Title: M2-branes and AdS/CFT Correspondence

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Abstract: TBA

M2 Branes and AdS/CFT

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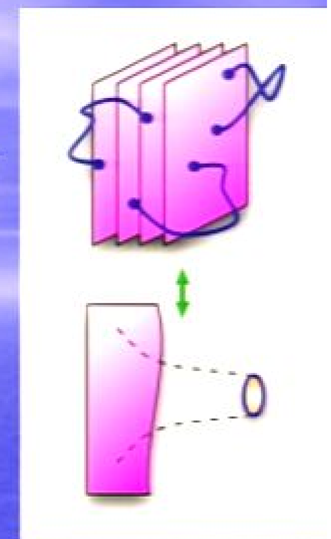
Introduction

- The gauge theory on coincident M2 branes has been a hot topic over the last few months.
- This is a long-standing problem: how to find the world volume theory on coincident super-membranes in 11-dimensional M-theory. This is harder than the description of D-branes in string theory that is known explicitly at small string coupling.
- But M-theory is inherently strongly coupled: one can think of it as the strong coupling limit of a 10-dimensional superstring theory. What to do?

D-Branes vs. Geometry

Dirichlet branes (Polchinski) realize maximally supersymmetric gauge theories.

A stack of N Dirichlet 3-branes realizes $\mathcal{N}=4$ supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)



$$ds^2 = \left(1 - \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) - \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 - r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by $L^4 = g_{YM}^2 N\alpha'$

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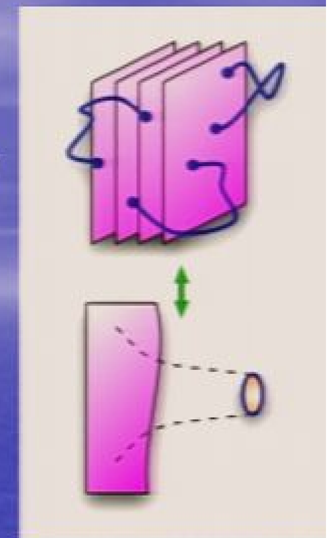

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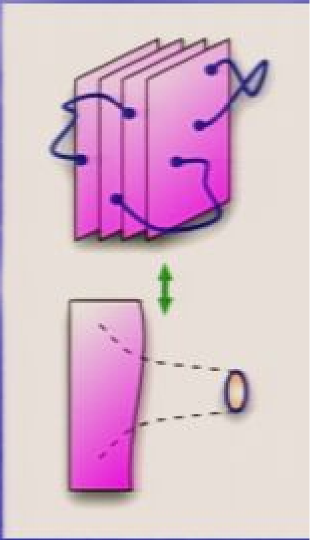

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Super-Conformal Invariance

- In the $\mathcal{N}$=4 SYM theory there are 6 scalar fields (it is useful to combine them into 3 complex scalars: Z, W, V) and 4 gluinos interacting with the gluons. All the fields are in the adjoint representation of the SU(N) gauge group.
- Comparing with QCD, the Asymptotic Freedom is canceled by the extra fields; the gauge coupling g_{YM} does not depend on the Energy. The theory is invariant under scale transformations x^μ -> a x^μ . It is also invariant under space-time inversions. Such a theory is called (super) conformal.

The AdS_5/CFT_4 Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}=4$ SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS_d space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \, .$$

- Its metric is $$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum^{d-2} (dx^i)^2 \right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\rm YM}^2 N}$

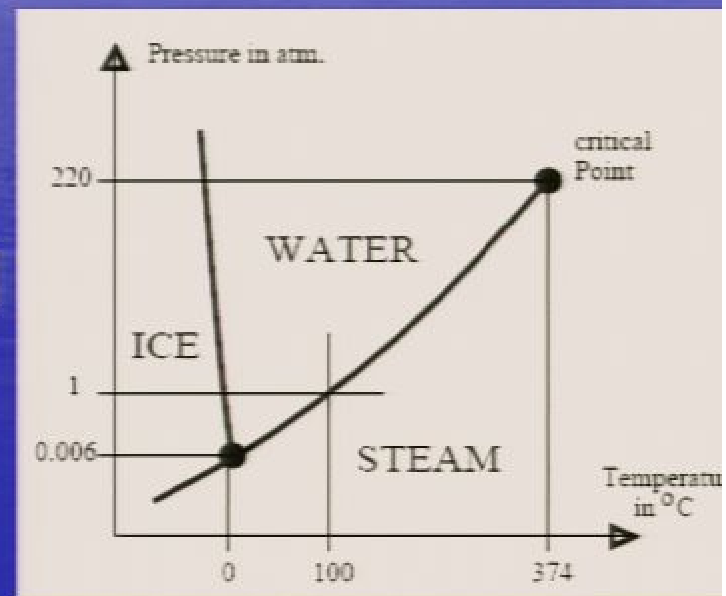
- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- The research on AdS_5/CFT_4 has rekindled interest in the maximally super-symmetric 4-d gauge theory and provided a host of information about its strongly coupled limit.
- This conformal gauge theory is becoming `The Harmonic Oscillator of 4-d Gauge Theory' in that it may be exactly solvable.
- It is a `cousin of Quantum Chromo-Dynamics' and has served as a guide, for example, to some phenomena observed in Heavy Ion Colliders.

AdS_4/CFT_3

- Besides describing all of known particle physics, Quantum Field Theory is important for understanding the vicinity of certain phase transitions, such as the all-important water/vapor transition.



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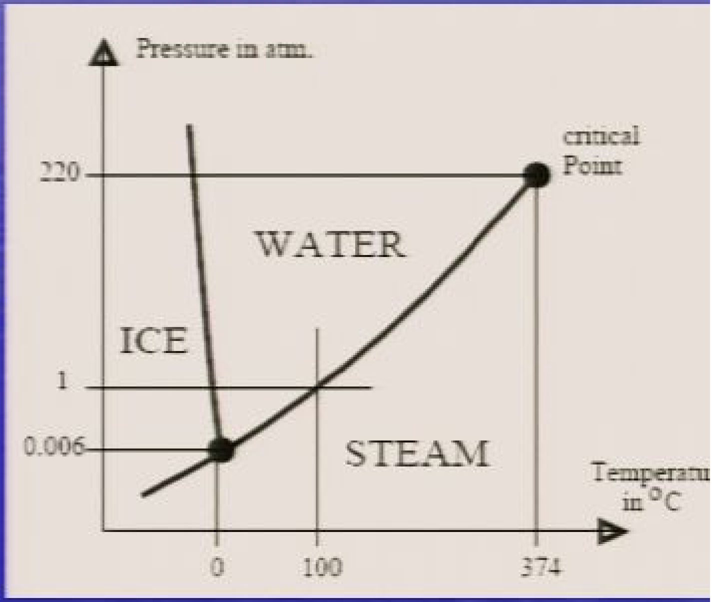

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- This transition is in the 3-d Ising Model Universality Class.
- Other common transitions are described by 3-d QFT with O(N) symmetry.
- 3-d theories are also very important in describing 2-d quantum systems, such as those in the Quantum Hall effect, high-Tc superconductors, etc.
- Can we find a `Harmonic Oscillator' of 3-d Conformal Field Theory ?

O(N) Sigma Model

- Describes 2nd order phase transitions in statistical systems with O(N) symmetry.

$$S = \int d^3x \left[\frac{1}{2}(\partial_\mu \phi^a)^2 + \frac{\lambda}{2N}(\phi^a \phi^a)^2 \right]$$

- IR fixed point can be studied using the Wilson-Fisher expansion in ϵ=4-d.
- The model simplifies in the large N limit since it possesses conserved currents

$$J_{(\mu_1 \cdots \mu_s)} = \phi^a \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^a + \ldots$$

Higher Spin Gauge Theory

- An AdS_4 dual of the large N sigma model was proposed. IK, Polyakov (2002)
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M2 Brane Theory

- The theory on N coincident M2-branes has N=8, the maximum possible supersymmetry in 3 dimensions.
- When N is large, its dual description is provided by the weakly curved $AdS_4 \times S^7$ background in 11-dimensional M-theory which is essentially described by Einstein gravity coupled to other fields.
- This dual description is tractable and makes many non-trivial predictions.

- A general prediction of the AdS/CFT duality is that the number of degrees of freedom on a large number N of coincident M2-branes scales as $N^{3/2}$

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- This is much smaller than the N^2 scaling found in the 4-d SYM theory on N coincident D3-branes (as described by the dual gravity). Gubser, I.K., Peet (1996)

Black Holes

- At finite temperature and density the dual description is provided by certain black holes inside AdS_4. Interestingly, they can carry both electric and magnetic charges, corresponding to charge density and magnetic field in the M2 brane theory. This has provided a way of modeling various phenomena in condensed matter physics, such as the Nernst effect observed by Ong in high T_c superconductors. Hartnoll, Herzog (2007); Hartnoll, Kovtun, Mueller, Sachdev (2007)

What is the M2 Brane Theory?

- It is the Infrared limit of the D2-brane theory, the N=8 supersymmetric Yang-Mills theory in 2+1 dimensions, i.e. it describes the degrees of freedom at energy much lower than $(g_{YM})^2$
- The number of such degrees of freedom $\sim N^{3/2}$ is much lower than the number of UV degrees of freedom $\sim N^2$.
- Is there a more direct way to characterize the Infrared Scale-Invariant Theory?

The BLG Theory

- In a remarkable recent development, Bagger and Lambert, and Gustavsson formulated an SO(4) Chern-Simons Gauge Theory with manifest N=8 superconformal gauge theory. In Van Raamsdonk's SU(2)xSU(2) formulation,

$$X^* = -\varepsilon X \varepsilon$$

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The ABJM Theory

- Aharony, Bergman, Jafferis and Maldacena argued that the correct description of a pair of M2-branes is slightly different. It involves U(2) x U(2) gauge theory.
- The SU(4) flavor symmetry is not manifest because of the choice of complex combinations

$$Z^1 = X^1 + iX^5\,, \qquad W_1 = X^{3\dagger} + iX^{7\dagger}$$
$$Z^2 = X^2 + iX^6\,, \qquad W_2 = X^{4\dagger} + iX^{8\dagger}$$

- The manifest flavor symmetry is SU(2)xSU(2)

$$\mathrm{W} = \frac{1}{4}\epsilon_{AC}\epsilon^{BD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{W}_B\mathcal{Z}^C\mathcal{W}_D$$

N=2 Superspace Formulation

- Define bi-fundamental superfields rotated by $SU(4)_{\text{flavor}}$ symmetry

$$\mathcal{Z} = Z(x_L) + \sqrt{2}\theta\zeta(x_L) + \theta^2 F(x_L) ,$$
$$\bar{\mathcal{Z}} = Z^\dagger(x_R) - \sqrt{2}\bar{\theta}\zeta^\dagger(x_R) - \bar{\theta}^2 F^\dagger(x_R)$$

$$Z^{\ddagger A} := -\varepsilon(Z^A)^{\mathrm{T}}\varepsilon = X^{\dagger A} + iX^{\dagger A+4}$$

- The superpotential is Benna, IK, Klose, Smedback,

$$\mathrm{W} = \frac{1}{4!}\epsilon_{ABCD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{Z}^{\ddagger B}\mathcal{Z}^C\mathcal{Z}^{\ddagger D}$$

- Using SO(4) gauge group notation,

$$\mathrm{W} = -\frac{1}{8\cdot 4!}\,\epsilon_{ABCD}\epsilon^{abcd}\,\mathcal{Z}_a^A\mathcal{Z}_b^B\mathcal{Z}_c^C\mathcal{Z}_d^D$$

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$$V^{\text{bos}} = -\frac{L^2}{48}\,\text{tr}\Big[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big]$$

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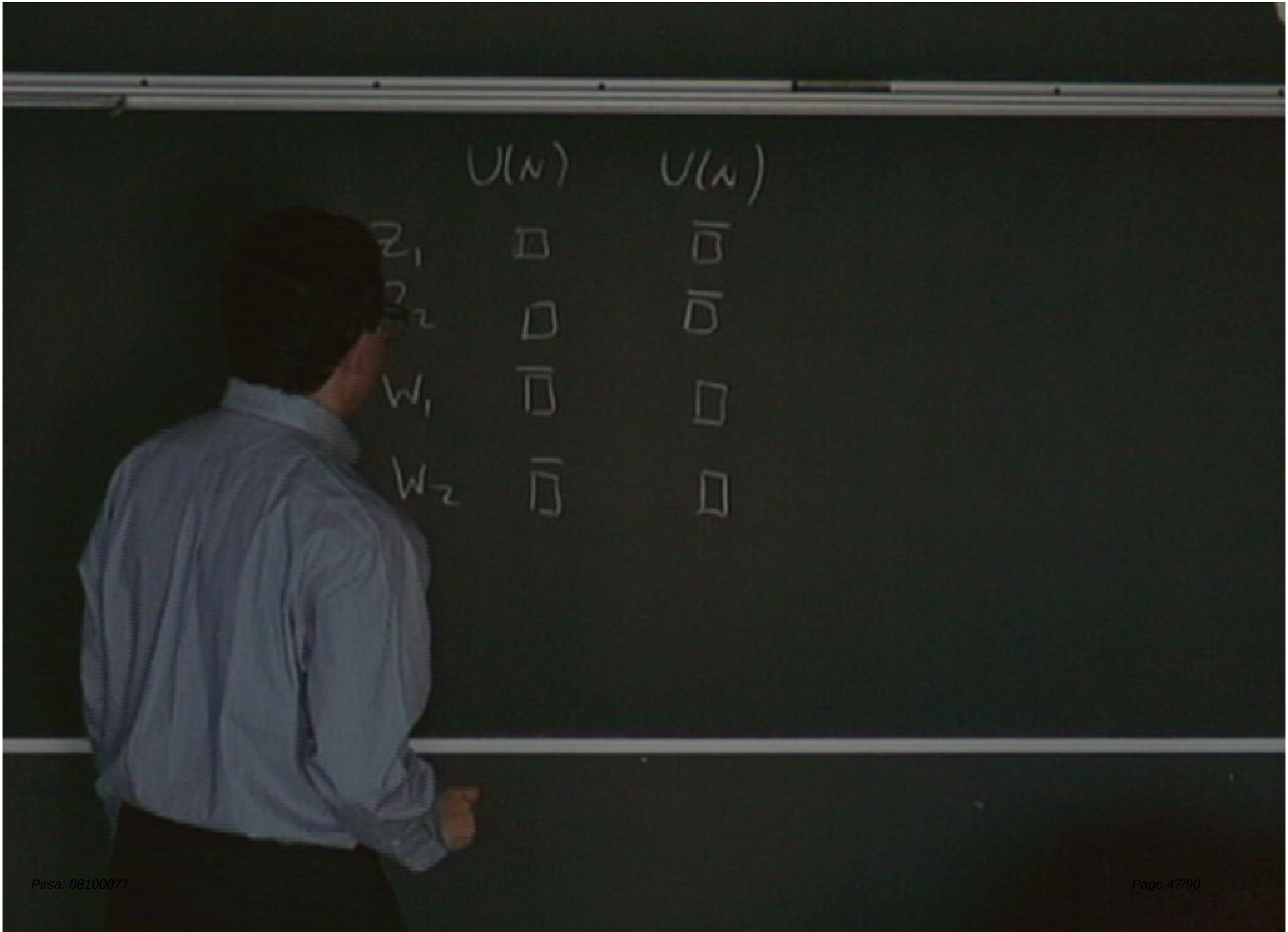
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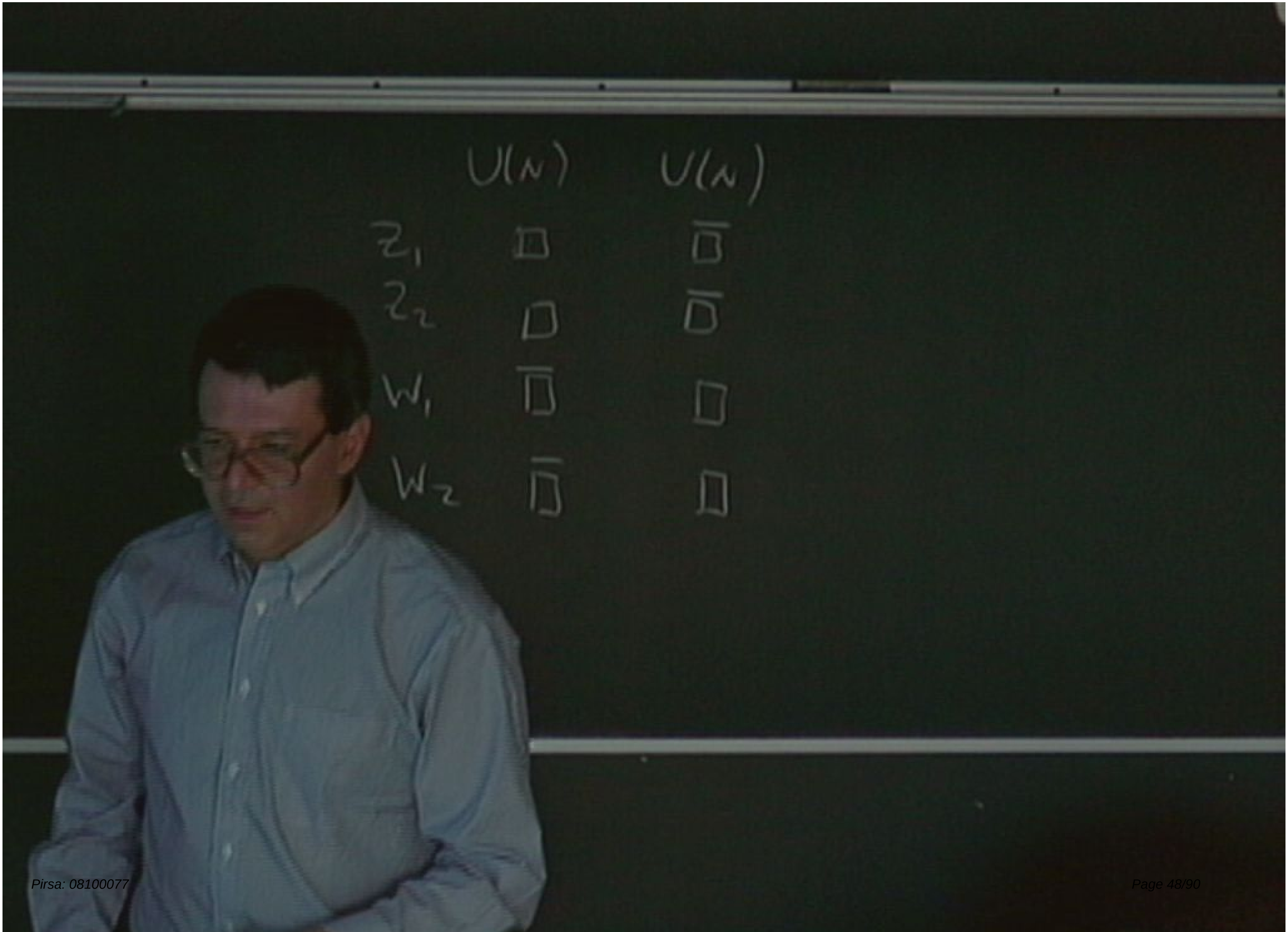
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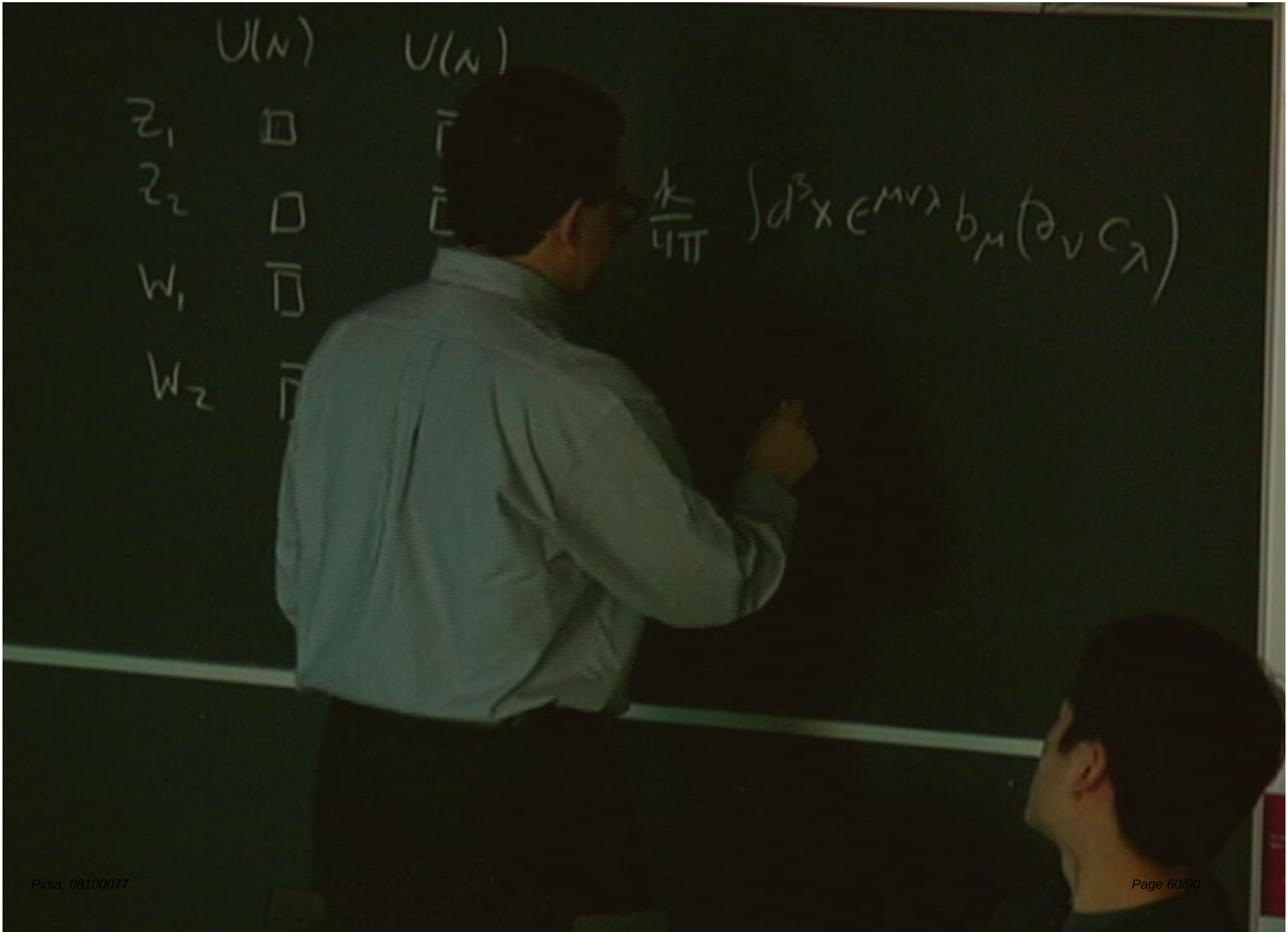
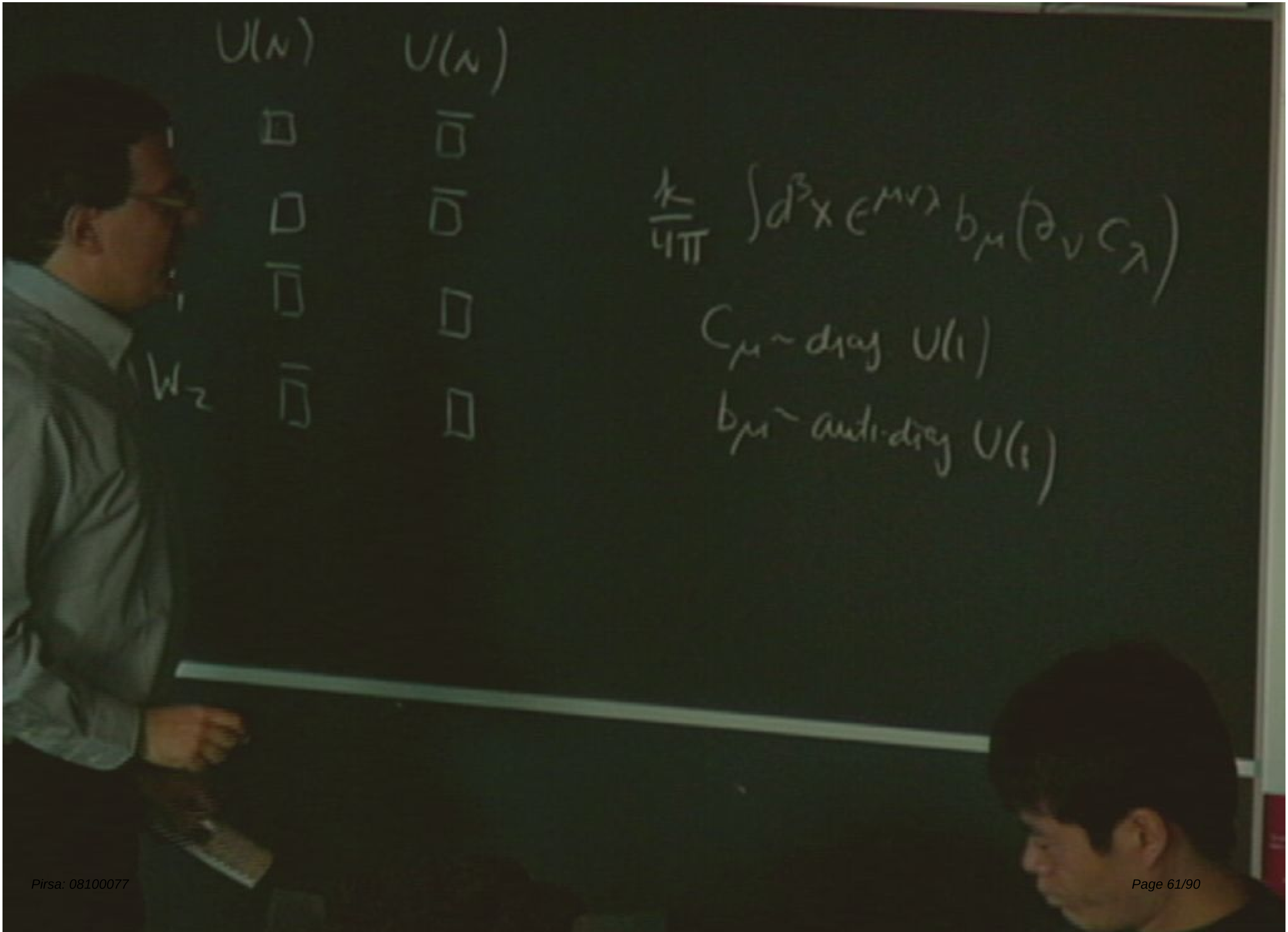
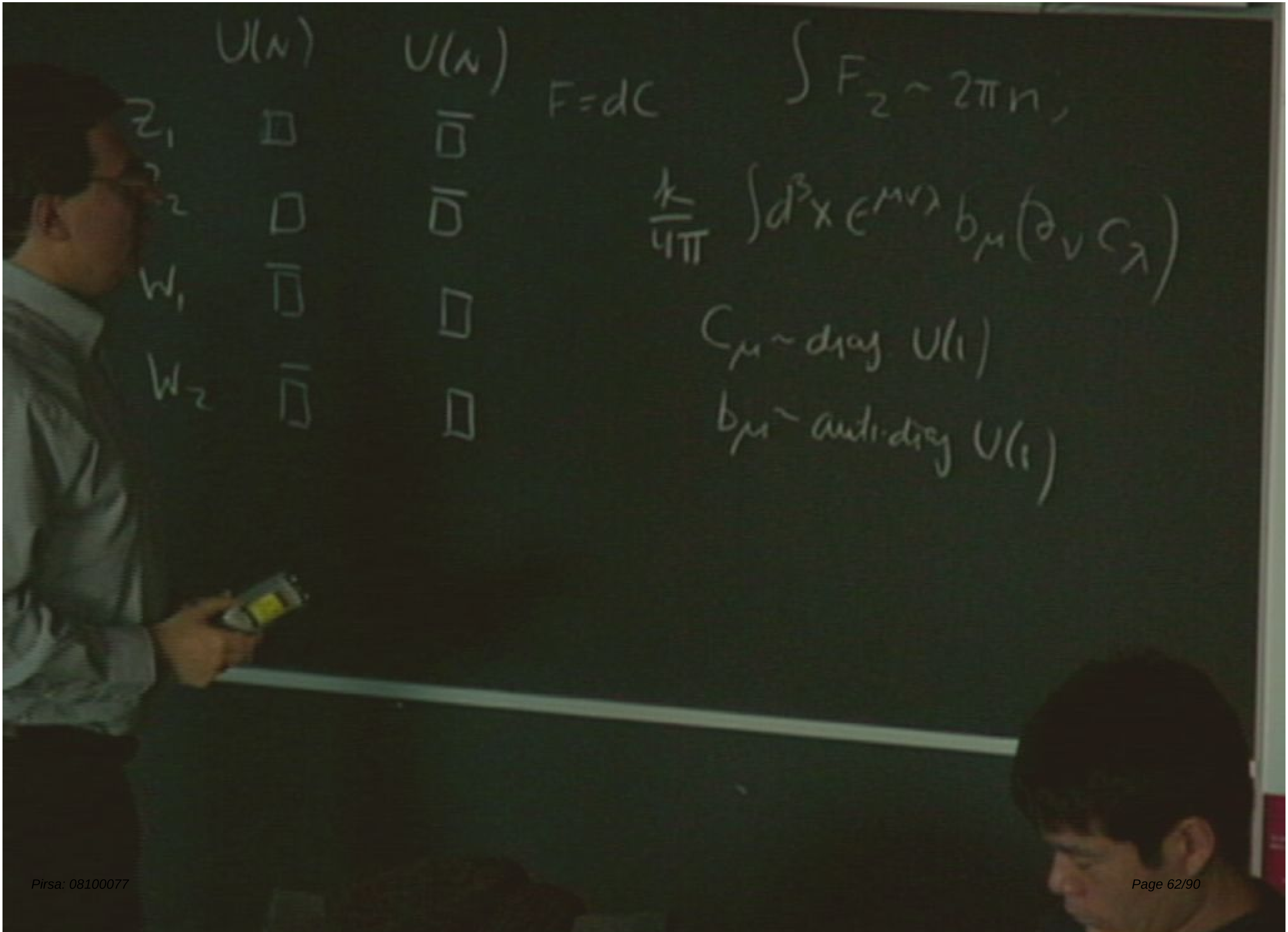

U(N)
U(N)
Z1
Z2
W1
W2
k/4π ∫d³x ε^μνλ b_μ(∂_ν c_λ)


U(N)
U(N)
W_2
k/4π ∫ d³x ε^{μνλ} b_μ (∂_ν c_λ)
c_μ ~ diag U(1)
b_μ ~ anti-diag U(1)


U(N) U(N)
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F = dC
$\int F_2 \sim 2\pi n$,
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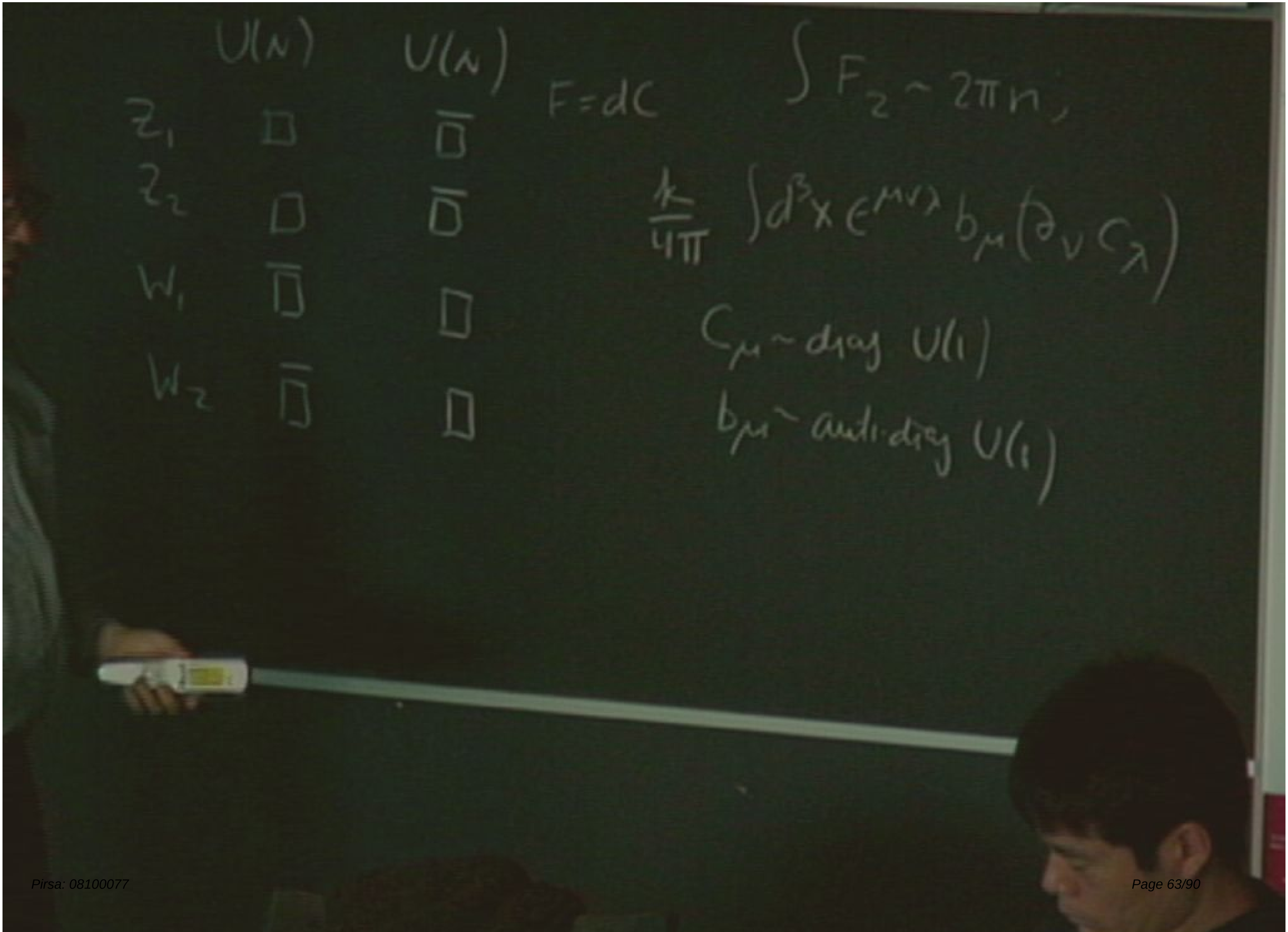

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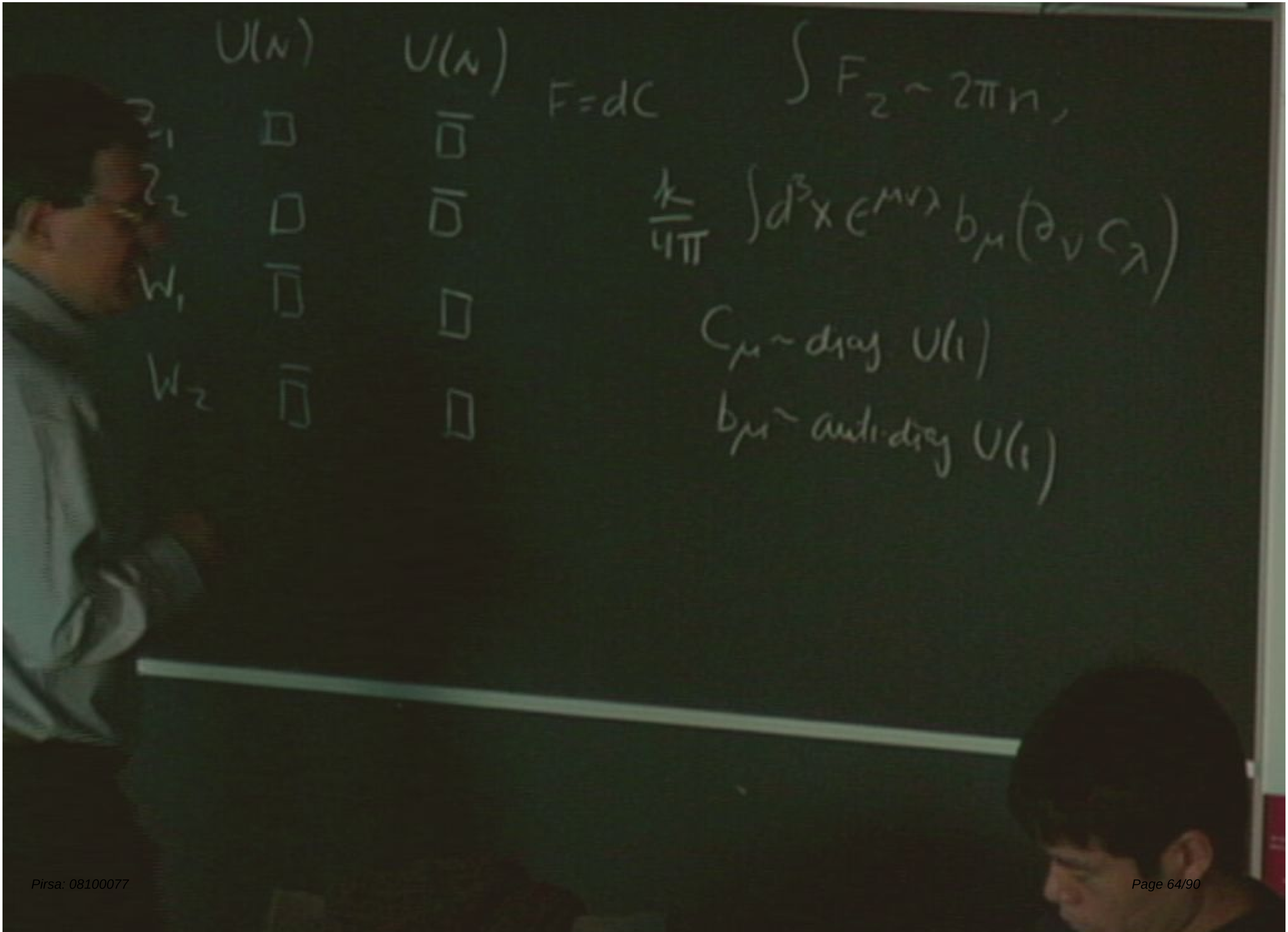
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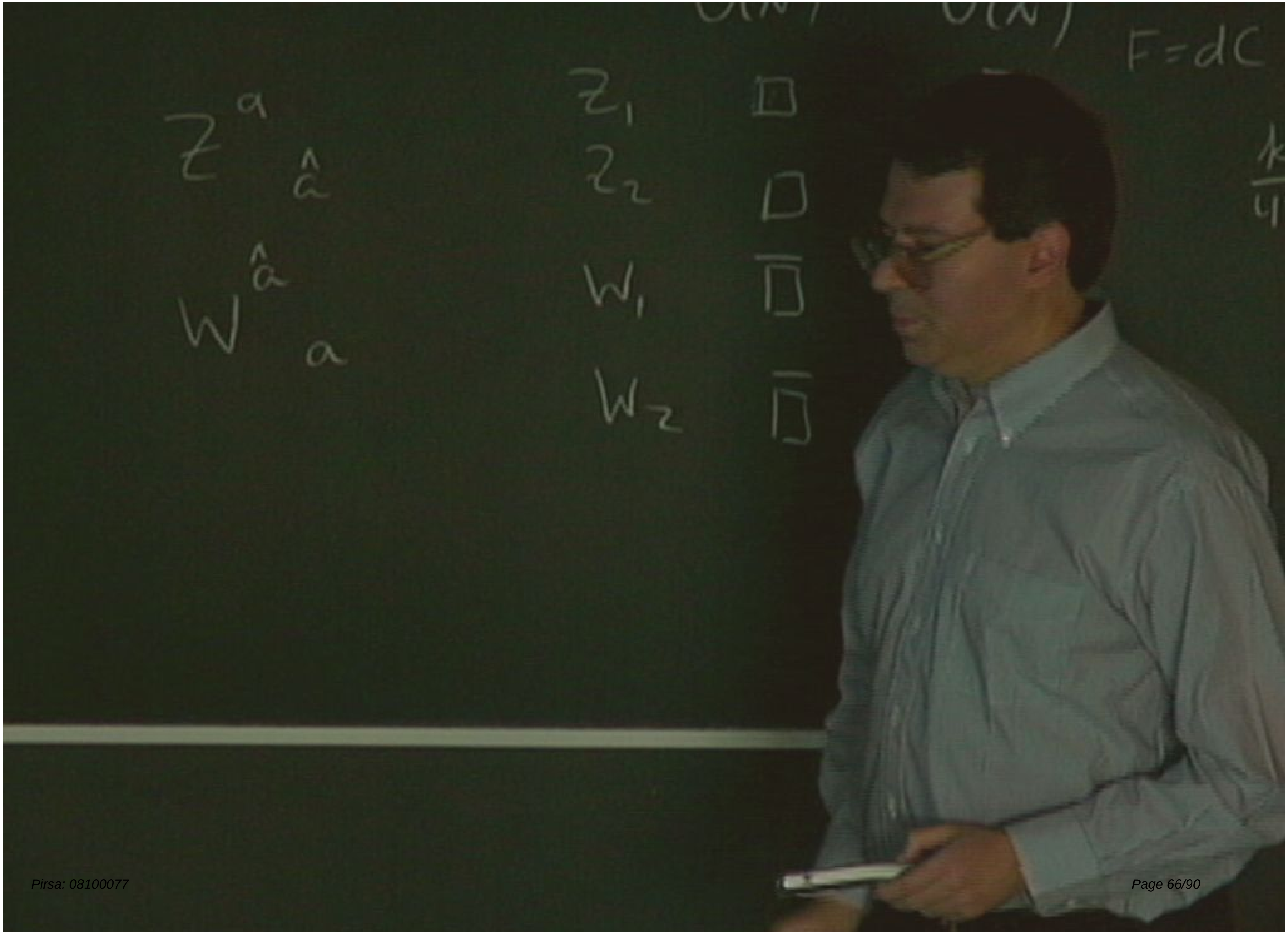
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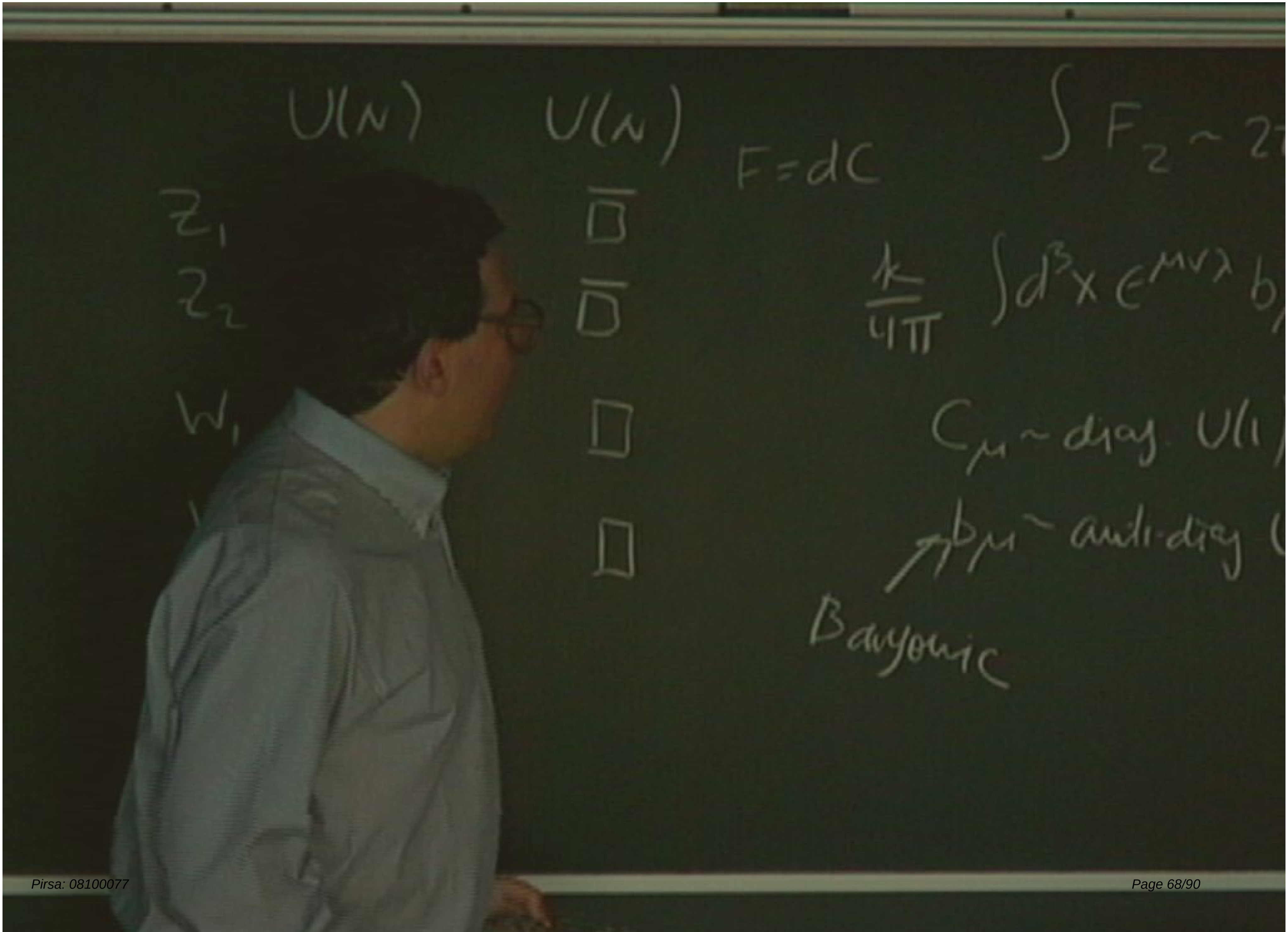
U(N) U(N)
F=dC
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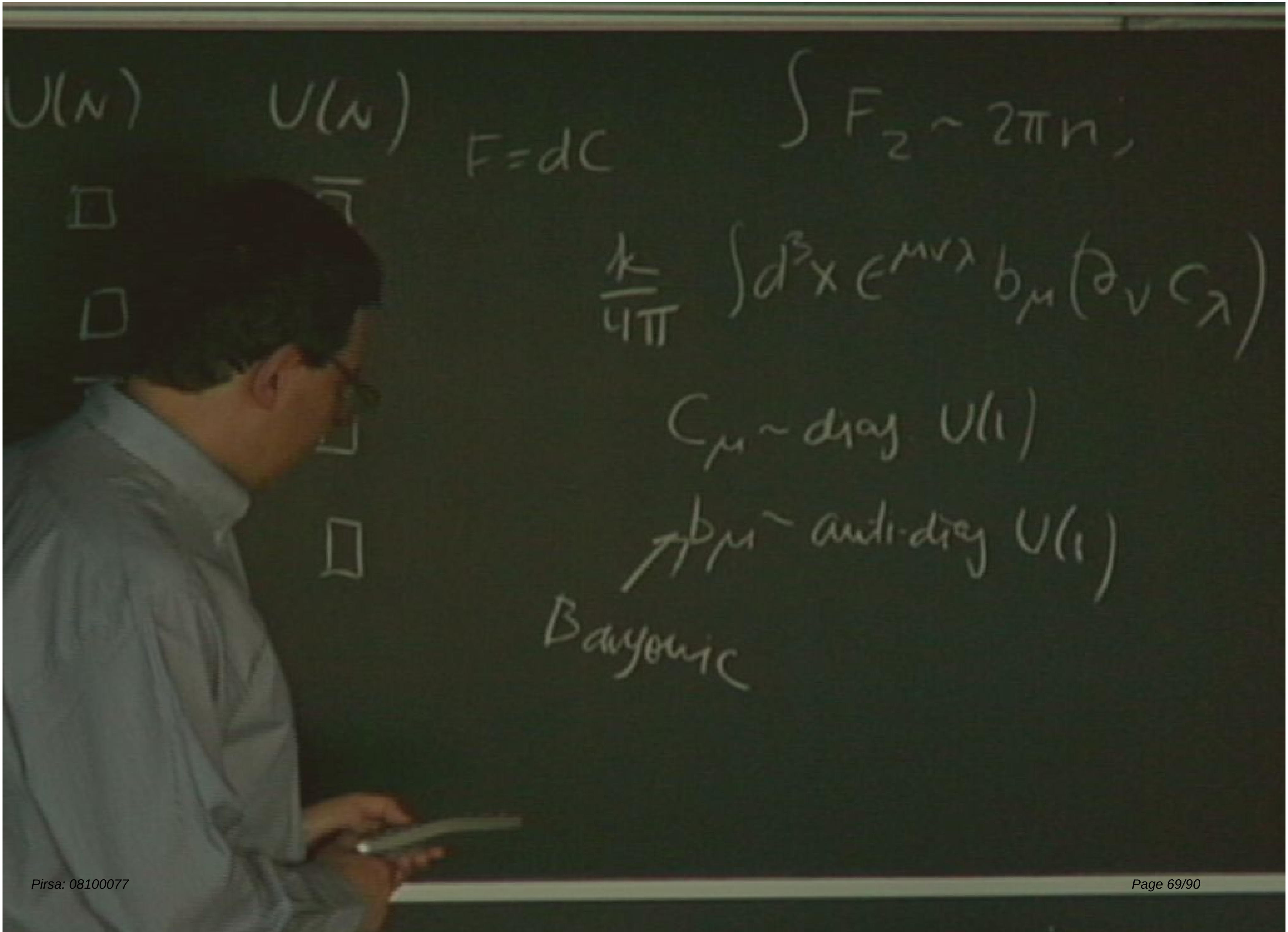
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Relevant Deformations

- The M2-brane theory may be perturbed by relevant operators that cause it to flow to new fixed points with reduced supersymmetry. Benna, IK, Klose, Smedback; IK, Klose, Murugan; Ahn
- For example, a quadratic superpotential deformation, allowed for k=1, 2, may preserve SU(3) flavor symmetry

$$\Delta W = m(\mathcal{Z}^4)^a{}_{\hat{a}}(\mathcal{Z}^4)^b{}_{\hat{b}}(e^{-2\tau})^{\hat{a}\hat{b}}_{ab}$$

- For U(2) x U(2) ABJM theory, this becomes

$$\Delta W = m T^{-2} \,\mathrm{tr}\, \mathcal{Z}^4 \mathcal{Z}^{4\ddagger}$$

- To understand the IR SCFT, we integrate out

$$\mathcal{Z}^4 = -\frac{T^{-2}}{12m} \epsilon_{ABC} \, \mathcal{Z}^A \mathcal{Z}^{\ddagger B} \mathcal{Z}^C$$

  to obtain

$$W_{\text{eff}} = \frac{T^{-6}}{144m} \epsilon_{ABC} \epsilon_{DEF} \,\mathrm{tr}\, \mathcal{Z}^A \mathcal{Z}^{\ddagger B} \mathcal{Z}^C \mathcal{Z}^{\ddagger D} \mathcal{Z}^E \mathcal{Z}^{\ddagger F}$$

  where A=1,2,3
- The R-charge and dimension of these 3 bi-fundamental superfields is 1/3

Squashed, stretched and warped

- The dual AdS_4 background of M-theory should also preserve N=2 SUSY and SU(3) flavor symmetry. Such an extremum of gauge SUGRA was found 25 years ago by Warner. Upon uplifting to 11-d we find a warped product of AdS_4 and of a `stretched and squashed' 7-sphere.
- Spectrum of mutliplets in gauged SUGRA may be compared with the gauge theory

- For U(2) x U(2) ABJM theory, this becomes

$$\Delta \mathrm{W} = m T^{-2}\, \mathrm{tr}\, \mathcal{Z}^4 \mathcal{Z}^{4\ddagger}$$

- To understand the IR SCFT, we integrate out

$$\mathcal{Z}^4 = -\frac{T^{-2}}{12m}\, \epsilon_{ABC}\, \mathcal{Z}^A \mathcal{Z}^{\ddagger B} \mathcal{Z}^C$$

to obtain

$$\mathrm{W}_{\mathrm{eff}} = \frac{T^{-6}}{144m}\, \epsilon_{ABC}\epsilon_{DEF}\, \mathrm{tr}\, \mathcal{Z}^A \mathcal{Z}^{\ddagger B} \mathcal{Z}^C \mathcal{Z}^{\ddagger D} \mathcal{Z}^E \mathcal{Z}^{\ddagger F}$$

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- Spectrum of mutliplets in gauged SUGRA may be compared with the gauge theory

	Scenario I	Scenario II
Hyper	$[n+2,0]_{\frac{n+2}{3}}$, $[0,n+2]_{-\frac{n+2}{3}}$	$[n+2,0]_{-\frac{2n+4}{3}}$, $[0,n+2]_{\frac{2n+4}{3}}$
Vector	$[n+1,1]_{\frac{n}{3}}$, $[1,n+1]_{-\frac{n}{3}}$	$[n+1,1]_{-\frac{2n}{3}}$, $[1,n+1]_{\frac{2n}{3}}$
Gravitino	$[n+1,0]_{\frac{n+1}{3}}$, $[0,n+1]_{-\frac{n+1}{3}}$	$[n+1,0]_{-\frac{2n-1}{3}}$, $[0,n+1]_{\frac{2n-1}{3}}$
Graviton	$[0,0]_{n}$, $[0,0]_{-n}$	$[0,0]_{0}$, $[0,0]_{0}$

- We find that Scenario I gives SU(3)xU(1)$_R$ quantum numbers in agreement with the proposed gauge theory dual where they are schematically given by

	Z^A	ζ^A	$Z_A^\dagger$	$\zeta_A^\dagger$	Z^4	ζ^4	$Z_4^\dagger$	$\zeta_4^\dagger$	x	θ	$\bar{\theta}$
SU(3)	**3**	**3**	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	**1**	**1**	**1**	**1**	**1**	**1**	**1**
Dimension	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{3}{2}$	1	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
R-charge	$+\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$+1$	0	-1	0	0	$+1$	-1

Hypermultiplets

- For even n, the operators are

$$H^{(n)A_1\ldots A_{n+2}} = \mathrm{tr}\, \mathcal{Z}^{(A_1} \mathcal{Z}^{A_2} e^{-2\tau}\, \mathcal{Z}^{A_3} \mathcal{Z}^{A_4} e^{-2\tau} \ldots \mathcal{Z}^{A_{n+1}} \mathcal{Z}^{A_{n+2})} e^{-2\tau}$$

- Their explicit form for U(2) x U(2) is

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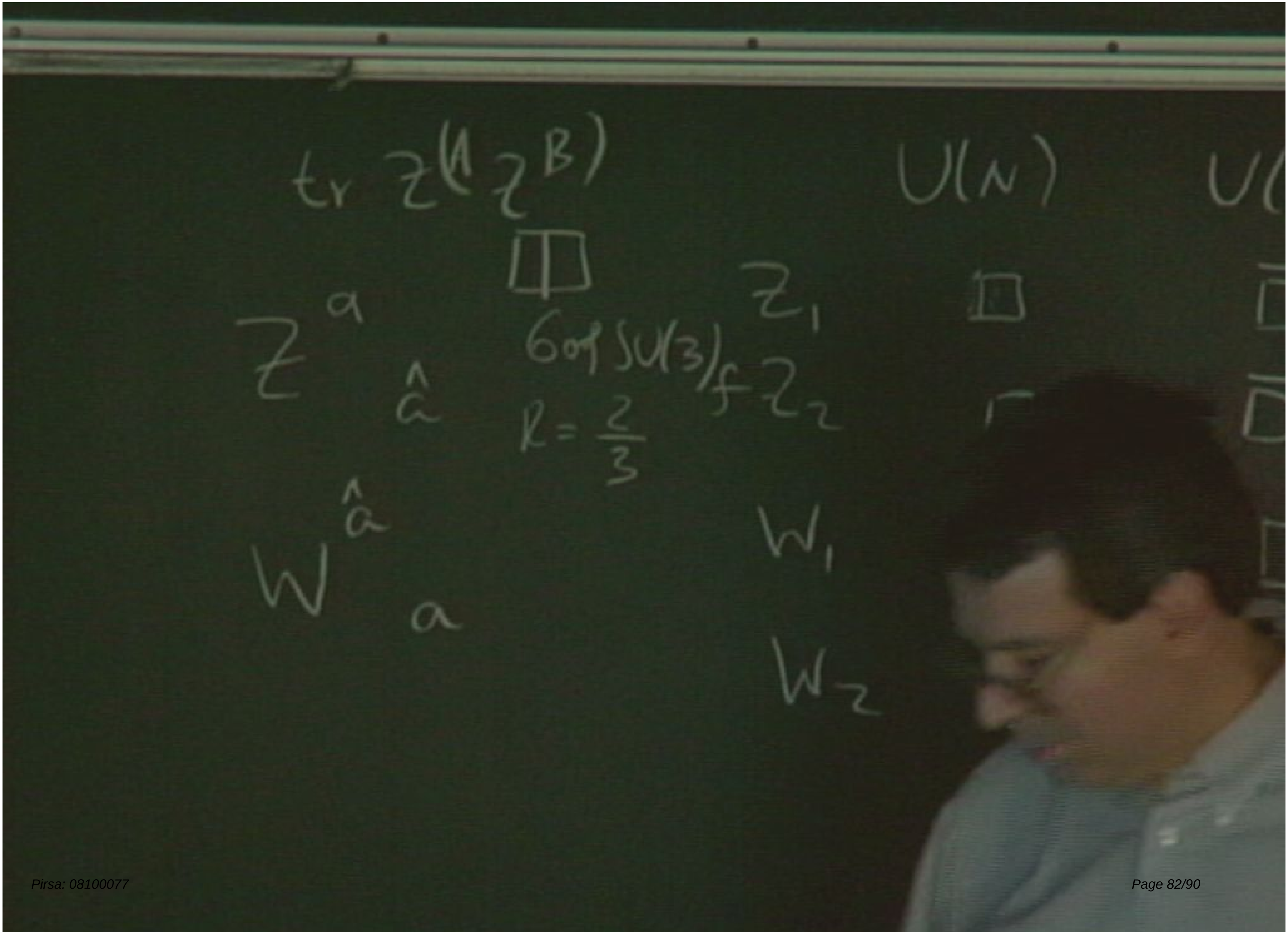
- For odd n, we need the single monopole operator $H^{(n)A_1\ldots A_{n+2}} = \mathrm{tr}\, \mathcal{Z}^{(A_1} \mathcal{Z}^{A_2} e^{-2\tau} \ldots \mathcal{Z}^{A_n} \mathcal{Z}^{A_{n+1}} e^{-2\tau} \mathcal{Z}^{A_{n+2})} e^{-\tau}$
- The quantum numbers are in agreement with SUGRA: $[n+2, 0]$ $\Delta_0 = y_0 = \frac{n+2}{3}$



U(N)
Z
W
Z1
Z2
W1
W2
R = 2/3

	Scenario I	Scenario II
Hyper	$[n+2,0]_{\frac{n+2}{3}}$, $[0,n+2]_{-\frac{n+2}{3}}$	$[n+2,0]_{-\frac{2n+4}{3}}$, $[0,n+2]_{\frac{2n+4}{3}}$
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Gravitino	$[n+1,0]_{\frac{n+1}{3}}$, $[0,n+1]_{-\frac{n+1}{3}}$	$[n+1,0]_{-\frac{2n-1}{3}}$, $[0,n+1]_{\frac{2n-1}{3}}$
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SU(3)	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Dimension	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{3}{2}$	1	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
R-charge	$+\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$+1$	0	-1	0	0	$+1$	-1

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Graviton	$[0,0]_n$, $[0,0]_{-n}$	$[0,0]_0$, $[0,0]_0$

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Dimension	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{3}{2}$	1	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
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Further Directions

- Other examples of AdS_4/CFT_3 dualities with N=1,2,3,... supersymmetry are being studied by many groups.
- Ultimate Physics Goal: to find a `simple' dual of a 3-d fixed point realized in Nature.

- Remarkably, the `M2 Mini-revolution' may have percolated even to the study of recently observed Charmonium decays:

  M2 signatures in psi(2S) radiative decays.
  Jonathan L. Rosner . EFI-08-24, Sep 2008. 8pp. Temporary entry
  e-Print: **arXiv:0809.0471** [hep-ph]
- But this is an entirely different story...

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