Title: The emergence of photons, electrons, and gravitons from quantum qbit systems

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URL: http://pirsa.org/08110003

Abstract: We show that if a condensed matter system (a quantum qbit system) is in a string-net condensed state, then the low energy excitation in such a system can be gauge bosons (such as photons) and fermions (such as electrons). Such a system is actually the ether that we have been looking for 150 years. We will also discuss a quantum qbit system that may even give rise to emergent gravitons.

The emergence of photons, electrons, and gravitons

Xiao-Gang Wen

Nov, 2008, Perimeter

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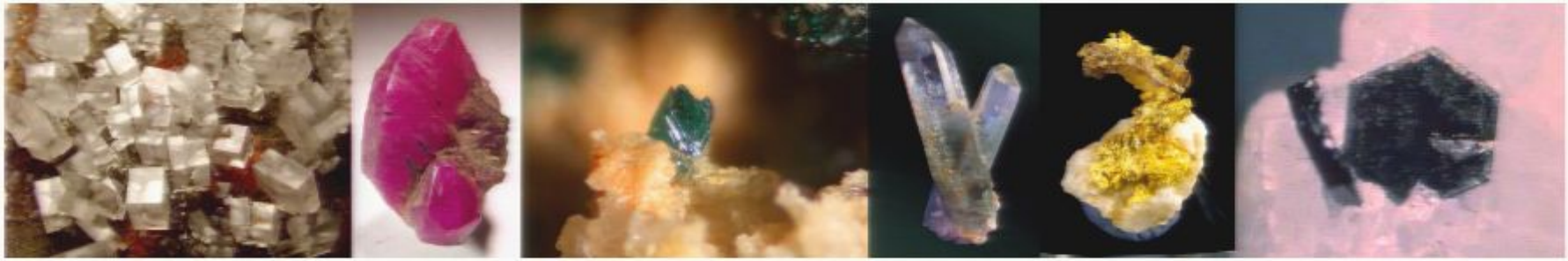
How to gain a deeper understanding of our world?



Reductionist approach

	LEPTONS		QUARKS	
Mass Particles	**Electron** Responsible for electricity and chemical reactions It has a charge of -1 Its anti-particle, the positron,has a charge of +1	**Electron Neutrino** Particle with no electric charge, and possibly no mass. Billions fly through your body every second.	**Up** It has an electric charge of +2/3. Protons contain 2, neutrons contain 1.	**Down** It has an electric charge of -1/3 Protons contain 1, neutrons contain 2.
All ordinary particles belong to this group				
These particles only existed just after the Big Bang. Now they are found in cosmic rays or produced in scientific laboratories such as CERN.	**Muon** It is heavier than the electron. It lives for two millionths of a second It has a charge of ±1	**Muon Neutrino** Created along with muons when some particles decay. It has no electric charge.	**Charm** Discovered in 1974. It is heavier than the Up. It has a charge of +2/3	**Strange** Discovered in 1963. It is heavier than the Down. It has a charge of -1/3
	Tau Heavier still; it is extremely unstable. It was discovered in 1975. It has a charge of ±1	**Tau Neutrino** Discovered in 2000. It has no electric charge.	**Top** Heavier still. Discovered in 1995. Electric charge +2/3	**Bottom** Heavier still; measuring bottom quarks is an important test of electroweak theory. Discovered in1977. Electric charge -1/3

Force Particles

These particles transmit the four fundamental forces of nature. Gravitons have so far not been discovered.

Gluons
Carriers of the strong force between quarks

Felt by: quarks and gluons

The explosive release of nuclear energy is the result of the strong force.

Photons
Particles that make up light. They carry the electromagnetic force

Felt by: charged particles

Electricity, magnetism and chemistry are all the results of electromagnetic force.

Intermediate vector bosons
Carriers of the weak force

Felt by: quarks and leptons

Some forms of radio-activity are the reult of the weak force.

Gravitons
Carriers of gravity

Felt by: all particles with mass

All the weight we experience is the result of the gravitational force.

We may want to go even deeper

What are the smaller parts of an electron or a photon?

What is the origin of elementary particles?

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Superstring theory (the old version):

photon graviton quark electron

Emergence approach

- Reductionist approach: space is EMPTY
- Emergence approach: space is a dynamical medium
  Elementary particles are the motions, the defects, the "whirlpools", etc in the medium.

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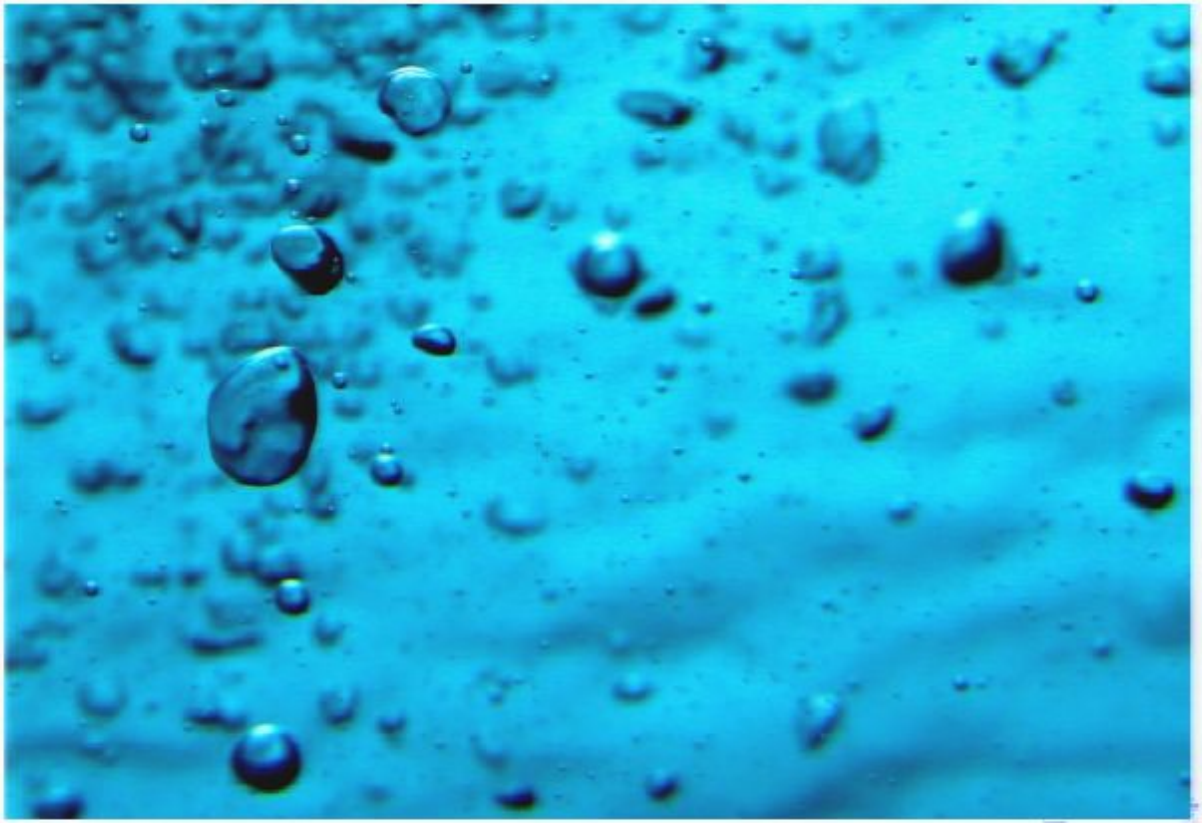
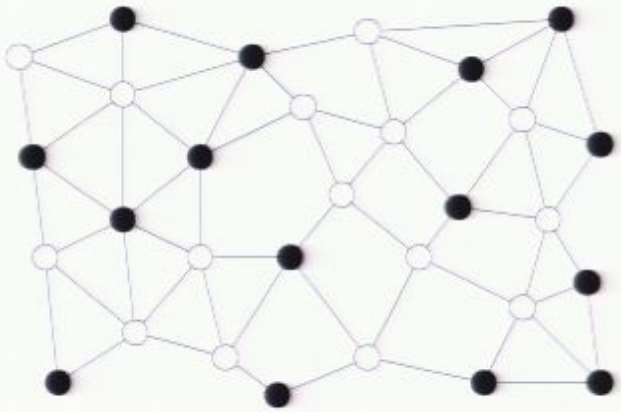
- Space $\sim$ Ocean Elementary particles $\sim$ Bubbles in ocean



What is an empty space?

- Space is formed by "degrees of freedom".
  No degrees of freedom, no space.
- Degrees of freedom = quantum states $|0\rangle, |1\rangle$ = quantum qbits (spins $|\downarrow\rangle, |\uparrow\rangle$)

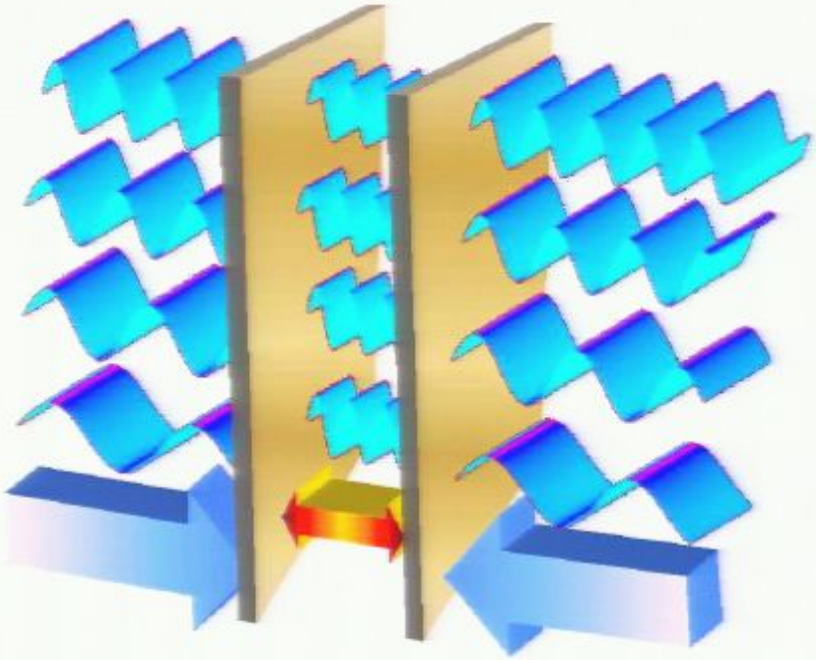
  Space = a quantum qbit system

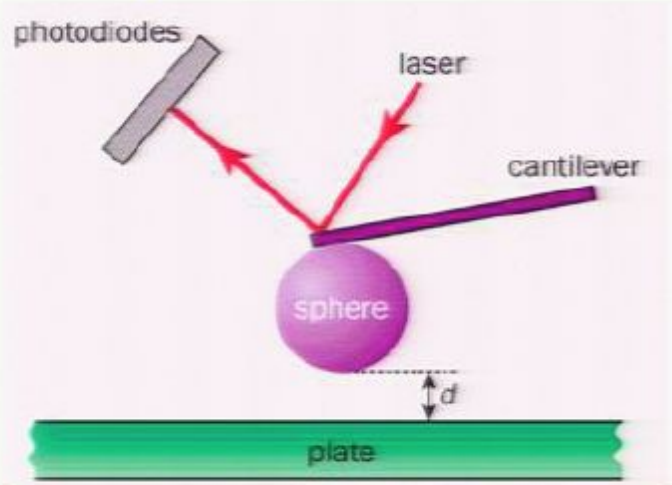


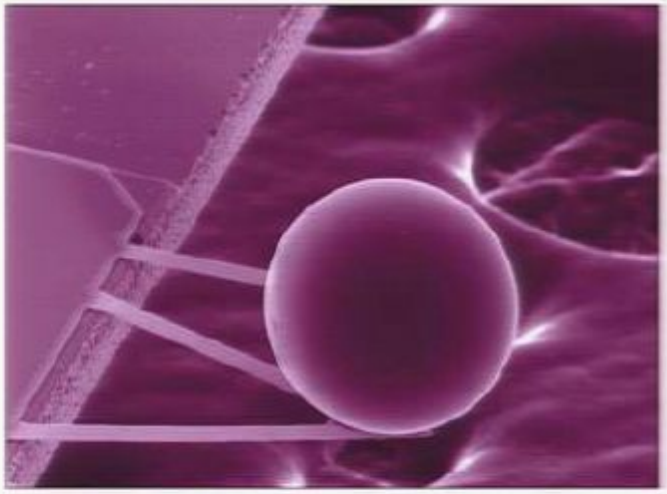
- "Empty" space = ground state of a quantum qbit system
- "Elementary particles" = motions/excitations above the ground state.

Experimental evidences of space as a dynamical medium

Casimire effect:
zero-point quantum fluctuations of a dynamical medium







Chen and Mohideen, Phys. Rev. Lett. 88, 101801 (2002)

Emergence of elementary particles

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- Particle-wave duality $\rightarrow$ Origin of particles = Origin of waves
- Empty space = ground state of the qbits
  Qbits in the ground state are organized in a certain way to minimize their energy.
- Excitations above the ground state = deformed qbit organization = propagating waves.
- If the motion of the wave satisfies the Maxwell equation, then we can view the wave as electromagnetic wave and claim that photons arise from the motions of qbits.

First try

$$H = -\sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

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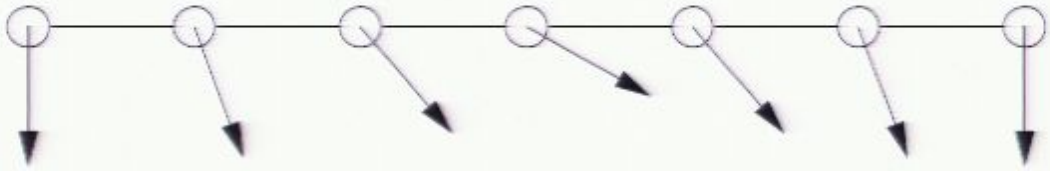

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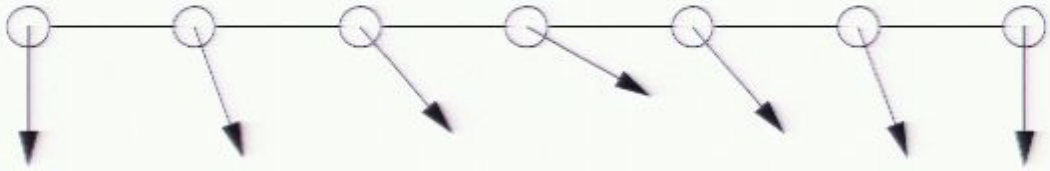

$\omega_k \propto k^2 \rightarrow$ The spin wave cannot satisfies the Maxwell equation

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Ground state = an anti-ferromagnetic state
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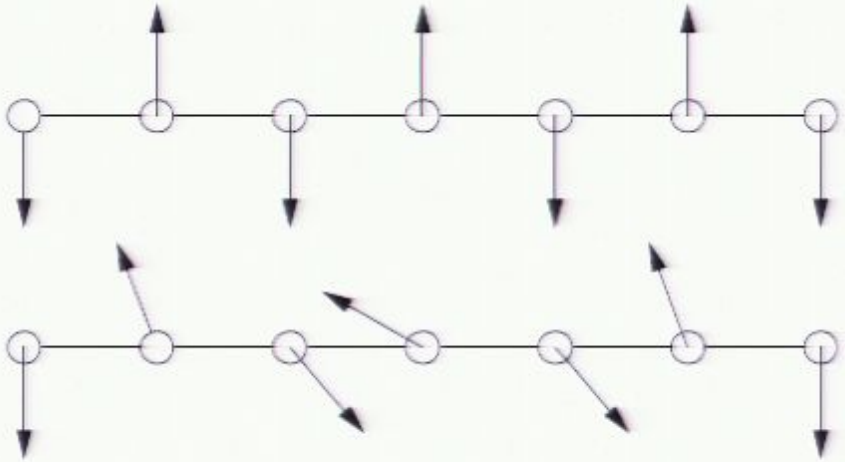

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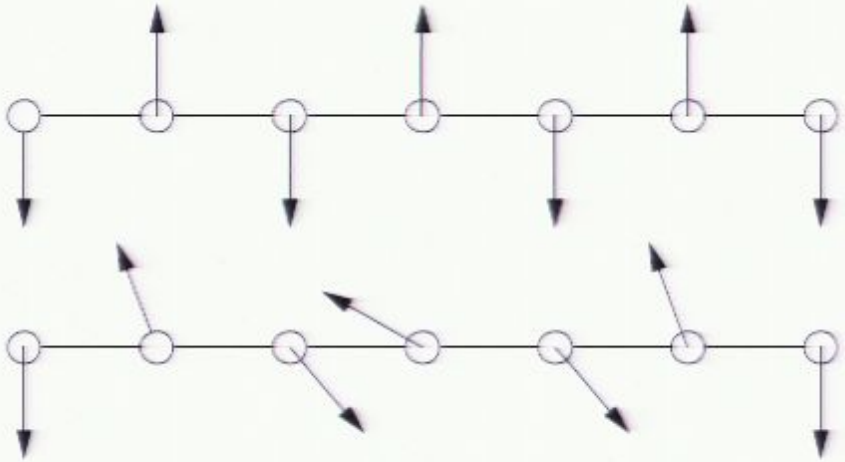

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  The wave equation $\frac{d^2\mathbf{S}}{dt^2} - \frac{d^2\mathbf{S}}{d\mathbf{x}^2} = 0$ is Euler equation, not Maxwell equation.

Principle of emergence

- Spins have different organizations (orders) in the ferromagnetic and anti-ferromagnetic ground states.
- Different spin organizations lead to different waves:
  Ferro: $\frac{d\mathbf{S}}{dt} - \mathbf{S} \times \frac{d^2\mathbf{S}}{d\mathbf{x}^2} = 0$ Anti-ferro: $\frac{d^2\mathbf{S}}{dt^2} - \frac{d^2\mathbf{S}}{d\mathbf{x}^2} = 0$
- In quantum theory, different waves $\longrightarrow$ different kinds of "elementary particles" or emergent particles.

Organization in ground state $\longrightarrow$ type of waves $\longrightarrow$ type of particles

What organization of spins give rise to Maxwell equation and photons?

Symmetry breaking theory of states of matter

- We used to believe that all organizations of qbits (spins) are described by symmetry breaking.
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- But non of those Goldstone modes satisfy the Maxwell equation. **Non emergent photons**.
- Non of those Goldstone modes satisfy the Dirac equation. **Non emergent electrons**.
- Non of those Goldstone modes satisfy the Einstein equation. **Non emergent gravitons**.
- Maybe ether really does not exists and photons, electrons, and gravitons are really elementary.

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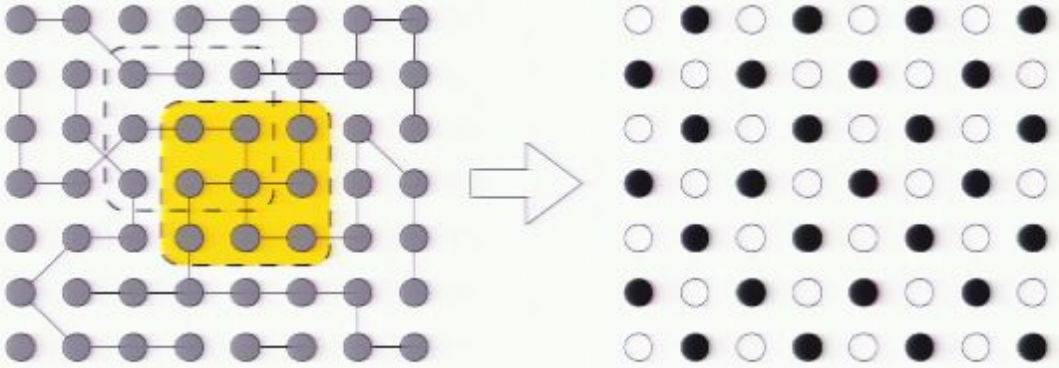
New states of matter beyond symmetry breaking – Long range entanglements

But the symmetry breaking states are basically direct product states: $\cdots|\uparrow\rangle\otimes|\downarrow\rangle\otimes|\uparrow\rangle\otimes|\downarrow\rangle\otimes\cdots$
with only short ranged entanglement



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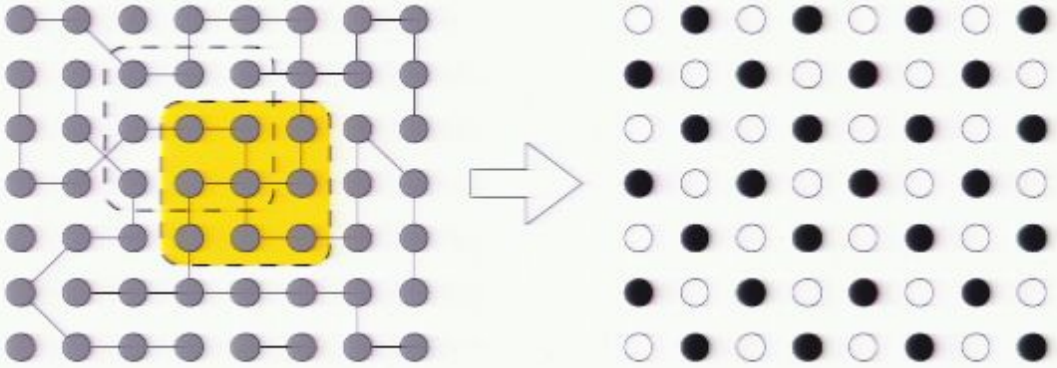
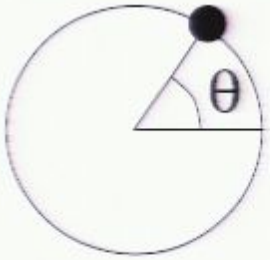
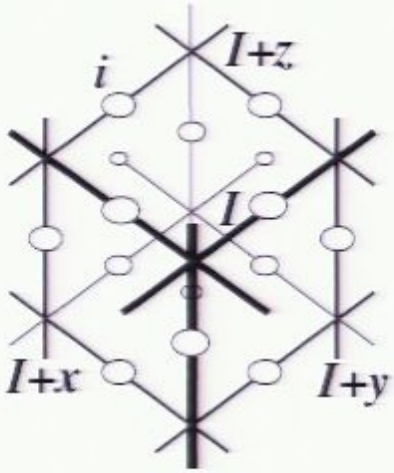


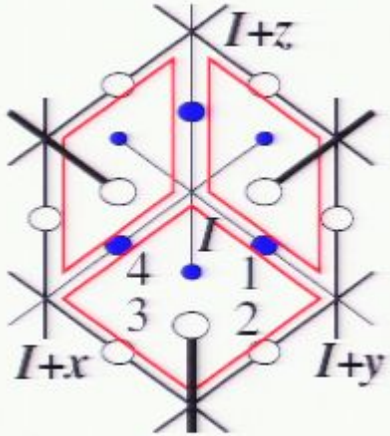
- States with long range entanglements do exist and represent new states of matter (new kind of order – *topological order*)
- Their collective modes may satisfy Maxwell equation, Dirac equation, and/or Einstein equation.

Third try: a rotor model on cubic lattice



one rotor





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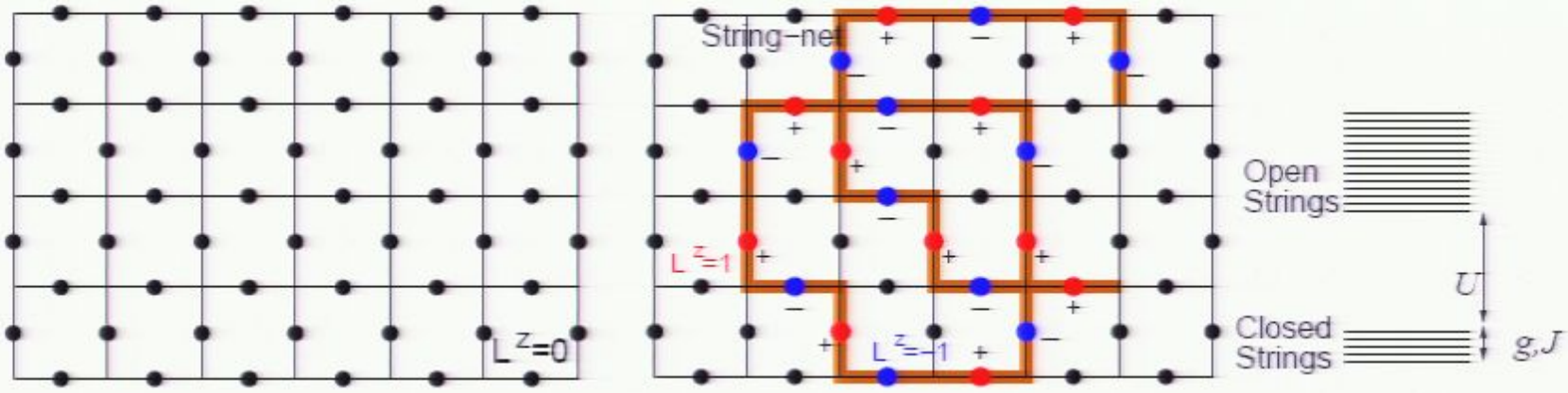
String-net condensation



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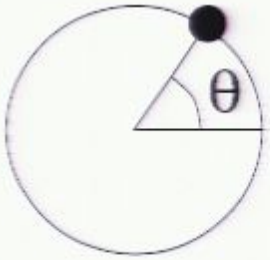

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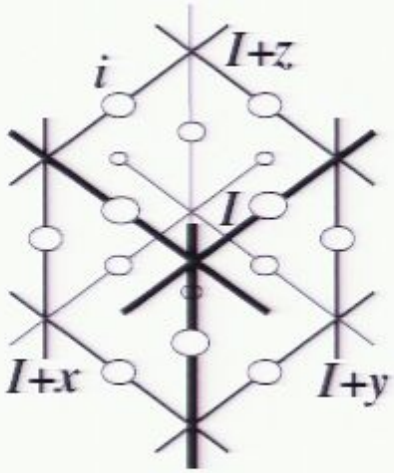

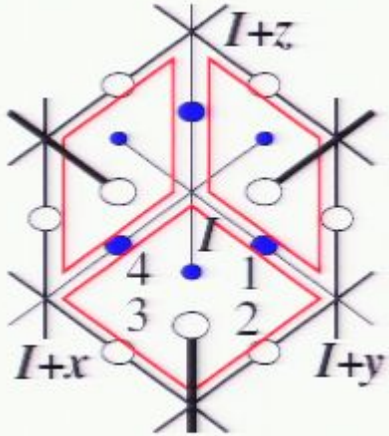

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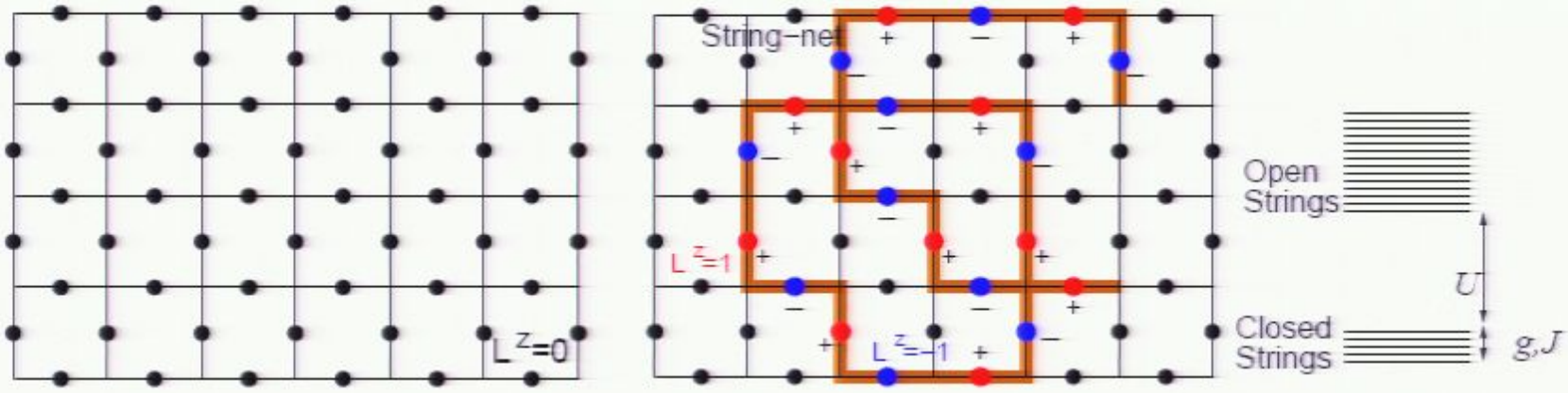

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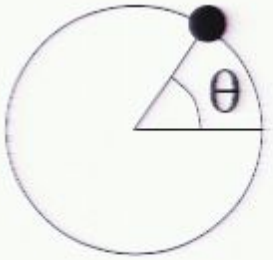

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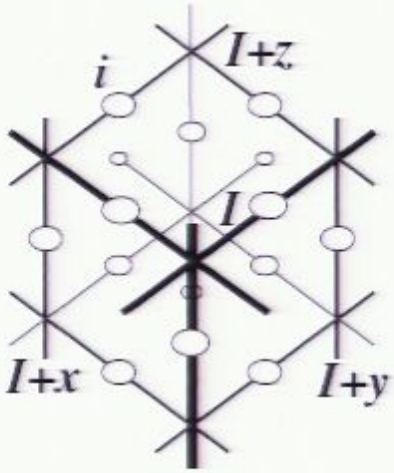

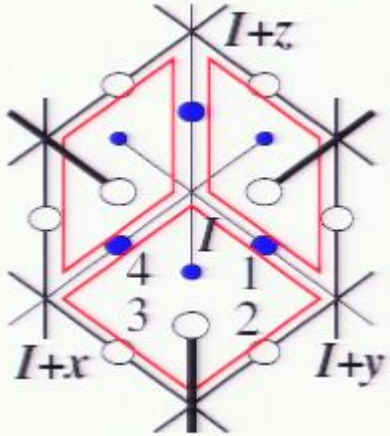

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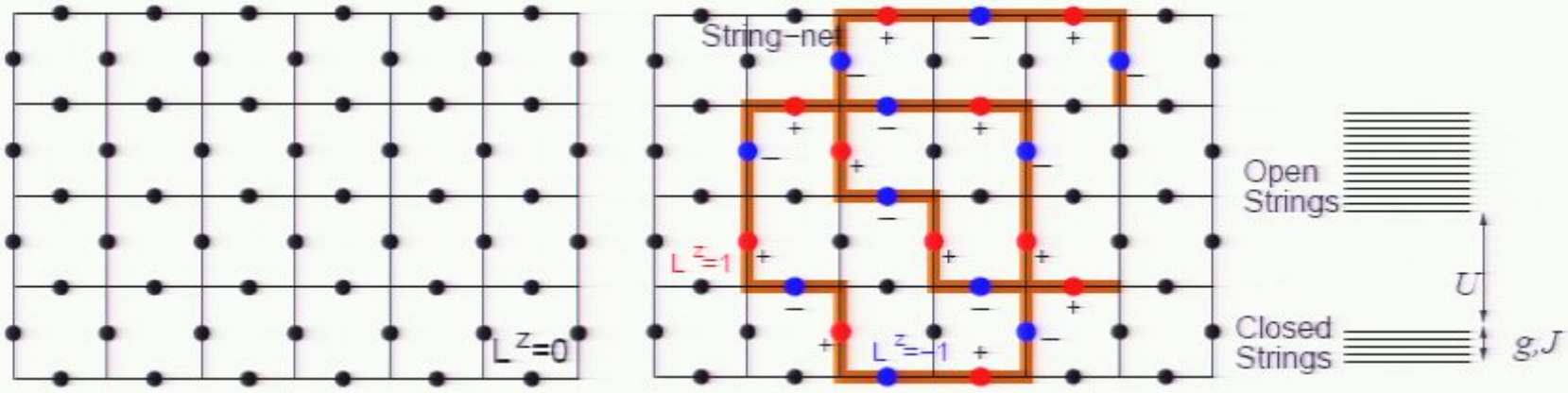

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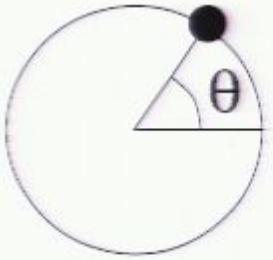

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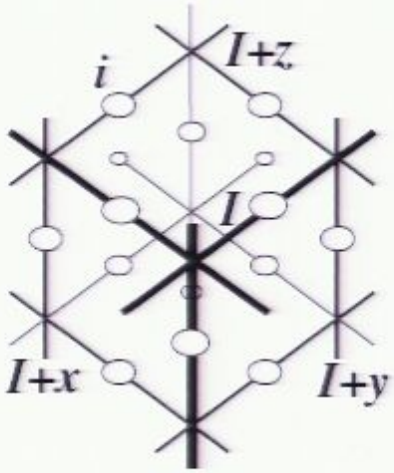

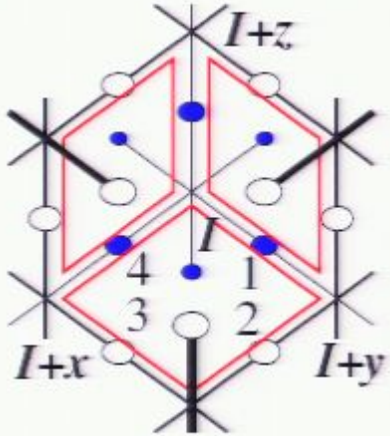

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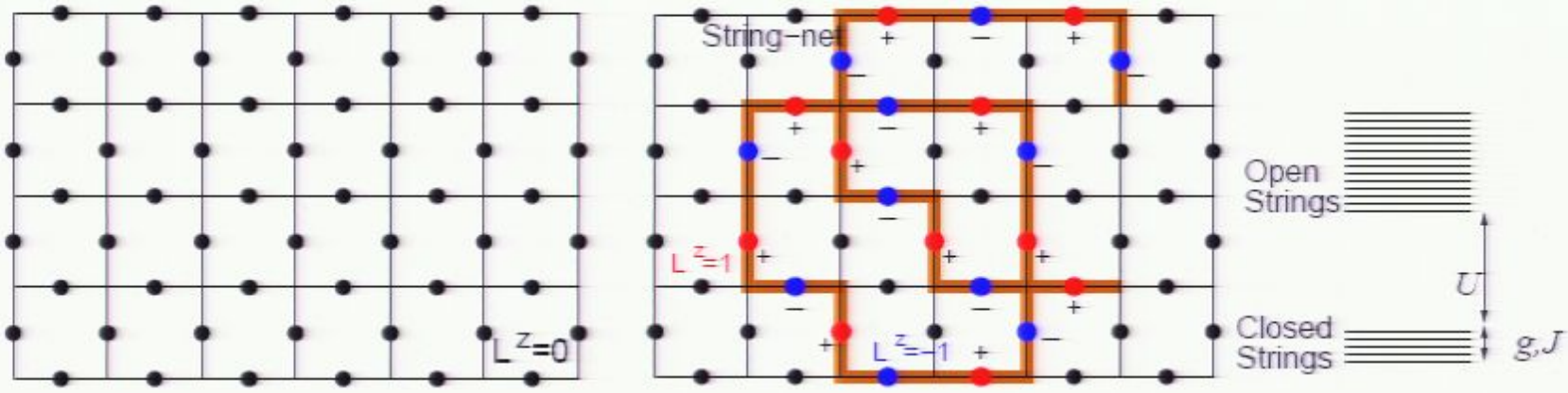

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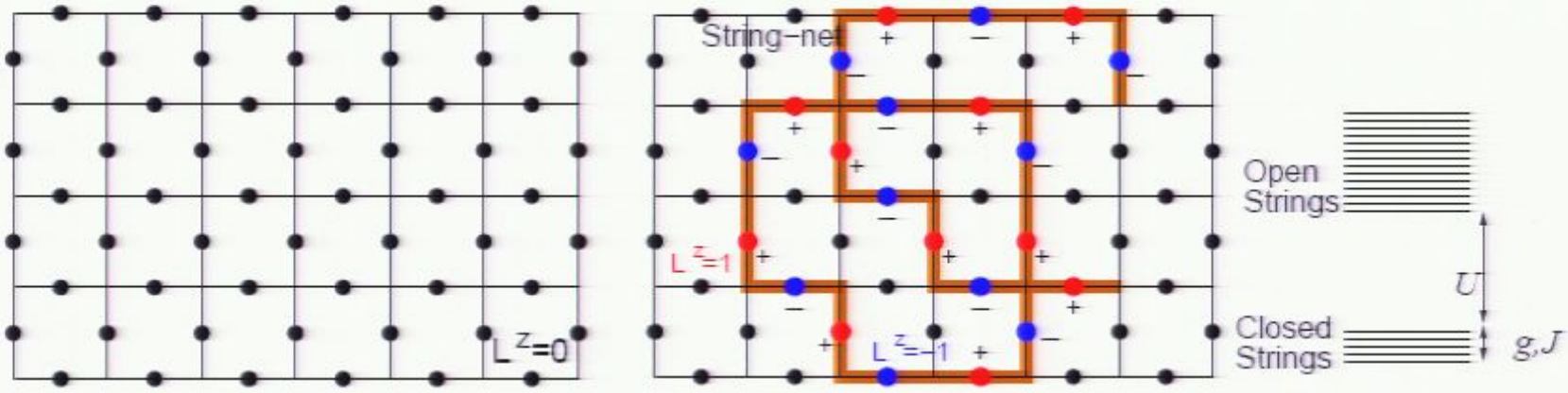

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- The ground state of the rotor Hamiltonian H when $U \gg g \gg J$

$$|\text{String-net liquid}\rangle = \sum_{\text{all closed string conf.}} \left| \quad \right\rangle$$

- The string-net liquid is new state of quantum matter with long range entanglement

The waves in string-net liquid satisfy Maxwell equation

Zero-point fluctuations in ground state Waves in string-net liquid

- String density wave: "string density" $\mathbf{E}(\mathbf{r}, t)$ satisfies $\partial \cdot \mathbf{E} = 0$

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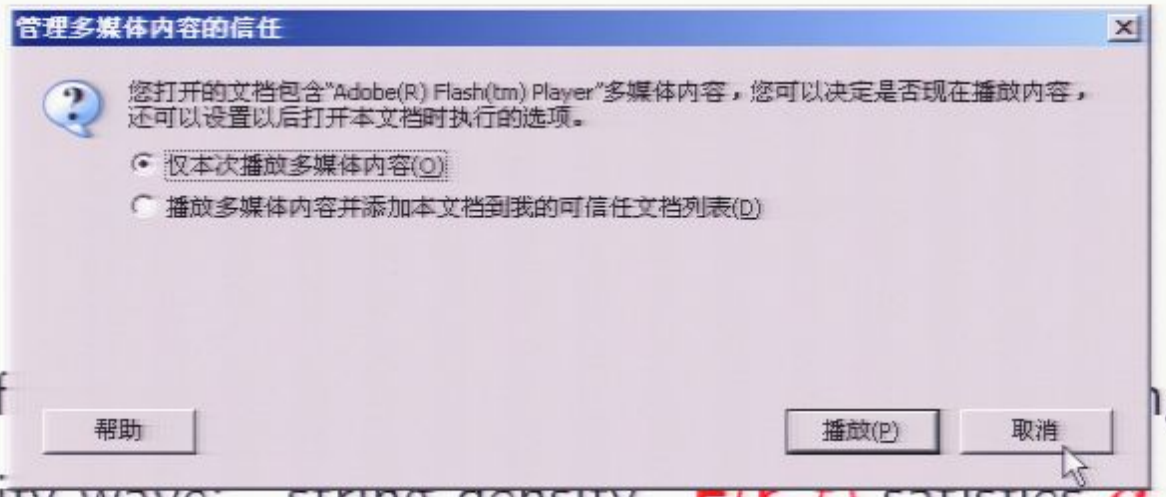

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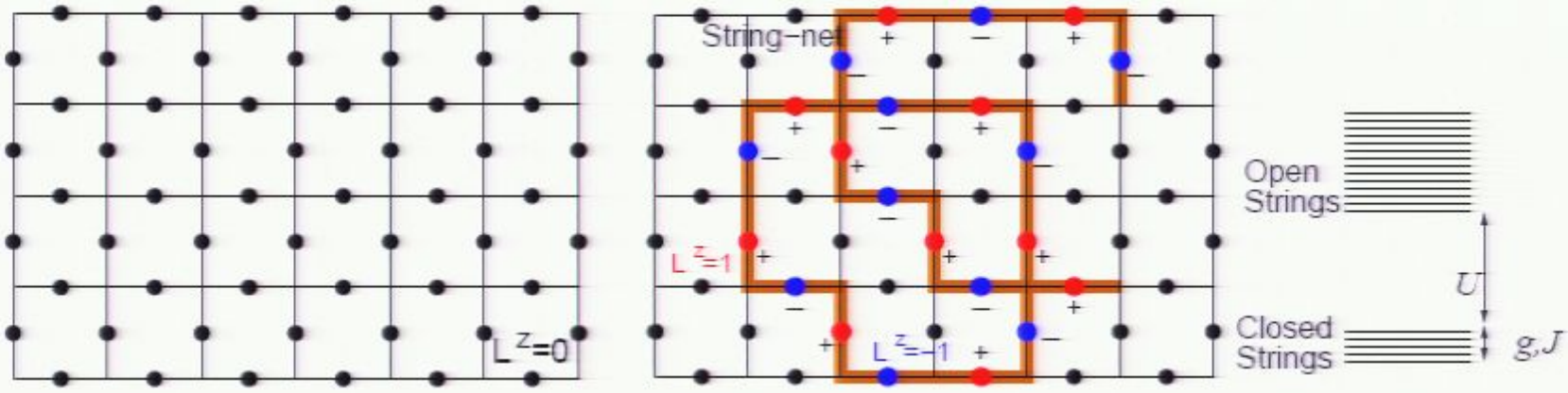

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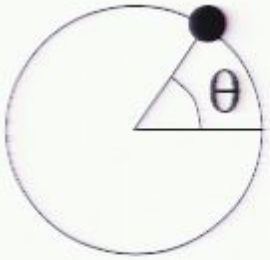

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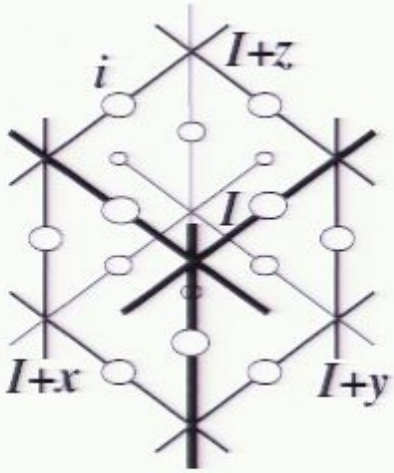
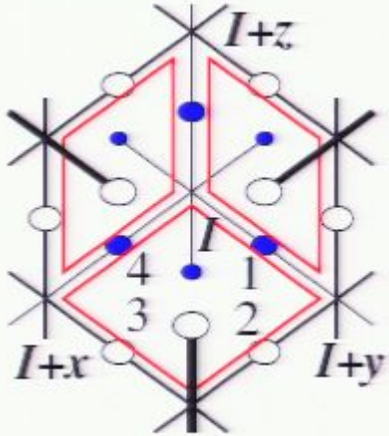




A rotor θ_i on every link of the cubic lattice:

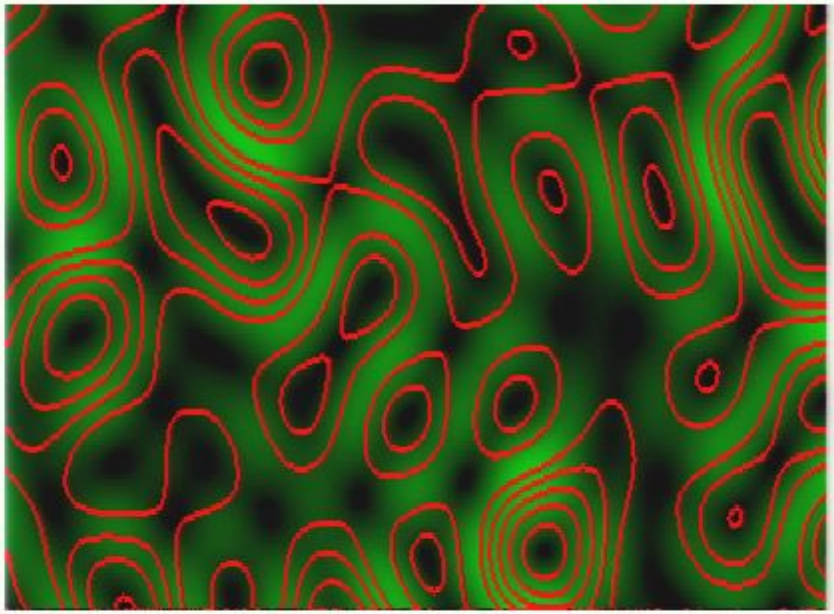
$$H = U \sum_{\mathbf{I}} Q_{\mathbf{I}}^2 - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{i}} (L_{\mathbf{i}})^2$$

$$Q_{\mathbf{I}} = \sum_{\mathbf{i} \text{ next to } \mathbf{I}} L_{\mathbf{i}}, \quad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

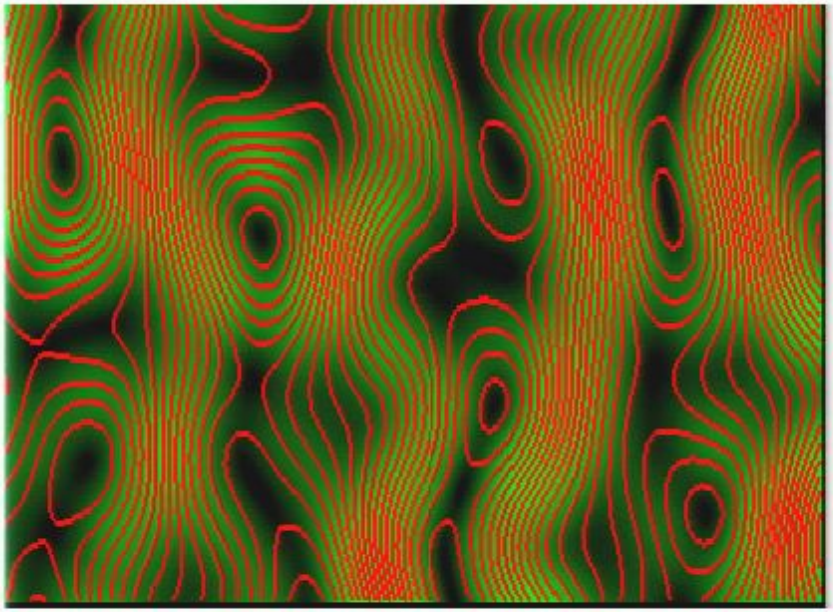
$L = i\partial_\theta$: the angular momentum of the rotor

$L^\pm = e^{\pm i\theta}$: the raising/lowering operators of L

The waves in string-net liquid satisfy Maxwell equation



Zero-point fluctuations in ground state



Waves in string-net liquid

- String density wave: "string density" $\mathbf{E}(\mathbf{r}, t)$ satisfies $\partial \cdot \mathbf{E} = 0$

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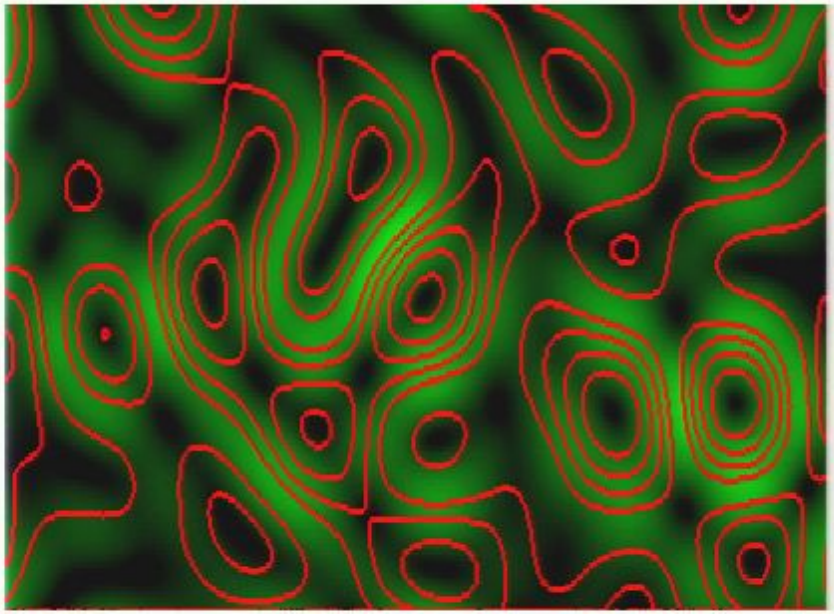

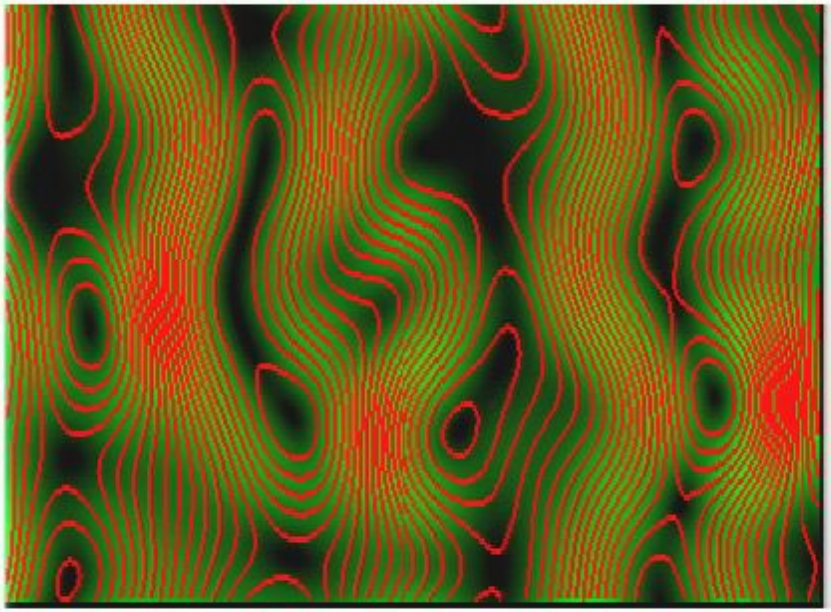

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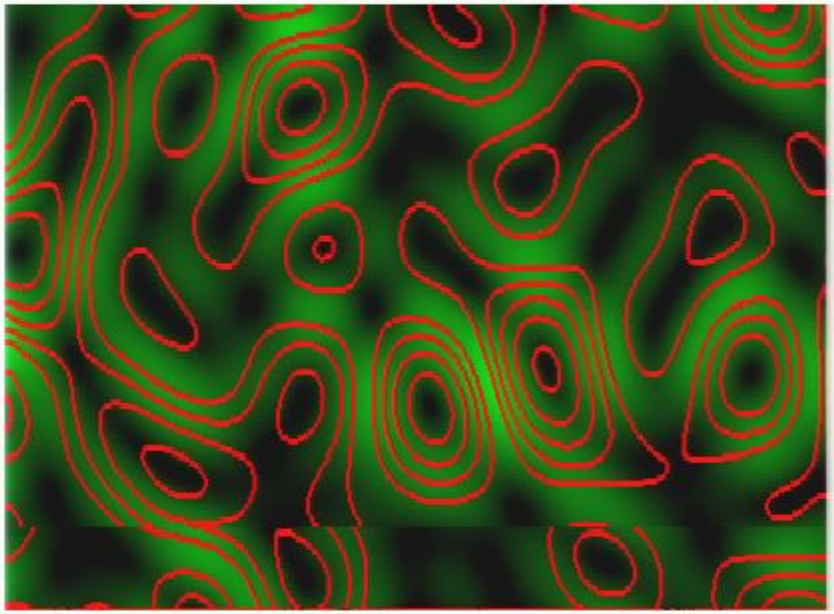

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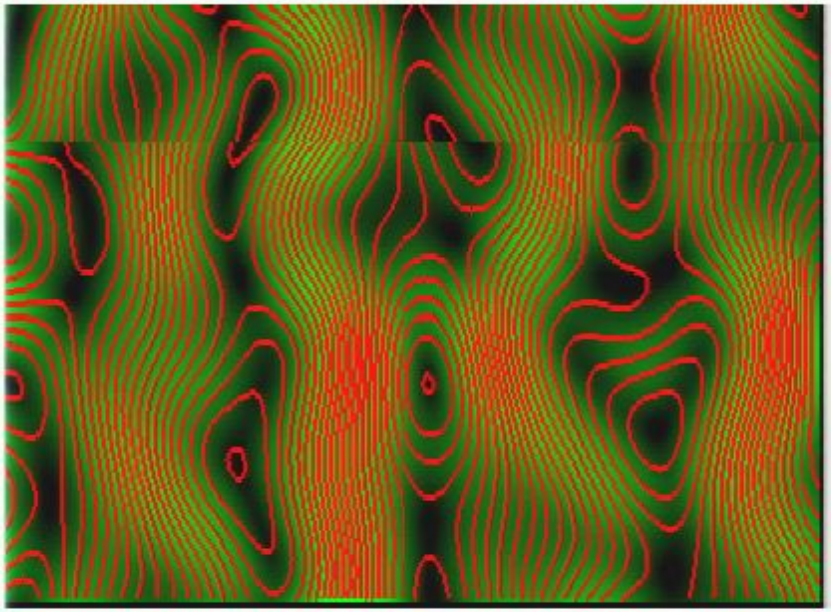

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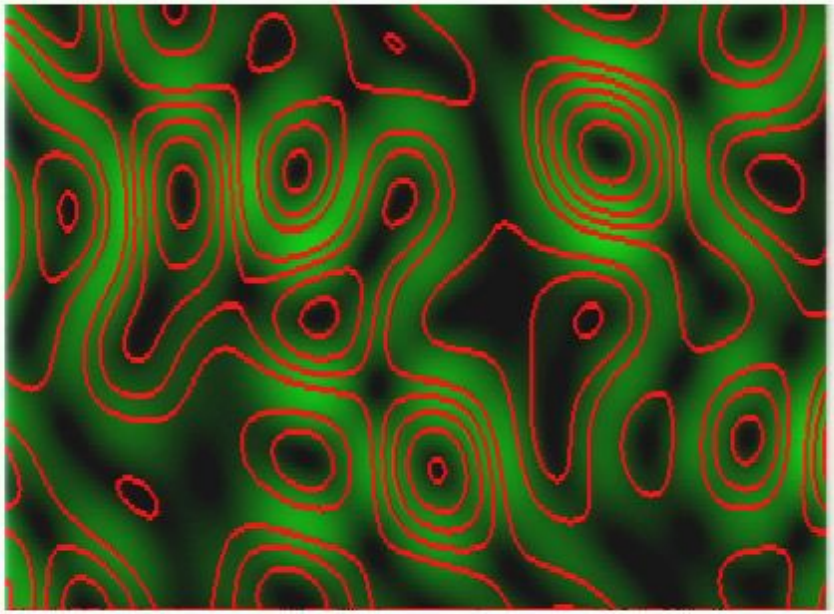
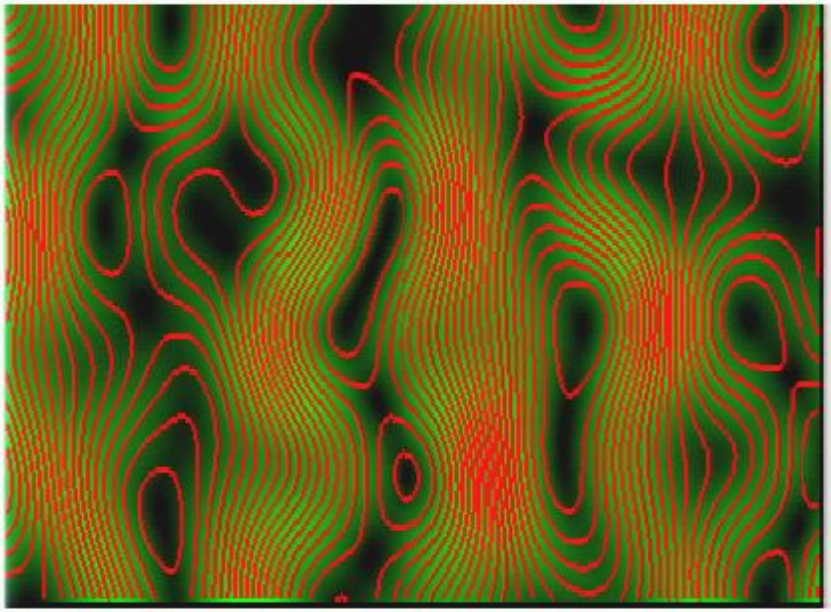


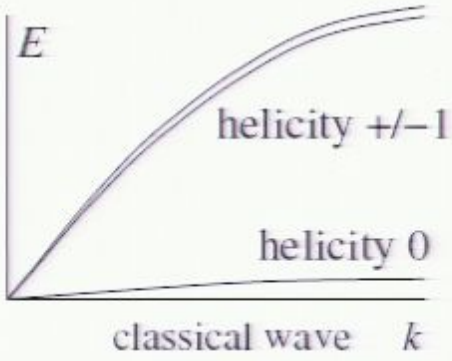


Zero-point fluctuations in ground state Waves in string-net liquid

- String density wave: "string density" $\mathbf{E}(\mathbf{r}, t)$ satisfies $\partial \cdot \mathbf{E} = 0$
- 150 years after Maxwell wrote down his equation, we finally find out what kind of medium produce Maxwell equation.
  Ether may be a string-net liquid
  Light may be collective motions of string-nets

Semi-classical picture

- Each cube has three rotors on the links in the x-, y-, and z-directions. The three θ's form the three component of a vector field $\mathbf{A} = (\theta^x, \theta^y, \theta^z)$.
- If we treat the rotor system as a classical system, the classical equation of motion is determined from the phase-space Lagrangian $\mathcal{L} = \sum L_i \dot{\theta}_i - H(L_i, \theta_i)$
- Dispersions of three modes is designed to have the following form



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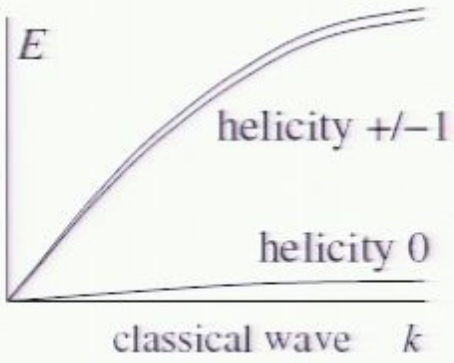

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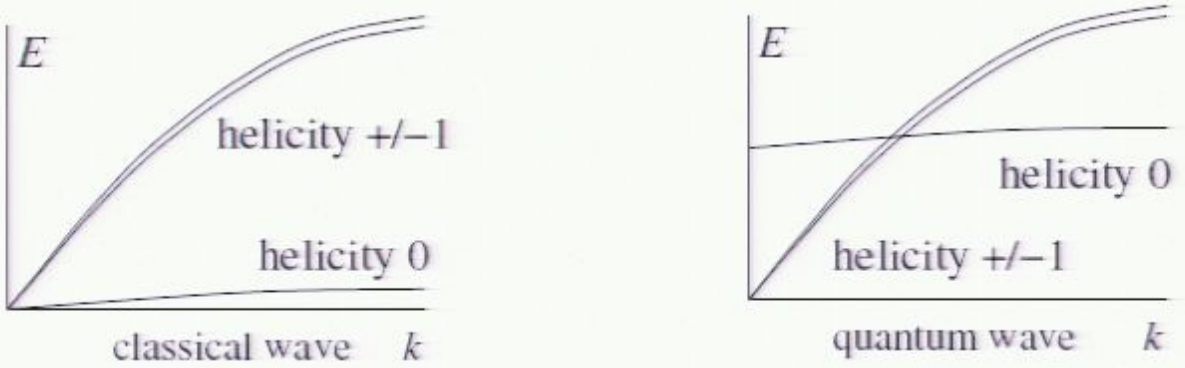

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Is space really a string-net liquid?

Even after knowing that string-net liquid explains the origin of photons, we are still not sure if the space really is a string-net liquid or not.

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String-net picture also explains the origin of electrons and Fermi statistics Levin & Wen 04

- Strings are unobservable in string-net condensed state.
- Ends of strings behave like independent particles.

They are *charges* of the gauge field.

Origin of Fermi statistics

- $H = U \sum_{\mathbf{l}} Q_{\mathbf{l}}^2 - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{i}} (L_{\mathbf{i}})^2$

  gives rise to the string-net condensed state $\sum_{\text{all conf.}}$ 

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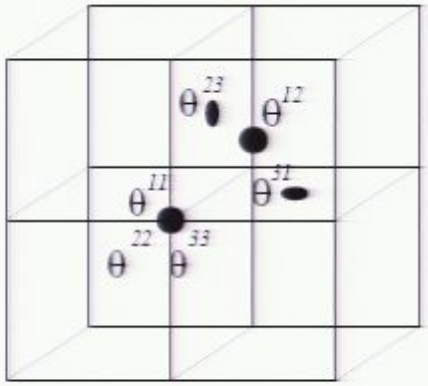
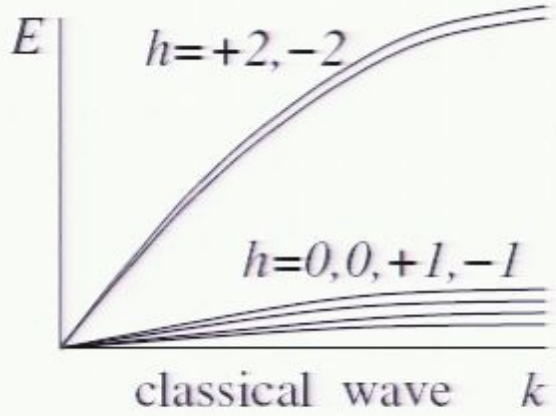
The different statistics of ends of strings is determined by the different kinds of string-net condensation.

The string-net picture unifies the gauge interactions and Fermi statistics

Emergence of "gravitons"

- Lattice model: each vertex has three Z_n-rotors (θ^{aa}, L_{aa}), $aa = 11, 22, 33$, $L_{aa} \sim L_{aa} + n$.
  Each face has one Z_n-rotor (θ^{ab}, L_{ab}), $ab = 12, 23, 31$.
  $\mathcal{L} = \sum L_{ab}\dot{\theta}^{ab}$ – Complicated H
- Total six modes with helicity $0, 0, \pm 1, \pm 2$





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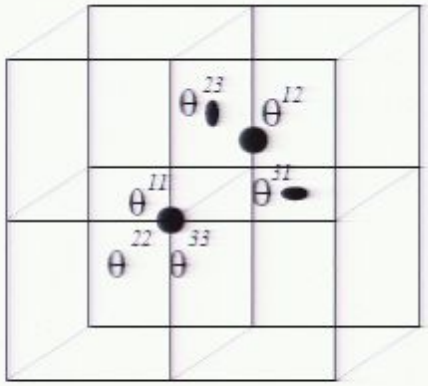
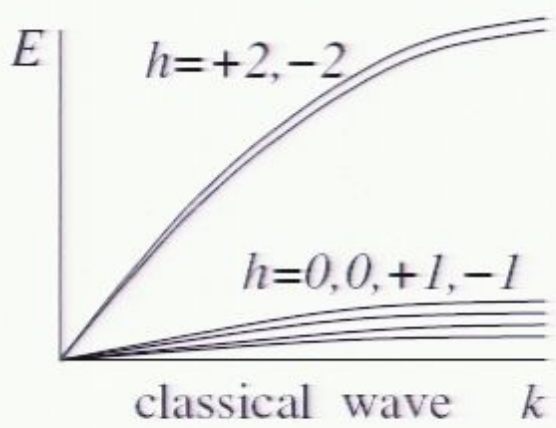
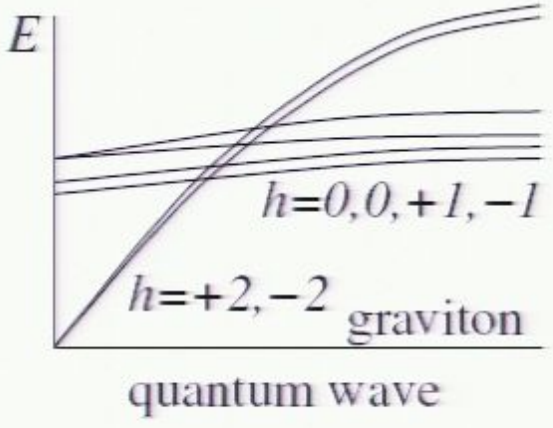


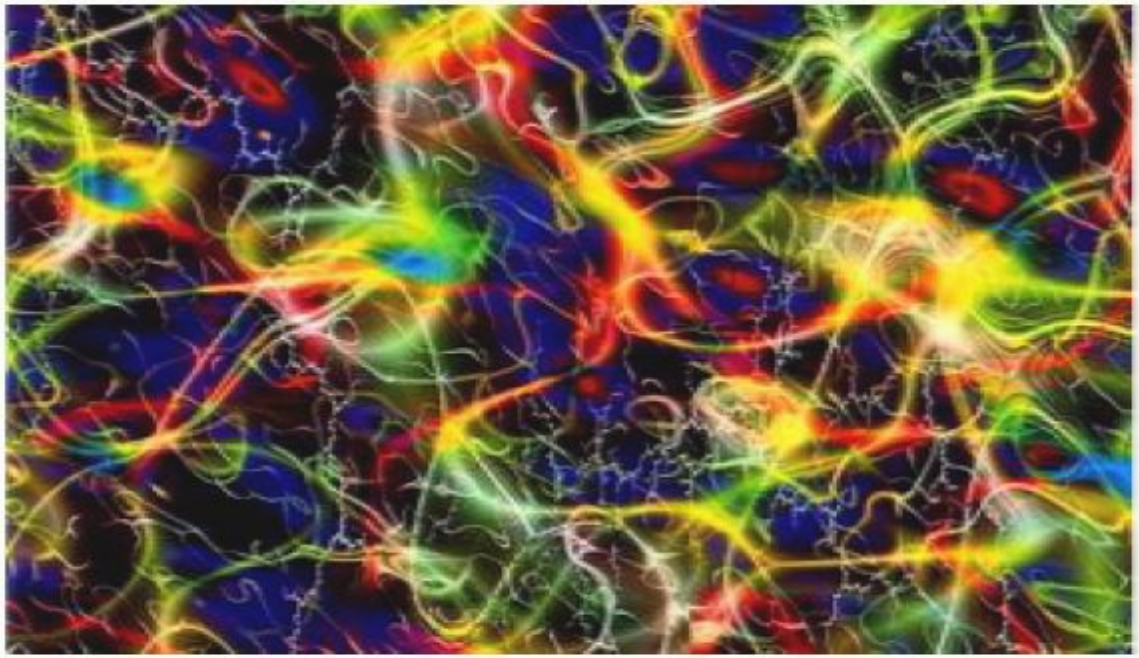




- The model is designed such that the helicity $0, 0, \pm 1$ modes have strong quantum fluctuations and are gapped in quantum theory.
- The helicity ± 2 modes have weak quantum fluctuations and remain gapless.

Summary

- Long-range entanglement $\longrightarrow$ new class of quantum matter beyond Landau symmetry breaking paradigm.
- String-net condensation $\longrightarrow$ a unification of light and electrons a unification of gauge interaction and Fermi statistics
- Long-range entanglement $\longrightarrow$ emergence of gravitons



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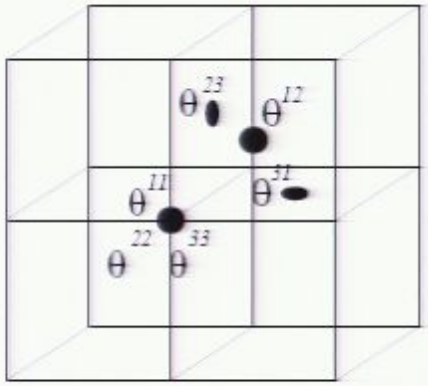

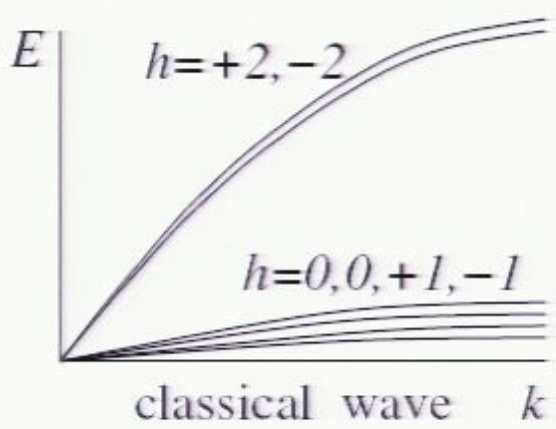

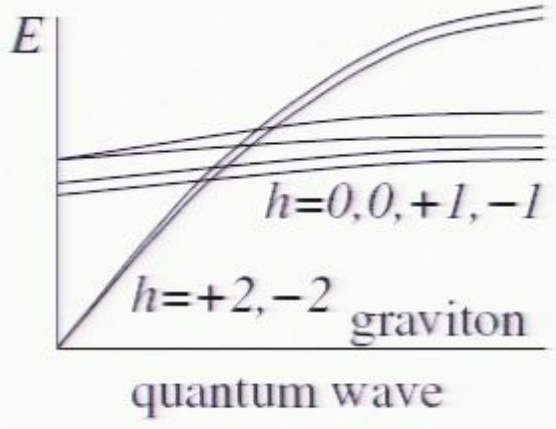

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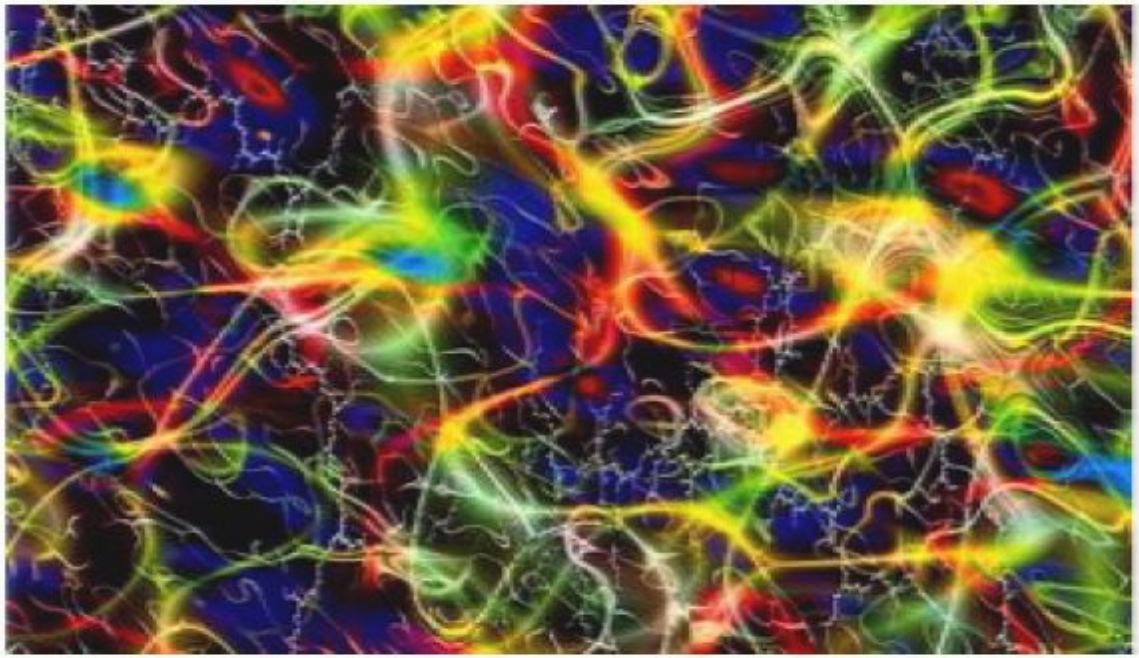
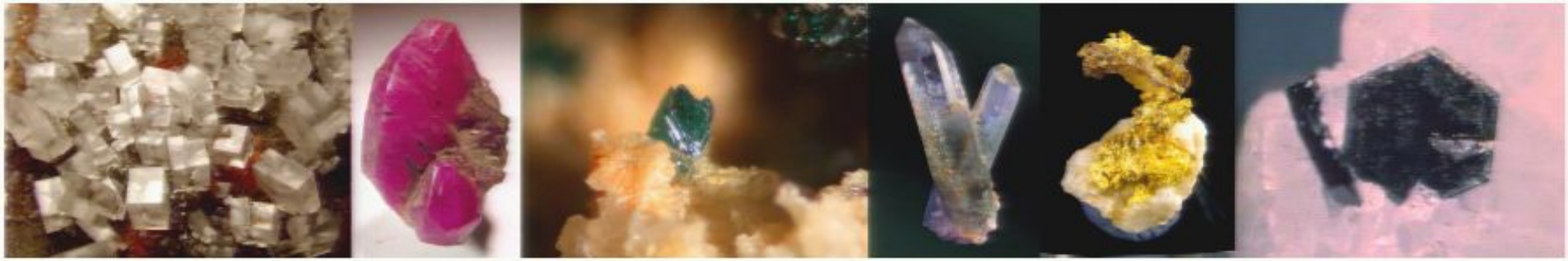


How to gain a deeper understanding of our world?



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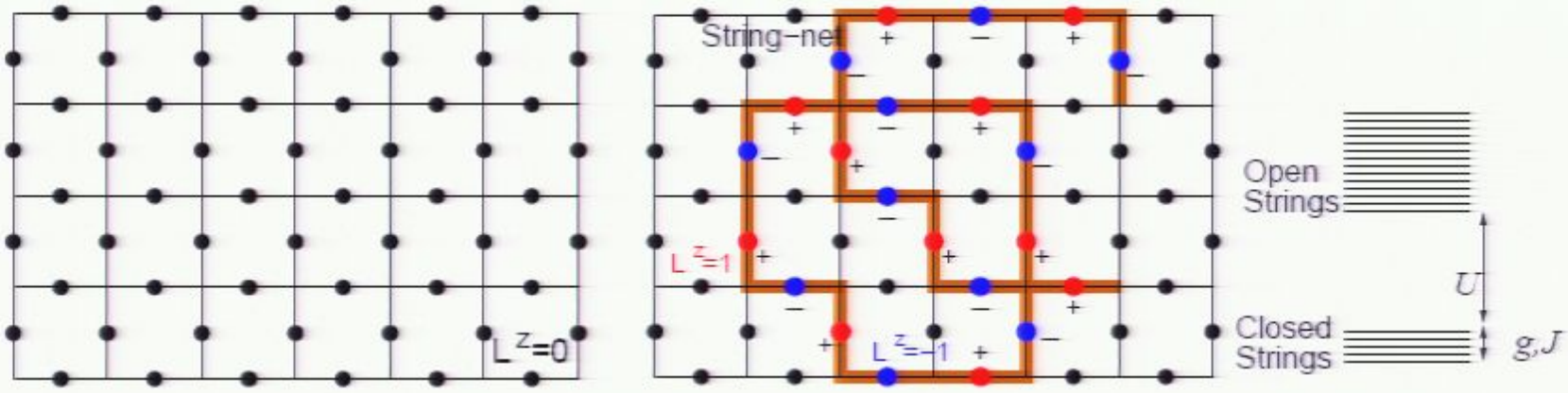

- the U-term $\longrightarrow$ closed strings. Open ends cost energy.
- J-term $\longrightarrow$ string tension
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- The ground state of the rotor Hamiltonian H when $U \gg g \gg J$

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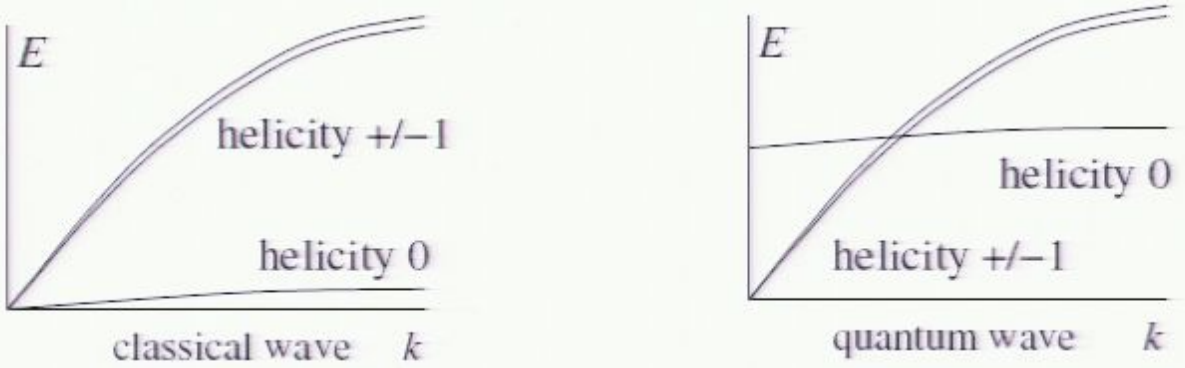

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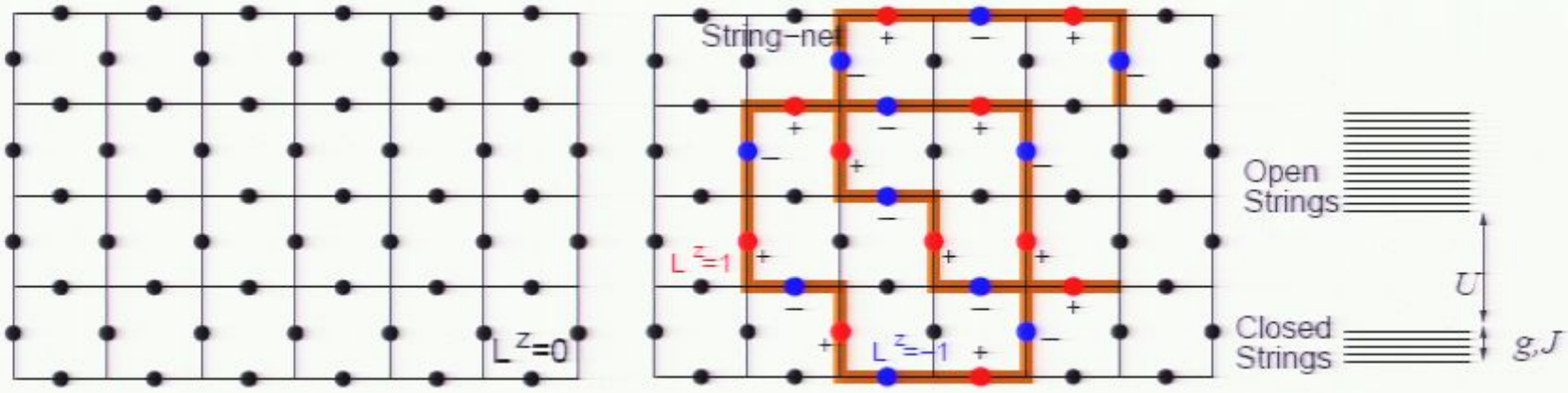

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