

Title: Quantum Field Theory 1 - Lecture 11A

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

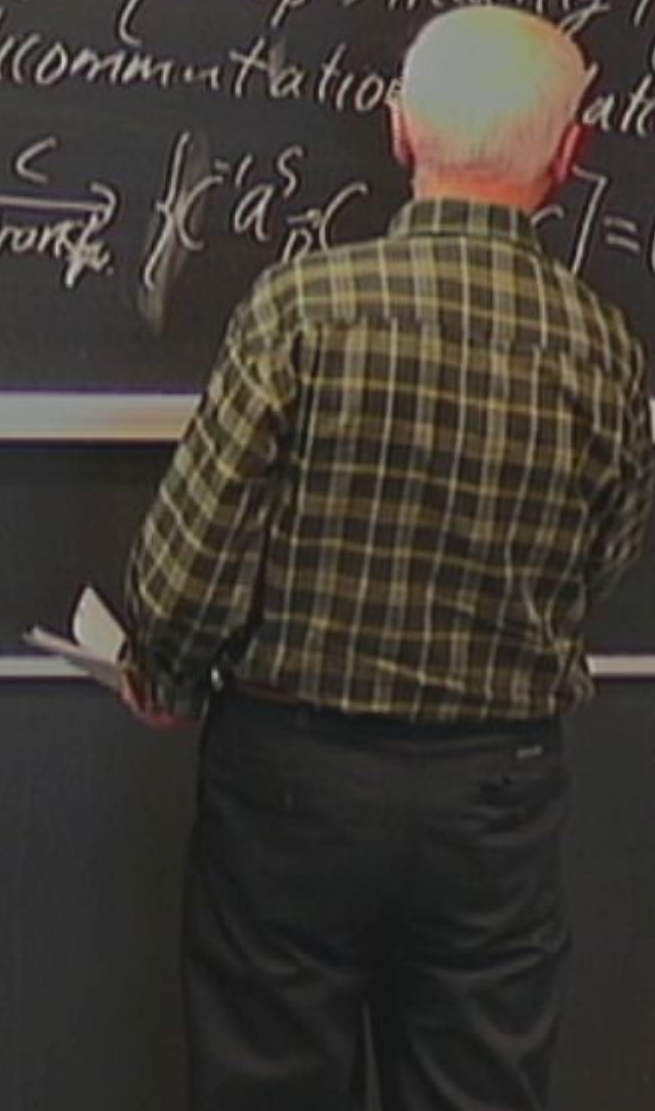
Charge Conjugation

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(Charge Conjugation
fermion $(\vec{p}, s) \rightarrow$ antifermion (\vec{p}, s) , and vice versa:
 $a_{\vec{p}}^s \mathcal{C} = \eta_c b_{\vec{p}}^s, \mathcal{C}^{-1} b_{\vec{p}}^s = \eta_c^* a_{\vec{p}}^s$

$c^{-1} a_{\vec{p}}^{s+} (= \eta^* b_{\vec{p}}^{s+}), c^{-1} b_{\vec{p}}^s (= \eta c a_{\vec{p}}^{s+})$, Actually $|\eta c|^2 = 1$ in
 order to keep same anticommutation relations.

$c^{-1} a_{\vec{p}}^{st} = \eta^x c b_{\vec{p}}^{st}$, $c^{-1} b_{\vec{p}}^s = \eta c a_{\vec{p}}^{st}$, Actually $|\eta c|^2 = 1$ in
 order to keep same anticommutation relations.
 $\{a_{\vec{p}}^s, a_{\vec{q}}^{st}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \xrightarrow{\text{transform}} \{c^{-1} a_{\vec{p}}^s, c^{-1} a_{\vec{q}}^{st}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$



$c' a_{\vec{p}}^s (= \eta_c^x b_{\vec{p}}^s), c' b_{\vec{p}}^s (= \eta_c a_{\vec{p}}^s)$, Actually $|\eta_c|^2 = 1$ in order to keep same anticommutation relations.

$\{a_{\vec{p}}^s, a_{\vec{q}}^{s'}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \xrightarrow{\text{transf.}} \{c' a_{\vec{p}}^s, c' a_{\vec{q}}^{s'}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \rightarrow$

$\rightarrow |\eta_c|^2 \{b_{\vec{p}}^s, b_{\vec{q}}^{s'}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \implies |\eta_c|^2 = 1$

Let us define actu

$$\rightarrow |\eta_{cl}|^2 \{b_{\vec{p}}, b_{\vec{q}}^{s+}\} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \implies |\eta_{cl}|^2 = 1$$

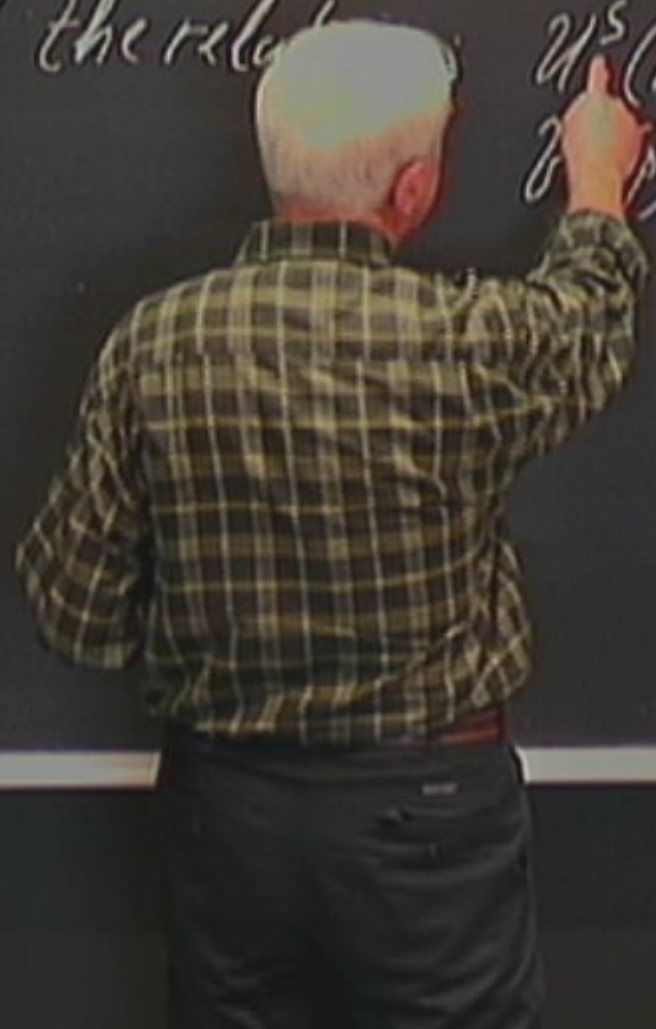
Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = ?$
 First

$$\rightarrow |\eta_C|^2 \{ b_{\vec{p}}, b_{\vec{q}}^{S^+} \} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \implies |\eta_C|^2 = 1$$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = ?$

First we need the relation

$$\begin{aligned} \psi^S(p) &= -i\gamma^2 (\psi^S(p))^* \quad (\text{without proof}) \\ \psi(p) &= -i\gamma^2 (\psi^S(p)) \end{aligned}$$



$$\rightarrow \langle \eta_{cl} | \{ b_{\vec{p}}, b_{\vec{q}}^{s+} \} = (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \implies |\eta_{cl}|^2 = 1$$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C^{-1}$?

First we need the relations:
$$\begin{aligned} \mathcal{U}^S(p) &= -i\gamma^2 (\mathcal{V}^S(p))^* & (\text{without proof}) \\ \mathcal{V}^S(p) &= -i\gamma^2 (\mathcal{U}^S(p))^* \end{aligned}$$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = \gamma^2 \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$

First we need the relations: $U^S(p) = -i\gamma^2 (\gamma^S(p))^*$ (without proof)
 $\gamma^S(p) = -i\gamma^2 (U^S(p))^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C^{-1} a_p^s C U^S(p) e^{-i p x} + C^{-1} b_p^s C \gamma^S(p) e^{i p x} \right)$$

order to keep same anticommutation relations

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = \gamma^2 \begin{pmatrix} 0 & \sigma^2 \\ \sigma^3 & 0 \end{pmatrix} \psi(x)$

First we need the relations: $u^s(p) = -i\gamma^2 (\gamma^s \psi)$ (without proof)
 $\psi^s(p) = -i\gamma^2 (u^s(p))$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C^{-1} a_p^s C u^s(p) e^{-ipx} + C^{-1} b_p^s C \psi^s(p) e^{ipx} \right) =$$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = \psi^c(x)$ $\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$

First we need the relations: $u^s(p) = -i\gamma^2 (v^s(p))^*$ (without proof)
 $v^s(p) = -i\gamma^2 (u^s(p))^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C^{-1} a_p^s C u^s(p) e^{-ipx} + C^{-1} b_p^s C v^s(p) e^{ipx} \right) =$$

$$= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s b_p^s$$



Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = \gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$

First we need the relations: $U^S(p) = -i\gamma^2 (\gamma^S(p))^*$ (without proof)

$\gamma^S(p) = -i\gamma^2 (U^S(p))^*$

$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(C^{-1} a_p^S C U^S(p) e^{ipx} + C^{-1} b_p^S C \gamma^S(p) e^{ipx} \right) =$$

$$= \eta C \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S b_p^S \left[-i\gamma^2 (\gamma^S(p))^* \right] + \dots$$

Let us define action of C on $\psi(x)$: $C^{-1}\psi(x)C = \gamma^2 \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$

First we need the relations: $u^s(p) = -i\gamma^2 (\gamma^s(p))^*$ (without proof)

$\gamma^s(p) = -i\gamma^2 (u^s(p))^*$

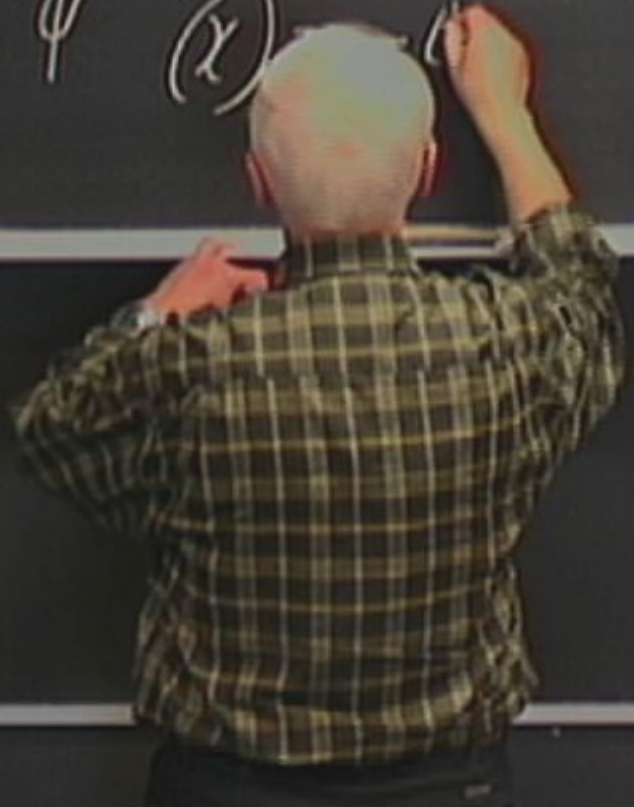
$$C^{-1}\psi(x)C = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C^{-1} a_p^s C u^s(p) e^{-i p x} + C^{-1} b_p^{s\dagger} C \gamma^s(p) e^{i p x} \right) =$$

$$= i \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(b_p^s [-i\gamma^2 (\gamma^s(p))^*] e^{-i p x} + a_p^{s\dagger} [-i\gamma^2 (u^s(p))^*] e^{i p x} \right)$$

Let us define action of C on $\psi(x)$. $C^{-1}\psi(x)C$ $\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$

First we need the relations: $a^S(p) = -i\gamma^2 (b^S(p))^*$ (without proof)
 $b^S(p) = -i\gamma^2 (a^S(p))^*$

$$\begin{aligned}
 C^{-1}\psi(x)C &= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(C^{-1} a_p^S C u(p)^S e^{-i p x} + C^{-1} b_p^{S\dagger} C v(p)^S e^{i p x} \right) \\
 &= i\gamma_C \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(b_p^S [-i\gamma^2 (b^S(p))^*] e^{-i p x} + a_p^{S\dagger} [-i\gamma^2 (a^S(p))^*] e^{i p x} \right) \\
 &= -i\gamma_C \gamma^2 \psi^*(x)
 \end{aligned}$$



First we need the relations:

$$\psi^{\dagger}(p) = -i\gamma^2 \psi^s(p) \quad (\text{without proof})$$

$$\psi^s(p) = -i\gamma^2 \psi^{\dagger}(p)$$

$$C^{-1} \psi(x) C = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C a_{\vec{p}}^s C \psi^s(p) e^{-ipx} + C^{-1} b_{\vec{p}}^{s\dagger} C \psi^s(p) e^{ipx} \right)$$

$$= i\gamma_c \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(b_{\vec{p}}^s [-i\gamma^2 \psi^s(p)]^* e^{-ipx} + a_{\vec{p}}^{s\dagger} [-i\gamma^2 \psi^s(p)]^* e^{ipx} \right)$$

$$= -i\gamma_c \gamma^2 \psi^{\dagger}(x) = -i\gamma_c \gamma^2 \psi^{\dagger}(x)$$



... and the relations:

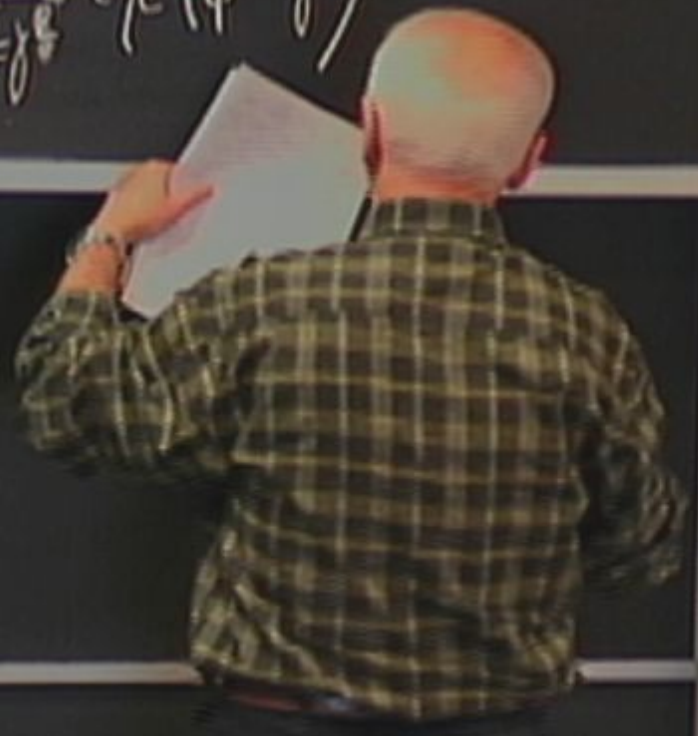
$$\psi^{\dagger}(p) = -i\gamma^2 (\gamma^5 \psi(p))^* \quad (\text{without proof})$$

$$\psi^{\dagger}(p) = -i\gamma^2 (\gamma^5 \psi(p))^*$$

$$C^{-1} \psi(x) C = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C a_{\vec{p}}^s C \psi(p) e^{-ipx} + C b_{\vec{p}}^s C \psi(p) e^{ipx} \right)$$

$$= i\gamma_c \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(b_{\vec{p}}^s [-i\gamma^2 (\gamma^5 \psi(p))^*] e^{-ipx} + a_{\vec{p}}^s [-i\gamma^2 (\gamma^5 \psi(p))^*] e^{ipx} \right)$$

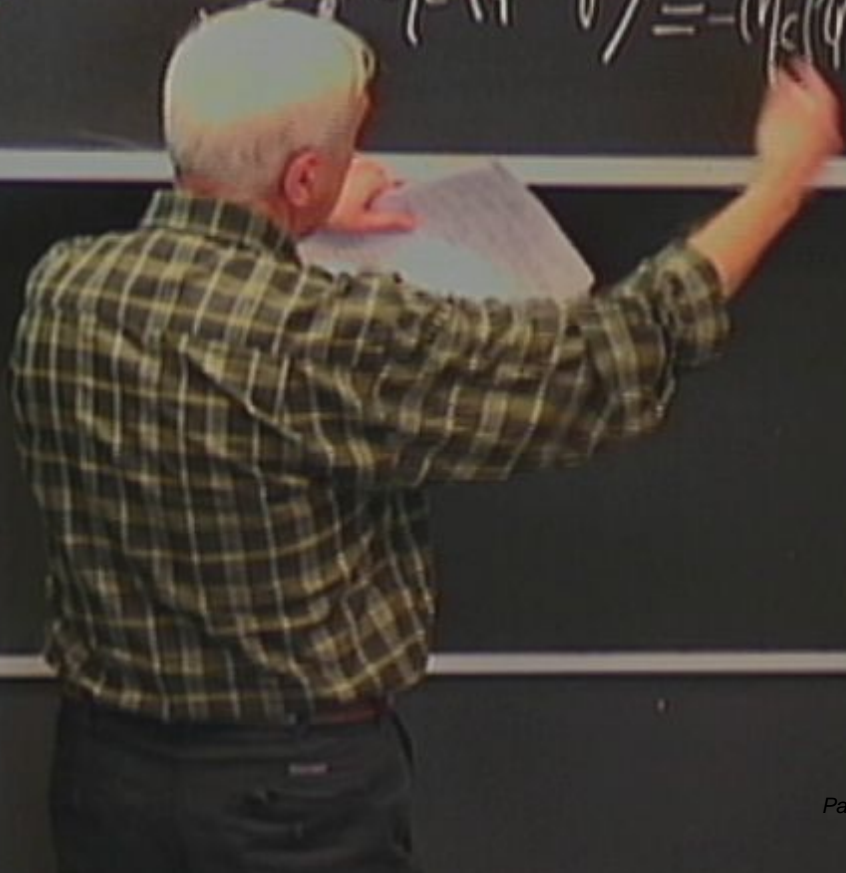
$$= -i\eta_c \gamma^2 \psi^*(x) = -i\eta_c \gamma^2 \psi^{\dagger T}(x) \quad \underline{\underline{\gamma^2 T = \gamma^0}}$$



... and the relations:

$$\begin{aligned}
 \psi^{\dagger}(\mathbf{p}) &= -i\gamma^2 (\psi^S(\mathbf{p}))^* && \text{(without proof)} \\
 \psi^S(\mathbf{p}) &= -i\gamma^2 (\psi^{\dagger}(\mathbf{p}))^*
 \end{aligned}$$

$$\begin{aligned}
 C^{-1} \psi(x) C &= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\mathbf{p}}}} \sum_s \left(C a_{\mathbf{p}}^s C^{-1} u(\mathbf{p}) e^{-ipx} + C b_{\mathbf{p}}^{s\dagger} C^{-1} v^s(\mathbf{p}) e^{ipx} \right) \\
 &= i\gamma^2 \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\mathbf{p}}}} \sum_s \left(b_{\mathbf{p}}^s [-i\gamma^2 (\psi^S(\mathbf{p}))^*] e^{-ipx} + a_{\mathbf{p}}^{s\dagger} [-i\gamma^2 (\psi^{\dagger}(\mathbf{p}))^*] e^{ipx} \right) \\
 &= -i\gamma^2 \psi^{\dagger}(x) = -i\gamma^2 \psi^{\dagger}(x)
 \end{aligned}$$



$$\begin{aligned}
 C^{-1} \psi(x) C &= \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(C a_{\vec{p}}^s C^{-1} U(p) e^{-i p x} + C b_{\vec{p}}^{s\dagger} C^{-1} V^s(p) e^{i p x} \right) \\
 &= \eta_C \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left(b_{\vec{p}}^s [-i \gamma^2 (\gamma^s(p))^*] e^{-i p x} + a_{\vec{p}}^{s\dagger} [-i \gamma^2 (a^s(p))^*] e^{i p x} \right) \\
 &= -i \eta_C \gamma^2 \psi^*(x) = -i \eta_C \gamma^2 \psi^\dagger{}^T \\
 &= \frac{1}{\gamma^{2T} = \gamma^2} -i \eta_C (\psi^\dagger \gamma^2)^T = -i \eta_C (\bar{\psi} \gamma^0 \gamma^2)^T
 \end{aligned}$$

(without proof)



$$\begin{aligned}
 \psi(x) &= \frac{1}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(\bar{a}_p^S \psi(x) + \bar{b}_p^{S\dagger} \psi(x) \right) \\
 &= \eta_c \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_S \left(\bar{b}_p^S [-\gamma^2 (\psi^S(p))^*] e^{-i p x} + a_p^{S\dagger} [-\gamma^2 (\psi^S(p))^*] e^{-i p x} \right) \\
 &= -i \eta_c \gamma^2 \psi^T(x) = -i \eta_c \gamma^2 \psi^{\dagger T} \stackrel{\gamma^{2T} = \gamma^2}{=} -i \eta_c (\psi^{\dagger} \gamma^2)^T = -i \eta_c (\bar{\psi} \gamma^0 \gamma^2)^T \\
 &\equiv \eta_c \hat{C} \bar{\psi}^T, \text{ where } \hat{C} \equiv -i (\gamma^0 \gamma^2)^T
 \end{aligned}$$



$$\begin{aligned}
 & \left(\frac{1}{2m} \nabla^2 \psi + \frac{1}{2} \gamma^0 \gamma^1 \gamma^2 \gamma^3 \psi \right) = \left(\frac{1}{2m} \nabla^2 \psi + \frac{1}{2} \gamma^0 \gamma^1 \gamma^2 \gamma^3 \psi \right) \\
 & = -i \eta_c \gamma^2 \psi^* \quad \text{where } \hat{C} \equiv -i (\gamma^0 \gamma^2)^T
 \end{aligned}$$

Properties of \hat{C} : $\hat{C}^\dagger = \hat{C}^{-1}$ (unitary), $\hat{C}^T = -\hat{C}$ (antisymmetric), $\hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^{\mu T}$, $\hat{C}^{-1} \gamma^5 \hat{C} = \gamma^{5T}$; $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$

$$\begin{aligned}
 & \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \left(\frac{1}{2m} \nabla^2 E_{\vec{p}} \right) \\
 & = -i \eta_c \gamma^2 \psi^* \\
 & \equiv \eta_c \hat{C} \Psi^T, \text{ where } \hat{C} \equiv -i (\gamma^0 \gamma^2)^T
 \end{aligned}$$

Properties of \hat{C} : $\hat{C}^\dagger = \hat{C}^{-1}$ (unitary), $\hat{C}^T = -\hat{C}$ (antisymmetric), $\hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^{\mu T}$, $\hat{C}^{-1} \gamma^5 \hat{C} = \gamma^{5T}$; $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -\mathbf{I} & 0 \\ 0 & \mathbf{I} \end{pmatrix}$

$$\begin{aligned}
 &= -i\eta_c \gamma^2 \psi^* \\
 &\equiv \eta_c \hat{C} \Psi^T, \text{ where } \hat{C} \equiv -i(\gamma^0 \gamma^2)^T
 \end{aligned}$$

Properties of \hat{C} : $\hat{C}^\dagger = \hat{C}^{-1}$ (unitary), $\hat{C}^T = -\hat{C}$ (antisymmetric), $\hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^{\mu T}$, $\hat{C}^{-1} \gamma^5 \hat{C} = \gamma^{5T}$; $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -\mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix}$

These properties of \hat{C} are valid for all representations of γ^μ . In our (chiral) representation \hat{C} is real, $\hat{C} =$

$$\begin{aligned}
 &= -i \eta_c \gamma^2 \psi^* \\
 &= \eta_c \hat{C} \psi^T, \text{ where } \hat{C} = -i (\gamma^0 \gamma^2)^T
 \end{aligned}$$

Properties of \hat{C} : $\hat{C}^\dagger = \hat{C}^{-1}$ (unitary), $\hat{C}^T = -\hat{C}$ (antisymmetric), $\hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^{\mu T}$, $\hat{C}^{-1} \gamma^5 \hat{C} = \gamma^{5T}$; $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$

These properties of \hat{C} are valid for all representations of γ^μ . In our (chiral) representation \hat{C} is real, $\hat{C} = -i \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\equiv \eta_c \hat{C} \Psi^T, \text{ where } \hat{C} \equiv -i(\gamma^0 \gamma^2)^T$$

Properties of \hat{C} : $\hat{C}^\dagger = \hat{C}^{-1}$ (unitary), $\hat{C}^T = -\hat{C}$ (antisymmetric), $\hat{C}^{-1} \gamma^m \hat{C} = -\gamma^{mT}$, $\hat{C}^{-1} \gamma^5 \hat{C} = \gamma^{5T}$; $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -\mathbf{I} & 0 \\ 0 & \mathbf{I} \end{pmatrix}$

These properties of \hat{C} are valid for all representations of γ^m . For a chiral representation \hat{C} is real, $\hat{C} = -i\gamma^0 \gamma^2$, therefore, $\hat{C}^\dagger = \hat{C}^{*T} = \hat{C}^T = -\hat{C}$, $\hat{C}^{-1} = -\hat{C}$

$$\equiv \eta_c \hat{C} \Psi^T, \text{ where } \hat{C} \equiv -i(\gamma^0 \gamma^2)^T$$

Properties of \hat{C} : $\hat{C}^\dagger = \hat{C}^{-1}$ (unitary), $\hat{C}^T = -\hat{C}$ (antisymmetric), $\hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^{\mu T}$, $\hat{C}^{-1} \gamma^5 \hat{C} = \gamma^{5T}$; $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 =$

These properties of \hat{C} are valid for all representations of γ^μ . In our (chiral) representation \hat{C} is real, $\hat{C} = -i \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$, and, therefore, $\hat{C}^\dagger = \hat{C}^{*T} = \hat{C}^T = -\hat{C}$, $\hat{C}^{-1} = -\hat{C}$

$$\bar{c}^{-1} \bar{\psi} c = (\bar{c}^{-1} \psi c)^{\dagger} \gamma^0 = (\eta c^{\dagger} \bar{\psi}^{\dagger}) \gamma^0 = \eta^* \bar{\psi}^{\dagger} c^{\dagger} =$$

$$= \eta$$

$$\begin{aligned}
 \bar{C}^{-1} \bar{A} \bar{C} &= (\bar{C}^{-1} \Psi C)^+ \gamma_0 = (\eta_c C^+ \bar{\Psi}^T)^+ \gamma_0 = \eta_c^* \bar{\Psi}^T C^+ \gamma_0 = \\
 &= \eta_c^* (\bar{\Psi}^T \gamma_0)^T C^+ \gamma_0 = \eta_c^* \bar{\Psi}^T \gamma_0^T C^+ \gamma_0
 \end{aligned}$$

$$\begin{aligned}
 \bar{C}^{-1} \bar{\Psi} C &= (\bar{C}^{-1} \Psi C)^+ \gamma^0 = (\eta_C C \bar{\Psi}^T)^+ \gamma^0 = \eta_C^* \bar{\Psi}^T{}^+ C \\
 &= \eta_C^* (\bar{\Psi}^+ \gamma^0)^T C^+ \gamma^0 = \eta_C^* \bar{\Psi}^T \gamma^0{}^T C^+ \gamma^0 = \eta_C^* \bar{\Psi}^T \gamma^0{}^T
 \end{aligned}$$

$$\begin{aligned}
 \bar{C}^{-1} \bar{\Psi} C &= (\bar{C}^{-1} \Psi C)^+ \gamma^0 = (\eta_C \bar{C} \bar{\Psi}^T)^+ \gamma^0 = \eta_C^* \bar{\Psi}^T C^+ \gamma^0 = \\
 &= \eta_C^* (\bar{\Psi}^+ \gamma^0)^T C^+ \gamma^0 = \eta_C^* \bar{\Psi}^T \gamma^0 T C^+ \gamma^0 = \eta_C^* \bar{\Psi}^T \gamma^0 T C^+
 \end{aligned}$$

$$\begin{aligned}
 \bar{c}^{-1} \bar{c} c &= (c^{-1} \psi c)^+ y_0 = (\eta_c \hat{c} \bar{\psi}^T)^+ y_0 = \eta_c^* \bar{\psi}^T \hat{c}^+ y_0 = \\
 &= \eta_c^* (\bar{\psi}^T \hat{c}^+ y_0) = \eta_c^* \bar{\psi}^T y_0 \hat{c}^+ y_0 = \eta_c^* \bar{\psi}^T y_0 \hat{c}^{-1} y_0 =
 \end{aligned}$$

$$\begin{aligned}
\bar{C}^{-1} \bar{\Psi} C &= (\bar{C}^{-1} \Psi C)^+ \gamma_0 = (\eta_C \hat{C} \bar{\Psi}^T)^+ \gamma_0 = \eta_C^* \bar{\Psi}^T \hat{C}^+ \gamma_0 = \\
&= \eta_C^* (\bar{\Psi}^+ \gamma_0)^+ \hat{C}^+ \gamma_0 = \eta_C^* \bar{\Psi}^T \gamma_0^T \hat{C}^+ \gamma_0 = \eta_C^* \bar{\Psi}^T \gamma_0^T \hat{C}^{-1} \gamma_0 = \\
&= \eta_C^* \bar{\Psi}^T \gamma_0^T \underbrace{\hat{C}^{-1} \gamma_0 \hat{C}}_{\hat{C}^{-1} \gamma_0 \hat{C}} \hat{C}^{-1} = \eta_C^* \bar{\Psi}^T \gamma_0^T \hat{C}^{-1} \gamma_0 \hat{C}^{-1}
\end{aligned}$$

$$\begin{aligned}
\bar{C}^{-1} \bar{\Psi} C &= (\bar{C}^{-1} \Psi C)^+ \gamma^0 = (\eta_c \hat{C} \bar{\Psi}^T)^+ \gamma^0 = \eta_c^* \bar{\Psi}^T \hat{C}^{-1} \\
&= \eta_c^* (\Psi^+ \gamma^0)^+ \hat{C}^{-1} \gamma^0 = \eta_c^* \Psi^T \gamma^0 \hat{C}^{-1} \gamma^0 = \eta_c^* \Psi^T \gamma^0 \hat{C}^{-1} \gamma^0 \\
&= \eta_c^* \Psi^T \gamma^0 \hat{C}^{-1} \gamma^0 \hat{C}^{-1} = \eta_c^* \Psi^T \gamma^0 \gamma^0 \hat{C}^{-1} = \eta_c^* \Psi^T \hat{C}^{-1}
\end{aligned}$$

$$\begin{aligned}
C^{-1} \bar{\psi} C &= (C^{-1} \psi C)^{\dagger} \gamma_0 = (\eta_C C^{\dagger} \bar{\psi}^{\dagger}) \gamma_0 = \eta_C^* \bar{\psi}^{\dagger} C^{\dagger} \gamma_0 = \\
&= \eta_C^* (\bar{\psi}^{\dagger} \gamma_0)^{\dagger} C^{\dagger} \gamma_0 = \eta_C^* \psi^T \gamma_0^T C^{\dagger} \gamma_0 = \eta_C^* \psi^T \gamma_0^T C^{-1} \gamma_0 = \\
&= \eta_C^* \psi^T \gamma_0^T \underbrace{C^{-1} \gamma_0 C}_{\gamma_0} C^{-1} = \eta_C^* \psi^T \gamma_0^T \gamma_0^T C^{-1} = -\eta_C^* \psi^T \gamma_0^T C^{-1} = -\eta_C^* \psi^T C^{-1}
\end{aligned}$$

all represent.

chiral

Transfor

$(2 \ 4 \ 7 \ 3)$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Transformations of bilinear fields:

$$\bar{\psi}\psi; \quad \bar{\psi}'\psi' = \bar{\psi}C^{-1}C'\psi = \bar{\psi}^* \psi^T C^{-1}$$

Transformations of bilinear fields:

$$\bar{\psi}\psi; \quad \bar{\psi}'\psi' = \bar{\psi}C^{-1}\psi' = \bar{\psi}C^{-1}C^{-1}\psi = -\frac{\hbar c}{c} \bar{\psi}C^{-1}C^{-1}\psi$$

$$-\frac{\hbar c}{c} \psi^T \bar{\psi}^T = -\frac{\hbar c}{c} \psi^T \bar{\psi}^T$$

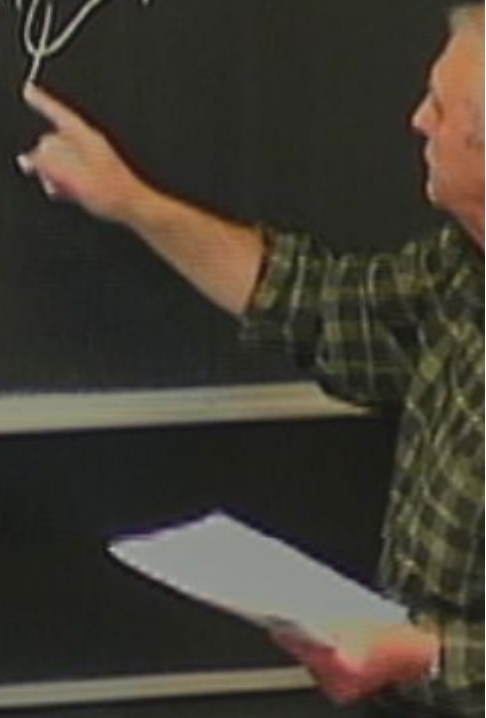
Transformations of bilinear fields:

$$\begin{aligned}
 \bar{\psi}\psi; \quad \bar{\psi}'\psi' &= \bar{\psi}'C\psi' = -\bar{\psi}'C^{-1}C\psi' = -\bar{\psi}'C^{-1}C\psi' = -\bar{\psi}'C^{-1}C\psi' = \\
 &= -\bar{\psi}'C^{-1}C\psi' = -\sum_a \psi_a \bar{\psi}_a \quad \text{anticommuting} \quad + \bar{\psi}_a \psi_a = \bar{\psi}\psi
 \end{aligned}$$

Transformations of bilinear fields:

$$\begin{aligned}
 \bar{\psi}\psi; \quad \bar{\psi}'\psi' &= \bar{C}'\bar{\psi}C' = \bar{C}'\bar{\psi}C' C^{-1}\psi C = -\frac{1}{|C|} \bar{\psi}^T C^{-1} C \psi^T = \\
 &= -\frac{|C|^{-2}}{|C|} \psi^T \bar{\psi}^T = -\sum_a \psi_a \bar{\psi}_a \quad \text{anticommuting} \quad + \bar{\psi}_a \psi_a = \bar{\psi}\psi
 \end{aligned}$$

$$\begin{aligned}
 \bar{\psi}\psi &: \bar{\psi}^T C^{-1} \psi C = \bar{\psi}^T C^{-1} C C^{-1} \psi C = -i \frac{\hbar c}{\lambda} \bar{\psi}^T C^{-1} C \psi^T = \\
 &= -\frac{\hbar c}{\lambda} \psi^T \bar{\psi}^T = -\sum_a \psi_a \bar{\psi}_a \quad \text{anticommuting} \quad + \bar{\psi}_a \psi_a = \bar{\psi}\psi \\
 i \bar{\psi} \gamma_5 \psi &: \bar{\psi}^T C^{-1} \gamma_5 \psi C = -i \psi^T C^{-1} \gamma_5 C \bar{\psi}^T
 \end{aligned}$$



$$= -\frac{|\hbar c|^2}{T} \psi^T \bar{\psi}^T = -\sum_a \psi_a \bar{\psi}_a \quad \text{anticommuting} \quad + \bar{\psi}_a \psi_a = \bar{\psi} \psi$$

$$i \bar{\psi} \gamma^5 \psi = \bar{\psi}^T \gamma^5 \psi^T = -i \psi^T \gamma^5 \bar{\psi}^T = -i \psi^T \gamma^5 \bar{\psi}^T =$$

$$= +i \bar{\psi} \gamma^5 \psi$$

$\bar{\psi} \gamma^\mu \psi$

$$= -\frac{|\hbar c|^2}{1} \psi^T \bar{\psi}^T = -\sum_{a=1}^4 \psi_a \bar{\psi}_a \quad \text{anticommuting} \quad + \bar{\psi}_a \psi_a = \bar{\psi} \psi$$

$$i \bar{\psi} \gamma^5 \psi: \quad \bar{\psi}^T \gamma^5 \psi^T = -i \psi^T \bar{\psi}^T \gamma^5 \psi^T = -i \psi^T \bar{\psi}^T \gamma^5 \psi^T = -i \psi^T \bar{\psi}^T \gamma^5 \psi^T =$$

$$= +i \bar{\psi} \gamma^5 \psi.$$

$$\bar{\psi} \gamma^M \psi: \quad \bar{\psi}^T \gamma^M \psi^T = -\psi^T \bar{\psi}^T \gamma^M \psi^T$$

$\sum_{a=1}^4 \gamma_a \psi_a$

$$i \bar{\psi} \gamma_5 \psi: C' \bar{\psi} \gamma_5 \psi C = -i \psi^T C^{-1} \gamma_5 C \bar{\psi}^T = -i \psi^T \gamma_5 \bar{\psi}^T = +i \bar{\psi} \gamma_5 \psi$$

$$\bar{\psi} \gamma_\mu \psi: C' \bar{\psi} \gamma_\mu \psi C = -\psi^T C^{-1} \gamma_\mu C \bar{\psi}^T = \psi^T \gamma_\mu \bar{\psi}^T =$$



$$\bar{\psi} \gamma^M \psi = \bar{\psi} \gamma^5 \psi = -\psi^T C \gamma^M \psi = \psi^T C \gamma^M \psi = \psi^T \gamma^M C \psi = \psi^T \gamma^M \psi$$

$$= \left\{ \right.$$

$$\bar{\psi} \gamma^m \psi : C^{-1} \bar{\psi} \gamma^5 \psi C = -\psi^T C \gamma^m C^{-1} \bar{\psi}^T = \psi^T \gamma^m T \bar{\psi}^T =$$

$$= -\bar{\psi} \gamma^m \psi$$

$$\bar{\psi} \gamma^m \gamma^5 \psi : C^{-1} \bar{\psi} \gamma^m \gamma^5 \psi C = -\psi^T$$

$$\bar{\psi} \gamma^{\mu} \psi : \bar{\psi} \gamma^{\mu} \psi = -\psi^T C \gamma^{\mu} C^{-1} \bar{\psi}^T = \psi^T \gamma^{\mu T} \bar{\psi}^T =$$

$$= -\bar{\psi} \gamma^{\mu} \psi$$

$$\bar{\psi} \gamma^{\mu} \gamma^5 \psi : \bar{\psi} \gamma^{\mu} \gamma^5 \psi = -\psi^T C \gamma^{\mu} \gamma^5 C^{-1} \bar{\psi}^T =$$

$$= \psi^T \gamma^{\mu T} \gamma^5 T \bar{\psi}^T = -\bar{\psi} \gamma^{\mu} \gamma^5 \psi = +\bar{\psi} \gamma^{\mu} \gamma^5 \psi$$

$$\bar{\psi} \gamma^m \psi : \bar{\psi} \gamma^m \psi = -\psi^T \gamma^m \psi = \psi^T \gamma^m \psi = \bar{\psi} \gamma^m \psi$$

$$\bar{\psi} \gamma^m \gamma^5 \psi : \bar{\psi} \gamma^m \gamma^5 \psi = -\psi^T \gamma^m \gamma^5 \psi = \psi^T \gamma^m \gamma^5 \psi = -\bar{\psi} \gamma^m \gamma^5 \psi = + \bar{\psi} \gamma^m \gamma^5 \psi$$

$$\bar{\psi} \gamma^m \psi : C^{-1} \bar{\psi} \gamma^5 \psi C = -\psi^T C \gamma^m C^{-1} \bar{\psi}^T = \psi^T \gamma^m T \bar{\psi}^T =$$

$$= -\bar{\psi} \gamma^m \psi$$

$$\bar{\psi} \gamma^m \gamma^5 \psi : C^{-1} \bar{\psi} \gamma^m \gamma^5 \psi C = -\psi^T C \gamma^m \gamma^5 C^{-1} \bar{\psi}^T =$$

$$= \psi^T \gamma^m T \gamma^5 T \bar{\psi}^T = -\bar{\psi} \gamma^5 \gamma^m \psi = + \bar{\psi} \gamma^m \gamma^5 \psi$$

Neutrino mass: $\bar{\psi} \gamma^m \frac{(1-\gamma^5)}{2} \psi$

Time Reversal (E. Wigner).

Time Reversal (E. Wigner).
 $(t, \vec{x}) \rightarrow (-t, \vec{x}) \Rightarrow \vec{p} \rightarrow -\vec{p}$ (like under P) and
 $S \rightarrow$

$$\vec{J}_{orb} = \vec{x} \times \vec{p} \xrightarrow[\substack{t \rightarrow -t \\ \vec{x} \rightarrow \vec{x}}]{\substack{x \rightarrow -x \\ I}} -\vec{x} \times \vec{p}$$

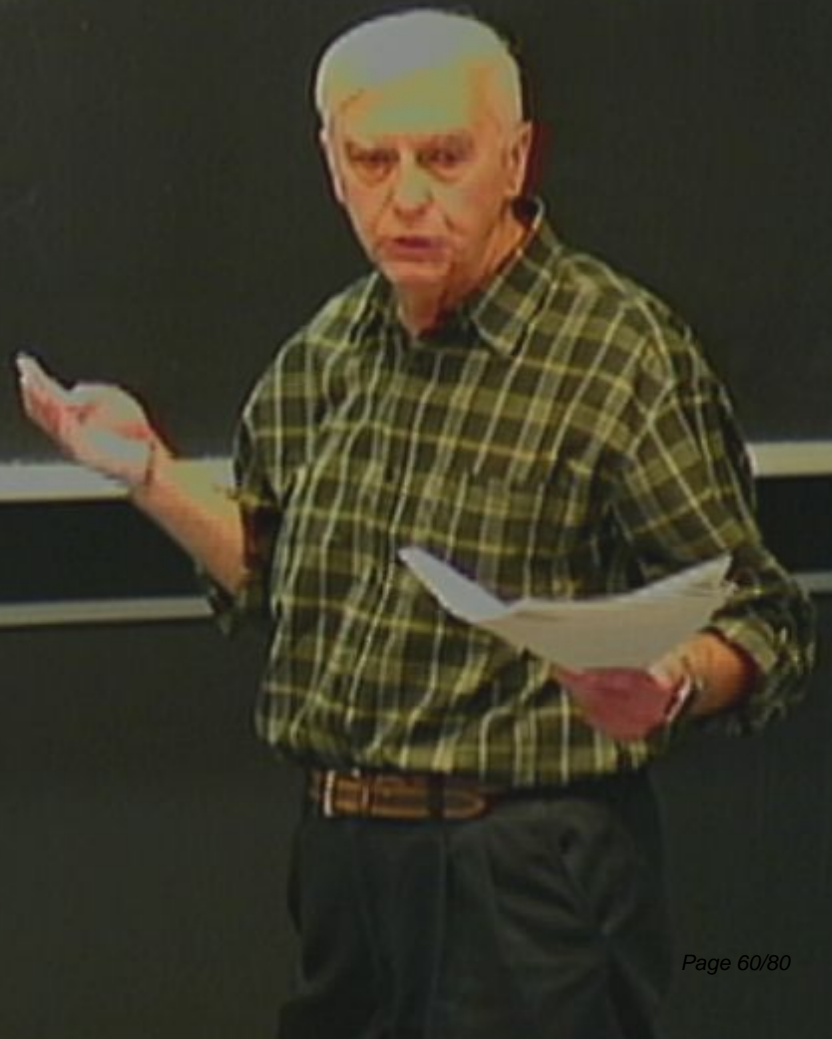
Because of that, we want that under T on:

$$T^{-1} \mathbf{a}_{\vec{p}}^S T = \lambda_T \mathbf{a}_{-\vec{p}}^{-S}, \quad T^{-1} \mathbf{b}_{\vec{p}}^S T = \mathbf{b}_{\vec{p}}^S$$

$$T^{-1} a_{\vec{p}}^S T = \eta_T a_{-\vec{p}}^{-S}, \quad T^{-1} b_{\vec{p}}^S T = \eta_T^* b_{-\vec{p}}^{-S}; \quad |\eta_T|^2 = 1$$

$$T^{-1} a_{\vec{p}}^s T = \eta_T a_{-\vec{p}}^{-s}, \quad T^{-1} b_{\vec{p}}^s T = \eta_T^* b_{-\vec{p}}^{-s}; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator.



$$T^{-1} a_{\vec{p}}^s T = \eta_T a_{-\vec{p}}^{-s}, \quad T^{-1} b_{\vec{p}}^s T = \eta_T^* b_{-\vec{p}}^{-s}; \quad |\eta_T|^2 = 1$$

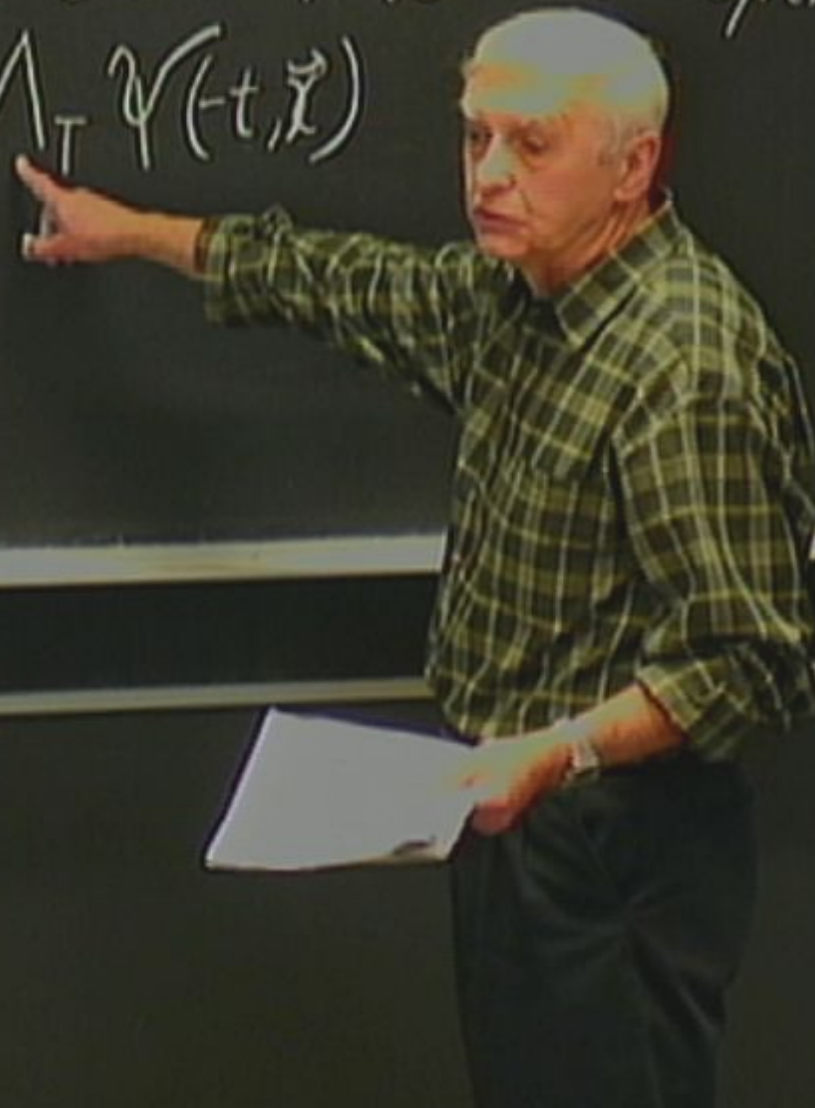
T cannot be a unitary linear operator:
Let us assume that T is a lin. operator:

$$T^{-1} \psi(t) =$$

$$T^{-1} a_{\vec{p}}^s T = \eta_T a_{-\vec{p}}^{-s}, \quad T^{-1} b_{\vec{p}}^s T = \xi_T b_{-\vec{p}}^{-s}; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator;
Let us assume that T is a lin. operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x})$$



$$T^{-1} a_{\vec{p}}^s T = \eta_T a_{-\vec{p}}^{-s}, \quad T^{-1} b_{\vec{p}}^s T = \xi_T b_{-\vec{p}}^{-s}; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator;
 Let us assume that T is a lin. operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} [T^{-1} a_{\vec{p}}^s]$$

$$T^{-1} \vec{a}_{\vec{p}} T = \eta_T \vec{a}_{-\vec{p}}, \quad T^{-1} \vec{b}_{\vec{p}} T = \eta_T^* \vec{b}_{-\vec{p}}; \quad -|\eta_T|^2 = 1$$

T cannot be a unitary linear operator.
 Let us assume that T is a lin. operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \left[T^{-1} \vec{a}_{\vec{p}} e^{-iE_{\vec{p}}t + i\vec{p}\cdot\vec{x}} + T^{-1} \vec{b}_{\vec{p}} T \vec{b}^S(\vec{p}) e^{iE_{\vec{p}}t - i\vec{p}\cdot\vec{x}} \right] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \left[\vec{a}_{-\vec{p}} e^{-iE_{\vec{p}}t + i\vec{p}\cdot\vec{x}} + \vec{a}_{\vec{p}}^S e^{iE_{\vec{p}}t - i\vec{p}\cdot\vec{x}} \right]$$

$$T^{-1} \vec{a}_{\vec{p}} T = \eta_T \vec{a}_{-\vec{p}}, \quad T^{-1} \vec{b}_{\vec{p}} T = \xi_T \vec{b}_{-\vec{p}}; \quad -|\eta_T|^2 = 1$$

T cannot be a unitary linear operator.
 Let us assume that T is a lin. operator:

$$T^{-1} \Psi(t, \vec{x}) = \int_{E_{\vec{p}} > 0} \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [T^{-1} \vec{a}_{\vec{p}} T u^s(\vec{p}) e^{-i(E_{\vec{p}} t + \vec{p} \cdot \vec{x})} + T^{-1} \vec{b}_{\vec{p}} T v^s(\vec{p}) e^{-i(E_{\vec{p}} t - \vec{p} \cdot \vec{x})}] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [a_{\vec{p}}^s(\Lambda_T u^s(\vec{p})) e^{-i(-E_{\vec{p}} t + \vec{p} \cdot \vec{x})} + b_{\vec{p}}^s(\Lambda_T v^s(\vec{p})) e^{-i(-E_{\vec{p}} t - \vec{p} \cdot \vec{x})}]$$

$$T^{-1} \vec{a}_{\vec{p}} T = \eta_T \vec{a}_{-\vec{p}}, \quad T^{-1} \vec{b}_{\vec{p}} T = \eta_T^* \vec{b}_{-\vec{p}}; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator.
 Let us assume that T is a unitary linear operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [T^{-1} \vec{a}_{\vec{p}} T u^S(\vec{p}) e^{-i(E_{\vec{p}} t + \vec{p} \cdot \vec{x})} + T^{-1} \vec{b}_{\vec{p}} T v^S(\vec{p}) e^{i(E_{\vec{p}} t + \vec{p} \cdot \vec{x})}] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [\vec{a}_{\vec{p}} (\Lambda_T u^S(\vec{p})) e^{-i(-E_{\vec{p}} t + \vec{p} \cdot \vec{x})} + \vec{b}_{\vec{p}} (\Lambda_T v^S(\vec{p})) e^{i(-E_{\vec{p}} t + \vec{p} \cdot \vec{x})}]$$



Because of that, we want that under 1. condition:

$$T^{-1} a_{\vec{p}}^s T = \eta_T \tilde{a}_{-\vec{p}}^{-s}, \quad T^{-1} b_{-\vec{p}}^{-s} T = \eta_T^* \tilde{b}_{\vec{p}}^{-s}; \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator.
 Let us assume that T is a linear operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \left[\frac{1}{\sqrt{2E_{\vec{p}}}} [T^{-1} a_{\vec{p}}^s T u^s(t)] e^{i(\vec{p}\vec{x} - E_{\vec{p}}t)} + \frac{1}{\sqrt{2E_{\vec{p}}}} [T^{-1} b_{-\vec{p}}^{-s} T v^s(t)] e^{i(\vec{p}\vec{x} - E_{\vec{p}}t)} \right] + \Lambda_T \left[\frac{1}{\sqrt{2E_{\vec{p}}}} [a_{\vec{p}}^s (\Lambda_T u^s(t))] e^{-i(\vec{p}\vec{x} - E_{\vec{p}}t)} + \frac{1}{\sqrt{2E_{\vec{p}}}} [b_{-\vec{p}}^{-s} (\Lambda_T v^s(t))] e^{-i(\vec{p}\vec{x} - E_{\vec{p}}t)} \right]$$

$$T^{-1} a_{\vec{p}}^s T = \eta_T a_{-\vec{p}}^{-s}, \quad T^{-1} b_{\vec{p}}^s T = \xi_T b_{-\vec{p}}^{-s}; \quad -|\eta_T|^2 = 1$$

T cannot be a unitary linear operator:
 Let us assume that T is a unitary linear operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \left[T^{-1} a_{\vec{p}}^s T u^s(\vec{p}) e^{-i(E_{\vec{p}}t + \vec{p}\cdot\vec{x})} + \right. \\
\left. T^{-1} b_{\vec{p}}^{s\dagger} T v^s(\vec{p}) e^{i(E_{\vec{p}}t + \vec{p}\cdot\vec{x})} \right] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \left[a_{\vec{p}}^s(\Lambda_T u^s(\vec{p})) e^{-i(-E_{\vec{p}}t + \vec{p}\cdot\vec{x})} + \right. \\
\left. + b_{\vec{p}}^{s\dagger}(\Lambda_T v^s(\vec{p})) e^{i(-E_{\vec{p}}t + \vec{p}\cdot\vec{x})} \right]$$

$+ b_{\vec{p}}^{\dagger} (A_T v^S(\vec{p})) e^{i(-L_p t + \vec{p} \cdot \vec{r})} \Big] \frac{(2\pi)^3 \sqrt{2E_{\vec{p}}}}{L} \Big[a_{\vec{p}}^{\dagger} (A_T u^S(\vec{p})) e^{i(L_p t - \vec{p} \cdot \vec{r})} + \dots \Big]$

it is impossible to satisfy at all t .



$$\begin{aligned}
 T^{-1} \psi(t, \vec{x}) T &= \Lambda_T \psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \left[T^{-1} a_{\vec{p}}^S T u^S(p) e^{-i(E_{\vec{p}}t + \vec{p}\vec{x})} + \right. \\
 &+ \left. T^{-1} b_{\vec{p}}^{S\dagger} T v^S(p) e^{i(E_{\vec{p}}t + \vec{p}\vec{x})} \right] - \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \left[a_{\vec{p}}^S (\Lambda_T u^S(p)) e^{-i(-E_{\vec{p}}t + \vec{p}\vec{x})} + \right. \\
 &+ \left. b_{\vec{p}}^{S\dagger} (\Lambda_T v^S(p)) e^{i(-E_{\vec{p}}t + \vec{p}\vec{x})} \right]
 \end{aligned}$$

for: $-i(E_{\vec{p}}t + \vec{p}\vec{x})$

it is impossible to satisfy at all t .

T cannot be a unitary linear operator:
 Let us assume that T is un. lin. operator:

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [T^{-1} a_{\vec{p}}^s T u^s(\vec{p}) e^{-i(E_{\vec{p}}t + \vec{p}\cdot\vec{x})} + T^{-1} b_{\vec{p}}^{s\dagger} T v^s(\vec{p}) e^{i(E_{\vec{p}}t + \vec{p}\cdot\vec{x})}] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [a_{\vec{p}}^s (\Lambda_T u^s(\vec{p})) e^{-i(-E_{\vec{p}}t + \vec{p}\cdot\vec{x})} + b_{\vec{p}}^{s\dagger} (\Lambda_T v^s(\vec{p})) e^{i(-E_{\vec{p}}t + \vec{p}\cdot\vec{x})}]$$

it is impossible to satisfy

$$\bar{\psi} \gamma^m \gamma^5 \psi; \quad \bar{\psi} \gamma^m \gamma^5 \psi = -\psi^T \bar{C} \gamma^m \gamma^5 C \bar{\psi}^T =$$

$$= \psi^T \gamma^{mT} \gamma^{5T} \bar{\psi}^T = -\bar{\psi} \gamma^5 \gamma^m \psi = + \underline{\bar{\psi} \gamma^m \gamma^5 \psi}.$$

Neutr. cur: $\bar{\psi} \gamma^m \frac{(1-\gamma^5)}{2} \psi$

Linear

$$T(\alpha |\psi\rangle + \beta |\psi'\rangle) = \alpha T|\psi\rangle + \beta T|\psi'\rangle$$

+ $b_{\vec{p}}^{\dagger} (A_T U^S(p))$] it is impossible to satisfy at all t .

Solution (E. Wigner): T is antiunitary:

unitary operator U : 1) $U a |\psi\rangle = a U |\psi\rangle$; 2) $U (|\psi\rangle + |\phi\rangle) = U |\psi\rangle + U |\phi\rangle$; 3) $U^{\dagger} = U^{-1} \Rightarrow (U\phi, U\psi) = (\phi, \psi)$

$$+ b_{\vec{p}}^{s\dagger} (A_T v^s(p)) e^{i(-E_p t + \vec{p} \cdot \vec{r})} \left. \right\} \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \left[a_{\vec{p}}^s (A_T u^s(p)) e^{-i(-E_p t + \vec{p} \cdot \vec{r})} + \right. \\ \left. + b_{\vec{p}}^{s\dagger} (A_T v^s(p)) e^{i(-E_p t + \vec{p} \cdot \vec{r})} \right] \text{ it is impossible to satisfy at all } t.$$

solution (E. Wigner). U is unitary.

unitary operator U : 1) $U a |\psi\rangle = a U |\psi\rangle$; 2) $U (|\psi\rangle + |\phi\rangle) = U |\psi\rangle + U |\phi\rangle$; 3) $U^\dagger = U^{-1} \Rightarrow (U\phi, U\psi) = (\phi, U^\dagger U\psi) = (\phi, \psi)$

antiunitary operator T : 1) $T a |\psi\rangle = a^* T |\psi\rangle$; 2) $T (|\psi\rangle + |\phi\rangle) = T |\psi\rangle + T |\phi\rangle$; 3) $(T\phi, T\psi) = (\psi, \phi) = (\phi, \psi)^*$

$$\left[b_{\vec{p}}^\dagger T U^S(p) e^{i(-E_p t + \vec{p} \cdot \vec{r})} + b_{\vec{p}}^S (A_T U^S(p)) e^{i(-E_p t + \vec{p} \cdot \vec{r})} \right] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \left[a_{\vec{p}}^S (A_T U^S(p)) e^{-i(-E_p t + \vec{p} \cdot \vec{r})} + \dots \right]$$

it is impossible to satisfy at all t .

unitary operator U : 1) $U a |\psi\rangle = a U |\psi\rangle$; 2) $U(|\psi\rangle + |\phi\rangle) = U|\psi\rangle + U|\phi\rangle$; 3) $U^\dagger = U^{-1} \Rightarrow (U\phi, U\psi) = (\phi, U^\dagger U\psi) = (\phi, \psi)$

antiunitary operator T : 1) $T a |\psi\rangle = a^* T |\psi\rangle$; 2) $T(|\psi\rangle + |\phi\rangle) = T|\psi\rangle + T|\phi\rangle$; 3) $(T\phi, T\psi) = (\psi, \phi) = (\phi, \psi)^* \Rightarrow$ probability

$$\left[b_{\vec{p}}^\dagger T U^S(p) e^{i(-E_p t + \vec{p} \cdot \vec{r})} + b_{\vec{p}}^S (A_T U^S(p)) e^{i(-E_p t + \vec{p} \cdot \vec{r})} \right] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \left[a_{\vec{p}}^S (A_T U^S(p)) e^{-i(-E_p t + \vec{p} \cdot \vec{r})} + \dots \right]$$

it is impossible to satisfy at all t.

Unitary operator U : 1) $U a |\psi\rangle = a U |\psi\rangle$; 2) $U(|\psi\rangle + |\phi\rangle) = U|\psi\rangle + U|\phi\rangle$; 3) $U^\dagger = U^{-1} \Rightarrow (U\phi, U\psi) = (\phi, U^\dagger U\psi) = (\phi, \psi)$

antiunitary operator T : 1) $T a |\psi\rangle = a^* T |\psi\rangle$; 2) $T(|\psi\rangle + |\phi\rangle) = T|\psi\rangle + T|\phi\rangle$; 3) $(T\phi, T\psi) = (\psi, \phi) = (\phi, \psi)^*$ \Rightarrow probability $|\langle \phi, \psi \rangle|^2$ still does not change under T

Transformations of bilinear fields.

$$\bar{\psi}\psi: \quad C^{-1} \bar{\psi}' \psi' = C^{-1} \bar{\psi} C C^{-1} \psi = -\frac{1}{|c|} \bar{\psi}^T \underline{C}^{-1} \underline{C} \psi^T =$$

$$= -\frac{|c|^{-2}}{|c|} \bar{\psi}^T \psi^T = -\sum_{a=1}^4 \psi_a \bar{\psi}_a \quad \text{anticommuting} \quad + \bar{\psi}_a \psi_a = \bar{\psi} \psi$$

$$i \bar{\psi} \gamma^5 \psi: \quad C^{-1} \bar{\psi}' \gamma^5 \psi' = -i \psi^T \underline{C}^{-1} \gamma^5 \underline{C} \psi^T = -i \bar{\psi}^T \gamma^5 \psi^T =$$

$$= +i \bar{\psi} \gamma^5 \psi$$

$$\bar{\psi} \gamma^\mu \psi: \quad C^{-1} \bar{\psi}' \gamma^\mu \psi' = -\psi^T \underline{C}^{-1} \gamma^\mu \underline{C} \psi^T = \psi^T \gamma^{\mu T} \psi^T =$$

$$= -\bar{\psi} \gamma^\mu \psi \quad (\psi) = (\psi, T^+ T \psi) = (\psi, \gamma^0)$$

Linear

$$T(\alpha|\psi\rangle + \beta|\chi\rangle) = \alpha T|\psi\rangle + \beta T|\chi\rangle$$

Transformations of bilinear fields.

$$\bar{\psi}\psi: \quad \bar{\psi}'\psi' = (\bar{\psi}'\psi)C^{-1}\psi' = -\bar{\psi}'\psi^T C^{-1}C\psi' = -\frac{1}{|c|} \bar{\psi}'\psi^T = -\sum_{a=1}^4 \psi'_a \bar{\psi}'_a \quad \text{anticommuting} \quad + \bar{\psi}'_a \psi'_a = \bar{\psi}'\psi'$$

$$i\bar{\psi}\gamma^5\psi: \quad \bar{\psi}'\gamma^5\psi' = -i\psi'^T C^{-1}\gamma^5 C\psi' = -i\psi'^T \gamma^5 \psi' = +i\bar{\psi}'\gamma^5\psi'$$

$$\bar{\psi}\gamma^\mu\psi: \quad \bar{\psi}'\gamma^\mu\psi' = -\psi'^T C^{-1}\gamma^\mu C\psi' = \psi'^T \gamma^\mu \psi' = \bar{\psi}'\gamma^\mu\psi'$$

$= (\psi', \psi) \quad (T\psi, T\psi) = (\psi, T^+T\psi) = (\psi, \psi)$

Neutrino: $\psi \gamma^\mu \frac{(1-\gamma^5)}{2} \psi$

Linear $T(\alpha|\psi\rangle + \beta|\psi'\rangle) = \alpha T|\psi\rangle + \beta T|\psi'\rangle$

$$\begin{aligned}
 & + T^{-1} b_{\vec{p}}^S T v^S(p) e^{i(pX - E_p t)} \\
 & + b_{\vec{p}}^{S\dagger} (A_T v^S(p)) e^{i(-E_p t + \vec{p} \cdot \vec{x})} \Bigg] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \left[a_{\vec{p}}^S (A_T u^S(p)) e^{-i(-E_p t + \vec{p} \cdot \vec{x})} + \right.
 \end{aligned}$$

it is impossible to satisfy at all t .

Solution (E. Wigner): T is antiunitary.

unitary operator U : 1) $U a |\psi\rangle = a U |\psi\rangle$; 2) $U(|\psi\rangle + |\varphi\rangle) = U|\psi\rangle + U|\varphi\rangle$; 3) $U^\dagger = U^{-1} \Rightarrow (U\varphi, U\psi) = (\varphi, \psi)$

antiunitary operator T : 1) $T a |\psi\rangle = a^* T |\psi\rangle$; 2) $T(|\psi\rangle + |\varphi\rangle) = T|\psi\rangle + T|\varphi\rangle$; 3) $(T\varphi, T\psi) = (\psi, \varphi) = (|\varphi, \psi\rangle|^2)$

$$T a_{\vec{p}} T = \eta_T a_{-\vec{p}}, \quad T b_{\vec{p}} T = \eta_T^* b_{-\vec{p}}, \quad |\eta_T|^2 = 1$$

T cannot be a unitary linear operator.
 Let us assume that T is anti-linear operator.

$$T^{-1} \Psi(t, \vec{x}) T = \Lambda_T \Psi(-t, \vec{x}) \Rightarrow \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [T^{-1} a_{\vec{p}}^S T (U^S) e^{i(E_{\vec{p}}t + \vec{p}\cdot\vec{x})} + T^{-1} b_{\vec{p}}^{S\dagger} T (U^S) e^{i(p_x x - E_{\vec{p}}t)}] = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} [a_{-\vec{p}}^S (\Lambda_T U^S) e^{-i(-E_{\vec{p}}t + \vec{p}\cdot\vec{x})} + b_{-\vec{p}}^{S\dagger} (\Lambda_T U^S) e^{-i(-E_{\vec{p}}t + \vec{p}\cdot\vec{x})}]$$

it is impossible to satisfy at a

antiunitary operator T :
 1) $T a |\psi\rangle = \eta T |\psi\rangle$, 2) $T |\psi\rangle = \eta |\psi\rangle$
 = $T |\psi\rangle + T |\psi\rangle$, 3) $(T\psi, T\psi) = (\psi, \psi) = (\psi, \psi)^* \Rightarrow \text{prob}$
 $|\langle \psi, \psi \rangle|^2$ still does not