Title: Problems in higher genus superstring amplitudes

Date: Oct 09, 2008 02:00 PM

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Abstract: I would like to provide a short, possibly elementary, introduction to the problem of computing string amplitudes at higher genus for superstrings. Essentially, I will recall which is the mathematical problem in defining the path integral measure (which has a well defined algebraic geometry realization for bosonic strings) and the solution proposed by d~@~YHocker and Phong for the genus 2 case. Their main results are the chiral splitted form of the measure, and its explicit expression in genus two. They proposed the splitting form to work at any genus and assumed some restriction for the explicit form which however did not permitted them to find a solution for genera higher then 2. I will tell something about the technology which permitted us to find explicit solutions for genus 3 and four. Indeed, we showed that the restrictions imposed by d~@~YHocker and Phong have no solution whereas the most general form compatible with modular invariance and clustering provide a unique solution, at least for genus 3 and 4. I will try to be as less technical as possible.

$$X : \Sigma_g \longrightarrow \mathbb{R}^D$$

$$Z_{Bos} = \int \frac{Dg_{mn}\, DX\, e^{-I(X,g)}}{\mathrm{Vol}}$$

$$I = \frac{1}{4\pi} \int \sqrt{g}\, g^{mn}\, \partial_m X^\mu \partial_n X^\mu\, d^2\xi$$

$$X : \Sigma_g \longrightarrow \mathbb{R}^D \qquad G = \mathrm{Diff}(\Sigma) \times \mathrm{Weyl}(\Sigma)$$

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$$Z_{Bos} = \int \frac{Dg_{mn}\, DX\, e^{-I(X,g)}}{\mathrm{Vol}(G)} \qquad \mathcal{M}_g = \{g_{mn}\} / \mathrm{Diff}(\Sigma) \times \mathrm{Weyl}(\Sigma)$$

$$I = \frac{1}{4\pi} \int \sqrt{g}\, g^{mn}\, \partial_m X^\mu \partial_n X^\mu \, d^2\xi$$

$g_{z\bar{z}}\, dz\, d\bar{z}$

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Beltrami diff. $\mu_{\bar{z}}{}^{z}$ $(\bar{\mu}_{z}{}^{\bar{z}})$

$g_{z\bar{z}} \longrightarrow$

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Beltrami diff. $\mu_{\bar{z}}{}^{z}$ $(\bar{\mu}_{\bar{z}}{}^{z})$

$$g_{z\bar{z}} \longrightarrow g_{z\bar{z}} + \sum_i t^i \mu_{\bar{z}}{}^{z} g_{zz}$$

$g_{z\bar{z}}\, dz\, d\bar{z}$

Beltrami d.ff. $\mu_{\bar{z}}{}^{z}$ $(\bar{\mu}_{z}{}^{\bar{z}})$

$$g_{\bar{z}\bar{z}} \longrightarrow g_{z\bar{z}} + \sum_i t^i \mu_{\bar{z}}{}^{z} g_{zz}$$

$$\tilde{\mu}_{\bar{z}}{}^{z} - \mu_{\bar{z}}{}^{z} = \partial_{\bar{z}} \delta v^{z}$$

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Beltrami diff. $\mu_{\bar z}{}^{z}$ $(\bar\mu_{z}{}^{\bar z})$. $T_g M_g = \{\text{Beltrami}\}/$

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$g_{zz\bar{z}} \longrightarrow g_{z\bar{z}} + \sum t^i \mu_{\bar{z}}{}^{z} g_{zz}$ $\quad \bar{\partial}_m \psi_{\underbrace{z \ldots z}_{m}} = \partial_{\bar{z}} \psi_{z \ldots z}$

$\tilde{\mu}_{\bar{z}}{}^{z} - \mu_{\bar{z}}{}^{z} = \partial_{\bar{z}} \delta v^z$ $\quad \partial_{\bar{z}} \to \tilde{\partial}_{\bar{z}} + \mu_{\bar{z}}{}^{z} \partial_z$

Φ_{zz}

$$\langle \Phi | \mu \rangle = \int \Phi_{zz}\, \mu_{\bar z}{}^{z}\, d^2\xi$$

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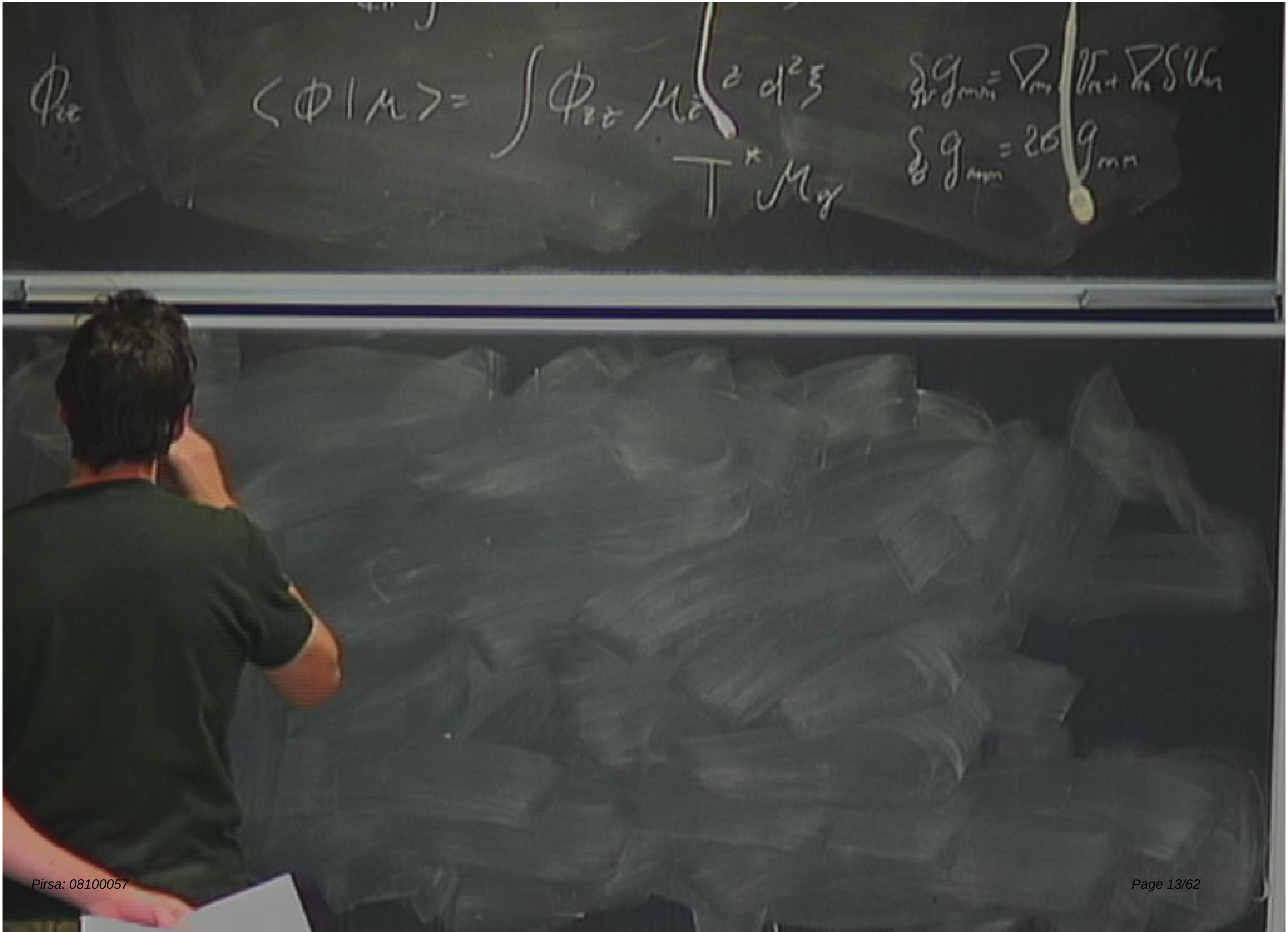
$$T^* \mathcal{M}_g$$

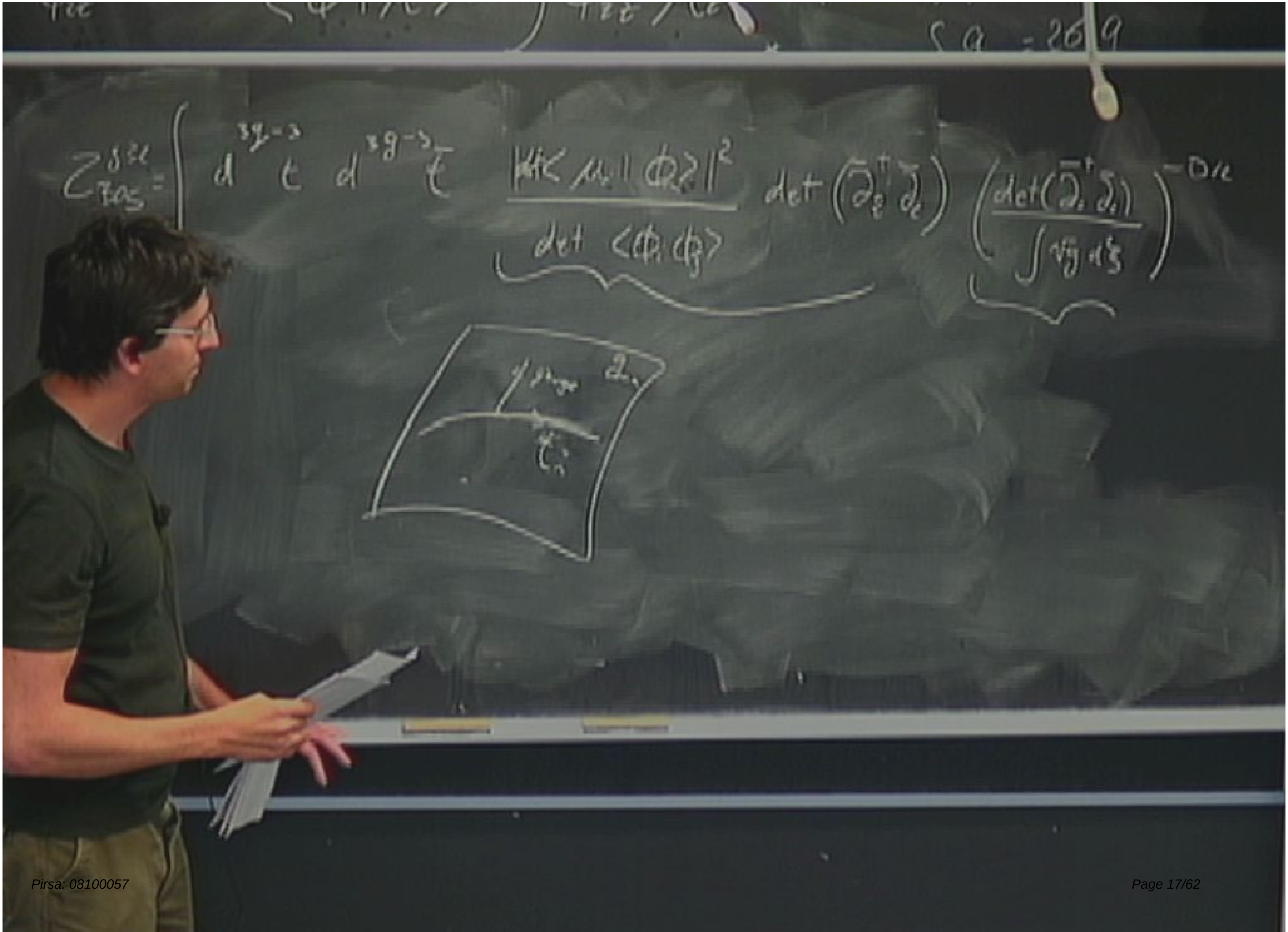

Φ_{zz}
$\langle \Phi | \mu \rangle = \int \Phi_{zz}\, \mu_{\bar z}^{\;z}\, d^2\xi$
$T^* \mathcal{M}_g$
$\delta g_{mn} = \nabla_m \delta v_n + \nabla_n \delta v_m$
$\delta g_{mn} = 2\delta\sigma\, g_{mn}$

$$Z_{Bos} = \int$$

g	$\dim \mathcal{M}_g$
0	0
1	1
$g \geq 2$	$3g-3$

$$T^{*}\mathcal{M}_{g} \qquad \delta g_{mn} = 2\sigma g_{mn}$$

$$Z^{g\geq 2}_{Bos} = \int d^{3g-3}t \, d^{3g-3}\bar{t} \, \frac{|\langle \mu | \phi \rangle|^2}{\det \langle \phi_i \phi_j \rangle} \det(\bar{\partial}_2^{\dagger} \bar{\partial}_2) \left(\frac{\det(\bar{\partial}_1^{\dagger} \bar{\partial}_1)}{\int \sqrt{g}\, d^2 \xi} \right)^{-D/2}$$



$$T^{*}\mathcal{M}_g \qquad \delta g_{mn} = 2\sigma g_{mn}$$

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ϕ_i

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ϕ_i

$$\delta g = t^i \mu_i + \epsilon \, g_{zz}$$

$$\int g^{z\bar z} g^{z\bar z} \Phi_{zz} \Phi_{\bar z\bar z} \sqrt{g}$$

$g_{z\bar z}\, dz\, d\bar z$

Beltrami diff. $\mu_{\bar z}{}^{z}$ $\left(\bar\mu_{z}{}^{\bar z}\right)$ $T_g M_g = \{\text{Beltrami}\}/\{\text{Im}\, \partial_{\bar z} v^z\}$

$g_{z\bar z} \longrightarrow g_{z\bar z} + \sum t^i \mu_{\bar z}{}^{z} g_{zz}$ $\quad \bar\partial_m \psi_{\underbrace{z\ldots z}_{m}} = \partial_{\bar z} \psi_{z\ldots z}$

$\tilde\mu_{\bar z}{}^{z} - \mu_{\bar z}{}^{z} = \partial_{\bar z} \delta v^z$ $\quad \partial_{\bar z} \to \tilde\partial_{\bar z} + \mu_{\bar z}{}^{z} \partial_z$

$$v_1^2 = GM$$

$$\delta_\sigma \log \frac{\det \bar{\partial}_m \partial_m}{\det \langle \phi_i^{(n)} | \phi_j^{(n)} \rangle \det \langle \phi^{(1-n)} \phi^{(1-n)} \rangle} = -\frac{6m^2 - 6m + 1}{6} \int d^2\xi \sqrt{g}\, R\, \delta\sigma$$

$v^2 = GM$

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$$-\frac{1}{6}\Big[\, 13$$

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$$-\frac{1}{6}\left[13-\frac{D}{2}\right]\int \ldots$$

$v_1^2 = GM$

$$\delta_\sigma \log \frac{\det \bar\partial_m \partial_m}{\det \langle \phi_i^{(m)} | \phi_j^{(m)} \rangle \det \langle \phi^{(1-m)} \phi^{(1-m)} \rangle} = -\frac{6m^2-6m+1}{6} \int d^2\xi \sqrt{g} R \delta\sigma$$

$D=26$

$$-\frac{1}{6}\left[13 - \frac{D}{2}\right] \int \ldots$$

$$v_{\phi}^2 = \frac{GM}{\ }$$

$$\partial_m \delta_m \log \frac{\det \bar{\partial}_m \partial_m}{\det \langle \phi_i^{(n)} | \phi_j^{(n)} \rangle \det \langle \phi^{(1-n)} \phi^{(1-n)} \rangle} = - \frac{6m^2 - 6m + 1}{12} \int d^2\xi \sqrt{g}\, \nabla_{\bar{z}} \mu \nabla_z \bar{\mu}$$

$D = 26$

$$-\frac{1}{6}\left[13 - \frac{D}{2} \right] \int \dots$$

$$v_t^2 = \frac{GM}{}$$

$$\delta_m \delta_{\bar m} \log \frac{\det \bar\partial_m \partial_m}{\det\langle \Phi_i^{(m)} | \phi_j^{(m)}\rangle \, \det\langle \bar\phi^{(1-m)} \bar\phi^{(1-m)}\rangle} = -\frac{6m^2-6m+1}{12}\int d^2\xi \sqrt{g}\, \nabla_{\bar z} \mu \nabla_{z}\bar\mu$$

$D=26$

$$-\frac{1}{6}\left[13 - \frac{D}{2}\right]\int \dots$$

I. Mumford ($\sim$ 1975)

$\mathcal{M}_g$

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$\mathcal{M}_g$ $\qquad K = (T^*\mathcal{M}_g)^{\otimes(3g-3)}$

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$\lambda =$

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$\mathcal{M}_g$

$$K = (T^{*}\mathcal{M}_g)^{\otimes(3g-3)}$$

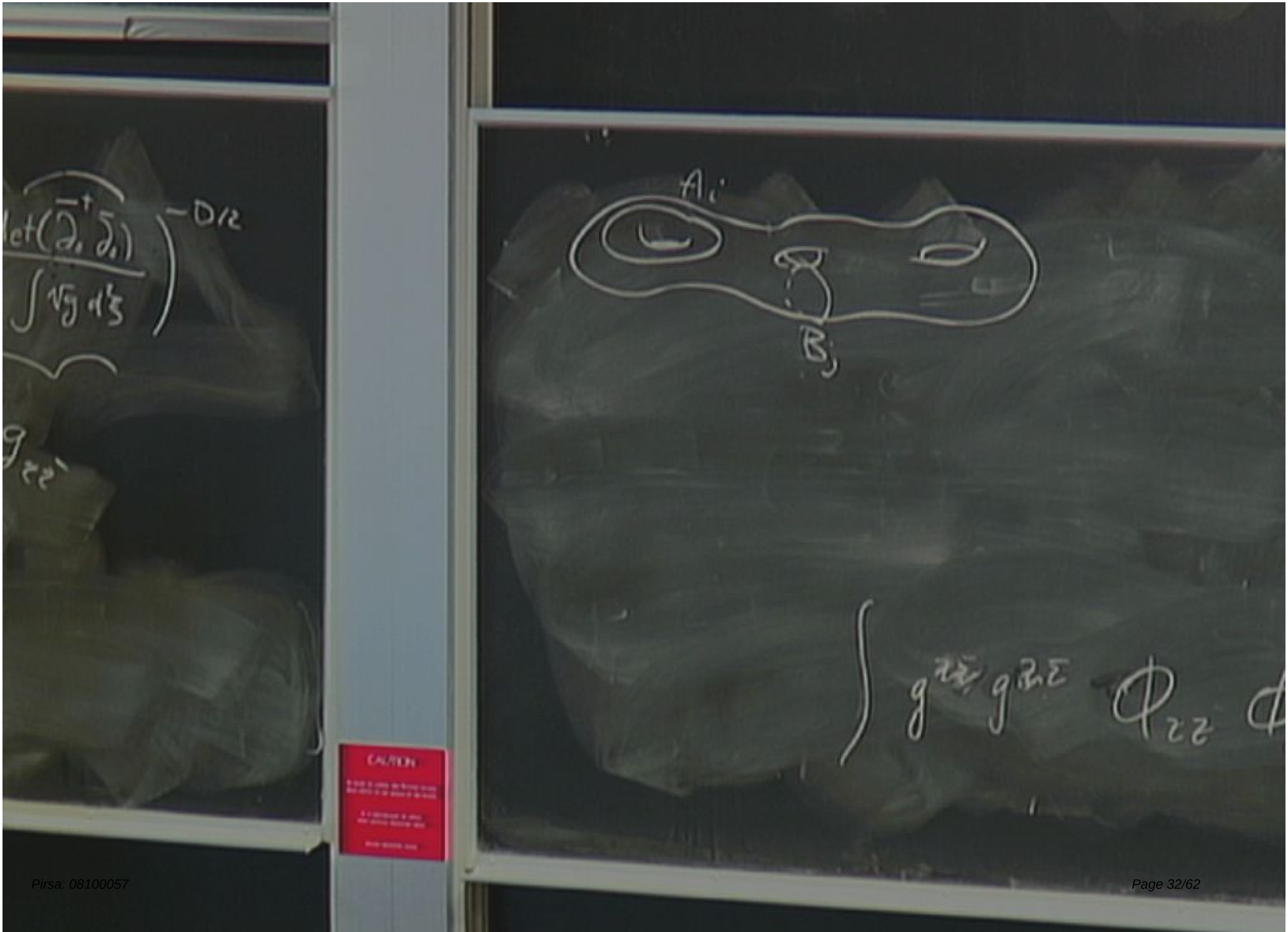
$$T^{*\,01}\mathcal{M}$$

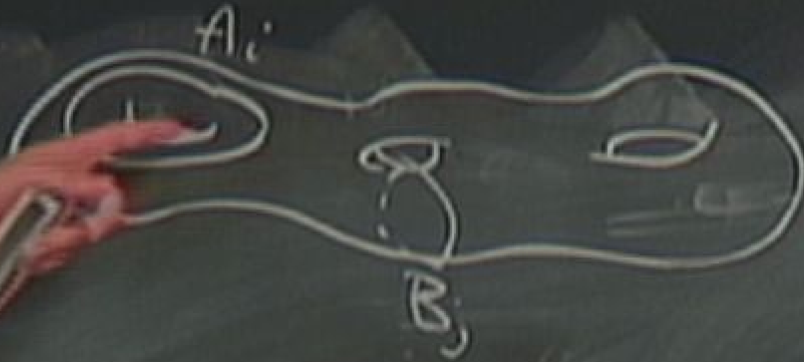
$$\lambda = (T^{*\,01}\mathcal{M}_g)^{g}$$

$\omega_I \quad I = 1, \dots g$

$$\eta_d = K \otimes \lambda^{-d}$$

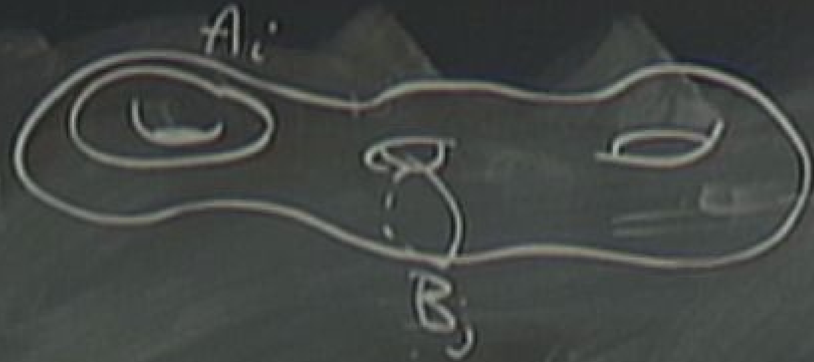
$d = 13$


-D/2
A_i
B_j



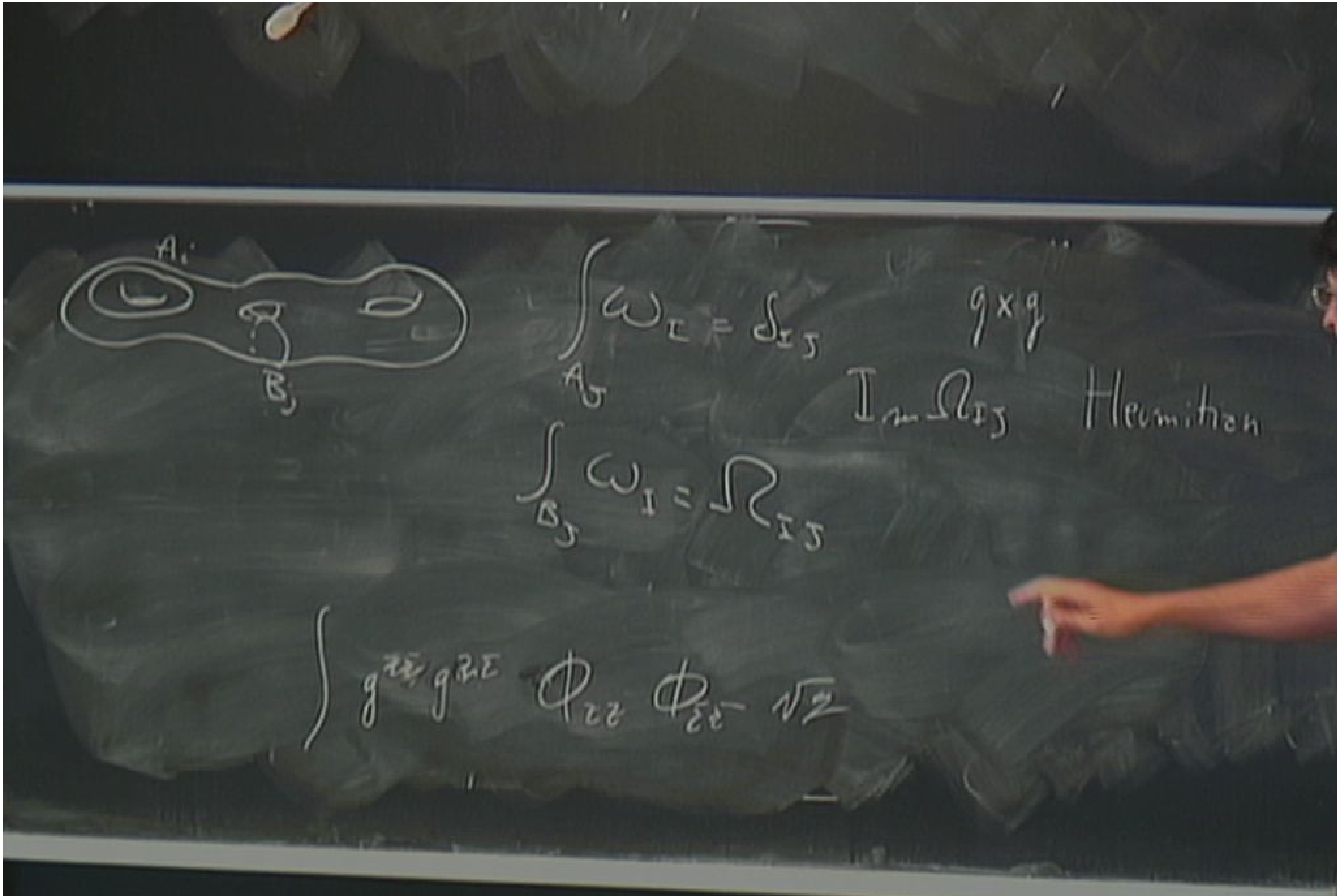
$$\int_{A_J} \omega_I = \delta_{IJ}$$

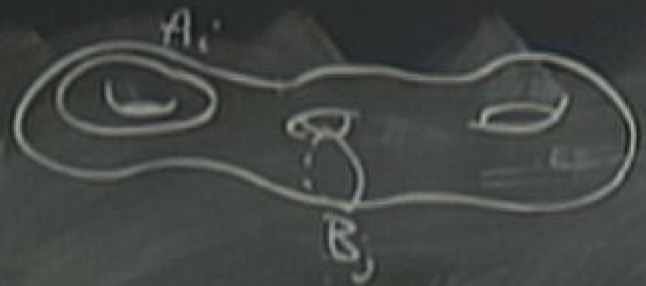
$$\int g^{z\bar{z}} g^{z\bar{z}} \Phi_{zz} \Phi_{\bar{z}\bar{z}} \sqrt{g}$$



$$\int_{A_J} \omega_I = \delta_{IJ}$$

$$\int g^{z\bar{z}} g^{\bar{z}z} \, \Phi_{zz} \, \Phi_{\bar{z}\bar{z}} \, \sqrt{g}$$


A_i
B_j
$\int_{A_J} \omega_I = \delta_{IJ}$
$g \times g$
$\mathrm{Im}\,\Omega_{IJ}$ Hermitian
$\int_{B_J} \omega_I = \Omega_{IJ}$
$\int g^{z\bar z} g^{z\bar z} \Phi_{zz} \Phi_{\bar z\bar z} \sqrt{g}$

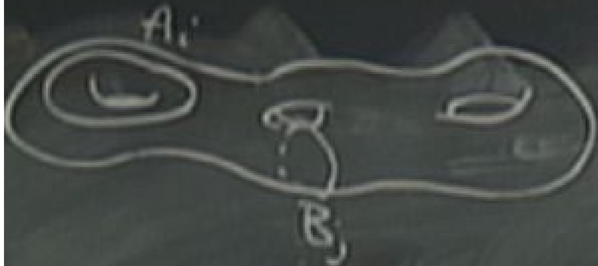


$$\int_{A_J} \omega_I = \delta_{IJ} \qquad g \times g$$

$$\mathrm{Im}\,\Omega_{IJ} \quad \text{Hermitian} >$$

$$\int_{B_J} \omega_I = \Omega_{IJ} \qquad \Omega_{IJ} \in H_g$$

$$\int g^{z\bar{z}} g^{z\bar{z}} \, \Phi_{zz} \Phi_{\bar{z}\bar{z}} \sqrt{g}$$

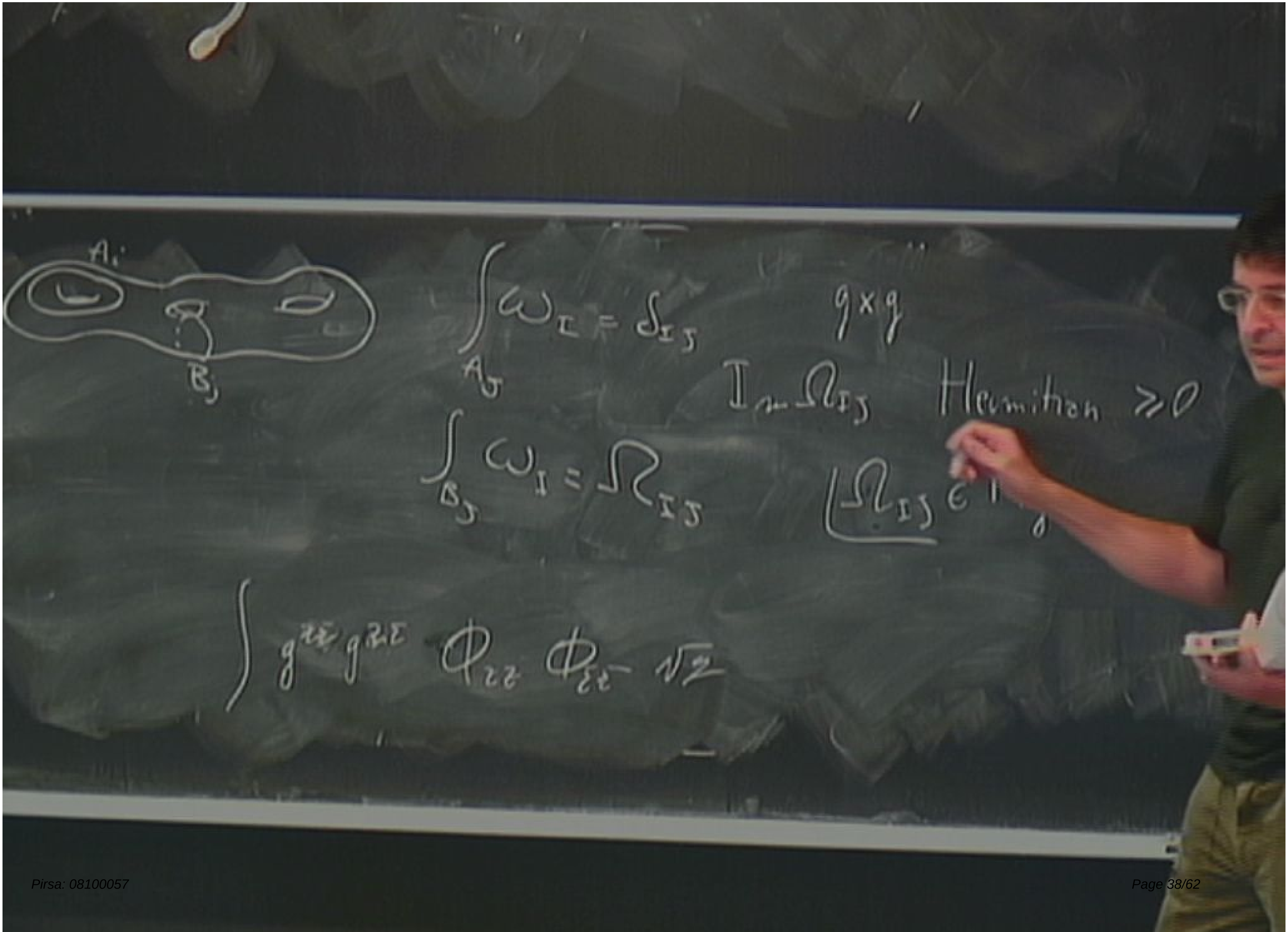


$$\int_{A_J} \omega_I = \delta_{IJ} \qquad g \times g$$

$$\mathrm{Im}\, \Omega_{IJ} \quad \text{Hermitian} \geq 0$$

$$\int_{B_J} \omega_I = \Omega_{IJ} \qquad \Omega_{IJ} \in H_g$$

$$\int g^{z\bar{z}} g^{z\bar{z}} \, \Phi_{zz} \, \Phi_{\bar{z}\bar{z}} \, \sqrt{g}$$



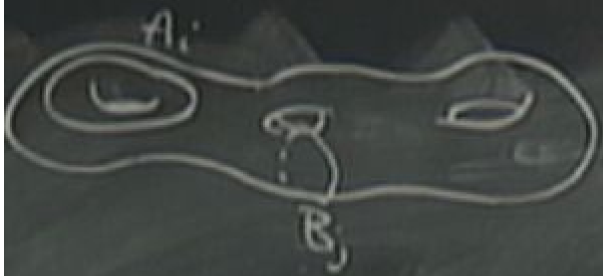
A_i
B_j
$\int_{A_J} \omega_I = \delta_{IJ}$
$g \times g$
$\mathrm{Im}\,\Omega_{IJ}$ Hermitian ≥ 0
$\int_{B_J} \omega_I = \Omega_{IJ}$
$\Omega_{IJ} \in$
$\int g^{z\bar{z}} g^{z\bar{z}} \Phi_{zz} \Phi_{\bar{z}\bar{z}} \sqrt{g}$

$$\int_{A_J} \omega_I = \delta_{IJ} \qquad g \times g$$

$$\mathrm{Im}\,\Omega_{IJ} \quad \text{Hermitian} \geq 0$$

$$\int_{B_J} \omega_I = \Omega_{IJ} \qquad \Omega_{IJ} \in H_g$$

$$\int g^{z\bar{z}} g^{z\bar{z}} \, \Phi_{zz} \, \Phi_{\bar{z}\bar{z}} \, \sqrt{g}$$



$$\int_{A_J} \omega_I = \delta_{IJ} \qquad g \times g$$

$$\mathrm{Im}\,\Omega_{IJ} \quad \text{Hermitian} \geq 0$$

$$\int_{B_J} \omega_I = \Omega_{IJ} \qquad \Omega_{IJ} \in H_g$$

$$\int g^{z\bar{z}} g^{z\bar{z}} \, \Phi_{zz} \, \Phi_{\bar{z}\bar{z}} \sqrt{g}$$

g	$\mathcal{M}_g$ ($3g-3$)	$\mathcal{H}_g$ ($\frac{g(g+1)}{2}$)
0	0	0
1	1	1
2	3	3
3	6	6
4	9	10

$$X = x + \theta \Psi + \bar{\theta} \psi + \dots$$

$$E = e + \theta \gamma \chi + \dots$$

$$s\mathcal{M} = \{\text{sup } \Sigma\} / s\mathrm{Weyl}(\Sigma) \times s\mathrm{Diff}(\Sigma)$$

$$(3g-3 \,|\, 2g-2)$$

$a, b \in \mathbb{Z}_2^{\,g}$

$$X = x + \theta\Psi + \bar{\theta}\psi + \dots$$

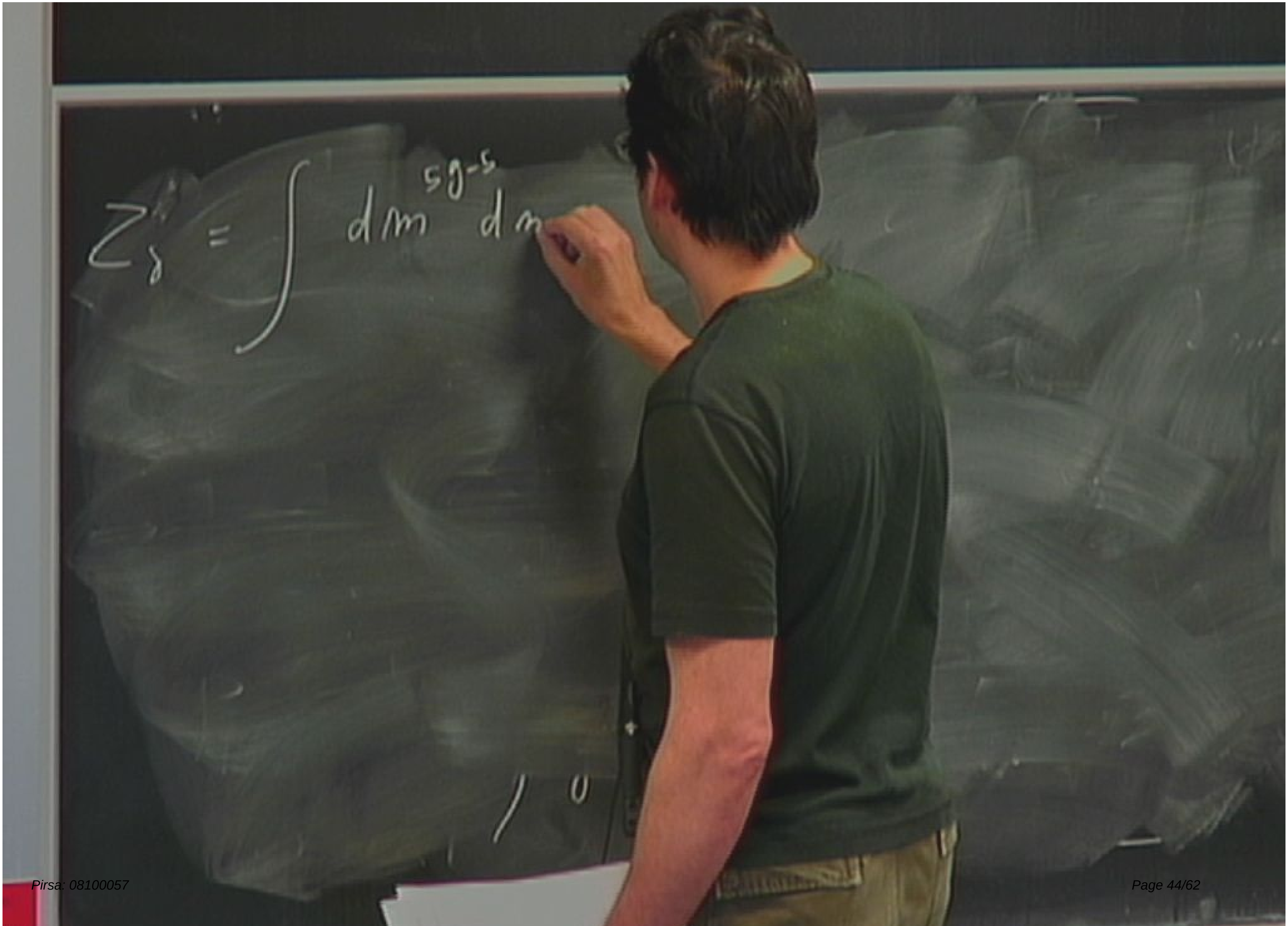
$$E = e + \theta\gamma\chi + \dots \qquad \delta = \begin{bmatrix} a \\ b \end{bmatrix}$$

e^{iq}

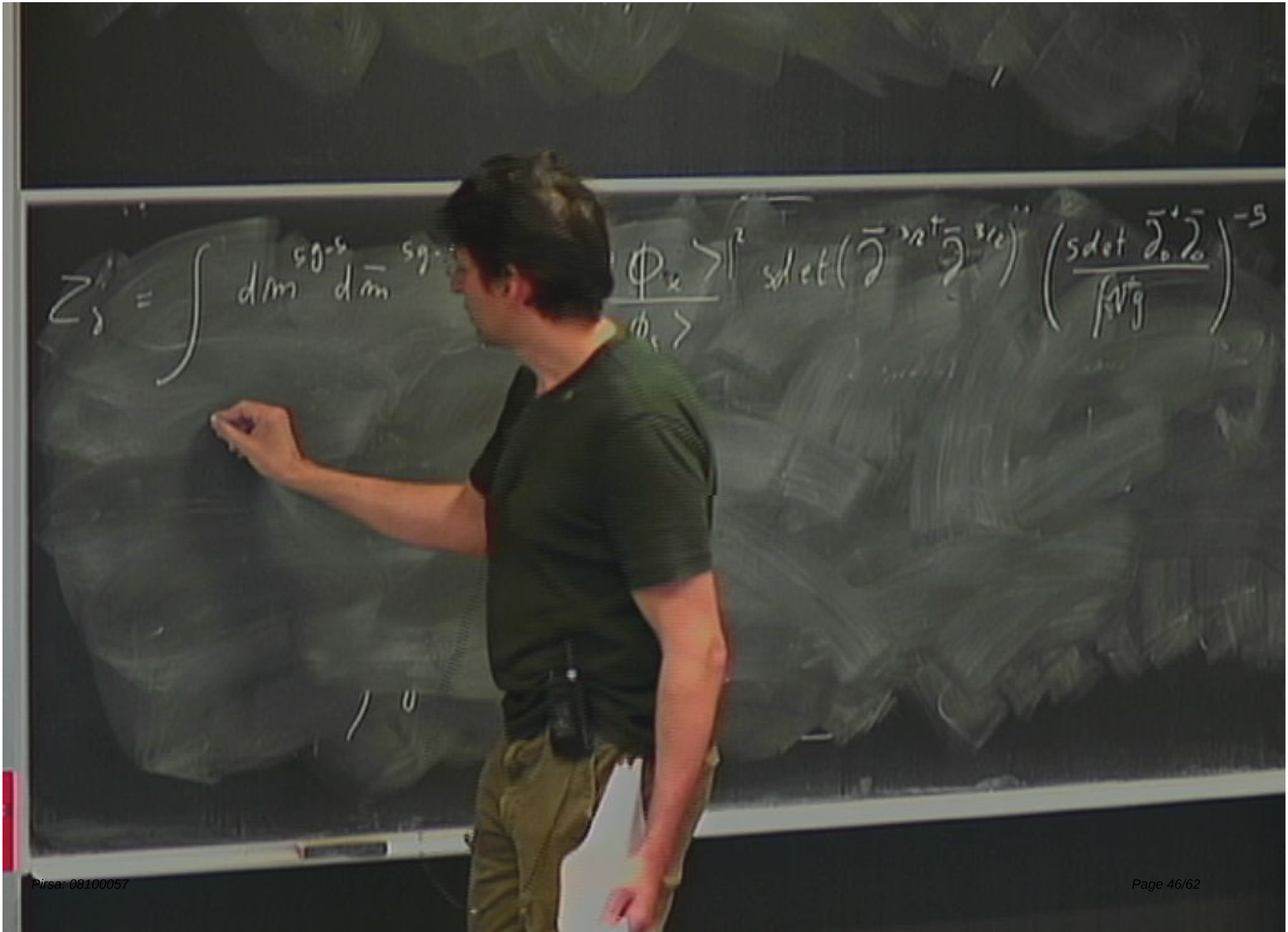
δ

$$s\mathcal{M} = \{\text{sup } E_\delta\} / s\mathrm{Weyl}(\Sigma) \times s\mathrm{Diff}(\Sigma)$$

$$(3g-3 \,\|\, 2g-2)$$


$Z_g = \int dm^{5g-5} \, dm$

$$Z_g = \int dm^{5g-5}\, d\bar{m}^{5g-g} \langle \mu$$

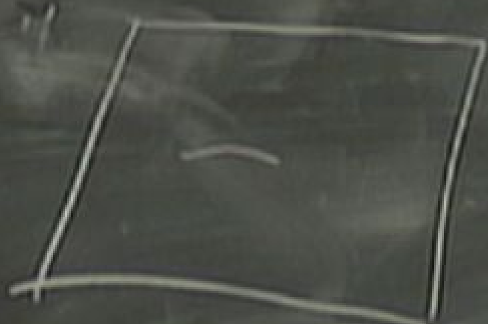


$$Z_g = \int dm^{5g-5}\, d\bar{m}^{5g-5}\, \frac{|\langle \mu_I | \Phi_x \rangle|^2}{\mathrm{sdet}\langle \Phi_i\, \Phi_j \rangle}\, \mathrm{sdet}\left(\bar{\partial}^{\dagger}_{3/2}\bar{\partial}_{3/2}\right)^{-1} \left(\frac{\mathrm{sdet}\, \bar{\partial}_0^{\dagger}\bar{\partial}_0}{\int \sqrt{g}}\right)^{-5}$$

$$Z_g' = \int dm^{5g-5}\, d\bar{m}^{5g-5}\, \frac{|\langle \mu_I | \Phi_\alpha \rangle|^2}{\mathrm{sdet}\langle \Phi_i\ \Phi_j \rangle}\, \mathrm{sdet}(\bar{\partial}^{-3/2\dagger}\bar{\partial}^{3/2}) \left(\frac{\mathrm{sdet}\ \bar{\partial}_0^\dagger \bar{\partial}_0}{\int \sqrt{g}}\right)^{-5}$$

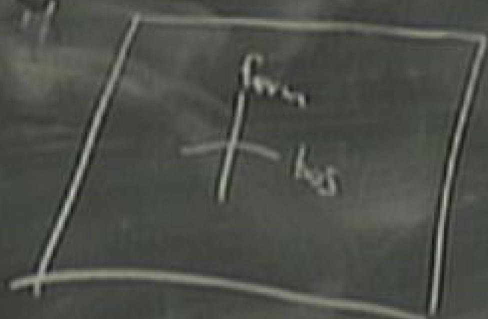
$d = 10$

$$Z_g = \int dm^{5g-5}\, d\bar{m}^{5g-5}\, \frac{|\langle \mu_I \,|\, \Phi_a \rangle|^2}{\mathrm{sdet}\langle \Phi_a\ \Phi_b \rangle}\, \mathrm{sdet}\left(\bar{\partial}_{3/2}^{\dagger}\bar{\partial}_{3/2}\right)\left(\frac{\mathrm{sdet}\,\bar{\partial}_0^{\dagger}\bar{\partial}_0}{\int\sqrt{g}}\right)^{-5}$$



$s\mathcal{M}_g$

$$Z_g = \int dm^{5g-5}\, d\bar{m}^{5g-5}\, \frac{|\langle \mu_I | \Phi_x \rangle|^2}{\mathrm{sdet}\langle \Phi_i\, \Phi_j \rangle}\, \mathrm{sdet}\left(\bar{\partial}_{3/2}^{\dagger}\bar{\partial}_{3/2}\right) \left(\frac{\mathrm{sdet}\, \bar{\partial}_0^{\dagger}\bar{\partial}_0}{\int \sqrt{g}}\right)^{-5}$$



$s\mathcal{M}_g$

$$Z_g' = \int dm^{5g-5}\, d\bar{m}^{5g-g}\, \frac{|\langle \mu_I | \Phi_\alpha \rangle|^2}{\mathrm{sdet}\langle \Phi_i\, \Phi_j \rangle}\, \mathrm{sdet}\left(\bar{\partial}^{3/2\,\dagger}\bar{\partial}^{3/2}\right)\left(\frac{\mathrm{sdet}\, \bar{\partial}_0^{\dagger}\bar{\partial}_0}{\int\sqrt{g}}\right)^{-5}$$

$s\mathcal{M}_g$

$$Z_g = \int dm^{5g-5}\, d\bar{m}^{5g-5}\, \frac{|\langle \mu_I | \Phi_\alpha \rangle|^2}{\mathrm{sdet}\langle \Phi_\alpha\, \Phi_\beta \rangle}\, \mathrm{sdet}\left(\bar{\partial}_{3/2}^{\dagger}\bar{\partial}_{3/2}\right) \left(\frac{\mathrm{sdet}\, \bar{\partial}_0^{\dagger}\bar{\partial}_0}{\int \sqrt{g}}\right)^{-5}$$

$s\mathcal{M}_g \quad \mu_I$

$$
\begin{tikzcd}
(g_{mn}, x) \arrow[r] \arrow[d] & (\tilde{g}_{mn}, \tilde{x}) \arrow[d] \\
g_{mn} \arrow[r, leftrightarrow] & \hat{g}_{mn}
\end{tikzcd}
$$

$$\hat{\Omega}_{IJ} = \Omega_{IJ} + \int S \ldots$$

$$\begin{tikzcd}
\hat{\Omega} \arrow[r] \arrow[d] & \hat{\Omega} \arrow[d] \\
\Omega \ldots \arrow[r] & \Omega \ldots
\end{tikzcd}$$

$$Z_g = \int dm^{5g-5}\, d\bar{m}^{5g-5}\, \frac{|\langle \mu_I | \Phi_x \rangle|^2}{\mathrm{sdet}\langle \Phi_i\, \Phi_j \rangle}\, \mathrm{sdet}(\bar{\partial}^{3/2\dagger}\bar{\partial}$$

$$Z_g = c_g \int c_{\Delta\Delta'}\, d\mu_\Delta \wedge d\bar{\mu}_{\Delta'}$$

$$Z_g^{1} = \int dm^{5g-5}\, d\bar{m}^{5g-5} \frac{|\langle \mu_I \,|\, \Phi_\alpha \rangle|^2}{\mathrm{sdet}\langle \Phi_I\, \Phi_J \rangle}\, \mathrm{sdet}(\bar{\partial}_{3/2}^{\dagger} \bar{\partial}_{3/2}) \left(\frac{\mathrm{sdet}\, \bar{\partial}_0^{\dagger} \bar{\partial}_0}{\int \sqrt{g}} \right)^{-5}$$

$$d\mu_{\Delta}\; d\mu_{B}\; \bar{\zeta}(\Delta)[\Omega]$$

$$Z_g = C_g \int \Xi_{\Delta\Delta'}\, d\bar{\mu}_{\Delta}' \wedge d\mu_{\Delta} \frac{1}{\det(\mathrm{Im}\,\Omega)^5}$$

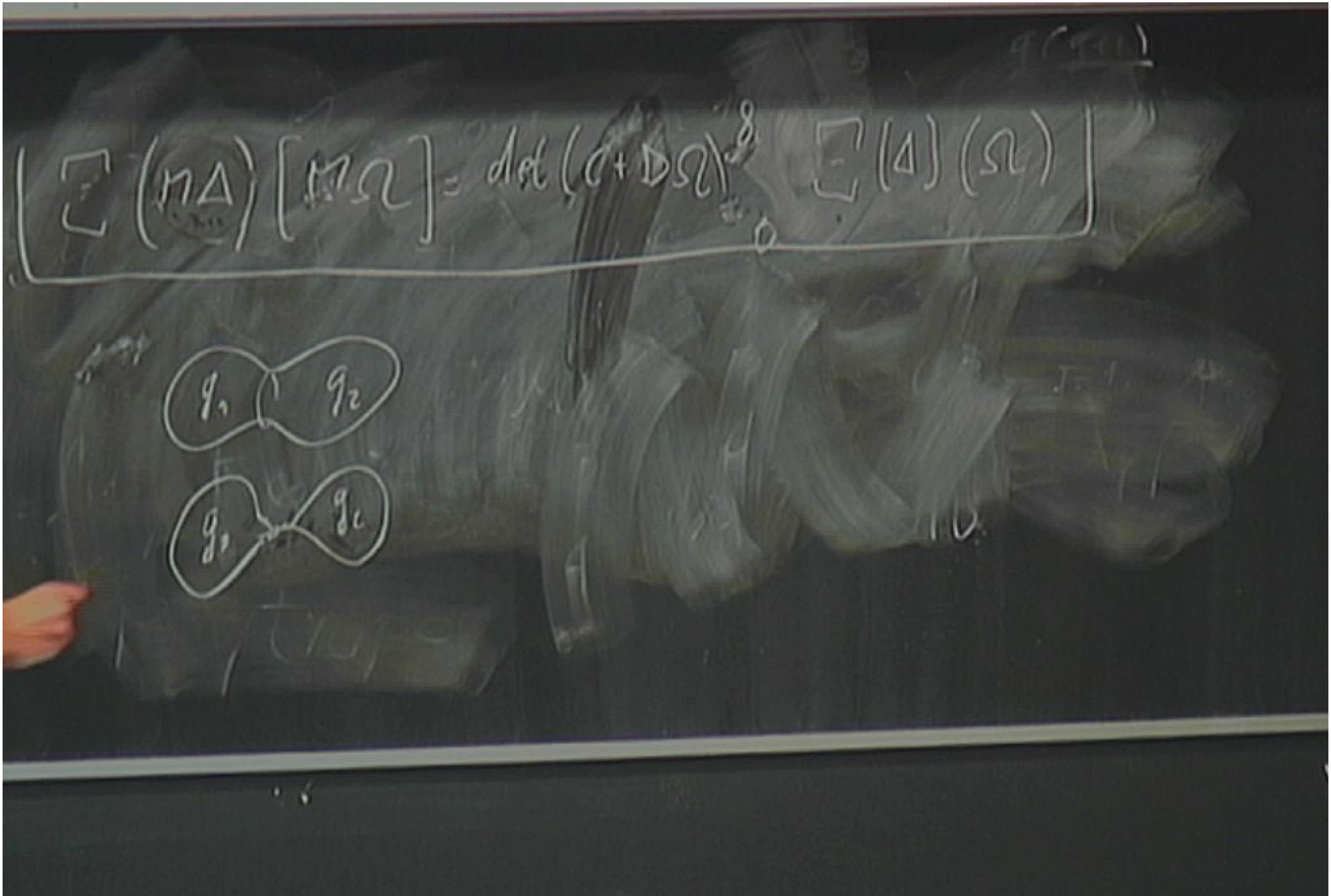
$$Z_g^1 = \int dm^{5g-5}\, d\bar{m}^{5g-5} \frac{|\langle \mu_I | \Phi_\alpha \rangle|^2}{\mathrm{sdet}\langle \Phi_i\, \Phi_j \rangle}\, \mathrm{sdet}(\bar{\partial}^{3/2\,\dagger} \bar{\partial}^{3/2}) \left(\frac{\mathrm{sdet}\, \bar{\partial}_0^{\dagger} \bar{\partial}_0}{\int \sqrt{g}} \right)^{-5}$$

$$d\mu_{\Delta_0}\; d\mu_\delta \quad \overline{\Xi}(\Delta)[\Omega]$$

$$Z_g = C_g \int \zeta_{\Delta\Delta'}\, d\bar{\mu}_{\Delta}' \wedge d\mu_{\Delta}\, \frac{1}{\det(\operatorname{Im}\Omega)^5}\, \mathcal{J}_P(2g, Z)$$

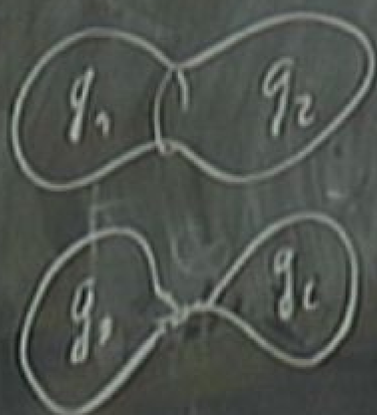
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\boxed{\Xi(n\Delta)[n\Omega] = \det(C+D\Omega)^{\delta}\,\Xi(\Delta)(\Omega)}$$



$Z(n\Delta)[n\Omega] = \det(C + D\Omega)^{\delta}\, Z(\Delta)(\Omega)$
g_1 g_2
g_1 g_2

$$\boxed{\Xi(n\Delta)[n\Omega] = \det(C + D\Omega)^{8}\, \Xi[\Delta](\Omega)}$$



$\hat{\varepsilon}$ $\hat{\mu}$

$\hat{\omega}$ $\boxed{\hat{\Omega}}$

$\hat{\Sigma}$ $\hat{\mu}$

$\hat{\omega}$ $\boxed{\hat{\Omega}}$