Title: Quantum Field Theory 1 - Lecture 9A

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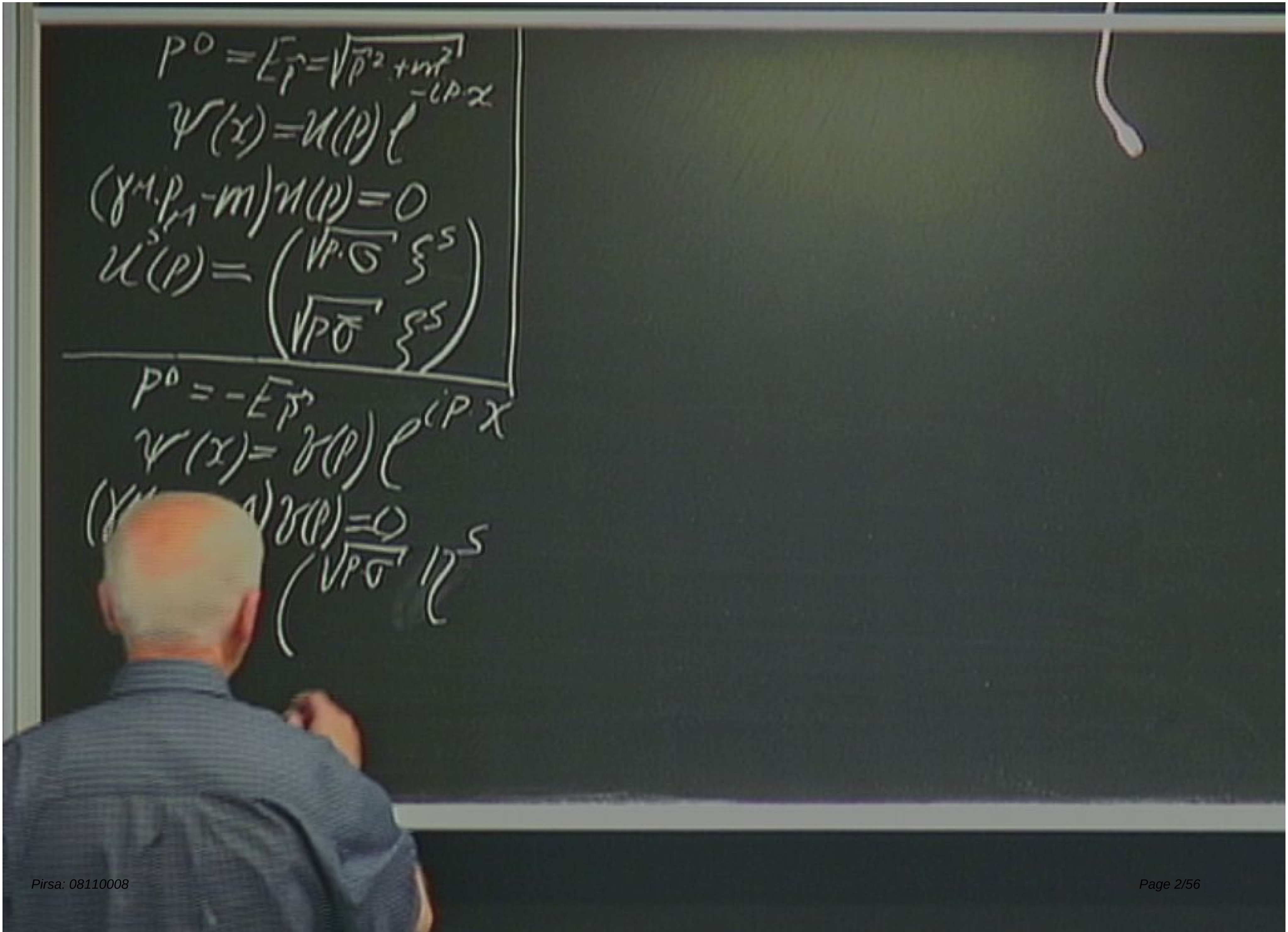
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario



$$p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$$\psi(x) = u(p)\, e^{-ip\cdot x}$$

$$(\gamma^\mu p_\mu - m)\, u(p) = 0$$

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\, \xi^s \\ \sqrt{p\cdot\bar{\sigma}}\, \xi^s \end{pmatrix}$$

$$p^0 = -E_{\vec{p}}$$

$$\psi(x) = v(p)\, e^{ipx}$$

$(\gamma$ [illegible] $)\, v(p) = 0$

$\Big(\sqrt{p\cdot\sigma}\, \eta^s$

$$\psi(x) = u(p)\, e$$

$$(\gamma^{\mu} \cdot p_{\mu} - m)\, u(p) = 0$$

$$u^{s}(p) = \begin{pmatrix} \sqrt{p \cdot \sigma}\, \xi^{s} \\ \sqrt{p \bar{\sigma}}\, \xi^{s} \end{pmatrix}$$

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Quantization of Dirac Field

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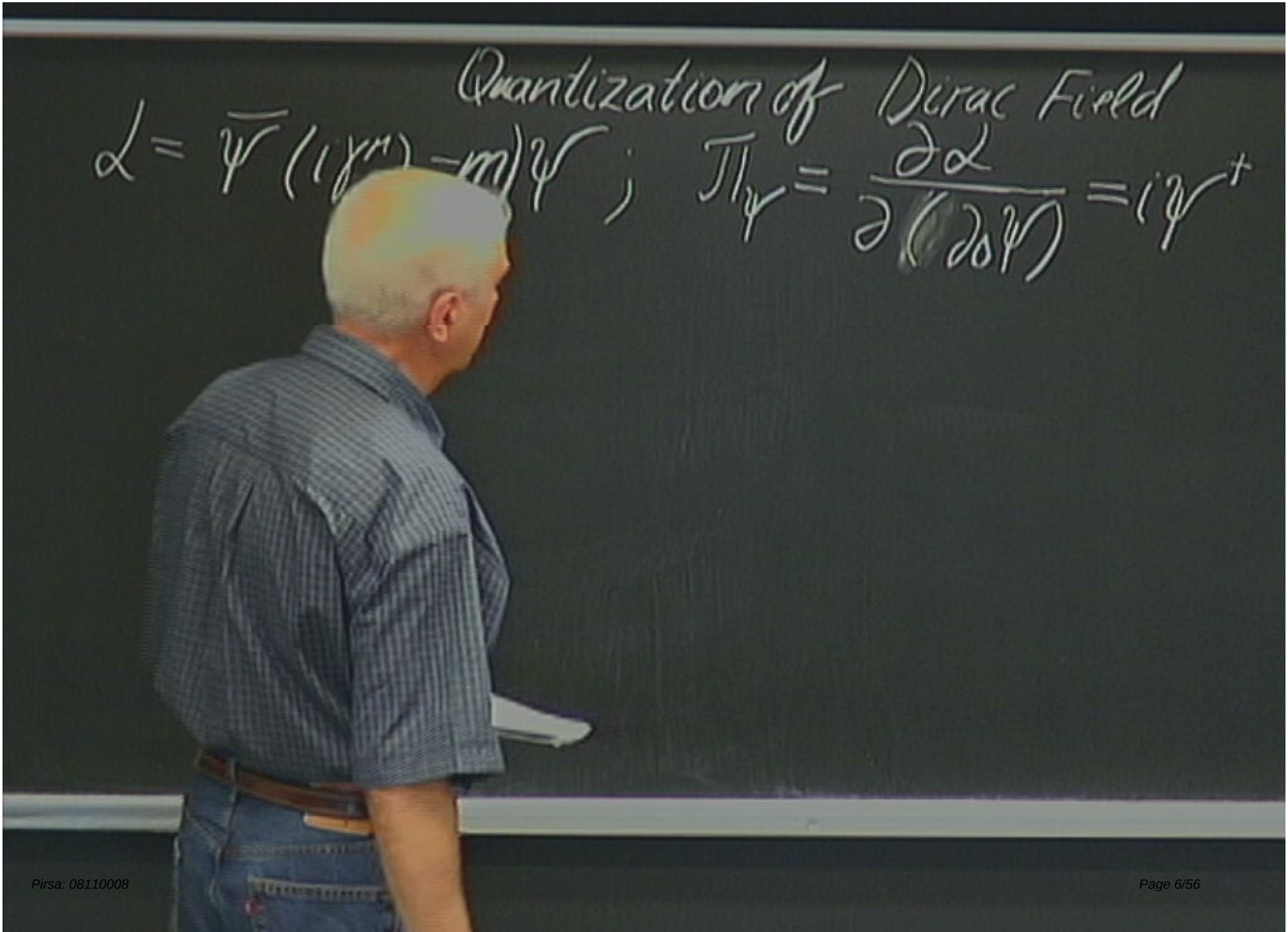
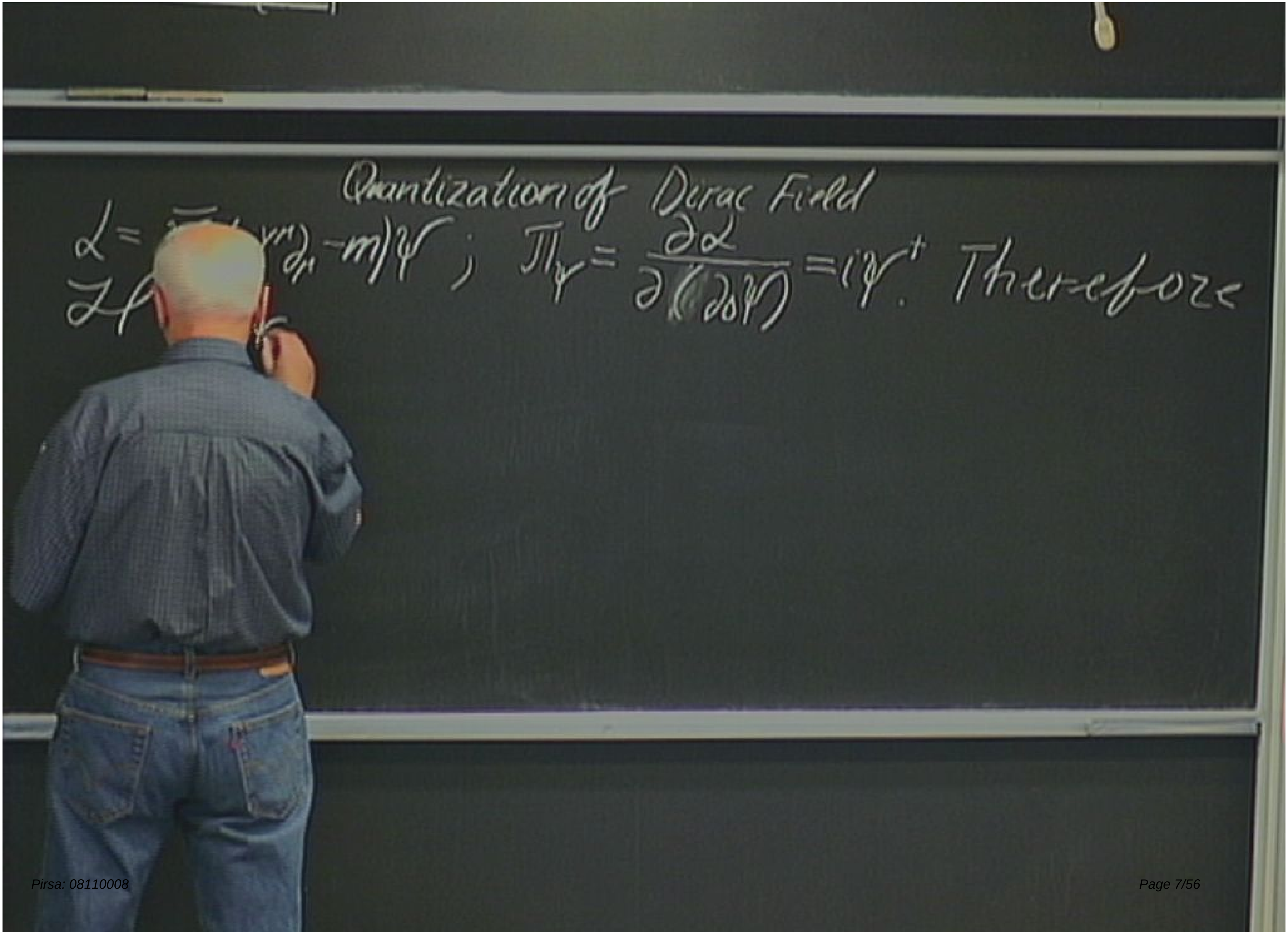
$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$;



Quantization of Dirac Field

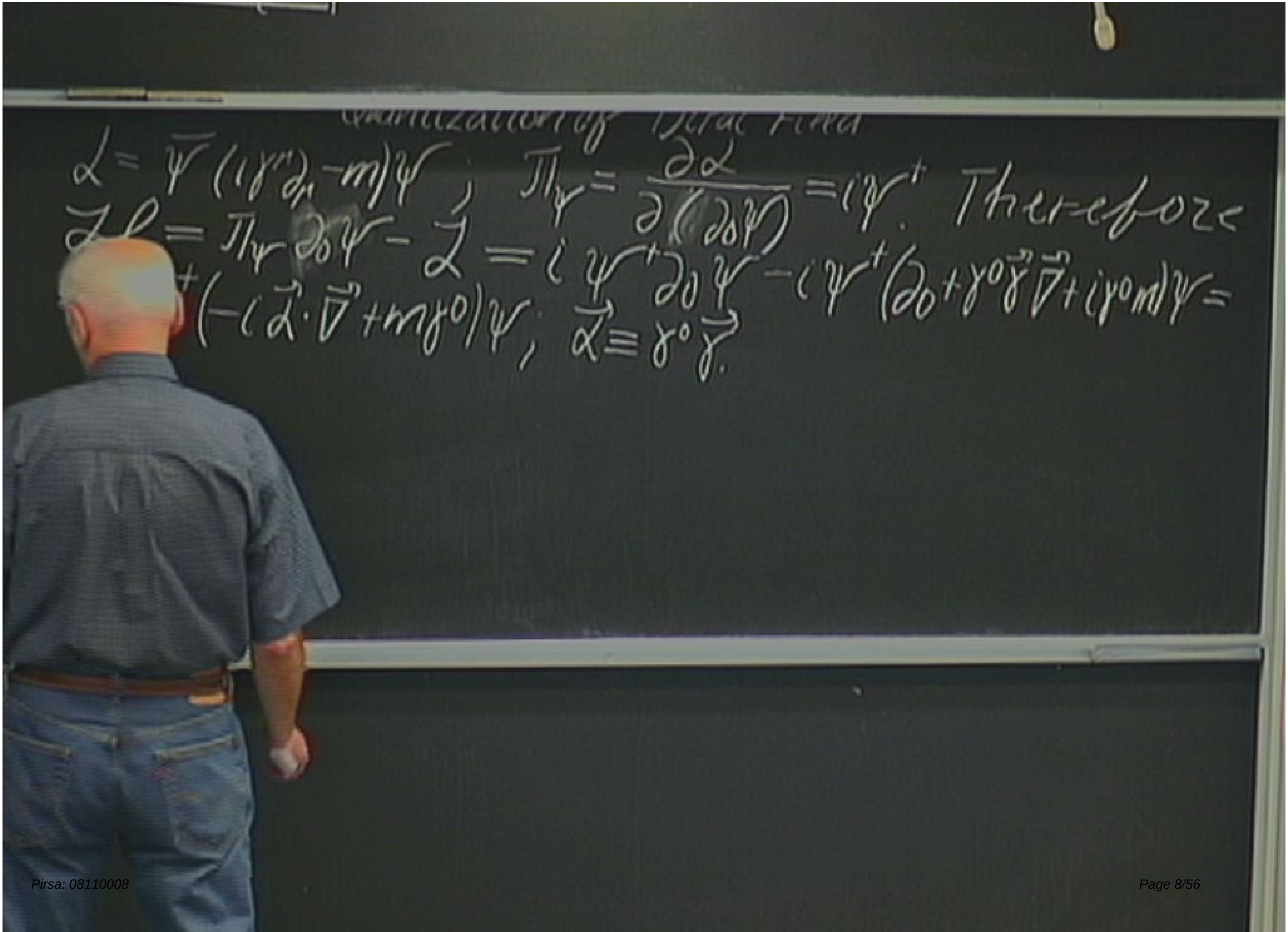
$\mathcal{L} = \bar{\psi}\,(i\gamma^{\mu}$ [illegible] $-m)\psi$; $\Pi_{\psi} = \dfrac{\partial \mathcal{L}}{\partial(\partial_0 \psi)} = i\psi^{\dagger}$



Quantization of Dirac Field

$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$; $\pi_\psi = \frac{\partial\mathcal{L}}{\partial(\partial_0\psi)} = i\psi^\dagger$. Therefore

$\mathcal{H}$



Quantization of Dirac Field

$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$; $\Pi_\psi = \frac{\partial \mathcal{L}}{\partial(\partial_0\psi)} = i\psi^\dagger$. Therefore

$\mathcal{H} = \Pi_\psi \partial_0\psi - \mathcal{L} = i\psi^\dagger\partial_0\psi - i\psi^\dagger(\partial_0 + \gamma^0\vec{\gamma}\vec{\nabla} + i\gamma^0 m)\psi =$

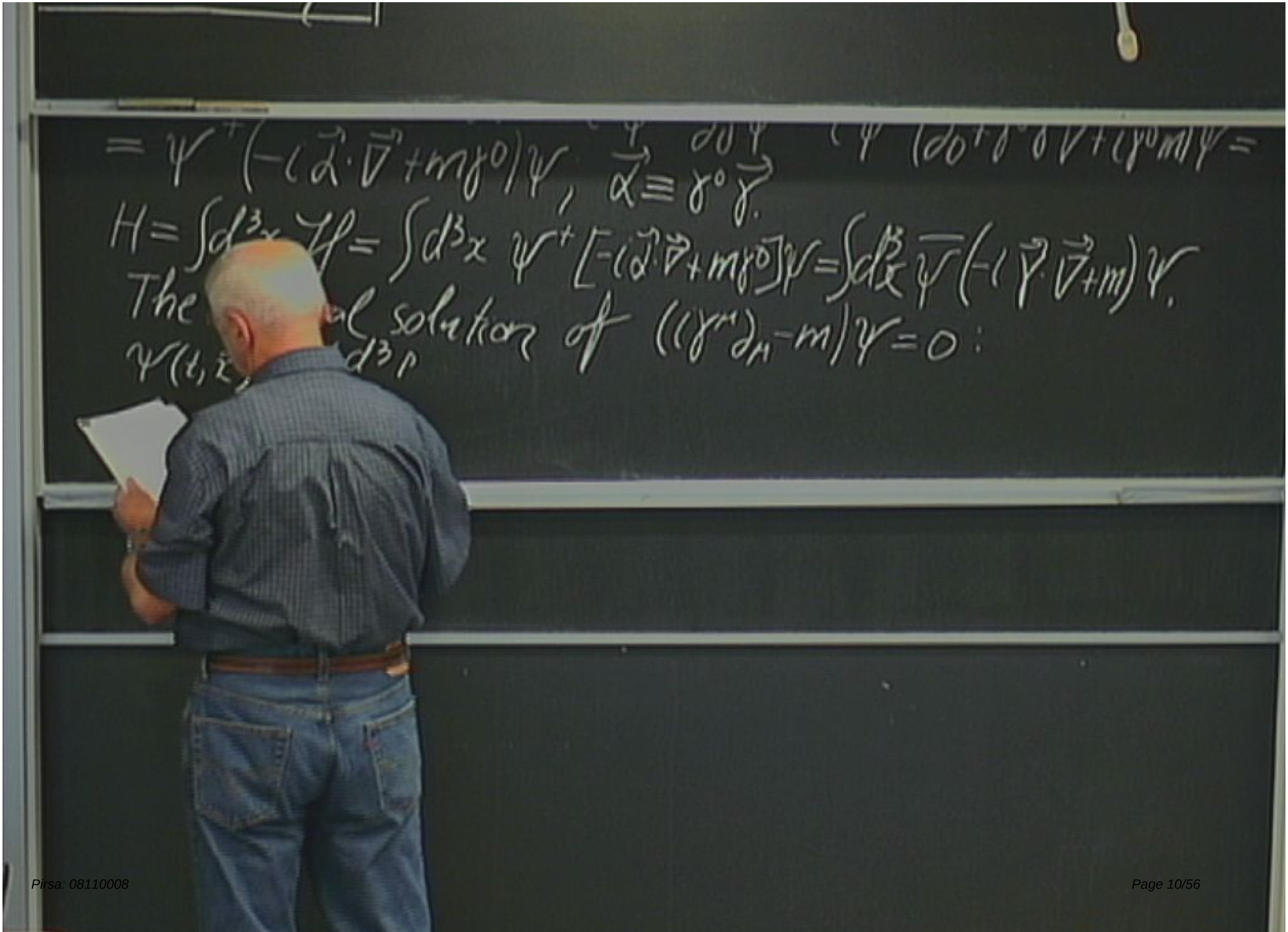
$^\dagger(-i\vec{\alpha}\cdot\vec{\nabla} + m\gamma^0)\psi$, $\vec{\alpha} \equiv \gamma^0\vec{\gamma}$.

$\mathcal{L} = \psi(i\gamma\partial_\mu - m)\psi$, $\pi_\psi = \frac{\partial\mathcal{L}}{\partial(\partial_0\psi)} = i\psi^+$. Therefore

$\mathcal{H} = \pi_\psi\partial_0\psi - \mathcal{L} = i\psi^+\partial_0\psi - i\psi^+(\partial_0 + \gamma^0\vec{\gamma}\vec{\nabla} + i\gamma^0 m)\psi =$

$= \psi^+(-i\vec{\alpha}\cdot\vec{\nabla} + m\gamma^0)\psi$, $\vec{\alpha} \equiv \gamma^0\vec{\gamma}$.

$H = \int d^3x\,\mathcal{H} = \int d^3x\,\psi^+[-i\vec{\alpha}\cdot\vec{\nabla} + m\gamma^0]\psi = \int d^3x\,\bar{\psi}(-i\vec{\gamma}\cdot\vec{\nabla} + m)\psi.$

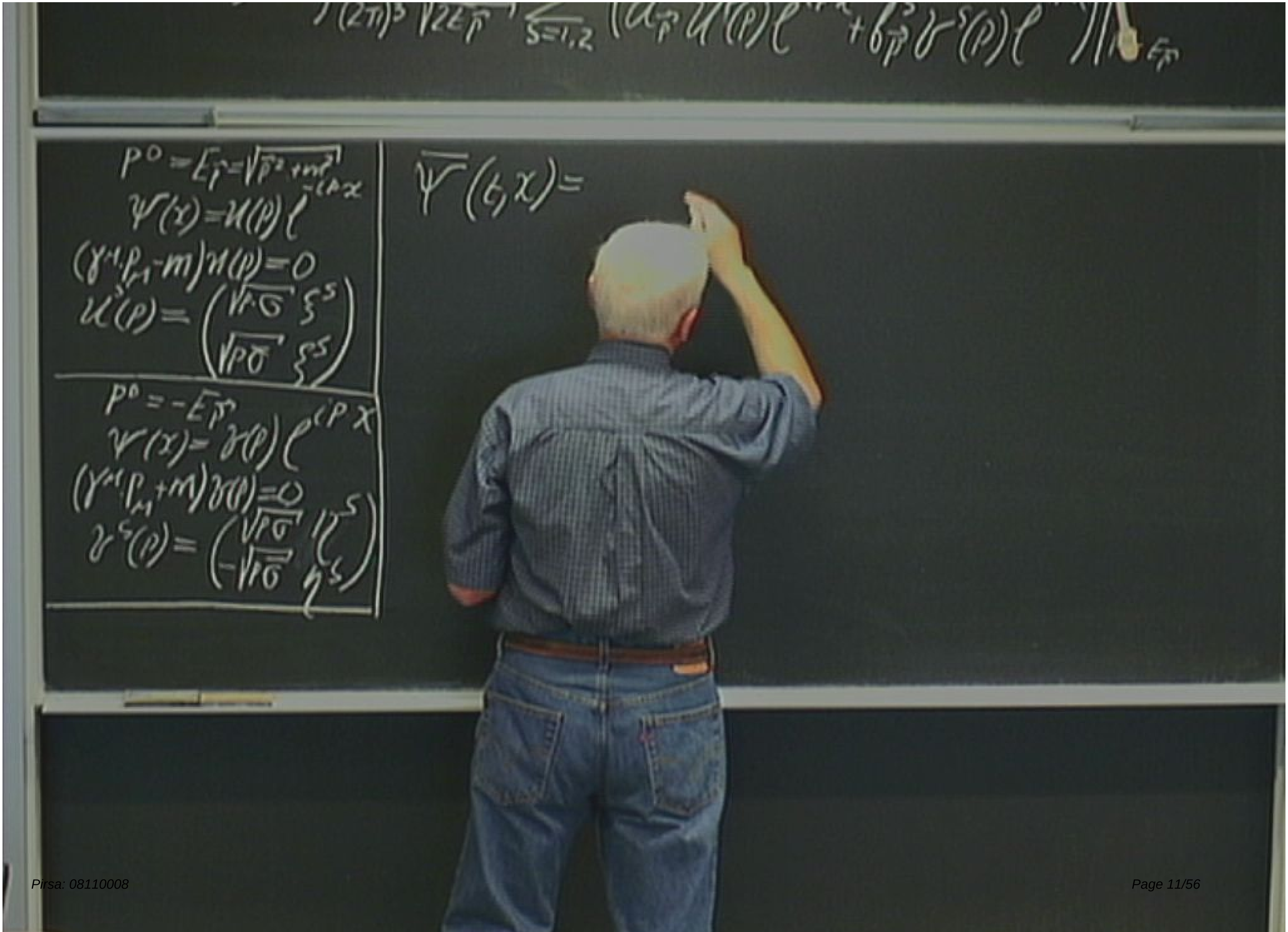


$= \psi^{\dagger}(-i\vec{\alpha}\cdot\vec{\nabla} + m\gamma^{0})\psi, \quad \vec{\alpha} \equiv \gamma^{0}\vec{\gamma}.$

$H = \int d^3x\, \mathcal{H} = \int d^3x\, \psi^{\dagger}[-i\vec{\alpha}\cdot\vec{\nabla} + m\gamma^{0}]\psi = \int d^3x\, \bar{\psi}(-i\vec{\gamma}\cdot\vec{\nabla} + m)\psi.$

The [illegible]al solution of $((i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$:

$\psi(t, \vec{x}$ [illegible] d^3p



$$(2\pi)^3 \sqrt{2E_{\vec p}} \sum_{s=1,2} \left(a_{\vec p}\, u(p)\, e \quad + b_{\vec p}^{s}\, v^{s}(p)\, e \right)\Big|_{E_{\vec p}}$$

$$p^0 = E_{\vec p} = \sqrt{\vec p^2 + m^2}$$

$$\psi(x) = u(p)\, e^{-ip\cdot x}$$

$$(\gamma^\mu p_\mu - m)\, u(p) = 0$$

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \xi^s \\ \sqrt{p\cdot\bar\sigma}\ \xi^s \end{pmatrix}$$

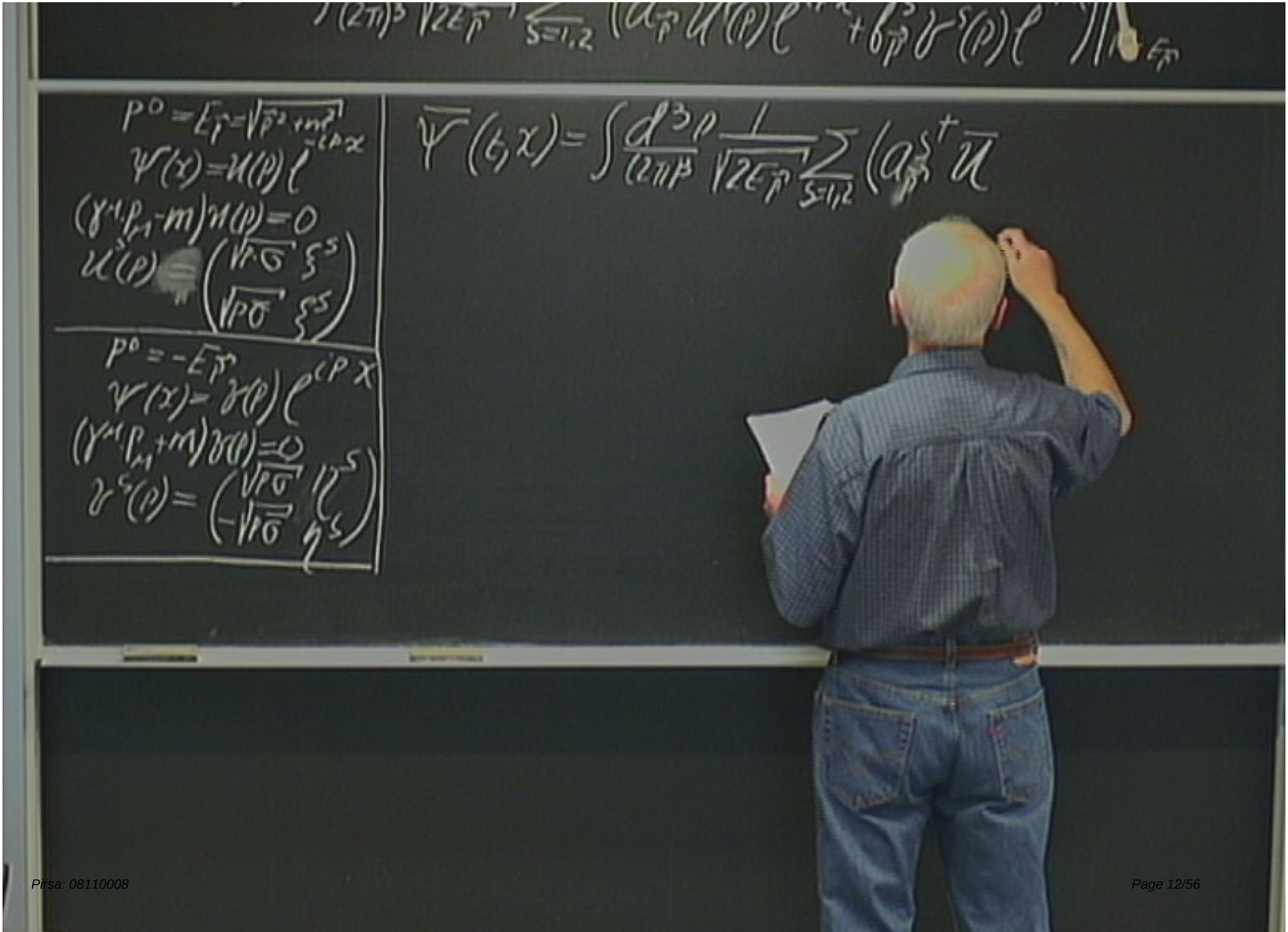
$$p^0 = -E_{\vec p}$$

$$\psi(x) = v(p)\, e^{ip\cdot x}$$

$$(\gamma^\mu p_\mu + m)\, v(p) = 0$$

$$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \eta^s \\ -\sqrt{p\cdot\bar\sigma}\ \eta^s \end{pmatrix}$$

$$\bar\psi(t,x) =$$



$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\sum_{s=1,2}\left(a_{\vec p}^s u^s(p)e^{-ipx} + b_{\vec p}^{s\dagger} v^s(p)e^{ipx}\right)\Big|_{p^0=E_{\vec p}}$$

$$p^0 = E_{\vec p} = \sqrt{\vec p^2 + m^2}$$

$$\psi(x) = u(p)e^{-ipx}$$

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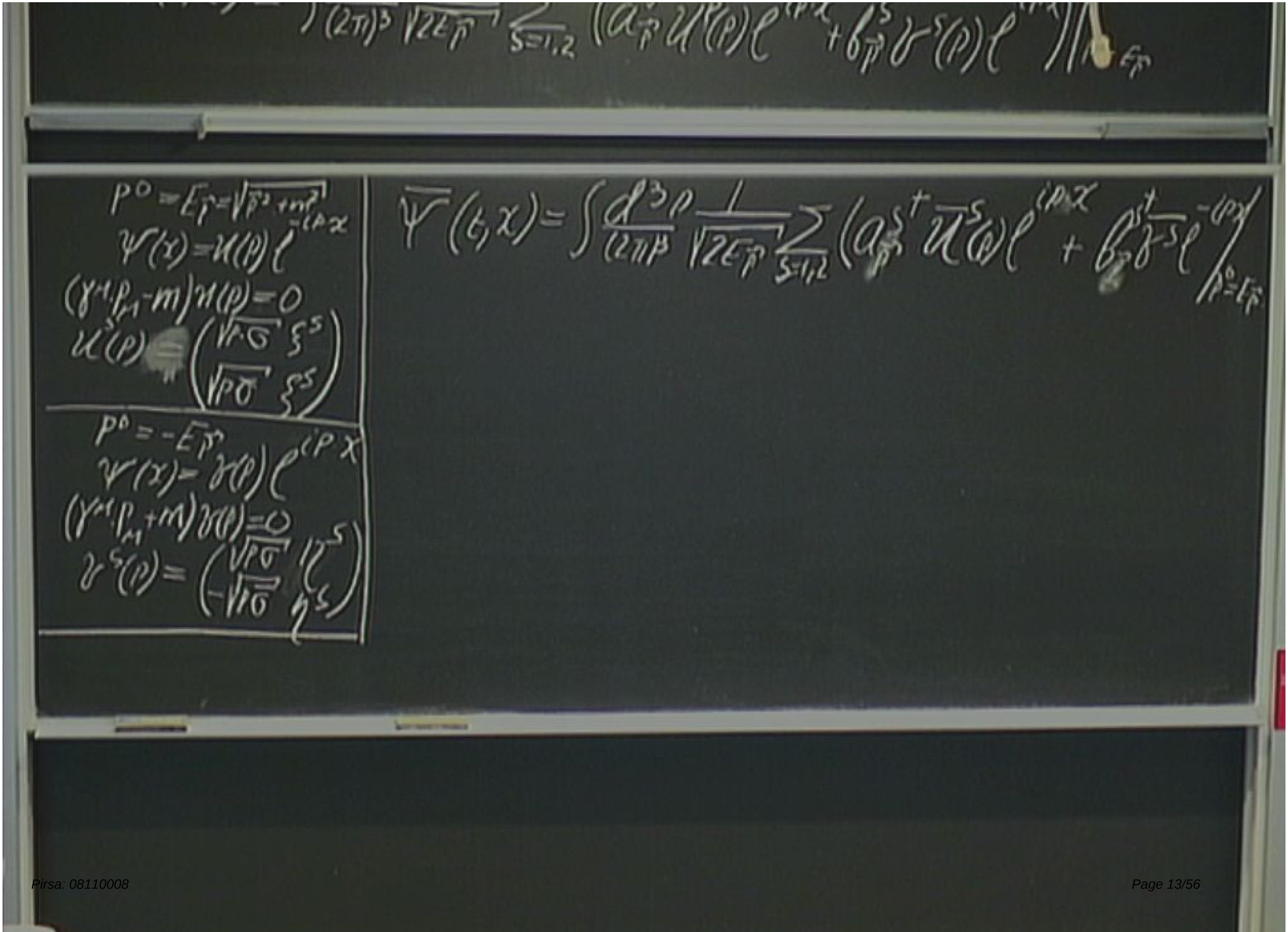
$$\bar\psi(t,x) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\sum_{s=1,2}\left(a_{\vec p}^{s\dagger}\bar u\right.$$



$$\frac{}{(2\pi)^3\,\sqrt{2E_{\vec p}}}\sum_{s=1,2}\left(a^s_{\vec p}\,u^s(p)\,e^{-ipx} + b^{s\dagger}_{\vec p}\,v^s(p)\,e^{ipx}\right)\Big|_{E_{\vec p}}$$

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$$\bar\Psi(t,x) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\sum_{s=1,2}\left(a^{s\dagger}_{\vec p}\,\bar u^s(p)\,e^{ipx} + b^{s\dagger}_{\vec p}\,\bar v^s\,e^{-ipx}\right)\Big|_{p^0=E_{\vec p}}$$

$$(2\pi)^3 \sqrt{2E_{\vec p}} \sum_{s=1,2} \left(a_{\vec p}\, u^s(p)\, e \; + b^s_{\vec p}\, v^s(p)\, e \right)\Big|_{E_{\vec p}}$$

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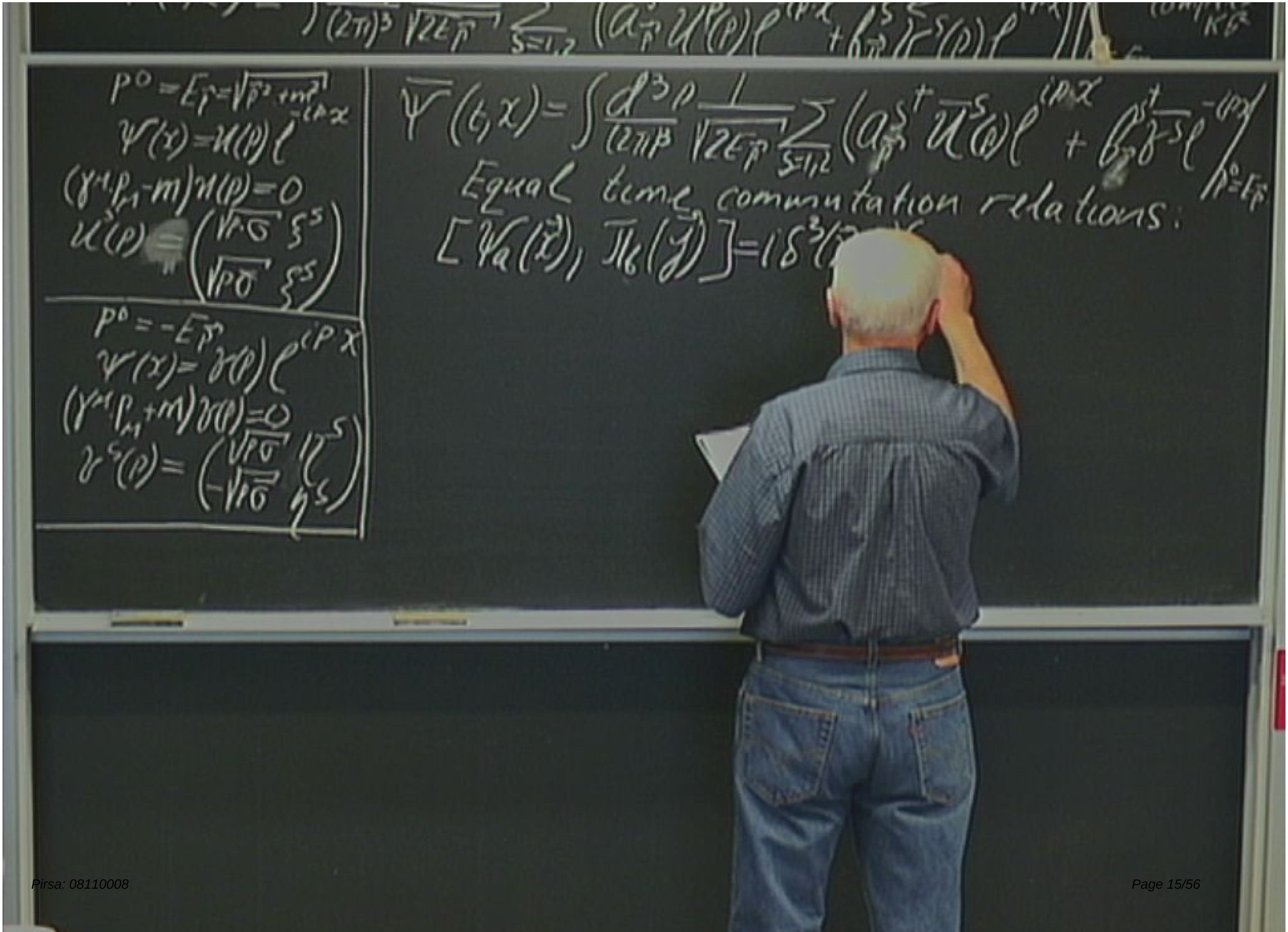

$$\frac{}{(2\pi)^3 \sqrt{2E_{\vec p}}} \sum_{s=1,2} \left(a_{\vec p}^s u^s(p) e^{ipx} + b_{\vec p}^{s\dagger} v^s(p) e \right)$$

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Equal time commutation relations:

$[\psi_a(\vec x), \pi_b(\vec y)] = i\delta^3($

$$\frac{}{(2\pi)^3}\frac{}{\sqrt{2E_{\vec p}}}\sum_{s=1,2}\left(a^s_{\vec p}u^s(p)e^{-ipx}+b^{s\dagger}_{\vec p}v^s(p)e^{ipx}\right)$$

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Equal time commutation relations:

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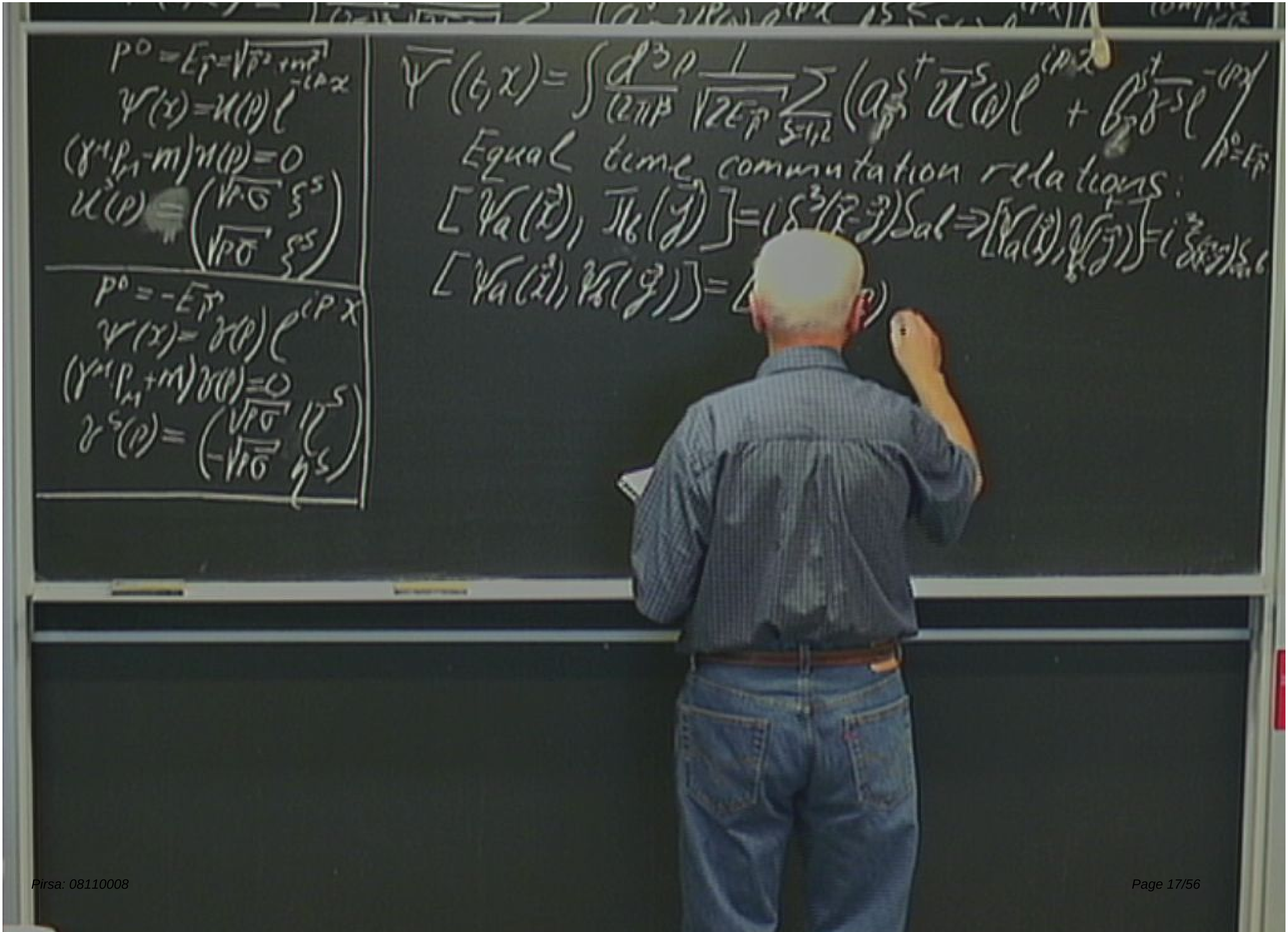

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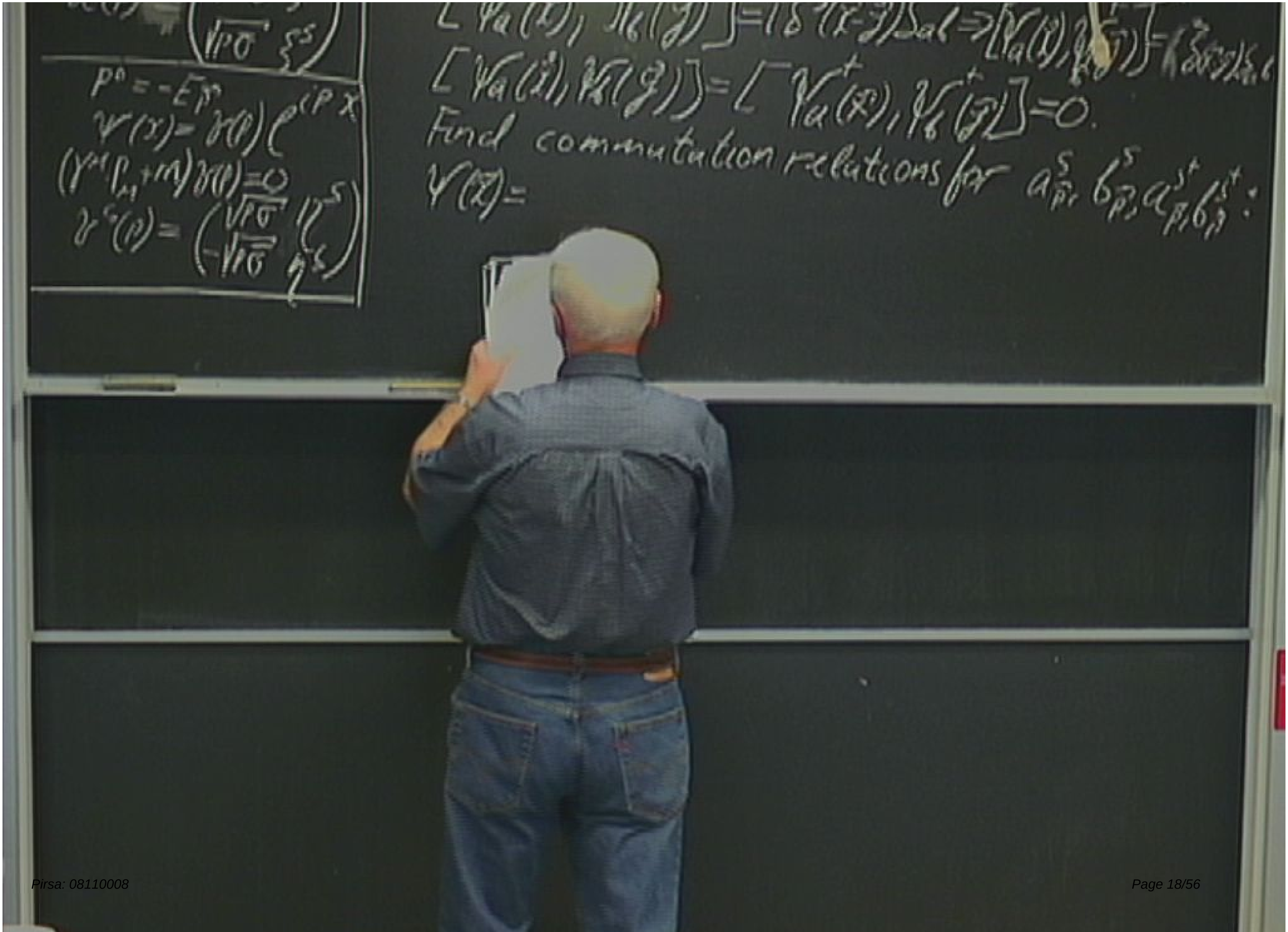
Equal time commutation relations:

$$[\psi_a(\vec x), \pi_b(\vec y)] = i\delta^3(\vec x-\vec y)\delta_{ab} \Rightarrow [\psi_a(\vec x), \psi_b^\dagger(\vec y)] = i\delta^3(\vec x-\vec y)\delta_{ab}$$

$$[\psi_a(\vec x), \psi_b(\vec y)] = \text{[illegible]}$$



$$\sqrt{p\sigma}\,\xi^s$$

$p^0 = -E_{\vec p}$

$\psi(x) = v(p)\,e^{ipx}$

$(\gamma^\mu p_\mu + m)\,v(p) = 0$

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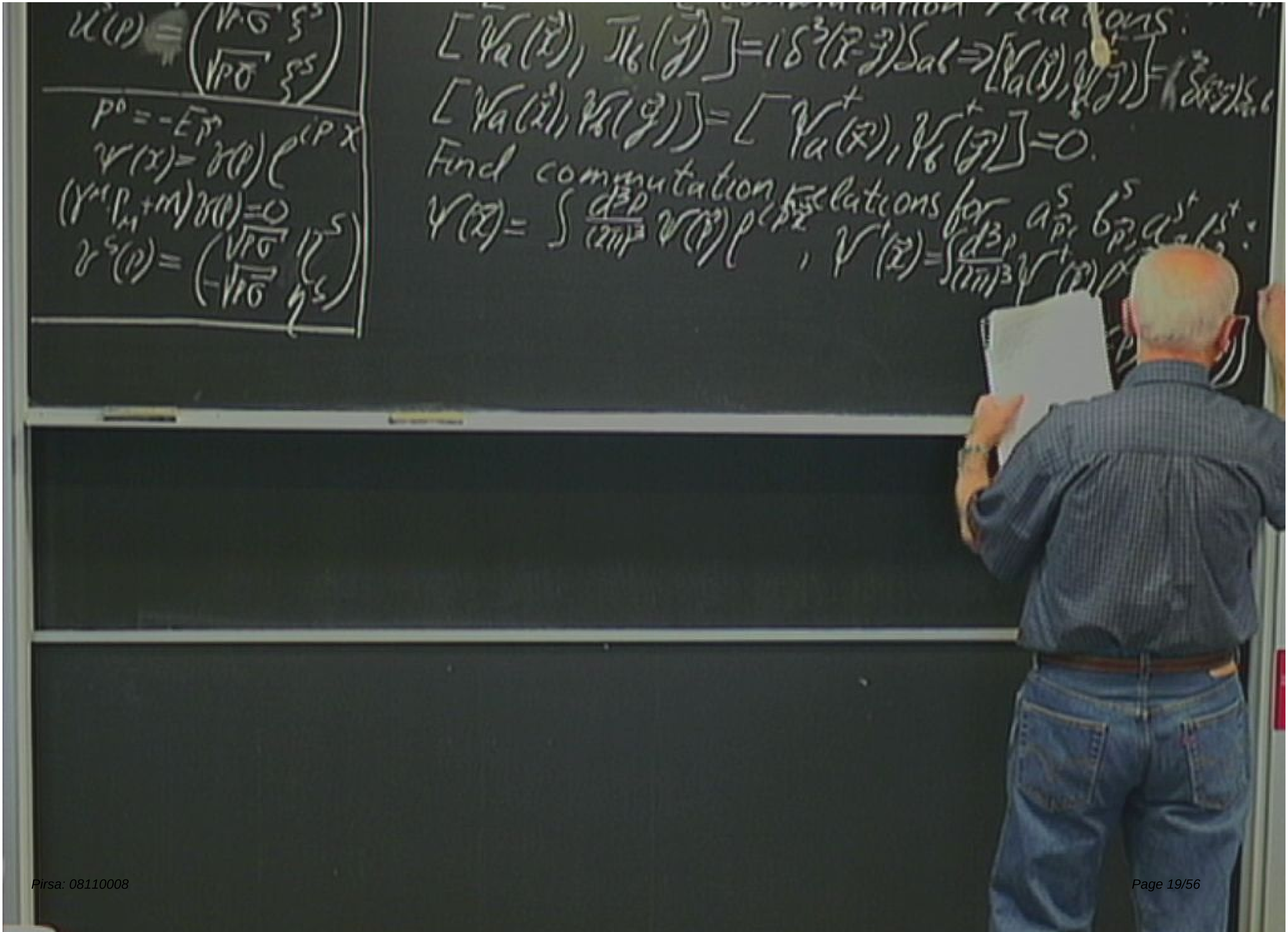
$[\psi_a(\vec x), \pi_b(\vec y)] = i\delta^3(\vec x - \vec y)\delta_{ab} \Rightarrow [\psi_a(\vec x), \psi_b^\dagger(\vec y)] = \hbar\delta^3(\vec x - \vec y)\delta_{ab}$

$[\psi_a(\vec x), \psi_b(\vec y)] = [\psi_a^\dagger(\vec x), \psi_b^\dagger(\vec y)] = 0.$

Find commutation relations for $a^s_{\vec p}, b^s_{\vec p}, a^{s\dagger}_{\vec p}, b^{s\dagger}_{\vec p}$:

$\psi(\vec x) =$



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commutation relations:

$$[\psi_a(\vec x), \pi_b(\vec y)] = i\delta^3(\vec x - \vec y)\delta_{ab} \Rightarrow [\psi_a(\vec x), \psi_b^\dagger(\vec y)] = \delta^3(\vec x - \vec y)\delta_{ab}$$

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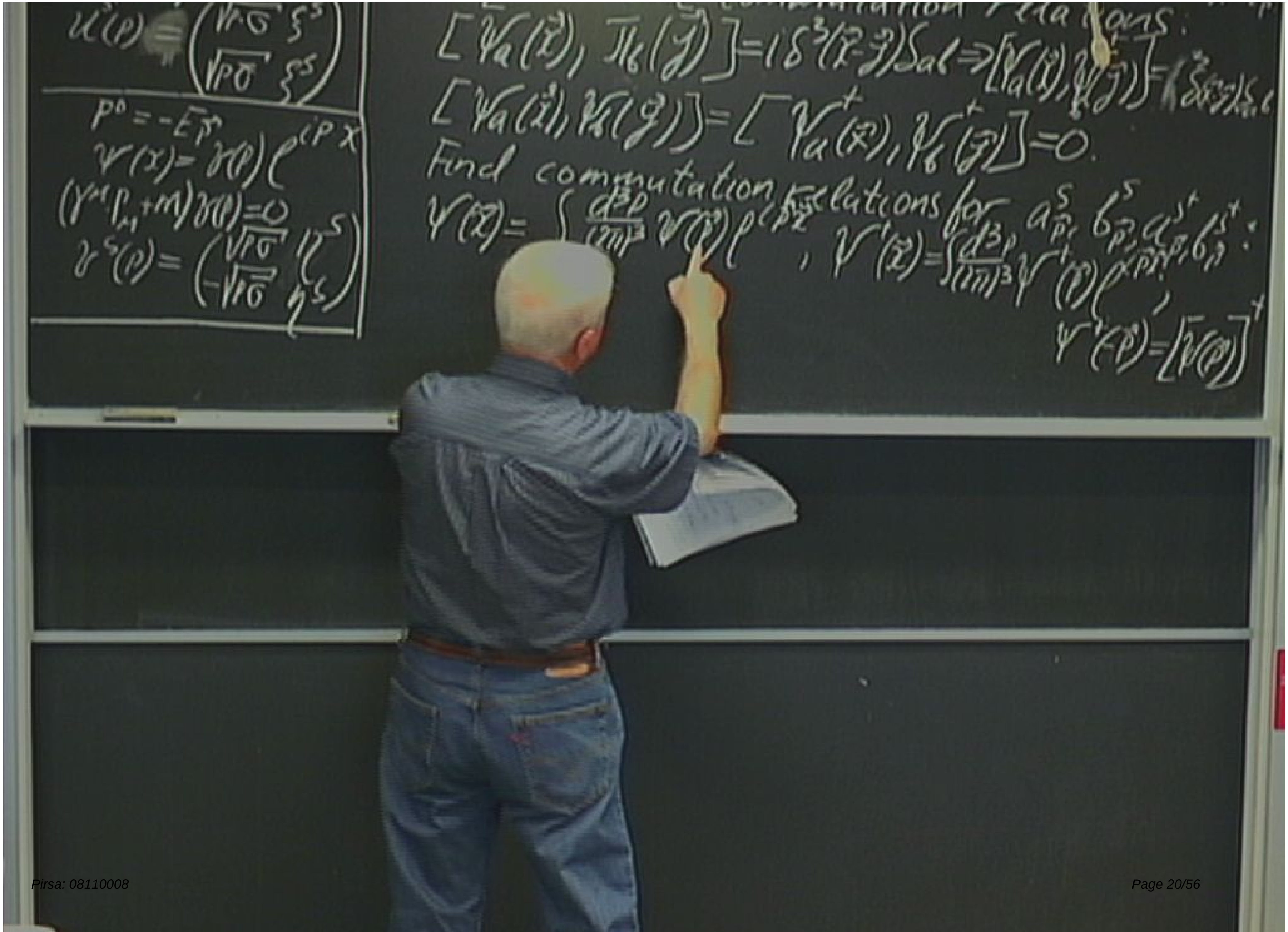
Find commutation relations for $a^s_{\vec p}, b^s_{\vec p}, a^{s\dagger}_{\vec p}, b^{s\dagger}_{\vec p}$:

$$\psi(\vec x) = \int \frac{d^3p}{(2\pi)^3}\, \psi(\vec p)\, e^{i\vec p\vec x}, \quad \psi^\dagger(\vec x) = \int \frac{d^3p}{(2\pi)^3}\, \psi^\dagger(\vec p)\, e$$



$u^s(p) = \begin{pmatrix} \sqrt{p\sigma}\,\xi^s \\ \sqrt{p\bar\sigma}\,\xi^s \end{pmatrix}$

$p^0 = -E_{\vec p}$

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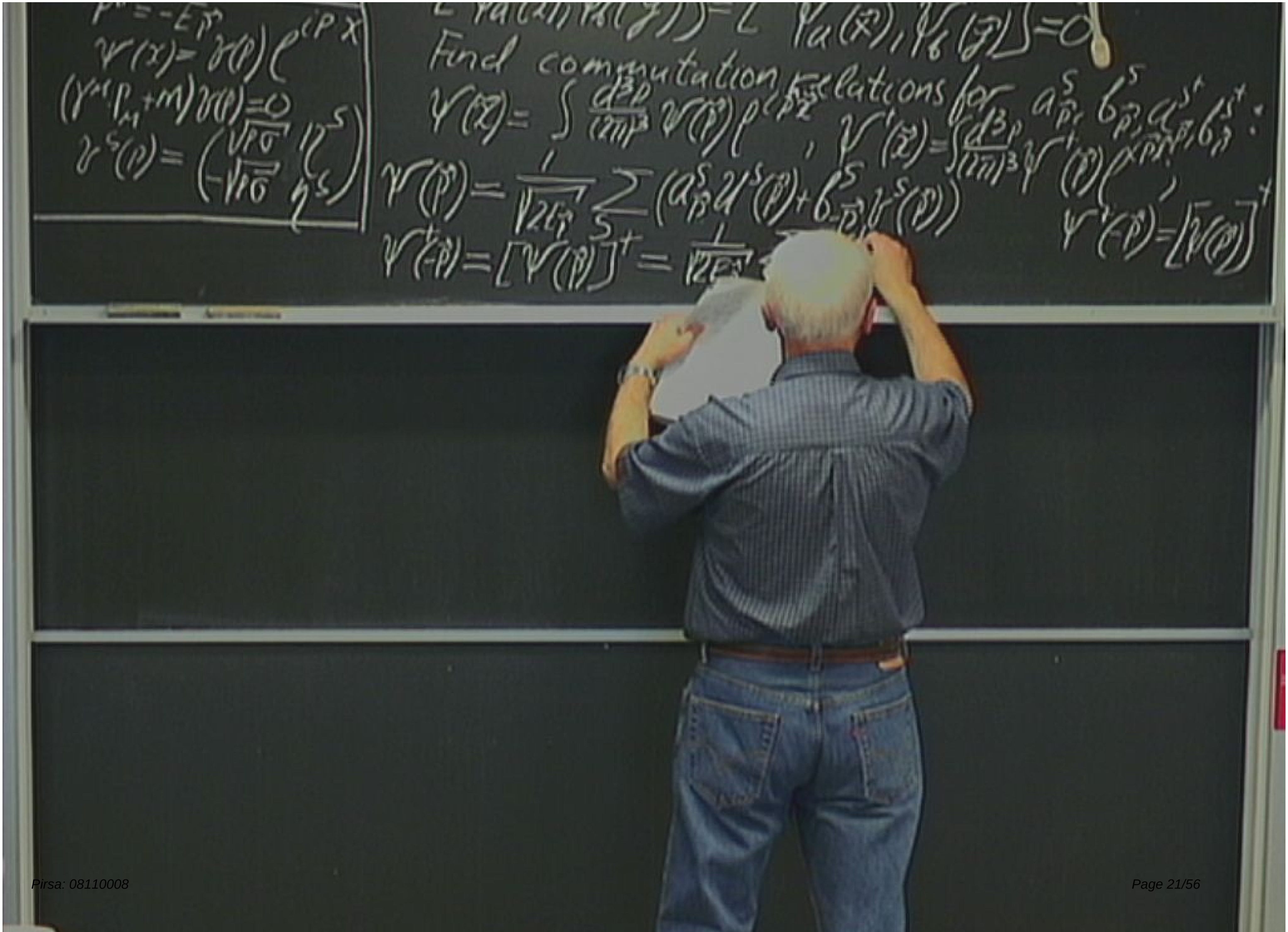
$[\psi_a(\vec x), \psi_b(\vec y)] = [\psi_a^\dagger(\vec x), \psi_b^\dagger(\vec y)] = 0.$

Find commutation relations for $a^s_{\vec p}, b^s_{\vec p}, a^{s\dagger}_{\vec p}, b^{s\dagger}_{\vec p}$:

$\psi(\vec x) = \int \frac{d^3p}{(2\pi)^3}\, \psi(\vec p)\, e^{i\vec p\vec x}$, $\psi^\dagger(\vec x) = \int \frac{d^3p}{(2\pi)^3}\, \psi^\dagger(\vec p)\, e^{-i\vec p\vec x}$,

$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger$



$p^\mu = -E, \vec{p}$

$\psi(x) = v(p) e^{ipx}$

$(\gamma^\mu p_\mu + m) v(p) = 0$

$v^s(p) = \begin{pmatrix} \sqrt{p\sigma}\,\eta^s \\ -\sqrt{p\bar\sigma}\,\eta^s \end{pmatrix}$

$\sum \psi_a(x)\psi_b(y)\} = \sum \{\psi_a(\vec{x}), \psi_b(\vec{y})\} = 0$

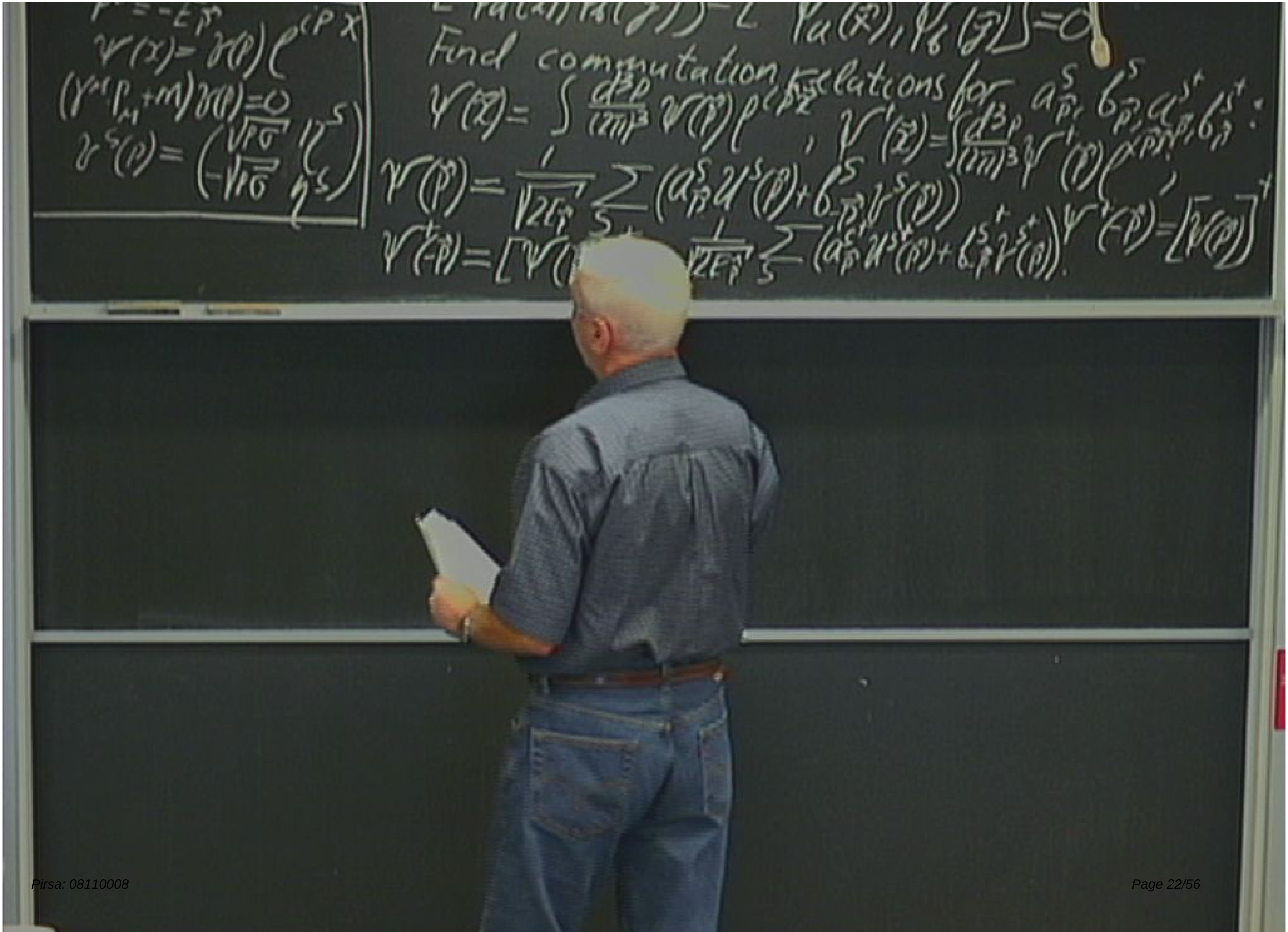
Find commutation relations for $a^s_{\vec p}, b^s_{\vec p}, a^{s\dagger}_{\vec p}, b^{s\dagger}_{\vec p}$:

$\psi(\vec x) = \int \frac{d^3p}{(2\pi)^3} \psi(\vec p) e^{i\vec p \vec x}$, $\psi^\dagger(\vec x) = \int \frac{d^3p}{(2\pi)^3} \psi^\dagger(\vec p) e^{-i\vec p \vec x}$,

$\psi(\vec p) = \frac{1}{\sqrt{2E_p}} \sum_s (a^s_{\vec p} u^s(\vec p) + b^s_{-\vec p} v^s(\vec p))$

$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger$

$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_p}}$



$p^0 = -E_{\vec p}$

$\psi(x) = v(p)\, e^{ipx}$

$(\gamma^\mu p_\mu + m)\, v(p) = 0$

$v^s(p) = \begin{pmatrix} \sqrt{p\sigma}\, \eta^s \\ -\sqrt{p\bar\sigma}\, \eta^s \end{pmatrix}$

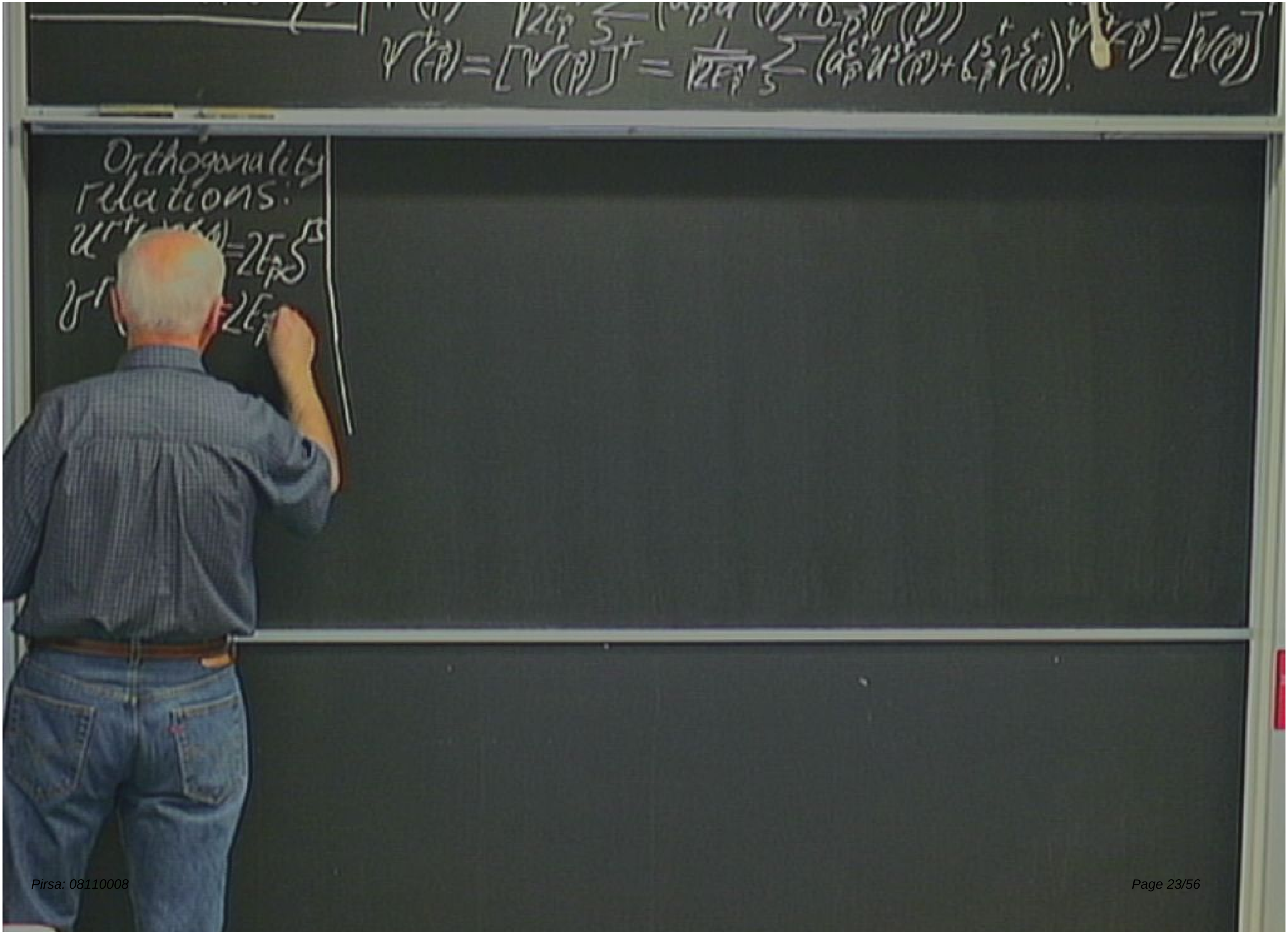
$\{\psi_a(\vec x), \psi_b(\vec y)\} = \{\psi_a^\dagger(\vec x), \psi_b^\dagger(\vec y)\} = 0$

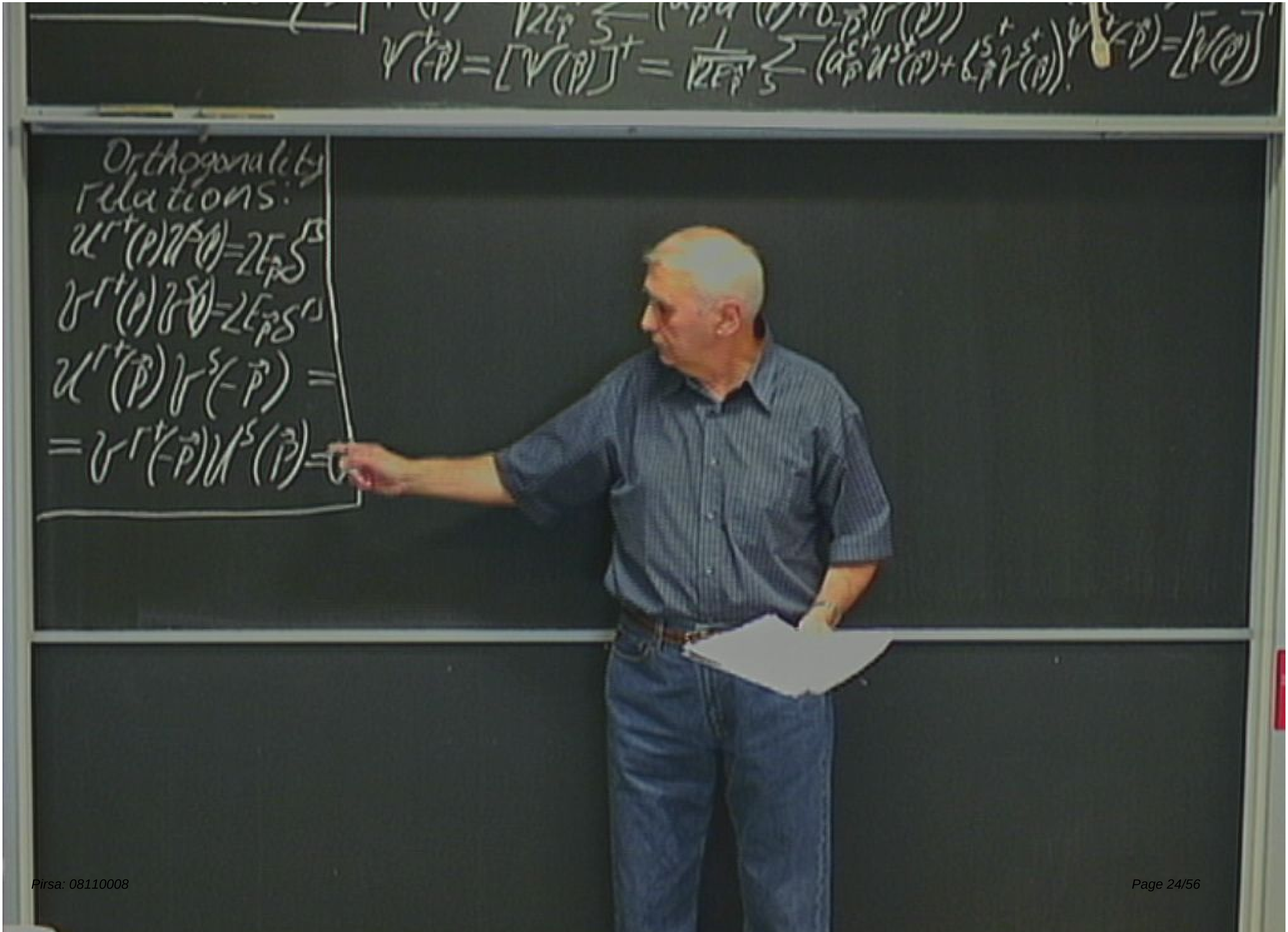
Find commutation relations for $a_{\vec p}^s, b_{\vec p}^s, a_{\vec p}^{s\dagger}, b_{\vec p}^{s\dagger}$:

$\psi(\vec x) = \int \frac{d^3p}{(2\pi)^3}\, \psi(\vec p)\, e^{i\vec p\vec x}, \quad \psi^\dagger(\vec x) = \int \frac{d^3p}{(2\pi)^3}\, \psi^\dagger(\vec p)\, e^{-i\vec p\vec x},$

$\psi(\vec p) = \frac{1}{\sqrt{2E_{\vec p}}} \sum_s \left(a_{\vec p}^s u^s(\vec p) + b_{-\vec p}^s v^s(\vec p)\right)$

$\psi^\dagger(-\vec p) = [\psi(\ldots = \frac{1}{\sqrt{2E_{\vec p}}} \sum_s \left(a_{\vec p}^s u^s(\vec p) + b_{\vec p}^{s\dagger} v^s(\vec p)\right). \quad \psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger$


$\psi^\dagger(\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s (a^{s\dagger}_{\vec p} u^{s\dagger}(\vec p) + b^{s}_{-\vec p} v^{s\dagger}(\vec p))$
Orthogonality relations:
$u^{r\dagger}$ $= 2E_{\vec p}\delta^{rs}$
$v^{r\dagger}$ $= 2E_{\vec p}$



$$\psi^{\dagger}(-\vec{p})=[\psi(\vec{p})]^{\dagger}=\frac{1}{\sqrt{2E_{\vec{p}}}}\sum_{s}\left(a_{\vec{p}}^{s\dagger}u^{s\dagger}(\vec{p})+b_{-\vec{p}}^{s}v^{s\dagger}(\vec{p})\right)$$

$$\bar{\psi}(-\vec{p})=[\bar{\psi}(\vec{p})]$$

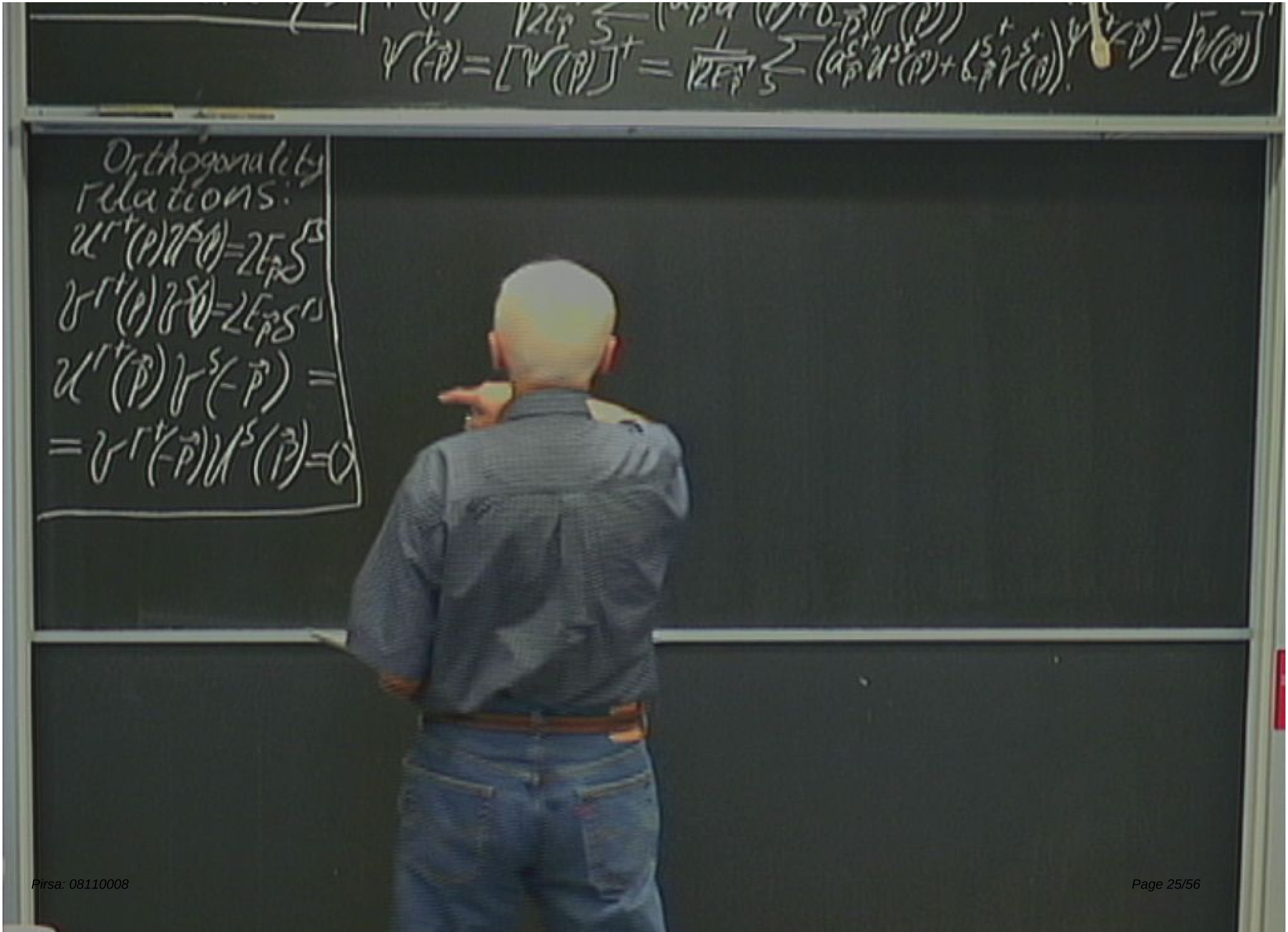
Orthogonality relations:

$$u^{r\dagger}(p)u^{s}(p)=2E_{\vec{p}}\delta^{rs}$$

$$v^{r\dagger}(p)v^{s}(p)=2E_{\vec{p}}\delta^{rs}$$

$$u^{r\dagger}(\vec{p})v^{s}(-\vec{p})=$$

$$=v^{r\dagger}(-\vec{p})u^{s}(\vec{p})=0$$



$\psi^{\dagger}(-\vec{p}) = [\psi(\vec{p})]^{\dagger} = \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_{s} (a^{s\dagger}_{\vec{p}} u^{s\dagger}(\vec{p}) + b^{s}_{-\vec{p}} v^{s\dagger}(\vec{p}))$.

$\bar{\psi}(-\vec{p}) = [\bar{\psi}(\vec{p})]$

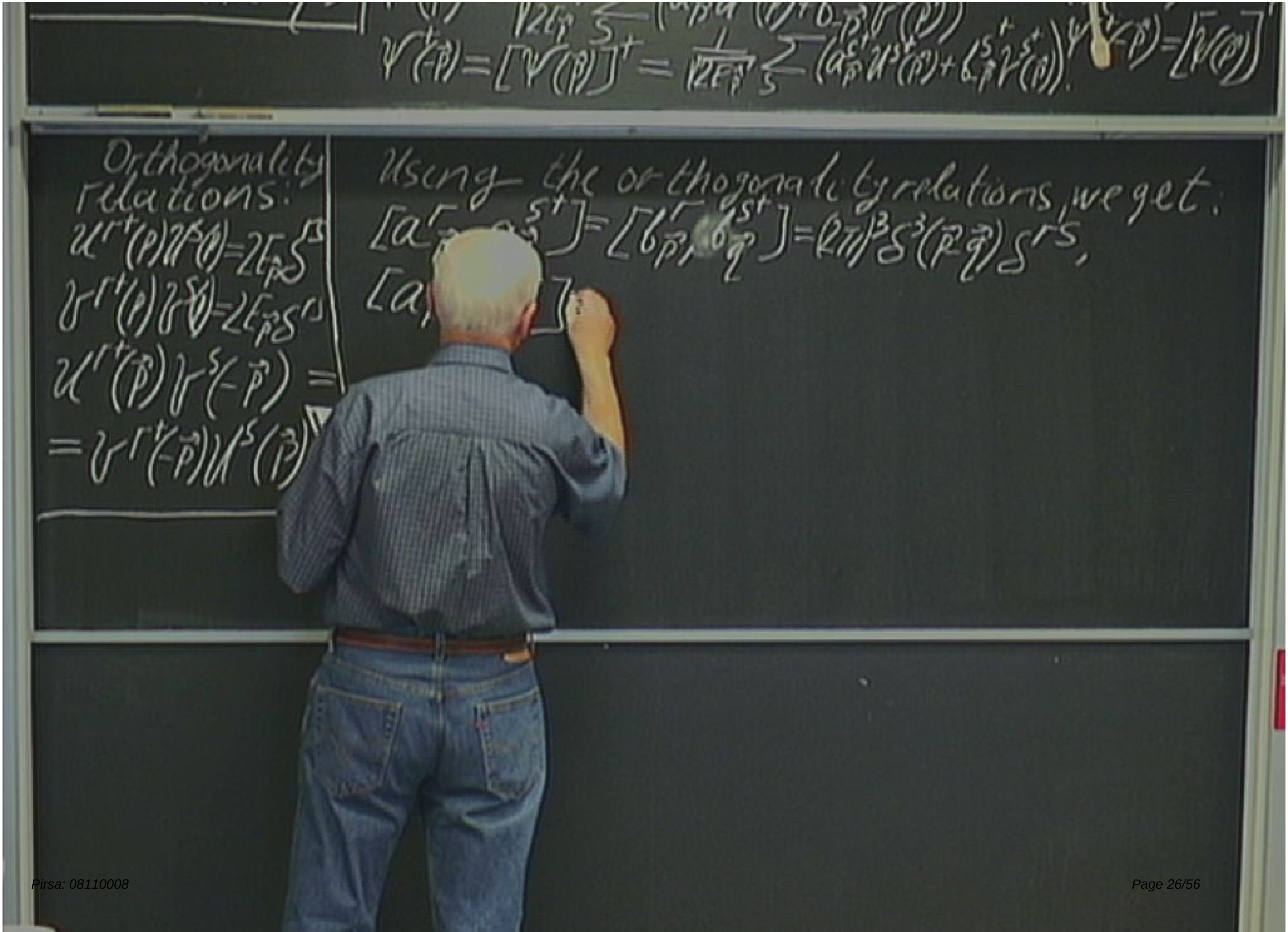
Orthogonality relations:

$u^{r\dagger}(\vec{p}) u^{s}(\vec{p}) = 2E_{\vec{p}} \delta^{rs}$

$v^{r\dagger}(\vec{p}) v^{s}(\vec{p}) = 2E_{\vec{p}} \delta^{rs}$

$u^{r\dagger}(\vec{p}) v^{s}(-\vec{p}) =$

$= v^{r\dagger}(-\vec{p}) u^{s}(\vec{p}) = 0$



$$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a_{\vec p}^{s\dagger} u^{s\dagger}(\vec p) + b_{-\vec p}^{s} v^{s\dagger}(\vec p)\right).$$

Orthogonality relations:

$u^{r\dagger}(p)u^s(p) = 2E_{\vec p}\delta^{rs}$

$v^{r\dagger}(p)v^s(p) = 2E_{\vec p}\delta^{rs}$

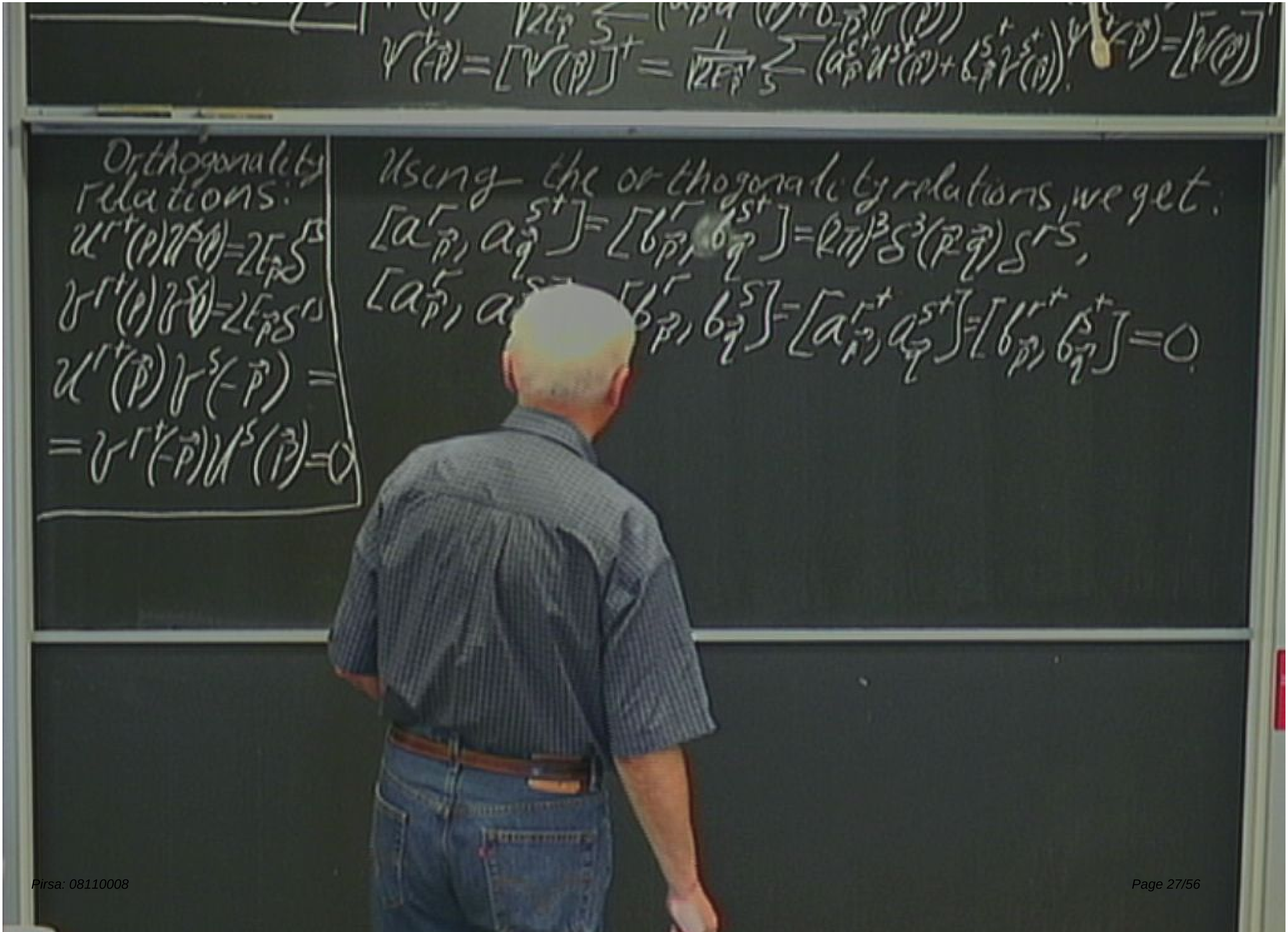
$u^{r\dagger}(\vec p)v^s(-\vec p) =$

$= v^{r\dagger}(-\vec p)u^s(\vec p)$

Using the orthogonality relations, we get:

$[a_{\vec p}^r, a_{\vec q}^{s\dagger}] = [b_{\vec p}^r, b_{\vec q}^{s\dagger}] = (2\pi)^3\delta^3(\vec p - \vec q)\delta^{rs}$,

$[a_{\vec p}\quad]$



$$\frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a^s_{\vec p}u^s(\vec p)+b^{s\dagger}_{-\vec p}v^s(\vec p)\right)$$

$$\psi^{\dagger}(-\vec p)=[\psi(\vec p)]^{\dagger}=\frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a^{s\dagger}_{\vec p}u^{s\dagger}(\vec p)+b^{s}_{-\vec p}v^{s\dagger}(\vec p)\right).\quad \psi^{\dagger}(-\vec p)=[\bar{\psi}(\vec p)]$$

Orthogonality relations:

$$u^{r\dagger}(\vec p)u^s(\vec p)=2E_{\vec p}\delta^{rs}$$

$$v^{r\dagger}(\vec p)v^s(\vec p)=2E_{\vec p}\delta^{rs}$$

$$u^{r\dagger}(\vec p)v^s(-\vec p)=$$

$$=v^{r\dagger}(-\vec p)u^s(\vec p)=0$$

Using the orthogonality relations, we get:

$$[a^r_{\vec p},a^{s\dagger}_{\vec q}]=[b^r_{\vec p},b^{s\dagger}_{\vec q}]=(2\pi)^3\delta^3(\vec p-\vec q)\delta^{rs},$$

$$[a^r_{\vec p},a\ \text{[illegible]}\ b^r_{\vec p},b^s_{\vec q}]=[a^{r\dagger}_{\vec p},a^{s\dagger}_{\vec q}]=[b^{r\dagger}_{\vec p},b^{s\dagger}_{\vec q}]=0.$$

$$\psi^{\dagger}(-\vec p) = [\psi(\vec p)]^{\dagger} = \frac{1}{\sqrt{2E_{\vec p}}}\sum_{s}\left(a^{s\dagger}_{\vec p}u^{s\dagger}(\vec p) + b^{s}_{-\vec p}v^{s\dagger}(\vec p)\right). \quad \psi^{\dagger}(-\vec p) = [\bar\psi(\vec p)]$$

Orthogonality relations:

$$u^{r\dagger}(\vec p)u^{s}(\vec p) = 2E_{\vec p}\delta^{rs}$$

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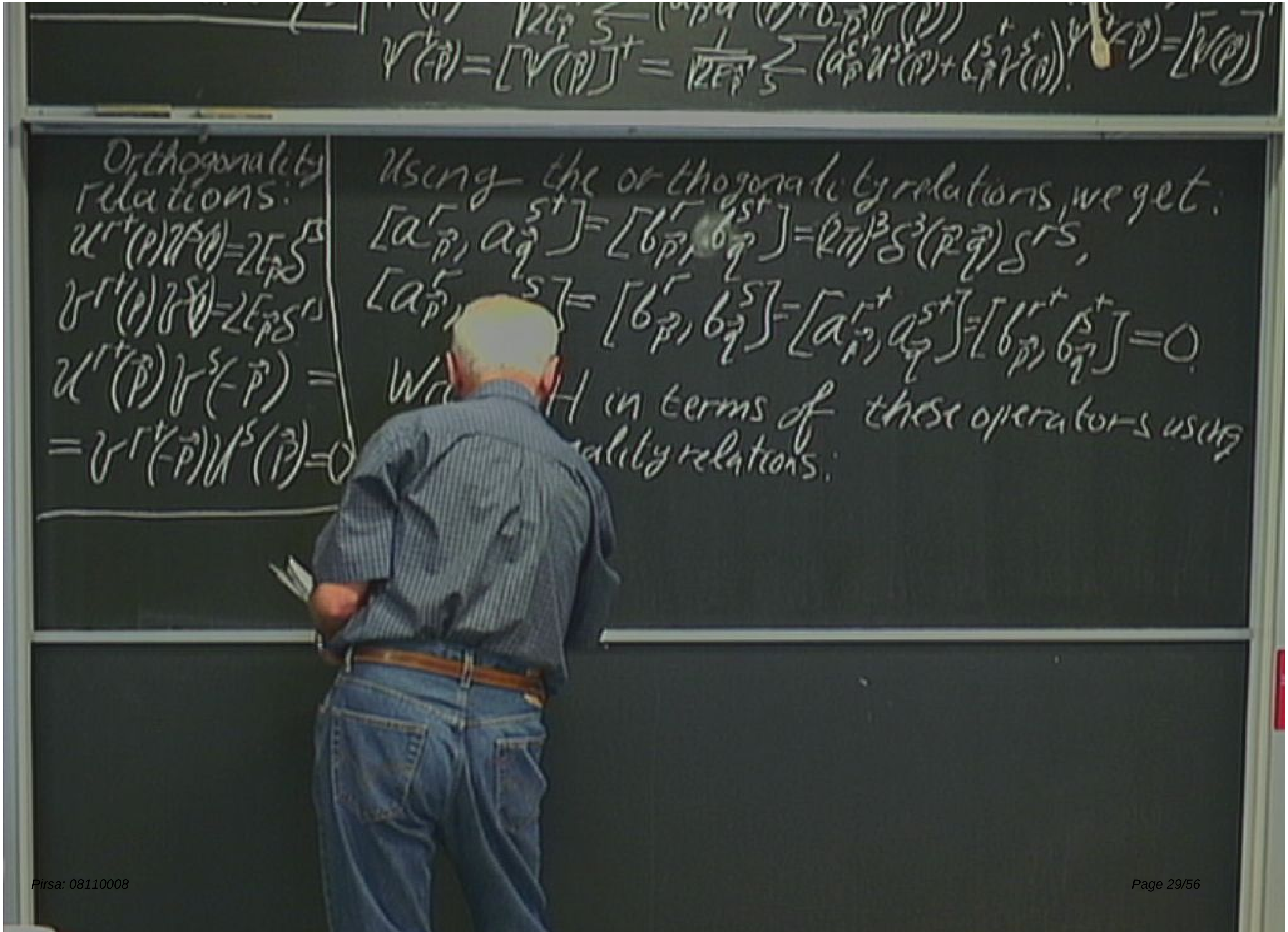

$$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a^{s\dagger}_{\vec p} u^{s\dagger}(\vec p) + b^{s}_{-\vec p} v^{s\dagger}(\vec p)\right). \quad \bar\psi(-\vec p) = [\bar\psi(\vec p)]$$

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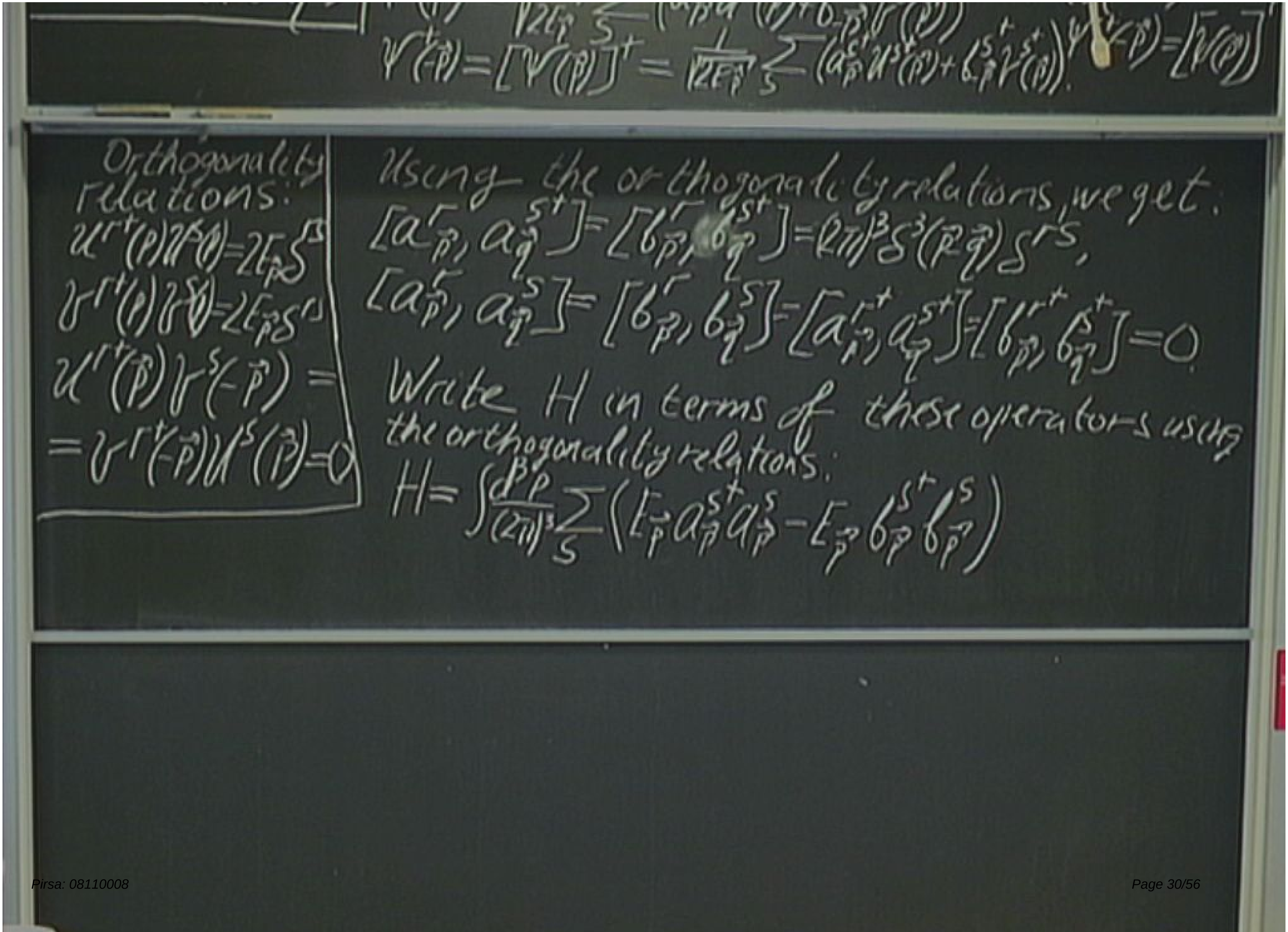
W[illegible] H in terms of these operators using [illegible]ality relations.



$$\psi^\dagger(-\vec{p}) = [\psi(\vec{p})]^\dagger = \frac{1}{\sqrt{2E_{\vec{p}}}}\sum_s \left(a_{\vec{p}}^{s\dagger} u^{s\dagger}(\vec{p}) + b_{-\vec{p}}^{s} v^{s\dagger}(\vec{p})\right).$$

Orthogonality relations:

$$u^{r\dagger}(p)u^s(p) = 2E_{\vec{p}}\delta^{rs}$$

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$$u^{r\dagger}(\vec{p})v^s(-\vec{p}) = v^{r\dagger}(-\vec{p})u^s(\vec{p}) = 0$$

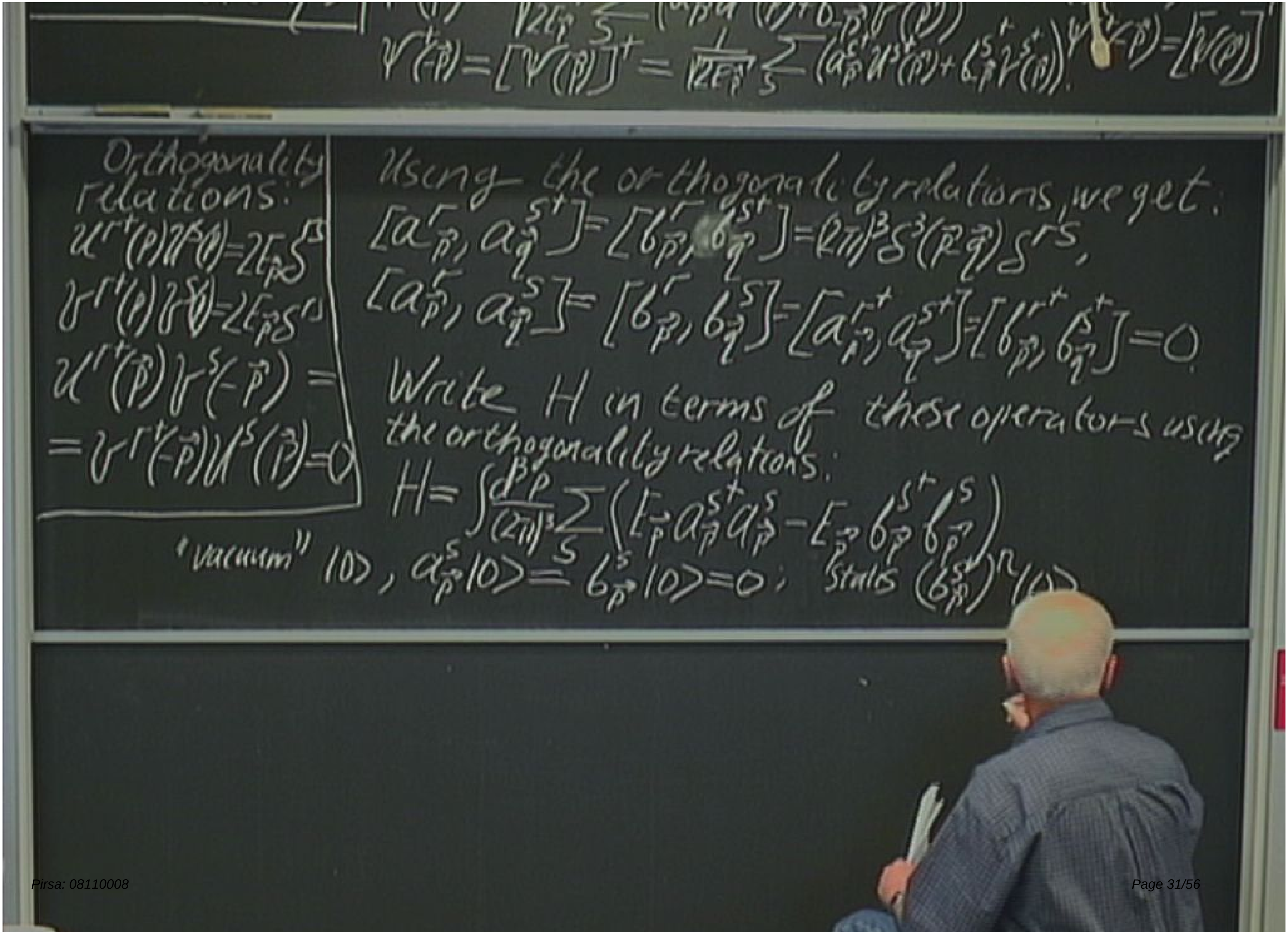
Using the orthogonality relations, we get:

$$[a_{\vec{p}}^r, a_{\vec{q}}^{s\dagger}] = [b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}] = (2\pi)^3\delta^3(\vec{p}-\vec{q})\delta^{rs},$$

$$[a_{\vec{p}}^r, a_{\vec{q}}^s] = [b_{\vec{p}}^r, b_{\vec{q}}^s] = [a_{\vec{p}}^{r\dagger}, a_{\vec{q}}^{s\dagger}] = [b_{\vec{p}}^{r\dagger}, b_{\vec{q}}^{s\dagger}] = 0.$$

Write H in terms of these operators using the orthogonality relations:

$$H = \int \frac{d^3p}{(2\pi)^3}\sum_s \left(E_{\vec{p}}\, a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s - E_{\vec{p}}\, b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s\right)$$



$$\psi^\dagger(-\vec p) = [\psi(\vec p)]^\dagger = \frac{1}{\sqrt{2E_{\vec p}}}\sum_s \left(a^{s\dagger}_{\vec p} u^{s\dagger}(\vec p) + b^{s}_{-\vec p} v^{s\dagger}(\vec p)\right). \quad \bar\psi(-\vec p) = [\bar\psi(\vec p)]$$

Orthogonality relations:

$u^{r\dagger}(p)u^s(p) = 2E_{\vec p}\delta^{rs}$

$v^{r\dagger}(p)v^s(p) = 2E_{\vec p}\delta^{rs}$

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Using the orthogonality relations, we get:

$[a^r_{\vec p}, a^{s\dagger}_{\vec q}] = [b^r_{\vec p}, b^{s\dagger}_{\vec q}] = (2\pi)^3\delta^3(\vec p - \vec q)\delta^{rs}$,

$[a^r_{\vec p}, a^s_{\vec q}] = [b^r_{\vec p}, b^s_{\vec q}] = [a^{r\dagger}_{\vec p}, a^{s\dagger}_{\vec q}] = [b^{r\dagger}_{\vec p}, b^{s\dagger}_{\vec q}] = 0$.

Write H in terms of these operators using the orthogonality relations:

$$H = \int \frac{d^3p}{(2\pi)^3}\sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s\dagger}_{\vec p} b^s_{\vec p}\right)$$

"vacuum" $|0\rangle$, $a^s_{\vec p}|0\rangle = b^s_{\vec p}|0\rangle = 0$; states $(b^{s\dagger}_{\vec p})^n|0\rangle$

$$\frac{1}{\sqrt{2E_{\vec p}}}\sum_s\left(a^s_{\vec p}u^s(\vec p)+b^{s\dagger}_{-\vec p}v^s(\vec p)\right)$$

$$\psi^\dagger(-\vec p)=[\psi(\vec p)]^\dagger=\frac{1}{\sqrt{2E_{\vec p}}}\sum_s\left(a^{s\dagger}_{\vec p}u^{s\dagger}(\vec p)+b^{s}_{-\vec p}v^{s\dagger}(\vec p)\right). \qquad \psi^\dagger(-\vec p)=[\psi(\vec p)]$$

Orthogonality relations:

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Using the orthogonality relations, we get:

$$[a^r_{\vec p},a^{s\dagger}_{\vec q}]=[b^r_{\vec p},b^{s\dagger}_{\vec q}]=(2\pi)^3\delta^3(\vec p-\vec q)\delta^{rs},$$

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Write H in terms of these operators using the orthogonality relations:

$$H=\int\frac{d^3p}{(2\pi)^3}\sum_s\left(E_{\vec p}a^{s\dagger}_{\vec p}a^s_{\vec p}-E_{\vec p}b^{s\dagger}_{\vec p}b^s_{\vec p}\right)$$

There is ground state "vacuum" $|0\rangle$, $a^s_{\vec p}|0\rangle=b^s_{\vec p}|0\rangle=0$; (vacuum) in this theory. States $(b^{s\dagger}_{\vec p})^n|0\rangle$ have $E^{(n)}(\vec p)<0$

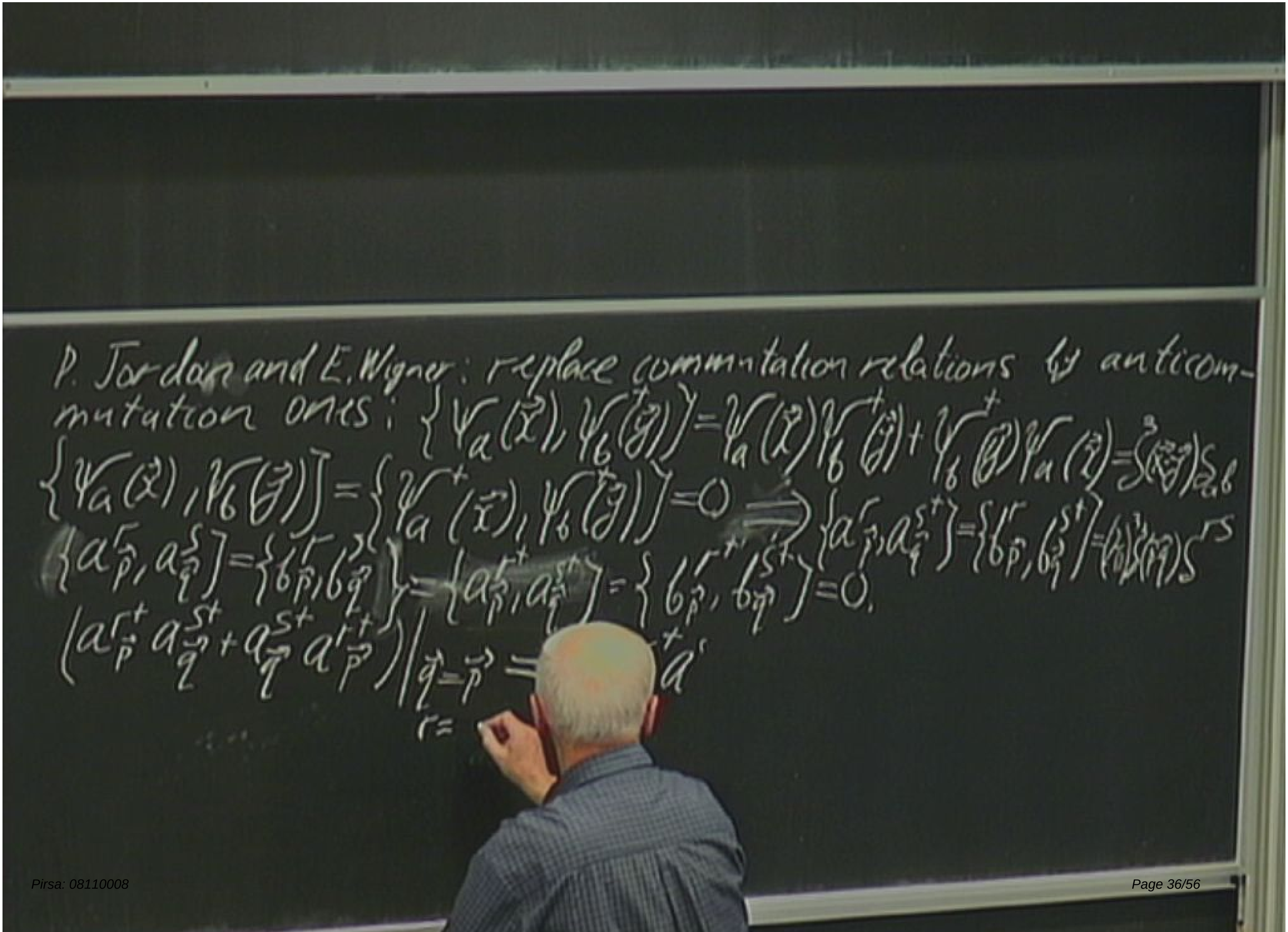
P. Jordan and E. Wigner: replace commutation relations by anticommutation ones:

P. Jordan and E. Wigner: replace commutation relations by anticommutation ones: $\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \psi_a(\vec{x})\psi_b^\dagger(\vec{y}) + \psi_b^\dagger(\vec{y})\psi_a(\vec{x}) = \delta^3(\vec{x}-\vec{y})\delta_{ab}$

$\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^\dagger(\vec{x}), \psi_b(\vec{y})\} = 0.$

P. Jordan and E. Wigner: replace commutation relations by anticommutation ones: $\{\psi_a(\vec{x}), \psi_b^{\dagger}(\vec{y})\} = \psi_a(\vec{x})\psi_b^{\dagger}(\vec{y}) + \psi_b^{\dagger}(\vec{y})\psi_a(\vec{x}) = \delta^3(\vec{x}-\vec{y})\delta_{ab}$

$\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^{\dagger}(\vec{x}), \psi_b^{\dagger}(\vec{y})\} = 0 \Rightarrow$

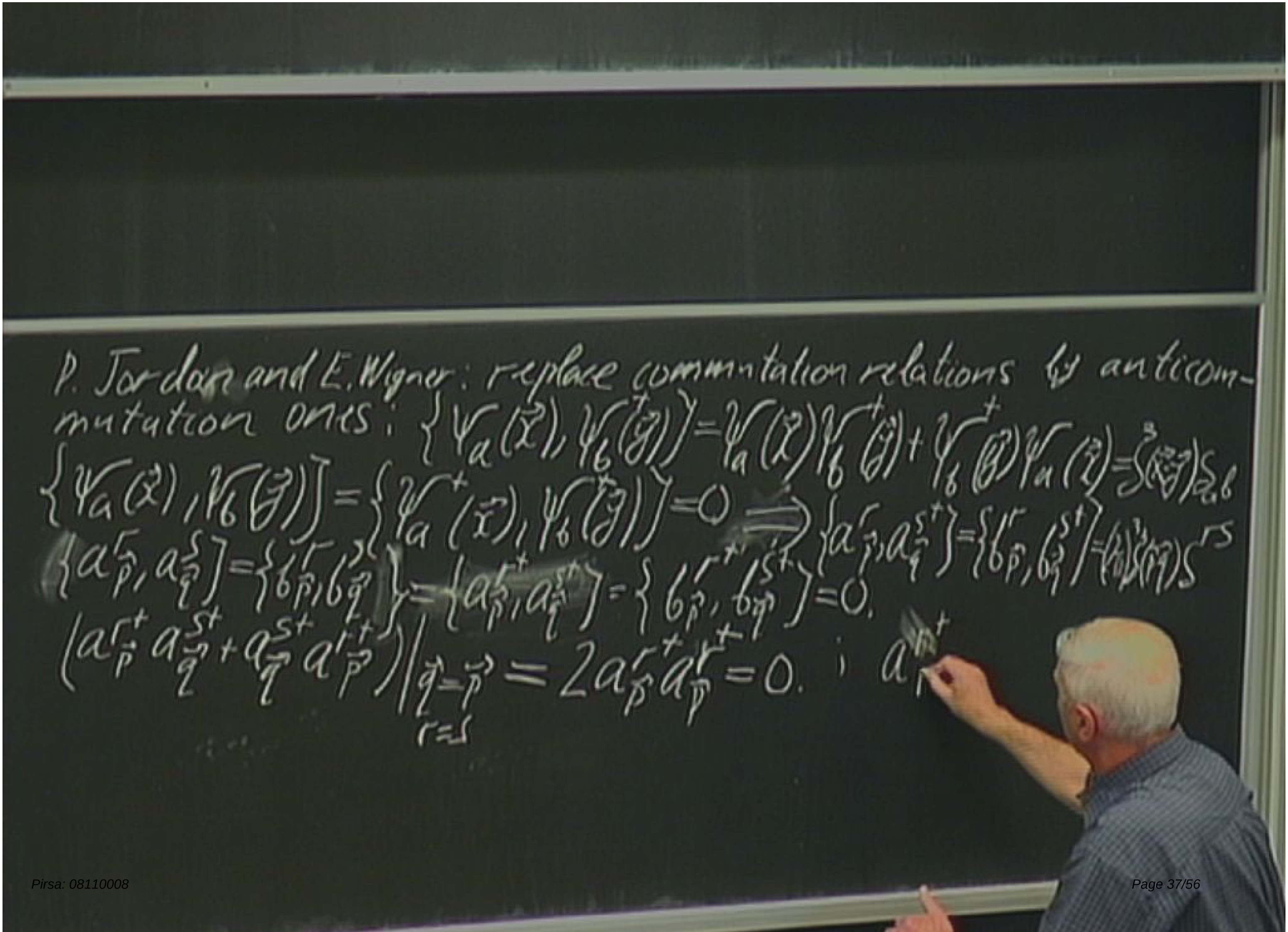


P. Jordan and E. Wigner: replace commutation relations by anticommutation ones: $\{\psi_a(\vec{x}),\psi_b^{\dagger}(\vec{y})\}=\psi_a(\vec{x})\psi_b^{\dagger}(\vec{y})+\psi_b^{\dagger}(\vec{y})\psi_a(\vec{x})=\delta^3(\vec{x}-\vec{y})\delta_{ab}$

$$\{\psi_a(\vec{x}),\psi_b(\vec{y})\}=\{\psi_a^{\dagger}(\vec{x}),\psi_b^{\dagger}(\vec{y})\}=0 \Longrightarrow \{a^r_{\vec{p}},a^{s\dagger}_{\vec{q}}\}=\{b^r_{\vec{p}},b^{s\dagger}_{\vec{q}}\}=(2\pi)^3\delta^3(\vec{p}-\vec{q})\delta^{rs}$$

$$\{a^r_{\vec{p}},a^s_{\vec{q}}\}=\{b^r_{\vec{p}},b^s_{\vec{q}}\}=\{a^{r\dagger}_{\vec{p}},a^{s\dagger}_{\vec{q}}\}=\{b^{r\dagger}_{\vec{p}},b^{s\dagger}_{\vec{q}}\}=0.$$

$$\left(a^{r\dagger}_{\vec{p}}a^{s\dagger}_{\vec{q}}+a^{s\dagger}_{\vec{q}}a^{r\dagger}_{\vec{p}}\right)\Big|_{\substack{\vec{q}=\vec{p}\\ r=}} = \quad {}^{\dagger}a$$



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$\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^\dagger(\vec{x}), \psi_b(\vec{y})\} = 0 \Longrightarrow \{a^r_{\vec{p}}, a^{s\dagger}_{\vec{q}}\} = \{b^r_{\vec{p}}, b^{s\dagger}_{\vec{q}}\} = (2\pi)^3\delta^3(\vec{p}-\vec{q})\delta^{rs}$

$\{a^r_{\vec{p}}, a^s_{\vec{q}}\} = \{b^r_{\vec{p}}, b^s_{\vec{q}}\} = \{a^{r\dagger}_{\vec{p}}, a^{s\dagger}_{\vec{q}}\} = \{b^{r\dagger}_{\vec{p}}, b^{s\dagger}_{\vec{q}}\} = 0.$

$\left(a^{r\dagger}_{\vec{p}} a^{s\dagger}_{\vec{q}} + a^{s\dagger}_{\vec{q}} a^{r\dagger}_{\vec{p}}\right)\Big|_{\substack{\vec{q}=\vec{p}\\ r=s}} = 2a^{r\dagger}_{\vec{p}}a^{r\dagger}_{\vec{p}} = 0.$; $a^{\dagger}_{p}$

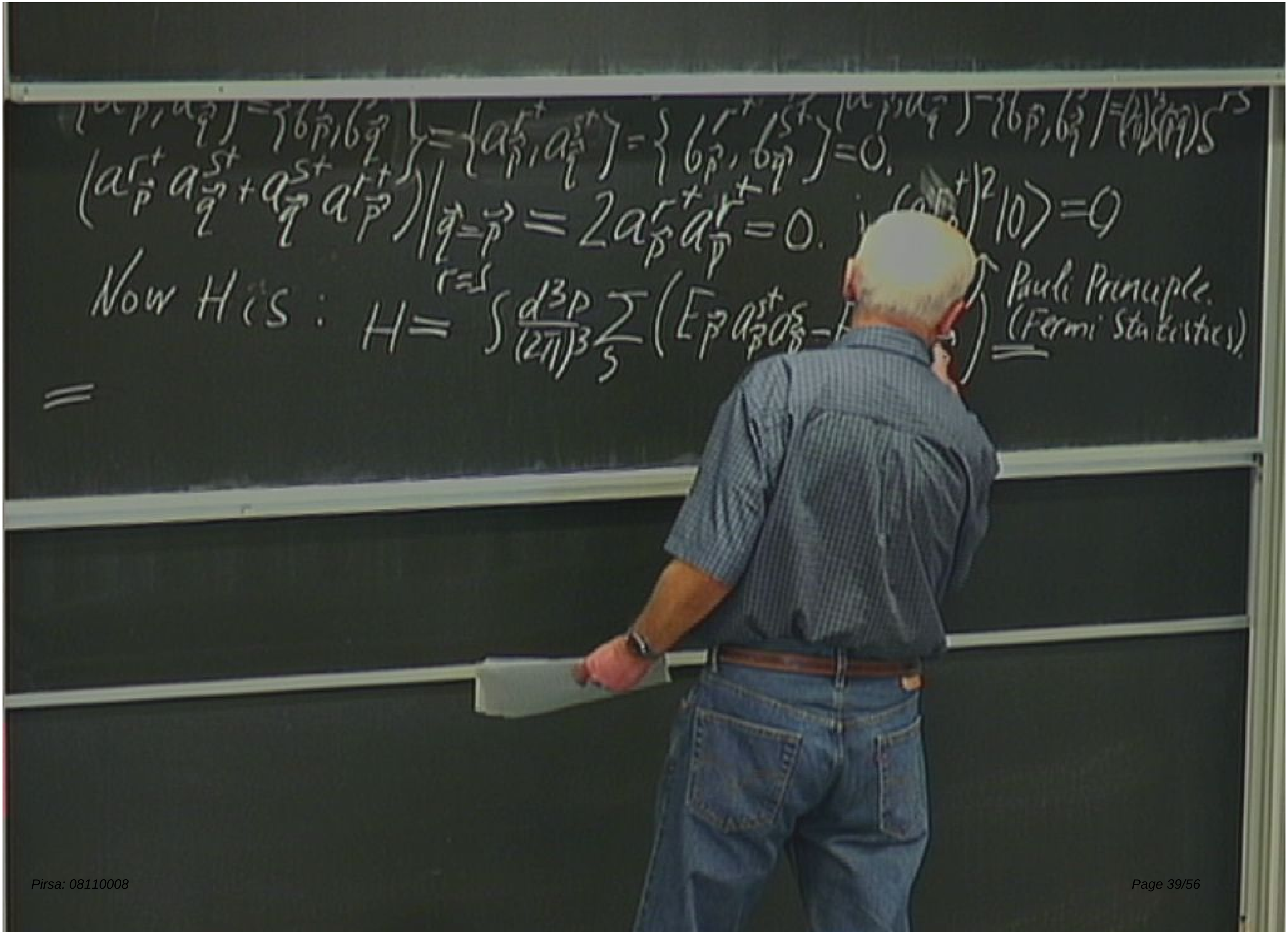
P. Jordan and E. Wigner: replace commutation relations by anticommutation ones: $\{\psi_a(\vec{x}), \psi_b^\dagger(\vec{y})\} = \psi_a(\vec{x})\psi_b^\dagger(\vec{y}) + \psi_b^\dagger(\vec{y})\psi_a(\vec{x}) = \delta^3(\vec{x}-\vec{y})\delta_{ab}$

$\{\psi_a(\vec{x}), \psi_b(\vec{y})\} = \{\psi_a^\dagger(\vec{x}), \psi_b^\dagger(\vec{y})\} = 0 \Longrightarrow \{a^r_{\vec{p}}, a^{s\dagger}_{\vec{q}}\} = \{b^r_{\vec{p}}, b^{s\dagger}_{\vec{q}}\} = (2\pi)^3\delta^3(\vec{p}-\vec{q})\delta^{rs}$

$\{a^r_{\vec{p}}, a^s_{\vec{q}}\} = \{b^r_{\vec{p}}, b^s_{\vec{q}}\} = \{a^{r\dagger}_{\vec{p}}, a^{s\dagger}_{\vec{q}}\} = \{b^{r\dagger}_{\vec{p}}, b^{s\dagger}_{\vec{q}}\} = 0.$

$(a^{r\dagger}_{\vec{p}} a^{s\dagger}_{\vec{q}} + a^{s\dagger}_{\vec{q}} a^{r\dagger}_{\vec{p}})\Big|_{\substack{\vec{q}=\vec{p} \\ r=s}} = 2a^{r\dagger}_{\vec{p}} a^{r\dagger}_{\vec{p}} = 0.$; $(a^{r\dagger}_{\vec{p}})^2|0\rangle = 0$

↑ Pauli Principle.
(Fermi Statistics)



$\{a_{\vec p}, a_{\vec q}\} = \{b_{\vec p}, b_{\vec q}\} = \{a^{r\dagger}_{\vec p}, a^{s\dagger}_{\vec q}\} = \{b^{r\dagger}_{\vec p}, b^{s\dagger}_{\vec q}\} = 0.$ $\{a^r_{\vec p}, a^{s\dagger}_{\vec q}\} = \{b^r_{\vec p}, b^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p-\vec q)\delta^{rs}$

$(a^{r\dagger}_{\vec p}\, a^{s\dagger}_{\vec q} + a^{s\dagger}_{\vec q}\, a^{r\dagger}_{\vec p})\Big|_{\substack{\vec q=\vec p\\ r=s}} = 2a^{r\dagger}_{\vec p}\, a^{r\dagger}_{\vec p} = 0.$ $\therefore (a^{\dagger}_{\vec p})^2|0\rangle = 0$ ↑ Pauli Principle. (Fermi Statistics)

Now H is: $H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - \ldots \right)$

$=$

$$\{a^r_{\vec p}, a^s_{\vec q}\} = \{b^r_{\vec p}, b^s_{\vec q}\} = \{a^{r\dagger}_{\vec p}, a^{s\dagger}_{\vec q}\} = \{b^{r\dagger}_{\vec p}, b^{s\dagger}_{\vec q}\} = 0. \qquad \{a^r_{\vec p}, a^{s\dagger}_{\vec q}\} = \{b^r_{\vec p}, b^{s\dagger}_{\vec q}\} = (2\pi)^3\delta^3(\vec p - \vec q)\delta^{rs}$$

$$\left(a^{r\dagger}_{\vec p} a^{s\dagger}_{\vec q} + a^{s\dagger}_{\vec q} a^{r\dagger}_{\vec p}\right)\Big|_{\substack{\vec q = \vec p \\ r=s}} = 2a^{r\dagger}_{\vec p} a^{r\dagger}_{\vec p} = 0. \; ; \; (a^{r\dagger}_{\vec p})^2|0\rangle = 0$$

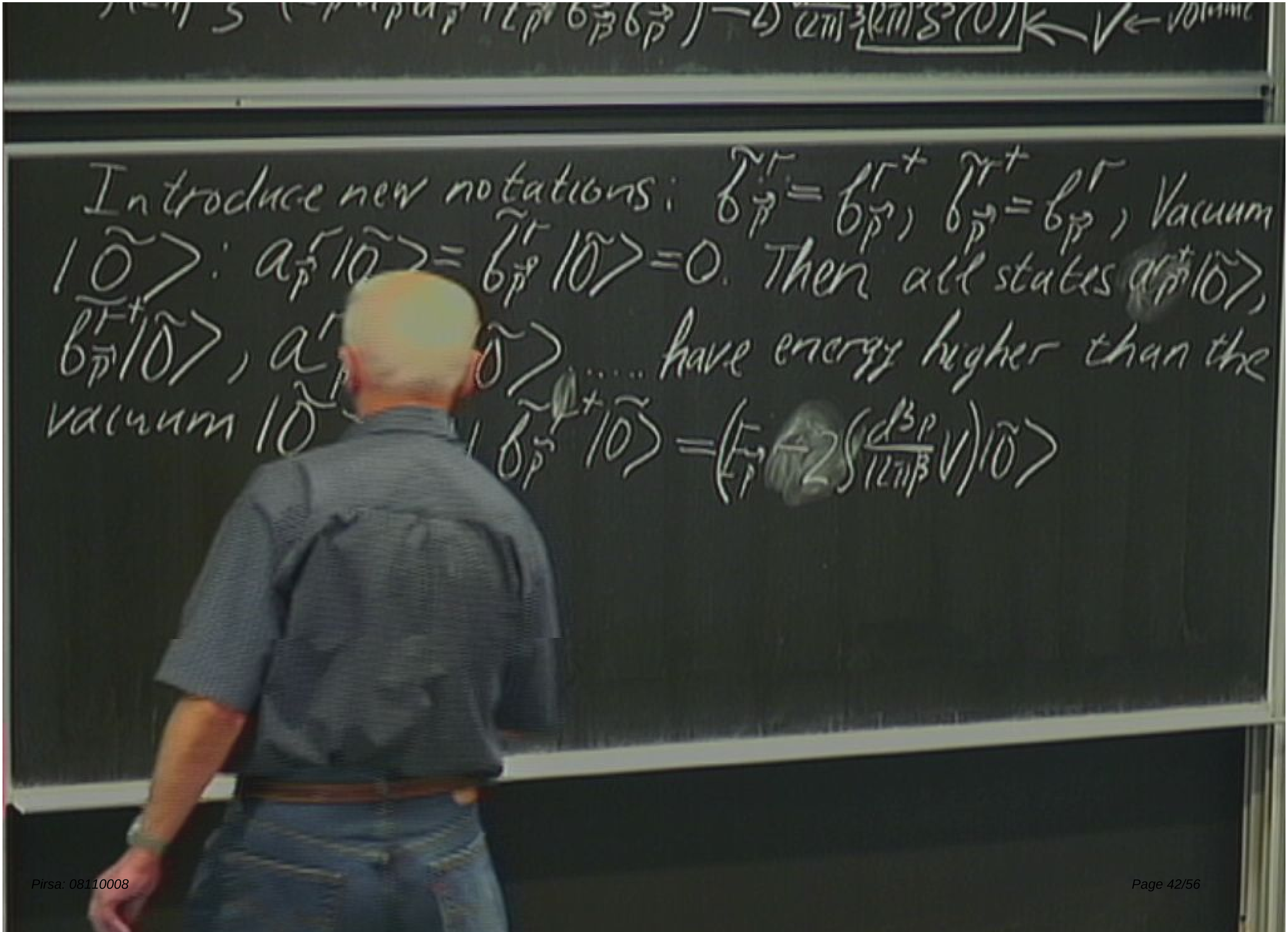
↑ Pauli Principle. (Fermi Statistics)

Now H is : $H = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} - E_{\vec p}\, b^{s}_{\vec p} b^{s\dagger}_{\vec p}\right)$

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_{\vec p}\, a^{s\dagger}_{\vec p} a^s_{\vec p} + E_{\vec p}\, b^{s}_{\vec p} b^{s\dagger}_{\vec p}\right) - 2\int \frac{d^3p}{(2\pi)^3} \underbrace{(2\pi)^3 \delta^3(0)} \leftarrow V \leftarrow \text{volume}$$

$\to \boxed{(2\pi)^3\delta(0)} \leftarrow V \leftarrow$ volume

Introduce new notations: $\tilde{b}^{r}_{\vec p} = b^{r\dagger}_{\vec p}$, $\tilde{b}^{r\dagger}_{\vec p} = b^{r}_{\vec p}$



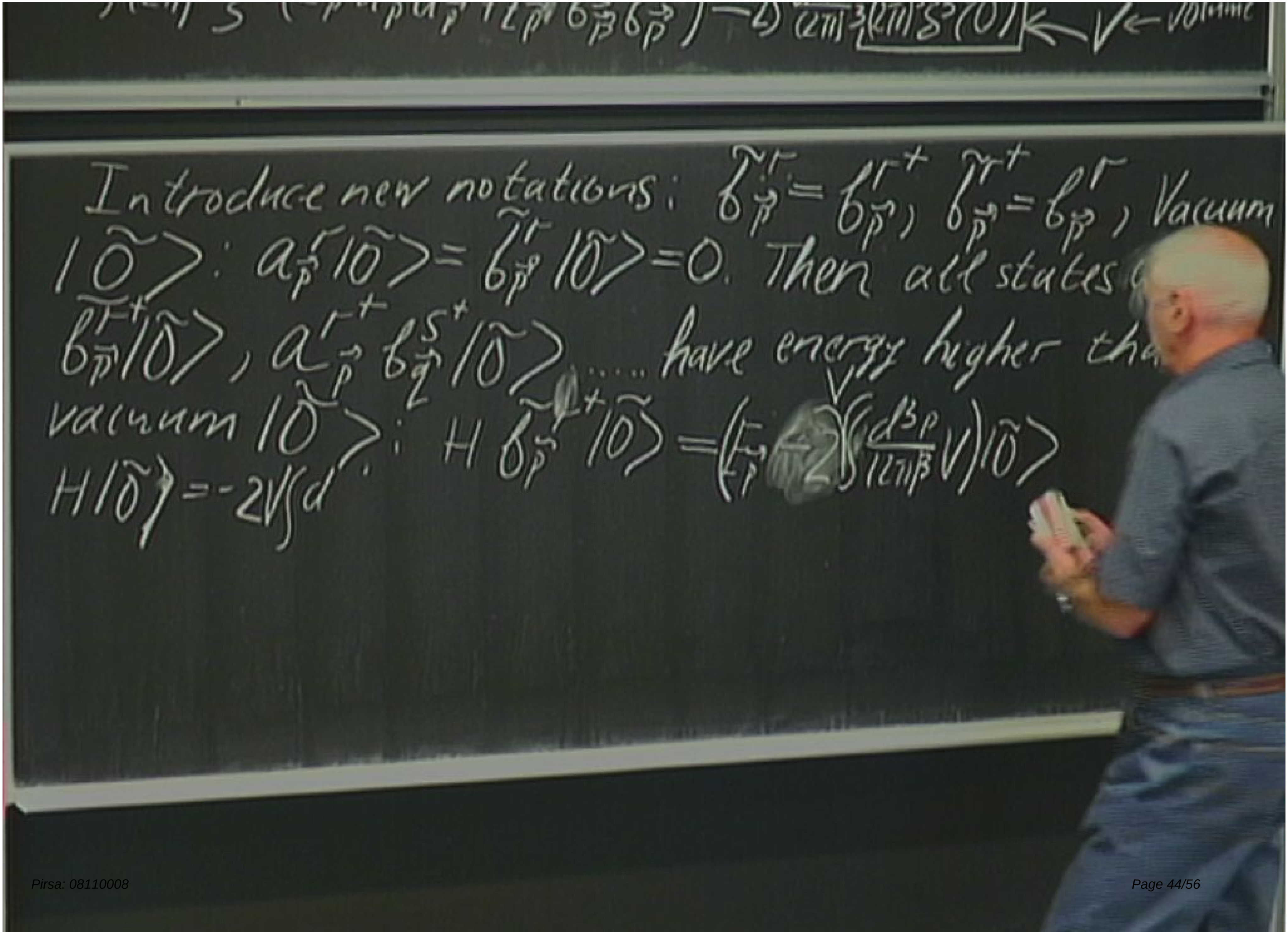
$\ldots \int (\ldots a_{\vec p} a_{\vec p}^{+} + E_{\vec p}\, b_{\vec p} b_{\vec p}^{+}) - 2 \ldots \frac{}{(2\pi)^3} \boxed{(2\pi)^3 \delta^3(0)} \leftarrow V \leftarrow$ volume

Introduce new notations: $\tilde b^{r}_{\vec p} = b^{r+}_{\vec p}$, $\tilde b^{r+}_{\vec p} = b^{r}_{\vec p}$, Vacuum $|\tilde 0\rangle$: $a^{r}_{\vec p}|\tilde 0\rangle = \tilde b^{r}_{\vec p}|\tilde 0\rangle = 0$. Then all states $a^{r+}_{\vec p}|\tilde 0\rangle$, $\tilde b^{r+}_{\vec p}|\tilde 0\rangle$, a^{r+} [illegible] $|\tilde 0\rangle$ …… have energy higher than the vacuum $|\tilde 0\rangle$ [illegible] $\tilde b^{r+}_{\vec p}|\tilde 0\rangle = (E_{\vec p} - 2\int \frac{d^3p}{(2\pi)^3} V)|\tilde 0\rangle$

$$\ldots \sum (\ldots a_{\vec p} a^{\dagger}_{\vec p} + \epsilon_{\vec p}\, b_{\vec p} b_{\vec p}) - 2 \frac{\ldots}{(2\pi)^3} \boxed{(2\pi)^3 \delta^3(0)} \leftarrow V \leftarrow \text{volume}$$

Introduce new notations: $\tilde b^{r}_{\vec p} = b^{r\,+}_{\vec p}$, $\tilde b^{r\,+}_{\vec p} = b^{r}_{\vec p}$, Vacuum $|\tilde 0\rangle$: $a^{r}_{\vec p}|\tilde 0\rangle = \tilde b^{r}_{\vec p}|\tilde 0\rangle = 0$. Then all states $a^{r\,+}_{\vec p}|\tilde 0\rangle$, [illegible]$\rangle$, $a^{r\,+}_{\vec p}\, b^{s\,+}_{\vec q}|\tilde 0\rangle$ ...... have energy higher than the [illegible]m $|\tilde 0\rangle$:

$$H\, \tilde b^{r\,+}_{\vec p}|\tilde 0\rangle = \left(E_{\vec p} - 2\int \frac{d^3p}{(2\pi)^3} V\right)|\tilde 0\rangle$$



$\frac{}{(2\pi)^3}\boxed{(2\pi)^3\delta^3(0)}$ ← V ← volume

Introduce new notations: $\tilde{b}^{r}_{\vec p} = b^{r\dagger}_{\vec p}$, $\tilde{b}^{r\dagger}_{\vec p} = b^{r}_{\vec p}$, Vacuum $|\tilde{0}\rangle$: $a^{r}_{\vec p}|\tilde{0}\rangle = \tilde{b}^{r}_{\vec p}|\tilde{0}\rangle = 0$. Then all states

$\tilde{b}^{r\dagger}_{\vec p}|\tilde{0}\rangle$, $a^{r\dagger}_{\vec p}\, b^{s\dagger}_{\vec q}|\tilde{0}\rangle$ …… have energy higher tha

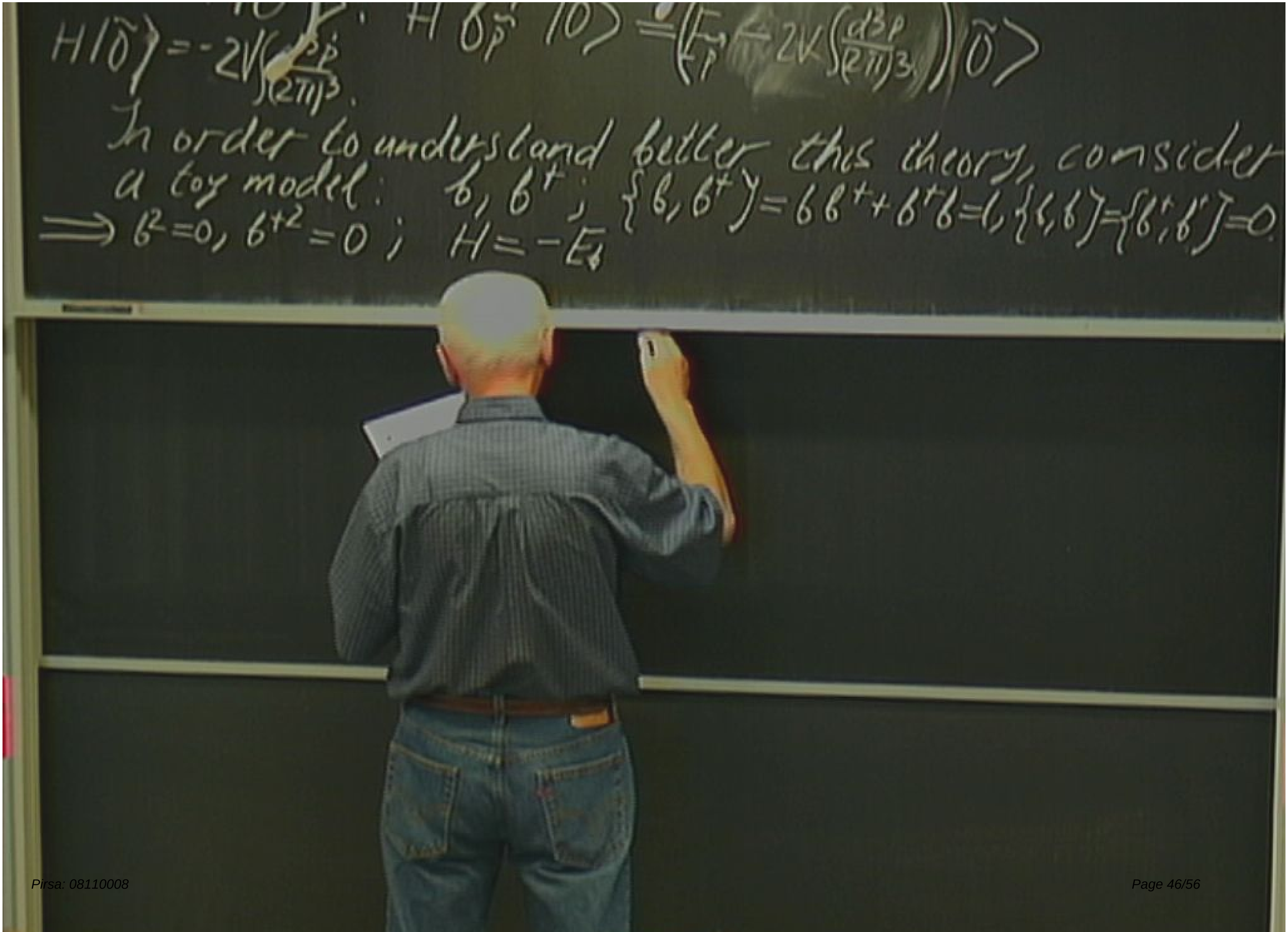
vacuum $|\tilde{0}\rangle$: $H\,\tilde{b}^{r\dagger}_{\vec p}|\tilde{0}\rangle = \left(E_{\vec p} \overset{V}{-2}\int \frac{d^3p}{(2\pi)^3}V\right)|\tilde{0}\rangle$

$H|\tilde{0}\rangle = -2V\int d$

$\ldots (2\pi)^3 \boxed{(2\pi)^3\delta^3(0)} \leftarrow V \leftarrow$ volume

Introduce new notations: $\tilde{b}^{r}_{\vec{p}} = b^{r\dagger}_{\vec{p}}$, $\tilde{b}^{r\dagger}_{\vec{p}} = b^{r}_{\vec{p}}$, Vacuum $|\tilde{0}\rangle$: $a^{r}_{\vec{p}}|\tilde{0}\rangle = \tilde{b}^{r}_{\vec{p}}|\tilde{0}\rangle = 0$. Then all states $a^{r\dagger}_{\vec{p}}|\tilde{0}\rangle$, $\tilde{b}^{r\dagger}_{\vec{p}}|\tilde{0}\rangle$, $a^{r\dagger}_{\vec{p}}\, b^{s\dagger}_{\vec{q}}|\tilde{0}\rangle$ …… have energy higher than the vacuum $|\tilde{0}\rangle$: $H\,\tilde{b}^{r\dagger}_{\vec{p}}|\tilde{0}\rangle = \left(E_{\vec{p}} - 2V\int\frac{d^3p}{(2\pi)^3}\right)|\tilde{0}\rangle$

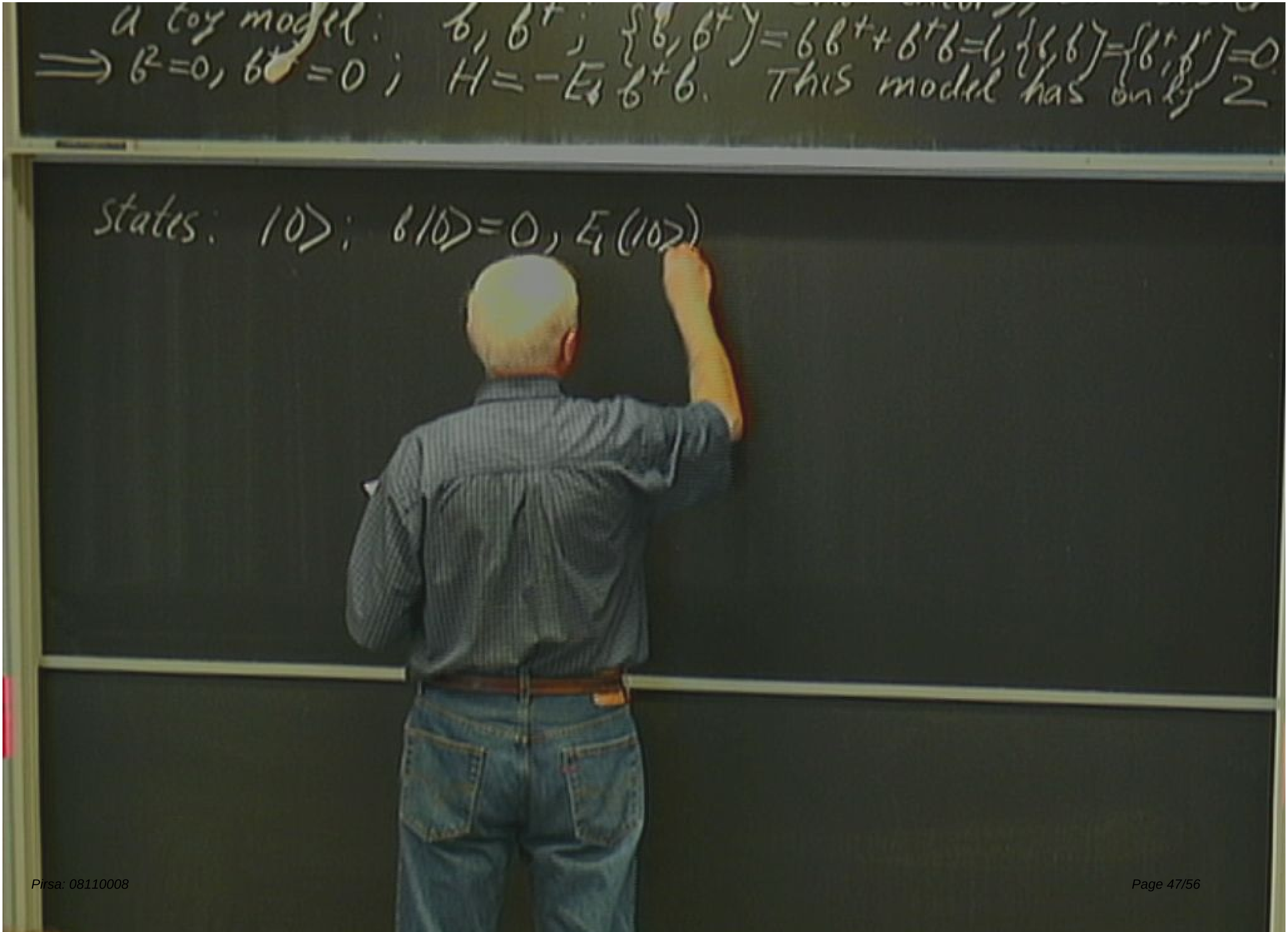
$H|\tilde{0}\rangle = -2V\int\frac{d^3p}{(2\pi)^3}$.



$H|\tilde{0}\rangle = -2V\int\frac{d^3p}{(2\pi)^3}$; $\quad H\, b^+_{\vec{p}}\, |\tilde{0}\rangle = \left(E_{\vec{p}} - 2V\int\frac{d^3p}{(2\pi)^3}\right)|\tilde{0}\rangle$

In order to understand better this theory, consider a toy model: b, b^+ ; $\{b, b^+\} = bb^+ + b^+b = 1, \{b, b\} = \{b^+, b^+\} = 0.$

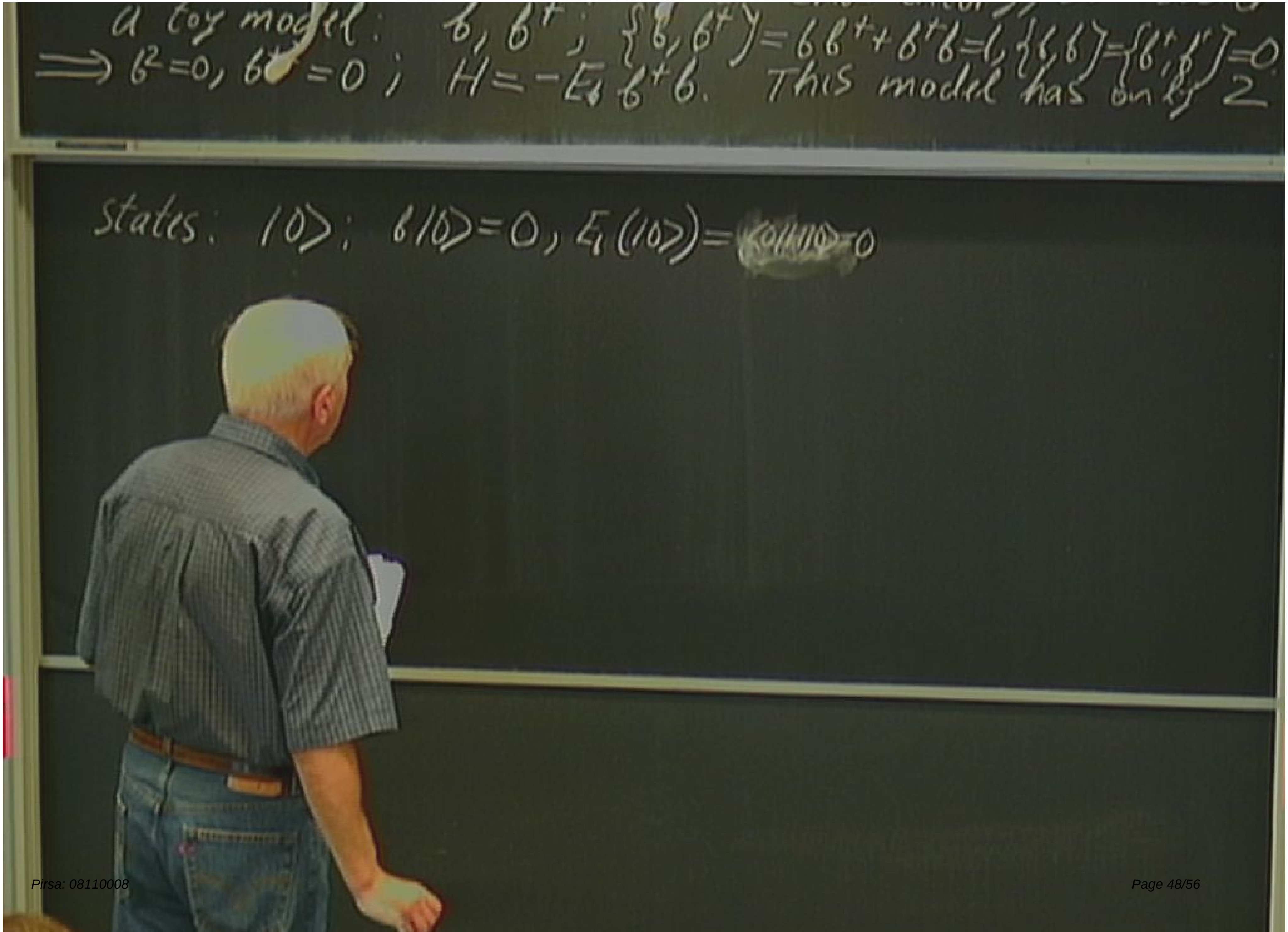
$\Longrightarrow b^2 = 0, \; b^{+2} = 0 \; ; \quad H = -E_0$



a toy model: b, b^+; $\{b, b^+\} = bb^+ + b^+b = 1$, $\{b, b\} = \{b^+, b^+\} = 0$

$\Longrightarrow b^2 = 0$, $b^{+2} = 0$; $H = -E_1 b^+ b$. This model has only 2

States: $|0\rangle$; $b|0\rangle = 0$, $E_1(|0\rangle)$



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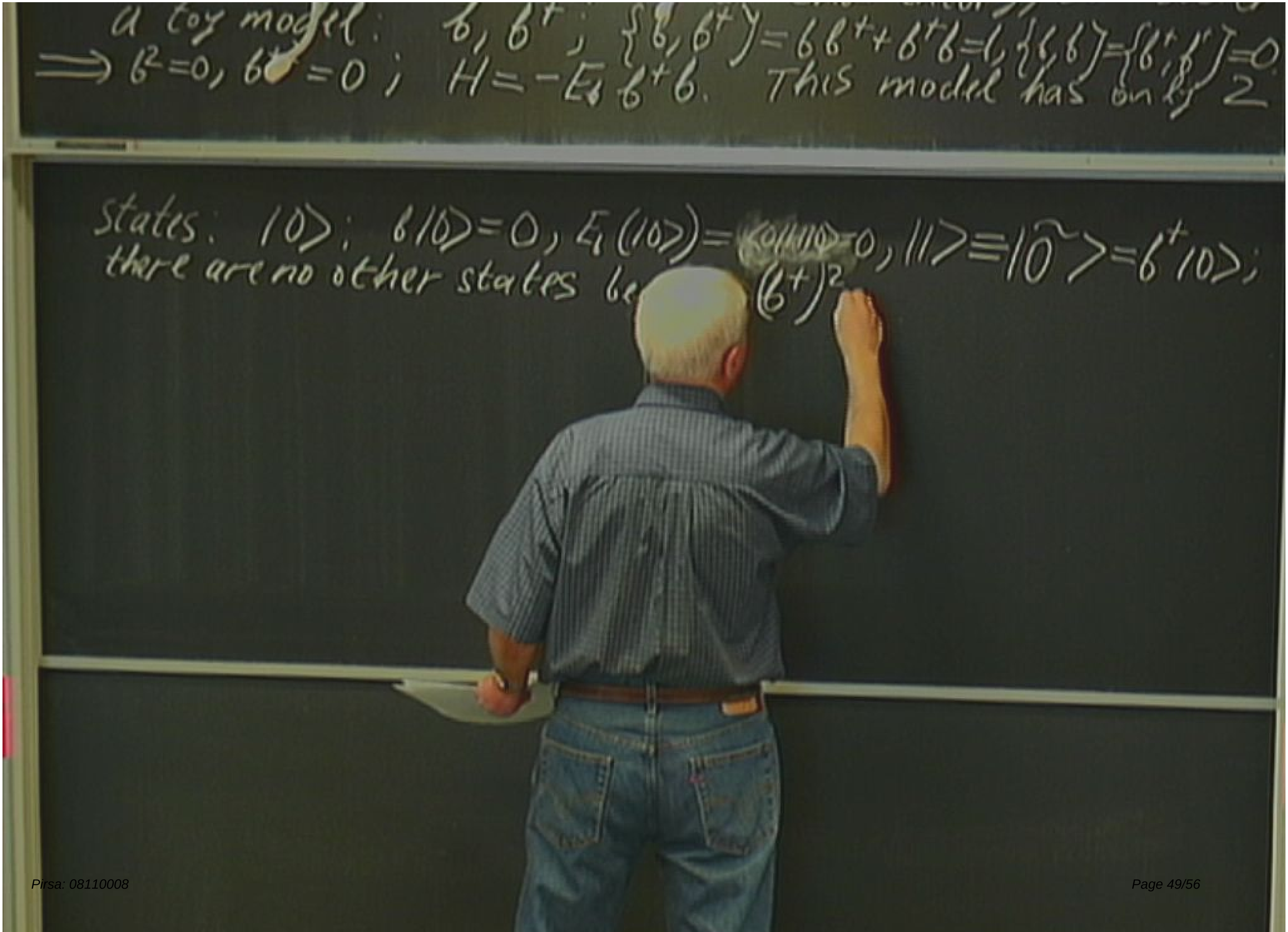
$\Longrightarrow b^2 = 0$, $b^{+2} = 0$; $H = -E_1 b^+ b$. This model has only 2

States: $|0\rangle$; $b|0\rangle = 0$, $E_1(|0\rangle) =$ ~~$\langle 0|H|0\rangle$~~ $= 0$



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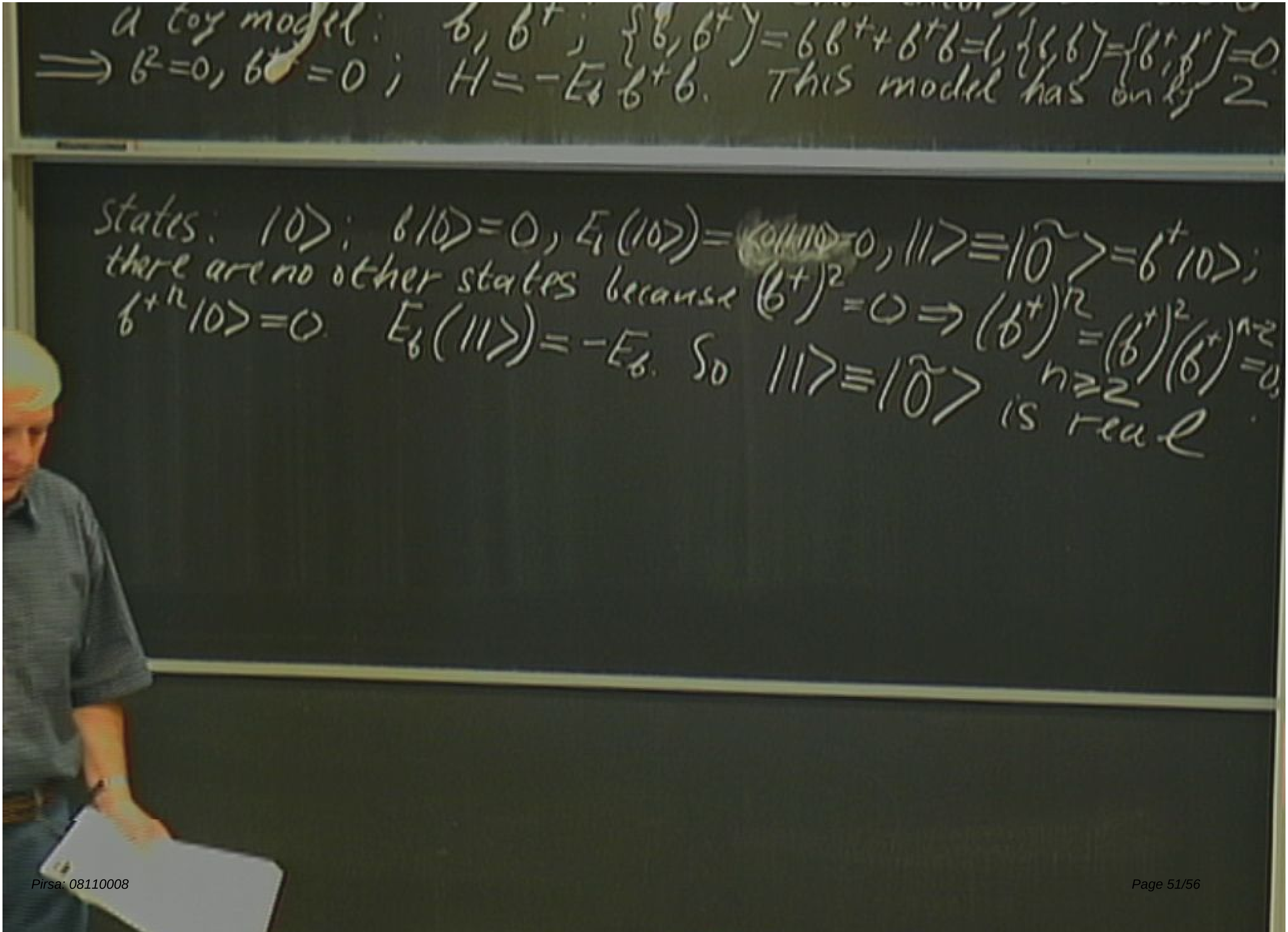

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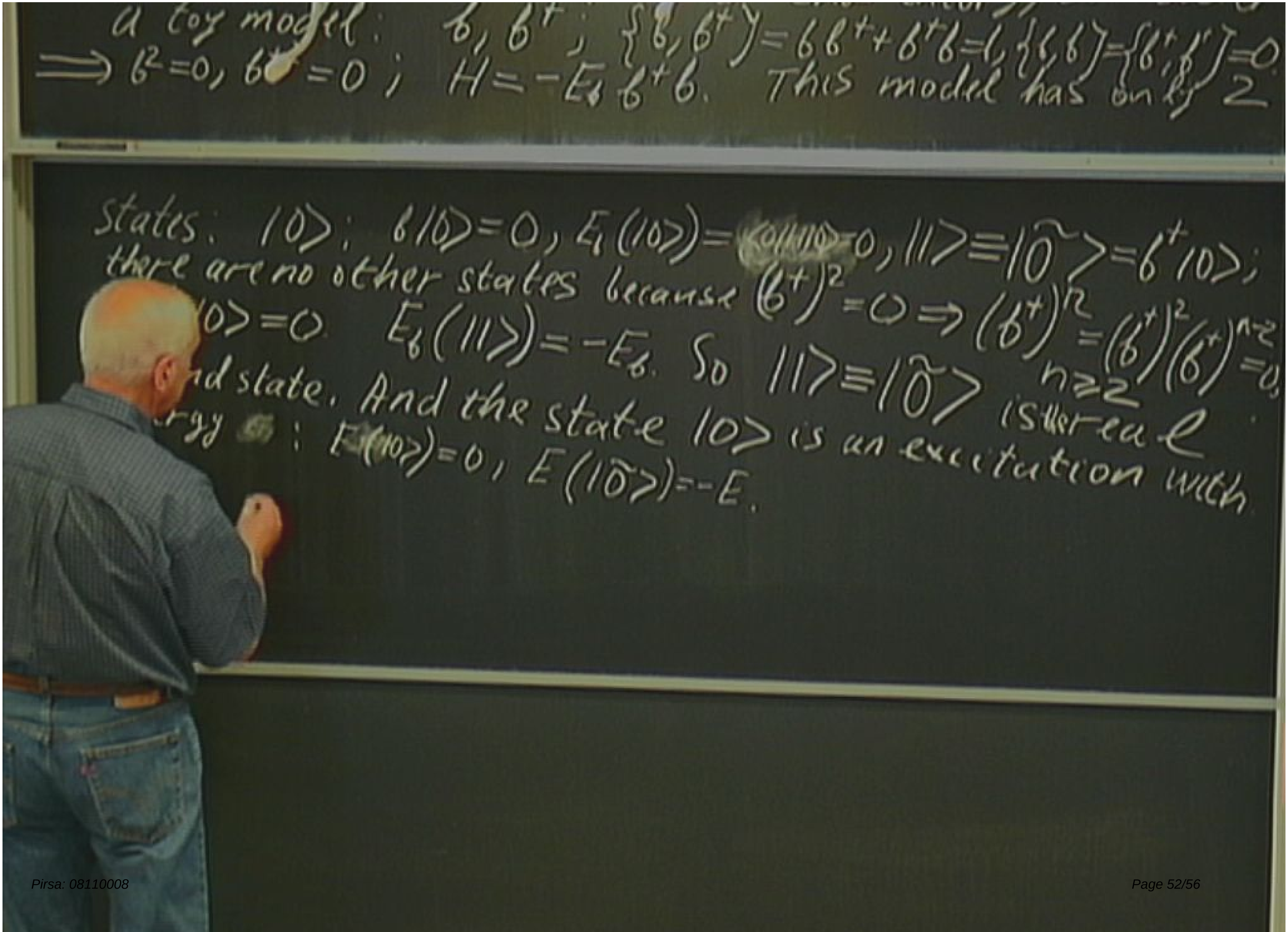

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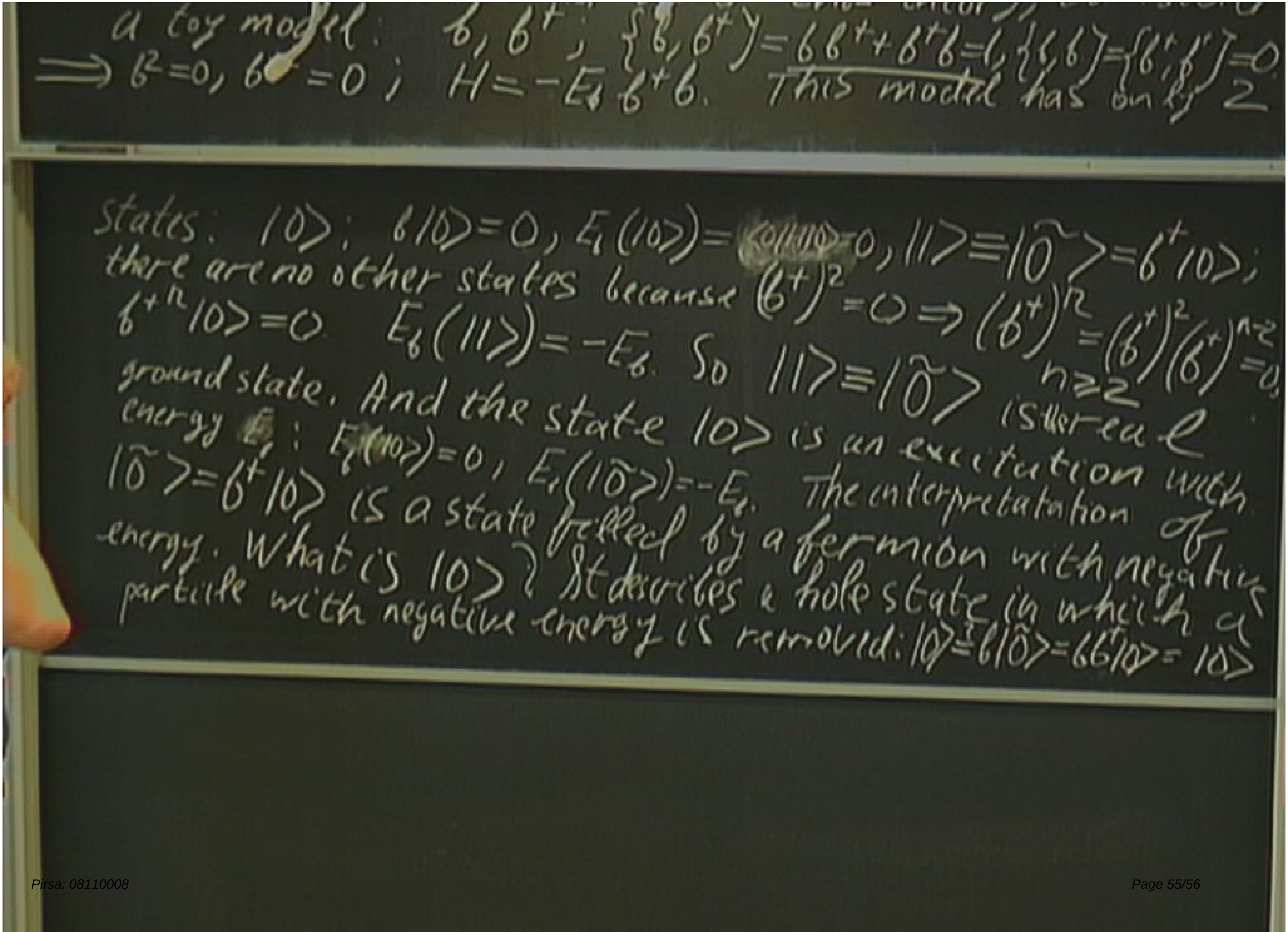

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As before, I could introduce $\tilde{b} = b^+$ and $\tilde{b}^+ = b$; then $\tilde{b}|\tilde{0}\rangle = 0$ and $H = -E_b b^+ b = E_b b b^+ - E_b = E_b \tilde{b}^+ \tilde{b} - E_b$.