

Title: Quantum Field Theory 1 - Lecture 9B

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

$$\psi(\vec{p}) = \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s u^s(\vec{p}) + b_{-\vec{p}}^{s\dagger} v^s(\vec{p}))$$

$$\psi^\dagger(-\vec{p}) = [\psi(\vec{p})]^\dagger = \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^{s\dagger} u^{s\dagger}(\vec{p}) + b_{-\vec{p}}^s v^{s\dagger}(\vec{p}))$$

Orthogonality relations:

$$u^{r\dagger}(\vec{p}) u^s(\vec{p}) = 2E_{\vec{p}} \delta^{rs}$$

$$v^{r\dagger}(\vec{p}) v^s(\vec{p}) = 2E_{\vec{p}} \delta^{rs}$$

$$u^{r\dagger}(\vec{p}) v^s(-\vec{p}) = 0$$

$$v^{r\dagger}(-\vec{p}) u^s(\vec{p}) = 0$$

Using the orthogonality relations, we get:

$$[a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}] = [b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}] = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \delta^{rs}$$

$$[a_{\vec{p}}^r, a_{\vec{q}}^s] = [b_{\vec{p}}^r, b_{\vec{q}}^s] = [a_{\vec{p}}^{r\dagger}, a_{\vec{q}}^{s\dagger}] = [b_{\vec{p}}^{r\dagger}, b_{\vec{q}}^{s\dagger}] = 0$$

Write H in terms of these operators using the orthogonality relations:

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s (E_{\vec{p}} a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s - E_{\vec{p}} b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s)$$

There is "vacuum"  $|0\rangle$ ,  $a_{\vec{p}}^s |0\rangle = b_{\vec{p}}^s |0\rangle = 0$ . States  $(b_{\vec{p}}^{s\dagger})^n |0\rangle$  have  $E(\vec{p}) < 0$  in this theory.

$\sum_{r=s} \langle a_{\vec{p}}^r a_{\vec{p}}^r \rangle = 0, \quad (a_{\vec{p}}^r |0\rangle = 0)$   
 Now  $H(S)$   

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left( \sum_r \left( \epsilon_{\vec{p}} a_{\vec{p}}^{r\dagger} a_{\vec{p}}^r - E_{\vec{p}} b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s \right) \right)$$
 Pauli Principle (Fermi Statistics)  

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s \left( \epsilon_{\vec{p}} a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + E_{\vec{p}} b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s \right) - \int \frac{d^3p}{(2\pi)^3} (2\pi)^3 \delta^3(0) \leftarrow \text{volume}$$

Introduce new notations:  $\tilde{b}_{\vec{p}}^r = b_{\vec{p}}^{r\dagger}, \tilde{b}_{\vec{p}}^{r\dagger} = b_{\vec{p}}^r$ , Vacuum  $|0\rangle$ :  
 $a_{\vec{p}}^r |0\rangle = \tilde{b}_{\vec{p}}^r |0\rangle = 0$ . Then all states  $a_{\vec{p}}^{r\dagger} |0\rangle, a_{\vec{p}}^{r\dagger} a_{\vec{q}}^{s\dagger} |0\rangle, \dots$  have energy higher than the vacuum state  $|0\rangle$ :  
 $H \tilde{b}_{\vec{p}}^r |0\rangle = (E_{\vec{p}} - 2V \int \frac{d^3p}{(2\pi)^3}) |0\rangle$

To understand better this theory, consider a model:  $b, b^\dagger; \{b, b^\dagger\} = bb^\dagger + b^\dagger b = 1, [b, b^\dagger] = [b, b] = 0$ .  
 $H = -E_b b^\dagger b$ . This model has only 2

As before, I could introduce  $\tilde{b} = b^\dagger$  and  $\tilde{b}^\dagger = b$ ; then  $\tilde{b} |0\rangle = 0$   
and  $H = -E_b b^\dagger b = E_b b b^\dagger - E_b = E_b \tilde{b}^\dagger \tilde{b} - E_b$ .

Back to Dirac theory:  
 $|0\rangle$

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$$\text{and } H = -E_0 b^\dagger b = E_0 b b^\dagger - E_0 = E_0 \tilde{b}^\dagger \tilde{b} - E_0.$$

Back to Dirac theory:

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Back to Dirac theory:

$|\tilde{0}\rangle : \tilde{b}_p^\dagger |\tilde{0}\rangle = 0 \implies |\tilde{0}\rangle = \prod_p \prod_{\uparrow} b_p^\dagger |0\rangle$

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$$|0\rangle : b_p^r |0\rangle = 0 \implies |0\rangle = \prod_r \prod_p b_p^{r+} |0\rangle.$$

Henceforth, we will use  $b_q^{r+} \rightarrow b_q^+$ ,  $b_q^r \rightarrow b_q^-$ .



Henceforth, we will use  $b_{\vec{q}}^{\dagger} \rightarrow b_{\vec{q}}^{\dagger}$ ,  $b_{\vec{q}} \rightarrow b_{\vec{q}}$ .

Use now (I) and (II)  $\psi(x), \bar{\psi}(x)$ . Equal time anticommutation relations at

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 anticommutation relations are:

$$[\psi_a(\vec{x}), \psi_b(\vec{y})] = \delta^3(\vec{x} - \vec{y}) S_{ab}, \quad \{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^\dagger(\vec{x}), \psi_b(\vec{y})\} = 0$$

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$$\Rightarrow \{a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}\} = (2\pi)^3$$

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~~anticommutation relations are:~~  $b_{\vec{q}} \rightarrow b'_{\vec{q}}, b'_{\vec{q}} \rightarrow b_{\vec{q}}$ .

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 \Rightarrow & \{a_{\vec{p}}^r, a_{\vec{q}}^{s+}\} = \{b_{\vec{p}}^r, b_{\vec{q}}^{s+}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}); \text{ The rest of anticommutators are zero. The ground state (vacuum): } a_{\vec{q}}^r |0\rangle = 0
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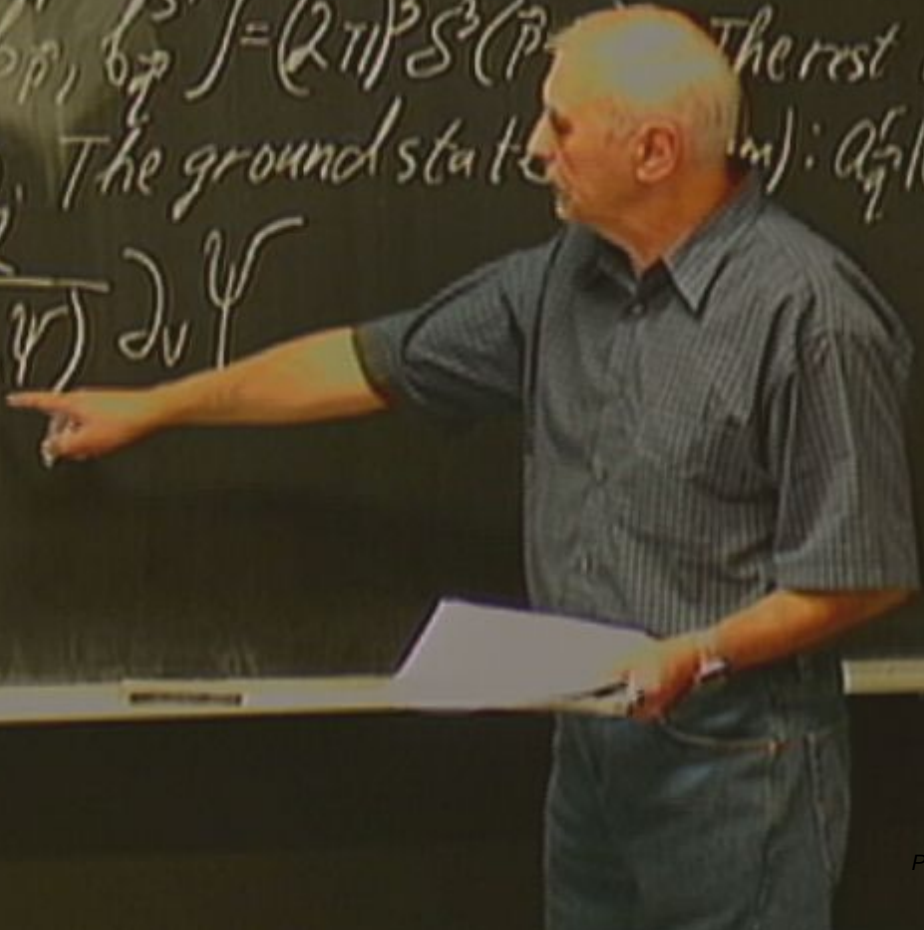
$$\Rightarrow \{a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}\} = \{b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}), \quad \text{The rest of anti-commutators are zero. The ground state (vacuum): } a_{\vec{q}}^r |0\rangle = b_{\vec{q}}^r |0\rangle = 0$$

$T^M_{\nu} - ?$

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$$T^M_{\nu} \text{ -? } \quad T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \partial_\nu \psi$$



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 \end{aligned}$$

The ground state  $|0\rangle$  is defined by  $a_{\vec{q}}|0\rangle = b_{\vec{q}}|0\rangle = 0$ .

$T^M_{\nu} = ?$   $T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \partial_\nu \psi - \mathcal{L} \delta_{\mu\nu}$

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$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \partial_\nu \psi - \mathcal{L} \delta^{\mu\nu}, \quad \text{However } \mathcal{L} = 0$$

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$$[\psi_a(x), \psi_b^\dagger(y)] = \delta^3(x-y) \delta_{ab}, \quad \{\psi_a(x), \psi_b(y)\} = \{\psi_a^\dagger(x), \psi_b^\dagger(y)\} = 0$$

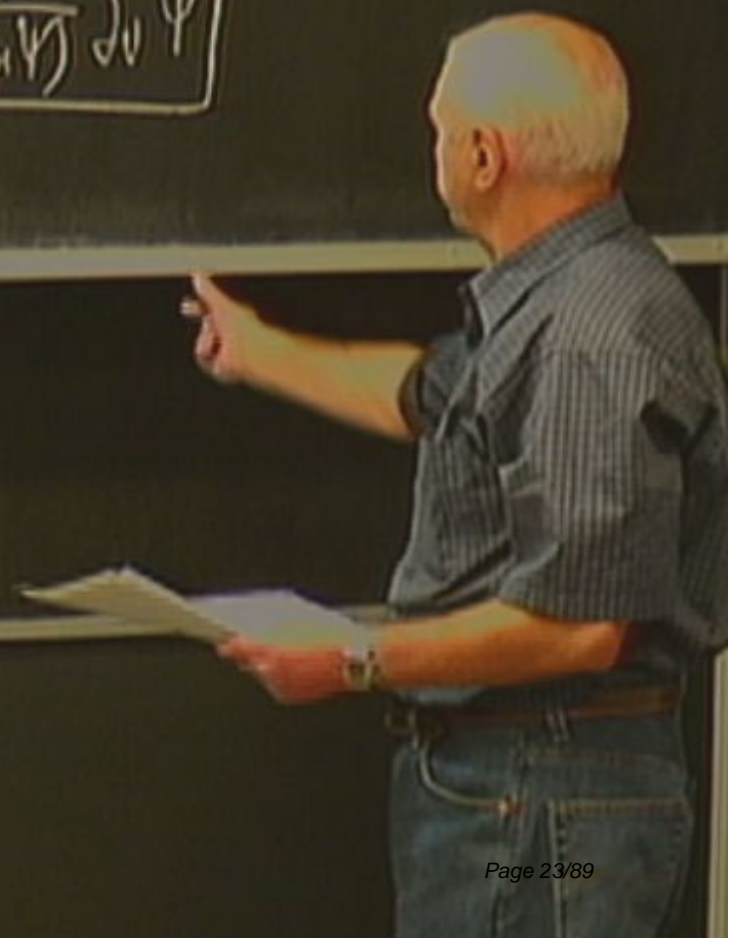
$$\Rightarrow \{a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}\} = \{b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}),$$

The rest of anticommutators are zero. The ground state (vacuum):  $a_{\vec{q}}^r |0\rangle = b_{\vec{q}}^r |0\rangle = 0$

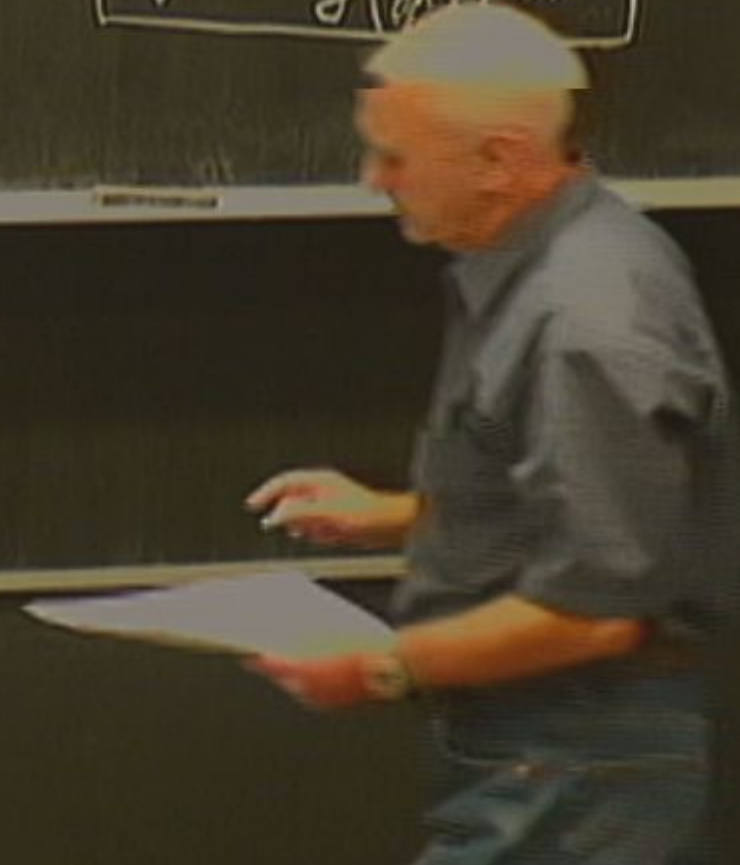
$$T^M_{\nu} = ? \quad T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \partial_\nu \psi - \mathcal{L} \delta^{\mu\nu}, \quad \text{However } \mathcal{L} = 0$$

for Dirac solutions

$\{ \psi_a(x), \psi_b(y) \} = \delta(x-y) \delta_{ab}, \{ \psi_a(x), \psi_b(y) \} = \{ \psi_a(x), \psi_b(y) \} = 0$   
 $\Rightarrow \{ a_{\vec{p}}, a_{\vec{q}}^{s\dagger} \} = \{ b_{\vec{p}}, b_{\vec{q}}^{s\dagger} \} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}),$  The rest of anti-commutators are zero. The ground state (vacuum):  $a_{\vec{q}}|0\rangle = b_{\vec{q}}|0\rangle = 0$   
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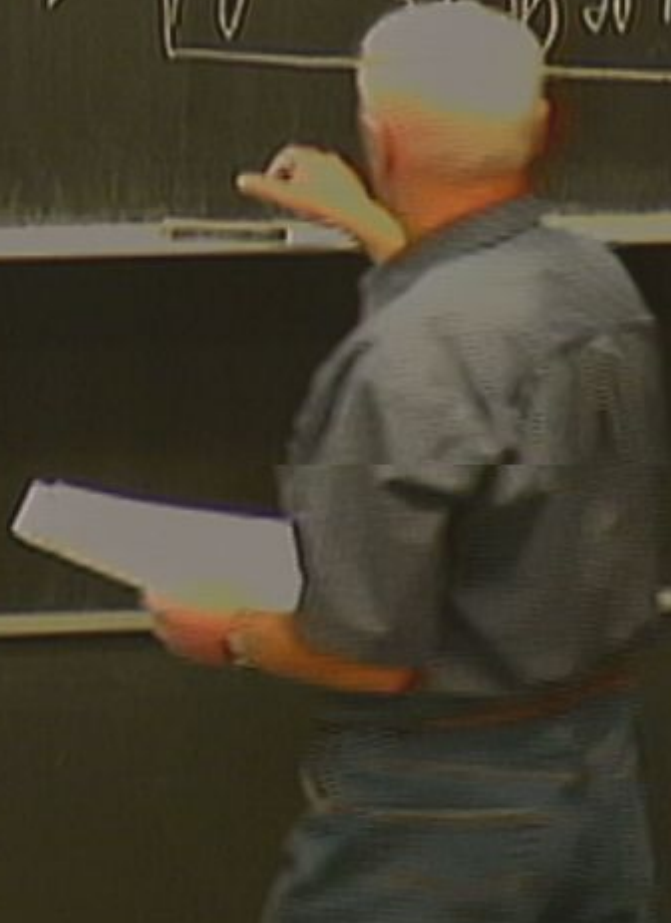


$\{a(\vec{p}), a(\vec{q})\} = \{b(\vec{p}), b(\vec{q})\} = \{\psi_a(\vec{x}), \psi_b(\vec{z})\} = 0$   
 $\Rightarrow \{a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}\} = \{b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$ , The rest of anti-commutators are zero. The ground state (vacuum):  $a_{\vec{q}}^r|0\rangle = b_{\vec{q}}^r|0\rangle = 0$   
 $T_{\nu}^{\mu} = ?$   $T_{\nu}^{\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\psi)} \partial_{\nu}\psi - \mathcal{L} \delta_{\nu}^{\mu}$ , However  $\mathcal{L} = 0$   
 for Dirac solutions  $\Rightarrow T_{\nu}^{\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\psi)} \partial_{\nu}\psi$   
 $\mathcal{H} =$





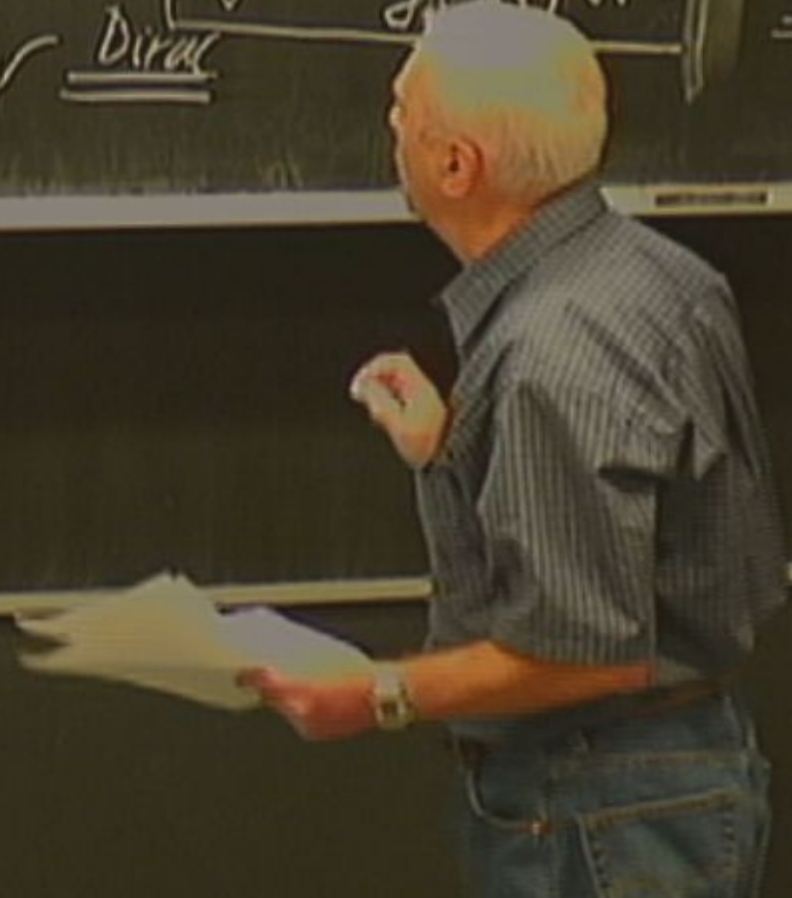
$\{a_{\vec{p}}^r, a_{\vec{q}}^{s+}\} = \{b_{\vec{p}}^r, b_{\vec{q}}^{s+}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$ ; The rest of anti-commutators are zero. The ground state (vacuum):  $a_{\vec{q}}^r |0\rangle = b_{\vec{q}}^r |0\rangle = 0$ .  
 $T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_{\nu} \psi)} \partial_{\nu} \psi - \mathcal{L}$ , However  $\mathcal{L} = 0$   
 for Dirac solutions  $\Rightarrow T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_{\nu} \psi)} \partial_{\nu} \psi = \bar{\psi} \gamma^{\mu} \partial_{\nu} \psi$   
 $\mathcal{H} = T^{00} =$



$\Rightarrow \{a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}\} = \{b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$ ; The rest of anti-commutators are zero. The ground state (vacuum):  $a_{\vec{q}}^r |0\rangle = b_{\vec{q}}^r |0\rangle = 0$ .

$T^M_{\nu} = ?$   $T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\psi)} \partial_{\nu}\psi - \mathcal{L} \delta^M_{\nu}$ , However  $\mathcal{L} = 0$  for Dirac solutions  $\Rightarrow T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\psi)} \partial_{\nu}\psi = \bar{\psi} \gamma^M \partial_{\nu}\psi$

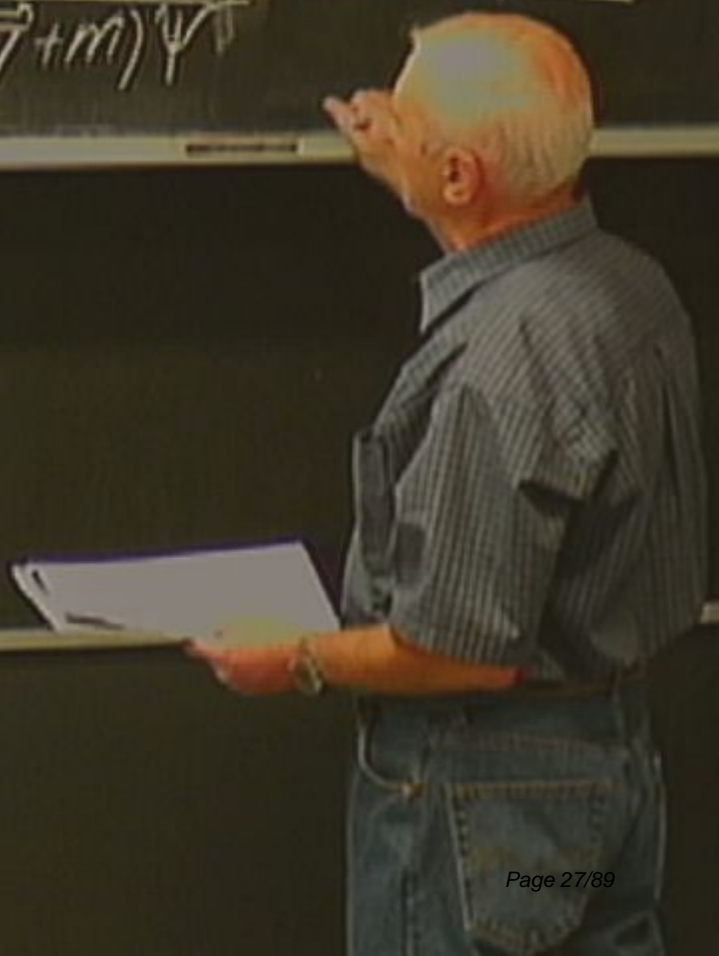
$\mathcal{H} = T^{00} = \bar{\psi} \gamma^0 \partial_0 \psi$  Dirac



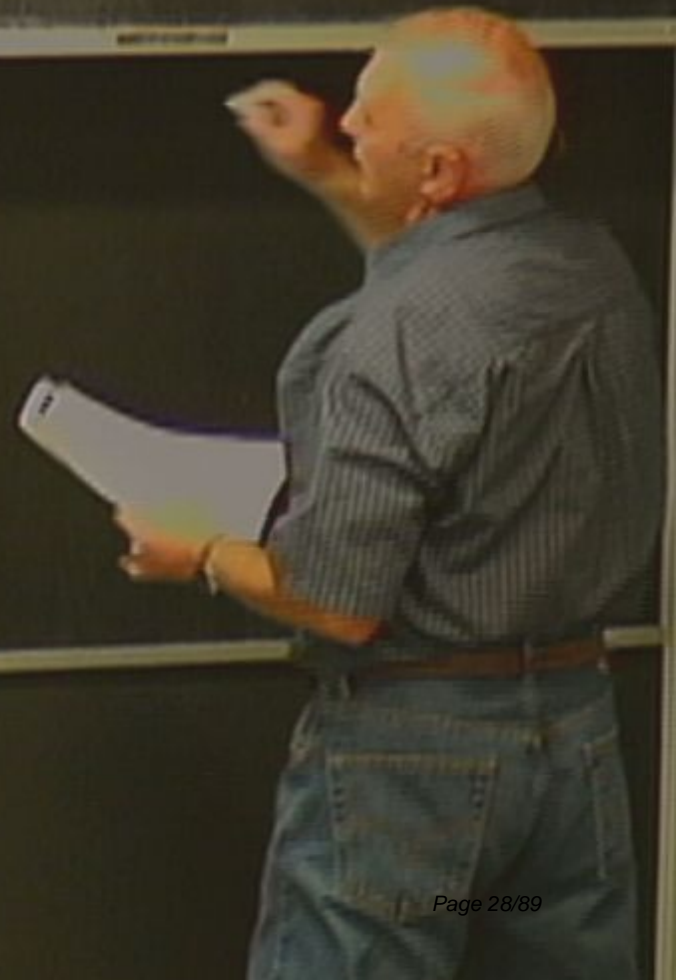
$\rightarrow \{a_{\vec{p}}, a_{\vec{q}}\} = \{b_{\vec{p}}, b_{\vec{q}}\} = 2\pi\delta^3(\vec{p}-\vec{q})$ , The rest of anti-commutators are zero. The ground state (vacuum):  $a_{\vec{q}}|0\rangle = b_{\vec{q}}|0\rangle = 0$

$T^M_{\nu} = ? \quad T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\Psi)} \partial_{\nu}\Psi - \mathcal{L} \delta^M_{\nu}$ , However  $\mathcal{L} = 0$  for Dirac solutions  $\Rightarrow T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\Psi)} \partial_{\nu}\Psi = \bar{\Psi} \gamma^M \partial_{\nu}\Psi$

$\mathcal{H} = T^{00} = \bar{\Psi} \gamma^0 \partial_0 \Psi \stackrel{\text{Dirac equation}}{=} \bar{\Psi} (-i\vec{\gamma} \cdot \vec{\nabla} + m) \Psi$



$T^M_{\nu} - ? \quad T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\Psi)} \partial_{\nu}\Psi - \mathcal{L} \delta^M_{\nu}$ , However  $\mathcal{L} = 0$   
 for Dirac solutions  $\Rightarrow T^M_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu}\Psi)} \partial_{\nu}\Psi = \bar{\Psi} i \gamma^M \partial_{\nu}\Psi$   
 $\mathcal{H} = T^{00} = \bar{\Psi} i \gamma^0 \partial_0 \Psi$  Dirac equation  $\Psi(-i \vec{\gamma} \cdot \vec{\nabla} + m)\Psi$ ,  $H = \int d^3x \mathcal{H}$



for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2} \frac{\delta \mathcal{L}}{\delta (\partial_\mu \Psi)} \partial_\nu \Psi = \bar{\Psi} (\gamma^\mu \partial_\nu + m) \Psi$

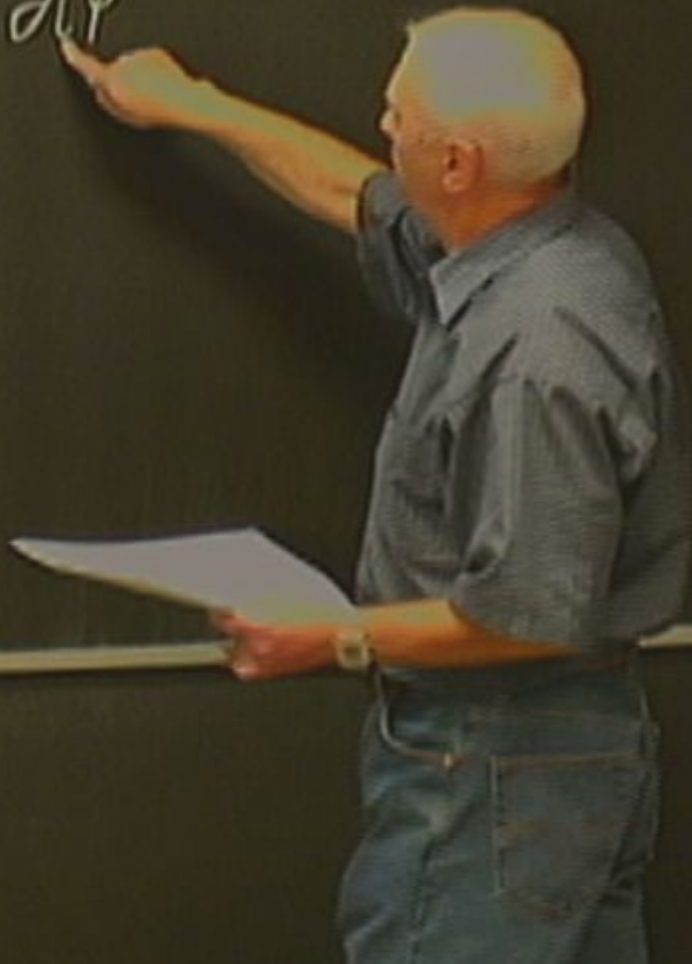
$\mathcal{L} = T^{00} = \bar{\Psi} (\gamma^0 \partial_0 + m) \Psi$  Dirac equation  $\bar{\Psi} (-i \vec{\gamma} \cdot \vec{\nabla} + m) \Psi$ ,  $H = \int d^3x \mathcal{H}$

$P^i = T^{0i} =$

for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2} \frac{\delta \mathcal{L}}{\delta (\partial_\mu \psi)} \partial_\nu \psi = \bar{\psi} \gamma^\mu \partial_\nu \psi$

$\mathcal{L} = T^{00} = \bar{\psi} i \gamma^0 \partial_0 \psi$  Dirac equation  $\bar{\psi} (-i \vec{\gamma} \cdot \vec{\nabla} + m) \psi$ ,  $H = \int d^3x \mathcal{H}$

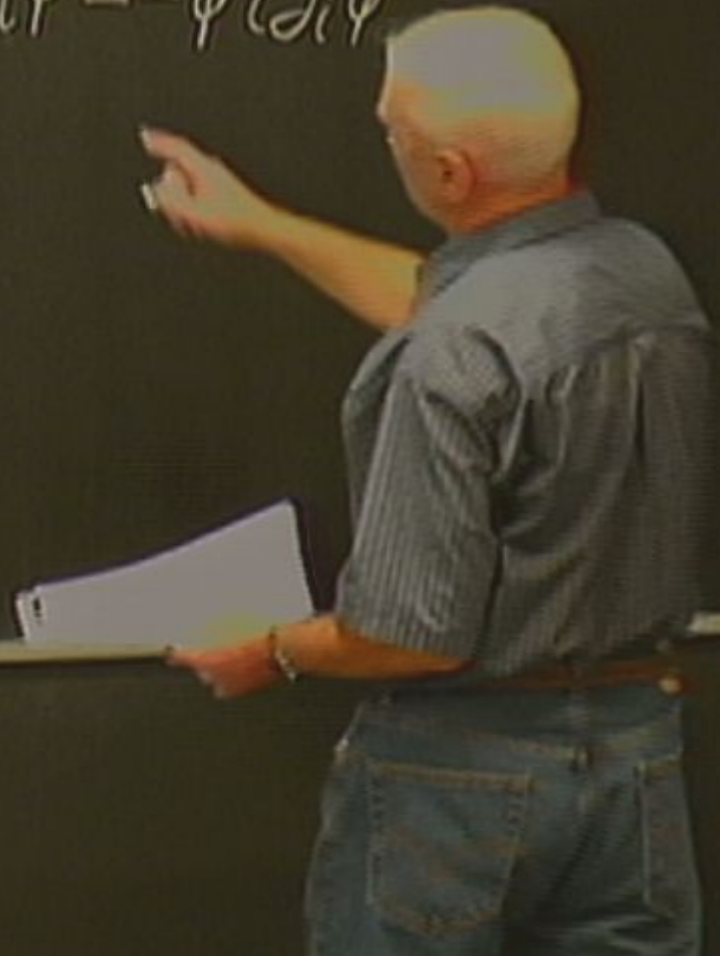
$P^i = T^{0i} = -\bar{\psi} i \gamma^0 \partial_i \psi$



for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2(\partial_\mu \Psi)} \partial_\nu \Psi = \bar{\Psi} \gamma^\mu \partial_\nu \Psi$

$\mathcal{H} = T^{00} = \bar{\Psi} i \gamma^0 \partial_0 \Psi$  Dirac equation  $\bar{\Psi} (-i \vec{\gamma} \cdot \vec{\nabla} + m) \Psi$ ,  $H = \int d^3x \mathcal{H}$

$P^i = T^{0i} = -\bar{\Psi} i \gamma^0 \partial_i \Psi = -\Psi \dot{\partial}_i \Psi$



for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2} \frac{\delta \mathcal{L}}{\delta (\partial_\mu \psi)} \partial_\nu \psi = \bar{\psi} \gamma^\mu \partial_\nu \psi$

$\mathcal{L} = T^{00} = \bar{\psi} (i \gamma^0 \partial_0 - i \vec{\gamma} \cdot \vec{\nabla} + m) \psi$  Dirac equation  $\psi$ ,  $H = \int d^3x \mathcal{H}$

$P^i = T^{0i} = -\bar{\psi} i \gamma^0 \partial_i \psi = -\psi^\dagger \partial_i \psi \Rightarrow \vec{P} = \psi^\dagger (-i \vec{\nabla}) \psi$



for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2} \frac{\delta \mathcal{L}}{\delta (\partial_\mu \psi)} \partial_\nu \psi = \bar{\psi} \gamma^{\mu\nu} \psi$

$\mathcal{H} = T^{00} = \bar{\psi} (i\gamma^0 \partial_0 + m) \psi$  Dirac equation  $\bar{\psi} (-i\vec{\gamma} \cdot \vec{\nabla} + m) \psi$ ,  $H = \int d^3x \mathcal{H}$

$\mathcal{P}^i = T^{0i} = -\bar{\psi} i\gamma^0 \partial_i \psi = -\psi^\dagger \partial_i \psi \Rightarrow \vec{\mathcal{P}} = \psi^\dagger (-i\vec{\nabla}) \psi$

$\vec{P} = \int d^3x \vec{\mathcal{P}} = \int d^3x \psi^\dagger (-i\vec{\nabla}) \psi$

for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2(2\pi)^4} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} = \frac{\bar{\psi} \gamma^\mu \partial^\nu \psi}{2(2\pi)^4}$

$\mathcal{L} = T^{00} = \bar{\psi} (i\gamma^0 \partial_0 \psi) \stackrel{\text{Dirac equation}}{=} \bar{\psi} (-i\vec{\gamma} \cdot \vec{\nabla} + m) \psi$ ,  $H = \int d^3x \mathcal{H}$

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$\vec{P} = \int d^3x \psi^\dagger (-i \vec{\nabla}) \psi$

$H = \int \frac{d^3p}{(2\pi)^3} (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s) - 2V \int \frac{d^3p}{(2\pi)^3}, E(10) = 0$



for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2(2\pi)^3} \partial_0 \psi = \bar{\psi} \gamma^\mu \partial^\nu \psi$

$$\mathcal{H} = T^{00} = \bar{\psi} i \gamma^0 \partial_0 \psi \stackrel{\text{Dirac equation}}{=} \bar{\psi} (-i \vec{\gamma} \cdot \vec{\nabla} + m) \psi, \quad H = \int d^3x \mathcal{H}$$

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for Dirac solutions  $\Rightarrow T^{\mu\nu} = \frac{2}{2(2\pi)^3} \partial^\mu \bar{\psi} \partial^\nu \psi = \bar{\psi} \gamma^\mu \partial^\nu \psi$

$\mathcal{L} = T^{00} = \bar{\psi} (i\gamma^0 \partial_0) \psi \stackrel{\text{Dirac equation}}{=} \bar{\psi} (-i\vec{\gamma} \cdot \vec{\nabla} + m) \psi, H = \int d^3x \mathcal{H}$

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$H = \int \frac{d^3p}{(2\pi)^3} \sum_s E_p (a_p^{s\dagger} a_p^s + b_p^{s\dagger} b_p^s) - 2 \int \frac{d^3p}{(2\pi)^3} E_p E(10) = 0$

$\vec{P} = \int \frac{d^3p}{(2\pi)^3} \sum_s \vec{p}$

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$H = \int \frac{d^3p}{(2\pi)^3} \sum_s E_p (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s) - 2 \int \frac{d^3p}{(2\pi)^3} \sum_s E_p (107) = 0$

$\vec{P} = \int \frac{d^3p}{(2\pi)^3} \sum_s \vec{p} (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s)$

Orthogonality relations

Using the orthogonality relations we get

$\mathcal{L}$  is invariant under  $\Psi(x) \rightarrow e^{i\alpha} \Psi(x), \overline{\Psi(x)} \rightarrow e^{-i\alpha} \overline{\Psi(x)}$

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Con current is  $j^\mu(x) = \bar{\psi} \gamma^\mu \psi$   
electric current in QED.

$$\psi(-p) = [\psi(p)]^\dagger = \frac{1}{\sqrt{2E_p}} \left[ (a_p^\dagger u^s(p) + b_p v^s(p))^\dagger \right] \quad (-p) = [\psi(p)]$$

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Conserved current is  $j^\mu(x) = \bar{\psi} \gamma^\mu \psi$  ← electric current in QED

$$Q = \int d^3x j^0(x) = \int d^3x \bar{\psi} \gamma^0 \psi = \int d^3x \psi^\dagger \psi = \int d^3x \psi^\dagger \psi$$

$$\psi(-p) = [\psi(p)]^\dagger = \frac{1}{\sqrt{2E_p}} \sum_s (a_{p,s}^\dagger u^{s\dagger}(-p) + b_{-p,s} v^s(p)) \quad (-p) = [\psi(p)]^\dagger$$

$\mathcal{L}$  is invariant under  $\psi(x) \rightarrow e^{i\alpha} \psi(x), \bar{\psi}(x) \rightarrow e^{-i\alpha} \bar{\psi}(x)$   
 Conserved current is  $j^\mu(x) = \bar{\psi} \gamma^\mu \psi$

$$Q = \int d^3x j^0(x) = \int d^3x \bar{\psi} \gamma^0 \psi = \int d^3x \psi^\dagger \psi$$

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s (a_{p,s}^\dagger a_{p,s}^\dagger + b_{-p,s} b_{-p,s}^\dagger)$$

$$\psi(-p) = [\psi(p)]^\dagger = \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^{s\dagger} u^s(p) + b_{-\vec{p}}^s v^s(p)) \quad (-p) = [\psi(p)]^\dagger$$

$\mathcal{L}$  is invariant under  $\psi(x) \rightarrow e^{i\alpha} \psi(x), \bar{\psi}(x) \rightarrow e^{-i\alpha} \bar{\psi}(x)$ .  
 Conserved current is  $j^\mu(x) = \bar{\psi} \gamma^\mu \psi$  ← electric current in QED.

$$Q = \int d^3x j^0(x) = \int d^3x \bar{\psi} \gamma^0 \psi = \int d^3x \psi^\dagger \psi = \int \frac{d^3p}{(2\pi)^3} \sum_s (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{-\vec{p}}^s b_{-\vec{p}}^{s\dagger}) = \int \frac{d^3p}{(2\pi)^3} \sum_s (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s - b_{-\vec{p}}^{s\dagger} b_{-\vec{p}}^s) +$$



$$\psi(-p) = [\psi(p)]^\dagger = \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^{s\dagger} u^s(p) + b_{-\vec{p}}^s v^s(p)) \quad (-p) = [\psi(p)]^\dagger$$

$\mathcal{L}$  is invariant under  $\psi(x) \rightarrow e^{i\alpha} \psi(x), \bar{\psi}(x) \rightarrow e^{-i\alpha} \bar{\psi}(x)$ .  
 Conserved current is  $j^\mu(x) = \bar{\psi} \gamma^\mu \psi$  ← electric current in QED.

$$Q = \int d^3x j^0(x) = \int d^3x \bar{\psi} \gamma^0 \psi = \int d^3x \psi^\dagger \psi = \int d^3x \psi^\dagger (i\not{\partial} + m) \psi =$$

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{-\vec{p}}^s b_{-\vec{p}}^{s\dagger}) = \int \frac{d^3p}{(2\pi)^3} \sum_s (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s - b_{-\vec{p}}^{s\dagger} b_{-\vec{p}}^s) +$$

$$+ 2V \int \frac{d^3p}{(2\pi)^3} \dots$$

Orthogonality relations:

$$\begin{aligned}
 u^{r\dagger}(\mathbf{p})u^s(\mathbf{p}) &= 2E_{\mathbf{p}}\delta^{rs} \\
 \bar{u}^{r\dagger}(\mathbf{p})\bar{u}^s(\mathbf{p}) &= 2E_{\mathbf{p}}\delta^{rs} \\
 u^{r\dagger}(\mathbf{p})\bar{u}^s(-\mathbf{p}) &= \\
 &= \bar{u}^{r\dagger}(-\mathbf{p})u^s(\mathbf{p}) = 0
 \end{aligned}$$

Using the orthogonality relations, we get:

$$\begin{aligned}
 [a_{\mathbf{p}}^r, a_{\mathbf{q}}^{s\dagger}] &= [b_{\mathbf{p}}^r, b_{\mathbf{q}}^{s\dagger}] = (2\pi)^3 \delta^3(\mathbf{p}-\mathbf{q}) \delta^{rs} \\
 [a_{\mathbf{p}}^r, a_{\mathbf{q}}^s] &= [b_{\mathbf{p}}^r, b_{\mathbf{q}}^s] = [a_{\mathbf{p}}^{r\dagger}, a_{\mathbf{q}}^{s\dagger}] = [b_{\mathbf{p}}^{r\dagger}, b_{\mathbf{q}}^{s\dagger}] = 0
 \end{aligned}$$

Write  $H$  in terms of these operators using the orthogonality relations:

$$H = \int d^3p \sum_s \left( E_{\mathbf{p}} a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^s - E_{\mathbf{p}} b_{\mathbf{p}}^{s\dagger} b_{\mathbf{p}}^s \right)$$

"vacuum"  $|0\rangle$ ,  $a_{\mathbf{p}}^s |0\rangle = b_{\mathbf{p}}^s |0\rangle = 0$ , states  $(b_{\mathbf{p}}^{s\dagger})^n |0\rangle$  have  $E(\mathbf{p}) < 0$   
 There is no ground state in this theory.

$$\begin{aligned}
 p^0 &= -E_{\vec{p}} \\
 \psi(x) &= \psi(\vec{p}) e^{i\vec{p}\cdot\vec{x}} \\
 (\gamma^{\mu} p_{\mu} + m)\psi(\vec{p}) &= 0 \\
 \psi^s(\vec{p}) &= \begin{pmatrix} \sqrt{E_{\vec{p}}} \chi^s \\ -\sqrt{E_{\vec{p}}} \eta^s \end{pmatrix}
 \end{aligned}$$

$$\psi^{\dagger}(\vec{p}) = [\psi(\vec{p})]^{\dagger} = \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s u^s(\vec{p}) + b_{\vec{p}}^{s\dagger} v^s(\vec{p}))$$

$H$  in terms of these operators using  
 commutation relations.

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s (E_{\vec{p}} a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s - E_{\vec{p}} b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s)$$

vacuum  $|0\rangle$ ,  $a_{\vec{p}}^s |0\rangle = b_{\vec{p}}^s |0\rangle = 0$ ; states  $(b_{\vec{p}}^{s\dagger})^n |0\rangle$  have  $E(\vec{p}) < 0$   
 ground state (vacuum)  $|0\rangle$  in this theory.

$$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$$\psi(x) = U(p) \begin{pmatrix} \psi^+(p) \\ \psi^-(p) \end{pmatrix}$$

$$(i \not{\partial} - m) \psi(x) = 0$$

$$U(p) = \begin{pmatrix} \psi^+(p) \\ \psi^-(p) \end{pmatrix}$$


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$$p^0 = \dots$$

$$\psi(x) = \dots$$

One-particle states:

$$\psi(\vec{p}) = [\psi(\vec{p})]^+ = \frac{1}{\sqrt{2E_{\vec{p}}}} \left[ a_{\vec{p}}^s u^s(\vec{p}) + b_{\vec{p}}^{\bar{s}} v^{\bar{s}}(\vec{p}) \right]$$

$$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$$\psi(x) = u(p) e^{-i p \cdot x}$$

$$(\gamma^\mu p_\mu - m) u(p) = 0$$

$$u(p) = \begin{pmatrix} \sqrt{E_{\vec{p}}} \chi \\ \sqrt{p^0} \sigma \cdot \hat{p} \chi \end{pmatrix}$$

$$p^0 = -E_{\vec{p}}$$

$$\psi(x) = v(p) e^{i p \cdot x}$$

$$(\gamma^\mu p_\mu + m) v(p) = 0$$

$$v(p) = \begin{pmatrix} \sqrt{E_{\vec{p}}} \chi \\ -\sqrt{p^0} \sigma \cdot \hat{p} \chi \end{pmatrix}$$

One-particle states:

$$| \vec{p}, s \rangle = \sqrt{2E_{\vec{p}}} a_{\vec{p}}^{s\dagger} | 0 \rangle$$

$$\psi(x) = \frac{1}{\sqrt{2\pi^3}} \int d^3p \left[ a_{\vec{p}} u^s(\vec{p}) e^{-i p \cdot x} + b_{\vec{p}} v^s(\vec{p}) e^{i p \cdot x} \right]$$

$$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$$\psi(x) = u(p) e^{-i p \cdot x}$$

$$(\gamma^\mu p_\mu - m) u(p) = 0$$

$$u(p) = \begin{pmatrix} \sqrt{E_{\vec{p}}} \xi^s \\ \sqrt{p^0} \end{pmatrix}$$

$$p^0 = -E_{\vec{p}}$$

$$\psi(x) = v(p) e^{i p \cdot x}$$

$$(\gamma^\mu p_\mu + m) v(p) = 0$$

$$v(p) = \begin{pmatrix} \sqrt{E_{\vec{p}}} \xi^s \\ -\sqrt{p^0} \end{pmatrix}$$

One-particle states:

$$|\vec{p}, s\rangle = \sqrt{2E_{\vec{p}}} a_{\vec{p}}^{s\dagger} |0\rangle, \quad \langle \vec{p}, r | \vec{q}, s \rangle =$$

$$= 2E_{\vec{p}} \delta^{rs} (2\pi)^3 \delta^3(\vec{p} - \vec{q})$$

$$\psi^{\dagger}(\vec{p}) = [\psi(\vec{p})]^\dagger = \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s u^s(\vec{p}) + b_{\vec{p}}^{s\dagger} v^s(\vec{p}))$$

$$\begin{aligned}
 & (a_{\vec{p}} + a_{-\vec{p}}) |q\rangle = 2a_{\vec{p}} a_{\vec{p}}^{\dagger} |q\rangle = 2|q\rangle \\
 & \text{Now H is } H = \int \frac{d^3p}{(2\pi)^3} \sum (E_p a_{\vec{p}}^{\dagger} a_{\vec{p}} - E_p b_{\vec{p}}^{\dagger} b_{\vec{p}}) \\
 & = \int \frac{d^3p}{(2\pi)^3} \sum (E_p a_{\vec{p}}^{\dagger} a_{\vec{p}} + E_p b_{\vec{p}}^{\dagger} b_{\vec{p}}) - \int \frac{d^3p}{(2\pi)^3} \sum (E_p a_{\vec{p}}^{\dagger} a_{\vec{p}} - E_p b_{\vec{p}}^{\dagger} b_{\vec{p}})
 \end{aligned}$$

Introduce new notations.  $\tilde{b}_{\vec{p}} = b_{\vec{p}}^{\dagger}$ ,  $\tilde{b}_{\vec{p}}^{\dagger} = b_{\vec{p}}$ , Vacuum  $|\tilde{0}\rangle$ .  $a_{\vec{p}} |\tilde{0}\rangle = \tilde{b}_{\vec{p}} |\tilde{0}\rangle = 0$ . Then all states  $a_{\vec{p}}^{\dagger} |\tilde{0}\rangle$ ,  $\tilde{b}_{\vec{p}}^{\dagger} |\tilde{0}\rangle$ ,  $a_{\vec{p}}^{\dagger} \tilde{b}_{\vec{p}}^{\dagger} |\tilde{0}\rangle$  have energy higher than the vacuum  $|\tilde{0}\rangle$ .  $H |\tilde{0}\rangle = -2V \int \frac{d^3p}{(2\pi)^3} \dots$

In order to understand better this theory, consider a toy model.  $b, b^{\dagger}, \{b, b^{\dagger}\} = b b^{\dagger} + b^{\dagger} b = 1, \{b, b\} = \{b^{\dagger}, b^{\dagger}\} = 0$   
 $\Rightarrow b^2 = 0, b^{\dagger 2} = 0; H = -E_1 b^{\dagger} b$ . This model has only 2

$$\begin{aligned}
 T^{\mu\nu} & \rightarrow T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} \psi)} \partial^{\nu} \psi - \delta^{\mu\nu} \mathcal{L} \\
 \text{for linear solutions } \Rightarrow T & = \int \frac{d^3x}{(2\pi)^3} \psi^{\dagger} (-i \vec{\nabla} \cdot \vec{\nabla} + m) \psi \\
 T^0 & = T^{00} = \int \frac{d^3x}{(2\pi)^3} \psi^{\dagger} (i \vec{\nabla} \cdot \vec{\nabla} + m) \psi
 \end{aligned}$$

$$\begin{aligned}
 \vec{P} & = T^{0i} = -\int \frac{d^3x}{(2\pi)^3} \psi^{\dagger} i \partial_i \psi \\
 \vec{P} & = \int \frac{d^3x}{(2\pi)^3} \vec{P} = \int \frac{d^3x}{(2\pi)^3} \psi^{\dagger} (-i \vec{\nabla} \psi) \\
 H & = \int \frac{d^3x}{(2\pi)^3} \sum E_p (a_{\vec{p}}^{\dagger} a_{\vec{p}} + b_{\vec{p}}^{\dagger} b_{\vec{p}}) \\
 \vec{P} & = \int \frac{d^3x}{(2\pi)^3} \sum \vec{p} (a_{\vec{p}}^{\dagger} a_{\vec{p}} + b_{\vec{p}}^{\dagger} b_{\vec{p}})
 \end{aligned}$$



Pauli Exclusion Principle  
(Fermi Statistics)  
 $\psi(\vec{x}) \leftarrow \psi(\vec{x}')$

$\psi^{\dagger} = b_p^{\dagger}$ , Vacuum states  $a_p^{\dagger} |0\rangle$ ,  
higher than the

$|0\rangle$   
theory, consider  
 $[b_i, b_j] = [b_i^{\dagger}, b_j^{\dagger}] = 0$   
total has only 2

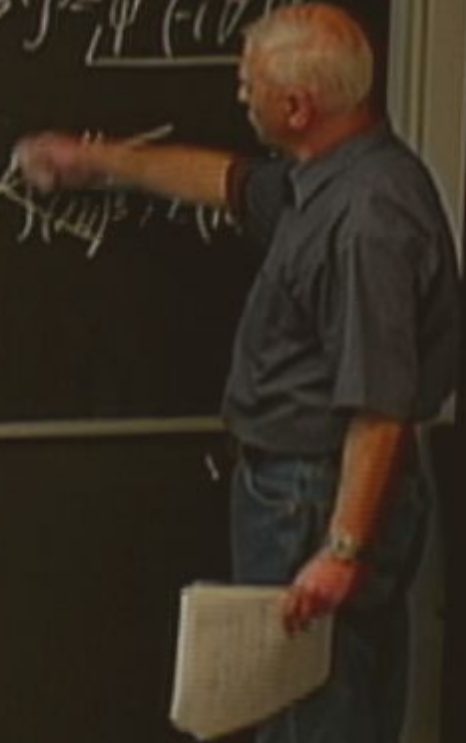
$$\mathcal{L} = T^{00} = \bar{\psi} (\gamma^0 \partial_0 \psi) \frac{\text{Dirac Equation}}{\text{Lagrangian}} \bar{\psi} (-i \vec{\gamma} \cdot \vec{\nabla} + m) \psi, \quad H = \int d^3x \mathcal{L}$$

$$P^i = T^{0i} = -\bar{\psi} \gamma^0 \partial_i \psi = -\psi \partial_i \psi \Rightarrow \vec{P} = \int d^3x \bar{\psi} (-i \vec{\nabla}) \psi$$

$$\vec{P} = \int d^3x \bar{\psi} (-i \vec{\nabla}) \psi$$

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s [p (a_p^{s\dagger} a_p^s + b_p^{s\dagger} b_p^s) - 2m \dots]$$

$$\vec{P} = \int \frac{d^3p}{(2\pi)^3} \sum_s \vec{p} (a_p^{s\dagger} a_p^s + b_p^{s\dagger} b_p^s)$$





$$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$$\psi(x) = u(p) e^{-i p \cdot x}$$

$$(\gamma^0 p_0 - m) u(p) = 0$$

$$u(p) = \begin{pmatrix} \sqrt{p_0} \xi^s \\ \sqrt{p_0} \xi^s \end{pmatrix}$$

$$p^0 = -E_{\vec{p}}$$

$$\psi(x) = v(p) e^{i p \cdot x}$$

$$(\gamma^0 p_0 + m) v(p) = 0$$

$$v(p) = \begin{pmatrix} \sqrt{p_0} \xi^s \\ -\sqrt{p_0} \xi^s \end{pmatrix}$$

One-particle states:

$$| \vec{p}, s \rangle = \sqrt{2 E_{\vec{p}}} a_{\vec{p}}^{s\dagger} | 0 \rangle, \quad \langle \vec{p}, r | \vec{q}, s \rangle =$$

$$= 2 E_{\vec{p}} \delta^{rs} (2\pi)^3 \delta^3(\vec{p} - \vec{q})$$

Lorentz transformations of  $\psi(x)$ :

$$[\psi(\vec{p})]^\dagger = \frac{1}{\sqrt{2 E_{\vec{p}}}} \left[ a_{\vec{p}}^s u^s(\vec{p}) + b_{\vec{p}}^{s\dagger} v^s(\vec{p}) \right]$$

$$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$$\psi(x) = u(p) e^{-i p \cdot x}$$

$$(\gamma^\mu p_\mu - m) u(p) = 0$$

$$u(p) = \begin{pmatrix} \sqrt{E_{\vec{p}}} \xi^s \\ \sqrt{p^0} \xi^s \end{pmatrix}$$

$$p^0 = -E_{\vec{p}}$$

$$\psi(x) = v(p) e^{i p \cdot x}$$

$$(\gamma^\mu p_\mu + m) v(p) = 0$$

$$v^s(p) = \begin{pmatrix} \sqrt{E_{\vec{p}}} \xi^s \\ -\sqrt{p^0} \xi^s \end{pmatrix}$$

One-particle states:

$$| \vec{p}, s \rangle = \sqrt{2 E_{\vec{p}}} a_{\vec{p}}^{s\dagger} | 0 \rangle, \quad \langle \vec{p}, r | \vec{q}, s \rangle =$$

$$= 2 E_{\vec{p}} \delta^{rs} (2\pi)^3 \delta^3(\vec{p} - \vec{q})$$

Lorentz transformations of  $\psi(x)$ :

$$U(\Lambda) \psi(x) U(\Lambda)^{-1} = \Lambda_{\frac{1}{2}} \psi(\Lambda^{-1} x)$$

$b_{\vec{p}} | 0 \rangle = 0$ , states  $(b_{\vec{p}}) | 0 \rangle$  have  $E(\vec{p}) < 0$   
 but in this theory

$$(\gamma^0 p_0 - m) u(p) = 0$$

$$u(p) = \begin{pmatrix} \sqrt{E_p} \xi^s \\ \sqrt{E_p} \xi^s \end{pmatrix}$$

$$p^0 = E_p$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \psi(p)$$

$$(\gamma^0 p_0 + m) v(p)$$

$$v^s(p) =$$

$$|p, s\rangle = \sqrt{2E_p} a_p^{s\dagger} |0\rangle, \langle p, r | q, s \rangle =$$

$$= 2E_p \delta^{rs} (2\pi)^3 \delta^3(p - q)$$

Lorentz transformations of  $\psi(x)$ :

$$U(\Lambda) \psi(x) U(\Lambda)^{-1} = \Lambda_{\frac{1}{2}} \psi(\Lambda^{-1}x)$$

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s (E_p a_p^{s\dagger} a_p^s - E_p b_p^{s\dagger} b_p^s)$$

$a_p^s |0\rangle = b_p^s |0\rangle = 0$ , states  $(b_p^{s\dagger})^n |0\rangle$  have  $E(p) < 0$   
 (vacuum)  $p$  in this theory.

$$u(p) = \begin{pmatrix} \sqrt{p^0} \xi^s \\ \sqrt{p^0} \eta^s \end{pmatrix}$$

$$p^0 = -E_{\vec{p}}$$

$$\psi(x) = \int d^3p \, e^{i p \cdot x}$$

$$(Y^{\mu\nu} p_{\mu} + m) \psi(p) = 0$$

$$\psi^s(p) = \begin{pmatrix} \sqrt{p^0} \xi^s \\ -\sqrt{p^0} \eta^s \end{pmatrix}$$

Lorentz transformations of  $\psi(x)$ :

$$U^{-1}(\Lambda) \psi(x) U(\Lambda) = \Lambda_{\frac{1}{2}} \psi(\Lambda^{-1}x)$$

$$U^{-1}(\Lambda) \psi(x) U(\Lambda) = U^{-1} \left( \frac{d^3p}{(2\pi)^3} \sum_s (a_{\vec{p}}^s \psi^s(p)) e^{-i p \cdot x} + \dots \right)$$

$$U(p) U(-p) = U^{-1}(-p) U^s(p) = 0$$

Write  $H$  in terms of the orthogonality relations

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s (E_{\vec{p}} a_{\vec{p}}^s \dots)$$

There is "vacuum"  $|0\rangle$ ,  $a_{\vec{p}}^s |0\rangle = b_{\vec{p}}^s |0\rangle = 0$  for  $E(\vec{p}) < 0$

There is ground state (vacuum) in this theory

$$\begin{aligned}
 \psi(x) &= \psi(p) \\
 (\gamma^{\mu} p_{\mu} + m) \psi(p) &= 0 \\
 \psi^s(p) &= \begin{pmatrix} \sqrt{p^0} \chi^s \\ -\sqrt{p^0} \eta^s \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 U(A) \psi(x) U(A)^{-1} &= \Lambda_{\frac{1}{2}} \psi(\Lambda^{-1}x) \\
 U^{-1}(A) \psi(x) U(A) &= U^{-1} \left( \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s \chi^s(p) e^{-ipx} + b_{\vec{p}}^{s\dagger} \psi^s(p) e^{ipx} \right) U
 \end{aligned}$$

$$\begin{aligned}
 U^{\dagger}(p) U^s &= 2E_p \delta^{rs} \\
 U^{\dagger}(\vec{p}) U^s(-\vec{p}) &= \\
 &= U^{\dagger}(-\vec{p}) U^s(\vec{p}) =
 \end{aligned}$$

$$[a_{\vec{p}}^{r\dagger}, a_{\vec{q}}^{s\dagger}] = [b_{\vec{p}}^{r\dagger}, b_{\vec{q}}^{s\dagger}] = 0$$

There is "vacuum"  $|0\rangle$  ground state  
 $|0\rangle = 0$  this theory. States  $(b_{\vec{p}}^{s\dagger})^n |0\rangle$  have  $E(\vec{p}) < 0$

$$\psi(x) = \psi(p) e^{i p x}$$

$$(p^2 - m^2) \psi(p) = 0$$

$$\psi^s(p) = \begin{pmatrix} \sqrt{E_p} \chi^s \\ -\sqrt{E_p} \eta^s \end{pmatrix}$$

$$U^{-1}(A) \psi(x) U(A) = \Lambda_{\frac{1}{2}} \psi(\Lambda^{-1} x)$$

$$U^{-1}(A) \psi(x) U(A) = U^{-1} \left( \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s \chi^s(p) e^{-i p x} + b_{\vec{p}}^{s\dagger} \psi^s(p) e^{i p x}) U^{-1} \right)$$

use  $U^{-1}(A) a_{\vec{p}}^s U(A) = \frac{\sqrt{E_{\Lambda^{-1} \vec{p}}}}{\sqrt{E_p}} a_{\Lambda^{-1} \vec{p}}^s$

$$\psi^{\dagger}(\vec{p}) \psi(\vec{p}) = 2E_p \delta^{ss}$$

$$U^{-1}(\vec{P}) \psi^s(-\vec{P}) = \psi^{\dagger}(-\vec{P}) U^s(\vec{P}) = 0$$

$$[a_{\vec{p}}, a_{\vec{q}}^s] = [b_{\vec{p}}, b_{\vec{q}}^s] = 0$$

$$[a_{\vec{p}}, a_{\vec{q}}^{s\dagger}] = [b_{\vec{p}}, b_{\vec{q}}^{s\dagger}] = \delta_{\vec{p}\vec{q}} \delta^{ss}$$

Write  $H$  in terms of these operators using the orthogonality

$$H = \int \frac{d^3 p}{(2\pi)^3} \sum_s$$

$$\begin{pmatrix} b_{\vec{p}}^{s\dagger} & b_{\vec{p}}^s \\ b_{\vec{p}}^s & b_{\vec{p}}^{s\dagger} \end{pmatrix} \begin{pmatrix} a_{\vec{p}}^s \\ a_{\vec{p}}^{s\dagger} \end{pmatrix} |0\rangle$$

have  $E(\vec{p}) < 0$

"vacuum"  $|0\rangle$ ,  $a_{\vec{p}}^s |0\rangle = b_{\vec{p}}^s |0\rangle = 0$   
 There is no ground state (vacuum)

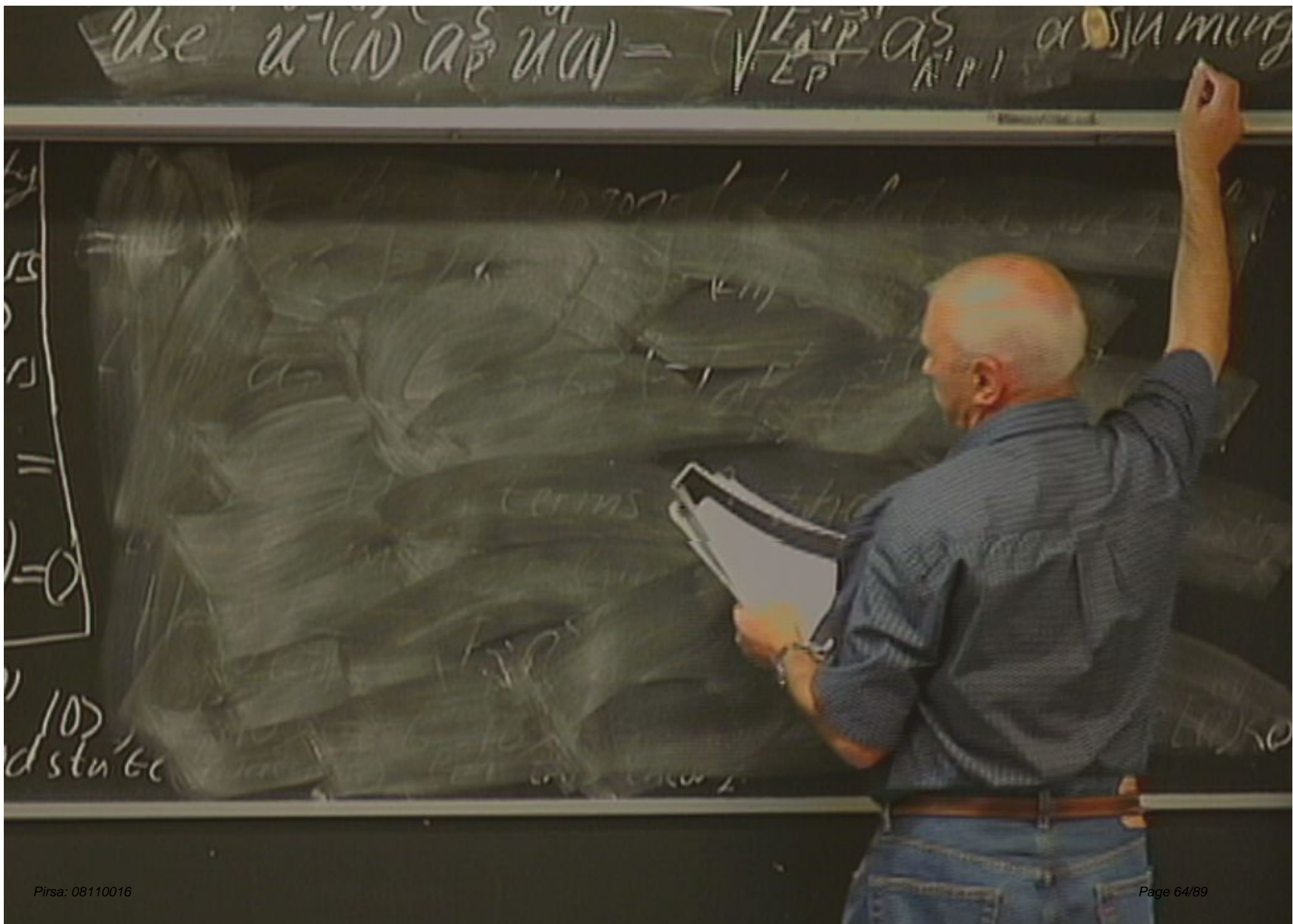
transformations of  $\psi(x)$

$$U(\Lambda)\psi(x)U(\Lambda)^{-1} = \Lambda_{\frac{1}{2}}\psi(\Lambda^{-1}x)$$

$$U^{-1}(\Lambda)\psi(x)U(\Lambda) = U^{-1} \left( \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s u^s(p) e^{-ipx} + b_{\vec{p}}^{s\dagger} v^s(p) e^{ipx}) \right) U$$

$$U^{-1}(\Lambda) a_{\vec{p}}^s U(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1}\vec{p}}}{E_p}} a_{\Lambda^{-1}\vec{p}}^s$$





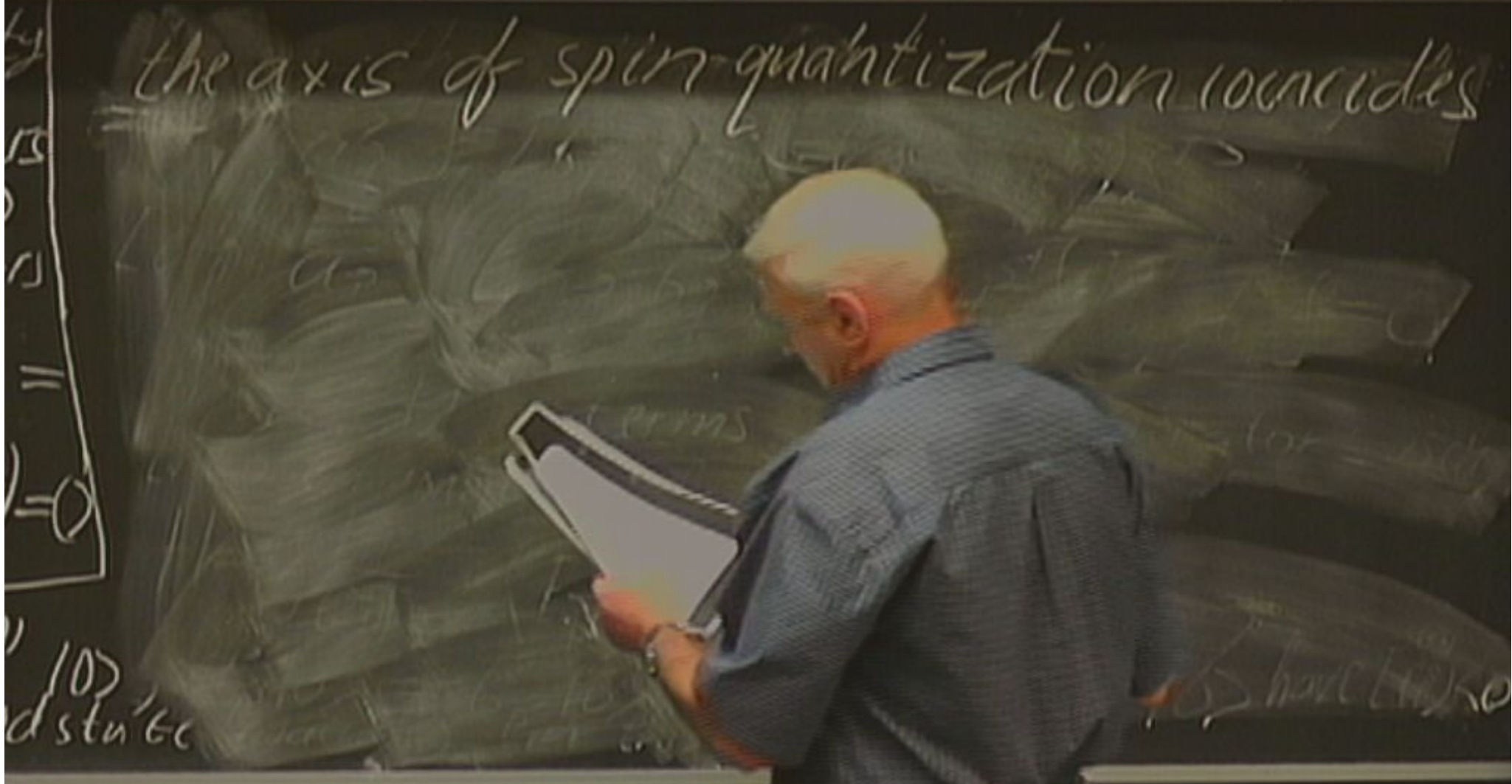
$$\text{Use } u^T(N) a_P^S u(N) = \frac{\sqrt{E_A^T P^T} a_P^S}{\sqrt{E_P}} a_{P|P}^S \text{ assuming}$$

*[The main body of the chalkboard contains numerous scribbles and faint, mostly illegible handwritten notes. Some visible words include 'terms' and 'dist'.]*



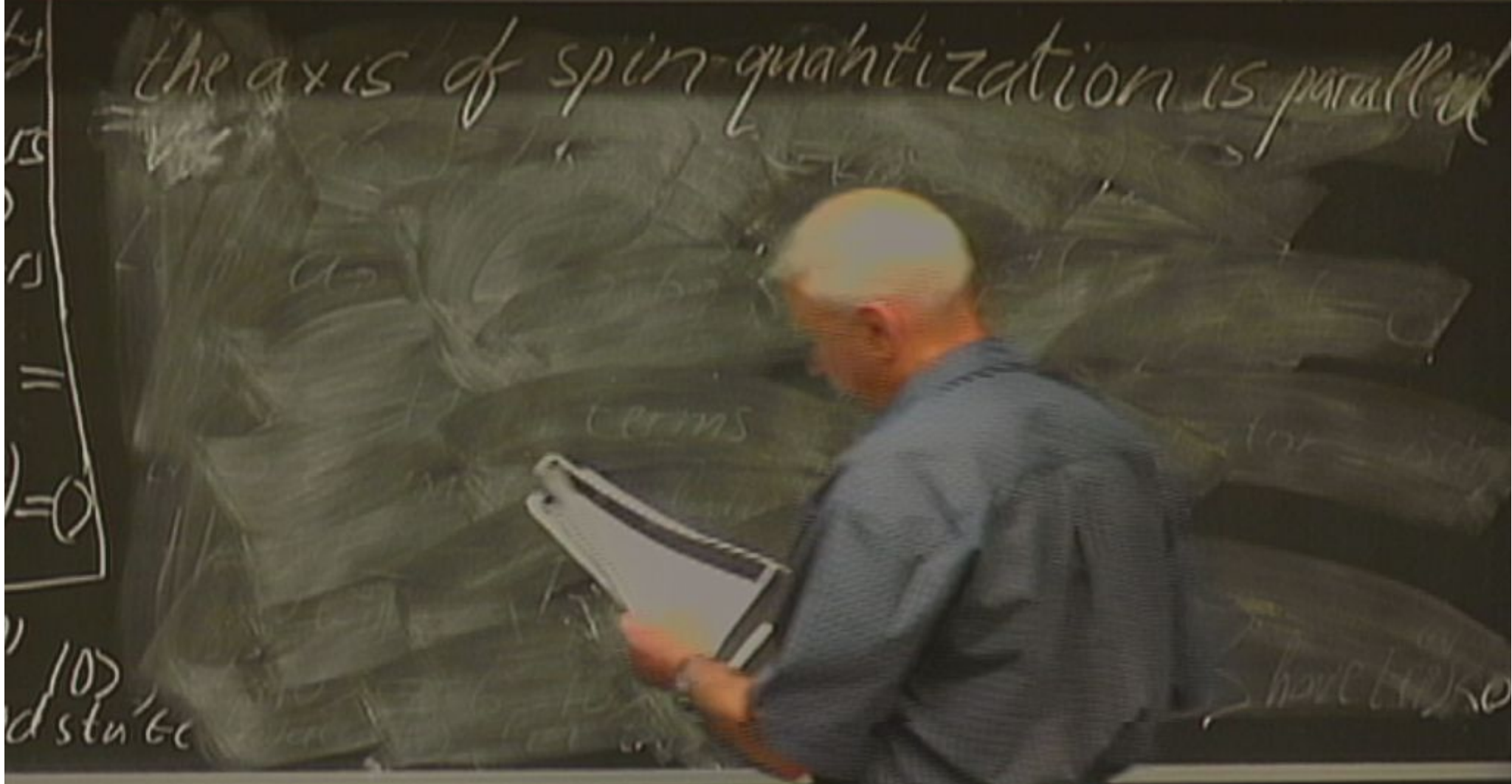
Use  $u^{-1}(N) a_{\vec{p}}^{\pm} u(N) = \sqrt{\frac{E_{\vec{p}} \mp p_z}{E_{\vec{p}}}} a_{N^{\pm} \vec{p}}^{\pm}$  assuming

the axis of spin quantization coincides



Use  $u^\dagger(N) a_P^s u(N) = \sqrt{\frac{E_P + m c^2}{2 E_P}} a_{N|P|}^s$  assuming

the axis of spin quantization is parallel



Use  $u^{-1}(N) a_{\vec{p}}^s u(N) = \sqrt{\frac{E + \vec{p} \cdot \vec{\sigma}}{2E}} a_{N, \vec{p}, 1}^s$  assuming

the axis of spin quantization is parallel  
to the rotation or boost axis

terms of the ... for ...  
to have these

$$\begin{aligned}
 & a_{\vec{p}} + b_{\vec{p}}^\dagger \hat{p}^s(p) e^{i(\vec{p} \cdot \vec{x} - \omega t)} \\
 & \text{Use } U^{-1}(N) a_{\vec{p}}^s U(N) = \frac{\sqrt{E_{\vec{p}} - p^s}}{E_{\vec{p}}} a_{\vec{p}}^s \quad \text{assuming}
 \end{aligned}$$

to the rotated and boost axis  
 Rewrite

$$\frac{1}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \sum_{\vec{p}} (a_{\vec{p}}^s) e^{-i(\vec{p} \cdot \vec{x} - \omega t)}$$

$$\frac{1}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \sqrt{E_{\vec{p}} - p^s} a_{\vec{p}}^s$$

... have there

Use  $u^{-1}(N) a_{\vec{p}}^s u(N) = \sqrt{\frac{E_{N'} + p_z}{E_{N'}}} a_{N'\vec{p}}^s$  assuming

the axis of spin quantization is parallel  
to the rotation or boost axis

$$K_{\vec{p}} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}}^s = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sqrt{\frac{E_{N'} + p_z}{E_{N'}}} a_{N'\vec{p}}^s$$

terms of the creation and annihilation operators  
for the boosted frame

Use  $u^{-1}(N) a_{\vec{p}}^s u(N) = \sqrt{\frac{E_{\vec{p}}}{E_{\vec{p}'}}} a_{\vec{p}'1}^s$  assuming

the axis of spin quantization is parallel  
to the rotation or boost axis

Rewrite  $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}}$

$\int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sqrt{\frac{E_{\vec{p}'}}{E_{\vec{p}}}} a_{\vec{p}'1}^s$

Then  $u^{-1}(N) \psi(x) u(N) =$

$|0\rangle$   
disturb

have the

Use  $u^{-1}(N) a_{\vec{p}}^s u(N) = \sqrt{\frac{E_{\vec{p}} + N \cdot \vec{p}}{E_{\vec{p}}}} a_{\vec{p}}^s$  assuming

the axis of spin quantization is parallel  
to the rotation or boost axis

Then 
$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} a_{\vec{p}}^s = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \sqrt{\frac{E_{\vec{p}} + N \cdot \vec{p}}{E_{\vec{p}}}} a_{\vec{p}}^s$$

Then 
$$\langle N | \psi(x) | N \rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \sqrt{\frac{E_{\vec{p}} + N \cdot \vec{p}}{E_{\vec{p}}}} a_{\vec{p}}^s u(\vec{p}) e^{-ipx}$$

Use  $u^{-1}(\Lambda) a_{\vec{p}}^s u(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1}\vec{p}}}{E_{\vec{p}}}} a_{\Lambda^{-1}\vec{p}}^s$  assuming

the axis of spin quantization is parallel  
to the rotation or boost axis

Rewrite  $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}}^s = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sqrt{\frac{E_{\Lambda^{-1}\vec{p}}}{E_{\vec{p}}}} a_{\Lambda^{-1}\vec{p}}^s$

$$u^{-1}(\Lambda) \psi(x) u(\Lambda) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sqrt{\frac{E_{\Lambda^{-1}\vec{p}}}{E_{\vec{p}}}} a_{\Lambda^{-1}\vec{p}}^s u(\vec{p}) e^{-ipx}$$



Use  $u^{-1}(\Lambda) a_{\vec{p}}^s u(\Lambda) = \sqrt{\frac{E_{\Lambda'P'}}{E_P}} a_{\Lambda'P'}^s$  assuming

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} = \int \frac{d^3p}{(2\pi)^3 2E_p} \sqrt{\frac{E_{\Lambda'P'}}{E_P}} a_{\vec{p}}^s$

Then  $u^{-1}(\Lambda) \psi(x) u(\Lambda) = \dots$

use  $\tilde{P} = \Lambda^{-1} P$

10) distance

$\sqrt{2E_{\Lambda'P'}} a_{\Lambda'P'}^s u(\vec{p}) (+)$

Use  $u^{-1}(\Lambda) a_{\vec{p}}^s u(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1}\vec{p}}}{E_{\vec{p}}}} a_{\Lambda^{-1}\vec{p}}^s$  assuming

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s = \int \frac{d^3p}{(2\pi)^3} a_{\vec{p}}^s$

Then  $u^{-1}(\Lambda) \psi(x) u(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2E_p}$

use  $\tilde{p} = \Lambda^{-1} p$   
 $\int \frac{d^3\tilde{p}}{(2\pi)^3 2E_{\tilde{p}}} \sqrt{2E_{\tilde{p}}} a_{\tilde{p}}^s$

Use  $u^{-1}(\Lambda) a_{\vec{p}}^s u(\Lambda) = \sqrt{\frac{E_{\Lambda' \vec{p}}}{E_{\vec{p}}}} a_{\Lambda' \vec{p}}^s$  assuming

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sqrt{\frac{E_{\Lambda' \vec{p}}}{E_{\vec{p}}}} a_{\Lambda' \vec{p}}^s$

Then  $u^{-1}(\Lambda) \psi(x) u(\Lambda) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sqrt{\frac{E_{\Lambda' \vec{p}}}{E_{\vec{p}}}} a_{\Lambda' \vec{p}}^s u(\vec{p}) e^{-i p \cdot x}$

use  $\vec{p} = \Lambda' \vec{p}$   $\int \frac{d^3 \tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} \sqrt{2E_{\tilde{p}}} a_{\tilde{p}}^s u(\tilde{p}) e^{-i \tilde{p} \cdot x}$

10) distance

$$\begin{aligned}
 & \psi^{\dagger}(\vec{p}, +m) \psi(\vec{p}, -m) = 0 \\
 & \psi^{\dagger}(\vec{p}) = \begin{pmatrix} \sqrt{E_p} \chi^{\dagger} \\ -\sqrt{E_p} \eta^{\dagger} \end{pmatrix} \\
 & U^{-1}(N) V(\alpha) U(N) = U^{-1} \left( \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s \psi^{\dagger}(\vec{p}, s) e^{-i\vec{p}\cdot\vec{x}} + b_{\vec{p}}^{s\dagger} \psi^{\dagger}(\vec{p}, s) e^{i\vec{p}\cdot\vec{x}}) \right) \\
 & \text{Use } U^{-1}(N) a_{\vec{p}}^s U(N) = \sqrt{\frac{E_{\vec{p}}}{E_p}} a_{\vec{p}}^s \text{ assuming }
 \end{aligned}$$

Orthogonality relations:

$$\begin{aligned}
 & \psi^{\dagger}(\vec{p}) \psi(\vec{p}) = 2E_p \chi^{\dagger} \chi \\
 & \psi^{\dagger}(\vec{p}) \psi(\vec{q}) = 2E_p \delta^3(\vec{p}-\vec{q}) \chi^{\dagger} \chi \\
 & \psi^{\dagger}(\vec{p}) \psi^{\dagger}(-\vec{p}) = 0 \\
 & \psi^{\dagger}(-\vec{p}) \psi(\vec{p}) = 0
 \end{aligned}$$

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3 2E_p} \sqrt{2E_p} a_{\vec{p}}^s$

Then  $U^{-1}(N) V(\alpha) U(N) = \int \frac{d^3 p}{(2\pi)^3 2E_p} \sqrt{2E_p} a_{\vec{p}}^s \psi^{\dagger}(\vec{p}, s) e^{-i\vec{p}\cdot\vec{x}} + \dots$

use  $\vec{p} = \vec{p}'$   $\int \frac{d^3 \vec{p}'}{(2\pi)^3 2E_{\vec{p}'}} \sqrt{2E_{\vec{p}'}} a_{\vec{p}'}^s U^{-1}(N) \psi^{\dagger}(\vec{p}', s) e^{-i\vec{p}'\cdot\vec{x}}$

"vacuum"  $|0\rangle$   
There is no ground state

$$e^{-iP \cdot x} = e^{-i\vec{P} \cdot \vec{x}} = e^{-iP \cdot \vec{x}} \quad \vec{b}_{\vec{p}} = \vec{b}_{\vec{p}}^{\dagger}, \quad \vec{b}_{\vec{p}} = \vec{b}_{\vec{p}}^{\dagger}, \quad \text{Vacuum}$$

Then all states  $a_{\vec{p}}^{\dagger} |0\rangle$  have energy higher than the

$$H|0\rangle = -2V \left( \frac{d^3p}{(2\pi)^3} \right) H_{\text{or}} \left( \frac{d^3p}{(2\pi)^3} \right) |0\rangle$$

In order to understand better this theory, consider a toy model:  $b, b^{\dagger}, \{b, b^{\dagger}\} = b b^{\dagger} + b^{\dagger} b = 1, \{b, b\} = \{b^{\dagger}, b^{\dagger}\} = 0$   
 $\Rightarrow b^2 = 0, b^{\dagger 2} = 0; \quad H = -E_b b^{\dagger} b.$  This model has only 2

Rewrite  $\int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3 2E_p} \sqrt{2E_p} a_{\vec{p}}^s$

Then  $U^{-1}(\Lambda) V(x) U(\Lambda) = \int \frac{d^3 p}{(2\pi)^3 2E_p} \sqrt{2E_p} a_{\vec{p}}^s U(\Lambda \vec{p}) e^{-i(\Lambda \vec{p} \cdot x)} + \dots$

use  $\vec{p} = \Lambda^{-1} \vec{p}'$

$\int \frac{d^3 \tilde{p}}{(2\pi)^3 2E_{\tilde{p}}} \sqrt{2E_{\tilde{p}}} a_{\tilde{p}}^s U(\Lambda \tilde{p}) e^{-i(\tilde{p} \cdot \tilde{x})} + \dots$

$\vec{p} = 0$   
 $s(\vec{p}) = 0$

um" 10)  
 found stu' 6c

$$U^S(p) = \begin{pmatrix} \sqrt{p_0} & 1 \\ -\sqrt{p_0} & \eta^S \end{pmatrix}$$

$$U^{-1}(\Lambda) \psi(x) U(\Lambda) = U^{-1} \int \frac{d^3 p}{(2\pi)^3} + b_{\vec{p}}^{\dagger} U^S(p) e^{i p x} U = \sqrt{\frac{E_{\Lambda^{-1} \vec{p}}}{E_{\vec{p}}}}$$

$$U^S(\Lambda \vec{p}) = \Lambda_{11} U^S(\vec{p})$$

$$\text{Use } U^{-1}(\Lambda) a_{\vec{p}}^S U(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1} \vec{p}}}{E_{\vec{p}}}}$$

$$U^{-1}(\vec{p}) \psi(\vec{p}) = 2E_{\vec{p}} S^1$$

$$U^{-1}(\vec{p}) \psi(\vec{p}) = 0$$

vacuum  $|0\rangle$   
 is ground state

Rewrite  $\int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}}^S =$

$$\text{Then } U^{-1}(\Lambda) \psi(x) U(\Lambda) = \int \frac{d^3 p}{(2\pi)^3} 2E_{\vec{p}}$$

$$\text{use } \vec{p} = \Lambda^{-1} \vec{p} \int \frac{d^3 \tilde{p}}{(2\pi)^3} 2E_{\tilde{p}} \sqrt{2E_{\tilde{p}}} a_{\tilde{p}}^S U$$

$$p^0 = -E_{\vec{p}} \\ \psi(x) = \delta(p) e^{i p x} \\ (Y^{\mu\nu} p_{\mu} + m) \delta(p) = 0 \\ \delta^s(p) = \begin{pmatrix} \sqrt{V p^0} & \eta^s \\ -\sqrt{V p^0} & \eta^s \end{pmatrix} \\ \psi^s(\Lambda \tilde{p}) = \Lambda_{1/2} \psi^s(\tilde{p}) \quad (10)$$

Lorentz transformations of  $\psi(x)$ :

$$U^{-1}(\Lambda) \psi(x) U(\Lambda) = \Lambda_{1/2} \psi(\Lambda^{-1} x)$$

$$U^{-1}(\Lambda) \psi(x) U(\Lambda) = U^{-1} \left( \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s \eta^s p) e^{-i p x} + b_{\vec{p}}^{s\dagger} \delta^s(p) e^{i p x} \right) U = \sqrt{\frac{E_{\vec{p}}}{E_{\vec{p}'}}} a_{\vec{p}'}^s U(\Lambda \tilde{p}) e^{-i p' x} + \dots$$

assuming

$$U^{-1}(\tilde{p}) \psi^s(-\tilde{p}) = \psi^s(-\tilde{p}) U^{-1}(\tilde{p}) = 0$$

"vacuum"  $|0\rangle$   
 There is ground state

Then  $U^{-1}(\tilde{p}) \psi^s(-\tilde{p}) = \dots$

$$U^{-1}(\tilde{p}) \psi^s(-\tilde{p}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s (a_{\vec{p}}^s \eta^s p) e^{-i p x} + \dots$$

use  $\tilde{p} = \Lambda^{-1} p$



(IV)  $U^{-1}(\Lambda) a_{\vec{p}}^s U(\Lambda) = \sqrt{\frac{E_{\Lambda'P}}{E_P}} a_{\Lambda^{-1}\vec{p}}^s$  a summary

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3P}{(2\pi)^3} \frac{1}{\sqrt{2E_P}} a_{\vec{p}}^s = \int \frac{d^3P}{(2\pi)^3 2E_P} \sqrt{2E_P} a_{\vec{p}}^s$

Then  $U^{-1}(\Lambda) \psi(x) U(\Lambda) = \int \frac{d^3P}{(2\pi)^3 2E_P} \sqrt{2E_{\Lambda'P}} a_{\Lambda^{-1}\vec{p}}^s U^s(\Lambda P) e^{-i(\Lambda P) \cdot x}$

use  
 $\tilde{P} = \Lambda^{-1} P$

$\int \frac{d^3\tilde{P}}{(2\pi)^3 2E_{\tilde{P}}} \sqrt{2E_{\tilde{P}}} a_{\tilde{p}}^s U^s(\Lambda P) e^{-i(\tilde{P}) \cdot \Lambda^{-1}x}$  use (IV)

Now  $H(S)$

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_{\vec{s}} \left( \epsilon_{\vec{p}} a_{\vec{p}}^{\vec{s}} a_{\vec{p}}^{\vec{s}} - E_{\vec{p}} b_{\vec{p}}^{\vec{s}} b_{\vec{p}}^{\vec{s}} \right)$$

Pauli Principle. (Fermi Statistics)

$$= \int \frac{d^3p}{(2\pi)^3} \sum_{\vec{s}} \left( \epsilon_{\vec{p}} a_{\vec{p}}^{\vec{s}} a_{\vec{p}}^{\vec{s}} + E_{\vec{p}} b_{\vec{p}}^{\vec{s}} b_{\vec{p}}^{\vec{s}} \right) - \int \frac{d^3p}{(2\pi)^3} (2\pi)^3 \delta(0) \leftarrow V \leftarrow \text{volume}$$

$$= \int \frac{d^3p}{(2\pi)^3}$$

energy high

$$H = -2V \int \frac{d^3p}{(2\pi)^3} \dots$$

$$H = \int \frac{d^3p}{(2\pi)^3} |0\rangle$$

In order to understand better this theory, consider a toy model:  $b, b^\dagger$ ;  $\{b, b^\dagger\} = b b^\dagger + b^\dagger b = 1$ ,  $[b, b] = [b^\dagger, b^\dagger] = 0$

$\Rightarrow b^2 = 0, b^{\dagger 2} = 0$ ;  $H = -E_b b^\dagger b$ . This model has only 2

Now  $H(S)$

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_{\vec{s}} \left( \epsilon_{\vec{p}} a_{\vec{p}}^{\vec{s}} a_{\vec{p}}^{\vec{s}} - E_{\vec{p}} b_{\vec{p}}^{\vec{s}} b_{\vec{p}}^{\vec{s}} \right)$$

Pauli Principle (Fermi Statistics)

$$= \int \frac{d^3p}{(2\pi)^3} \sum_{\vec{s}} \left( \epsilon_{\vec{p}} a_{\vec{p}}^{\vec{s}} a_{\vec{p}}^{\vec{s}} + E_{\vec{p}} b_{\vec{p}}^{\vec{s}} b_{\vec{p}}^{\vec{s}} \right) - \int \frac{d^3p}{(2\pi)^3} (2\pi)^3 \delta^3(0) \leftarrow V \leftarrow \text{volume}$$

$$= \int \frac{d^3p}{(2\pi)^3} \sqrt{2E_{\vec{p}}}$$

energy high

$$|0\rangle = -2V \int \frac{d^3p}{(2\pi)^3} H_{\text{or}} \left( \frac{d^3p}{(2\pi)^3} \right) |0\rangle$$

In order to understand better this theory, consider a toy model:  $b, b^\dagger$ ;  $\{b, b^\dagger\} = b b^\dagger + b^\dagger b = 1$ ,  $\{b, b\} = \{b^\dagger, b^\dagger\} = 0$   
 $\Rightarrow b^2 = 0, b^{\dagger 2} = 0$ ;  $H = -E_b b^\dagger b$ . This model has only 2

$$\frac{1}{(2\pi)^3} \int d^3p \left( -p \cdot \vec{u}_{\vec{p}} \vec{u}_{\vec{p}} + E_{\vec{p}} \vec{b}_{\vec{p}} \vec{b}_{\vec{p}} \right) \rightarrow \frac{1}{(2\pi)^3} \int d^3p \delta^3(\vec{p}) \leftarrow$$

$$= \int \frac{d^3\vec{p}}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} a_{\vec{p}}^S \Lambda_{1/2} U^S(\vec{p})$$

energy high

$$H|0\rangle = -2V \int \frac{d^3p}{(2\pi)^3} H_{op} \left( \frac{d^3p}{(2\pi)^3} \right) |0\rangle$$

In order to understand better this theory,  
a toy model:  $b, b^\dagger$ ;  $\{b, b^\dagger\} = b b^\dagger + b^\dagger b = 1$

$$\Rightarrow b^2 = 0, b^{\dagger 2} = 0; \quad H = -E_b b^\dagger b. \quad \text{This model has}$$

$$\frac{1}{(2\pi)^3} \int d^3p \left( -p^\alpha \tilde{a}_{\vec{p}}^\dagger \tilde{a}_{\vec{p}} + E_{\vec{p}} \tilde{b}_{\vec{p}}^\dagger \tilde{b}_{\vec{p}} \right) \rightarrow \frac{1}{(2\pi)^3} \int d^3p \delta^3(\vec{p}) \leftarrow$$

$$= \int \frac{d^3\tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} a_{\tilde{p}}^s \Lambda_{1/2} U^s(\tilde{p}) e^{-i\tilde{p} \cdot \vec{x}}$$

$$H|0\rangle = -2V \int \frac{d^3p}{(2\pi)^3} \dots$$

In order to understand what  
 a toy model:  $b, b^\dagger$

$$\Rightarrow b^2 = 0, b^{\dagger 2} = 0; \quad H = -E_b b^\dagger b$$

Now HCS

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_{\vec{s}} \left( \epsilon_{\vec{p}} a_{\vec{p}}^{\vec{s}\dagger} a_{\vec{p}}^{\vec{s}} - E_{\vec{p}} b_{\vec{p}}^{\vec{s}\dagger} b_{\vec{p}}^{\vec{s}} \right)$$

Pauli Principle (Fermi Statistics)

$$= \int \frac{d^3p}{(2\pi)^3} \sum_{\vec{s}} \left( \epsilon_{\vec{p}} a_{\vec{p}}^{\vec{s}\dagger} a_{\vec{p}}^{\vec{s}} + E_{\vec{p}} b_{\vec{p}}^{\vec{s}\dagger} b_{\vec{p}}^{\vec{s}} \right) - \int \frac{d^3p}{(2\pi)^3} (2\pi)^3 \delta^3(0) \leftarrow V \leftarrow \text{volume}$$

$$= \int \frac{d^3\tilde{p}}{(2\pi)^3 \sqrt{2E_{\tilde{p}}}} a_{\tilde{p}}^{\vec{s}} \Lambda_{1/2} U^{\vec{s}}(\tilde{p}) e^{-i\tilde{p}\cdot\tilde{A}x} + \dots = \Lambda_{1/2} \Psi(\tilde{A}x)$$

Vacuum

$$H|0\rangle = -2V \int \frac{d^3p}{(2\pi)^3} H_{or} = \int \frac{d^3p}{(2\pi)^3} |0\rangle$$

In order to understand better this theory, consider a toy model:  $b, b^\dagger$ ;  $\{b, b^\dagger\} = b b^\dagger + b^\dagger b = 1$ ,  $\{b, b\} = \{b^\dagger, b^\dagger\} = 0$   
 $\Rightarrow b^2 = 0, b^{\dagger 2} = 0$ ;  $H = -E_b b^\dagger b$ . This model has only 2

$$\begin{aligned}
 & \psi(\vec{p}, m) \psi(\vec{p}) = 0 \\
 & \psi^s(\vec{p}) = \begin{pmatrix} \sqrt{E_p} \chi^s \\ -\sqrt{E_p} \eta^s \end{pmatrix} \\
 & u^s(\Lambda \vec{p}) = \Lambda_{1/2} u^s(\vec{p}) \\
 & u^s(\Lambda \vec{p}) u(\Lambda x) = \Lambda_{1/2} \psi(\Lambda x) \\
 & + b_{\vec{p}}^s \psi^s(\vec{p}) e^{i\vec{p} \cdot \vec{x}} u^s \\
 & u^s(\Lambda \vec{p}) a_{\vec{p}}^s u(\Lambda x) = \sqrt{\frac{E_{\Lambda \vec{p}}}{E_{\vec{p}}}} a_{\Lambda \vec{p}}^s \text{ assuming}
 \end{aligned}$$

Orthogonality relations:

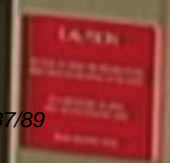
$$\begin{aligned}
 u^{r'}(\vec{p}) u^s(\vec{p}) &= 2E_p \delta^{rs} \\
 \bar{u}^{r'}(\vec{p}) \bar{u}^s(\vec{p}) &= 2E_p \delta^{rs} \\
 u^{r'}(\vec{p}) \bar{u}^s(-\vec{p}) &= 0 \\
 \bar{u}^{r'}(-\vec{p}) u^s(\vec{p}) &= 0
 \end{aligned}$$

"vacuum" ground state

The axis of spin quantization is parallel to the direction of boost axis

Rewrite  $a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3} \sqrt{2E_p} a_{\vec{p}}^s$

Then  $a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3} \sqrt{2E_p} a_{\vec{p}}^s$

$$\int \frac{d^3 p}{(2\pi)^3} \sqrt{2E_p} a_{\vec{p}}^s u^s(\Lambda \vec{p}) e^{-i\vec{p} \cdot \vec{x}} + \dots$$


$$\begin{aligned}
 & \psi(\vec{p}, m) \psi(\vec{0}) = 0 \\
 & \psi^s(\vec{p}) = \begin{pmatrix} \sqrt{E_p} \chi^s \\ -\sqrt{E_p} \eta^s \end{pmatrix} \\
 & u^s(\Lambda \vec{p}) = \Lambda_{1/2} u^s(\vec{p}) \quad u^s(\Lambda \vec{p}) = \Lambda_{1/2} u^s(\vec{p}) \\
 & u^s(\Lambda \vec{p}) = \Lambda_{1/2} u^s(\vec{p}) \quad u^s(\Lambda \vec{p}) = \Lambda_{1/2} u^s(\vec{p})
 \end{aligned}$$

Orthogonal relations:

$$\begin{aligned}
 u^r(\vec{p})^\dagger u^s(\vec{p}) &= 2E_p \delta^{rs} \\
 u^r(\vec{p})^\dagger u^s(-\vec{p}) &= 0 \\
 v^r(\vec{p})^\dagger v^s(\vec{p}) &= -2E_p \delta^{rs} \\
 v^r(\vec{p})^\dagger v^s(-\vec{p}) &= 0
 \end{aligned}$$

"vacuum"  $|0\rangle$   
 There is no ground state

The axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sqrt{2E_p} a_{\vec{p}}^s$

Then  $u^s(\Lambda \vec{p}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sqrt{2E_p} a_{\vec{p}}^s u^s(\Lambda \vec{p}) e^{-i\vec{p}\cdot\vec{x}} + \dots$



$$\begin{aligned}
 & \psi^\dagger(p, m) \psi(p) = 0 \\
 & \psi^\dagger(p) = \begin{pmatrix} \sqrt{E_p} & 0 \\ -\sqrt{E_p} & 0 \end{pmatrix} \\
 & \psi^\dagger(\Lambda \vec{p}) = \Lambda_{1/2} \psi^\dagger(\vec{p}) \Lambda^{-1} \\
 & \psi^\dagger(\Lambda \vec{p}) \psi(\Lambda \vec{x}) = \Lambda_{1/2} \psi^\dagger(\vec{p}) \psi(\Lambda \vec{x}) \\
 & \psi^\dagger(\Lambda \vec{p}) \psi(\Lambda \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s \psi(\Lambda \vec{x}) e^{-i p \cdot \Lambda \vec{x}} + b_{\vec{p}}^{s\dagger} \psi(\Lambda \vec{x}) e^{i p \cdot \Lambda \vec{x}}) \\
 & \psi^\dagger(\Lambda \vec{p}) \psi(\Lambda \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s \psi(\Lambda \vec{x}) e^{-i p \cdot \Lambda \vec{x}} + b_{\vec{p}}^{s\dagger} \psi(\Lambda \vec{x}) e^{i p \cdot \Lambda \vec{x}}) \\
 & \psi^\dagger(\Lambda \vec{p}) \psi(\Lambda \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s \psi(\Lambda \vec{x}) e^{-i p \cdot \Lambda \vec{x}} + b_{\vec{p}}^{s\dagger} \psi(\Lambda \vec{x}) e^{i p \cdot \Lambda \vec{x}})
 \end{aligned}$$

Orthogonality relations:

$$\begin{aligned}
 & u^{r\dagger}(p) u^s(p) = 2E_p \delta^{rs} \\
 & v^{r\dagger}(p) v^s(p) = 2E_p \delta^{rs} \\
 & u^{r\dagger}(p) v^s(-p) = 0 \\
 & v^{r\dagger}(-p) u^s(p) = 0
 \end{aligned}$$

"vacuum"  
There is no ground state

The axis of spin quantization is parallel to the rotation or boost axis

Rewrite  $\int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s$

Then  $\psi^\dagger(\Lambda \vec{p}) \psi(\Lambda \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec{p}}^s \psi(\Lambda \vec{x}) e^{-i p \cdot \Lambda \vec{x}} + \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} b_{\vec{p}}^{s\dagger} \psi(\Lambda \vec{x}) e^{i p \cdot \Lambda \vec{x}}$