Title: SICs, Convex Cones, and Algebraic Sets

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Abstract: The question whether SICs exist can be viewed as a question about the structure of the convex set of quantum measurements, or turned into one about quantum states, asserting that they must have a high degree of symmetry. I\'ll address Chris Fuchs\' contrast of a \'probability first\' view of the issue with a \'generalized probabilistic theories\' view of it. I\'ll review some of what\'s known about the structure of convex state and measurement spaces with symmetries of a similar flavor, including the quantum one, and speculate on connections to recent SIC triple product results. And I\'ll present some old calculations, which will look familiar to old hands but may be worth contemplating yet again, reducing the Heisenberg-symmetric-SIC existence problem to the existence of solutions to a set of simultaneous polynomials in unit-modulus complex variables.

SICs, convexity and algebraic sets

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Oct. 29, 2008 / Perimeter Institute

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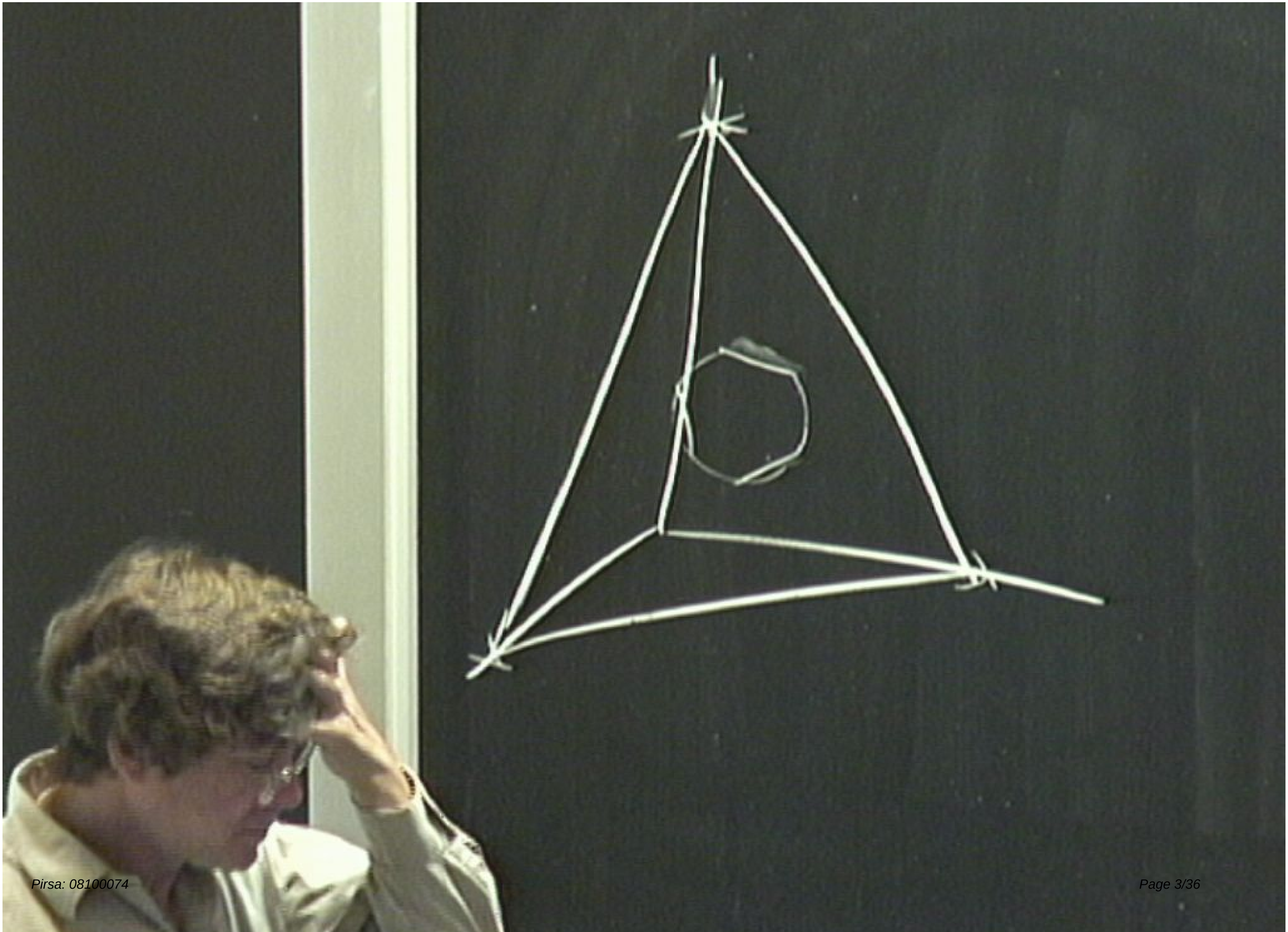
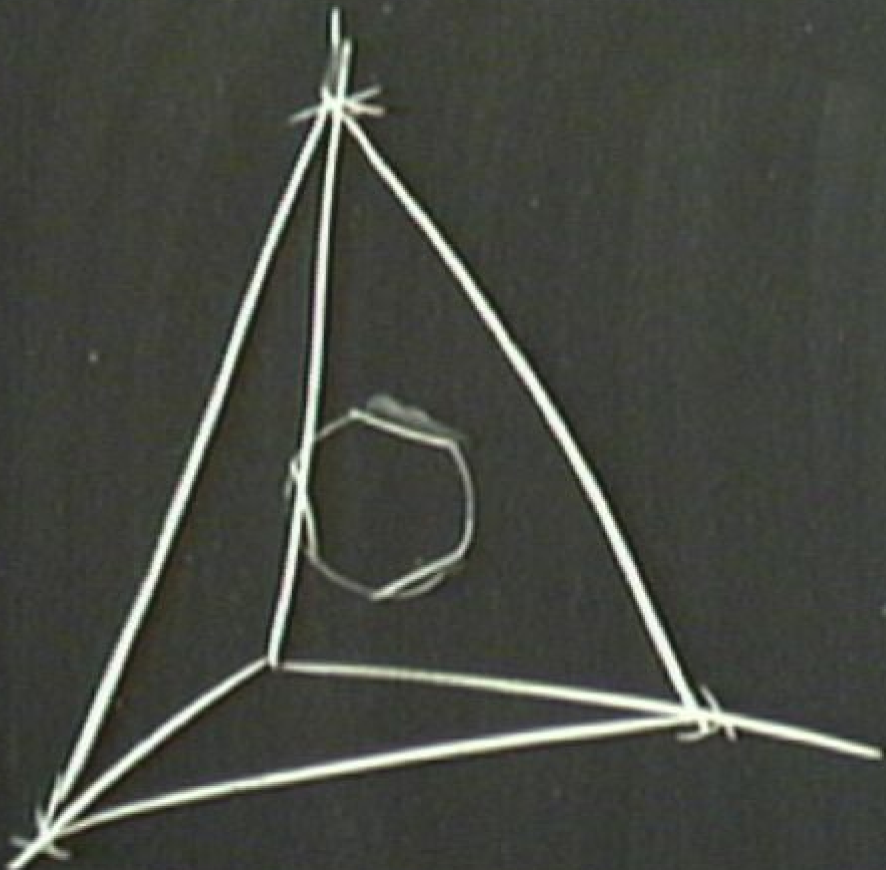
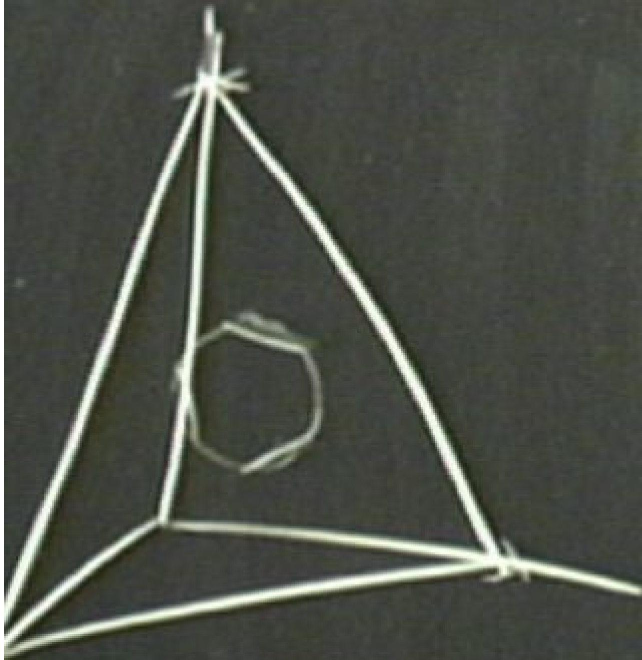
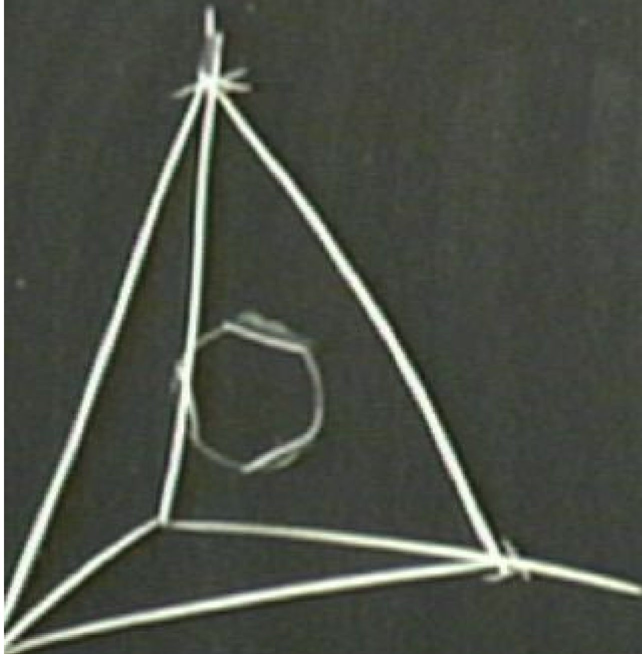




$$\sum_i p(i)\, q(i)$$

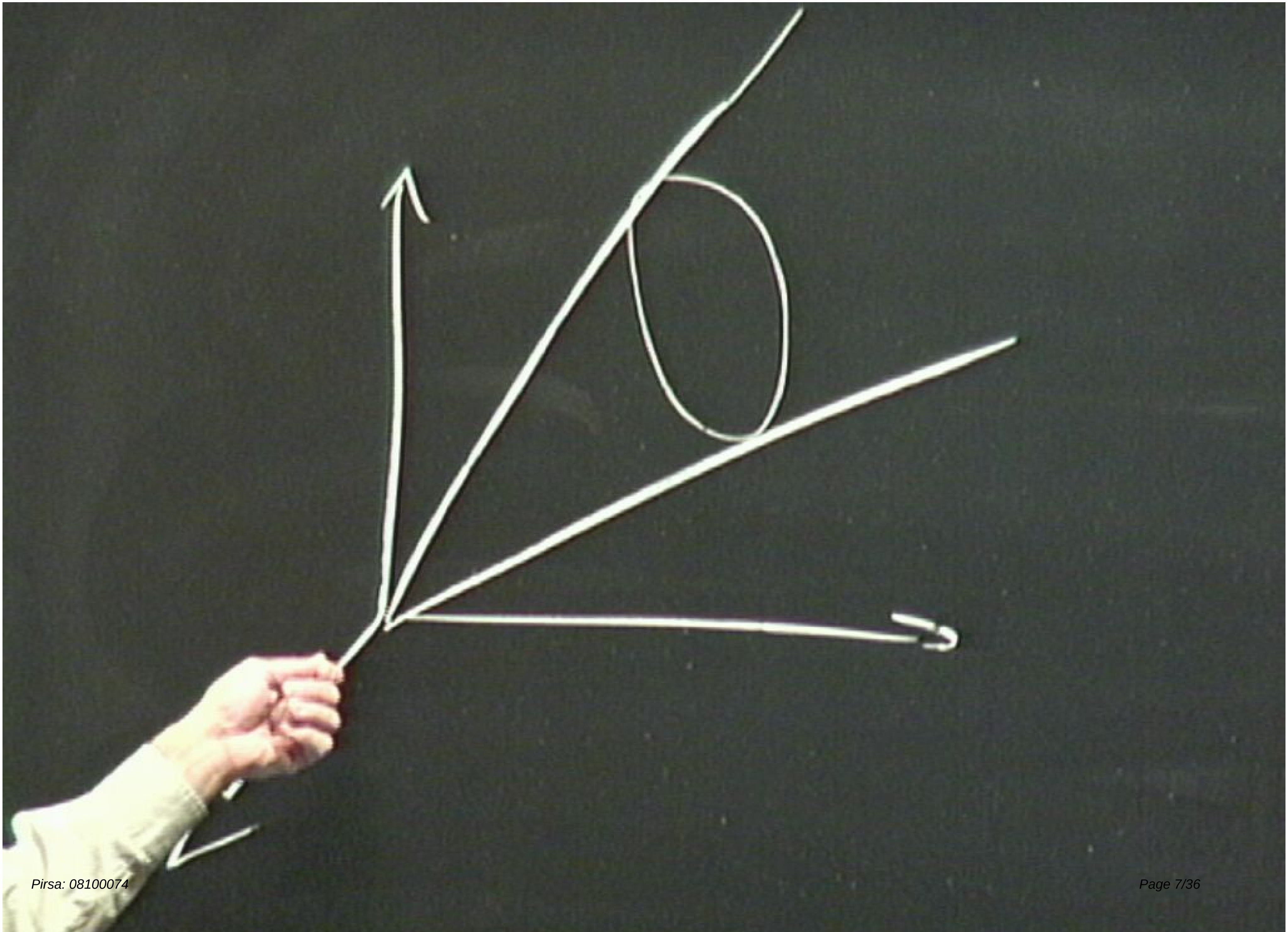


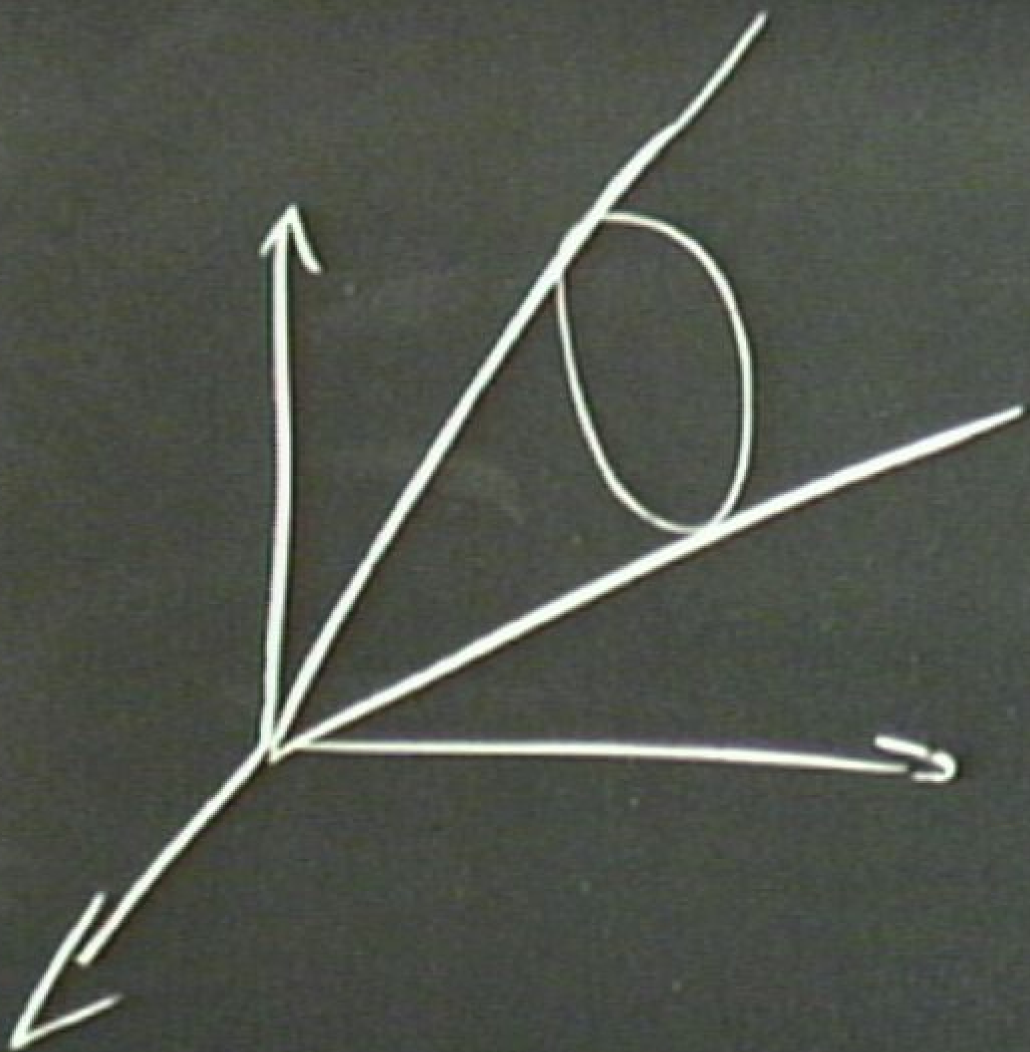
$$\frac{1}{d(d+1)} \leq \sum_i p(i)\, q(i) \leq \frac{2}{d(d+1)}$$



$$\frac{1}{d(d+1)} \leq \sum_i p(i)\, q(i) \leq \frac{2}{d(d+1)}$$

$$i = 1, \ldots, d^2$$





$K \subseteq \mathbb{R}^n$

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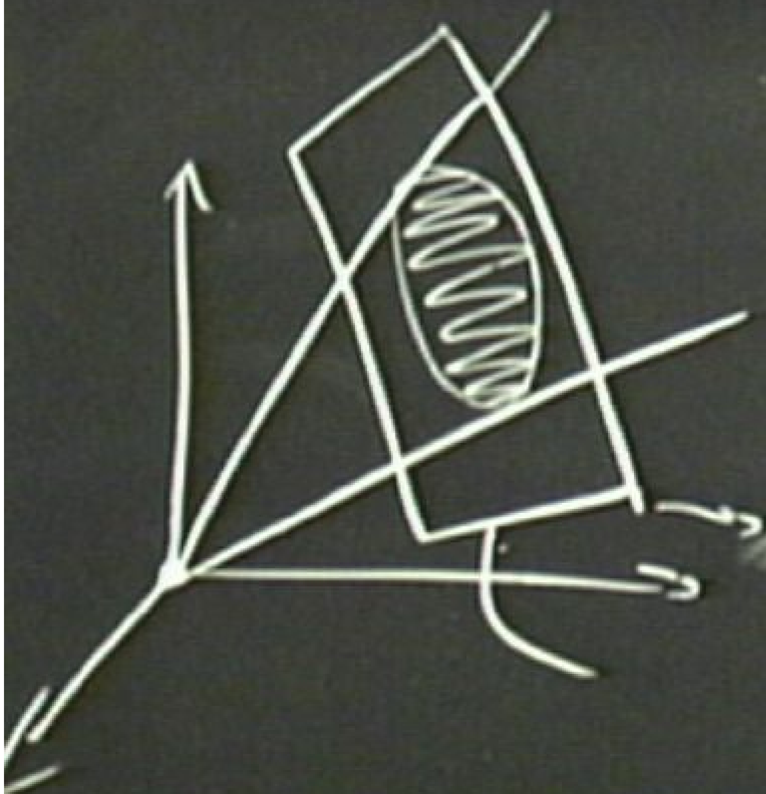
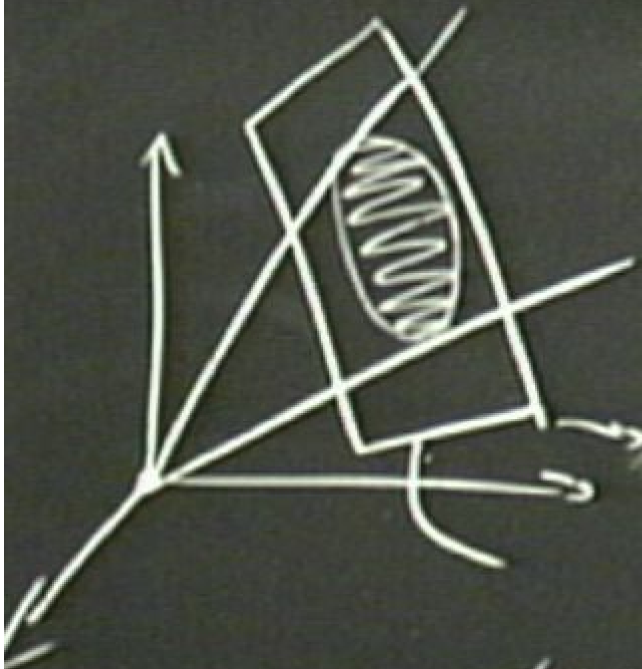


$$K \subseteq \mathbb{R}^n$$

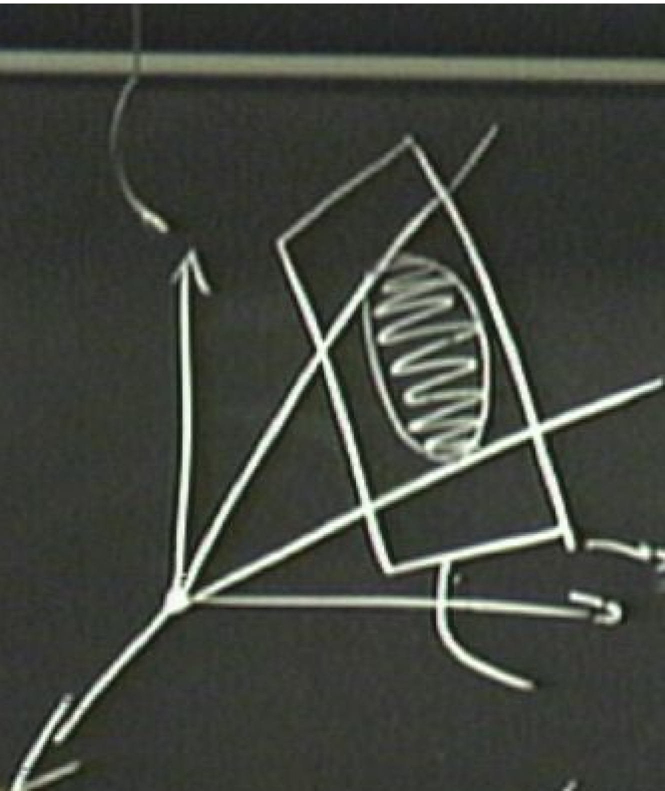
$$\{\omega \in \mathbb{R}^n : \underset{\substack{\uparrow \\ (\mathbb{R}^n)^*}}{u}(\omega) = 1\}$$



$$K \subseteq \mathbb{R}^n$$

$$\Big\{\omega \in \mathbb{R}^n : \underset{\substack{\uparrow \\ (\mathbb{R}^n)^*}}{u}(\omega) = 1\Big\} = A$$

$\omega \in A \cap K$ - normalized state



$K \subseteq \mathbb{R}^n$

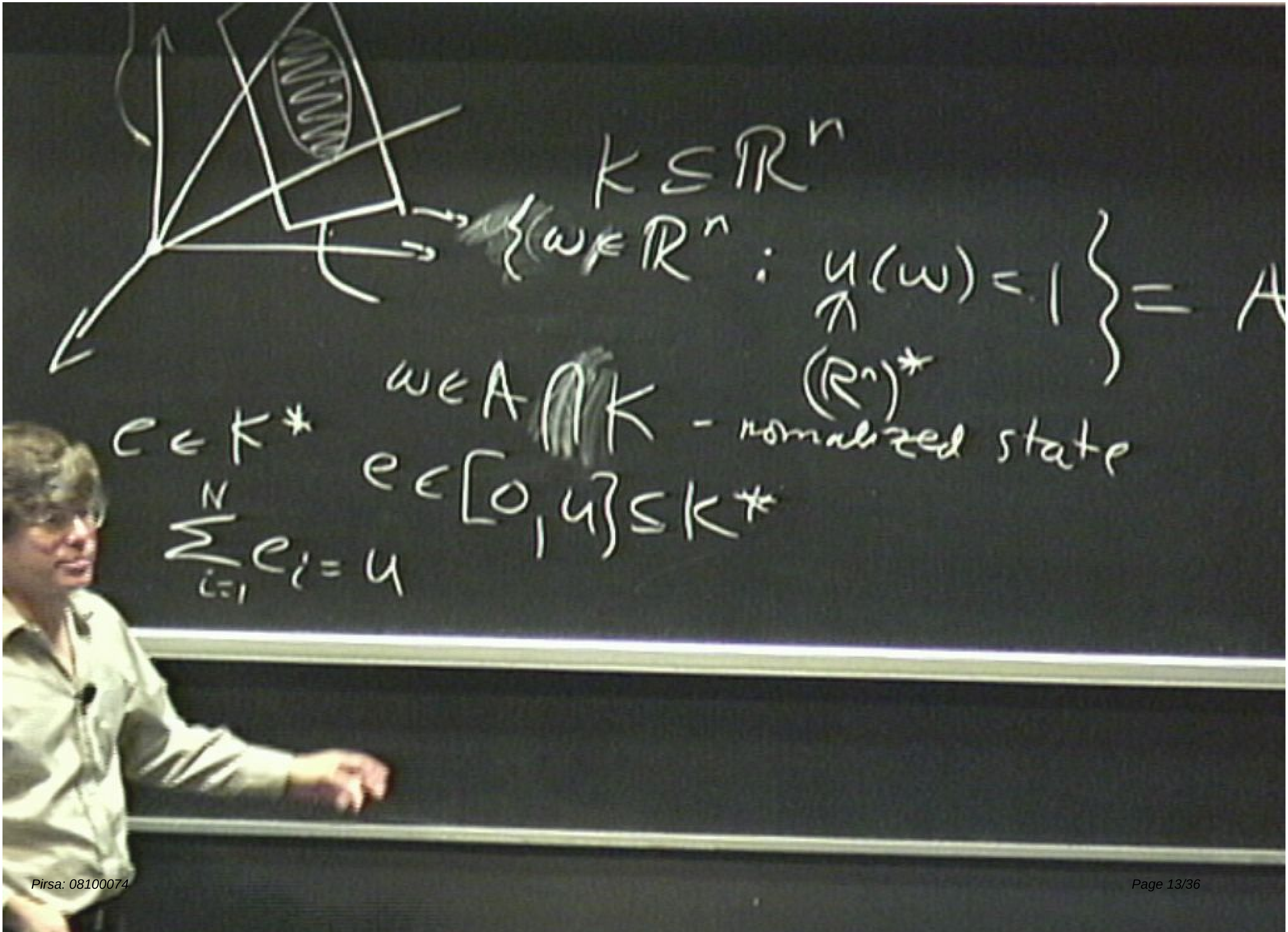
$\{\omega \in \mathbb{R}^n : u(\omega) = 1\} = A$

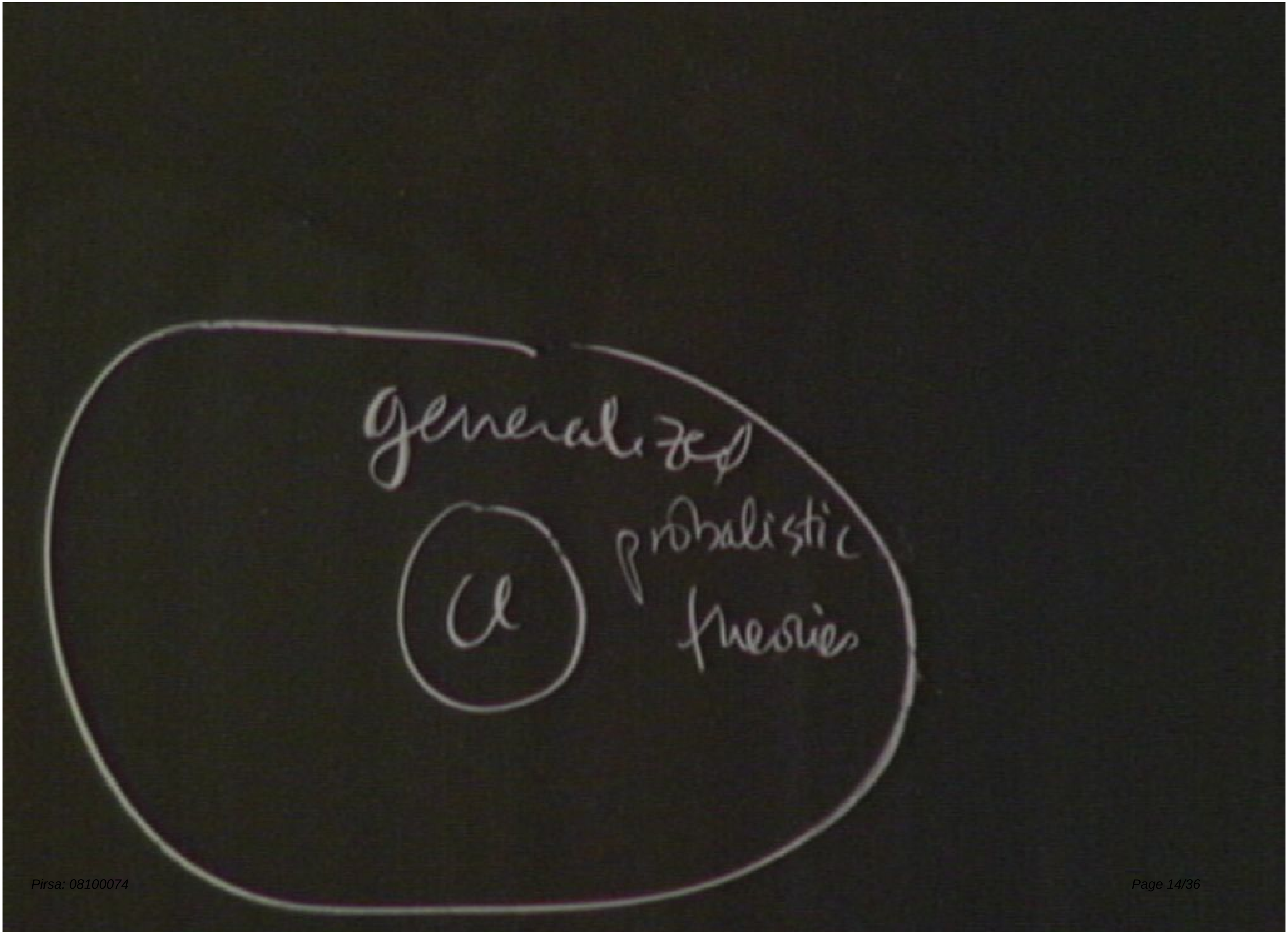
$u \in (\mathbb{R}^n)^*$

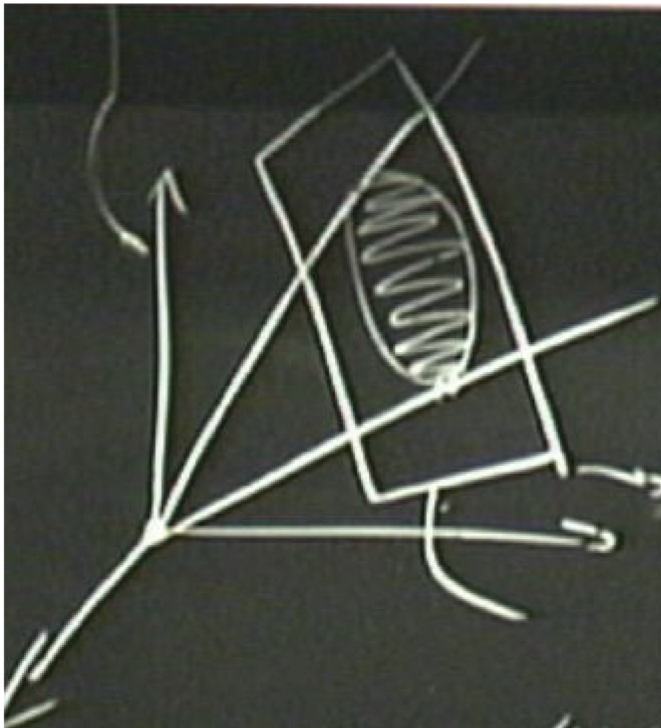
$e \in K^*$

$\omega \in A \cap K$ - normalized state

$e \in [0, u] \subseteq K^*$


$K \subseteq \mathbb{R}^n$
$\{\omega \in \mathbb{R}^n : u(\omega) = 1\} = A$
$(\mathbb{R}^n)^*$
$\omega \in A \cap K$ - normalized state
$e \in K^*$
$e \in [0, u] \subseteq K^*$
$\sum_{i=1}^{N} e_i = u$


generalized
probalistic
theories



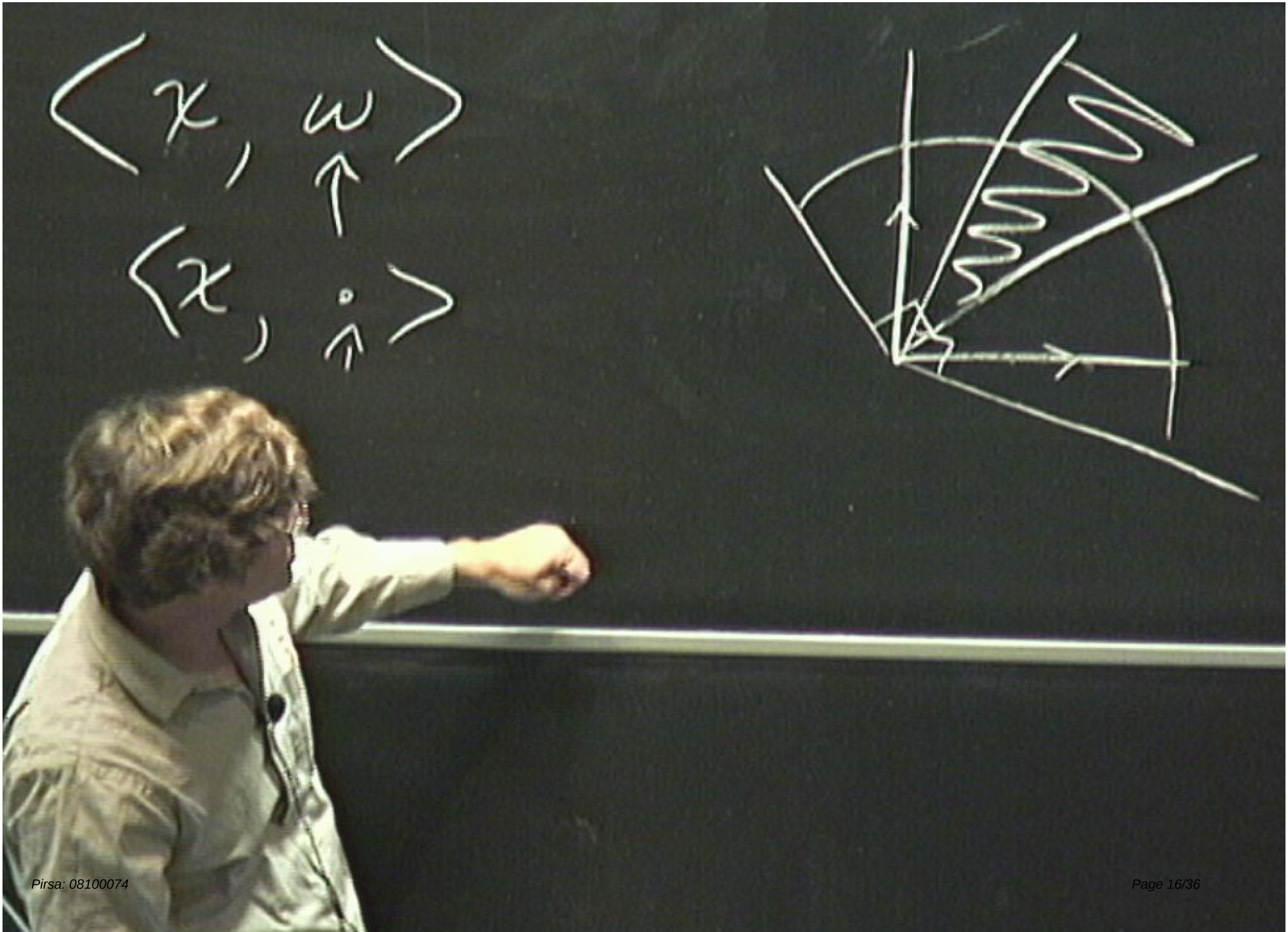
$$K \subseteq \mathbb{R}^n$$

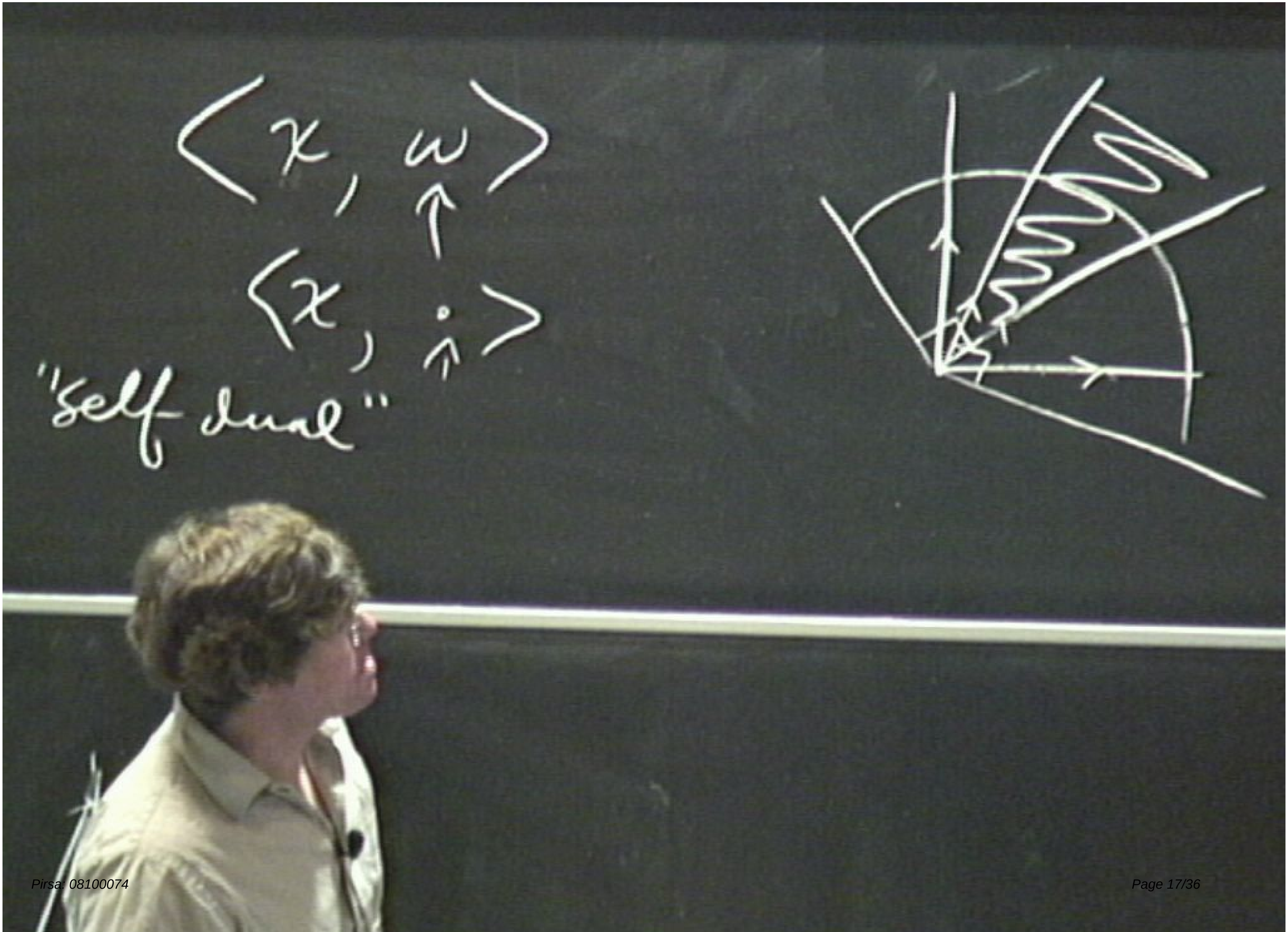
$$\{\omega \in \mathbb{R}^n : \underset{\substack{\uparrow \\ (\mathbb{R}^n)^*}}{u}(\omega) = 1\} = A$$

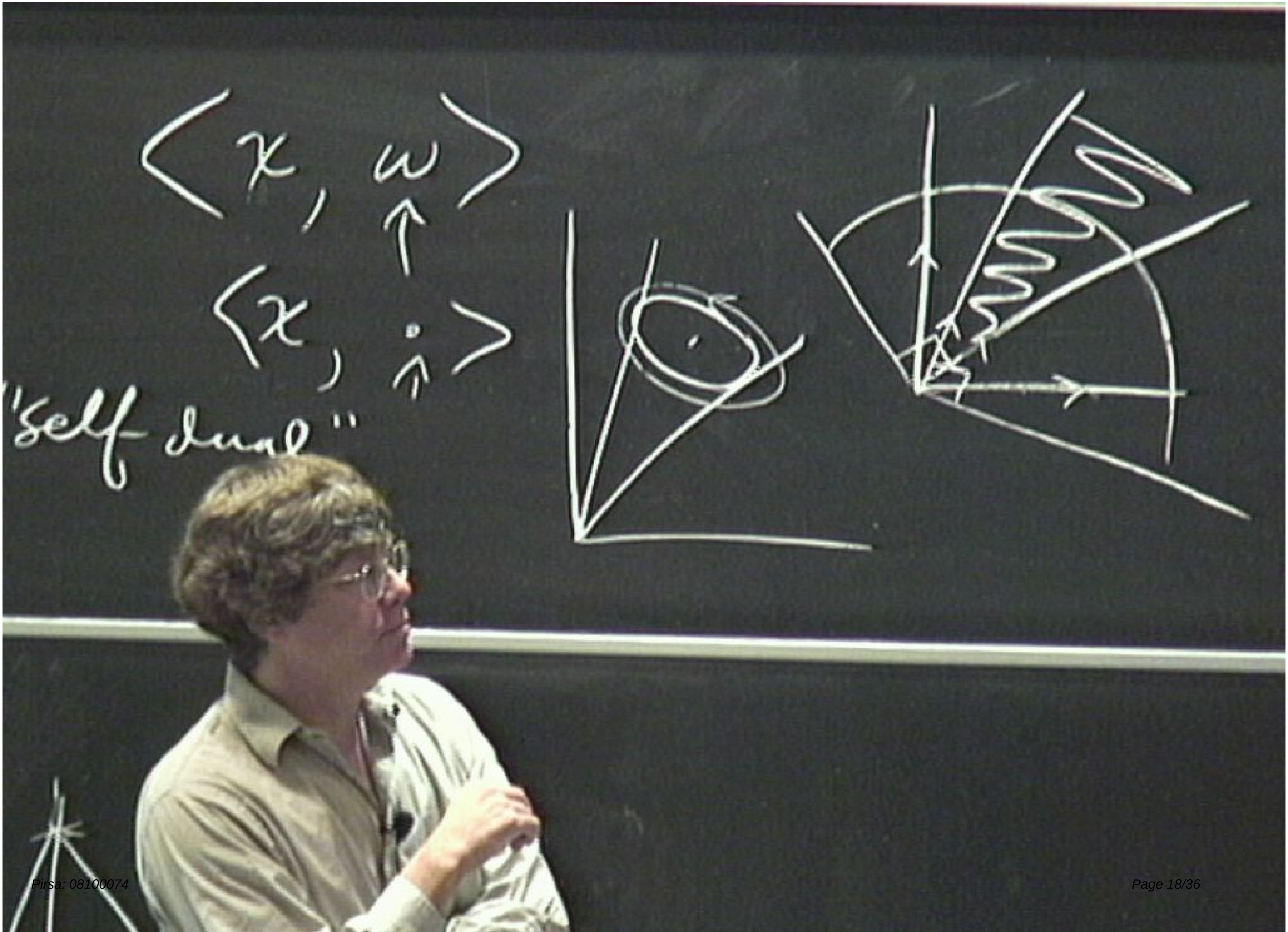
$\omega \in A \cap K$ - normalized state

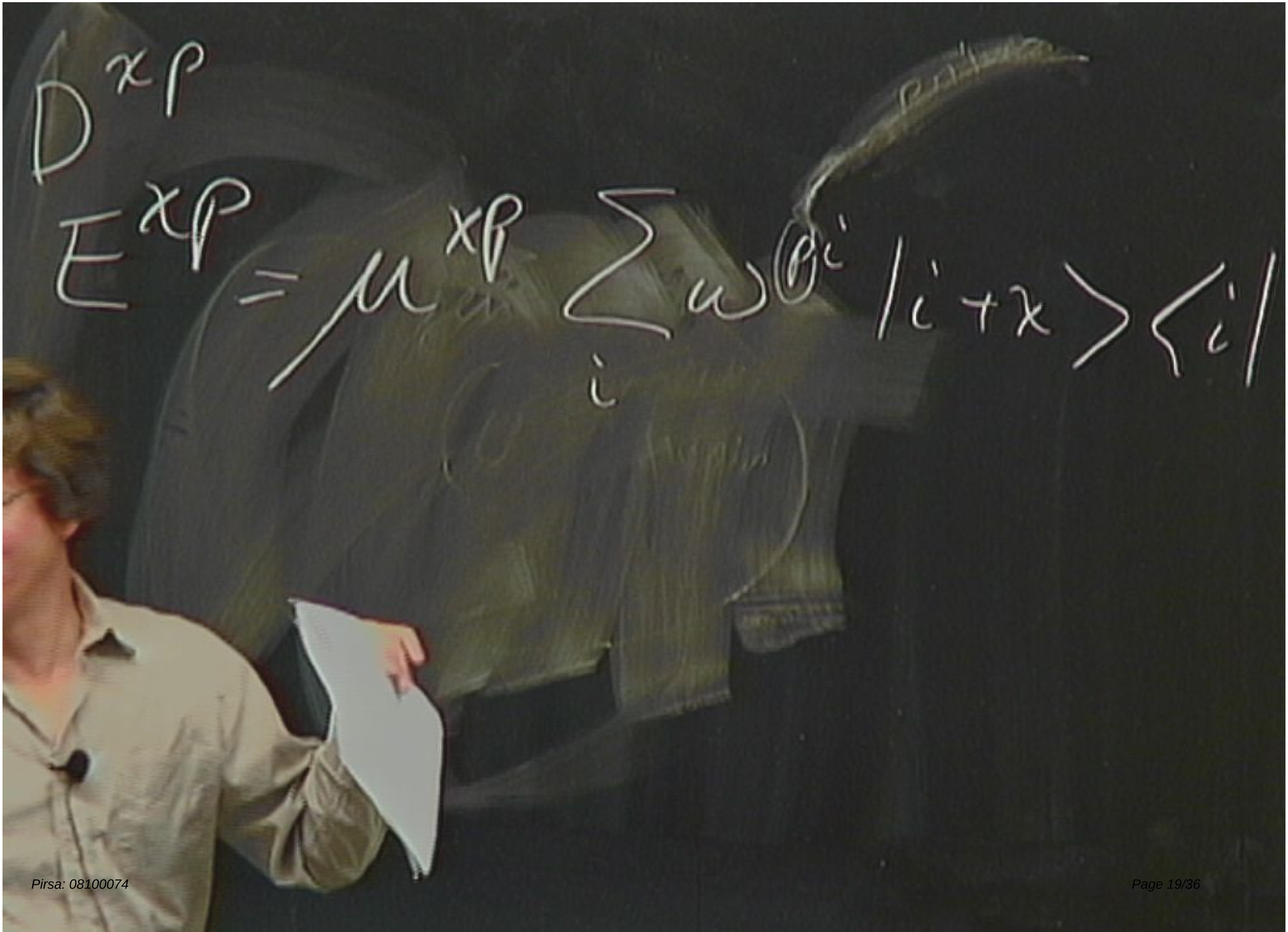
$e \in K^*$ $\quad$ $e \in [0, u] \subseteq K^*$

$$\sum_{i=1}^{N} e_i = u$$


$\langle x, \omega \rangle$


⟨x, ω⟩
⟨x, ·⟩
"self dual"


$\langle x, \omega \rangle$
$\langle x, \cdot \rangle$
"self dual"


D^{xp}
$E^{xp} = \mu^{xp} \sum_i \omega^{\theta i} |i+x\rangle\langle i|$

D^{xp}

$$E^{xp} = \mu^{xp} \sum_i \omega^{pi} |i+x\rangle\langle i|$$

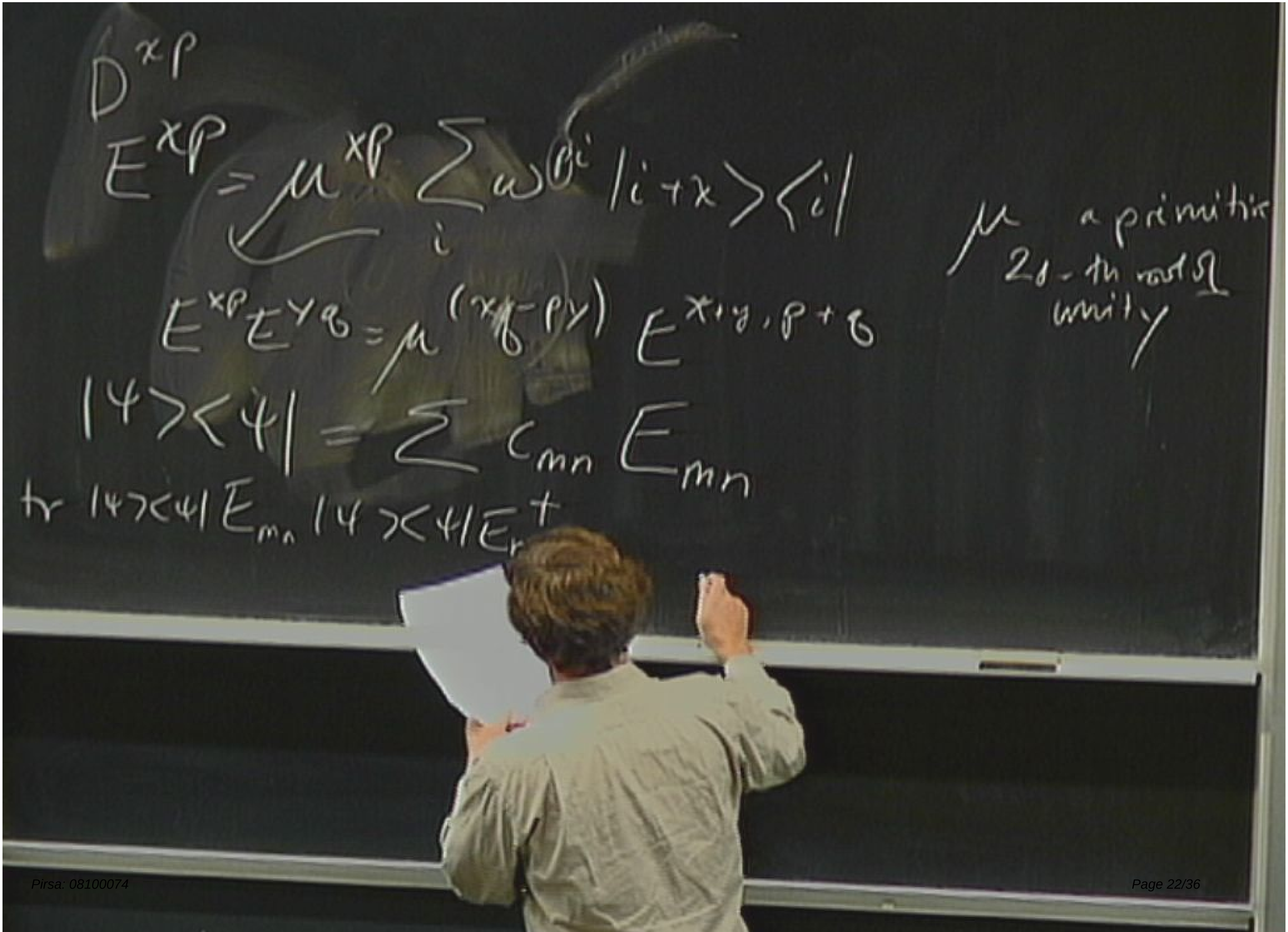
μ a primitive $2d$-th root of unity

$$D^{x\rho}$$

$$E^{x\rho} = \mu^{x\rho} \sum_i \omega^{\rho i} |i+x\rangle\langle i|$$

μ a primitive $2d$-th root of unity

$$E^{x\rho} E^{y\sigma} = \mu^{(x\sigma - \rho y)} E^{x+y,\, \rho+\sigma}$$



D^{xp}
$E^{xp} = \mu^{xp} \sum_i \omega^{pi} |i+x\rangle\langle i|$
μ a primitive $2d$-th root of unity
$E^{xp}E^{yq} = \mu^{(xq-py)} E^{x+y,\,p+q}$
$|\psi\rangle\langle\psi| = \sum c_{mn} E_{mn}$
$\mathrm{tr}\, |\psi\rangle\langle\psi| E_{mn} |\psi\rangle\langle\psi| E_{n}^{\dagger}$

$$D^{xp}$$

$$E^{xp} = \mu^{xp} \sum_i \omega^{pi} |i+x\rangle\langle i|$$

μ a primitive $2d$-th root of unity

$$E^{xp}E^{yq} = \mu^{(xq-py)} E^{x+y,\,p+q}$$

$$|\psi\rangle\langle\psi| = \sum c_{mn} E_{mn}$$

$$|c_{mn}|^2 = \frac{1}{d^2(d+1)}$$

$mn \neq 00$

$$\left|\mathrm{tr}\, |\psi\rangle\langle\psi| E_{mn} |\psi\rangle\langle\psi| E_{mn}^{\dagger}\right| = \frac{1}{d+1}$$

$$D^{x\rho}$$

$$E^{x\rho} = \mu^{x\rho} \sum_i \omega^{\rho i} |i+x\rangle\langle i|$$

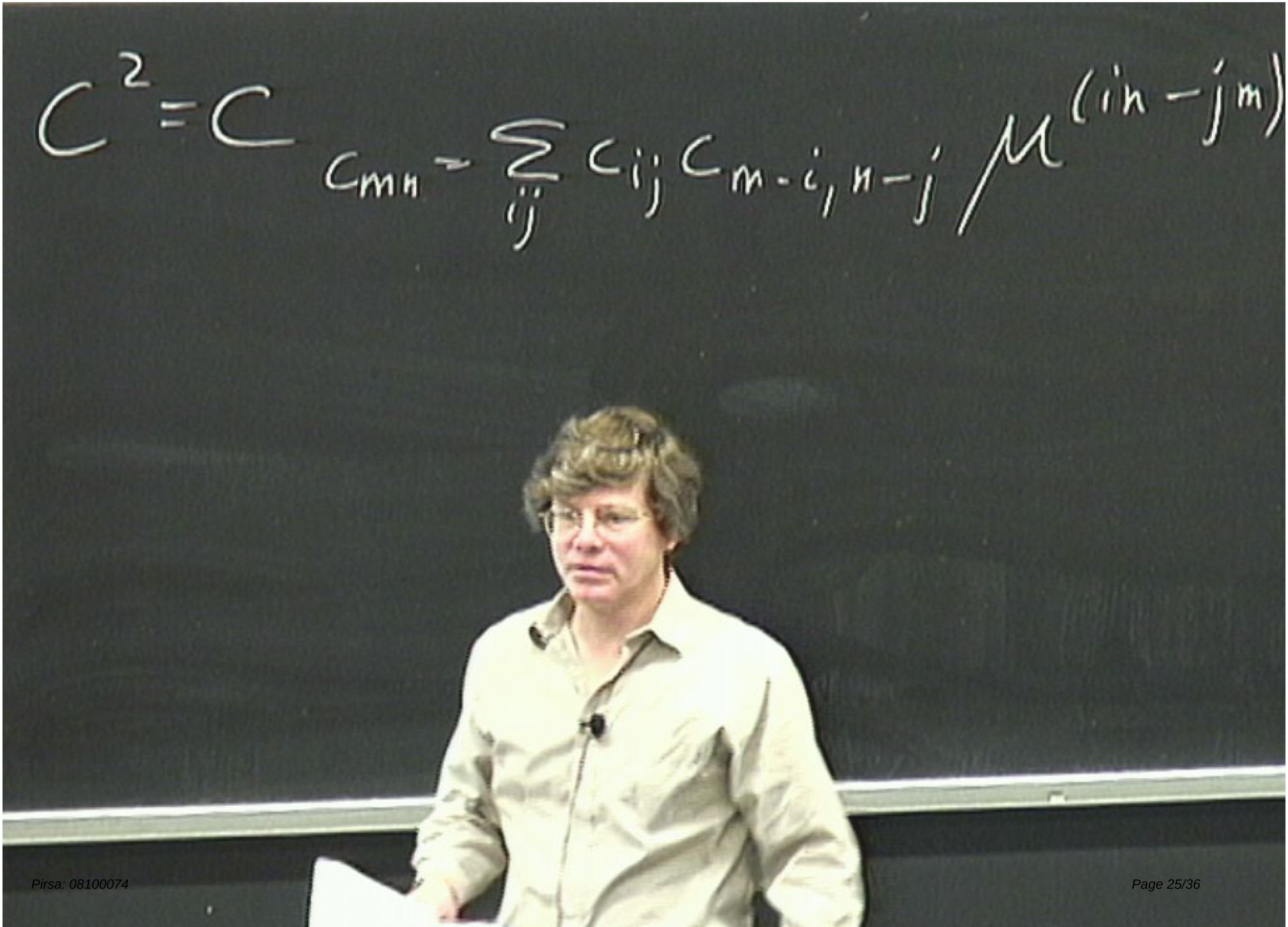
μ a primitive $2d$-th root of unity

$$E^{x\rho} E^{y\sigma} = \mu^{(x\sigma - \rho y)} E^{x+y,\, \rho+\sigma}$$

$$|\psi\rangle\langle\psi| = \sum c_{mn} E_{mn}$$

$$|c_{mn}|^2 = \frac{1}{d^2(d+1)} \quad mn \neq 00$$

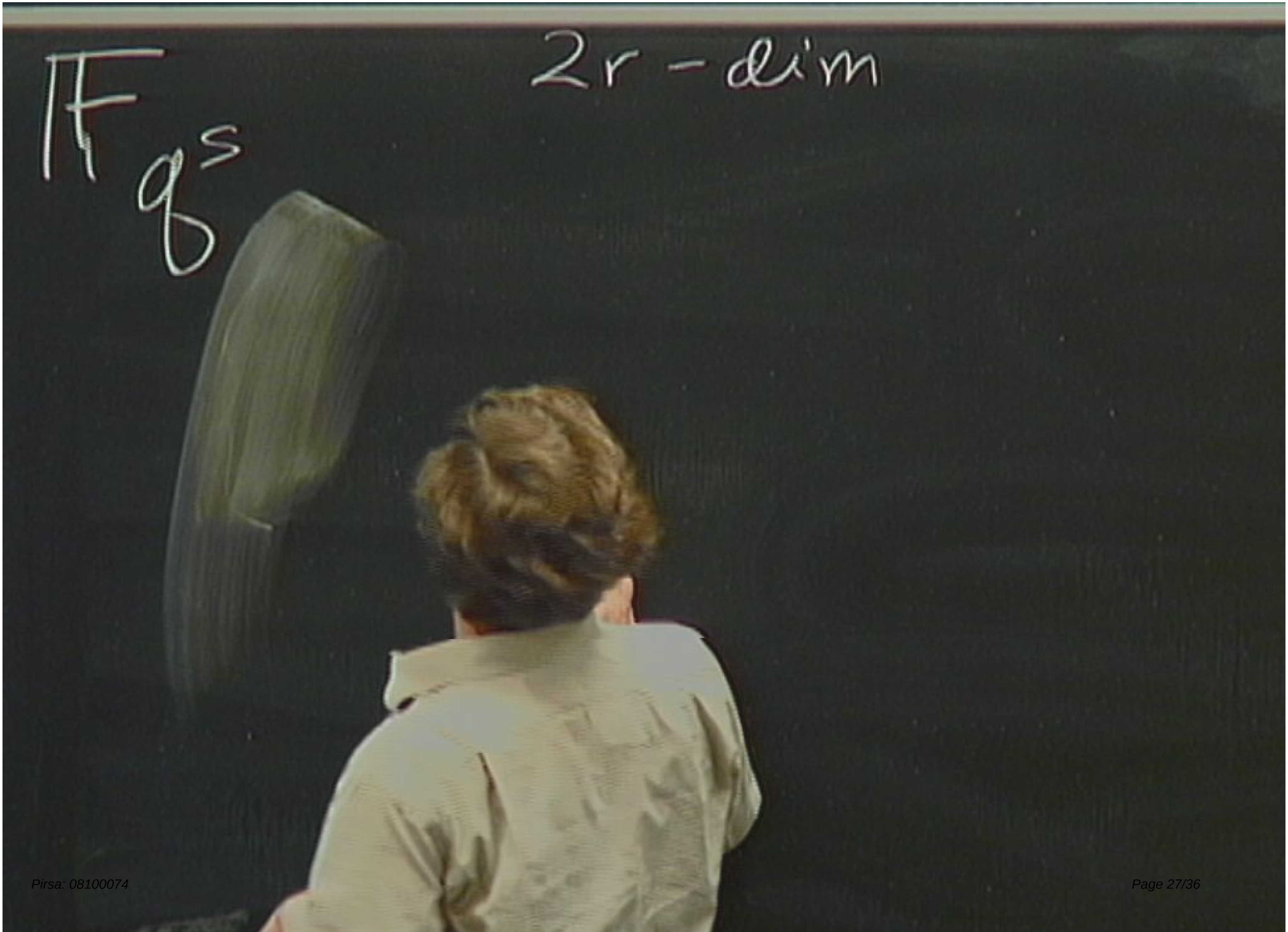
$$\left| \mathrm{tr}\, |\psi\rangle\langle\psi| E_{mn} |\psi\rangle\langle\psi| E_{mn}^{\dagger} \right| = \frac{1}{d+1}$$

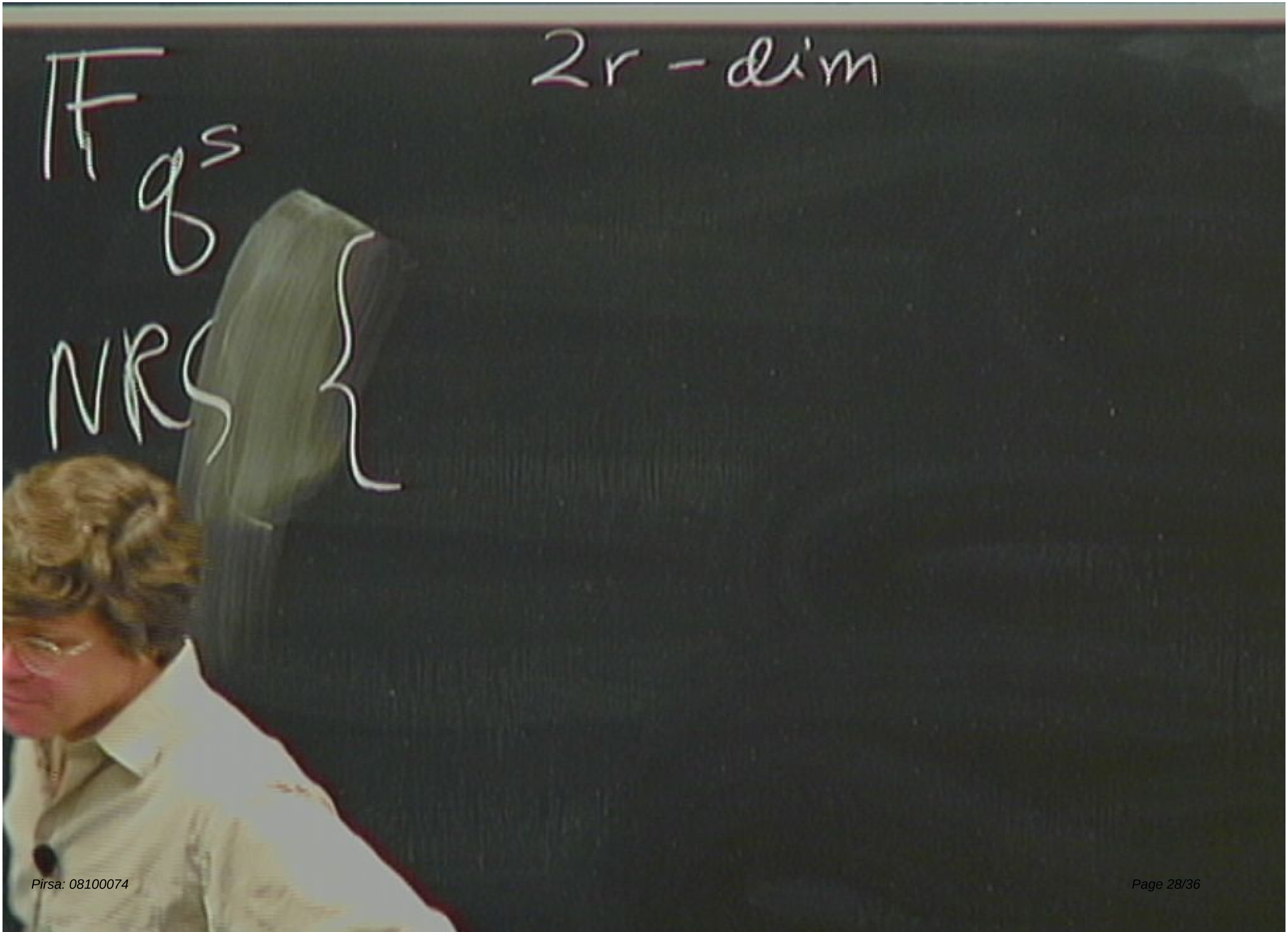

$C^2 = C$
$c_{mn} = \sum_{ij} c_{ij}\, c_{m-i,n-j}\, \mu^{(in-jm)}$

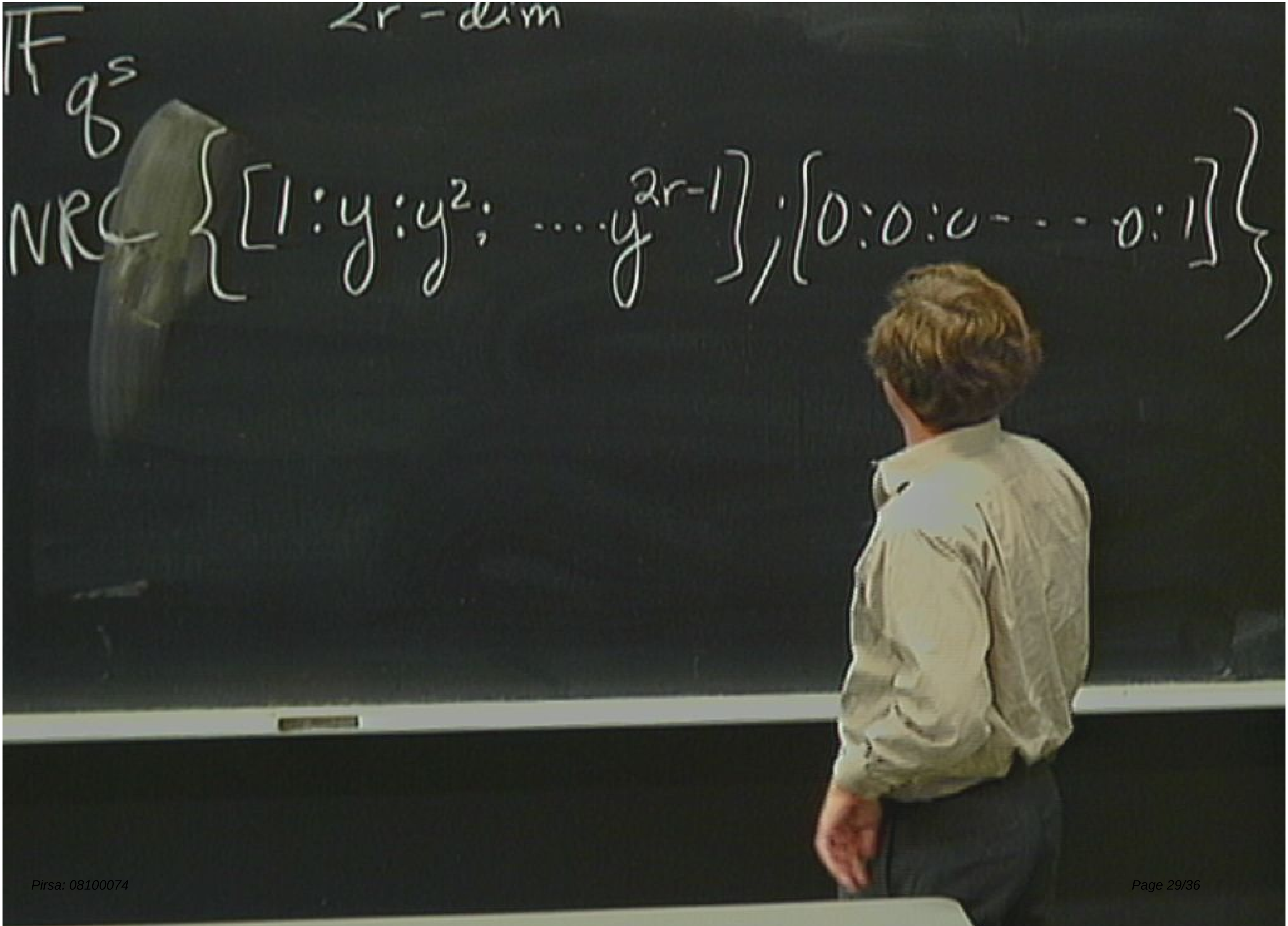
$$z_{mn} = c_{mn}\, d\sqrt{d+1}$$

$$(d-2)\sqrt{d+1}\, z_{mn} = \sum_{(i,j)\in\{00,mn\}} z_{ij}\, z_{m-i,n-j}\, \mu^{in-jm}$$

$$z_{-m,-n} = z^{*}_{mn}$$


$\mathbb{F}_{q^s}$
2r - dim


$\mathbb{F}_{q^s}$
2r - dim
NRS

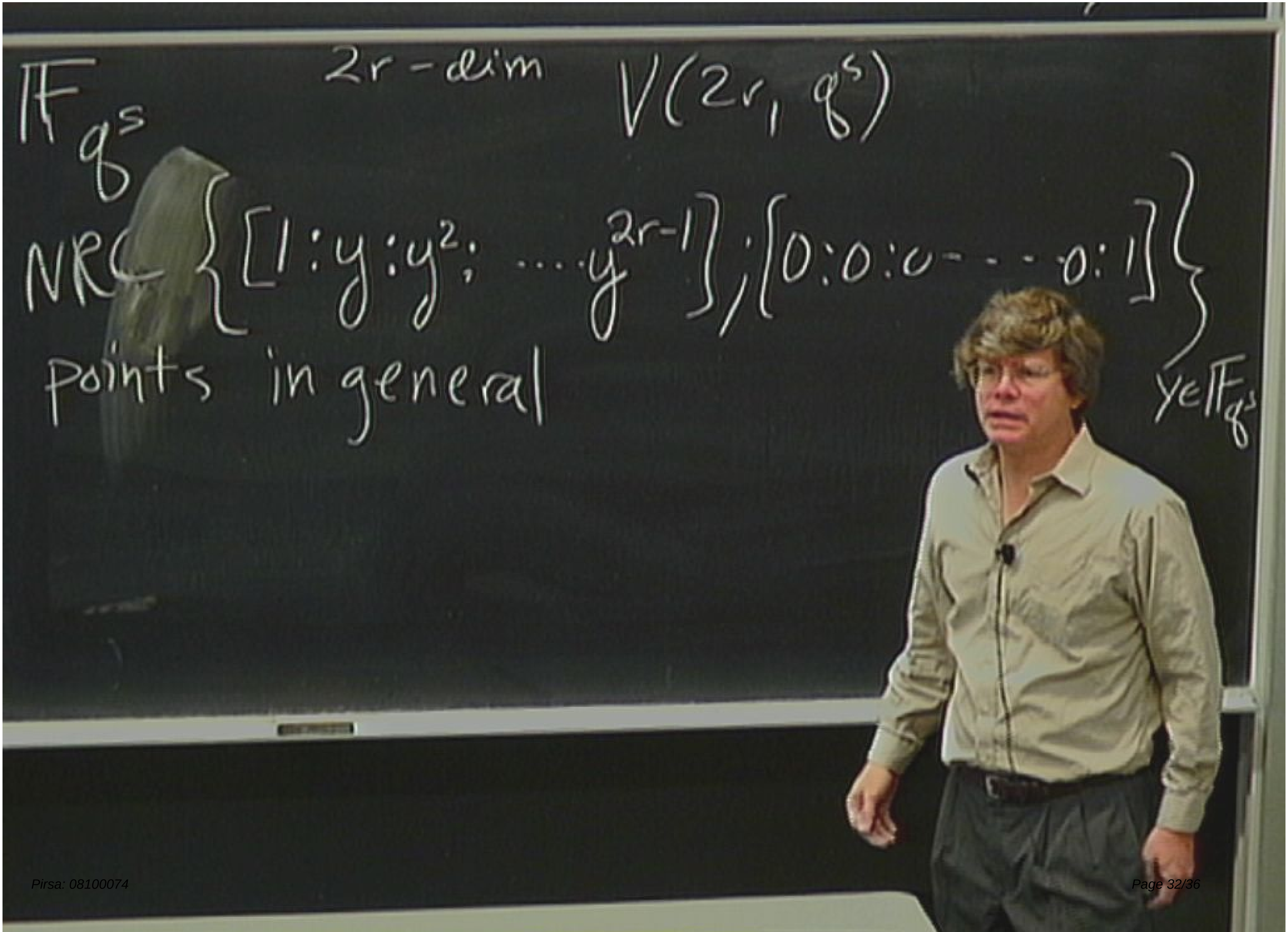

2r - dim
$\mathbb{F}_{q^s}$
NRC $\{[1:y:y^2: \ldots y^{2r-1}]; [0:0:0 \cdots 0:1]\}$

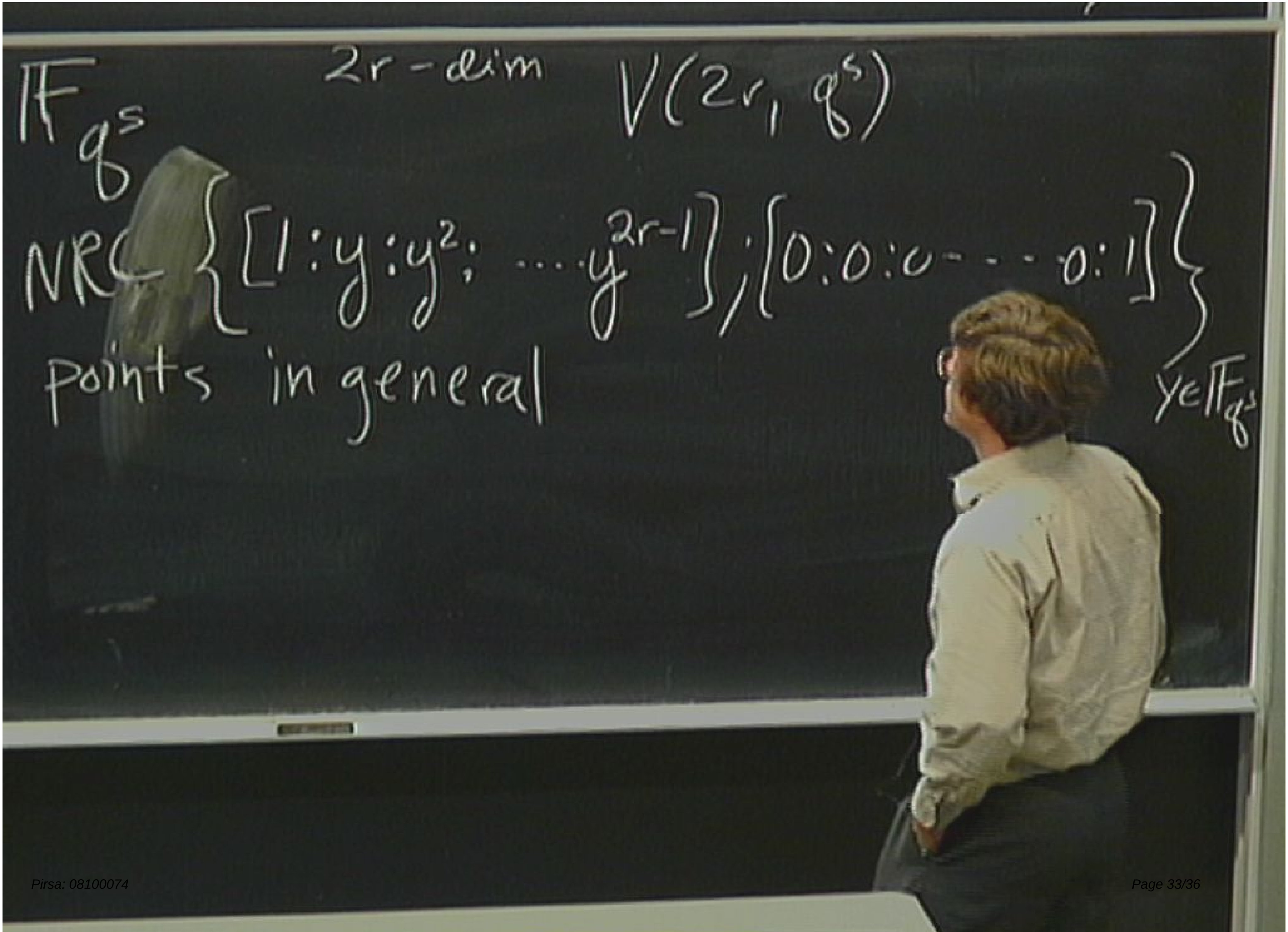
$\mathbb{F}_{q^s}$ 2r - dim

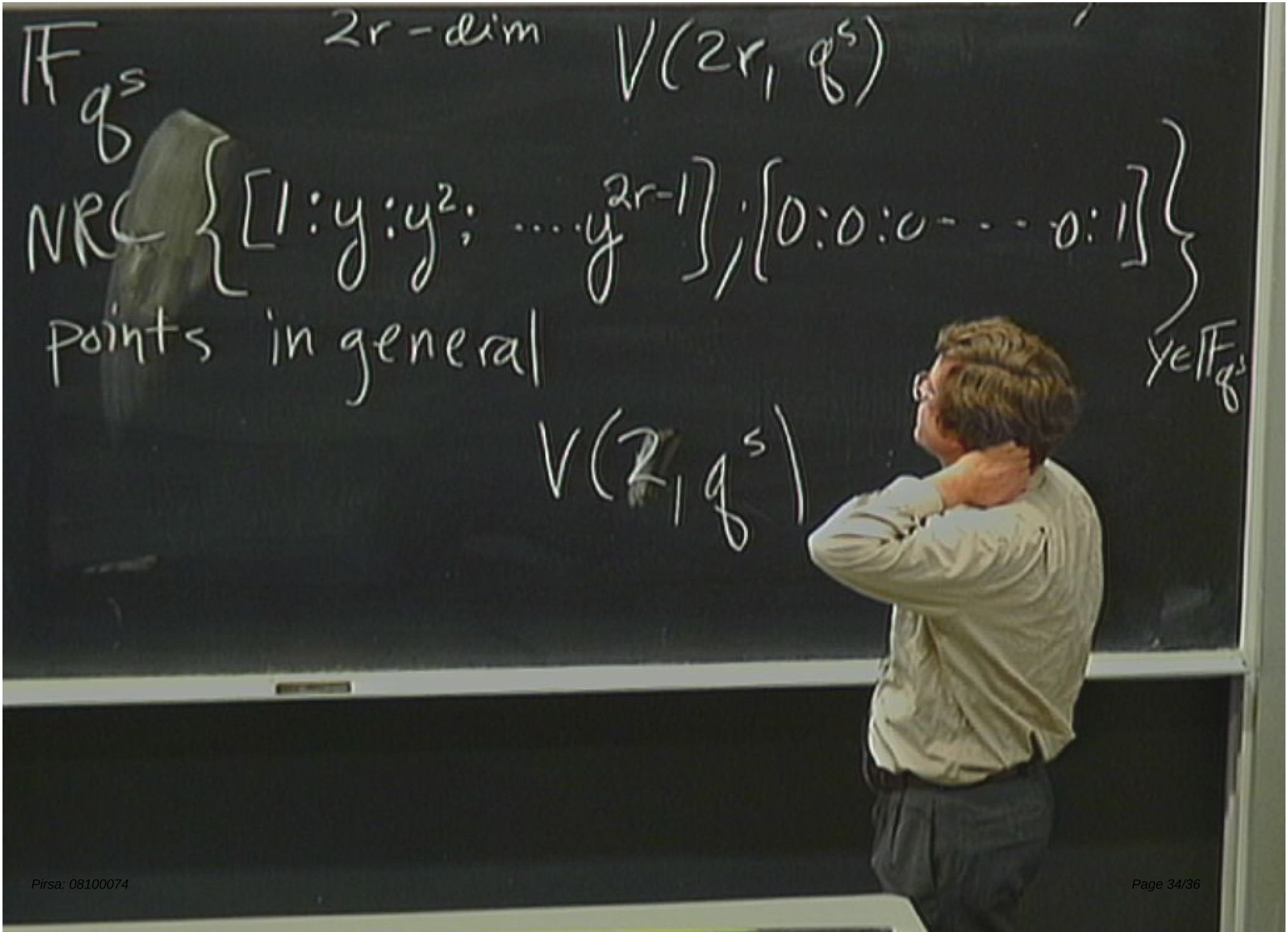
$$NRC\left\{[1:y:y^2:\cdots y^{2r-1}],[0:0:0\cdots 0:1]\right\}_{y\in\mathbb{F}_{q^s}}$$

$\mathbb{F}_{q^s}$ $\quad$ 2r - dim $\quad V(2r, q^s)$

$$\text{NRC} \left\{ [1 : y : y^2 : \ldots\ldots y^{2r-1}], [0:0:0 \cdots 0:1] \right\}_{y \in \mathbb{F}_{q^s}}$$


$\mathbb{F}_{q^s}$
2r - dim
$V(2r, q^s)$
NRC $\{[1:y:y^2: \ldots y^{2r-1}];[0:0:0 \cdots 0:1]\}$
$y \in \mathbb{F}_{q^s}$
points in general


$\mathbb{F}_{q^s}$
2r - dim
$V(2r, q^s)$
NRC $\{[1:y:y^2:\dots y^{2r-1}];[0:0:0\cdots 0:1]\}$
$y \in \mathbb{F}_{q^s}$
points in general


$\mathbb{F}_{q^s}$
2r - dim
$V(2r, q^s)$
NRC $\{[1:y:y^2: \ldots y^{2r-1}]; [0:0:0 \cdots 0:1]\}_{y \in \mathbb{F}_{q^s}}$
points in general
$V(2, q^s)$

$\mathbb{F}_{q^s}$ 2r-dim $V(2r, q^s)$

NRC $\left\{ [1 : y : y^2 : \cdots y^{2r-1}] ; [0:0:0 \cdots 0:1] \right\}_{y \in \mathbb{F}}$

points in general

$V(2, q^s)$ q^s+1

points in general

FOCS '02 $V(\mathbb{Z}, q^s)$: $(\mathbb{Q}^s$

HB, Crepeau, Gottesman, Smith, Tapp