

Title: Time and the big bang

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Abstract: The evidence for the big bang is now overwhelming. However, the basic question of what caused the bang remains open. One possibility is that time somehow \emerged,\ placing the universe in an inflationary state. Another, perhaps more conservative possibility, is that the big bang was a violent event in a pre-existing universe. I will describe model calculations employing the AdS/CFT correspondence which show how this is possible, and which point to a new explanation for the origin of large scale structure in the universe.

①

Time and the Big Bang

Neil Turok, PI

work in progress w/ B. Craps, T. Hertog

- * the big question
- * AdS/CFT
- * Dual description
- * Quantum resolution
- * Particle creation + backreaction
- * GLASSy perturbations
- * Outlook + problems

Was the **big bang** the beginning? ²

YES → seem to require inflation
to smooth, flatten universe,
generate density variations

NO → inflation not needed,
epoch of slow contraction $w > 1$
can play the same role
- ekpyrotic/cyclic cosmology

Can we learn something from
string theory?

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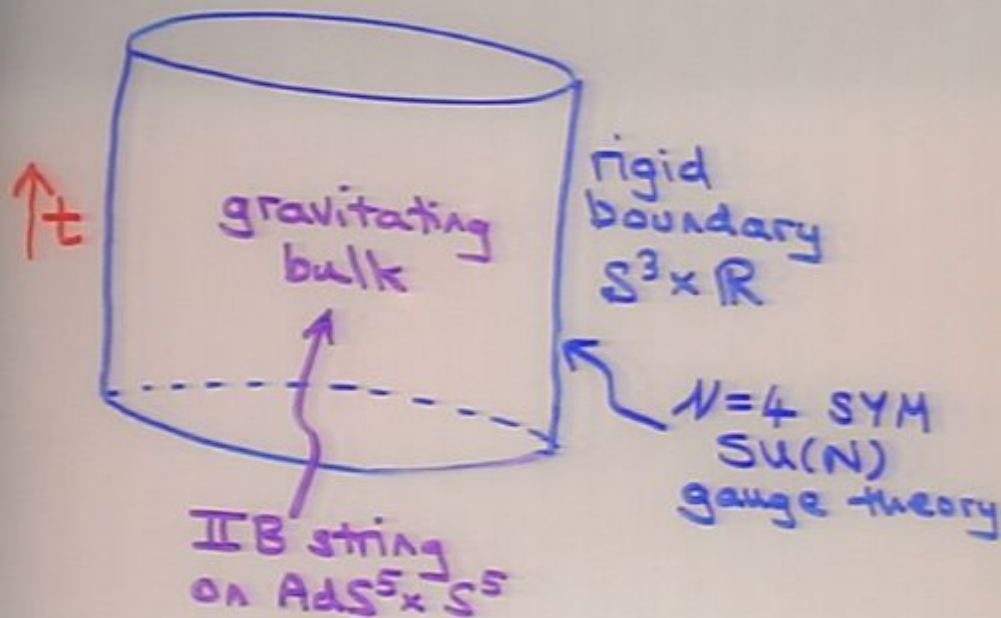
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AdS/CFT : a powerful tool for 3
quantum gravity (Maldacena, Witten...)

conformal picture :

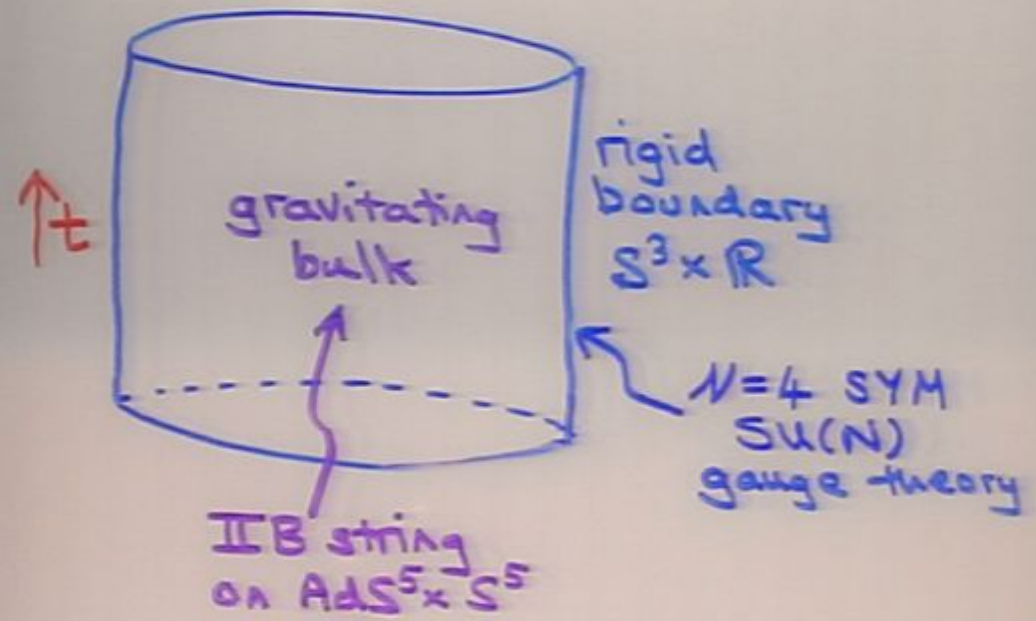


$$g_s \longleftrightarrow \frac{1}{N}$$

$(R_{AdS})^4$

AdS/CFT : a powerful tool for quantum gravity (Maldacena, Witten...)

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$$\left(\frac{R_{AdS}}{l_s}\right)^4 \longleftrightarrow g_{YM}^2 N \equiv g_t$$

't Hooft

④

Idea:

bulk cosmology \leftrightarrow boundary dual

Caveat:

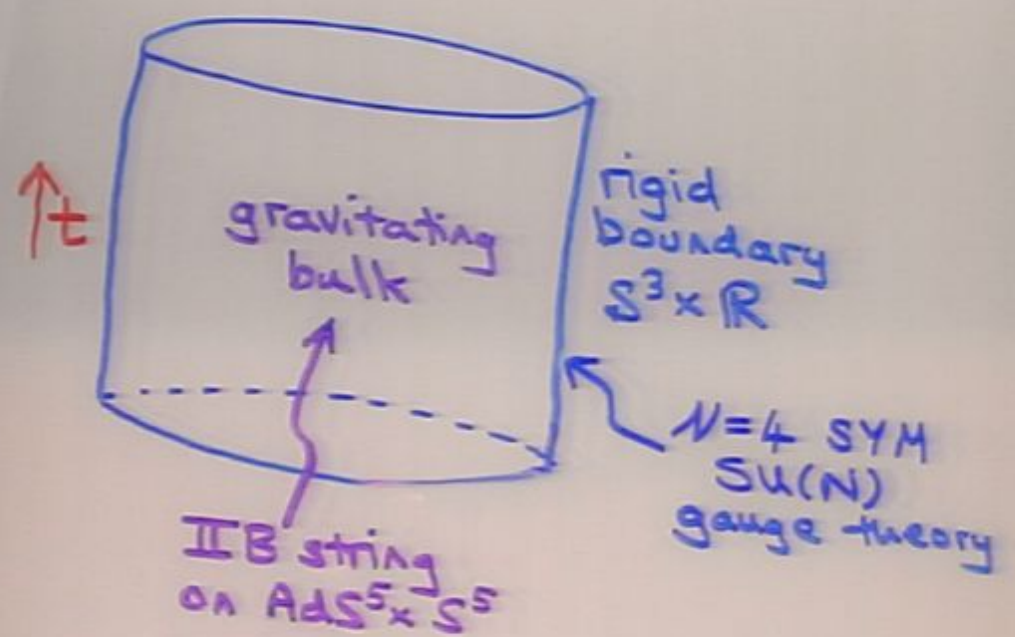
dual theory tractable for $g_t \ll 1$.

stringy corrections to Einstein gravity are large in this limit

not yet clear how to extend to $g_t \gg 1$

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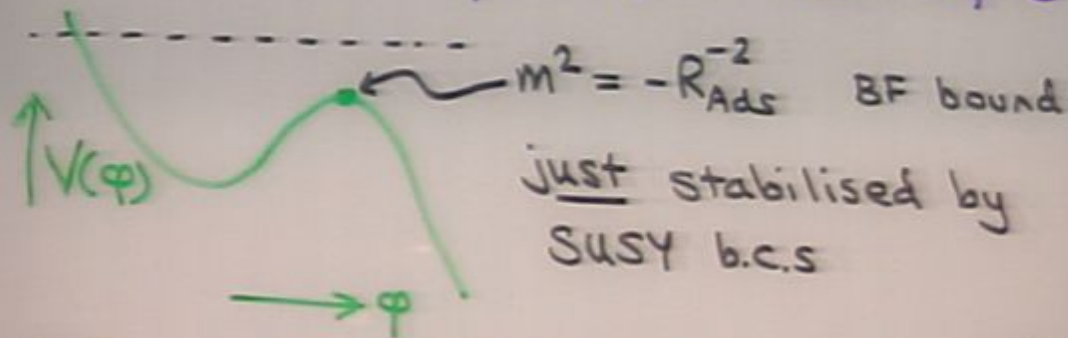
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SUGRA description of bulk :

truncate to just one scalar φ -
a particular quadrupole distⁿ of S^5



$$\text{AdS: } ds^2 = R_{\text{AdS}}^2 \left[-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2 \right]$$

$$r \rightarrow \infty \quad \varphi \sim \alpha r^{-2} \ln r + \beta r^{-2}$$

α, β fns on $\mathbb{R} \times S^3$

SUSY b.c.s $\alpha = 0 \rightarrow$ stable

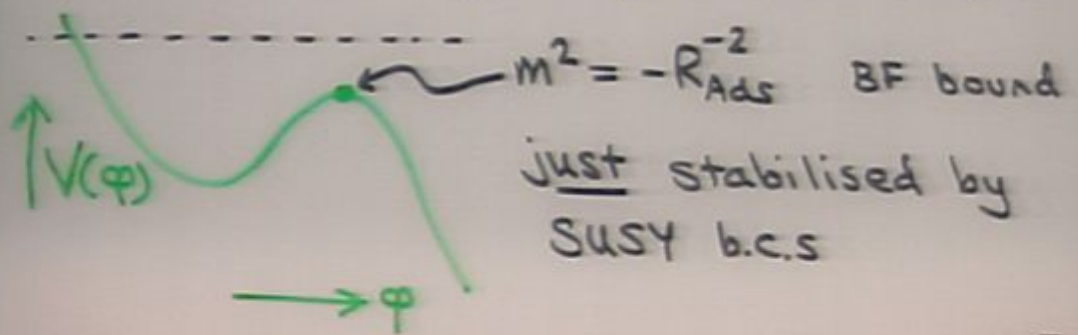
Generalised b.c.s $\alpha = f\beta,$ cosmological
 $f > 0 \rightarrow$ dynamics

\rightarrow big crunch : what is it? \dots

5

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\rightarrow big crunch : what next? Hertog + Horowitz

Dual theory:

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AdS/CFT
dictionary

$$\varphi \leftrightarrow \mathcal{O} = \frac{1}{N} \text{Tr} \left[\Phi_1^2 - \frac{1}{5} \sum_{i=2}^6 \Phi_i^2 \right]$$

$$\beta \leftrightarrow \langle \mathcal{O} \rangle$$

$$d \leftrightarrow \text{source for } \mathcal{O}$$

$$S_{\text{SYM}} \rightarrow S_{\text{SYM}} + \int d^4x f \mathcal{O}^2$$

$$f > 0 \rightarrow \underline{V \sim -f \phi^4}, \quad \phi^2 \equiv \mathcal{O}$$

General feature:

Cosmological
Dynamics



Unbounded
negative scalar
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→ no ground state, nevertheless,

theory is *perturbatively
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General feature:

Cosmological Dynamics \leftrightarrow Unbounded negative scalar potential

\rightarrow no ground state, nevertheless,

theory is * perturbatively renormalisable

* asymptotically free

Renormalized potential

(7)

$$V_{\text{ren}} \sim -\frac{1}{N^2} \frac{\phi^4}{\ln(\phi^2/M^2)}$$

exact at large $N \rightarrow$ quantum corrections do not turn potential around.

Can we make sense of this theory?

$V \sim -\lambda \phi^4$: homogeneous mode on S^3 ,
 $\bar{\phi}$ runs to infinity in finite time
(agrees w/ bulk cosmology for α, β).

But S^3 finite $\rightarrow \bar{\phi}$ is quantum

\rightarrow can define a self-adjoint extension

\rightarrow well-defined quantum evolution

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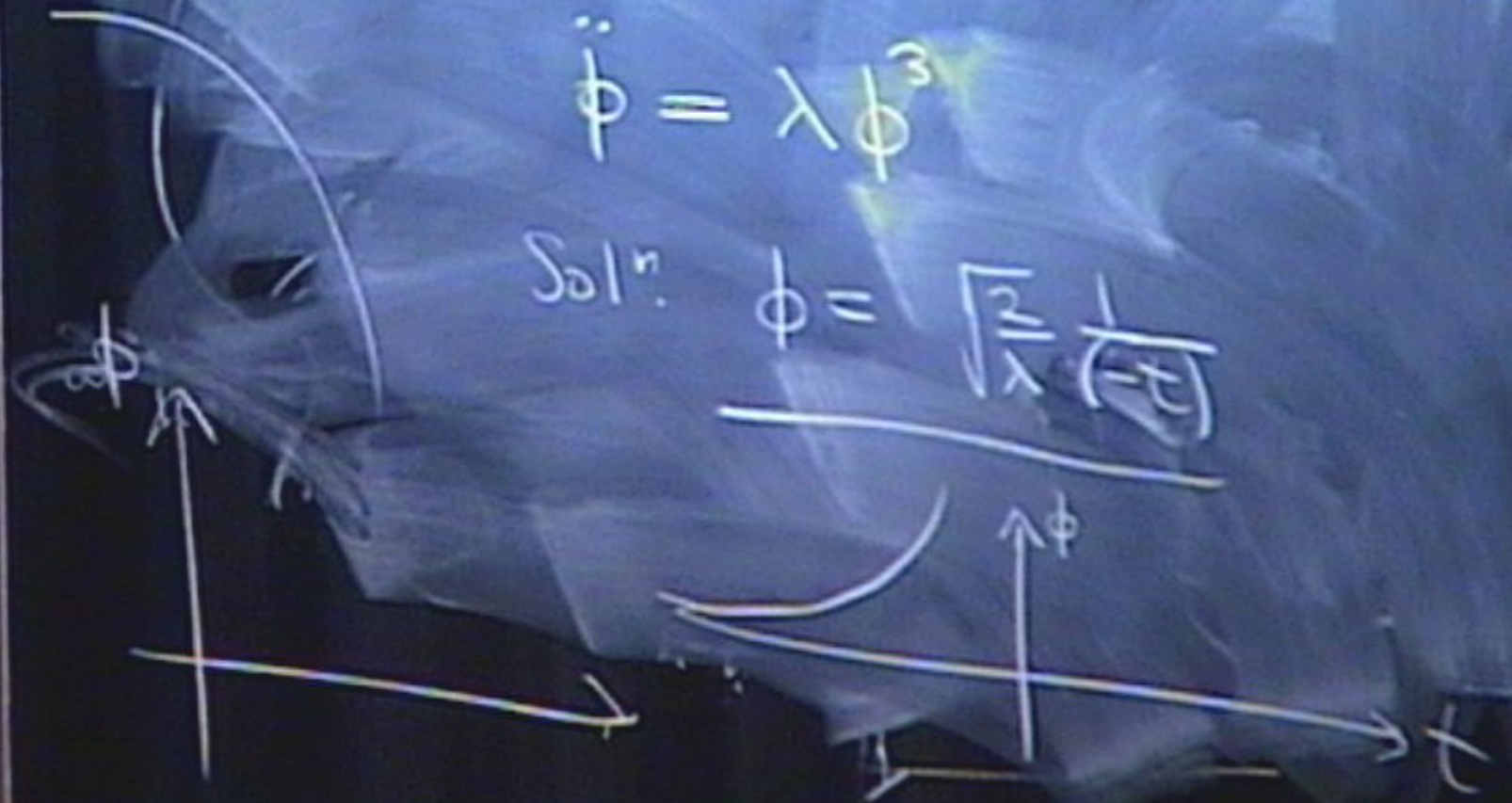
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$$V_3 \int dt \left(\frac{1}{2} \dot{\phi}^2 + \frac{\lambda}{4} \phi^4 \right)$$

$$\ddot{\phi} = \lambda \phi^3$$

$$\text{Sol}^n. \quad \phi = \sqrt{\frac{2}{\lambda}} \frac{1}{t}$$



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e.g. energy eigenstates



$$\Psi_E^\pm \sim \frac{e^{\pm i\sqrt{\lambda}\bar{\phi}^3}}{\bar{\phi}}$$

WKB becomes exact as $\bar{\phi} \rightarrow \infty$

- Hamiltonian is self-adjoint if
restrict Hilbert space to


$$\Psi_E^+ + e^{i\alpha} \Psi_E^-$$

with α independent of E .

Key point: spread in wf^a keeps
you away from the singularity

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Notes:

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non-positivity of gravitational

$$KE \quad , \quad \tilde{S}_{EH} \sim - \int dt a \dot{a}^2$$

\leftrightarrow non-positivity of dual PE

$$\tilde{S}_{\phi} \sim \int dt \left(\frac{1}{2} \dot{\phi}^2 + \lambda \phi^4 \right) \quad \text{i.e. negative}$$

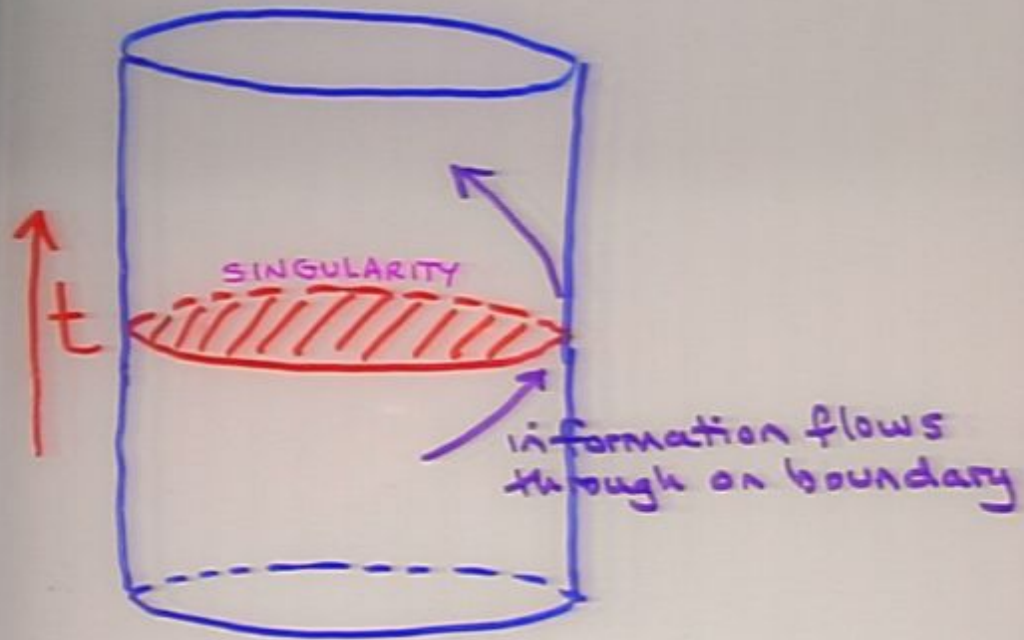
but dual theory has no ordering problems, and role of time is completely conventional

— space is "emergent"

— time is not !

96

General idea:



9a

Note': unconventional features

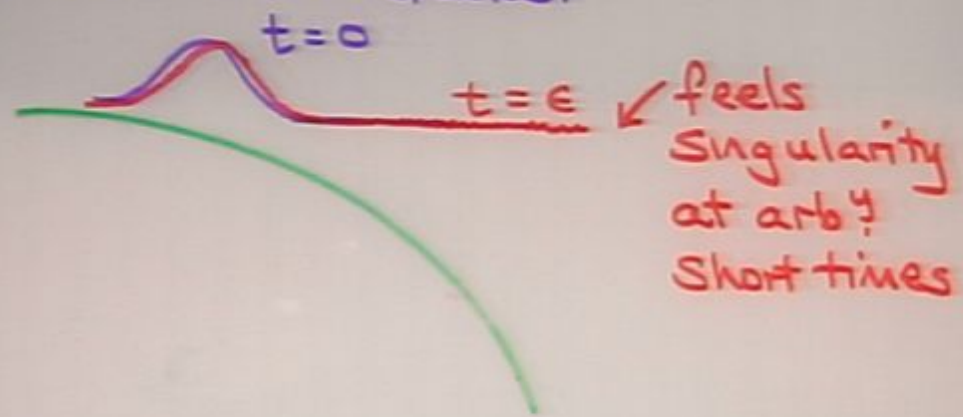
- * Field theory has no ground state
- * Canonical ensemble doesn't exist
- * Expectation values generally ill-defined $\Psi \sim \frac{1}{x} \cos(x^3)$
 $\langle x \rangle = \int dx \frac{1}{x^2} \cos^2(x^3) \cdot x = \infty!$

Note:

quantum premonition

due to nonlocality (omni-science!)

eg. localised wavepacket



- the universe knows the singularity is coming!
- have to impose BC there, at all times

Key points

(11)

- * Semiclassical approxⁿ becomes exact as $x \rightarrow \infty$ ($x = \sqrt{V_3 \bar{\phi}}$)
- * Field evolution becomes ultralocal near singularity - spacelike separated points decouple

~~spacelike separated points decouple~~



- these allow us to solve the quantum dynamics of ϕ .

Complex Classical Solutions 12

Semiclassical (\hbar) expansion $\Psi = A e^{iS/\hbar}$ A, S series in \hbar

leading order $S = S_{cl}$

$$S_{cl}(x, t) = \int_{t_i, x_i}^{t_f, x} (p\dot{x} - V) dt + \frac{\hbar}{i} \log \mathcal{F}(x_i)$$

e.g. Gaussian wpkt

$$x + 2i \frac{pL^2}{\hbar} = x_c + 2i \frac{p_c L^2}{\hbar} \quad t = t_i$$
$$\left(\rightarrow \Psi \sim e^{ip_c x} e^{-(x-x_c)^2 / 4L^2} \right)$$

$$x = x_f$$

$$t = t_f$$

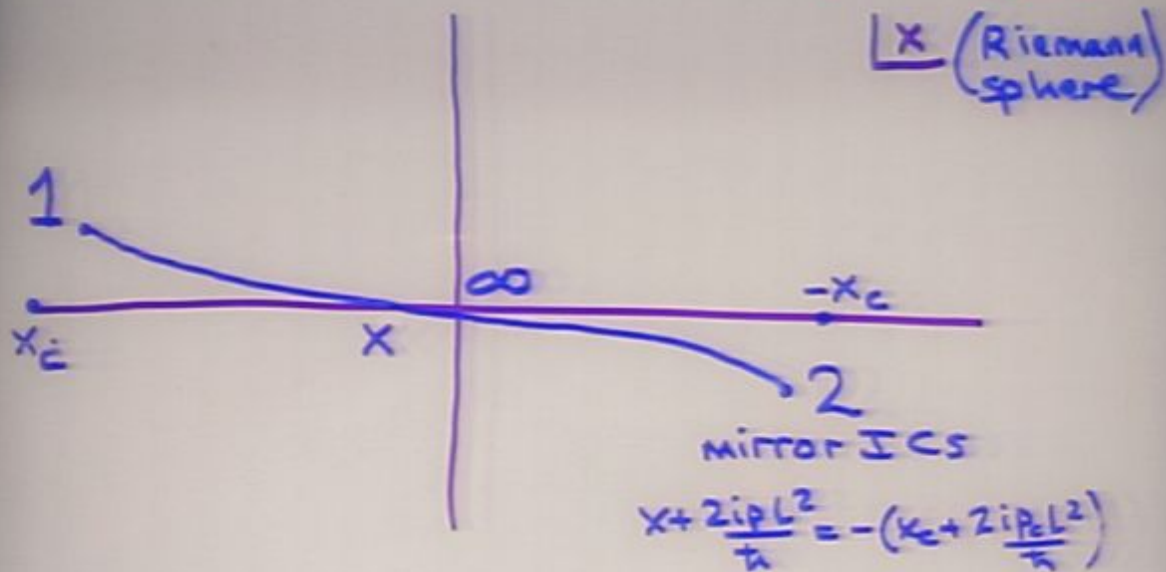
- line in complexified phase space

Note: two limits in which solution

is real: $L=0$ and $L=\infty$

Generically solution is complex

We can implement SA bc 13
 at $x \rightarrow \infty$ via method of images:



Symm $x \rightarrow -x$ guarantees no flux
 through $x = \infty$.

(3a)

⇒

$$H \sim \frac{1}{x} \left(e^{ix^3} e^{-\frac{1}{2}(\frac{1}{x}+t)^2} + e^{it} e^{-ix^3} e^{-\frac{1}{2}(\frac{1}{x}-t)^2} \right)$$

Gaussian
running past
 $x = \infty$

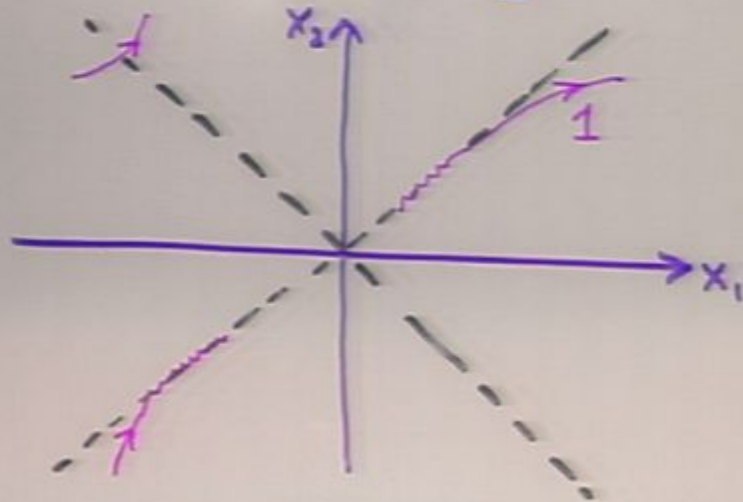
Gaussian
entering
at $x = \infty$

$$G = \frac{1}{L t_i^2} \frac{1 + iL^2/\pi t_i}{1 + 6iL^2/\pi t_i}$$

2 site model

(14)

$$S = \int \frac{1}{2} \dot{x}_1^2 + \frac{1}{2} \dot{x}_2^2 - \frac{(x_1 - x_2)^2}{\Delta^2} + \lambda x_1^4 + \lambda x_2^4$$



$$\Psi = \left(A_1 e^{iS_1/\hbar} + e^{i\theta} A_2 e^{iS_2/\hbar} \right) + e^{i\theta} \left(A_3 e^{iS_3/\hbar} + A_4 e^{iS_4/\hbar} \right)$$

IC: $x_1 \rightarrow -x_1$ $x_3 \rightarrow x_2$ $x_1 \rightarrow -x_1$
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ignore coupling $\frac{1}{\Delta^2} \rightarrow$ sat SA bcs exactly

$\frac{1}{\Delta^2}$ small \rightarrow need to adjust ICs for

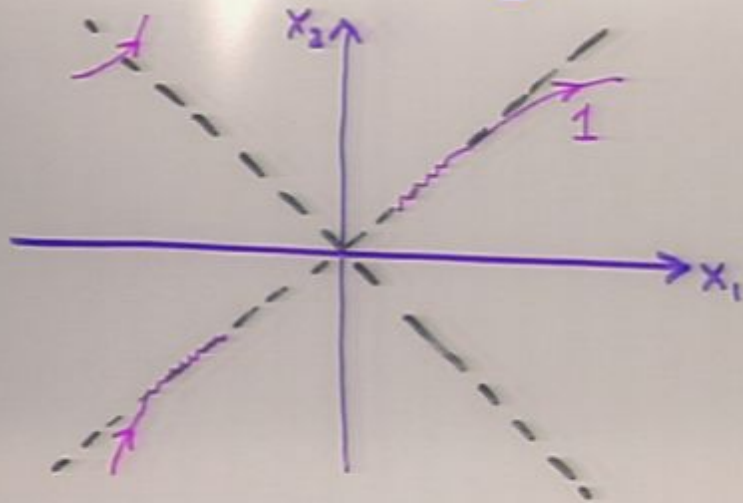
2, 3 to ensure SA bcs satisfied

Note: new phase for each site: $\pm \text{transl}^N$

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$$\Psi = \left(A_1 e^{i\beta_1 x_1 / \Delta} + e^{i\alpha} A_2 e^{i\beta_2 x_2 / \Delta} \right) + e^{i\beta} \left(A_3 e^{i\beta_3 x_1 / \Delta} + A_4 e^{i\beta_4 x_2 / \Delta} \right)$$

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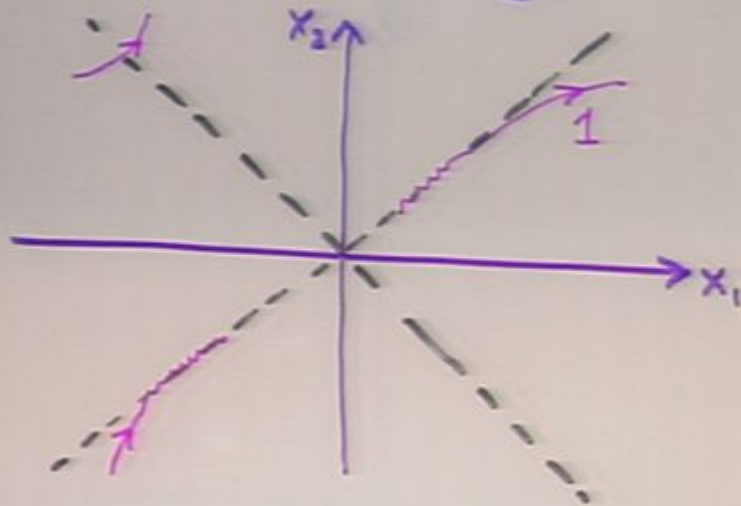
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invariance sets $\alpha = \beta$

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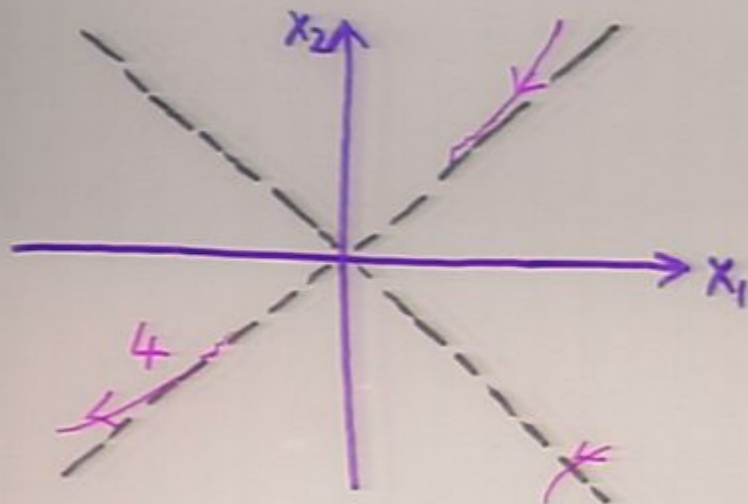
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Upshot : solⁿ 4 dominates well (15)
after the bounce:



Particle Creation

(16)

homog. mode $\bar{x} = x_1 + x_2$

pertⁿ $\delta x = x_1 - x_2$

ICs : at $t = t_i$ $\bar{x} + 2i\frac{\bar{p}L^2}{\hbar} = x_c + 2i\frac{p_c L^2}{\hbar}$

- gaussian wpkt centred on $x_c = -\frac{1}{t_i \lambda}$

and $\delta x + \frac{i}{\omega} \delta \dot{x} = 0$ ($\omega^2 = 2/\Delta^2$)

- δx in its adiabatic vacuum $\delta x \sim e^{i\omega t}$
pos. freq.

$$S = \int \dot{\bar{x}}^2 + \delta \dot{x}^2 + \lambda \bar{x}^4 + \lambda \bar{x}^2 \delta x^2 + \lambda \delta x^4 - \frac{2}{\Delta^2} \delta x^2$$

background $\ddot{\bar{x}} = \lambda \bar{x}^3 \rightarrow \bar{x} \sim \frac{1}{\sqrt{\lambda}(t-i\epsilon)}$
near $t=0$

pertⁿs $\delta \ddot{x} = \lambda \bar{x}^2 \delta x - \frac{2}{\Delta^2} \delta x + o(\delta x^3) \cdot \lambda$
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ϵ related to λ

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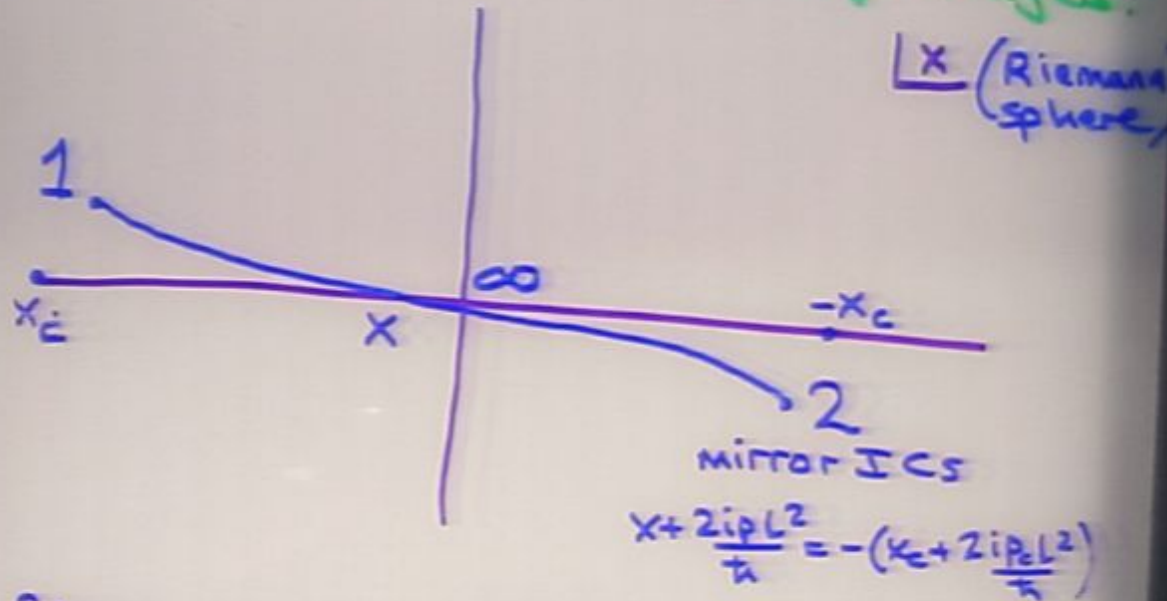
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ϵ related to L : $x_i = 0$ if $\text{Re}(t-i\epsilon) > 0$

Solution: Hankel : pos. freq. in
→ pos. freq. out
→ no particle production at
lowest order!

(17)

(+ Tolley
+ NT)

Full nonlinear treatment:

conclusion remains, particle
production very small, back
-reaction negligible at small λ .

- this is a result of the coupling
 λ being constant, in turn a result
of classical scale-invariance

BUT:

(18)

in QFT for $AdS^5 \times S^5$, λ FUNS

$$\rightarrow \omega^2 = k^2 - \frac{6}{(t-i\epsilon)^2} \left(1 + \frac{5}{12} \frac{1}{\log(Mt)} + o\left(\frac{1}{\log^2}\right) \dots \right)$$

particle production

$$\text{Bogoliubov } \beta \sim \frac{1}{\log(k_M)} e^{-2k\epsilon}$$

$$\Rightarrow P_{\text{particles}} \sim \int d^3k \cdot k \frac{1}{\log^2(k_M)} e^{-4k\epsilon}$$

$$\sim \frac{1}{\epsilon + \log^2(k\epsilon)}$$

Est $\epsilon \sim \frac{R_{\text{AdS}}}{\sqrt{\log(MR_{\text{AdS}})}} \quad (\text{minimal sprd wpkt, typical } X_f)$

$$\rightarrow P_{\text{particles}} \sim \frac{1}{R_{\text{AdS}}^4} \quad \text{c.f. } P_{\text{class}} \sim \frac{\log(MR_{\text{AdS}})}{R_{\text{AdS}}^4}$$

- backreaction small

N-dependence

(19)

$$\rho_{\text{particles}} \sim N^5 \quad \rho_{\text{classical}} \sim \lambda^{-1} N^2$$

BR small if $\lambda < N^{-3}$, but $V(\phi)$ isn't under control there (yet!)

$$V_{\text{1-loop}} \sim \lambda^2 \quad \text{large } N \text{ corr's} \sim \frac{\lambda g_t^2}{N^2} (?)$$

- can neglect if $g_t^2 < \frac{1}{N}$, but this is getting further from gravity regime

models with UV fixed point at $\lambda < 0$ may bounce in a calculable regime

e.g. Dymarsky, Klebanov, Roiban : quiver gauge theories obtained by orbifolding S^5

Origin of Perturbations (20)

Observations: perts are approx:

Gaussian

Linear

Adiabatic

: GLASSY.

Scalar

Scale-invariant

- a prediction of simple inflation models,
but perhaps more generic?

Here, bulk curvature pert'n's $\delta h_{\mu\nu}$
are related via AdS/CFT to
boundary stress-tensor correlators

$$\langle \delta T_{\mu\nu}(x) \delta T_{\rho\lambda}(x') \rangle \text{ etc.}$$

Conformally (invariant)

(21)

Straightforward calculations show these are scale-invariant up to log corrections e.g.

$$\langle \delta T_{00}(r,t) \delta T_{00}(0,t) \rangle \sim \frac{1}{r^6 t^2} \frac{\ln(\frac{t}{\mu t})}{\ln(\mu r)^2}$$

- and slightly red due to asympt. freedom.

Conjecture: observed GLASSY pert's due to asymptotic conformal invariance of dual field theory

Outlook + Problems

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- * AdS/CFT with generalised BCs + deformed dual theory provide a laboratory for studying cosmic sing⁴'s
- * Simplest model doesn't bounce, but more realistic models might
→ Selection on the 'landscape'?
- * Need model with UV fixed point - asymptotic conformal invariance
- * Need to translate boundary correlators into the bulk
- * Need to extend this to large g_t