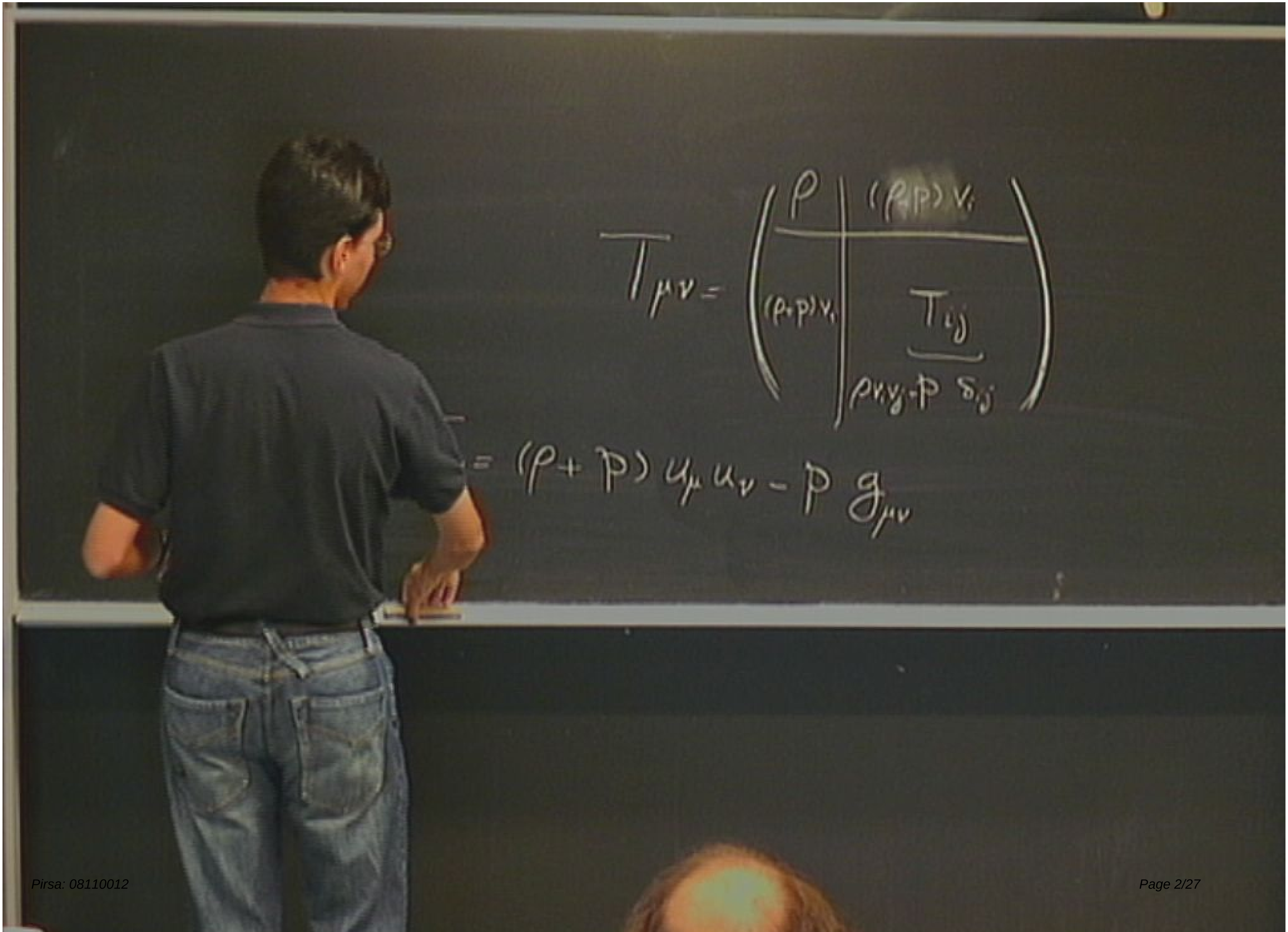
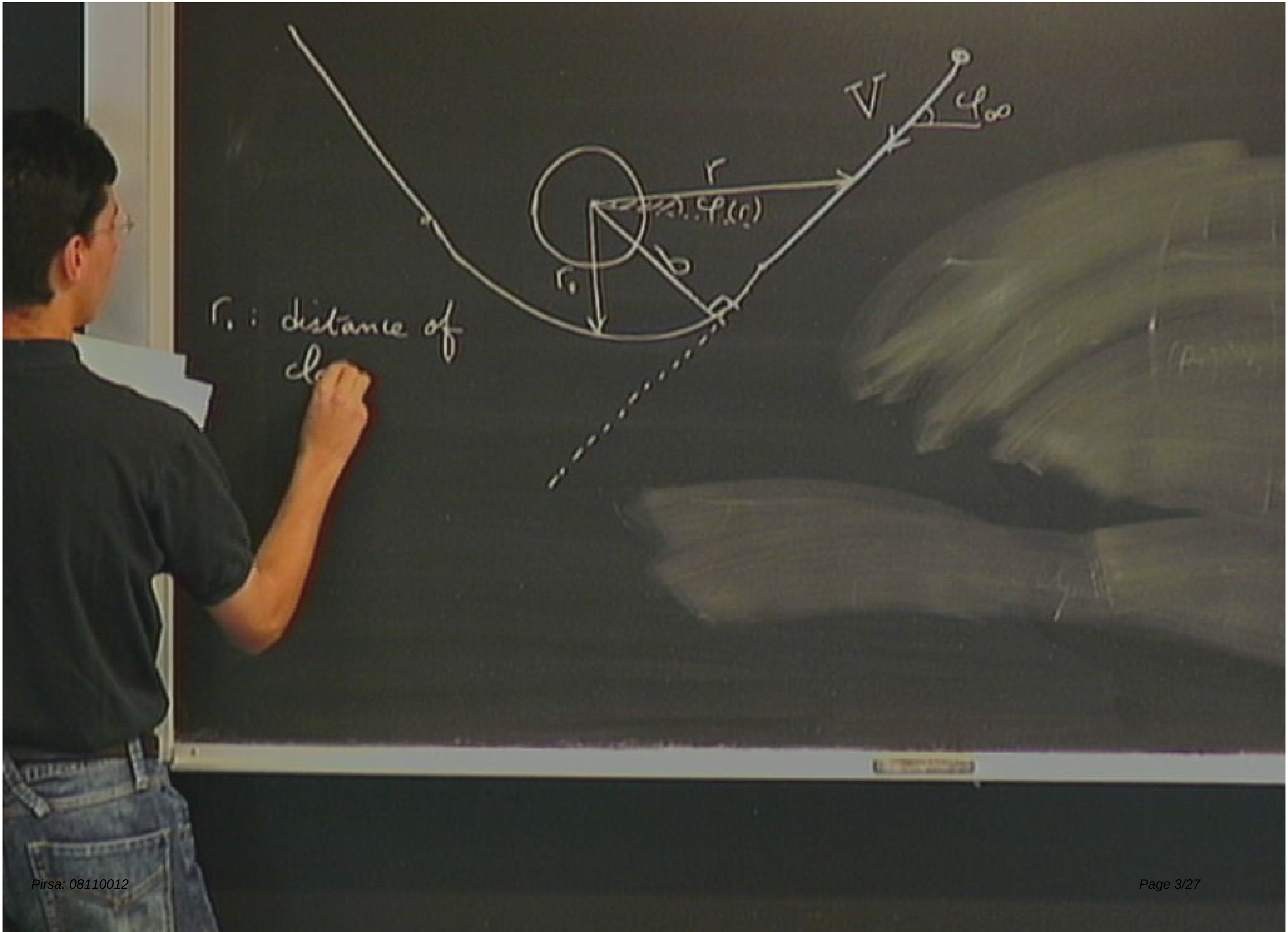
Title: Astrophysics and Cosmology through Problems - 10B

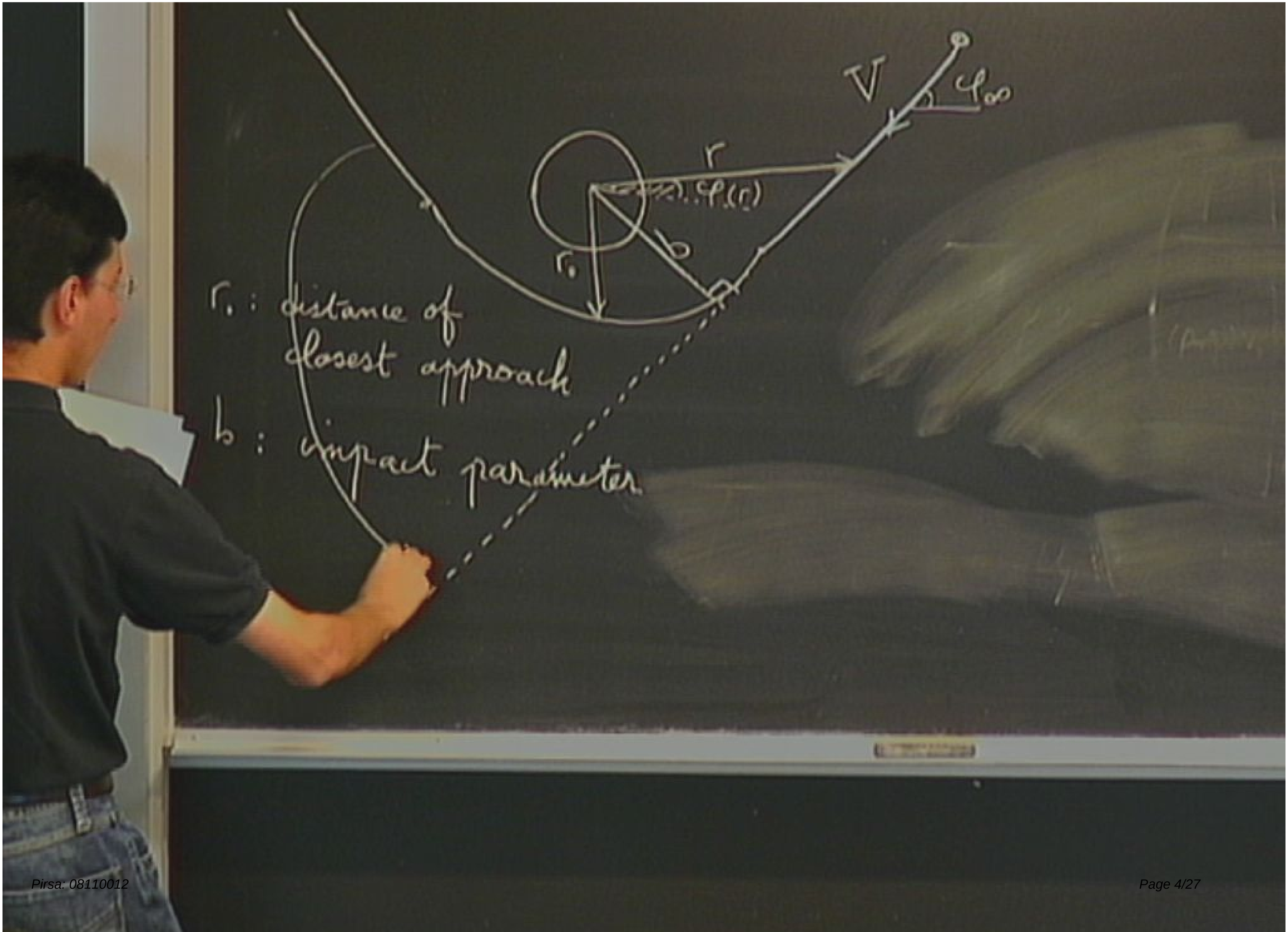
Date: Nov 06, 2008 12:30 PM

URL: http://pirsa.org/08110012

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.


$T_{\mu\nu} = \left(\begin{array}{c|c} \rho & (\rho+p)v_i \\ \hline (\rho+p)v_i & T_{ij} \end{array}\right)$
$T_{ij} = \rho v_i v_j - p\,\delta_{ij}$
$= (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}$


V
φ_∞
r
$\varphi_0(r)$
b
r_0
r_0 : distance of


V
φ_∞
r
$\varphi(r)$
b
r_0
r_0 : distance of closest approach
b : impact parameter

$$\frac{d\varphi}{d\lambda} = \dots \qquad \frac{dr}{d\lambda} = \dots$$

$$\frac{d\varphi}{dr} = \frac{d\varphi/d\lambda}{dr/d\lambda} = \alpha(r) \qquad \varphi = \int \alpha(r)\, dr$$

$$\Rightarrow \varphi(r) = \int \frac{A^{1/2}(r)\, dr}{r^2\left(\frac{1}{L^2 B(r)} - \frac{E}{L^2} - \frac{1}{r^2}\right)^{1/2}}$$

$$\left.\frac{dr}{dt}\right|_{\infty} = -V$$

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$$\left.\frac{A}{B}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{r^2} - \frac{1}{B} = -E\,\right|_{\infty} \longrightarrow E = 1 - V^2$$

$$\left.\frac{dr}{dt}\right|_{\infty} = -V \qquad \frac{dr/d\lambda}{d\varphi/d\lambda}$$

$$\left.\frac{A}{B}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{r^2} - \frac{1}{B} = -E \;\right|_{\infty} \longrightarrow \boxed{E = 1 - V^2}$$

$$\frac{A}{r^4}\left(\frac{dr}{d\varphi}\right)^2 + \frac{1}{r^2} - \frac{1}{L^2 B} = \frac{-E}{L^2}$$

$$\left.\frac{dr}{dt}\right|_{\infty} = -V \qquad \frac{dr/d\lambda}{d\varphi/d\lambda}$$

$$\left.\frac{A}{B}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{r^2} - \frac{1}{B} = -E\ \right|_{\infty} \longrightarrow \boxed{E = 1 - V^2}$$

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$$\left.\frac{dr}{dt}\right|_\infty = -V \qquad \left.\frac{dr/d\lambda}{d\varphi/d\lambda}\right|_\infty \qquad r^2\frac{d\varphi}{dt} = LB(r)$$

$$\frac{A}{B}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{r^2} - \frac{1}{B} = -E \longrightarrow \boxed{E = 1 - V^2}$$

$$\frac{A}{r^4}\left(\frac{dr}{d\varphi}\right)^2 + \frac{1}{r^2} - \frac{1}{L^2 B} = -\frac{E}{L^2}$$

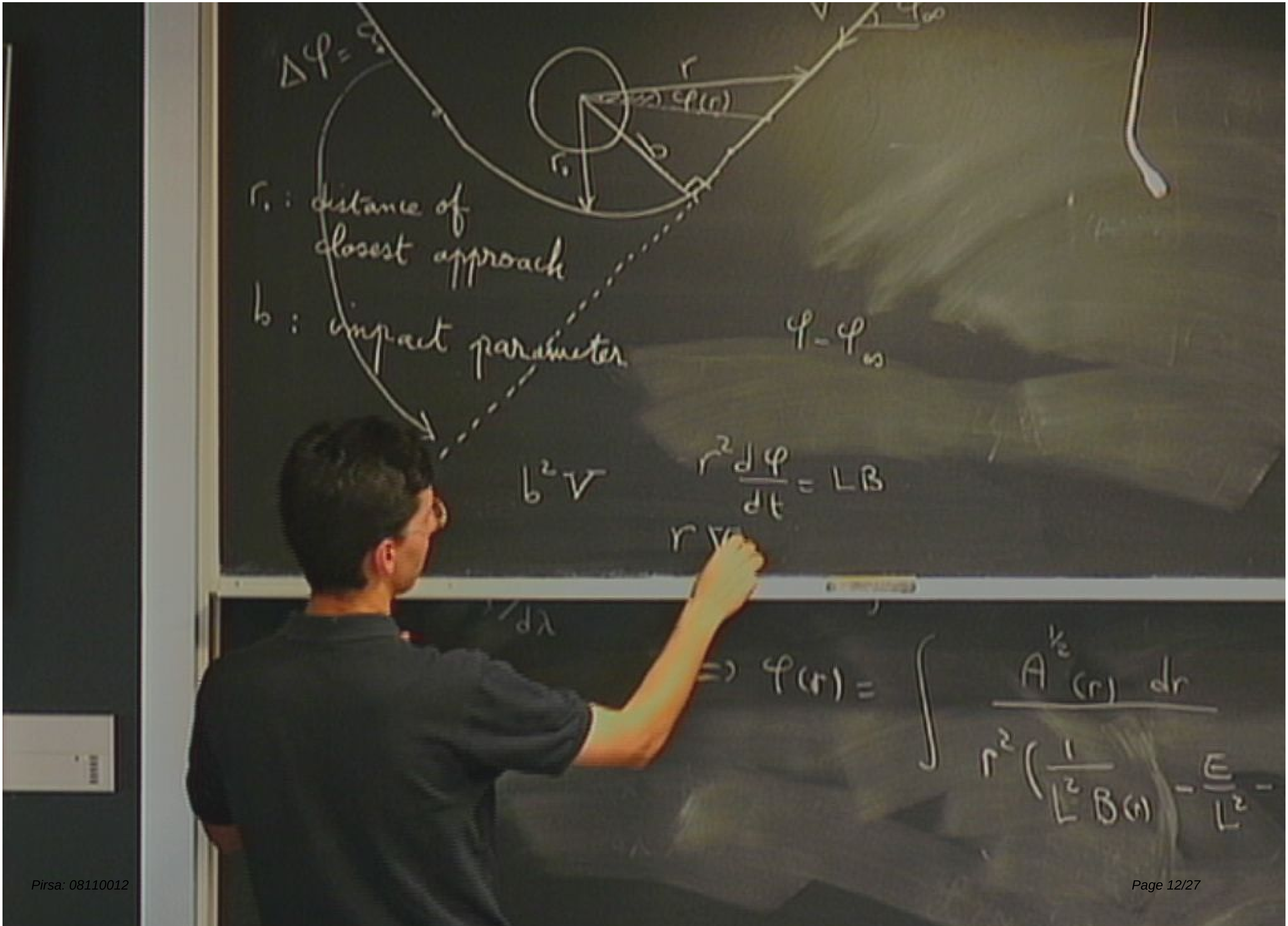
$$\left.\frac{dr}{dt}\right|_{\infty} = -V$$

$$\frac{dr/d\lambda}{d\varphi/d\lambda}$$

$$r^2 \frac{d\varphi}{dt} = LB(r)$$

$$\left.\frac{A}{B}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{r^2} - \frac{1}{B} = -E \;\right|_{\infty} \longrightarrow \boxed{E = 1 - V^2}$$

$$b \sim r\varphi - r\varphi_{\infty}$$


r
φ(r)
b
r₀
r₀ : distance of closest approach
b : impact parameter
φ - φ∞
b²V
r² dφ/dt = LB
r V
dλ
⇒ φ(r) = ∫ A^½(r) dr / r²(1/(L² B(r)) - E/L² -

closest approach

b : impact parameter

$\varphi - \varphi_\infty$

$b^2 V$

$r^2 \frac{d\varphi}{dt} = LB$

$r V = L$

$\frac{d\varphi}{d\lambda} = \dots \qquad \frac{dr}{d\lambda} = \dots$

$$\frac{d\varphi}{dr} = \frac{d\varphi/d\lambda}{dr/d\lambda} = \alpha(r) \qquad \varphi = \int \alpha(r)\, dr$$

$$\Rightarrow \varphi(r) = \int \frac{A^{1/2}(r)\, dr}{r^2\left(\frac{1}{L^2 B(r)} - \frac{E}{L^2} - \right.}$$

$$\frac{d\varphi}{d\lambda} \Big/ \frac{dt}{d\lambda} =$$

$$\varphi = \int \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2 B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$$

$$\varphi = \int_{r_0}^{\infty} \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2 B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$$

$$\Delta\varphi = 2\int_{r_0}^{\infty} \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2\,B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$$

$$\pi - \Delta\varphi = ?$$

$$ds^2 = c^2 dt^2 - dx^2$$

$$\Delta\varphi = 2\int_{r_0}^{\infty} \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2 B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$$

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$1/r = u$

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$ds^2 = c^2dt^2 - dx^2$

$b\ V$

$\frac{\quad}{dt} = LB$

$r V = L$

$dr \qquad dr/d\lambda$

$$\Rightarrow \varphi(r) = \int \frac{A^{1/2}(r)\, dr}{r^2 \left(\frac{1}{L^2 B(r)} - \frac{E}{L^2} - \frac{1}{r^2} \right)^{1/2}}$$

$x = \sin\theta$

$$\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}} = \theta = \sin^{-1} x$$

$$\frac{}{dt} = LB$$

$$r V = L$$

dr $\quad d/d\lambda$

$$\Rightarrow \varphi(r) = \int \frac{A^{1/2}(r)\, dr}{r^2 \left(\frac{1}{L^2 B(r)} - \frac{E}{L^2} - \frac{1}{r^2} \right)^{1/2}}$$

$x = \sin\theta$

$x \rightarrow \frac{x}{\sqrt{A}}$

$$\int \frac{dx}{\sqrt{A - x^2}} = \int \frac{\cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}} = \theta = \sin^{-1} x$$

$b\,V \qquad \frac{\quad}{dt} = LB$

$rV = L$

$dr \qquad df/d\lambda$

$$\Rightarrow \varphi(r) = \int \frac{A^{1/2}(r)\, dr}{r^2\left(\frac{1}{L^2 B(r)} - \frac{E}{L^2} - \frac{1}{r^2}\right)^{1/2}}$$

$x = \sin\theta$

$x \to \frac{x}{\sqrt{A}}$

$$\int \frac{dx}{\sqrt{1 - x^2}} = \int \frac{\cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}} = \theta = \sin^{-1} x$$

$$\frac{1}{r^2}\Big)^2$$

$$1/r = u$$

$$A(r) = 1 + 2\gamma \frac{MG}{r} + \dots$$

$$B(r) = 1 - 2\frac{MG}{r} + \dots$$

$$ds^2 = c^2 dt^2 - dx^2$$

$$\frac{1}{r^2}\Big)^2$$

$$1/r = u$$

$$A(r) = 1 + 2\gamma \frac{MG}{r} + \dots$$

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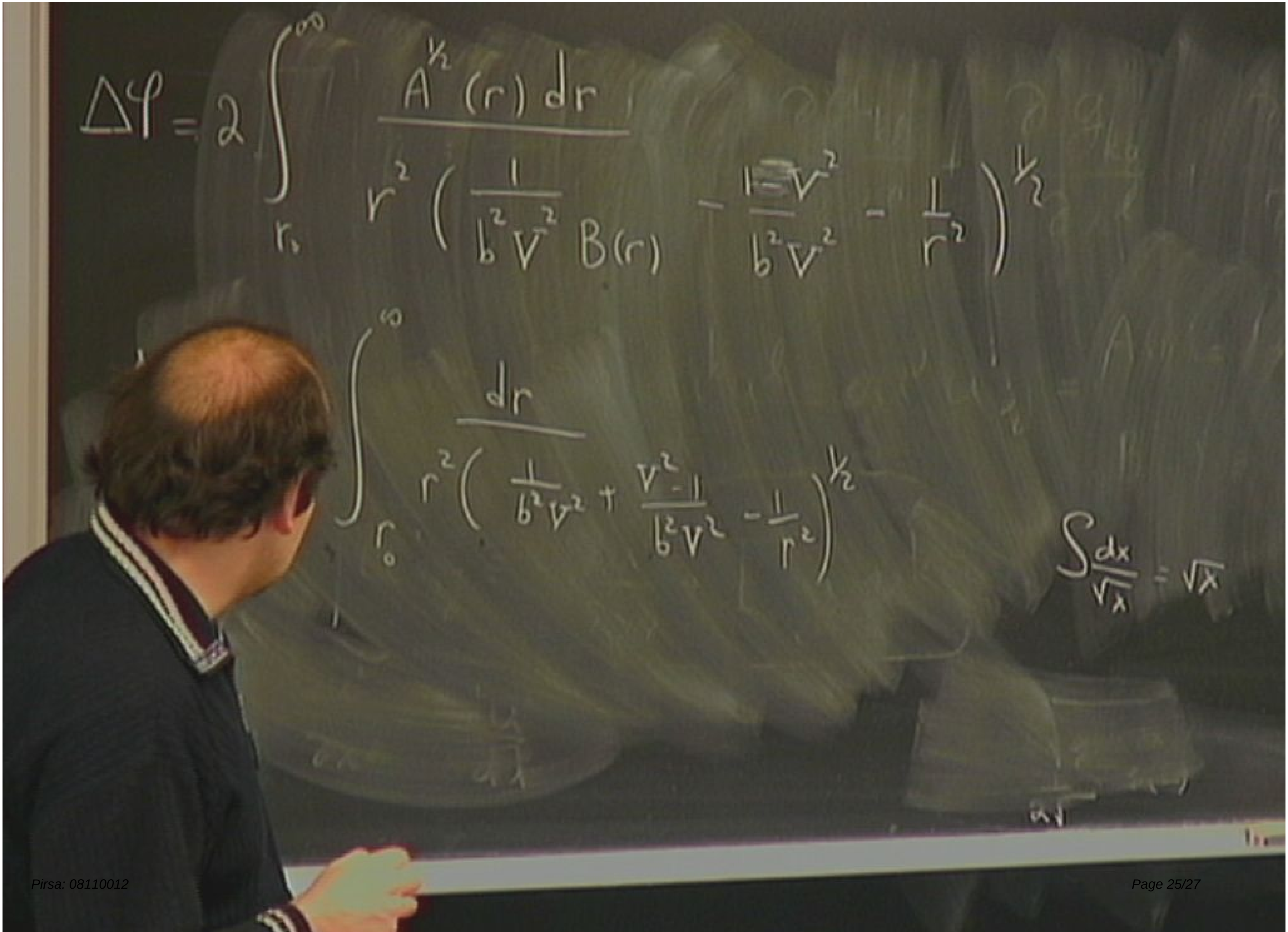
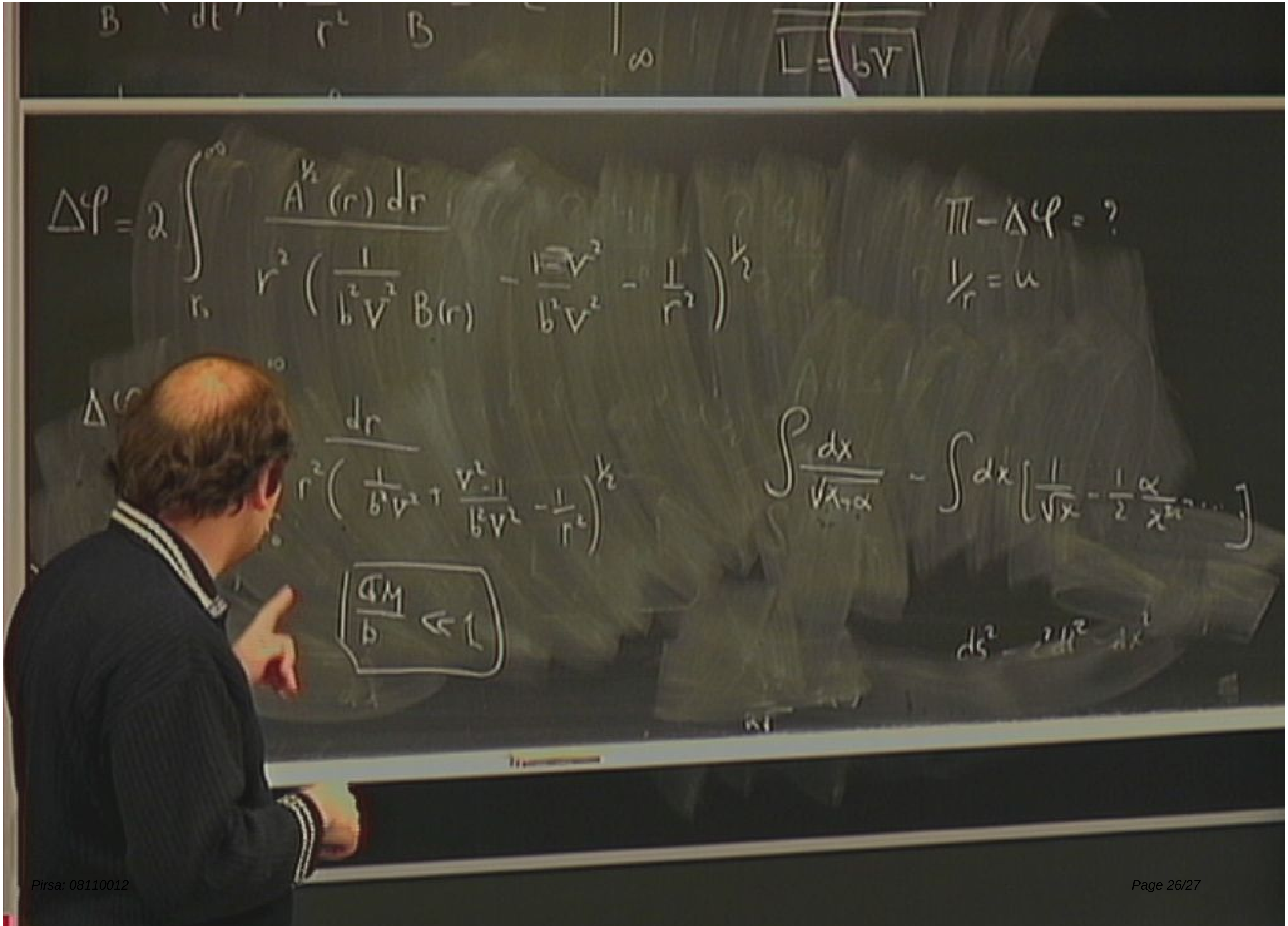
$$ds^2 - c^2 dt^2 - dx^2$$


$\Delta\varphi = 2\int_{r_0}^{\infty} \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2 B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$
$\int_{r_0}^{\infty} \frac{dr}{r^2\left(\frac{1}{b^2V^2} + \frac{V^2-1}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$
$\int \frac{dx}{\sqrt{x}} = \sqrt{x}$



$L = bV$
$\Delta\varphi = 2\int_{r_0}^{\infty} \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$
$\pi - \Delta\varphi = ?$
$1/r = u$
$\int \frac{dx}{\sqrt{x+\alpha}} \sim \int dx\left[\frac{1}{\sqrt{x}} - \frac{1}{2}\frac{\alpha}{x^{3/2}} \dots\right]$
$\boxed{\frac{GM}{b} \ll 1}$

$B \quad dt \quad r^2 \quad B \qquad \infty \qquad \boxed{L = bV}$

$$\Delta\varphi = 2\int_{r_0}^{\infty} \frac{A^{1/2}(r)\,dr}{r^2\left(\frac{1}{b^2V^2 B(r)} - \frac{1-V^2}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$$

$\pi - \Delta\varphi = ?$

$1/r = u$

$$\Delta\varphi = 2\int_{r_0}^{\infty} \frac{dr}{r^2\left(\frac{1}{b^2V^2} + \frac{V^2-1}{b^2V^2} - \frac{1}{r^2}\right)^{1/2}}$$

$$\int \frac{dx}{\sqrt{x+\alpha}} \sim \int dx \left[\frac{1}{\sqrt{x}} - \frac{1}{2}\frac{\alpha}{x^{3/2}} \cdots\right]$$

$$\boxed{\frac{GM}{b} \ll 1}$$

$ds^2 \ldots dt^2 \ldots dx^2$