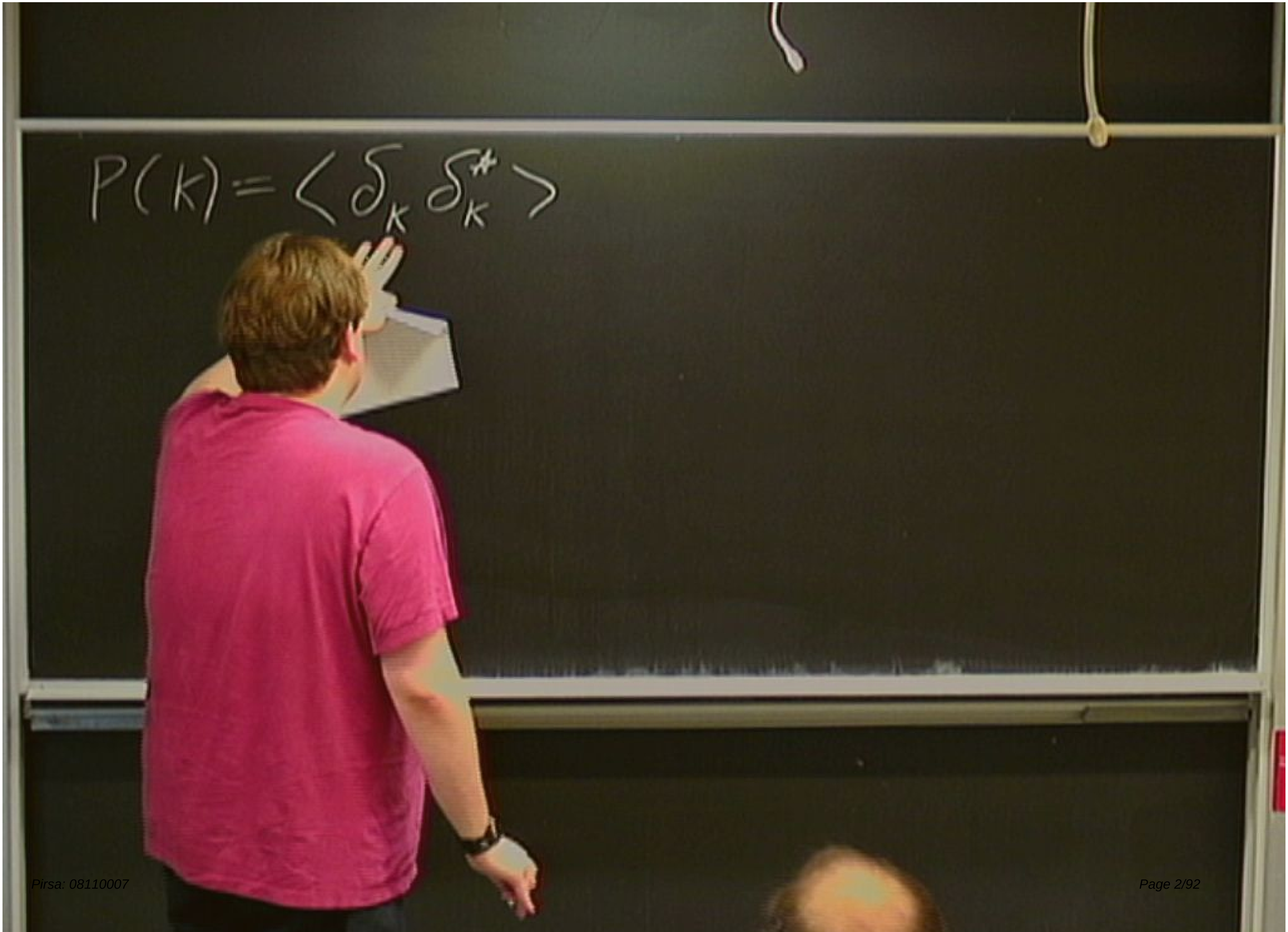
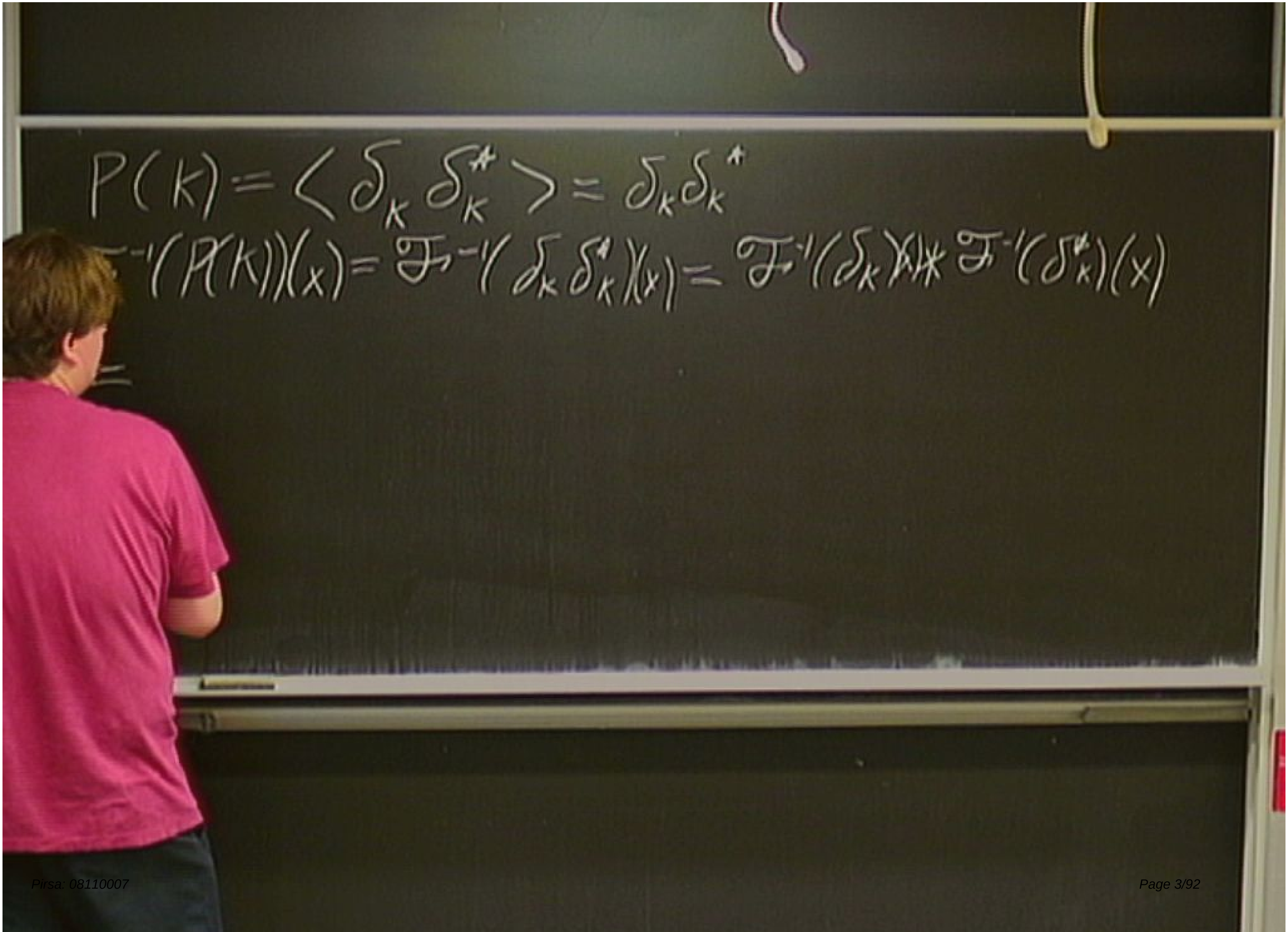
Title: Astrophysics and Cosmology through Problems - 13A

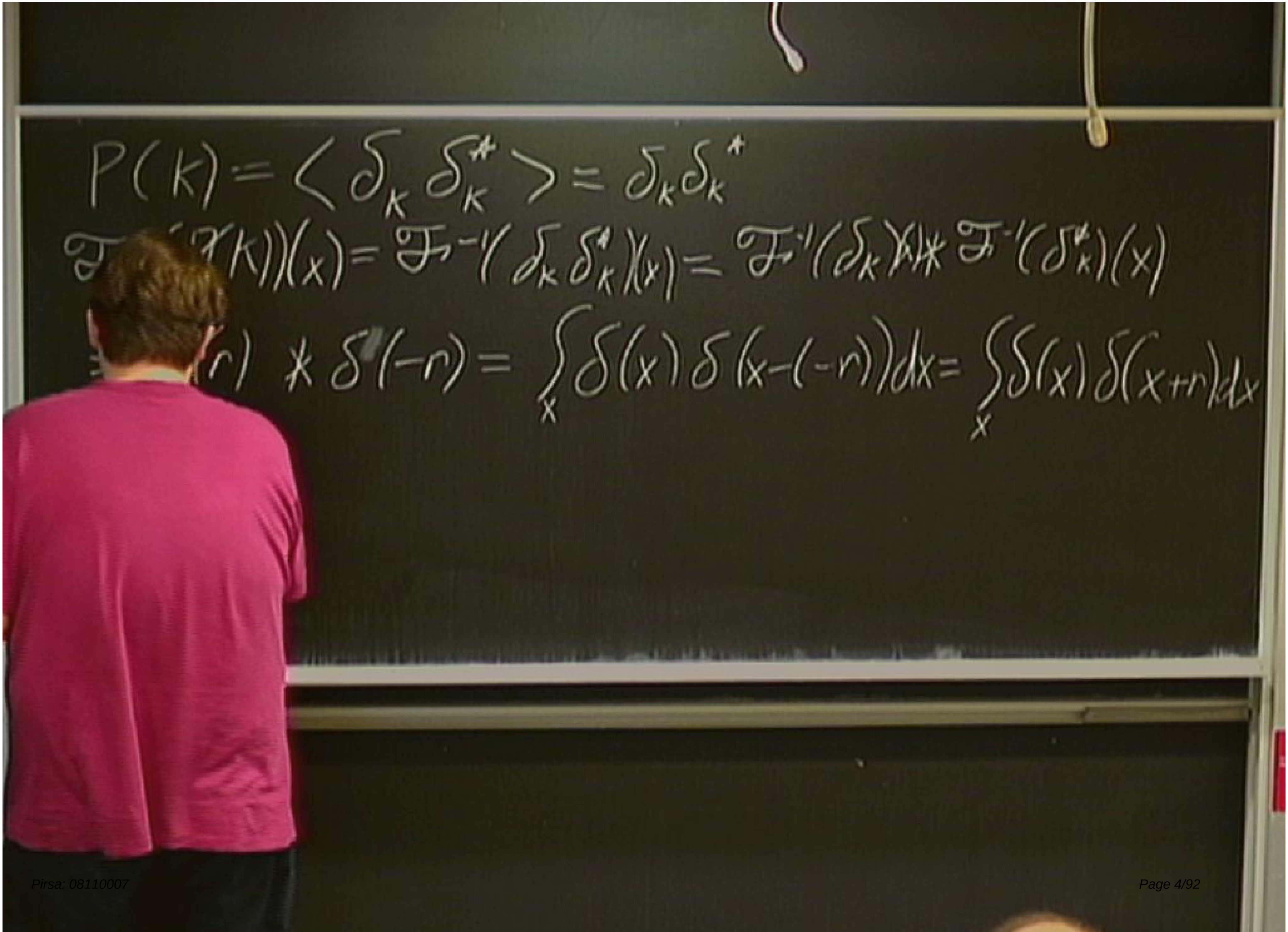
Date: Nov 27, 2008 10:00 AM

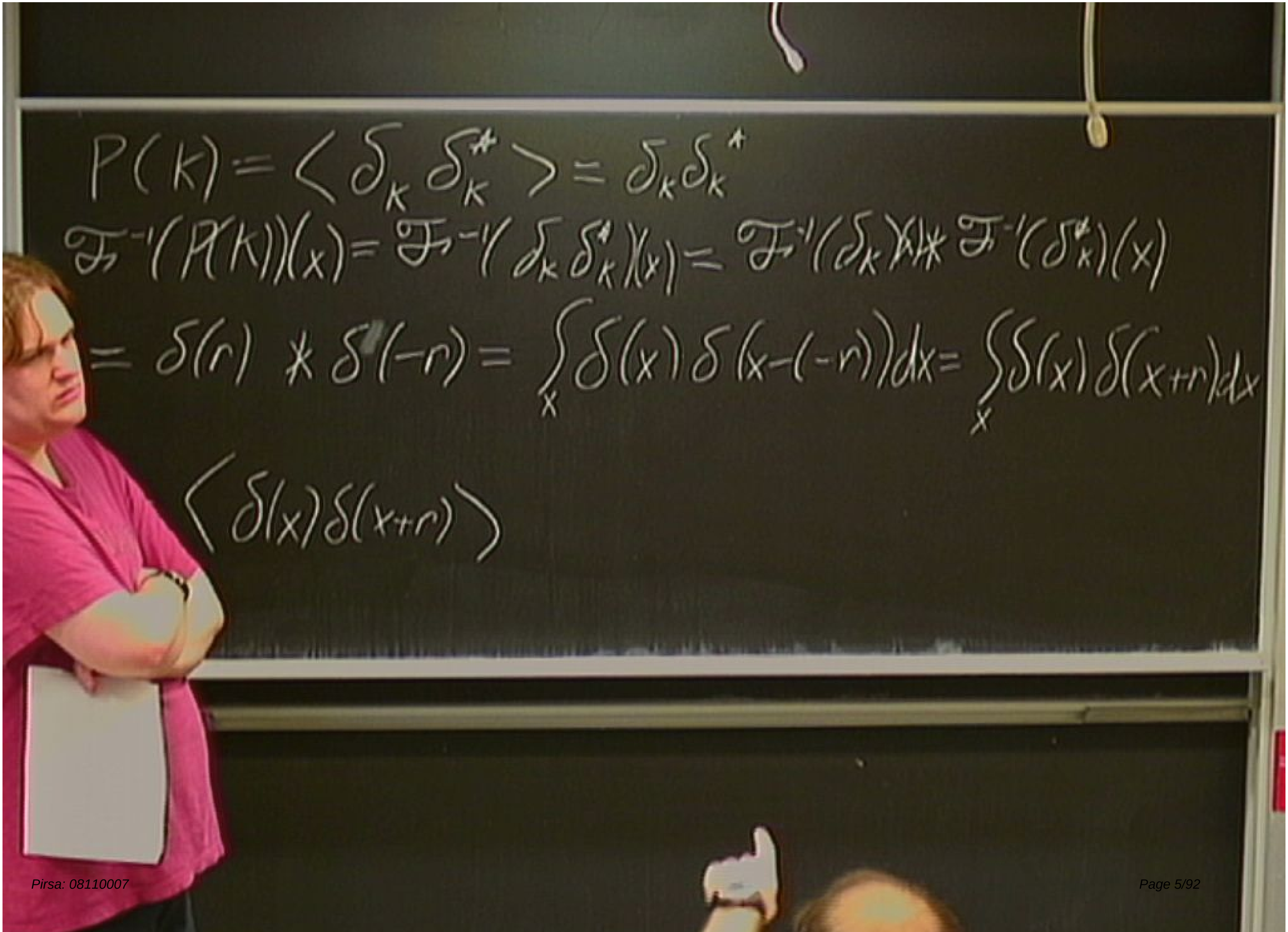
URL: http://pirsa.org/08110007

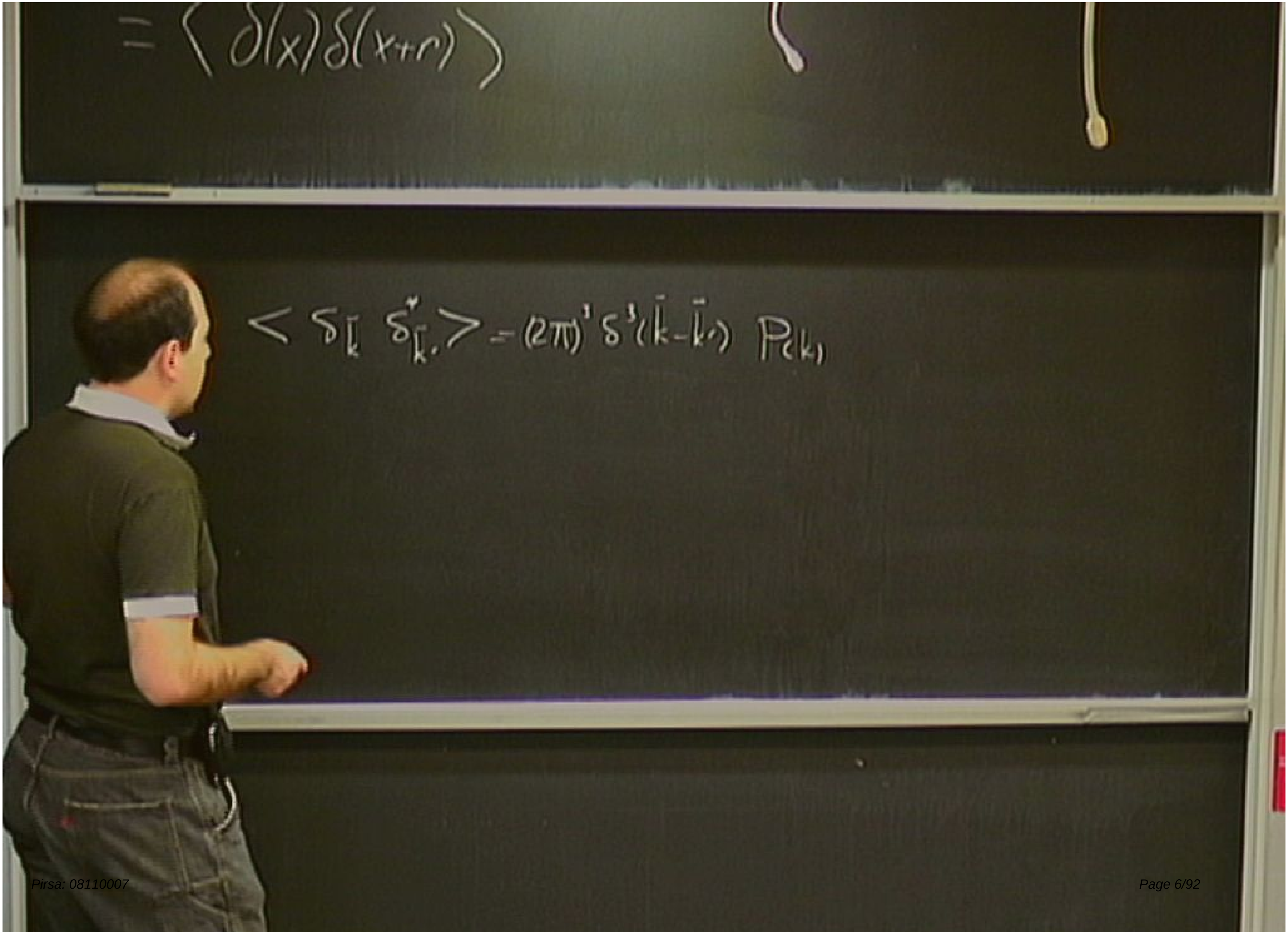
Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

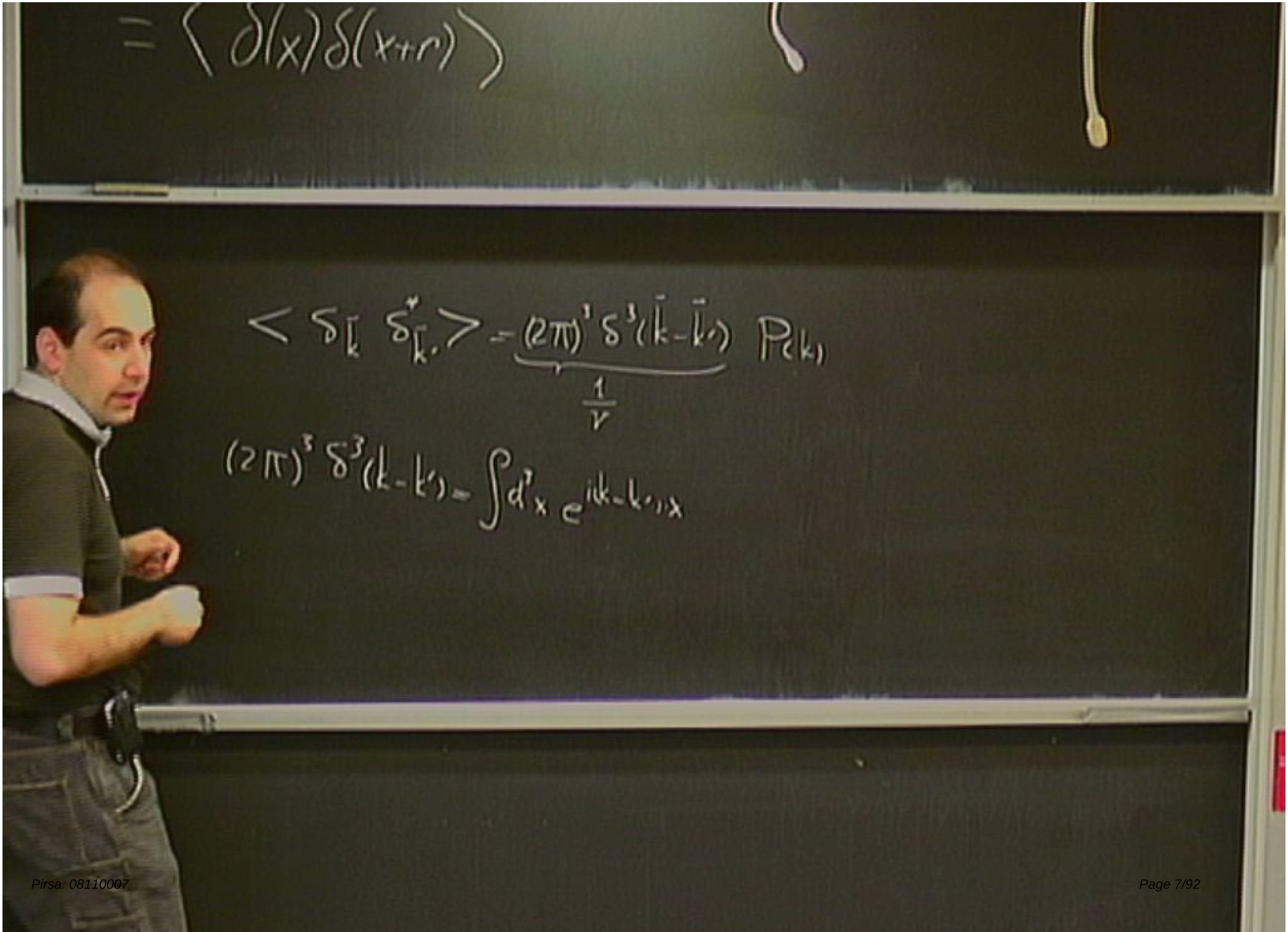

$P(k) = \langle \delta_k \delta_k^* \rangle$


$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$
$\mathcal{F}^{-1}(P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k)(x) * \mathcal{F}^{-1}(\delta_k^*)(x)$
$=$

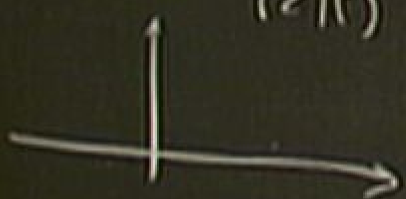

$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$
$(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k)(x) * \mathcal{F}^{-1}(\delta_k^*)(x)$
$(r) * \delta(-r) = \int_x \delta(x)\, \delta(x-(-r))\, dx = \int_x \delta(x)\, \delta(x+r)\, dx$


$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$
$\mathcal{F}^{-1}(P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k)(x) * \mathcal{F}^{-1}(\delta_k^*)(x)$
$= \delta(r) * \delta^*(-r) = \int_x \delta(x)\,\delta(x-(-r))\,dx = \int_x \delta(x)\,\delta(x+r)\,dx$
$\langle \delta(x)\,\delta(x+r) \rangle$

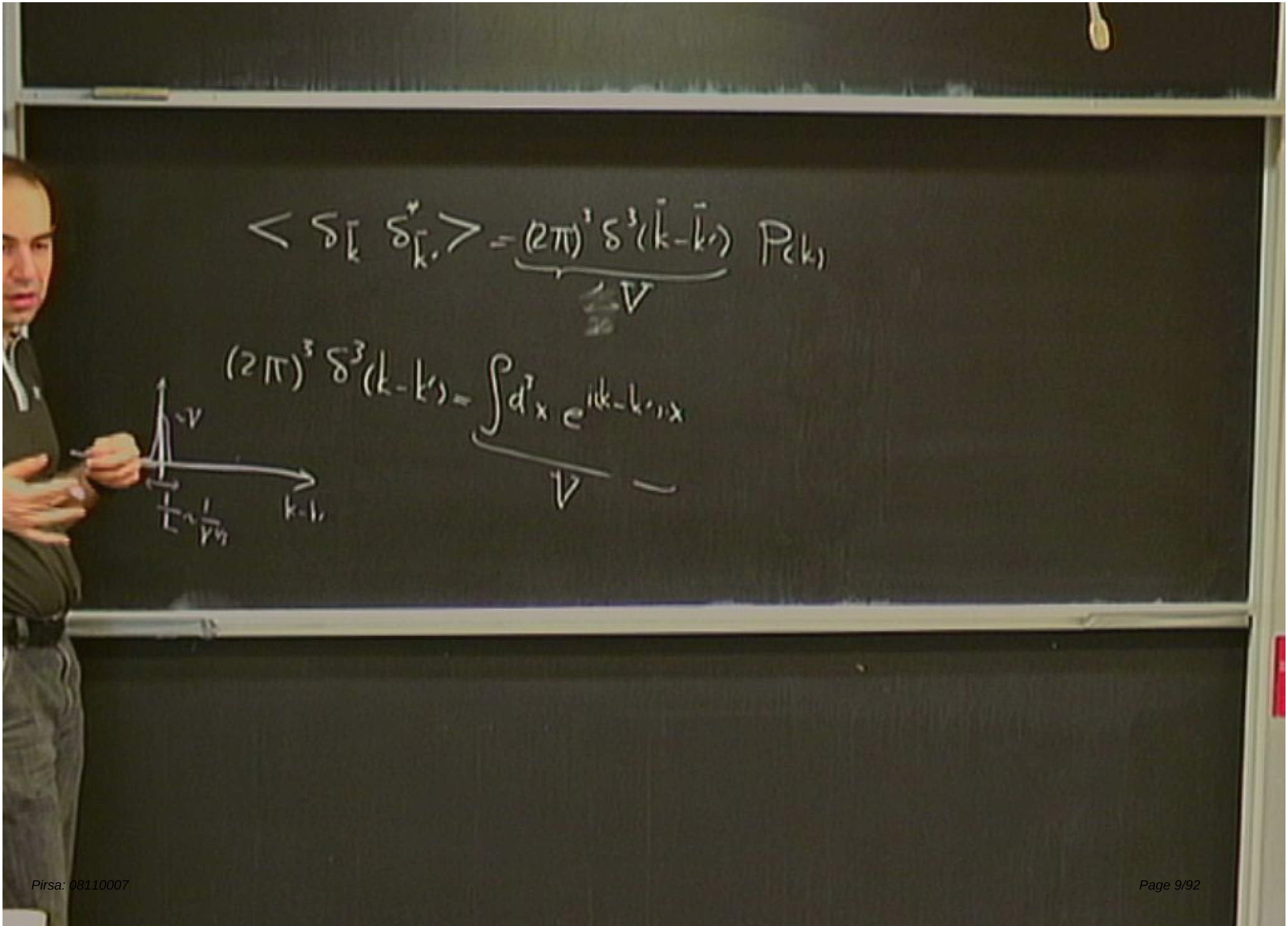

$= \langle \delta(x)\delta(x+r) \rangle$
$\langle \delta_{\vec{k}} \, \delta^*_{\vec{k}'} \rangle = (2\pi)^3 \delta^3(\vec{k}-\vec{k}') \, P(k)$


$\equiv \langle \delta(x)\delta(x+r)\rangle$
$\langle \delta_{\vec{k}}\, \delta^*_{\vec{k}'} \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k}-\vec{k}')}_{\frac{1}{V}} P(k)$
$(2\pi)^3 \delta^3(k-k') = \int d^3x \, e^{i(k-k')x}$

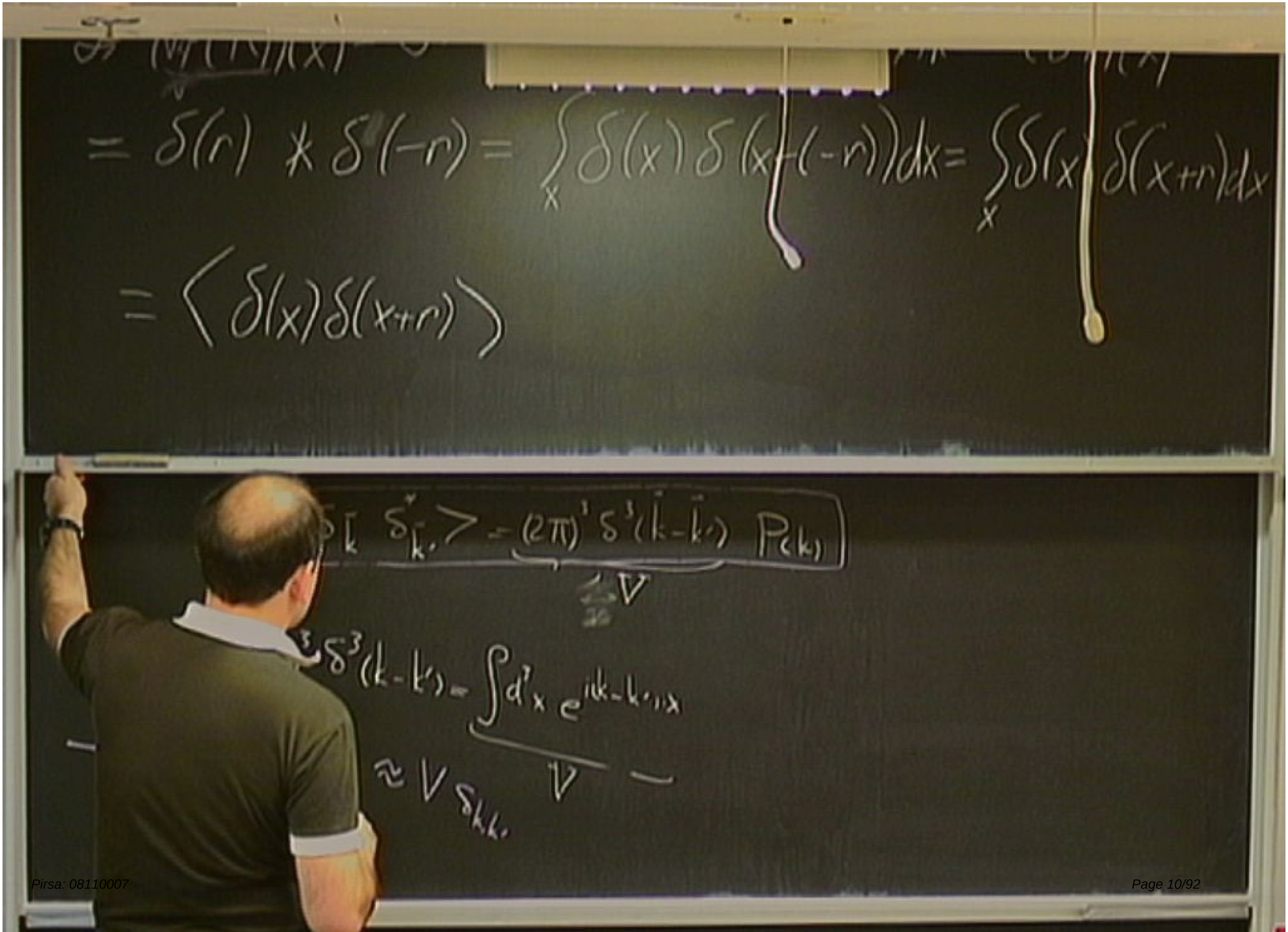
$$\langle \delta_{\vec{k}} \, \delta^*_{\vec{k}'} \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k}-\vec{k}')}_{V} P(k)$$



$$(2\pi)^3 \delta^3(k-k') = \underbrace{\int d^3x \, e^{i(k-k')x}}_{V}$$



$\langle \delta_{\vec{k}} \, \delta^*_{\vec{k}'} \rangle = \underbrace{(2\pi)^3 \delta^3(\vec{k}-\vec{k}')}_{V} P(k)$
$(2\pi)^3 \delta^3(k-k') = \underbrace{\int d^3x \, e^{i(k-k')x}}_{V}$
$\sim V$
$k-k'$
$L \sim \frac{1}{V^{1/3}}$


= δ(r) * δ(-r) = ∫ δ(x) δ(x-(-r)) dx = ∫ δ(x) δ(x+r) dx
= ⟨ δ(x) δ(x+r) ⟩
δk δ*k' > = (2π)³ δ³(k-k') P(k)
V
δ³(k-k') = ∫d³x e^i(k-k')x
≈ V δkk'
V

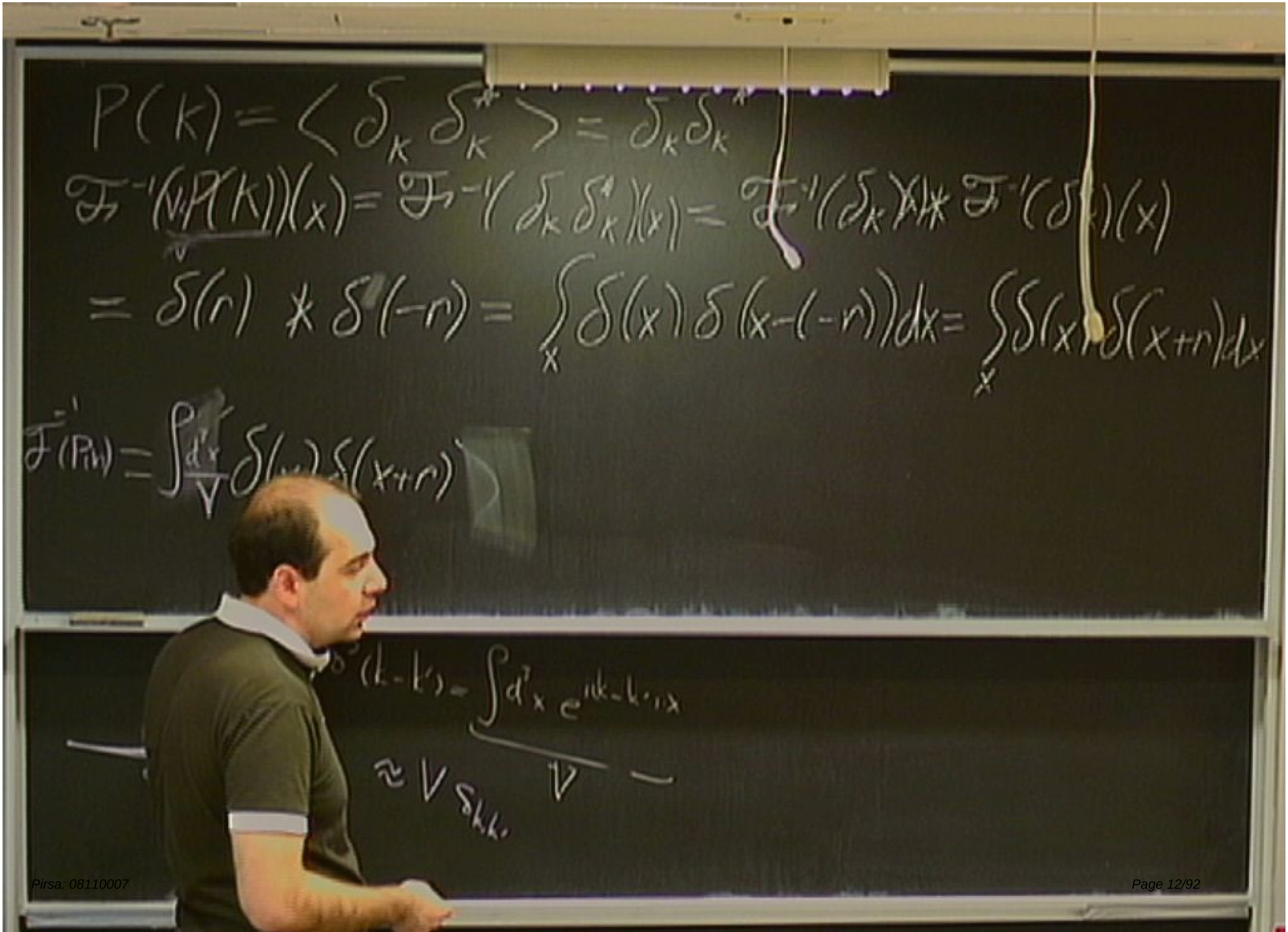
$$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$$

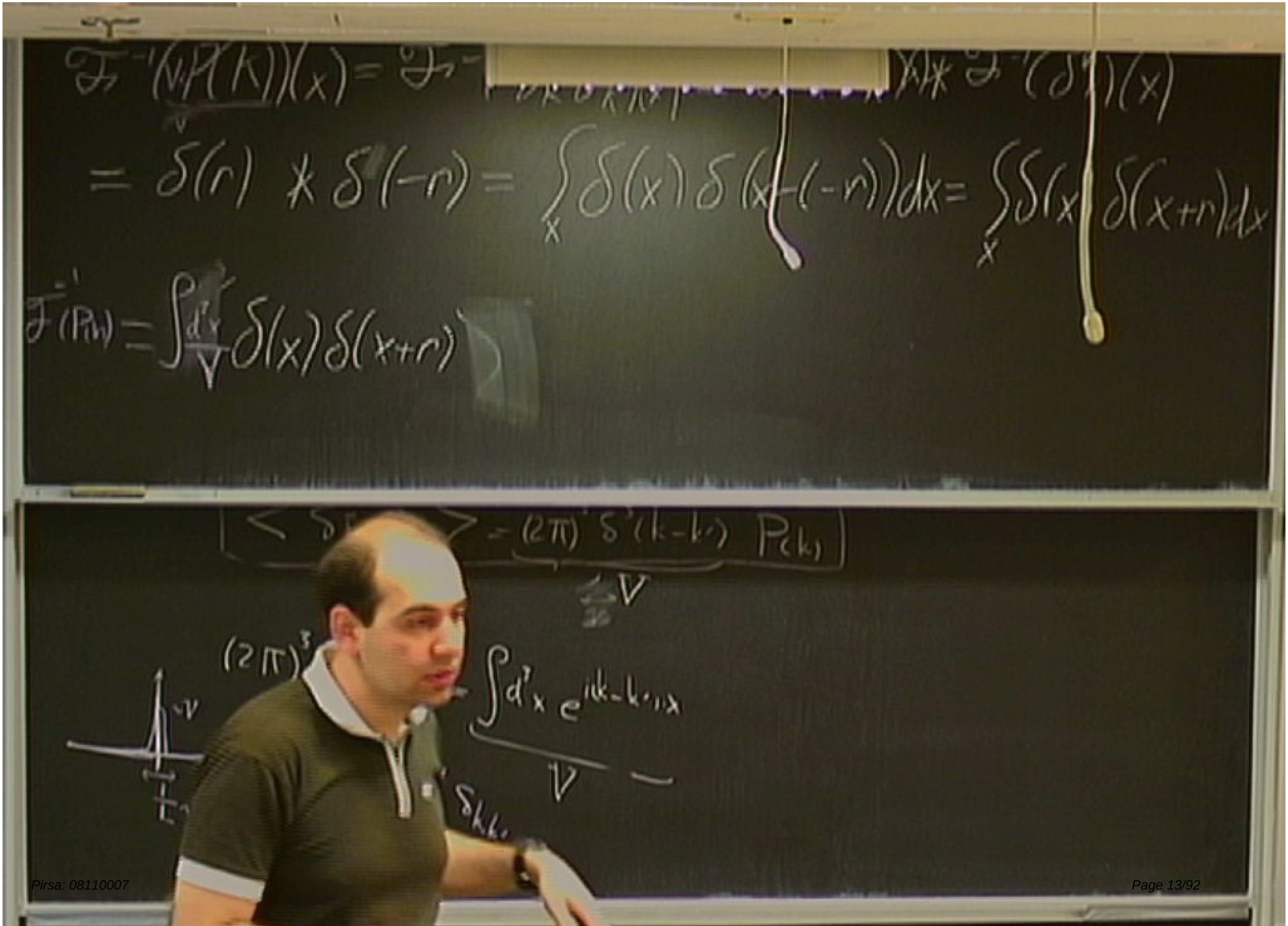
$$\mathcal{F}^{-1}(V P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k)(x) * \mathcal{F}^{-1}(\delta_k^*)(x)$$

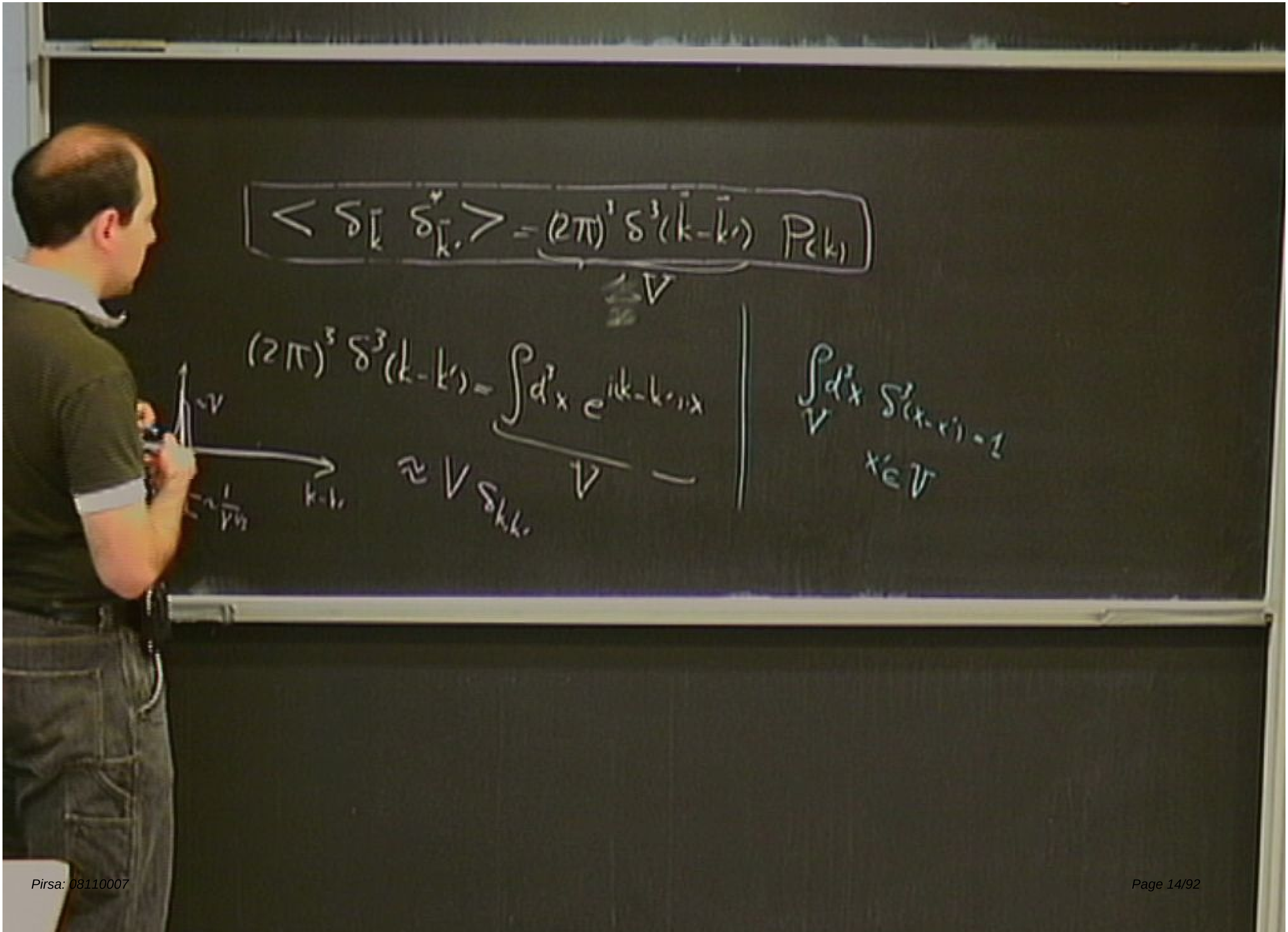
$$= \delta(r) * \delta^*(-r) = \int_x \delta(x)\,\delta(x-(-r))\,dx = \int_x \delta(x)\,\delta(x+r)\,dx$$

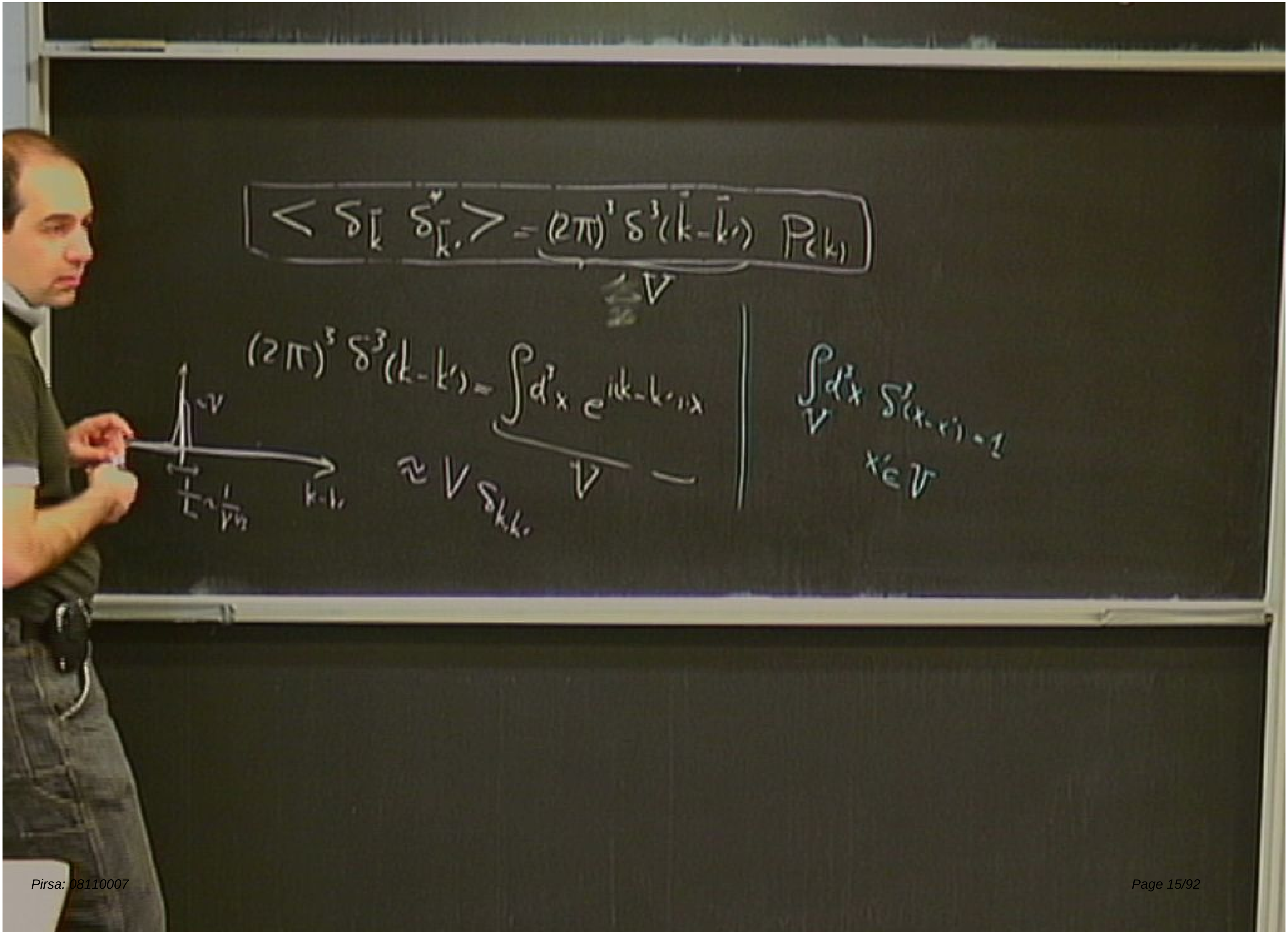
$$\langle \delta(x)\,\delta(x+r) \rangle$$

$$\approx V \delta_{kk'}$$

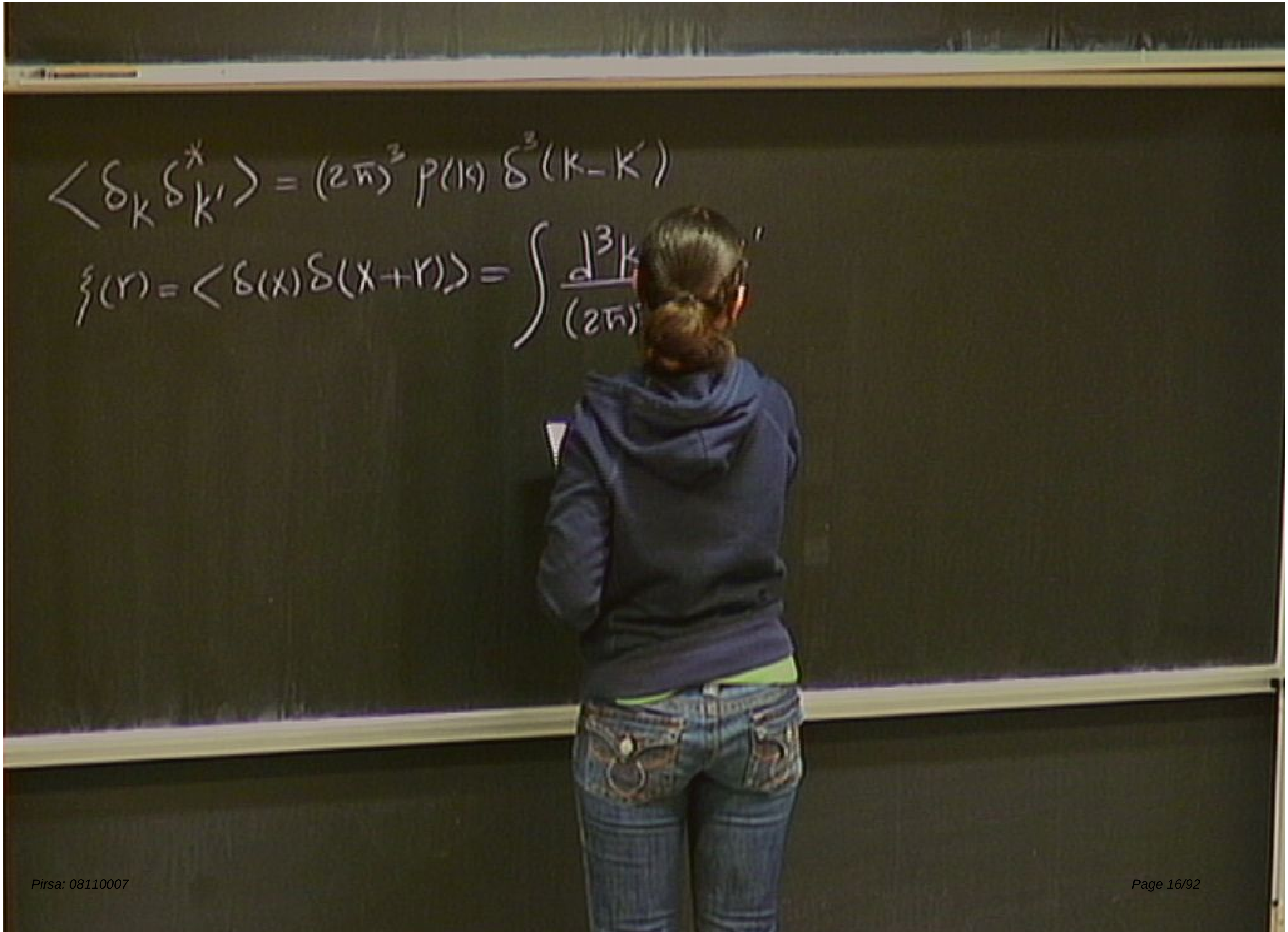

$P(k) = \langle \delta_k \delta_k^* \rangle = \delta_k \delta_k^*$
$\mathcal{F}^{-1}(V P(k))(x) = \mathcal{F}^{-1}(\delta_k \delta_k^*)(x) = \mathcal{F}^{-1}(\delta_k)(x) * \mathcal{F}^{-1}(\delta_k^*)(x)$
$= \delta(r) * \delta(-r) = \int_x \delta(x)\,\delta(x-(-r))\,dx = \int_x \delta(x)\,\delta(x+r)\,dx$
$\mathcal{F}^{-1}(P(k)) = \int \frac{d^3x}{V}\,\delta(x)\,\delta(x+r)$
$\delta^3(k-k') = \frac{\int d^3x\, e^{i(k-k')x}}{V}$
$\approx V \delta_{kk'}$

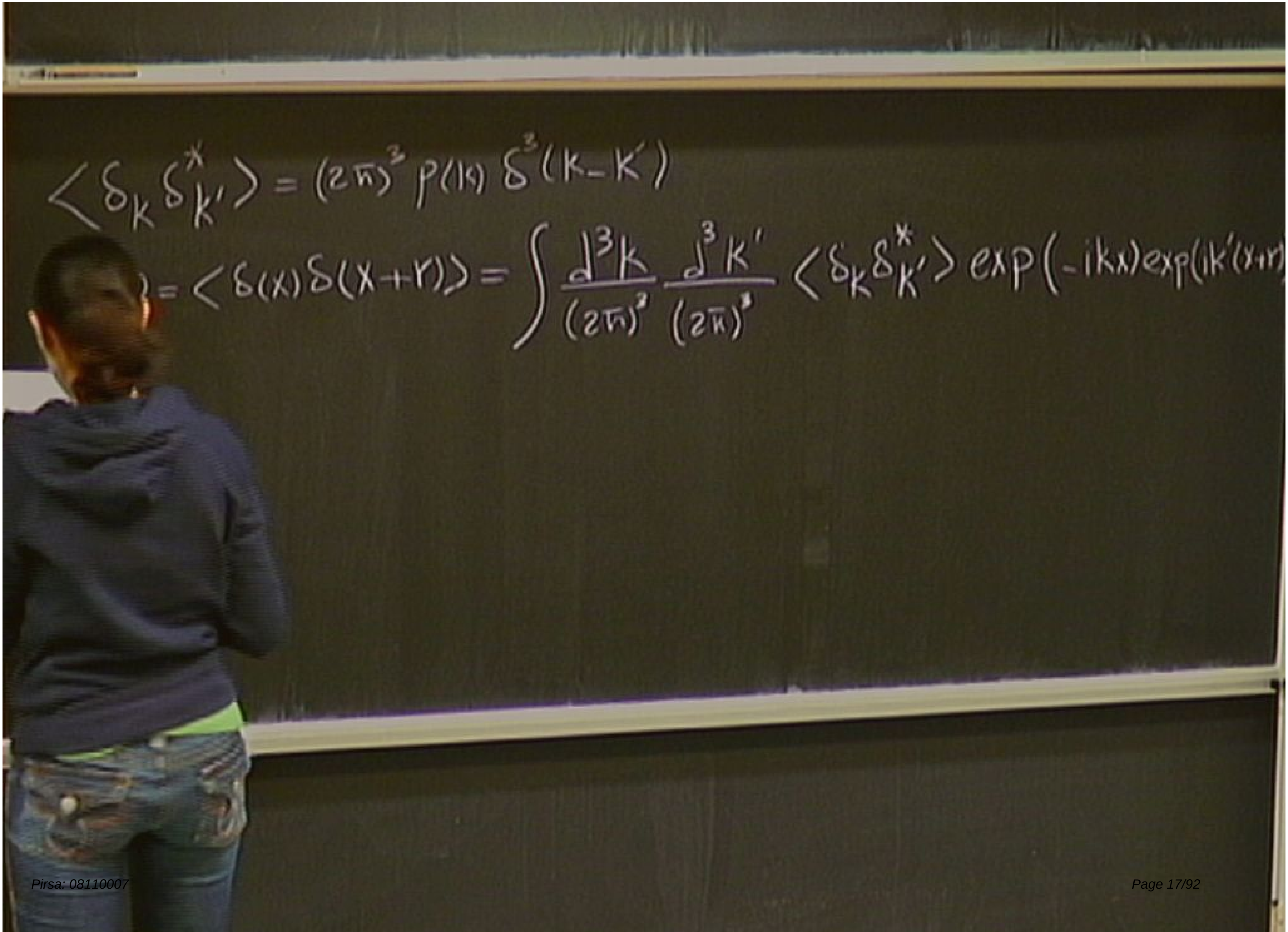

= δ(r) * δ(-r) = ∫ δ(x) δ(x-(-r))dx = ∫ δ(x) δ(x+r) dx
x x
∫ d³x/V δ(x) δ(x+r)
(2π)³
∫ d³x e^{i(k-k')·x}
V

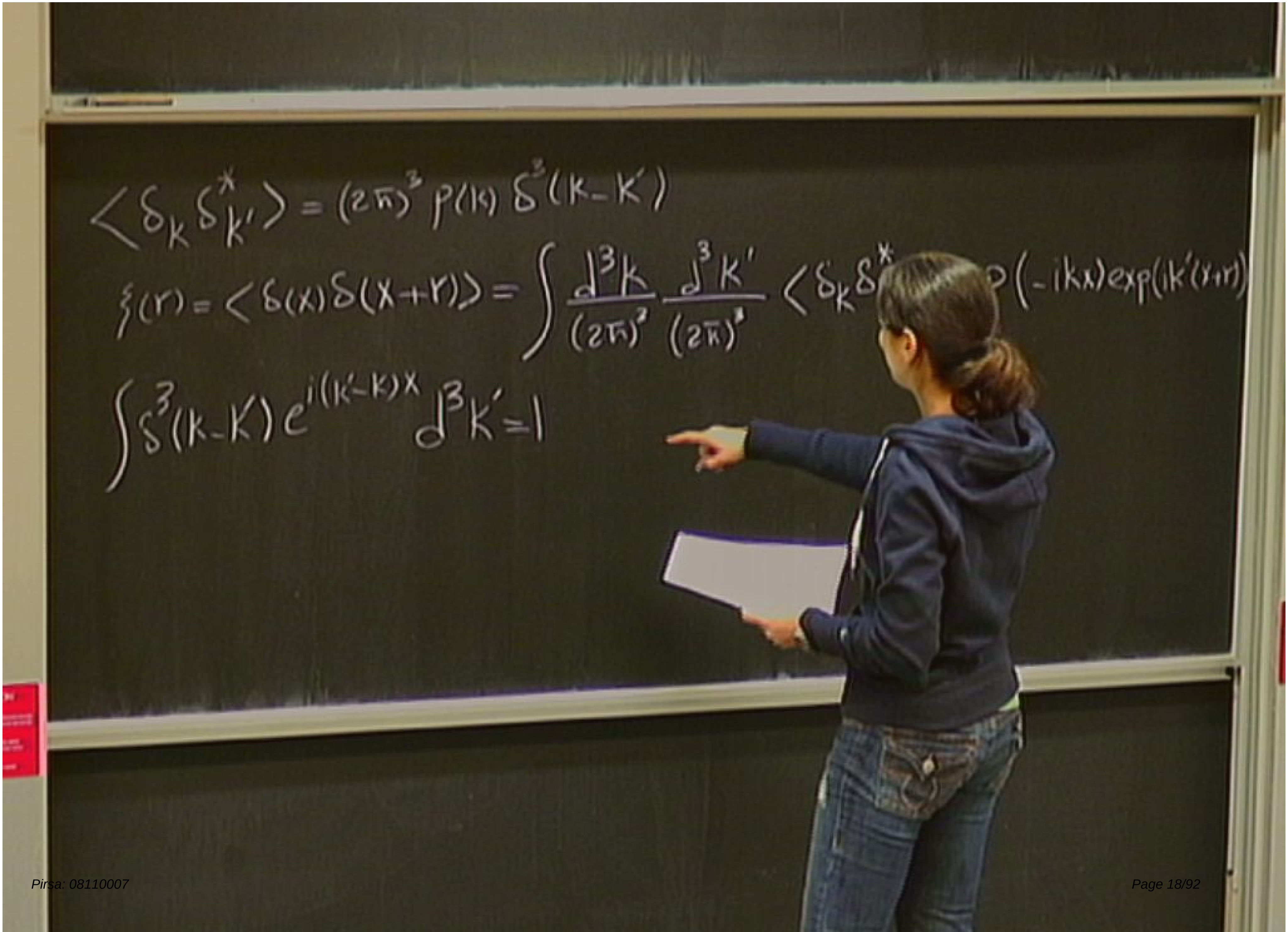

$\boxed{\langle \delta_{\vec{k}}\, \delta^*_{\vec{k}'} \rangle = (2\pi)^3 \delta^3(\vec{k}-\vec{k}')\, P(k)}$
$\sim V$
$(2\pi)^3 \delta^3(k-k') = \int d^3x\, e^{i(k-k')x}$
V
$\approx V \delta_{kk'}$
$\sim V$
$\sim \frac{1}{V^{1/3}}$
$k-k'$
$\int_V d^3x\, \delta^3(x-x') = 1$
$x' \in V$



$\langle \delta_{\vec{k}} \, \delta^*_{\vec{k}'} \rangle = (2\pi)^3 \delta^3(\vec{k}-\vec{k}') \, P(k)$
$\approx V$
$(2\pi)^3 \delta^3(k-k') = \int d^3x \, e^{i(k-k')x}$
$\approx V \delta_{kk'}$
V
$\sim V$
$k-k'$
$\frac{1}{L} \sim \frac{1}{V^{1/3}}$
$\int_V d^3x \, \delta^3(x-x') = 1$
$x' \in V$


$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k - k')$
$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3}$


$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k - k')$
$) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r)$


$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(k-k')$
$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*$
$p(-ikx)\exp(ik'(x+r))$
$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r)\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r))$$

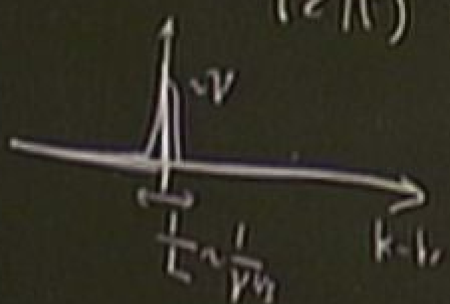
$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$\ldots = \int \frac{\ldots}{V}\,\delta(x)\,\delta(x+r)$$

$$\boxed{\langle \delta_{\vec k}\, \delta^*_{\vec k'} \rangle = \underbrace{(2\pi)^3 \delta^3(\vec k - \vec k')}_{V}\; P(k)}$$

$$\left(\sum_i a_i\right)\left(\sum_j b_j\right) = \sum_{ij} a_i b_j$$

$$(2\pi)^3 \delta^3(k-k') = \underbrace{\int d^3x\, e^{i(k-k')x}}_{V}$$



$$\approx V\, \delta_{k k'}$$

$$\int_V d^3x\, \delta^3(x-x') = 1 \qquad x' \in V$$

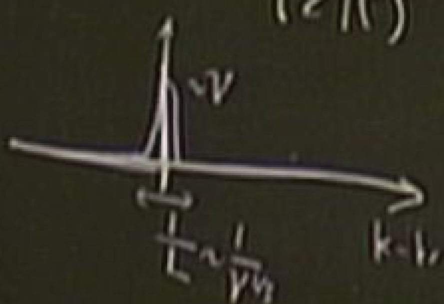
$$= \int \frac{d^3x}{V}\, \delta(x)\,\delta(x+r)$$

$$\boxed{\langle \delta_{\vec k}\, \delta^*_{\vec k'} \rangle = \underbrace{(2\pi)^3 \delta^3(\vec k - \vec k')}_{\approx V} P(k)}$$

$$(\sum_i a_i)(\sum_j b_j) = \sum_{ij} a_i b_j$$

$$(2\pi)^3 \delta^3(k-k') = \underbrace{\int d^3x\, e^{i(k-k')\cdot x}}_{V}$$

$$\approx V\, \delta_{k,k'}$$



$$\int_V d^3x\, F(x)\, \delta^3(x-x') = F(x')$$
$$x' \in V$$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k-k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r)$$

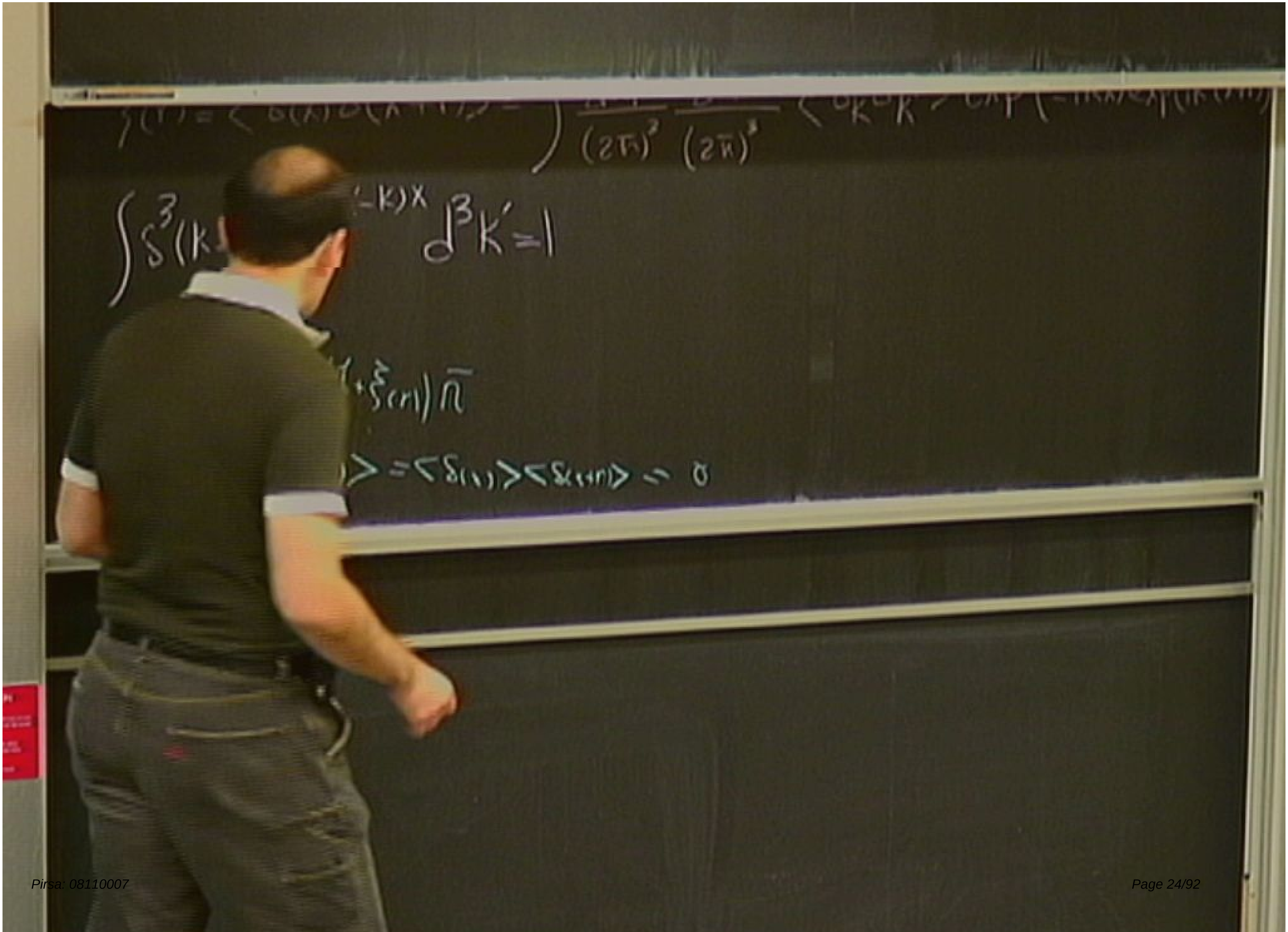
$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

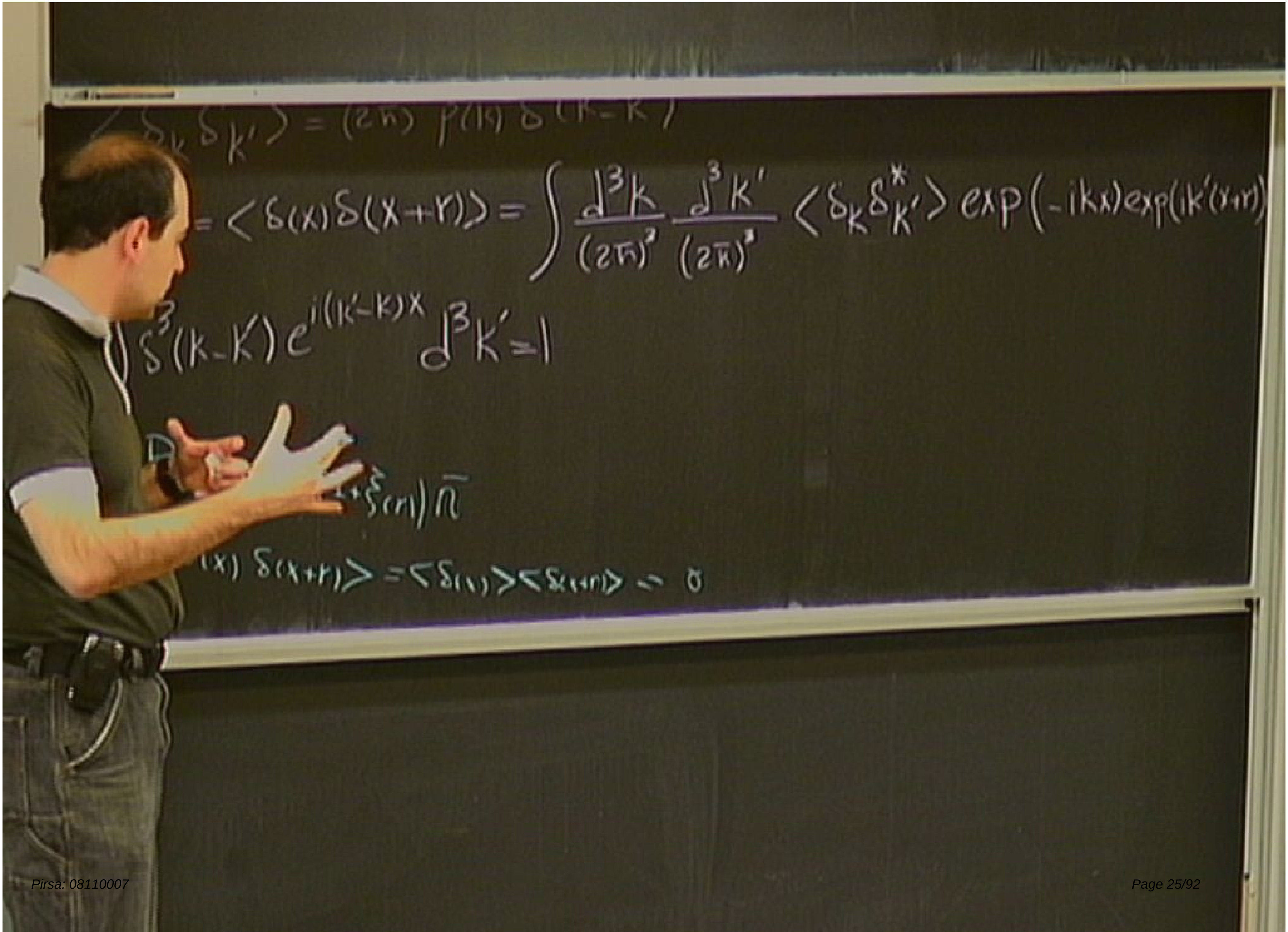
$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(k-k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx) \exp(ik'(x+r))$$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$P(\Delta^3 r_j; r) = (1+\xi(r))\,\bar{n}$$





< δk δk' > = (2π) P(k) δ (k − k')
= < δ(x) δ(x+r) > = ∫ d³k/(2π)³ d³k'/(2π)³ < δk δ*k' > exp(−ikx) exp(ik'(x+r))
∫ δ³(k − k') e^{i(k'−k)x} d³k' = 1
ξ(r)) n̄
(x) δ(x+r) > = < δ(x) > < δ(x+r) > = 0

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(k-k')$$

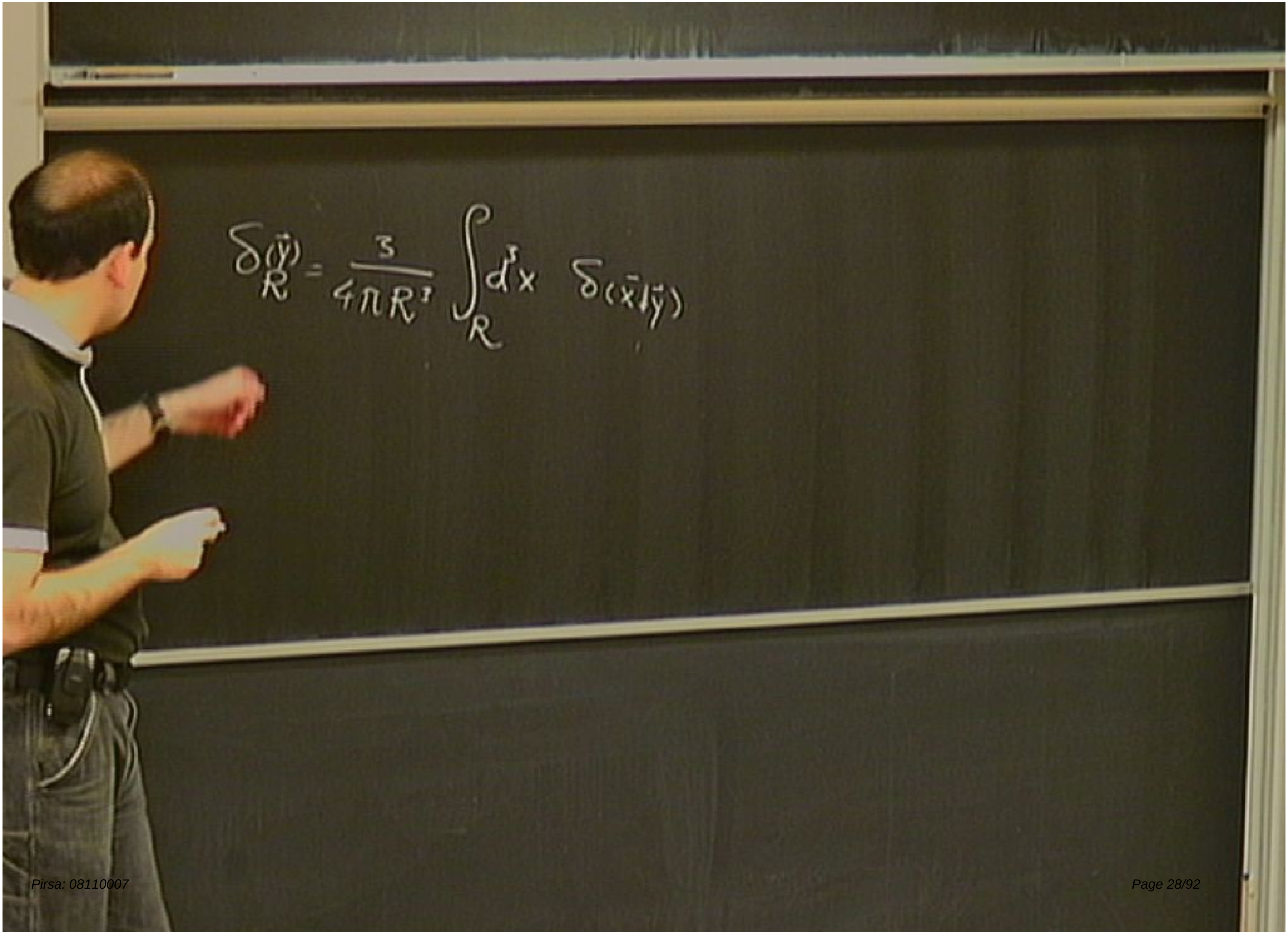
$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r))$$

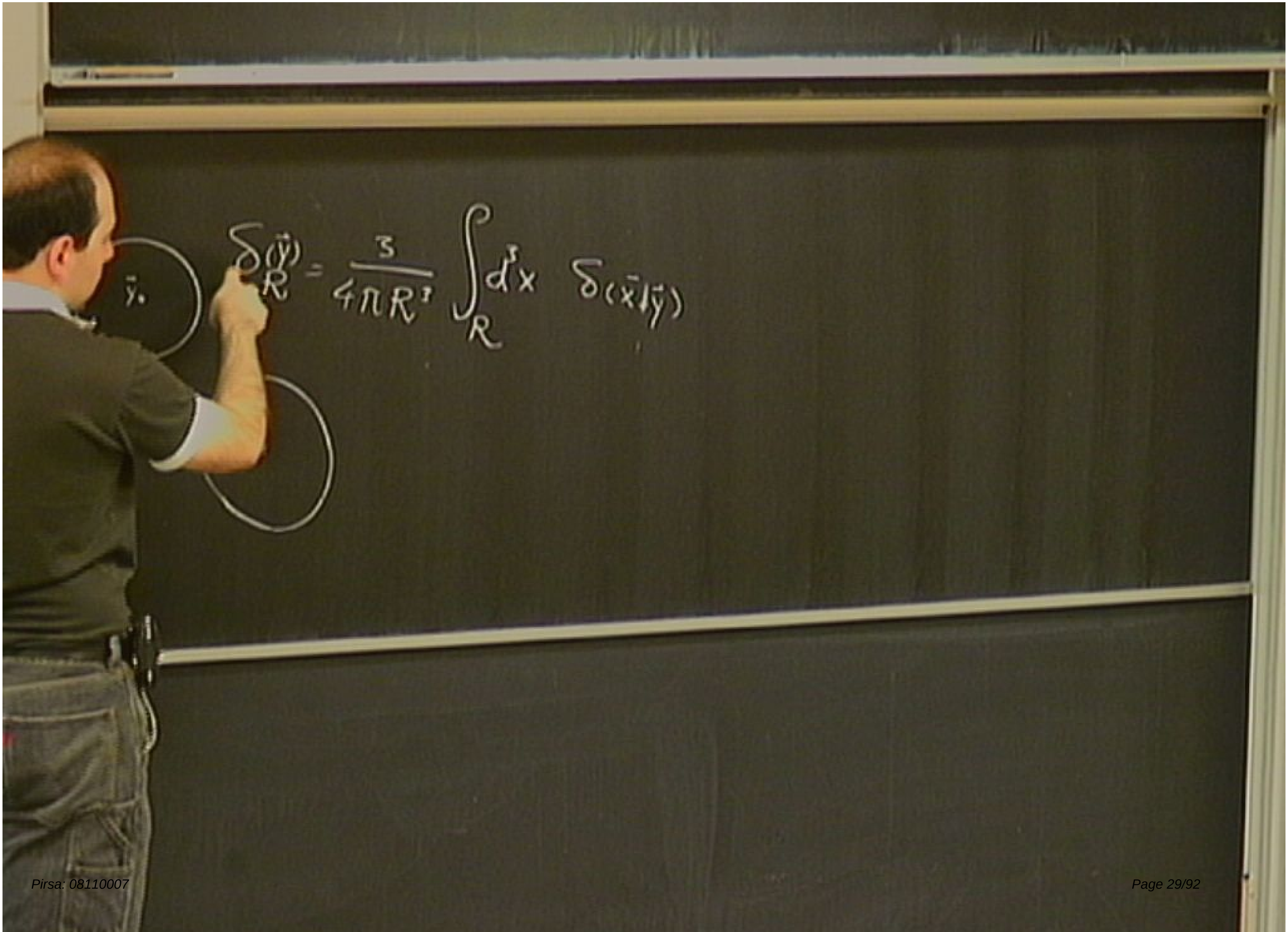
$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$P(\Delta^3 r; r) = (1+\xi(r))\bar{n}$$

$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \;\; \delta(\vec{x})$$

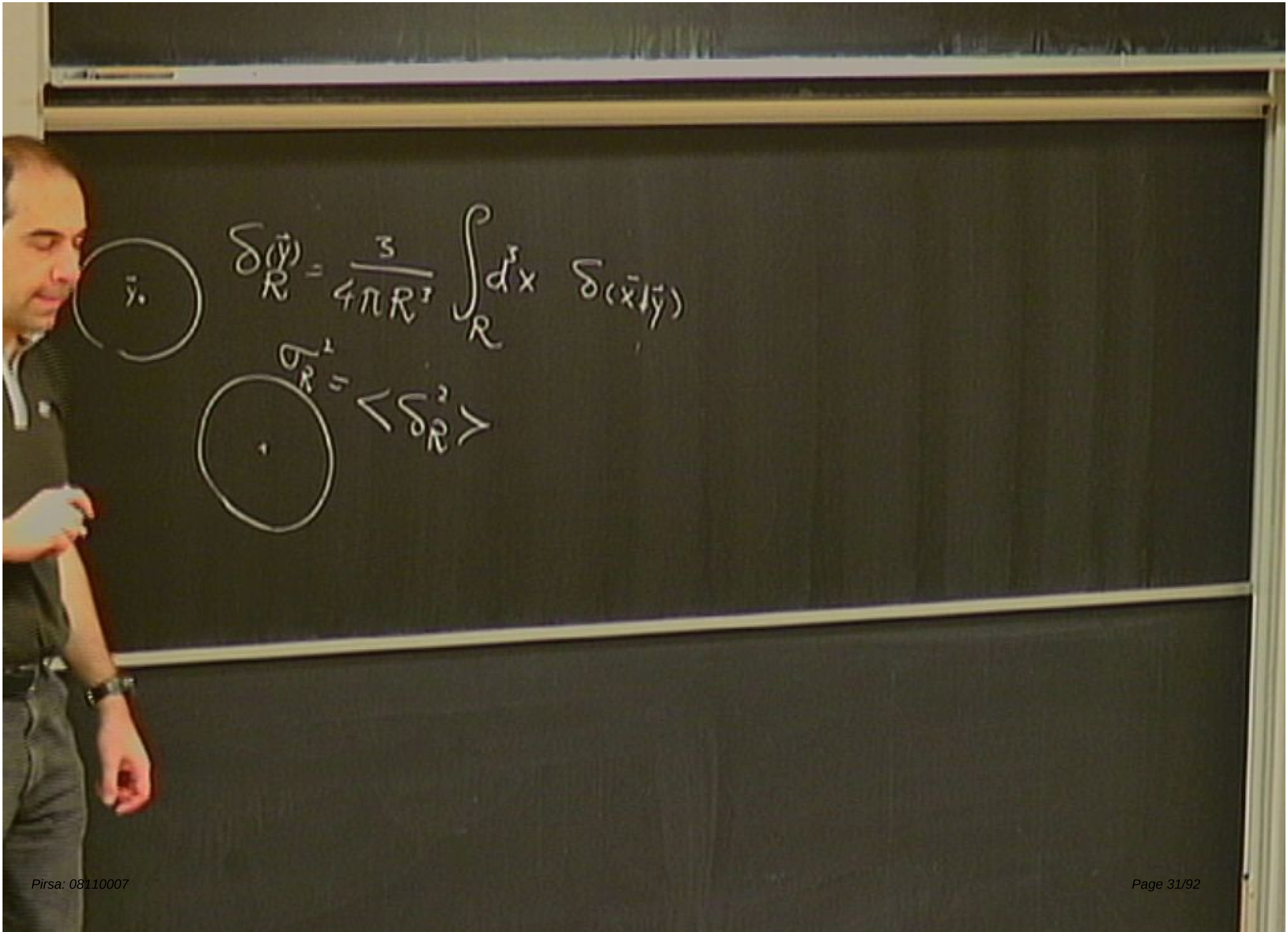

$\delta_R(\vec{y}) = \frac{3}{4\pi R^3} \int_R d^3x \ \delta(\vec{x}+\vec{y})$

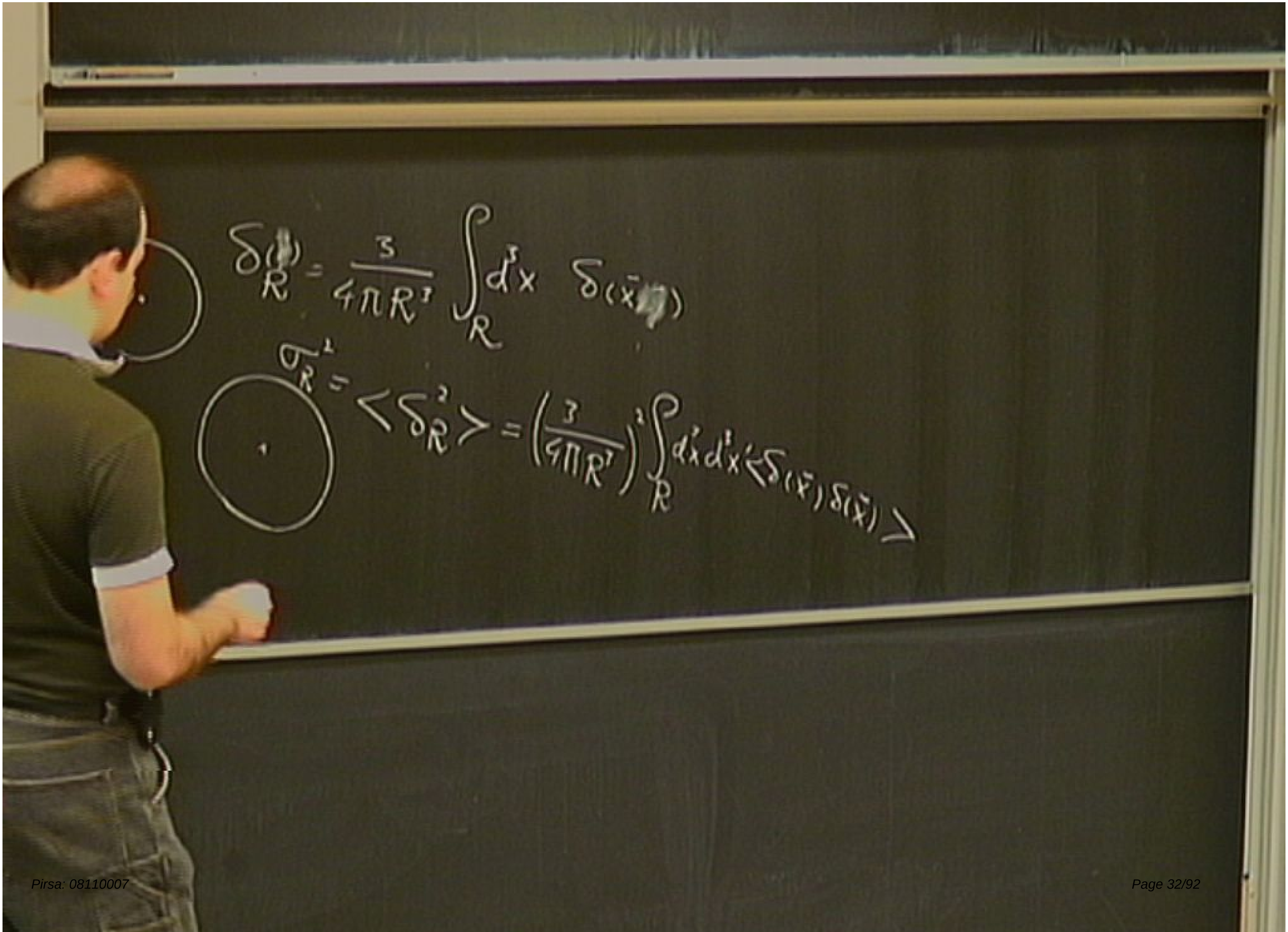


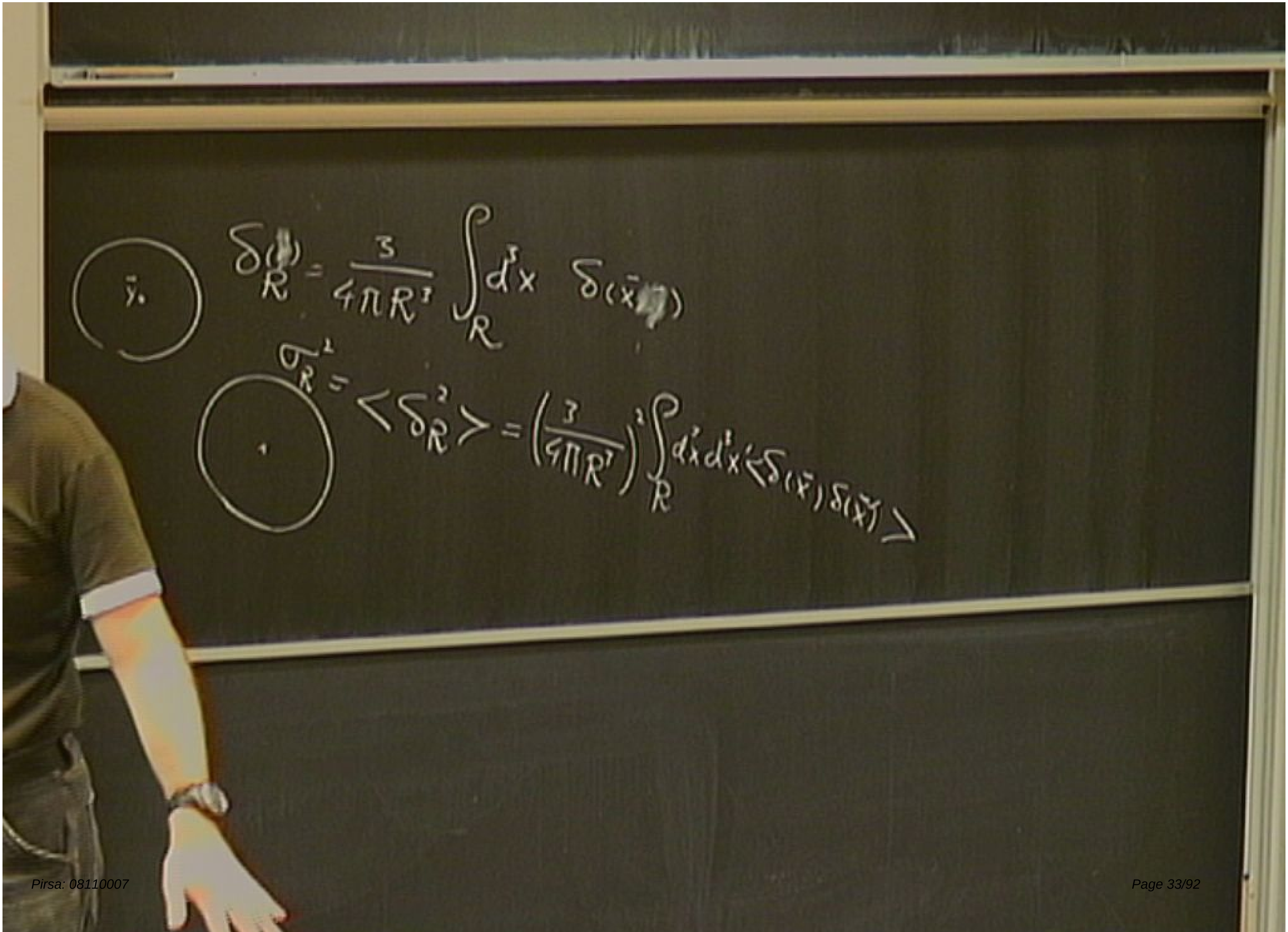
$\delta_R(\vec{y}) = \frac{3}{4\pi R^3} \int_R d^3x \; \delta(\vec{x}|\vec{y})$
$\vec{y}.$

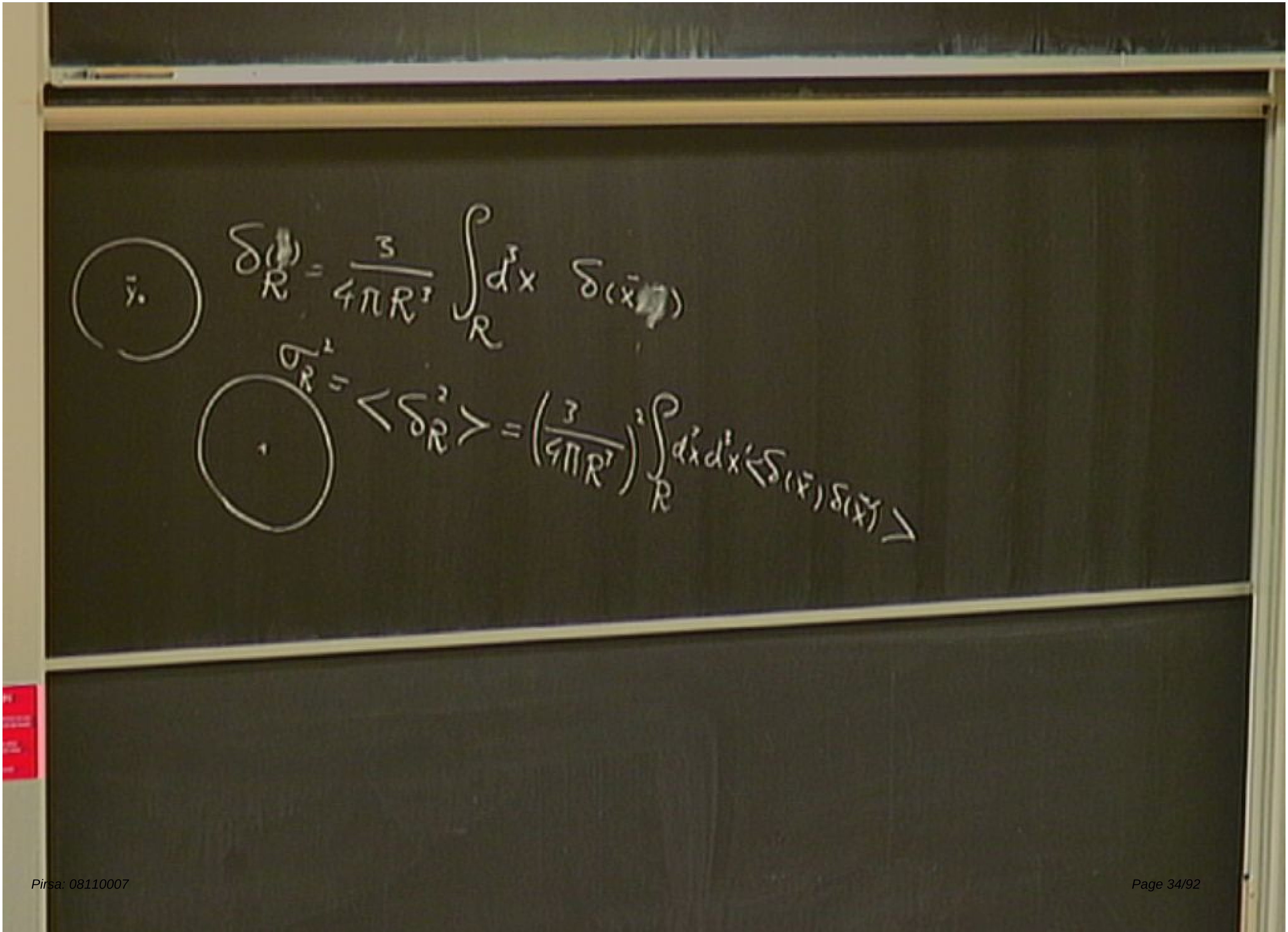
$$\delta_R(\vec{y}) = \frac{3}{4\pi R^3} \int_R d^3x \;\; \delta(\vec{x}+\vec{y})$$

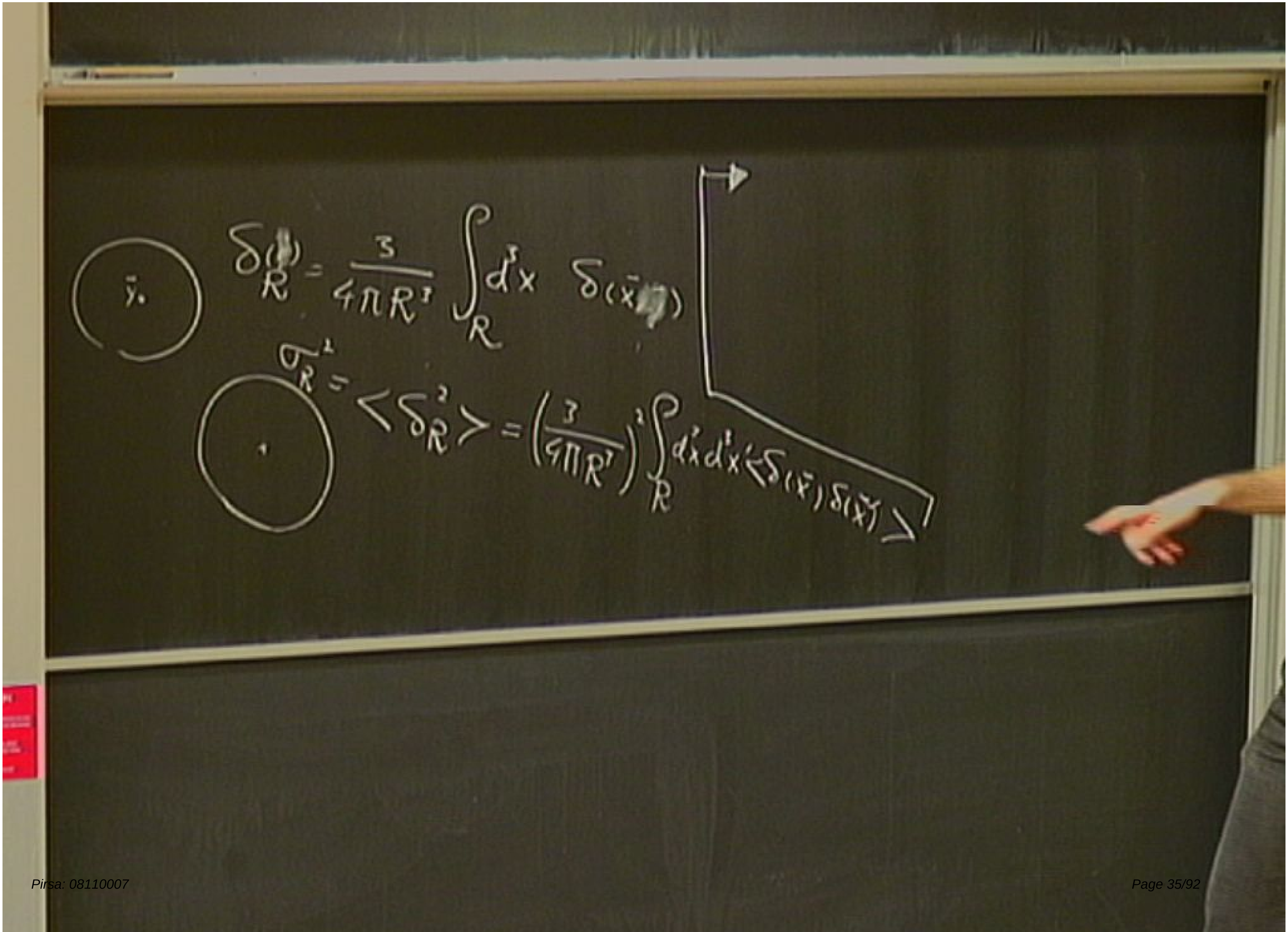
$$\langle \delta_R^2 \rangle$$

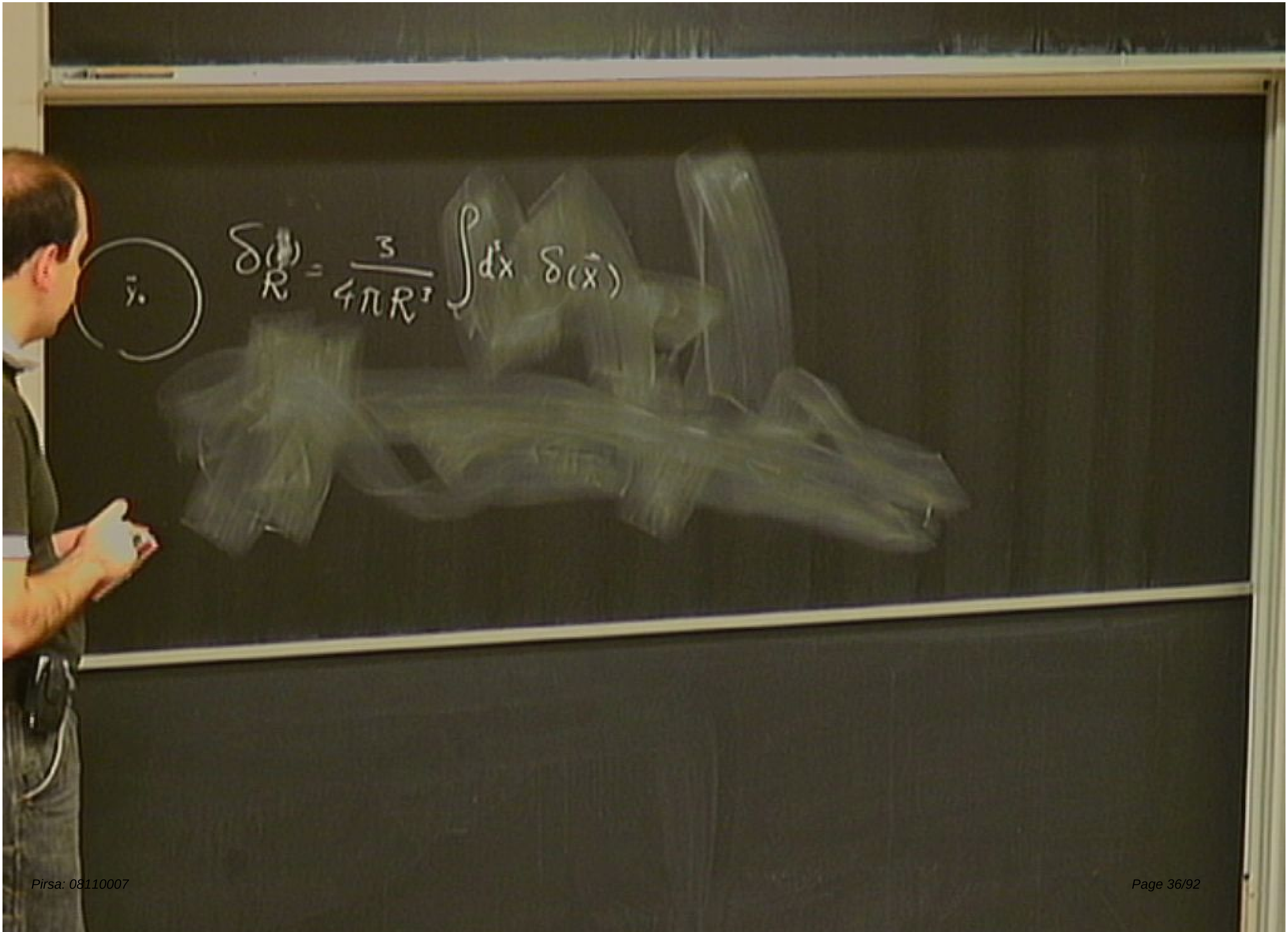

$\vec{y}$
$\delta_R(\vec{y}) = \frac{3}{4\pi R^3} \int_R d^3x \; \delta(\vec{x}|\vec{y})$
$\sigma_R^2 = \langle \delta_R^2 \rangle$

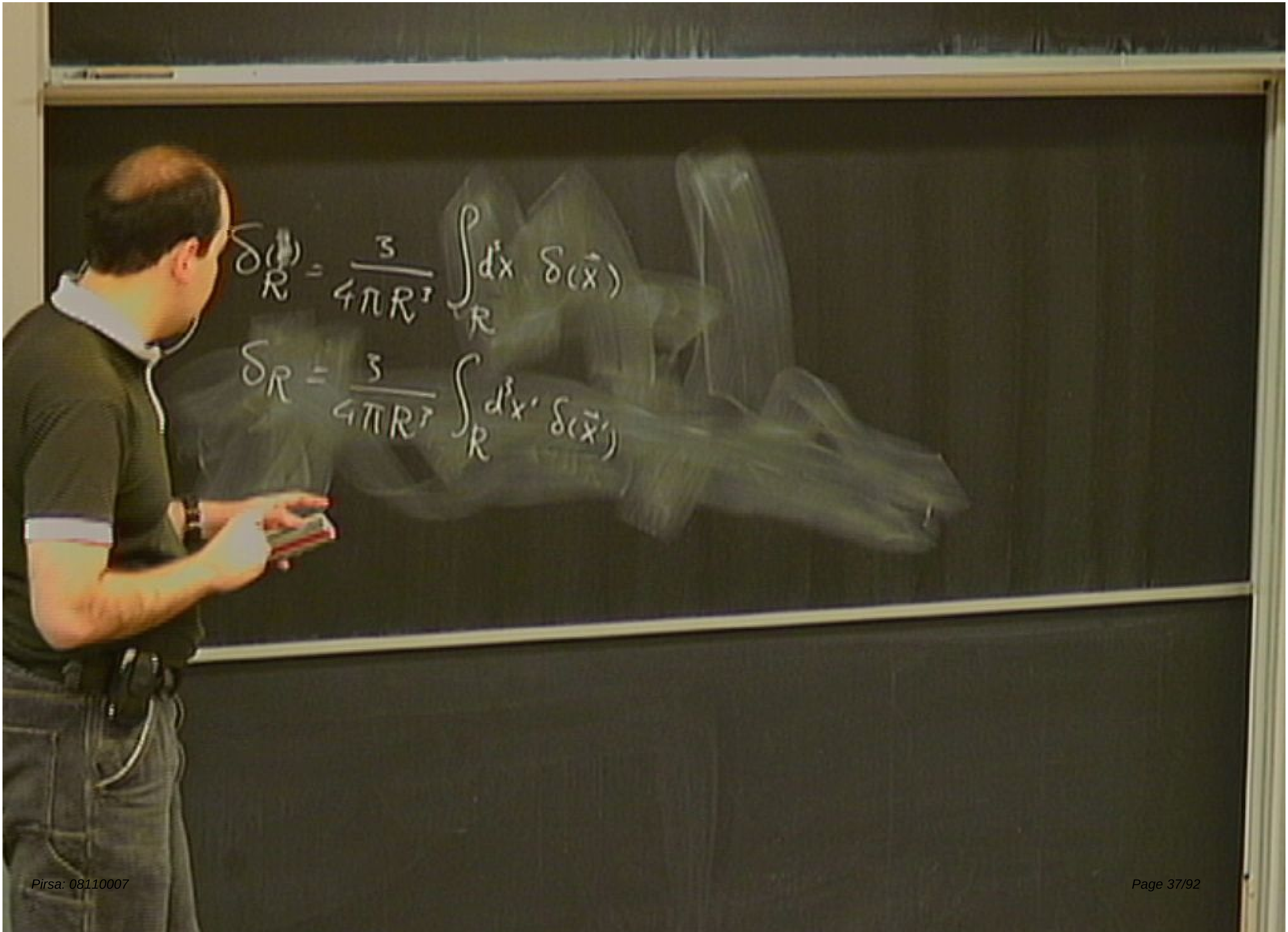

$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \; \delta(\vec{x})$
$\sigma_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$


$\vec{y}$
$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \; \delta(\vec{x})$
$\sigma_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$


$\vec{y}$
$\delta_R(\ ) = \frac{3}{4\pi R^3}\int_R d^3x\ \delta(\vec{x}\ )$
$\sigma_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x\, d^3x' \langle \delta(\vec{x})\, \delta(\vec{x}') \rangle$


$\vec{y}$
$\delta_R(\ ) = \frac{3}{4\pi R^3}\int_R d^3x \ \delta(\vec{x}\ )$
$\sigma_R^2 = \langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x\, d^3x' \langle \delta(\vec{x})\, \delta(\vec{x}') \rangle$

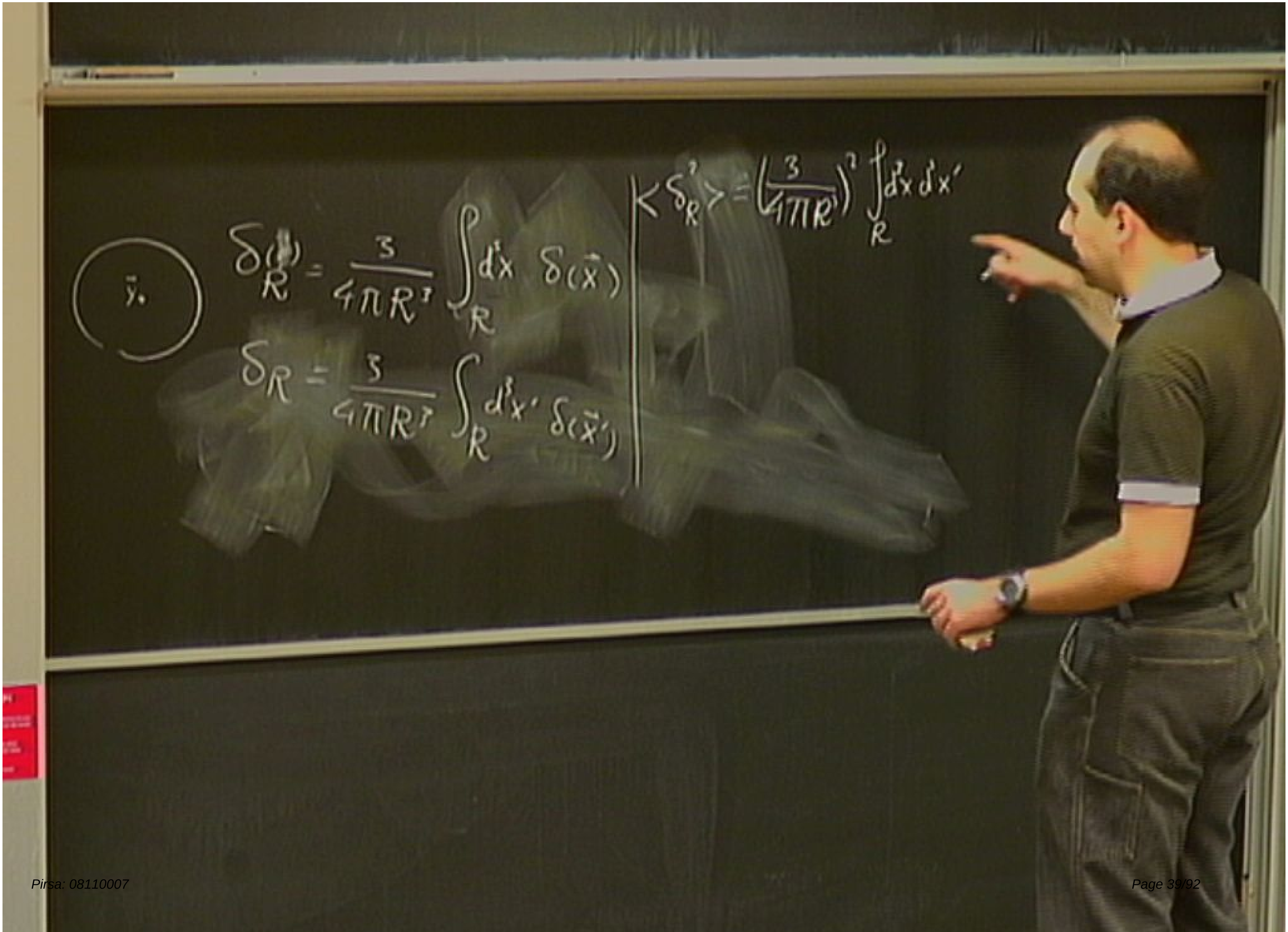

$\vec{y}_0$
$\delta_R = \frac{3}{4\pi R^3}\int d^3x \; \delta(\vec{x})$

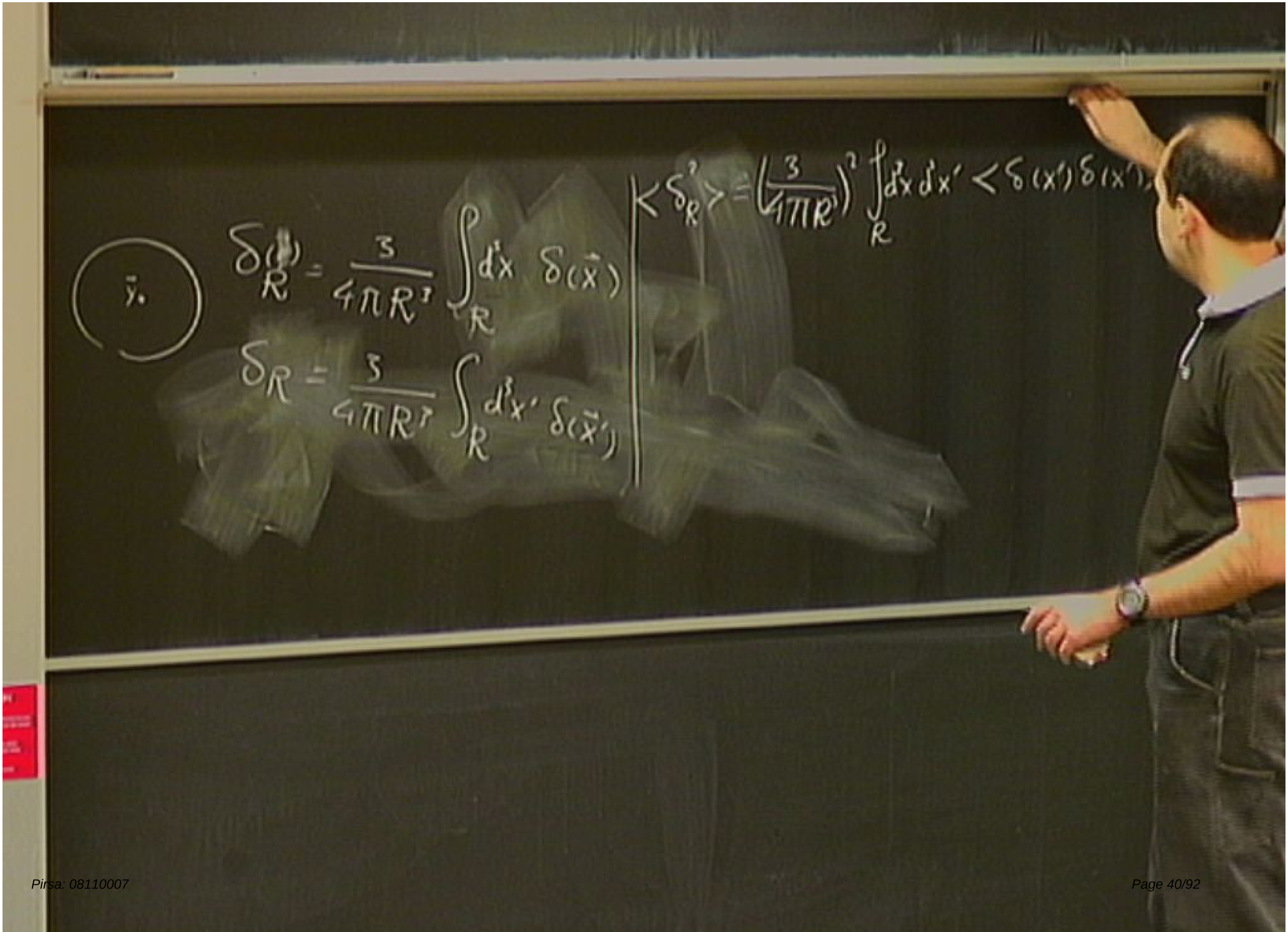

$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x \; \delta(\vec{x})$
$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \; \delta(\vec{x}')$

$\vec{y}_0$

$$\delta_R(\,\cdot\,) = \frac{3}{4\pi R^3}\int_R d^3x \;\delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \;\delta(\vec{x}')$$


$\vec{y}_0$
$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x \; \delta(\vec{x})$
$\delta_R = \frac{3}{4\pi R^3} \int_R d^3x' \; \delta(\vec{x}')$
$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x'$


$\vec{y}$
$\delta_R(\vec{y}) = \frac{3}{4\pi R^3}\int_R d^3x\ \delta(\vec{x})$
$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x'\ \delta(\vec{x}')$
$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x\, d^3x' \langle \delta(x')\delta(x')$

$$\delta_R(\vec{y}) = \frac{3}{4\pi R^3}\int_R d^3x \; \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \; \delta(\vec{x}')$$

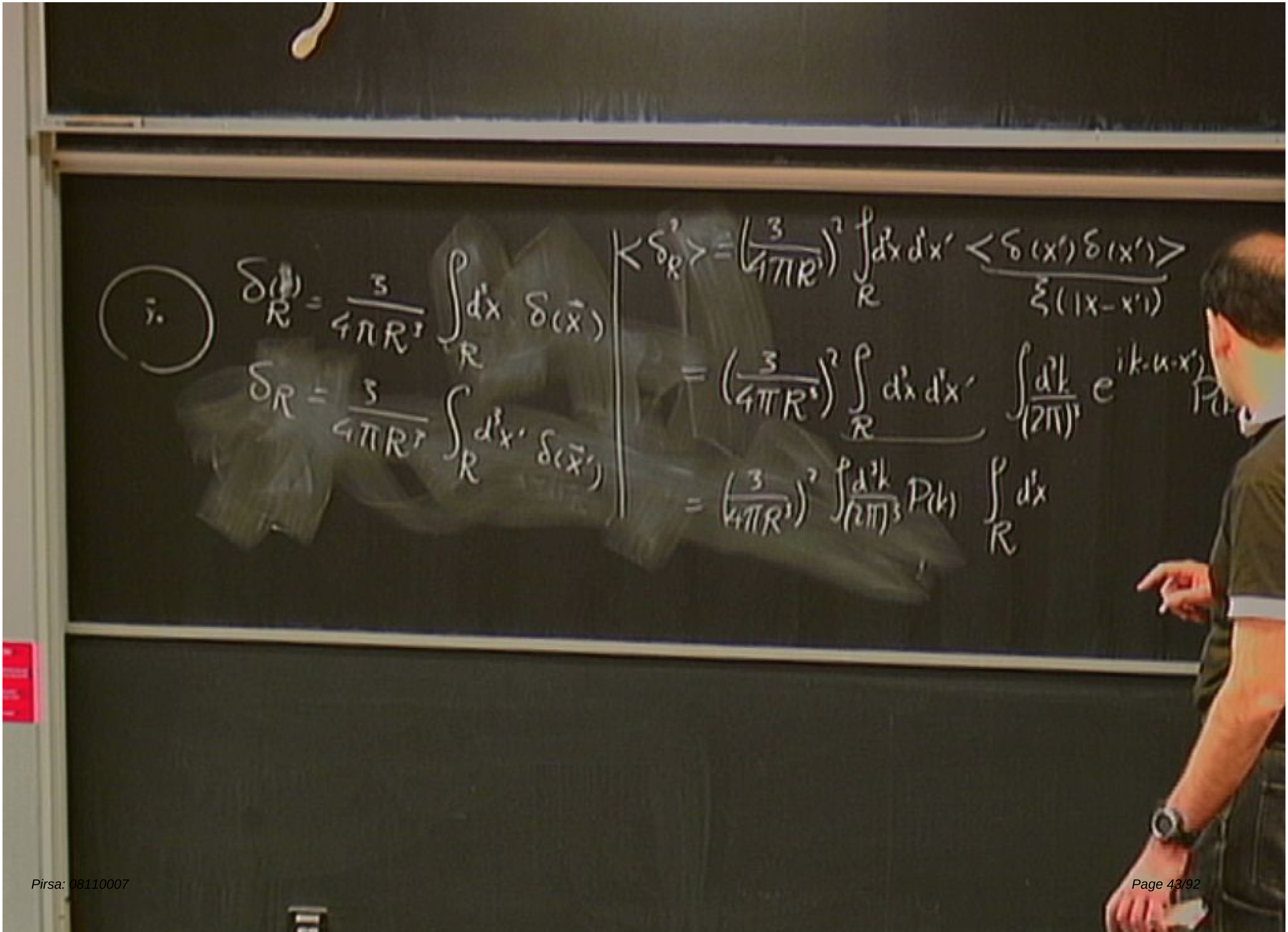
$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \; \underbrace{\langle \delta(x')\,\delta(x') \rangle}_{\xi(|x-x'|)}$$

$$\delta_R(\,) = \frac{3}{4\pi R^3}\int_R d^3x \; \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \; \delta(\vec{x}')$$

$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \; \underbrace{\langle \delta(x') \delta(x') \rangle}_{\xi(|x-x'|)}$$

$$= \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \; \int \frac{d^3k}{(2\pi)^3} \, e^{ik\cdot(x-x')} P(k)$$


$\delta_R(\vec{x}) = \frac{3}{4\pi R^3}\int_R d^3x\ \delta(\vec{x})$
$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x'\ \delta(\vec{x}')$
$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x\, d^3x' \underbrace{\langle \delta(x')\delta(x')\rangle}_{\xi(|x-x'|)}$
$= \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x\, d^3x' \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')} P(k)$
$= \left(\frac{3}{4\pi R^3}\right)^2 \int \frac{d^3k}{(2\pi)^3} P(k) \int_R d^3x$

$$\delta_R(\vec{x}) = \frac{3}{4\pi R^3}\int_R d^3x \; \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \; \delta(\vec{x}')$$

$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \underbrace{\langle \delta(x') \delta(x') \rangle}_{\xi(|x-x'|)}$$

$$= \left(\frac{3}{4\pi R^3}\right)^2 \underbrace{\int_R d^3x \, d^3x'}\; \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')} P(k)$$

$$= \left(\frac{3}{4\pi R^3}\right)^2 \int \frac{d^3k}{(2\pi)^3} P(k) \left(\int_R d^3x \; e^{ik\cdot x}\right)\left(\int_R d^3x \; e^{-ik\cdot x'}\right)$$

$$\delta_R(\vec{x}) = \frac{3}{4\pi R^3}\int_R d^3x \; \delta(\vec{x})$$

$$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \; \delta(\vec{x}')$$

$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x\, d^3x' \underbrace{\langle \delta(x')\delta(x')\rangle}_{\xi(|x-x'|)}$$

$$= \left(\frac{3}{4\pi R^3}\right)^2 \underbrace{\int_R d^3x\, d^3x'}\; \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')} P(k)$$

$$= \int \frac{d^3k}{(2\pi)^3} P(k) \underbrace{\left(\int_R \frac{d^3x}{V} e^{ik\cdot x}\right)}_{W(kR)}\underbrace{\left(\int_R \frac{d^3x}{V} e^{-ik\cdot x}\right)}_{W}$$

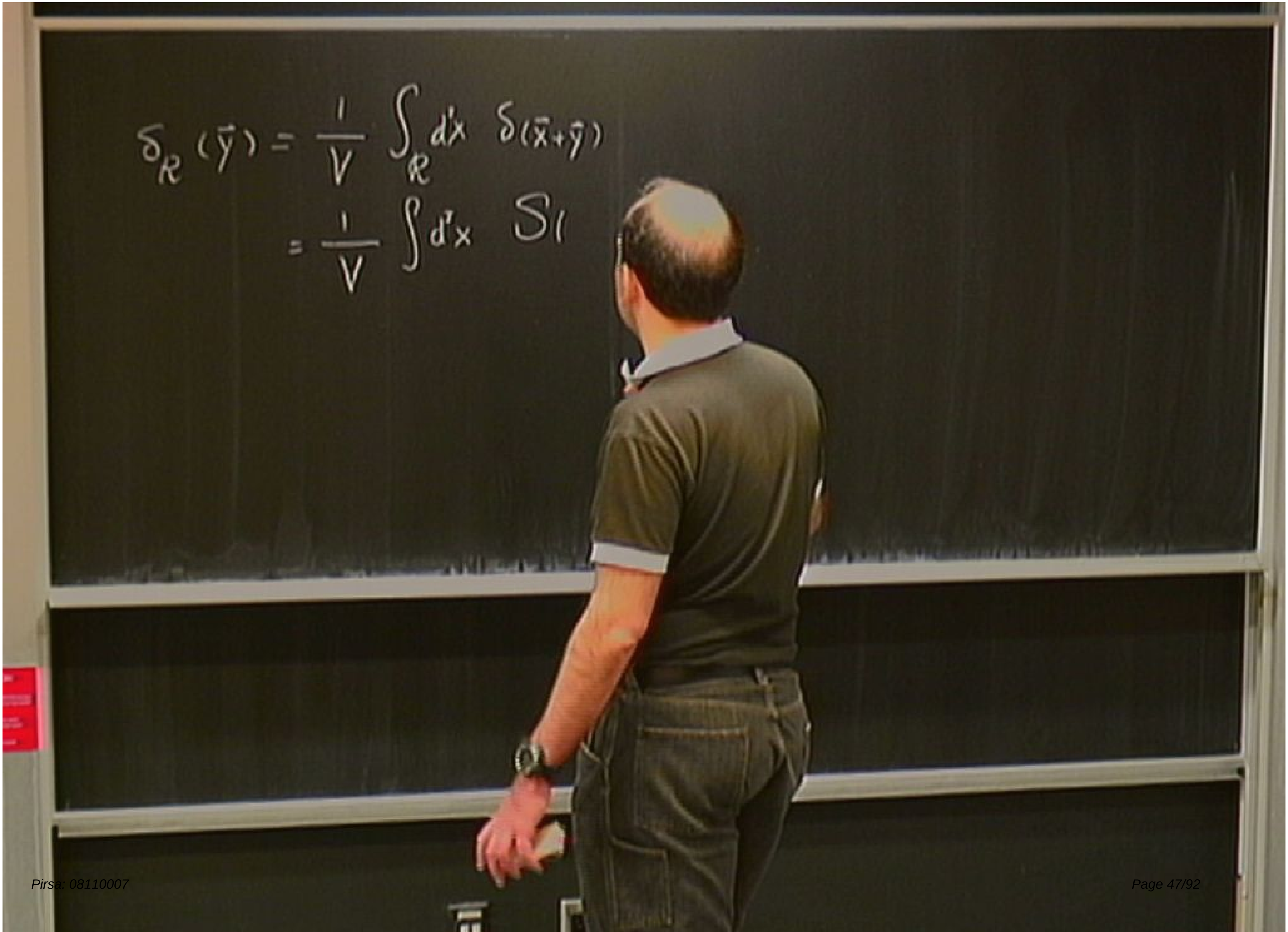
$$\delta_R(\vec{x}_0) = \frac{3}{4\pi R^3}\int_R d^3x \; \delta(\vec{x})$$

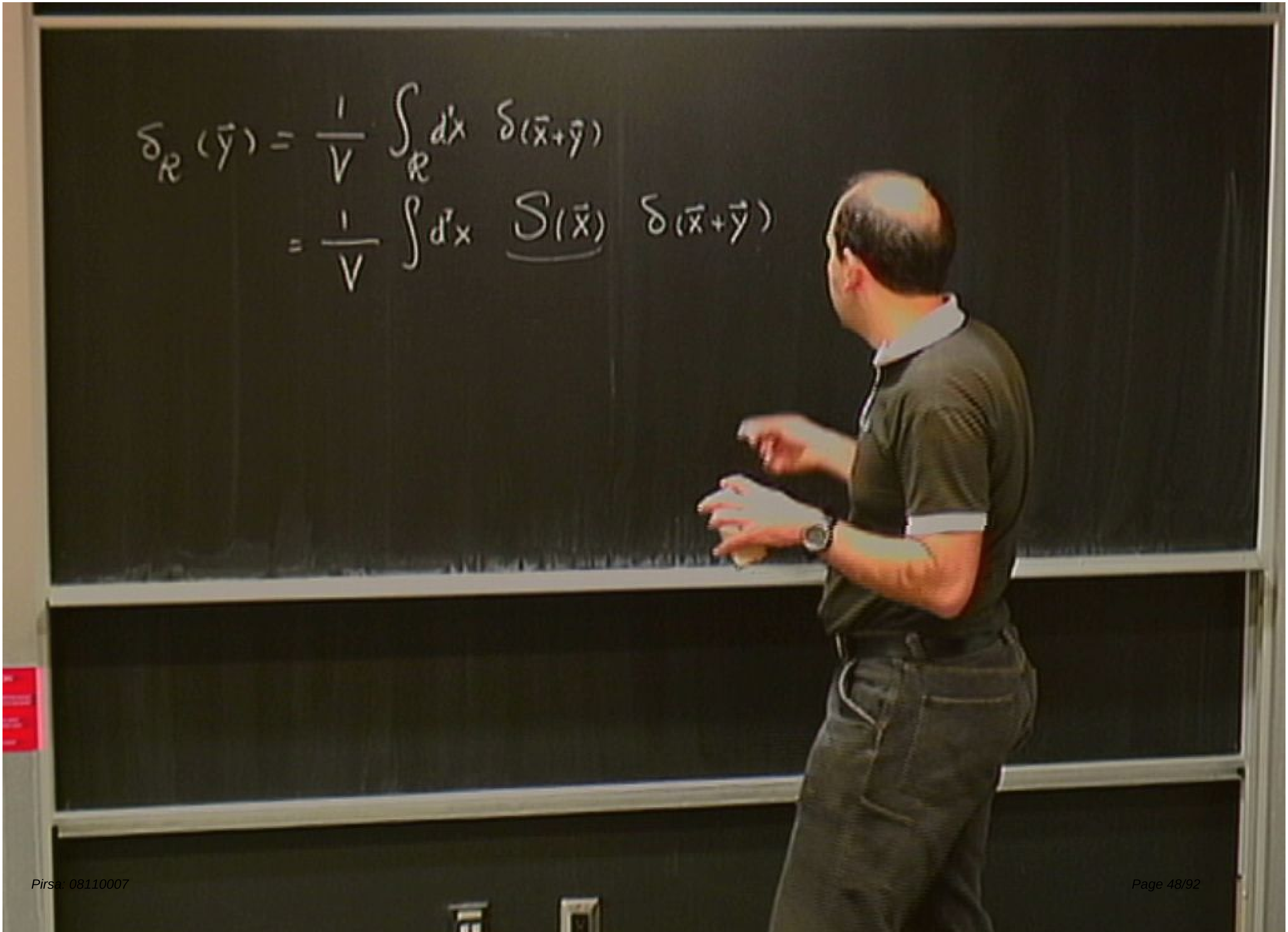
$$\delta_R = \frac{3}{4\pi R^3}\int_R d^3x' \; \delta(\vec{x}')$$

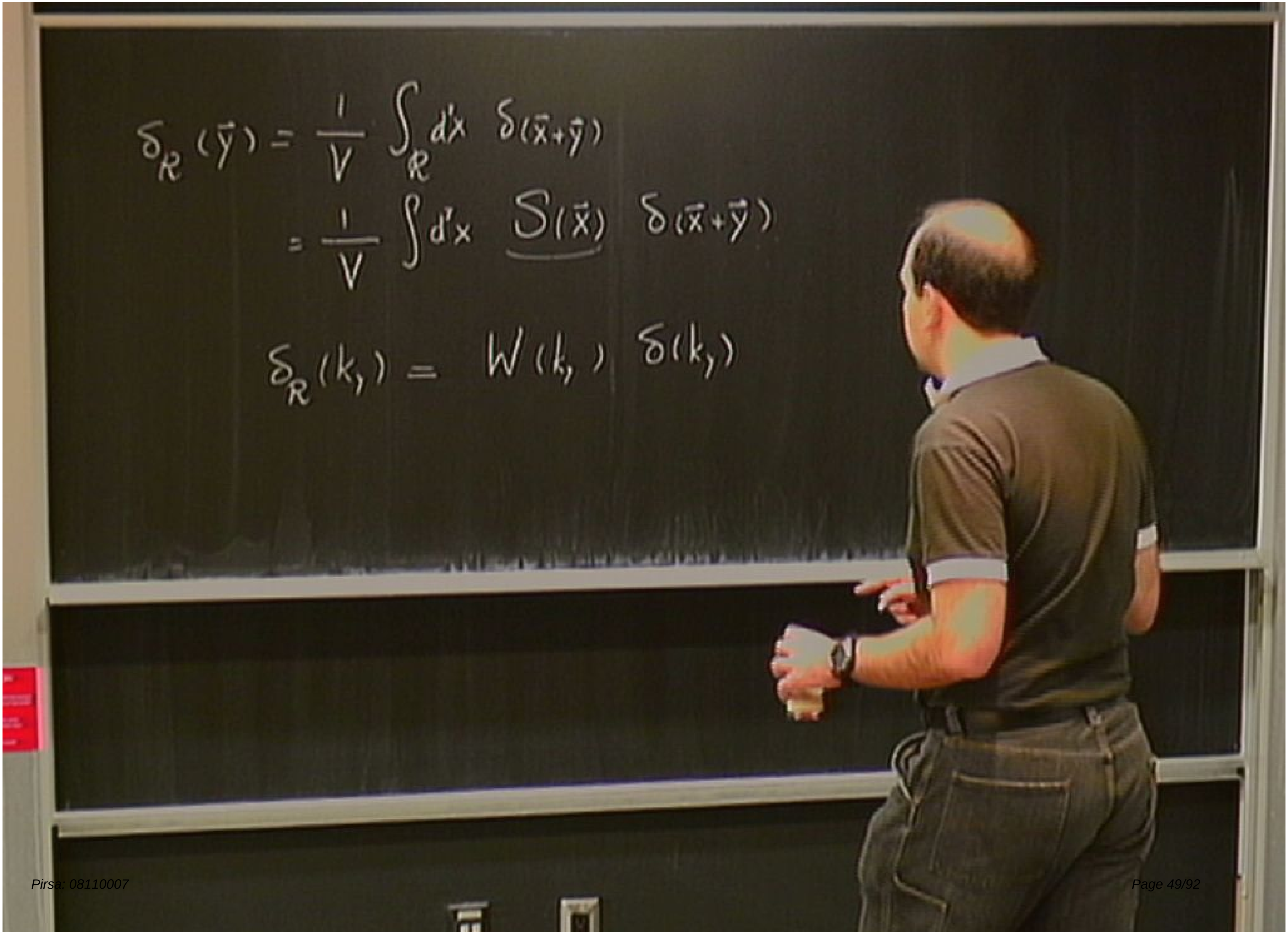
$$\langle \delta_R^2 \rangle = \left(\frac{3}{4\pi R^3}\right)^2 \int_R d^3x \, d^3x' \; \underbrace{\langle \delta(x')\delta(x') \rangle}_{\xi(|x-x'|)}$$

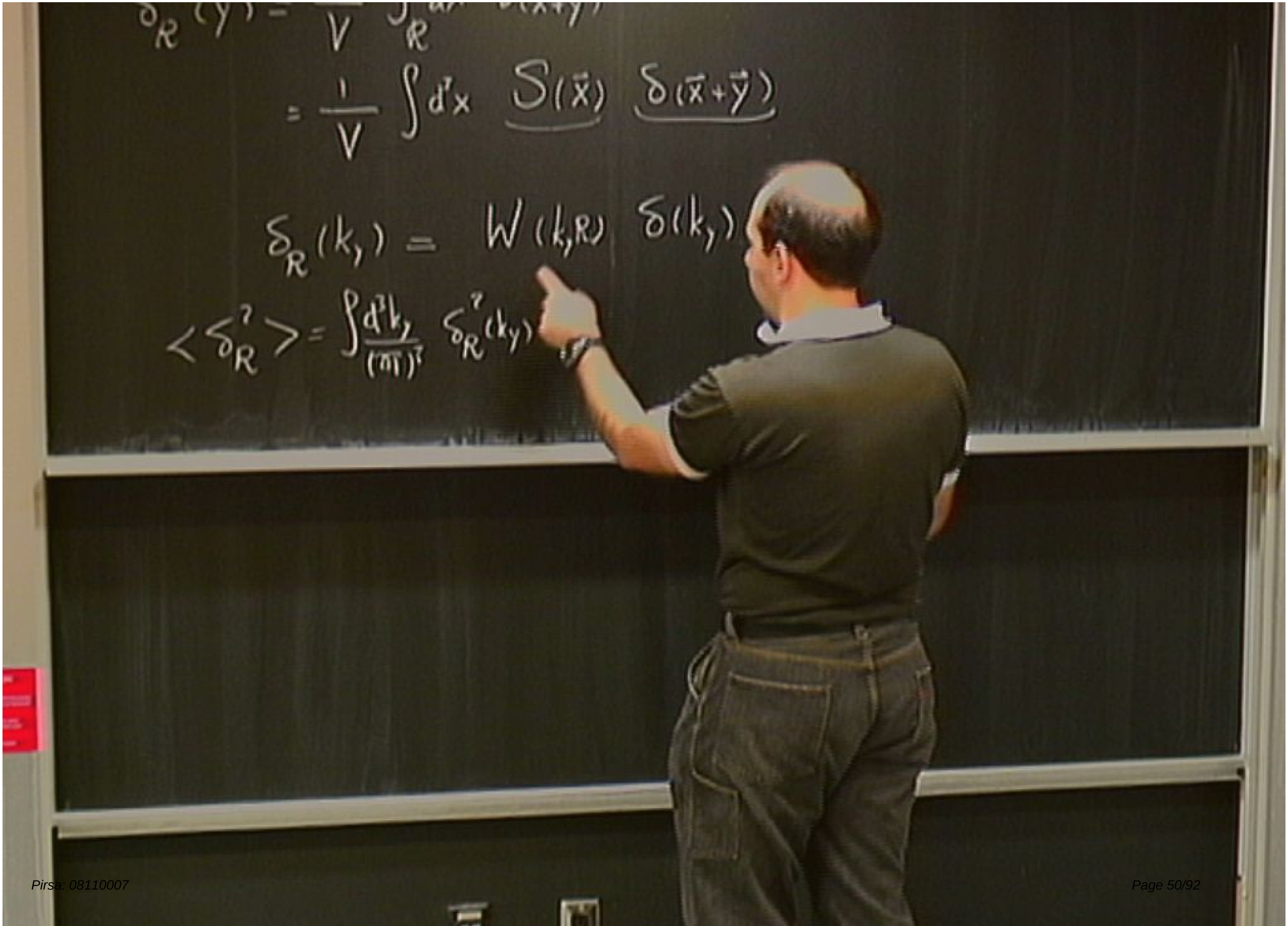
$$= \left(\frac{3}{4\pi R^3}\right)^2 \underbrace{\int_R d^3x \, d^3x'}\; \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')} P(k)$$

$$= \int \frac{d^3k}{(2\pi)^3} P(k) \underbrace{\left(\int_R \frac{d^3x}{V} e^{ik\cdot x}\right)}_{W(kR)} \underbrace{\left(\int_R \frac{d^3x}{V} e^{-ik\cdot x'}\right)}_{W^*(kR)}$$

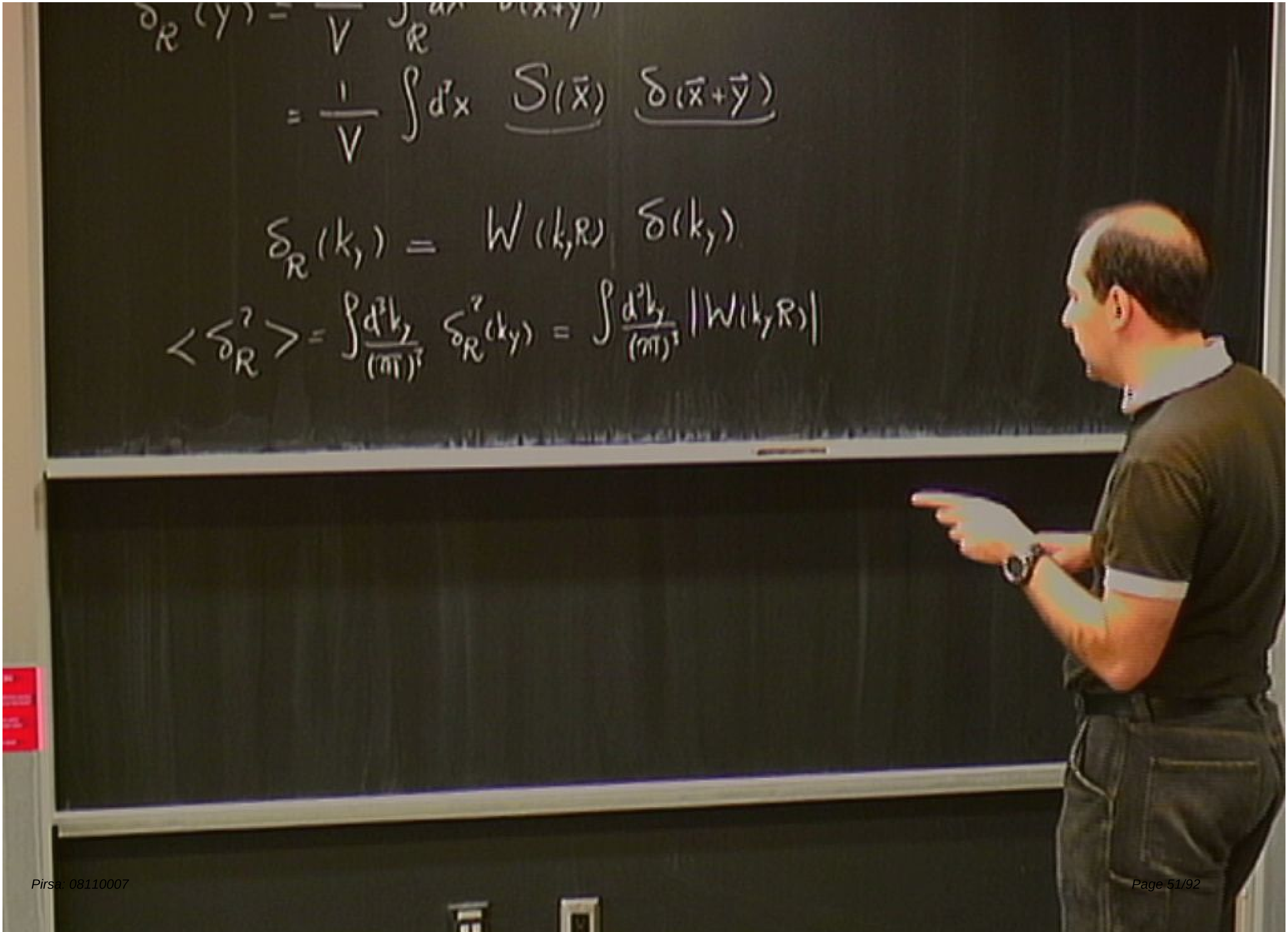

$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x \; \delta(\vec{x}+\vec{y})$
$= \frac{1}{V}\int d^3x \; \delta($


$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x \, \delta(\vec{x}+\vec{y})$
$= \frac{1}{V}\int d^3x \, S(\vec{x}) \, \delta(\vec{x}+\vec{y})$


$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x\ \delta(\vec{x}+\vec{y})$
$= \frac{1}{V}\int d^3x\ \underline{S(\vec{x})}\ \delta(\vec{x}+\vec{y})$
$\delta_R(k_y) = W(k_y)\ \delta(k_y)$



$= \frac{1}{V}\int d^3x \; S(\vec{x}) \; \delta(\vec{x}+\vec{y})$
$\delta_R(k_y) = W(k_y R) \; \delta(k_y)$
$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \; \delta_R^2(k_y)$

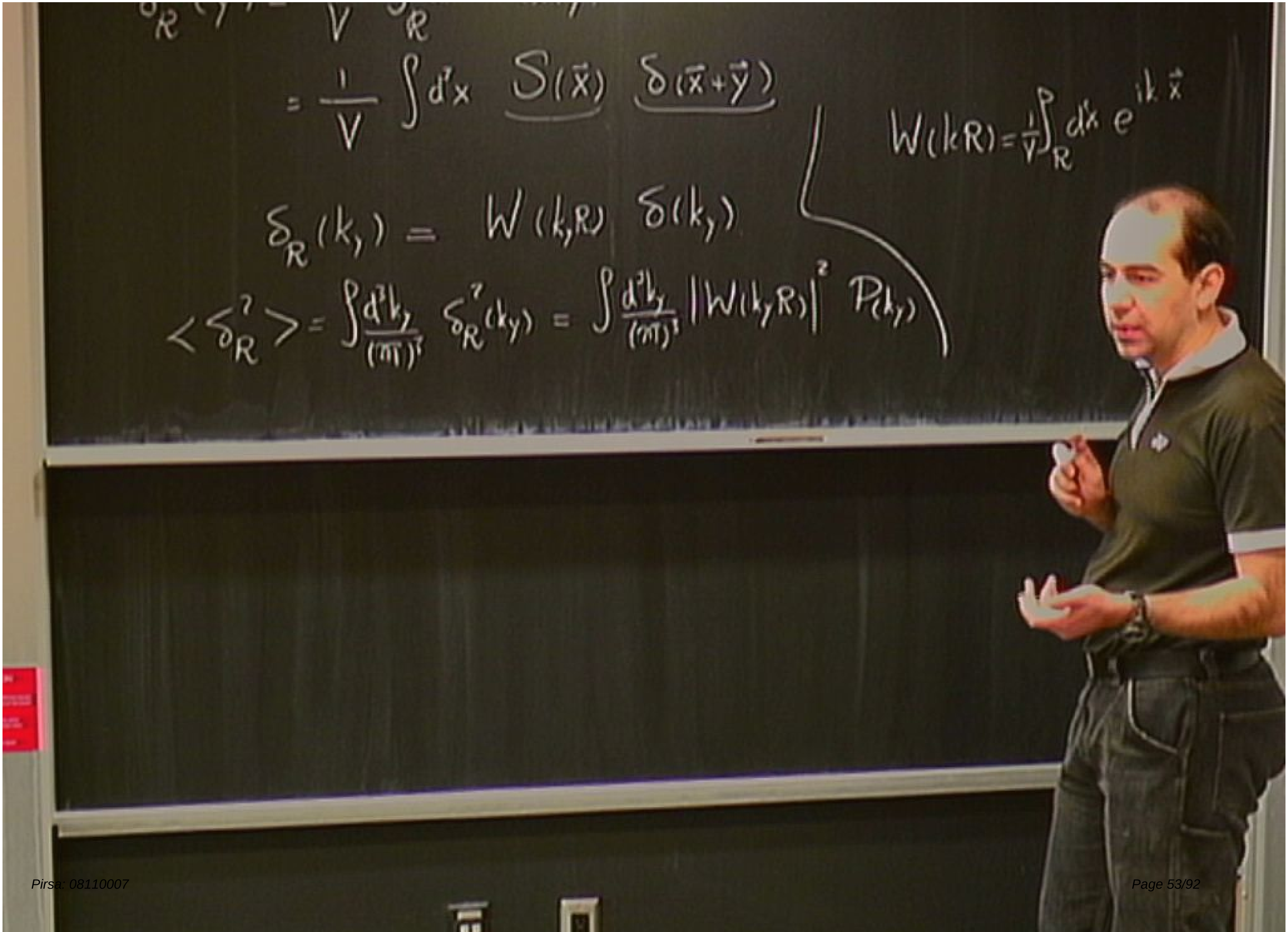

$= \frac{1}{V} \int d^3x \; S(\vec{x}) \; \delta(\vec{x}+\vec{y})$
$\delta_R(k_y) = W(k,R) \; \delta(k_y)$
$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3} \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y R)|$

$$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x\ \delta(\vec{x}+\vec{y})$$

$$= \frac{1}{V}\int d^3x\ \underline{S(\vec{x})}\ \underline{\delta(\vec{x}+\vec{y})}$$

$$\delta_R(k_y) = W(k_y R)\ \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\ \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y R)|^2\ P(k_y)$$


$= \frac{1}{V}\int d^3x\, \delta(\vec{x})\, \delta(\vec{x}+\vec{y})$
$W(kR) = \frac{1}{V}\int_R d^3x\, e^{ik\cdot\vec{x}}$
$\delta_R(k_y) = W(k_y R)\, \delta(k_y)$
$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\, \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y R)|^2\, P(k_y)$

$$\delta_R(\vec{y}) = \frac{1}{V}\int_R$$

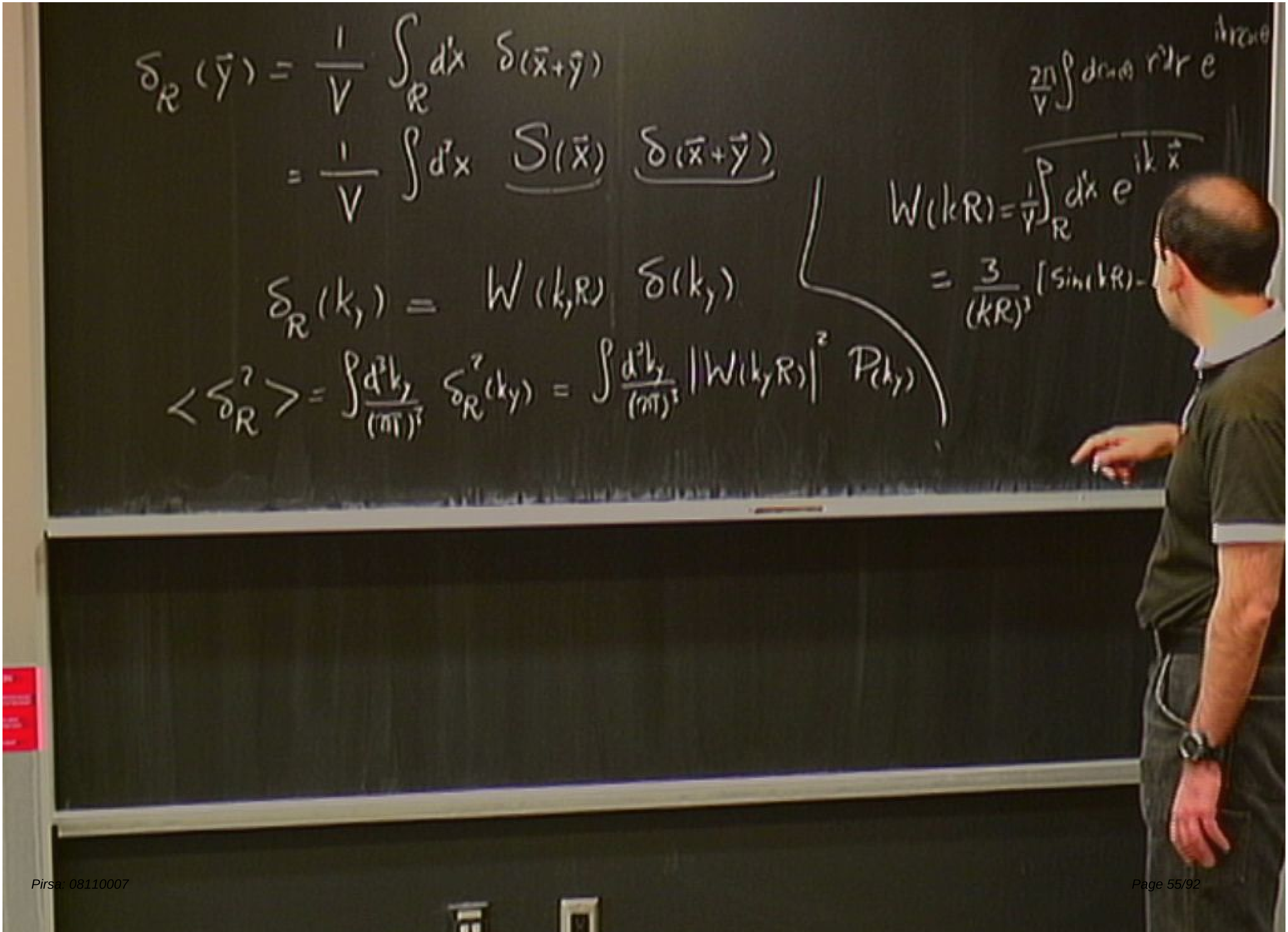
$$= \frac{1}{V}\int d^3x \; \underline{\delta(\vec{x})} \; \underline{\delta(\vec{x}+\vec{y})}$$

$$W(kR) = \frac{1}{V}\int_R d^3x \, e^{i\vec{k}\cdot\vec{x}}$$

$$= \frac{3}{(kR)^3}\left[\sin(kR) - kR\cos(kR)\right]$$

$$\delta_R(k_y) = W(k_y R)\,\delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\,\delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3}\,|W(k_y R)|^2\, P(k_y)$$



$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x\ \delta(\vec{x}+\vec{y})$
$= \frac{1}{V}\int d^3x\ S(\vec{x})\ \delta(\vec{x}+\vec{y})$
$\delta_R(k_y) = W(k_yR)\ \delta(k_y)$
$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\ \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_yR)|^2\ P(k_y)$
$\frac{2\pi}{V}\int d\cos\theta\ r^2 dr\ e^{ikr\cos\theta}$
$W(kR) = \frac{1}{V}\int_R d^3x\ e^{i\vec{k}\cdot\vec{x}}$
$= \frac{3}{(kR)^3}[\sin(kR) -$

$$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x\; \delta(\vec{x}+\vec{y})$$

$$= \frac{1}{V}\int d^3x\; \underline{S(\vec{x})}\; \underline{\delta(\vec{x}+\vec{y})}$$

$$\delta_R(k_y) = W(k_y R)\; \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\, \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3}\, |W(k_y R)|^2\, P(k_y)$$

$$\frac{2\pi}{V}\int d\cos\theta\; r^2 dr\; e^{ikr\cos\theta}$$

$$W(kR) = \frac{1}{V}\int_R d^3x\; e^{i\vec{k}\cdot\vec{x}}$$

$$= \frac{3}{(kR)^3}\left[\sin(kR) - kR\cos(kR)\right]$$

$$\delta_R(\vec{y}) = \frac{1}{V}\int_R d^3x \; \delta(\vec{x}+\vec{y})$$

$$= \frac{1}{V}\int d^3x \; \underline{S(\vec{x})} \; \underline{\delta(\vec{x}+\vec{y})}$$

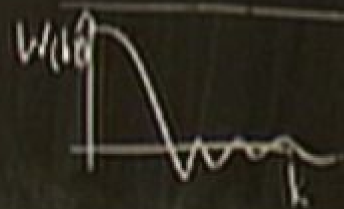
$$\delta_R(k_y) = W(k_y R)\; \delta(k_y)$$

$$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\, \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3}\, |W(k_y R)|^2 \, P(k_y)$$

$$\frac{2\pi}{V}\int d\cos\theta \; r^2 dr \; e^{ikr\cos\theta}$$

$$W(kR) = \frac{1}{V}\int_R d^3x \; e^{i\vec{k}\cdot\vec{x}}$$

$$= \frac{3}{(kR)^3}\left[\sin(kR) - kR\cos(kR)\right]$$



$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(k-k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r)\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\langle \delta_k\delta^*_{k'}\rangle \exp(-ikx)\exp(ik'(x+r$$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$P(\Delta^3 r; r) = (1+\xi(r))\,\bar{n}$$

$$\langle \delta(x)\,\delta(x+r)\rangle = \langle \delta(x)\rangle\langle \delta(x+r)\rangle \sim 0$$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r))$$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

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$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k-k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp$$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$\xi(r) = A\, \delta^3(\vec{r})$$

$$P(\Delta^3 r; r) = (1+\xi(r))\,\bar{n}$$

$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k-k')$$

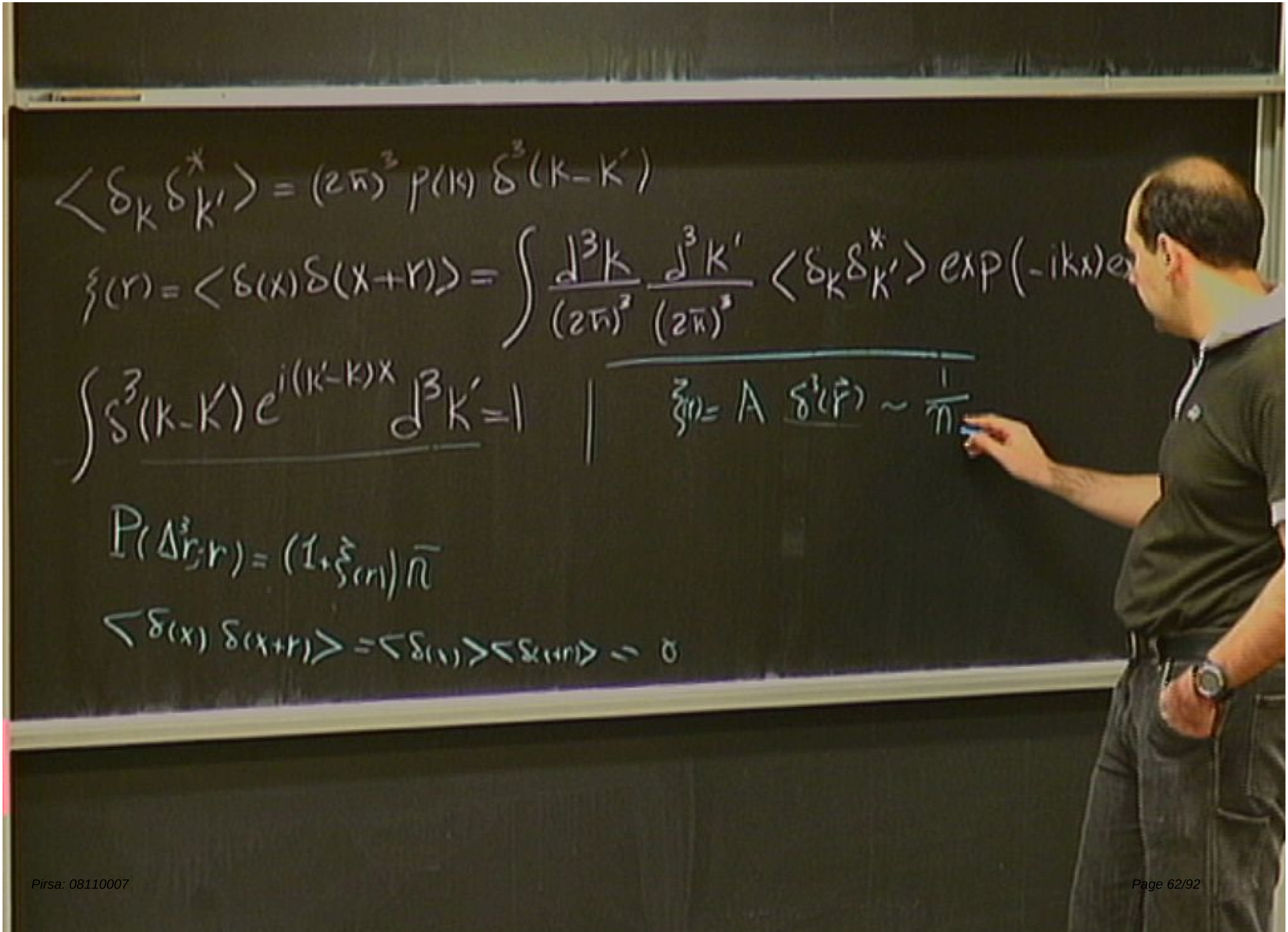
$$\xi(r) = \langle \delta(x)\,\delta(x+r)\rangle = \int \frac{d^3k}{(2\pi)^3}\,\frac{d^3k'}{(2\pi)^3}\,\langle \delta_k \delta^*_{k'}\rangle \exp(-ikx)\exp(ik'(x+r))$$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$\xi(r) = A\,\delta^3(\vec{r})$$

$$P(\Delta^3 r; r) = (1+\xi(r))\,\bar{n}$$

$$\langle \delta(x)\,\delta(x+r)\rangle = \langle \delta(x)\rangle\langle \delta(x+r)\rangle = 0$$


$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(k-k')$
$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)e$
$\int \delta^3(k-k)\, e^{i(k'-k)x}\, d^3k' = 1$
$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{\bar{n}}$
$P(\Delta^3 r; r) = (1+\xi(r))\, \bar{n}$
$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k-k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r))$$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{n_g}\delta^3(r)$$

$$P(\Delta^3 r; r) = (1+\xi(r))\,\bar{n}$$

$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k-k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r))$$

$$\int \delta^3(k-k') e^{i(k'-k)x} d^3k' = 1$$

$$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{\bar{n}_g} \delta^3(r)$$

$$P(\Delta^3 r; r) = (1+\xi(r))\bar{n}$$

$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\langle \delta_k \delta^*_{k'} \rangle = (2\pi)^3 P(|k|)\, \delta^3(k - k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \delta_k \delta^*_{k'} \rangle \exp(-ikx)\exp(ik'(x+r))$$

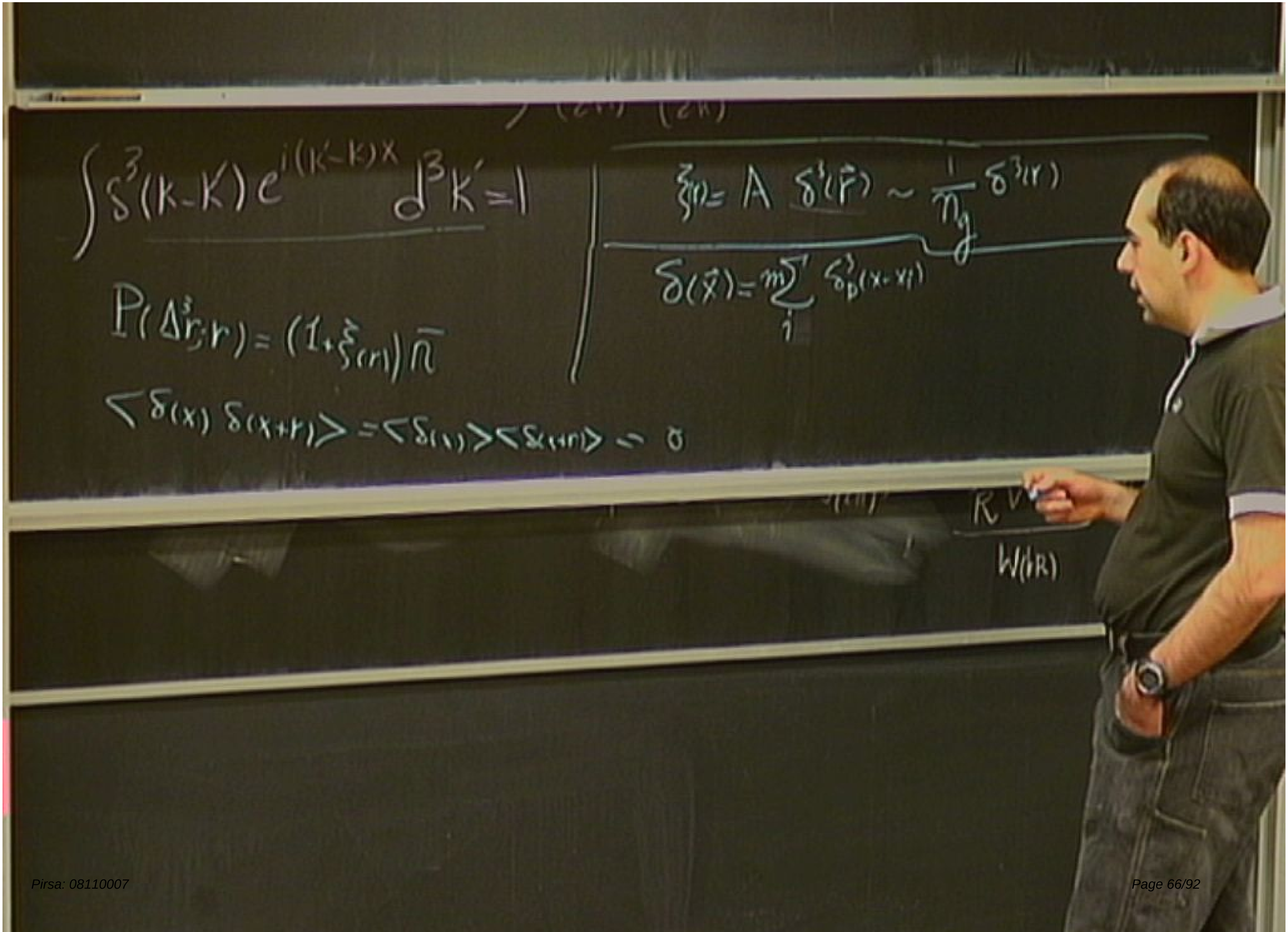
$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$P(\Delta^3 r_j; r) = (1 + \xi(r))\,\bar{n}$$

$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{n_g} \delta^3(r)$$

$$\delta(\vec{x}) = \sum_i \delta^3_D(x - x_i)$$


∫ δ³(k−k′) e^{i(k′−k)x} d³k′ = 1
P(Δ³r; r) = (1 + ξ(r)) n̄
⟨δ(x) δ(x+r)⟩ = ⟨δ(x)⟩⟨δ(x+r)⟩ = 0
ξ(r) = A δ³(r⃗) ~ 1/n_g δ³(r)
δ(x⃗) = m Σ_i δ³_D(x − x_i)
W(kR)

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$P(\Delta^3 r_j; r) = (1+\xi(r))\,\bar{n}$$

$$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$$

$$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{n_g}\, \delta^3(r)$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}} \sum_i \delta_D^3(x - x_i) - 1$$

$$\frac{R^V}{W(kR)} \quad \frac{R^V}{W'(kR)}$$

$$\int \delta^3(k'-k)\, e^{i(k'-k)x}\, d^3k' = 1$$

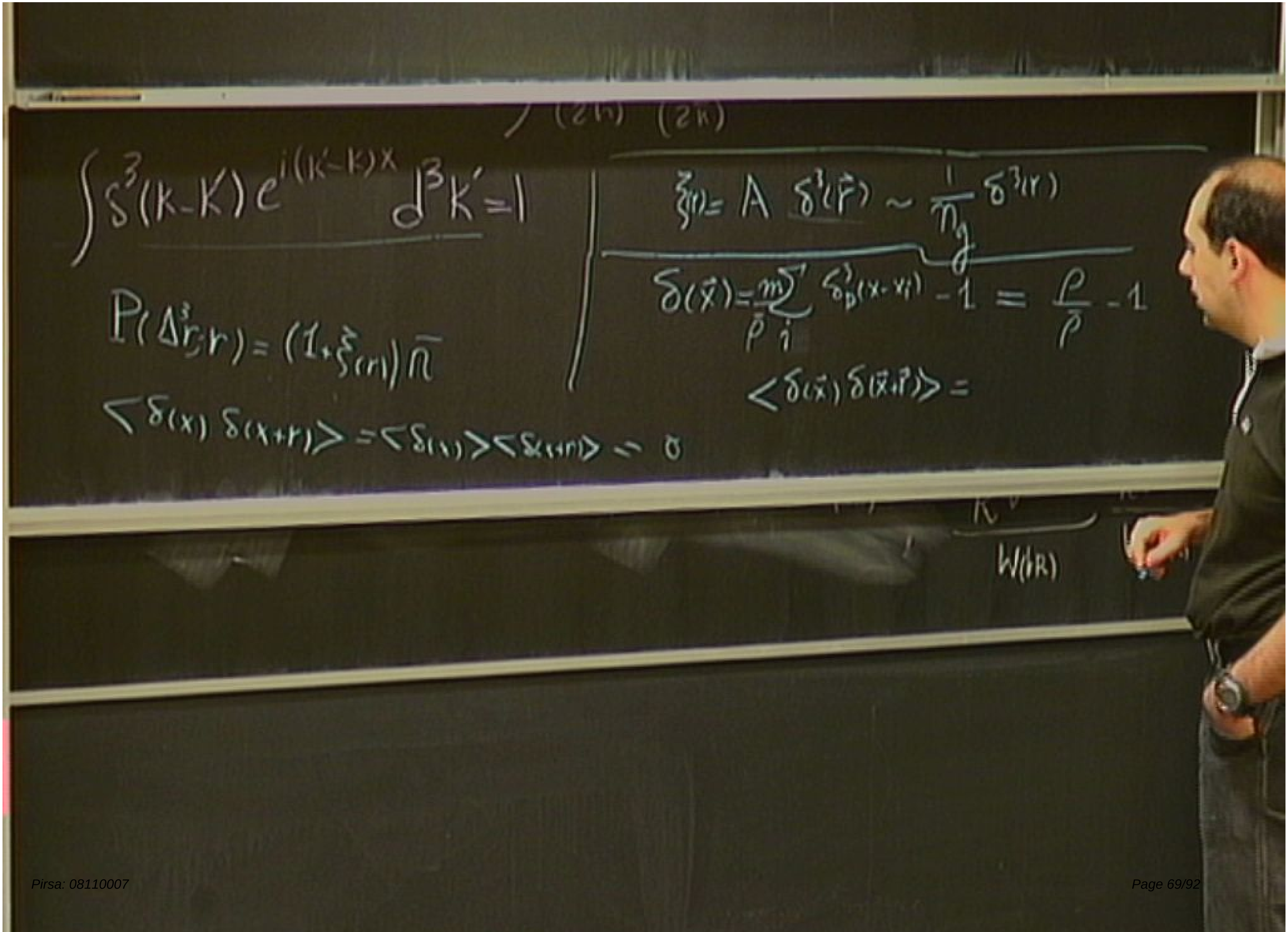
$$P(\Delta^3 r_j; r) = (1+\xi(r))\,\bar{n}$$

$$\langle \delta(x)\,\delta(x+r)\rangle = \langle \delta(x)\rangle\langle \delta(x+r)\rangle = 0$$

$$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{n_g}\delta^3(r)$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}}\sum_i \delta_D^3(x-x_i) - 1 = \frac{\rho}{\bar{\rho}} - 1$$

$$\frac{K^{\nu}}{W(kR)} \qquad \frac{K^{\nu}}{W'(kR)}$$


$\int \delta^3(k-k') e^{i(k'-k)x} d^3k' = 1$
$\xi(r) = A\, \delta^3(\vec{r}) \sim \frac{1}{\bar{n}_g} \delta^3(r)$
$P(\Delta^3 r_j; r) = (1+\xi(r))\, \bar{n}$
$\delta(\vec{x}) = \frac{m}{\bar{\rho}} \sum_i \delta_D^3(x - x_i) - 1 = \frac{\rho}{\bar{\rho}} - 1$
$\langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle =$
$\langle \delta(x)\, \delta(x+r) \rangle = \langle \delta(x) \rangle \langle \delta(x+r) \rangle = 0$
$W(kR)$

$$\int \delta^3(k-k')\, e^{i(k'-k)x}\, d^3k' = 1$$

$$P(\Delta^3 r_j; r) = (1+\xi(r))\,\bar{n}$$

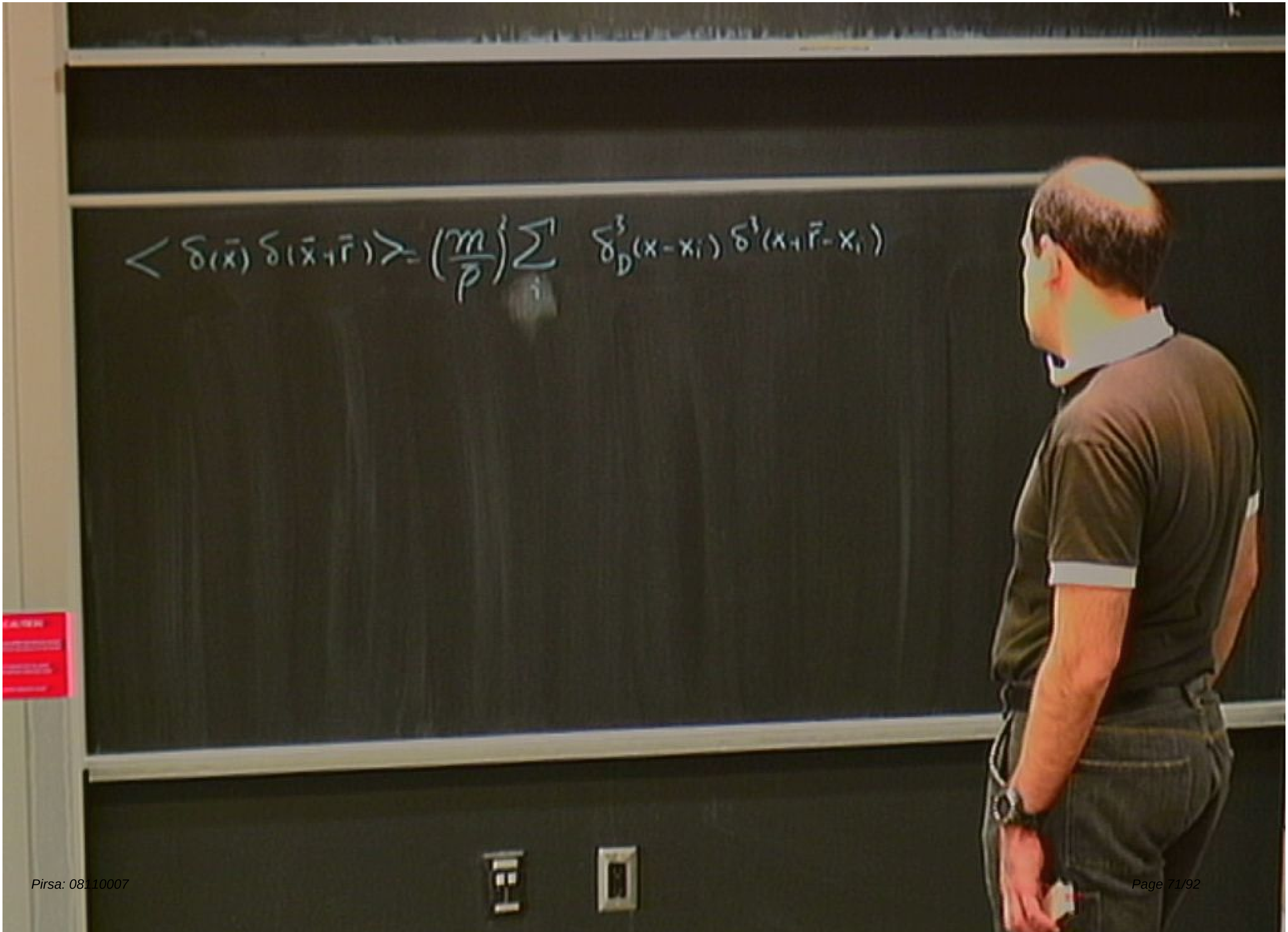
$$\langle \delta(x)\,\delta(x+r)\rangle = \langle \delta(x)\rangle \langle \delta(x+r)\rangle = 0$$

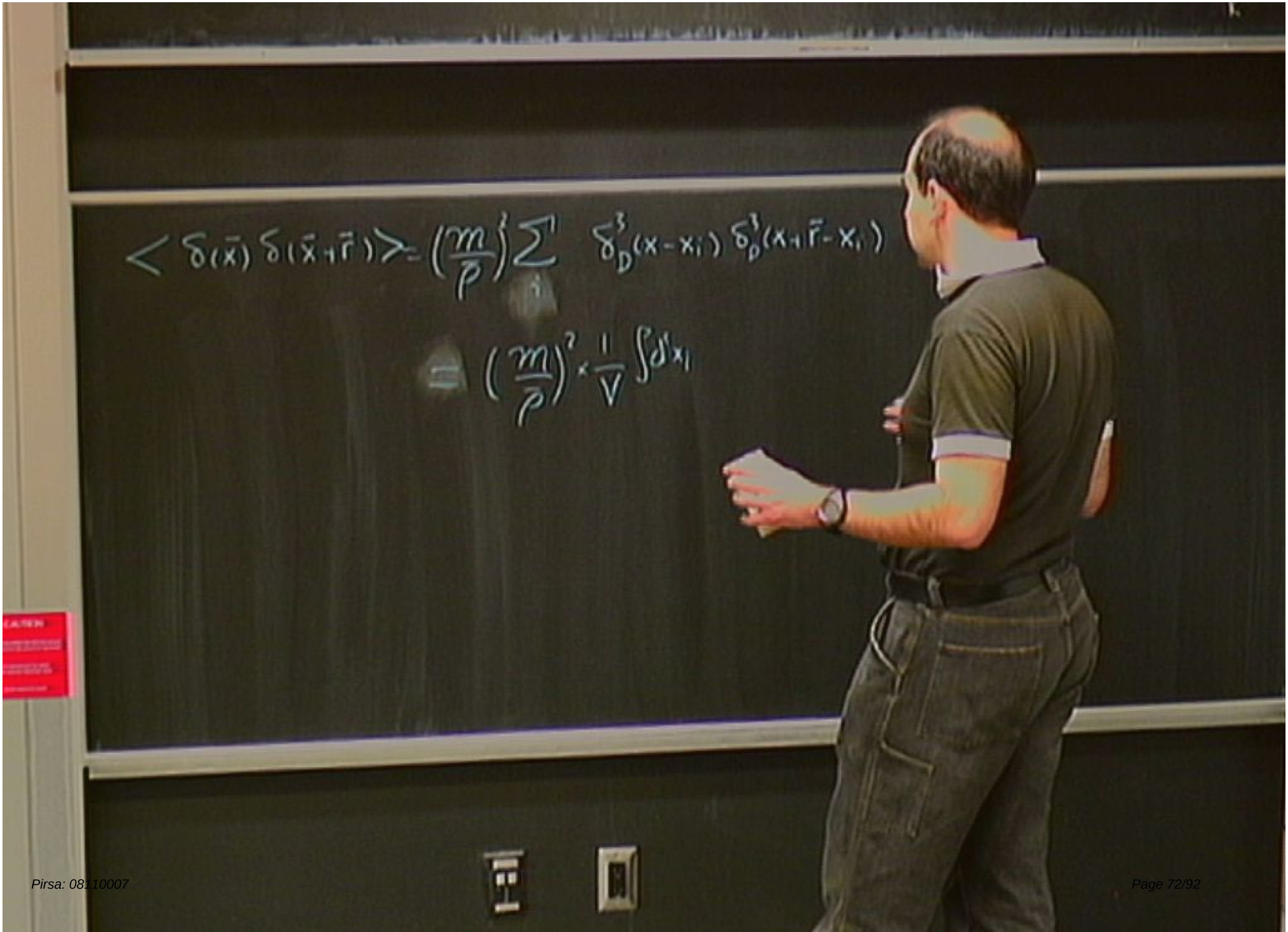
$$\xi(r) = A\,\delta^3(\vec{r}) \sim \frac{1}{n_g}\,\delta^3(r)$$

$$\delta(\vec{x}) = \frac{m}{\bar{\rho}}\sum_i \delta_D^3(x-x_i) - 1 = \frac{\rho}{\bar{\rho}} - 1$$

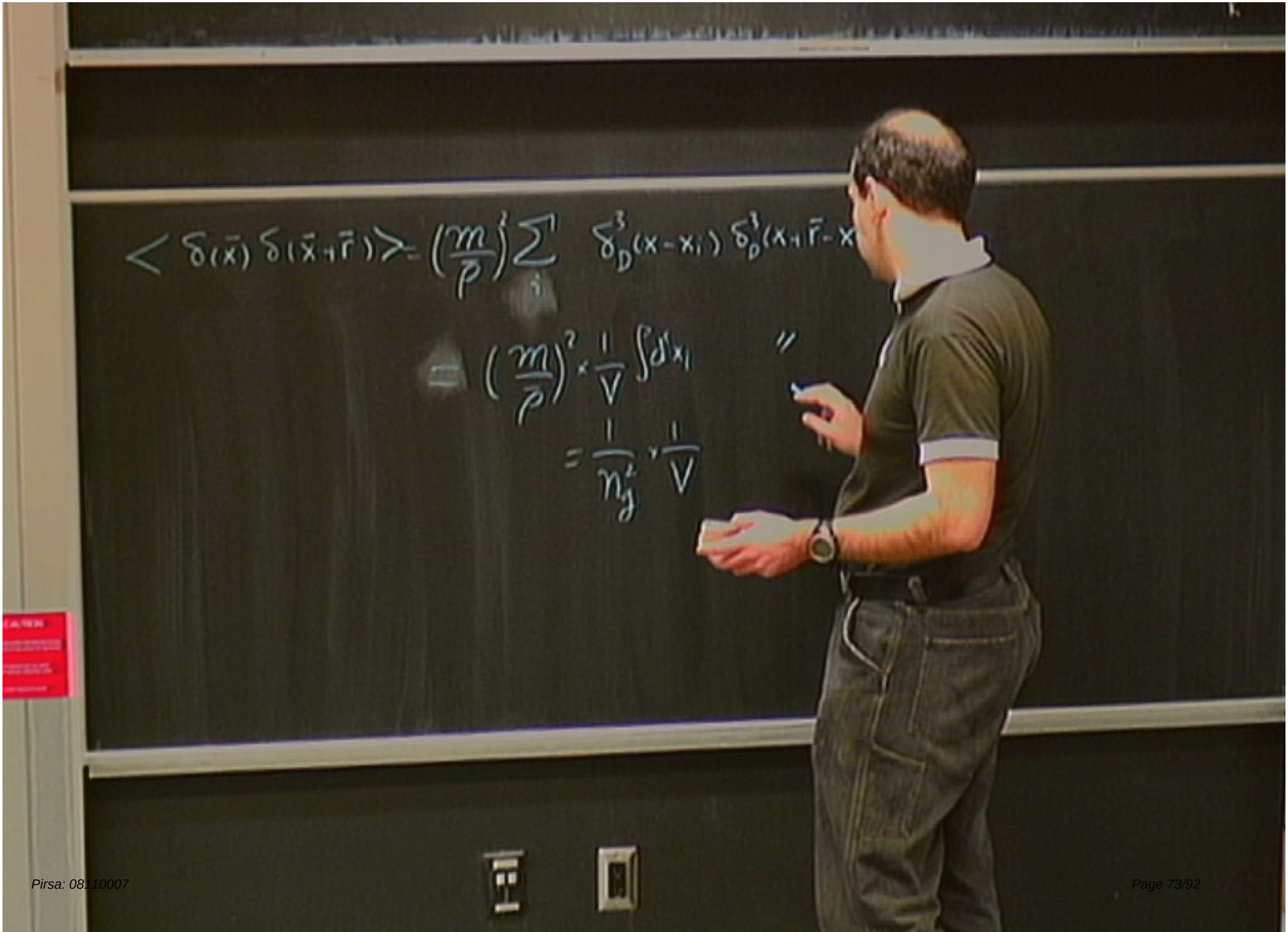
$$\langle \delta(\vec{x})\,\delta(\vec{x}+\vec{r})\rangle = \left(\frac{m}{\bar{\rho}}\right)^2 \sum_i \left[\delta_D^3(x-x_i)\right]^2$$

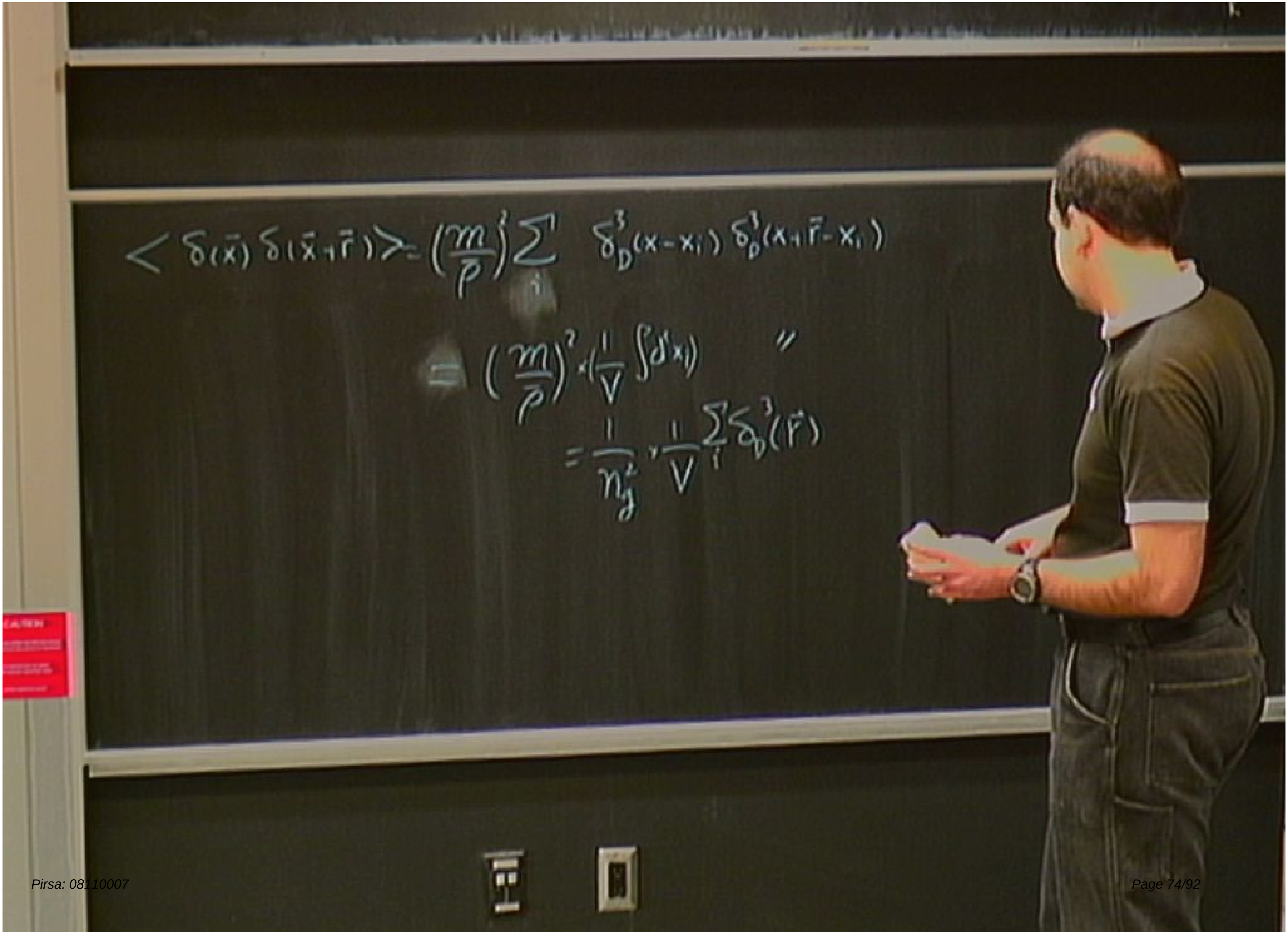
$W(kR)$ $\quad W^2(kR)$

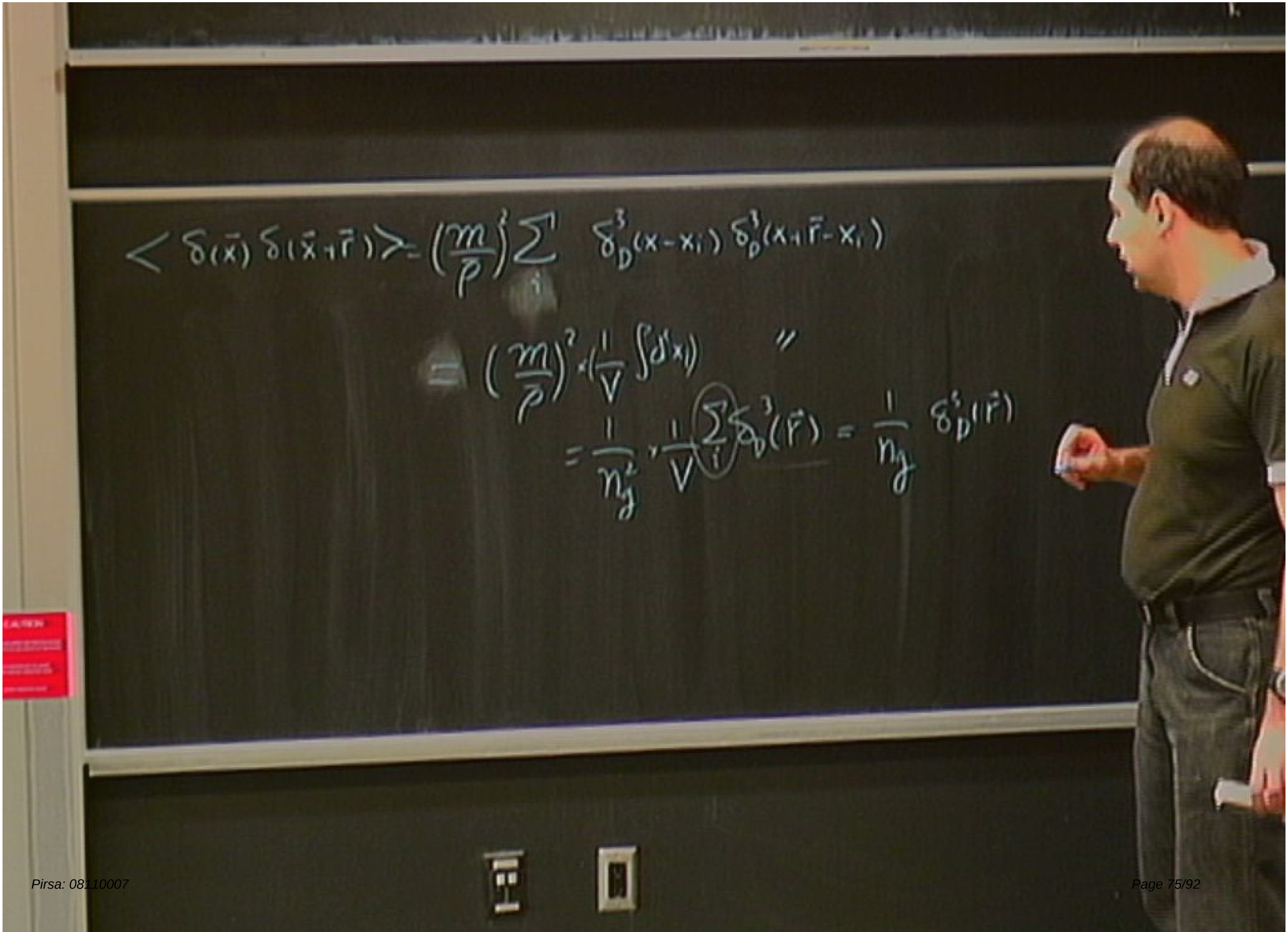

$\langle \delta(\vec{x})\,\delta(\vec{x}+\vec{r})\rangle = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(x-x_i)\,\delta^3(x+\vec{r}-x_i)$

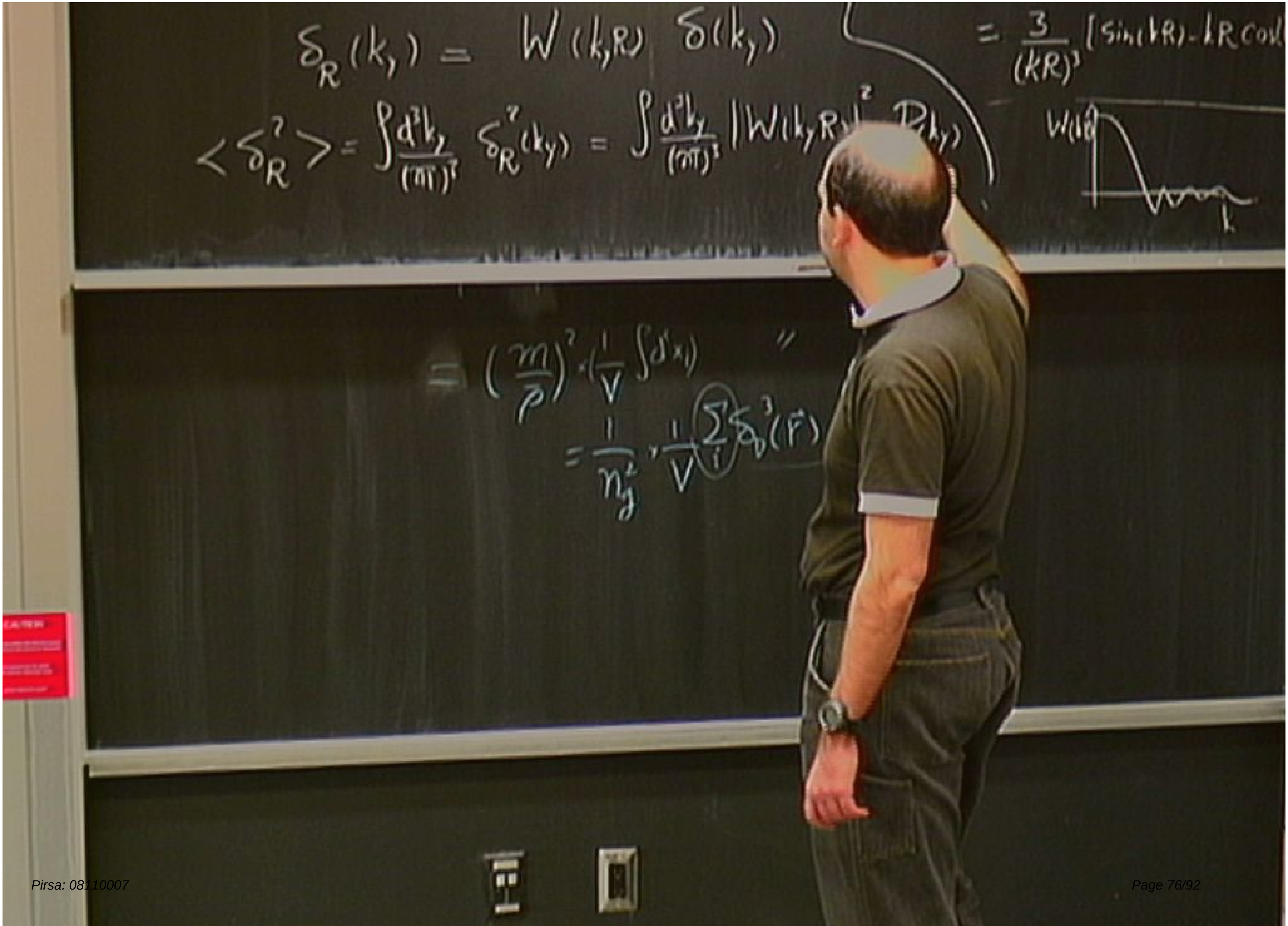


$< \delta(\bar{x})\, \delta(\bar{x}+\bar{r}) > = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(x-x_i)\, \delta_D^3(x+\bar{r}-x_i)$
$= \left(\frac{m}{\rho}\right)^2 \times \frac{1}{V} \int d^3x_i$

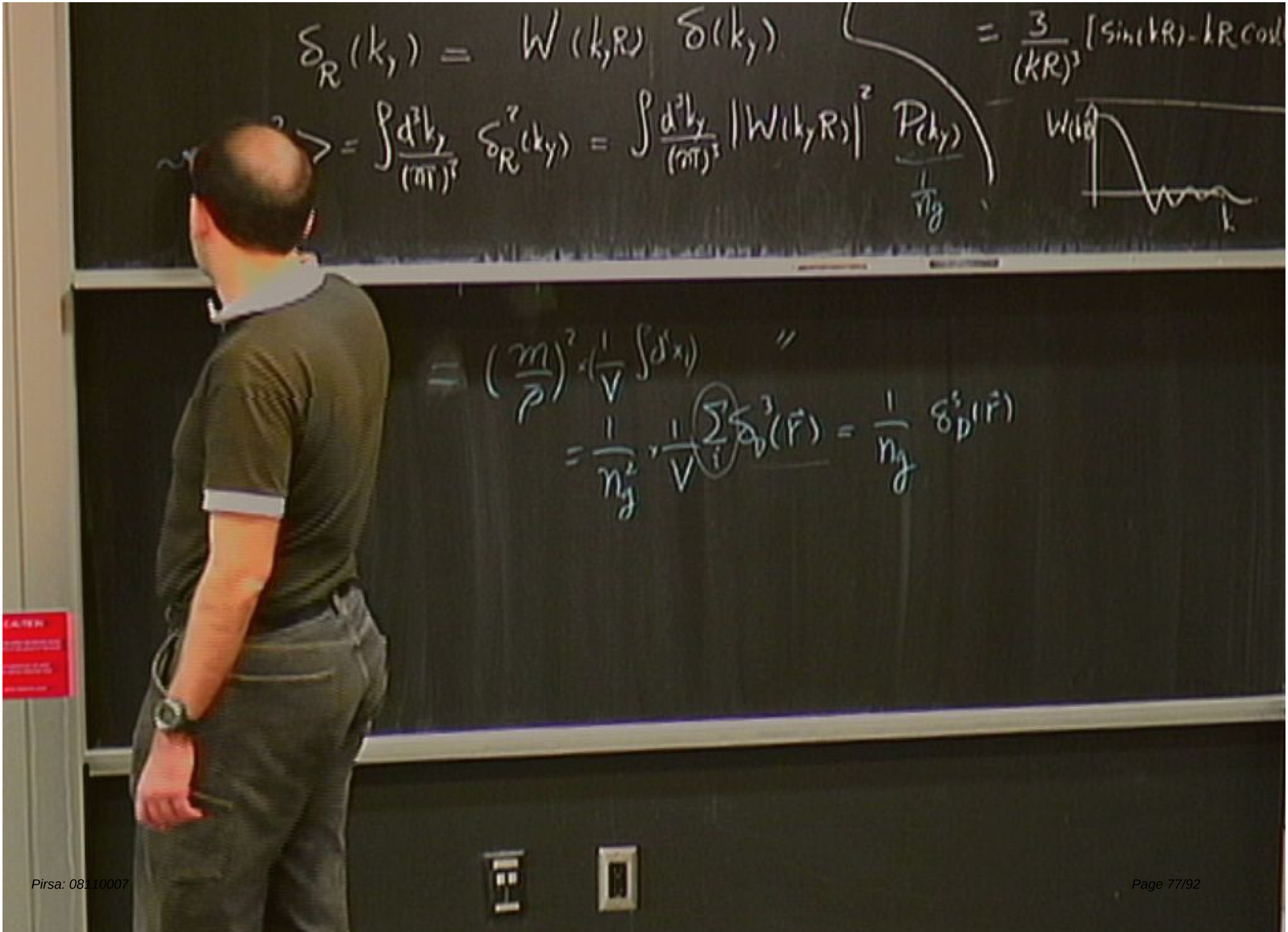

$\langle \delta(\vec{x})\,\delta(\vec{x}+\vec{r})\rangle = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(x-x_i)\,\delta_D^3(x+\vec{r}-x$
$= \left(\frac{m}{\rho}\right)^2 \times \frac{1}{V}\int d^3x_1$ "
$= \frac{1}{n_g^2}\times\frac{1}{V}$

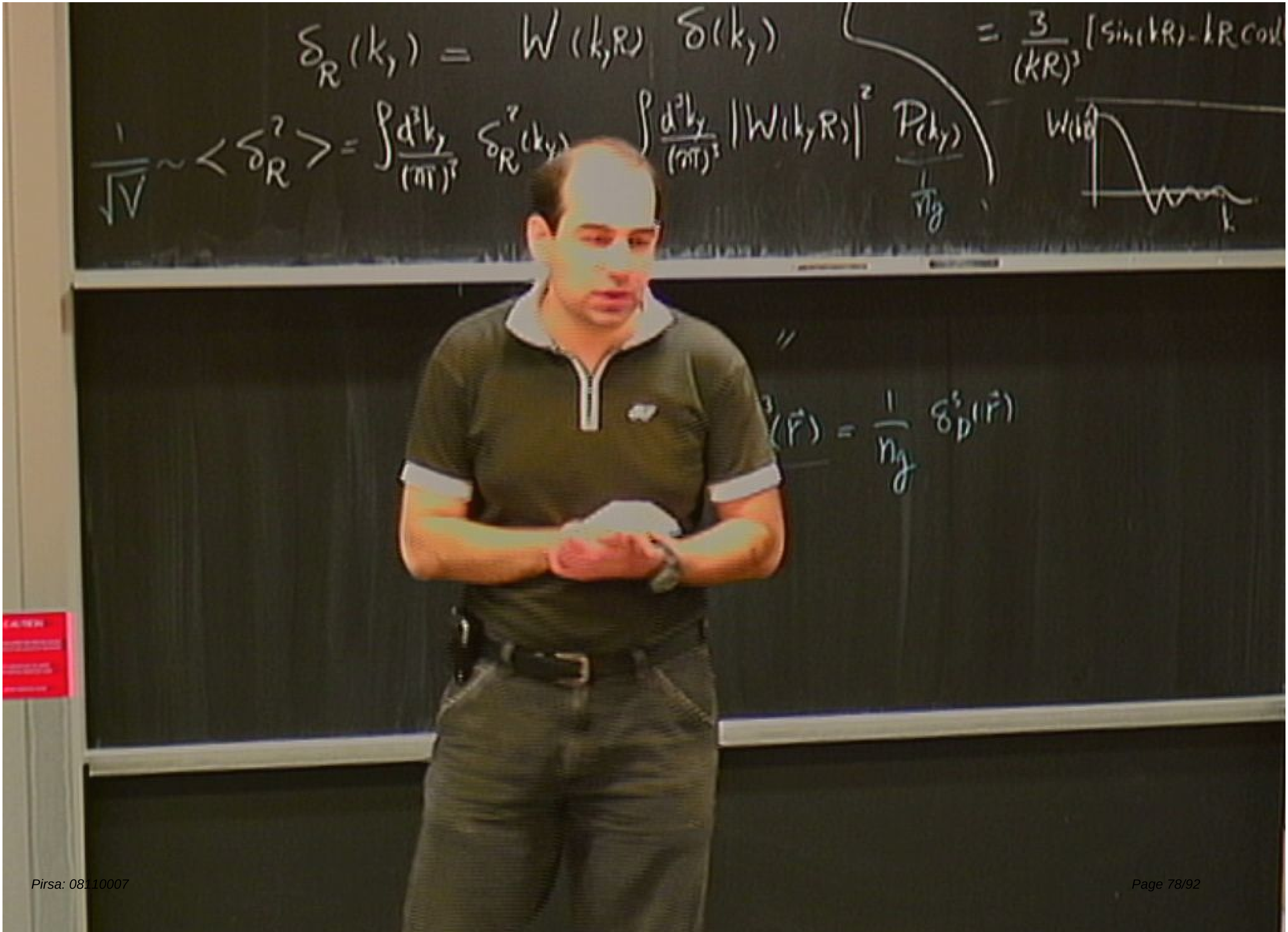

$< \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) > = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(x-x_i)\, \delta_D^3(x+\vec{r}-x_i)$
$= \left(\frac{m}{\rho}\right)^2 \times \left(\frac{1}{V}\int d^3x_i\right)$ "
$= \frac{1}{n_g^2} \times \frac{1}{V} \sum_i \delta_D^3(\vec{r})$


$\langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(x-x_i)\, \delta_D^3(x+\vec{r}-x_i)$
$= \left(\frac{m}{\rho}\right)^2 \left(\frac{1}{V}\int d^3x_i\right)$ "
$= \frac{1}{n_g^2} \cdot \frac{1}{V} \sum_i \delta_D^3(\vec{r}) = \frac{1}{n_g}\, \delta_D^3(\vec{r})$



$\delta_R(k_y) = W(k,R)\,\delta(k_y)$
$= \frac{3}{(kR)^3}[\sin(kR) - kR\cos$
$\langle \delta_R^2 \rangle = \int \frac{d^3k_y}{(2\pi)^3}\,\delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3}\,|W(k_y R)|^2\,P(k_y)$
$W(k)$
k
$= \left(\frac{m}{\rho}\right)^2 \left(\frac{1}{V}\int d^3x_i\right)$
$= \frac{1}{n_g^2}\,\frac{1}{V}\sum_i \delta_D^3(\vec{r})$

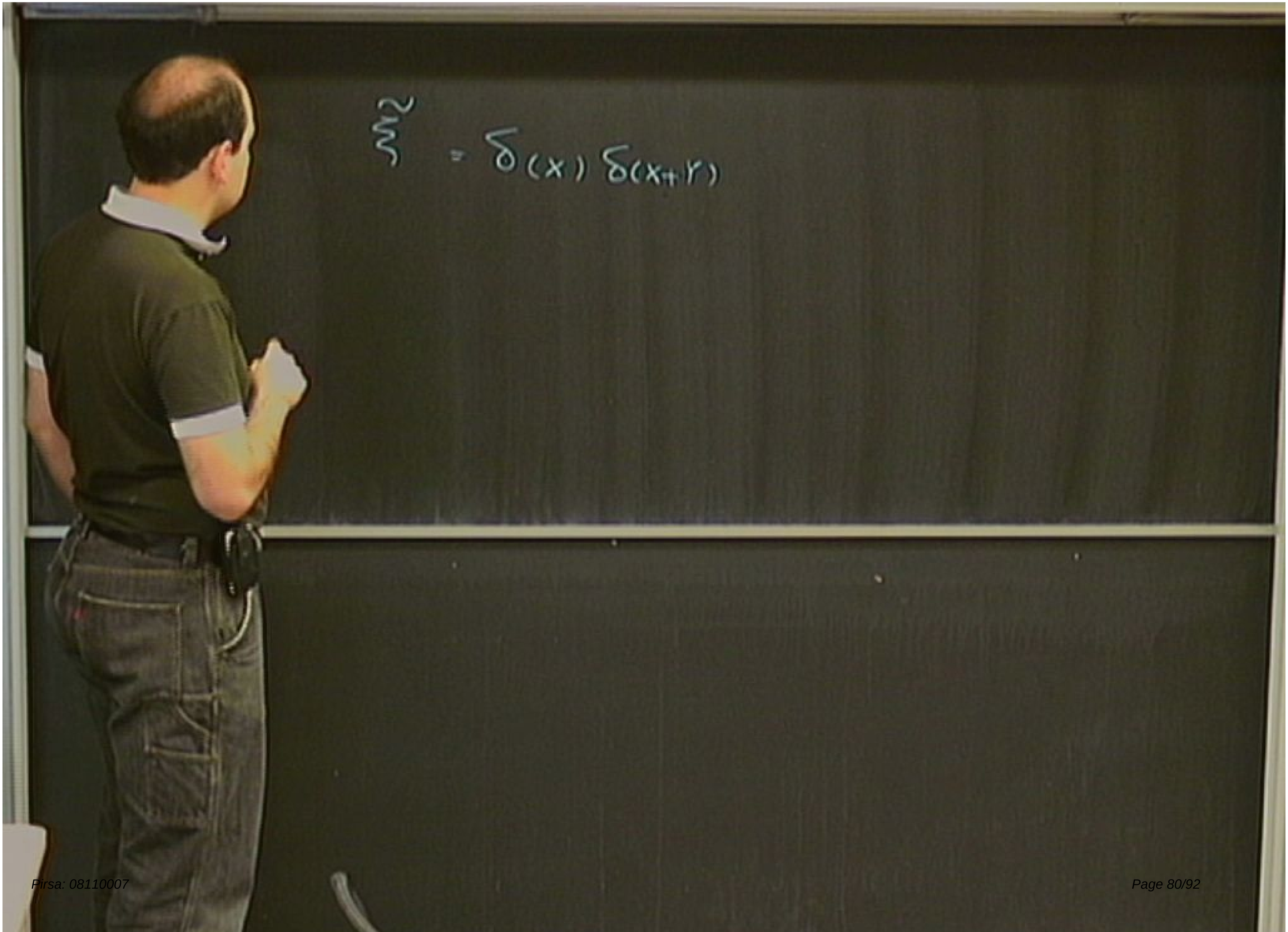

$\delta_R(k_y) = W(k_y R)\ \delta(k_y)$
$= \frac{3}{(kR)^3}[\sin(kR) - kR\cos$
$= \int \frac{d^3k_y}{(2\pi)^3}\ \delta_R^2(k_y) = \int \frac{d^3k_y}{(2\pi)^3} |W(k_y R)|^2\ P(k_y)$
$\frac{1}{n_g}$
$W(kR)$
k
$= \left(\frac{m}{\rho}\right)^2 \cdot \left(\frac{1}{V}\int d^3x_1\right)$
$= \frac{1}{n_g^2} \cdot \frac{1}{V} \sum_i \delta_D^3(\vec{r}) = \frac{1}{n_g}\ \delta_D^3(\vec{r})$

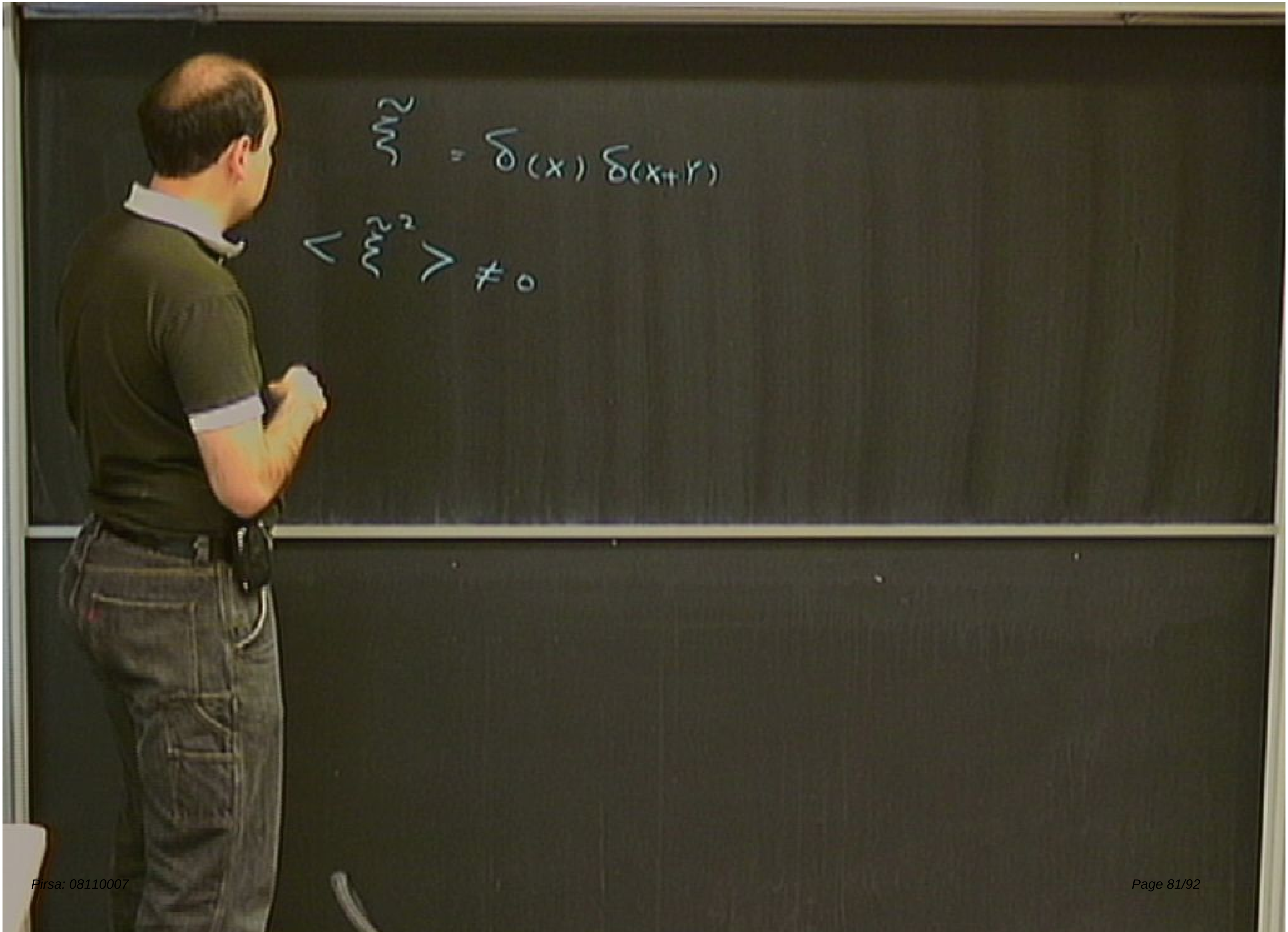

δR(ky) = W(k,R) δ(ky)
= 3/(kR)³ [sin(kR) - kR co
1/√V ~ <δR²> = ∫d³ky/(2π)³ δR²(ky)
∫d³ky/(2π)³ |W(ky,R)|² P(ky)
1/ng
W(kR)
k
(r) = 1/ng δ³D(r)

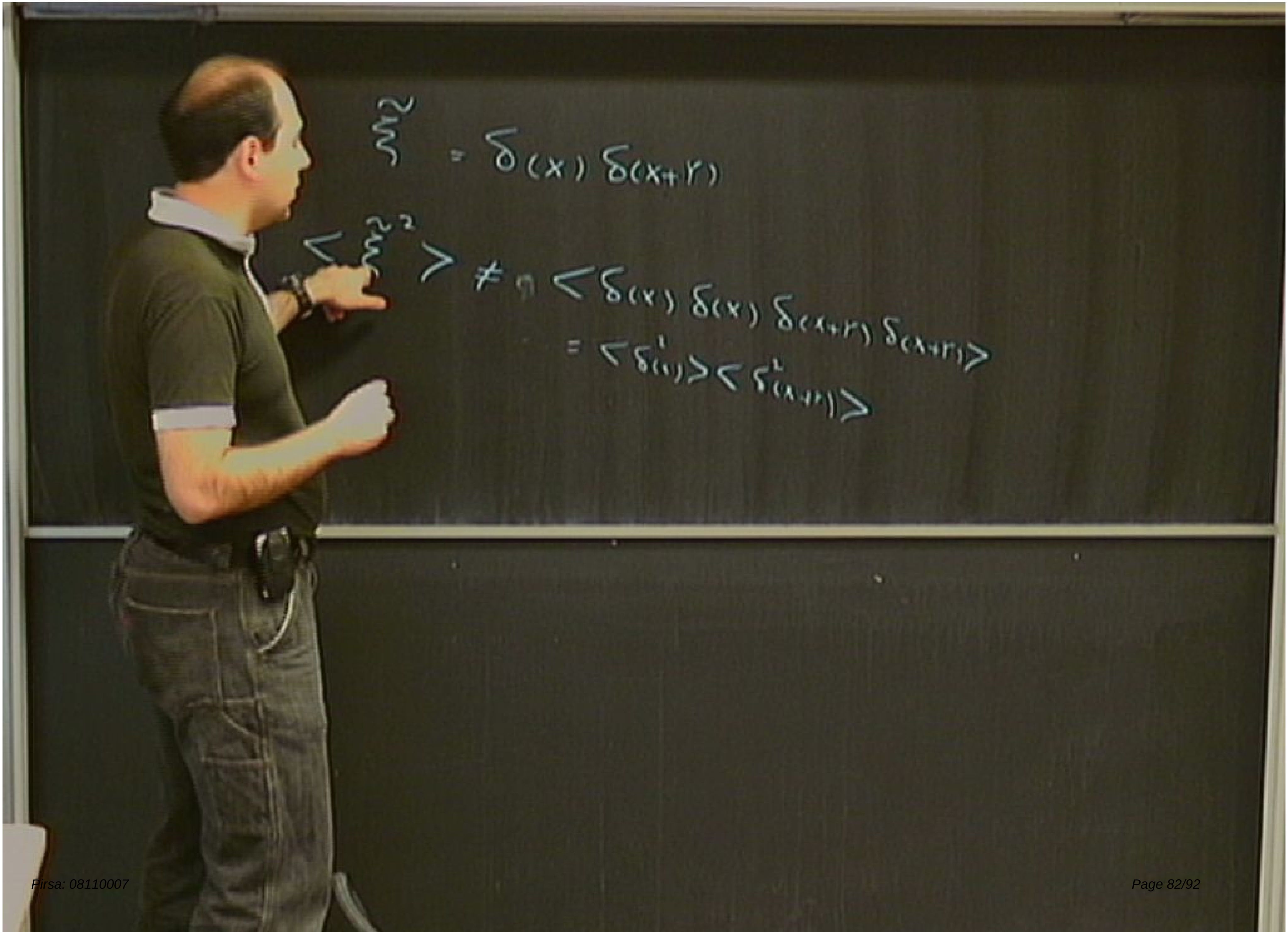
$$\langle \delta(\vec{x})\,\delta(\vec{x}+\vec{r})\rangle = \left(\frac{m}{\rho}\right)^2 \sum_i \delta_D^3(x-x_i)\,\delta_D^3(x+\vec{r}-x_i)$$

$$= \left(\frac{m}{\rho}\right)^2 \cdot \left(\frac{1}{V}\int d^3x_i\right) \quad ''$$

$$= \frac{1}{n_g^2}\cdot\frac{1}{V}\sum_i \delta_D^3(\vec{r}) = \frac{1}{n_g}\,\delta_D^3(\vec{r})$$


$\tilde{\xi} = \delta(x)\,\delta(x+r)$

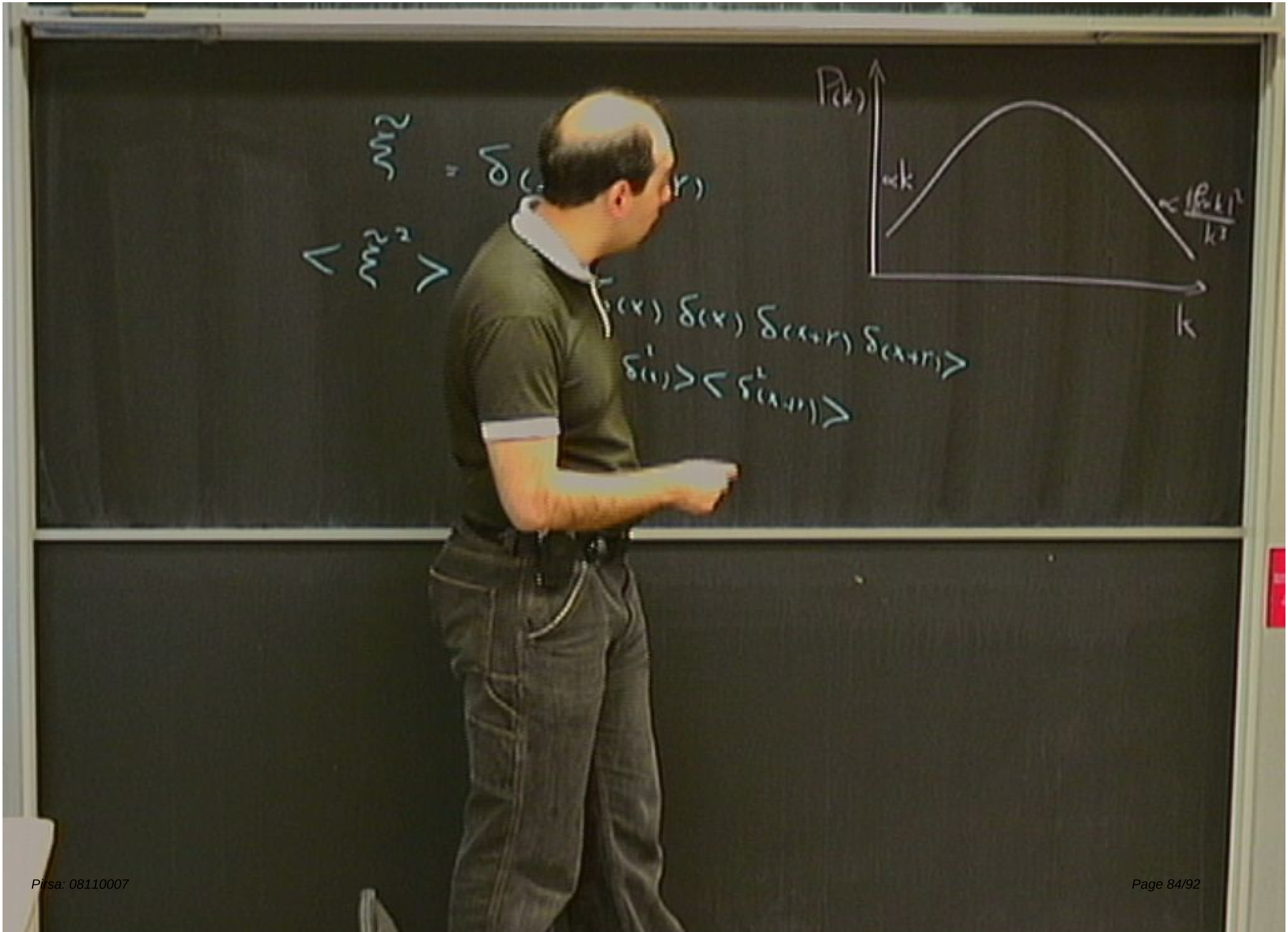

$\tilde{\xi} = \delta(x)\,\delta(x+r)$
$\langle \tilde{\xi}^2 \rangle \neq 0$

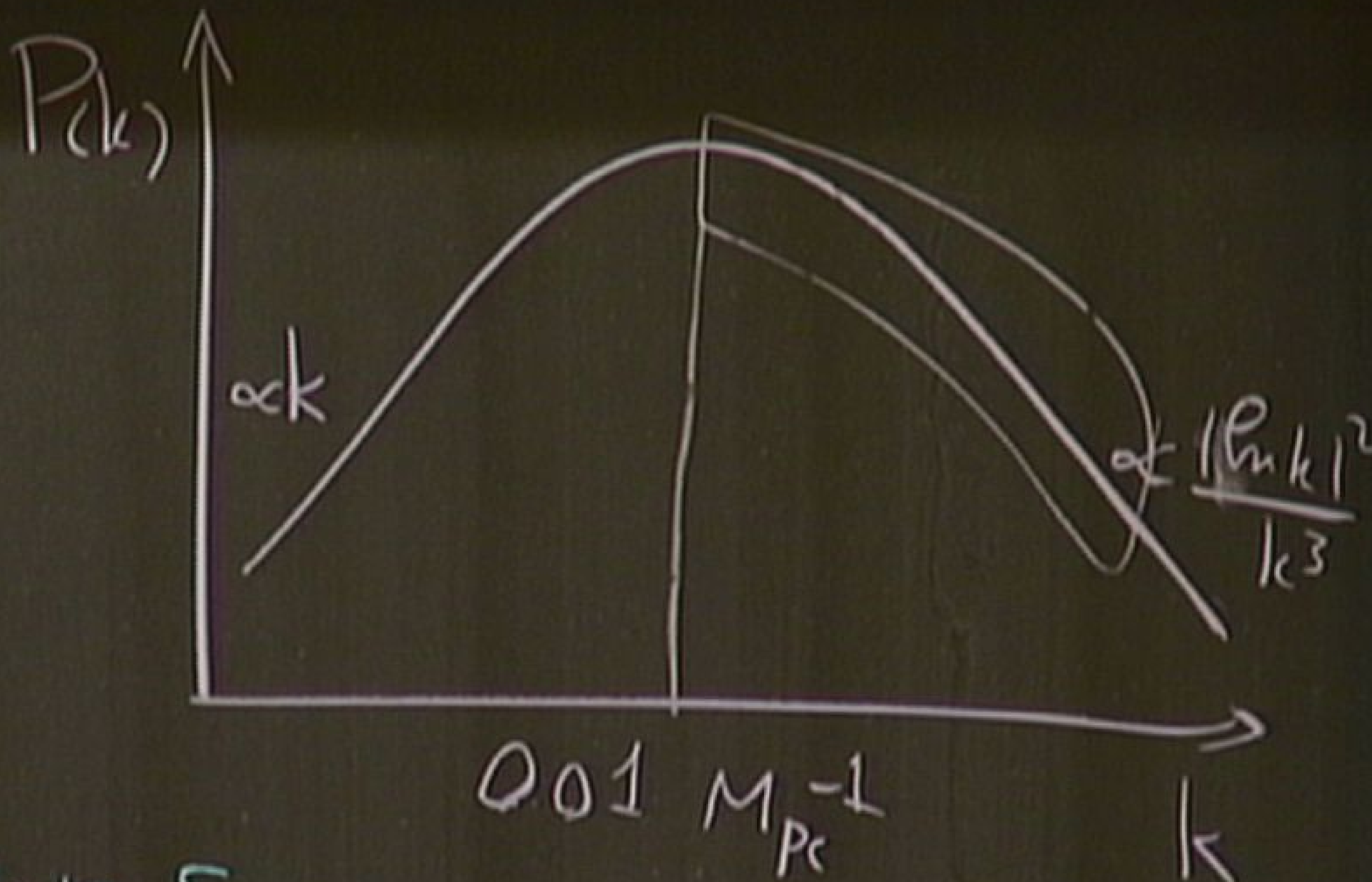

$\tilde{\xi} = \delta(x)\,\delta(x+r)$
$\langle \tilde{\xi}^2 \rangle \neq \langle \delta(x)\,\delta(x)\,\delta(x+r)\,\delta(x+r) \rangle$
$= \langle \delta^2(x) \rangle \langle \delta^2(x+r) \rangle$

$$\tilde{\xi} = \delta(x)\,\delta(x+r)$$

$$\langle \tilde{\xi}^2 \rangle \neq \langle \delta(x)\,\delta(x)\,\delta(x+r)\,\delta(x+r) \rangle$$

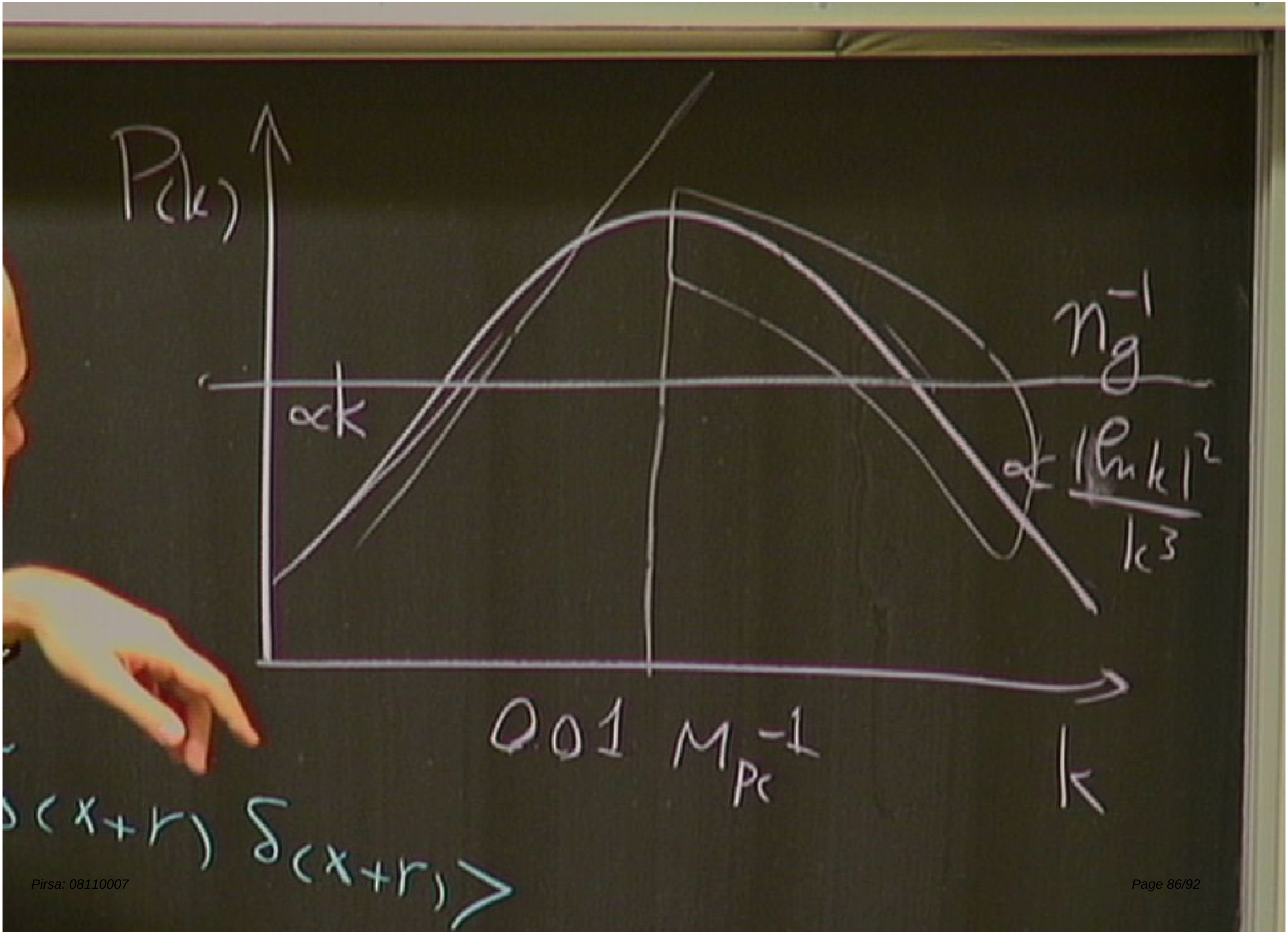
$$= \langle \delta^2(x) \rangle \langle \delta^2(x+r) \rangle$$

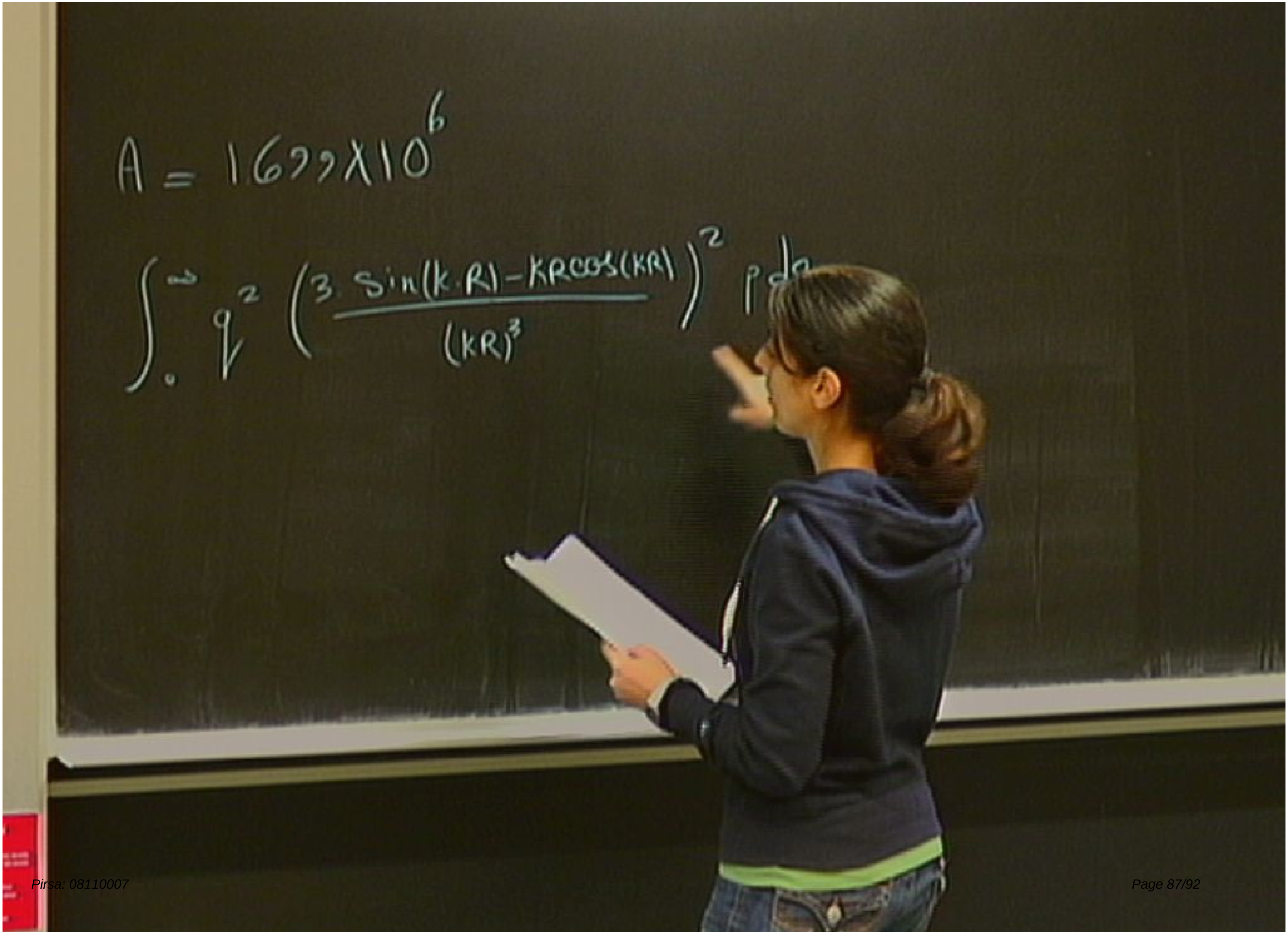

P(k)
∝k
k



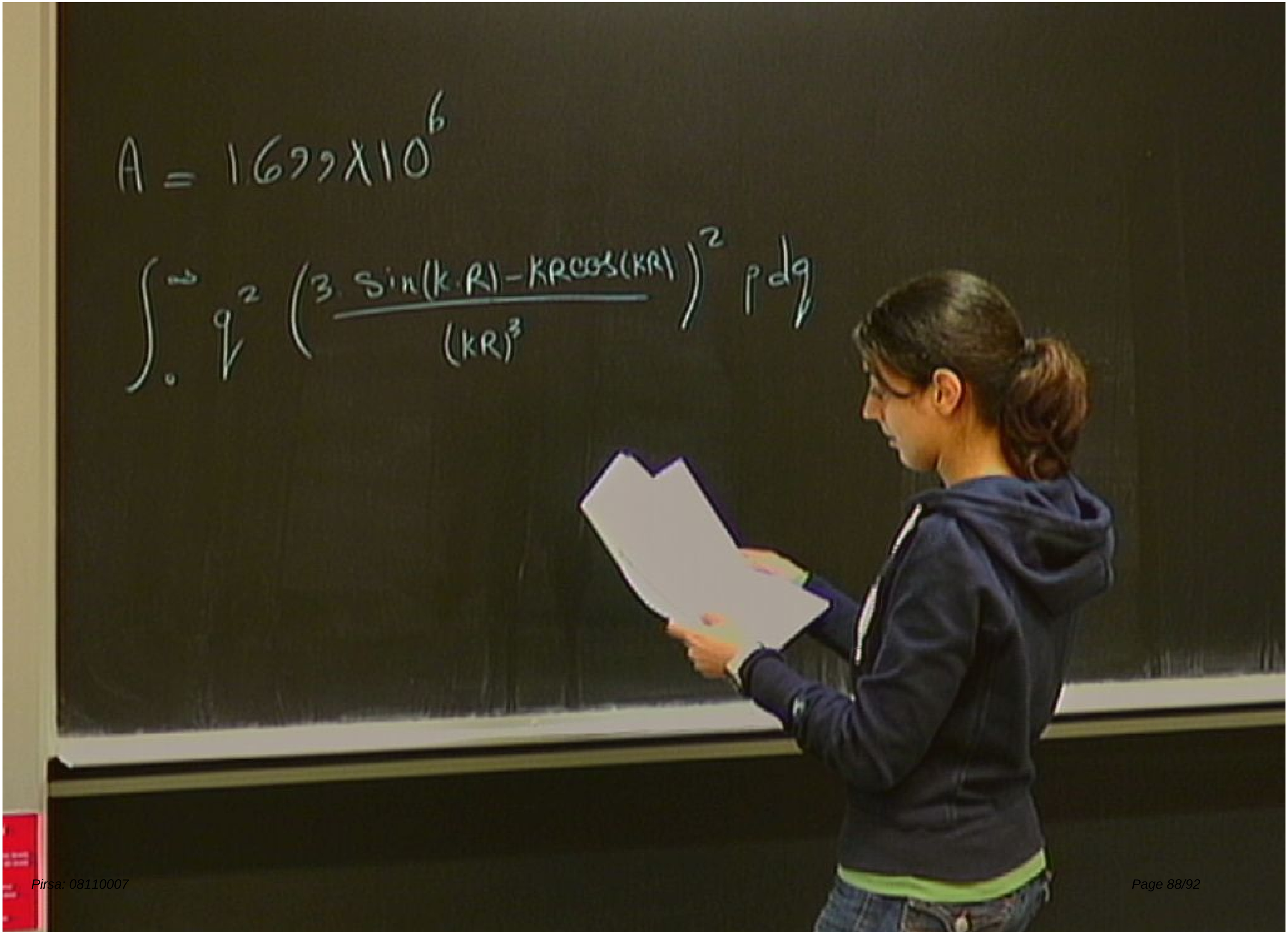
$) \, \delta(x+r) \, \delta(x+r)>$

$\delta^2(x+r)>$


P(k)
∝k
n_g^-1
∝ |φ_k|^2 / k^3
0.01 Mpc^-1
k
δ(x+r) δ(x+r)>



A = 1677 X 10^6
∫ q^2 (3 Sin(k R) − KR cos(KR) / (KR)^3)^2 ρ dq


$A = 1.677 \times 10^{6}$
$\int_0^{\infty} q^2 \left(3\,\frac{\sin(k\cdot R) - kR\cos(kR)}{(kR)^3}\right)^2 \rho\, dq$

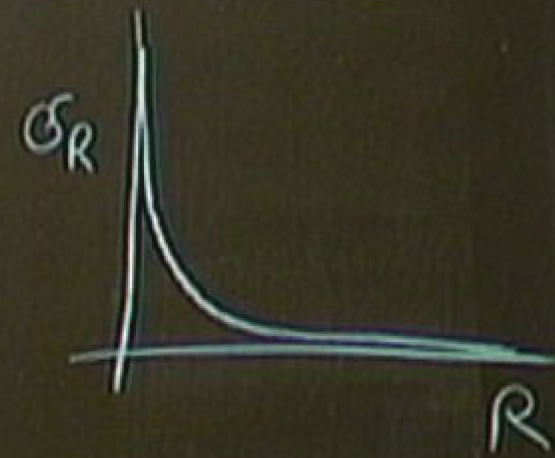
$$A = 1.677 \times 10^{6}$$

$$\int_0^{\infty} q^2 \left(\frac{3 \cdot \sin(k \cdot R) - kR\cos(kR)}{(kR)^3} \right)^2 P \, dq$$

$$K = q \, \Omega \, h^2$$

$$A = 1677 \times 10^{6}$$

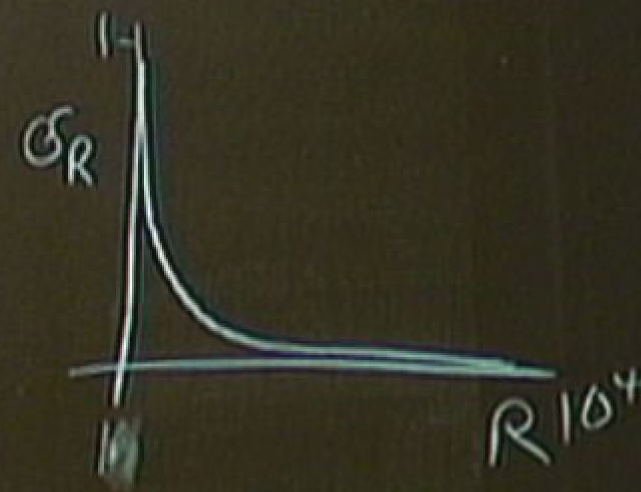
$$\int_0^{\infty} q^2 \left(3 \cdot \frac{\sin(k \cdot R) - kR\cos(kR)}{(kR)^3} \right)^2 P \, dq$$



$$K = q\,\Omega\, h^2$$

$$A = 1.677 \times 10^{6}$$

$$\int_0^{\infty} q^2 \left(\frac{3\,(\sin(k R) - kR\cos(kR))}{(kR)^3} \right)^2 P\, dq$$



$$K = q\,\Omega\,h^2$$

$$A = 1.677 \times 10^{6}$$

$$\int_0^{\infty} q^2 \left(3\,\frac{\sin(kR) - kR\cos(kR)}{(kR)^3}\right)^2 P\,dq$$

$$K = q\,\Omega\,h^2$$