

Title: Astrophysics and Cosmology through Problems - 12B

Date: Nov 20, 2008 12:30 PM

URL: <http://pirsa.org/08110014>

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

$$= \frac{0}{T} + \frac{na^3 dT}{T^2} - \frac{a(n\mu a^3)}{T} + a^3 n d\left(\frac{\mu}{T}\right)$$

$$= \frac{na^3 dT}{T^2} + \frac{pd(na^3)}{T} + \frac{na^3 d\mu}{T} + a^3 n d\left(\frac{\mu}{T}\right)$$

$$dP = \frac{1}{a^3} d((p+p)a^3)$$

$$\frac{a^3}{T} (p+p-n\mu) = \frac{d}{dT} ((p+p)a^3)$$

$$s = (p+p)/T$$

$$\left(\frac{\mu}{T}\right) d(na^3) = d\left(\frac{a^3}{T} ((p+p)a^3)\right)$$

$$T d(s a^3) = T d\left(\frac{(p+p)a^3}{T}\right)$$

$$= d((p+p)a^3) \frac{dT}{T}$$

$$= d((p+p)a^3) - a^3 dp = d(a^3) + p d(a^3)$$

$$T d(s a^3) = d$$

$$T d(s V) = d$$

$$\Rightarrow T dS = dE + p dV - \mu dN$$

$$P = \int E f(k) \beta k = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{3/2} E^2 dE}{e^{(E-\mu)/T} + 1}$$

$$P = \int \frac{1}{3} \frac{|k|^2}{E} f(k) d^3 = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{3/2} E^2 dE}{e^{(E-\mu)/T} + 1}$$

$$d(Sa^3) = d\left(\frac{g^3}{T} (P + P - n\mu)\right) + \left(\frac{\mu}{T}\right) d(na^3)$$

$$TdS = dE + PdV - Nd\mu ; T$$

$$\frac{g}{2\pi^2} \int_m^\infty \frac{E^2 dE}{e^{E/T} + 1} = \left\{ \frac{3}{4} \left(\frac{5(3)}{\pi^2} \right) g_{\Gamma} T \right\} F$$

$$P = \int \frac{1}{3} \frac{|k|^2}{E} f(k) d^3 = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{3/2}}{E} \frac{dE}{e^{(E-\mu)/T} + 1}$$

$$d(\Omega a^3) = d\left(\frac{g^3}{T} (P + P - n\mu)\right) + \left(\frac{g^3}{T}\right) d(na^3)$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$n = \frac{g}{(2\pi)^3} \int \frac{E^2 dE}{e^{(E-\mu)/T} + 1}$$

$$P = \int \frac{1}{3} \frac{|k|^2}{E} f(k) d^3 = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{3/2}}{(E - \mu)^{\pm 1}} dE$$

$$d(SA^3) = d\left(\frac{g^3}{T} (\rho + P - n\mu)\right) F\left(\frac{\mu}{T}\right) d(na^3)$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$\frac{E^2 dE}{e^{\frac{E}{T} \pm 1}} = \begin{cases} \left(\frac{5(3)}{\pi^2}\right) g_b T^3 & B \\ \frac{3}{4} \left(\frac{5(3)}{\pi^2}\right) g_F T^3 & F \end{cases}$$

$$d(\underbrace{sa^3}) = d\left(\frac{g^3}{T} (\rho + P - n\mu)\right) + \left(\frac{\mu}{T}\right) d(na^3)$$

$$\underbrace{T dS} = dE + P dV - \underbrace{N d\mu} ; TS = E + PV - N\mu$$

$$\frac{2\pi^2}{15} g T^3$$

$$g = g_{bc}$$

$$dE = \begin{cases} \left(\frac{5(3)}{\pi^2}\right) g_b T^3 & B \\ \frac{3}{4} \left(\frac{5(3)}{\pi^2}\right) g_F T^3 & F \end{cases}$$

$$\langle E \rangle = \frac{1}{n} = \begin{cases} 3.15 T & F \end{cases}$$

$$p_e = G$$

$$\mu_p + \mu_e = \mu_H$$

ρ

$$\rho \propto \frac{1}{a}$$

$$a\rho = \text{const}$$

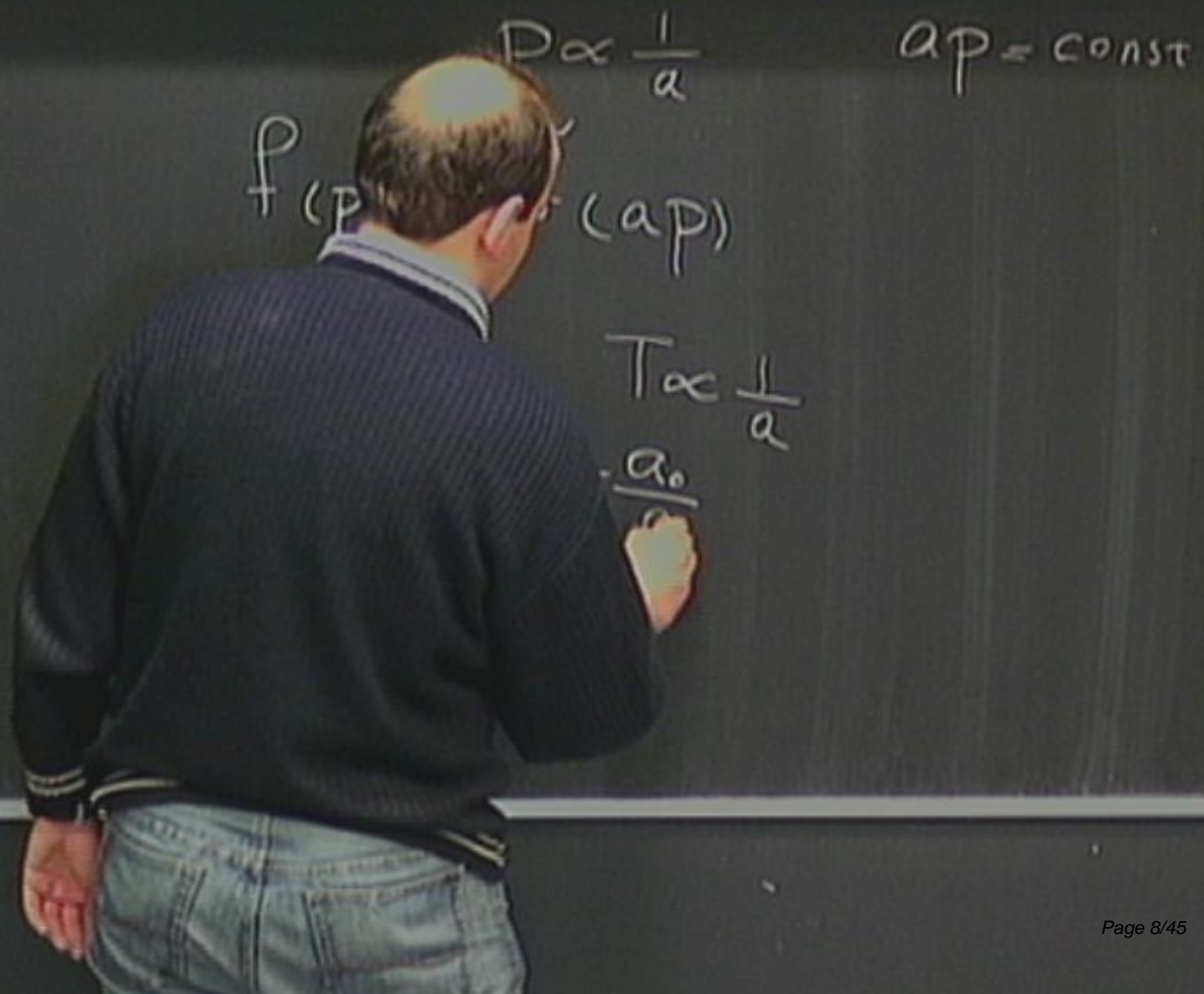
$p_e = G$ $\mu_p + \mu_e = \mu_H$

$p \propto \frac{1}{a}$ $ap = \text{const}$

$f(p) = (ap)$

$T \propto \frac{1}{a}$

$\frac{a_0}{a}$



$$p_e = G$$

$$\mu_p + \mu_e = \mu_H$$

$$p \propto \frac{1}{a}$$

$$ap = \text{const}$$

$$f(p, t) = f(ap)$$

$$T \propto \frac{1}{a}$$

$$L_T Z = \frac{a_0}{a}$$

$$\underbrace{\dots} = ac + \underbrace{Fdv - N d\mu}$$

$$\rho = \rho_b + \epsilon \rho$$

$$H^2 = \frac{8\pi G}{3}$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \quad H = \frac{\dot{a}}{a}$$

$$H^2 = \frac{8\pi G}{3} \rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_b + \delta\rho)$$

$$\left. \begin{array}{l} H^2 = \frac{8\pi G}{3} \rho_b \\ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_b + \delta\rho) \end{array} \right\} \delta\rho \frac{8\pi G}{3} = \frac{k}{a^2} \Rightarrow \delta\rho = \frac{3k}{8\pi G a^2 \rho_b}$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho \quad H = \frac{\dot{a}}{a}$$

$$H^2 = \frac{8\pi G}{3} \rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_b + \delta\rho)$$

$$\left. \begin{array}{l} H^2 = \frac{8\pi G}{3} \rho_b \\ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_b + \delta\rho) \end{array} \right\} \delta\rho \frac{8\pi G}{3} = \frac{k}{a^2} \Rightarrow \frac{\delta\rho}{\rho_b} = \frac{3k}{8\pi G a^2 \rho_b}$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$\rho = \rho_b + \delta\rho$$

$$H = \frac{\dot{a}}{a}$$

$$\frac{\Delta H}{(H^2 - H^2_0)} = \frac{8\pi G \delta\rho}{3} - \frac{k}{a^2}$$

$$H^2 = \frac{8\pi G}{3} \rho_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_b + \delta\rho)$$

$$\delta\rho \frac{8\pi G}{3} - \frac{k}{a^2} \Rightarrow \frac{\delta\rho}{\rho_b} = \frac{3k}{8\pi G a^2 \rho_b}$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$P = P_b + \delta P$$

$$H = \frac{a^3}{a}$$

$$\frac{\Delta H}{(H^2 - H^3)} = \frac{8\bar{u}G \delta P - \frac{k}{a^2}}{3} \quad \delta = \frac{\delta P}{P_b}$$

$$H^2 = \frac{8\bar{u}G}{3} P_b$$

$$2H \Delta H = \frac{8\bar{u}G \delta P - \frac{k}{a^2}}{3}$$

$$H^2 + \frac{k}{a^2} = \frac{8\bar{u}G}{3} (P_b + \delta P)$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$P = P_b + \delta P$$

$$H = \frac{a^2}{a}$$

$$\frac{\Delta H}{(H^2 - H^2)} = \frac{8\bar{u}G \delta P - \frac{k}{a^2}}{3} \quad \delta = \frac{\delta P}{P_b}$$

$$H^2 = \frac{8\bar{u}G}{3} P_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\bar{u}G}{3} (P_b + \delta P)$$

$$2H \Delta H = \frac{8\bar{u}G \delta P - \frac{k}{a^2}}{3}$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$P = P_b + \delta P$$

$$H = \frac{\dot{a}}{a}$$

$$\frac{\Delta H}{(H^2 - H^2)} = \frac{8\bar{u}G \delta P}{3} - \frac{k}{a^2} \quad \delta = \frac{\delta P}{P_b}$$

$$H^2 = \frac{8\bar{u}G}{3} P_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\bar{u}G}{3} (P_b + \delta P)$$

$$2H \Delta H = \frac{8\bar{u}G \delta P}{3} - \frac{k}{a^2}$$

$$\dot{P} + 3H P = 0 \Rightarrow \delta \dot{P} + 3H \delta P + 3H \dot{P}$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$P = P_b + \delta P$$

$$H = \frac{\dot{a}}{a}$$

$$\frac{\Delta H}{(H^2 - H^3)} = \frac{8\bar{u}G \delta P}{3} - \frac{k}{a^2} \quad \delta = \frac{\delta P}{P_b}$$

$$H^2 = \frac{8\bar{u}G}{3} P_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\bar{u}G}{3} (P_b + \delta P)$$

$$2H \Delta H = \frac{8\bar{u}G \delta P}{3} - \frac{k}{a^2}$$

$$\dot{P} + \beta H P = 0 \Rightarrow \delta \dot{P} + 3 \delta (H P) = 0$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$P = P_b + \delta P$$

$$H = \frac{\dot{a}}{a}$$

$$\frac{\Delta H}{(H^2 - H^3)} = \frac{8\pi G}{3} \delta \rho - \frac{k}{a^2} \quad \delta = \frac{\delta P}{P_b}$$

$$H^2 = \frac{8\pi G}{3} P_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (P_b + \delta P)$$

$$2H \Delta H = \frac{8\pi G}{3} \delta P - \frac{k}{a^2}$$

$$2a^2 H \Delta H = \frac{8\pi G a^2}{3} \delta P - k$$

$$\frac{2a^2 H \Delta H}{P_b} = \delta \frac{8\pi G a^2}{3} - \frac{k}{P_b}$$

$$\dot{\rho} + 3H\rho = 0 \Rightarrow \delta \dot{\rho} + 3\delta(H\rho) = 0$$

$$T dS = dE + P dV - N d\mu ; TS = E + PV - N\mu$$

$$P = P_b + \delta P$$

$$H = \frac{\dot{a}}{a}$$

$$\frac{\Delta H}{(H^2 - H^3)} = \frac{8\bar{n}G}{3} \delta P - \frac{k}{a^2} \quad \delta = \frac{\delta P}{P_b}$$

$$H^2 = \frac{8\bar{n}G}{3} P_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\bar{n}G}{3} (P_b + \delta P)$$

$$2H \Delta H = \frac{8\bar{n}G}{3} \delta P - \frac{k}{a^2}$$

$$2a^2 H \Delta H = \frac{8\bar{n}G a^2}{3} \delta P - k$$

$$2a^2 H \Delta H = \delta \frac{8\bar{n}G a^2}{3} P_b - k$$

$$4a^2 H \Delta H +$$

$$\dot{P} + \beta H P = 0 \Rightarrow \delta \dot{P} + 3\delta (H P) = 0$$

$$4a\alpha H\Delta H + 2aH\Delta H + 2a^2 H(\Delta H) = -\frac{8\pi G\delta a}{3a^2} + \frac{68\pi G}{3a} \rho_0$$

$$4a^2 H \Delta H + 2a H \Delta H + 2a^2 H (\Delta H) = -\frac{2\pi G \delta \rho_p}{3a^2} + \frac{6\pi G}{3a} \rho_0$$

$$\underbrace{dS} = dE + PdV - Nd\mu ; \quad TS = E + PV - N\mu$$

$$P = P_b + \delta P \quad H = \frac{\dot{a}}{a}$$

$$H^2 = \frac{8\pi G}{3} P_b$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (P_b + \delta P)$$

$$\dot{P} + 3HP = 0 \Rightarrow \delta \dot{P} + 3\delta(HP) = 0$$

$$\boxed{\dot{\delta} + 3\delta H = 0}$$

$$\frac{\Delta H}{(H^2 - H^2)} = \frac{8\pi G}{3} \delta P - \frac{k}{a^2} \quad \delta = \frac{\delta P}{P_b}$$

$$2H\Delta H = \frac{8\pi G}{3} \delta P - \frac{k}{a^2}$$

$$2\dot{a}^2 H \Delta H = \frac{8\pi G}{3} \dot{a}^2 \delta P - k$$

$$2\dot{a}^2 H \Delta H = \delta \frac{8\pi G \dot{a}^2}{3} \cdot k$$

$\frac{\delta P}{P_b}$

$$4a\dot{a}H\Delta H + 2aH\Delta H + 2a^2H(\Delta H) = -\frac{8\pi G\delta a}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2aH\left(-\frac{\dot{\delta}}{3}\right) + 2a^2H(\Delta H) = -\frac{8\pi G\delta a}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$



$$4a\dot{a}H\Delta H + 2aH\Delta H + 2a^2 H(\Delta H) = -\frac{8\pi G \delta \dot{a}}{3a^2} \rho_0 + \frac{\delta 8\pi G}{3a} \rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2aH\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\left(\underbrace{\Delta H}_{-\frac{\dot{\delta}}{3}}\right) = -\frac{8\pi G \delta \dot{a}}{3a^2} \rho_0 + \frac{\delta 8\pi G}{3a} \rho_0$$

$$\frac{k}{a^2} \quad \delta = \frac{sp}{P_b}$$

$$\frac{k}{a^2}$$

$$k$$

$$k$$

$$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta H) = -8$$

$$+ a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\left(\frac{\delta}{3}\right) = -$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2}$$

$$\frac{k}{a^2} \quad \delta = \frac{sp}{P_b}$$

$$\frac{k}{a^2}$$

k

$$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta H) = -8$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\left(\frac{-\dot{\delta}}{3}\right) = -8$$

$$H = \frac{2}{3t}$$

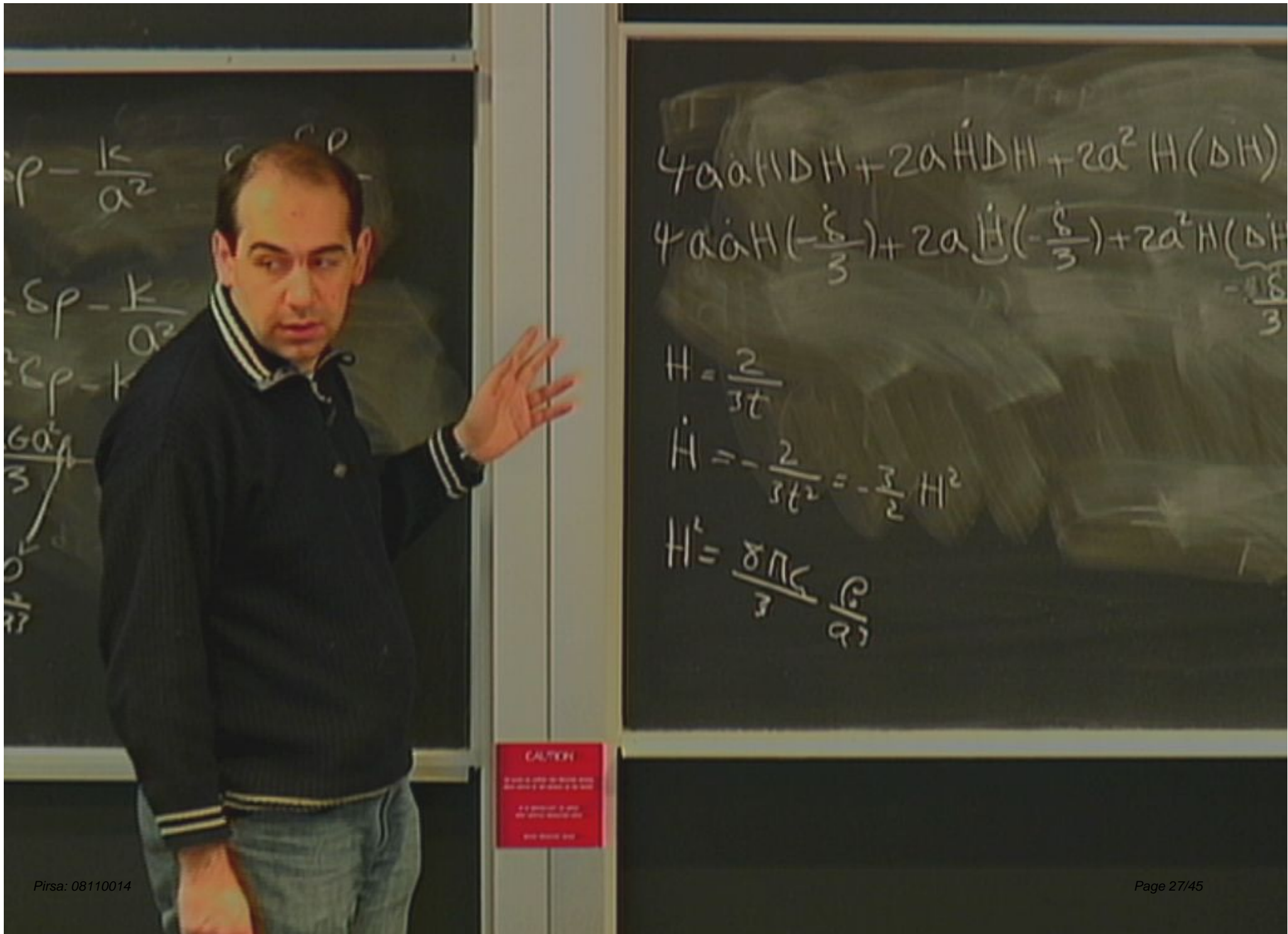
$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2} H^2$$

CAUTION

Do not use the word "and" in the title of a paper.

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$$\rho - \frac{k}{a^2}$$

$$\rho - \frac{k}{a^2}$$

$$\frac{6a^2}{3}$$

$$\frac{2}{3}$$

$$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta H)$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\left(\Delta H\right)$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2} H^2$$

$$H^2 = \frac{8\pi G}{3} \frac{\rho}{a^2}$$

CAUTION
DO NOT TOUCH THE BOARD
IF YOU TOUCH THE BOARD
YOU WILL BE FINED

$$4a\dot{a}H\Delta H + 2a\dot{H}\Delta H + 2a^2 H(\Delta H) = -\frac{8\pi G \delta \dot{\rho}}{3a^2} + \frac{\delta 8\pi G}{3a} \rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2 H\left(\Delta H\right) = \frac{\delta a}{\rho_0} + \frac{\delta 8\pi G}{3a} \rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2} H^2$$

$$H' = \frac{8\pi G}{3} \frac{\rho}{a^3}$$

$$-\frac{4}{3} a^2 H^2 \dot{\delta} - \frac{2}{3}$$

$$4a\dot{a}H\Delta H + 2a^2\dot{H}\Delta H + 2a^2H(\Delta H) = -\frac{8\pi G\delta a}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2H\left(\frac{\delta}{3}\right) = -\frac{8\pi G\delta a}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\dot{\delta} =$$

$$4a\dot{a}H\Delta H + 2a^2\dot{H}\Delta H + 2a^2H(\Delta H) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho + \frac{\delta 8\pi G}{3a}\rho$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2H\left(\frac{\delta}{3}\right) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho + \frac{\delta 8\pi G}{3a}\rho$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\ddot{\delta} = -\dot{\delta}$$

$$4a\dot{a}H\Delta H + 2a^2\dot{H}\Delta H + 2a^2H(\Delta H) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\delta}{3}\right) + 2a^2H\left(\frac{\delta}{3}\right) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\delta - \frac{2}{3}a^2H\dot{\delta} = -\delta H^3 a$$

$$\dot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\delta$$

$$4a\dot{a}H\Delta H + 2a^2\dot{H}\Delta H + 2a^2H(\Delta H) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2H\left(\frac{-\dot{\delta}}{3}\right) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^2}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\dot{\delta} - \frac{2}{3}a^2H\dot{\delta} = -\delta H^3 a^2 + \dot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\dot{\delta}$$

$$4a\dot{a}H\Delta H + 2a^2\dot{H}\Delta H + 2a^2H(\Delta H) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$4a\dot{a}H\left(-\frac{\dot{\delta}}{3}\right) + 2a^2\dot{H}\left(-\frac{\dot{\delta}}{3}\right) + 2a^2H\left(\frac{\delta}{3}\right) = -\frac{8\pi G\delta\dot{a}}{3a^2}\rho_0 + \frac{\delta 8\pi G}{3a}\rho_0$$

$$H = \frac{2}{3t}$$

$$\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$$

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^2}$$

$$-\frac{4}{3}a^2H^2\dot{\delta} - \frac{2}{3}a^2\left(-\frac{3}{2}H^2\right)\delta - \frac{2}{3}a^2H\dot{\delta} = -\delta H^3 a^2 + \dot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\delta$$

$$\frac{1}{\lambda^2} + (1 + \lambda^2) \dots$$

$$\nabla^2 \phi = 4\pi G \rho$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot \mathbf{v}}{a} = 0$$

$$\dot{\mathbf{v}} + H\mathbf{v} = -\nabla\phi \cdot \mathbf{a}$$

$$\int dS \dots$$

$$\frac{1}{a^2} + (1 + \lambda^2) \dots$$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$$

$$\dot{v} + H v = -\nabla \phi / a$$

$$\int dS = \int dV - \int dV$$

$$\frac{1}{\lambda^2} + (1 + \lambda^2) \dots$$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2$$

$$\nabla \cdot (\dot{v} + H v) = -\nabla^2 \phi / a$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$$

$$\dot{v} + H v = -\nabla \phi / a$$

$$\int dS = a^2 \int P dV - \int \rho dV$$

$$\frac{1}{\lambda^2} + (1 + \lambda^2) \dots$$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2 \quad \nabla \cdot (\dot{v} + H v) = -\nabla^2 \phi / a$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$$

$$\nabla \cdot \dot{v} + H \nabla \cdot v = 4\pi G \rho \delta a$$

$$\dot{v} + H v = -\nabla \phi / a$$

$$\nabla \cdot (\dot{v}) + H \nabla \cdot v = -4\pi G \rho \delta a$$

$$\dot{\delta} + H a \delta = \dots$$

$$\nabla \cdot v = -a \frac{d\delta}{dt}$$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2 \quad \nabla \cdot (\dot{v} + H v) = -\nabla^2 \phi / a$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$$

$$\nabla \cdot \dot{v} + H \nabla \cdot v = 4\pi G \rho \delta a$$

$$\frac{d}{dt}(\nabla \cdot v) + H \nabla \cdot v = -4\pi G \rho \delta a$$

$$\dot{v} + H v = -\nabla \phi / a$$

$$\frac{d}{dt}(a \delta) + H a \delta = \dots$$

$$\dot{\delta} + \delta + H \delta = 4\pi G \rho \delta$$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \rho \delta$$

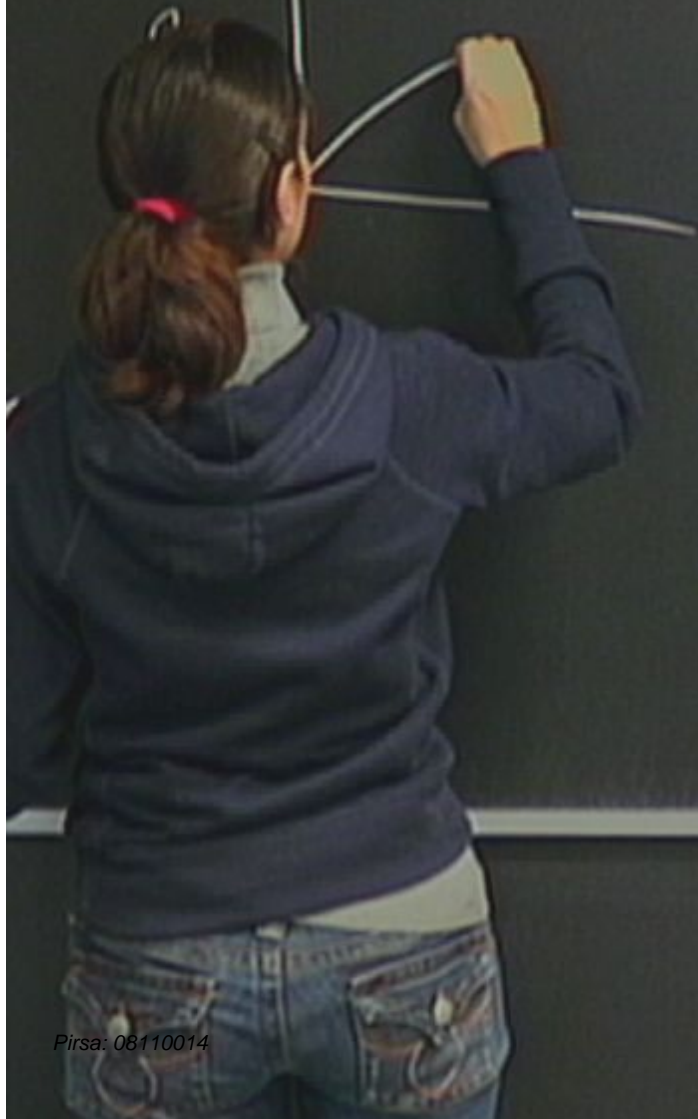
$$\rightarrow \nabla \cdot v = -a \frac{d\delta}{dt}$$

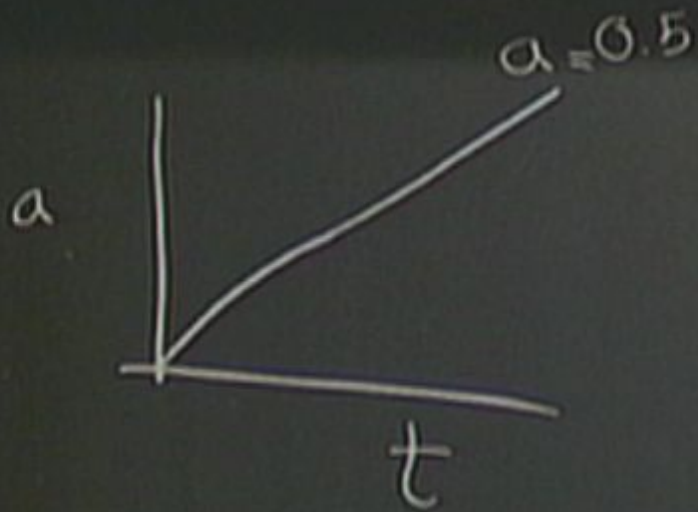
$$[dS = a^2 + P a \dot{v} - N \dot{a}]$$

$$a = 0.5$$

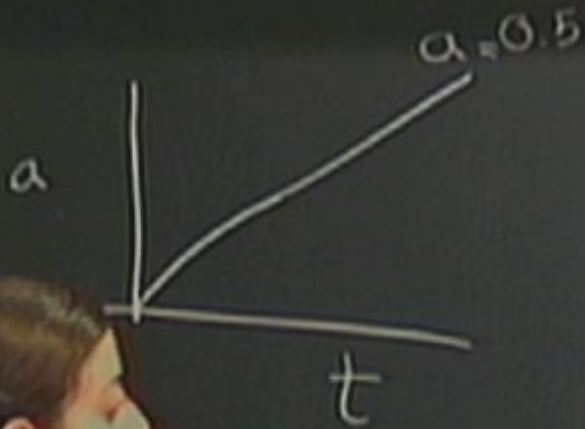
$$\sigma_A = 0.7$$

$$\sigma_m = 0.3$$





$$\Omega_{\Lambda} = 0.7$$
$$\Omega_m = 0.3$$

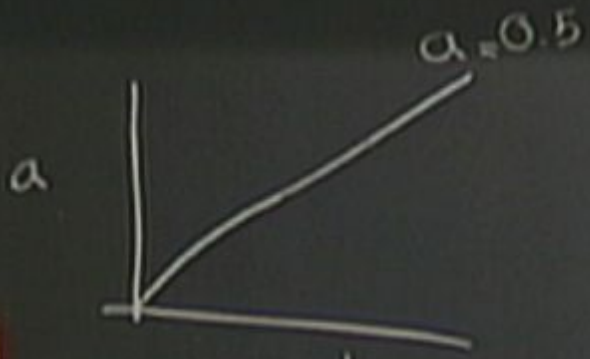


$$\Omega_\Lambda = 0.7$$

$$\Omega_m = 0.3$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$

$$H^2(a) = H_0^2 \left[\Omega_0 a^{-3} + \Omega_\Lambda + (1 - \Omega_0) a^2 \right]$$

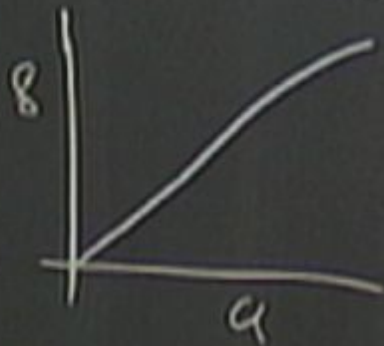
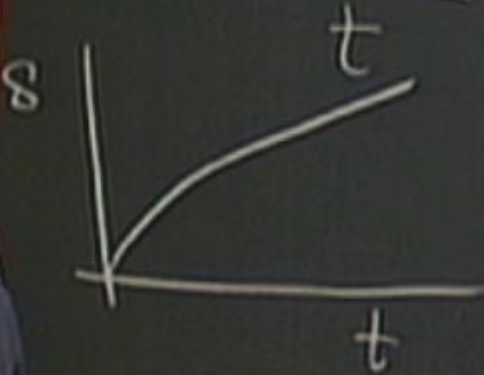


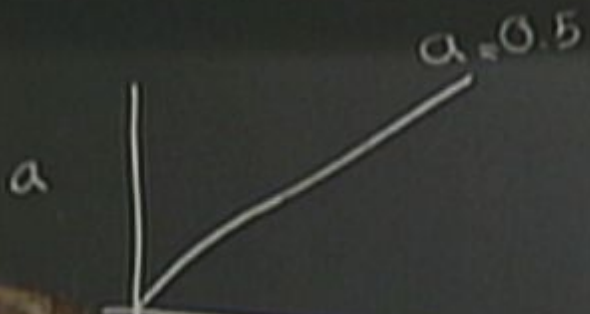
$$\Omega_{\Lambda} = 0.7$$

$$\Omega_m = 0.3$$

$$H^2(a) = H_0^2 \left[\Omega_0 a^{-3} + \Omega_{\Lambda} + (1 - \Omega_0) a^2 \right]$$

$$\Omega_{\Lambda} = \frac{\Lambda}{3H^2}$$





$$\Omega_\Lambda = 0.7$$

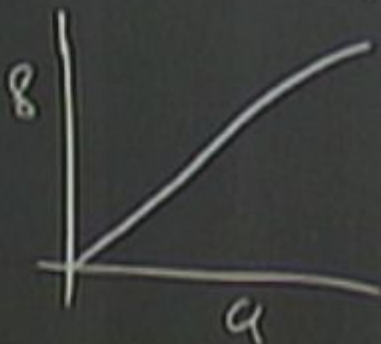
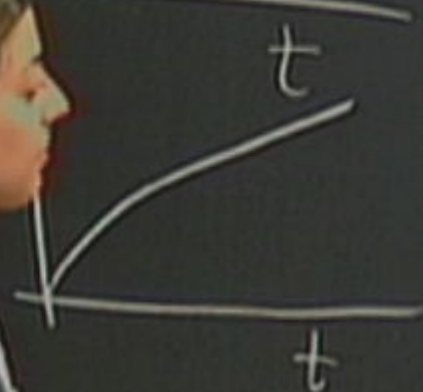
$$\Omega_m = 0.3$$

$$H^2(a) = H_0^2 \left[\Omega_0 a^{-3} + \Omega_\Lambda + (1 - \Omega_0) a^{-2} \right]$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$

$$a \propto t^{2/3}$$

$$\delta \propto t^{2/3}$$



$$(1 - \Omega_0) \ddot{a}^2$$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2 \quad \nabla \cdot (\dot{v} + H v) = -\nabla^2 \phi / a$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$$

$$\dot{v} + H v = -\nabla \phi / a$$

$$\nabla \cdot \dot{v} + H \nabla \cdot v = 4\pi G \rho \delta a$$

$$\frac{d}{dt} (\nabla \cdot v) + H \nabla \cdot v = 4\pi G \rho \delta a$$

$$\rightarrow \nabla \cdot v = -a \frac{d\delta}{dt}$$

$$\frac{d}{dt} \left[\frac{\dot{p}}{\rho} + \nabla \cdot v = 0 \right]$$

$$\frac{d}{dt} \left[\frac{h_p}{b} + h_{(125)} \right]$$



CAUTION
 Please do not touch the board
 as it is very hot
 and may cause injury

$$(1 - \Omega_0) \ddot{a}^2$$

$$\nabla^2 \phi = 4\pi G \rho \delta a^2 \quad \nabla \cdot (\dot{v} + H v) = -\nabla^2 \phi / a$$

$$\frac{d\delta}{dt} + \frac{\nabla \cdot v}{a} = 0$$

$$\dot{v} + H v = -\nabla \phi / a$$

$$\rightarrow \nabla \cdot v = -a \frac{d\delta}{dt}$$

$$\frac{\dot{\rho}}{\rho} + \nabla \cdot v = 0$$

$$\frac{d}{dt} [\rho_b + \rho(\delta)] = \dots$$

$$\nabla \cdot \dot{v} + H \nabla \cdot v = 4\pi G \rho \delta a$$

$$\frac{d}{dt} (\nabla v) + H \nabla \cdot v = 4\pi G \rho \delta a$$

$$\frac{d}{dt} (a \delta) + H a \delta = \dots$$

$$\frac{\dot{\delta}}{a} + \delta + H \delta = 4\pi G \rho \delta$$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \rho \delta$$

CAUTION
 DO NOT TOUCH THE BOARD
 OR THE EQUIPMENT
 IN THE CLASSROOM