Title: Was Spacetime a Glorious Historical Accident?

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Abstract: Exactly half a century after Minkowskiâ€™s justly famous lecture, Diracâ€™s efforts to quantize gravity led him â€œto doubt how fundamental the four-dimensional requirement in physics isâ€•. Dirac does not appear to have explored this doubt further, but I shall argue that it needs to be considered seriously. The fact is that Einstein and Minkowski fused space and time into a four-dimensional continuum but never directly posed the two most fundamental questions in dynamics: What is time? What is motion? It was an historical accident that Einstein attempted to implement Machâ€™s principle after he had created special relativity; otherwise he would have been forced to address these questions, which have never been properly considered. I shall show how they can be answered and suggest that: 1) time and space are utterly different; 2) the dynamical law of the universe may define absolute simultaneity in a manner that is still consistent with local validity of Minkowskiâ€™s marvellous notion of spacetime.

Sept 08

10 David Kaiser, history of science
17 no talk
24 Jos Uffink foundations of quantum mechanics

Oct

1 Julian Barbour, the nature of time
8 Introduction of new postdocs, Bistro
15 Chris Eilasmith neuroscience
22 Andy Albrecht, cosmology and time
29 Sara Diamond, art and science

Nov

5 Vincent Rivasseau, quantum field theory
12 Xiao-Gang Wen, statistical physics
19 Bistro session
26 Catherine Kallin, condensed matter theory

Dec

3 Abhay Ashtekar, quantum gravity
10 Keith Dines, high energy theory

WAS SPACETIME A GLORIOUS HISTORICAL ACCIDENT?

Julian Barbour (with Bruno Bertotti, Edward Anderson, Brendan Foster, Bryan Kelleher, Karel Kuchař, and Niall Ó Murchadha)

Space and Time 100 Years after Minkowski, Bad Honnef, 7-12 September 2008

Minkowski (1908): "Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Dirac (1958): [The $3+1$ canonical decomposition of general relativity] "has led me to doubt how fundamental the four-dimensional requirement in physics is."

Wheeler (1979): "General relativity 'reverses' special relativity, in that it provides a preferred time coordinate."

OVERVIEW OF TALK

1. Historical Review
2. Formulation of Mach's Principle
3. The Theory of Time and Clocks
4. Defining Motion by Best Matching
5. Machian Particle Mechanics
6. 'Derivation' of GR, SR, and Gauge Theory
7. Scale-Invariant Geometrodynamics
8. Conclusions

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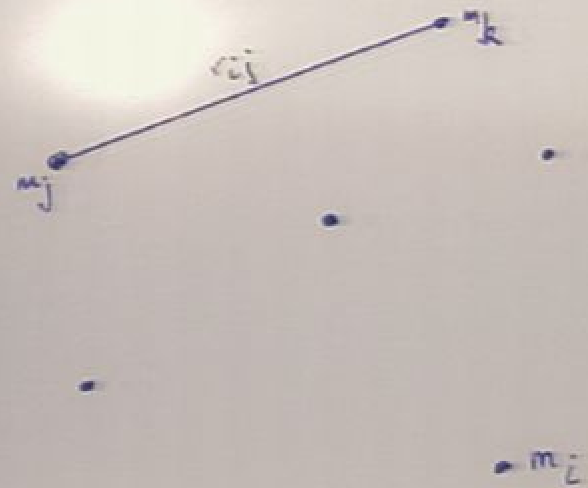
Newton (~ 1670): "Velocity depends on the distance passed over in a given time"

$$v = \frac{\delta x}{\delta t}$$



Only separations and (relative) masses m_i/M are physical.

For more than 5 particles the r_{ij} satisfy algebraic relations that permit their embedding in Euclidean space.

Leibniz: "Space is the order of coexisting things"

The order defines an instant.

Time is the succession of such instants.

The universe swims in nothing.
But *how* does it swim?

This is the question behind the issue of background independence.

3. CONCEPTUAL PRINCIPLES

1. *Three-dimensional configurations* of a dynamically closed universe evolve.

2. For particles, only the ratios $\hat{r}_{ij}$ of their separations r_{ij} are physical: $\hat{r}_{ij} = r_{ij}/\sqrt{\sum_{i<j} r_{ij}^2}$

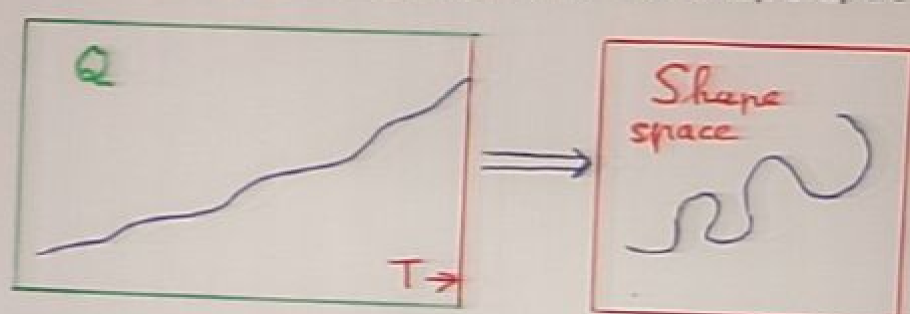
3. 3-geometries and not 3-metrics are physical.

4. Time is derived from change.

The 3N+1 dimensional configuration space QT of an 'island' universe contains non-physical data. Quotient out the unphysical data

$$\boxed{\frac{\mathrm{Q}}{\mathrm{Euclid}} = \mathcal{R}, \quad \frac{\mathrm{Q}}{\mathrm{Similarity}} = \mathcal{S}}$$

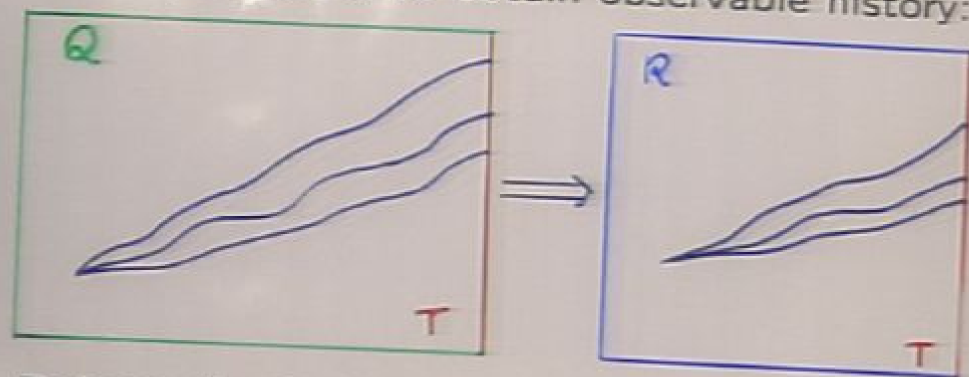
$\mathcal{R}$ is the *relative space*, $\mathcal{S}$ is the *shape space*



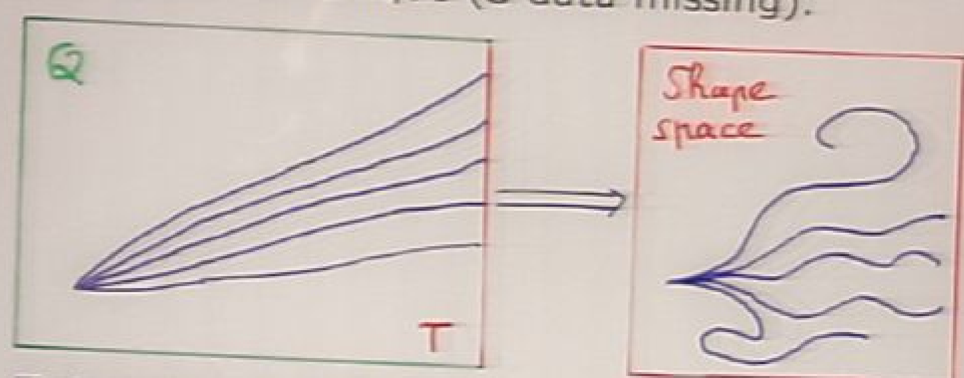
The space T of absolute times is redundant.
All objective data are in $\mathcal{S}$.

THE SHORTCOMINGS OF NEWTONIAN THEORY

Project Newtonian histories in QT down to $\mathcal{R}$T or, better, $\mathcal{S}$ to obtain observable history:



Poincaré's Analysis in 1902: A point and a direction in $\mathcal{R}$T fail to determine initial magnitude and direction of angular momentum **L**. Evolution not unique (3 data missing).



Extension in 2002: A point and direction in S fail to determine initial L, T/V, and fraction of T in expansion (5 data missing).

THE THEORY OF TIME AND CLOCKS

"It is utte... ...ossible to measure change by time."

$$A_J = 2\int_A^B d\lambda \sqrt{(E-V)\sum_i \frac{m_i}{2}\frac{d\mathbf{x}_i}{d\lambda}\cdot\frac{d\mathbf{x}_i}{d\lambda}} \approx$$

$$\boxed{\sqrt{2}\int_A^B \sqrt{(E-V)\sum_i m_i\,\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}\ \text{(Jacobi 1843)}}$$

Lipschitz, Darboux

The **canonical momenta** $\mathbf{p}_i = \sqrt{\frac{E-V}{T}}\, m_i \frac{d\mathbf{x}_i}{d\lambda}$

$$\approx \boxed{\sqrt{\frac{E-V}{2}}\,\frac{m_i\,\delta\mathbf{x}_i}{\sqrt{m_i\,\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}}}$$ satisfy the **constraint**

'Hypotenuse'

$\sum_i \frac{\mathbf{p}_i\cdot\mathbf{p}_i}{2m_i} = E-V,$ and equations of motion are

$$\frac{d}{d\lambda}\left(\sqrt{\frac{E-V}{T}}\, m_i \frac{d\mathbf{x}_i}{d\lambda}\right) = -\sqrt{\frac{T}{E-V}}\frac{\partial V}{\partial \mathbf{x}_i}.$$

Choose λ to make $T = E-V$ and recover

Newton's $m_i \frac{d^2\mathbf{x}_i}{dt^2} = -\frac{\partial V}{\partial \mathbf{x}_i}$ with

'Hypotenuse'

ephemeris time $\boxed{\delta t \approx \sqrt{\frac{\frac{1}{2}\sum m_i\,\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}{E-V}}}$

"Time is deduced from the...

"Constraints and gauge transformations"

Julian Barbour and Brendan Foster

Dirac's theorem that all first-class primary constraints generate gauge transformations is not universally true. Jacobi's principle is a counter example

arXiv: 0808 : 1223 [gr-qc]

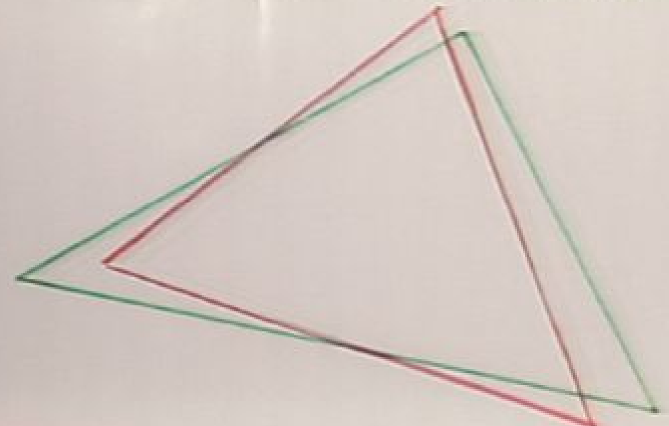
Brendan is giving a seminar on this here next week.

Clarification vis à vis talks on relational time (Milburn) and quantum reference frames (Bartlett).

The Cerne Abbas giant critique was not directed at that work.

DEFINING MOTION BY BEST MATCHING

Define a Metric on Shape Space $\mathcal{S}$



Shuffle triangles into **best-matched** position

An *equilocality relation* is established by the **Principle of Least Incongruence**

Minimize

$$\sqrt{U \sum_{i,\ a,\ \alpha} (\delta x_i^a - \delta\omega^\alpha\, t_{\alpha b}^a\, x_i^b)\, (\delta x_i^a - \delta\omega^\alpha\, t_{\alpha b}^a\, x_i^b)}$$

wrt group parameters $\delta\omega^\alpha$, $\alpha = 1 - 7$, where $t_{\alpha j}^i$ are the seven Euclidean generators, U is a function on $\mathcal{S}$, and $\mathbf{x}_i = (x_i^a, x_i^b, x_i^c)$.

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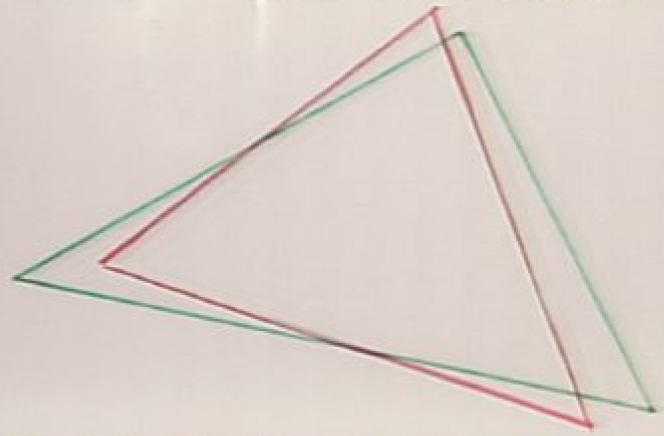

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MACHIAN PARTICLE DYNAMICS

M

$$\boxed{A_{\mathsf{M}} = \int_{\mathsf{A}}^{\mathsf{B}} \mathrm{d}\lambda\, \mathcal{L},\ \mathcal{L} = 2\sqrt{U T_{\mathsf{M}}},}$$

$$T_{\mathsf{M}} = \frac{1}{2}\sum_{i,a,\alpha} m_i (\dot{x}_i^a - \dot{\omega}^\alpha t^a_{\alpha b} x_i^b)(\dot{x}_i^a - \dot{\omega}^\alpha t^a_{\alpha b} x_i^b).$$

$$p_i^a = \frac{\partial \mathcal{L}}{\partial \dot{x}_i^a} = \sqrt{\frac{U}{T}}\, m_i (\dot{x}_i^a - \dot{\omega}^\alpha t^a_{\alpha b} x_i^b),\ \dot{x}_i^a = \frac{\mathrm{d} x_i^a}{\mathrm{d}\lambda}$$

Variation wrt $\dot{\omega}^a$ leads to constraints:

$$\boxed{\begin{gathered}\mathbf{P} = 0,\ \mathbf{L} = 0,\ D = 0,\ \text{where} \\ \mathbf{P} \equiv \sum_i \mathbf{p}_i,\ \mathbf{L} \equiv \sum_i \mathbf{x}_i \times \mathbf{p}_i,\ D \equiv \sum_i \mathbf{x}_i \cdot \mathbf{p}_i\end{gathered}}$$

$$\mathbf{x}_i = (x_i^a,\ x_i^b,\ x_i^c),\ \mathbf{p}_i = (p_i^a,\ p_i^b,\ p_i^c)$$

Euler–Lagrange equations $\dfrac{\mathrm{d}\mathbf{p}_i}{\mathrm{d}\lambda} = -\sqrt{\dfrac{T}{U}}\dfrac{\partial V}{\partial \mathbf{x}_i}$

propagate constraints if and only if

$$\boxed{U = U(r_{ij}) \text{ and } U \text{ is homogeneous of degree -2}}$$

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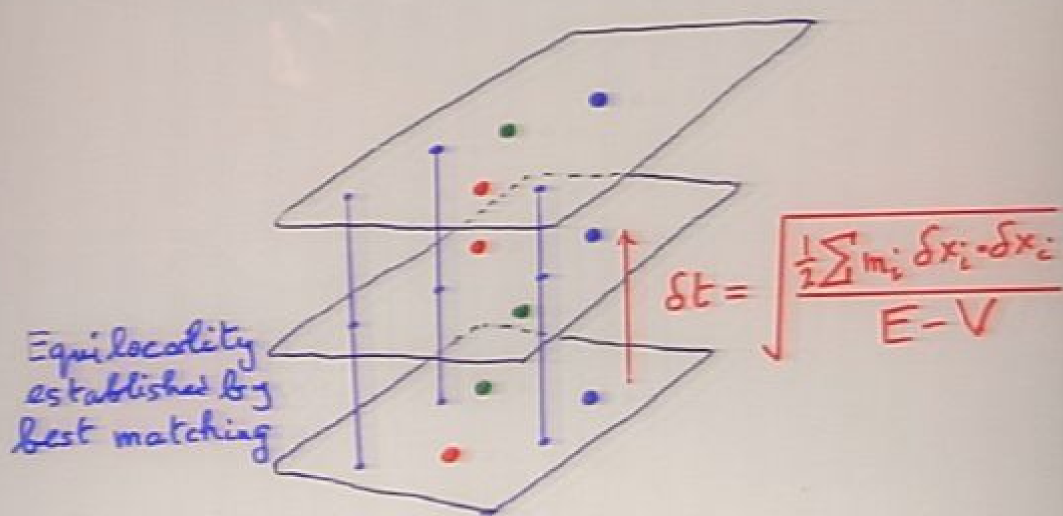
Distinguished representation:

$$\boxed{\begin{gathered}\frac{\mathrm{d}\mathbf{p}_i}{\mathrm{d}t} = -\frac{\partial V}{\partial \mathbf{x}_i} \textit{ in cms inertial system} \\ \text{with } \mathbf{P}=0,\ \mathbf{L}=0,\ D=0,\ E=0, \\ I = \sum_{i<j} m_i r_{ij}^2 = \text{Const } [\dot{I} = 2D].\end{gathered}}$$

Subsystems of the Universe obey *unconstrained* Newtonian dynamics.

Construction of Newtonian absolute space and time out of Machian 'raw' data

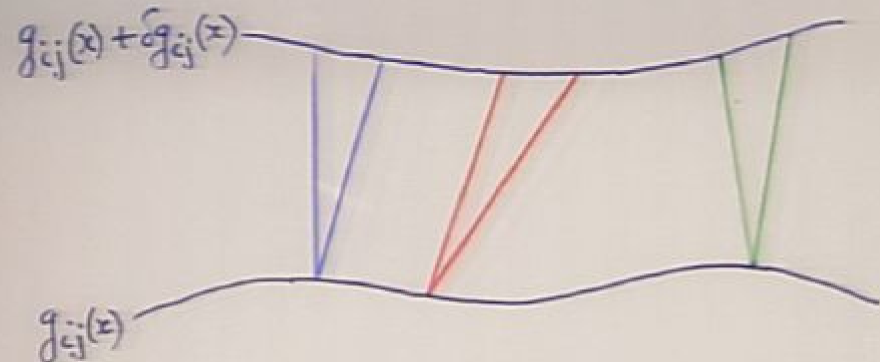


MACHIAN 'DERIVATION' OF GENERAL RELATIVITY

Riem: $=$ Space of Riemannian 3-Metrics

g_{ij} on a spatially closed 3-manifold

$$\textbf{Superspace} = \frac{\textbf{Riem}}{\textbf{3-Diffeomorphisms}}$$



To define metric on superspace

best match Jacobi-type action $\int \delta\lambda \int d^3x\,\mathcal{L}$

$$\boxed{\begin{gathered}\mathcal{L}=\sqrt{gU\,G_A^{ijkl}\{\dot g_{ij}-\dot\xi_{(i;j)}\}\{\dot g_{kl}-\dot\xi_{(k;l)}\}}\\ g=\det(g_{ij}),\; G_A^{ijkl}=g^{ik}g^{jl}-Ag^{ij}g^{kl}\end{gathered}}$$

Machian Geometrodynamics (Ó Murchadha)

$$\boxed{A=\int \mathrm{d}\lambda \int \mathrm{d}^3x\,\mathcal{L},\ \mathcal{L}=\sqrt{gUT}}$$

M

Local

$$T=G_A^{ijkl}\{\dot g_{ij}-\dot\xi_{(i;j)}\}\{\dot g_{kl}-\dot\xi_{(k;l)}\}$$

$$g=\det(g_{ij}),\ G_A^{ijkl}=g^{ik}g^{jl}-Ag^{ij}g^{kl},$$

and U is a 3-scalar formed from g_{ij}.

The canonical momenta p^{ij} and trace

$$\frac{\partial\mathcal{L}}{\partial\dot g_{ij}}=p^{ij}=\sqrt{\frac{gU}{T}}\,G_A^{ijkl}\{\dot g_{kl}-\dot\xi_{(i;j)}\},\ p=g_{ij}p^{ij}$$

must satisfy at each space point the **primary constraint** $\boxed{p^{ij}p_{ij}-\frac{A}{3A-1}p^2-gU=0}$

and the **secondary constraint** $\boxed{p^{ij}{}_{;j}=0}$ arising from variation wrt $\dot\xi_i$.

EL Eqs. propagate quadratic constraint only if

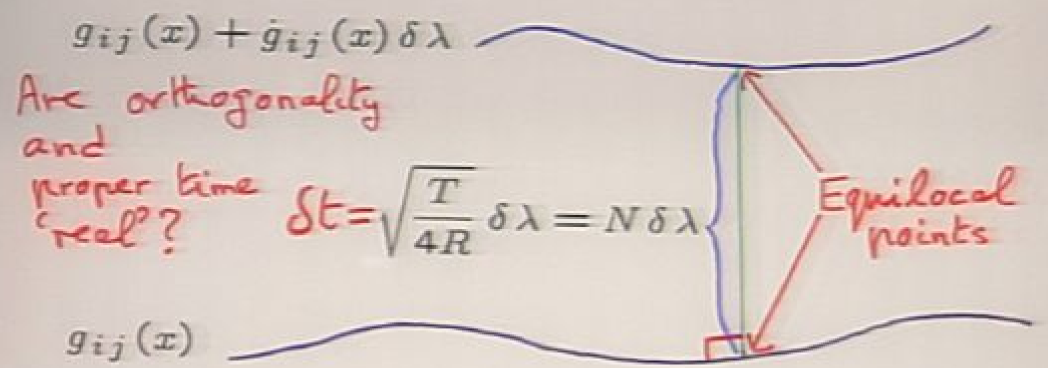
$\boxed{A=1 \text{ and } U=\Lambda+kR,\ \Lambda=\text{Const},\ k=0,\pm 1}$

where R is the 3d scalar curvature.

Relativity without Relativity

The two Machian requirements needed to define relational momenta p^{ij}, natural simplicity assumptions, the 'magic' square root, and **consistency** lead to the Baierlein–Sharp–Wheeler action (1962) for GR.

Construction of Spacetime



Connections between best-matched (*equilocal*) points are hypersurface-orthogonal in the constructed spacetime and have 'vertical' separation equal to local proper-time.

The local square root gives GR but takes away unique evolution in superspace ('sheaf' of geodesics More anon.

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'DERIVATION' OF SPECIAL RELATIVITY

Attempt to couple scalar field to Machian g_{ij}:

$$\boxed{A_{g+\phi} = \int \mathrm{d}\lambda \int \mathrm{d}^3x \sqrt{g\,(R + \underset{\text{Freedom}}{U_\phi})(T_g + \underset{\text{`Fixed'}}{T_\phi})},}$$

Fixed

where

$$T_\phi = (\dot\phi - \phi_{;i}\dot\xi^i)^2,\ U_\phi = \sum_n A_n\phi^n - \frac{C}{4} g^{ij}\phi_{;i}\phi_{;j}$$

with C an arbitrary constant.

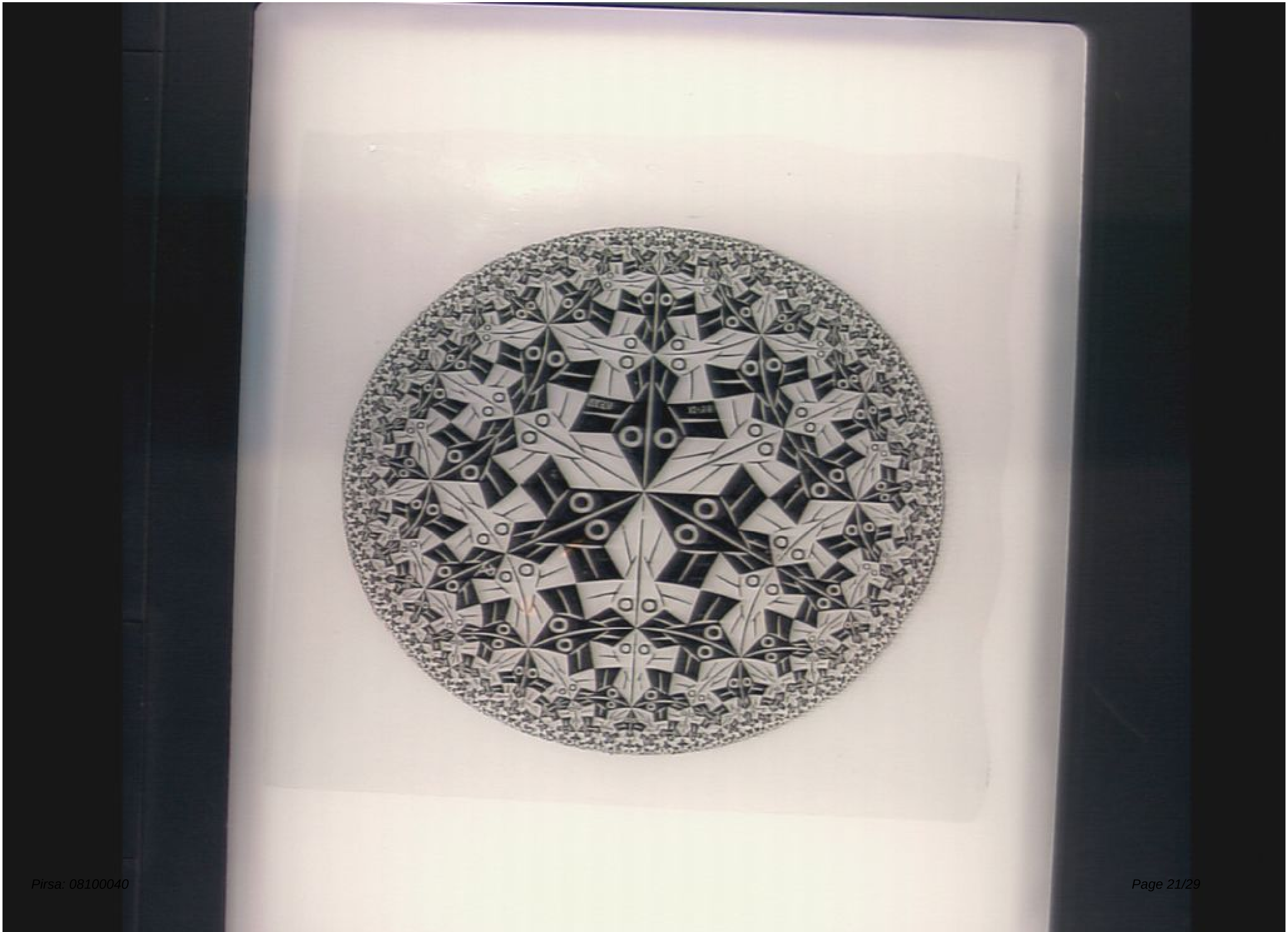
The modified constraints are

$$\boxed{p^{ij}p_{ij} - \frac{1}{2}p^2 + \pi^2 - gR + U_\phi(C) = 0}$$

and $\boxed{p^{ij}{}_{;j} - \frac{1}{2}\pi\phi^{;i} = 0}$, where

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \frac{1}{2N}\{\dot\phi - \phi_{;i}\dot\xi^i\},\ 2N = \sqrt{\frac{g\,(R + U_\phi)}{T_g + T_\phi}}$$

The EL Eqs. only propagate the quadratic constraint if $\boxed{C = 1}$. This happens for all fields and enforces a <u>universal light cone</u>. The Machian approach reverses things: GR is derived first, *then* SR (Barbour, Foster, Ó Murchadha).



Beltrami (-Poincaré) repn.

SCALE-INVARIANT g_{ij} – DYNAMICS

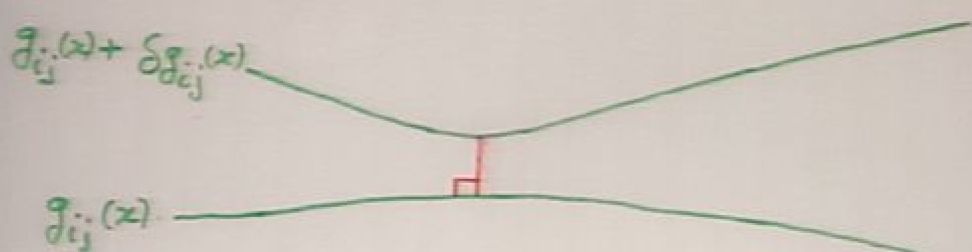
$$\text{Superspace} = \frac{\text{Riem}}{3\text{-diffeomorphisms}}$$

$$\text{CS} + V = \frac{\text{Superspace}}{\text{Volume-preserving } g_{ij} \to \phi^4 g_{ij}}$$

$$\text{Conformal Superspace (CS)} = \frac{\text{Superspace}}{g_{ij} \to \phi^4 g_{ij}}$$

Best matching of Jacobi-type actions wrt 3-diffeomorphisms and conformal transformations leads to theories in which *a point and a direction* in CS or CS+V determine unique curves.

Action is invariant under $g_{ij} \to \omega^4 g_{ij}$, $\phi \to \phi/\omega$ but theory is unconventional because orthogonal separation between conformal-group orbits is not constant:



DYNAMICS IN CS+V

the BSW action

Nothing better!

Best match wrt 3-diffeomorphisms and volume-preserving conformal transformations

The action becomes

where

with

where

Best matching wrt $\dot{\phi} \Rightarrow p_\phi = 0$.

B . $p \equiv \frac{4}{\phi}(\pi - \sqrt{g} < p >)$, so that

$$p = \frac{\ddot{}}{\sqrt{g}} = \lambda\text{-dependent spatial } Const.$$

This is York's CMC condition

Also $\pi^{ij}{}_{;j} = 0$ and with $\sigma^{ij} = \pi^{ij} - \frac{1}{3} g^{ij} \pi$

$$\sigma^{ij}\sigma_{ij} - \frac{\pi^2 \hat{\phi}^{12}}{6} - g\hat{\phi}^8 \left(R - 8\frac{\nabla^2 \hat{\phi}}{\hat{\phi}} \right) = 0.$$

This is the Lichnerowicz–York equation.

Variation wrt ϕ using $p = Const$ and

$$N = \frac{1}{2}\sqrt{\frac{\hat{T}}{\hat{\phi}^{-4}(R - 8\nabla^2 \phi/\phi)}} \text{ in gauge } \hat{\phi} = 1$$

gives the CMC lapse-fixing condition

$$NR - \nabla^2 N + \frac{Np^2}{4} = D = \text{spatial } Const$$

Res. matching wrt $\dot{\phi} \Rightarrow p_\phi = 0$.

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Machian Quantum Gravity (MQG)

Details of research project funded by the Foundational Questions Institute (fqxi.org) will shortly be available on my website www.platonia.com or can be requested from me at

julian.barbour@physics.ox.ac.uk

Offers of collaboration will be enthusiastically considered. Some travel and accommodation funds available, especially for young researchers and even students at undergraduate level. Mini-workshops at and visits to College Farm in DNO (Deepest North Oxfordshire).

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CONCLUSIONS & QUANTUM-GRAVITY IMPLICATIONS

(non-discrete)

1. Case for configuration-space approach seems relatively strong though not decisive.

2. Expansion of the universe is mysterious.

3. Action as incongruence of shapes is attractive, especially in view of special properties of CMC initial data.

4. Quantum gravity may break relativity by favouring CMC data.

5. Mach may yet have the last word.

PS. Mystery of complex numbers and real linear Wheeler-DeWitt equation.