Title: Triviality from the Exact Renormalization Group

Date: Oct 30, 2008 02:00 PM

URL: http://pirsa.org/08100078

Abstract: After reviewing Wilson\'s picture of renormalization, and the associated Exact Renormalization Group, I will show that no (physically acceptable) non-trivial fixed points exist for scalar field theory in D>=4. Consequently, an asymptotic safety scenario is ruled out, and the triviality of the theory is confirmed.

Triviality from The Exact Renormalization Group

Oliver J. Rosten

Sussex U.

October 2008

Outline of this Lecture

1. Qualitative Aspects of the ERG
   - The Basic Ideas
2. Renormalizability
   - Continuum Limits
3. ERG Equations
4. Triviality
   - Correlation Functions
   - Technicalities
   - Application to Fixed Points
5. Conclusion

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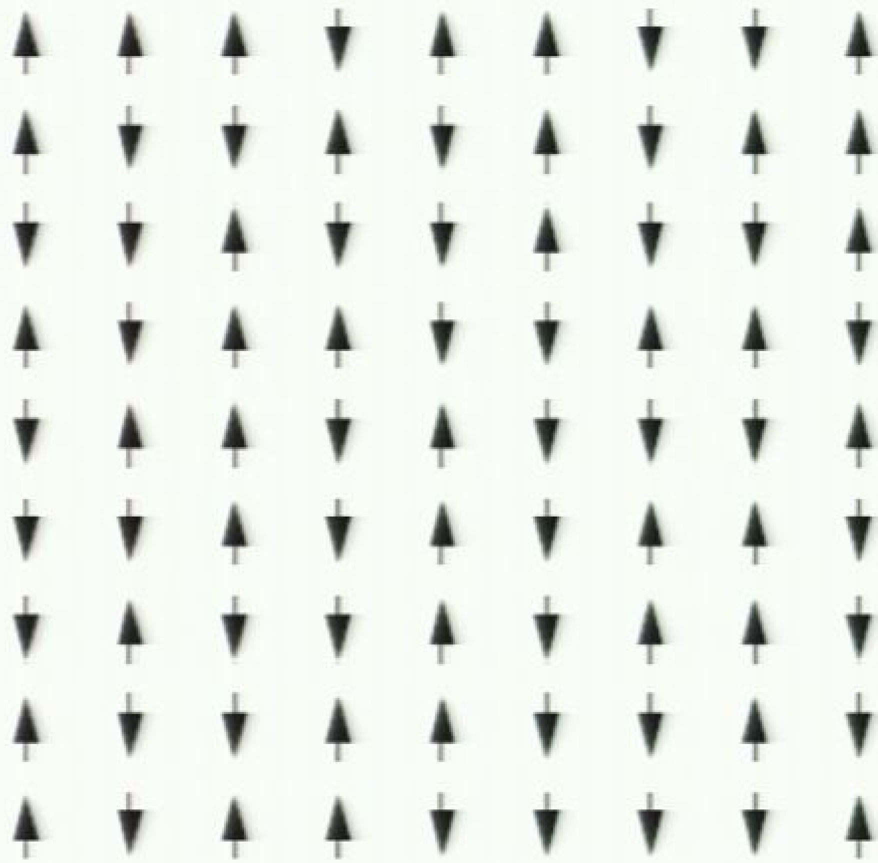

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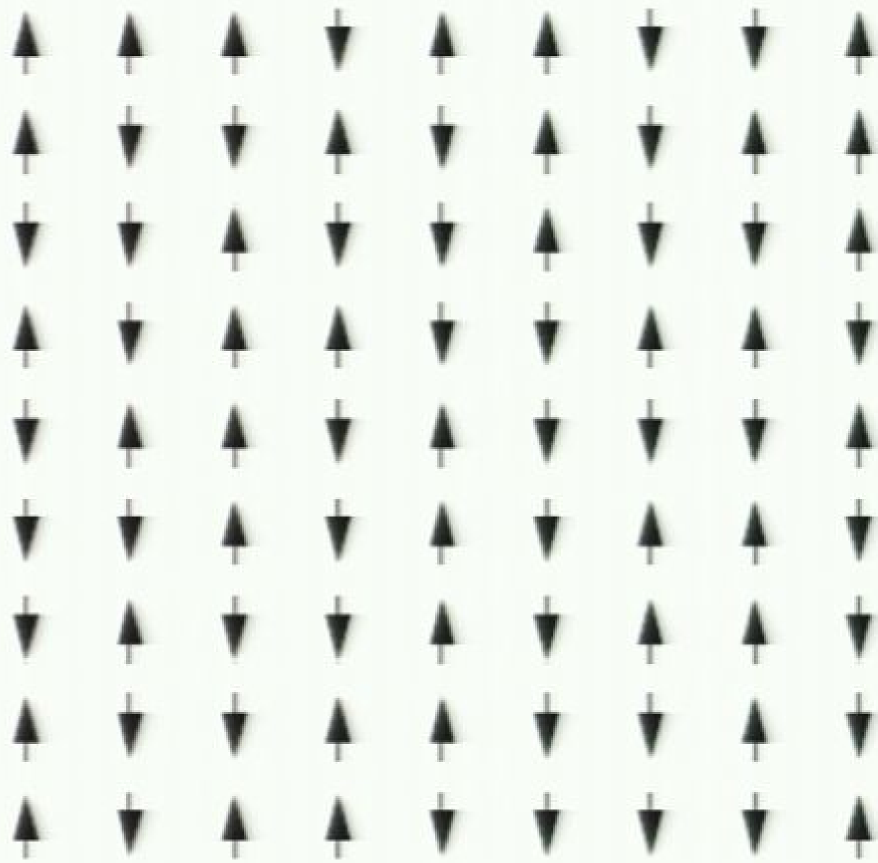

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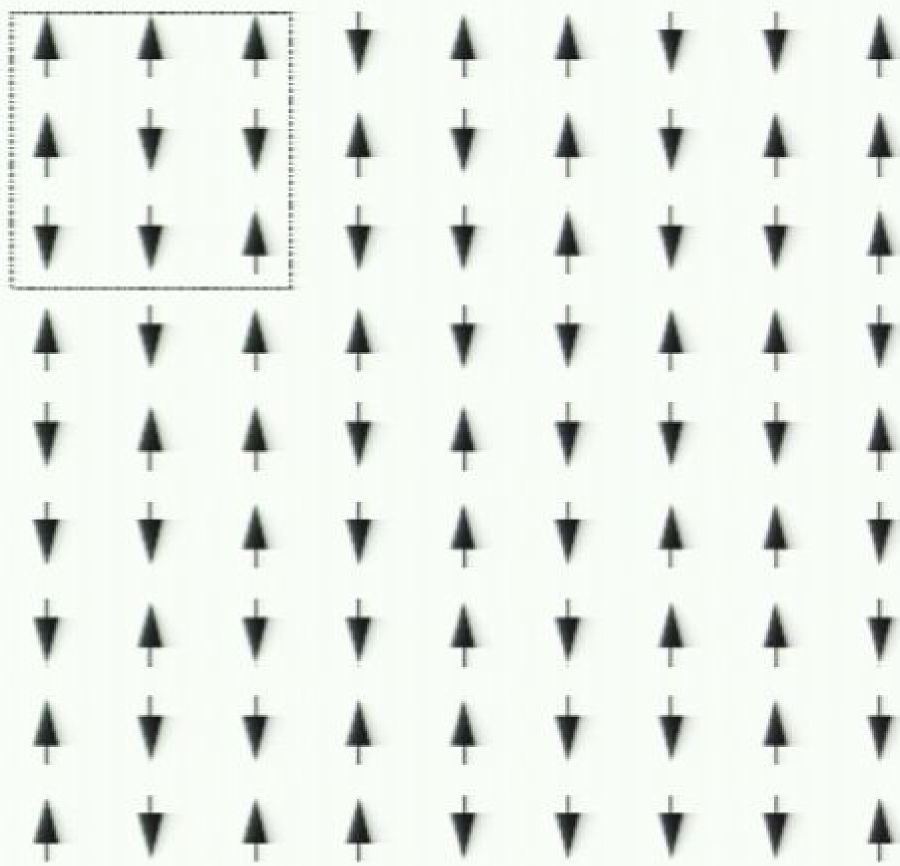

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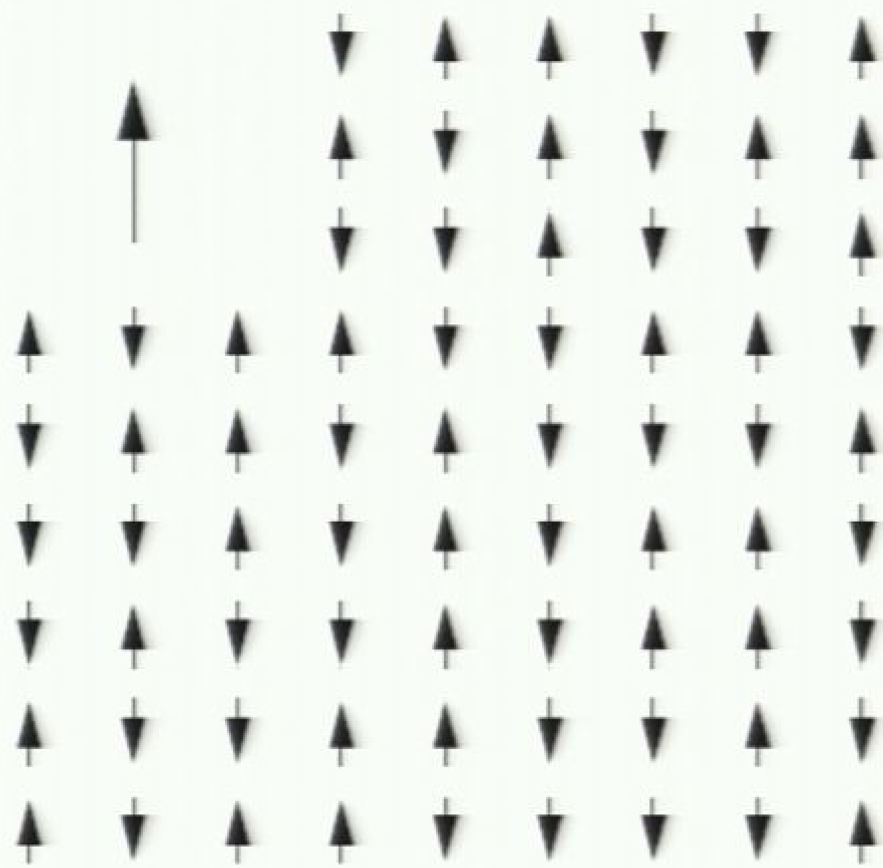

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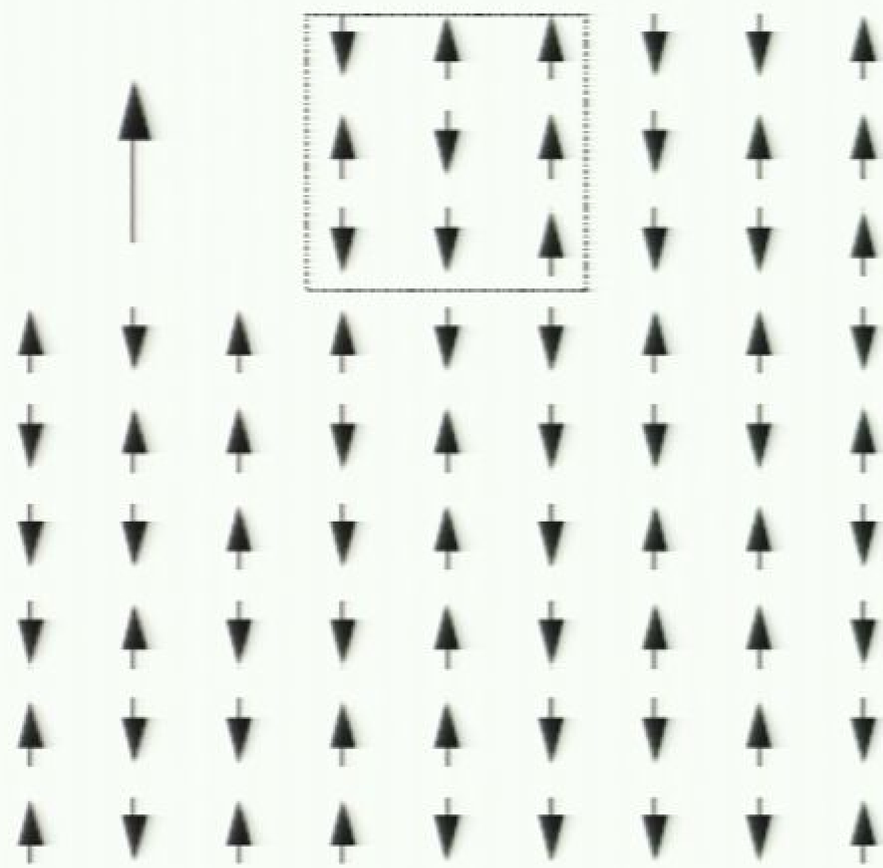

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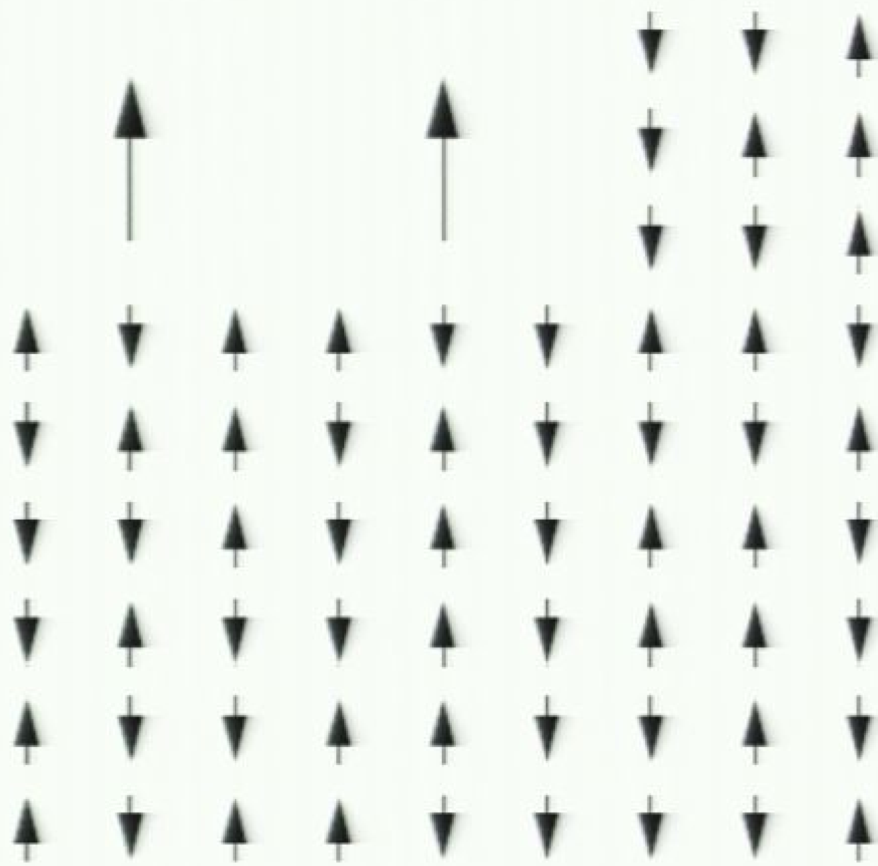

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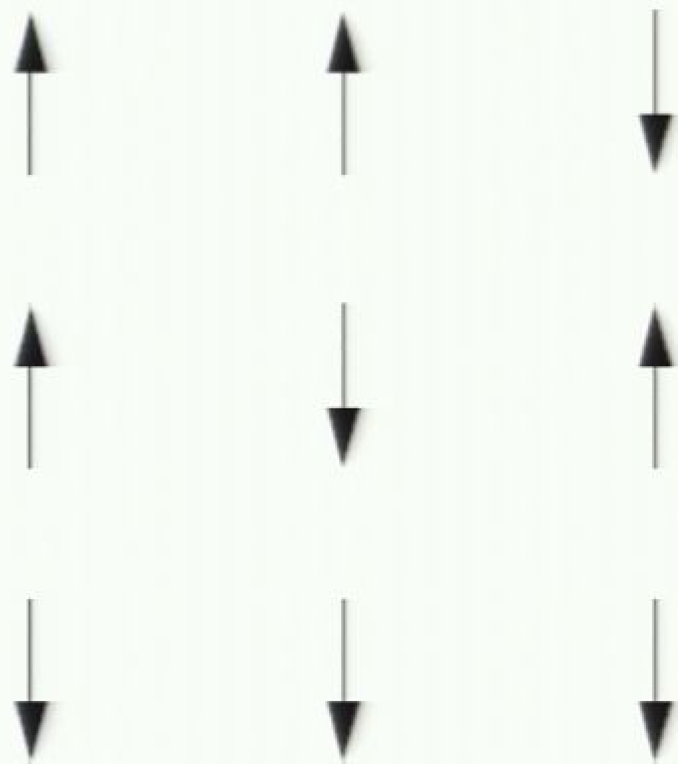

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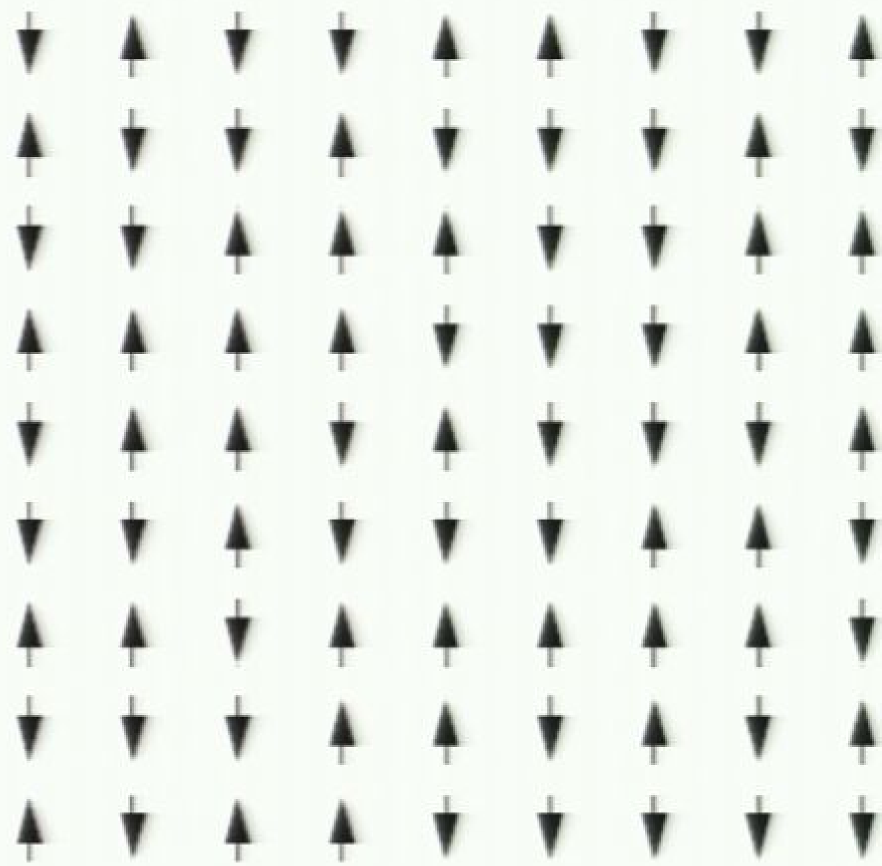

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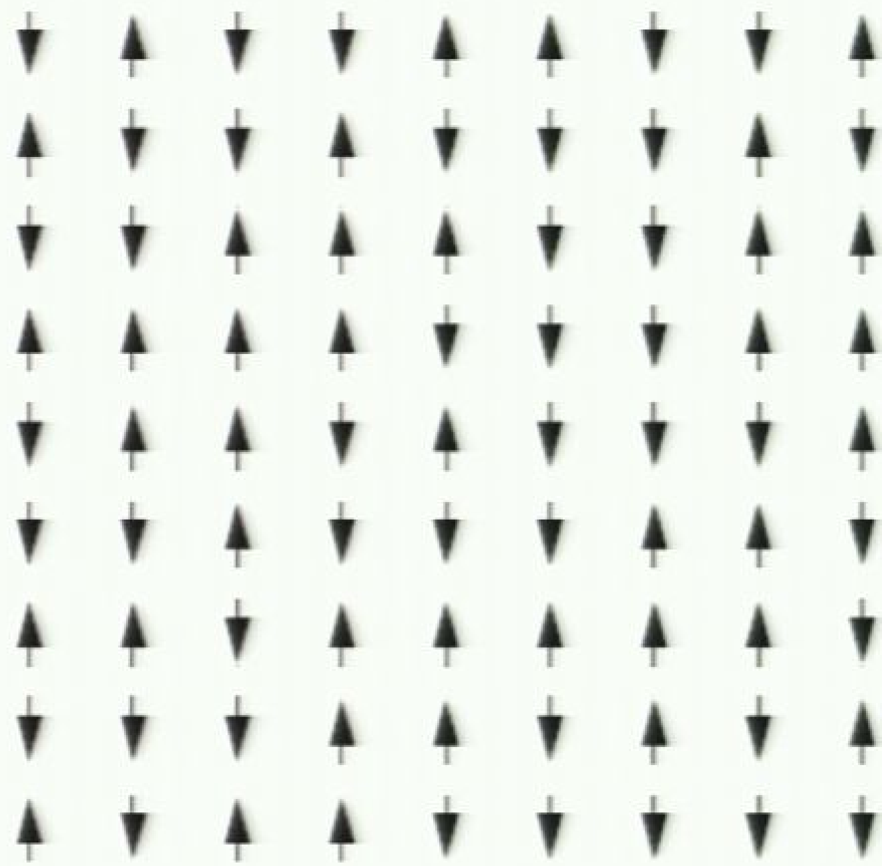

The ERG implements the continuous version of blocking

Flows in Parameter Space

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- The transformation can have fixed points

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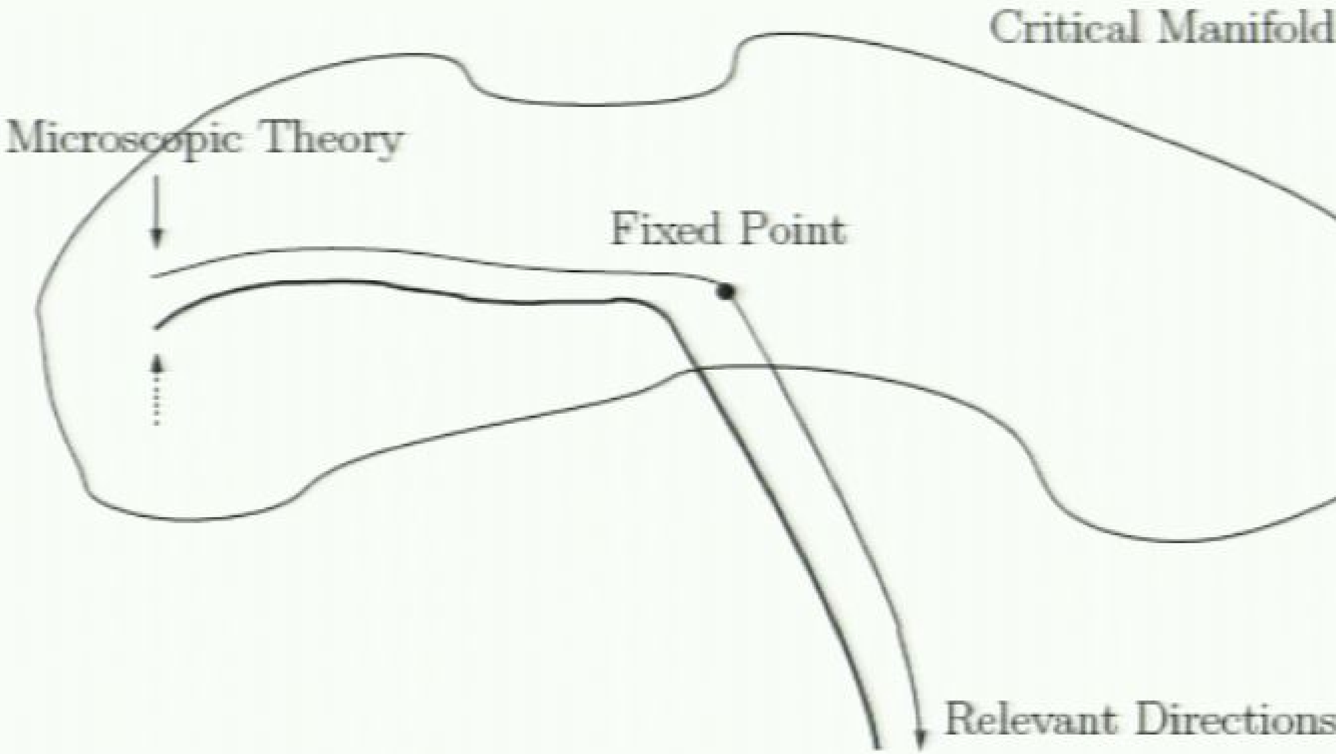

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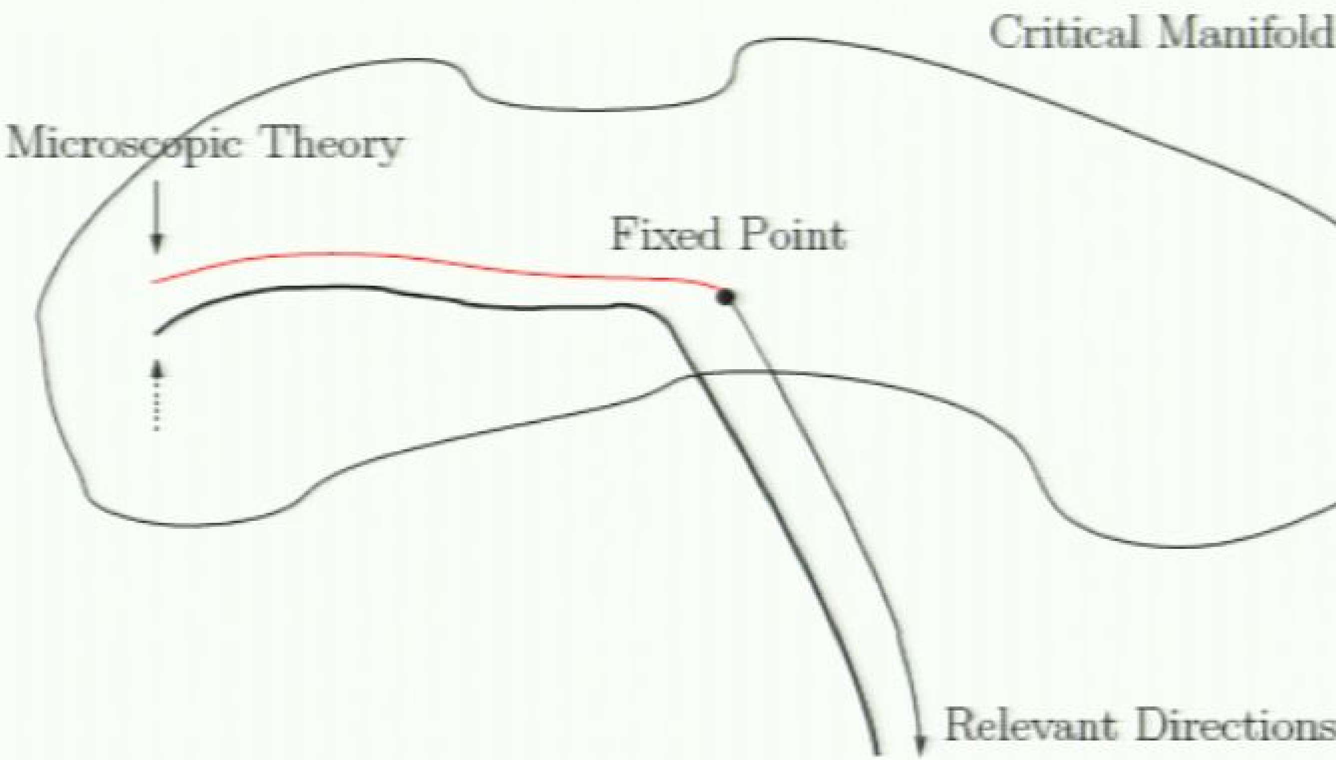

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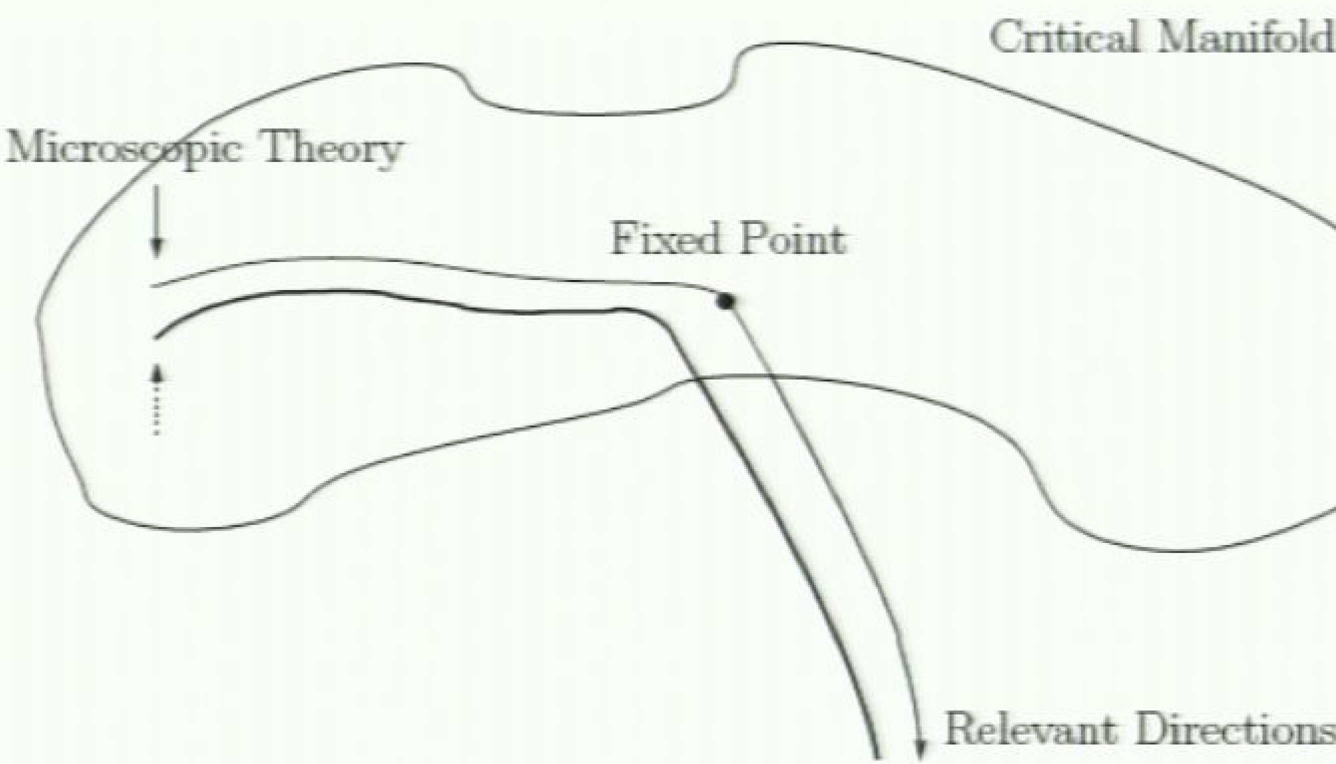

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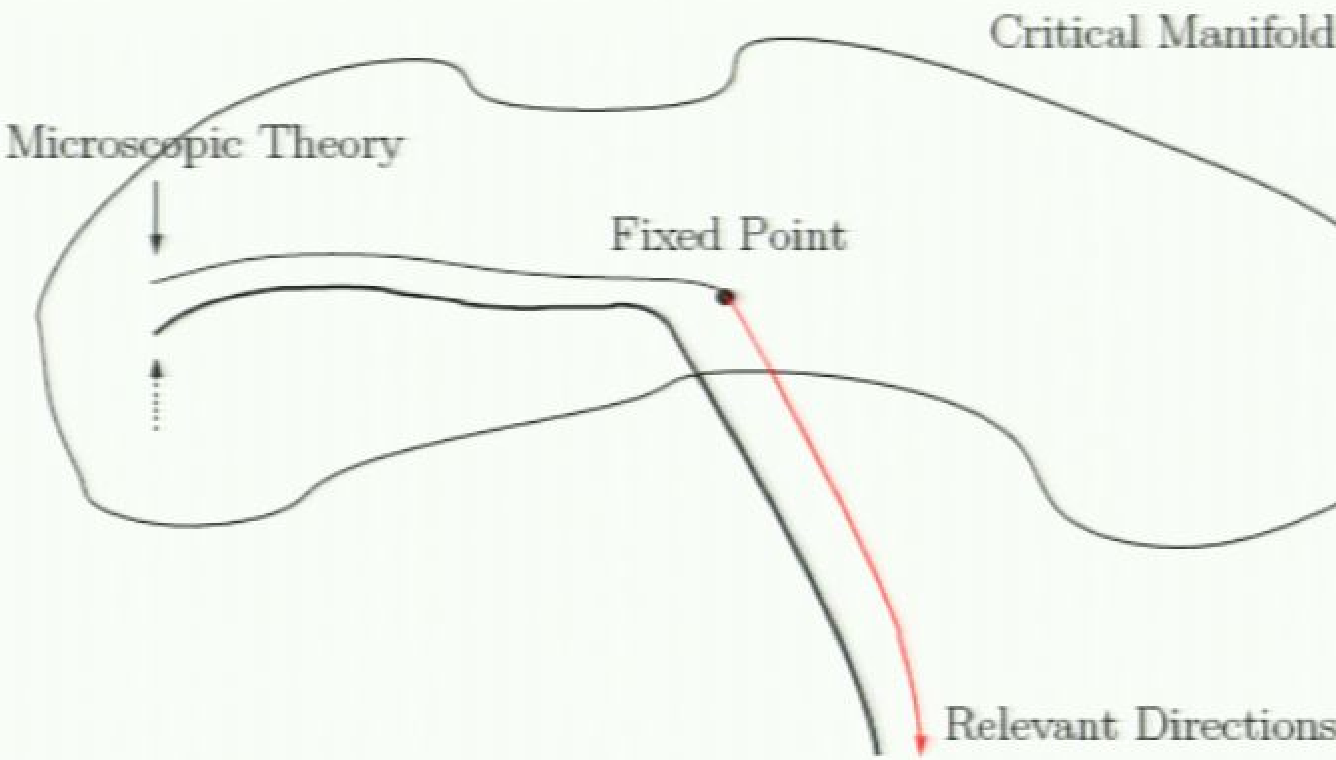

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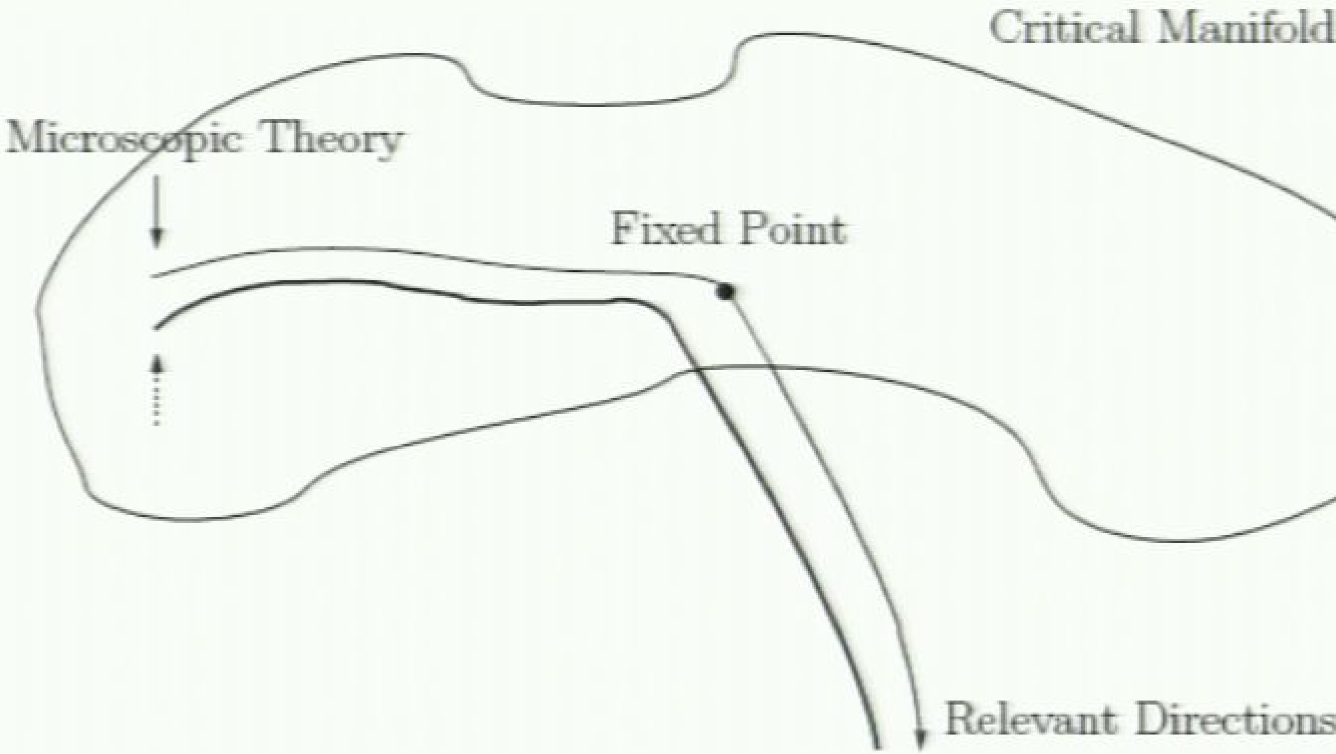

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- The critical manifold is spanned by the irrelevant operators
- Flows along the relevant directions leave the critical surface
- If there are *n* relevant directions, then we must tune *n* quantities to get on to the critical surface

The Wilsonian Effective Action

Start with the partition function

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$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

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Ingredients of ERG Transformation

- Blocking (coarse-graining)
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Implementing Rescaling

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- Remember to take account of anomalous dimensions!
- Ie $X \to X\Lambda^{\text{full scaling dimension}}$
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- Fixed points: $\partial_t S_\star = 0$

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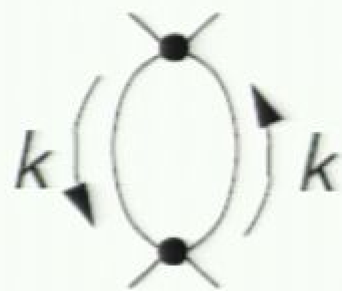

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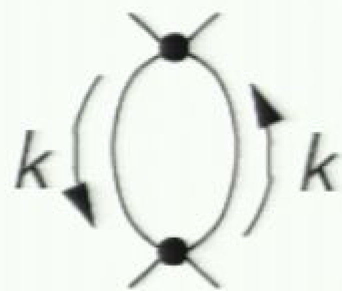

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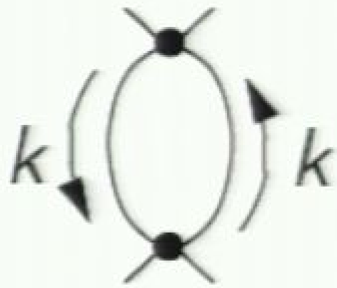

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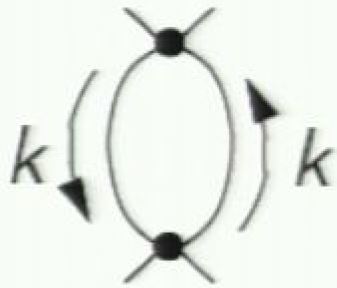

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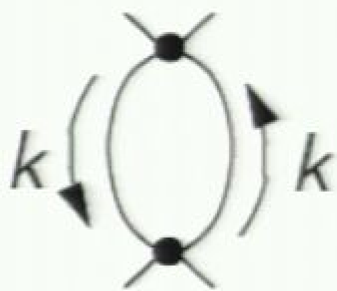

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- The ERG is a natural tool to study renormalizability
  - It has a built in cutoff
  - It relates physics at different scales

Continuum Limits I

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The Question

Are there effective actions $S_{\Lambda,\Lambda_0}[\varphi]$ for which we can safely send $\Lambda_0 \to \infty$?

The Simplest Answer

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Are there effective actions $S_{\Lambda,\Lambda_0}[\varphi]$ for which we can safely send $\Lambda_0 \rightarrow \infty$?

The Simplest Answer

- Rescale all quantities, using Λ
- Only dimensionless variables appear
- Fixed points of the ERG correspond to continuum limits!

$$\partial_t S_\star[\varphi] = 0$$

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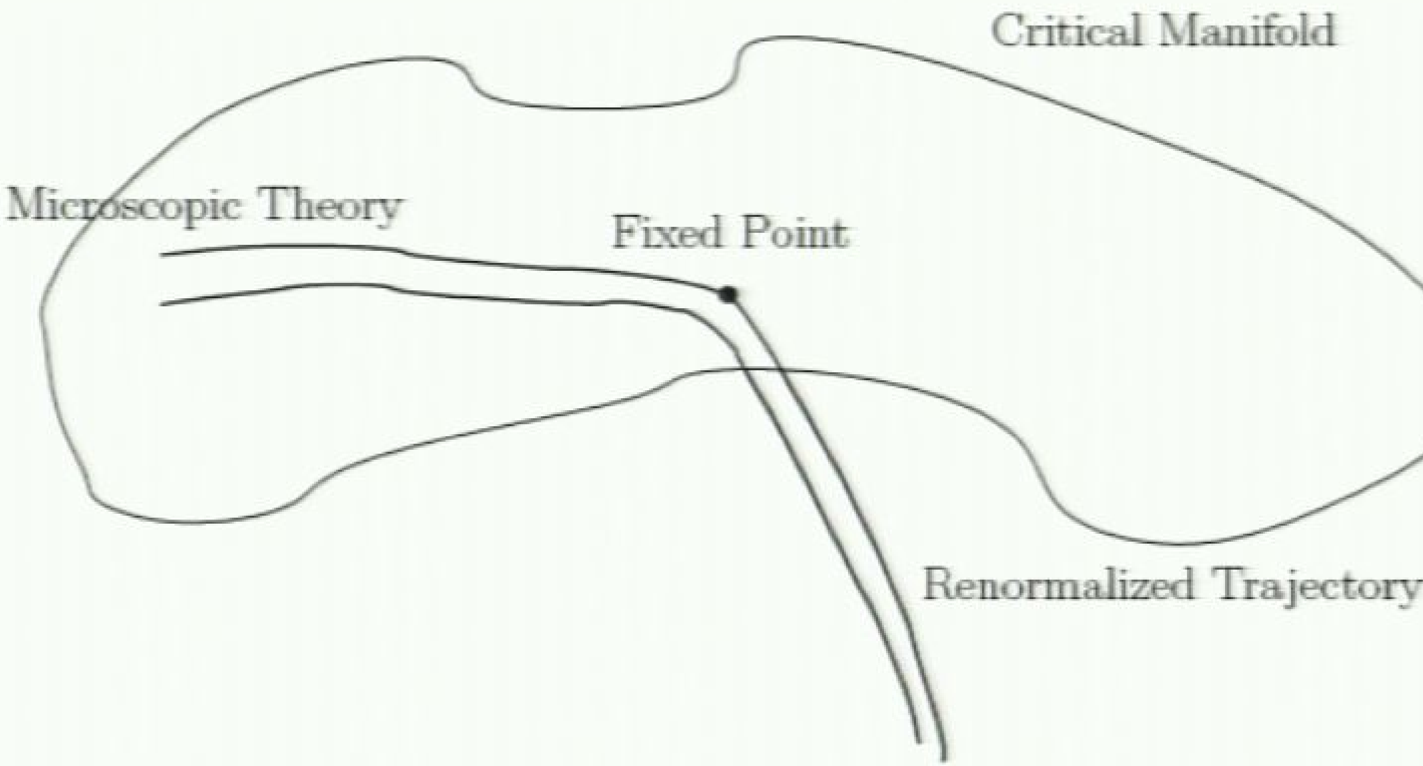
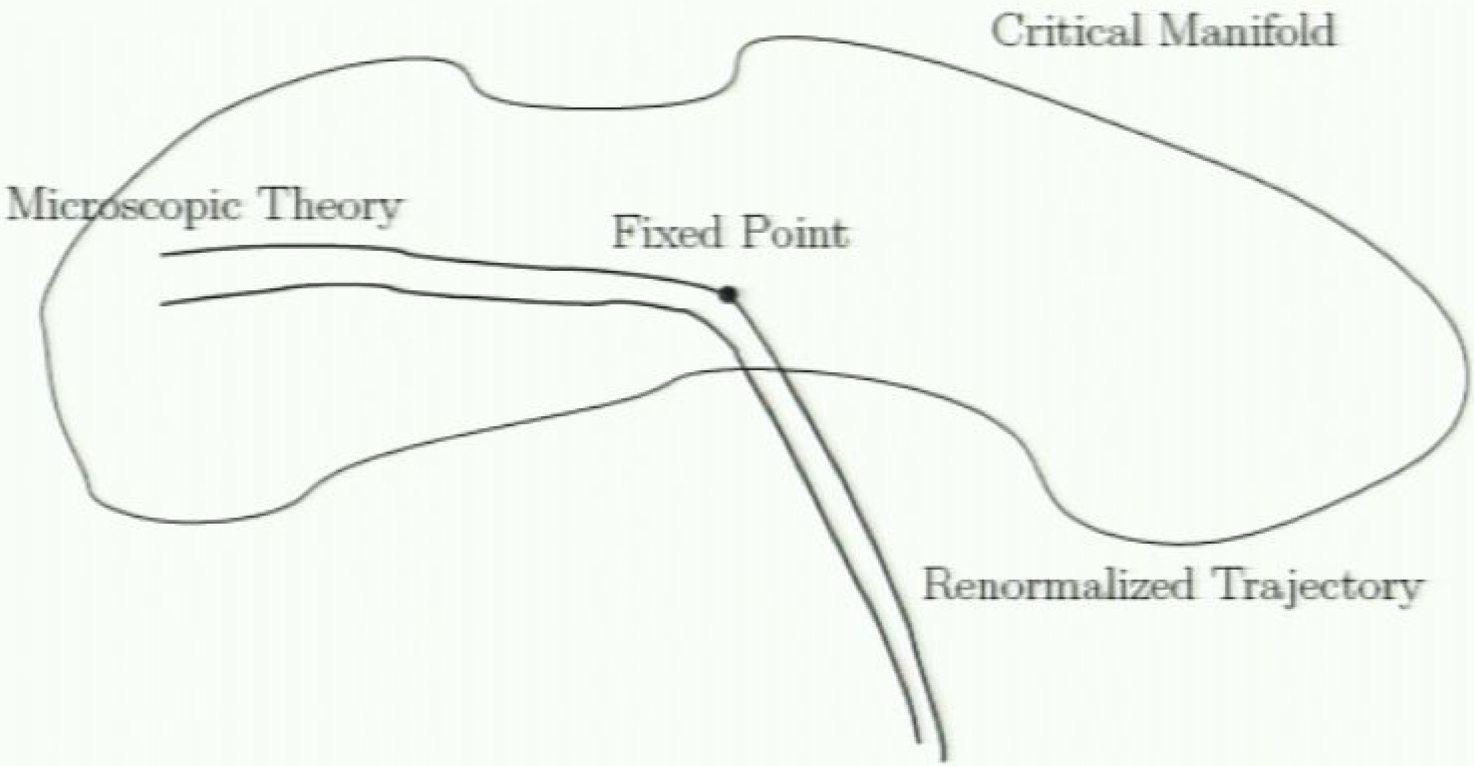


Continuum Limits II



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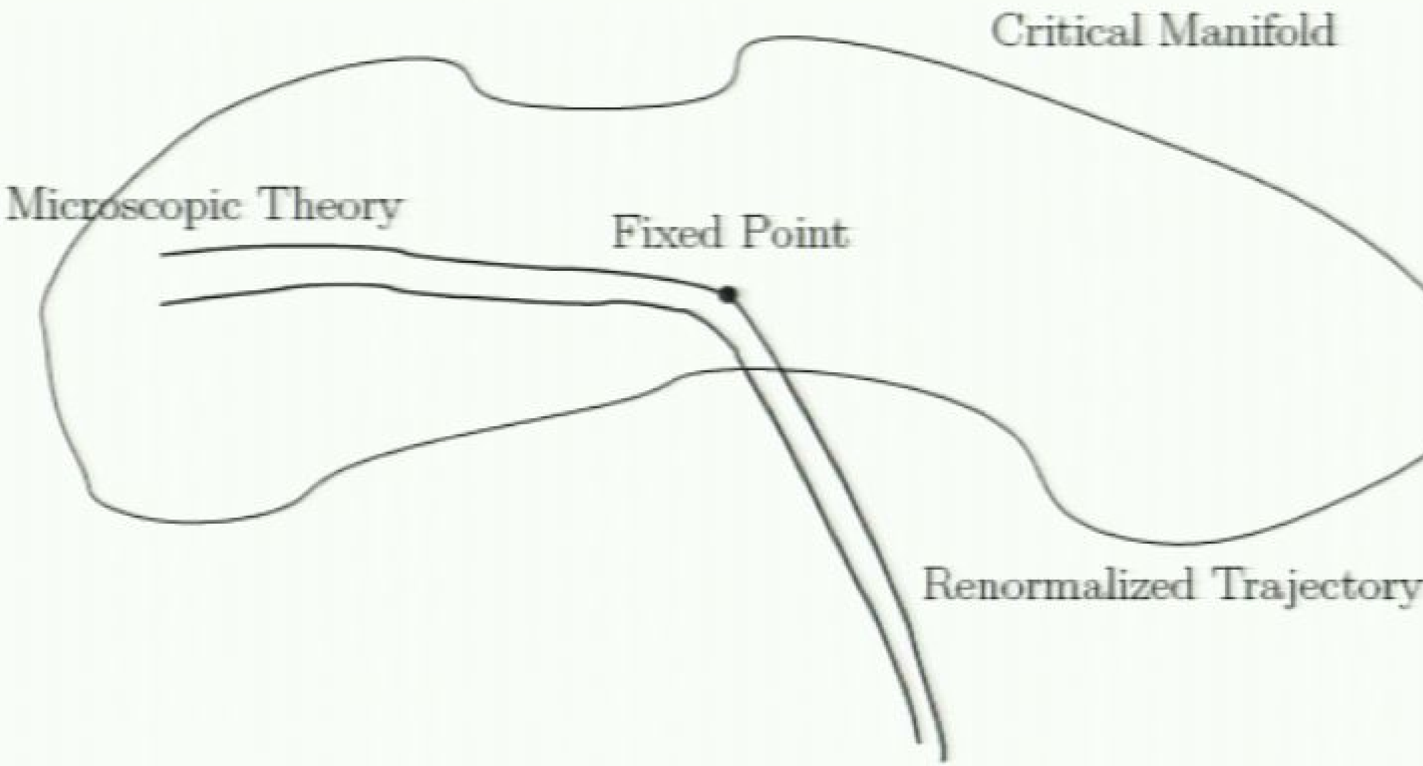

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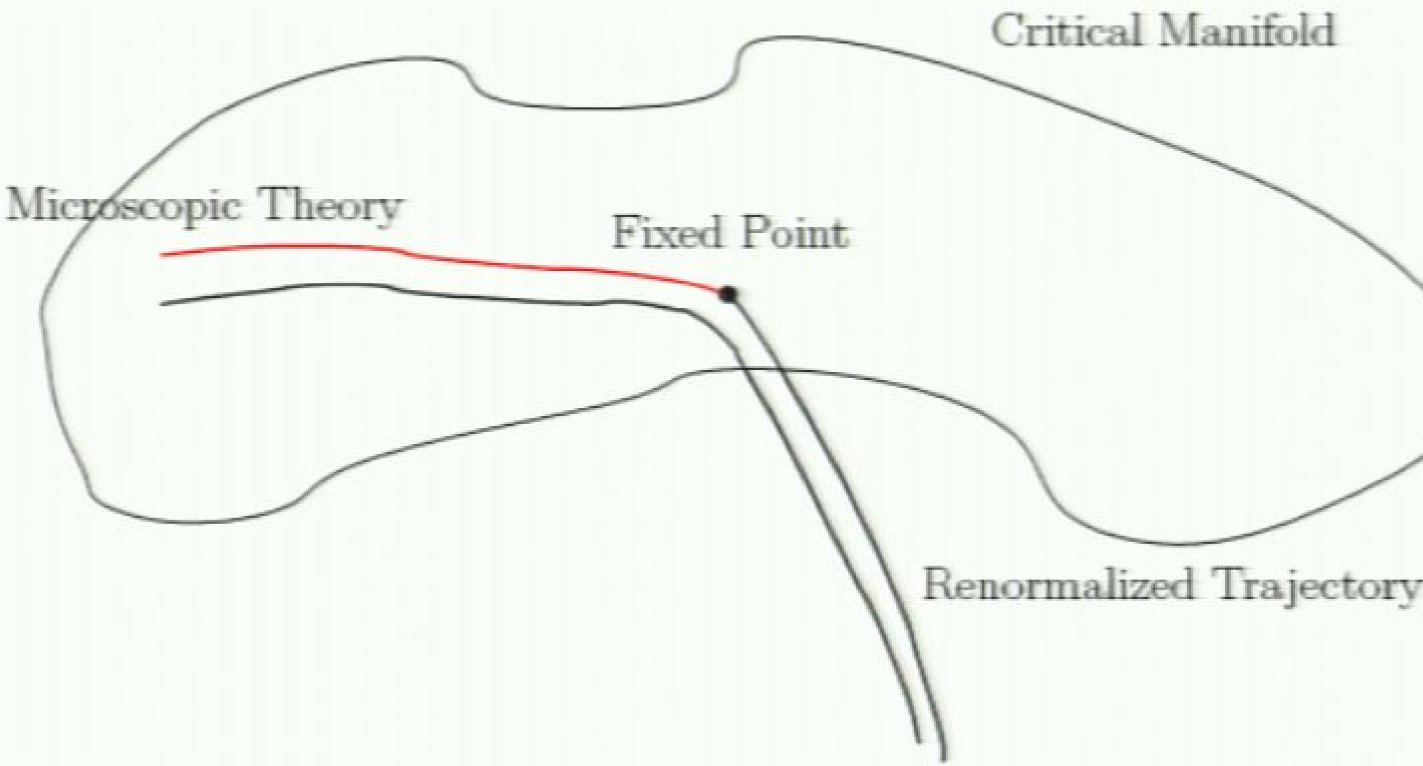

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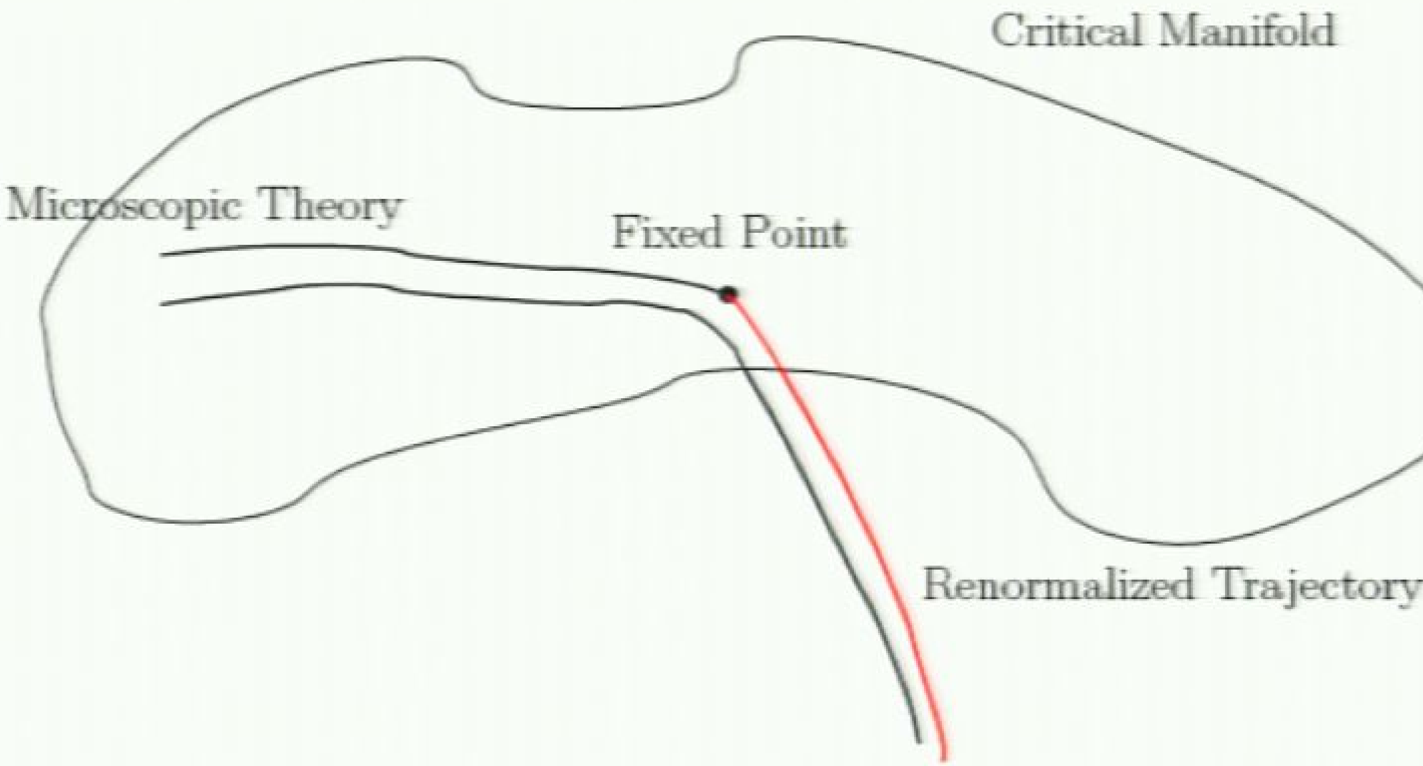

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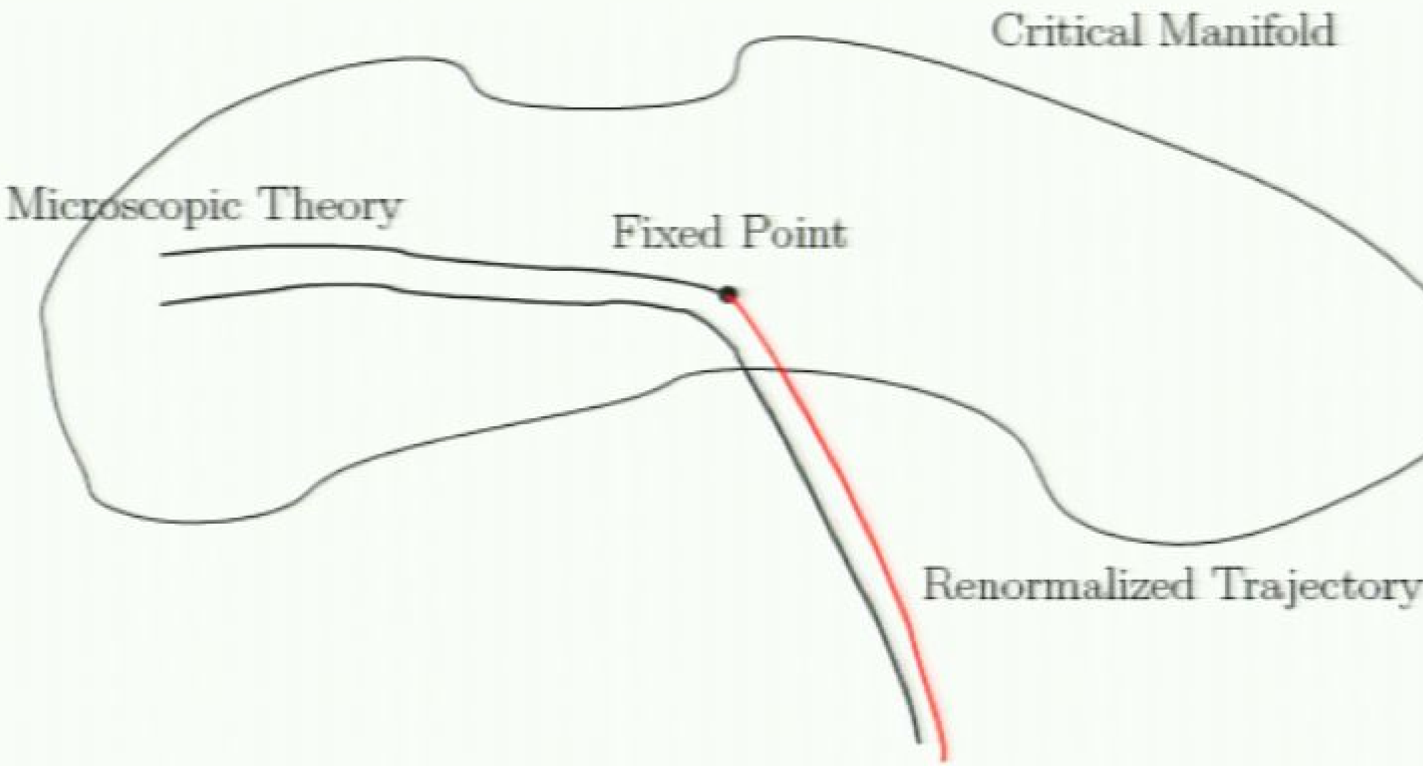

- Tune the trajectory towards the critical surface, as $\Lambda_0 \to \infty$
- The trajectory splits in two:
  - One part sinks into the fixed point
  - One part emanates out
- Actions on the RT are renormalizable

The Key Point

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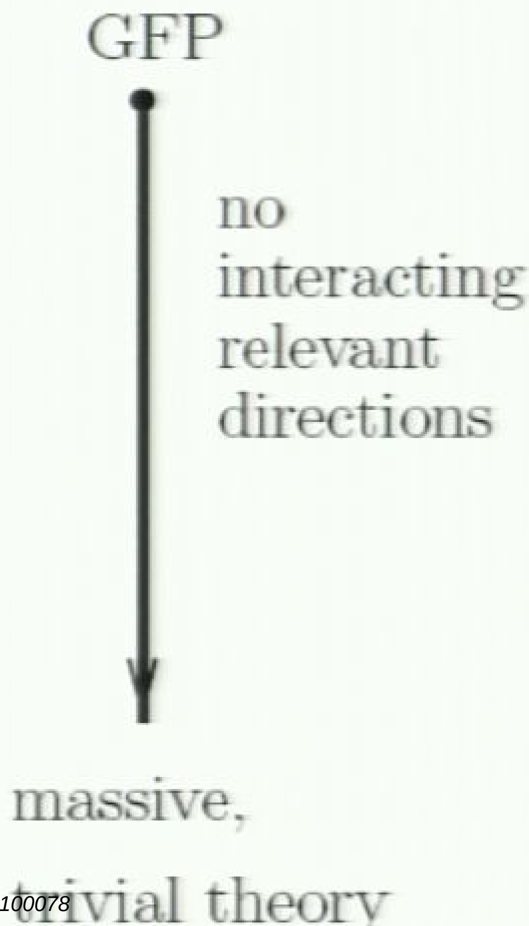
Triviality   Asymptotic Freedom   Asymptotic Safety



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GFP

no interacting relevant directions

massive, trivial theory

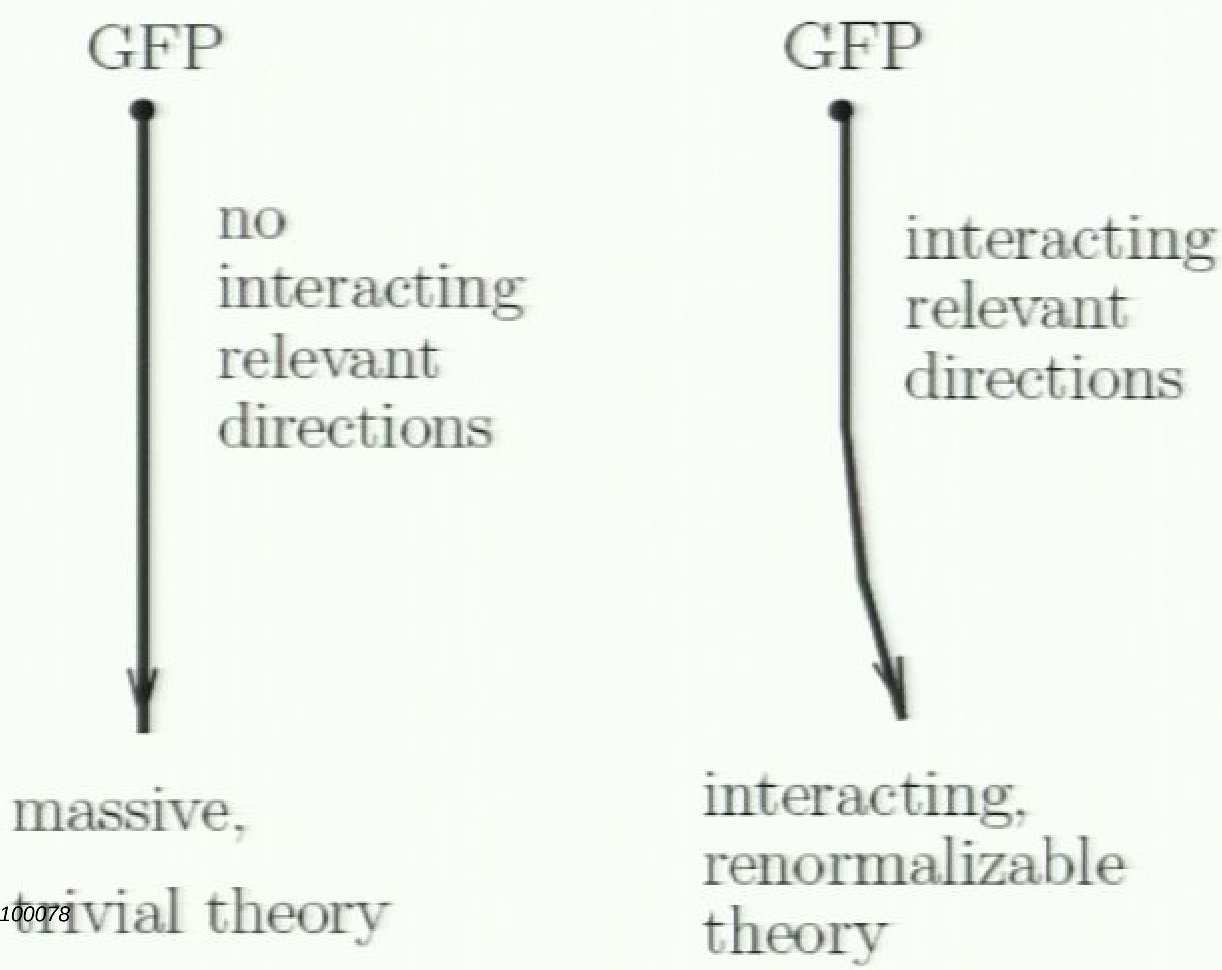
GFP

interacting relevant directions

interacting, renormalizable theory

Asymptotic Freedom etc.

Triviality | Asymptotic Freedom | Asymptotic Safety



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interacting, renormalizable theory

Asymptotic Safety

NT FP

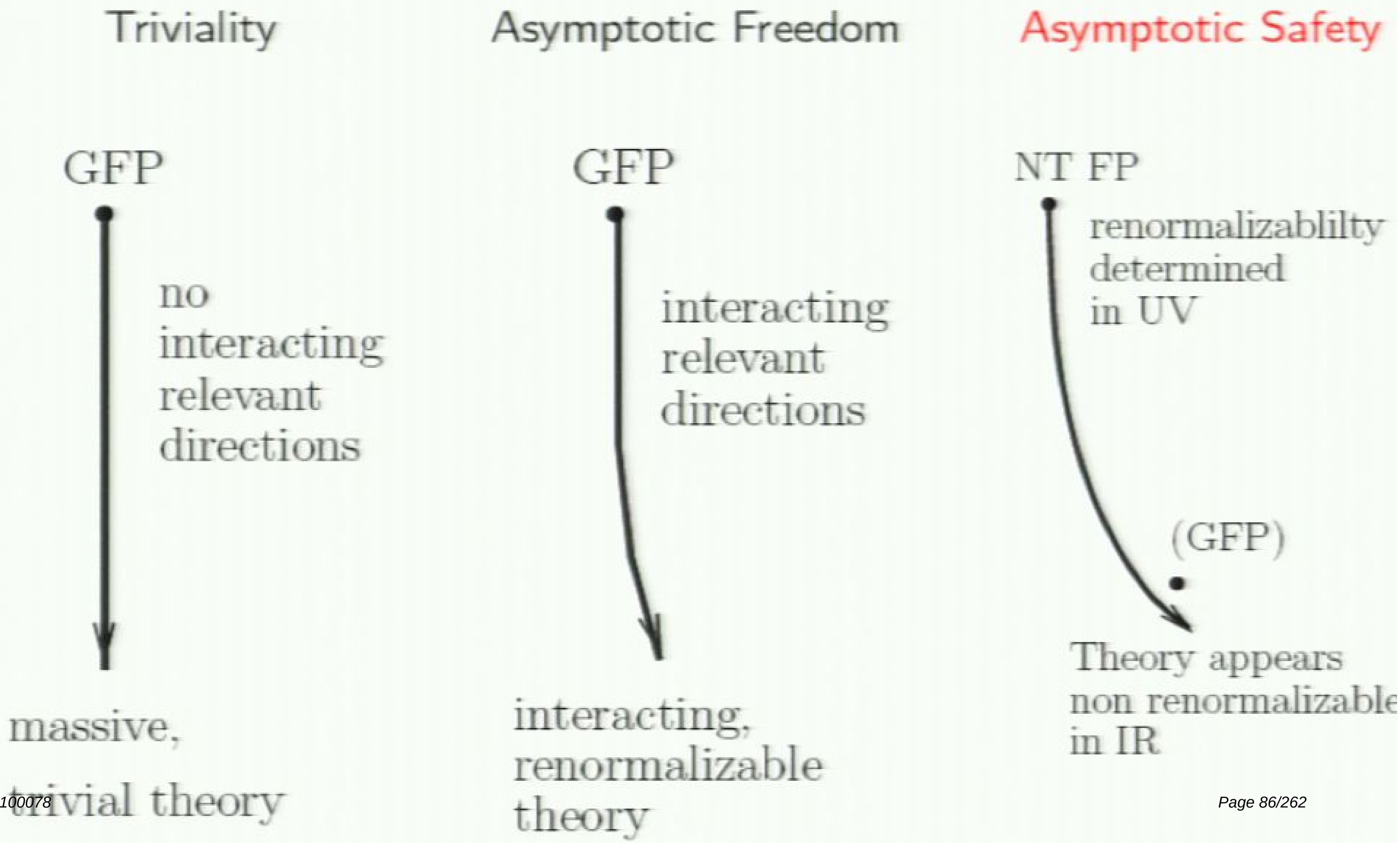
renormalizablilty determined in UV

(GFP)

Theory appears non renormalizable in IR

Scalar Field Theory in $D = 4$

Asymptotic Freedom etc.



Scalar Field Theory in $D = 4$

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The Gaussian Fixed Point

- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant

Other Fixed Points

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Implementing a Cutoff

1. Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} - \frac{C(p,\Lambda_0)}{p^2}$$

2. Introduce the effective cutoff, Λ

$$C(p,\Lambda_0) = C_{\mathrm{UV}}(p,\Lambda) + C_{\mathrm{IR}}(p,\Lambda,\Lambda_0)$$

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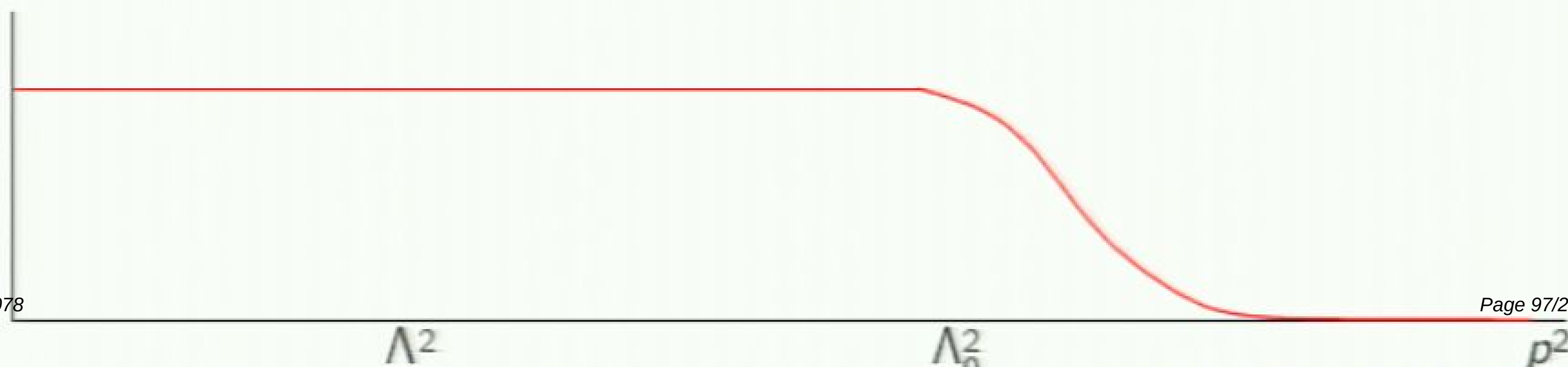

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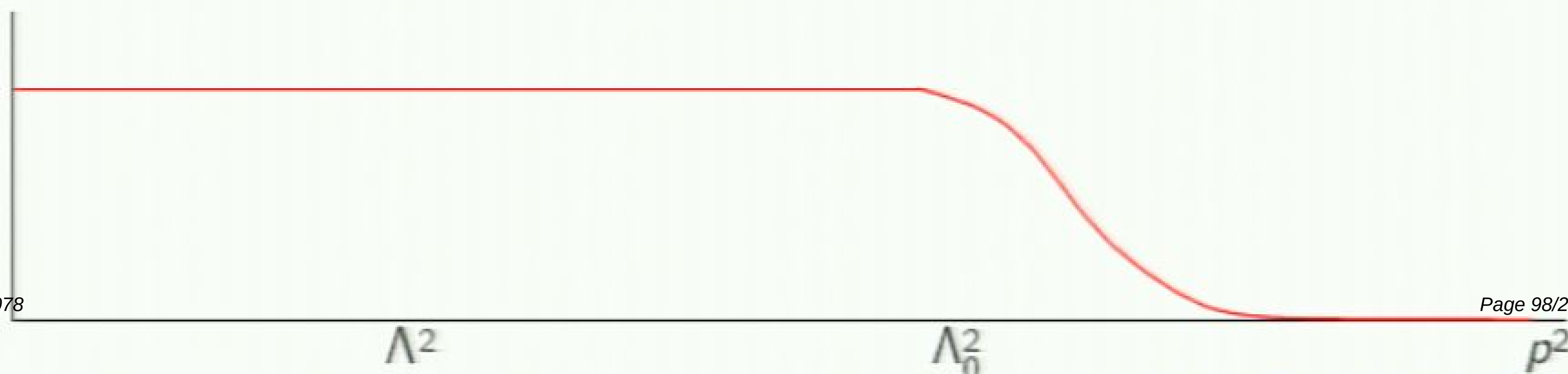

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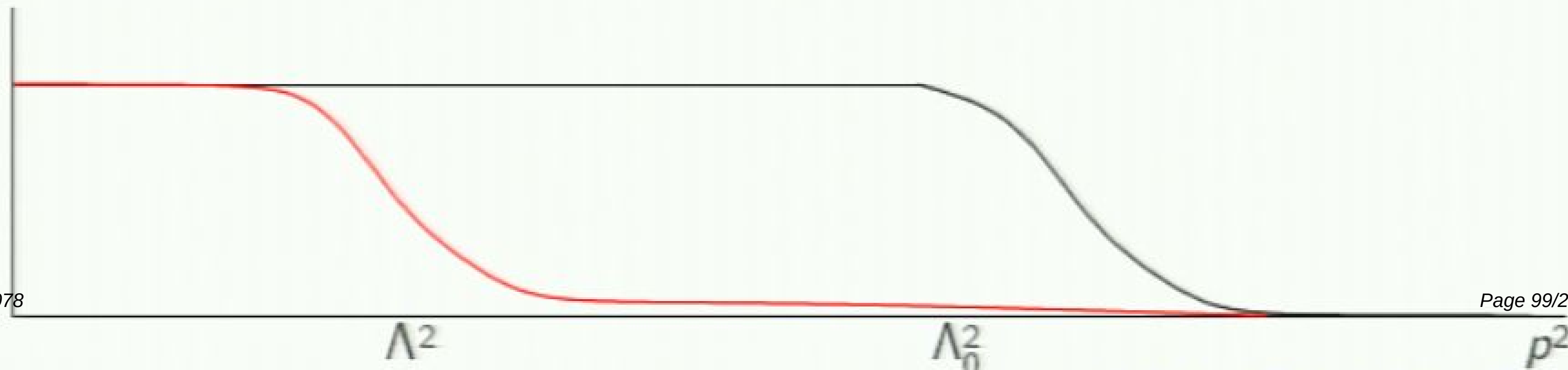

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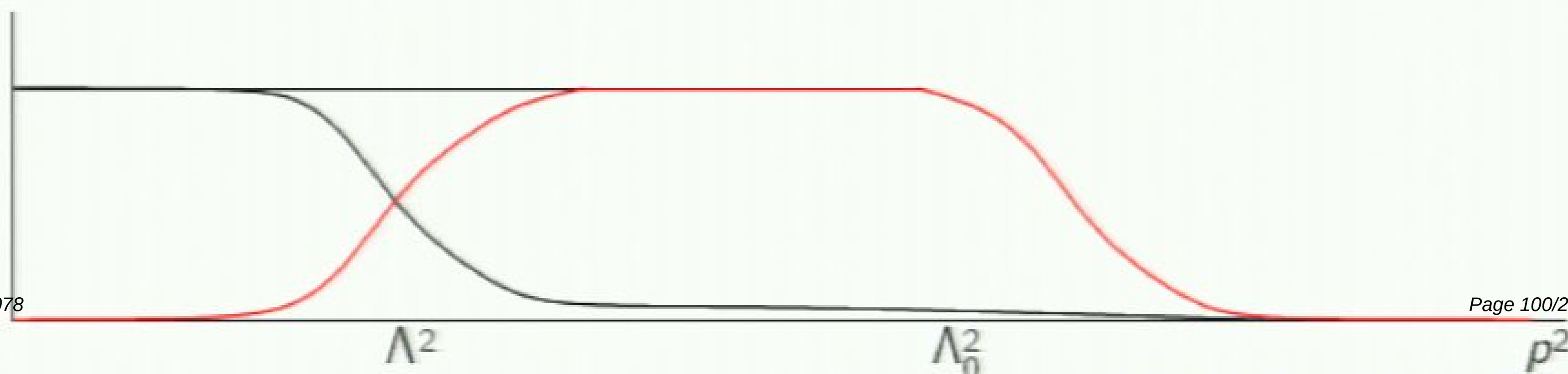

Polchinski Equation

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda\partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2}\frac{\delta S_\Lambda^{\text{int}}}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta S_\Lambda^{\text{int}}}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta S_\Lambda^{\text{int}}}{\delta\varphi}$$

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&= \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi} + \frac{1}{2}\varphi\cdot\Delta^{-1}\cdot\dot{\Delta}\cdot\Delta^{-1}\cdot\varphi \\
&= \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi} + \frac{1}{2}\varphi\cdot\Lambda\partial_\Lambda\Delta^{-1}\cdot\varphi
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- Up to a discarded vacuum energy term:

$$-\Lambda\partial_\Lambda S = \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi}$$

Recasting the Polchinski Equation II

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Very General ERGs

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- defines our ERG

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1. Qualitative Aspects of the ERG
   - The Basic Ideas
2. Renormalizability
   - Continuum Limits
3. ERG Equations
4. Triviality
   - Correlation Functions
   - Technicalities
   - Application to Fixed Points
5. Conclusion

The Dual Action

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Definition

- $-\mathcal{D}[\varphi] = \ln\left[\exp\left(\frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\Delta\cdot\frac{\delta}{\delta\varphi}\right)e^{-S_{\Lambda}^{\text{int}}[\varphi]}\right]$

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Philosophy

- I want to manipulate the expression

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- I will expand the exponentials in the dual action
- I will expand the Wilsonian effective action in powers of the field
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- None of the series are ever truncated
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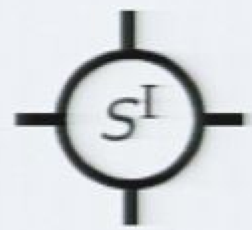
- Wilsonian Effective Action

$S^{\text{int}}_{\Lambda}[\varphi] = \frac{1}{2}$ 

$\varphi^2 + \frac{1}{4!}$ 

$\varphi^4 + \cdots$

Diagrammatics

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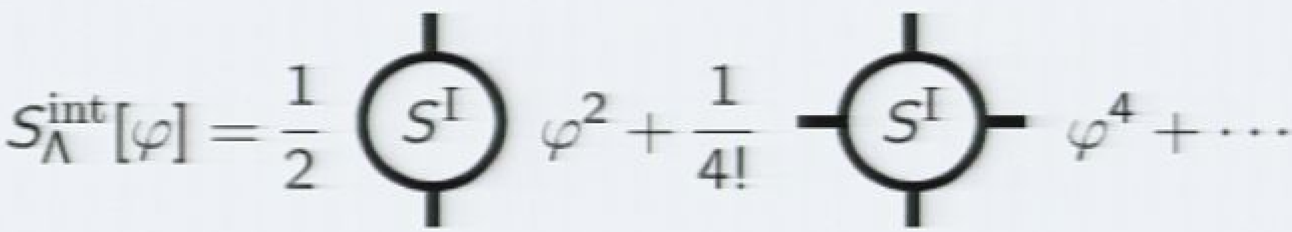

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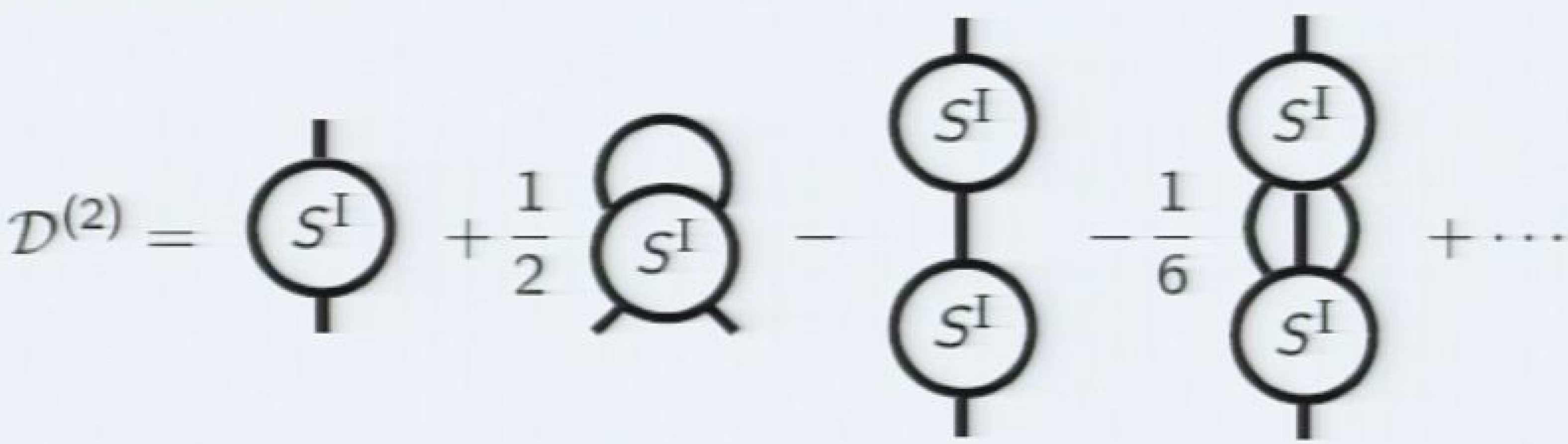

$n > 2$-Point Correlation Functions

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- Consider constructing $n > 2$-point connected correlation functions from the bare action

Example

- $G(p_1, \ldots, p_n) = -\mathcal{D}^{(n)}(p_1, \ldots, p_n) \prod_{i=1}^{n} \Delta_b(p_i), \qquad n > 2.$
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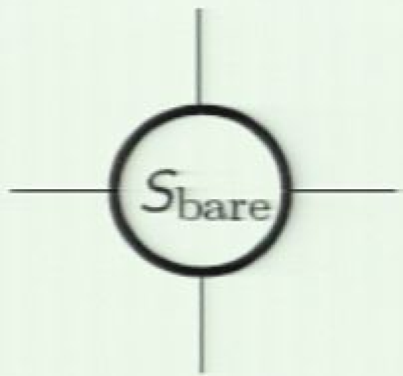
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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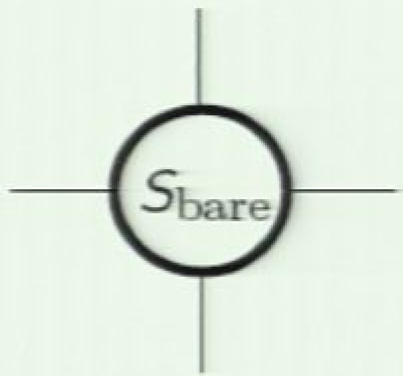
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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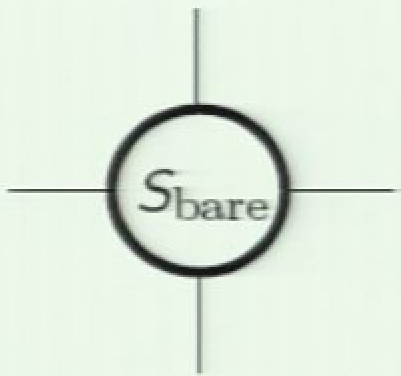
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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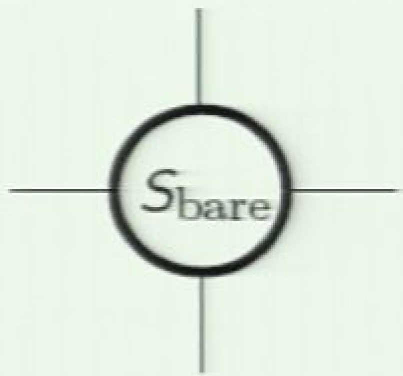
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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- Consider constructing the 2-point connected correlation functions from the bare action
- The first contribution is Δ_b
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Notation

- Define $\overline{\mathcal{D}}^{(n)}$ to be the 1PI pieces of $\mathcal{D}^{(n)}$

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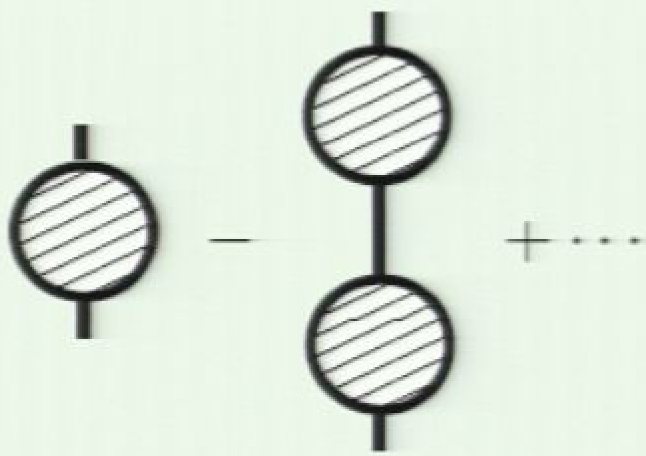
- $\mathcal{D}^{(2)} =$ 

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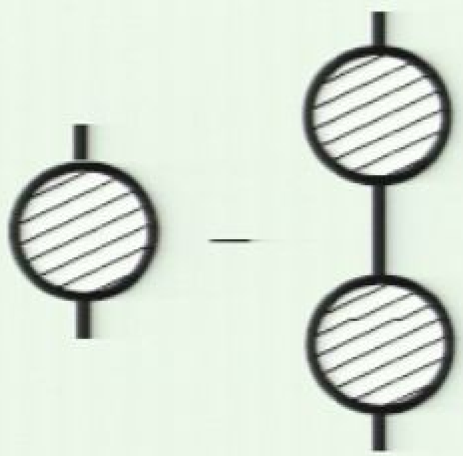
- $\mathcal{D}^{(2)} =$  $+ \cdots$

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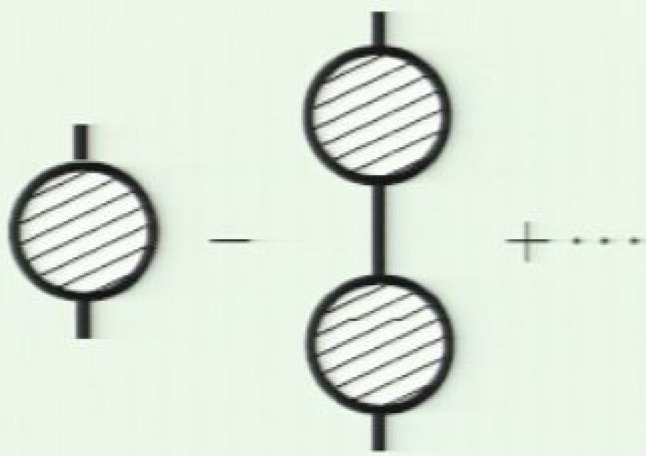
- $\mathcal{D}^{(2)} =$ 

- $$\mathcal{D}^{(2)}(p) = \frac{\overline{\mathcal{D}}^{(2)}(p)}{1 + \Delta(p)\overline{\mathcal{D}}^{(2)}(p)}$$

Dressed Effective Propagator

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Definition

Interpretation

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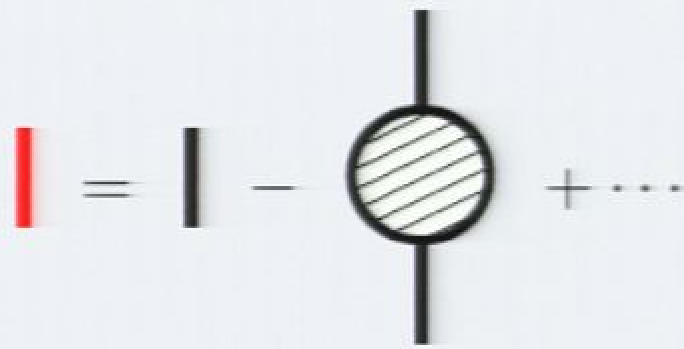
- 

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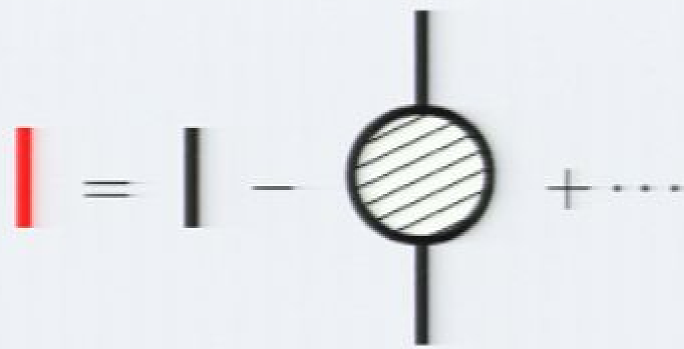
- 
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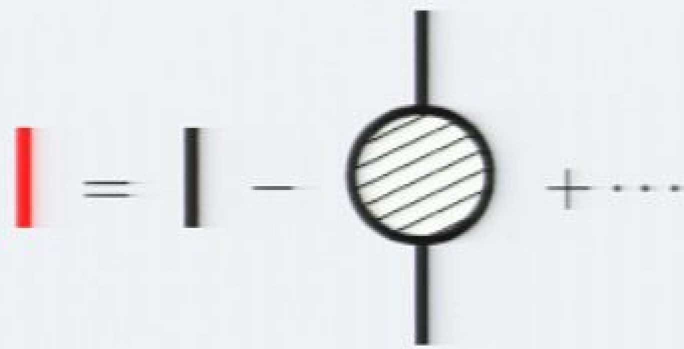
- 

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- We want to investigate fixed points
- It is convenient to rescale to dimensionless variables

$$\varphi \to \varphi\sqrt{Z}\Lambda^{(D-2)/2}$$

$$p \to p\Lambda$$

- But scaling out the anomalous dimension produces an annoying change to the Polchinski equation!
- This factor of Z also appears in the dual action

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  - $\eta \equiv \Lambda\frac{d\ln Z}{d\Lambda}$
- The dual action is defined as before
- But its flow is different

$$-\left(\Lambda\partial_\Lambda + \frac{\eta}{2}\varphi\cdot\frac{\delta}{\delta\varphi}\right)\mathcal{D}[\varphi] = -\frac{\eta}{2}\varphi\cdot\Delta^{-1}\cdot\varphi$$

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A More Convenient Flow Equation

- After rescaling $\varphi \to \varphi\sqrt{Z}$ choose the blocking functional, Ψ, such that

$$\left(-\Lambda\partial_\Lambda + \frac{\eta}{2}\varphi \cdot \frac{\delta}{\delta\varphi}\right) S = \frac{1}{2}\frac{\delta S}{\delta\varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta\varphi}$$

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$$\Delta_\partial \equiv D + \int \frac{d^D p}{(2\pi)^D}\varphi(p) p \cdot \frac{\partial}{\partial p}\frac{\delta}{\delta\varphi(p)}.$$

- The rescaled effective propagator is independent of t

$$\Delta(p) = \frac{c(p^2)}{p^2}$$

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- Set $\partial_t \mathcal{D}_\star^{(2)}(p) = 0$
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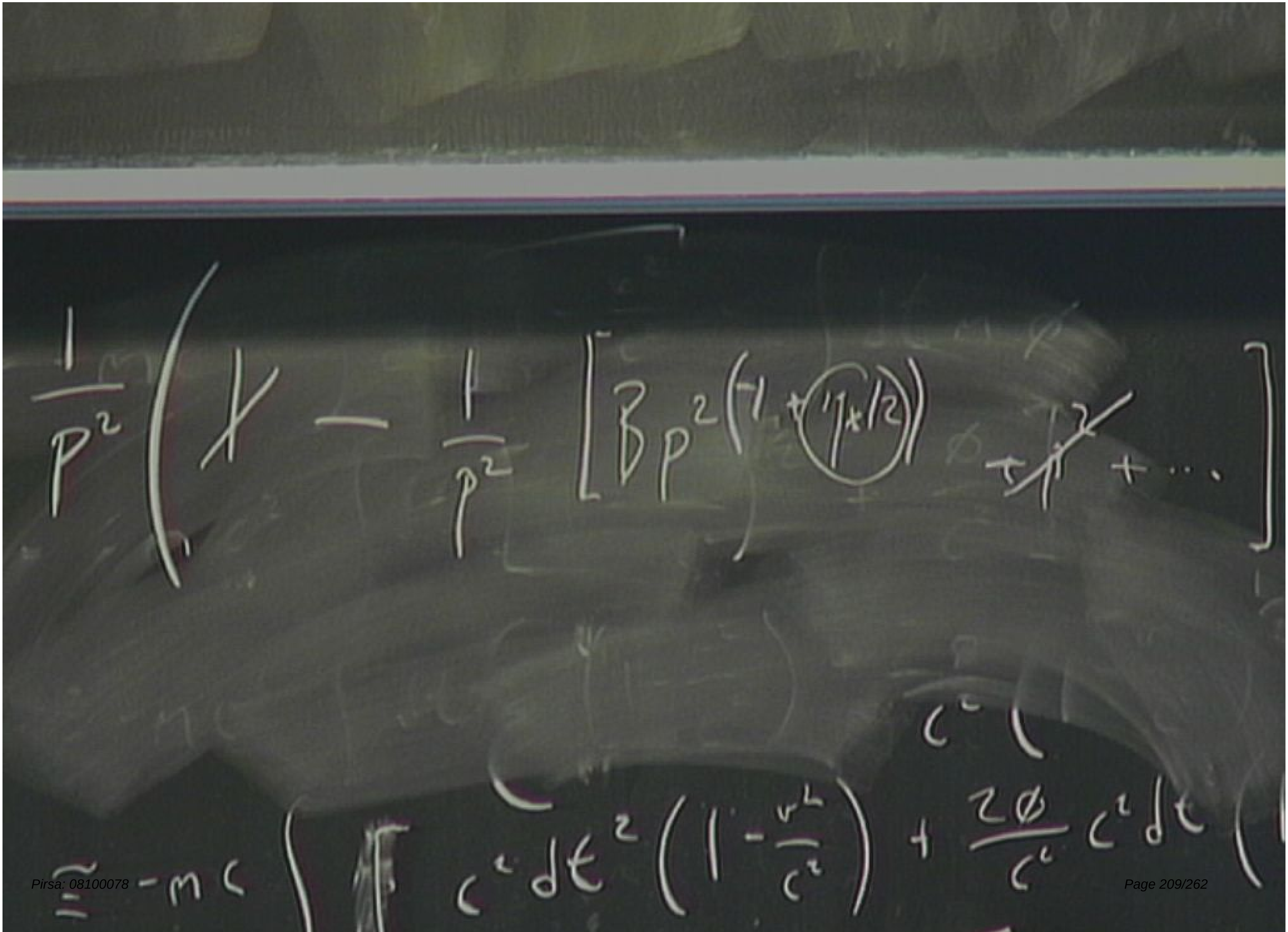
1/p² (1 − 1/p² [B p² (1 + 1/2) + …]
≅ −mc ∫ [c² dt² (1 − v²/c²) + 2φ/c² c² dt

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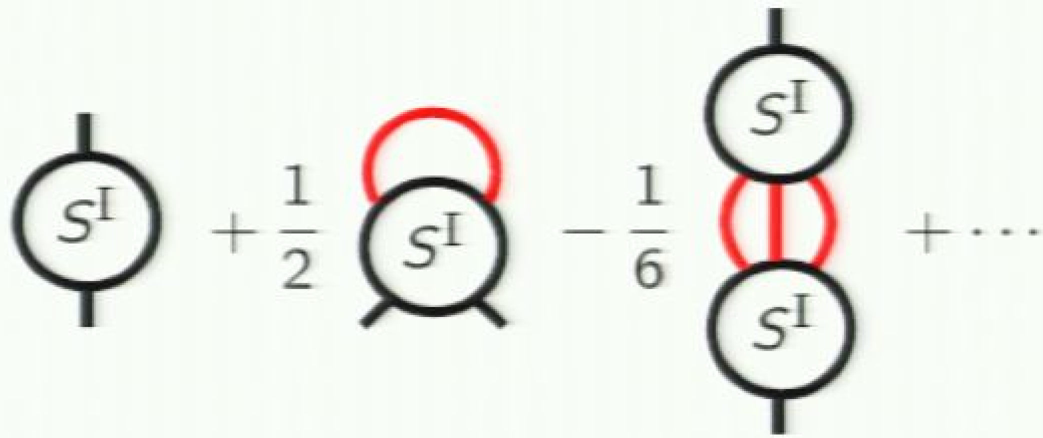

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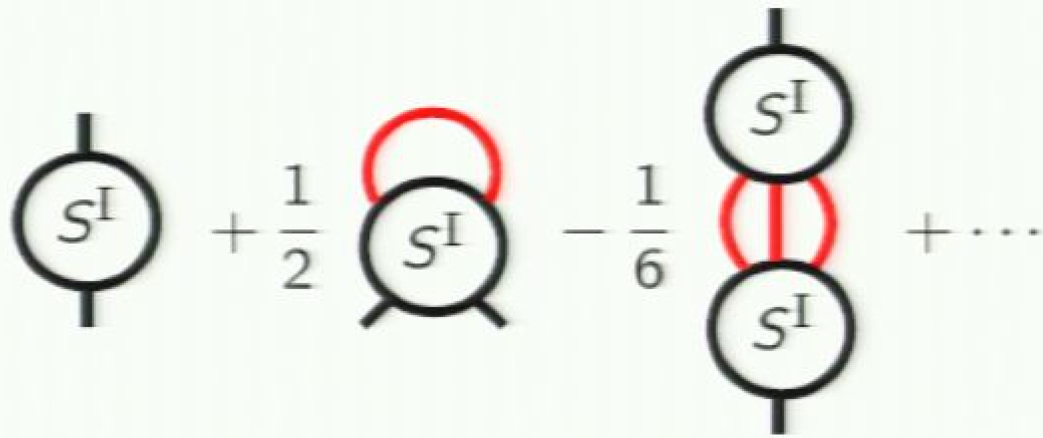

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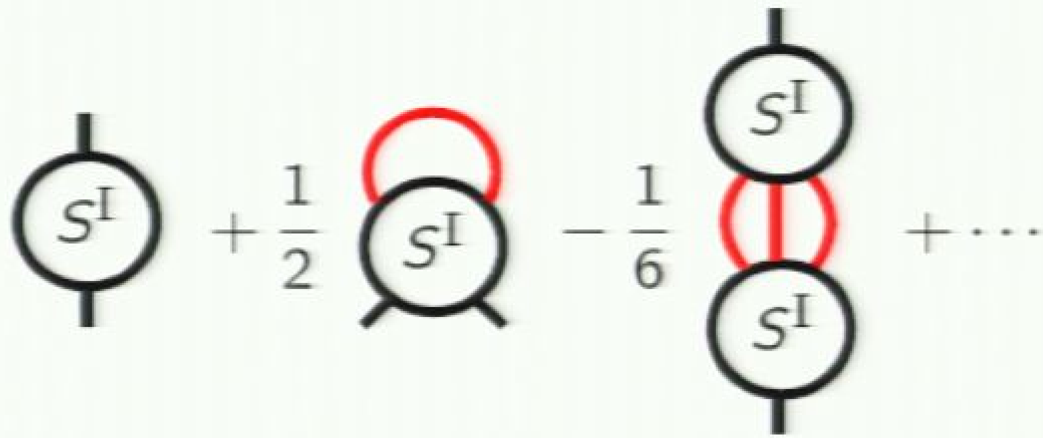

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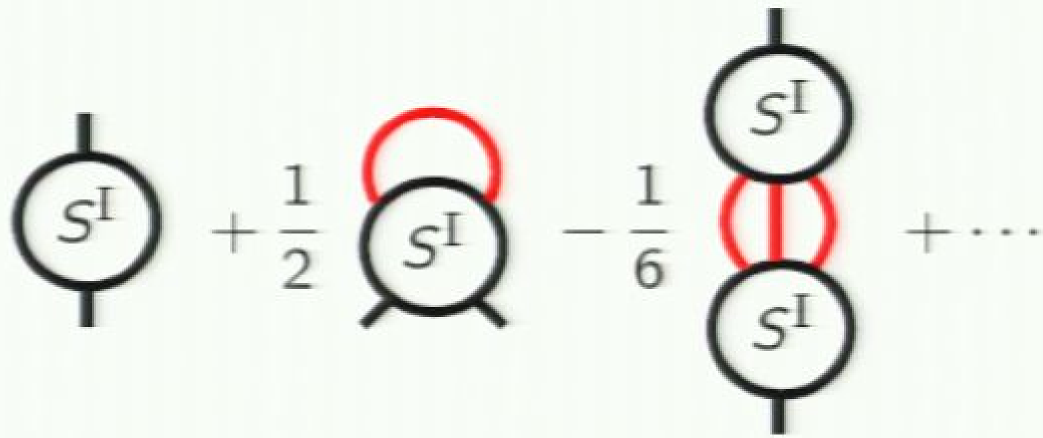

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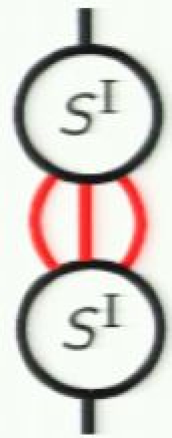

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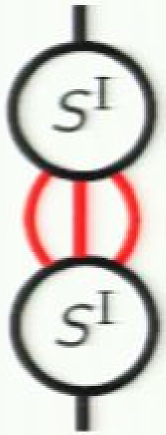
$X(p) \equiv$ 

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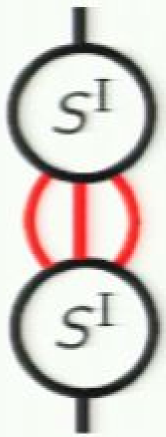
$X(p) \equiv$  $\sim \int \frac{d^D l}{(2\pi)^D} \int \frac{d^D k}{(2\pi)^D} \frac{1}{[l^2(l+k)^2(k+p)^2]^{1-\eta_\star/2}}$

IR Power Counting

- Consider $X(p)$ for small p
- Check for IR divergences
- Net power of momenta $= 2D - 6(1 - \eta_\star/2)$
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- $X(p) = a'' + \ldots$

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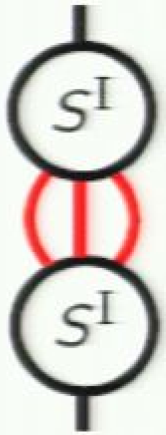
$X(p) \equiv$ 

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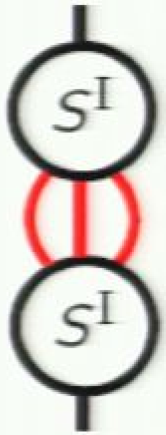

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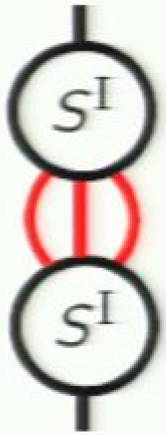
$$X(p) \equiv$$ 

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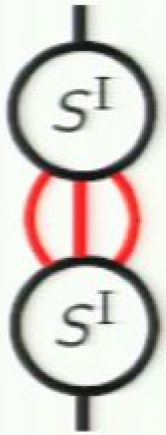
$X(p) \equiv$ 

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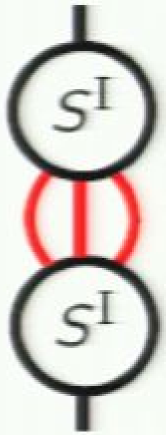
$$X(p) \equiv$$ 

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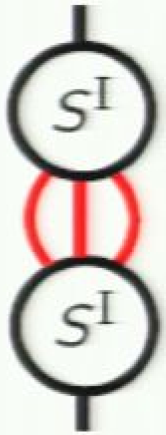
$X(p) \equiv$ 

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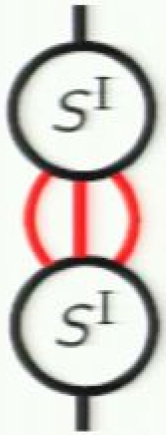
$X(p) \equiv$ 

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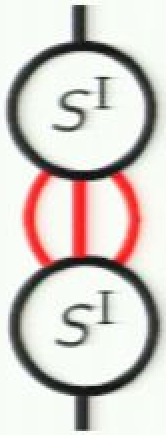
$X(p) \equiv$ 

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IR Power Counting

- Power counting generalizes to all diagrams
- For $\epsilon = 0$ and $\eta_\star > 0$ OR $\epsilon > 0$ and $\eta_\star \geq 0$

$$\overline{\mathcal{D}}_\star^{(2)}(p) = c + dp^2 + \ldots$$

- But $\overline{\mathcal{D}}_\star^{(2)}(p) \sim p^{2(1-\eta_\star/2)}$
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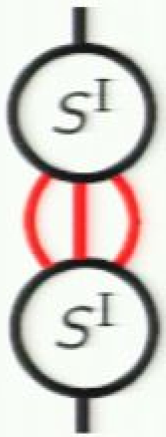

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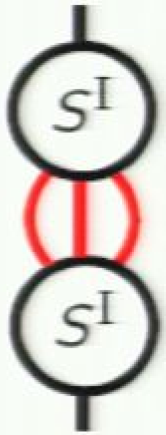
$X(p) \equiv$ 

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Triviality in $D \geq 4$

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- Consider $D = 4 + \epsilon$, $\eta_\star$ non-negative
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- Fixed points with $\eta_\star < 0$ certainly exist
- Wegner found a family with

$$S_\star \sim \frac{1}{2}\varphi \cdot p^{2(1-\eta_\star/2)} \cdot \varphi$$

  with $\eta_\star = -2, -4, \ldots$
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Thank you for listening

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