Title: MUBS in infinite dimensions: the problematic analogy between L2(R) and C^d

Date: Oct 30, 2008 10:30 AM

URL: http://pirsa.org/08100072

Abstract:

0802.0394
Weigert, Wilkinson

0707.2071
Appleby, Dang, Fuchs

394

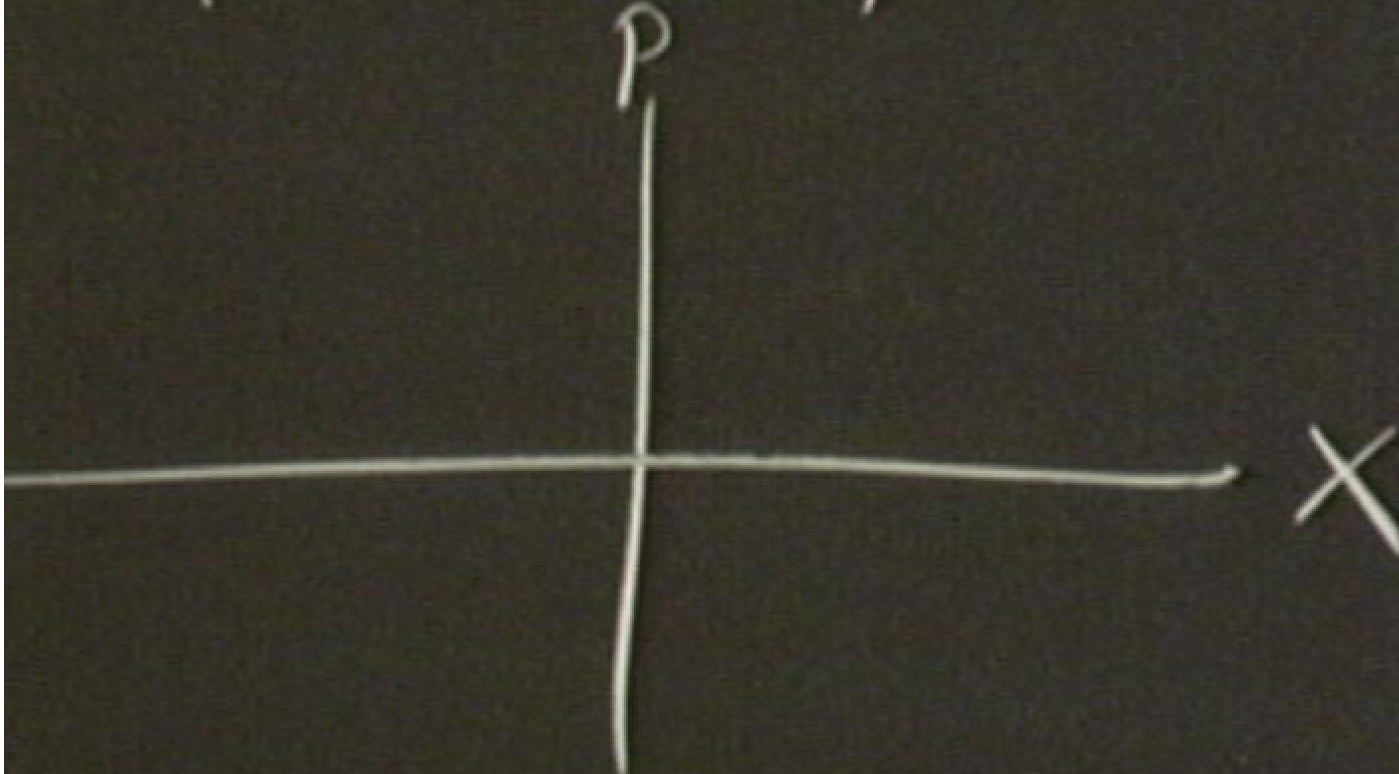
kinson

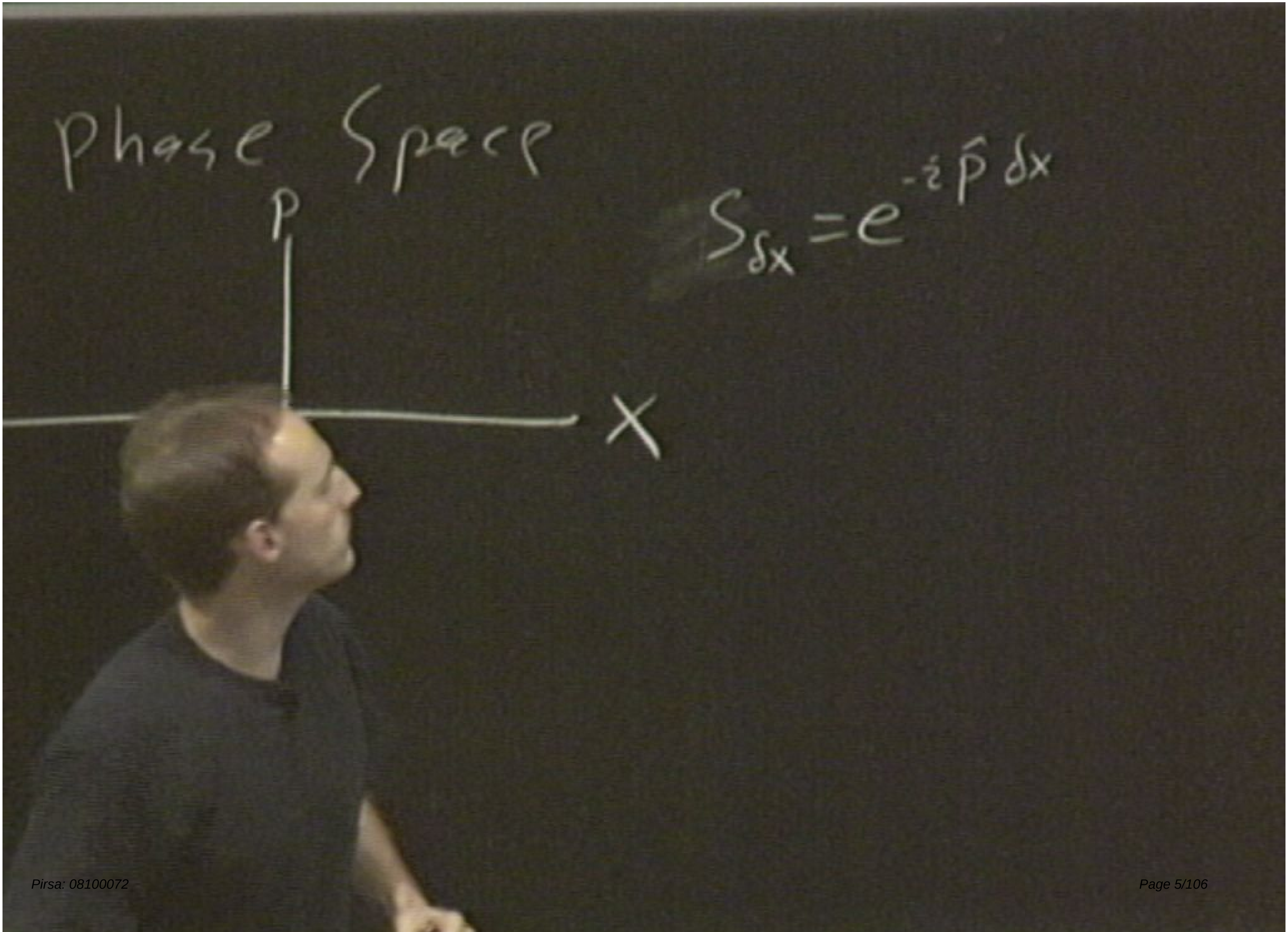
What about continuous variables?

0707

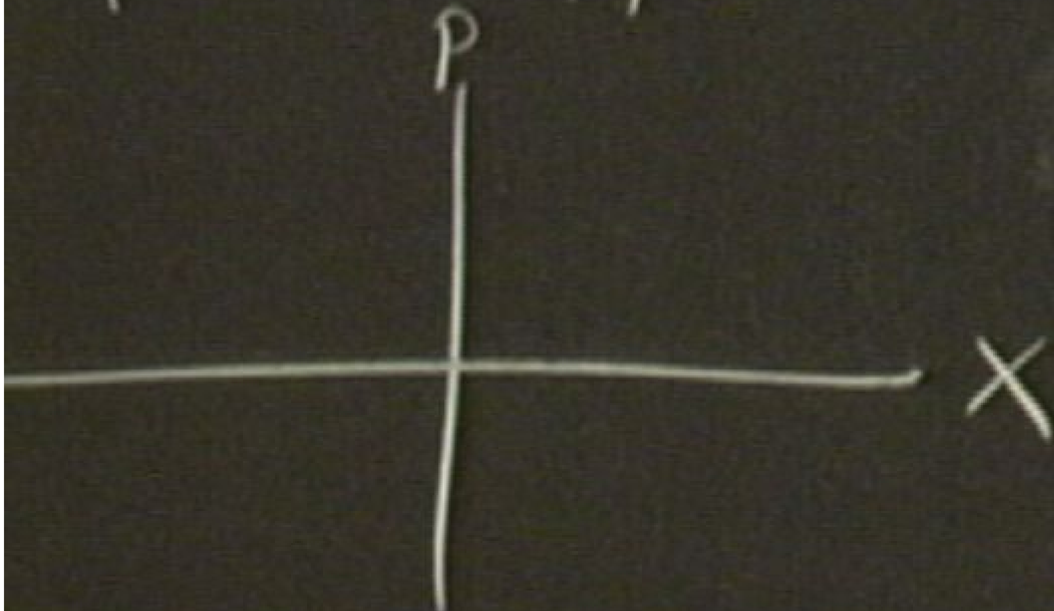
Appleby,

Phase Space




Phase Space
p
x
$S_{\delta x} = e^{-i\hat{p}\,\delta x}$

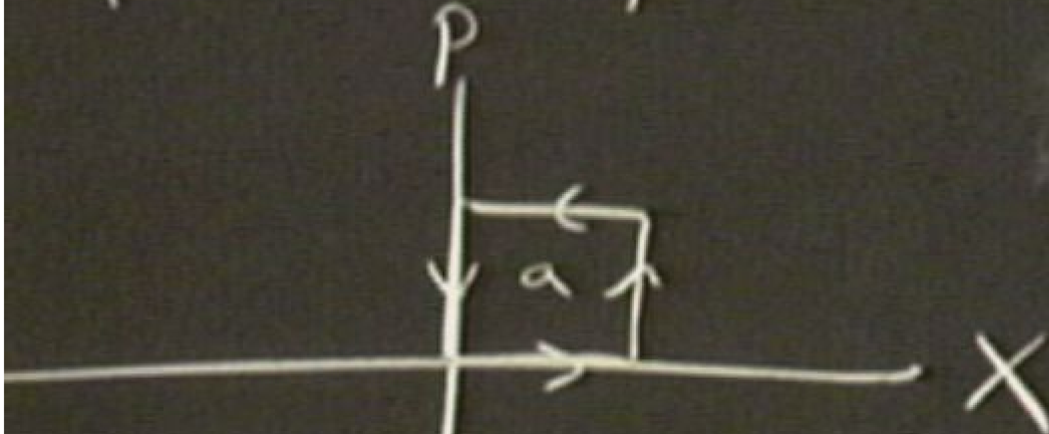
Phase Space



$$S_{\delta x} = e^{-i\hat{p}\,\delta x}$$

$$A_{\delta p} = e^{-i\hat{x}\,\delta p}$$

Phase Space

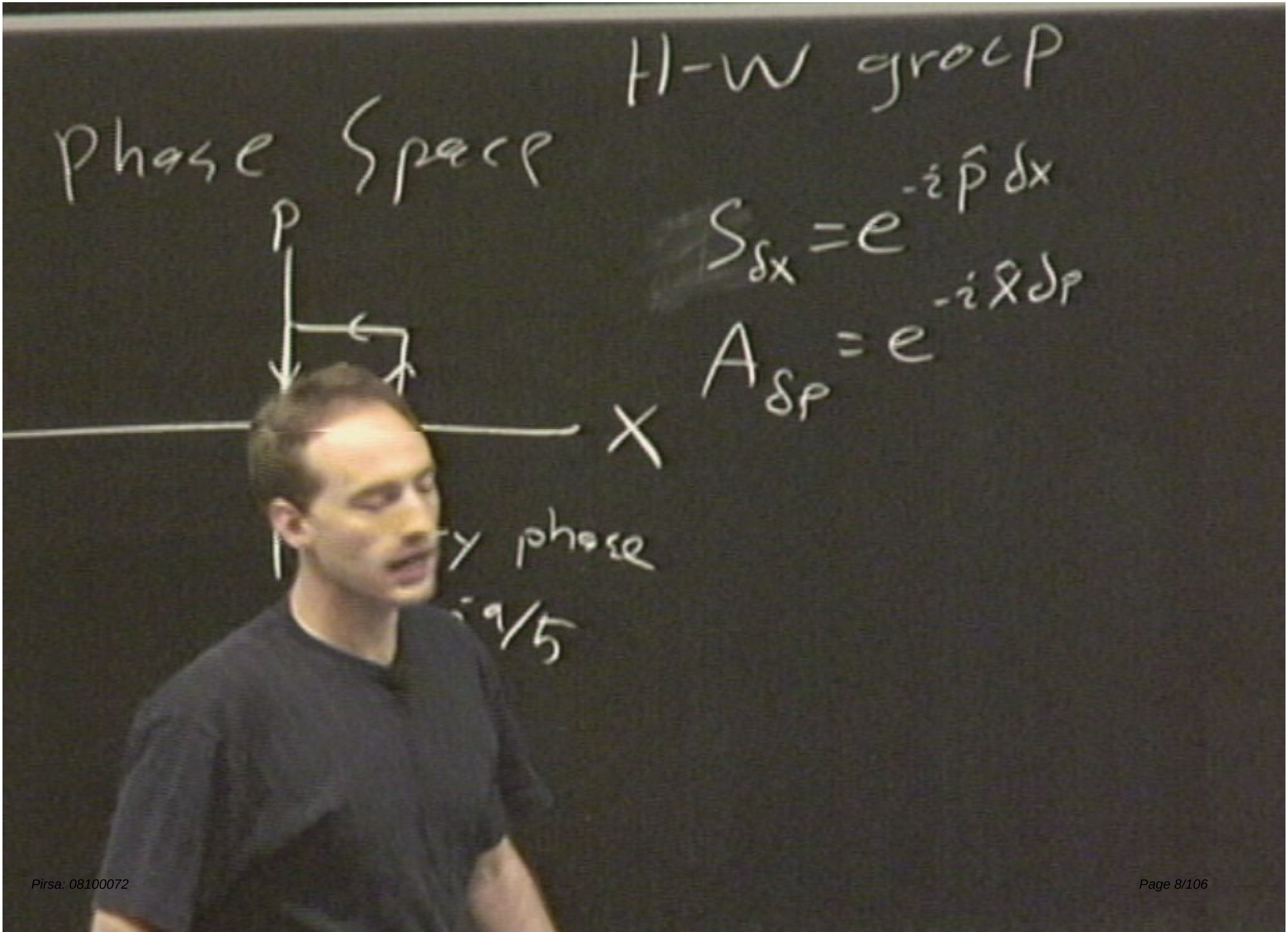


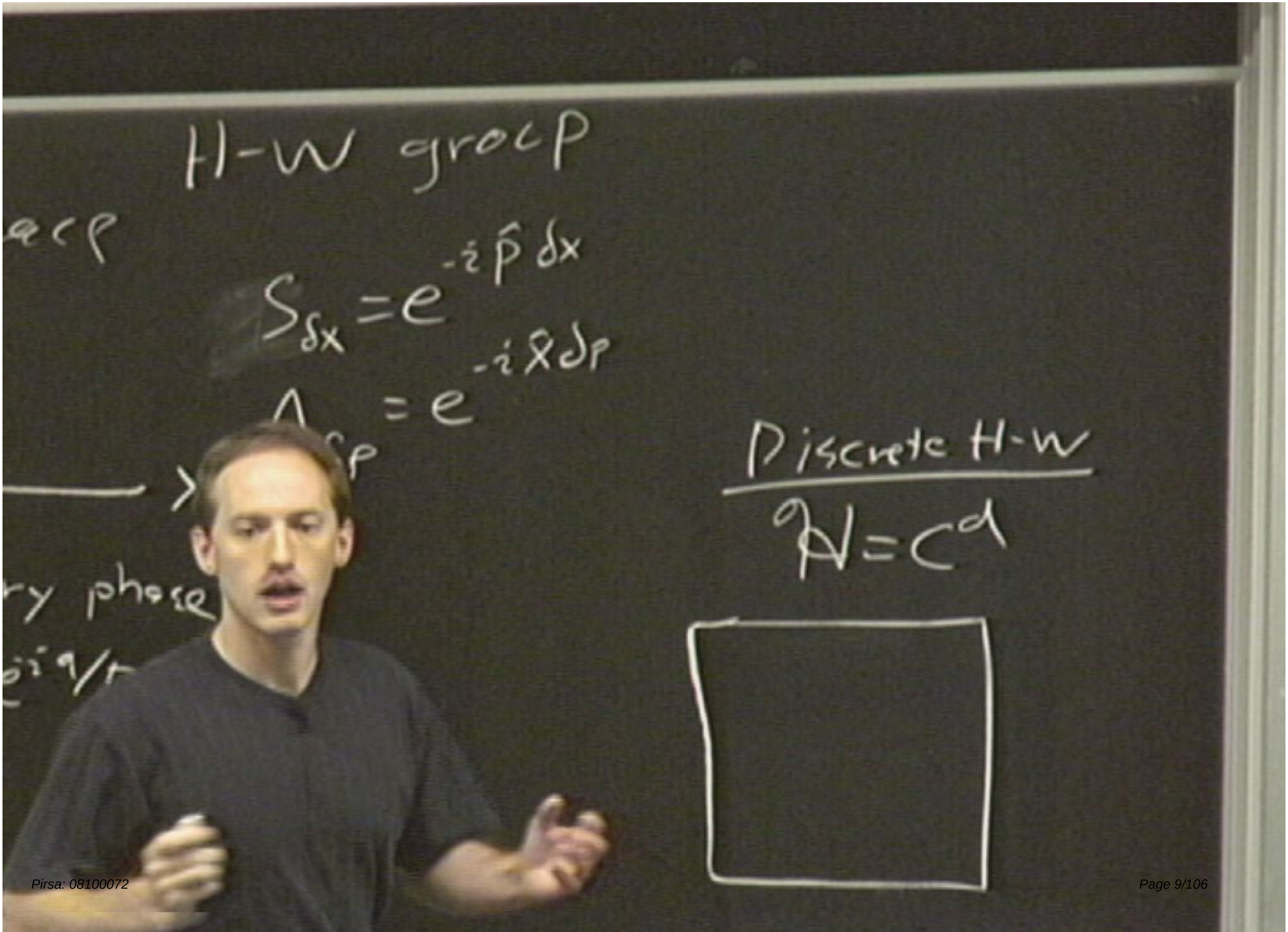
$$S_{\delta x} = e^{-i\hat{p}\,\delta x}$$

$$A_{\delta p} = e^{-i\hat{x}\,\delta p}$$

Berry phase

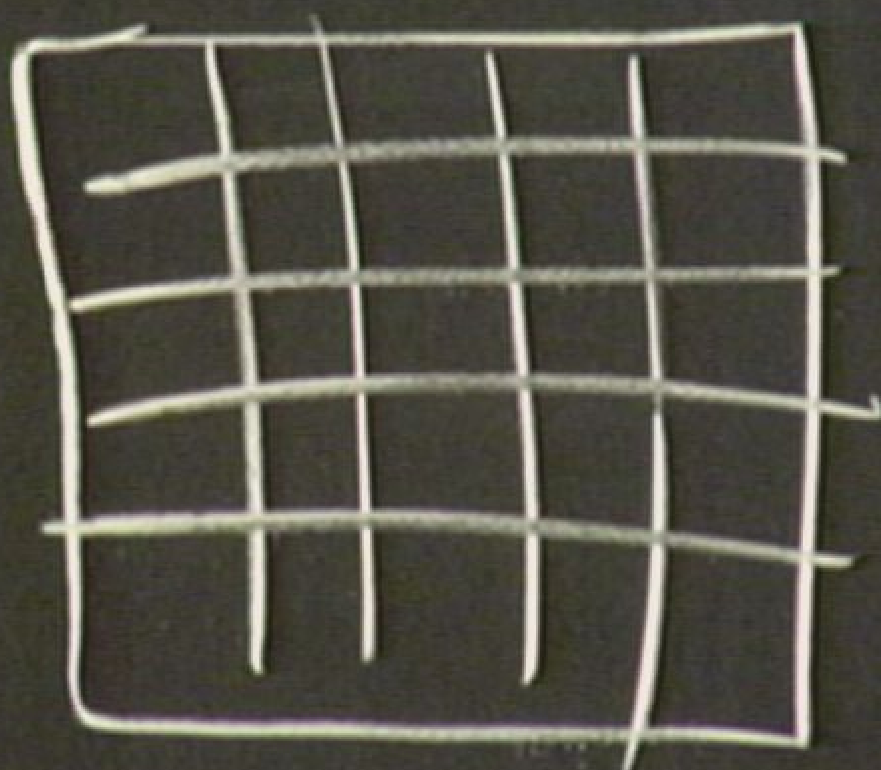
$$e^{ia/5}$$

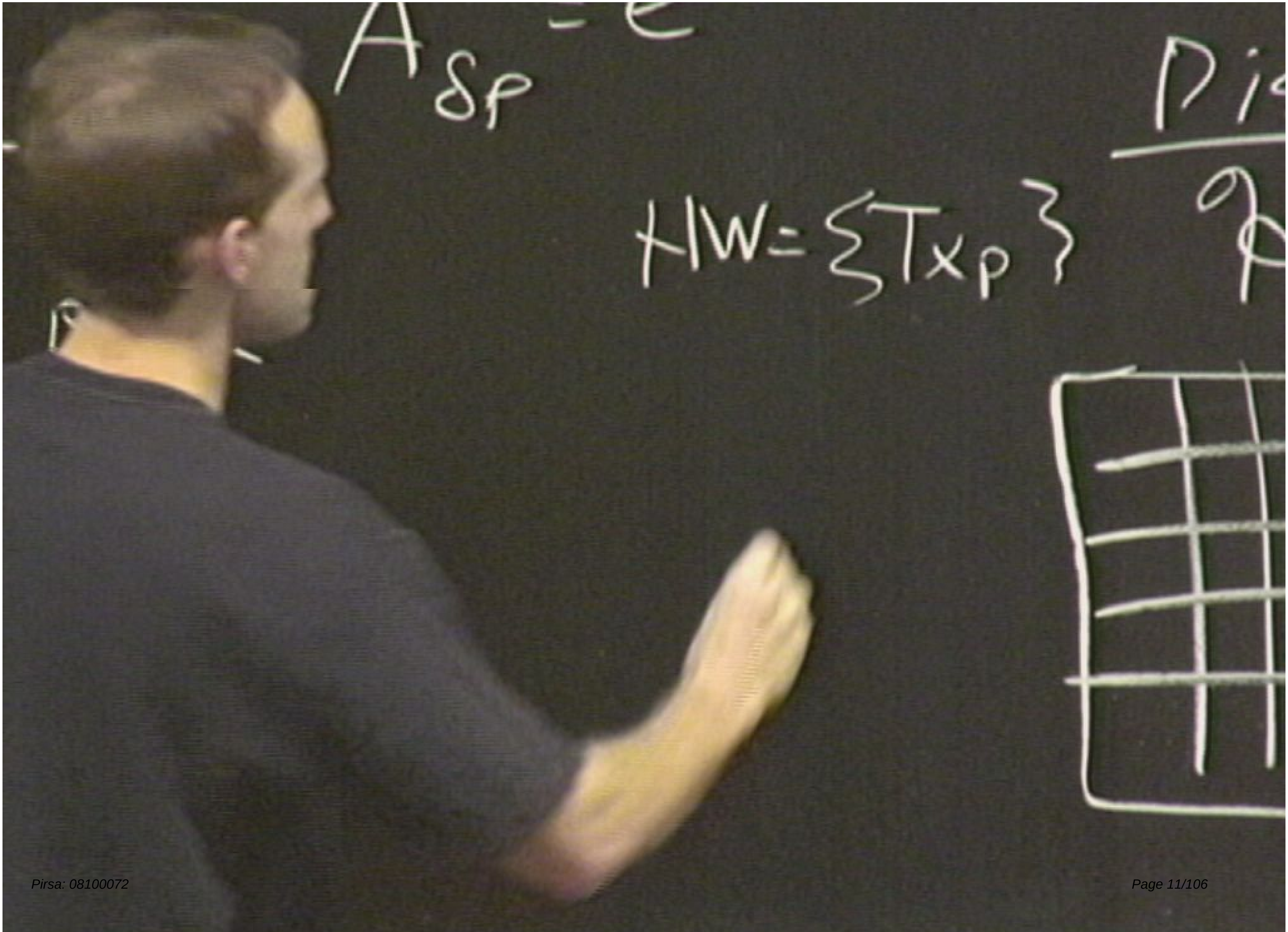

H-W group
Phase Space
p
x
$S_{\delta x} = e^{-i\hat{p}\,\delta x}$
$A_{\delta p} = e^{-i\hat{x}\,\delta p}$
y phase


H-W group
$S_{\delta x} = e^{-i\hat{p}\,\delta x}$
$= e^{-i\hat{x}\,\delta p}$
Discrete H-W
$\mathcal{H} = C^d$
phase

Discrete H-W

$$\mathcal{H} = \mathbb{C}^d$$




A δp
HW = {Txp}

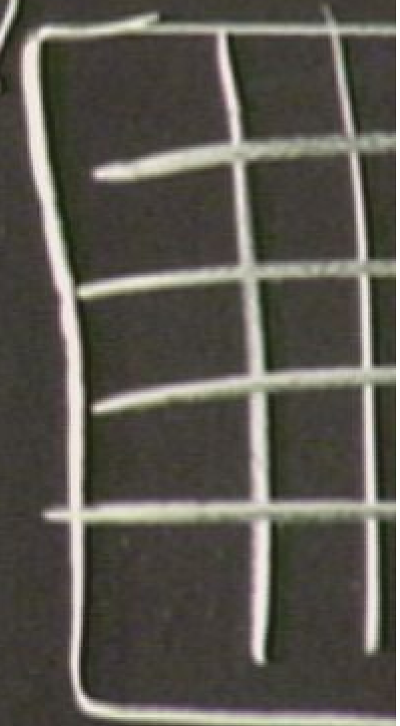
$A_{\delta p} = e$

$\times$

phase

$HW = \{T_{xp}\}$

$x, p \in [0..d\text{-}1]$



$$S_{\delta x} = e^{-i\hat{p}\,\delta x}$$

$$A_{\delta p} = e^{-i\hat{x}\,\delta p}$$

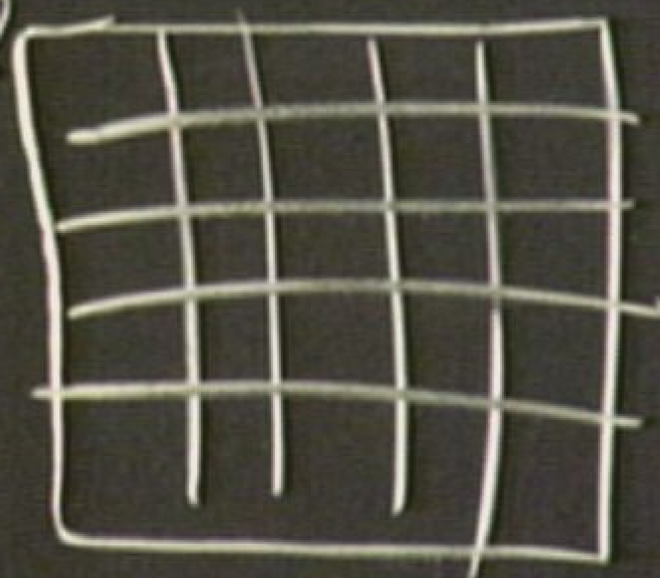
$$\boxed{d = \text{Prime}}$$

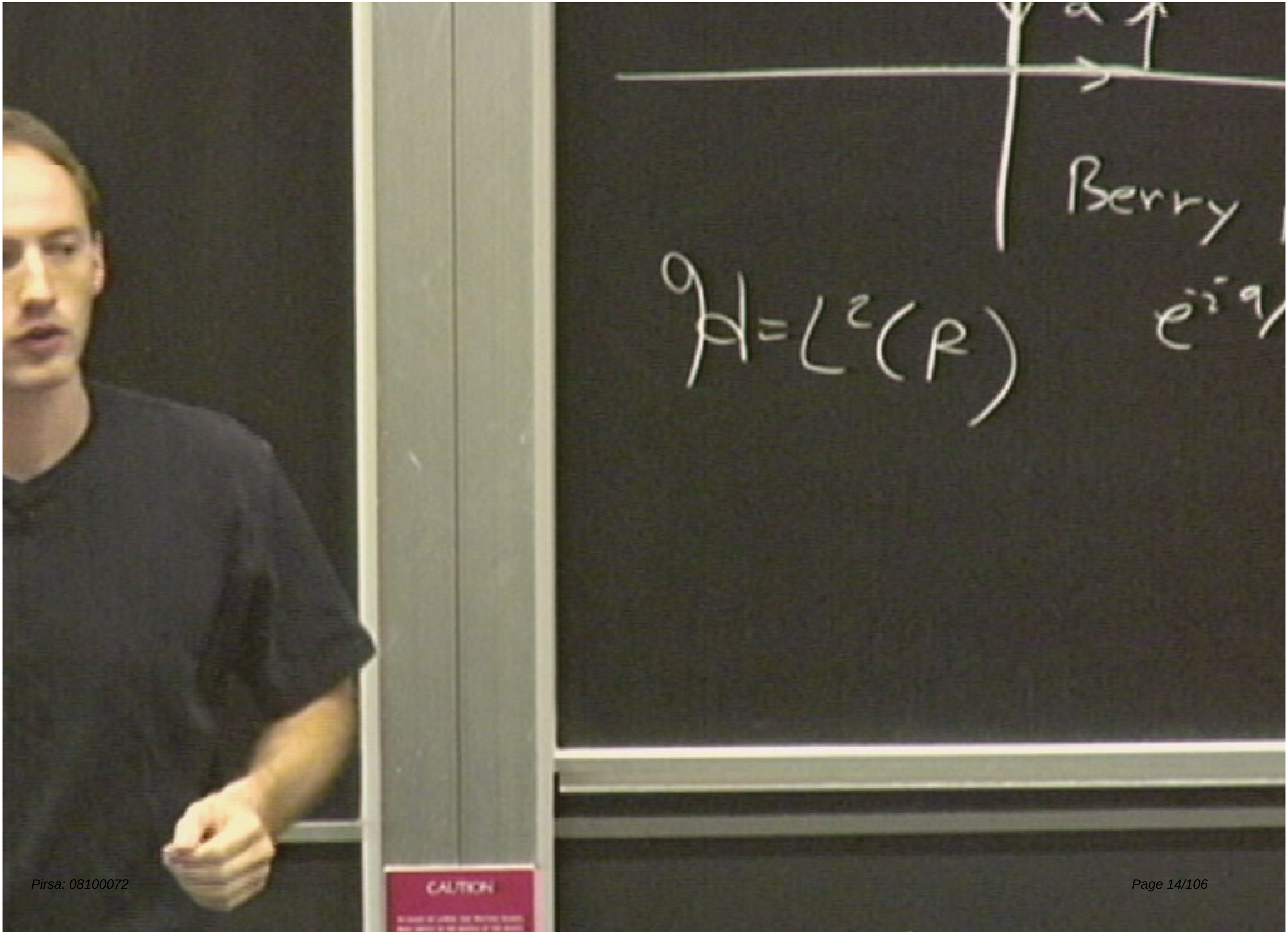
Discrete H-W

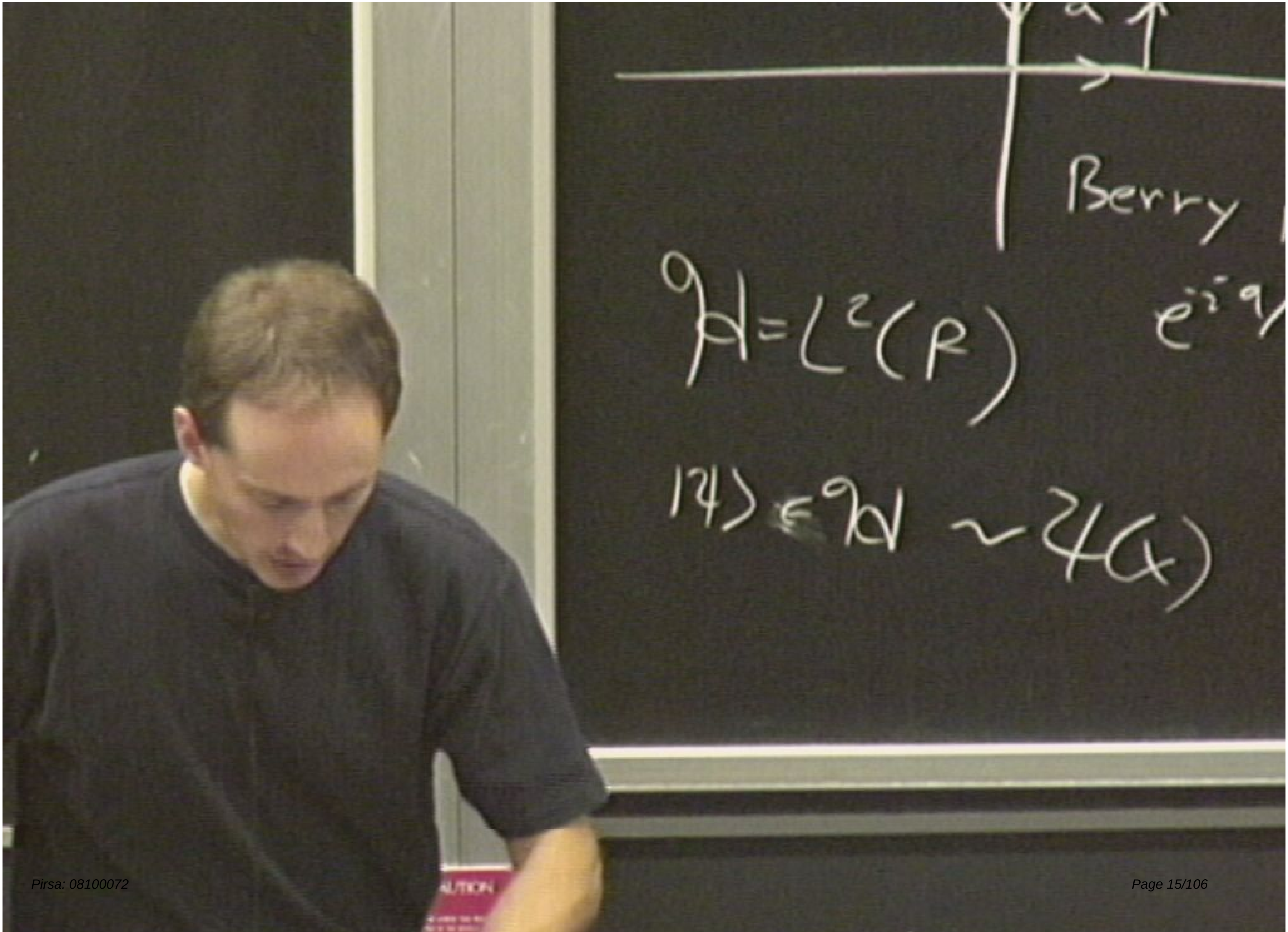
$$\mathcal{H} = \mathbb{C}^d$$

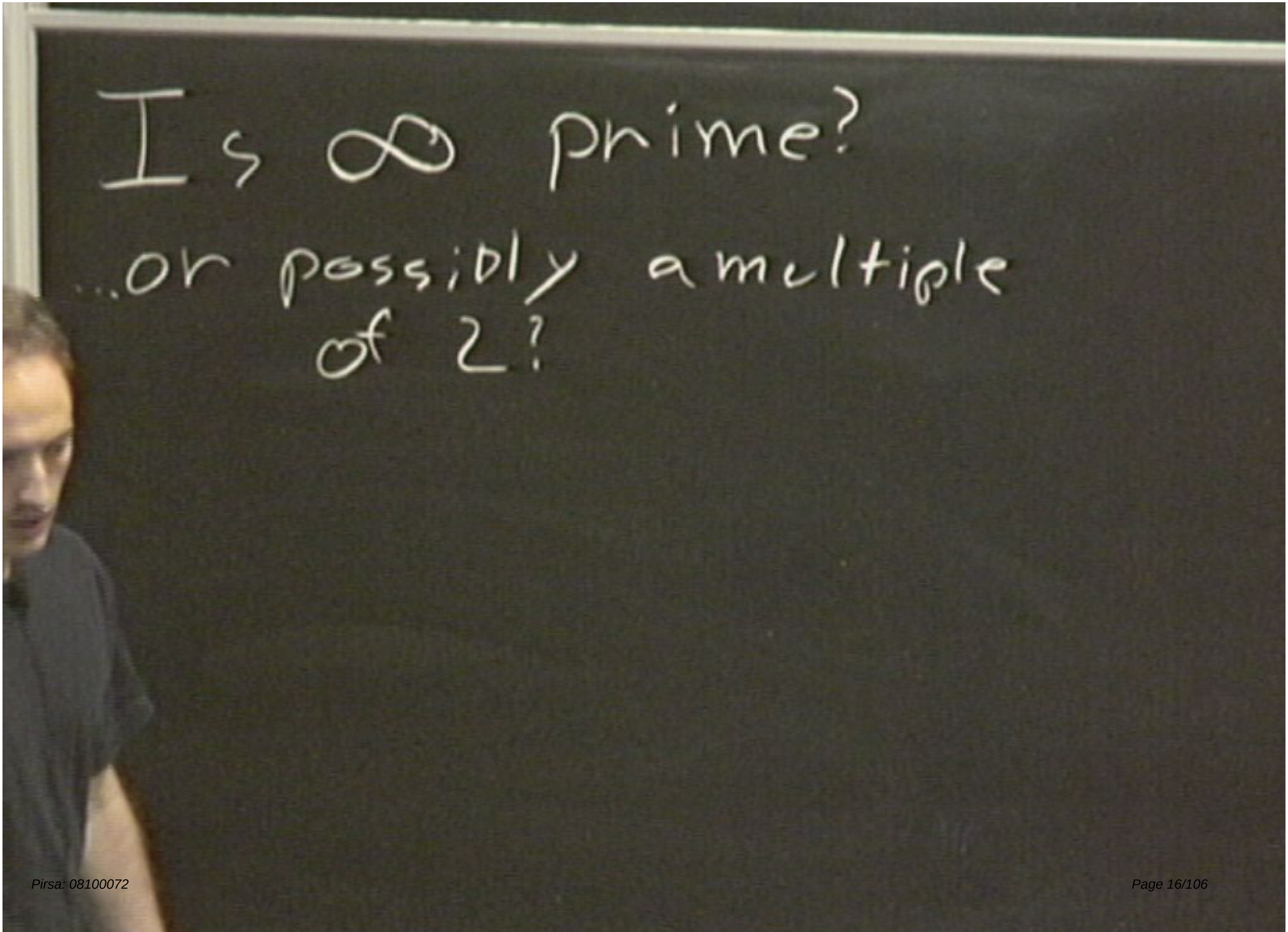
$$HW = \{T_{xp}\}$$

$$x, p \in [0..d-1]$$




Berry
$\mathcal{H} = L^2(R)$
CAUTION


Berry
$\mathcal{H} = L^2(\mathbb{R})$
$|\psi\rangle \in \mathcal{H} \sim \psi(x)$


Is ∞ prime?
...or possibly a multiple
of 2?

Is ∞ prime?

...or possibly a multiple of 2?

If d = prime

then

Is ∞ prime?

...or possibly a multiple of 2?

If d = prime

then $\exists$ $d+1$ MUBs

Is ∞ prime?

...or possibly a multiple of 2?

If d = prime

then $\exists$ $d+1$ mUBs

If $d = 2 \times$ odd #

then $\exists$ 3 mUBs

.?

.ltiple

If $d = d_1^{r_1} d_2^{r_2} \cdots$

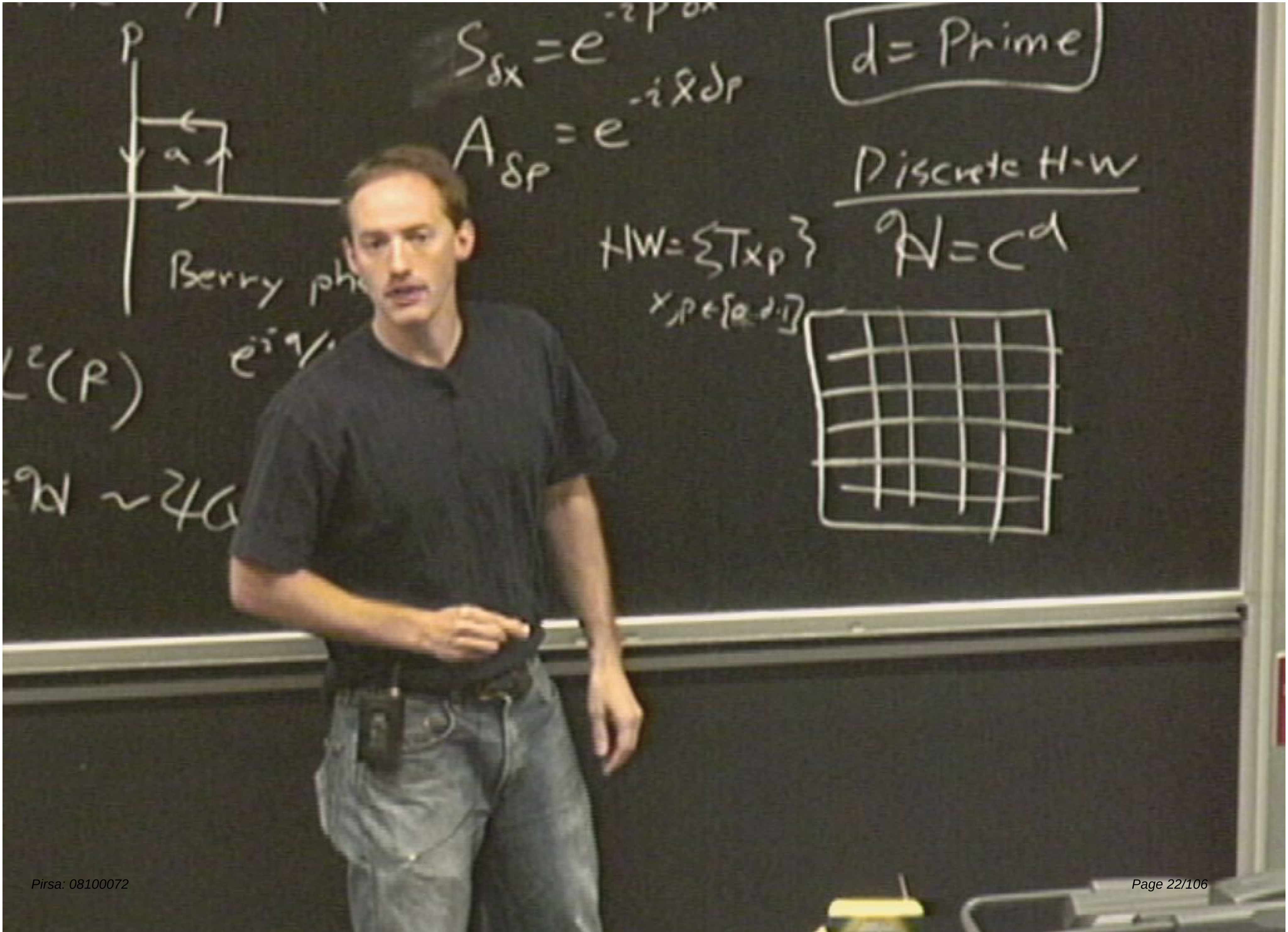
$\# MUBs \geq \min_i (d_i^{r_i} + 1)$

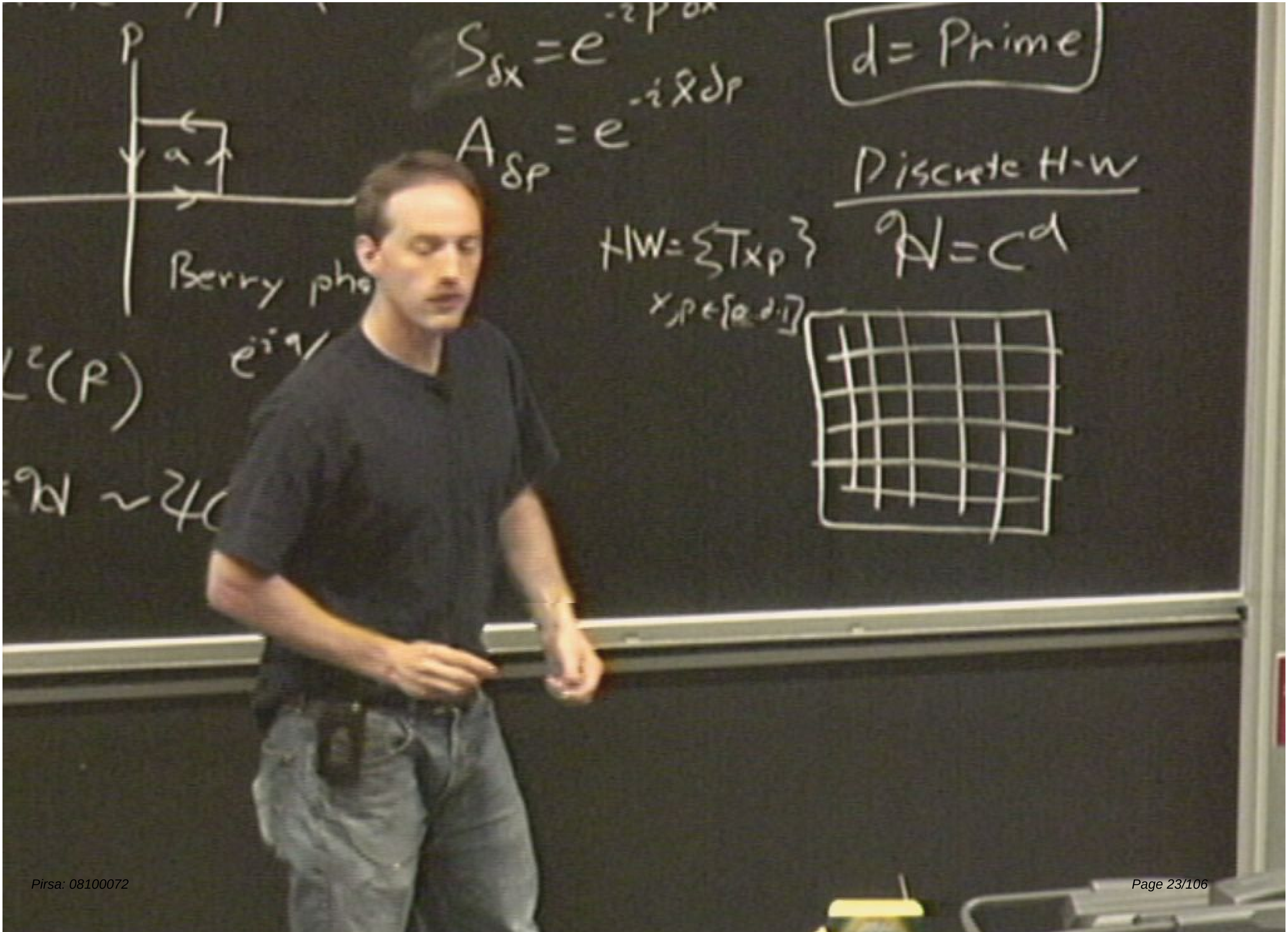
...?

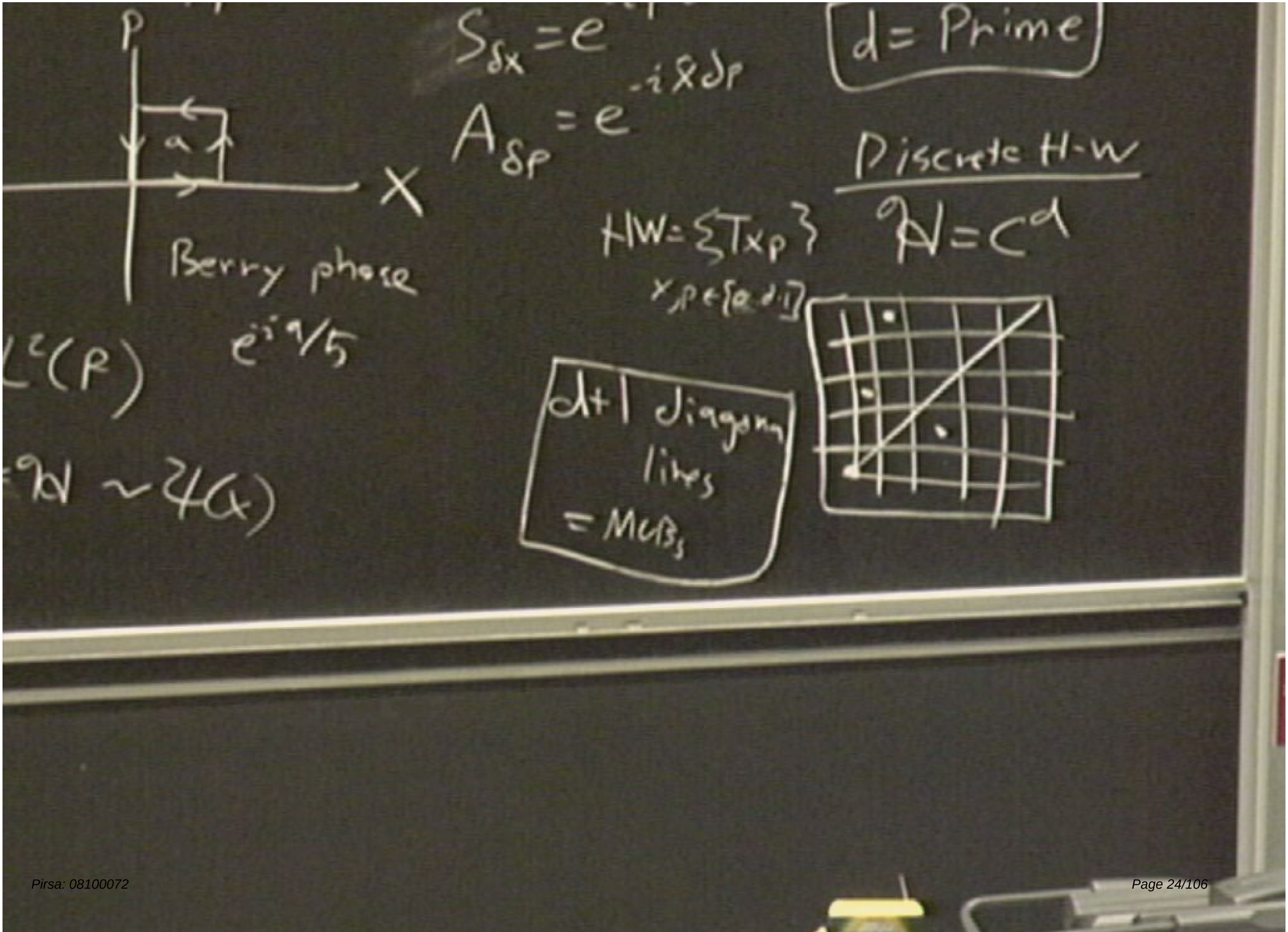
...ltiple

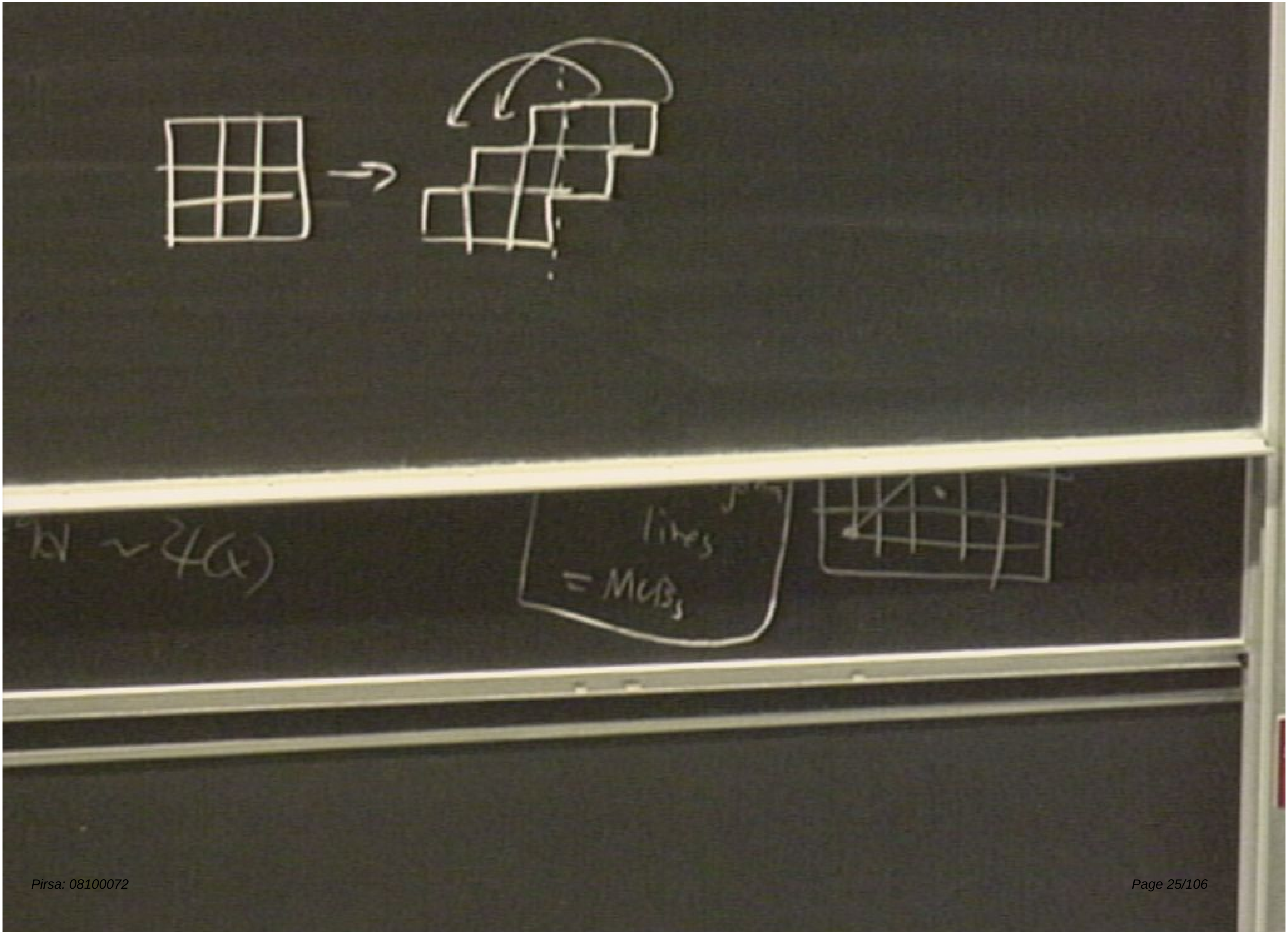
If $d = d_1^{r_1} d_2^{r_2} \cdots$

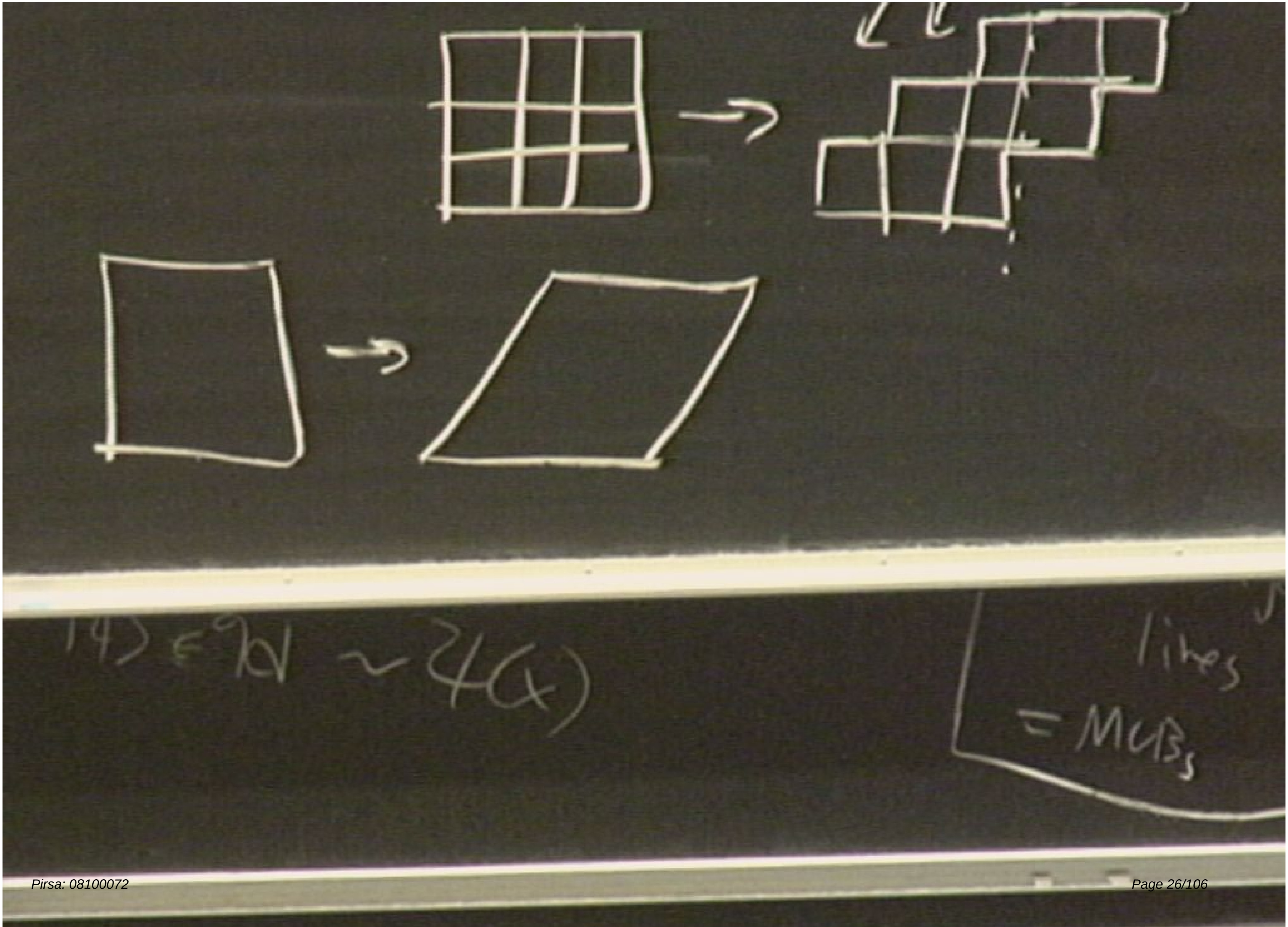
$\# \text{MUBs} \geq \min_i (d_i^{r_i} + 1)$


d= Prime
Discrete H-W
Berry

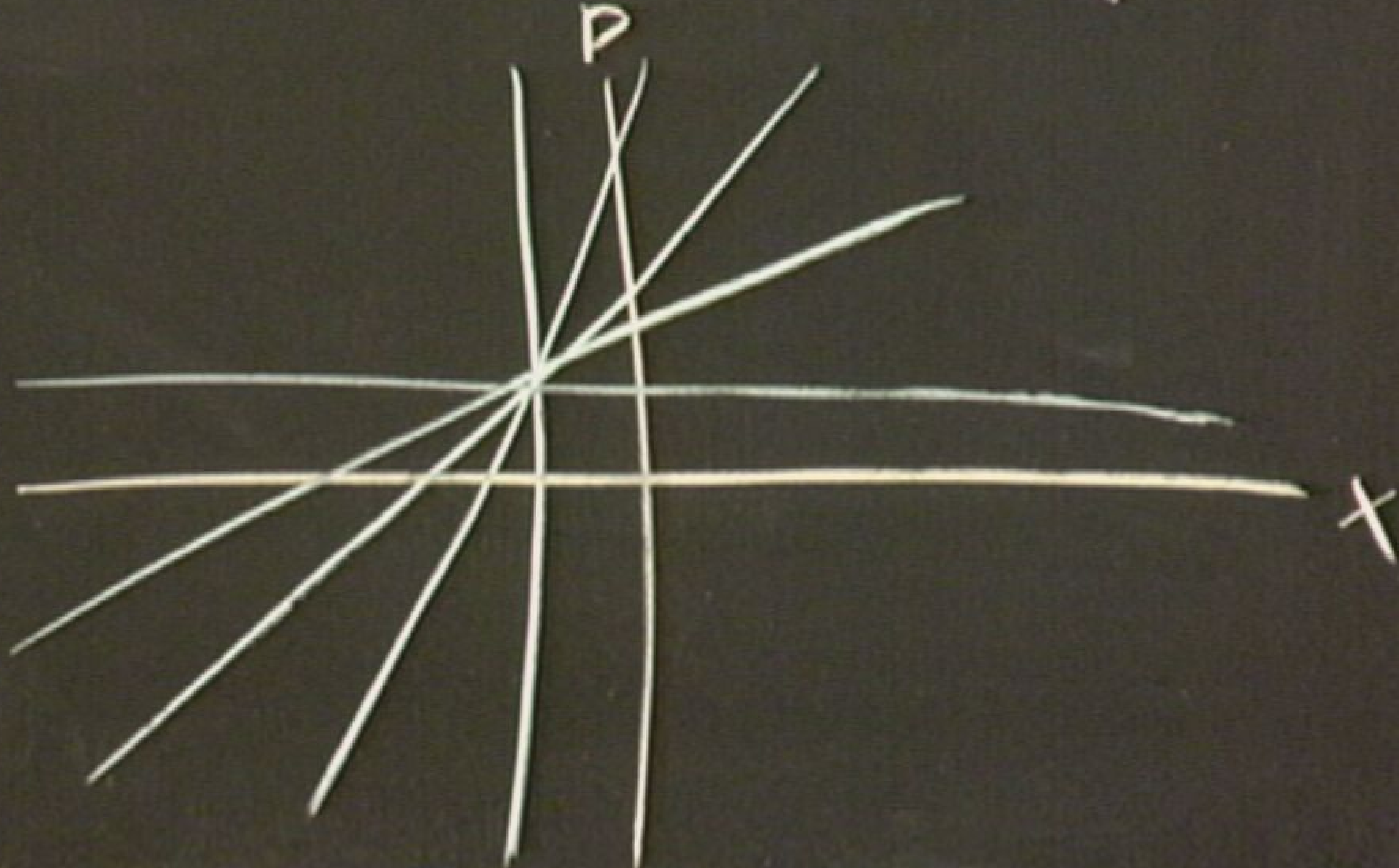

S_δx = e
A_δp = e
d= Prime
Discrete H-W
HW={T_xp}
Berry pha
L²(R)

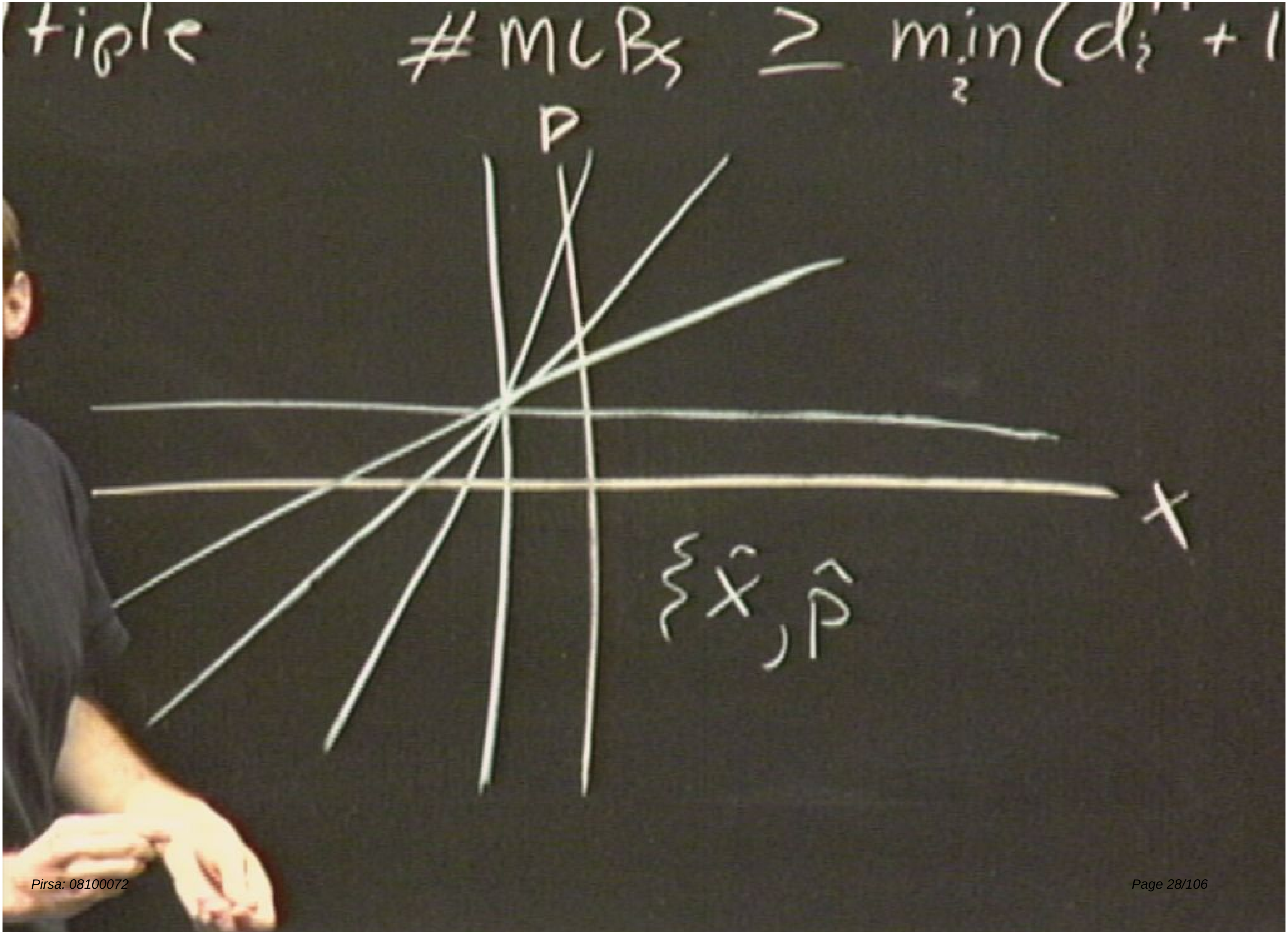

d = Prime
Discrete H-W
Berry phase
d+1 diagonal lines
= MUBs


lines
= MCBs

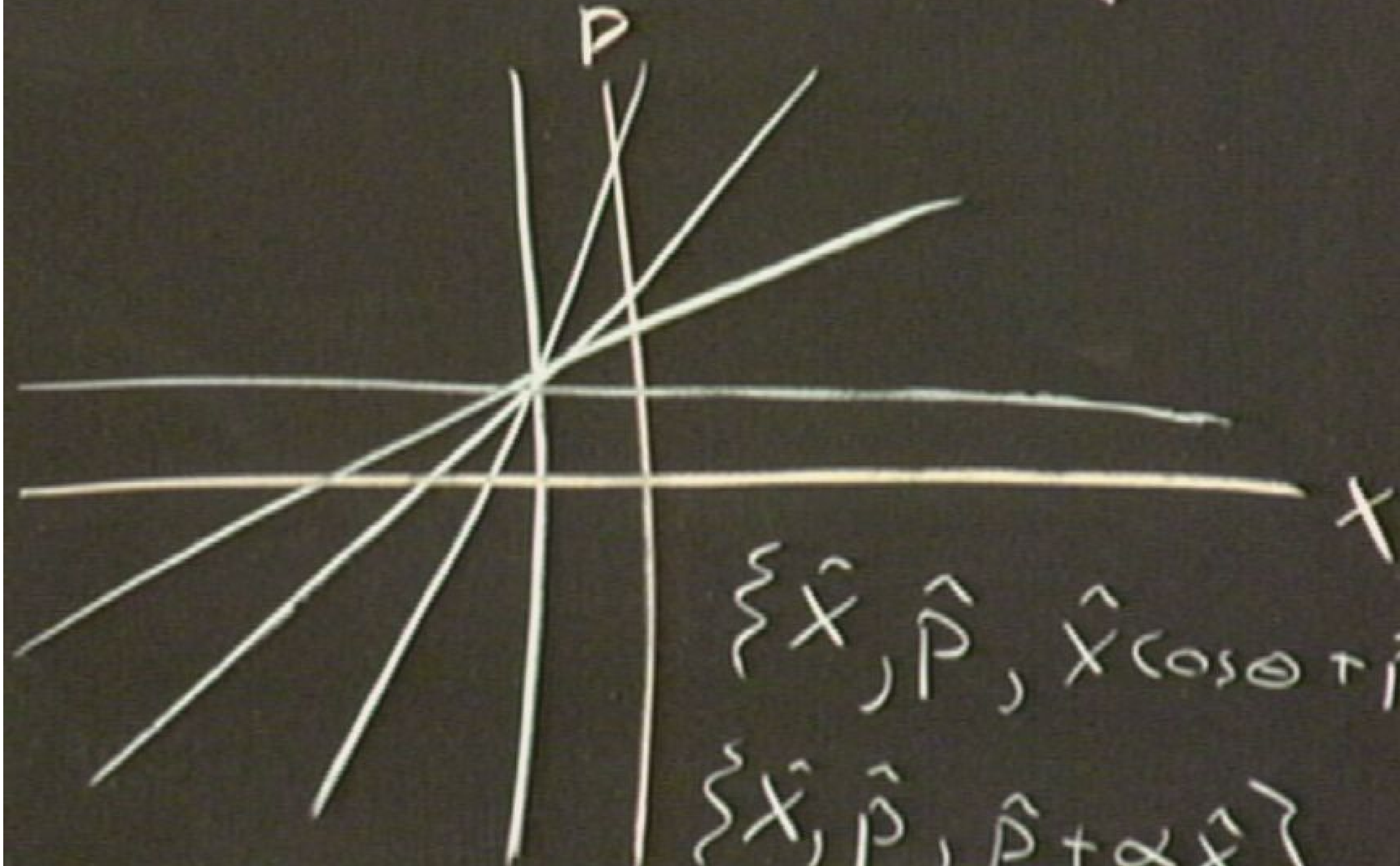

|ψ⟩ ∈ ℋ ~ ψ(x)
lines
= MUBs

-ltiple $\# MCBs \geq \min_i (d_i +$




tiple
#MUBs ≥ min(d + 1
p
x
{x̂, p̂

iple $\#\,\mathrm{MUBs} \geq \min_i (d_i + 1)$



$$\{\hat{x},\ \hat{p},\ \hat{x}\cos\theta + \hat{p}\sin\theta\}$$

$$\{\hat{x},\ \hat{p},\ \hat{p} + \alpha\hat{x}\}$$

There are only 3 MUBs in $L^2(P)$

There are only 3 MUBs in $L^2(\mathbb{R})$

$$\hat{X} \longrightarrow \{|x\rangle\}$$

$$\hat{P} \longrightarrow \{|p\rangle\}$$

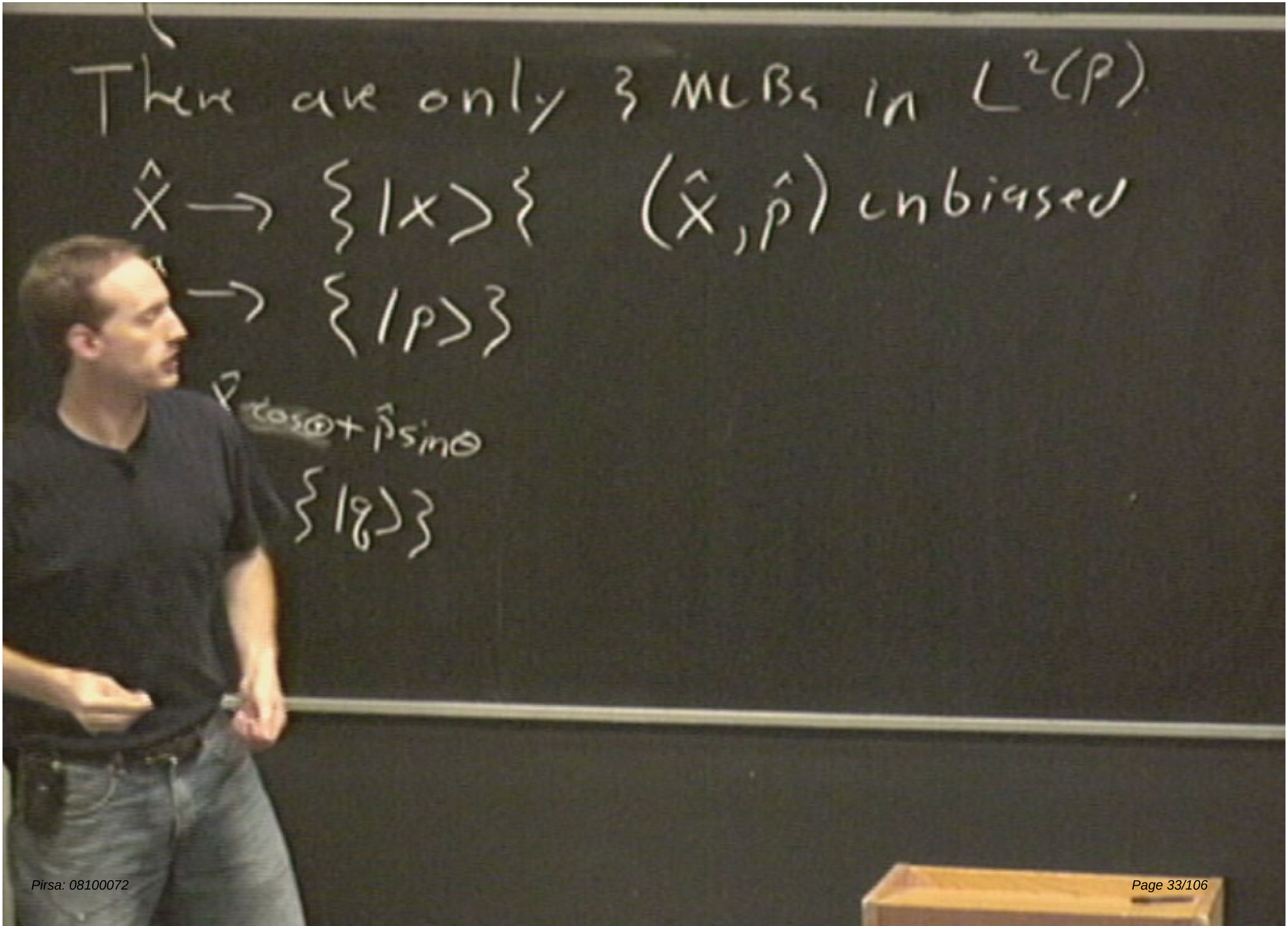
There are only 3 MUBs in $L^2(\mathbb{R})$

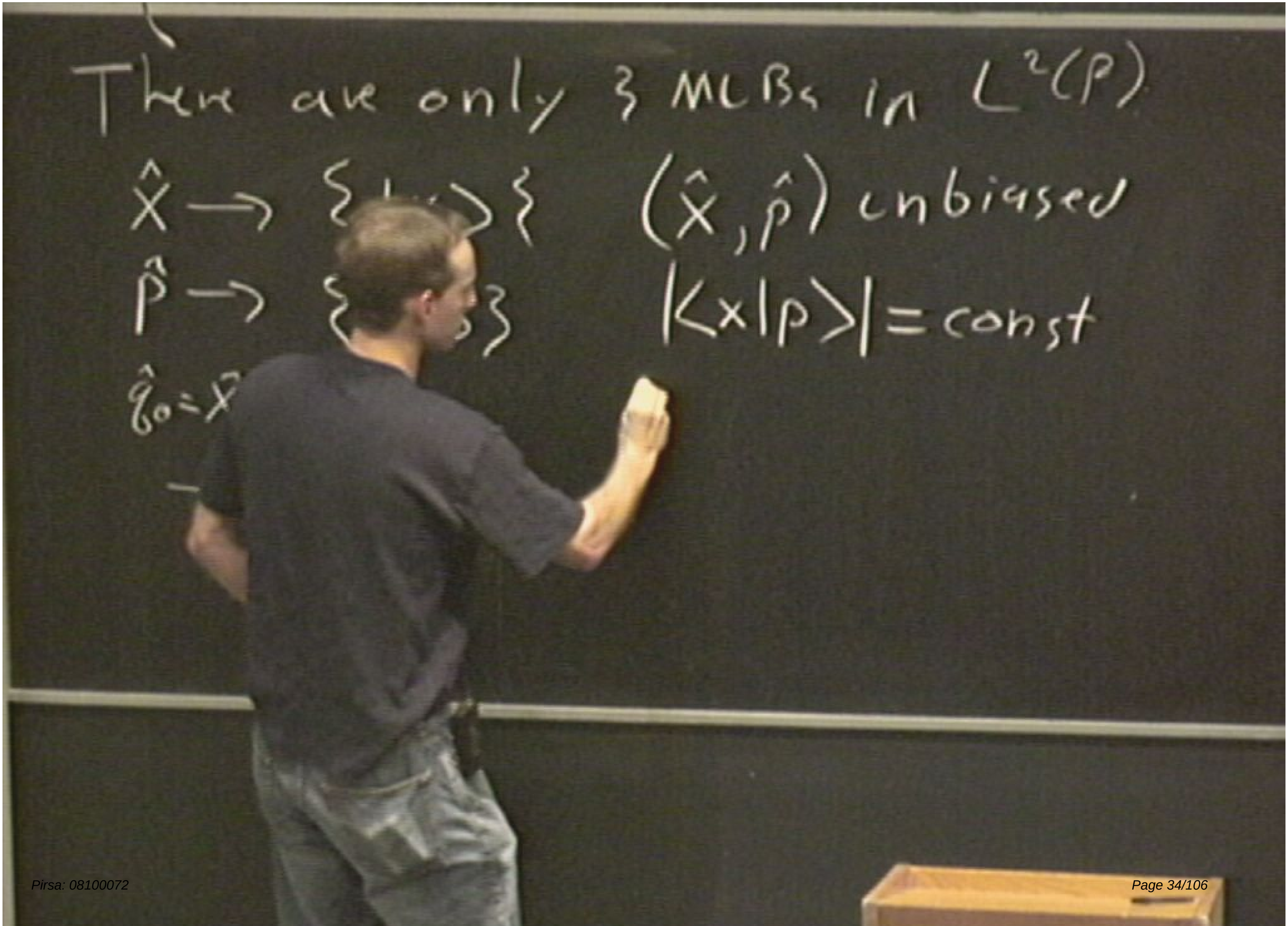
$\hat{X} \longrightarrow \{|x\rangle\}$

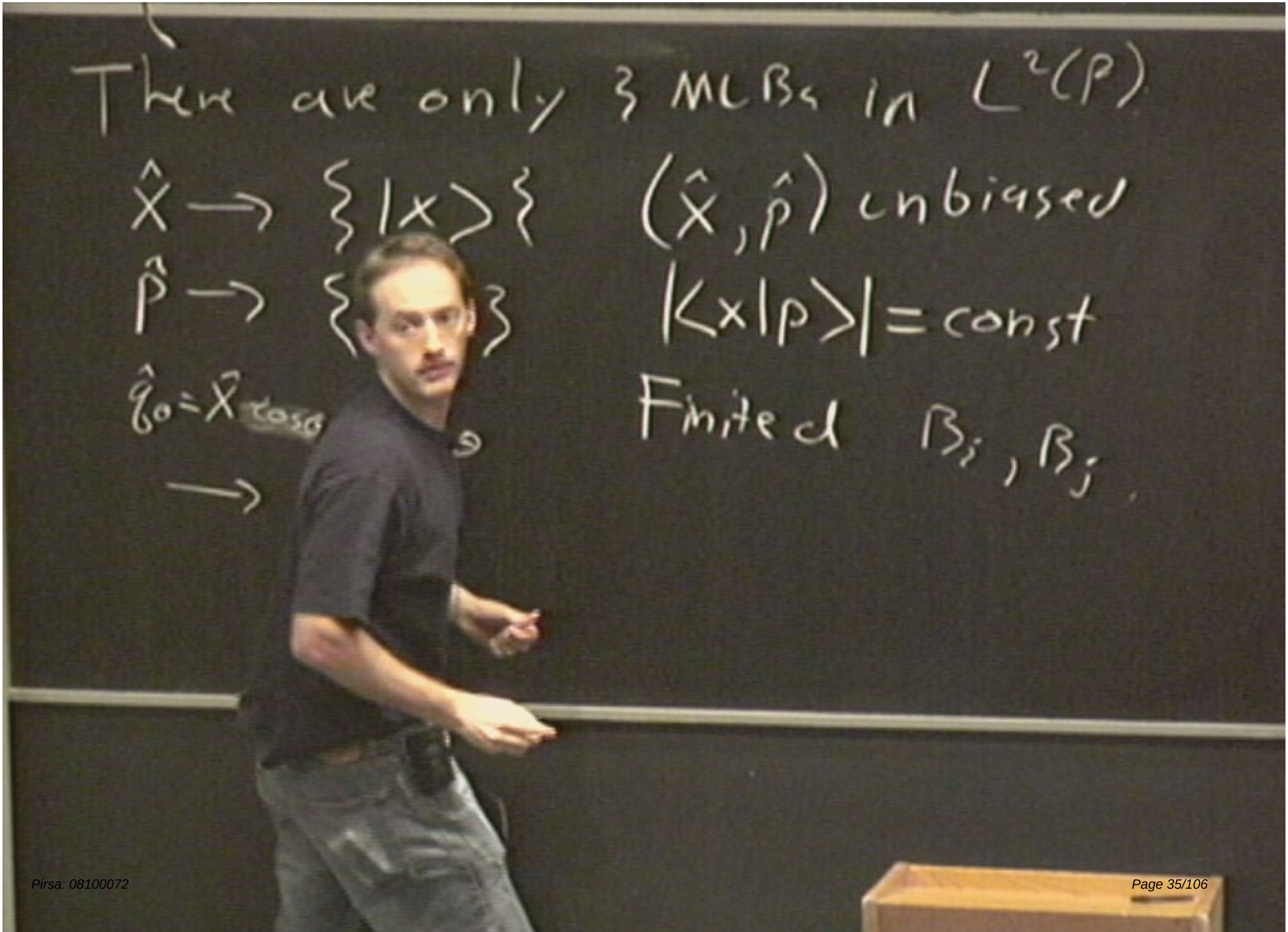
$\hat{P} \longrightarrow \{|p\rangle\}$

$\hat{q}_\theta = \hat{X}\cos\theta + \hat{P}\sin\theta$

$\longrightarrow \{|q\rangle\}$


There are only 3 MUBs in $L^2(\mathbb{R})$
$\hat{x} \to \{|x\rangle\}$
$(\hat{x}, \hat{p})$ unbiased
$\hat{p} \to \{|p\rangle\}$
$\hat{x}\cos\theta + \hat{p}\sin\theta$
$\{|q\rangle\}$


There are only 3 MUBs in $L^2(\mathbb{R})$
$(\hat{x}, \hat{p})$ unbiased
$|\langle x|p\rangle| = \text{const}$


There are only 3 MUBs in L²(ℝ)
x̂ → {|x⟩}
(x̂, p̂) unbiased
p̂ → {
}
|⟨x|p⟩| = const
q̂₀ = x̂
Finite d
Bᵢ, Bⱼ
→

There are only 3 MUBs in $L^2(\mathbb{R})$

$\hat{X} \to \{|x\rangle\}$

$(\hat{X}, \hat{p})$ unbiased

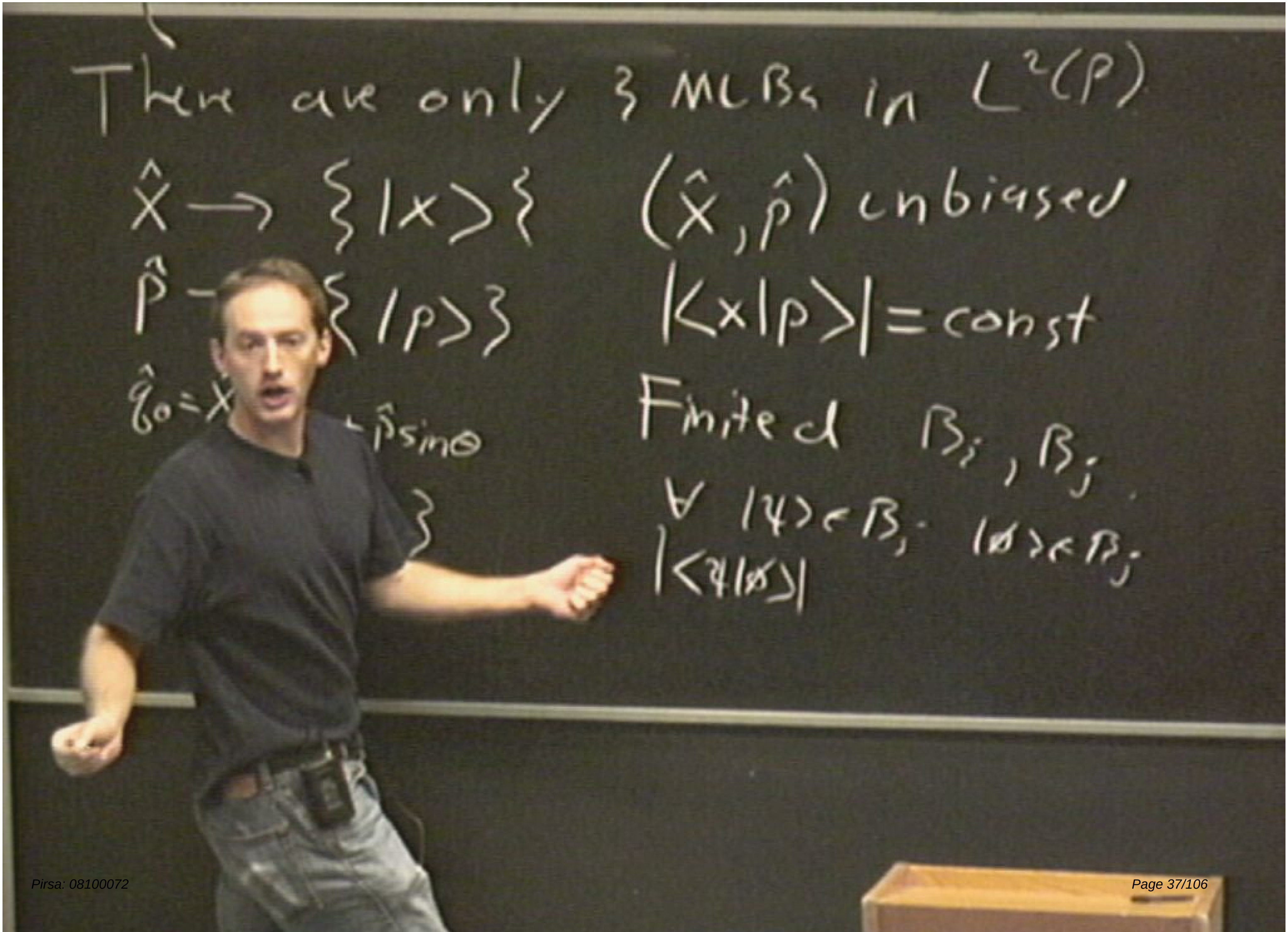
$\hat{P} \to \{| \ldots \}$

$|\langle x|p\rangle| = \text{const}$

$\hat{q}_0 = \hat{X}$ ...

Finite d B_i, B_j

$\forall\, |\psi\rangle \in B_i : \; |\phi\rangle \in B_j$


There are only 3 MUBs in $L^2(\mathbb{R})$
$\hat{X} \longrightarrow \{|x\rangle\}$
$(\hat{X}, \hat{p})$ unbiased
$\hat{p} \rightarrow \{|p\rangle\}$
$|\langle x|p\rangle| = \text{const}$
$\hat{q}_\theta = \hat{X}$ … $+ \hat{p}\sin\theta$
Finite d B_i, B_j
$\forall\ |\psi\rangle \in B_i \quad |\phi\rangle \in B_j$
$|\langle\psi|\phi\rangle|$

There are only 3 MUBs in $L^2(\mathbb{R})$

$\hat{X} \longrightarrow \{|x\rangle\}$

$\hat{P} \longrightarrow \{|p\rangle\}$

$\hat{q}_\theta = \hat{X}\cos\theta + \hat{p}\sin\theta$

$\longrightarrow \{|q\rangle\}$

$(\hat{x}, \hat{p})$ unbiased

$|\langle x|p\rangle| = \text{const}$

Finite d $\quad B_i, B_j$

$\forall\ |\psi\rangle \in B_i,\ |\phi\rangle \in B_j$

$|\langle\psi|\phi\rangle| = \text{const} = 1/\sqrt{d}$

$(\hat{x}, \hat{p})$ unbiased

$$|\langle x|p\rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar}}$$

Finite d $\quad B_i, B_j$

$\forall\ |\psi\rangle \in B_i,\ |\phi\rangle \in B_j$

$$|\langle\psi|\phi\rangle| = \text{const} = 1/\sqrt{d}$$

$(\hat{X}, \hat{q})$ are also unbiased

$$|\langle x|q\rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar}}\sin\theta$$

$(\hat{X}, \hat{q})$ are also unbiased

$$|\langle x|q\rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar}\,\sin\theta}$$

$(\hat{p}, \hat{q})$ are unbiased

$$|\langle q|p\rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar}\,\cos\theta}$$

$(\hat{X}, \hat{q})$ are also unbiased

$$|\langle x|q\rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar}\,\sin\theta}$$

$(\hat{p}, \hat{q})$ are unbiased

$$|\langle q|p\rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar}\cos\theta}.$$

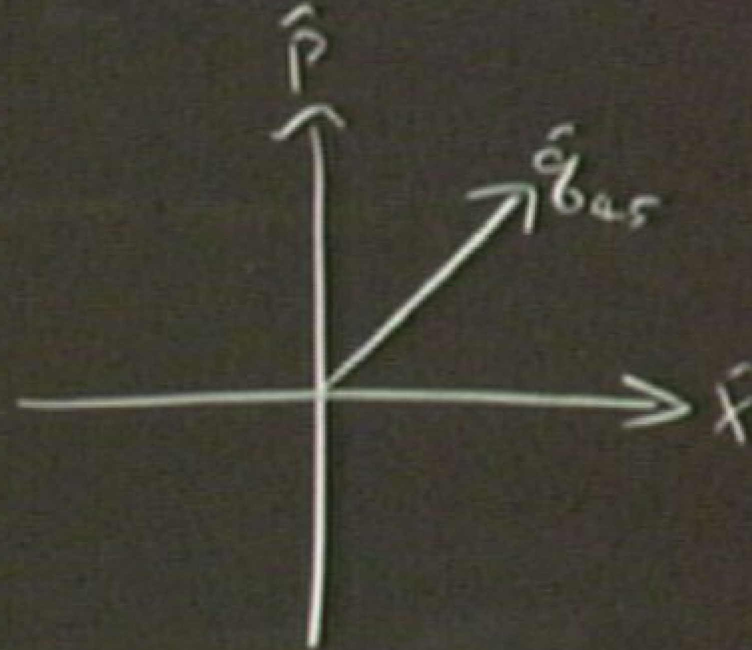
$(\hat{p}, \hat{q})$ are unbiased

$$|\langle q | p \rangle| = \text{const} = \frac{1}{\sqrt{2\pi\hbar \cos\theta}}.$$

$\{\hat{x}, \hat{p}, \hat{q}_\theta\}$ not MUB

because $|\langle q | x \rangle| \neq |\langle x | p \rangle|$

$= \text{const} = \frac{1}{\sqrt{2\pi\hbar}} \sin\theta$



unbiased

onst $= \frac{1}{\sqrt{2\pi\hbar}\cos\theta}$.

MCB

$\rangle| \quad \langle x|p\rangle|$

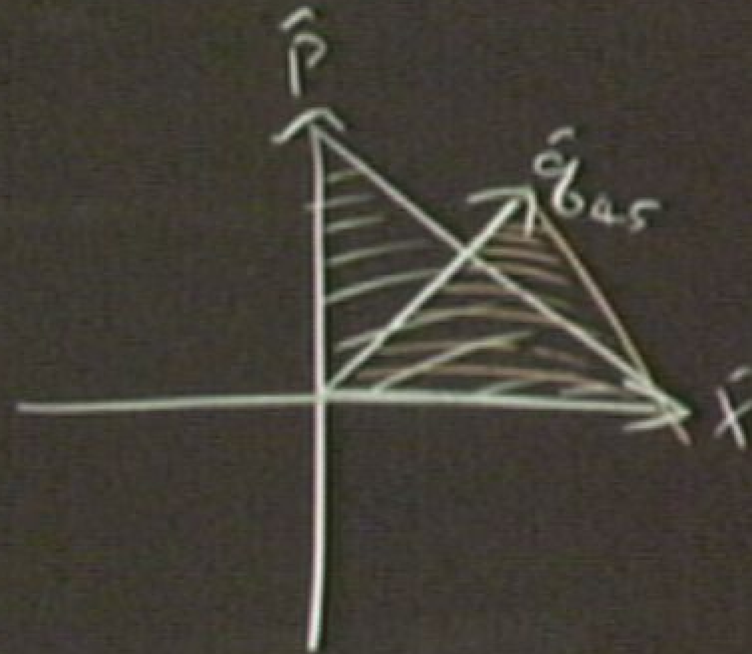
$= \text{const} = \frac{1}{\sqrt{2\pi\hbar}} \sin\theta$

unbiased

$\text{onst} = \frac{1}{\sqrt{2\pi\hbar}\cos\theta}.$

not MUB

$x\rangle| \neq |\langle x|p\rangle|$



$$\langle x|x'\rangle = \delta(x-x')$$

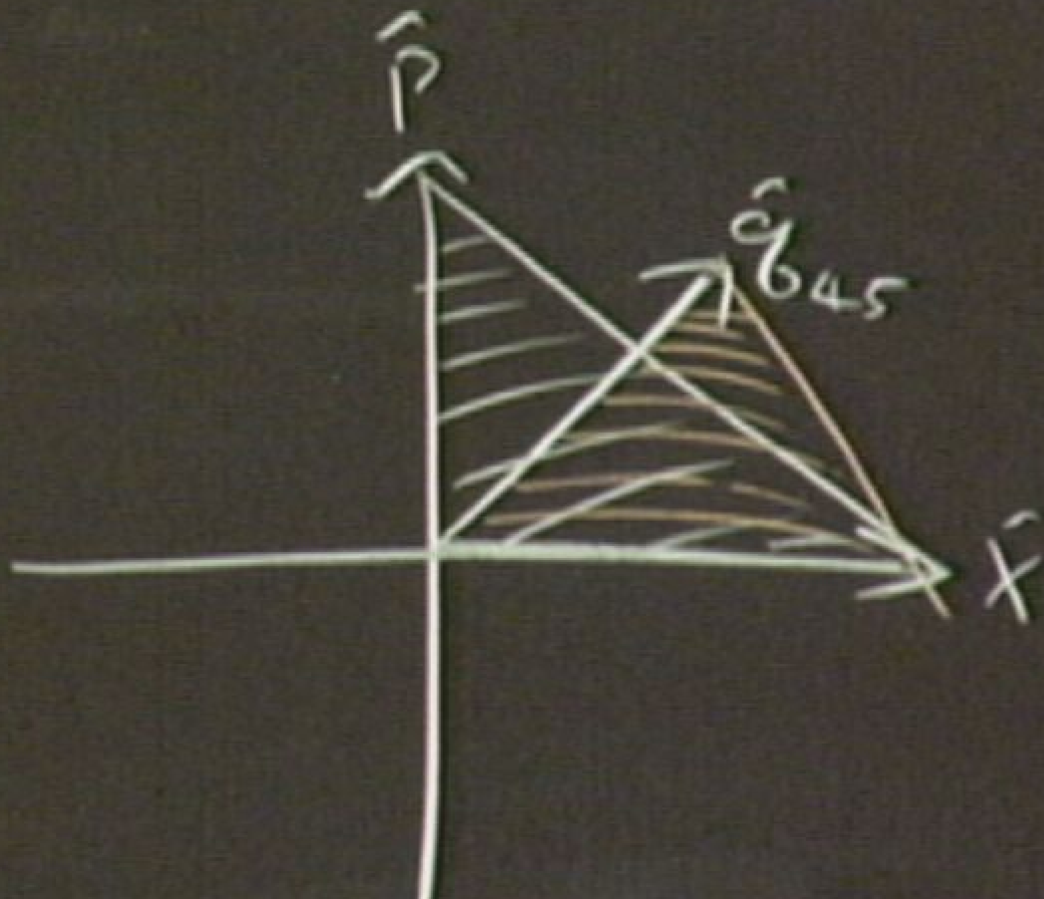
$$\langle p|p'\rangle = \delta(p-p')$$

$$\langle q|q'\rangle = \delta(q-q')$$

used

$\langle x|\psi\rangle$

$= \frac{1}{\sqrt{2\pi\hbar}}\sin\theta$



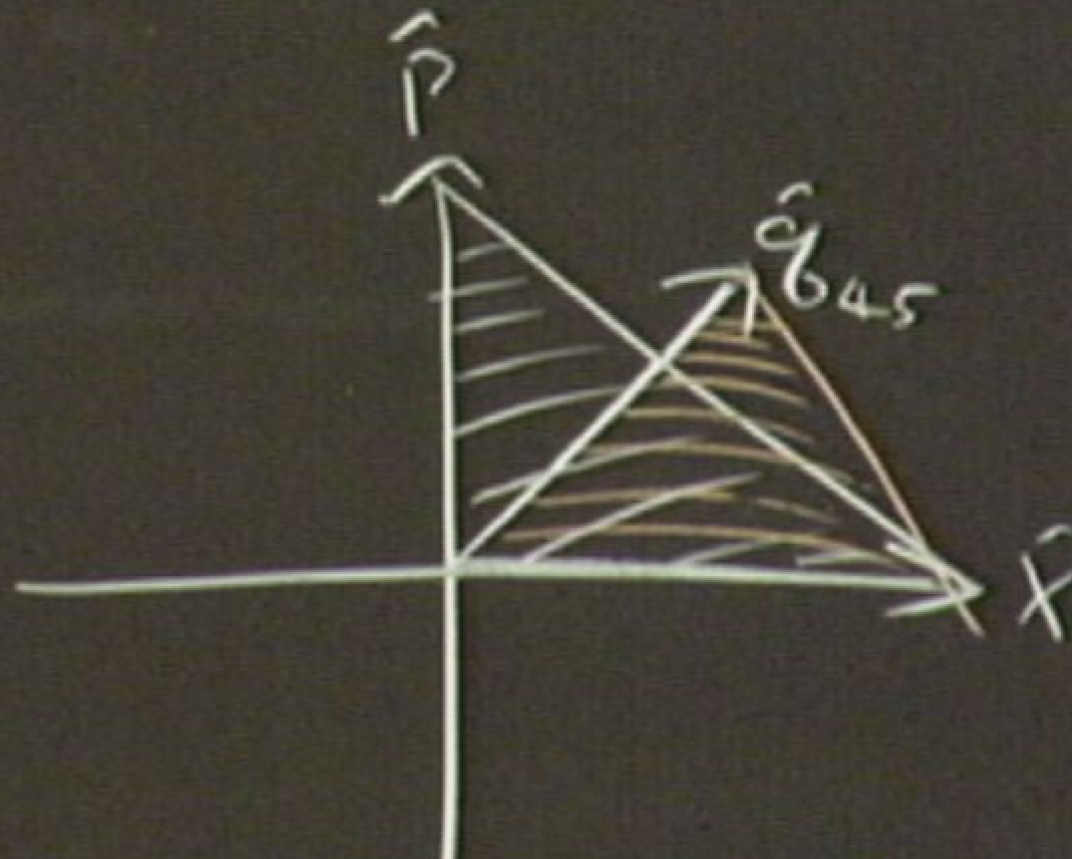
d

$= \frac{1}{\sqrt{2\pi\hbar}\cos\theta}.$

$\langle x|x'\rangle = \delta(x-x')$

$$\langle x|\psi\rangle = \psi(x)$$

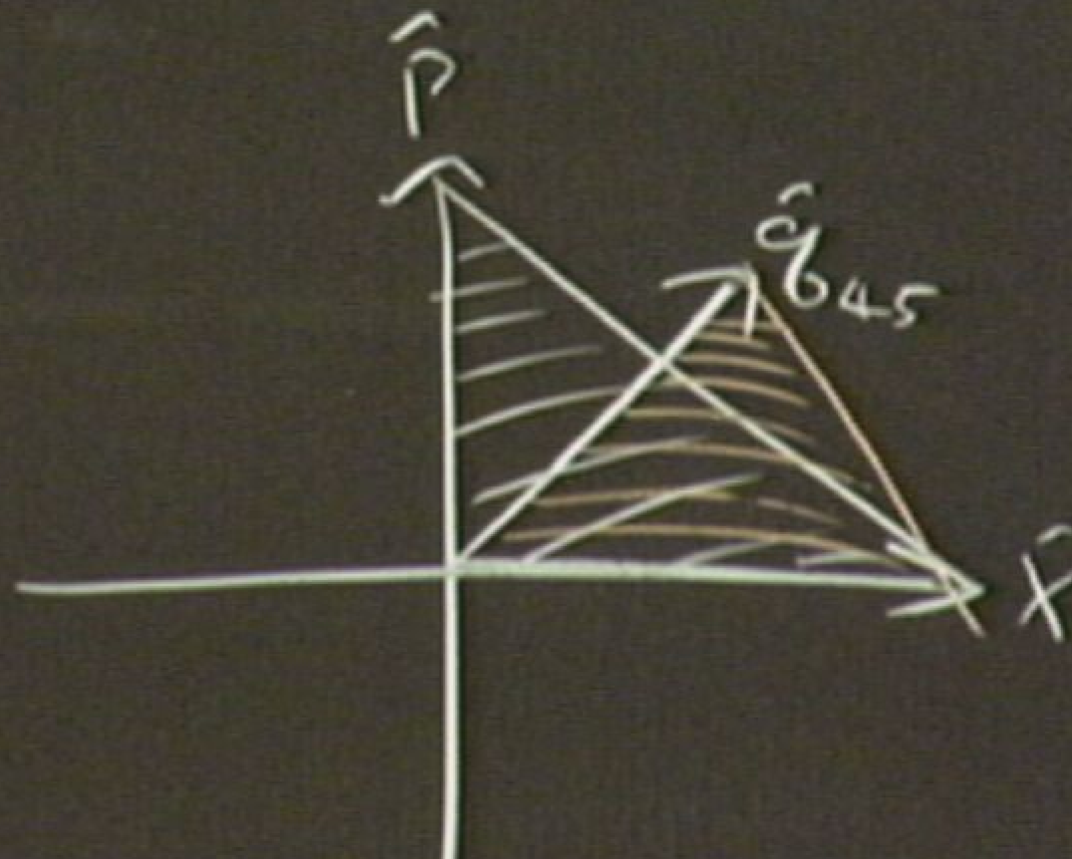
$= \sin\theta$



$$\langle x|\psi\rangle = \psi(x)$$

$$p(x) = |\psi(x)|^2$$

$\sin\theta$



$$\left\{ \hat{x},\ \hat{p},\ \frac{\hat{x}+\hat{p}}{\sqrt{2}} \right\} \neq \text{MCB}$$

$$\left\{ \hat{x},\ \hat{p},\ \hat{x}+\hat{p} \right\}$$

$$\left\{\hat{x},\ \hat{p},\ \frac{\hat{x}+\hat{p}}{\sqrt{2}}\right\} \neq MUB$$

$$\{\hat{x},\ \hat{p},\ \hat{x}+\hat{p}\}$$

$$\hat{x} \longrightarrow \{|x\rangle\} \Longrightarrow \langle x|x'\rangle = \delta(x-x')$$

$$\hat{\tilde{x}} = 2\hat{x} \longrightarrow \{|\tilde{x}\rangle\} \Longrightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$$

$$= \delta(2(x-x'))$$

$$\left\{\hat{x}, \hat{p}, \frac{\hat{x}+\hat{p}}{\sqrt{2}}\right\} \neq MUB$$

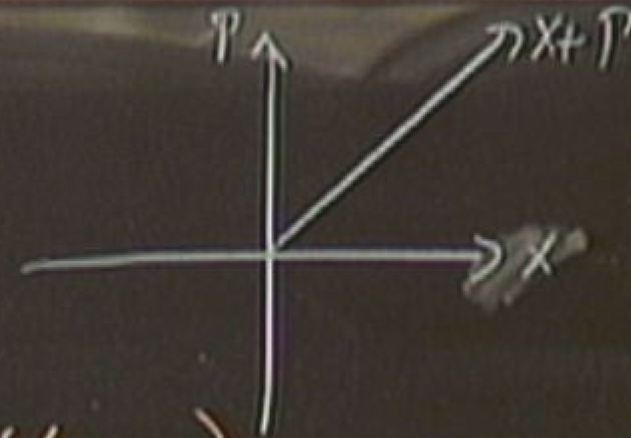
$$\{\hat{x}, \hat{p}, \hat{x}+\hat{p}\}$$

$$\hat{x} \longrightarrow \{|x\rangle\} \Longrightarrow \langle x|x'\rangle = \delta(x-x')$$

$$\hat{\tilde{x}} = 2\hat{x} \rightarrow \{|\tilde{x}\rangle\} \Longrightarrow \langle\tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$$

$$= \delta(2(x-x'))$$

$$\left\{\hat{x},\ \hat{p},\ \frac{\hat{x}+\hat{p}}{\sqrt{2}}\right\} \neq MCB$$

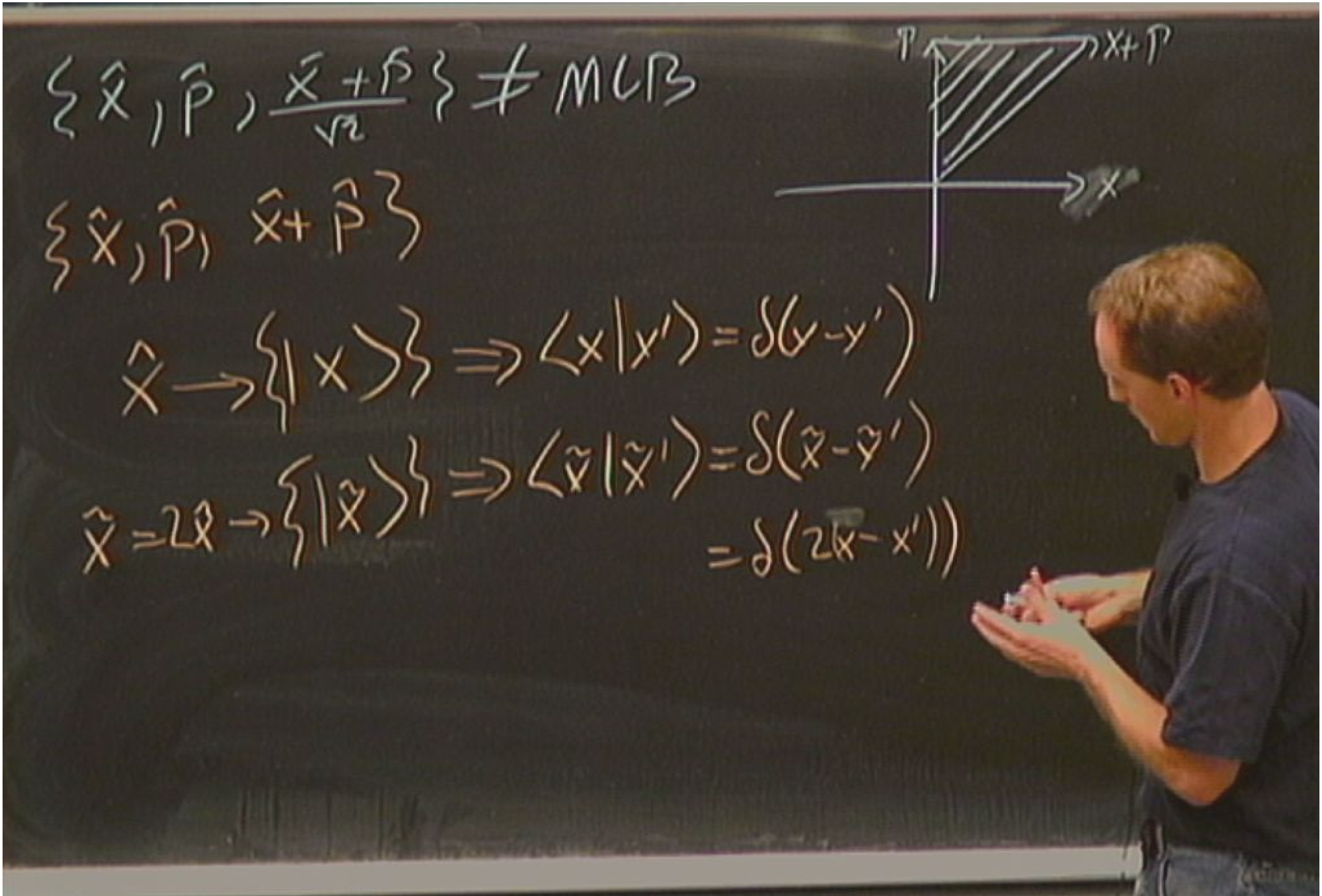


$$\{\hat{x},\ \hat{p},\ \hat{x}+\hat{p}\}$$

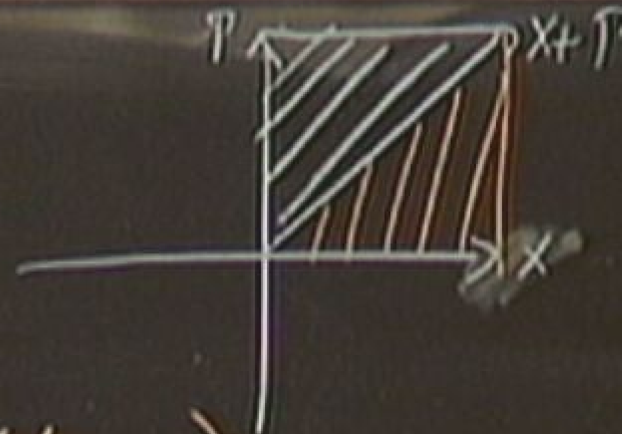
$$\hat{x} \longrightarrow \{|x\rangle\} \Longrightarrow \langle x|x'\rangle = \delta(x-x')$$

$$\tilde{x} = 2\hat{x} \rightarrow \{|\tilde{x}\rangle\} \Longrightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$$

$$= \delta(2x - x')$$


$\{\hat{x}, \hat{p}, \frac{\hat{x}+\hat{p}}{\sqrt{2}}\} \neq MCB$
p
x+p
x
$\{\hat{x}, \hat{p}, \hat{x}+\hat{p}\}$
$\hat{x} \rightarrow \{|x\rangle\} \Rightarrow \langle x|x'\rangle = \delta(x-x')$
$\{|\tilde{x}\rangle\} \Rightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$
$\hat{\tilde{x}} = 2\hat{x} \rightarrow \{|\tilde{x}\rangle\}$
$= \delta(2(x-x'))$

$$\left\{\hat{x}, \hat{p}, \frac{\hat{x}+\hat{p}}{\sqrt{2}}\right\} \neq MUB$$



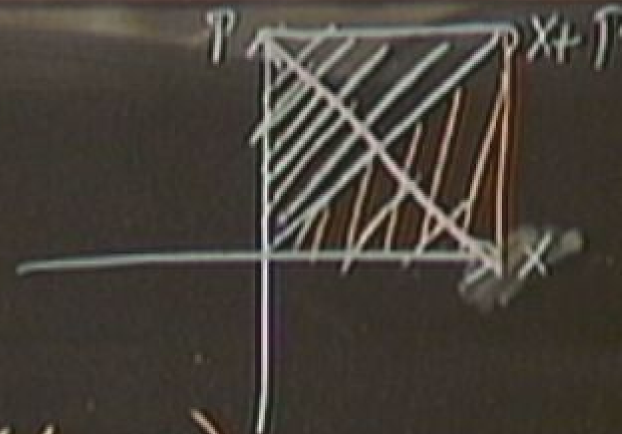
$$\{\hat{x}, \hat{p}, \hat{x}+\hat{p}\}$$

$$\hat{x} \longrightarrow \{|x\rangle\} \Rightarrow \langle x|x'\rangle = \delta(x-x')$$

$$\tilde{x} = 2\hat{x} \rightarrow \{|\tilde{x}\rangle\} \Rightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$$

$$= \delta(2(x-x'))$$

$$\left\{\hat{x},\ \hat{p},\ \frac{\hat{x}+\hat{p}}{\sqrt{2}}\right\} \neq MCB$$

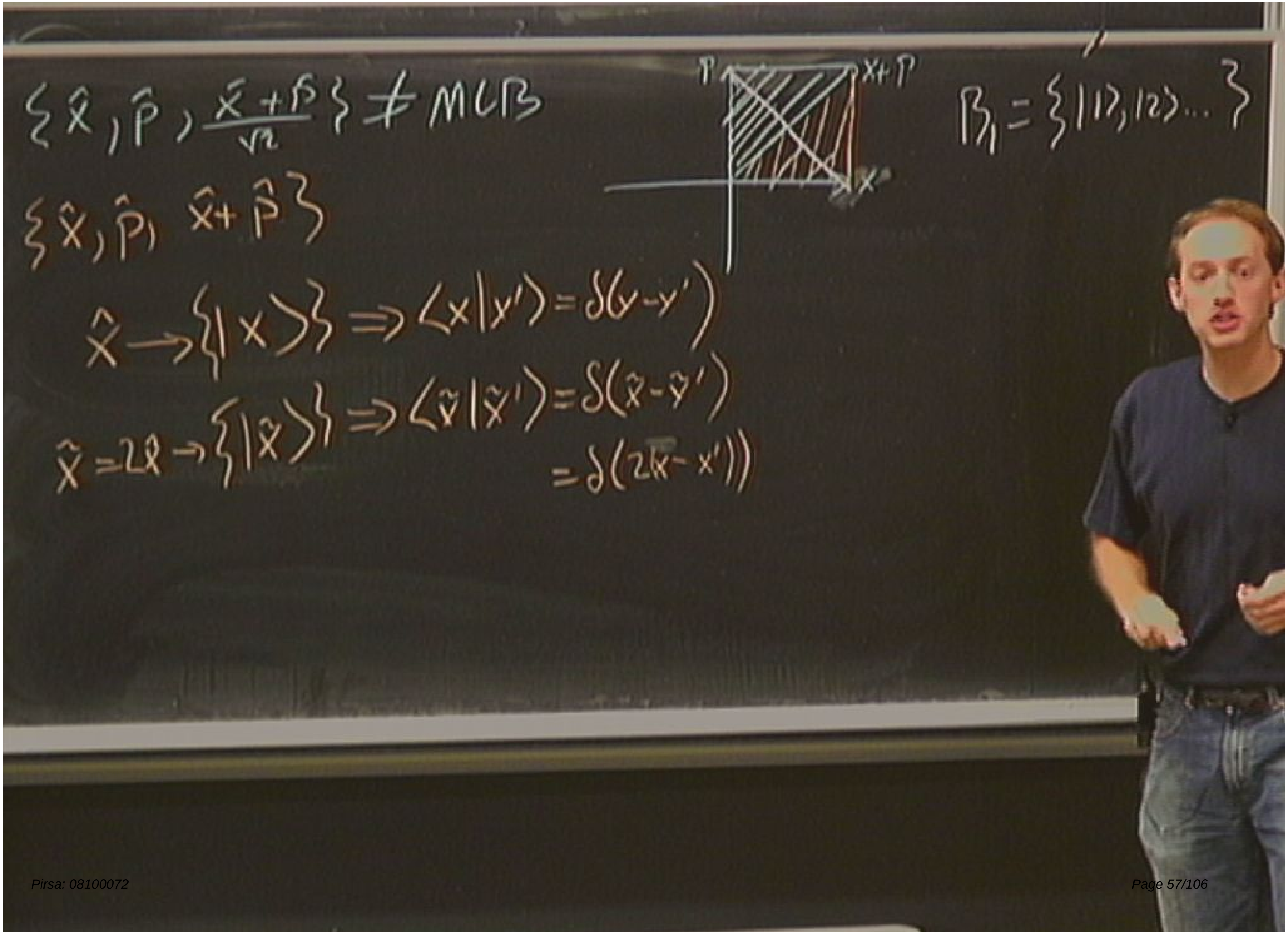


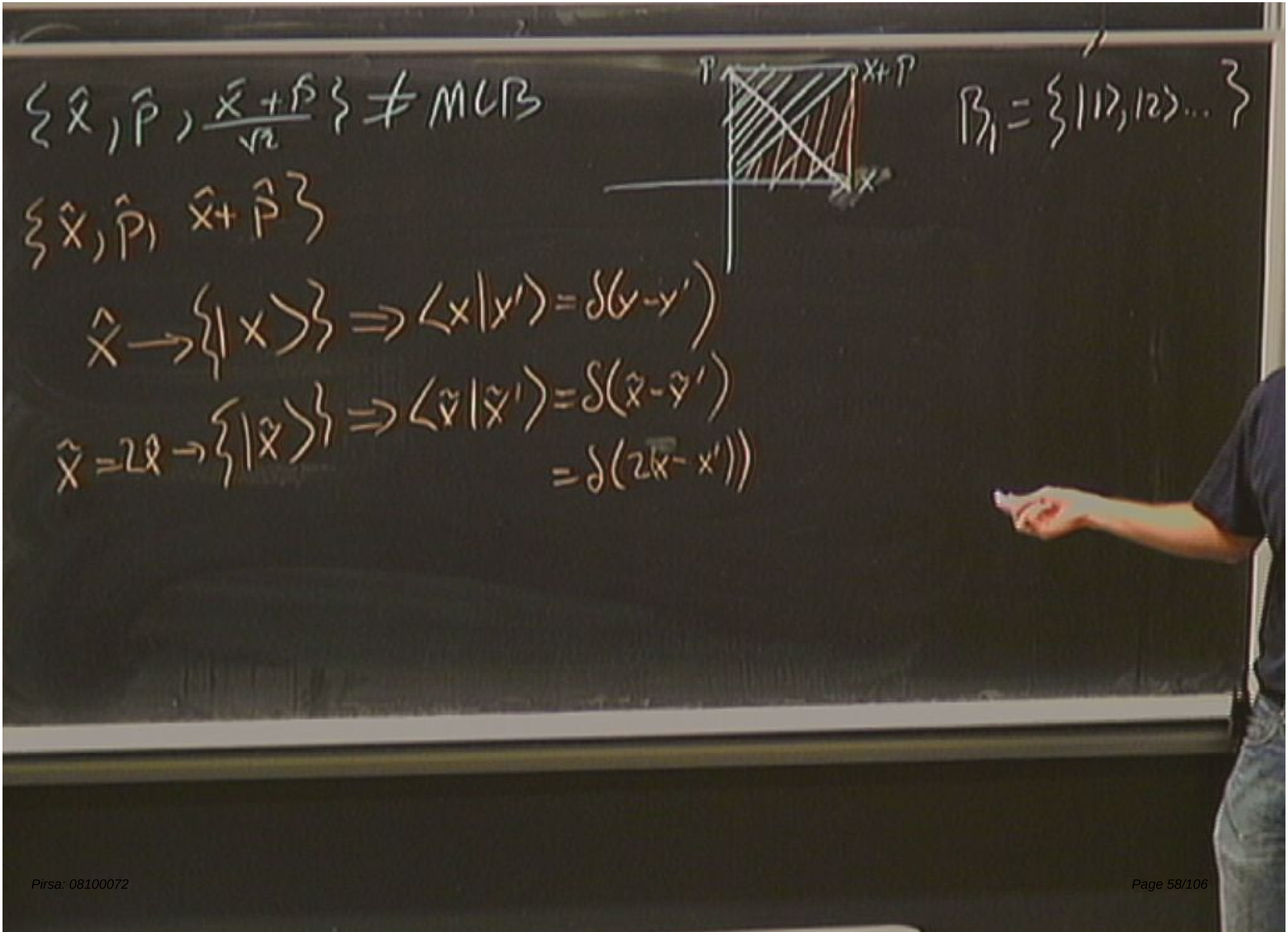
$$\left\{\hat{x},\ \hat{p},\ \hat{x}+\hat{p}\right\}$$

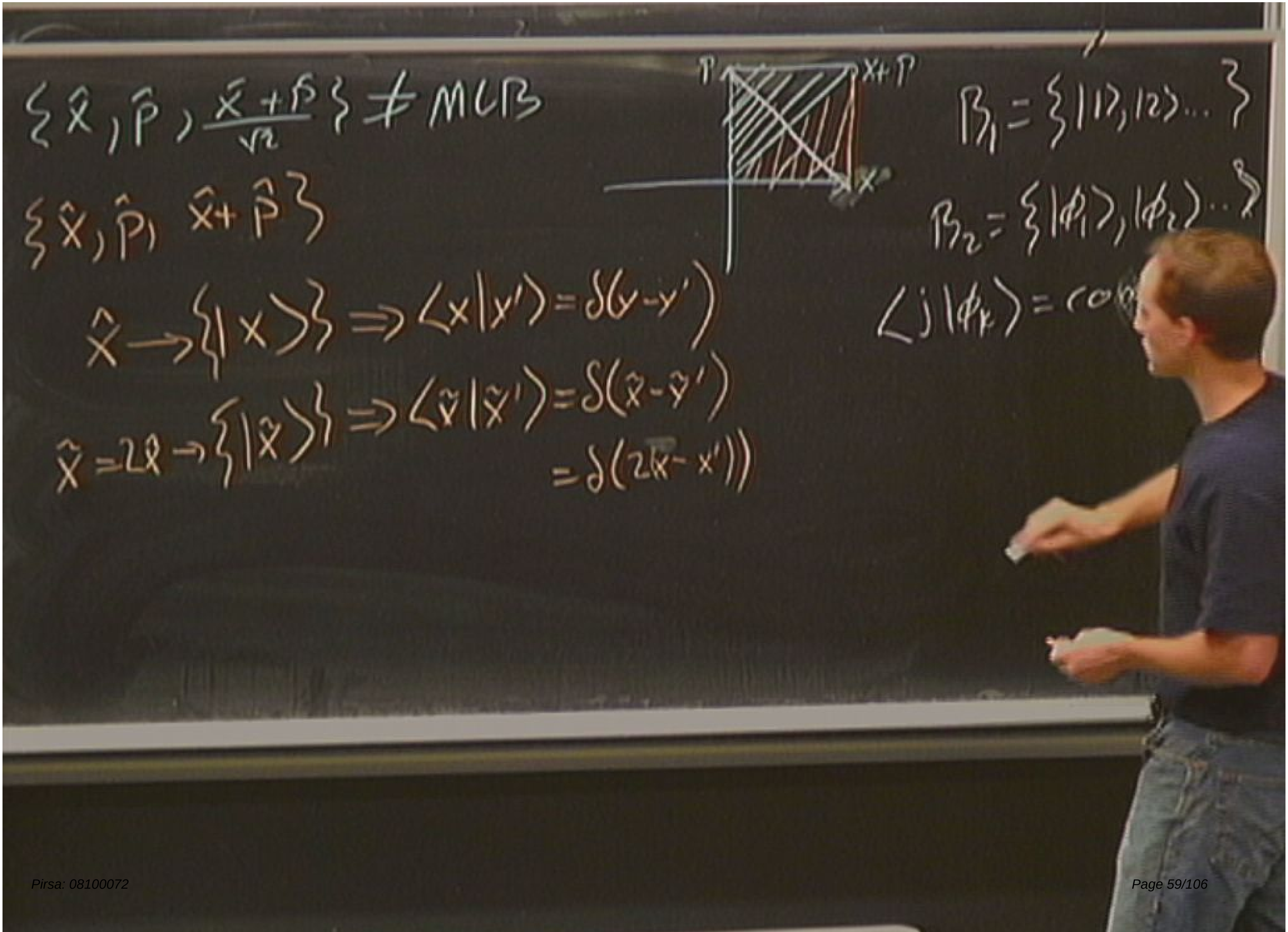
$$\hat{x} \longrightarrow \{|x\rangle\} \Longrightarrow \langle x|x'\rangle = \delta(x-x')$$

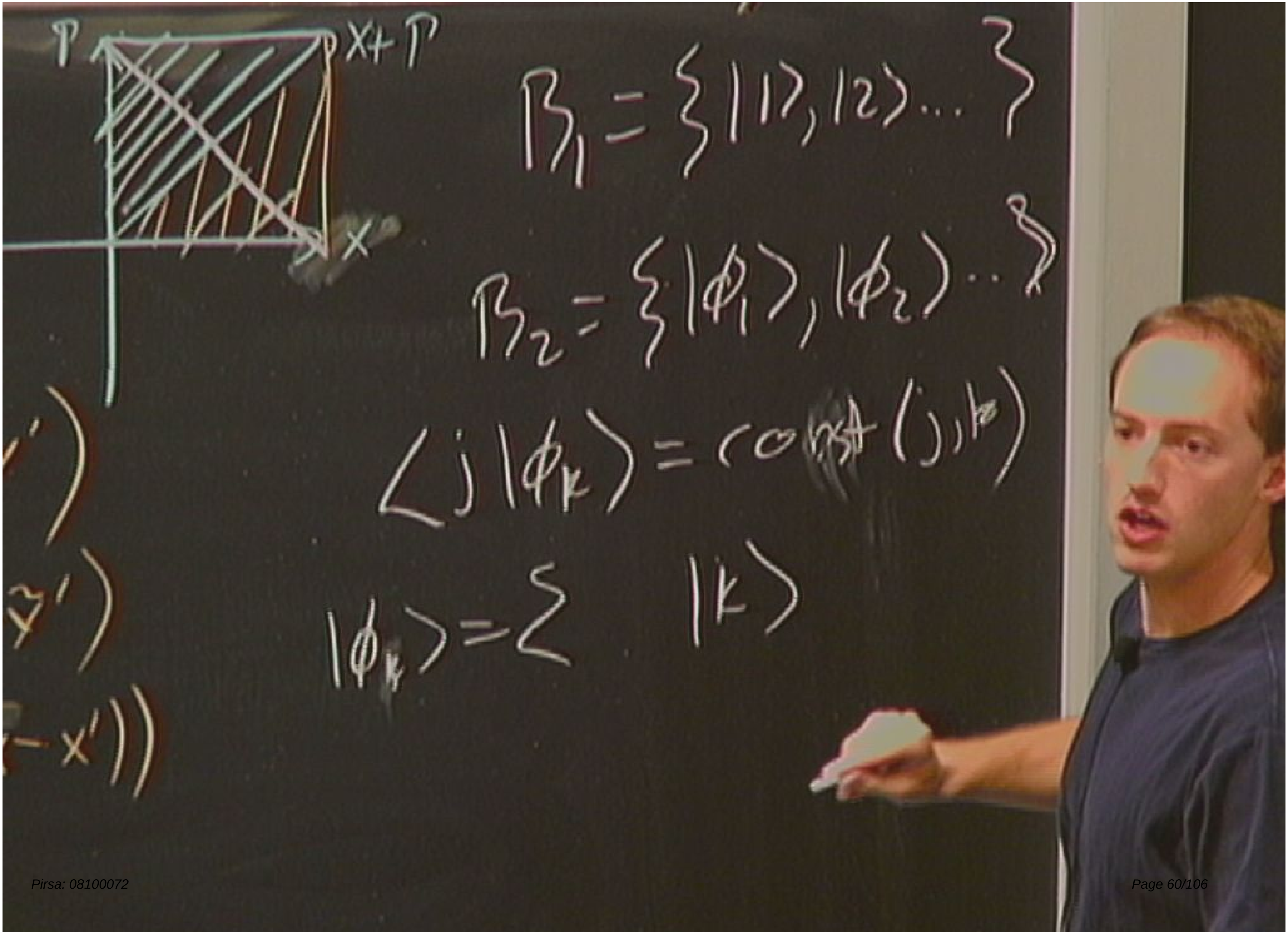
$$\hat{\tilde{x}} = 2\hat{x} \rightarrow \{|\tilde{x}\rangle\} \Longrightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$$

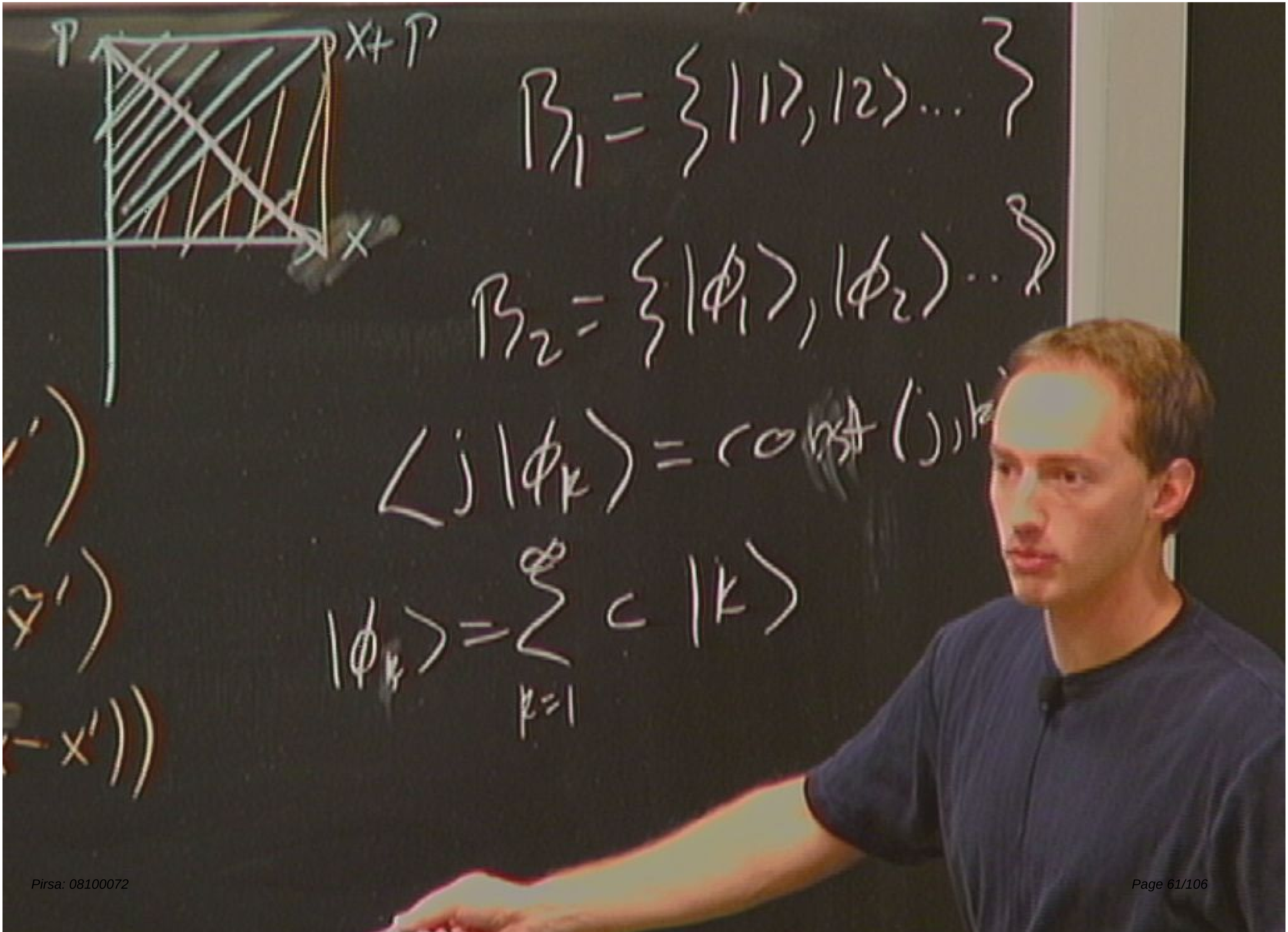
$$= \delta(2(x-x'))$$

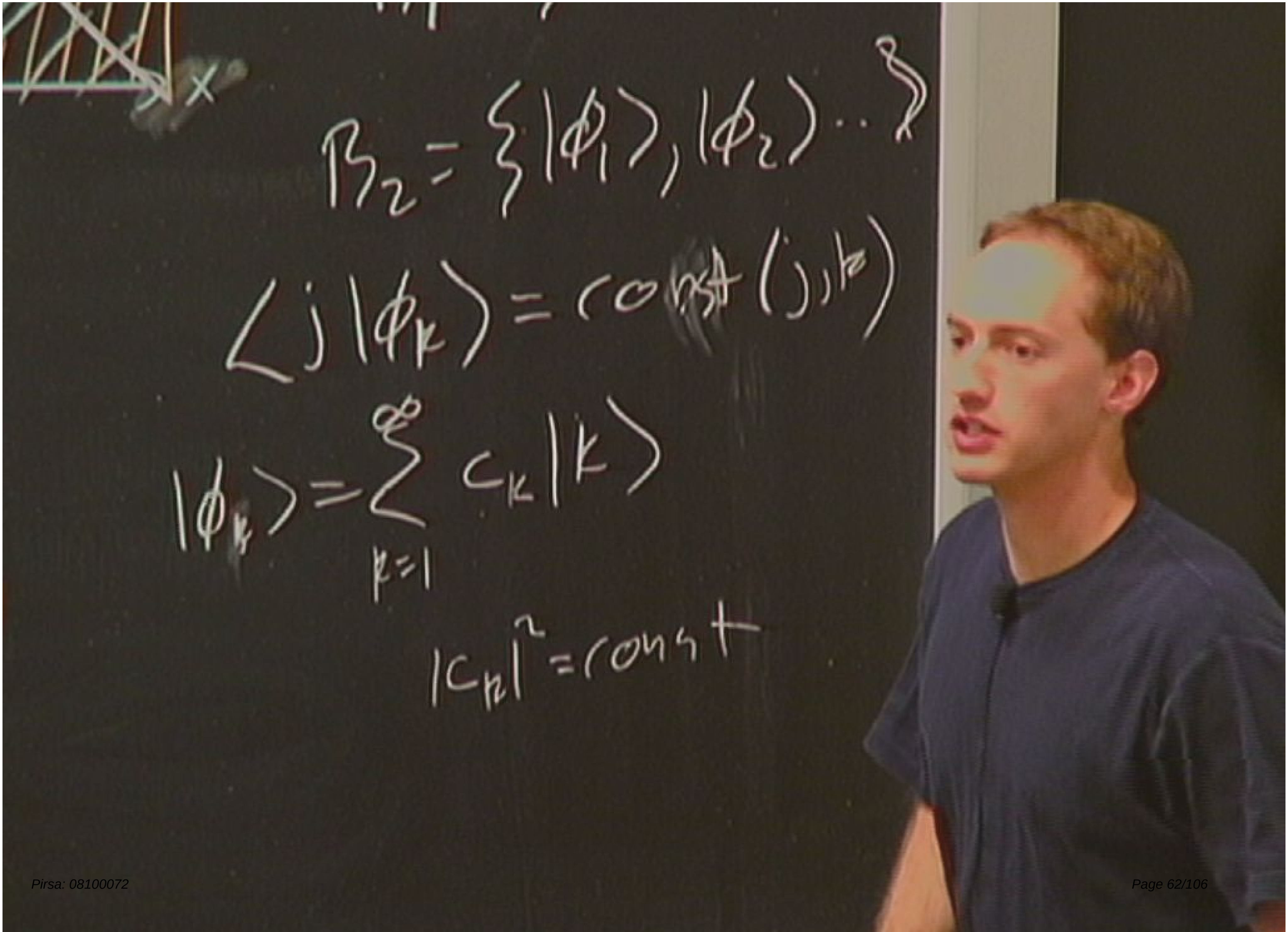

$\{\hat{x}, \hat{p}, \frac{\hat{x}+\hat{p}}{\sqrt{2}}\} \neq$ MCB
p
$x+p$
x
$B_1 = \{|1\rangle, |2\rangle \ldots\}$
$\{\hat{x}, \hat{p}, \hat{x}+\hat{p}\}$
$\hat{x} \rightarrow \{|x\rangle\} \Rightarrow \langle x|y'\rangle = \delta(y-y')$
$\{|\tilde{x}\rangle\} \Rightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$
$\tilde{x} = 2x \rightarrow \{|\tilde{x}\rangle\}$
$= \delta(2(x-x'))$


$\{\hat{x}, \hat{p}, \frac{\hat{x}+\hat{p}}{\sqrt{2}}\} \neq$ MUB
$\{\hat{x}, \hat{p}, \hat{x}+\hat{p}\}$
$\hat{x} \to \{|x\rangle\} \Rightarrow \langle x|y'\rangle = \delta(y-y')$
$\hat{\tilde{x}} = 2\hat{x} \to \{|\tilde{x}\rangle\} \Rightarrow \langle \tilde{x}|\tilde{x}'\rangle = \delta(\tilde{x}-\tilde{x}')$
$= \delta(2(x-x'))$
p
x+p
x
$B_1 = \{|1\rangle, |2\rangle \ldots\}$

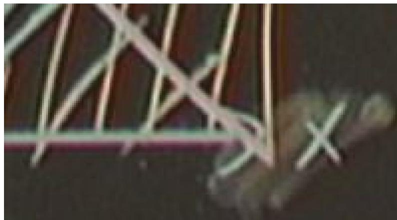

{x̂, p̂, (x̂+p̂)/√2} ≠ MUB
{x̂, p̂, x̂+p̂}
x̂ → {|x⟩} ⇒ ⟨x|y'⟩ = δ(y−y')
{|x̃⟩} ⇒ ⟨x̃|x̃'⟩ = δ(x̃−x̃')
x̃ = 2x → {|x̃⟩}
= δ(2x−x'))
p
x+p
x
B1 = {|1⟩, |2⟩ ... }
B2 = {|φ1⟩, |φ2⟩ ... }
⟨j|φk⟩ = co


p
x+p
x
B1 = { |1>, |2> ... }
B2 = { |φ1>, |φ2> ... }
<j|φk> = const(j,k)
|φk> = Σ |k>


P
X+P
X
B1 = { |1>, |2> ... }
B2 = { |φ1>, |φ2> ... }
<j|φk> = const(j,k
|φk> = Σ c |k>
k=1



$B_2 = \{|\phi_1\rangle, |\phi_2\rangle \ldots\}$
$\langle j|\phi_k\rangle = const(j,k)$
$|\phi_k\rangle = \sum_{k=1}^{\infty} c_k |k\rangle$
$|c_k|^2 = const$

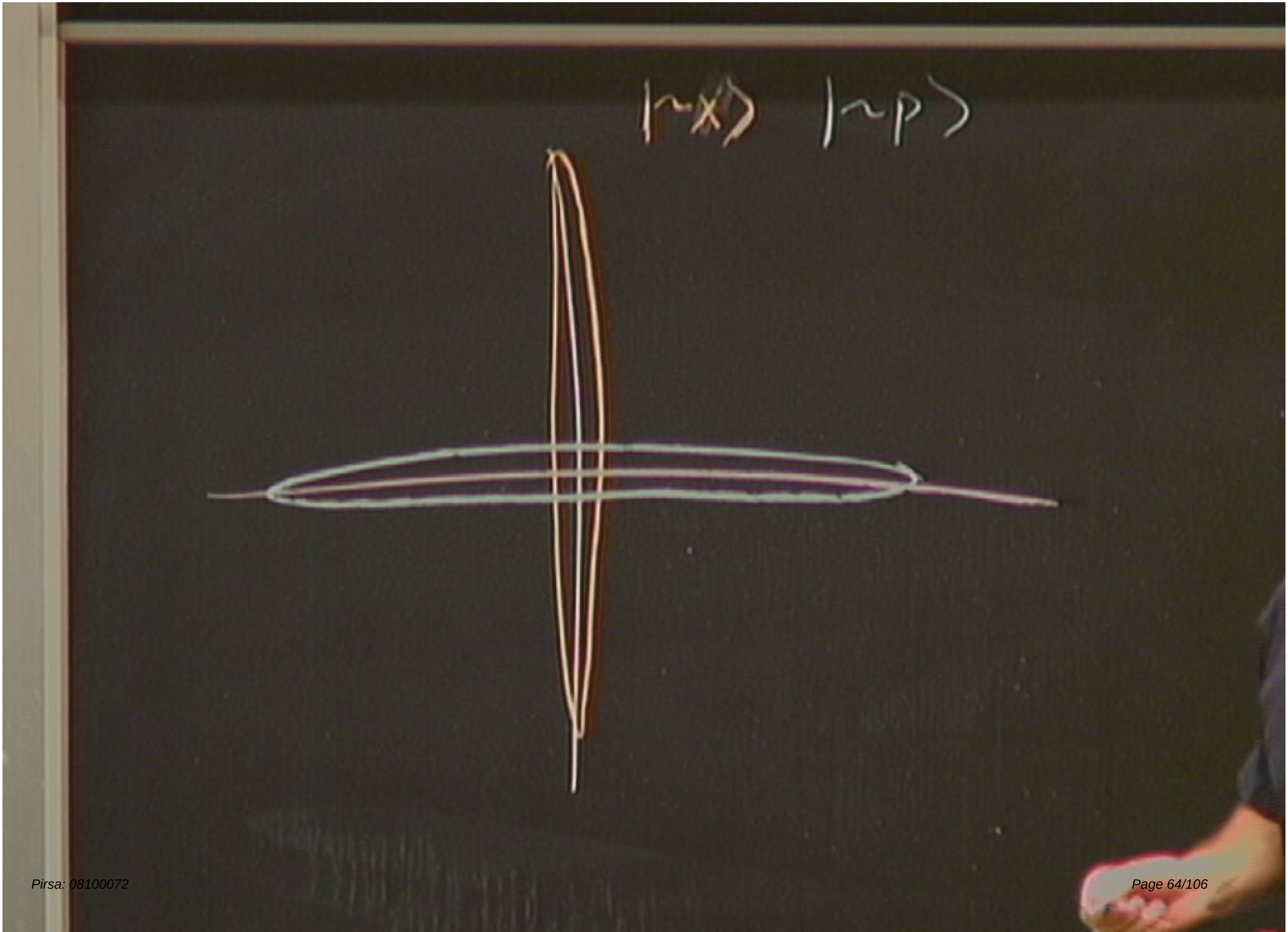


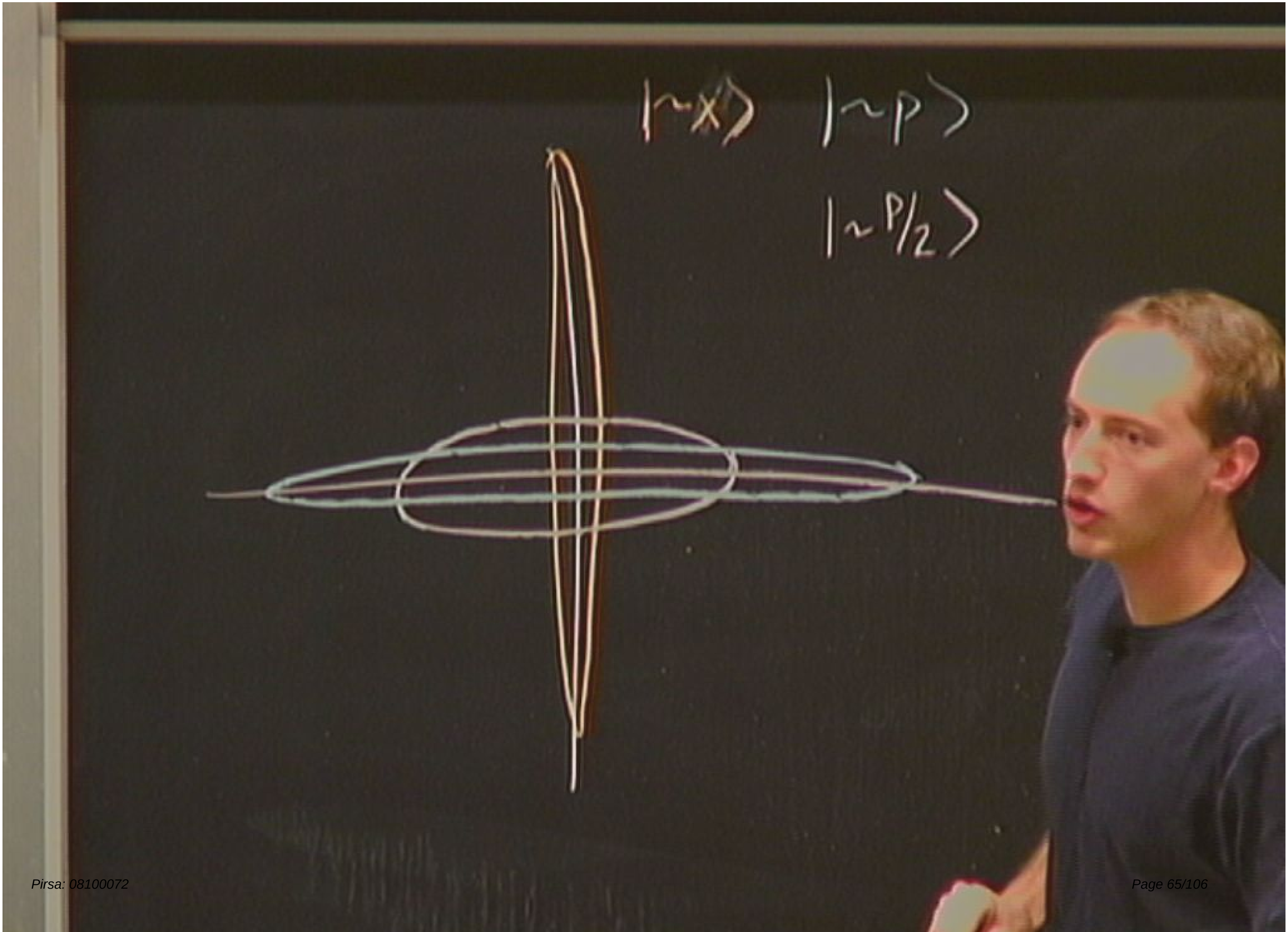
$$B_2 = \{|\phi_1\rangle, |\phi_2\rangle \dots\}$$

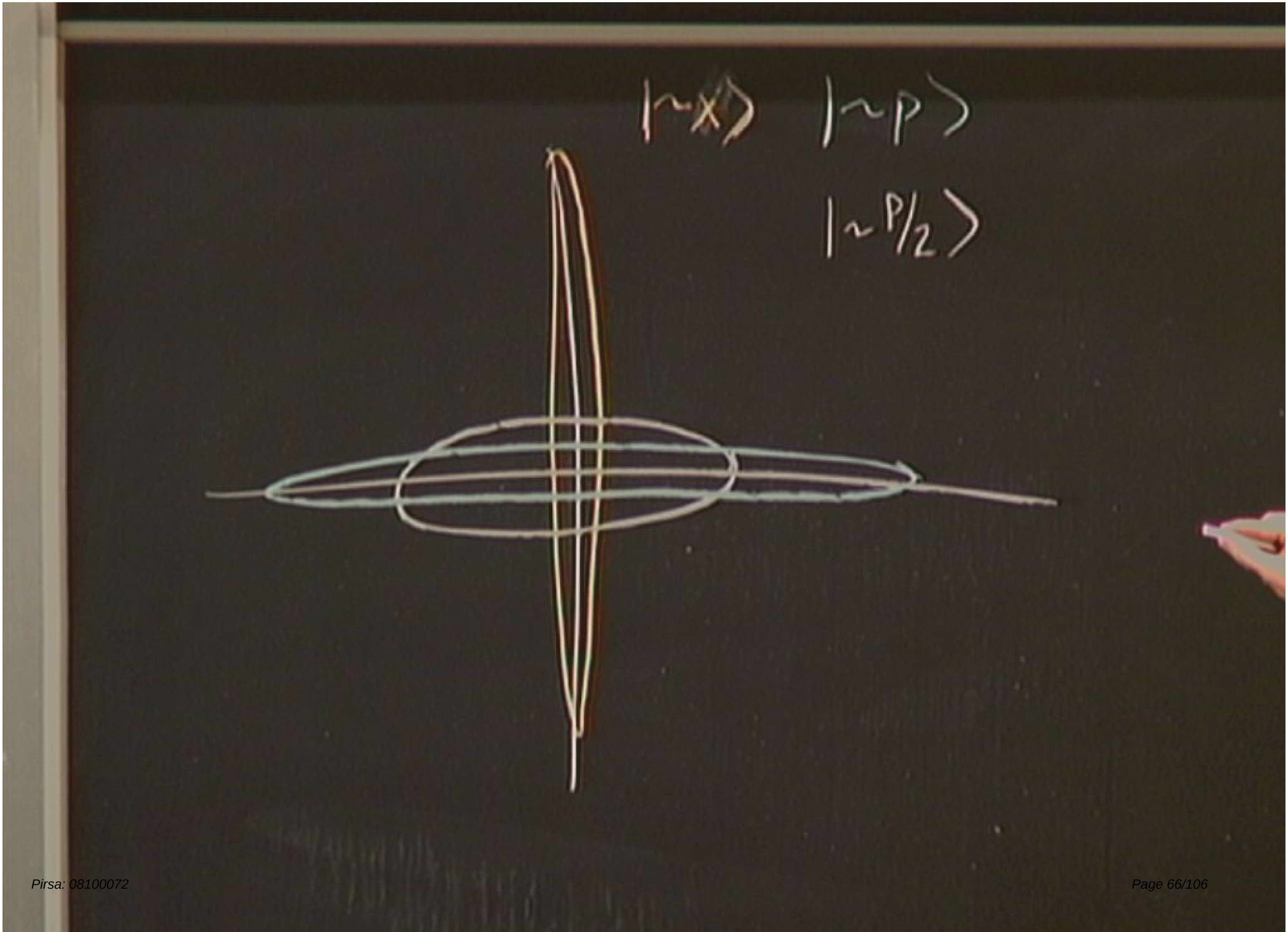
$$\langle j|\phi_k\rangle = \text{const}(j,k)$$

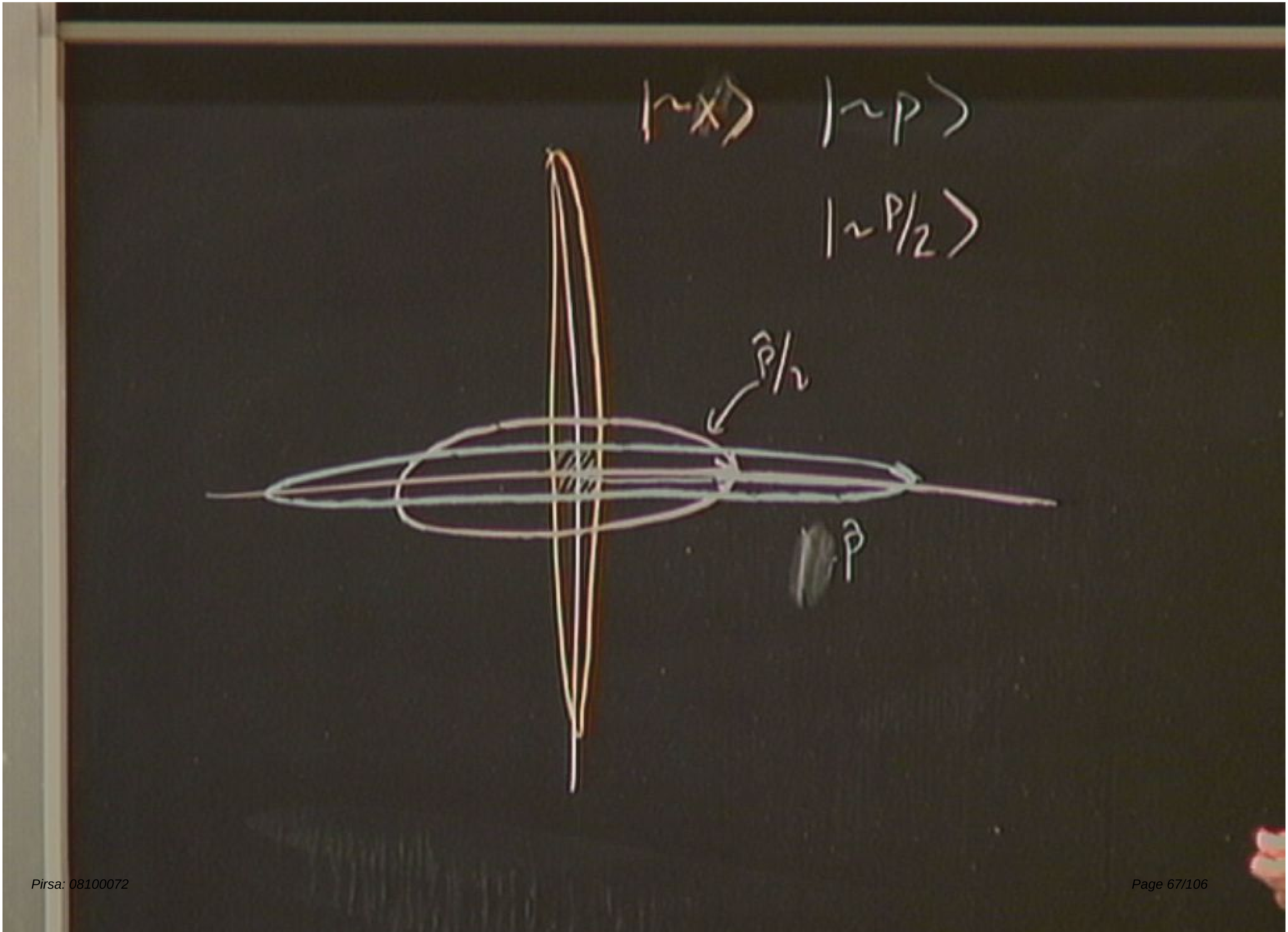
$$|\phi_k\rangle = \sum_{k=1}^{\infty} c_k |k\rangle$$

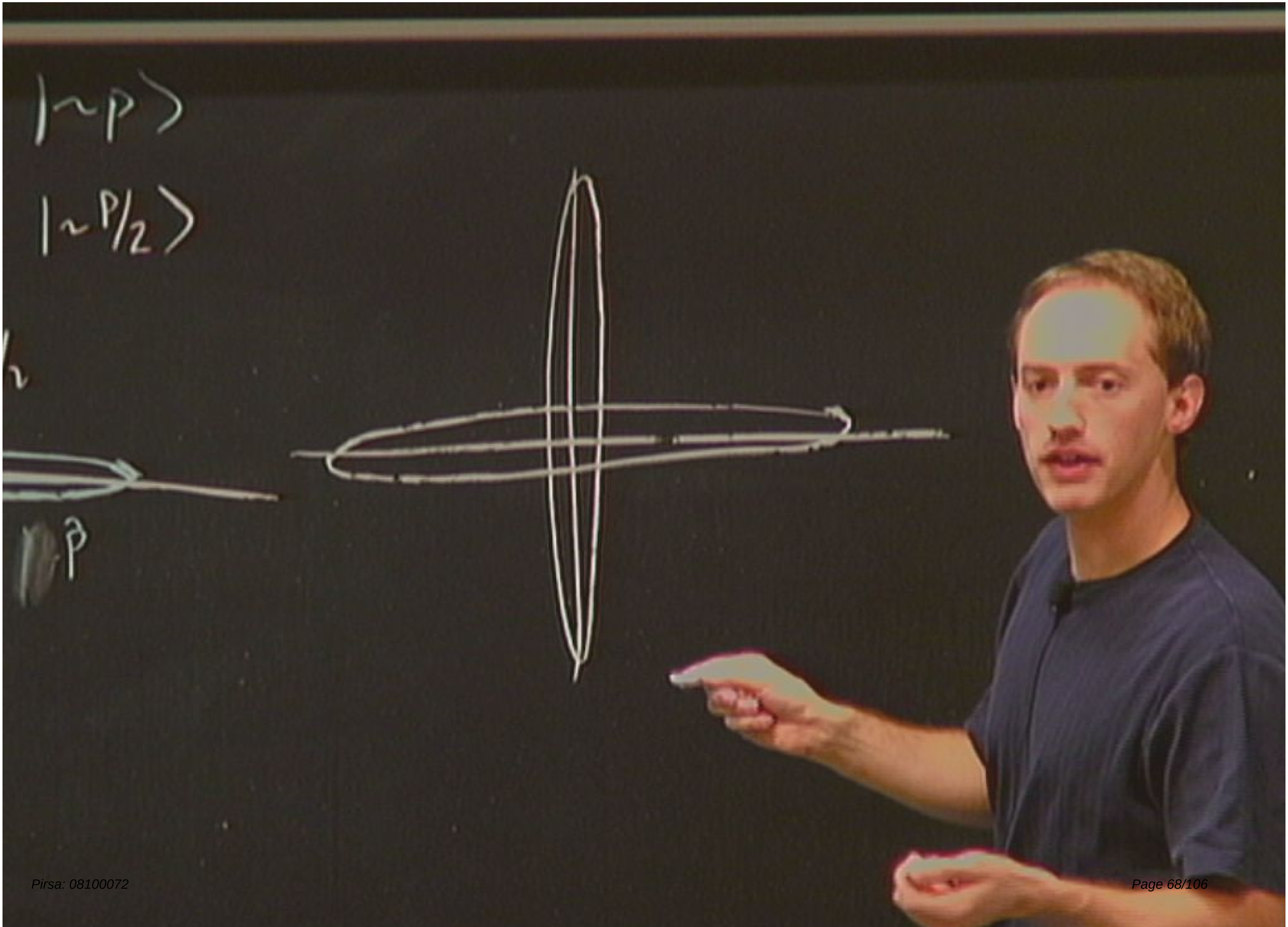
$$|c_k|^2 = \text{const}$$

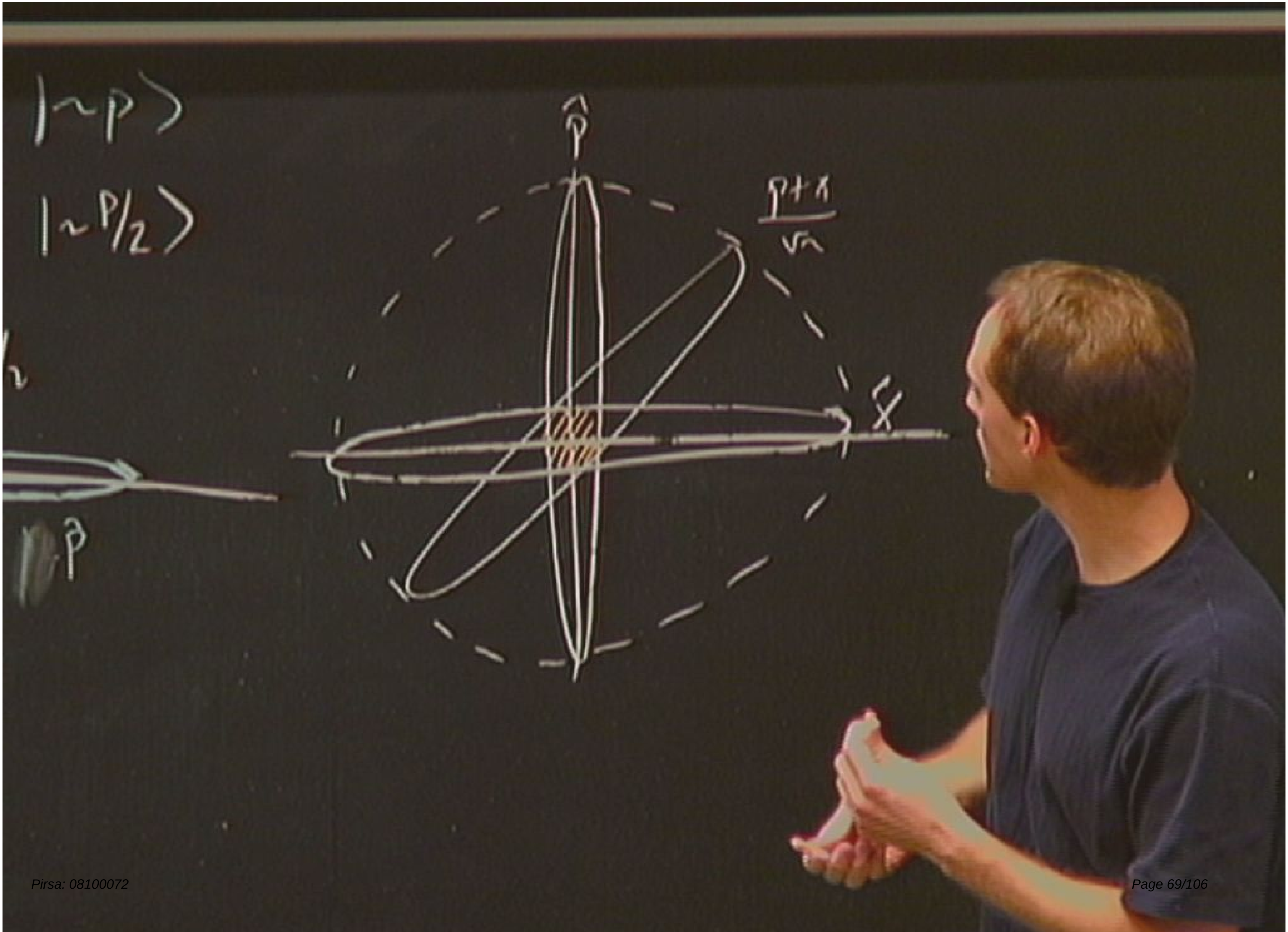

$|\sim x\rangle$ $|\sim p\rangle$

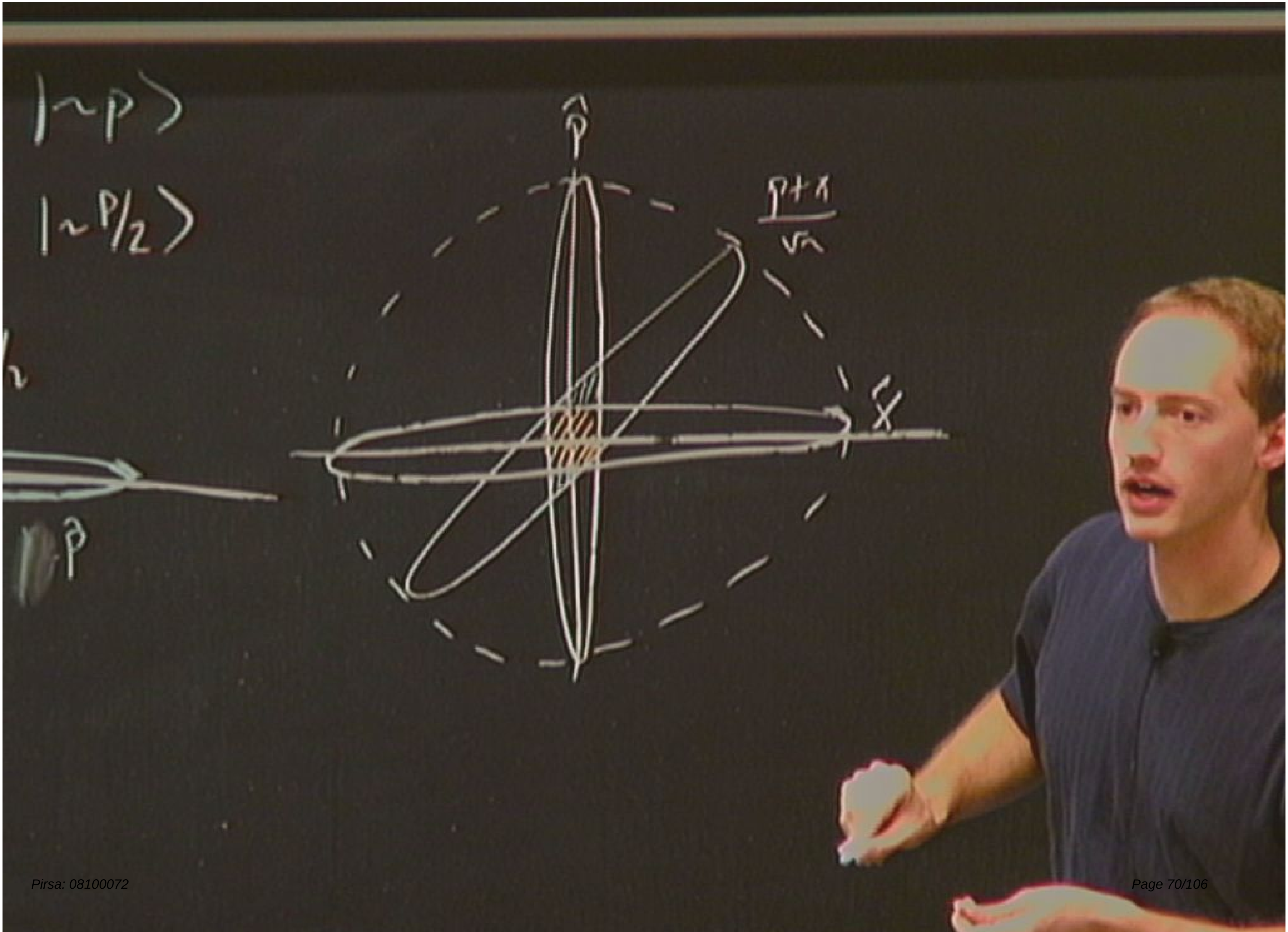

|~x⟩
|~p⟩
|~P/2⟩


$|\sim x\rangle$
$|\sim p\rangle$
$|\sim p/2\rangle$

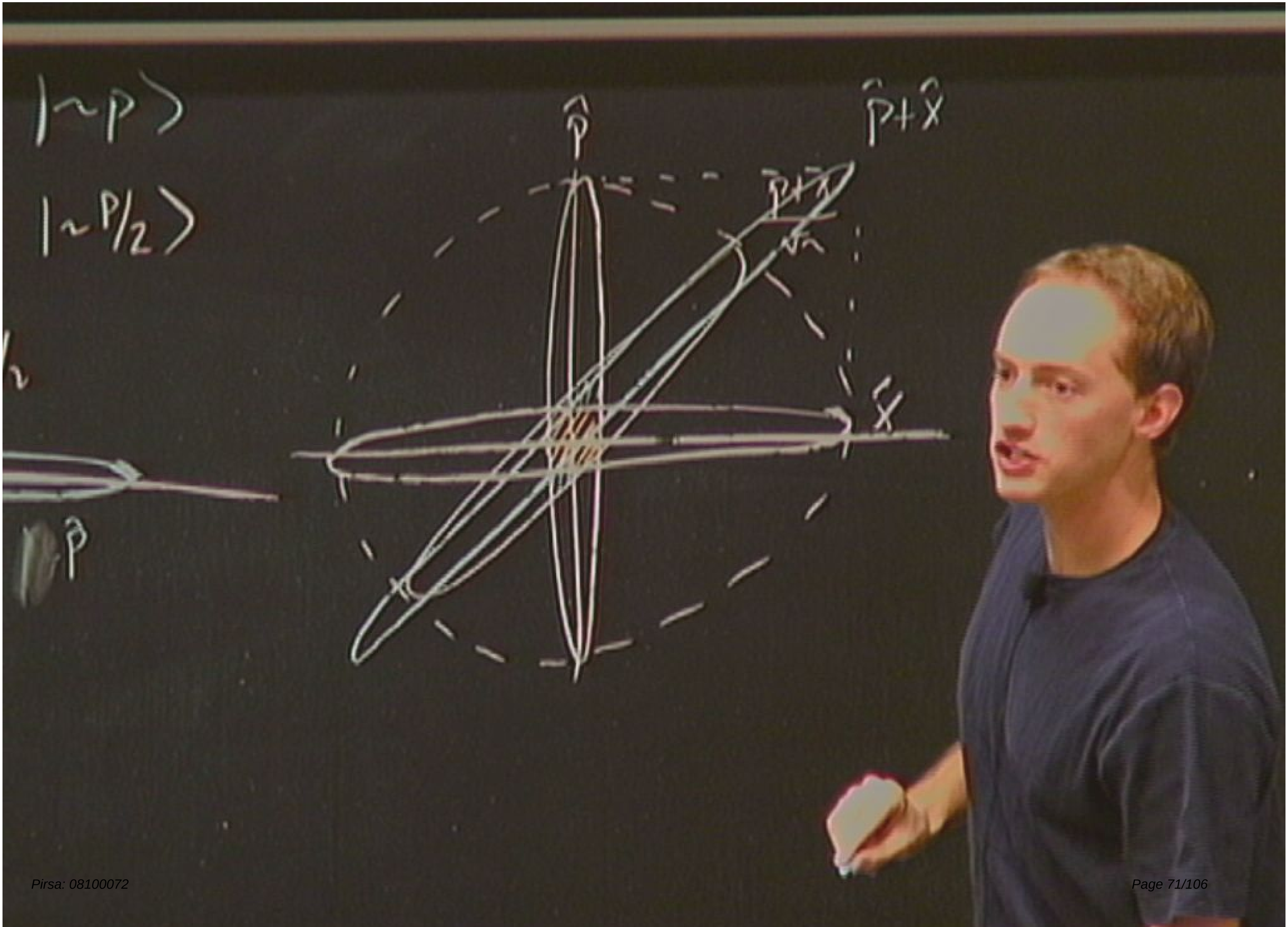

|~x⟩
|~p⟩
|~p/2⟩
p̂/2
p̂

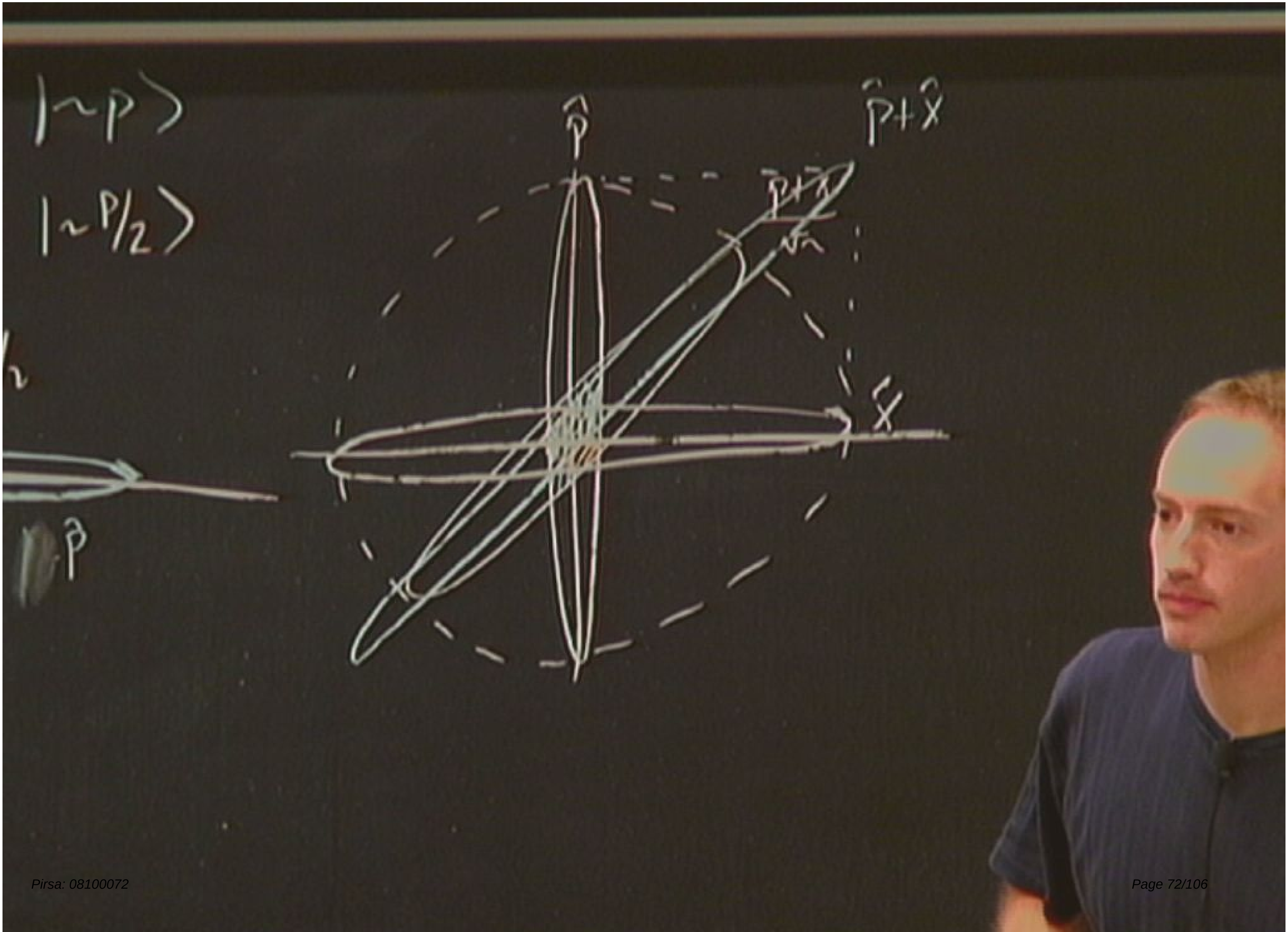

|~p⟩
|~p/2⟩

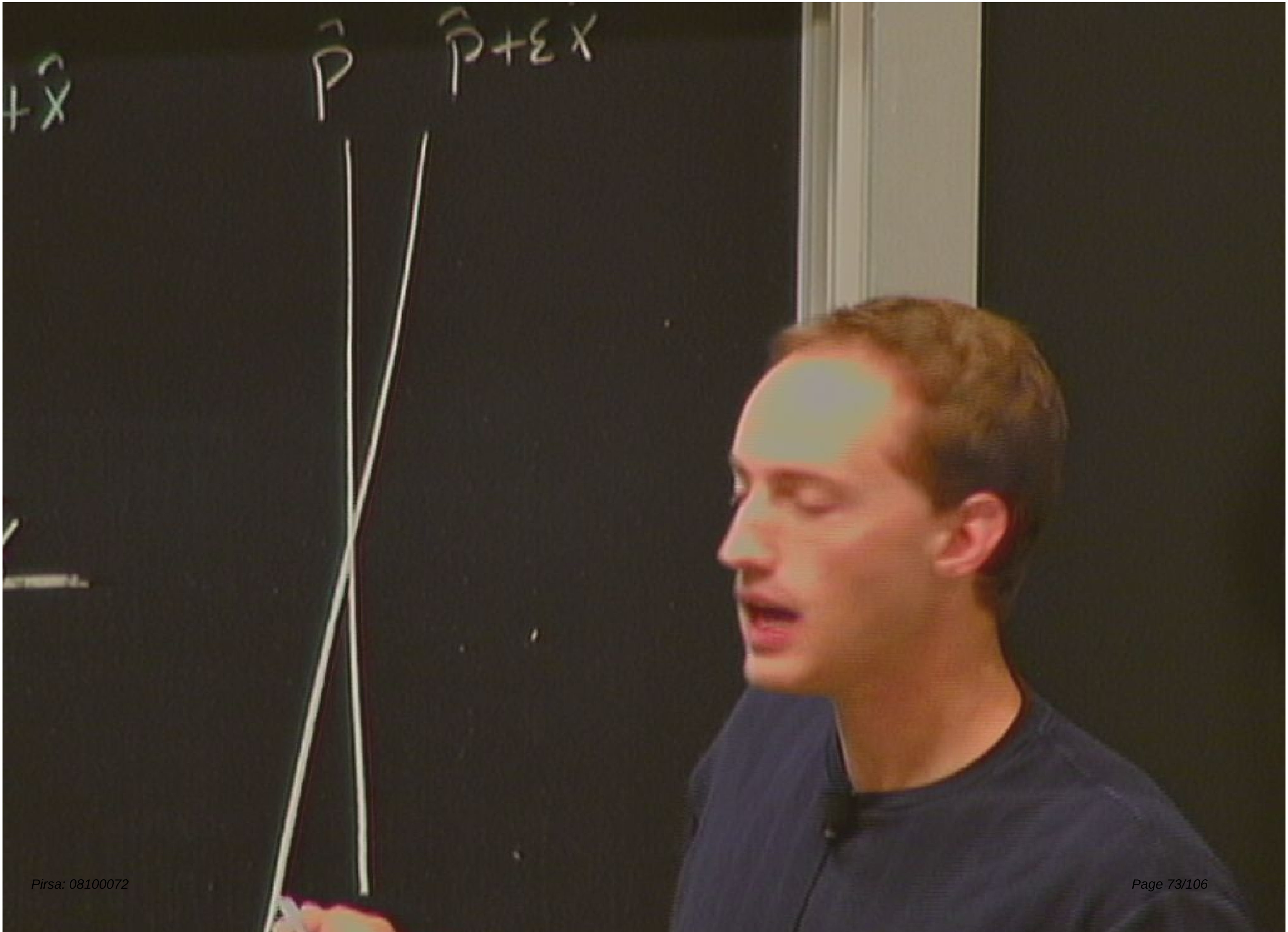

$|\sim p\rangle$
$|\sim p/2\rangle$
$\hat{p}$
$\frac{p+x}{\sqrt{2}}$
$\hat{x}$

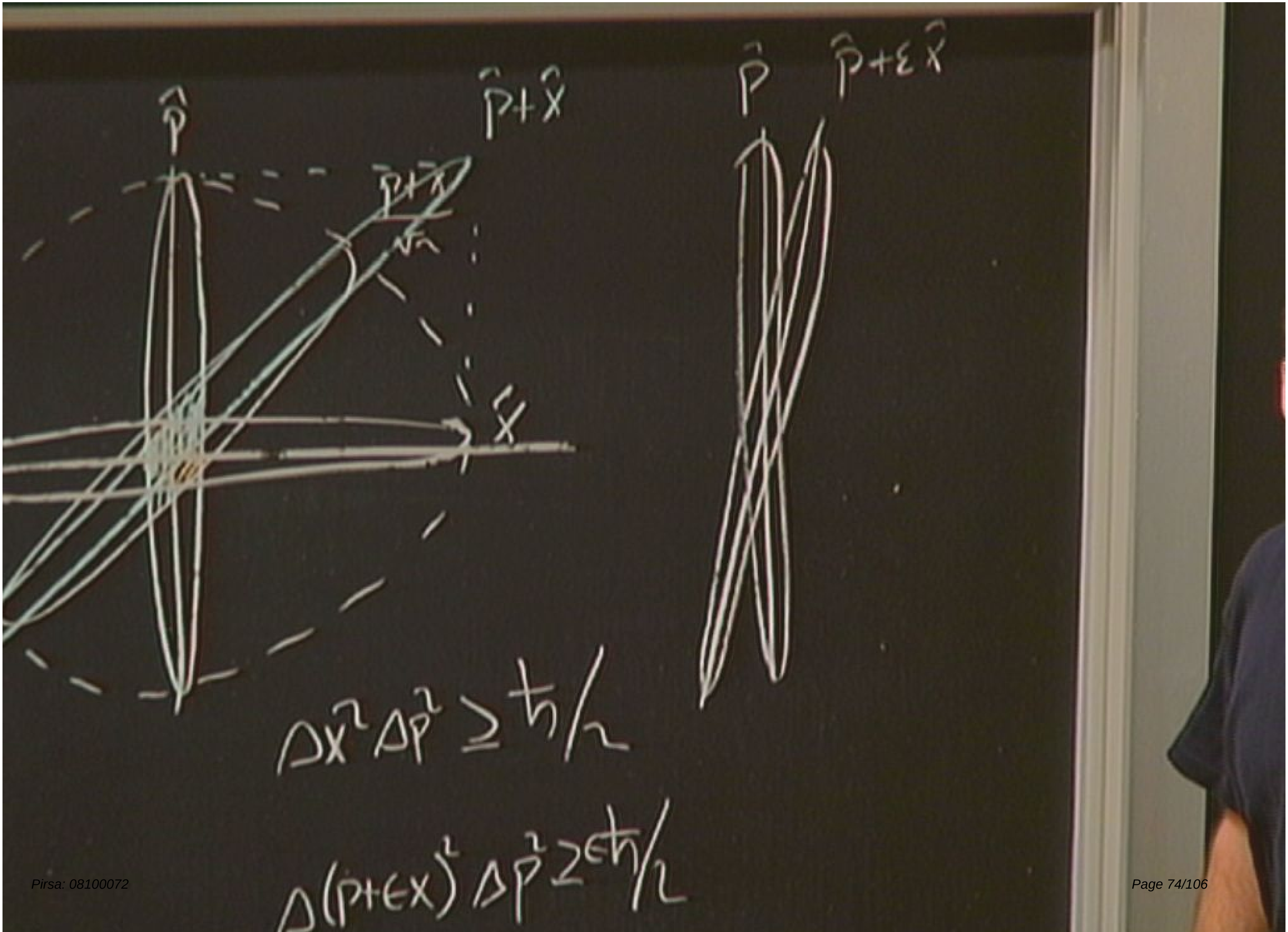


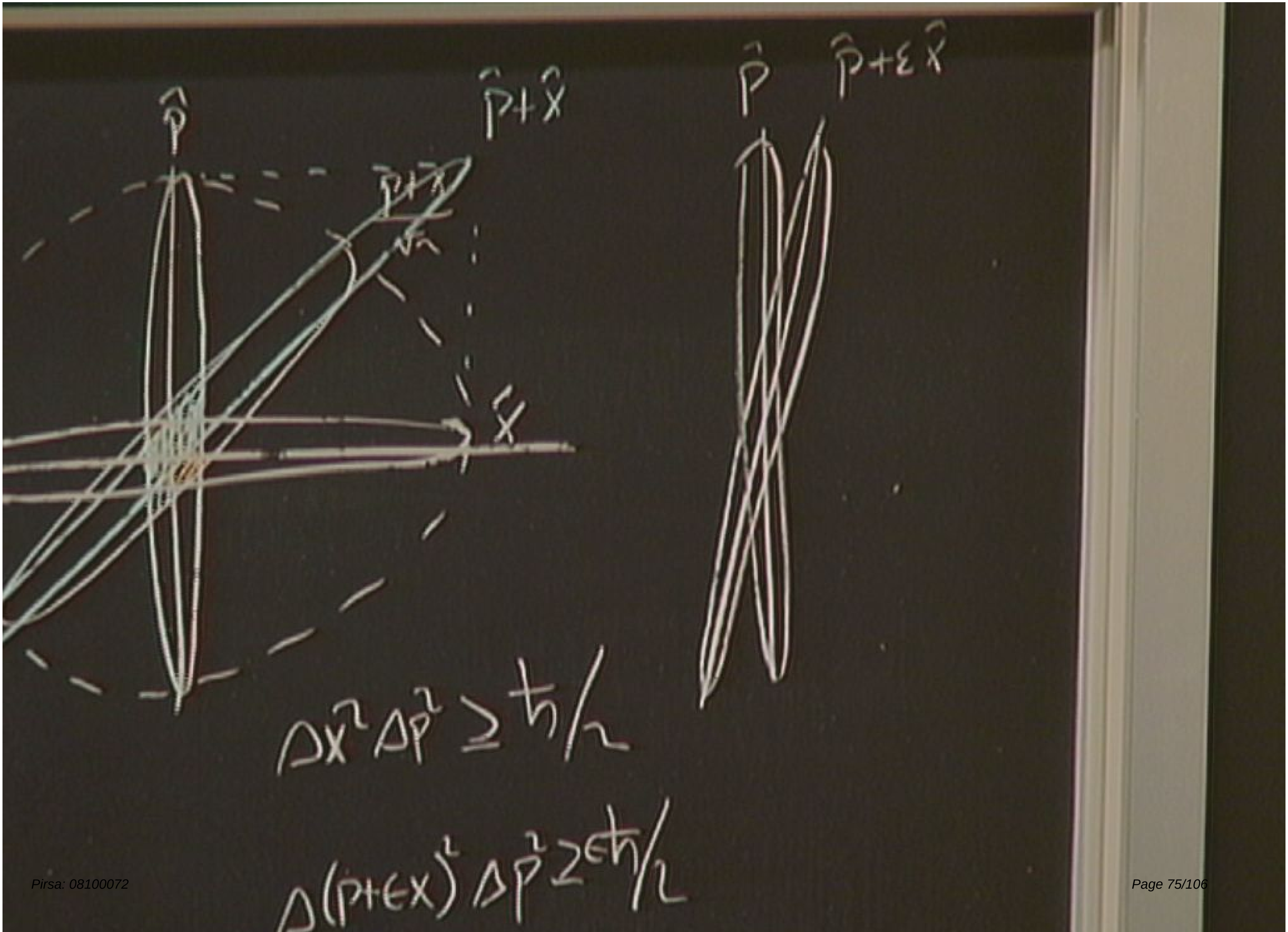
$|\sim p\rangle$
$|\sim p/2\rangle$
$\hat{p}$
$\frac{p+x}{\sqrt{2}}$
$\hat{x}$

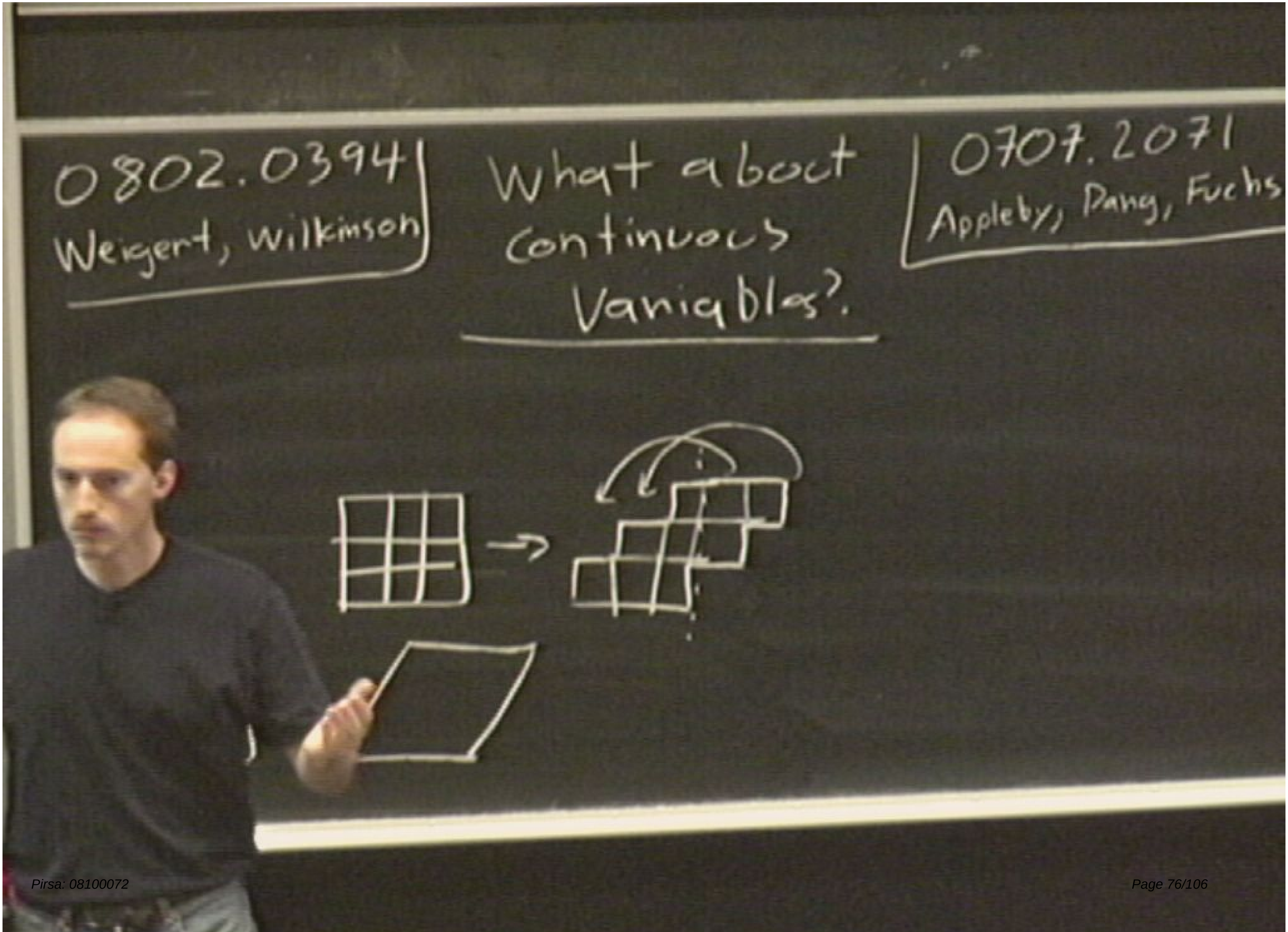

$|\sim p\rangle$
$|\sim p/2\rangle$
$\hat{p}$
$\hat{p}+\hat{x}$
$\hat{x}$
$\hat{p}$


|~p⟩
|~p/2⟩
p̂
p̂+x̂
x̂


$\hat{p}$
$\hat{p}+\varepsilon \hat{x}$


$\hat{p}$
$\hat{p}+\hat{x}$
$\hat{p}+\hat{x}$
$\sqrt{2}$
$\hat{x}$
$\hat{p}$
$\hat{p}+\varepsilon\hat{x}$
$\Delta x^2 \Delta p^2 \geq \hbar/2$
$\Delta(p+\epsilon x)^2 \Delta p^2 \geq \epsilon\hbar/2$

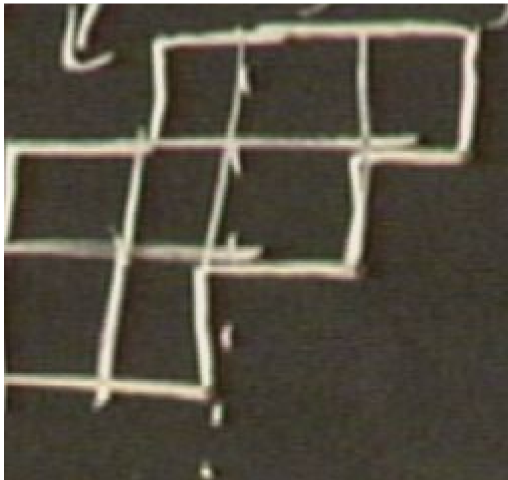

$\hat{p}$
$\hat{p}+\hat{x}$
$\hat{x}$
$\hat{p}$
$\hat{p}+\varepsilon\hat{x}$
$\Delta x^2 \Delta p^2 \geq \hbar/2$
$\Delta(p+\epsilon x)^2 \Delta p^2 \geq \epsilon\hbar/2$


0802.0394
Weigert, Wilkinson
What about continuous Variables?
0707.2071
Appleby, Dang, Fuchs

about
ous
ables?.

0707.2071
Appleby, Dang, Fuchs

SIC states
are like
coherent
states.



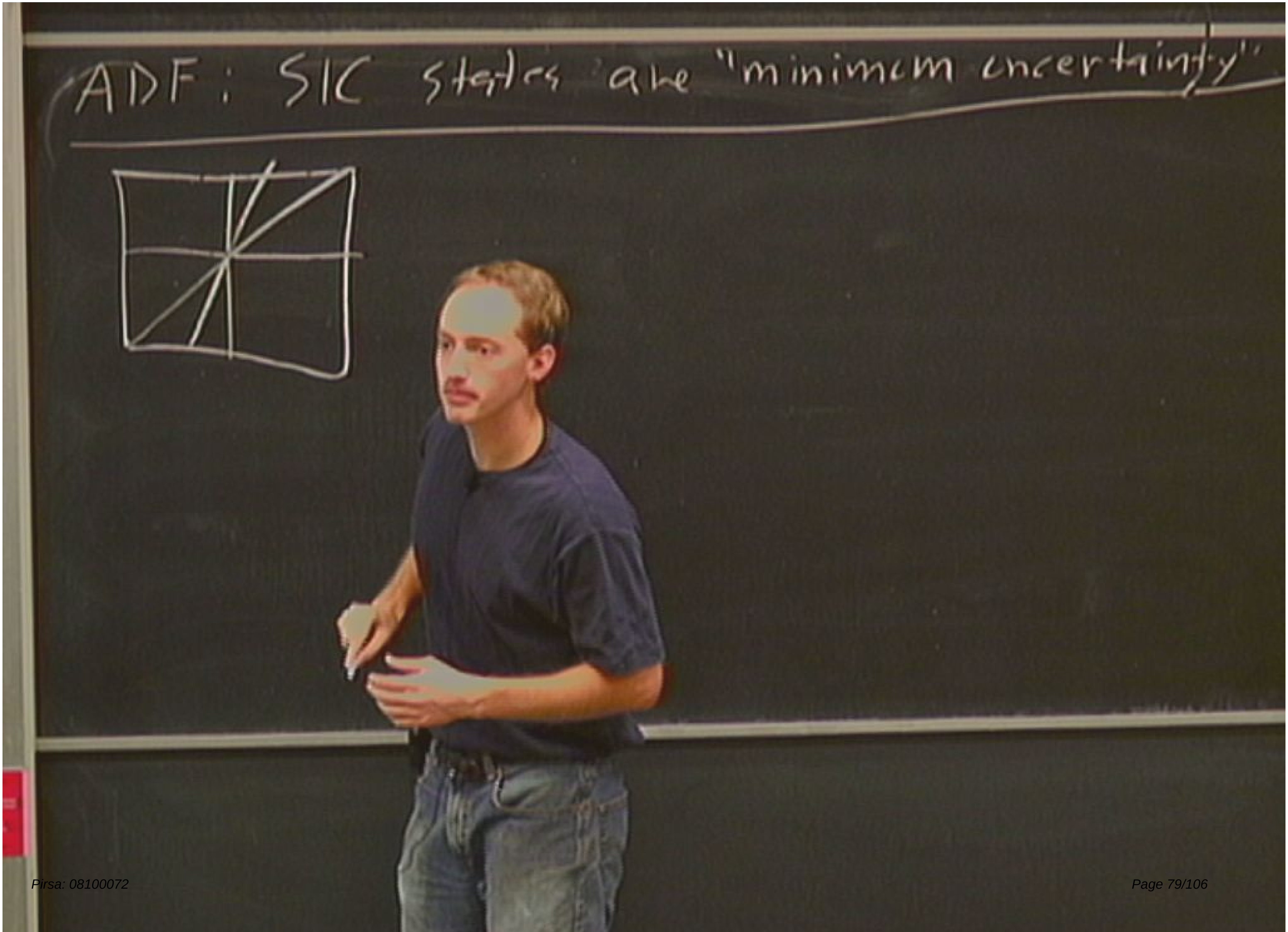
states.

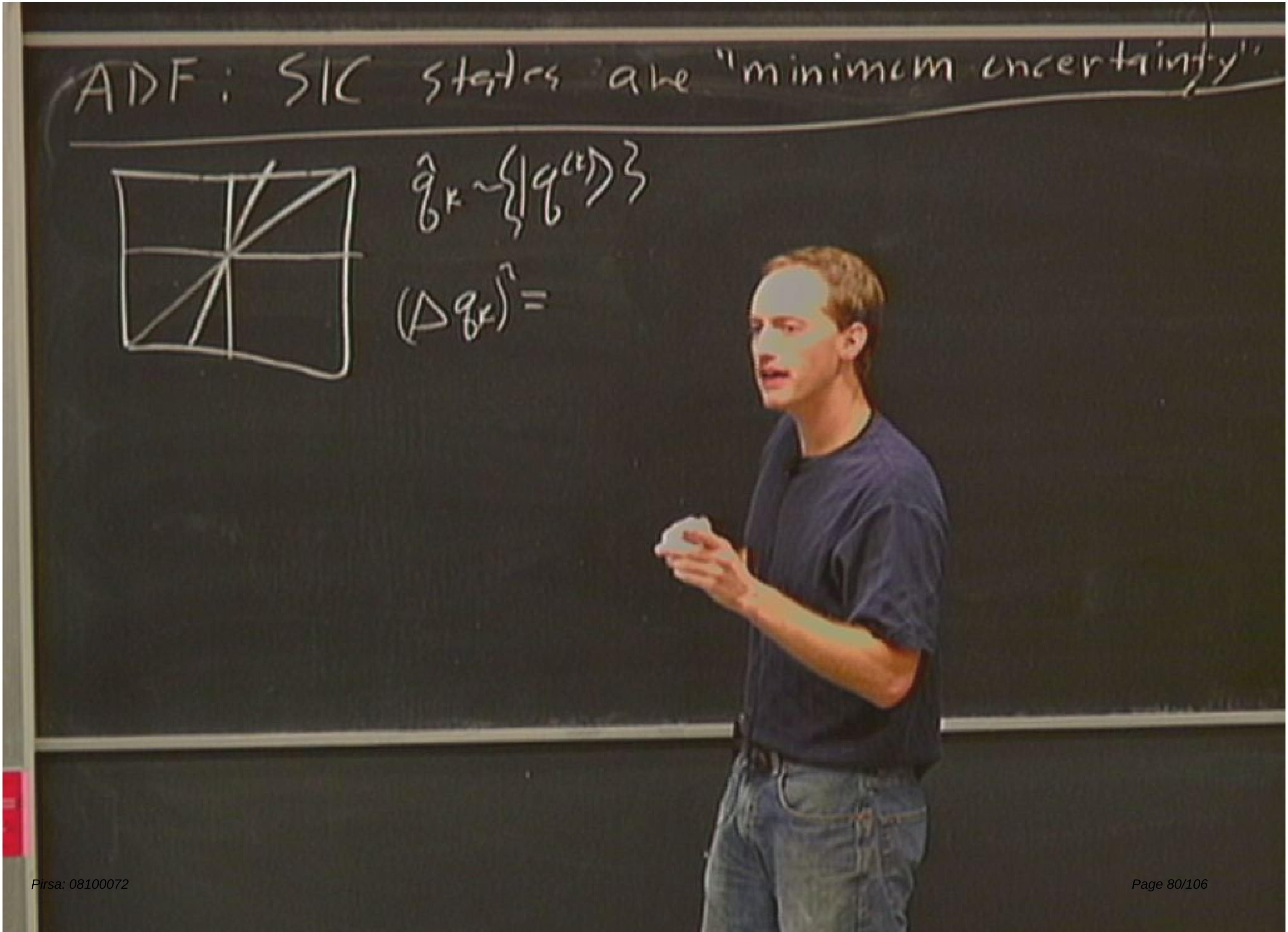
SICs are horribly unlike coherent states.

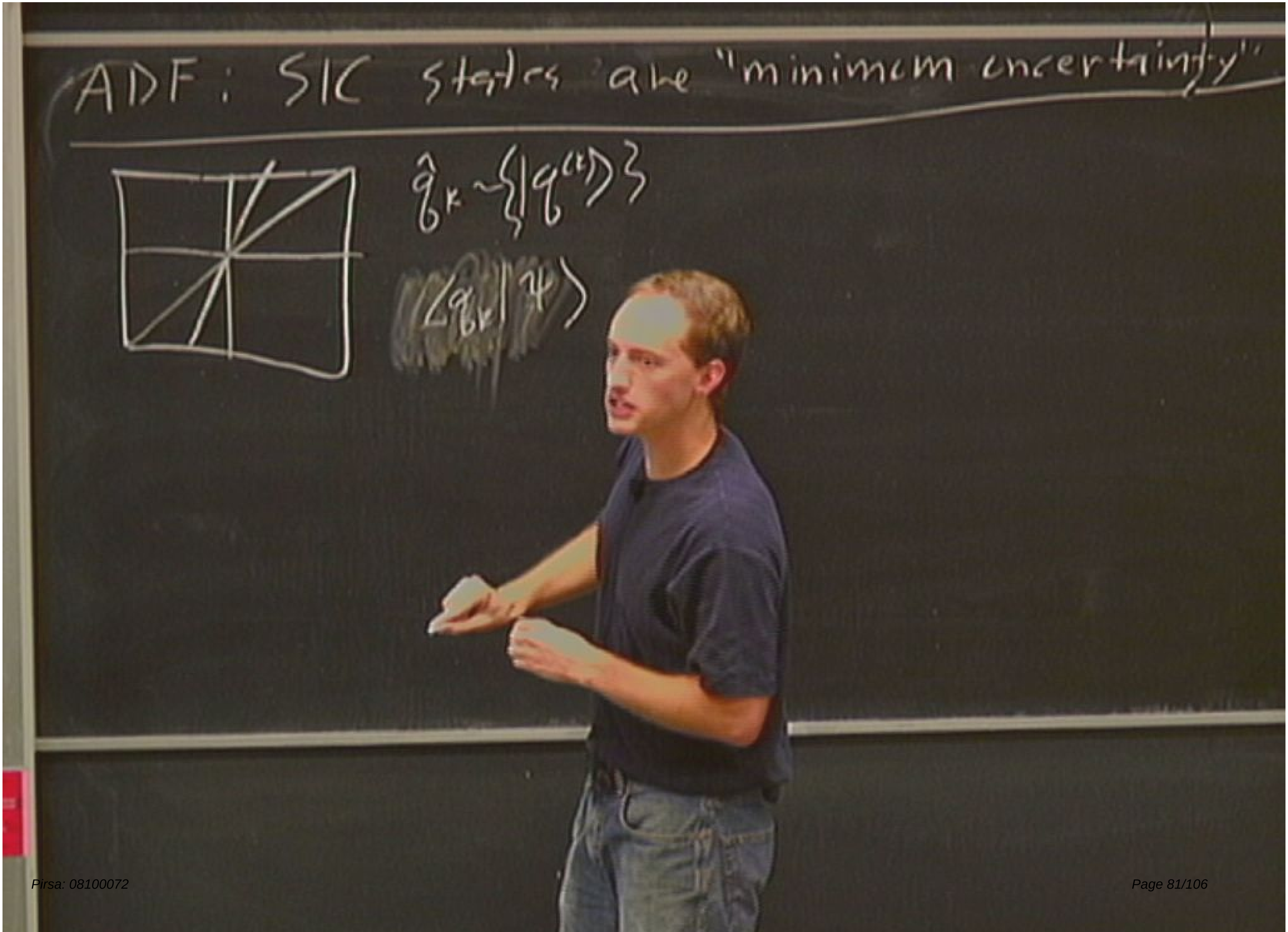
$$\langle x|x'\rangle = \delta(x-x')$$

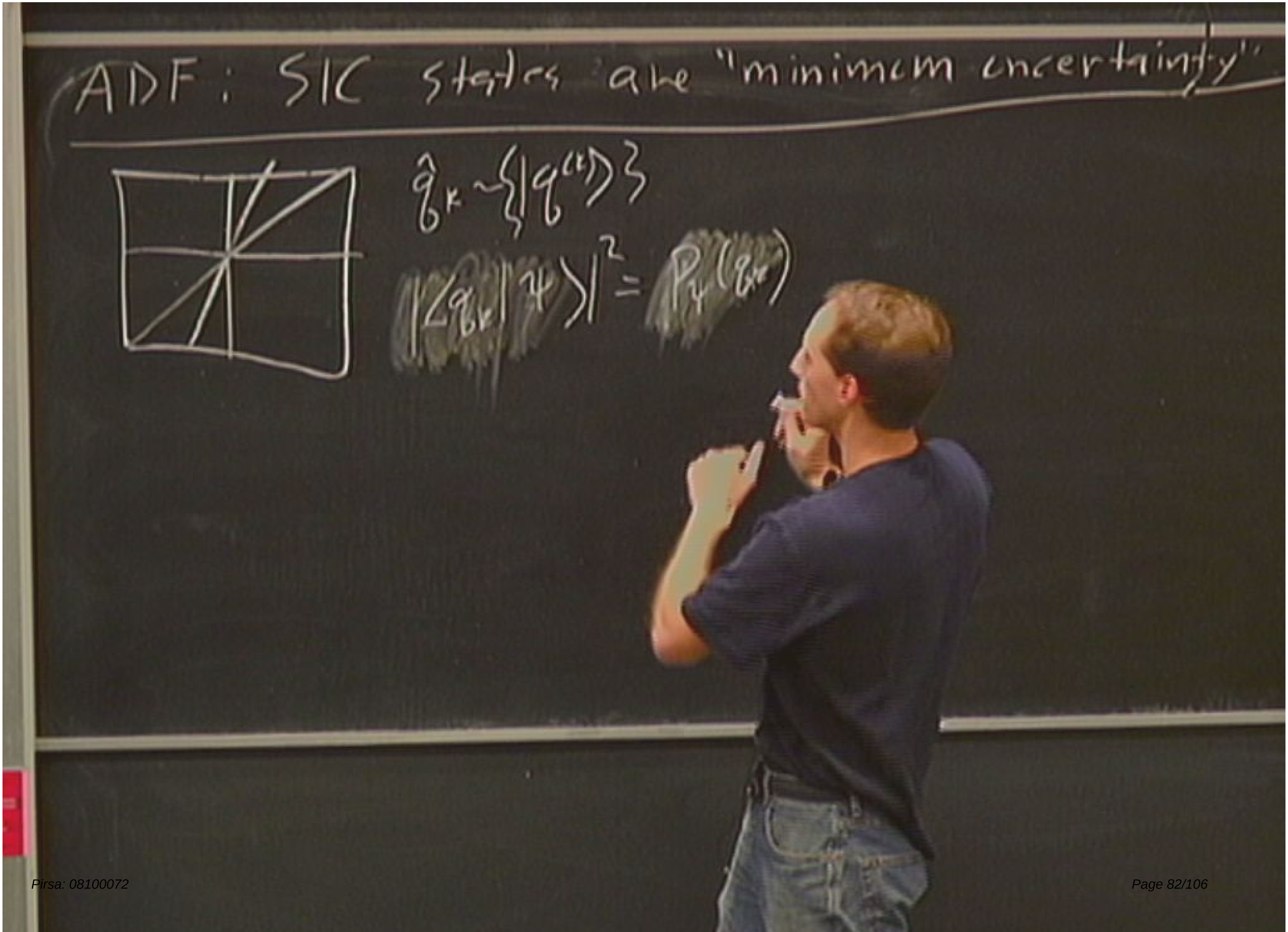
$$\langle p|p'\rangle = \delta(p-p')$$

$$\langle q|q'\rangle = \delta(q-q')$$

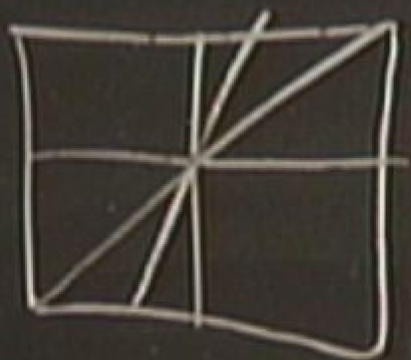

ADF: SIC states are "minimum uncertainty"


ADF: SIC states are "minimum uncertainty"
q̂k ~ {|q(k)⟩}
(Δqk)² =


ADF: SIC states are "minimum uncertainty"
$\hat{q}_k \sim \{|q^{(k)}\rangle\}$
$\langle q_k|\psi\rangle$


ADF: SIC states are "minimum uncertainty"
$\hat{q}_k \sim \{|q^{(k)}\rangle\}$
$|\langle q_k | \psi \rangle|^2 = P_\psi(q_k)$

ADF: SIC states are "minimum uncertainty"

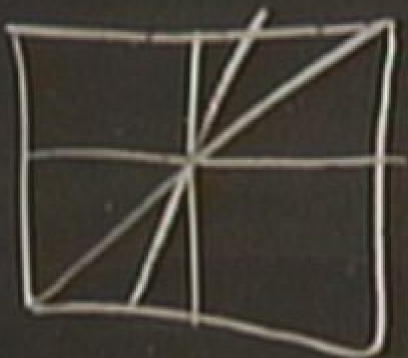


$\hat{q}_k \sim \{|q^{(k)}\rangle\}$

$|\langle q_k|\psi\rangle|^2 = P_\psi(q_k)$

$\log\left[\left(\sum P^2\right)_1 \left(\sum P^2\right)_2 \cdots \left(\sum P^2\right)_{d+1}\right]$

ADF: SIC states are "minimum uncertainty"

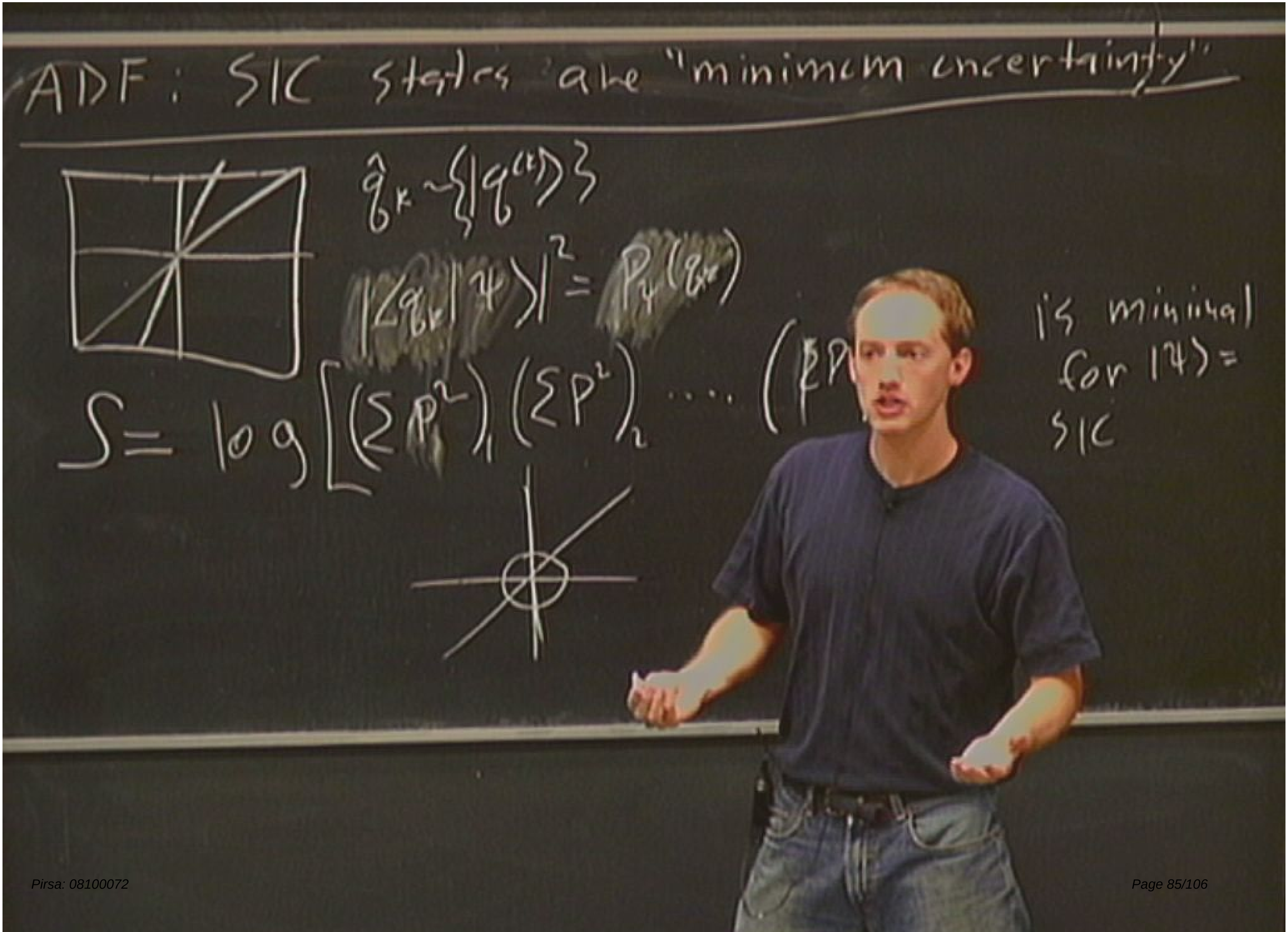


$$\hat{q}_k \sim \{|q^{(k)}\rangle\}$$

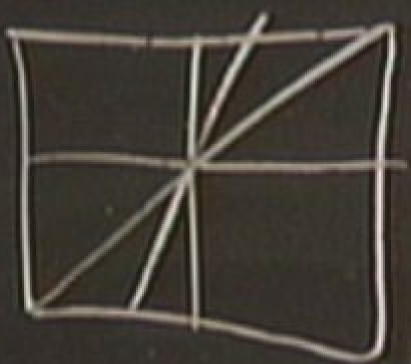
$$|\langle q_k|\psi\rangle|^2 = P_\psi(q_k)$$

$$S = \log\left[\left(\sum P^2\right)_1 \left(\sum P^2\right)_2 \cdots \left(\sum P^2\right)_{d+1}\right]$$

is minimal for $|\psi\rangle =$ SIC


ADF: SIC states are "minimum uncertainty"
$\hat{q}_k \sim \{|q^{(k)}\rangle\}$
$|\langle q_k|\psi\rangle|^2 = P_\psi(q_k)$
$S = \log\left[(\sum P^2)_1 (\sum P^2)_2 \cdots\right]$
is minimal for $|\psi\rangle$ = SIC

ADF: SIC states are "minimum uncertainty"



$\hat{q}_k \sim \{|q^{(k)}\rangle\}$

$|\langle q_k|\psi\rangle|^2 = P_\psi(q_k)$

$$S = \log\left[\left(\sum P^2\right)_1 \left(\sum P^2\right)_2 \cdots \left(\sum P^2\right)_{d+1}\right]$$

is minimal for $|\psi\rangle =$ SIC

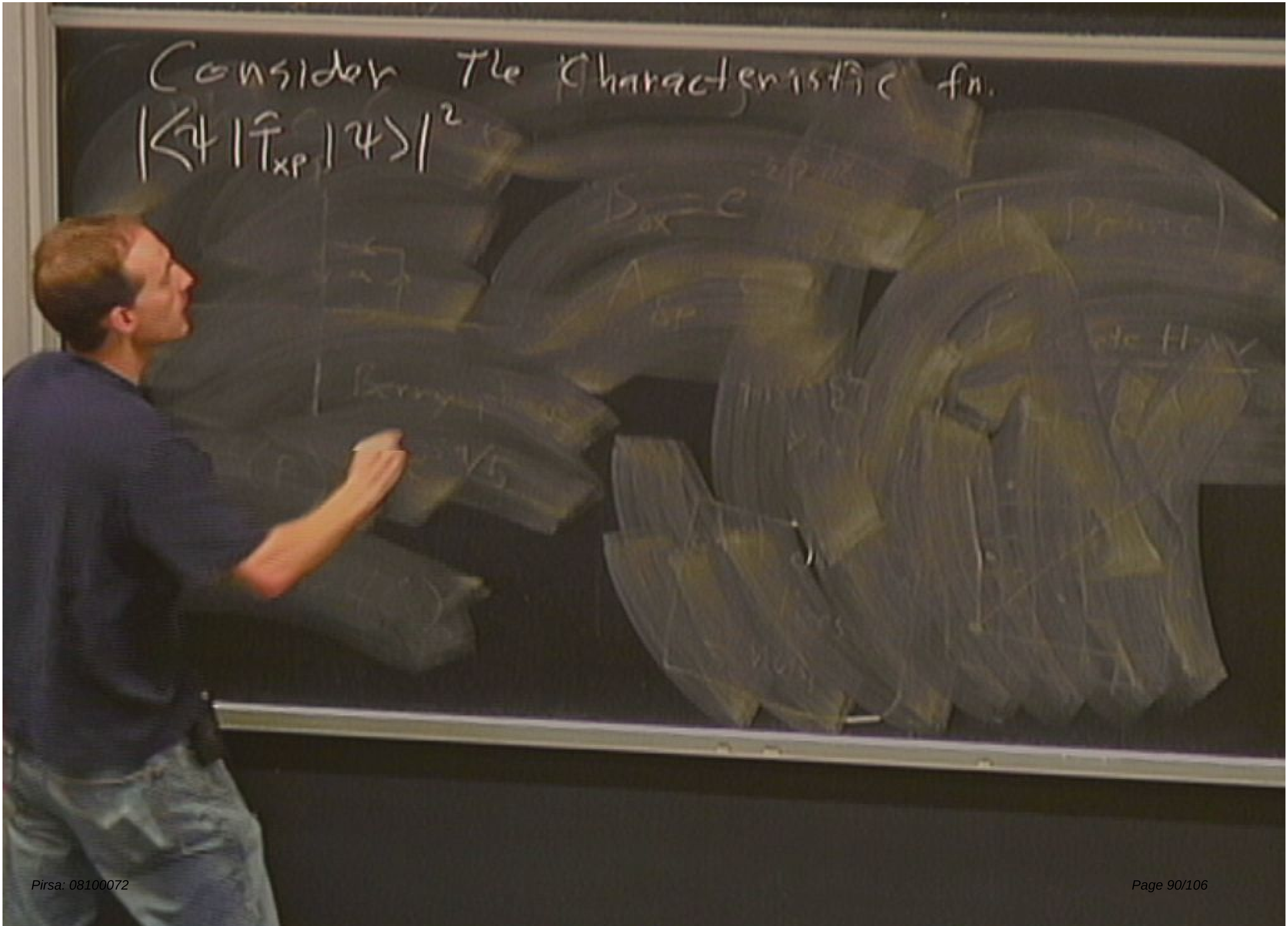
Consider The Characteristic fn.

Consider the Characteristic fn.

$|\psi\rangle$

Consider the Characteristic fn.

$T_{xp}|\psi\rangle$

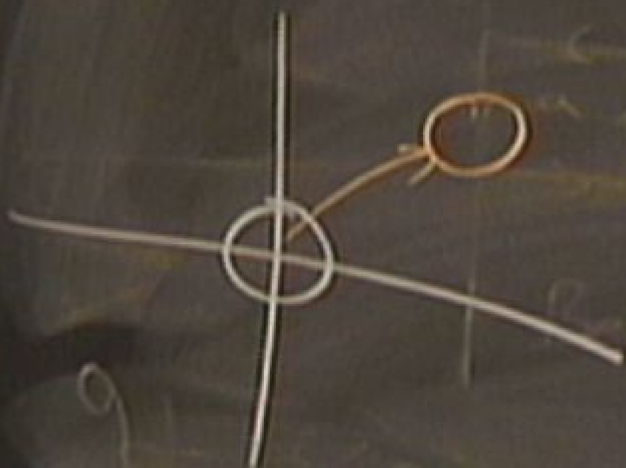

Consider The Characteristic fn.
$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2$

Consider the characteristic fn.

$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$

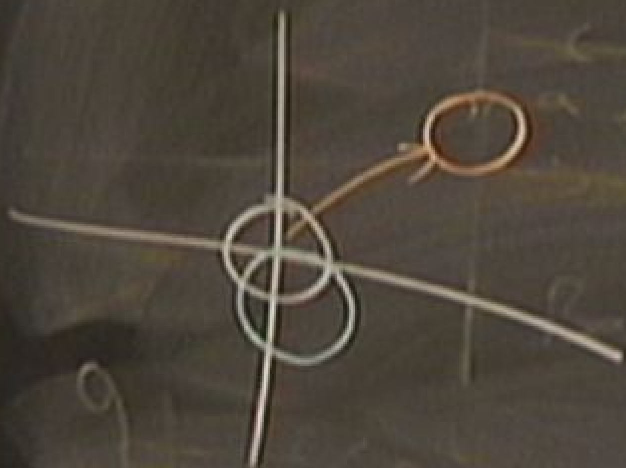
Consider the Characteristic fn.

$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$



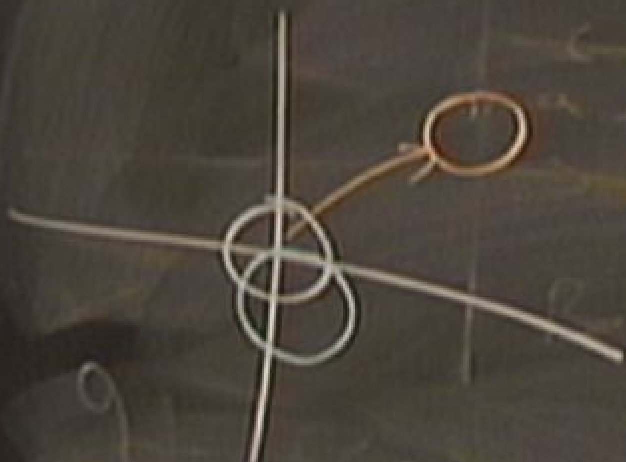
Consider the characteristic fn.

$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$



Consider the characteristic fn.

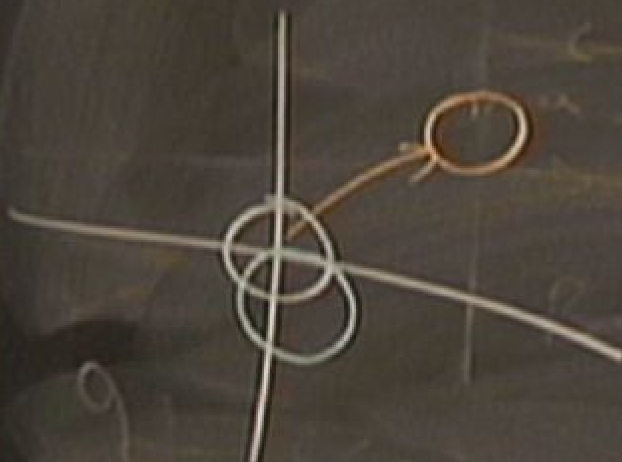
$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$



$$C(x,p) = e^{-(x^2+p^2)/\hbar}$$

Consider the characteristic fn.

$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$

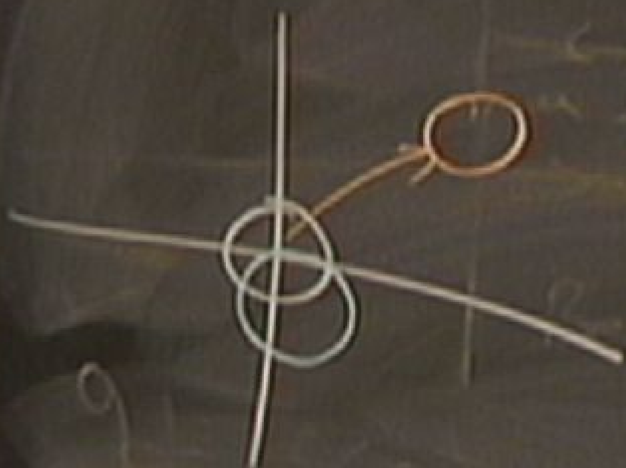


$$C(x,p) = e^{-(x^2+p^2)/\hbar}$$

Let $|\psi\rangle = \xi|c$

Consider the characteristic fn.

$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$



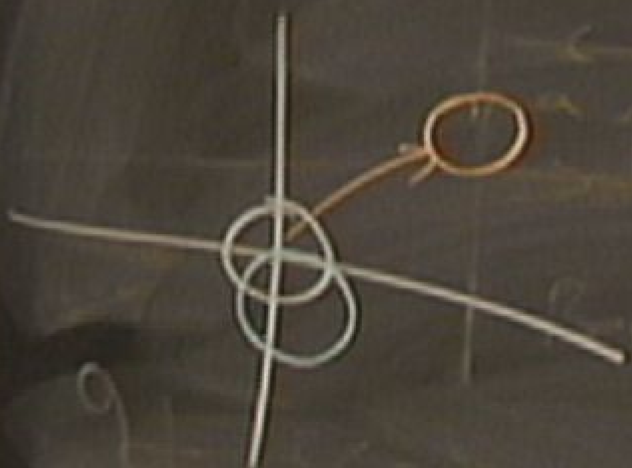
$$C(x,p) = e^{-(x^2+p^2)/\hbar}$$

Let $|\psi\rangle = \sum |c$

$$\langle\psi|T_{xp}|\psi\rangle$$

Consider the characteristic fn.

$$|\langle\psi|\hat{T}_{xp}|\psi\rangle|^2 = C(x,p)$$



$$C(x,p) = e^{-(x^2+p^2)/\hbar}$$

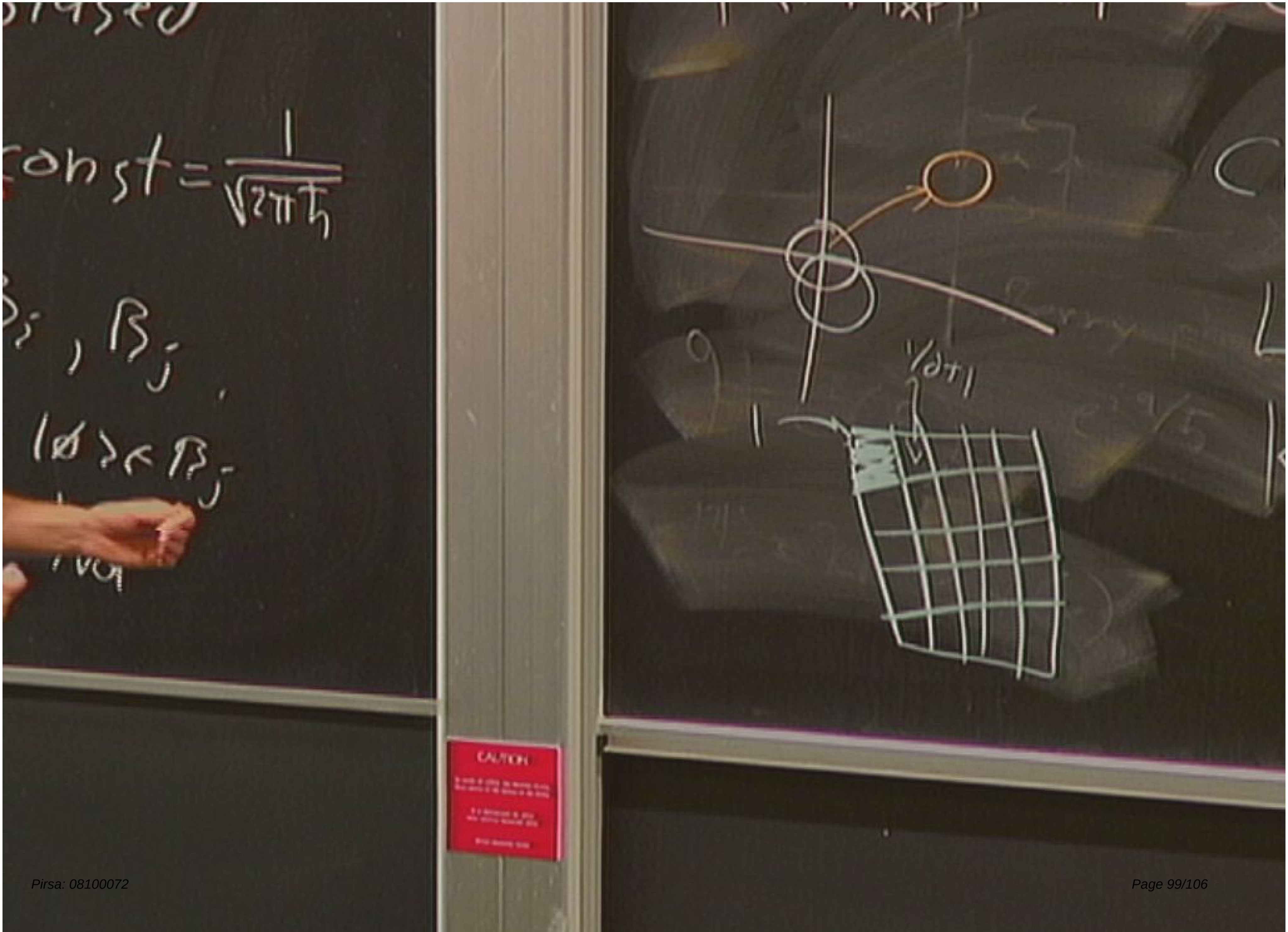
Let $|\psi\rangle = \xi/c$

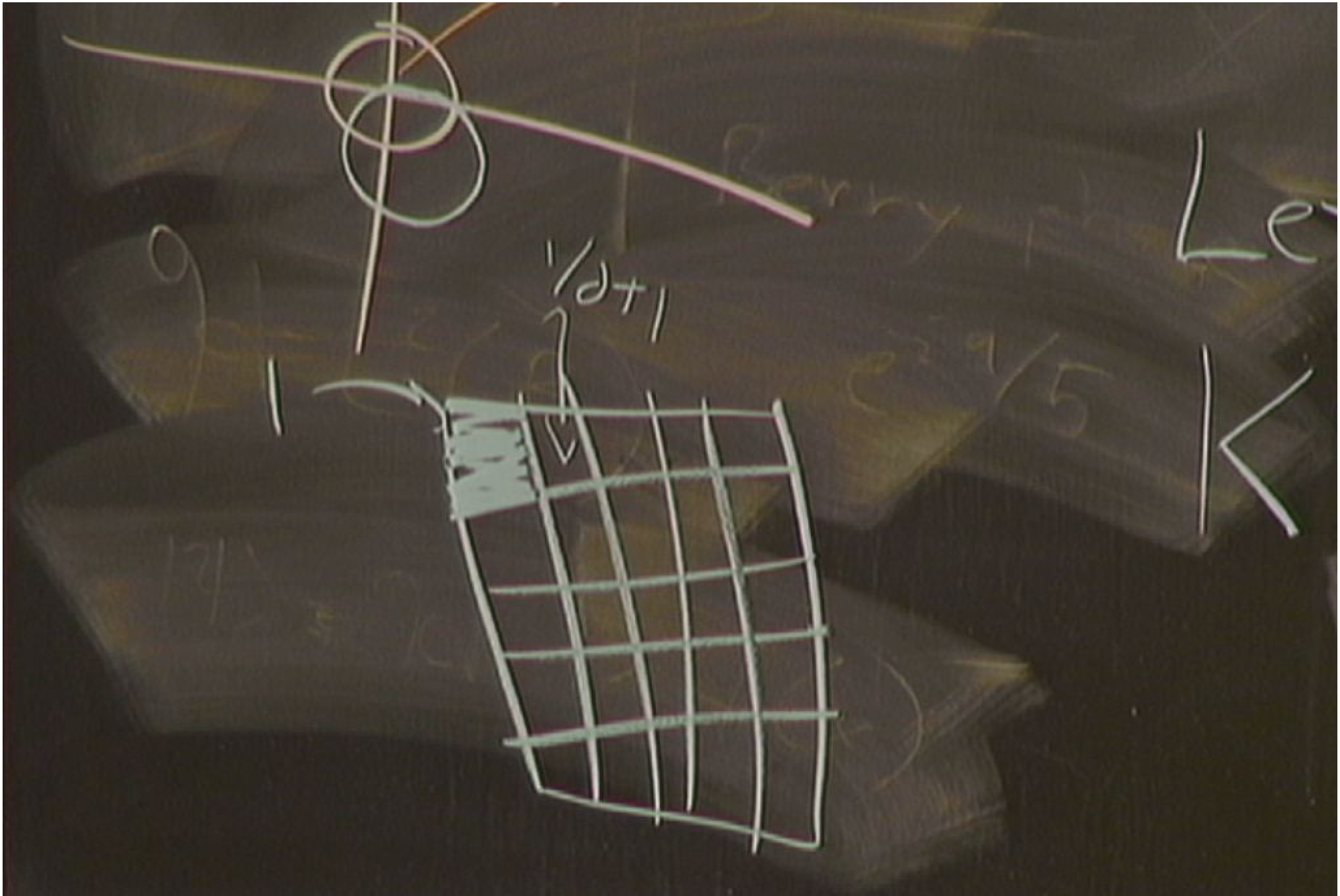
$$\langle\psi|T_{xp}|\psi\rangle = \langle\psi|\psi_{xp}\rangle$$

$$C(x,p) = e^{-(x^2+p^2)/\hbar}$$

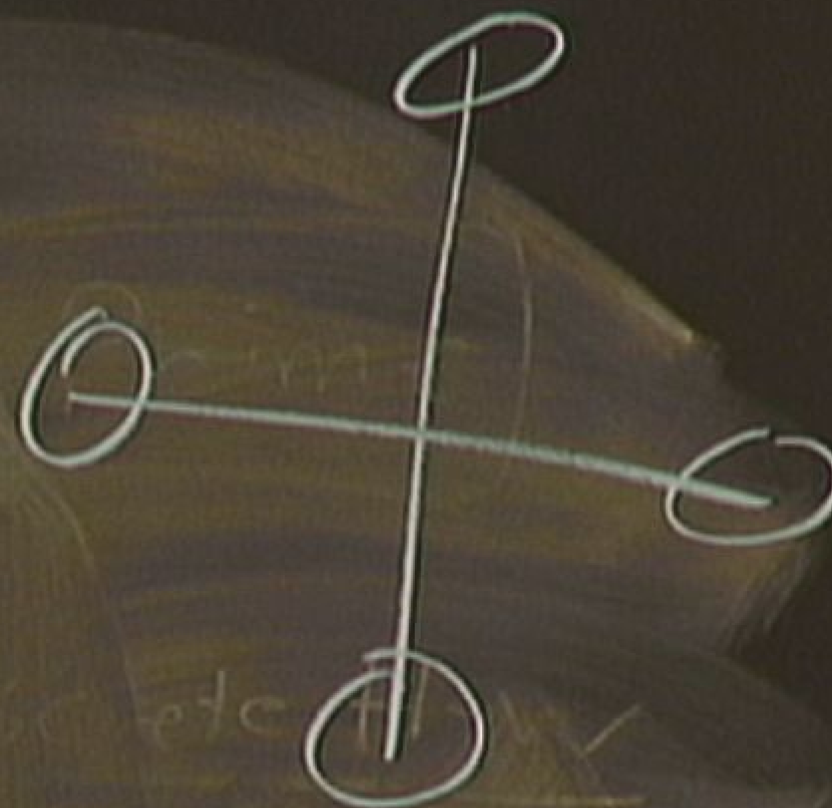
Let $|\psi\rangle = \xi|C$

$$|\langle\psi|T_{xp}|\psi\rangle|^2 = |\langle\psi|\psi_{xp}\rangle|^2 = 1/\alpha+1$$


const=
CAUTION


1/d+1
1

cteristic fn.



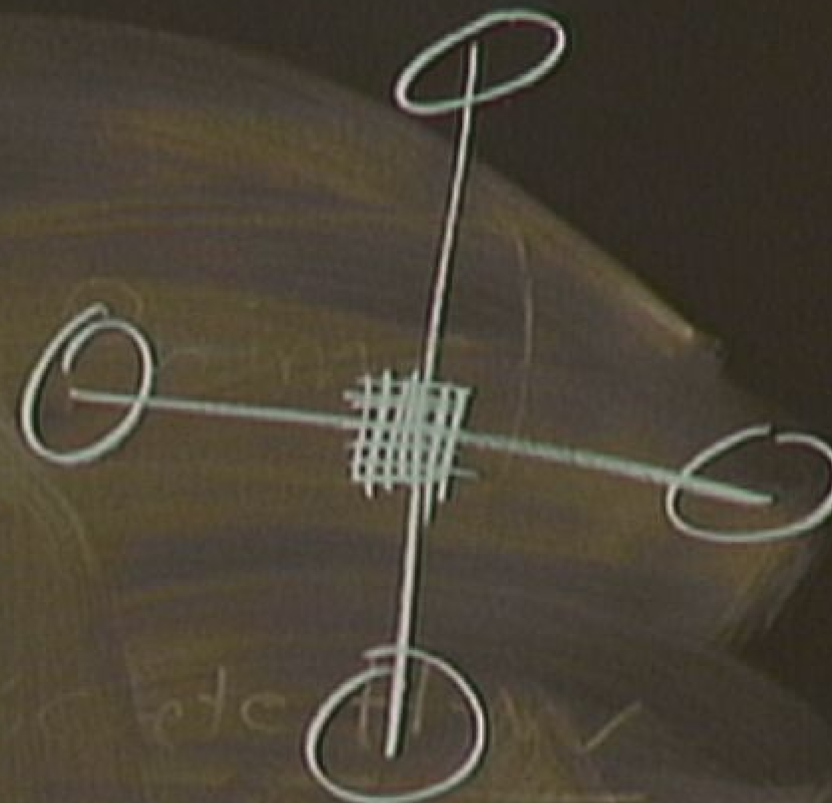
$$= e^{-(x^2+p^2)/\hbar}$$

$\psi\rangle = \sum |c.$

$|\psi\rangle|^2 = |\langle\psi|\psi_0\rangle|^2 = 1/$

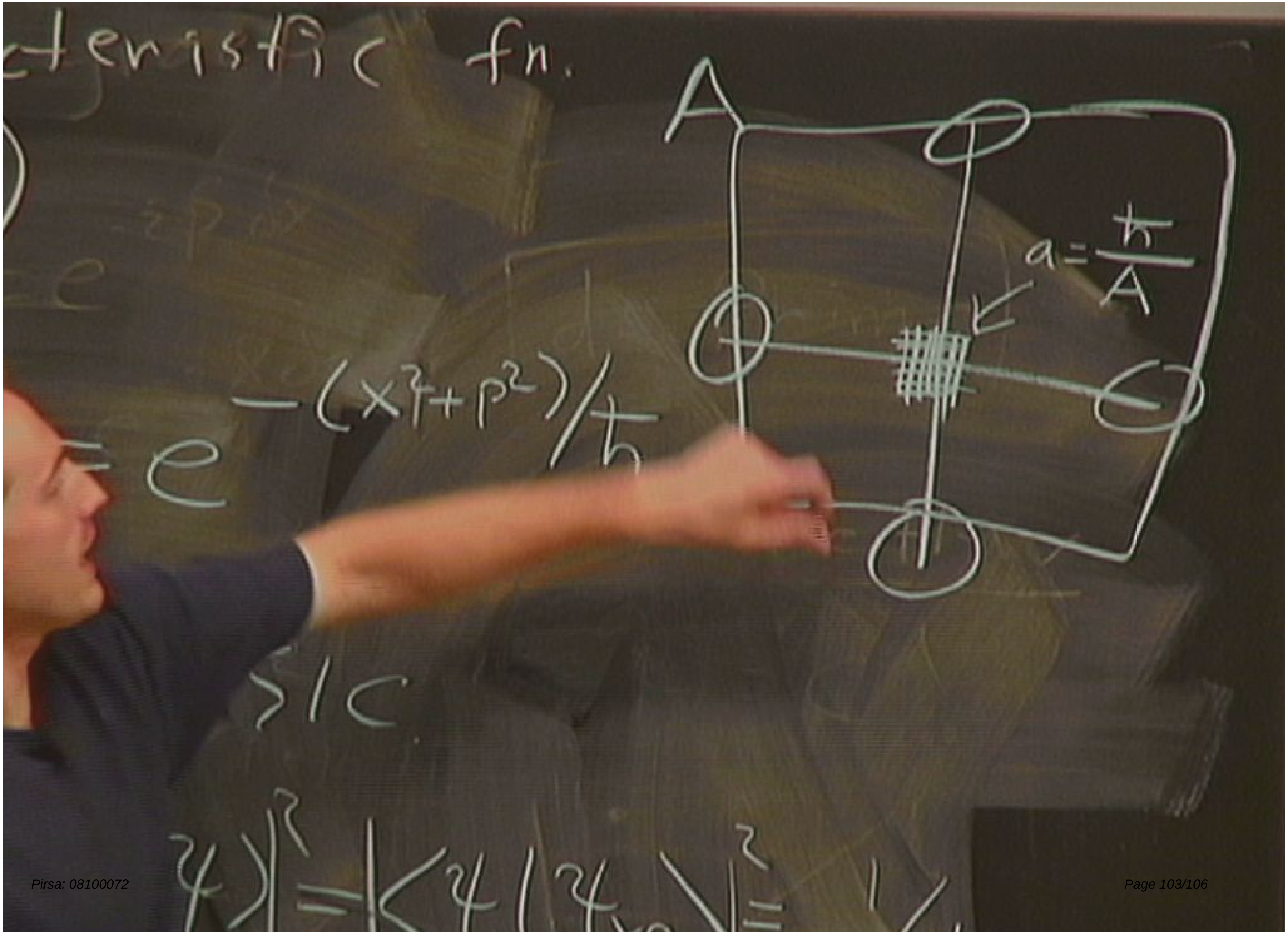
cteristic fn.

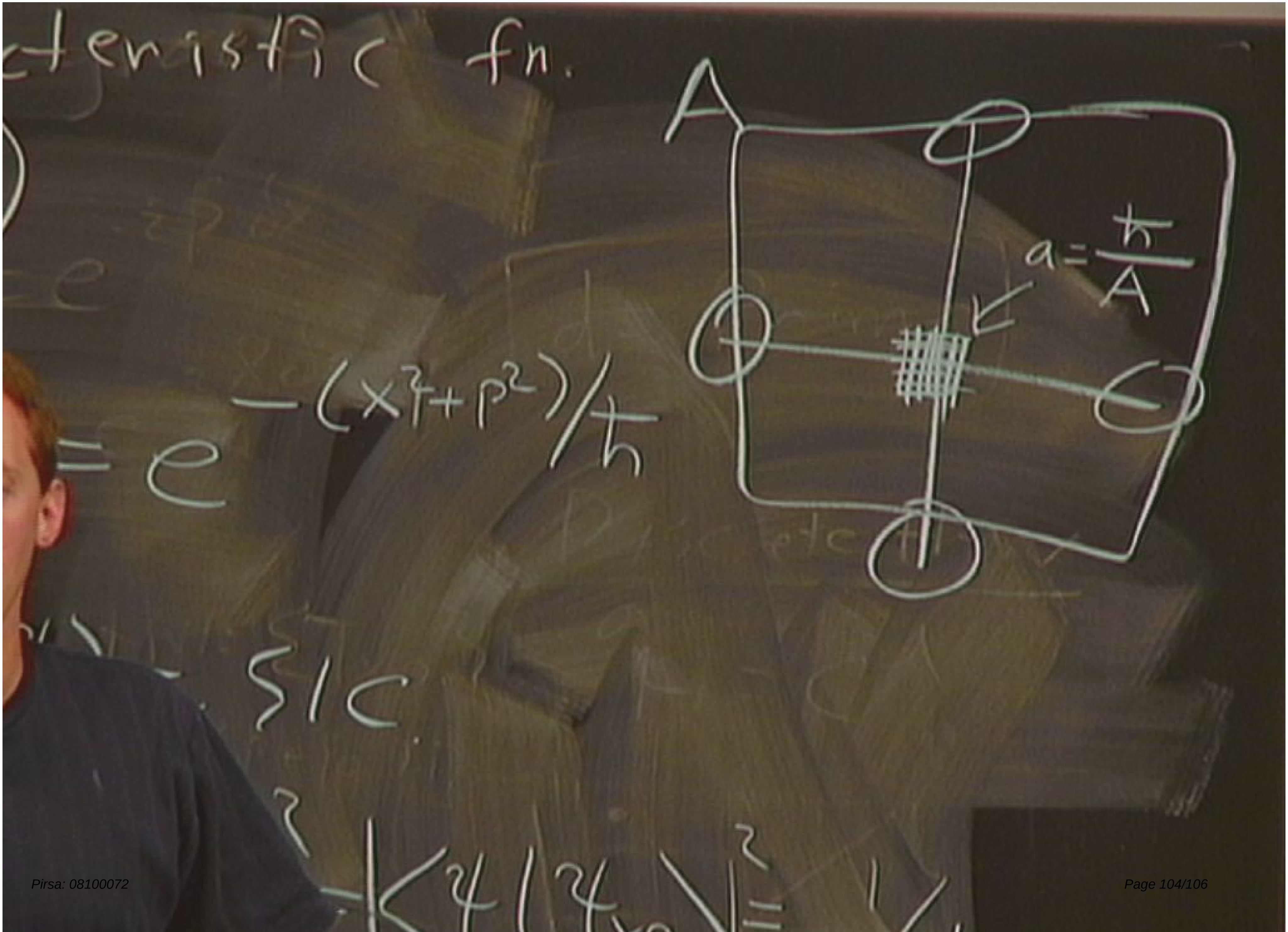
$= e^{-(x^2+p^2)/\hbar}$

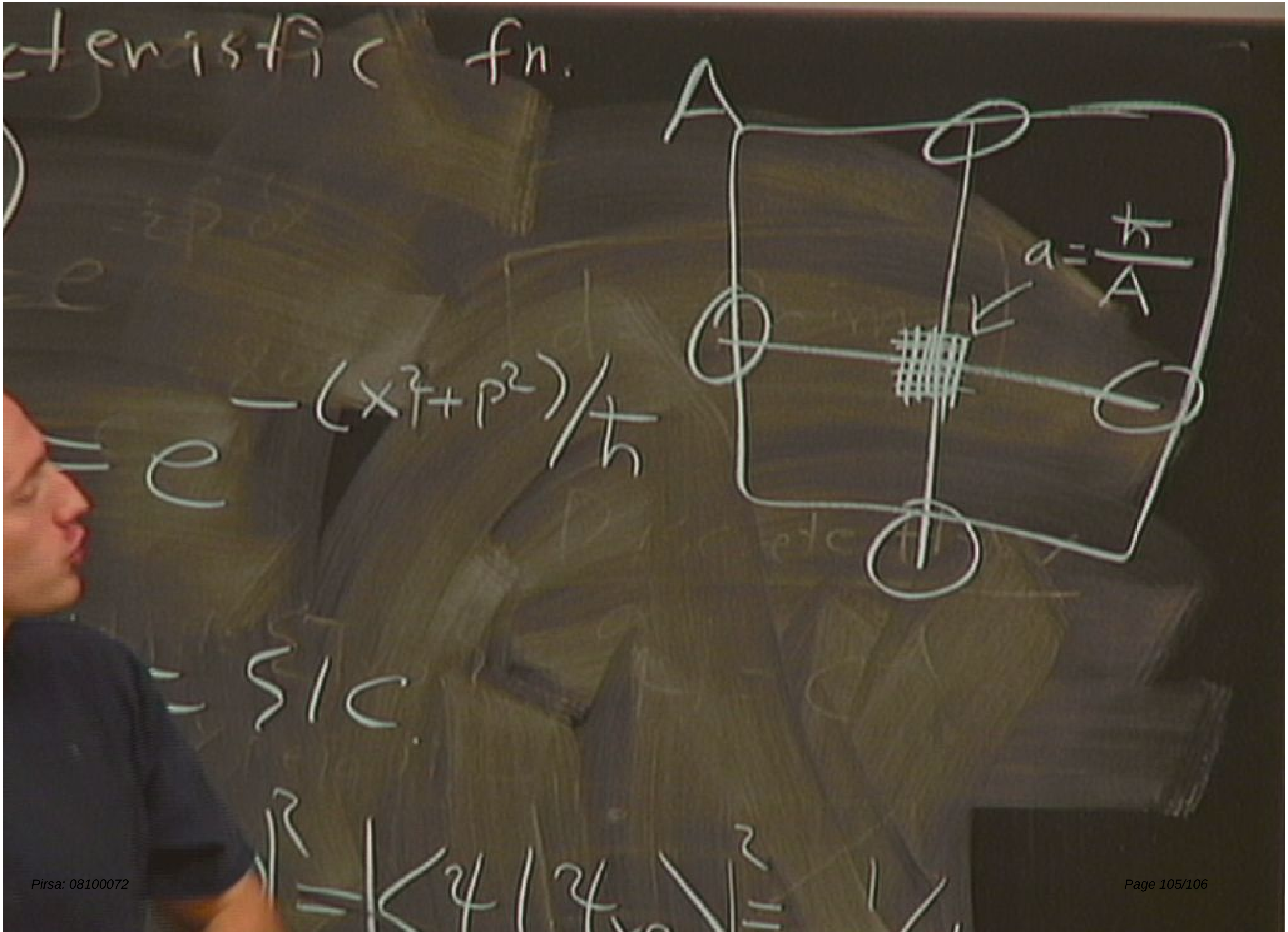


$\psi\rangle = \sum | c$

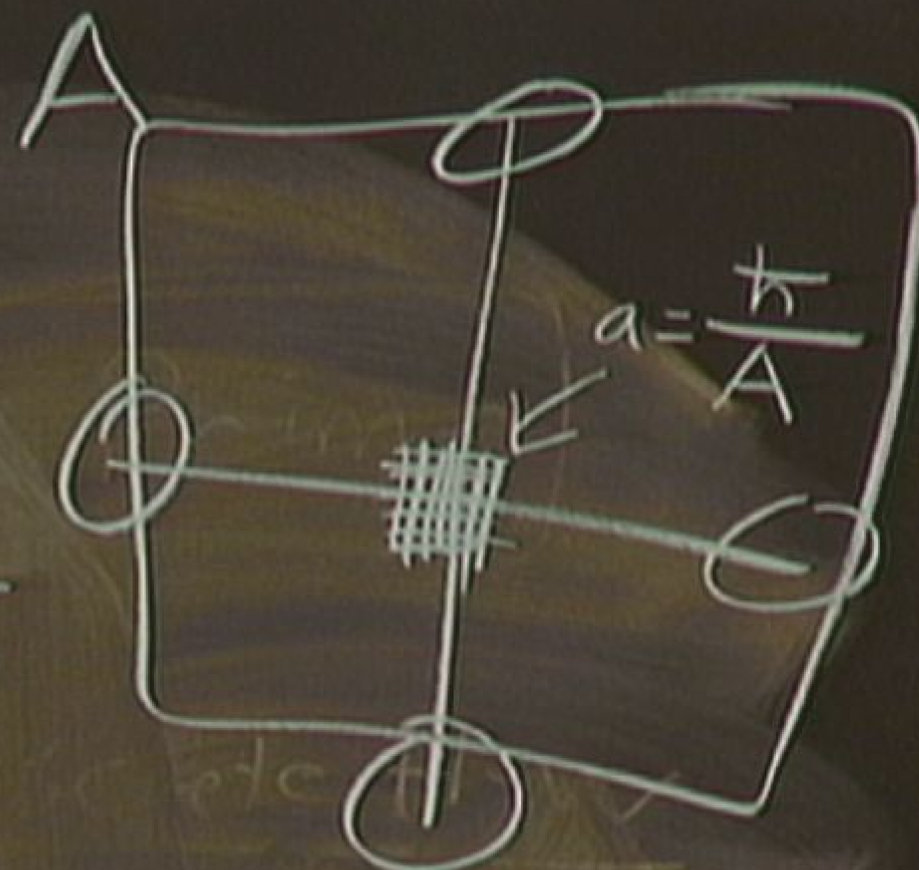
$\langle p|\psi\rangle|^2 = |\langle\psi|\psi_0\rangle|^2 =$


cteristic fn.
A
a = ħ/A


cteristic fn.
A
a = ħ/A
= e^{-(x²+p²)/ħ}
ξ/C


cteristic fn.
A
a = ħ/A
= e^{-(x²+p²)/ħ}

cteristic fn.



$$= e^{-(x^2+p^2)/\hbar}$$

$\psi\rangle = \sum | c$

$xp|\psi\rangle|^2 = |\langle\psi|\psi_{xp}\rangle|^2 =$