Title: IR modification of gravity and the forbidden mass range of graviton in de Sitter Space

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Abstract: One of the most challenging problems in theoretical physics today is the so called cosmological constant problem. While current observational constraints are consistent with the predictions of GR with a tiny cosmological constant, often referred to as the dark energy, it remains possible that it\'s the deviation of the law of gravity at large distance from Einstein\'s theory that resolves the puzzle. In this talk, I will briefly review some of the theoretical attempts made along this line, including the simple massive gravity, large extra dimensions, Unimodular gravity, classically constrained gravity, as well as their difficulties. I will then focus on some most recent study on the theory of massive graviton in de Sitter space, which may be more closely related to the reality both today and during the inflationary epoch. In particular, I would describe a model, in which one is able to open up the forbidden mass range of the graviton on a de Sitter background discovered by Higuchi.

# IR modification of Gravity and the forbidden mass range of graviton in de Sitter space

Yanwen Shang

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Department of Physics
University of Toronto

Perimeter Institute

# Outline

1. Motivation
2. Introduction of massive gravity
3. "Classically constrained" models
4. Massive gravity on de Sitter background
   - The special case with $m^2 = 2H^2$
   - More general case for $m^2 < 2H^2$
   - Solutions
   - Some more discussions
5. Summary and outlook

# Outline

1. Motivation

# Motivation

- Cosmological constant problem
- Degravitation of the dark energy, G. Dvali, S. Hofmann, J. Khoury
- Cosmic expansion

Andrew's talk yesterday gave a much better summary.

- de Sitter background might be relevant to both late time cosmology and inflationary epoch
- No vDVZ discontinuity for massive gravity in (A)dS space, M. Porratti.

$$\frac{2\Lambda - 2m^2}{2(3m^2 - 2\Lambda)} = \begin{cases} -\frac{1}{2} & m^2 \to 0, \Lambda \neq 0 \\ -\frac{1}{3} & m^2 \neq 0, \Lambda \to 0 \end{cases}$$

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2 Introduction of massive gravity

# FP theory of linearized massive gravity

- Most trivially, introduce a mass term in the Linearized EH action:

$$\mathcal{L}_{\rm FP} = \frac{1}{4}m^2(h_{\mu\nu}h^{\mu\nu} - \alpha h^2).$$

  $\alpha = -\frac{1}{2} \Rightarrow$ cosmological constant term. Any other values of $\alpha \neq 1$ introduces a ghost into the theory.

- $h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + \nabla_\mu A^{\rm T}_\nu + \nabla_\mu A^{\rm T}_\nu + \gamma_{\mu\nu}\sigma + \nabla_\mu\nabla_\nu\tau \Rightarrow$

$$\begin{aligned} h^2_{\mu\nu} =& h^{\rm TT\,2}_{\mu\nu} - 2A^{\rm T}_\nu\Box A^{\rm T\nu} - 6H^2A^{\rm T}_\nu A^{\rm T\nu} + 4\sigma^2 \\ &+ \tau\Box^2\tau + 3H^2\tau\Box\tau + 2\sigma\Box\tau \end{aligned}$$

$$h^2 = 16\sigma^2 + 8\sigma\Box\tau + \tau\Box^2\tau$$

  The only way to avoid introducing $\tau\Box^2\tau$ in the mass term is by choosing $\alpha = 1$.

# Problems

- vDVZ discontinuity: Graviton propagator is given by

$$h_{\mu\nu} = -\frac{T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu}}{\Box + m^2}$$

Instead of that in the pure EH gravity:

$$h_{\mu\nu} = -\frac{T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}}{\Box}$$

The discrepancy can be easily observed by comparing the Newton's constant measured by light bending and planetary orbits.

# FP massive gravity

No consistent nonlinear completion without extra dimensions

- Naively it propagates six degrees of freedom, but incidentally eom+Bianchi demands
$$\frac{3}{2}m^2 h = T$$
eliminating one additional degree of freedom
- Alternatively,
$$\mathcal{L}_{\rm PF} = 3m^2\sigma^2 + \frac{3}{2}m^2\sigma\Box\tau + \dots$$
eom of $\tau$ leads to $\Box\sigma = 0$.
- No longer true beyond the linearized level
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3. "Classically constrained" models

# Jnimodular Gravity

Initially proposed to "solve" the cosmological constant problem. J.J. van der Bij, H. van Dam, F. Wilczek, A. Zee and so on

- Remove one "degree of freedom" at the classical level.

$$\mathcal{L} = -\frac{\sqrt{-g}}{2}(R-\Lambda) - \lambda(\sqrt{-g}-1) + \mathcal{L}_M$$

Equation of motion:

$$G_{\mu\nu} + (\Lambda - \lambda)g_{\mu\nu} = T_{\mu\nu} \quad (1)$$

$$\sqrt{-g} - 1 = 0 \quad (2)$$

Bianchi identities $\Rightarrow$

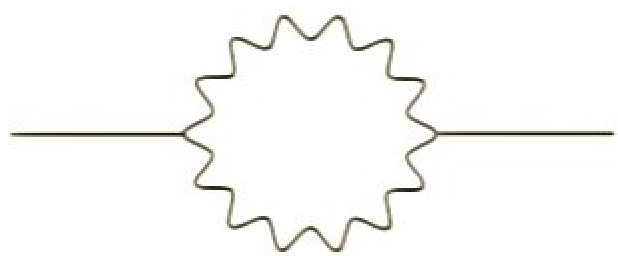
$$\partial_\mu \lambda = 0. \qquad \Rightarrow \lambda = \text{const.}$$

$\lambda$ is a constant of integration

- Flat Minkowski space is a solution even if $\Lambda \neq 0$.

# Problems

- There are infinite number of solutions parametrized by a constant scalar curvature $R$.
  No reason to prefer the flat solution.
- Difficult to put into a covariant framework
- Radiatively unstable. $\lambda$ picks up mass term and kinetic term from loops.



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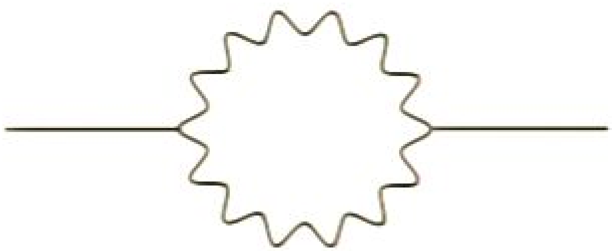

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# "Classically constrained" model

So, can we improve this model by removing all gauge degrees of freedom at the classical level?
G. Gabadadze, Y. Shang, arXiv:hep-th/0506040.

- Initially aiming at constructing a self-consistent massive gravity within the 4-d framework.
- Think of it as a gauge fixed Lagrangian of GR (or any gauge theory)
- Allowing zero modes of the FP operator leads to non-trivial background
- BRST completed theory in path-integral guarantees transversality of the two-point functions $\Rightarrow$ radiative stability

# Action and the constraints

The following gauge fixing version appears most interesting

- $$\mathcal{L} = -\frac{\sqrt{-g}}{2}R + \sqrt{-g}g^{\mu\nu}\partial_\mu\lambda_\nu + \mathcal{L}_M$$

  $\lambda_\mu$ are four Lagrangian multipliers, imposing four constraints, a gauge fixing scheme.
- Equations of motion

$$\begin{aligned} G_{\mu\nu} - (\partial_\mu\lambda_\nu + \partial_\nu\lambda_\mu) + g_{\mu\nu}\partial^\sigma\lambda_\sigma =& T_{\mu\nu} \\ \partial_\mu(\sqrt{-g}g^{\mu\nu}) =& 0 \end{aligned}$$

# The many sides of this constraint

The condition

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$$

has many interesting consequences. It's equivalent to

- $$\Gamma^\alpha_{\mu\nu}g^{\mu\nu} = 0$$
- And, also equivalent to say that EH Lagrangian is correct only while expressed in the *harmonic coordinates* $\{x^\mu\}$ that satisfy

$$D^\alpha D_\alpha x^\mu = 0$$

- which also implies

$$\nabla^\nu A_\nu = g^{\mu\nu}\partial_\mu A_\nu$$

# Equation of motion for the Lagrangian multiplier $\lambda_\mu$

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Therefore, given any $\Lambda$

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \partial_\mu\lambda_\nu + \partial_\nu\lambda_\mu = \Lambda\eta_{\mu\nu}$$

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The Lagrangian multipliers $\lambda_\mu$ in our model is radiatively stable

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$$\gamma^{\mu\nu} = \sqrt{-g} g^{\mu\nu}, \quad g_{\mu\nu} = \frac{g_{\mu\nu}}{\sqrt{-g}} \tag{3}$$

$$\nabla^{\mu\nu}_{\alpha} \equiv \gamma^{\mu\tau}\delta^{\nu}_{\alpha}\partial_\tau - \gamma^{\nu\tau}\delta^{\mu}_{\alpha}\partial_\tau - \partial_\alpha(\gamma^{\mu\nu} \quad ) \tag{4}$$

$$\mathcal{L} = -\frac{1}{2}\gamma^{\mu\nu}(R_{\mu\nu}(\gamma) - \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu) + \frac{i}{2}(\partial_\mu\bar{c}_\nu + \partial_\nu\bar{c}_\mu)\nabla^{\mu\nu}_{\alpha}c^\alpha \tag{5}$$

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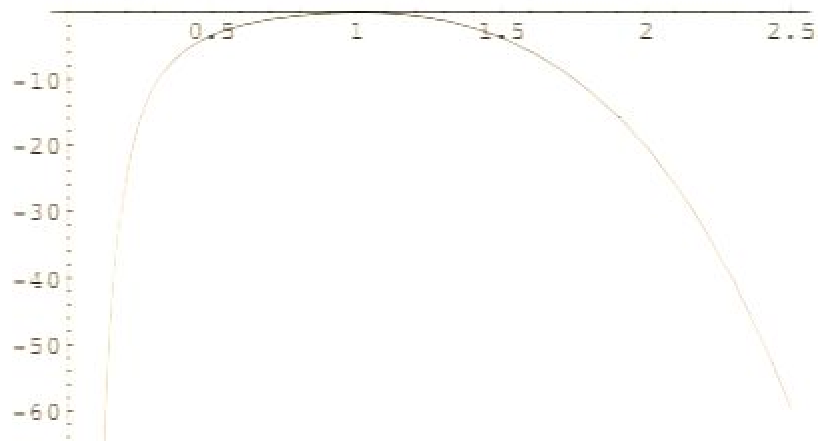
# Problems

- Flat solution is unstable.




G. Gabadadze and Y. Shang, "Quantum cosmology of classically constrained gravity," Phys. Lett. B **635**, 235 (2006) [arXiv:hep-th/0511137].

- Non-Linear completion not known
- Massive deformation leads to ghost at the linear level

# Outline

4 Massive gravity on de Sitter background

# Massive spin-2 on de Sitter background

The action is given by:

- $$\mathcal{L} = \mathcal{L}^{(2)}_{\text{EH}}(h_{\mu\nu}) - \frac{1}{4}m^2(h^2_{\mu\nu} - h^2)$$

  where $\mathcal{L}^{(2)}_{\text{EH}}$ is the second order EH action around de Sitter background with cosmological constant $\Lambda = 3H^2$:

  $$\mathcal{L}^{(2)}_{\text{EH}} = \frac{1}{4}h_{\mu\nu}\Box h^{\mu\nu} + \frac{1}{2}(\nabla_\mu h^{\mu\nu})^2 + \frac{1}{2}h_{\mu\nu}\nabla^\mu\nabla^\nu h - \frac{1}{4}h\Box h - \frac{1}{2}H^2\left(h^2_{\mu\nu} + \frac{1}{2}h^2\right)$$

- This theory is consistent when $H = 0$.

# The Higuchi ghost

When $H^2 \neq 0$, there is a ghost if $m^2 < 2H^2$.
Higuchi, Deser, Waldron ...

- If we decompose $h_{\mu\nu}$ as:

$$h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + \nabla_\mu A^{\rm T}_\nu + \nabla_\mu A^{\rm T}_\nu + \gamma_{\mu\nu}\sigma + \nabla_\mu \nabla_\nu \tau$$

- we find

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm EH} + \mathcal{L}_{\rm FP} = & \frac{1}{4} h^{\rm TT}_{\mu\nu}(\Box - 2H^2) h^{\rm TT\mu\nu} - \frac{3}{2}\sigma(\Box + 4H^2)\sigma \\ & - \frac{1}{4} m^2 h^{\rm TT\,2}_{\mu\nu} + 3m^2\sigma^2 - \frac{3}{4} H^2 m^2 \tau \Box \tau + \frac{3}{2} m^2 \sigma \Box \tau \\ & + \frac{1}{2} m^2 A^{\rm T}_\nu \Box A^{\rm T\nu} + \frac{3}{2} H^2 m^2 A^{\rm T}_\nu A^{\rm T\nu}\end{aligned}$$

# Effective action

- Integrate out the non-dynamical fields $\tau$ and $A^{\mathrm{T}}_{\mu}$:

$$\mathcal{L}^{(2)}_{\mathrm{EH+PF}} = \frac{1}{4} h^{\mathrm{TT}}_{\mu\nu} \Box h^{\mathrm{TT}\mu\nu} - \frac{1}{4}(m^2 + 2H^2) h^{\mathrm{TT}\,2}_{\mu\nu} + \frac{3(m^2 - 2H^2)}{4H^2}\left(\sigma\Box\sigma + 4H^2\sigma^2\right)$$

- $\sigma$ is a ghost when $m^2 < 2H^2$.
- At the special point when $m^2 = 2H^2$, $\sigma$ disappears in the action and the theory contains extra local symmetry:

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\rho(x)$$

- Equation of motion demands

$$(m^2 - 2H^2) h = T$$

when $m^2 = 2H^2$, the theory can not couple to the trace of $T_{\mu\nu}$.

# A summary of massive graviton on de Sitter background:

- when $m^2 < 2H^2$ there's a ghost
- when $m^2 > 2H^2$ there both spin-2 and spin-0 degrees of freedom propagating but no ghost
- when $m^2 = 2H^2$ Propagating only the spin-2 modes, no ghost or tachyon, but enhanced local symmetry forbid coupling with non-vanishing trace of $T_{\mu\nu}$

# Where was this study initiated

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- Despite its origin from the DGP model, the form of this operator $\mathcal{O}$ can be motivated by the requirement that the Bianchi identities are remains intact:

$$\nabla^\mu \mathcal{O}_{\mu\nu}\phi = 0$$

- Equations of motion are

$$\mathcal{O}^{\mu\nu} h_{\mu\nu} = 0$$

$$G^{\mathrm{dS}}_{\mu\nu} - H^2(h_{\mu\nu} - \gamma_{\mu\nu} h) - \mathcal{O}_{\mu\nu}\phi = -T_{\mu\nu}$$

- Using Bianchis+trace as usual, we find

$$\frac{1}{2} h_{\mu\nu} = \frac{1}{\Delta_L - 4H^2} T^{(1/3)}_{\mu\nu} - \frac{\gamma_{\mu\nu}}{12} \frac{T}{\Box + 4H^2} + \frac{1}{3}\left(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box\right) \frac{T}{(\Box + 4H^2)^2}$$

where $\Delta_L$ is the Lichnerowicz operator:

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- The amplitude:

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# Move on to region $m^2 < 2H^2$

- Consider the action for a graviton of mass $m$ and a scalar field $\phi$, and as before, with kinetic mixing

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\text{EH}}(h_{\mu\nu}) - \frac{1}{4}m^2(h^2_{\mu\nu} - h^2) - \phi\mathcal{O}^{\mu\nu}h_{\mu\nu} + \phi\mathcal{K}\phi + h_{\mu\nu}T^{\mu\nu} + q\phi T$$

We keep the operator $\mathcal{O}$

$$\mathcal{O}_{\mu\nu} = \nabla_\mu\nabla_\nu - \gamma_{\mu\nu}\Box - 3H^2\gamma_{\mu\nu}$$

- Again we have left the kinetic term of $\phi$ undetermined, but simply assume that

$$\mathcal{K} = A\Box + B$$

Just to remind you:

$$Q \equiv -\Box - 4H^2$$

while $A$ and $B$ are assumed to be constants and $\mathcal{K}$ commutes with $Q$.

- Equation of motion:

$$G^{\rm ds}_{\mu\nu} - \frac{m^2}{2}(h_{\mu\nu} - \gamma_{\mu\nu}h) - \mathcal{O}_{\mu\nu}\phi = -T_{\mu\nu}$$
$$\mathcal{O}^{\mu\nu}h_{\mu\nu} - 2\mathcal{K}\phi = qT$$

Bianchi $\Rightarrow$

$$\nabla^\mu h_{\mu\nu} = \nabla_\nu h$$

which can be used to reduce the eom to:

$$\frac{1}{2}\left[\Box h_{\mu\nu} - (2H^2 + m^2)h_{\mu\nu} - \gamma_{\mu\nu}(H^2 - m^2)h - \nabla_\mu\nabla_\nu h\right] = -T_{\mu\nu} + \mathcal{O}_{\mu\nu}\phi$$

and the trace of this equation:

$$(3H^2 - \frac{3}{2}m^2)h + 3Q\phi = T$$

- EOM of $\phi$ implies that

$$-3H^2 h - 2\mathcal{K}\phi = qT$$

All together, we find:

$$\phi = \frac{(1+q)H^2 - \frac{1}{2}qm^2}{3H^2 Q + (m^2 - 2H^2)\mathcal{K}} T$$

$$h = \frac{qQ + \frac{2}{3}\mathcal{K}}{\frac{1}{2}qm^2 - (1+q)H^2} \phi$$

- and the amplitude:

$$\frac{1}{2}(\Delta_L - 6H^2 + m^2)h_{\mu\nu} = T_{\mu\nu} - \mathcal{O}_{\mu\nu}\phi - \frac{1}{2}\mathcal{M}_{\mu\nu}h$$

where

$$\mathcal{M}_{\mu\nu} \equiv (3H^2 - m^2)\gamma_{\mu\nu} + \nabla_\mu \nabla_\nu$$

# The physical metric perturbation

- Again, let's focus on the physical metric perturbation

$$h^{\rm phy}_{\mu\nu} = h_{\mu\nu} - q\gamma_{\mu\nu}\phi \; .$$

which couples to $T_{\mu\nu}$

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$$\begin{aligned}\frac{1}{2}h^{\rm phy}_{\mu\nu} &= \frac{1}{\Delta_L - 6H^2 + m^2}\left\{T_{\mu\nu} - \mathcal{O}_{\mu\nu}\phi - \frac{1}{2}\mathcal{M}_{\mu\nu}h\right\} + \frac{1}{2}\gamma_{\mu\nu}q\phi \\ &= \frac{1}{\Delta_L - 6H^2 + m^2}\left\{T_{\mu\nu} - \mathcal{O}_{\mu\nu}\phi - \frac{1}{2}\mathcal{M}_{\mu\nu}h\right. \\ &\qquad \left. + \frac{q(Q - 2H^2 + m^2)}{2}\gamma_{\mu\nu}\phi\right\} \; .\end{aligned}$$

- $q$ and $\mathcal{K}$ are still to be determined.
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# The master equation for $\mathcal{K}$

- This leads to the magic equation that determines $\mathcal{K}$:

$$(2H^2 - m^2)\mathcal{K} = -\frac{3q}{2}\left(2H^2 - m^2\right)Q + 3\left(2H^2 - m^2\right)\left[(1+q)H^2 - \frac{1}{2}qm^2\right]$$

- Notice that when $m^2 = 2H^2$, the above equation is automatically satisfied and $\mathcal{K}$ remains completely arbitrary. That is why for this special case one finds additional freedom as discussed earlier.
- Things are very different when $m^2 \neq 2H^2$. Here $\mathcal{K}$ is fixed to be

$$\mathcal{K} = -\frac{3q}{2}Q + 3\left[(1+q)H^2 - \frac{1}{2}qm^2\right]$$

Such a choice of $\mathcal{K}$ gives rise to a series of surprising simplifications of the solutions.

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# Summary

- "Classically constrained model"
  - May be considered as a gauge fixing version of the GR(gauge theory). Allowing for non-normalizable modes leads to interesting novel background.
  - Non-linear completion? Boundary condition and the instability of flat solution?
- Massive spin-2 on de Sitter background
  - $\mathcal{O} = \nabla_\mu \nabla_\nu - \Box\gamma_{\mu\nu} - 3H^2\gamma_{\mu\nu}$ is very special
  - Found a particular choice of $\mathcal{K}$ that brings in more simplifications when $m^2 \neq 2H^2$
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  - More physical explanation of this weird looking action?
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# A curious coincidence

- Therefore we find

$$\mathcal{L}^{(2)}_{\text{eff}} = \mathcal{L}^{(2)}_{\text{EH+PF}}(\tilde{h}_{\mu\nu}) - \left[\frac{1}{2}m^2 c + (1+q)\right](\phi\nabla_\mu\nabla_\nu\tilde{h}_{\mu\nu} - \phi\Box\tilde{h}) + \left[3(1+q)H^2 - \frac{3}{2}qm^2\right]\tilde{h}\phi - \left[\frac{3}{2}q(q+1+m^2c) + \frac{3}{4}m^2H^2c^2\right]\phi\Box\phi + \frac{3}{2}(2q-1)\left[qm^2 - 2(q+1)H^2\right]\phi^2 + \tilde{h}_{\mu\nu}T^{\mu\nu}.$$

- It is not completely trivial that we can set the value of a single constant $c$ to remove all the derivative mixings between $\tilde{h}$ and $\phi$:

$$c = -\frac{2(1+q)}{m^2}$$

# $h$ is $\phi$

- An immediate consequence of the given choice of $\mathcal{K}$ is that $h$ is directly proportional to $\phi$:

$$h = -2\phi$$

removing of of the degree of freedom among the two.

- Furthermore, we find

$$\phi = -\frac{1}{3(\Box + 6H^2 - m^2)}T$$

- The single poles inside the curly brackets, besides the $q(Q - 2H^2 + m^2)\phi$ term, cancel exactly. and the final result:

$$\frac{1}{2}h^{\text{phy}}_{\mu\nu} = \frac{1}{\Delta_L - 6H^2 + m^2}\left(T^{(1/2)}_{\mu\nu} + \frac{1+q}{6}\gamma_{\mu\nu}T\right)$$

# Jnfortunately we can't get rid of tachyons

- Both $\phi$ and the scalar component of $h_{\mu\nu}^{\rm phy}$, for $q$ satisfying the bound given above, have positive mass squared if $m^2 > 6H^2$,
- they become tachyonic when $m^2 < 6H^2$, one of which is decoupled from the matter stress-tensor and the other couples to it with the strength suppressed as $m^2/(m^2 + 2H^2)$.
- Such a tachyon is an improvement over the ghost if $2H^2 > m^2$ but $H \sim m$, so that the tachyon instability time is of the order of the age of the Universe.
- In our model there is a continuous transition to the massless limit $m \to 0$ and the vDVZ discontinuity is absent for some choices of $q$.

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# Some more discussions

We discuss the effective Lagrangian in term of $h^{\rm phy}_{\mu\nu}$:

- Recall $h_{\mu\nu} = h^{\rm phy}_{\mu\nu} - q\gamma_{\mu\nu}\phi$:

$$\mathcal{L}_{eff} = \mathcal{L}^{(2)}_{EH+PF}(h^{\rm phy}_{\mu\nu}) - (1+q)\phi\mathcal{O}^{\mu\nu}h^{\rm phy}_{\mu\nu} - \frac{3}{2}q(q+1)\phi\Box\phi - \frac{3}{2} qm^2 h^{\rm phy}\phi + + \frac{3}{2}(2q-1)(qm^2 - 2(q+1)H^2)\phi^2 + h^{\rm phy}_{\mu\nu}T^{\mu\nu}$$

- Let us perform the following transformation

$$h^{\rm phy}_{\mu\nu} = \tilde{h}_{\mu\nu} + c\nabla_\mu\nabla_\nu\phi.$$

- Clearly: $\mathcal{L}^{(2)}_{\rm EH}(\tilde{h}_{\mu\nu}) = \mathcal{L}^{(2)}_{\rm EH}(h^{\rm phy}_{\mu\nu})$,
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# Are there ghosts?

- Recall the amplitude:

$$\frac{1}{2}h^{\rm phy}_{\mu\nu} = \frac{1}{\Delta_L - 6H^2 + m^2}\left(T^{(1/2)}_{\mu\nu} + \frac{1+q}{6}\gamma_{\mu\nu}T\right)$$

No ghost if $1 + q > 0 \Rightarrow q > -1$.
- How about the extra degree of freedom not sourced by $T_{\mu\nu}$? To determine that, we use the following trick: temporarily set to zero $T_{\mu\nu}$ and add a putative source $J$ via the term $+J\phi$ in the action.
- $J$ should excite a different combinations of the helicity-0 mode and $\phi$; if there is a ghost not sourced by $T$, it should be sourced by $J$.
- Performing this analysis in a similar way, we find that for

$$q > \frac{2H^2}{m^2 - 2H^2}$$

no ghosts are excited by $J$ either.
- If $m^2 < 2H^2$, $\frac{2H^2}{m^2-2H^2} < -1$.

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- Therefore we find

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{(2)} =& \mathcal{L}_{\text{EH+PF}}^{(2)}(\tilde{h}_{\mu\nu}) - \left[\frac{1}{2}m^2 c + (1+q)\right](\phi\nabla_\mu\nabla_\nu\tilde{h}_{\mu\nu} - \phi\Box\tilde{h}) \\ &+ \left[3(1+q)H^2 - \frac{3}{2}qm^2\right]\tilde{h}\phi \\ &- \left[\frac{3}{2}q(q+1+m^2 c) + \frac{3}{4}m^2 H^2 c^2\right]\phi\Box\phi \\ &+ \frac{3}{2}(2q-1)\left[qm^2 - 2(q+1)H^2\right]\phi^2 + \tilde{h}_{\mu\nu}T^{\mu\nu}.\end{aligned}$$

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# f we remove the kinetic mixings ...

We must choose:

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- The theory reduces to

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{(2)} =& \mathcal{L}_{\text{EH+PF}}^{(2)}(\tilde{h}_{\mu\nu}) - \frac{3}{2}\left[q(m^2 - 2H^2) - 2H^2\right]\tilde{h}\phi \\ &+ \frac{3(q+1)}{2m^2}\left[q(m^2 - 2H^2) - 2H^2\right]\phi\Box\phi \\ &+ \frac{3}{2}(2q-1)\left[q(m^2 - 2H^2) - 2H^2\right]\phi^2 + \tilde{h}_{\mu\nu}T^{\mu\nu}.\end{aligned}$$

- Amazingly, all the mixing terms share a common factor

$$q(m^2 - 2H^2) - 2H^2$$

# One more curious result

All the mixing terms contain a common factor.

- By choosing
$$q = \frac{2H^2}{m^2 - 2H^2}.$$
All the mixings disappear simultaneously. The theory becomes a massive spin-2 on de Sitter background without any extra degrees of freedom.
- With this choice of $q$ and the special form of $\mathcal{O}^{\mu\nu}$ and $\mathcal{K}$, the theory is nothing but a pure linearized gravity on de Sitter background expressed after a certain conformal and gauge transformations.
- This of course contains a ghost when $m^2 < 2H^2$. Notice that in this case $q < -1$, consistent with our conclusions given above.
- When $q = -1$, $\phi$ is not dynamical

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$$c = -\frac{2(1+q)}{m^2}$$

- The theory reduces to

$$\begin{aligned}\mathcal{L}^{(2)}_{\text{eff}} =& \mathcal{L}^{(2)}_{\text{EH+PF}}(\tilde{h}_{\mu\nu}) - \frac{3}{2}\left[q(m^2-2H^2)-2H^2\right]\tilde{h}\phi \\ &+ \frac{3(q+1)}{2m^2}\left[q(m^2-2H^2)-2H^2\right]\phi\Box\phi \\ &+ \frac{3}{2}(2q-1)\left[q(m^2-2H^2)-2H^2\right]\phi^2 + \tilde{h}_{\mu\nu}T^{\mu\nu}.\end{aligned}$$

- Amazingly, all the mixing terms share a common factor

$$q(m^2-2H^2)-2H^2$$

# One more curious result

All the mixing terms contain a common factor.

- By choosing
$$q = \frac{2H^2}{m^2 - 2H^2}.$$
All the mixings disappear simultaneously. The theory becomes a massive spin-2 on de Sitter background without any extra degrees of freedom.
- With this choice of $q$ and the special form of $\mathcal{O}^{\mu\nu}$ and $\mathcal{K}$, the theory is nothing but a pure linearized gravity on de Sitter background expressed after a certain conformal and gauge transformations.
- This of course contains a ghost when $m^2 < 2H^2$. Notice that in this case $q < -1$, consistent with our conclusions given above.
- When $q = -1$, $\phi$ is not dynamical

# Outline

5 Summary and outlook

# Summary

- "Classically constrained model"
  - May be considered as a gauge fixing version of the GR(gauge theory). Allowing for non-normalizable modes leads to interesting novel background.
  - Non-linear completion? Boundary condition and the instability of flat solution?
- Massive spin-$2$ on de Sitter background
  - $\mathcal{O} = \nabla_\mu \nabla_\nu - \Box \gamma_{\mu\nu} - 3H^2 \gamma_{\mu\nu}$ is very special
  - Found a particular choice of $\mathcal{K}$ that brings in more simplifications when $m^2 \neq 2H^2$
  - When $q > -1$, the theory appears free of ghost
  - More physical explanation of this weird looking action?
  - Non-linear completion, extra dimension?
  - Tachyonic instabilities?

THANK YOU