Title: Gauge-Invariant Summation of All QCD Virtual Gluon Exchanges

Date: Oct 31, 2008 01:00 PM

URL: http://pirsa.org/08100052

Abstract: The interpretation of virtual gluons as ghosts in the non-linear gluonic structure of QCD permits the formulation and realization of a manifestly gauge-invariant and Lorentz covariant theory of interacting quarks/antiquarks, for all values of coupling. The simplest example of quark/antiquark scattering in a high-energy, quenched, eikonal model at large coupling is shown to be expressable as a set of finite, local integrals which may be evaluated numerically; and before evaluation, it is clear that the result will be dependent only on, and damped by increasing momentum transfer, while displaying physically-reasonable color dependence in a manner underlying the MIT Bag Model and an effective, asymptotic freedom. These results are compatible with an earlier, instanton, field-strength analysis of Reinhardt, et.al.

Virtual Gluon – Ghost QCD

(by HMF, Y. Gabellini, Th. Grandou, Y.-M. Sheu)

- A new approach to QCD: Combination of older, non-perturbative, field-strength formalism with eikonal models for scattering and production. The essential adjectives are: NP, MGI, MLC.

- How this approach began: Abelian eikonal models generalized to non-Abelian QCD (J. Avan, HMF, Y. Gabellini)

→ Essentially a combinatoric problem:
NP, Abelian theories → coherent exchange of neutral (virtual) quanta; but coherence is lost if ∃ isotopic/color restrictions of absorption and emission by relevant fermions.

- Why study Eikonal Models?

→ Definition of "Eikonal Model": When 4-momentum k of every real or virtual quanta emitted or absorbed by relevant fermion $\ll p_\mu$ of fermion (in rel. sense: $|p\cdot k| \gg k^2$, or $|k_i \cdot k_j|$).

→ Began as "soft-photon" Physics, e.g., removal of all IR divs. in QED.

→ For 30 years, ∃ed Strong-Coupling Abelian Physics; and its zeroth approximation is the Eikonal Model. Systematic corrections defined years ago (HMF, Th. Grandou). Eikonal models are Unitary, with agreement with exp. data across 30 years, from electromagnetic form factors of the nucleon to the highest energy p-p, π-p, p̄-p scattering. (HMF, T. Gasser; Cheng + Wu; Bourrely, Soffer, and Wu)

• **In QED**: No $\mathcal{L}'$, no color indices, $F^2 \to f^2$.

→ "Proper quantization" of QED in Coulomb gauge: Not MLC.
$A_i(x)$ dynamical variables, $A_0(x)$ given by constraint eq.

→ To obtain QED in MLC form:

a) follow Schwinger, adjoin operator gauge transfs, so that all Green's functions take on MLC form; or:

b) as here: Replace exact: $-\frac{1}{4}\int f^2 = -\frac{1}{2}\int(\partial_\nu A_\mu)^2 + \frac{1}{2}\int(\partial_\mu A_\mu)^2$
by gauge-dependent:

$$-\frac{1}{4}\int f^2 = -\frac{1}{2}\int(\partial_\nu A_\mu)^2 + \frac{\lambda}{2}\int(\partial_\mu A_\mu)^2,$$

and treat the $\frac{\lambda}{2}\int(\partial_\mu A_\mu)^2$ term as an effective, $g=0$, "interaction": Starting from the $\lambda=0$ formalism, the new, free-photon generating functional is given by

$$\mathcal{Z}_0^{(\lambda)}[j] = \underbrace{e^{\frac{i\lambda}{2}\int(\partial_\mu A_\mu)^2}\Big|_{A \to \frac{1}{i}\frac{\delta}{\delta j}}}_{e^{-\frac{i}{2}\int\frac{\delta}{\delta j_\mu}(-\lambda\partial_\mu\partial_\nu)\frac{\delta}{\delta j_\nu}}} \cdot \underbrace{\mathcal{Z}_0^{(0)}[j]}_{e^{\frac{i}{2}\int j_\mu D^{(0)}_{c,\mu\nu} j_\nu}}, \quad D^{(0)}_{c,\mu\nu} = \delta_{\mu\nu} D_c^{(0)},$$

where $D_c^{(0)}(x-y) \Rightarrow$ free-photon Feynman gauge prop.

$$\therefore \mathcal{Z}_0^{(\lambda)}[j] \Rightarrow e^{\frac{i}{2}\int j_\mu D^{(\rho)}_{c,\mu\nu} j_\nu} \cdot e^{-\frac{1}{2}\mathrm{Tr}\ln(1+\lambda\partial\partial D_c^{(0)})},$$

where: $\tilde{D}^{(\rho)}_{c,\mu\nu}(k) = \frac{1}{k^2-i\epsilon}\left[\delta_{\mu\nu} - \rho\frac{k_\mu k_\nu}{k^2-i\epsilon}\right], \quad \rho = \frac{\lambda}{\lambda-1};$

and: $\mathrm{Tr}\ln(1+\lambda\partial\partial D_c^{(0)}) \to \mathrm{Tr}\ln(1-\lambda)$.

Clearly, any value of λ is acceptable, except $\lambda = 1$ ...
because, in QED, MGI is incompatible with MLC.

$\therefore$ In QED, $\mathfrak{Z}^{(\lambda)}[j,\eta,\bar{\eta}] = N\, e^{i\int \bar{\eta}\, G_c[A]\,\eta + L[A]}\Big|_{A \to \frac{1}{i}\frac{\delta}{\delta j}} \cdot e^{\frac{i}{2}\int j\, D_c^{(\lambda)}\, j}$,

with the choice of gauge, λ or $\xi = \frac{\lambda}{\lambda - 1}$, specified at the outset.
Here, no MGI; but GI can be shown, indirectly ...

In QCD, because the $g = 0$ theory is $\Leftrightarrow$ as QED (with a color index), do the same thing:

$$\mathfrak{Z}[j,\eta,\bar{\eta}] \to N'\, e^{i\int \bar{\eta}\, G_c[A]\,\eta + L[A]} \cdot e^{i\int \mathcal{L}'[A]}\Big|_{A \to \frac{1}{i}\frac{\delta}{\delta j}} \cdot e^{\frac{i}{2}\int j^a_\mu\, D_{c\,\mu\nu}^{(\lambda)\,ab}\, j^b_\nu},$$

with the $D_c^{(\lambda)}$, and $\therefore$ the choice of gauge, specified initially.
(Can be generalized to other gauges, e.g., the axial gauges.)
$\to$ $\therefore$ In this form, no Gribov copies to worry about!

Both forms satisfy MLC, but not MGI ...

• A Useful Rearrangement, $\Rightarrow$ to Schwinger solns, but more easily understood:

$$\mathcal{F}\left[\frac{1}{i}\frac{\delta}{\delta j}\right] e^{\frac{i}{2}\int j D_c j} = e^{\frac{i}{2}\int j D_c j} \cdot e^{-\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}}\, \mathcal{F}[A]\Big|_{A = \int D_c j}.$$

Easy to prove!

Then: $\mathfrak{Z}_{QED}[j,\eta,\bar{\eta}] \to N\, e^{\frac{i}{2}\int j D_c j} \cdot \underbrace{e^{-\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}}}_{\text{(all virtual photon fluctuations)}} \cdot \underbrace{e^{i\int \bar{\eta}\, G_c[A]\,\eta + L[A]}}_{\text{(rel. Pot. Theory)}}\Big|_{A \to \int D_c j}$, and:

$$Z_{QCD}[j,\eta,\bar{\eta}] \to N' e^{\frac{i}{2}\int j D_c j} \cdot e^{-\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}} \cdot e^{i\int \mathcal{L}(A)}\, e^{i\int \bar{\eta} G_c(A) \eta + L(A)} \Big|_{A = \int D_c j} .$$

Here, $e^{-\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}} \equiv$ "Linkage operator" $\Rightarrow e^{\mathfrak{D}_A}$.

Its main feature is easy to see:

$$e^{\mathfrak{D}_A}\, \mathcal{G}_I[A] \cdot \mathcal{G}_{II}[A] = \left(e^{\mathfrak{D}_A} \mathcal{G}_I[A]\right) e^{\overleftrightarrow{\mathfrak{D}}_A} \left(e^{\mathfrak{D}_A} \mathcal{G}_{II}[A]\right),$$

where $\overleftrightarrow{\mathfrak{D}}_A \equiv -i \int \overleftarrow{\frac{\delta}{\delta A}} D_c \overrightarrow{\frac{\delta}{\delta A}}$.

Obvious generalization to multiple products ...

E.G., in QED:

$$N^{-1} = e^{\mathfrak{D}_A} \cdot e^{L[A]} \Big|_{A\to 0} . \to \langle S \rangle$$

$S_c'(x,y) = N e^{\mathfrak{D}_A} \cdot G_c(x,y|A) e^{L[A]} \Big|_{A\to 0}$; and in quenched approximation,

$S_c'(x,y) \to e^{\mathfrak{D}_A} G_c(x,y|A) \Big|_{A\to 0}$ $\qquad L \to 0,\ N \to 1,$

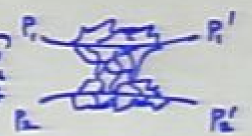
For pp scattering:

$$= N \cdot e^{\mathfrak{D}_A} G_c^{I}(x_1,y_1|A)\, G_c^{II}(x_2,y_2|A)\, e^{L[A]} \Big|_{A\to 0},$$

(plus $x_1 \leftrightarrow x_2$ asym)

and in quenched approx.:

$$\left(e^{\mathfrak{D}_A} G_c^{I}[A]\right) e^{\overleftrightarrow{\mathfrak{D}}_A} \left(e^{\mathfrak{D}_A} G_c^{II}[A]\right)\Big|_0 = \sum_{FG}$$



A most efficient way of expressing the sum over all Feynman graphs ... which is necessary for strong coupling.

• **Question** : When can one sum over all such graphs ?

→ In Abelian eikonal limit, when k of A wave $\ll p$ of $G_c(x,y|A)$.

Then, each $G_c[A] \sim \exp\left[ig p_\mu \int_{-\infty}^{0} ds\, A_\mu(y-sp) + ig p'_\mu \int_{0}^{\infty} ds\, A_\mu(y-sp')\right]$,

or, for simplicity : $\simeq \exp\left[ig p_\mu \int_{-\infty}^{+\infty} ds\, A_\mu(y-sp)\right]$, since $p-p' = q \ll p$ or p'.

The functional operation is immediate : $e^{\mathfrak{D}_A} \cdot e^{i\int f \cdot A}\Big|_{A\to 0} = e^{\frac{i}{2}\int f \cdot D_c \cdot f}$,

where : $f_\mu(w) = g \int_{-\infty}^{+\infty} ds\, p_\mu\, \delta(w - y + sp)$.

→ Even if the exponential was quadratic in A, the linkage operation can be performed exactly (equivalent to Gaussian functional integration):

$$e^{\mathfrak{D}_A} \cdot e^{\frac{i}{2}\int A K A + i\int f \cdot A}\Big|_{A\to 0} = e^{\frac{i}{2}\int f \cdot \left(D_c \frac{1}{1 - K D_c}\right) \cdot f} \cdot e^{-\frac{1}{2} \mathrm{Tr} \ln (1 - K D_c)}$$

• Now return to Schwingerian QCD, in its original form :

$$Z^{(\lambda)}_{QCD}[j,\eta,\bar{\eta}] = N\, e^{i\int \bar{\eta} G_c[A]\eta + L[A]} \cdot e^{i\int \mathcal{L}'[A]} \cdot \underbrace{e^{\frac{i}{2}\lambda \int (\partial_\mu A_\mu)^2}\Big|_{A\to \frac{1}{i}\frac{\delta}{\delta j}} \cdot e^{\frac{i}{2}\int j D_c^{(\lambda)} j}}_{e^{\frac{i}{2}\int j D_c^{(\lambda)} j} \cdot e^{-\mathrm{Tr}\ln(1-\lambda)}}$$

and consider the exact relation :

$$\int \mathcal{L}' = -\frac{1}{4}\int F^2 + \frac{1}{4}\int f^2 = -\frac{1}{4}\int F^2 + \frac{1}{2}\int (\partial_\nu A_\mu)^2 - \frac{1}{2}\int (\partial_\mu A_\mu)^2$$

$$\to -\frac{1}{4}\int F^2 + \frac{1}{2}\int A_\mu (-\partial^2) A_\mu - \frac{1}{2}\int (\partial_\mu A_\mu)^2 .$$

7

Substitute that $\int \mathcal{L}'[A]$ into the Schwinger sol'ú :

- $$\mathcal{Z}^{(\lambda)}_{QCD}[j,\eta,\bar\eta] \to N\, e^{i\int \bar\eta G_c[A]\eta + L[A]} \cdot e^{\frac{i}{2}\int A(-\partial^2)A - \frac{i}{2}(1-\lambda)\int(\partial_\mu A_\mu)^2} \cdot e^{-\frac{i}{4}\int F^2} \Big|_{A \to \frac{1}{i}\frac{\delta}{\delta j}} \cdot e^{\frac{i}{2}\int j D_c^{(0)} j}$$

→ In QED one cannot choose $\lambda = 1$; but in QCD : Yes!
And $\lambda = 1 \Rightarrow$ MGI. We obtain :

$$\mathcal{Z}_{QCD}[j,\eta,\bar\eta] \to N\, e^{i\int \bar\eta G_c[A]\eta + L[A]} \cdot e^{-\frac{i}{4}\int F^2} \cdot e^{\frac{i}{2}\int A(-\partial^2)A} \Big|_{A \to \frac{1}{i}\frac{\delta}{\delta j}} \cdot e^{\frac{i}{2}\int j D_c^{(0)} j},$$

or, after rearrangement :

$$\mathcal{Z}_{QCD} \to N\, e^{\frac{i}{2}\int j D_c^{(0)} j} \cdot e^{\mathcal{D}_A} \cdot e^{-\frac{i}{4}\int F^2} \cdot e^{\frac{i}{2}\int A(-\partial^2)A} \cdot e^{i\int \bar\eta G_c[A]\eta + L[A]} \Big|_{A = \int D j}.$$

→ You cannot see it just yet, but something extraordinary has occured...

- What to do about the F^2-term? Follow Halpern, who in the '70s wrote :

$$e^{-\frac{i}{4}\int F^2} = N' \int d[\chi]\, e^{+\frac{i}{4}\int \chi^2 + \frac{i}{2}\int F\cdot\chi}$$

, where $\int d[\chi]$ is a F.I. :

$$\int d[\chi] = \prod_i \prod_a \prod_{\mu\nu} \int_{-\infty}^{+\infty} d\chi^a_{\mu\nu}[w_i], \quad \chi^2 = \sum \left[\chi^a_{\mu\nu}(w)\right]^2, \quad F\cdot\chi = \sum F^a_{\mu\nu}(w)\,\chi^a_{\mu\nu}(w).$$

Why? Because $F^a_{\mu\nu}$ is quadratic in A, and eikonal amplitudes can be obtained non-perturbatively.

- Consider QQ, or $Q\bar Q$ scattering (at high energies). Suppress initial and final color degrees of freedom, and (long) kinematical analysis which leads to :

$$T(s,t) \sim i \int d^2b\, e^{i\vec q\cdot\vec b}\left[1 - e^{i\chi(s,b)}\right],$$

and : $e^{i\chi} = N'' \int d[\alpha_I] \int d[\alpha_{II}] \int d[\Omega_I] \int d[\Omega_{II}] \, e^{i\int(\alpha_I \Omega_I + \alpha_{II}\Omega_{II})} \left(e^{i\int \lambda_1 \alpha_I(s)}\right)_+ \left(e^{i\lambda_2 \cdot \int \alpha_{II}}\right)_+$

$\cdot \int d[\overline{X}] \cdot e^{\mathfrak{D}_A} \, e^{\frac{i}{2}\int A\cdot(K+\Delta K)\cdot A + i\int R\cdot A} \Big|_{A\to 0}$, where : $K^{ab}_{\mu\nu} = g f_{abc} \overline{X}^c_{\mu\nu}(w)$,

$\Delta K^{ab}_{\mu\nu} = g_{\mu\nu} \delta_{ab} (-\partial^2)$,

$R^a_\mu = \Omega^a_{I\mu} + \Omega^a_{II\mu} + \partial_\lambda \overline{X}^a_{\mu\lambda}$, $\Omega^a_{I,II\,\mu}(w) = g \int_{-\infty}^{+\infty} ds \, p^\mu_{1,2} \, \delta^{(4)}(w - y + s p_{1,2}) \, \Omega_{I,II}(s)$,

and the $\int d[\alpha] \cdot \int d[\Omega]$ arise when extracting the A-dependence of each $G_c[A]$ from under its Ordered Exponential :

$$\left(e^{ig\int_{-\infty}^{+\infty} ds \, p_\mu A^a_\mu(y - sp)\lambda^a}\right)_+ = \int_{-\infty}^{+\infty} d[\alpha(s)] \, \delta[\alpha^a(s) - g\, p\cdot A^a(y - sp)] \cdot \left(e^{i\int_{-\infty}^{+\infty} \lambda\cdot\alpha(s)}\right)_+$$

$$\Rightarrow N' \int d[\Omega(s)] \int d[\alpha(s)] \, e^{i\int ds\, \Omega_a(s)[\alpha^a(s) - g\, p\cdot A^a(y - sp)]} \cdot \left(e^{i\lambda\int ds\, \alpha(s)}\right)_+$$

- Now, focus attention on the linkage operation, which can be done exactly :

$$e^{\frac{i}{2}\int\int dz_1 dz_2 \, R(z_1)\cdot\left[D_c^{(0)} \frac{1}{1 - (K+\Delta K) D_c^{(0)}}\right]\cdot R(z_2)} \cdot e^{-\frac{1}{2}\mathrm{Tr}\ln\left[1 - (K+\Delta K) D_c^{(0)}\right]}$$

NB : this, under all the other F.I.s, contains all virtual-gluon interactions, including cubic and quartic ints., between QQ / $Q\bar{Q}$.

- Now, observe that : $\Delta K \cdot D_c^{(0)} = (-\partial^2) D_c^{(0)} \Rightarrow 1$, and this 1 cancels the 1 of the denominator :

$$[1 - (K+\Delta K) D_c^{(0)}] \Rightarrow -[K\cdot D_c^{(0)}]$$, so that :

$$\frac{i}{2}\int R\cdot D_c^{(0)}\left[1-(K+\delta K)D_c^{(0)}\right]^{-1}\cdot R \to \frac{i}{2}\int R\cdot D_c^{(0)}\left[-K\cdot D_c^{(0)}\right]^{-1}\cdot R$$

$$\Rightarrow \frac{i}{2}\int R\cdot D_c^{(0)}\cdot (D_c^{(0)})^{-1}(-K)^{-1}\cdot R \Rightarrow -\frac{i}{2}\int R\cdot K^{-1}\cdot R$$

What has happened? The choice $\lambda = 1 \to$ MGI $\Rightarrow$ treats all virtual gluons as "ghost particles" whose gauge-dep. propagators disappear from the final answer.
[This argument used the Feynman $D_c^{(0)}$; but it can be repeated with a prop. in an arbitrary gauge.]

This is GI with a vengeance! No gluon props. in final answer!

NB: All subsequent corrections (remove quenching, corrections to the eikonal model, ...) will display the same property. The K, of $[K^{-1}]$, will be more complicated, but all gluon props. will disappear.

NB': This is true for all (non-pert.) values of g; and for large g, the most important contributions of R arise from the $Q_{I,II}$ dependence $\sim$ each prop. to g.

$\therefore$ Instead of conventional, Abelian result, where g-dep. has the form: $g\cdot(a+bg^2+cg^4+\ldots)\cdot g$, we here have effectively summed all the virtual g^2 into the $(\frac{1}{g})$ from $[K^{-1}]$; we get in this exponential $\sim g\cdot\frac{1}{g}\cdot g = g$.

If, as usual in eikonal models, we suppress self-interactions then: $\frac{i}{2}\int Q\cdot[-K]^{-1}\cdot Q \to i\int Q_I\cdot[-K]^{-1}\cdot Q_{II}$, retaining only the cross-term, the interaction between I and II.

Another benefit of HGI : Since the $Q_{I,II}[w] \sim \delta^{(4)}(w - y + sP_{1,2})$, the "action-at-a-distance" of $D_c(w-u)$ in ord. gauge-dep. theory, is replaced by an "effectively local" interaction, mediated by the Halpern variable, $\overline{X}$.

So Entire integral is evaluated at some "fixed" point : $w_\mu \to w_\mu^0$, while the $s_{1,2}$ are fixed $\simeq 0$, corresponding to a "distance of closest approach" of the quarks ← not exactly 0, but almost...

Then, the F.I.s $\int d[\alpha_I] \int d[\alpha_{II}] \int d[\Omega_I] \int d[\Omega_{II}] \to$ ordinary integrals at $s_{1,2} \simeq 0$: $\int d^n\alpha_I \int d^n\alpha_{II} \int d^n\Omega_I \int d^n\Omega_{II}$, n = no. of color DoF ;
($n=3$, $SU(2)$; $n=8$, $SU(3)$)

Simplifications!

What remains of the $Q_I \cdot (-K)^{-1} \cdot Q_{II}$ expon. factor, after some rescaling, and in the CM frame :

$$ig\varphi(b)\, P_\mu^{I} P_\nu^{II} \left(f \cdot \overline{X}\right)^{-1}\Big|_{\mu\nu}^{ab} \Omega_I^a(t)\, \Omega_{II}^b(0) \quad ,$$

where, $\overline{X} = \overline{X}(w^0)$, and $\varphi(b)$ arises from a

$$\delta^{(2)}(b) = \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} \to \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} \cdot e^{-\vec{k}^2/M^2} \equiv \varphi(b) \sim \varphi(0)\, e^{-b^2M^2/4} \quad ,$$

$$M^2 \sim O((P_1+P_2)^2) .$$

Why? Because the δ-f of the eikonal $Q_I \cdot Q_{II}$ assumes strictly 0 transverse mom. fluctuations, which is modified (in the usual cases) by the $D_c(w-u)$ factors. But here, $\nexists$ no D_c ; and this correction to the eik. approx. must be inserted "by hand", corresponding to bound quarks, with $\langle k_\perp \rangle < M \sim O(E)$.

NB : Even for large g, as $b \uparrow$, our exp. factor $\downarrow$; and v.v.