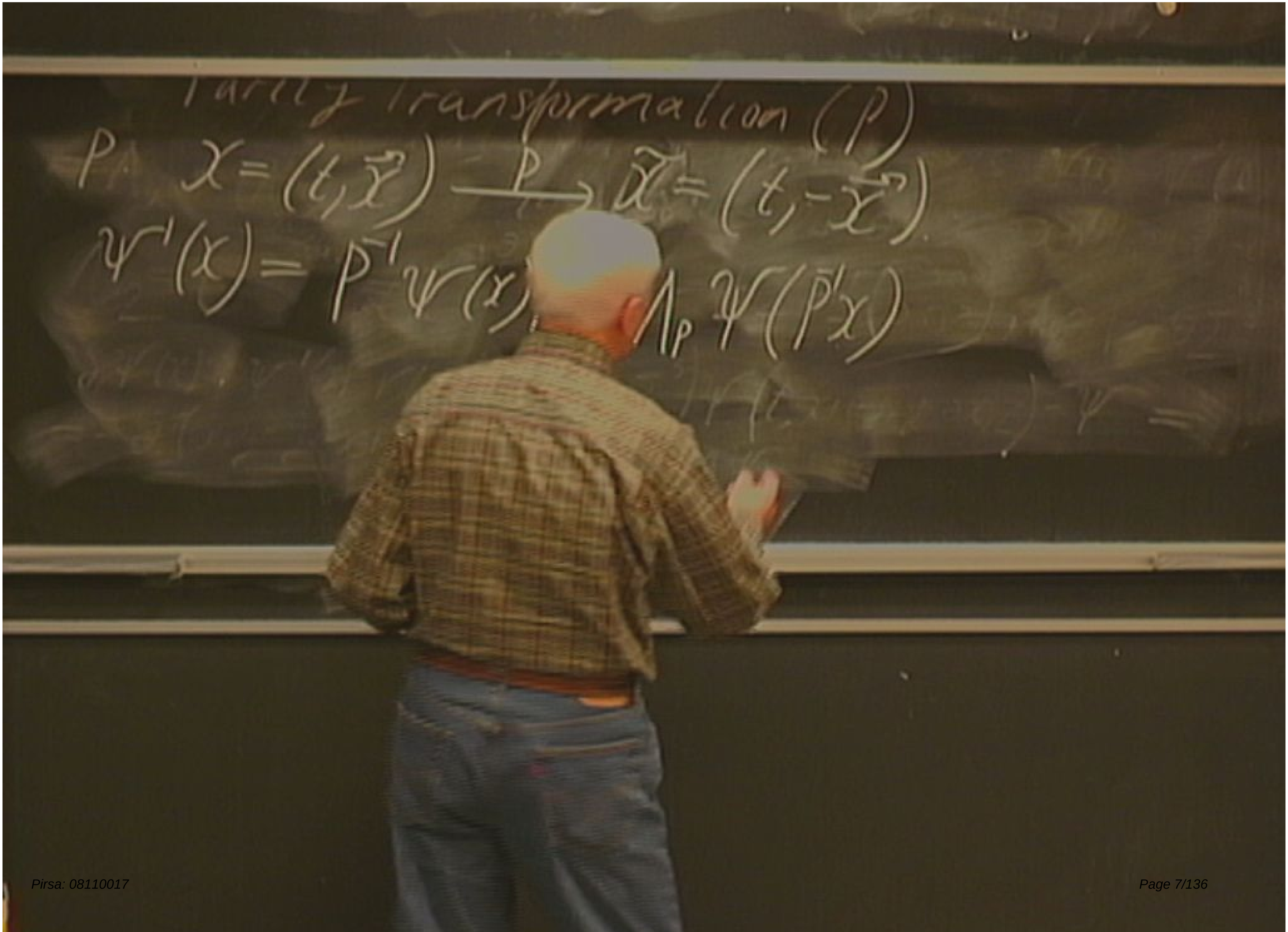

J^{012} = I_z = i ψ̄ γ^0 Δψ = −i ψ̄ γ^0 (x∂_y − y∂_x + ½Σ^3) ψ
orbital
J⃗ = ψ̄ γ^0 (x⃗ × (−i∇⃗) + ½Σ⃗) ψ = ψ^† (x⃗ × (−i∇⃗) + ½Σ⃗) ψ
ψ^†

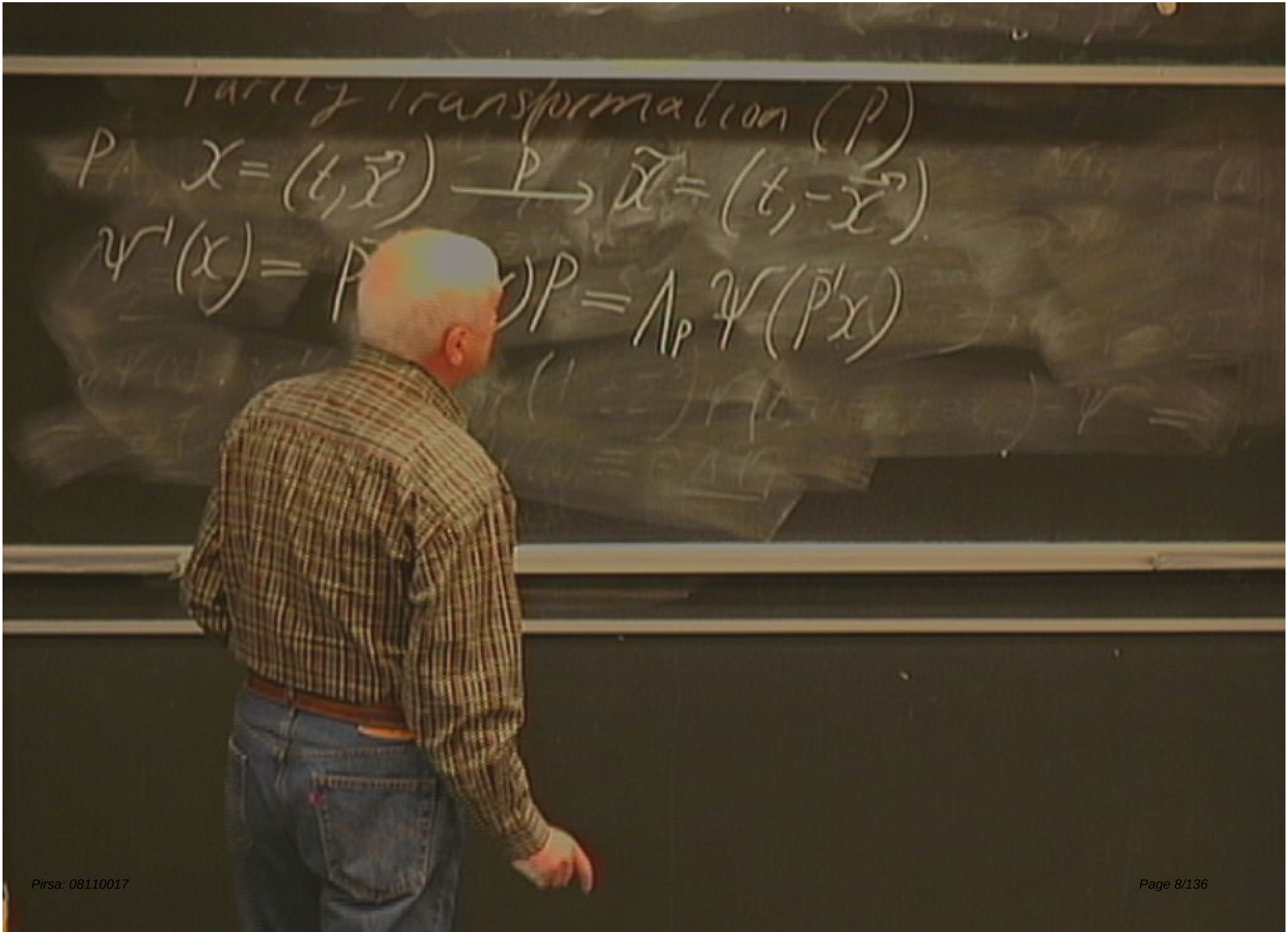

P, C, T Transformations

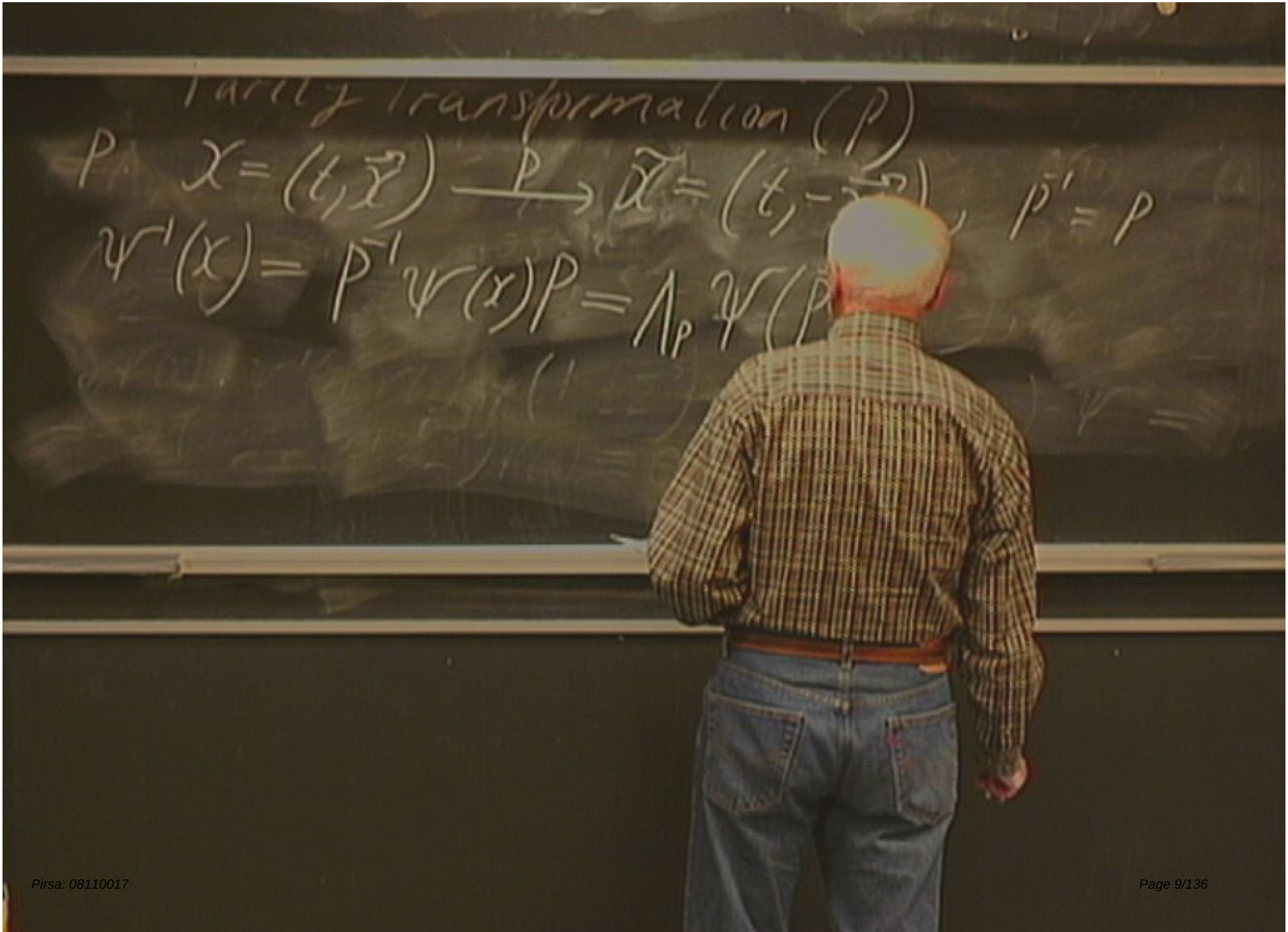

P, C, T Transformations
Parity Transformation (P)
x = (t, x⃗)

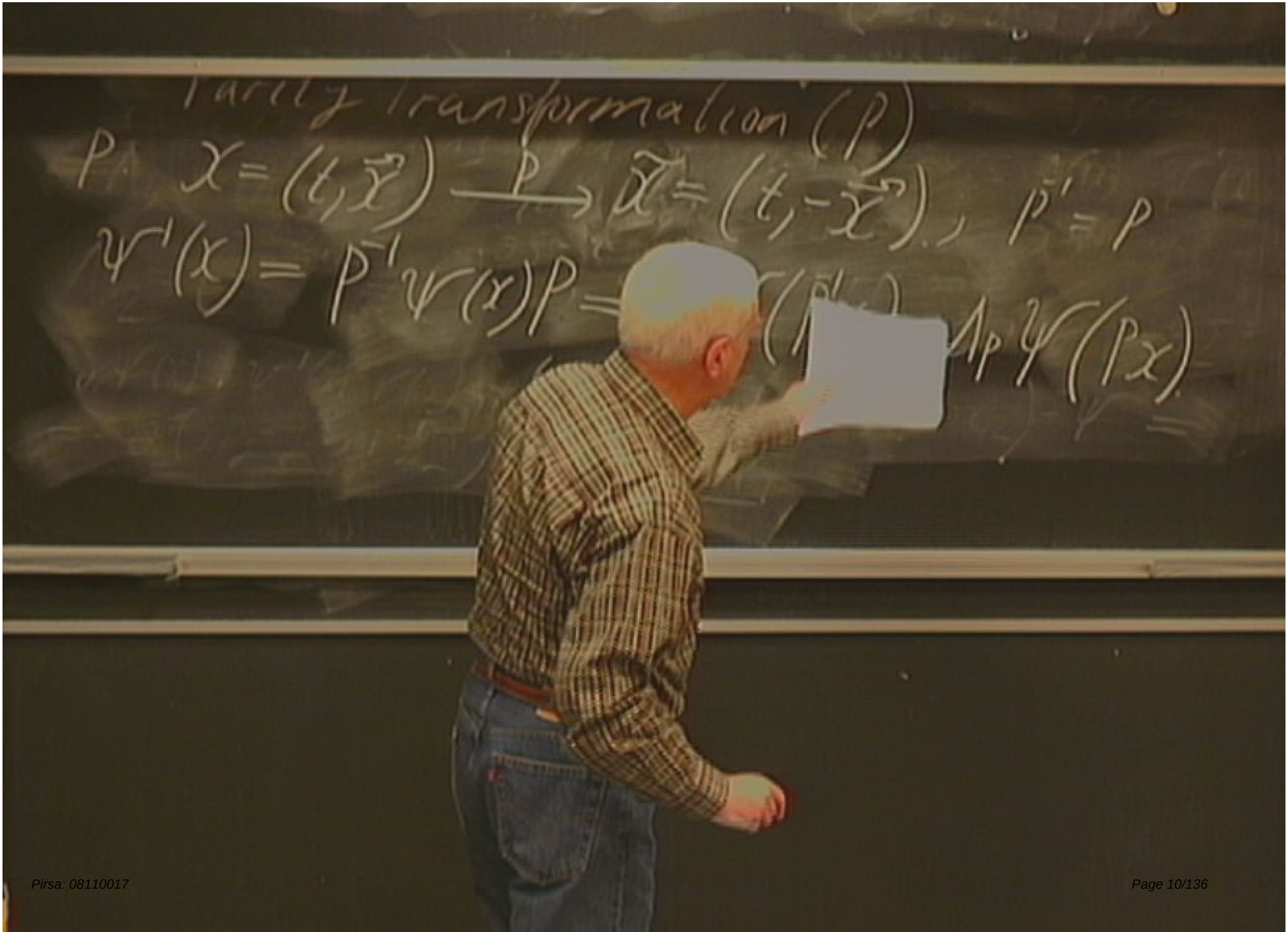

P x = (t, x⃗) —P→ x̃ = (t, −x⃗).
ψ′(x) = P⁻¹ ψ(x) P


Parity Transformation (P)
P: x=(t,x⃗) —P→ x̃=(t,−x⃗)
ψ'(x) = P⁻¹ψ(x)P =
(P'x)

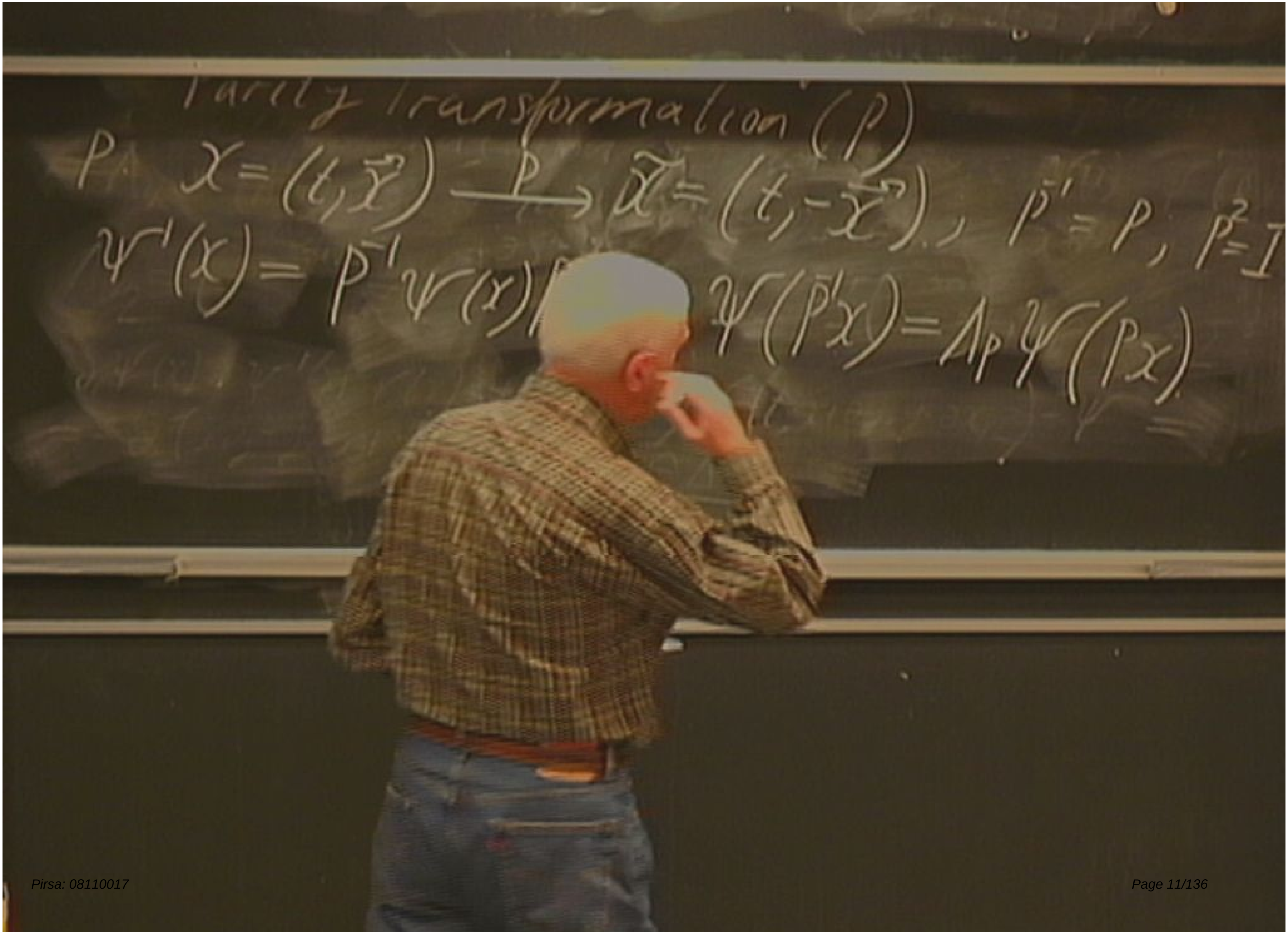

Parity Transformation (P)
P: x=(t,x⃗) —P→ x̃=(t,−x⃗)
ψ'(x) = P⁻¹ ψ(x) = Λ_P ψ(P⁻¹x)

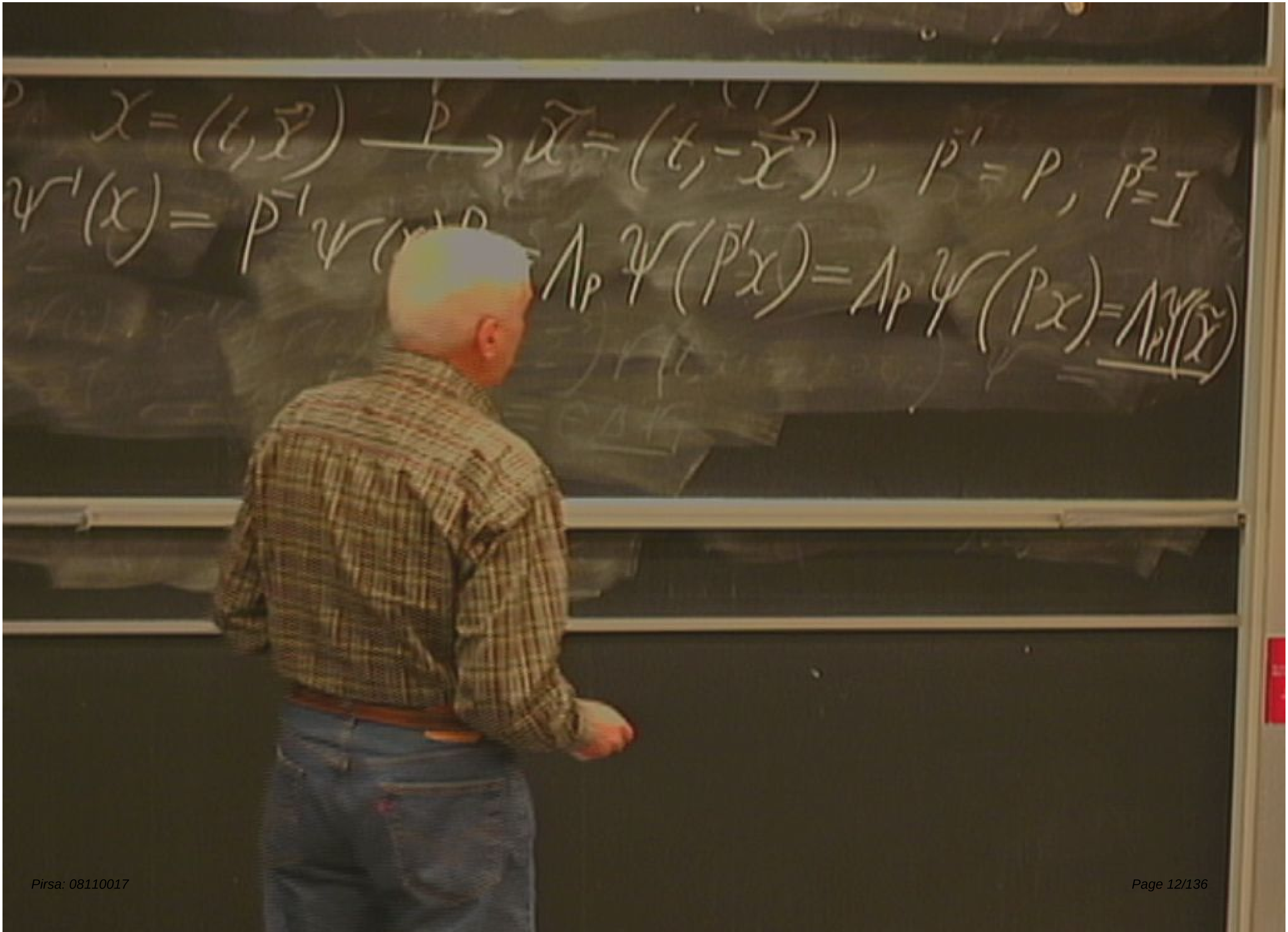

Parity Transformation (P)
P: x=(t,x) P x̃=(t,-x)
ψ'(x)=
P=Λp ψ(P'x)

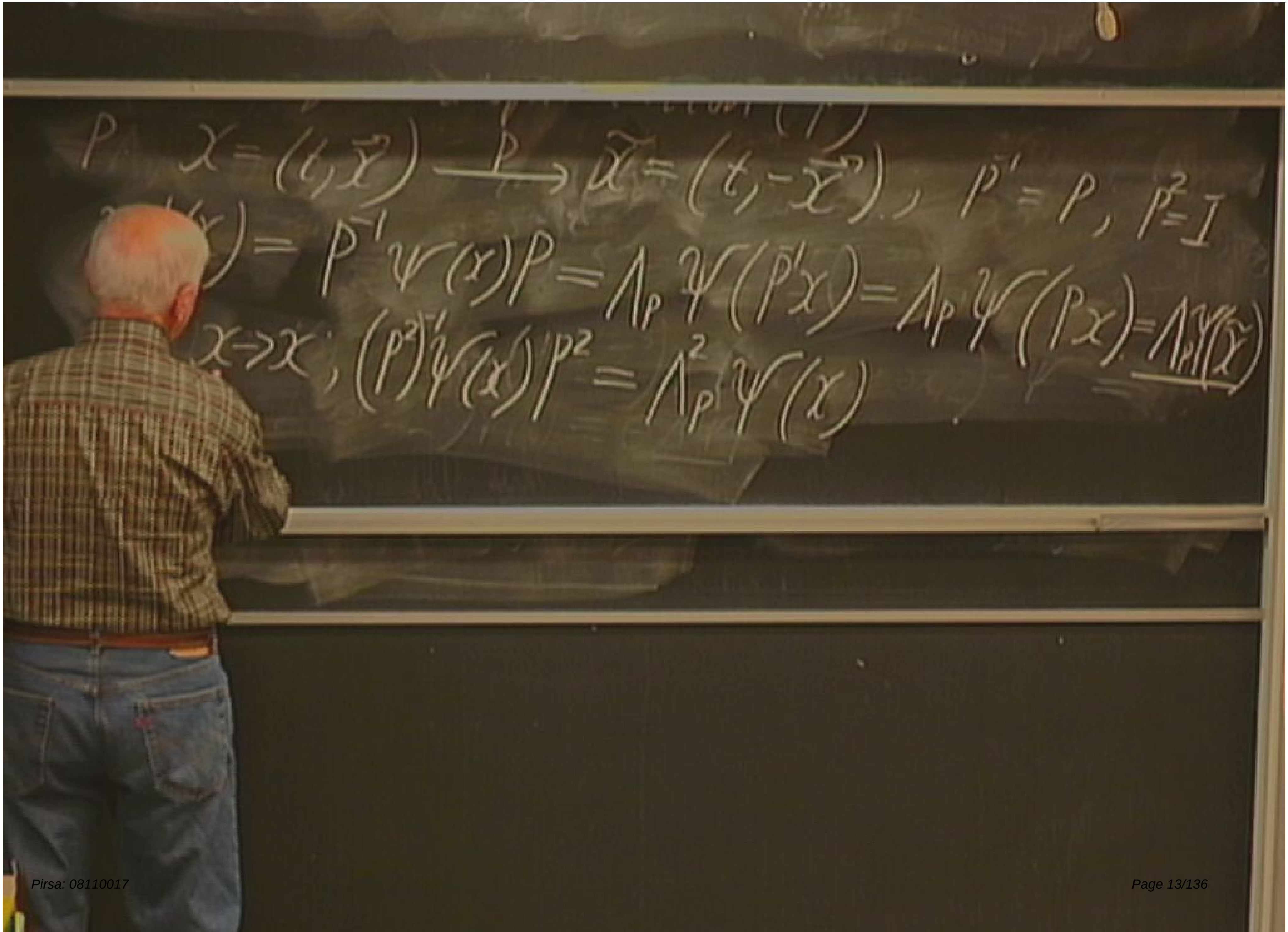

Parity Transformation (P)

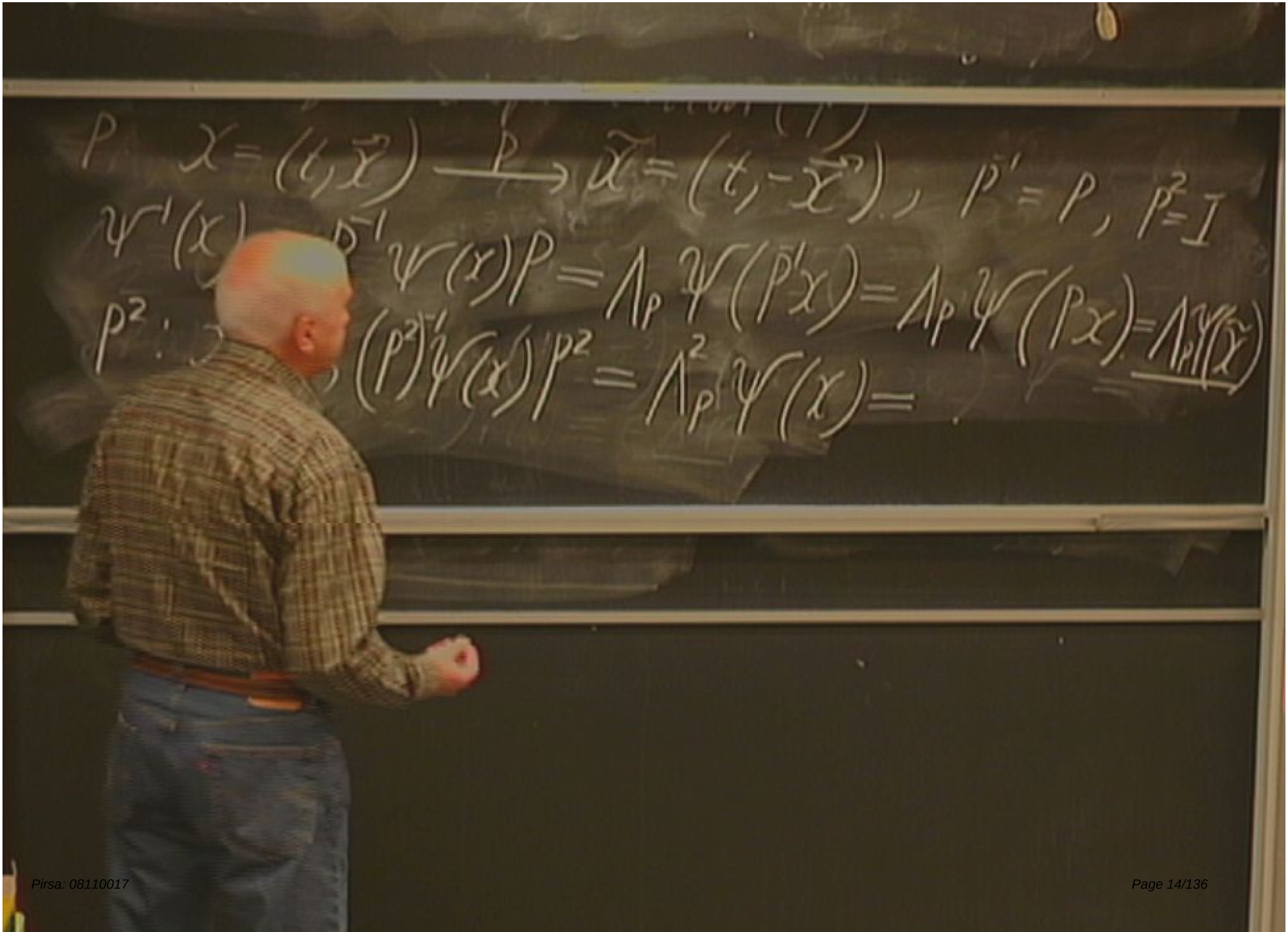


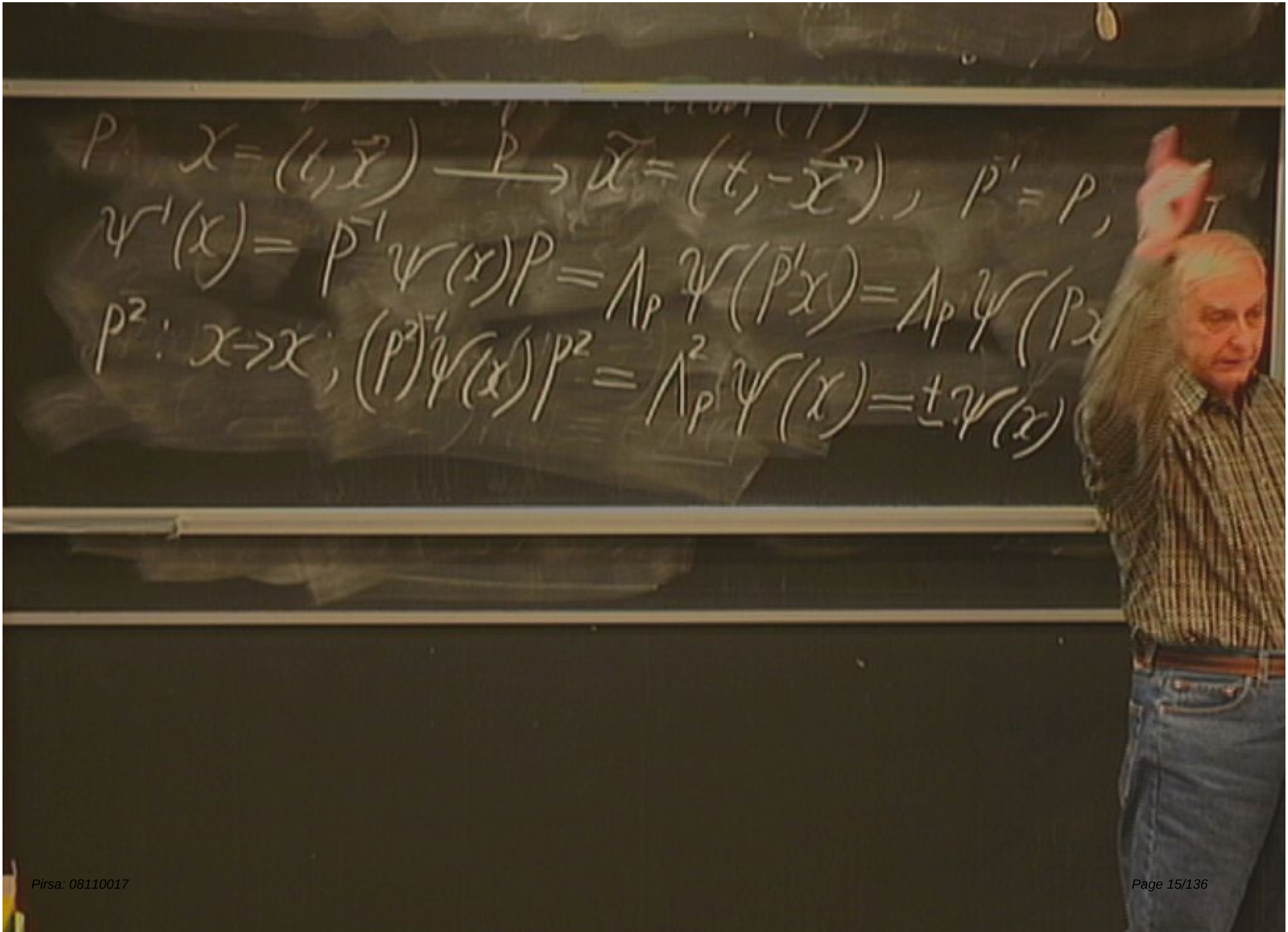
Parity Transformation (P)
P: x = (t, x⃗) —P→ x̃ = (t, −x⃗), P⁻¹ = P
ψ'(x) = P⁻¹ψ(x)P =
Λ_P ψ(Px)

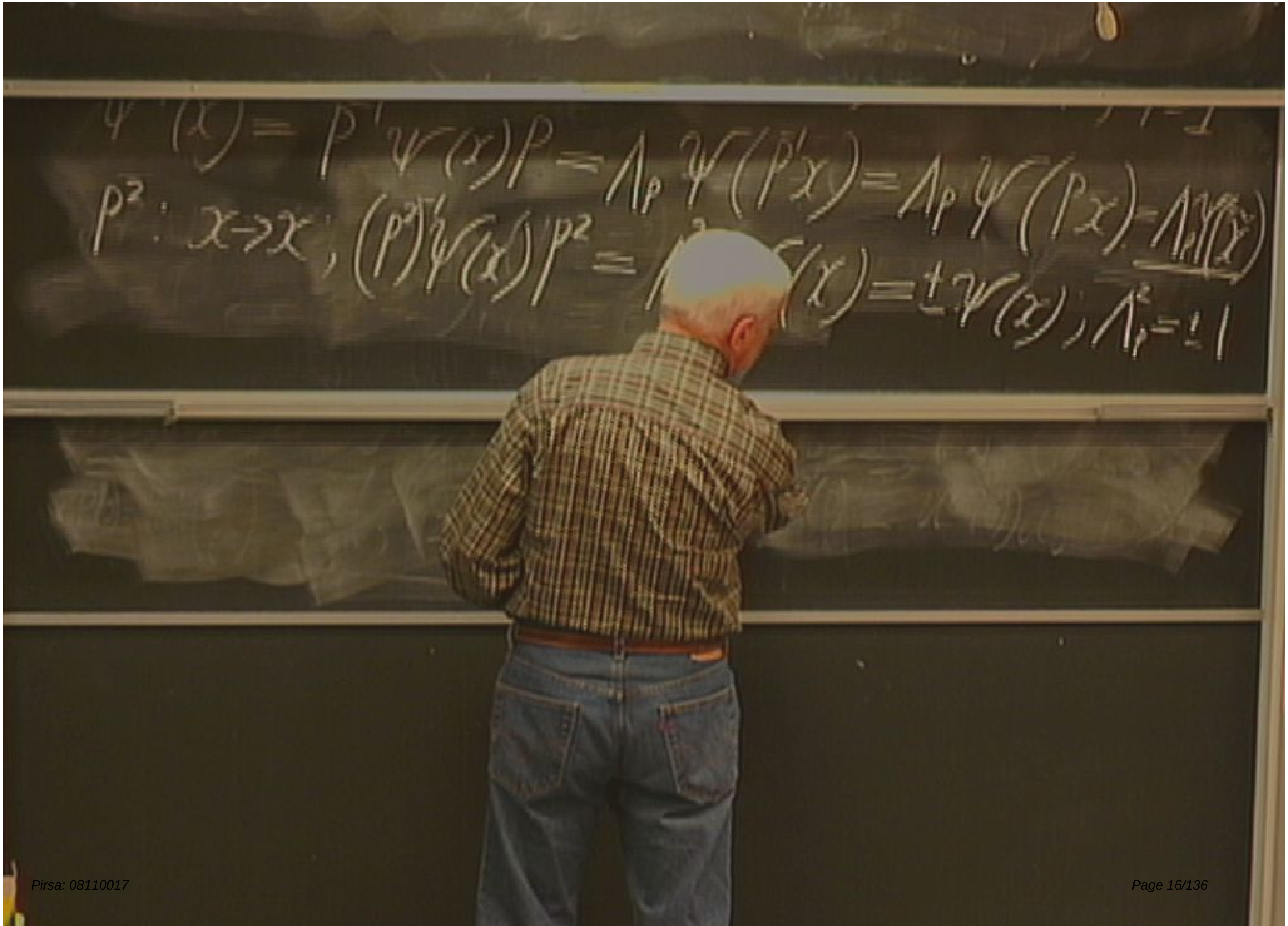

Parity Transformation (P)
P: x=(t,x) → x̃=(t,-x), P⁻¹=P, P²=I

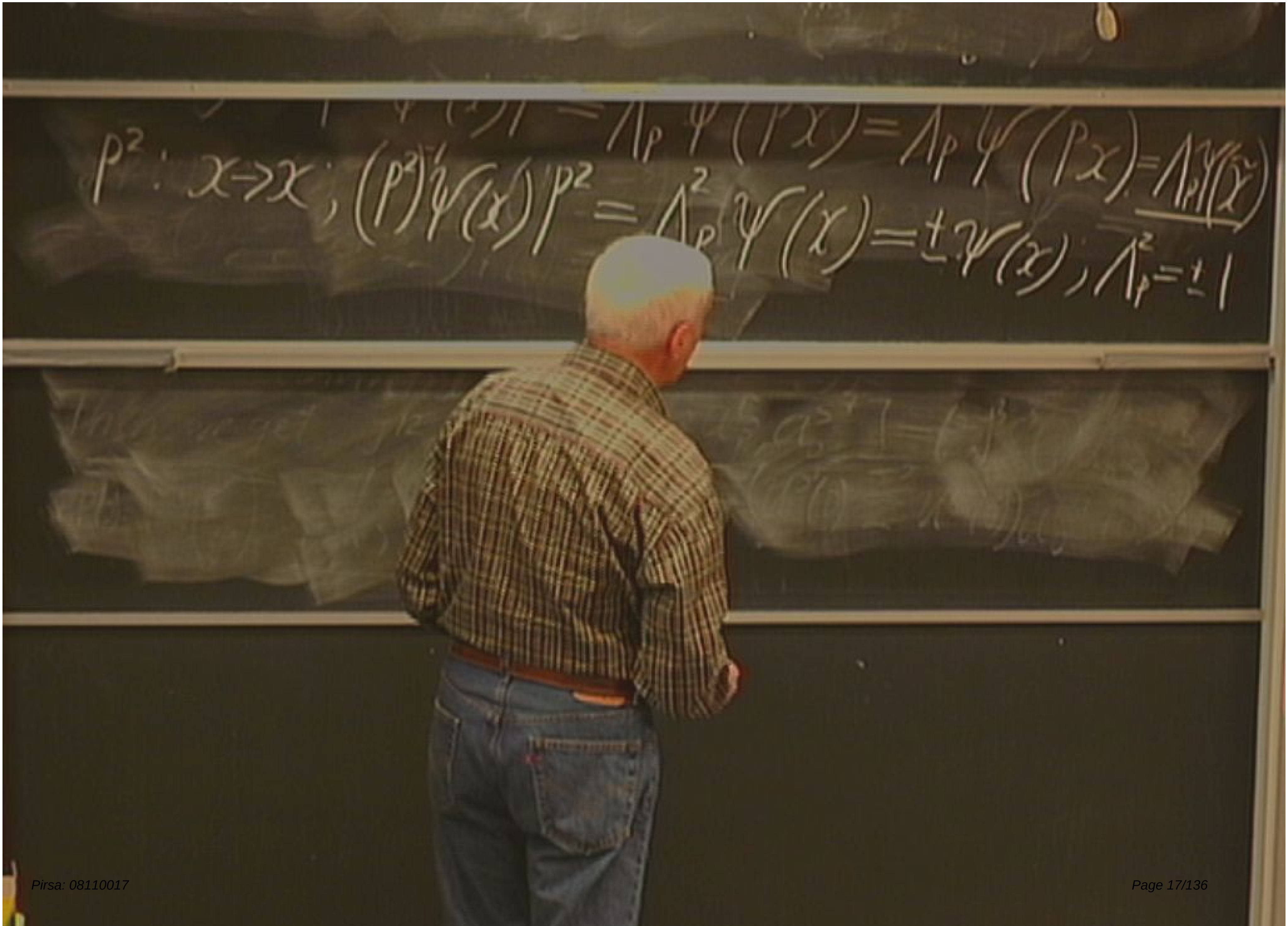

x = (t, x⃗) —P→ x̃ = (t, −x⃗), P' = P, P² = I
ψ'(x) = P⁻¹ψ(x)P = Λ_P ψ(P'x) = Λ_P ψ(Px) = Λ_P ψ(x̃)

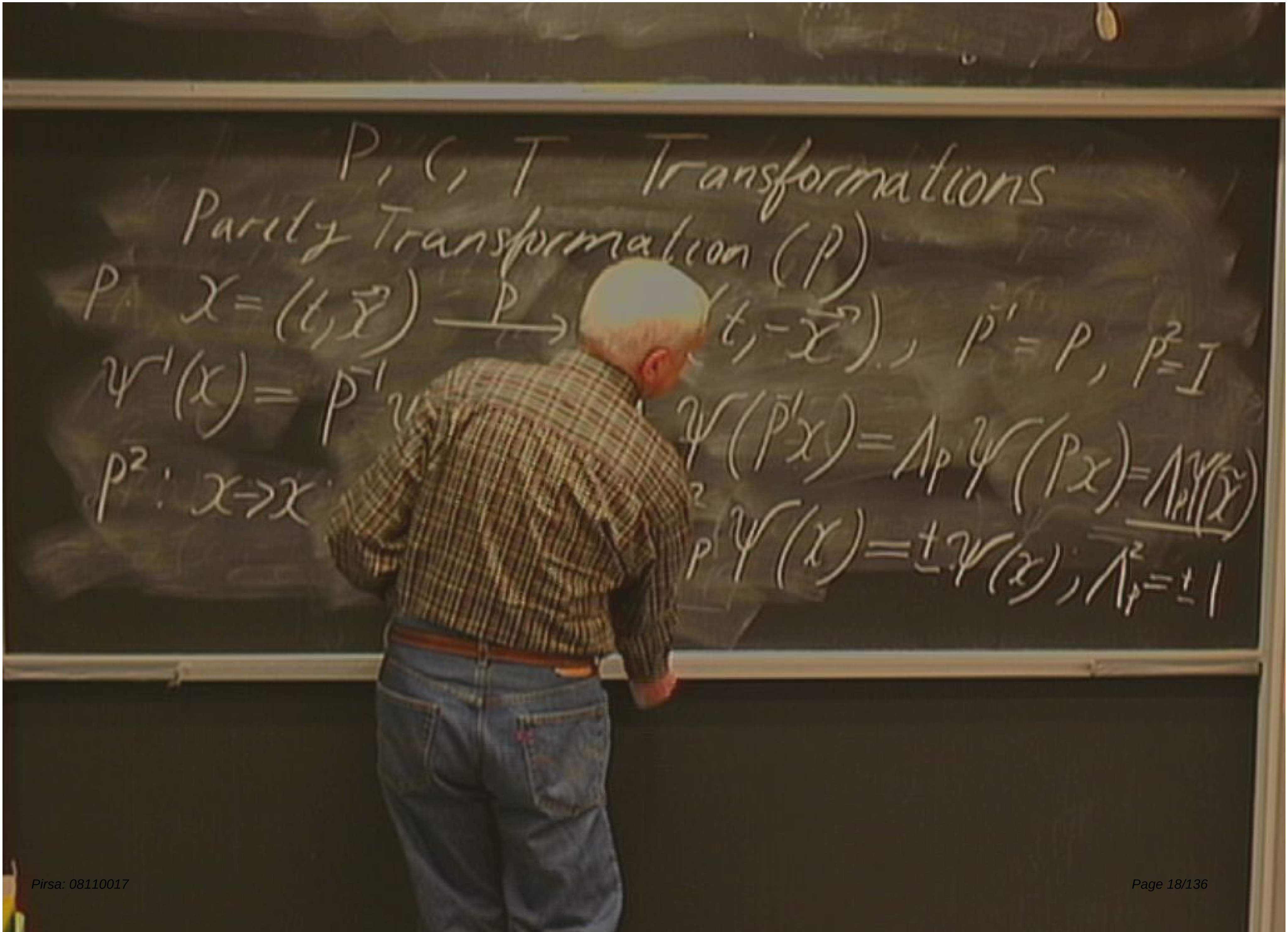

P: x = (t, x⃗) →P→ x̃ = (t, −x⃗), P' = P, P² = I
(x) = P⁻¹ ψ(x) P = Λ_P ψ(P'x) = Λ_P ψ(Px) = Λ_P ψ(x̃)
x → x; (P²)⁻¹ ψ(x) P² = Λ_P² ψ(x)


P x = (t, x⃗) —P→ x̃ = (t, −x⃗), P' = P, P² = I
ψ'(x) P⁻¹ ψ(x) P = Λp ψ(P'x) = Λp ψ(Px) = Λp ψ(x̃)
P²: (P²)⁻¹ ψ(x) P² = Λp² ψ(x) =

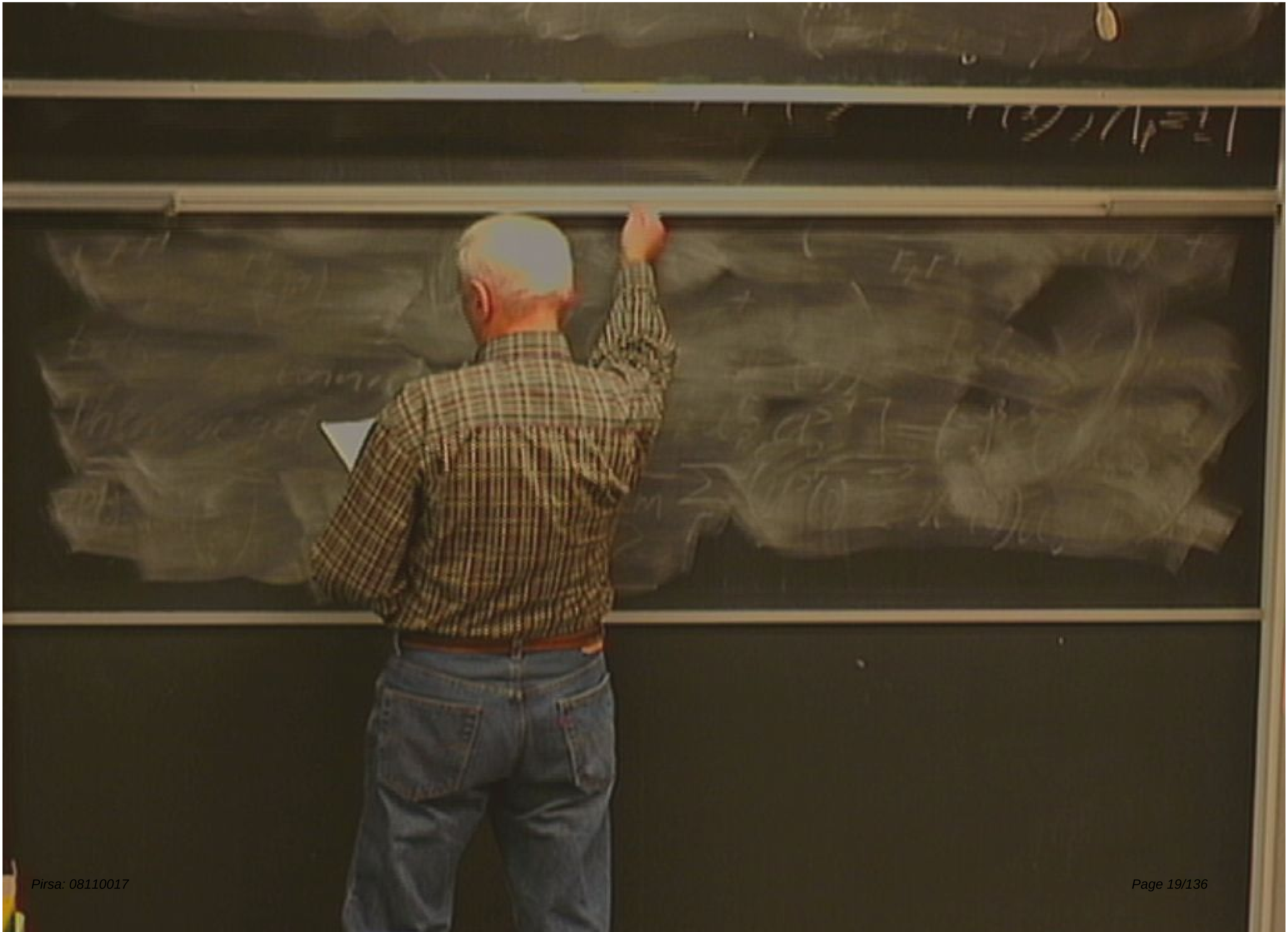

P x = (t, x⃗) —P→ x̃ = (t, −x⃗), P′ = P,
ψ′(x) = P⁻¹ψ(x)P = Λ_P ψ(P′x) = Λ_P ψ(P
P² : x→x ; (P²)⁻¹ψ(x)P² = Λ_P² ψ(x) = ±ψ(x)




= Λp ψ(Px) = Λp ψ(Px) = Λp ψ(x̃)
P² : x→x ; (P²)⁻¹ψ(x) P² = Λp² ψ(x) = ± ψ(x) ; Λp² = ±1



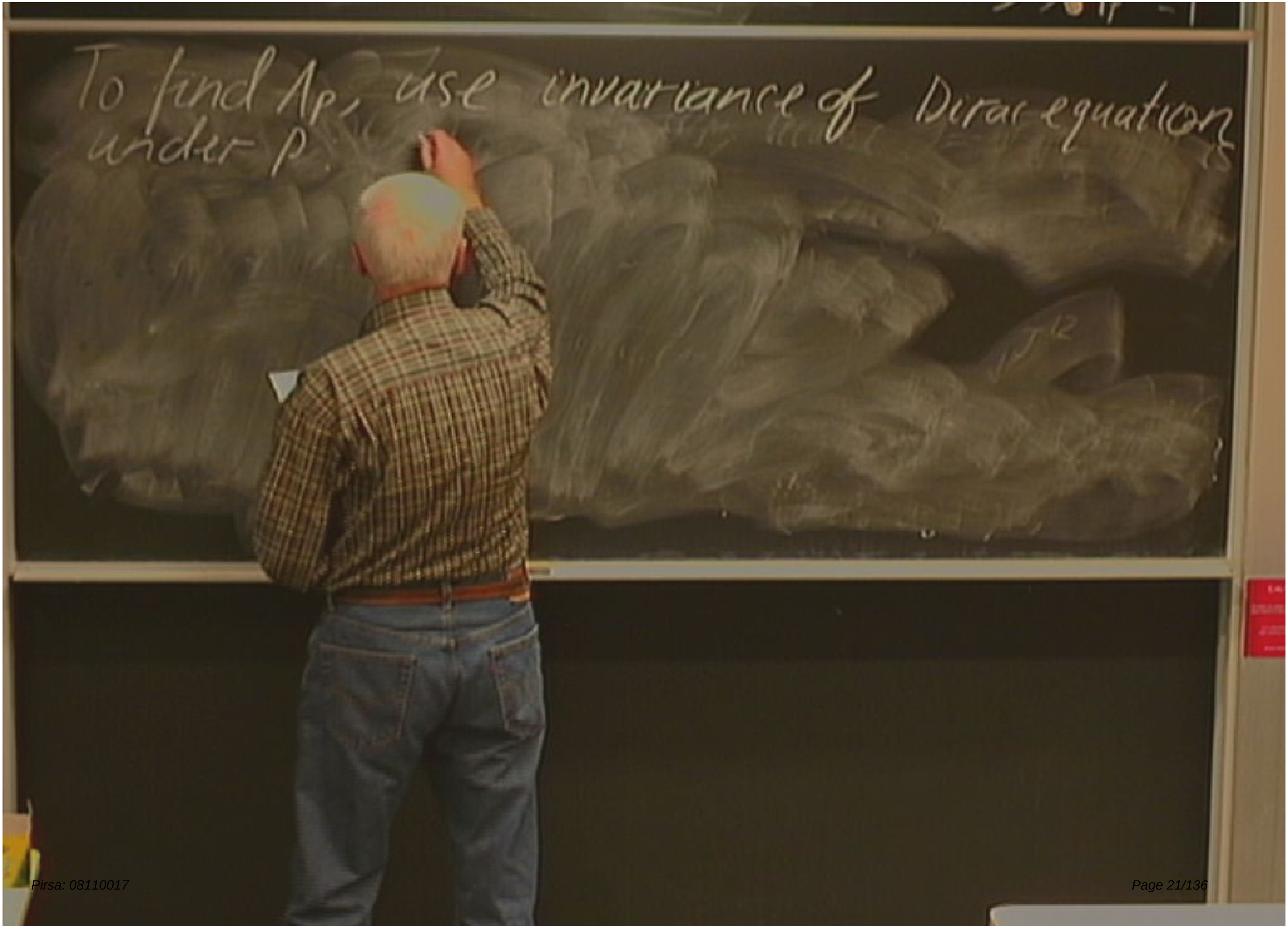
P, C, T Transformations
Parity Transformation (P)

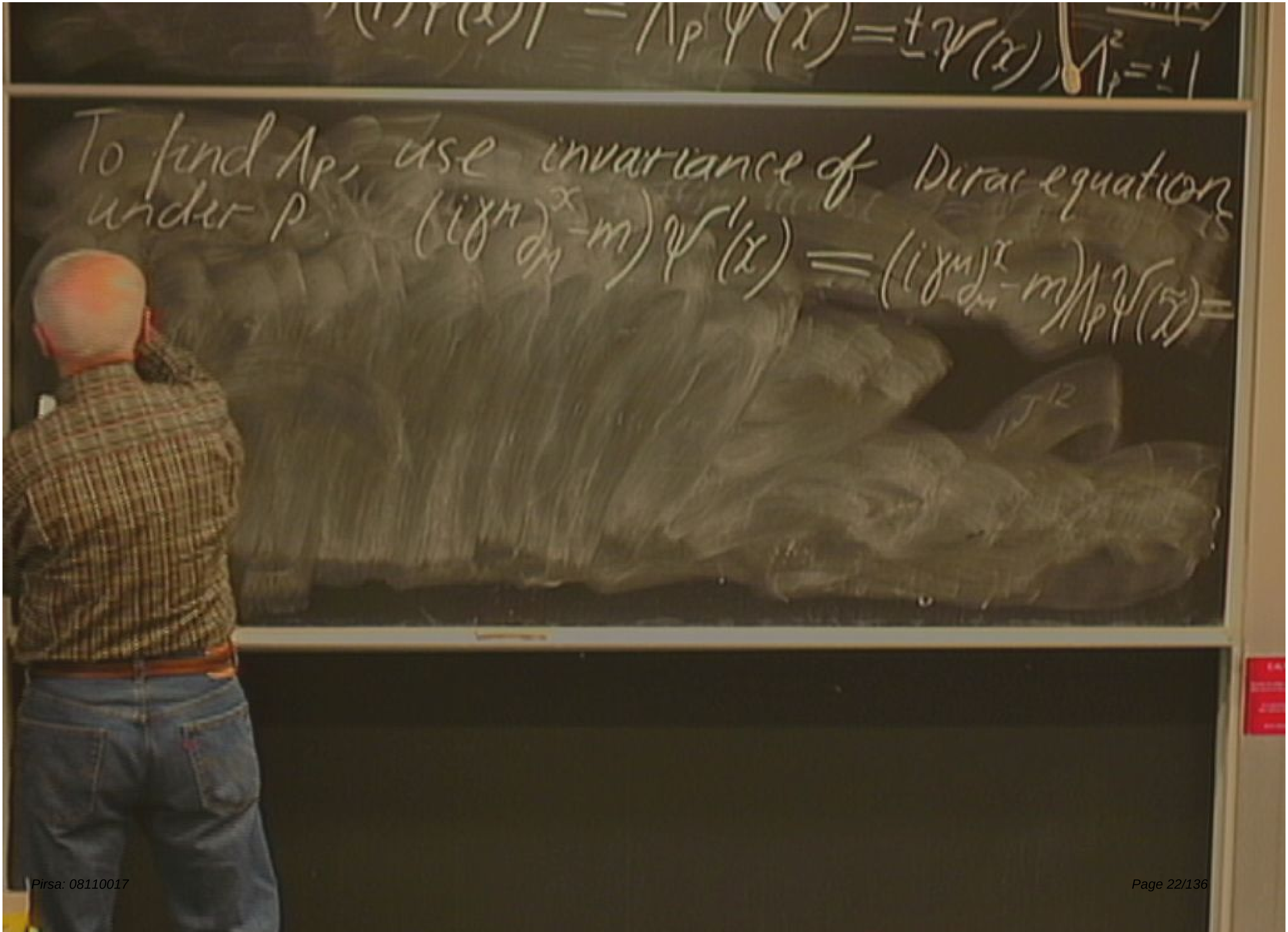


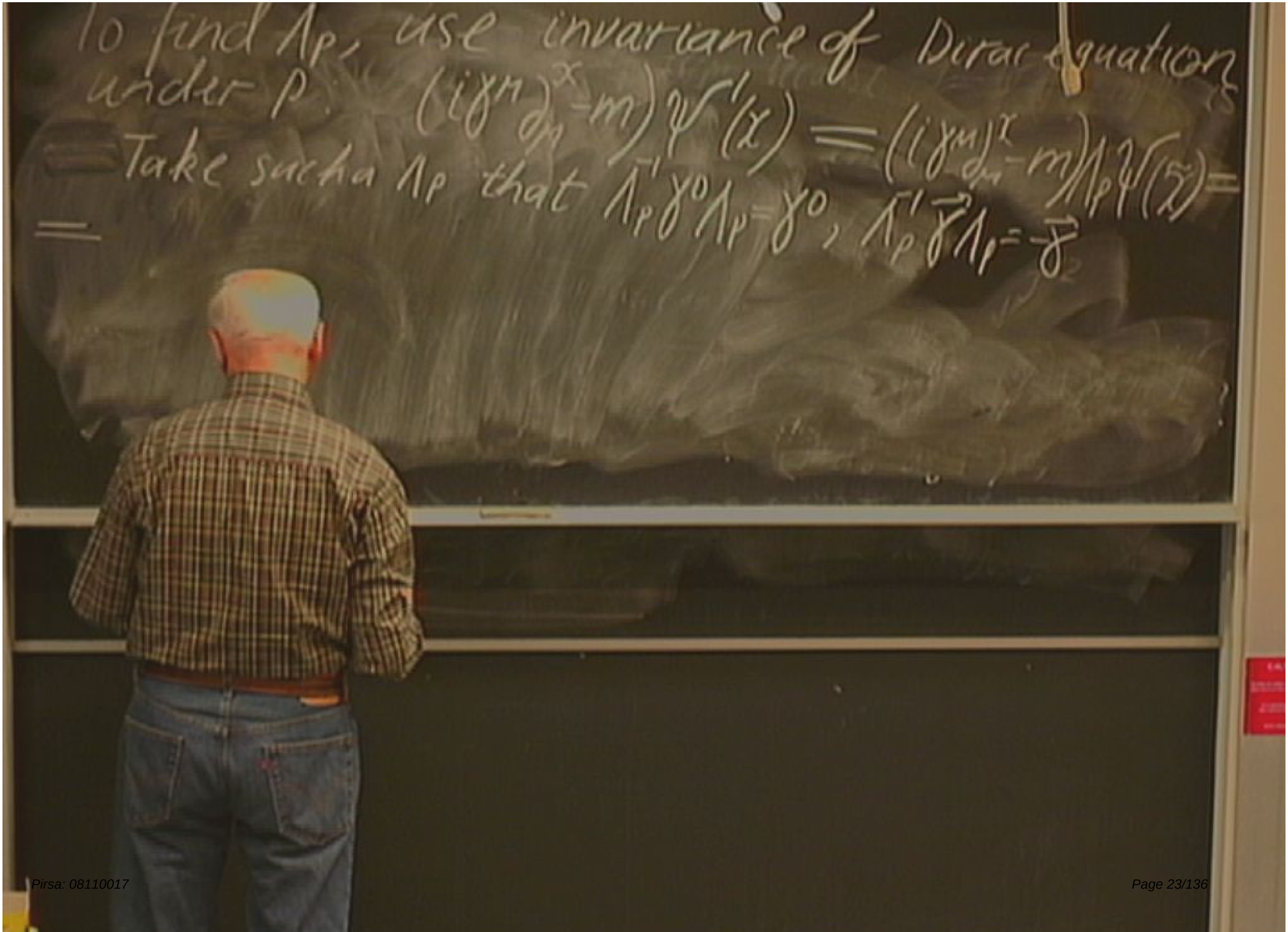
$$J^{12} = I_z = i\bar{\psi}\gamma^0 \Delta\psi = -i\bar{\psi}\gamma^0\left(x\partial_y - y\partial_x + \frac{i}{2}\Sigma^3\right)\psi$$

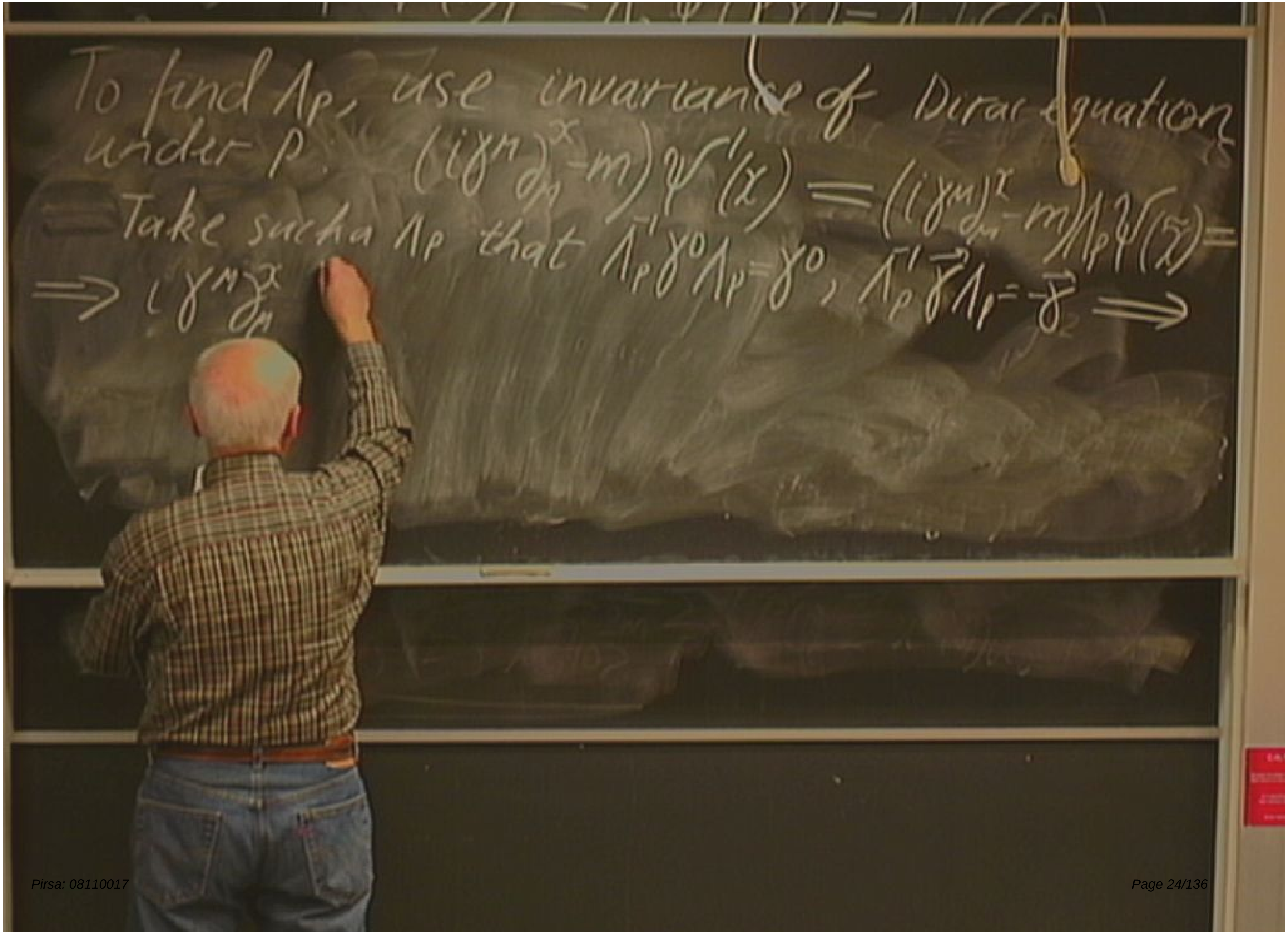
(orbital ang. mom.)

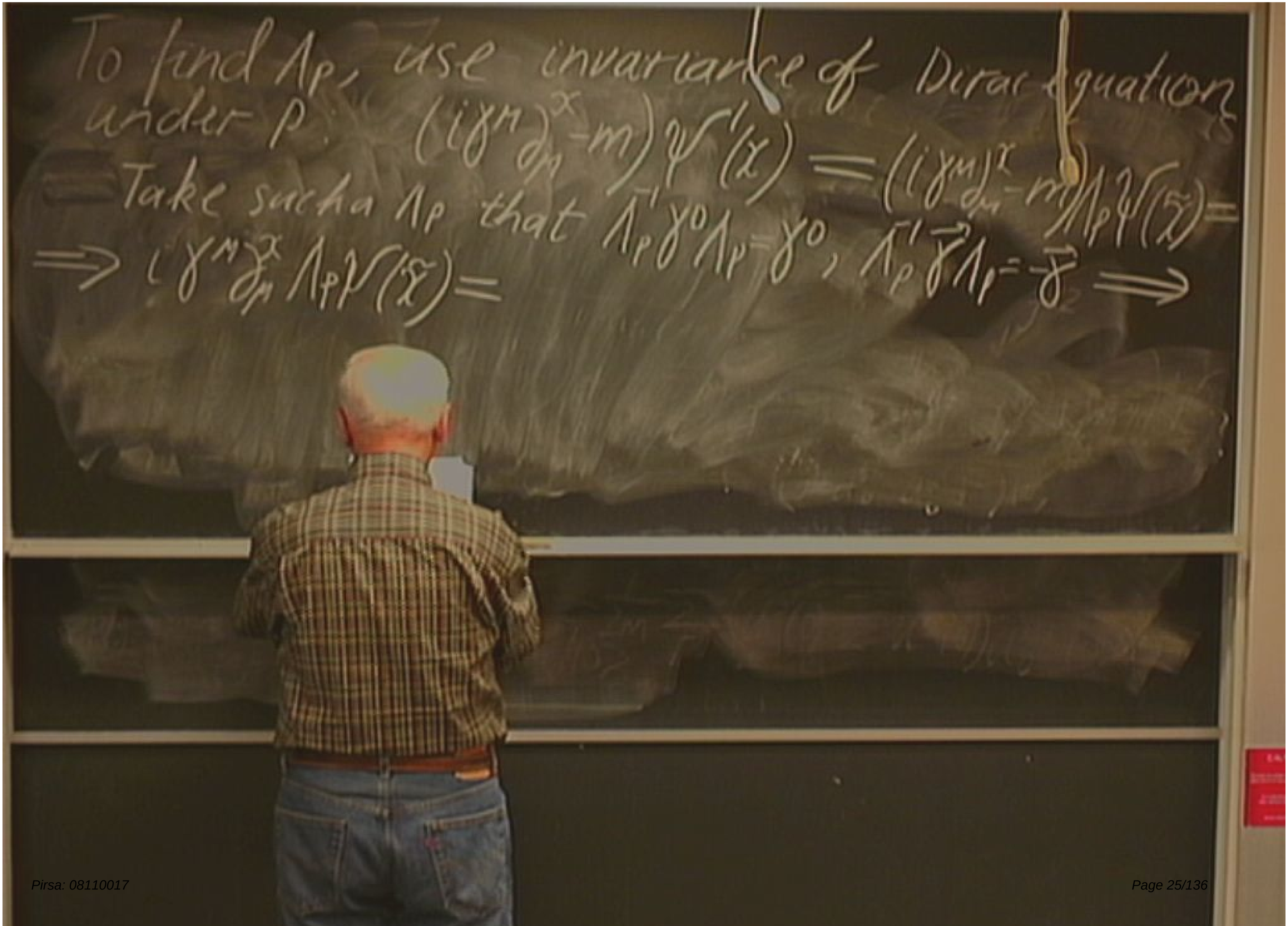
$$\vec{J} = \underbrace{\bar{\psi}\gamma^0}_{\psi^\dagger}\left(\vec{x}\times(-i\vec{\nabla}) + \frac{1}{2}\vec{\Sigma}\right)\psi = \psi^\dagger\left(\vec{x}\times(-i\vec{\nabla}) + \frac{1}{2}\vec{\Sigma}\right)\psi$$

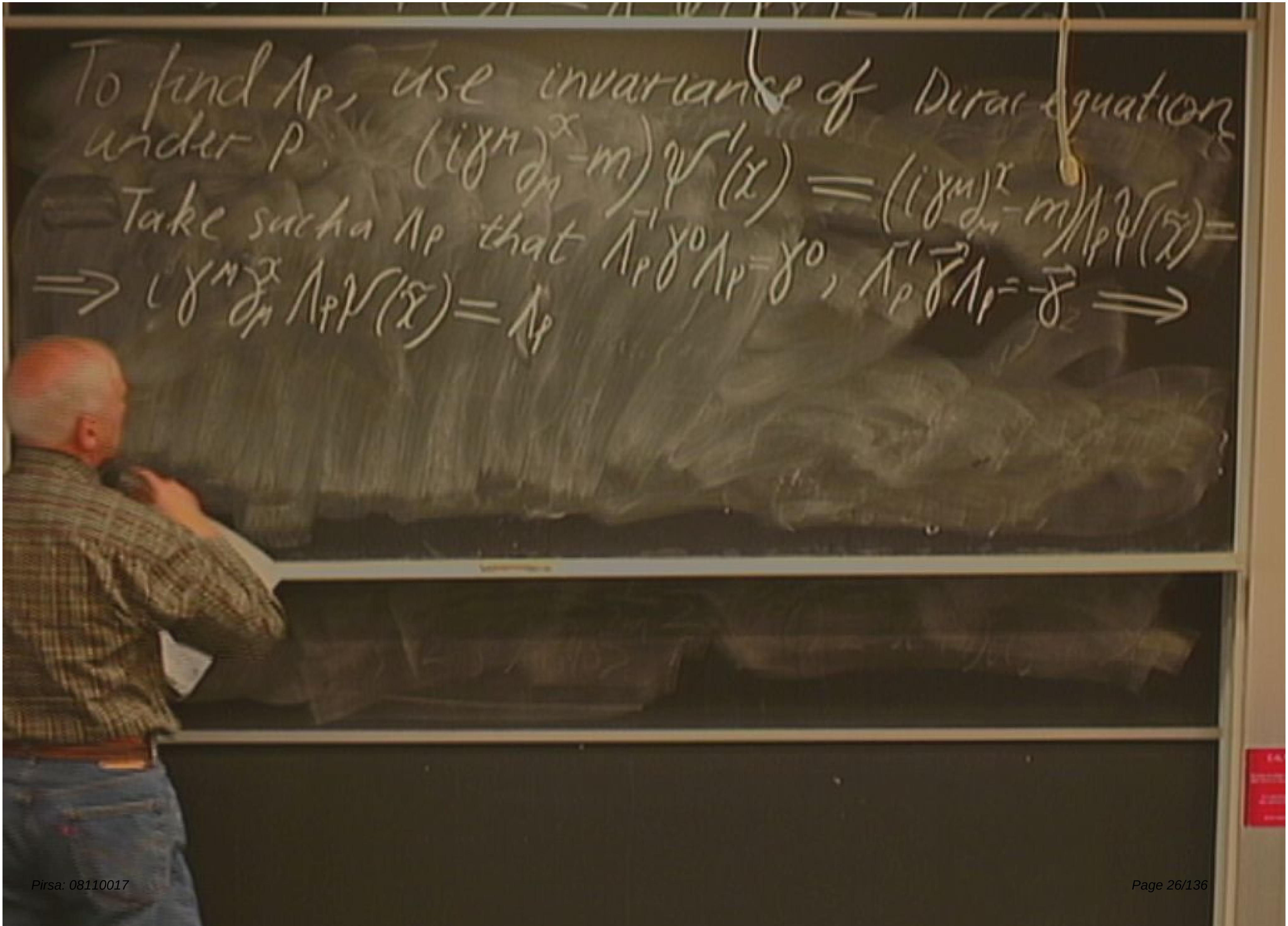

To find A_P, use invariance of Dirac equation
under P

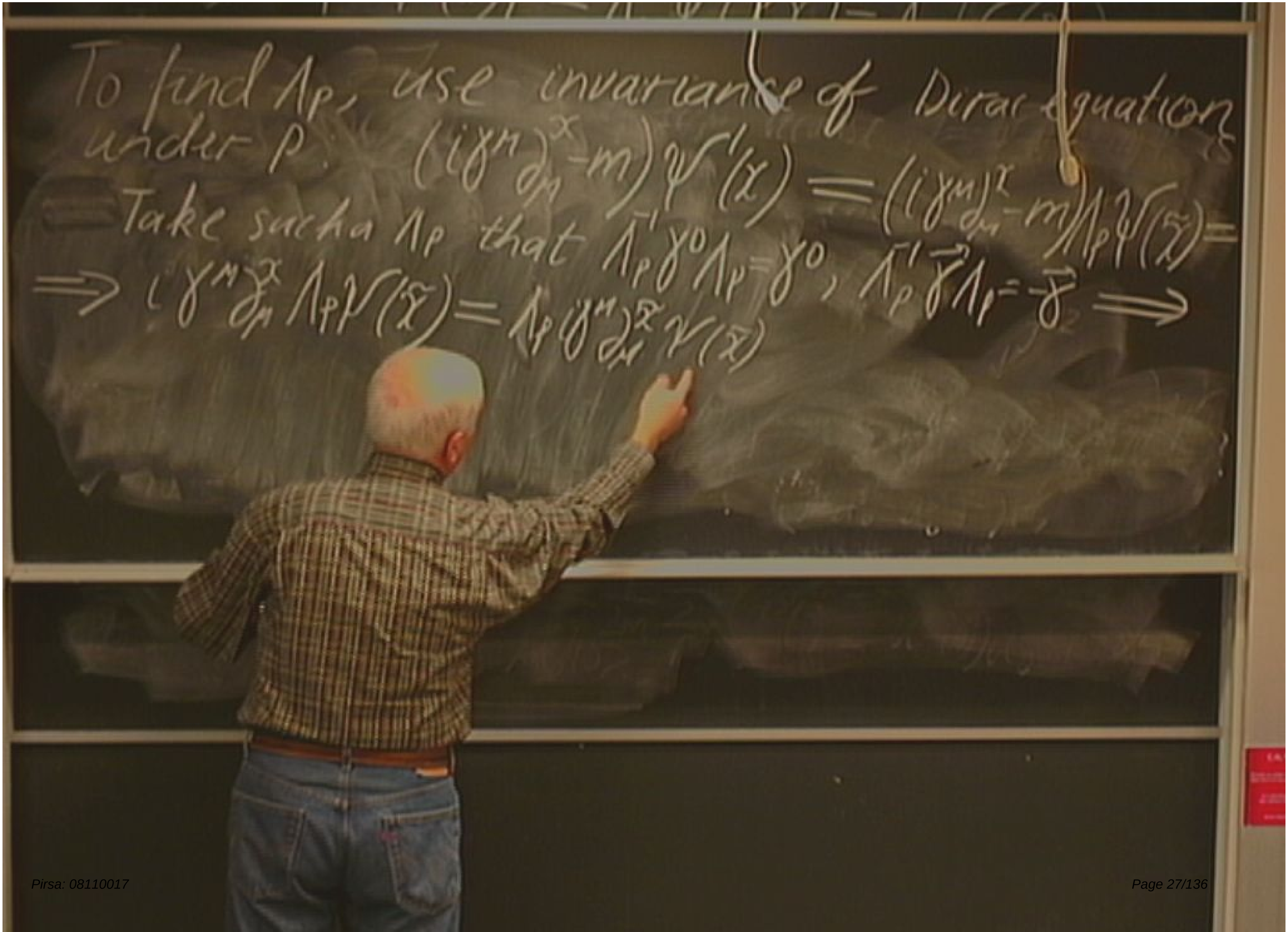

$= \Lambda_P \psi(\tilde{x}) = \pm \psi(\tilde{x})$, $\Lambda_P^2 = \pm 1$
To find Λ_P, use invariance of Dirac equation
under P: $(i\gamma^\mu \partial_\mu^x - m)\psi'(x) = (i\gamma^\mu \partial_\mu^x - m)\Lambda_P \psi(\tilde{x}) =$


To find Λp, use invariance of Dirac equation
under P: (iγ^μ ∂_μ^x − m)ψ'(x) = (iγ^μ ∂_μ^x − m)Λp ψ(x̃) =
Take such a Λp that Λp^-1 γ^0 Λp = γ^0, Λp^-1 γ⃗ Λp = −γ⃗
=


To find Λ_P, use invariance of Dirac equation
under P: $(i\gamma^\mu \partial_\mu^x - m)\psi'(x) = (i\gamma^\mu \partial_\mu^x - m)\Lambda_P \psi(\bar{x}) =$
Take such a Λ_P that $\Lambda_P^{-1}\gamma^0\Lambda_P = \gamma^0$, $\Lambda_P^{-1}\vec{\gamma}\Lambda_P = -\vec{\gamma} \Longrightarrow$
$\Longrightarrow i\gamma^\mu \partial_\mu^x$


To find Λp, use invariance of Dirac equation
under P: (iγ^μ ∂_μ^x − m)ψ'(x) = (iγ^μ ∂_μ^x − m)Λp ψ(x̃) =
Take such a Λp that Λp^{-1} γ^0 Λp = γ^0, Λp^{-1} γ⃗ Λp = −γ⃗ ⟹
⟹ iγ^μ ∂_μ^x Λp ψ(x̃) =


To find Λ_P, use invariance of Dirac equation
under P: $(i\gamma^\mu \partial_\mu^x - m)\psi'(x) = (i\gamma^\mu \partial_\mu^x - m)\Lambda_P \psi(\tilde{x}) =$
Take such a Λ_P that $\Lambda_P^{-1}\gamma^0\Lambda_P = \gamma^0$, $\Lambda_P^{-1}\vec{\gamma}\Lambda_P = -\vec{\gamma}$ $\Rightarrow$
$\Rightarrow i\gamma^\mu \partial_\mu^x \Lambda_P \psi(\tilde{x}) = \Lambda_P$


To find Λ_P, use invariance of Dirac equation
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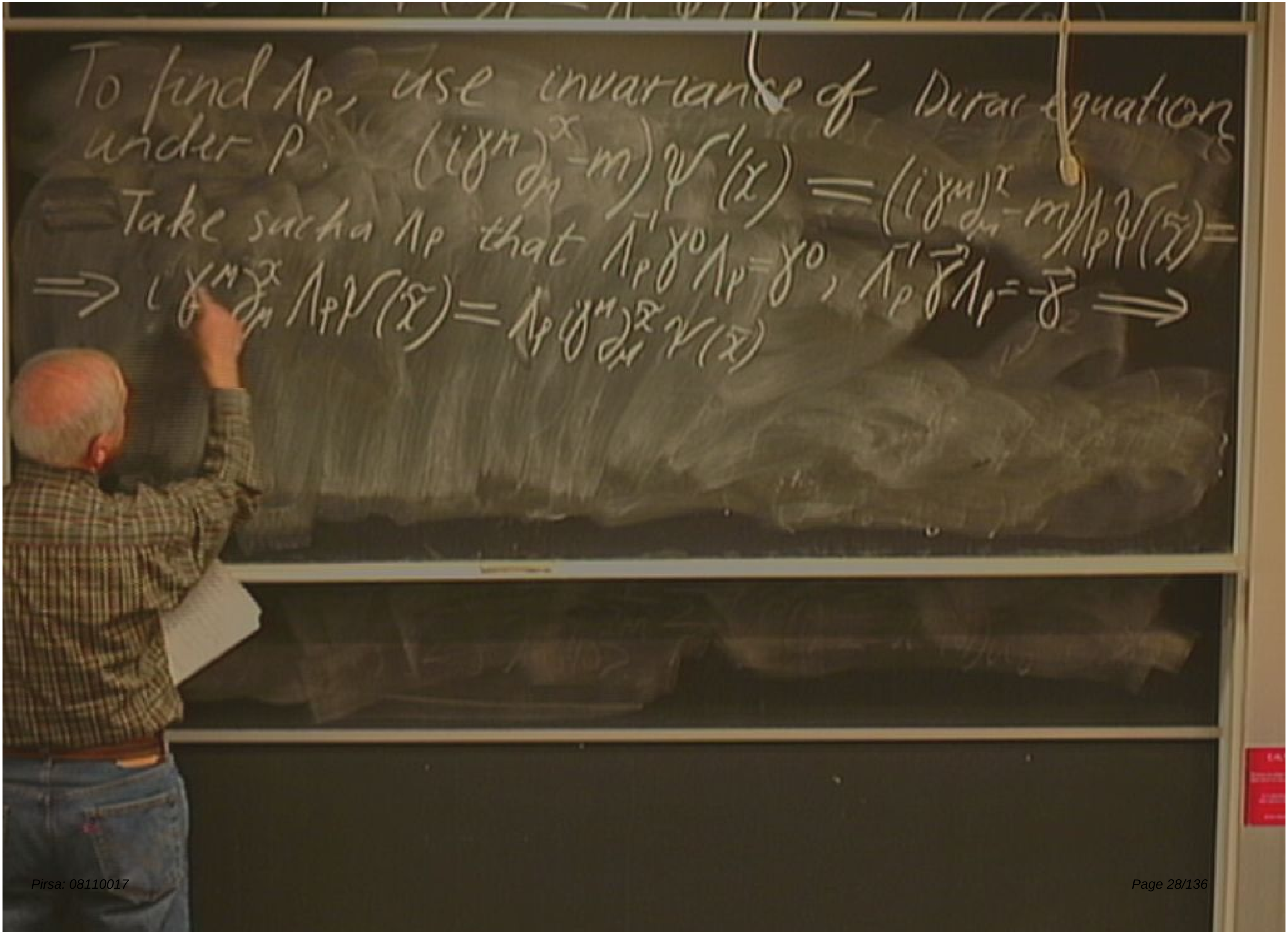
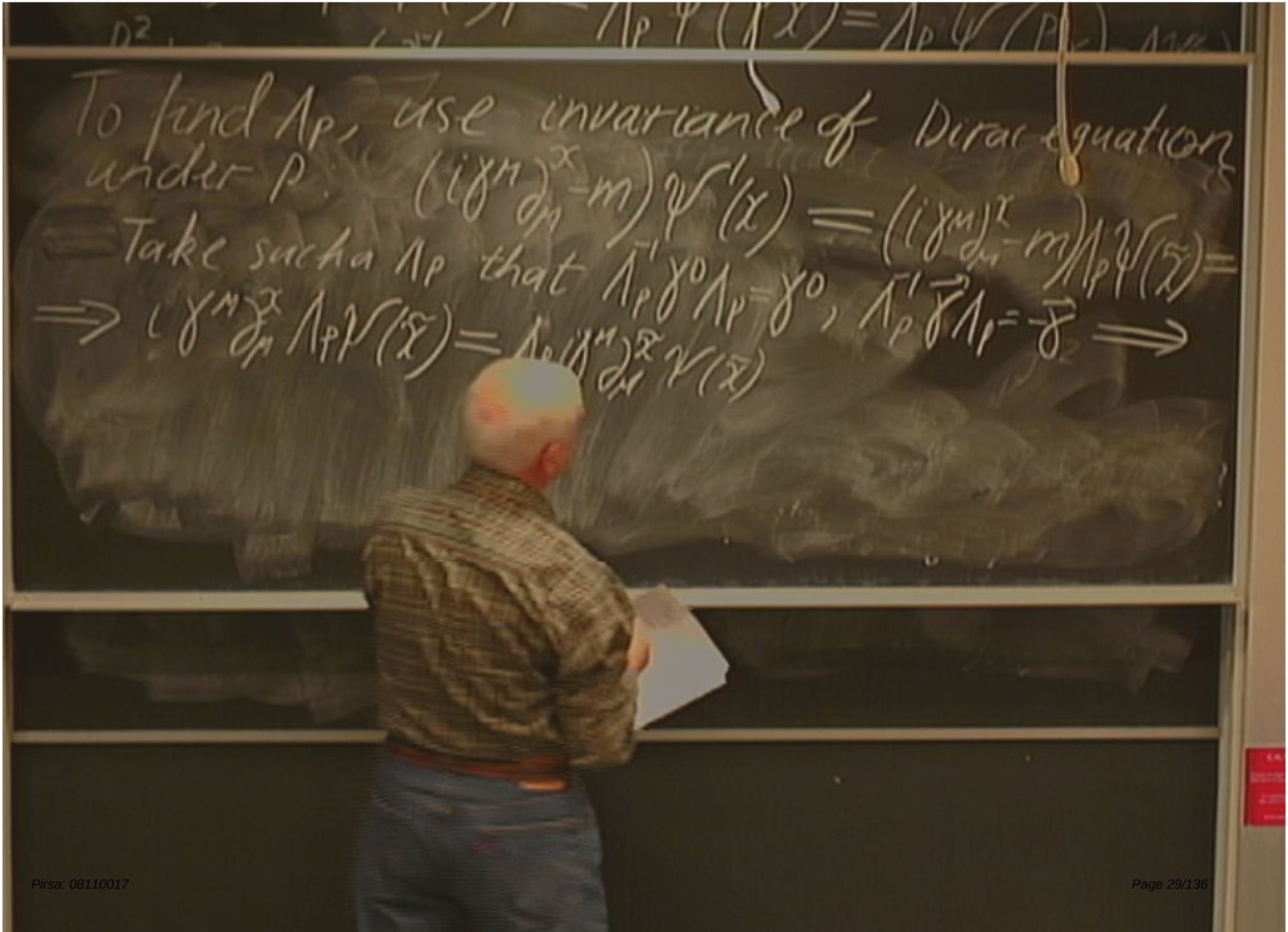
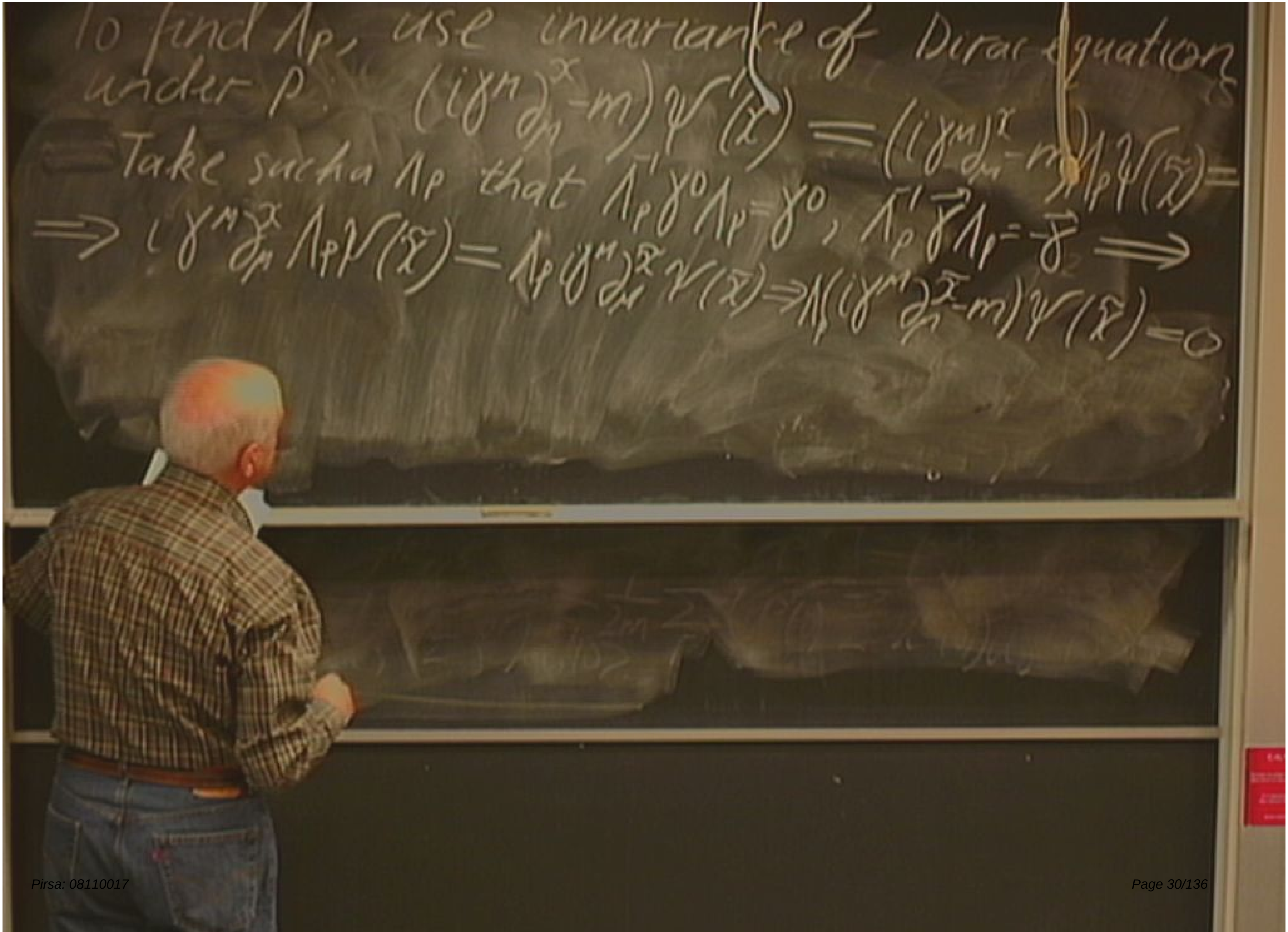
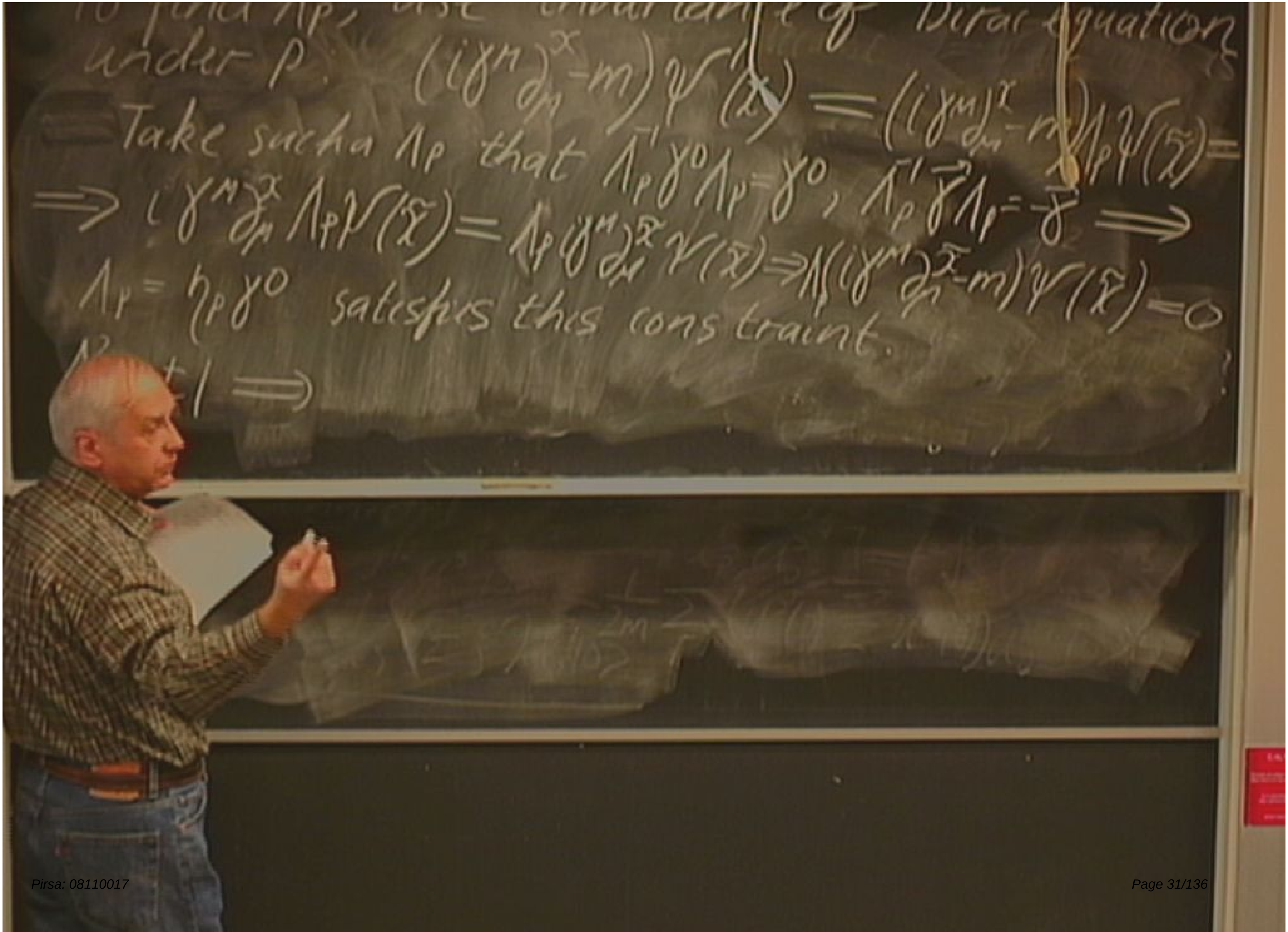
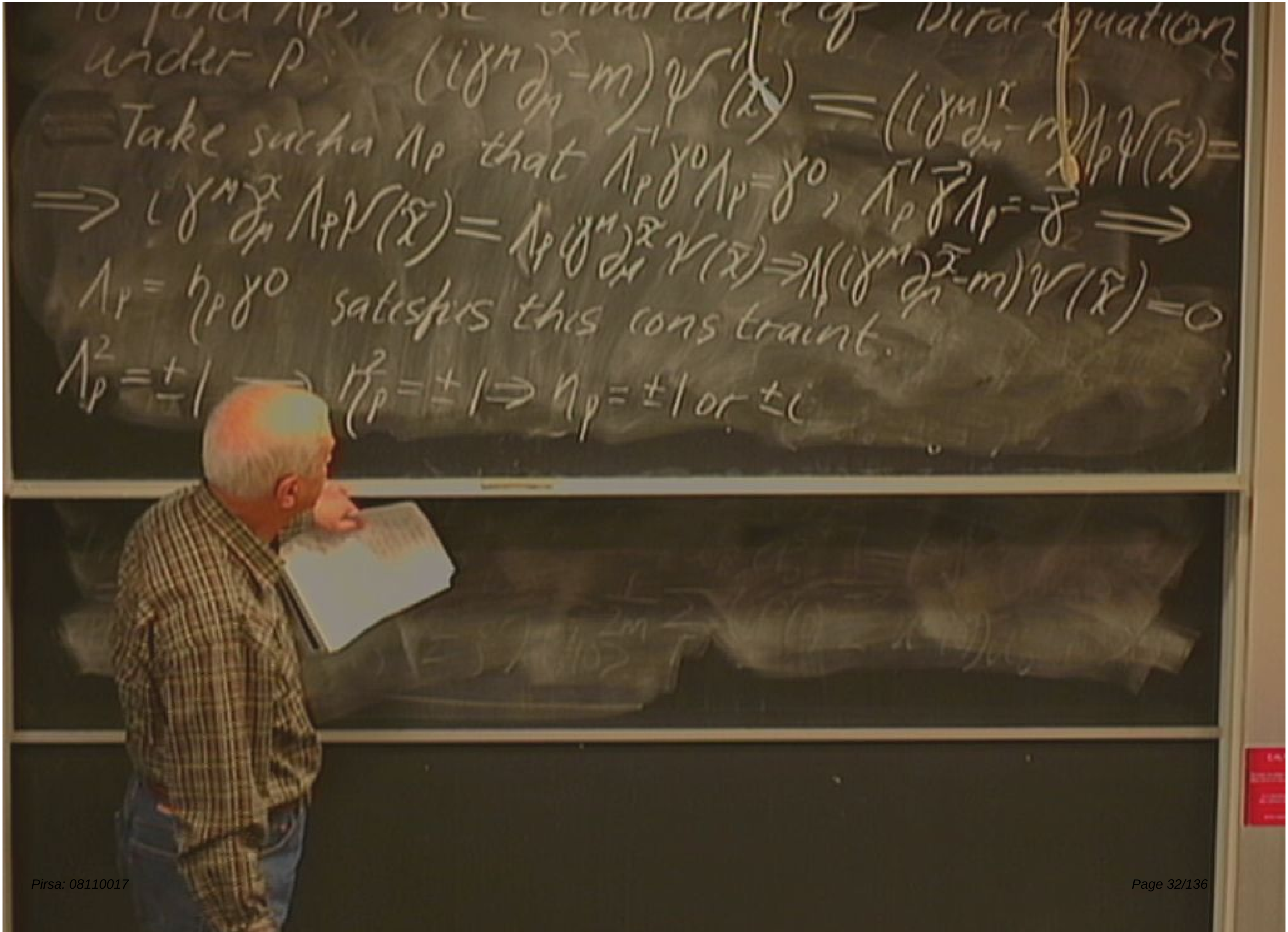
Take such a Λ_P that $\Lambda_P^{-1} \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$ $\Rightarrow$
$\Rightarrow i\gamma^\mu \partial_\mu^x \Lambda_P \psi(\tilde{x}) = \Lambda_P (i\gamma^\mu \partial_\mu^{\tilde{x}}) \psi(\tilde{x})$

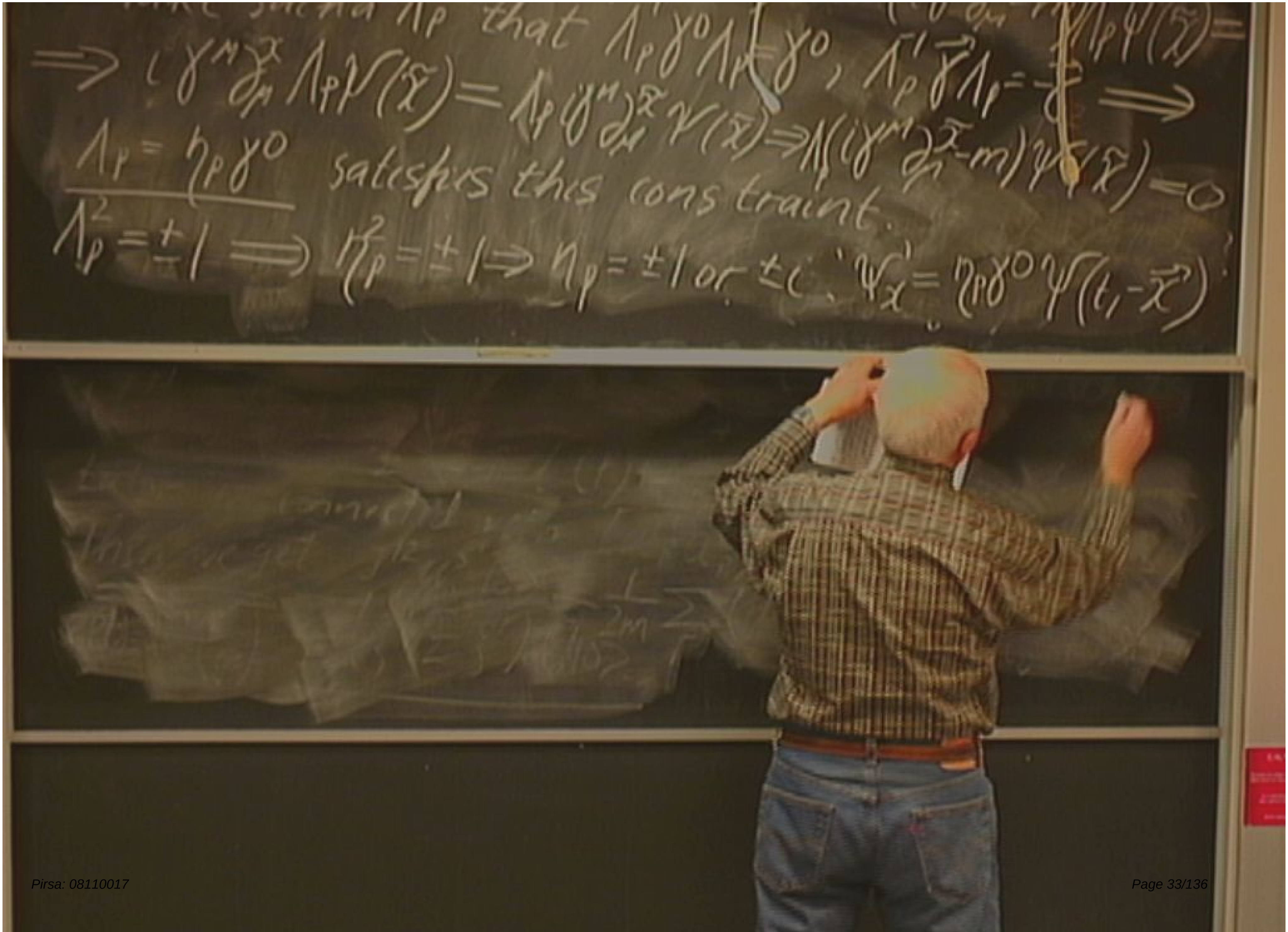

To find Λ_P, use invariance of Dirac equation
under P: (iγ^μ ∂_μ^x − m) ψ'(x) = (iγ^μ ∂_μ^x̃ − m) Λ_P ψ(x̃) =
Take such a Λ_P that Λ_P^{-1} γ^0 Λ_P = γ^0, Λ_P^{-1} γ⃗ Λ_P = −γ⃗ ⇒
⇒ i γ^μ ∂_μ^x Λ_P ψ(x̃) = Λ_P (iγ^μ ∂_μ^x̃) ψ(x̃)

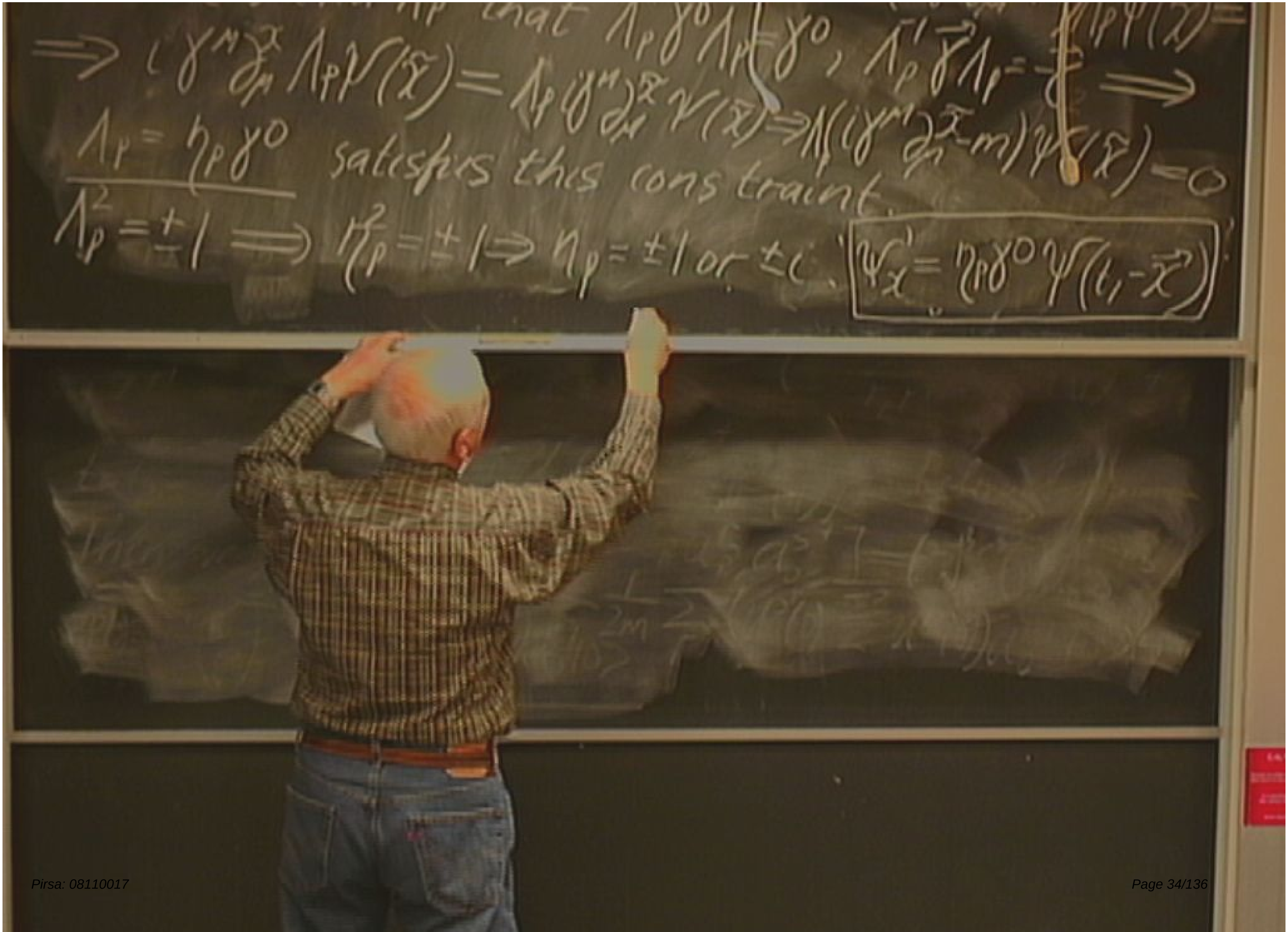

To find Λp, use invariance of Dirac equation
under P: (iγ^μ ∂_μ^x − m) ψ'(x) = ((iγ^μ) ∂_μ^x − m) Λp ψ(x̃) =
Take such a Λp that Λp^{-1} γ^0 Λp = γ^0, Λp^{-1} γ⃗ Λp = −γ⃗ ⟹
⟹ iγ^μ ∂_μ^x Λp ψ(x̃) = Λp (iγ^μ ∂_μ^x̃ ψ(x̃))

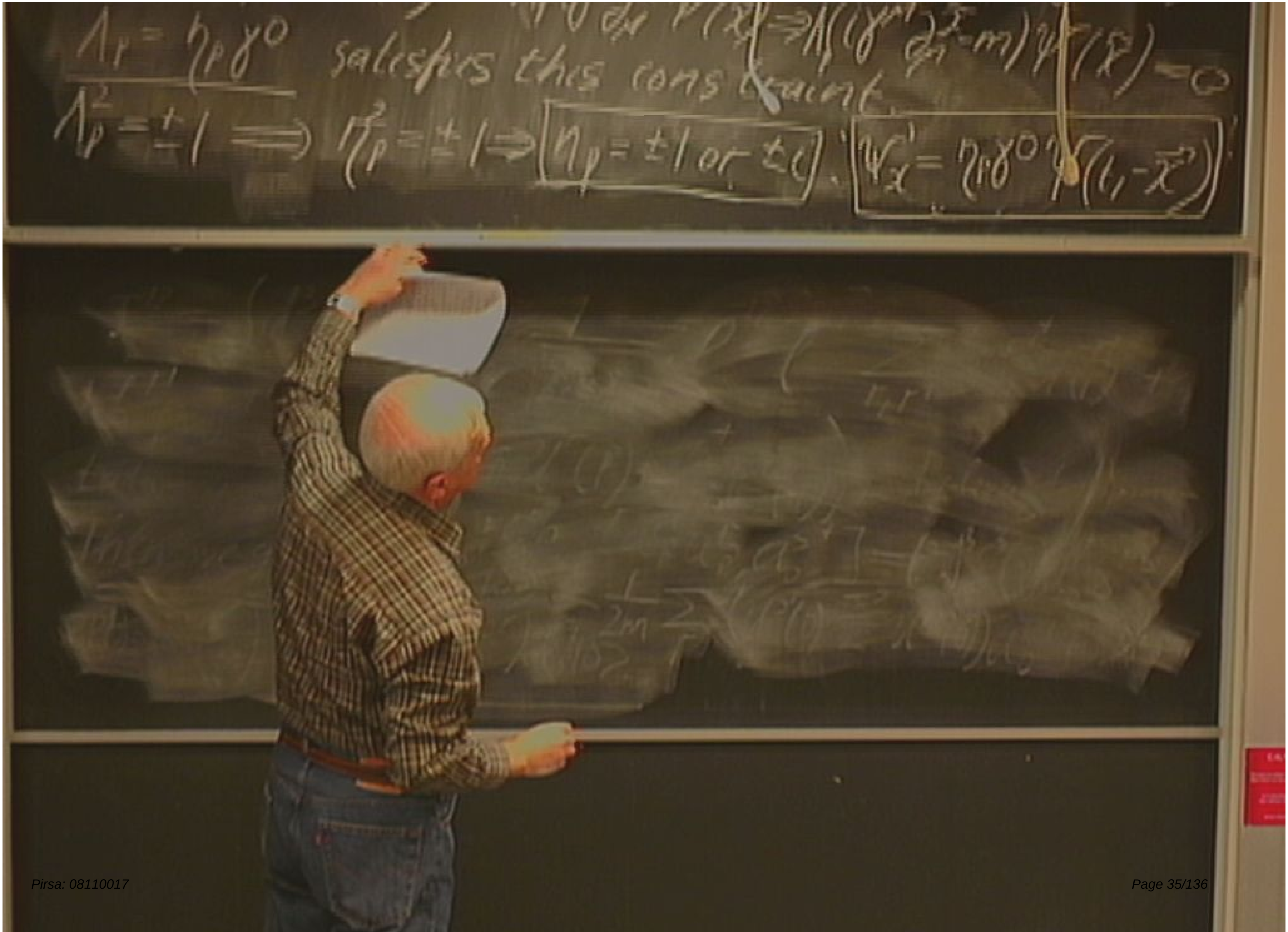

To find Λp, use invariance of Dirac equation
under P: (iγ^μ ∂_μ^x − m) ψ'(x) = (iγ^μ ∂_μ^x − m) Λp ψ(x̄) =
Take such a Λp that Λp^-1 γ^0 Λp = γ^0, Λp^-1 γ⃗ Λp = −γ⃗ ⇒
⇒ iγ^μ ∂_μ^x Λp ψ(x̄) = Λp (iγ^μ ∂_μ^x̄) ψ(x̄) ⇒ Λp (iγ^μ ∂_μ^x̄ − m) ψ(x̄) = 0


under P: (iγ^μ ∂_μ^x − m)ψ'(x) = (iγ^μ ∂_μ^x − m)Λp ψ(x̄) =
Take such a Λp that Λp^{-1} γ^0 Λp = γ^0, Λp^{-1} γ⃗ Λp = −γ⃗ ⇒
⇒ iγ^μ ∂_μ^x Λp ψ(x̄) = Λp (iγ^μ) ∂_μ^x̄ ψ(x̄) ⇒ Λp (iγ^μ ∂_μ^x̄ − m) ψ(x̄) = 0
Λp = ηp γ^0 satisfies this constraint.

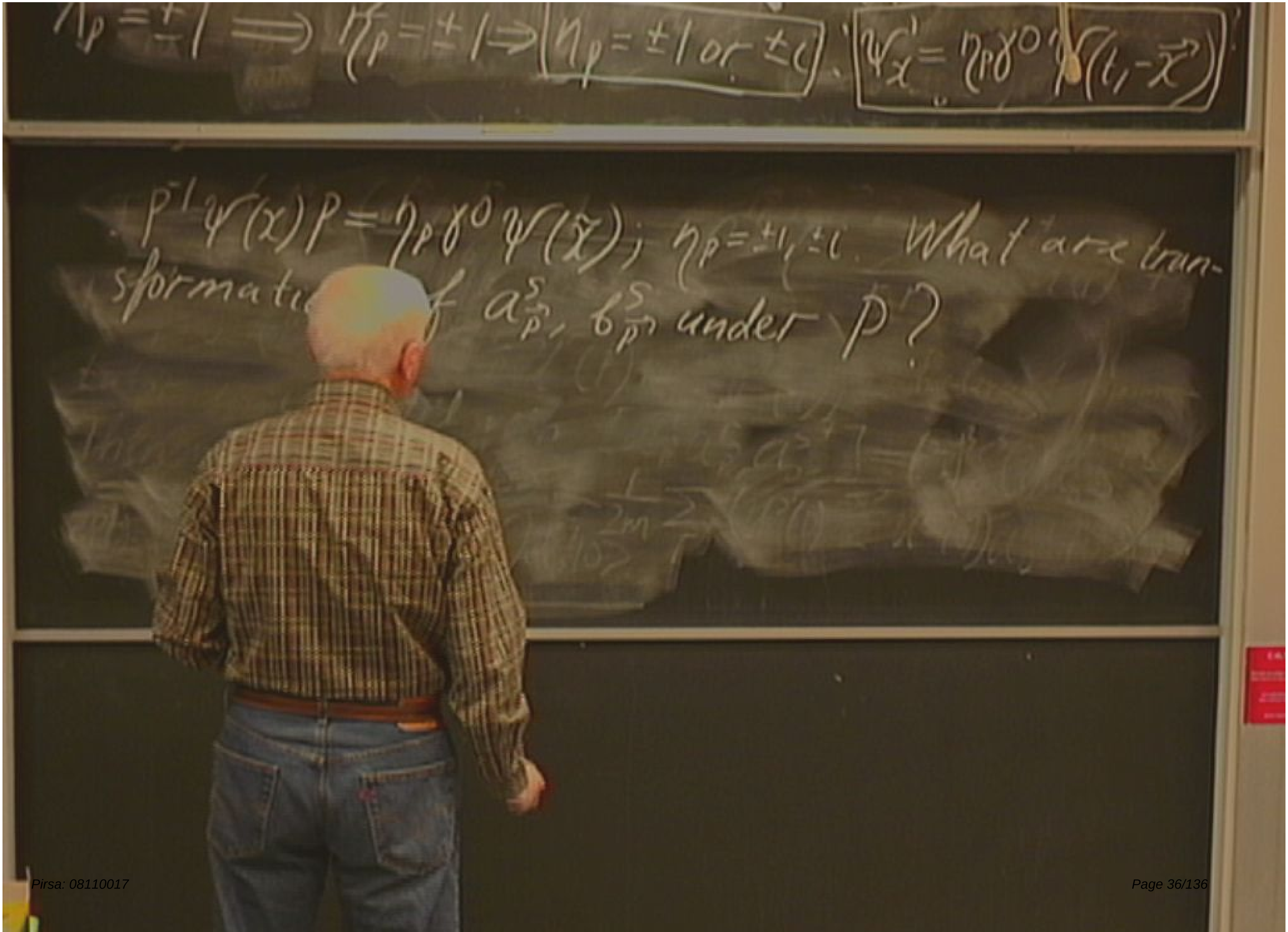

To find Λ_P, use invariance of Dirac equation
under P: $(i\gamma^\mu \partial_\mu^x - m)\psi'(x) = (i\gamma^\mu \partial_\mu^x - m)\Lambda_P \psi(\bar{x}) =$
Take such a Λ_P that $\Lambda_P^{-1}\gamma^0\Lambda_P = \gamma^0$, $\Lambda_P^{-1}\vec{\gamma}\Lambda_P = -\vec{\gamma} \Rightarrow$
$\Rightarrow i\gamma^\mu \partial_\mu^x \Lambda_P \psi(\bar{x}) = \Lambda_P (i\gamma^\mu \partial_\mu^{\bar{x}})\psi(\bar{x}) \Rightarrow \Lambda_P(i\gamma^\mu \partial_\mu^{\bar{x}} - m)\psi(\bar{x}) = 0$
$\Lambda_P = \eta_P \gamma^0$ satisfies this constraint.
$\Lambda_P^2 = \pm 1 \Rightarrow \eta_P^2 = \pm 1 \Rightarrow \eta_P = \pm 1$ or $\pm i$


that $\Lambda_P^{-1}\gamma^0\Lambda_P = \gamma^0$, $\Lambda_P^{-1}\vec{\gamma}\Lambda_P = -\vec{\gamma}$ $\Rightarrow$
$\Lambda_P = \eta_P \gamma^0$ satisfies this constraint.
$\Lambda_P^2 = \pm 1 \Rightarrow \eta_P^2 = \pm 1 \Rightarrow \eta_P = \pm 1$ or $\pm i$

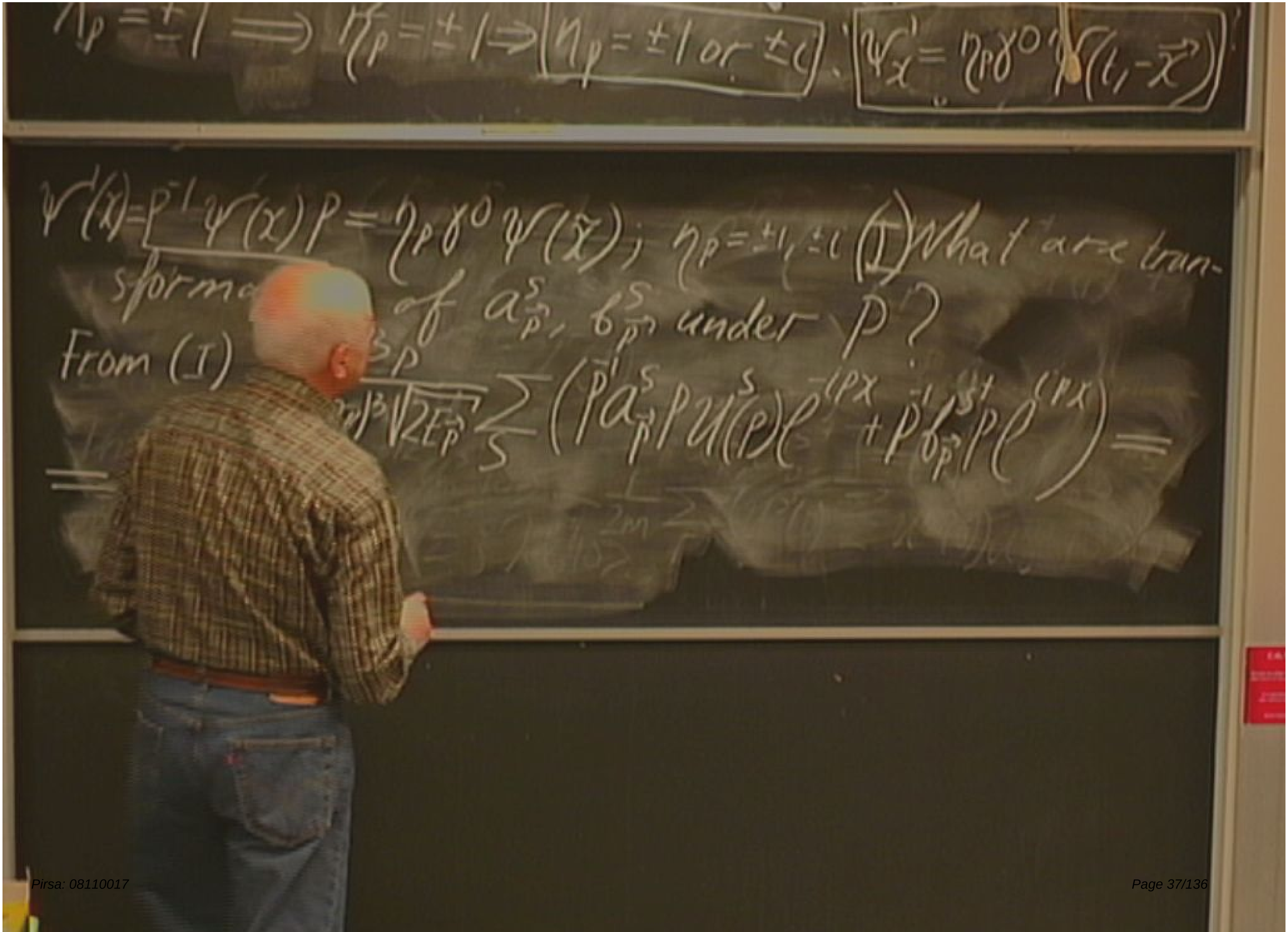

that $\Lambda_P \gamma^0 \Lambda_P = \gamma^0$, $\Lambda_P^{-1} \vec{\gamma} \Lambda_P = -\vec{\gamma}$
$\Rightarrow i\gamma^\mu \partial^{\tilde{x}}_\mu \Lambda_P \psi(\tilde{x}) = \Lambda_P (i\gamma^\mu \partial^{\tilde{x}}_\mu) \psi(\tilde{x}) \Rightarrow \Lambda_P (i\gamma^\mu \partial^{\tilde{x}}_\mu - m) \psi(\tilde{x}) = 0$ $\Rightarrow$
$\Lambda_P = \eta_P \gamma^0$ satisfies this constraint.
$\Lambda_P^2 = \pm 1 \Rightarrow \eta_P^2 = \pm 1 \Rightarrow \eta_P = \pm 1$ or $\pm i$. $\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})$

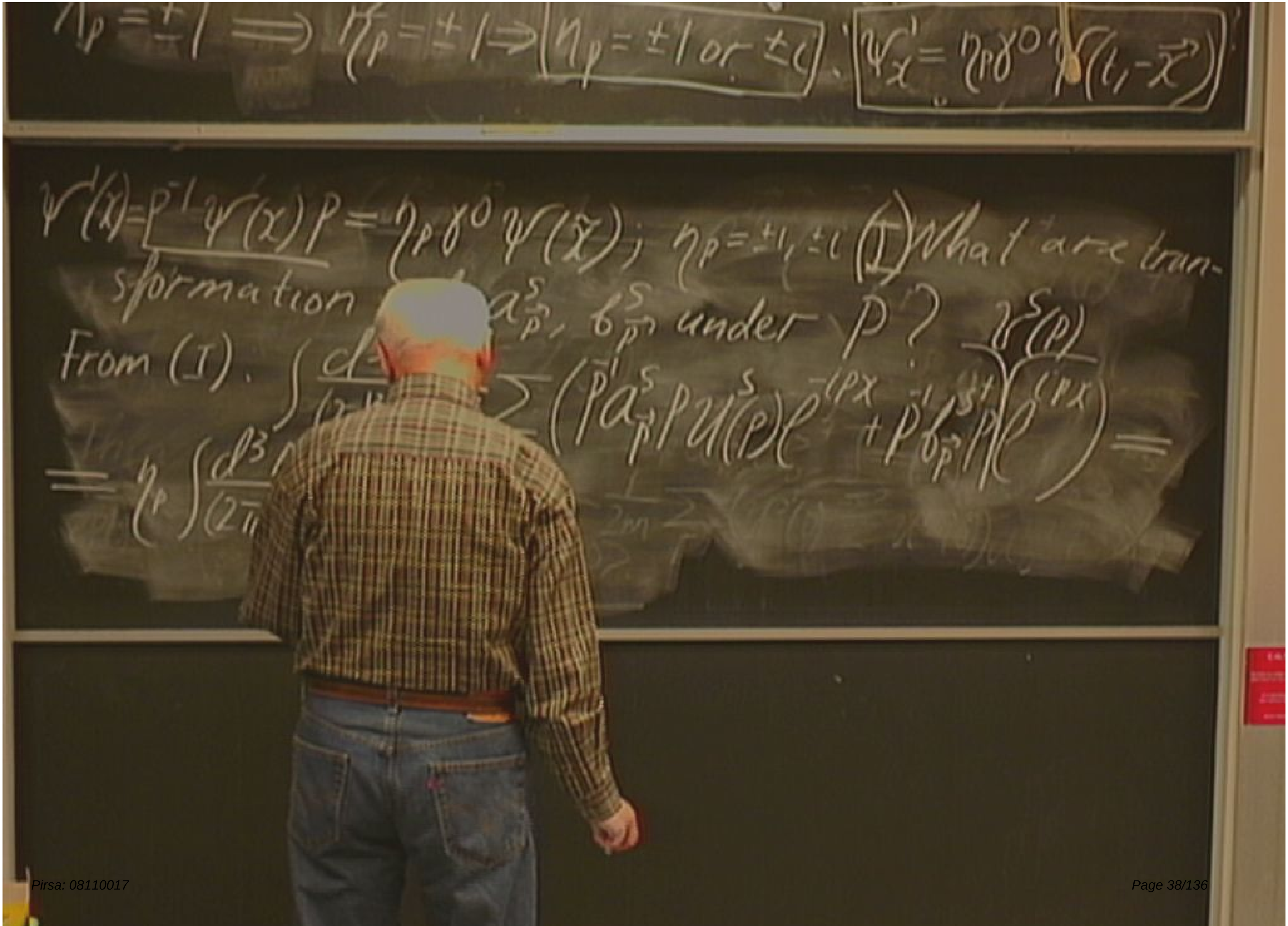


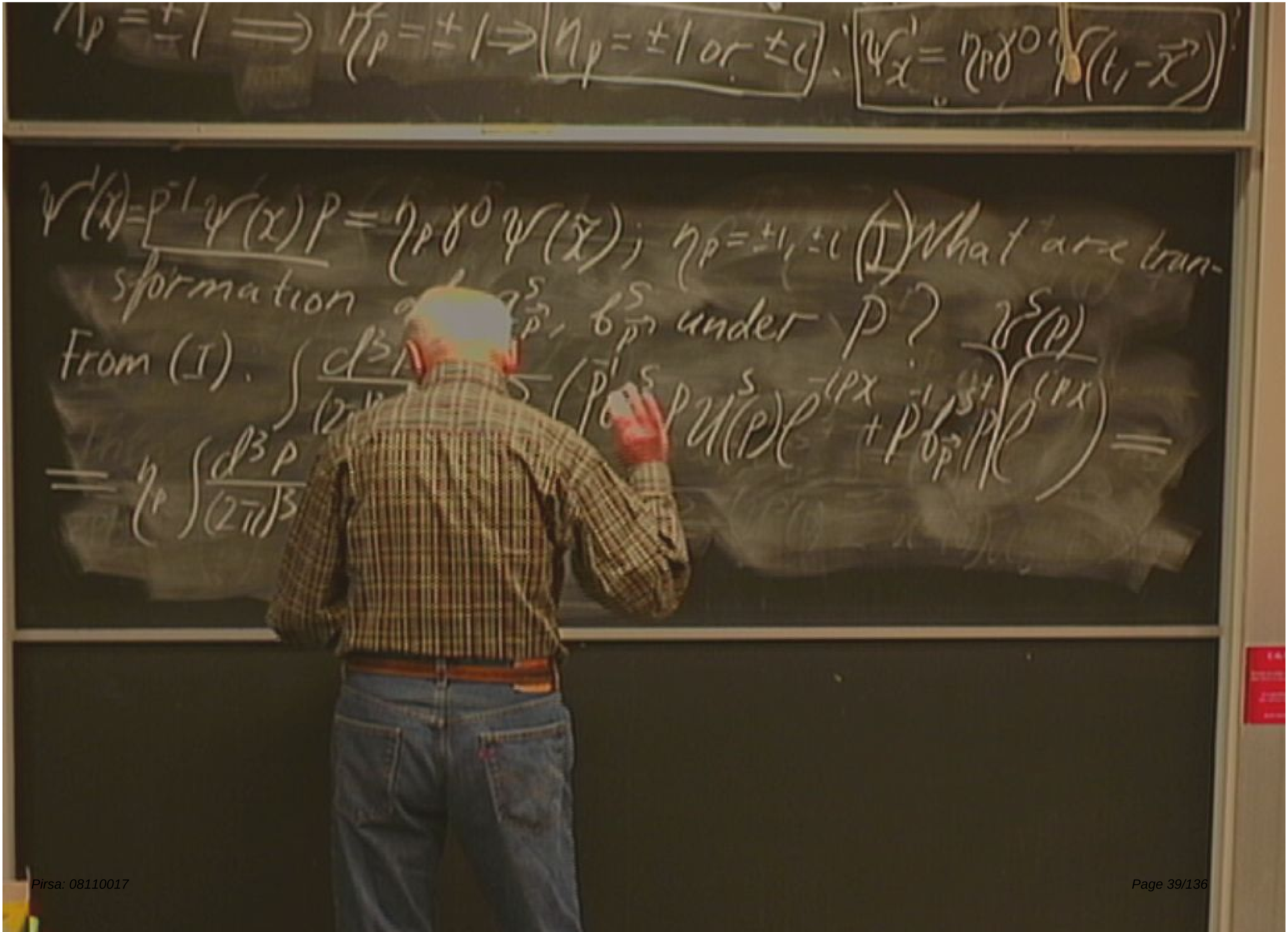
Λp = ηp γ0 satisfies this constraint
Λp² = ±1 ⟹ ηp² = ±1 ⟹ ηp = ±1 or ±i

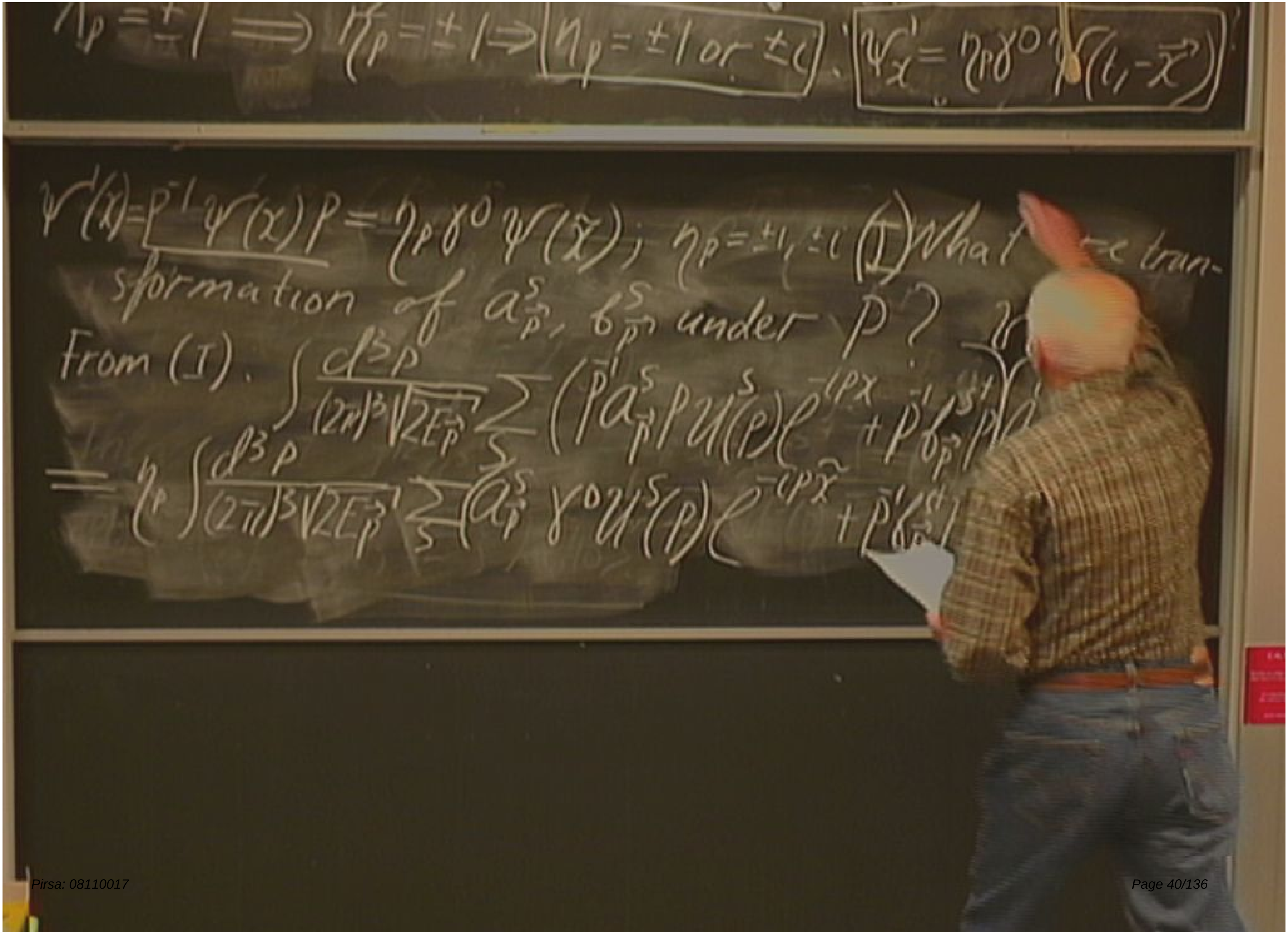


η_p = ±1 ⟹ η_p = ±1 ⟹ η_p = ±1 or ±i
ψ'_x = η_p γ^0 ψ(t, -x⃗)
P^{-1} ψ(x) P = η_P γ^0 ψ(x̃); η_P = ±1, ±i What are tran-
sformati of a^s_p, b^s_p under P?


η_P = ±1 ⟹ η_P = ±1 ⟹ η_P = ±1 or ±i . ψ'_x = η_P γ^0 ψ(t, −x⃗)
ψ'(x) = P^{-1} ψ(x) P = η_P γ^0 ψ(x̃) ; η_P = ±1, ±i (I) What are trans-
formations of a^s_p⃗, b^s_p⃗ under P ?
From (I)
Σ_s (P^{-1} a^s_p⃗ P u^s(p) e^{-ipx} + P^{-1} b^{s†}_p⃗ P e^{ipx}) =


$\eta_P = \pm 1 \Longrightarrow \eta_P = \pm 1 \Rightarrow \eta_P = \pm 1$ or $\pm i$. $\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})$
$\psi'(x) = P^{-1} \psi(x) P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What are tran-
sformation
$a^s_{\vec{p}}$, $b^s_{\vec{p}}$ under P?
From (I).


What are transformation of
under P?
From (I).


η_p = ±1 ⟹ η_p = ±1 ⟹ η_p = ±1 or ±i
ψ'_x = η_p γ^0 ψ(t, -x)
ψ'(x) = P^{-1} ψ(x) P = η_p γ^0 ψ(x̃); η_p = ±1, ±i (I)
What ... transformation of a^s_p, b^s_p under P?
From (I).

$\eta_p = \pm 1 \Longrightarrow \eta_p = \pm 1 \Rightarrow \boxed{\eta_p = \pm 1 \text{ or } \pm i}$. $\boxed{\psi'_x = \eta_p \gamma^0 \psi(t, -\vec{x})}$

$\psi'(x) = \underline{P^{-1}\psi(x)P} = \eta_p \gamma^0 \psi(\tilde{x})$; $\eta_p = \pm 1, \pm i$ (I) What are transformation of $a^s_{\vec{p}}$, $b^s_{\vec{p}}$ under P? $\overline{v^s(p)}$

From (I). $\int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(P^{-1} a^s_{\vec{p}} P u^s(p) e^{-ipx} + P^{-1} b^{s+}_{\vec{p}} P v^s(p) e^{ipx} \right) =$

$= \eta_p \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{\vec{p}} \gamma^0 u^s(p) e^{-ip\tilde{x}} + P^{-1} b^{s+}_{\vec{p}} P \gamma^0 v^s(p) e^{ip\tilde{x}} \right)$

$\eta_P = \pm 1 \Longrightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i}$. $\boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$

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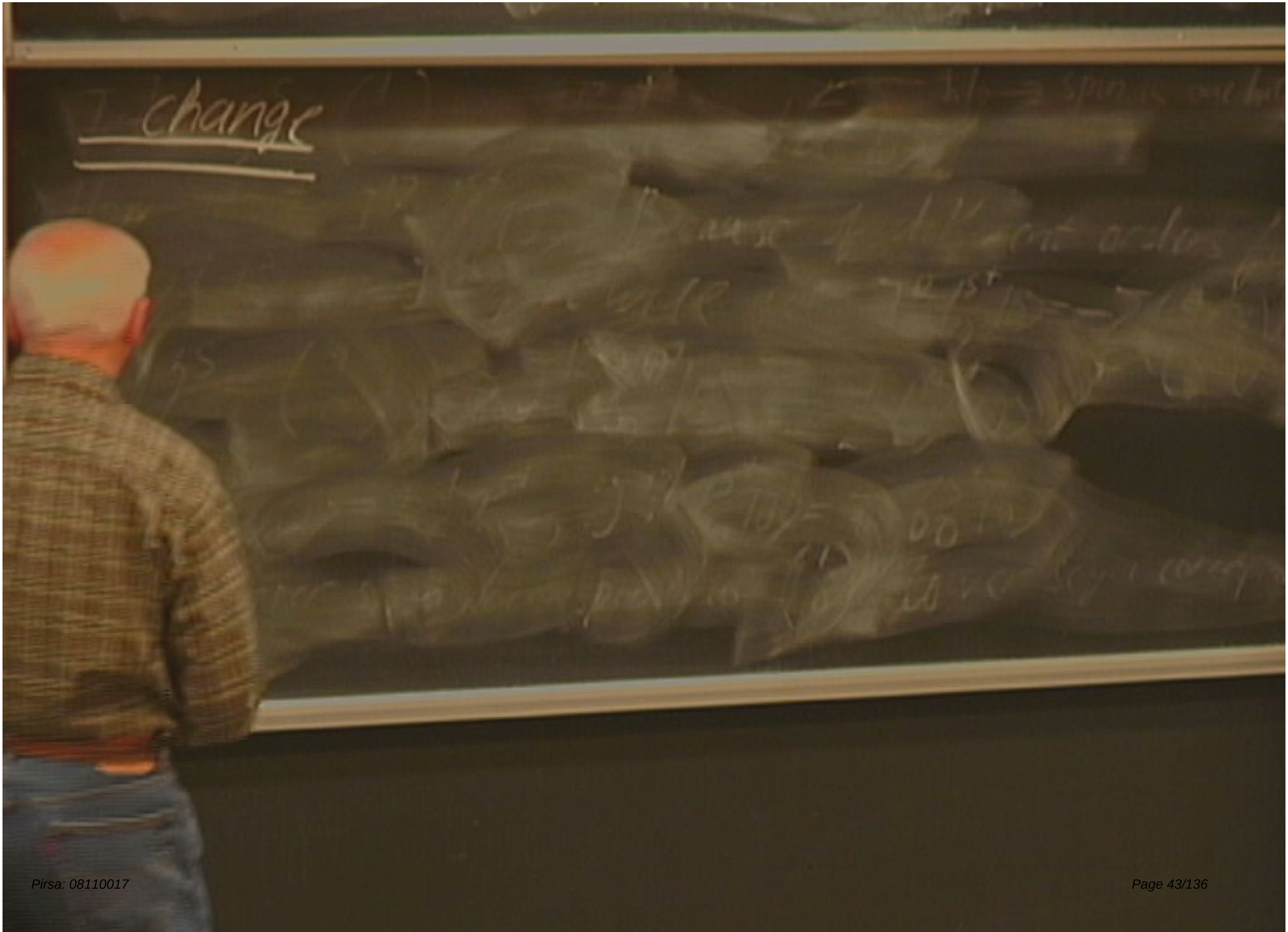
From (I). $\int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(P^{-1} a^s_{\vec{p}} P u^s(p) e^{-ipx} + P^{-1} b^{s+}_{\vec{p}} P v^s(p) e^{ipx} \right) =$

$= \eta_P \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{\vec{p}} \gamma^0 u^s(p) e^{-ip\tilde{x}} + P^{-1} b^{s+}_{\vec{p}} P \gamma^0 v^s(p) e^{ip\tilde{x}} \right) =$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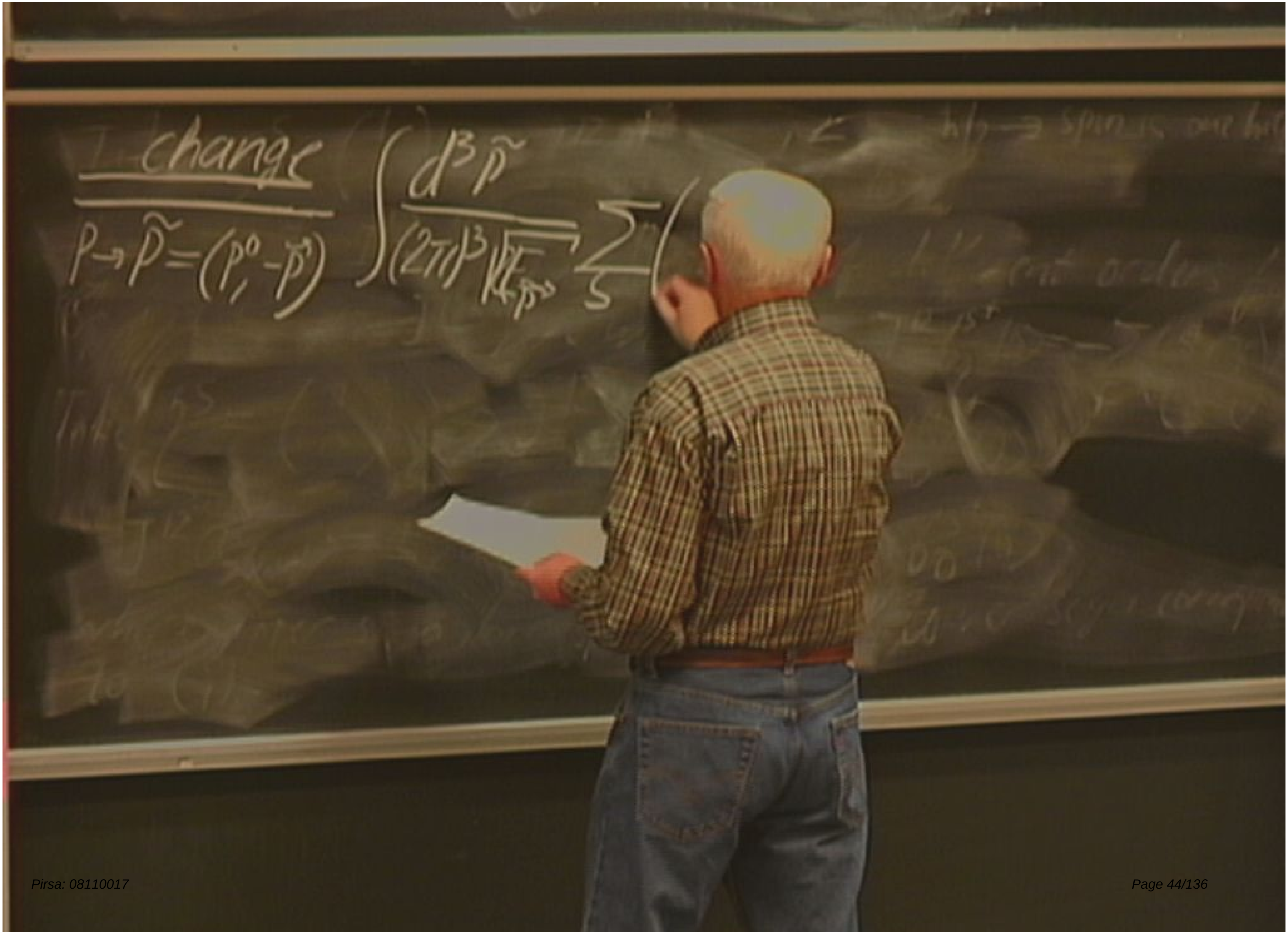
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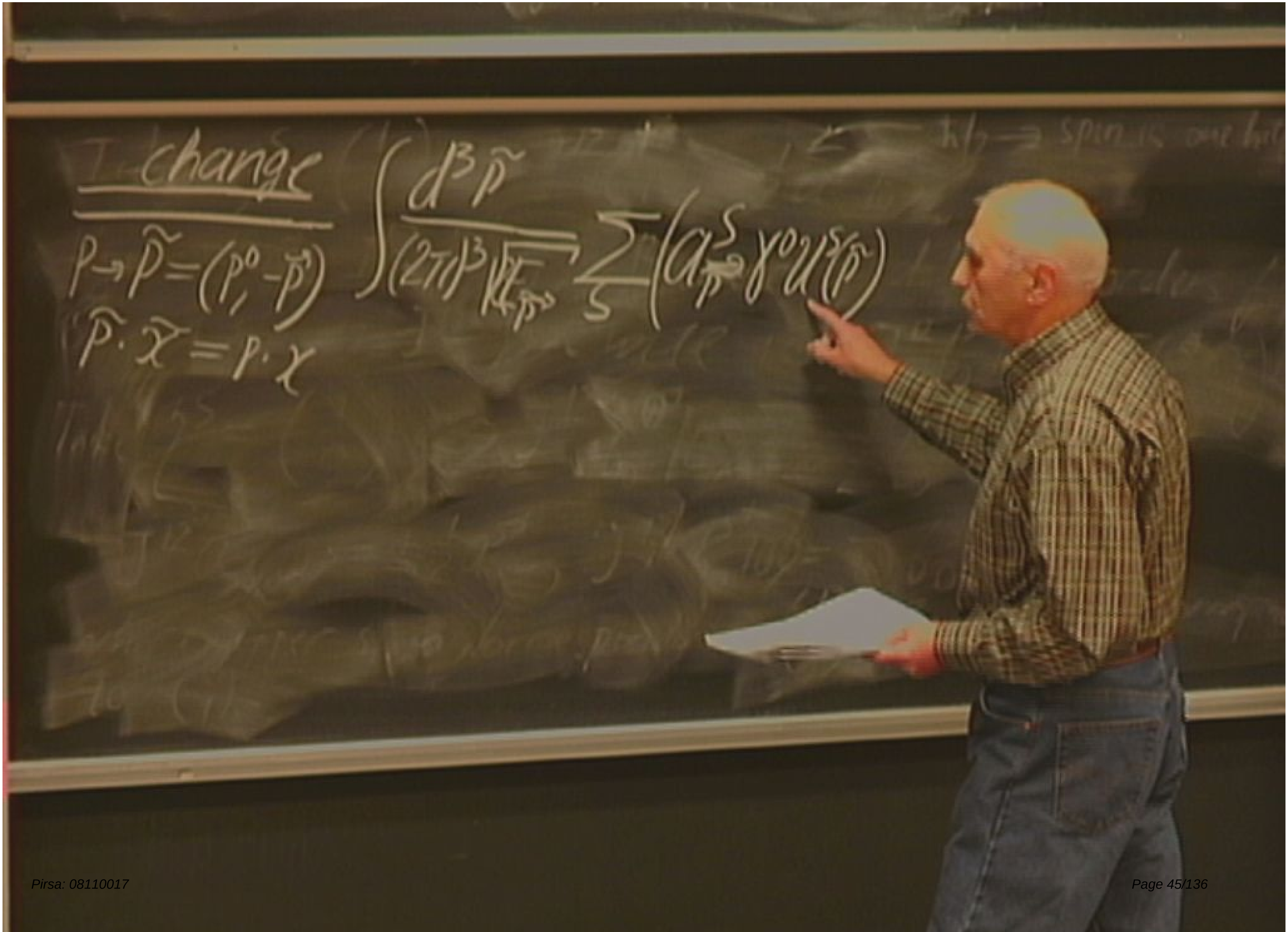
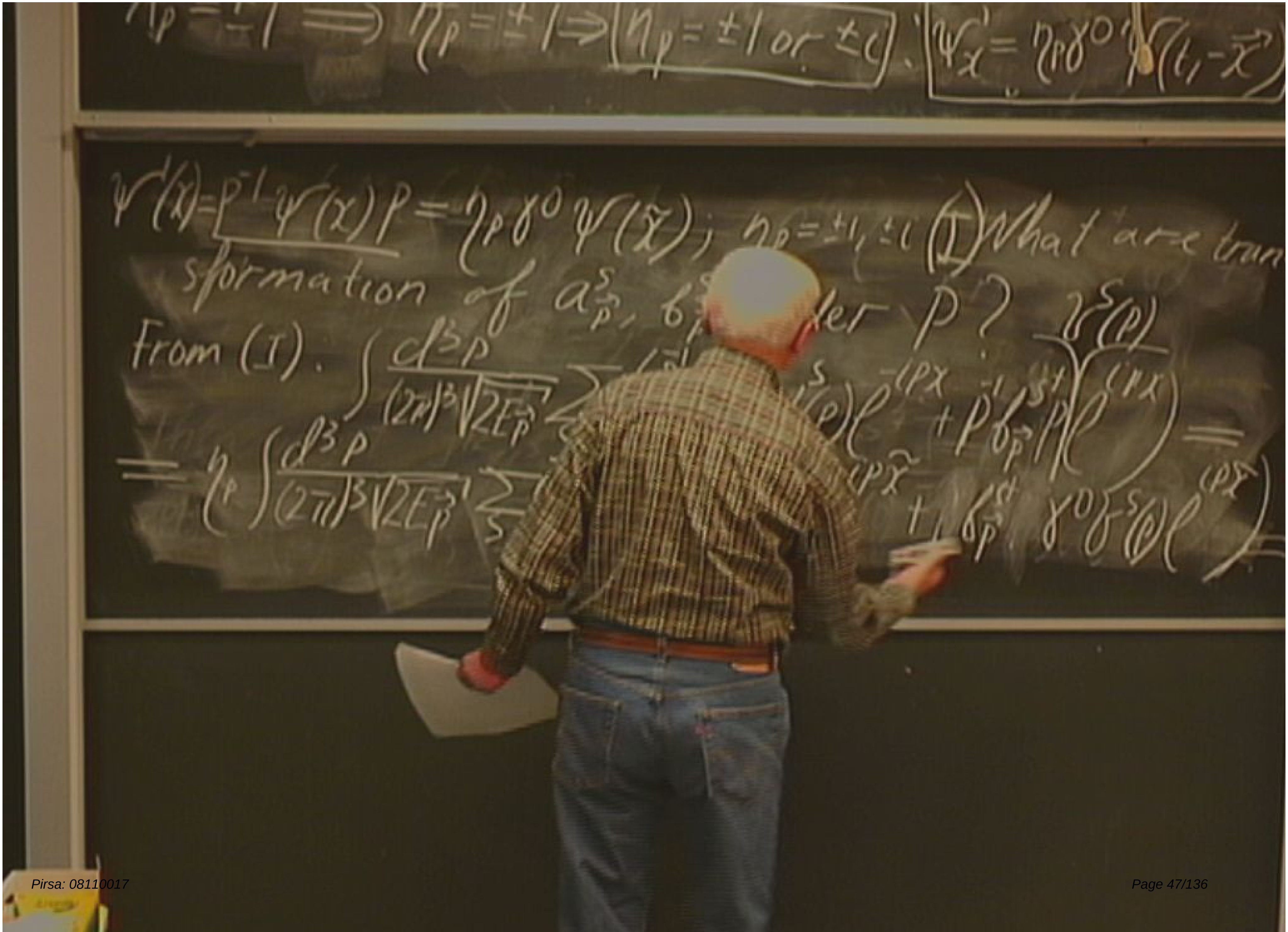

change

change

$p \to \tilde{p} = (p^0, -\vec{p})$

$\tilde{p} \cdot \tilde{x} = p \cdot x$

$$\int \frac{d^3\tilde{p}}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{\vec{p}}\, \gamma^0 u^s(\tilde{p})\right) e^{-i\tilde{p}\tilde{x}} +$$

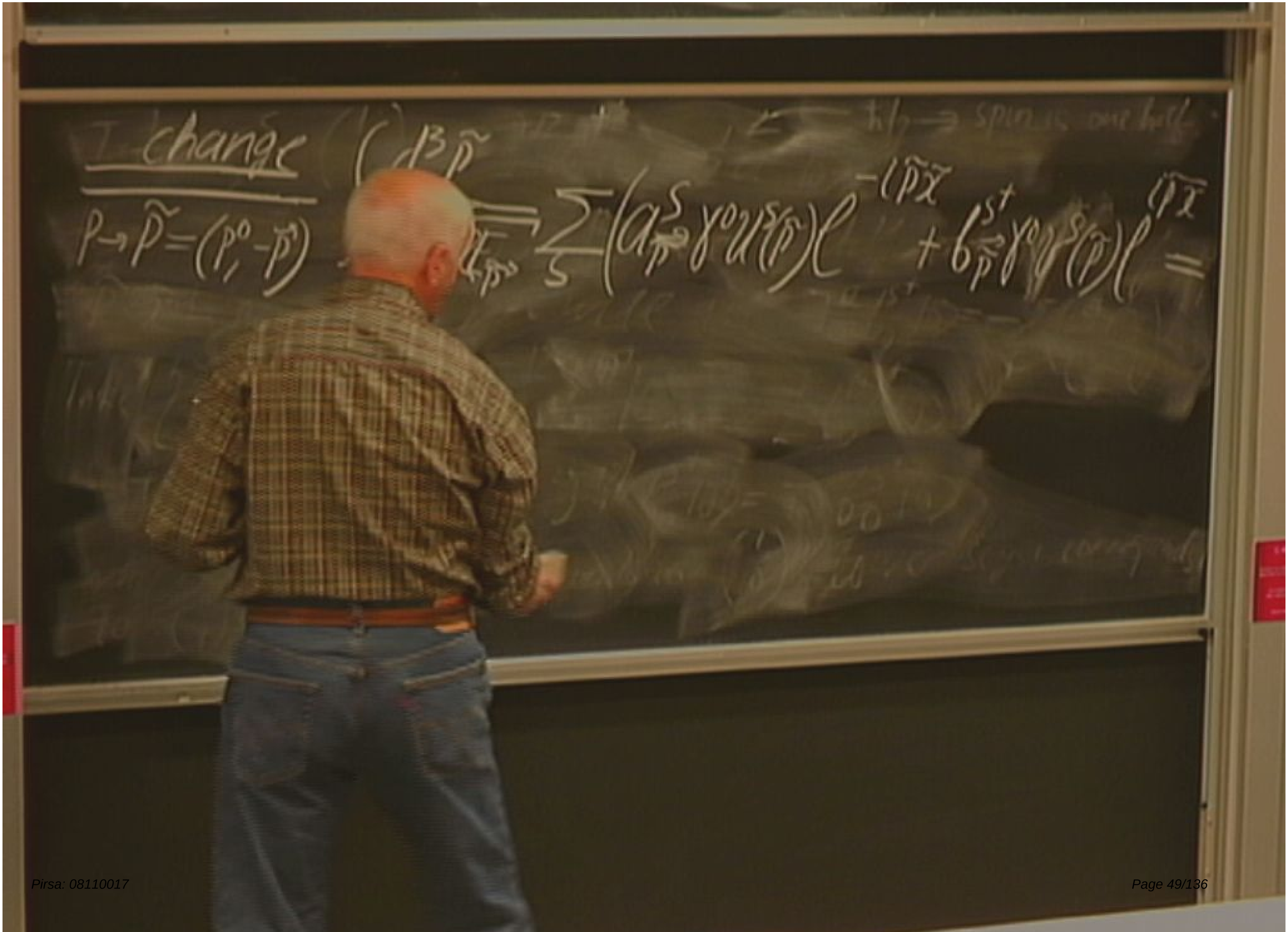

What are tran-
sformation of
under P ?
From (I).

$\eta_P = \pm 1 \Rightarrow \eta_P = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i}$. $\boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$

$\psi'(x) = P^{-1}\psi(x)P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What are transformation of $a^s_{\vec{p}}, b^s_{\vec{p}}$ under P?

From (I). $\int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(P^{-1} a^s_{\vec{p}} P u^s(p) e^{-ipx} + P^{-1} b^{s\dagger}_{\vec{p}} P v^s(p) e^{ipx} \right) =$

$= \eta_P \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{\vec{p}} \gamma^0 u^s(p) e^{-ip\tilde{x}} + b^{s\dagger}_{\vec{p}} \gamma^0 v^s(p) e^{ip\tilde{x}} \right) =$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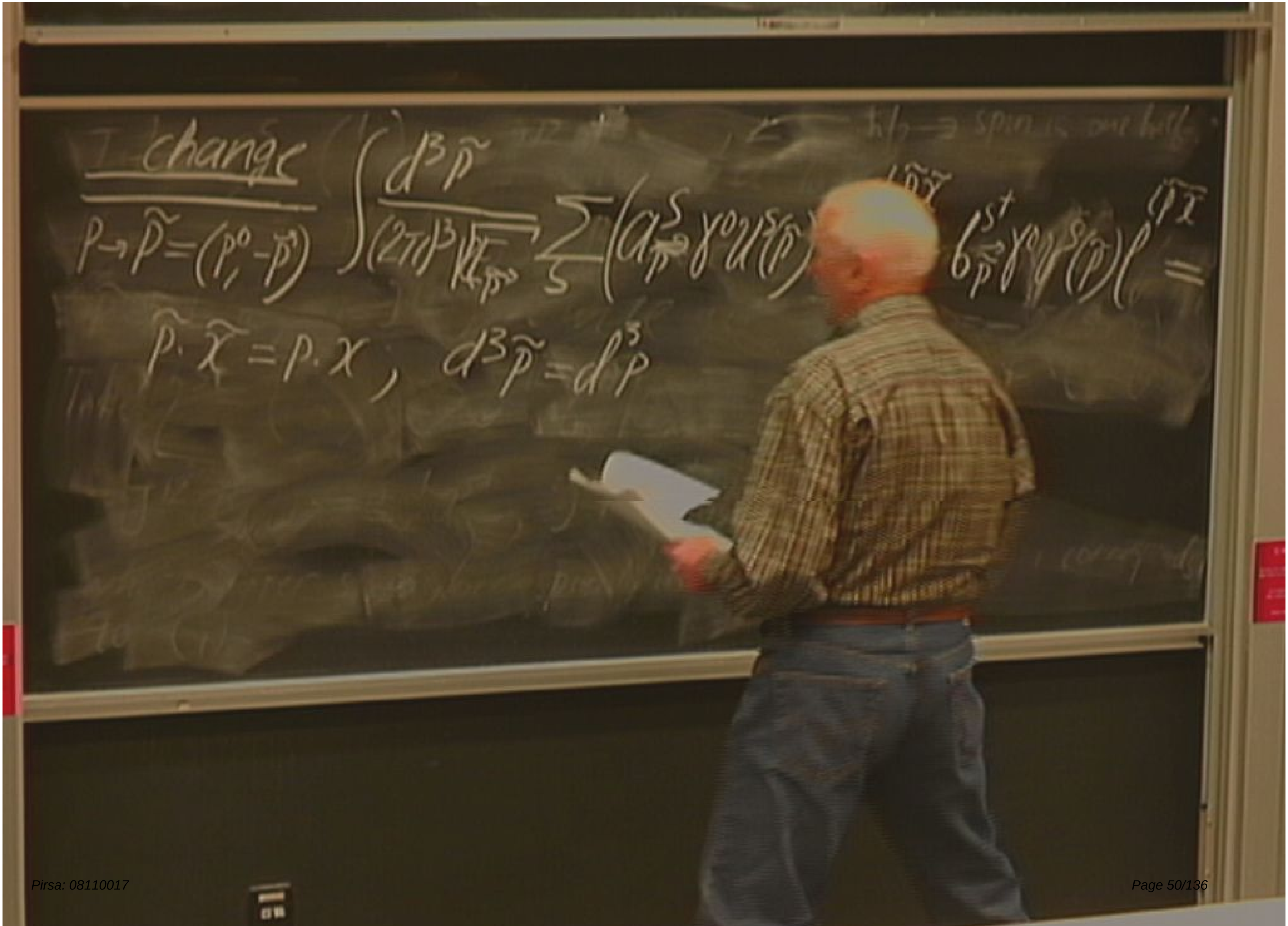
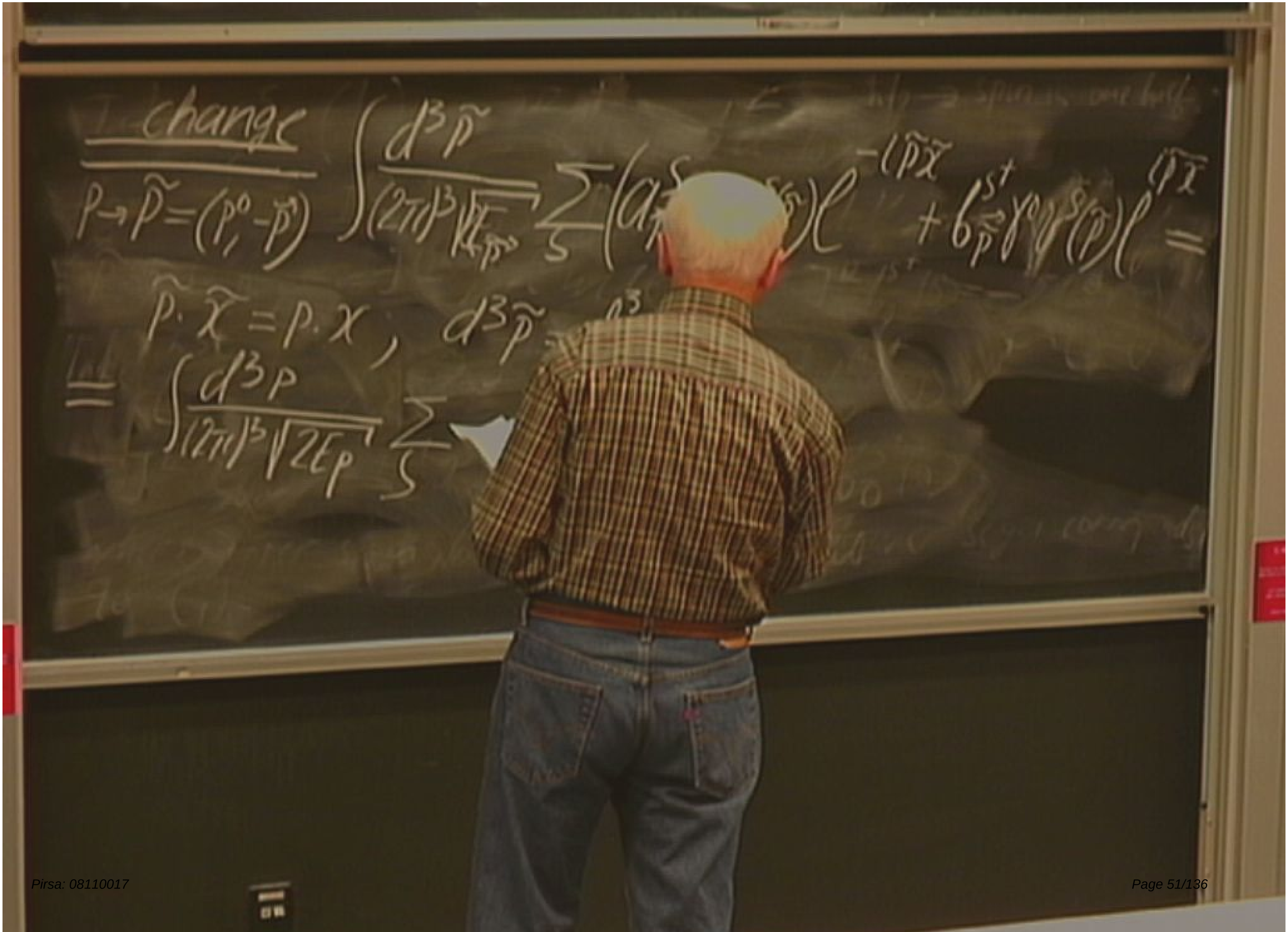
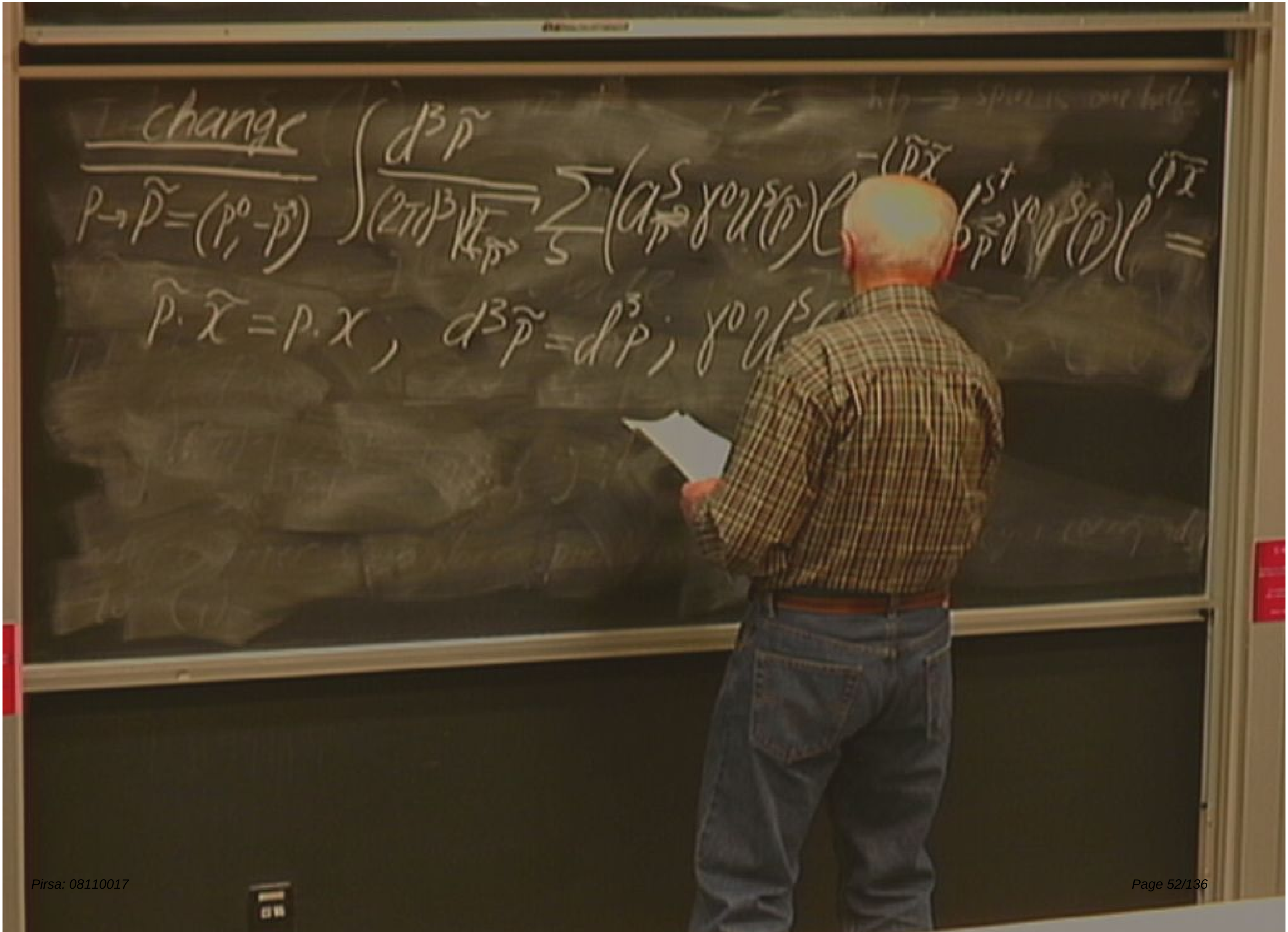
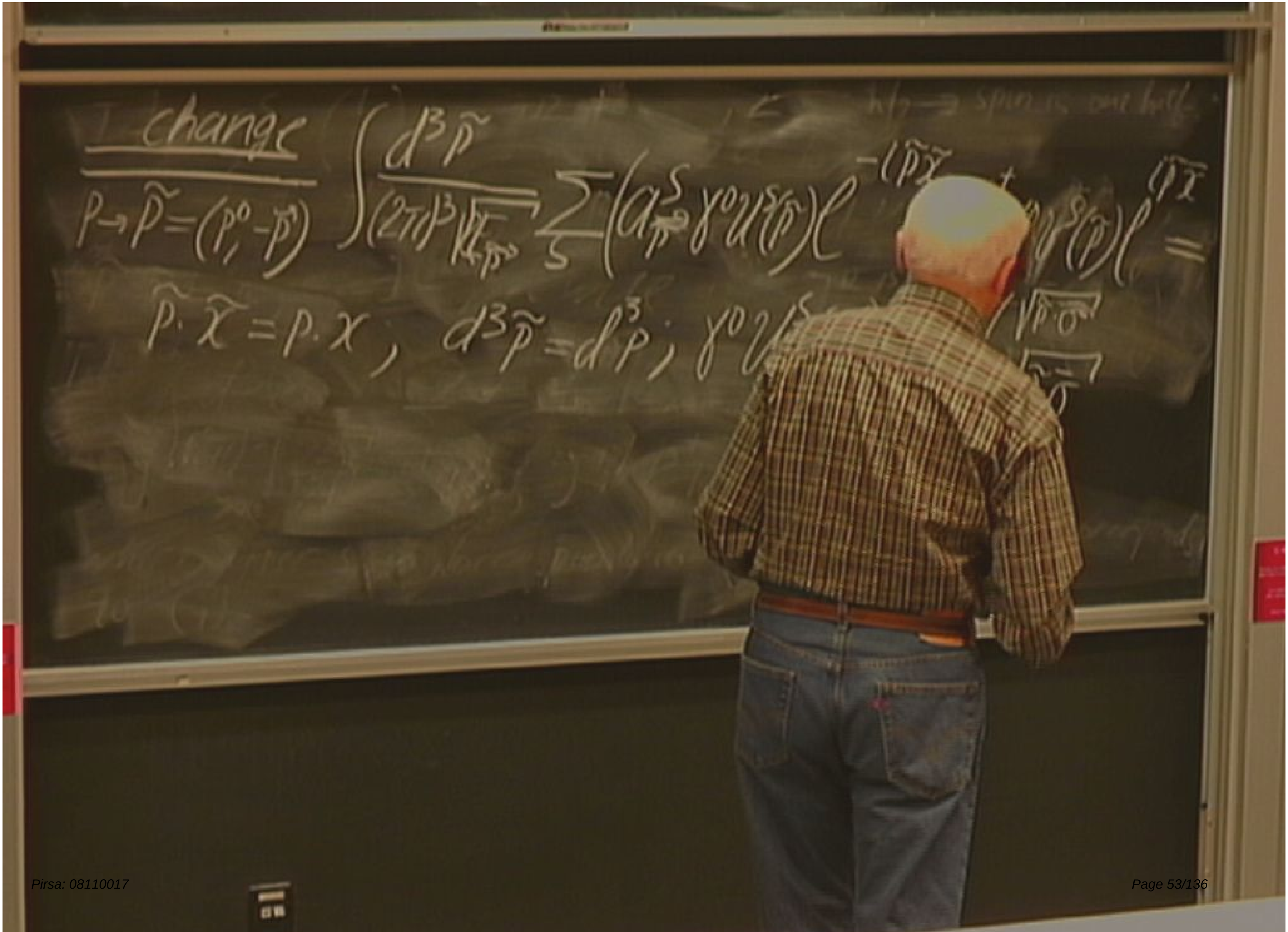
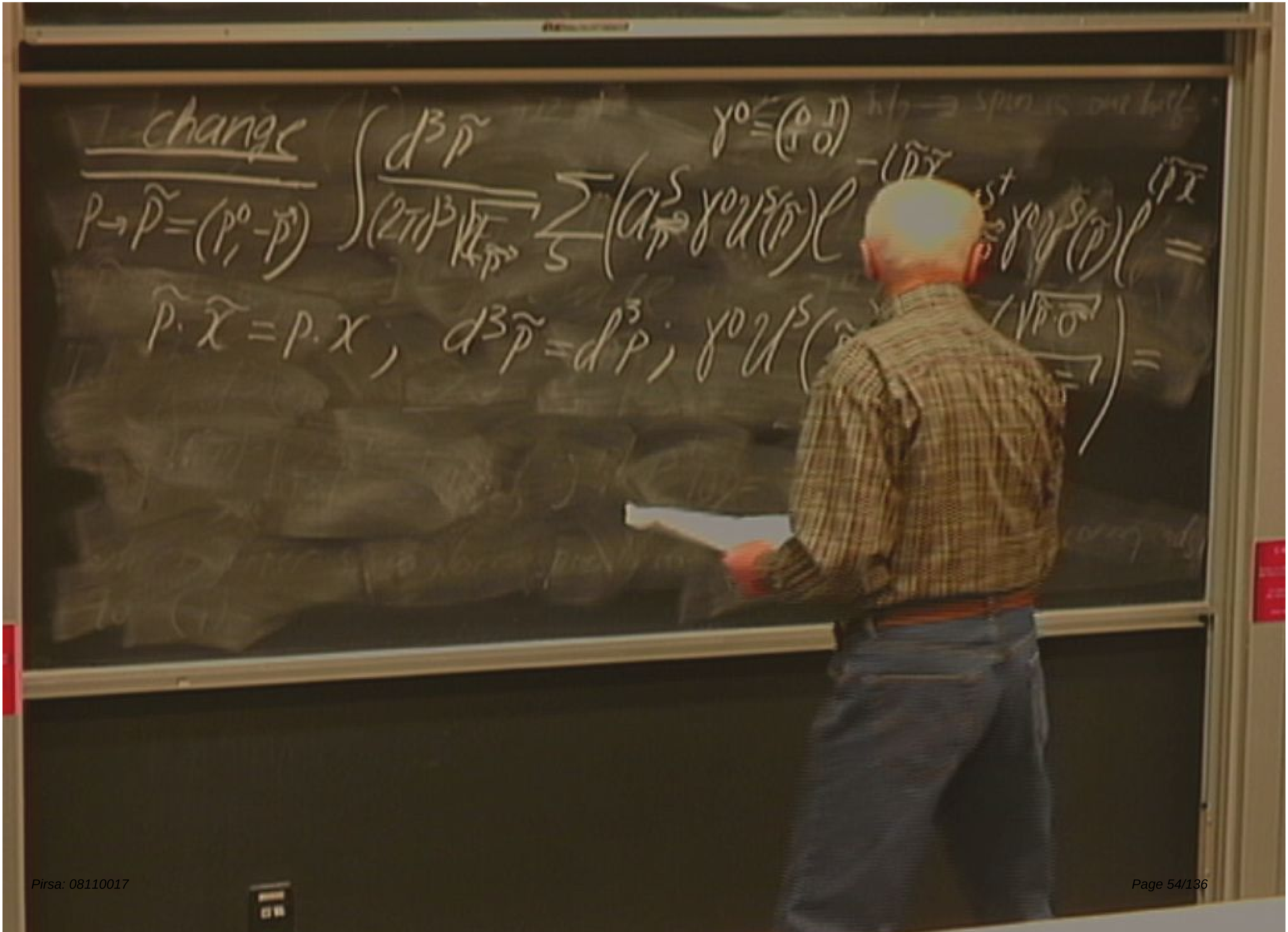
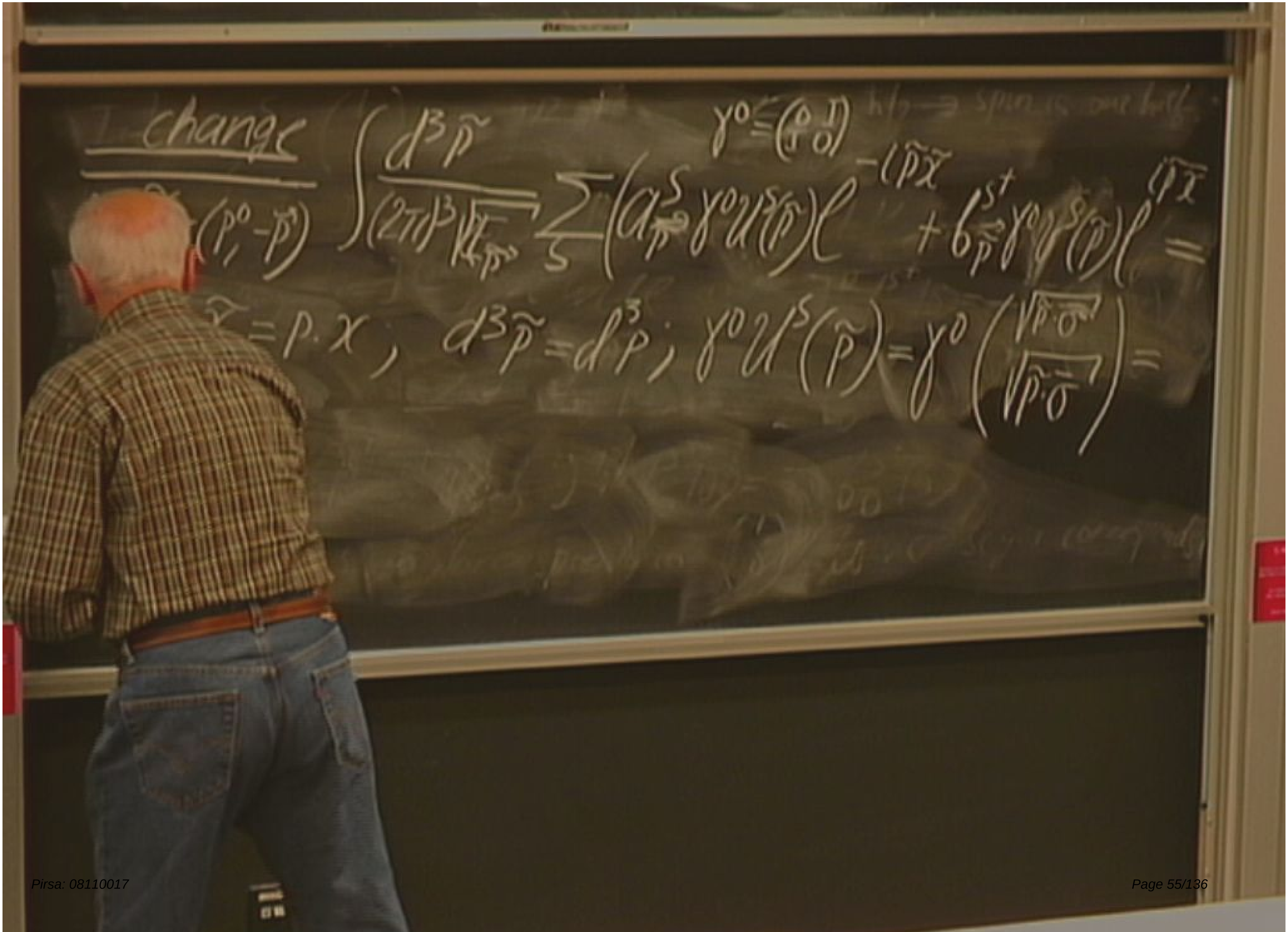

change
$p \to \tilde{p} = (p^0, -\vec{p})$
$\tilde{p} \cdot \tilde{x} = p \cdot x, \quad d^3\tilde{p} = d^3 p$


change
p→p̃=(p⁰,-p⃗)
p̃·x̃ = p·x ,


change


change
P→P̃=(P⁰,−p⃗)
p̃·x̃=p·x ; d³p̃=d³p ;


change
P → P̃ = (p⁰, −p⃗)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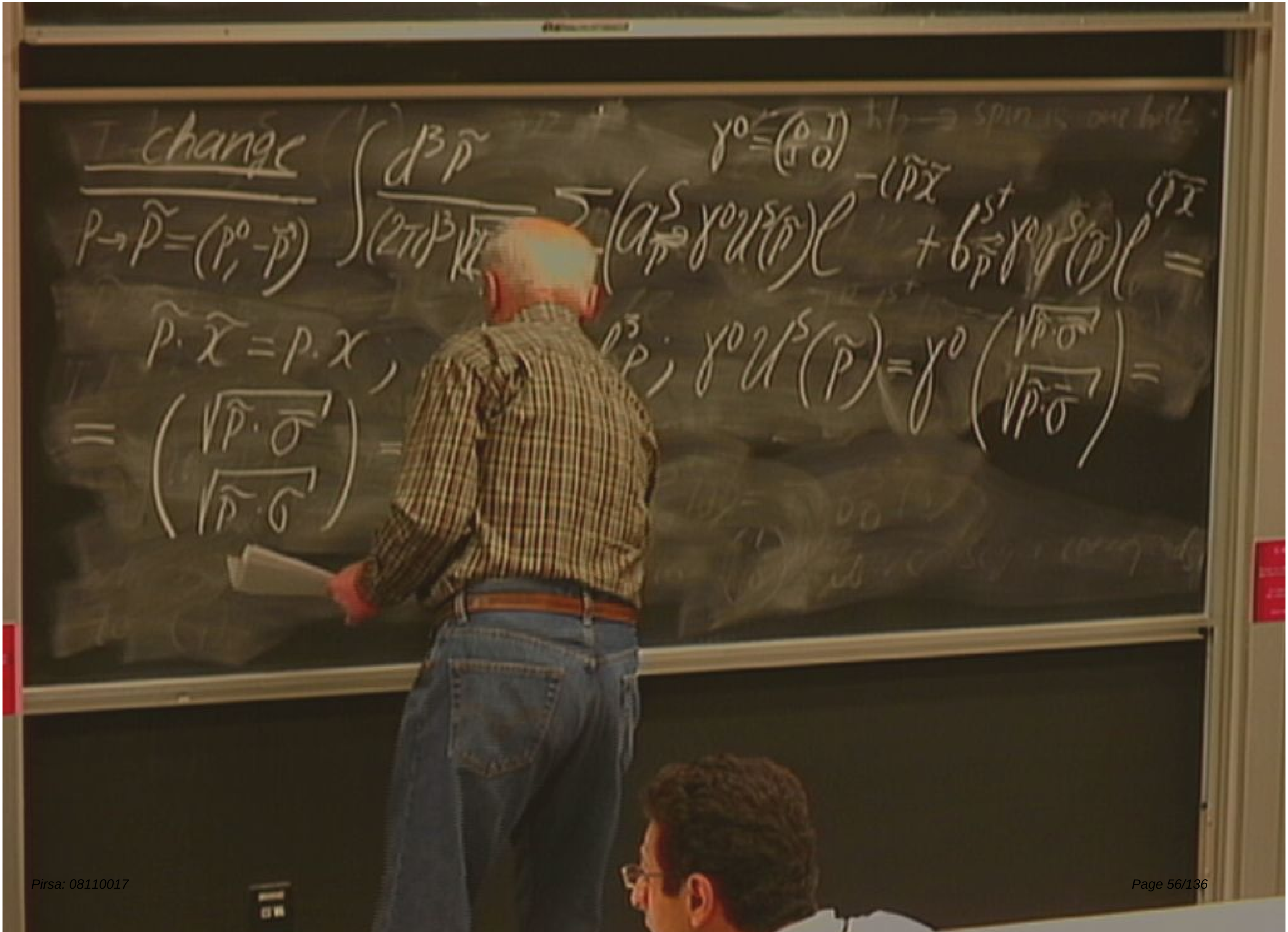
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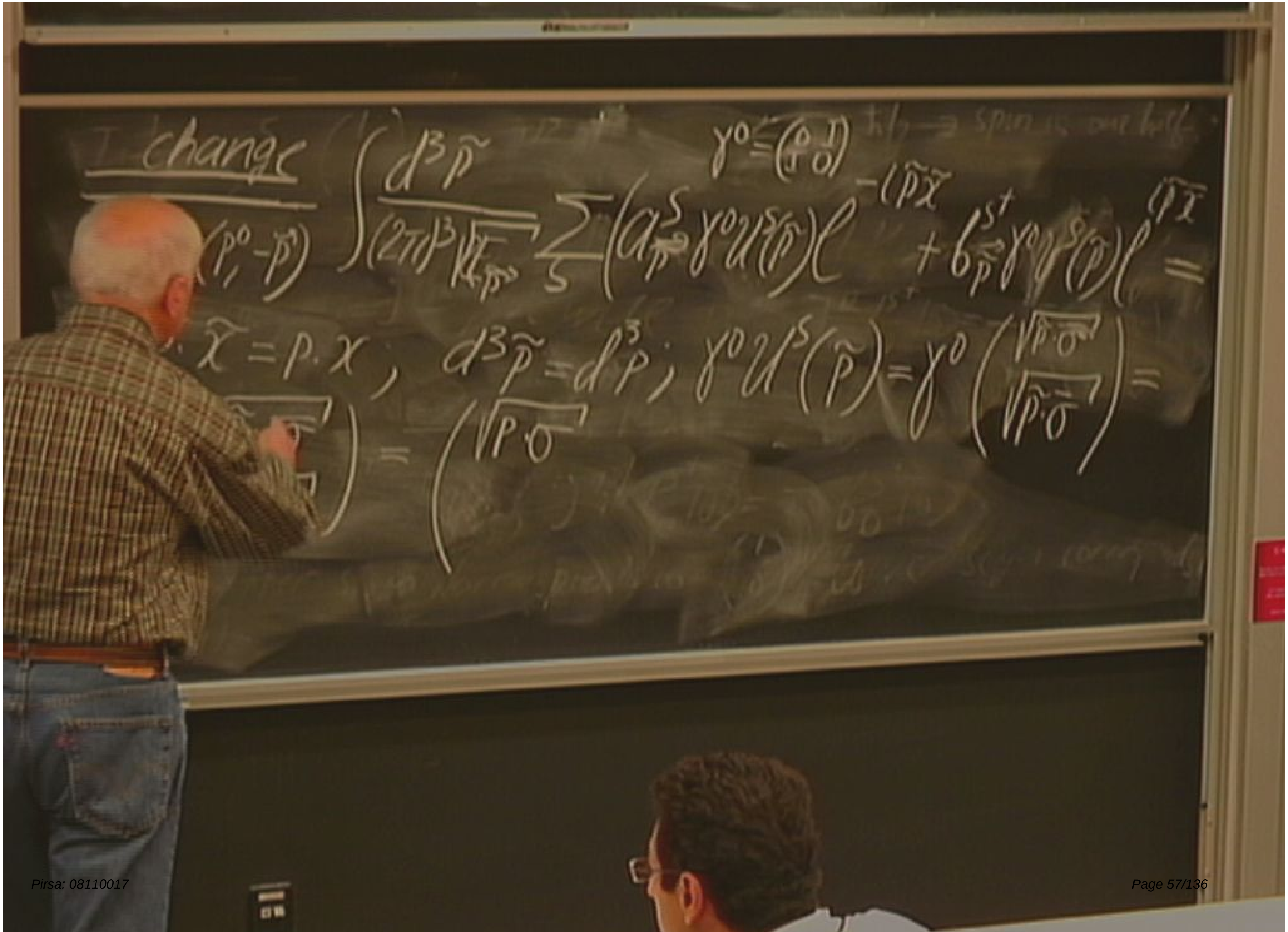
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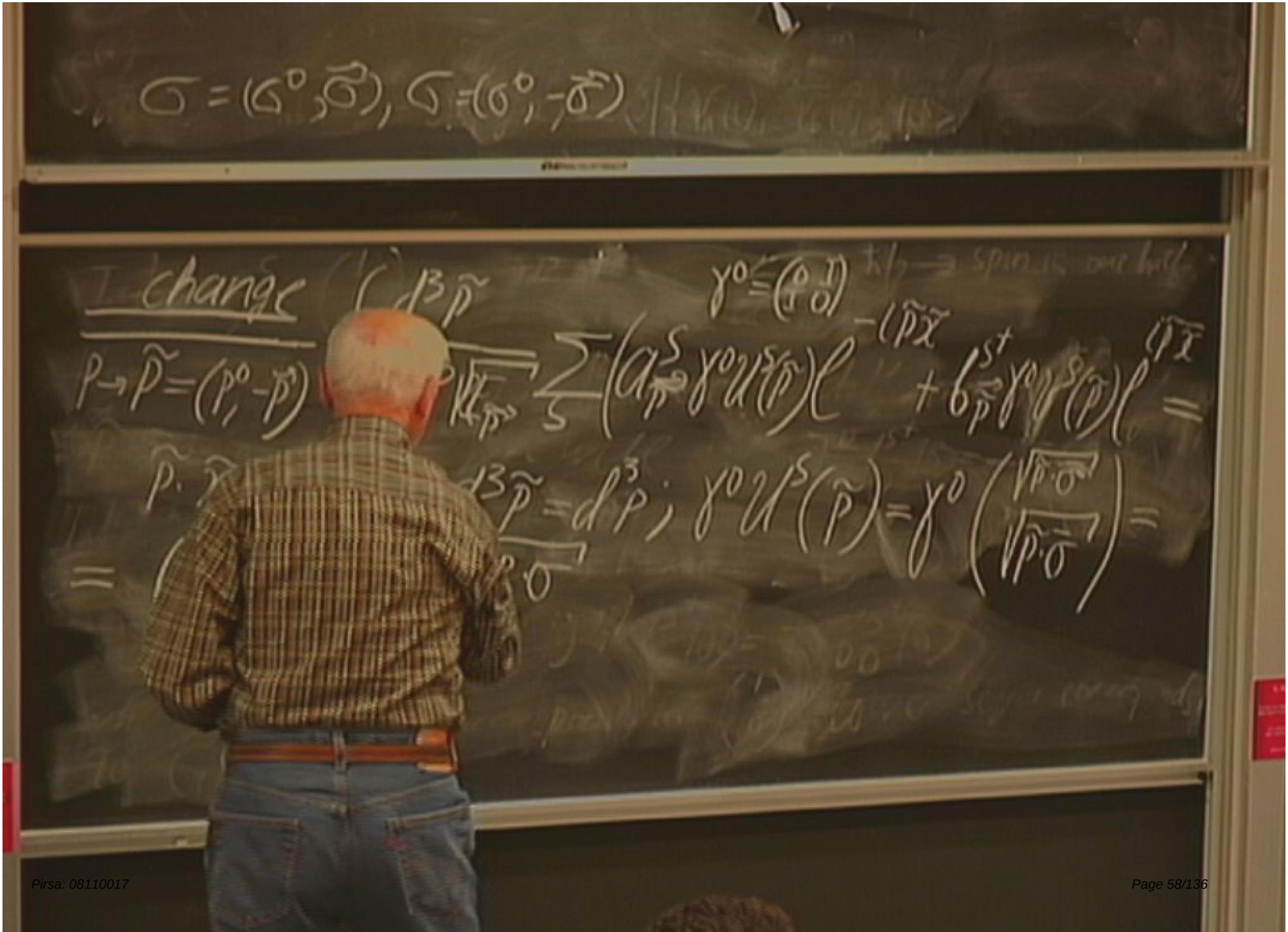
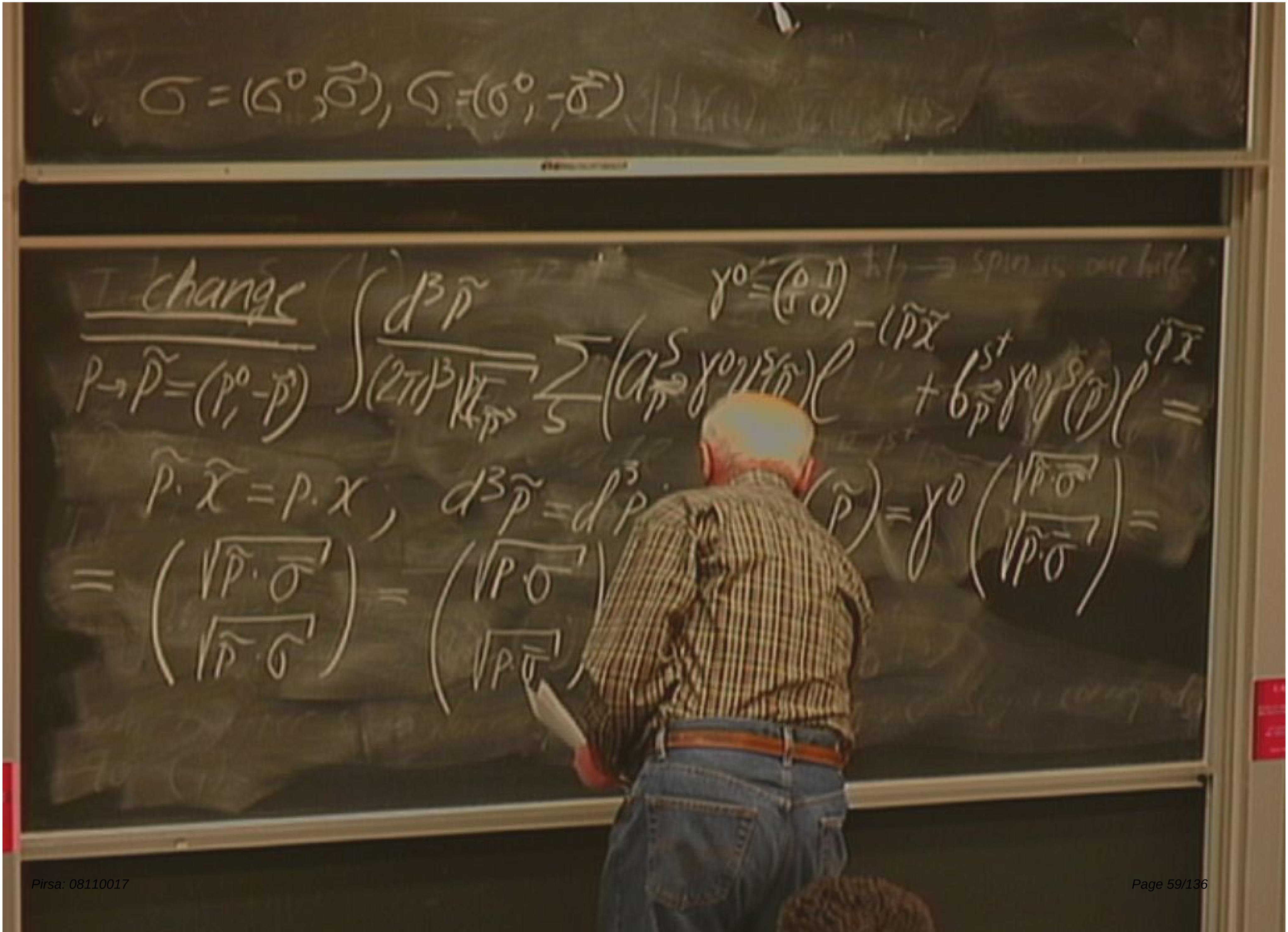


I change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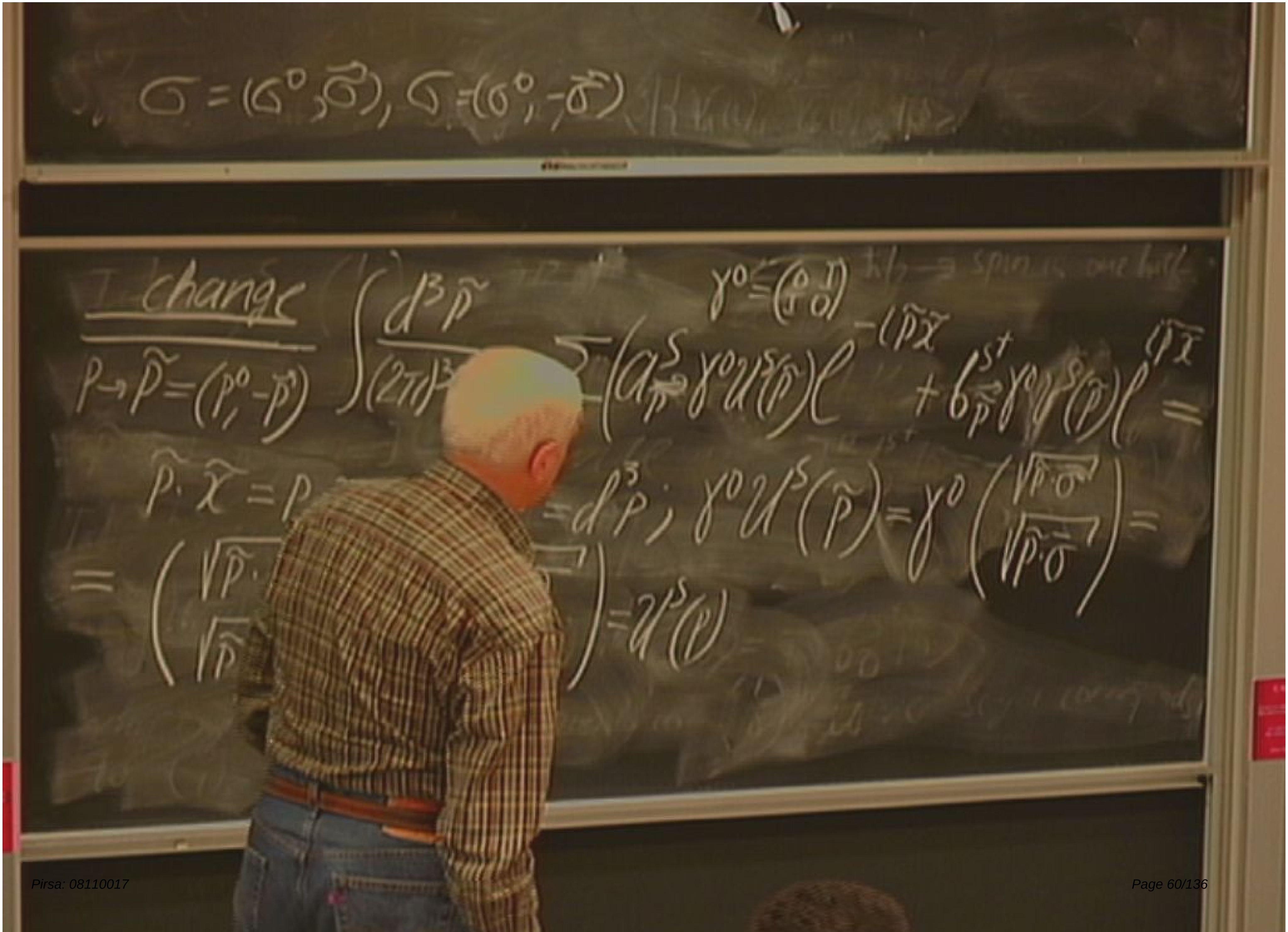
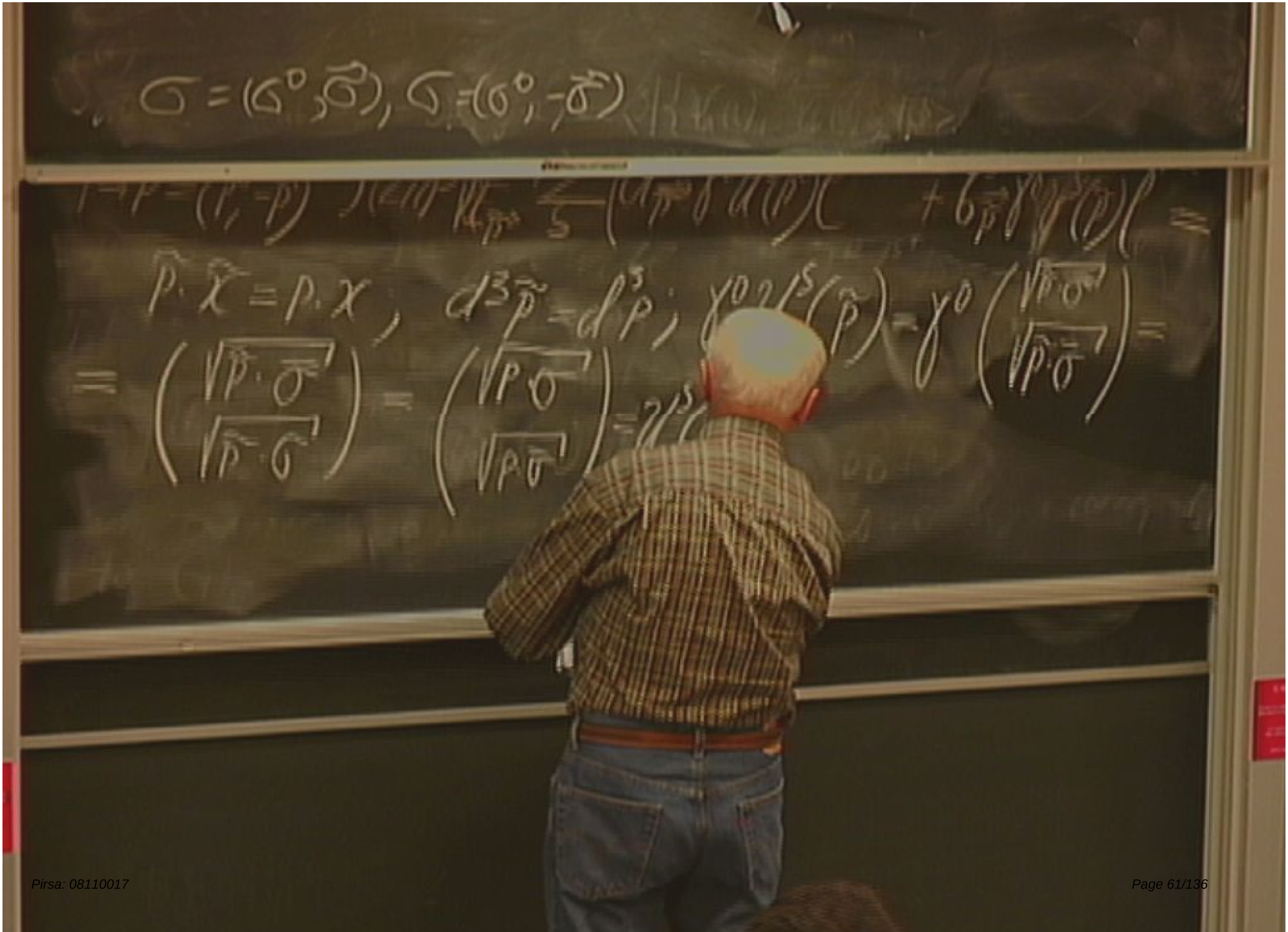
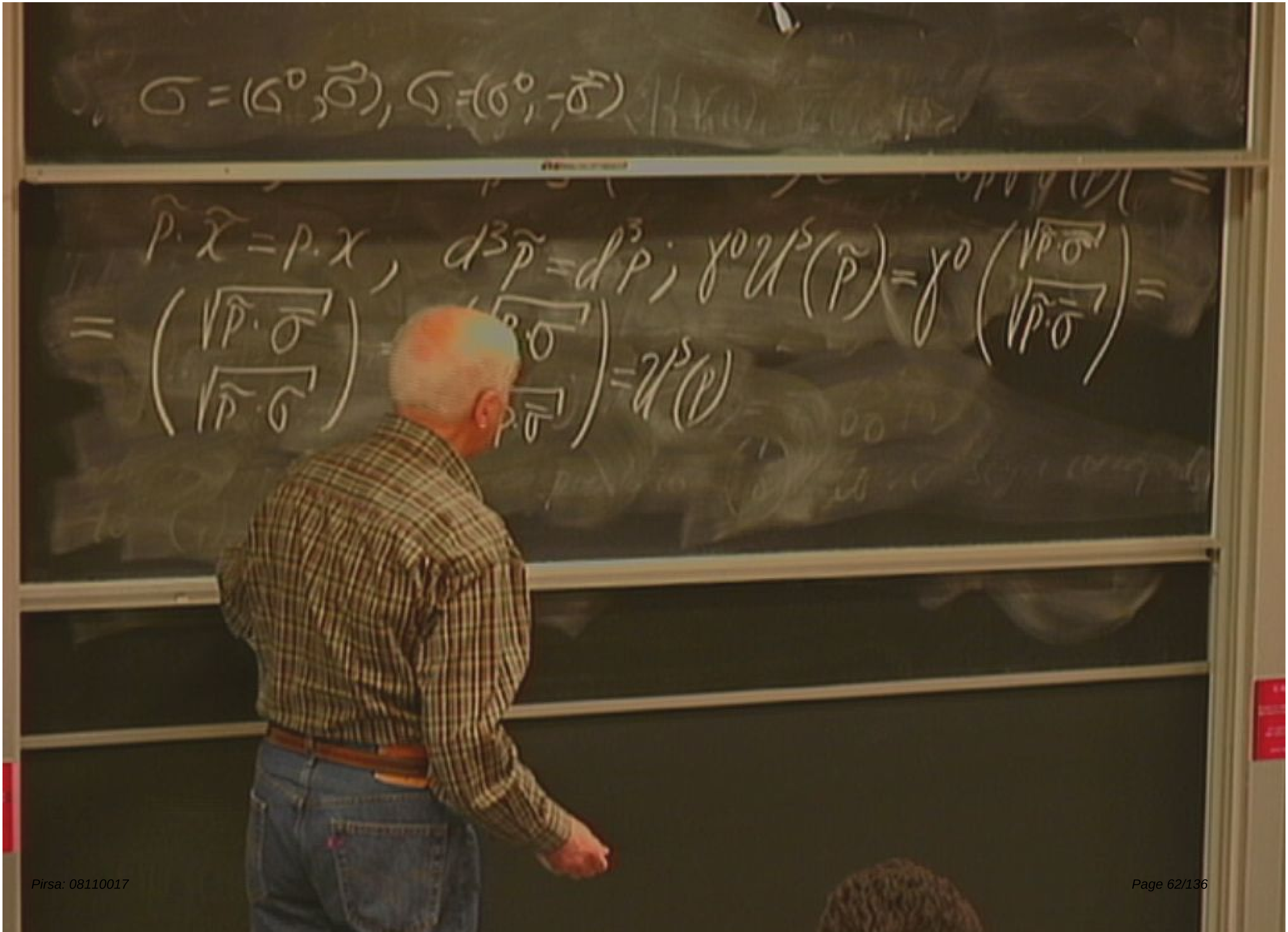
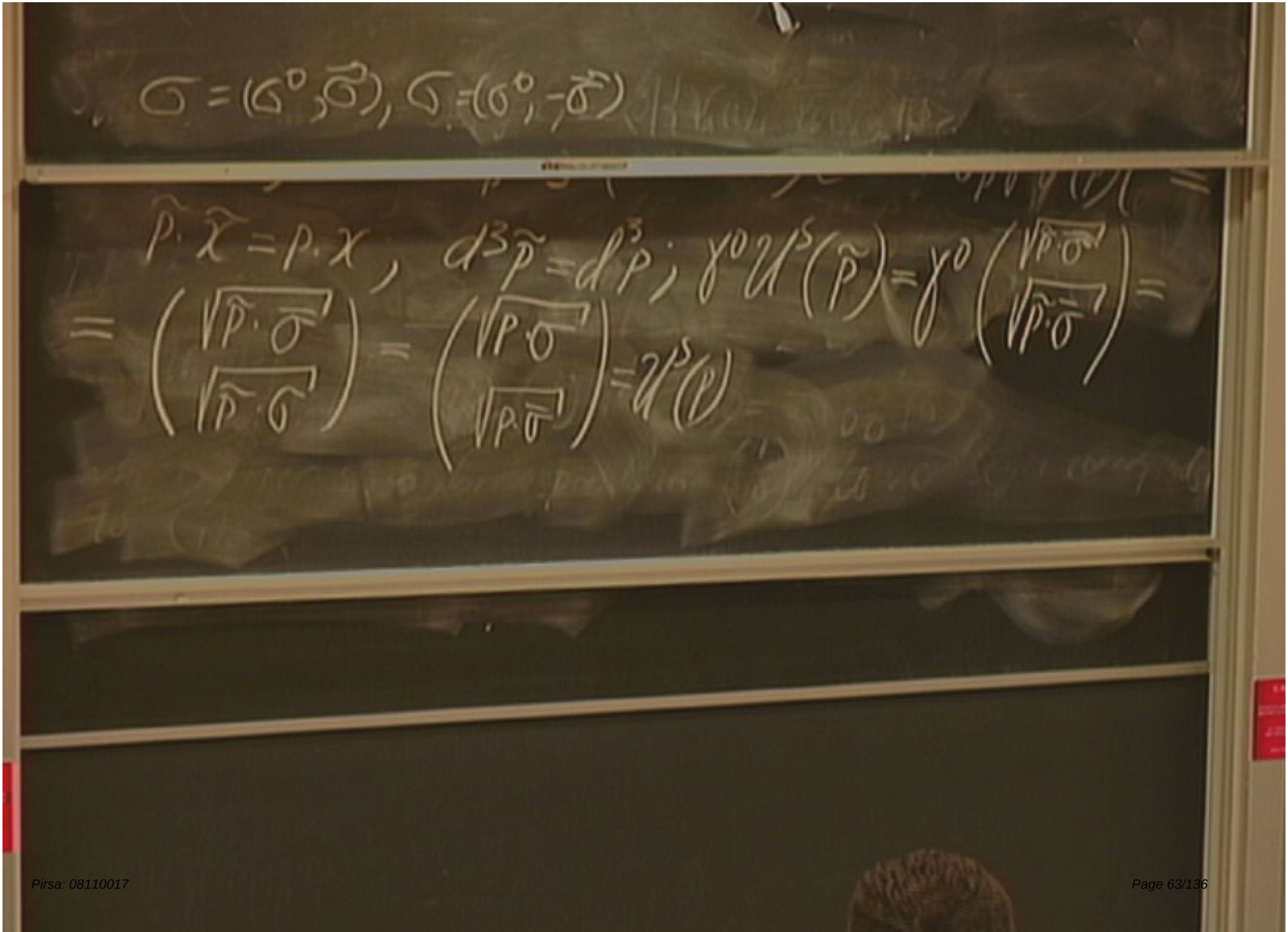
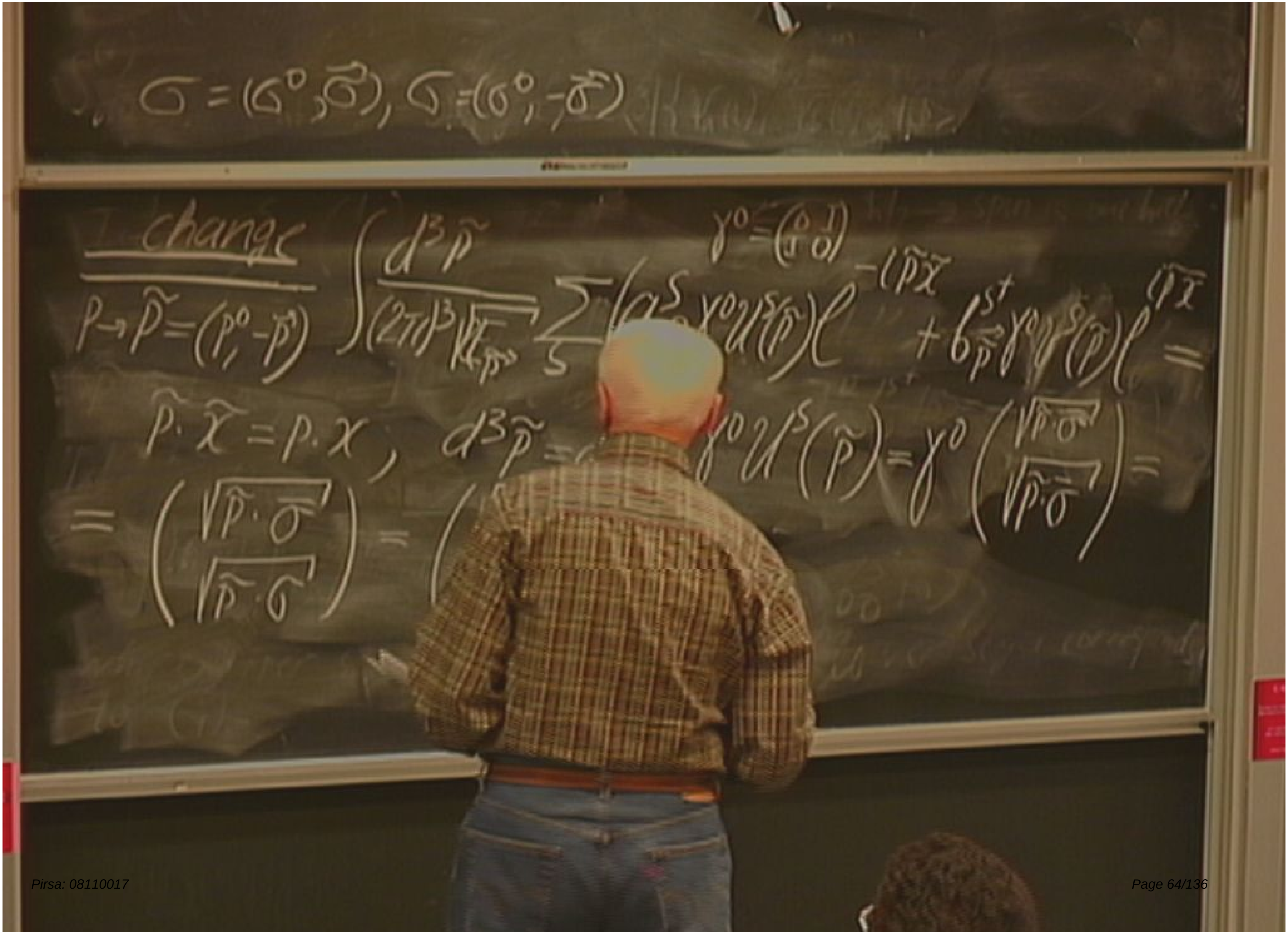
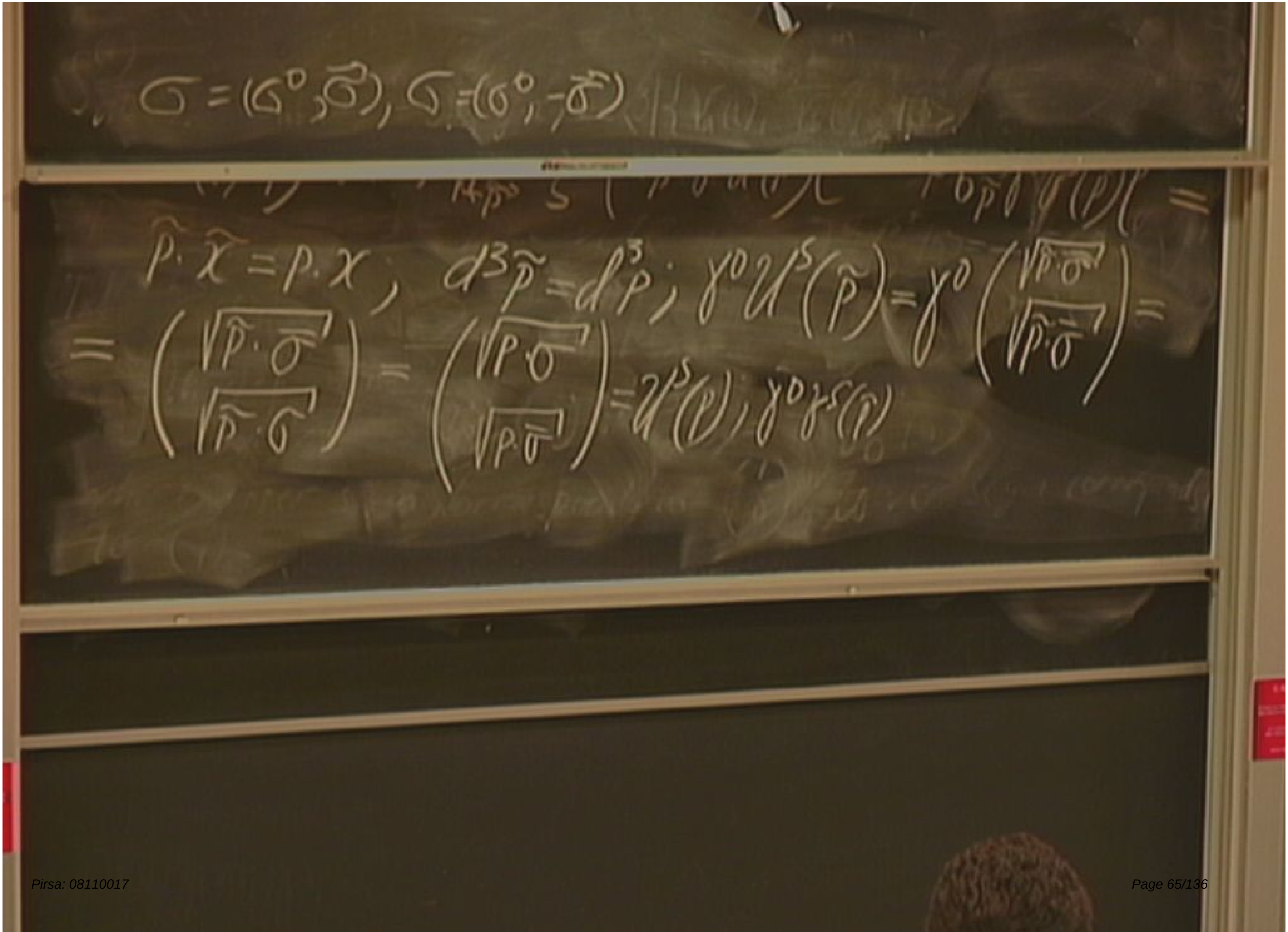

change

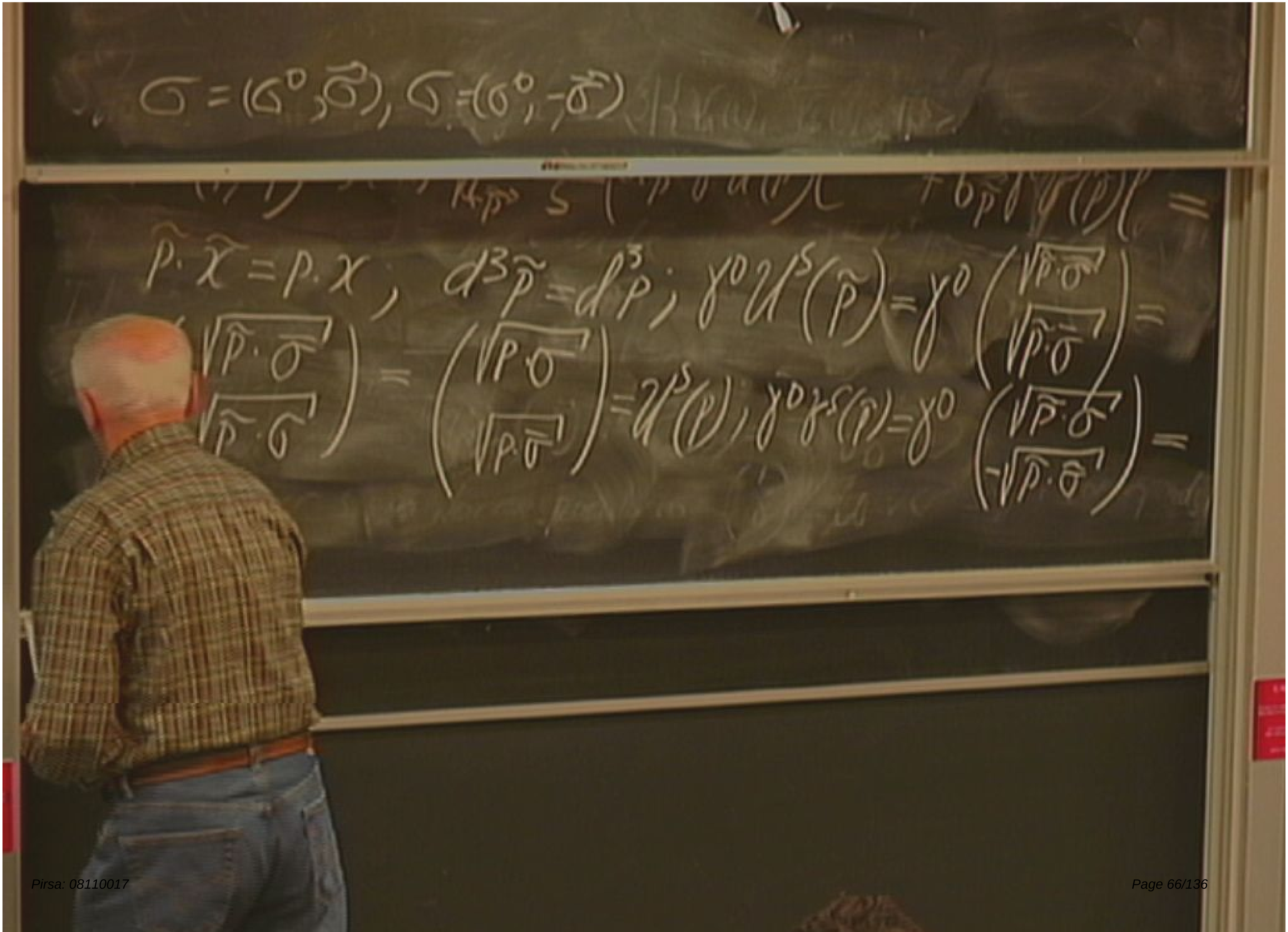


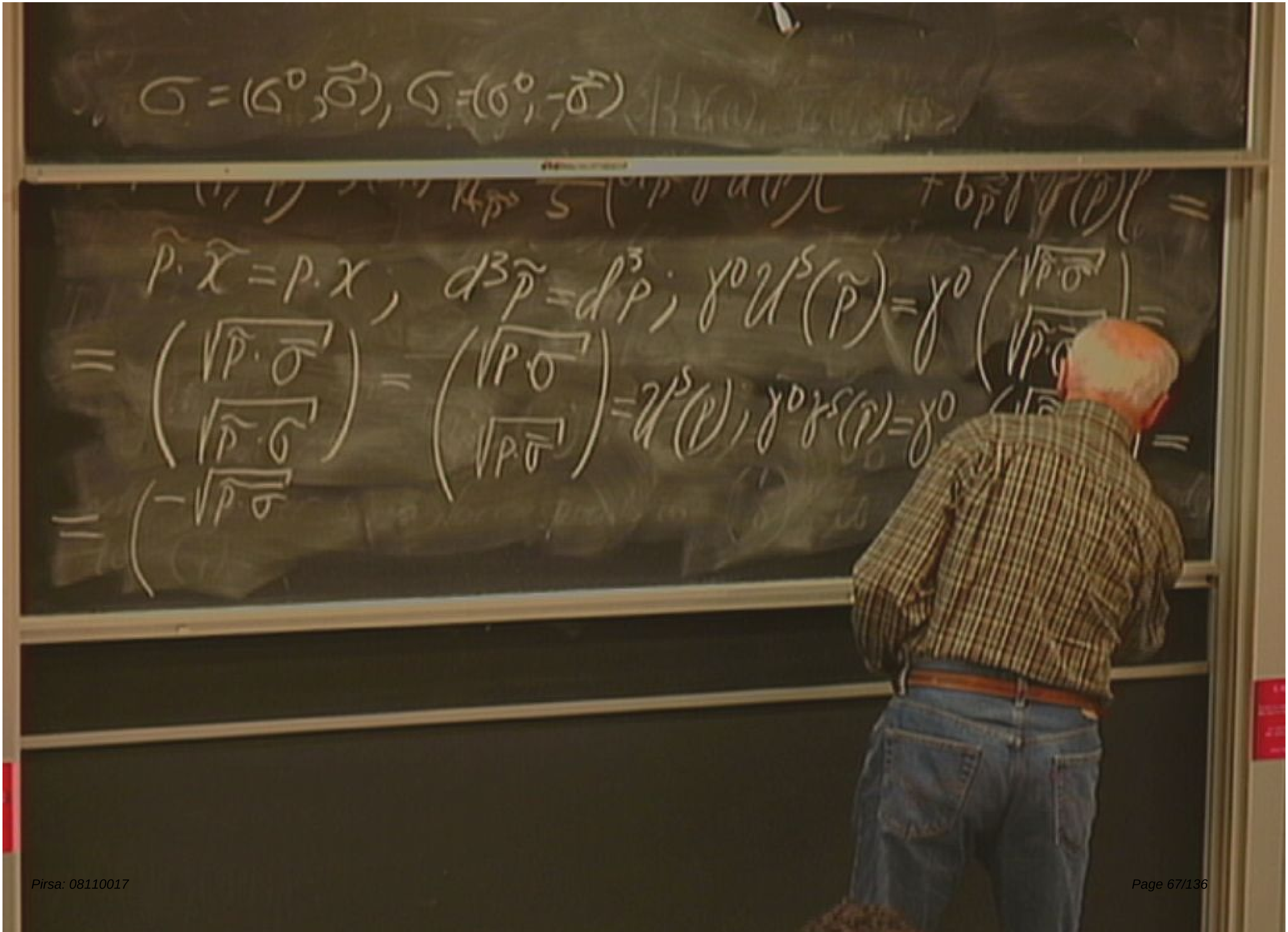

$\sigma = (\sigma^0, \vec{\sigma}),\ \bar{\sigma} = (\sigma^0, -\vec{\sigma})$
$\tilde{p}\cdot\tilde{x} = p\cdot x,\ d^3\tilde{p} = d^3p;\ \gamma^0 u^s(\tilde{p}) = \gamma^0 \begin{pmatrix}\sqrt{\tilde{p}\cdot\sigma} \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}\end{pmatrix} =$

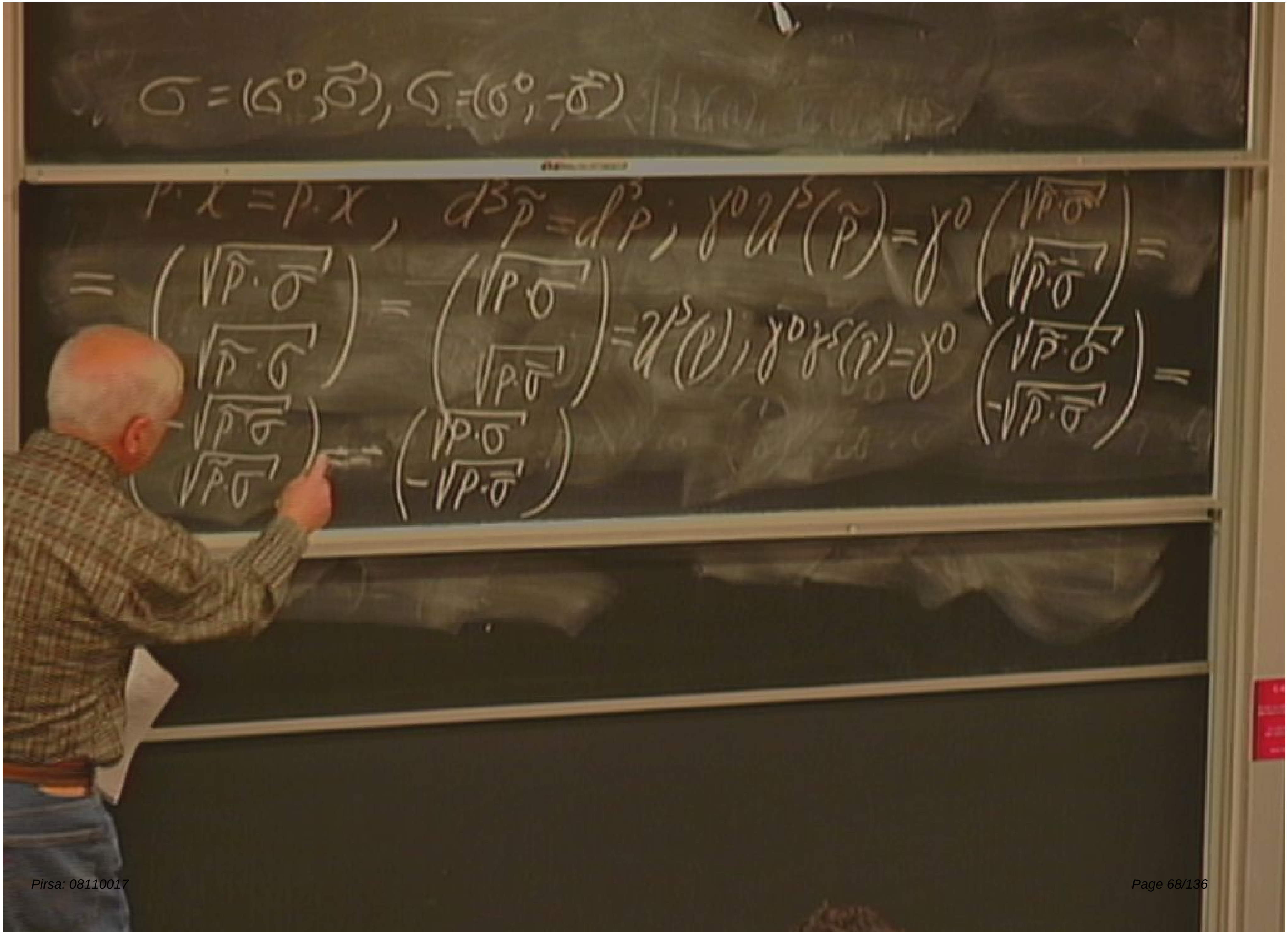

$\sigma = (\sigma^0, \vec{\sigma}), \; \bar{\sigma} = (\sigma^0, -\vec{\sigma})$
$\tilde{p} \cdot \tilde{x} = p \cdot x, \; d^3\tilde{p} = d^3p; \; \gamma^0 u^s(\tilde{p}) = \gamma^0 \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} =$
$= \begin{pmatrix} \sqrt{\tilde{p} \cdot \bar{\sigma}} \\ \sqrt{\tilde{p} \cdot \sigma} \end{pmatrix} = \begin{pmatrix} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \bar{\sigma}} \end{pmatrix} = u^s(p)$

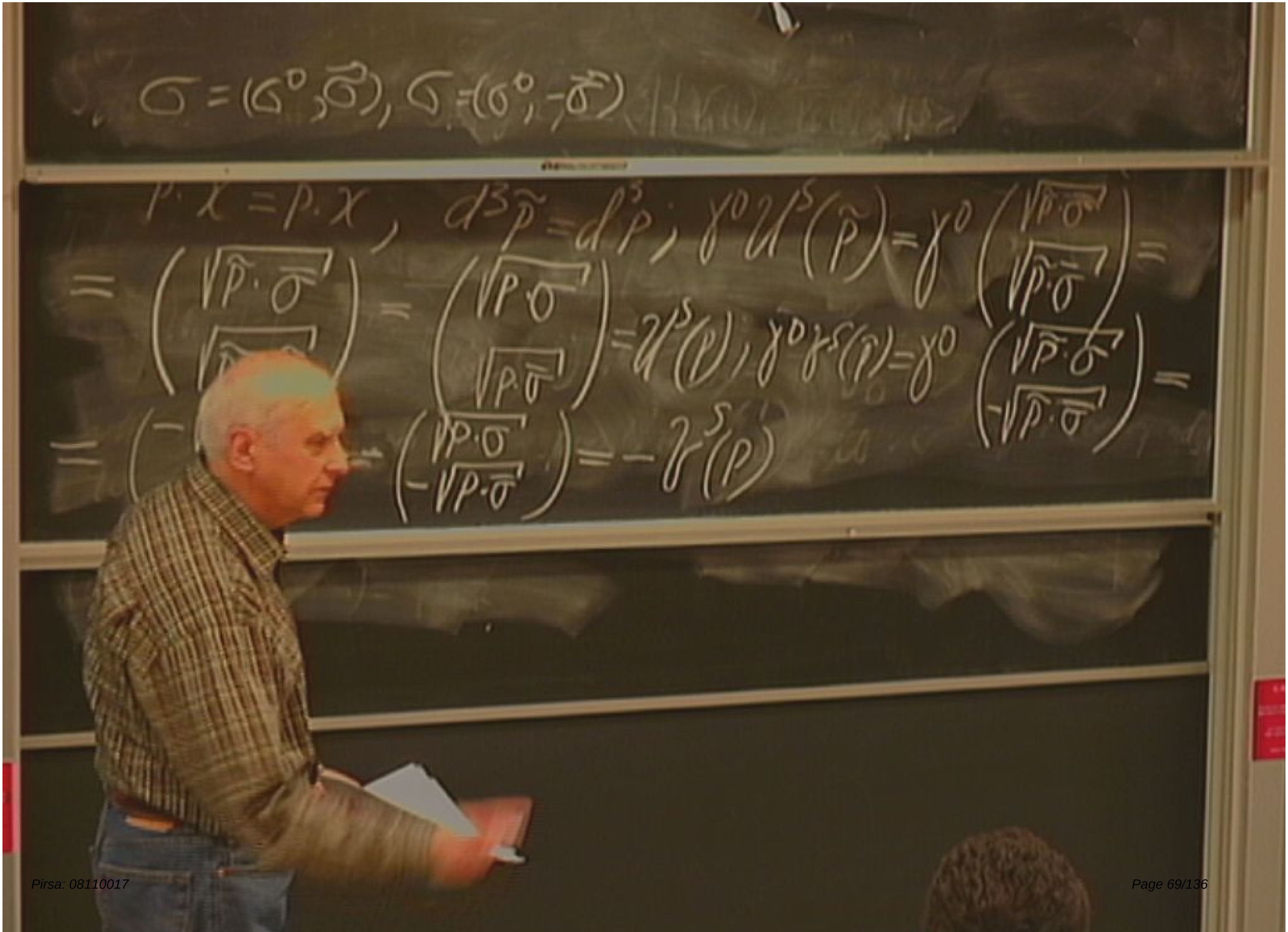


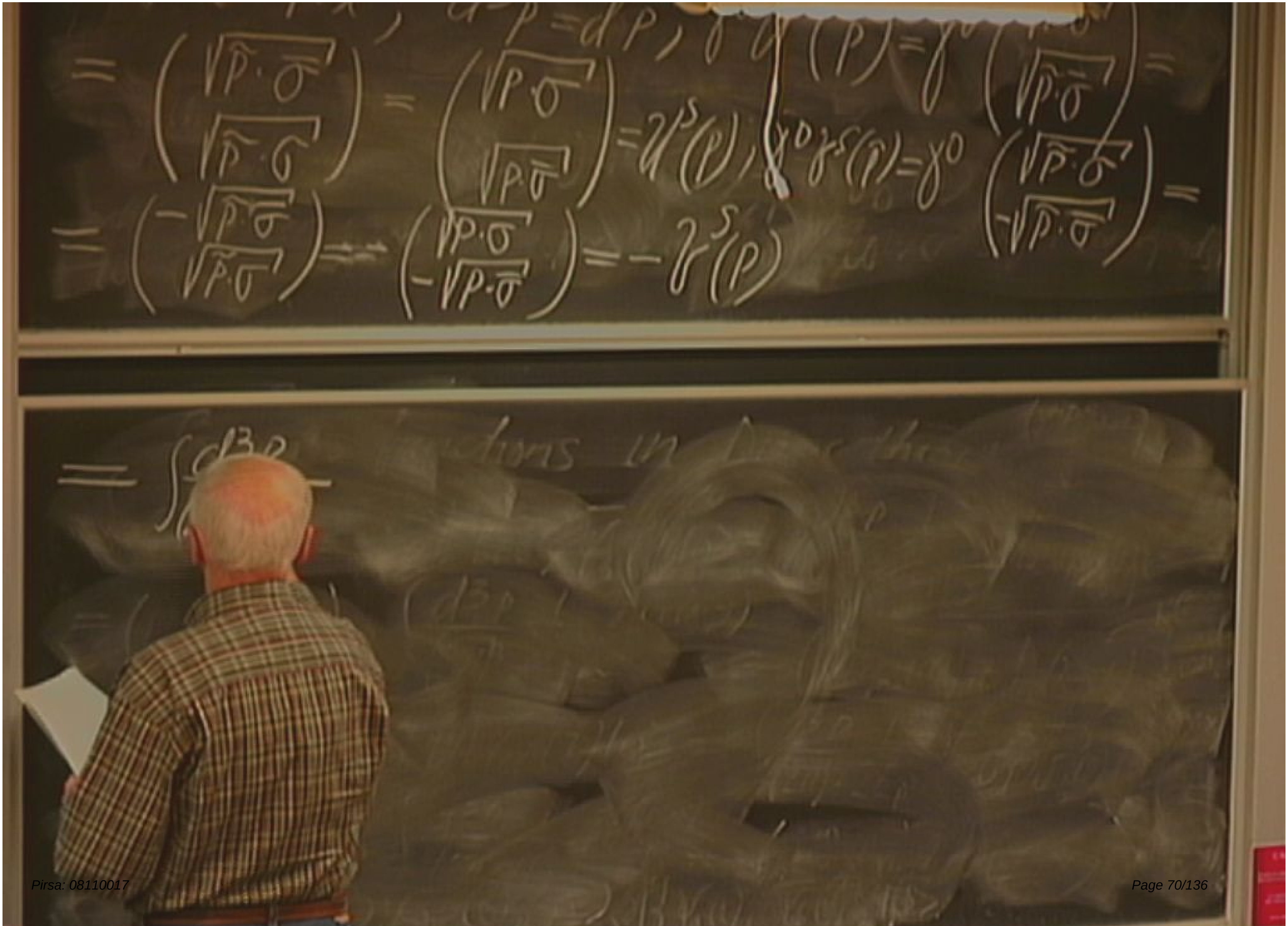

σ = (σ⁰, σ⃗), σ̄ = (σ⁰, −σ⃗)
p̃·x̃ = p·x ; d³p̃ = d³p ; γ⁰u^s(p̃) = γ⁰ (√(p̃·σ) ; √(p̃·σ̄)) =
= (√(p̃·σ̄) ; √(p̃·σ)) = (√(p·σ) ; √(p·σ̄)) = u^s(p) ; γ⁰v^s(p̃)

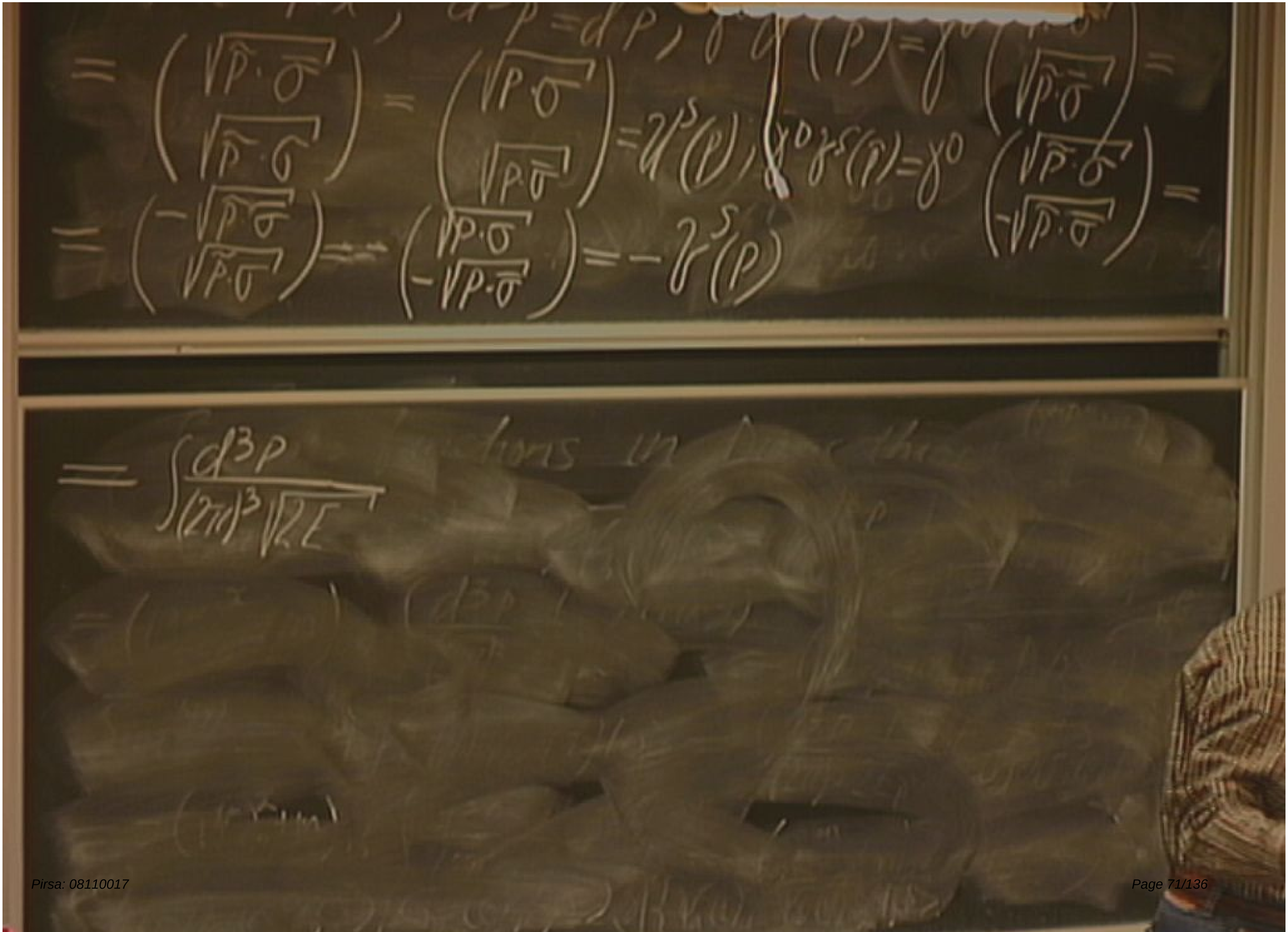

σ = (σ⁰, σ⃗), σ̄ = (σ⁰, −σ⃗)
p̃·x̃ = p·x ; d³p̃ = d³p ; γ⁰u^s(p̃) = γ⁰
= u^s(p) ; γ⁰v^s(p̃) = γ⁰

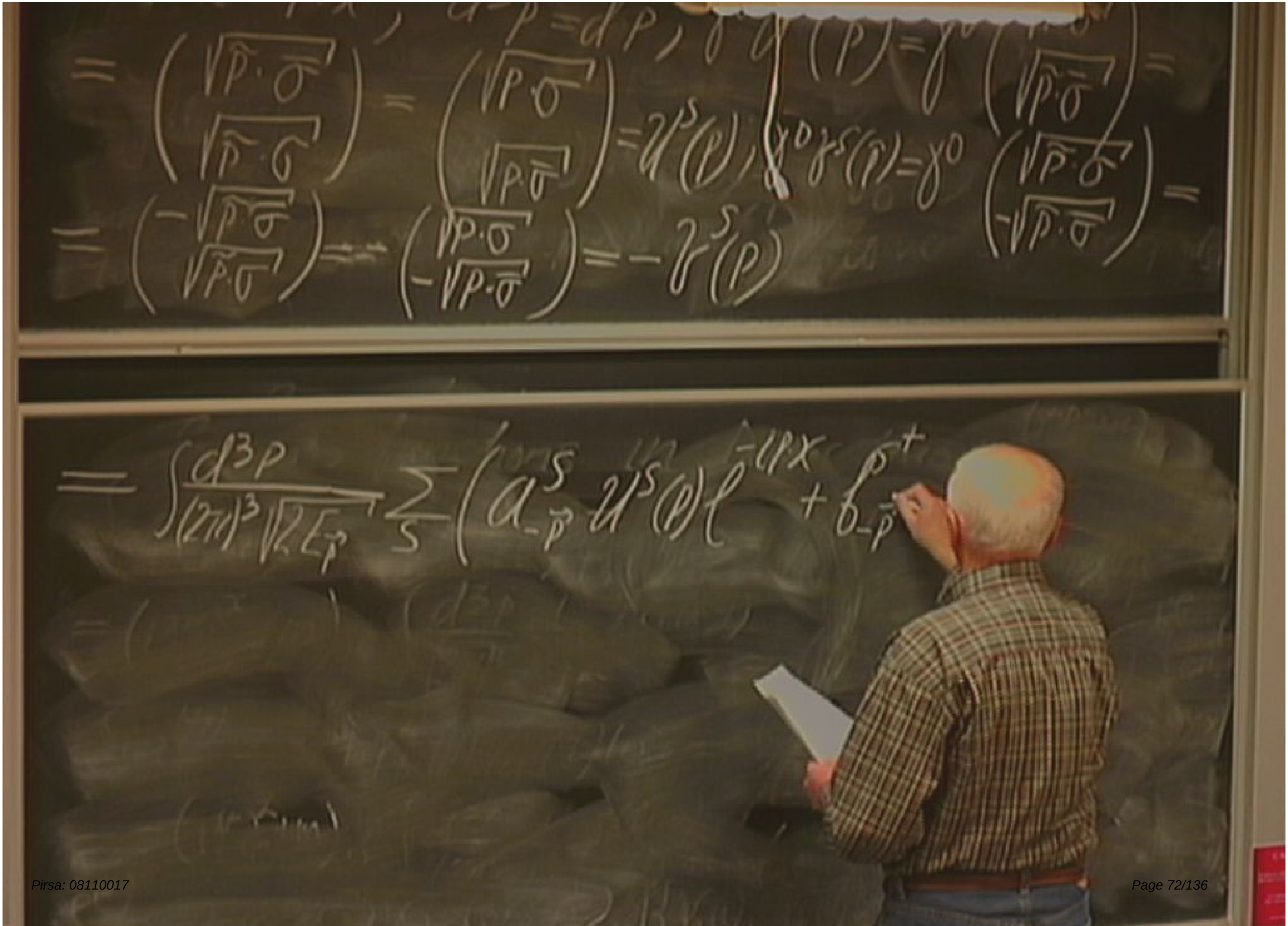

σ = (σ⁰, σ⃗), σ̃ = (σ⁰, −σ⃗)
p̃ · x̃ = p · x ; d³p̃ = d³p ;

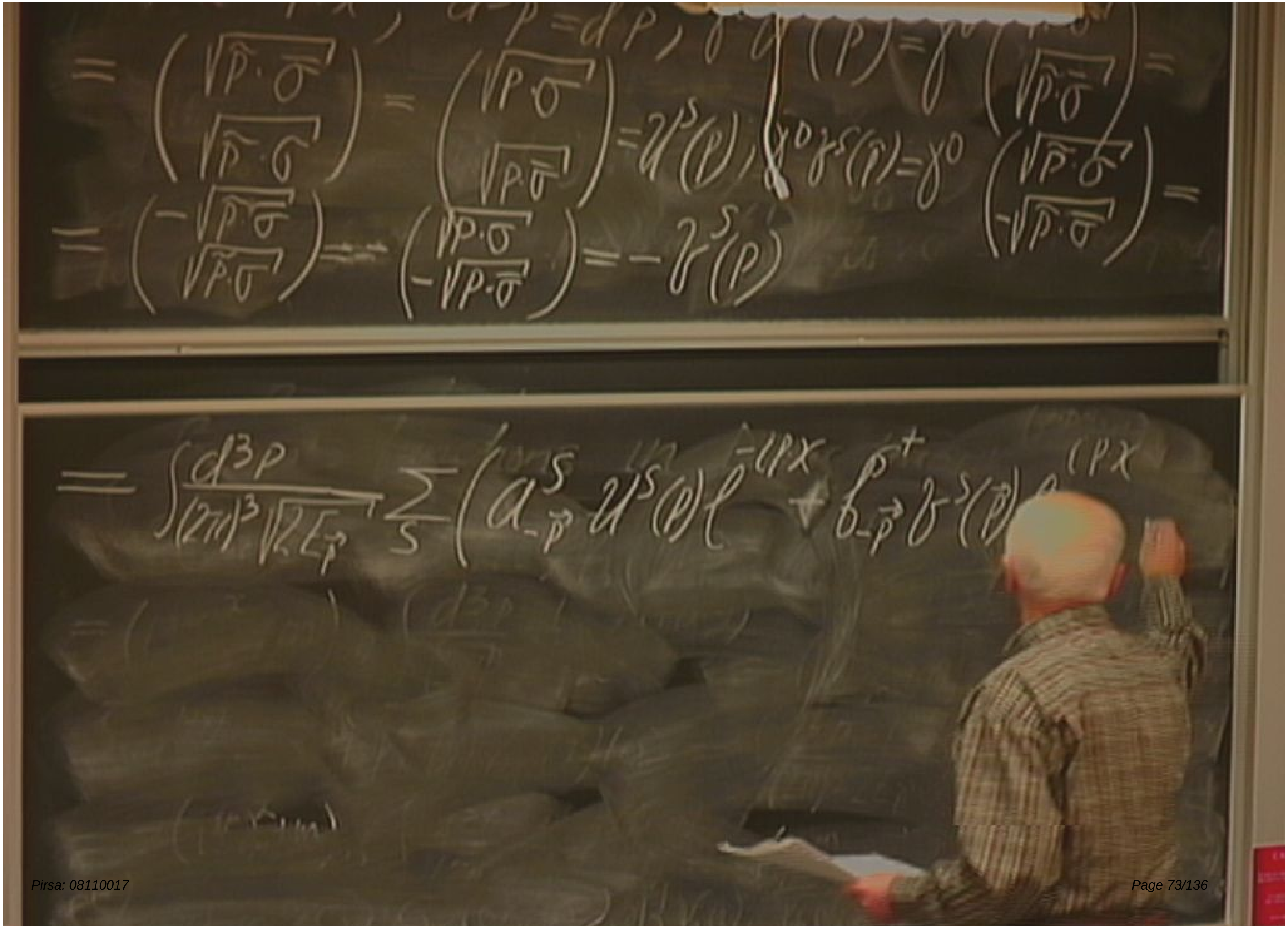

σ = (σ⁰, σ⃗), σ̄ = (σ⁰, −σ⃗)
p̃·x̃ = p·x, d³p̃ = d³p ; γ⁰u^s(p̃) = γ⁰ (√(p̃·σ) ; √(p̃·σ̄)) =
= (√(p̃·σ̄) ; √(p̃·σ)) = (√(p·σ) ; √(p·σ̄)) = u^s(p) ; γ⁰v^s(p̃) = γ⁰ (√(p̃·σ) ; −√(p̃·σ̄)) =
= (√(p·σ) ; −√(p·σ̄))


σ = (σ⁰, σ⃗), σ̄ = (σ⁰, −σ⃗)
p̃·x̃ = p·x, d³p̃ = d³p, γ⁰u^s(p̃) = γ⁰ (√(p̃·σ) ; √(p̃·σ̄)) =
= (√(p̃·σ̄) ; √(p̃·σ)) = (√(p·σ) ; √(p·σ̄)) = u^s(p), γ⁰v^s(p̃) = γ⁰ (√(p̃·σ) ; −√(p̃·σ̄)) =
= (√(p·σ) ; −√(p·σ̄)) = −v^s(p)




= −γ5(p)
= ∫ d3p / (2π)3 √2E

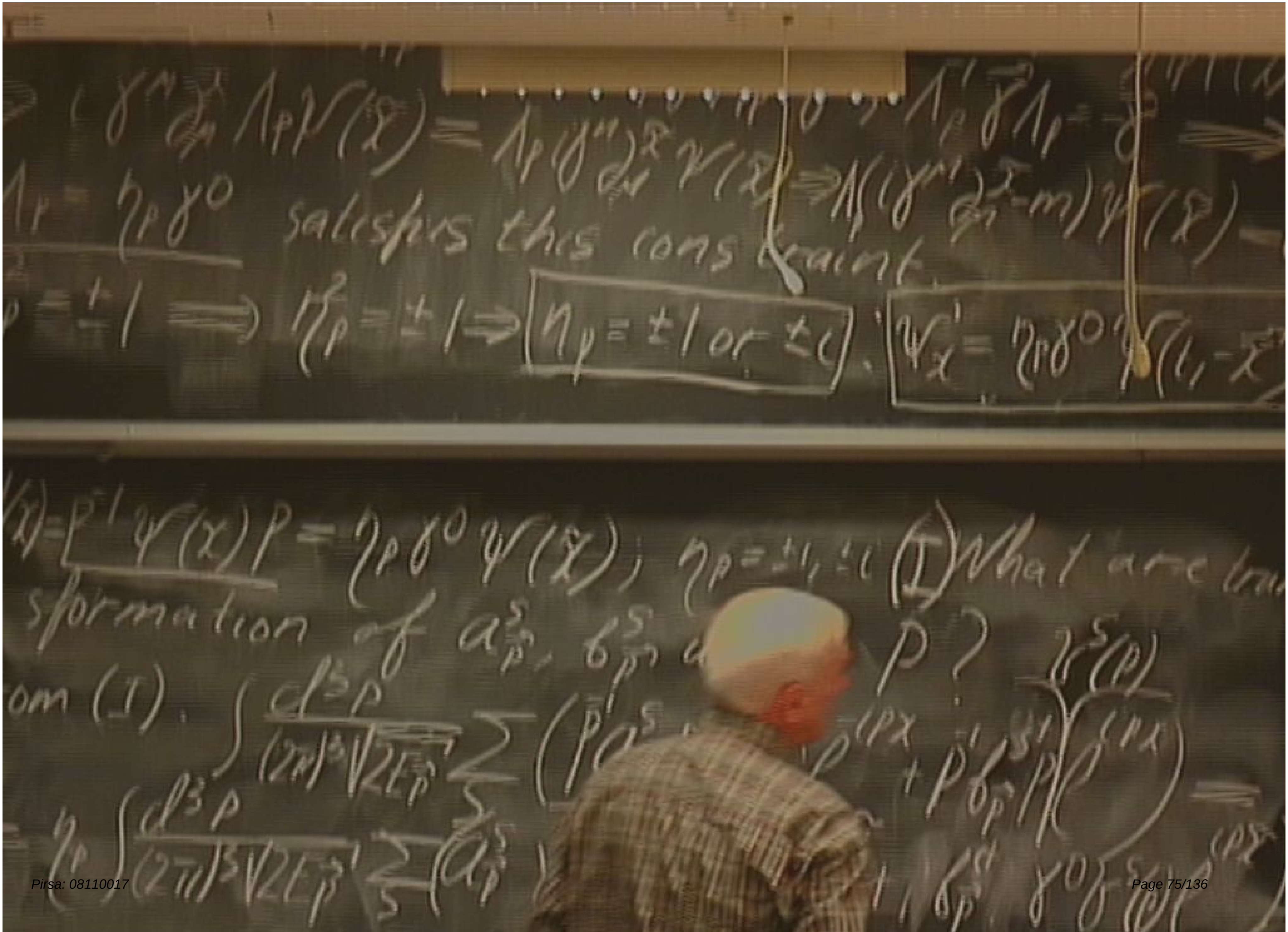




$$= \begin{pmatrix}\sqrt{\hat{p}\cdot\sigma}\\ \sqrt{\hat{p}\cdot\bar{\sigma}}\end{pmatrix} = \begin{pmatrix}\sqrt{p\cdot\sigma}\\ \sqrt{p\cdot\bar{\sigma}}\end{pmatrix} = u^s(p), \quad \gamma^0 v^s(\hat{p}) = \gamma^0 \begin{pmatrix}\sqrt{\hat{p}\cdot\sigma}\\ -\sqrt{\hat{p}\cdot\bar{\sigma}}\end{pmatrix} =$$

$$= \begin{pmatrix}-\sqrt{\hat{p}\cdot\bar{\sigma}}\\ \sqrt{\hat{p}\cdot\sigma}\end{pmatrix} = -\begin{pmatrix}\sqrt{p\cdot\sigma}\\ -\sqrt{p\cdot\bar{\sigma}}\end{pmatrix} = -v^s(p)$$

$$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{-\vec{p}}\, u^s(p)\, e^{-ipx} + b^{s\dagger}_{-\vec{p}}\, v^s(p)\, e^{ipx} \right)$$


satisfies this constraint
What are transformation of

$\ldots \partial_\mu \Lambda_P \psi(\tilde{x}) = \Lambda_P (i\gamma^\mu) \partial_\mu^{\tilde{x}} \psi(\tilde{x}) \Rightarrow \Lambda_P (i\gamma^\mu \partial_\mu^{\tilde{x}} - m)\psi(\tilde{x}) = \ldots$

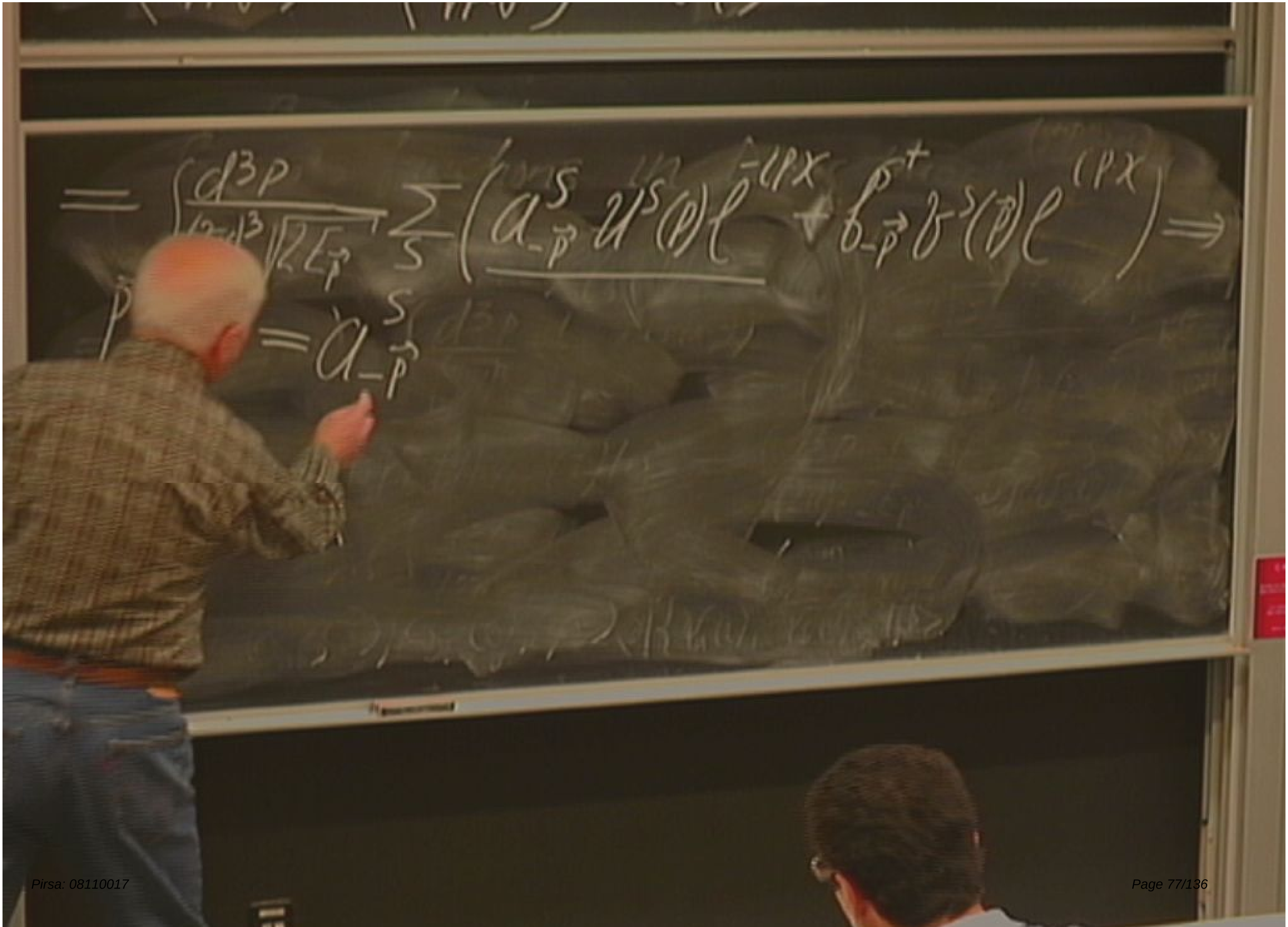
$\Lambda_P = \eta_P \gamma^0$ satisfies this constraint.

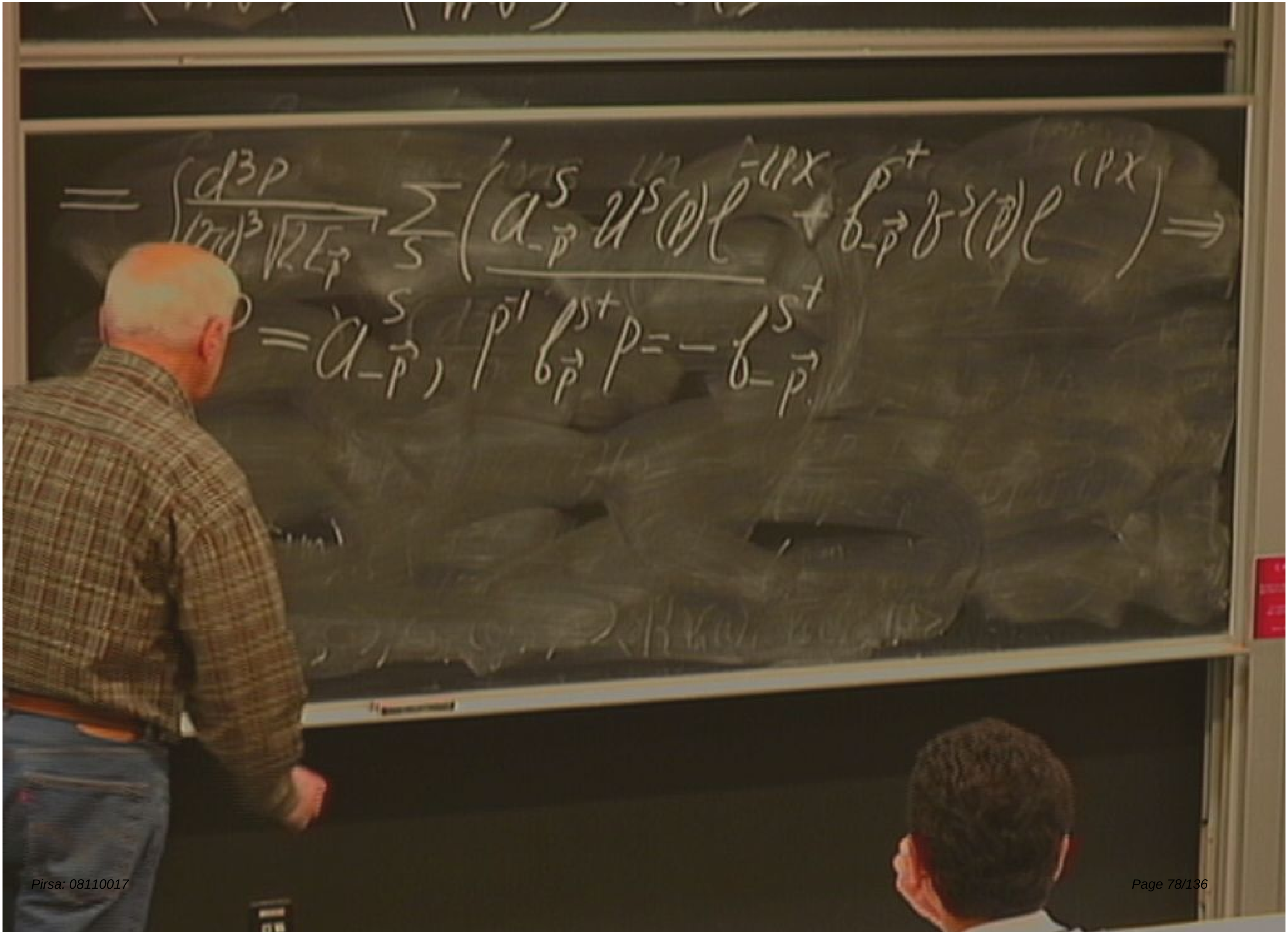
$\Lambda_P^2 = \pm 1 \Longrightarrow \eta_P^2 = \pm 1 \Rightarrow \boxed{\eta_P = \pm 1 \text{ or } \pm i}$; $\boxed{\psi'_x = \eta_P \gamma^0 \psi(t, -\vec{x})}$

$\psi'(x) = P^{-1}\psi(x) P = \eta_P \gamma^0 \psi(\tilde{x})$; $\eta_P = \pm 1, \pm i$ (I) What are transformation of $a^s_{\vec{p}}$, $b^s_{\vec{p}}$ under P?

From (I): $\int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(P^{-1} a^s_{\vec{p}} P u^s(p) e^{-ipx} + P^{-1} b^{s\dagger}_{\vec{p}} P v^s(p) e^{ipx} \right) =$

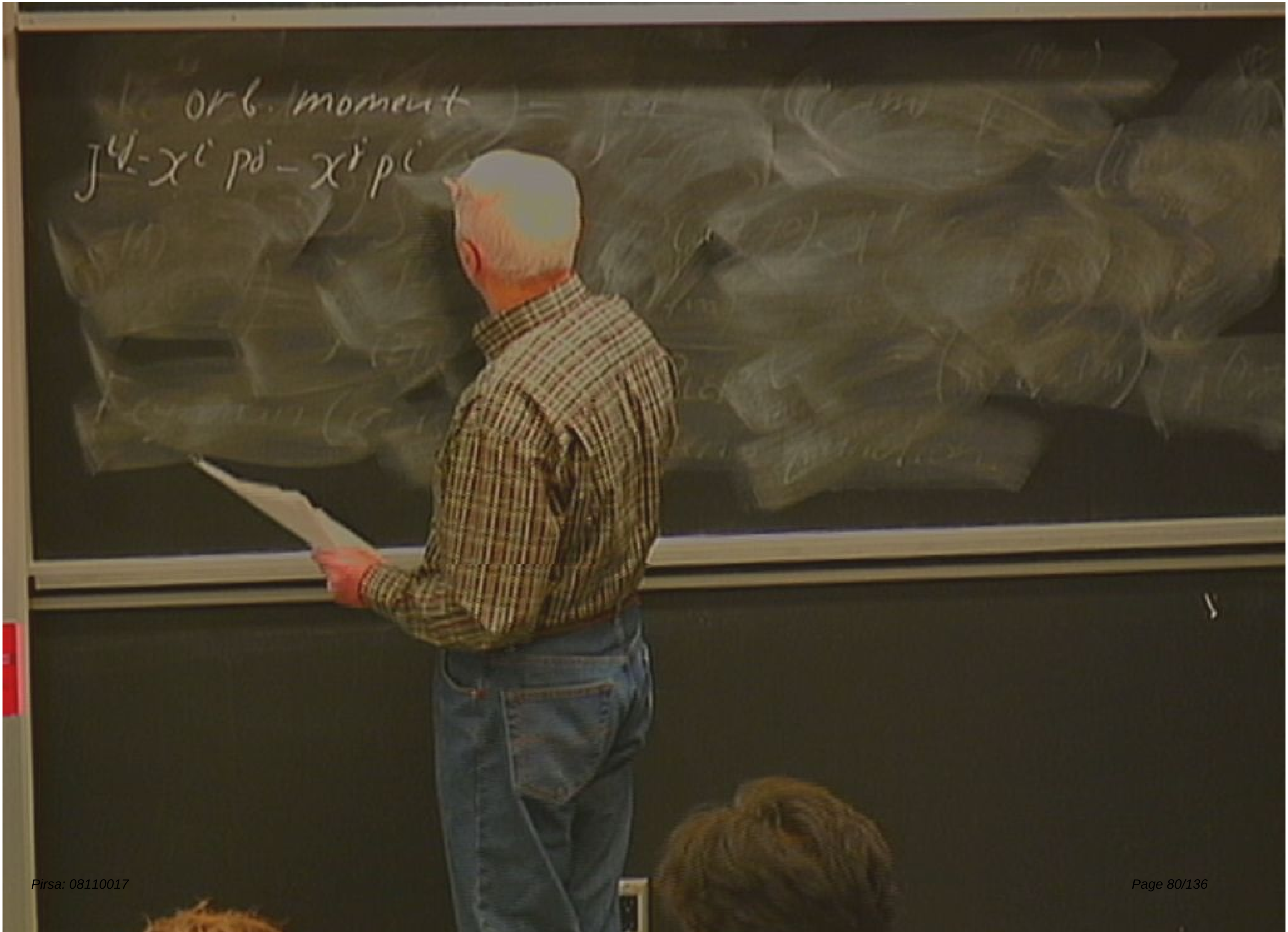
$= \eta_P \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{\vec{p}} \gamma^0 u^s(p) e^{-ip\tilde{x}} + b^{s\dagger}_{\vec{p}} \gamma^0 v^s(p) e^{ip\tilde{x}} \right)$

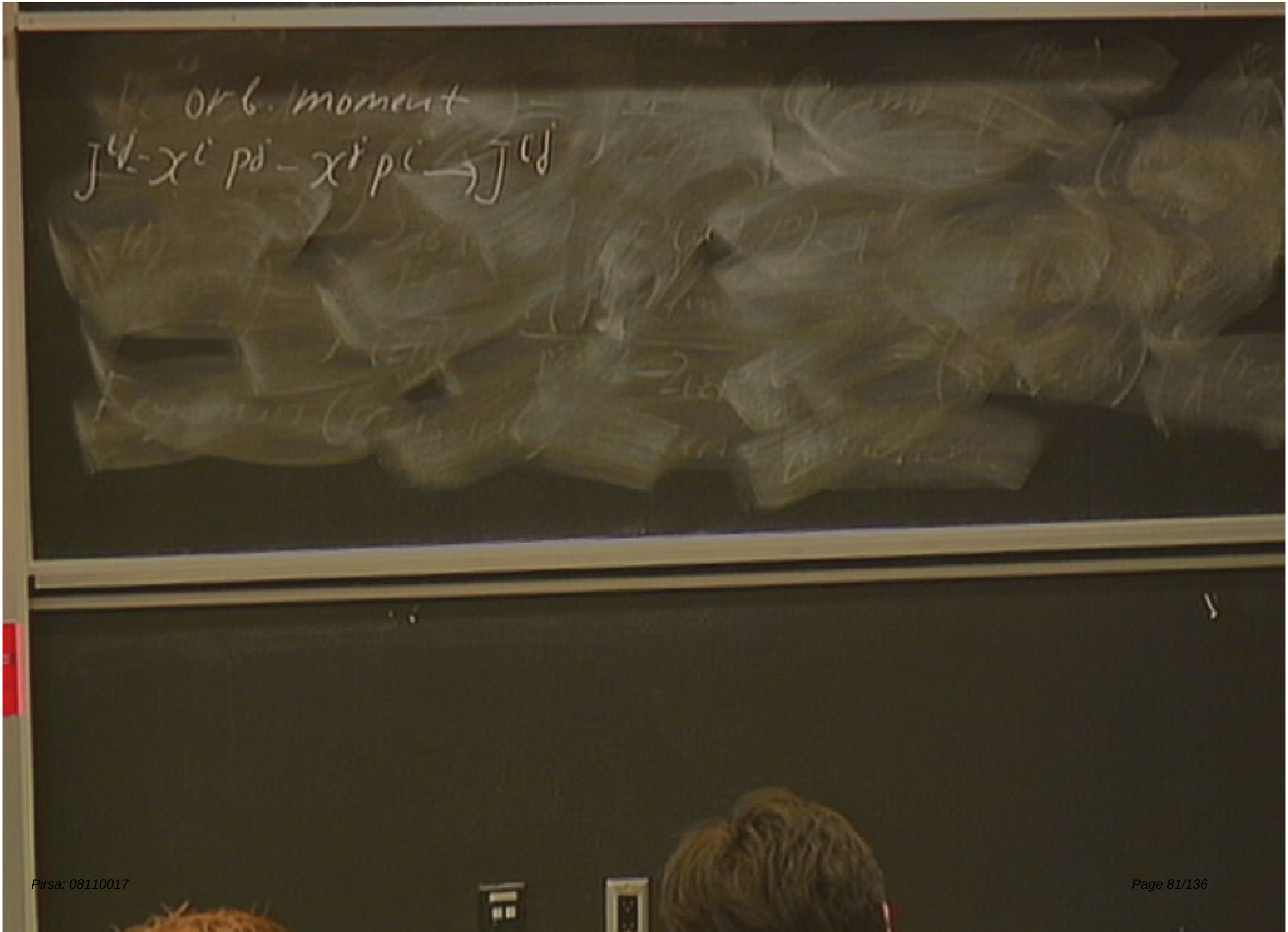



$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} \sum_s \left(a^s_{-\vec p}\, u^s(p)\, e^{-ipx} + b^{s\dagger}_{-\vec p}\, v^s(p)\, e^{ipx} \right) \Longrightarrow$
$= a^s_{-\vec p}, \quad P^{-1} b^{s\dagger}_{\vec p} P = -b^{s\dagger}_{-\vec p}$

$$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{-\vec{p}}\, u^s(p)\, e^{-ipx} + b^{s\dagger}_{-\vec{p}}\, v^s(p)\, e^{ipx} \right) \Longrightarrow$$

$$P^{-1} a^s_{\vec{p}} P = a^s_{-\vec{p}}, \quad P^{-1} b^{s\dagger}_{\vec{p}} P = -b^{s\dagger}_{-\vec{p}}$$

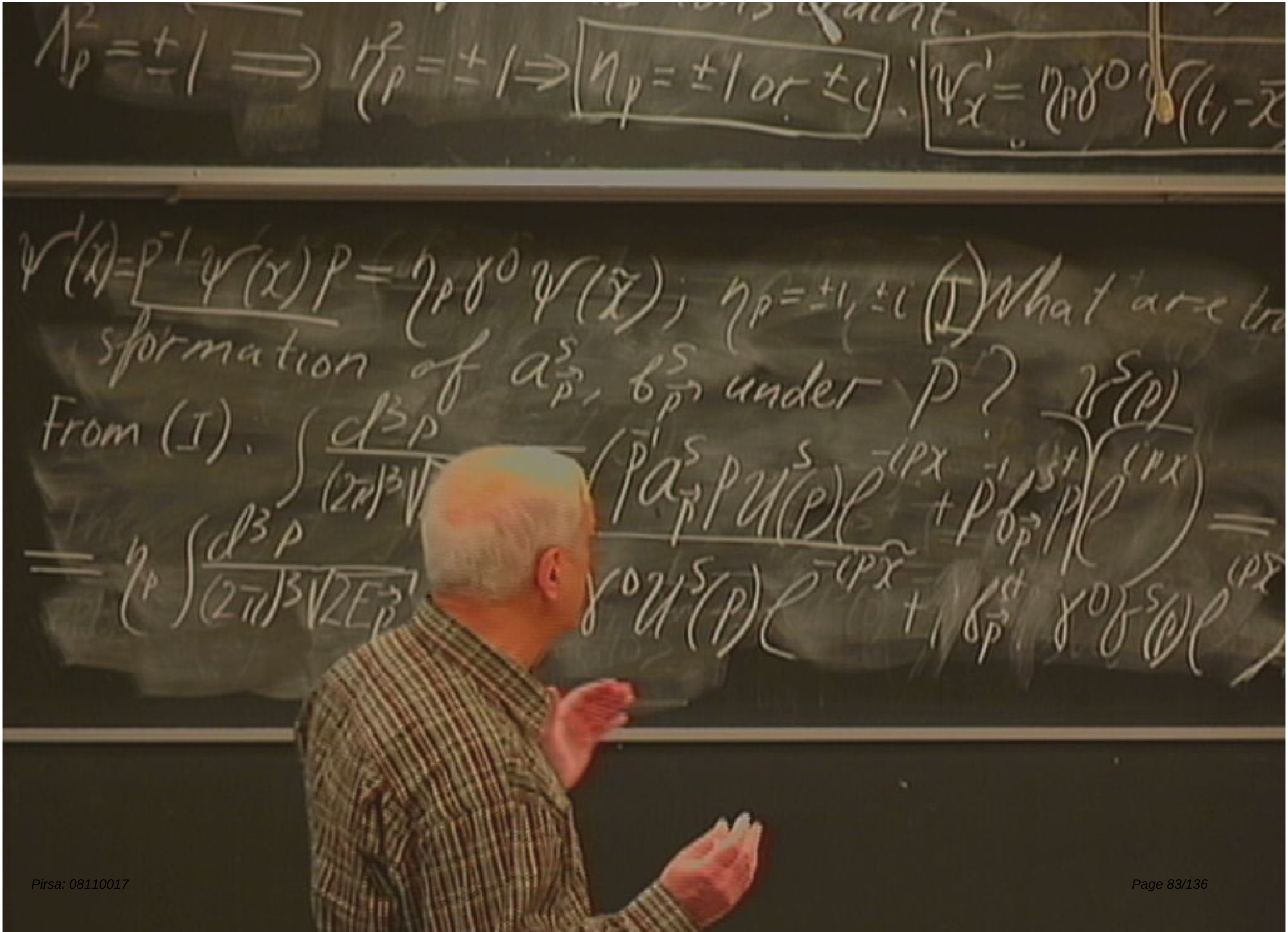

orb. moment
J^ij = x^i p^j − x^j p^i

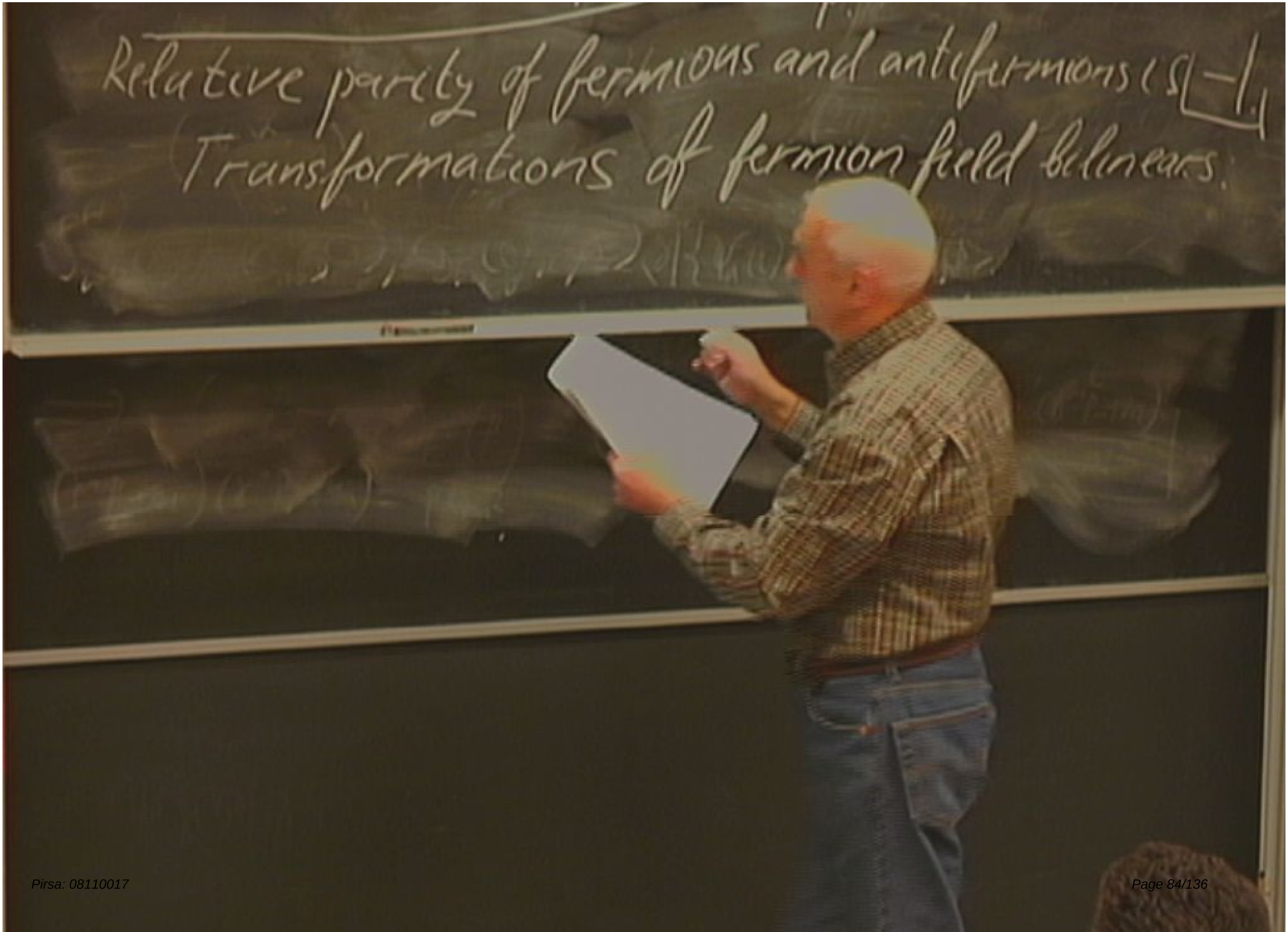

orb. moment
J^ij = x^i p^j − x^j p^i → J^ij

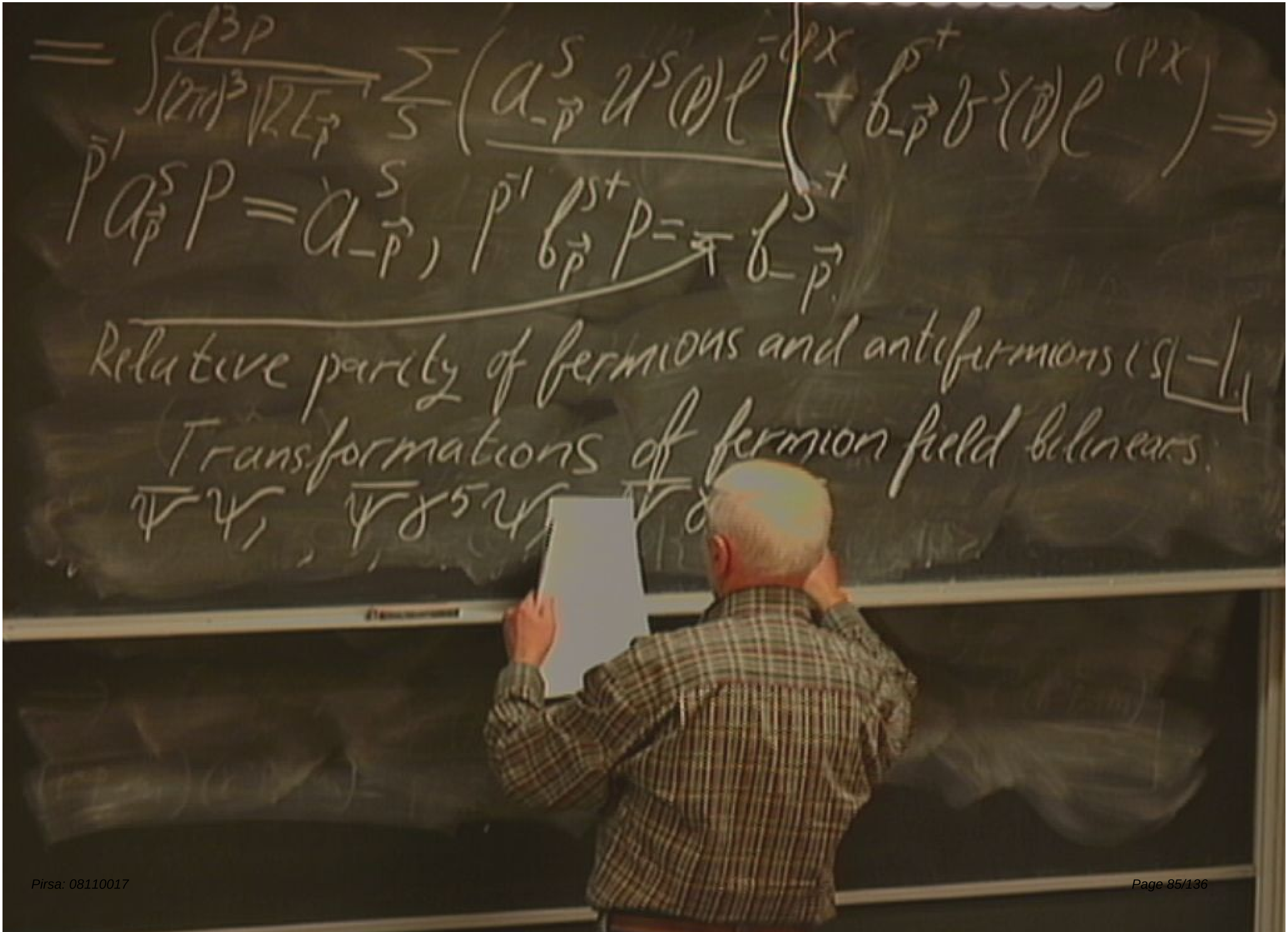
$$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} \sum_s \left(a^s_{-\vec p}\, u^s(p)\, e^{-ipx} + b^{s\dagger}_{-\vec p}\, v^s(p)\, e^{ipx} \right) \Rightarrow$$

$$P'\, a^s_{\vec p}\, P = a^s_{-\vec p}, \quad P'\, b^{s\dagger}_{\vec p}\, P = -b^{s\dagger}_{-\vec p}$$

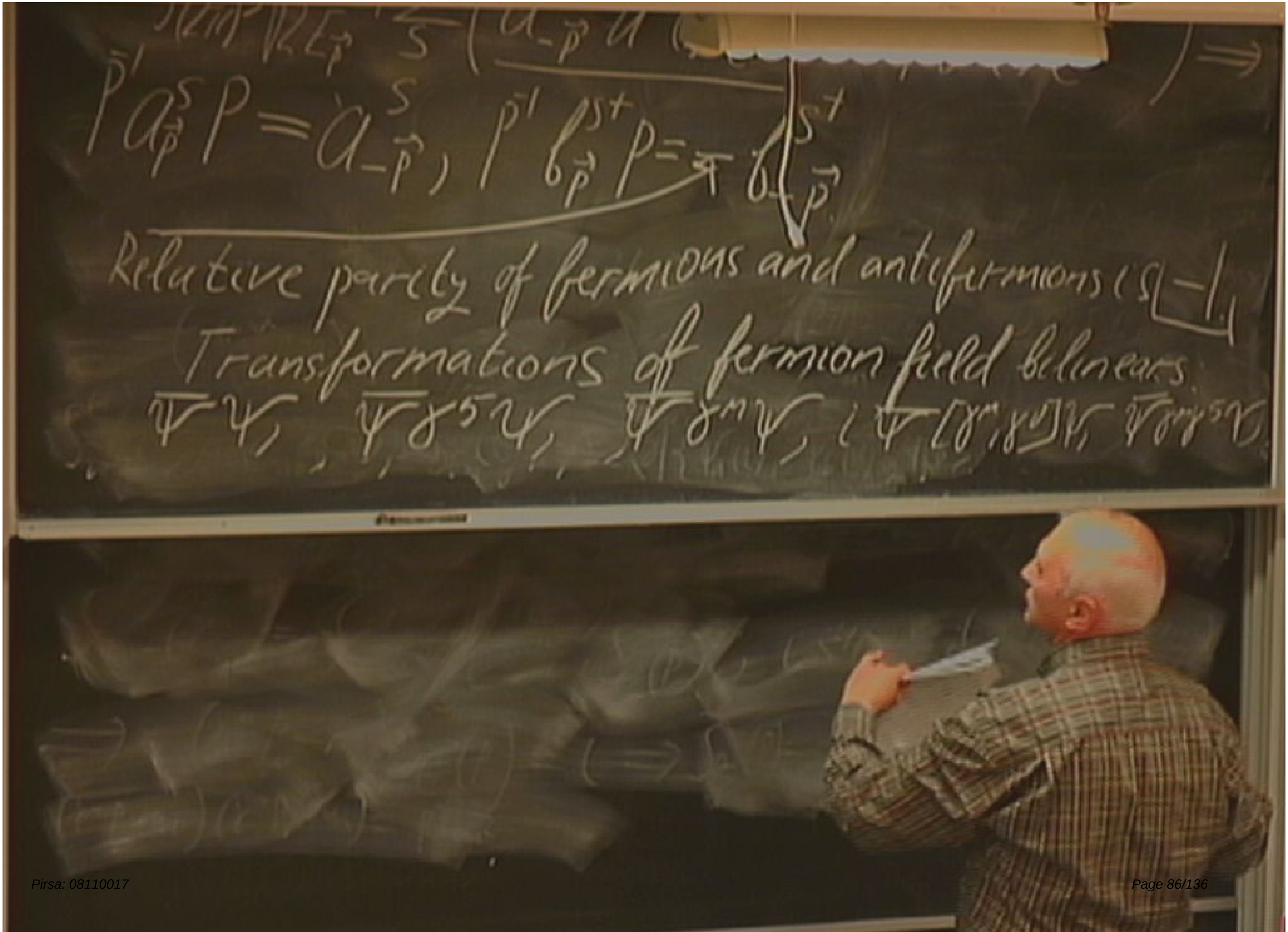
Relative parity of fermions and antifermions is -1.

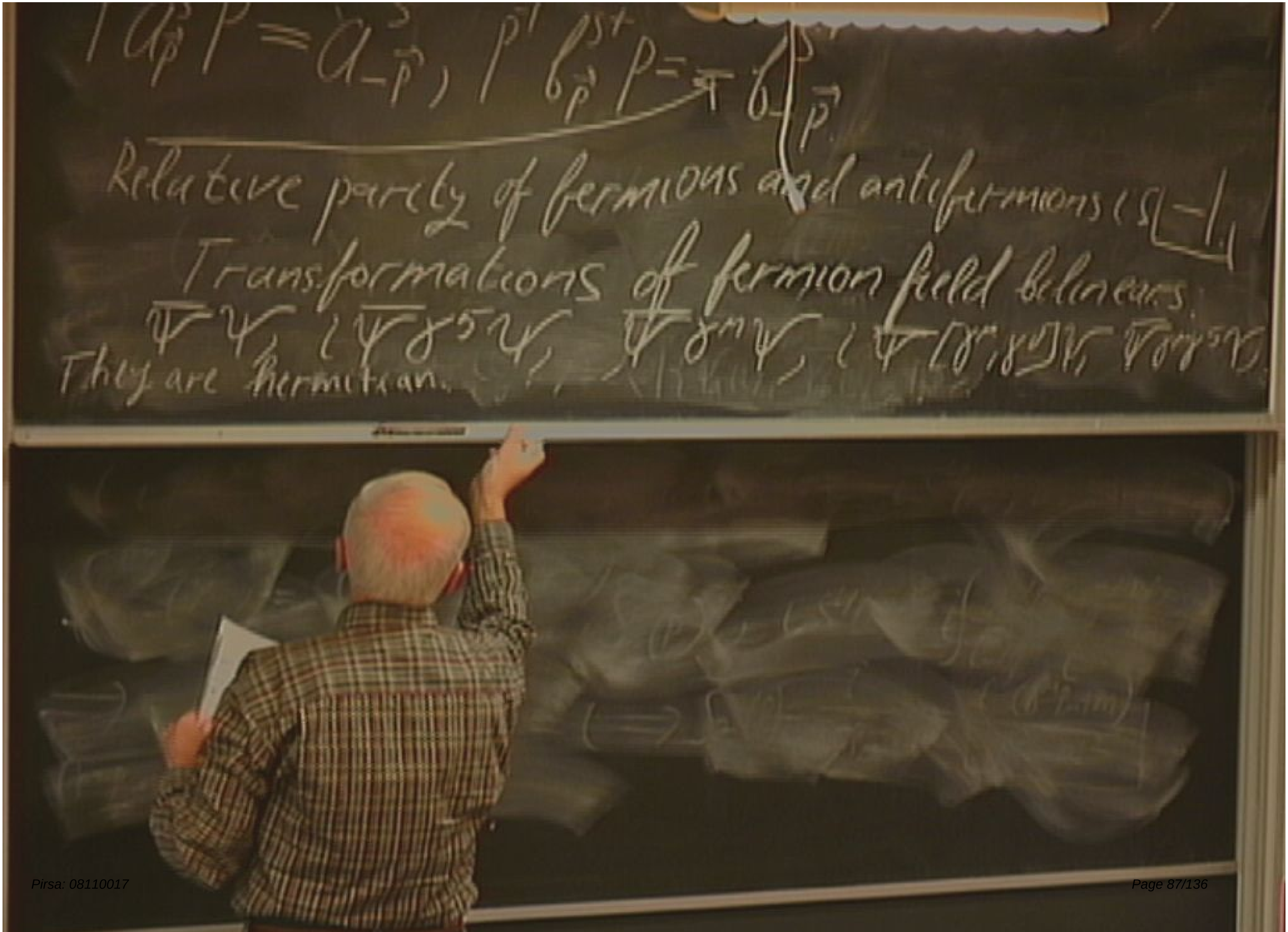

$\Lambda_p^2 = \pm 1 \Longrightarrow \eta_p^2 = \pm 1 \Rightarrow \eta_p = \pm 1$ or $\pm i$
$\psi'_x = \eta_p \gamma^0 \psi(t, -\vec{x}$
$\psi'(x) = P^{-1} \psi(x) P = \eta_p \gamma^0 \psi(\tilde{x})$; $\eta_p = \pm 1, \pm i$ (I) What are t
sformation of $a^s_{\vec{p}}$, $b^s_{\vec{p}}$ under P?
From (I).

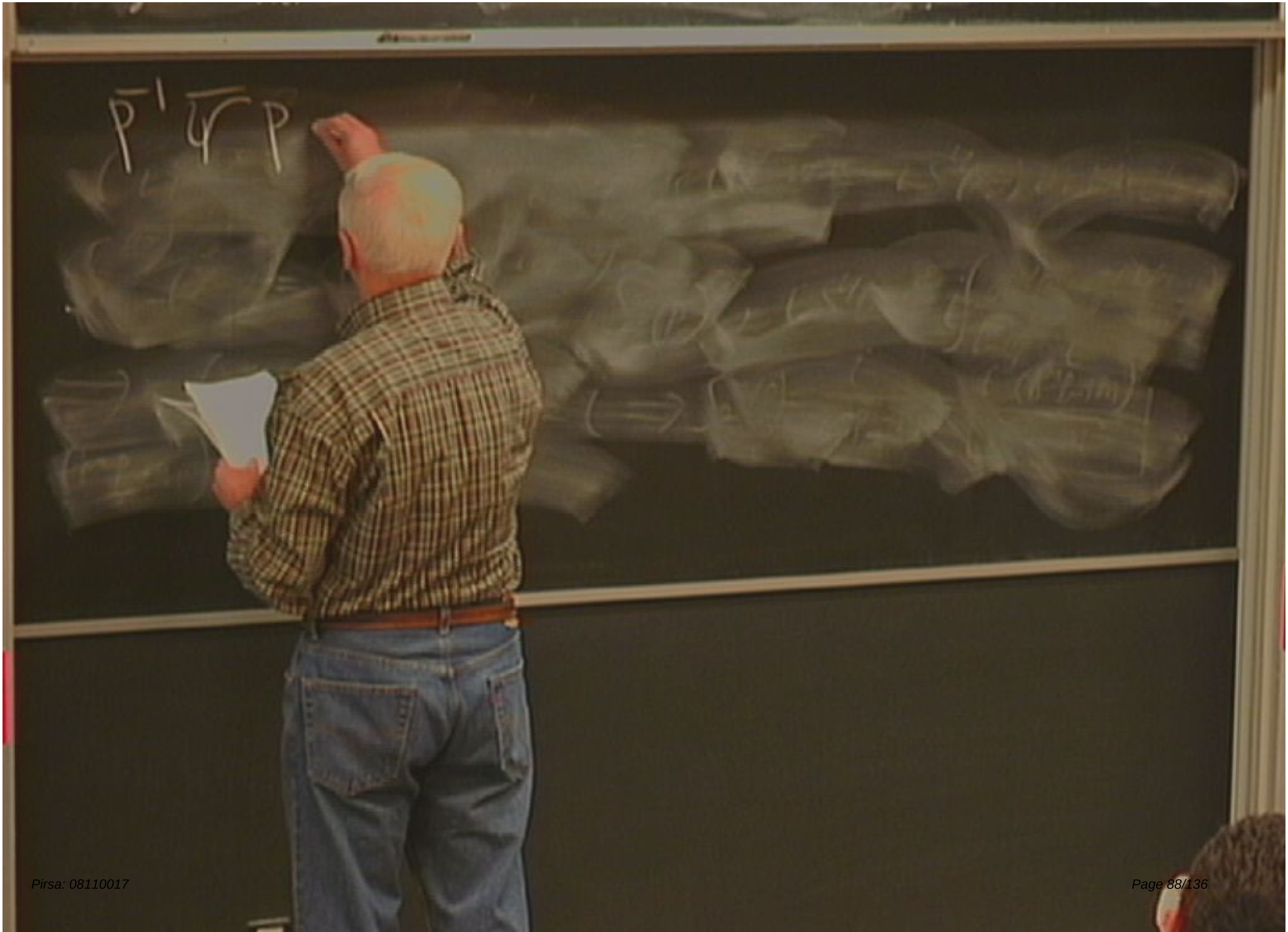

Relative parity of fermions and antifermions is -1.
Transformations of fermion field bilinears.

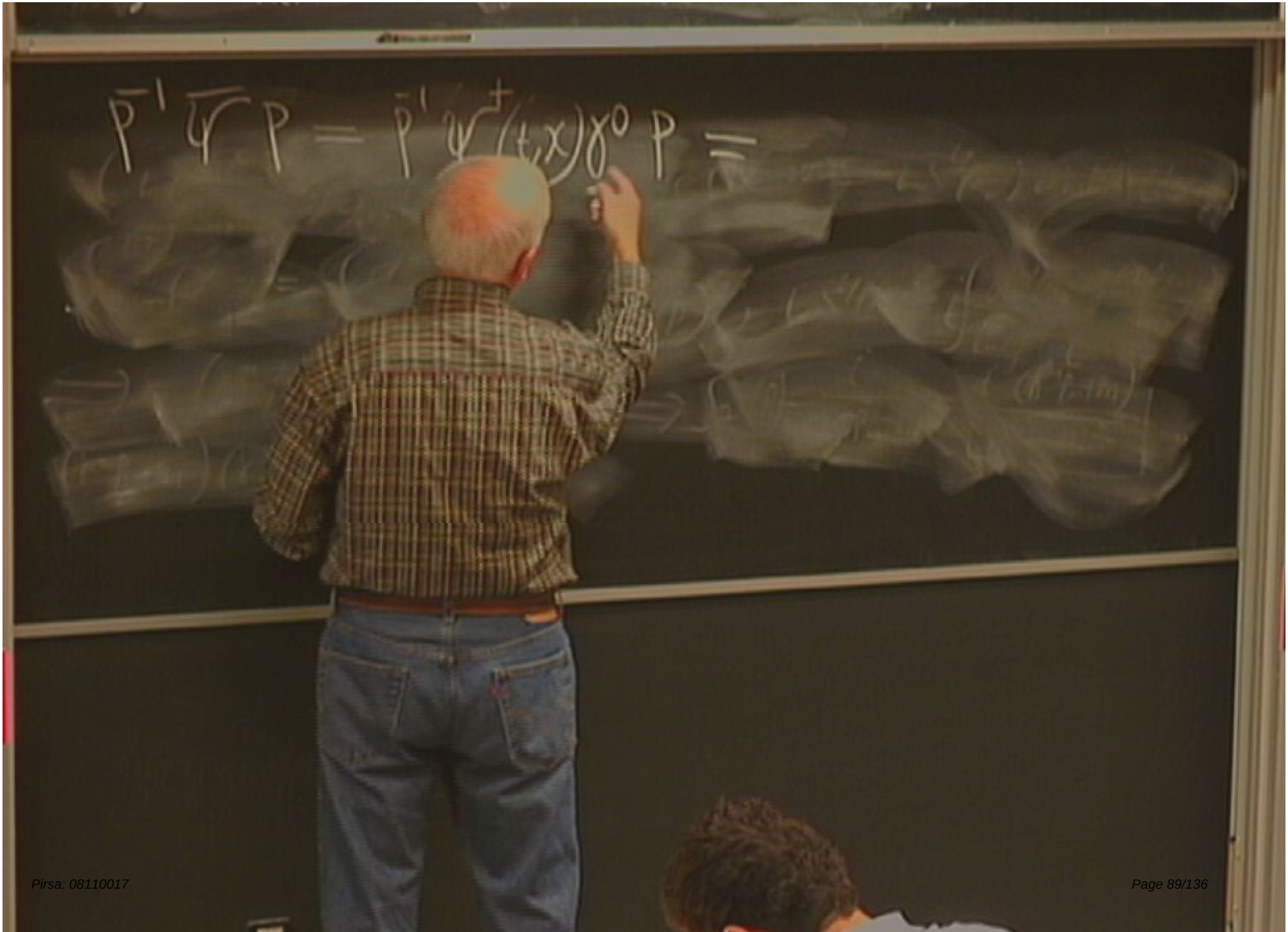


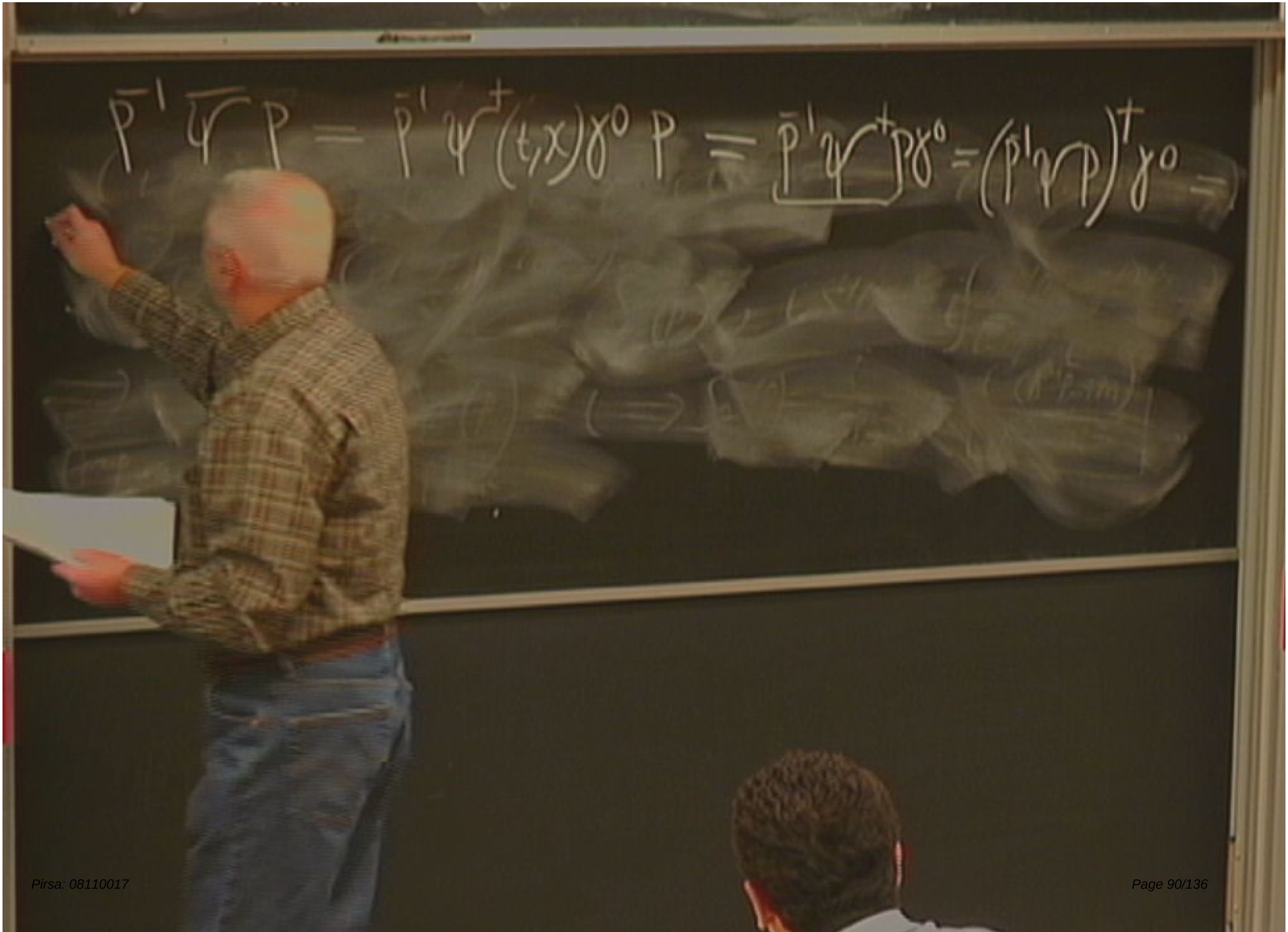
Relative parity of fermions and antifermions is −1.
Transformations of fermion field bilinears.

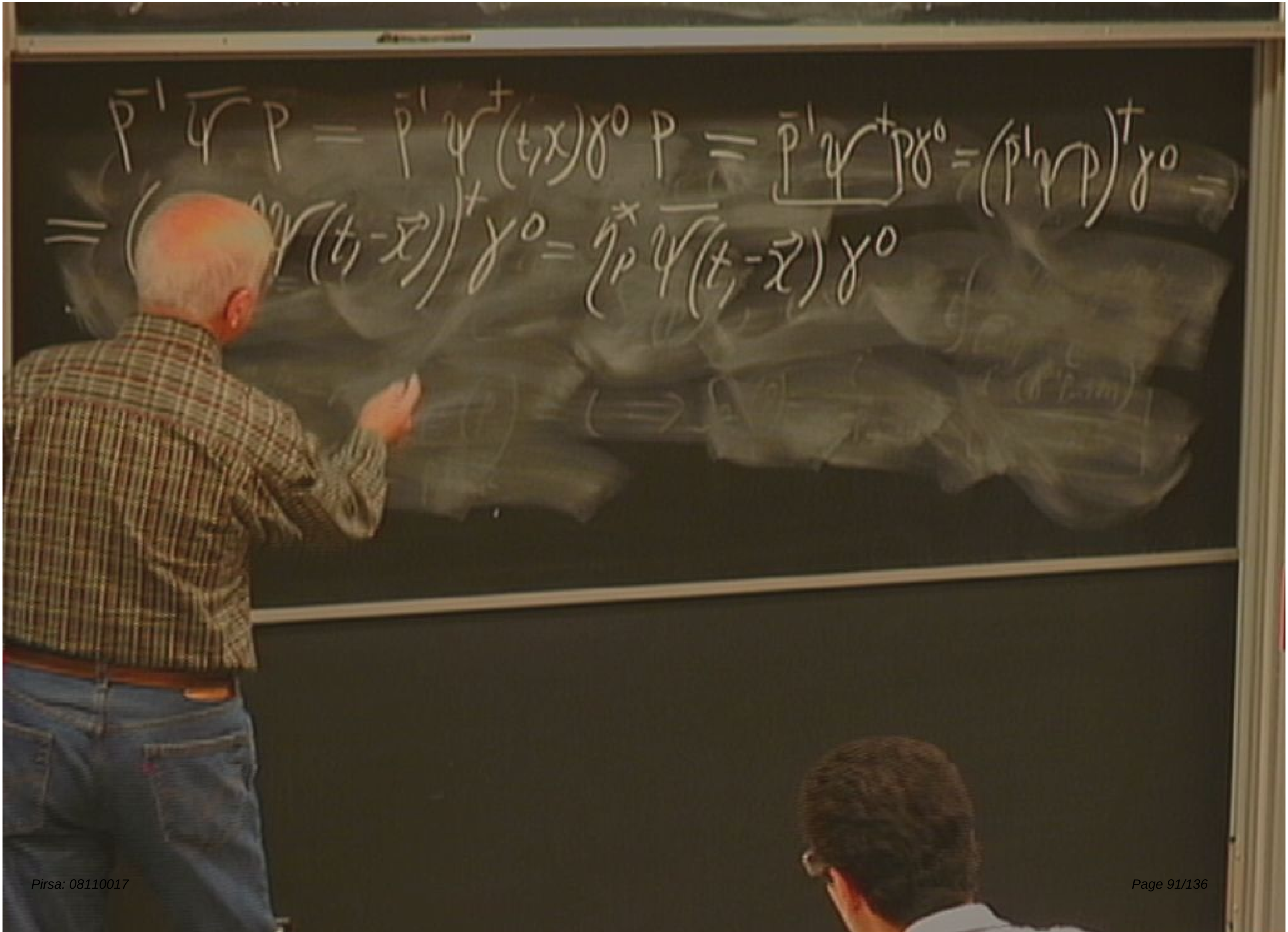

Relative parity of fermions and antifermions is −1
Transformations of fermion field bilinears

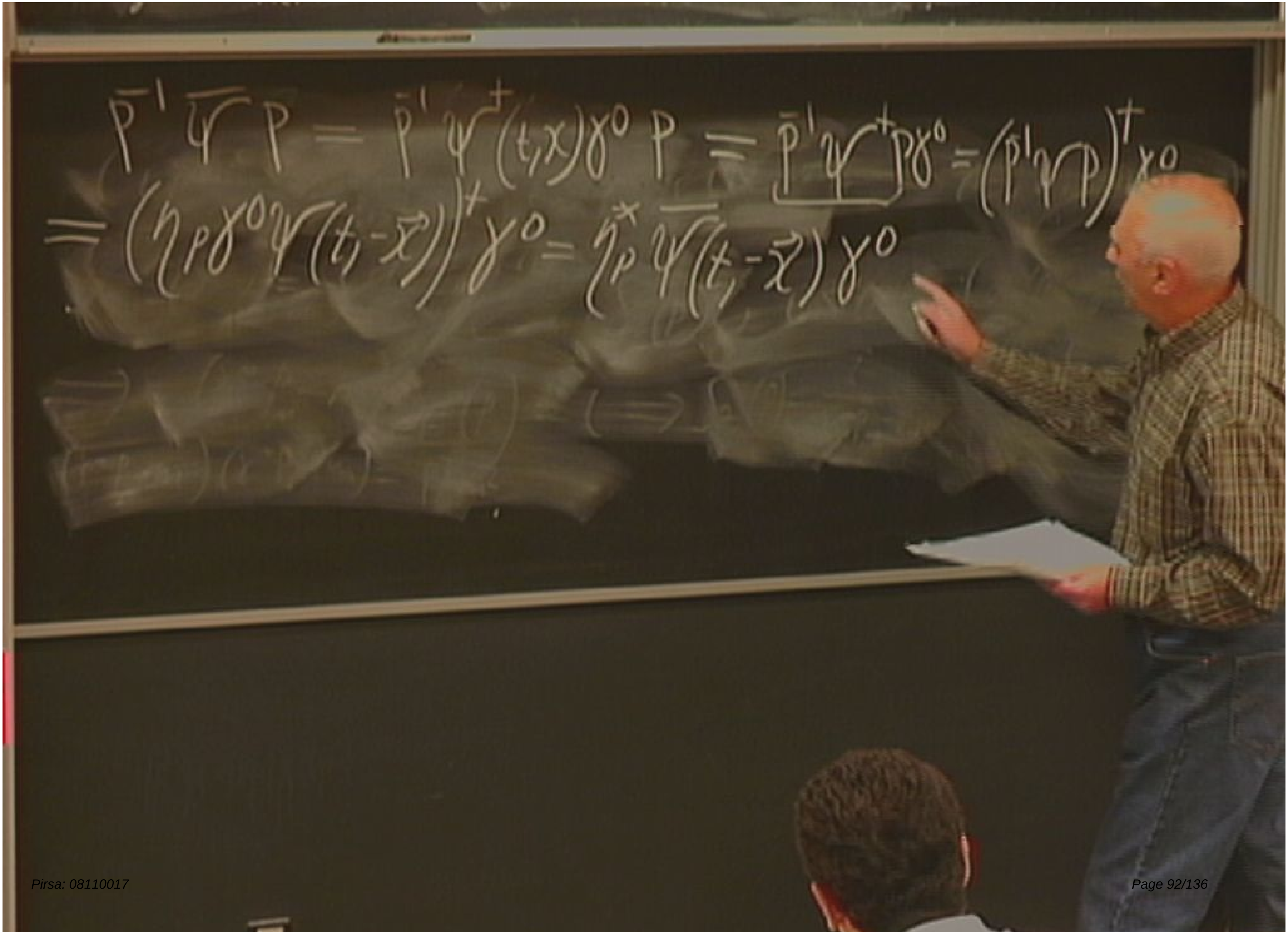

Relative parity of fermions and antifermions is -1.
Transformations of fermion field bilinears
They are hermitian.

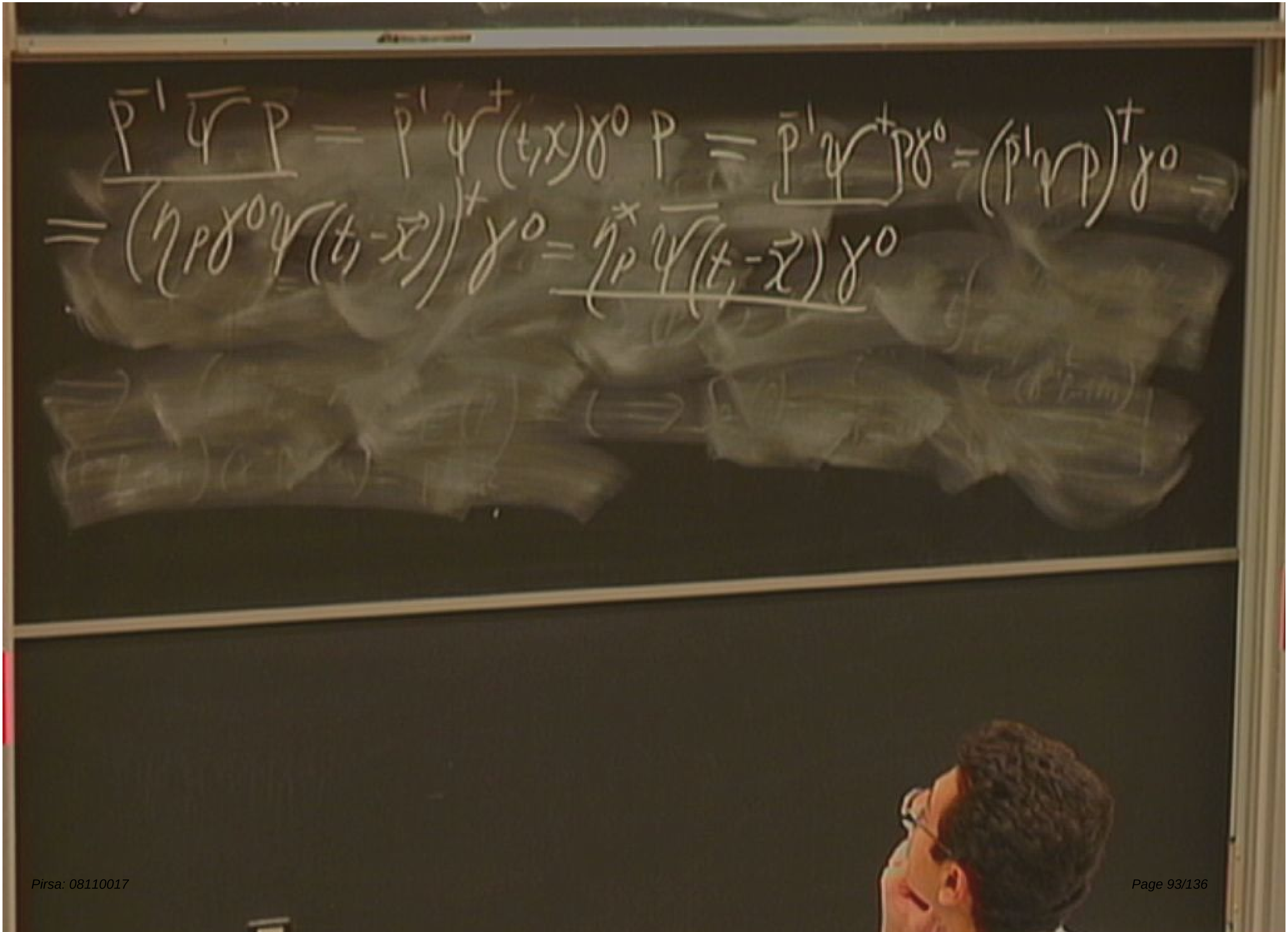


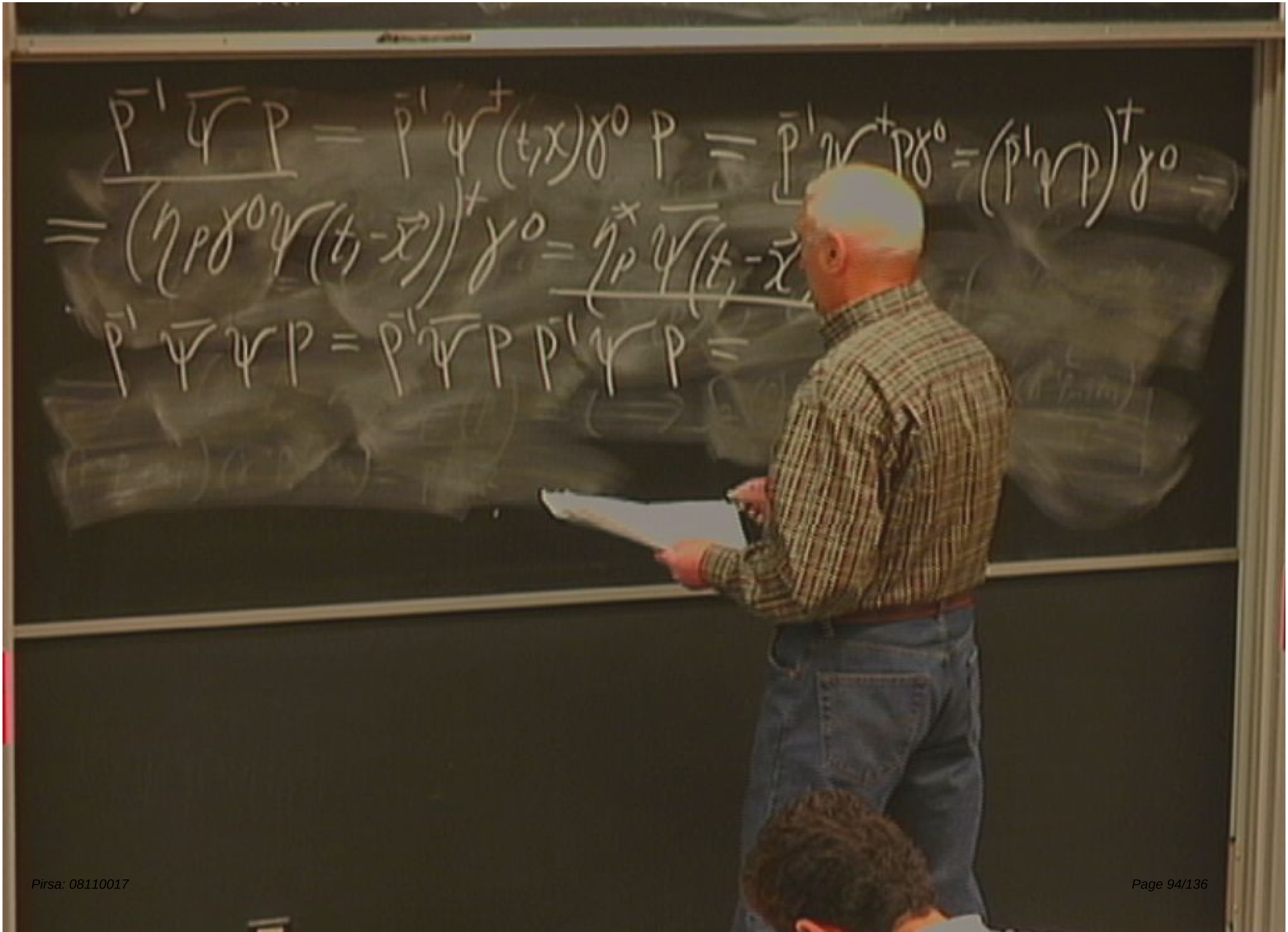

$\bar{p}' \bar{\psi} p = \bar{p}' \psi^\dagger(t,x)\gamma^0 p =$




P̄⁻¹ ψ̄ P = P̄⁻¹ ψ†(t,x) γ⁰ P = P̄⁻¹ψ†P γ⁰ = (P̄⁻¹ψ P)† γ⁰ =
= (ηγψ(t,−x⃗))† γ⁰ = η*P ψ̄(t,−x⃗) γ⁰


P⁻¹ ψ̄ P = P⁻¹ ψ†(t,x) γ⁰ P = P⁻¹ ψ† P γ⁰ = (P⁻¹ ψ P)† γ⁰
= (η_P γ⁰ ψ(t,-x))† γ⁰ = η_P* ψ̄(t,-x) γ⁰

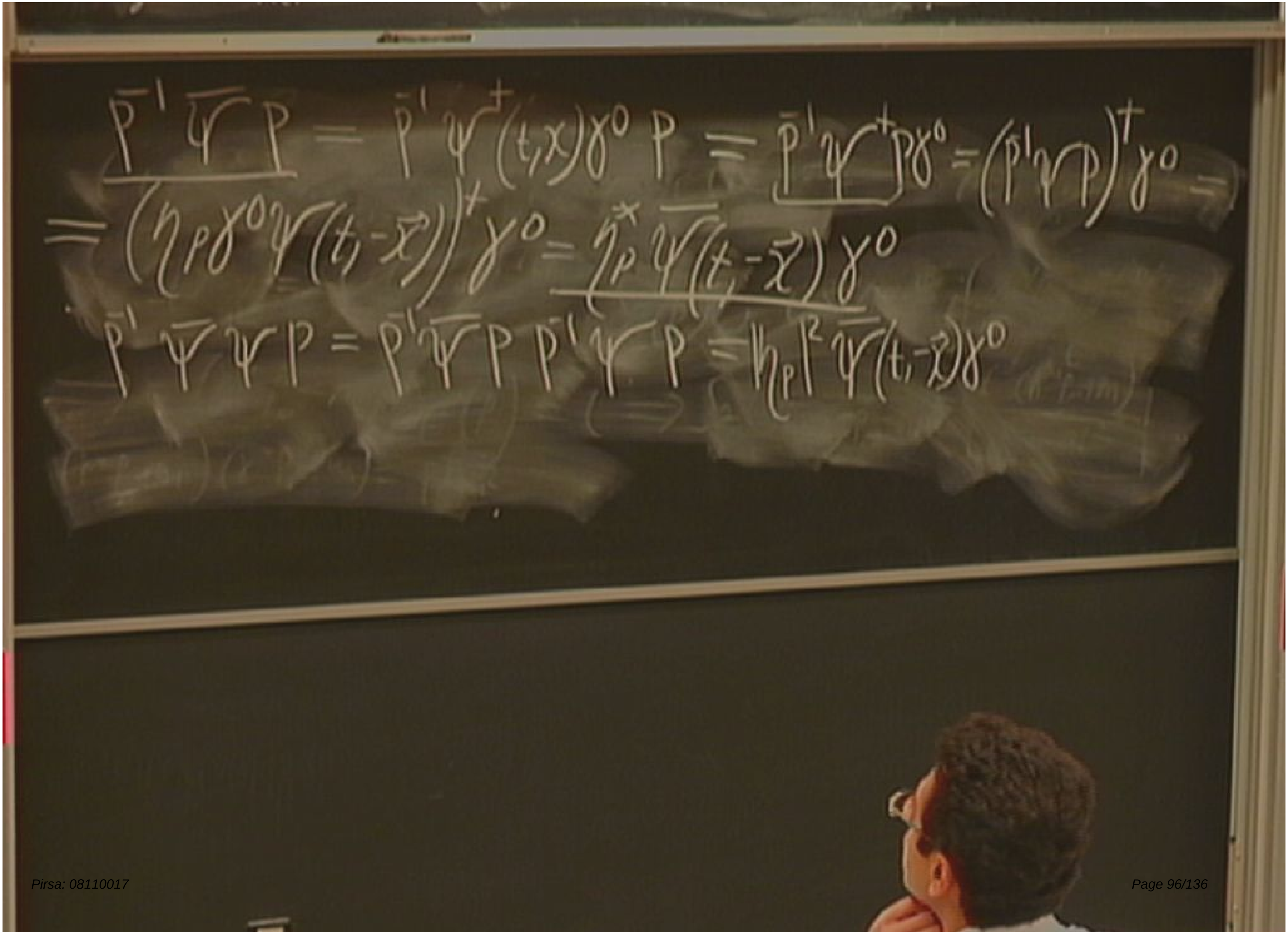

$P^{-1}\bar{\psi}P = P^{-1}\psi^{\dagger}(t,x)\gamma^0 P = P^{-1}\psi^{\dagger}P\gamma^0 = (P^{-1}\psi P)^{\dagger}\gamma^0 =$
$= (\eta_P\gamma^0\psi(t,-\vec{x}))^{\dagger}\gamma^0 = \eta_P^{*}\bar{\psi}(t,-\vec{x})\gamma^0$

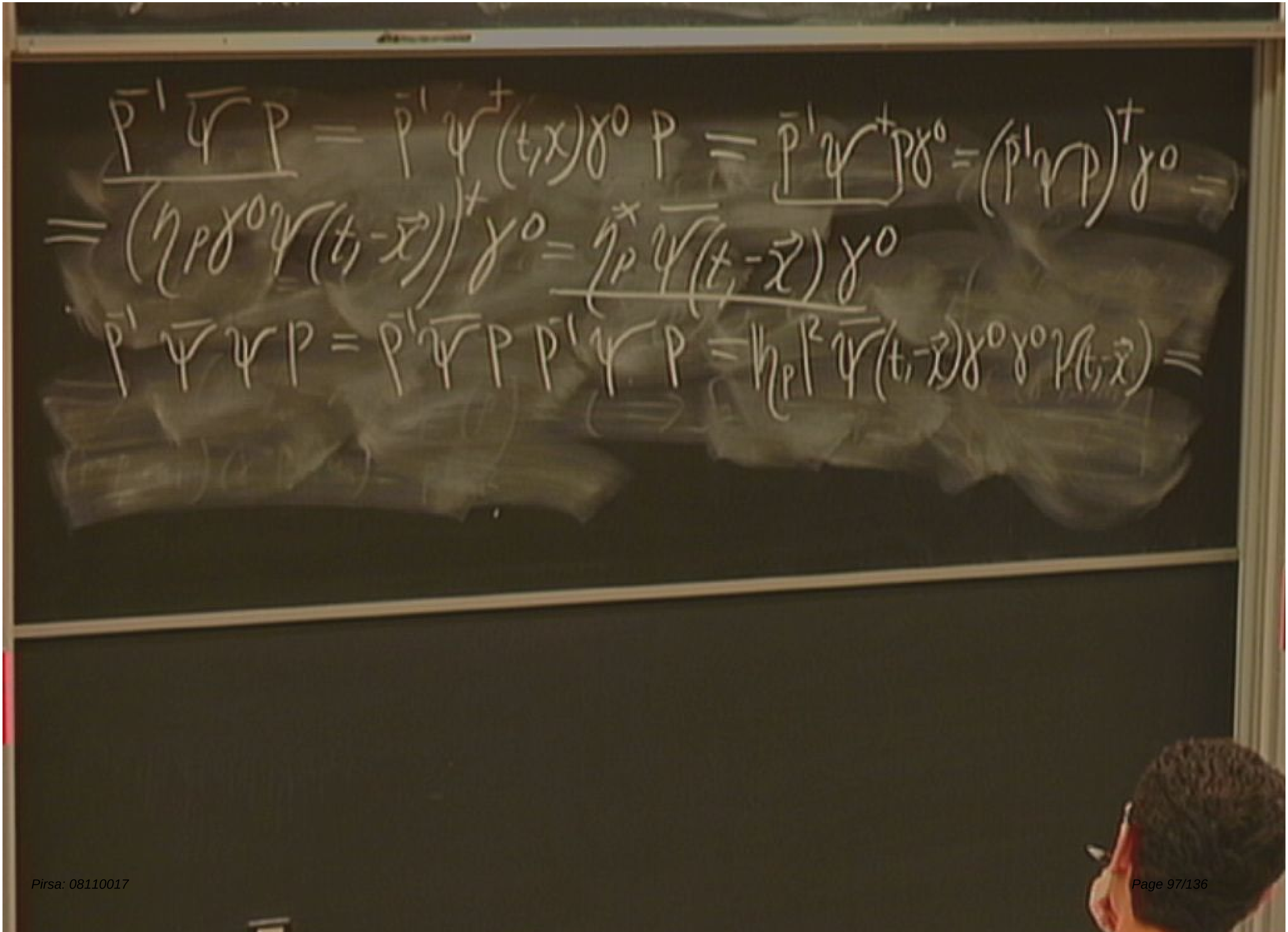


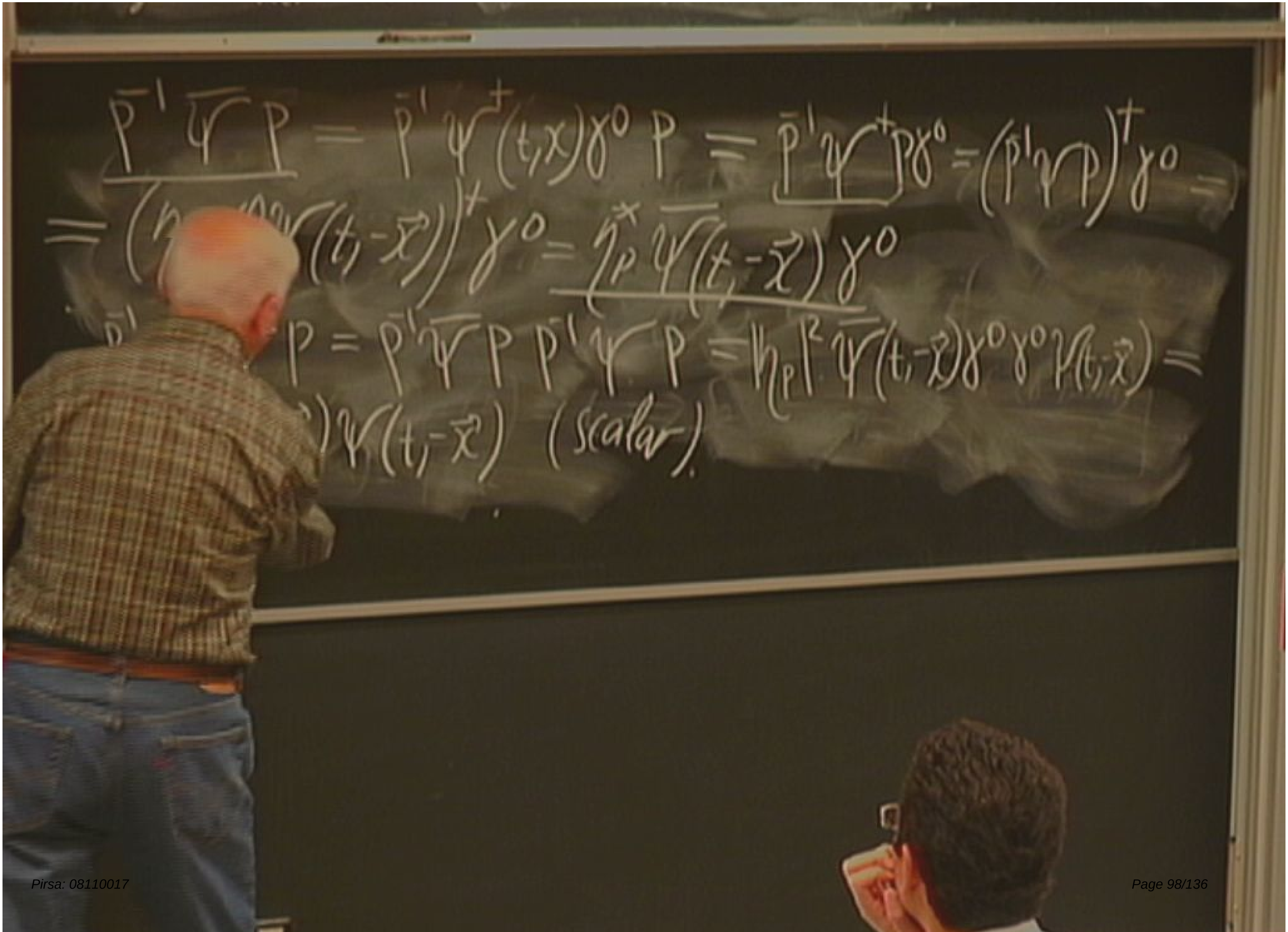
$$P^{-1}\bar{\psi}P = P^{-1}\psi^{\dagger}(t,x)\gamma^0 P = P^{-1}\psi^{\dagger}P\gamma^0 = (P^{-1}\psi P)^{\dagger}\gamma^0 =$$

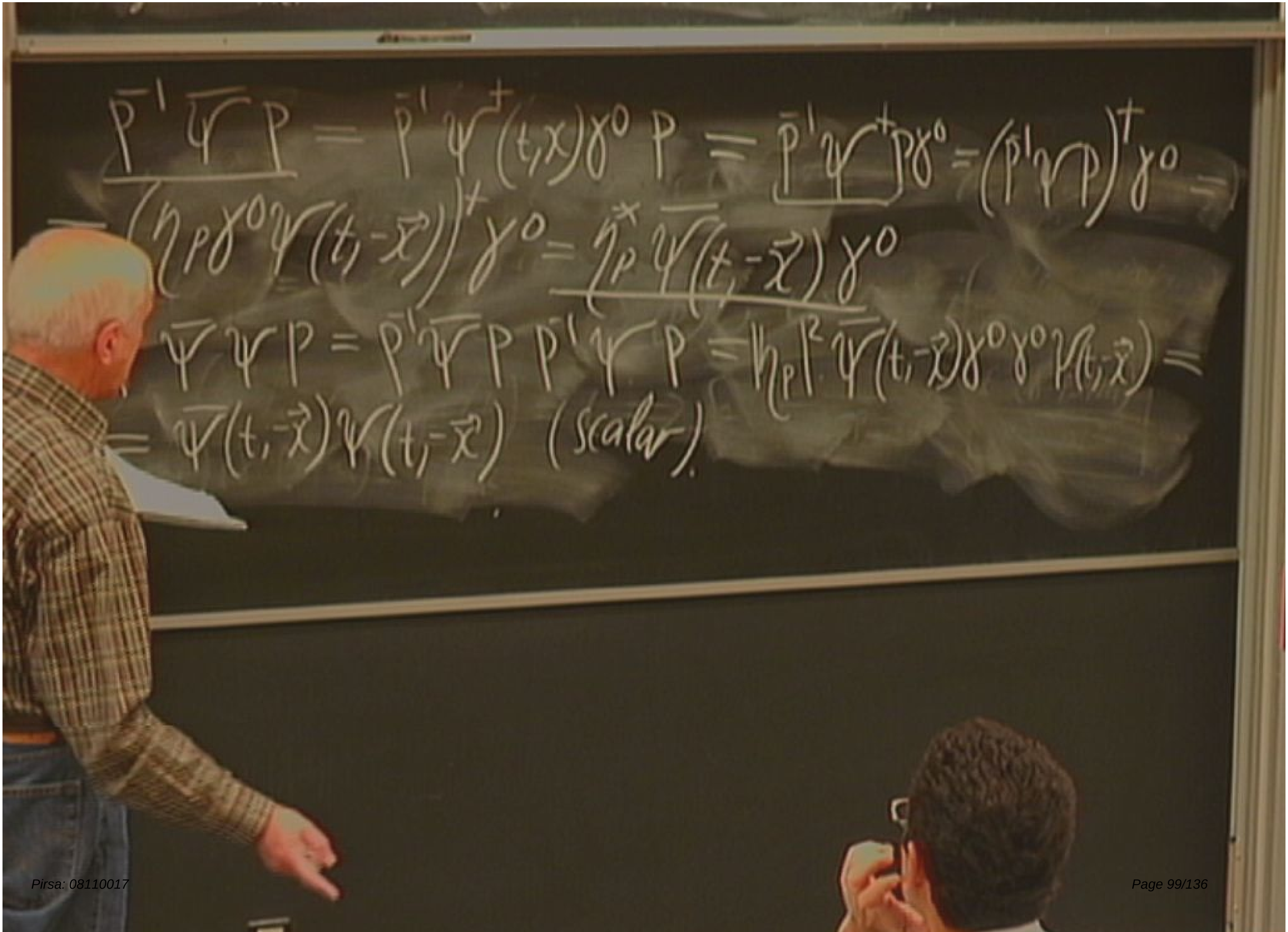
$$= (\eta_P\gamma^0\psi(t,-\vec{x}))^{\dagger}\gamma^0 = \eta_P^{*}\bar{\psi}(t,-\vec{x})\gamma^0$$

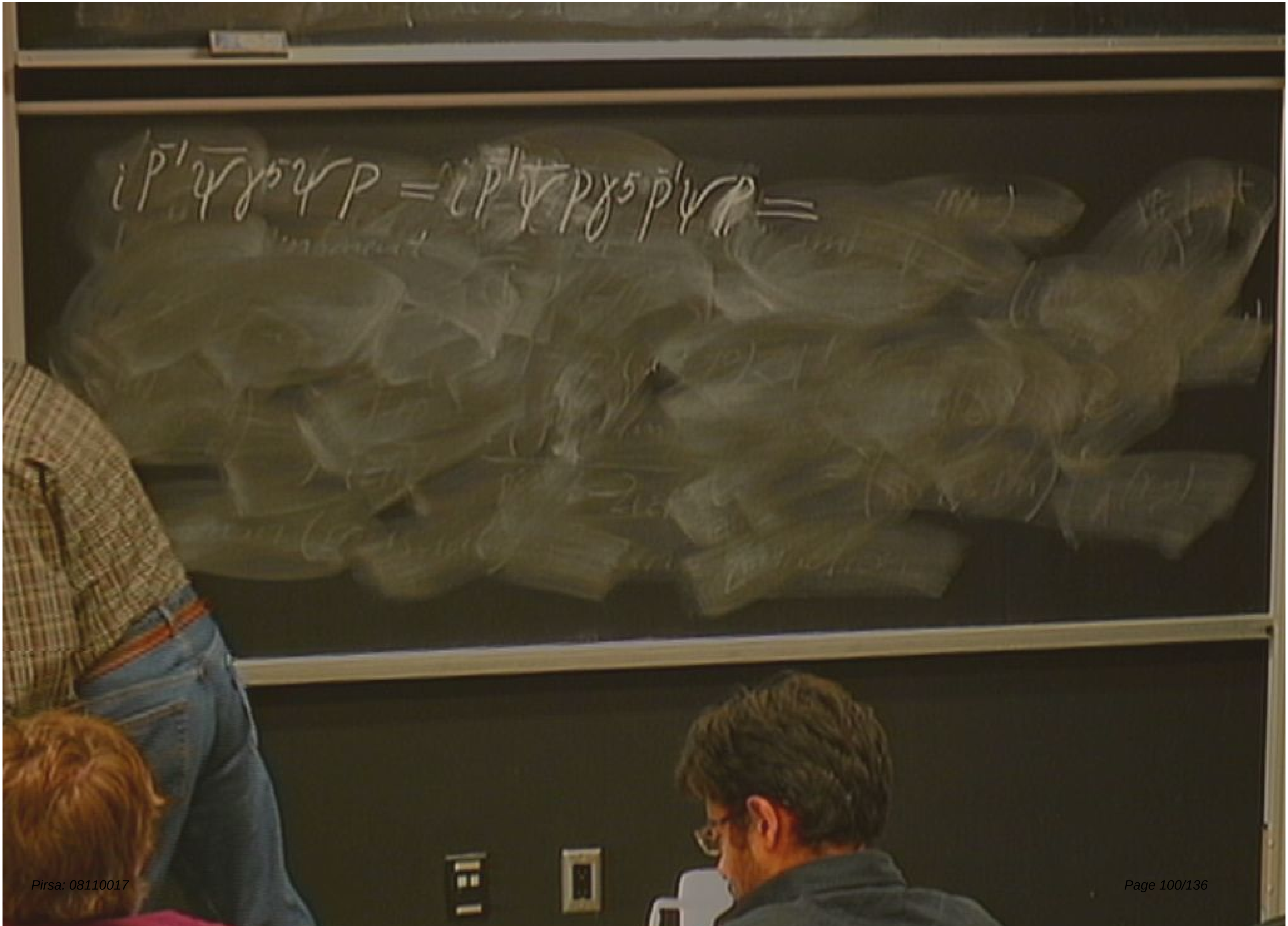
$$P^{-1}\bar{\psi}\psi P = P^{-1}\bar{\psi}P\,P^{-1}\psi P = |\eta_P|^2$$


$\bar{P}^{-1}\bar{\psi}P = \bar{P}^{-1}\psi^{\dagger}(t,x)\gamma^0 P = \bar{P}^{-1}\psi^{\dagger}P\gamma^0 = (\bar{P}^{-1}\psi P)^{\dagger}\gamma^0 =$
$= (\eta_P\gamma^0\psi(t,-\vec{x}))^{\dagger}\gamma^0 = \eta_P^{*}\bar{\psi}(t,-\vec{x})\gamma^0$
$\bar{P}^{-1}\bar{\psi}\psi P = \bar{P}^{-1}\bar{\psi}P\,\bar{P}^{-1}\psi P = |\eta_P|^2\bar{\psi}(t,-\vec{x})\gamma^0$

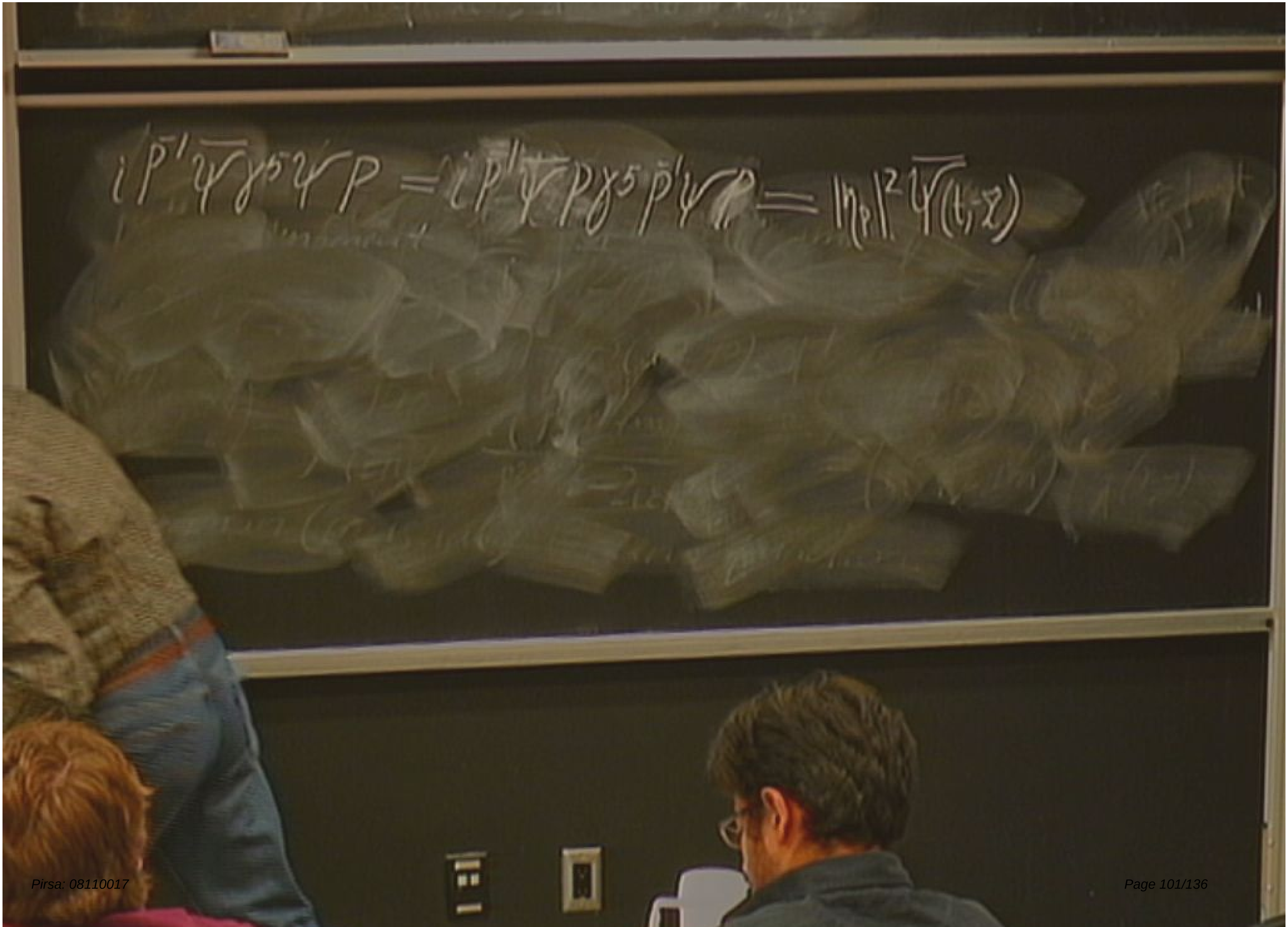

$P^{-1}\bar{\psi}P = P^{-1}\psi^{\dagger}(t,x)\gamma^0 P = P^{-1}\psi^{\dagger}P\gamma^0 = (P^{-1}\psi P)^{\dagger}\gamma^0 =$
$= (\eta_P\gamma^0\psi(t,-\vec{x}))^{\dagger}\gamma^0 = \eta_P^{*}\bar{\psi}(t,-\vec{x})\gamma^0$
$P^{-1}\bar{\psi}\psi P = P^{-1}\bar{\psi}P\,P^{-1}\psi P = |\eta_P|^2\bar{\psi}(t,-\vec{x})\gamma^0\gamma^0\psi(t,-\vec{x}) =$

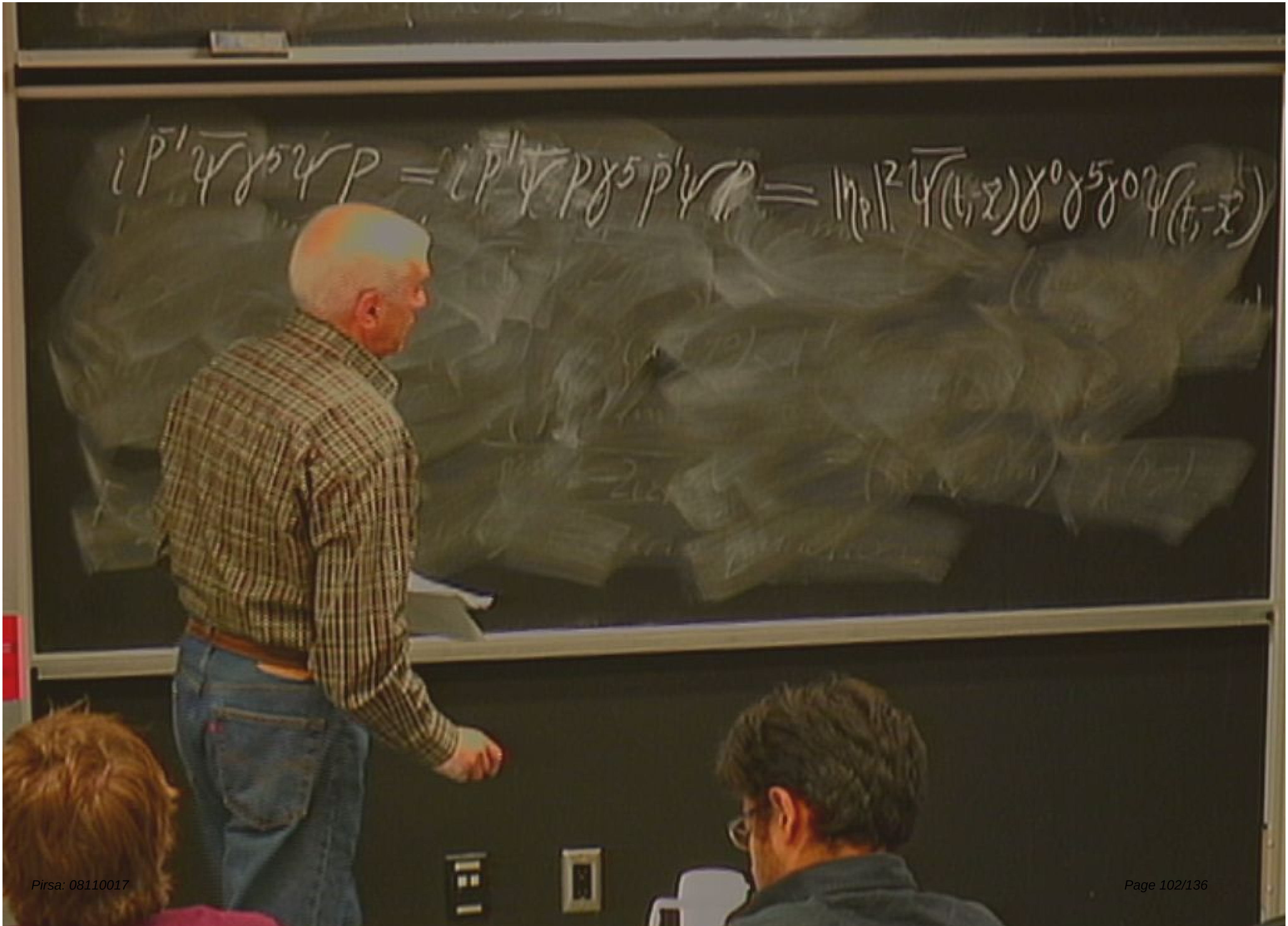

(scalar)

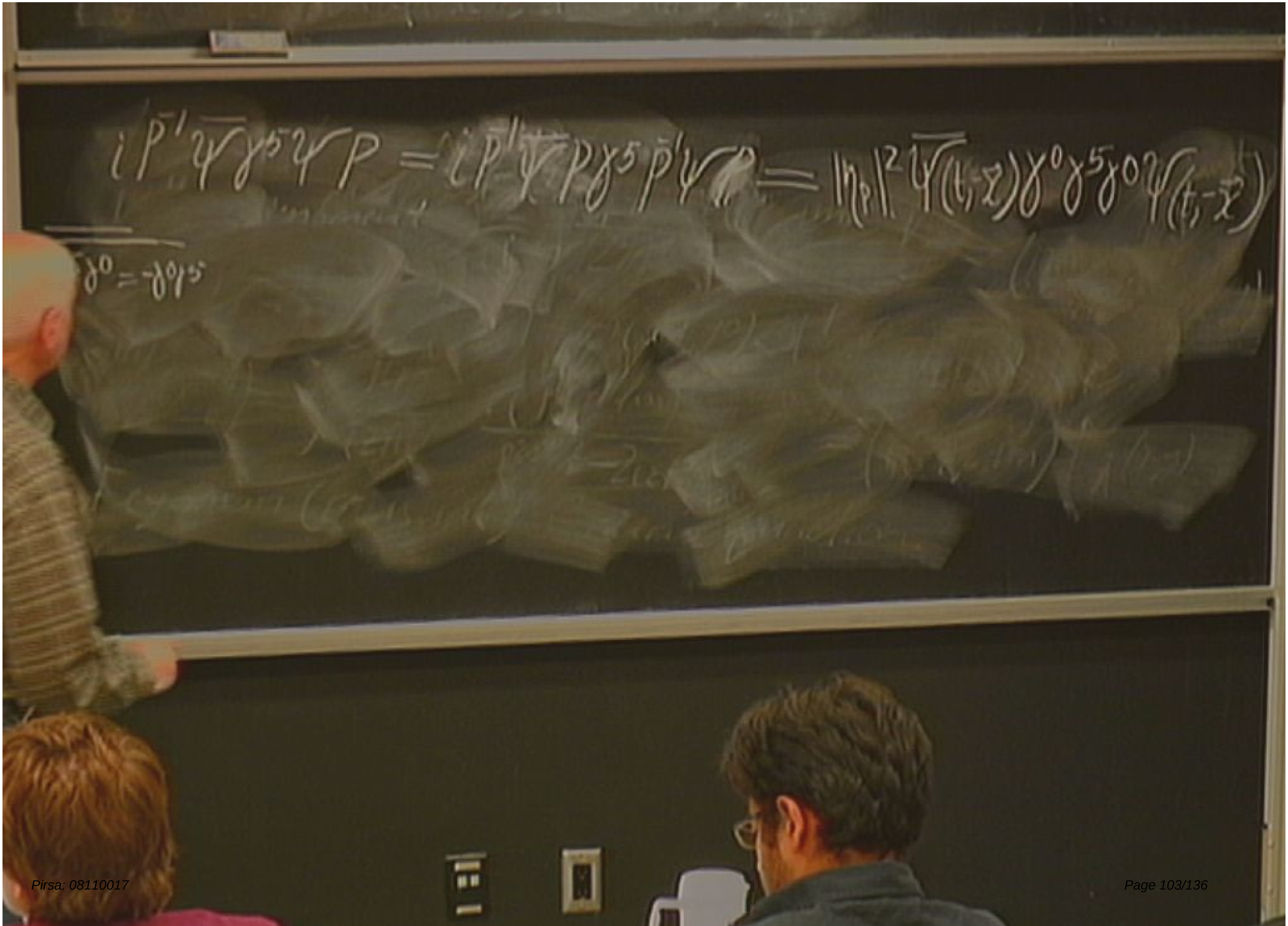

(scalar)

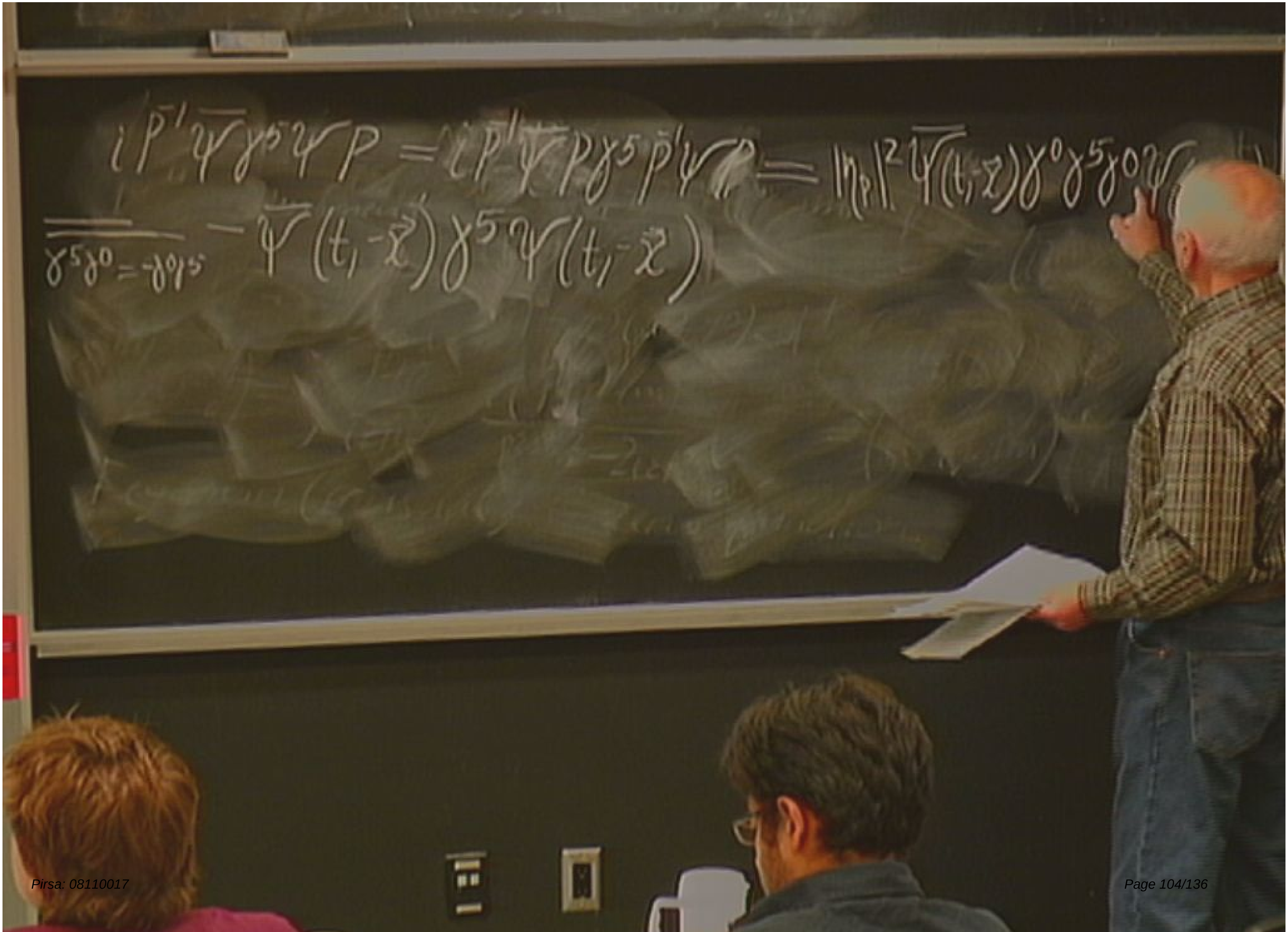


$i\bar{p}'\bar{\psi}\gamma^5\psi p = i\bar{p}'\bar{\psi} p \gamma^5 \bar{p}'\psi p =$

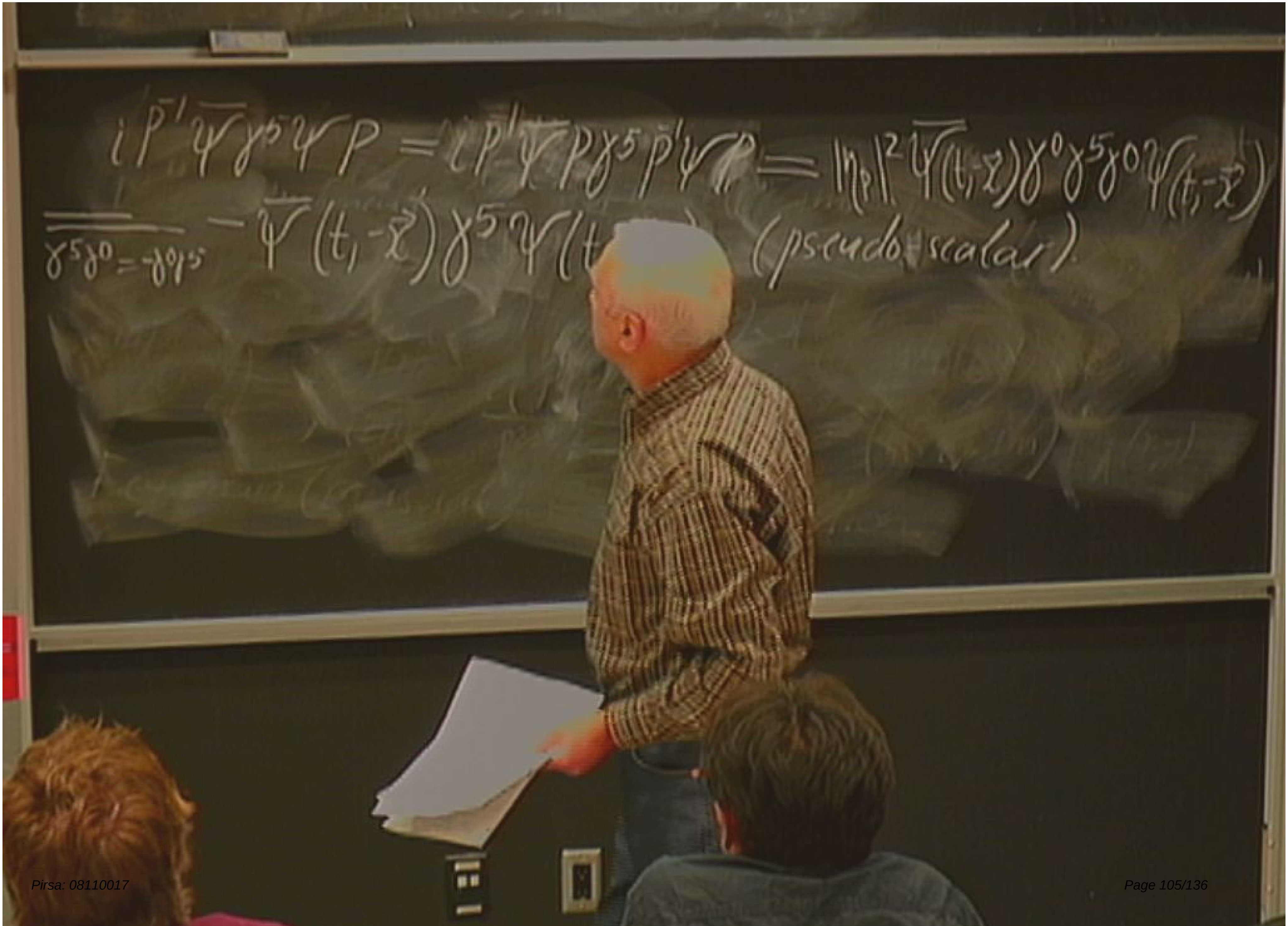

$i\bar{p}'\bar{\psi}\gamma^5\psi p = i\bar{p}'\bar{\psi}p\gamma^5\bar{p}'\psi p = |n_p|^2\bar{\psi}(t,x)$

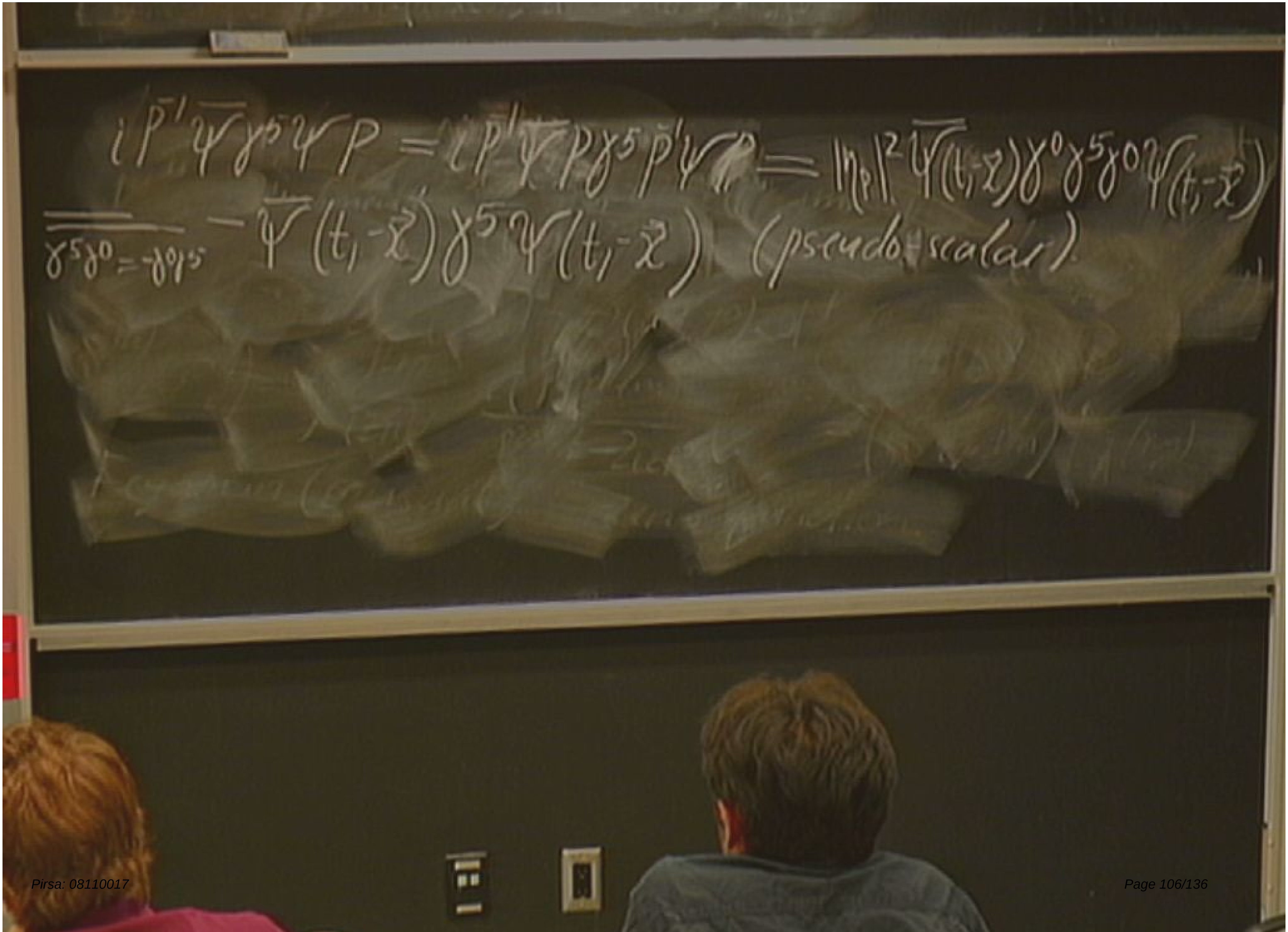

$i\bar{P}'\bar{\psi}\gamma^5\psi P = i\bar{P}'\bar{\psi}P\gamma^5\bar{P}'\psi P = |\eta_P|^2\bar{\psi}(t,\vec{x})\gamma^0\gamma^5\gamma^0\psi(t,-\vec{x})$

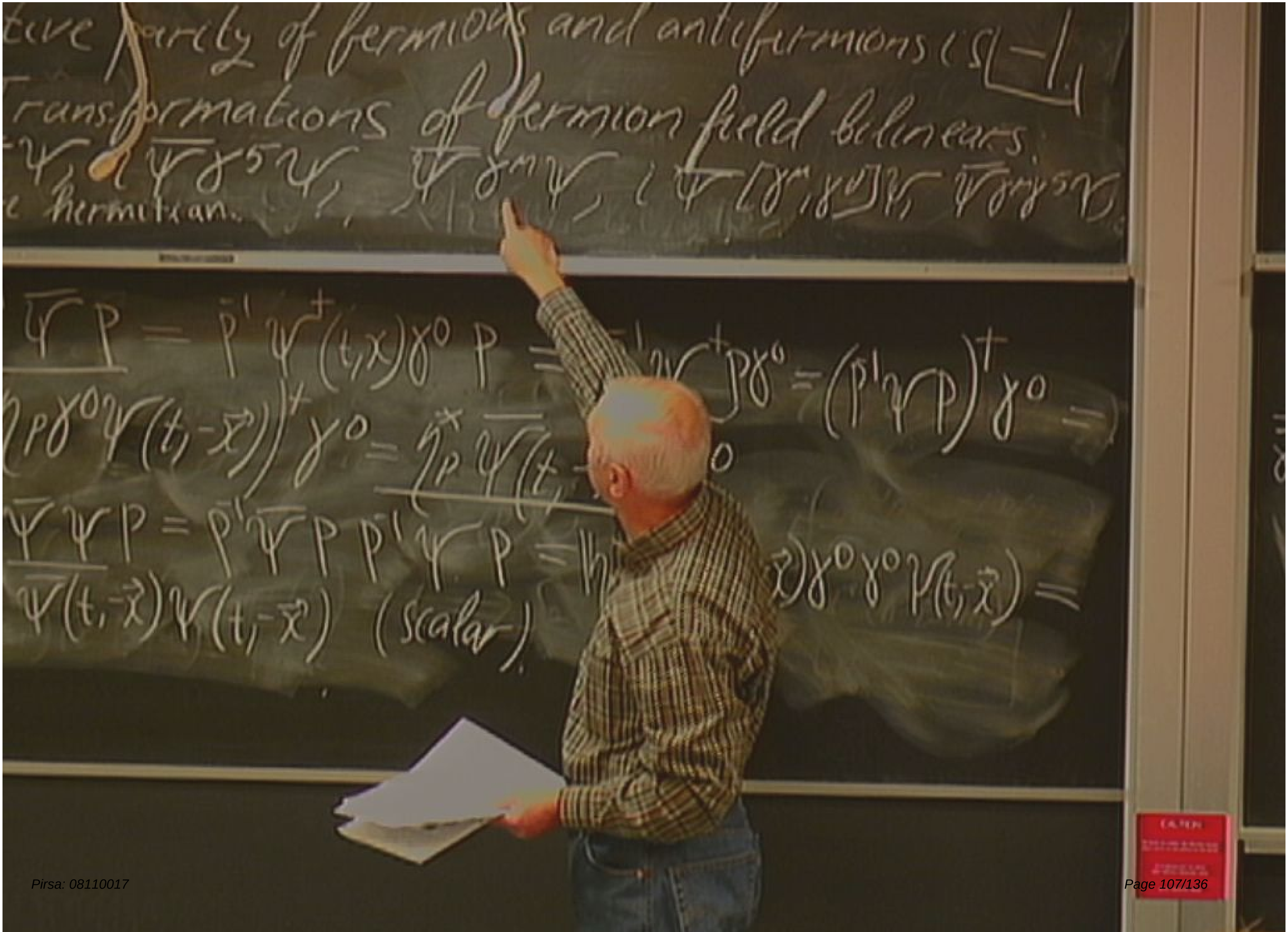

iP⁻¹ ψ̄ γ⁵ ψ P = iP⁻¹ ψ̄ P γ⁵ P⁻¹ ψ P = |η_P|² ψ̄(t,-x) γ⁰ γ⁵ γ⁰ ψ(t,-x)
γ⁵γ⁰ = -γ⁰γ⁵

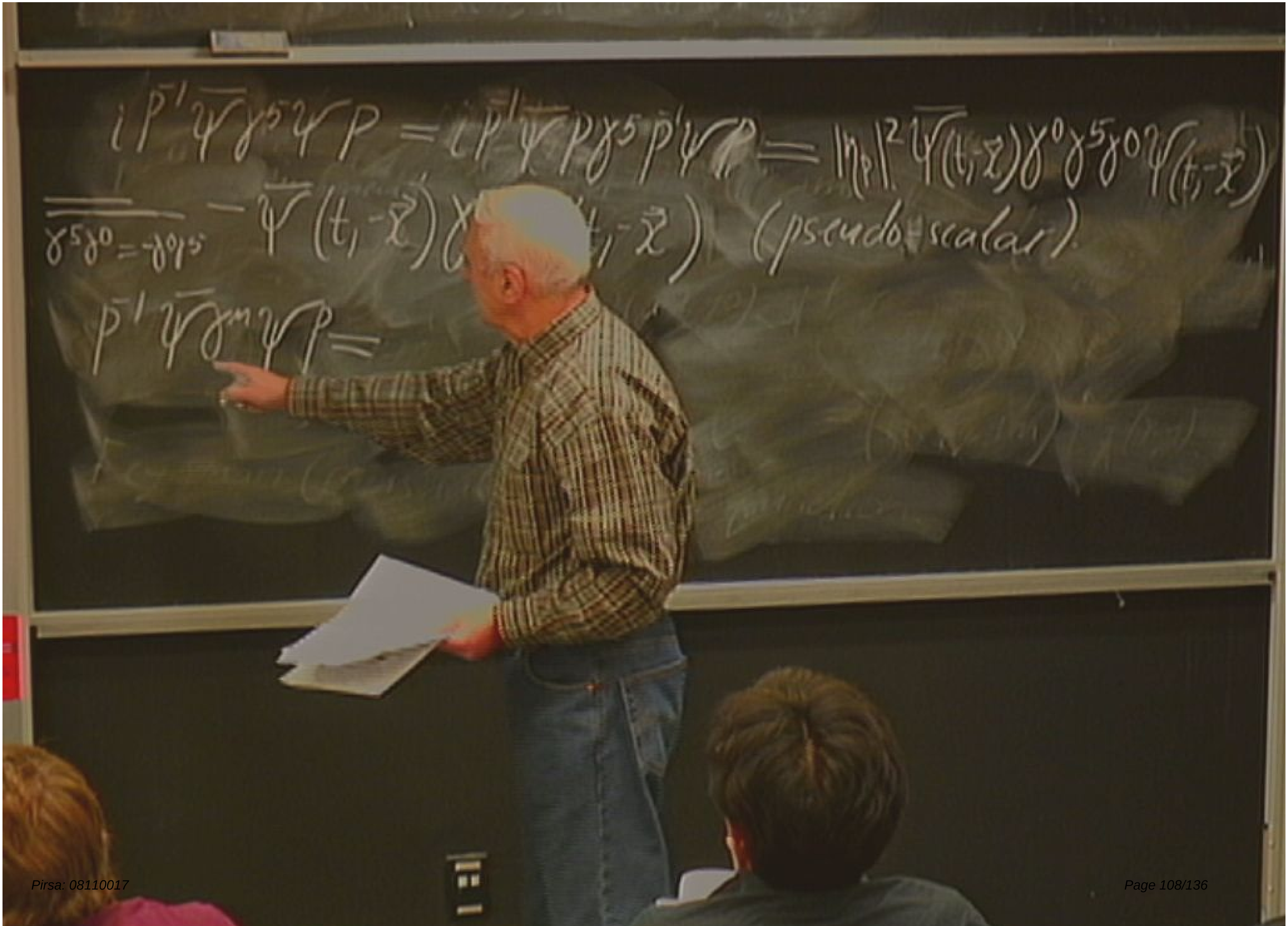


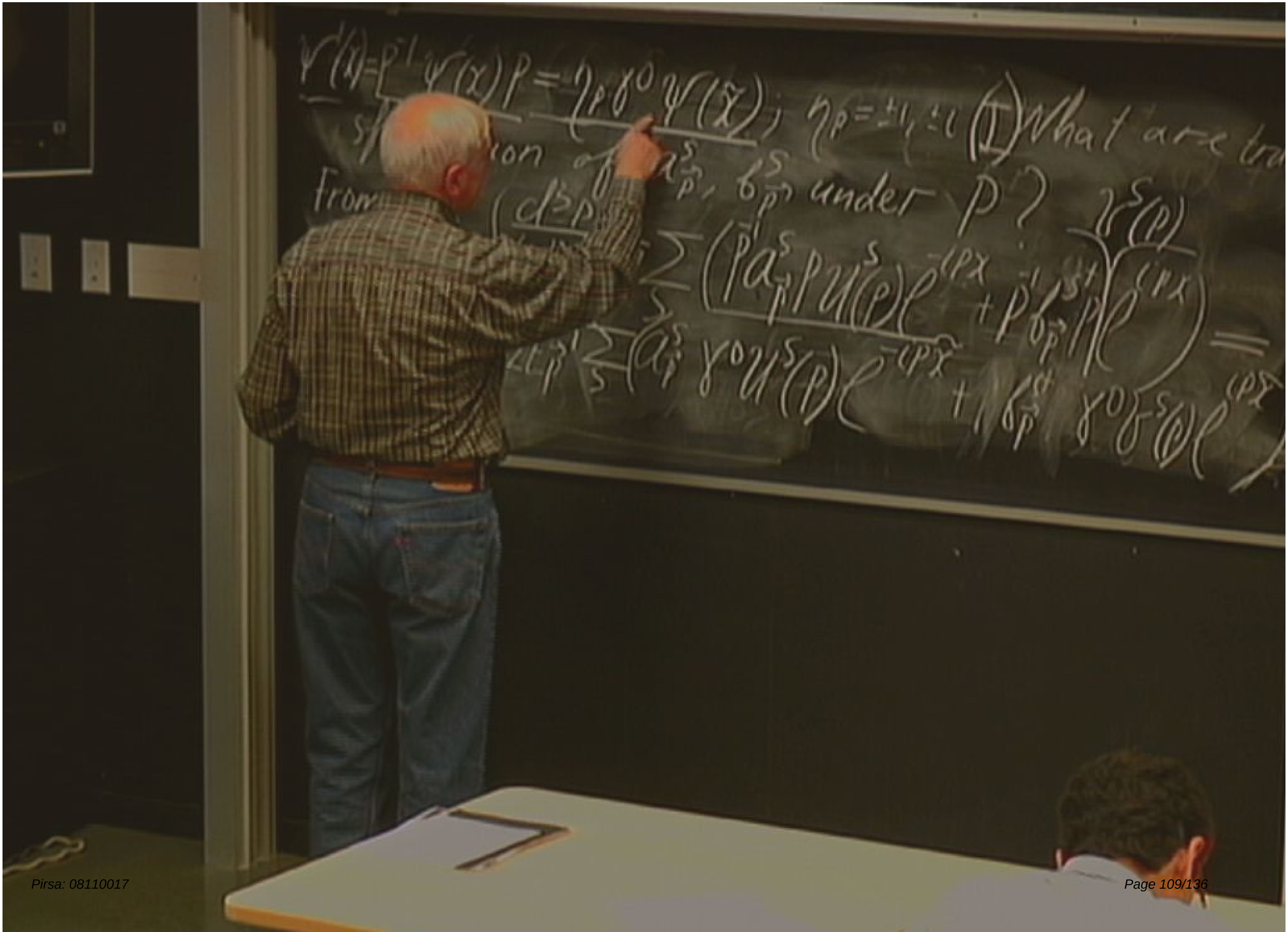
$iP^{-1}\bar{\psi}\gamma^5\psi P = iP^{-1}\bar{\psi}P\gamma^5P^{-1}\psi P = |\eta_P|^2\bar{\psi}(t,-\vec{x})\gamma^0\gamma^5\gamma^0\psi$
$\gamma^5\gamma^0 = -\gamma^0\gamma^5$
$-\bar{\psi}(t,-\vec{x})\gamma^5\psi(t,-\vec{x})$

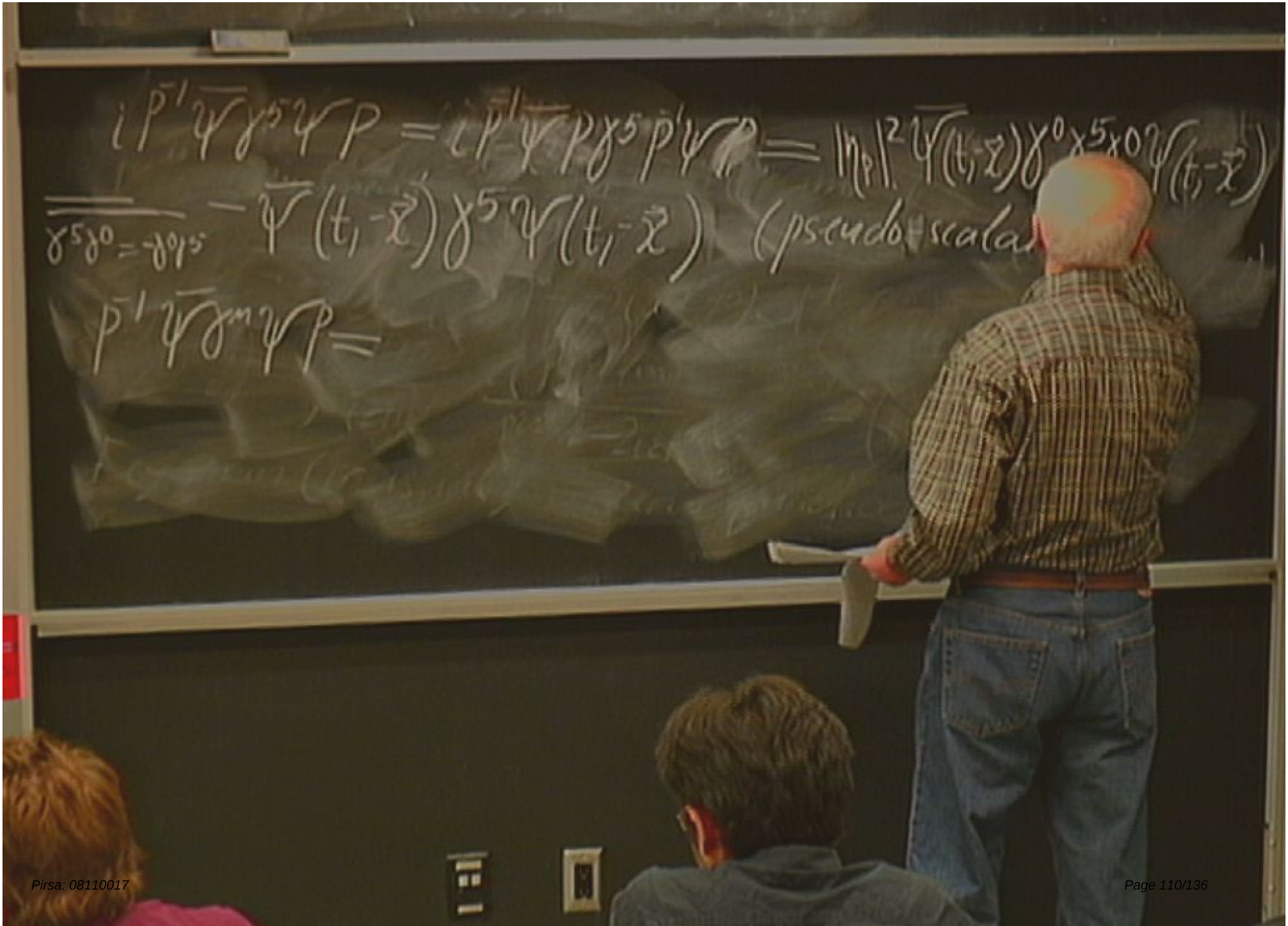

(pseudo scalar).

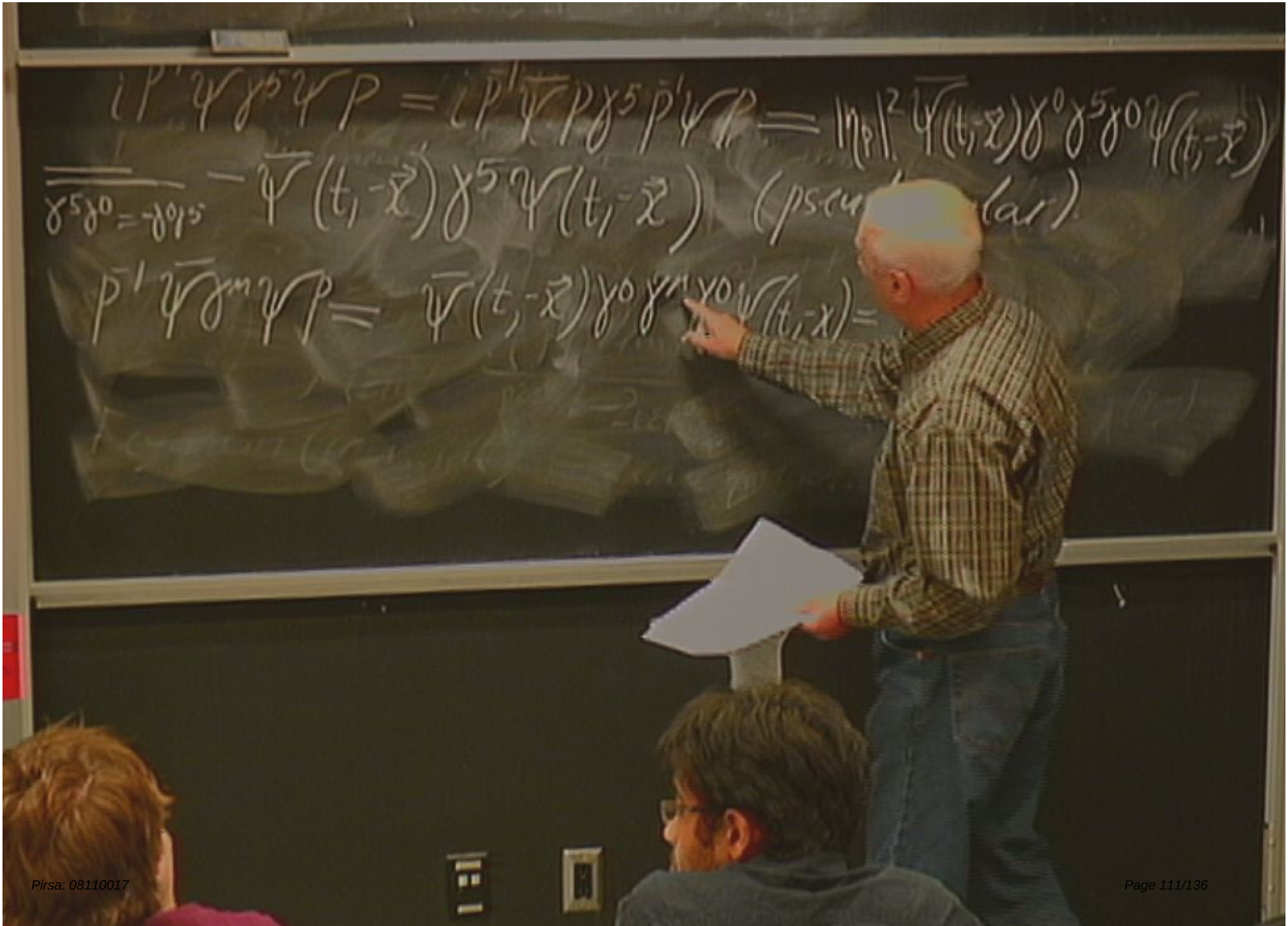

$i P^{-1} \bar{\psi} \gamma^5 \psi P = i P^{-1} \bar{\psi} P \gamma^5 P^{-1} \psi P = |\eta_P|^2 \bar{\psi}(t,-\vec{x}) \gamma^0 \gamma^5 \gamma^0 \psi(t,-\vec{x})$
$\gamma^5 \gamma^0 = -\gamma^0 \gamma^5$
$- \bar{\psi}(t,-\vec{x}) \gamma^5 \psi(t,-\vec{x})$ (pseudo-scalar).

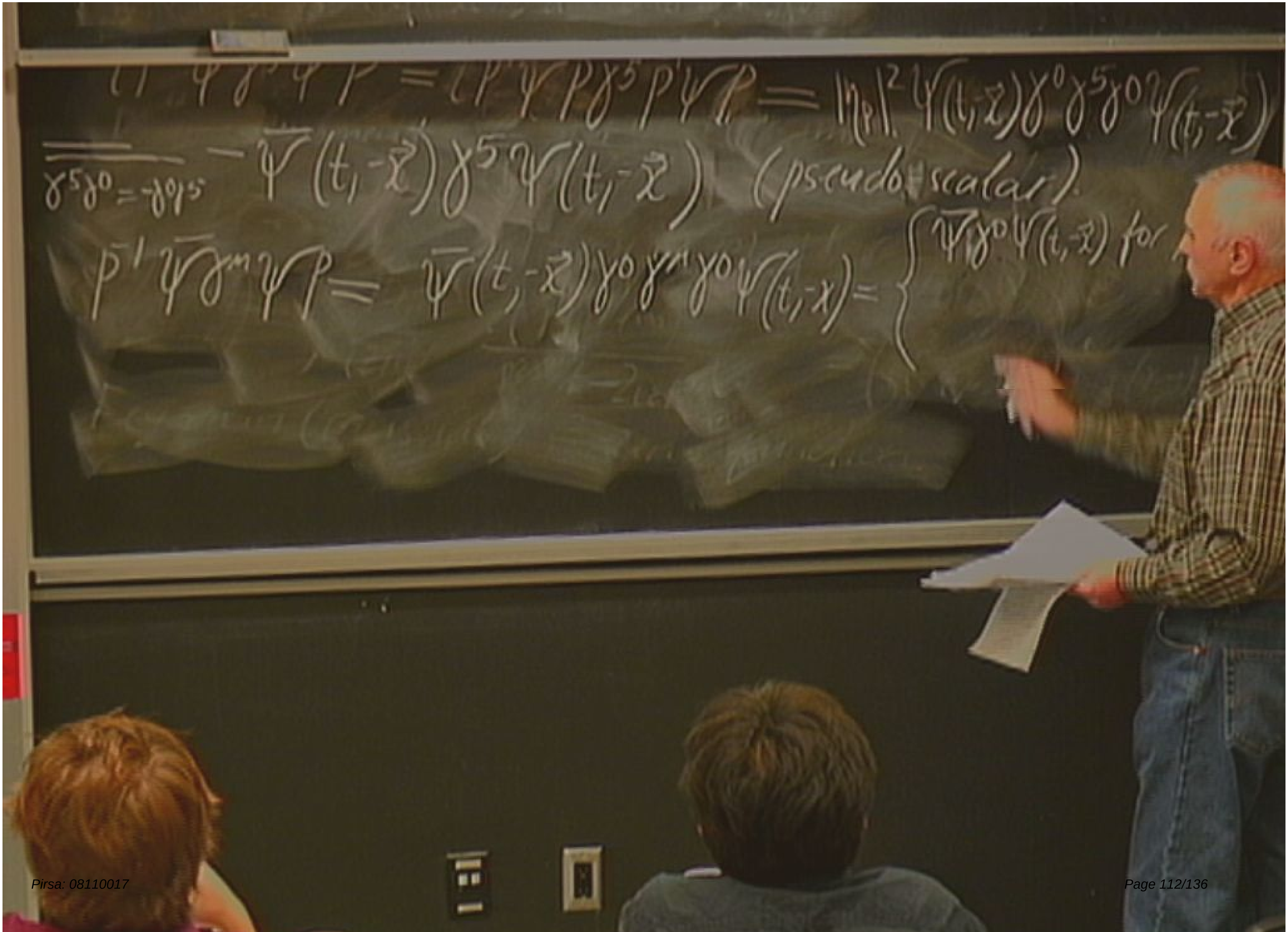

tive parity of fermions and antifermions is
Transformations of fermion field bilinears
hermitean.
(scalar)

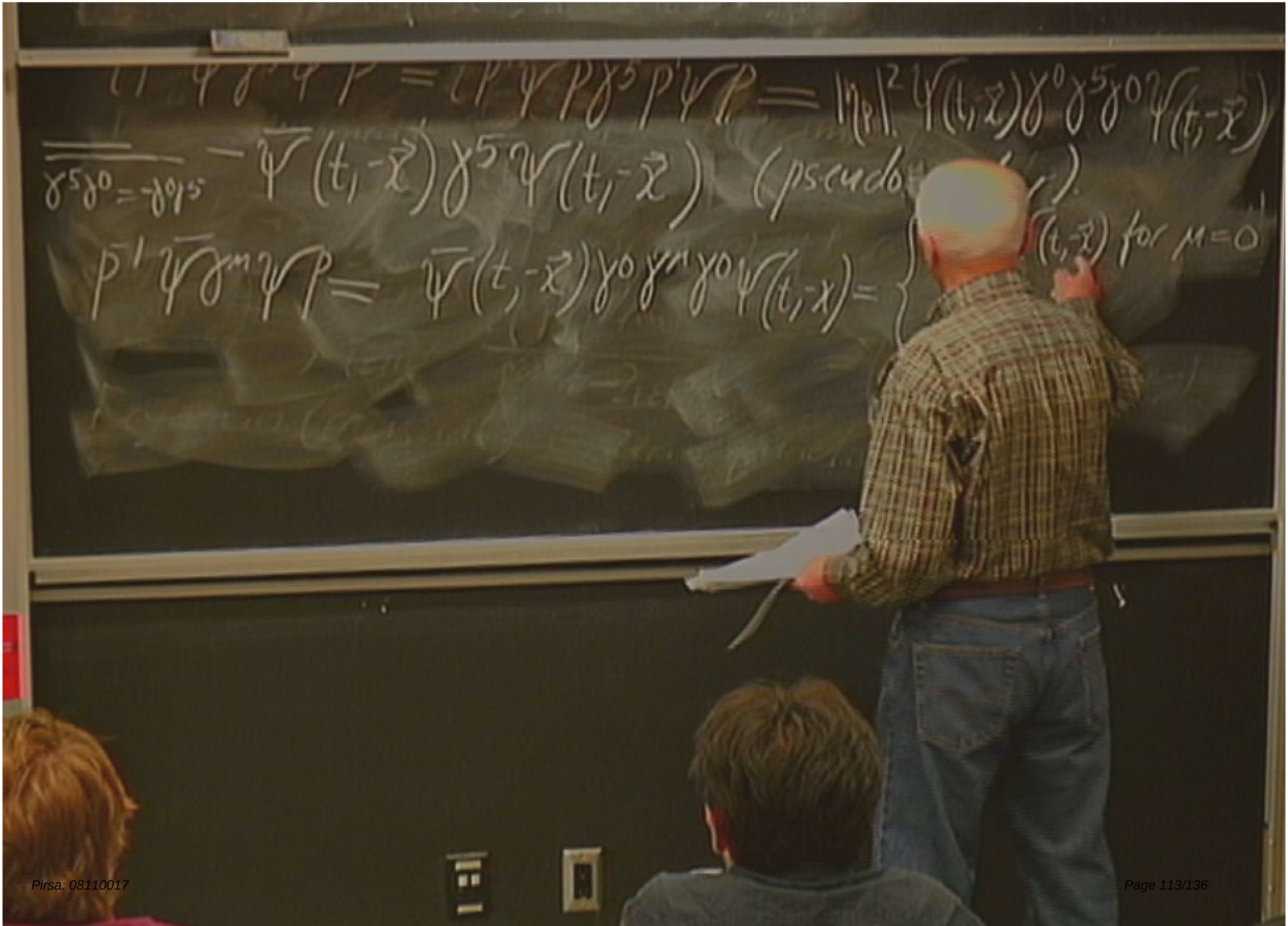

(pseudo-scalar)

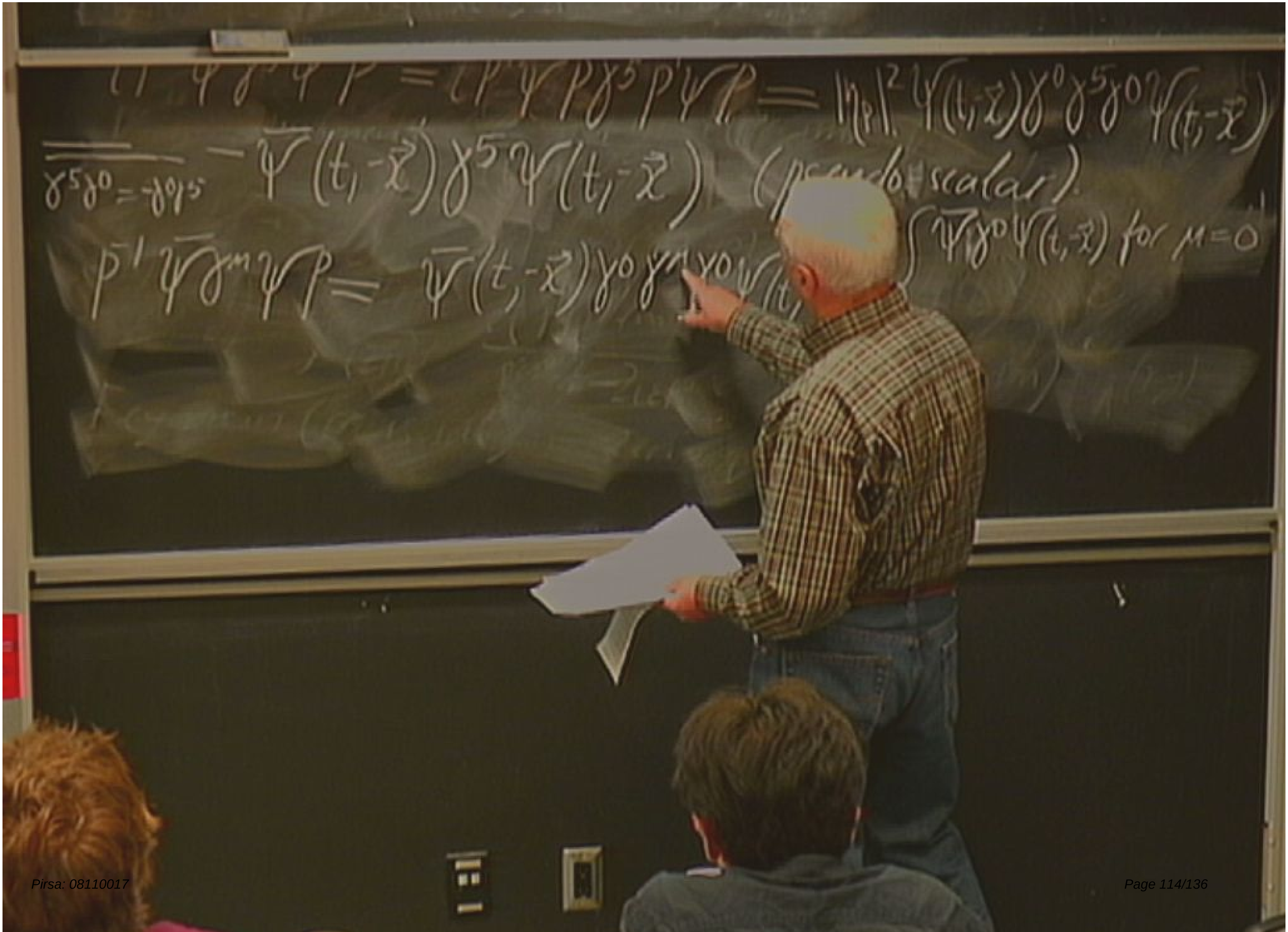

What
under

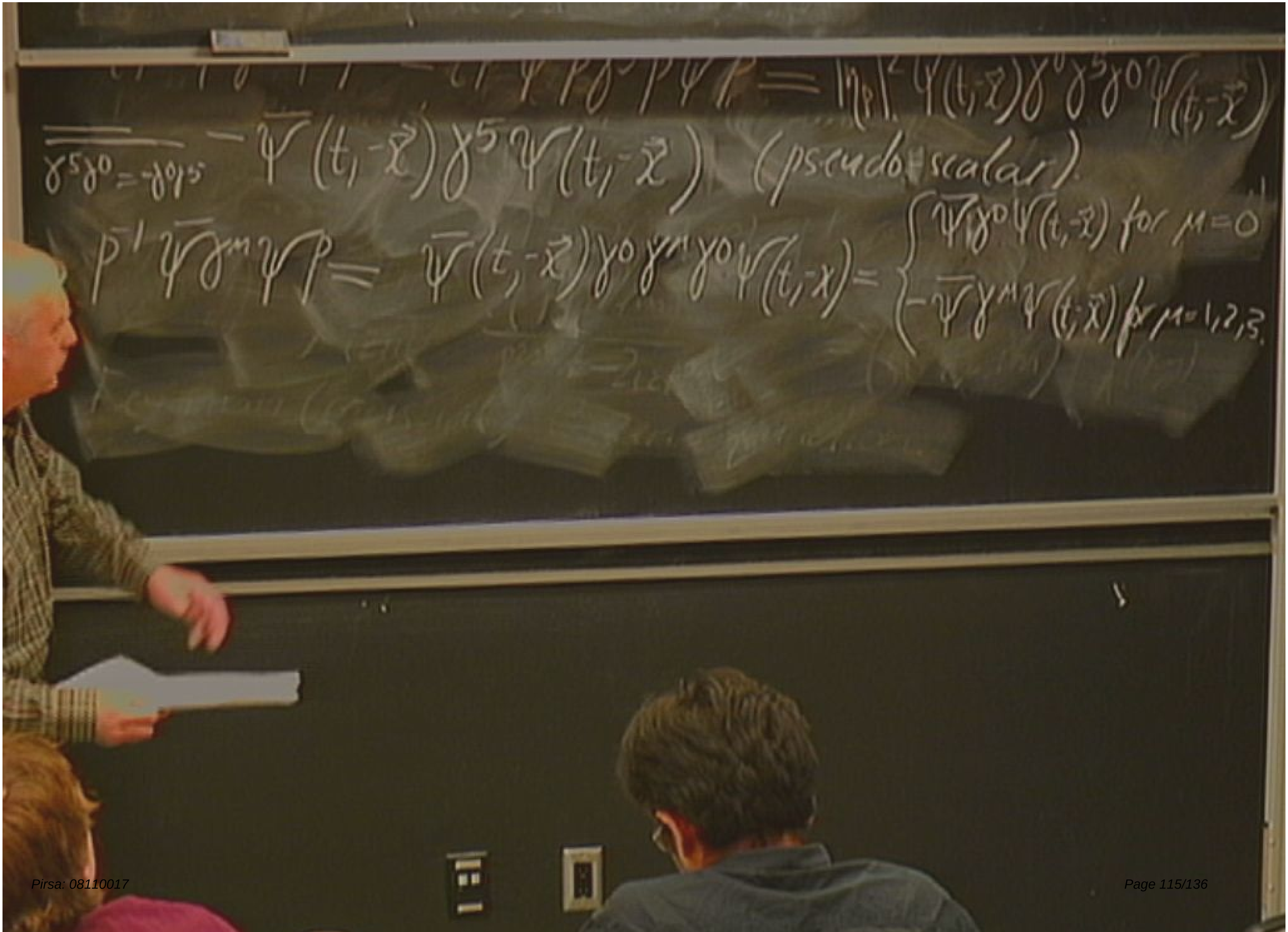

(pseudo-scalar

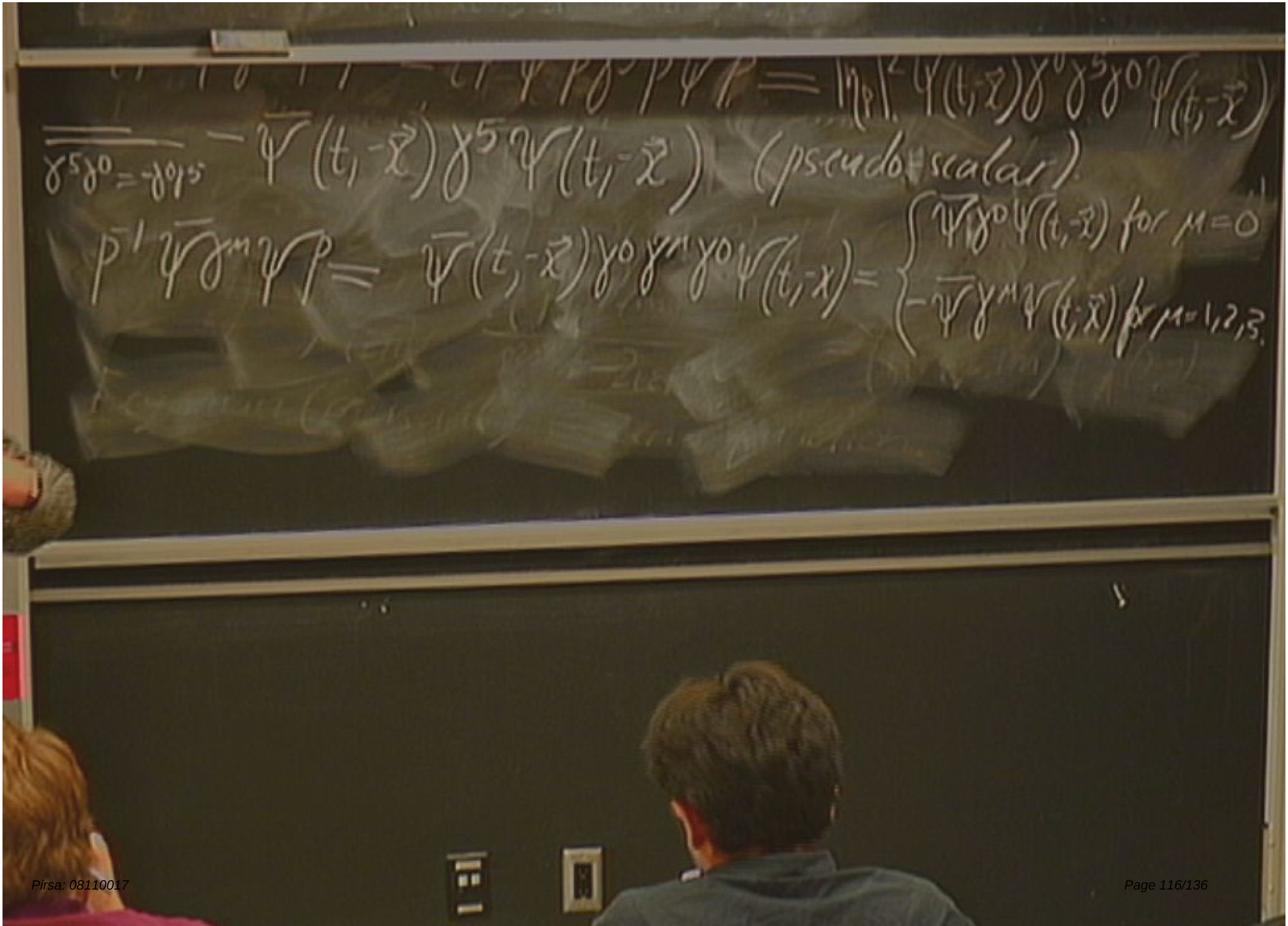

$iP^{-1}\bar{\psi}\gamma^5\psi P = iP^{-1}\bar{\psi}P\gamma^5 P^{-1}\psi P = |\eta_P|^2\bar{\psi}(t,-\vec{x})\gamma^0\gamma^5\gamma^0\psi(t,-\vec{x})$
$\gamma^5\gamma^0 = -\gamma^0\gamma^5$
$-\bar{\psi}(t,-\vec{x})\gamma^5\psi(t,-\vec{x})$
$P^{-1}\bar{\psi}\gamma^\mu\psi P = \bar{\psi}(t,-\vec{x})\gamma^0\gamma^\mu\gamma^0\psi(t,-x) =$

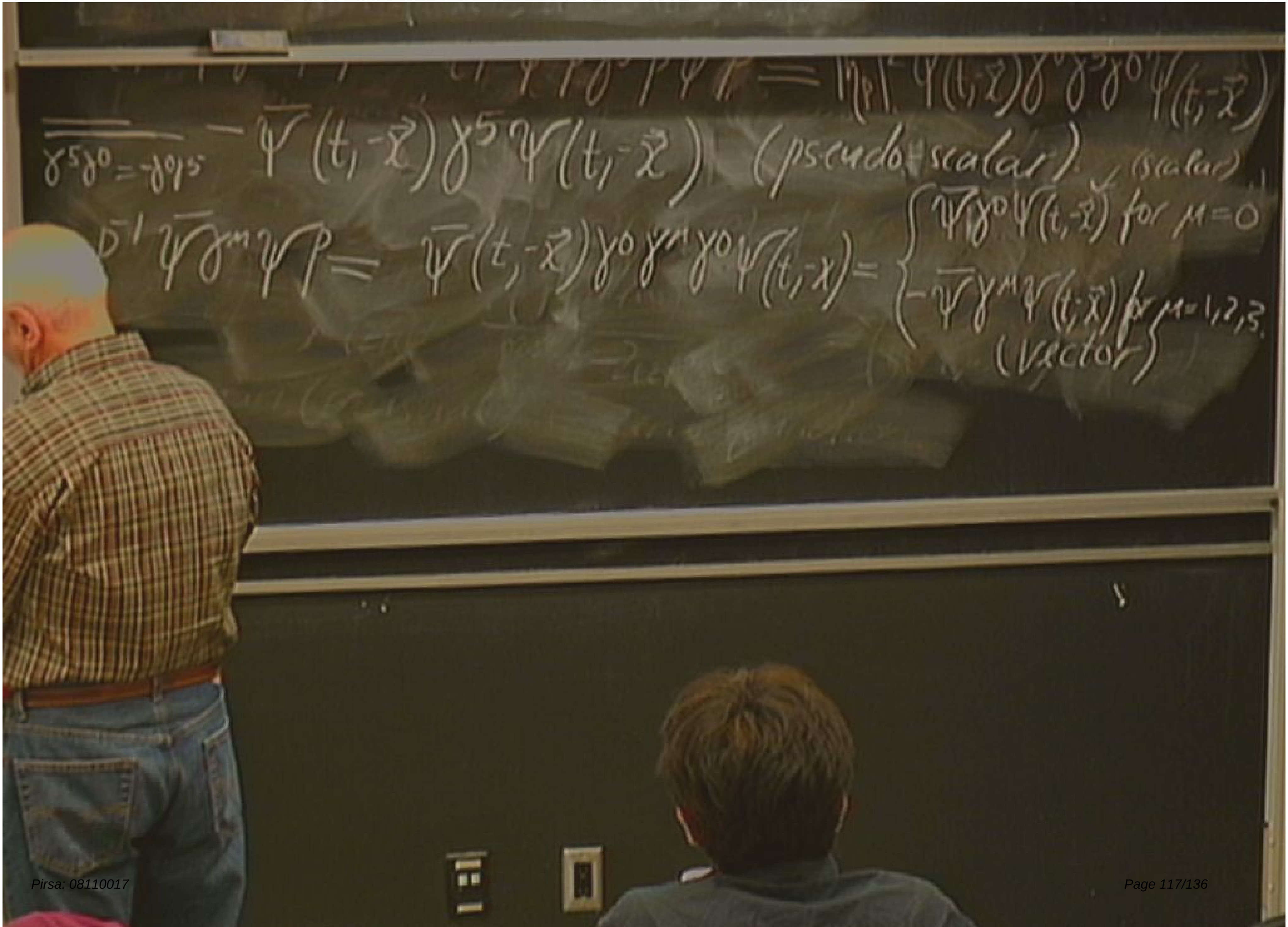

(pseudo scalar)

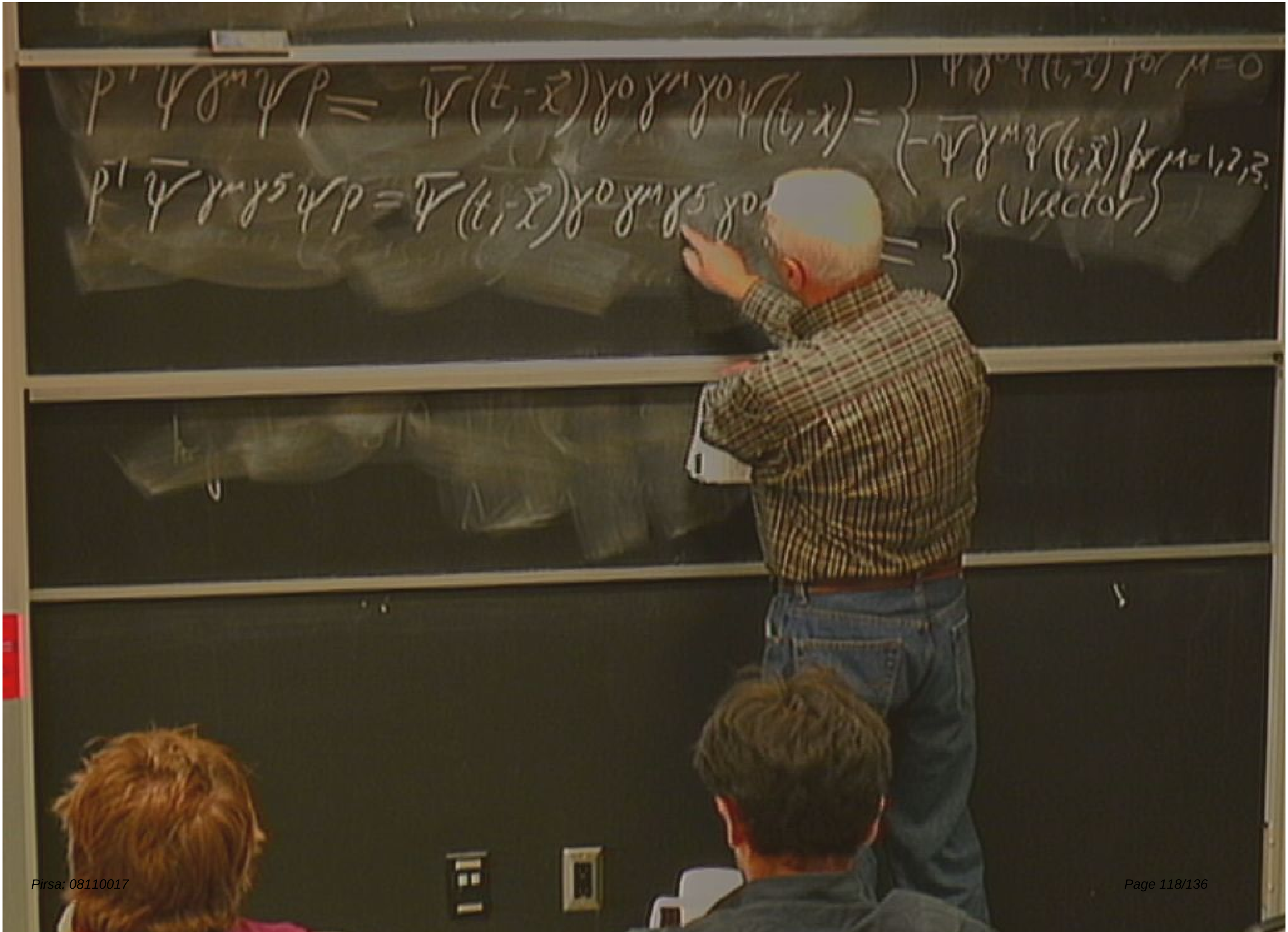


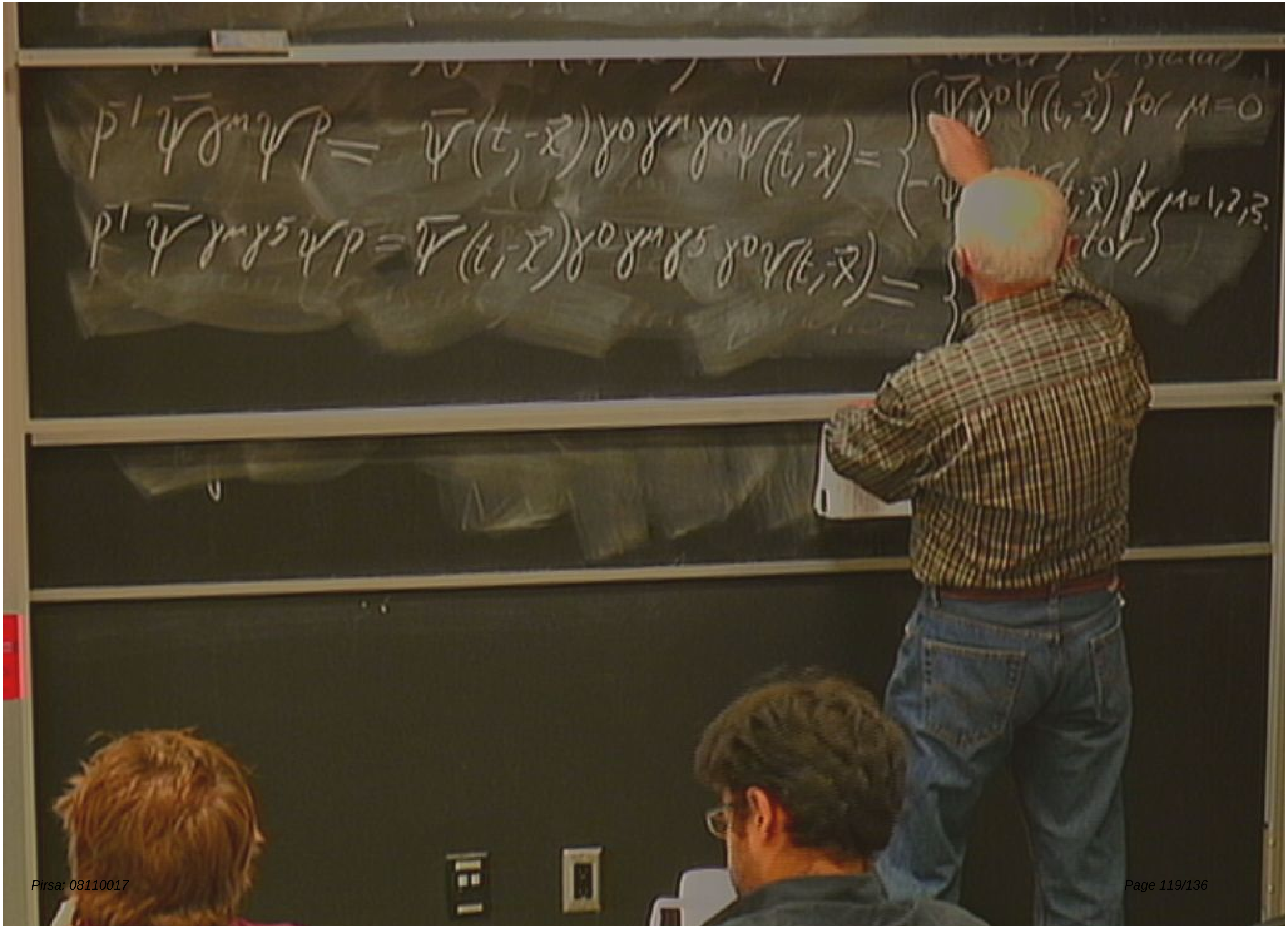

(pseudo scalar)
for μ=0

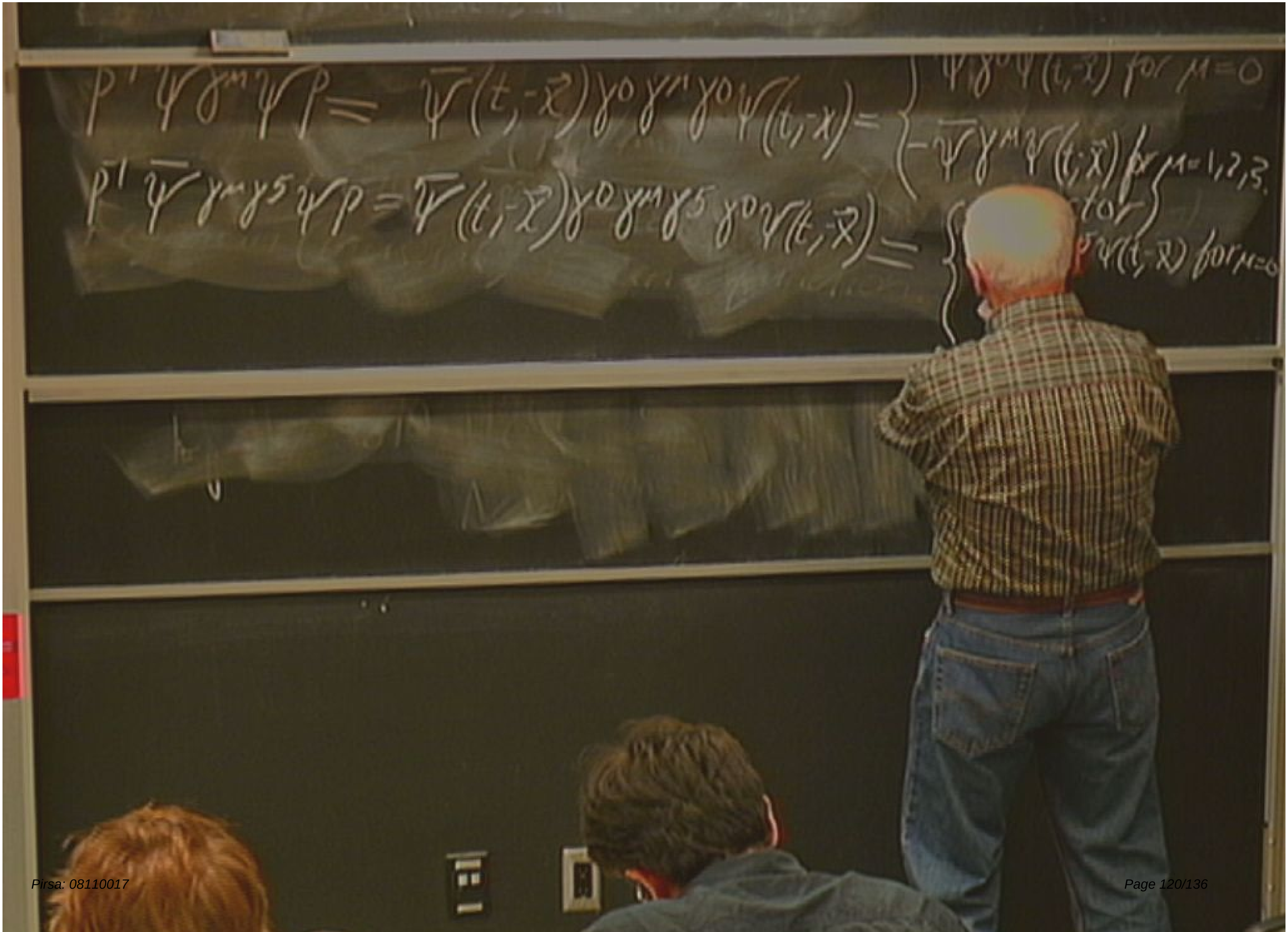

= |η_P|^2 ψ̄(t,-x)γ^0γ^5γ^0ψ(t,-x)
γ^5γ^0 = -γ^0γ^5
= -ψ̄(t,-x)γ^5ψ(t,-x) (pseudo-scalar)
P^-1 ψ̄γ^μψP = ψ̄(t,-x)γ^0γ^μγ^0ψ(t,-x) =
ψ̄γ^0ψ(t,-x) for μ=0
-ψ̄γ^μψ(t,-x) for μ=1,2,3.

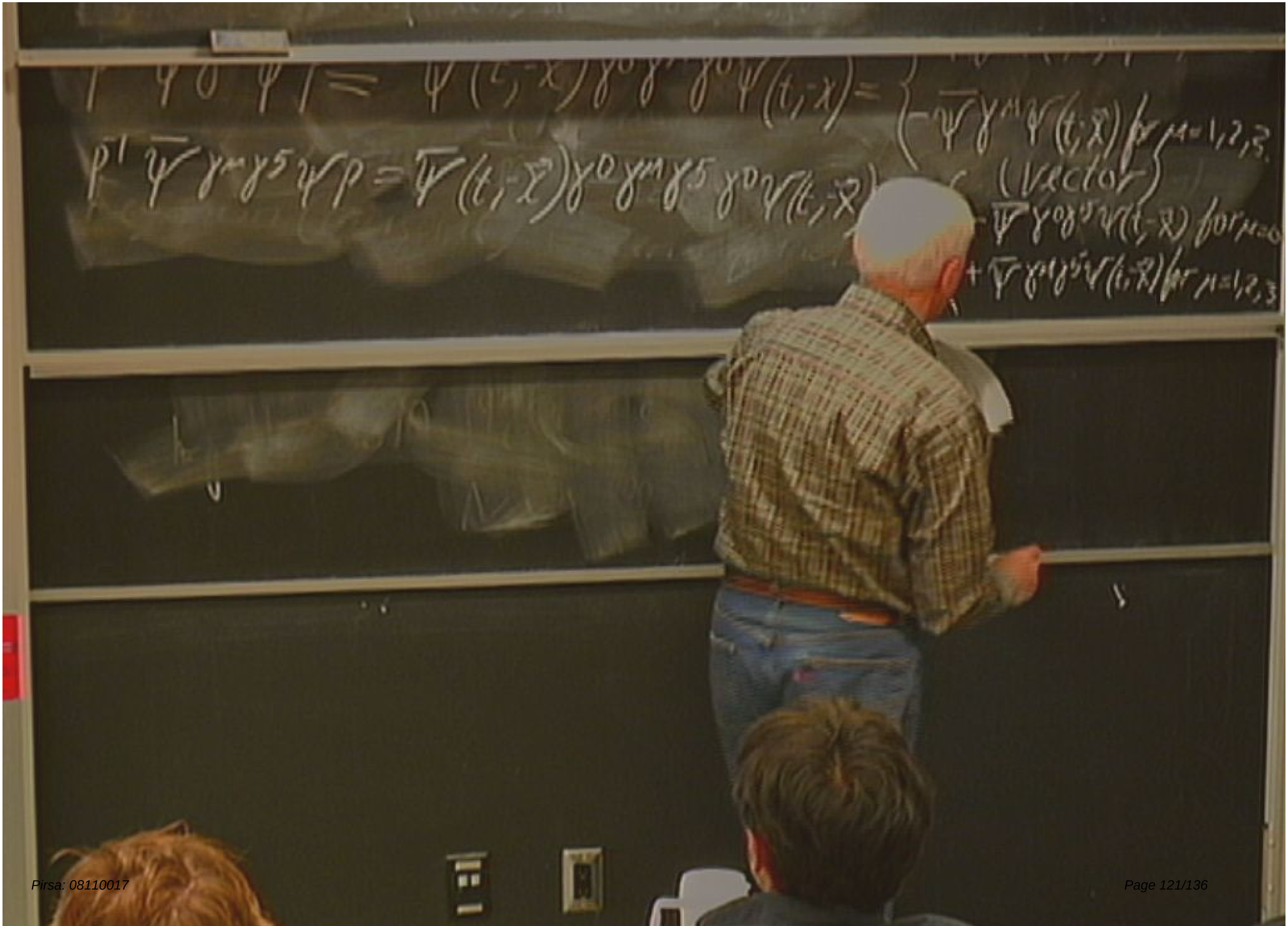

$= |\eta_P|^2 \bar\psi(t,-\vec{x})\gamma^0\gamma^5\gamma^0\psi(t,-\vec{x})$
$\gamma^5\gamma^0 = -\gamma^0\gamma^5$
$-\bar\psi(t,-\vec{x})\gamma^5\psi(t,-\vec{x})$ (pseudo-scalar)
$P^{-1}\bar\psi\gamma^\mu\psi P = \bar\psi(t,-\vec{x})\gamma^0\gamma^\mu\gamma^0\psi(t,-x) =$
$\bar\psi\gamma^0\psi(t,-\vec{x})$ for $\mu=0$
$-\bar\psi\gamma^\mu\psi(t,-\vec{x})$ for $\mu=1,2,3.$

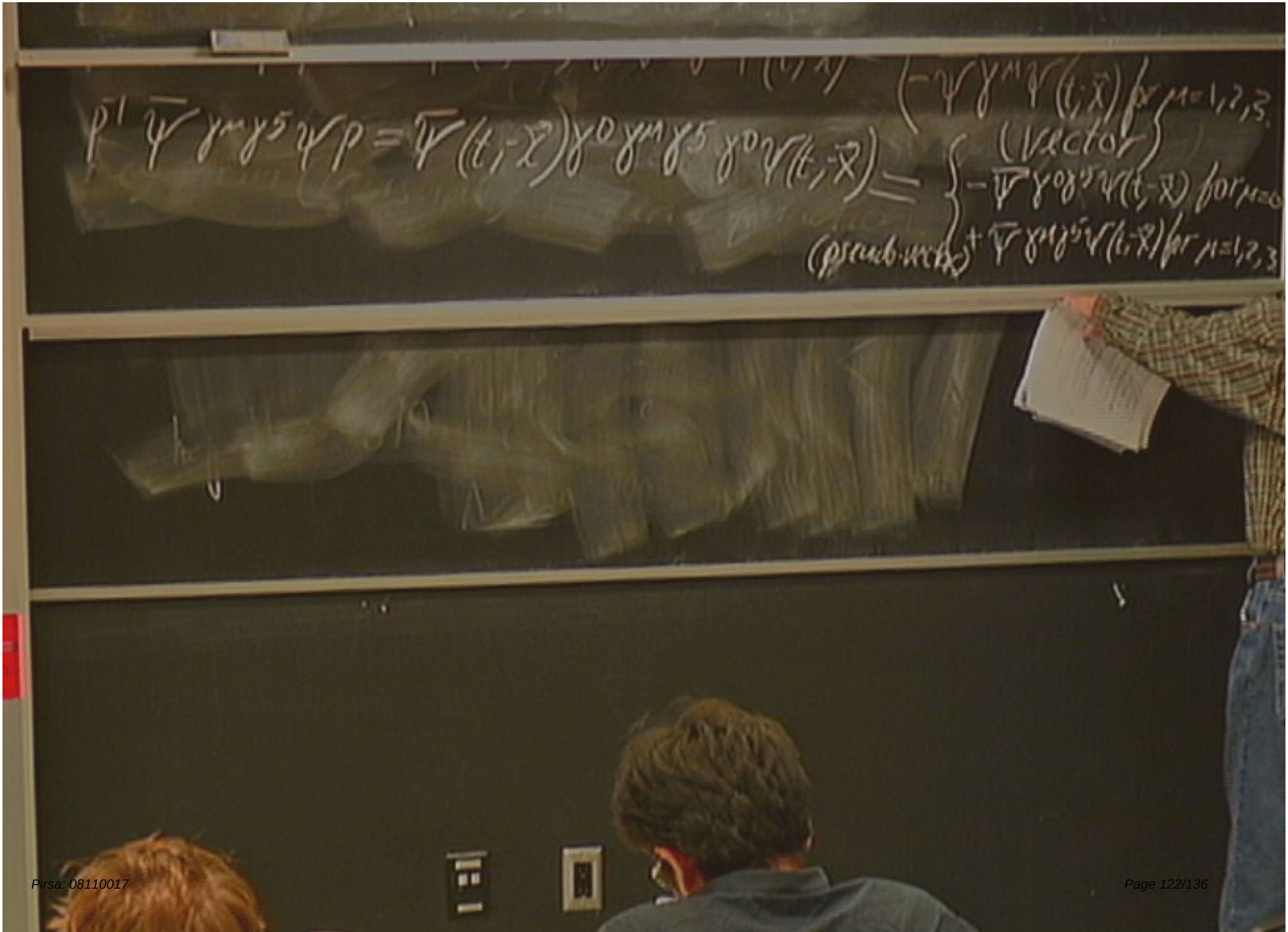

γ5γ0 = -γ0γ5
-ψ̄(t,-x)γ5ψ(t,-x) (pseudo scalar)
(scalar)
ψ̄γ0ψ(t,-x) for μ=0
-ψ̄γμψ(t,-x) for μ=1,2,3
(vector)


(vector)


for μ=0
for μ=1,2,3


for μ=0
for μ=1,2,3.
for μ=0


(vector)


(vector)

...elative parity of fermions and antifermions (-1)

Transformations of fermion field bilinears

$\bar\psi\psi$, $i\bar\psi\gamma^5\psi$, $\bar\psi\gamma^\mu\psi$, $i\bar\psi[\gamma^\mu,\gamma^\nu]\psi$, $\bar\psi\gamma^\mu\gamma^5\psi$

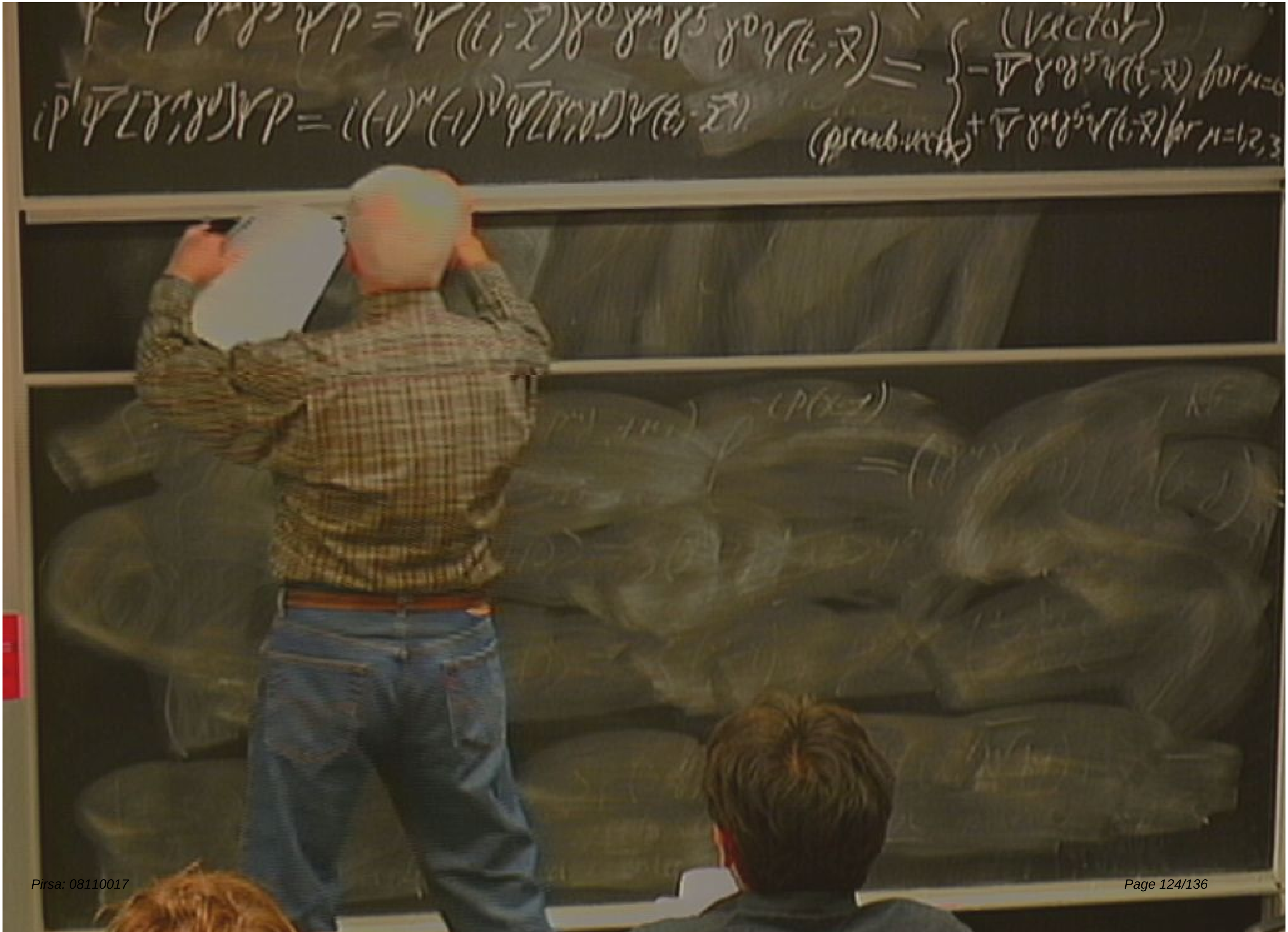
...y are hermitian.

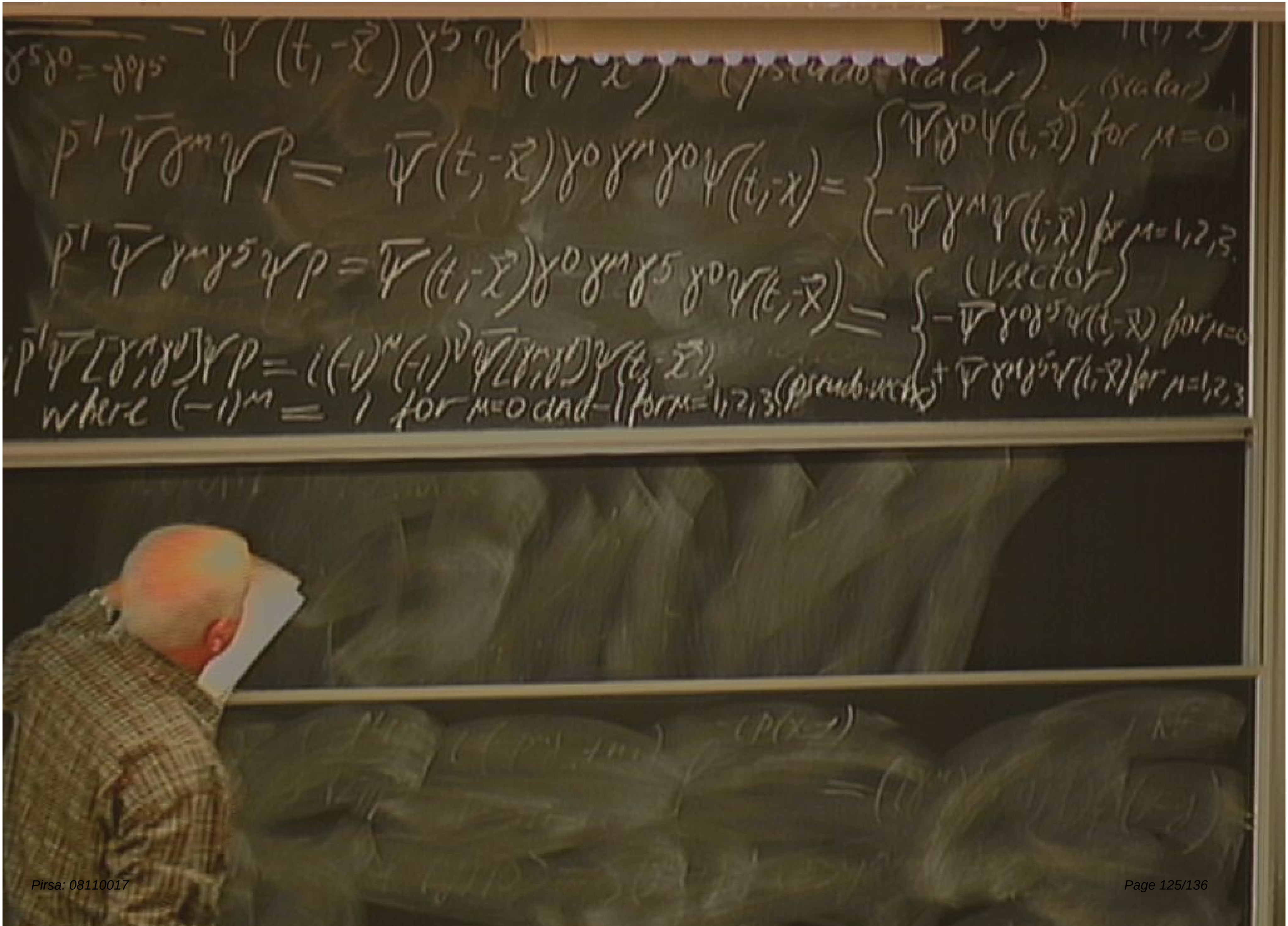
$$P^{-1}\bar\psi P = P^{-1}\psi^\dagger(t,x)\gamma^0 P = P^{-1}\psi^\dagger P\gamma^0 = (P^{-1}\psi P)^\dagger\gamma^0 =$$

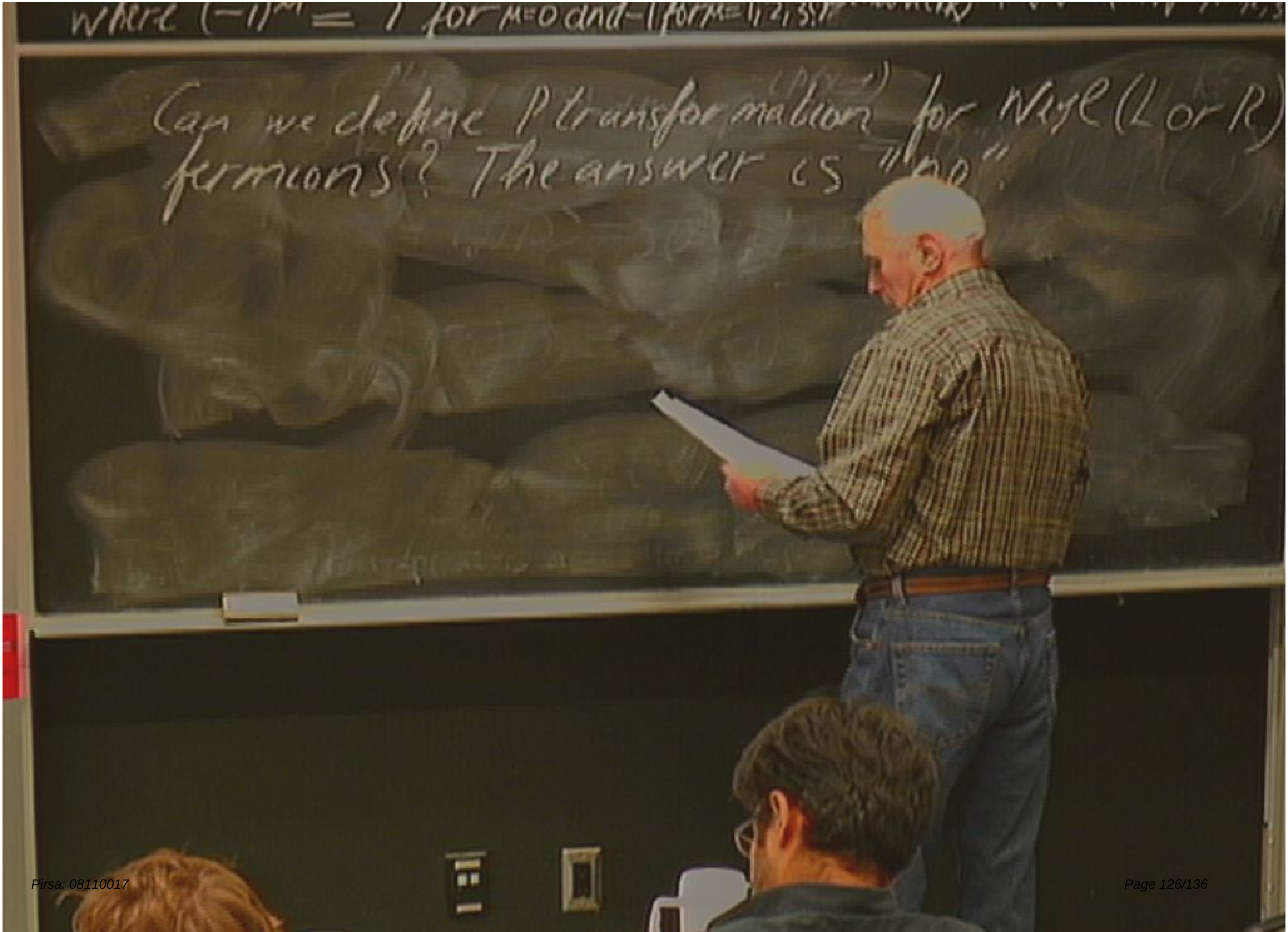
$$= (\eta_P\gamma^0\psi(t,-\vec{x}))^\dagger\gamma^0 = \eta_P^*\bar\psi(t,-\vec{x})\gamma^0$$

$$P^{-1}\bar\psi\psi P = P^{-1}\bar\psi P P^{-1}\psi P = |\eta_P|^2\bar\psi(t,-\vec{x})\gamma^0\gamma^0\psi(t,-\vec{x}) =$$

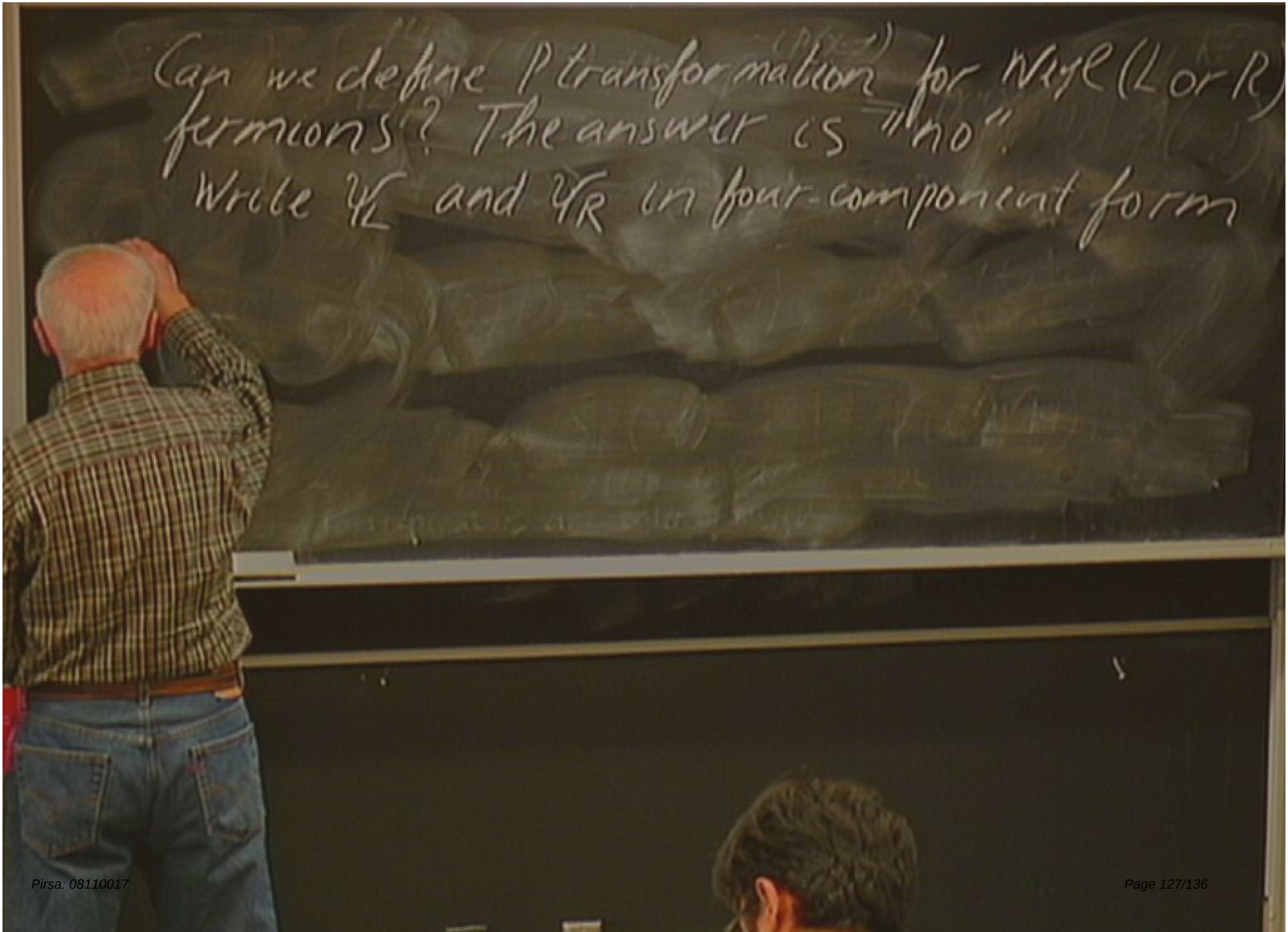
$$= \bar\psi(t,-\vec{x})\psi(t,-\vec{x}) \quad (\text{scalar})$$

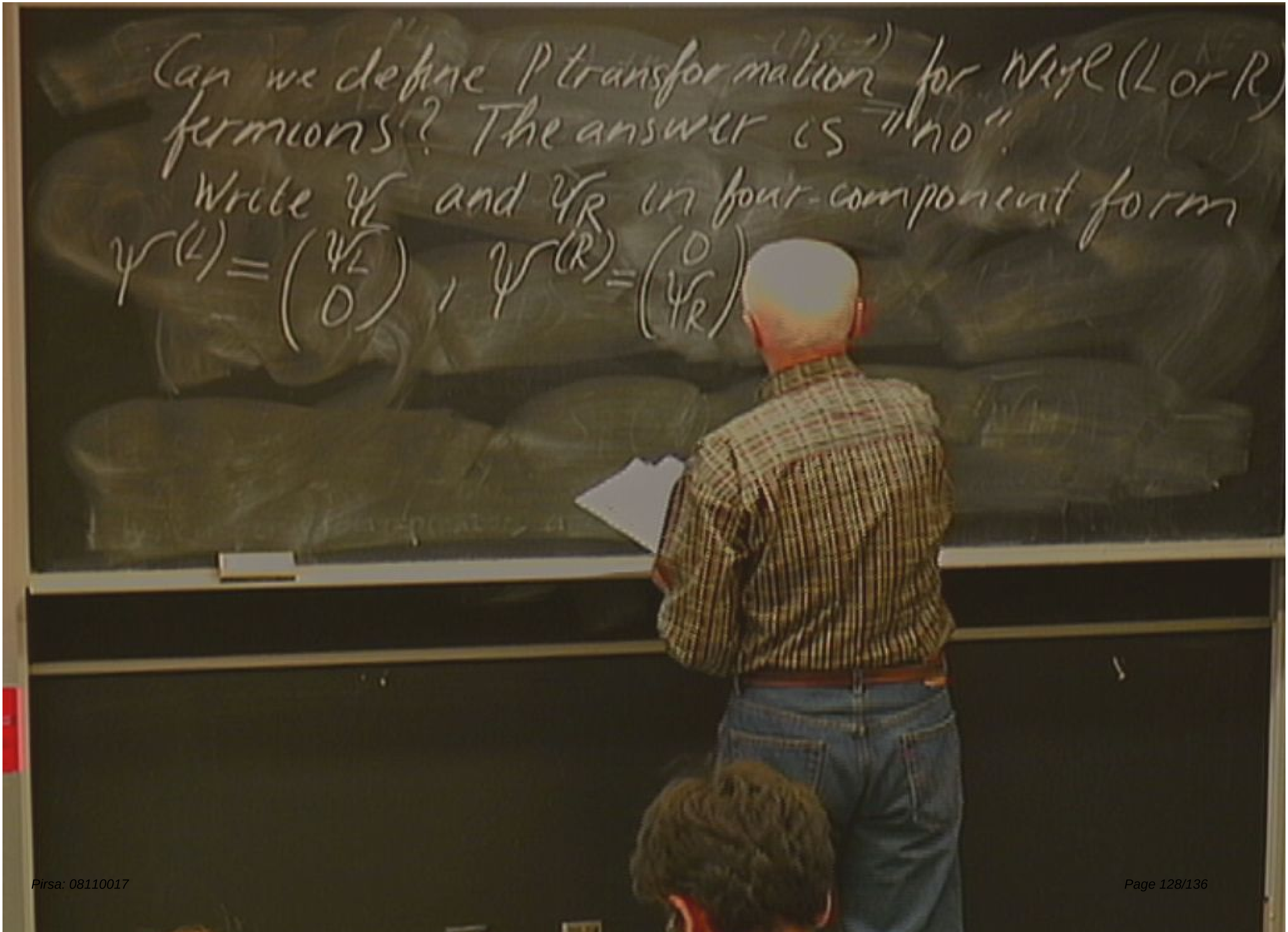

(vector)
for μ=0
(pseudo-vector)
for μ=1,2,3


γ5γ0 = -γ0γ5   ψ̄(t,-x)γ5ψ(t,x) (pseudo-scalar)
(scalar)
P^-1 ψ̄γ^μψP = ψ̄(t,-x)γ0γ^μγ0ψ(t,-x) =
ψ̄γ0ψ(t,-x) for μ=0
-ψ̄γ^μψ(t,-x) for μ=1,2,3
(vector)
P^-1 ψ̄γ^μγ5ψP = ψ̄(t,-x)γ0γ^μγ5γ0ψ(t,-x) =
-ψ̄γ0γ5ψ(t,-x) for μ=0
+ψ̄γ^μγ5ψ(t,-x) for μ=1,2,3
(pseudo-vector)
P^-1 ψ̄[γ^μ,γ^ν]ψP = i(-1)^μ(-1)^ν ψ̄[γ^μ,γ^ν]ψ(t,-x),
where (-1)^μ = 1 for μ=0 and -1 for μ=1,2,3,



Can we define P transformation for Weyl (L or R)
fermions? The answer is "no".


Can we define P transformation for Weyl (L or R) fermions? The answer is "no".
Write ψ_L and ψ_R in four-component form


Can we define P transformation for Weyl (L or R)
fermions? The answer is "no".
Write ψL and ψR in four-component form
ψ(L) = (ψL, 0), ψ(R) = (0, ψR)

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

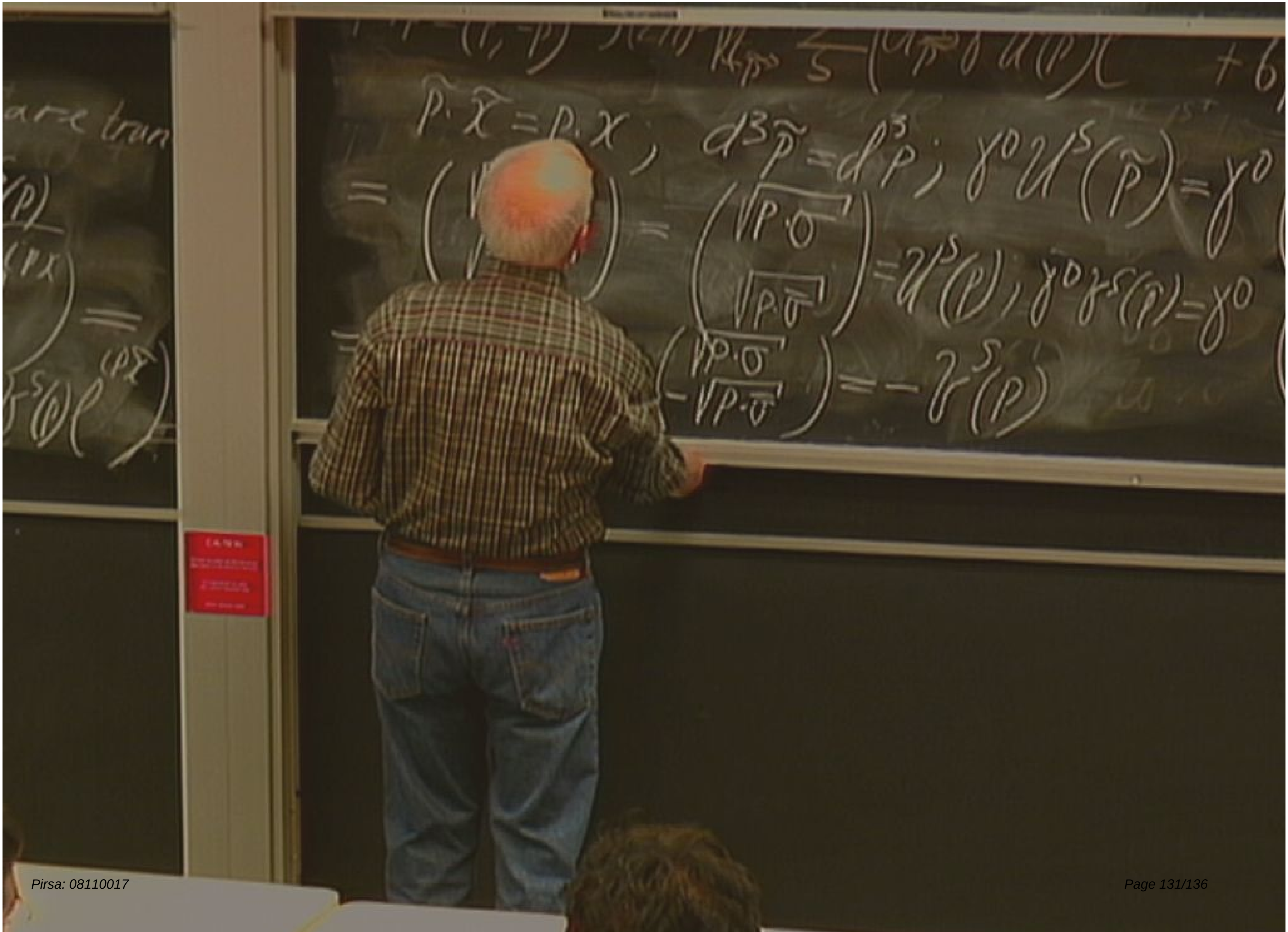
Write ψ_L and ψ_R in four-component form:

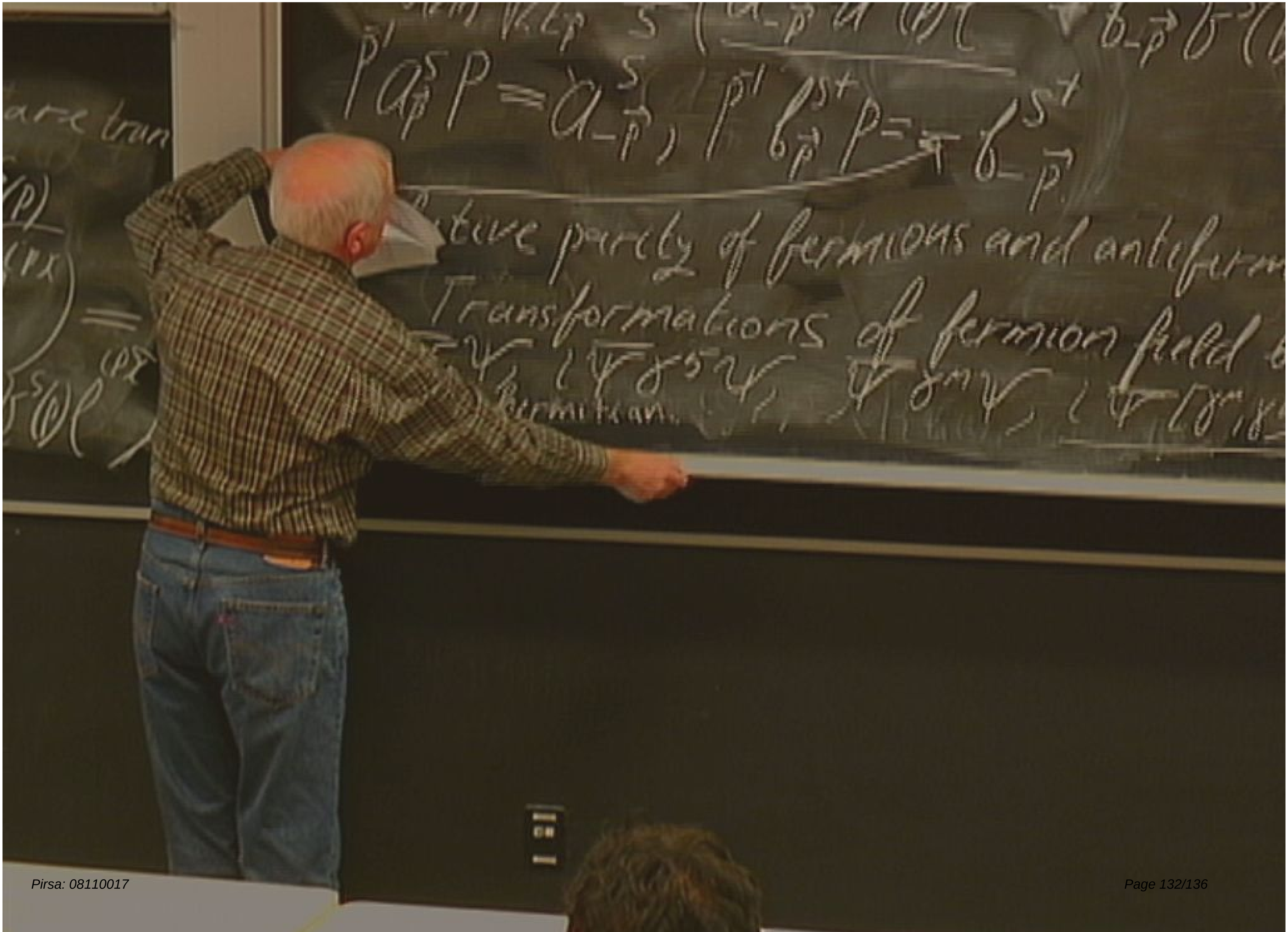
$$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}. \quad \text{P-transformation:}$$

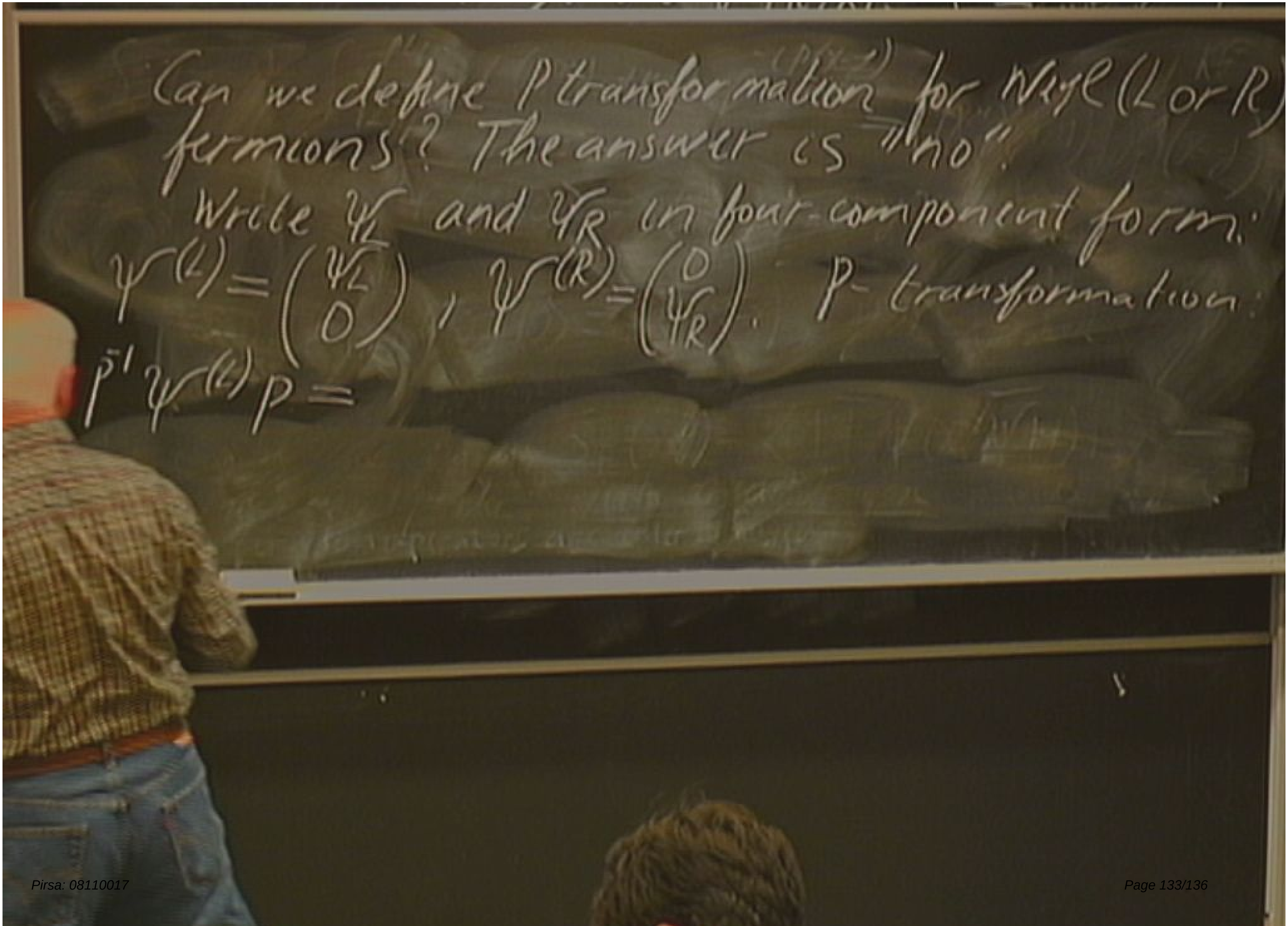
Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

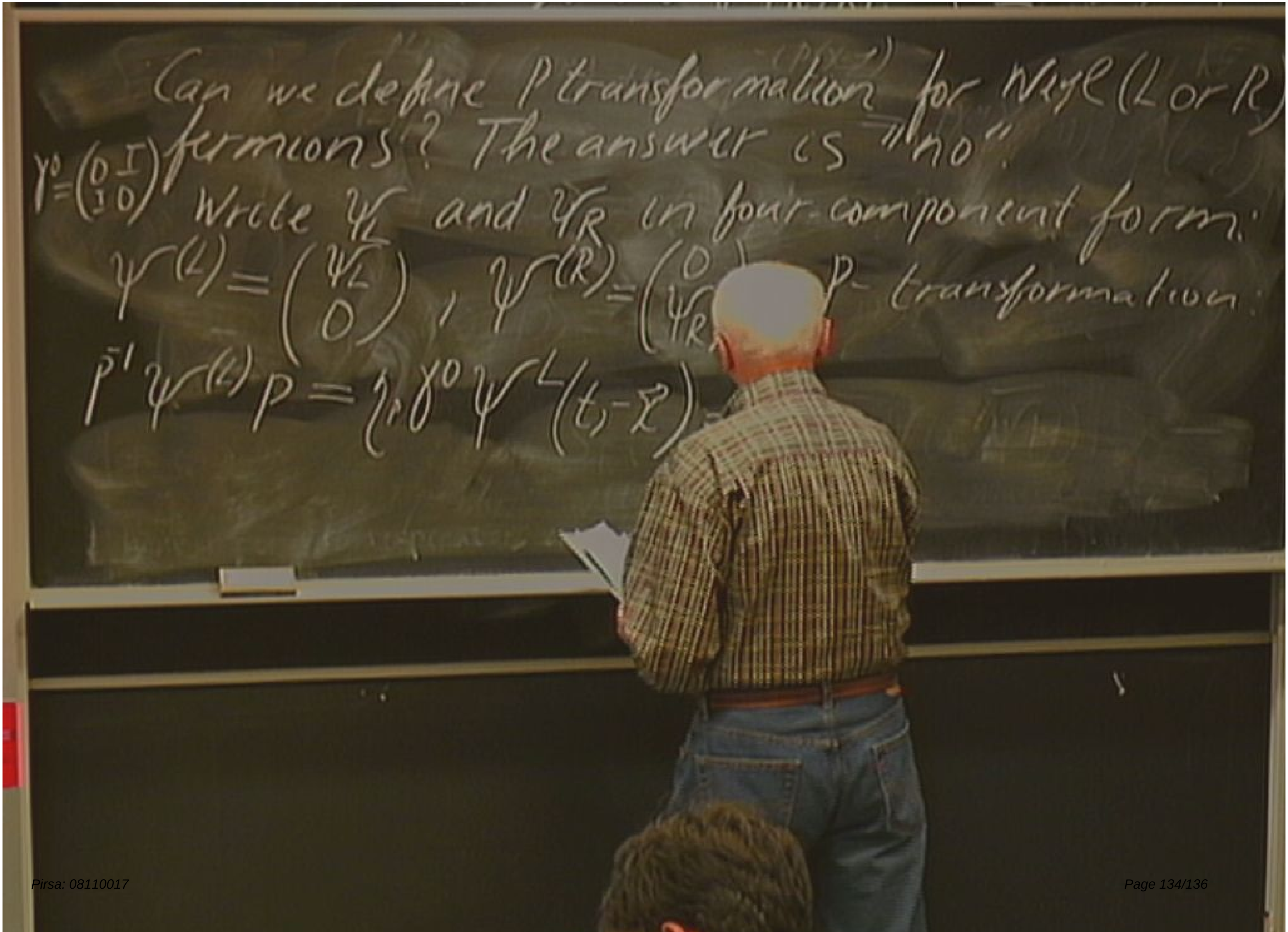
Write ψ_L and ψ_R in four-component form:

$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$, $\psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$. P-transformation:




tive parity of fermions and antiferm
Transformations of fermion field
hermitean.


Can we define P transformation for Weyl (L or R) fermions? The answer is "no".
Write ψ_L and ψ_R in four-component form:
$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$, $\psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$. P-transformation:
$P^{-1} \psi^{(L)} P =$


Can we define P transformation for Weyl (L or R)
fermions? The answer is "no".
γ⁰ = (0 I / I 0)
Write ψL and ψR in four-component form:
ψ(L) = (ψL / 0), ψ(R) = (0 / ψR). P-transformation:
P⁻¹ ψ(L) P = ηP γ⁰ ψL(t, −x)

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

Write ψ_L and ψ_R in four-component form:

$$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}.$$

P-transformation:

$$P^{-1} \psi^{(L)} P = \eta_P \gamma^0 \psi^L(t, -\vec{x}) = \eta_P \begin{pmatrix} 0 \\ \psi_L \end{pmatrix} \leftarrow \text{right-handed fermion}$$

Can we define P transformation for Weyl (L or R) fermions? The answer is "no".

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Write ψ_L and ψ_R in four-component form:

$$\psi^{(L)} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \psi^{(R)} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}.$$

P-transformation:

$$P^{-1} \psi^{(L)} P = \eta_P \gamma^0 \psi^L(t, -\underline{x}) = \eta_P \begin{pmatrix} 0 \\ \psi_L \end{pmatrix} \leftarrow \text{right-handed fermion}$$

$$P^{-1} \psi^{(R)} P = \eta_P \begin{pmatrix} \psi_R \\ 0 \end{pmatrix} \leftarrow \text{left-handed fermion.}$$