Title: Quantum Field Theory 1 - Lecture 9B

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

$$v^s(\vec{p}) = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a^s_{\vec p} u^s(\vec p) + b^{s\dagger}_{-\vec p} v^s(\vec p)\right)$$

$$v^\dagger(-\vec p) = [v(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a^{s\dagger}_{\vec p} u^{s\dagger}(\vec p) + b^s_{-\vec p} v^{s\dagger}(\vec p)\right)$$

Orthogonality relations:

$$u^{r\dagger}(p)u^s(p) = 2E_p\delta^{rs}$$

$$v^{r\dagger}(p)v^s(p) = 2E_{\vec p}\delta^{rs}$$

$$u^{r\dagger}(\vec p)v^s(-\vec p) = v^{r\dagger}(-\vec p)u^s(\vec p) = 0$$

Using the orthogonality relations, we get:

$$[a^r_{\vec p}, a^{s\dagger}_{\vec q}] = [b^r_{\vec p}, b^{s\dagger}_{\vec q}] = (2\pi)^3\delta^3(\vec p - \vec q)\delta^{rs},$$

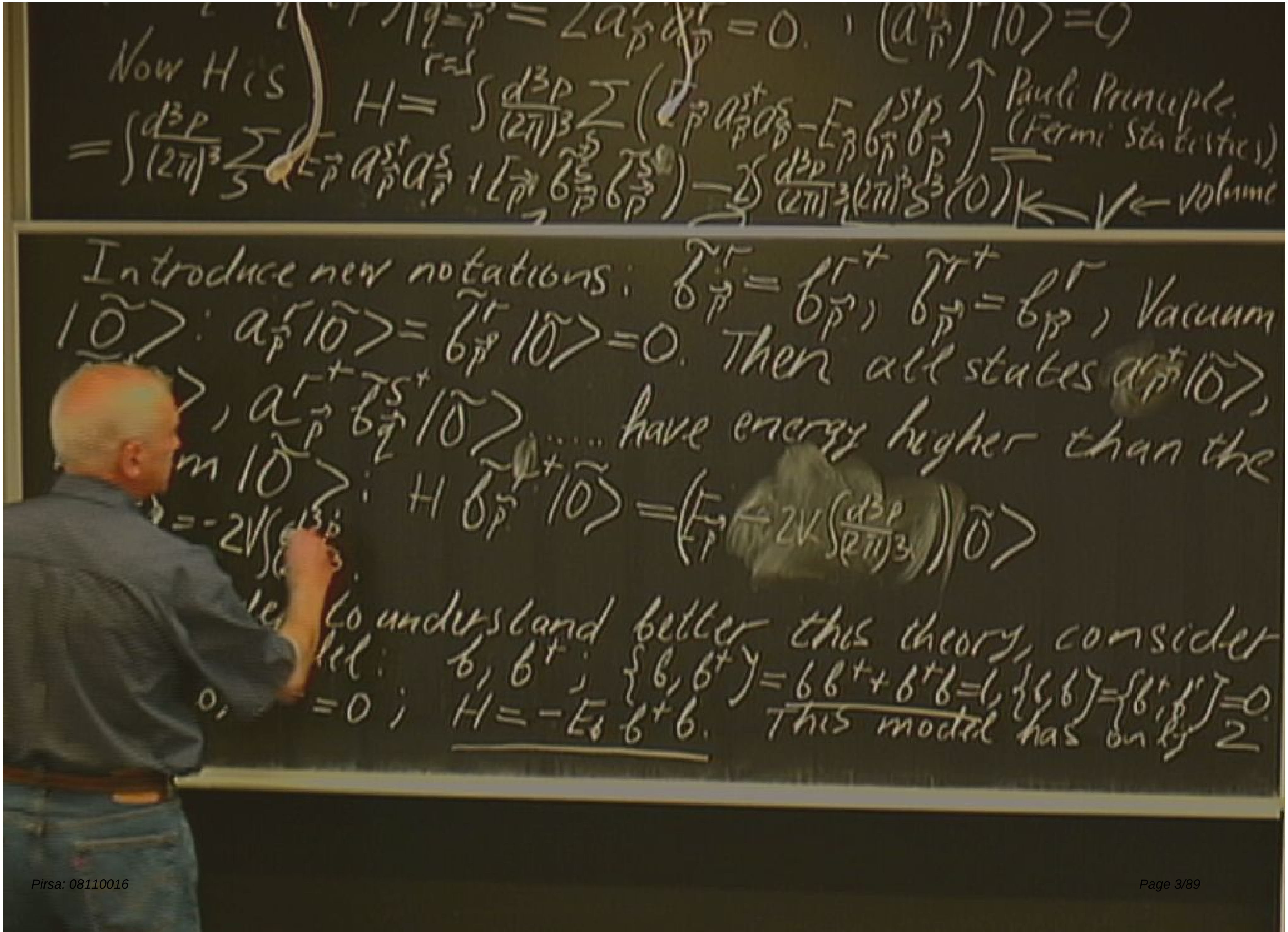
$$[a^r_{\vec p}, a^s_{\vec q}] = [b^r_{\vec p}, b^s_{\vec q}] = [a^{r\dagger}_{\vec p}, a^{s\dagger}_{\vec q}] = [b^{r\dagger}_{\vec p}, b^{s\dagger}_{\vec q}] = 0$$

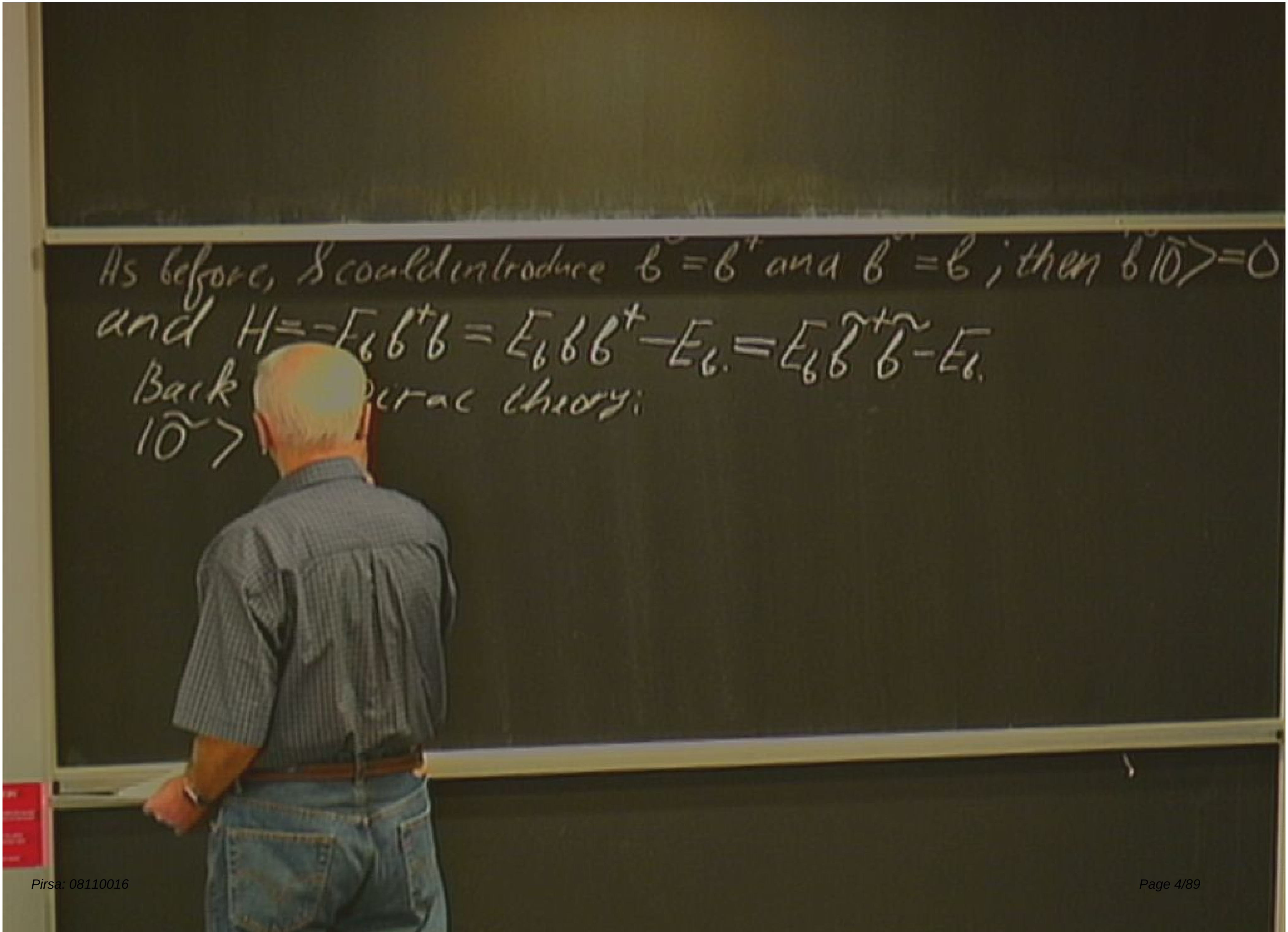
Write H in terms of these operators using the orthogonality relations:

$$H = \int \frac{d^3p}{(2\pi)^3}\sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s\dagger}_{\vec p} b^s_{\vec p}\right)$$

"vacuum" $|0\rangle$, $a^s_{\vec p}|0\rangle = b^s_{\vec p}|0\rangle = 0$; states $(b^{s\dagger}_{\vec p})^n|0\rangle$ have $E(\vec p) < 0$

There is no ground state (vacuum) in this theory


$= \sum a_{\vec p}^{r} a_{\vec p}^{r} = 0 \; ; \; (a_{\vec p}^{r\dagger})^2 |0\rangle = 0$
Now H is
$H = \int \frac{d^3p}{(2\pi)^3} \sum_{r,s} \left(E_{\vec p}\, a_{\vec p}^{s\dagger} a_{\vec p}^{s} - E_{\vec p}\, b_{\vec p}^{s\dagger} b_{\vec p}^{s} \right)$
Pauli Principle (Fermi Statistics)
$= \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a_{\vec p}^{s\dagger} a_{\vec p}^{s} + E_{\vec p}\, \tilde b_{\vec p}^{s\dagger} \tilde b_{\vec p}^{s} \right) - 2\int \frac{d^3p}{(2\pi)^3} (2\pi)^3 \delta^3(0)$
$V \leftarrow$ volume
Introduce new notations: $\tilde b_{\vec p}^{r} = b_{\vec p}^{r\dagger}$, $\tilde b_{\vec p}^{r\dagger} = b_{\vec p}^{r}$, Vacuum
$|\tilde 0\rangle$: $a_{\vec p}^{r}|\tilde 0\rangle = \tilde b_{\vec p}^{r}|\tilde 0\rangle = 0$. Then all states $a_{\vec p}^{r\dagger}|\tilde 0\rangle$,
$\rangle$, $a_{\vec p}^{r\dagger} \tilde b_{\vec q}^{s\dagger}|\tilde 0\rangle$ ...... have energy higher than the
m $|\tilde 0\rangle$: $H \tilde b_{\vec p}^{r\dagger}|\tilde 0\rangle = \left(E_{\vec p} - 2V\int \frac{d^3p}{(2\pi)^3}\right)|\tilde 0\rangle$
$= -2V\int \frac{d^3p}{(2\pi)^3}$
to understand better this theory, consider
del: $b, b^\dagger$; $\{b, b^\dagger\} = bb^\dagger + b^\dagger b = 1$, $\{b,b\} = \{b^\dagger, b^\dagger\} = 0$
$= 0$; $H = -E_0\, b^\dagger b$. This model has only 2

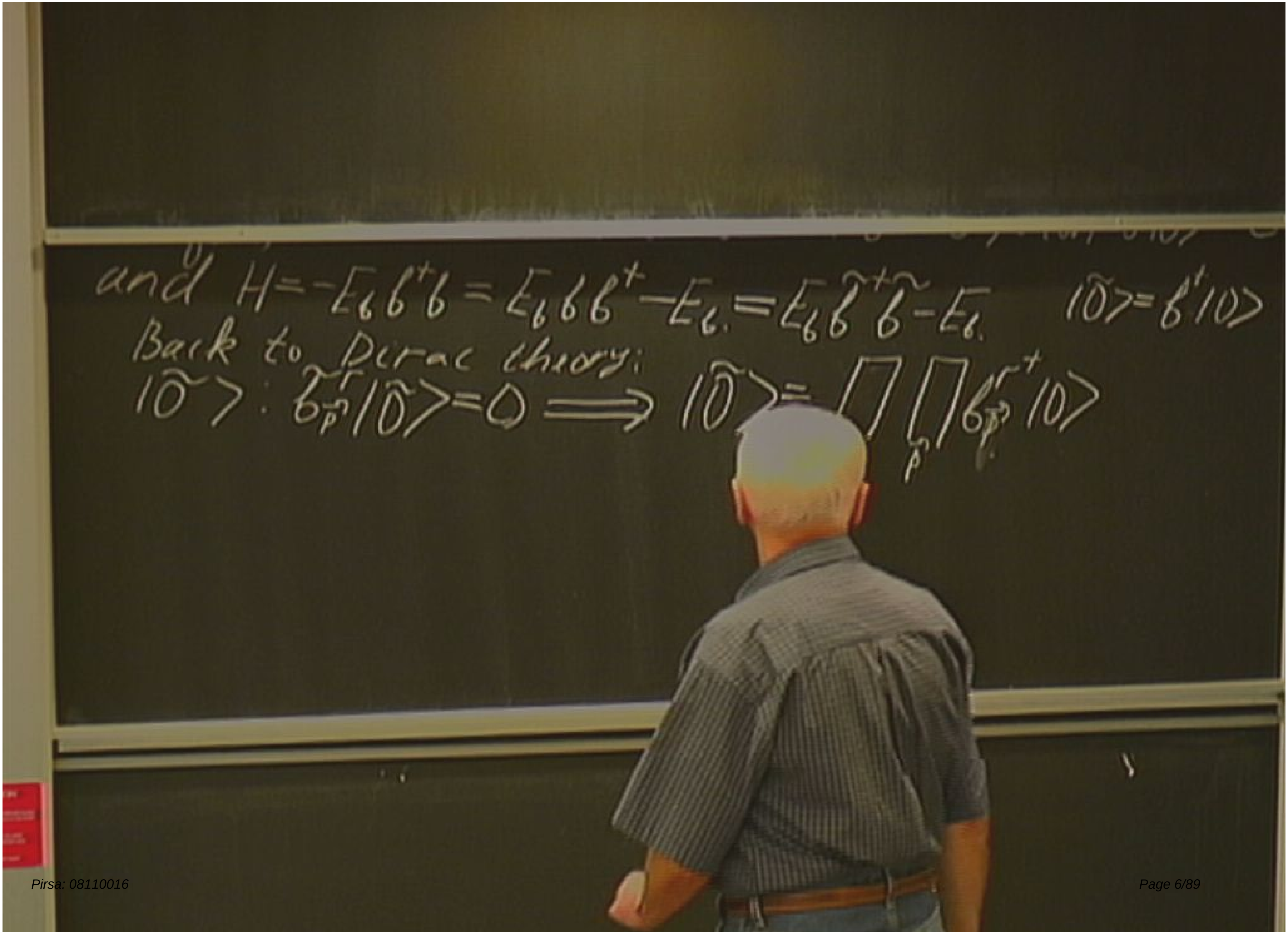

As before, I could introduce $\tilde{b} = b^{\dagger}$ and $\tilde{b}^{\dagger} = b$; then $\tilde{b}|\tilde{0}\rangle = 0$
and $H = -E_b b^{\dagger} b = E_b b b^{\dagger} - E_b = E_b \tilde{b}^{\dagger} \tilde{b} - E_b$.
Back
irac theory:
$|\tilde{0}\rangle$

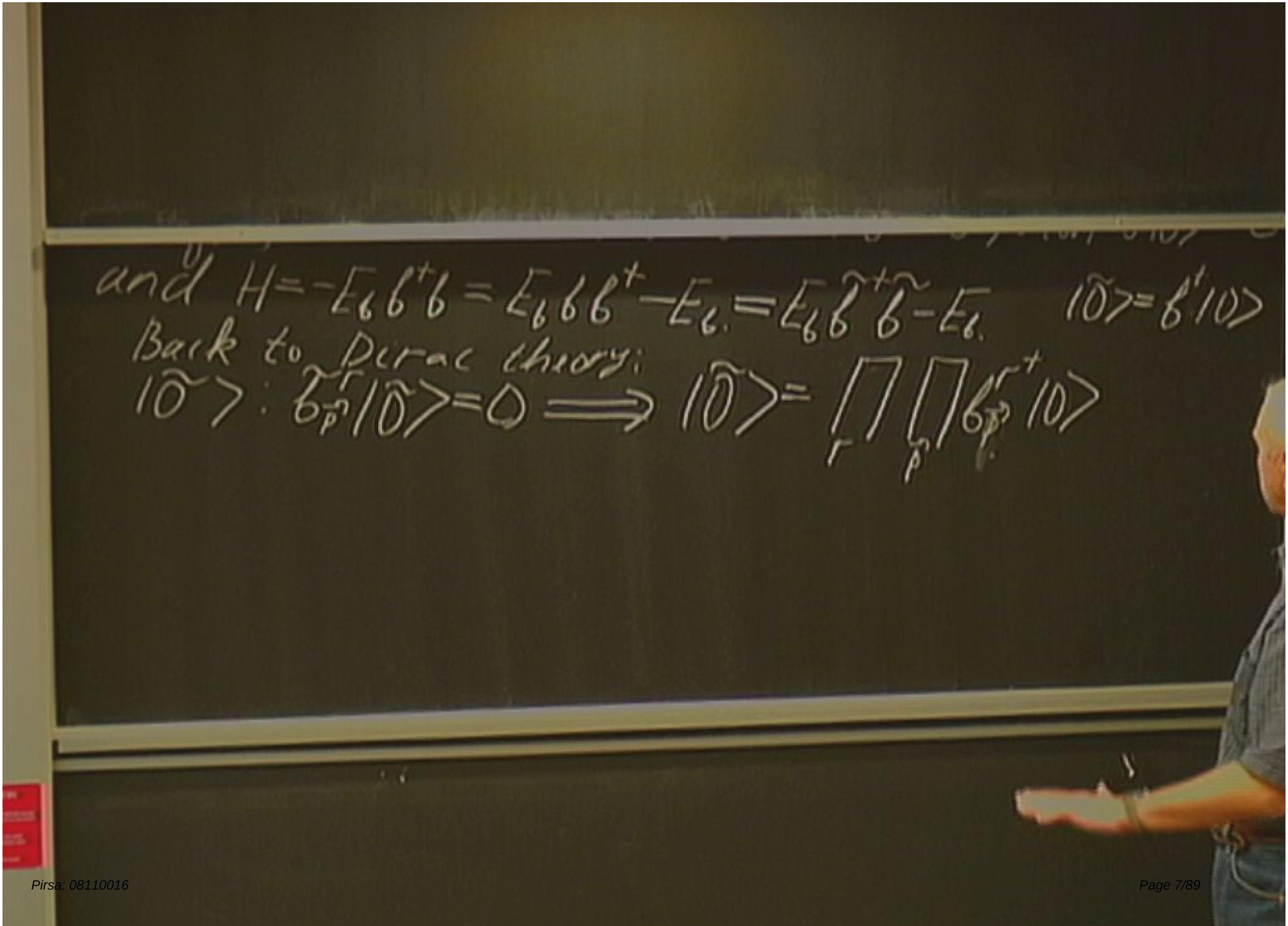
As before, I could introduce $b = b$ and $b = b$, then $b|0\rangle = 0$

and $H = -E_b b^\dagger b = E_b b b^\dagger - E_b = E_b \tilde{b}^\dagger \tilde{b} - E_b.$

Back to Dirac theory:

$|\tilde{0}\rangle : \tilde{b}_p^\dagger |\tilde{0}\rangle = 0$


and $H = -E_b b^+ b = E_b b b^+ - E_b = E_b \tilde{b}^+ \tilde{b} - E_b$ $|\tilde{0}\rangle = b^+|0\rangle$
Back to Dirac theory:
$|\tilde{0}\rangle : \tilde{b}_p^+ |\tilde{0}\rangle = 0 \Longrightarrow |\tilde{0}\rangle = \prod \prod_p b_p^+ |0\rangle$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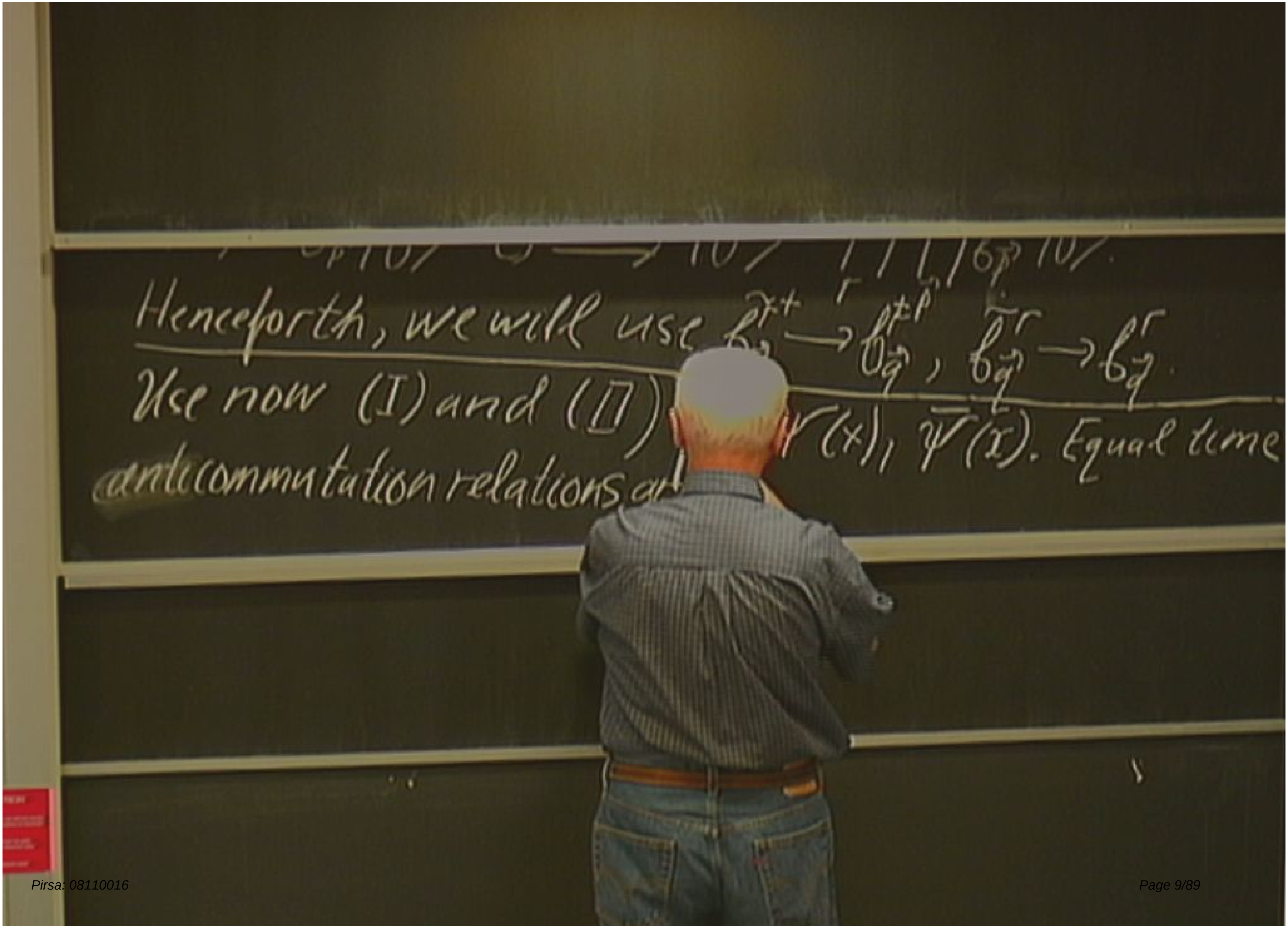
Back to Dirac theory:
$|\tilde{0}\rangle : \tilde{b}_p^{r}|\tilde{0}\rangle = 0 \Longrightarrow |\tilde{0}\rangle = \prod_r \prod_p b_p^{r\dagger}|0\rangle$

$|\tilde{0}\rangle : \tilde{b}^r_{\vec{p}}|\tilde{0}\rangle = 0 \Longrightarrow |\tilde{0}\rangle = \prod_r \prod_{\vec{q}} b^{r\dagger}_{\vec{q}} |0\rangle.$

Henceforth, we will use $\tilde{b}^{r\dagger}_{\vec{q}} \longrightarrow b^{\dagger r}_{\vec{q}}$, $\tilde{b}^{r}_{\vec{q}} \longrightarrow b^{r}_{\vec{q}}$.


Henceforth, we will use $\tilde{b}^{+r}_{\vec{q}} \to b^{+r}_{\vec{q}}$, $\tilde{b}^{r}_{\vec{q}} \to b^{r}_{\vec{q}}$.
Use now (I) and (II)
$\psi(x)$, $\bar{\psi}(x)$. Equal time
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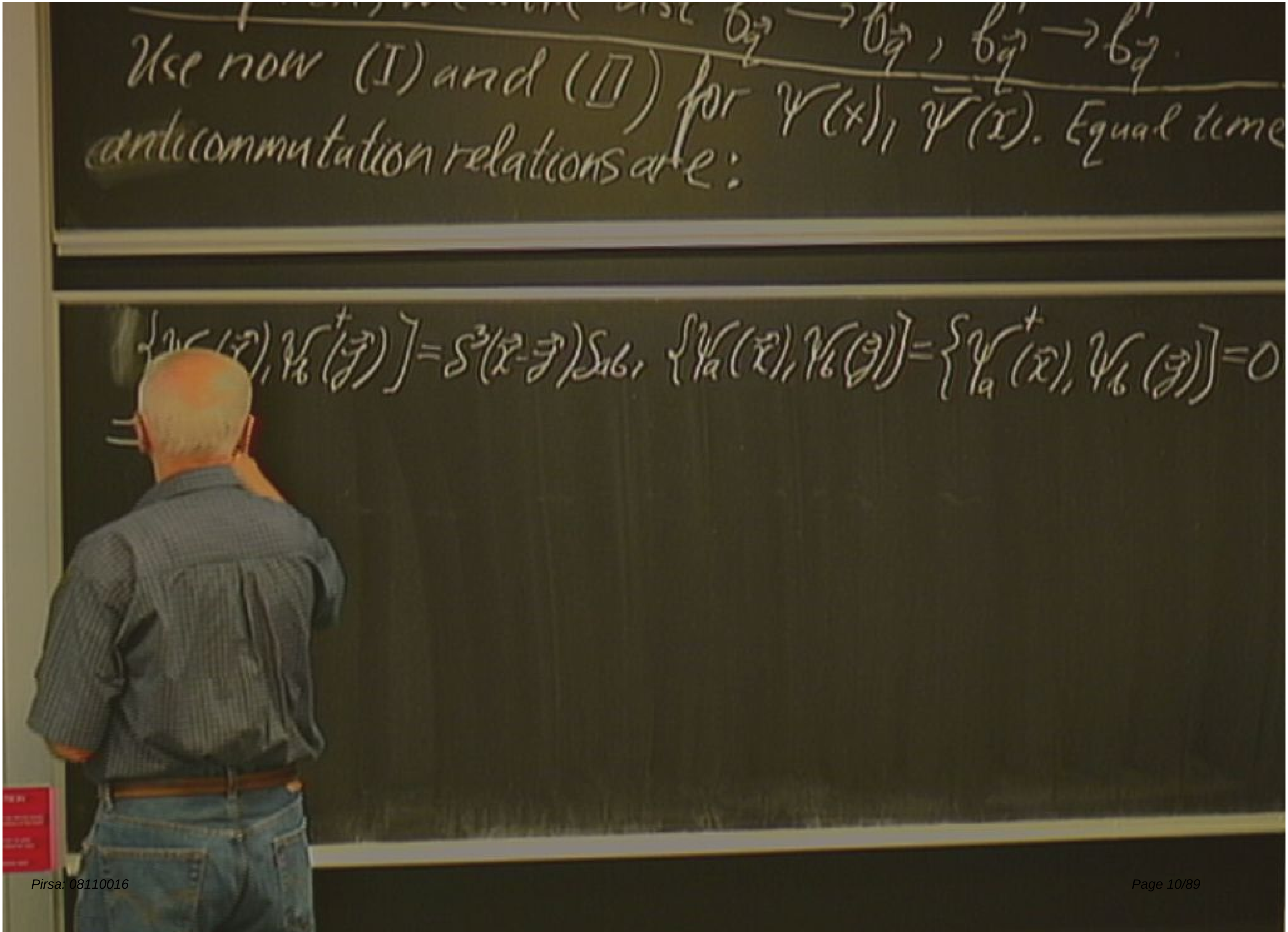
Use now (I) and (II) for $\psi(x)$, $\bar{\psi}(x)$. Equal time
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$\{\psi_a(\vec{x}), \psi_b^\dagger(\vec{y})\} = \delta^3(\vec{x}-\vec{y})\delta_{ab}$, $\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^\dagger(\vec{x}), \psi_b(\vec{y})\} = 0$

$b_{\vec q} \to b^{\dagger}_{\vec q}\,,\; b^{\dagger}_{\vec q} \to b_{\vec q}\,.$

Use now (I) and (II) for $\psi(x)$, $\bar{\psi}(x)$. Equal time anticommutation relations are:

$$\{\psi_a(\vec x), \psi_b^{\dagger}(\vec y)\} = \delta^3(\vec x - \vec y)\,\delta_{ab}\,, \quad \{\psi_a(\vec x), \psi_b(\vec y)\} = \{\psi_a^{\dagger}(\vec x), \psi_b(\vec y)\} = 0$$

$\Longrightarrow$

~~first $b_{\vec q} \to b'_{\vec q}$, $b_{\vec q} \to b'_{\vec q}$.~~

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$$\Longrightarrow \{a^r_{\vec{p}}, a^{s\dagger}_{\vec{q}}\} = (2\pi)^3$$

~~$b_{\vec q} \to b^{\dagger}_{\vec q}$, $b^{\dagger}_{\vec q} \to b_{\vec q}$~~

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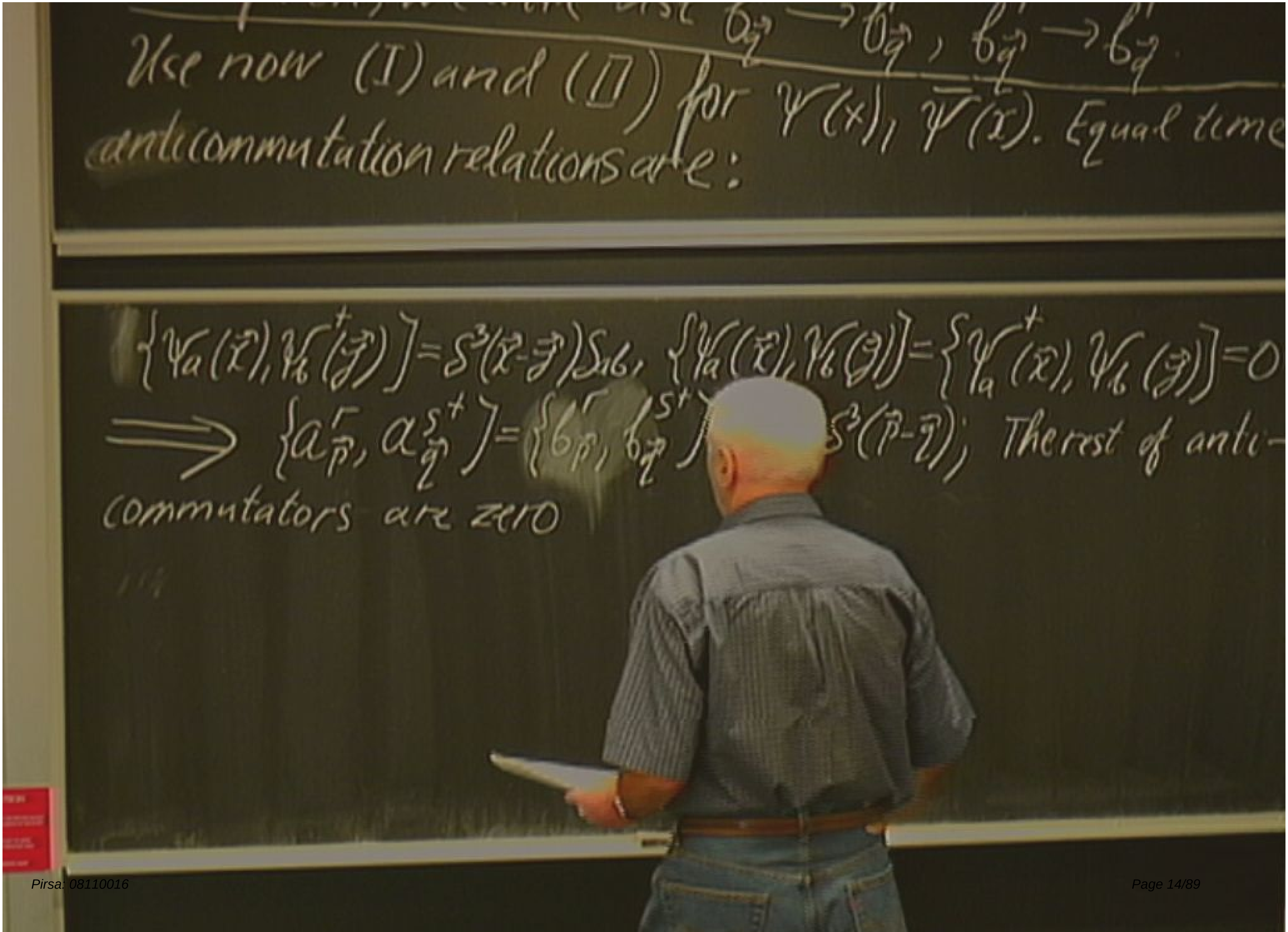
$$\Longrightarrow \{a^r_{\vec p}, a^{s\dagger}_{\vec q}\} = \{b^r_{\vec p}, b^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q);$$



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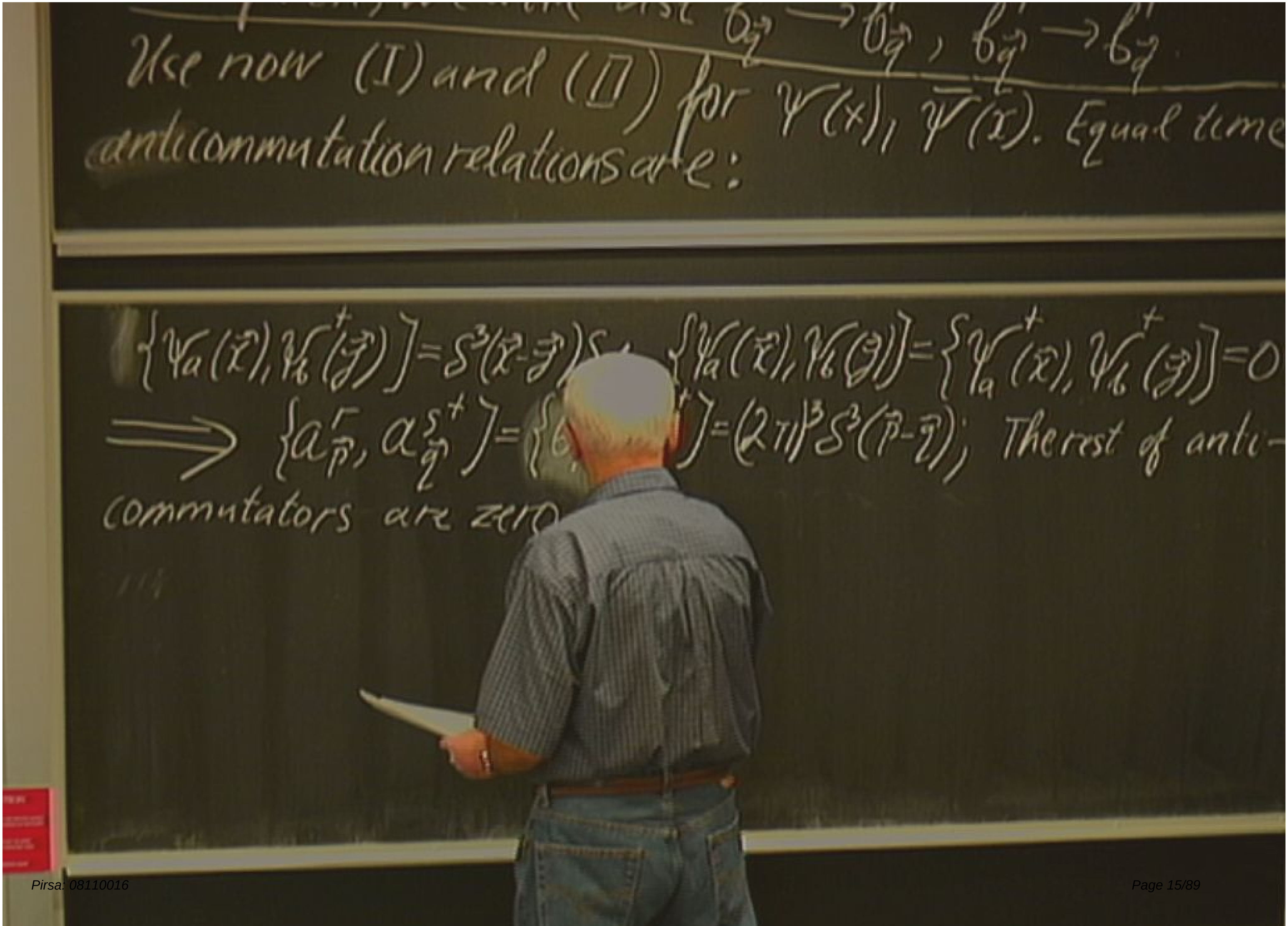
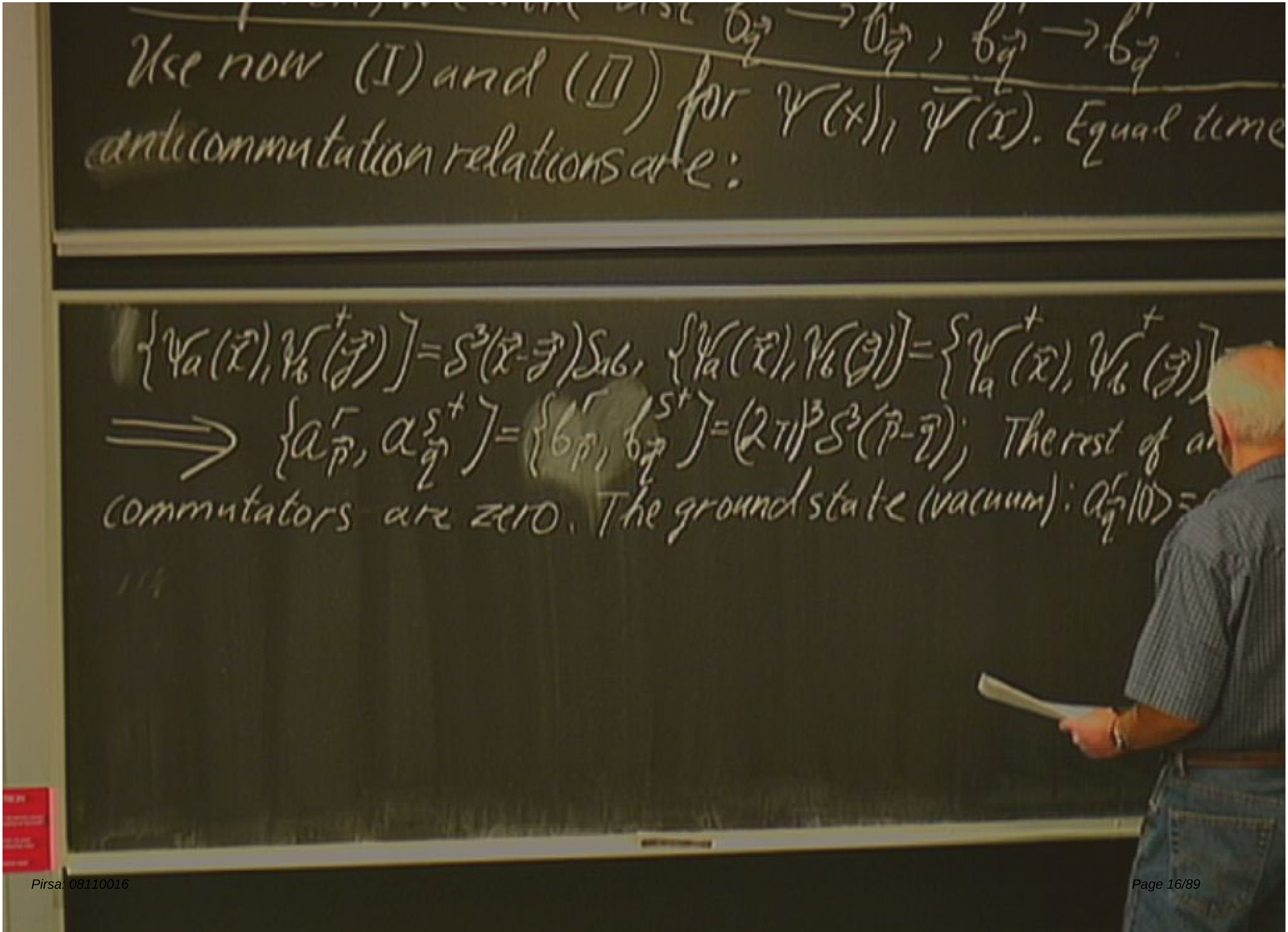
$\{\psi_a(\vec{x}), \psi_b^{+}(\vec{y})\} = \delta^3(\vec{x}-\vec{y})\delta_{ab}$, $\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^{+}(\vec{x}), \psi_b(\vec{y})\} = 0$
$\Longrightarrow \{a_{\vec{p}}^{r}, a_{\vec{q}}^{s+}\} = \{b_{\vec{p}}^{r}, b_{\vec{q}}^{s+}\}$ … $\delta^3(\vec{p}-\vec{q})$; The rest of anti-commutators are zero


Use now (I) and (II) for ψ(x), ψ̄(x). Equal time
anticommutation relations are:
{ψ_a(x⃗), ψ_b^†(y⃗)} = δ³(x⃗−y⃗)δ_ab , {ψ_a(x⃗), ψ_b(y⃗)} = {ψ_a^†(x⃗), ψ_b^†(y⃗)} = 0
⟹ {a_p⃗^r, a_q⃗^s†} = {b … } = (2π)³δ³(p⃗−q⃗); The rest of anti-
commutators are zero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$\Longrightarrow \{a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}] = \{b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}] = (2\pi)^3\delta^3(\vec{p}-\vec{q})$; The rest of an
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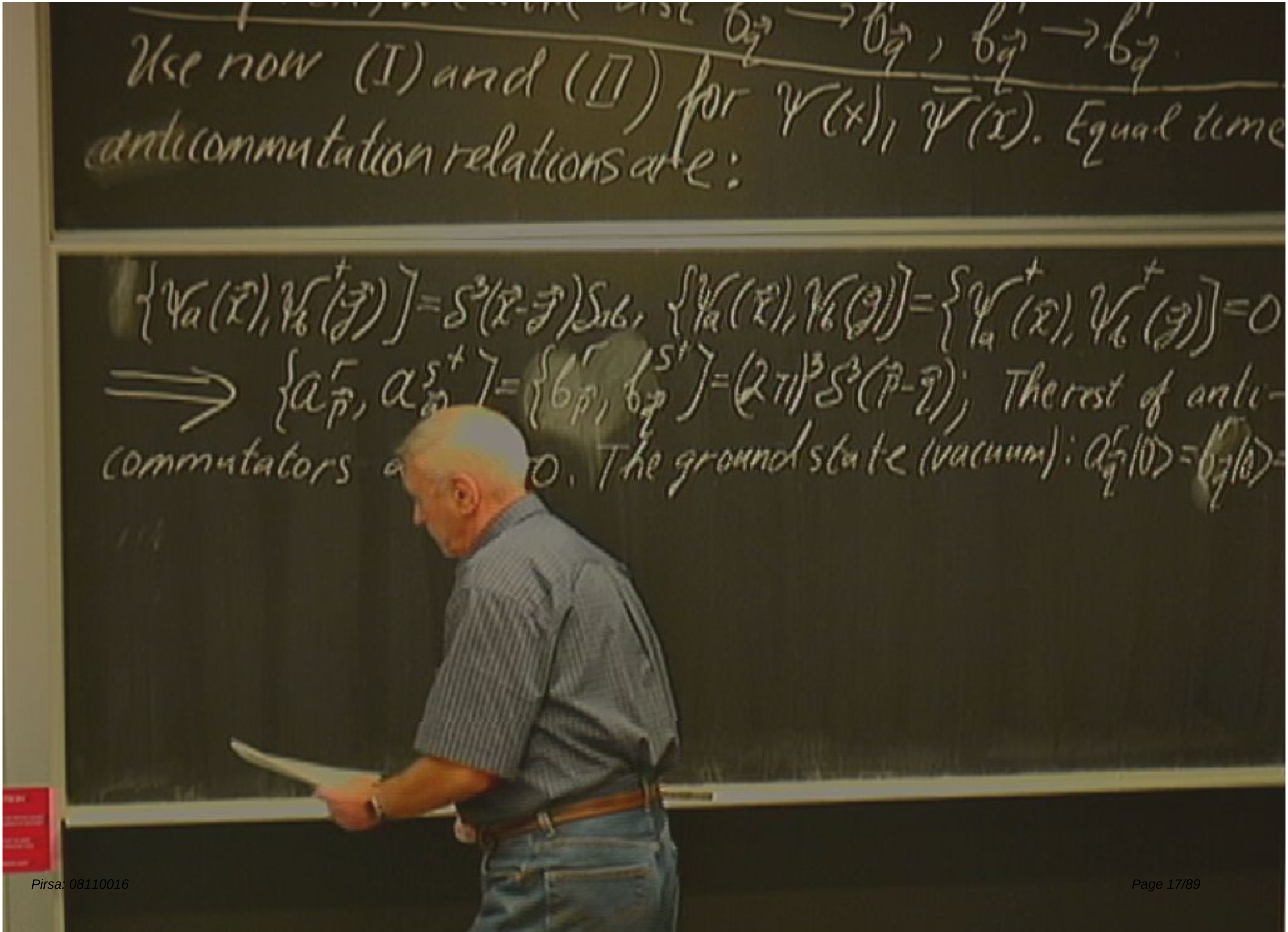
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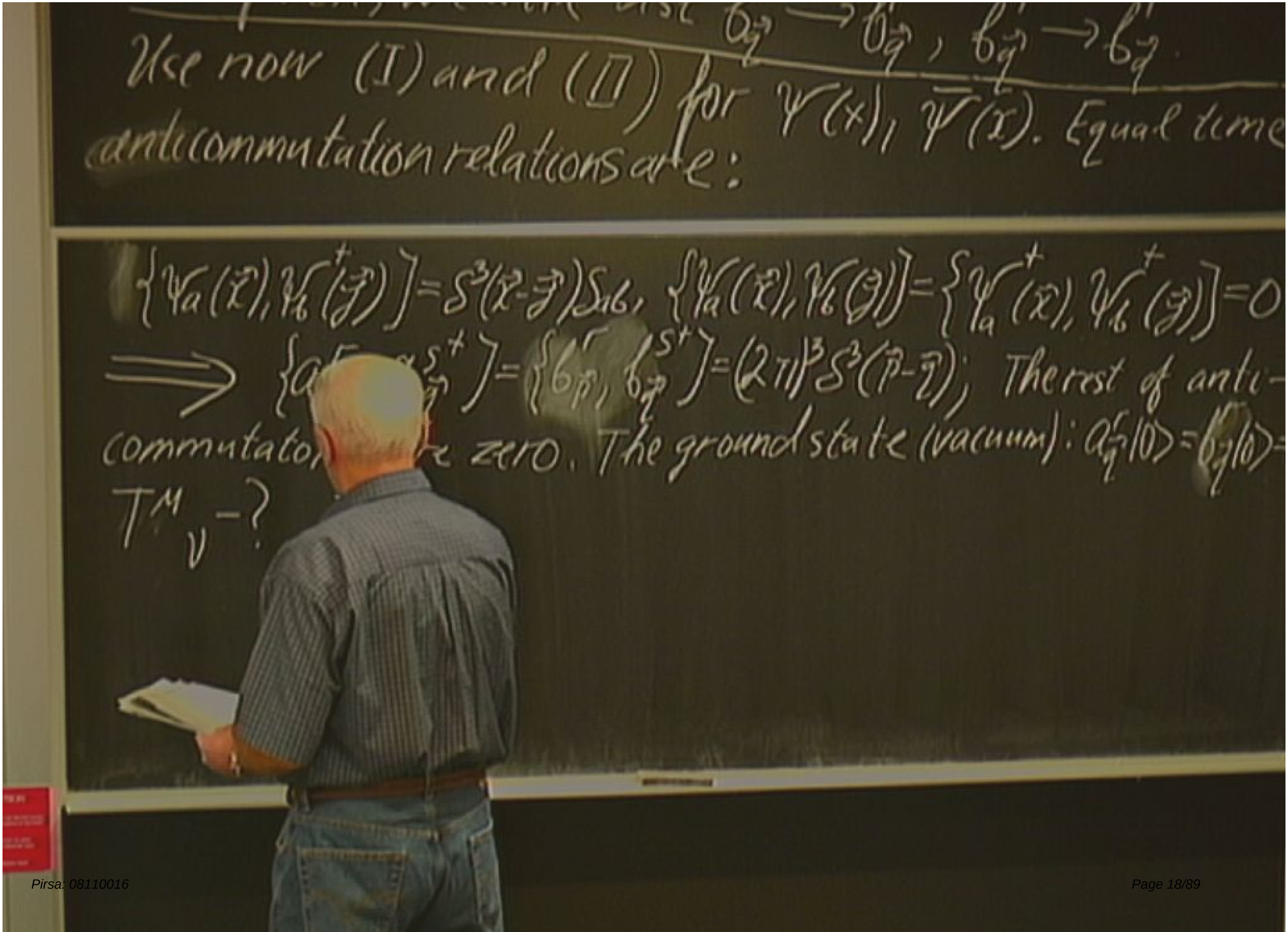
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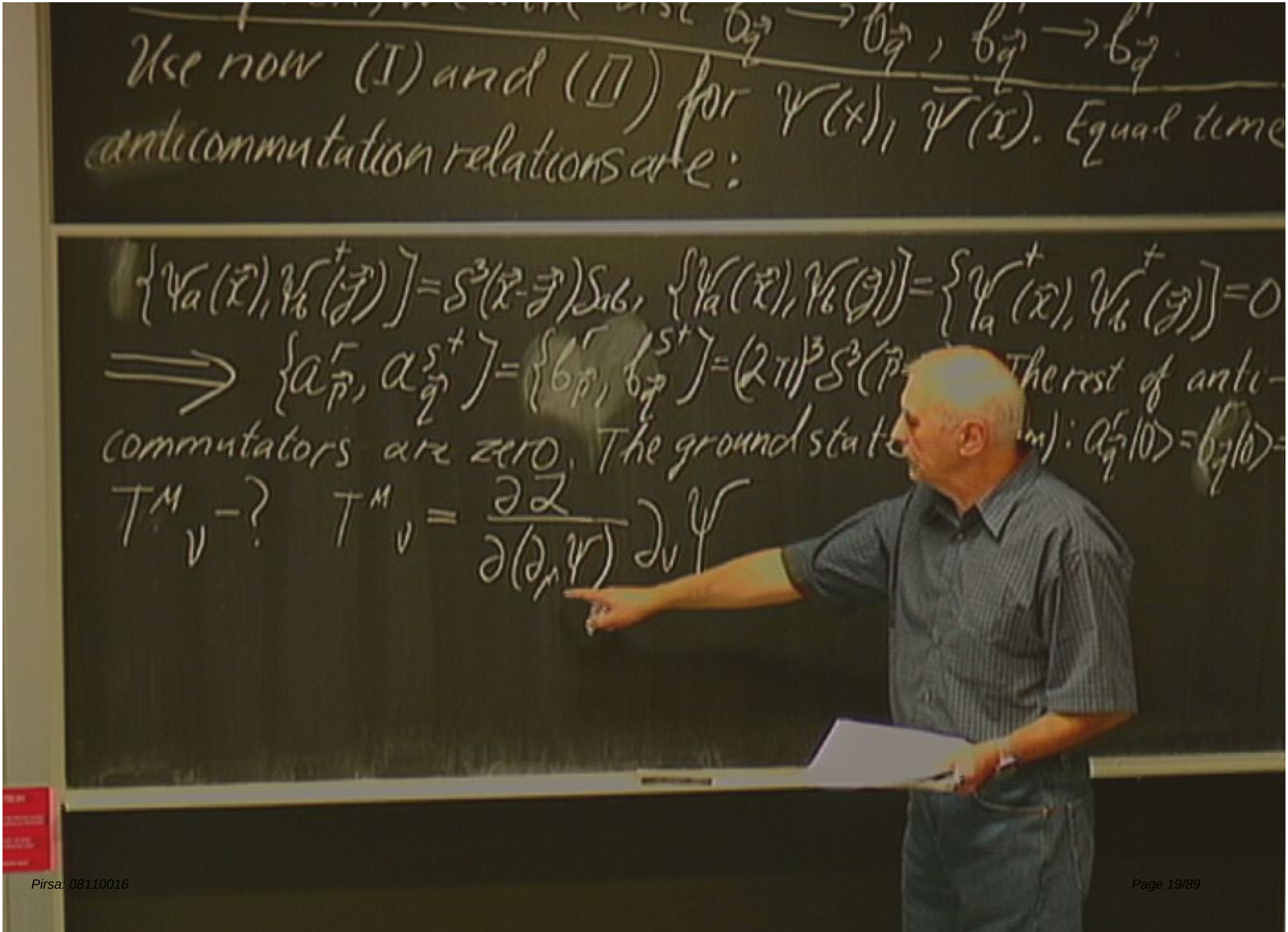
$T^{\mu}{}_{\nu}$ – ?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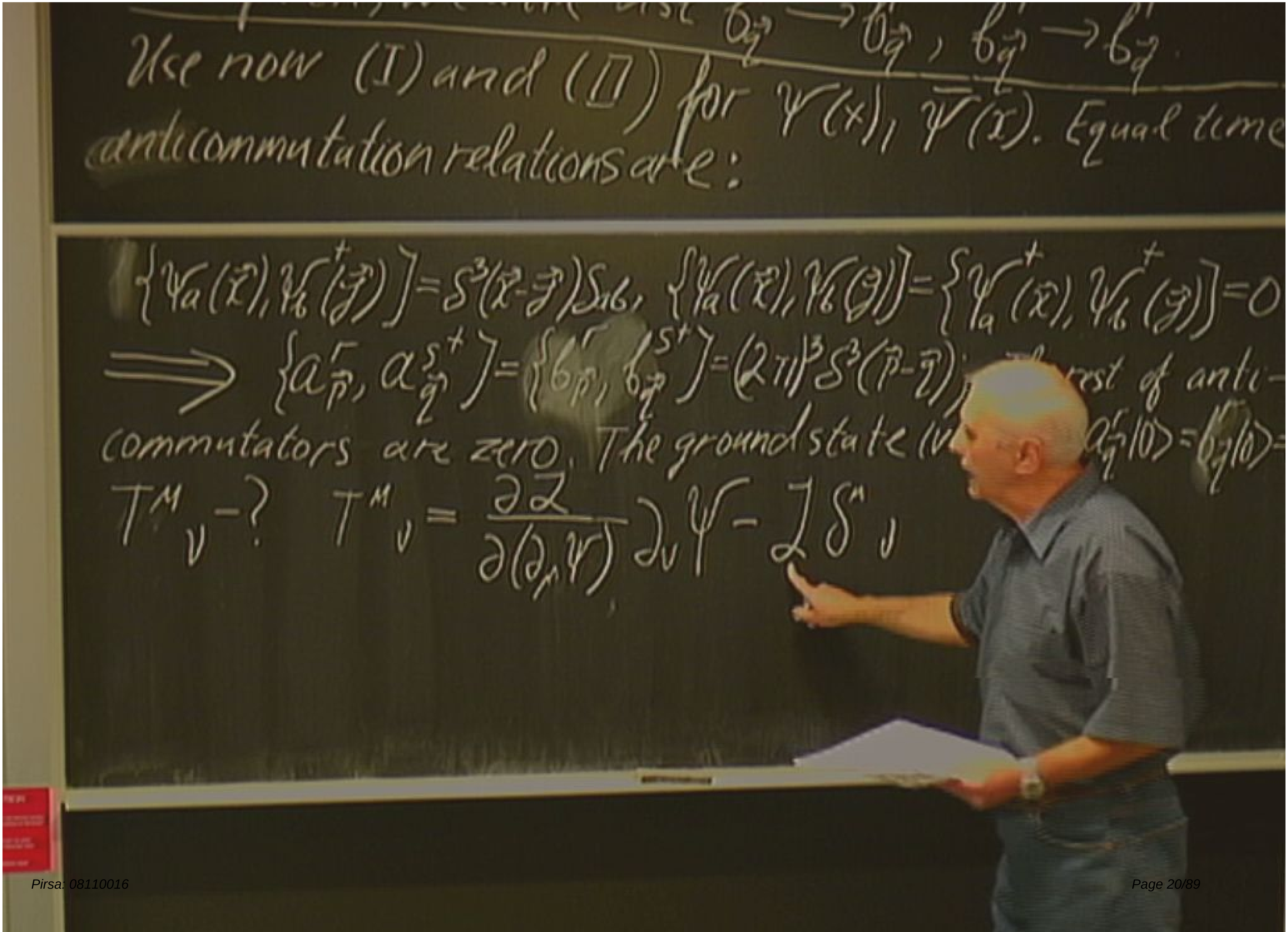
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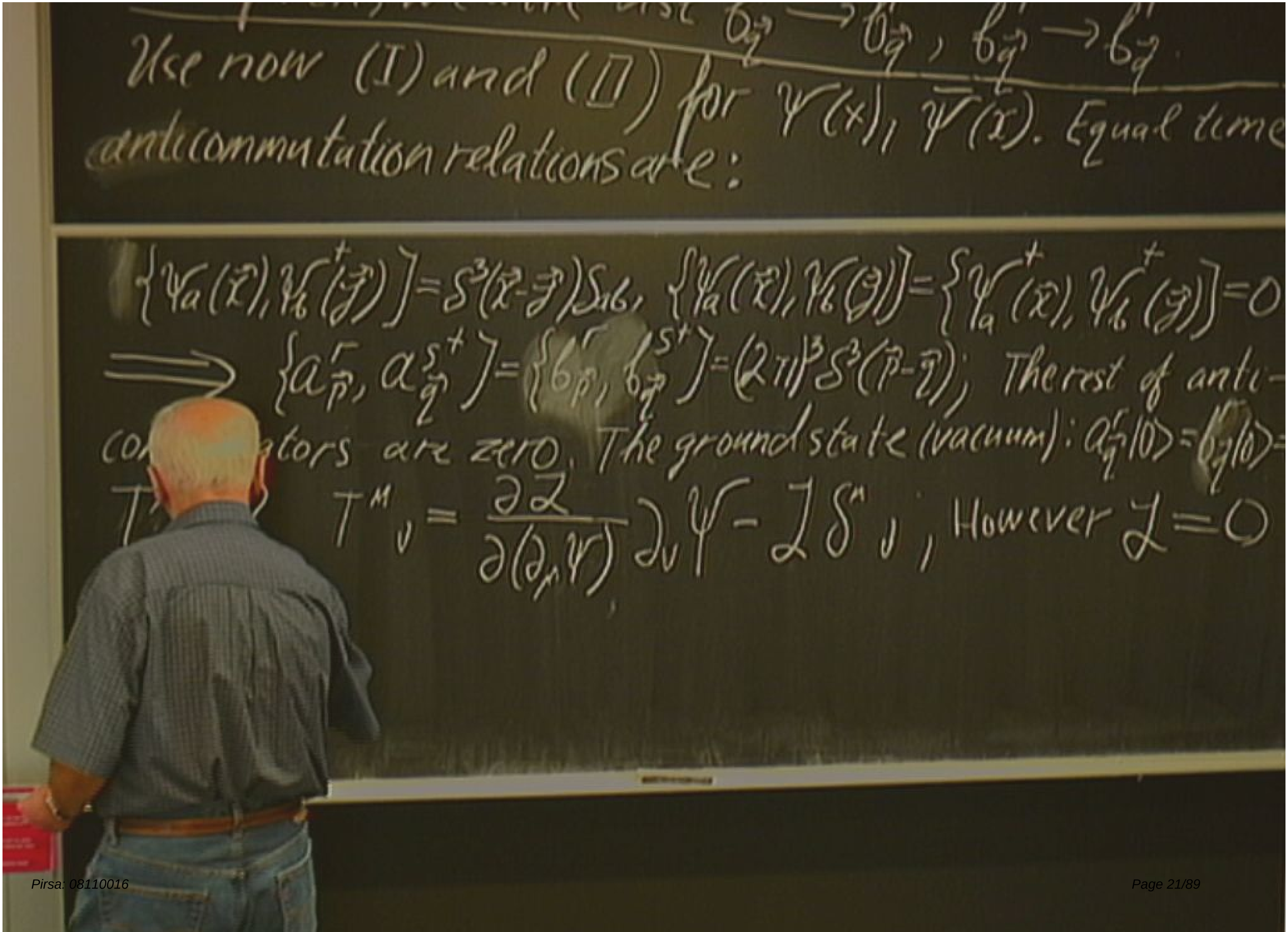
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$T^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\partial_\nu \psi - \mathcal{L}\delta^{\mu}{}_{\nu}$; However $\mathcal{L} = 0$

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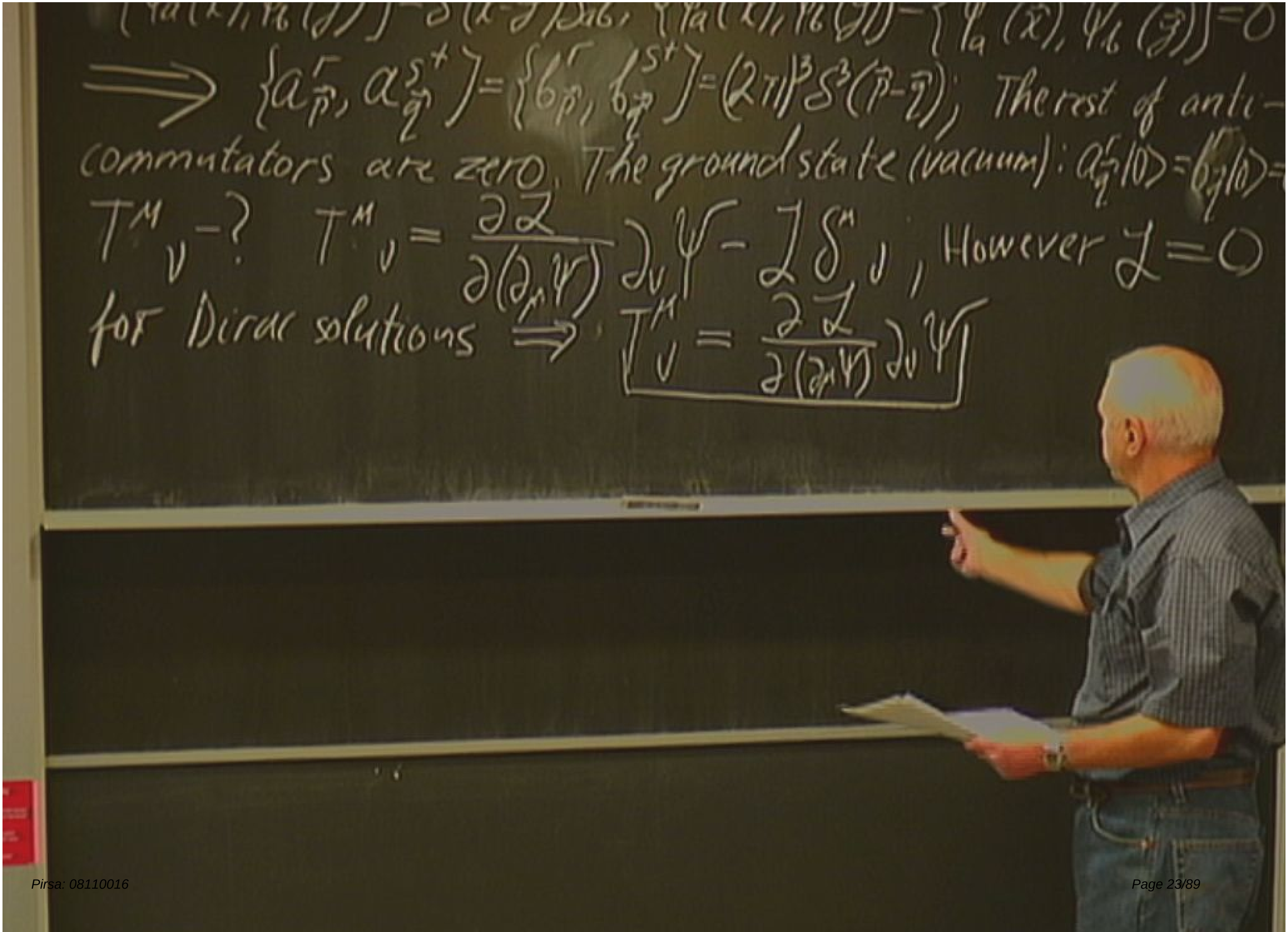
$T^{\mu}{}_{\nu}$ -? $T^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\partial_\nu \psi - \mathcal{L}\delta^{\mu}{}_{\nu}$; However $\mathcal{L}=0$ for Dirac solutions


$\Rightarrow \{a^r_{\vec p}, a^{s\dagger}_{\vec q}\} = \{b^r_{\vec p}, b^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q)$; The rest of anti-
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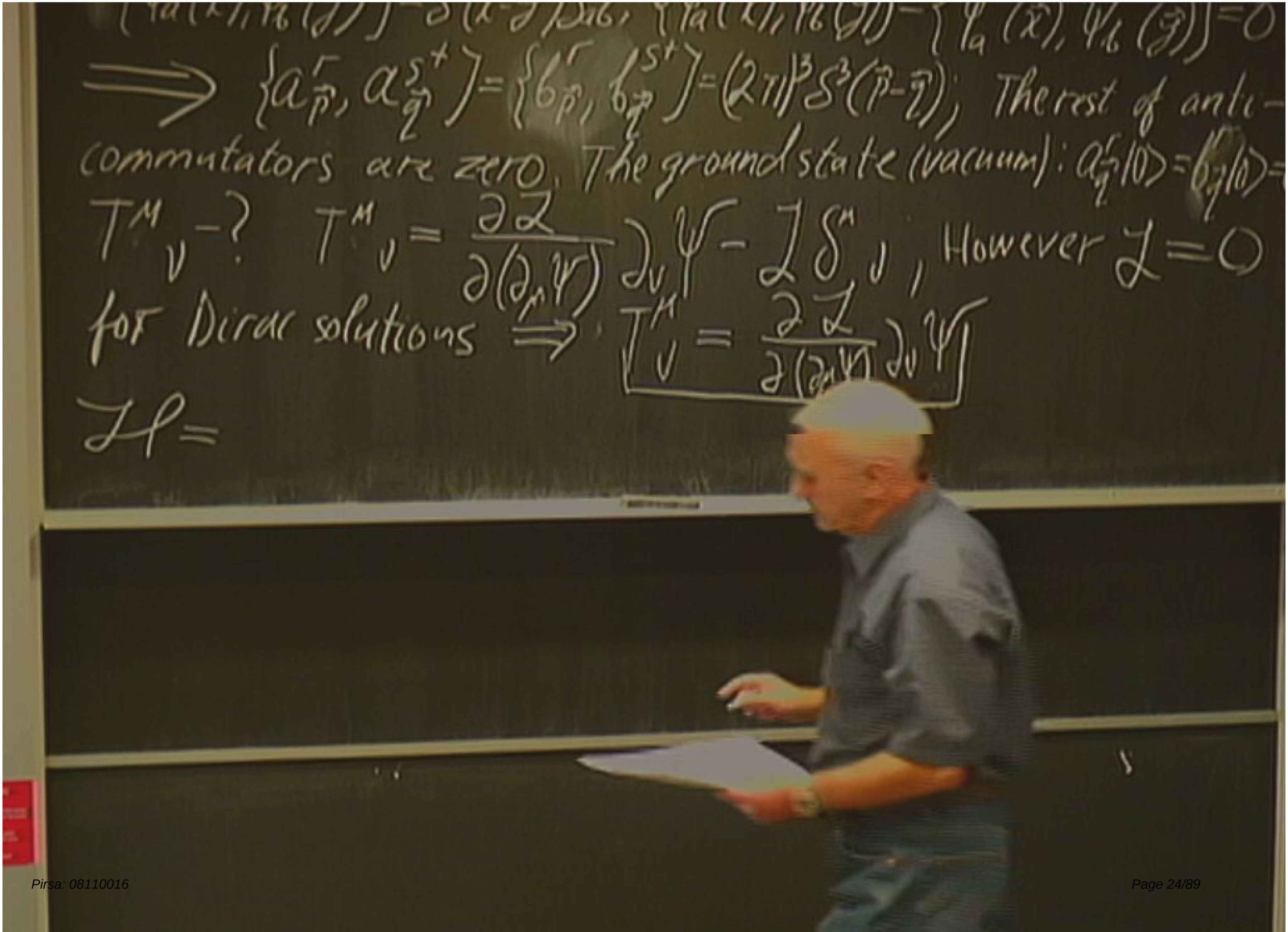

$\Longrightarrow \{a^r_{\vec p}, a^{s+}_{\vec q}\} = \{b^r_{\vec p}, b^{s+}_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q)$; The rest of anti-
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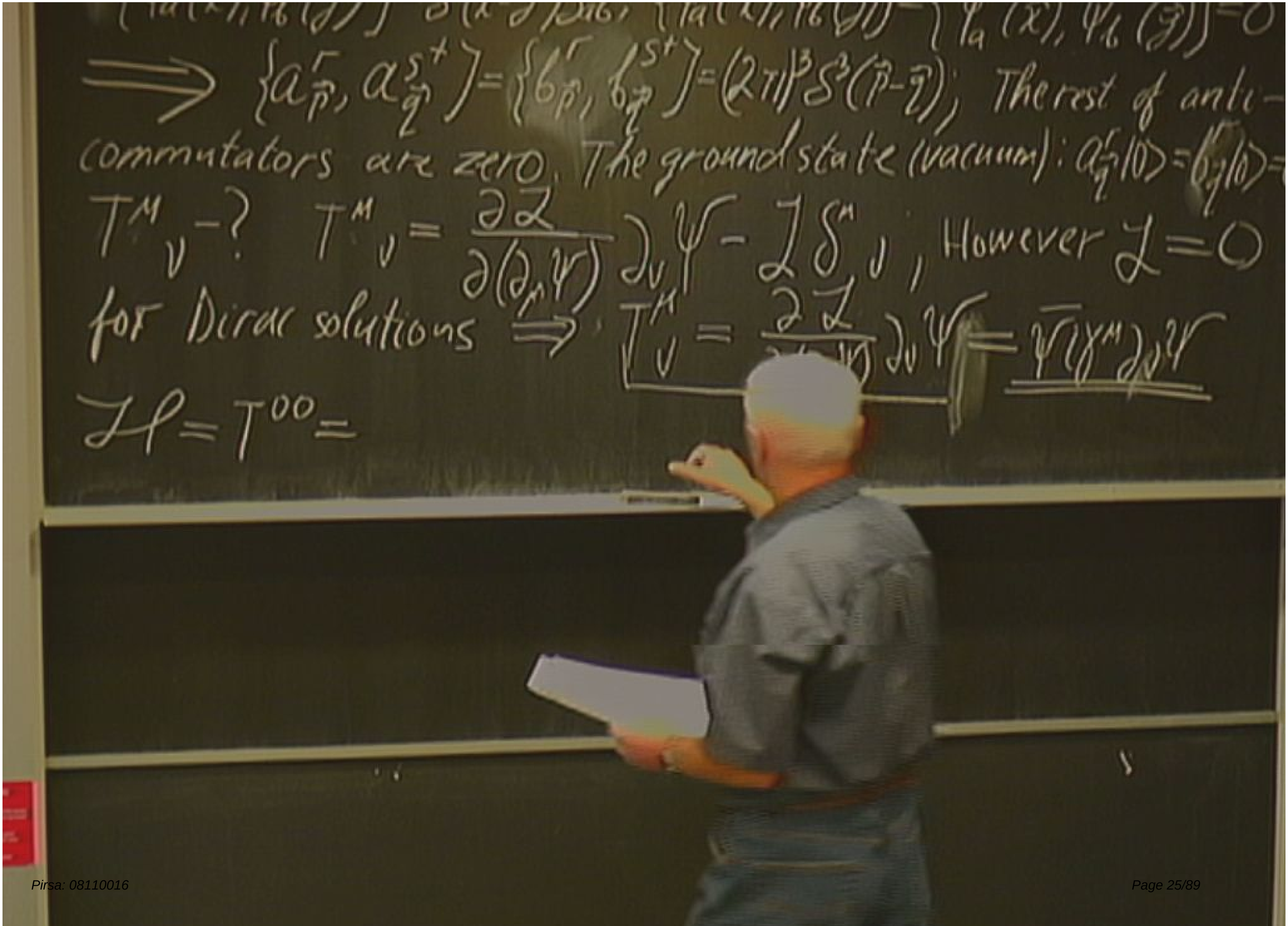
$\mathcal{H} =$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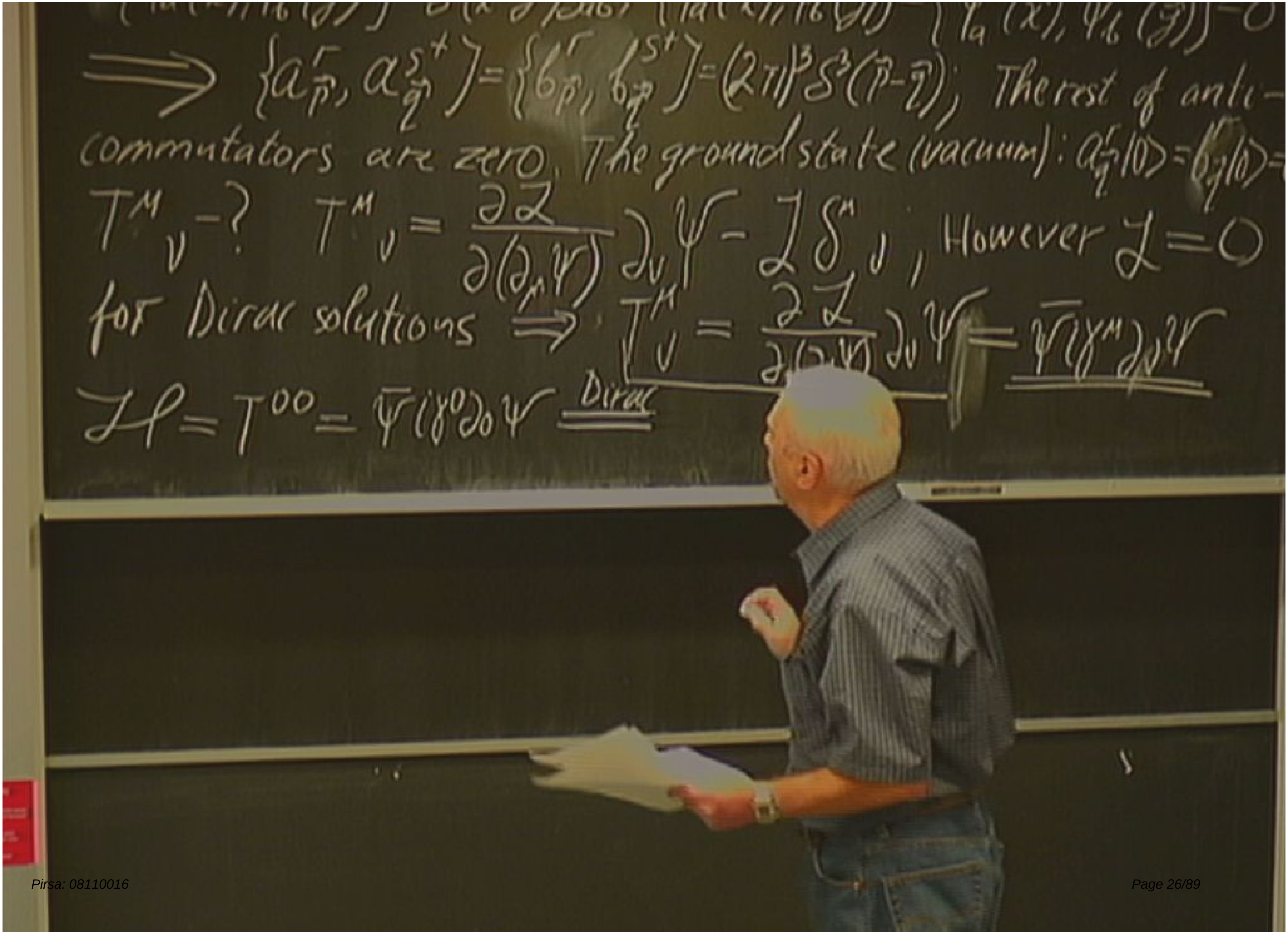
$\mathcal{H} = T^{00} =$


$\Longrightarrow \{a^r_{\vec p}, a^{s+}_{\vec q}\} = \{b^r_{\vec p}, b^{s+}_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q)$; The rest of anti-
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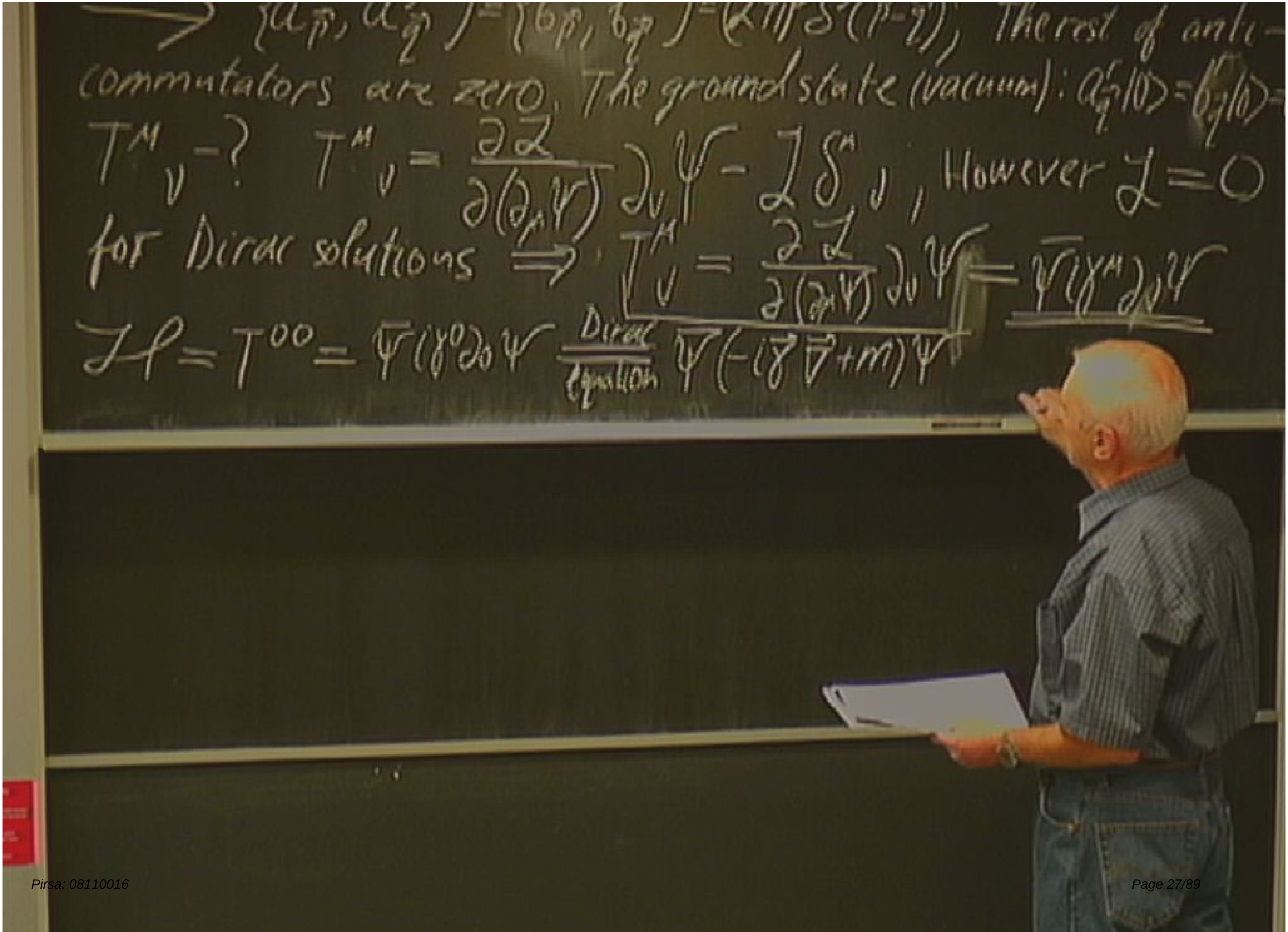
for Dirac solutions $\Longrightarrow T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi)}\partial_\nu\psi = \bar\psi i\gamma^\mu\partial_\nu\psi$
$\mathcal{H} = T^{00} = \bar\psi i\gamma^0\partial_0\psi$ Dirac



$\Rightarrow \{a_{\vec p}, a^\dagger_{\vec q}\} = \{b_{\vec p}, b^\dagger_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q)$, The rest of anti-
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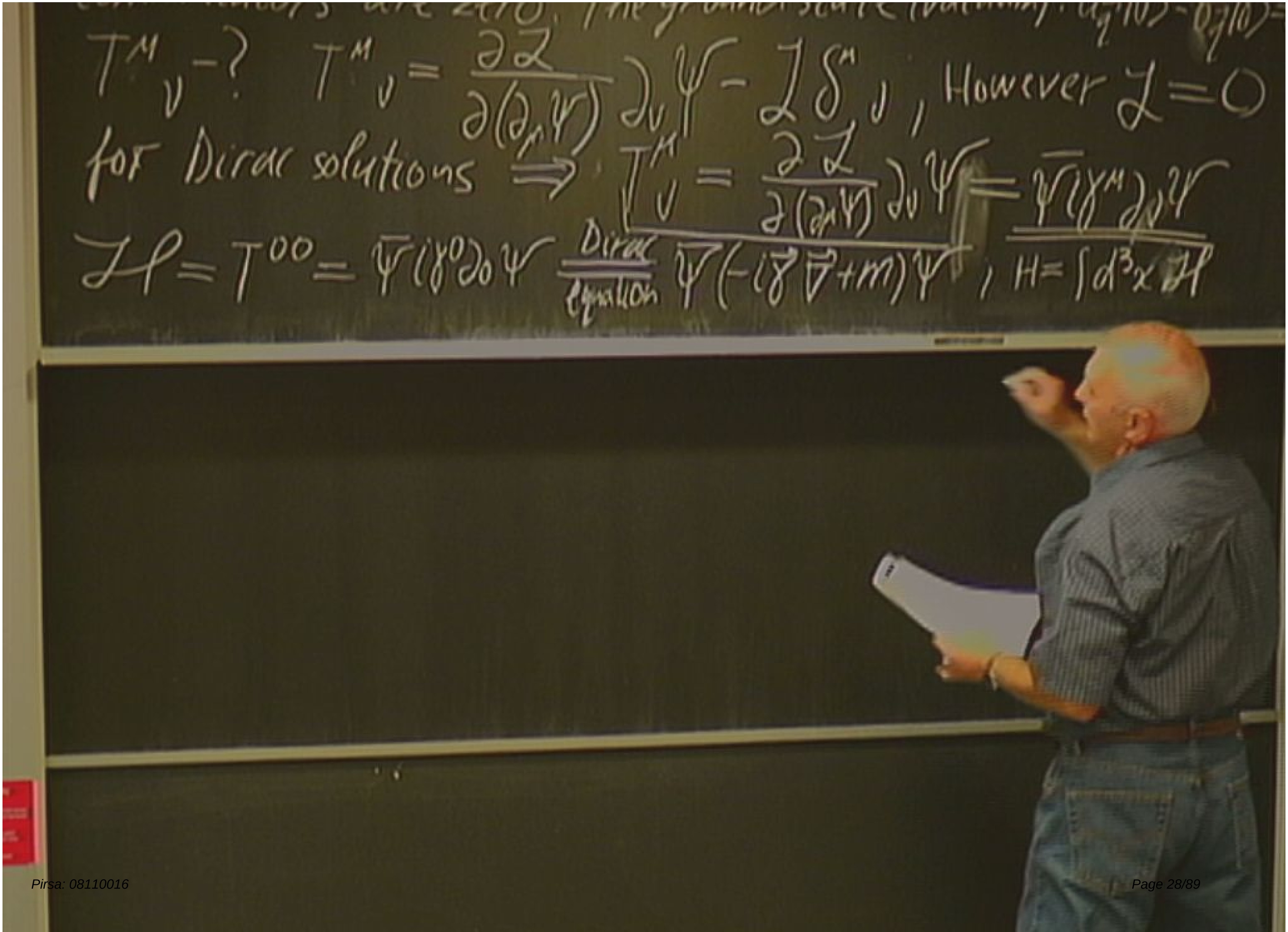
for Dirac solutions $\Rightarrow T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi)}\partial_\nu\psi = \bar\psi i\gamma^\mu\partial_\nu\psi$
$\mathcal{H} = T^{00} = \bar\psi i\gamma^0\partial_0\psi \overset{\text{Dirac equation}}{=} \bar\psi(-i\vec\gamma\cdot\vec\nabla + m)\psi$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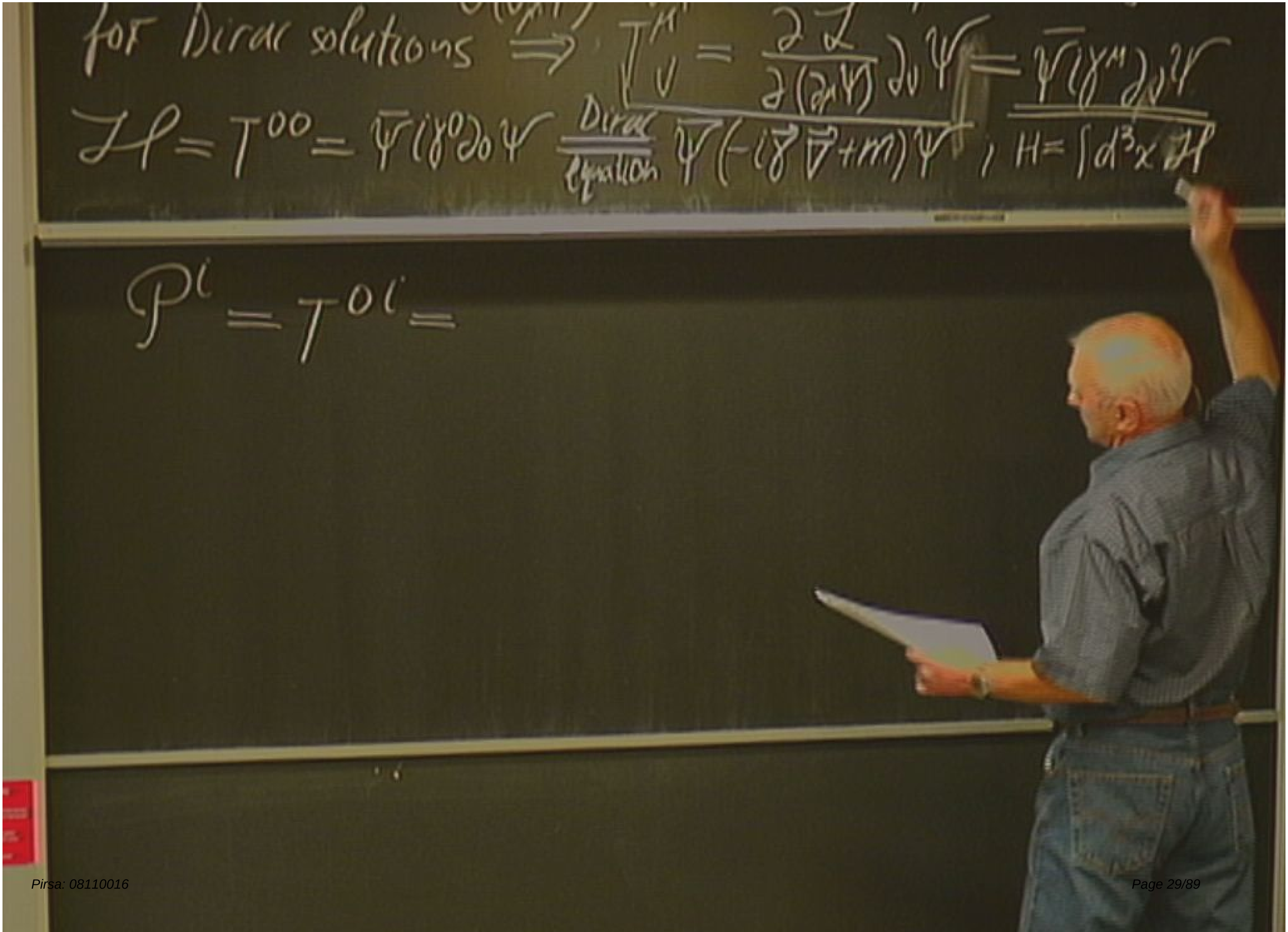
for Dirac solutions $\Rightarrow$ $T^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\partial_\nu \psi = \bar{\psi} i\gamma^{\mu}\partial_\nu \psi$
$\mathcal{H} = T^{00} = \bar{\psi} i\gamma^{0}\partial_0 \psi \underset{\text{Dirac equation}}{=} \bar{\psi}(-i\vec{\gamma}\cdot\vec{\nabla}+m)\psi$, $H=\int d^3x\, \mathcal{H}$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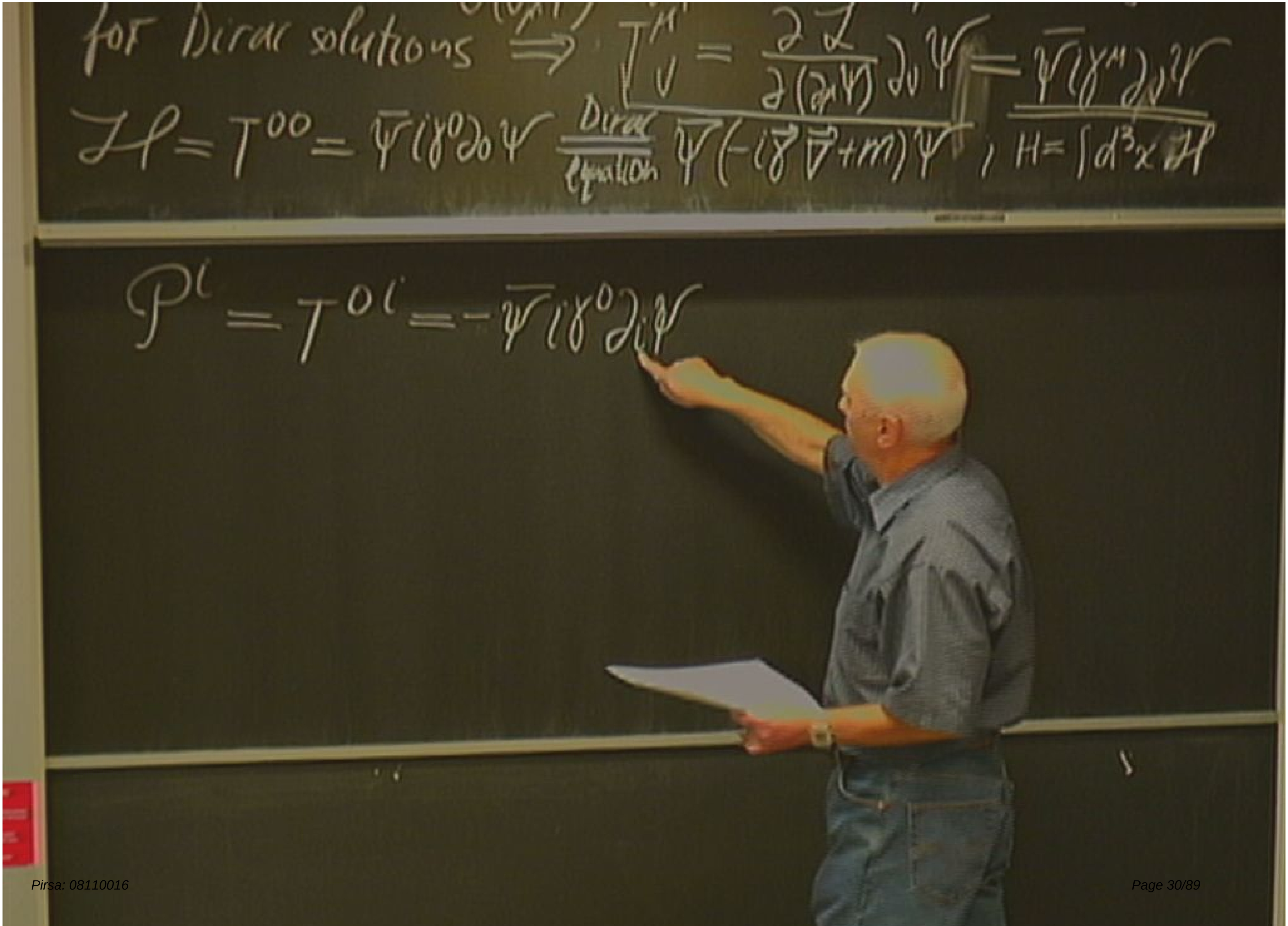
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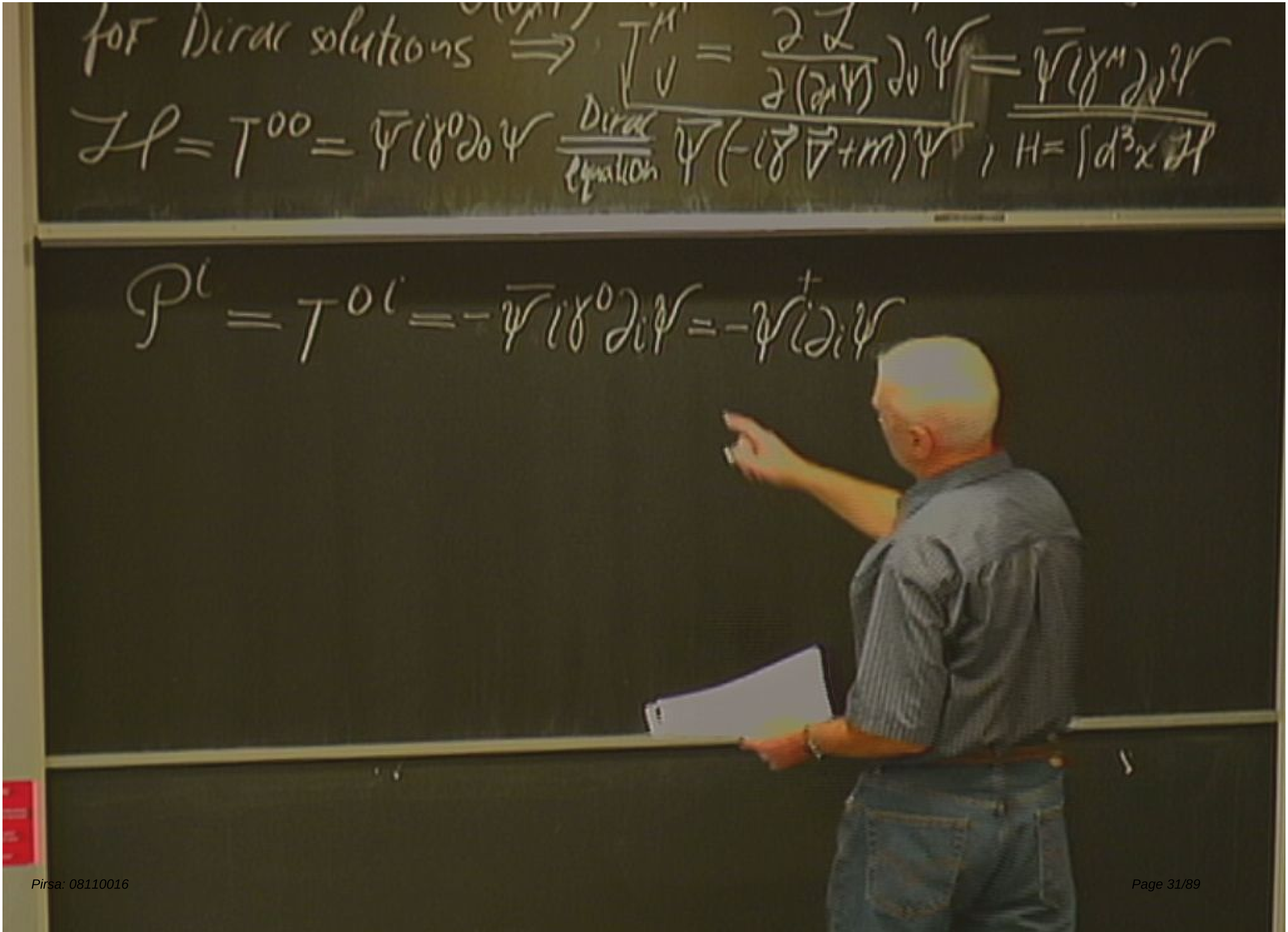
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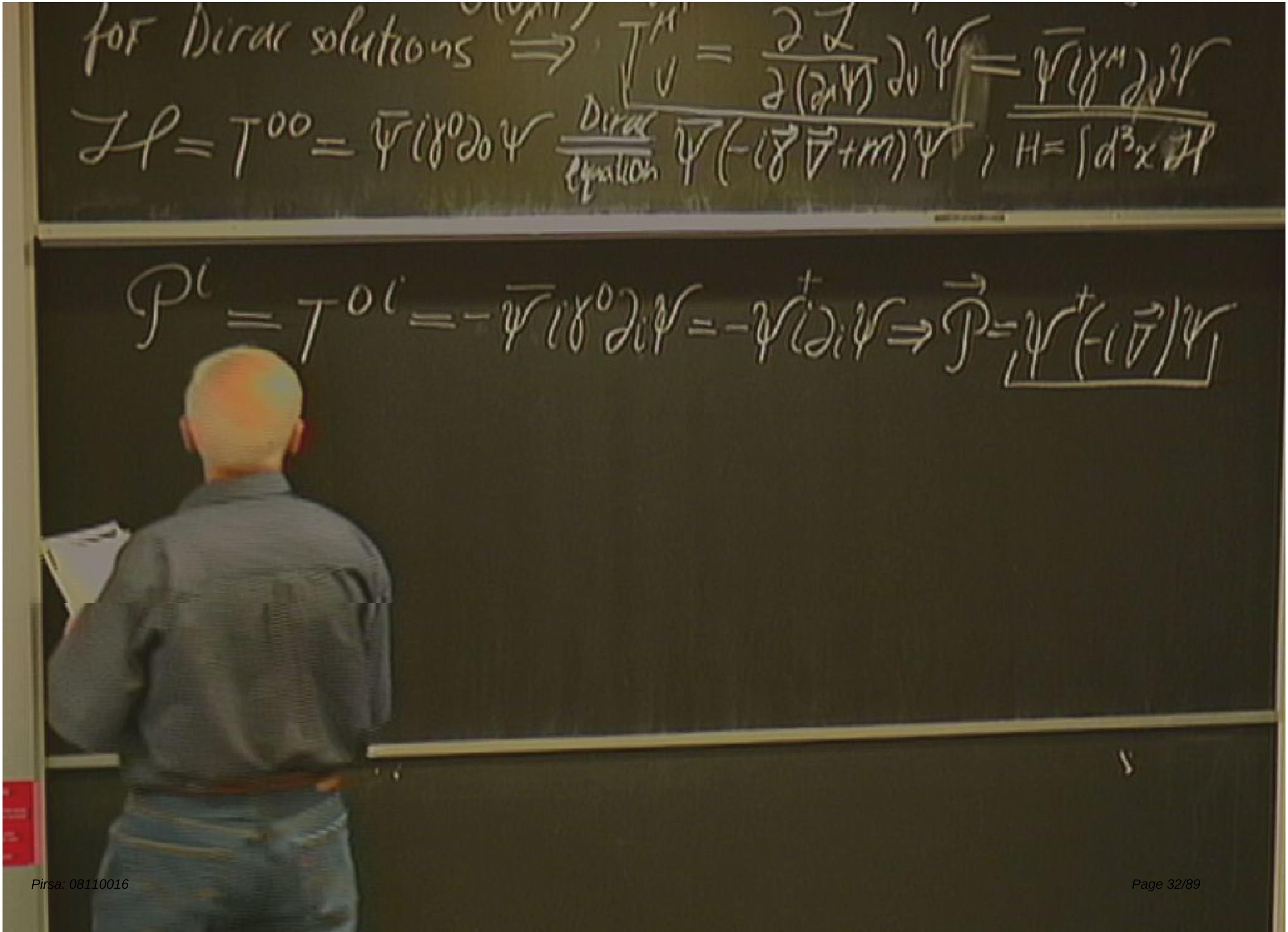
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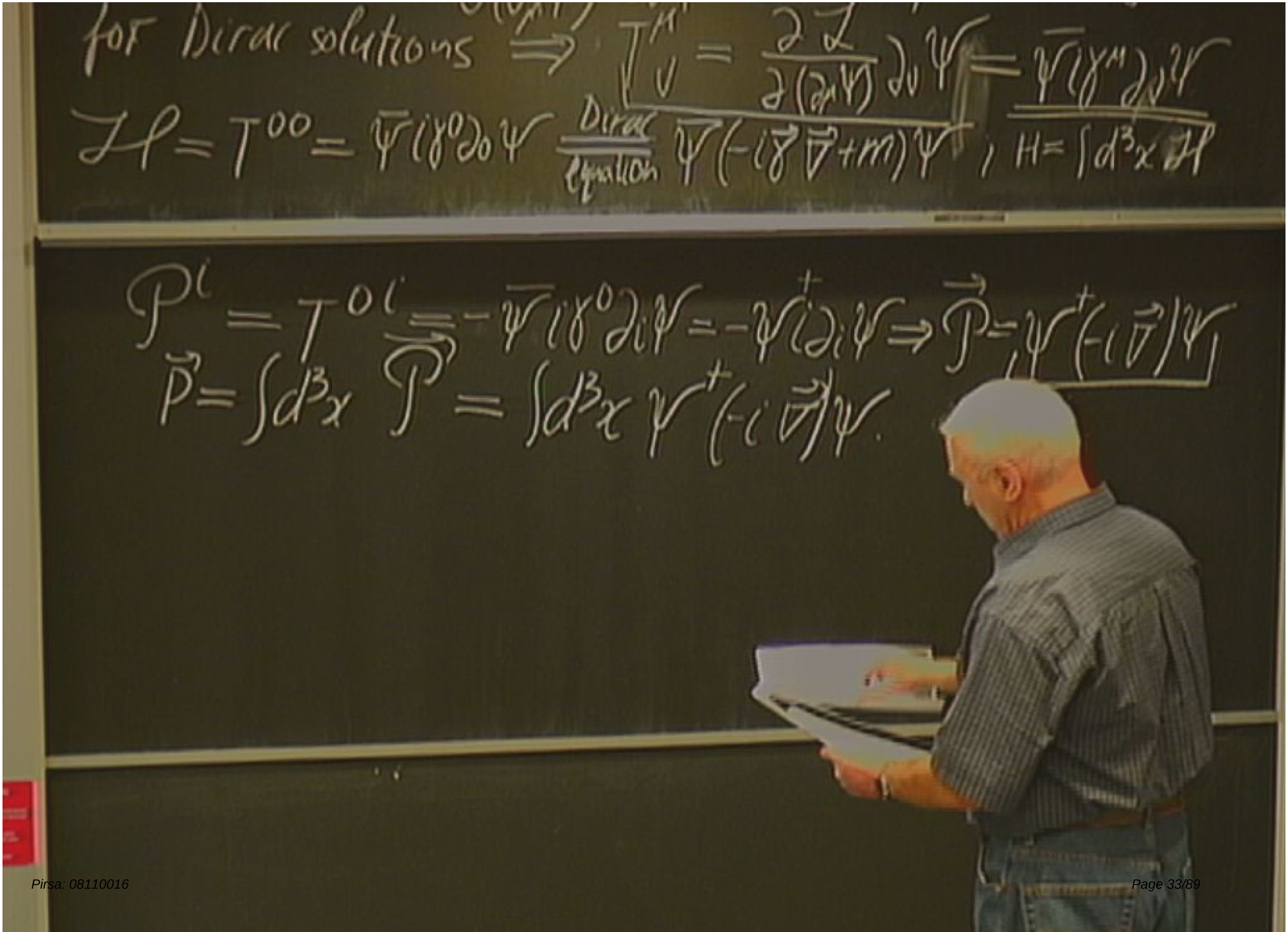
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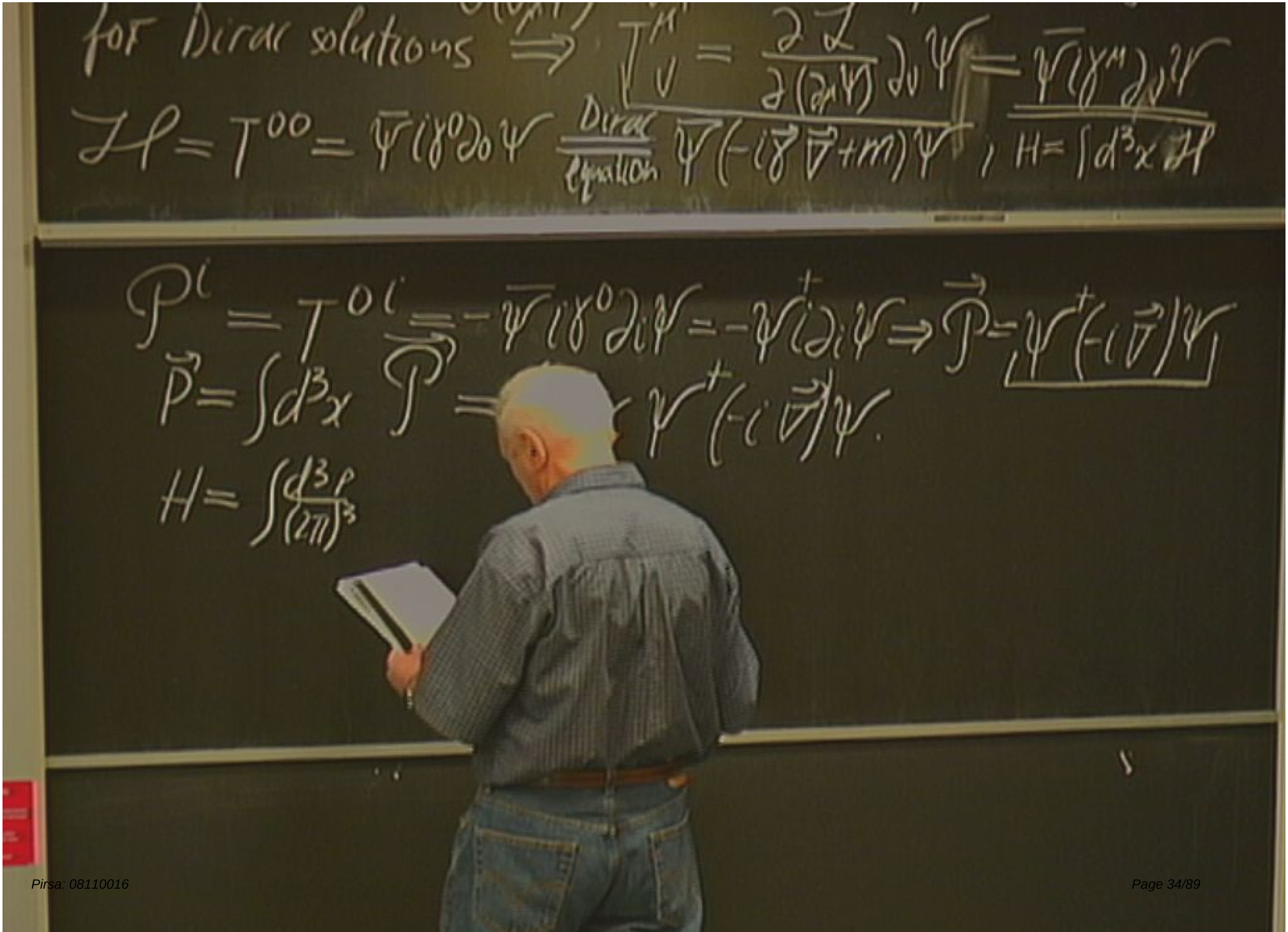
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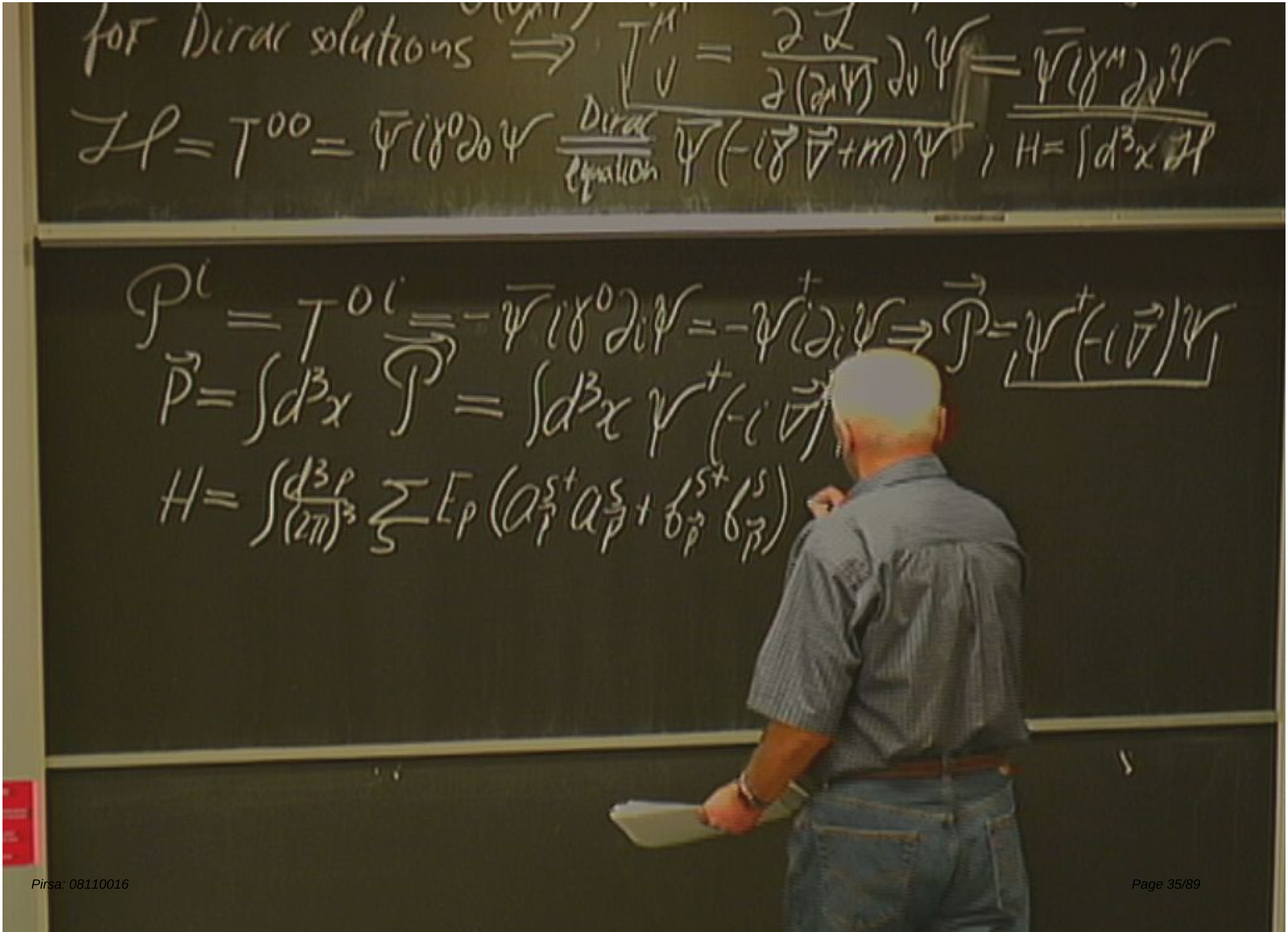

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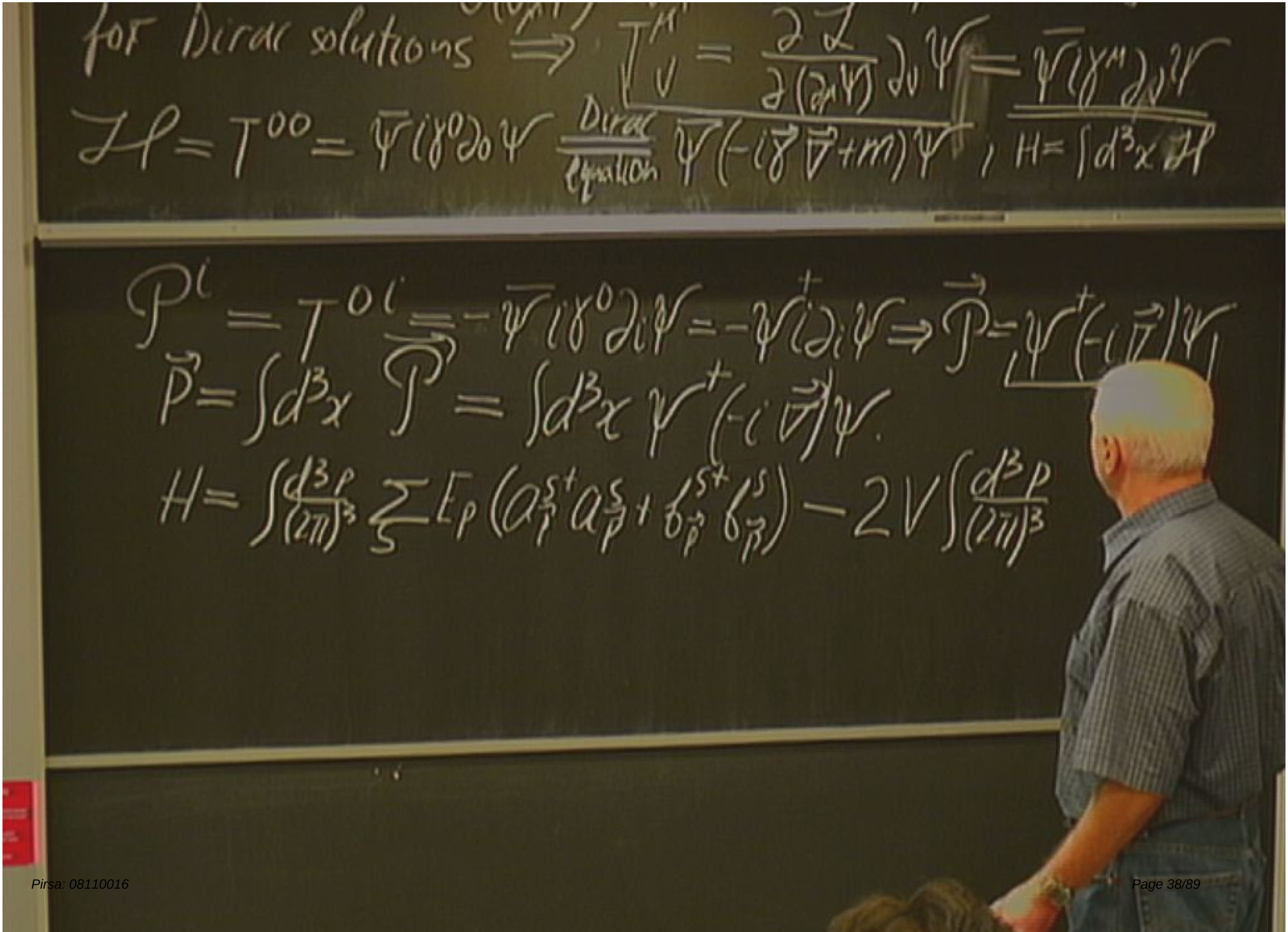
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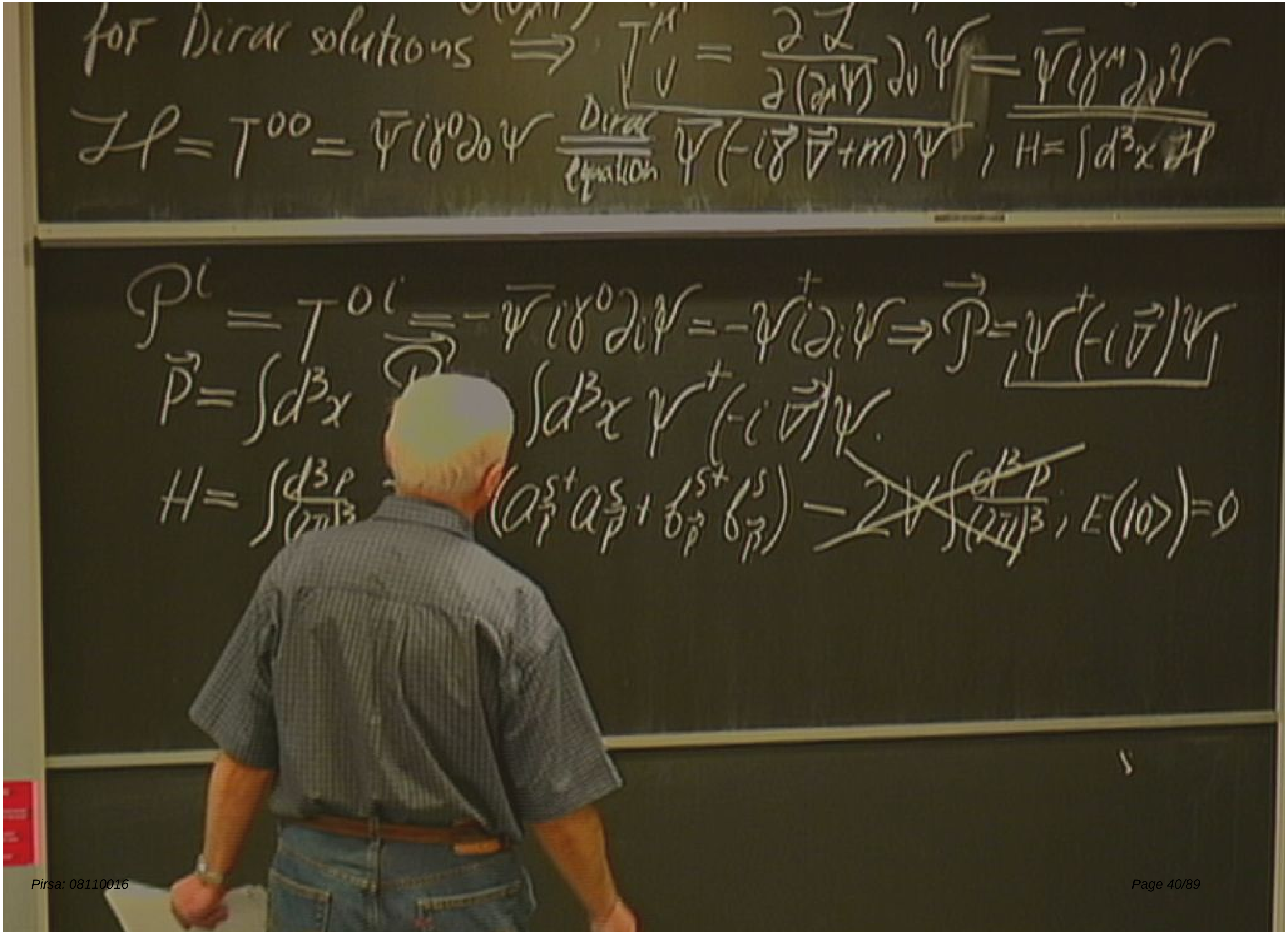
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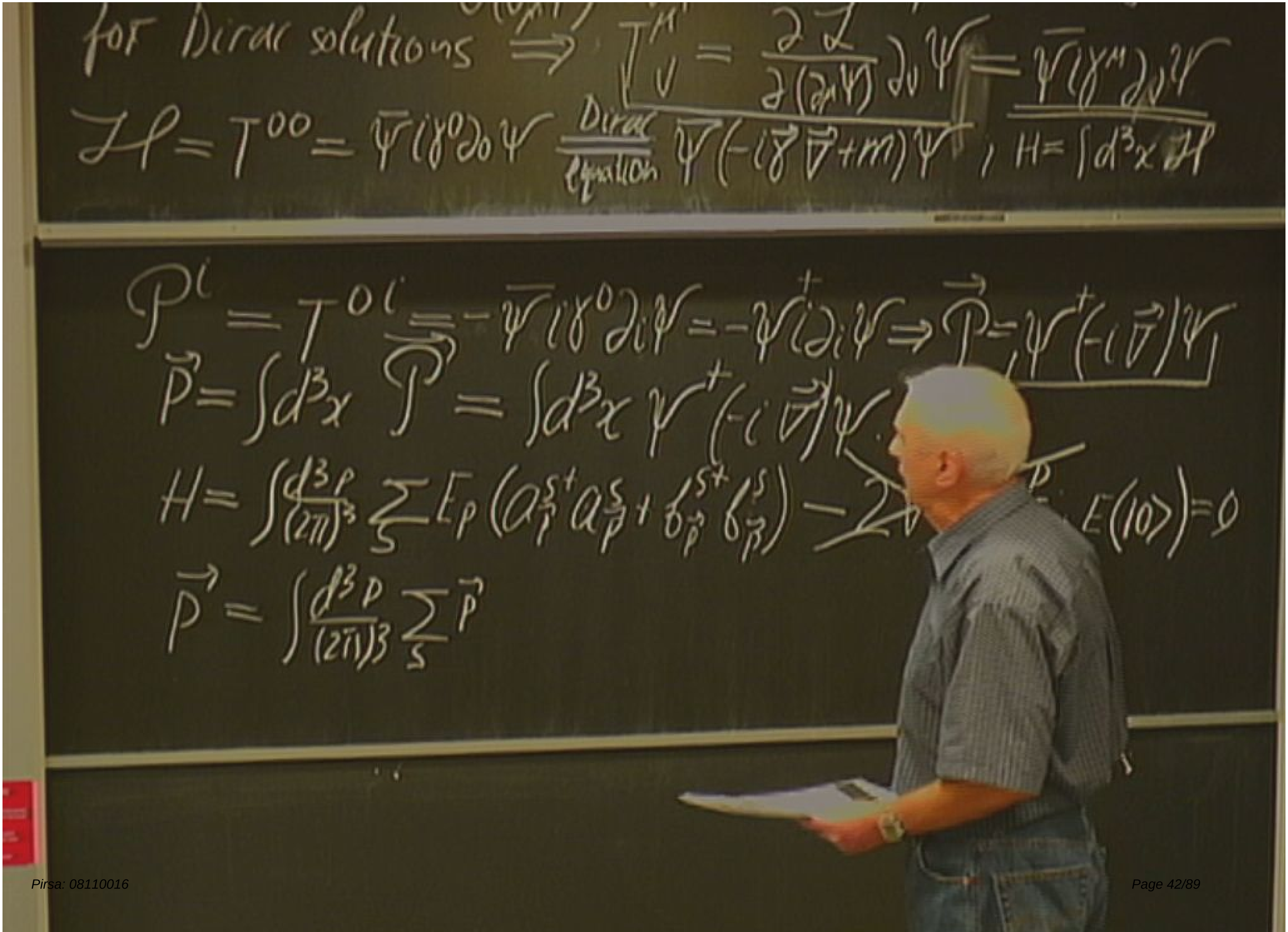
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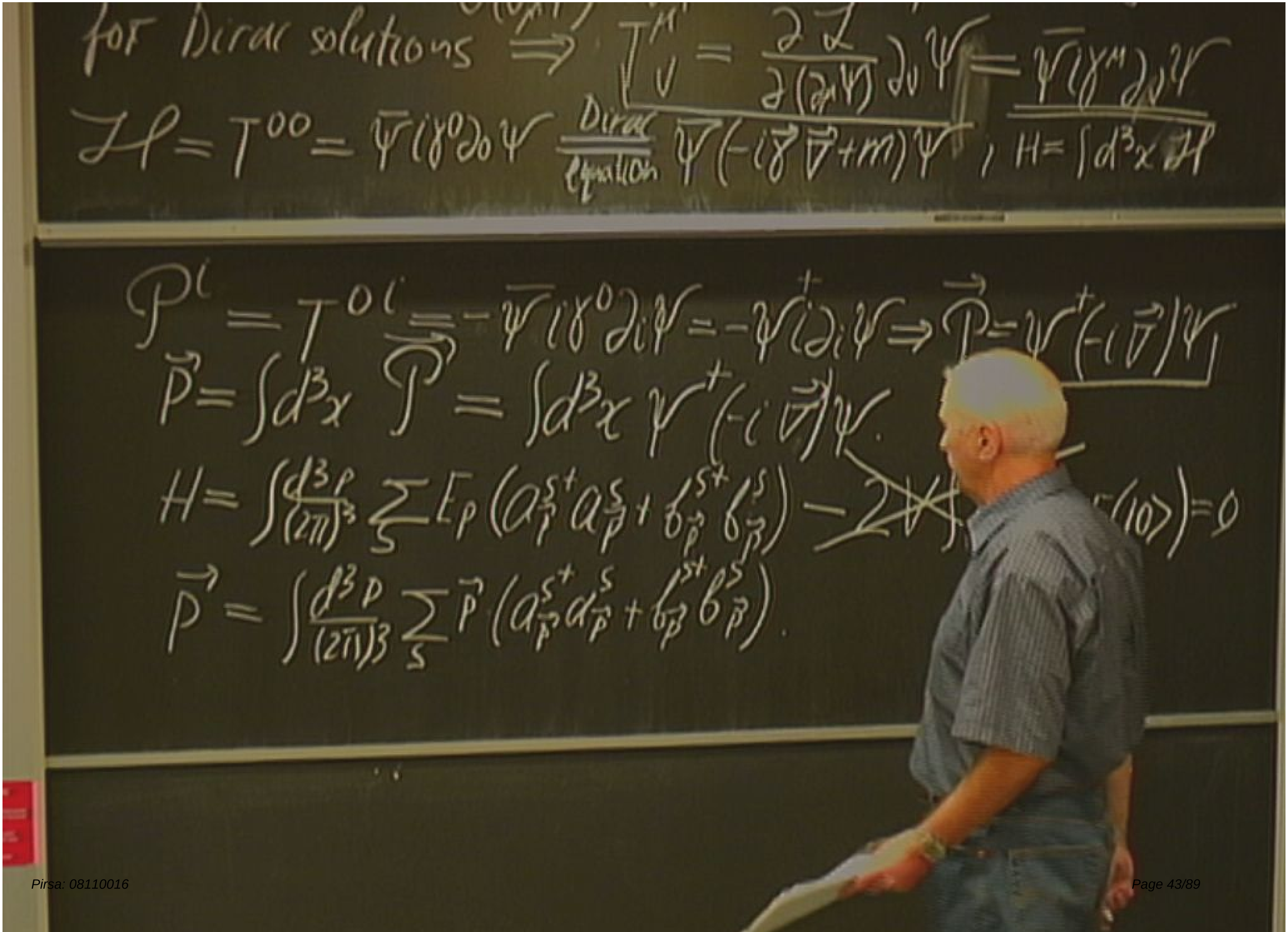
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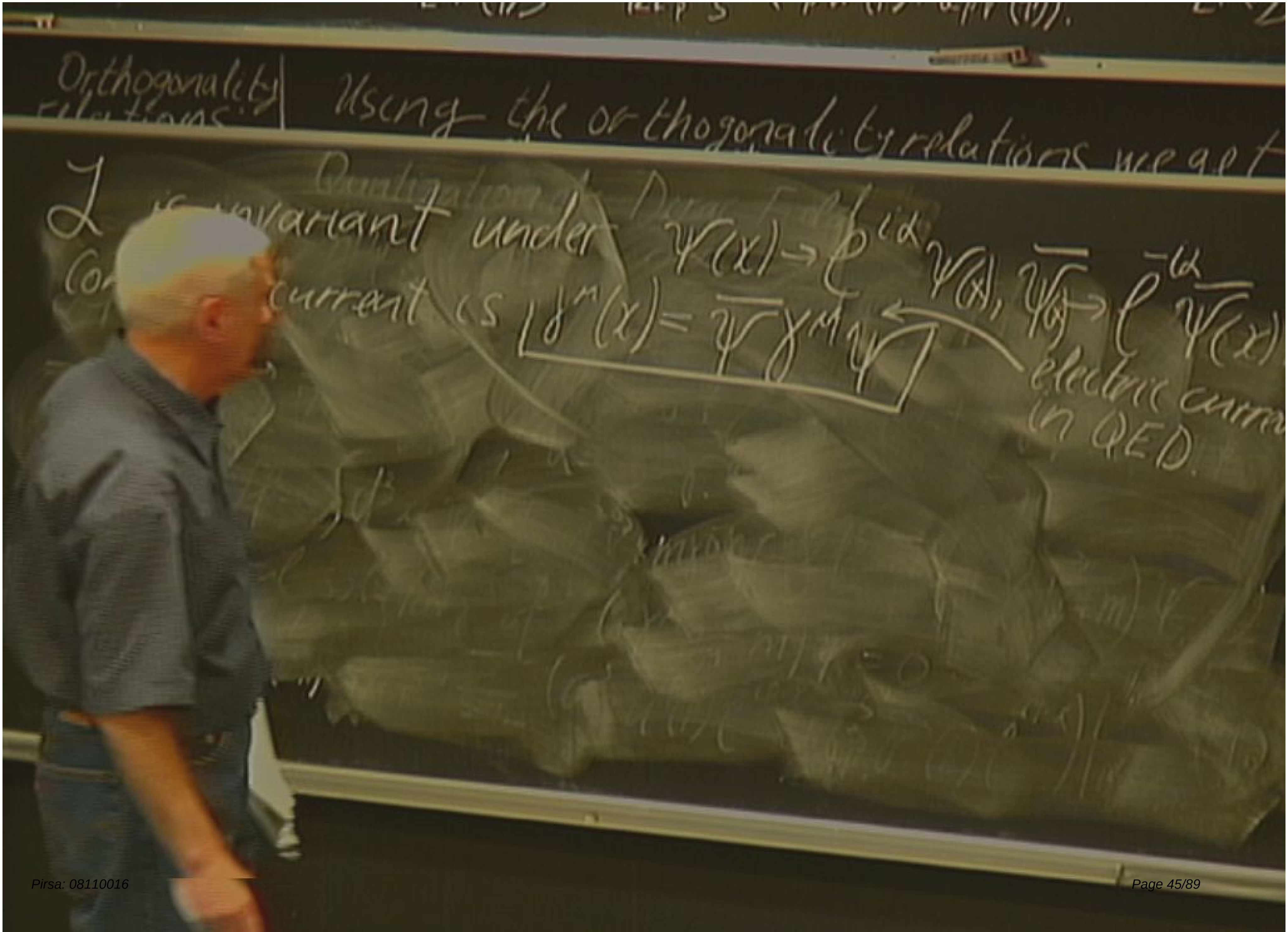
Orthogonality relations:

Using the orthogonality relations we get

$\mathcal{L}$ is invariant under $\psi(x) \to e^{i\alpha}\psi(x)$, $\bar{\psi}(x) \to e^{-i\alpha}\bar{\psi}(x)$.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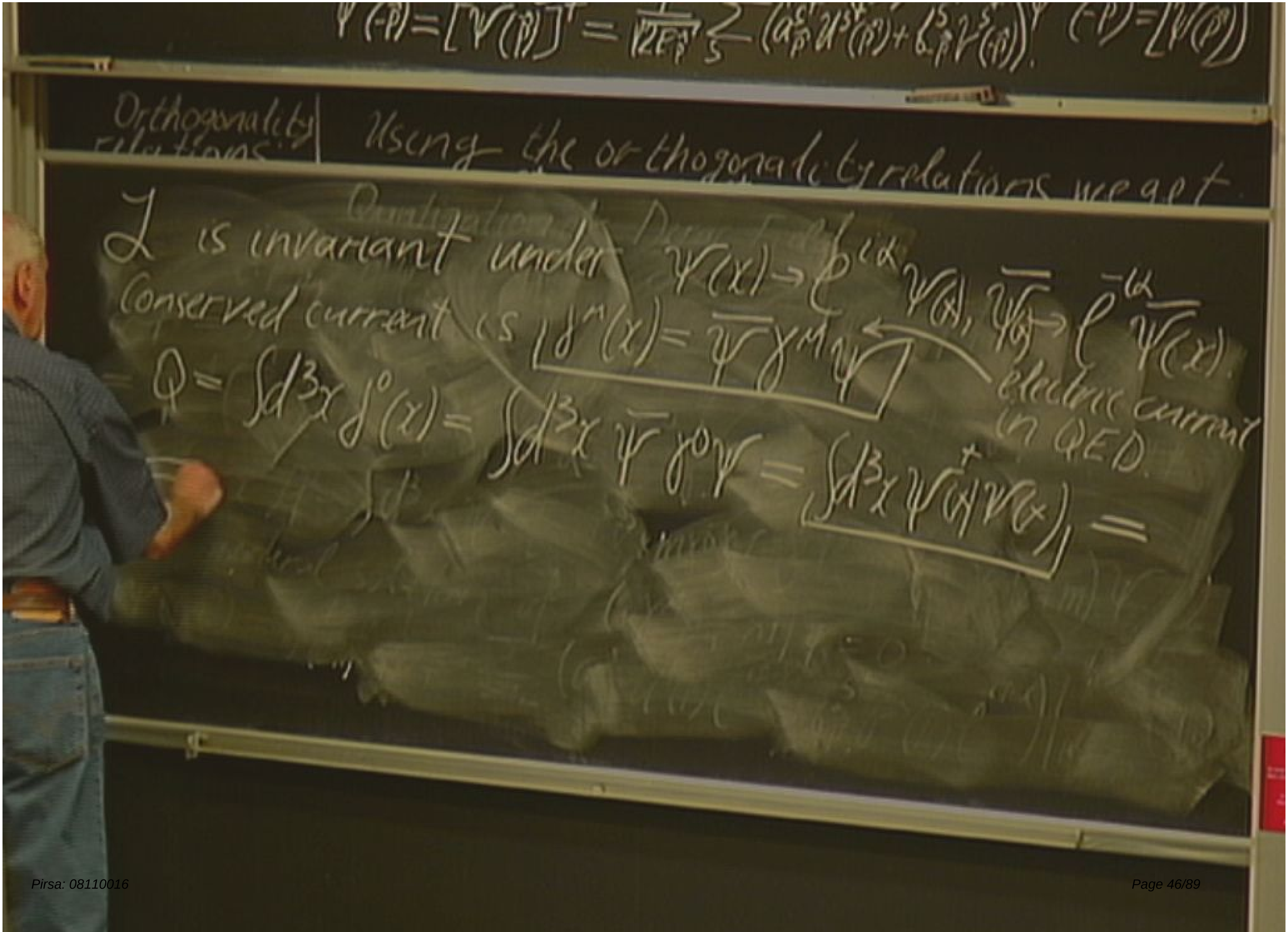
current is $\boxed{j^\mu(x) = \overline{\psi}\gamma^\mu\psi}$
electric current in QED.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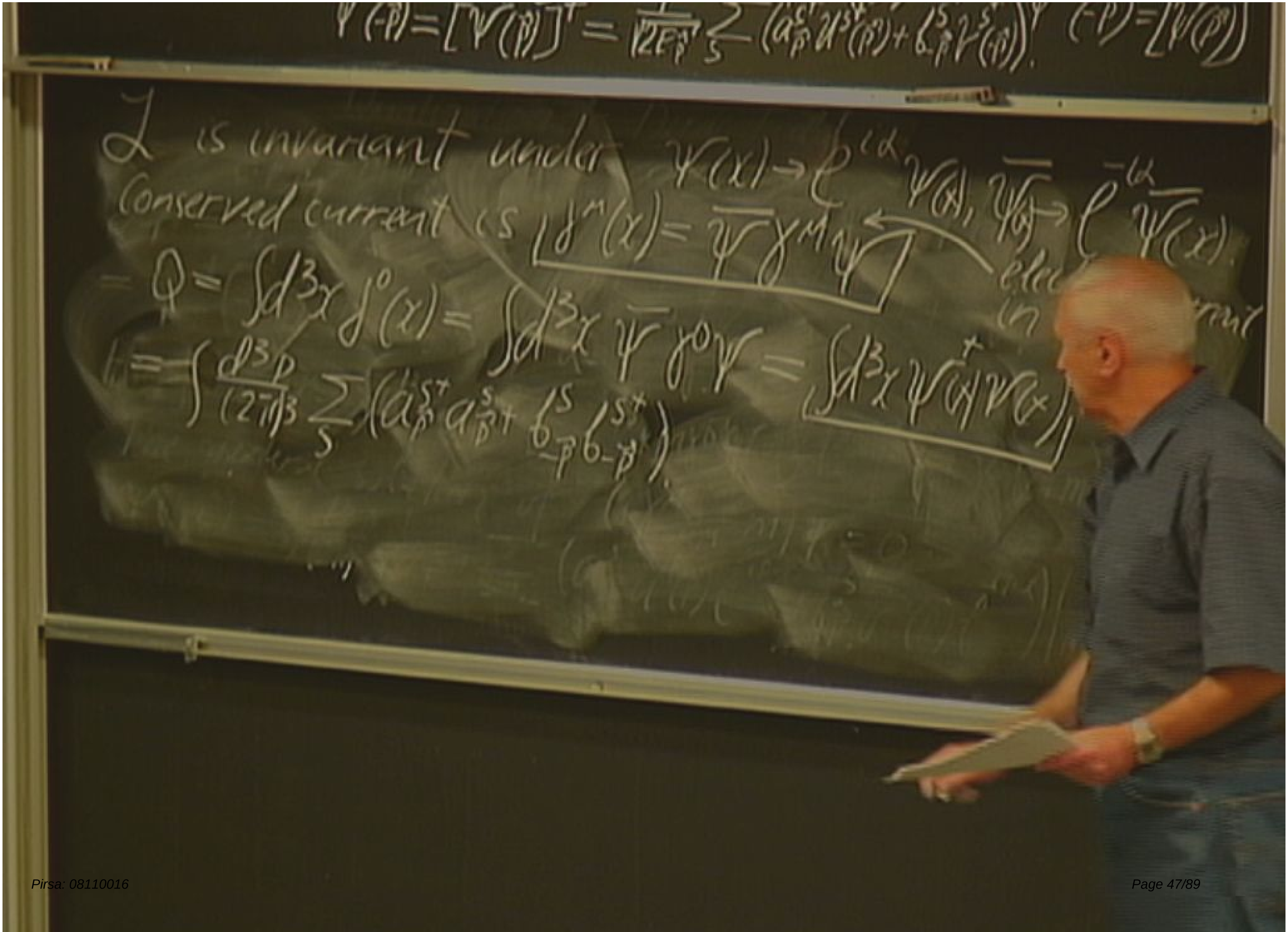
electric current in QED.
$Q = \int d^3x\, j^0(x) = \int d^3x\, \bar\psi\gamma^0\psi = \int d^3x\, \psi^\dagger(x)\psi(x) =$



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$Q = \int d^3x\, j^0(x) = \int d^3x\, \bar{\psi}\gamma^0\psi = \boxed{\int d^3x\, \psi^\dagger(x)\psi(x)}$
$= \int \frac{d^3p}{(2\pi)^3} \sum_s (a_p^{s\dagger} a_p^s + b_{-p}^s b_{-p}^{s\dagger})$.

$$\bar\psi(-\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a_{\vec p}^{s\dagger} u^{s\dagger}(\vec p) + b_{-\vec p}^{s} v^{s\dagger}(-\vec p)\right) \quad \bar\psi(-\vec p) = [\psi(\vec p)]$$

$\mathcal{L}$ is invariant under $\psi(x)\to e^{i\alpha}\psi(x)$, $\bar\psi(x)\to e^{-i\alpha}\bar\psi(x)$.

Conserved current is $\boxed{j^\mu(x) = \bar\psi\gamma^\mu\psi}$ electric current in QED.

$$Q = \int d^3x\, j^0(x) = \int d^3x\, \bar\psi\gamma^0\psi = \int d^3x\, \psi^\dagger(x)\psi(x) =$$

$$= \int \frac{d^3p}{(2\pi)^3}\sum_s \left(a_{\vec p}^{s\dagger} a_{\vec p}^{s} + b_{-\vec p}^{s} b_{-\vec p}^{s\dagger}\right) = \int \frac{d^3p}{(2\pi)^3}\sum_s \left[a_{\vec p}^{s\dagger} a_{\vec p}^{s} - b_{-\vec p}^{s\dagger} b_{-\vec p}^{s}\right) +$$

$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}} \sum_s (a^{s\dagger}_{\vec p} u^{s\dagger}(\vec p) + b^{s}_{-\vec p} v^{s\dagger}(-\vec p))$. $\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger$

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$$= \int \frac{d^3p}{(2\pi)^3} \sum_s (a^{s\dagger}_{\vec p} a^s_{\vec p} + b^s_{-\vec p} b^{s\dagger}_{-\vec p}) = \int \frac{d^3p}{(2\pi)^3} \sum_s (a^{s\dagger}_{\vec p} a^s_{\vec p} - b^{s\dagger}_{-\vec p} b^s_{-\vec p}) +$$

$$+ 2V \int \frac{d^3p}{(2\pi)^3}$$

Orthogonality relations:

$$u^{r\dagger}(p)\,u^s(p) = 2E_{\vec p}\,\delta^{rs}$$

$$v^{r\dagger}(p)\,v^s(p) = 2E_{\vec p}\,\delta^{rs}$$

$$u^{r\dagger}(\vec p)\,v^s(-\vec p) = v^{r\dagger}(-\vec p)\,u^s(\vec p) = 0$$

Using the orthogonality relations, we get:

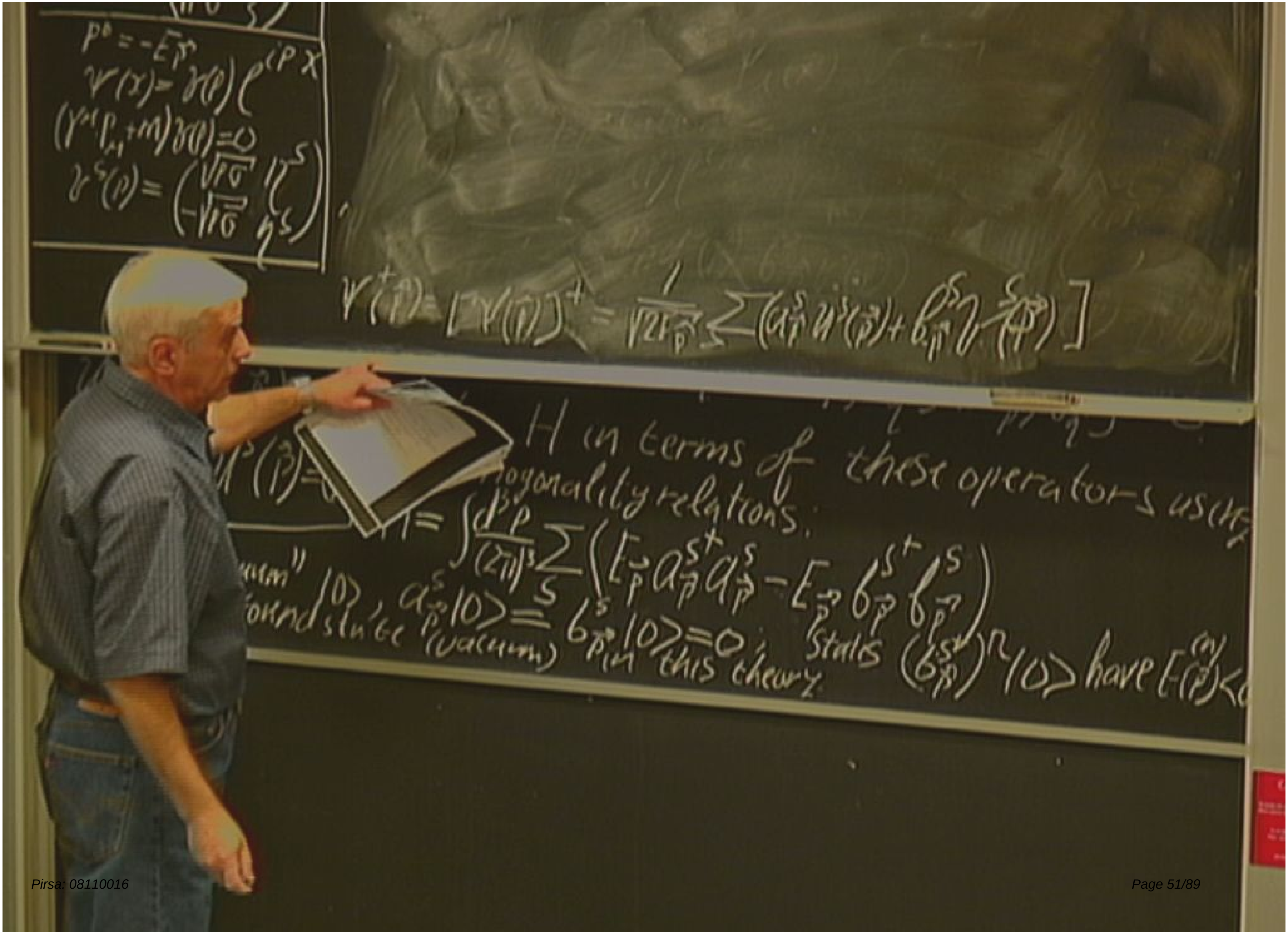
$$[a^r_{\vec p}, a^{s\dagger}_{\vec q}] = [b^r_{\vec p}, b^{s\dagger}_{\vec q}] = (2\pi)^3\delta^3(\vec p-\vec q)\,\delta^{rs},$$

$$[a^r_{\vec p}, a^s_{\vec q}] = [b^r_{\vec p}, b^s_{\vec q}] = [a^{r\dagger}_{\vec p}, a^{s\dagger}_{\vec q}] = [b^{r\dagger}_{\vec p}, b^{s\dagger}_{\vec q}] = 0$$

Write H in terms of these operators using the orthogonality relations:

$$H = \int \frac{d^3p}{(2\pi)^3}\sum_s \left(E_{\vec p}\,a^{s\dagger}_{\vec p}a^s_{\vec p} - E_{\vec p}\,b^{s\dagger}_{\vec p}b^s_{\vec p}\right)$$

There is no ground state (vacuum) in this theory: "vacuum" $|0\rangle$, $a^s_{\vec p}|0\rangle = b^s_{\vec p}|0\rangle = 0$; States $(b^{s\dagger}_{\vec p})^n|0\rangle$ have $E(\vec p)<0$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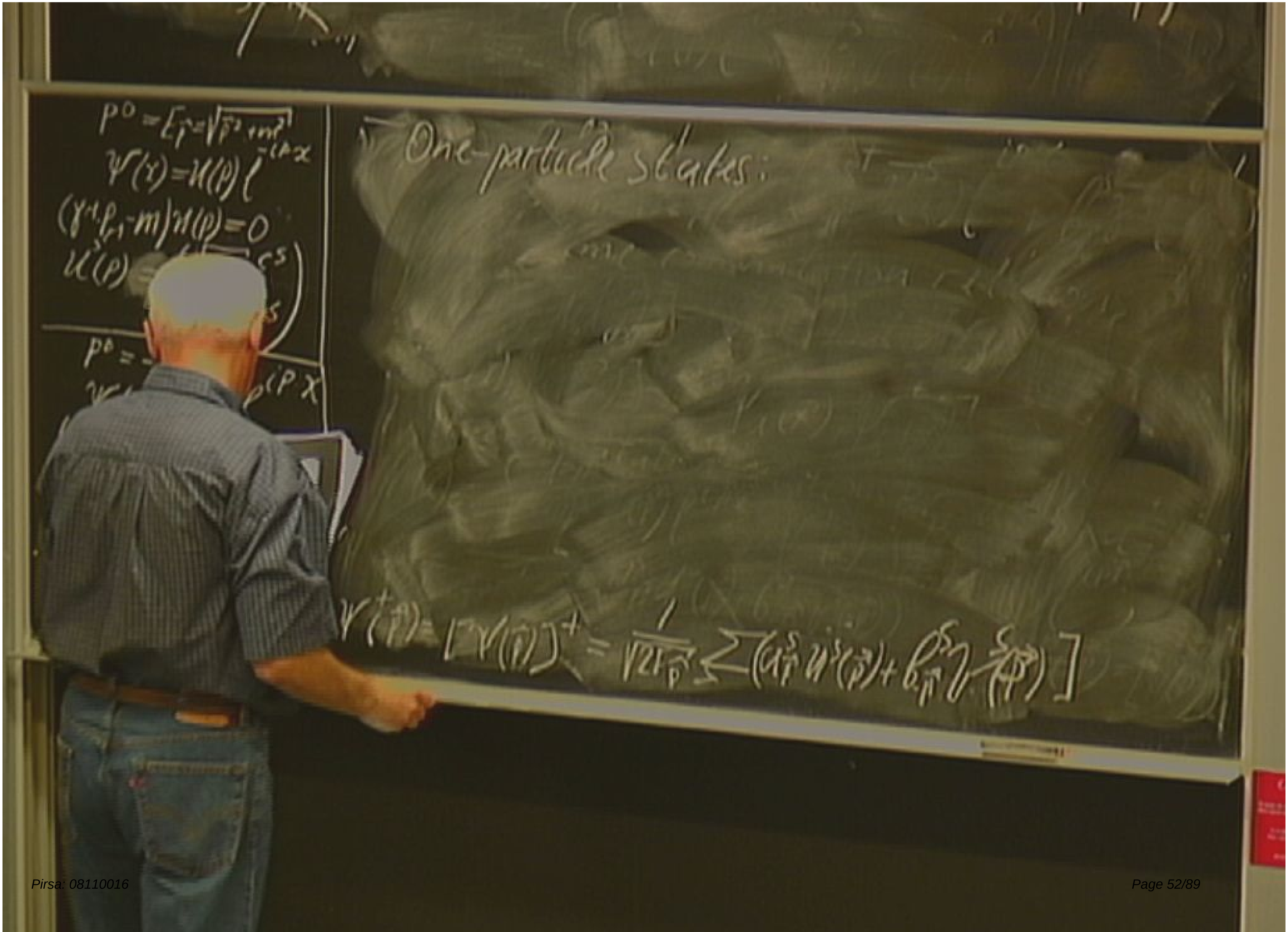
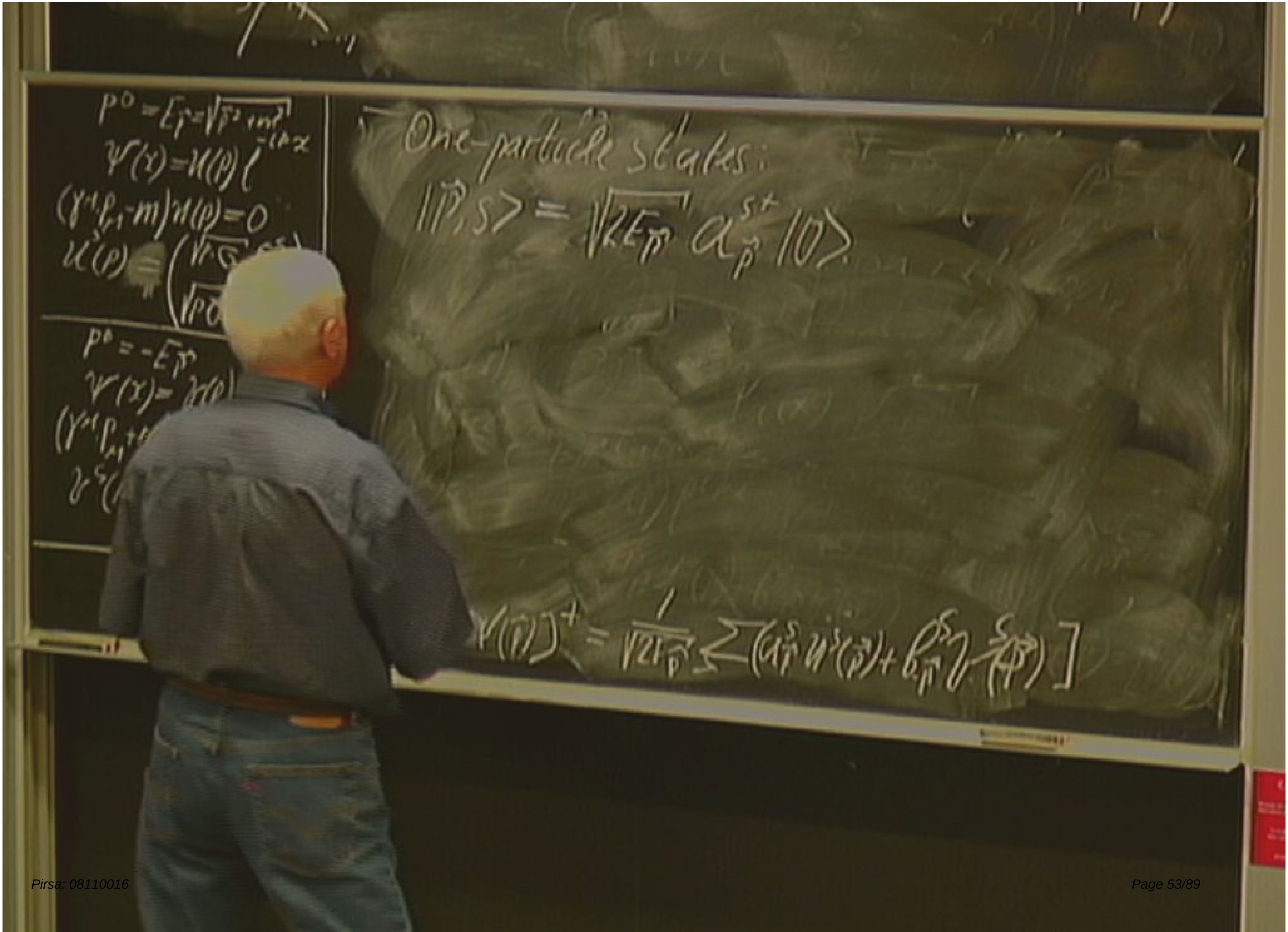
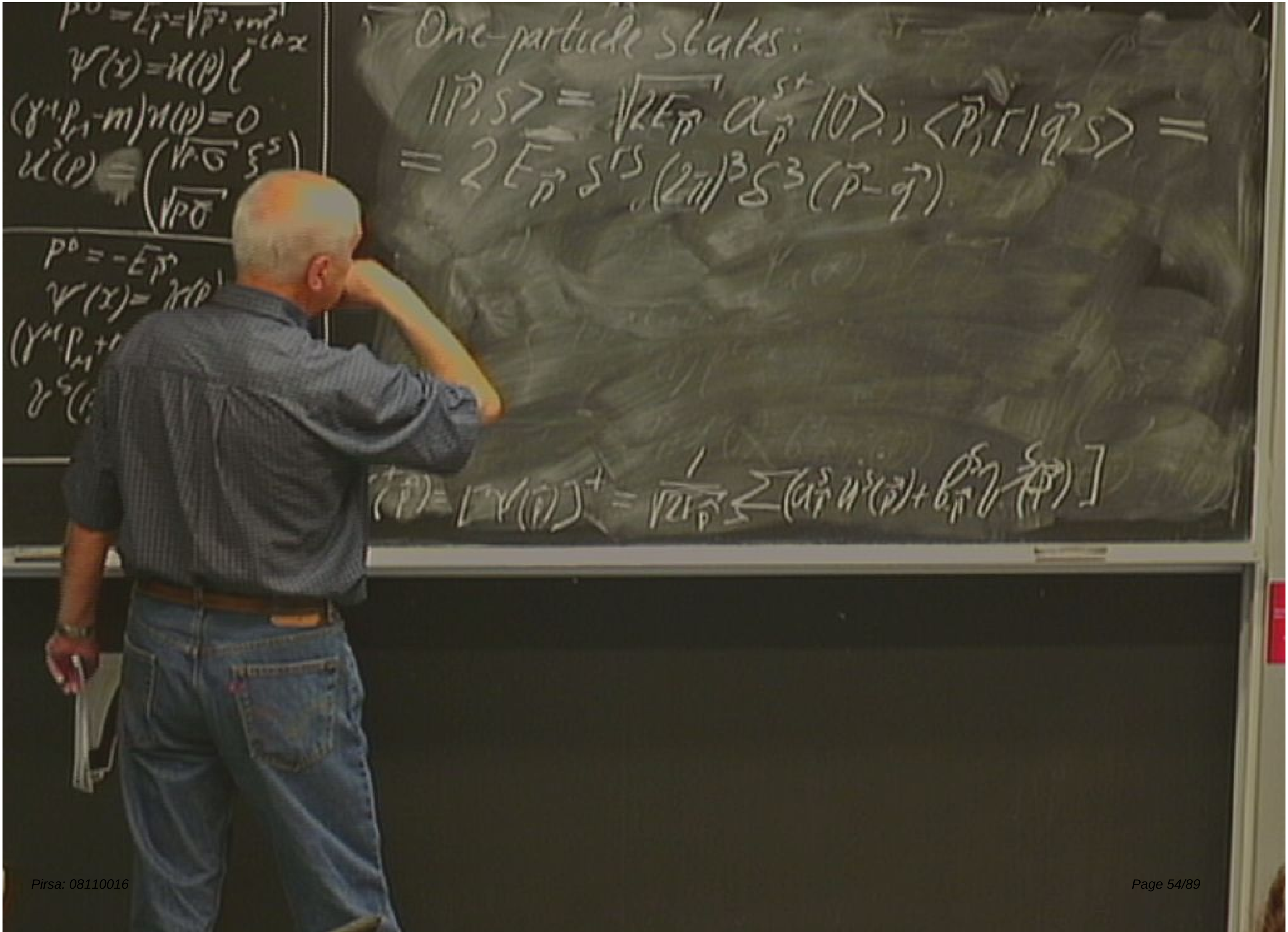
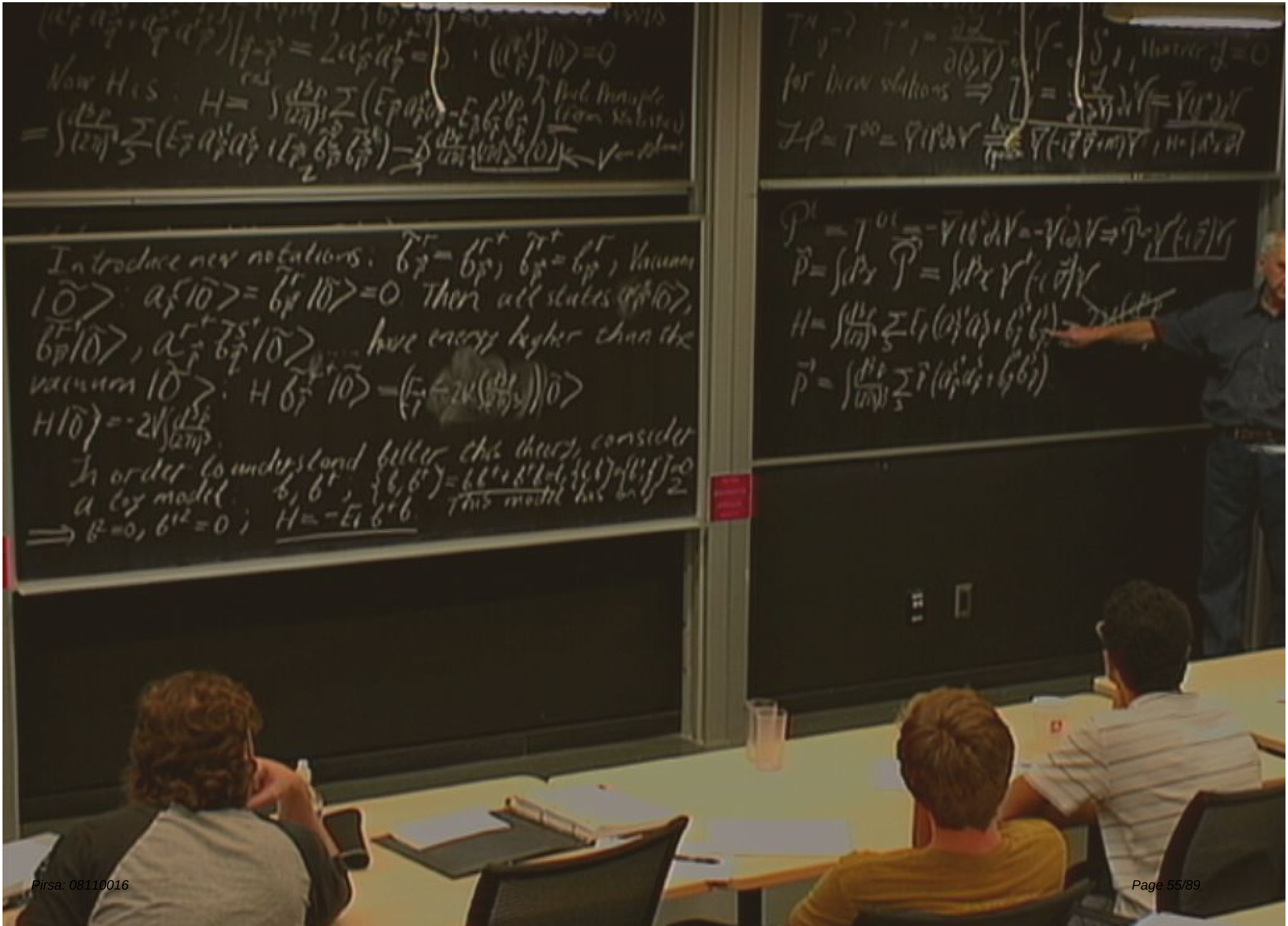
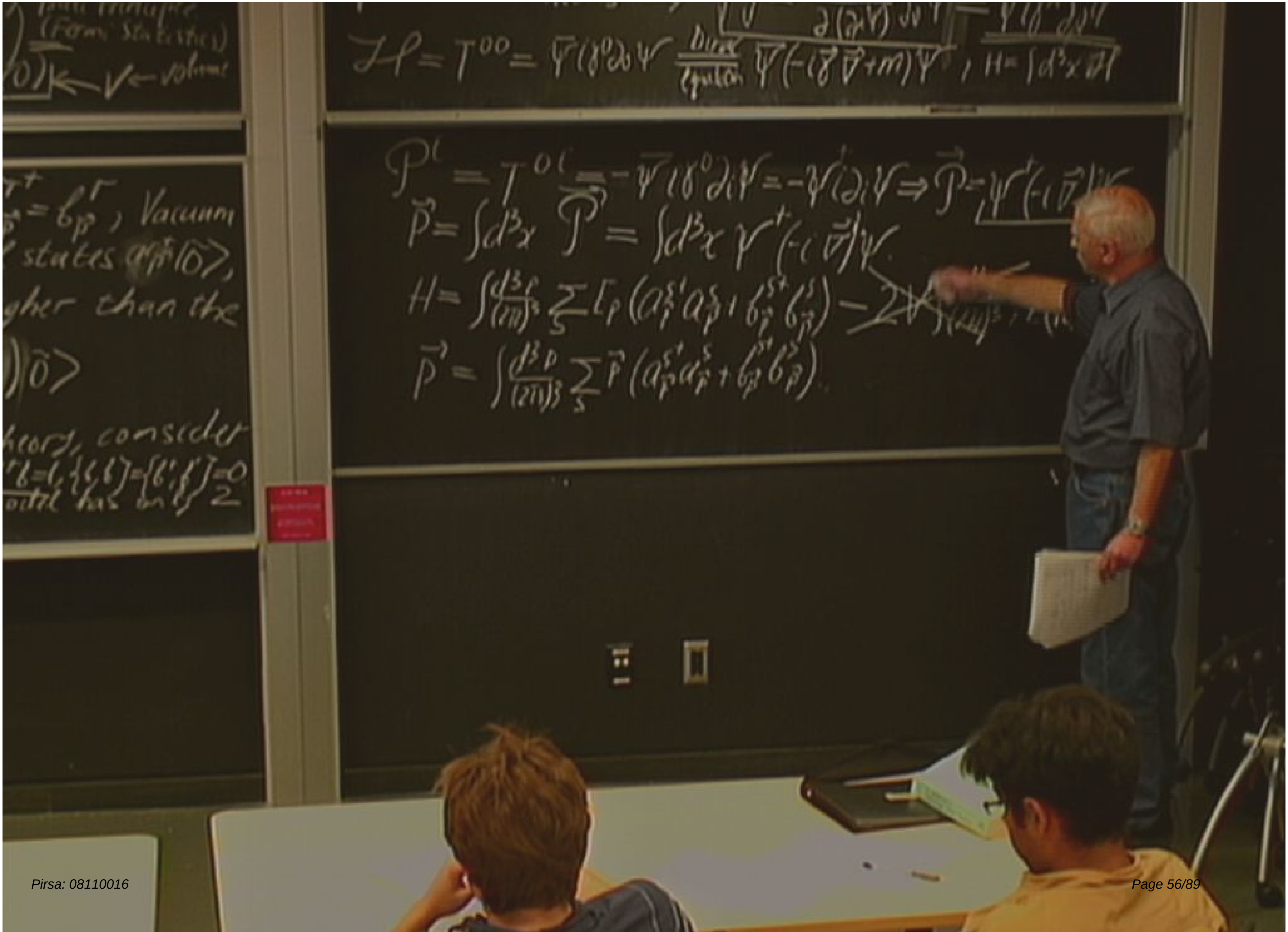
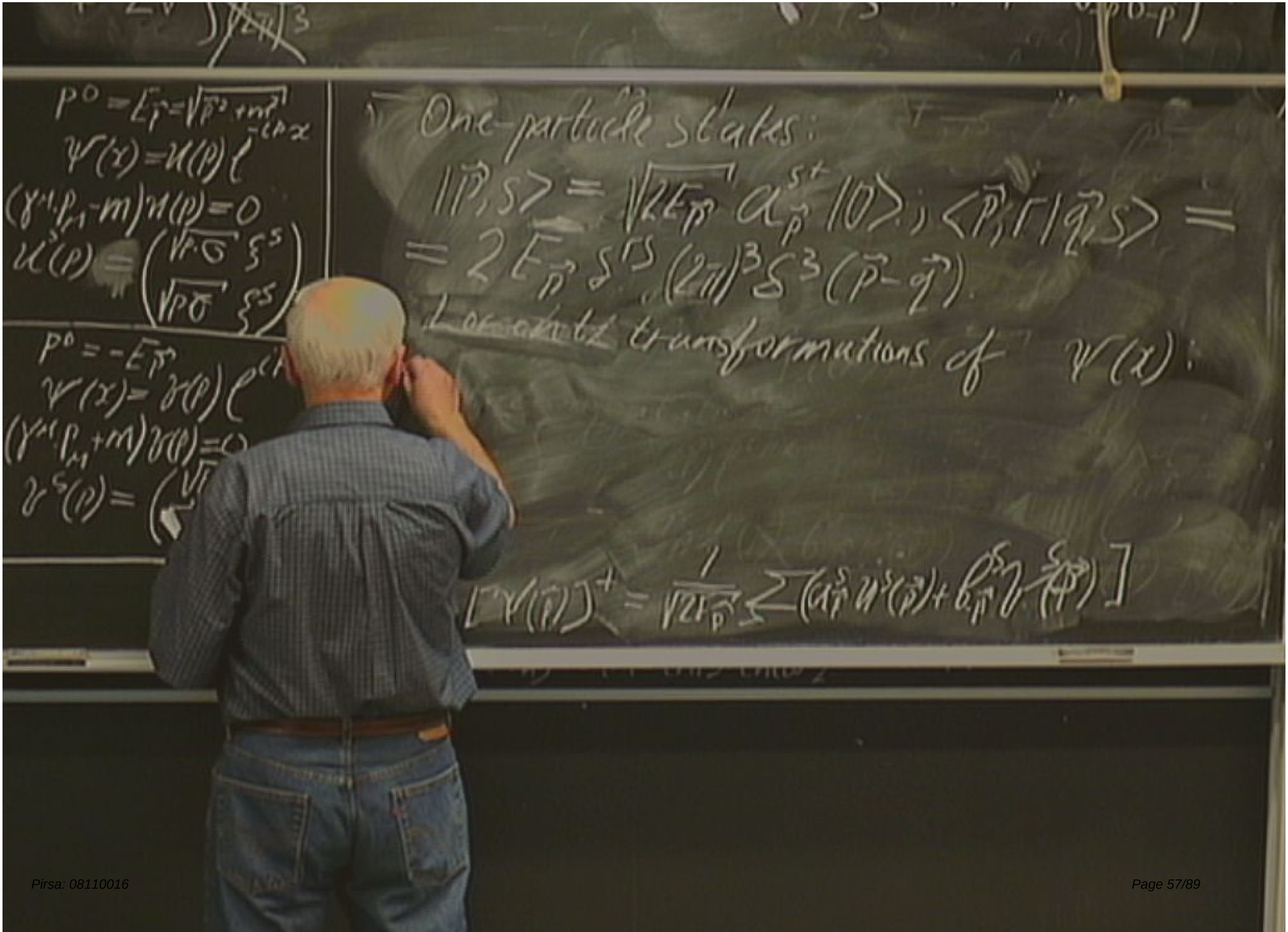

One-particle states:

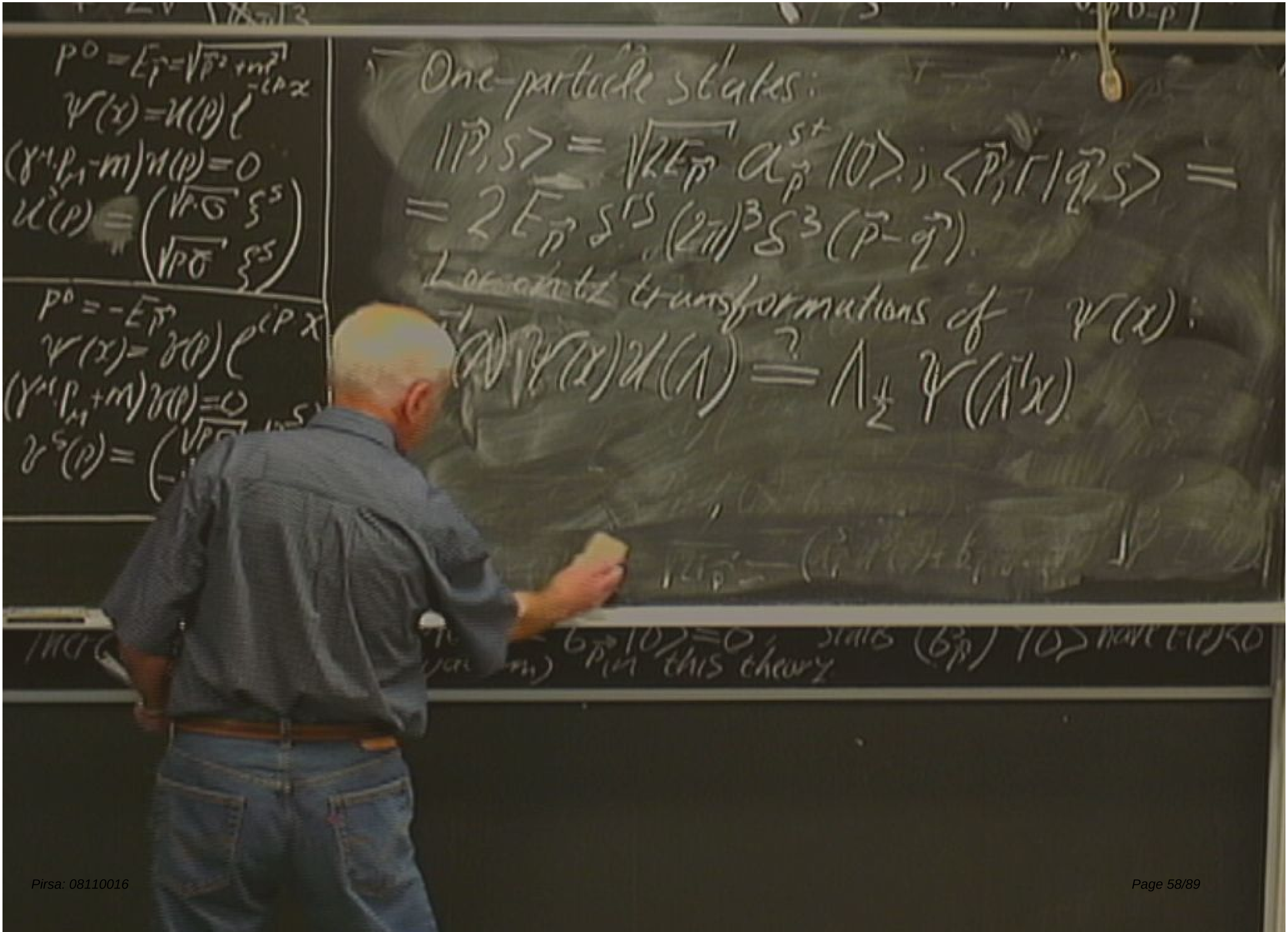

One-particle states:
$|\vec{p}, s\rangle = \sqrt{2E_{\vec{p}}}\, a^{s\dagger}_{\vec{p}} |0\rangle$
$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$
$\psi(x) = u(p)\, e^{-ipx}$
$(\gamma^\mu p_\mu - m) u(p) = 0$
$p^0 = -E_{\vec{p}}$


One-particle states:
$|\vec{p}, s\rangle = \sqrt{2E_{\vec{p}}}\, a^{s+}_{\vec{p}}\, |0\rangle \, ; \, \langle \vec{p}, r | \vec{q}, s\rangle =$
$= 2E_{\vec{p}}\, \delta^{rs} (2\pi)^3 \delta^3(\vec{p} - \vec{q})$
$\psi(x) = u(p)\, e^{-ipx}$
$(\gamma^\mu p_\mu - m)\, u(p) = 0$
$p^0 = -E_{\vec{p}}$


Now H is:
Introduce new notations.
Then all states
have energy higher than the vacuum
In order to understand better this theory, consider a toy model
This model has only 2
for these solutions




$p^0 = E_{\vec p} = \sqrt{\vec p^2 + m^2}$
$\psi(x) = u(p)e^{-ipx}$
$(\gamma^\mu p_\mu - m)u(p) = 0$
$u^s(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar\sigma}\,\xi^s\end{pmatrix}$
$p^0 = -E_{\vec p}$
$\psi(x) = v(p)e$
$(\gamma^\mu p_\mu + m)v(p) = 0$
$v^s(p) =$
One-particle states:
$|\vec p, s\rangle = \sqrt{2E_{\vec p}}\, a^{s+}_{\vec p}|0\rangle$; $\langle \vec p, r|\vec q, s\rangle =$
$= 2E_{\vec p}\,\delta^{rs}(2\pi)^3\delta^3(\vec p - \vec q)$.
Lorentz transformations of $\psi(x)$:


One-particle states:
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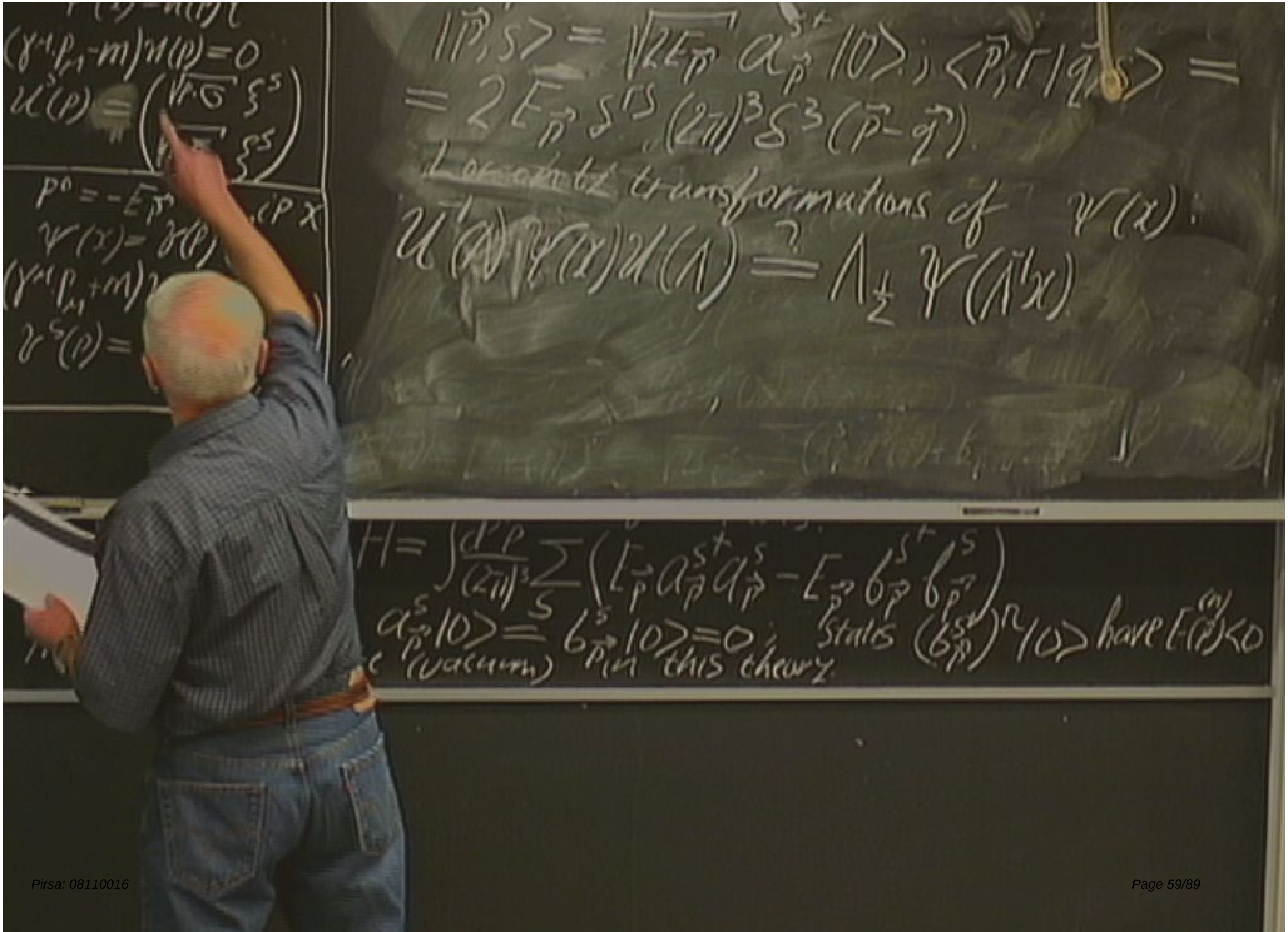
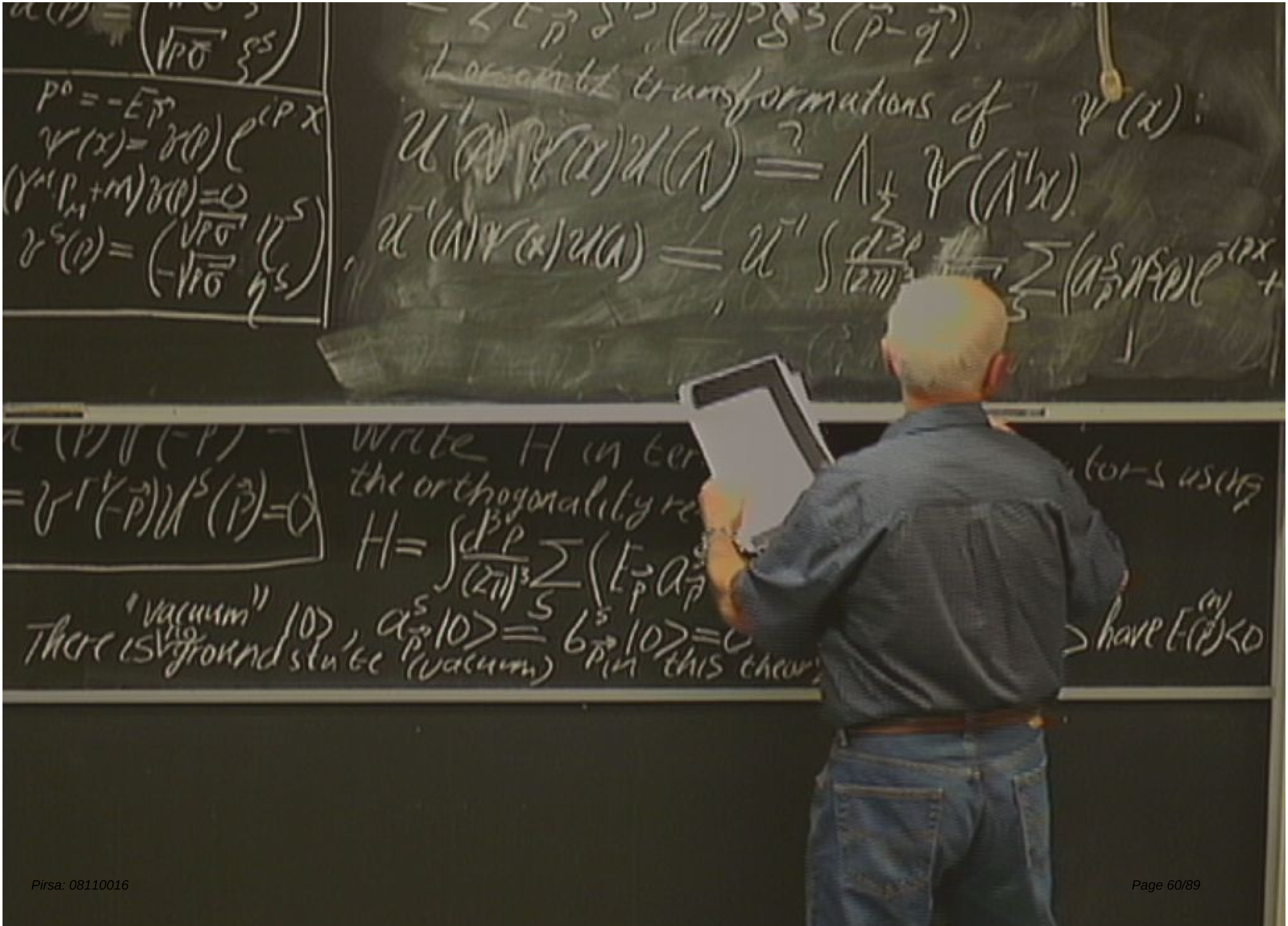
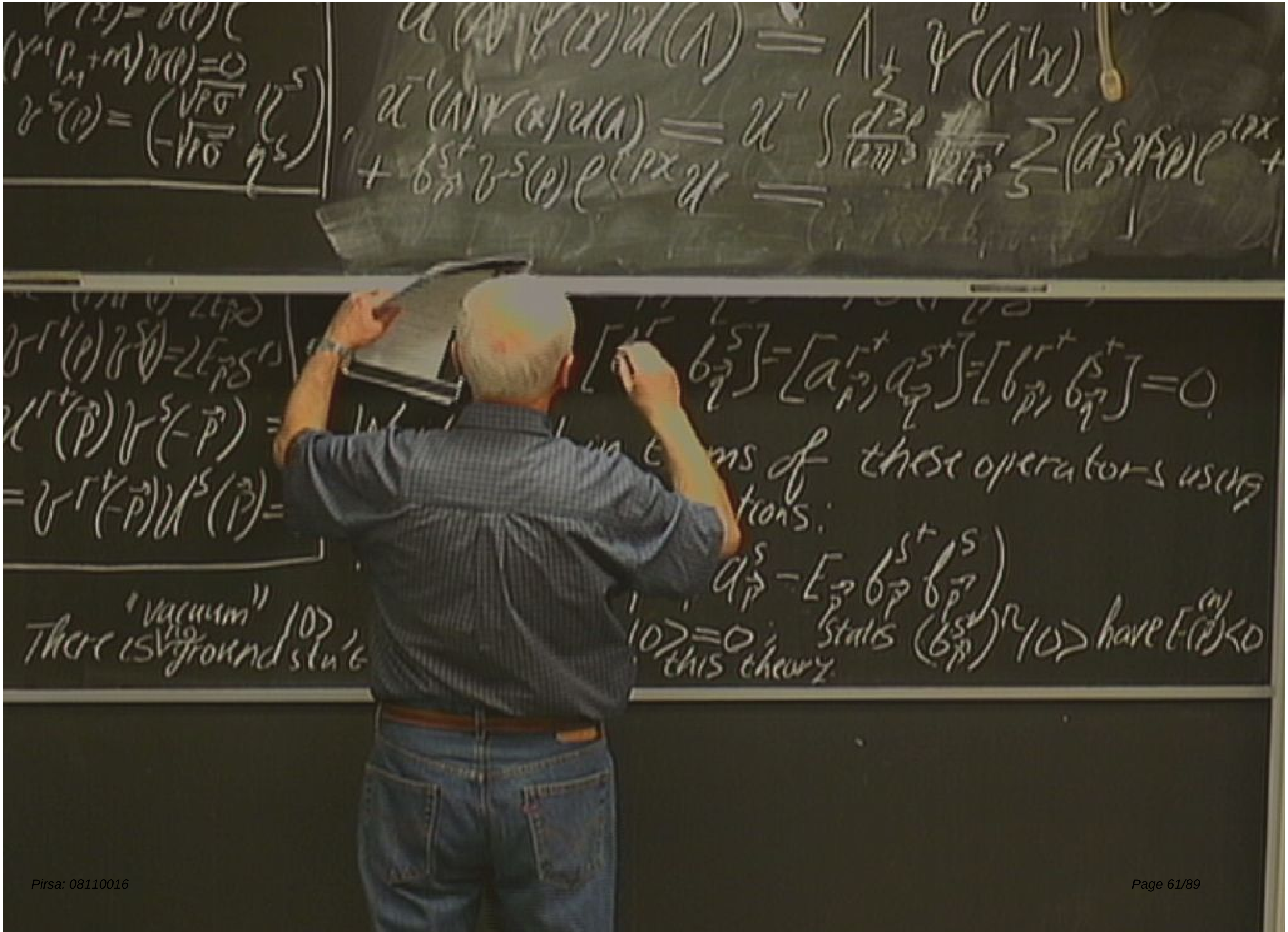
Lorentz transformations of $\psi(x)$:
$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$
$\psi(x) = u(p)\, e^{-ipx}$
$(\gamma^\mu p_\mu - m) u(p) = 0$
$p^0 = -E_{\vec{p}}$
$\psi(x) = v(p)\, e^{ipx}$
$(\gamma^\mu p_\mu + m) v(p) = 0$
this theory

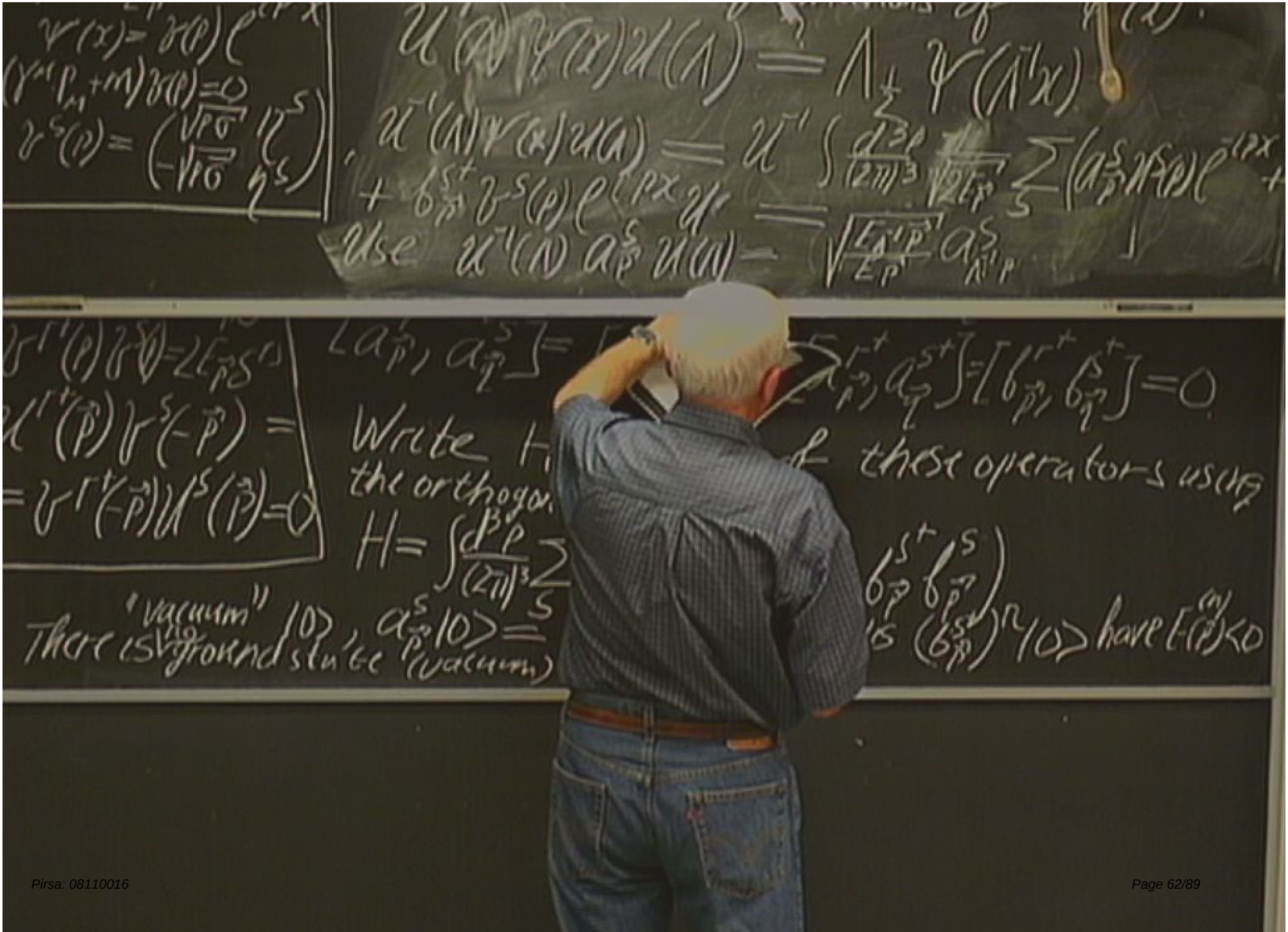


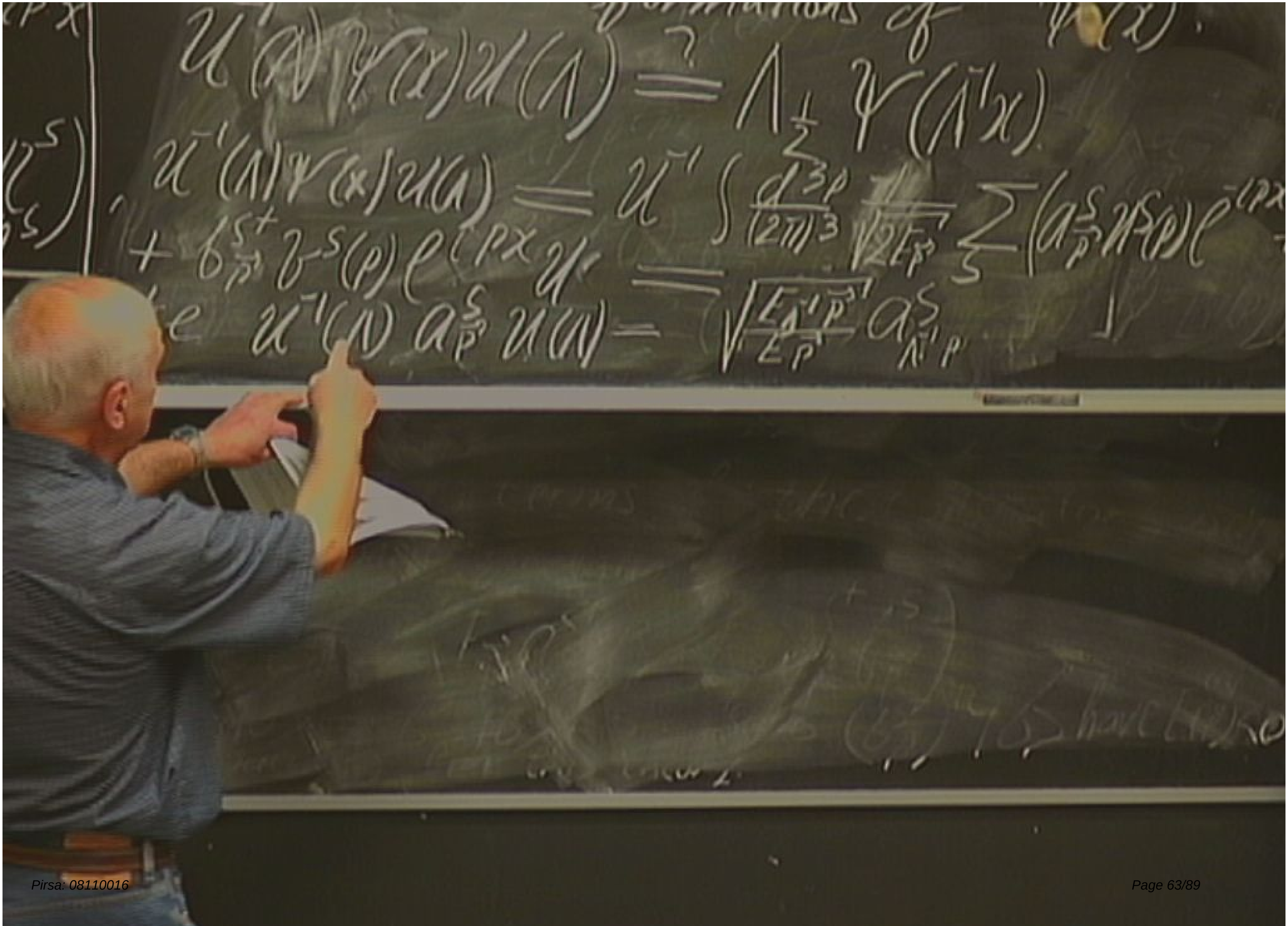
$(\gamma^\mu p_\mu - m)u(p) = 0$
$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar\sigma}\,\xi^s \end{pmatrix}$
$p^0 = -E_{\vec p}$
$(\gamma^\mu p_\mu + m)$
$v^s(p) =$
$|\vec p, s\rangle = \sqrt{2E_{\vec p}}\, a^{s+}_{\vec p} |0\rangle$; $\langle \vec p, r | \vec q, s\rangle =$
$= 2E_{\vec p}\, \delta^{rs} (2\pi)^3 \delta^3(\vec p - \vec q)$
Lorentz transformations of $\psi(x)$:
$U^{-1}(\Lambda)\psi(x)U(\Lambda) \stackrel{?}{=} \Lambda_{\frac12}\psi(\Lambda^{-1}x)$
$H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s+}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s+}_{\vec p} b^s_{\vec p}\right)$
$a^s_{\vec p}|0\rangle = b^s_{\vec p}|0\rangle = 0$; states $(b^{s+}_{\vec p})^n|0\rangle$ have $E(\vec p) < 0$
(vacuum) in this theory

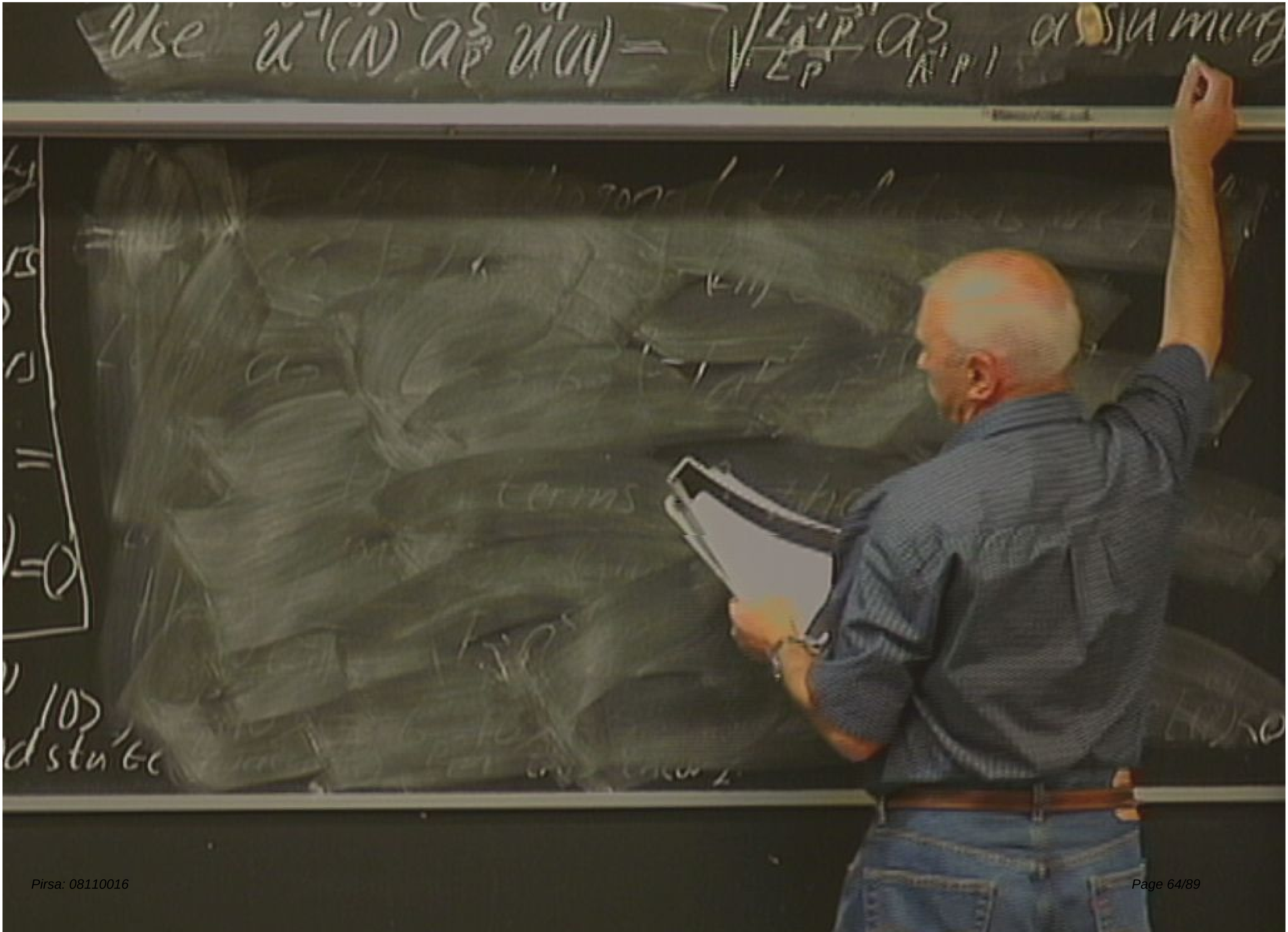

Lorentz transformations of
Write H in ter
the orthogonality re
"vacuum"
There is no ground state (vacuum) in this theor
have

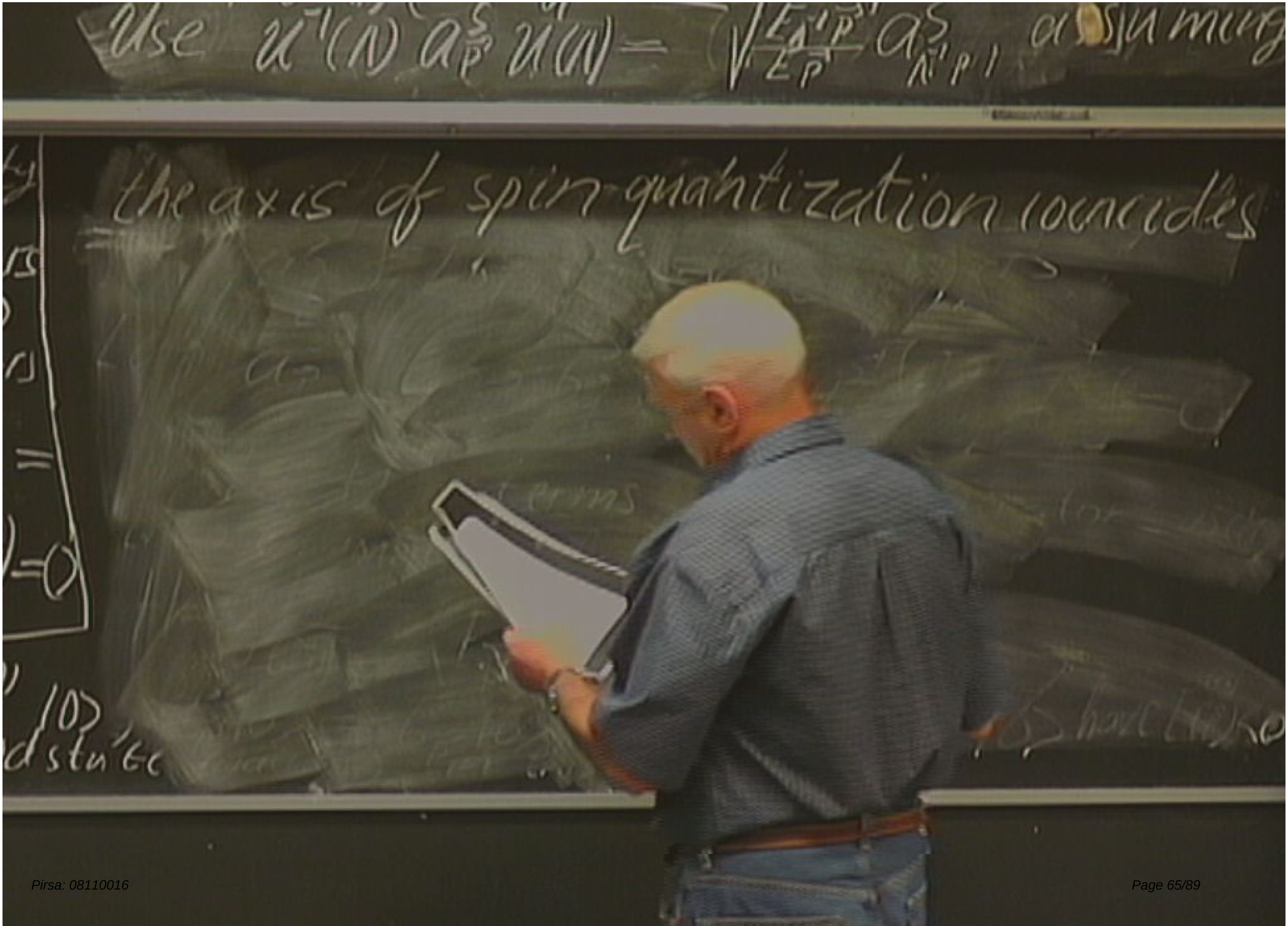


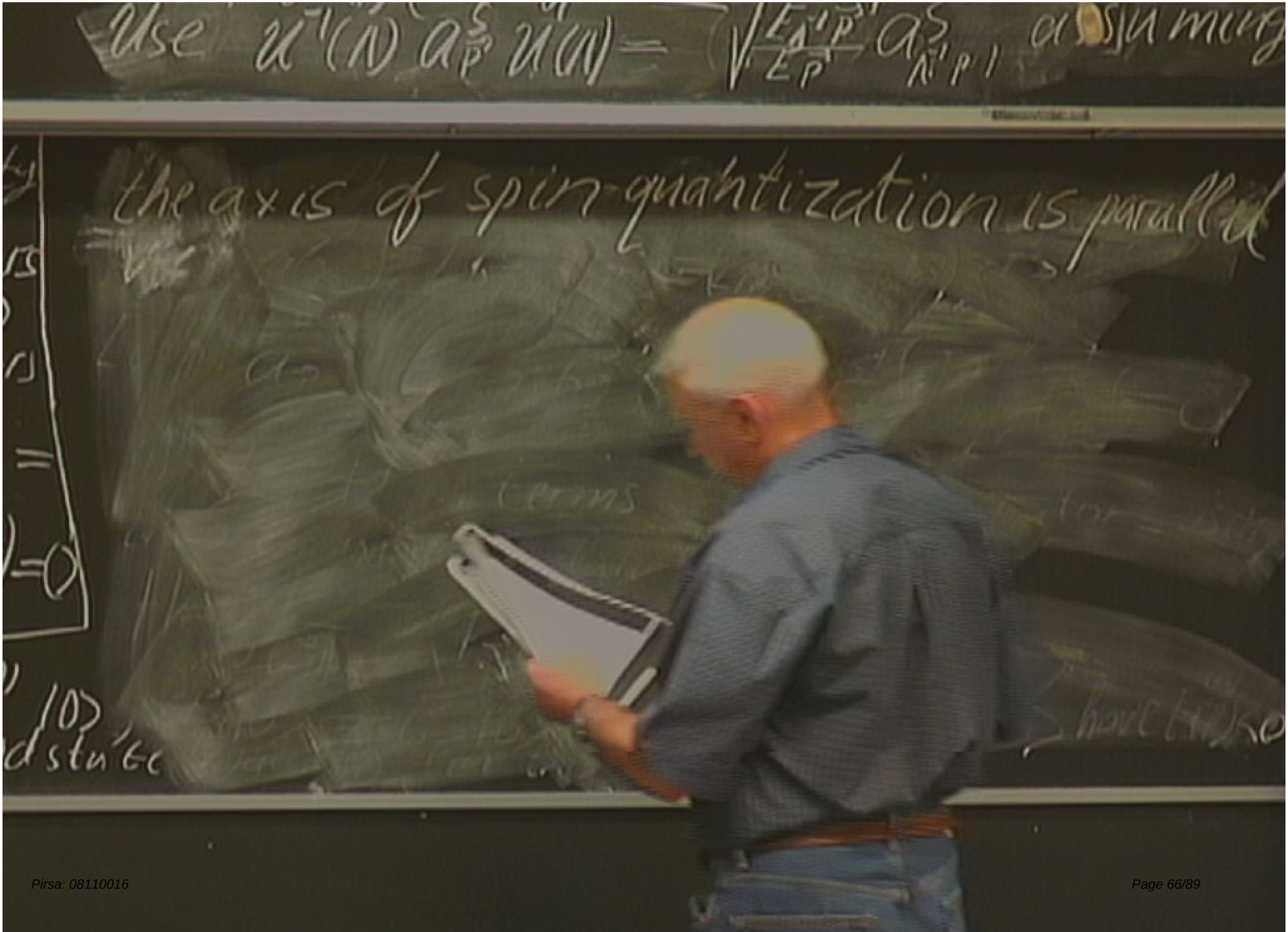
these operators using
"vacuum"
There is no ground state
this theory

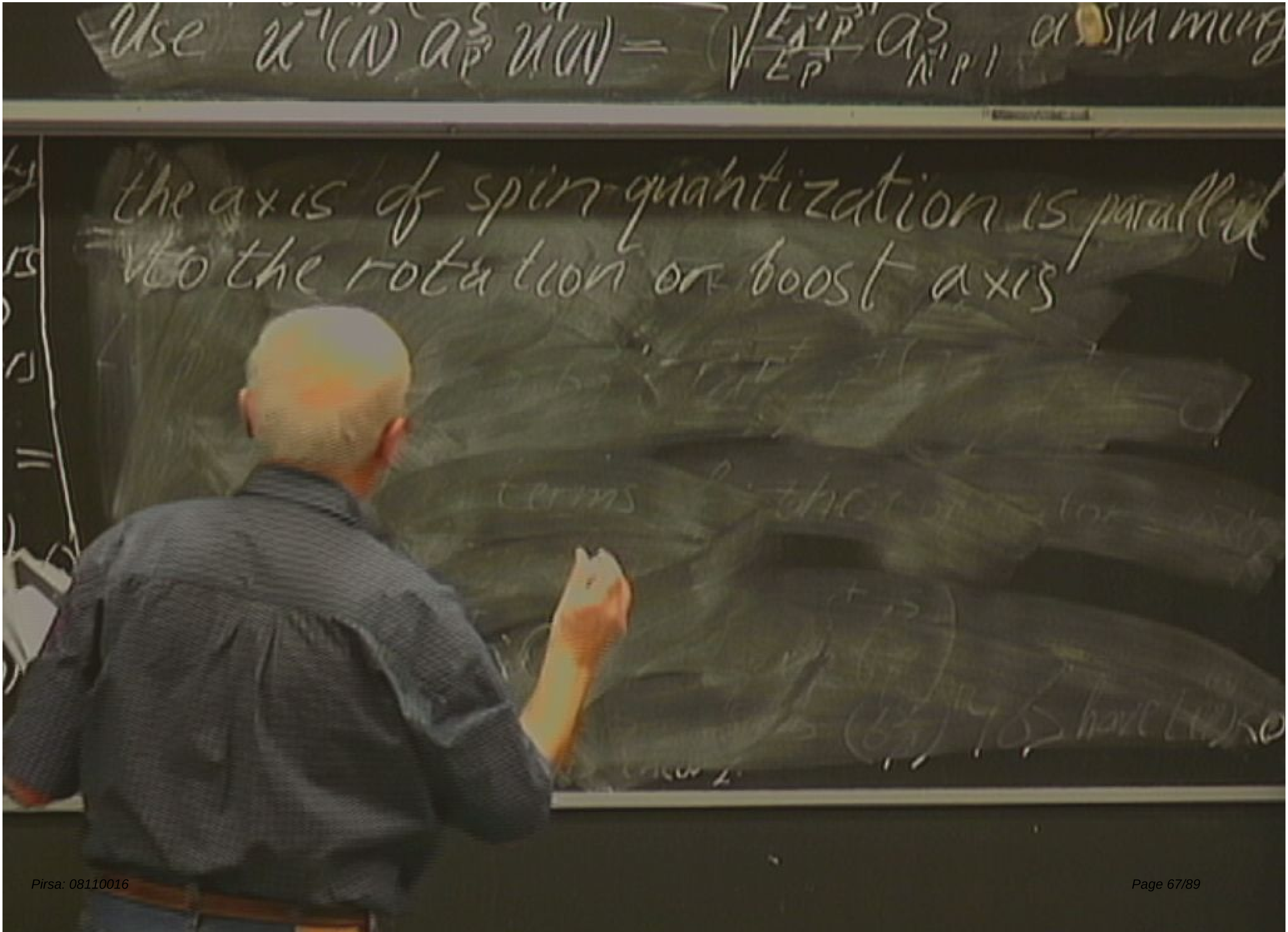

Use
Write H
these operators using
the orthogon
"vacuum"
There is no ground state (vacuum)
have

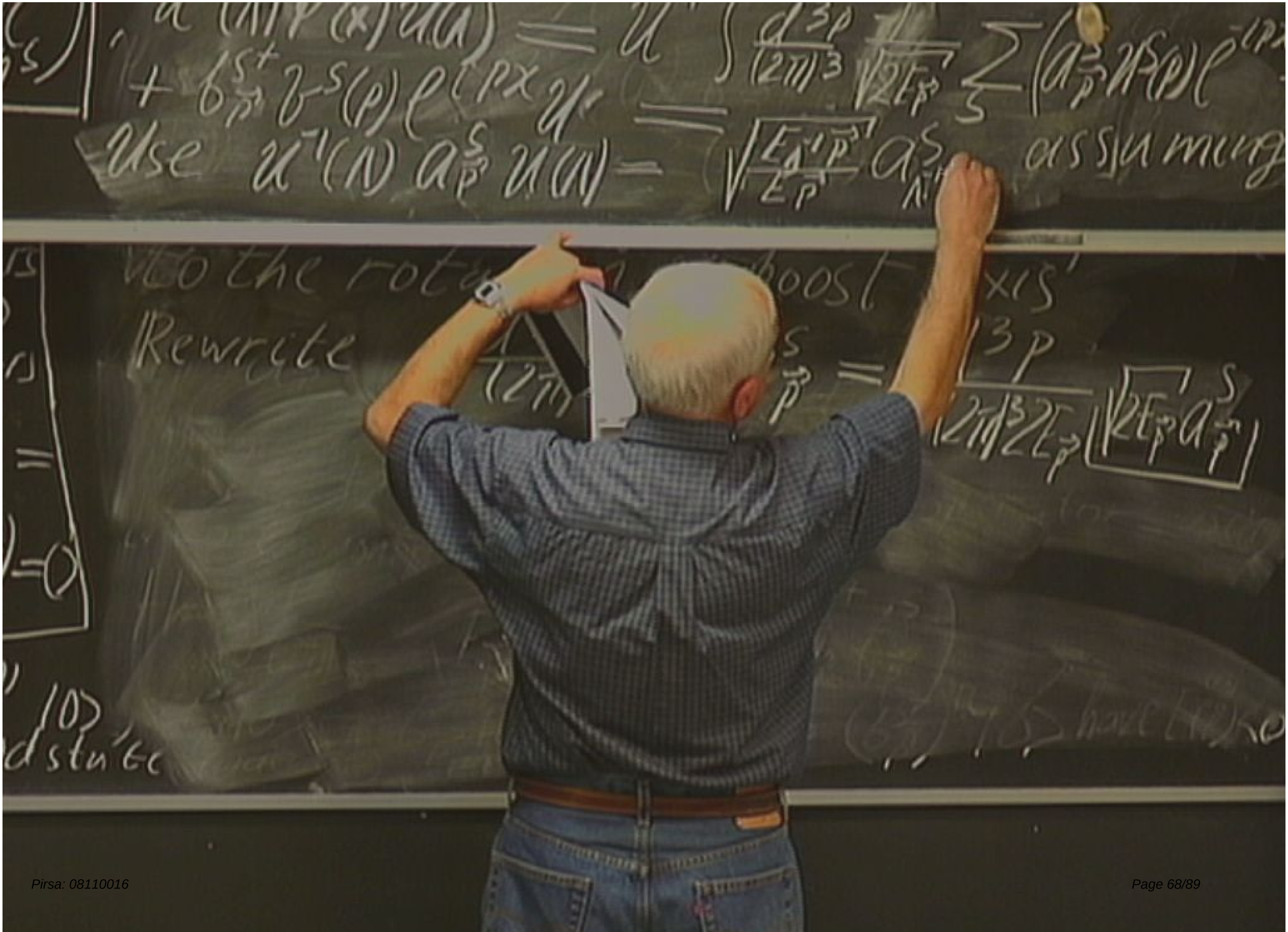


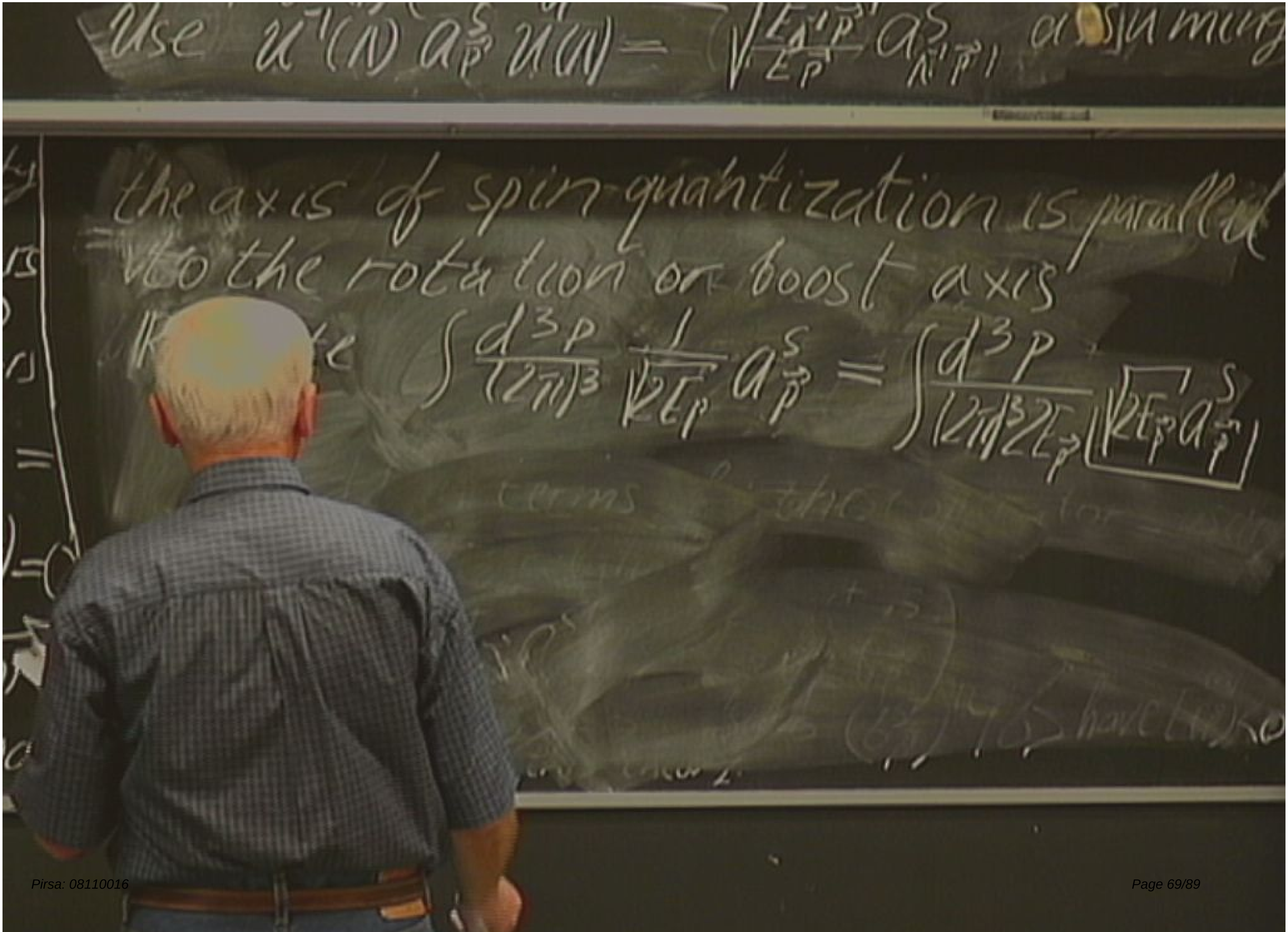

Use

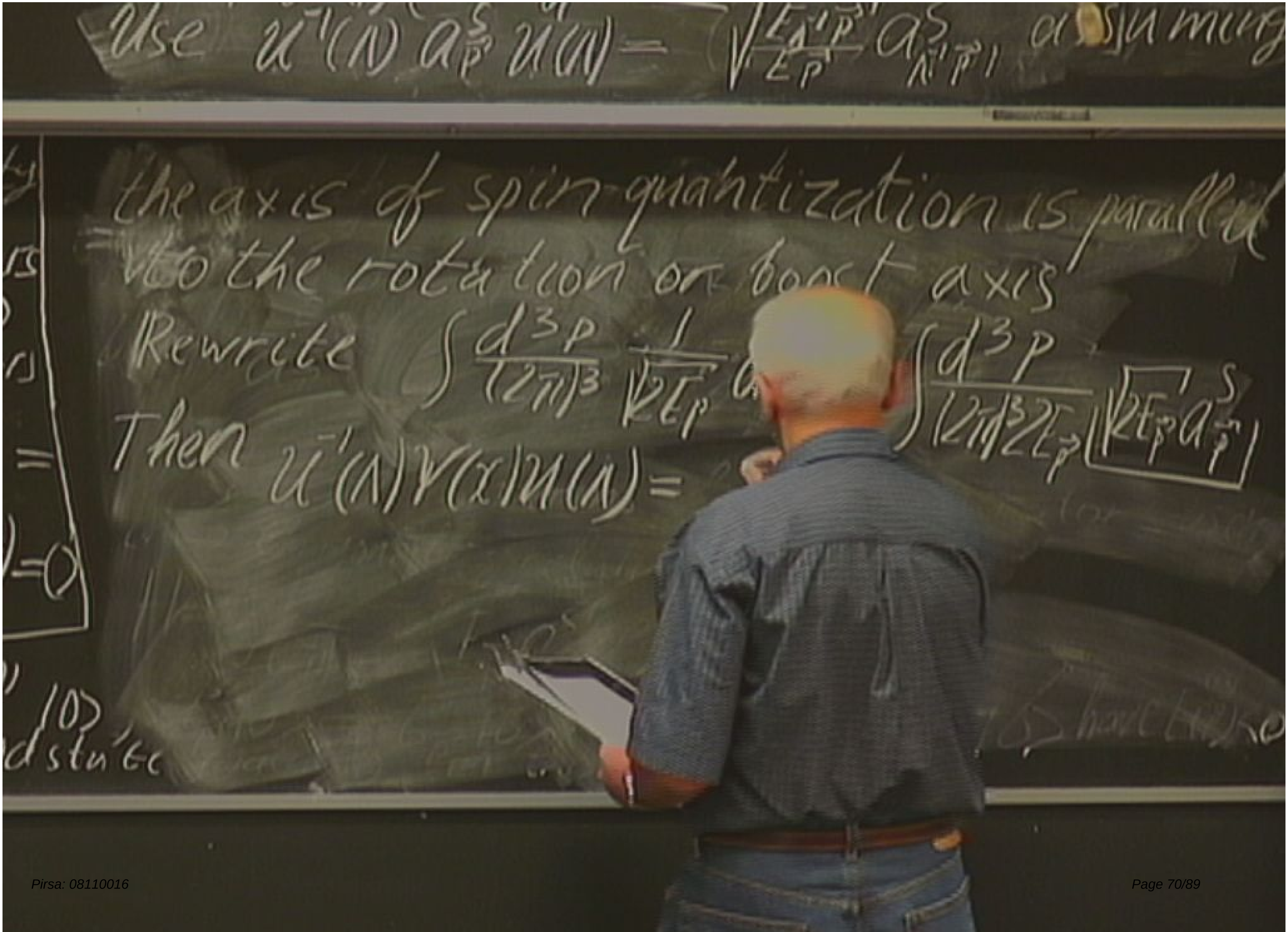

Use
the axis of spin quantization coincides

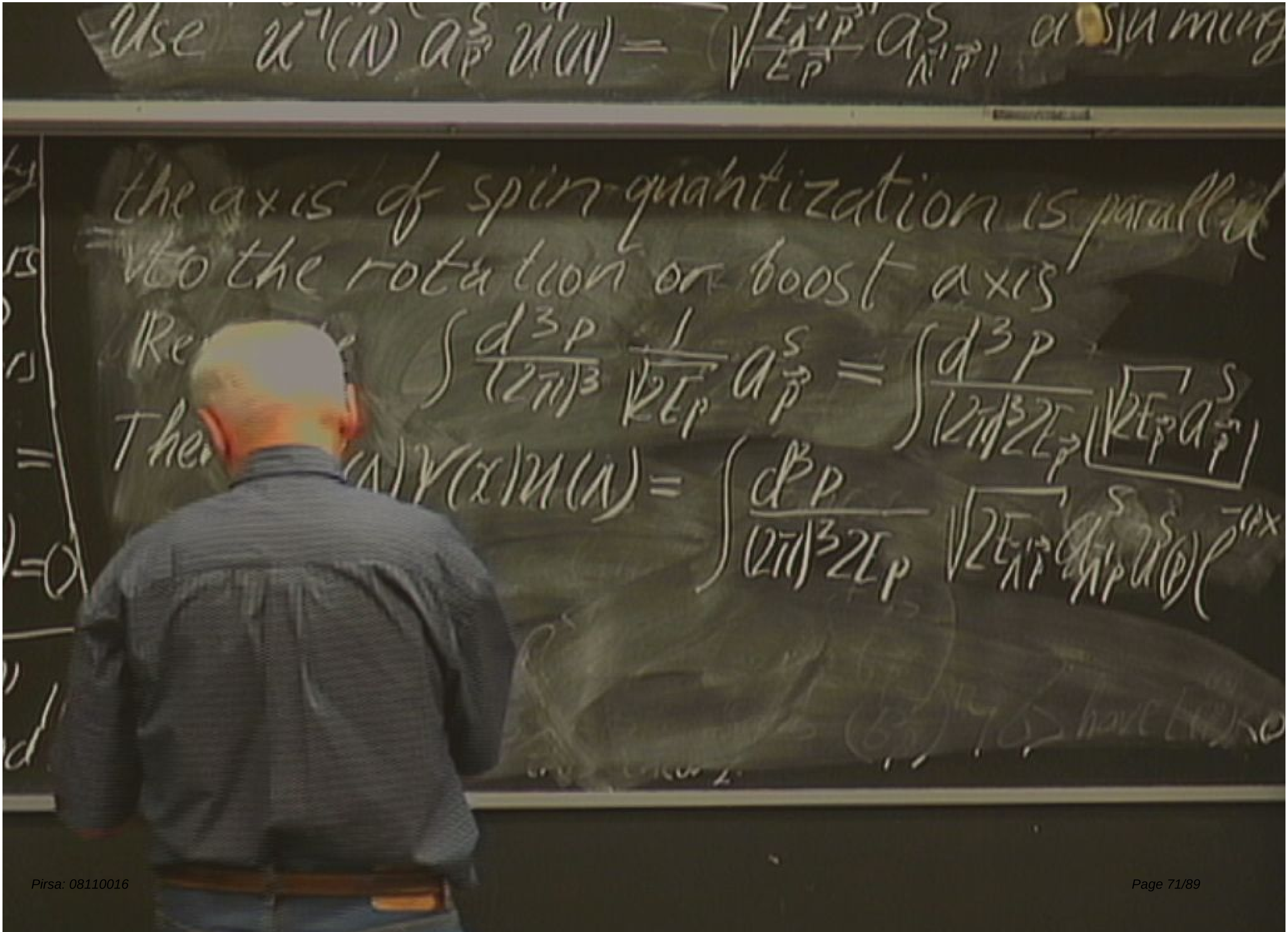

Use $u^{-1}(\Lambda)\, a^{\xi}_{p}\, u(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}}\, a^{\xi}_{\Lambda p}$ assuming
the axis of spin quantization is parallel


Use u^-1(Λ) a_p^ξ u(Λ) = √(E_Λp / E_p) a^ξ_Λp assuming
the axis of spin quantization is parallel
to the rotation or boost axis

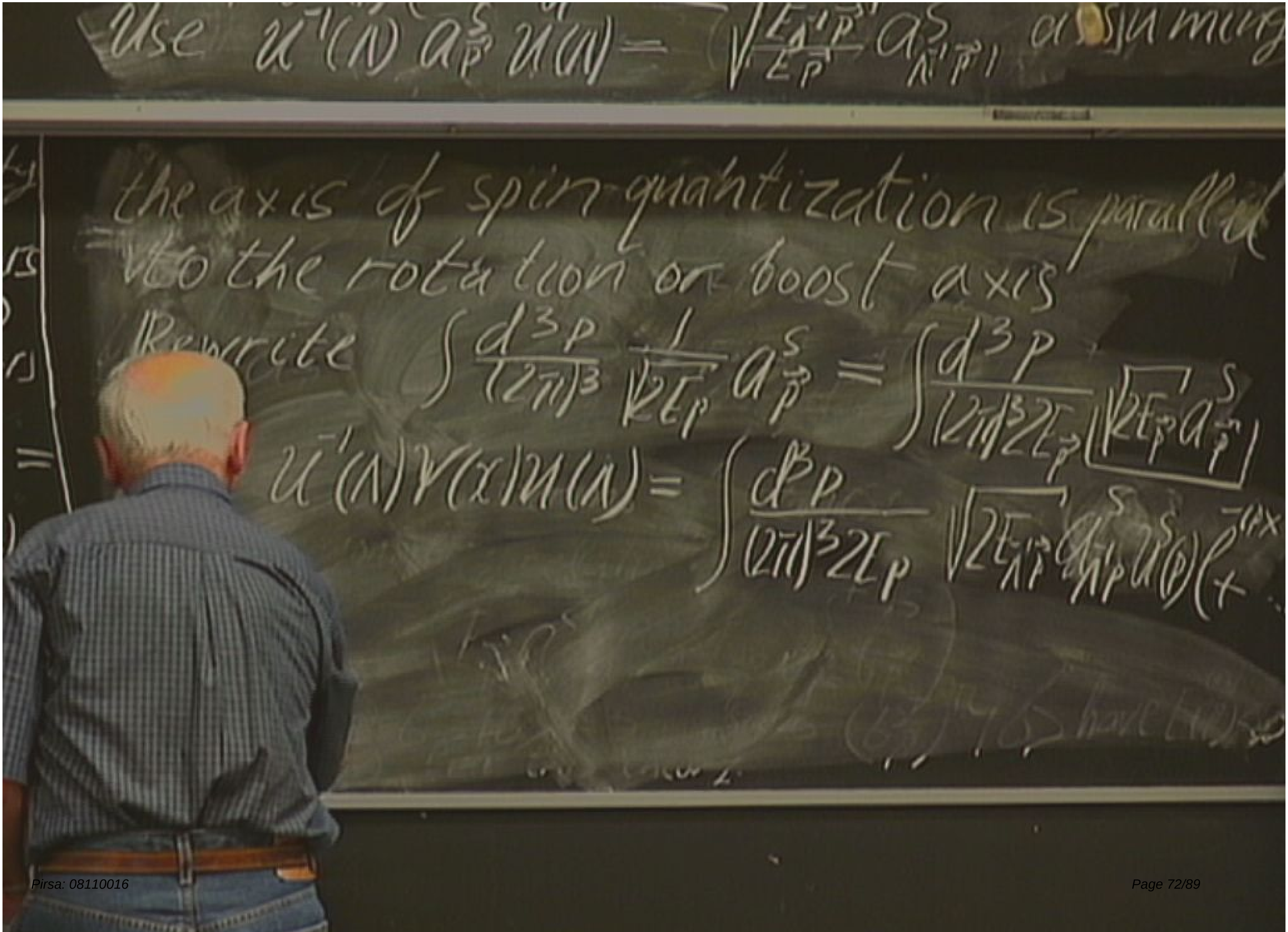

Use
Rewrite

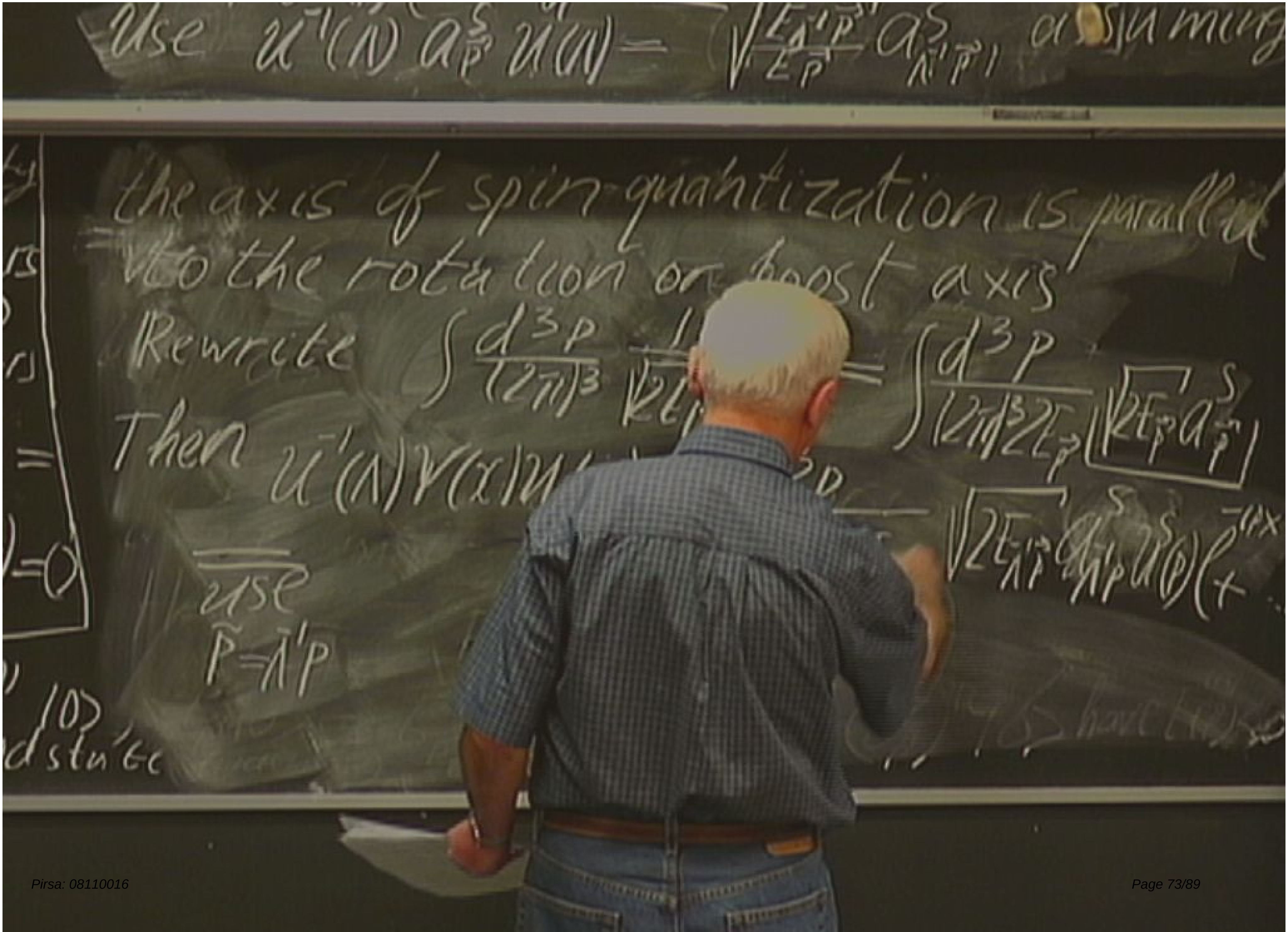

Use $U^{-1}(\Lambda)\, a^{s}_{\vec p}\, U(\Lambda) = \sqrt{\frac{E_{\Lambda \vec p}}{E_{\vec p}}}\, a^{s}_{\Lambda \vec p}$ assuming
the axis of spin quantization is parallel to the rotation or boost axis
$\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a^{s}_{\vec p} = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} \boxed{\sqrt{2E_{\vec p}}\, a^{s}_{\vec p}}$


Use
assuming
the axis of spin quantization is parallel
to the rotation or boost axis
Rewrite
Then

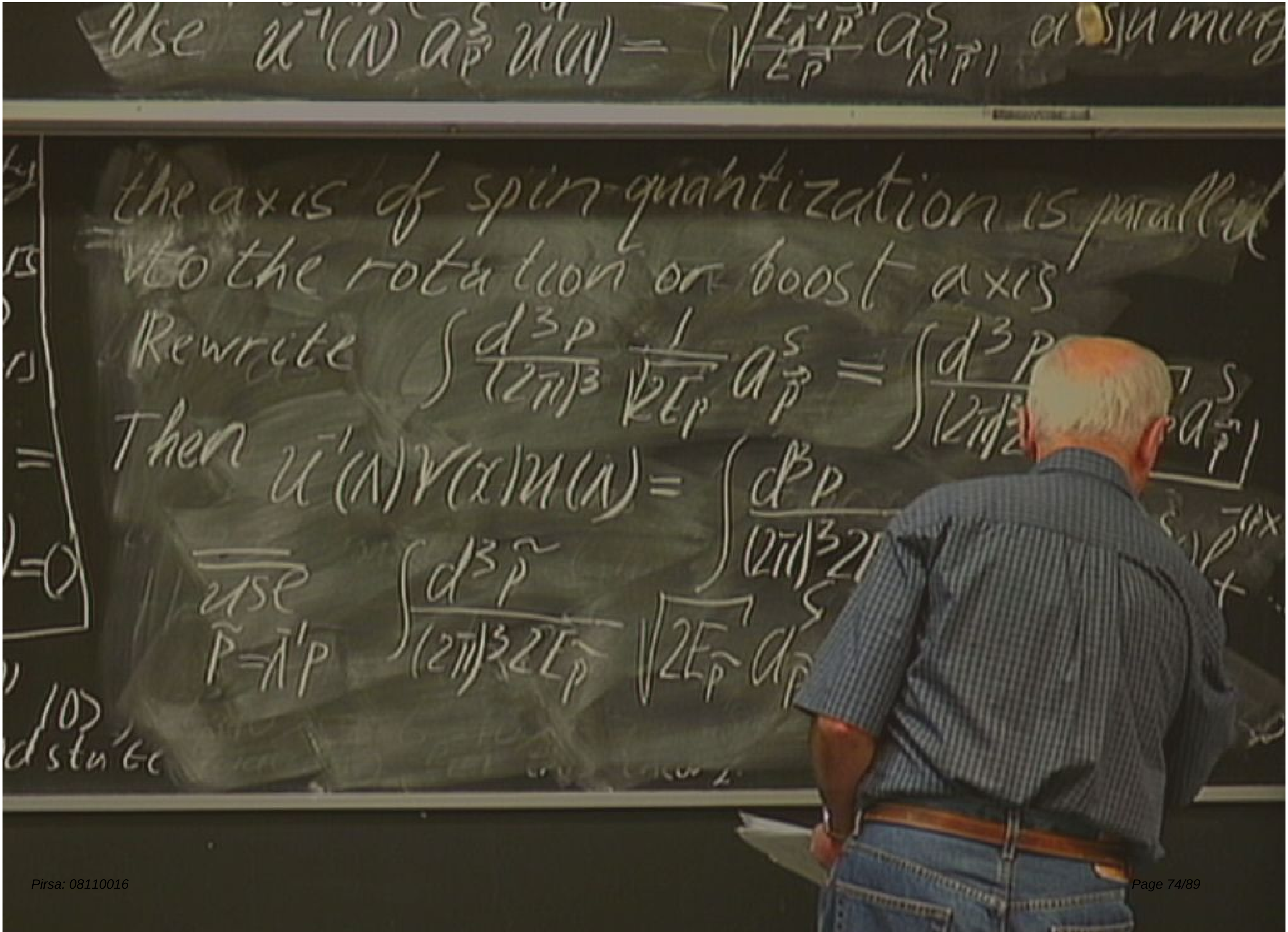


Use $u^{-1}(\Lambda)\, a^s_{\vec p}\, u(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}}\, a^s_{\Lambda \vec p}$
the axis of spin quantization is parallel to the rotation or boost axis
$\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a^s_{\vec p} = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} \boxed{\sqrt{2E_{\vec p}}\, a^s_{\vec p}}$
$\psi(x) u(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sqrt{2E_{\Lambda p}}\, a^s_{\Lambda p}\, u^s(p)\, e^{-ipx}$


Use $U^{-1}(\Lambda)\, a^{s}_{\vec p}\, U(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1}p}}{E_{\vec p}}}\, a^{s}_{\Lambda^{-1}\vec p}$ assuming
the axis of spin quantization is parallel
to the rotation or boost axis
Rewrite $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}}\, a^{s}_{\vec p} = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} \boxed{\sqrt{2E_{\vec p}}\, a^{s}_{\vec p}}$
$U^{-1}(\Lambda)\psi(x)U(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sqrt{2E_{\Lambda^{-1}p}}\, a^{s}_{\Lambda^{-1}p}\, u^{s}(p)\, e^{-ipx} + \dots$



Use
the axis of spin quantization is parallel
to the rotation or boost axis
Rewrite
Then
use


Use $u^{-1}(\Lambda)\, a^{s}_{\vec p}\, u(\Lambda) = \sqrt{\frac{E_{\Lambda \vec p}}{E_{\vec p}}}\, a^{s}_{\Lambda \vec p}$
the axis of spin quantization is parallel
to the rotation or boost axis
Rewrite $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a^s_{\vec p} = \int \frac{d^3p}{(2\pi)^3}$
Then $u^{-1}(\Lambda)\psi(x)u(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2}$
use
$\tilde p = \Lambda^{-1} p$
$\int \frac{d^3\tilde p}{(2\pi)^3 2E_{\tilde p}} \sqrt{2E_{\tilde p}}\, a$

Use $U^{-1}(\Lambda)\, a^s_{\vec p}\, U(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1}p}}{E_p}}\, a^s_{\Lambda^{-1}\vec p}$ assuming

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}}\, a^s_{\vec p} = \int \frac{d^3p}{(2\pi)^3\, 2E_p} \boxed{\sqrt{2E_{\vec p}}\, a^s_{\vec p}}$

Then $U^{-1}(\Lambda)\,\psi(x)\,U(\Lambda) = \int \frac{d^3p}{(2\pi)^3\, 2E_p} \sqrt{2E_{\Lambda^{-1}p}}\, a^s_{\Lambda^{-1}p}\, u^s(p)\, e^{-ipx} + \dots$

use $\tilde p = \Lambda^{-1}p$ $\quad \int \frac{d^3\tilde p}{(2\pi)^3\, 2E_{\tilde p}} \sqrt{2E_{\tilde p}}\, a^s_{\tilde p}\, u^s(\Lambda\tilde p)\, e^{-}$

$(\gamma^\mu p_\mu + m) v(p) = 0$

$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\, \eta^s \\ -\sqrt{p\cdot\bar\sigma}\, \eta^s \end{pmatrix}$

$\mathcal{U}^{-1}(\Lambda)\psi(x)\mathcal{U}(\Lambda) = \Lambda_{\frac{1}{2}}\psi(\Lambda^{-1}x)$

$\mathcal{U}^{-1}(\Lambda)\psi(x)\mathcal{U}(\Lambda) = \mathcal{U}^{-1}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s \left(a_p^s u^s(p) e^{-ipx} + b_p^{s\dagger} v^s(p) e^{ipx}\right)\mathcal{U} =$

Use $\mathcal{U}^{-1}(\Lambda)\, a_p^s\, \mathcal{U}(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}}\, a^s_{\Lambda \vec p}$ assuming

the axis of spin quantization is parallel to the rotation or boost axis

Orthogonality relations:

$u^{r\dagger}(p) u^s(p) = 2E_p \delta^{rs}$

$v^{r\dagger}(p) v^s(p) = 2E_p \delta^{rs}$

$u^{r\dagger}(\vec p) v^s(-\vec p) =$

$= v^{r\dagger}(-\vec p) u^s(\vec p) = 0$

Rewrite $\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}} a_p^s = \int \frac{d^3p}{(2\pi)^3 2E_p}\boxed{\sqrt{2E_p}\, a_{\vec p}^s}$

Then $\mathcal{U}^{-1}(\Lambda)\psi(x)\mathcal{U}(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2E_p}\sqrt{2E_{\Lambda p}}\sum_s a^s_{\Lambda p} u^s(p) e^{-ipx} + \ldots$

use $\tilde p = \Lambda p$: $\int \frac{d^3\tilde p}{(2\pi)^3 2E_{\tilde p}}\sqrt{2E_{\tilde p}}\, a_{\tilde p}^s\, u^s(\Lambda^{-1}\tilde p)\, e^{-i\tilde p x}$

There is a "vacuum" $|0\rangle$ ground state

$$e^{-ip\cdot x} = e^{-i\tilde{\Lambda}p\cdot\tilde{\Lambda}x} = e^{-i\tilde{p}\cdot\tilde{\Lambda}x}; \quad \tilde{b}^{r} = b^{r+}_{\vec{p}}, \quad \tilde{b}^{r+}_{\vec{p}} = b^{r}_{\vec{p}}, \text{ Vacuum}$$

...=0 Then all states $a^{+}_{p}|\tilde{0}\rangle$

...ergy higher than th...

$$H|\tilde{0}\rangle = -2V\int\frac{d^3p}{(2\pi)^3} \ldots; \quad H \ldots \int\frac{d^3p}{(2\pi)^3}\,|\tilde{0}\rangle$$

In order to understand better this theory, conside...
a toy model: b, b^{+}; $\{b, b^{+}\} = bb^{+} + b^{+}b = 1$, $\{b,b\} = \{b^{+},b^{+}\} = 0$

$\Longrightarrow b^2 = 0,\ b^{+2} = 0$; $\underline{H = -E_0 b^{+}b}$. This model has only 2...

Rewrite $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a^s_{\vec p} = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} \boxed{\sqrt{2E_{\vec p}}\, a^s_{\vec p}}$

Then $U^{-1}(\Lambda)\psi(x)U(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sqrt{2E_{\Lambda \vec p}}\, a^s_{\Lambda p} u^s(p) e^{-ipx} + \dots$

use $\tilde p = \Lambda' p$: $\int \frac{d^3\tilde p}{(2\pi)^3 2E_{\tilde p}} \sqrt{2E_{\tilde p}}\, a^s_{\tilde p}\, u^s(\Lambda^{-1}\tilde p) e^{-i\tilde p \cdot \Lambda x} + \dots =$

$$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar\sigma}\,\eta^s \end{pmatrix}$$

$$u^s(\Lambda\tilde p) = \Lambda_{1/2}\, u^s(\tilde p)$$

$$\mathcal{U}^{-1}(\Lambda)\,\psi(x)\,\mathcal{U}(\Lambda) = \mathcal{U}^{-1}\int \frac{d^3p}{(2\pi)^3} \ldots$$

$$+\, b^{s\dagger}_{\vec p}\, v^s(p)\, e^{ipx}\, \mathcal{U} =$$

Use $\mathcal{U}^{-1}(\Lambda)\, a^s_{\vec p}\, \mathcal{U}(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}} \ldots$

$\ldots = 2E_{\vec p} \ldots$

$\ldots(\vec p) \ldots = 2E_{\vec p}\,\delta^{rs}$

$u^{r\dagger}(\vec p)\, v^s(-\vec p) =$

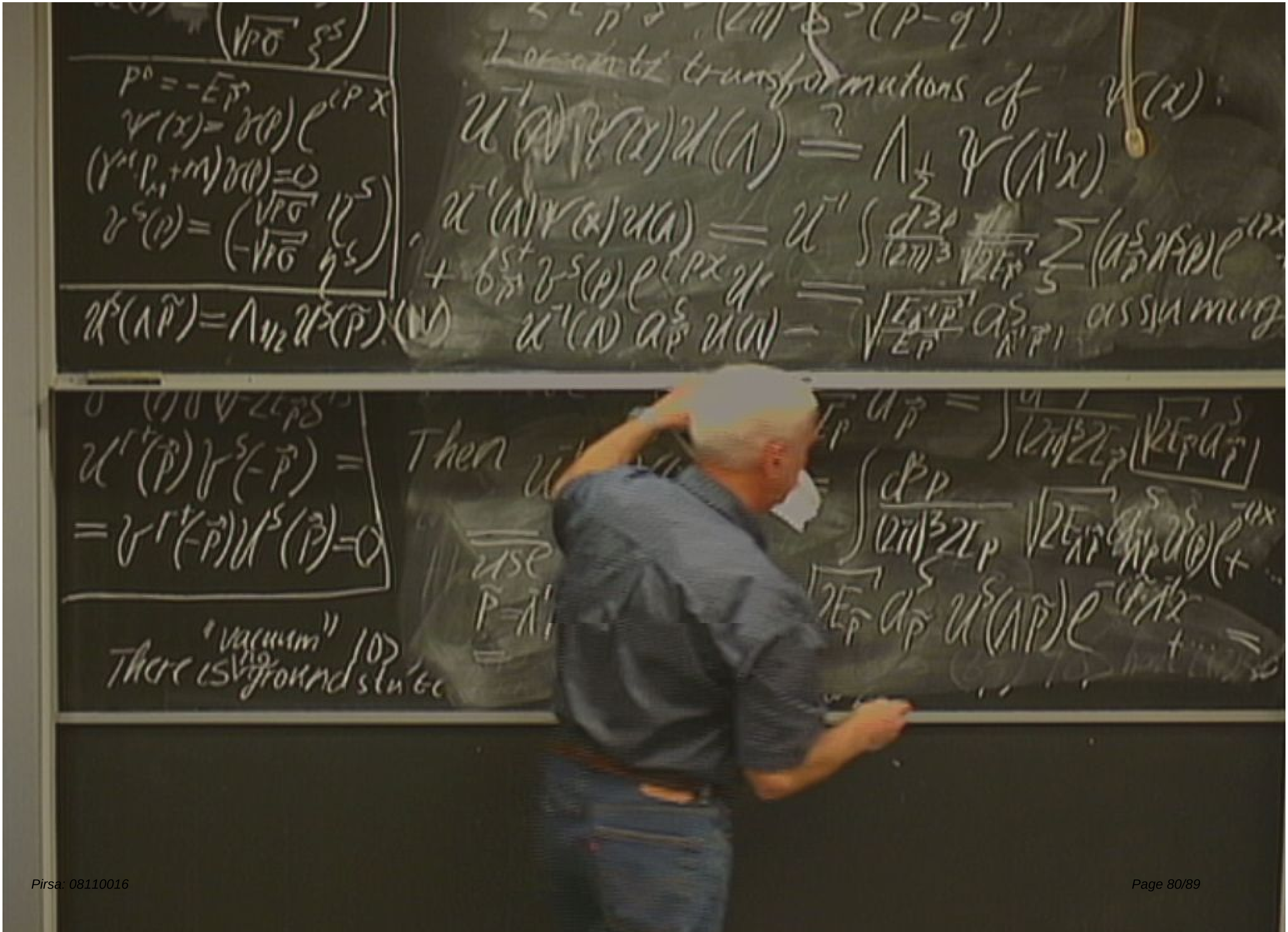
$v^{r\dagger}(-\vec p)\, u^s(\vec p) = 0$

"vacuum" $|0\rangle$ … is no ground state

Rewrite $\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\, a^s_{\vec p} =$

Then $\mathcal{U}^{-1}(\Lambda)\,\psi(x)\,\mathcal{U}(\Lambda) = \int \frac{d^3p}{(2\pi)^3\, 2E \ldots}$

use $\tilde p = \Lambda^{-1} p$ $\quad \int \frac{d^3\tilde p}{(2\pi)^3\, 2E_{\tilde p}} \sqrt{2E_{\tilde p}}\; a^s_{\tilde p}\; u \ldots$


Lorentz transformations of ψ(x):
Then
assuming
"vacuum" |0⟩
There is ground state

$$U^{-1}(\Lambda)\, a^{s}_{\vec p}\, U(\Lambda) = \sqrt{\frac{E_{\Lambda^{-1}\vec p}}{E_{\vec p}}}\, a^{s}_{\Lambda^{-1}\vec p}$$

the axis of spin quantization is parallel to the rotation or boost axis

Rewrite $\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a^s_{\vec p} = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} \boxed{\sqrt{2E_{\vec p}}\, a^s_{\vec p}}$

Then $U^{-1}(\Lambda)\psi(x)U(\Lambda) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sqrt{2E_{\Lambda^{-1}\vec p}}\, a^s_{\Lambda^{-1}\vec p}\, u^s(p)\, e^{-ipx} + \ldots$

use $\tilde p = \Lambda^{-1} p$

$= \int \frac{d^3\tilde p}{(2\pi)^3 2E_{\tilde p}} \sqrt{2E_{\tilde p}}\, a^s_{\tilde p}\, u^s(\Lambda\tilde p)\, e^{-i(\Lambda\tilde p)\cdot x}$ use $+ \ldots$ (IV)

Now H is

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s}_{\vec p} b^{s\dagger}_{\vec p} \right)$$

Pauli Principle (Fermi Statistics)

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} + E_{\vec p}\, \tilde b^{s\dagger}_{\vec p} \tilde b^{s}_{\vec p} \right) - 2 \int \frac{d^3p}{(2\pi)^3} E_{\vec p}\, \boxed{(2\pi)^3 \delta^3(0)} \leftarrow V \leftarrow \text{volume}$$

$$= \int \frac{d^3p}{(2\pi)^3}$$

... energy higher ...

; H ... $= -2V \int \frac{d^3p}{(2\pi)^3}$ ... $\int \frac{d^3p}{(2\pi)^3}$ ... $|\tilde 0\rangle$

In order to understand better this theory, consider a toy model: $b, b^\dagger$; $\{b, b^\dagger\} = bb^\dagger + b^\dagger b = 1$, $\{b,b\} = \{b^\dagger, b^\dagger\} = 0$

$\Rightarrow b^2 = 0,\ b^{\dagger 2} = 0$; $\underline{H = -E_0\, b^\dagger b}$. This model has only 2

Now H is

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s}_{\vec p} b^{s\dagger}_{\vec p} \right)$$

Pauli Principle. (Fermi Statistics)

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} + E_{\vec p}\, \tilde b^{s\dagger}_{\vec p} \tilde b^{s}_{\vec p} \right) - \sum_s \int \frac{d^3p}{(2\pi)^3} \boxed{(2\pi)^3 \delta^3(0)} \leftarrow V \leftarrow \text{volume}$$

$$= \int \frac{d^3\tilde p}{(2\pi)^3 \sqrt{2E_{\vec p}}}$$

...ergy higher...

$$H|\tilde 0\rangle = -2V \int \frac{d^3p}{(2\pi)^3} \ldots \left(\frac{d^3p}{(2\pi)^3} \right) |\tilde 0\rangle$$

In order to understand better this theory, consider a toy model: $b, b^\dagger$; $\{b, b^\dagger\} = bb^\dagger + b^\dagger b = 1$, $\{b, b\} = \{b^\dagger, b^\dagger\} = 0$

$\Rightarrow b^2 = 0,\ b^{\dagger 2} = 0$; $\underline{H = -E_b\, b^\dagger b}$. This model has only 2

$$\boxed{(2\pi)^3\delta^3(0)}$$

$$= \int \frac{d^3\tilde{p}}{(2\pi)^3\sqrt{2E_{\tilde{p}}}} a^{s}_{\tilde{p}} \Lambda_{1/2} u^{s}(\tilde{p})$$

...ergy highe...

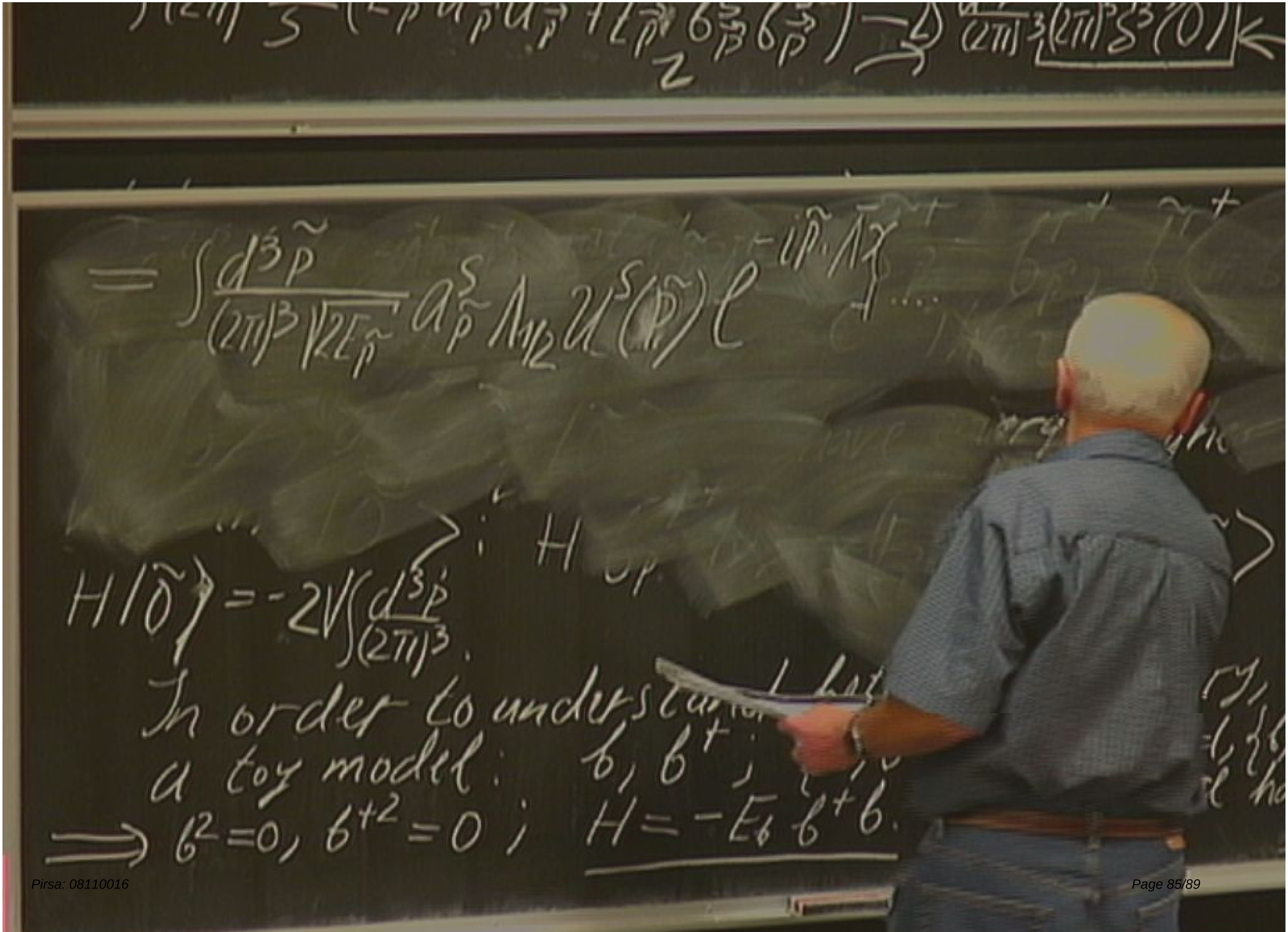
$$H|\tilde{0}\rangle = -2V\int\frac{d^3p}{(2\pi)^3} \ldots \int\frac{d^3p}{(2\pi)^3}\Big)|\tilde{0}\rangle$$

In order to understand better this theory,

a toy model: b, b^+; $\{b, b^+\} = bb^+ + b^+b = 1$, $\{b$...

This model ha...

$$\Longrightarrow b^2 = 0,\ b^{+2} = 0\ ;\ \underline{H = -E_b\, b^+ b.}$$


In order to understand
a toy model: b, b+ ;
⇒ b2=0, b+2=0 ; H = −Eb b+b.

Now H is

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s}_{\vec p} b^{s\dagger}_{\vec p} \right)$$ Pauli Principle. (Fermi Statistics)

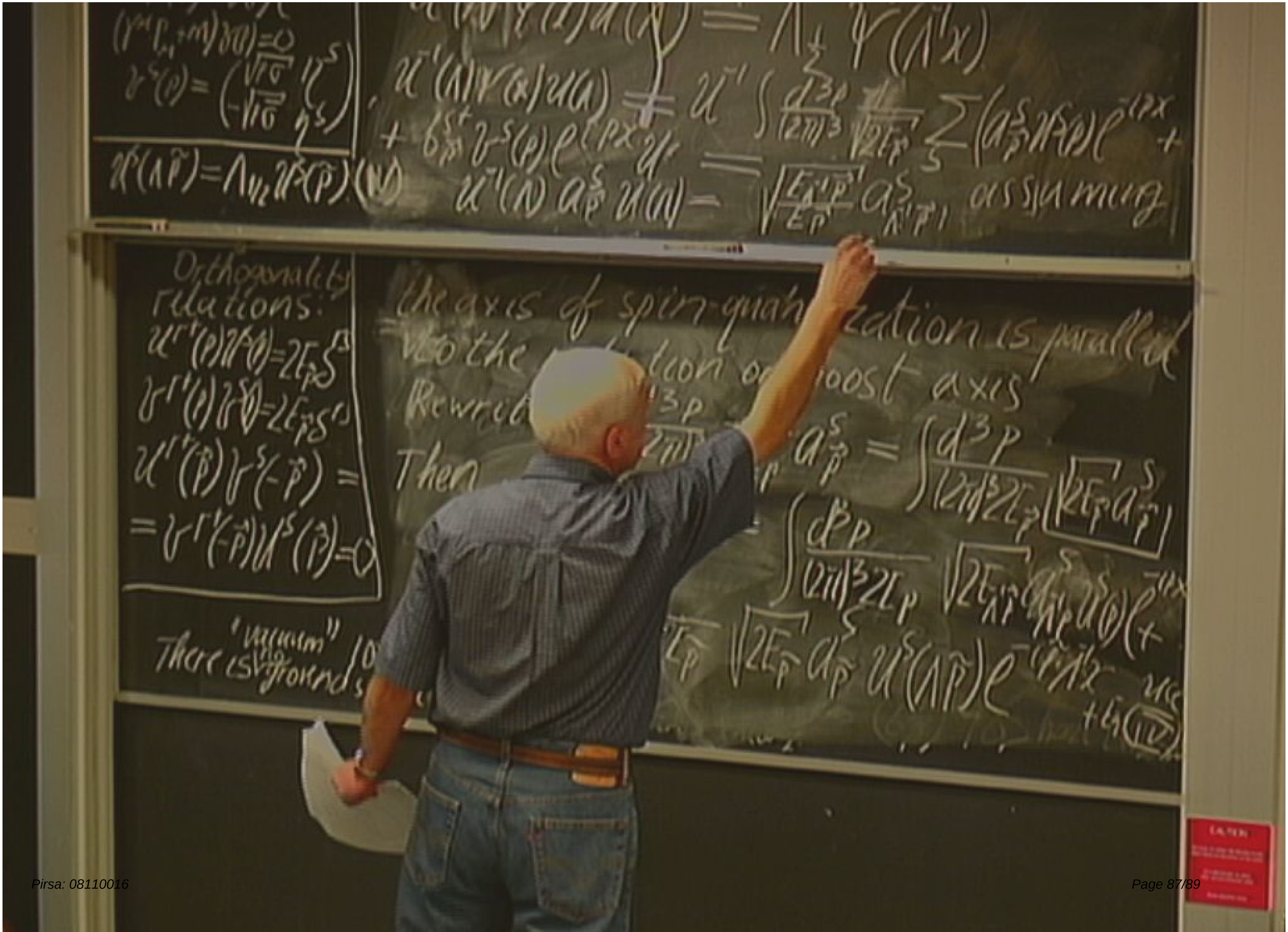
$$= \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} + E_{\vec p}\, b^{s\dagger}_{\vec p} b^{s}_{\vec p} \right) - 2 \int \frac{d^3p}{(2\pi)^3} \boxed{(2\pi)^3 \delta^3(0)} \leftarrow V \leftarrow \text{volume}$$

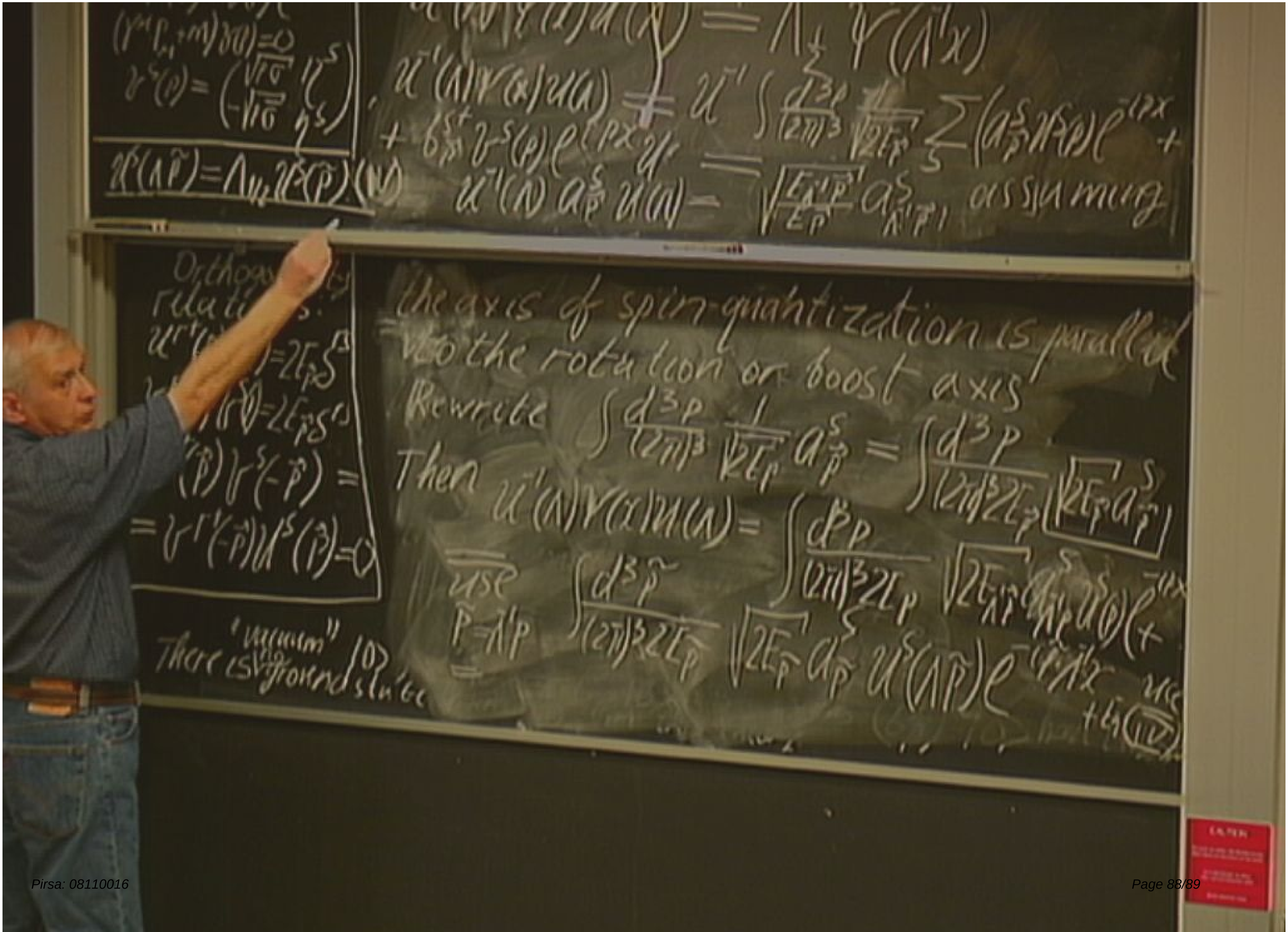
$$= \int \frac{d^3\tilde p}{(2\pi)^3 \sqrt{2E_{\tilde p}}}\, a^s_{\tilde p}\, \Lambda_{1/2}\, u^s(\tilde p)\, e^{-i\tilde p \cdot \Lambda^{-1} x} + \dots = \boxed{\Lambda_{1/2}\, \psi(\Lambda^{-1} x)}$$

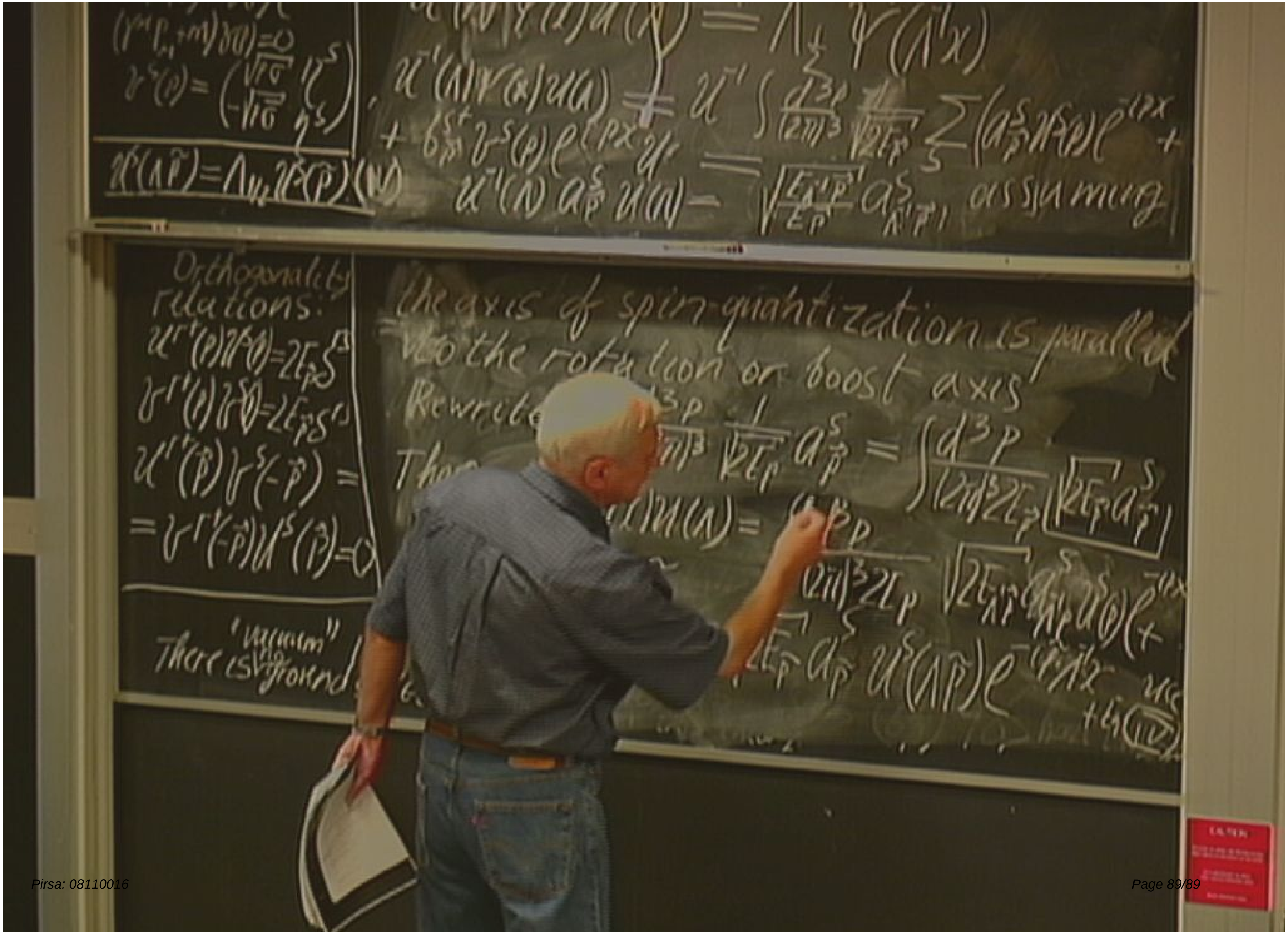
$$H|\tilde 0\rangle = -2V \int \frac{d^3p}{(2\pi)^3} \dots \left(\int \frac{d^3p}{(2\pi)^3} \right) |\tilde 0\rangle$$

In order to understand better this theory, consider a toy model: $b, b^\dagger$; $\{b, b^\dagger\} = b b^\dagger + b^\dagger b = 1$, $\{b, b\} = \{b^\dagger, b^\dagger\} = 0$

$\Longrightarrow b^2 = 0$, $b^{\dagger 2} = 0$; $H = -E_0\, b^\dagger b$. This model has only 2


assuming
Orthogonality relations:
the axis of spin-quantization is parallel
to the
axis
Then
"vacuum"


assuming
the axis of spin-quantization is parallel
to the rotation or boost axis
Rewrite
Then
use


assuming
Orthogonality relations:
the axis of spin quantization is parallel
to the rotation or boost axis
Rewrite
Then