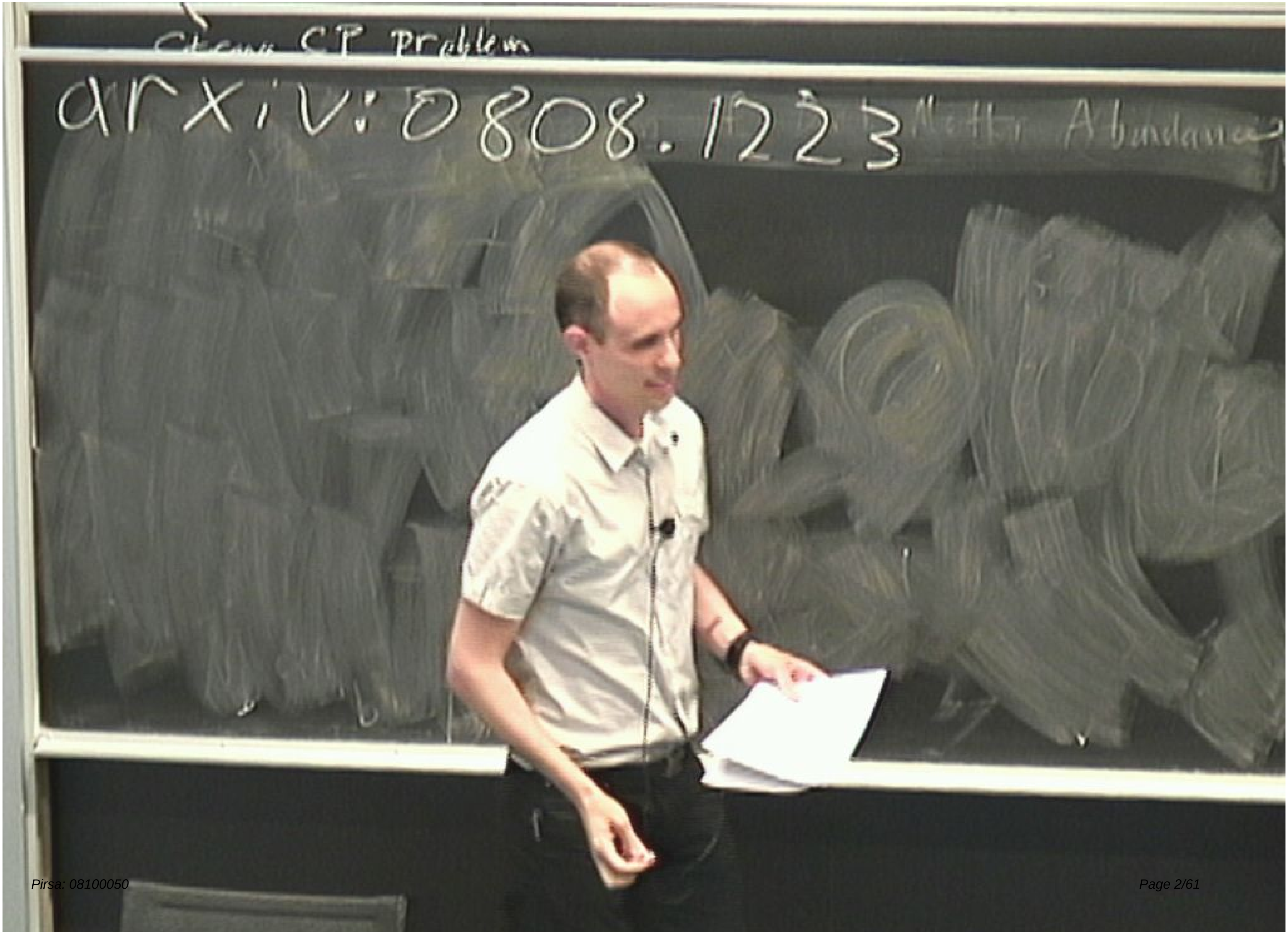
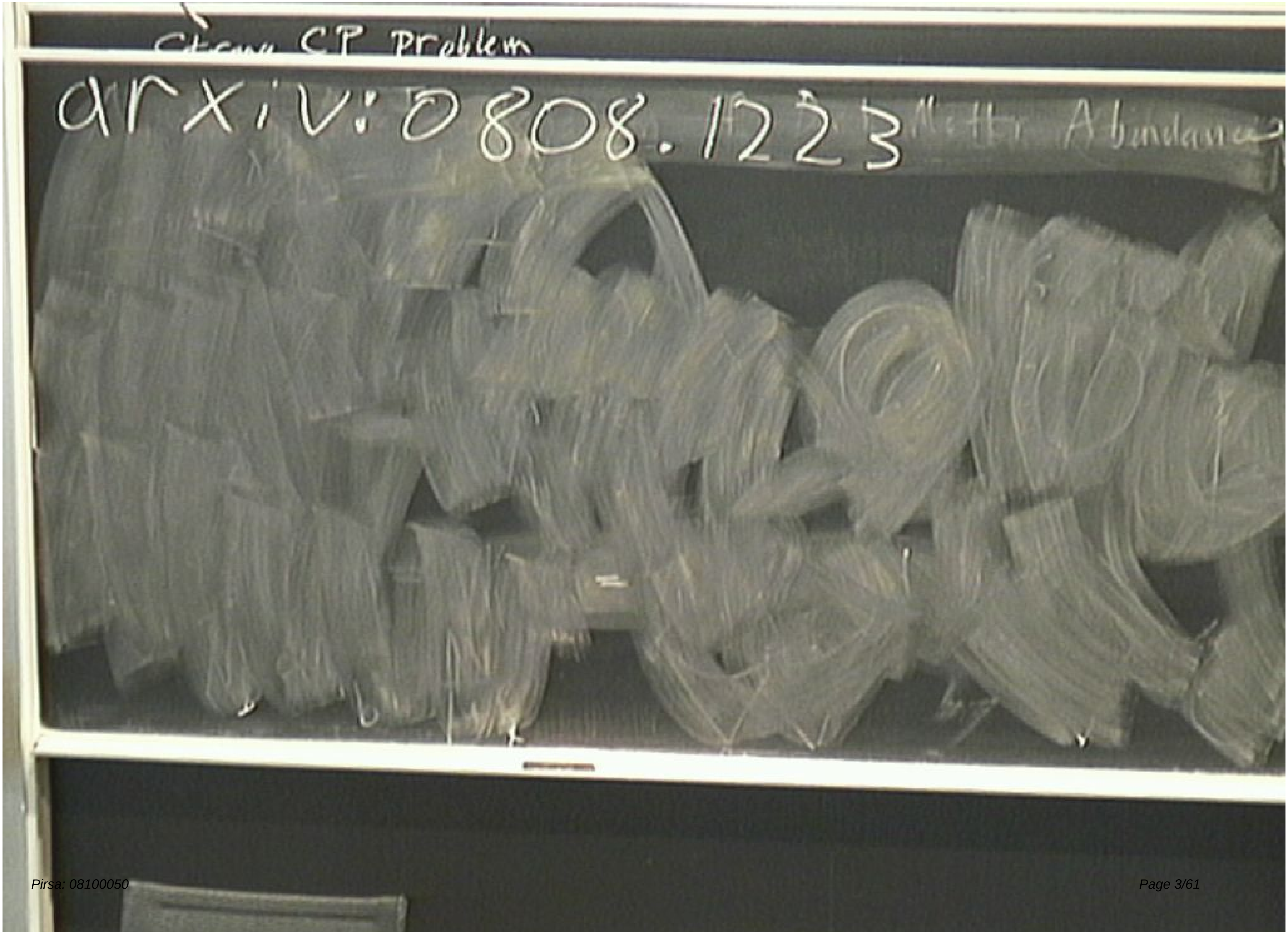
Title: Dirac\'s penumbra: constraints and gauge transformations in reparametrization invariant theories

Date: Oct 07, 2008 04:00 PM

URL: http://pirsa.org/08100050

Abstract: A simple theorem of Dirac identifies primary first-class constraints as generators of transformations, \'that do not affect the physical state\'. This result has profound implications for the definition of physical states and observables in the quantization of constrained systems, and leads to one aspect of the infamous \'problem of time\' in quantum gravity. As I will discuss, a close look at the theorem reveals that it depends crucially on the assumption of an absolute time. This assumption does not hold for reparametrization invariant theories, such as parametrized particle mechanics, and in these theories, the primary Hamiltonian constraint does generate physical change. I will also look at just what Dirac did and did not say about this case, and what has been said by reviewers since.


arxiv:0808.1223


CP Problem
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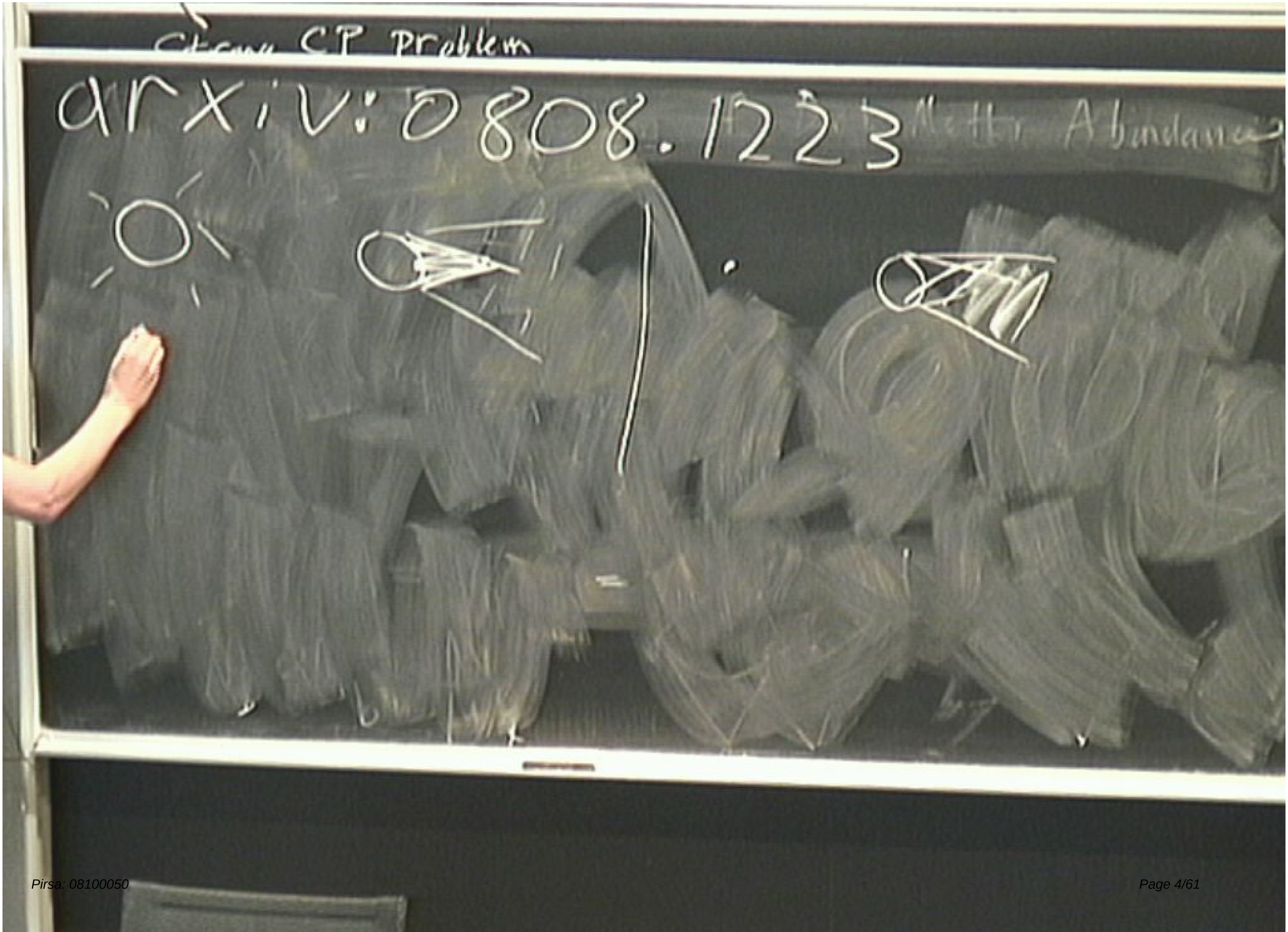
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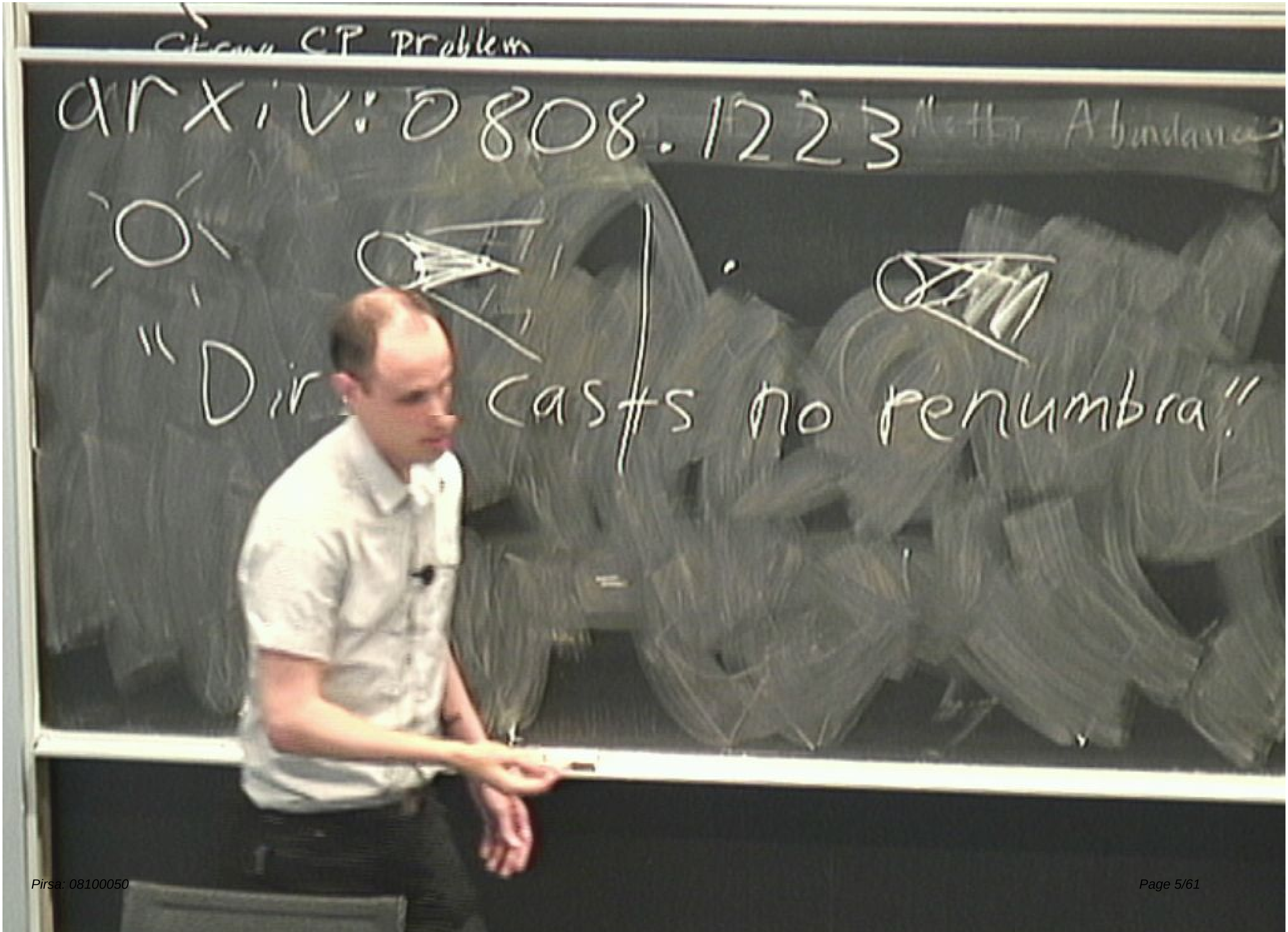
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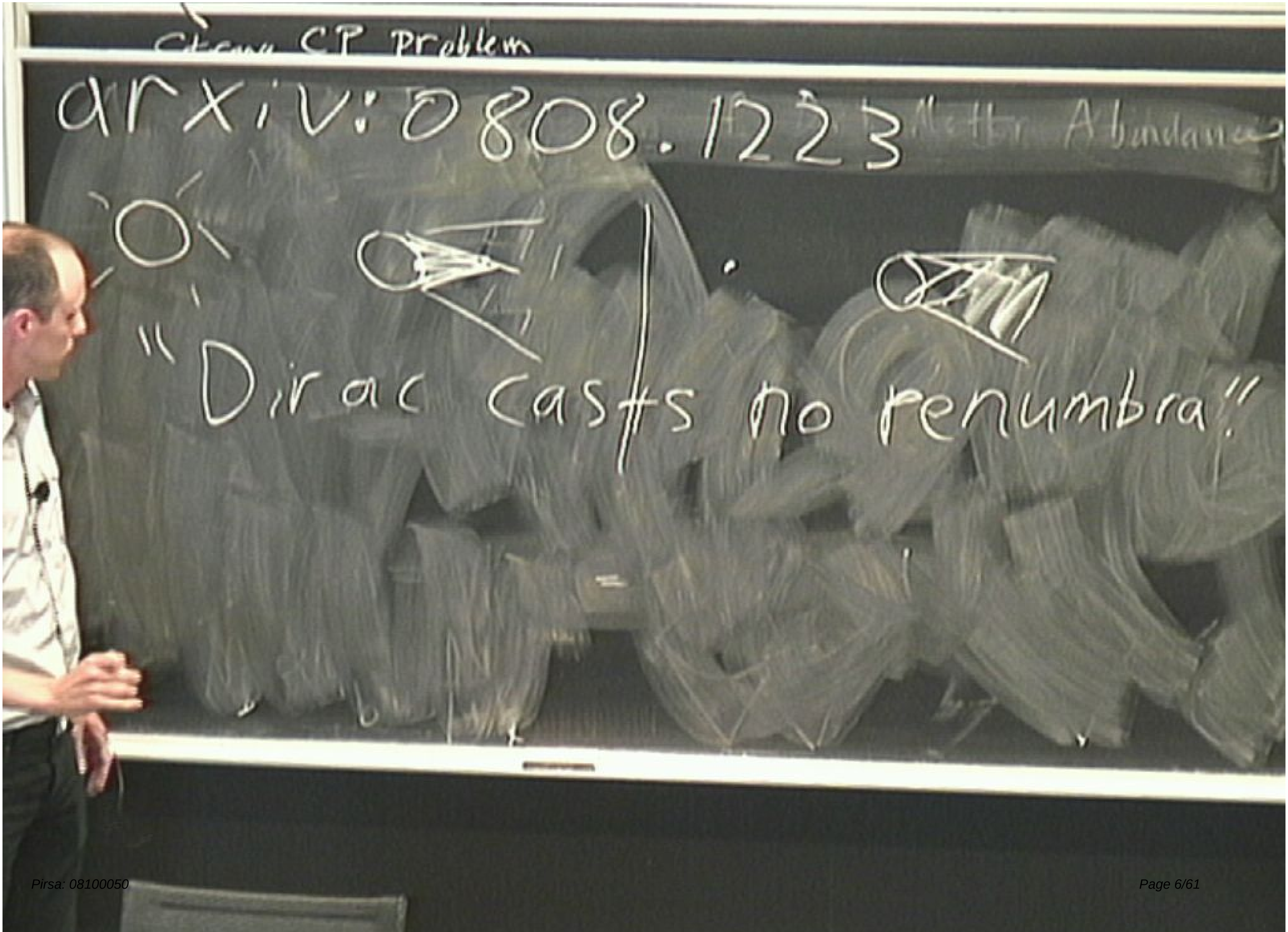
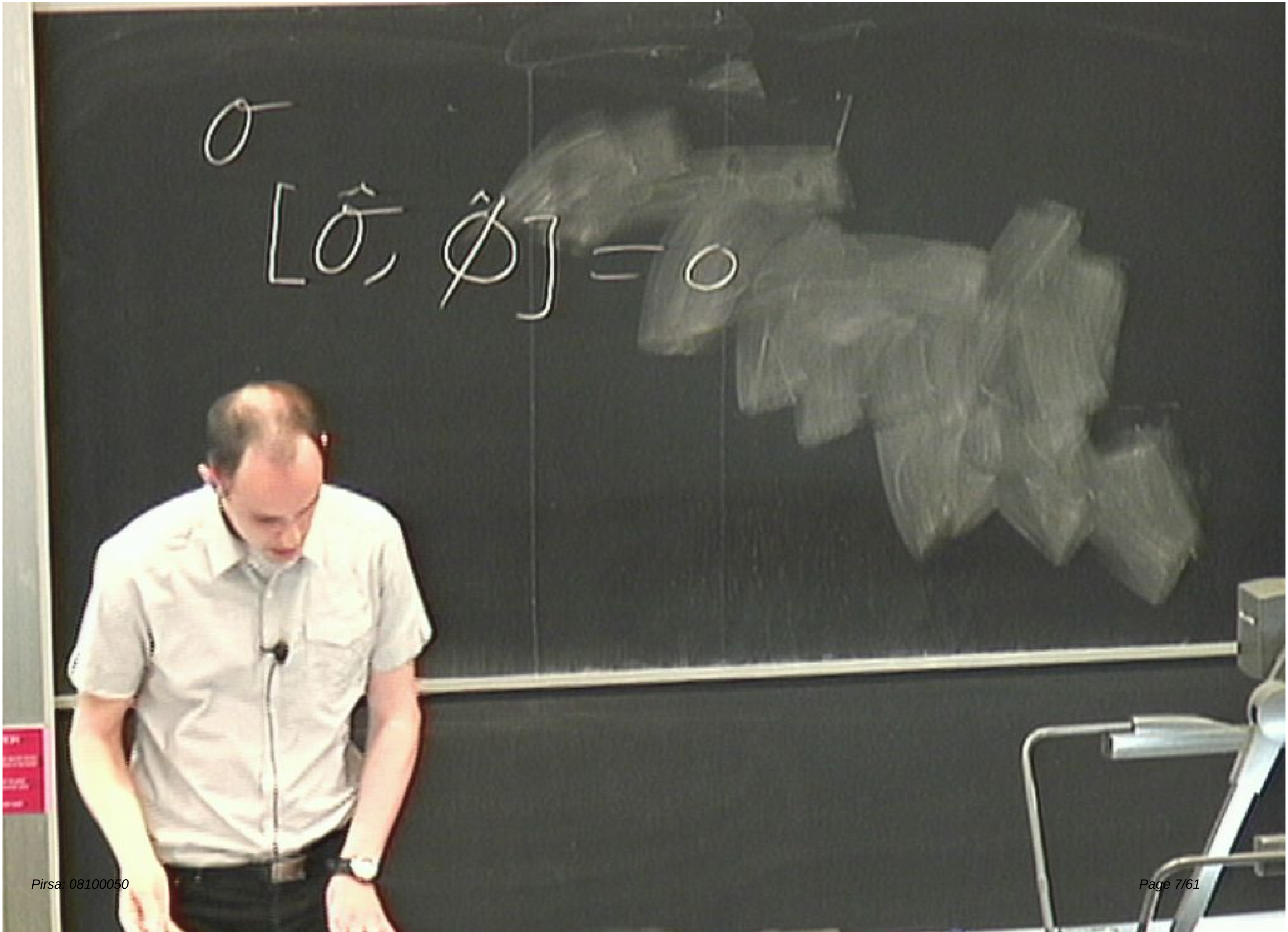
"Dir
casts no penumbra!"


Strong CP Problem
arxiv:0808.1223
"Dirac casts no penumbra!"


σ
$[\hat{\sigma}, \hat{\phi}] = 0$

σ

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$$\dot{\sigma} \propto [\sigma, H] = 0$$

σ

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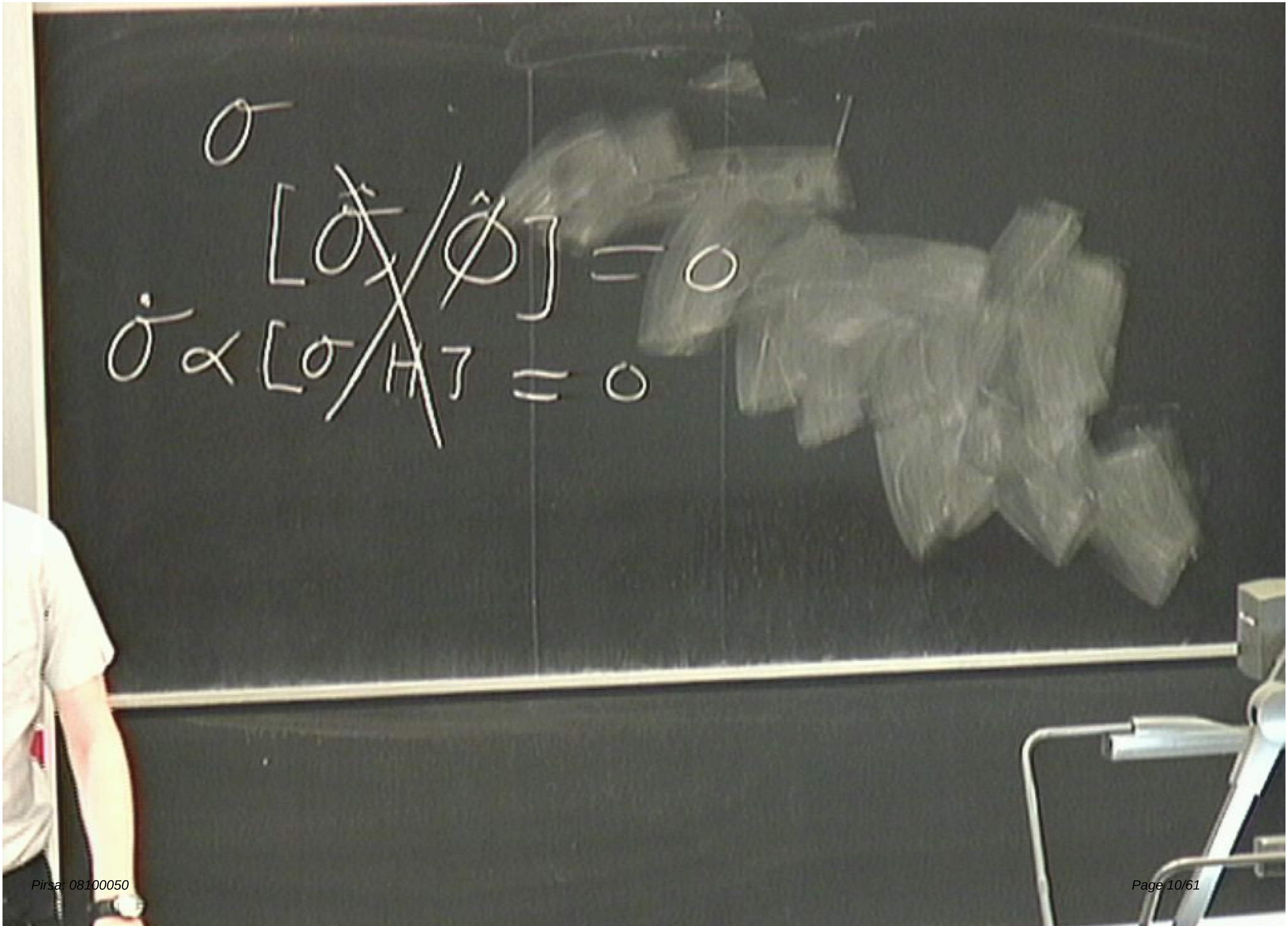
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$\cancel{[\hat{\sigma}, \hat{\phi}] = 0}$
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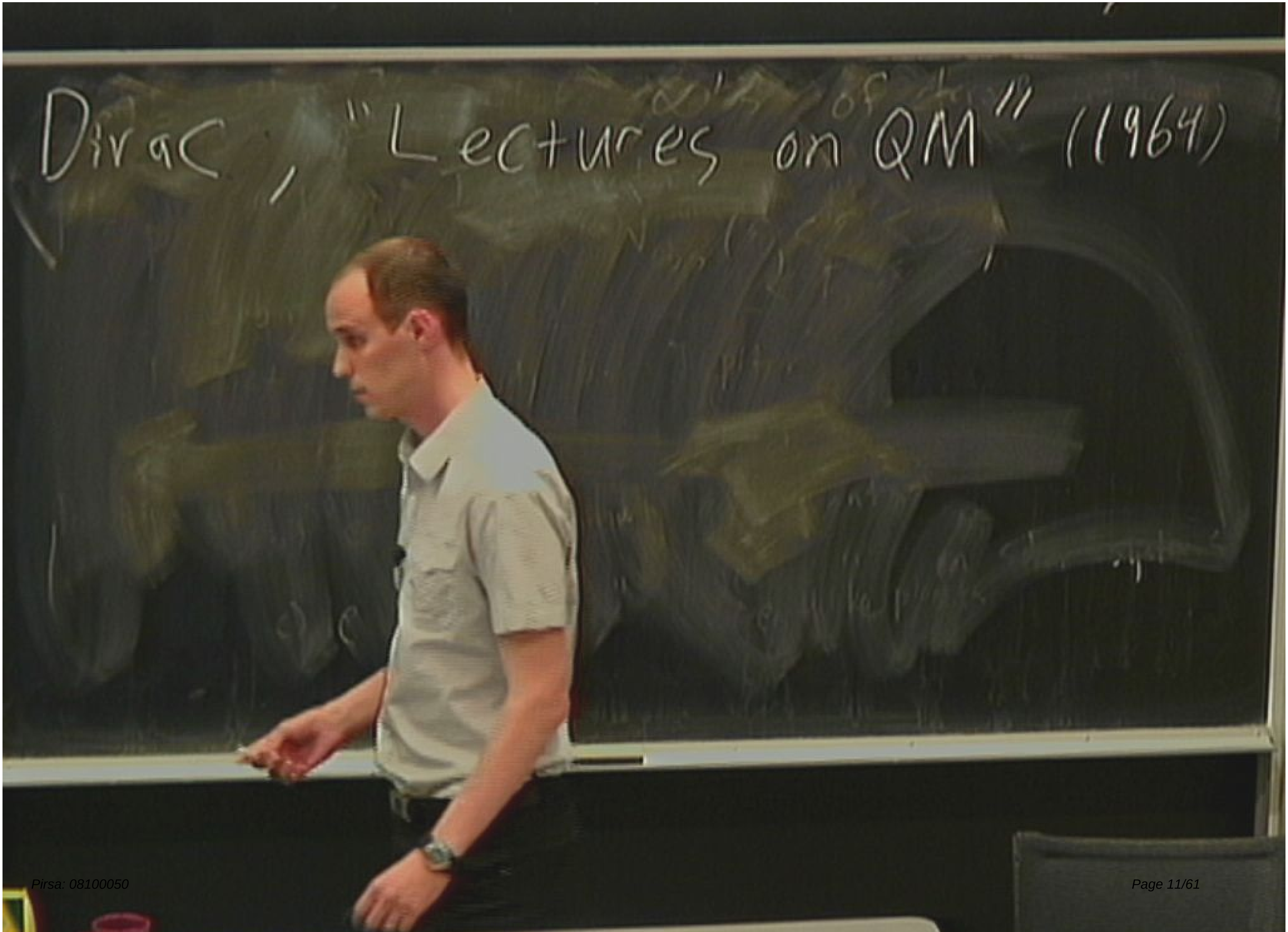
Dirac, "Lectures on QM" (1964)

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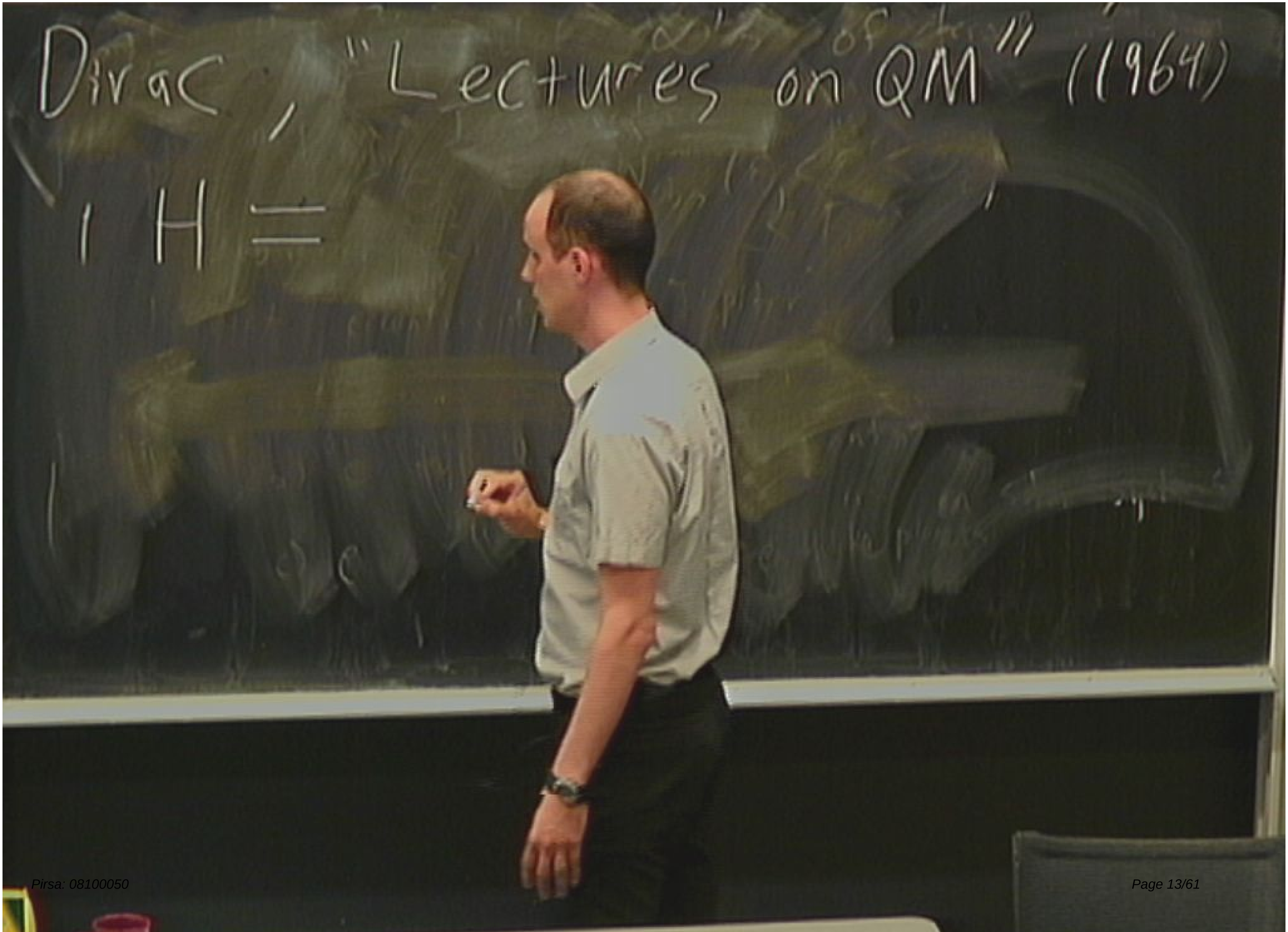
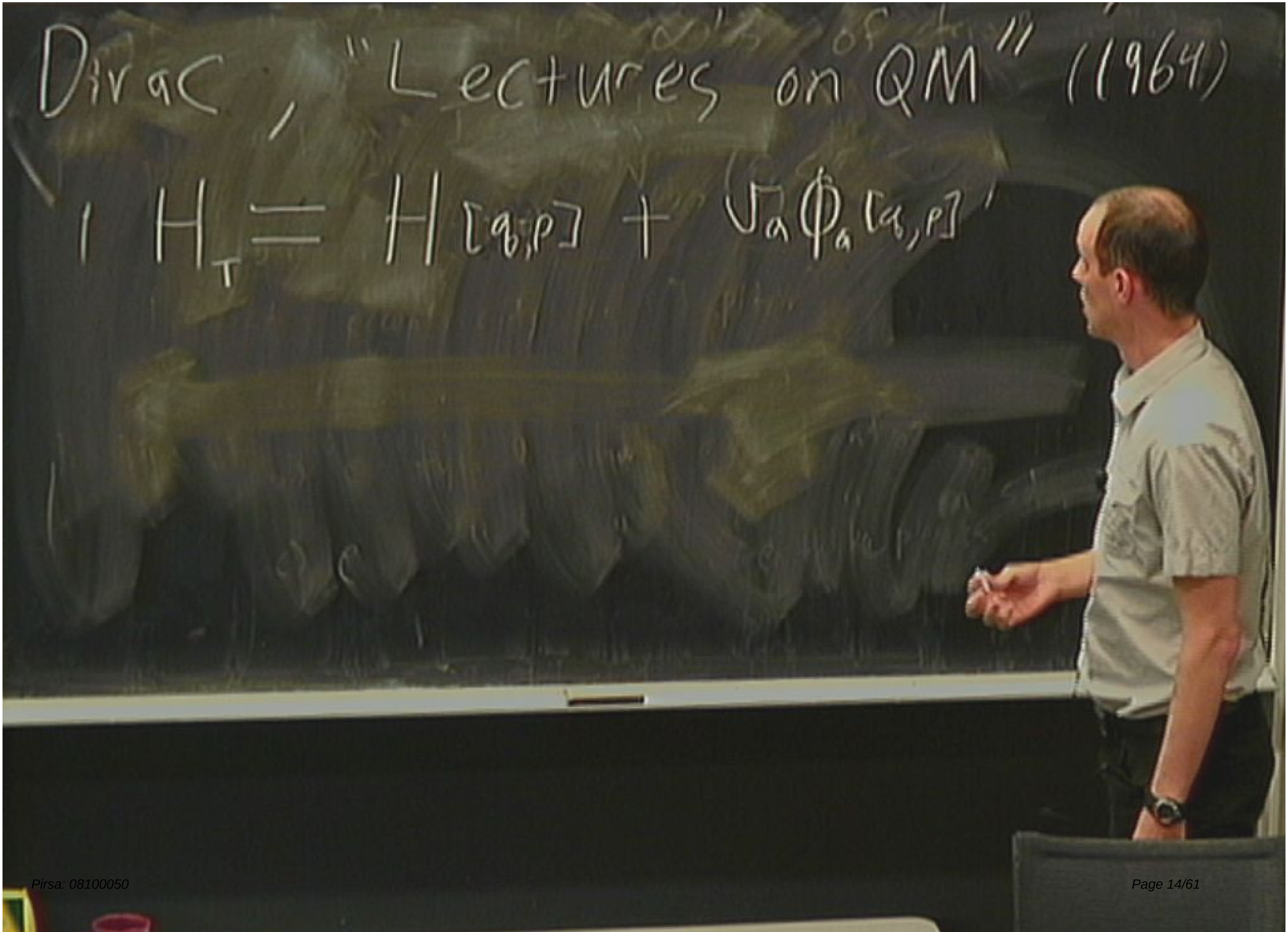

Dirac, "Lectures on QM" (1964)
1 H =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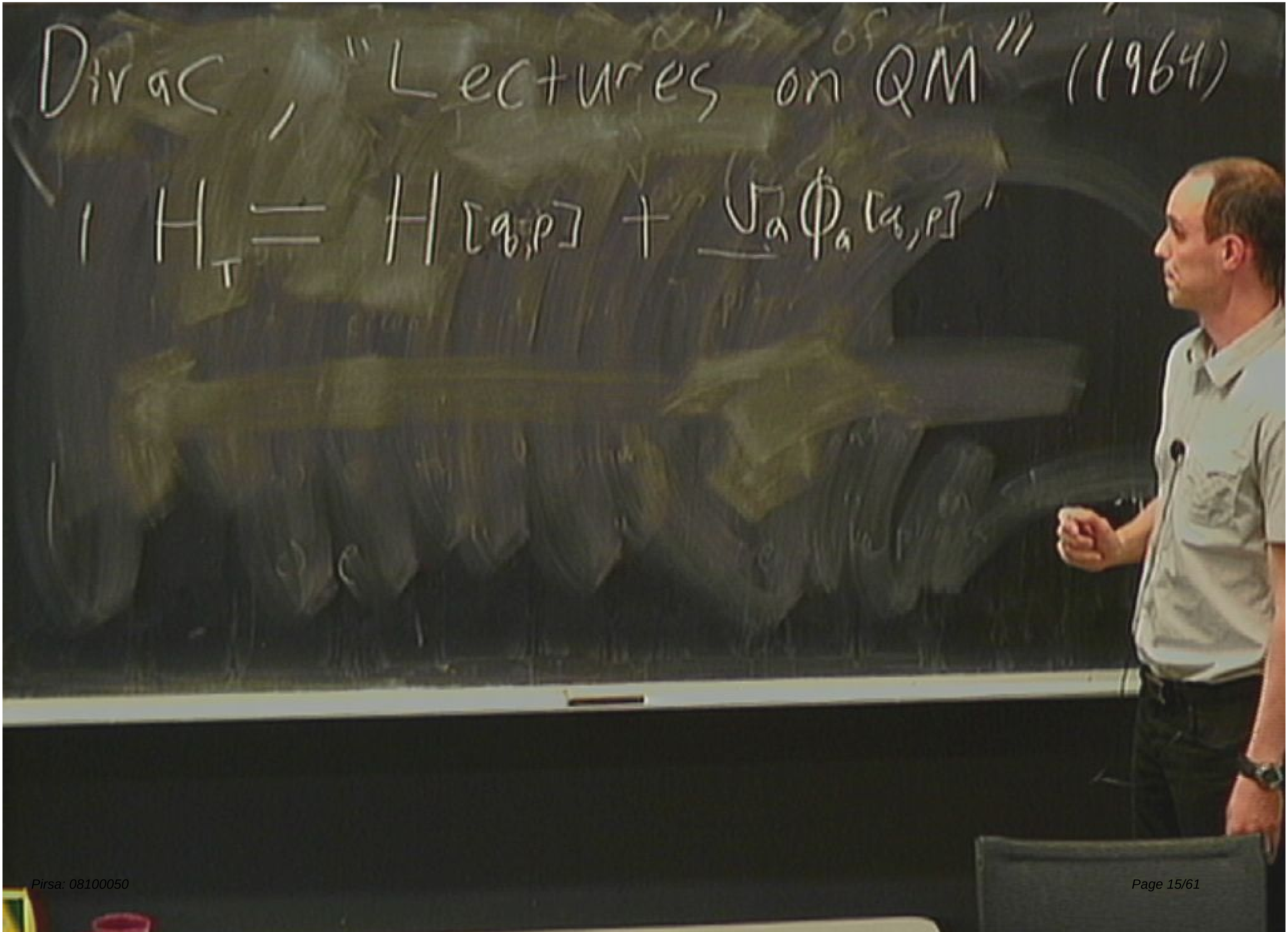
$H_T = H[q,p] + v_a \phi_a[q,p]$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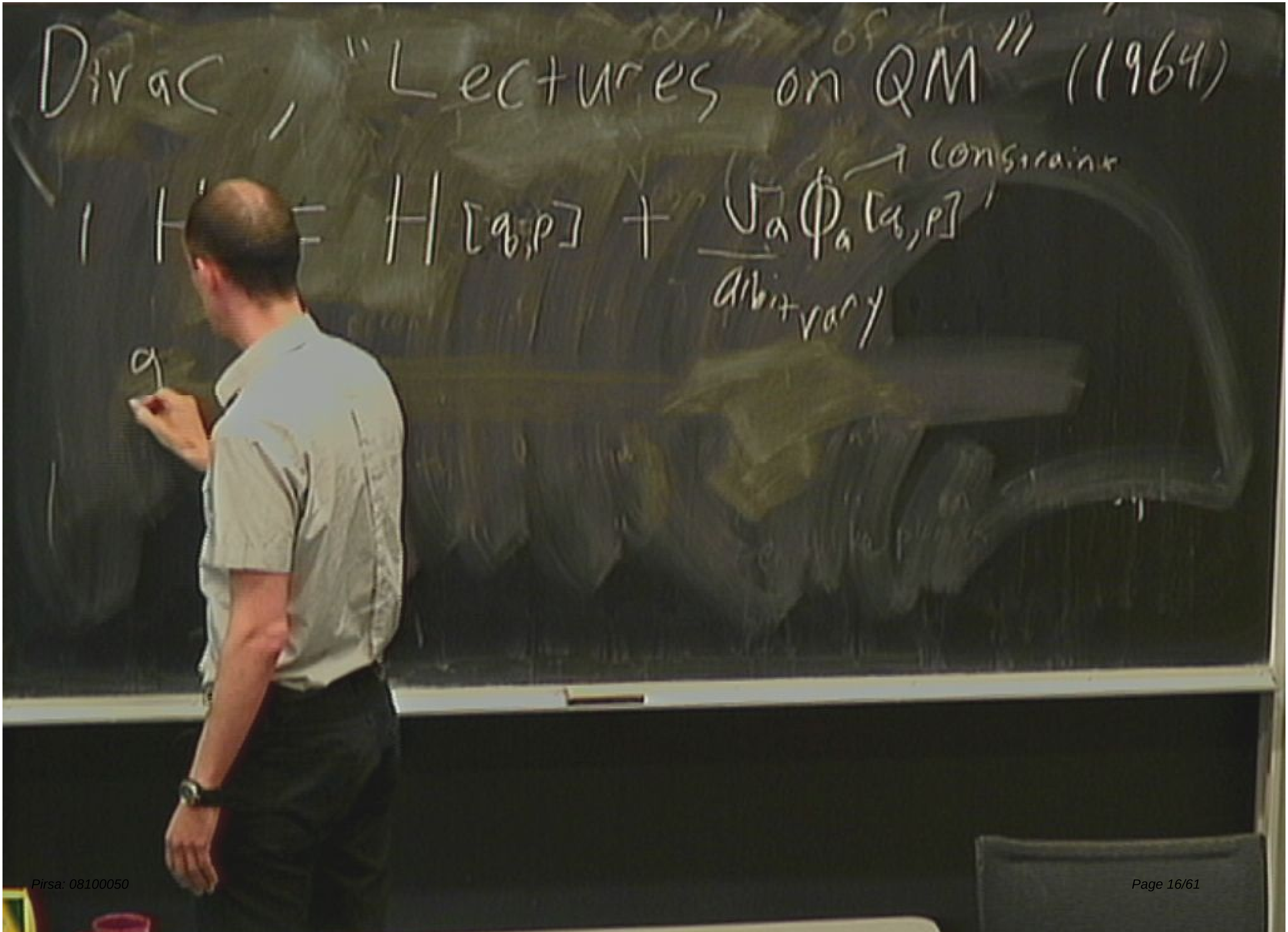
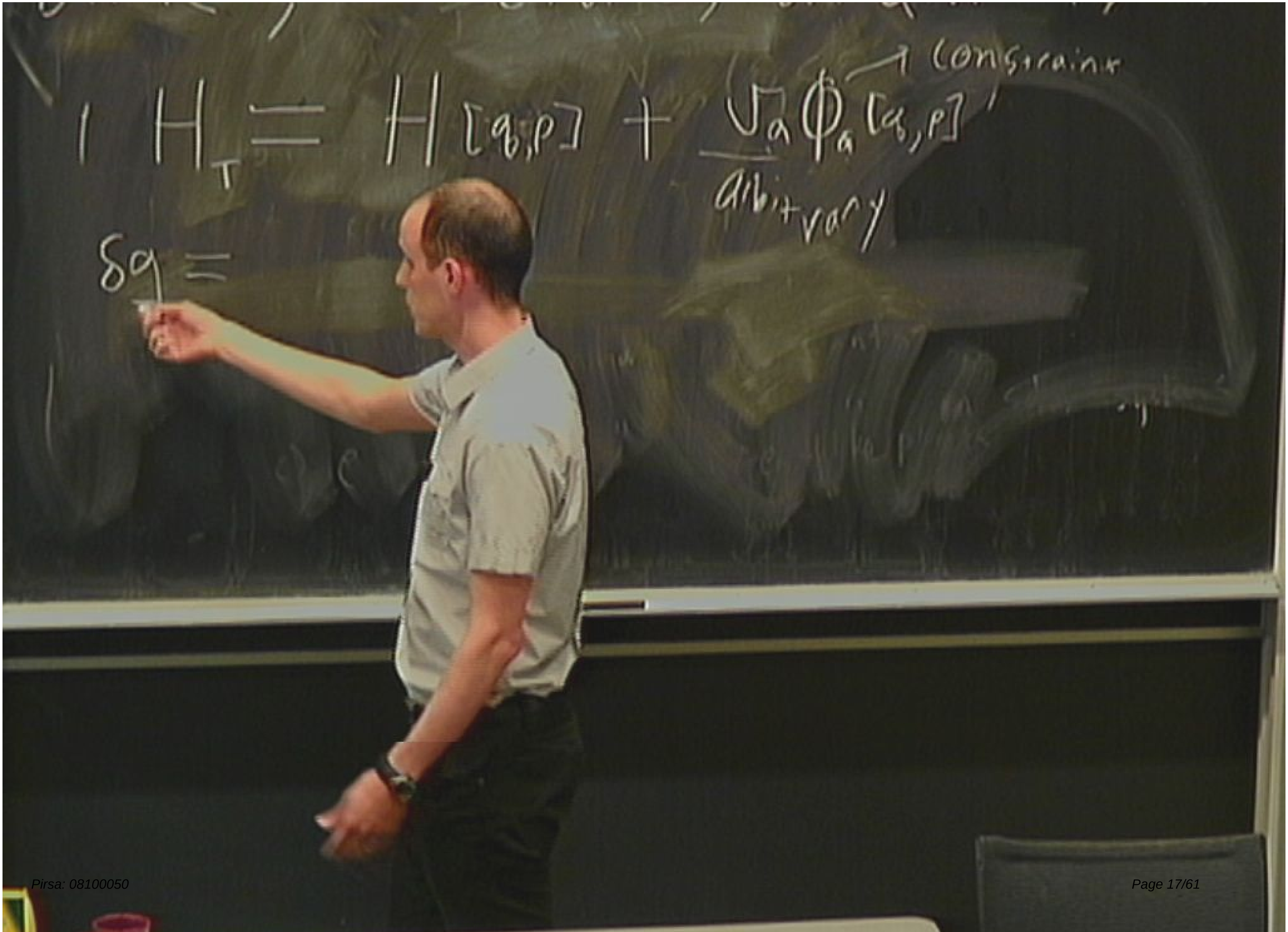

Dirac, "Lectures on QM" (1964)
constraint
H = H[q,p] + v_a φ_a[q,p]
arbitrary
q


constraint
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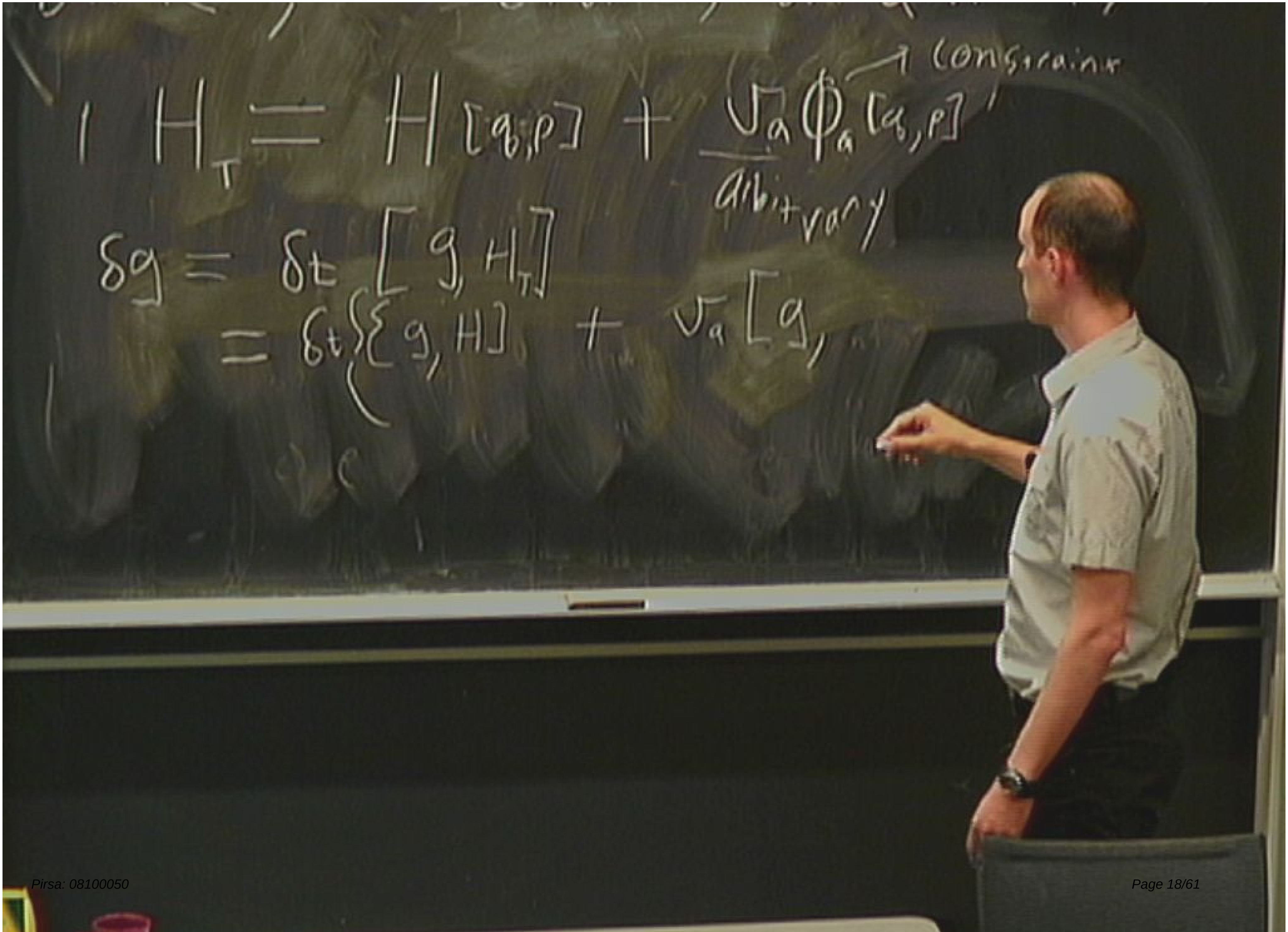
arbitrary
$\delta q =$


constraint
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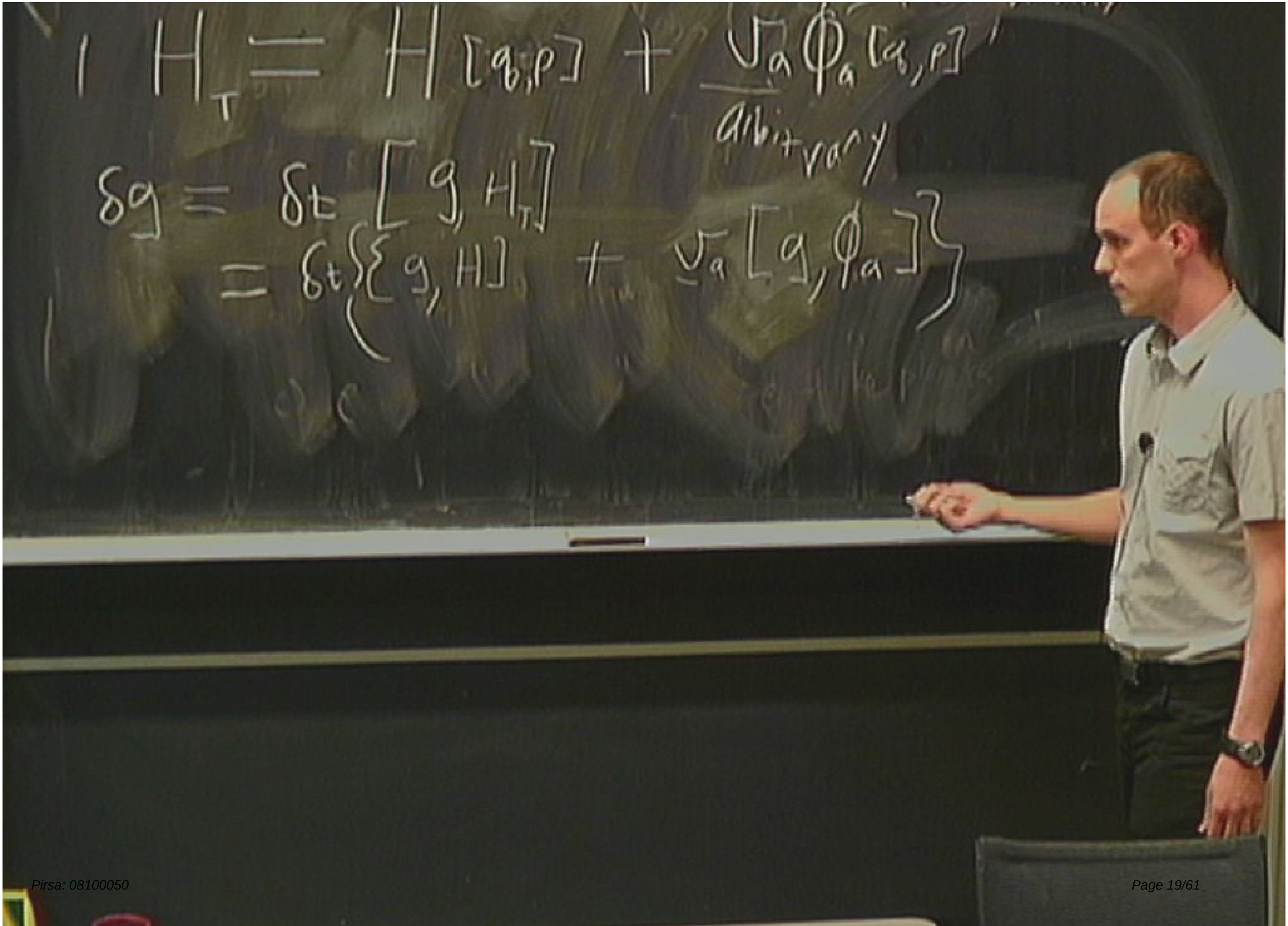
arbitrary
$\delta g = \delta t\, [g, H_T]$
$= \delta t \{ [g, H] + v_a [g,$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$\delta g = \delta t\,[g, H_T]$
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Dirac, "Lectures on QM" (1964)

$$H_T = H[q,p] + \underbrace{v_a}_{\text{arbitrary}} \phi_a[q,p]$$ constraint

$$\delta g = \delta t\,[g, H_T]$$

$$= \delta t\{[g,H] + v_a[g,\phi_a]\}$$

$$\Delta g(\delta t) = \epsilon_a [g, \phi_a], \quad \epsilon_a = \delta t\,[v_a - v'_a$$

Dirac, "Lectures on QM" (1964)

$$H_T = H[q,p] + \underbrace{v_a}_{\text{arbitrary}} \phi_a[q,p] \quad \text{constraint}$$

$$\delta g = \delta t\, [g, H_T]$$

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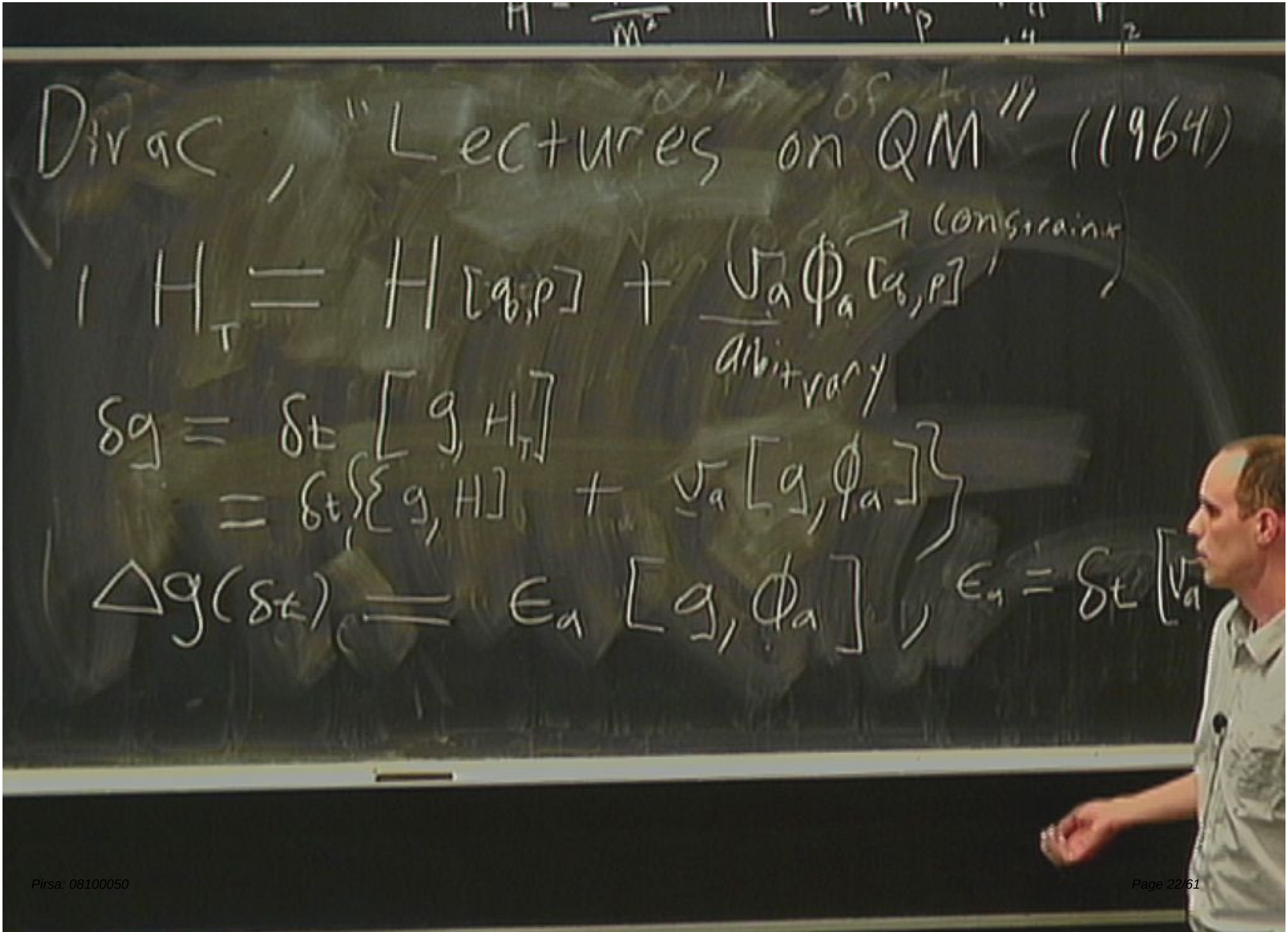
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(constraint)
arbitrary
$\delta g = \delta t\, [g, H_T]$
$= \delta t \{ [g, H] + v_a [g, \phi_a] \}$
$\Delta g(\delta t) = \epsilon_a [g, \phi_a]$, $\epsilon_a = \delta t\, [v_a$

Dirac, "Lectures on QM" (1964)

$$H_T = H[q,p] + \underbrace{v_a}_{\text{arbitrary}} \phi_a[q,p] \quad (\phi_a \to \text{constraint})$$

$$\delta g = \delta t\,[g, H_T]$$

$$= \delta t\,\{[g,H] + v_a[g,\phi_a]\}$$

$$\Delta g(\delta t) = \epsilon_a [g, \phi_a], \quad \epsilon_a = \delta t\,[v$$

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$$H_T = H[q,p] + \underbrace{v_a}_{\text{arbitrary}} \overbrace{\phi_a}^{\text{constraint}}[q,p]$$

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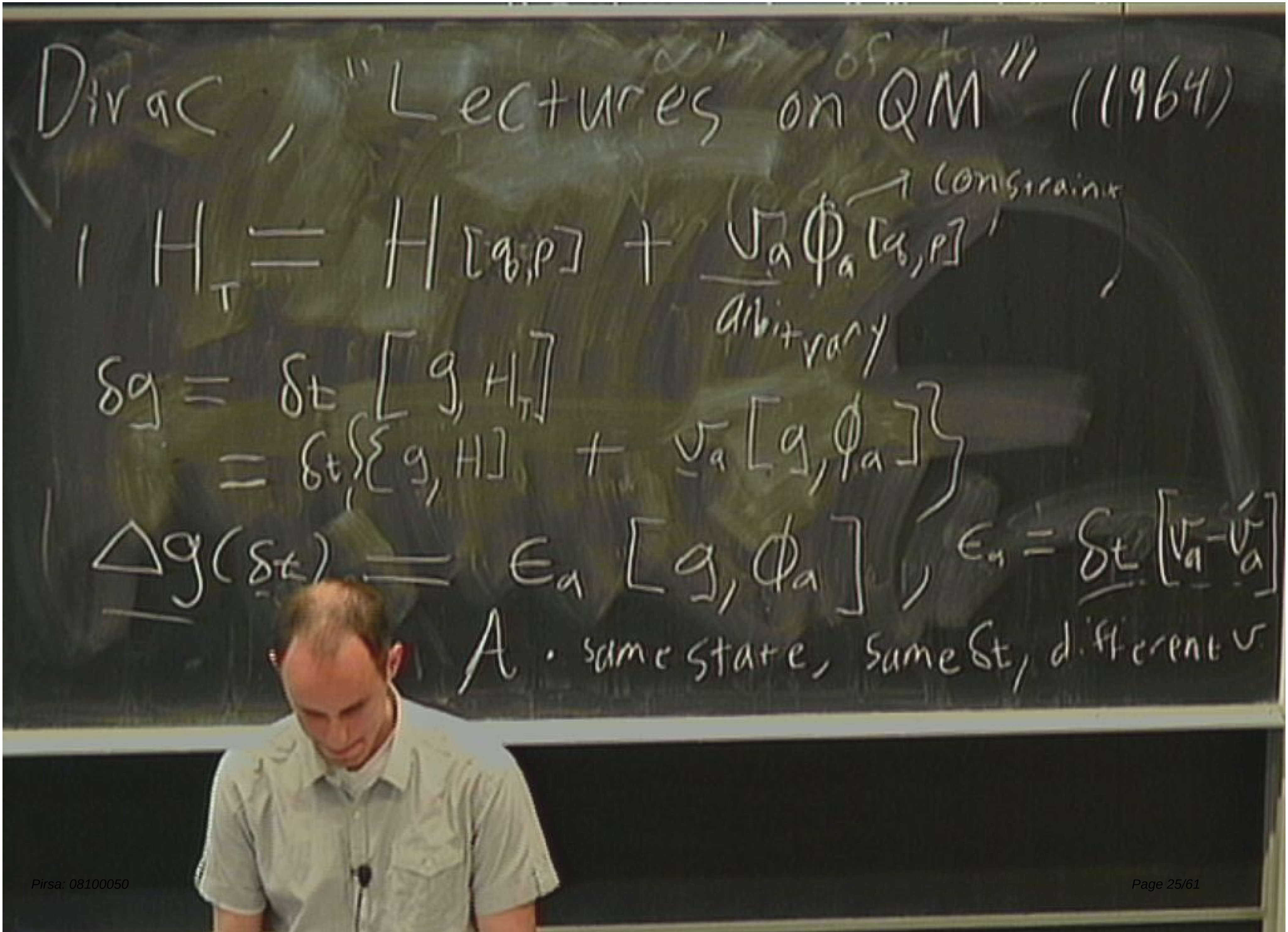
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A · same state, same δt, different v

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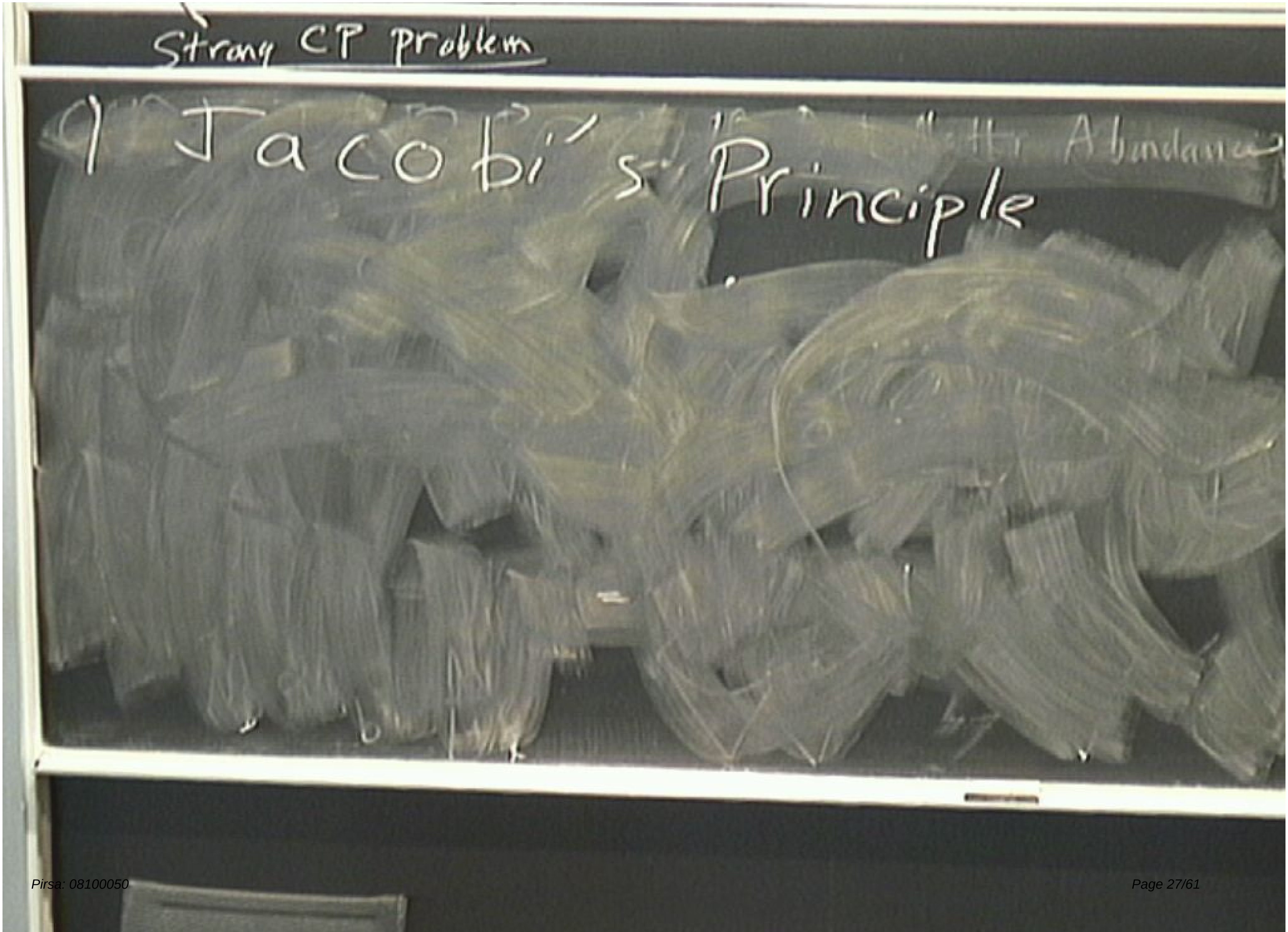
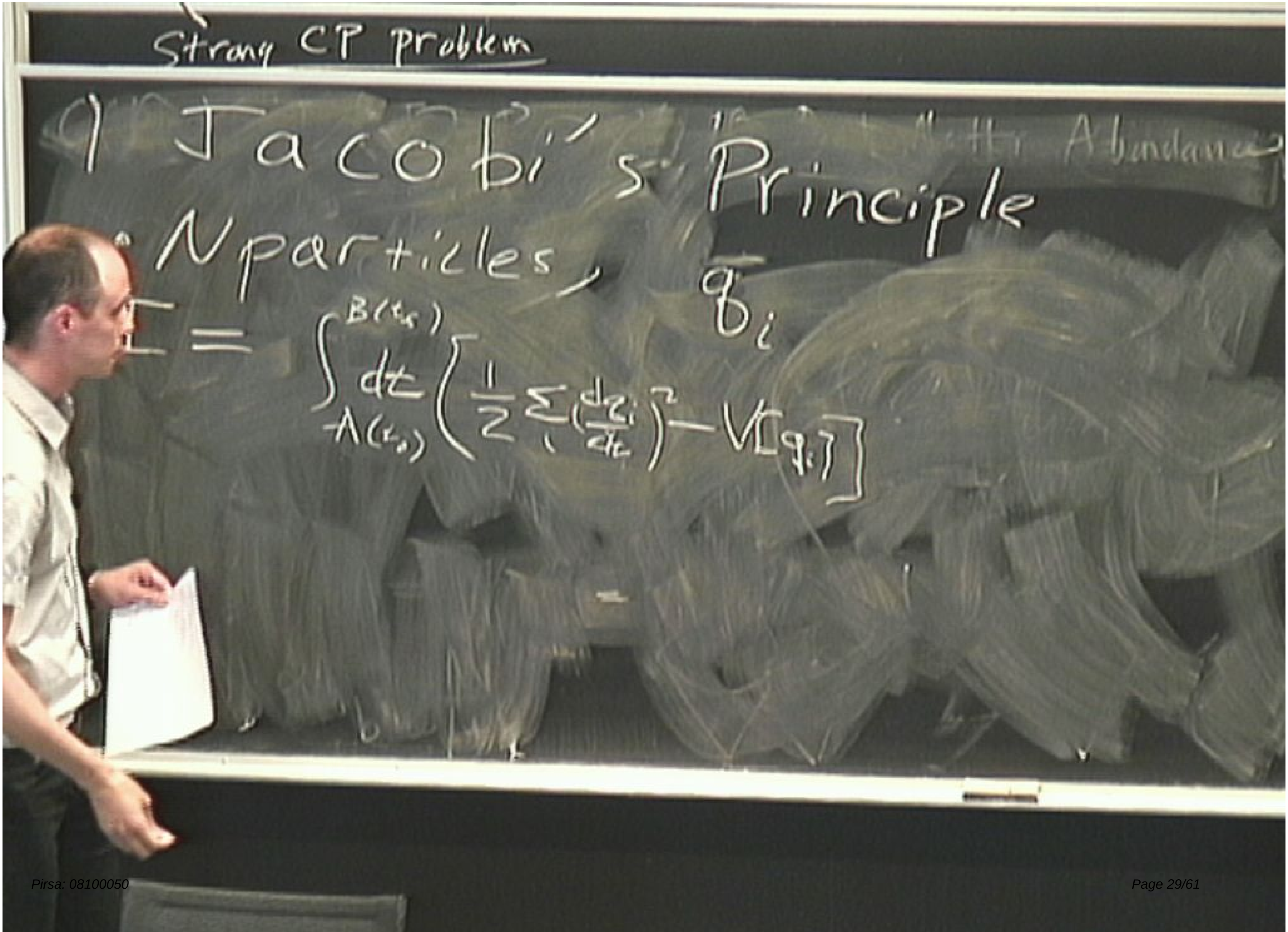

Strong CP problem
1) Jacobi's Principle

Strong CP problem

1) Jacobi's Principle

- N particles, q_i


Strong CP problem
1) Jacobi's Principle
• N particles, q_i
$I = \int_{A(t_0)}^{B(t_1)} dt \left(\frac{1}{2}\sum_i \left(\frac{dq_i}{dt}\right)^2 - V[q_i]\right)$

1) Jacobi's Principle

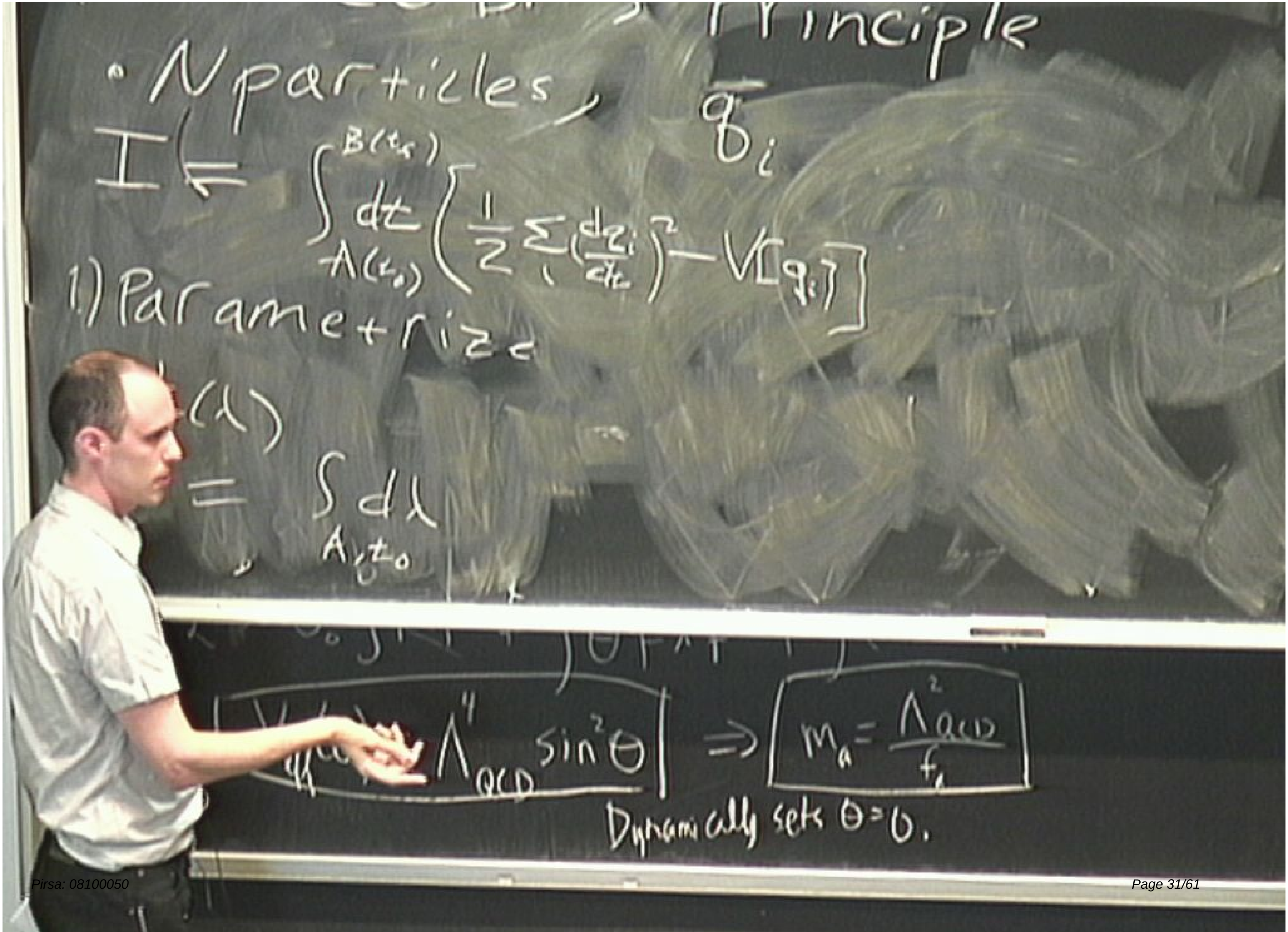
• N particles, q_i

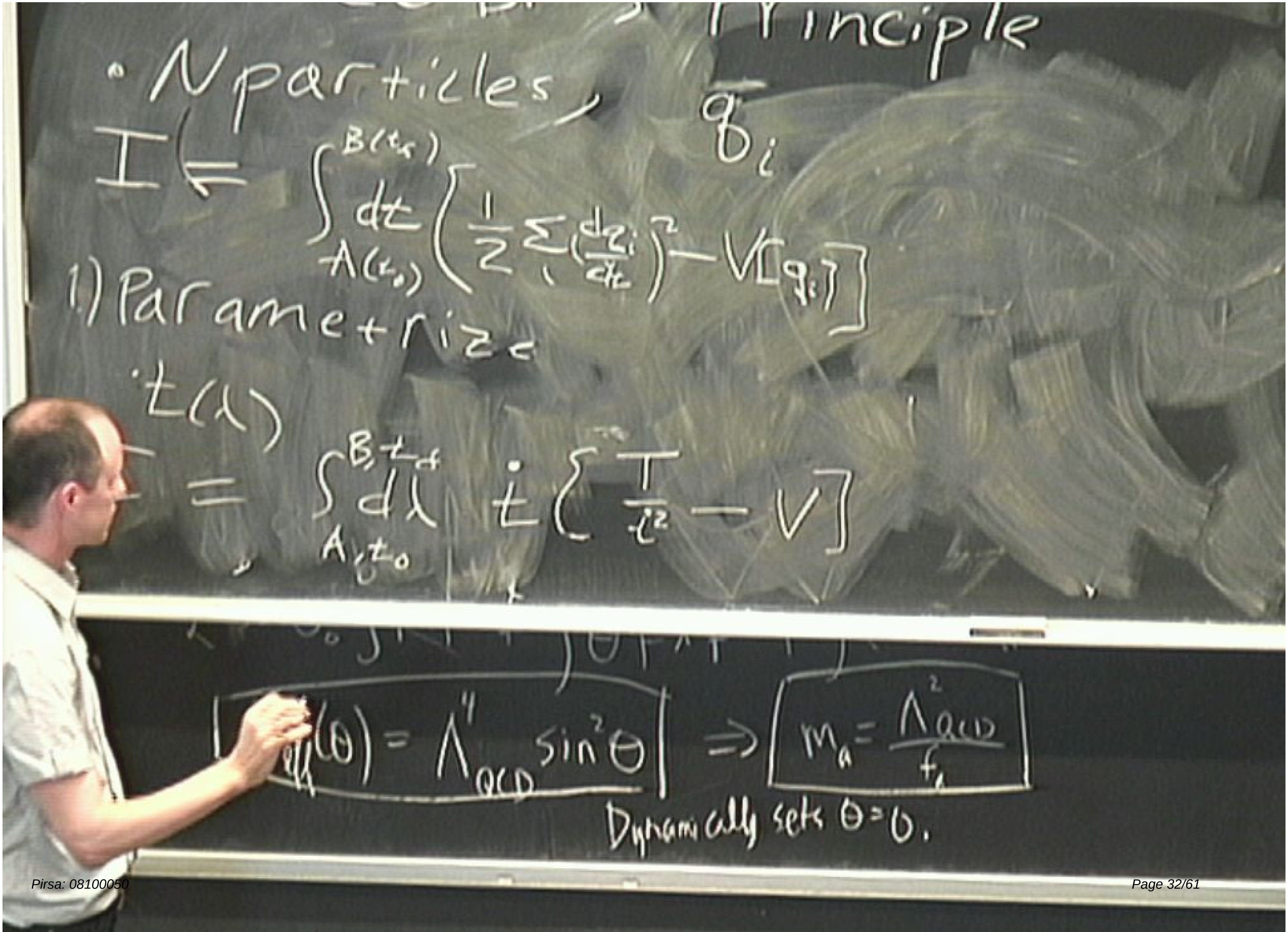
$$I = \int_{A(t_0)}^{B(t_f)} dt \left[\frac{1}{2}\sum_i \left(\frac{dq_i}{dt}\right)^2 - V[q_i]\right]$$

1) Parametrize

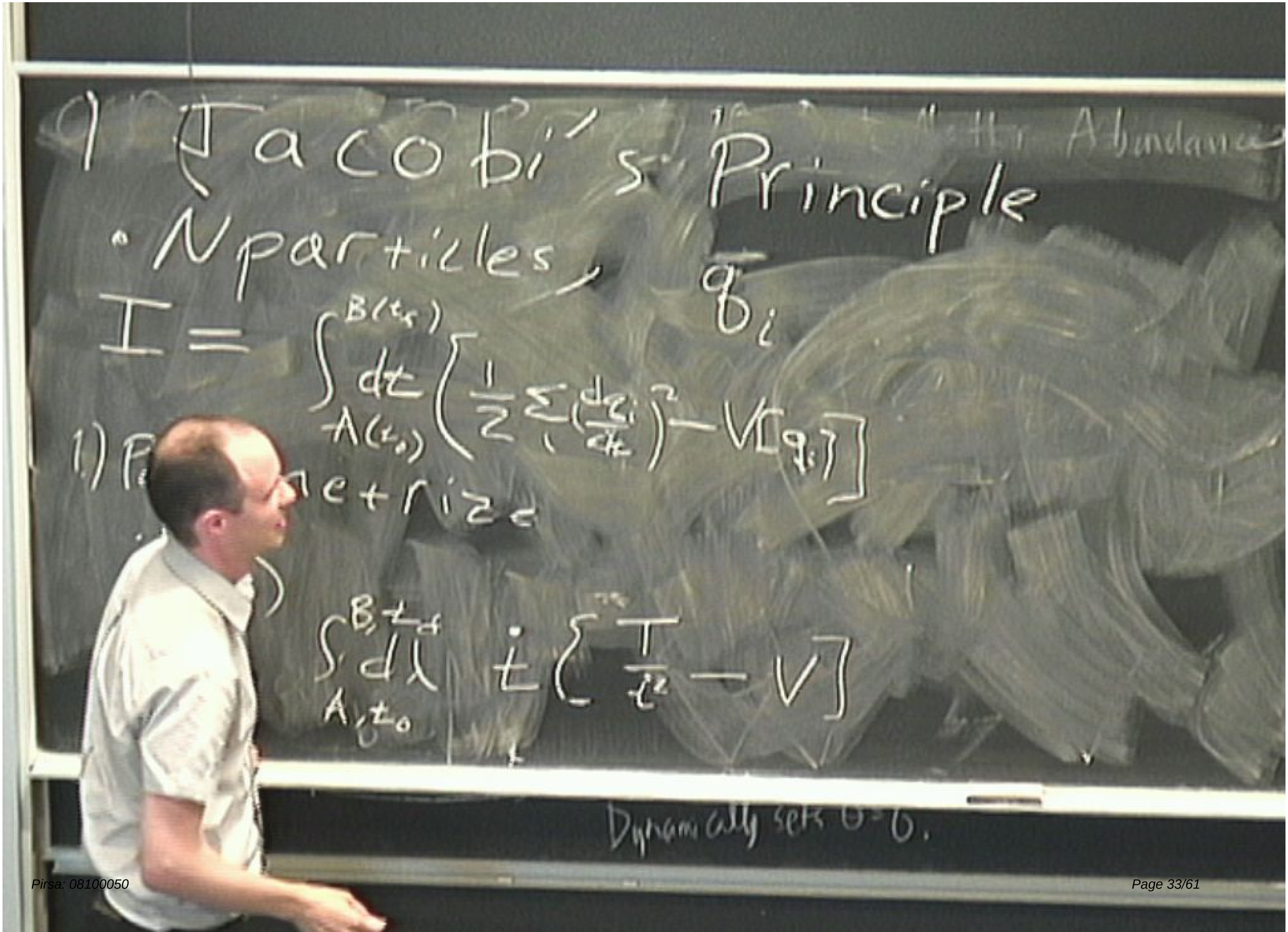
• t

Dynamically sets $\theta = 0$.


Principle
• N particles, q_i
I = ∫ dt (1/2 Σ_i (dq_i/dt)^2 − V[q_i])
1) Parametrize
= ∫ dλ
m_a = Λ_QCD^2 / f_a
Dynamically sets θ = 0.



Principle
• N particles, q_i
$I = \int_{A(t_0)}^{B(t_f)} dt \left[\frac{1}{2} \sum_i \left(\frac{dq_i}{dt}\right)^2 - V[q_i] \right]$
1) Parametrize
• $t(\lambda)$
$= \int_{A, t_0}^{B, t_f} d\lambda \, \dot{t} \left[\frac{T}{\dot{t}^2} - V \right]$
$V(\theta) = \Lambda_{QCD}^4 \sin^2\theta \Rightarrow m_a = \frac{\Lambda_{QCD}^2}{f_a}$
Dynamically sets $\theta = 0$.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1) Parametrize
$\int_{A,t_0}^{B,t_f} d\lambda \; \dot{t} \left[\frac{T}{\dot{t}^2} - V \right]$
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1) Jacobi's Principle

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1) Parametrize

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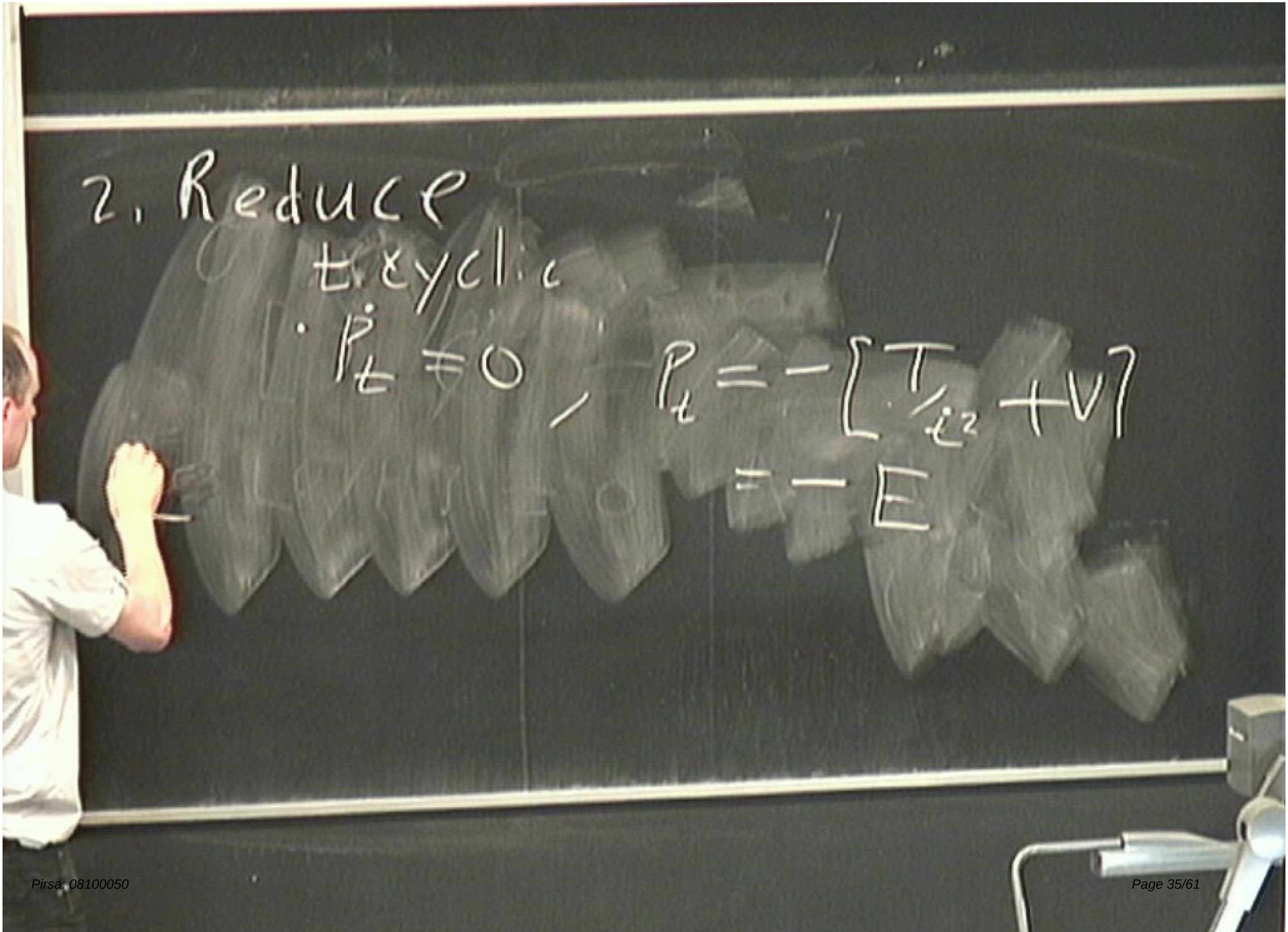
2. Reduce
t cyclic
· $\dot{p}_t = 0$, $p_t = -[T/\dot{t}^2 + V]$
$= -E$

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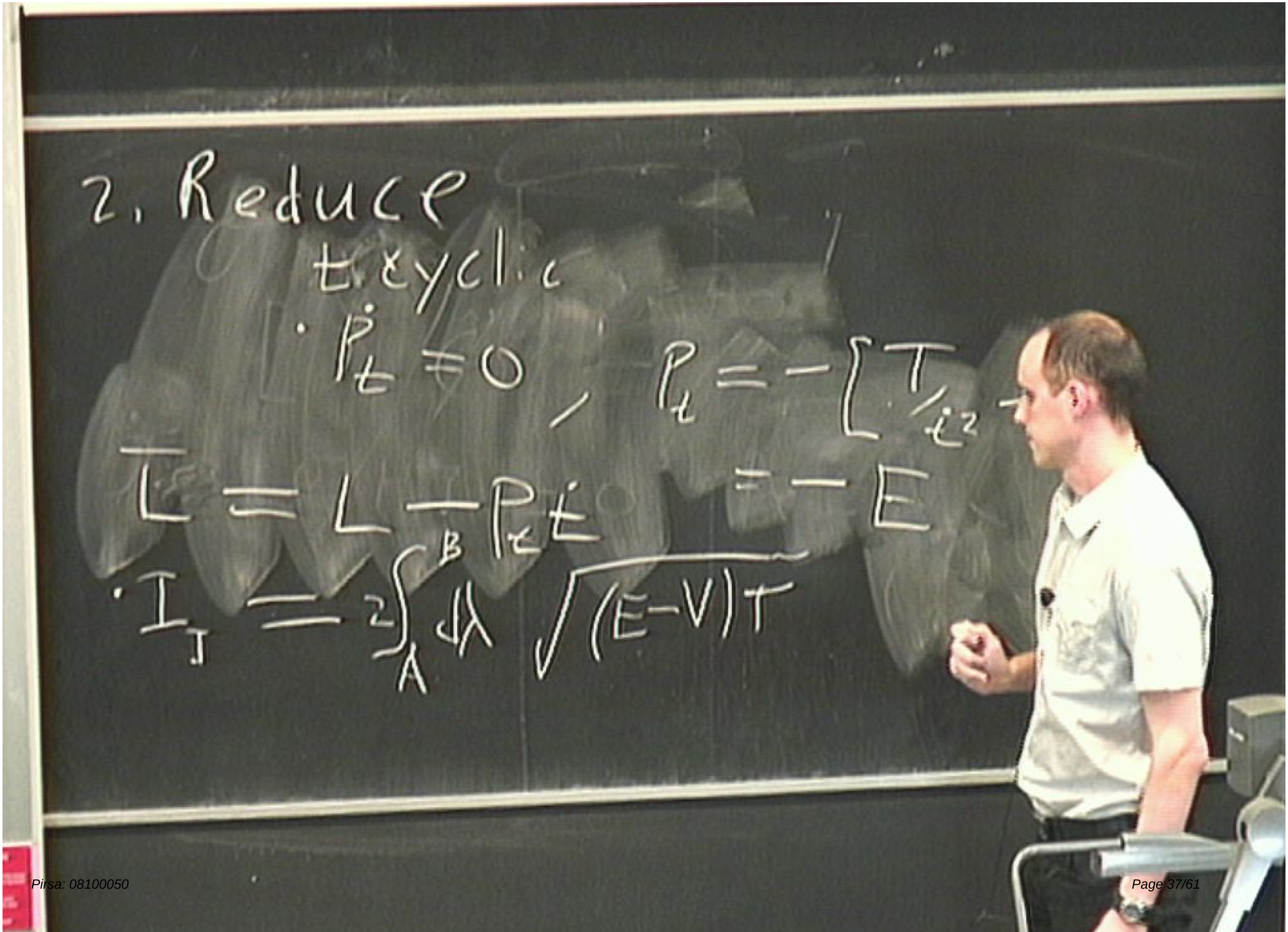
$\bar{L} = L - p_t \dot{t}$


2. Reduce
t cyclic
· $\dot{P}_t = 0$, $P_t = -[T/\dot{t}^2$
$\bar{L} = L - P_t \dot{t} = -E$
· $I_J = 2\int_A^B d\lambda \sqrt{(E-V)T}$

2. Reduce

t cyclic

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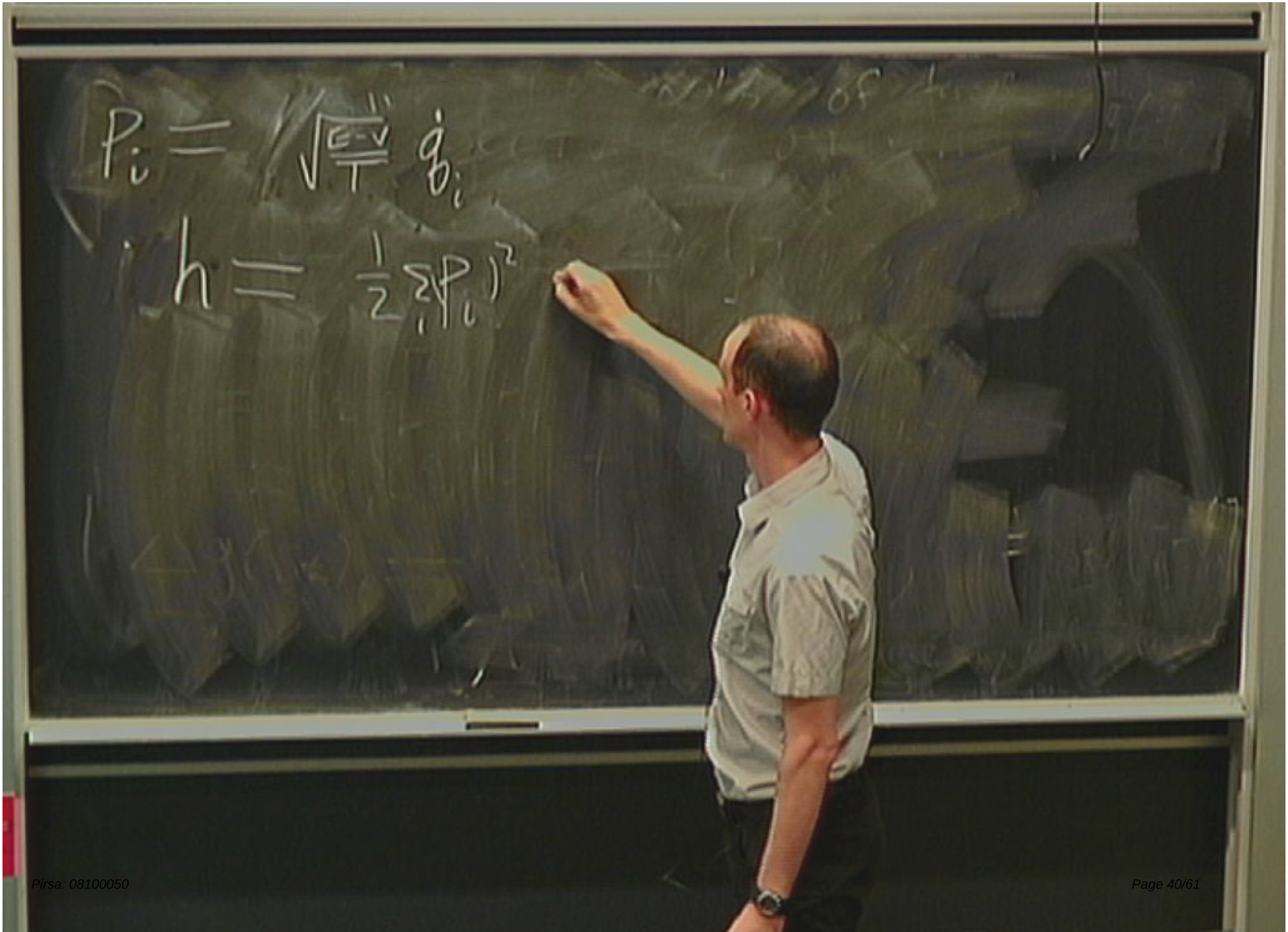
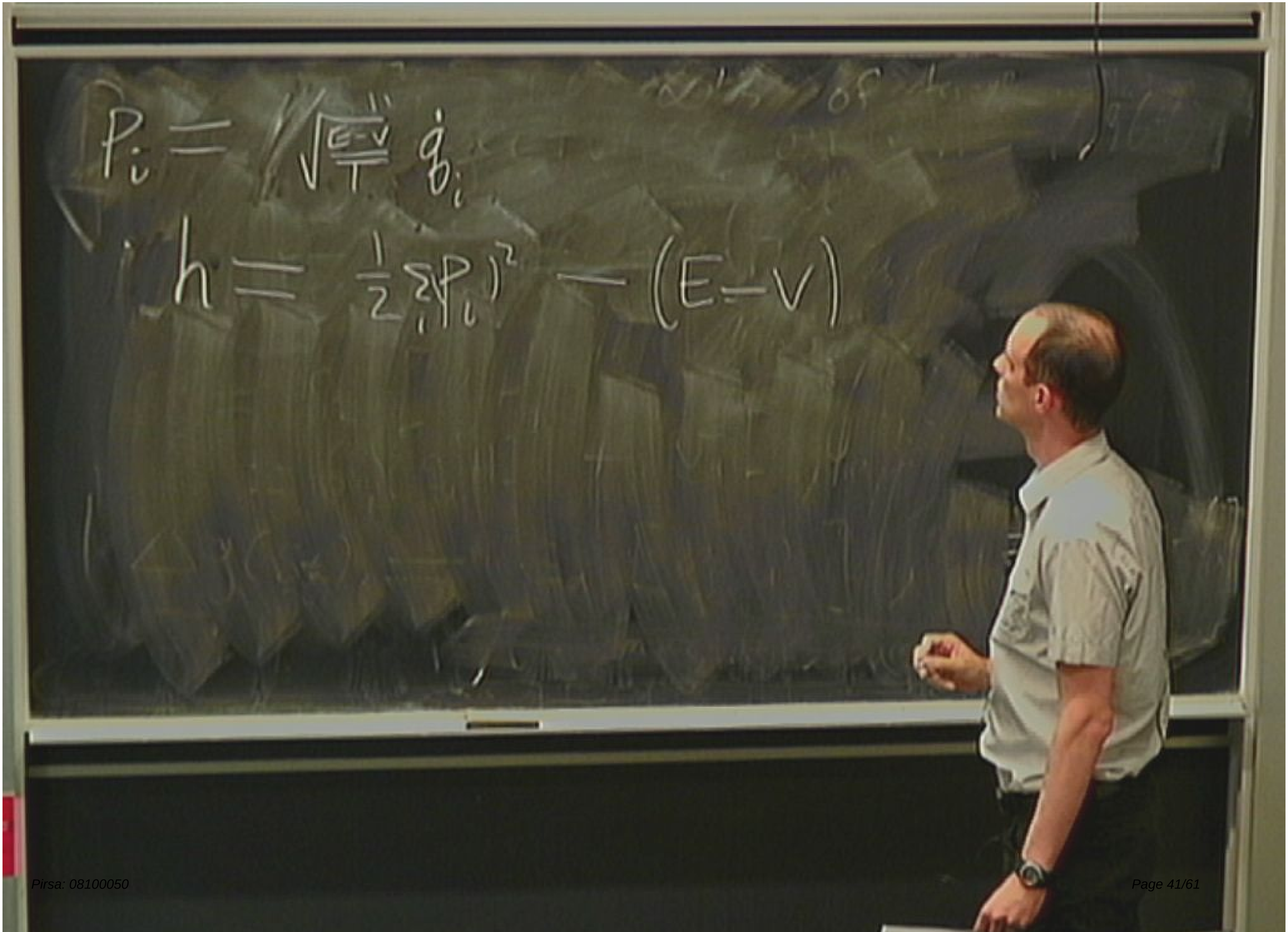
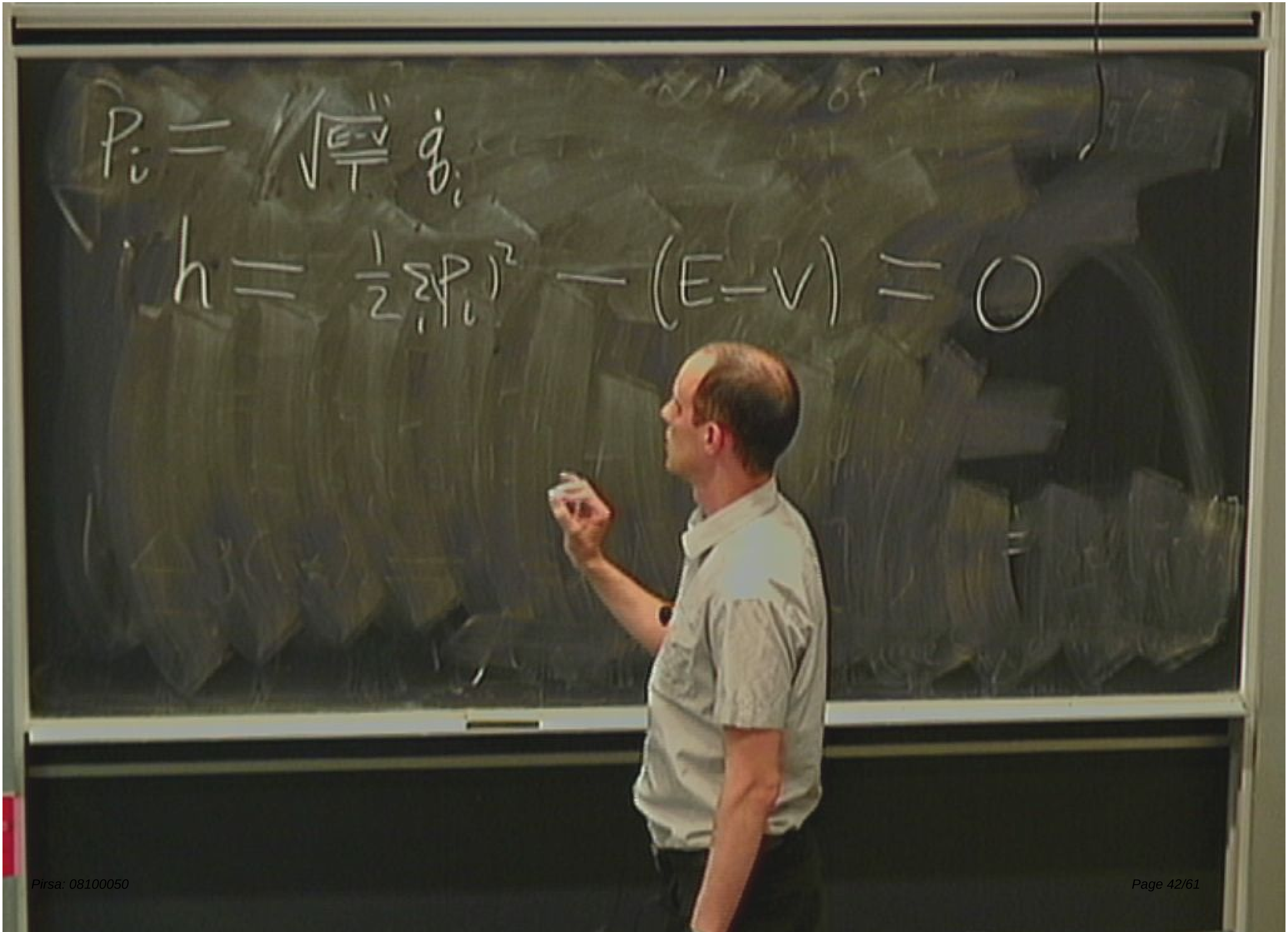
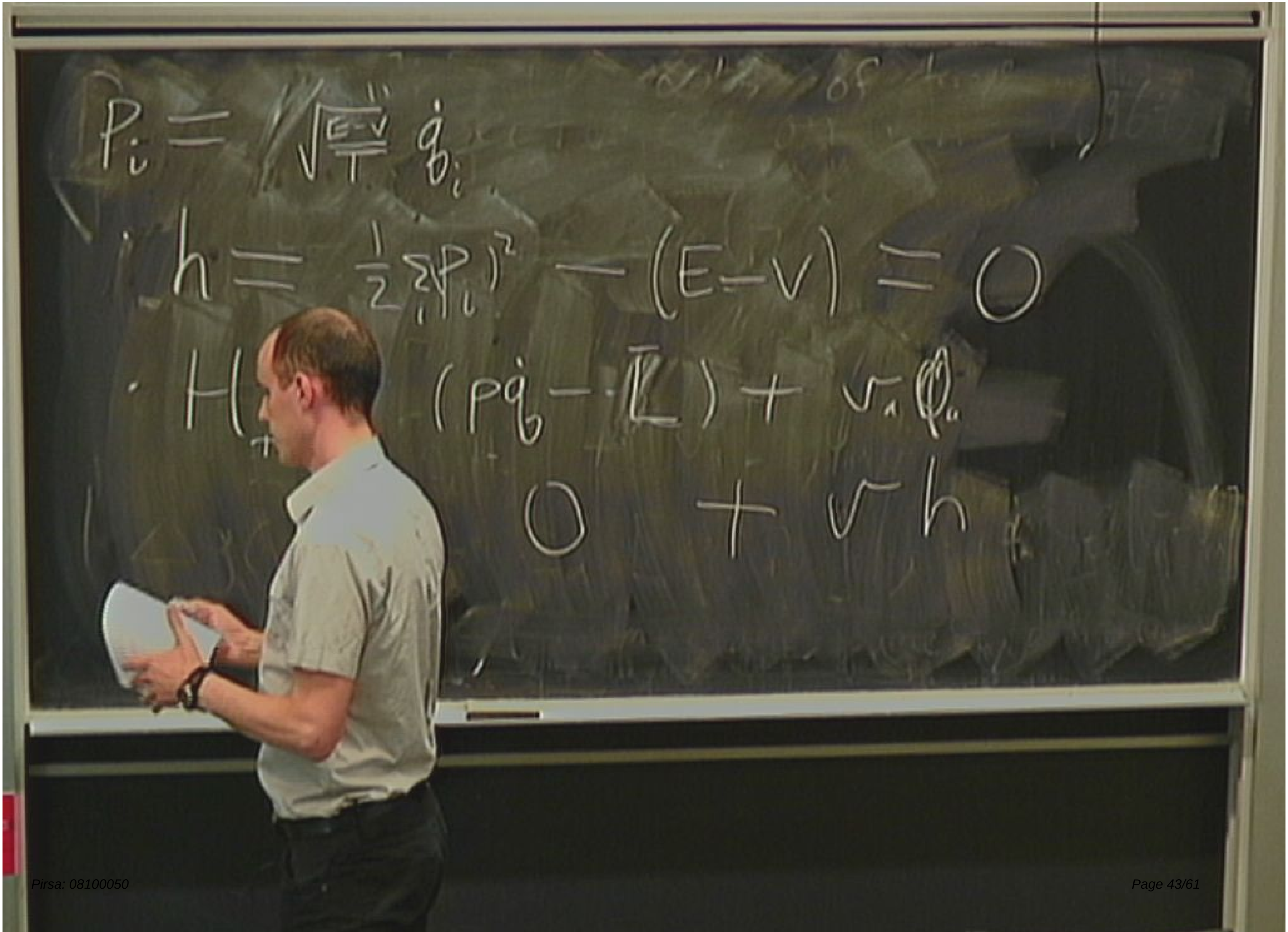

$p_i = \sqrt{\frac{E-V}{T}}\,\dot{q}_i$
$h = \frac{1}{2}\sum_i p_i^2$


$p_i = \sqrt{\frac{E-V}{T}}\,\dot{q}_i$
$h = \frac{1}{2}\sum_i p_i^2 - (E-V)$


$p_i = \sqrt{\frac{E-V}{T}}\,\dot{q}_i$
$h = \frac{1}{2}\sum_i p_i^2 - (E-V) = 0$

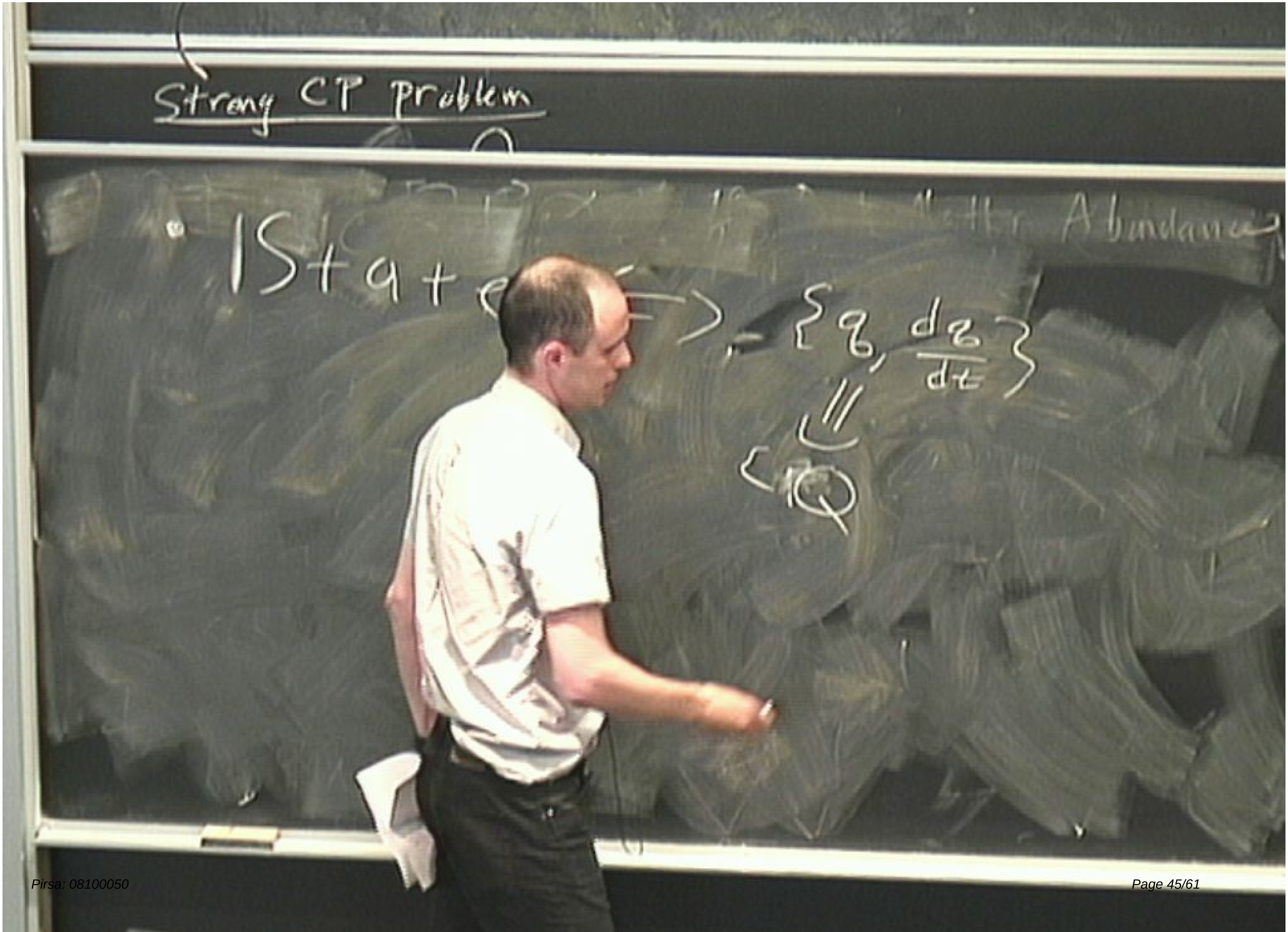

$p_i = \sqrt{\frac{E-V}{T}}\, \dot{q}_i$
$h = \frac{1}{2}\sum_i (p_i)^2 - (E-V) = 0$
$(p\dot{q} - L) + v_n \phi_n$
$0 + v h$

$$p_i = \sqrt{\frac{E-V}{T}}\,\dot{q}_i$$

$$h = \frac{1}{2}\sum_i p_i^2 - (E-V) = 0$$

$$H_T = (p\dot{q} - L) + v_a \phi_a$$

$$= 0 + v\,h$$


Strong CP problem
Abundance

Strong CP problem

Abundance

$$|\text{State}\rangle \Longleftrightarrow \left\{ q, \frac{dq}{dt} \right\}$$

$$\| $$

$$[Q, d_Q]$$

Abundance

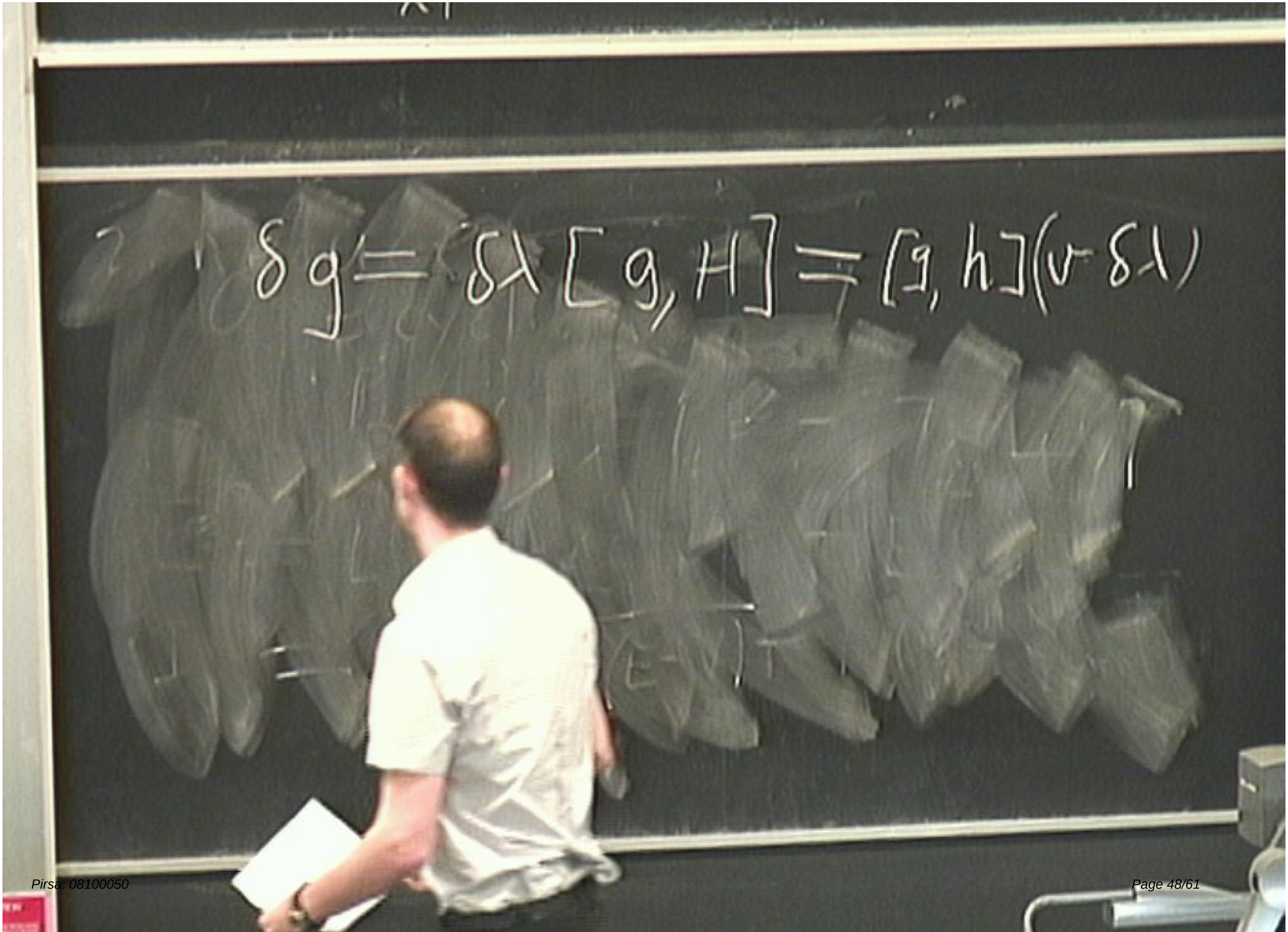
$$|\text{State}\rangle \Longleftrightarrow \left\{q, \frac{dq}{dt}\right\}$$

$$\| $$

$$[Q, d_Q]$$

$$\|$$

unique $[q, p]$


δg = δλ [g, H] = [g, h](v·δλ)

$$\delta g = \delta\lambda \, [g, H] = [g, h](v\,\delta\lambda)$$

$$I = \int d\lambda \, [p\dot{q} - H]$$

$$\delta I = 0 \iff \delta(v\,d\lambda) = 0$$

$$\delta g = \delta\lambda \, [g, H] = [g, h](\sqrt{\ }\,\delta\lambda)$$

$$I = \int d\lambda \, [p\dot{q} - H]$$

$$\delta I = 0 \iff \delta(\underbrace{\sqrt{\ }\,d\lambda}_{=\,d\tau}) = 0$$

$$\delta g = \delta\lambda\,[g, H] = [g, h](\underline{v}\,\delta\lambda)$$

$$I = \int d\lambda\,[p\dot{q} - H]$$

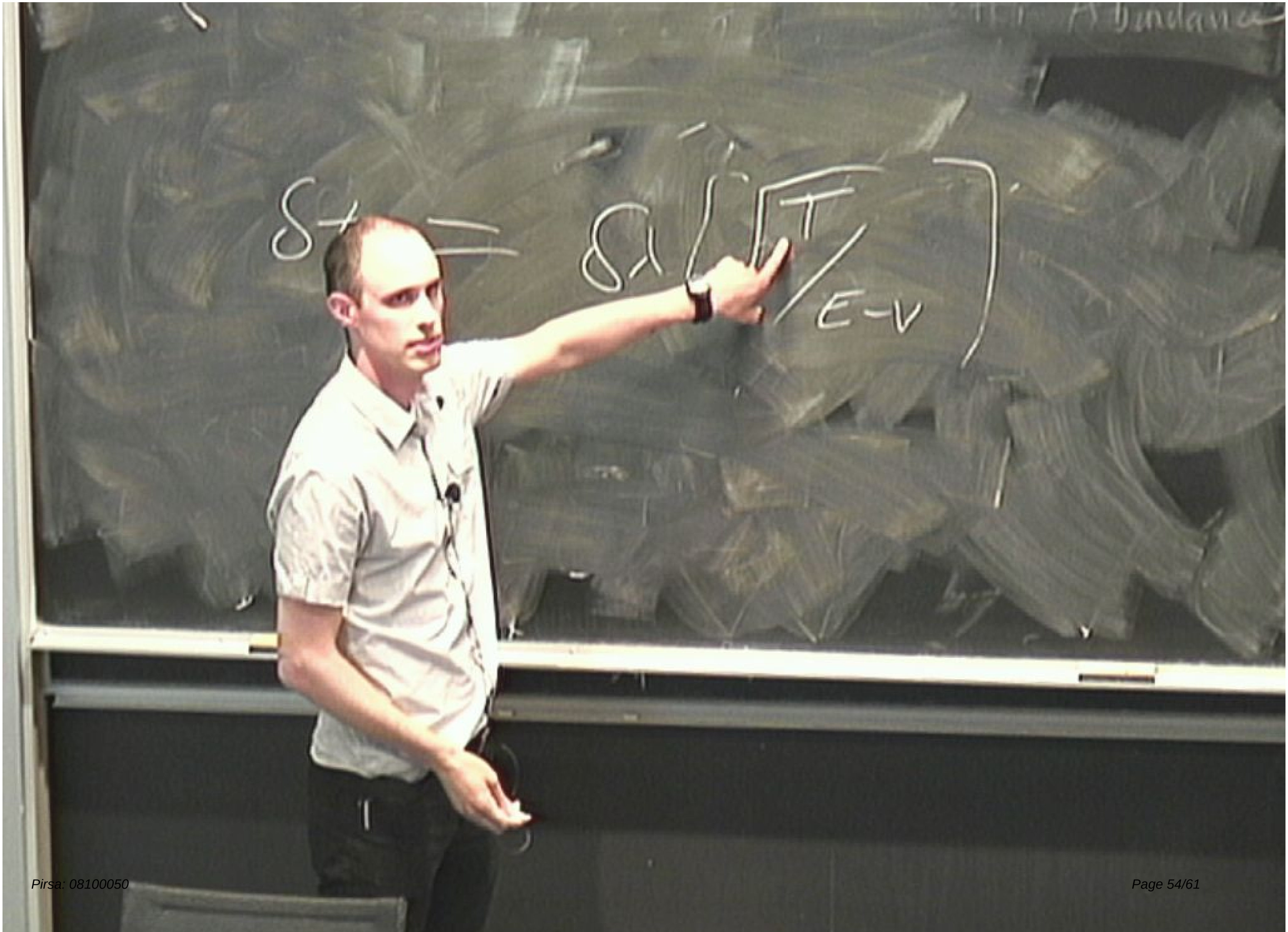
$$\delta I = 0 \iff \delta(\underbrace{\underline{v}\,d\lambda}_{= dt}) = 0$$

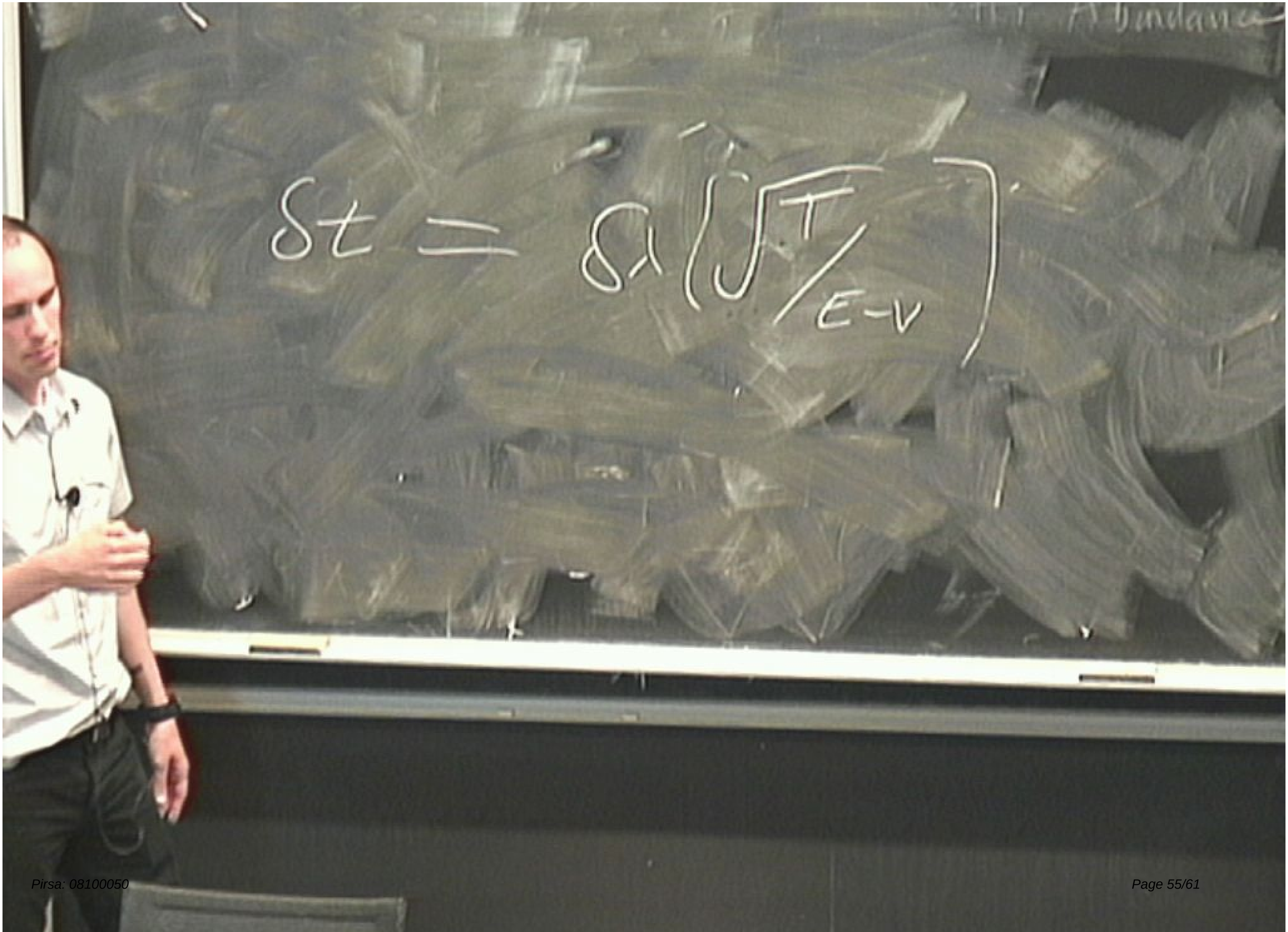
“We now have to face the problem of seeing how we can ensure that our quantum theory shall be a relativistic theory. For that purpose ***we have to go back to first principles…I would like to go back to the beginning of our Hamiltonian development*** and consider a special case.”

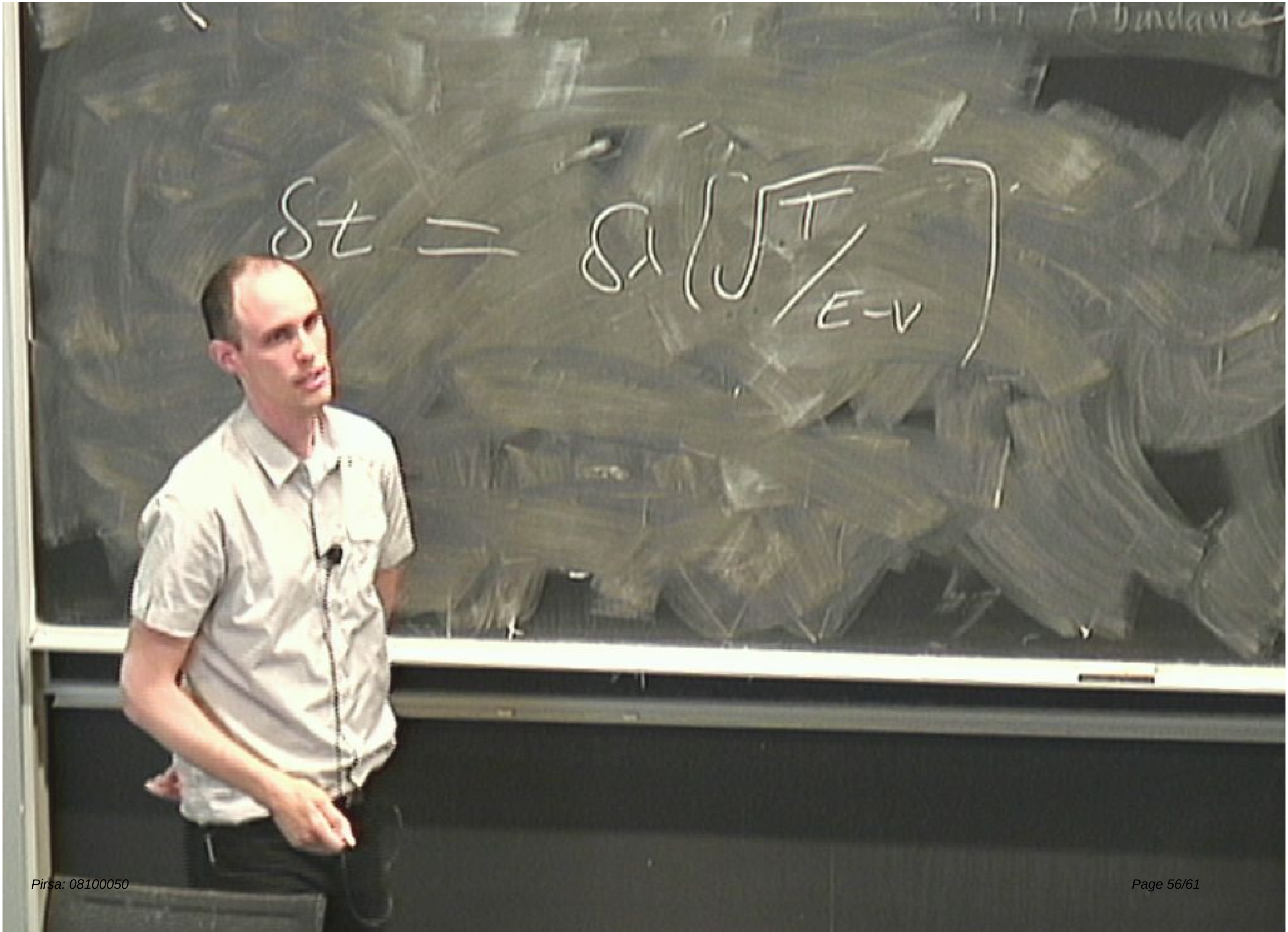
(pp. 45-46)

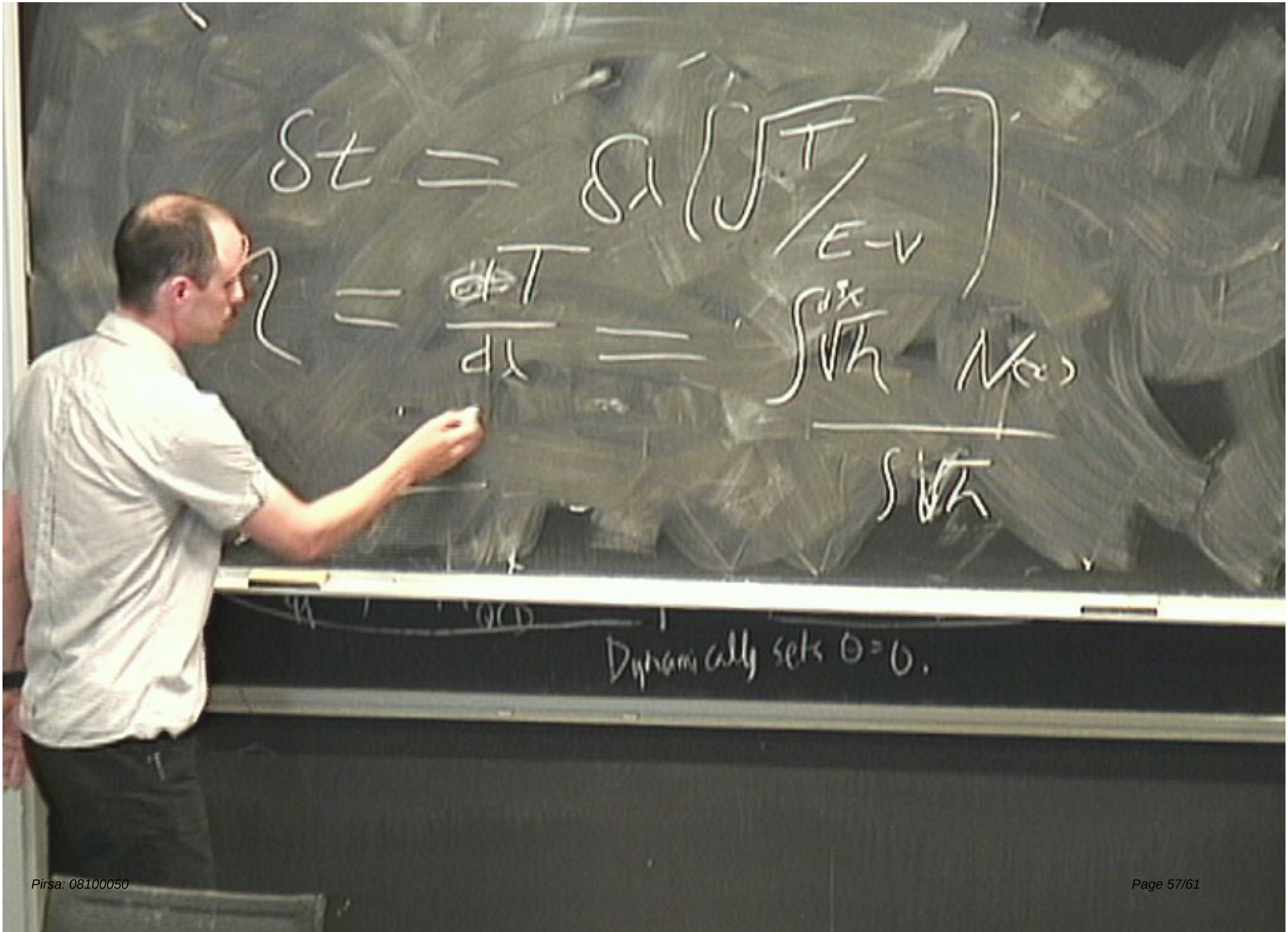
"One can see in this way that the [tangential component] is something which is not of real physical importance, it is just concerned with the mathematical technique. The quantity which is of real physical importance is the [normal component]. This [normal component] gives us the first-class constraint which is associated with a motion of the surface normal to itself. ***That is something which is of dynamical importance.***"

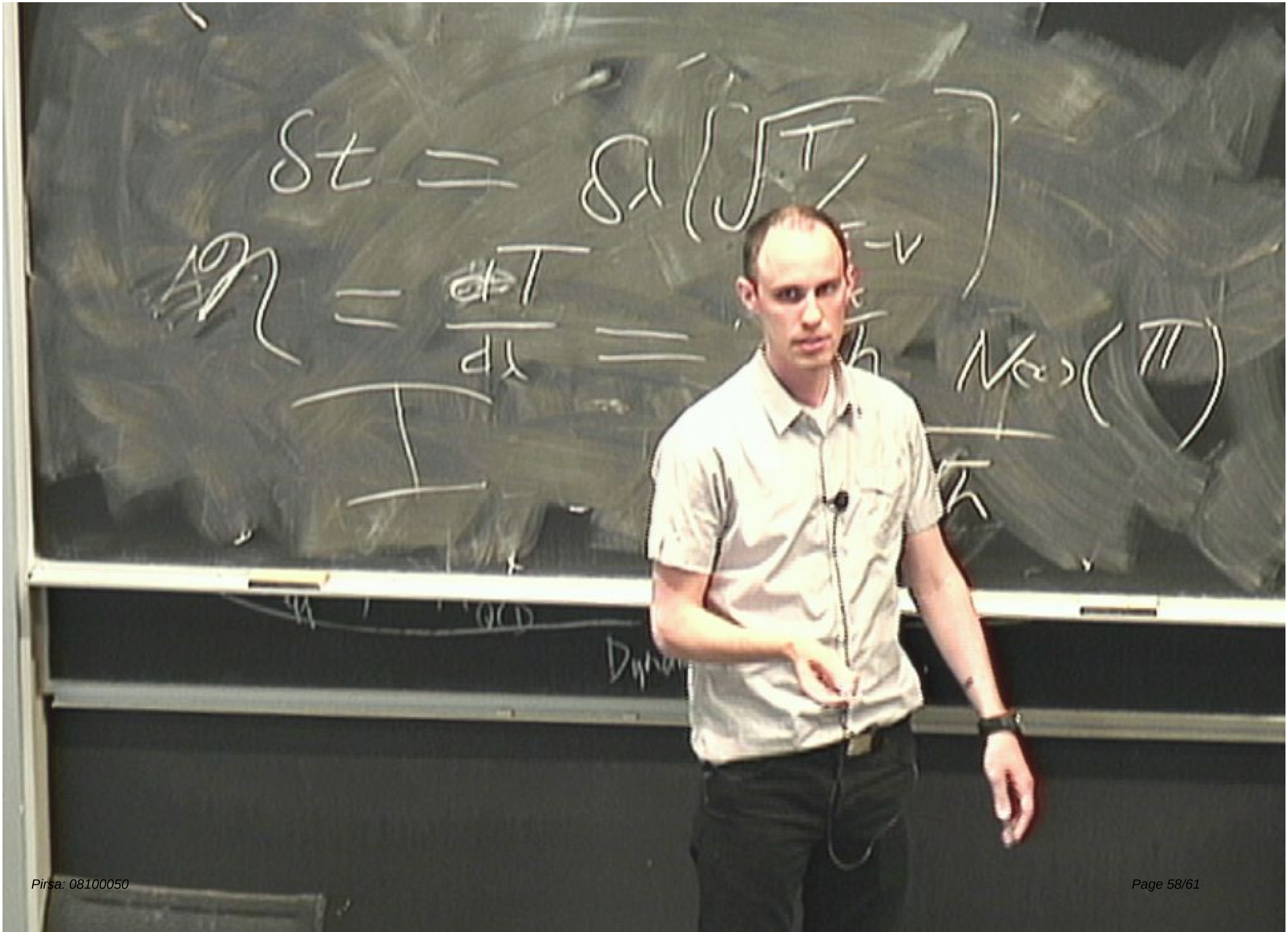
(p. 60)

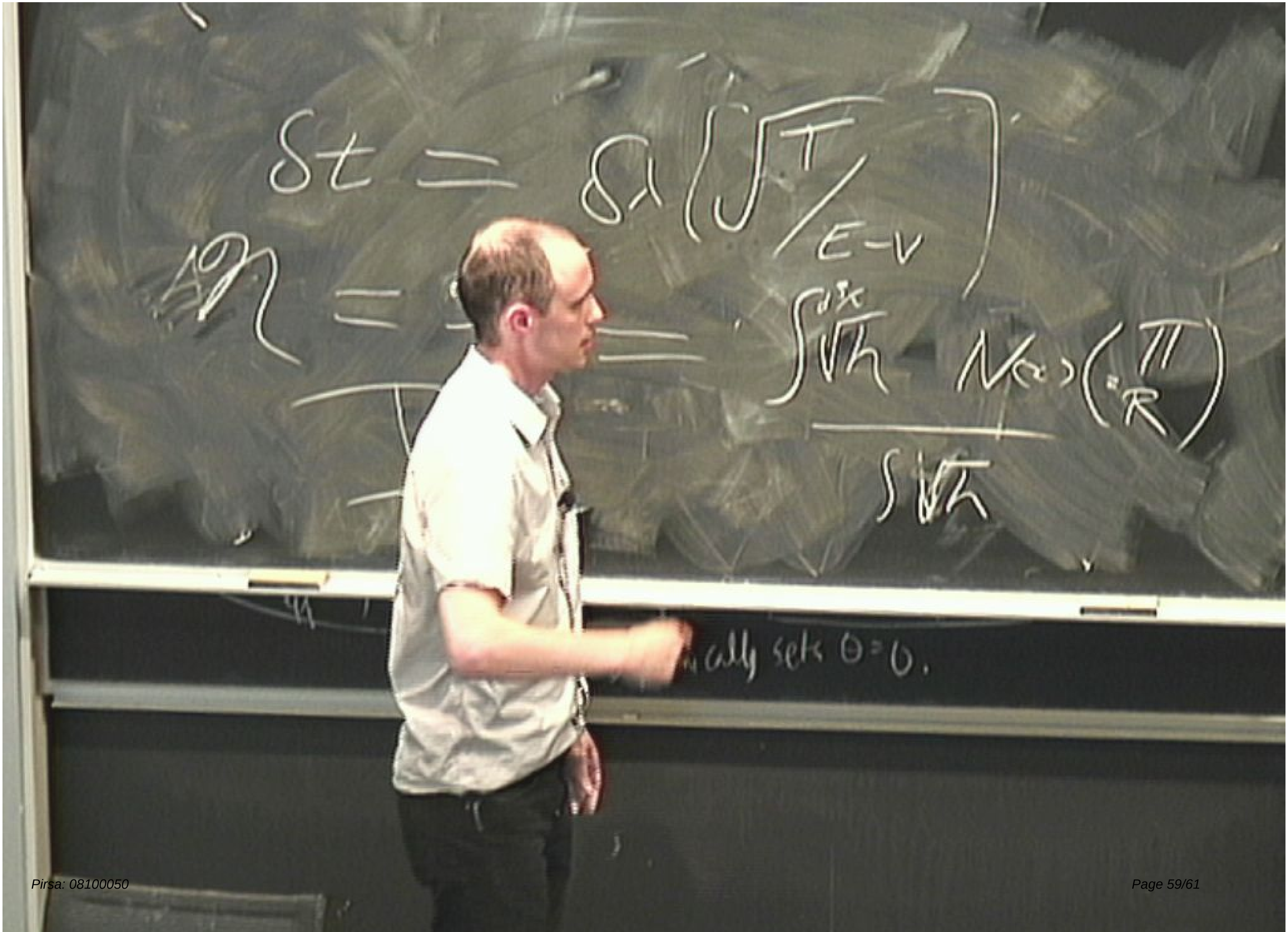



$\delta t = \delta a \left(\int T / _{E-V} \right)$


δt = δa (∫ T / E−V


Dynamically sets θ = 0.

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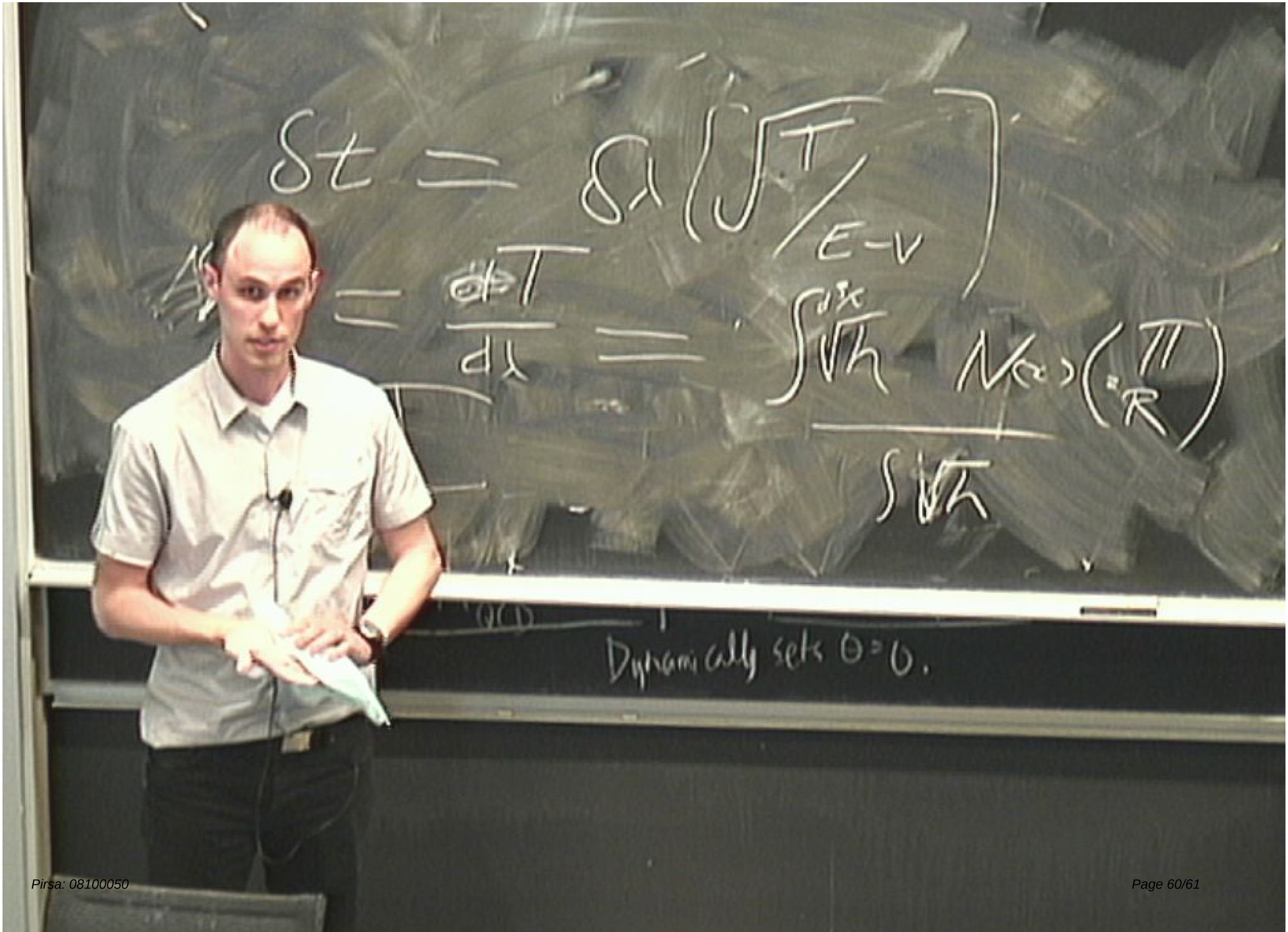
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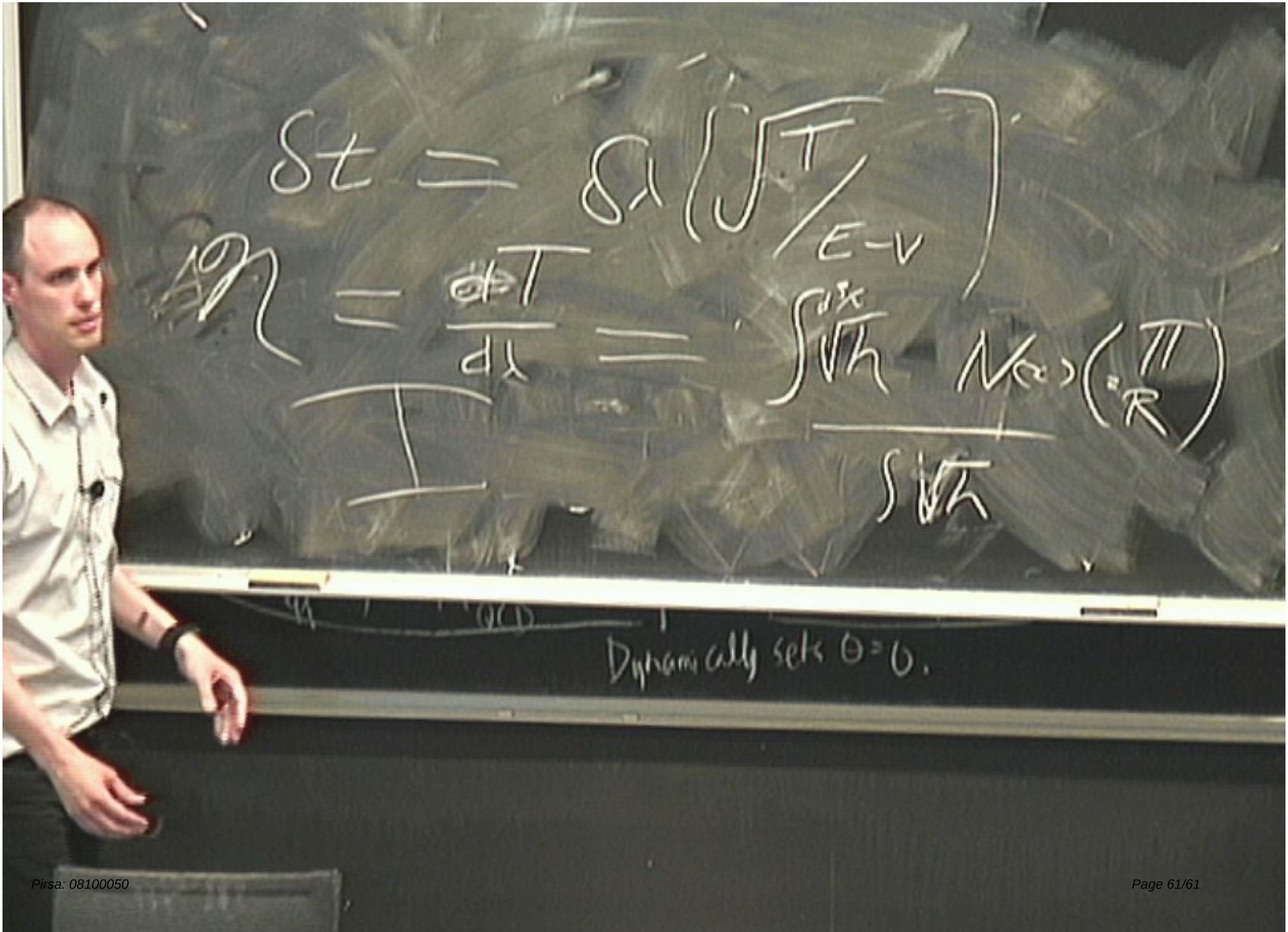
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