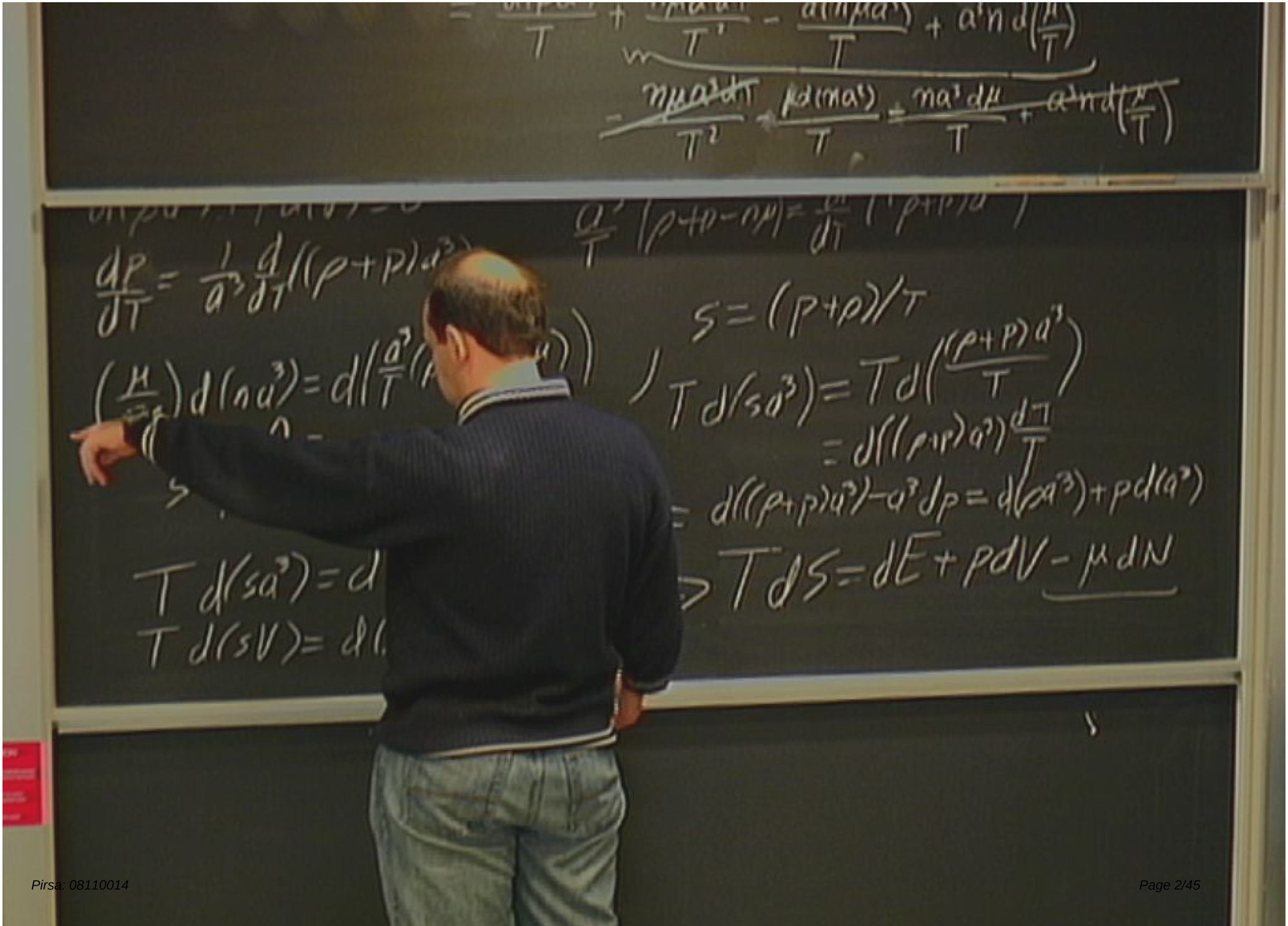
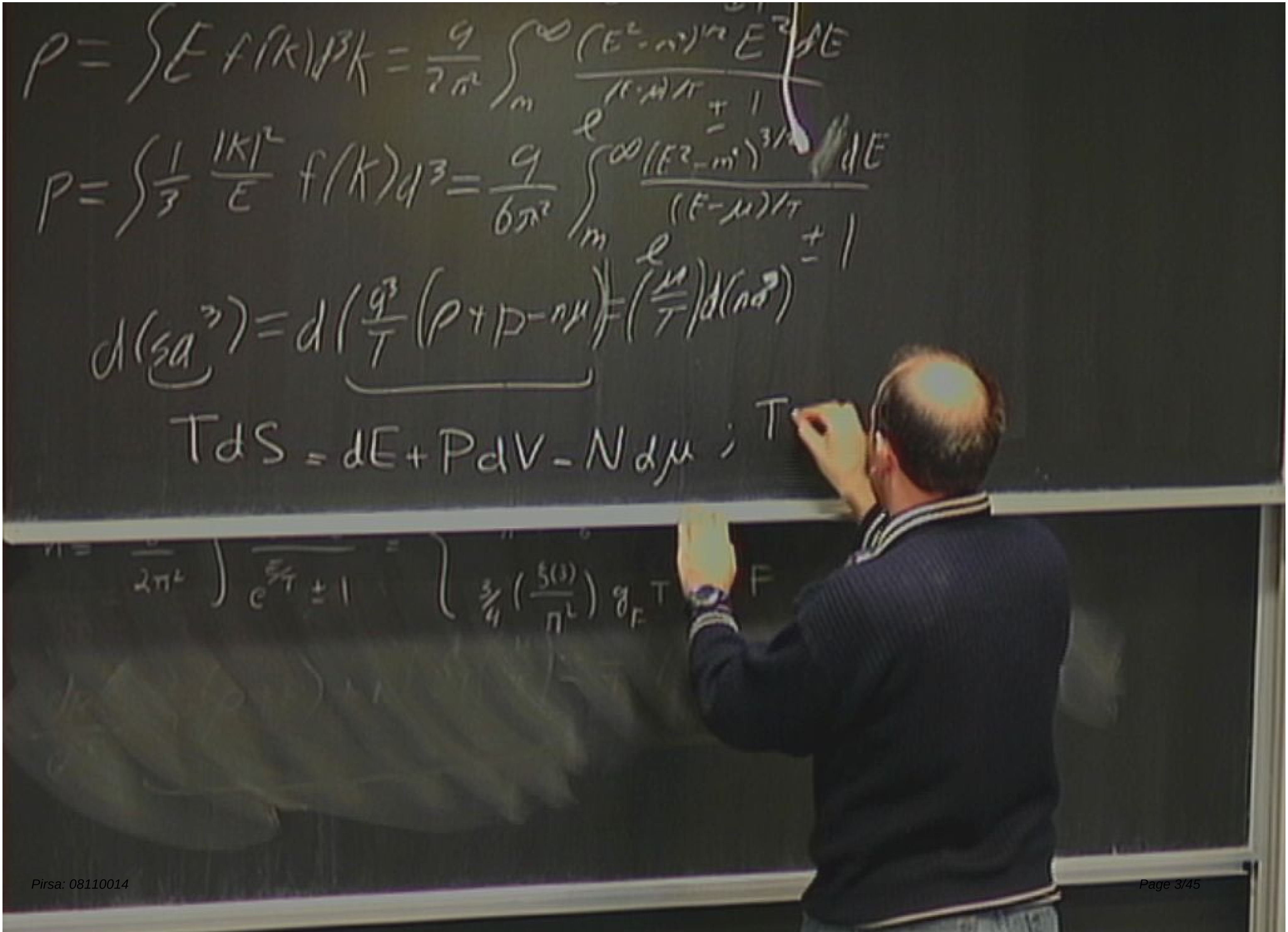
Title: Astrophysics and Cosmology through Problems - 12B

Date: Nov 20, 2008 12:30 PM

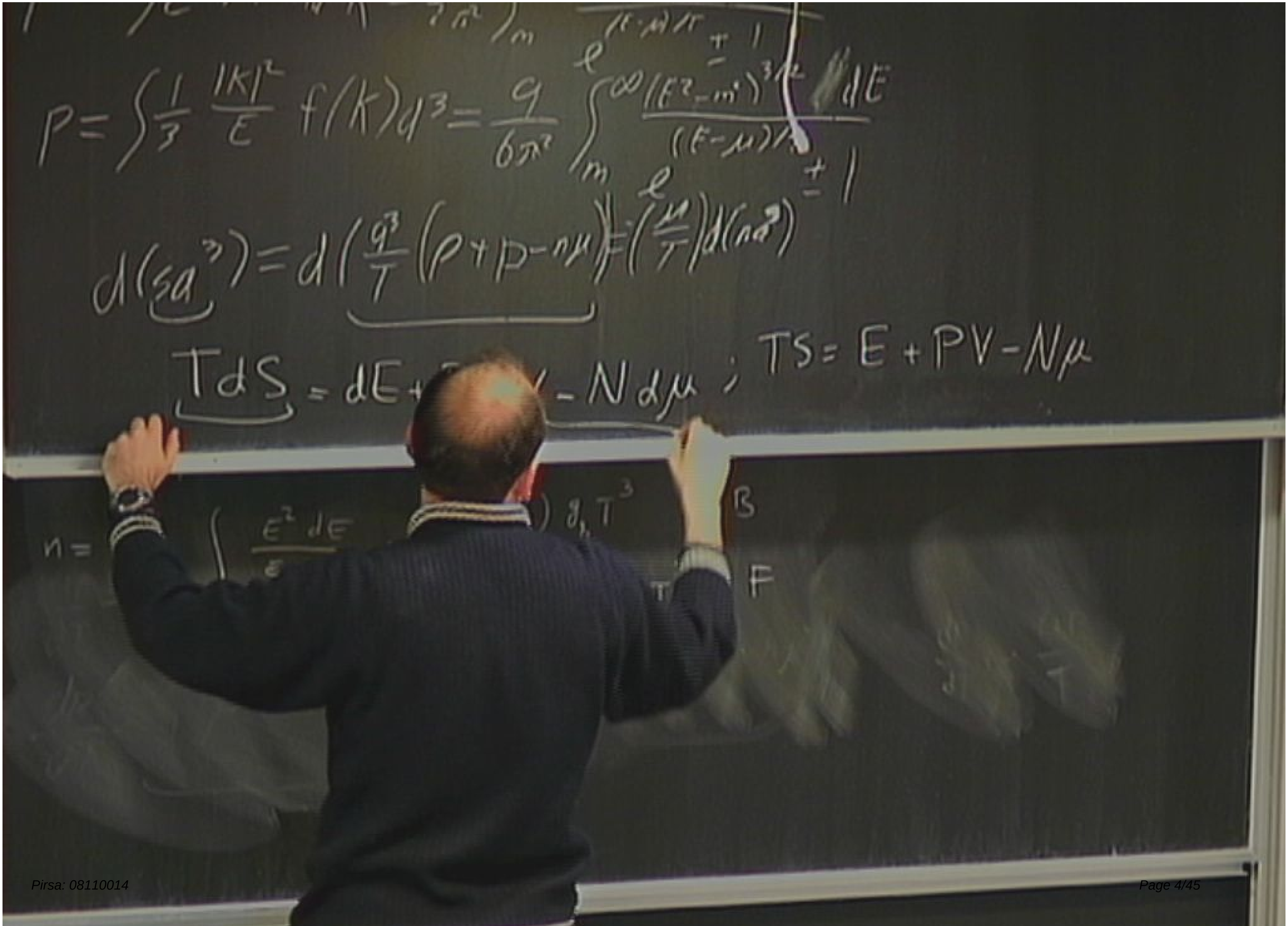
URL: http://pirsa.org/08110014

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.


s = (ρ+p)/T
T d(sa³) = T d((ρ+p) a³ / T)
= d((ρ+p)a³) − a³ dp = d(ρa³) + p d(a³)
⇒ T dS = dE + p dV − μ dN
T d(sa³) = d
T d(sV) = d



$\rho = \int E f(k) d^3k = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2-m^2)^{1/2} E^2 dE}{e^{(E-\mu)/T} \pm 1}$
$P = \int \frac{1}{3} \frac{|k|^2}{E} f(k) d^3 = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2-m^2)^{3/2} dE}{e^{(E-\mu)/T} \pm 1}$
$d(sa^3) = d\left(\frac{a^3}{T}(\rho + p - n\mu)\right) + \left(\frac{\mu}{T}\right) d(na^3)$
$TdS = dE + PdV - N d\mu \ ; \ T$
$\frac{3}{4}\left(\frac{\zeta(3)}{\pi^2}\right) g_F T$

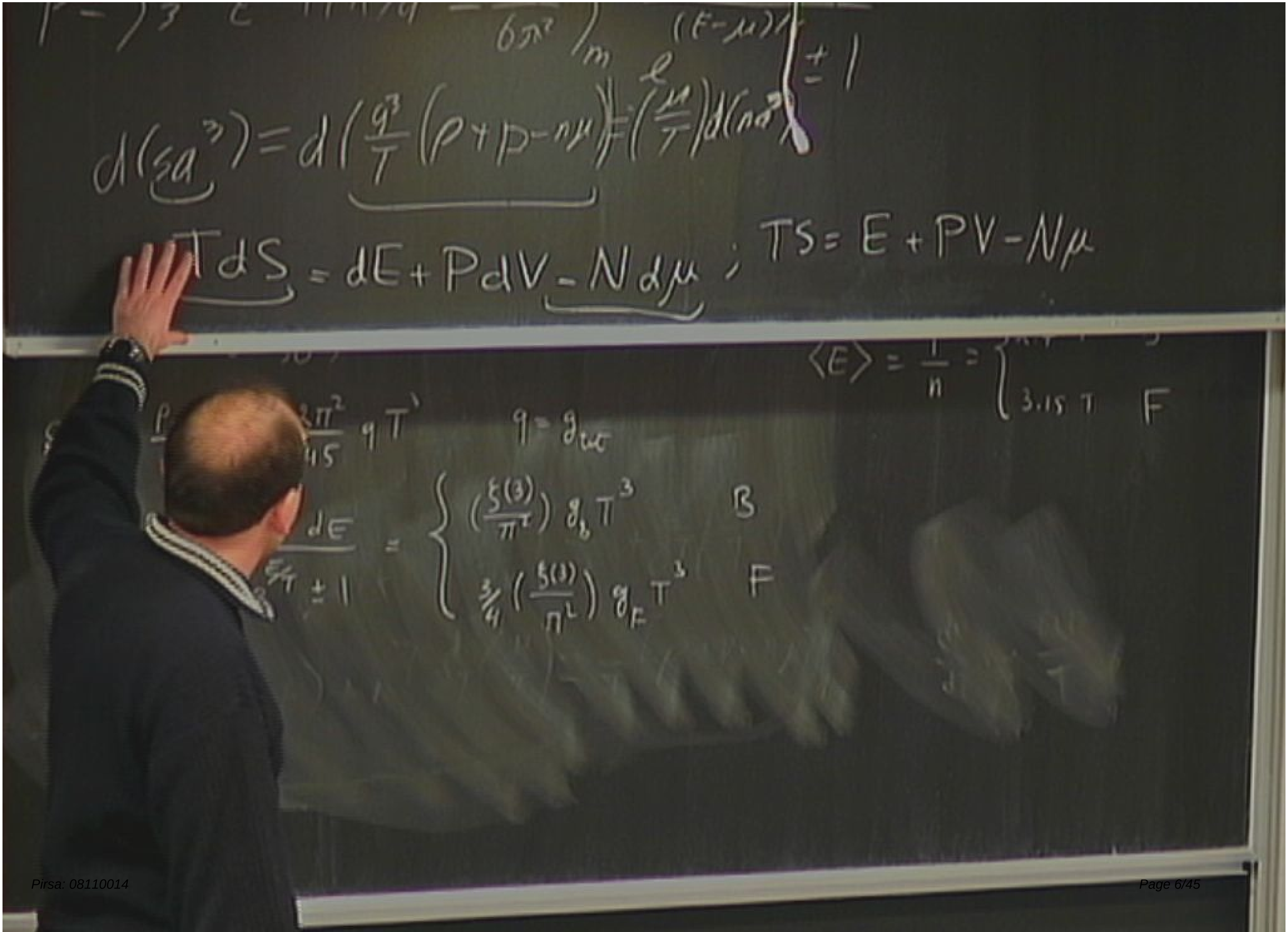

P = ∫ 1/3 |k|²/E f(k) d³ = g/6π² ∫_m^∞ (E² − m²)^{3/2} dE / (e^{(E−μ)/T} ± 1)
d(sa³) = d(g³/T (ρ + p − nμ)) − (μ/T) d(na³)
TdS = dE + … − N dμ ; TS = E + PV − Nμ
B
F

$$P = \int \frac{1}{3}\frac{|k|^2}{E} f(k)\, d^3 = \frac{g}{6\pi^2}\int_m^\infty \frac{(E^2-m^2)^{3/2}\, dE}{e^{(E-\mu)/T} \pm 1}$$

$$d(\underbrace{sa^3}) = d\Big(\underbrace{\frac{a^3}{T}(\rho + p - n\mu)}\Big) + \left(\frac{\mu}{T}\right) d(na^3)$$

$$\underbrace{T\,dS} = dE + P\,dV \underbrace{- N\,d\mu}\ ; \quad TS = E + PV - N\mu$$

$$\int \frac{E^2\, dE}{e^{E/T} \pm 1} = \begin{cases} \left(\frac{\zeta(3)}{\pi^2}\right) g_b T^3 & B \\ \frac{3}{4}\left(\frac{\zeta(3)}{\pi^2}\right) g_F T^3 & F \end{cases}$$

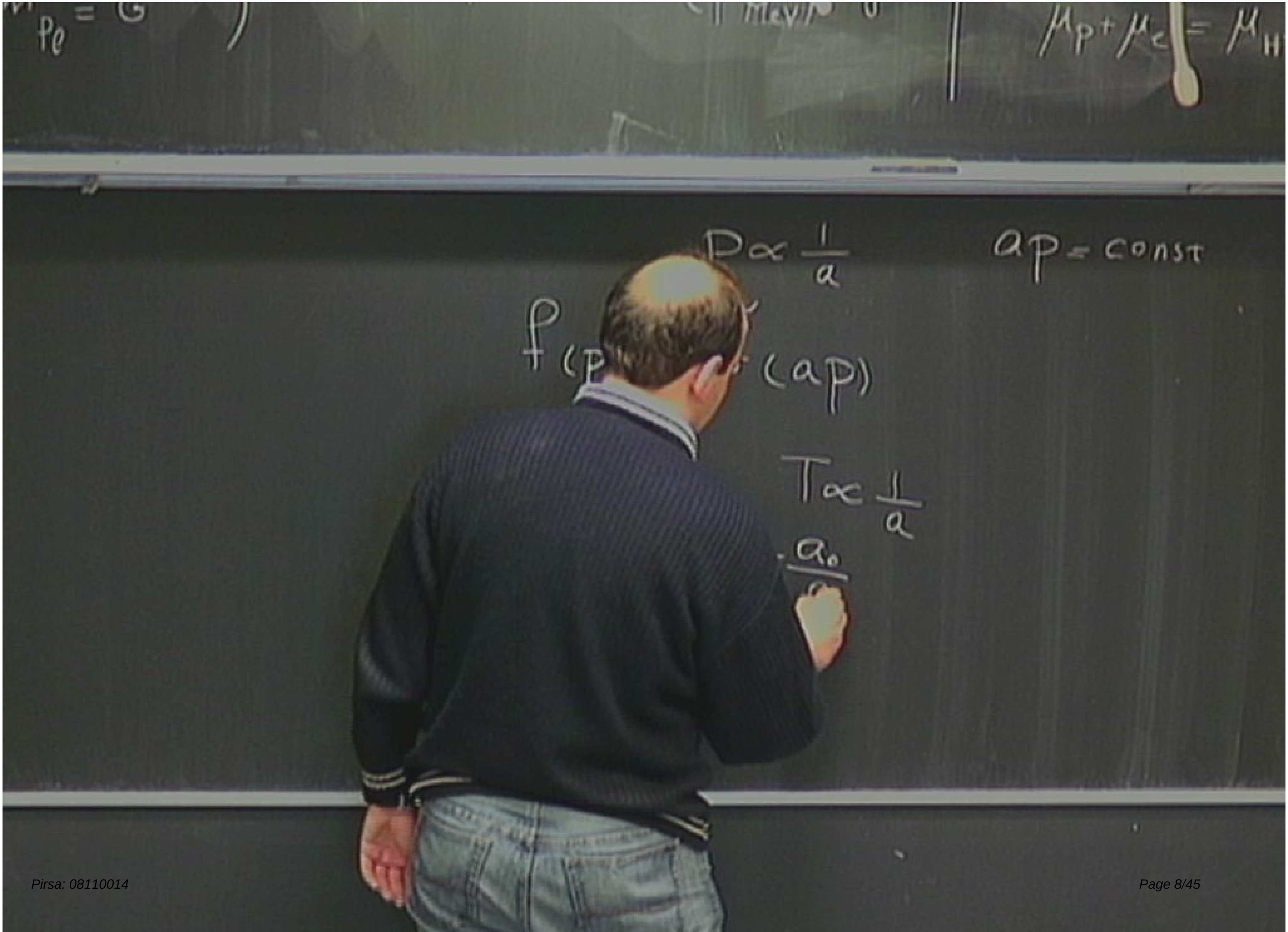

d(sa³) = d(a³/T (ρ + p − nμ)) − (μ/T) d(na³)
TdS = dE + PdV − N dμ ; TS = E + PV − Nμ
B
F

$P_e = G$)

$\mu_p + \mu_e = \mu_H$

$p \propto \frac{1}{a}$ $\quad$ $ap = \text{const}$

ρ


$\mu_p + \mu_e = \mu_H$
$p \propto \frac{1}{a}$
$ap = \mathrm{const}$
(ap)
$T \propto \frac{1}{a}$

$p_e = G$)

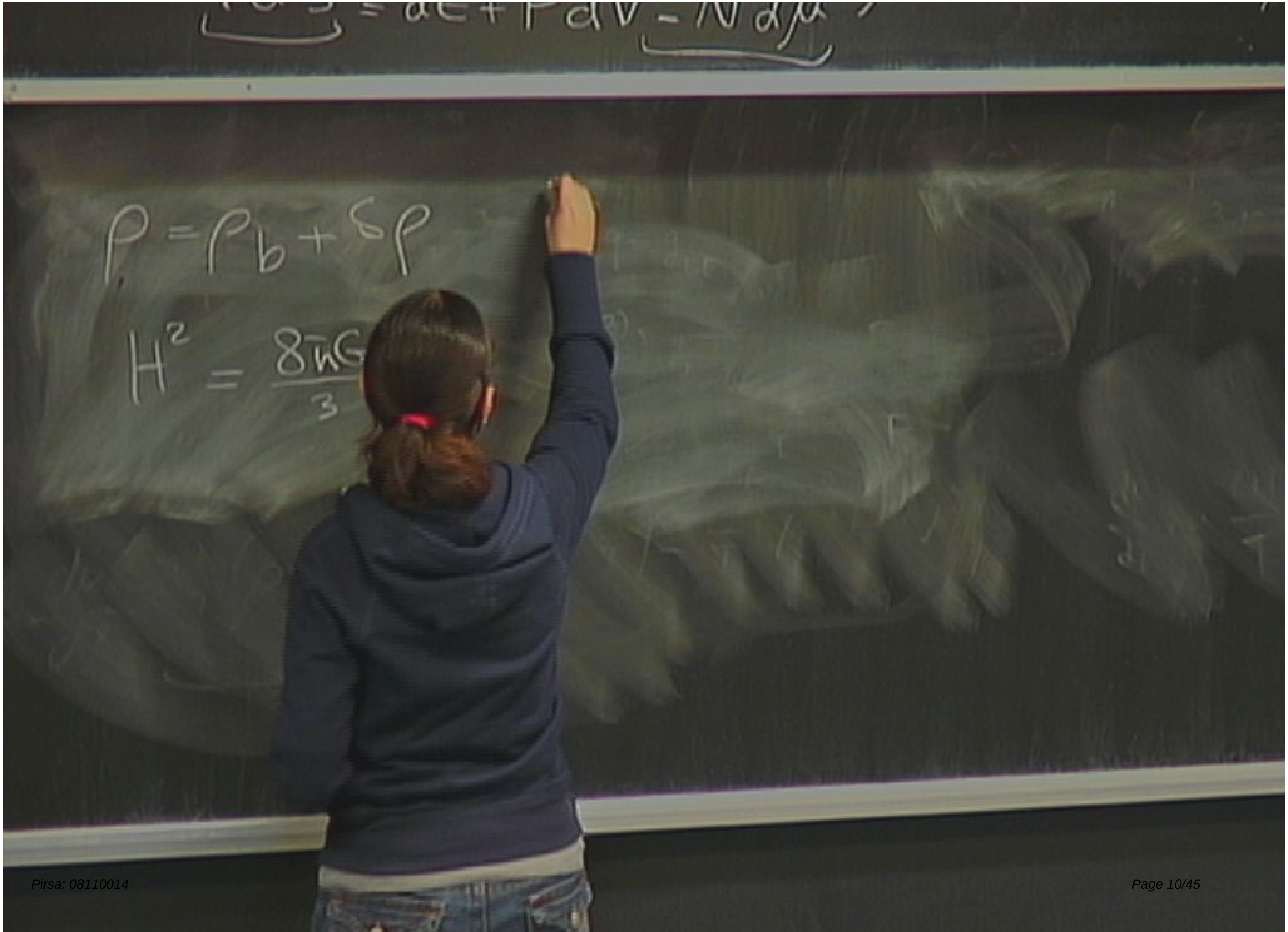
$\mu_p + \mu_e = \mu_H$

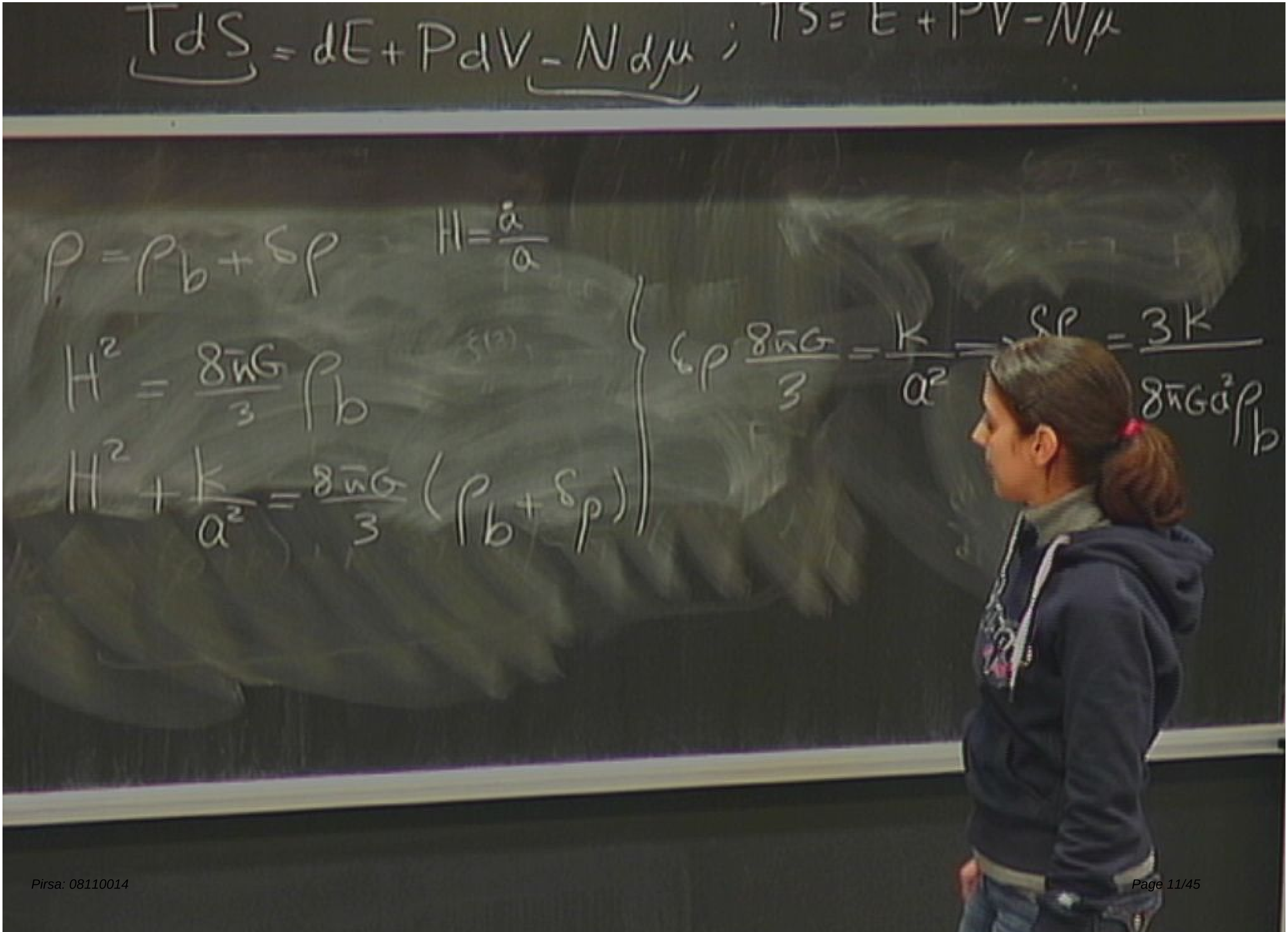
$$p \propto \frac{1}{a} \qquad ap = \text{const}$$

$$f(p,t) = \tilde{f}(ap)$$

$$T \propto \frac{1}{a}$$

$$1 + z = \frac{a_0}{a}$$


ρ = ρb + δρ
H² = 8πG / 3


$TdS = dE + PdV - N d\mu \; ; \; TS = E + PV - N\mu$
$\rho = \rho_b + \delta\rho$
$H = \frac{\dot{a}}{a}$
$H^2 = \frac{8\pi G}{3}\rho_b$
$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$
$\delta\rho \frac{8\pi G}{3} = \frac{k}{a^2} \rightarrow \frac{\delta\rho}{\rho_b} = \frac{3k}{8\pi G a^2 \rho_b}$

$$\underbrace{TdS} = dE + PdV \underbrace{- N d\mu} \; ; \; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot{a}}{a}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$$

$$\delta\rho \frac{8\pi G}{3} = \frac{k}{a^2} \Rightarrow \frac{\delta\rho}{\rho_b} = \frac{3k}{8\pi G a^2 \rho_b}$$

$$\underbrace{TdS} = dE + PdV \underbrace{- N d\mu} \; ; \; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot a}{a}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$$

$$\underbrace{\Delta H}_{(H'^2 - H^2)} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$$

$$\delta\rho \frac{8\pi G}{3} = \frac{k}{a^2} \Longrightarrow \frac{\delta\rho}{\rho_b} = \frac{3k}{8\pi G a^2 \rho_b}$$

$$\underbrace{TdS} = dE + PdV \underbrace{- N d\mu} \; ; \; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot{a}}{a}$$

$$\frac{\Delta H}{(H'^2 - H^2)} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2} \qquad \delta = \frac{\delta\rho}{\rho_b}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$$

$$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$$

$$\underbrace{TdS} = dE + PdV \underbrace{- N\,d\mu}\ ; \quad TS = E + PV - N\mu$$

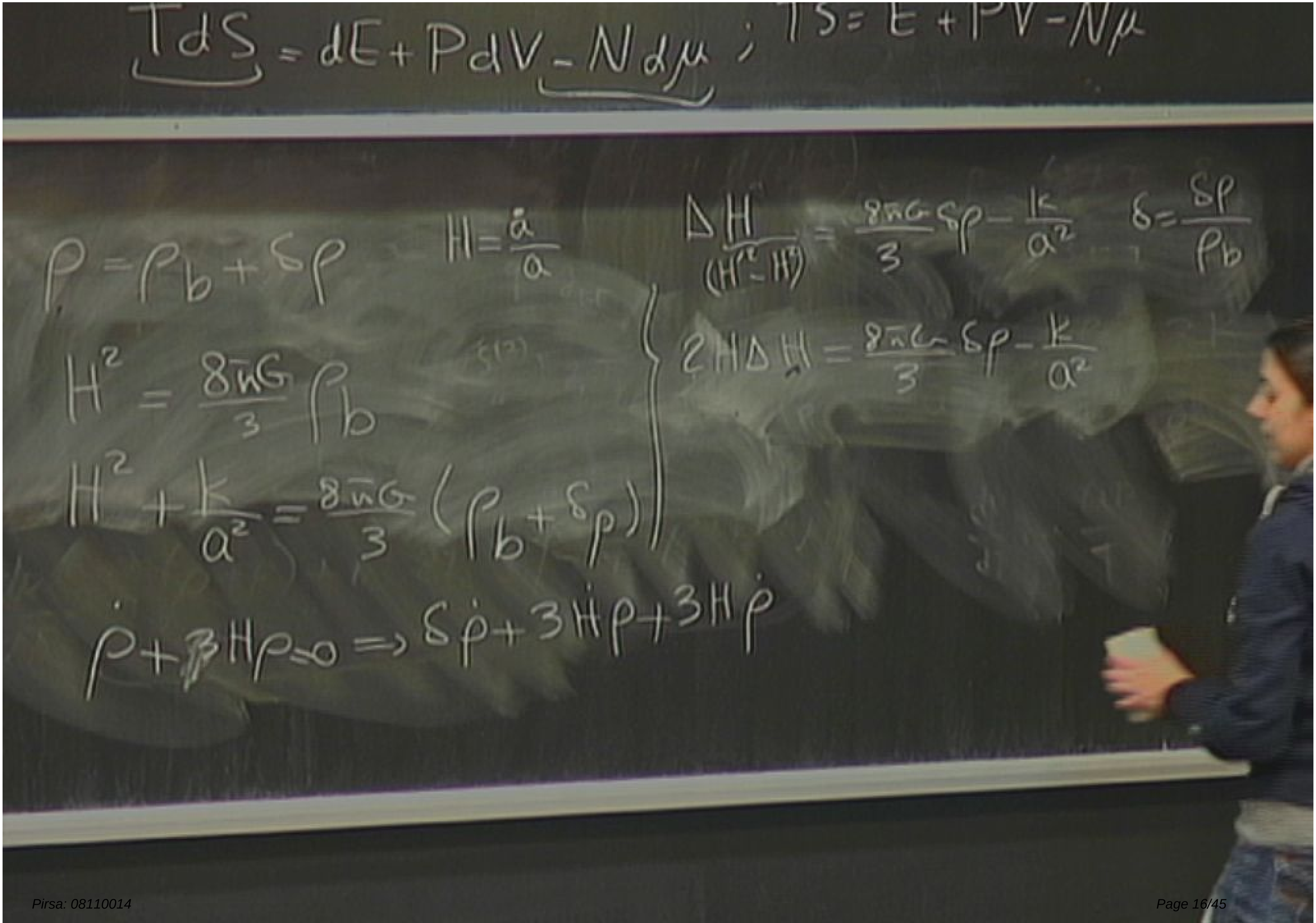
$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot a}{a}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_b + \delta\rho\right)$$

$$\underbrace{\Delta H}_{(H'^2 - H^2)} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2} \qquad \delta = \frac{\delta\rho}{\rho_b}$$

$$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$$


$TdS = dE + PdV - Nd\mu \;;\; TS = E + PV - N\mu$
$\rho = \rho_b + \delta\rho$
$H = \frac{\dot a}{a}$
$\frac{\Delta H}{(H'^2 - H^2)} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$
$\delta = \frac{\delta\rho}{\rho_b}$
$H^2 = \frac{8\pi G}{3}\rho_b$
$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$
$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$
$\dot\rho + 3H\rho = 0 \Rightarrow \delta\dot\rho + 3\dot H\rho + 3H\dot\rho$

$$\underbrace{TdS} = dE + PdV \underbrace{- N d\mu} \; ; \; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot{a}}{a}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$$

$$\Delta H \underset{(H'^2 - H^2)}{} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2} \qquad \delta = \frac{\delta\rho}{\rho_b}$$

$$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3H\rho = 0 \Rightarrow \delta\dot{\rho} + 3\delta(H\rho) = 0$$

$$\underbrace{TdS} = dE + PdV \underbrace{- Nd\mu} \; ; \; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot a}{a}$$

$$\underbrace{\Delta H}_{(H'^2 - H^2)} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2} \qquad \delta = \frac{\delta\rho}{\rho_b}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

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$$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$$

$$2a^2 H\Delta H = \frac{8\pi G a^2}{3}\delta\rho - k$$

$$\frac{2a^2}{\rho_b} H\Delta H = \delta\,\frac{8\pi G a^2}{3} - \frac{k}{\rho_b}$$

$$\dot\rho + 3H\rho = 0 \Rightarrow \delta\dot\rho + 3\delta(H\rho) = 0$$

$$\underbrace{TdS} = dE + PdV \underbrace{- N d\mu} \; ; \; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \qquad H = \frac{\dot a}{a}$$

$$H^2 = \frac{8\pi G}{3}\rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$$

$$\underset{(H'^2 - H^2)}{\Delta H} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2} \qquad \delta = \frac{\delta\rho}{\rho_b}$$

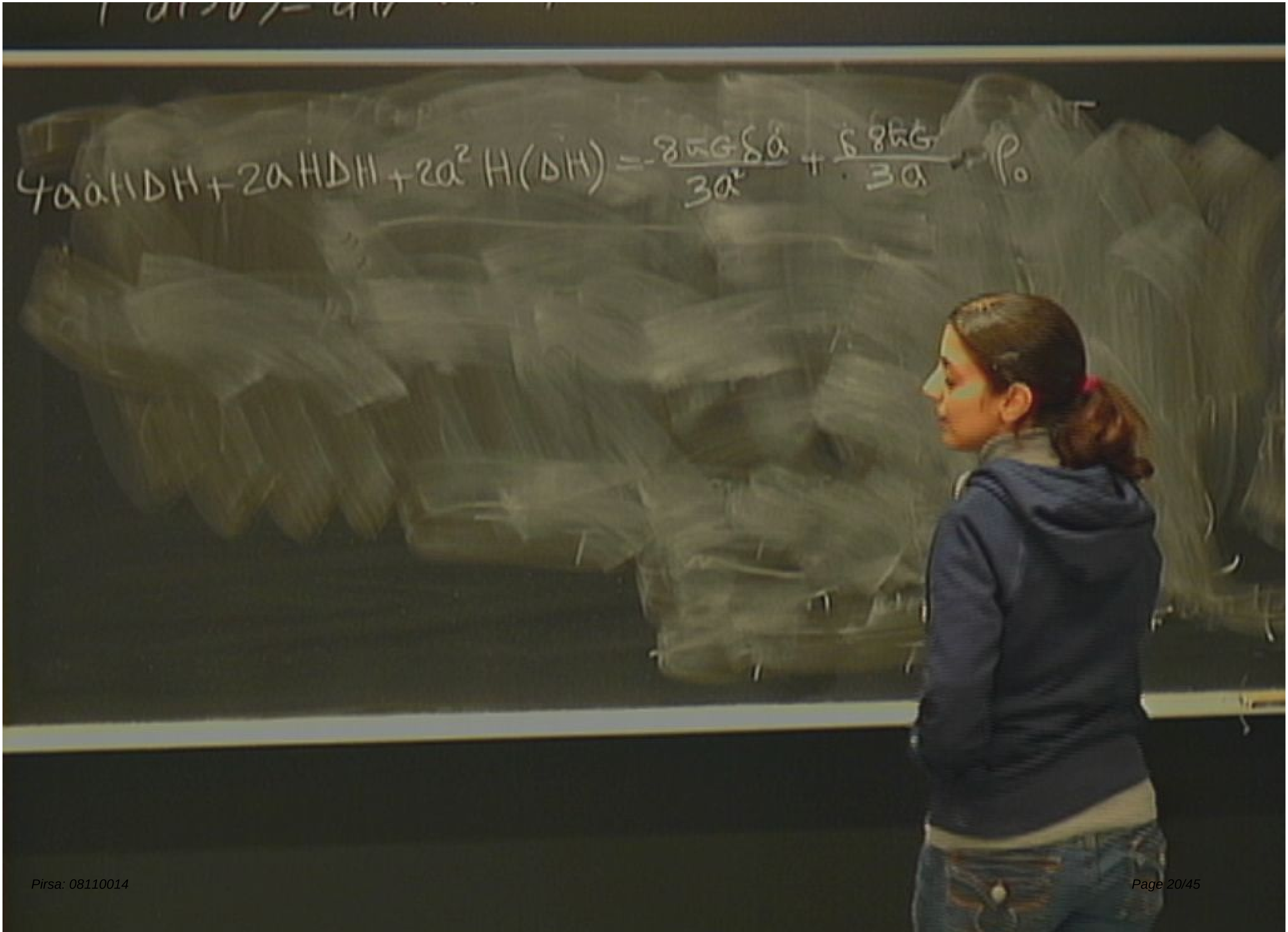
$$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$$

$$2a^2 H\Delta H = \frac{8\pi G a^2}{3}\delta\rho - k$$

$$2a^2 H\Delta H = \delta\,\frac{8\pi G a^2}{3}\rho_b - k$$

$$4\dot a a H\Delta H +$$

$$\dot\rho + 3H\rho = 0 \Rightarrow \delta\dot\rho + 3\,\delta(H\rho) = 0$$


$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\delta\dot{a}}{3a^2} + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$

$$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta\,8\pi G}{3a}\dot{\rho}_0$$

$TdS = dE + PdV - Nd\mu$; $TS = E + PV - N\mu$

$\rho = \rho_b + \delta\rho$ $\quad H = \frac{\dot a}{a}$ $\quad \underbrace{\Delta H}_{(H'^2 - H^2)} = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$ $\quad \delta = \frac{\delta\rho}{\rho_b}$

$H^2 = \frac{8\pi G}{3}\rho_b$

$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_b + \delta\rho)$

$2H\Delta H = \frac{8\pi G}{3}\delta\rho - \frac{k}{a^2}$

$2a^2 H\Delta H = \frac{8\pi G a^2}{3}\delta\rho - k$

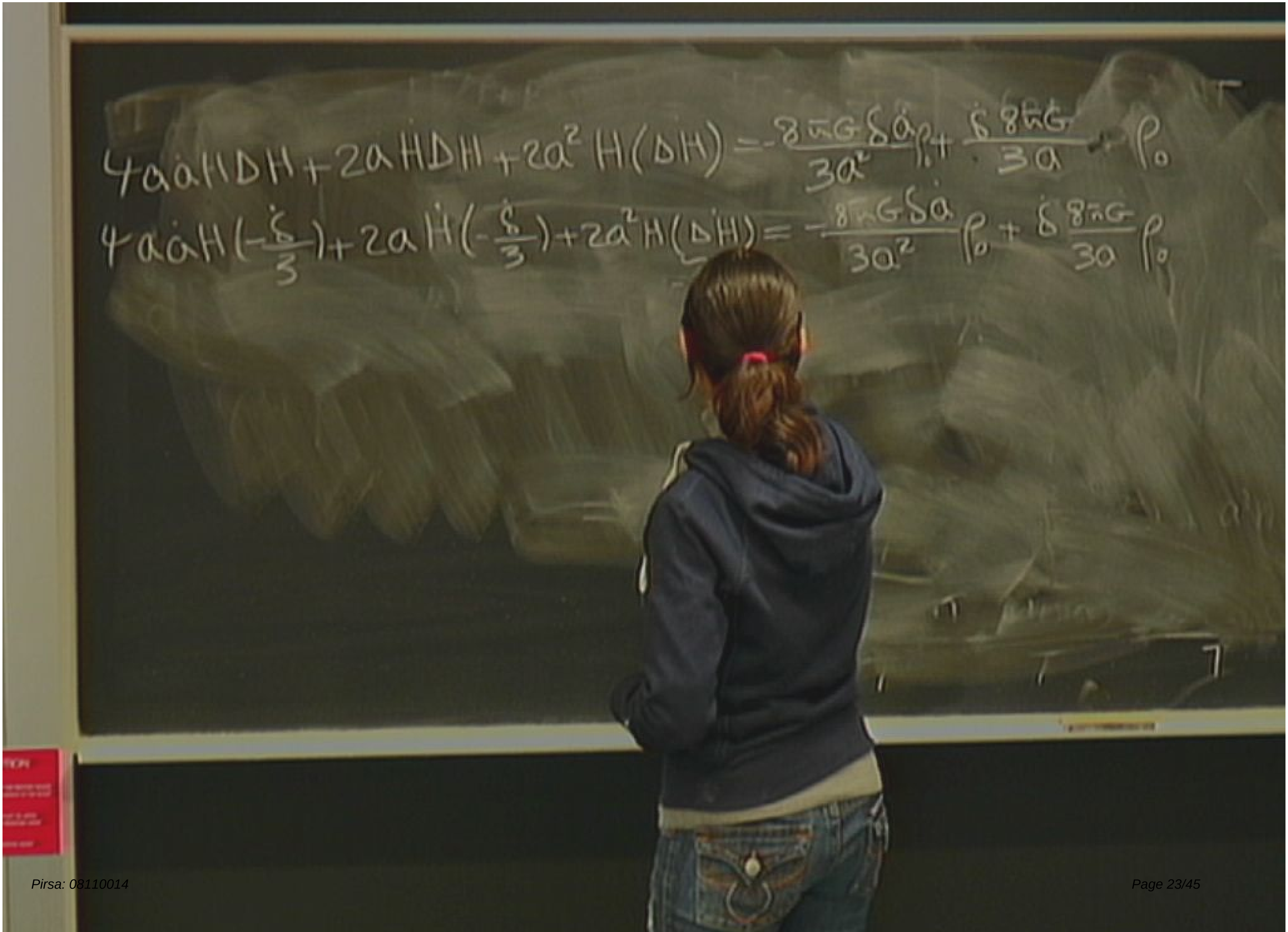
$2a^2 H\Delta H = \delta\, \frac{8\pi G a^2}{3}\rho - k$

$4\dot a a H\Delta H +$

$\frac{\rho}{a^3}$

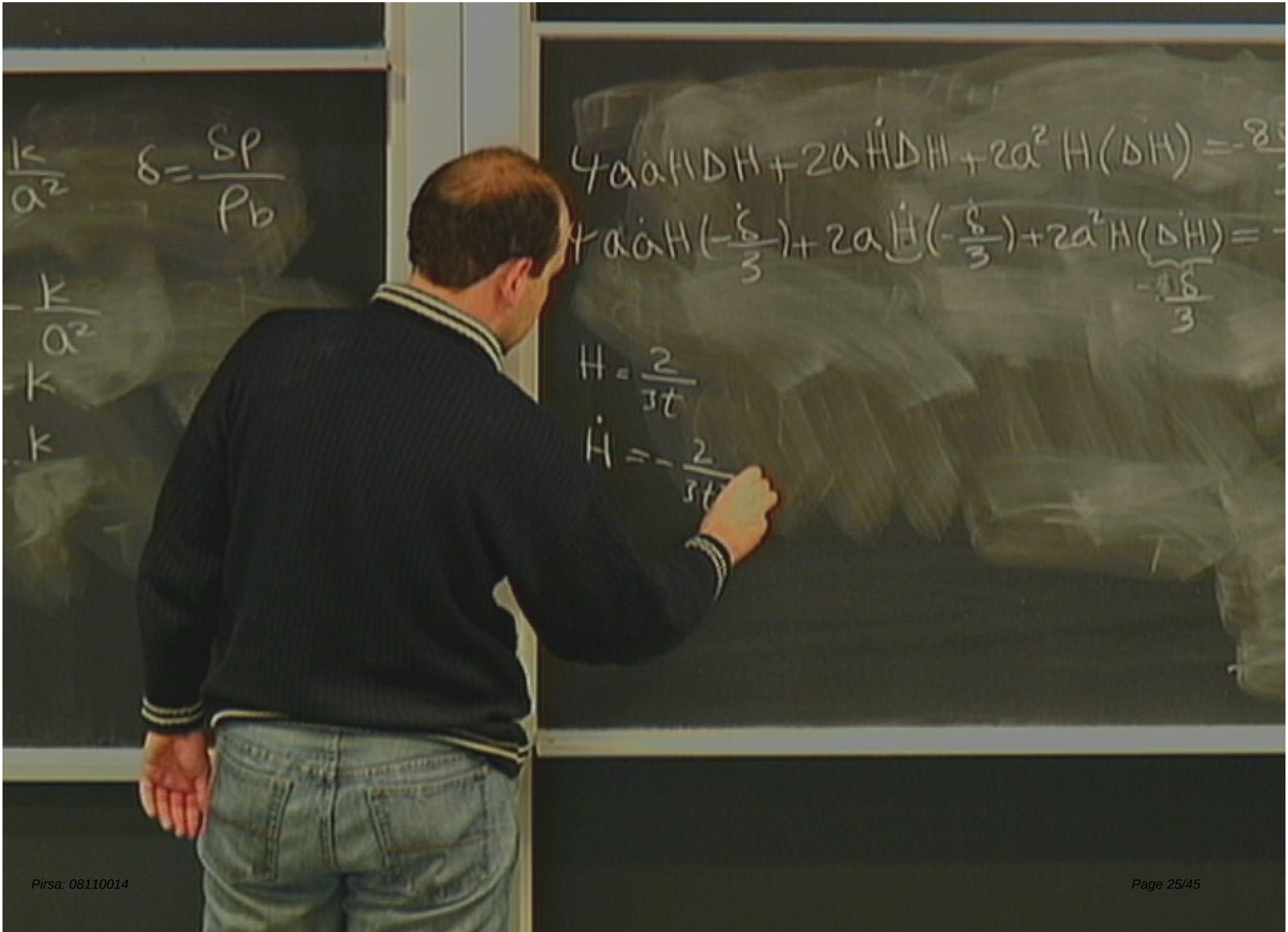
$\dot\rho + 3H\rho = 0 \Rightarrow \delta\dot\rho + 3\delta(H\rho) = 0$

$\boxed{\dot\delta + 3\delta H = 0}$


$4a\dot{a}\dot{H}\Delta H + 2a\dot{H}\Delta\dot{H} + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$
$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\dot{a}}{3a^2}\rho_0 + \dot{\delta}\,\frac{8\pi G}{3a}\rho_0$

$$4a\dot{a}H\,\Delta H + 2a\dot{H}\,\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\Delta\dot{H})}_{-\frac{\ddot{\delta}}{3}} = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \dot{\delta}\,\frac{8\pi G}{3a}\rho_0$$


$\delta = \frac{\delta\rho}{\rho_b}$
$H = \frac{2}{3t}$

$$\frac{k}{a^2} \qquad \delta = \frac{\delta\rho}{\rho_b}$$

$$-\frac{k}{a^2}$$

$$-k$$

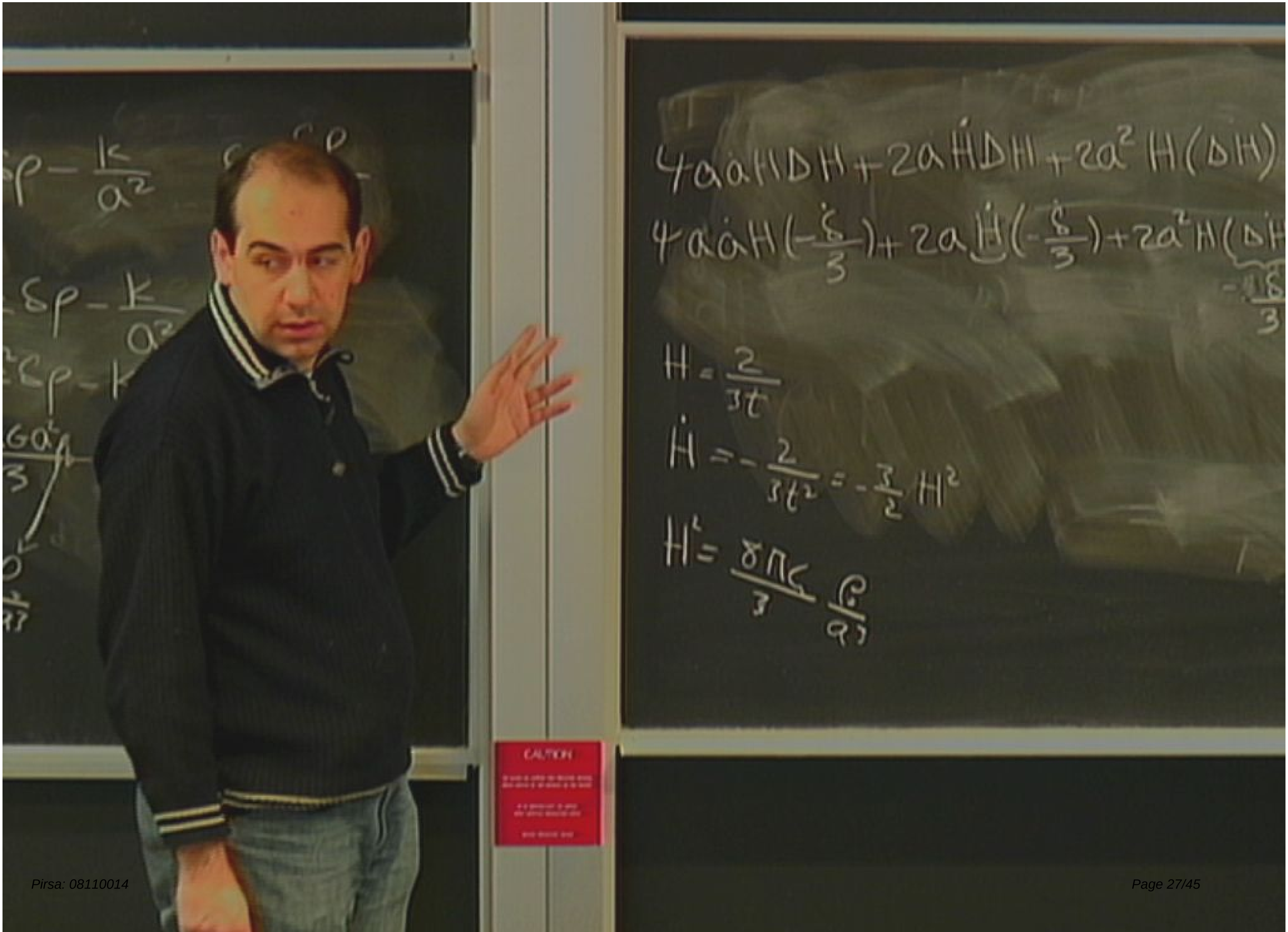
$$-k$$

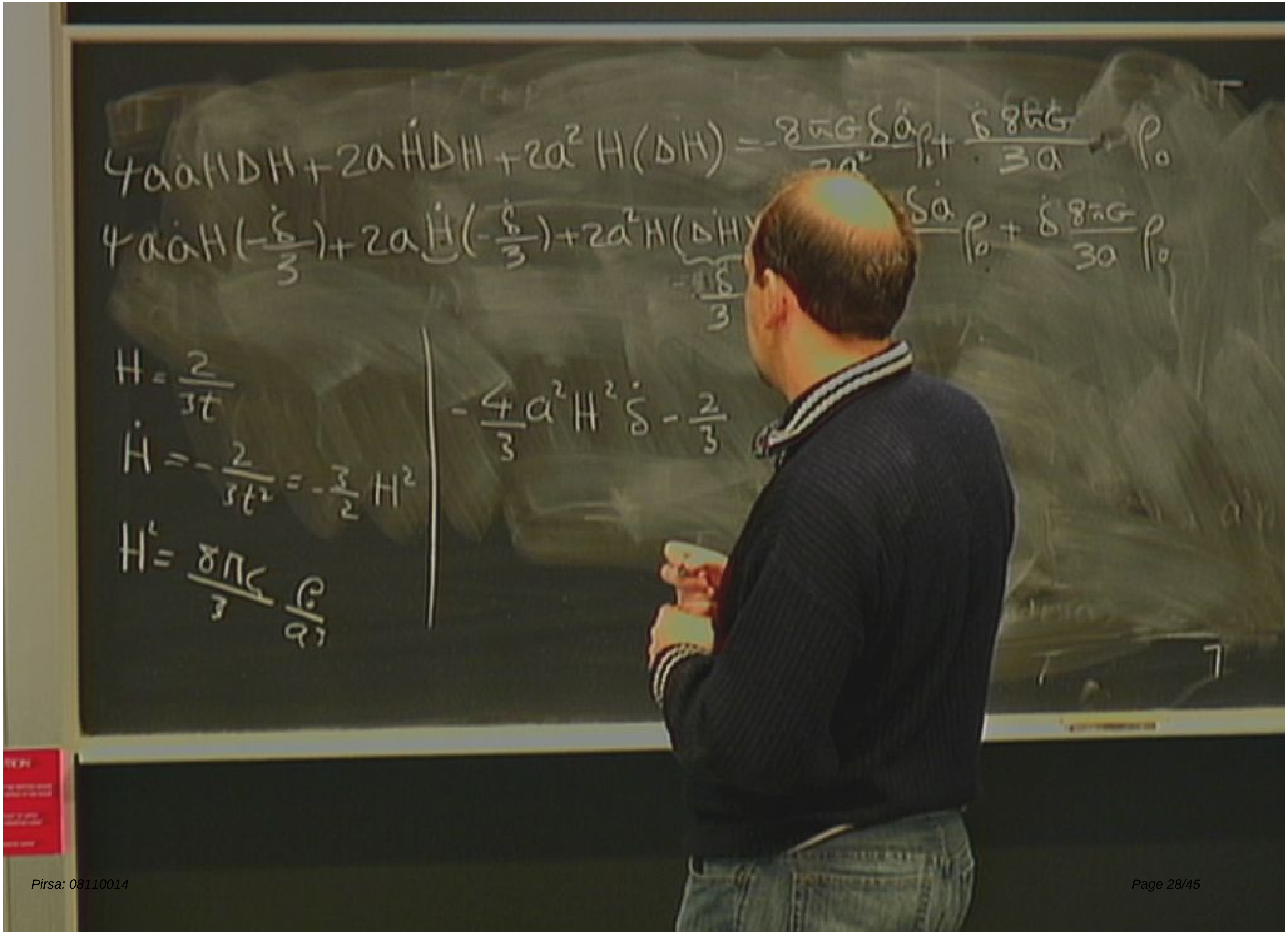
$$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta \dot{H}) = -8\ldots$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\Delta\dot{H})}_{-\frac{\ddot{\delta}}{3}} = -\ldots$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$


4aȧHΔH + 2aḢΔH + 2a²H(ΔH)
H = 2/3t
Ḣ = −2/3t² = −3/2 H²
H² = 8πG/3 ρ₀/a³
CAUTION


H = 2/3t
Ḣ = -2/3t² = -3/2 H²
H² = 8πG/3 ρ₀/a³

$$4a\dot{a}H\,\Delta H + 2a^2\dot{H}\,\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\Delta\dot{H})}_{-\frac{\ddot{\delta}}{3}} = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \dot{\delta}\,\frac{8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\ddot{\delta} =$$

$$4a\dot{a}H\,\Delta H + 2a^2\dot{H}\,\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\Delta\dot{H})}_{-\frac{\ddot{\delta}}{3}} = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \dot{\delta}\,\frac{8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\ddot{\delta} = -\,\delta$$

$$4a\dot{a}H\Delta\dot{H} + 2a^2\dot{H}\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\Delta\dot{H})}_{-\frac{\ddot{\delta}}{3}} = -\frac{8\pi G\,\delta(\dot{a})}{3a}\rho_0 + \dot{\delta}\,\frac{8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\ddot{\delta} = -\delta H^2 a^2$$

$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\delta$$

$$4a\dot{a}H\Delta H + 2a^2\dot{H}\Delta H + 2a^2 H(\Delta\dot{H}) = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\Delta\dot{H})}_{-\frac{\ddot{\delta}}{3}} = -\frac{8\pi G\,\delta\,(\dot{a}a)}{3a^3}\rho_0 + \dot{\delta}\,\frac{8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\ddot{\delta} = -\delta H^2 a^2 +$$

$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\delta$$

$$4a\dot{a}H\,\Delta H + 2a^2\dot{H}\,\Delta H + 2a^2 H(\dot{\Delta H}) = -\frac{8\pi G\,\delta\,\dot{a}}{3a^2}\rho_0 + \frac{\dot{\delta}\,8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\underbrace{(\dot{\Delta H})}_{-\frac{\ddot{\delta}}{3}} = -\frac{8\pi G\,\delta\,(\dot{a})}{3a}\rho_0 + \dot{\delta}\frac{8\pi G}{3a}\rho_0$$

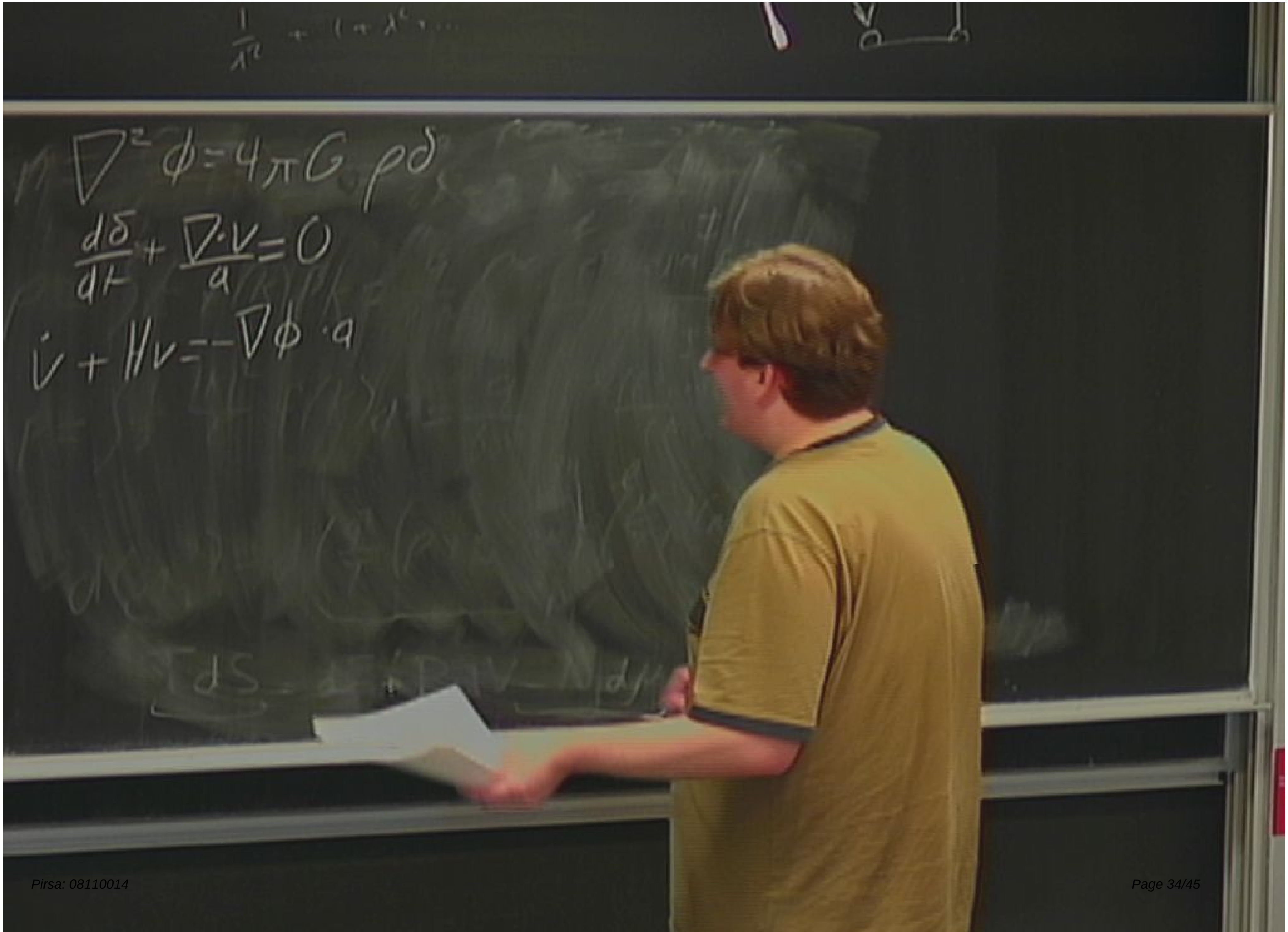
$$H = \frac{2}{3t}$$

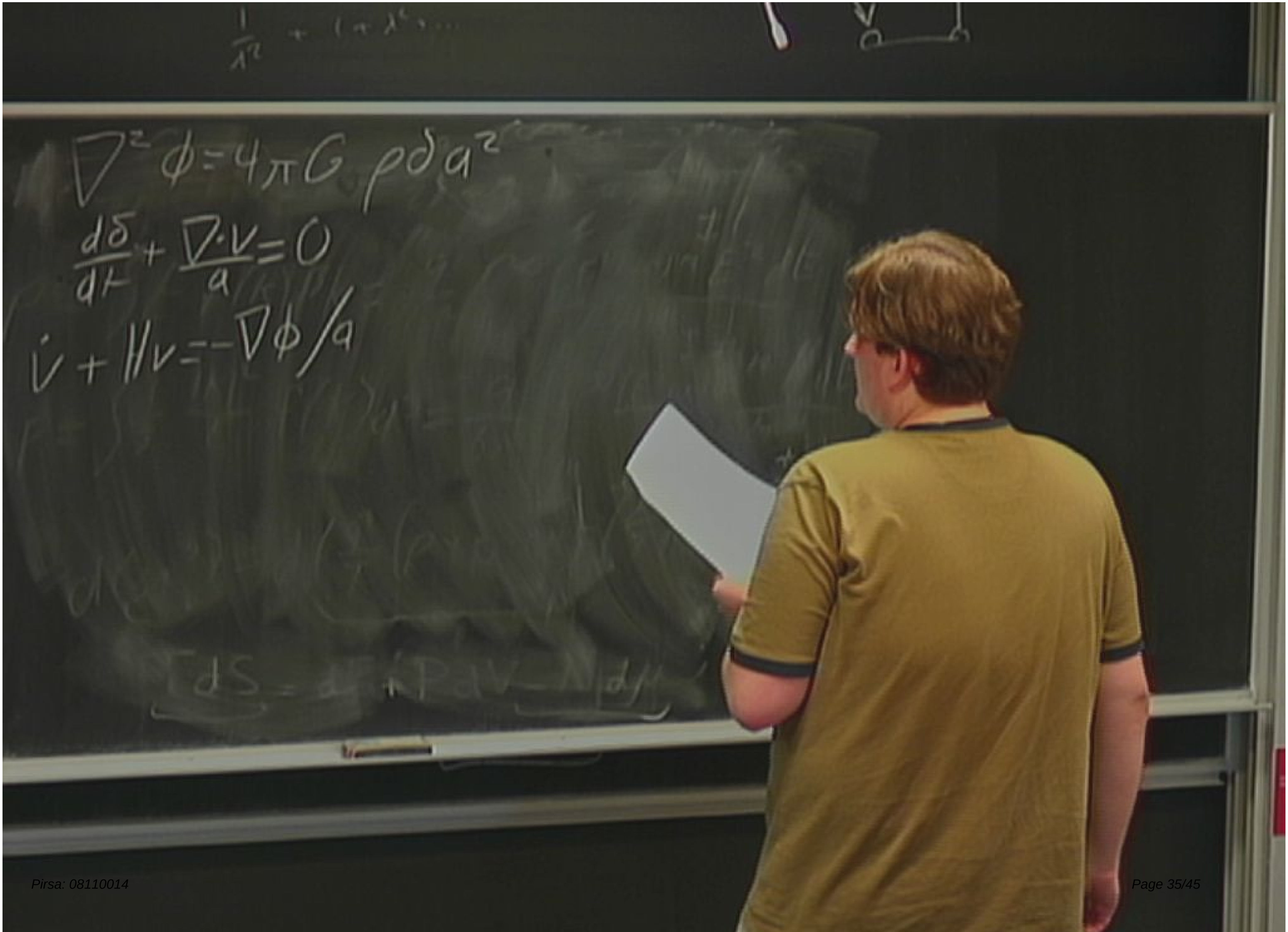
$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

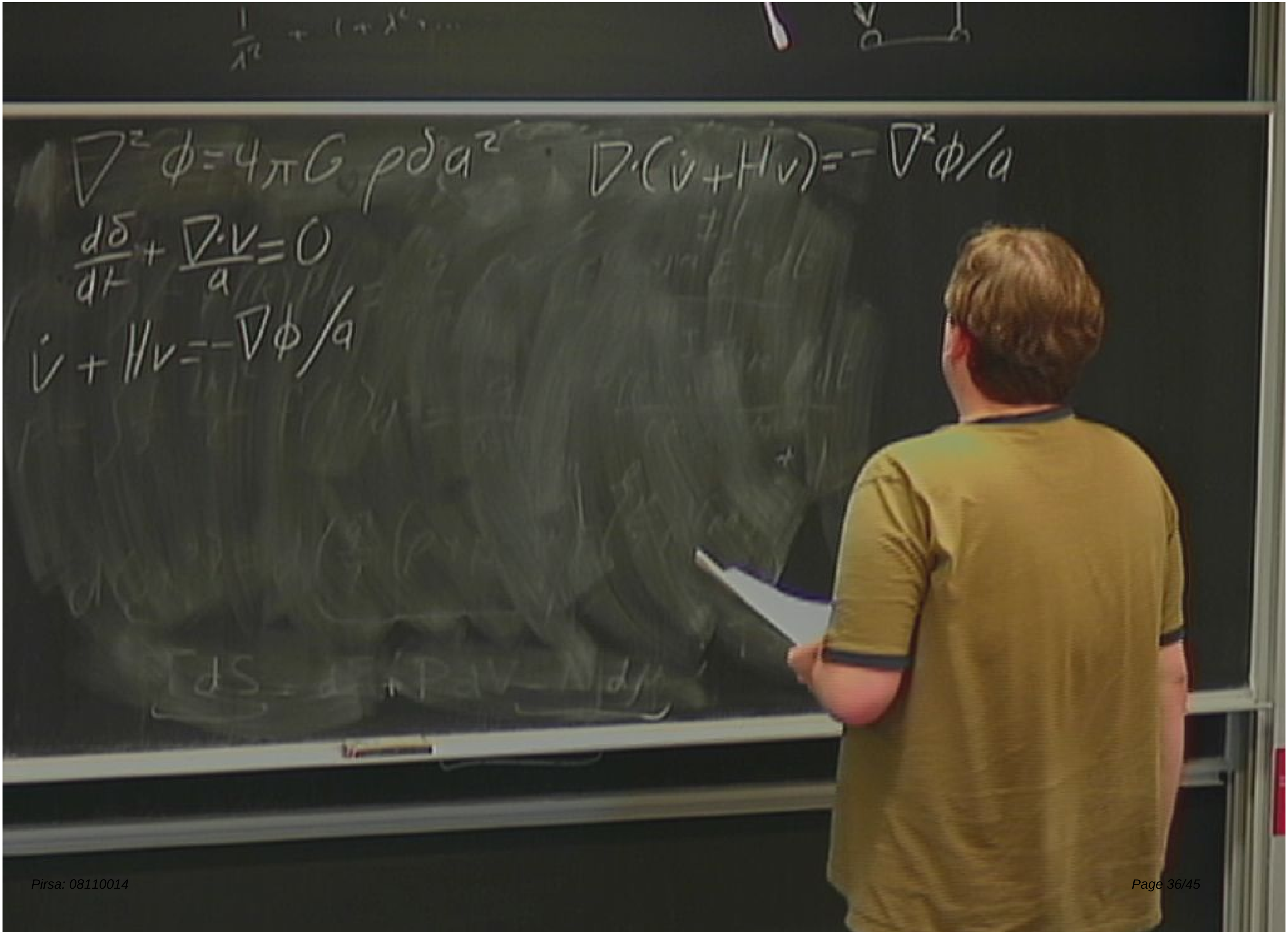
$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

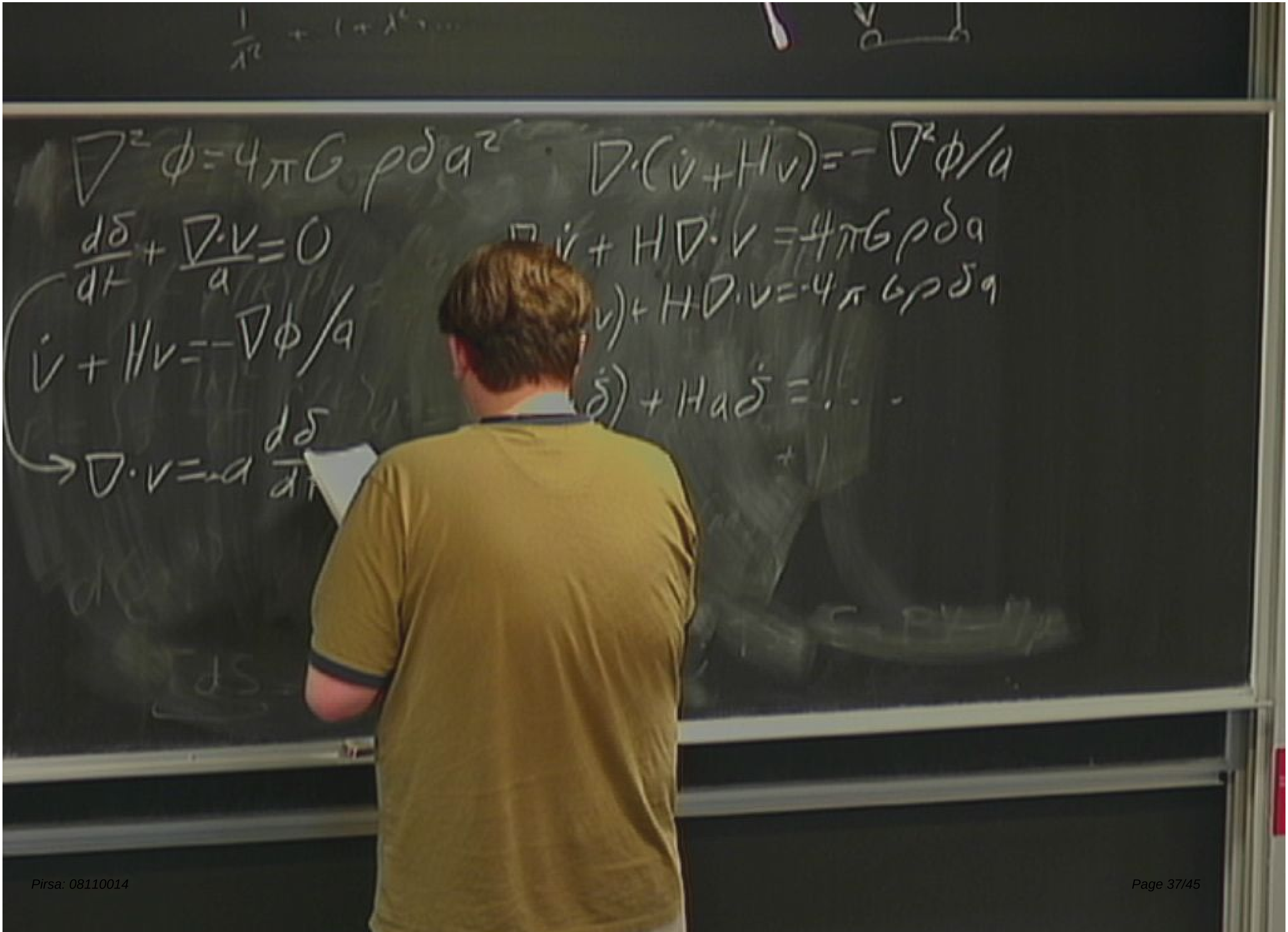
$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\ddot{\delta} = -\delta H^2 a^2 +$$

$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\delta$$


$\nabla^2 \phi = 4\pi G \rho\delta$
$\frac{d\delta}{dt} + \frac{\nabla\cdot v}{a} = 0$
$\dot{v} + Hv = -\nabla\phi \cdot a$


$\nabla^2 \phi = 4\pi G \rho \delta a^2$
$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$
$\dot{v} + Hv = -\nabla \phi / a$


$\nabla^2 \phi = 4\pi G \rho \delta a^2$
$\nabla\cdot(\dot{v} + Hv) = -\nabla^2\phi/a$
$\frac{d\delta}{dt} + \frac{\nabla\cdot v}{a} = 0$
$\dot{v} + Hv = -\nabla\phi/a$


$\nabla^2\phi = 4\pi G\rho\delta a^2$
$\nabla\cdot(\dot{v}+Hv) = -\nabla^2\phi/a$
$\frac{d\delta}{dt} + \frac{\nabla\cdot v}{a} = 0$
$\nabla\cdot\dot{v} + H\nabla\cdot v = 4\pi G\rho\delta a$
$\dot{v} + Hv = -\nabla\phi/a$
$v) + H\nabla\cdot v = -4\pi G\rho\delta a$
$\nabla\cdot v = -a\frac{d\delta}{dt}$
$\delta) + Ha\dot{\delta} = \ldots$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2$$

$$\frac{d\delta}{dt} + \frac{\nabla\cdot v}{a} = 0$$

$$\dot{v} + Hv = -\nabla\phi/a$$

$$\nabla\cdot v = -a\frac{d\delta}{dt}$$

$$\nabla\cdot(\dot{v} + Hv) = -\nabla^2\phi/a$$

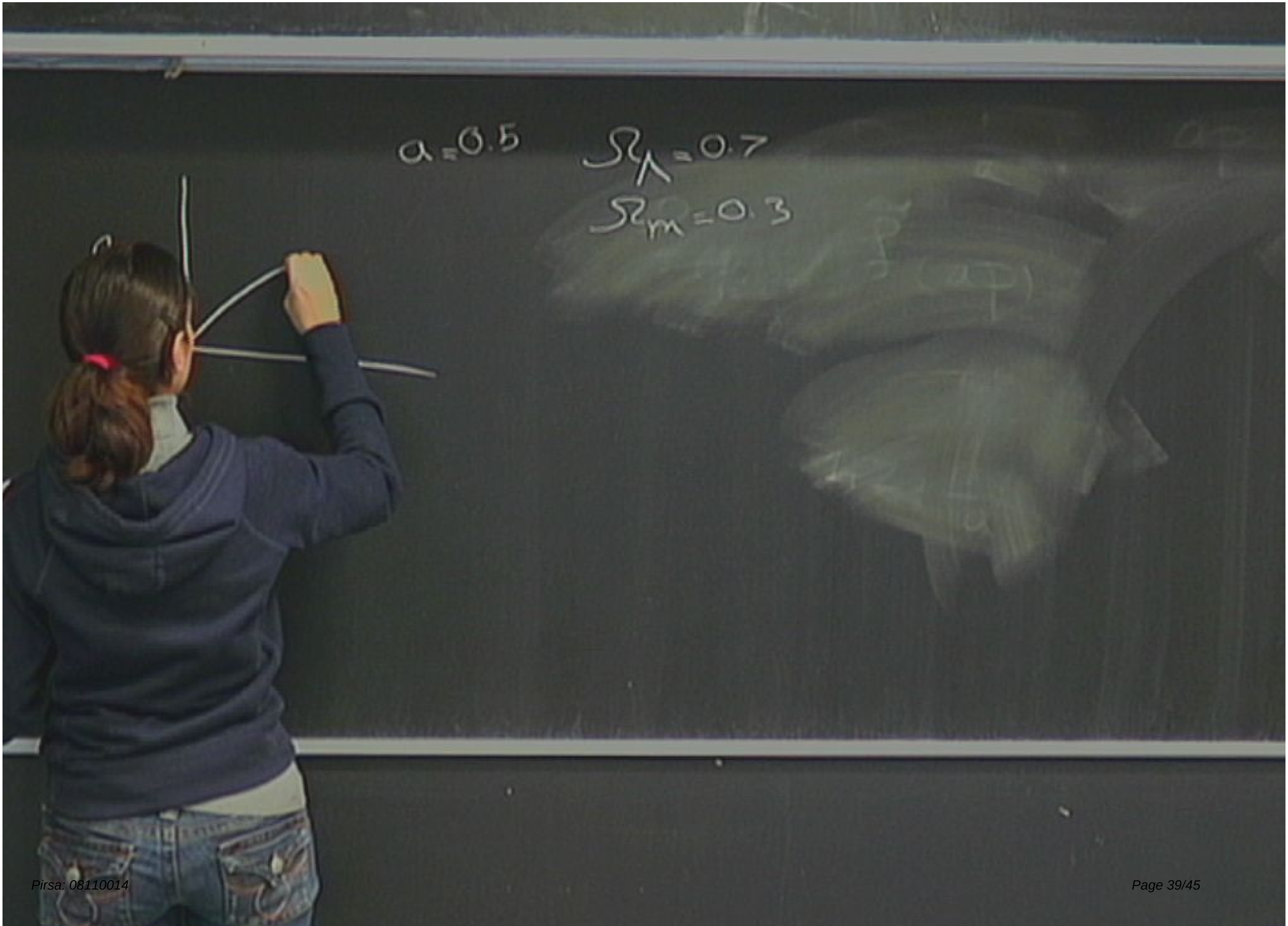
$$\nabla\cdot\dot{v} + H\nabla\cdot v = -4\pi G\rho\delta a$$

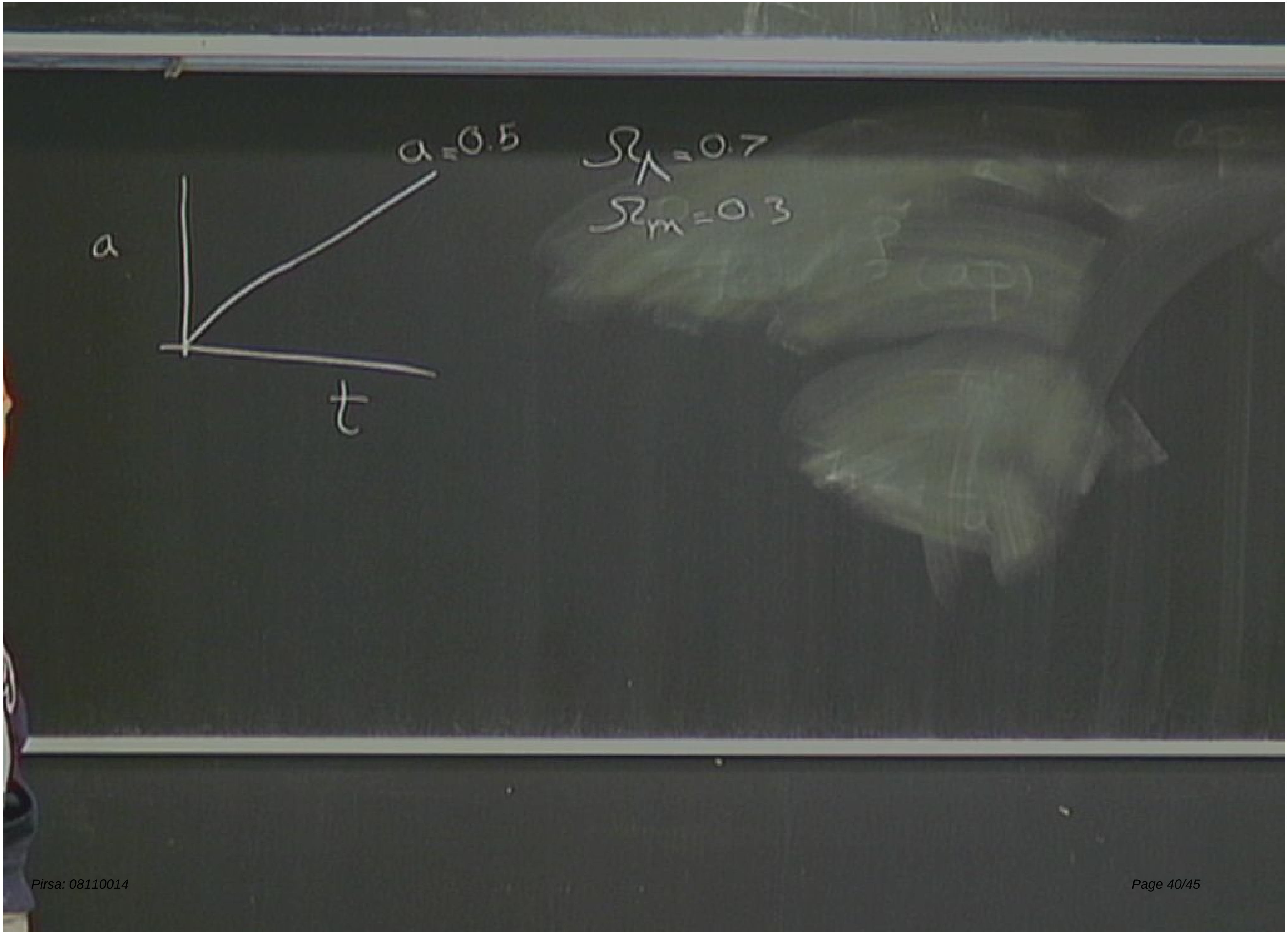
$$\frac{d}{dt}(\nabla\cdot v) + H\nabla\cdot v = -4\pi G\rho\delta a$$

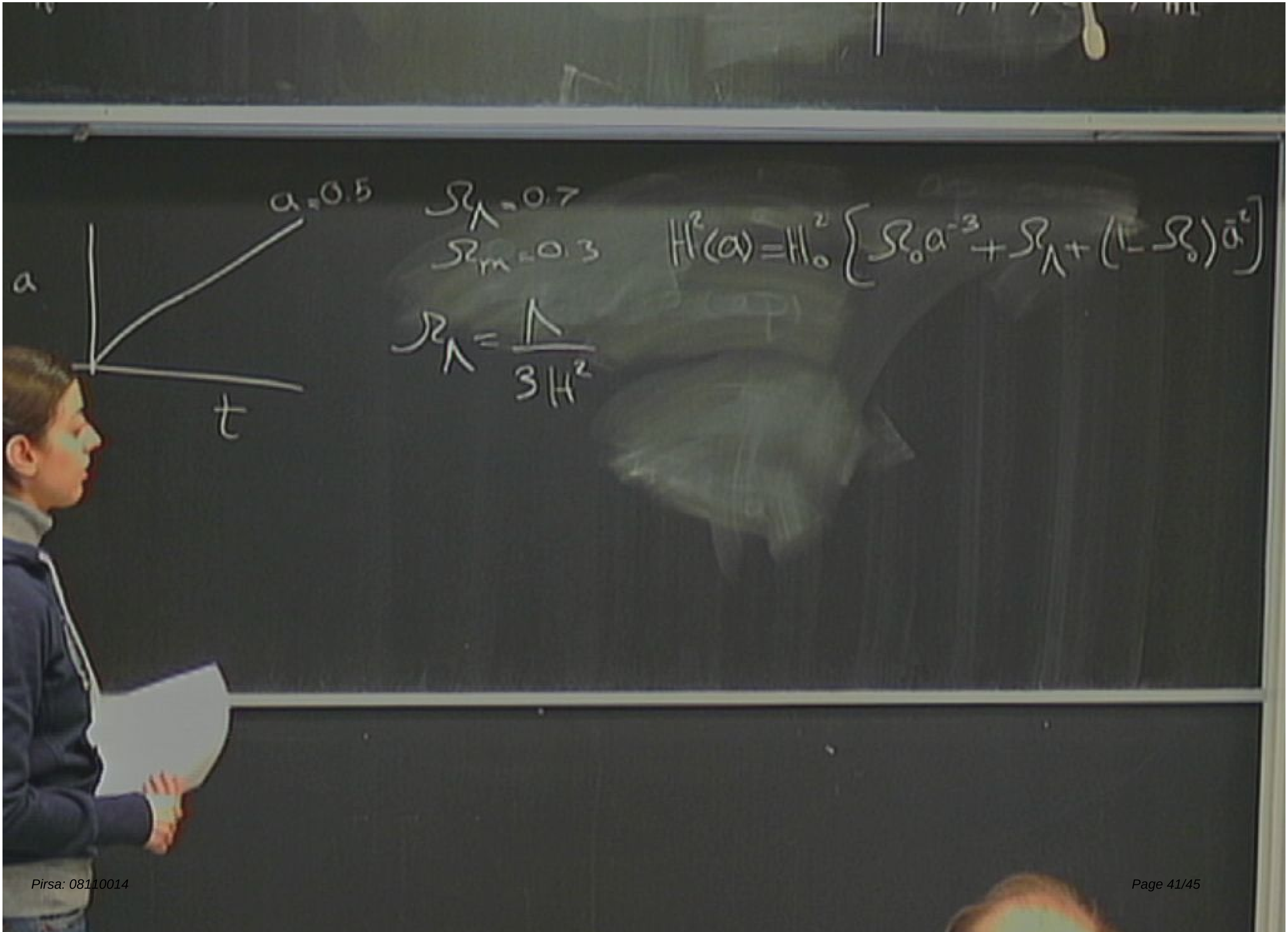
$$\frac{d}{dt}(a\dot{\delta}) + Ha\dot{\delta} = \ldots$$

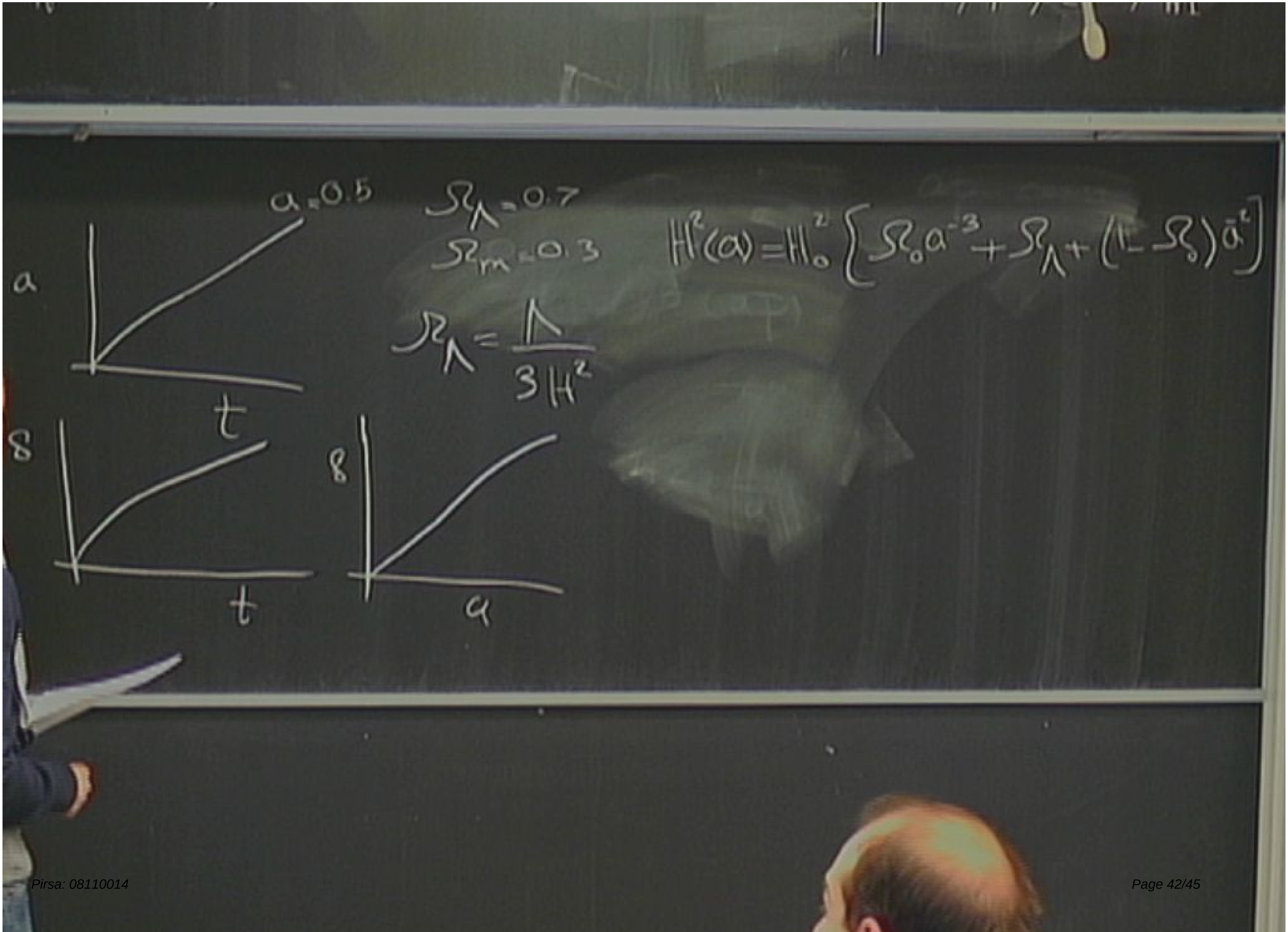
$$\frac{\dot{a}}{a}\dot{\delta} + \ddot{\delta} + H\dot{\delta} = 4\pi G\rho\delta$$

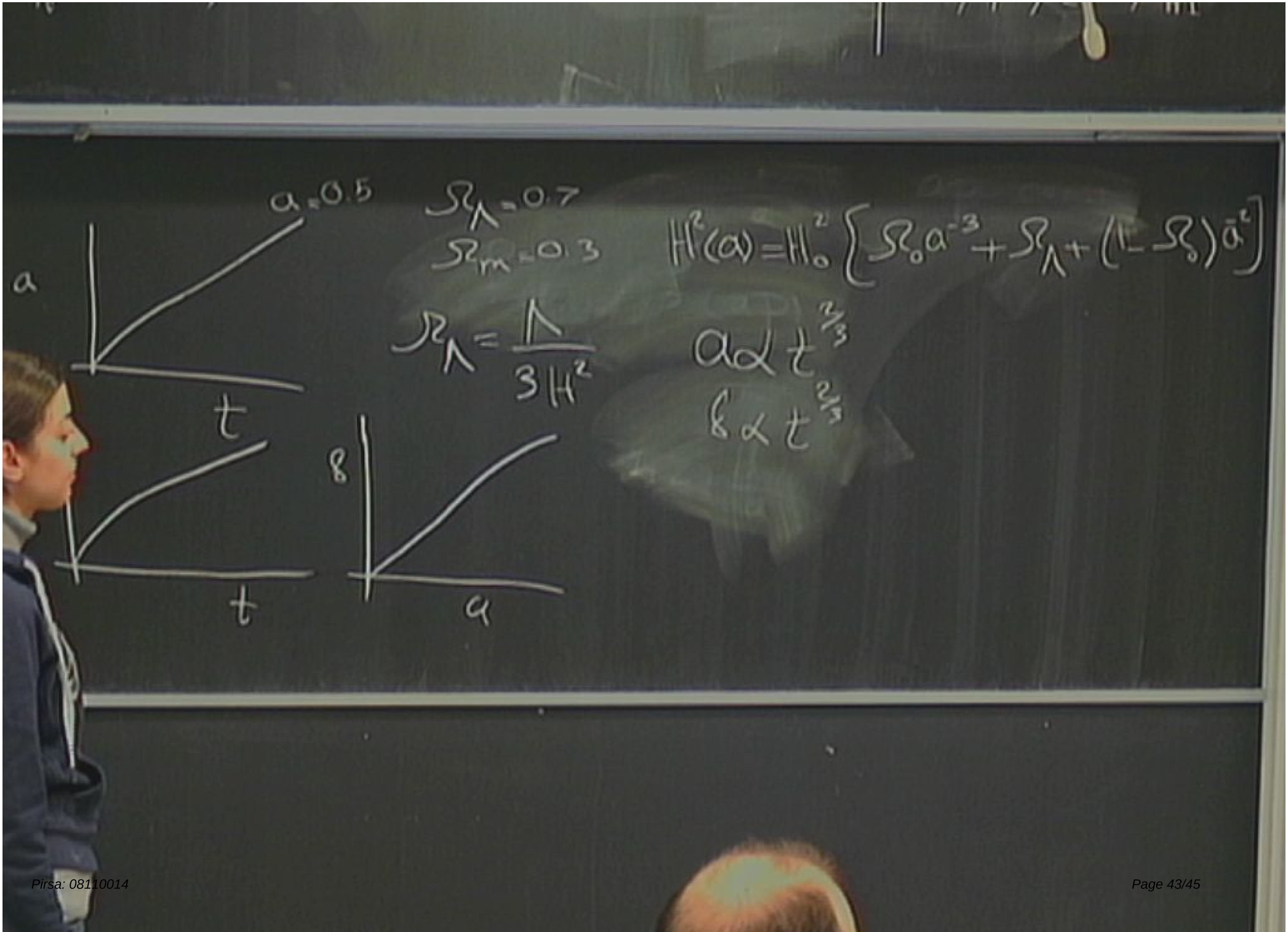
$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho\delta$$

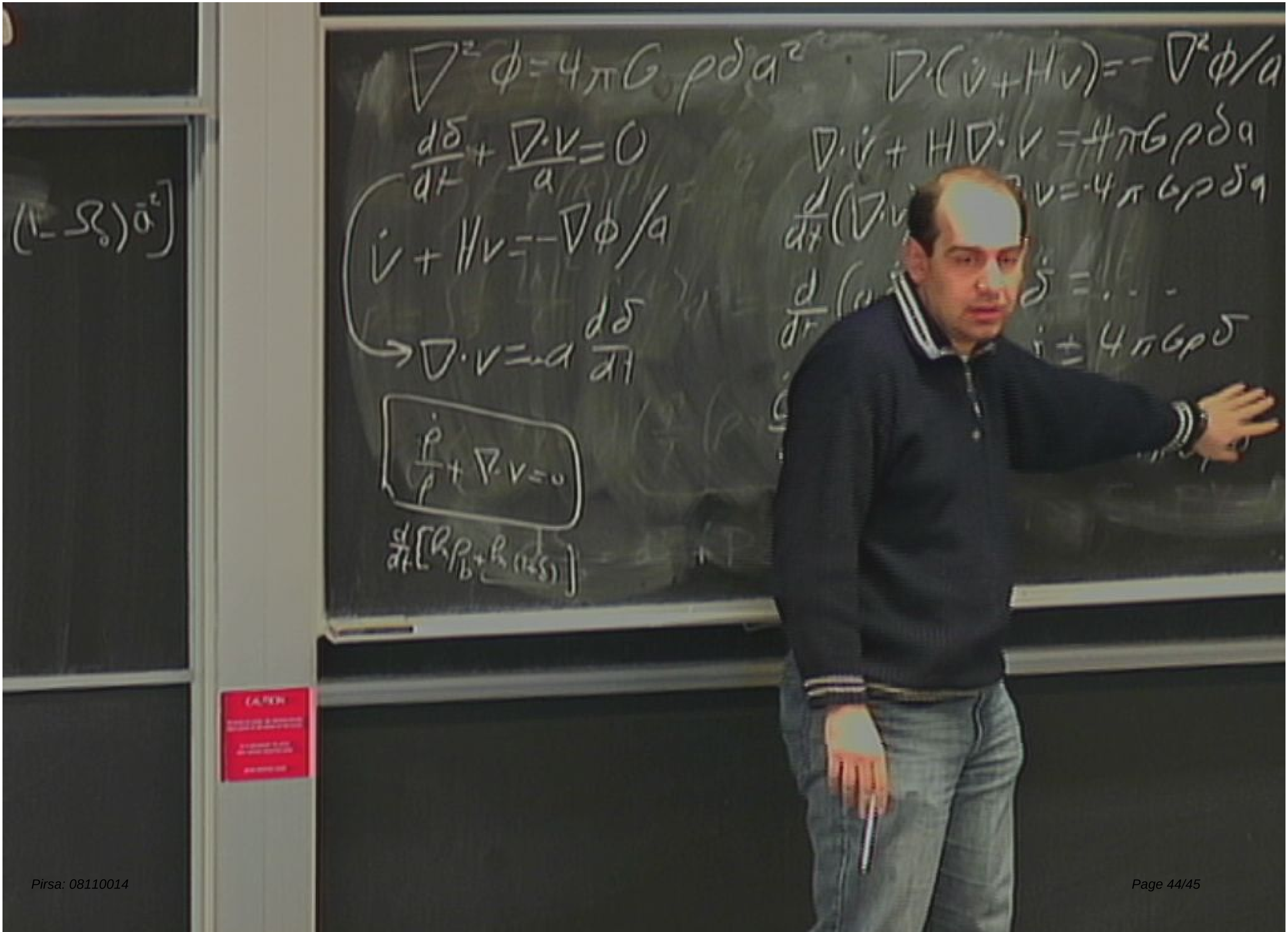

a = 0.5
$\Omega_\Lambda = 0.7$
$\Omega_m = 0.3$

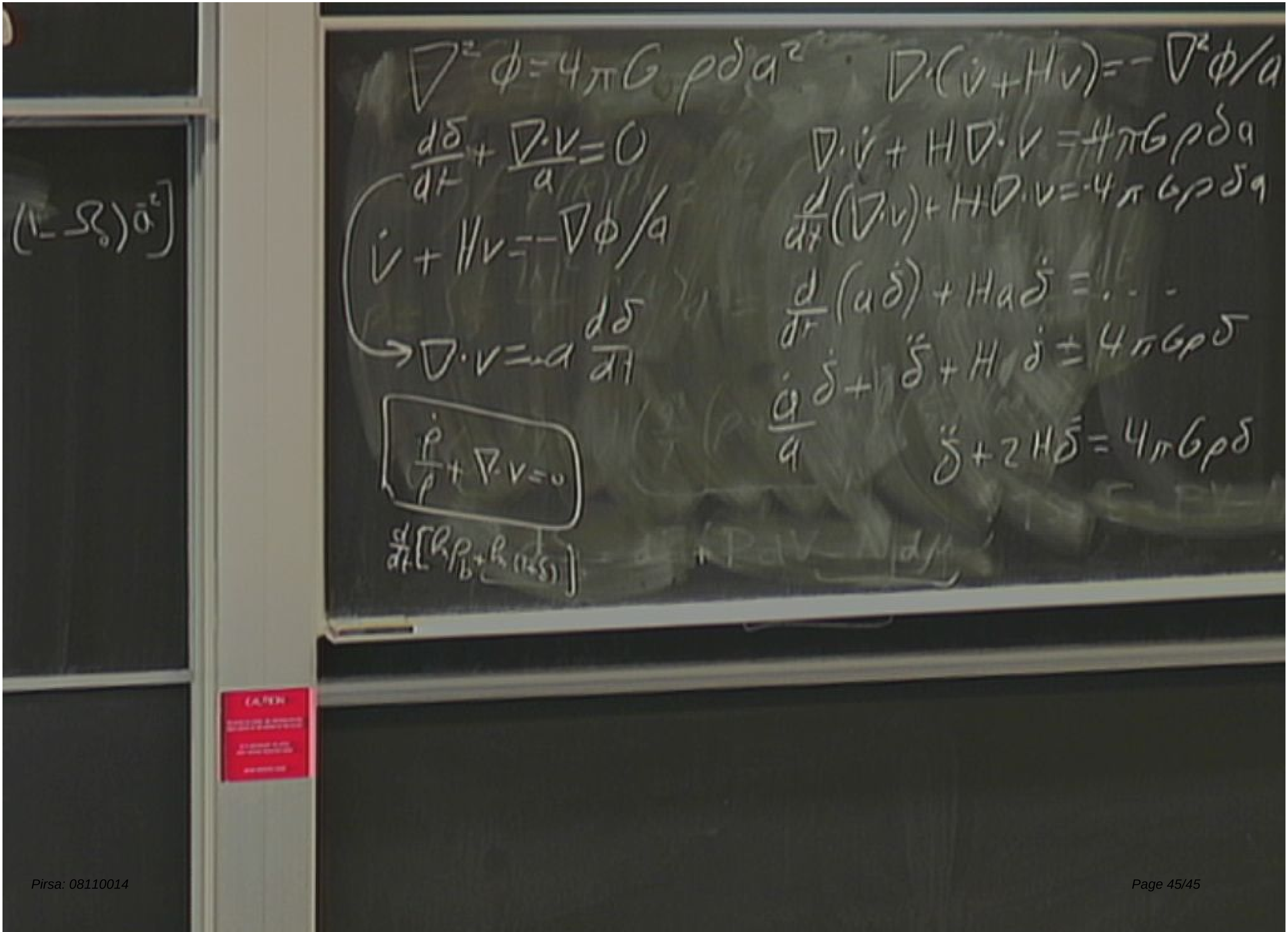

a = 0.5
Ω_Λ = 0.7
Ω_m = 0.3
a
t


a
t
$a = 0.5$
$\Omega_\Lambda = 0.7$
$\Omega_m = 0.3$
$\Omega_\Lambda = \frac{\Lambda}{3H^2}$
$H^2(a) = H_0^2 \left\{ \Omega_0 a^{-3} + \Omega_\Lambda + (1 - \Omega_0) a^{-2} \right\}$


a = 0.5
Ω_Λ = 0.7
Ω_m = 0.3
H²(a) = H₀² {Ω₀ a⁻³ + Ω_Λ + (1 − Ω₀) a⁻²}
Ω_Λ = Λ / 3H²
a
t
δ
t
δ
a


$a = 0.5$
$\Omega_\Lambda = 0.7$
$\Omega_m = 0.3$
$H^2(a) = H_0^2 \left[\Omega_0 a^{-3} + \Omega_\Lambda + (1 - \Omega_0) a^{-2} \right]$
$\Omega_\Lambda = \frac{\Lambda}{3H^2}$
$a \propto t^{2/3}$
$\delta \propto t^{2/3}$
a
t
δ
t
a


∇²φ = 4πG ρδa²
∇·(v̇ + Hv) = −∇²φ/a
dδ/dt + ∇·v/a = 0
∇·v̇ + H∇·v = 4πGρδa
v̇ + Hv = −∇φ/a
∇·v = −a dδ/dt
ρ̇/ρ + ∇·v = 0
CAUTION


$(1-\Omega_0)\,a^2]$
$\nabla^2\phi = 4\pi G\,\rho\delta a^2$
$\nabla\cdot(\dot v + Hv) = -\nabla^2\phi/a$
$\frac{d\delta}{dt} + \frac{\nabla\cdot v}{a} = 0$
$\nabla\cdot\dot v + H\nabla\cdot v = 4\pi G\rho\delta a$
$\frac{d}{dt}(\nabla\cdot v) + H\nabla\cdot v = 4\pi G\rho\delta a$
$\dot v + Hv = -\nabla\phi/a$
$\frac{d}{dt}(a\dot\delta) + Ha\dot\delta = \ldots$
$\nabla\cdot v = -a\frac{d\delta}{dt}$
$\dot\delta + \ddot\delta + H\dot\delta = 4\pi G\rho\delta$
$\frac{\dot a}{a}\dot\delta +$
$\frac{\dot\rho}{\rho} + \nabla\cdot v = 0$
$\ddot\delta + 2H\dot\delta = 4\pi G\rho\delta$
$\frac{d}{dt}[\ldots]$