Title: Physical Limits of Inference

Date: Nov 11, 2008 04:00 PM

URL: http://pirsa.org/08110000

Abstract: I show that physical devices that perform observation, prediction, or recollection share an underlying mathematical structure. I call devices with that structure ``inference devices\'\'. I present a set of existence and impossibility results concerning inference devices. These results hold independent of the precise physical laws governing our universe. In a limited sense, the impossibility results establish that Laplace was wrong to claim that even in a classical, non-chaotic universe the future can be unerringly predicted, given sufficient knowledge of the present. Alternatively, these impossibility results can be viewed as a non-quantum mechanical ``uncertainty principle\'\'. Next I explore the close connections between the mathematics of inference devices and of Turing Machines. I end by informally discussing the philosophical implications of these results, e.g., for whether the universe ``is\'\' a computer.

PHYSICAL LIMITS OF INFERENCE

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NASA-ARC-03-0667

ROADMAP

1) *Formalize observation, prediction, and memory*

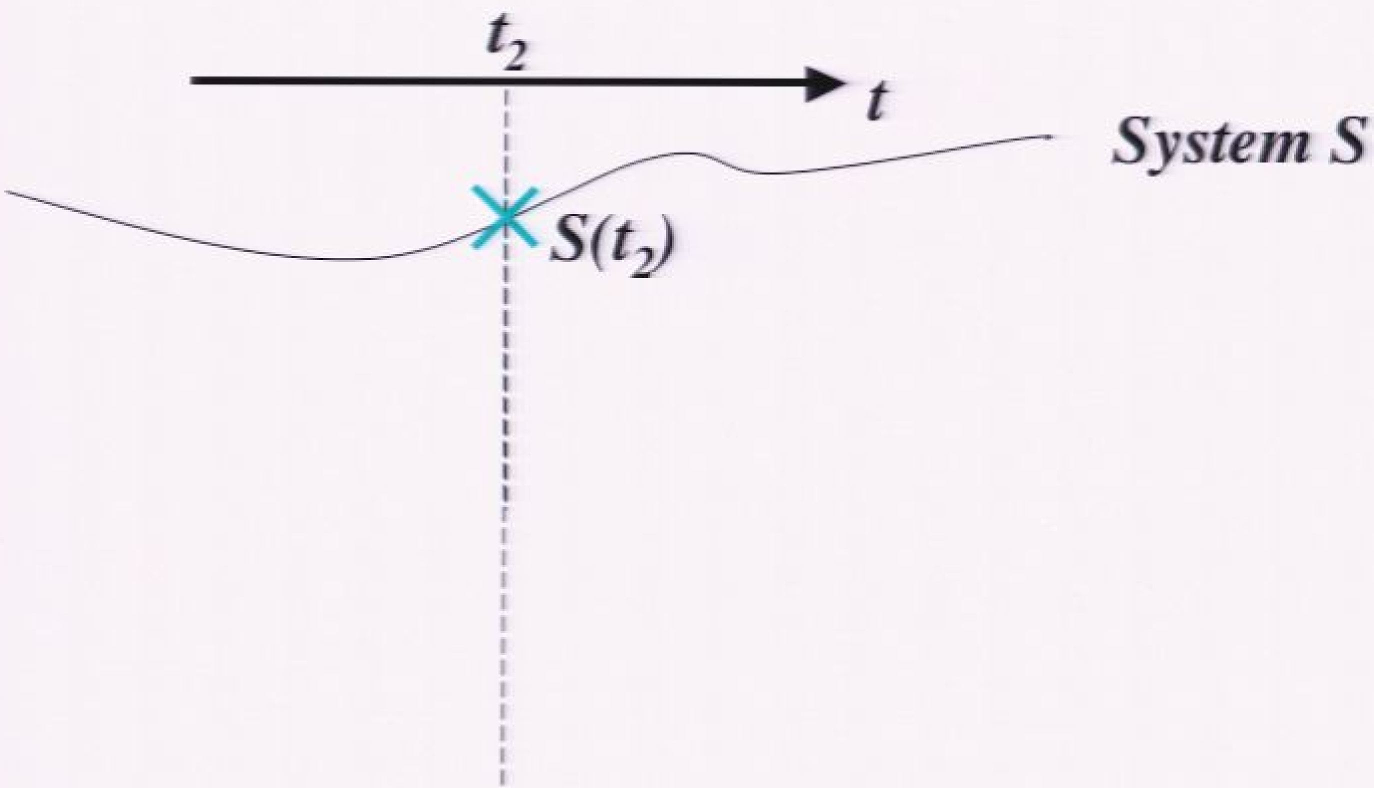
2) *Extract what's in common:* **inference devices**

3) *Elementary properties of inference devices*

4) *Inference devices and Turing machine theory:* **strong inference** *and* **inference complexity**

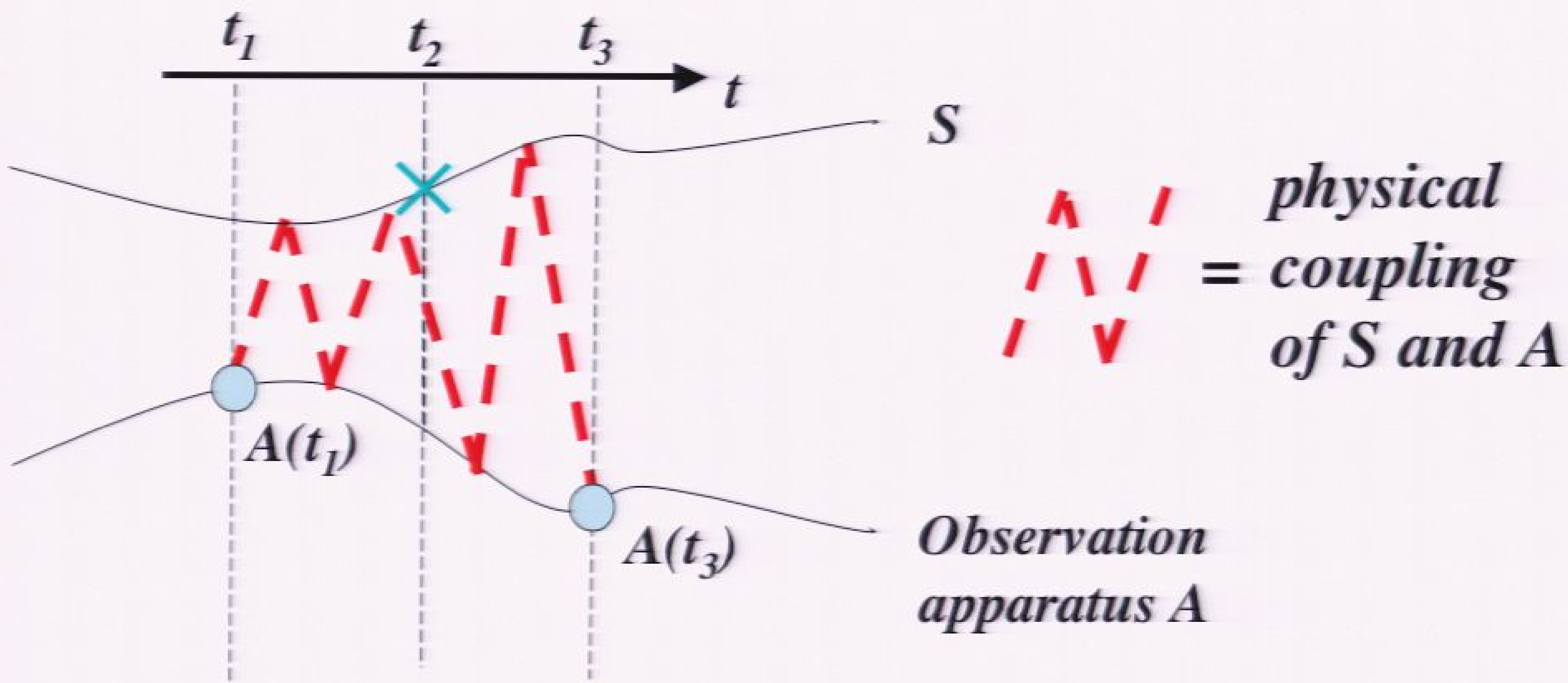
5) **Stochastic inference**

OBSERVATION



1) $S(t_2)$ is what we want to observe.

OBSERVATION

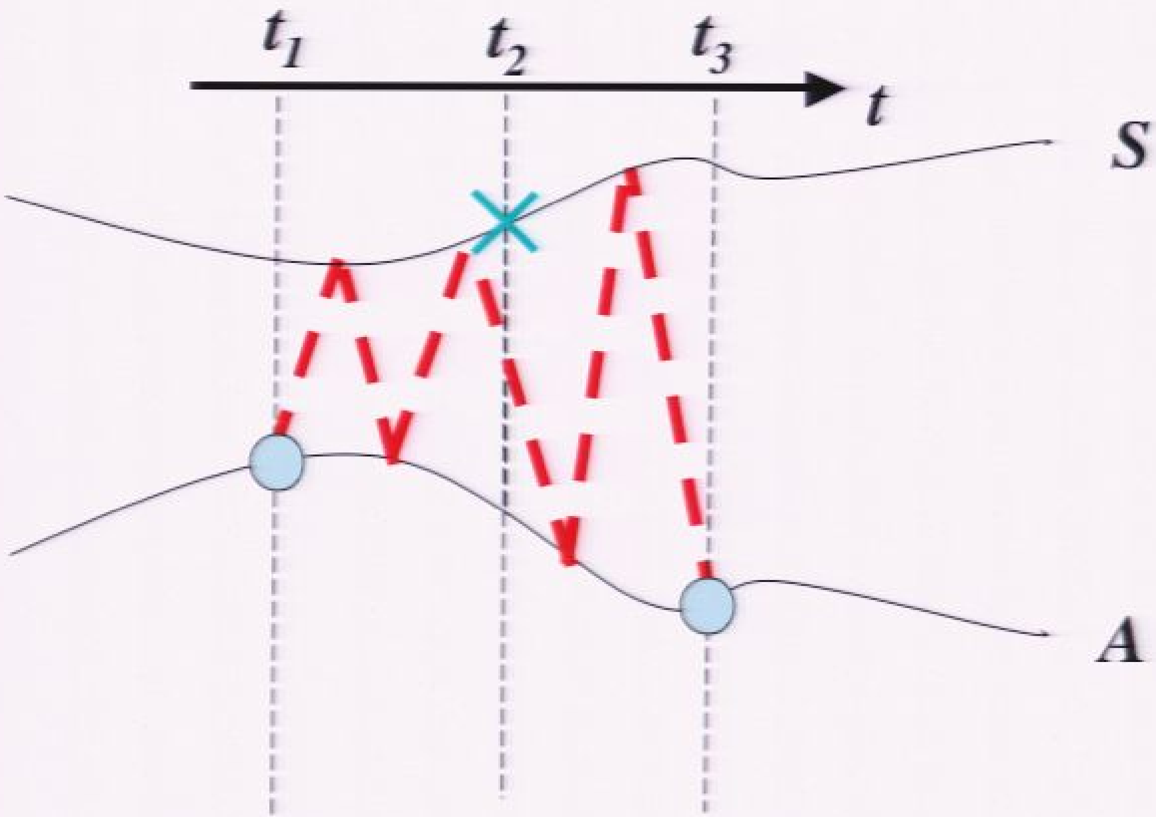


1) $S(t_2)$ is what we want to observe.

2) To do that we setup $A(t_1)$ appropriately.

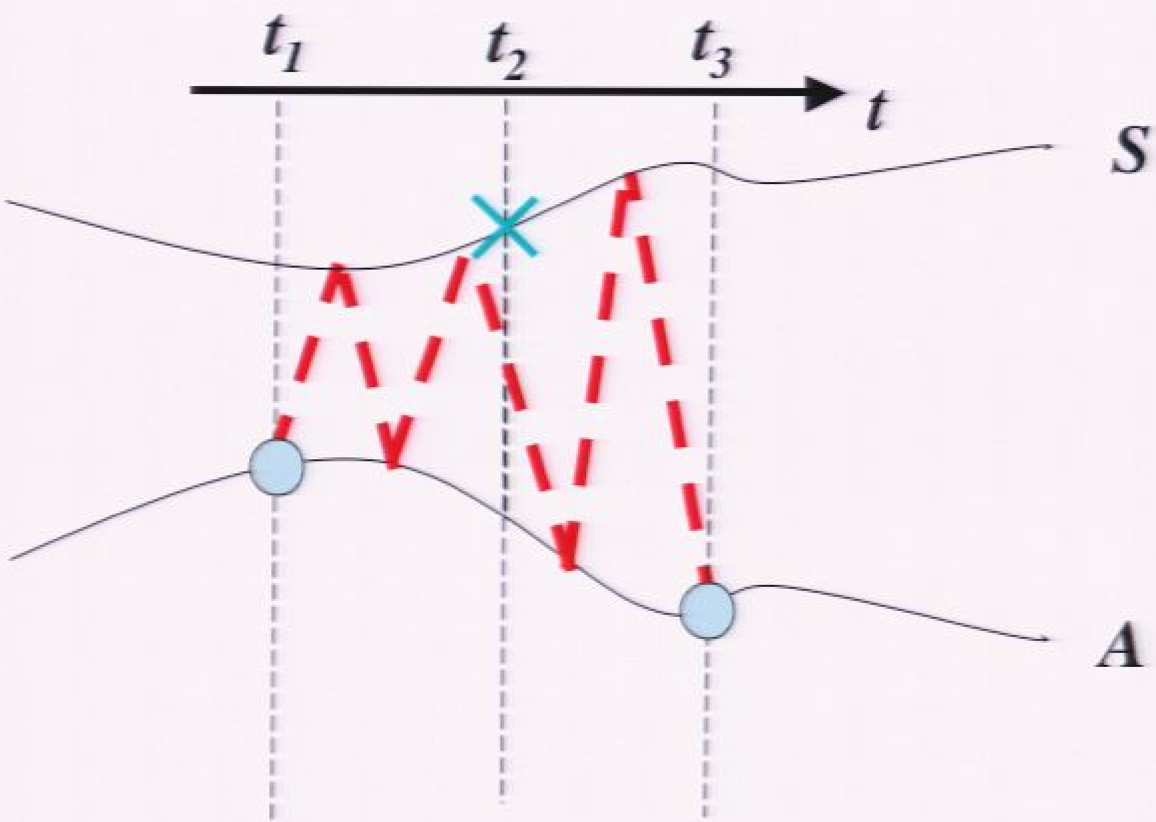
3) The conclusion of the observation is $A(t_3)$.

OBSERVATION



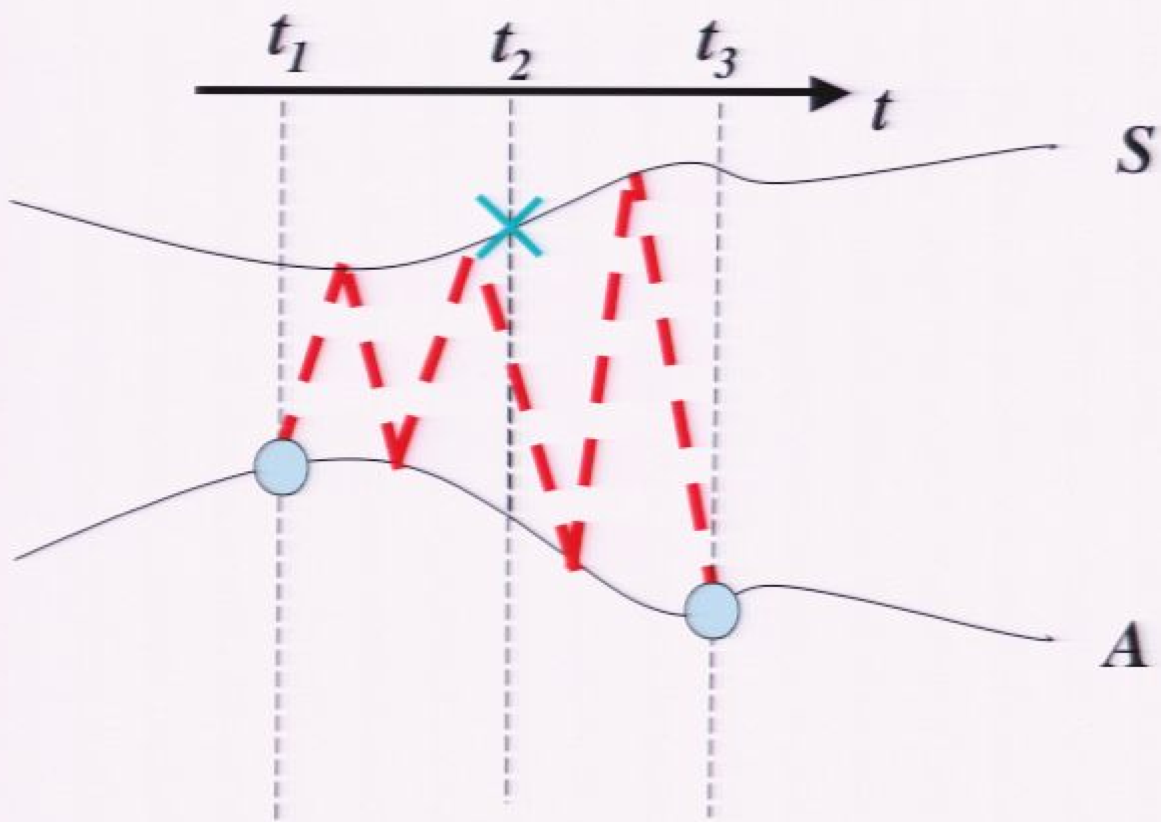
- *Just having A(t_3) and S(t_2) correlated is NOT observation*
- *If a tree falls, but the camera aimed at it encrypts the image, the image on that camera is not an "observation"*
- *How imbue A(t_3) with semantic meaning?*

OBSERVATION



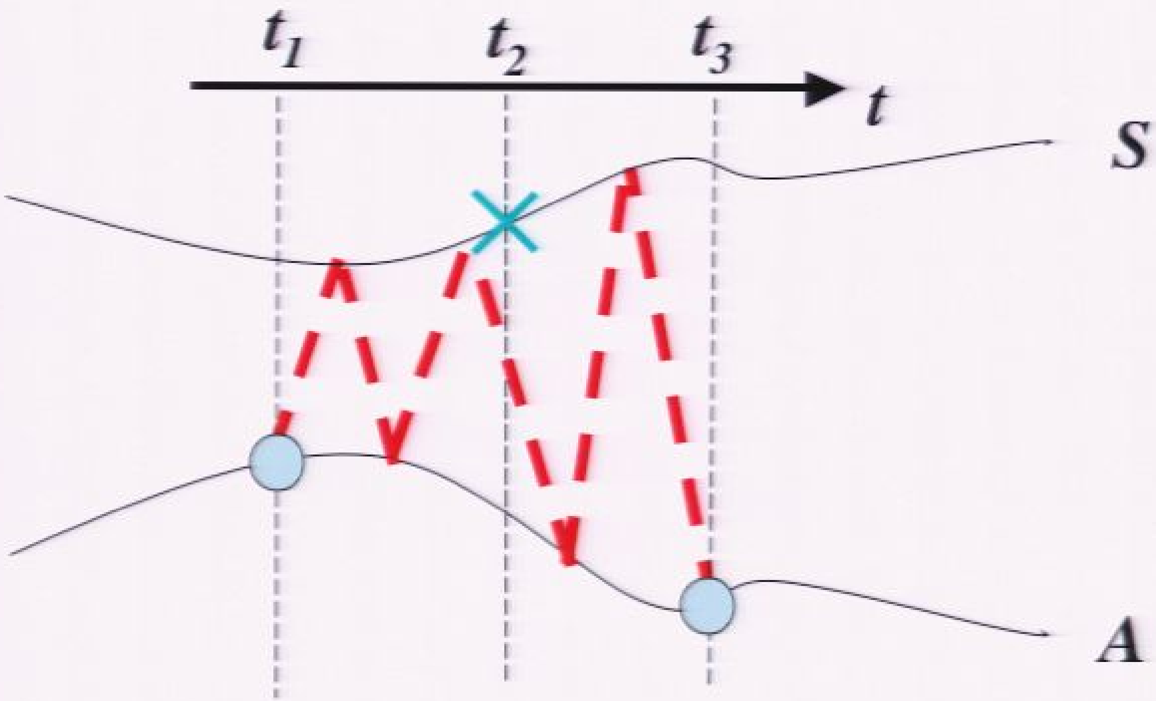
Suppose $A(t_3)$ says
$S(t_2) = K$.

OBSERVATION



Suppose $A(t_3)$ *says* $S(t_2) = K$.

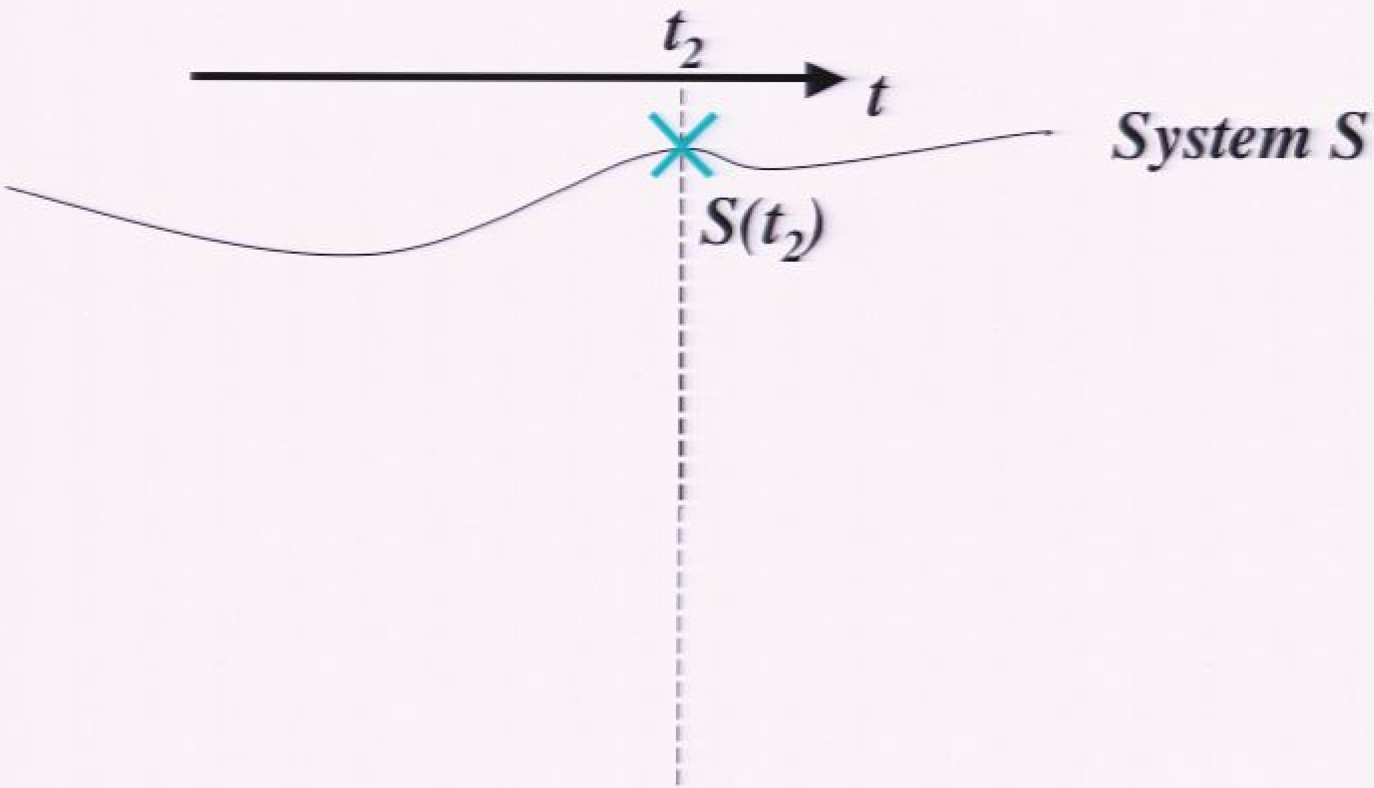
OBSERVATION



1) $A(t_3)$ *has "semantic meaning" if there is a scientist who can read it to answer, "Does* $S(t_2) = L$*?"* $\forall L$.

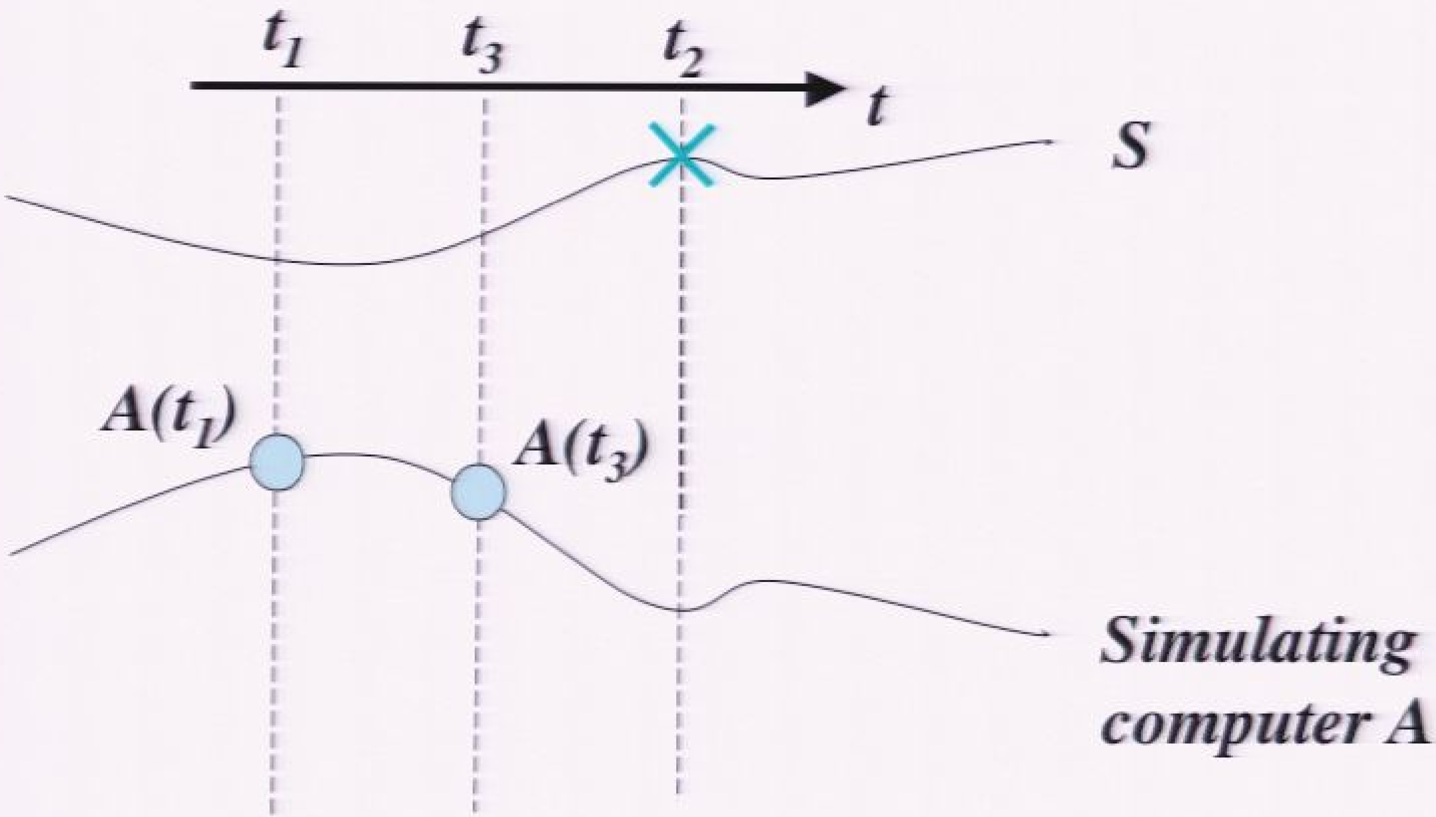
2) **Successful** *observation is when the scientist's answer to any such question is correct.*

PREDICTION



1) $S(t_2)$ is what we want to predict.

PREDICTION

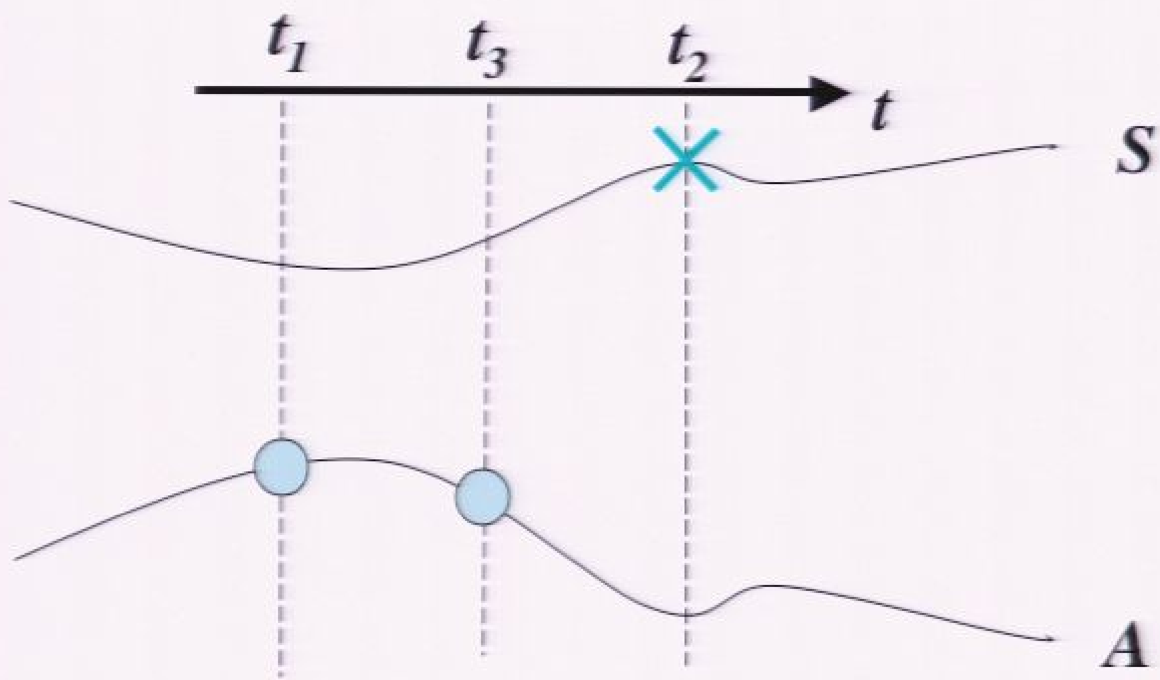


1) $S(t_2)$ is what we want to predict.

2) To do that we <u>setup</u> $A(t_1)$ appropriately.

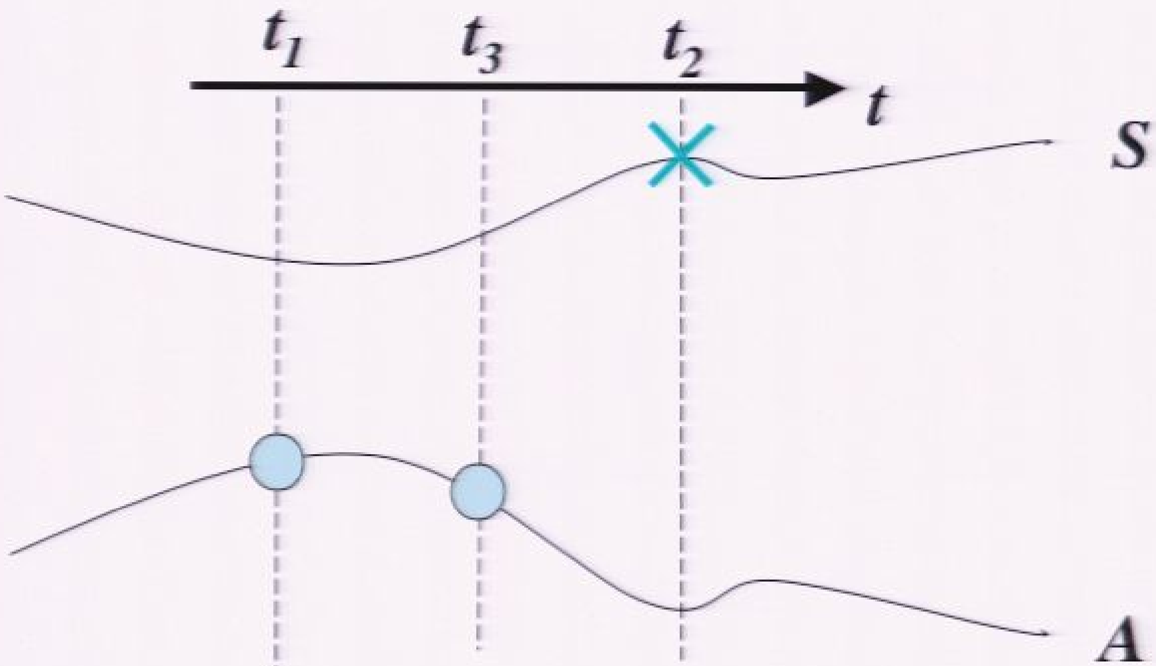
3) The <u>conclusion</u> of the prediction is $A(t_3)$.

PREDICTION



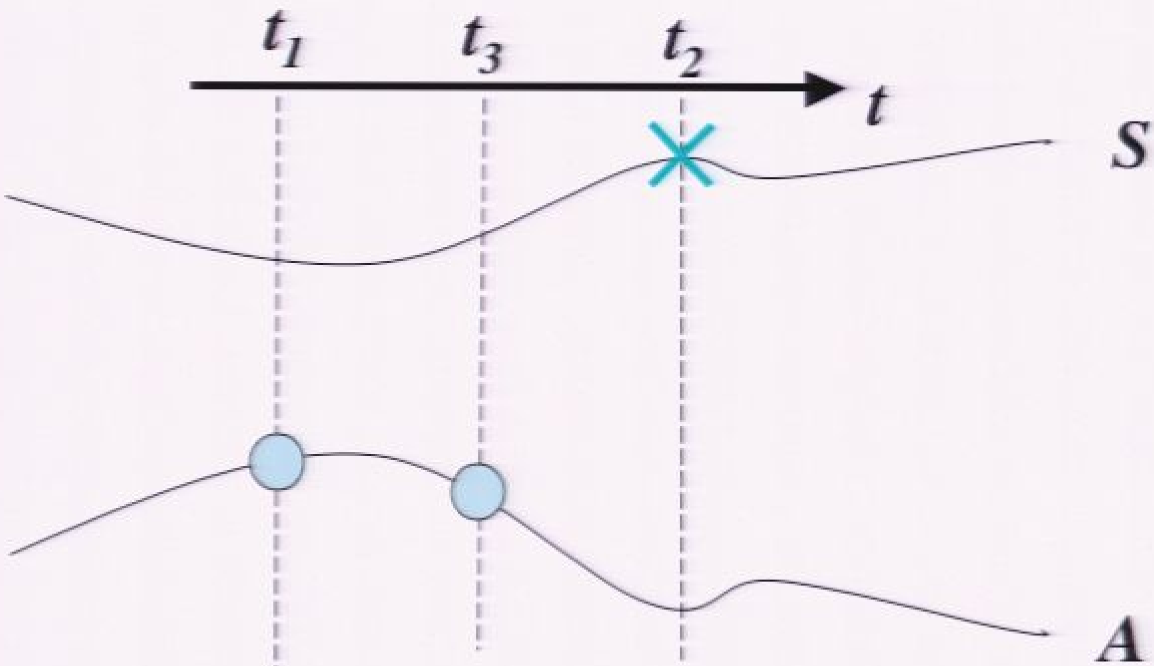
- *Just having $A(t_3)$ and $S(t_2)$ correlated is NOT prediction*
- *If a tree will fall, but the results of a simulation of tree dynamics is encrypted, the results aren't a "prediction"*
- *How imbue $A(t_3)$ with semantic meaning?*

PREDICTION



Suppose $A(t_3)$ says $S(t_2) = K$.

PREDICTION



Suppose $A(t_3)$ *says* $S(t_2) = K$.

1) $A(t_3)$ *has "semantic meaning" if there is a scientist who can read it to answer, "Does* $S(t_2) = L$*?"* $\forall L$.

2) **Successful** *prediction is when the scientist's answer to any such question is correct.*

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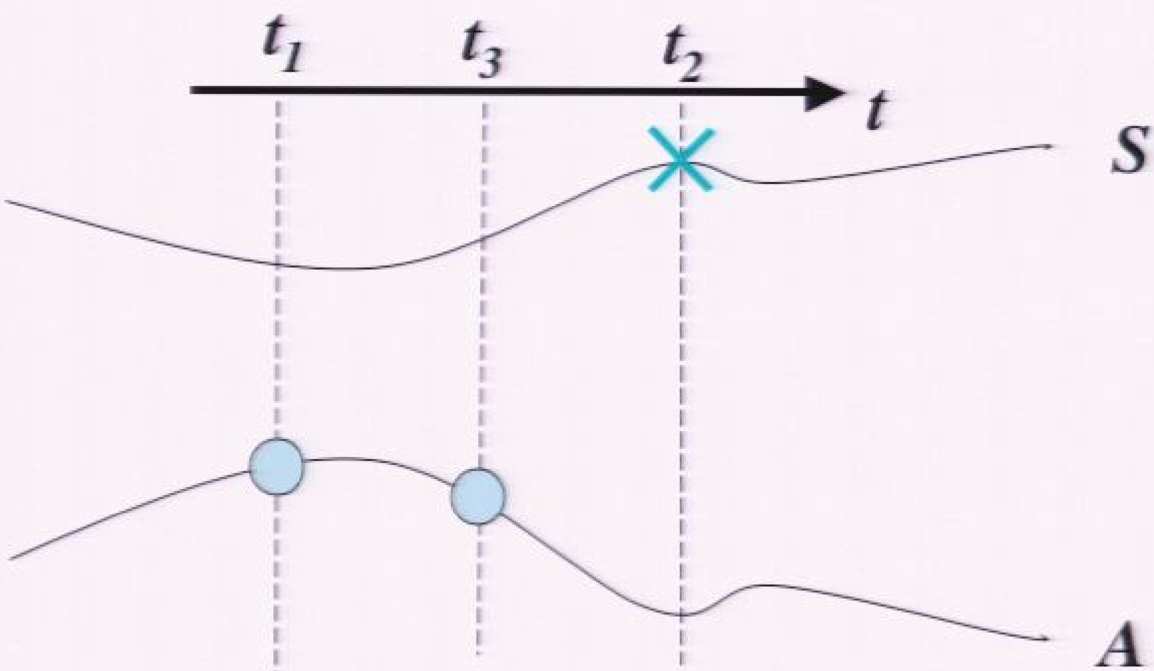

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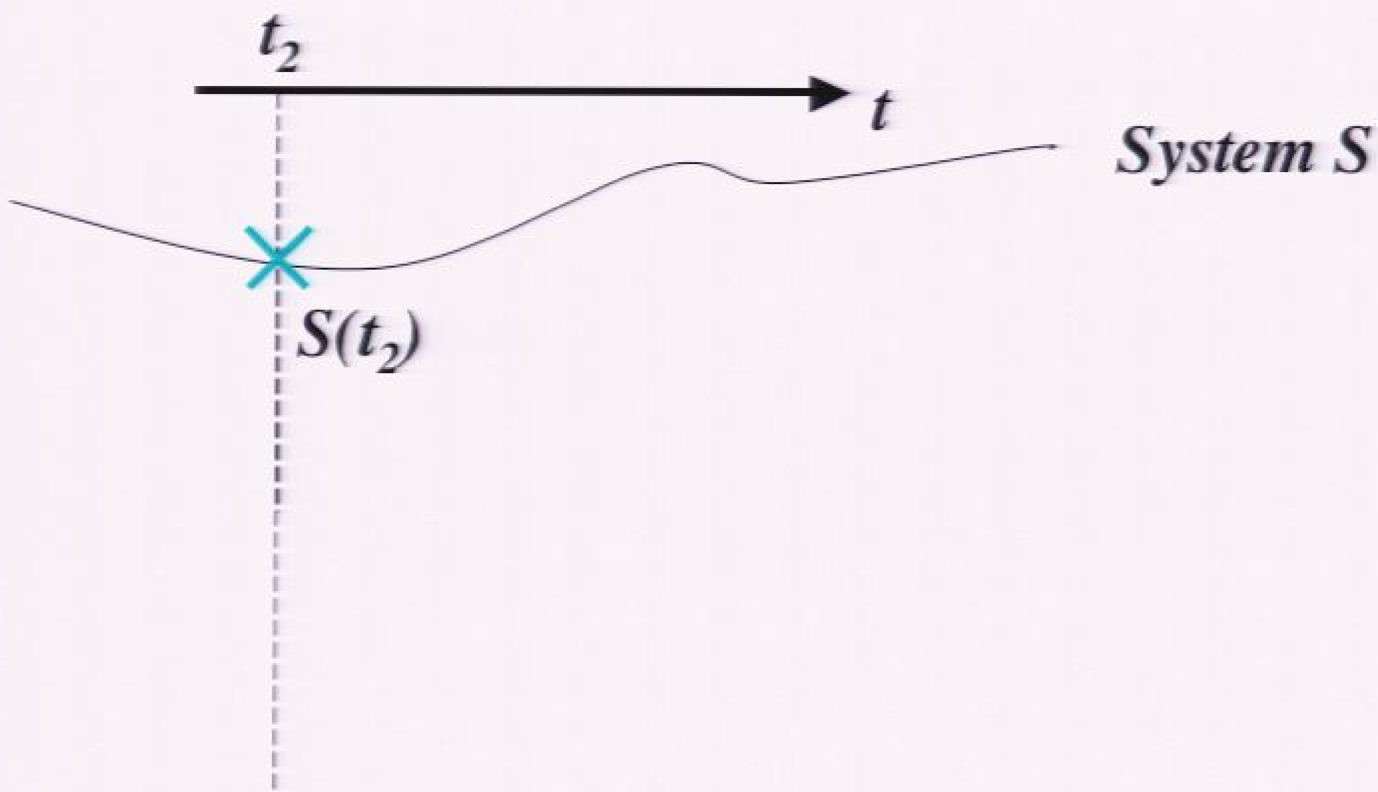
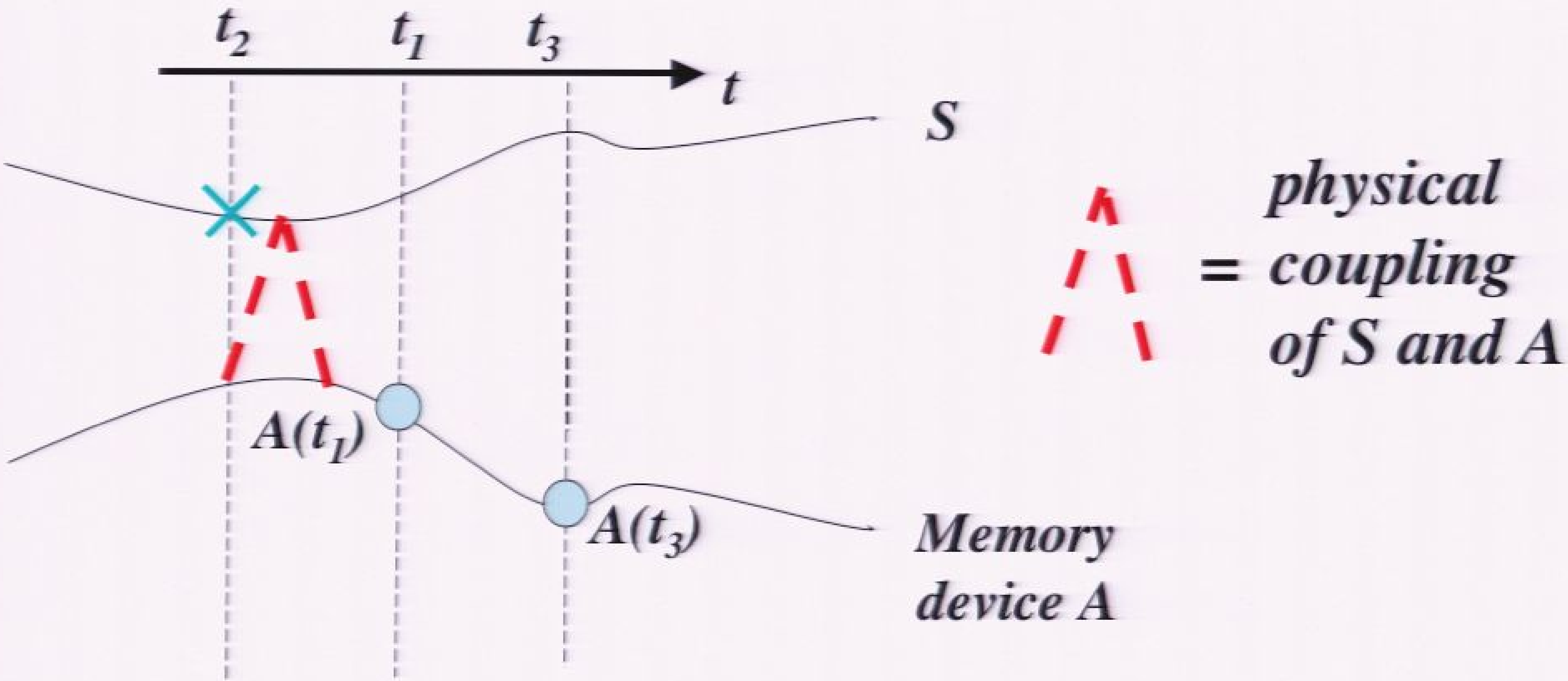
MEMORY



1) $S(t_2)$ is an event (out of many) we want to recollect.

MEMORY

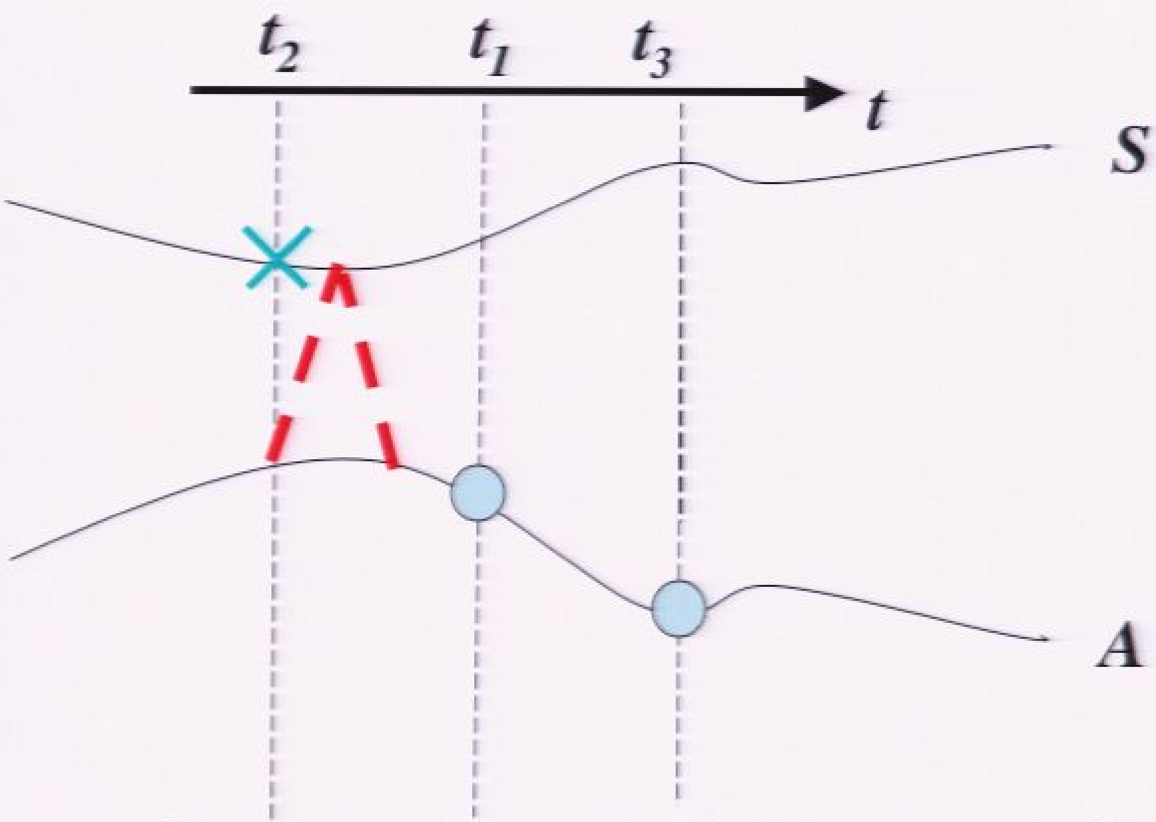


1) $S(t_2)$ is an event (out of many) we want to recollect.

2) To do that we <u>setup</u> $A(t_1)$ appropriately.

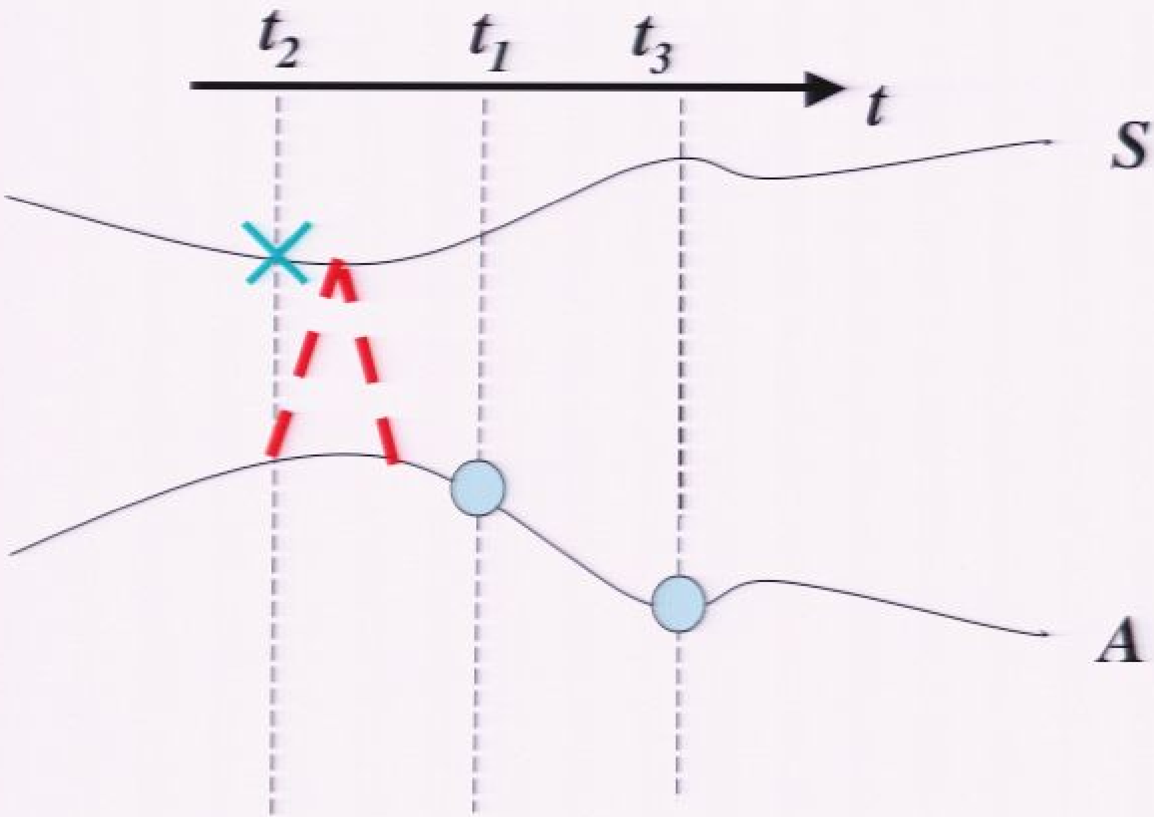
3) The <u>conclusion</u> of our recollection request is $A(t_3)$.

MEMORY



- *Just having $A(t_3)$ and $S(t_2)$ correlated is NOT memory*
- *If a tree fell, but the video of its fall is encrypted in our memory device, that video isn't a "memory"*
- *How imbue $A(t_3)$ with semantic meaning?*

MEMORY



Suppose $A(t_3)$ says $S(t_2) = K$.

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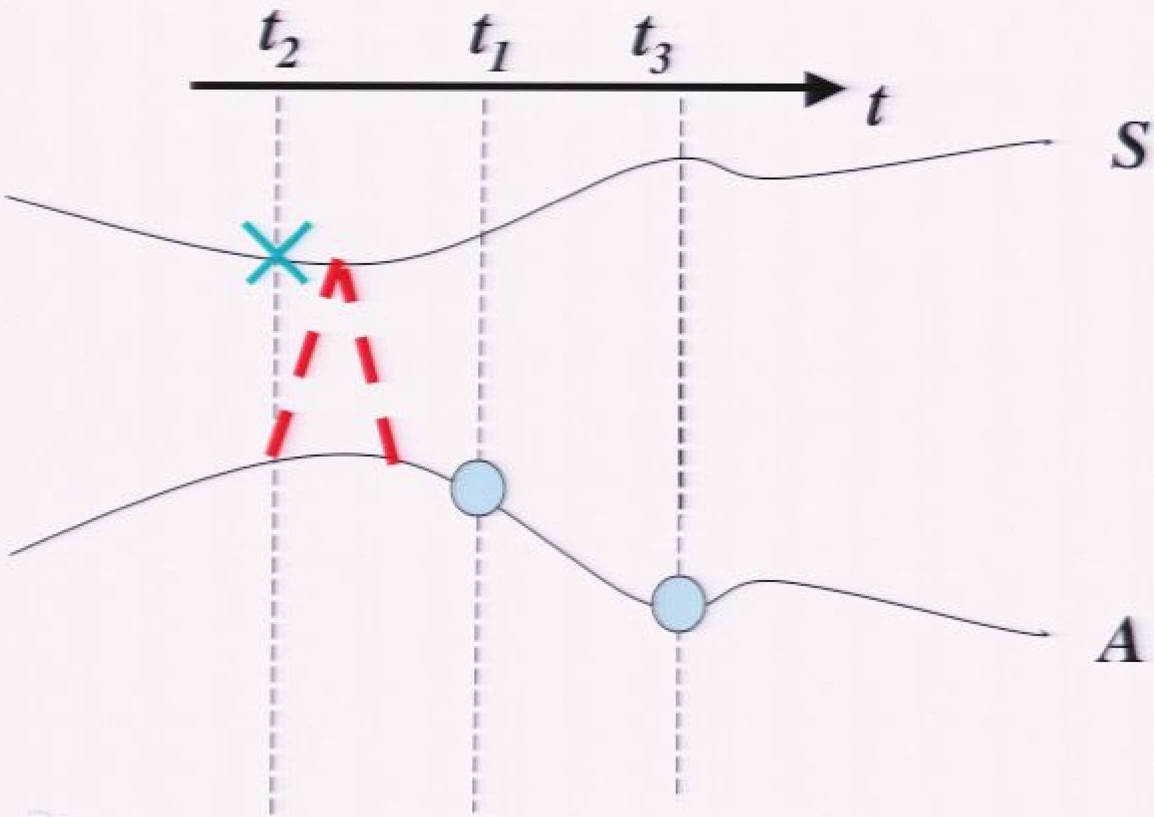

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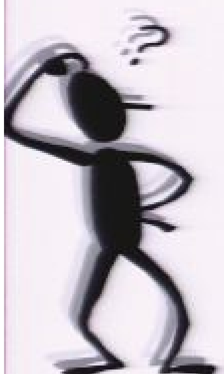

1) *$A(t_3)$ has "semantic meaning" if there is a scientist who can read it to answer, "Does $S(t_2) = L$?" $\forall L$.*

2) **Successful** *recollection is when the scientist's answer to any such question is correct.*

ROADMAP

1) *Formalize observation, prediction, and memory*

2) *Extract what's in common:* **inference devices**

3) *Elementary properties of inference devices*

4) *Inference devices and Turing machine theory:* **strong inference** *and* **inference complexity**

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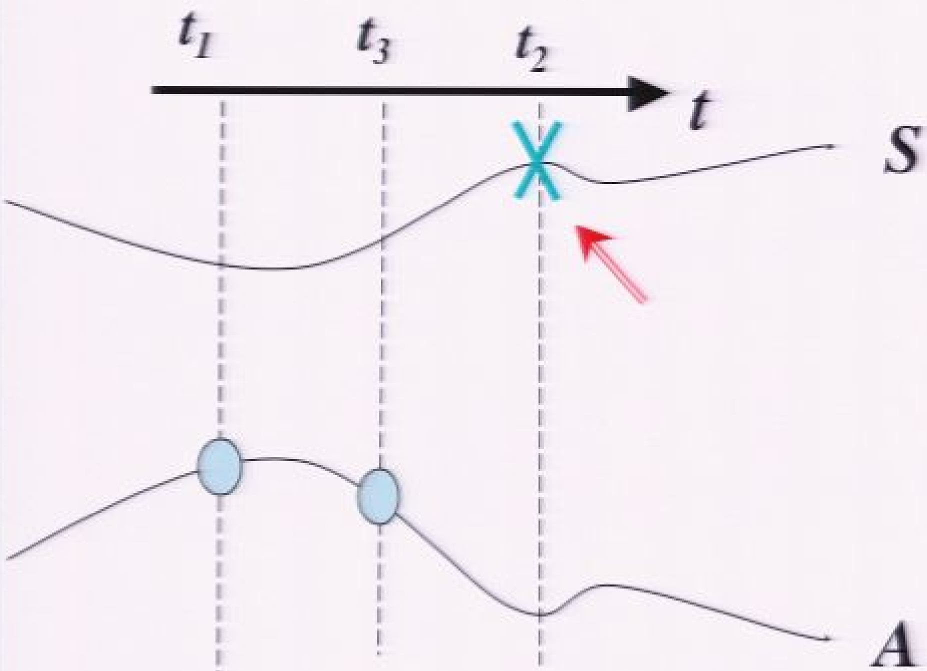
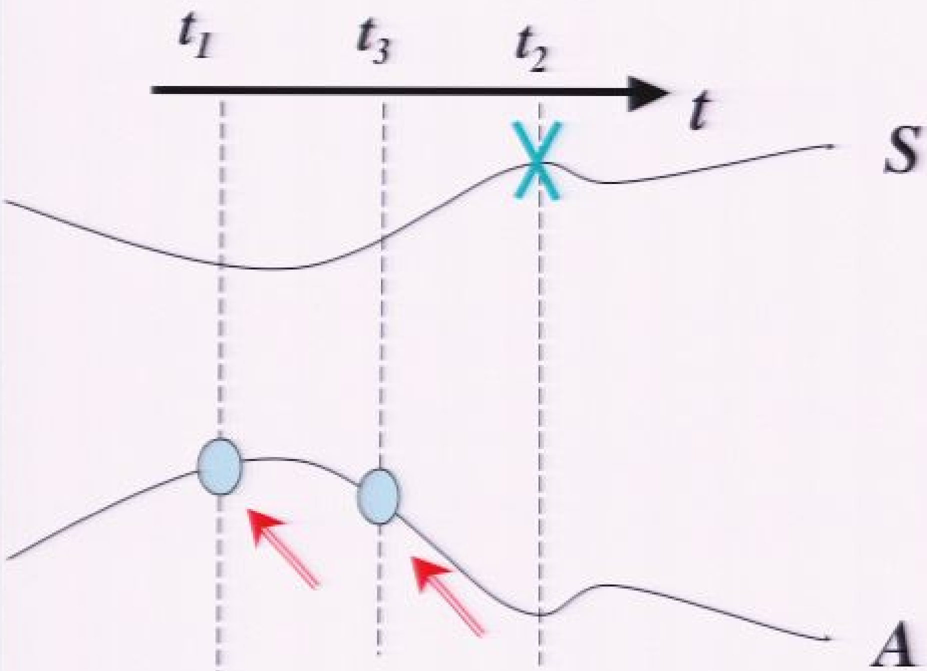
INFERENCE



- *S exists in the physical universe*
- *So $S(t_2)$ is a function of u, the universe's worldline*
- *Write that function as $\Gamma(u)$*

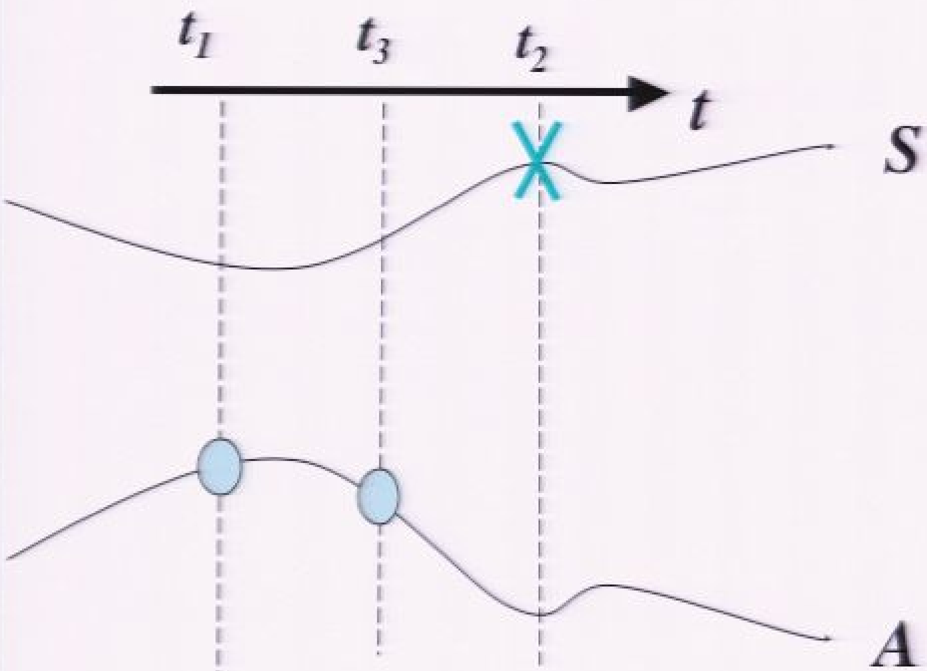
INFERENCE



1) $\Gamma(u) = S(t_2)$

- *A exists in the physical universe*
- *So $A(t_1)$ and $A(t_3)$ are functions of u*
- *Write them as $\zeta(u)$ and $\chi(u)$*

INFERENCE

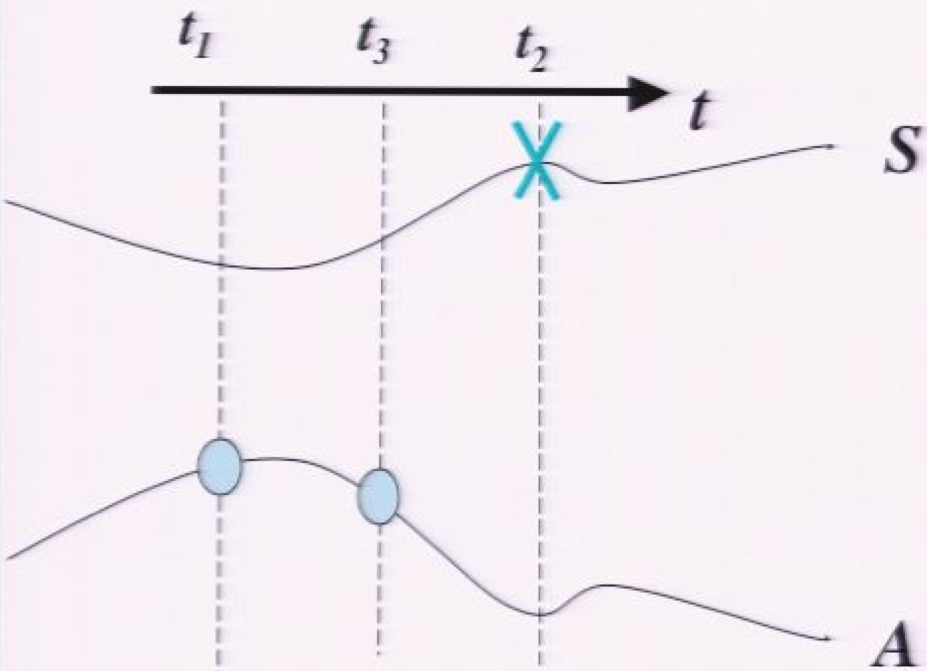


1) $\Gamma(u) = S(t_2)$

2) $\chi(u) = A(t_1)$, $\zeta(u) = A(t_3)$

- *For every L, "$S(t_2) = L$?" is a binary-valued function of $S(t_2)$.*
- *Write that function as q_L*

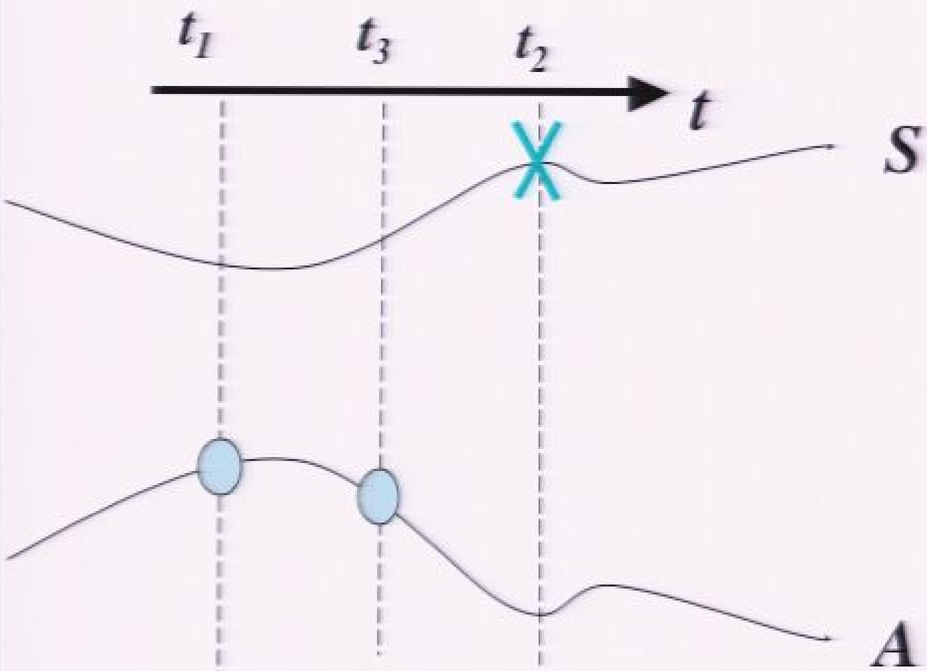
INFERENCE



1) $\Gamma(u) = S(t_2)$
2) $\chi(u) = A(t_1)$, $\zeta(u) = A(t_3)$
3) $q_L(\gamma) = 1$ iff $\gamma = L$

- *The scientist exists in the physical universe*
- *So which question they are asking is a function of u*
- *Write that function as Q(u)*

INFERENCE



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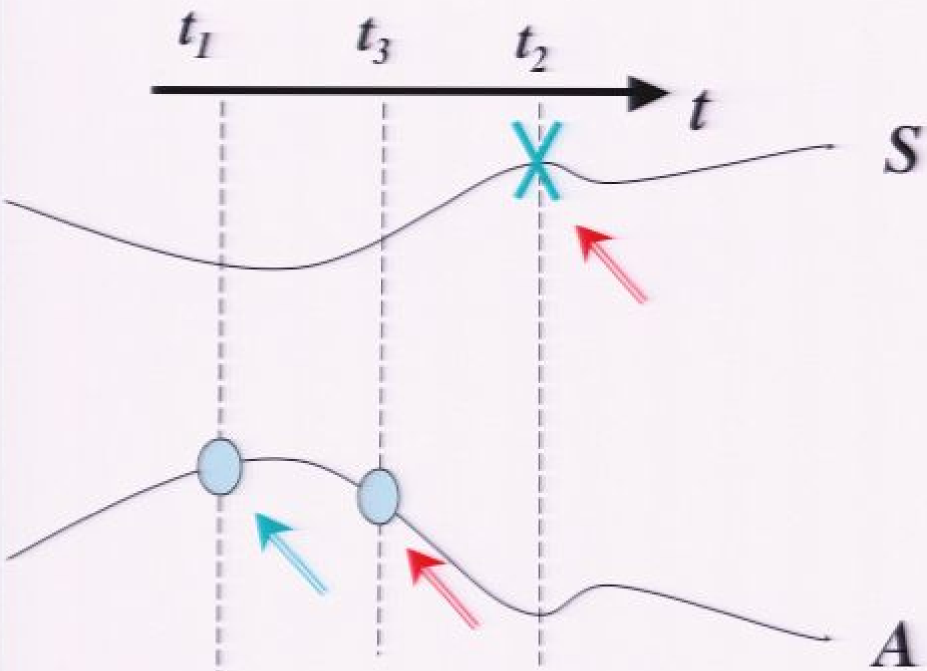
3) $q_L(\gamma) = 1$ iff $\gamma = L$
4) $Q(u)$

- *The scientist exists in the physical universe*
- *So what answer for Q they read from $\zeta(u)$ is a function of u*
- *Write that function as $Y(u)$*

INFERENCE



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3) $q_L(\gamma) = 1$ iff $\gamma = L$

4) $Q(u)$

5) $Y(u)$

<u>Successful inference</u>: $\forall q_L$,

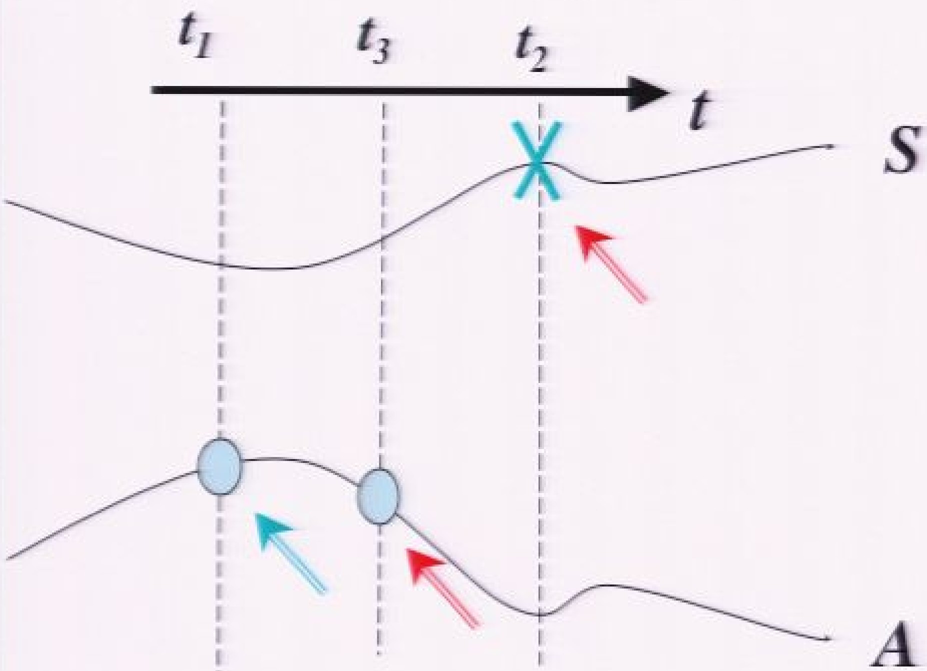
$\exists$ a, b such that

$$(Q(u), \chi(u)) = (a, b)$$

$$\Rightarrow$$

$$Y(u) = q_L(\Gamma(u))$$

INFERENCE



<u>Successful inference</u>: $\forall q_L$,

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$$(Q(u), \chi(u)) = (a, b)$$

$$\Rightarrow$$

$$Y(u) = q_L(\Gamma(u))$$

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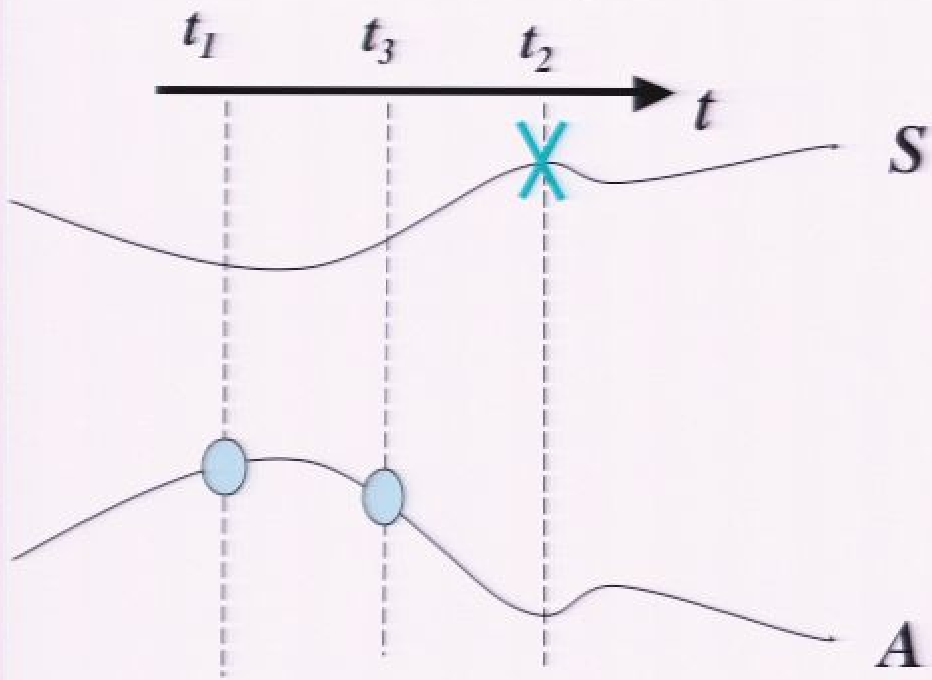
1) *Don't even require $a = q_L$*
2) *Allow any b; even $b = q_L(\Gamma(u))$*
3) *No restrictions on power of A, or power of the scientist*
4) *No restrictions on coupling between S and A*

INFERENCE SIMPLIFIED



1) $\Gamma(u) = S(t_2)$

2) $X(u) = (\chi(u), Q(u))$

3) $q_L(\gamma) = 1 \text{ iff } \gamma = L$

4) $Y(u)$

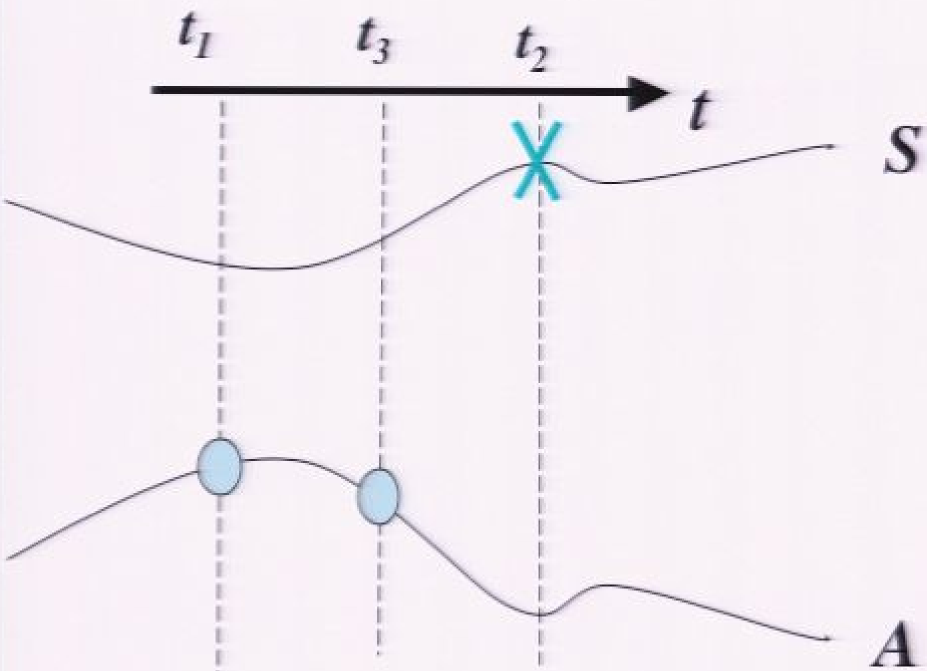
$\forall q_L, \exists \mathbf{x} : \forall u \in U,$

$$X(u) = x$$

$$\Rightarrow$$

$$Y(u) = q_L(\Gamma(u))$$

INFERENCE



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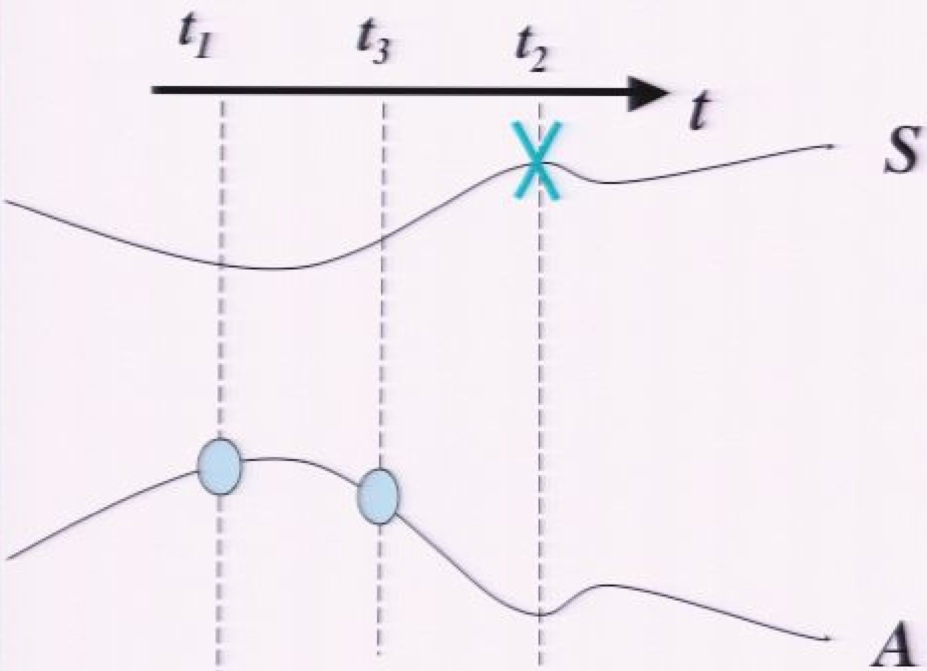
3) $q_L(\gamma) = 1$ iff $\gamma = L$
4) $Q(u)$

- *The scientist exists in the physical universe*
- *So what answer for Q they read from $\zeta(u)$ is a function of u*
- *Write that function as Y(u)*

INFERENCE



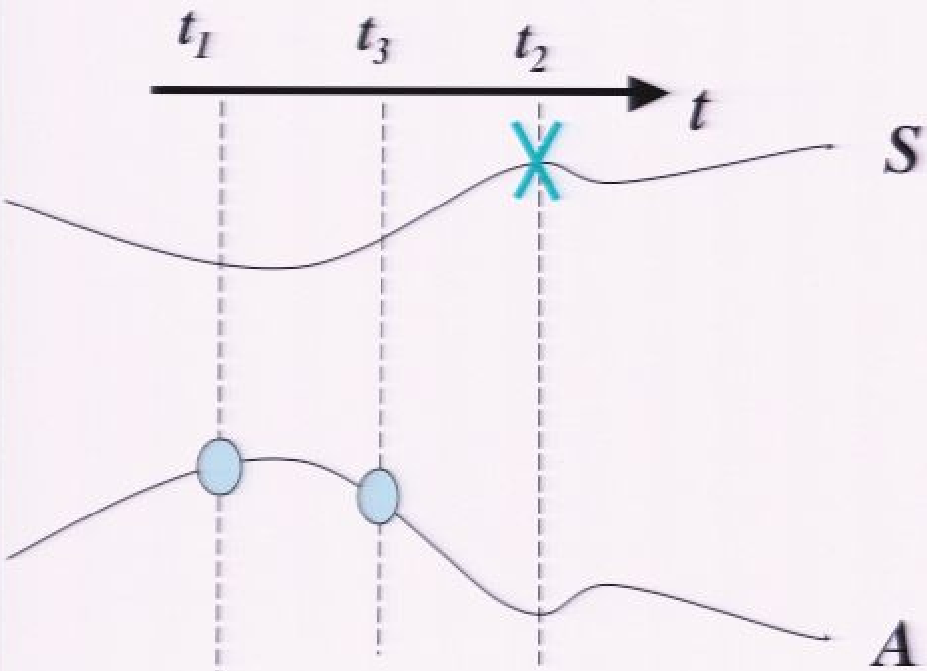
1) $\Gamma(u) = S(t_2)$

2) $\chi(u) = A(t_1)$, $\zeta(u) = A(t_3)$

3) $q_L(\gamma) = 1$ *iff* $\gamma = L$

- ***The scientist exists in the physical universe***
- ***So which question they are asking is a function of u***
- ***Write that function as Q(u)***

INFERENCE



1) $\Gamma(u) = S(t_2)$

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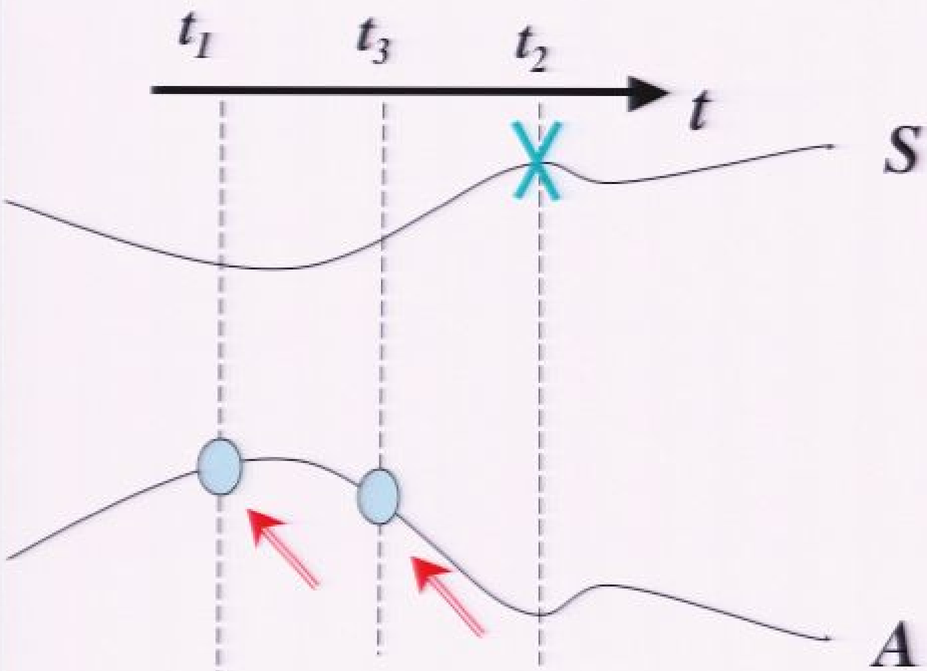
- For every L, "$S(t_2) = L$?" is a binary-valued function of $S(t_2)$.
- Write that function as q_L

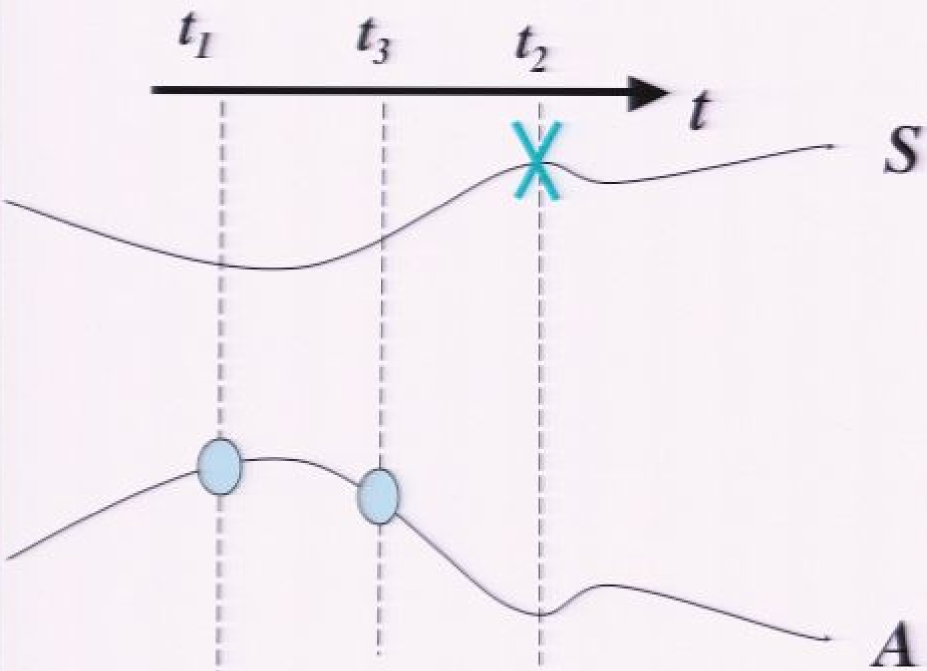
INFERENCE



1) $\Gamma(u) = S(t_2)$

- A exists in the physical universe
- So $A(t_1)$ and $A(t_3)$ are functions of u
- Write them as $\zeta(u)$ and $\chi(u)$

INFERENCE



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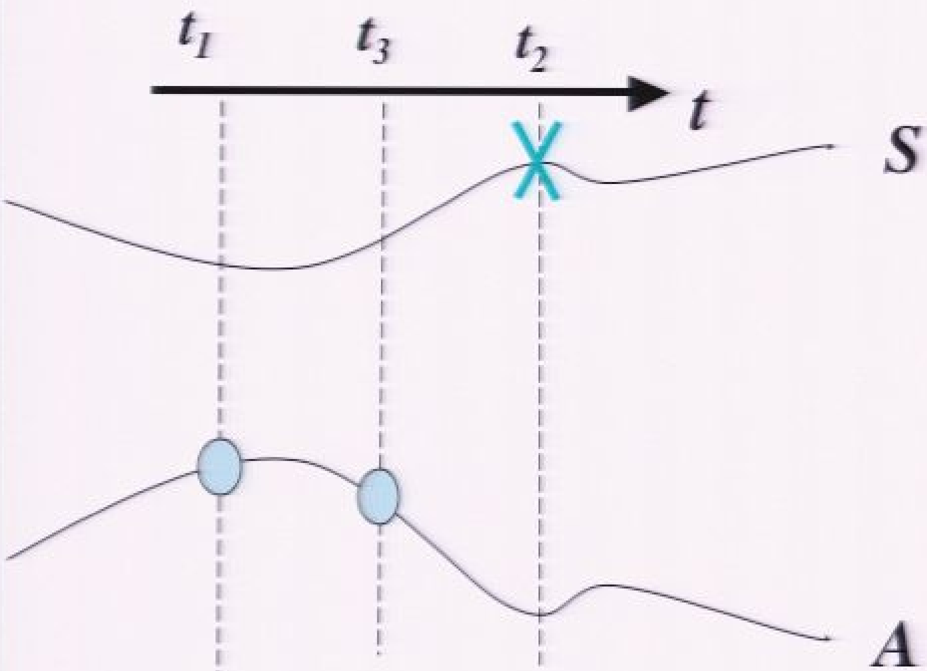


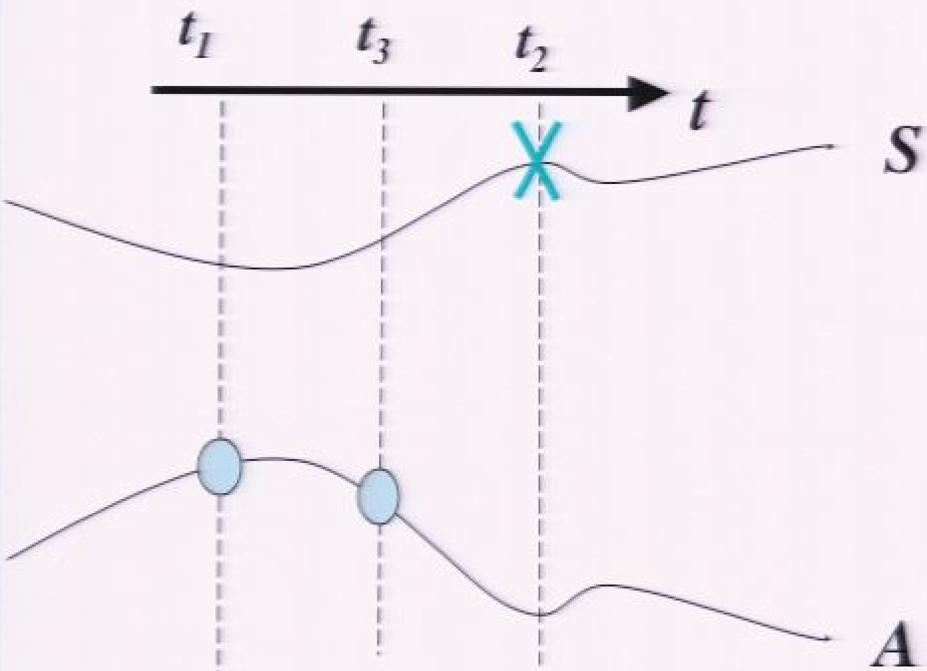
1) $\Gamma(u) = S(t_2)$

2) $\chi(u) = A(t_1)$, $\zeta(u) = A(t_3)$

3) $q_L(\gamma) = 1$ iff $\gamma = L$

- *The scientist exists in the physical universe*
- *So which question they are asking is a function of u*
- *Write that function as Q(u)*

INFERENCE



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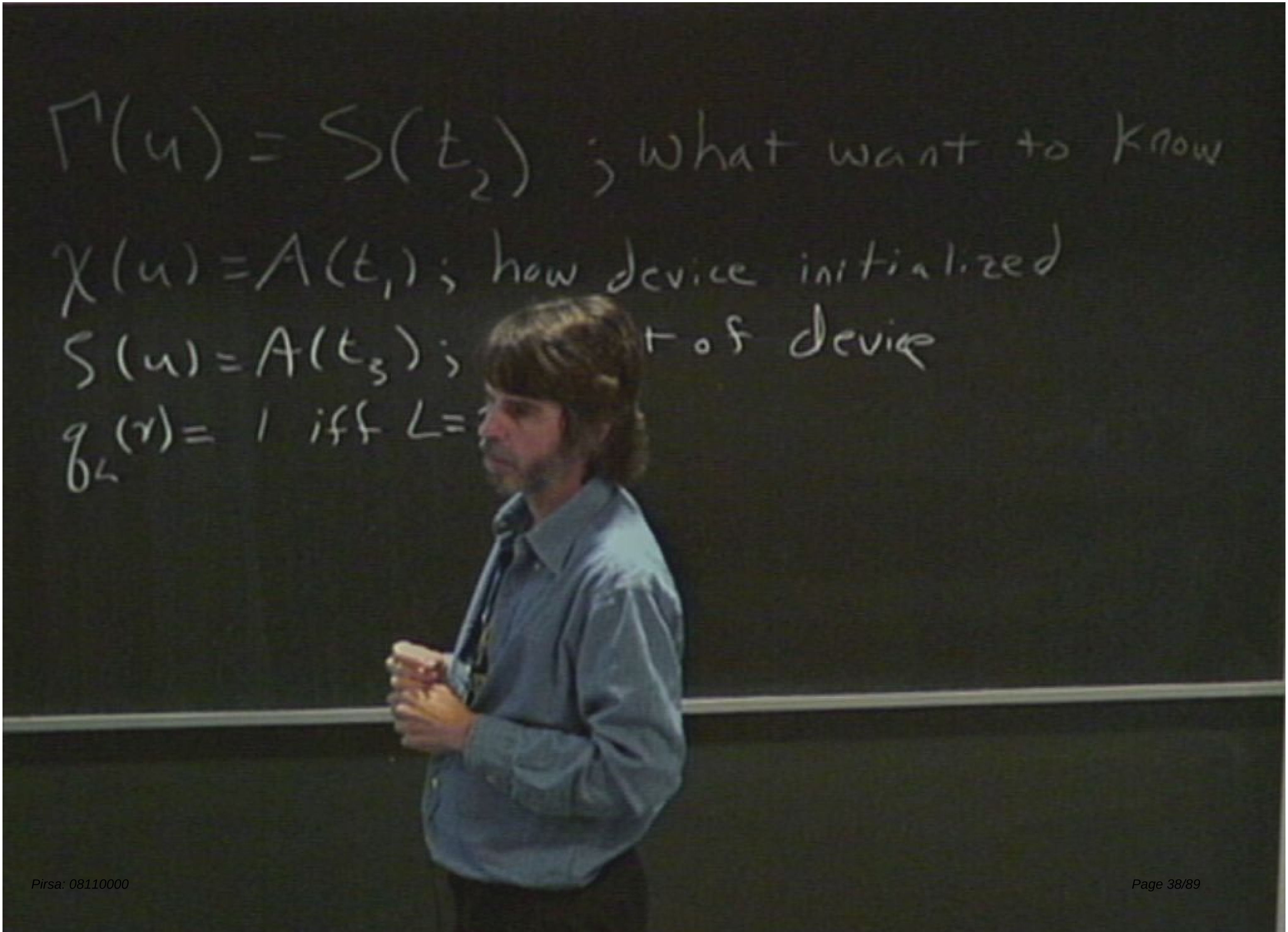
4) $Q(u)$

- ***The scientist exists in the physical universe***
- ***So what answer for Q they read from $\zeta(u)$ is a function of u***
- ***Write that function as Y(u)***


Γ(u) = S(t2) ; what want to know
χ(u) = A(t1) ; how device initialized
S(u) = A(t3) ;
t of device
qL(r) = 1 iff L =

$\Gamma(u) = S(t_2)$; what want to know

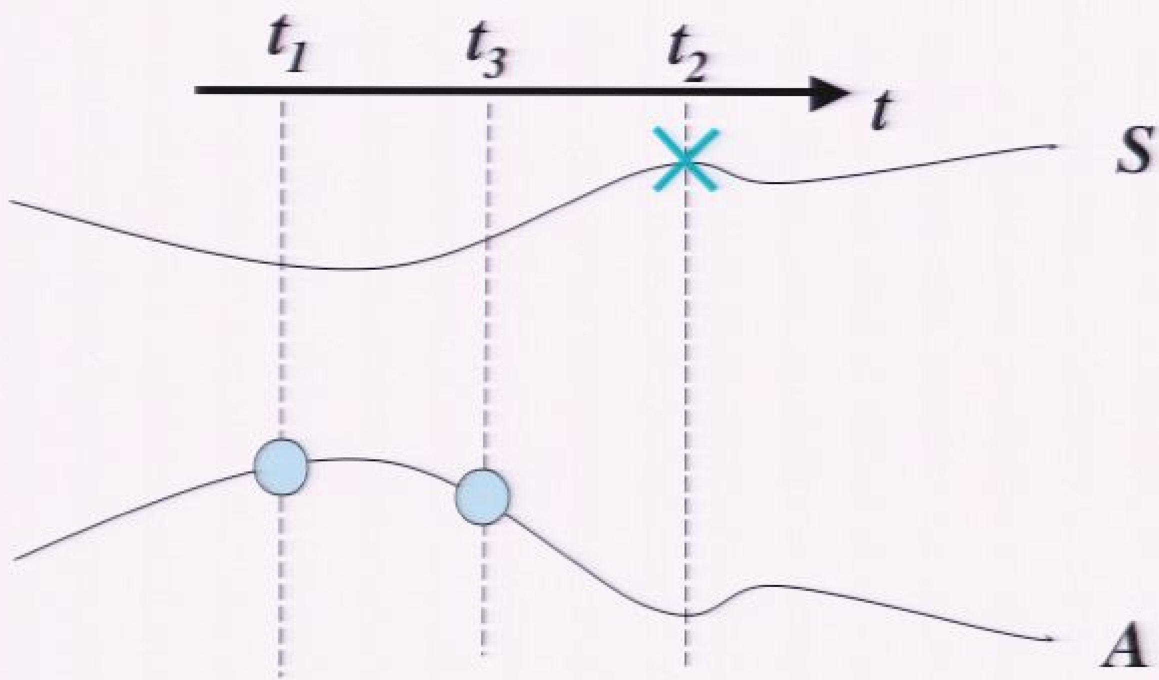
$\chi(u) = A(t_1)$; how device initialized

$S(u) = A(t_3)$; output of device

$q_L(\gamma) = 1$ iff $L = \gamma$

$Q(u) =$

PREDICTION



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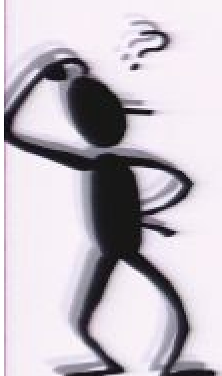


1) *$A(t_3)$ has "semantic meaning" if there is a scientist who can read it to answer, "Does $S(t_2) = L$?" $\forall L$.*

2) **Successful** *prediction is when the scientist's answer to any such question is correct.*

$\Gamma(u) = S(t_2)$; what want to know

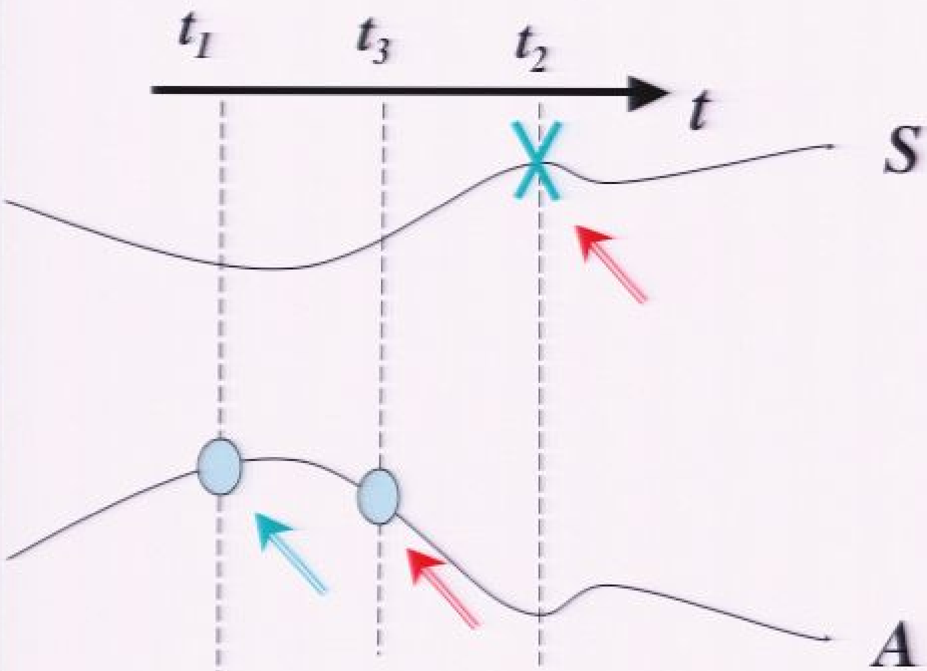
$\chi(u) = A(t_1)$; how device initialized

$\varsigma(u) = A(t_3)$; output of device

$q_L(\gamma) = 1$ iff $L = \gamma$

$Q(u) =$ Scientist's question

INFERENCE



1) $\Gamma(u) = S(t_2)$

2) $\chi(u) = A(t_1),\ \zeta(u) = A(t_3)$

3) $q_L(\gamma) = 1$ *iff* $\gamma = L$

4) $Q(u)$

5) $Y(u)$

Successful inference: $\forall q_L$,

$\exists$ **a, b such that**

$$(\mathbf{Q(u)}, \chi(\mathbf{u})) = (\mathbf{a}, \mathbf{b})$$

$$\Rightarrow$$

$$\mathbf{Y(u)} = \mathbf{q_L}(\Gamma(\mathbf{u}))$$

$\Gamma(u) = S(t_2)$; what want to know

$\chi(u) = A(t_1)$; how device initialized

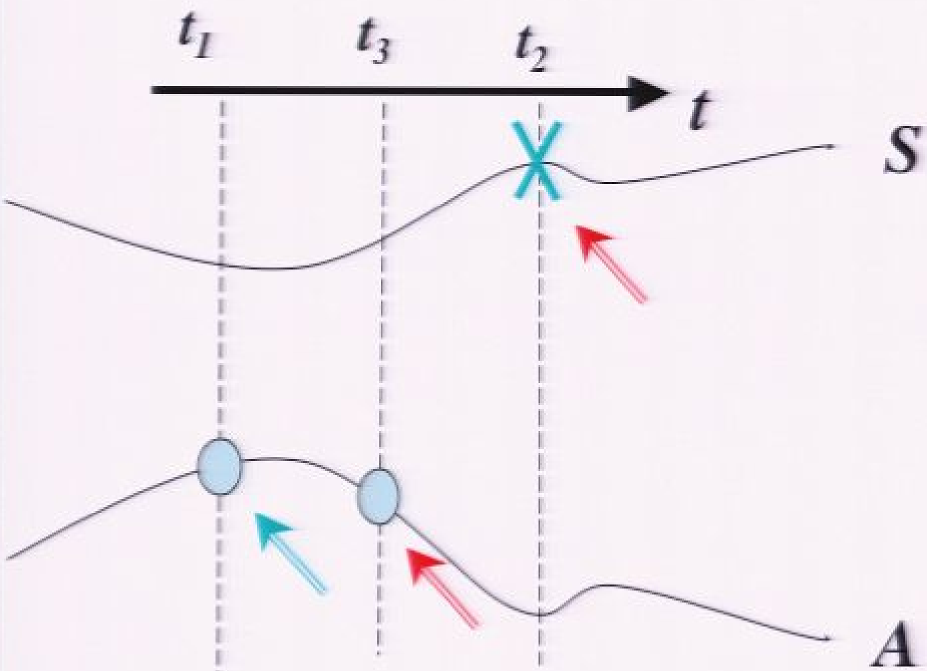
$\varsigma(u) = A(t_3)$; output of device

$q_L(\gamma) = 1$ iff $L = \gamma$

$Q(u) =$ Scientist's question

$Y(u) =$ " answer

INFERENCE



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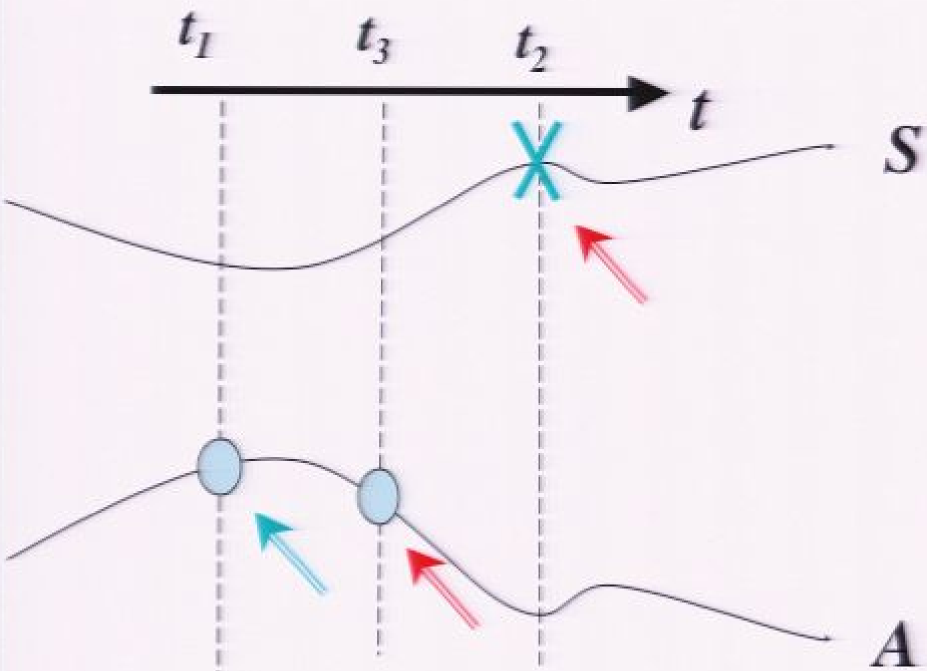
$$(Q(u), \chi(u)) = (a, b)$$

$$\Rightarrow$$

$$Y(u) = q_L(\Gamma(u))$$

INFERENCE



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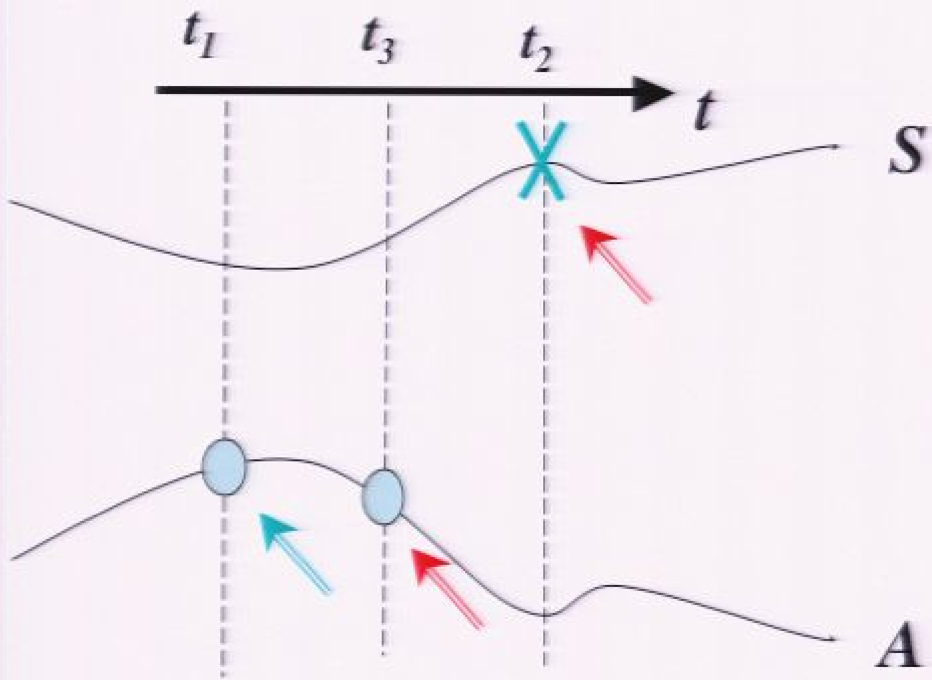

<u>Successful inference</u>: $\forall q_L$,

$\exists$ a, b such that

$$(Q(u), \chi(u)) = (a, b)$$

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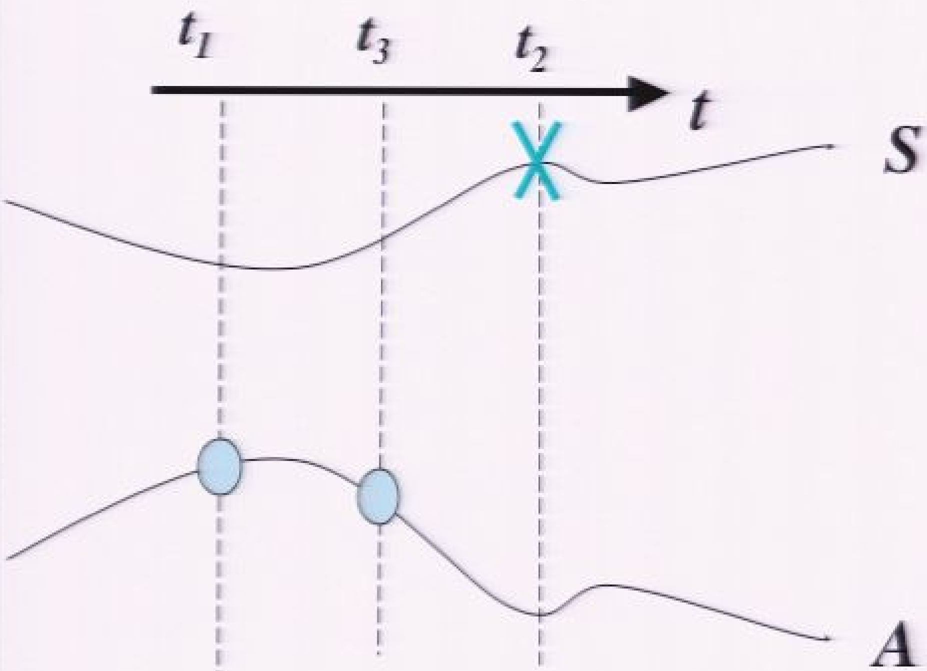

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3) $q_L(\gamma) = 1 \text{ iff } \gamma = L$

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$$\forall q_L, \exists x : \forall u \in U,$$

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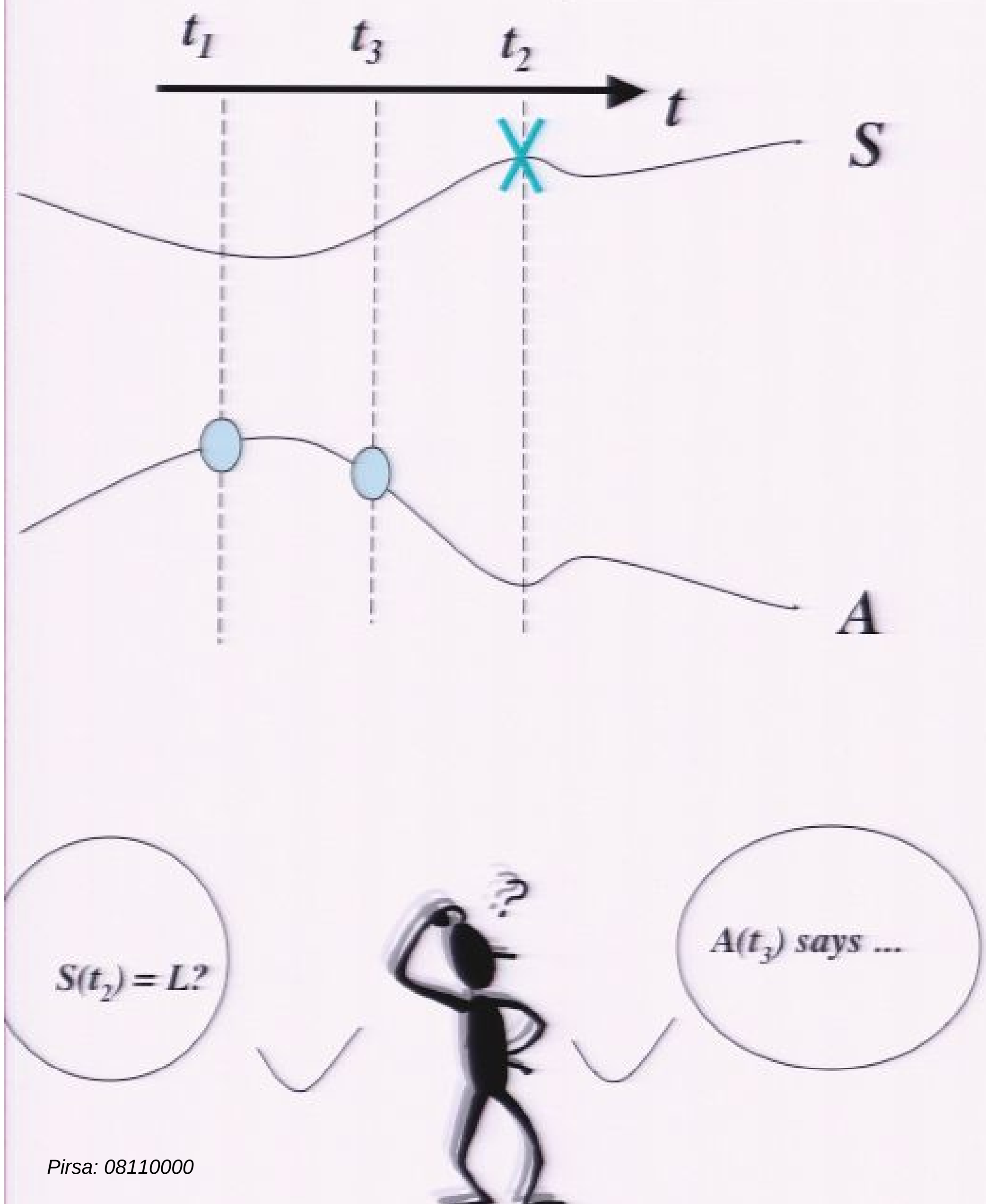
WEAK INFERENCE



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An *Inference Device* is

a pair of functions C = (X(u), Y(u))
where Y is binary-valued

C *Weakly infers* Γ ("C > Γ") iff

$\forall q_L, \exists x$ such that

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ROADMAP

1) *Formalize observation, prediction, and memory*

↓

2) *Extract what's in common:* **inference devices**

↓

3) *Elementary properties of inference devices*

↓

4) *Inference devices and Turing machine theory:*
strong inference *and* **inference complexity**

↓

5) **Stochastic inference**

ELEMENTARY PROPERTIES OF INFERENCE

1) For any Γ, $\exists$ a device that infers Γ.

2) For any device, $\exists$ a Γ it does not infer. (Impossibility result.)

3) Inference need not be transitive:

   $(X_1, Y_1) > Y_2$ and $(X_2, Y_2) > Y_3$ does not mean $(X_1, Y_1) > Y_3$

4) Let $\{\Gamma_i\}$ be a set of functions over U and $W \subset U$, where for all i, $|\Gamma_i(U)| = |\Gamma_i(W)|$. Then $\exists$ a device that infers each Γ_i.

ENGINEERING IMPLICATIONS OF IMPOSSIBILITY RESULT

1) *For any simulator, there is always a prediction that cannot be guaranteed correct.*
   - *Laplace was wrong.*

2) *For any recording apparatus, there is always a past event that cannot be guaranteed to have been correctly recorded.*

3) *For any observation apparatus, there is always an observation that cannot be guaranteed to be correct.*
   - *Non-quantum mechanical "uncertainty principle"*

BREADTH OF IMPOSSIBILITY RESULT

1) *No requirement that x specify the question being asked.*

2) *No restriction on information fed into the device via x.*
   - *Can even have correct answer specified in x.*

- *No restriction on coupling between a simulator and the "rest of the universe" while it is computing the simulation.*

4) *No restriction on how long a simulator takes to run.*
   - *Answer can even occur* after *event being simulated.*

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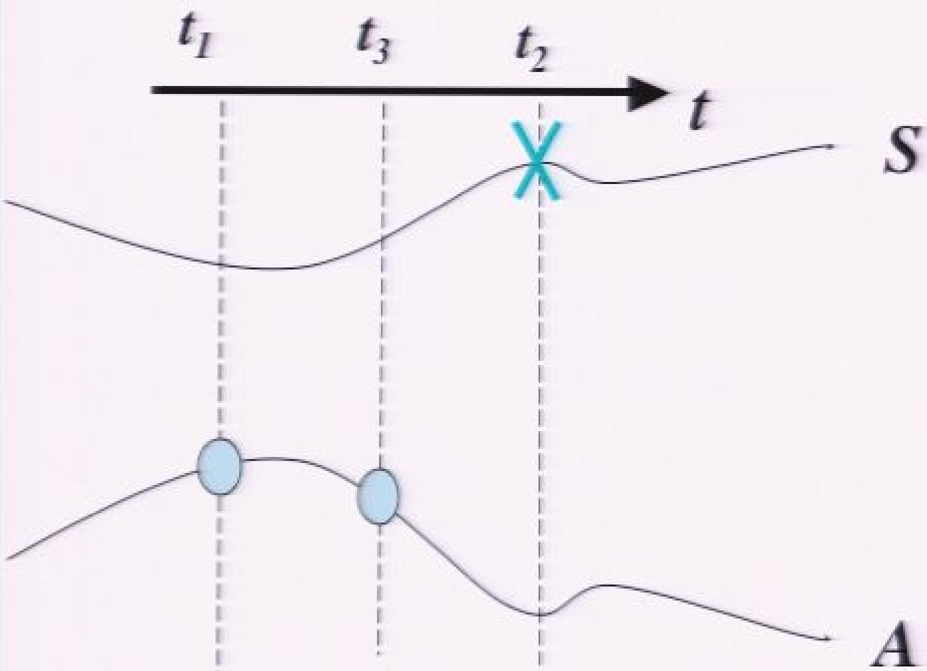

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INFERENCE RELATIONS BETWEEN DEVICES

- Often not interested in inference of arbitrary functions, but rather inference relation among a set of devices.

1) Two devices (X, Y), (X', Y') are ***pairwise distinguishable*** iff any pair (x, x') can occur

2) A set of devices $\{(X_i, Y_i)\}$ is ***mutually distinguishable*** iff any tuple $(x_1, x_2, ...)$ can occur

1) No device is distinguishable from itself.

2) Pairwise distinguishability may not be transitive.

INFERENCE RELATIONS BETWEEN DEVICES -2

3) If all pairs of devices from $\{C_i\}$ are pairwise distinguishable, $\exists$ at most one k : $C_k > C_j$ $\forall j \neq k$. "Monotheism" theorem. (N.b., control is a special type of inference.)

4) If all pairs of devices from $\{C_i\}$ are pairwise distinguishable, can have $C_1 > C_2 > \ldots C_1$.

5) If the set of devices $\{C_i\}$ is mutually distinguishable, can*not* have $C_1 > C_2 > \ldots C_1$.

ROADMAP

1) ***Formalize observation, prediction, and memory***

2) ***Extract what's in common:*** **inference devices**

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5) **Stochastic inference**

STRONG INFERENCE

- A universal Turing Machine T can emulate any other one, T'
- T does that by having its input be the program and input of T'

The analog with inference devices:

C = (X, Y) *strongly infers* C' = (X', Y') iff:

$\forall$ probe functions q_L of Y'(U), $\forall$ x',

$\exists$ x s.t.

$$X(u) = x \;\Rightarrow\; Y(u) = q_L(Y'[u]),\; X'(u) = x'$$

- "$C_1 >> C_2$" means C_1 strongly infers C_2

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PROPERTIES OF STRONG INFERENCE

1) $C_1 >> C_2$ and $C_2 > \Gamma \Rightarrow C_1 > \Gamma$
   - Just like with UTM's and TM's (contrast weak inference)

2) $C_1 >> C_2$ and $C_2 >> C_3 \Rightarrow C_1 >> C_3$
   - Just like with UTM's (contrast weak inference)

3) For any C_1, $\exists\, C_2$ that C_1 does not strongly infer

4) If $\forall x_1$, $|X_1^{-1}(x_1)| > 2$, then $\exists\, C_2$ such that $C_2 >> C_1$

5) No two devices can strongly infer each other
   - Distinguishability irrelevant (contrast weak inference)
   - Even if C_1 and C_2 are same system at different moments in time: "intelligent design theorem".

PROPERTIES OF STRONG INFERENCE - 3

6) Cannot have $C_1 >> C_2 >> \ldots C_1$.

7) Let D be a finite set of devices, where:

- The associated digraph G of strong inferences among the devices in D is weakly connected;
- Any pair of devices (nodes) in G not connected by an edge is pairwise distinguishable

Then G has exactly one root.

PROPERTIES OF STRONG INFERENCE - 3

9) Let C_1 and C_2 be copies, i.e., implement the same logical mapping from X to Y.

E.g., same system at different moments in time.

i) Can have $C_1 > C_2$, even if $X_1(U)$, $X_2(U)$ are finite

ii) Can have $C_1 >> C_2$, but only if $X_1(U)$, $X_2(U)$ are infinite

(ii) says “intelligent design requires an infinite god”.

INFERENCE COMPLEXITY

1) Kolmogorov complexity of a string s with respect to TM T is smallest input string to T needed for T to produce s.

2) Inference complexity of Γ with respect to device (X, Y) is smallest set of setup values $x \in X$ needed for C to infer Γ.

- Size measured with -ln of phase space volume.

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2) There is a bound on "translation" inference complexity added in going from C_1 to C_2:

- *Given a finite function* Γ *and two devices* C_1 *and* C_2 : $C_1 >> C_2$,

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$$|\Gamma(U)| \left(\max_{x_2} \min_{x_1 : X_1 = x_1 \Rightarrow X_2 = x_2, Y_1 = Y_2} \ln \left[\frac{X_2^{-1}(x_2)}{X_1^{-1}(x_1)} \right] \right)$$

- *The RHS is non-negative, and independent of volume units*

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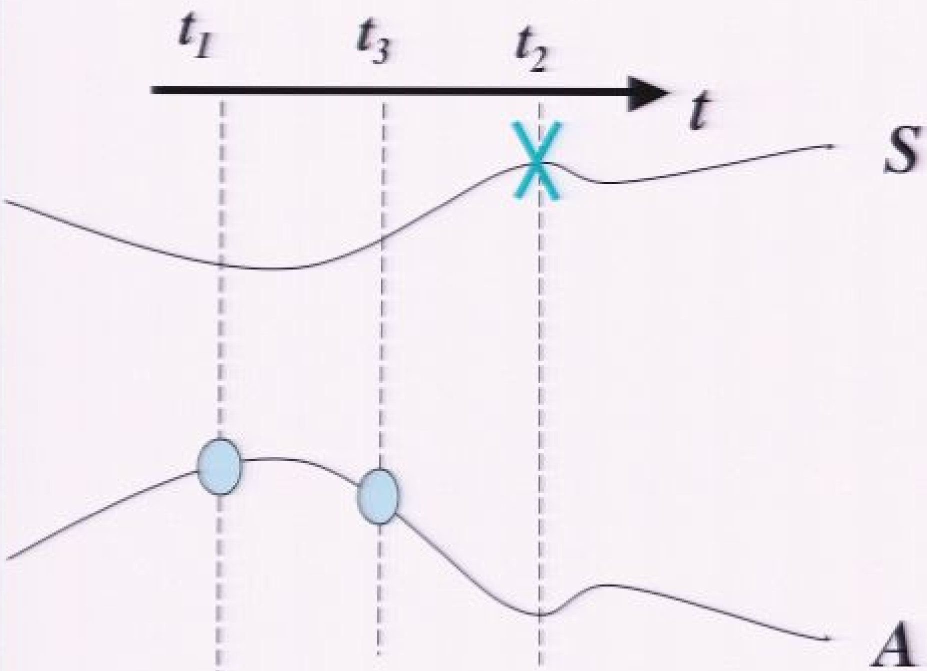

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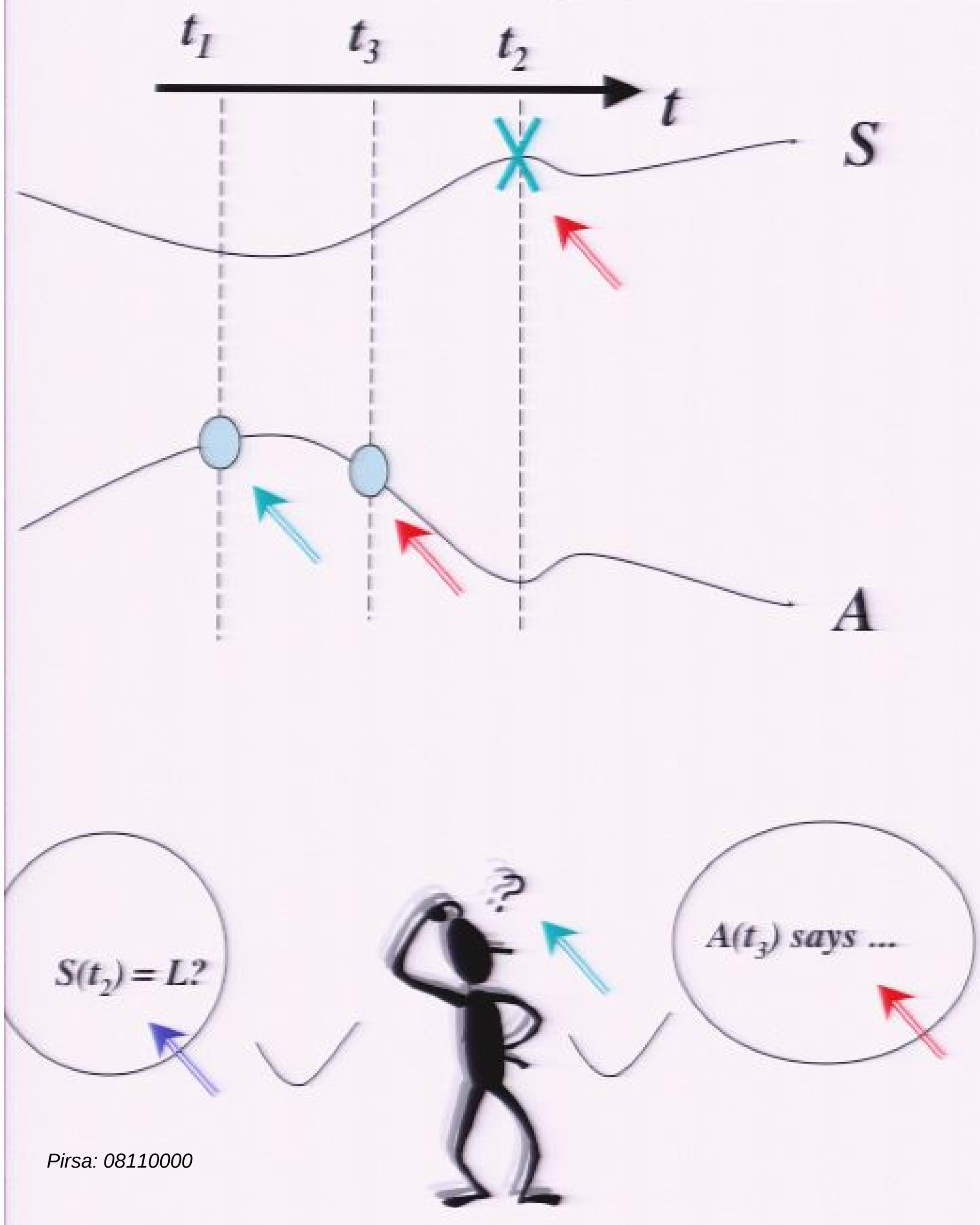

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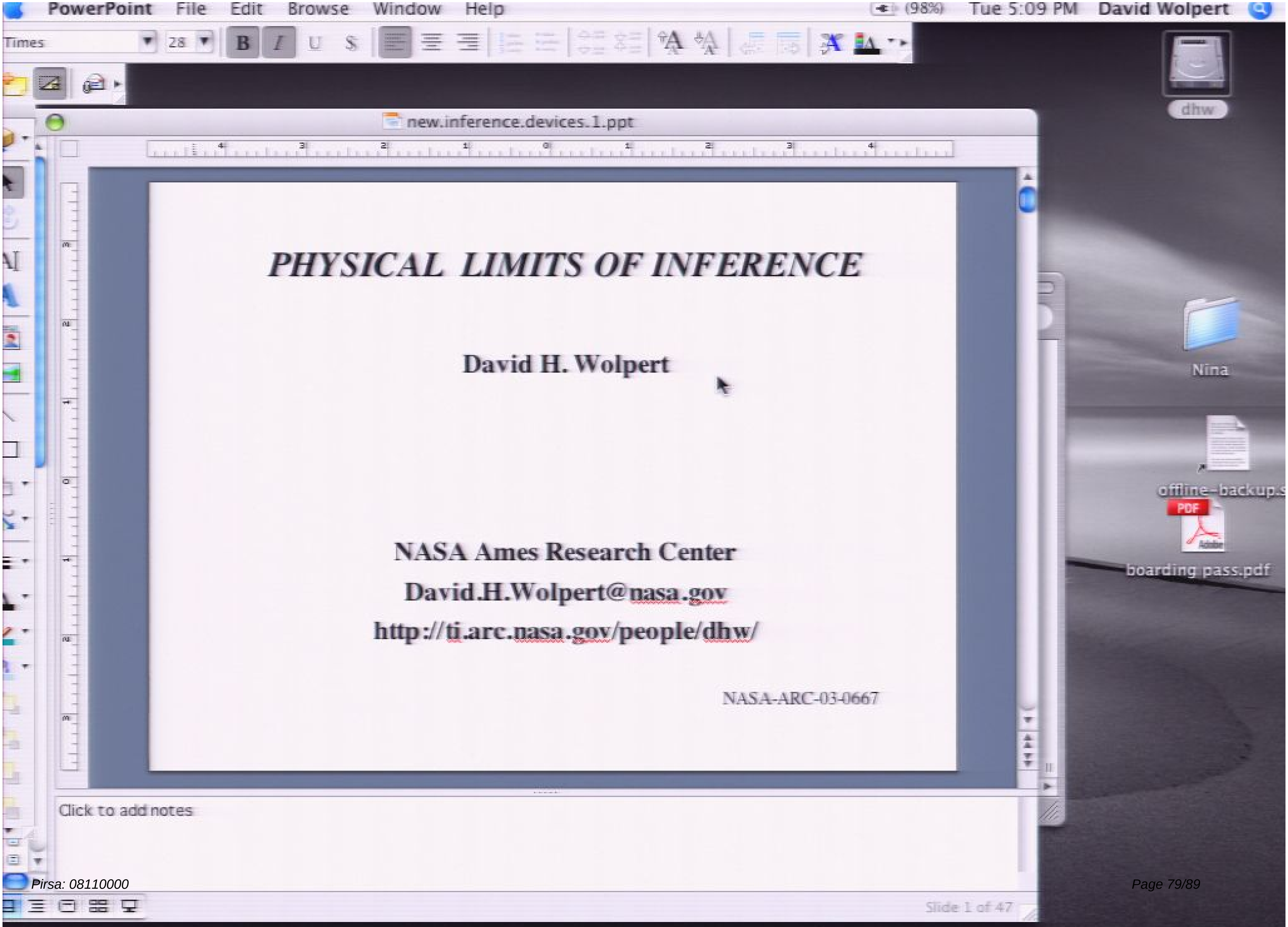
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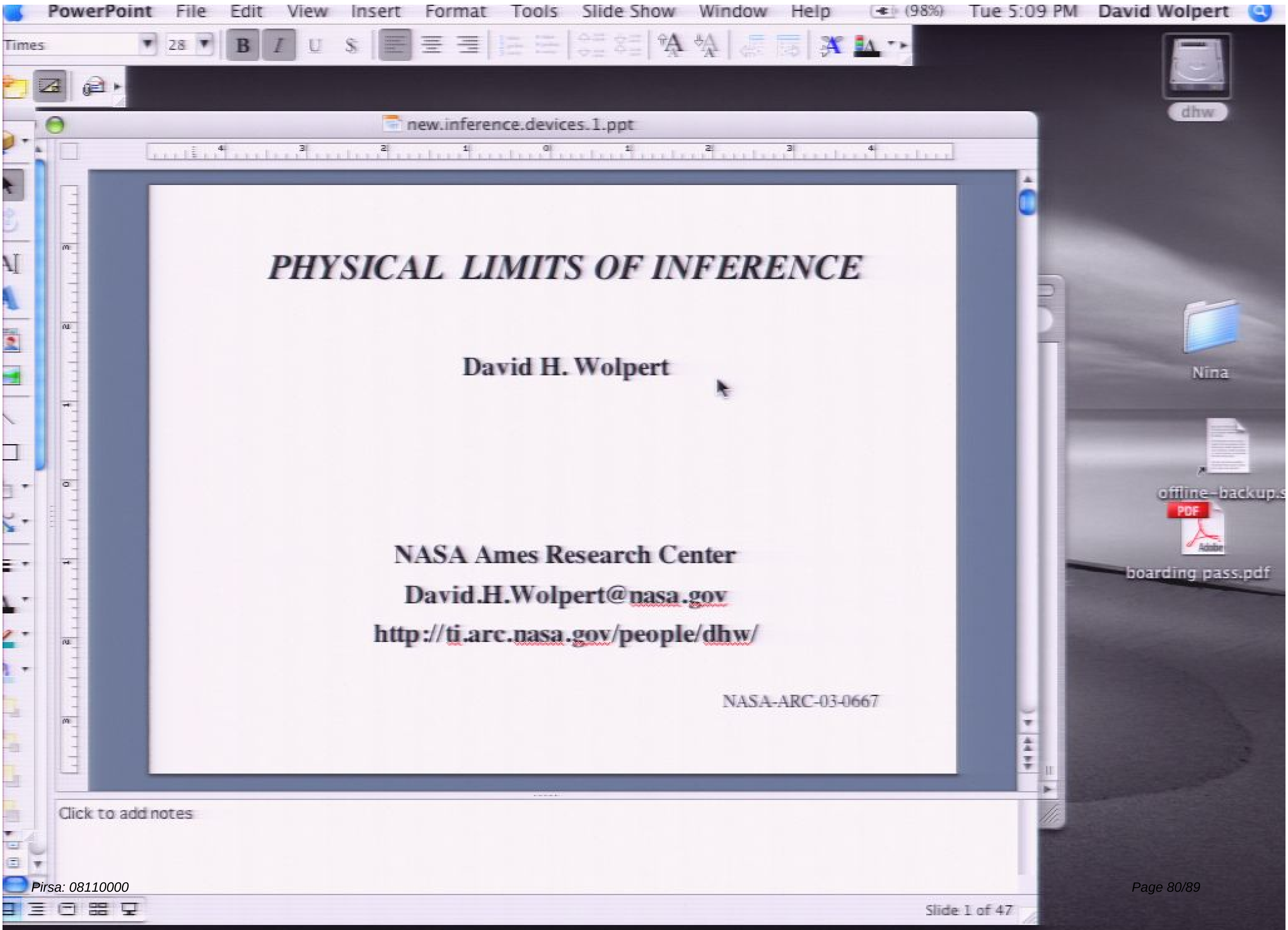
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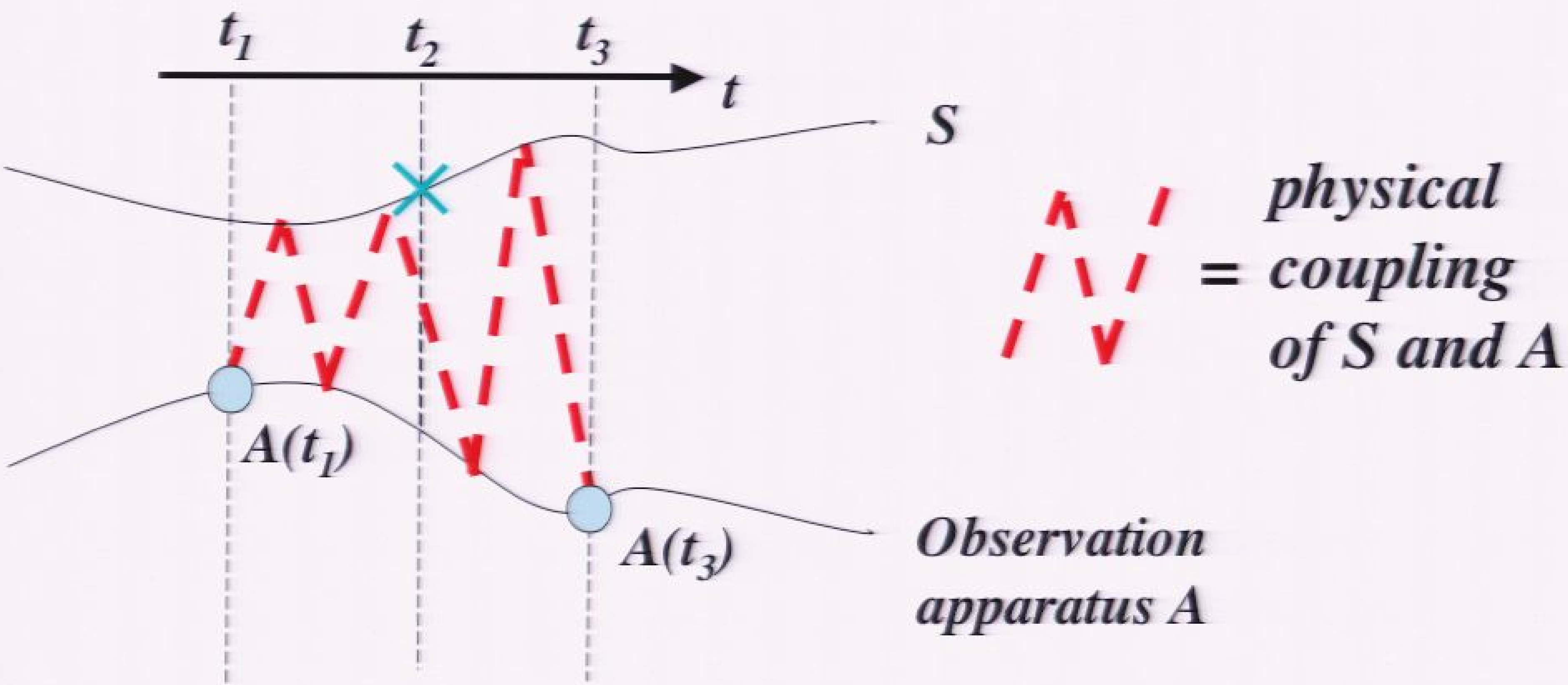
OBSERVATION



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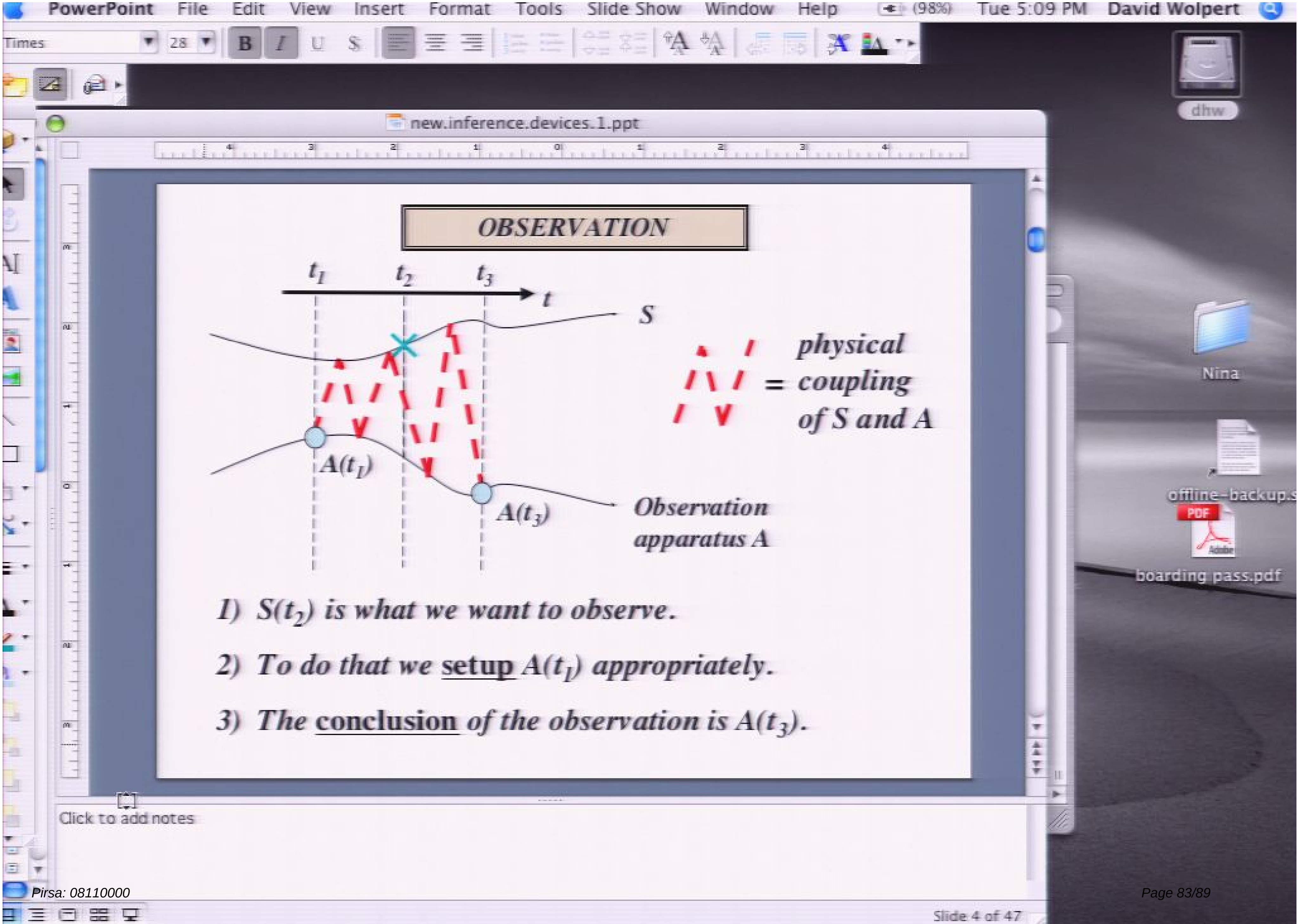
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OBSERVATION
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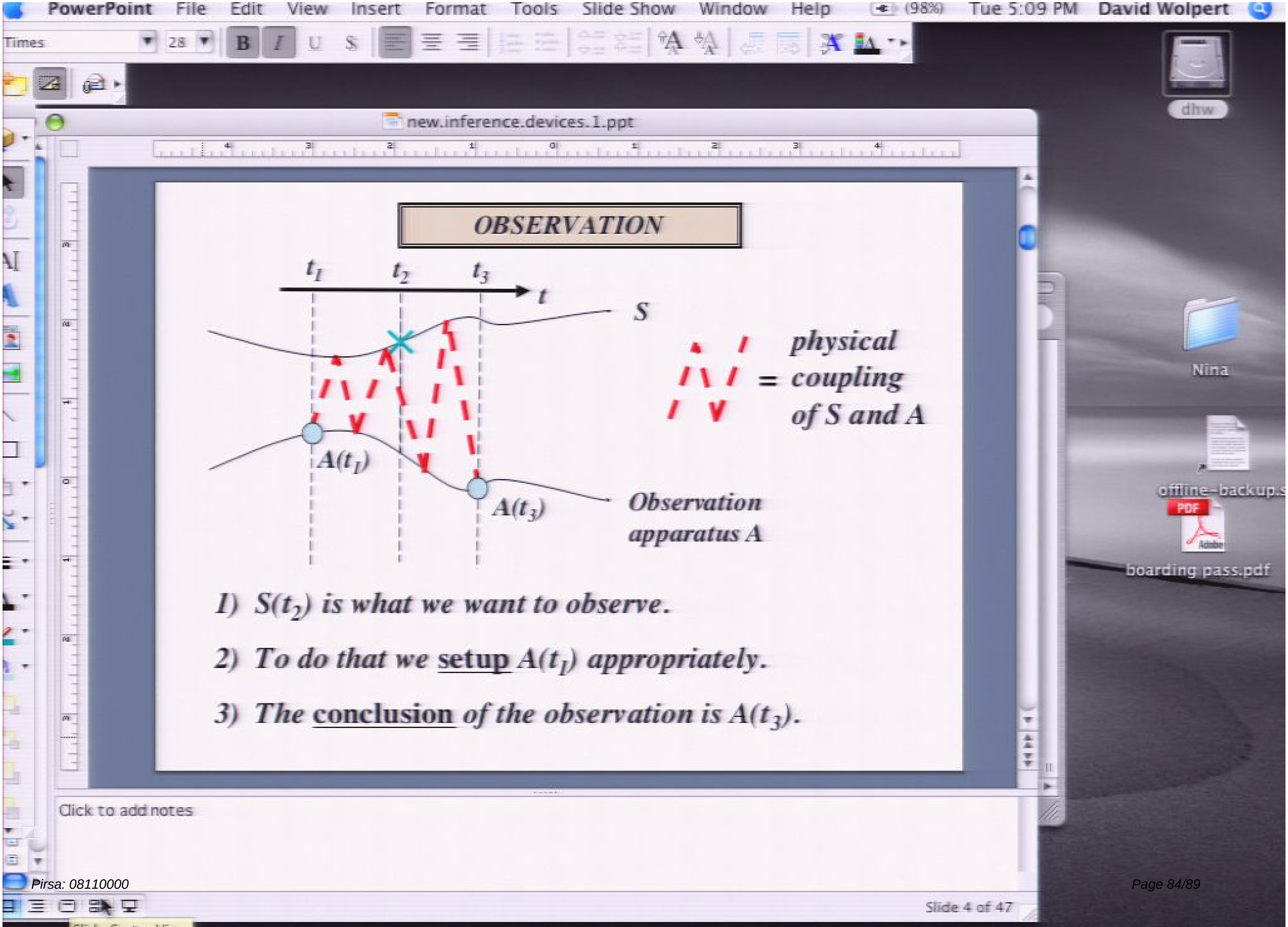
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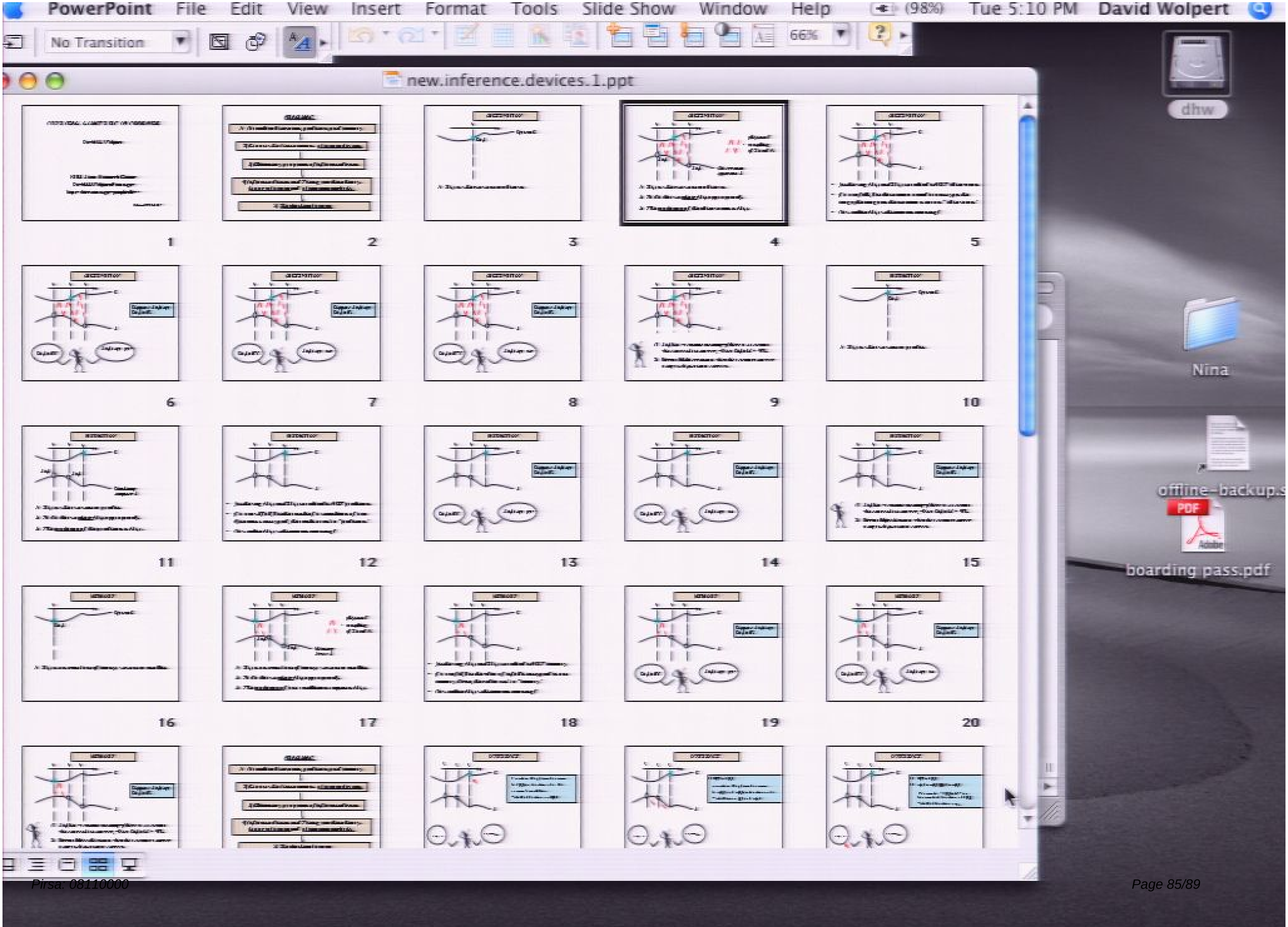
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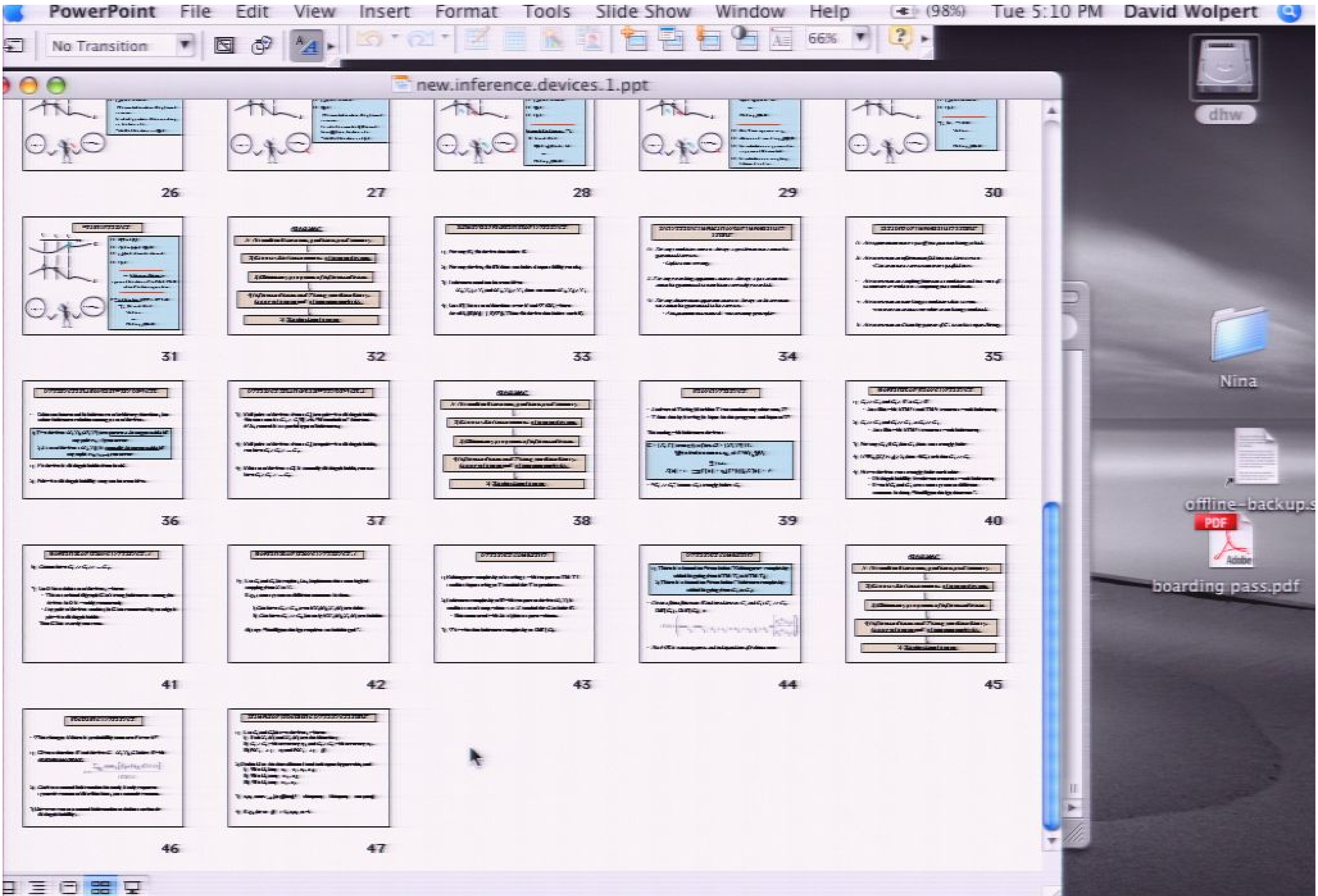
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STOCHASTIC INFERENCE

- **What changes if there is probability measure P over U?**

1) **Given a function Γ and device C = (X, Y), C infers Γ with *covariance accuracy***

$$\varepsilon = \frac{\sum_{q_L} \max_x \left[E_P(Y q_L(\Gamma) \mid x) \right]}{|\Gamma(U)|}$$

2) **Can't use mutual information instead; it only captures syntactic content of distributions, not semantic content.**

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EXAMPLE OF STOCHASTIC INFERENCE RESULT

1) Let C_1 and C_2 be two devices, where:
   i) Both $X_1(U)$ and $X_2(U)$ are the binaries;
   ii) $C_1 > C_2$ with accuracy ε_1, and $C_2 > C_1$ with accuracy ε_2.
   iii) $P(X_1 = -1) = \alpha$, and $P(X_2 = -1) = \beta$

2) Define H as the four-dimensional unit open hypercube, and
   i) $\forall z \in H,\ k(z) = z_1 + z_4 - z_2 - z_3$;
   ii) $\forall z \in H,\ m(z) = z_2 - z_4$;
   iii) $\forall z \in H,\ n(z) = z_3 - z_4$.

3) $\varepsilon_1\varepsilon_2 \le \max_{z\in H} |\alpha\, \beta[k(z)]^2 + ak(z)m(z) + bk(z)n(z) + m(z)n(z)|$

4) E.g., for $\alpha = \beta = 1/2$, $\varepsilon_1\varepsilon_2 \le 1/4$.