Title: One-loop Riemann correlators and dS invariance

Date: Oct 28, 2010 02:00 PM

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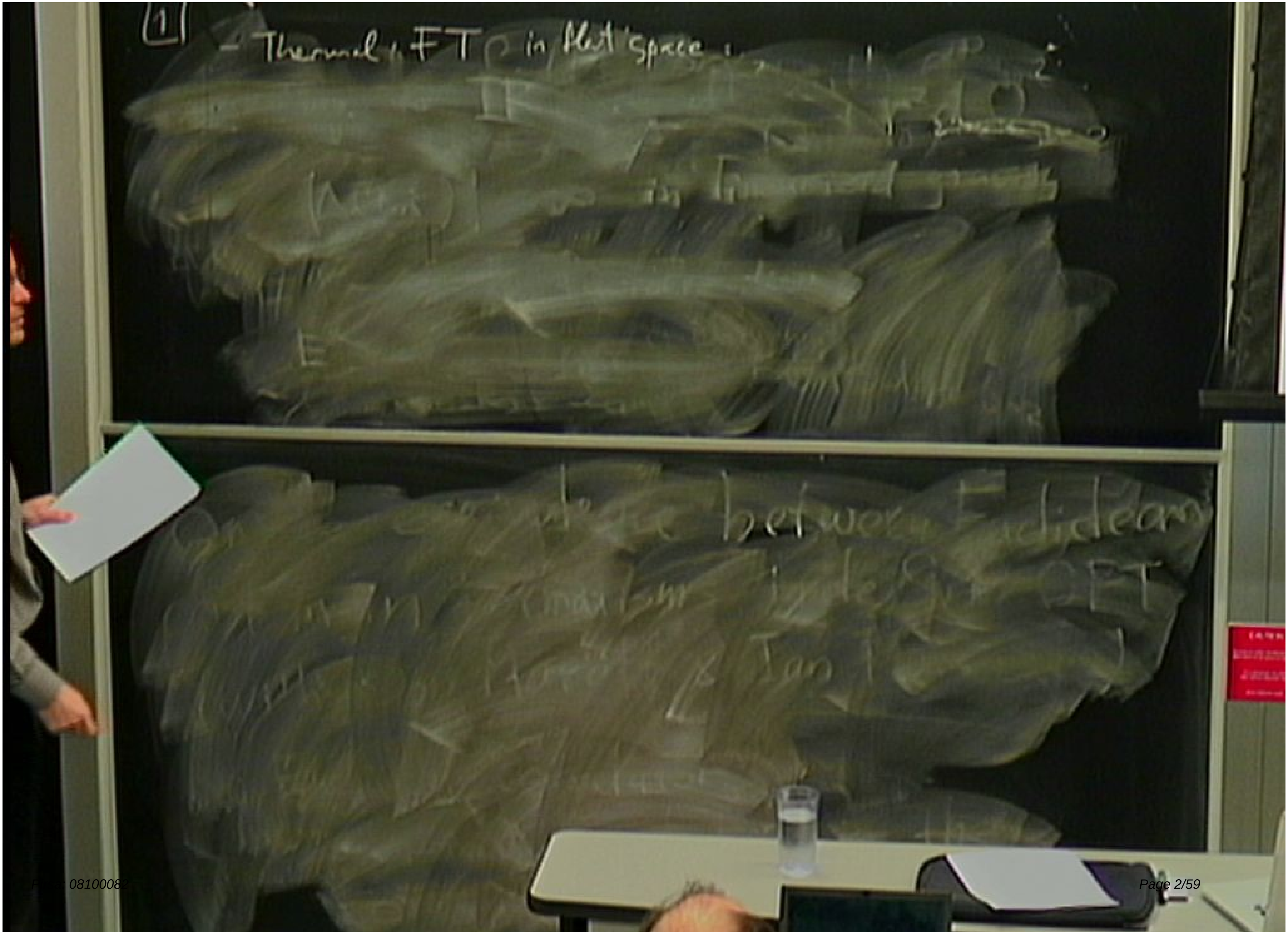
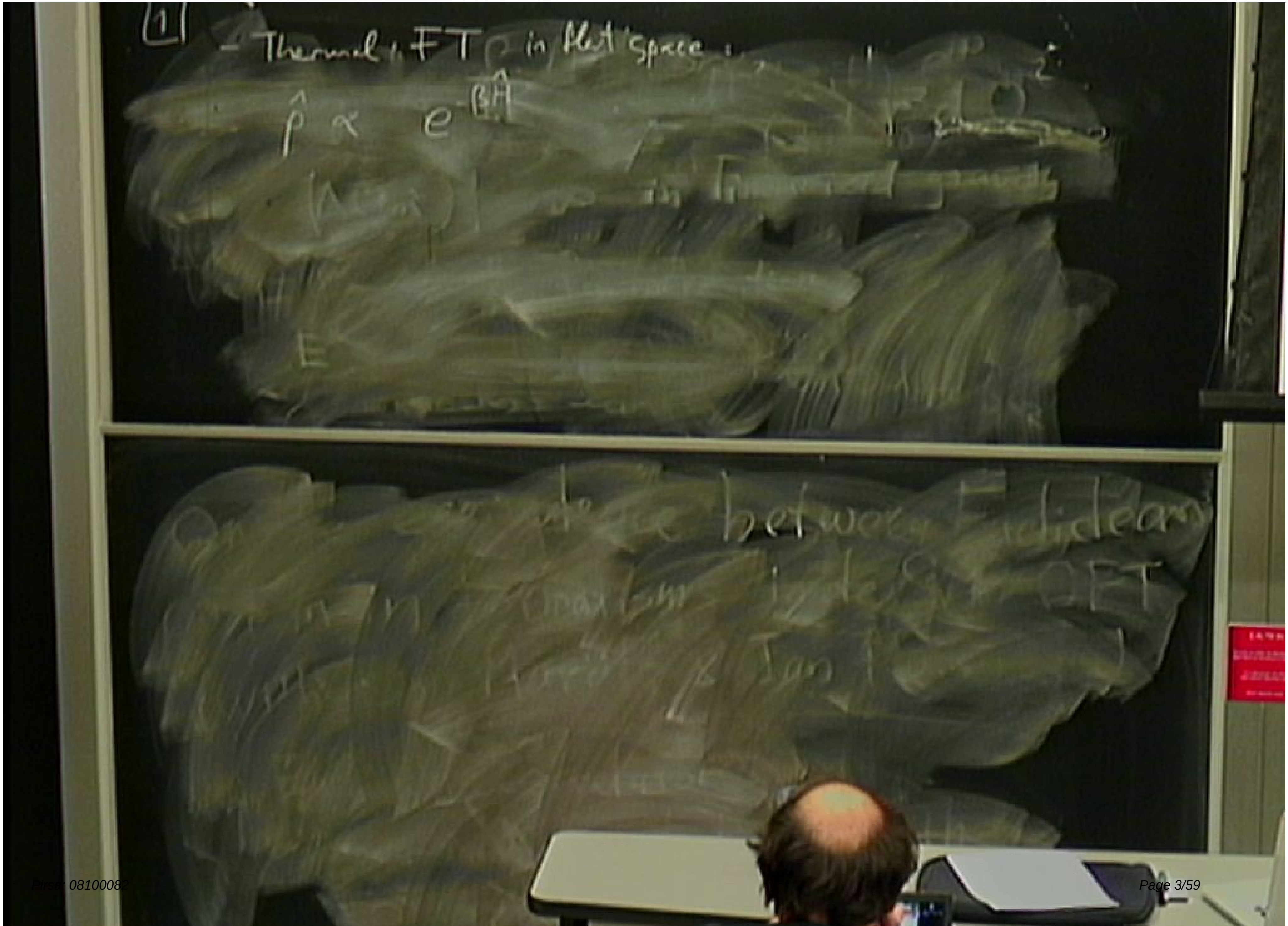
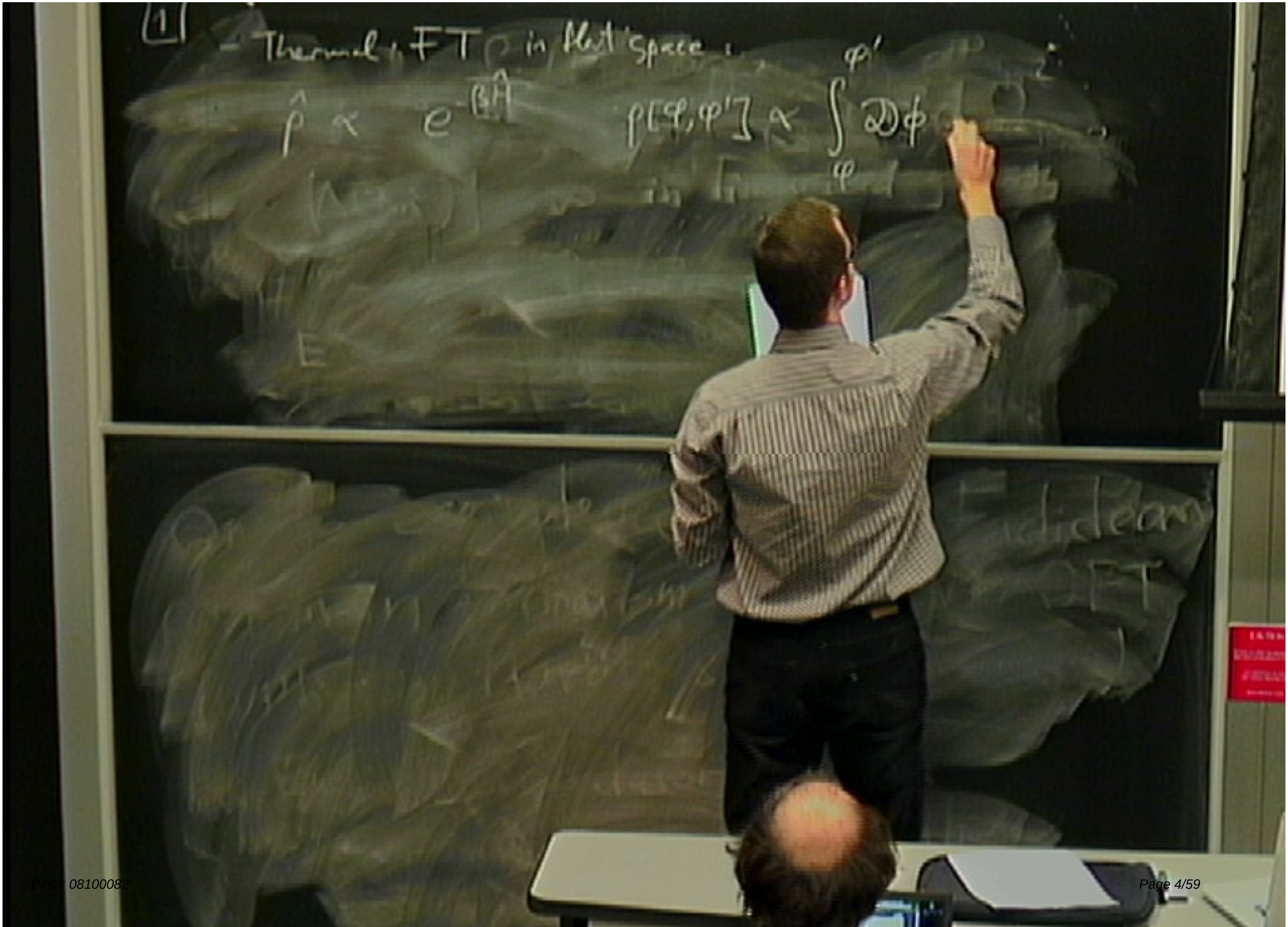
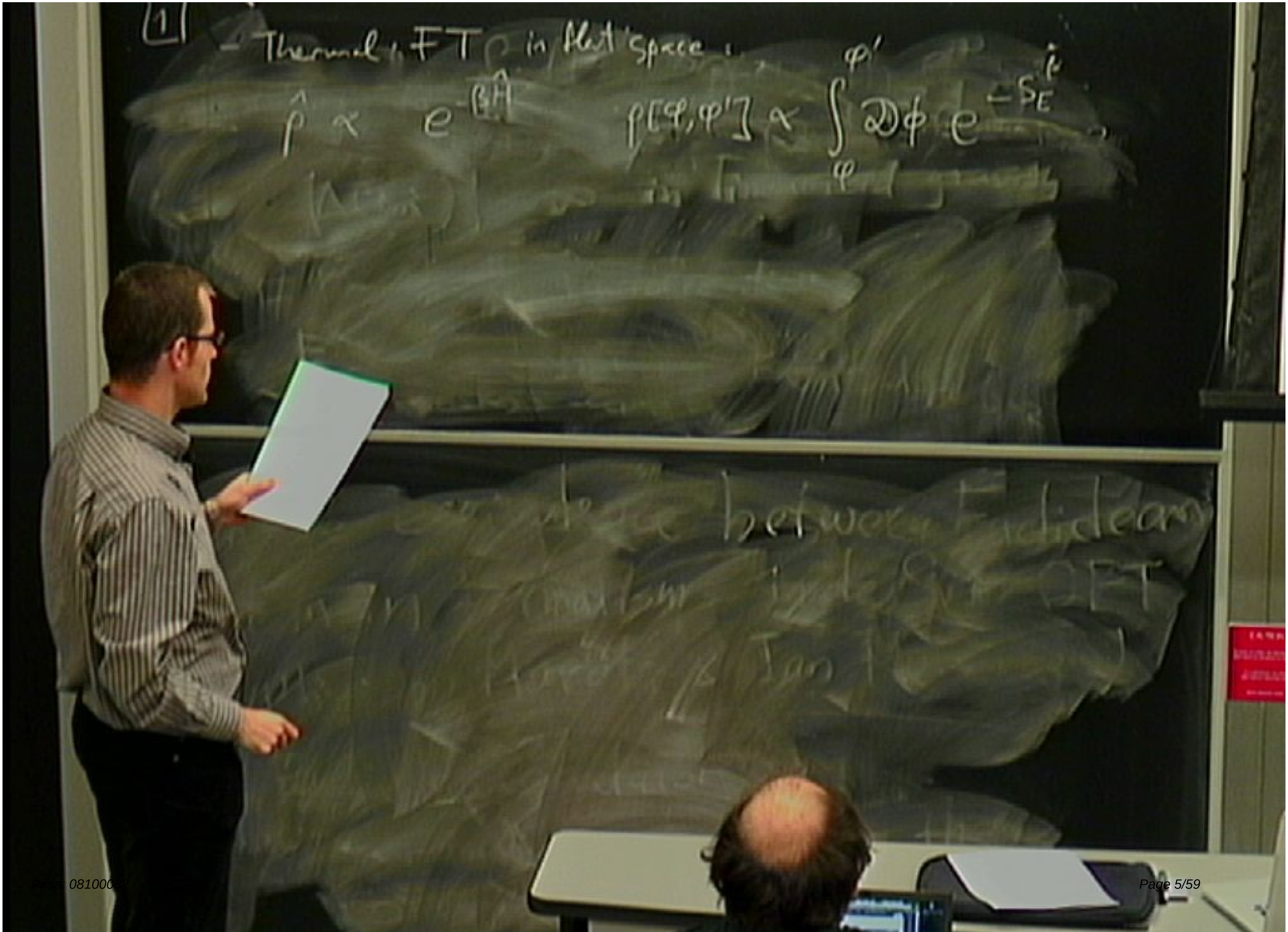
Abstract: I will start with a brief qualitative discussion of the construction of a dS-invariant state for interacting theories using Euclidean methods and its real-time evolution within the closed-time-path formalism, as well as of the closely related in-in formalism. Next, I will focus on the two-point quantum correlation function for the Riemann tensor of the metric perturbations around dS including the one-loop correction from matter fields. A key object is the stress tensor two-point function, from which the one-loop Ricci correlator follows straightforwardly. We have obtained the exact result for minimally coupled fields with arbitrary mass in terms of maximally symmetric bitensors, which makes dS invariance manifest. Long range correlations are present for sufficiently small (but nonvanishing) masses, and the discontinuity of the massless limit can be understood in a simple way. Finally, I will comment on the implications for the tensorial power spectrum and on the calculation of the Weyl correlations.


1
- Thermal FT in flat space

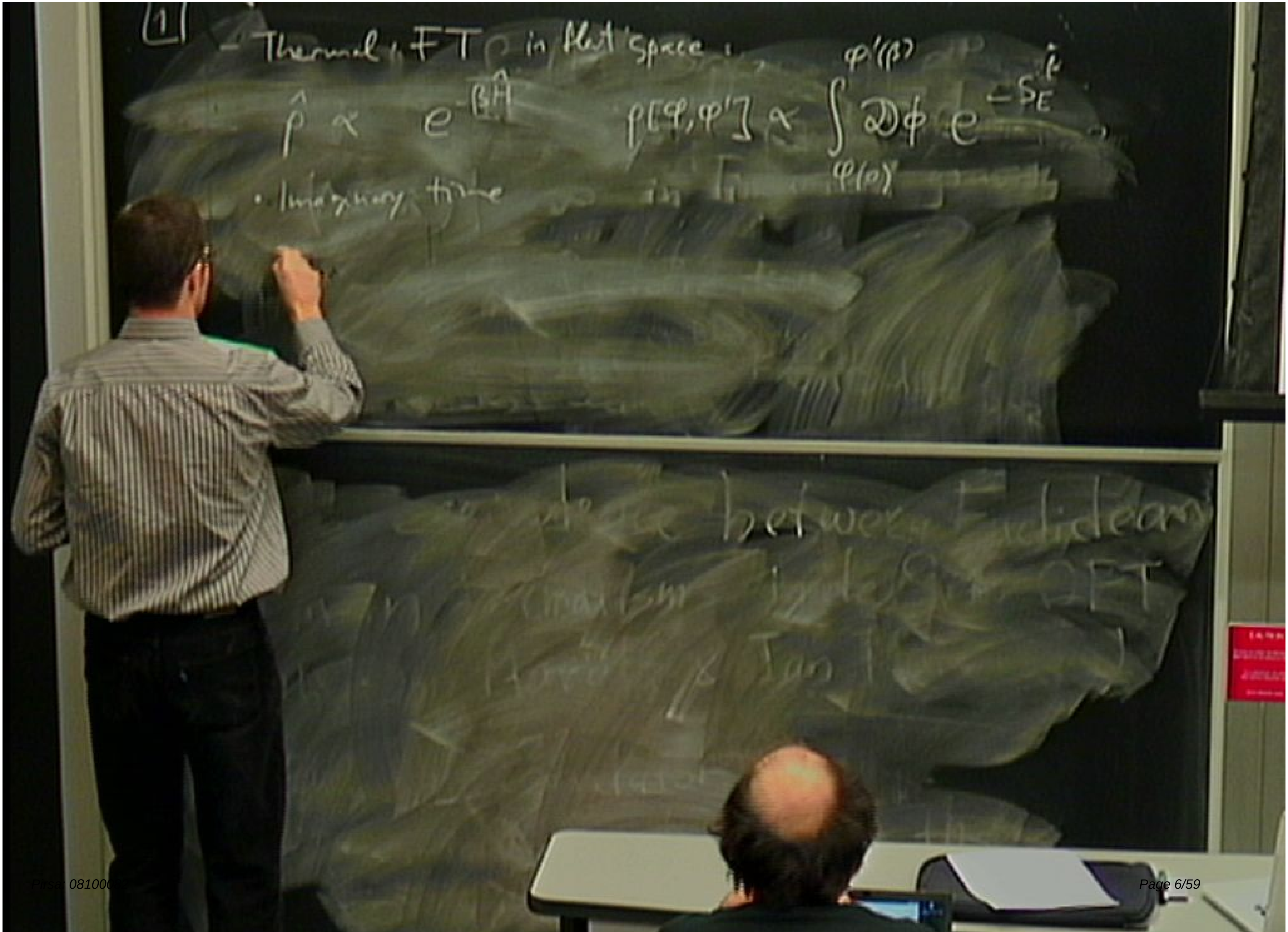


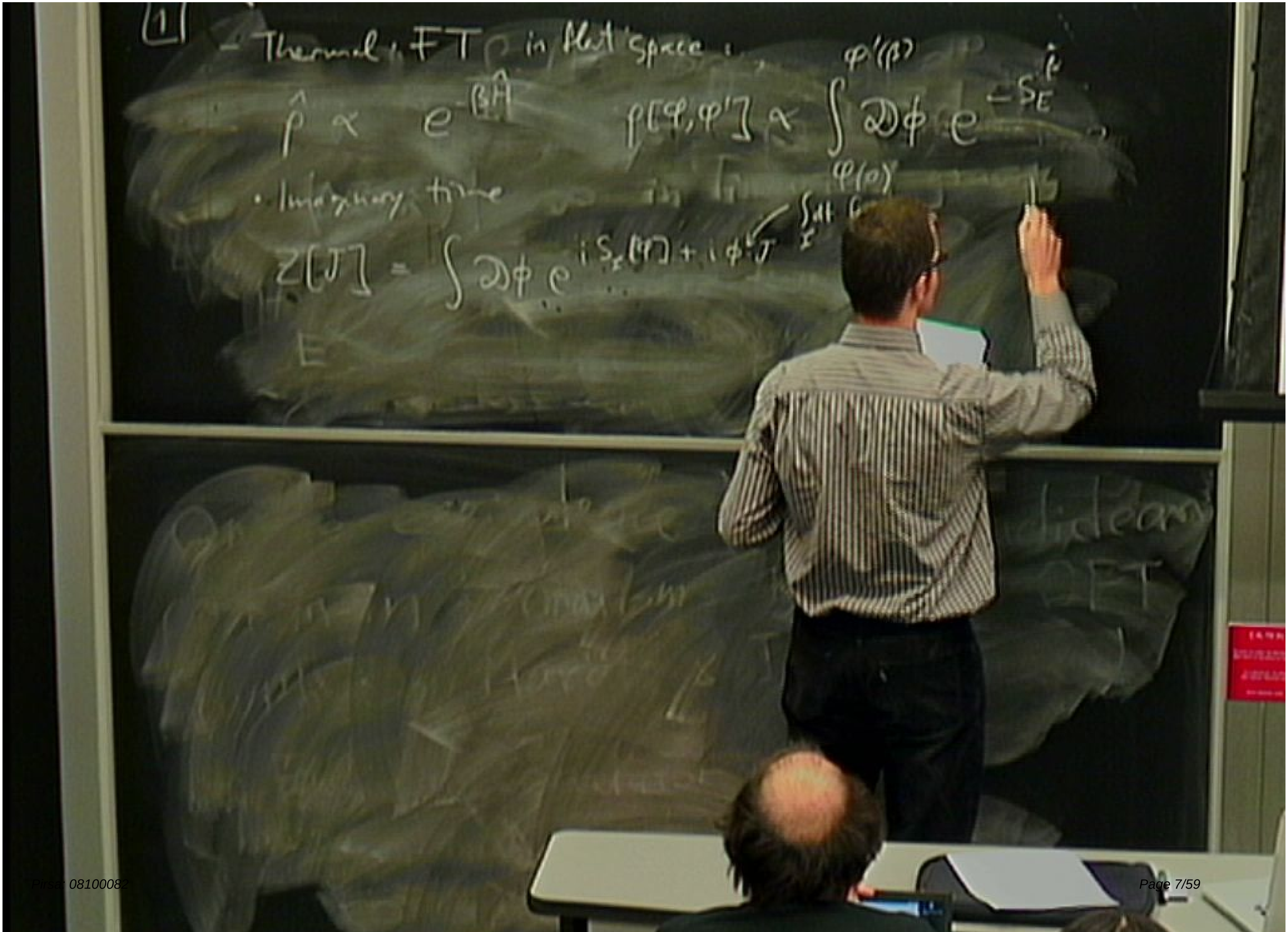
1
– Thermal QFT ρ in flat space
ρ̂ ∝ e^{-βĤ}
between Euclidean
QFT

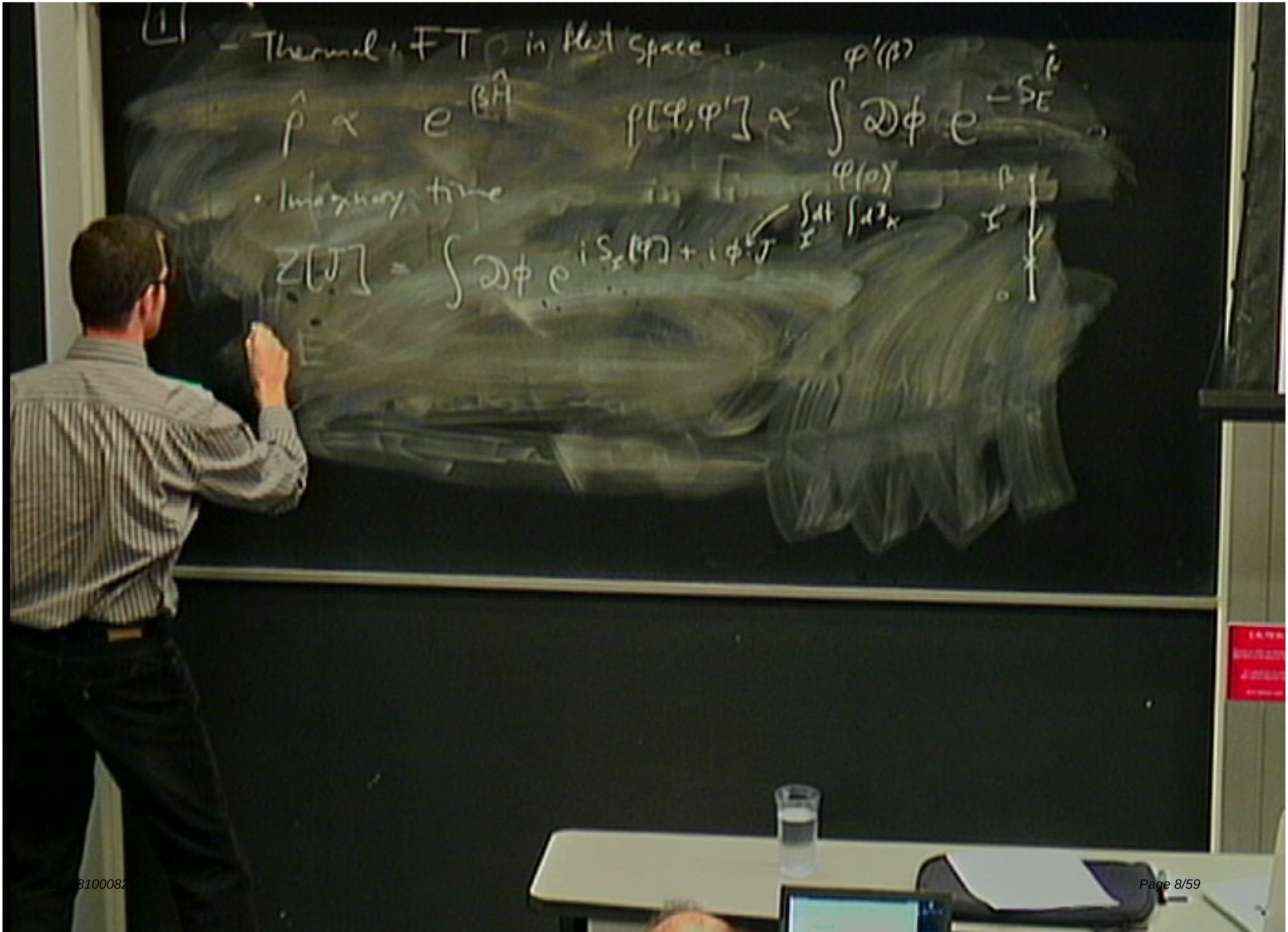

1
- Thermal FT in Hilb' space
Euclidean
QFT

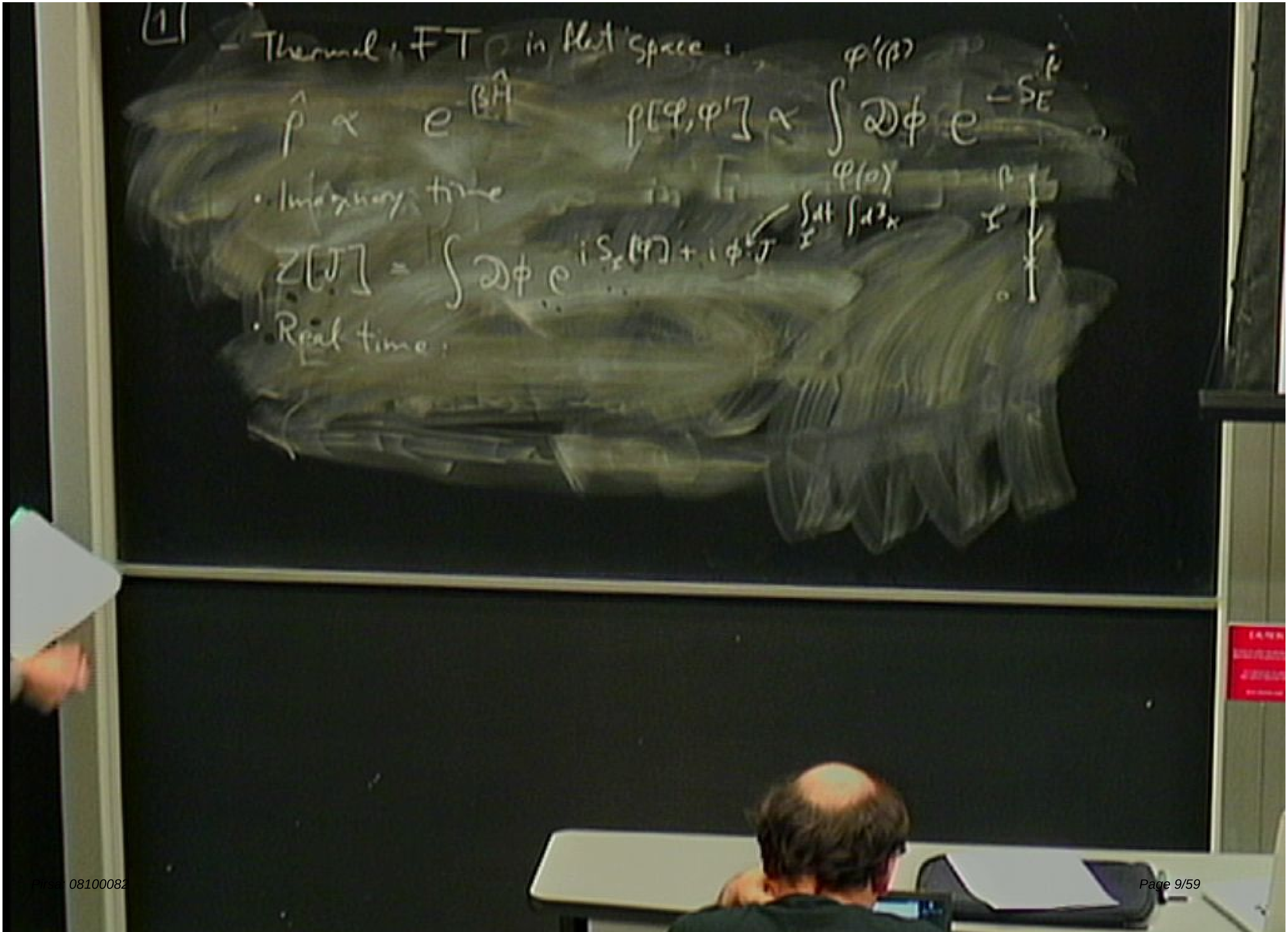


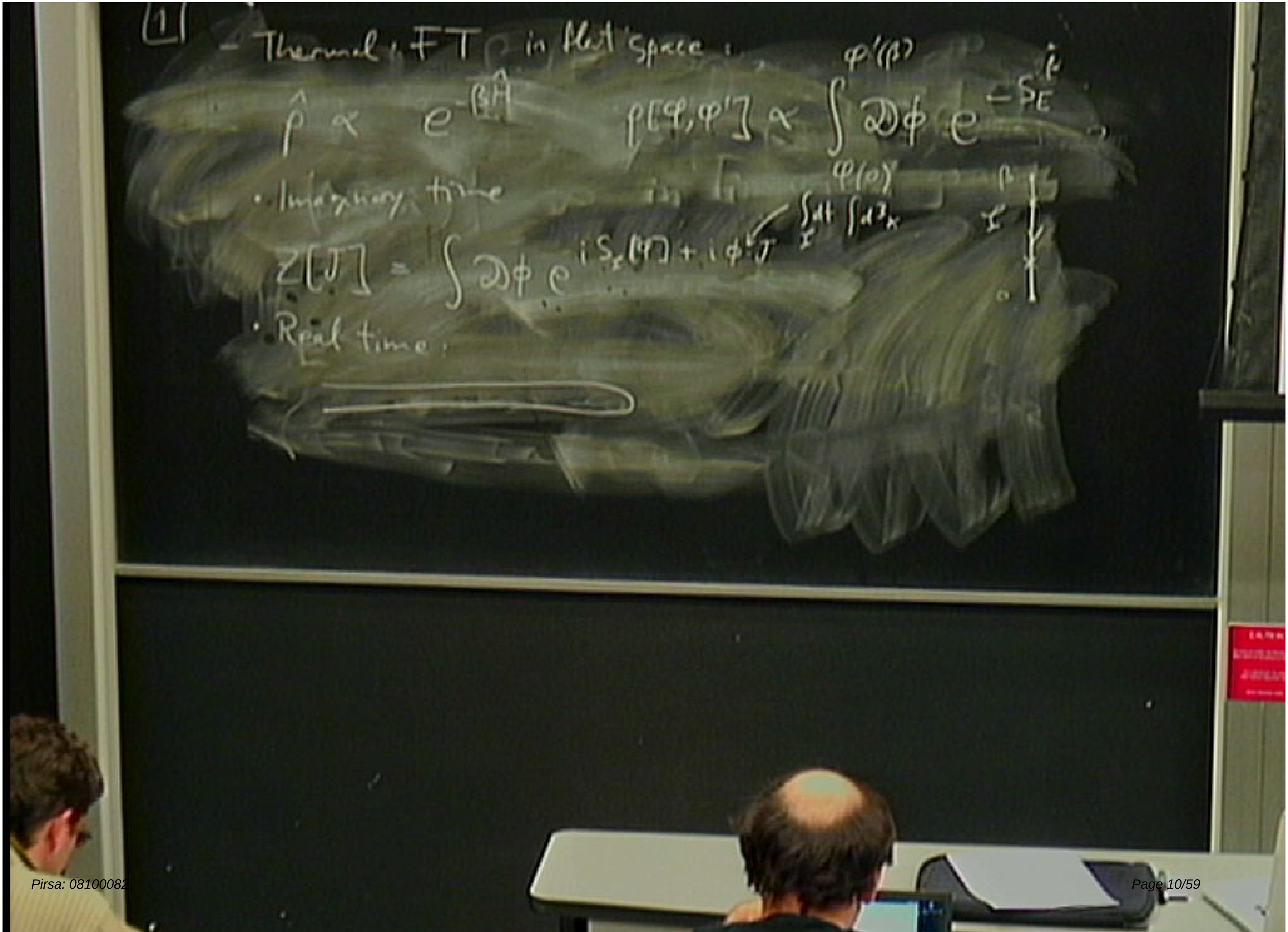
1
Thermal FT in flat space
$\hat{\rho} \propto e^{-\beta \hat{H}}$
$\rho[\varphi, \varphi'] \propto \int_{\varphi}^{\varphi'} \mathcal{D}\phi \, e^{-S_E}$

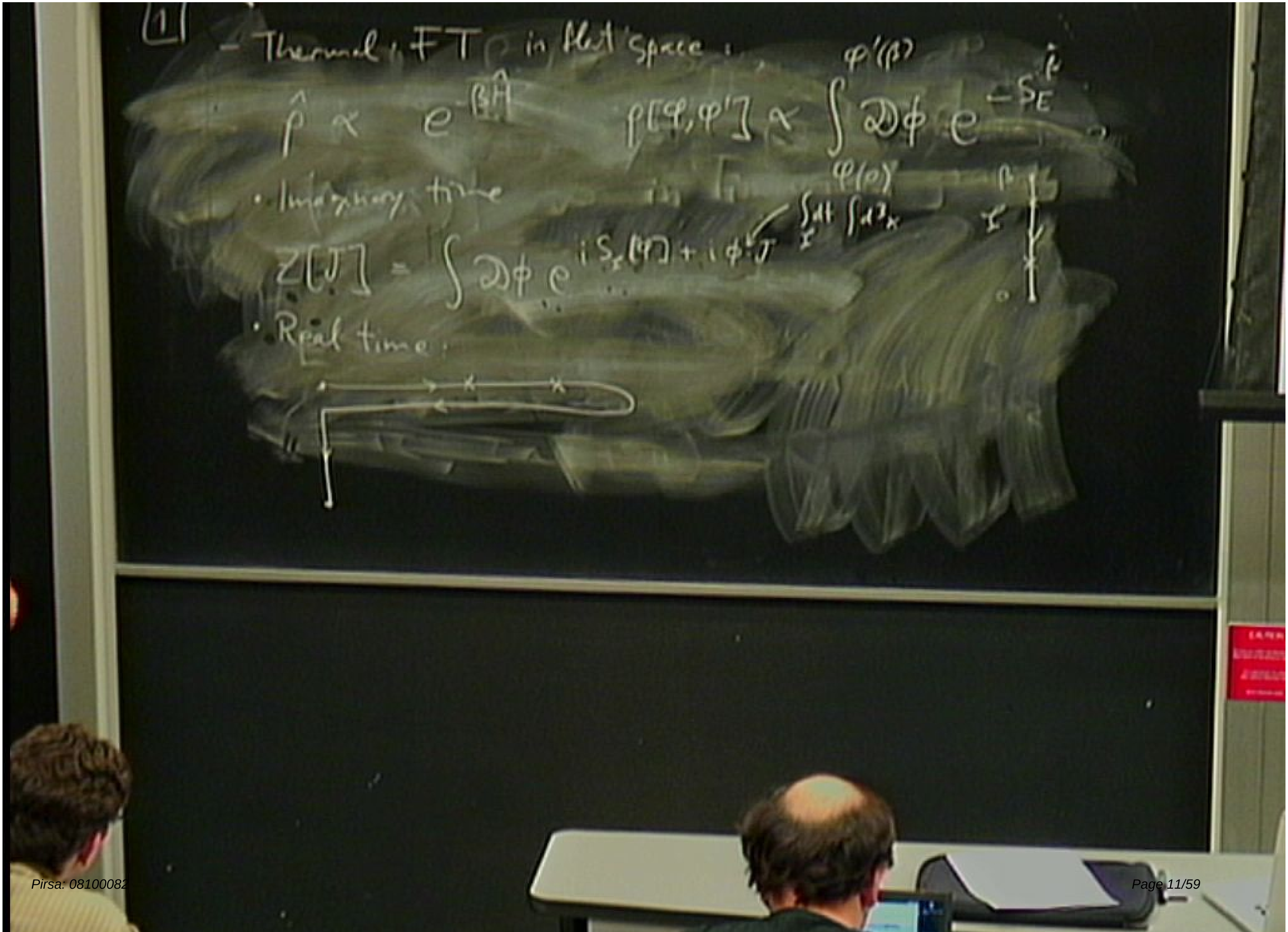

1
– Thermal FT in flat space:
$\hat{\rho} \propto e^{-\beta \hat{H}}$
$\rho[\varphi,\varphi'] \propto \int_{\varphi(0)}^{\varphi'(\beta)} \mathcal{D}\phi \, e^{-S_E}$
· Imaginary time


Thermal FT ρ in flat space
Imaginary time


Thermal FT in flat space
Imaginary time


Thermal
Imaginary time
Real time


1
– Thermal FT in flat space
• Imaginary time
• Real time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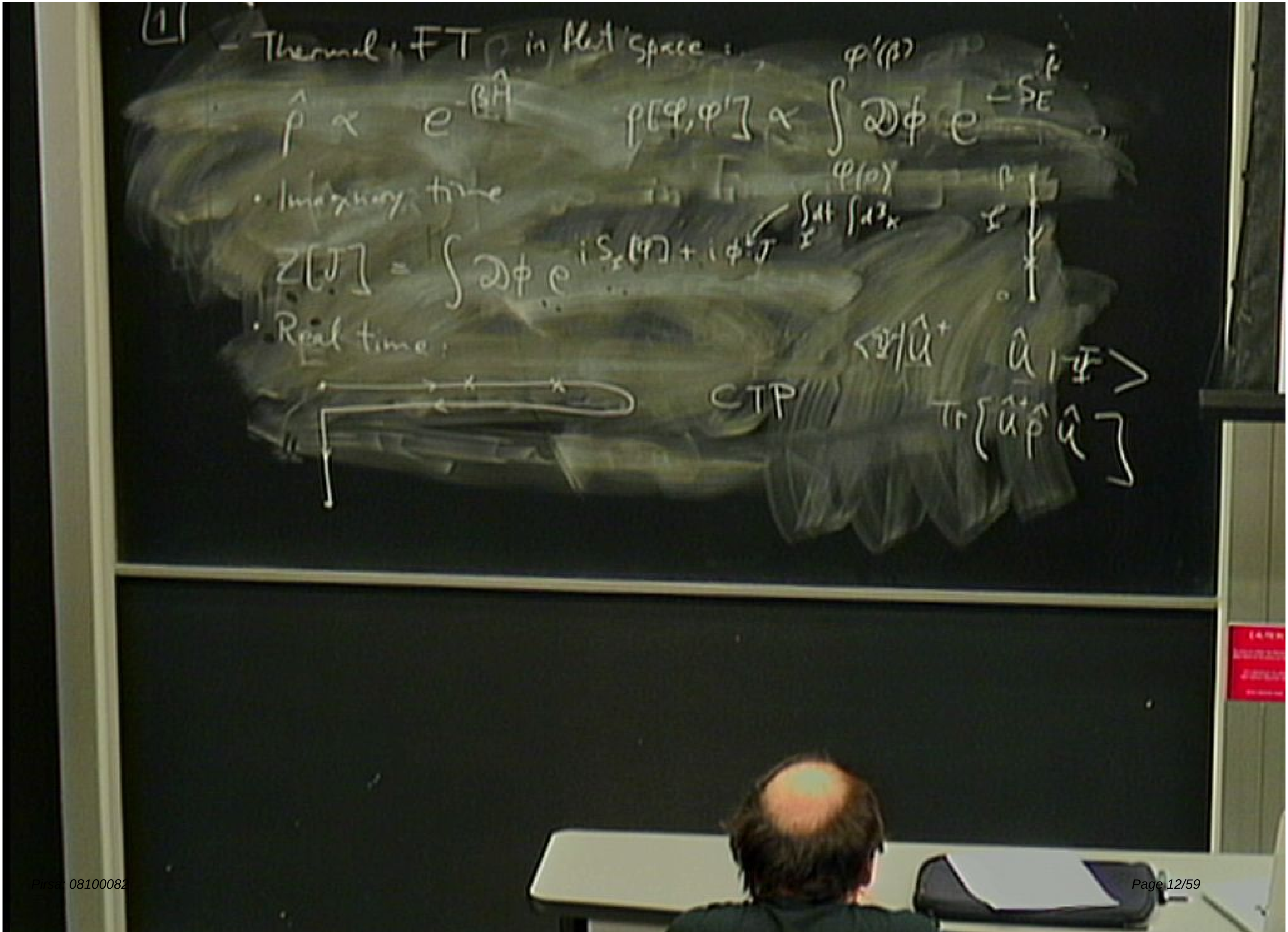
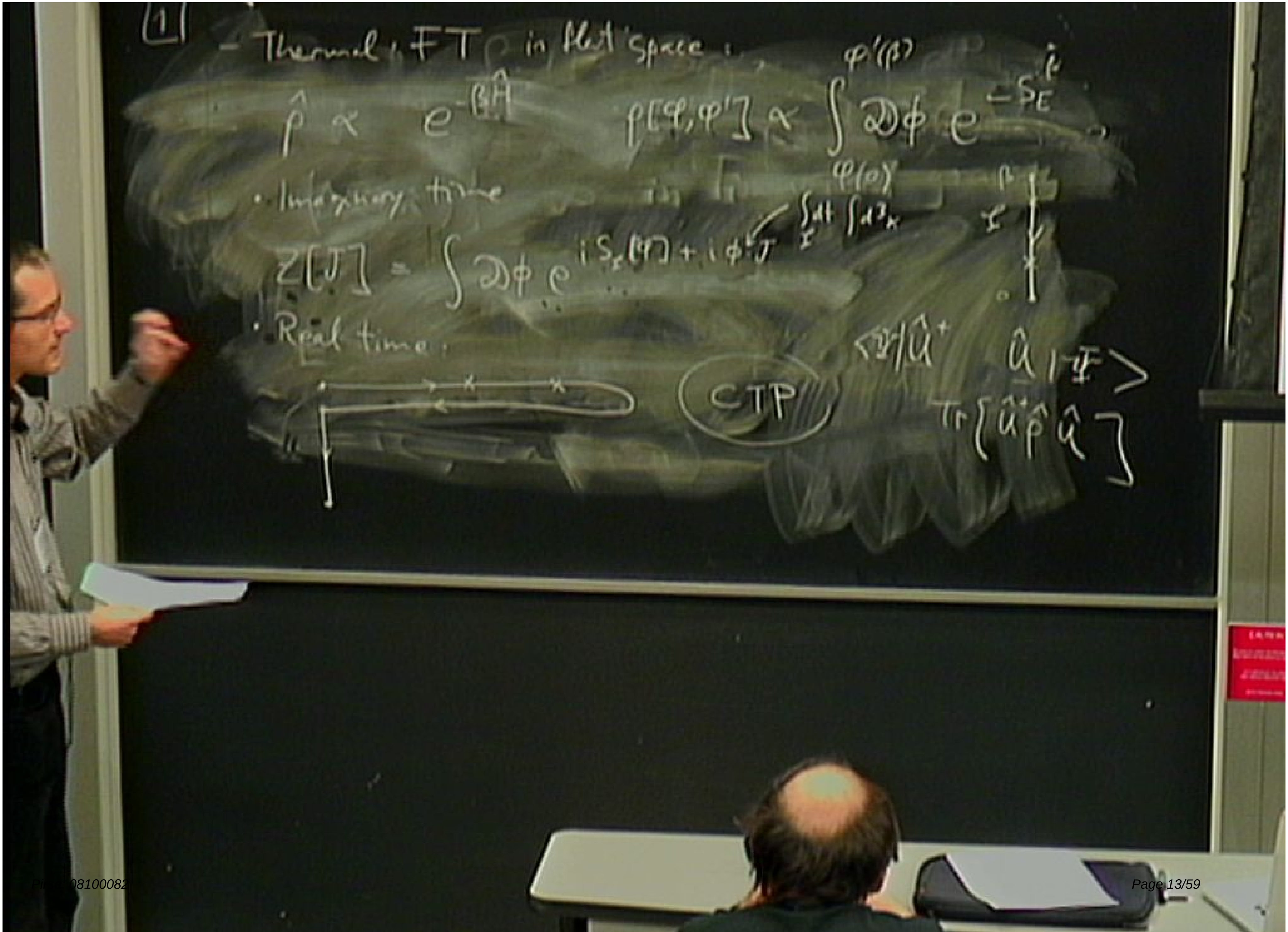
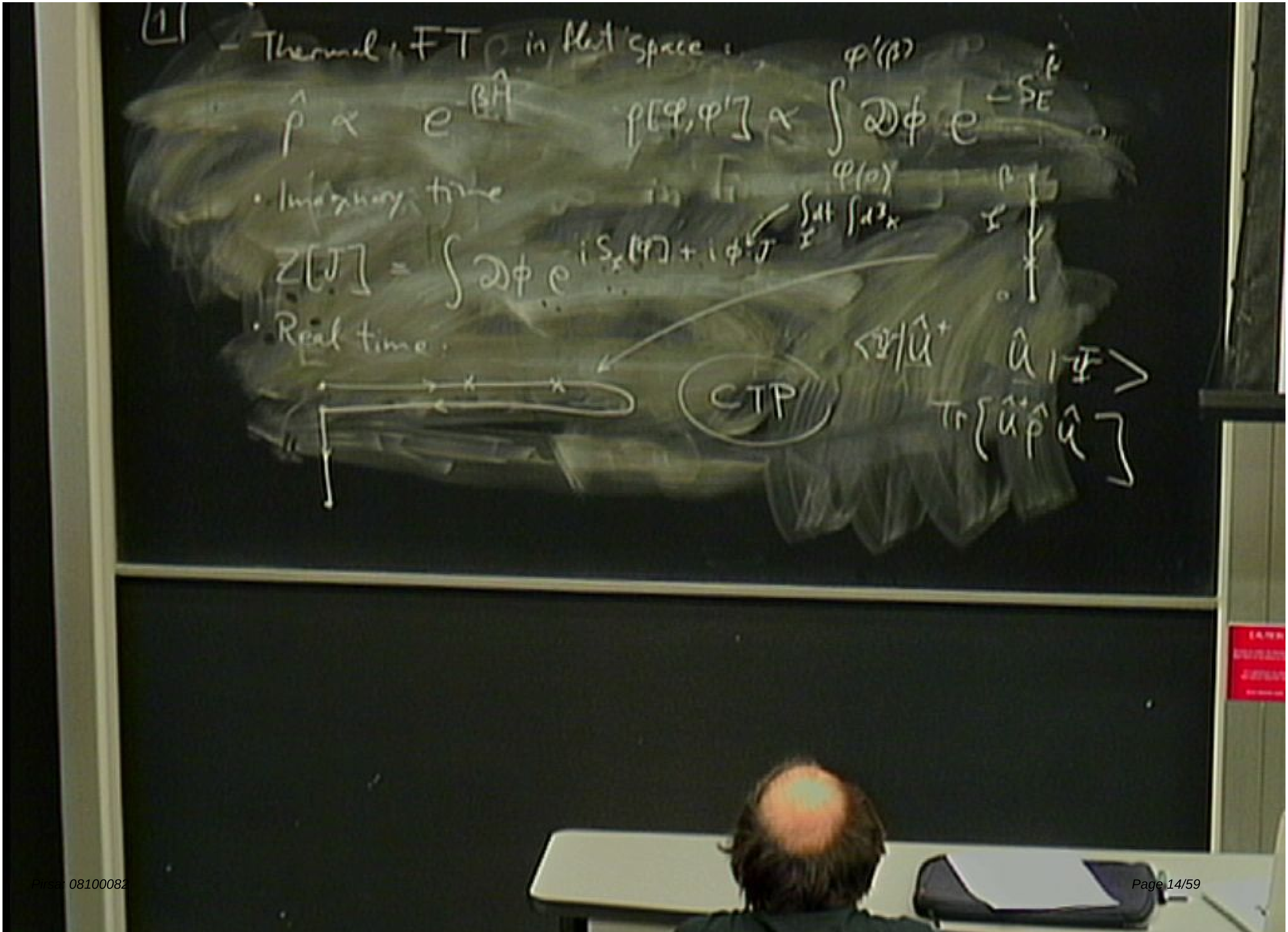
Thermal FT in flat space
Imaginary time
Real time

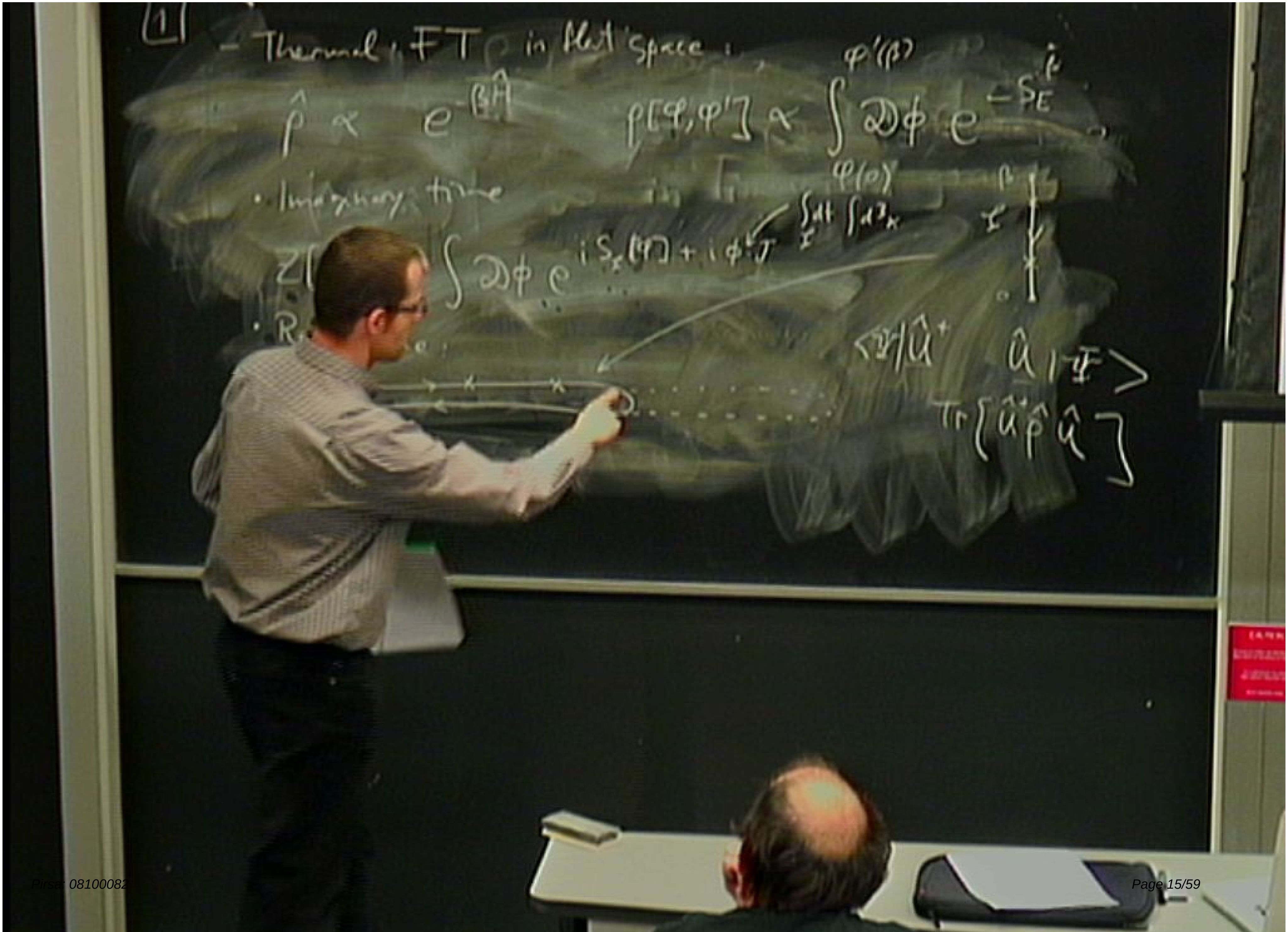


1
- Thermal FT in flat space:
• Imaginary time
• Real time:
CTP

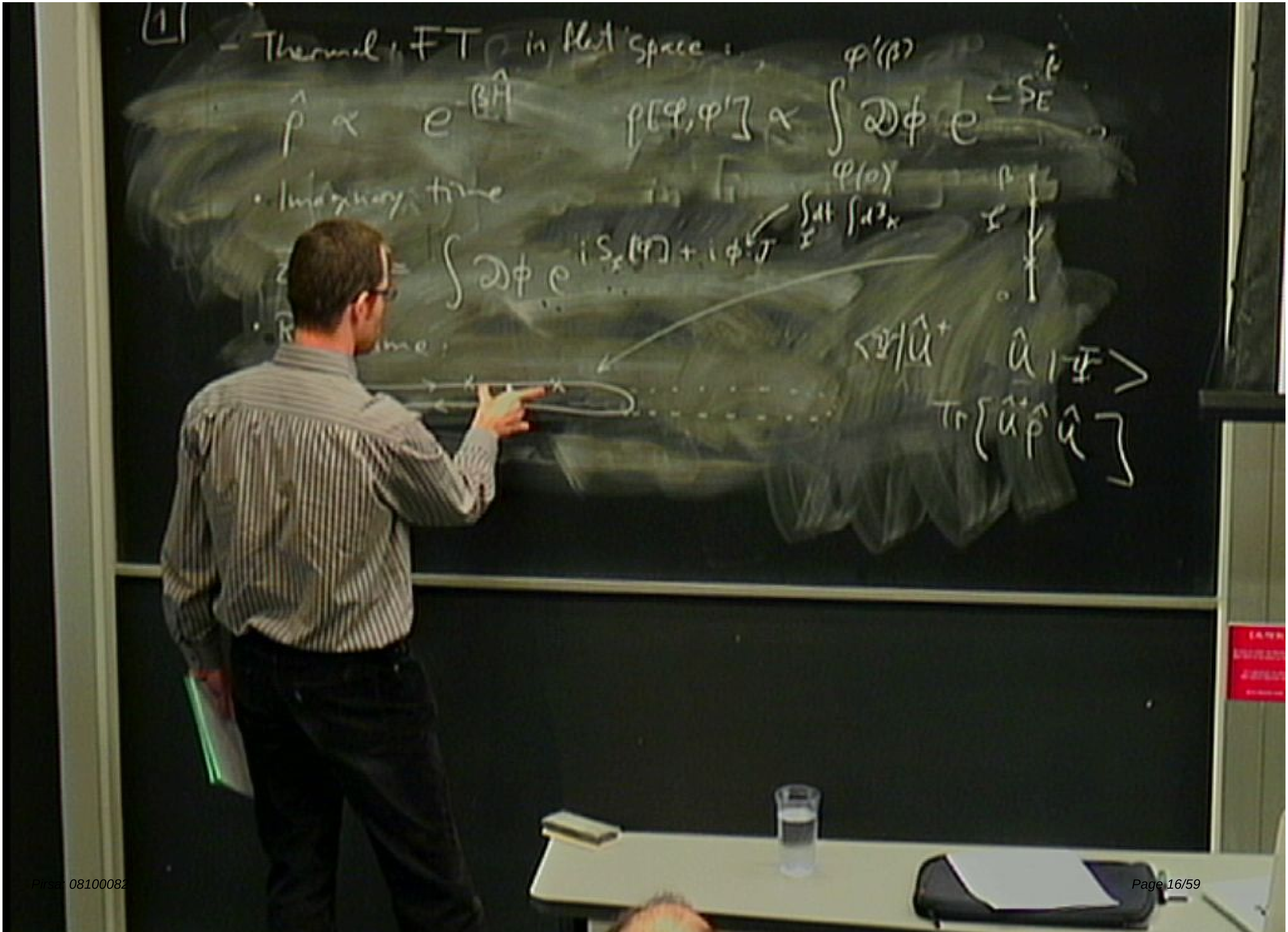


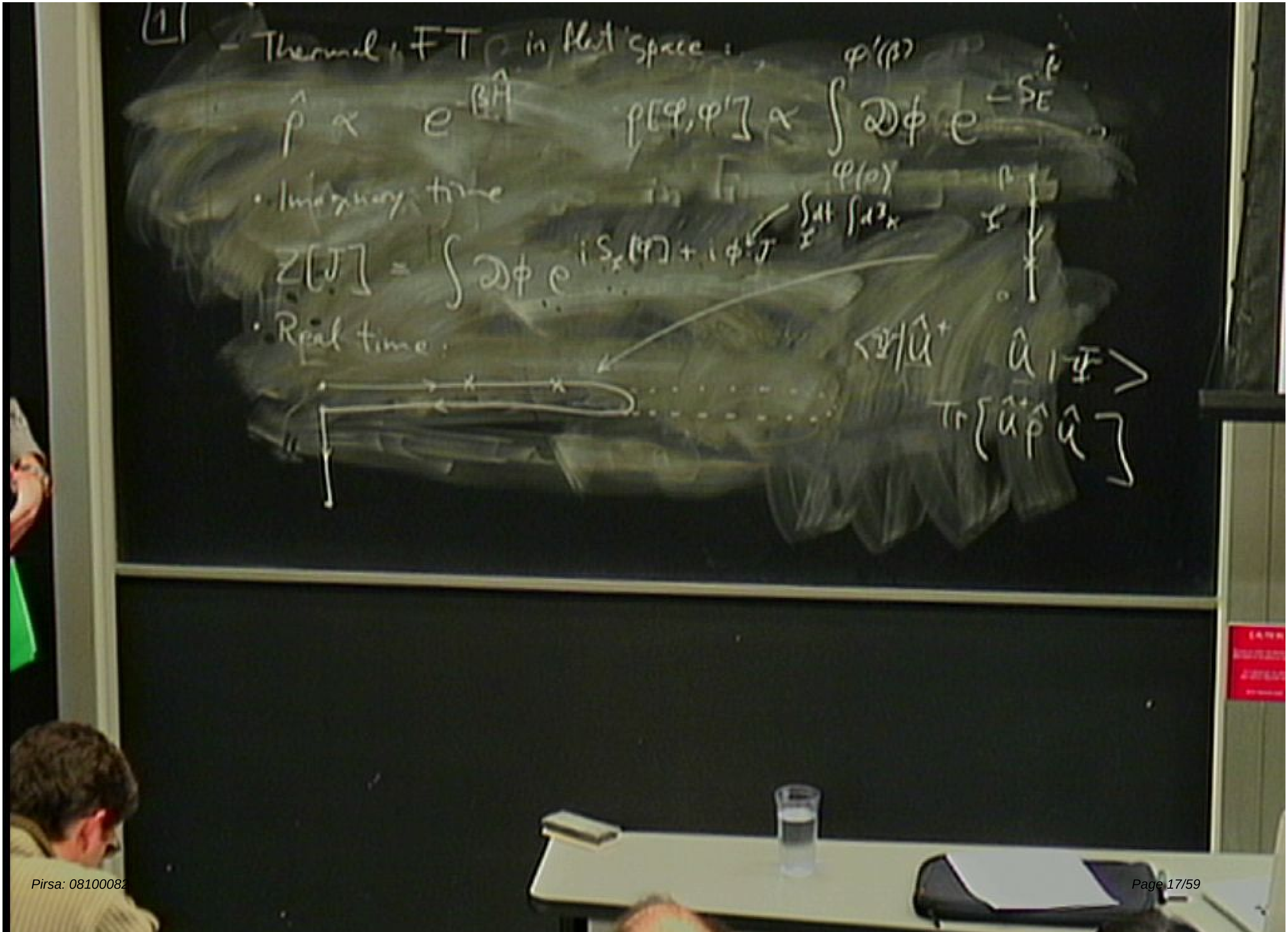
- Thermal FT in flat space
• Imaginary time
• Real time
CTP


Thermal FT in flat space
Imaginary time
Real time
CTP

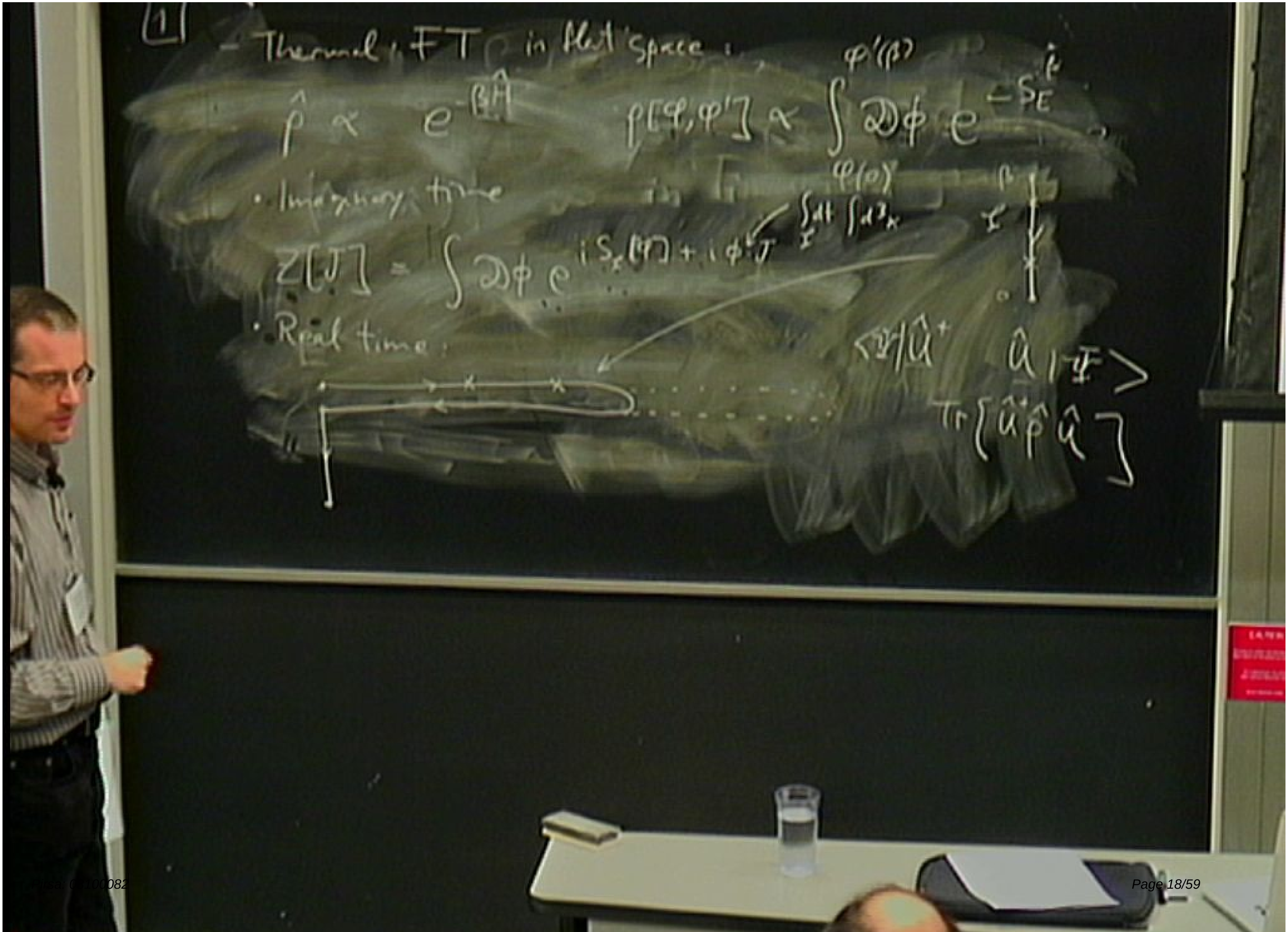


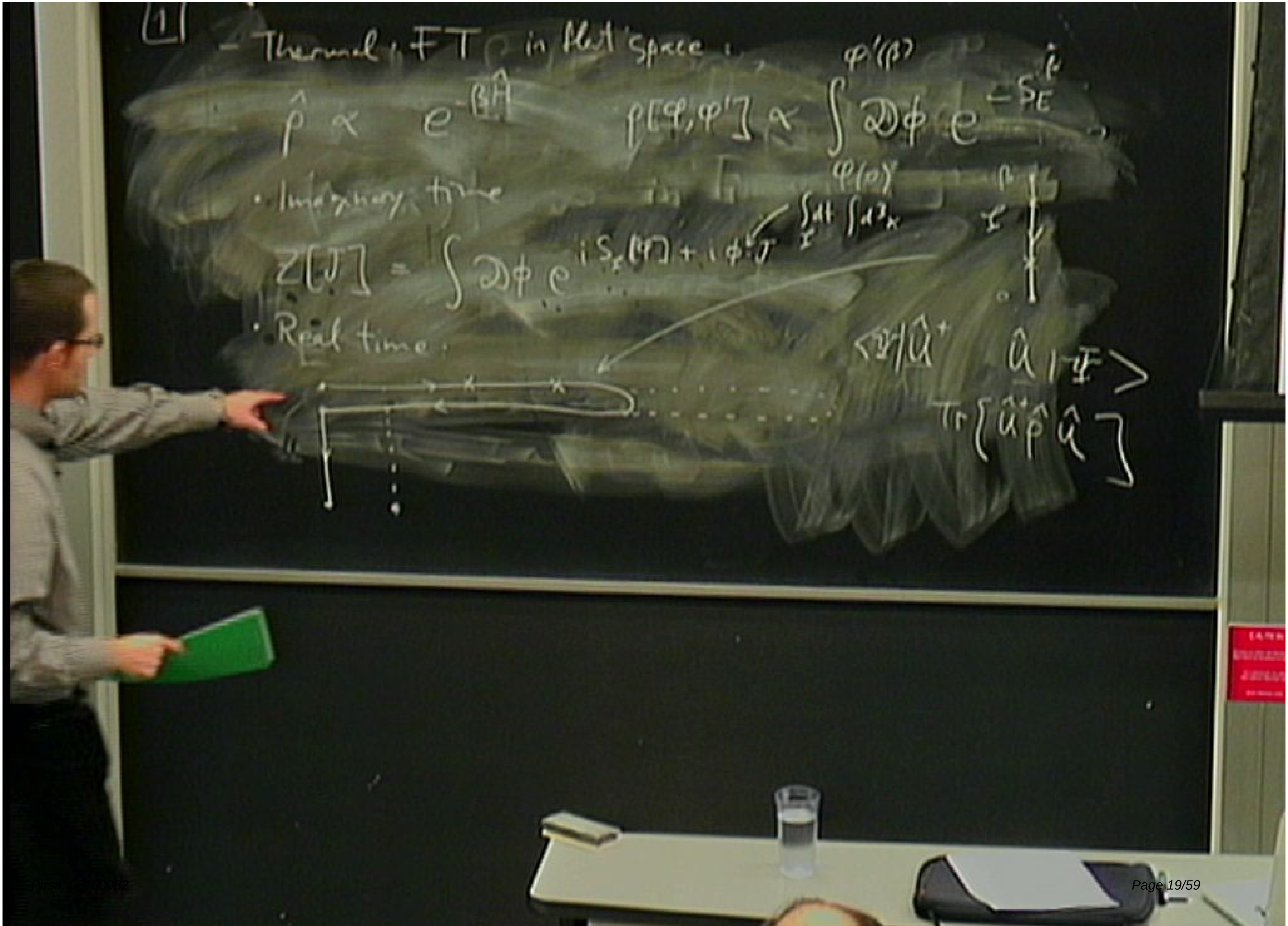
Thermal FT
Imaginary time

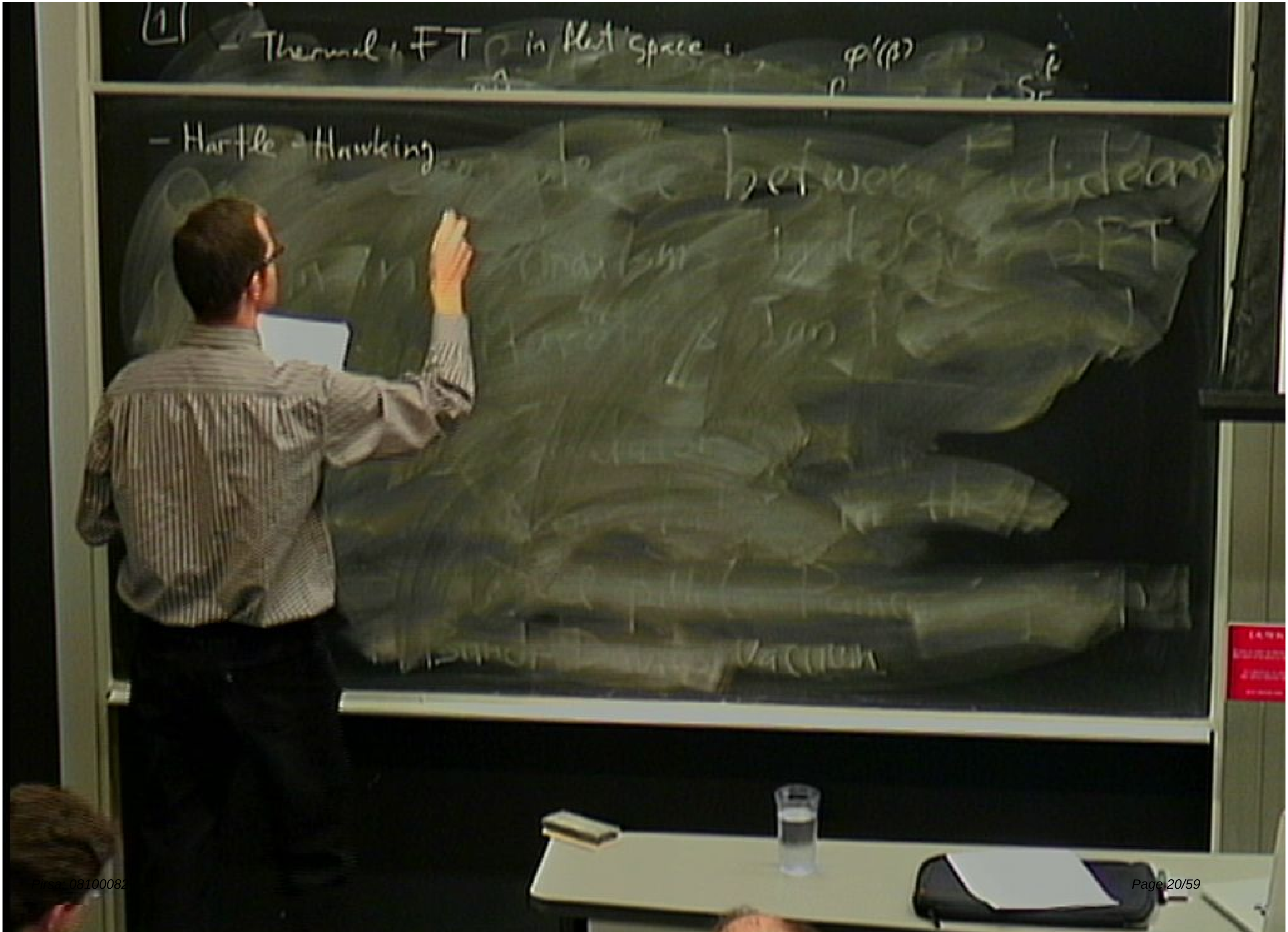

1
Thermal FT in flat space
Imaginary time
Tr[Û†ρ̂Û]

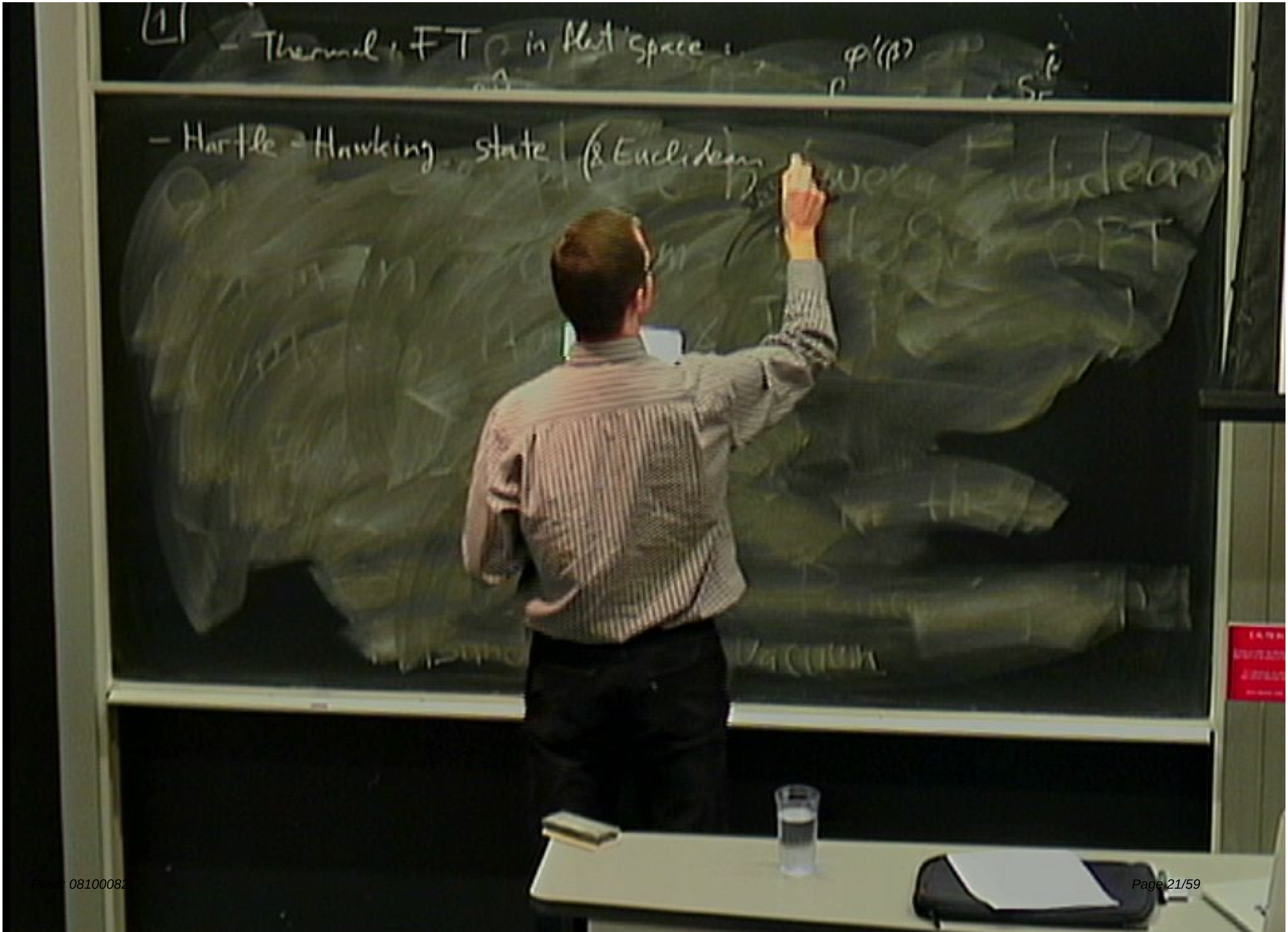


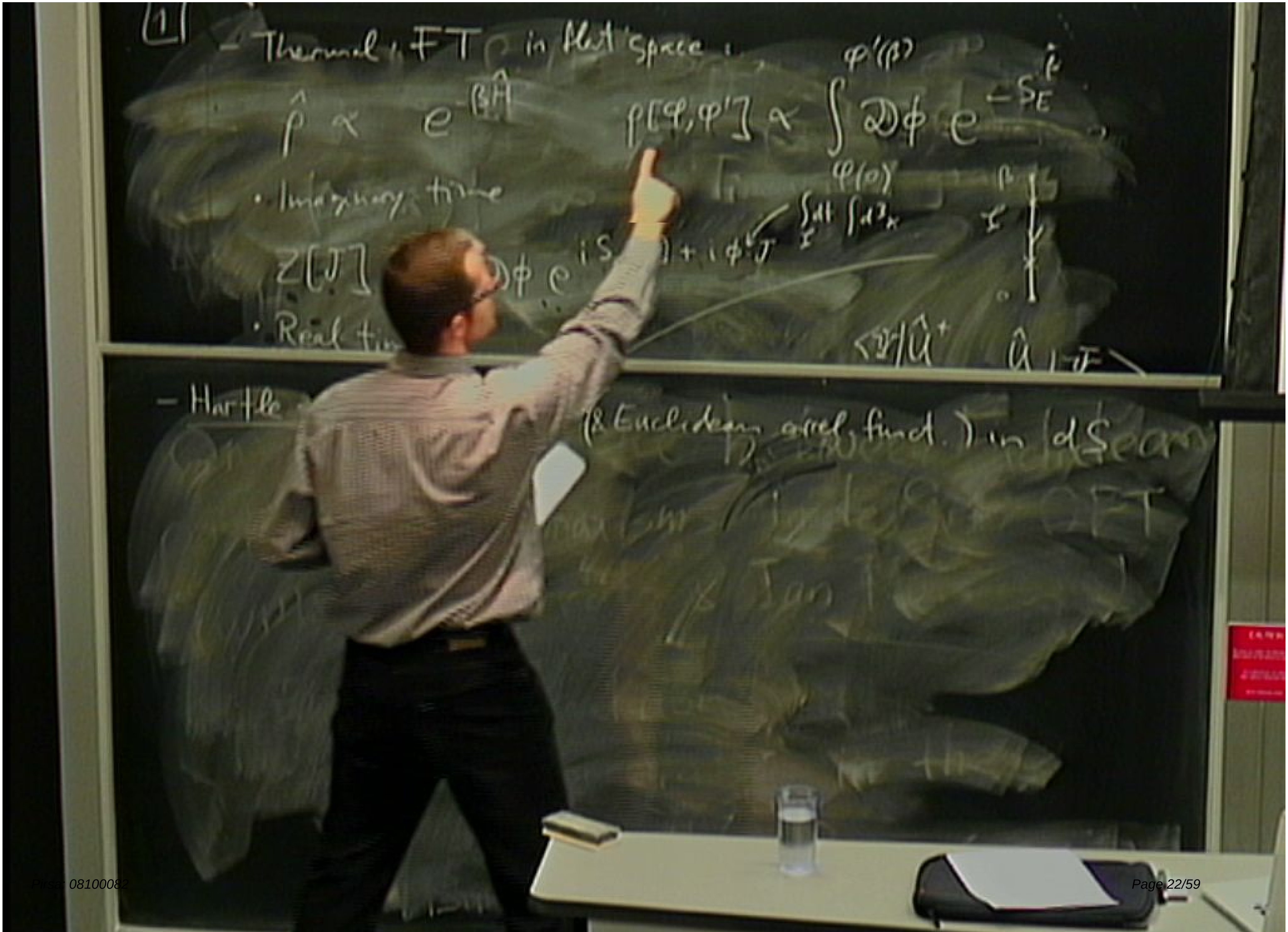
Thermal QFT in flat space
Imaginary time
Real time

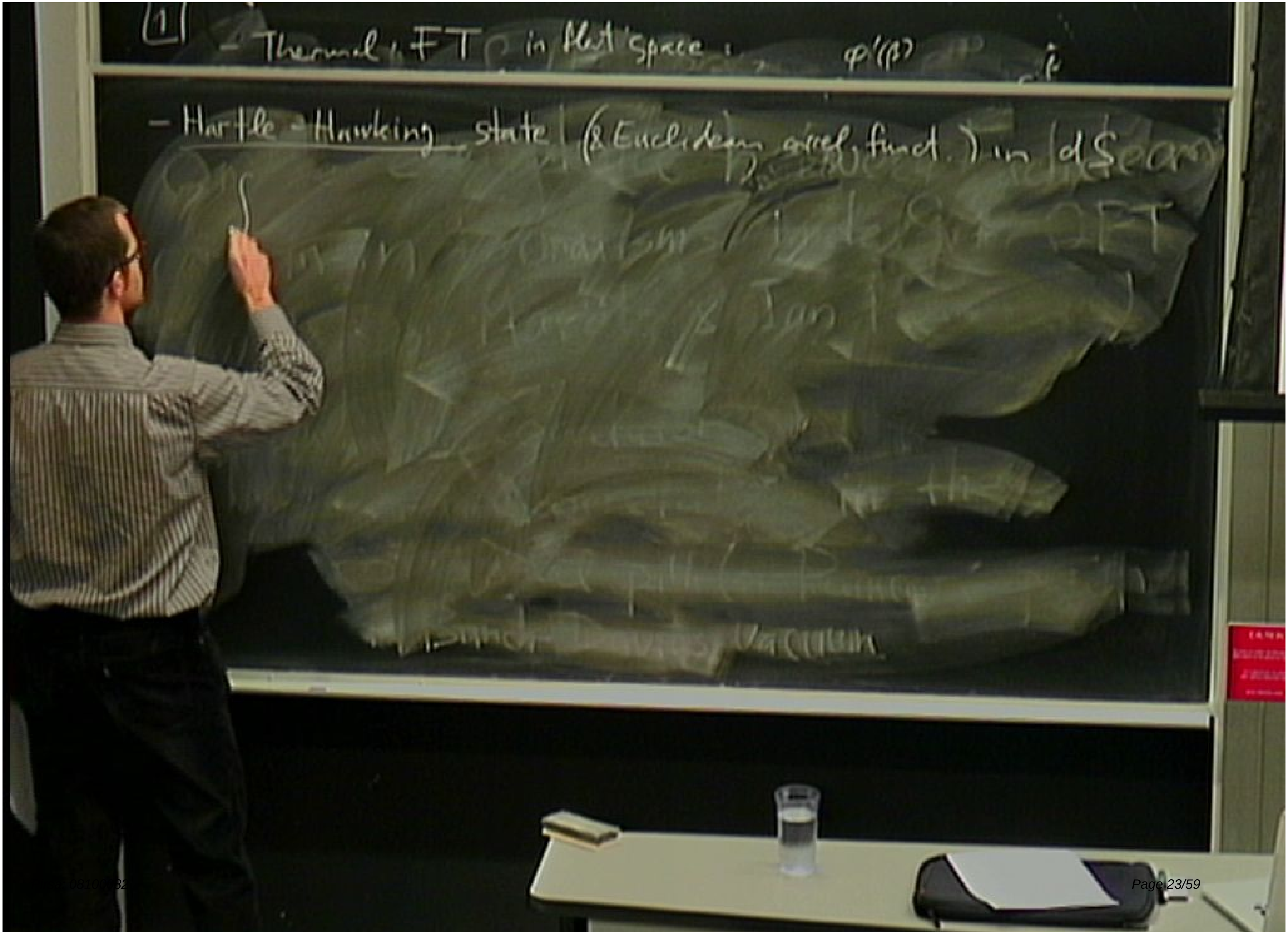

Thermal FT in flat space
Imaginary time
Real time

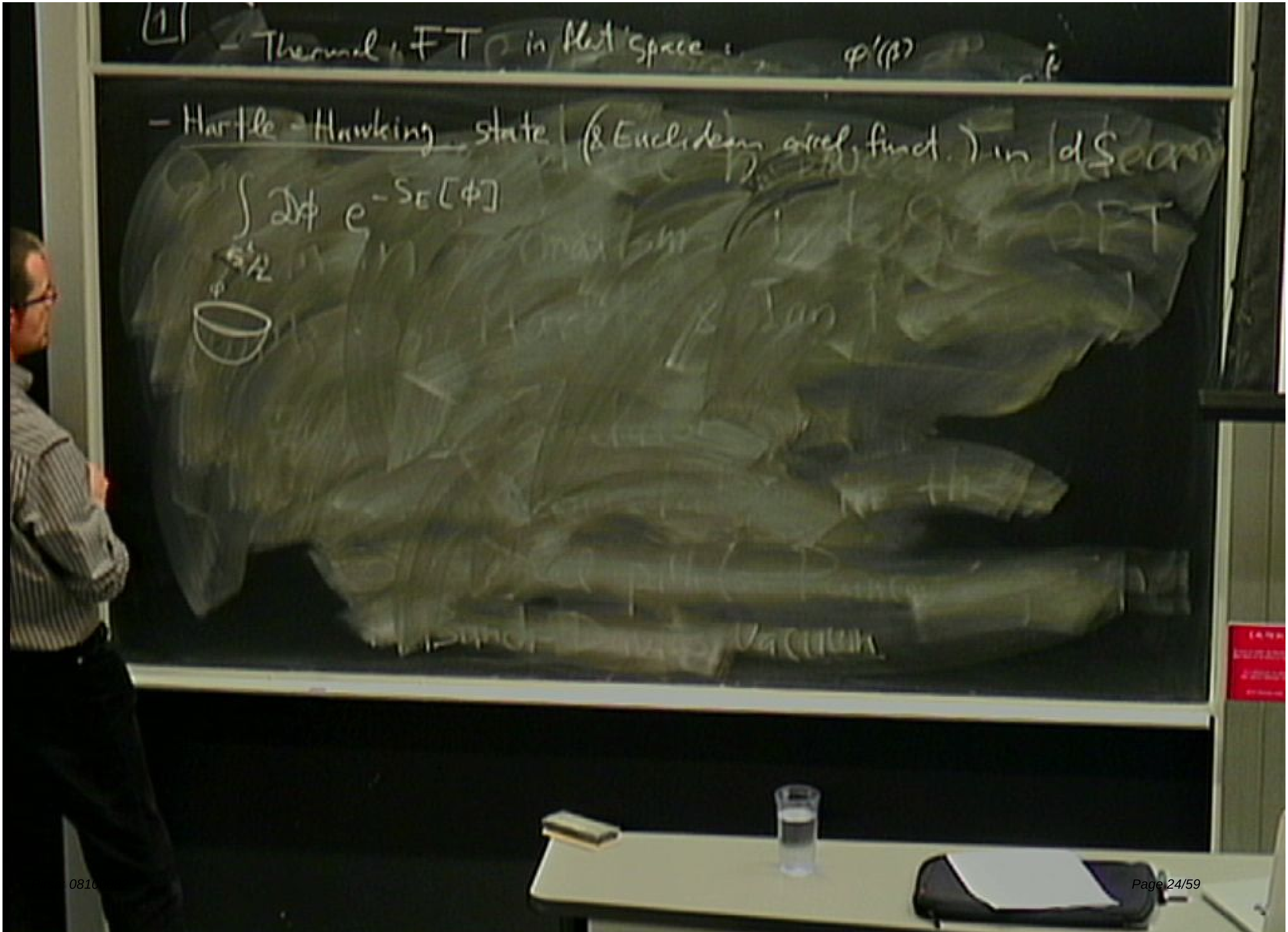

Thermal FT in flat space
Imaginary time
Real time

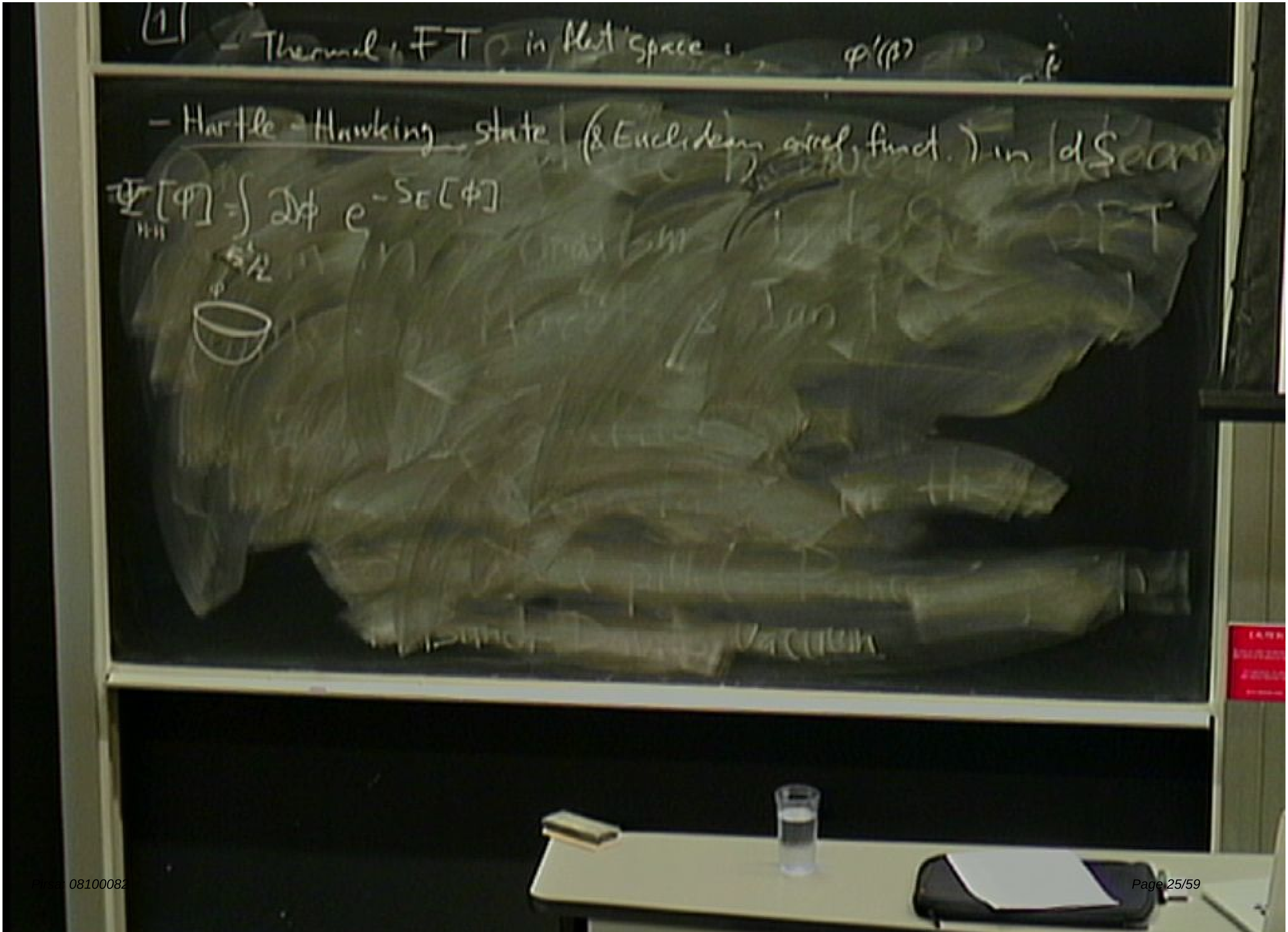

– Thermal FT in flat space
– Hartle – Hawking

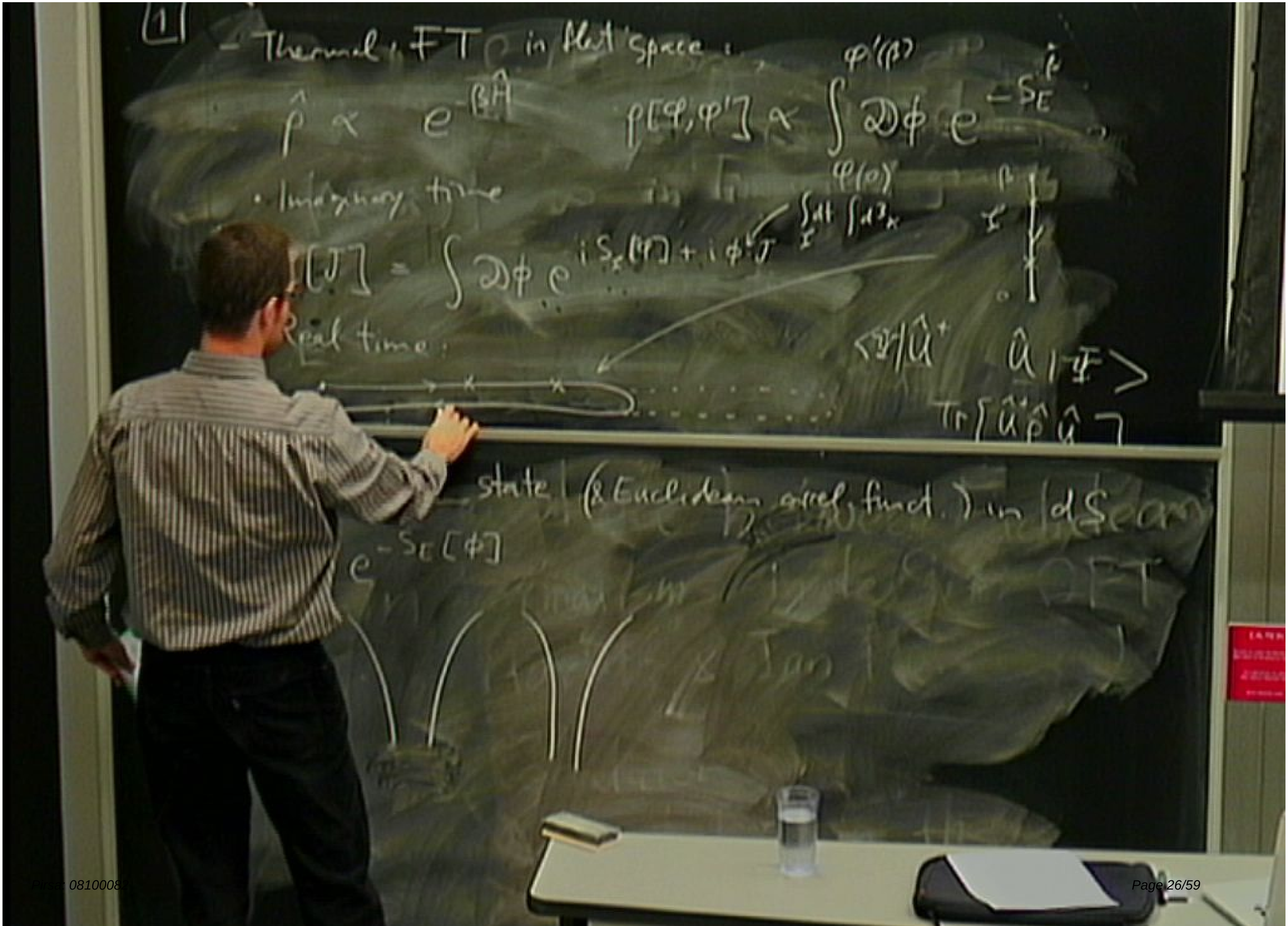

- Thermal QFT in flat space
- Hartle-Hawking state (& Euclidean

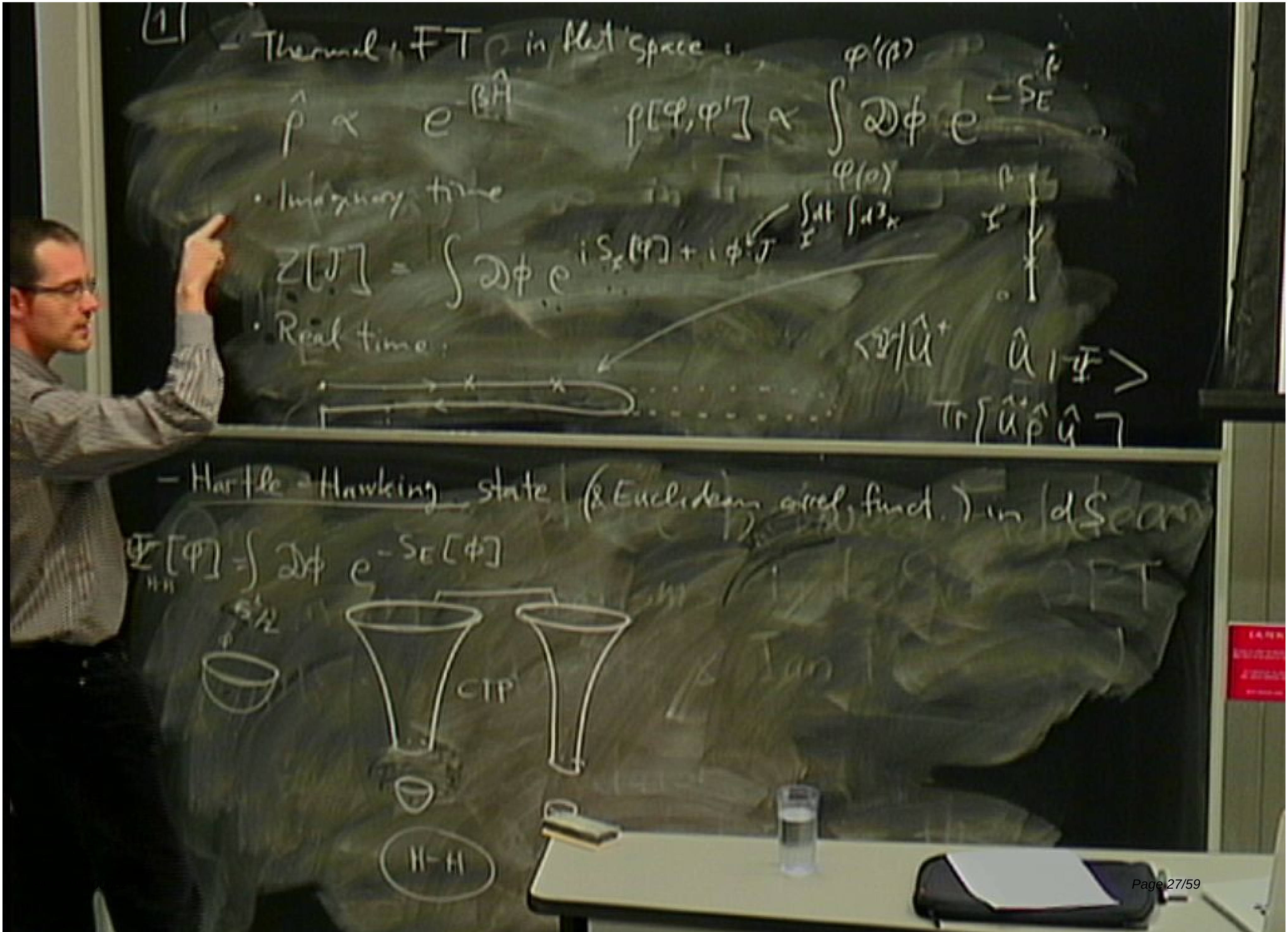

– Thermal QFT in flat space
· Imaginary time
· Real time
– Hartle


1
– Thermal FT in flat space
– Hartle-Hawking state (& Euclidean corrl. funct.) in dS


1
– Thermal QFT in flat space : φ'(β)
– Hartle-Hawking state (& Euclidean correl. funct.) in dS
∫ Dφ e^{-S_E[φ]}
vacuum


- Thermal FT in flat space
- Hartle - Hawking state (& Euclidean correl. funct.) in dS
Ψ[φ] = ∫ Dφ e^{-S_E[φ]}


Thermal FT in flat space
Imaginary time
Real time
state (& Euclidean correl. funct.) in dS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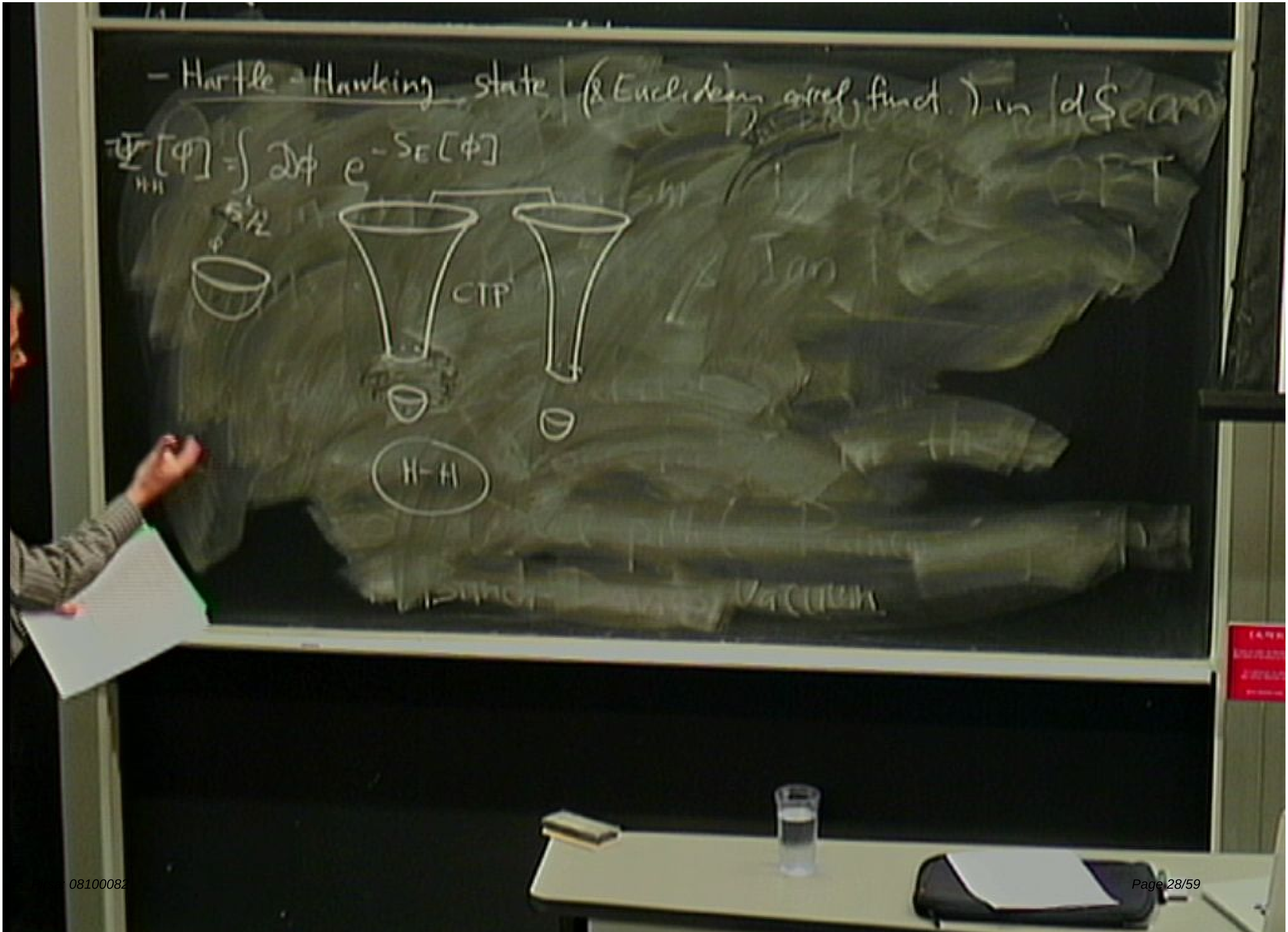
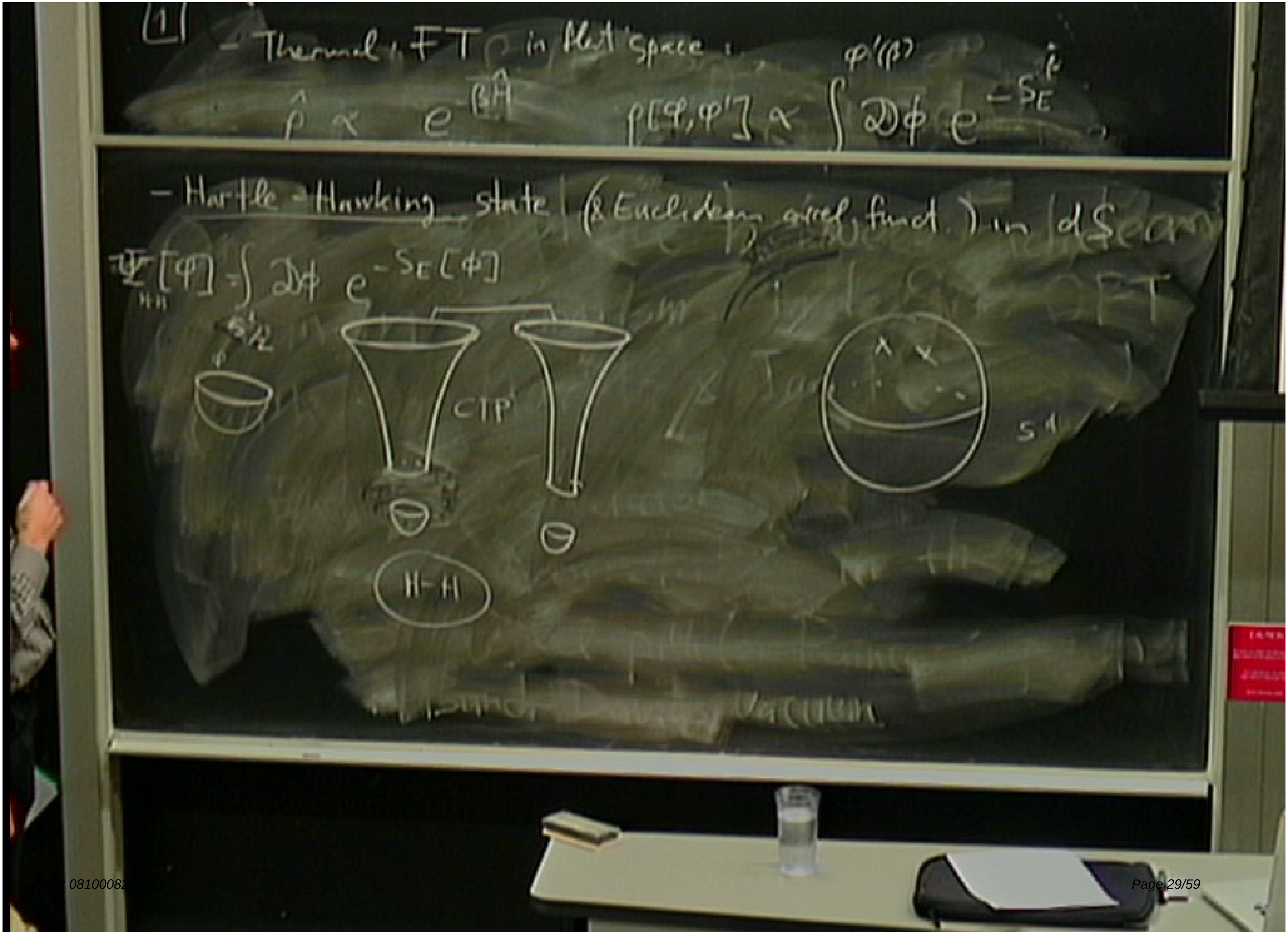
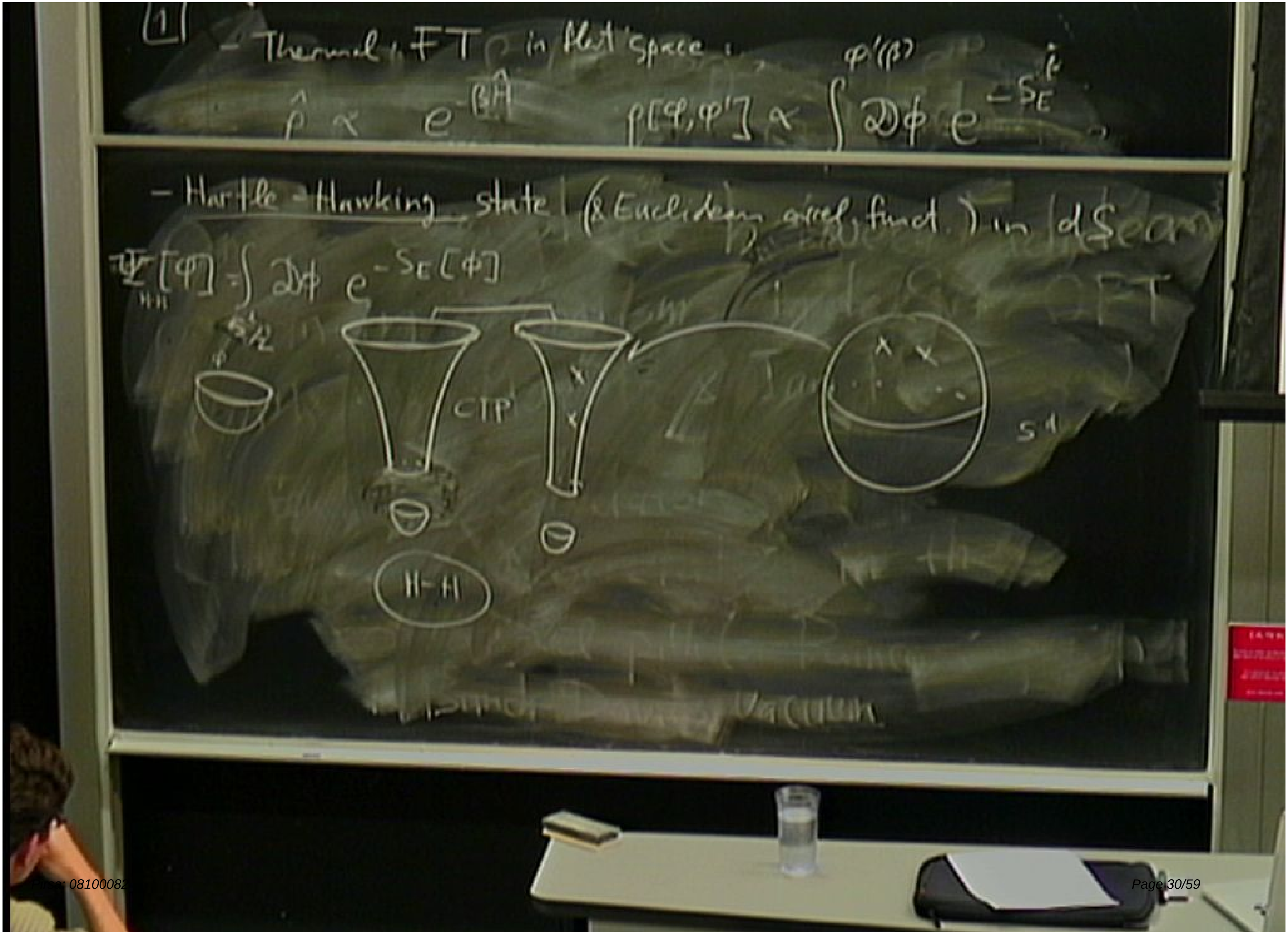
- Thermal FT in flat space
ρ̂ ∝ e^{-βĤ}
ρ[φ, φ'] ∝ ∫ 𝒟φ e^{-S_E}
• Imaginary time
Z[J] = ∫ 𝒟φ e^{iS_E[φ] + iφJ}
• Real time
Tr [Û⁺ ρ̂ Û]
- Hartle - Hawking state (& Euclidean wavef. funct.) in dS
Ψ_{HH}[φ] = ∫ 𝒟φ e^{-S_E[φ]}
CTP
H-H


– Hartle – Hawking state
CTP
H-H



- Thermal FT in flat space
$\hat{\rho} \propto e^{-\beta \hat{H}}$
$\rho[\varphi, \varphi'] \propto \int \mathcal{D}\phi \, e^{-S_E}$
- Hartle - Hawking state (& Euclidean correl. funct.) in dS
$\Psi_{HH}[\varphi] = \int \mathcal{D}\phi \, e^{-S_E[\phi]}$
CTP
S 4
H-H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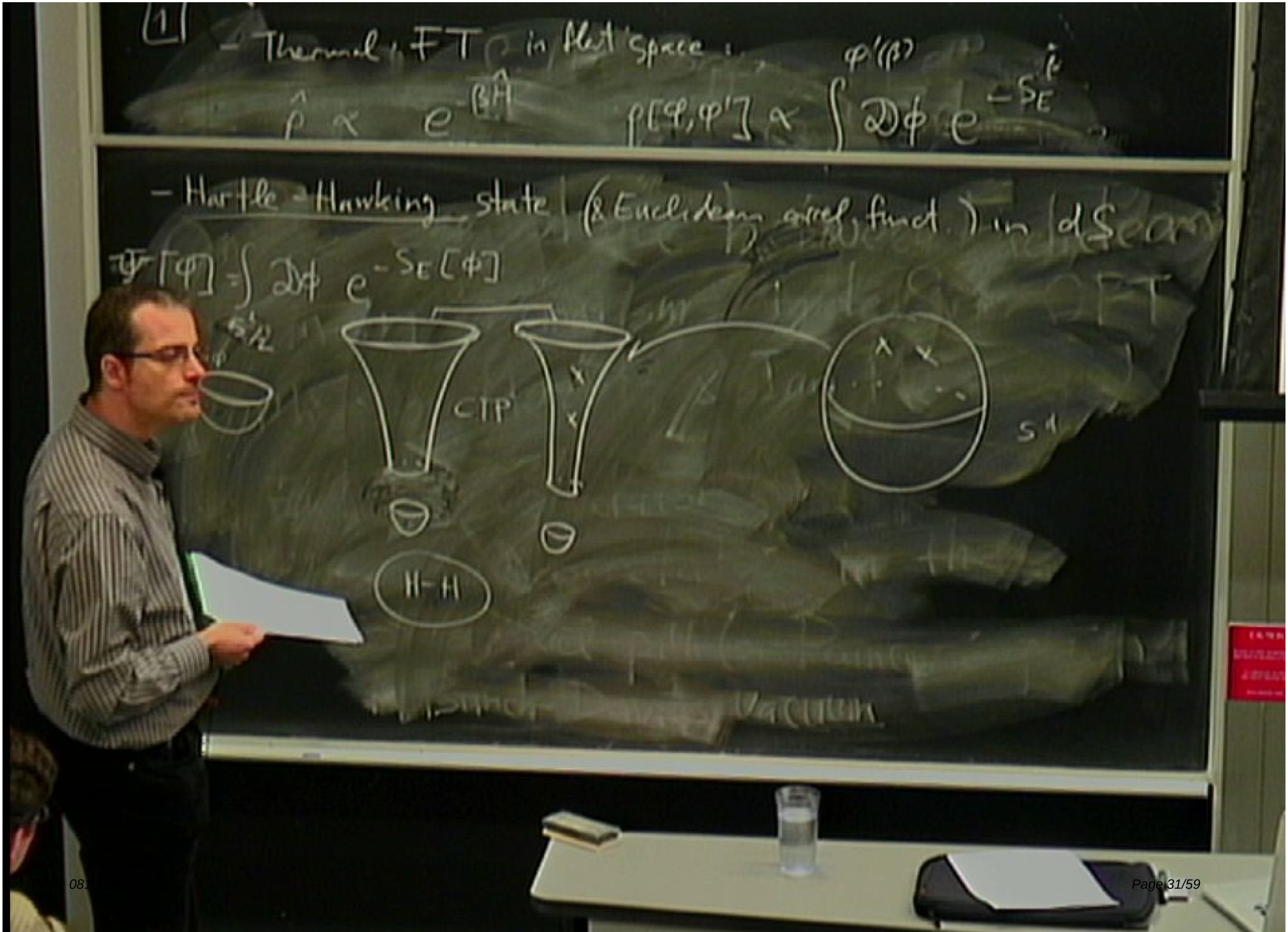
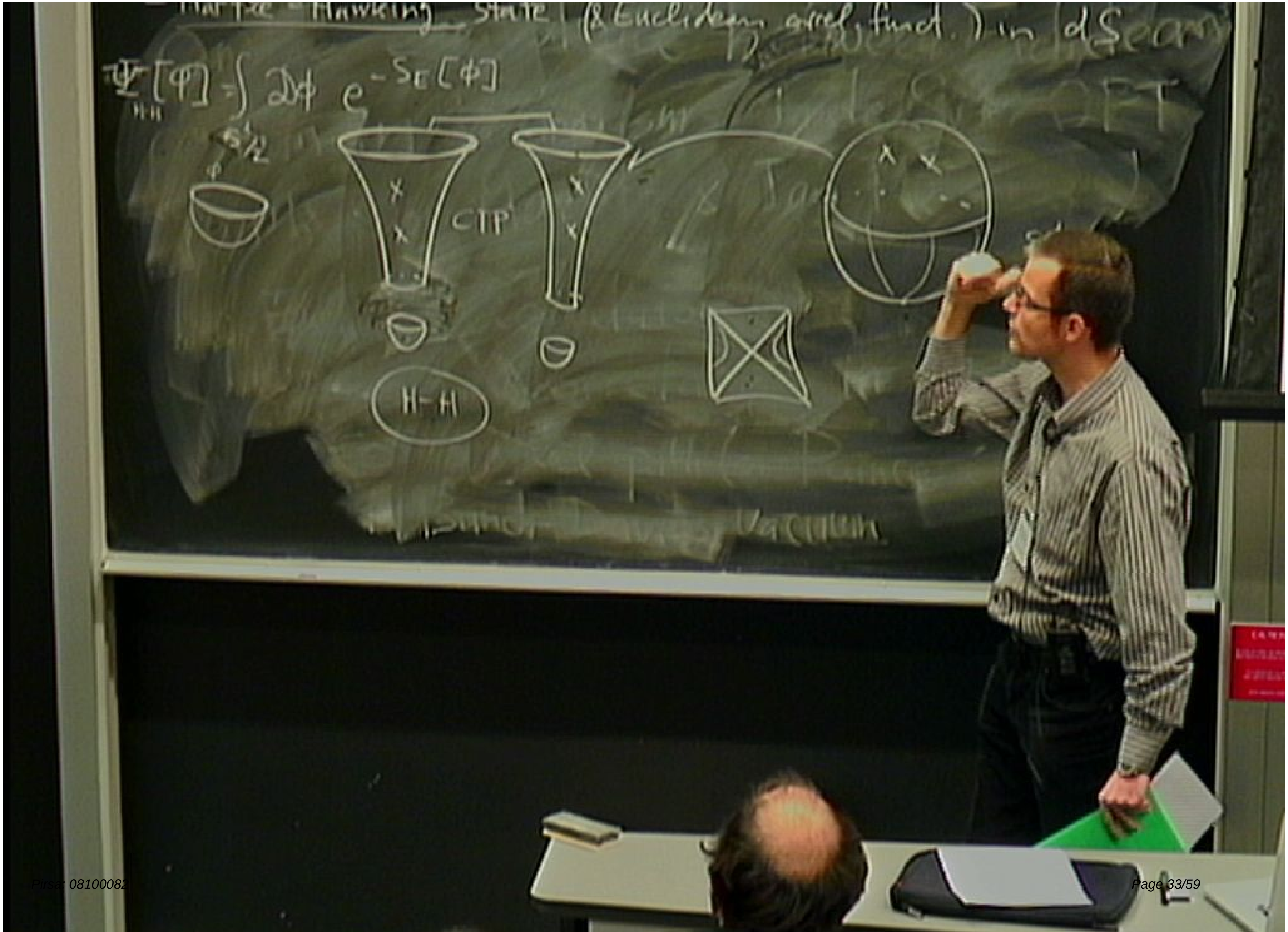
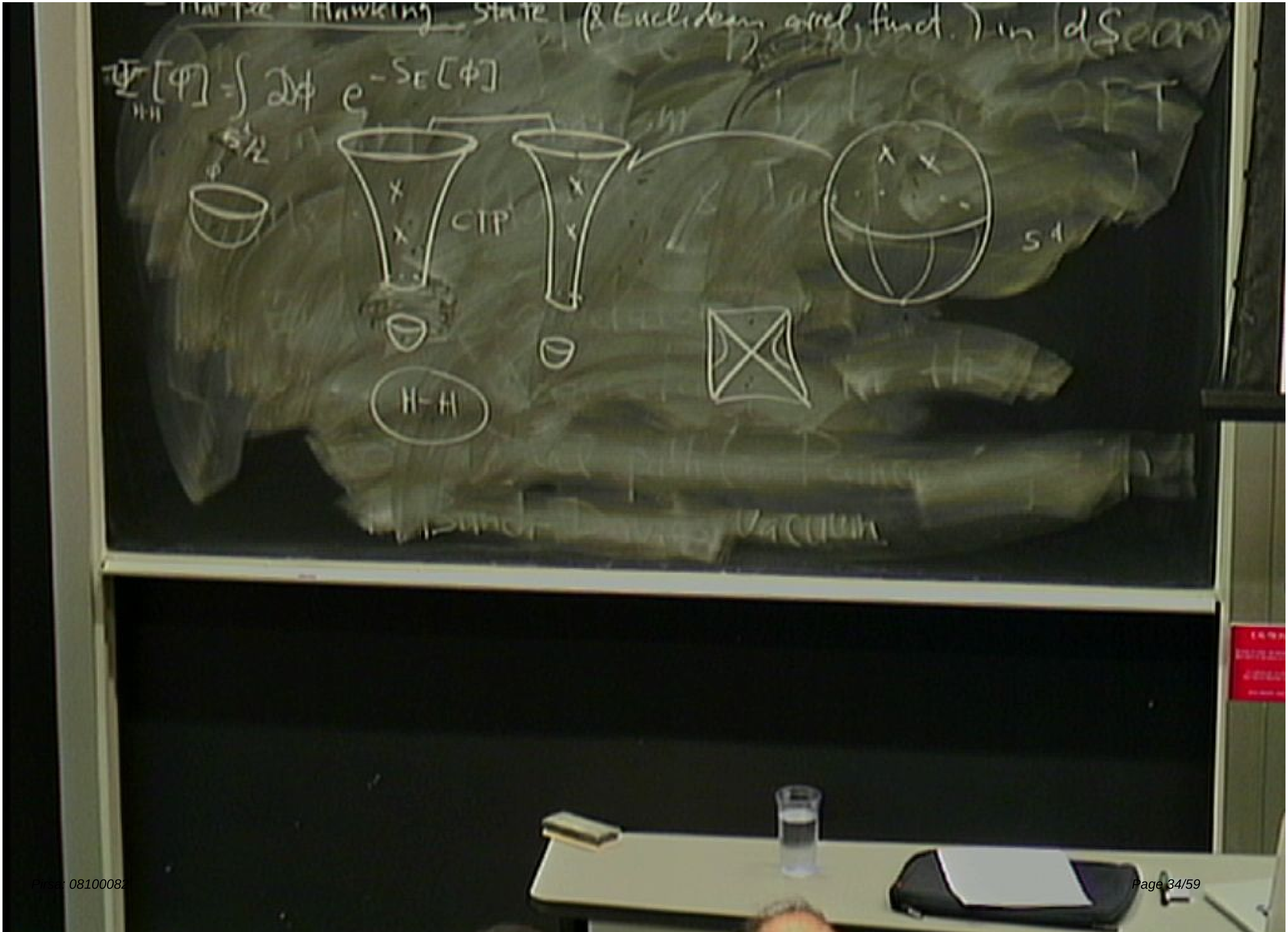
CTP
S^4
H-H

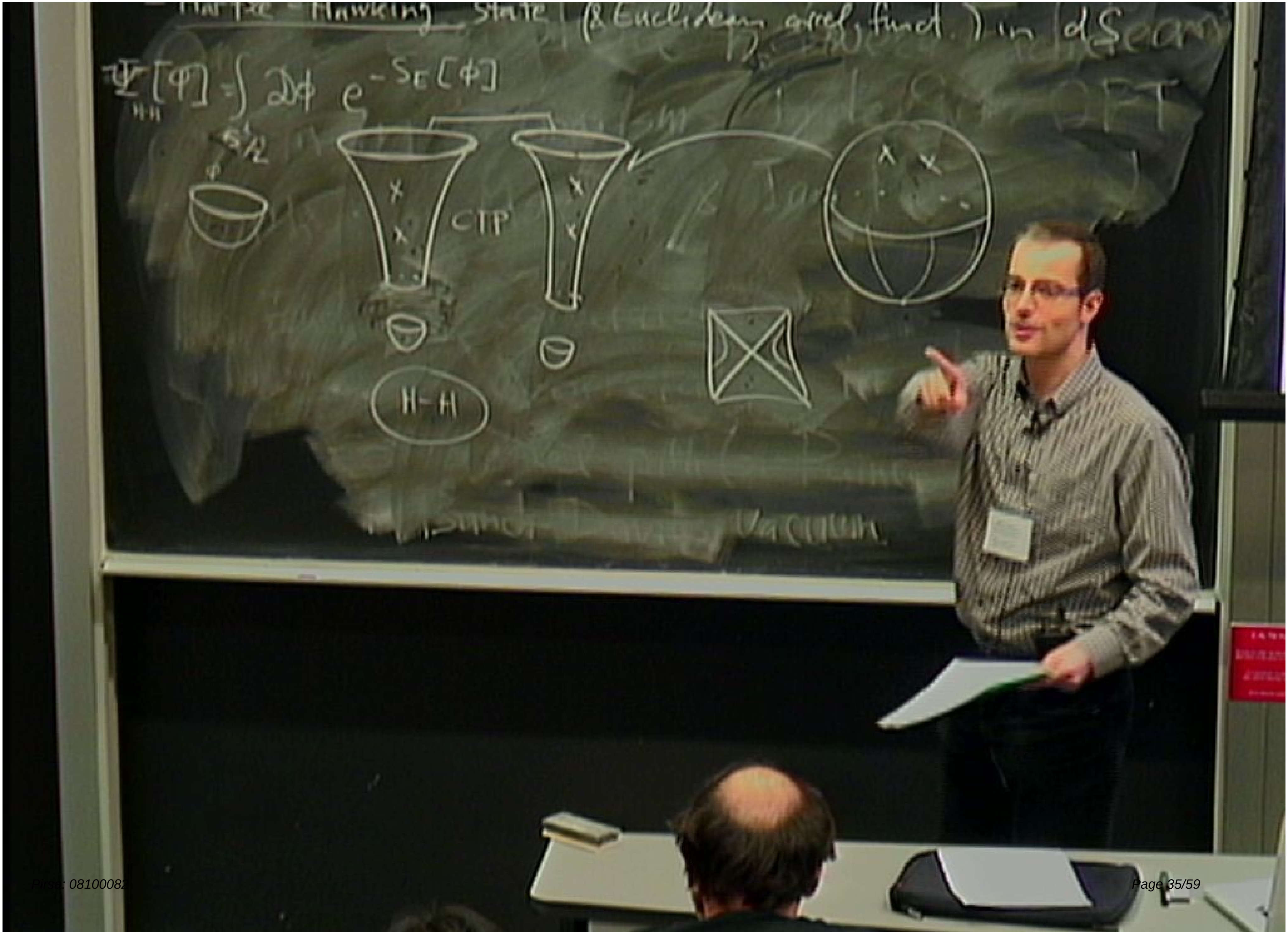


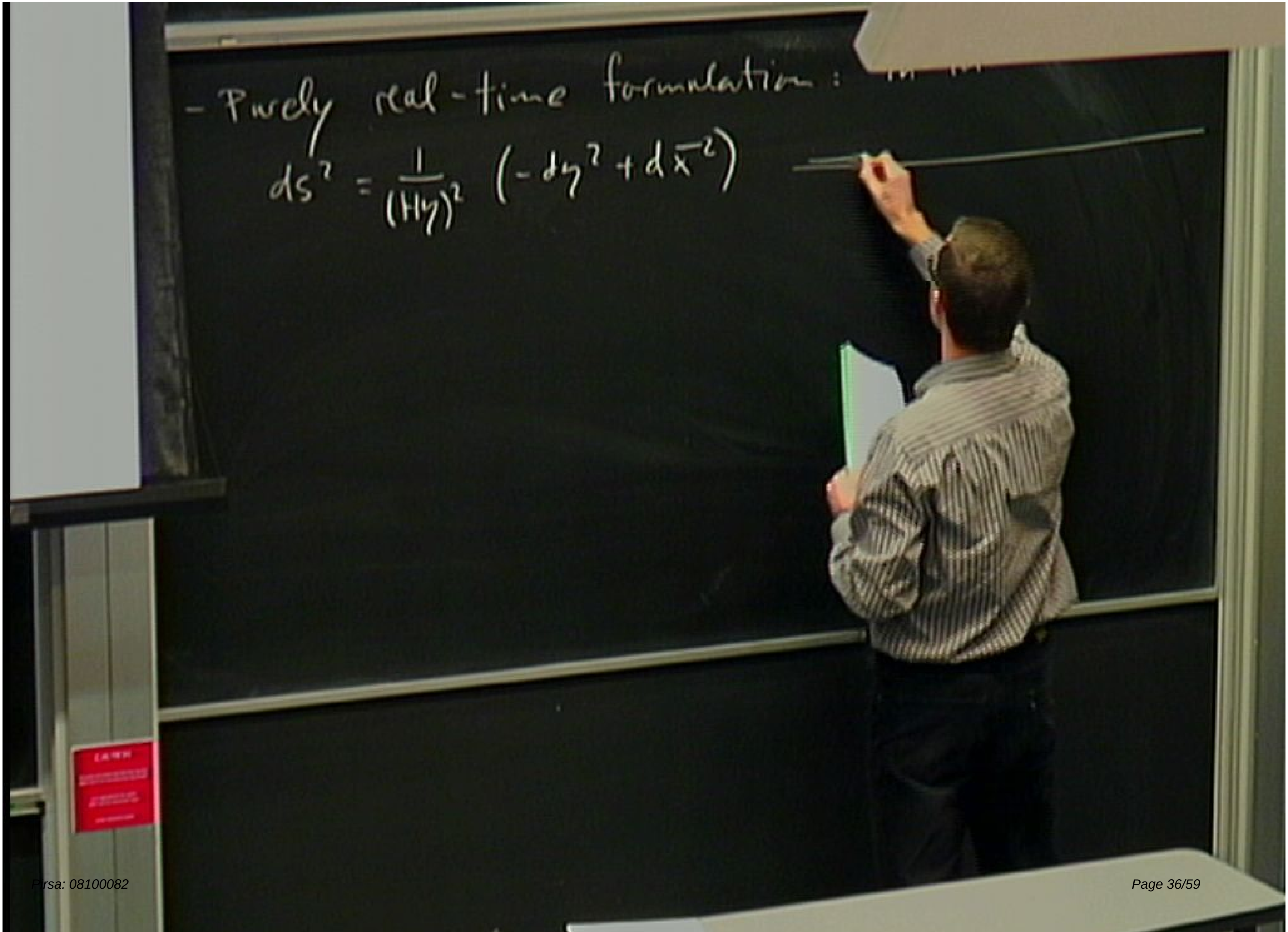
1
– Thermal FT in flat space
– Hartle – Hawking state
CTP
H-H

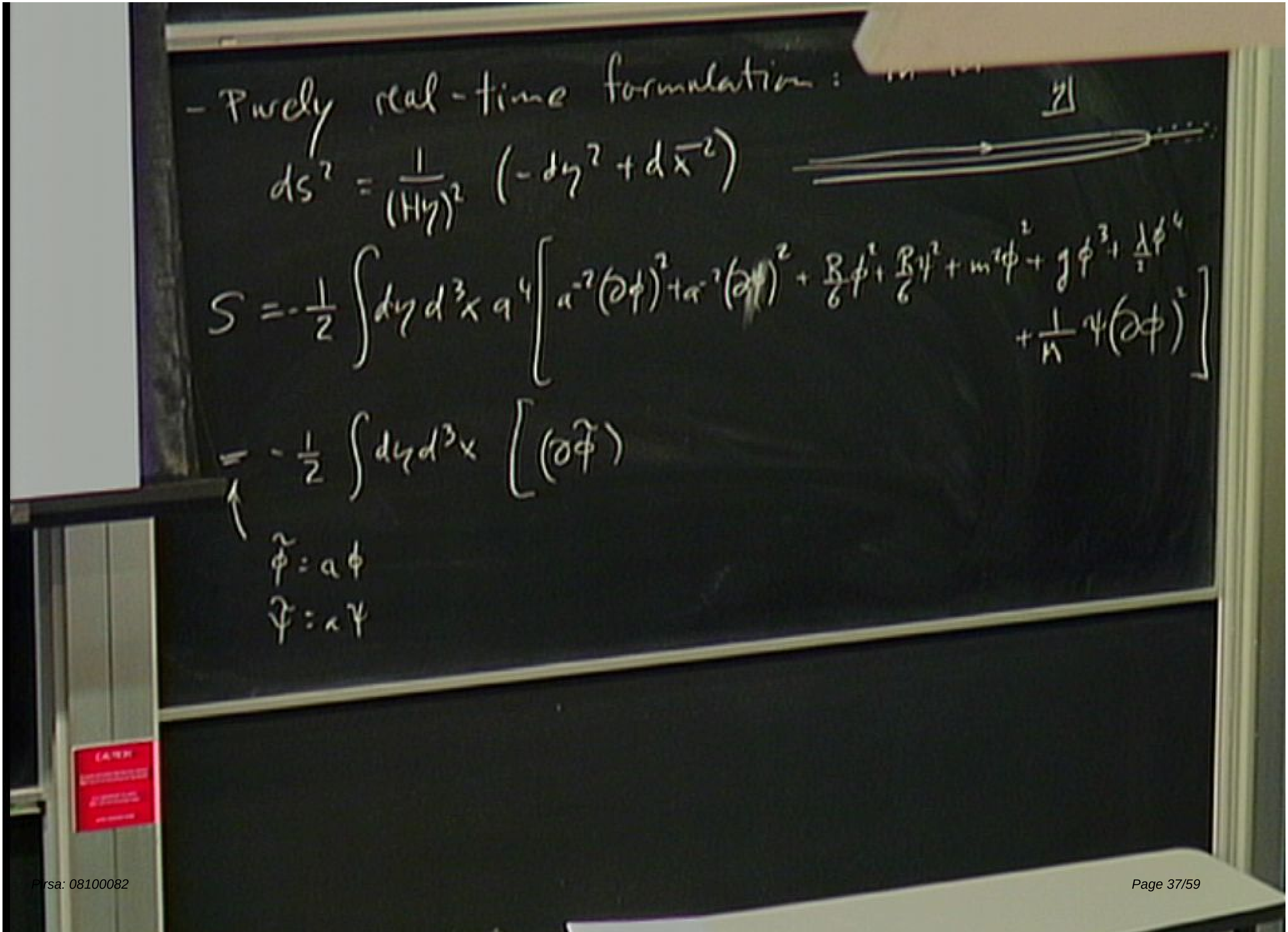

- Hartle - Hawking state (& Euclidean correl. funct.) in dS
CTP
S4
H-H

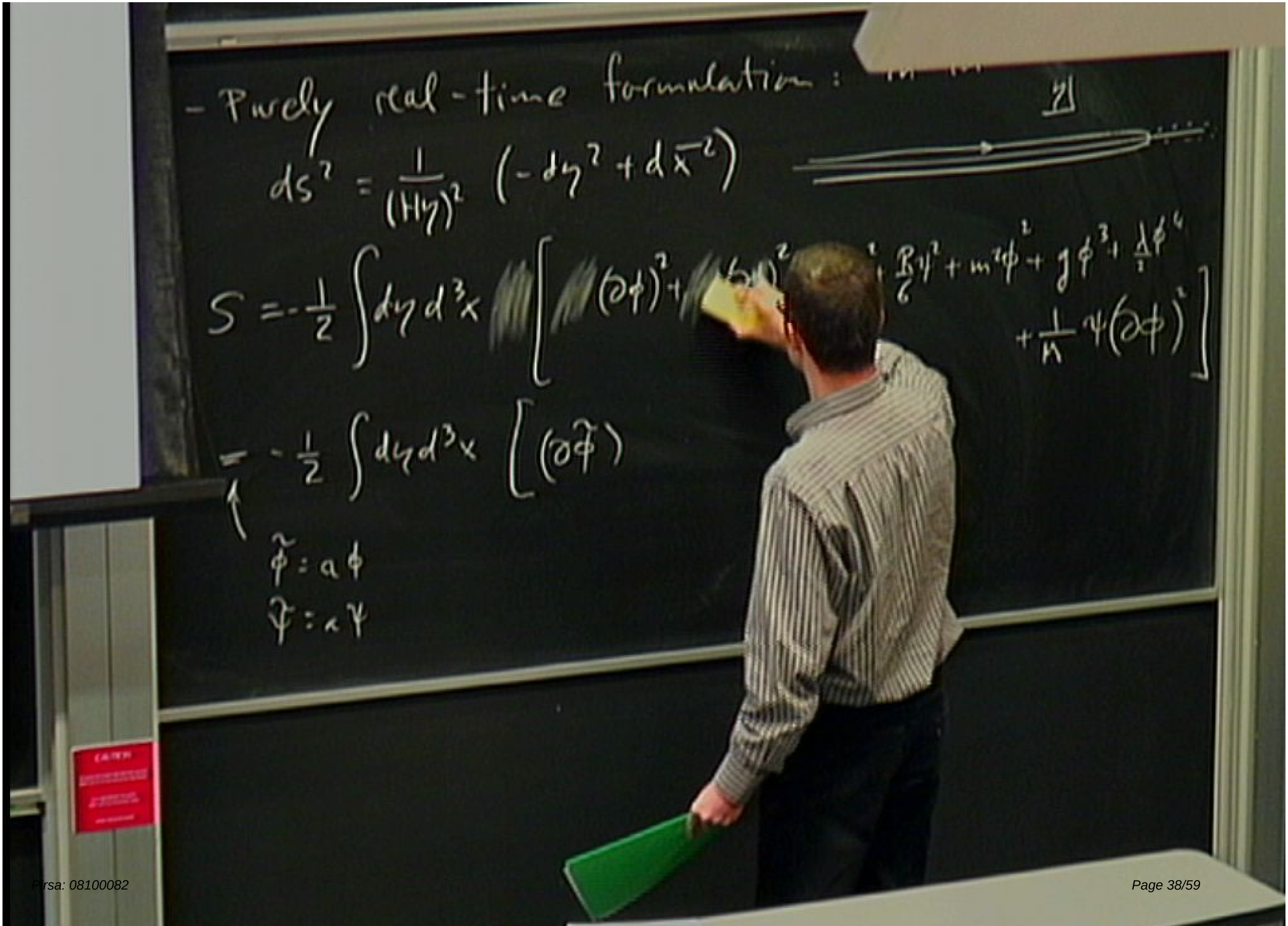

Hartle - Hawking state
CTP
H-H
vacuum

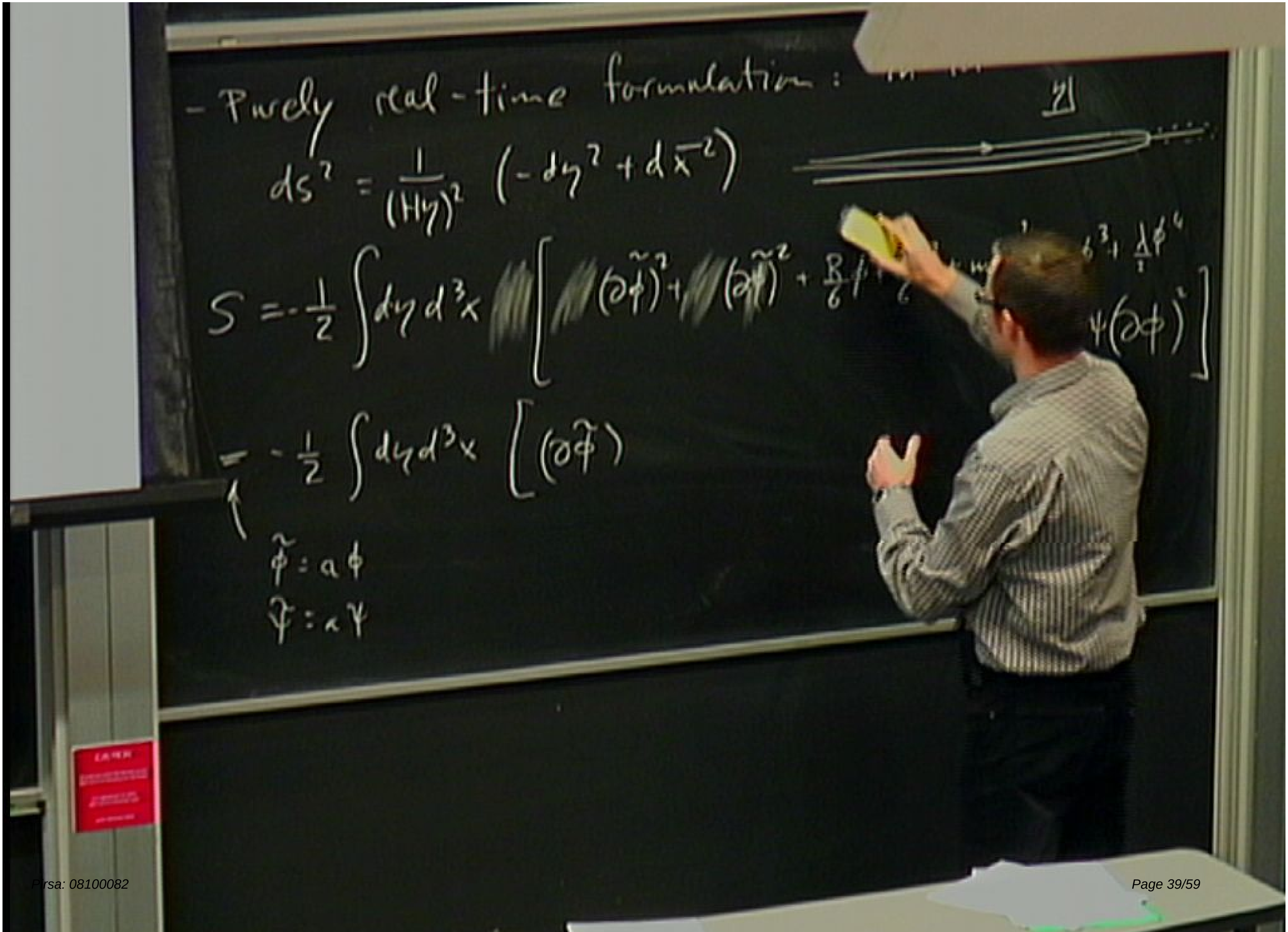

Hartle - Hawking state
CIP
H-H
S4
Vacuum


CTP
H-H


- Purely real-time formulation:
$ds^2 = \frac{1}{(H\eta)^2}\left(-d\eta^2 + d\vec{x}^2\right)$


- Purely real-time formulation:
$ds^2 = \frac{1}{(H\eta)^2}\left(-d\eta^2 + d\vec{x}^2\right)$
$S = -\frac{1}{2}\int d\eta\, d^3x\, a^4 \left[a^{-2}(\partial\phi)^2 + a^{-2}(\partial\psi)^2 + \frac{R}{6}\phi^2 + \frac{R}{6}\psi^2 + m^2\phi^2 + g\phi^3 + \frac{\lambda}{2}\phi^4 + \frac{1}{M}\psi(\partial\phi)^2 \right]$
$= -\frac{1}{2}\int d\eta\, d^3x \left[(\partial\tilde{\phi}) \right.$
$\tilde{\phi} = a\phi$
$\tilde{\psi} = a\psi$


- Purely real-time formulation :
ds² = 1/(Hη)² (-dη² + dx⃗²)
S = -1/2 ∫dη d³x [(∂φ)² +
+ 1/M ψ(∂φ)²]
= -1/2 ∫dη d³x [(∂φ̃)
φ̃ = aφ
ψ̃ = aψ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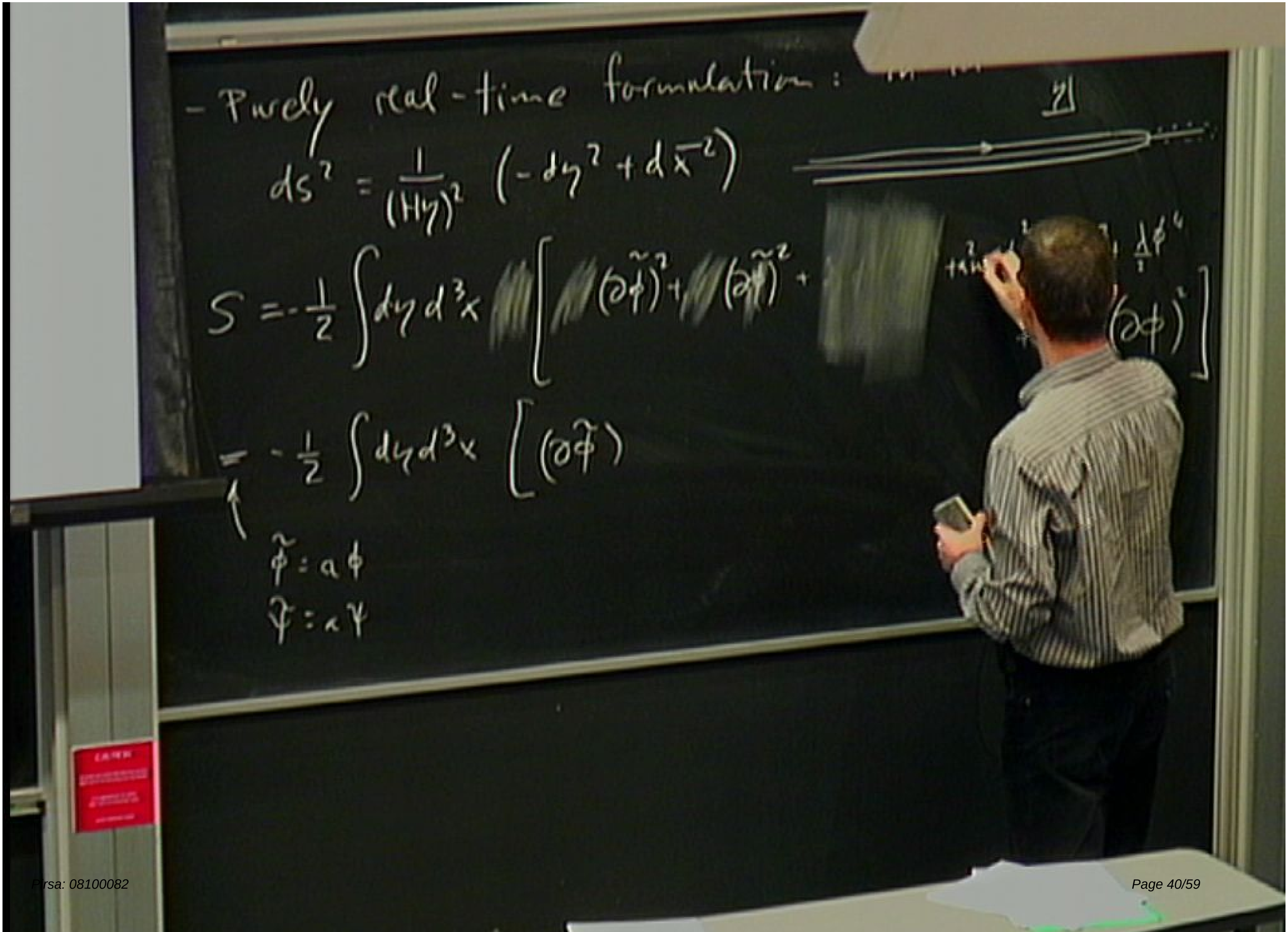
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$ds^2 = \frac{1}{(H\eta)^2}(-d\eta^2 + d\vec{x}^2)$
$S = -\frac{1}{2}\int d\eta\, d^3x \,[\,(\partial\tilde{\phi})^2 + (\partial\tilde{\psi})^2 + \ldots (\partial\phi)^2\,]$
$= -\frac{1}{2}\int d\eta\, d^3x \,[\,(\partial\tilde{\psi})$
$\tilde{\phi} = a\phi$
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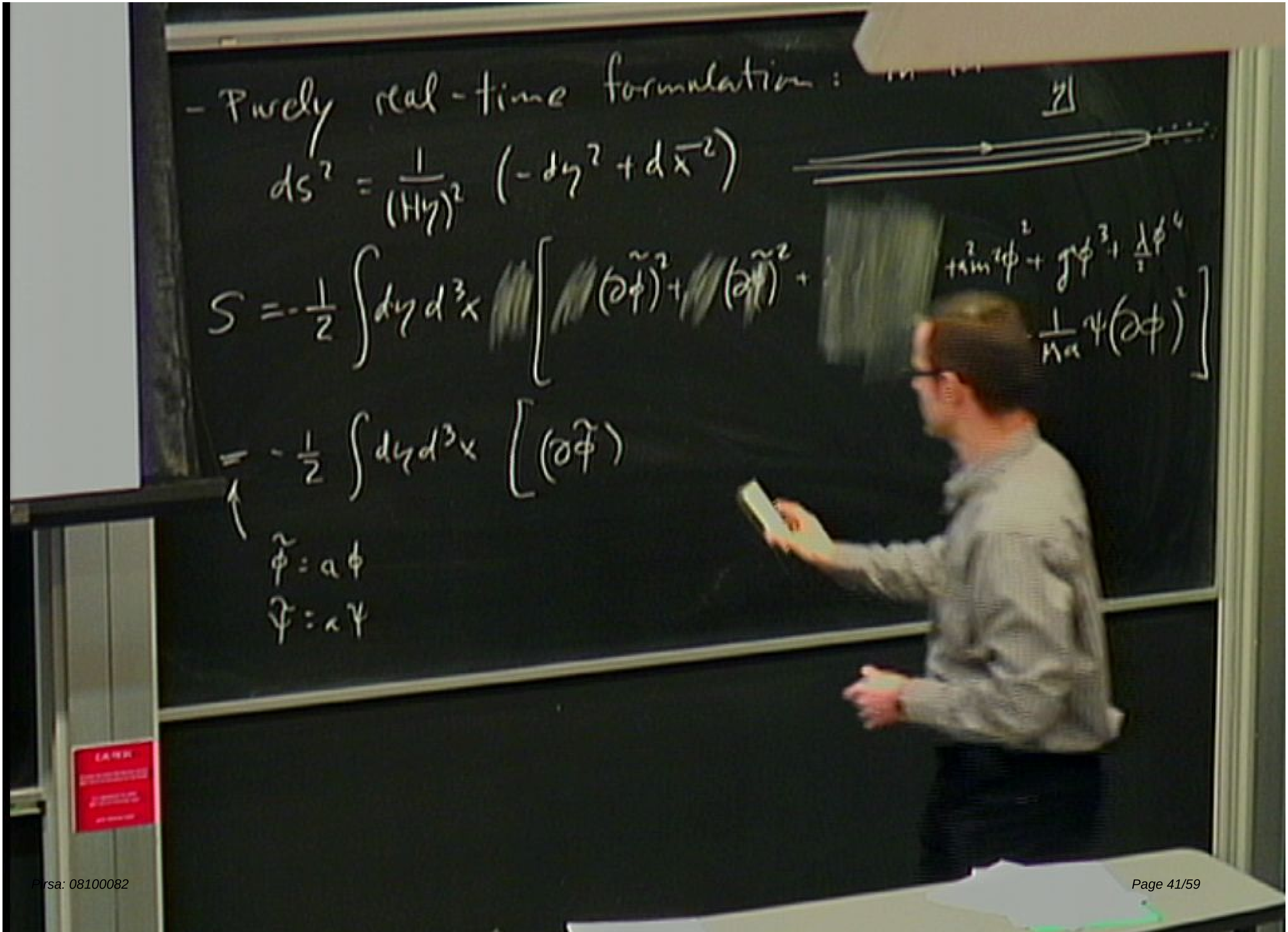
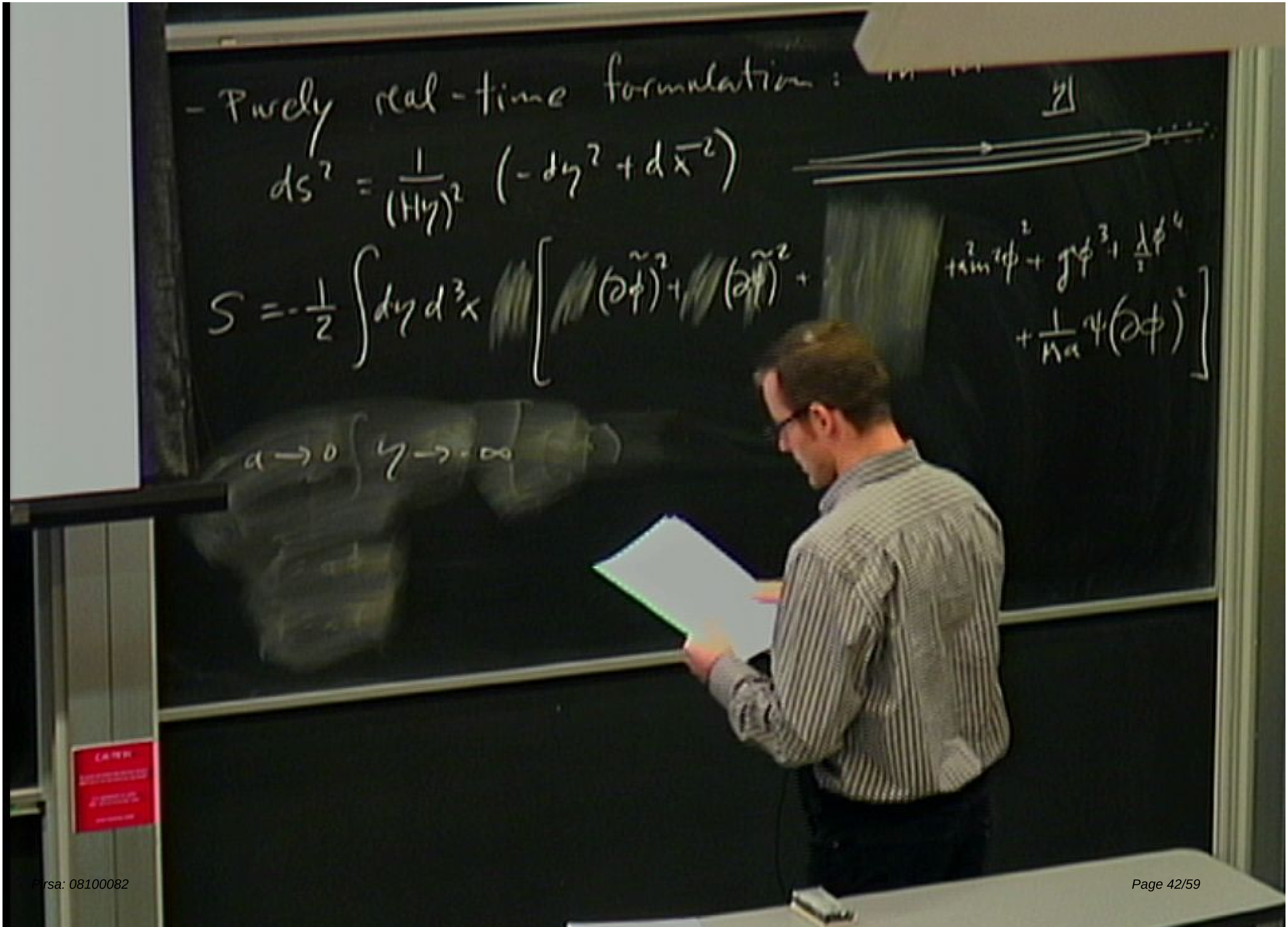

- Purely real-time formulation:
ds² = 1/(Hη)² (-dη² + dx²)
S = -1/2 ∫dη d³x [(∂φ̃)² + (∂φ̃)² +
+ m²φ² + gφ³ + λφ⁴
1/Ha ψ(∂φ)²]
= -1/2 ∫dη d³x [(∂φ̃)
φ̃ = aφ
ψ̃ = aψ

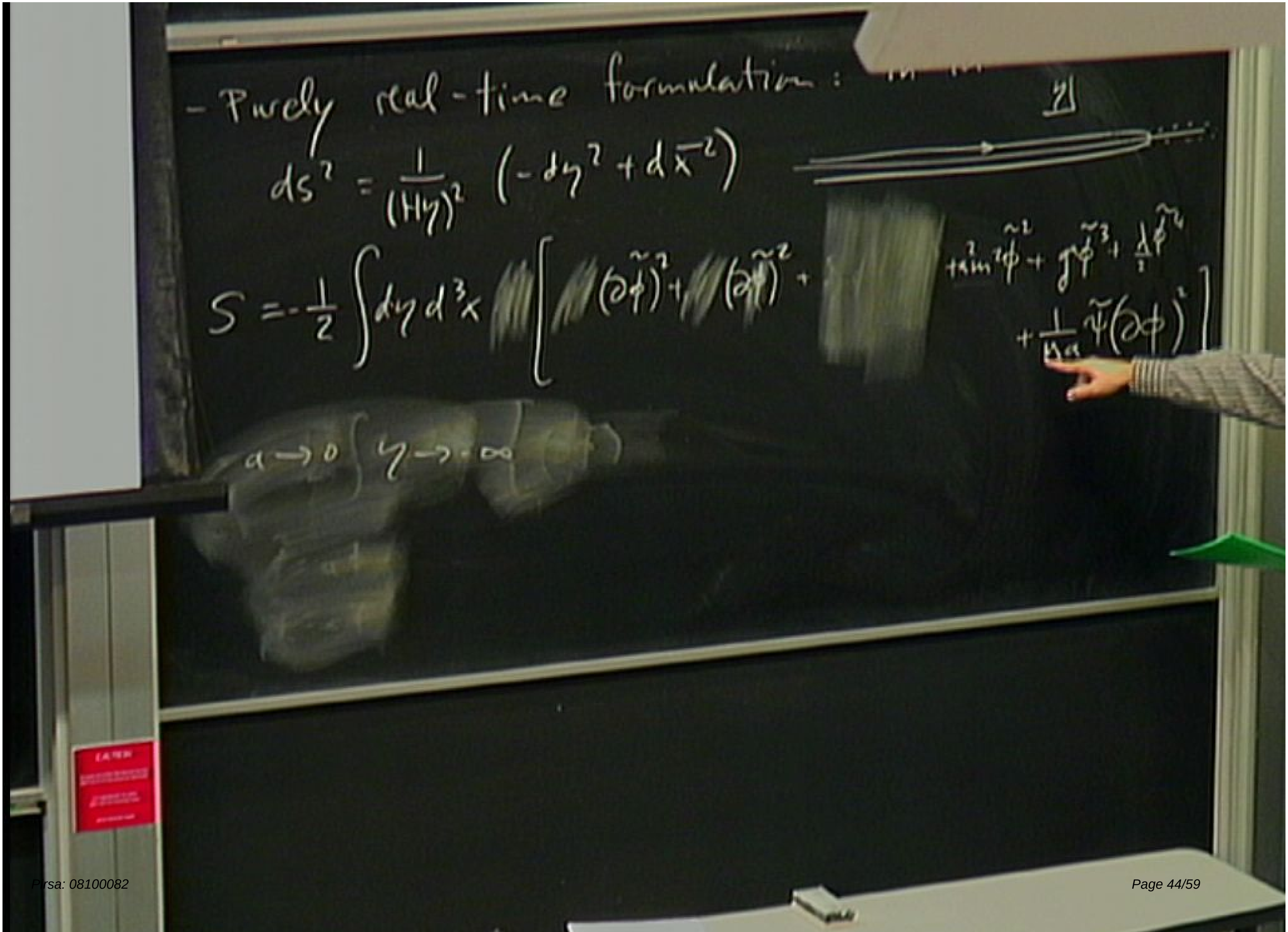

- Purely real-time formulation :
ds² = 1/(Hη)² (-dη² + dx⃗²)
S = -1/2 ∫dη d³x
a → 0 η → -∞

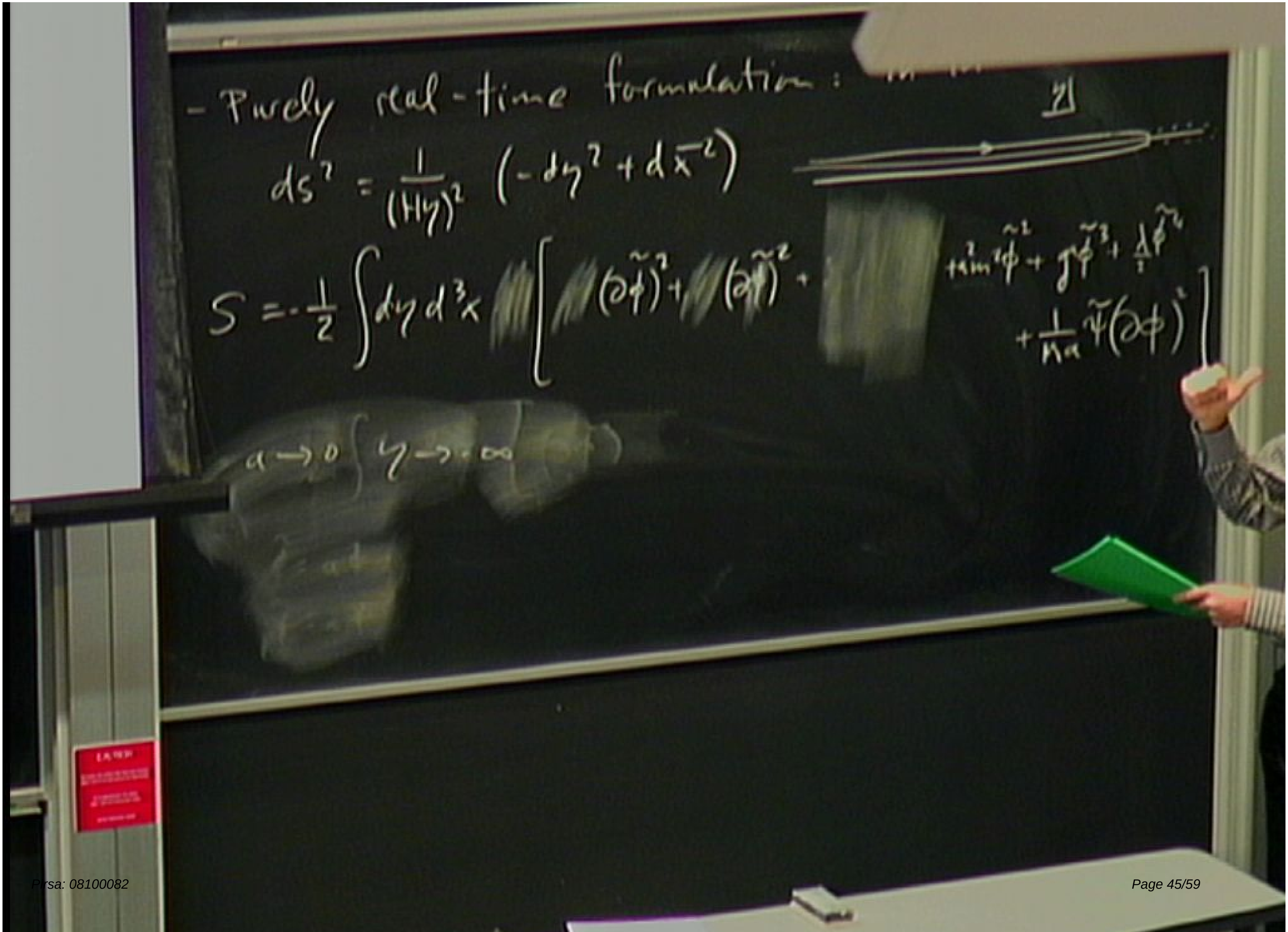
- Purely real-time formulation:

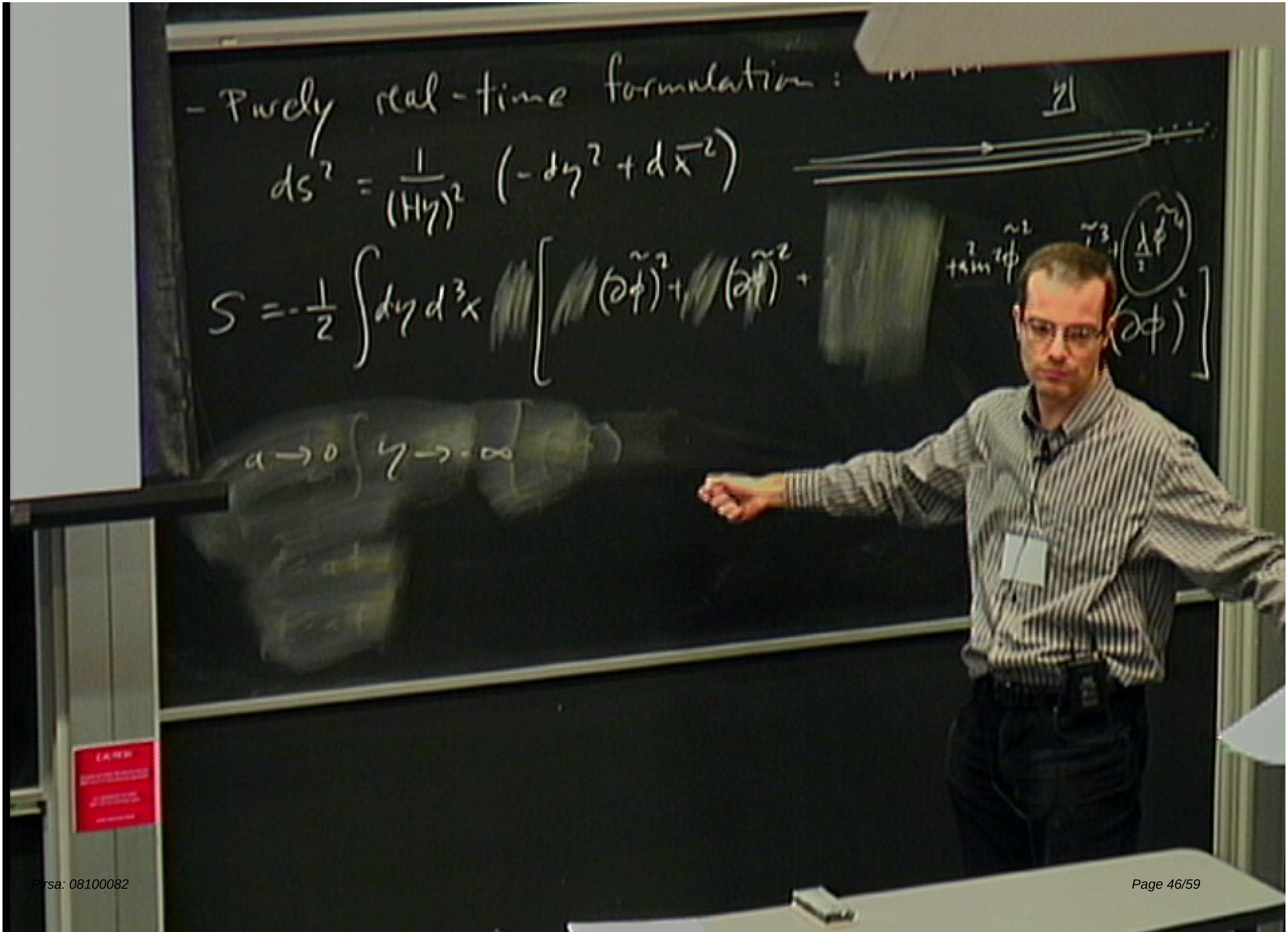
$$ds^2 = \frac{1}{(H\eta)^2}\left(-d\eta^2 + d\vec{x}^2\right)$$

$$S = -\frac{1}{2}\int d\eta\, d^3x \left[(\partial\tilde{\phi})^2 + (\partial\tilde{\phi})^2 + \ldots + a^2 m^2 \tilde{\phi}^2 + g\tilde{\phi}^3 + \frac{\lambda}{4}\tilde{\phi}^4 + \frac{1}{Ha}\tilde{\psi}(\partial\phi)^2 \right]$$

$a \to 0$; $\eta \to -\infty$


- Purely real-time formulation :
$ds^2 = \frac{1}{(H\eta)^2}(-d\eta^2 + d\vec{x}^2)$
$S = -\frac{1}{2}\int d\eta\, d^3x\, [\ (\partial\tilde{\phi})^2 + (\partial\tilde{\phi})^2 +$
$+ \frac{1}{2} m^2\tilde{\phi}^2 + g\tilde{\phi}^3 + \frac{\lambda}{2}\tilde{\phi}^2$
$+ \frac{1}{H\eta}\tilde{\psi}(\partial\phi)^2\ ]$
$a \to 0$
$\eta \to -\infty$


- Purely real-time formulation:
ds² = 1/(Hη)² (-dη² + dx²)
S = -½ ∫dη d³x [(∂φ̃)² + (∂ψ̃)² +
+a²m²φ̃² + gφ̃³ + λ/4 φ̃⁴
+ 1/Ha ψ̃(∂φ)²]
a → 0 { η → -∞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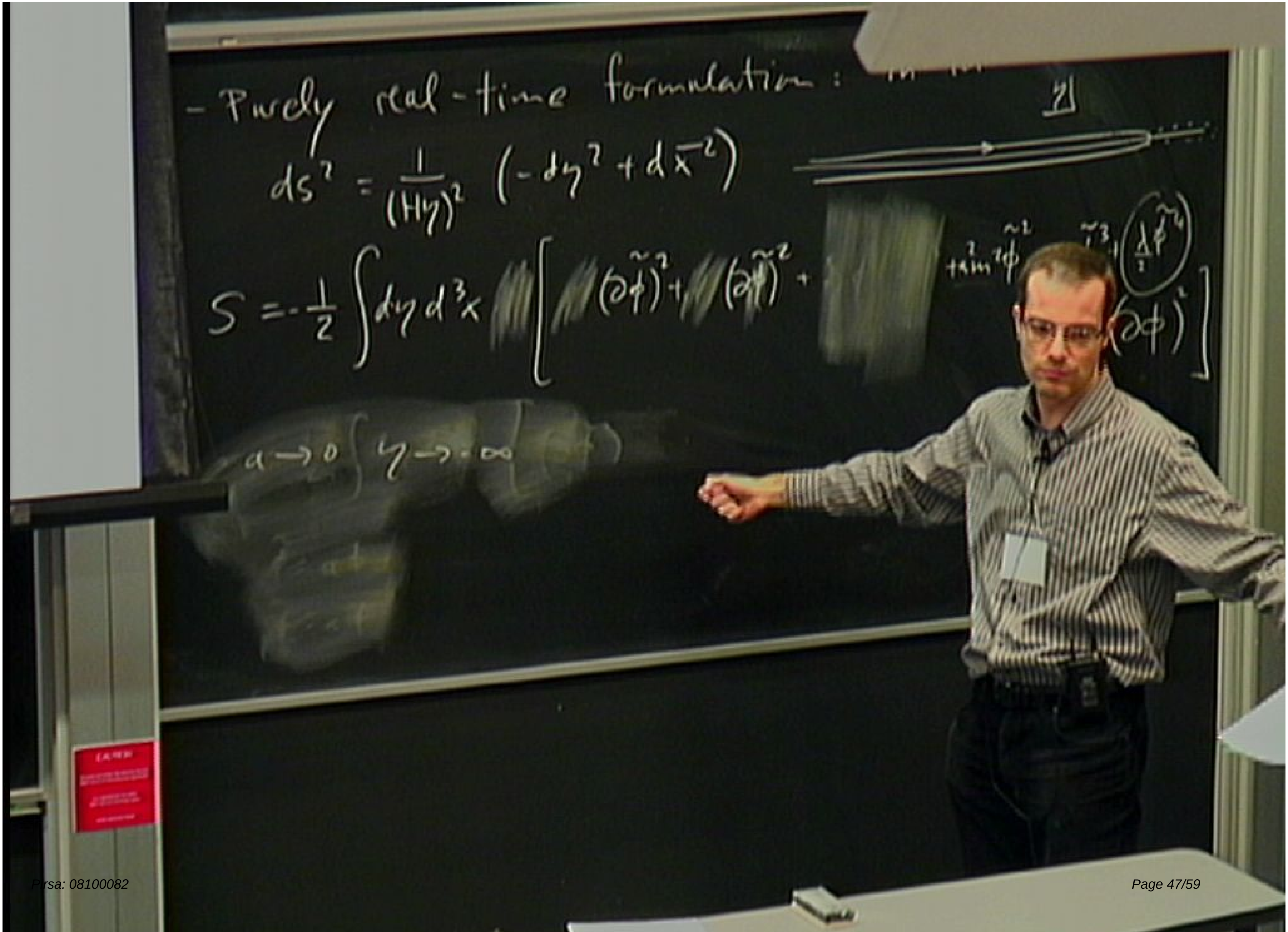
S = -1/2 ∫dη d³x [
a → 0
η → -∞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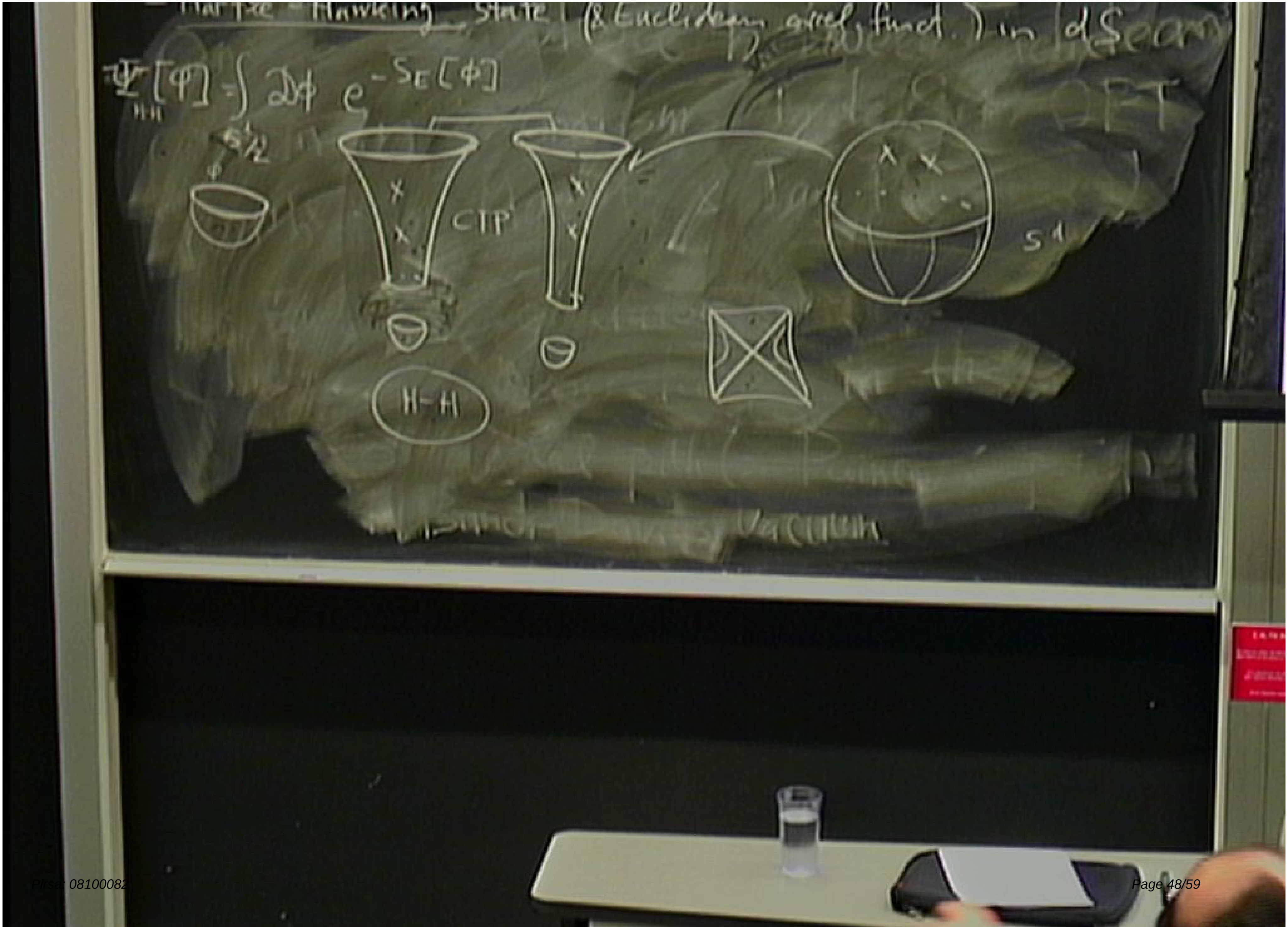
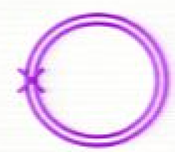

Hartle - Hawking state (& Euclidean green funct.) in dS
$\Psi_{HH}[\varphi] = \int \mathcal{D}\phi \, e^{-S_E[\phi]}$
CIP'
H-H
S¹
Vacuum

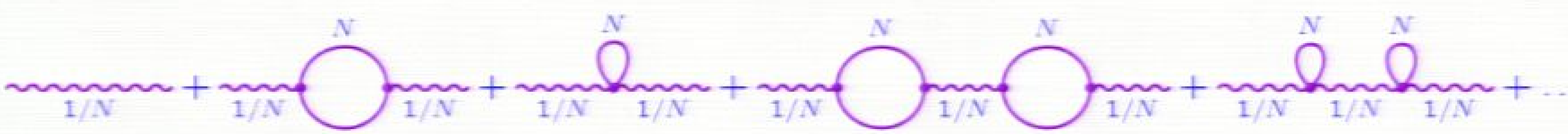
One-loop Riemann correlators

- $g_{ab}^{\rm dS} + h_{ab}$ → perturbative QG as an EFT
- N matter fields, $l_{\rm p}^2 = \bar{l}_{\rm p}^2/N$ → large N expansion

$$G_{ab}[g] = \frac{8\pi \bar{l}_{\rm p}^2}{N} \left\langle \hat{T}_{ab}[g] \right\rangle_{\rm ren} \sim O(1)$$



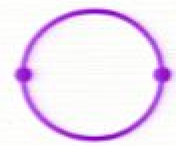
- CTP correlators for the metric perturbations:



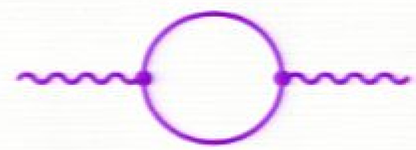
no *graviton loops* (no F-P ghosts)

Decomposition: Ricci + Weyl

$$\left\langle \hat{G}_a{}^b(x)\,\hat{G}_c{}^d(y)\right\rangle_{\rm c} = \left(8\pi \bar{l}_{\rm p}^2\right)^2 \left\langle \hat{t}_a{}^b(x)\,\hat{t}_c{}^d(y)\right\rangle \sim \frac{\bar{l}_{\rm p}^4}{N}$$

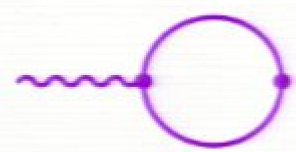


$$\left\langle \hat{C}_{abcd}(x)\,\hat{C}_{efgh}(y)\right\rangle_{\rm c} \sim \frac{\bar{l}_{\rm p}^2}{N}\left(1 + \mathcal{O}(\bar{l}_{\rm p}^2)\right)$$



Kouris

$$\left\langle \hat{C}_{abcd}(x)\,\hat{G}_e{}^f(y)\right\rangle_{\rm c} \sim \frac{\bar{l}_{\rm p}^4}{N}$$



Ricci (stress tensor) correlator

- Stress-tensor quantum correlations: $(\xi = 0)$

  $$\left\langle \left\{ \hat{t}_{ab}(x), \hat{t}_{cd}(y) \right\} \right\rangle \qquad \hat{T}_{ab} = \nabla_a \hat{\phi}\, \nabla_b \hat{\phi} + \frac{1}{2} g_{ab} \left(\nabla_c \hat{\phi}\, \nabla^c \hat{\phi} + m^2 \hat{\phi}^2 \right)$$

  Expansion in terms of 10 *max. symm. bitensors* (2 undet. functions of Z) $\longrightarrow$ dS invariance

- Exact result for arbitrary d (also for AdS).

- Long-distance behavior (tensorial prefactors):

$$\left. \begin{array}{ll} m = 0,\ \xi = 0 & \mathcal{N} \sim \dfrac{1}{Z^2} \\ m > 0,\ \xi = 0 & \mathcal{N} \sim \dfrac{1}{Z^{2(m/H)^2/(d-1)}} \end{array} \right\} \quad Z \ll -1$$

- Discontinuity of the *massless limit* and existence of long-range correlations for light fields:

$$\mathcal{N} \sim \frac{1}{Z^2} \qquad \text{v.s.} \qquad \mathcal{N} \sim \frac{1}{Z^{2(m/H)^2/(d-1)}}$$

- Intuitive explanation:

$$\hat{T}_{ab} = \nabla_a \hat{\phi} \nabla_b \hat{\phi} + \frac{1}{2} g_{ab} \left(\nabla_c \hat{\phi} \nabla^c \hat{\phi} + m^2 \hat{\phi}^2 \right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2 \phi + k^2/a^2 = 0 \qquad \begin{cases} \phi \sim e^{-3Ht} \\ \phi \sim e^{-\frac{m^2}{3H^2} Ht} \end{cases}$$

Discontinuity: $G^+(x,y) \sim \frac{1}{m^2} \frac{1}{Z^{m^2/(d-1)H^2}}$

Weyl correlator

- Sensitive to initial state corrections (*entanglement*) ($\langle \hat{R}_a{}^b(x)\, \hat{R}_c{}^d(y)\rangle_c$ not to this order).
- 2 vertex integrals:
  in-in formalism in *spatially flat* coordinates
  ($i\epsilon$ prescription → Wick rotation)
- For dS background → no dependence on $\ln \mu^2$

$$R^2 \ln \mu^2\,, \quad C^{abcd}C_{abcd} \ln \mu^2 \quad \longrightarrow\ 0$$

(GW power spectrum)

- Exact calculation with Markus Fröb (*U. Barcelona*):
  - No approximation, including all terms.
  - Correlator at different times.
  - $m^2 = 0 \qquad \xi = 1/6$

    $m^2 = 0 \qquad \xi = 0$

    $m^2 \neq 0 \qquad$ (?)

Conclusion and discussion

- One-loop corrections from matter fields to Riemann correlator in de Sitter.
- Manifestly dS-invariant result for Ricci correlator.
  - Long-range correlations for *light* fields: $1/Z^{2m^2/3H^2}$
  - Discontinuity of *massless* limit: $1/Z^2$
- Expect similar results for Weyl correlator.

- Implications for GW power spectrum (*spatially flat* coordinates):

$$\left\langle \hat{R}_{\alpha}{}^{\beta}(t,\vec{k})\,\hat{R}_{\mu}{}^{\nu}(t,-\vec{k})\right\rangle_{c}$$ → no tree level

$$\left\langle \hat{C}_{\alpha\beta\gamma\delta}(t,\vec{k})\,\hat{C}_{\mu\nu\rho\sigma}(t,-\vec{k})\right\rangle_{c}$$

tree level ↘ 1-loop ↙

$m^2 = 0$
$$\frac{H^2}{m_{\rm p}^2}\frac{1}{k^3}\left(\frac{k^2}{a^2(t)H^2}\right)^2\left[1+\#\left(\frac{H}{m_{\rm p}}\right)^2\right]$$

$0 < m^2 \ll H^2$
$$\frac{H^2}{m_{\rm p}^2}\frac{1}{k^3}\left(\frac{k^2}{a^2(t)H^2}\right)^2\left[1+\#\left(\frac{H}{m_{\rm p}}\right)^2\left(\frac{a(t)H}{k}\right)^{4-\frac{4m^2}{3H^2}}\right]$$

red-tilted
spectrum correction

- Exact calculation with Markus Fröb (*U. Barcelona*):
  - No approximation, including all terms.
  - Correlator at different times.
  - $m^2 = 0 \qquad \xi = 1/6$

    $m^2 = 0 \qquad \xi = 0$

    $m^2 \neq 0 \qquad$ (?)

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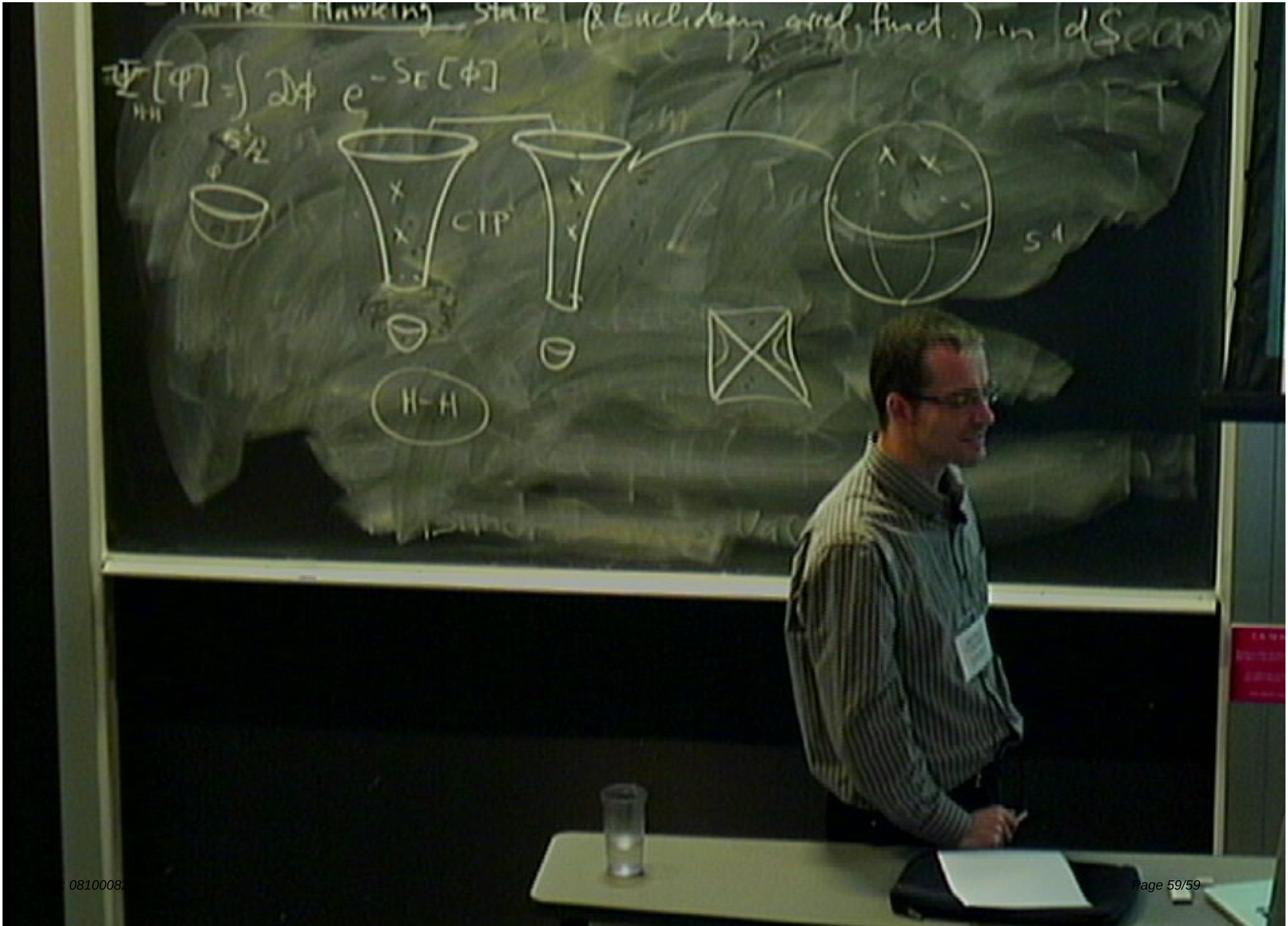
$$\hat{T}_{ab} = \nabla_a \hat{\phi} \nabla_b \hat{\phi} + \frac{1}{2} g_{ab} \left(\nabla_c \hat{\phi} \nabla^c \hat{\phi} + m^2 \hat{\phi}^2 \right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + k^2/a^2 = 0 \qquad \begin{cases} \phi \sim e^{-3Ht} \\ \phi \sim e^{-\frac{m^2}{3H^2}Ht} \end{cases}$$

Discontinuity: $G^+(x,y) \sim \dfrac{1}{m^2} \dfrac{1}{Z^{m^2/(d-1)H^2}}$


Hawking state
$\Psi_{HH}[\varphi] = \int \mathcal{D}\phi \, e^{-S_E[\phi]}$
CTP
H-H