Title: Gravitational waves from cosmological sources

Date: Oct 21, 2008 02:00 PM

URL: http://pirsa.org/08100029

Abstract: In this talk I will discuss gravitational wave production by early universe sources. I will focus on the gravitational waves produced by a network of cosmic strings and the bounds that can be placed on cosmic string model parameters using current and future experiments. I will also talk about recent work on gravitational waves produced by sources in the early universe when the expansion of the universe cannot be neglected. As an example of such a process I will consider the preheating epoch that may follow inflation.

Gravitational waves from cosmological sources

Xavier Siemens

Center for Gravitation and Cosmology

UNIVERSITY of WISCONSIN
UWMILWAUKEE

Gravitational waves from cosmological sources

Xavier Siemens

Center for Gravitation and Cosmology

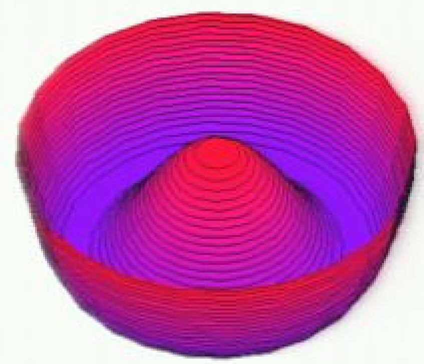
UNIVERSITY of WISCONSIN
UW MILWAUKEE

Summary

- Cosmic strings:
  - ⋆ Brief intro
  - ⋆ Detectability of bursts by LIGO
  - ⋆ Detectability of stochastic background by LIGO, LISA, and other experiments
  - ⋆ Bursts in LISA
- Preheating

Cosmic strings

- Two kinds of strings. Field-theoretic strings (old-school), and new cosmic superstrings
- Kibble (1976) realized that in theories with phase transitions defect formation is generic
- The most simple phase transition that leads to string formation



$$U(1) \xrightarrow{\eta} 1$$

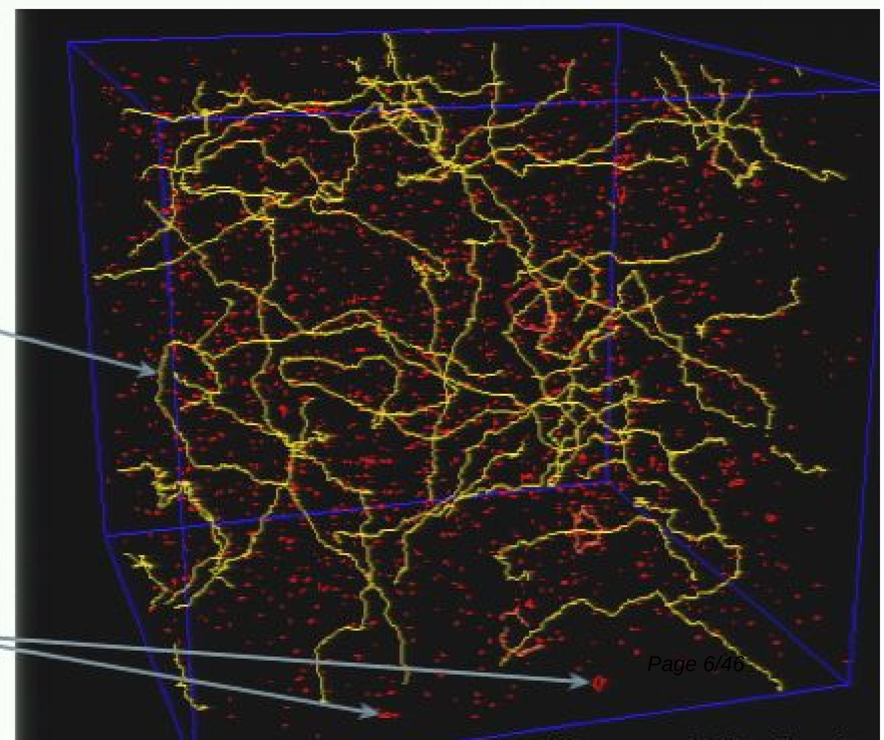
- Strings form with mass per unit length $\mu \sim \eta^2$
- Characterize strings using dimensionless quantity $G\mu = \left(\frac{\eta}{m_p}\right)^2$

Cosmic string evolution

- Cosmic strings do not lead to cosmological disasters (unlike monopoles and domain walls)
- Network evolves into a scaling solution where the energy density in strings is a small fraction of the matter or radiation energy density.
- Scaling occurs because network produces loops which decay gravitationally

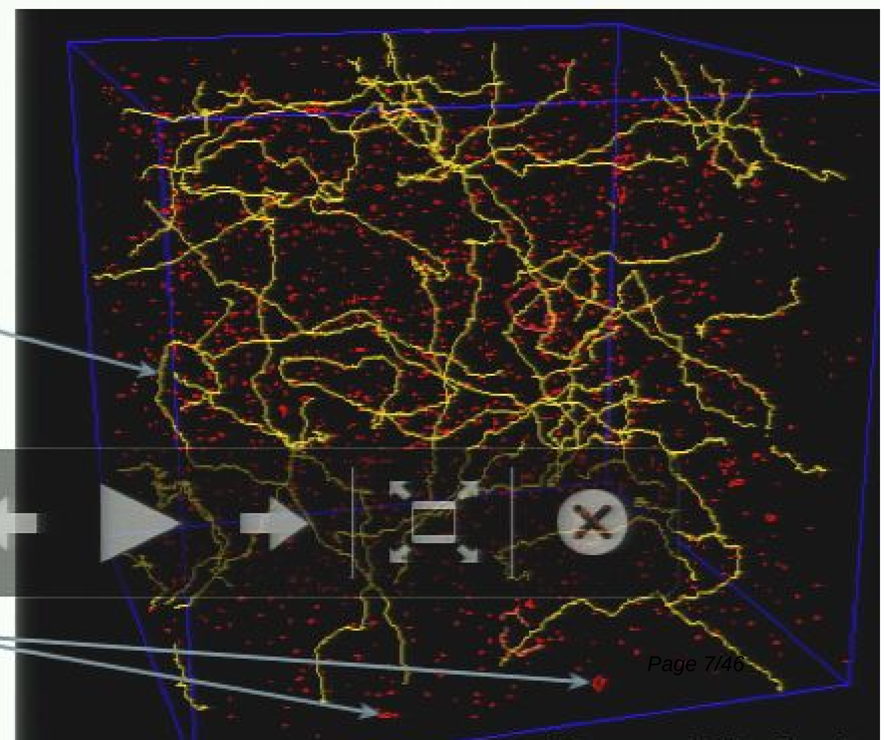
Long strings (yellow)

Loops (red)
How big are they?



Cosmic string evolution

- Cosmic strings do not lead to cosmological disasters (unlike monopoles and domain walls)
- Network evolves into a scaling solution where the energy density in strings is a small fraction of the matter or radiation energy density.
- Scaling occurs because network produces loops which decay gravitationally

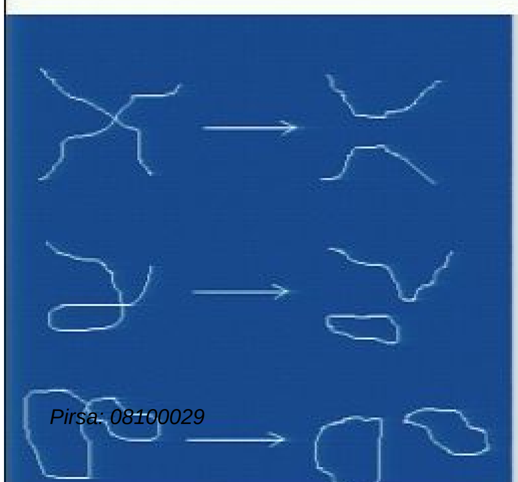


Long strings (yellow)

Loops (red)
How big are they?

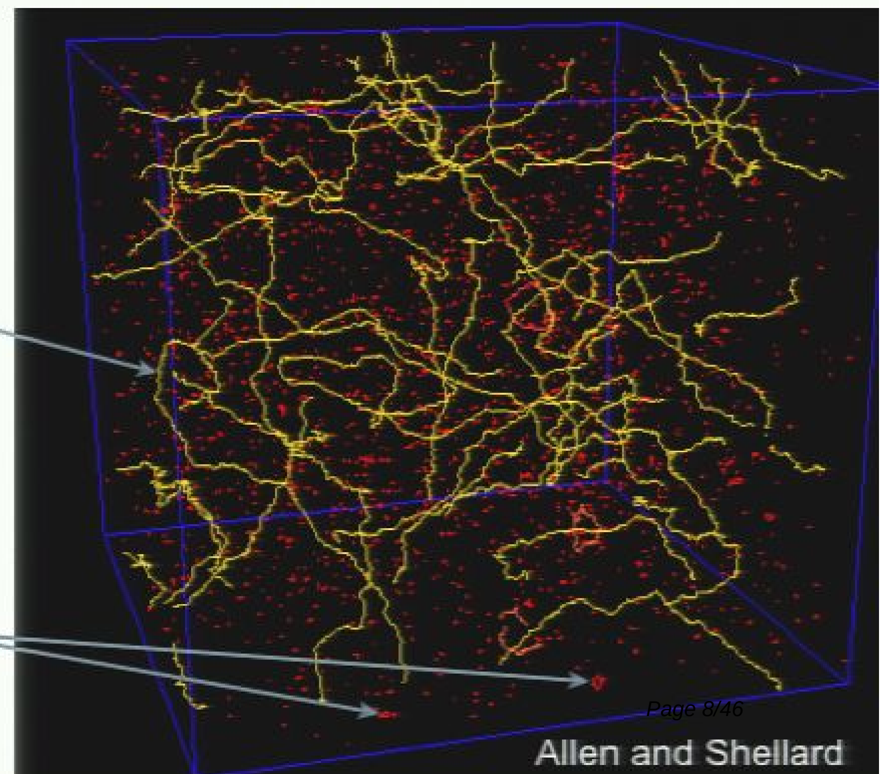
Cosmic string evolution

- Cosmic strings do not lead to cosmological disasters (unlike monopoles and domain walls)
- Network evolves into a scaling solution where the energy density in strings is a small fraction of the matter or radiation energy density.
- Scaling occurs because network produces loops which decay gravitationally



Long strings (yellow)

Loops (red)
How big are they?



Recent interest in cosmic strings

- Angular spectrum of CMB rules out values of the tension at the GUT scale

$$G\mu < 10^{-7}$$

- But for these and lighter strings [Damour & Vilenkin 00,01,05] bursts from cusps could be detectable by LIGO, VIRGO, and LISA

- Later it was realised [Sarangi & Tye, 02] that string theory inspired inflation models would also lead to cosmic string production and thus have consequences observable in the near future. [Sarangi, Tye, Polchinksi, Jones, Jackson, Copeland, Myers, Dvali, Vilenkin, Wyman, Leblond, Shlaer, Firouzjahi, Brandenberger,...]

Size of cosmic string loops

Loops decay gravitationally according to

$$l(t) = l_i - \Gamma G\mu(t - t_i), \;\; \Gamma \sim 50$$

If the size of loops at formation is $l_i = \alpha t_i$ then loops decay in a time

$$t_f = \left(\frac{\alpha}{\Gamma G\mu} + 1\right) t_i$$

Loops are long-lived when $\alpha \gg \Gamma G\mu$, short- and long-lived loops lead to very different loop populations

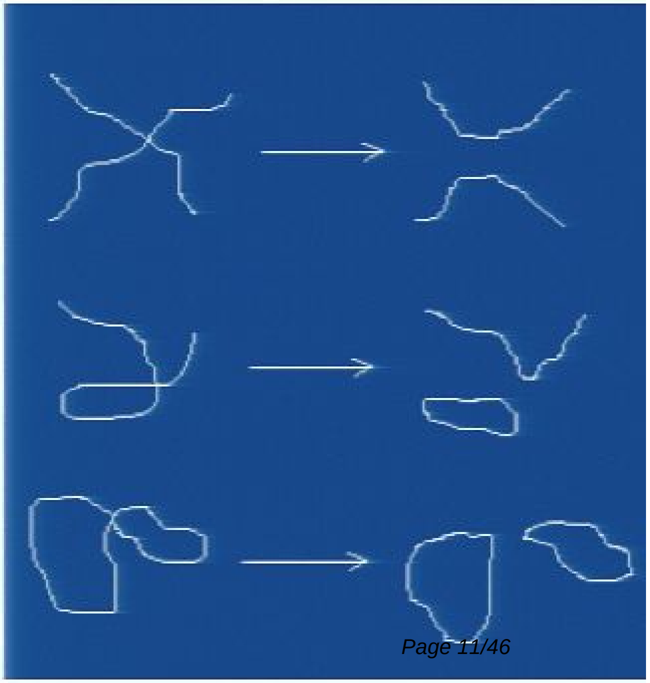
There is disagreement on the value of α

Olum, Vanchurin, Vilenkin	Shellard, Martins, Avgoustidis, Ringeval, Sakellariadou, Bouchet	Polchinski, Rocha, Dubath
$\alpha \sim 0.1$	$\alpha \ll 0.1$	$\alpha \sim 0.1, \alpha \sim (\Gamma G\mu)$

Cosmic strings vs. cosmic superstrings

- Same signal, different interpretation of results.
- Two differences:
  - More than one type of string can form (complicated interactions--nasty problem)
  - Re-connection probability

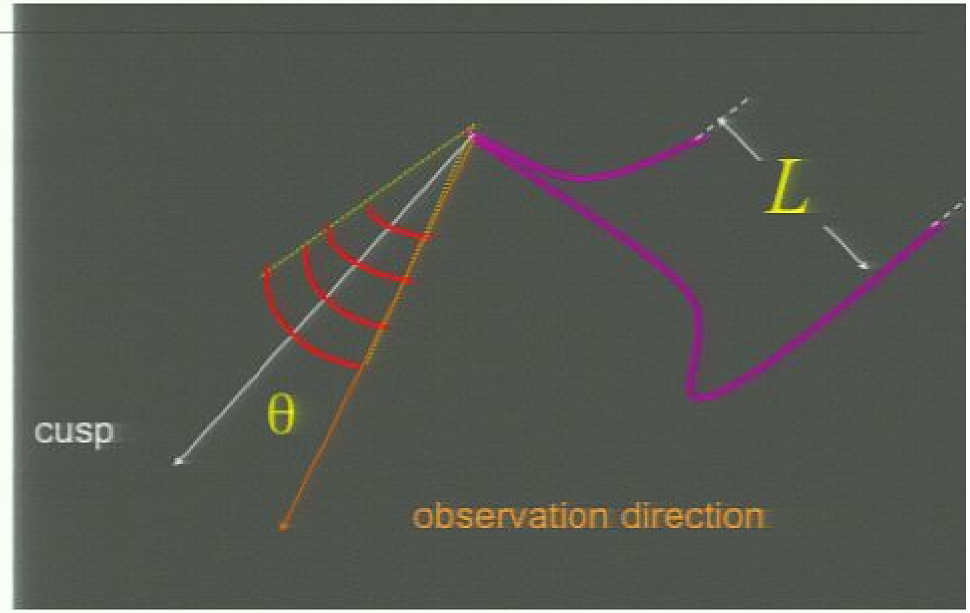
$$10^{-3} \leq p \leq 1$$



- Effect is to increase the density of string

$$\rho \propto p^{-1}$$

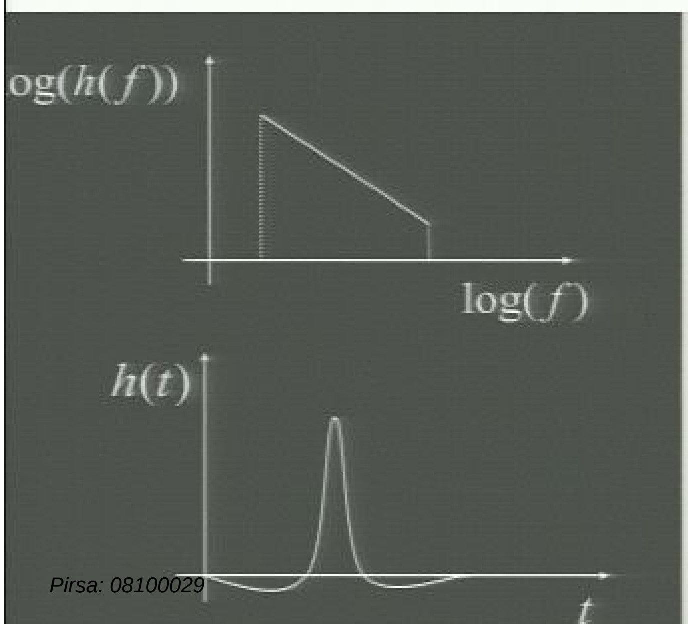
The gravitational signal produced by a cusp



Cusps are regions of string that instantaneously acquire huge Lorentz boosts

Metric perturbation is computed using linearized Einstein Eqs.

Waveform is generic: All cusps are the same



$$h(f) = A f^{-4/3} \Theta(f_h - f)\Theta(f - f_l)$$

- High frequency cutoff f_h depends on cusp direction
- Low frequency cutoff f_l is cosmological, in practice depends on instrument
- $A \sim \dfrac{G\mu L^{2/3}}{r}$

Computing the gravitational burst rate

- Burst rate calculation starts from the loop distribution; number density of loops with lengths in interval dl at time t

$$n(l,t)dl$$

- Each loop oscillates in time $l/2$ and has (on average) c cusps per oscillation

$$\frac{2c}{l}n(l,t)dl$$

- Write all quantities in the rate as cosmology dependent functions of the redshift

$$t = H_0^{-1}\phi_t(z) \quad r = H_0^{-1}\phi_r(z) \quad dV(z) = H_0^{-3}\phi_V(z)dz$$

- A loop of some length l at some redshift z produces a burst of amplitude $A(l,z)$ Can write the rate of bursts as a function of the amplitude dR/dA and integrate the rate above some amplitude deemed detectable

$$R(> A_{\min}) = \int_{A_{\min}}^{\infty} dA \frac{dR}{dA}$$

LIGO

4km Interferometer
&
2km Interferometer

4km Interferometer

Computing the gravitational burst rate

- Burst rate calculation starts from the loop distribution; number density of loops with lengths in interval dl at time t

$$n(l,t)dl$$

- Each loop oscillates in time $l/2$ and has (on average) c cusps per oscillation

$$\frac{2c}{l}n(l,t)dl$$

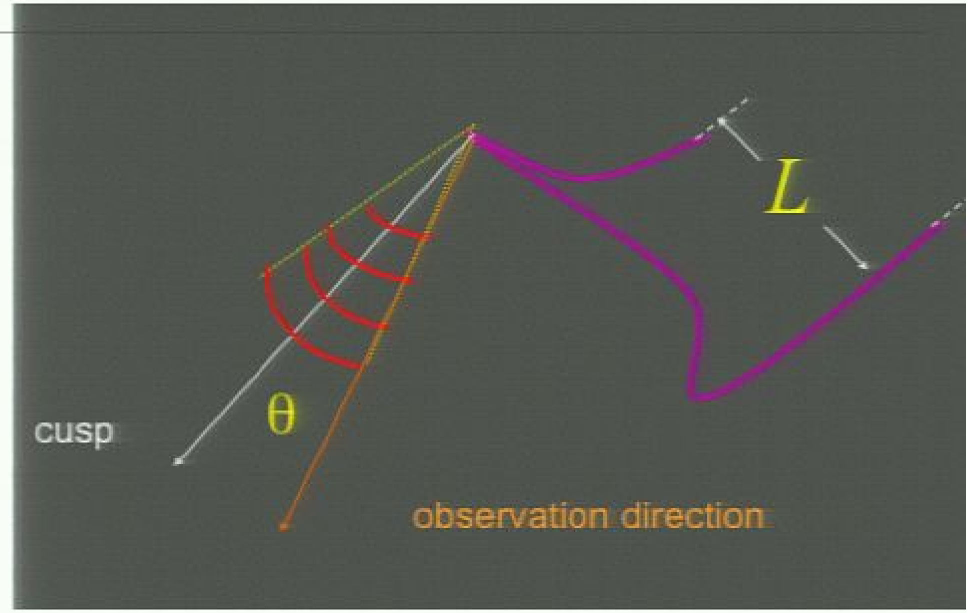
- Write all quantities in the rate as cosmology dependent functions of the redshift

$$t = H_0^{-1}\phi_t(z) \quad r = H_0^{-1}\phi_r(z) \quad dV(z) = H_0^{-3}\phi_V(z)dz$$

- A loop of some length l at some redshift z produces a burst of amplitude $A(l,z)$ Can write the rate of bursts as a function of the amplitude dR/dA and integrate the rate above some amplitude deemed detectable

$$R(> A_{\min}) = \int_{A_{\min}}^{\infty} dA \frac{dR}{dA}$$

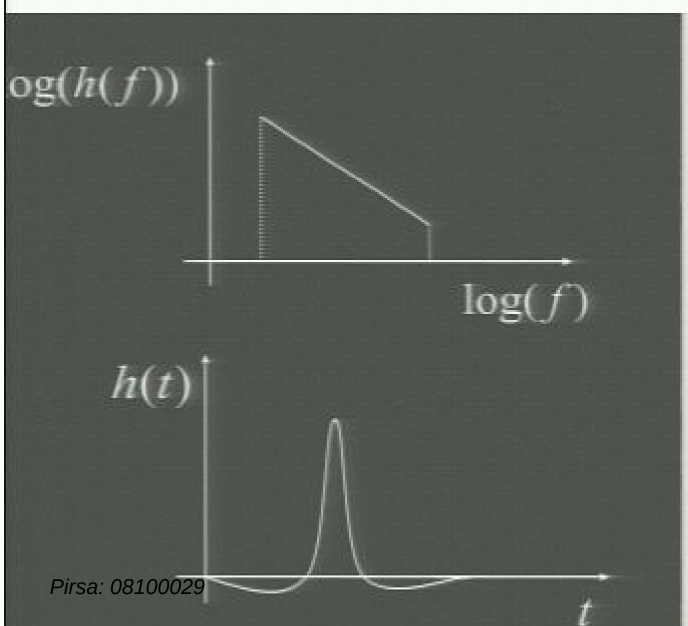
The gravitational signal produced by a cusp



Cusps are regions of string that instantaneously acquire huge Lorentz boosts

Metric perturbation is computed using linearized Einstein Eqs.

Waveform is generic: All cusps are the same



$$h(f) = A f^{-4/3} \Theta(f_h - f)\Theta(f - f_l)$$

- High frequency cutoff f_h depends on cusp direction
- Low frequency cutoff f_l is cosmological, in practice depends on instrument
- $A \sim \dfrac{G\mu L^{2/3}}{r}$

Computing the gravitational burst rate

- Burst rate calculation starts from the loop distribution; number density of loops with lengths in interval dl at time t

$$n(l,t)dl$$

- Each loop oscillates in time $l/2$ and has (on average) c cusps per oscillation

$$\frac{2c}{l}n(l,t)dl$$

- Write all quantities in the rate as cosmology dependent functions of the redshift

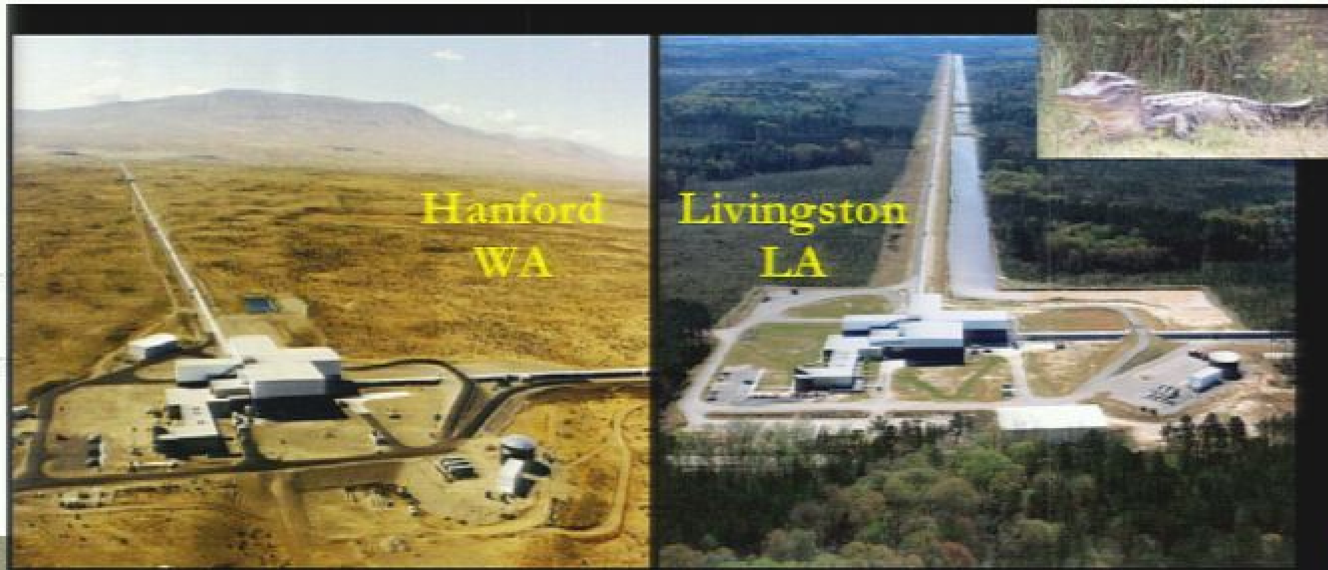
$$t = H_0^{-1}\phi_t(z) \quad r = H_0^{-1}\phi_r(z) \quad dV(z) = H_0^{-3}\phi_V(z)dz$$

- A loop of some length l at some redshift z produces a burst of amplitude $A(l,z)$ Can write the rate of bursts as a function of the amplitude dR/dA and integrate the rate above some amplitude deemed detectable

$$R(>A_{\min}) = \int_{A_{\min}}^{\infty} dA \frac{dR}{dA}$$

LIGO

km Interferometer
&
km Interferometer



4km Interferometer

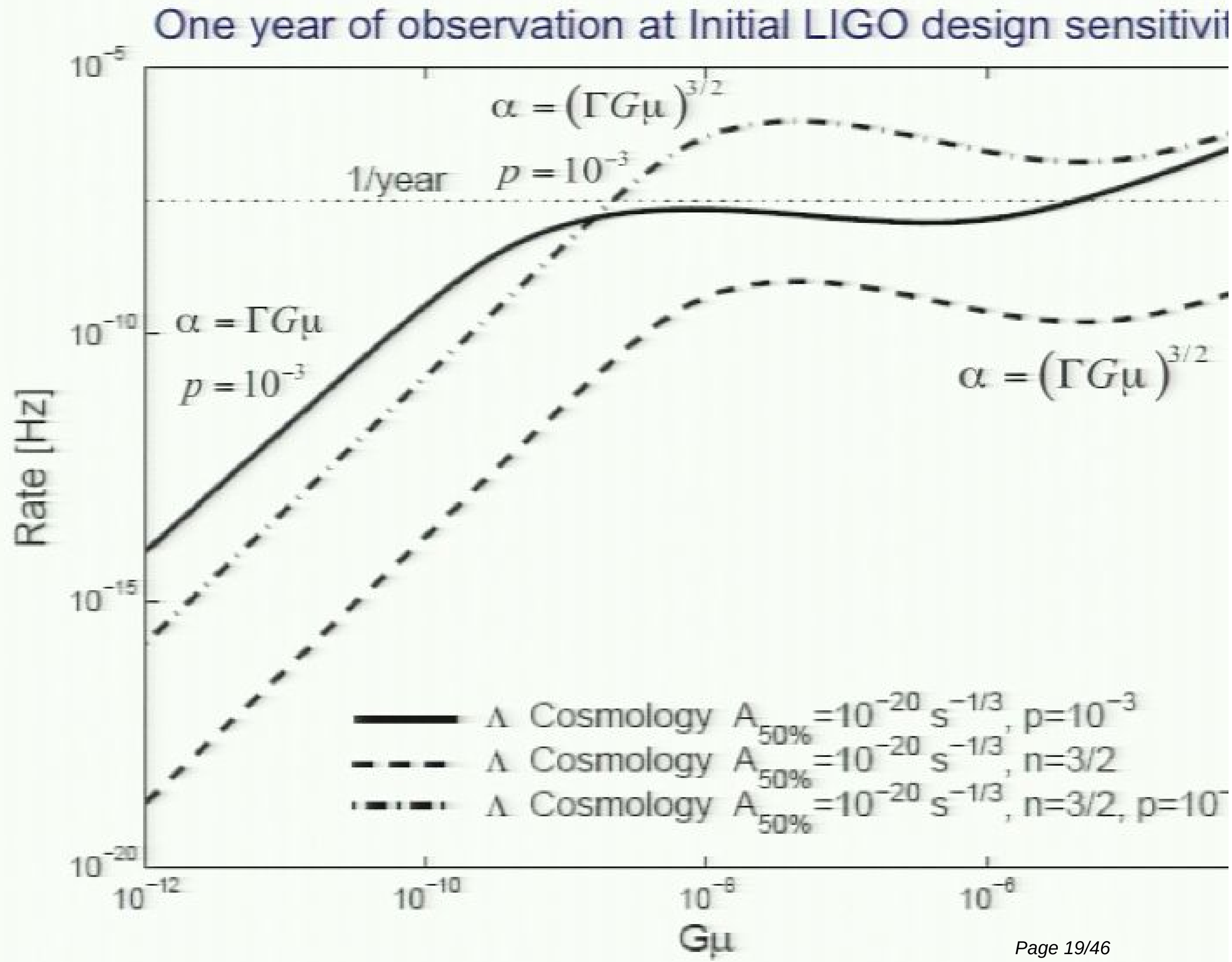
Detectability of cosmic string bursts by LIGO

For "small" loops (short-lived loops)

$$n(l,t) \propto \delta(l - \alpha t)$$

Results are less optimistic than Damour and Vilenkin. But superstrings could still be detected.

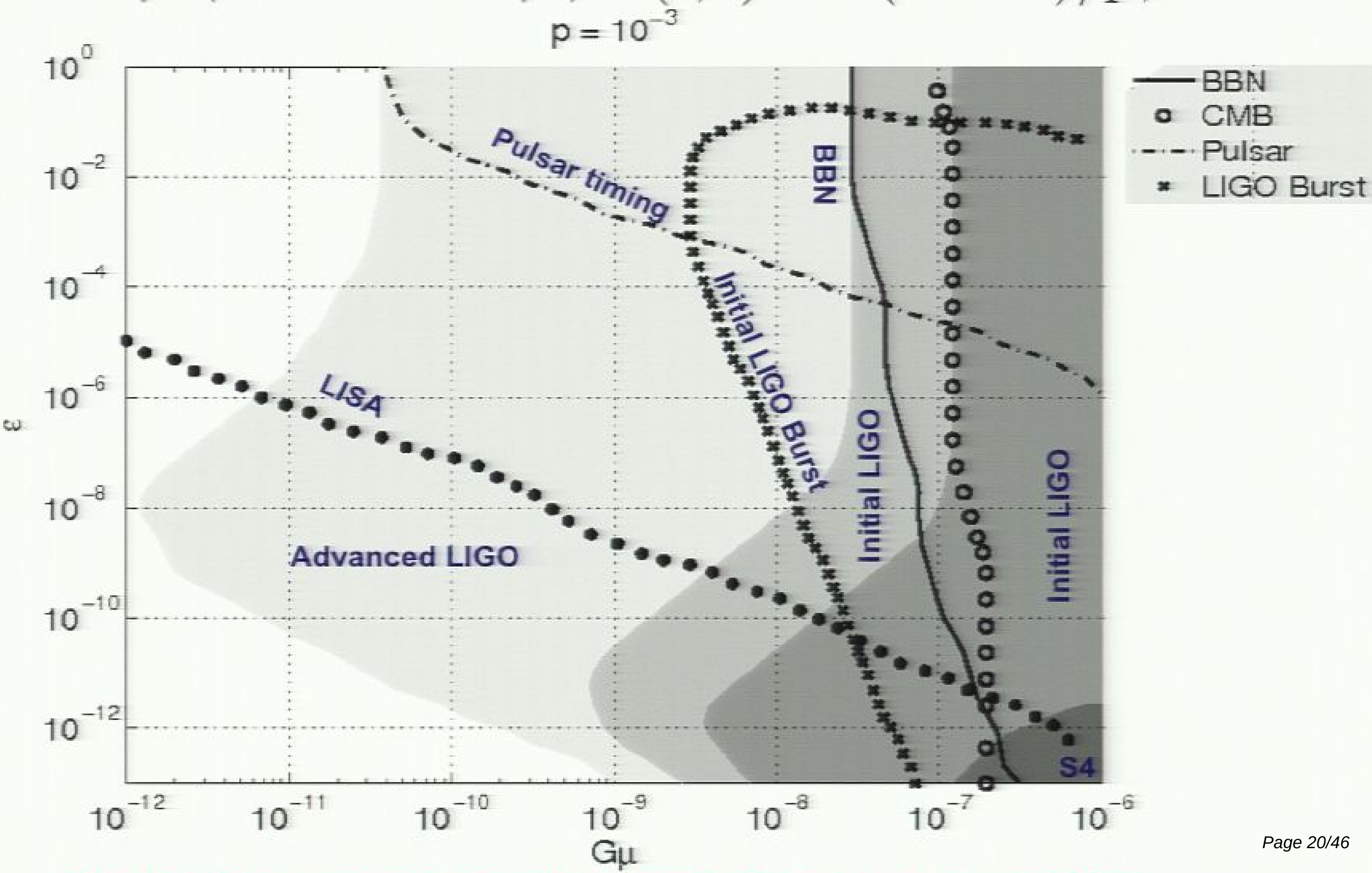
"Large" loops are not so interesting (we'll see)



XS, J. Creighton, I. Maor, K. Cannon, S. Majumder, J. Read, Phys. Rev. D73 (2006) 105001 (gr-qc/0603115)

Stochastic background of a cosmic string network

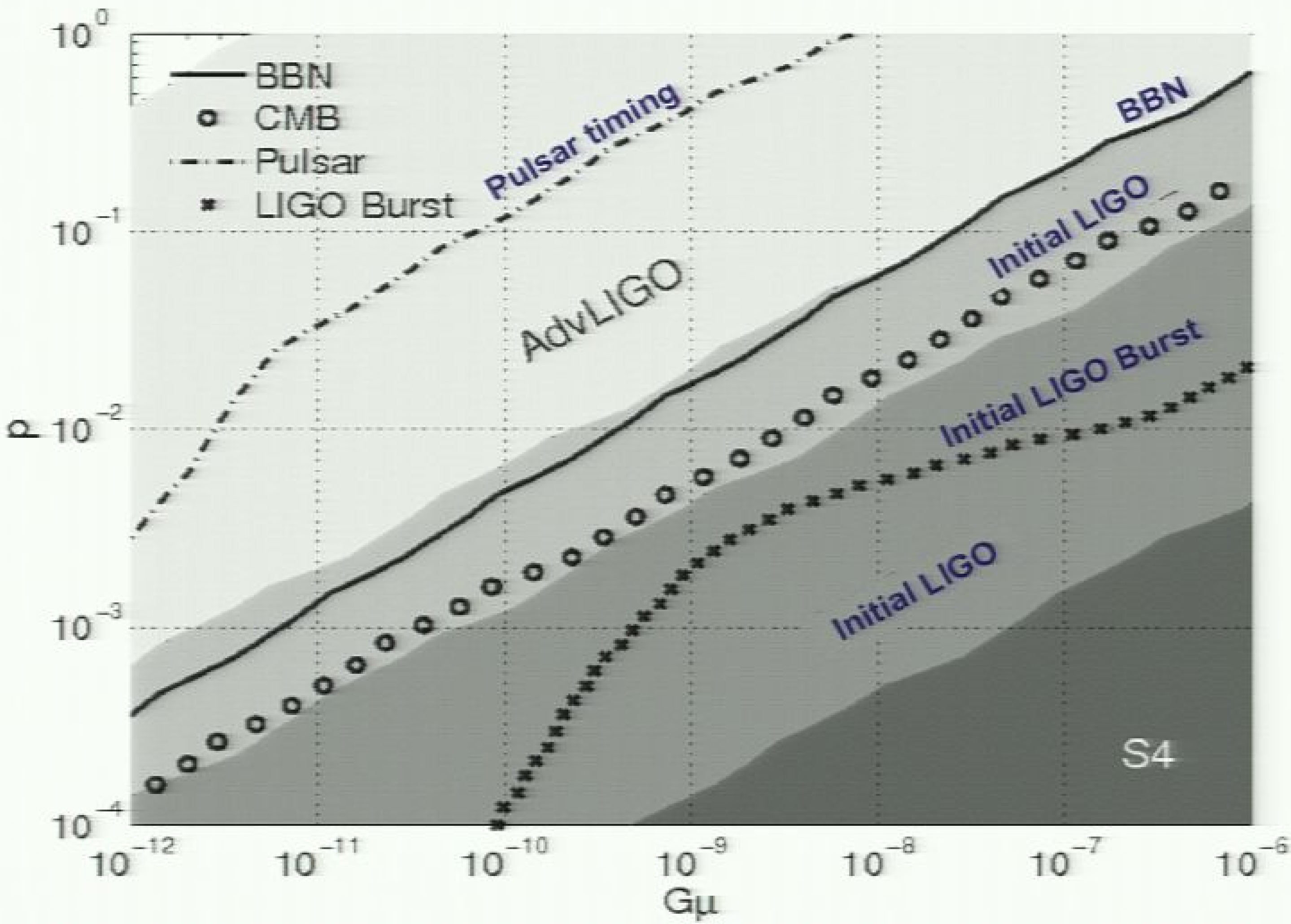
For "small" loops (short-lived loops) $n(l,t) \propto \delta(l - \alpha t)/p, \;\; \alpha = \varepsilon \Gamma G\mu$



XS, V. Mandic, J. Creighton, Phys. Rev. Lett. 98 (2007) 111101 (gr-qc/0610920)

Stochastic background of a cosmic string network

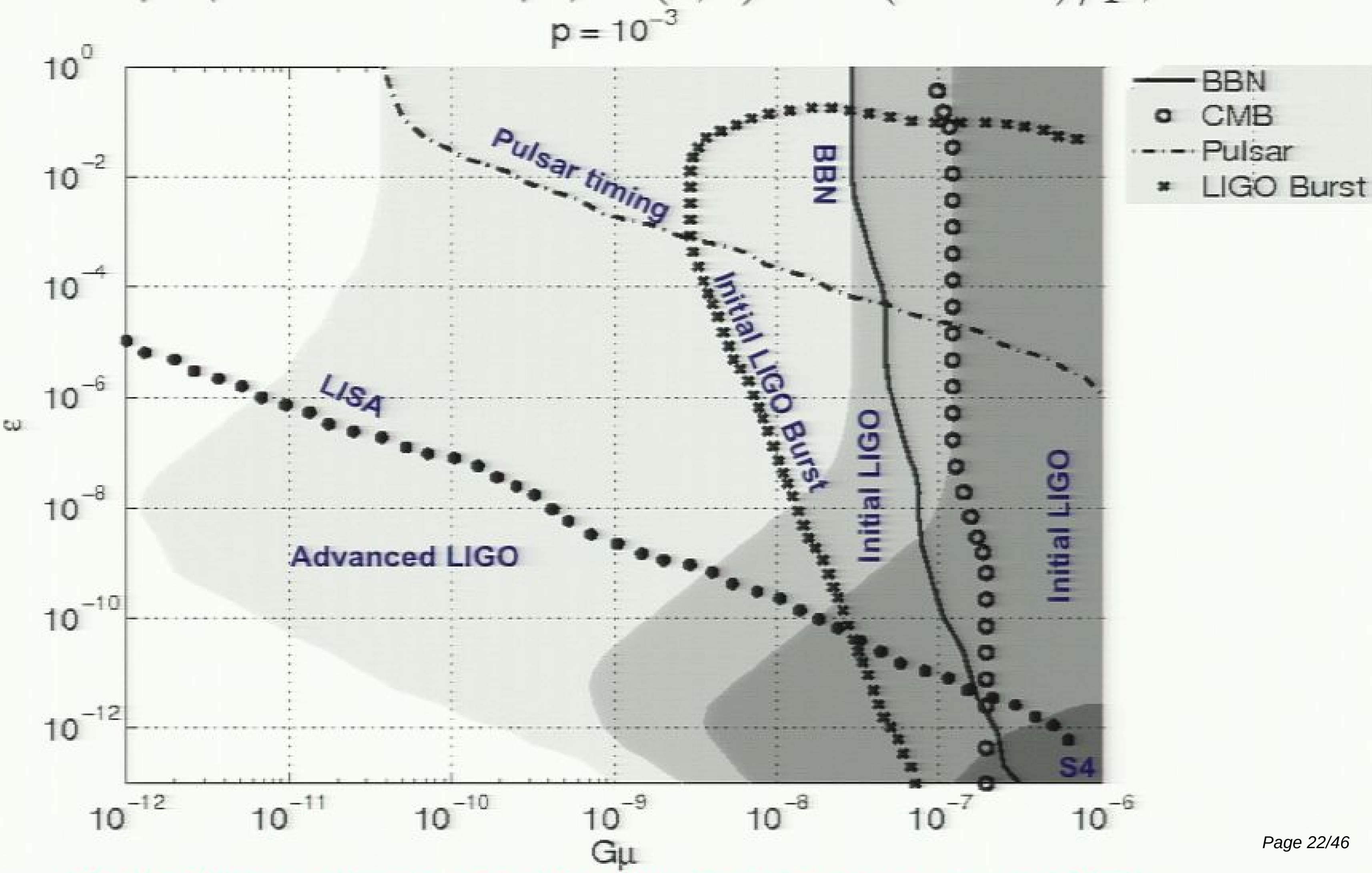
For “large” loops (long-lived loops) vary $p,\ G\mu$ set $\alpha = 0.1$



XS, V. Mandic, J. Creighton, Phys. Rev. Lett. 98 (2007) 111101 (gr-qc/0610920)

Stochastic background of a cosmic string network

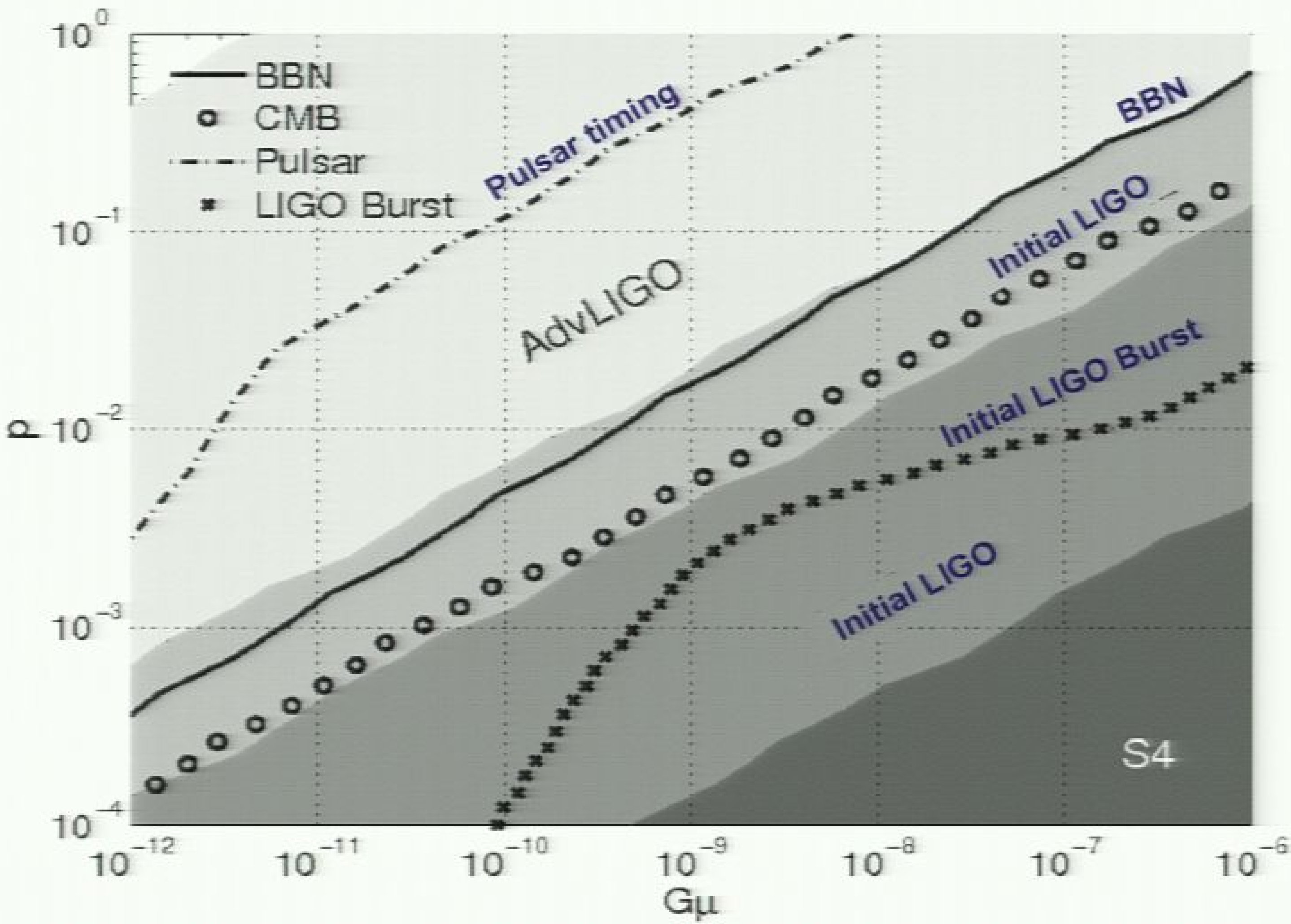
For "small" loops (short-lived loops) $n(l,t) \propto \delta(l - \alpha t)/p, \;\; \alpha = \varepsilon \Gamma G\mu$



XS, V. Mandic, J. Creighton, Phys. Rev. Lett. 98 (2007) 111101 (gr-qc/0610920)

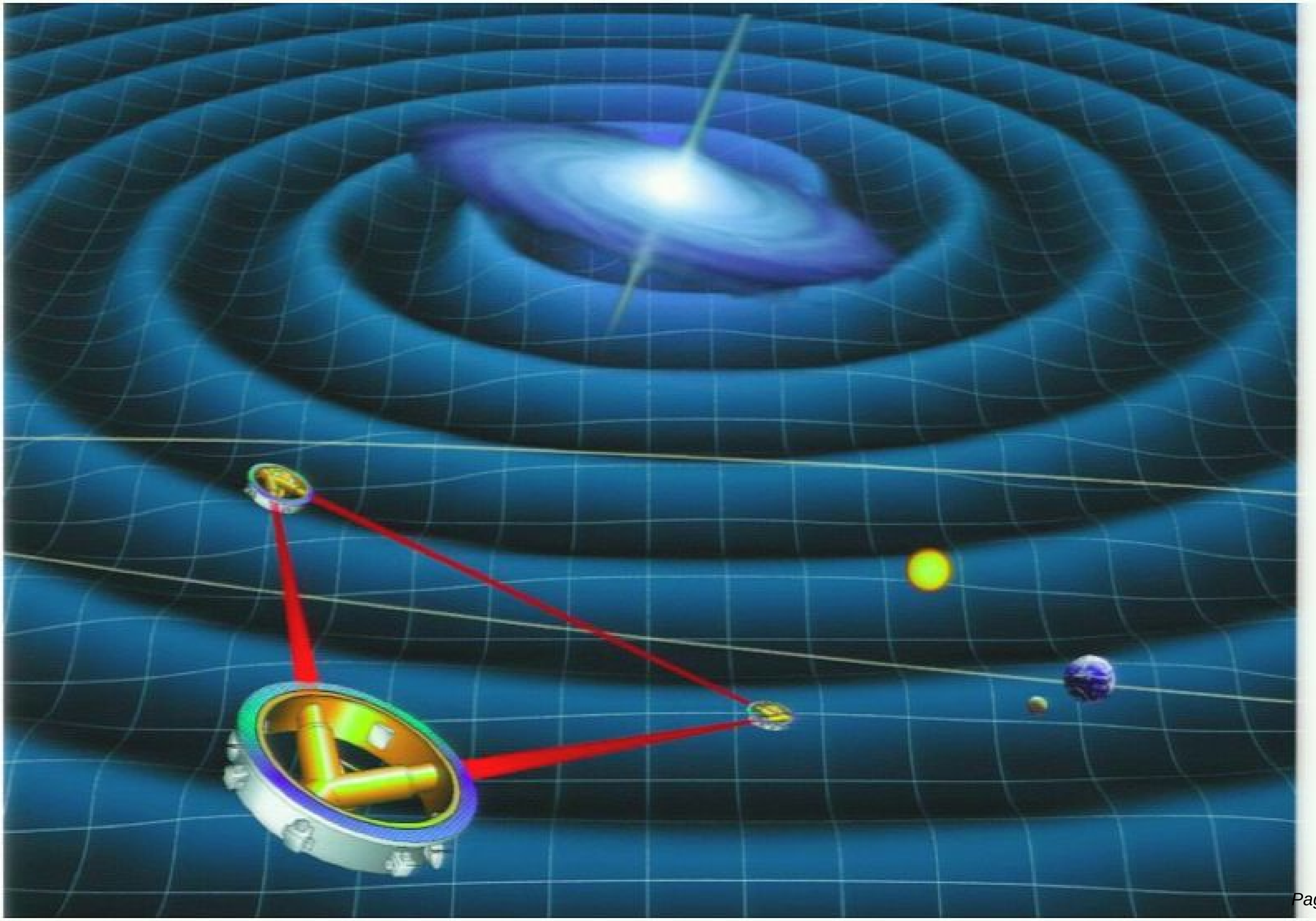
Stochastic background of a cosmic string network

For “large” loops (long-lived loops) vary p, $G\mu$ set $\alpha = 0.1$

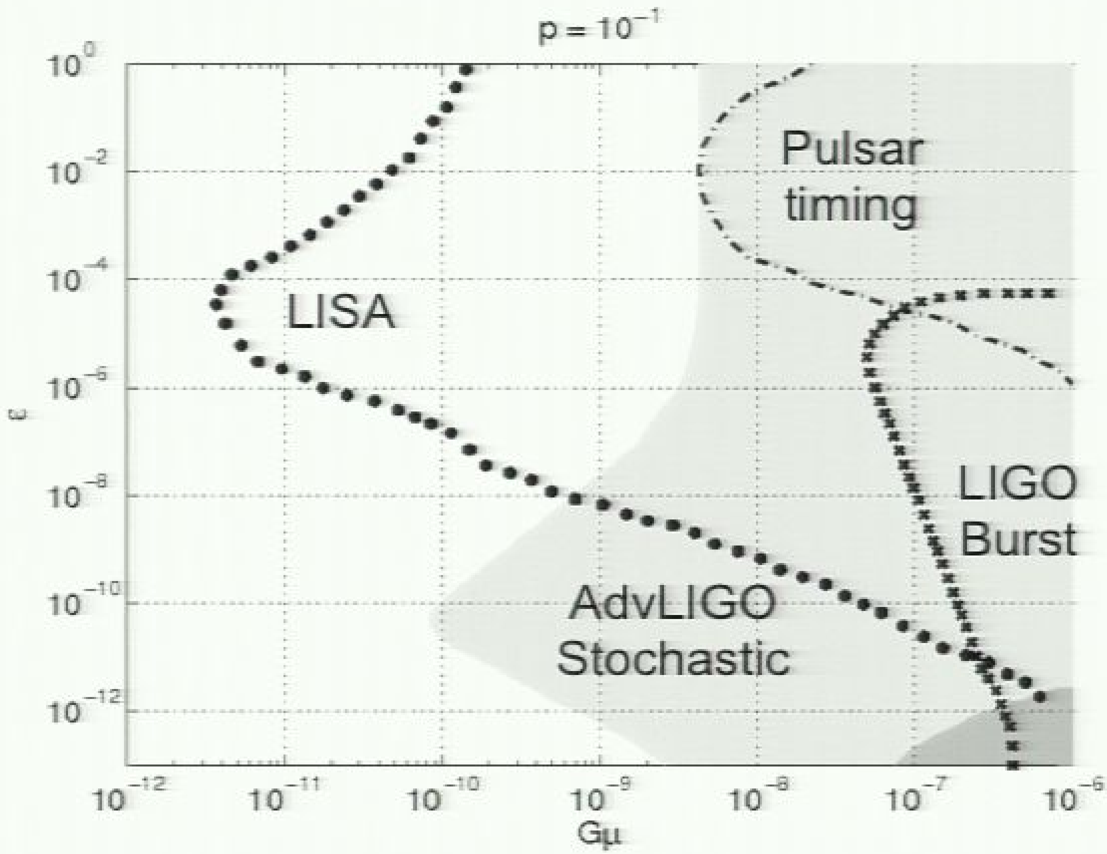


XS, V. Mandic, J. Creighton, Phys. Rev. Lett. 98 (2007) 111101 (gr-qc/0610920)

LISA

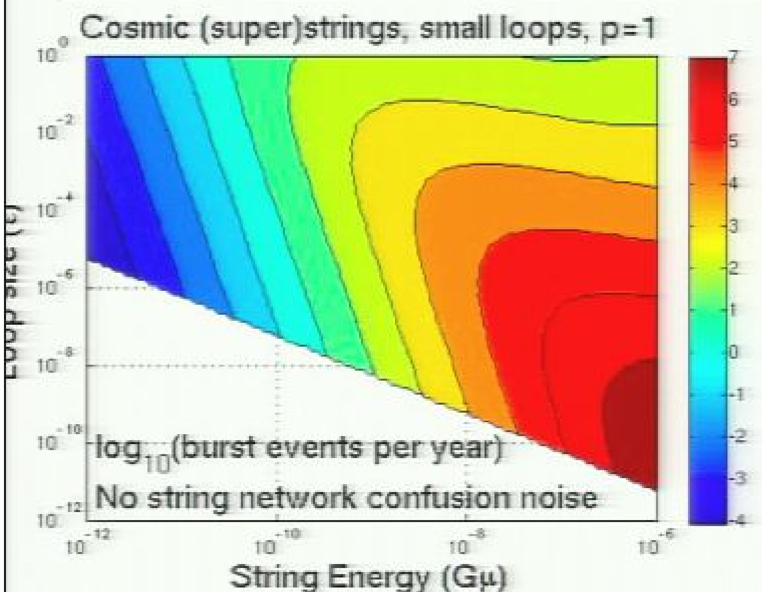


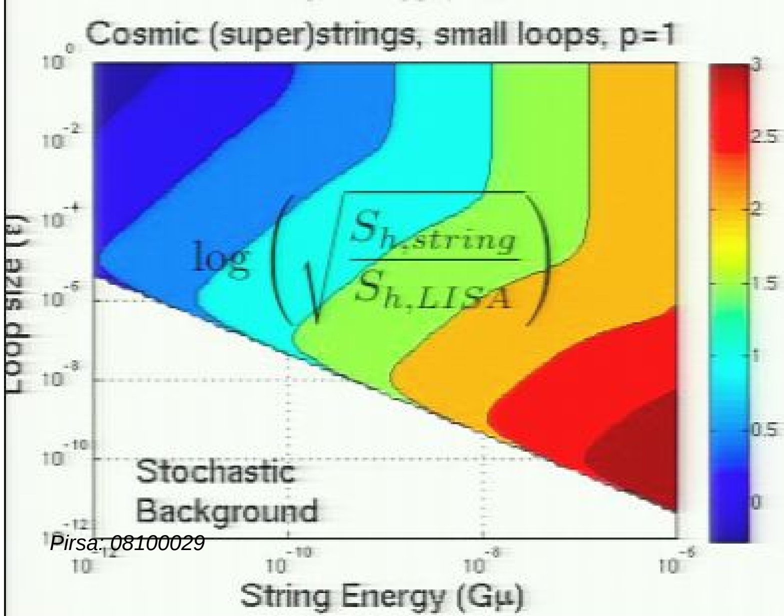
Detectability of cosmic string bursts by LISA

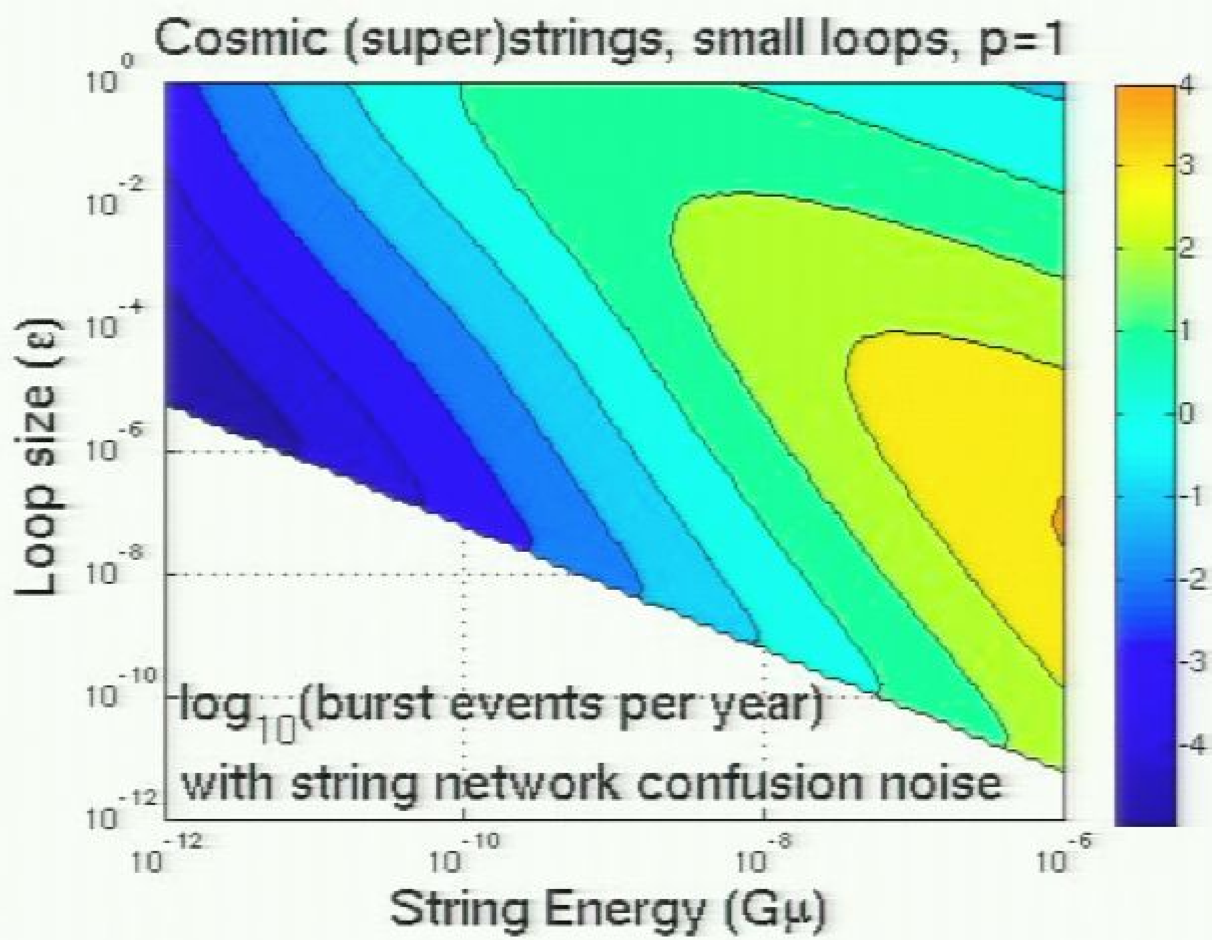


- LISA is so sensitive, for LISA estimates we just compute the noise added to the instrument by the network of cosmic strings
- Suppose we detect stochastic background using LISA; how do we know it's cosmic strings and not something else?

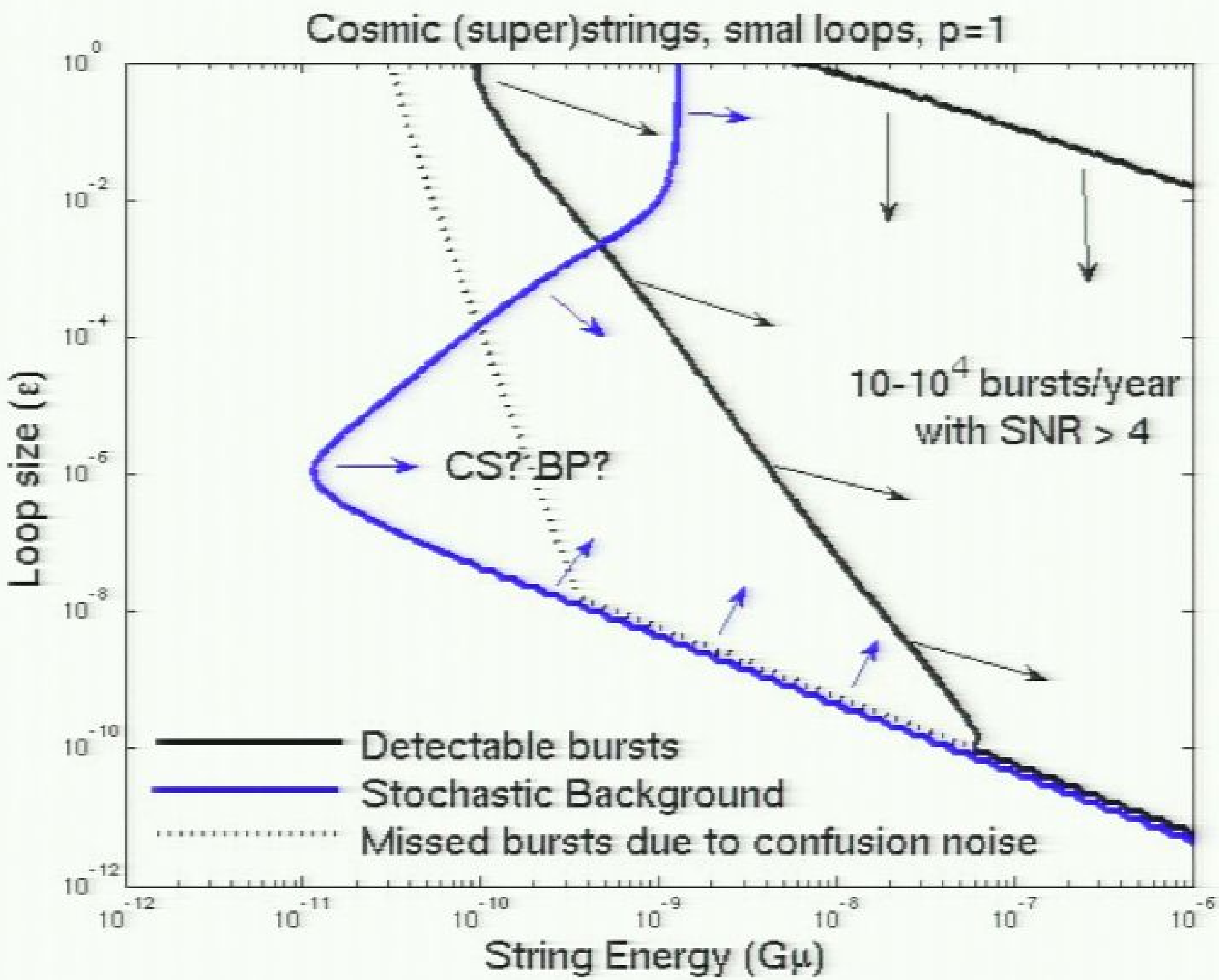
LISA and bursts from small loops







LISA and bursts from small loops



Gravitational waves from cosmic (super)strings

- For small loops current most constraining results are from pulsar timing and BBN, but LIGO will explore an area of parameter space complementary to these
- If loops are large stochastic searches are more effective than burst searches. Severe constraints on superstrings.
- Of all gravitational wave detectors LISA has the best chance of detecting a cosmic string generated signal (stochastic and burst)
- A combined LISA burst and stochastic detection could help pin down properties of cosmic strings: string tension, reconnection probability,...

Gravitational waves from preheating [w/. Larry Price]

- Read interesting paper by Easther and Lim on gravitational waves from preheating [JCAP 0604:010,2006. e-Print: astro-ph/0601617]

- Easther and Lim used LATTICEEASY [Felder & Tkachev], a code that evolve scalar fields in a background (say, expanding) self consistently:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{4}\phi^4 - \frac{1}{2}g^2\phi^2\chi^2$$

- They then used formula by Weinberg to estimate gravitational wave production (stochastic background):

$$\frac{dE_{\rm gw}}{d\Omega} = \pi^2 \sum_{i,j} \int_{-\infty}^{\infty} d\omega\, \omega^2 \left|T_{ij}^{\rm TT}(\omega, \mathbf{k})\right|^2$$

Gravitational waves from cosmic (super)strings

- For small loops current most constraining results are from pulsar timing and BBN, but LIGO will explore an area of parameter space complementary to these
- If loops are large stochastic searches are more effective than burst searches. Severe constraints on superstrings.
- Of all gravitational wave detectors LISA has the best chance of detecting a cosmic string generated signal (stochastic and burst)
- A combined LISA burst and stochastic detection could help pin down properties of cosmic strings: string tension, reconnection probability,...

Gravitational waves from preheating [w/. Larry Price]

- Read interesting paper by Easther and Lim on gravitational waves from preheating [JCAP 0604:010,2006. e-Print: astro-ph/0601617]

- Easther and Lim used LATTICEEASY [Felder & Tkachev], a code that evolve scalar fields in a background (say, expanding) self consistently:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{4}\phi^4 - \frac{1}{2}g^2\phi^2\chi^2$$

- They then used formula by Weinberg to estimate gravitational wave production (stochastic background):

$$\frac{dE_{\mathrm{gw}}}{d\Omega} = \pi^2 \sum_{i,j} \int_{-\infty}^{\infty} d\omega\, \omega^2 \left| T_{ij}^{\mathrm{TT}}(\omega, \mathbf{k}) \right|^2$$

Previous work

- Khlebnikov & Tkachev: Flat space approximation.
- Dufaux, Bergman, Felder, Kofman & Uzan: Approximate Green's function.
- Easther, Giblin & Lim: Evolve TT gauge metric perturbation.
- Garcia-Bellido, Figueroa & Sastre: Evolve metric then take TT part.
- Larry Price & XS: Exact Green's function.

Theoretical framework

- Background space-time $ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2)$

- Perturbations $$\ddot{h}_{ab}^{\mathrm{TT}} + 2\frac{\dot{a}(\eta)}{a(\eta)}\dot{h}_{ab}^{\mathrm{TT}} - \partial_x^2 h_{ab}^{\mathrm{TT}} = 16\pi T_{ab}^{\mathrm{TT}}$$

- Assume expansion $a(\eta) = \alpha\eta^n$

First tried Green's functions in configuration space [R. Caldwell, Phys. Rev. D 48, 4688 - 4692 (1993)]. Pretty ugly. Matter era Green's functions have support inside light-cone.

Theoretical framework

- Spatial Fourier transform $\ddot{h}_{ab}^{\rm TT} + \frac{2}{\eta}\dot{h}_{ab}^{\rm TT} + k^2 h_{ab}^{\rm TT} = 16\pi T_{ab}^{\rm TT}$

- Very easy solution $h_{ab}^{\rm TT} = \frac{16\pi}{k}\int_{\eta_i}^{\eta_f} d\eta' \, \frac{\eta'}{\eta} \sin[k(\eta - \eta')] T_{ab}^{\rm TT}$

$$\left(h_{ab}^{\rm TT} = \frac{16\pi}{k^3}\int_{\eta_i}^{\eta_f} \frac{\eta'}{\eta^3}\Big\{(1 + k^2\eta'\eta)\sin[k(\eta - \eta')] + k(\eta - \eta')\cos[k(\eta - \eta')]\Big\} T_{ab}^{\rm TT} \right)$$

Generally, for $a(\eta) = \alpha\eta^n$, solutions are spherical Bessel functions.

Theoretical framework (analytic)

- From the energy density in GWs $\rho_{\mathrm{gw}} = \dfrac{\left\langle \dot{h}_{ab}\dot{h}^{ab} \right\rangle}{32\pi a^2(\eta)}$
- Can derive a formula for the energy per unit solid angle which, for radiation era expansion, is

$$\frac{dE_{\mathrm{gw}}}{d\Omega} = \frac{16\pi^2}{\alpha^2\eta^6}\sum_{i,j}\int_{-\infty}^{\infty} d\omega \left[\sin(\omega\eta) - \omega\eta\cos(\omega\eta)\right]^2 \left|\frac{\partial}{\partial\omega}T_{ij}^{\mathrm{TT}}(\omega,\mathbf{k})\right|$$

- Weinberg's formula

$$\frac{dE_{\mathrm{gw}}}{d\Omega} = \pi^2\sum_{i,j}\int_{-\infty}^{\infty} d\omega\,\omega^2\left|T_{ij}^{\mathrm{TT}}(\omega,\mathbf{k})\right|^2$$

Theoretical framework (computational)

- From the energy density in GWs $\rho_{\rm gw} = \dfrac{\left\langle \dot{h}_{ab}\dot{h}^{ab} \right\rangle}{32\pi a^2(\eta)}$
- Can also derive a formula for the energy per unit log frequency interval

$$\frac{d\rho_{\rm gw}}{d\ln k} = \frac{k^3}{32\pi a^2(\eta)}\frac{1}{V}\int d\Omega \sum_{a,b} \dot{h}_{ab}(\eta,\mathbf{k})\dot{h}^*_{ab}(\eta,\mathbf{k})$$

- And on a computer use

$$\frac{d\rho_{\rm gw}}{d\ln k} = \frac{k^3}{8a^2(\eta)}\frac{1}{V}\sum_{a,b} \dot{h}_{ab}(\eta,k\hat{\mathbf{k}}_{\mathbf{p}})\dot{h}^*_{ab}(\eta,k\hat{\mathbf{k}}_{\mathbf{p}})$$

and average over several directions

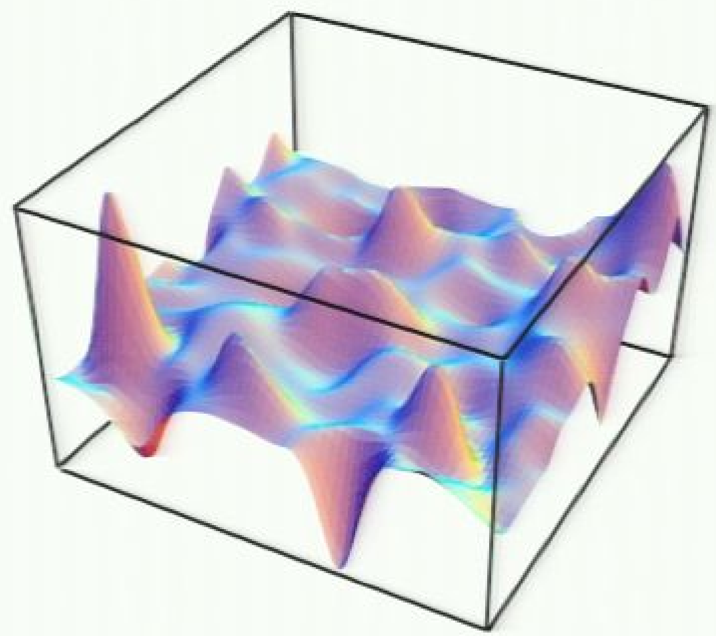
Computational framework

Source evolution with LATTICEEASY:
[Felder & Tkachev]

- Publicly available
- Easy to use
- Many pre-packaged models

To compute GWs wrote a code that piggybacks on LATTICEEASY and

- Use FFTW.
- Threading with OpenMP.
- Detailed optimization.



On a 4 core 3GHz machine 256^3 box takes 18 hours! (standard simulation)

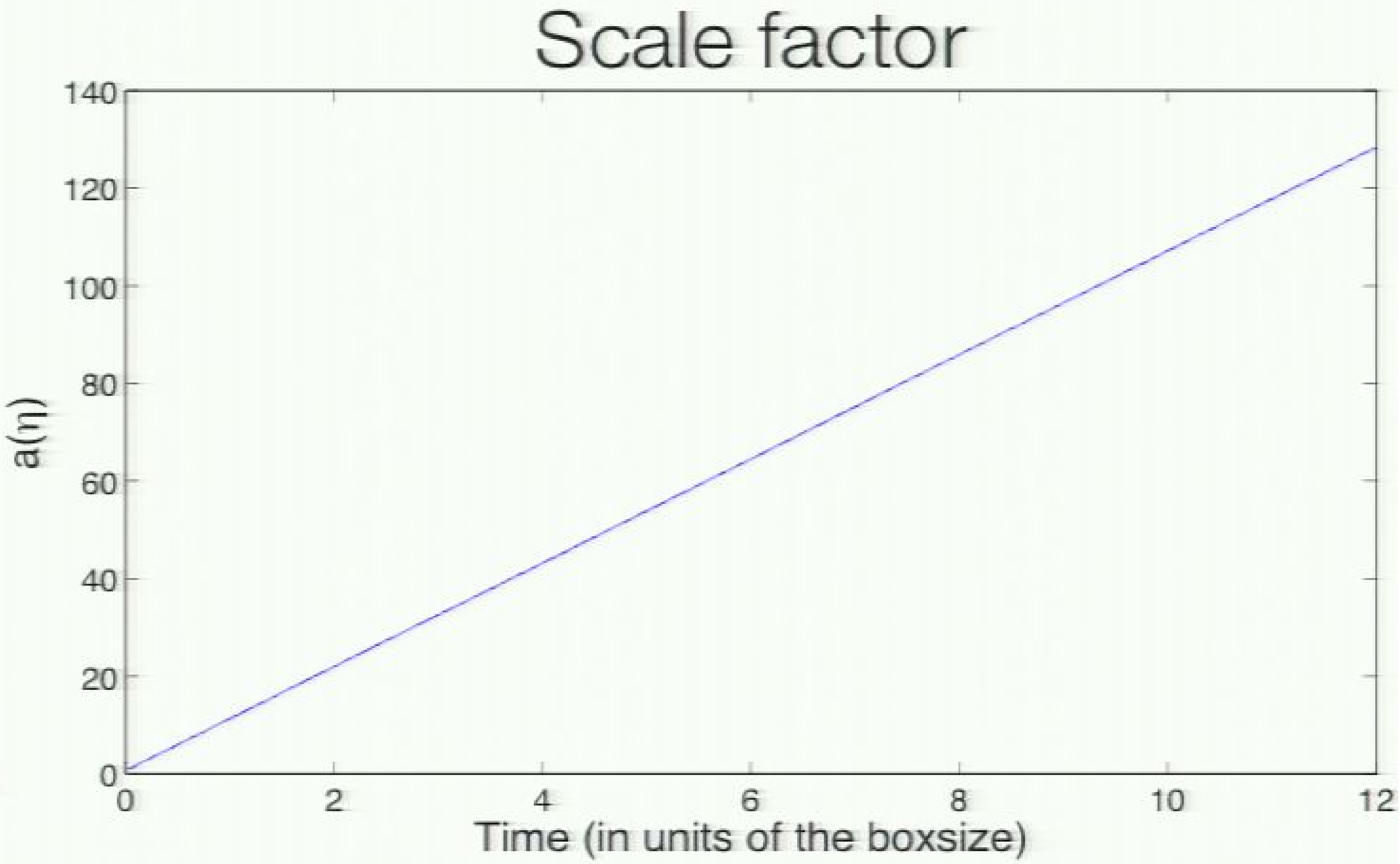
Dufaux, Bergman, Felder, Kofman & Uzan (200

Results

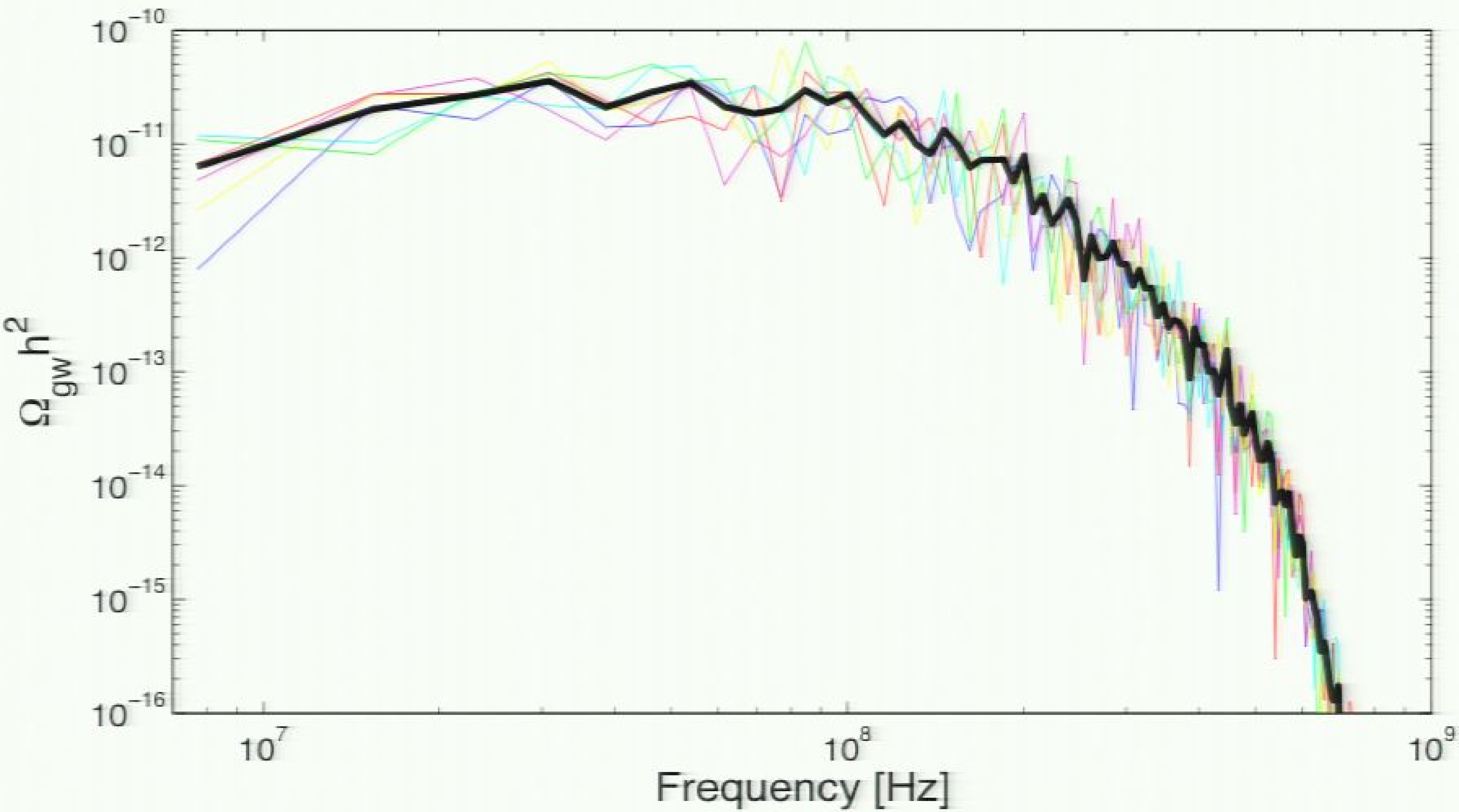
- Ran a standard simulation
- GUT scale inflation

$$\lambda = 10^{-14}$$

$$g^2 = 1.2 \times 10^{-12}$$



Results

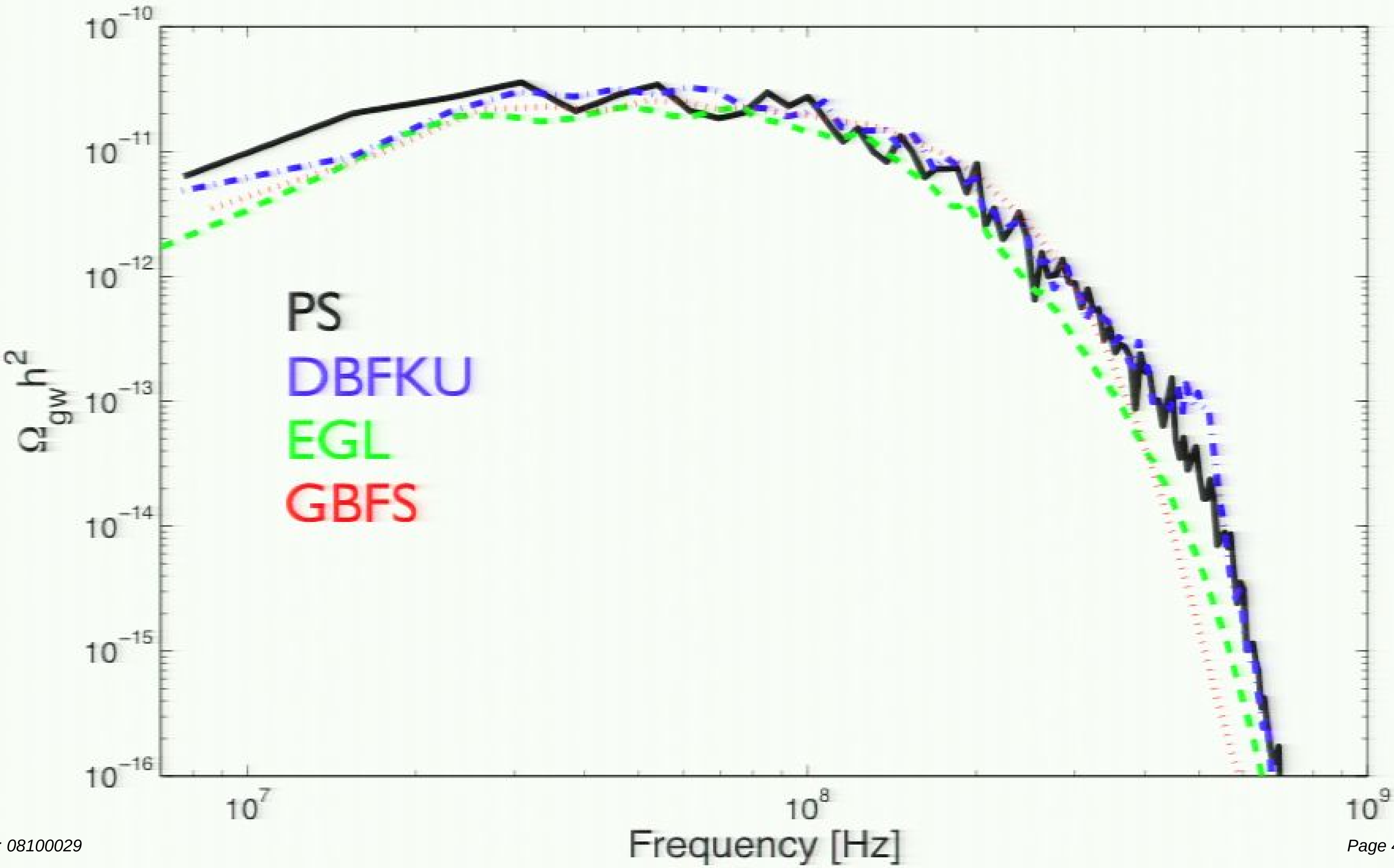


10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
$\Omega_{gw}h^2$
10^7
10^8
10^9
Frequency [Hz]

Future work

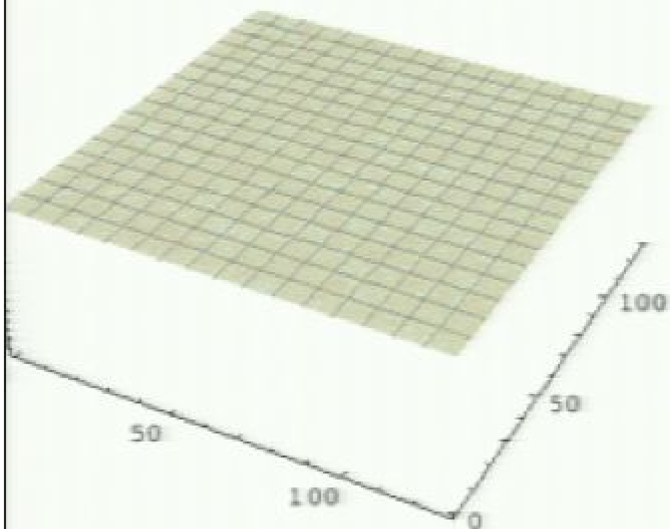
- More fields coupled to the inflaton (with John “Tom” Giblin, Larry Price)
- Second order phase transitions (with John “Tom” Giblin, Larry Price)
- Electroweak scale physics (LISA)
- Got a model?

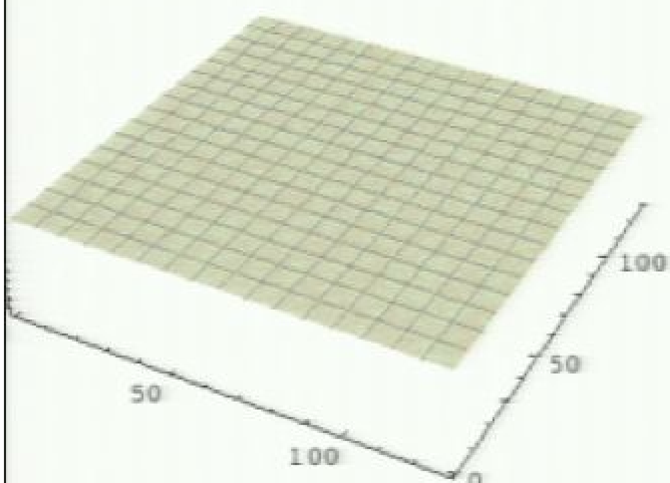
Results



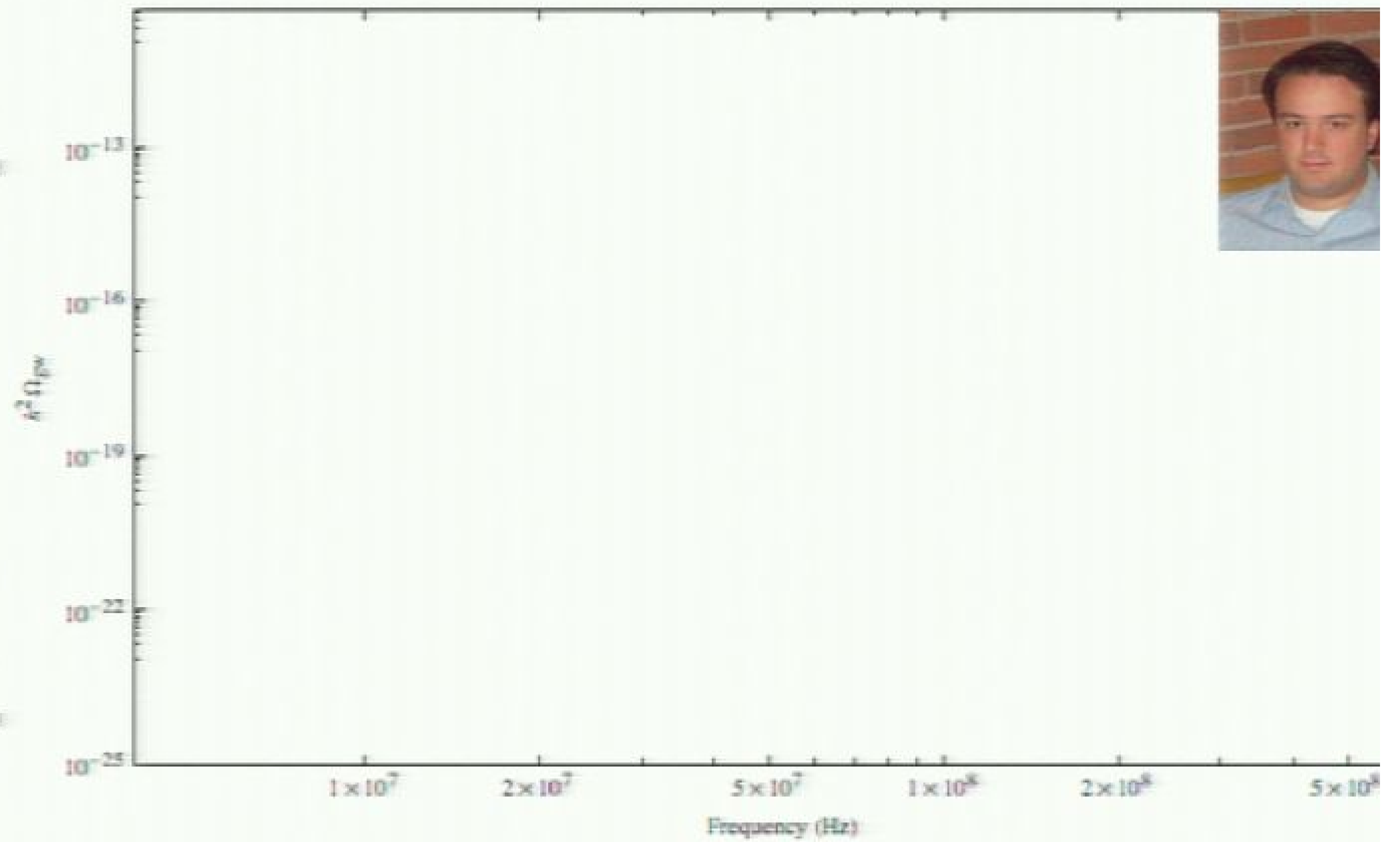
Larry Price, XS, Phys. Rev. D78 (2008) 063641 (arXiv:0805.3570)

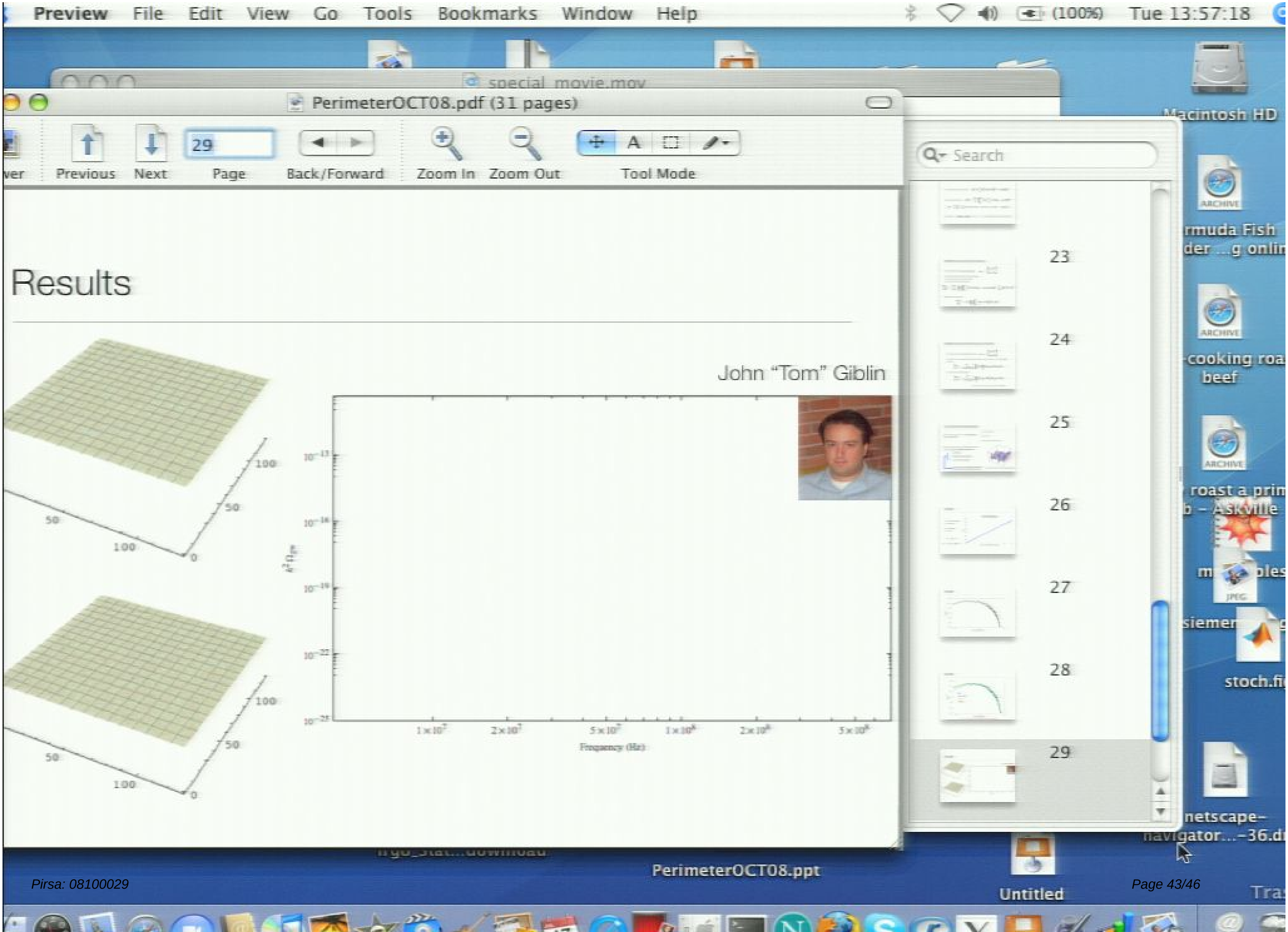
Results



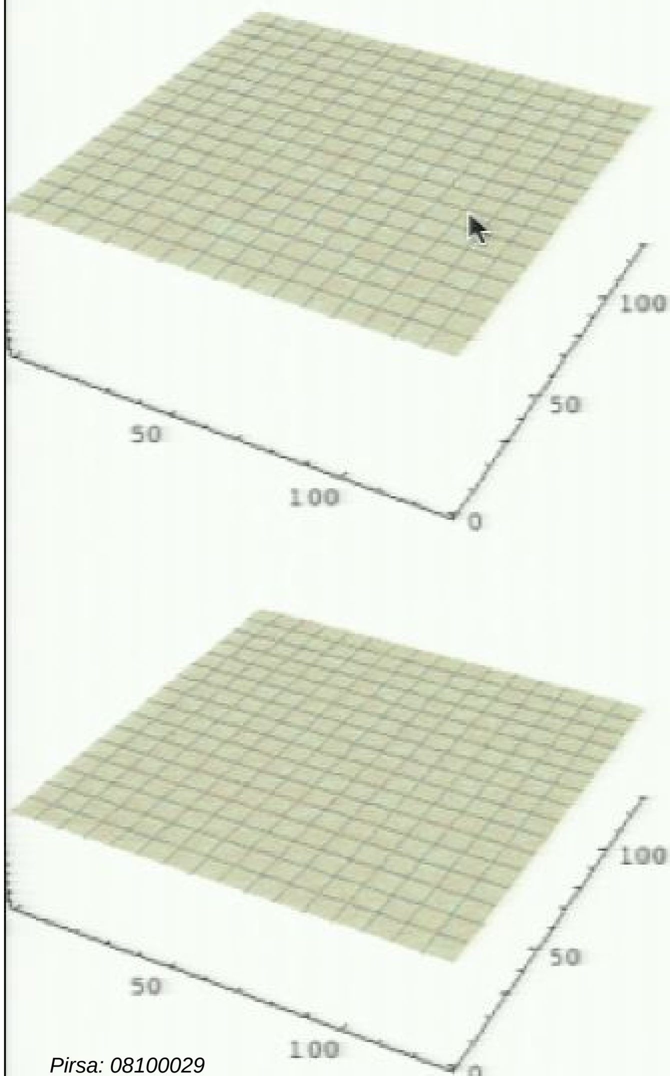


John “Tom” Gibl

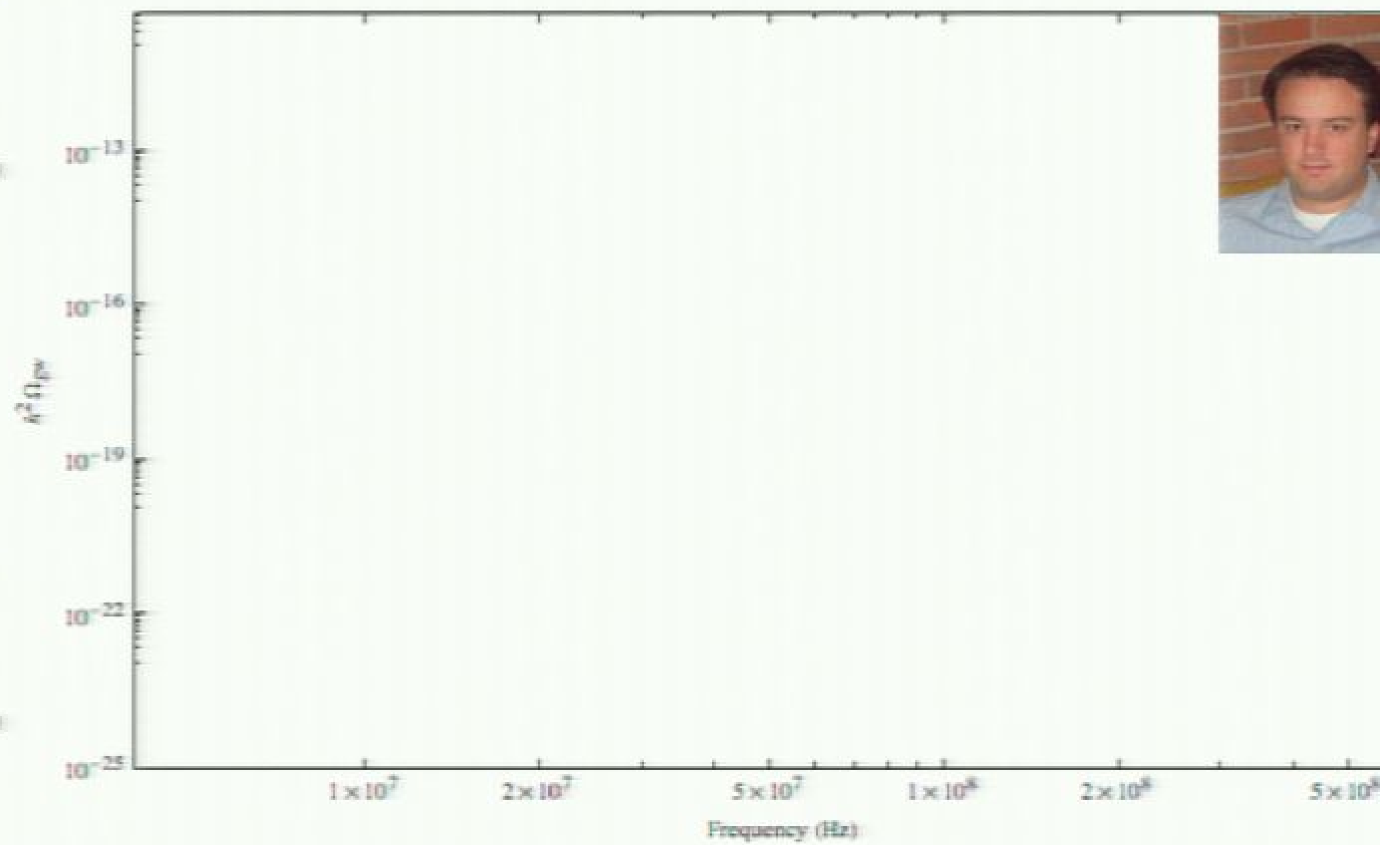



Preview File Edit View Go Tools Bookmarks Window Help
Tue 13:57:18
PerimeterOCT08.pdf (31 pages)
Previous
Next
Page
Back/Forward
Zoom In
Zoom Out
Tool Mode
Search
Results
John “Tom” Giblin
Frequency (Hz)
Macintosh HD
PerimeterOCT08.ppt
Untitled

Results



John "Tom" Gibl



Future work

- More fields coupled to the inflaton (with John “Tom” Giblin, Larry Price)
- Second order phase transitions (with John “Tom” Giblin, Larry Price)
- Electroweak scale physics (LISA)
- Got a model?

Future work

- More fields coupled to the inflaton (with John “Tom” Giblin, Larry Price)
- Second order phase transitions (with John “Tom” Giblin, Larry Price)
- Electroweak scale physics (LISA)
- Got a model?