Title: Semiclassical limit of general-triangulation Regge calculus and spin foams

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Abstract: The general boundary state formulation is a key tool for extracting the semiclassical limit of nonpertubative theories of quantum gravity. In this talk I will discuss how this formalism works in the context of four-dimensional quantum Regge calculus with a general triangulation. A Gaussian boundary state selects a classical internal solution and peaks the path integral on it. As a result boundary observables, in particular the two-point function, can be computed order by order in a semiclassical asymptotic expansion. When the same methods are applied to a modified Regge theory that substitutes the exponential of the action by its cosine at each simplex in the triangulation, as conjectured from the semiclassical limit of spin foam models, the contributions from the sign-reversed terms are suppressed and the results match those of conventional Regge calculus. This talk is based on the results published in arXiv:0808.1107 [gr-qc].

Semiclassical limit of general-triangulation Regge calculus and spin foams

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arXiv:0808.1107, to appear in NPB

PI, 10/10/2008

A general question for background-independent approaches to quantum gravity:

How can a semiclassical limit making connection with perturbative theory around a background classical spacetime be recovered?

A possible answer: General Boundary Formulation (Oeckl, Rovelli)

$$\langle \hat{\mathcal{O}}(\gamma) \rangle_q = \frac{1}{Z_q} \int \mathrm{D}\gamma \; \hat{\mathcal{O}}(\gamma) \; \Psi_q(\gamma) \int_{\partial g = \gamma} \mathrm{D}g \; \mathrm{e}^{iS[g]}$$

with boundary state Ψ_q peaked on classical 3-geometry q.

In spin foams, this has been used to compute the graviton propagator:

$$\langle \hat{\mathcal{O}}_1 \, \hat{\mathcal{O}}_2 \rangle_q = \frac{1}{Z_q} \sum_s \hat{\mathcal{O}}_1 \, \hat{\mathcal{O}}_2 \, \Psi_q[s] \, W[s] \, ,$$

$$W[s] = \sum_{F, \partial F = s} \prod_f A_f(F) \prod_e A_e(F) \prod_v A_v(F) \, .$$

Lots of results for Barrett-Crane, and driving force behind new models.

But so far, calculations mostly done with a single simplex.

Does it work for a general triangulation, with nontrivial summation over nonperturbative internal variables?

Related question: is the "cosine problem" of the semiclassical limit of SFs solved for many simplices as it is for one?

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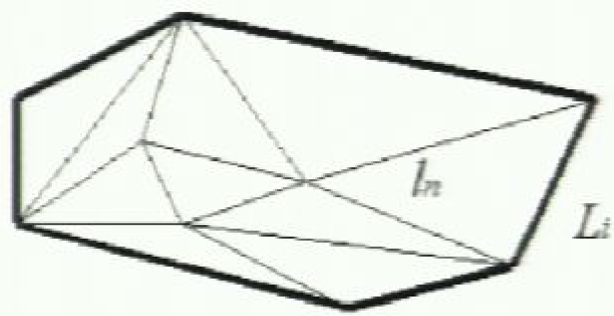
Use Regge Calculus as testing ground for this question



$$\langle \hat{\mathcal{O}} \rangle_q = \frac{1}{Z_q} \int \prod_i \mathrm{d}L_i \prod_n \mathrm{d}l_n \, \mu(l_n, L_i) \, \hat{\mathcal{O}} \, \Psi_q(L_i) \, \exp\left(\frac{i}{\hbar} S(l_n, L_i)\right)$$

where $S(l_n, L_i) = \frac{1}{\kappa} \sum_a A_a(l_n, L_i) \, \phi_a(l_n, L_i)$ is the Regge action for Riemannian 4d simplicial gravity, $\hat{\mathcal{O}}$ is a boundary geometry observable, and $\mu(l_n, L_i)$ is a suitable measure.

Construction of the boundary state

Given a classical solution (l_n^0, L_i^0), the associated semiclassical boundary state is

$$\Psi_q(L_i) = C \exp\left[-\frac{1}{2\hbar\kappa}\,\alpha_{ij}\,(L_i - L_i^0)\,(L_j - L_j^0)\right] \times \exp\left[-\frac{i}{\hbar}\,K_i^0\,(L_i - L_i^0)\right]$$

where α_{ij} has positive-definite real part, and K_i^0 is the extrinsic boundary curvature of the classical solution:

$$K_i^0 = \left.\frac{\partial S}{\partial L_i}\right|_{\substack{L_i = L_i^0 \\ l_n = l_n^0}}$$

Ψ_q is a Gaussian peaked both in intrinsic and extrinsic boundary geometry, with relative dispersion scaling as $\frac{\sqrt{\hbar\kappa}}{L_0}$.

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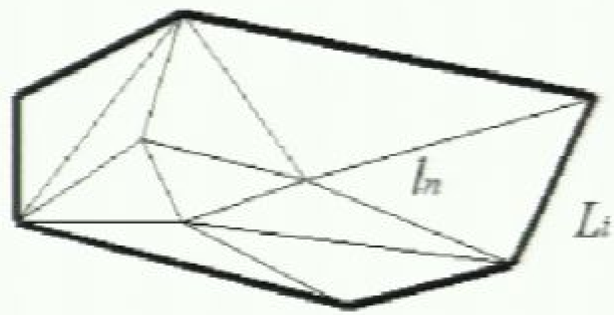

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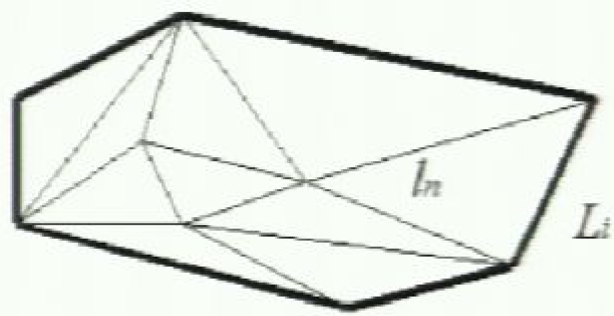

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Semiclassical evaluation of observables

$$\langle \mathcal{O}(L_i) \rangle_q = \frac{1}{Z_q} \int \prod_{n,i} \mathrm{d}l_n \, \mathrm{d}L_i \, \mu(l_n, L_i) \, \exp\left(\frac{i}{\hbar} S(l_n, L_i) \right) \mathcal{O}(L_i)$$
$$\times \exp\left[-\frac{1}{2\hbar\kappa} \alpha_{ij} \, (L_i - L_i^0) \, (L_j - L_j^0) \right] \exp\left[-\frac{i}{\hbar} K_i^0 \, (L_i - L_i^0) \right]$$

We use a stationary phase approximation for integrals of the form $F(\lambda) = \int \prod_i dx^i \; f(x) \, e^{i\lambda \, Q(x)}$ with complex Q. The domain is $x = (l_n, L_i)$ (restricted by μ) and the large parameter is $\lambda = \hbar^{-1}$. The expansion formula is:

$$F(\lambda) = \left(\frac{2\pi}{\lambda} \right)^{\frac{d}{2}} \frac{e^{-i \, \mathrm{ind}[H_{ij}](\bar{x})}}{\sqrt{|\mathrm{det} H_{ij}(\bar{x})|}} \, e^{i\lambda Q(\bar{x})} \; \left(\sum_{n=0}^{N} a_n \lambda^{-n} + o(\lambda^{-N}) \right)$$

6

We have:

$$Q(l_n, L_i) = S(l_n, L_i) + \frac{i}{2\kappa}\alpha_{ij}\left(L_i - L_i^0\right)\left(L_j - L_j^0\right) - K_i^0\left(L_i - L_i^0\right)$$

The expansion coefficients a_n are obtained from Q, f and their derivatives evaluated at the pont x_0 that simultaneously minimizes $\mathrm{Im}[Q]$ and extremizes $\mathrm{Re}[Q]$. If there is no such point the integral is supressed.

Minimum of $\mathrm{Im}[Q(x)] \Longrightarrow L_i = L_i^0$

$$\text{Extremum of } \mathrm{Re}[Q(x)] \Longrightarrow \begin{cases} \frac{\partial S(l_n, L_i^0)}{\partial l_n} & = 0 \\ \left.\frac{\partial S(l_n, L_i)}{\partial L_i}\right|_{L_i = L_i^0} & = K_i^0 \end{cases}$$

$\Longrightarrow l_n = l_n^0$, unique solution if no flat patches.

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Examples of observables:

One-point function: $\langle L_i \rangle_q = L_i^0 + O(\hbar)$

Connected two-point function:
$\langle\, L_i\, L_j\, \rangle_q - \langle\, L_i\, \rangle_q\, \langle\, L_j\, \rangle_q = \; i\hbar\, [\mathbf{H}^{-1}]_{ij} + O(\hbar^2)$

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 S}{\partial L_i \partial L_j} + \frac{i}{\kappa}\alpha_{ij} & \left[\frac{\partial^2 S}{\partial L_i \partial l_n}\right]^t \\ \frac{\partial^2 S}{\partial L_i \partial l_n} & \frac{\partial^2 S}{\partial l_n \partial l_p} \end{pmatrix}$$

$\mathbf{H}$ is invertible (if no flat patches). The boundary state breakes conformal invariance by selecting the physical scale of the "background" solution.

To the first order, all dependence on the nonperturbative measure μ cancels out.

8

The connected two-point function can be rewritten as

$$\langle\, L_i\, L_j\, \rangle_q - \langle\, L_i\, \rangle_q \,\langle\, L_j\, \rangle_q = i\hbar \left(\frac{\partial^2 S^H}{\partial L_i \partial L_j}(L_i^0) + \frac{i}{\kappa}\alpha_{ij} \right)^{-1} + O(\hbar^2)$$

Compare with expression from perturbative theory around l_n^0:

$$\langle\, \delta l_i\, \delta l_j\, \rangle = i\hbar \left(\frac{\partial^2 S}{\partial l \partial l} + i\epsilon \right)^{-1}_{ij}$$

If the boundary state is defined by integrating out the external variables, we obtain consistently $\alpha_{ij} = [-i\, H^{ext} + \epsilon]_{ij}$, and recover perturbative expression. Important because perturbative Regge calculus recovers the correct graviton propagator in continuum limit! (Rocek & Williams).

Expansion parameter

We have arrived at our expressions with a "small $\hbar$" stationary phase approximation. But what is the physical expansion parameter? Comparing the expectation value of a boundary observable to its classical value, we obtain:

$$\xi = \frac{\langle \mathcal{O}(L) \rangle_q - \mathcal{O}(L^0)}{\mathcal{O}(L^0)} \sim \frac{\hbar\kappa}{\lambda^2}$$

where λ is the typical lengthscale of the classical solution, which is in turn given by the typical lengthscale L_0 of the boundary state.

The physical expansion parameter is the ratio of the Planck scale to the boundary scale.

Flat space

When the "background" classical geometry includes flat regions, there is no unique solution for the internal edge lengths due to translational symmetries of the action. However, we can integrate by stationary phase over variables "orthogonal" to the flat hypersurface and then average the result over the translational variables.

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z_q} \int \prod_{n=1}^{4q} \mathrm{d}y_n \, \frac{\exp\left[\frac{i}{\hbar} Q(L_i^0, x_n^0(y_n), y_n)\right]}{\left[\det \mathcal{H}(L_i^0, x_n^0(y_n), y_n)\right]^{1/2}} \sum_j \hbar^j a_j(L_i^0, x_n^0(y_n), y_n)$$

To the lowest order, the result of the first integrals is *independent* of the redundant variables and the last integrations are trivial. The connected two-point function is unchanged:

$$\langle L_i L_j \rangle_q - \langle L_i \rangle_q \langle L_j \rangle_q = i\hbar \left(\frac{\partial^2 S^H}{\partial L_i \partial L_j}(L_i^0) + \frac{i}{\kappa} \alpha_{ij} \right)^{-1} + O(\hbar^2)$$

Incidentally, this proves *triangulation independence* for flat configurations is valid to the lowest order.

If we consider two classical solutions representing the same physical geometry, with different triangulations that are related via Pachner moves within flat patches, then first-order semiclassical boundary correlations are the same on both.

Relevant for "triangulation dependence" problem in spin foam models...?

Spin Foam models

For the Barrett-Crane vertex amplitude we have:

$$A_v(j_f, i_e) \sim P_v \cos\left[\left(\sum_{j_f} j_f\theta_f\right) + \frac{\pi}{4}\right] + D_v\,, \qquad j_f \to \infty$$

This suggests the following ansatz for the semiclassical regime of boundary observables:

$$\langle\hat{\mathcal{O}}\rangle_q = \frac{1}{Z_q}\int\prod_{i,n}\mathrm{d}l_n\,\mathrm{d}L_i\,\tilde{\mu}(l_n, L_i)\,\prod_\sigma\left[P_\sigma(l_n^\sigma, L_i^\sigma)\right.$$
$$\left.\times\cos\left(\frac{1}{\hbar}S_\sigma(l_n^\sigma, L_i^\sigma) + \frac{\pi}{4}\right) + D_\sigma(l_n^\sigma, L_i^\sigma)\right]\hat{\mathcal{O}}\,\Psi_q(L_i)$$

(Modifed Regge path integral with cosine structure at each simplex.)

Expand the product over simplices as a sum over sign assignments:

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$$\times \exp\left[-\frac{i}{\hbar} K^0_i \, (L_i - L^0_i) \right]$$

Now we may apply again the stationary phase approximation to each term.

The minimum of $Im[Q]$ again gives $L_i = L_i^0$, and the extremum equations for $\mathrm{Re}[Q]$ are:

$$\begin{aligned} \frac{\partial}{\partial l_n} S_\epsilon(l_n, L_m^0) &= 0 \\ \frac{\partial}{\partial L_i} S_\epsilon(l_n, L_i)\Big|_{L_i = L_i^0} &= K_i^0 \end{aligned} \qquad (1)$$

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Summary of results

- The boundary state formalism can be successfuly applied to extract a semiclassical regime from nonperturbative Regge calculus with an arbitrary triangulation. Results from the conventional perturbative formulation can be recovered.
- The flat space degeneracy does not affect the results because the boundary state peaks us in the desired internal *geometry*. Results are triangulation-independent to the first order.
- The cosine ansatz for the semiclassical regime of spin foam models gives the same results as ordinary Regge calculus. The sign-reversed and degenerate contributions are suppressed by the boundary state.

To do:

- Finish establishing the relation between the perturbative and the boundary state formulations, fully recover scaling with distance. (Some open questions left.)
- Apply to spin foam models. LF & FC path integral formulation?
- Understand better how triangulation refinements work. Connections to the renormalization group? Effective field theory...?

Sketching of "piecewise limit" research programme:

Full LQG/SFs ⟶ Semiclassical boundary state LQG/SFs ⟶ Boundary state Regge Calculus ⟶ Perturbative Regge calculus ⟶ Perturbative Continuum Quantum Gravity (including EFT corrections?)

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This suggests the following ansatz for the semiclassical regime of boundary observables:

$$\langle\hat{\mathcal{O}}\rangle_q = \frac{1}{Z_q}\int\prod_{i,n} \mathrm{d}l_n\,\mathrm{d}L_i\,\tilde{\mu}(l_n, L_i)\,\prod_\sigma\left[P_\sigma(l_n^\sigma, L_i^\sigma)\right.$$
$$\left.\times\cos\left(\frac{1}{\hbar}S_\sigma(l_n^\sigma, L_i^\sigma) + \frac{\pi}{4}\right) + D_\sigma(l_n^\sigma, L_i^\sigma)\right]\hat{\mathcal{O}}\,\Psi_q(L_i)$$

(Modifed Regge path integral with cosine structure at each simplex.)