

Title: Astrophysics and Cosmology through Problems - 13A

Date: Nov 27, 2008 10:00 AM

URL: <http://pirsa.org/08110007>

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

$$P(k) = \langle \delta_k \delta_k^* \rangle$$

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$$\mathcal{F}^{-1}(P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k)(x) * \mathcal{F}^{-1}(\delta_k^*)(x)$$

$$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$$

$$\mathcal{F}^{-1}(\mathcal{F}(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k) * \mathcal{F}^{-1}(\delta_k^*)(x)$$

$$\delta(r) * \delta(-r) = \int_x \delta(x) \delta(x - (-r)) dx = \int_x \delta(x) \delta(x + r) dx$$

$$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$$

$$\mathcal{F}^{-1}(P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k) * \mathcal{F}^{-1}(\delta_k^*)(x)$$

$$= \delta(r) * \delta(-r) = \int_x \delta(x) \delta(x - (-r)) dx = \int_x \delta(x) \delta(x + r) dx$$

$$\langle \delta(x) \delta(x+r) \rangle$$

$$= \langle \delta(x) \delta(x+r) \rangle$$

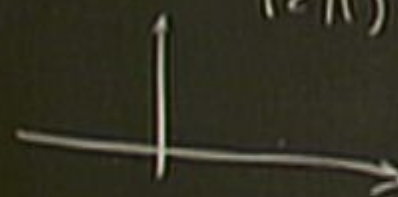
$$\langle s_{\vec{k}} s_{\vec{k}'}^* \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P(k)$$

$$= \langle \delta(x) \delta(x+r) \rangle$$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\frac{1}{V}} P(k)$$

$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}}$$

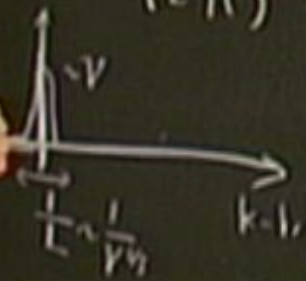
$$\langle \delta_{\vec{k}} \delta_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\substack{\approx V \\ \approx 2\pi}} P(k)$$



$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \underbrace{\int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}}}_V$$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\substack{\approx V \\ \approx 2\pi}} P_c(k)$$

$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \underbrace{\int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}}}_V$$



$$\begin{aligned}
 &= \delta(r) * \delta(-r) = \int_x \delta(x) \delta(x+(-r)) dx = \int_x \delta(x) \delta(x+r) dx \\
 &= \langle \delta(x) \delta(x+r) \rangle
 \end{aligned}$$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'} \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\approx V} P(k)$$

$$\begin{aligned}
 \delta^3(\vec{k} - \vec{k}') &= \int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}} \\
 &\approx V \delta_{\vec{k}, \vec{k}'}
 \end{aligned}$$

$$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$$

$$\mathcal{F}^{-1}(NP(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k) \ast \mathcal{F}^{-1}(\delta_k^*)(x)$$

$$= \delta(r) \ast \delta(-r) = \int_x \delta(x) \delta(x - (-r)) dx = \int_x \delta(x) \delta(x + r) dx$$

$$\langle \delta(x) \delta(x+r) \rangle$$

$$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$$

$$\mathcal{F}^{-1}(P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k) * \mathcal{F}^{-1}(\delta_k^*)(x)$$

$$= \delta(r) * \delta(-r) = \int_x \delta(x) \delta(x - (-r)) dx = \int_x \delta(x) \delta(x + r) dx$$

$$\mathcal{F}^{-1}(P(k)) = \int \frac{d^3x}{V} \delta(x) \delta(x + r)$$

$$\delta(k - k') = \int d^3x e^{i(k - k') \cdot x}$$

$$\approx \frac{V}{V} \delta_{kk'}$$

$$\mathcal{F}^{-1}(P(k))(x) = \mathcal{F}^{-1}(\delta(r)) * \mathcal{F}^{-1}(\delta^*(r))(x)$$

$$= \delta(r) * \delta(-r) = \int_x \delta(x) \delta(x - (-r)) dx = \int_x \delta(x) \delta(x+r) dx$$

$$\mathcal{F}^{-1}(P_m) = \int \frac{d^3x}{V} \delta(x) \delta(x+r)$$

$$\langle \delta^*(k) \delta(k') \rangle = (2\pi)^3 \delta^3(k-k') P(k)$$

$$\stackrel{\approx}{=} \frac{V}{(2\pi)^3}$$

$$\int d^3x e^{ik-k' \cdot x}$$

$$\frac{1}{V}$$

$$\delta_{kk'}$$

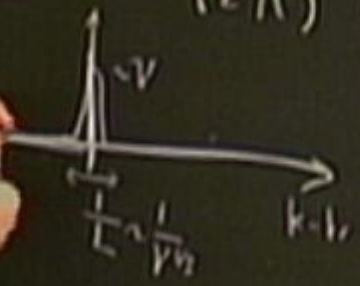


$$\langle S_{\vec{k}} S_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\approx V} P(k)$$

$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}} \quad \left| \quad \int_V d^3x S(\vec{x} - \vec{r}) = 1 \right.$$

$\xrightarrow{\vec{k} \approx \frac{1}{V} \sum_{\vec{k}'} \vec{k}'} \approx V \delta_{\vec{k}, \vec{k}'} \quad \quad \quad \vec{x} \in V$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\approx V} P(k)$$



$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}}$$

$$\approx V \delta_{\vec{k}, \vec{k}'}$$

$$\int_V d^3x \delta^3(\vec{x} - \vec{x}') = 1$$

$\vec{x}' \in V$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3}$$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\} = \langle \delta(x) \delta(x+y) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+y))$$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle p(-ikx) \exp(ik'(x+r))$$

$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1$$

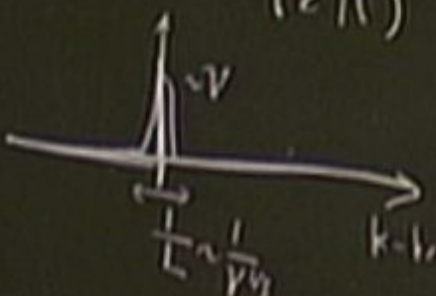
$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1$$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\frac{1}{V}} P(\vec{k})$$

$$\begin{aligned} & (\sum_i a_i) (\sum_j b_j) \\ &= \sum_{ij} a_i b_j \end{aligned}$$



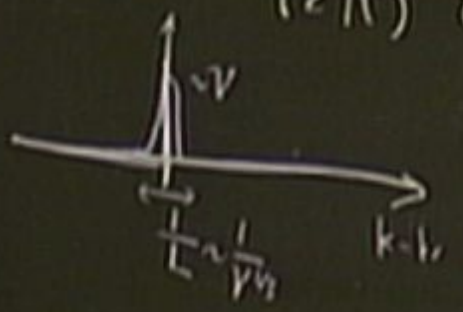
$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}}$$

$$\approx V \delta_{\vec{k}, \vec{k}'}$$

$$\int_V d^3x \delta^3(\vec{x} - \vec{x}') = 1 \quad \vec{x}' \in V$$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'}^* \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k} - \vec{k}')}_{\approx \frac{1}{V}} P(k)$$

$$\begin{aligned} & (\sum_i a_i) (\sum_j b_j) \\ &= \sum_{ij} a_i b_j \end{aligned}$$



$$(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \int d^3x e^{i(\vec{k} - \vec{k}') \cdot \vec{x}}$$

$$\approx V \delta_{\vec{k}, \vec{k}'}$$

$$\int_V d^3x \overbrace{\delta^3(\vec{x} - \vec{x}')}^{\vec{F}(\vec{x})} = \vec{F}(\vec{x})$$

$\vec{x} \in V$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1$$

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 P(k) \delta(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int d^3 k \delta(k - k') e^{i(k' - k)x} d^3 k' = 1$$

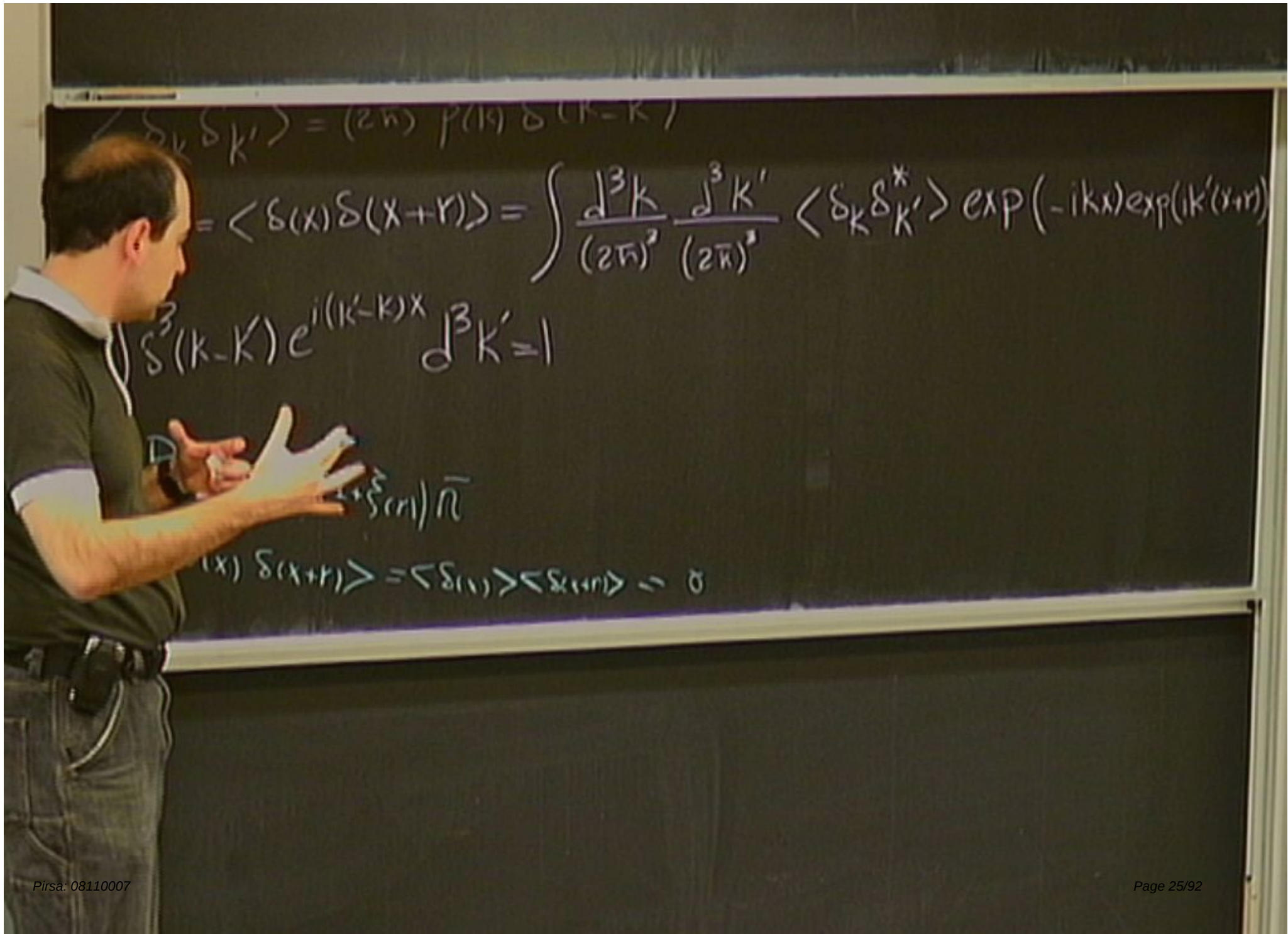
$$P(\Delta r, r) = (1 + \xi(r)) \bar{n}$$

$$\chi(t) = \langle \phi(x) \phi(x+t) \rangle = \frac{1}{(2\pi)^2} \frac{1}{(2\pi)^2} \langle \phi_K \phi_{-K} \rangle \exp(-iKx) \exp(iK(x+t))$$

$$\int d^3k \exp(-ikx) \int d^3k' = 1$$

$$(\vec{s}(r)) \cdot \vec{n}$$

$$\langle \vec{s}(r) \cdot \vec{n} \rangle = \langle \vec{s}(r) \rangle \cdot \langle \vec{n} \rangle = 0$$



$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 p(1) \delta(k - k')$$

$$= \langle \delta(x) \delta(x+y) \rangle = \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+y))$$

$$\delta^3(k - k') e^{i(k' - k)x} \int d^3 k' = 1$$

$$D(x) = \langle \delta(x) \delta(x+y) \rangle = \langle \delta(x) \rangle \langle \delta(x+y) \rangle - 0$$

$$\langle \delta(x) \delta(x+y) \rangle = \langle \delta(x) \rangle \langle \delta(x+y) \rangle - 0$$

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 P(k) \delta(k - k')$$

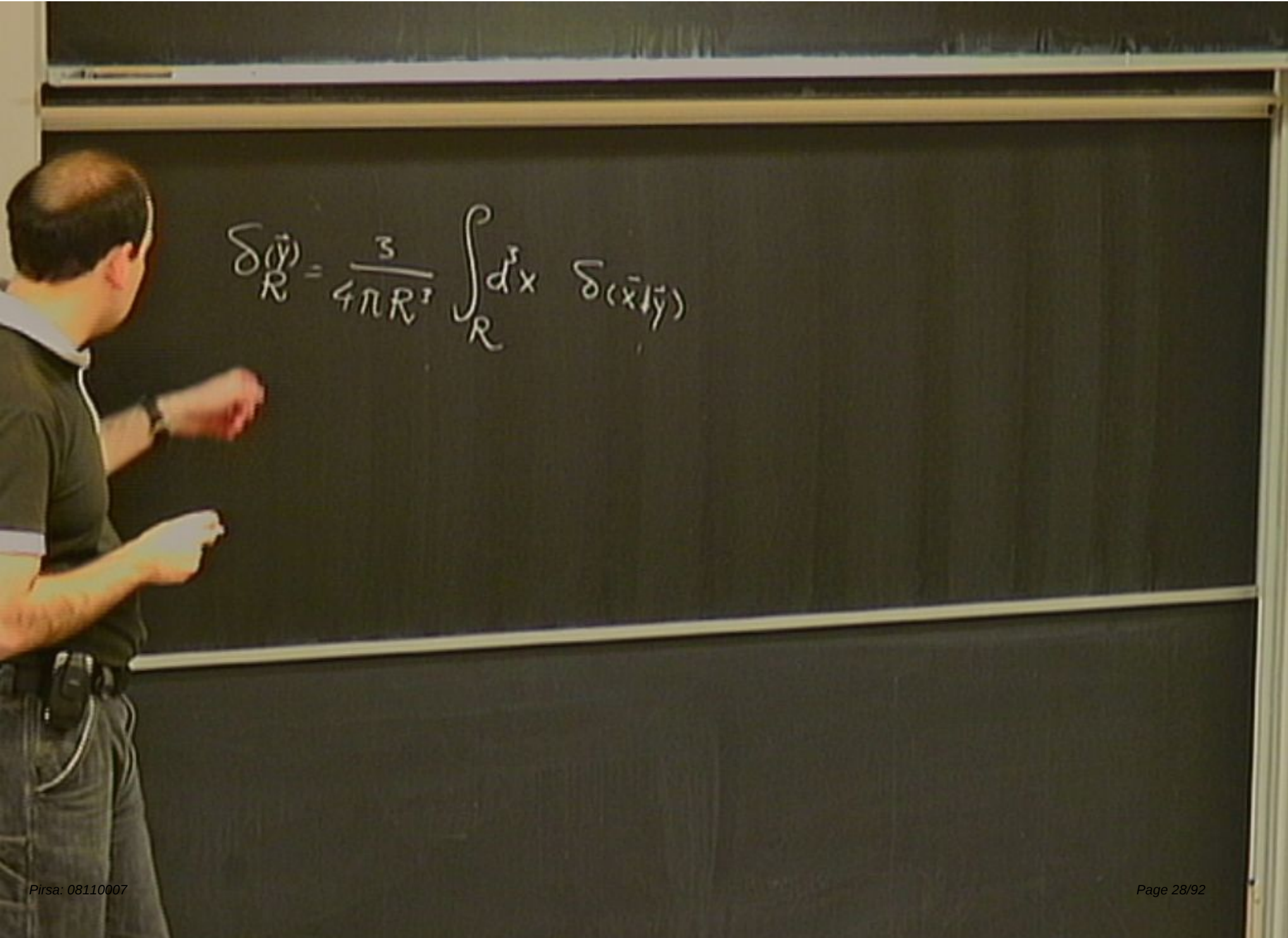
$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int d^3(k-k') e^{i(k'-k)x} d^3 k' = 1$$

$$P(\Delta r, r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \, \delta(\vec{x})$$


$$\delta_{\vec{R}}(\vec{y}) = \frac{3}{4\pi R^3} \int_{\vec{R}} d^3x \, \delta(\vec{x} - \vec{y})$$

A man in a dark shirt and jeans is writing on a chalkboard. He is using a piece of chalk to write the equation. To the left of the equation, there is a small circle containing the vector $\vec{y}$. Below the equation, there is a larger circle.
$$\delta_{\vec{R}}(\vec{y}) = \frac{3}{4\pi R^2} \int_{\vec{R}} d^3x \delta(\vec{x} - \vec{y})$$



$$\delta_{\vec{R}}(\vec{y}) = \frac{3}{4\pi R^3} \int_{\vec{R}} d^3x \delta(\vec{x} - \vec{y})$$



$$\langle \delta_{\vec{R}}^2 \rangle$$



$\vec{y}$

$$\delta_R(\vec{y}) = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x}|\vec{y})$$

$$q_R^2 = \langle \delta_R^2 \rangle$$

The chalkboard contains two hand-drawn circles. The top circle has a central dot and is labeled with $\vec{y}$. The bottom circle also has a central dot and is associated with the equation $q_R^2 = \langle \delta_R^2 \rangle$.



$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$



$$q_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \int_R d^3x' \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$$



$$\bar{y} \quad \delta_R^{(1)} = \frac{3}{4\pi R^2} \int_R d^3x \delta(\vec{x} - \bar{y})$$



$$q_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^2} \right)^2 \int_R d^3x d^3x' \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$$



$$\bar{y} \quad \delta_R^{(1)} = \frac{3}{4\pi R^2} \int_R d^3x \delta(\vec{x})$$



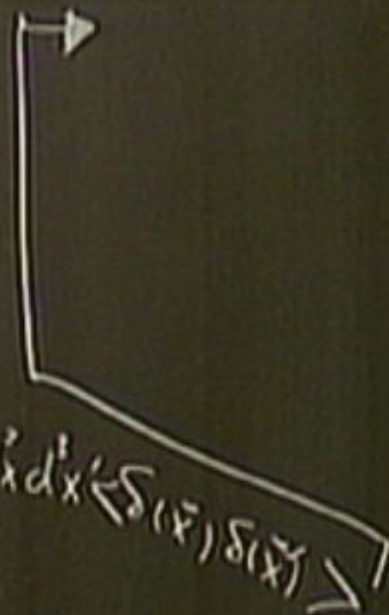
$$q_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^2} \right)^2 \int_R d^3x d^3x' \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$$



$$\delta_R(\vec{y}) = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x} - \vec{y}_c)$$

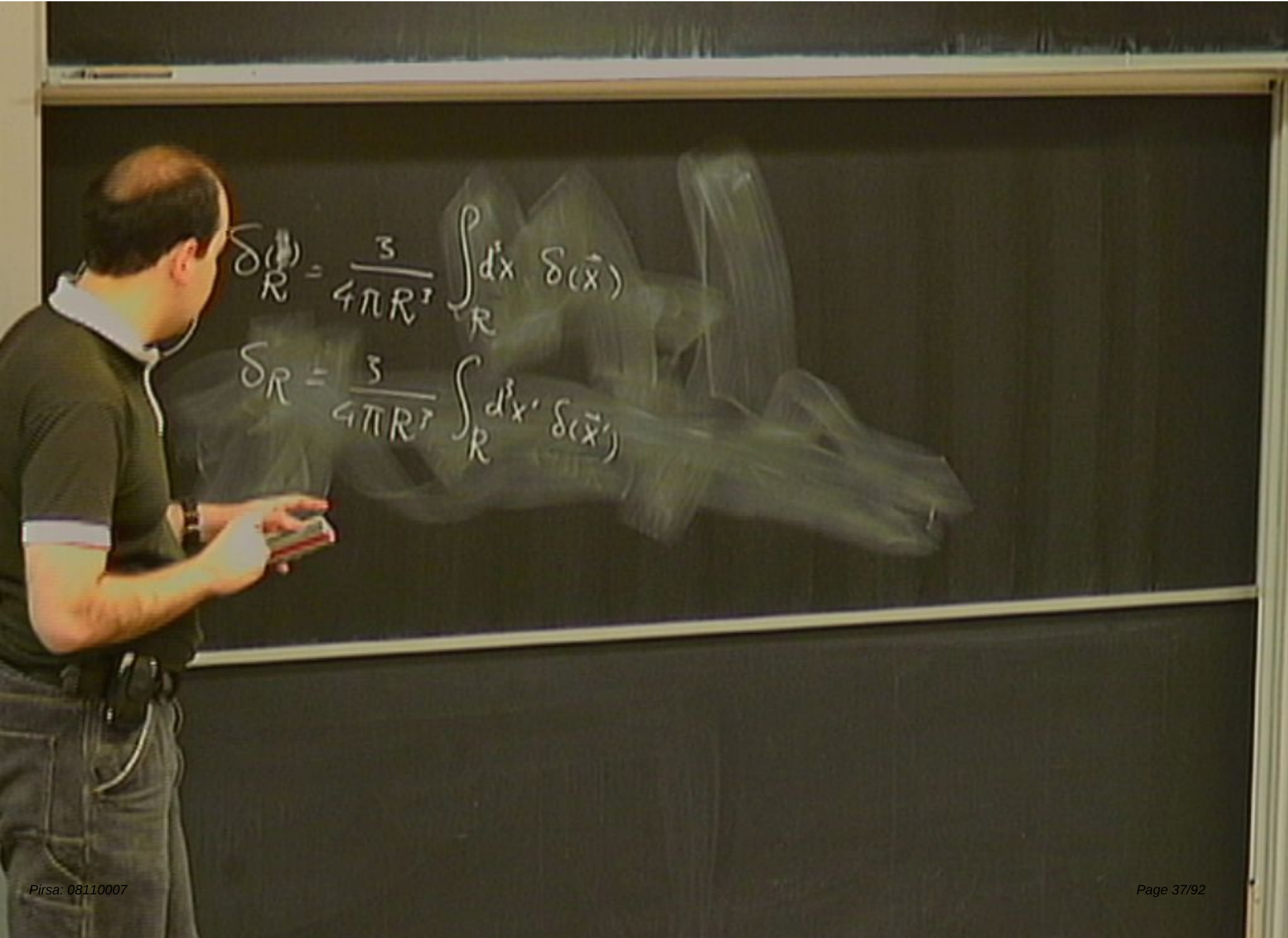


$$q_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$$



$\vec{y}_*$

$$\delta_R^{(1)} = \frac{3}{4\pi R^3} \int d^3x \delta(\vec{x})$$


$$\delta_R^{(1)} = \frac{3}{4\pi R^2} \int_R d^3x \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^2} \int_R d^3x' \delta(\vec{x}')$$

$\vec{y}$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x' \delta(\vec{x}')$$

$\vec{y}_i$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x' \delta(\vec{x}')$$

$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \int_R d^3x'$$

(y.)

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x' \delta(\vec{x}')$$

$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \langle \delta(\vec{x}') \delta(\vec{x}) \rangle$$

$\bar{y}_.$

$$\bar{\delta}_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$

$$\bar{\delta}_R = \frac{3}{4\pi R^3} \int_R d^3x' \delta(\vec{x}')$$

$$\langle \bar{\delta}_R^2 \rangle = \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \frac{\langle \delta(\vec{x}') \delta(\vec{x}') \rangle}{\xi(|\vec{x} - \vec{x}'|)}$$

(y.)

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x' \delta(\vec{x}')$$

$$\begin{aligned} \langle \delta_R^2 \rangle &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \frac{\langle \delta(\vec{x}') \delta(\vec{x}) \rangle}{\xi(|\vec{x} - \vec{x}'|)} \\ &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{u} \cdot \vec{x}'} P(k) \end{aligned}$$

ii.

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x' \delta(\vec{x}')$$

$$\begin{aligned} \langle \delta_R^2 \rangle &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \frac{\langle \delta(\vec{x}) \delta(\vec{x}') \rangle}{\xi(|\vec{x} - \vec{x}'|)} \\ &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x d^3x' \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} P(k) \\ &= \left(\frac{3}{4\pi R^3} \right)^2 \int \frac{d^3k}{(2\pi)^3} P(k) \int_R d^3x \end{aligned}$$

$$\begin{aligned}
 \textcircled{i.} \quad \delta_R^{(1)} &= \frac{3}{4\pi R^3} \int_R d^3x \, \delta(\vec{x}) \\
 \delta_R &= \frac{3}{4\pi R^3} \int_R d^3x' \, \delta(\vec{x}')
 \end{aligned}
 \left| \right.
 \begin{aligned}
 \langle \delta_R^2 \rangle &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \, d^3x' \, \frac{\langle \delta(\vec{x}') \delta(\vec{x}') \rangle}{\xi(|\vec{x} - \vec{x}'|)} \\
 &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \, d^3x' \, \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{u} \cdot \vec{x}'} P(k) \\
 &= \left(\frac{3}{4\pi R^3} \right)^2 \int \frac{d^3k}{(2\pi)^3} P(k) \left(\int_R d^3x \, e^{i\vec{k} \cdot \vec{x}} \right) \left(\int_R d^3x' \, e^{-i\vec{k} \cdot \vec{x}'} \right)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{i.} \quad \delta_R^{(1)} &= \frac{3}{4\pi R^3} \int_R d^3x \, \delta(\vec{x}) \\
 \delta_R &= \frac{3}{4\pi R^3} \int_R d^3x' \, \delta(\vec{x}')
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 \langle \delta_R^2 \rangle &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \, d^3x' \, \frac{\langle \delta(\vec{x}') \delta(\vec{x}') \rangle}{\xi(|\vec{x} - \vec{x}'|)} \\
 &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \, d^3x' \, \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} P(k) \\
 &= \int \frac{d^3k}{(2\pi)^3} P(k) \underbrace{\left(\int_R \frac{d^3x}{V} e^{i\vec{k} \cdot \vec{x}} \right)}_{W(kR)} \underbrace{\left(\int_R \frac{d^3x'}{V} e^{-i\vec{k} \cdot \vec{x}'} \right)}_{W(kR)}
 \end{aligned}
 \right.$$

$$\begin{aligned}
 \textcircled{1.} \quad \delta_R &= \frac{3}{4\pi R^3} \int_R d^3x \, \delta(\vec{x}) \\
 \delta_R &= \frac{3}{4\pi R^3} \int_R d^3x' \, \delta(\vec{x}')
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 \langle \delta_R^2 \rangle &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \, d^3x' \, \frac{\langle \delta(\vec{x}') \delta(\vec{x}') \rangle}{\xi(|\vec{x} - \vec{x}'|)} \\
 &= \left(\frac{3}{4\pi R^3} \right)^2 \int_R d^3x \, d^3x' \, \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{u} \cdot \vec{x}'} P(k) \\
 &= \int \frac{d^3k}{(2\pi)^3} P(k) \underbrace{\left(\int \frac{d^3x}{R^3} e^{i\vec{k} \cdot \vec{x}} \right)}_{W(kR)} \underbrace{\left(\int \frac{d^3x'}{R^3} e^{-i\vec{k} \cdot \vec{x}'} \right)}_{W^*(kR)}
 \end{aligned}
 \right.$$

$$\begin{aligned}\delta_{\mathcal{R}}(\vec{y}) &= \frac{1}{V} \int_{\mathcal{R}} d^3x \, \delta(\vec{x} + \vec{y}) \\ &= \frac{1}{V} \int d^3x \, S(\end{aligned}$$

$$\begin{aligned}\delta_{\mathcal{R}}(\vec{y}) &= \frac{1}{V} \int_{\mathcal{R}} d^3x \, \delta(\vec{x} + \vec{y}) \\ &= \frac{1}{V} \int d^3x \, \underline{S(\vec{x})} \, \delta(\vec{x} + \vec{y})\end{aligned}$$



$$\begin{aligned}\delta_{\mathcal{R}}(\vec{y}) &= \frac{1}{V} \int_{\mathcal{R}} d^3x \, \delta(\vec{x} + \vec{y}) \\ &= \frac{1}{V} \int d^3x \, \underline{S(\vec{x})} \, \delta(\vec{x} + \vec{y})\end{aligned}$$

$$\delta_{\mathcal{R}}(k_y) = W(k_y) \delta(k_y)$$

$$\delta_R(\vec{y}) = \frac{1}{V} \int_R d\vec{x} \delta(\vec{x} + \vec{y})$$

$$= \frac{1}{V} \int d^3x \, \underline{S(\vec{x})} \, \underline{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y)$$

$$\delta_R(\vec{y}) = \frac{1}{V} \int_R d\vec{x} \delta(\vec{x} + \vec{y})$$

$$= \frac{1}{V} \int d^3x \, \underline{S(\vec{x})} \, \underline{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(m)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(m)^3} |W(k, R)|^2$$

$$\delta_R(\vec{y}) = \frac{1}{V} \int_R d^3x \delta(\vec{x} + \vec{y})$$

$$= \frac{1}{V} \int d^3x \underline{S(\vec{x})} \underline{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k, R)|^2 P(k_y)$$

$$\phi_R(\vec{r}) = \frac{1}{V} \phi_R$$

$$= \frac{1}{V} \int d^3x \, \underline{S(\vec{x})} \, \underline{\delta(\vec{x} + \vec{y})}$$

$$\phi_R(k, y) = W(k, R) \, \delta(k, y)$$

$$W(k, R) = \frac{1}{V} \int_R d^3x \, e^{ik \cdot \vec{x}}$$

$$\langle \phi_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \phi_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y, R)|^2 P(k_y)$$



$$= \frac{1}{V} \int d^3x \, \underline{S(\vec{x})} \, \underline{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$W(k, R) = \frac{1}{V} \int_R d^3x \, e^{ik \cdot \vec{x}}$$

$$= \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)]$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y, R)|^2 P(k_y)$$

$$\delta_R(\vec{y}) = \frac{1}{V} \int_R d\vec{x} \delta(\vec{x} + \vec{y})$$

$$= \frac{1}{V} \int d^3x \underbrace{S(\vec{x})} \underbrace{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y, R)|^2 P(k_y)$$

$$W(k, R) = \frac{1}{V} \int_R d\vec{x} e^{i\vec{k} \cdot \vec{x}} = \frac{2\pi}{V} \int_0^R dr r e^{ikr} = \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)]$$

$$\delta_R(\vec{y}) = \frac{1}{V} \int_R d^3x \delta(\vec{x} + \vec{y})$$

$$= \frac{1}{V} \int d^3x \underbrace{S(\vec{x})} \underbrace{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k, R)|^2 P(k_y)$$

$$W(k, R) = \frac{1}{V} \int_R d^3x e^{i\vec{k} \cdot \vec{x}} = \frac{2\pi}{V} \int_0^R dr r^2 e^{ikr} = \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)]$$

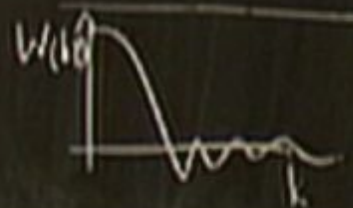
$$\delta_R(\vec{y}) = \frac{1}{V} \int_R d\vec{x} \delta(\vec{x} + \vec{y})$$

$$= \frac{1}{V} \int d^3x \underbrace{S(\vec{x})} \underbrace{\delta(\vec{x} + \vec{y})}$$

$$\delta_R(k_y) = W(k, R) \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y, R)|^2 P(k_y)$$

$$W(k, R) = \frac{1}{V} \int_R d\vec{x} e^{i\vec{k} \cdot \vec{x}} = \frac{2\pi}{V} \int_0^R dr r e^{ikr} = \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)]$$



$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1$$

$$P(\Delta r, r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

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$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1 \quad \Bigg| \quad \xi(r) = A \delta^3(\vec{r})$$

$$P(\Delta \vec{r}; r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 p(k) \delta^3(k - k')$$

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$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1 \quad \Bigg| \quad \xi(r) = A \delta^3(\vec{r})$$

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$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'x)$$

$$\int \delta^3(k - k') e^{i(k' - k)x} d^3k' = 1 \quad \left| \quad \xi(r) = A \delta^3(\vec{r}) \sim \frac{1}{r^3} \right.$$

$$P(\Delta \vec{r}; r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 \rho(k) \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int \delta^3(k - k') e^{i(k' - k)x} d^3 k' = 1 \quad \left| \quad \xi(r) = A \delta^3(\vec{r}) \sim \frac{1}{n_g} \delta^3(r) \right.$$

$$P(\Delta \vec{r}; r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

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$$\xi(r) = \langle \delta(x) \delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta_{k'}^* \rangle \exp(-ikx) \exp(ik'(x+r))$$

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$$\xi(r) = A \delta^3(\vec{r}) \sim \frac{1}{n_g} \delta^3(r)$$

$$\delta(\vec{r}) = \sum_i \delta_p^3(x - x_i)$$

$$P(\Delta \vec{r}; r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\int \delta^3(\mathbf{k}-\mathbf{k}') e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} d^3\mathbf{k}' = 1$$

$$P(\Delta\mathbf{r}, r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(\mathbf{x}) \delta(\mathbf{x}+\mathbf{r}) \rangle = \langle \delta(\mathbf{x}) \rangle \langle \delta(\mathbf{x}+\mathbf{r}) \rangle = 0$$

$$\xi(r) = A \delta^3(\mathbf{r}) \sim \frac{1}{n_g} \delta^3(\mathbf{r})$$

$$\delta(\mathbf{x}) = \sum_i \delta_p(\mathbf{x}-\mathbf{x}_i)$$

R^V

$W(R)$

$$\int \delta^3(k-k') e^{i(k-k')x} d^3k' = 1$$

$$P(\Delta \vec{r}, r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\xi(r) = A \delta^3(\vec{r}) \sim \frac{1}{n_g} \delta^3(r)$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}} \sum_i \delta_p^3(x - x_i) - 1$$

 R^V
 $W(kR)$
 R^V
 $W(kR)$

$$\int d^3 k' e^{i(k-k')x} d^3 k' = 1$$

$$P(\Delta \vec{r}, r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(x) \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\xi(r) = A \delta^3(\vec{r}) \sim \frac{1}{n_g} \delta^3(r)$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}} \sum_i \delta_D(x - x_i) - 1 = \frac{\rho}{\bar{\rho}} - 1$$

$$\int \delta^3(\mathbf{k}-\mathbf{k}') e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} d^3\mathbf{k}' = 1$$

$$P(\Delta\mathbf{r}; r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(\mathbf{x}) \delta(\mathbf{x}+\mathbf{r}) \rangle = \langle \delta(\mathbf{x}) \rangle \langle \delta(\mathbf{x}+\mathbf{r}) \rangle = 0$$

$$\xi(r) = A \delta^3(\mathbf{r}) \sim \frac{1}{n_g} \delta^3(\mathbf{r})$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}} \sum_i \delta_D(\mathbf{x} - \mathbf{r}_i) - 1 = \frac{\rho}{\bar{\rho}} - 1$$

$$\langle \delta(\vec{x}) \delta(\vec{x}+\vec{r}) \rangle =$$

$W(R)$

$$\int \delta^3(\mathbf{k}-\mathbf{k}') e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} d^3\mathbf{k}' = 1$$

$$P(\Delta\vec{r}, r) = (1 + \xi(r)) \bar{n}$$

$$\langle \delta(\mathbf{x}) \delta(\mathbf{x}+\mathbf{r}) \rangle = \langle \delta(\mathbf{x}) \rangle \langle \delta(\mathbf{x}+\mathbf{r}) \rangle = 0$$

$$\xi(r) = A \delta^3(\vec{r}) \sim \frac{1}{n_g} \delta^3(\vec{r})$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}} \sum_i \delta_D^3(\mathbf{x}-\mathbf{x}_i) - 1 = \frac{\rho}{\bar{\rho}} - 1$$

$$\langle \delta(\vec{x}) \delta(\vec{x}+\vec{r}) \rangle = \left(\frac{m}{\bar{\rho}}\right)^2 \sum_i \left[\delta_D^3(\mathbf{x}-\mathbf{x}_i) \right]^2$$

$$\frac{K^2}{W(kR)} \quad \frac{K^2}{W(kR)}$$

$$\langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle = \left(\frac{m}{\rho} \right)^2 \sum_i \delta_D^3(\vec{x} - \vec{x}_i) \delta_D^3(\vec{x} + \vec{r} - \vec{x}_i)$$

$$\begin{aligned} \langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle &= \left(\frac{m}{\rho} \right)^2 \sum_i \delta_D^3(\vec{x} - \vec{x}_i) \delta_D^3(\vec{x} + \vec{r} - \vec{x}_i) \\ &= \left(\frac{m}{\rho} \right)^2 \frac{1}{V} \int d^3x_i \end{aligned}$$

$$\langle \delta(\bar{x}) \delta(\bar{x} + \bar{r}) \rangle = \left(\frac{m}{\rho} \right)^2 \sum_i \delta_D^3(\mathbf{x} - \mathbf{x}_i) \delta_D^3(\mathbf{x} + \bar{r} - \mathbf{x}_i)$$

$$= \left(\frac{m}{\rho} \right)^2 \frac{1}{V} \int d^3x_i$$

$$= \frac{1}{n_g} \frac{1}{V}$$

$$\langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle = \left(\frac{m}{\rho} \right)^2 \sum_i \delta_D^3(\vec{x} - \vec{x}_i) \delta_D^3(\vec{x} + \vec{r} - \vec{x}_i)$$

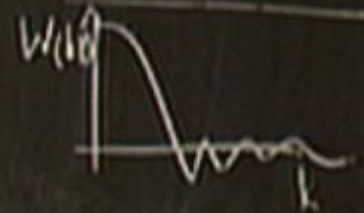
$$= \left(\frac{m}{\rho} \right)^2 \left(\frac{1}{V} \int d^3x_i \right) "$$

$$= \frac{1}{n_g} \cdot \frac{1}{V} \sum_i \delta_D^3(\vec{r})$$

$$\begin{aligned}
 \langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle &= \left(\frac{m}{\rho} \right)^2 \sum_i \delta_D^3(\vec{x} - \vec{x}_i) \delta_D^3(\vec{x} + \vec{r} - \vec{x}_i) \\
 &= \left(\frac{m}{\rho} \right)^2 \left(\frac{1}{V} \int d^3x_i \right) \quad " \\
 &= \frac{1}{n_g^2} \cdot \frac{1}{V} \left(\sum_i \delta_D^3(\vec{r}) \right) = \frac{1}{n_g} \delta_D^3(\vec{r})
 \end{aligned}$$

$$\delta_R(k_y) = W(k_y R) \delta(k_y) = \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)]$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3 k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3 k_y}{(2\pi)^3} |W(k_y R)|^2 P(k_y)$$

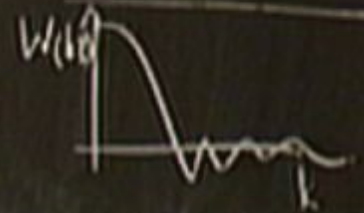


$$= \left(\frac{m}{\rho}\right)^2 \left(\frac{1}{V} \int d^3 x_i\right) "$$

$$= \frac{1}{n_g} \cdot \frac{1}{V} \left(\sum_i \delta^3(\vec{r}_i) \right)$$

$$\delta_R(k_y) = W(k_y, R) \delta(k_y) \quad \left(= \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)] \right)$$

$$\langle \delta_R^2(k_y) \rangle = \int \frac{d^3 k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3 k_y}{(2\pi)^3} |W(k_y, R)|^2 \underbrace{P(k_y)}_{\frac{1}{n_g}}$$

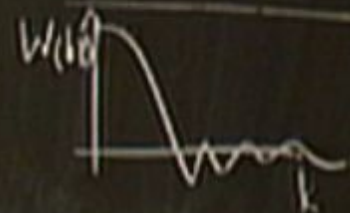


$$= \left(\frac{m}{\rho}\right)^2 \left(\frac{1}{V} \int d^3 x_i\right) = \frac{1}{n_g^2} \cdot \frac{1}{V} \left(\sum_i \delta_D^3(\vec{r}_i) \right) = \frac{1}{n_g} \delta_D^3(\vec{r})$$

$$\delta_R(k, r) = W(k, R) \delta(k, r)$$

$$= \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)]$$

$$\frac{1}{\sqrt{V}} \sim \langle \delta_R^2 \rangle = \int \frac{d^3k}{(2\pi)^3} \delta_R^2(k, r) \int \frac{d^3k}{(2\pi)^3} |W(k, R)|^2 \frac{P(k, r)}{\frac{1}{n_g}}$$

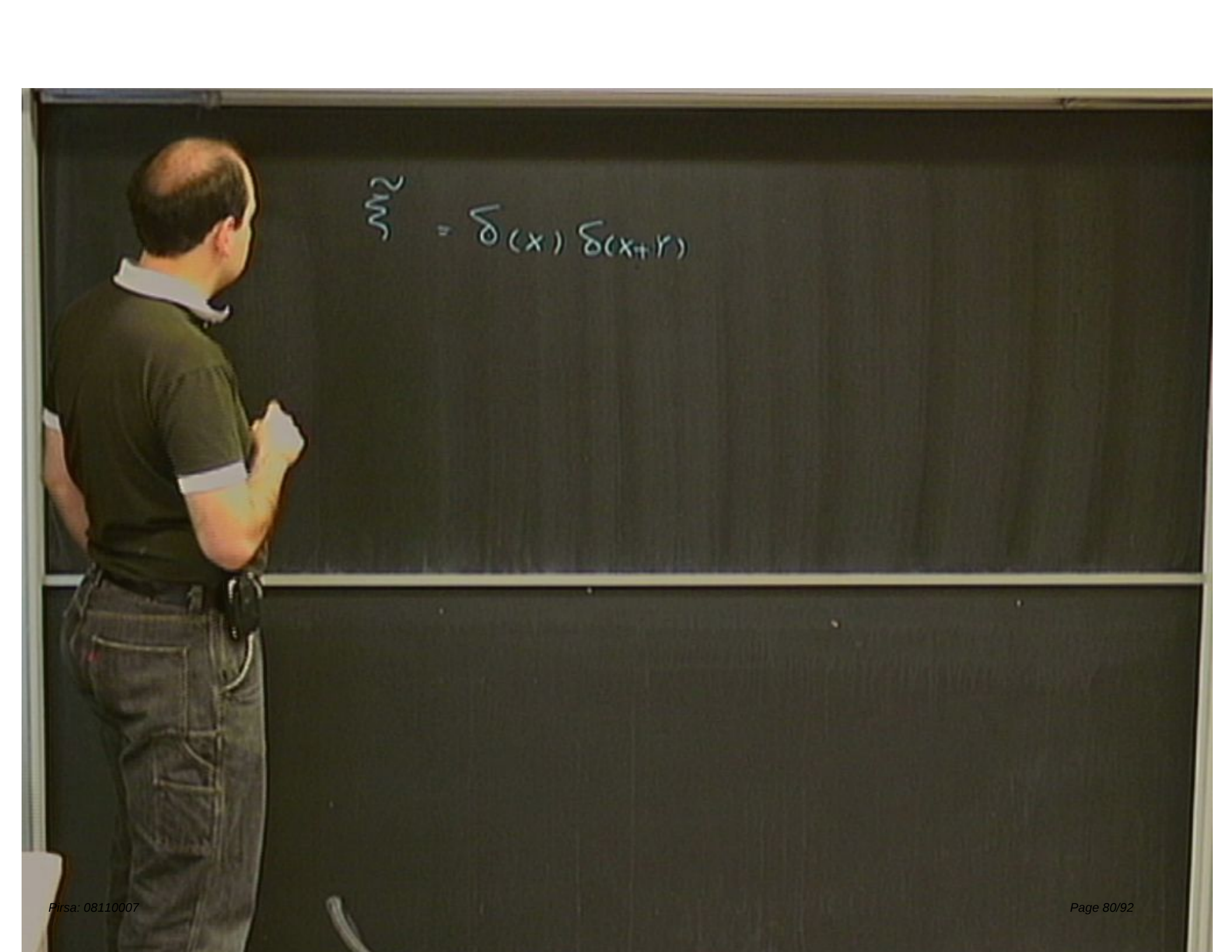


$$\delta(\vec{r}) = \frac{1}{n_g} \delta_D(\vec{r})$$

$$\langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(\vec{x} - \vec{x}_i) \delta_D^3(\vec{x} + \vec{r} - \vec{x}_i)$$

$$= \left(\frac{m}{\rho}\right)^2 \left(\frac{1}{V} \int d^3x\right) "$$

$$= \frac{1}{n_g^2} \cdot \frac{1}{V} \left(\sum_i \delta_D^3(\vec{r}) \right) = \frac{1}{n_g} \delta_D^3(\vec{r})$$


$$\zeta = \delta(x) \delta(x+r)$$


$$\tilde{m} = \delta(x) \delta(x+r)$$
$$\langle \tilde{m}^2 \rangle \neq 0$$

$$\tilde{u}_2 = \delta(x) \delta(x+r)$$

$$\langle \tilde{u}_2^2 \rangle \neq \langle \delta(x) \delta(x) \delta(x+r) \delta(x+r) \rangle$$

$$= \langle \delta^2(x) \rangle \langle \delta^2(x+r) \rangle$$

$$\tilde{w} = \delta(x) \delta(x+r)$$

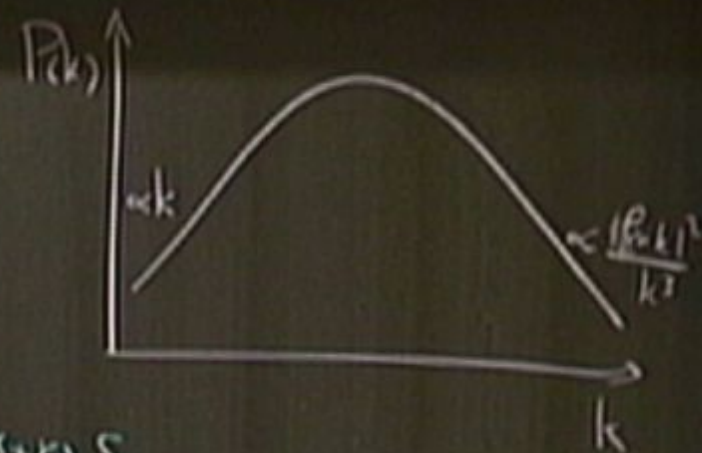
$$\begin{aligned} \langle \tilde{w}^2 \rangle &\neq \langle \delta(x) \delta(x) \delta(x+r) \delta(x+r) \rangle \\ &= \langle \delta^2(x) \rangle \langle \delta^2(x+r) \rangle \end{aligned}$$

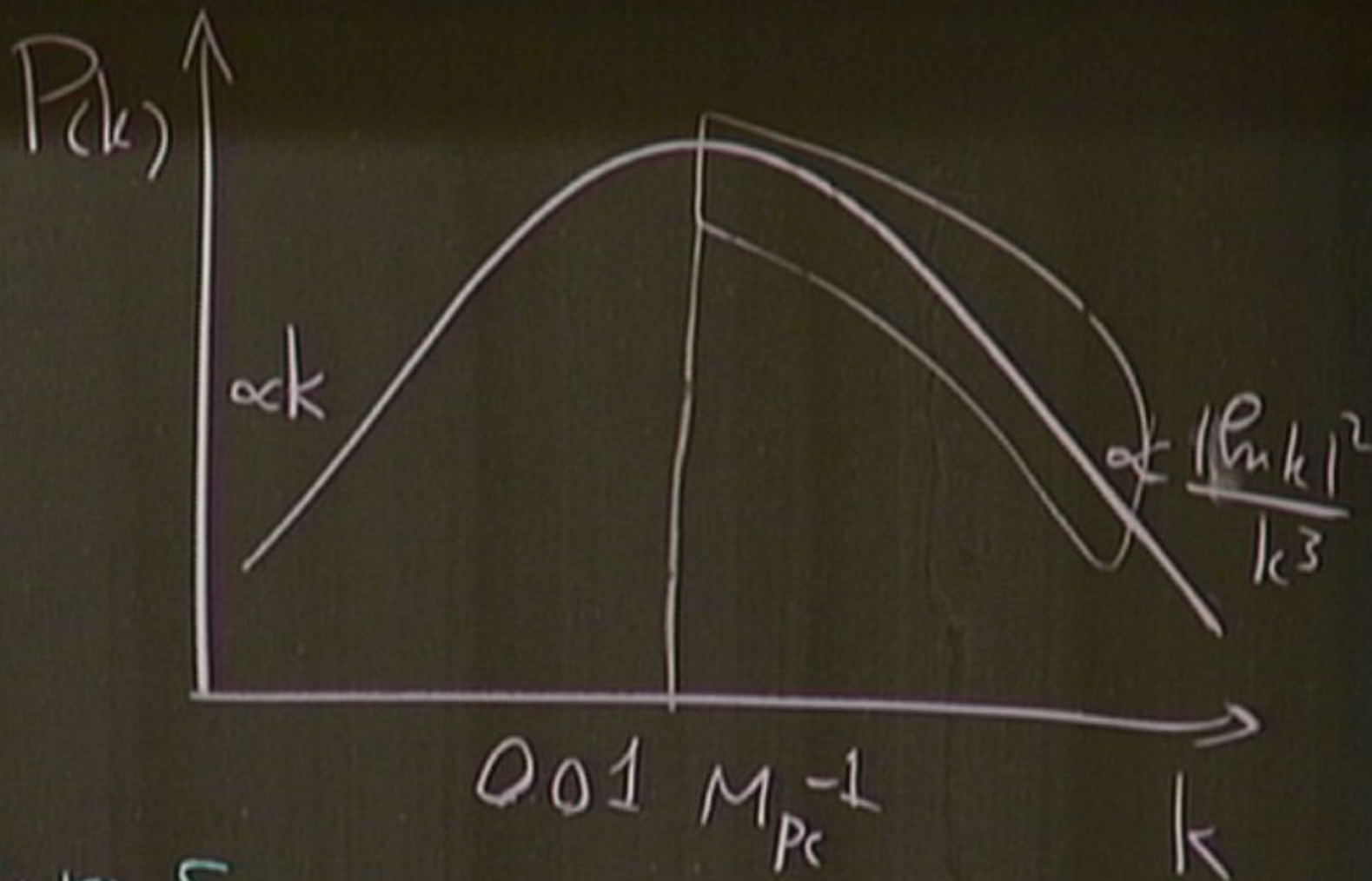
$$\tilde{u} = \delta(x-r)$$

$$\langle \tilde{u}^2 \rangle$$

$$\delta(x) \delta(x) \delta(x+r) \delta(x+r)$$

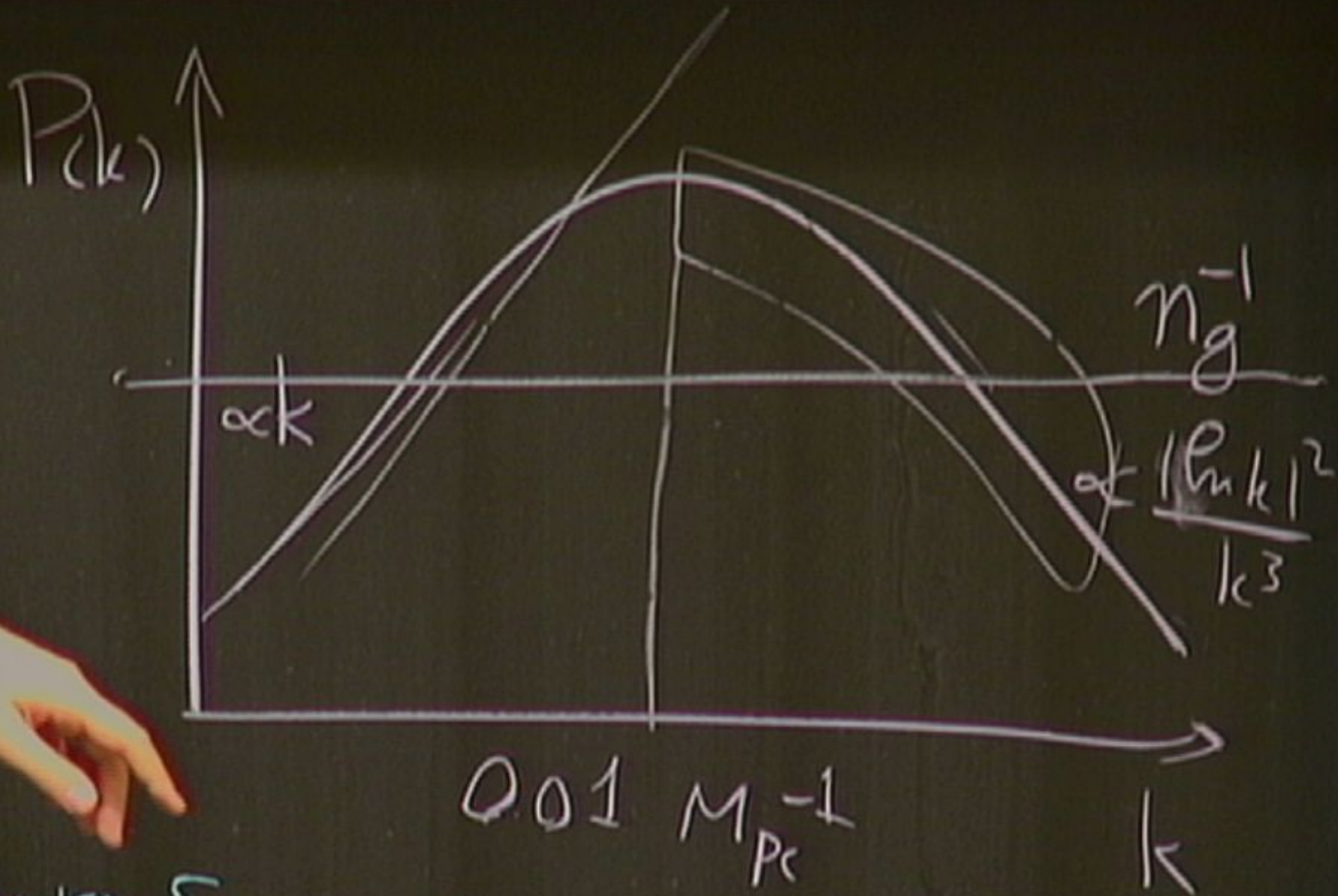
$$\delta(x) > \delta(x+r)$$





$$) \delta(x+r) \delta(x+r) >$$

$$\delta^2(x+r)$$



$$\delta(x+r) \delta(x+r) >$$

$$A = 1.677 \times 10^6$$

$$\int_0^\infty q^2 \left(\frac{3 \sin(kR) - kR \cos(kR)}{(kR)^3} \right)^2 p \, dk$$

$$A = 1.677 \times 10^6$$

$$\int_0^{\infty} q^2 \left(\frac{3 \sin(kR) - kR \cos(kR)}{(kR)^3} \right)^2 p dq$$

$$A = 1.677 \times 10^6$$

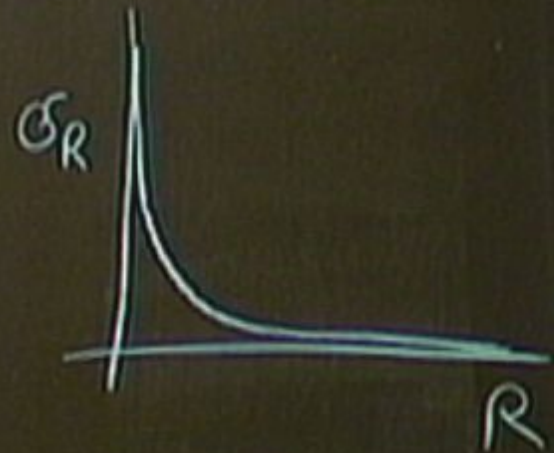
$$\int_0^\infty q^2 \left(\frac{3 \sin(kR) - kR \cos(kR)}{(kR)^3} \right)^2 p dq$$

$$K = q \hbar^2$$

$$A = 1677 \times 10^6$$

$$\int_0^\infty q^2 \left(3 \frac{\sin(kR) - kR \cos(kR)}{(kR)^3} \right)^2 p dq$$

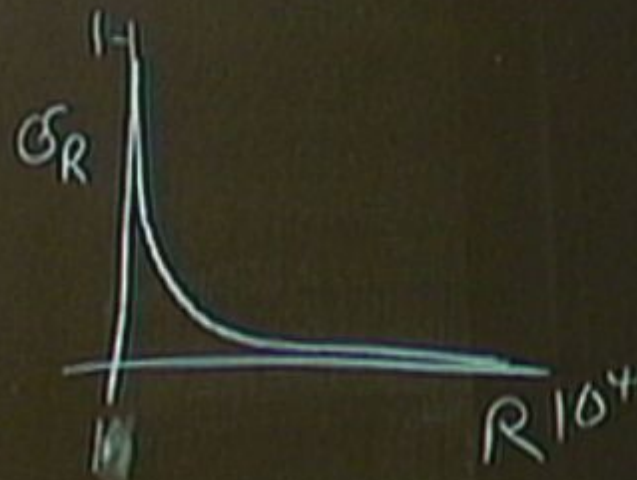
$$K = q \Omega \hbar^2$$



$$A = 1.677 \times 10^6$$

$$\int_0^\infty q^2 \left(3 \frac{\sin(kR) - kR \cos(kR)}{(kR)^3} \right)^2 p dq$$

$$K = q \Omega \hbar^2$$



$$A = 1677 \times 10^6$$

$$\int_0^\infty q^2 \left(3 \frac{\sin(kR) - kR \cos(kR)}{(kR)^3} \right)^2 p dq$$

$$K = q R h^2$$

