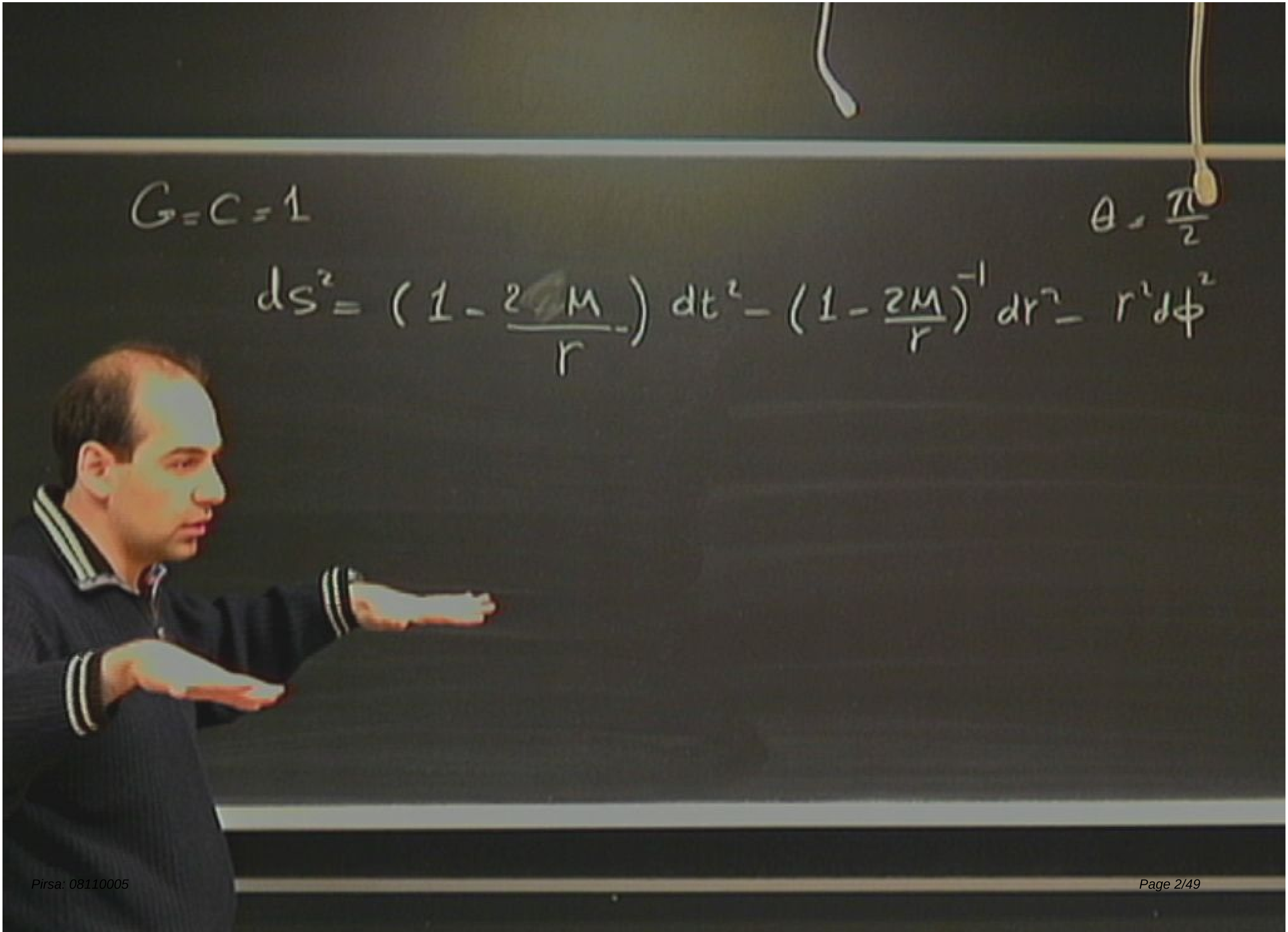
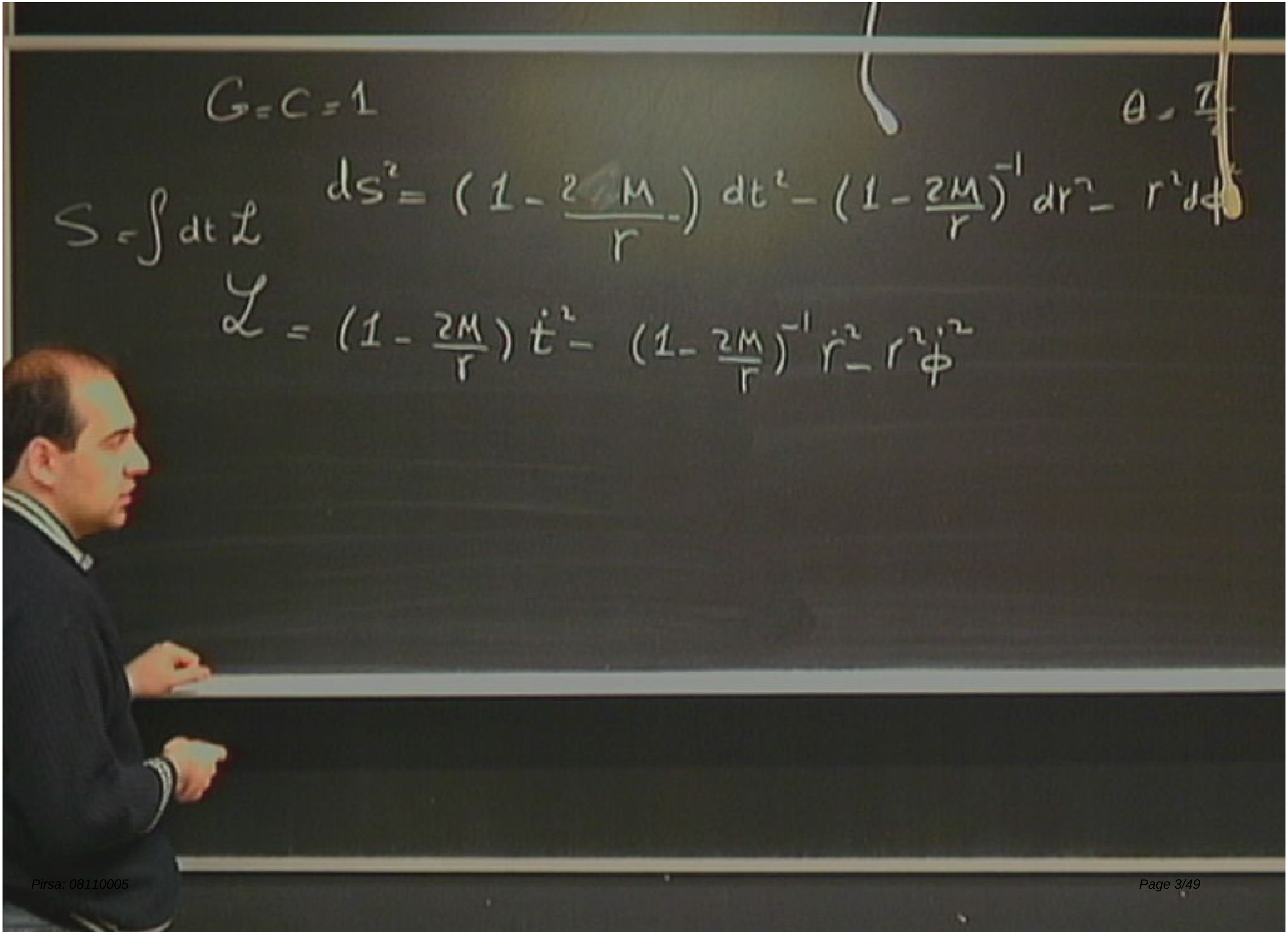
Title: Astrophysics and Cosmology through Problems - 11A

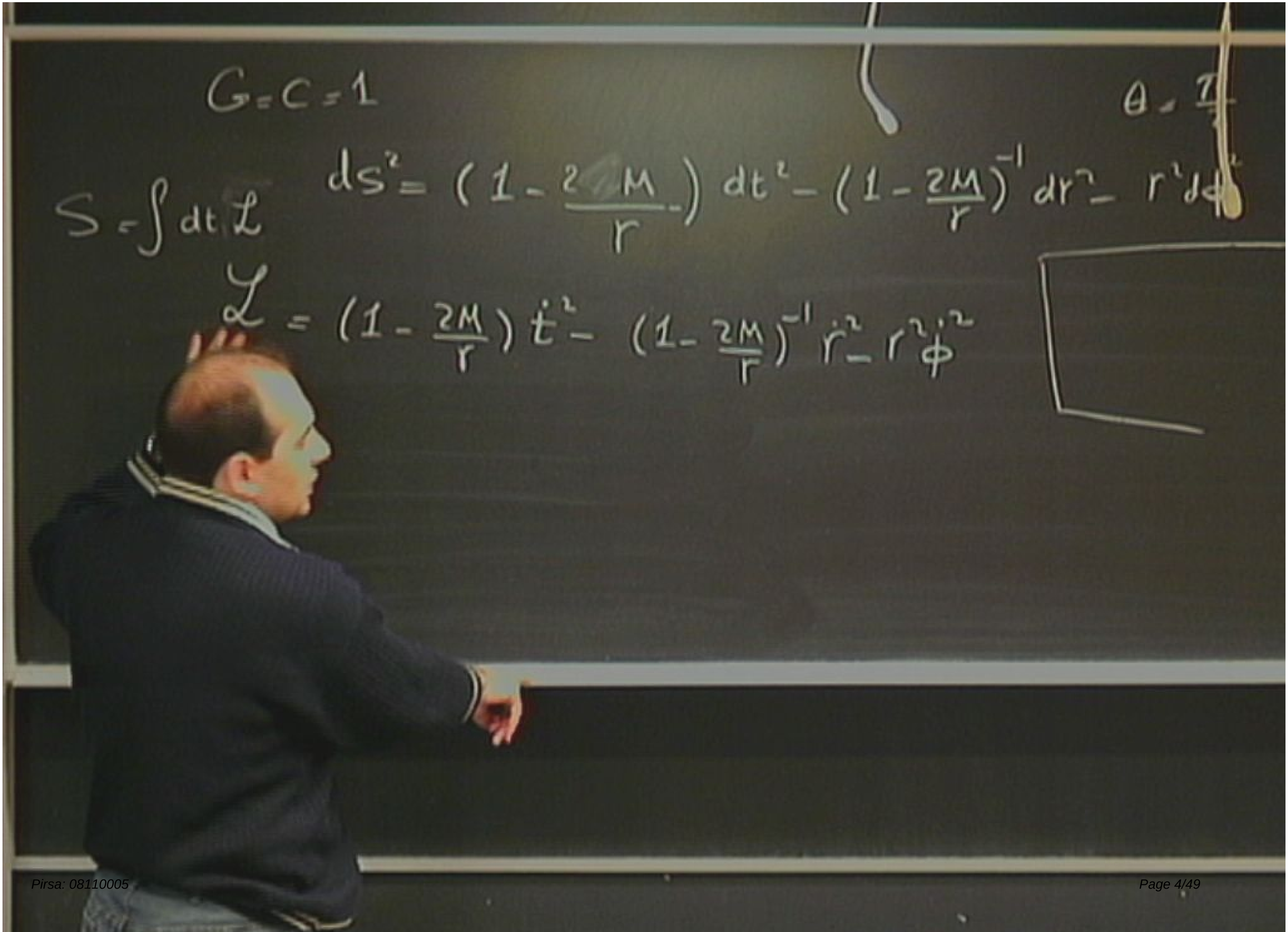
Date: Nov 13, 2008 10:00 AM

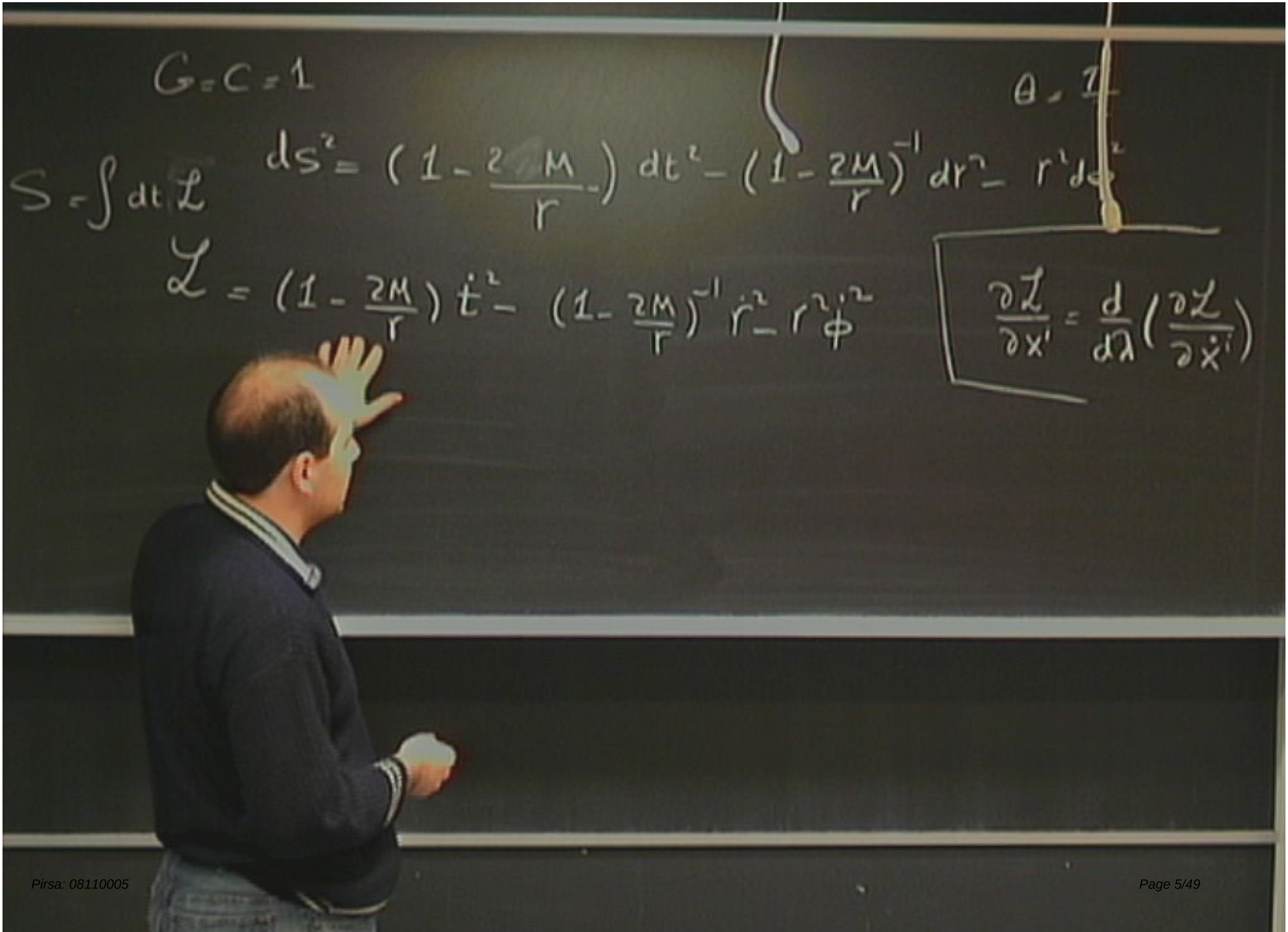
URL: http://pirsa.org/08110005

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.


$G = c = 1$
$\theta = \frac{\pi}{2}$
$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2$


$G = c = 1$
$\theta = \frac{\pi}{2}$
$S = \int dt\, \mathcal{L}$
$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2$
$\mathcal{L} = \left(1 - \frac{2M}{r}\right) \dot{t}^2 - \left(1 - \frac{2M}{r}\right)^{-1} \dot{r}^2 - r^2 \dot{\phi}^2$


G = C = 1
θ = π/2
S = ∫ dt 𝓛
ds² = (1 − 2M/r) dt² − (1 − 2M/r)⁻¹ dr² − r² dφ²
𝓛 = (1 − 2M/r) ṫ² − (1 − 2M/r)⁻¹ ṙ² − r² φ̇²


$G = c = 1$
$\theta =$
$S = \int dt\, \mathcal{L}$
$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2$
$\mathcal{L} = \left(1 - \frac{2M}{r}\right) \dot{t}^2 - \left(1 - \frac{2M}{r}\right)^{-1} \dot{r}^2 - r^2 \dot{\phi}^2$
$\boxed{\frac{\partial \mathcal{L}}{\partial x^i} = \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^i}\right)}$

$G = c = 1$

$\theta = \frac{\pi}{2}$

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2$$

$$S = \int dt\, \mathcal{L}$$

$$\mathcal{L} = \left(1 - \frac{2M}{r}\right) \dot{t}^2 - \left(1 - \frac{2M}{r}\right)^{-1} \dot{r}^2 - r^2 \dot{\phi}^2$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial x^i} = \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^i}\right)}$$

$$\cancel{2}\ell = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = -\cancel{2} r^2 \dot{\phi}$$

$$= \frac{\partial \mathcal{L}}{\partial \dot{t}}$$

$$G = c = 1 \qquad \theta = \frac{\pi}{2}$$

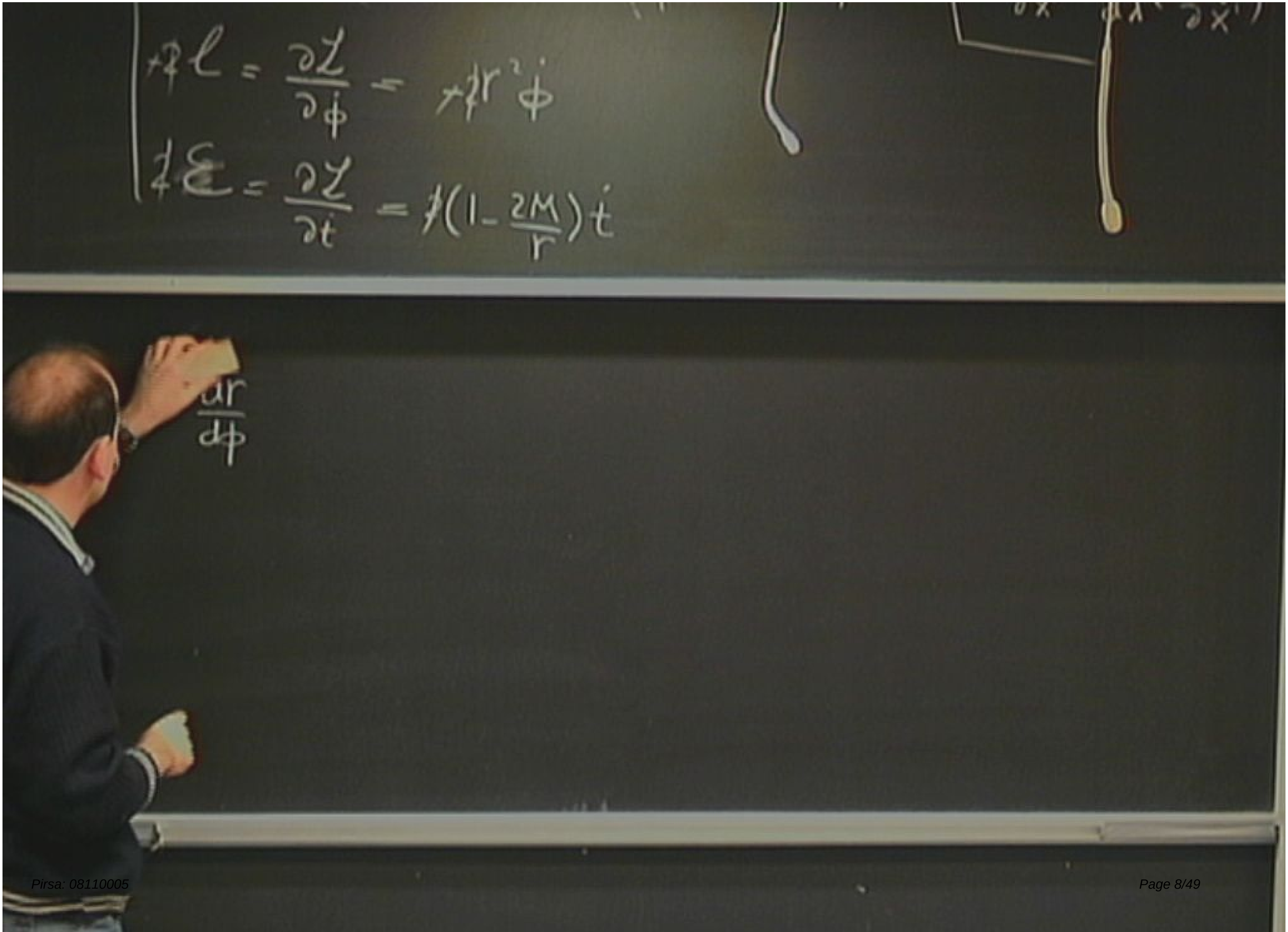
$$S = \int dt\, \mathcal{L} \qquad ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2$$

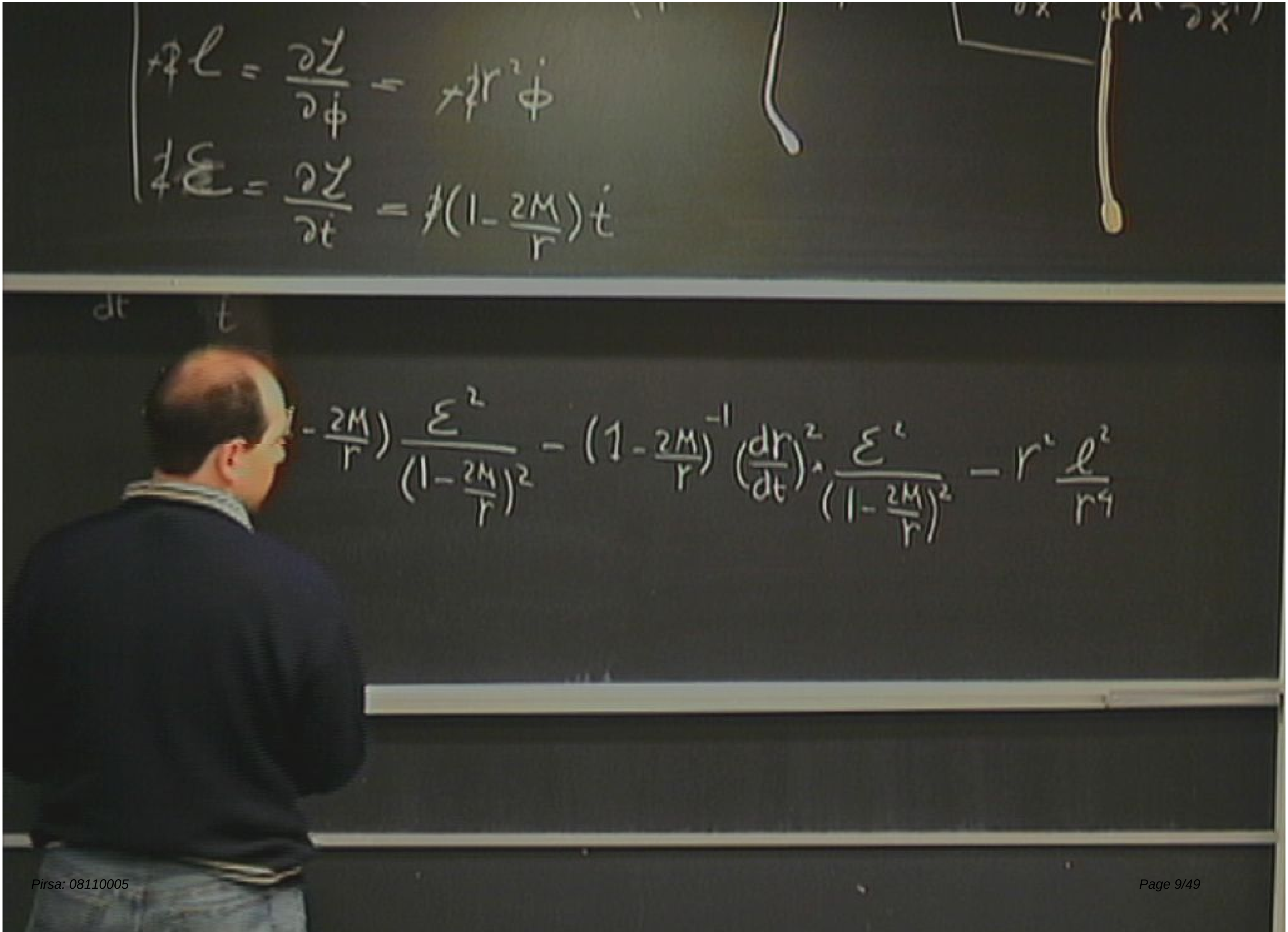
$$\mathcal{L} = \left(1 - \frac{2M}{r}\right)\dot t^2 - \left(1 - \frac{2M}{r}\right)^{-1}\dot r^2 - r^2 \dot\phi^2$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial x^i} = \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot x^i}\right)}$$

$$\ell = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \cancel{-2}\, r^2 \dot\phi$$

$$E = \frac{\partial \mathcal{L}}{\partial \dot t} = \cancel{2}\left(1 - \frac{2M}{r}\right)\dot t$$

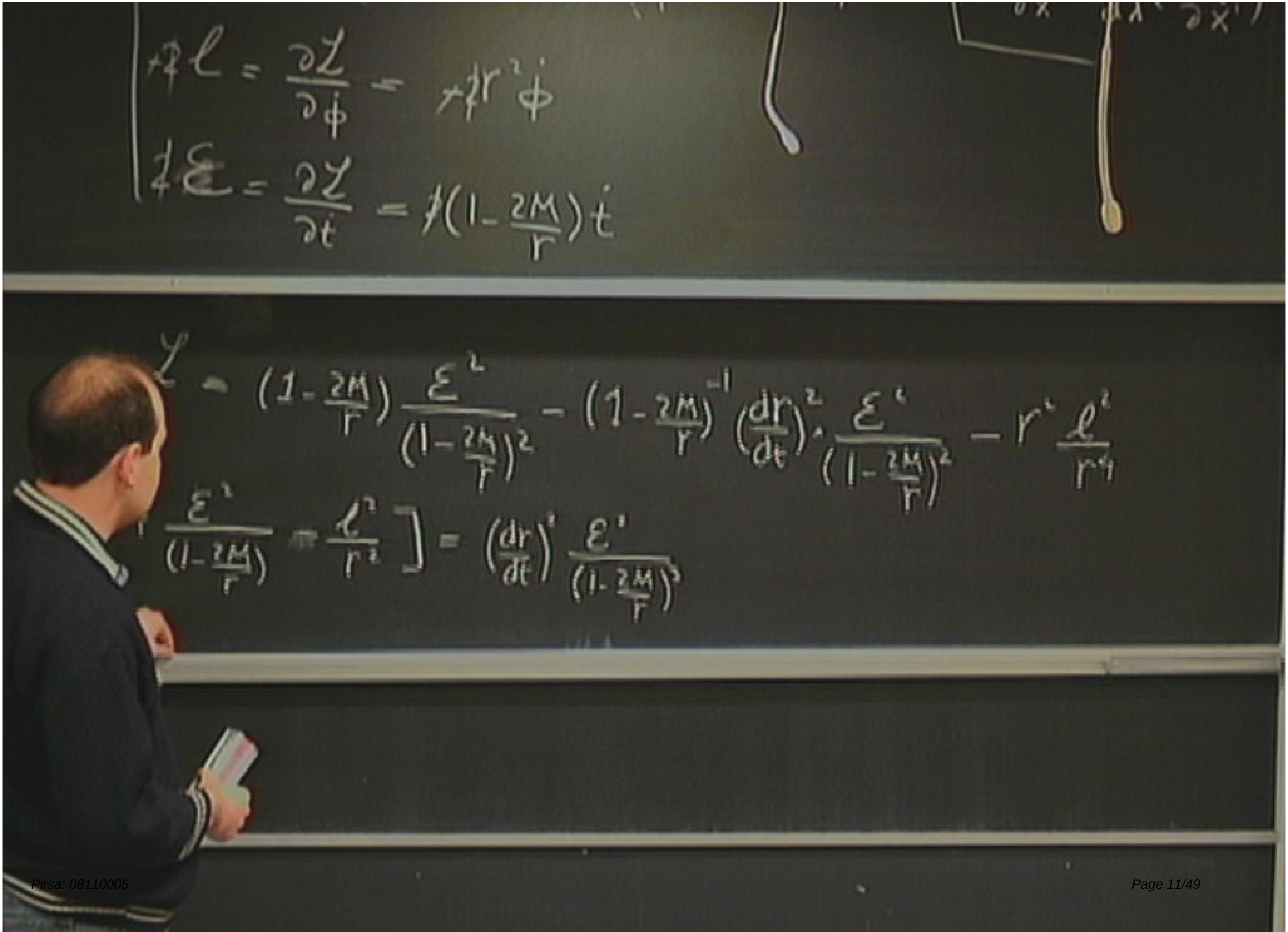

$\ell = \frac{\partial L}{\partial \dot\phi} = r^2 \dot\phi$
$E = \frac{\partial L}{\partial \dot t} = (1 - \frac{2M}{r}) \dot t$
$\frac{dr}{d\phi}$


$\cancel{+2}\,\ell = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \cancel{+2}\, r^2 \dot\phi$
$\cancel{2}\,\mathcal{E} = \frac{\partial \mathcal{L}}{\partial \dot t} = \cancel{2}\left(1 - \frac{2M}{r}\right)\dot t$
$-\frac{2M}{r})\frac{\mathcal{E}^2}{(1-\frac{2M}{r})^2} - \left(1-\frac{2M}{r}\right)^{-1}\left(\frac{dr}{dt}\right)^2 \frac{\mathcal{E}^2}{(1-\frac{2M}{r})^2} - r^2\frac{\ell^2}{r^4}$

$$\cancel{2}\,\ell = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \cancel{2}\,r^2\dot\phi$$

$$\cancel{2}\,\mathcal{E} = \frac{\partial \mathcal{L}}{\partial \dot t} = \cancel{2}\left(1-\frac{2M}{r}\right)\dot t$$

$$\mathcal{L} = \left(1-\frac{2M}{r}\right)\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^2} - \left(1-\frac{2M}{r}\right)^{-1}\left(\frac{dr}{dt}\right)^2\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^2} - r^2\frac{\ell^2}{r^4}$$


$\cancel{2}\,\ell = \frac{\partial L}{\partial \dot\phi} = \cancel{2}\, r^2 \dot\phi$
$\cancel{2}\,\mathcal{E} = \frac{\partial L}{\partial \dot t} = \cancel{2}\left(1-\frac{2M}{r}\right)\dot t$
$L = \left(1-\frac{2M}{r}\right)\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^2} - \left(1-\frac{2M}{r}\right)^{-1}\left(\frac{dr}{dt}\right)^2\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^2} - r^2\frac{\ell^2}{r^4}$
$\left[\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)} - \frac{\ell^2}{r^2}\right] = \left(\frac{dr}{dt}\right)^2\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^3}$

$$\cancel{2}\ell = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \cancel{2} r^2 \dot\phi$$

$$\cancel{2}\mathcal{E} = \frac{\partial \mathcal{L}}{\partial \dot t} = \cancel{2}\left(1 - \frac{2M}{r}\right)\dot t$$

$$\frac{dr}{dt} = \frac{\dot r}{\dot t}$$

$$\mathcal{L} = \left(1-\frac{2M}{r}\right)\frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^2} - \left(1-\frac{2M}{r}\right)^{-1}\left(\frac{dr}{dt}\right)^2 \frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^2} - r^2\frac{\ell^2}{r^4}$$

$$\left[-\mathcal{L} + \frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)} - \frac{\ell^2}{r^2}\right] = \left(\frac{dr}{dt}\right)^2 \frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^3}$$

$$\cancel{+2}\,\ell = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \cancel{+2}\, r^2 \dot\phi$$

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$$\left[-\mathcal{L} + \frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)} - \frac{\ell^2}{r^2}\right] = \left(\frac{dr}{dt}\right)^2 \frac{\mathcal{E}^2}{\left(1-\frac{2M}{r}\right)^3}$$

$$2E = \left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{r^2} - \frac{2GM}{r}$$

$$\left(\frac{dr}{dt}\right)^2 = 2E - \frac{\ell^2}{r^2} + \frac{2GM}{r}$$

$$2E = \left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{r^2} - \frac{2GM}{r}$$

$$\left(\frac{dr}{dt}\right)^2 = 2E - \frac{\ell^2}{r^2} + \frac{2GM}{r} \quad ; \text{ Newtonian}$$

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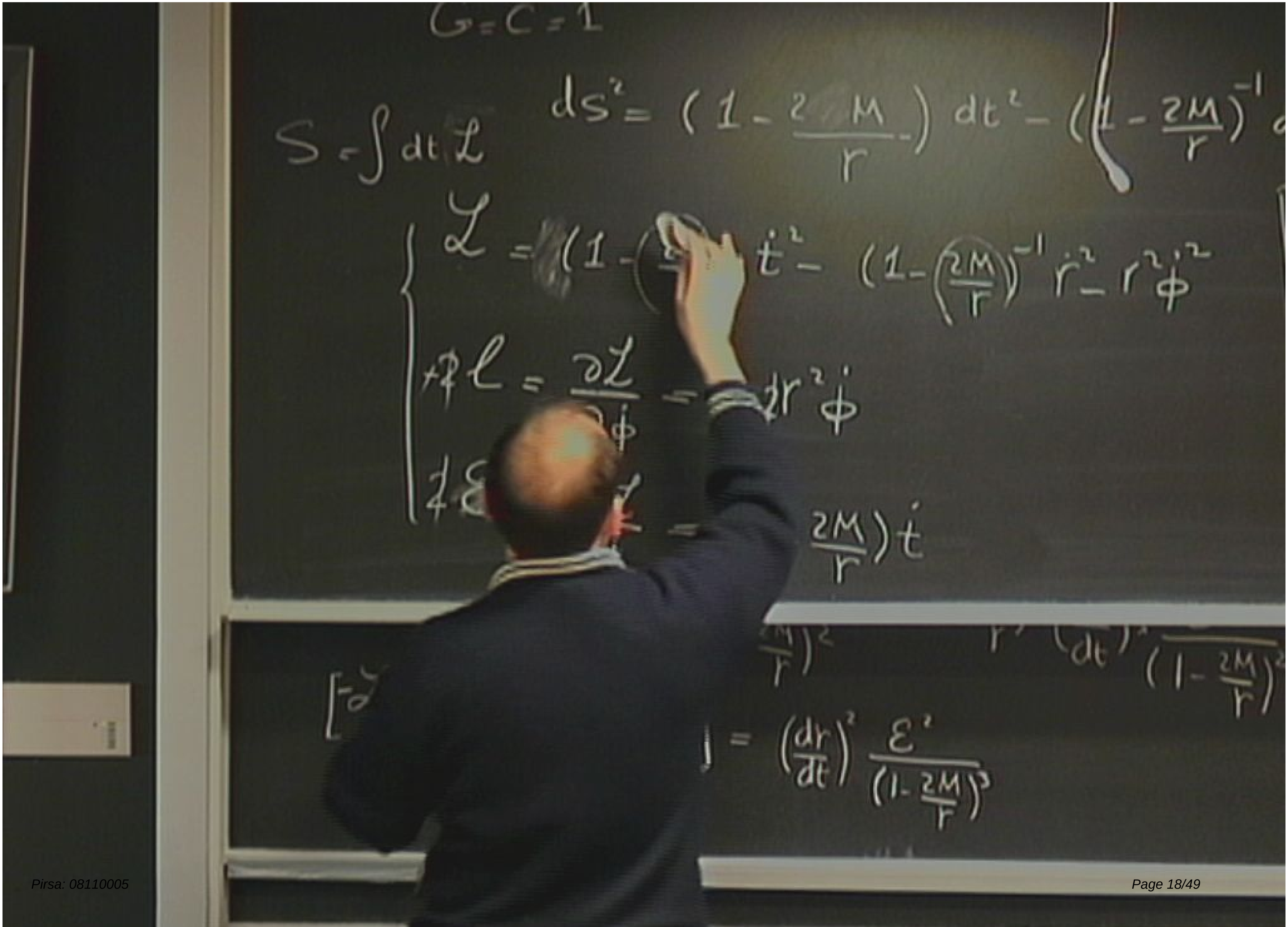
$$\left(\frac{dr}{dt}\right)^2 = 2E - \frac{\ell^2}{r^2} + \frac{2GM}{r} \qquad \text{; Newtonian}$$

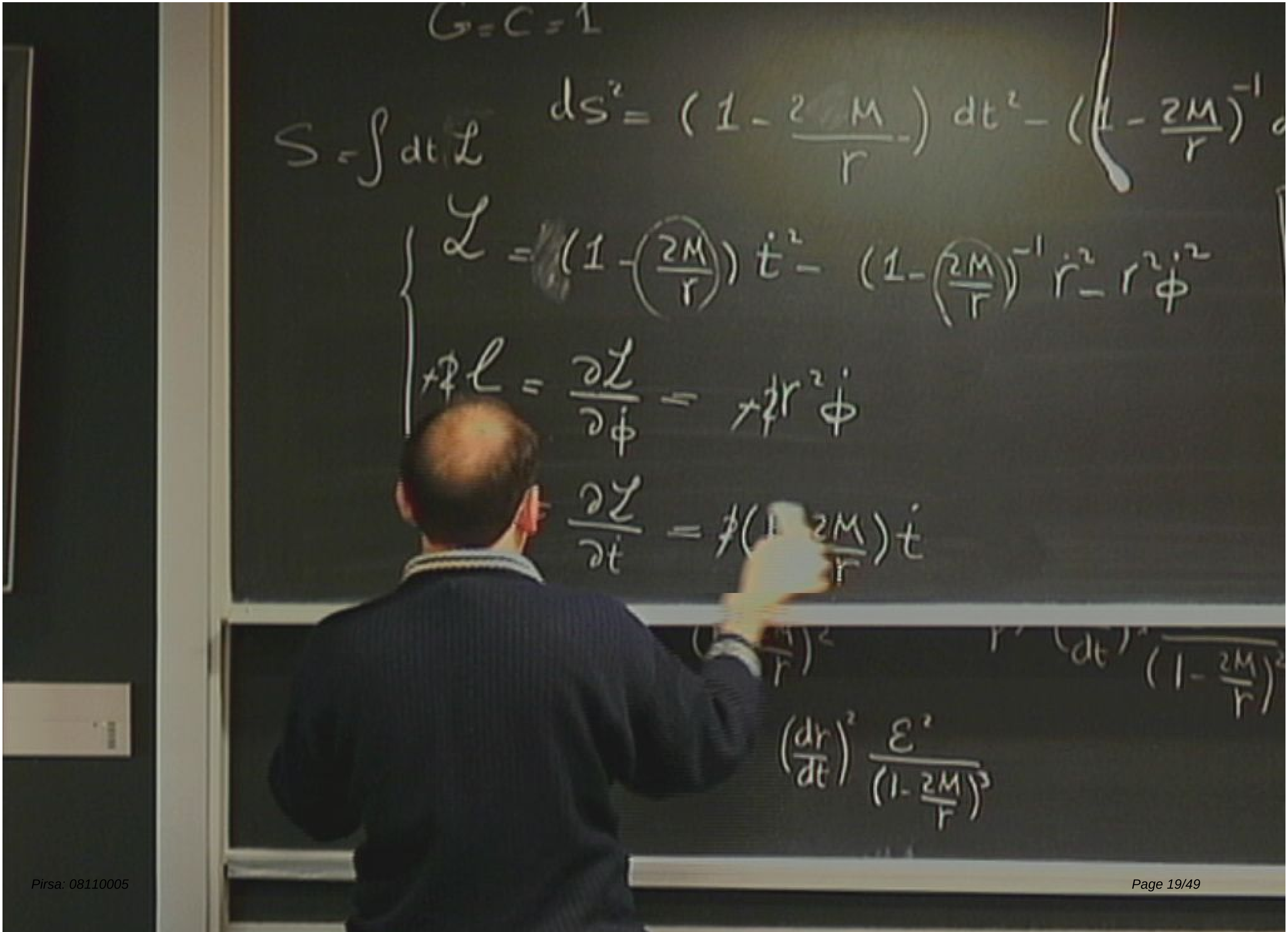
$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{6M}{r}\right)\left[1 + \frac{2M}{r} - \frac{\ell^2}{\mathcal{E}^2 r^2} - \frac{\mathcal{L}}{\mathcal{E}}\right]$$

$$2E = \left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{r^2} - \frac{2GM}{r}$$

$$\left(\frac{dr}{dt}\right)^2 = 2E - \frac{\ell^2}{r^2} + \frac{2GM}{r} \quad ; \text{Newtonian}$$

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{6M}{r}\right)\left[1 + \frac{2M}{r} - \frac{\ell^2}{\varepsilon^2 r^2} - \frac{\mathcal{L}}{\varepsilon}\right]$$


G = c = 1
$ds^2 = (1 - \frac{2M}{r}) dt^2 - (1 - \frac{2M}{r})^{-1}$
$S = \int dt \, \mathcal{L}$
$\mathcal{L} = (1 - \dot{t}^2 - (1 - \frac{2M}{r})^{-1} \dot{r}^2 - r^2 \dot{\phi}^2$
$\ell = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$
$\frac{2M}{r}) \dot{t}$
$= (\frac{dr}{dt})^2 \frac{\mathcal{E}^2}{(1 - \frac{2M}{r})^3}$


$G = c = 1$
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$\ell = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = r^2 \dot{\phi}$
$\frac{\partial \mathcal{L}}{\partial \dot{t}} = \left(1 - \frac{2M}{r}\right) \dot{t}$
$\left(\frac{dr}{dt}\right)^2 \frac{\mathcal{E}^2}{\left(1 - \frac{2M}{r}\right)^3}$

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{6M}{r}\right)\left[1 + \frac{2M}{r} - \frac{\ell^2}{\varepsilon^2 r^2} - \frac{\text{[illegible]}}{\varepsilon}\right]; \quad \text{massive part.}$$

Massless part.

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{6M}{r}\right)\left(1 + \frac{2M}{r} - \frac{L^2}{r^2}\right)$$

$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left[1+\frac{2M}{r}-\frac{\ell^2}{\varepsilon^2 r^2}-\frac{[illegible]}{\varepsilon}\right]; \text{ massive part.}$$

Massless part.

$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left(1+\frac{2M}{r}-\frac{L^2}{r^2}\right)$$

$$= 1-\frac{4M}{r}-\frac{L^2}{r^2}+O\left(\frac{M^2}{r^2}\right)$$

$$\left(\frac{dr}{dt}\right)^2 = \cancel{\left(1-\frac{6M}{r}\right)} \left[1 + \frac{2M}{r} - \frac{\ell^2}{\varepsilon^2 r^2} - \frac{\ell}{\varepsilon}\right] ;$$ massive part.

Massless part.

$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left(1+\frac{2M}{r}-\frac{L^2}{r^2}\right)$$

$$= 1 - \frac{4M}{r} - \frac{L^2}{r^2} + O\left(\frac{M^2}{r^2}\right)$$

$$\left(\frac{dr}{dt}\right)^2 = \cancel{\left(1-\frac{6M}{r}\right)} \left[1 + \frac{2M}{r} - \frac{\ell^2}{\mathcal{E}^2 r^2} - \frac{\ell}{\mathcal{E}}\right] ; \text{ massive part.}$$

Massless part.

$$L = b_* v_0$$

$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left(1+\frac{2M}{r}-\frac{L^2}{r^2}\right)$$

$$= 1 - \frac{4M}{r} - \frac{L^2}{r^2} + O\left(\frac{M^2}{r^2}\right)$$

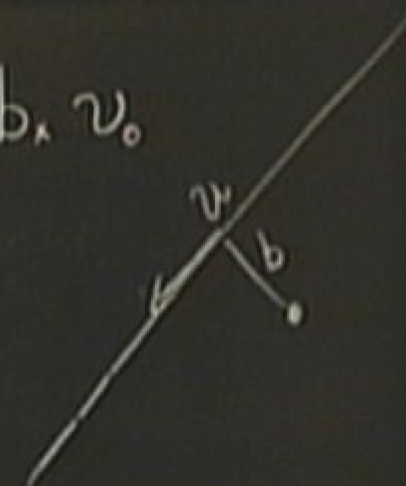
$$\left(\frac{dr}{dt}\right)^2 = \cancel{\left(1-\frac{6M}{r}\right)} \left[1 + \frac{2M}{r} - \frac{\ell^2}{\mathcal{E}^2 r^2} - \frac{\varepsilon}{\mathcal{E}}\right] \; ; \text{ massive part.}$$

Massless part.

$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left(1 + \frac{2M}{r} - \frac{L^2}{r^2}\right)$$

$$= 1 - \frac{4M}{r} - \frac{L^2}{r^2} + O\left(\frac{M^2}{r^2}\right)$$

$$L = b_\ast v_0$$



$$\left(\frac{dr}{dt}\right)^2 = \cancel{\left(1-\frac{6M}{r}\right)}\left[1+\frac{2M}{r}-\frac{\ell^2}{\varepsilon^2 r^2}-\frac{1}{\varepsilon}\right]; \text{ massive part.}$$

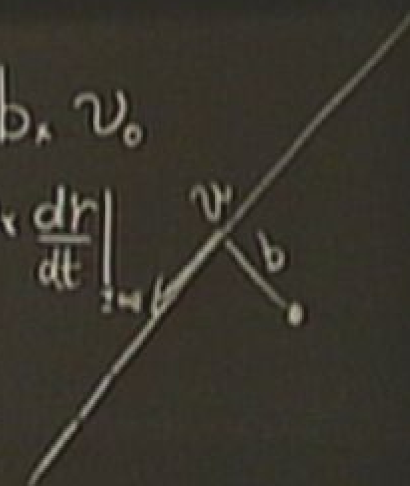
Massless part.

$$\ldots = \left(1-\frac{6M}{r}\right)\left(1+\frac{2M}{r}-\frac{L^2}{r^2}\right)$$

$$= 1-\frac{4M}{r}-\frac{L^2}{r^2}+O\left(\frac{M^2}{r^2}\right)$$

$$L = b_* v_0$$

$$= b_* \left.\frac{dr}{dt}\right|_{t=\infty}$$



$$\left(\frac{dr}{dt}\right)^2 = \cancel{\left(1-\frac{6M}{r}\right)}\left[1+\frac{2M}{r}-\frac{\ell^2}{\mathcal{E}^2 r^2}-\frac{\varepsilon}{\mathcal{E}}\right]; \text{ massive part.}$$

Massless part.

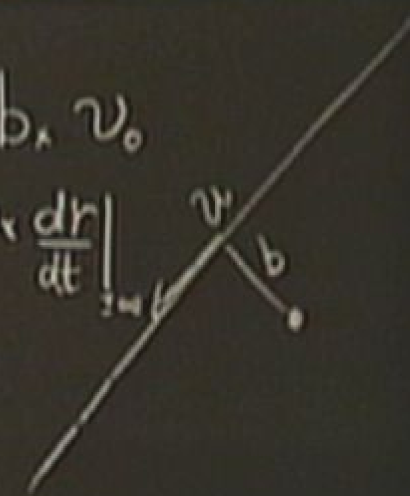
$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left(1+\frac{2M}{r}-\frac{L^2}{r^2}\right)$$

$$= 1-\frac{4M}{r}-\frac{L^2}{r^2}+O\left(\frac{M^2}{r^2}\right)$$

$$dt = \frac{\dot t}{\dot\phi}\, d\phi = \frac{\mathcal{E}}{\ell}\cdot\frac{r^2}{\left(1-\frac{2M}{r}\right)}\, d\phi$$

$$L = b_\times v_0$$

$$= b_\times \left.\frac{dr}{dt}\right|_{t=-\infty}$$



$$\left(\frac{dr}{dt}\right)^2 = \cancel{\left(1-\frac{6M}{r}\right)}\left[1+\frac{2M}{r}-\frac{\ell^2}{\mathcal{E}^2 r^2}-\frac{\varepsilon}{\mathcal{E}}\right] ;$$ massive part.

Massless part.

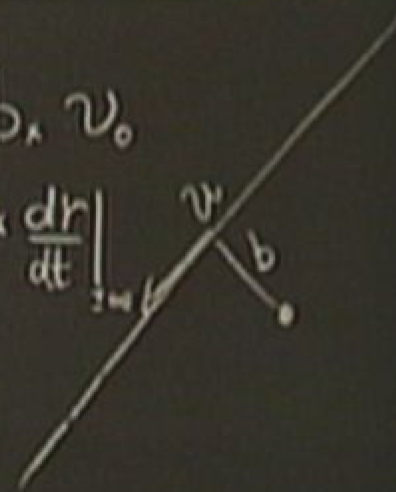
$$\left(\frac{dr}{dt}\right)^2 = \left(1-\frac{6M}{r}\right)\left(1+\frac{2M}{r}-\frac{L^2}{r^2}\right)$$

$$= 1-\frac{4M}{r}-\frac{L^2}{r^2}+O\left(\frac{M^2}{r^2}\right)$$

$$dt = \frac{\dot t}{\dot\phi}\, d\phi = \frac{\mathcal{E}}{\ell}\cdot\frac{r^2}{\left(1-\frac{2M}{r}\right)}\, d\phi$$

$$L = b\cdot v_0$$

$$= b\cdot \left.\frac{dr}{dt}\right|_{t=-\infty}$$



Newtonian

$$\left(\frac{dr}{d\phi}\right)^2 \frac{L^2\left(1-\cancel{\frac{2M}{r}}\right)^2}{r^2} = v_0^2 - \frac{L^2}{r^2} + \frac{2M}{r}$$

photon

$$\left(\frac{dr}{d\phi}\right)^2 \times \frac{L^2\left(1-\frac{2M}{r}\right)^2}{r^2} = 1 - \frac{4M}{r} - \frac{L^2}{r^2}$$

Newtonian

$$\left(\frac{dr}{d\phi}\right)^2 \frac{L^2\left(1-\cancel{\frac{2M}{r}}\right)^2}{r^4} = v_0^2 - \frac{L^2}{r^2} + \frac{2M}{r}$$

photon

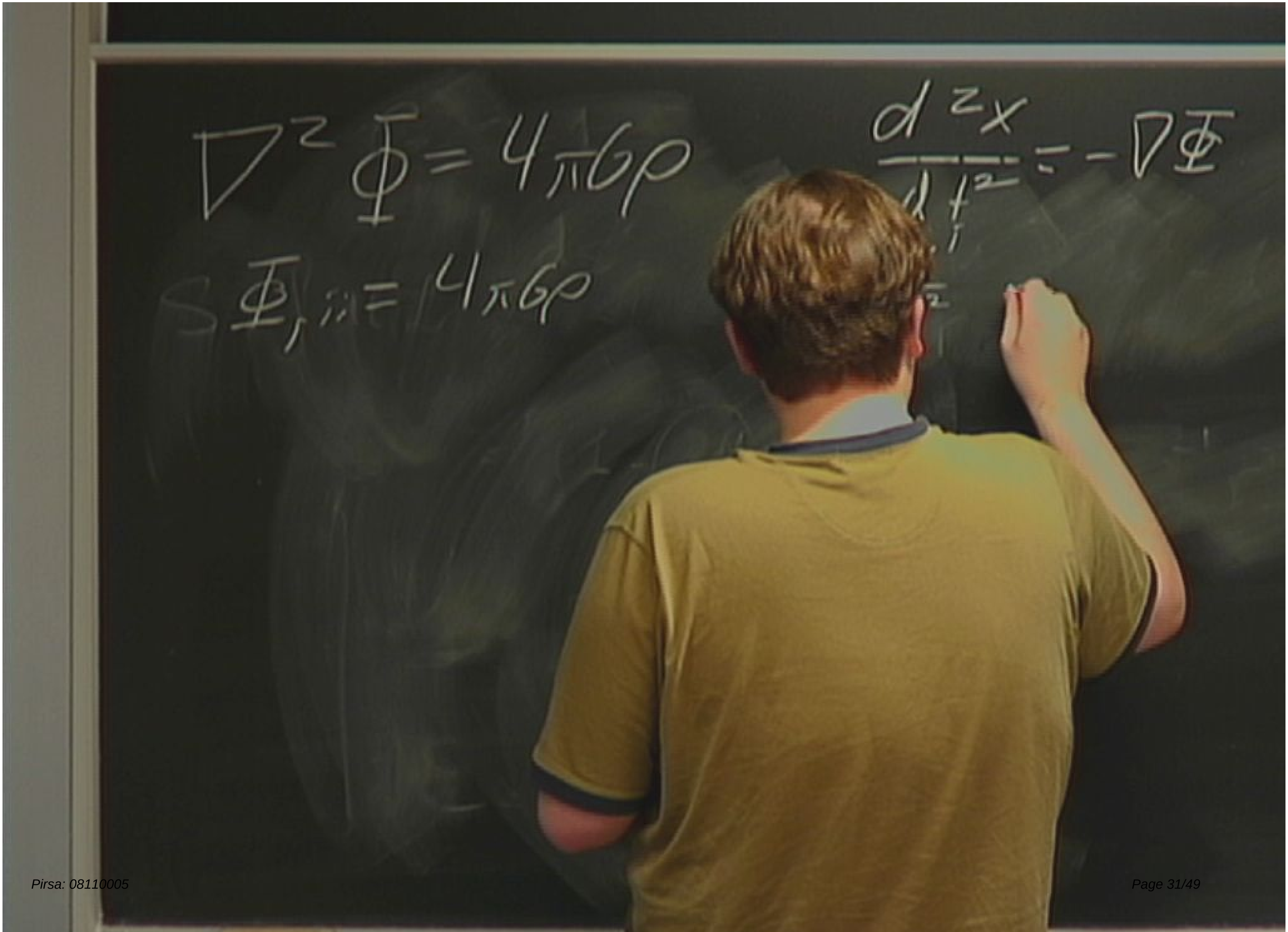
$$\left(\frac{dr}{d\phi}\right)^2 \times \frac{L^2\left(1-\frac{2M}{r}\right)^2}{r^4} = 1 - \frac{4M}{r} - \frac{L^2}{r^2}$$

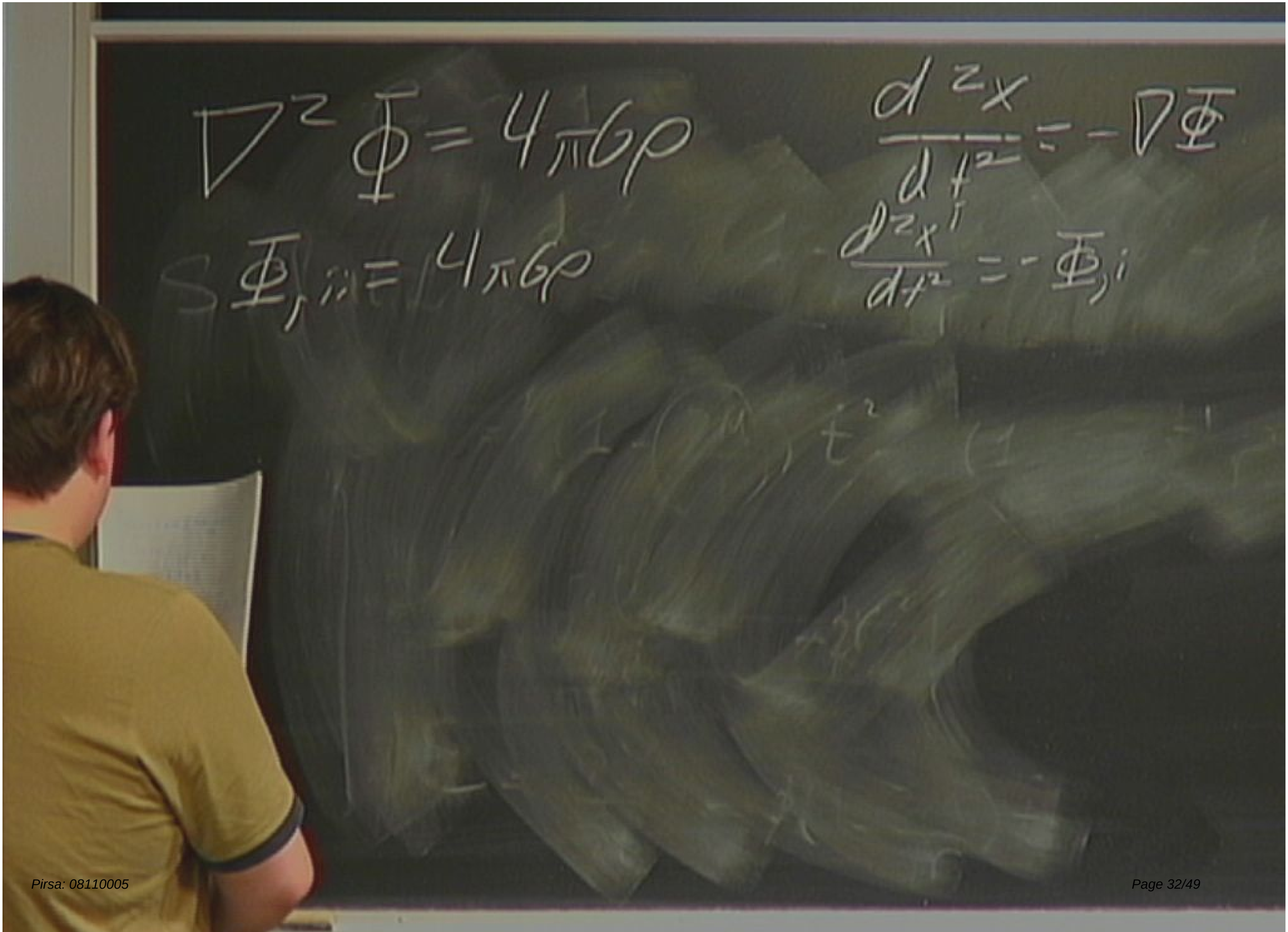
Newtonian

$$\left(\frac{dr}{d\phi}\right)^2 \frac{L^2\left(1-\cancel{\frac{2M}{r}}\right)^2}{r^4} = v_0^2 - \frac{L^2}{r^2} + \frac{2M}{r}$$

photon

$$\left(\frac{dr}{d\phi}\right)^2 \times \frac{L^2\left(1-\frac{2M}{r}\right)^2}{r^4} = 1 - \frac{4M}{r} - \frac{L^2}{r^2}$$


$\nabla^2 \Phi = 4\pi G\rho$
$\frac{d^2x}{dt^2} = -\nabla\Phi$
$\Phi_{,ii} = 4\pi G\rho$


$\nabla^2 \Phi = 4\pi G\rho$
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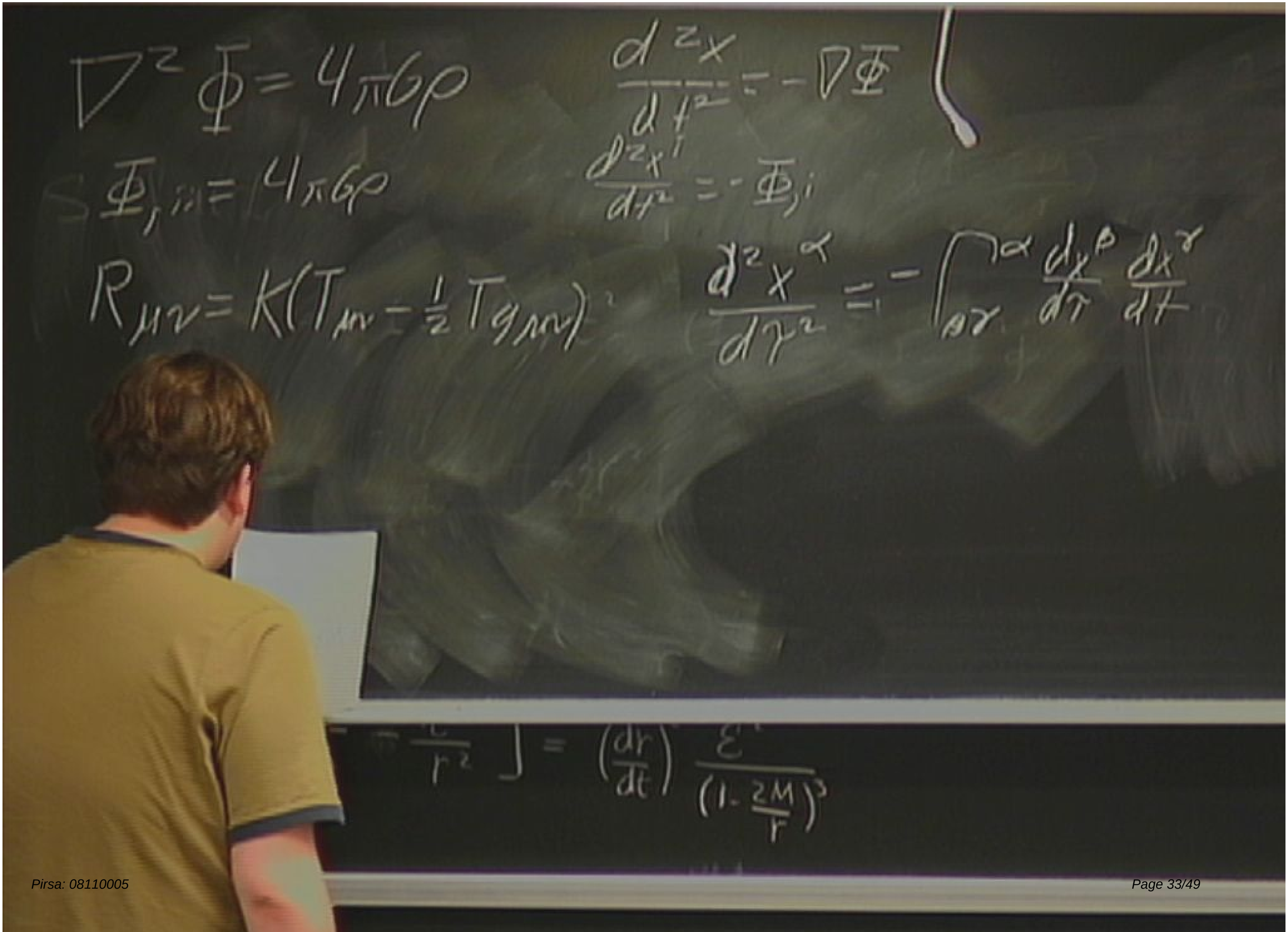
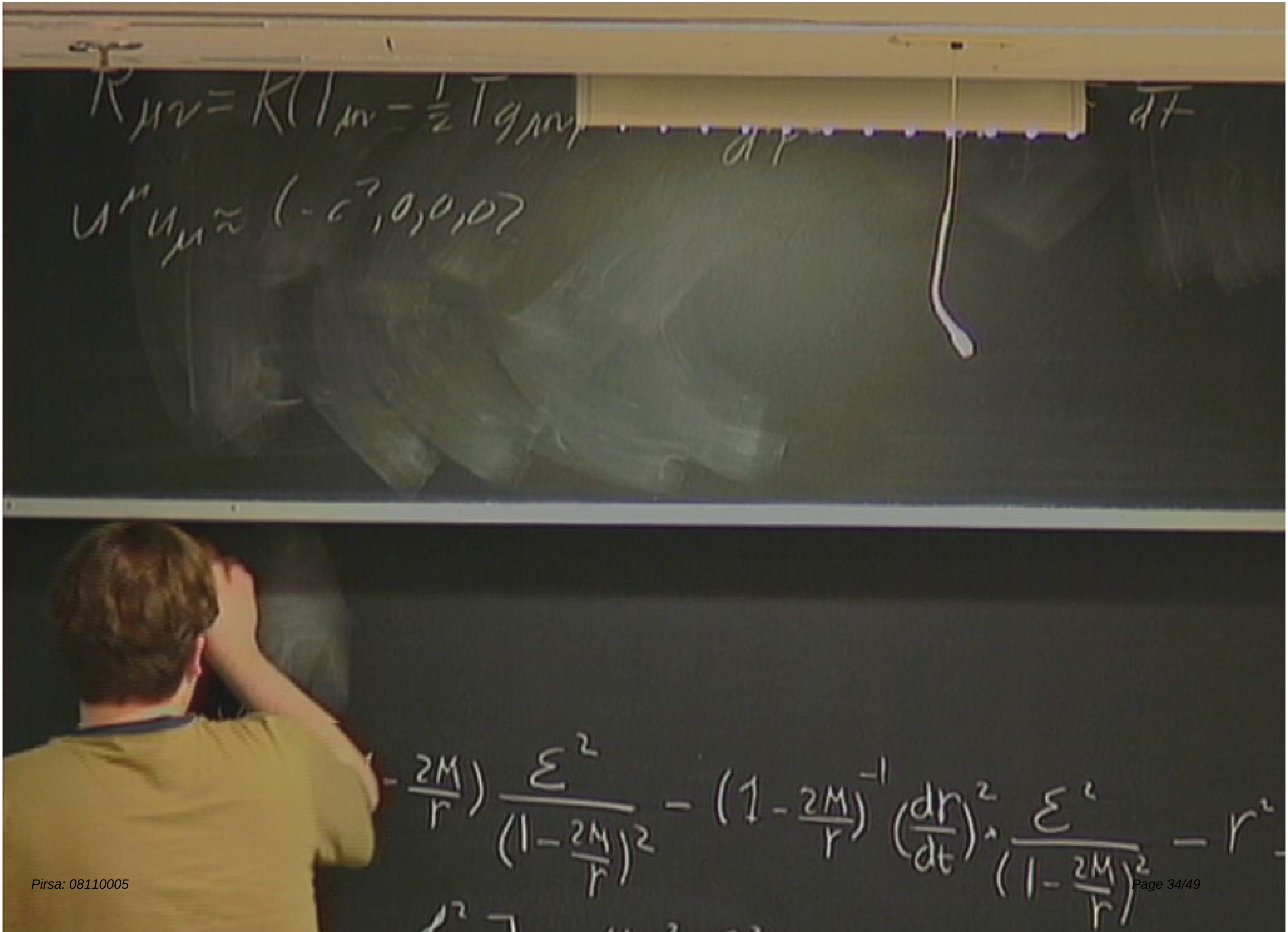
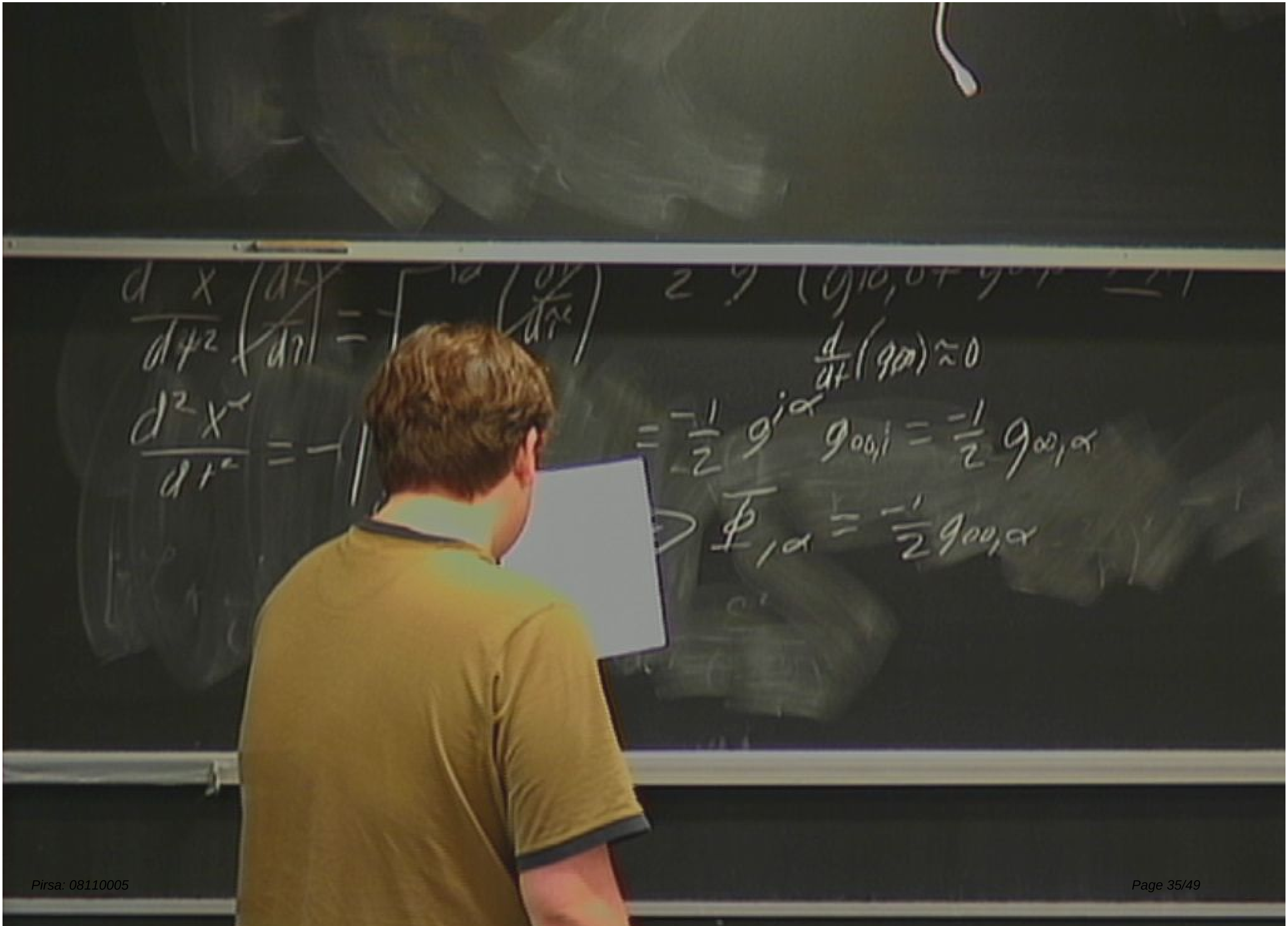
$\Phi_{,ii} = 4\pi G\rho$
$\frac{d^2x^i}{dt^2} = -\Phi_{,i}$


$\nabla^2 \Phi = 4\pi G\rho$
$\frac{d^2x}{dt^2} = -\nabla\Phi$
$\Phi_{,ii} = 4\pi G\rho$
$\frac{d^2x^i}{dt^2} = -\Phi_{,i}$
$R_{\mu\nu} = K(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})$
$\frac{d^2x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau}$


R_μν = K(T_μν − ½ T g_μν)
dτ
u^μ u_μ ≈ (−c², 0, 0, 0)


$\frac{d}{dt}(g_{00}) \approx 0$
$= -\frac{1}{2} g^{i\alpha} g_{00,i} = -\frac{1}{2} g_{00,\alpha}$
$\Rightarrow \Phi_{,\alpha} = -\frac{1}{2} g_{00,\alpha}$

$$\frac{d^2x}{dt^2}\left(\frac{dt}{d\tau}\right) = -\Gamma^{\alpha}_{00}\left(\frac{dt}{d\tau}\right)$$

$$\frac{d}{dt}(g_{00}) \approx 0$$

$$\frac{d^2x^{\alpha}}{dt^2} = -\Gamma^{\alpha}_{00}$$

$$= -\frac{1}{2} g^{i\alpha} g_{00,i} = -\frac{1}{2} g_{00,\alpha}$$

$$\Rightarrow \Phi_{,\alpha} = -\frac{1}{2} g_{00,\alpha}$$

$$g_{00} = -c^2 - 2\Phi$$

$$\frac{dx^\beta}{d\tau} = \left(\frac{dt}{d\tau}, 0, 0, 0\right) \qquad \Gamma^\alpha_{00} = \Phi_{,\alpha}$$

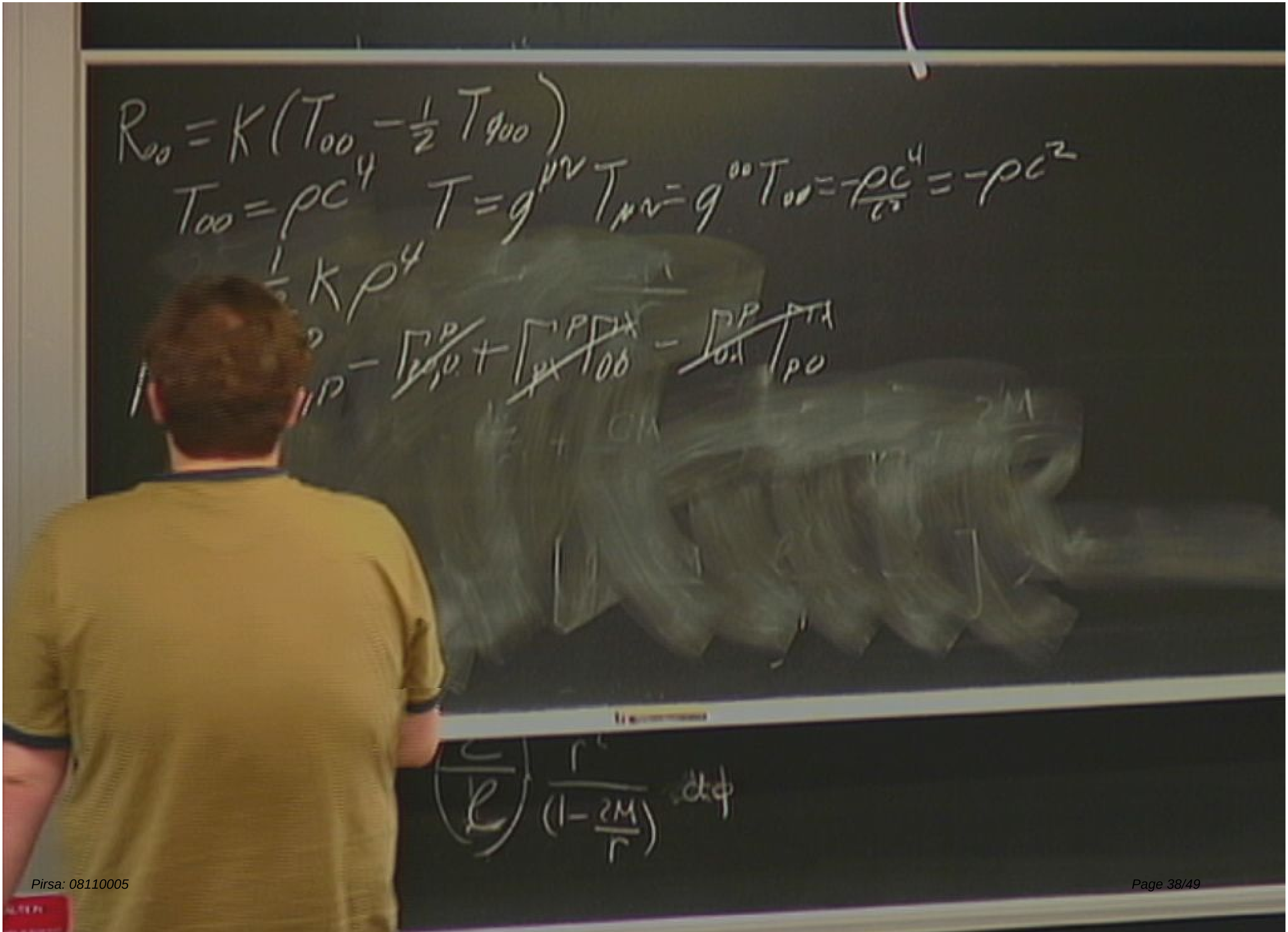
$$\frac{d^2 x^\alpha}{dt^2}\cancel{\left(\frac{dt}{d\tau}\right)} = -\Gamma^\alpha_{00}\cancel{\left(\frac{dt}{d\tau}\right)} = \frac{1}{2} g^{i\alpha}\left(g_{i0,0} + g_{0i,0} - \underline{g_{00,i}}\right)$$

$$\frac{d}{dt}(g_{00}) \approx 0$$

$$\frac{d^2 x^\alpha}{dt^2} = -\Gamma^\alpha_{00} \qquad = \frac{-1}{2} g^{i\alpha} g_{00,i} = \frac{-1}{2} g_{00,\alpha}$$

$$\Rightarrow \Phi_{,\alpha} = \frac{-1}{2} g_{00,\alpha}$$

$$g_{00} = -c^2 - 2\Phi$$


$R_{00} = K(T_{00} - \frac{1}{2} T g_{00})$
$T_{00} = \rho c^4 \quad T = g^{\mu\nu} T_{\mu\nu} = g^{00} T_{00} = -\frac{\rho c^4}{c^2} = -\rho c^2$
$\frac{1}{2} K \rho c^4$
$(1 - \frac{2M}{r})$

$$R_{00} = K\left(T_{00} - \frac{1}{2} T g_{00}\right)$$

$$T_{00} = \rho c^4 \qquad T = g^{\mu\nu} T_{\mu\nu} = g^{00} T_{00} = -\frac{\rho c^4}{c^2} = -\rho c^2$$

$$= \frac{1}{2} K \rho^4$$

$$R_{00} = \Gamma^{\rho}_{00,\rho} - \cancel{\Gamma^{\rho}_{\rho 0,0}} + \cancel{\Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{00}} - \cancel{\Gamma^{\rho}_{0\lambda}\Gamma^{\lambda}_{\rho 0}}$$

$$= \Phi_{,\rho\rho}$$

$$\Rightarrow \Phi_{,ii} = \frac{1}{2} K \rho c^4$$

$$= \left(\frac{c}{\ell}\right) \frac{r^2}{\left(1 - \frac{2M}{r}\right)} d\phi$$

$$R_{00} = K\left(T_{00} - \frac{1}{2} T g_{00}\right)$$

$$T_{00} = \rho c^4 \quad T = g^{\mu\nu} T_{\mu\nu} = g^{00} T_{00} = -\frac{\rho c^4}{c^2} = -\rho c^2$$

$$= \frac{1}{2} K \rho c^4$$

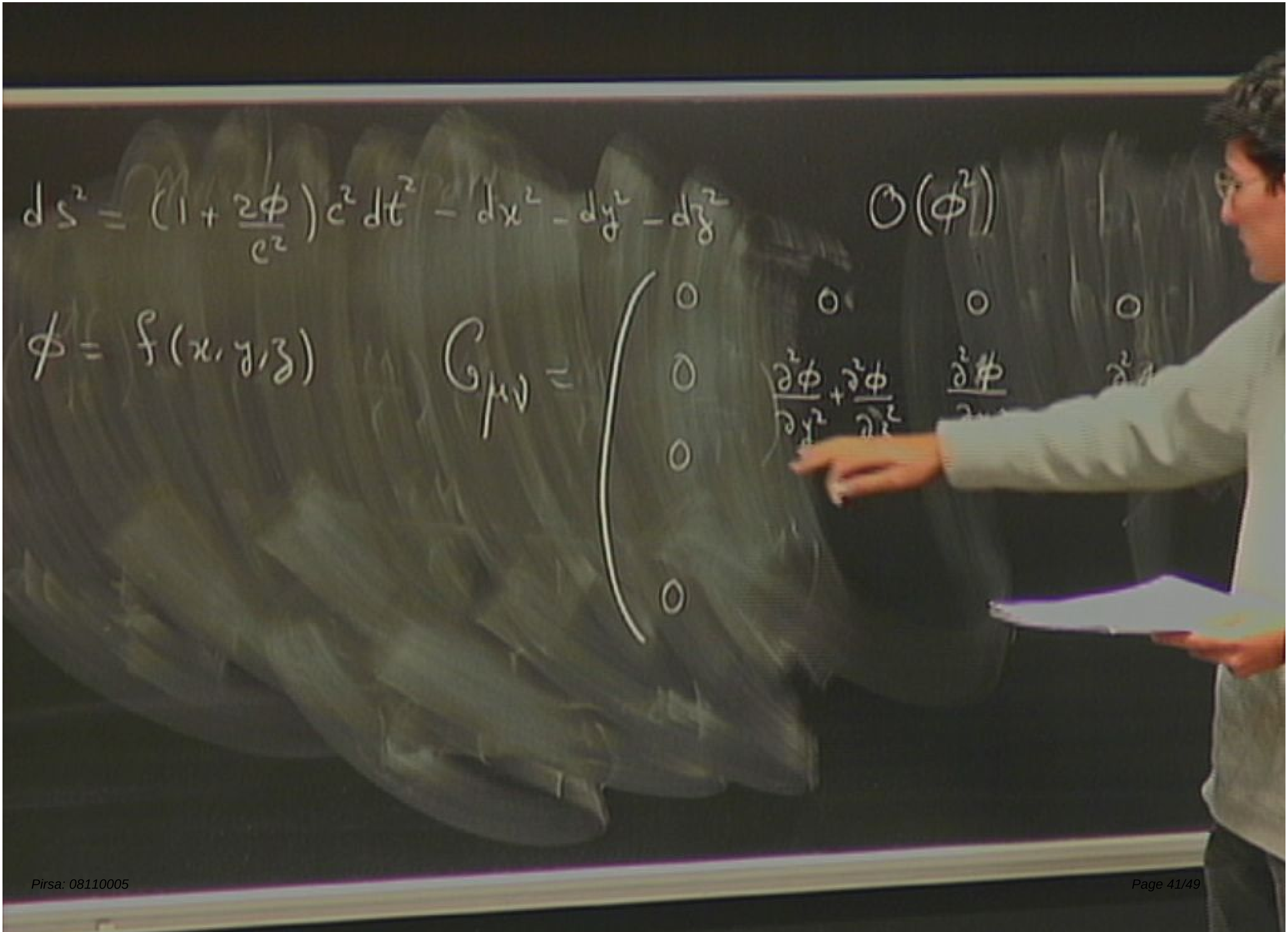
$$R_{00} = \Gamma^{\rho}_{00,\rho} - \cancel{\Gamma^{\rho}_{\rho 0,0}} + \cancel{\Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{00}} - \cancel{\Gamma^{\rho}_{0\lambda}\Gamma^{\lambda}_{\rho 0}}$$

$$= \Phi_{,\rho\rho}$$

$$K = \frac{8\pi G}{c^4} \Longrightarrow \Phi_{,ii} = 4\pi G\rho$$

$$\Longrightarrow \Phi_{,ii} = \frac{1}{2} K \rho c^4$$

$$= \left(\frac{L}{\ell}\right) \frac{r}{\left(1 - \frac{2M}{r}\right)} d\phi$$


$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - dx^2 - dy^2 - dz^2$
$O(\phi^2)$
$\phi = f(x, y, z)$
$G_{\mu\nu} =$
$0 \quad 0 \quad 0 \quad 0$
$0 \quad \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$
0
0

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right)c^2 dt^2 - dx^2 - dy^2 - dz^2 \qquad O(\phi^2)$$

$$\phi = f(x, y, z) \qquad G_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} & \frac{\partial^2\phi}{\partial y\,\partial x} & \frac{\partial^2\phi}{\partial x\,\partial z} \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right)c^2 dt^2 - dx^2 - dy^2 - dz^2 \qquad O(\phi^2)$$

$$\phi = f(x, y, z)$$

$$G_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} & \frac{\partial^2\phi}{\partial y\,\partial x} & \frac{\partial^2\phi}{\partial x\,\partial z} \\ 0 & & \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2 \zeta}{\partial} & \\ 0 & & & \end{pmatrix}$$

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right)c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$O(\phi^2)$$

$$2\nabla^2\phi =$$

$$\phi = f(x, y, z)$$

$$G_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} & \frac{\partial^2\phi}{\partial y\,\partial x} & \frac{\partial^2\phi}{\partial x\,\partial z} \\ 0 & & \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} & \\ 0 & & & \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

$$2\nabla^2\phi = 8\pi\rho(c^2+u^2)$$

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right)c^2dt^2 - dx^2 - dy^2 - dz^2 \qquad O(\phi^2)$$

$$\phi = f(x,y,z)$$

$$G_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} & \frac{\partial^2\phi}{\partial y\partial x} & \frac{\partial^2\phi}{\partial x\partial z} \\ 0 & & \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} & \\ 0 & & & \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

$$\left(1-\frac{2\phi}{c^2}\right)$$

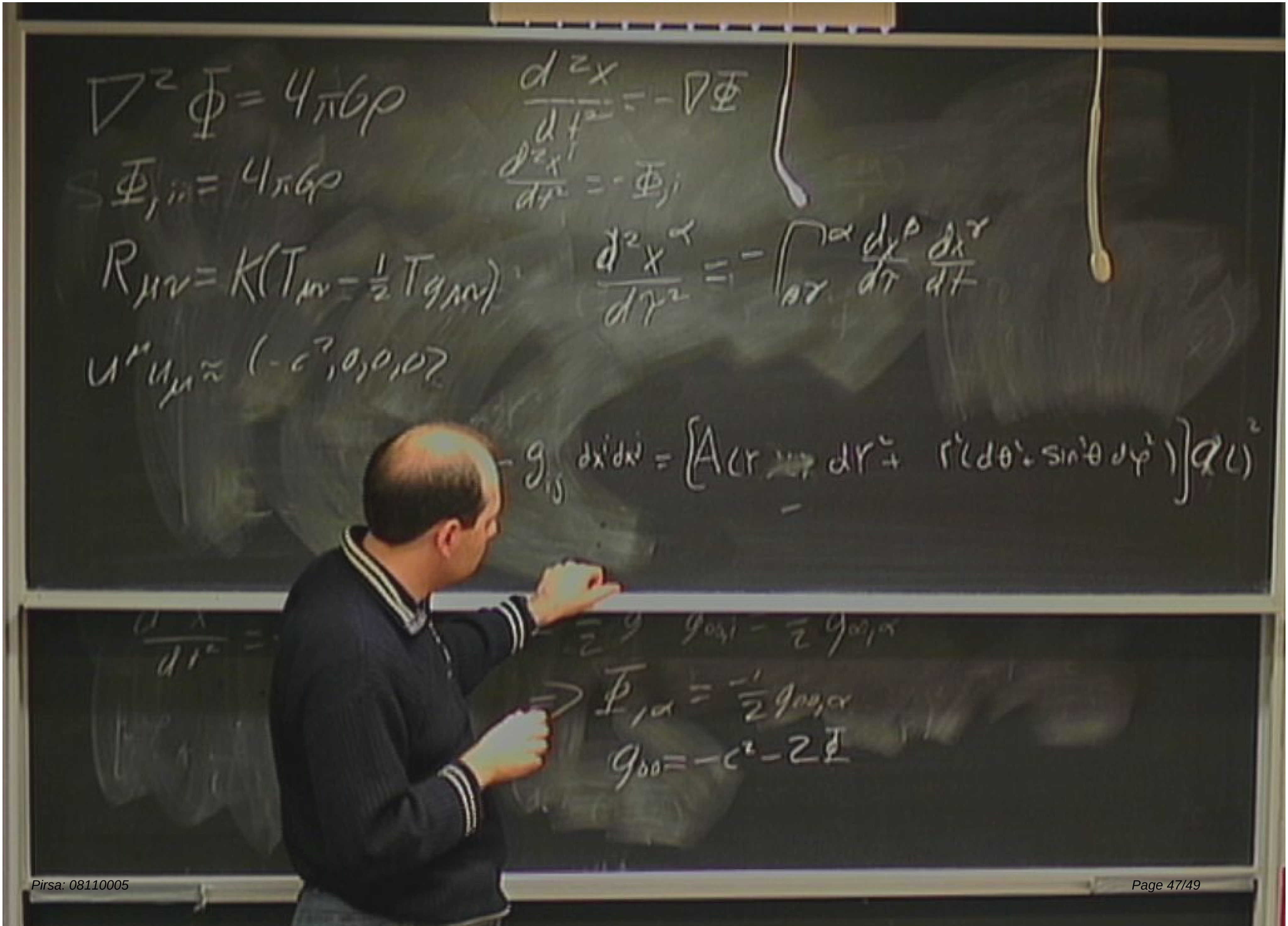
$$ds^2 = \left(1+\frac{2\phi}{c^2}\right)c^2dt^2 - \left[dx^2 - dy^2 - dz^2\right] \qquad O(\phi^2)$$

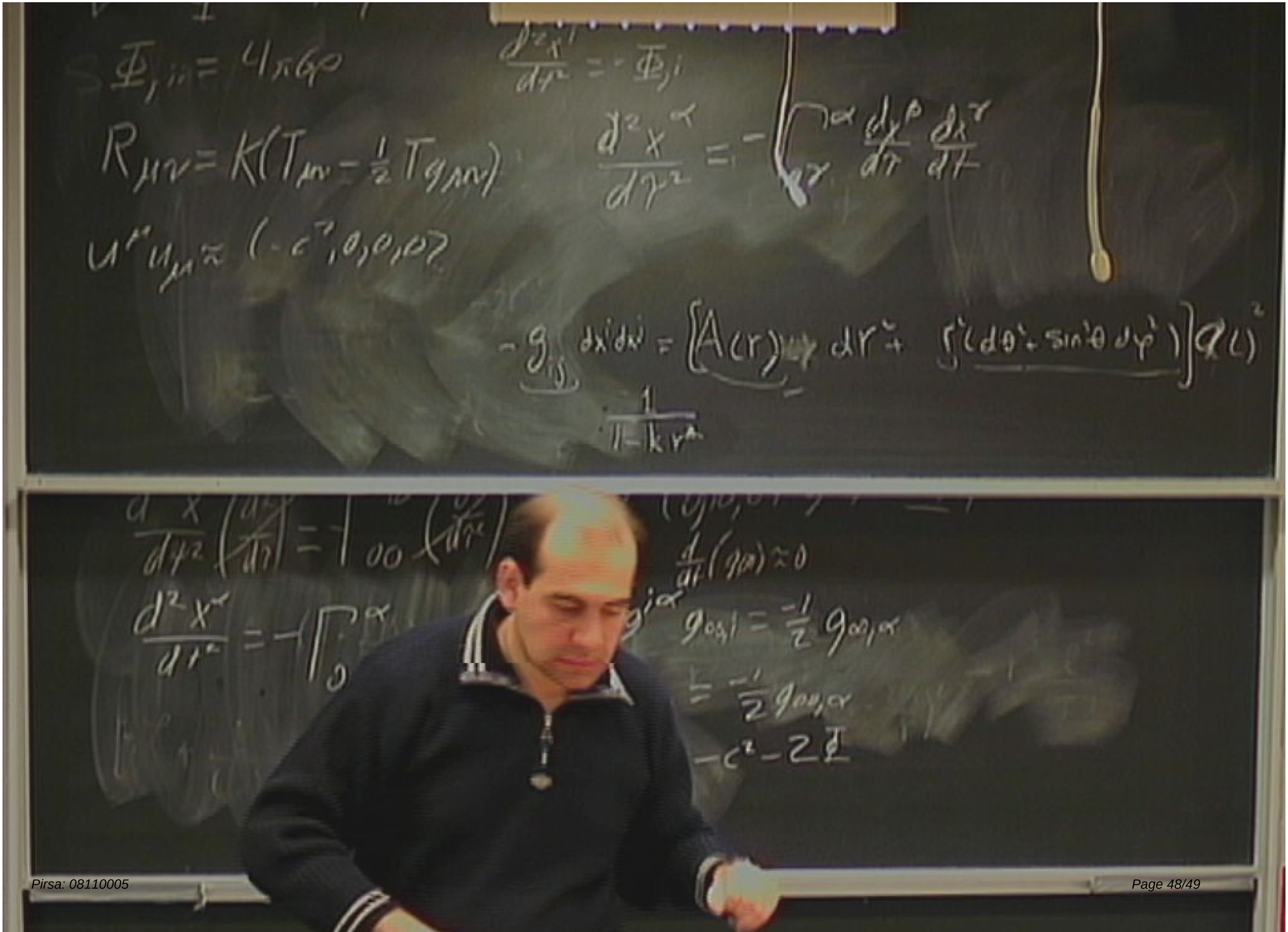
$$2\nabla^2\phi = 8\pi\rho(a^2+\cancel{v^2})$$

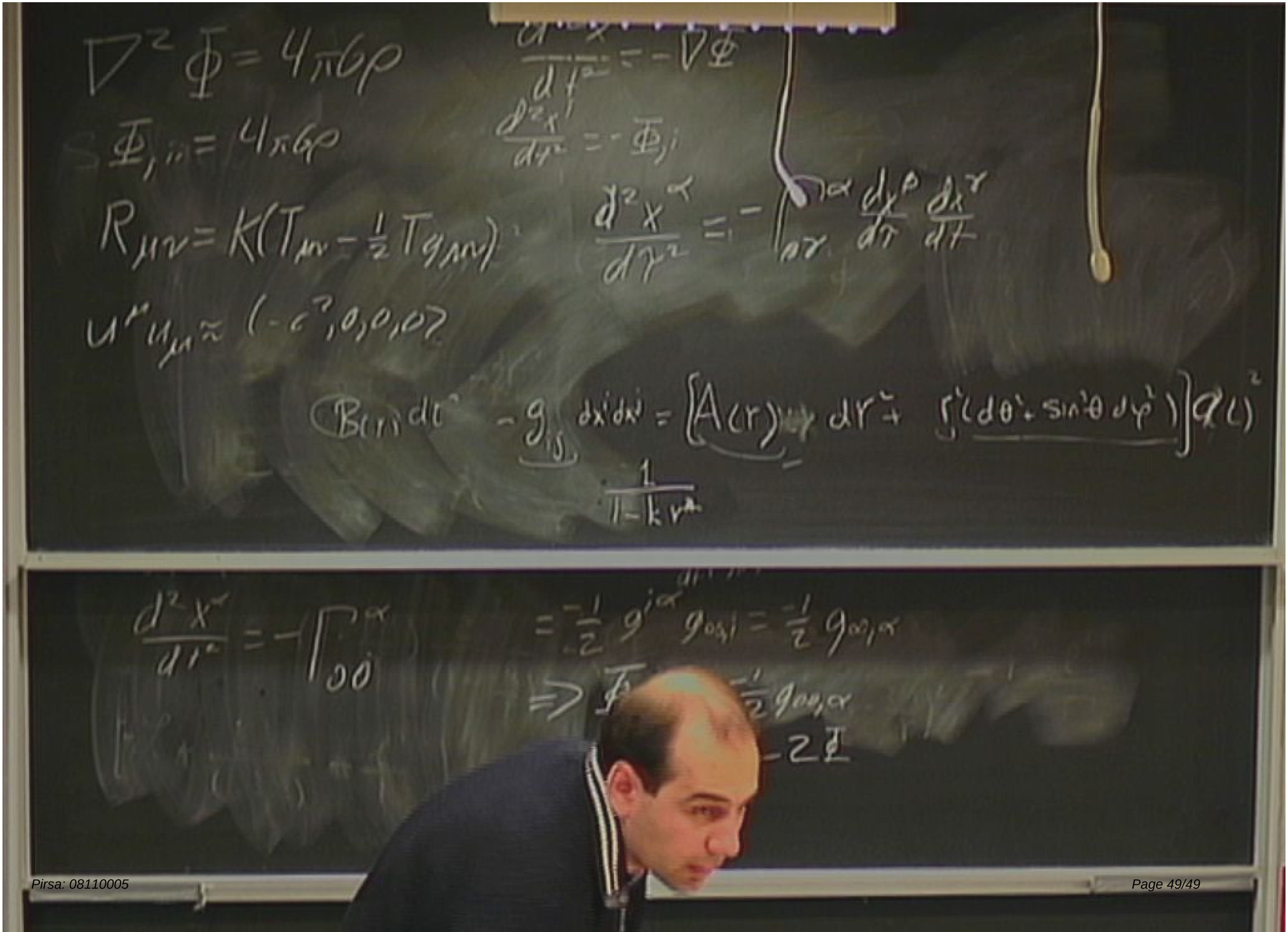
$$\nabla^2\phi = 4\pi\rho$$

$$\phi = f(x,y,z)$$

$$G_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} & \frac{\partial^2\phi}{\partial y\partial x} & \frac{\partial^2\phi}{\partial x\partial z} \\ 0 & & \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2} & \\ 0 & & & \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2} \end{pmatrix}$$


∇²Φ = 4πGρ
d²x/dt² = −∇Φ
Φ,ii = 4πGρ
d²x^i/dt² = −Φ,i
R_μν = K(T_μν − ½ T g_μν)
d²x^α/dτ² = −Γ^α_βγ dx^β/dτ dx^γ/dτ
u^μ u_μ ≈ (−c², 0, 0, 0)
g_ij dx^i dx^j = [A(r) dr² + r²(dθ² + sin²θ dφ²)] a(t)²
Φ,α = −½ g_00,α
g_00 = −c² − 2Φ


Φ,ii = 4πGρ
d²xⁱ/dt² = -Φ,i
Rμν = K(Tμν - ½ T gμν)
d²x^α/dτ² = -Γ^α_βγ dx^β/dτ dx^γ/dτ
u^μ u_μ = (-c², 0, 0, 0)
-gij dxⁱdxʲ = [A(r) dr² + r²(dθ² + sin²θ dφ²)] a(t)²
1/(1-kr²)
d/dτ(g00) ≈ 0
d²x^α/dτ² = -Γ^α_00
g00,i = -½ g00,α
= -½ g00,α
-c² - 2Φ


$\nabla^2 \Phi = 4\pi G\rho$
$\Phi_{,ii} = 4\pi G\rho$
$R_{\mu\nu} = K(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})$
$u^\mu u_\mu \approx (-c^2, 0, 0, 0)$
$\frac{d^2 x^i}{dt^2} = -\Phi_{,i}$
$\frac{d^2 x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau}$
$\frac{d^2 x^\alpha}{dt^2} = -\Gamma^\alpha_{00}$
$= -\frac{1}{2} g^{i\alpha} g_{00,i} = -\frac{1}{2} g_{00,\alpha}$
$- 2\Phi$