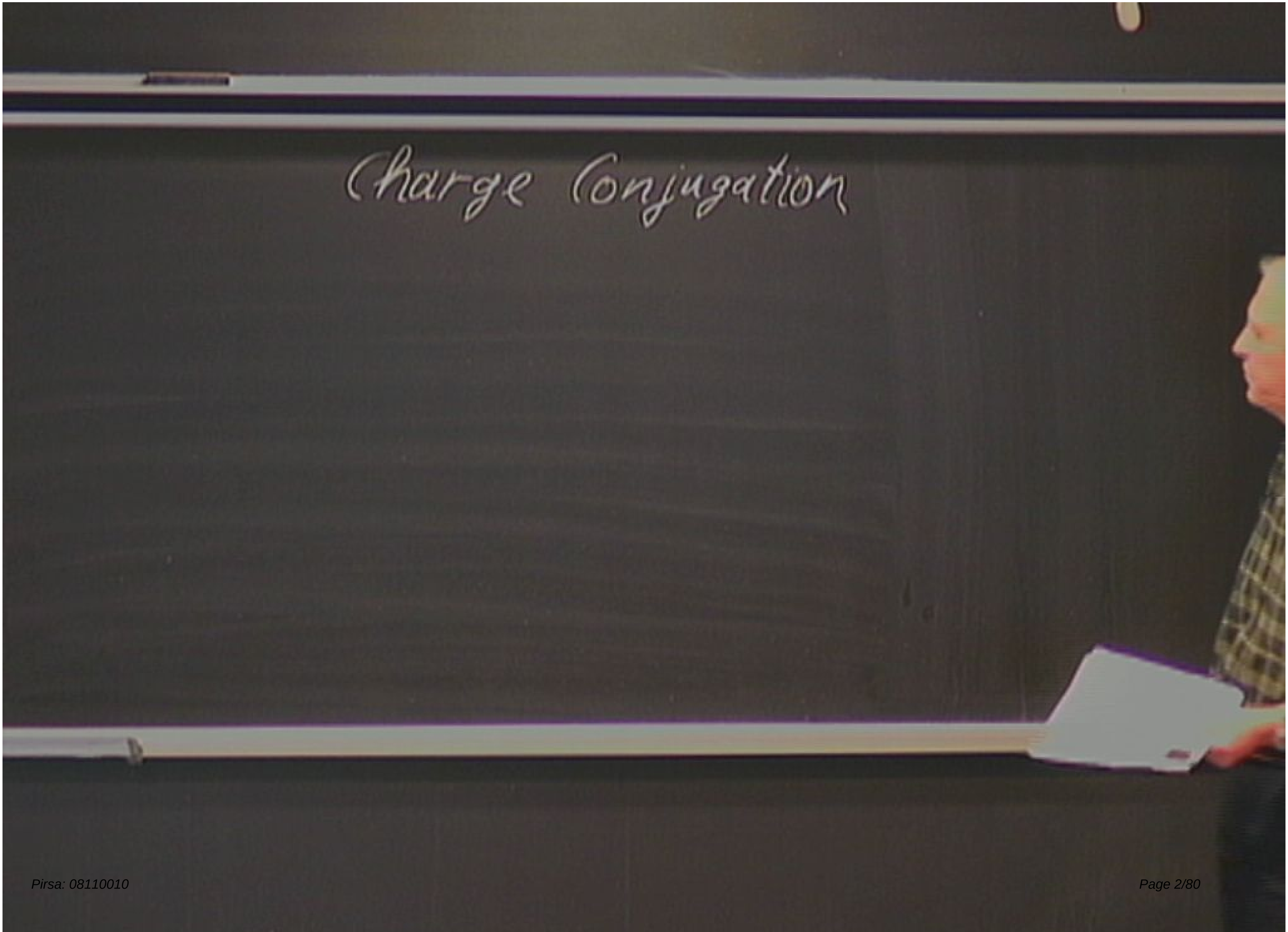
Title: Quantum Field Theory 1 - Lecture 11A

Date: Nov 19, 2008 10:30 AM

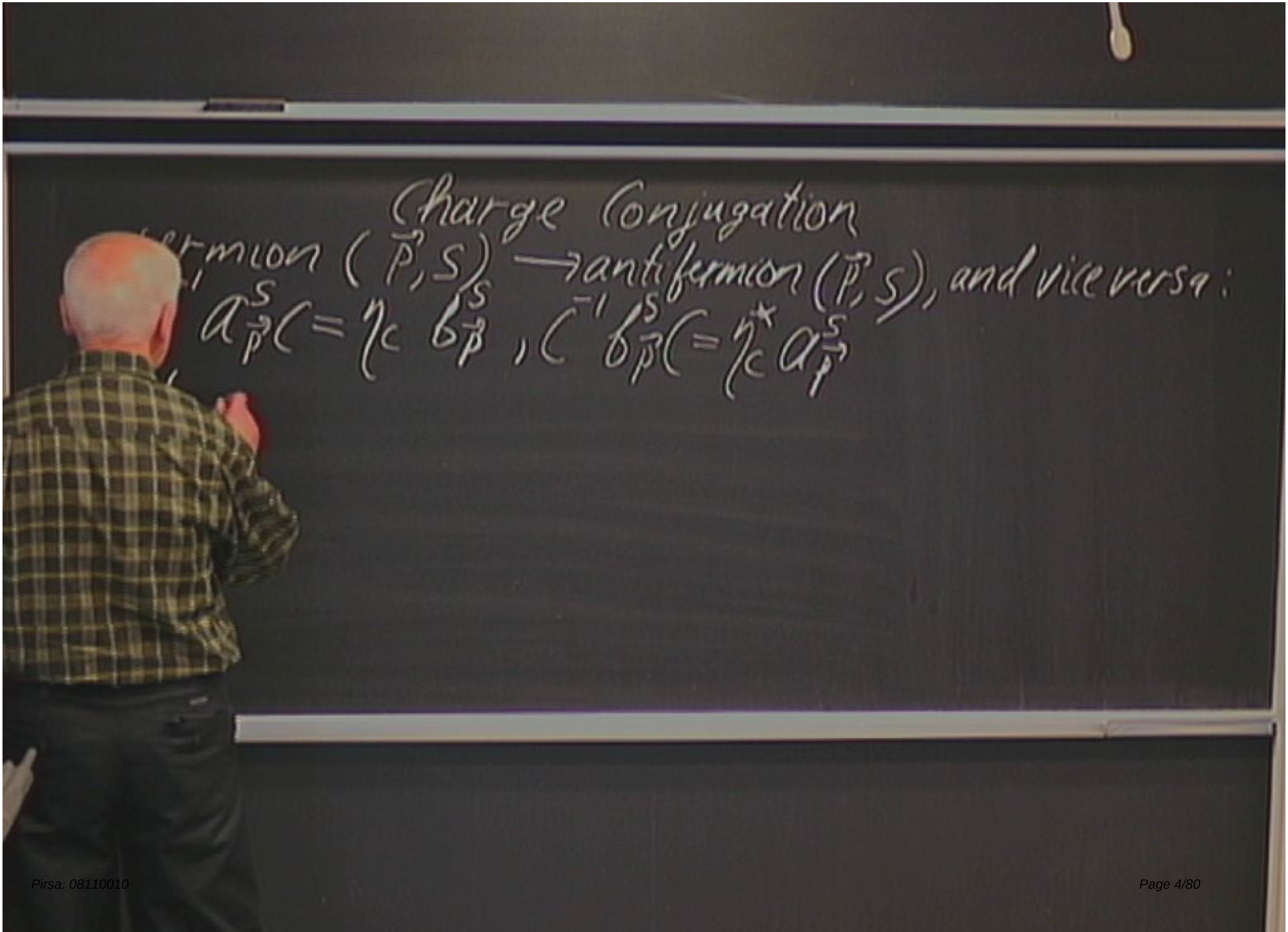
URL: http://pirsa.org/08110010

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario



Charge Conjugation

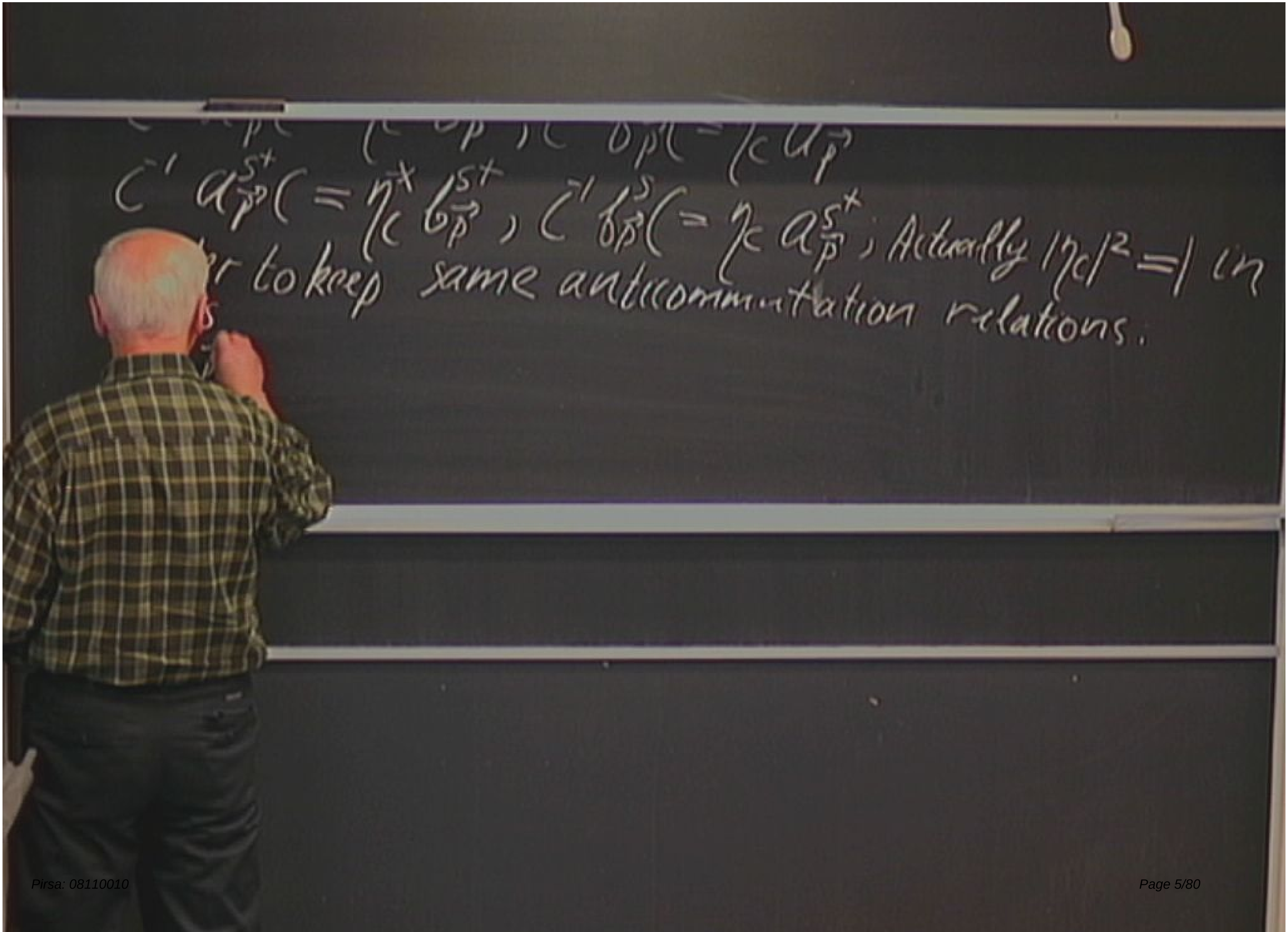
Charge Conjugation



Charge Conjugation

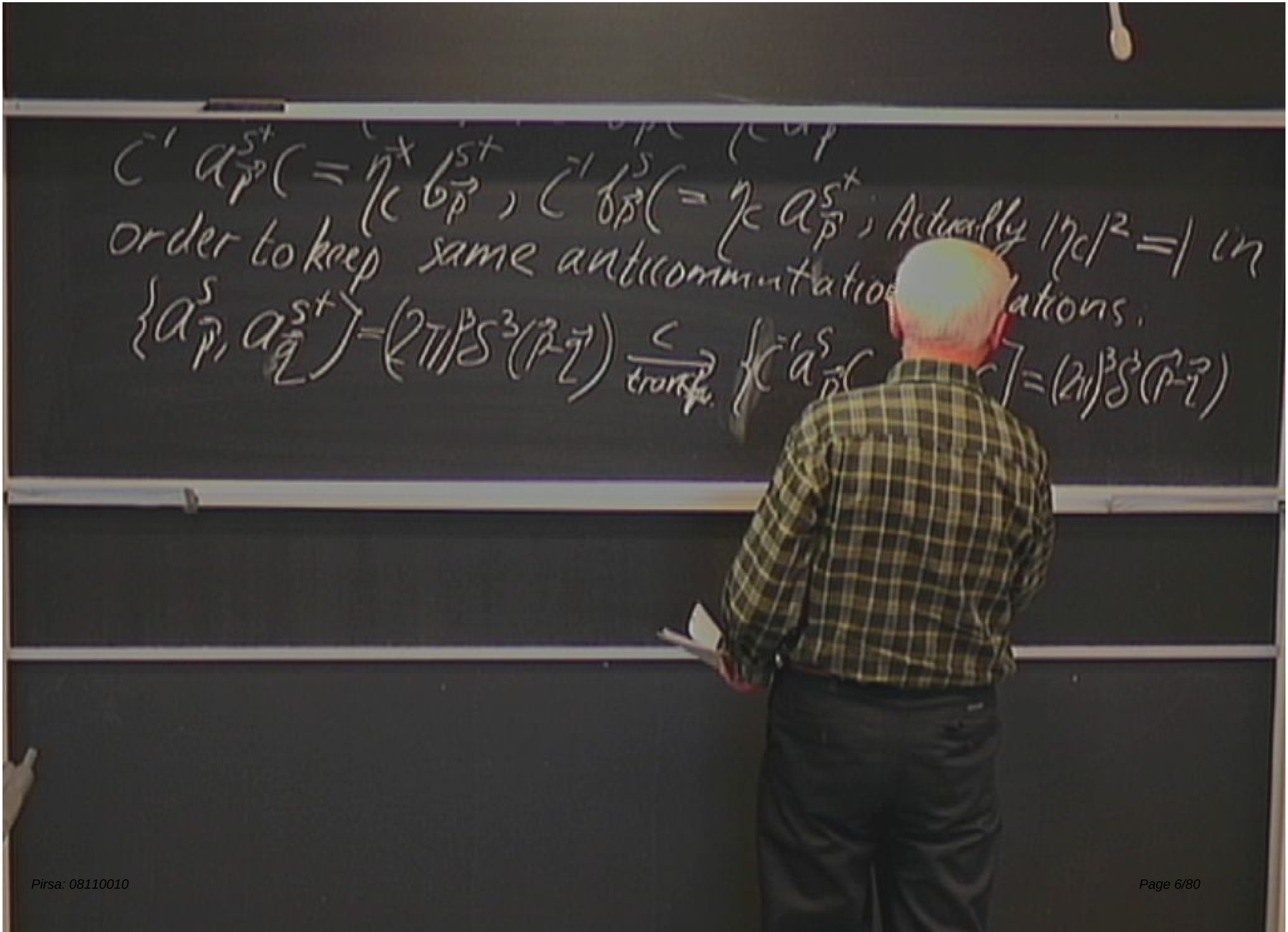
fermion $(\vec{p}, s) \longrightarrow$ antifermion $(\vec{p}, s)$, and vice versa:

$$a^{s}_{\vec{p}} C = \eta_c \, b^{s}_{\vec{p}} \, , \; C^{-1} b^{s}_{\vec{p}} C = \eta_c^{*} \, a^{s}_{\vec{p}}$$



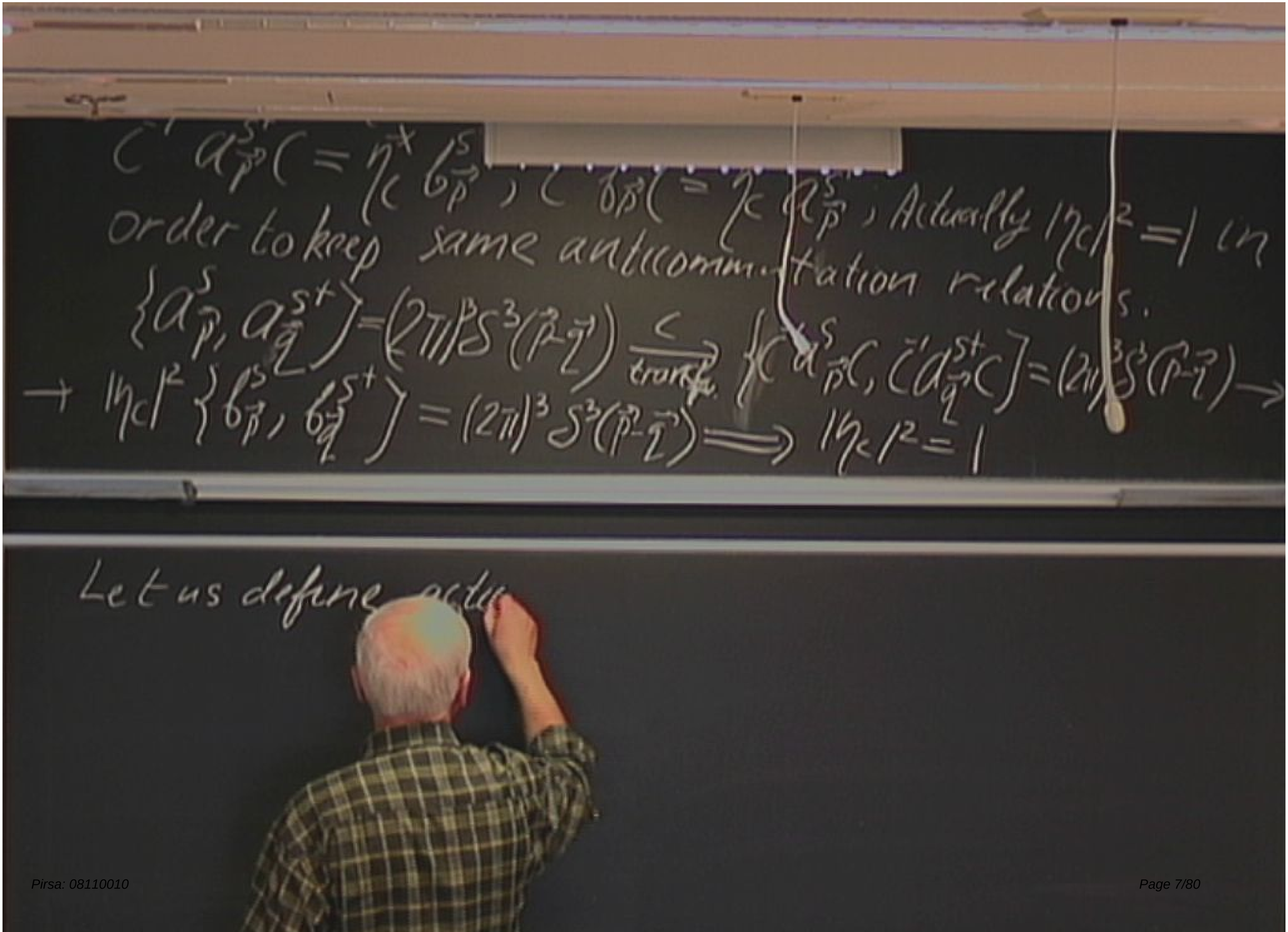
$C^{-1} a_{\vec{p}}^{s+} C = \eta_c^* b_{\vec{p}}^{s+}$, $C^{-1} b_{\vec{p}}^{s} C = \eta_c a_{\vec{p}}^{s+}$; Actually $|\eta_c|^2 = 1$ in

r to keep same anticommutation relations.



$C^{-1} a^{s\dagger}_{\vec p} C = \eta_c^{*} b^{s\dagger}_{\vec p}$, $C^{-1} b^{s}_{\vec p} C = \eta_c a^{s\dagger}_{\vec p}$; Actually $|\eta_c|^2 = 1$ in order to keep same anticommutatio ations.

$\{a^s_{\vec p}, a^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q) \xrightarrow[\text{transf.}]{C} \{C^{-1}a^s_{\vec p}C$ $C] = (2\pi)^3\delta^3(\vec p - \vec q)$

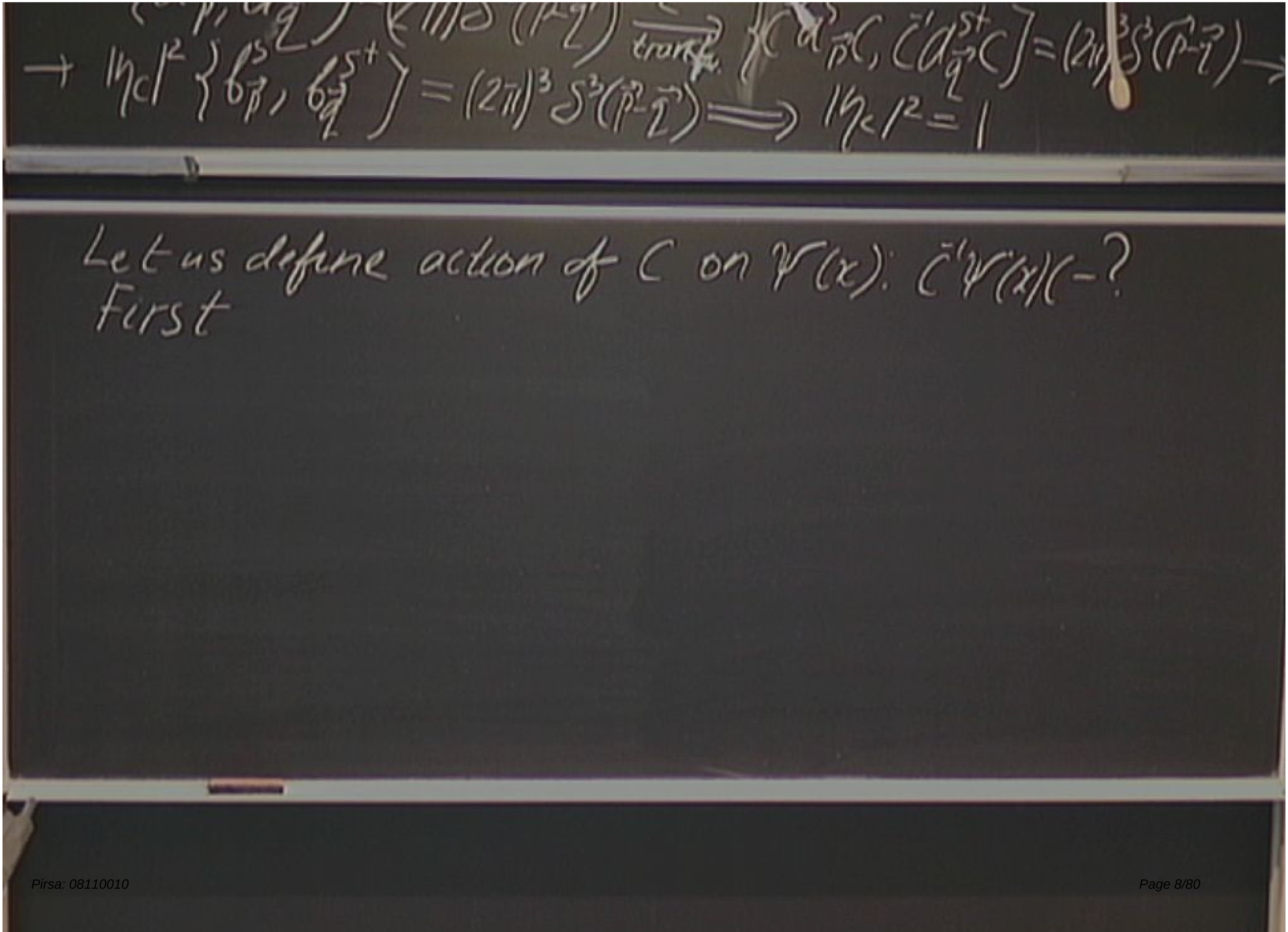


$C^{-1} a^s_{\vec p} C = \eta_c^* b^s_{\vec p}$, $C^{-1} b^s_{\vec p} C = \eta_c a^s_{\vec p}$, Actually $|\eta_c|^2 = 1$ in order to keep same anticommutation relations.

$$\{a^s_{\vec p}, a^{s+}_{\vec q}\} = (2\pi)^3 \delta^3(\vec p - \vec q) \xrightarrow[\text{transf.}]{C} \{C^{-1} a^s_{\vec p} C, C^{-1} a^{s+}_{\vec q} C\} = (2\pi)^3 \delta^3(\vec p - \vec q) \rightarrow$$

$$\rightarrow |\eta_c|^2 \{b^s_{\vec p}, b^{s+}_{\vec q}\} = (2\pi)^3 \delta^3(\vec p - \vec q) \Longrightarrow |\eta_c|^2 = 1$$

Let us define actu

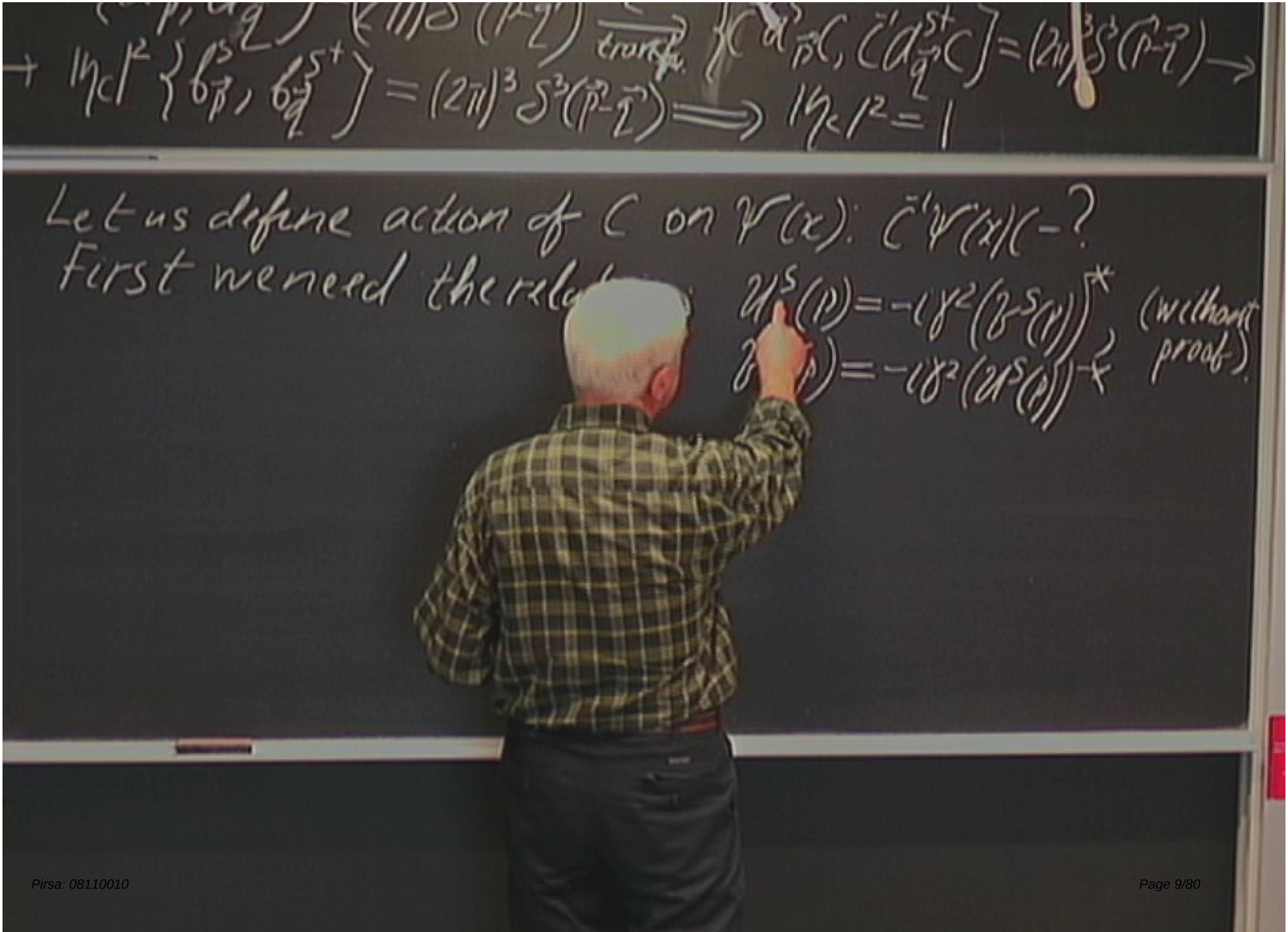


$$\{a_{\vec p}, a^{\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p-\vec q) \xrightarrow[\text{transf.}]{C} \{C^{-1}a_{\vec p}C, C^{-1}a^{s\dagger}_{\vec q}C\} = (2\pi)^3\delta^3(\vec p-\vec q) \to$$

$$\to |\eta_c|^2\{b^s_{\vec p}, b^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p-\vec q) \Longrightarrow |\eta_c|^2 = 1$$

Let us define action of C on $\Psi(x)$: $C^{-1}\Psi(x)C$ – ?

First



$$\ldots (2\pi)^3\delta^3(\vec p-\vec q) \xrightarrow[\text{transf.}]{C} \{C^{-1}a^s_{\vec p}C, C^{-1}a^{s\dagger}_{\vec q}C\} = (2\pi)^3\delta^3(\vec p-\vec q) \longrightarrow$$

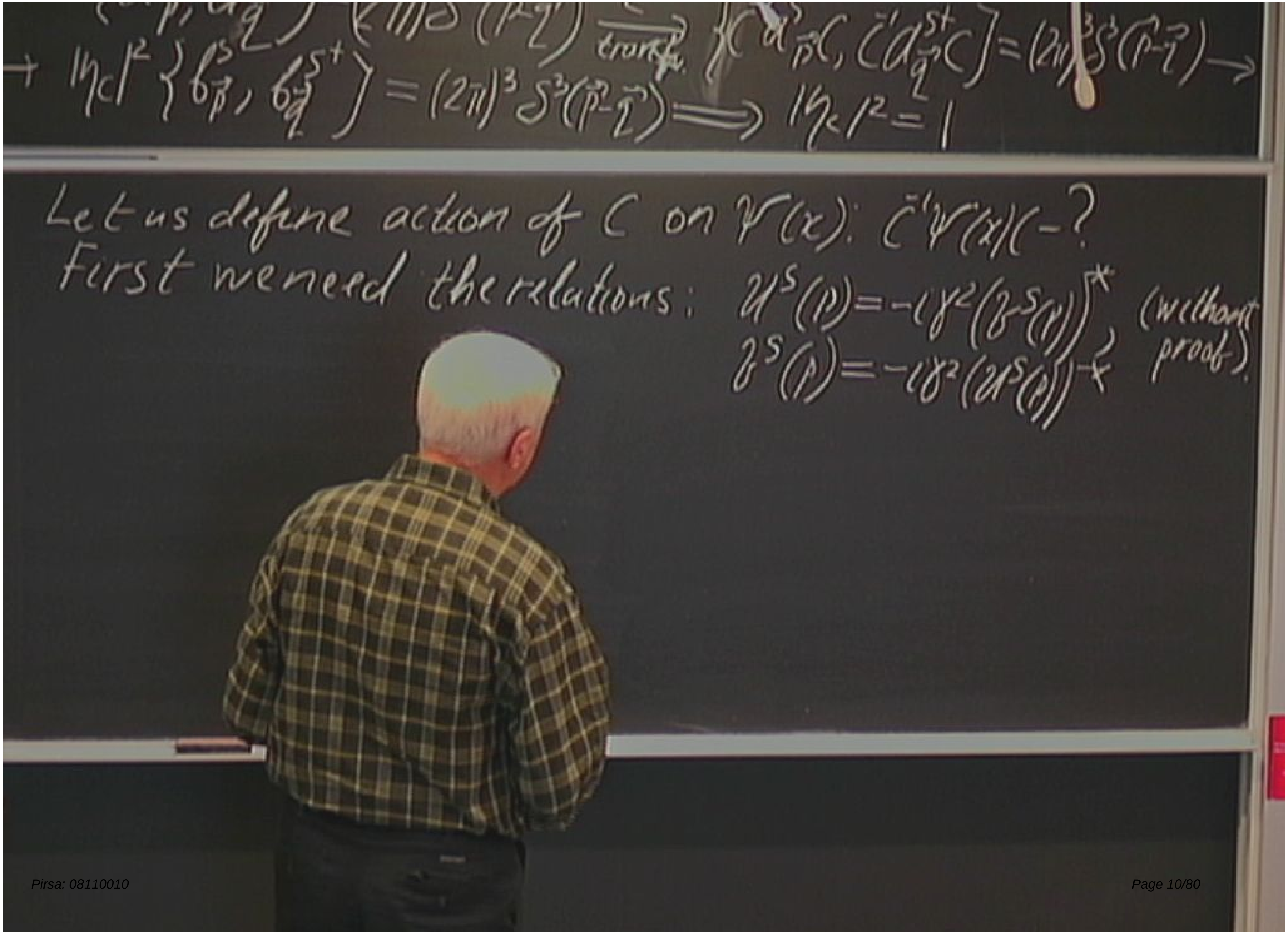
$$\to |\eta_C|^2\{b^s_{\vec p}, b^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p-\vec q) \Longrightarrow |\eta_C|^2 = 1$$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C$ – ?

First we need the relat

$$u^s(p) = -i\gamma^2(v^s(p))^*,$$
$$v^s(p) = -i\gamma^2(u^s(p))^*$$

(without proof).



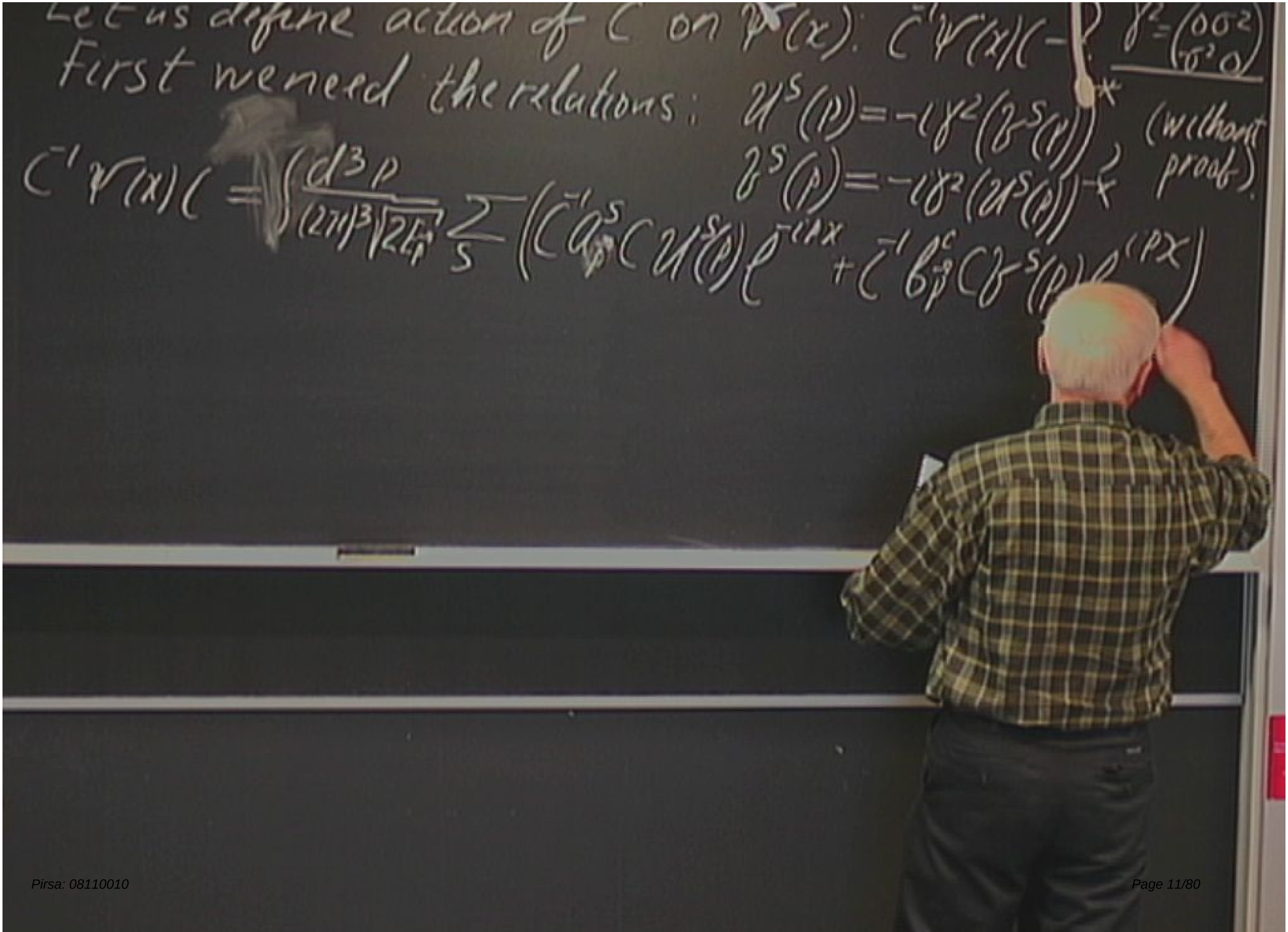
$\{a^s_{\vec p}, a^{s+}_{\vec q}\} = (2\pi)^3\delta^3(\vec p-\vec q) \xrightarrow[\text{transf.}]{C} \{C^{-1}a^s_{\vec p}C, C^{-1}a^{s+}_{\vec q}C\} = (2\pi)^3\delta^3(\vec p-\vec q) \to$

$\to |\eta_c|^2\{b^s_{\vec p}, b^{s+}_{\vec q}\} = (2\pi)^3\delta^3(\vec p-\vec q) \Longrightarrow |\eta_c|^2 = 1$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = ?$

First we need the relations: $u^s(p) = -i\gamma^2(v^s(p))^*$, (without proof).

$v^s(p) = -i\gamma^2(u^s(p))^*$



Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C =$

$$\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$$

First we need the relations: $u^s(p) = -i\gamma^2 (v^s(p))^*$

$v^s(p) = -i\gamma^2 (u^s(p))^*$ (without proof).

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \left(C^{-1}a_p^s C u^s(p) e^{-ipx} + C^{-1} b_p^{s} C v^s(p) e^{ipx} \right)$$

order to keep same anticommutation relations $= 1$ in

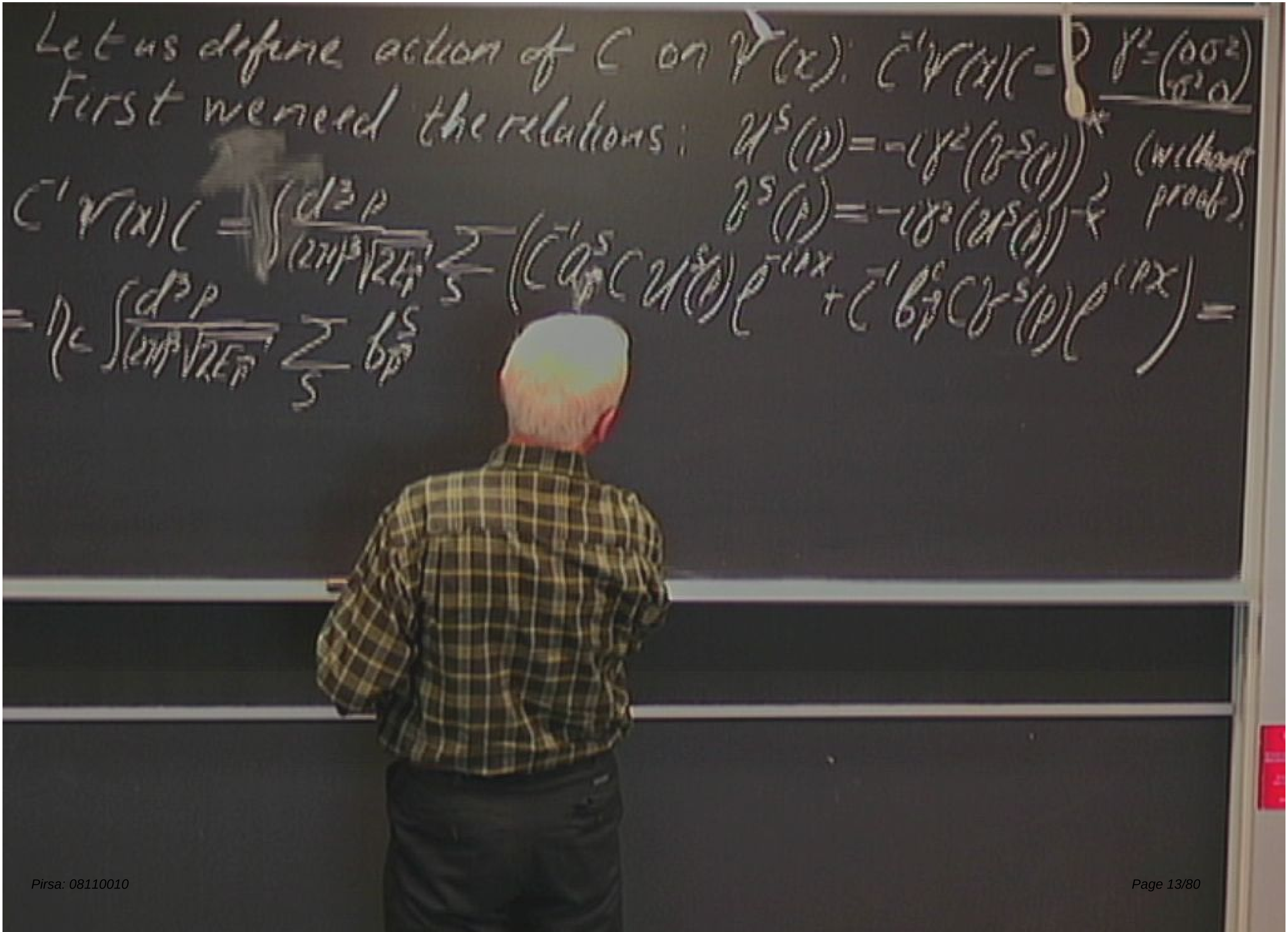
$\{a^s, a^{t}\}$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = $ $\quad \underline{\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}}$

First we need the relations: $u^s(p) = -i\gamma^2(v^s(p))^*$ (without proof).

$v^s(p) = -i\gamma^2(u^s(p))^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \left(C^{-1}a_p^s C\, u^s(p)\, e^{-ipx} + C^{-1} b_p^{s\dagger} C\, v^s(p)\, e^{ipx} \right) =$$



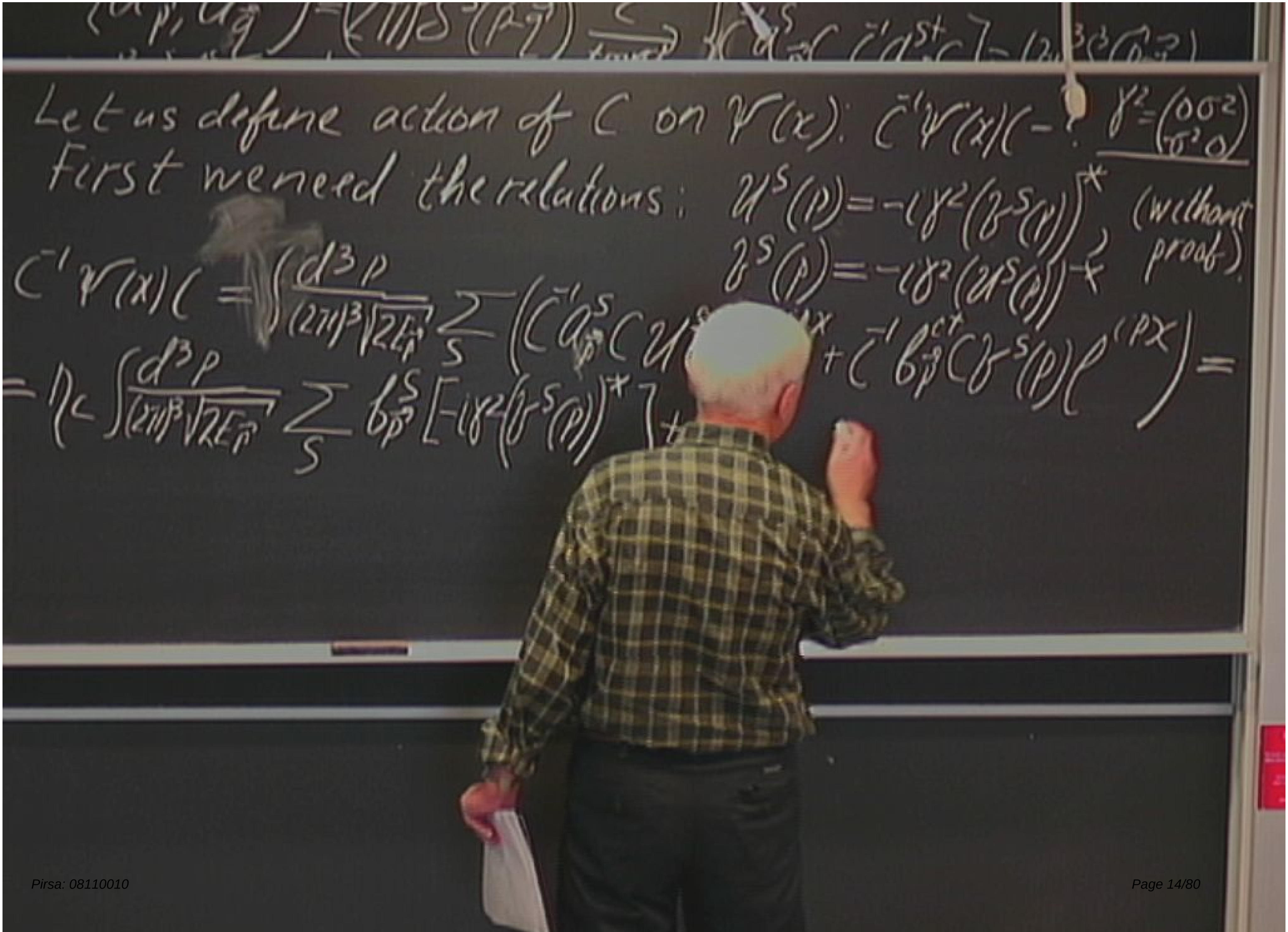
Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C =$ $\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$

First we need the relations: $u^s(p) = -i\gamma^2 (v^s(p))^*$ (without proof)

$v^s(p) = -i\gamma^2 (u^s(p))^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \left(C^{-1}a_p^s C u^s(p) e^{-ipx} + C^{-1} b_p^s C v^s(p) e^{ipx}\right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s b_p^s$$



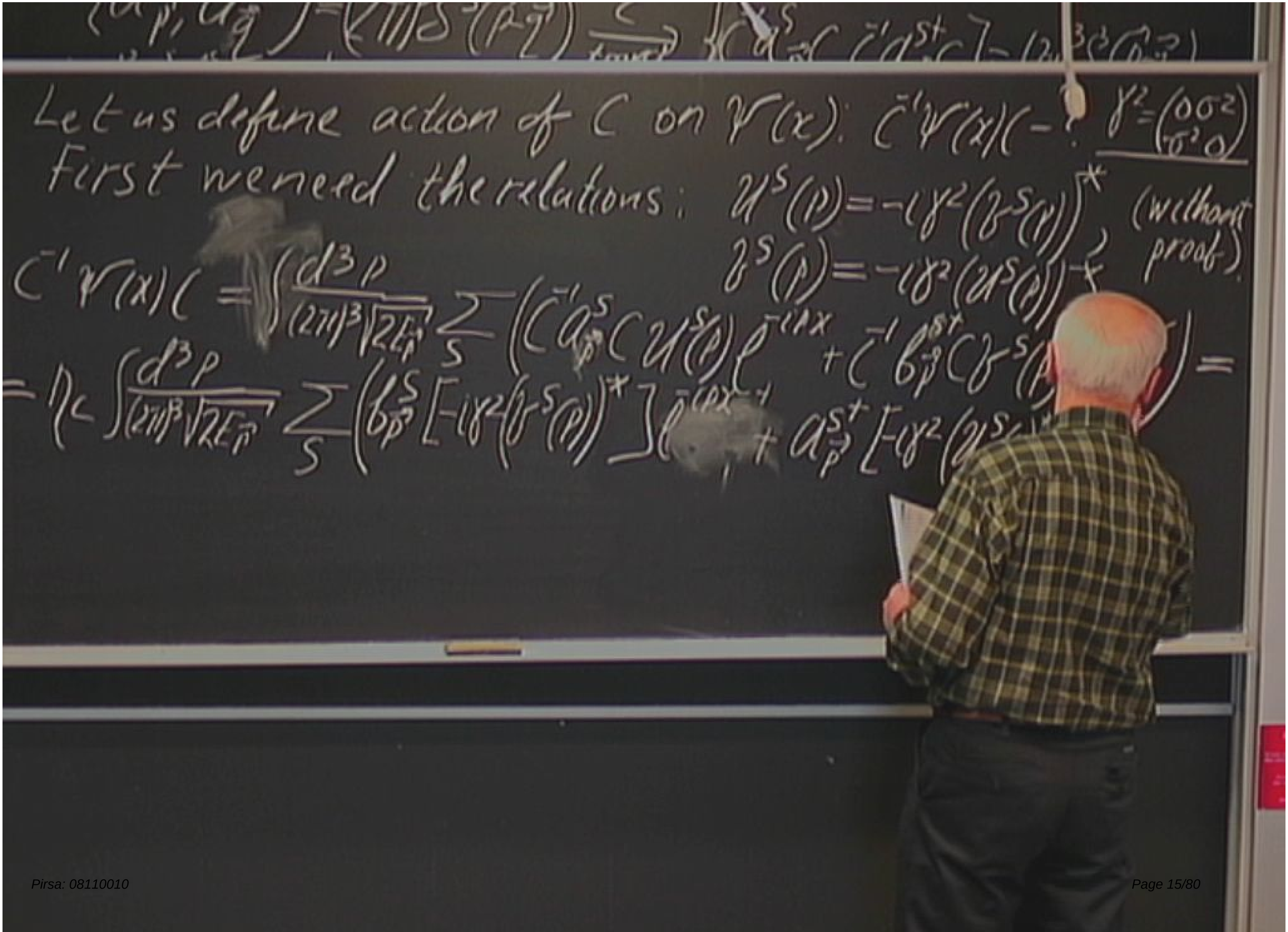
$\{a_p, a_q^\dagger\} = (2\pi)^3\delta^3(\vec p - \vec q) \xrightarrow[\text{transf.}]{C} \{C^{-1}a_p^s C, C^{-1}a_q^{s\dagger}C\} = (2\pi)^3\delta^3(\vec p - \vec q)$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = ?$ $\underline{\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}}$

First we need the relations: $u^s(p) = -i\gamma^2 (v^s(p))^*$, $v^s(p) = -i\gamma^2 (u^s(p))^*$ (without proof).

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \left(C^{-1}a_p^s C\, u^s \ldots e^{-ipx} + C^{-1} b_p^{s\dagger} C\, v^s(p)\, e^{ipx} \right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s b_p^s \left[-i\gamma^2 (v^s(p))^* \right] +$$



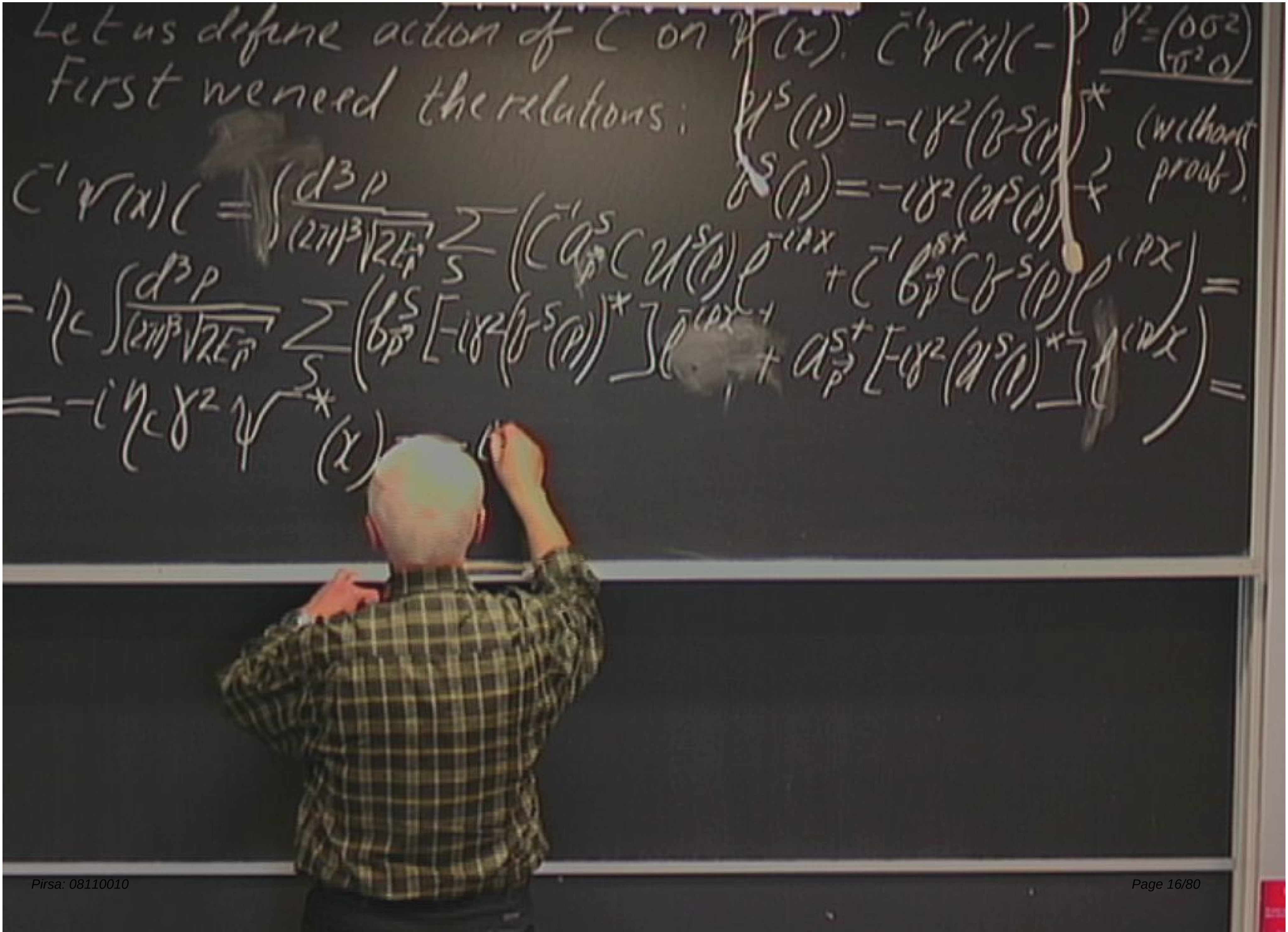
Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = ?$ $\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$

First we need the relations: $u^s(p) = -i\gamma^2\left(v^s(p)\right)^*$ (without proof).

$v^s(p) = -i\gamma^2\left(u^s(p)\right)^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \left(C^{-1}a^s_{\vec{p}}C\, u^s(p)\, e^{-ipx} + C^{-1} b^{s\dagger}_{\vec{p}} C\, v^s(p) \ldots \right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(b^s_{\vec{p}}\left[-i\gamma^2\left(v^s(p)\right)^*\right] e^{-ipx} + a^{s\dagger}_{\vec{p}}\left[-i\gamma^2\left(u^s \ldots\right.\right.\right.$$



Let us define action of C on ψ(x): $C^{-1}\psi(x)C -$

$\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$

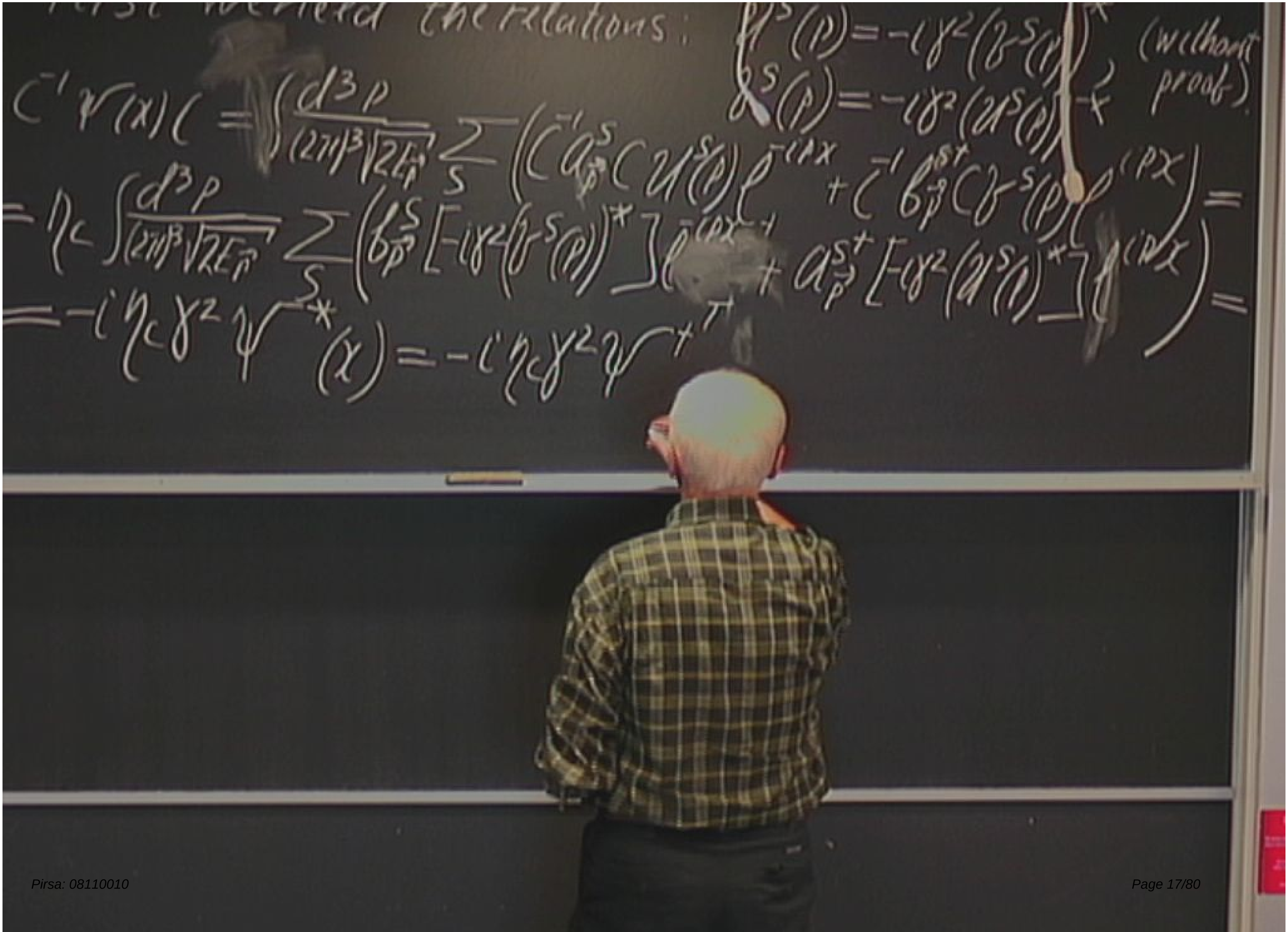
First we need the relations: $u^s(p) = -i\gamma^2(v^s(p))^*$ (without proof)

$v^s(p) = -i\gamma^2(u^s(p))^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\sum_s \left(C^{-1}a^s_{\vec p}C\,u^s(p)\,e^{-ipx} + C^{-1}b^{s\dagger}_{\vec p}C\,v^s(p)\,e^{ipx}\right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\sum_s \left(b^s_{\vec p}\left[-i\gamma^2(v^s(p))^*\right]e^{-ipx} + a^{s\dagger}_{\vec p}\left[-i\gamma^2(u^s(p))^*\right]e^{ipx}\right) =$$

$$= -i\eta_c\gamma^2\psi^*(x)$$



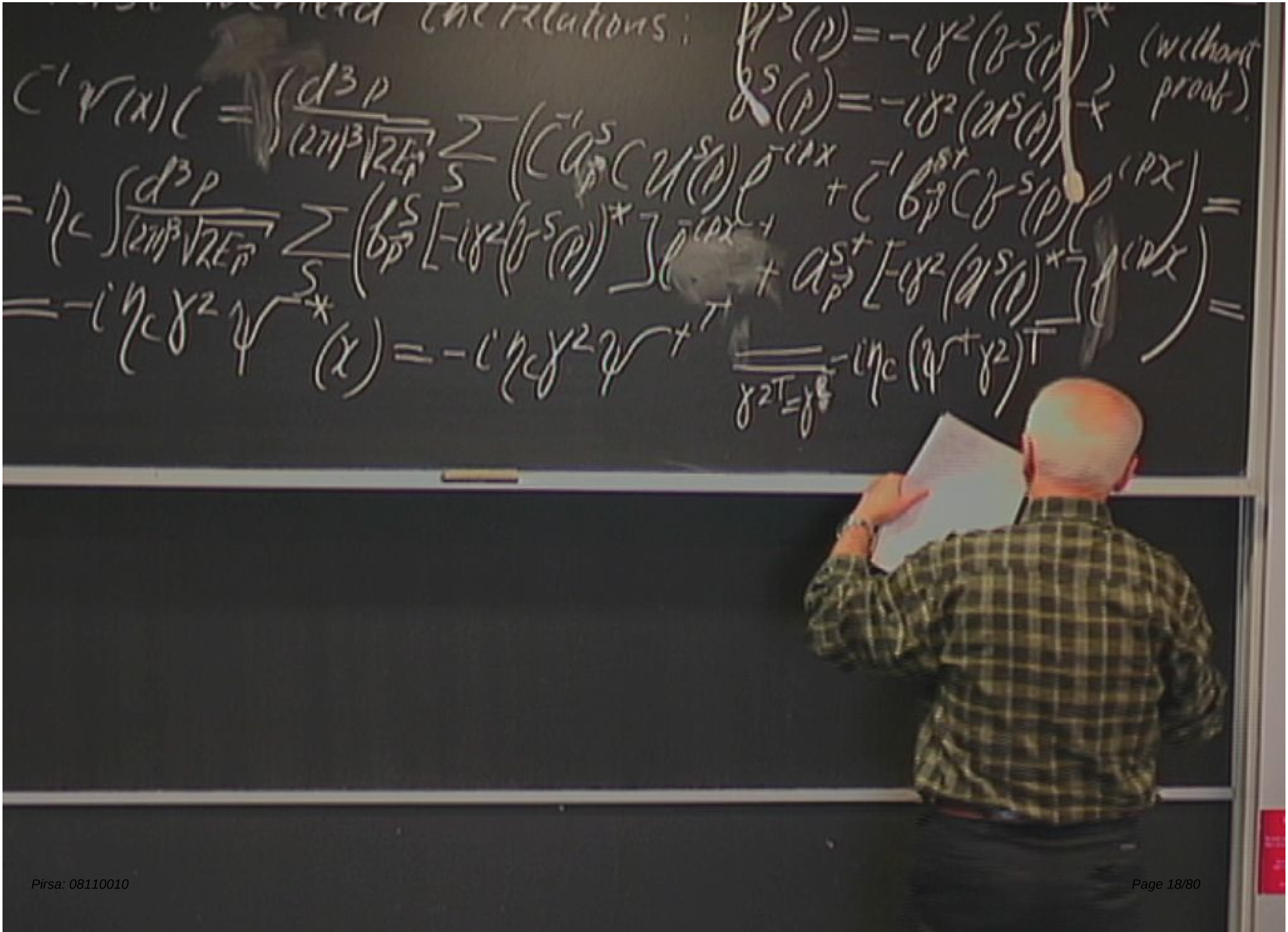
rst we need the relations: $u^s(p) = -i\gamma^2(v^s(p))^*$ (without proof).

$v^s(p) = -i\gamma^2(u^s(p))^*$

$$C^{-1}\psi(x)C = \int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\sum_s\left(C^{-1}a^s_{\vec p}C\,u^s(p)e^{-ipx} + C^{-1}b^{s\dagger}_{\vec p}C\,v^s(p)e^{ipx}\right) =$$

$$= \eta_c\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\sum_s\left(b^s_{\vec p}\left[-i\gamma^2(v^s(p))^*\right]e^{-ipx} + a^{s\dagger}_{\vec p}\left[-i\gamma^2(u^s(p))^*\right]e^{ipx}\right) =$$

$$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T}$$



... the relations:

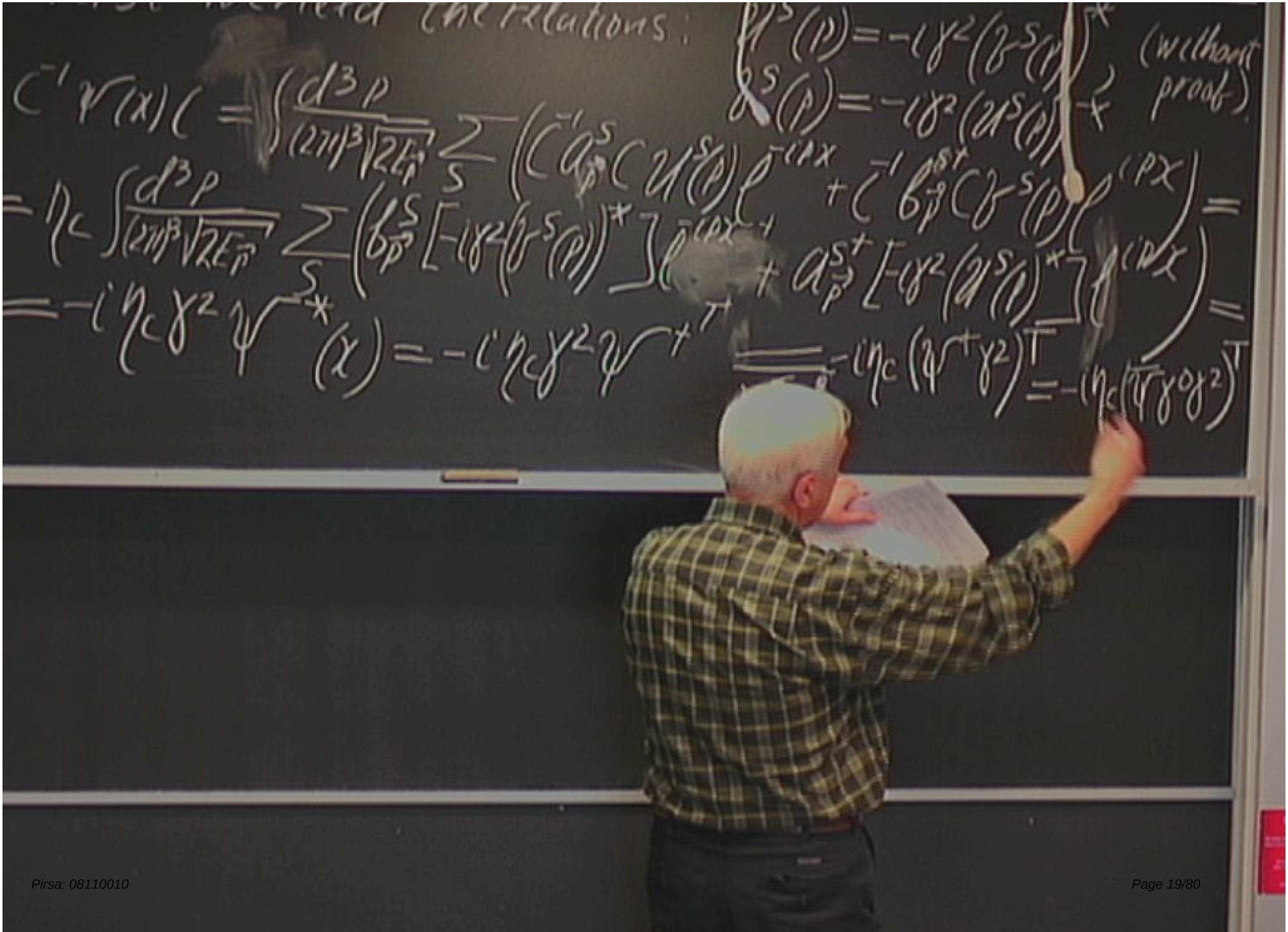
$u^s(p) = -i\gamma^2 (v^s(p))^*$

$v^s(p) = -i\gamma^2 (u^s(p))^*$ (without proof).

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} \sum_s \left(C^{-1}a^s_{\vec p}C\, u^s(p)\, e^{-ipx} + C^{-1} b^{s\dagger}_{\vec p} C\, v^s(p)\, e^{ipx} \right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} \sum_s \left(b^s_{\vec p} \left[-i\gamma^2 (v^s(p))^*\right] e^{-ipx} + a^{s\dagger}_{\vec p} \left[-i\gamma^2 (u^s(p))^*\right] e^{ipx} \right) =$$

$$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T} \underset{\gamma^{2T}=\gamma^2}{=} -i\eta_c(\psi^\dagger\gamma^2)^T$$



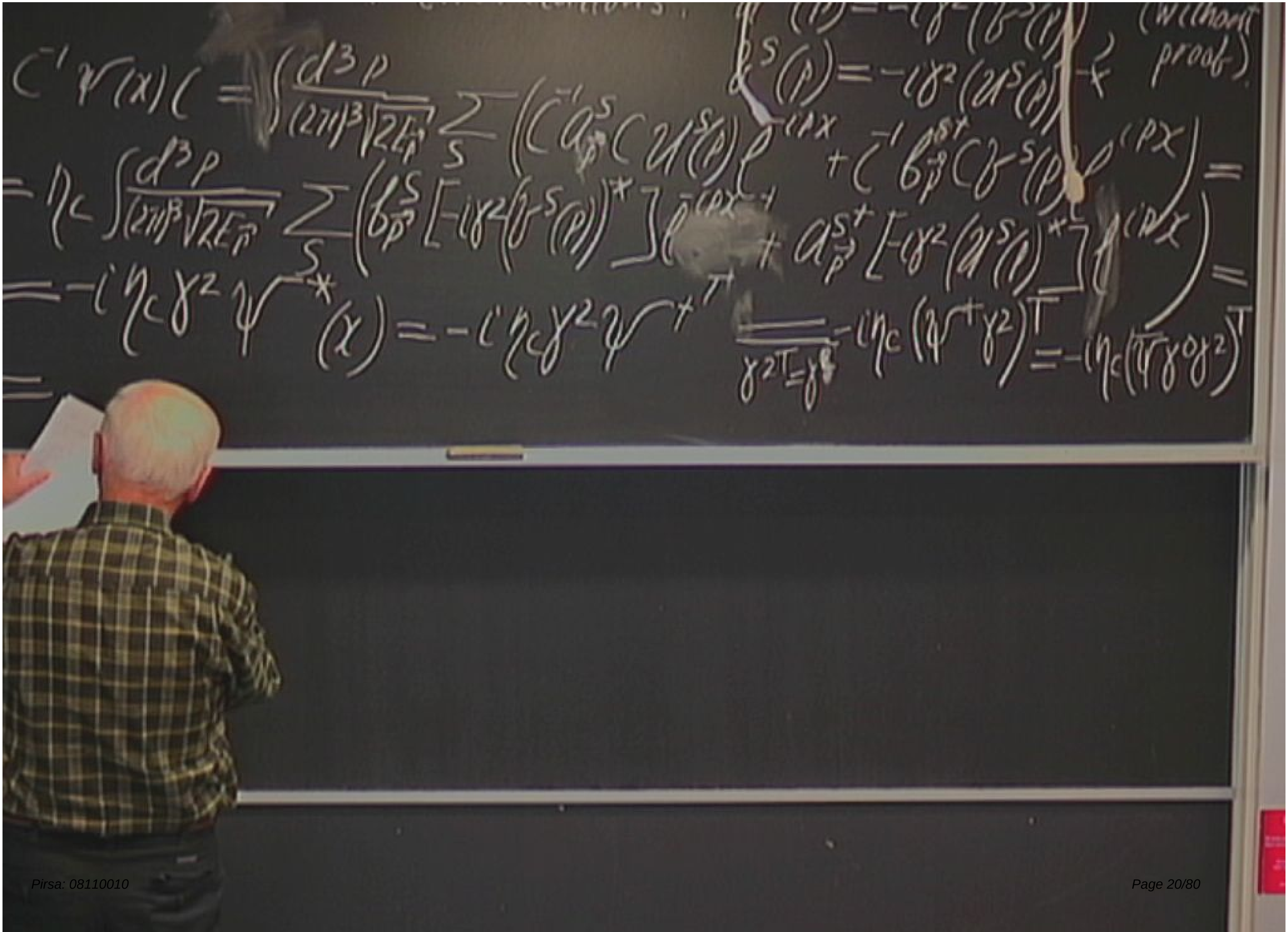
the relations:

$u^s(p) = -i\gamma^2 (v^s(p))^*$
$v^s(p) = -i\gamma^2 (u^s(p))^*$ (without proof).

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(C^{-1} a^s_{\vec{p}} C u^s(p) e^{-ipx} + C^{-1} b^{s+}_{\vec{p}} C v^s(p) e^{ipx} \right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left(b^s_{\vec{p}} \left[-i\gamma^2 (v^s(p))^*\right] e^{-ipx} + a^{s+}_{\vec{p}} \left[-i\gamma^2 (u^s(p))^*\right] e^{ipx} \right) =$$

$$= -i\eta_c \gamma^2 \psi^*(x) = -i\eta_c\gamma^2\psi^{+T} = -i\eta_c(\psi^+\gamma^2)^T = -i\eta_c(\bar{\psi}\gamma^0\gamma^2)^T$$



$$(b^{-s}(p))^* = -i\gamma^2 (b^s(p))^*$$

(without proof).

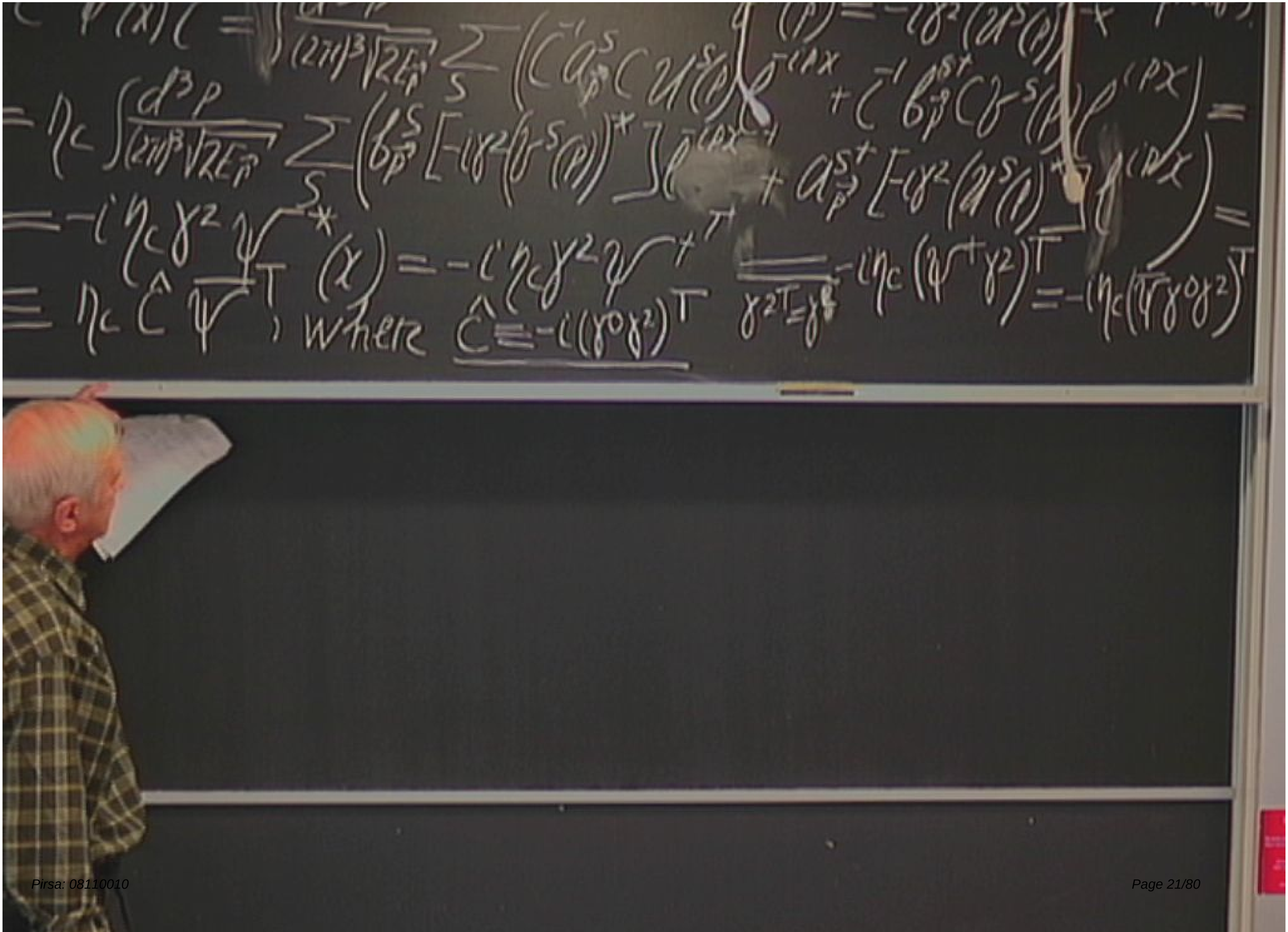
$$v^s(p) = -i\gamma^2 (u^s(p))^*$$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} \sum_s \left(C^{-1} a^s_{\vec p} C\, u^s(p)\, e^{-ipx} + C^{-1} b^{s\dagger}_{\vec p} C\, v^s(p)\, e^{ipx} \right) =$$

$$= \eta_c \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} \sum_s \left(b^s_{\vec p} \left[-i\gamma^2 (v^s(p))^* \right] e^{-ipx} + a^{s\dagger}_{\vec p} \left[-i\gamma^2 (u^s(p))^* \right] e^{ipx} \right) =$$

$$= -i\eta_c \gamma^2 \psi^*(x) = -i\eta_c \gamma^2 \psi^{\dagger T} \underset{\gamma^{2T} = \gamma^2}{=} -i\eta_c (\psi^\dagger \gamma^2)^T = -i\eta_c (\bar\psi \gamma^0 \gamma^2)^T$$

$$=$$



$$C\psi(x)C = \int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\sum_s\left(C^{-1}a^s_{\vec p}C\,u^s(p)\,e^{-ipx} + C^{-1}b^{s\dagger}_{\vec p}C\,v^s(p)\,e^{ipx}\right) =$$

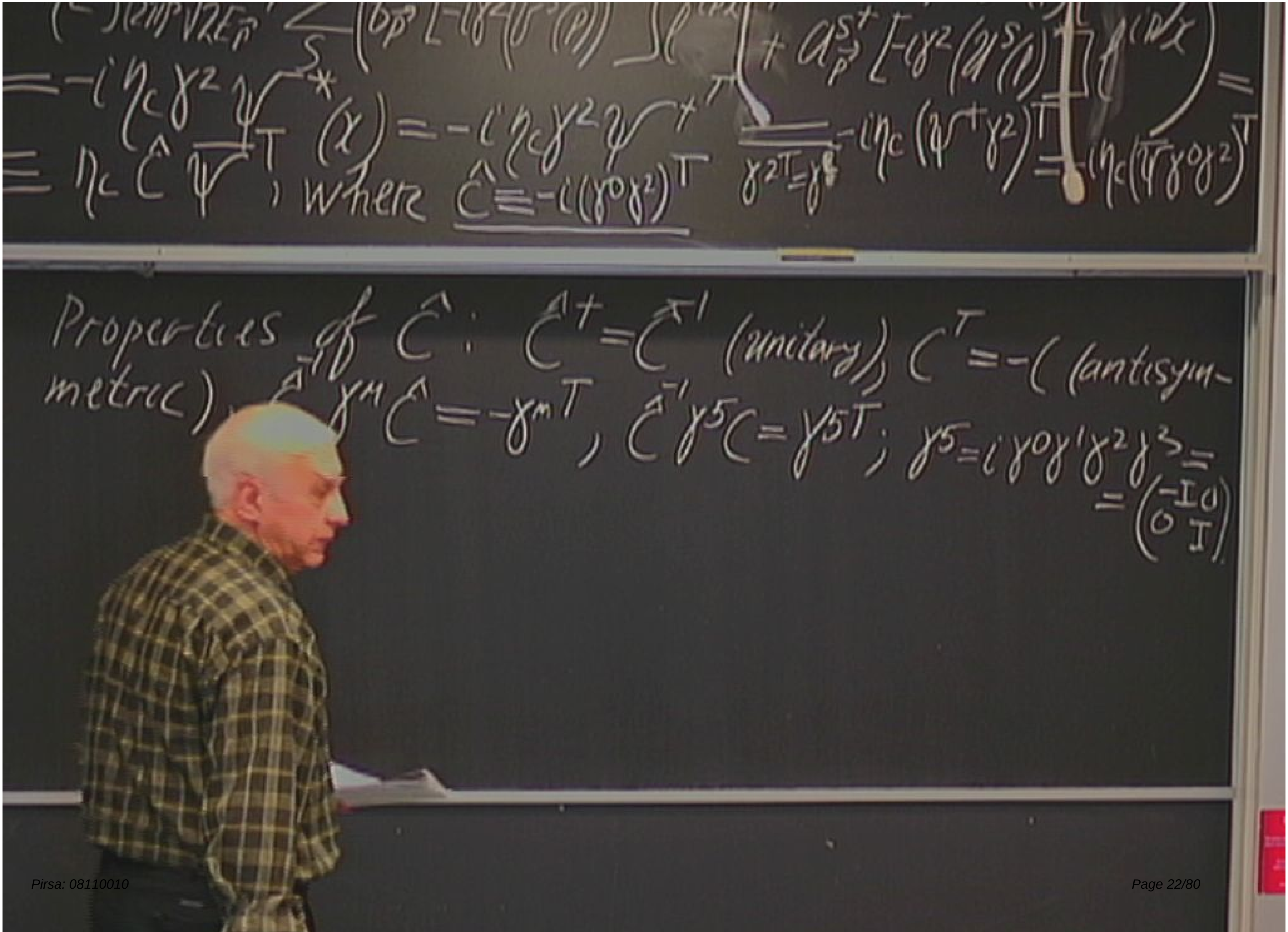
$$v(p) = -i\gamma^2(u^s(p))^*$$

$$= \eta_c\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\sum_s\left(b^s_{\vec p}\left[-i\gamma^2(v^s(p))^*\right]e^{-ipx} + a^{s\dagger}_{\vec p}\left[-i\gamma^2(u^s(p))^*\right]e^{ipx}\right) =$$

$$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T} = -i\eta_c(\psi^\dagger\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$$

$$\gamma^{2T} = \gamma^2$$

$$\equiv \eta_c\hat C\bar\psi^T, \text{ where } \underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$$

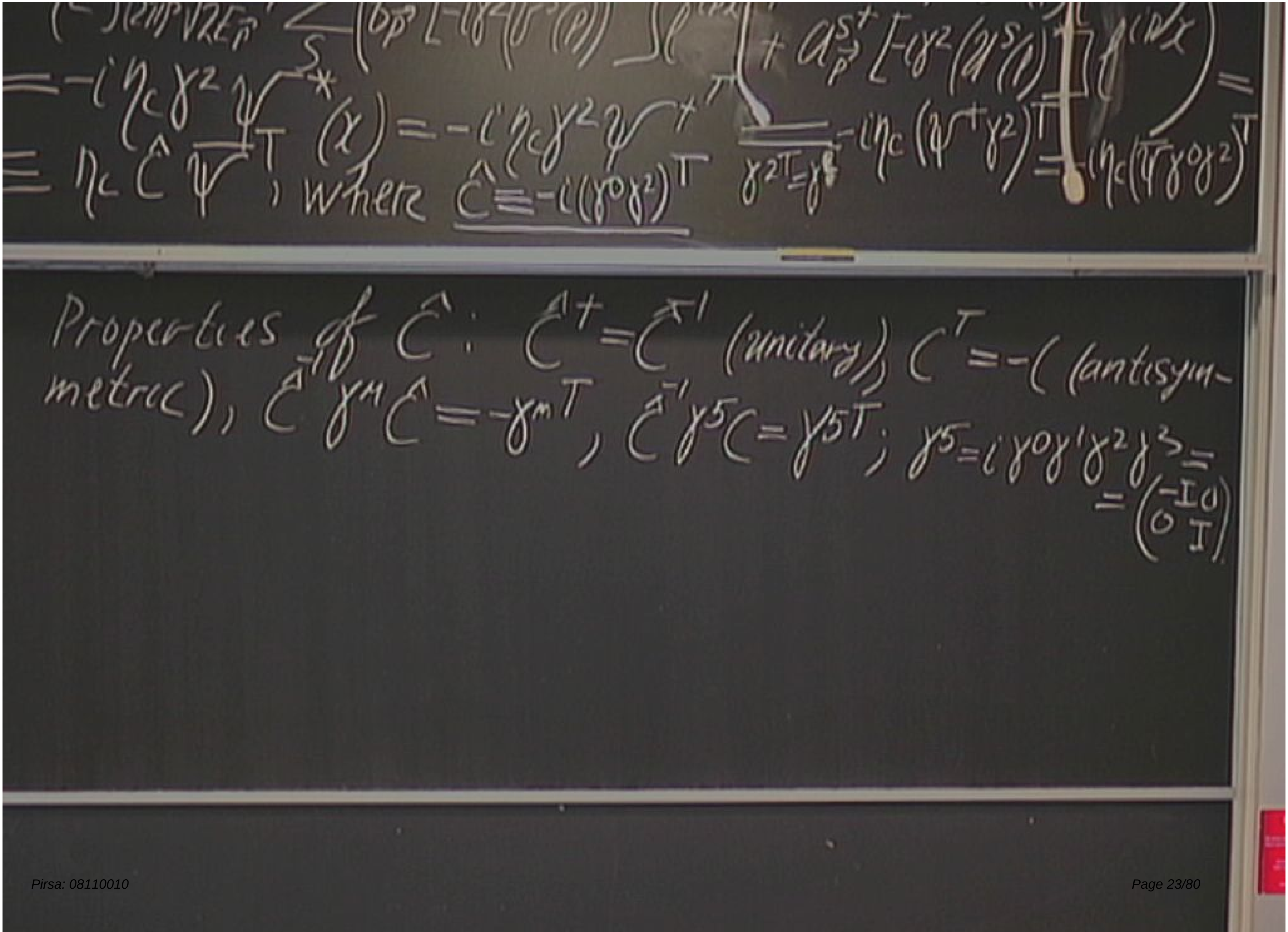


$$(\ldots)(2\pi)^3\sqrt{2E_{\vec p}}\sum_s \Big(b_{\vec p}^{s}[-i\gamma^2(v^s(p))^*]\,e^{\ldots} + a_{\vec p}^{s\dagger}[-i\gamma^2(u^s(p))^*]\,e^{ipx}\Big) =$$

$$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T} \underset{\gamma^{2T}=\gamma^2}{=} -i\eta_c(\psi^\dagger\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$$

$$\equiv \eta_c\hat C\bar\psi^T, \text{ where } \underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$$

Properties of $\hat C$: $\hat C^\dagger = \hat C^{-1}$ (unitary), $C^T = -C$ (antisymmetric), $\hat C^{-1}\gamma^\mu\hat C = -\gamma^{\mu T}$, $\hat C^{-1}\gamma^5 C = \gamma^{5T}$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix}-I & 0\\ 0 & I\end{pmatrix}$



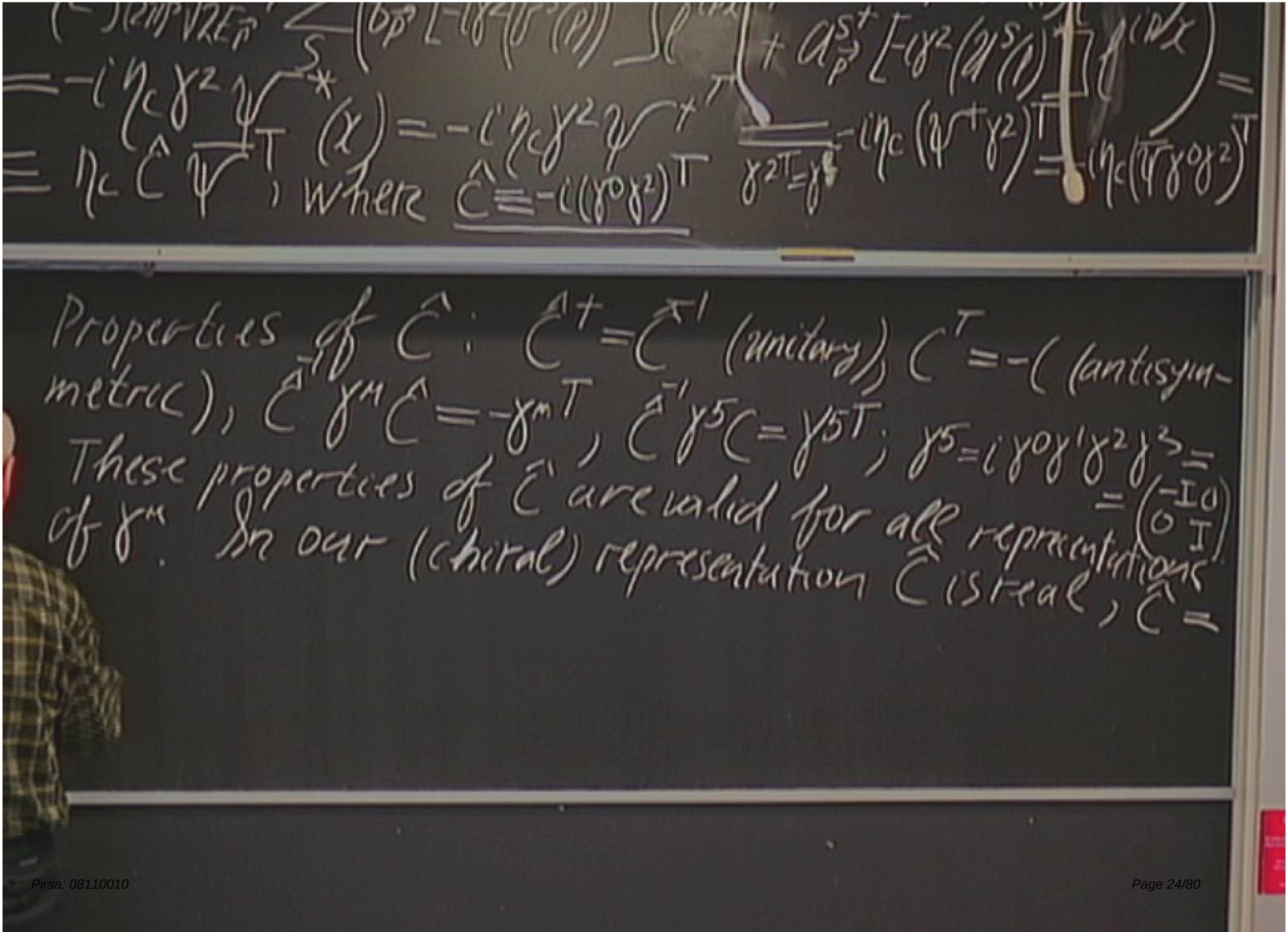
$(\ldots)(2\pi)^3\sqrt{2E_{\vec p}}\sum_s \Big(b_{\vec p}^s[-i\gamma^2(v^s(\vec p))^*]e^{-ipx} + a_{\vec p}^{s\dagger}[-i\gamma^2(u^s(\vec p))^*]e^{ipx}\Big) =$

$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T} \underset{\gamma^{2T}=\gamma^2}{=} -i\eta_c(\psi^\dagger\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$

$\equiv \eta_c\hat C\bar\psi^T$, where $\underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$

Properties of $\hat C$: $\hat C^\dagger = \hat C^{-1}$ (unitary), $C^T = -C$ (antisymmetric), $\hat C^{-1}\gamma^\mu\hat C = -\gamma^{\mu T}$, $\hat C^{-1}\gamma^5 C = \gamma^{5T}$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 =$

$= \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$



$(\ldots)(2\pi)^3\sqrt{2E_{\vec p}}\sum_s\Big(b_{\vec p}[-i\gamma^2(v^s(p))^*]e^{\ldots} + a_{\vec p}^{s+}[-i\gamma^2(u^s(p))^*]e^{ipx}\Big) =$

$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{+T} \underset{\gamma^{2T}=\gamma^2}{=} -i\eta_c(\psi^+\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$

$\equiv \eta_c\hat C\bar\psi^T$, where $\underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$

Properties of $\hat C$: $\hat C^+ = \hat C^{-1}$ (unitary), $C^T = -C$ (antisymmetric), $\hat C^{-1}\gamma^\mu\hat C = -\gamma^{\mu T}$, $\hat C^{-1}\gamma^5 C = \gamma^{5T}$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix}-I & 0\\ 0 & I\end{pmatrix}$

These properties of $\hat C$ are valid for all representations of γ^μ. In our (chiral) representation $\hat C$ is real, $\hat C =$

$(\ldots)(2\pi)^3\sqrt{2E_{\vec p}}\sum_s \Big(b_{\vec p}^{s}[-i\gamma^2(v^s(\vec p))^*]e^{-ipx} + a_{\vec p}^{s\dagger}[-i\gamma^2(u^s(\vec p))^*]e^{ipx}\Big) =$

$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T} \underset{\gamma^{2T}=\gamma^2}{=} -i\eta_c(\psi^\dagger\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$

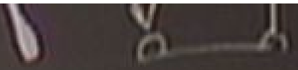
$\equiv \eta_c\hat C\bar\psi^T$, where $\underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$

Properties of $\hat C$: $\hat C^\dagger = \hat C^{-1}$ (unitary), $C^T = -C$ (antisymmetric), $\hat C^{-1}\gamma^\mu\hat C = -\gamma^{\mu T}$, $\hat C^{-1}\gamma^5 C = \gamma^{5T}$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$

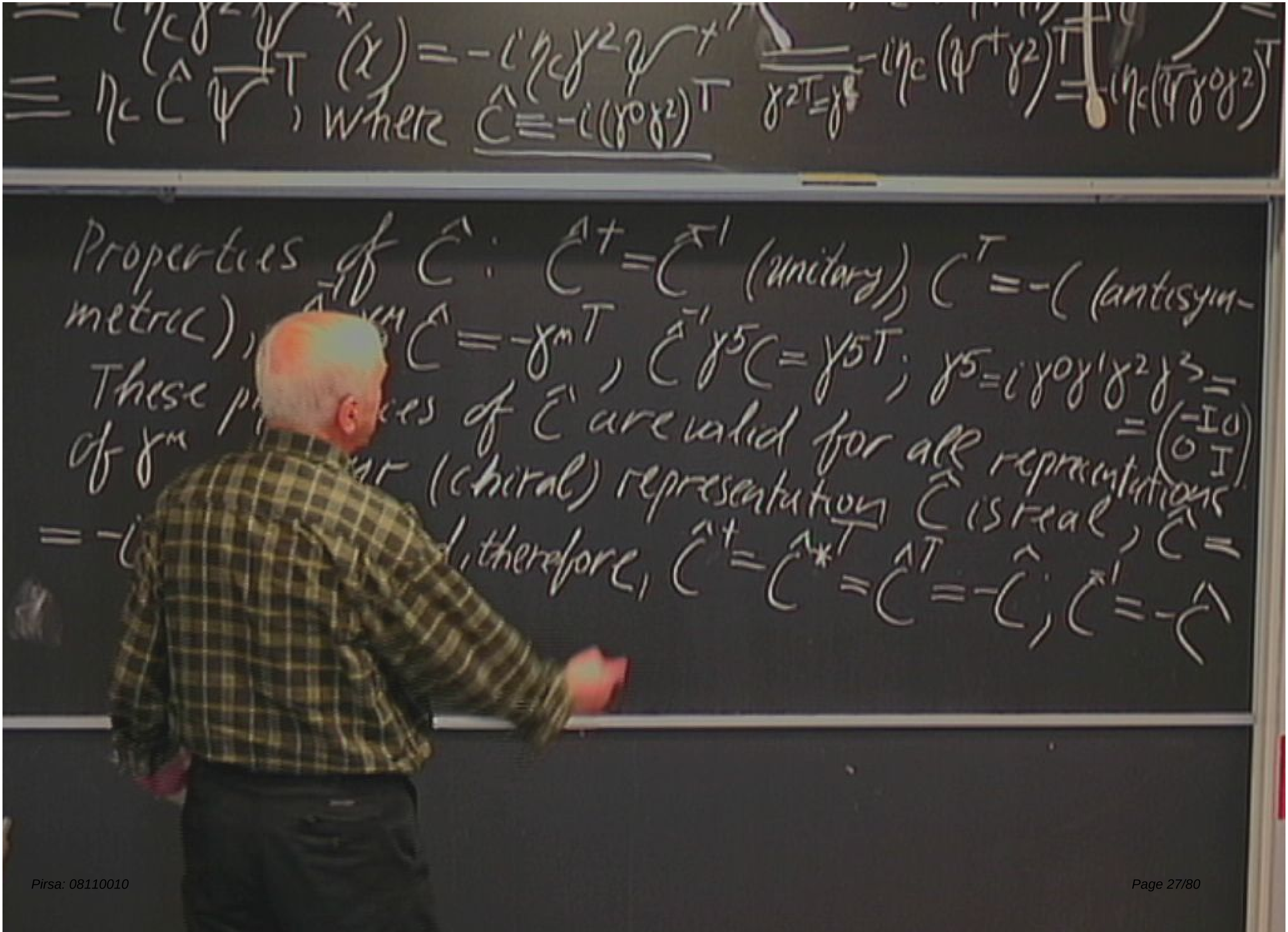
These properties of $\hat C$ are valid for all representations of γ^μ. In our (chiral) representation $\hat C$ is real, $\hat C =$

$= -i\begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$

$\frac{1}{\lambda^2} + (+\lambda^2 \dots$



$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$



$= -i\eta_c\gamma^2\psi^*$

$(\eta_c\gamma^2\psi^*)^T(x) = -i\eta_c\gamma^2\psi^{+T}$ $\underset{\gamma^{2T}=\gamma^2}{=}$ $-i\eta_c(\psi^+\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$

$\equiv \eta_c\hat C\bar\psi^T$, where $\underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$

Properties of $\hat C$: $\hat C^+ = \hat C^{-1}$ (unitary), $C^T = -C$ (antisymmetric), $\hat C^{-1}\gamma^m\hat C = -\gamma^{mT}$, $\hat C^{-1}\gamma^5 C = \gamma^{5T}$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$

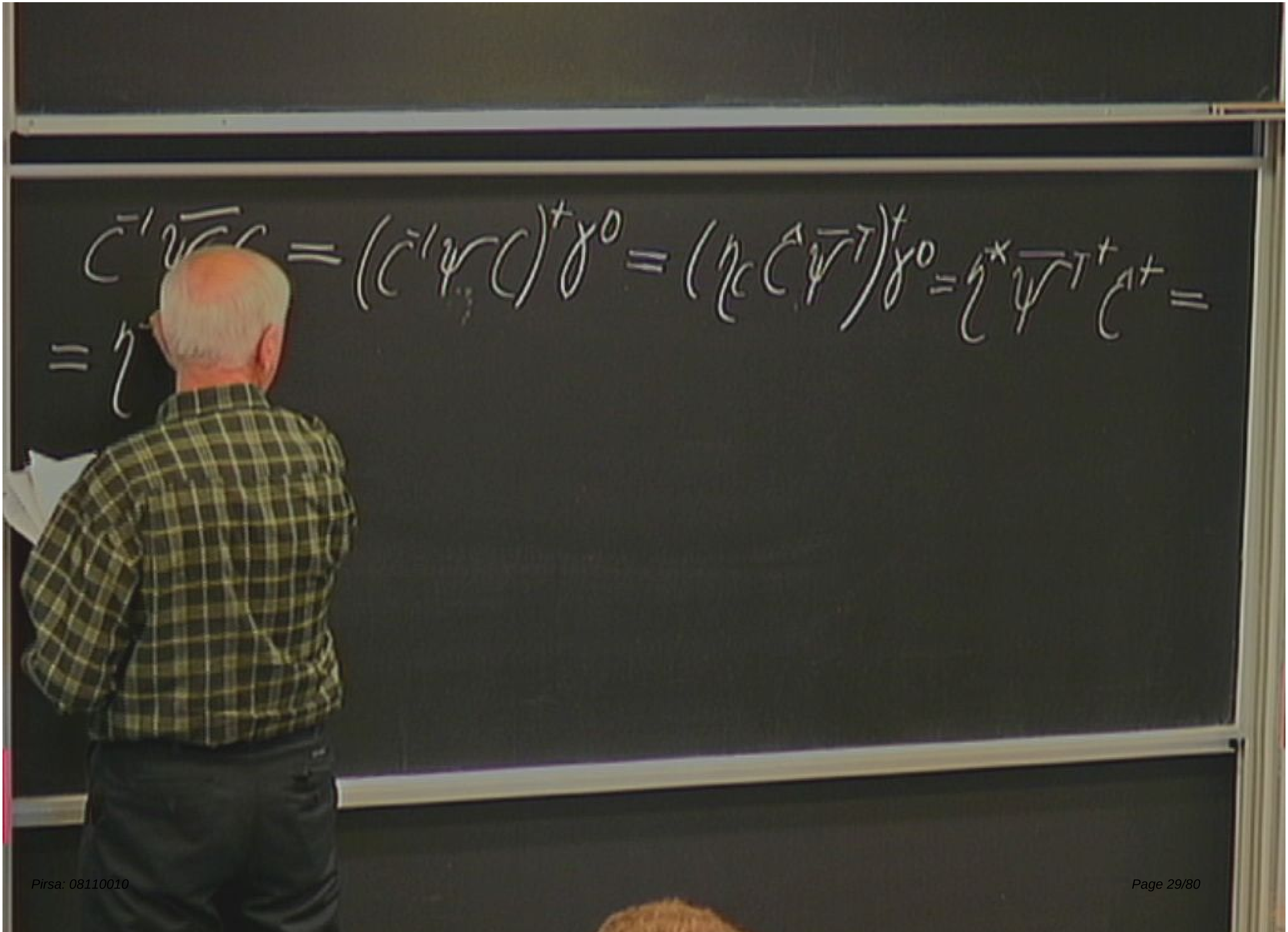
These pr…es of $\hat C$ are valid for all representations of γ^m … (chiral) representation $\hat C$ is real, $\hat C = -i$… , therefore, $\hat C^+ = \hat C^{*T} = \hat C^T = -\hat C$; $\hat C^{-1} = -\hat C$

$$= -i\eta_c\gamma^2\psi^*(x) = -i\eta_c\gamma^2\psi^{\dagger T} \underset{\gamma^{2T}=\gamma^2}{=} -i\eta_c(\psi^\dagger\gamma^2)^T = -i\eta_c(\bar\psi\gamma^0\gamma^2)^T$$

$$\equiv \eta_c\hat C\bar\psi^T, \text{ where } \underline{\hat C \equiv -i(\gamma^0\gamma^2)^T}$$

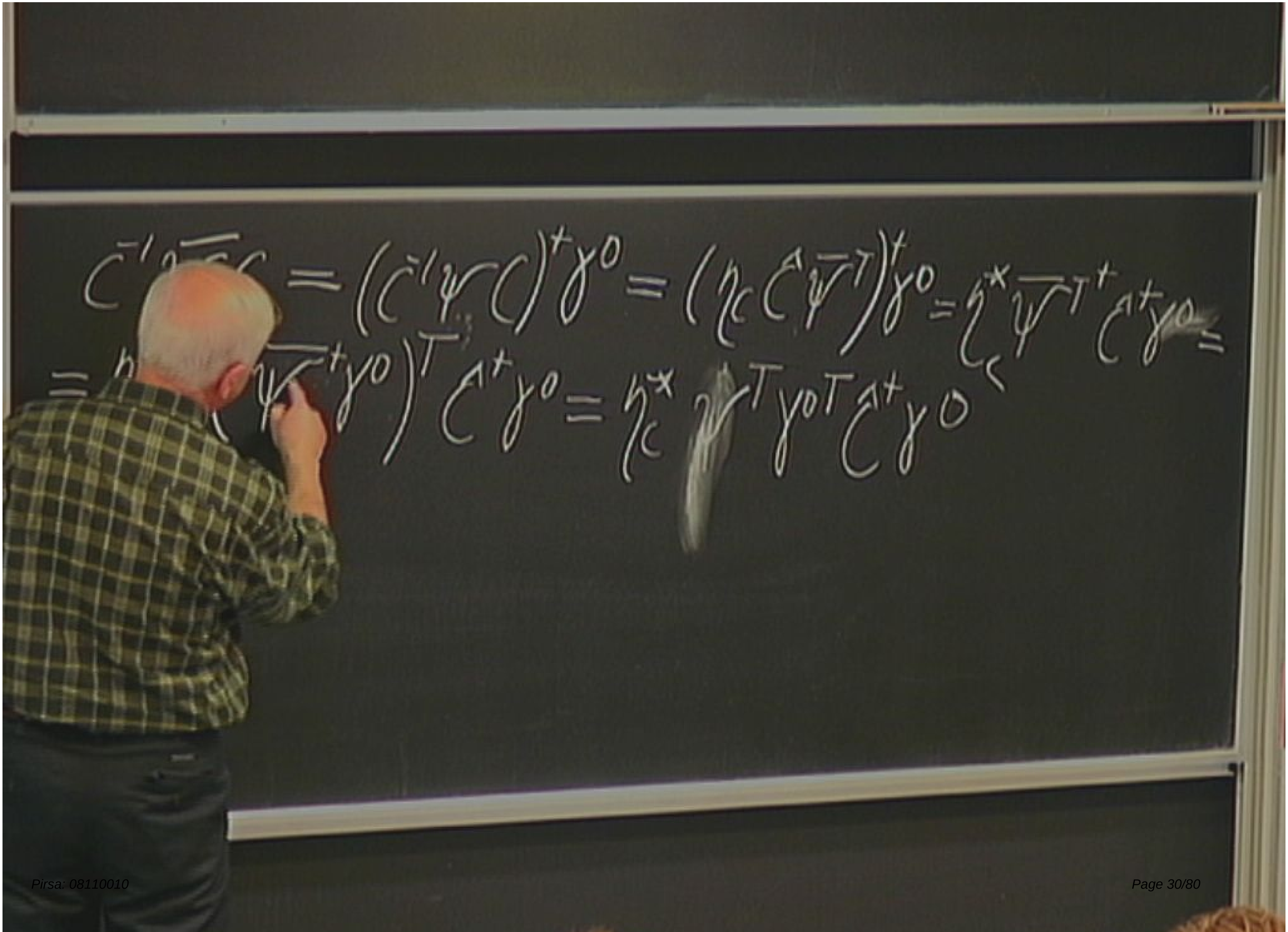
Properties of $\hat C$: $\hat C^\dagger = \hat C^{-1}$ (unitary), $C^T = -C$ (antisymmetric), $\hat C^{-1}\gamma^\mu\hat C = -\gamma^{\mu T}$, $\hat C^{-1}\gamma^5 C = \gamma^{5T}$; $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$

These properties of $\hat C$ are valid for all representations of γ^μ. In our (chiral) representation $\hat C$ is real, $\hat C = = -i\begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$, and, therefore, $\hat C^\dagger = \hat C^{*T} = \hat C^T = -\hat C$; $\hat C^{-1} = -\hat C$



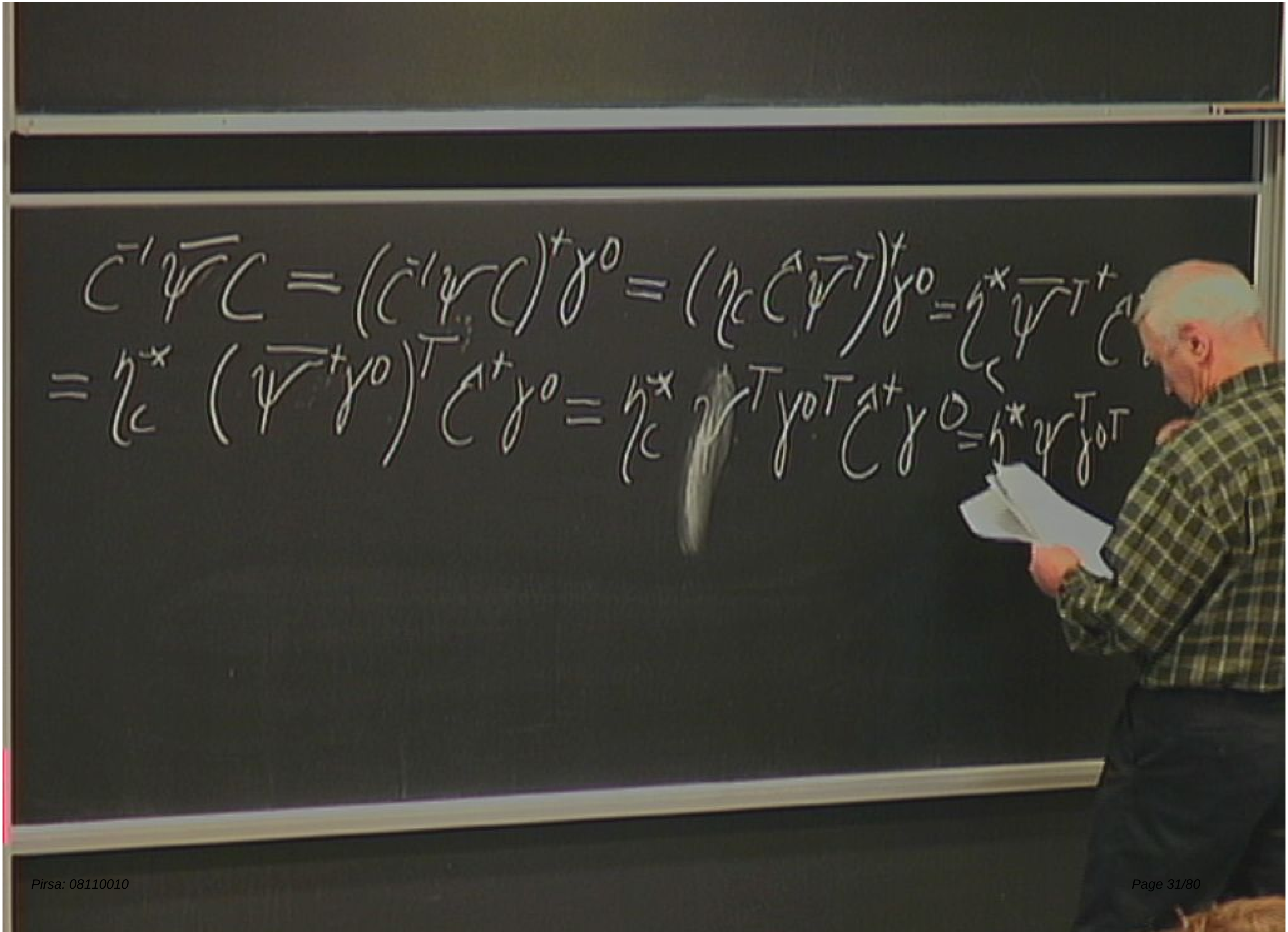
$$C^{-1}\bar{\psi}_c C = (C^{-1}\psi_c C)^\dagger\gamma^0 = (\eta_c C\bar{\psi}^T)^\dagger\gamma^0 = \eta^*\bar{\psi}^{T\dagger}C^\dagger =$$

$$= \eta$$



$$\hat{C}^{-1}\overline{\psi}\hat{C} = (\hat{C}^{-1}\psi\hat{C})^\dagger\gamma^0 = (\eta_c\hat{C}\overline{\psi}^T)^\dagger\gamma^0 = \eta_c^*\overline{\psi}^{T\dagger}\hat{C}^\dagger\gamma^0 =$$

$$= \eta_c^*(\overline{\psi}^\dagger\gamma^0)^T\hat{C}^\dagger\gamma^0 = \eta_c^*\psi^T\gamma^{0T}\hat{C}^\dagger\gamma^0$$



$$C^{-1}\bar{\psi}C = (C^{-1}\psi C)^{\dagger}\gamma^0 = (\eta_c \hat{C}\bar{\psi}^T)^{\dagger}\gamma^0 = \eta_c^* \bar{\psi}^{T\dagger}\hat{C}$$

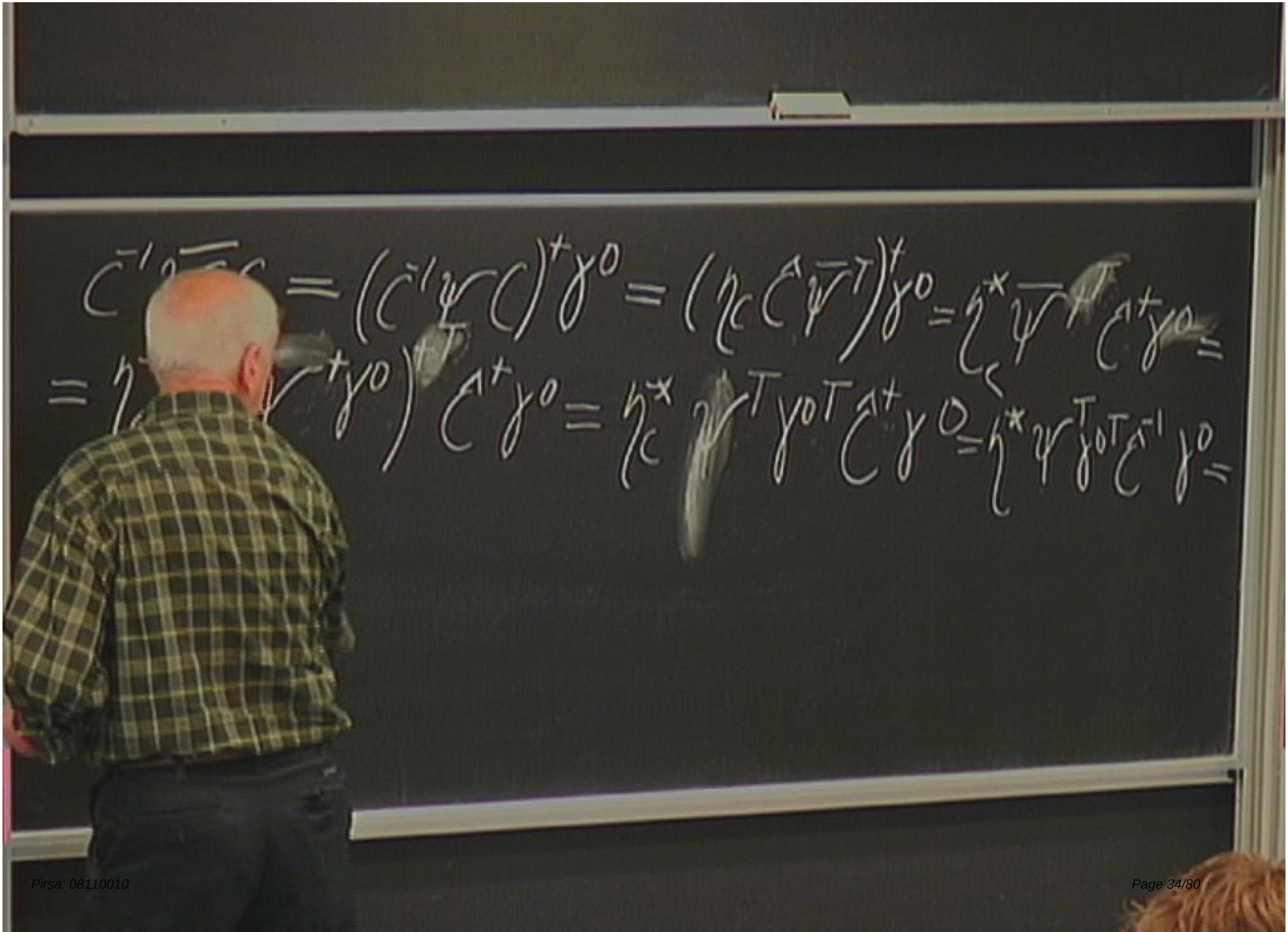
$$= \eta_c^* (\bar{\psi}^{\dagger}\gamma^0)^T \hat{C}^{\dagger}\gamma^0 = \eta_c^* \psi^T \gamma^{0T}\hat{C}^{\dagger}\gamma^0 = \eta^* \psi^T \gamma^{0T}$$

$$C^{-1}\bar{\psi}C = (C^{-1}\psi C)^{\dagger}\gamma^{0} = (\eta_c \hat{C}\bar{\psi}^{T})^{\dagger}\gamma^{0} = \eta_c^{*}\bar{\psi}^{T\dagger}\hat{C}^{\dagger}\gamma^{0} =$$

$$= \eta_c^{*}\,(\psi^{\dagger}\gamma^{0})^{T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\,\psi^{T}\gamma^{0T}\hat{C}^{\dagger}\gamma^{0} = \eta^{*}\psi^{T}\gamma^{0T}\hat{C}^{-1}$$

$$C^{-1}\bar{\psi}C = (C^{-1}\psi C)^{\dagger}\gamma^{0} = (\eta_c\hat{C}\bar{\psi}^{T})^{\dagger}\gamma^{0} = \eta_c^{*}\bar{\psi}^{T\dagger}\hat{C}^{\dagger}\gamma^{0} =$$

$$= \eta_c^{*}\,(\psi^{\dagger}\gamma^{0})^{T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\,\psi^{T}\gamma^{0T}\hat{C}^{\dagger}\gamma^{0} = \eta^{*}\psi^{T}\gamma^{0T}\hat{C}^{-1}$$



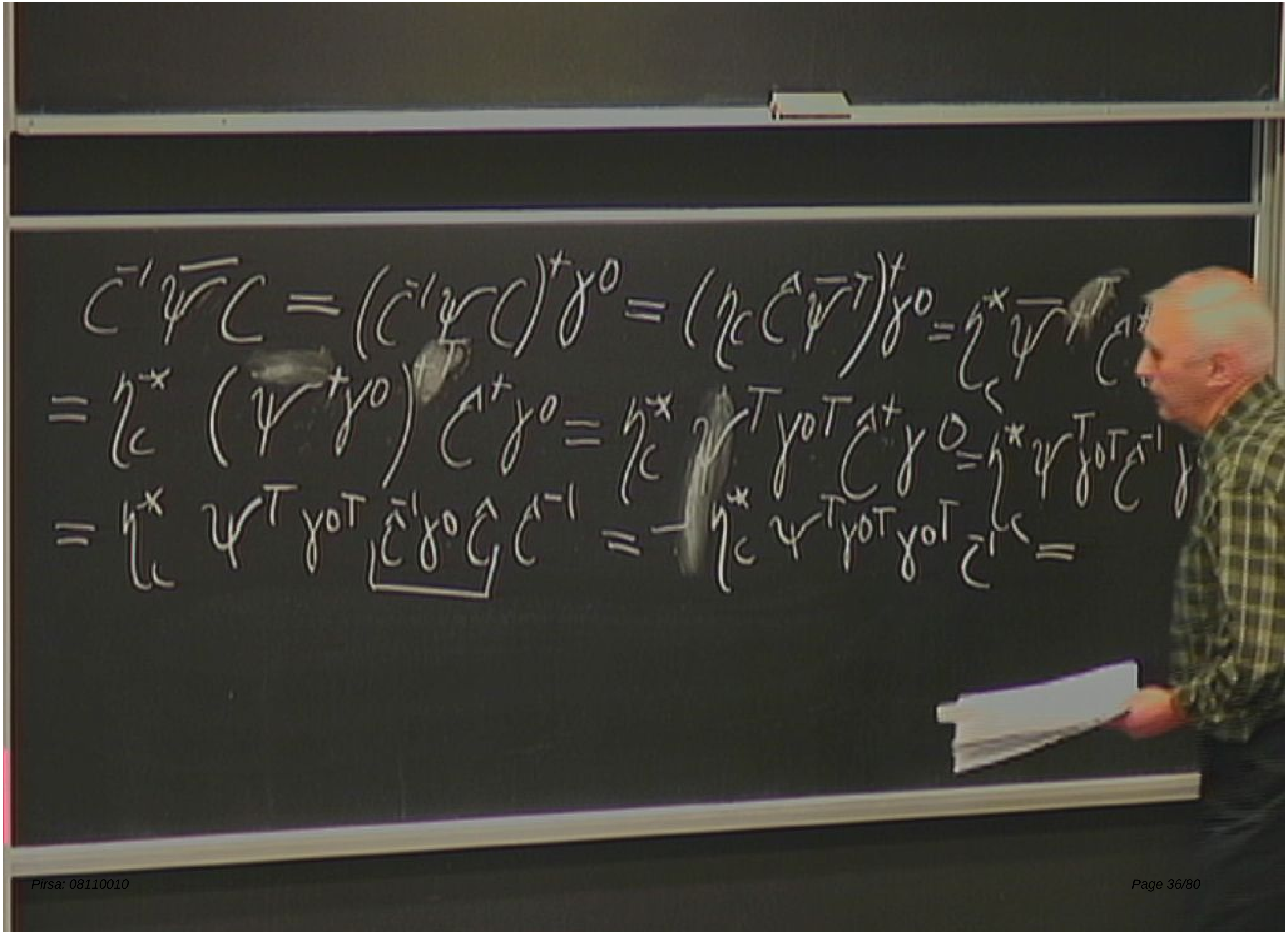
$$\hat{C}^{-1}\bar{\psi}\hat{C} = (\hat{C}^{-1}\psi\hat{C})^{\dagger}\gamma^{0} = (\eta_c\hat{C}\bar{\psi}^{T})^{\dagger}\gamma^{0} = \eta_c^{*}\bar{\psi}^{\dagger T}\hat{C}^{\dagger}\gamma^{0} =$$

$$= \eta_c^{*}(\psi^{\dagger}\gamma^{0})^{\dagger T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{-1}\gamma^{0} =$$

$$C^{-1}\bar{\psi}C = (C^{-1}\psi C)^{\dagger}\gamma^0 = (\eta_c\hat{C}\bar{\psi}^T)^{\dagger}\gamma^0 = \eta_c^*\bar{\psi}^{T}\hat{C}^{\dagger}\gamma^0 =$$

$$= \eta_c^*(\psi^{\dagger}\gamma^0)^T\hat{C}^{\dagger}\gamma^0 = \eta_c^*\psi^T\gamma^{0T}\hat{C}^{\dagger}\gamma^0 = \eta_c^*\psi^T\gamma^{0T}\hat{C}^{-1}\gamma^0 =$$

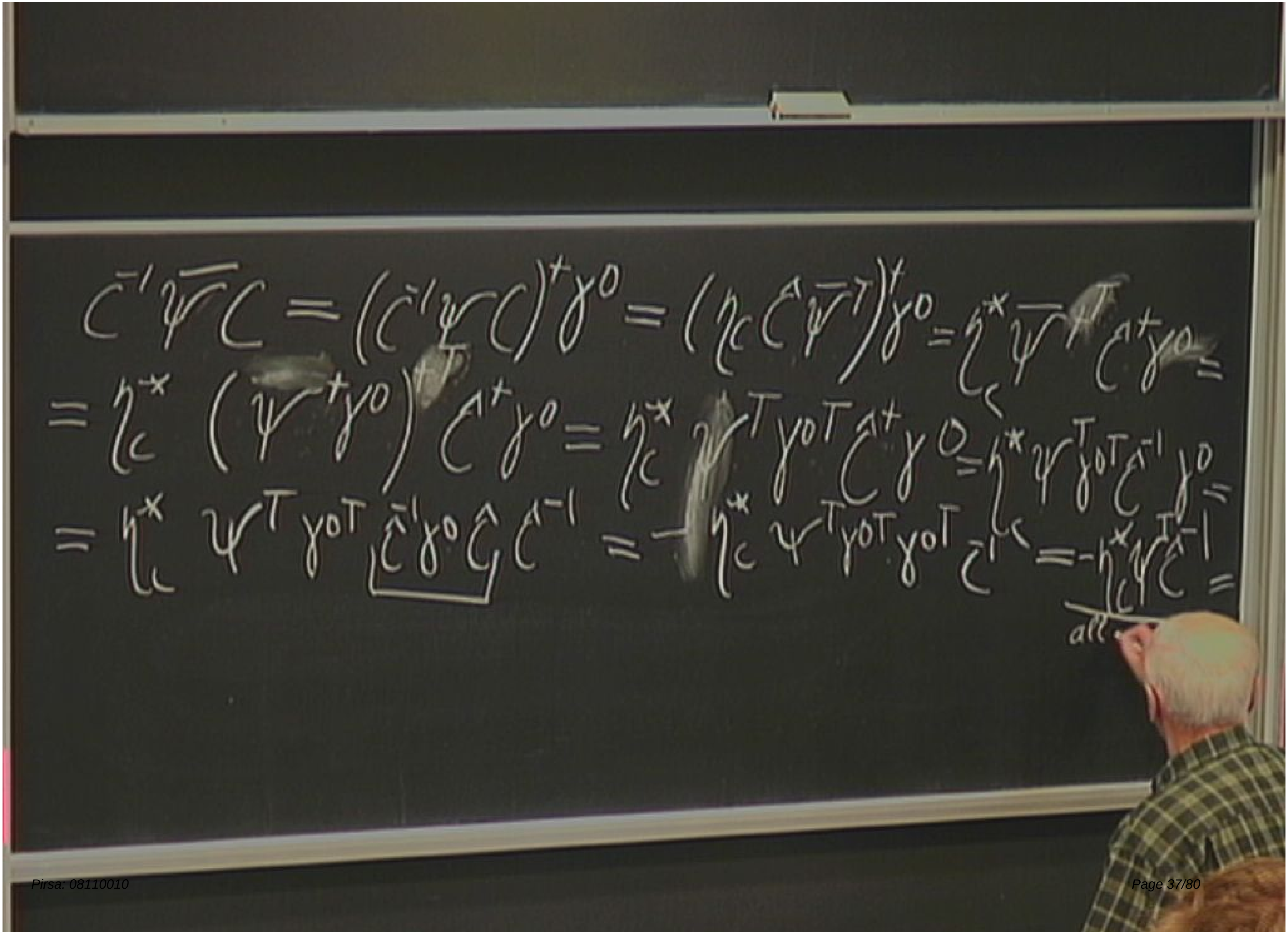
$$= \eta_c^*\psi^T\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^0\hat{C}}\hat{C}^{-1} =$$



$$\hat{C}^{-1}\bar{\psi}\hat{C} = (\hat{C}^{-1}\psi\hat{C})^{\dagger}\gamma^{0} = (\eta_c\hat{C}\bar{\psi}^{T})^{\dagger}\gamma^{0} = \eta_c^{*}\bar{\psi}^{T\dagger}\hat{C}^{\dagger}$$

$$= \eta_c^{*}(\psi^{\dagger}\gamma^{0})^{T\dagger}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{-1}\gamma$$

$$= \eta_c^{*}\psi^{T}\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^{0}\hat{C}}_{}\hat{C}^{-1} = -\eta_c^{*}\psi^{T}\gamma^{0T}\gamma^{0T}\hat{C}^{-1} =$$

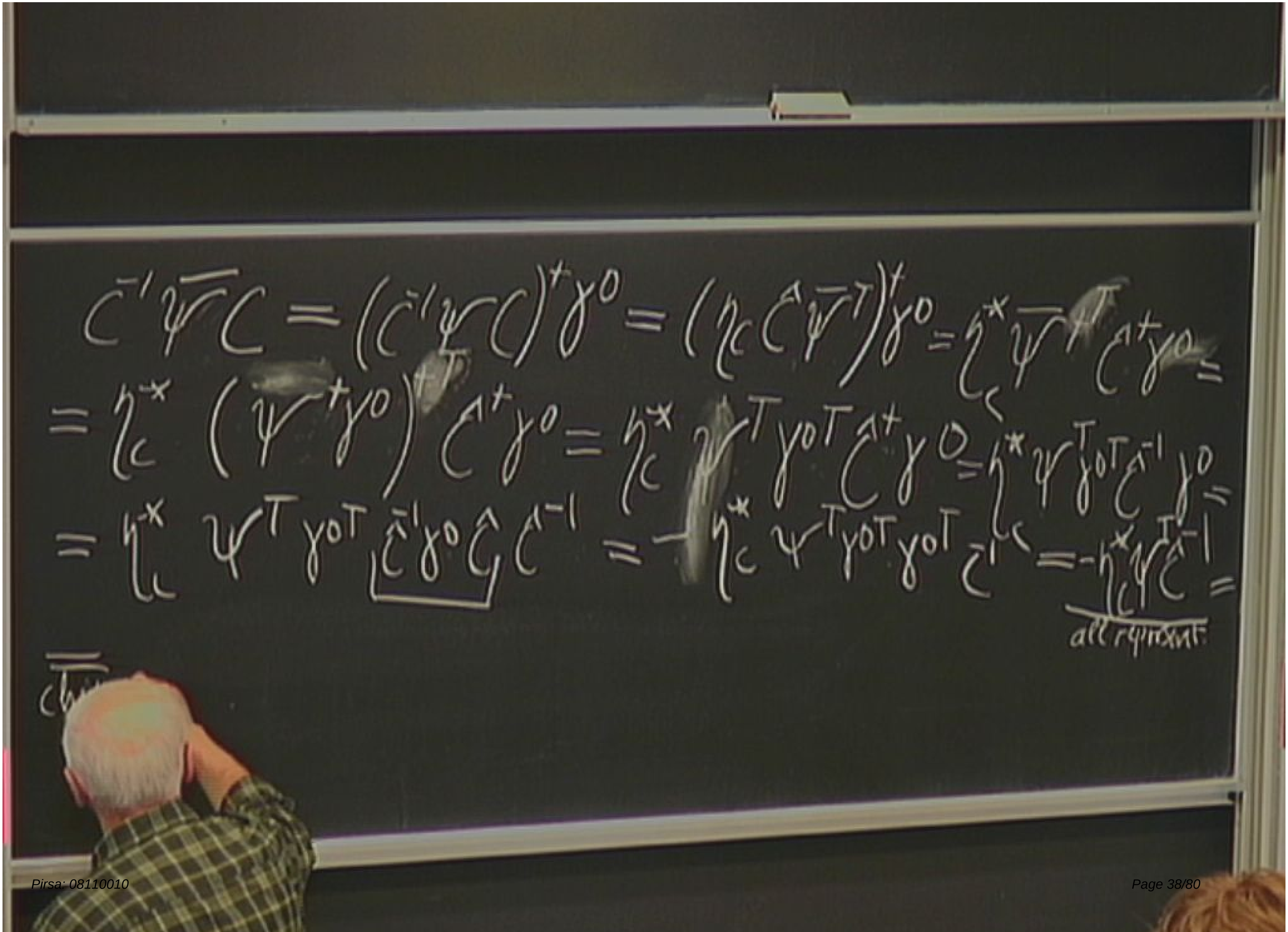


$$\hat{C}^{-1}\bar{\psi}\hat{C} = (\hat{C}^{-1}\psi\hat{C})^{\dagger}\gamma^0 = (\eta_c\hat{C}\bar{\psi}^T)^{\dagger}\gamma^0 = \eta_c^*\bar{\psi}^{T\dagger}\hat{C}^{\dagger}\gamma^0 =$$

$$= \eta_c^*(\psi^{\dagger}\gamma^0)^{T}\hat{C}^{\dagger}\gamma^0 = \eta_c^*\psi^T\gamma^{0T}\hat{C}^{\dagger}\gamma^0 = \eta_c^*\psi^T\gamma^{0T}\hat{C}^{-1}\gamma^0 =$$

$$= \eta_c^*\psi^T\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^0\hat{C}}\hat{C}^{-1} = -\eta_c^*\psi^T\gamma^{0T}\gamma^{0T}\hat{C}^{-1} = \underline{-\eta_c^*\psi^T\hat{C}^{-1}} =$$

all



$$\hat{C}^{-1}\bar{\psi}\hat{C} = (\hat{C}^{-1}\psi\hat{C})^{\dagger}\gamma^{0} = (\eta_c\hat{C}\bar{\psi}^{T})^{\dagger}\gamma^{0} = \eta_c^{*}\bar{\psi}^{T\dagger}\hat{C}^{\dagger}\gamma^{0} =$$

$$= \eta_c^{*}(\psi^{\dagger}\gamma^{0})^{T\dagger}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{-1}\gamma^{0} =$$

$$= \eta_c^{*}\psi^{T}\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^{0}\hat{C}}\hat{C}^{-1} = -\eta_c^{*}\psi^{T}\gamma^{0T}\gamma^{0T}\hat{C}^{-1} = \underbrace{-\eta_c^{*}\psi^{T}\hat{C}^{-1}}_{\text{all reprsnt.}} =$$

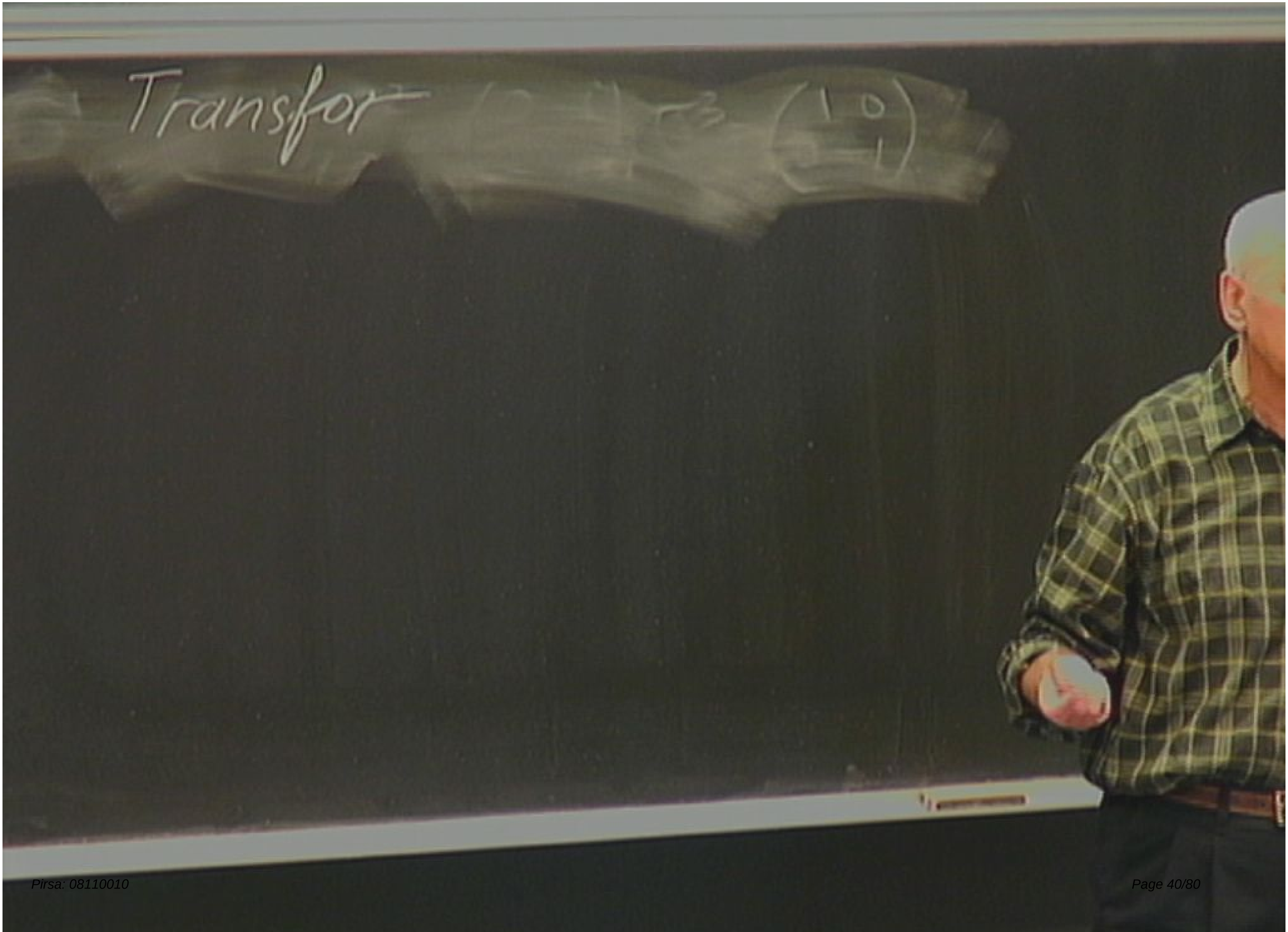
$\bar{\bar{\ }}$ ch

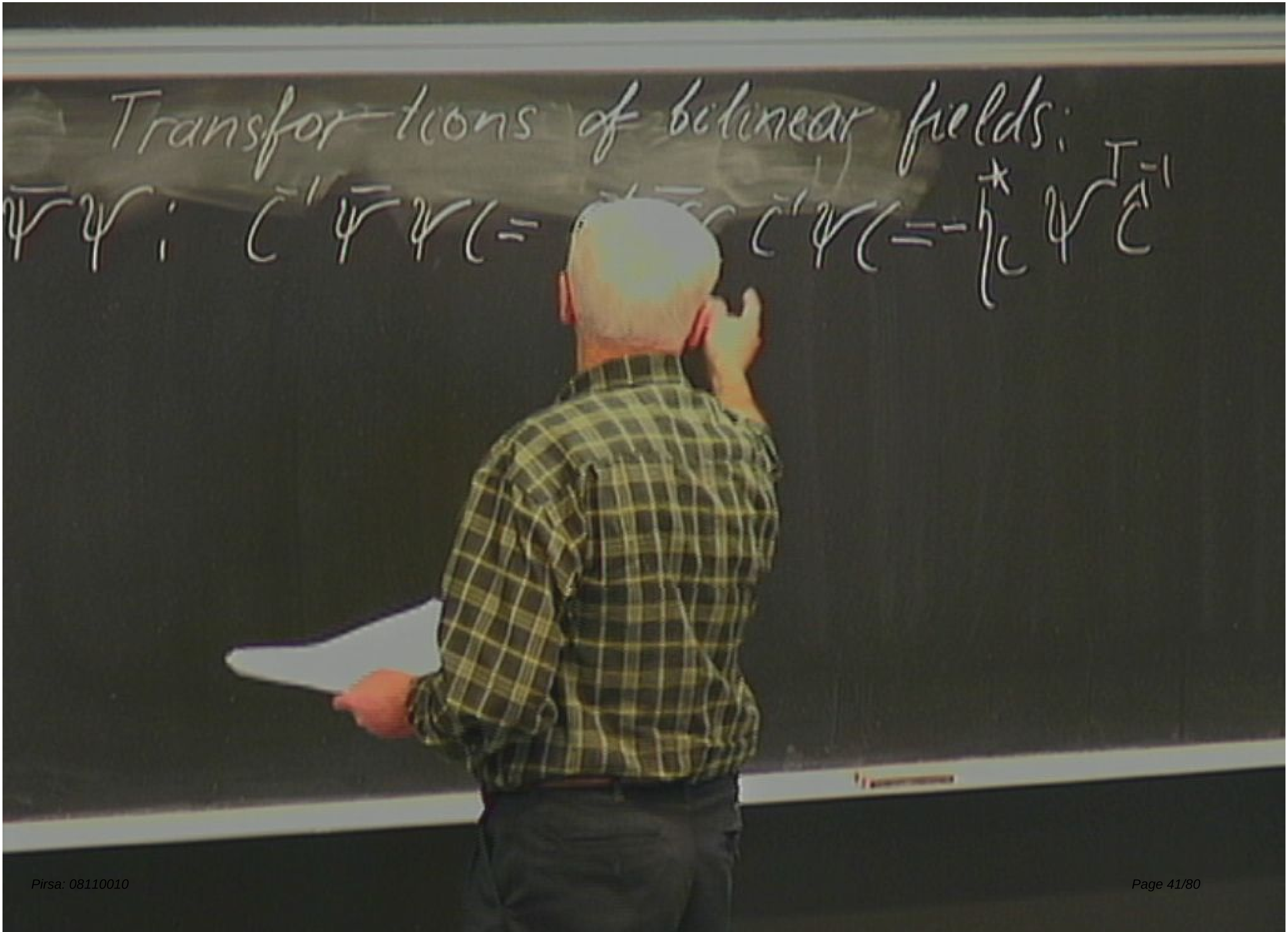
$$\hat{C}^{-1}\overline{\overline{\psi}}\hat{C} = (\hat{C}^{-1}\psi\hat{C})^{\dagger}\gamma^{0} = (\eta_c\hat{C}\overline{\psi}^{T})^{\dagger}\gamma^{0} = \eta_c^{*}\overline{\psi}^{T\dagger}\hat{C}^{\dagger}\gamma^{0} =$$

$$= \eta_c^{*}(\psi^{\dagger}\gamma^{0})^{T\dagger}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{\dagger}\gamma^{0} = \eta_c^{*}\psi^{T}\gamma^{0T}\hat{C}^{-1}\gamma^{0} =$$

$$= \eta_c^{*}\psi^{T}\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^{0}\hat{C}}\hat{C}^{-1} = -\eta_c^{*}\psi^{T}\gamma^{0T}\gamma^{0T}\hat{C}^{-1} = \underset{\text{all represent.}}{\underline{-\eta_c^{*}\psi^{T}\hat{C}^{-1}}} =$$

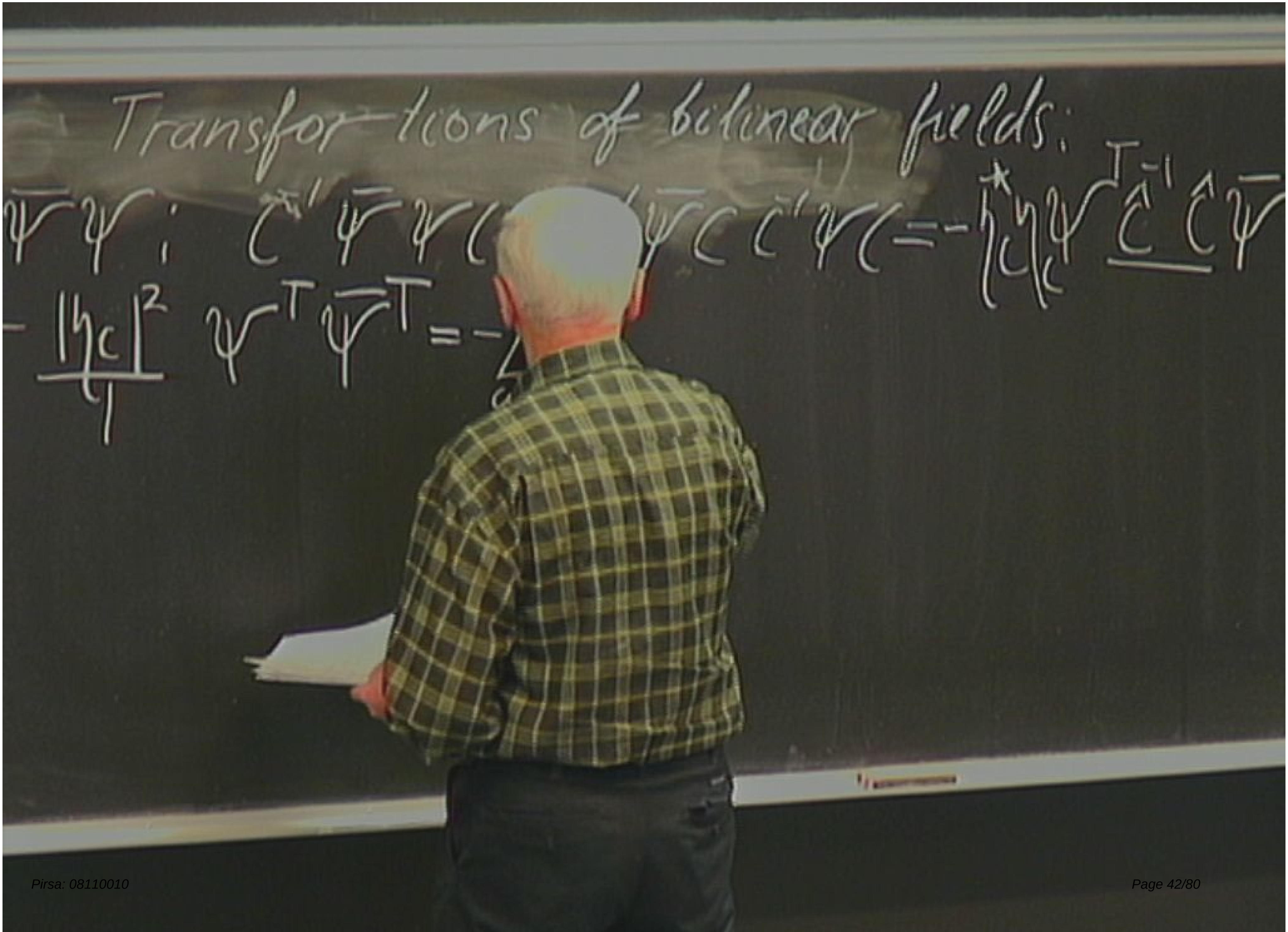
$$\underset{\text{chiral repres.}}{=} \underset{\text{Peskin}}{\underline{\eta_c^{*}\psi^{T}\hat{C}}}$$


Transfor



Transfor-tions of bilinear fields:

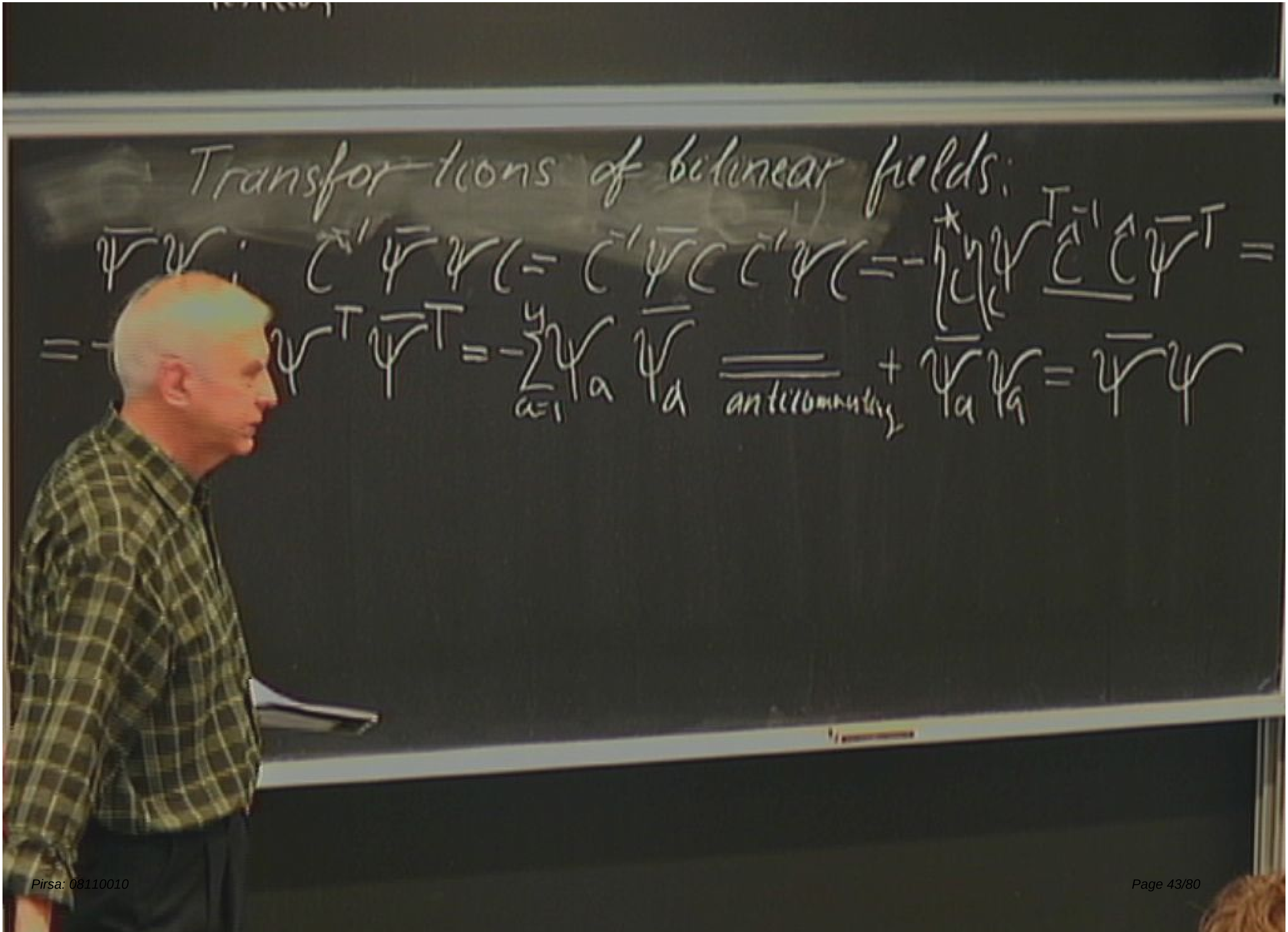
$\bar{\psi}\psi$: $C^{-1}\bar{\psi}\psi C =$ [illegible] $C^{-1}\psi C = -\eta_c^{*}\psi^{T}C^{-1}$



Transfor-tions of bilinear fields:

$\bar{\psi}\psi$: $C^{-1}\bar{\psi}\psi C$ $\bar{\psi} C C^{-1}\psi C = -\eta_c^{*}\eta_c \psi^{T} \underline{C^{-1}} C \bar{\psi}^{T}$

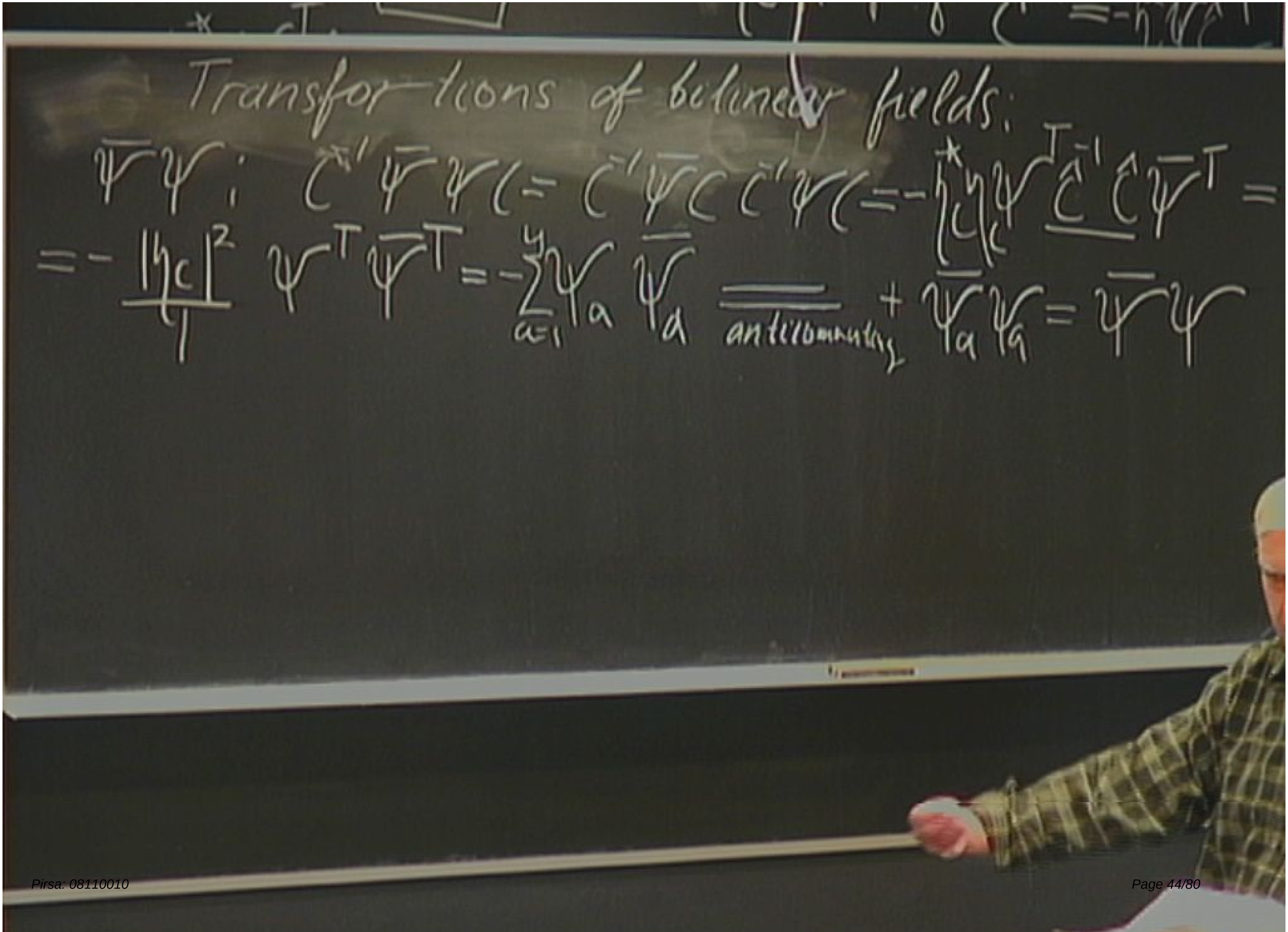
$-\underline{|\eta_c|^2}_{1} \psi^{T}\bar{\psi}^{T} = -$



Transfor-tions of bilinear fields:

$$\bar{\psi}\psi: \quad C^{-1}\bar{\psi}\psi C = C^{-1}\bar{\psi}C\,C^{-1}\psi C = -\eta_c^*\eta_c\,\psi^T \underline{C^{-1}C}\,\bar{\psi}^T =$$

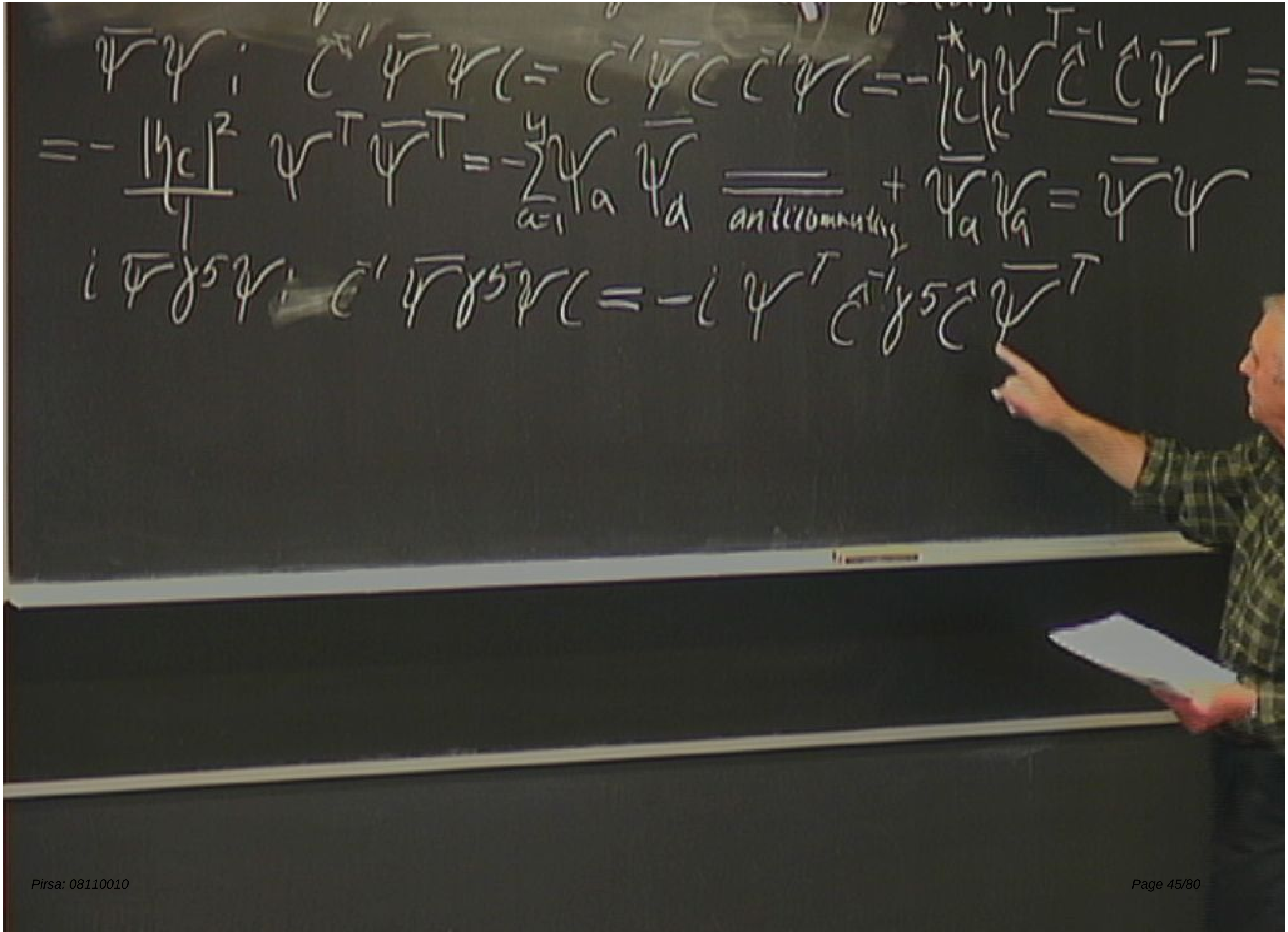
$$= - \psi^T\bar{\psi}^T = -\sum_{a=1}^{4}\psi_a\bar{\psi}_a \underset{\text{anticommuting}}{=} +\bar{\psi}_a\psi_a = \bar{\psi}\psi$$



Transfor-tions of bilinear fields:

$$\bar\psi\psi: \; C^{-1}\bar\psi\psi C = C^{-1}\bar\psi C\, C^{-1}\psi C = -\underbrace{\eta_c^*\eta_c}\,\psi^T \underline{C^{-1} C}\,\bar\psi^T =$$

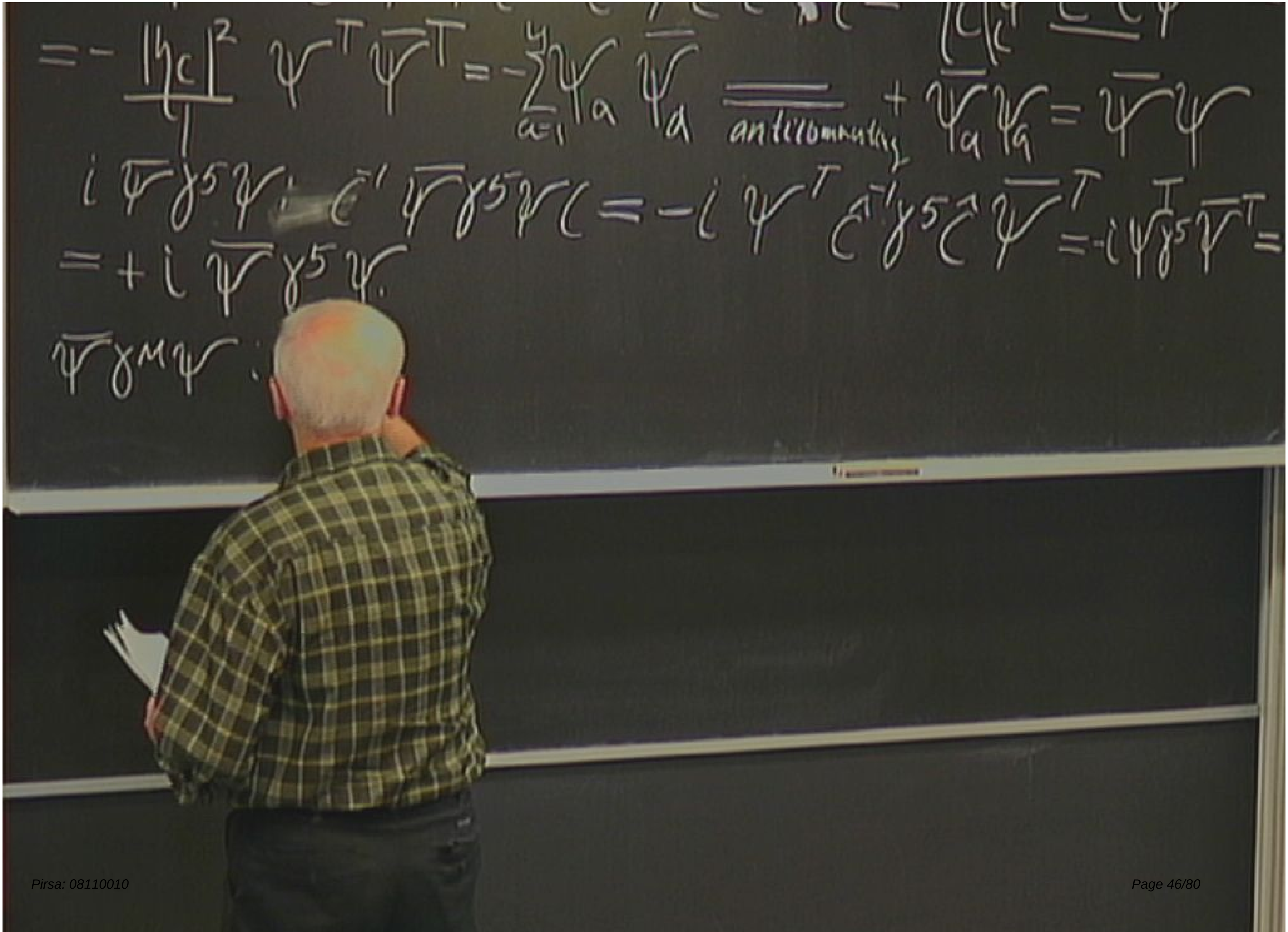
$$= -\underbrace{|\eta_c|^2}_{1}\,\psi^T\bar\psi^T = -\sum_{a=1}^{4}\psi_a\bar\psi_a \underset{\text{anticommuting}}{=} +\bar\psi_a\psi_a = \bar\psi\psi$$



$$\bar\psi\psi : \; C^{-1}\bar\psi\psi C = C^{-1}\bar\psi C C^{-1}\psi C = -\eta_c^{*}\eta_c \psi^T \underline{C^{-1} C}\, \bar\psi^T =$$

$$= -\underbrace{|\eta_c|^2}_{1} \psi^T \bar\psi^T = -\sum_{a=1}^{4} \psi_a \bar\psi_a \underset{\text{anticommuting}}{=} + \bar\psi_a \psi_a = \bar\psi\psi$$

$$i\bar\psi\gamma^5\psi : \; C^{-1}\bar\psi\gamma^5\psi C = -i\,\psi^T C^{-1}\gamma^5 C \bar\psi^T$$

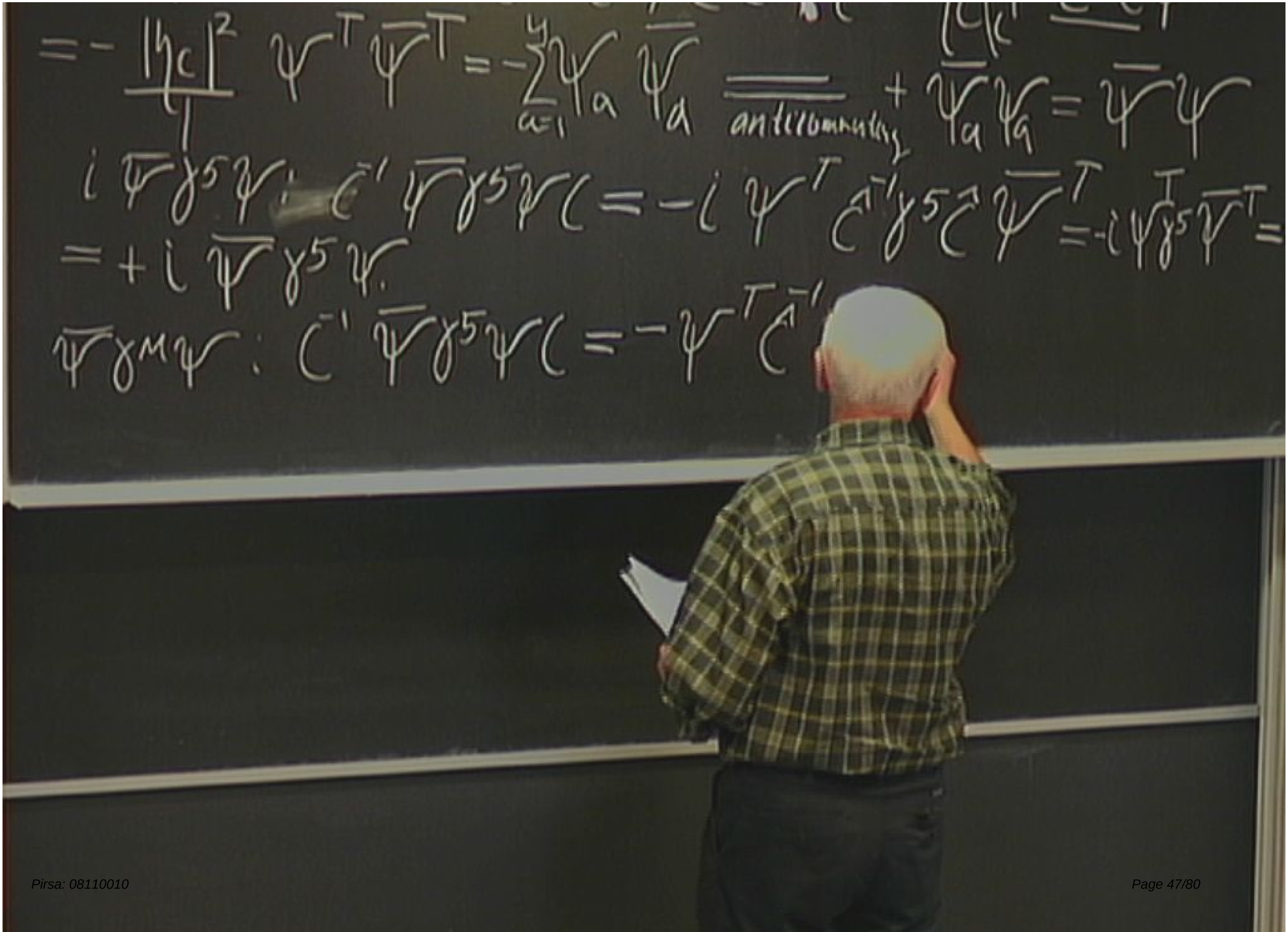


$$= -\underbrace{|\eta_c|^2}_{1} \psi^T \bar{\psi}^T = -\sum_{a=1}^{4} \psi_a \bar{\psi}_a \underset{\text{anticommuting}}{=} + \bar{\psi}_a \psi_a = \bar{\psi} \psi$$

$$i \bar{\psi} \gamma^5 \psi : C^{-1} \bar{\psi} \gamma^5 \psi C = -i \psi^T C^{-1} \gamma^5 C \bar{\psi}^T = -i \psi^T \gamma^5 \bar{\psi}^T =$$

$$= +i \bar{\psi} \gamma^5 \psi.$$

$$\bar{\psi} \gamma^\mu \psi :$$



$$= -\frac{|\eta_c|^2}{1} \psi^T \bar{\psi}^T = -\sum_{a=1}^{4} \psi_a \bar{\psi}_a \underset{\text{anticommuting}}{=} + \bar{\psi}_a \psi_a = \bar{\psi}\psi$$

$$i\bar{\psi}\gamma^5\psi : C^{-1}\bar{\psi}\gamma^5\psi C = -i\,\psi^T C^{-1}\gamma^5 C\,\bar{\psi}^T = -i\,\psi^T\gamma^{5T}\bar{\psi}^T =$$

$$= +i\,\bar{\psi}\gamma^5\psi.$$

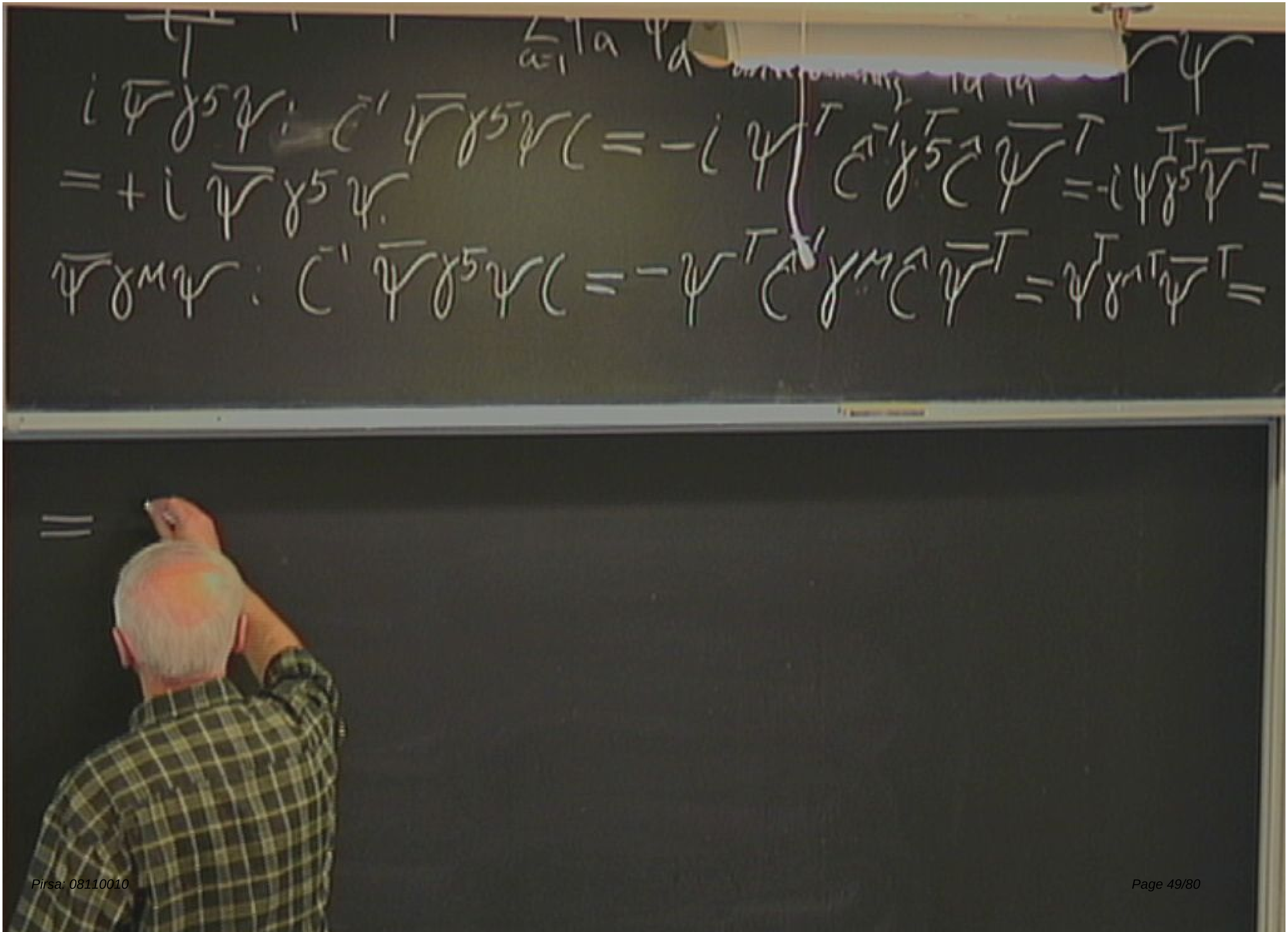
$$\bar{\psi}\gamma^\mu\psi : C^{-1}\bar{\psi}\gamma^5\psi C = -\psi^T C^{-1}$$

$$= -\underbrace{|\eta_c|^2}_{1} \psi^T \bar{\psi}^T = -\sum_{a=1}^{4} \psi_a \bar{\psi}_a \underset{\text{anticommuting}}{=} + \bar{\psi}_a \psi_a = \bar{\psi}\psi$$

$$i\bar{\psi}\gamma^5\psi : \; C^{-1}\bar{\psi}\gamma^5\psi C = -i\psi^T C^{-1}\gamma^5 C \bar{\psi}^T = -i\psi^T\gamma^5\bar{\psi}^T =$$

$$= +i\bar{\psi}\gamma^5\psi.$$

$$\bar{\psi}\gamma^\mu\psi : \; C^{-1}\bar{\psi}\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu C \bar{\psi}^T =$$



$\sum_{a=1} \eta_a \psi_a$

$i\bar{\psi}\gamma^5\psi$: $C^{-1}\bar{\psi}\gamma^5\psi C = -i\psi^T C^{-1}\gamma^{5T} C\bar{\psi}^T = -i\psi^T\gamma^{5T}\bar{\psi}^T =$

$= +i\bar{\psi}\gamma^5\psi.$

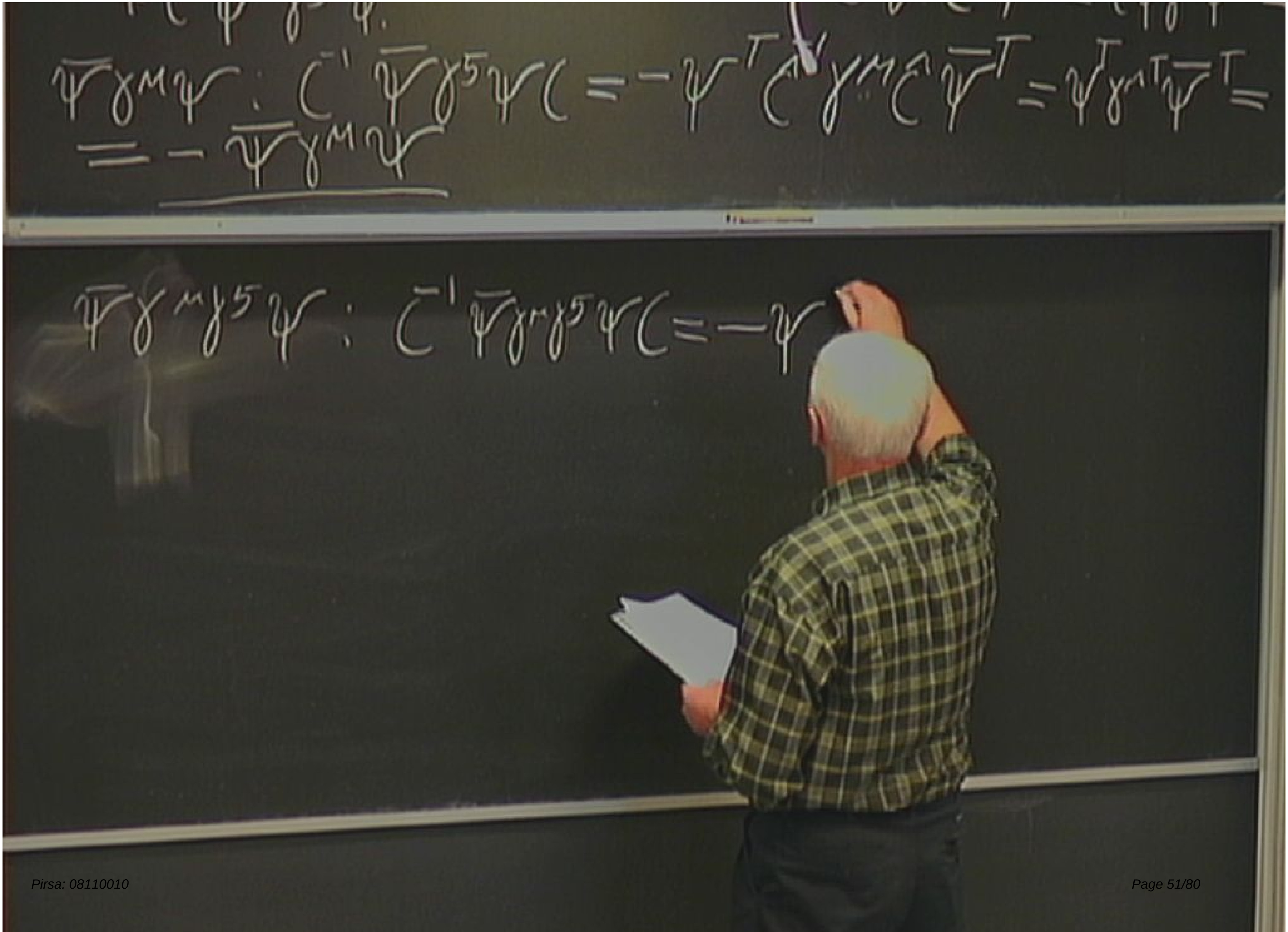
$\bar{\psi}\gamma^\mu\psi$: $C^{-1}\bar{\psi}\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu C\bar{\psi}^T = \psi^T\gamma^{\mu T}\bar{\psi}^T =$

$=$

$$\bar{\psi}\gamma^{\mu}\psi:\ C^{-1}\bar{\psi}\gamma^{5}\psi C=-\psi^{T}C^{-1}\gamma^{\mu}C\bar{\psi}^{T}=\psi^{T}\gamma^{\mu T}\bar{\psi}^{T}=$$

$$=\underline{-\bar{\psi}\gamma^{\mu}\psi}$$

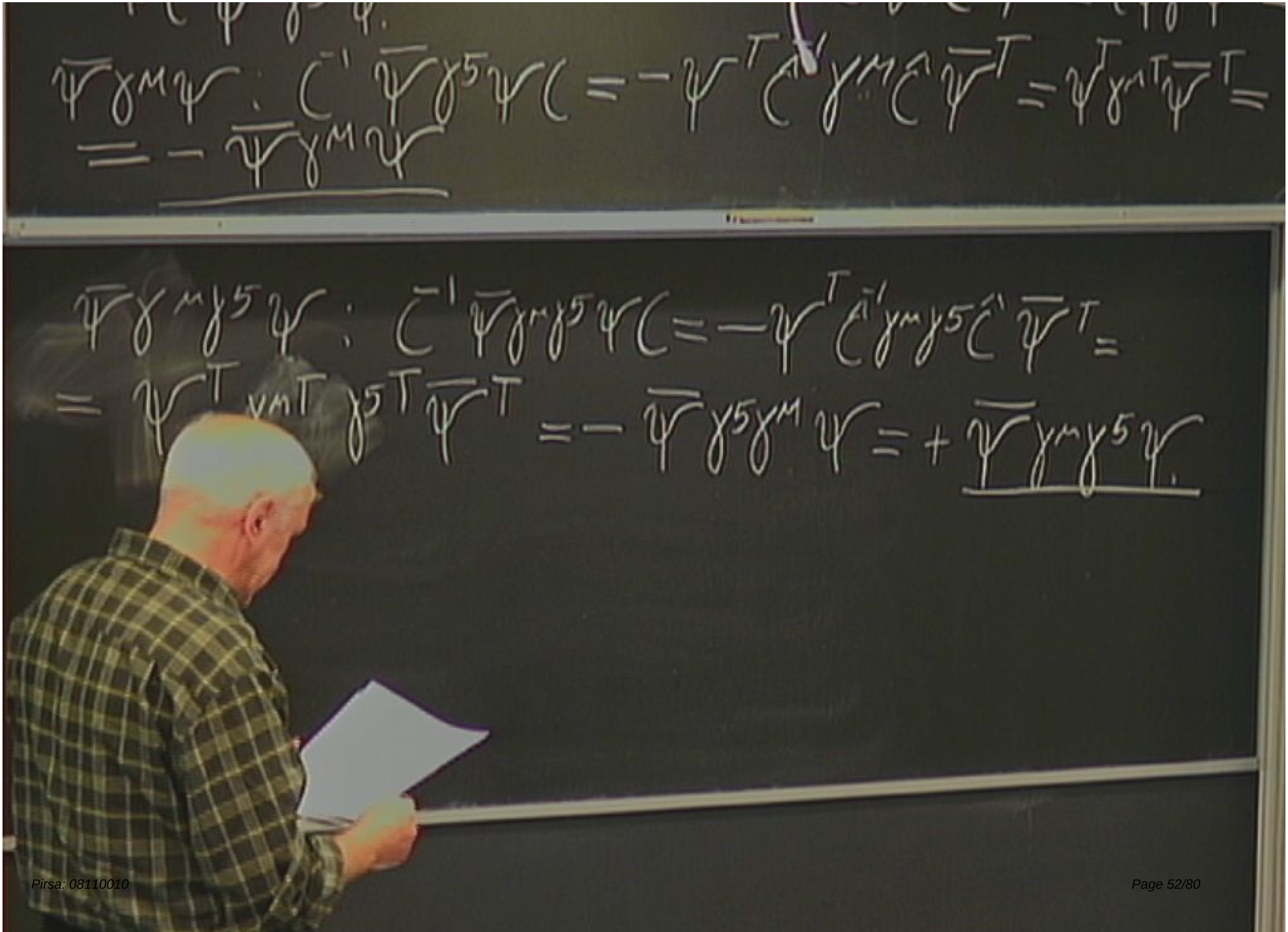
$$=\Bigg\{$$



$$\bar{\psi}\gamma^{\mu}\psi : \ C^{-1}\bar{\psi}\gamma^{5}\psi C = -\psi^{T}C^{-1}\gamma^{\mu}C\bar{\psi}^{T} = \psi^{T}\gamma^{\mu T}\bar{\psi}^{T} =$$

$$= -\underline{\bar{\psi}\gamma^{\mu}\psi}$$

$$\bar{\psi}\gamma^{\mu}\gamma^{5}\psi : \ C^{-1}\bar{\psi}\gamma^{\mu}\gamma^{5}\psi C = -\psi$$



$$\bar{\psi}\gamma^{\mu}\psi : \; C^{-1}\bar{\psi}\gamma^{5}\psi C = -\psi^{T}C^{-1}\gamma^{\mu}\hat{C}\bar{\psi}^{T} = \psi^{T}\gamma^{\mu T}\bar{\psi}^{T} =$$

$$= -\underline{\bar{\psi}\gamma^{\mu}\psi}$$

$$\bar{\psi}\gamma^{\mu}\gamma^{5}\psi : \; C^{-1}\bar{\psi}\gamma^{\mu}\gamma^{5}\psi C = -\psi^{T}C^{-1}\gamma^{\mu}\gamma^{5}\hat{C}\bar{\psi}^{T} =$$

$$= \psi^{T}\gamma^{\mu T}\gamma^{5T}\bar{\psi}^{T} = -\bar{\psi}\gamma^{5}\gamma^{\mu}\psi = +\underline{\bar{\psi}\gamma^{\mu}\gamma^{5}\psi.}$$

$$\bar\psi\gamma^\mu\psi : \; C^{-1}\bar\psi\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu C\bar\psi^T = \psi^T\gamma^{\mu T}\bar\psi^T =$$

$$= -\underline{\bar\psi\gamma^\mu\psi}$$

$$\bar\psi\gamma^\mu\gamma^5\psi : \; C^{-1}\bar\psi\gamma^\mu\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu\gamma^5 C\bar\psi^T =$$

$$= \psi^T\gamma^{\mu T}\gamma^{5T}\bar\psi^T = -\bar\psi\gamma^5\gamma^\mu\psi = +\underline{\bar\psi\gamma^\mu\gamma^5\psi}.$$

$$\bar\psi\gamma^\mu\psi : \; C^{-1}\bar\psi\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu C\bar\psi^T = \psi^T\gamma^{\mu T}\bar\psi^T =$$

$$= -\underline{\bar\psi\gamma^\mu\psi}$$

$$\bar\psi\gamma^\mu\gamma^5\psi : \; C^{-1}\bar\psi\gamma^\mu\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu\gamma^5 C\bar\psi^T =$$

$$= \psi^T\gamma^{\mu T}\gamma^{5T}\bar\psi^T = -\bar\psi\gamma^5\gamma^\mu\psi = +\underline{\bar\psi\gamma^\mu\gamma^5\psi.}$$

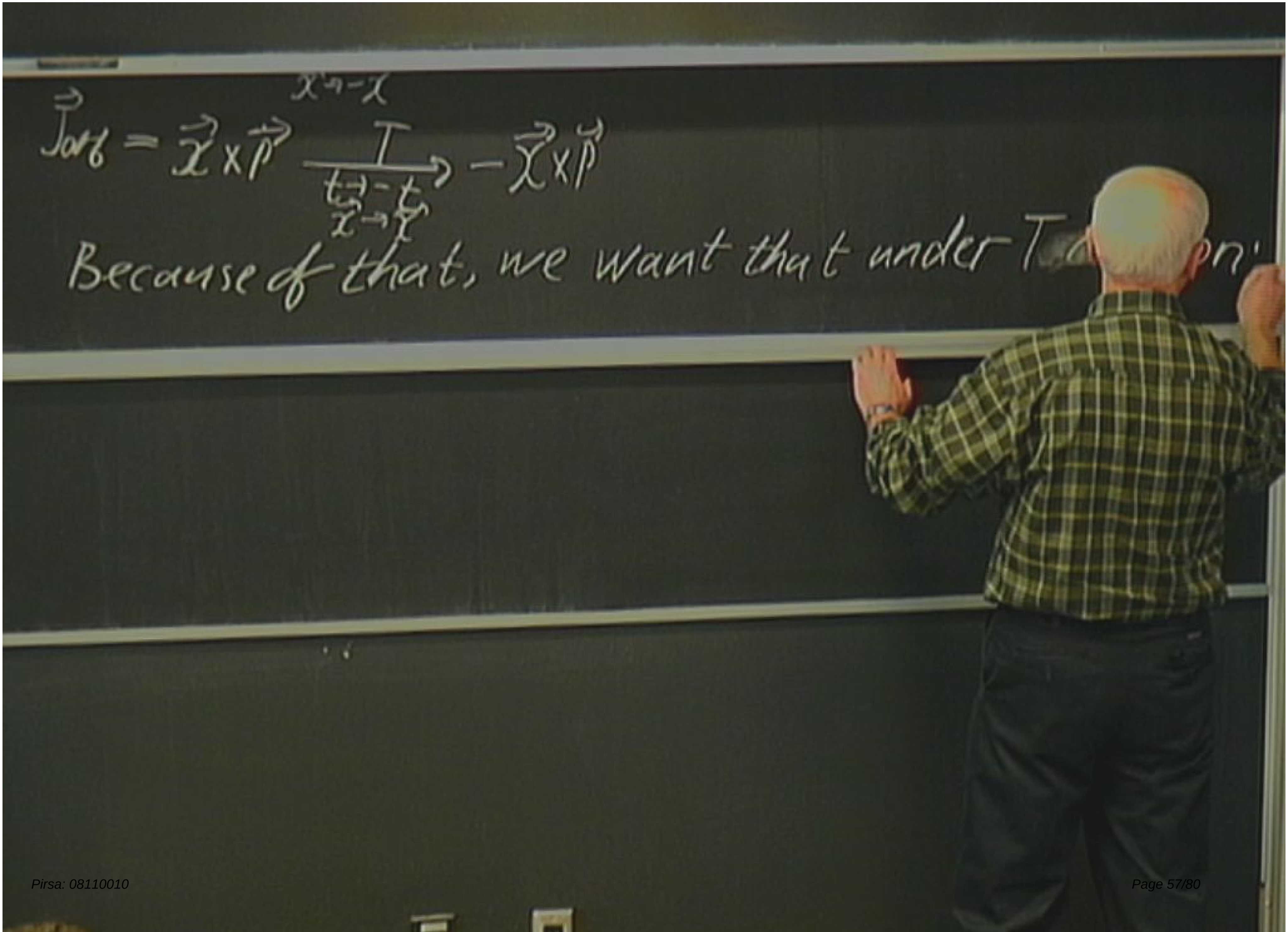
Neutr. curr.: $\bar\psi\gamma^\mu\frac{(1-\gamma^5)}{2}\psi$

Time Reversal (E. Wigner).

Time Reversal (E. Wigner).

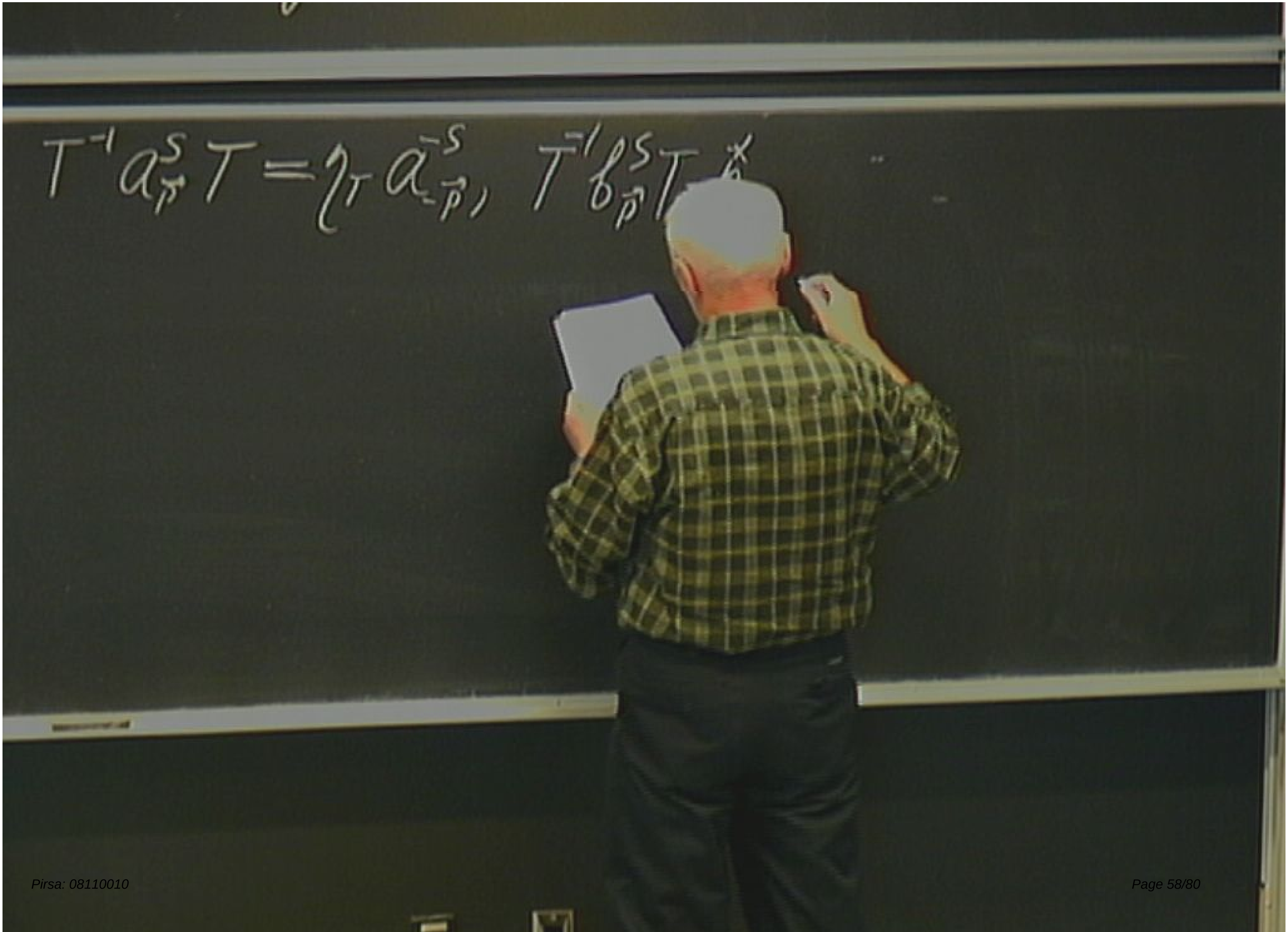
$(t, \vec{x}) \to (-t, \vec{x}) \Rrightarrow \vec{P} \to -\vec{P}$ (like under P) and

$S \to$

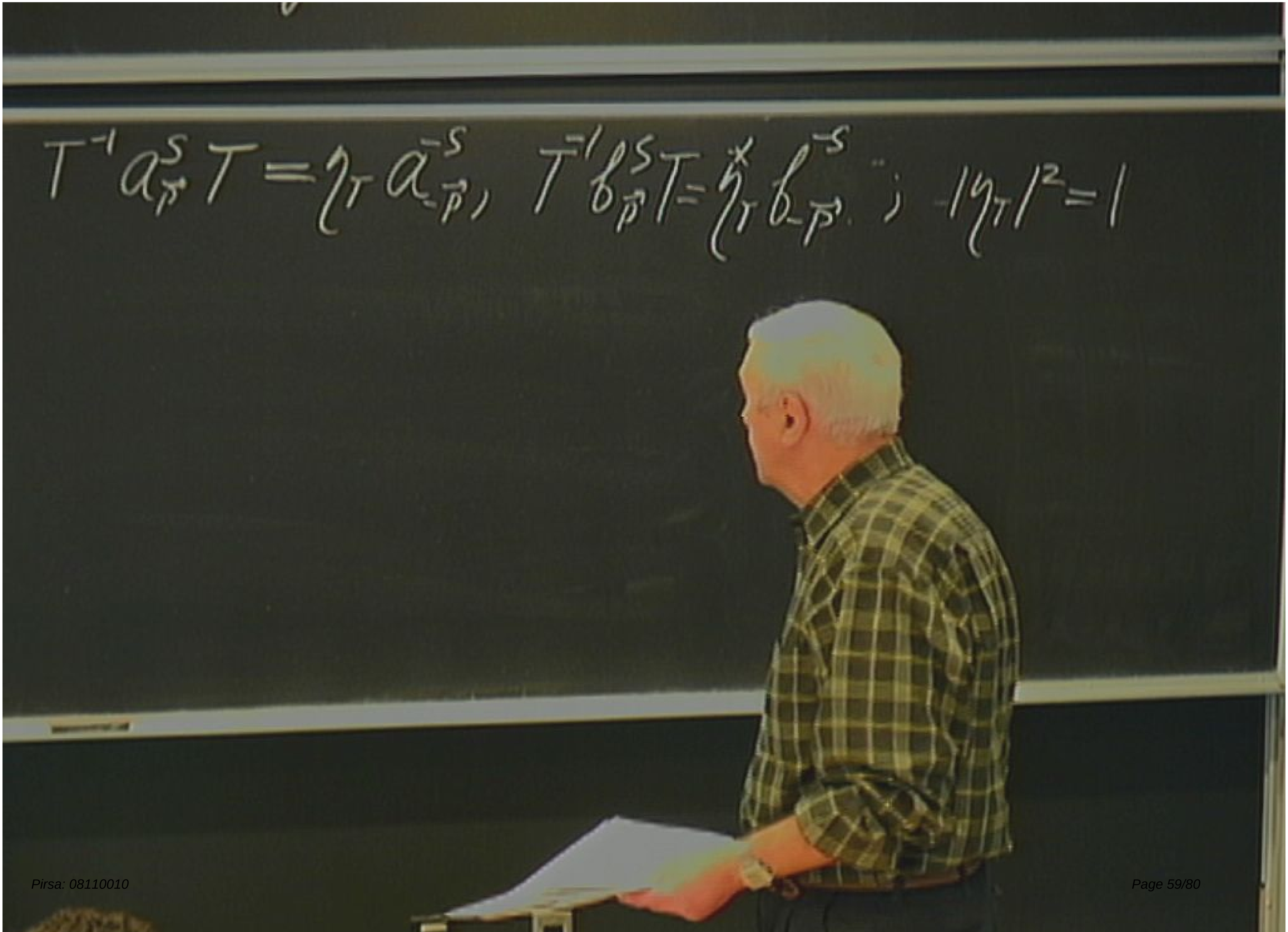


$$\vec{J}_{orb} = \vec{x}\times\vec{p} \xrightarrow[\substack{t\to -t \\ \vec{x}\to\vec{x}}]{\overset{\vec{x}\to-\vec{x}}{T}} -\vec{x}\times\vec{p}$$

Because of that, we want that under T [illegible]n:



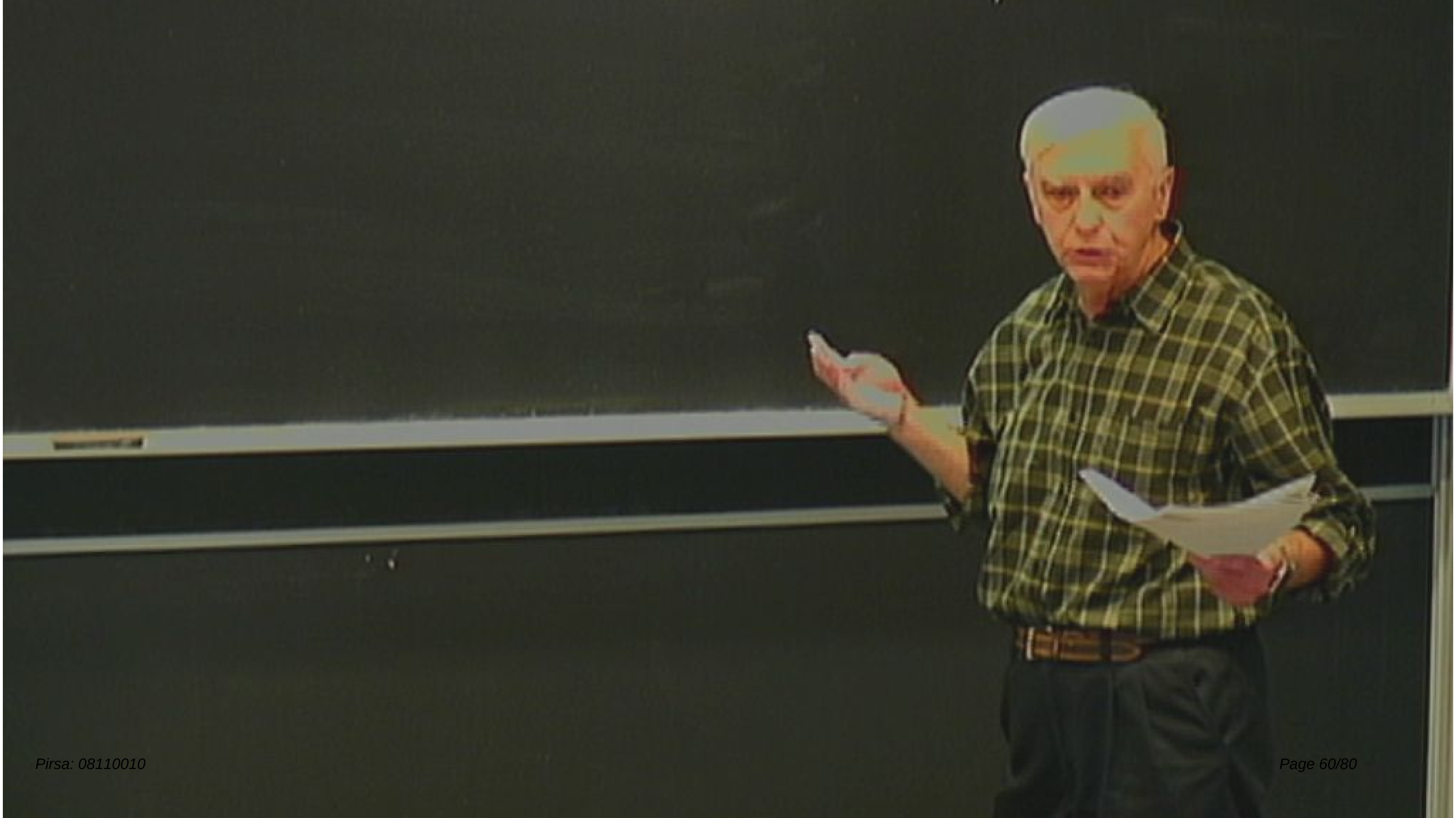
$$T^{-1} a^{s}_{\vec{p}} T = \eta_T \, a^{-s}_{-\vec{p}}, \quad T^{-1} b^{s}_{\vec{p}} T$$

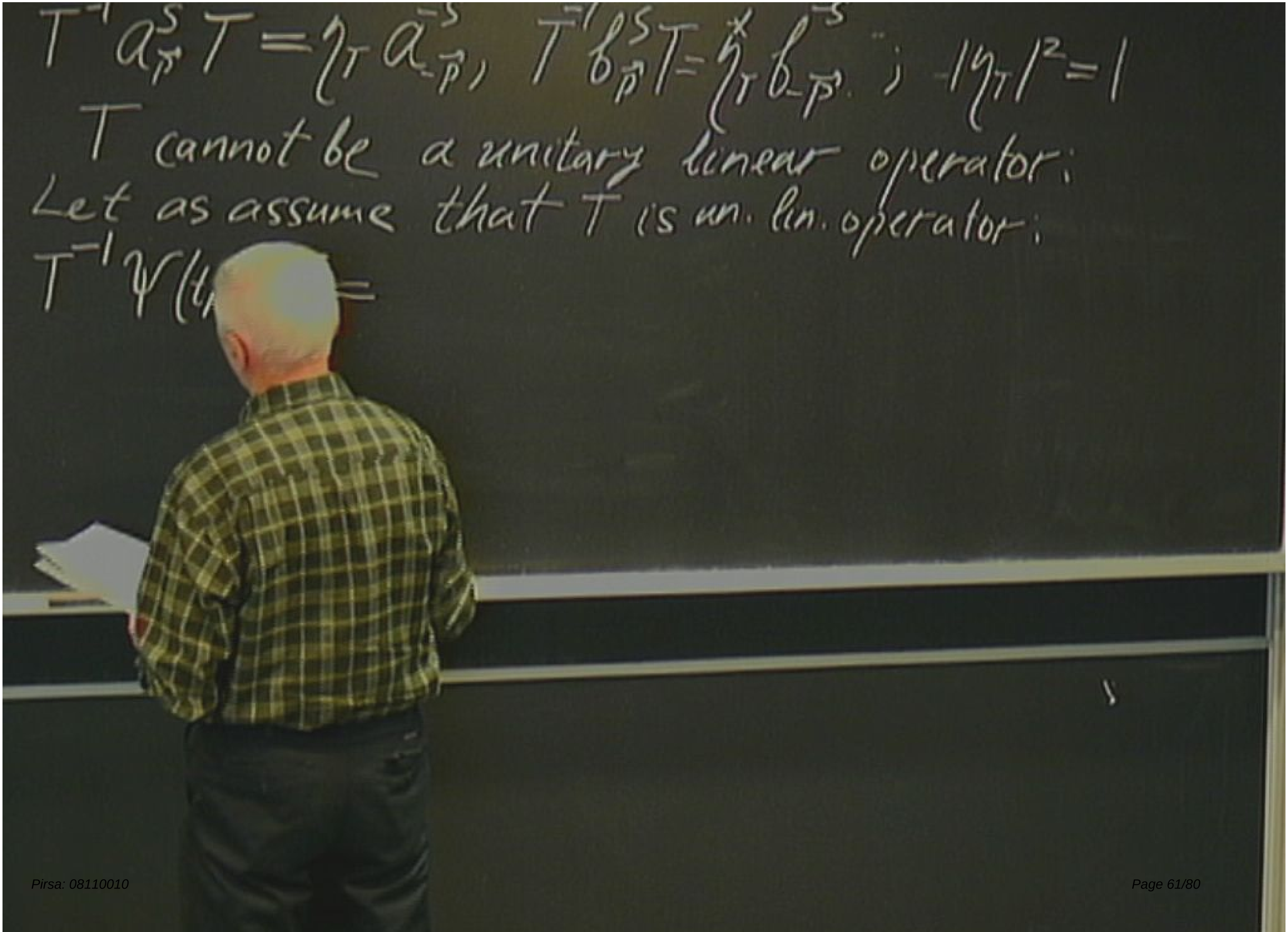


$$T^{-1} a^{s}_{\vec p}\, T = \eta_T\, a^{-s}_{-\vec p}, \quad T^{-1} b^{s}_{\vec p}\, T = \eta^*_T\, b^{-s}_{-\vec p}\ ;\ |\eta_T|^2 = 1$$

$$T^{-1} a^{s}_{\vec{p}} T = \eta_T a^{-s}_{-\vec{p}}, \quad T^{-1} b^{s}_{\vec{p}} T = \eta_T^{*} b^{-s}_{-\vec{p}} \; ; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator:



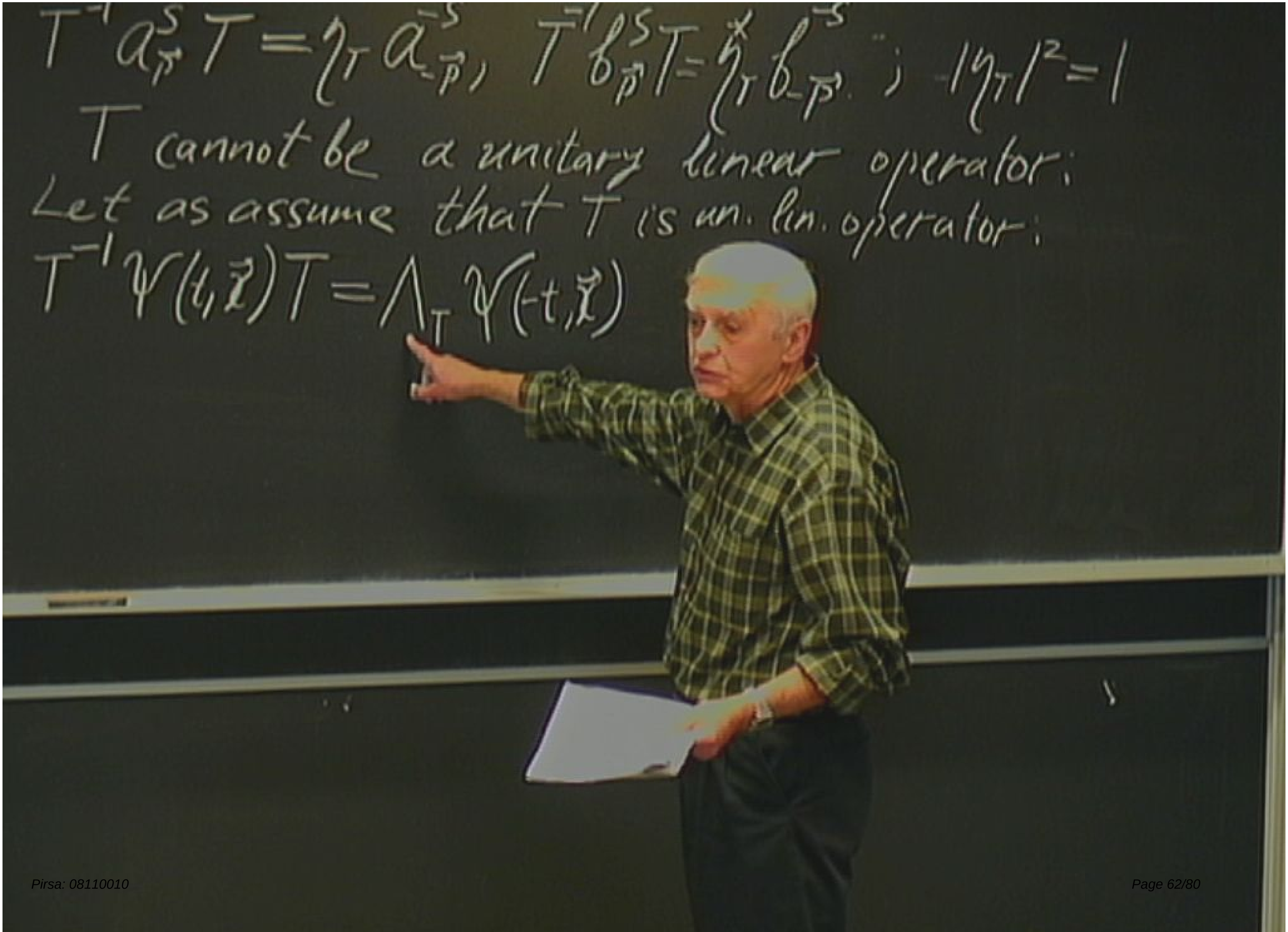


$T^{-1} a^s_{\vec p} T = \eta_T a^{-s}_{-\vec p}$, $T^{-1} b^s_{\vec p} T = \eta^*_T b^{-s}_{-\vec p}$; $|\eta_T|^2 = 1$

T cannot be a unitary linear operator:

Let as assume that T is un. lin. operator:

$T^{-1}\psi(t \quad =$

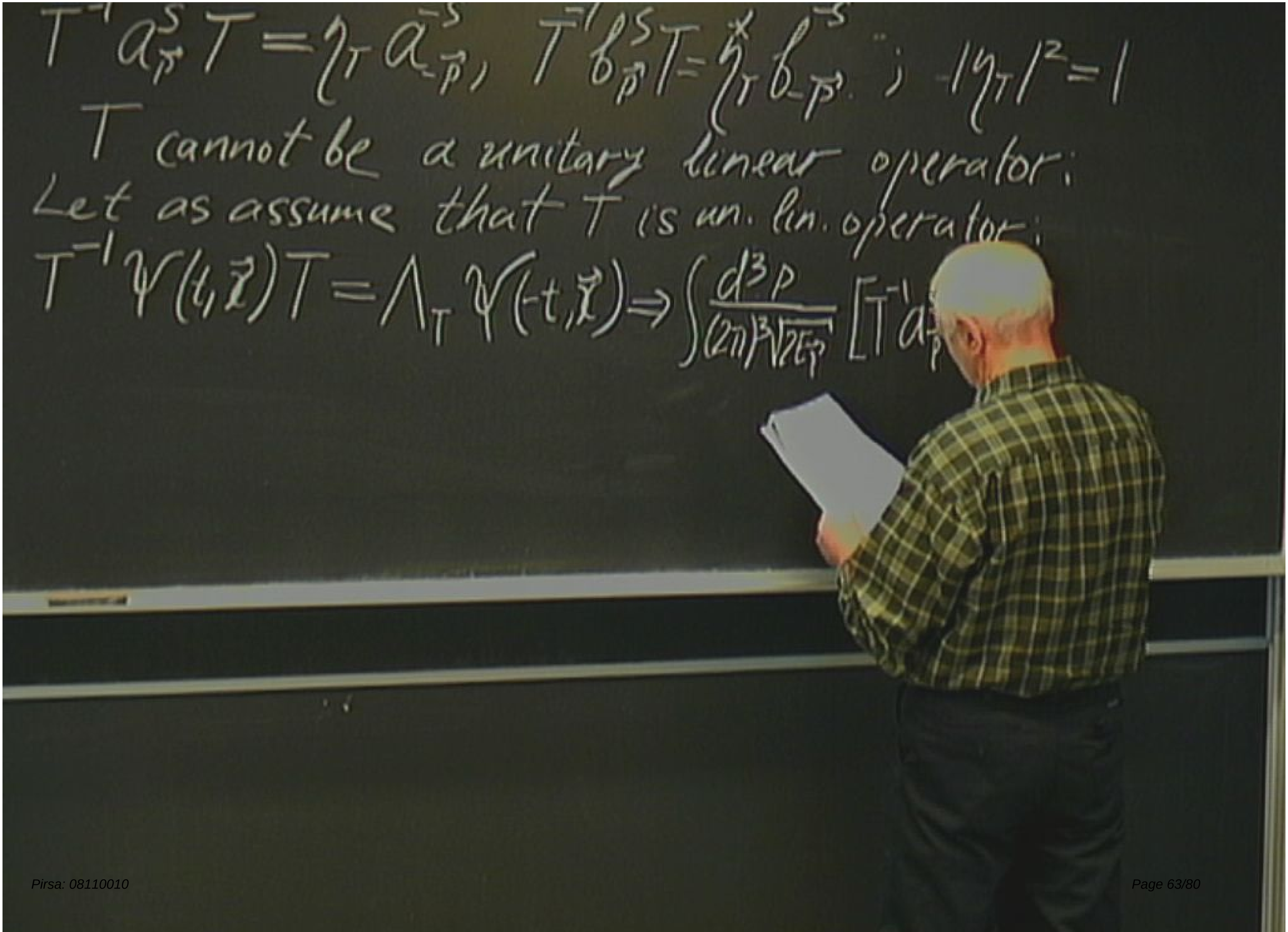


$T^{-1} a^{s}_{\vec{p}} T = \eta_T a^{-s}_{-\vec{p}}$, $T^{-1} b^{s}_{\vec{p}} T = \eta_T^{*} b^{-s}_{-\vec{p}}$; $|\eta_T|^2 = 1$

T cannot be a unitary linear operator:

Let as assume that T is un. lin. operator:

$$T^{-1} \psi(t,\vec{x}) T = \Lambda_T \psi(-t,\vec{x})$$

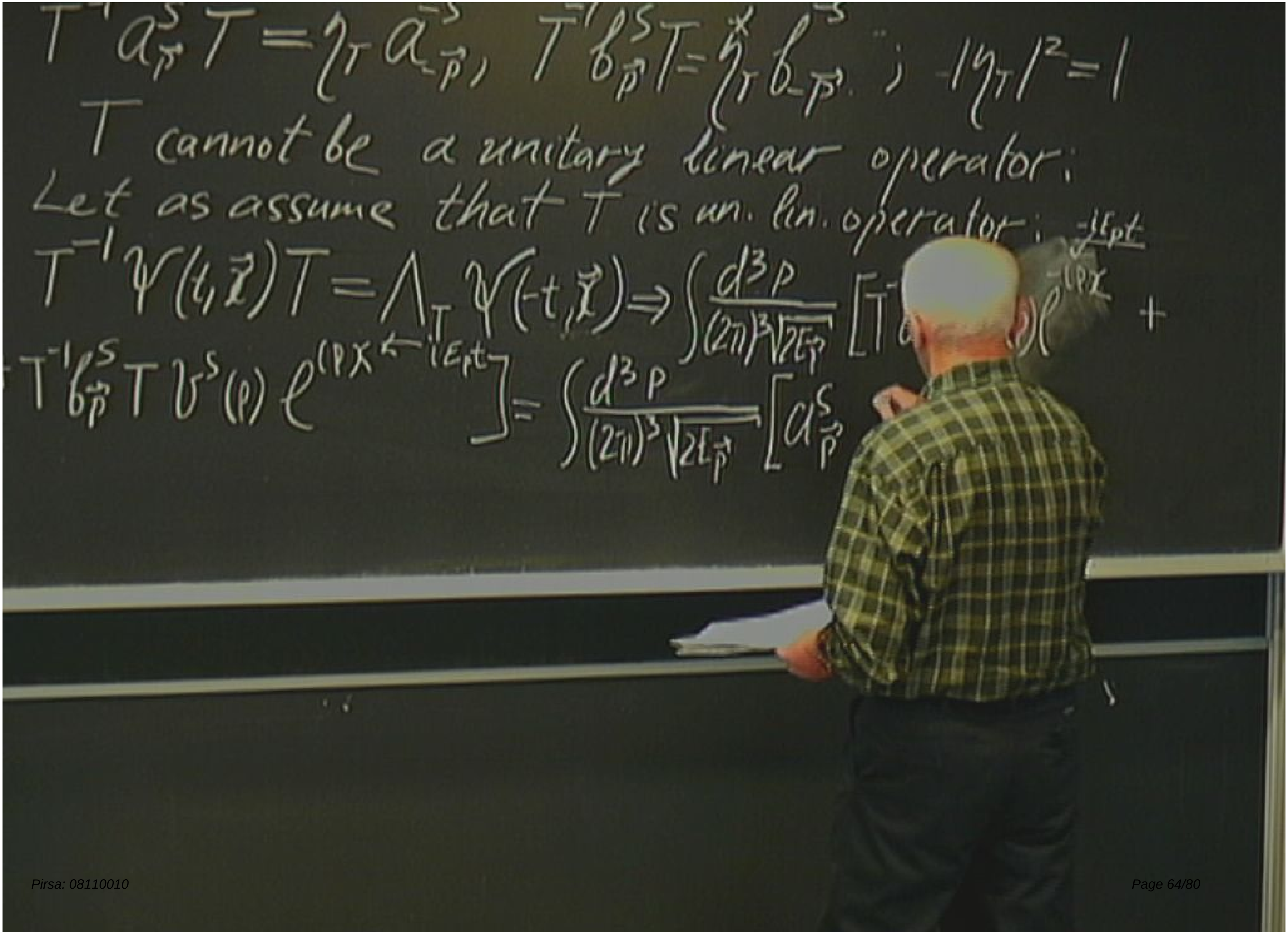


$$T^{-1}a^{s}_{\vec{p}}T=\eta_T\,a^{-s}_{-\vec{p}},\quad T^{-1}b^{s}_{\vec{p}}T=\overset{*}{\eta}_T\,b^{-s}_{-\vec{p}}\,;\quad |\eta_T|^2=1$$

T cannot be a unitary linear operator:

Let as assume that T is un. lin. operator:

$$T^{-1}\psi(t,\vec{x})T=\Lambda_T\,\psi(-t,\vec{x})\Rightarrow\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}[T^{-1}a_{\vec{p}}$$



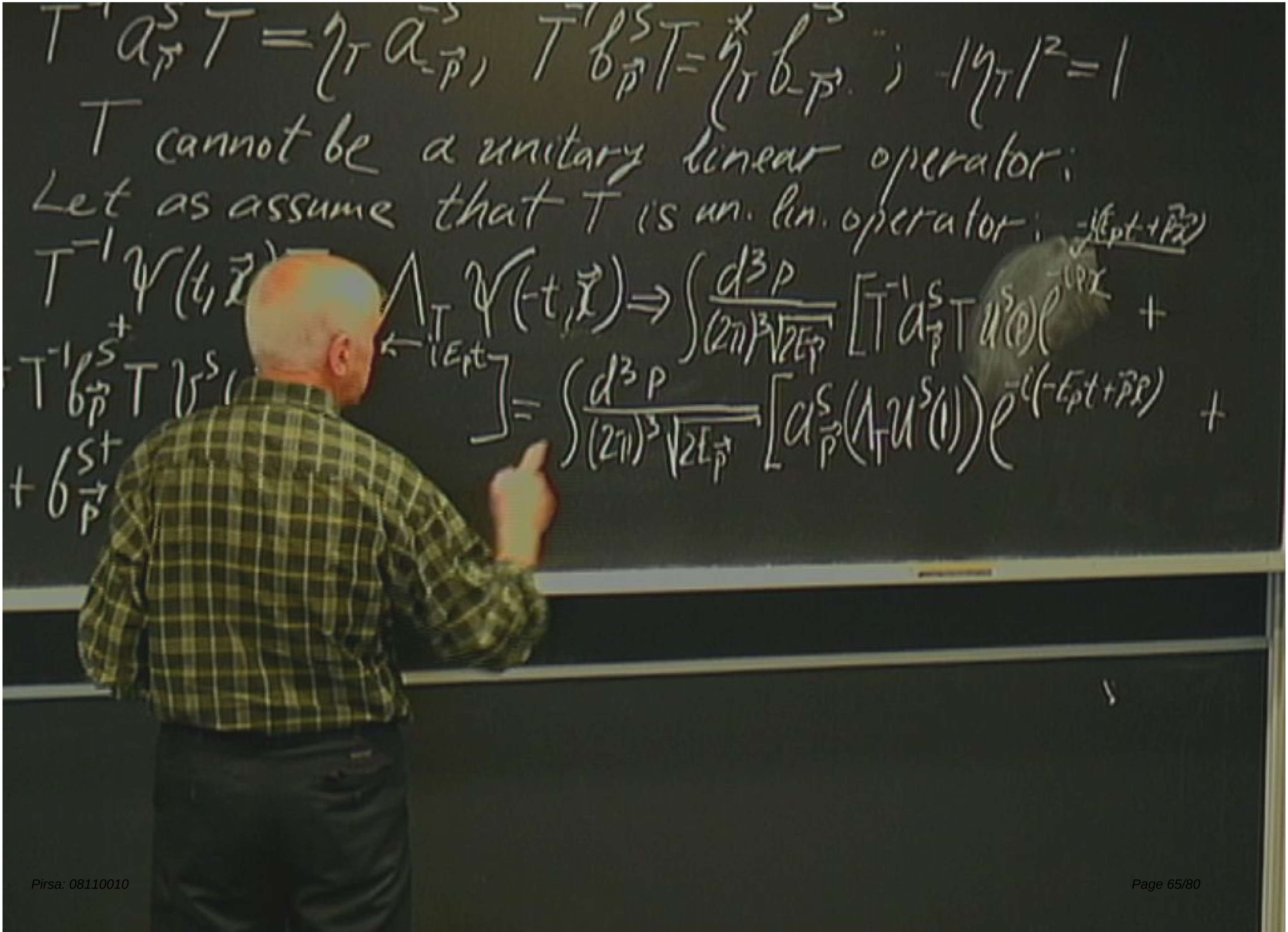
$$T^{-1} a^{s}_{\vec p}\, T = \eta_T\, a^{-s}_{-\vec p}, \quad T^{-1} b^{s}_{\vec p}\, T = \eta^{*}_T\, b^{-s}_{-\vec p}\,; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator:

Let as assume that T is un. lin. operator:

$$T^{-1}\psi(t,\vec x)\,T = \Lambda_T\,\psi(-t,\vec x) \Rightarrow \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[T^{-1}\,\text{[illegible]}\,\Big)\, e^{-ipx \leftarrow -iE_p t} +$$

$$+\, T^{-1} b^{s}_{\vec p}\, T\, v^{s}(p)\, e^{ipx \leftarrow iE_p t}\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[a^{s}_{\vec p}$$



$T^{-1} a^s_{\vec p} T = \eta_T a^{-s}_{-\vec p}$, $T^{-1} b^s_{\vec p} T = \eta_T^* b^{-s}_{-\vec p}$; $|\eta_T|^2 = 1$

T cannot be a unitary linear operator:

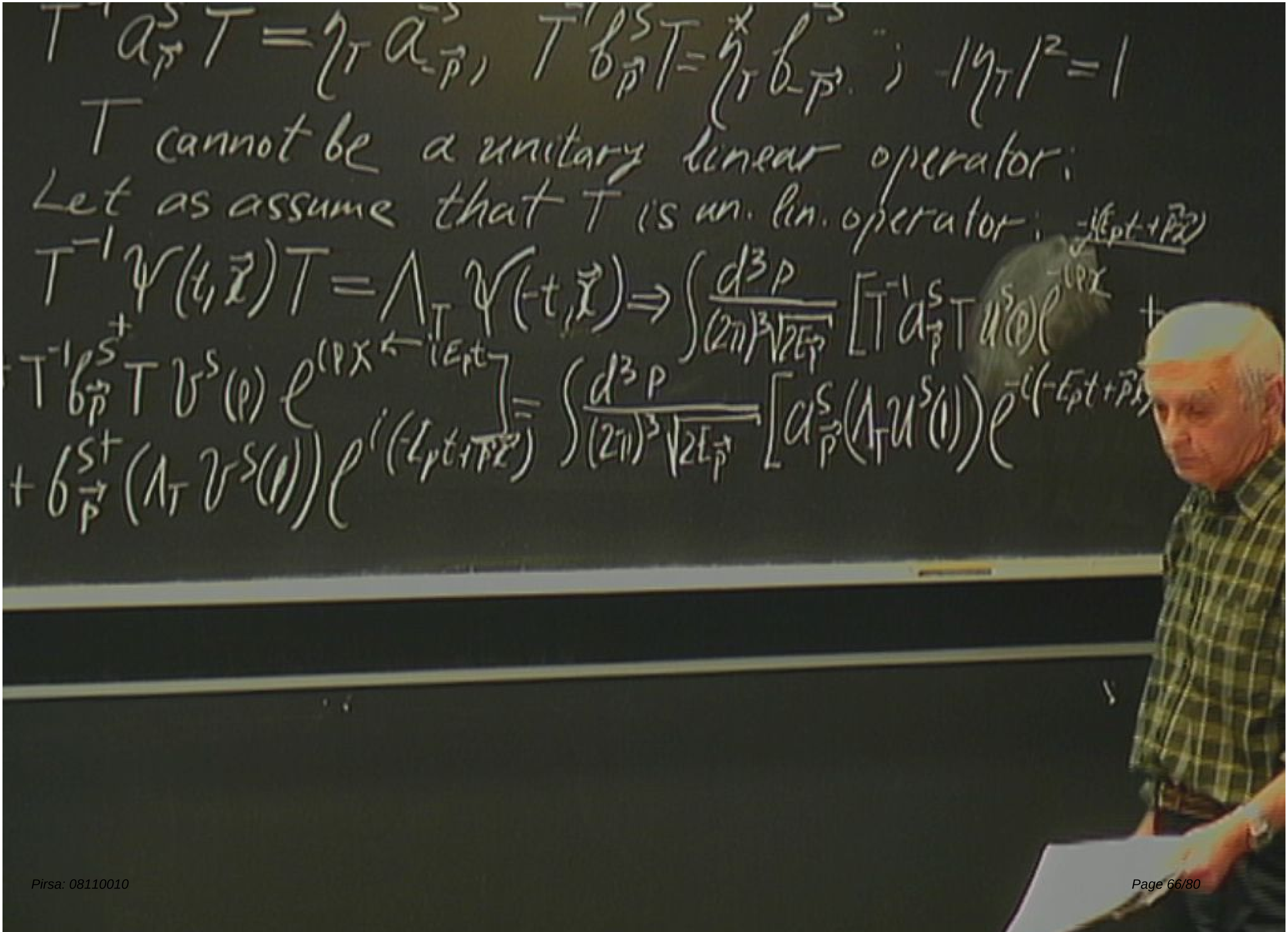
Let as assume that T is un. lin. operator:

$$T^{-1}\psi(t,\vec x) = \Lambda_T \psi(-t,\vec x) \Rightarrow \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\left[T^{-1} a^s_{\vec p} T u^s(p) e^{-ipx} \overset{-i(E_p t + \vec p \vec x)}{} + \right.$$

$\leftarrow iE_p t$

$$+ T^{-1} b^{s+}_{\vec p} T v^s(\ldots \Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\left[a^s_{\vec p}(\Lambda_T u^s(p)) e^{-i(-E_p t + \vec p \vec x)} + \right.$$

$$+ b^{s+}_{\vec p}$$



$$T^{-1} a^s_{\vec p} T = \eta_T a^{-s}_{-\vec p}, \quad T^{-1} b^s_{\vec p} T = \eta^*_T b^{-s}_{-\vec p} \; ; \; |\eta_T|^2 = 1$$

T cannot be a unitary linear operator:

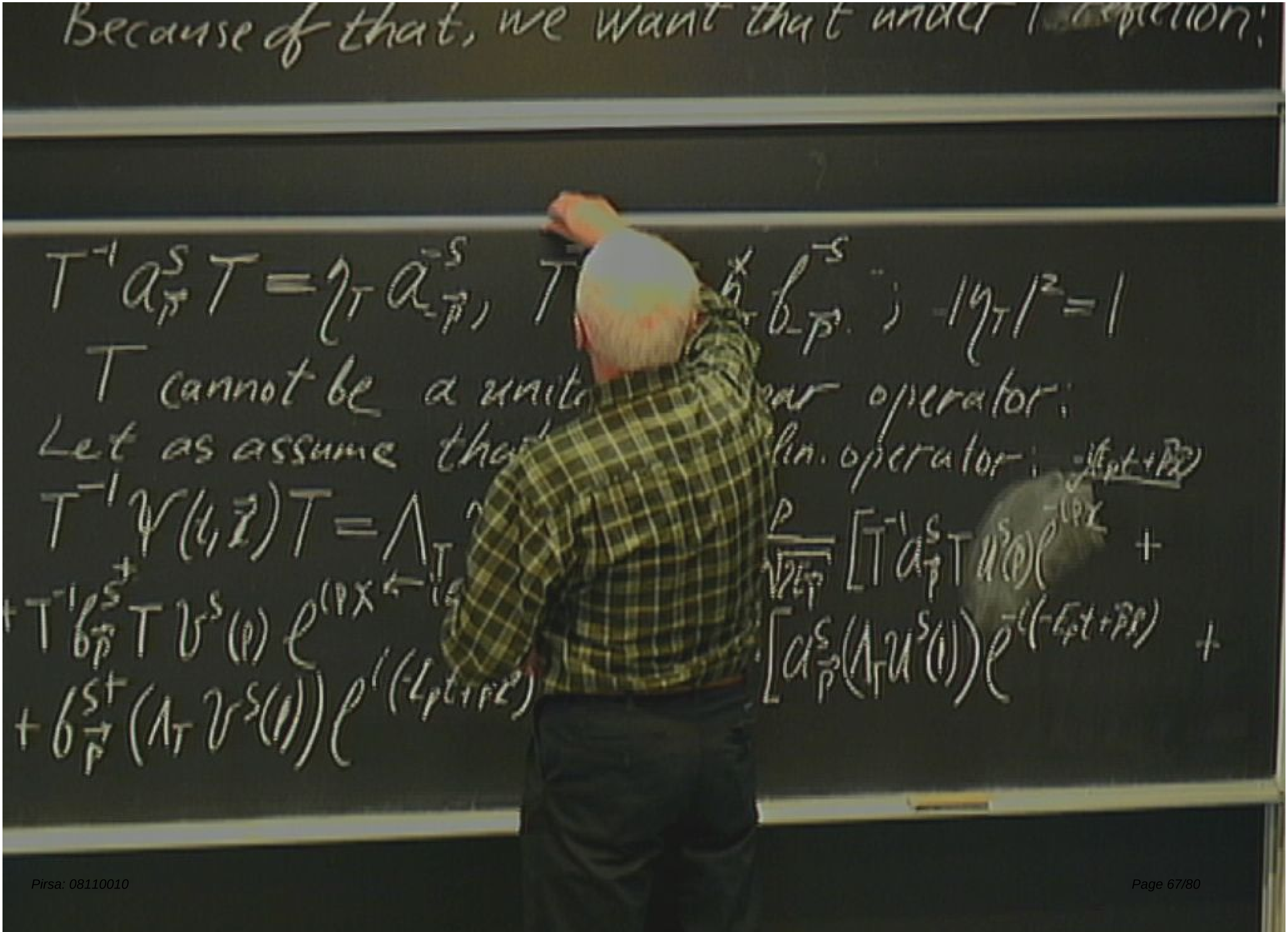
Let as assume that T is un. lin. operator:

$$\underline{-i(E_p t - \vec p \vec x)}$$

$$T^{-1}\psi(t,\vec x)T = \Lambda_T \psi(-t,\vec x) \Rightarrow \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\left[T^{-1}a^s_{\vec p}T\, u^s(p)\, e^{-ipx} + \right.$$

$$\left. + T^{-1} b^{s+}_{\vec p} T\, v^s(p)\, e^{ipx \leftarrow iE_p t}\right] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\left[a^s_{\vec p}(\Lambda_T u^s(p))\, e^{-i(-E_p t + \vec p \vec x)}\right.$$

$$+ b^{s+}_{\vec p}(\Lambda_T v^s(p))\, e^{i(-E_p t + \vec p \vec x)}$$



Because of that, we want that under T [illegible]eflection:

$T^{-1} a^s_{\vec p} T = \eta_T a^{-s}_{-\vec p}$, T [illegible] $\eta^*_T b^{-s}_{-\vec p}$; $|\eta_T|^2 = 1$

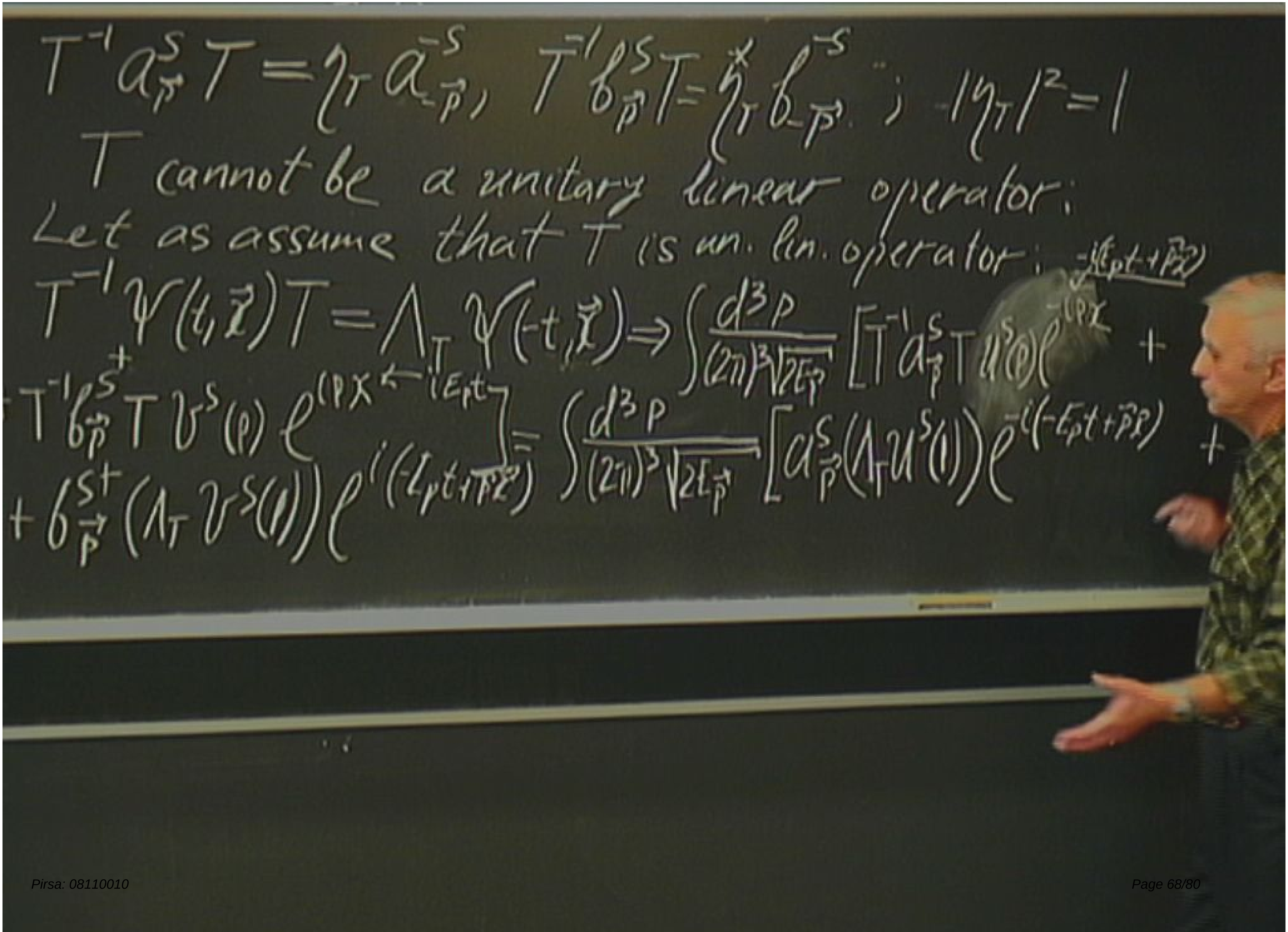
T cannot be a unit[illegible]ear operator:

Let as assume tha[illegible] lin. operator: $-i(E_p t+\vec p\vec x)$

$T^{-1}\psi(t,\vec x)T = \Lambda_T$ [illegible] $\frac{p}{\sqrt{2E_p}}[T^{-1}a^s_{\vec p}T u^s(p) e^{-ipx}$ +

$+T^{-1}b^{s+}_{\vec p} T v^s(p) e^{ipx} \leftarrow i($ [illegible] $[a^s_{-\vec p}(\Lambda_T u^s(p)) e^{-i(-E_p t+\vec p\vec x)}$ +

$+ b^{s+}_{\vec p}(\Lambda_T v^s(p)) e^{i(-E_p t+\vec p\vec x)}$



$$T^{-1} a^s_{\vec p} T = \eta_T a^{-s}_{-\vec p}, \quad T^{-1} b^s_{\vec p} T = \eta^*_T b^{-s}_{-\vec p} \; ; \; |\eta_T|^2 = 1$$

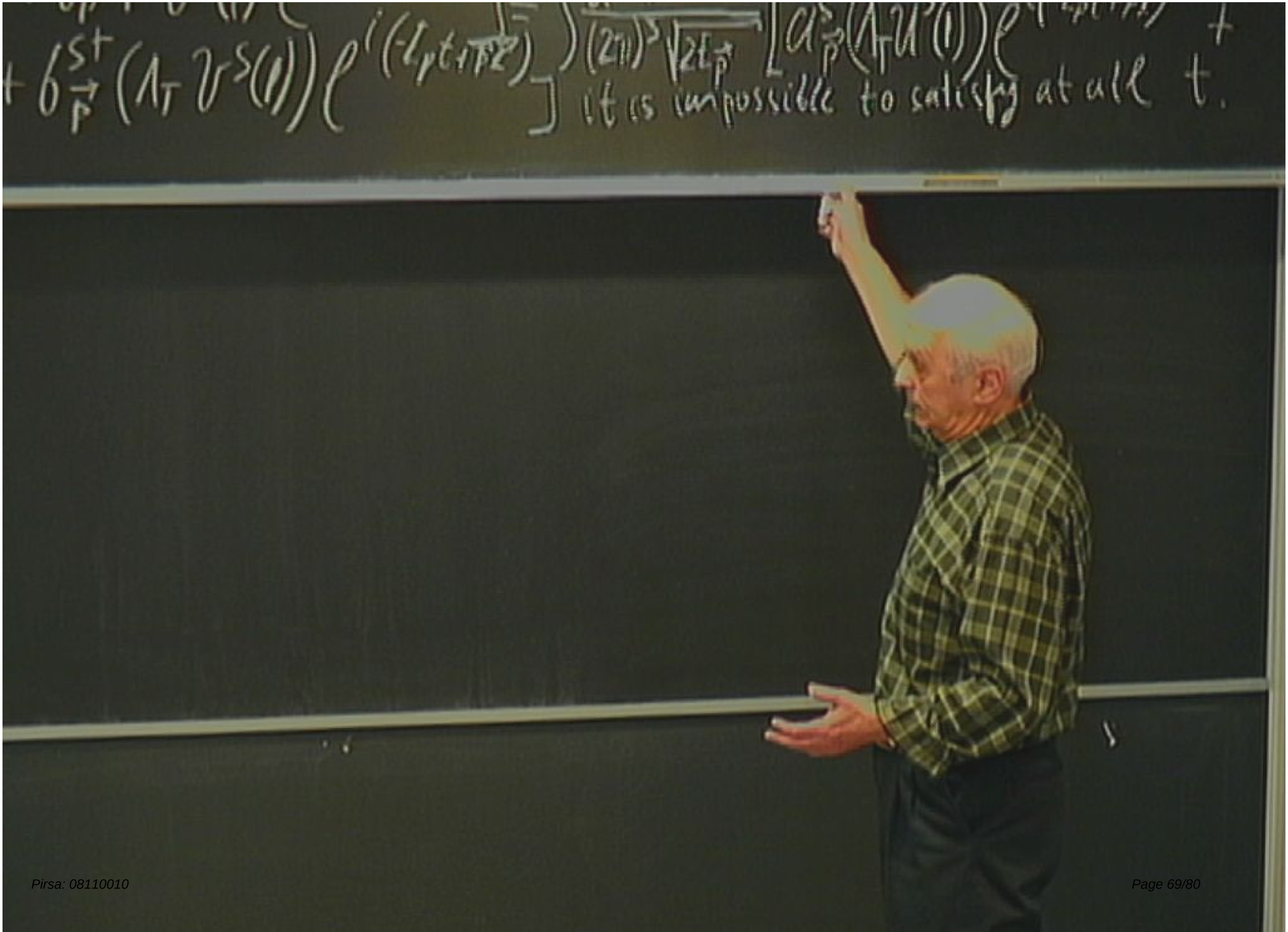
T cannot be a unitary linear operator:

Let as assume that T is un. lin. operator:

$$T^{-1}\psi(t,\vec x)T = \Lambda_T \psi(-t,\vec x) \Rightarrow \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[T^{-1}a^s_{\vec p} T u^s(p) e^{-ipx} \overset{-i(E_pt+\vec p\vec x)}{} +$$

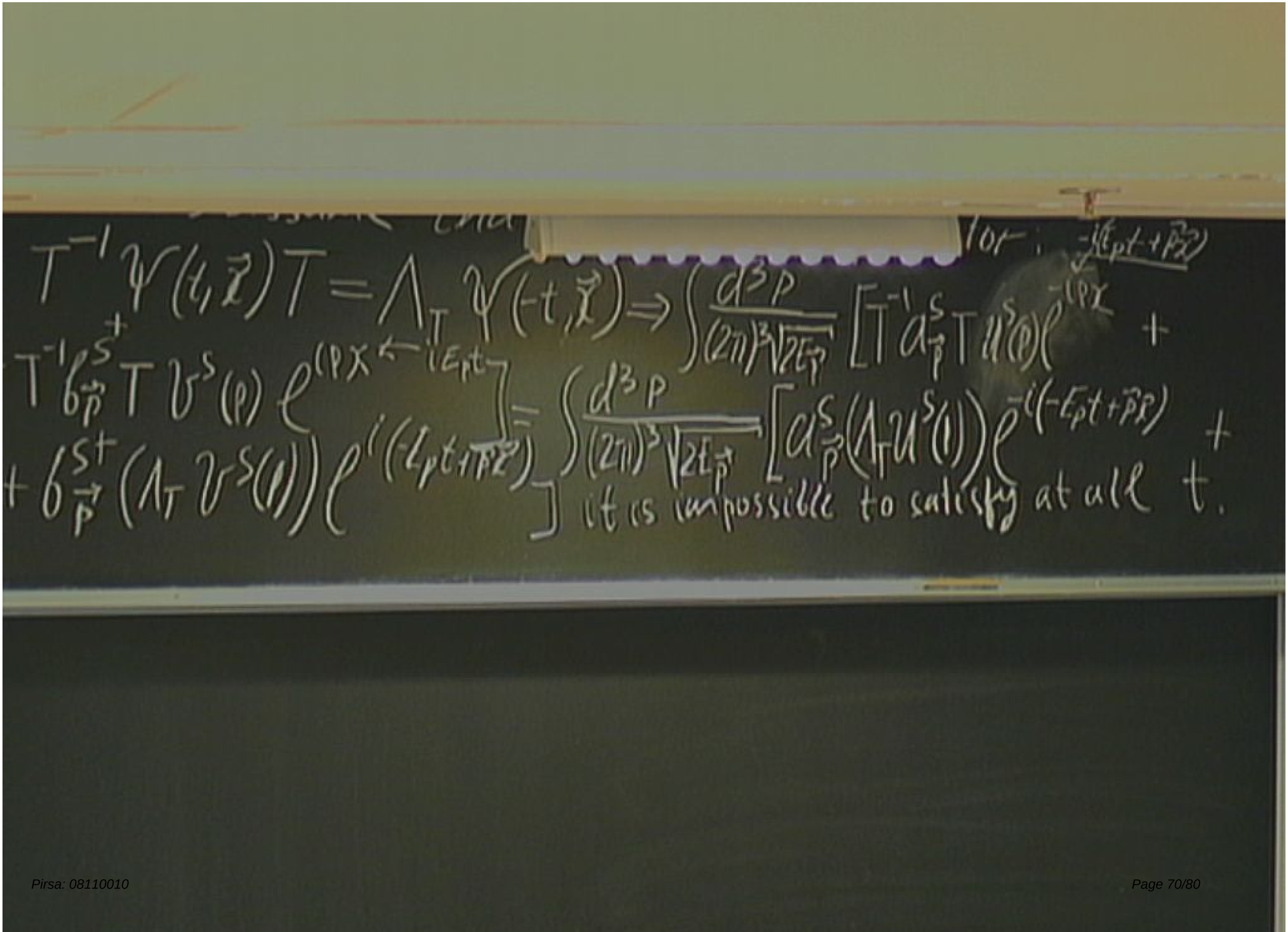
$$+ T^{-1} b^{s+}_{\vec p} T v^s(p) e^{ipx \leftarrow iE_pt}\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[a^s_{\vec p}(\Lambda_T u^s(p)) e^{-i(-E_pt+\vec p\vec x)} +$$

$$+ b^{s+}_{\vec p}(\Lambda_T v^s(p)) e^{i(-E_pt+\vec p\vec x)}$$



$\int \frac{}{(2\pi)^3 \sqrt{2E_{\vec p}}} \Big[a^{s}_{\vec p} (\Lambda_T u^s(0)) e^{\cdots} +$

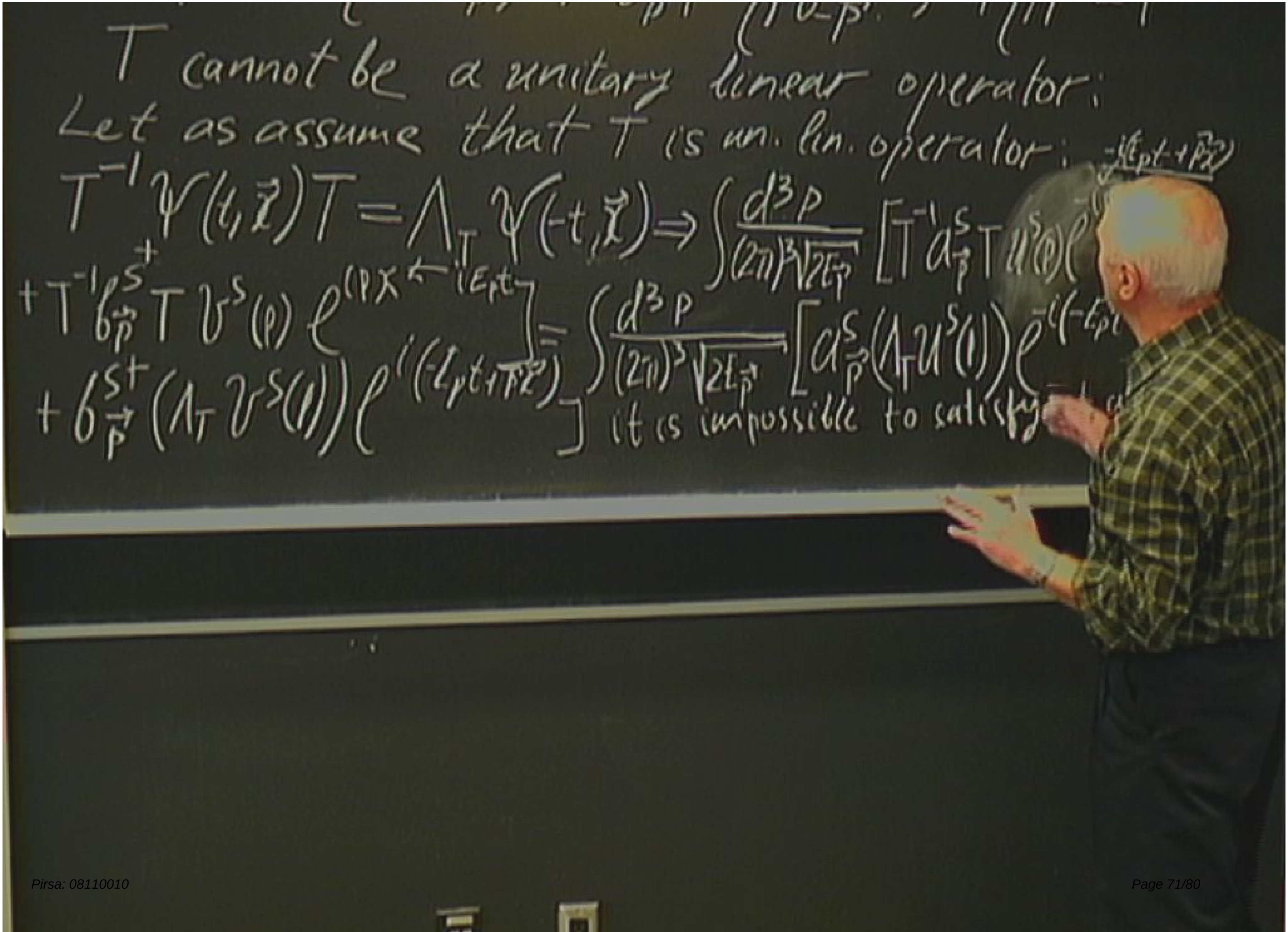
$+ b^{s\dagger}_{\vec p} (\Lambda_T v^s(0)) e^{i(-E_p t + \vec p \vec z)} \Big]$ it is impossible to satisfy at all t.



for: $\underline{-i(E_p t + \vec{p}\vec{x})}$

$$T^{-1}\psi(t,\vec{x})T = \Lambda_T\,\psi(-t,\vec{x}) \Rightarrow \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\Big[T^{-1}a^s_{\vec{p}}T\,u^s(p)\,e^{-ipx} + T^{-1}b^{s+}_{\vec{p}}T\,v^s(p)\,e^{ipx \leftarrow iE_pt}\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\Big[a^s_{\vec{p}}(\Lambda_T u^s(p))\,e^{-i(-E_pt+\vec{p}\vec{x})} + b^{s+}_{\vec{p}}(\Lambda_T v^s(p))\,e^{i(-E_pt+\vec{p}\vec{x})}\Big]$$

it is impossible to satisfy at all t.



T cannot be a unitary linear operator:

Let as assume that T is un. lin. operator:

$$T^{-1}\psi(t,\vec{x})T=\Lambda_T\,\psi(-t,\vec{x})\Rightarrow\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\Big[T^{-1}a^s_{\vec{p}}T\,u^s(p)\,e^{-i(E_pt-\vec{p}\vec{x})}$$

$$+T^{-1}b^{s+}_{\vec{p}}T\,v^s(p)\,e^{ipx\leftarrow iE_pt}\Big]=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\Big[a^s_{\vec{p}}(\Lambda_T u^s(p))\,e^{-i(-E_pt}$$

$$+b^{s+}_{\vec{p}}(\Lambda_T v^s(p))\,e^{i(-E_pt+\vec{p}\vec{x})}\Big]$$

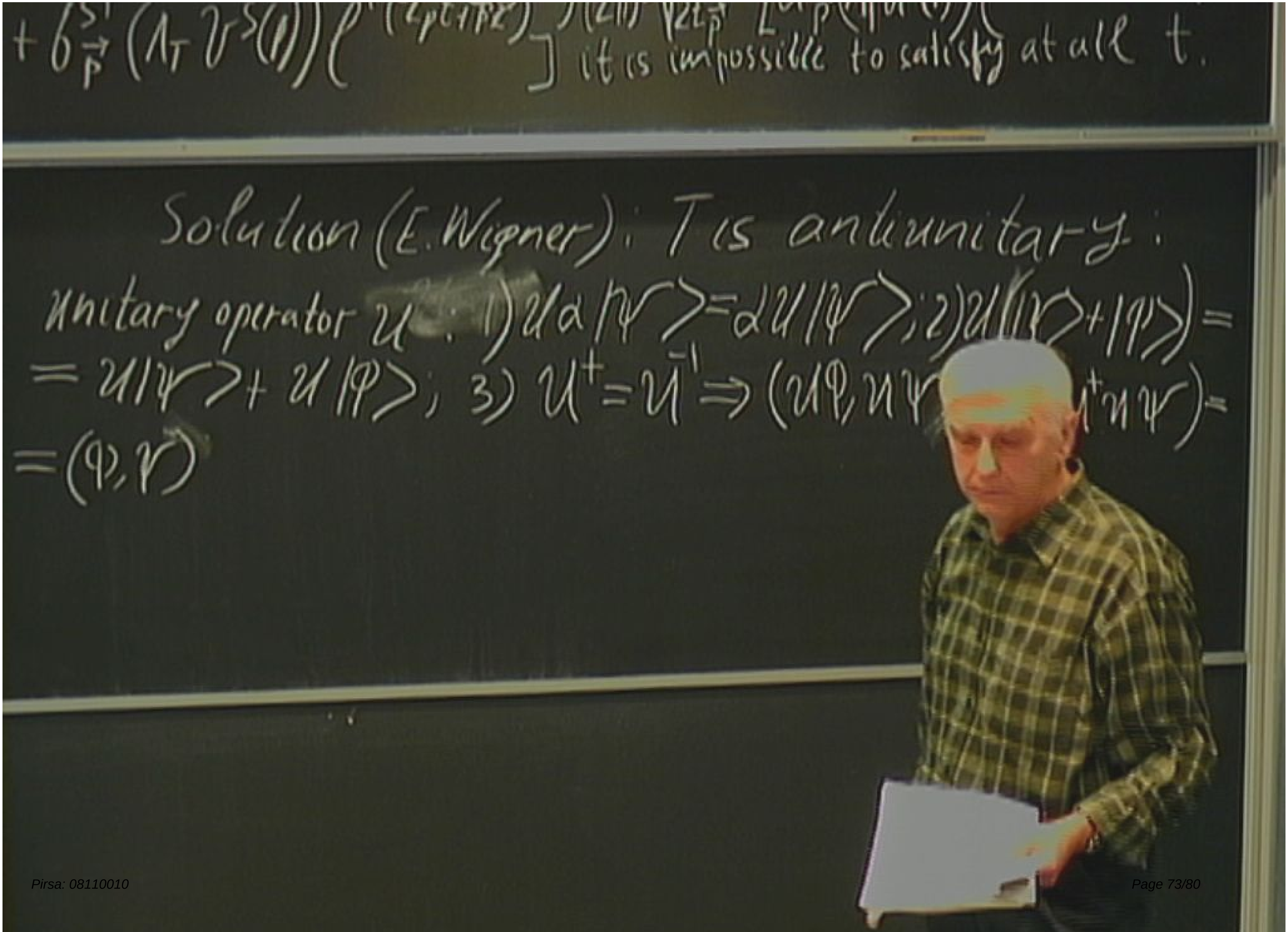
it is impossible to satisfy

$$\bar{\psi}\gamma^{\mu}\gamma^{5}\psi : C^{-1}\bar{\psi}\gamma^{\mu}\gamma^{5}\psi C = -\psi^{T}C^{-1}\gamma^{\mu}\gamma^{5}C\,\bar{\psi}^{T} =$$

$$= \psi^{T}\gamma^{\mu T}\gamma^{5T}\bar{\psi}^{T} = -\bar{\psi}\gamma^{5}\gamma^{\mu}\psi = +\underline{\bar{\psi}\gamma^{\mu}\gamma^{5}\psi.}$$

Neutr. curr. : $\bar{\psi}\gamma_{\mu}\frac{(1-\gamma^{5})}{2}\psi.$

Linear $T(\alpha|\psi\rangle + \beta|\psi\rangle) = \alpha T|\psi\rangle + \beta T|\varphi'\rangle$



$+ b^{s\dagger}_{\vec{p}}(\Lambda_T v^s(p))e$] it is impossible to satisfy at all t.

Solution (E. Wigner): T is antiunitary:

unitary operator U: 1) $U\alpha|\psi\rangle = \alpha U|\psi\rangle$; 2) $U(|\psi\rangle + |\varphi\rangle) =$
$= U|\psi\rangle + U|\varphi\rangle$; 3) $U^+ = U^{-1} \Rightarrow (U\varphi, U\psi$ [illegible] $^{+}U\psi) =$
$= (\varphi, \psi)$

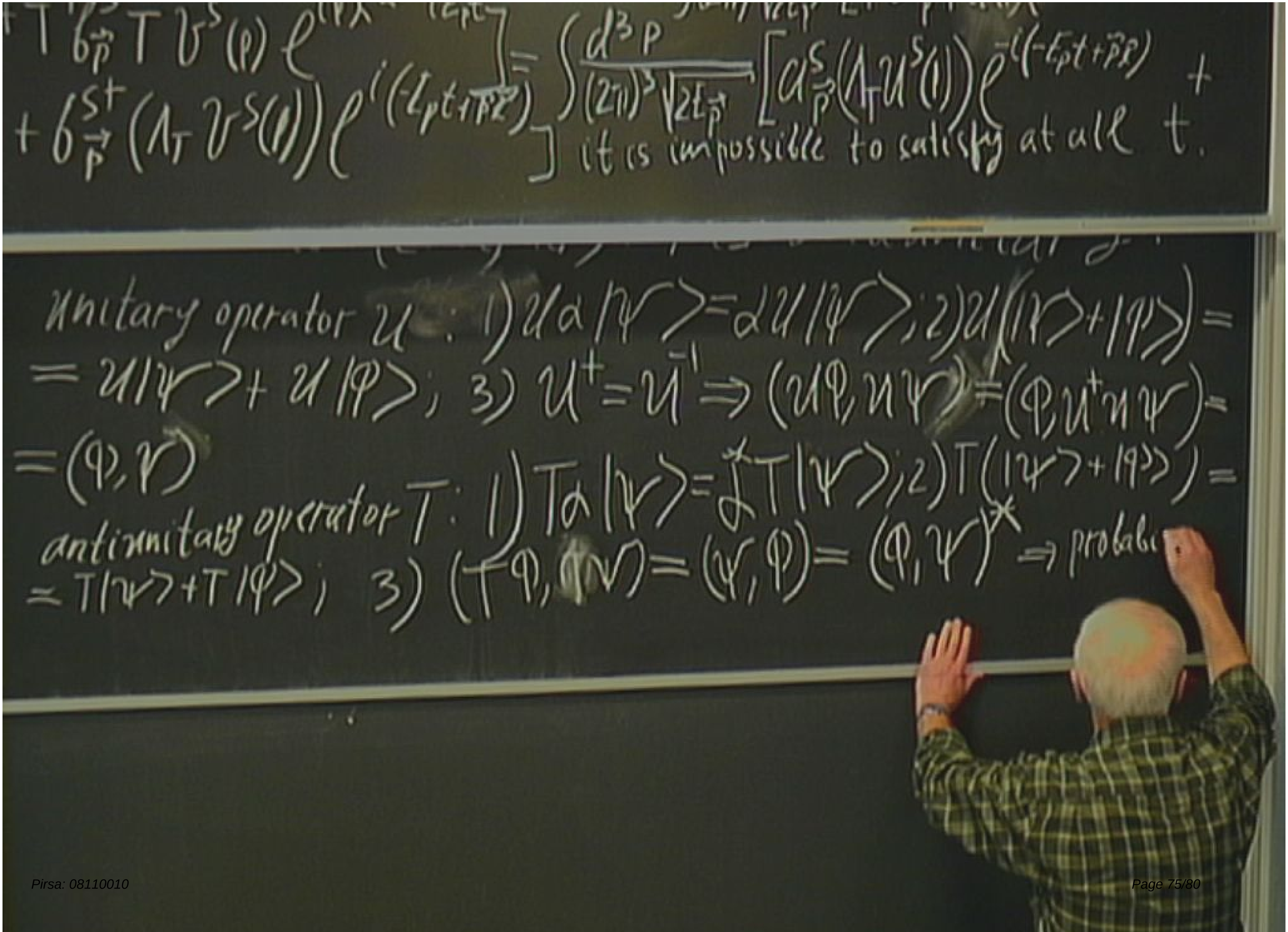
$+ b_{\vec p}^{\,+} v^{s}(p)\, e$ $\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[a_{\vec p}^{s}(\Lambda_T u^s(p))\, e^{-i(-E_p t+\vec p\vec x)} +$

$+ b_{\vec p}^{s+}(\Lambda_T v^s(p))\, e^{i(-E_p t+\vec p\vec x)}\Big]$ it is impossible to satisfy at all t.

Solution (E. Wigner): T is antiunitary.

unitary operator U: 1) $U\alpha|\psi\rangle = \alpha U|\psi\rangle$; 2) $U(|\psi\rangle+|\varphi\rangle) =$
$= U|\psi\rangle + U|\varphi\rangle$; 3) $U^+ = U^{-1} \Rightarrow (U\varphi, U\psi) = (\varphi, U^+U\psi) =$
$= (\varphi, \psi)$

antiunitary operator T: 1) $T\alpha|\psi\rangle = \alpha^* T|\psi\rangle$; 2) $T(|\psi\rangle+|\varphi\rangle) =$
$= T|\psi\rangle + T|\varphi\rangle$; 3) $(T\varphi, T\psi) = (\psi, \varphi) = (\varphi, \psi)^*$

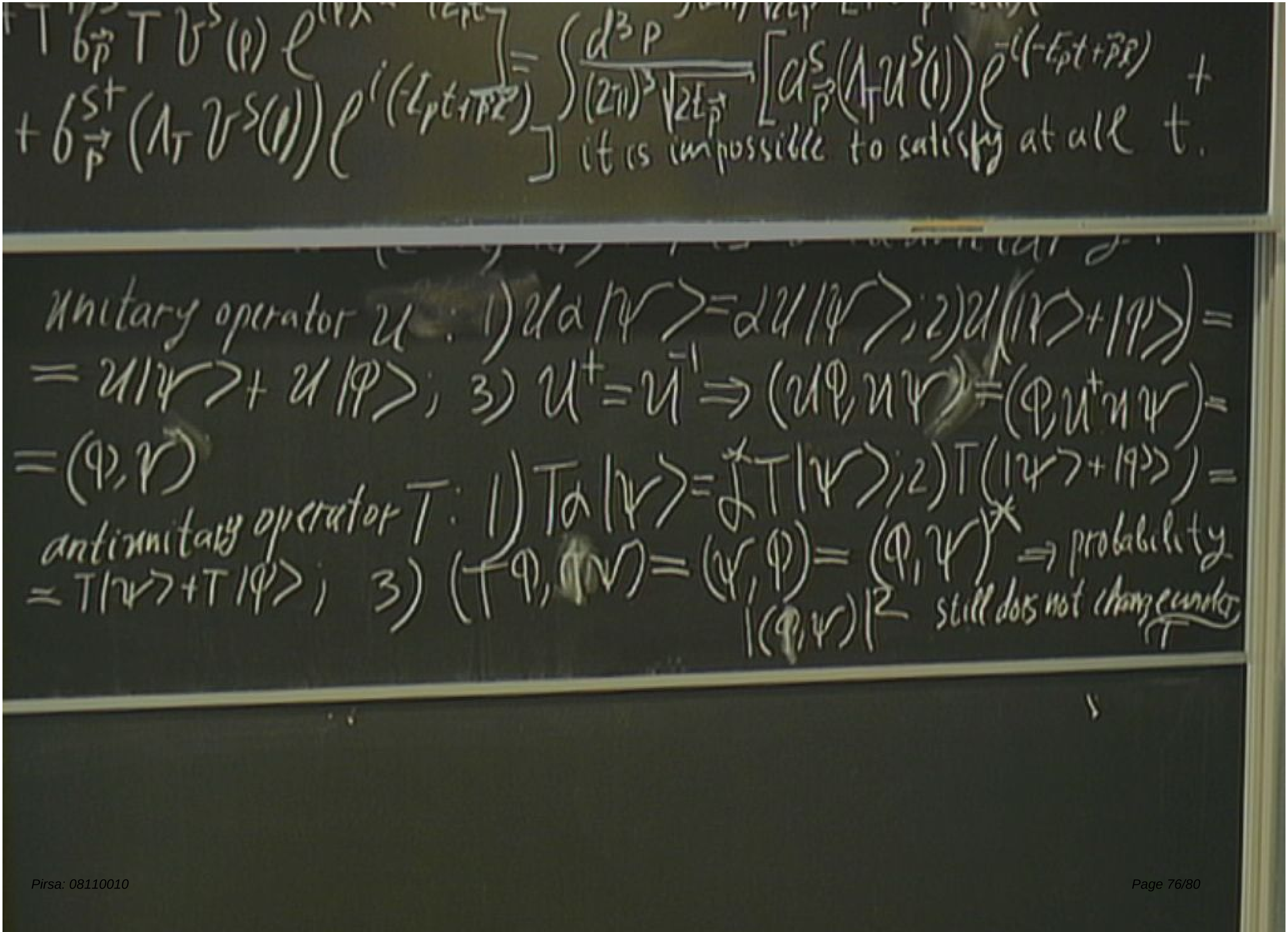


$+ T b_{\vec p}^{s+} T v^s(p) e^{i(px)}\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[a^s_{\vec p}(\Lambda_T u^s(p)) e^{-i(-E_pt+\vec p\vec x)} +$

$+ b^{s+}_{\vec p}(\Lambda_T v^s(p)) e^{i(-E_pt+\vec p\vec x)}\Big]$ it is impossible to satisfy at all t.

unitary operator $\mathcal{U}$: 1) $\mathcal{U}\alpha|\psi\rangle = \alpha\mathcal{U}|\psi\rangle$; 2) $\mathcal{U}(|\psi\rangle+|\varphi\rangle) =$
$= \mathcal{U}|\psi\rangle + \mathcal{U}|\varphi\rangle$; 3) $\mathcal{U}^+ = \mathcal{U}^{-1} \Rightarrow (\mathcal{U}\varphi, \mathcal{U}\psi) = (\varphi, \mathcal{U}^+\mathcal{U}\psi) =$
$= (\varphi, \psi)$

antiunitary operator T: 1) $T\alpha|\psi\rangle = \alpha^* T|\psi\rangle$; 2) $T(|\psi\rangle+|\varphi\rangle) =$
$= T|\psi\rangle + T|\varphi\rangle$; 3) $(T\varphi, T\psi) = (\psi, \varphi) = (\varphi, \psi)^* \Rightarrow$ probab

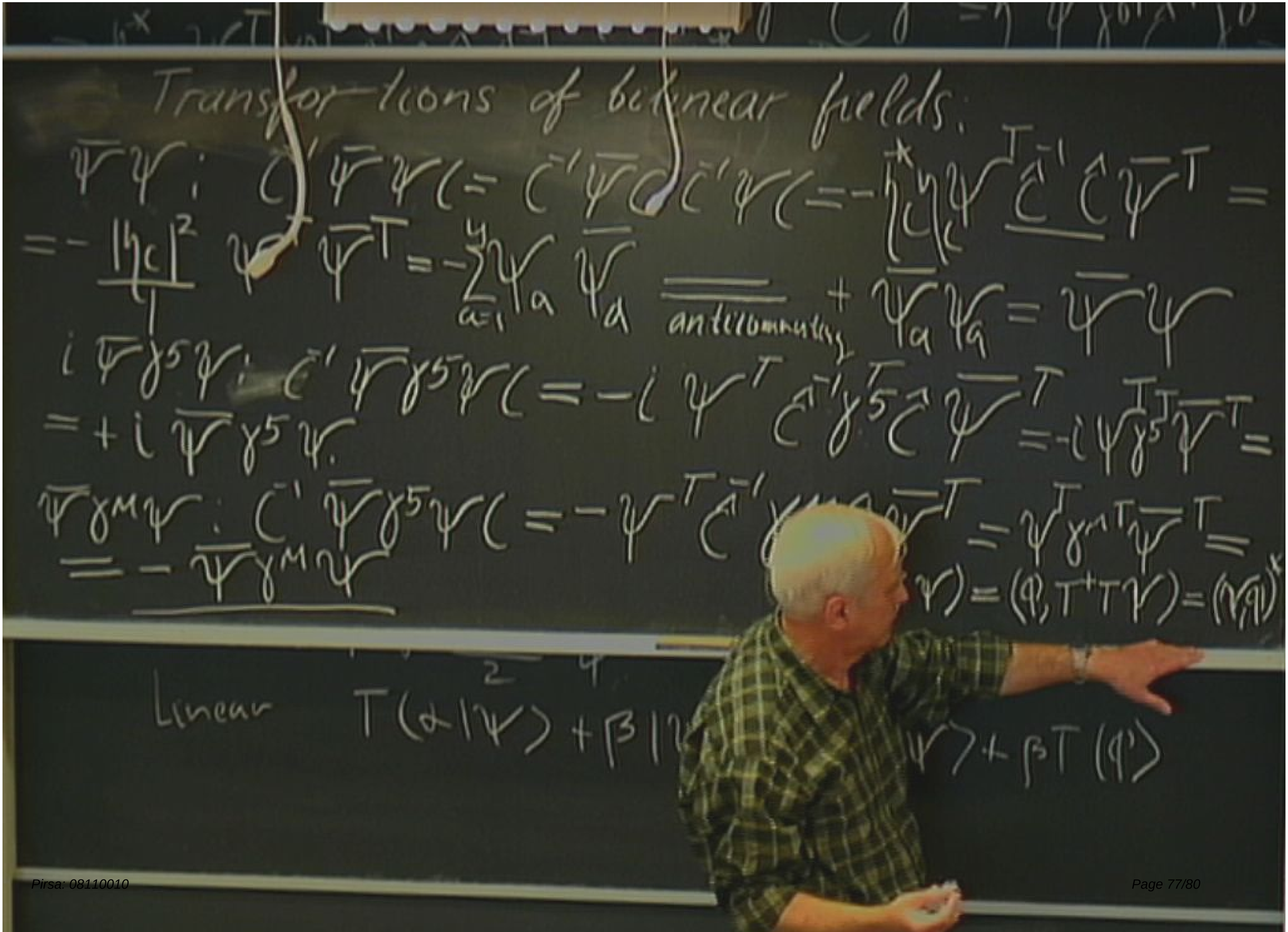


$+ T b_{\vec p} T v^s(p) e$ $\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[a^s_{\vec p}(\Lambda_T u^s(p))e^{-i(-E_p t+\vec p \vec x)} +$

$+ b^{s+}_{\vec p}(\Lambda_T v^s(p))e^{i(-E_p t+\vec p \vec x)}\Big]$ it is impossible to satisfy at all t.

unitary operator $\mathcal{U}$: 1) $\mathcal{U}\alpha|\psi\rangle = \alpha\,\mathcal{U}|\psi\rangle$; 2) $\mathcal{U}(|\psi\rangle + |\varphi\rangle) =$
$= \mathcal{U}|\psi\rangle + \mathcal{U}|\varphi\rangle$; 3) $\mathcal{U}^+ = \mathcal{U}^{-1} \Rightarrow (\mathcal{U}\varphi, \mathcal{U}\psi) = (\varphi, \mathcal{U}^+\mathcal{U}\psi) =$
$= (\varphi, \psi)$

antiunitary operator T: 1) $T\alpha|\psi\rangle = \alpha^* T|\psi\rangle$; 2) $T(|\psi\rangle + |\varphi\rangle) =$
$= T|\psi\rangle + T|\varphi\rangle$; 3) $(T\varphi, T\psi) = (\psi, \varphi) = (\varphi, \psi)^* \Rightarrow$ probability
$|(\varphi,\psi)|^2$ still does not change under T



Transfor tions of bilinear fields:

$\bar\psi\psi$: $C^{-1}\bar\psi\psi C = C^{-1}\bar\psi C C^{-1}\psi C = -\eta_c^*\eta_c \psi^T \underline{C^{-1} C} \bar\psi^T =$

$= -\underbrace{|\eta_c|^2}_{1} \psi \bar\psi^T = -\sum_{a=1}^{4} \psi_a \bar\psi_a \underset{\text{anticommuting}}{=} + \bar\psi_a \psi_a = \bar\psi\psi$

$i\bar\psi\gamma^5\psi$: $C^{-1}\bar\psi\gamma^5\psi C = -i\psi^T C^{-1}\gamma^{5T} C \bar\psi^T = -i\psi^T\gamma^{5T}\bar\psi^T =$

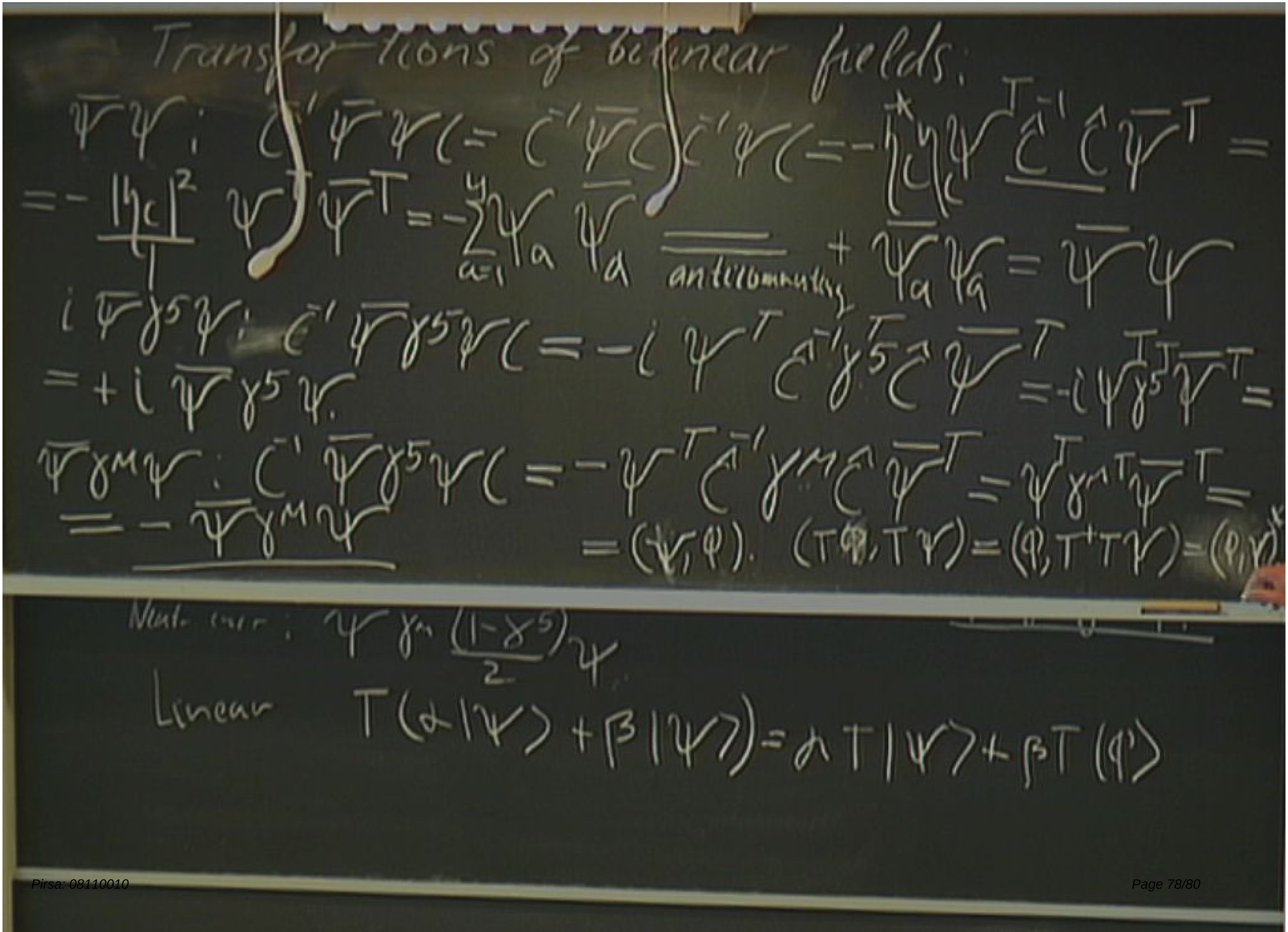
$= +i\bar\psi\gamma^5\psi.$

$\bar\psi\gamma^\mu\psi$: $C^{-1}\bar\psi\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu C \bar\psi^T = \psi^T\gamma^{\mu T}\bar\psi^T =$

$= -\underline{\bar\psi\gamma^\mu\psi}$

$\psi) = (\varphi, T^\dagger T\psi) = (\psi, \varphi)^*$

Linear $T(\alpha|\psi\rangle + \beta|\varphi\rangle) = \alpha T|\psi\rangle + \beta T|\varphi\rangle$



Transfor tions of bilinear fields:

$\overline{\psi}\psi$: $C^{-1}\overline{\psi}\psi C = C^{-1}\overline{\psi}C\,C^{-1}\psi C = -\eta_c^{*}\eta_c\,\psi^T \underline{C^{-1}C}\,\overline{\psi}^T =$

$= -\frac{|\eta_c|^2}{1}\psi^T\overline{\psi}^T = -\sum_{a=1}^{4}\psi_a\overline{\psi}_a \underset{\text{anticommuting}}{=} +\overline{\psi}_a\psi_a = \overline{\psi}\psi$

$i\overline{\psi}\gamma^5\psi$: $C^{-1}\overline{\psi}\gamma^5\psi C = -i\psi^T C^{-1}\gamma^{5T}C\overline{\psi}^T = -i\psi^T\gamma^{5T}\overline{\psi}^T =$

$= +i\overline{\psi}\gamma^5\psi.$

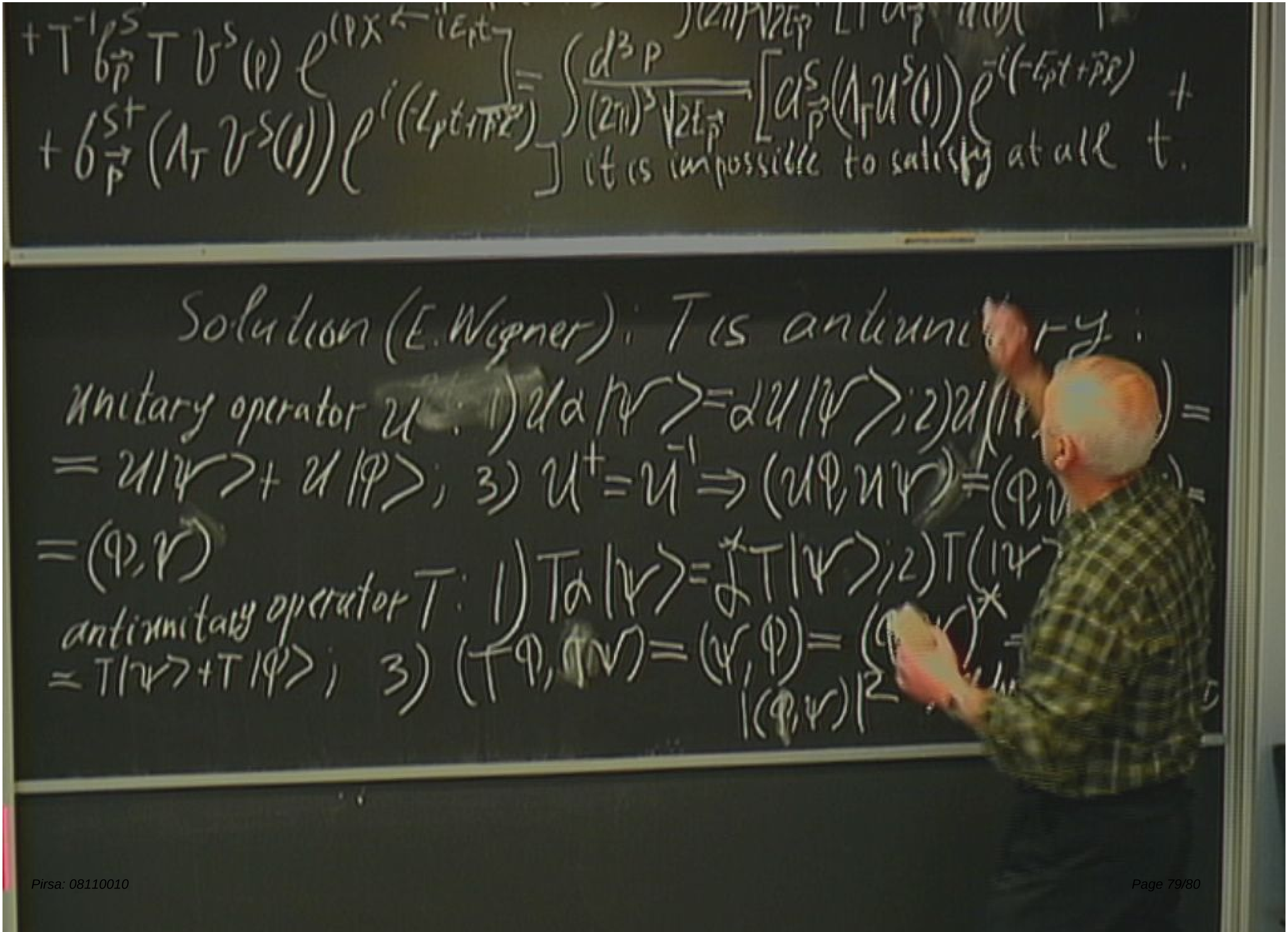
$\overline{\psi}\gamma^\mu\psi$: $C^{-1}\overline{\psi}\gamma^5\psi C = -\psi^T C^{-1}\gamma^\mu C\overline{\psi}^T = \psi^T\gamma^{\mu T}\overline{\psi}^T =$

$= -\underline{\overline{\psi}\gamma^\mu\psi}$

$= (\psi,\varphi)$. $(T\varphi, T\psi) = (\varphi, T^\dagger T\psi) = (\varphi,\psi)^*$

Neut. curr.: $\overline{\psi}\gamma_\mu\frac{(1-\gamma^5)}{2}\psi$

Linear $T(\alpha|\psi\rangle + \beta|\psi\rangle) = \alpha T|\psi\rangle + \beta T|\varphi\rangle$



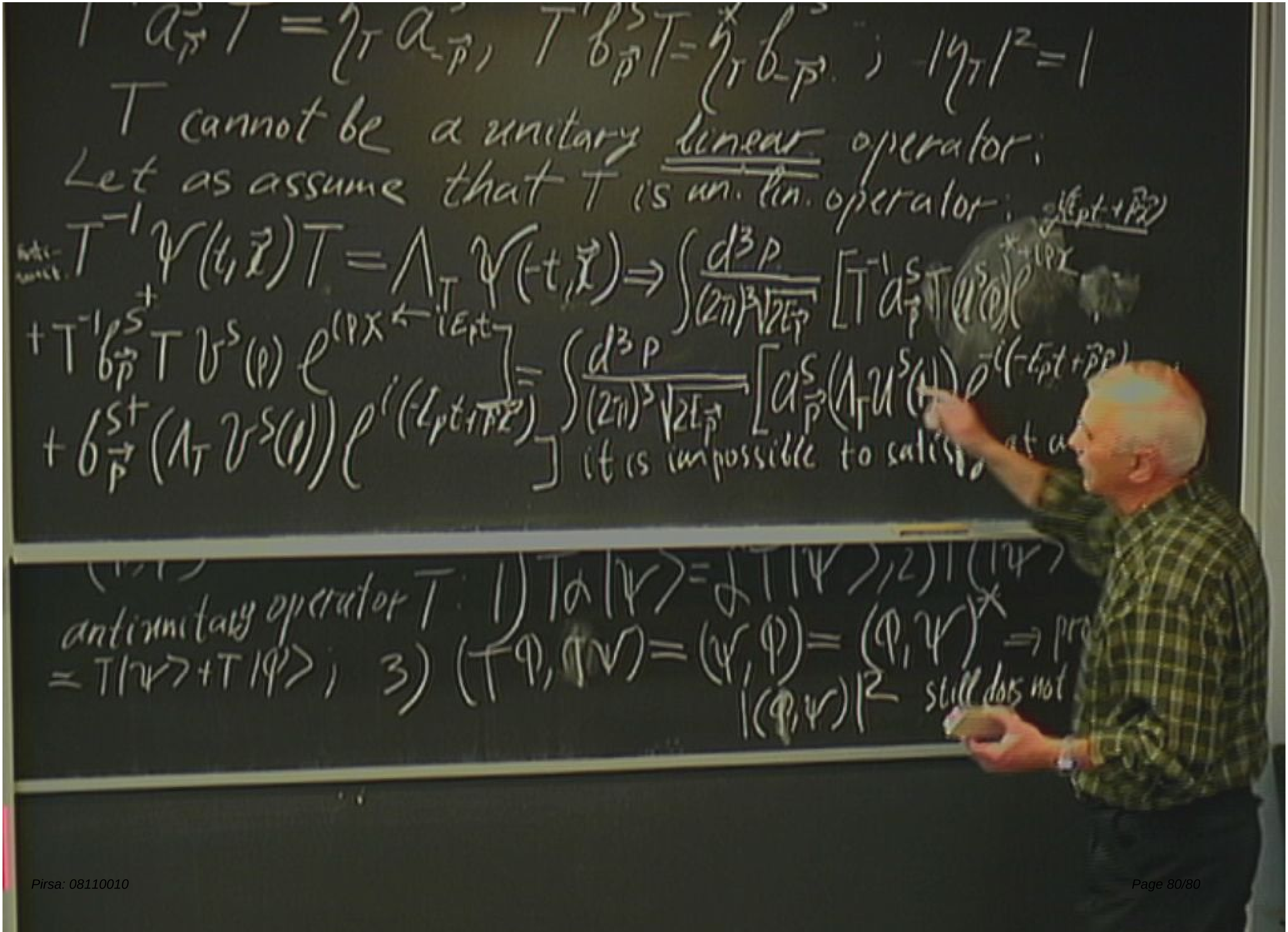
$+T^{-1}b^{s}_{\vec{p}}T\,v^{s}(p)\,e^{i(px \leftarrow iE_pt}\Big] = \int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}}\Big[a^{s}_{\vec{p}}(\Lambda_T u^{s}(p))\,e^{-i(-E_pt+\vec{p}\vec{x})} +$

$+\,b^{s+}_{\vec{p}}(\Lambda_T v^{s}(p))\,e^{i(-E_pt+\vec{p}\vec{x})}\Big]$ it is impossible to satisfy at all t.

Solution (E. Wigner): T is antiunitary:

unitary operator U: 1) $U\alpha|\psi\rangle = \alpha U|\psi\rangle$; 2) $U($[illegible]$) =$
$= U|\psi\rangle + U|\varphi\rangle$; 3) $U^{+} = U^{-1} \Rightarrow (U\varphi, U\psi) = (\varphi, U$[illegible]$) =$
$= (\varphi, \psi)$

antiunitary operator T: 1) $T\alpha|\psi\rangle = \alpha^{*}T|\psi\rangle$; 2) $T(|\psi\rangle$[illegible]
$= T|\psi\rangle + T|\varphi\rangle$; 3) $(T\varphi, T\psi) = (\psi, \varphi) = (\varphi, \psi)^{*}$ [illegible]
$|(\varphi,\psi)|^{2}$ [illegible]



$T^{-1} a^s_{\vec p} T = \eta_T a^s_{-\vec p}, \quad T^{-1} b^s_{\vec p} T = \eta^*_T b^s_{-\vec p}\ ; \quad |\eta_T|^2 = 1$

T cannot be a unitary <u>linear</u> operator:

Let as assume that T is un. lin. operator; $e^{i(E_pt+\vec p\vec x)}$

anti-unit. $T^{-1}\psi(t,\vec x)T = \Lambda_T \psi(-t,\vec x) \Rightarrow \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[T^{-1}a^s_{\vec p}T(u^s(p))e^{+ipx}$

$+T^{-1}b^{s+}_{\vec p}T\, v^s(p)\, e^{ipx \leftarrow iE_pt}\Big] = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}}\Big[a^s_{\vec p}(\Lambda_T u^s(p))e^{-i(-E_pt+\vec p\vec x)}$

$+b^{s+}_{\vec p}(\Lambda_T v^s(p))e^{i(-E_pt+\vec p\vec x)}\Big]$ it is impossible to satisfy at a

antiunitary operator T: 1) $T\alpha|\psi\rangle = \alpha T|\psi\rangle$, 2) $T(|\psi\rangle$

$= T|\psi\rangle + T|\varphi\rangle$; 3) $(T\varphi, T\psi) = (\psi,\varphi) = (\varphi,\psi)^* \Rightarrow$ pr

$|(\varphi,\psi)|^2$ still does not