Title: Simulation of quantum many-body systems with tensor network methods

Date: Oct 08, 2008 04:00 PM

URL: http://pirsa.org/08100025

Abstract: In this talk I will give an introduction to the simulation of quantum many-body systems using the so-called tensor networks. After a brief historical review, I will introduce the basics on tensor network representations of quantum states, and will explain some recent developments. In particular, in the last part of my talk I will focus on recent results obtained in the simulation of 2-dimensional quantum lattice systems of infinite size.

Quantum many-body systems and tensor networks: simulation methods and applications

Román Orús

School of Physical Sciences,

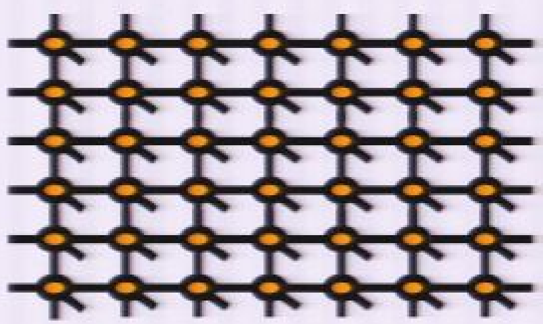
University of Queensland, Brisbane (Australia)

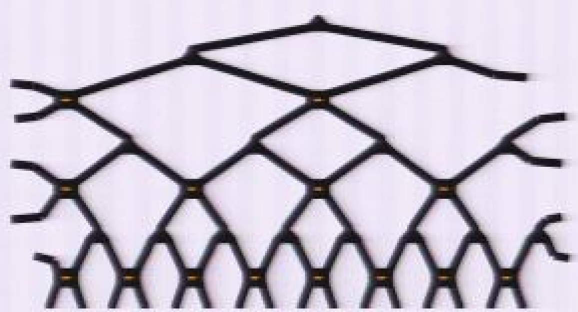
Perimeter Institute

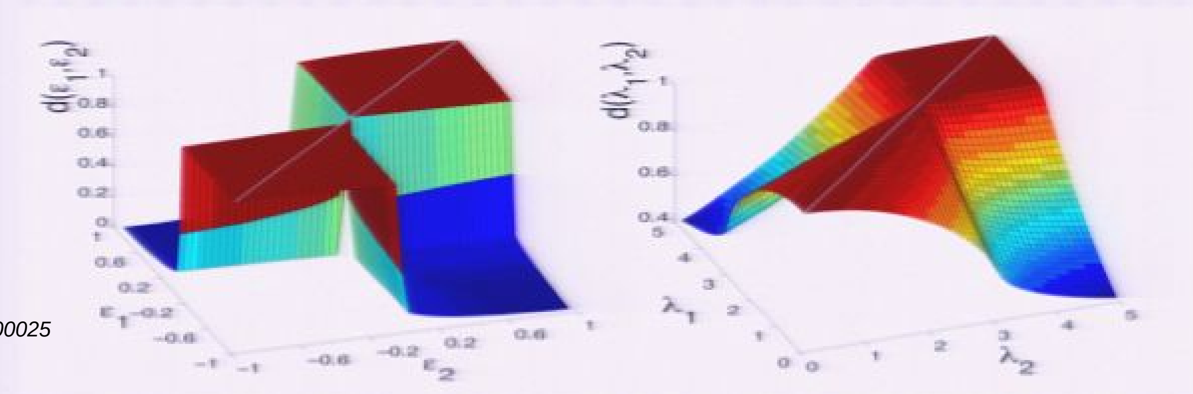
October 8th 2008

Goal of this talk:

Explain the background and basics of methods to simulate quantum many-body systems with tensor networks, together with some recent results and applications



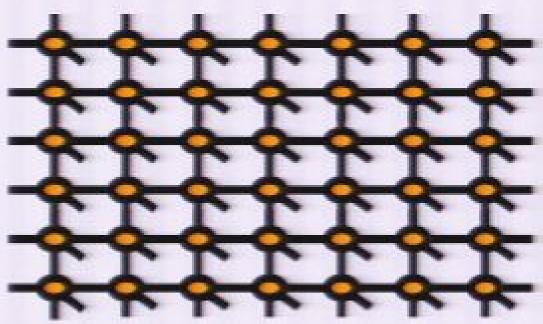


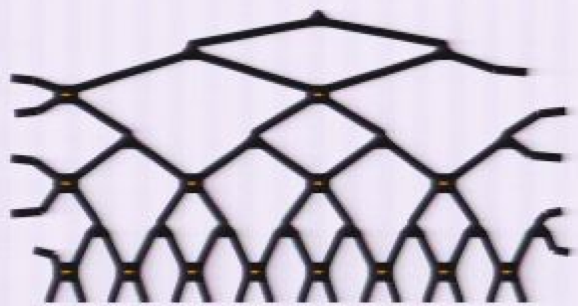


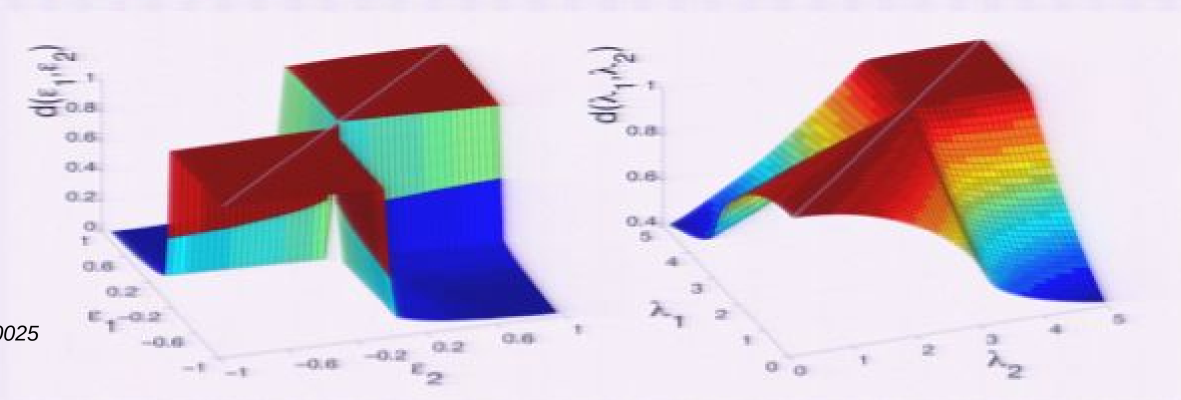
ROADMAP

Goal of this talk:

Explain the background and basics of methods to simulate quantum many-body systems with tensor networks, together with some recent results and applications





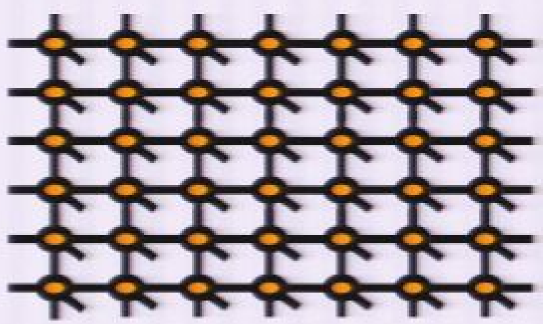


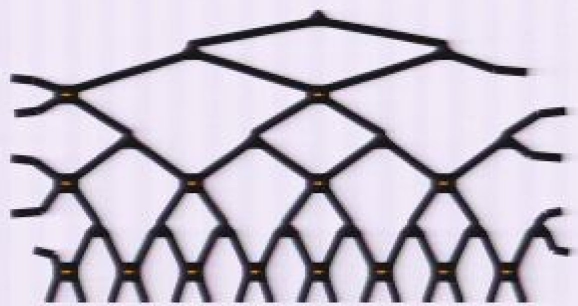
ROADMAP

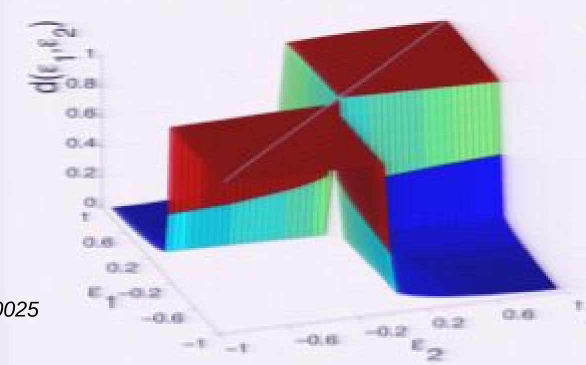
Some history

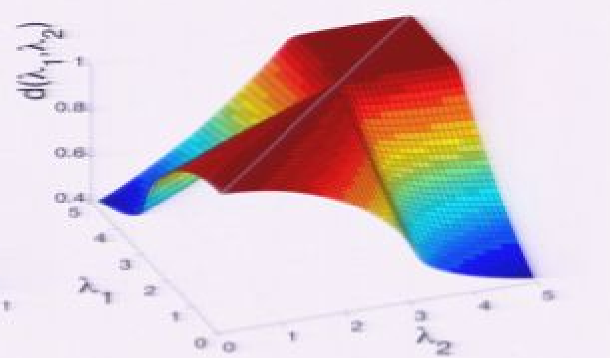
Goal of this talk:

Explain the background and basics of methods to simulate quantum many-body systems with tensor networks, together with some recent results and applications

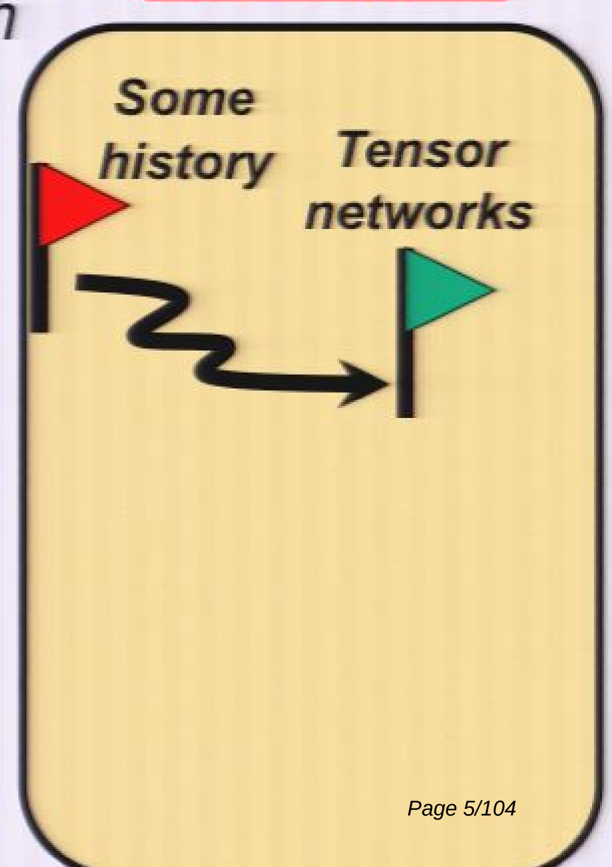






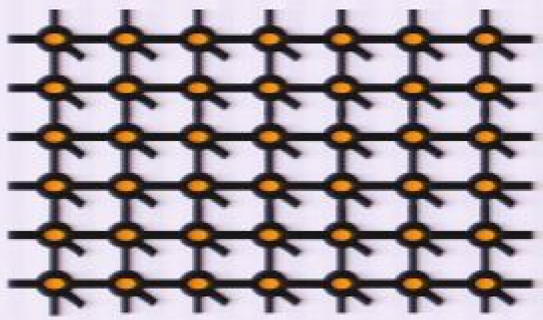


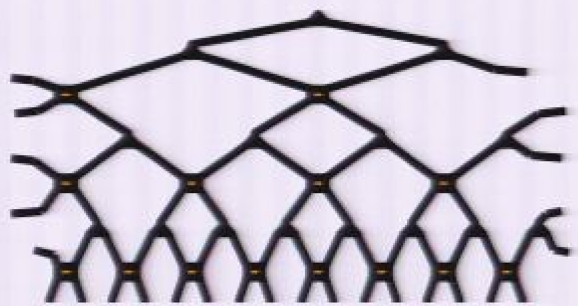
ROADMAP

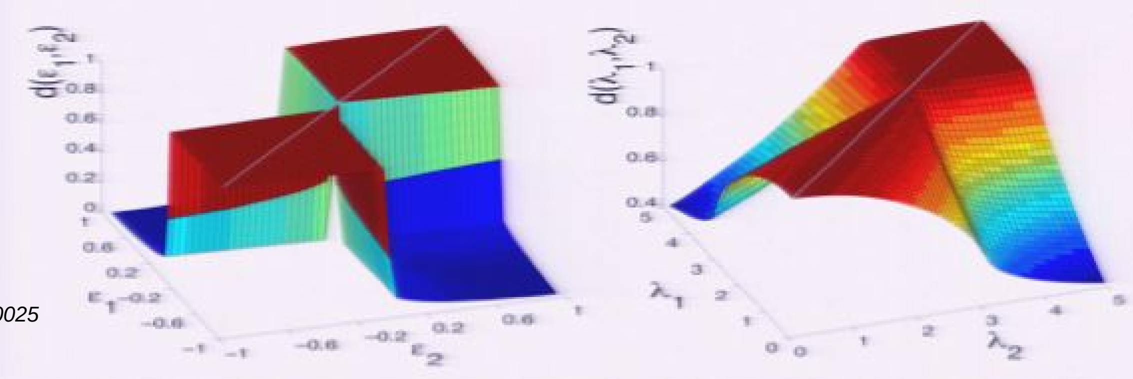


Goal of this talk:

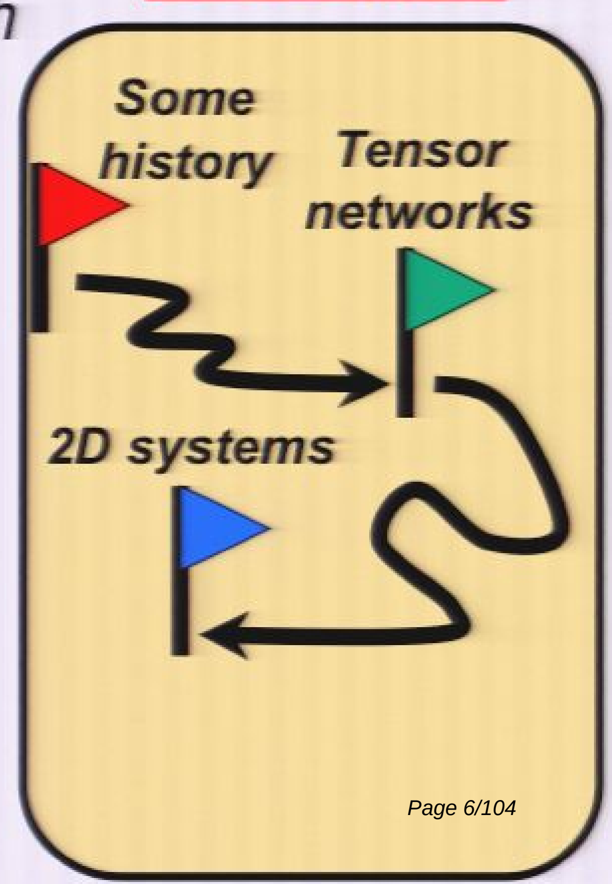
Explain the background and basics of methods to simulate quantum many-body systems with tensor networks, together with some recent results and applications





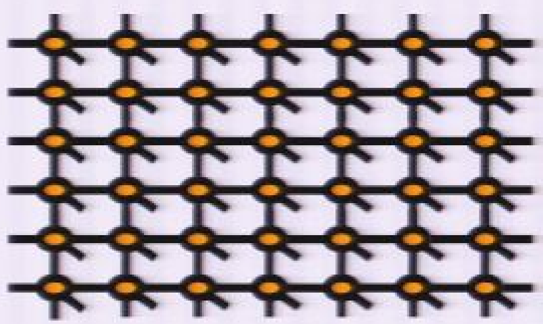


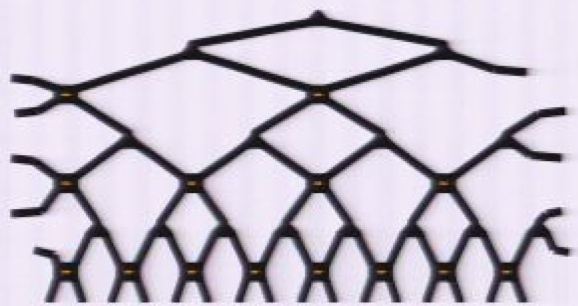
ROADMAP

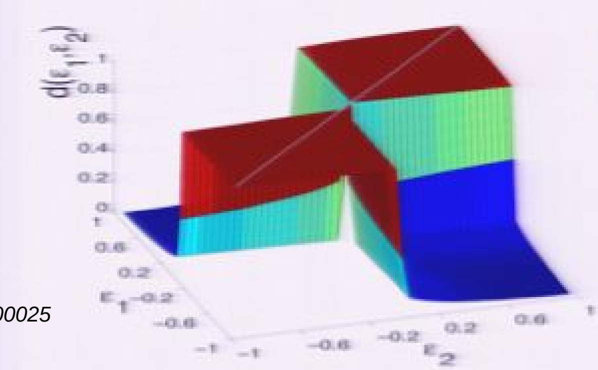


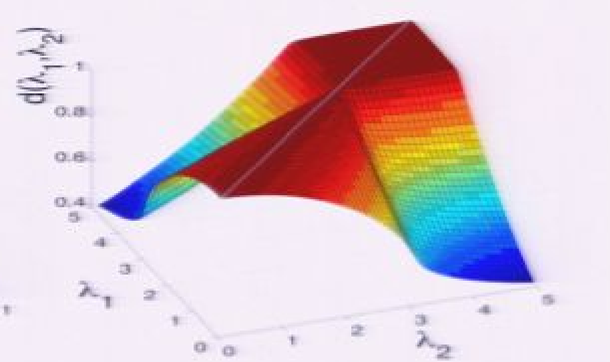
Goal of this talk:

Explain the background and basics of methods to simulate quantum many-body systems with tensor networks, together with some recent results and applications

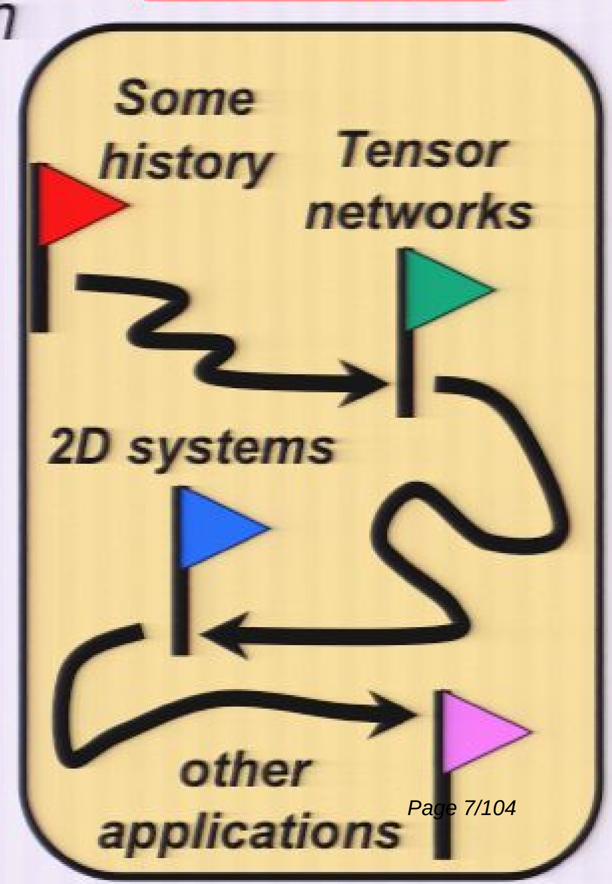








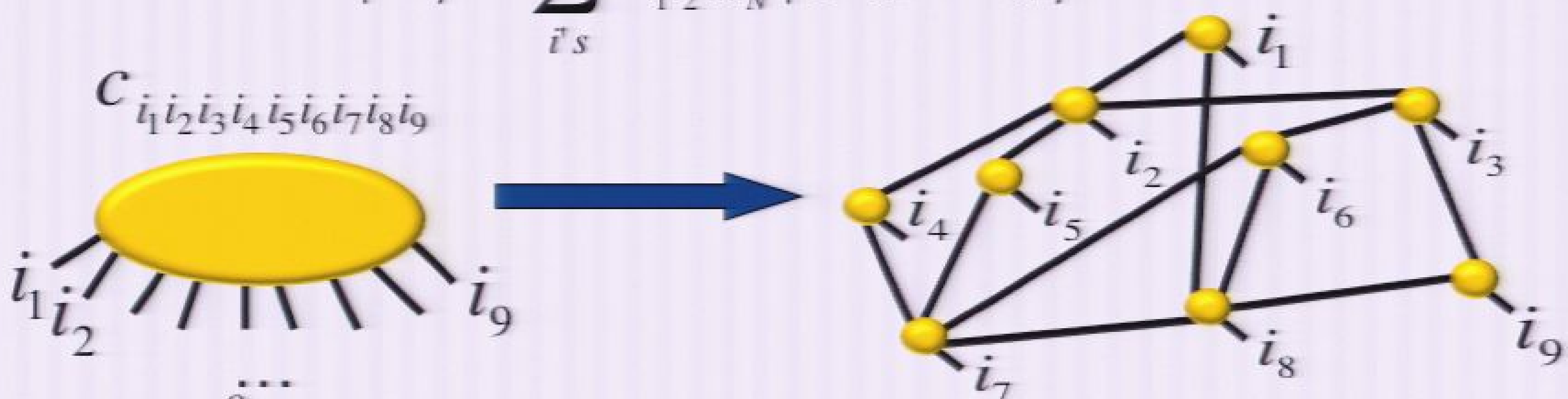
ROADMAP



Main idea

- **Problem:** how to represent efficiently quantum states of a quantum lattice system
- **Solution:** divide the coefficient of the quantum state in a *Tensor Network that only depends on a small number of parameters*

$$|\Psi\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle$$



$O(p^9)$ parameters

$O(9p)$ parameters

Outline

1.- Brief historical review

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

Outline

➡ **1.- Brief historical review**

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

How it all began…

K. Wilson

Renormalization Group (RG), 1971
(Nobel Prize in 1982)

Identification of the relevant degrees of freedom of a physical system at long distances

How it all began…

K. Wilson

Renormalization Group (RG), 1971
(Nobel Prize in 1982)

Identification of the relevant degrees of freedom of a physical system at long distances

About 20 years later…

S. R. White

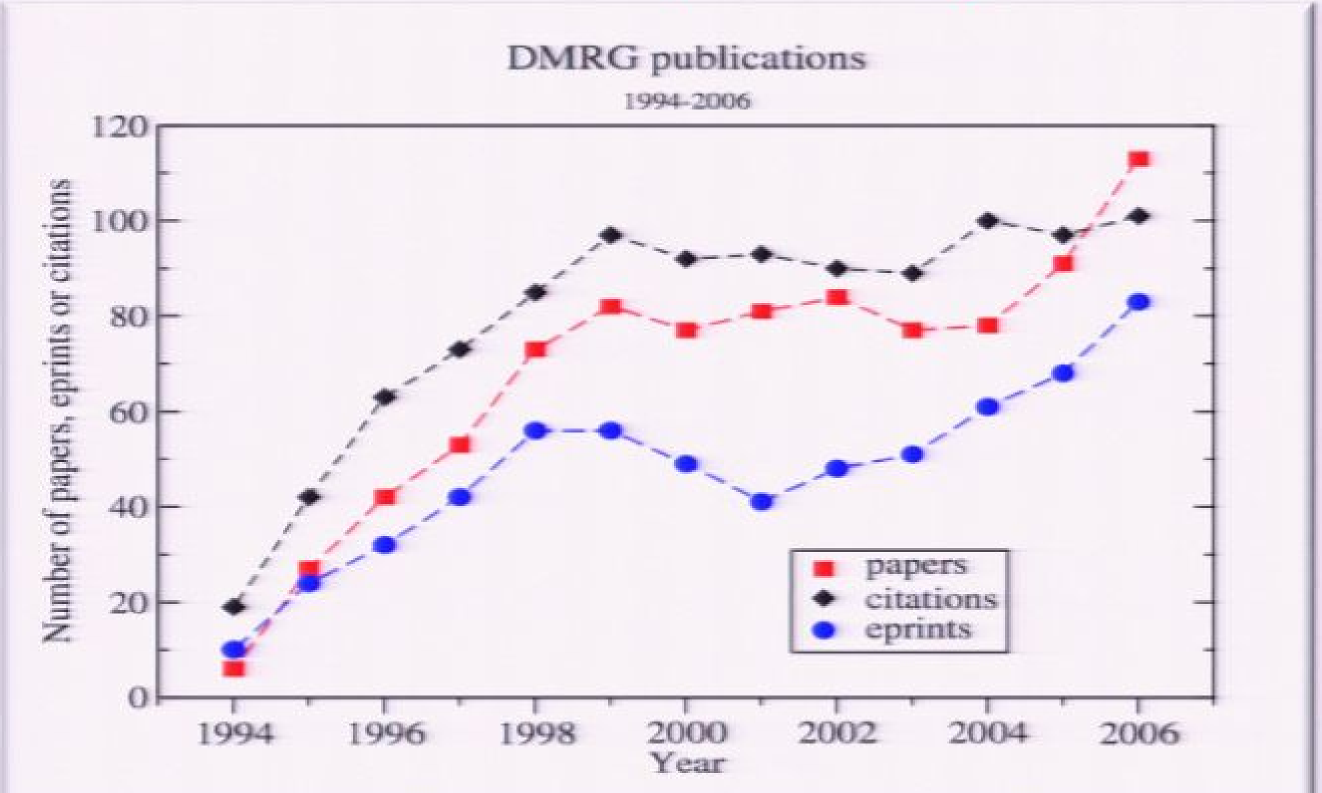
Density Matrix Renormalization Group (DMRG), 1992

Impact of DMRG

- DMRG is a numerical method to compute the low-energy properties of quantum systems, and has been the method of reference to simulate 1-dim quantum systems for the last 15 years.

Impact of DMRG

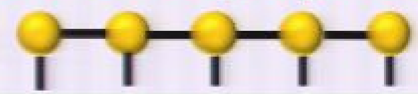
- DMRG is a numerical method to compute the low-energy properties of quantum systems, and has been the method of reference to simulate 1-dim quantum systems for the last 15 years.



What is behind DMRG?

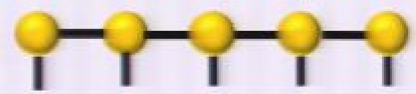
DMRG approximates the ground state of a physical system by a ***Matrix Product State*** 

What is behind DMRG?

DMRG approximates the ground state of a physical system by a ***Matrix Product State*** 

- 1987 I. Affleck, T. Kennedy, E. H. Lieb, H. Tasaki ⇨ AKLT model, 'Valence Bond States'

What is behind DMRG?

DMRG approximates the ground state of a physical system by a ***Matrix Product State*** 

- 1987 I. Affleck, T. Kennedy, E. H. Lieb, H. Tasaki ⇨ AKLT model, 'Valence Bond States'

- 1992 M. Fannes, B. Nachtergaele, R. F. Werner ⇨ Generalization to Matrix Product States (MPS)

What is behind DMRG?

DMRG approximates the ground state of a physical system by a ***Matrix Product State***

- 1987 I. Affleck, T. Kennedy, E. H. Lieb, H. Tasaki ⇨ AKLT model, 'Valence Bond States'
- 1992 M. Fannes, B. Nachtergaele, R. F. Werner ⇨ Generalization to Matrix Product States (MPS)

- 1995 S. Ostlund, S. Rommer ⇨ Connection between DMRG and Matrix Product States

Deep connection to quantum information theory

- 2003 G. Vidal

VOLUME 91, NUMBER 14 PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2003

Efficient Classical Simulation of Slightly Entangled Quantum Computations

Guifré Vidal
Institute for Quantum Information, California Institute of Technology, Pasadena, California 91125, USA

(Received 25 February 2003; published 1 October 2003)

We present a classical protocol to efficiently simulate any pure-state quantum computation that involves only a restricted amount of entanglement. More generally, we show how to classically simulate pure-state quantum computations on n qubits by using computational resources that grow linearly in n and exponentially in the amount of entanglement in the quantum computer. Our results imply that a necessary condition for an exponential computational speedup (with respect to classical computations) is that the amount of entanglement increases with the size n of the computation, and provide an explicit lower bound on the required growth.

DOI: 10.1103/PhysRevLett.91.147902 PACS numbers: 03.67.Lx, 03.65.Ud, 03.67.Hk, 03.67.Mn

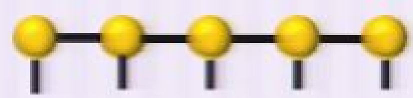
Classical simulation of quantum computations which involve a small amount of entanglement

Deep connection to quantum information theory

- 2003 G. Vidal ⇨ Connection between MPS and *quantum computing*

Deep connection to quantum information theory

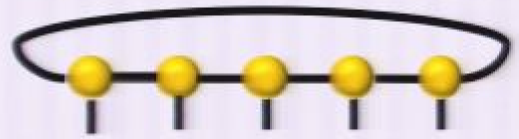
- 2003 G. Vidal ⇨ Connection between MPS and *quantum computing*
- 2003 G. Vidal ⇨ Time evolution of 1-dim systems with MPS: the Time Evolving Block Decimation (TEBD) algorithm



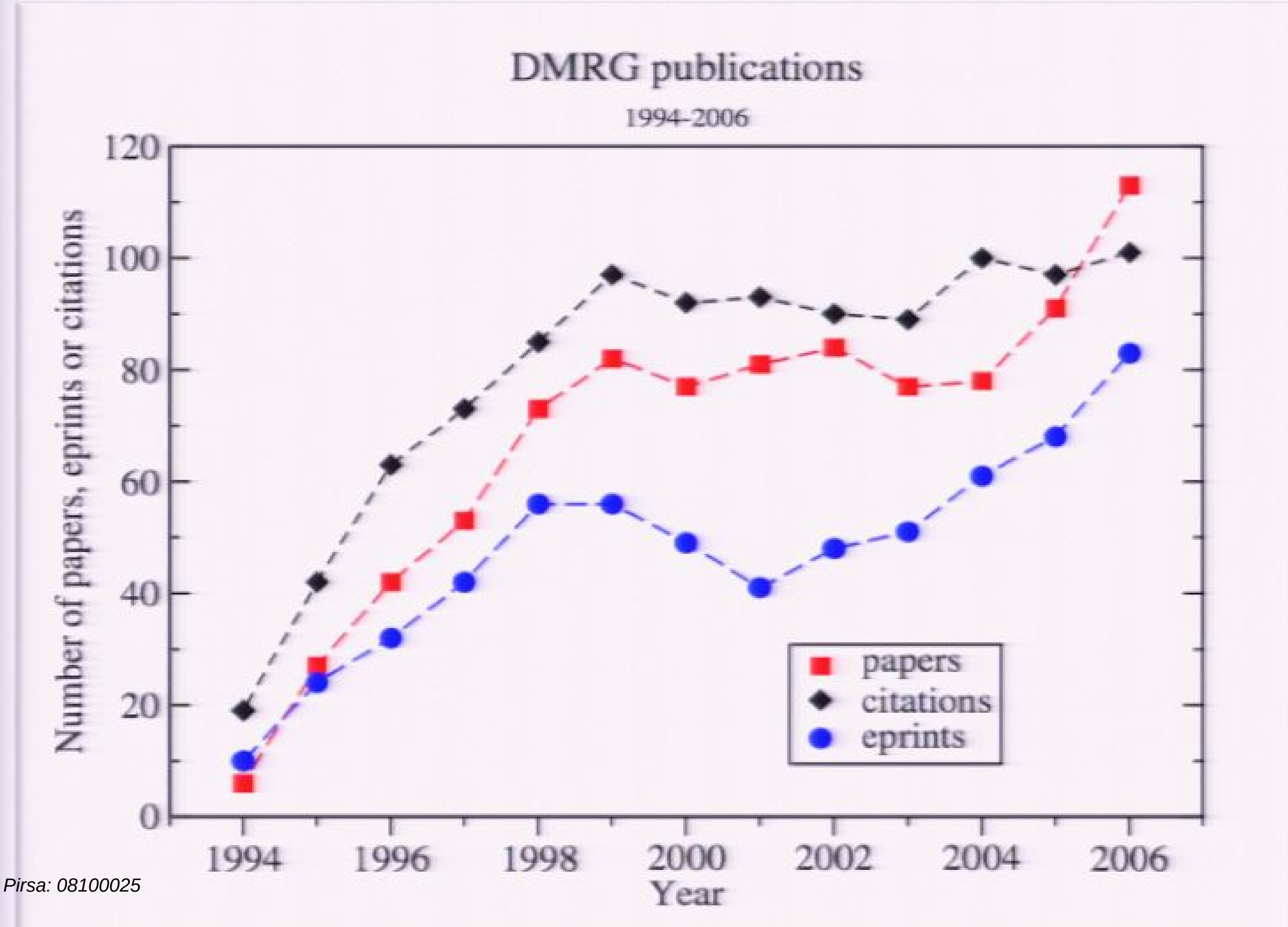
Deep connection to quantum information theory

- 2003 G. Vidal ⇨ Connection between MPS and *quantum computing*
- 2003 G. Vidal ⇨ Time evolution of 1-dim systems with MPS: the Time Evolving Block Decimation (TEBD) algorithm

- 2004 F. Verstraete, D. Porras, J. I. Cirac ⇨ Periodic boundary conditions



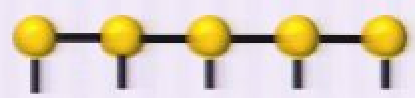
Deep connection to quantum information theory



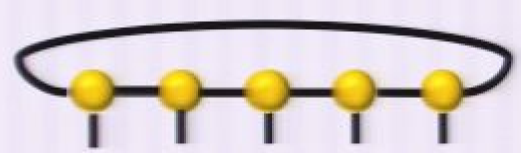
Deep connection to quantum information theory

- 2003 G. Vidal ⇨ Connection between MPS and *quantum computing*

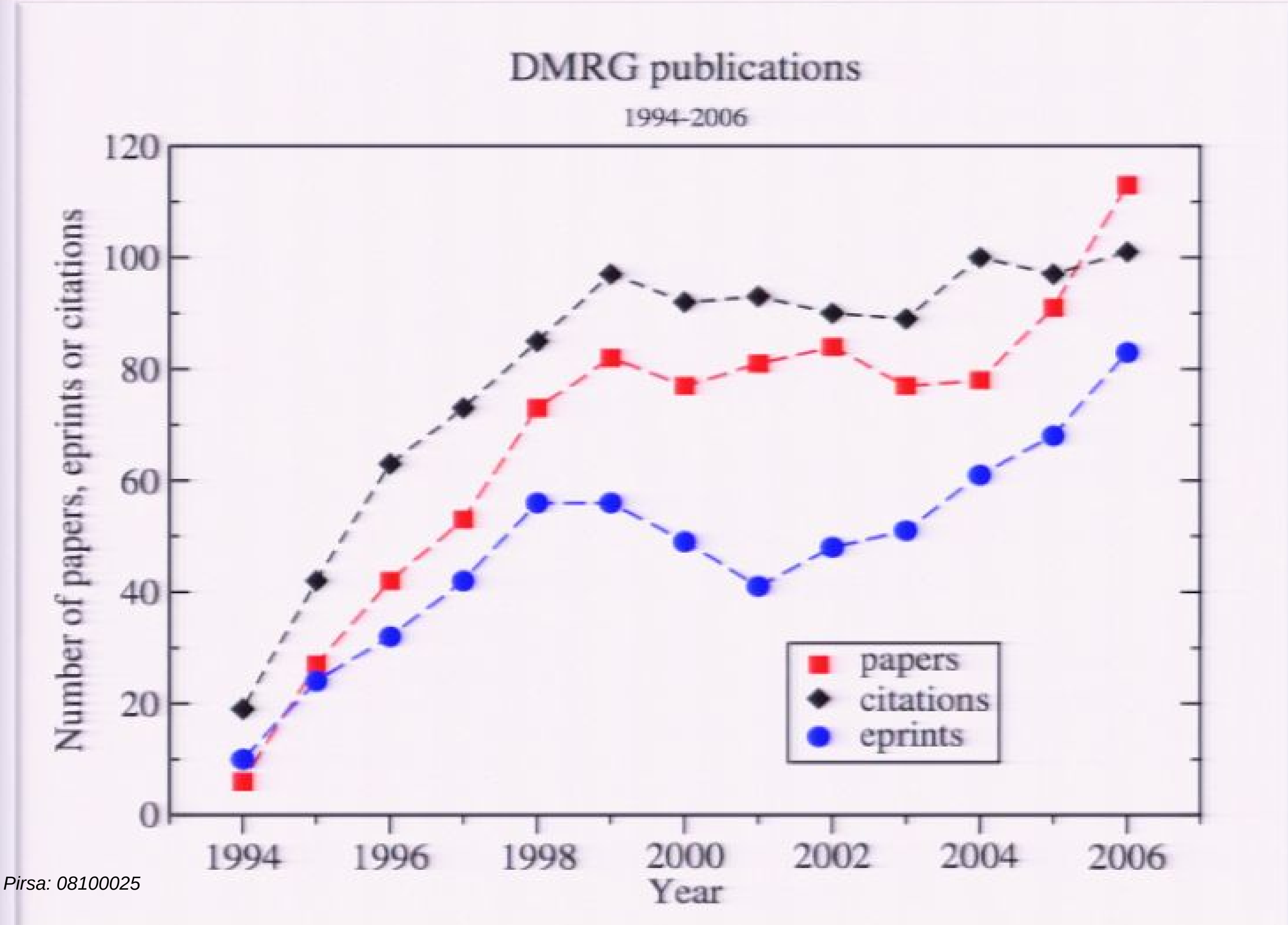
- 2003 G. Vidal ⇨ Time evolution of 1-dim systems with MPS: the Time Evolving Block Decimation (TEBD) algorithm



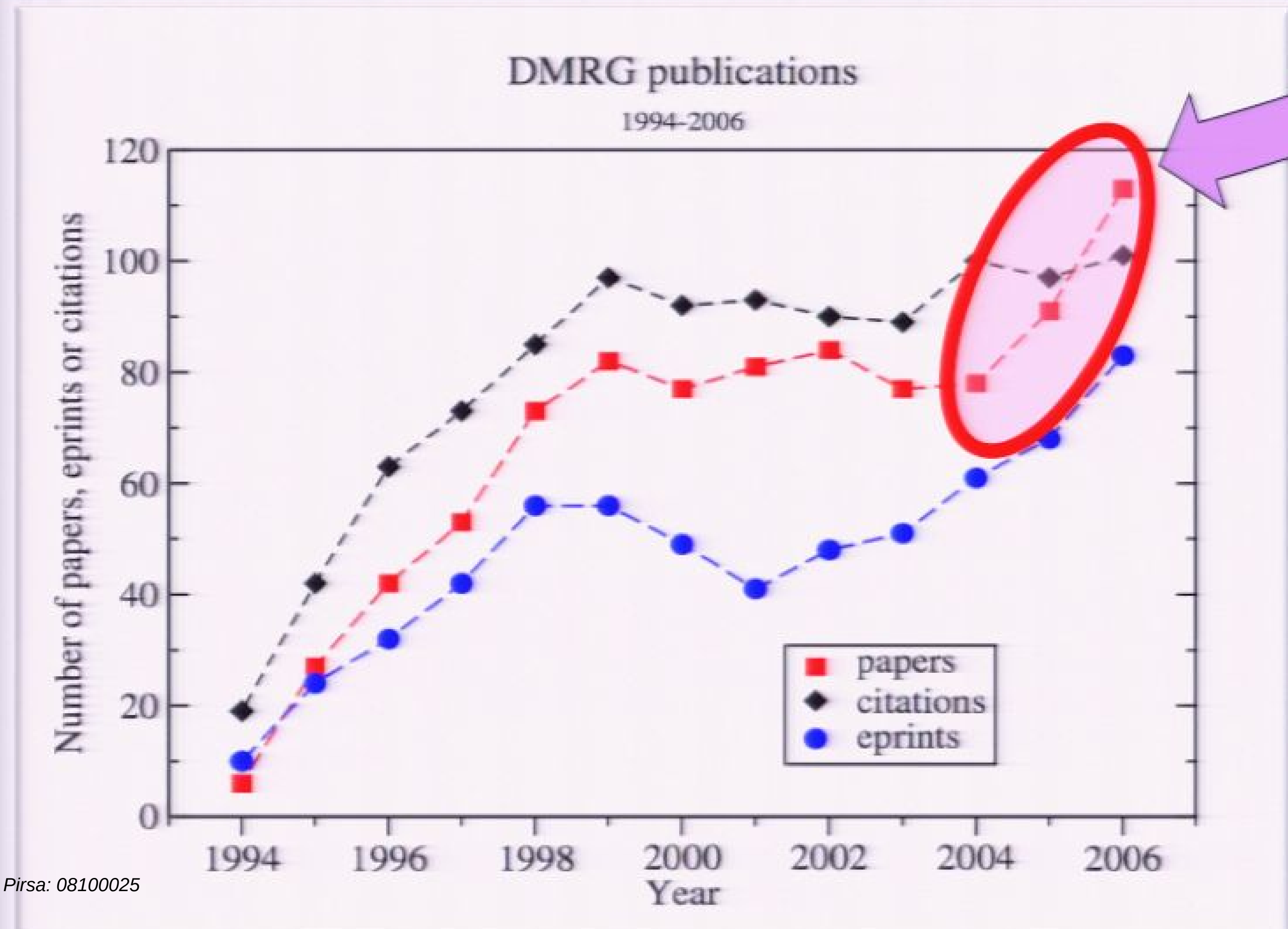
- 2004 F. Verstraete, D. Porras, J. I. Cirac ⇨ Periodic boundary conditions



Deep connection to quantum information theory



Deep connection to quantum information theory



quantum information 'jump'

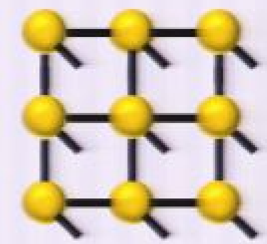
What about other systems?

- 2000 T. Nishino, K. Okunishi, Y. Hieida, N. Maeshima, Y. Akutsu ⇨ Tensor Product States (TPS) for d-dim systems

What about other systems?

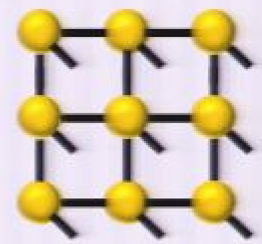
- 2000 T. Nishino, K. Okunishi, Y. Hieida, N. Maeshima, Y. Akutsu ⇨ Tensor Product States (TPS) for d-dim systems

- 2004 F. Verstraete, J. I. Cirac ⇨ Method of Projected Entangled Pair States (PEPS) for d-dim systems

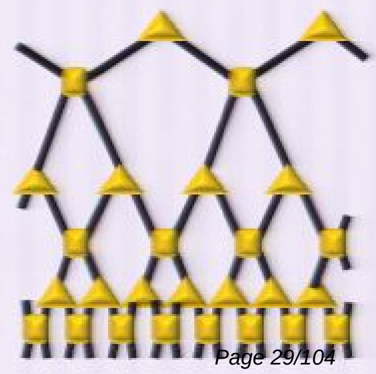


What about other systems?

- 2000 T. Nishino, K. Okunishi, Y. Hieida, N. Maeshima, Y. Akutsu ⇨ Tensor Product States (TPS) for d-dim systems
- 2004 F. Verstraete, J. I. Cirac ⇨ Method of Projected Entangled Pair States (PEPS) for d-dim systems



- 2005 G. Vidal ⇨ Multiscale Entanglement Renormalization Ansatz (MERA), for scale-invariant systems



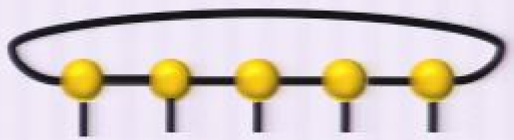
Summary of history

- 1-dim

MPS

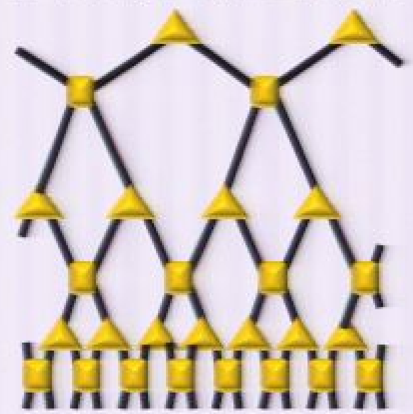
1992 & 2003

MPS PBC



2004

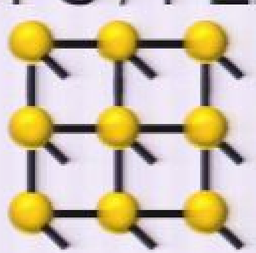
1-dim MERA



2005

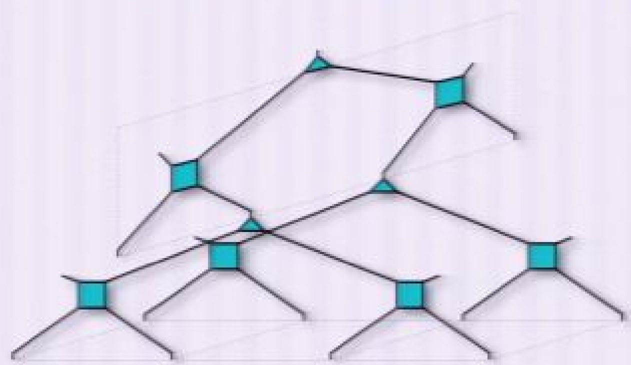
- d-dim

TPS / PEPS



2000 & 2004

d-dim MERA



2005

Outline

1.- **Brief historical review**

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

Outline

1.- Brief historical review

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

Tensor Networks (TN)

$$|\Psi\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle \in \left(\mathbf{C^p}\right)^{\otimes N}$$

p^N complex parameters

Tensor Networks (TN)

$$|\Psi\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle \in \left(\mathbf{C}^{\mathbf{p}}\right)^{\otimes N}$$

p^N complex parameters

$c_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$

$O(p^9)$ parameters

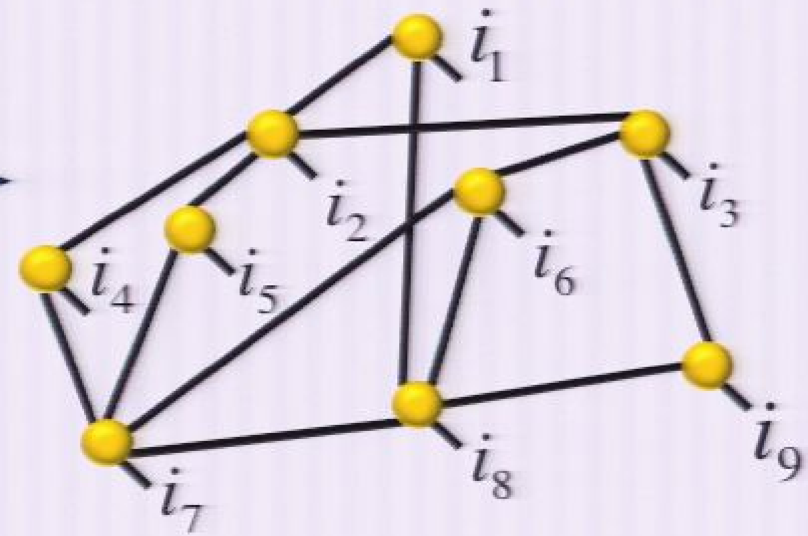
Tensor Networks (TN)

$$|\Psi\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle \in \left(\mathbf{C}^{\mathbf{p}}\right)^{\otimes N}$$

p^N complex parameters

$c_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$

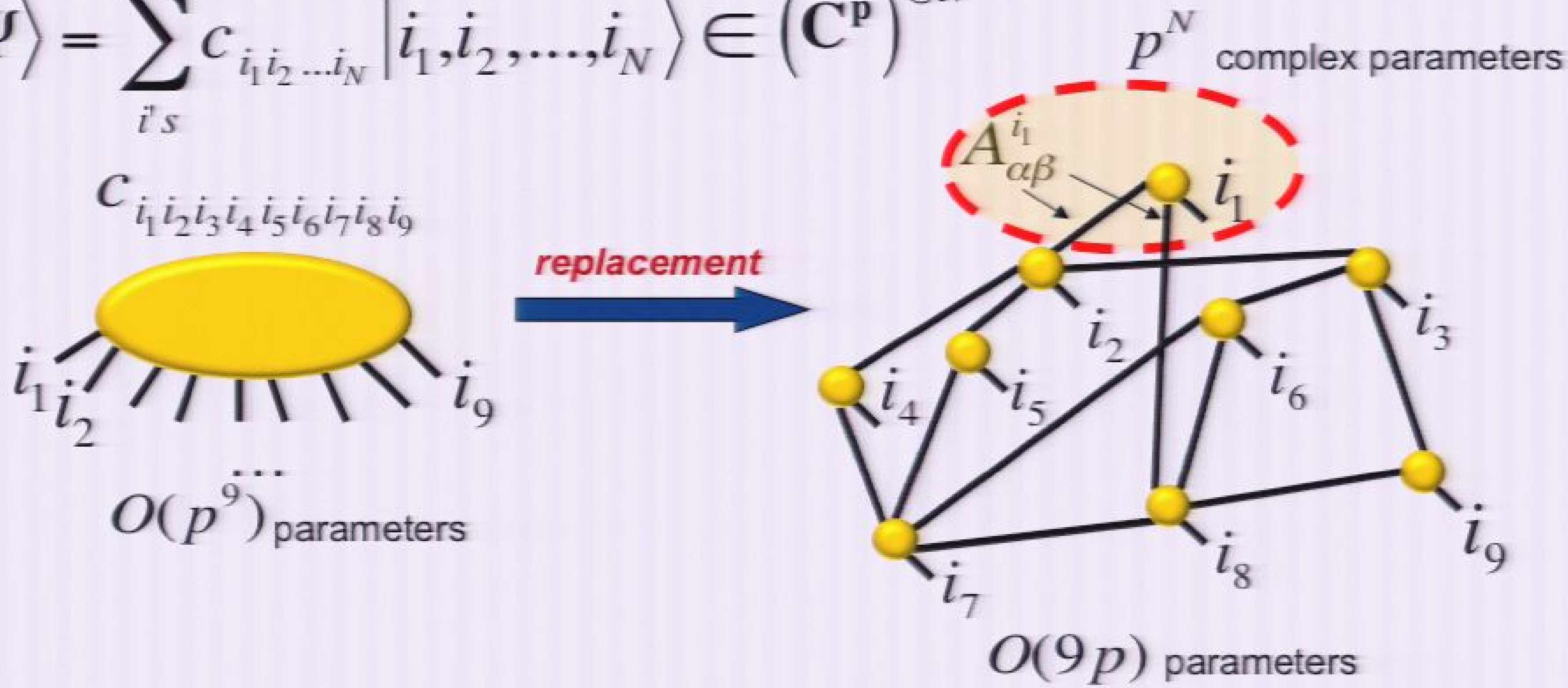
$O(p^9)$ parameters



$O(9p)$ parameters

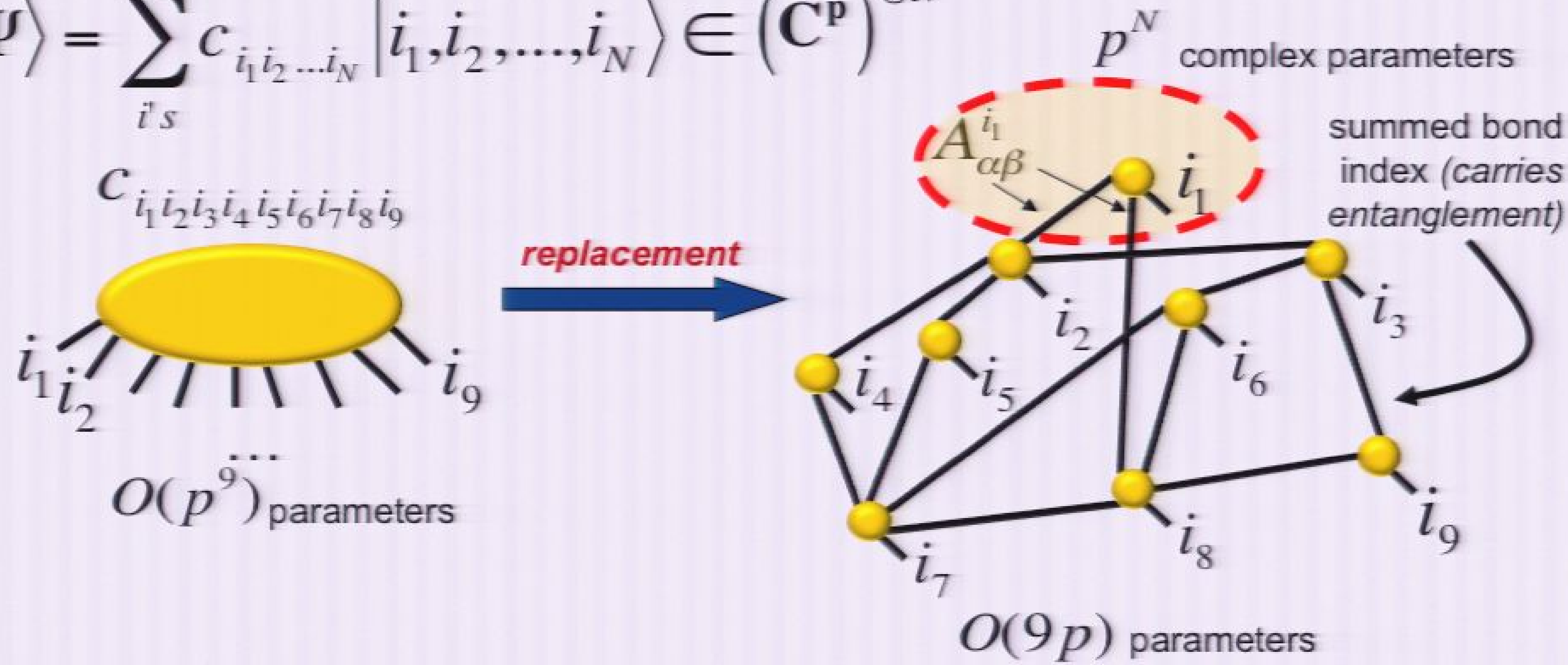
Tensor Networks (TN)

$$|\Psi\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle \in (\mathbf{C}^{\mathbf{p}})^{\otimes N}$$



Tensor Networks (TN)

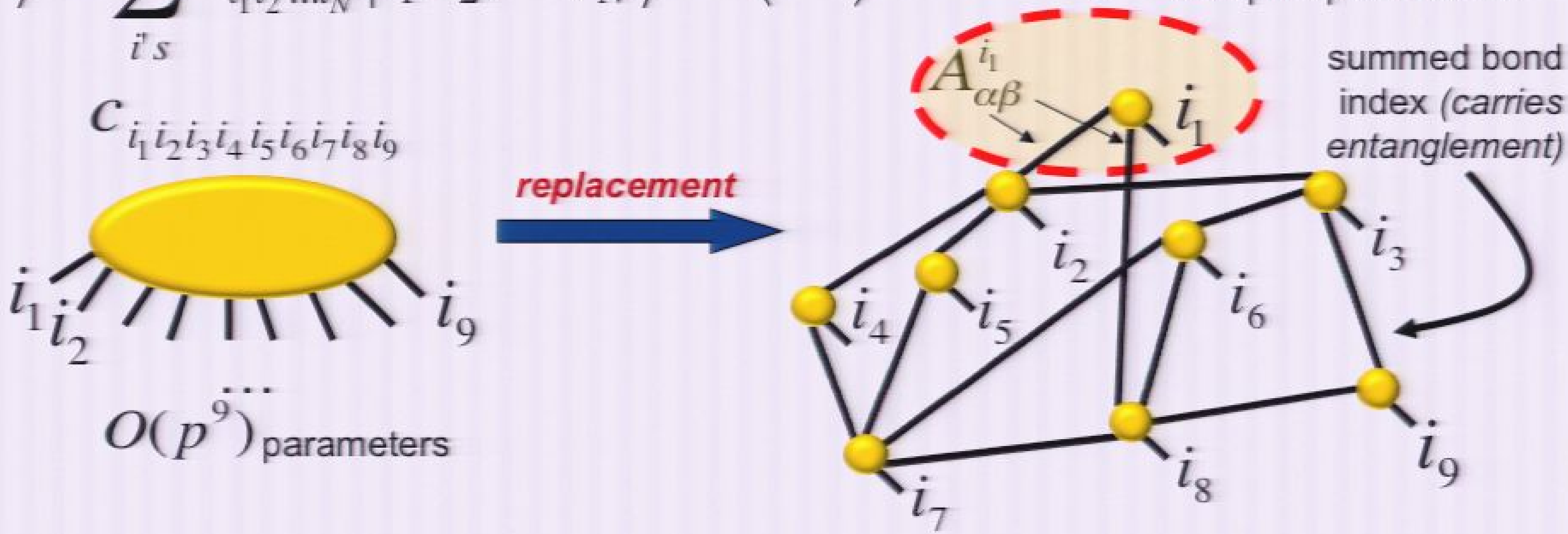
$$\left|\Psi\right\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} \left|i_1, i_2, \ldots, i_N\right\rangle \in \left(\mathbf{C^p}\right)^{\otimes N}$$



Tensor Networks (TN)

$$|\Psi\rangle = \sum_{i's} c_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle \in (\mathbf{C}^{\mathbf{p}})^{\otimes N}$$

p^N complex parameters



$O(p^9)$ parameters

$O(9p)$ parameters

range of the bond indices

maximum valence of the tensors

In general, $O(NpD^V)$ parameters

Justification

Generic state

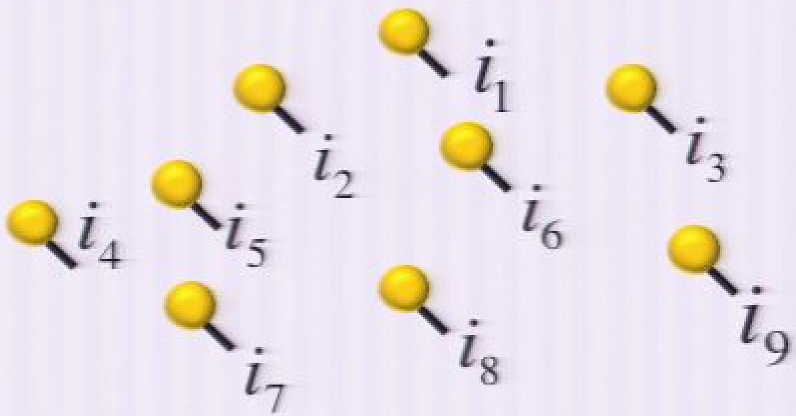
Correlations OK but not efficient (exact)

Justification

Generic state

Correlations OK
but not efficient
(exact)

Product state



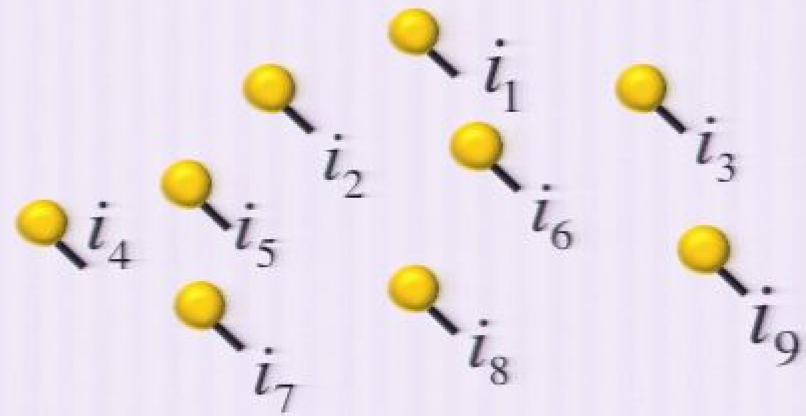
Efficiency OK
but not correlated
(mean field)

Justification

Generic state

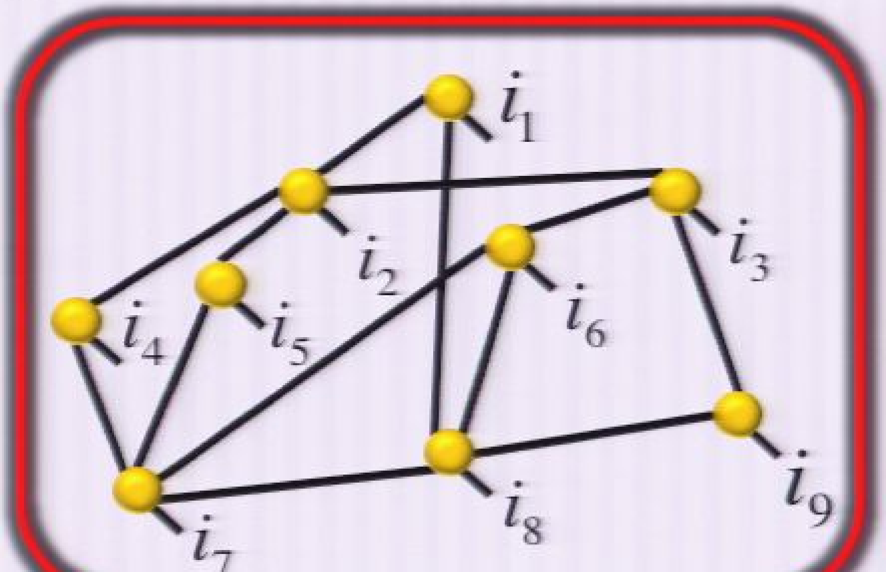
Correlations OK
but not efficient
(exact)

Product state



Efficiency OK
but not correlated
(mean field)

Tensor network



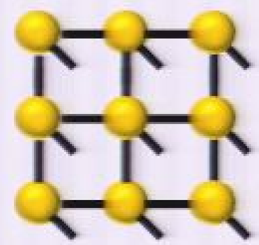
Efficient and correlated

Examples of TN

Matrix Product States (MPS)

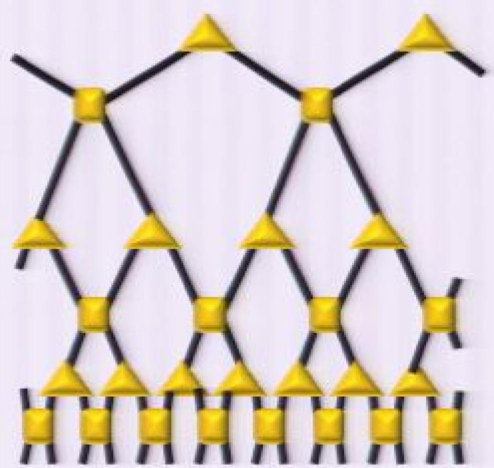
1-dim systems

Projected Entangled Pair States (PEPS)



d-dim systems, d > 1

Multiscale Entanglement Renormalization Ansatz (MERA)



Scale-invariant systems

Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)

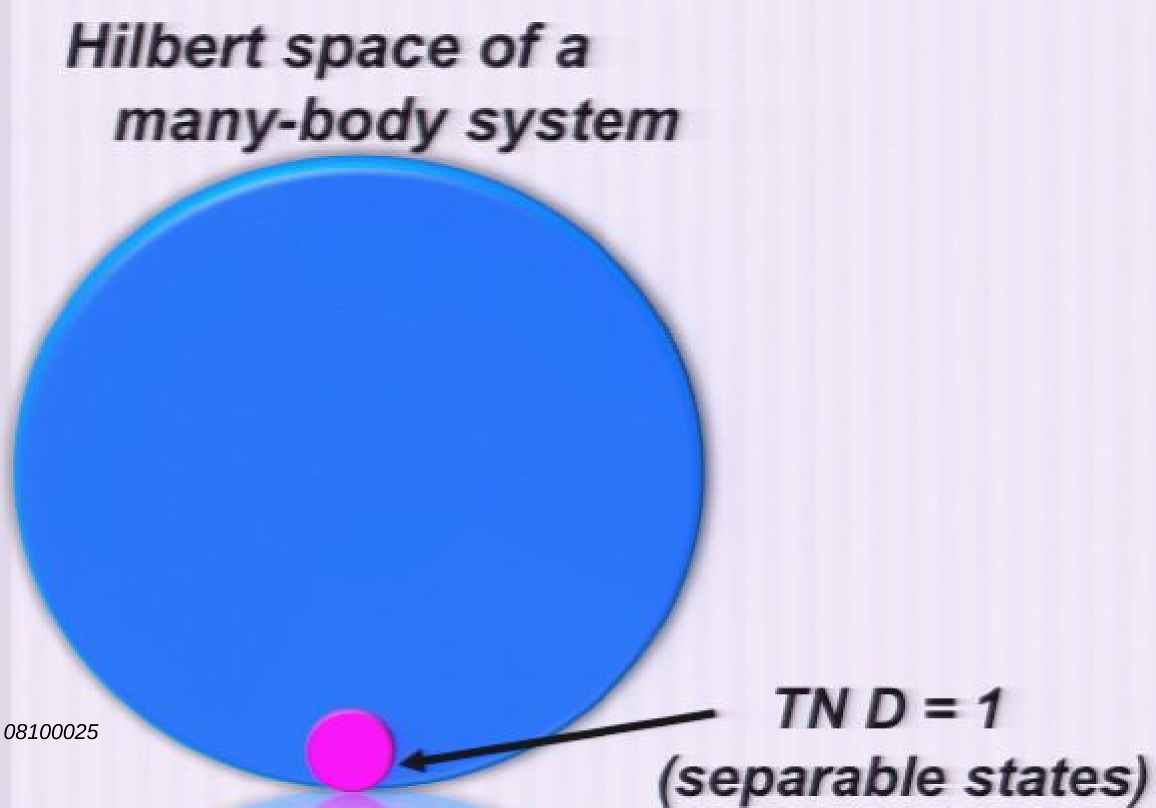
Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)

Hilbert space of a many-body system

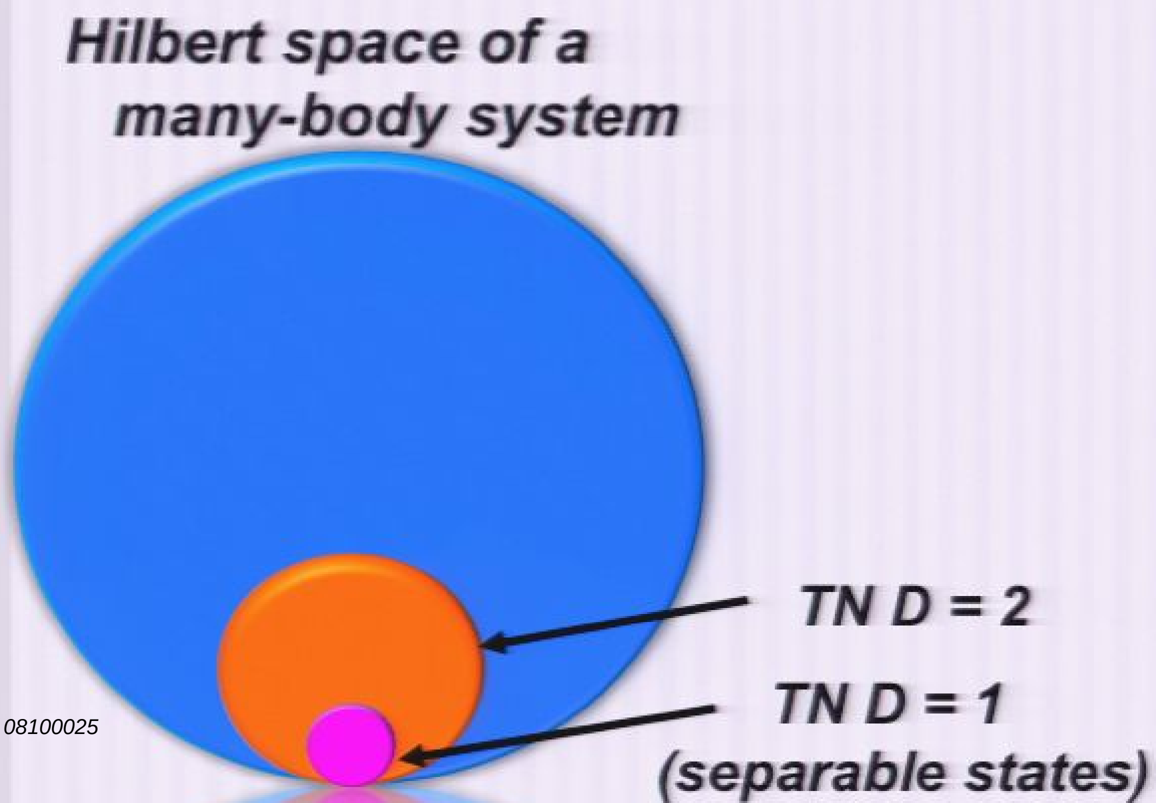
Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)



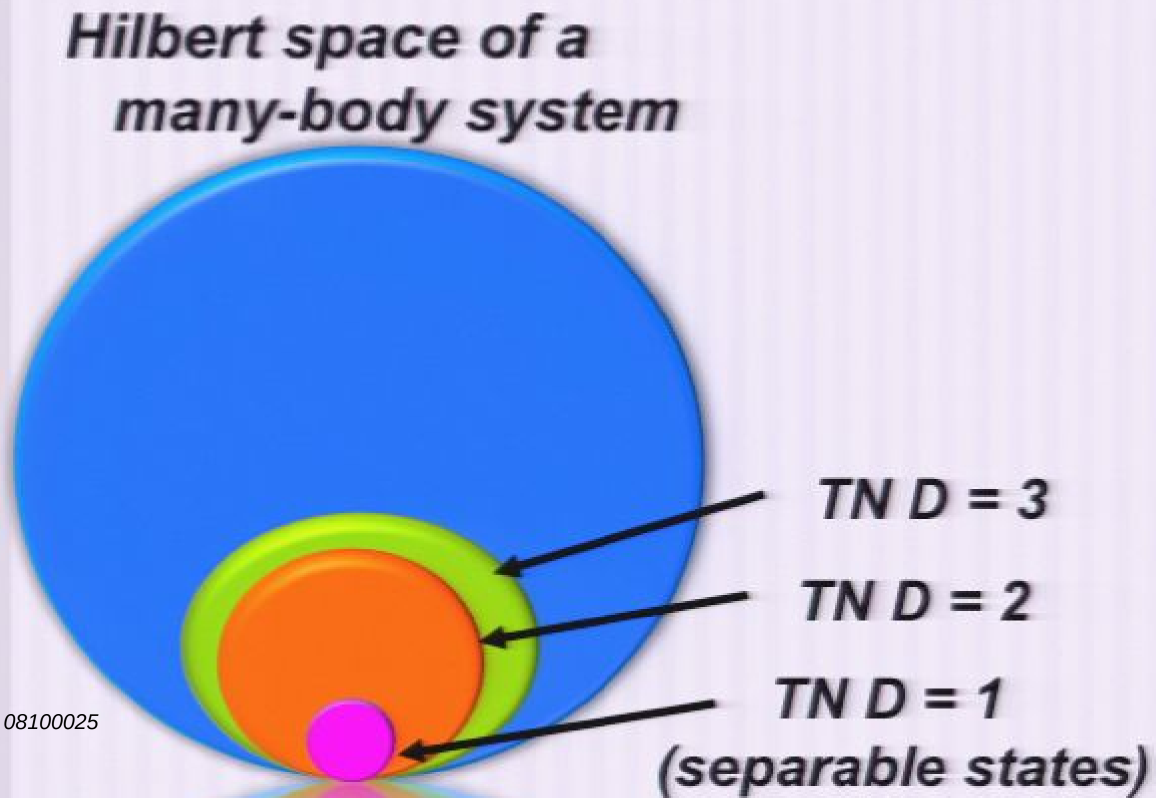
Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)



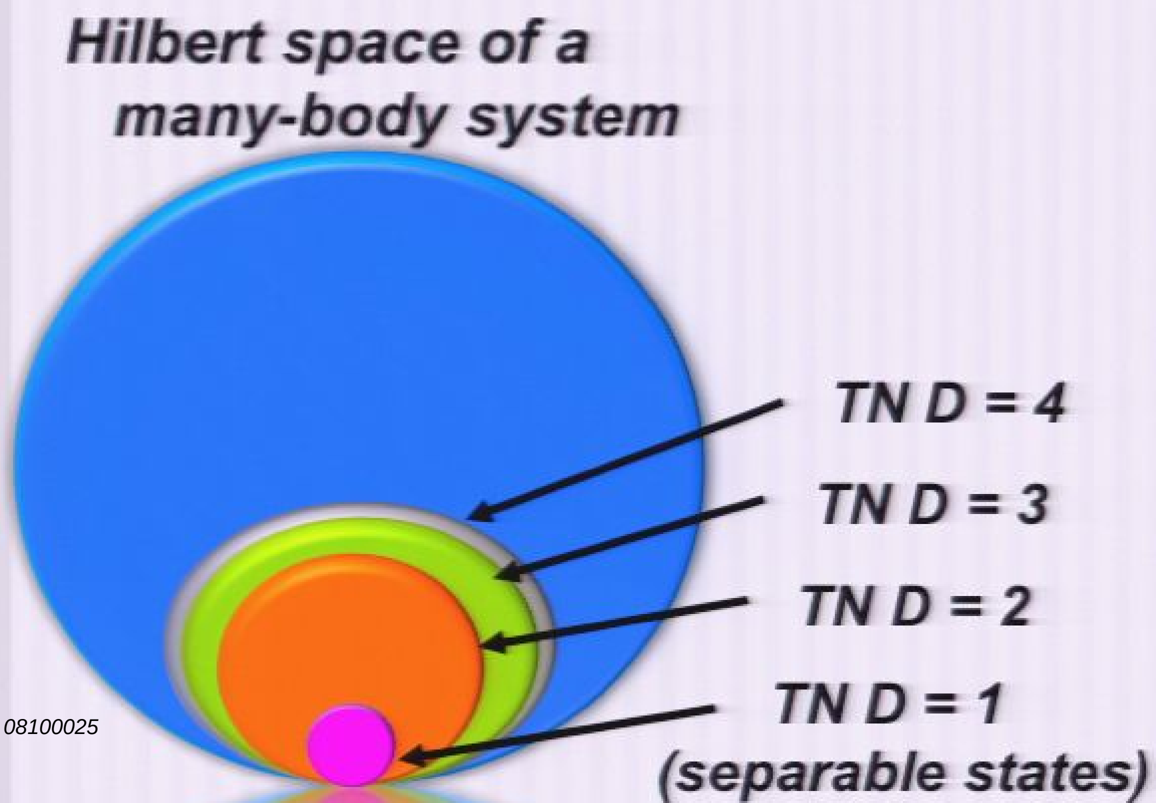
Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)



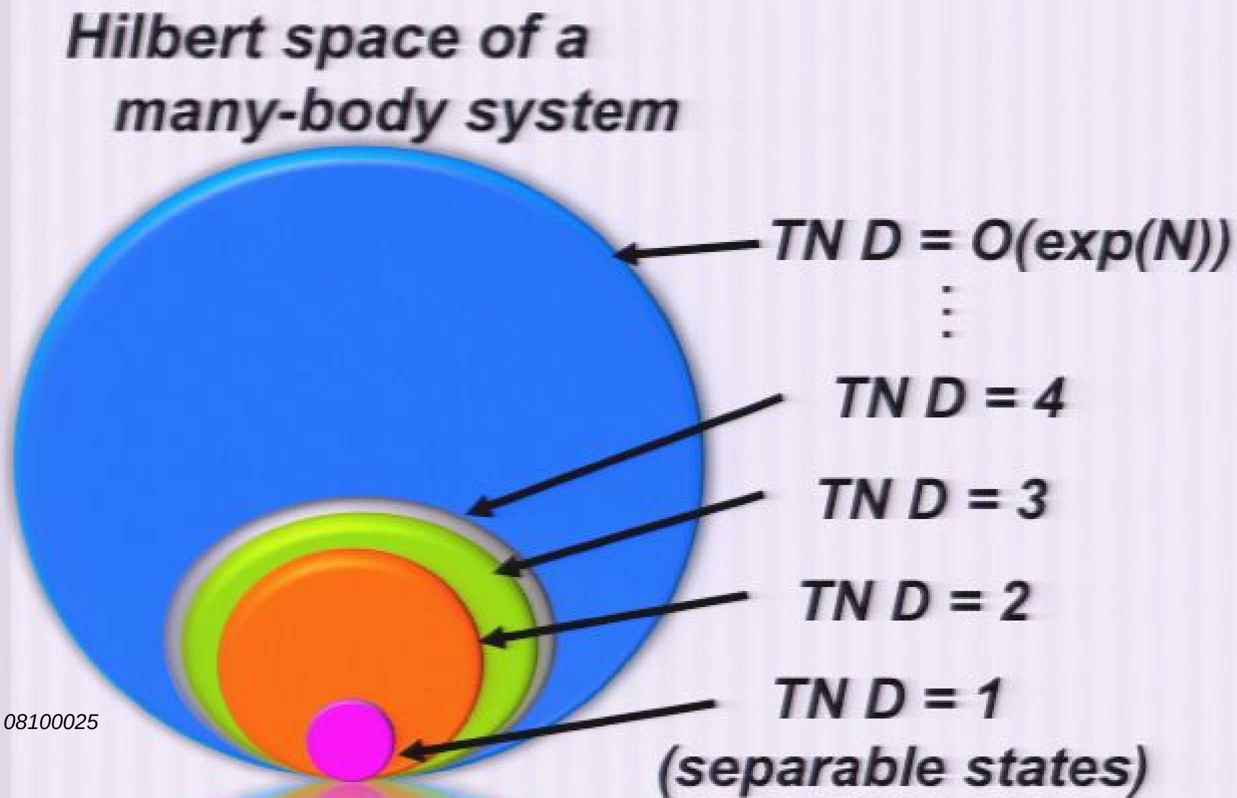
Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)



Tensor Network Algorithms

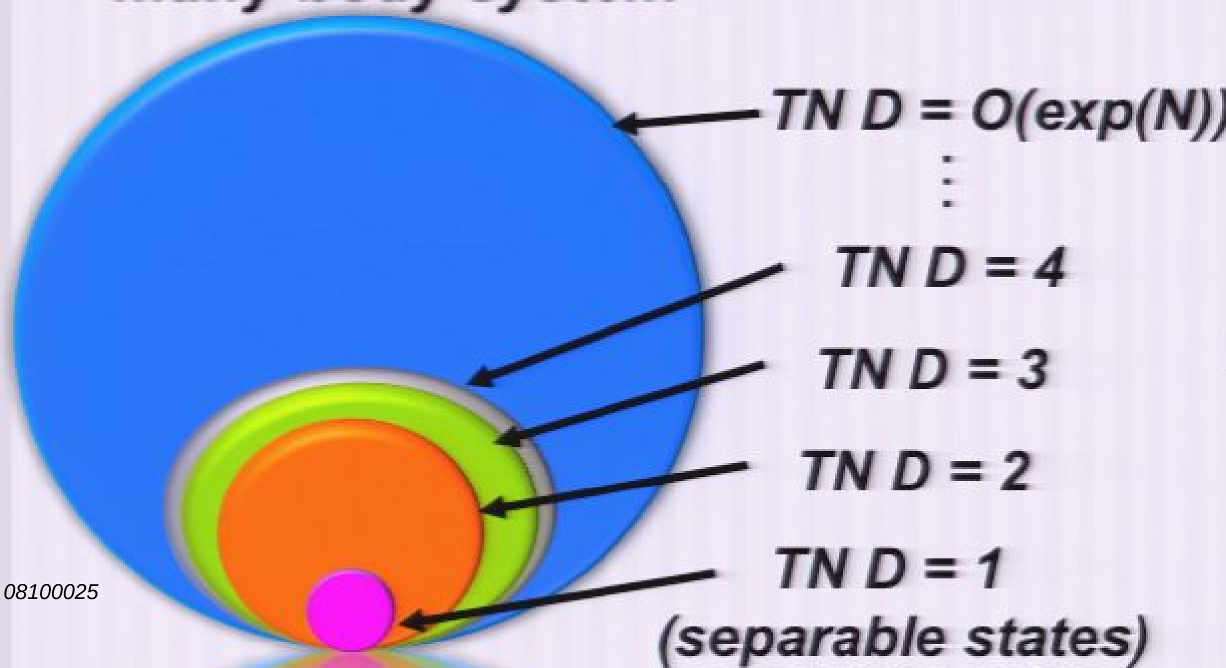
Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)



Tensor Network Algorithms

Observation: many relevant quantum states in Nature seem to be well represented by TN (e.g. ground states of some Hamiltonians with local interactions)

Hilbert space of a many-body system



Idea: use TN with fixed and increasing D to compute the properties of quantum many-body systems, e.g. ground states, dynamics ...

Why Tensor Networks?

Why Tensor Networks?

- Efficient ***description*** of the system

Why Tensor Networks?

- Efficient ***description*** of the system
- Evaluation of ***expectation values*** $\langle \Psi | O | \Psi \rangle$ (order parameters, correlators…) - exact in 1-dim, approximate in d-dim d > 1 (NP-Hard) -

Why Tensor Networks?

- Efficient ***description*** of the system
- Evaluation of ***expectation values*** $\langle \Psi | O | \Psi \rangle$ (order parameters, correlators…) - exact in 1-dim, approximate in d-dim d > 1 (NP-Hard) -
- Efficient ***updating*** after an operation, e.g.

$$e^{-iHt}|\Psi\rangle \qquad e^{-Ht}|\Psi\rangle$$

dynamics ***ground states***

Why Tensor Networks?

- Efficient ***description*** of the system
- Evaluation of ***expectation values*** $\langle \Psi | O | \Psi \rangle$ (order parameters, correlators…) - exact in 1-dim, approximate in d-dim d > 1 (NP-Hard) -
- Efficient ***updating*** after an operation, e.g.

$$e^{-iHt}|\Psi\rangle \qquad e^{-Ht}|\Psi\rangle$$

dynamics ***ground states***

- Efficient ***optimization of cost functions***, e.g.

$$\min_{|\Psi\rangle \in PEPS} \langle \Psi | H | \Psi \rangle \qquad \min_{|\Psi\rangle \in PEPS} \left\| |\Psi\rangle - |\Psi'\rangle \right\|^2$$

ground states

Why Tensor Networks?

- Efficient ***description*** of the system
- Evaluation of ***expectation values*** $\langle\Psi|O|\Psi\rangle$ (order parameters, correlators…) - exact in 1-dim, approximate in d-dim d > 1 (NP-Hard) -
- Efficient ***updating*** after an operation, e.g.

$$e^{-iHt}|\Psi\rangle \qquad e^{-Ht}|\Psi\rangle$$

dynamics ***ground states***

- Efficient ***optimization of cost functions***, e.g.

$$\min_{|\Psi\rangle\in PEPS}\langle\Psi|H|\Psi\rangle \qquad \min_{|\Psi\rangle\in PEPS}\left\||\Psi\rangle-|\Psi'\rangle\right\|^2$$

ground states

- Allow to deal with ***infinite lattices (thermodynanic limit)*** for translationally invariant systems

Why Tensor Networks?

- Efficient ***description*** of the system
- Evaluation of ***expectation values*** $\langle\Psi|O|\Psi\rangle$ (order parameters, correlators…) - exact in 1-dim, approximate in d-dim d > 1 (NP-Hard) -
- Efficient ***updating*** after an operation, e.g.

$$e^{-iHt}|\Psi\rangle \qquad e^{-Ht}|\Psi\rangle$$

dynamics ***ground states***

- Efficient ***optimization of cost functions***, e.g.

$$\min_{|\Psi\rangle\in PEPS}\langle\Psi|H|\Psi\rangle \qquad \min_{|\Psi\rangle\in PEPS}\left\||\Psi\rangle-|\Psi'\rangle\right\|^2$$

ground states

- Allow to deal with ***infinite lattices (thermodynanic limit)*** for translationally invariant systems
- **Fastest simulation algorithms.** E.g., 1 month of quantum Monte Carlo in a supercomputer vs 1 day of TN in a desktop PC

Outline

1.- Brief historical review

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

Outline

1.- Brief historical review

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

Infinite 2-dim quantum systems (iPEPS algorithm)

We developed an algorithm that uses infinite PEPS to evaluate the ground state properties of ***infinite-size quantum lattice systems in 2-dim***

J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac, cond-mat/0703788





Infinite 2-dim quantum systems (iPEPS algorithm)

We developed an algorithm that uses infinite PEPS to evaluate the ground state properties of ***infinite-size quantum lattice systems in 2-dim***





J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac, cond-mat/0703788

INPUT: 2-dim lattice Hamiltonian

$$H = \sum_{\langle \vec{r}, \vec{r}' \rangle} h^{[\vec{r}, \vec{r}']}$$

Infinite 2-dim quantum systems (iPEPS algorithm)

We developed an algorithm that uses infinite PEPS to evaluate the ground state properties of ***infinite-size quantum lattice systems in 2-dim***

J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac,
cond-mat/0703788

INPUT: 2-dim lattice Hamiltonian ➡ imaginary-time evolution

$$H = \sum_{\langle \vec{r},\vec{r}' \rangle} h^{[\vec{r},\vec{r}']} \quad \Rightarrow \quad e^{-Ht}|\Psi\rangle$$

Infinite 2-dim quantum systems (iPEPS algorithm)

We developed an algorithm that uses infinite PEPS to evaluate the ground state properties of ***infinite-size quantum lattice systems in 2-dim***

J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac, cond-mat/0703788

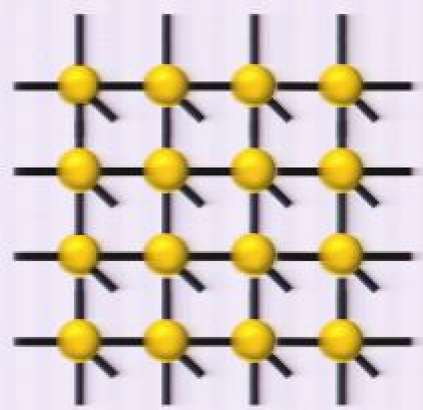
INPUT: 2-dim lattice Hamiltonian

$$H = \sum_{\langle \vec{r}, \vec{r}' \rangle} h^{[\vec{r}, \vec{r}']}$$

➡ imaginary-time evolution

$$e^{-Ht} |\Psi\rangle$$

➡ iPEPS for the ground state $|\Psi_0\rangle$



Infinite 2-dim quantum systems (iPEPS algorithm)

We developed an algorithm that uses infinite PEPS to evaluate the ground state properties of ***infinite-size quantum lattice systems in 2-dim***

J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac, cond-mat/0703788

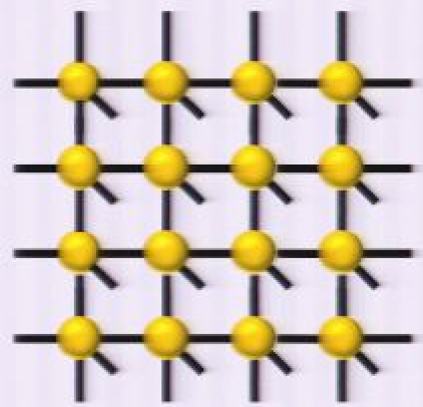
INPUT: 2-dim lattice Hamiltonian

$$H = \sum_{\langle \vec{r}, \vec{r}' \rangle} h^{[\vec{r}, \vec{r}']}$$

➡ imaginary-time evolution

$$e^{-Ht} |\Psi\rangle$$

➡ iPEPS for the ground state $|\Psi_0\rangle$



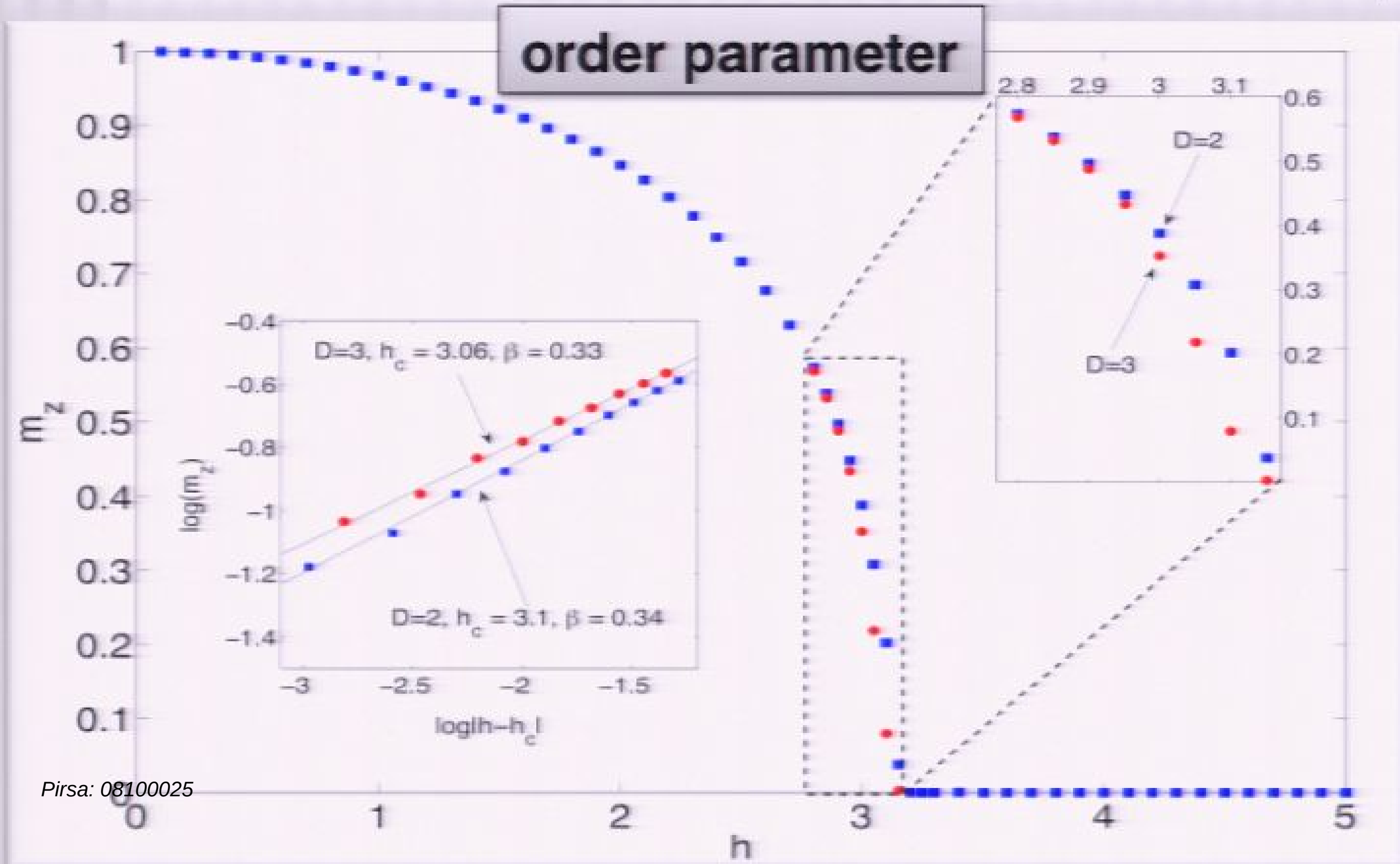
➡ **OUTPUT:** expectation values

$$\langle \Psi_0 | O | \Psi_0 \rangle$$

iPEPS: results

Benchmark: 2D quantum Ising model on the infinite square lattice

$$H = -\sum_{\langle \vec{r}, \vec{r}' \rangle} \sigma_z^{\vec{r}} \sigma_z^{\vec{r}'} + h \sum_{\vec{r}} \sigma_x^{\vec{r}}$$



Infinite 2-dim quantum systems (iPEPS algorithm)

We developed an algorithm that uses infinite PEPS to evaluate the ground state properties of ***infinite-size quantum lattice systems in 2-dim***

J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac,
cond-mat/0703788

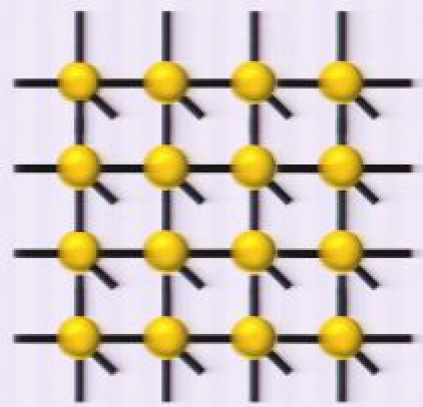
INPUT: 2-dim lattice Hamiltonian

$$H = \sum_{\langle \vec{r}, \vec{r}' \rangle} h^{[\vec{r}, \vec{r}']}$$

➡ imaginary-time evolution

$$e^{-Ht} |\Psi\rangle$$

➡ iPEPS for the ground state $|\Psi_0\rangle$



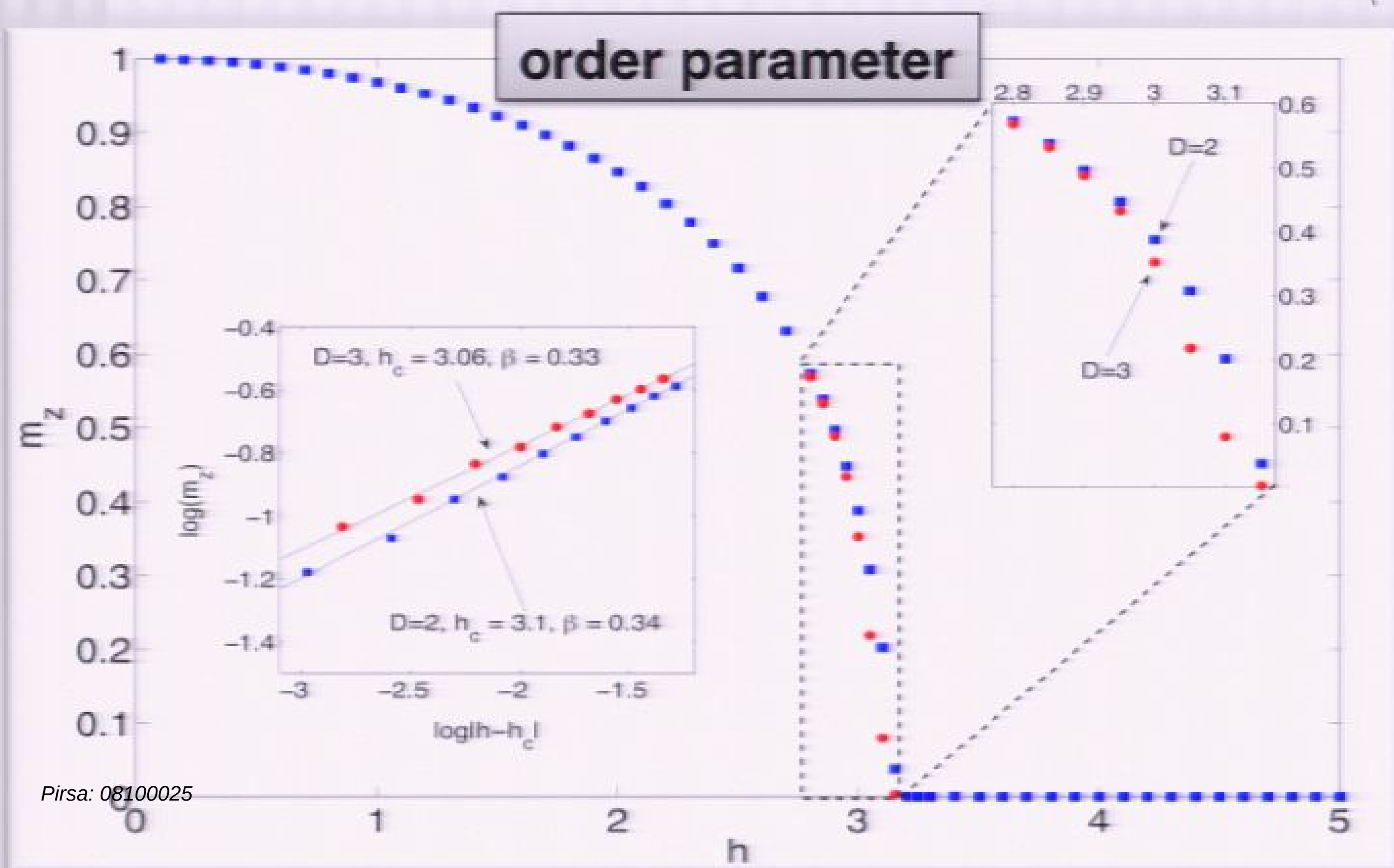
➡ **OUTPUT:** expectation values

$$\langle \Psi_0 | O | \Psi_0 \rangle$$

iPEPS: results

Benchmark: 2D quantum Ising model on the infinite square lattice

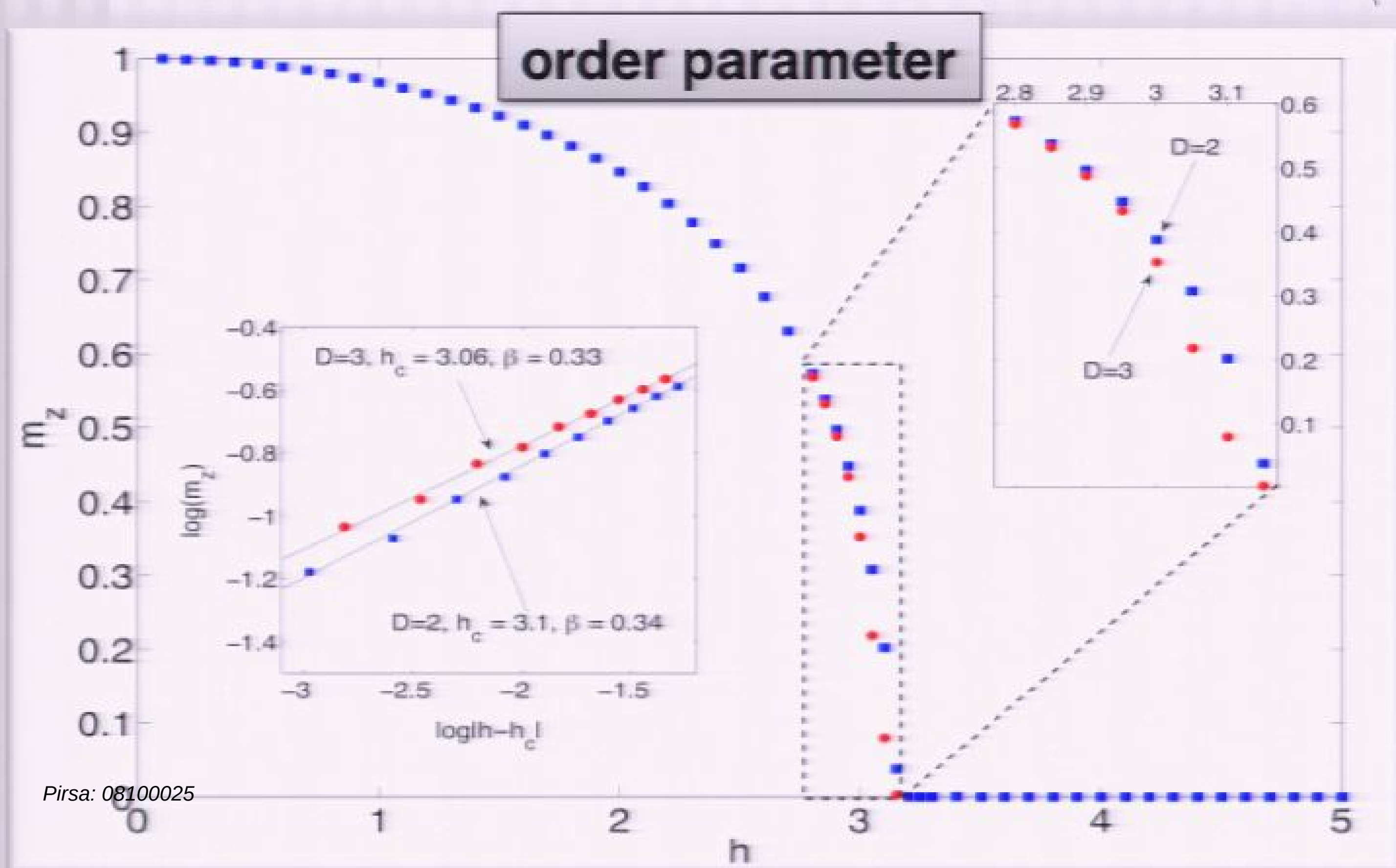
$$H = -\sum_{\langle \vec{r},\vec{r}' \rangle} \sigma_z^{\vec{r}} \sigma_z^{\vec{r}'} + h \sum_{\vec{r}} \sigma_x^{\vec{r}}$$



iPEPS: results

Benchmark: 2D quantum Ising model on the infinite square lattice

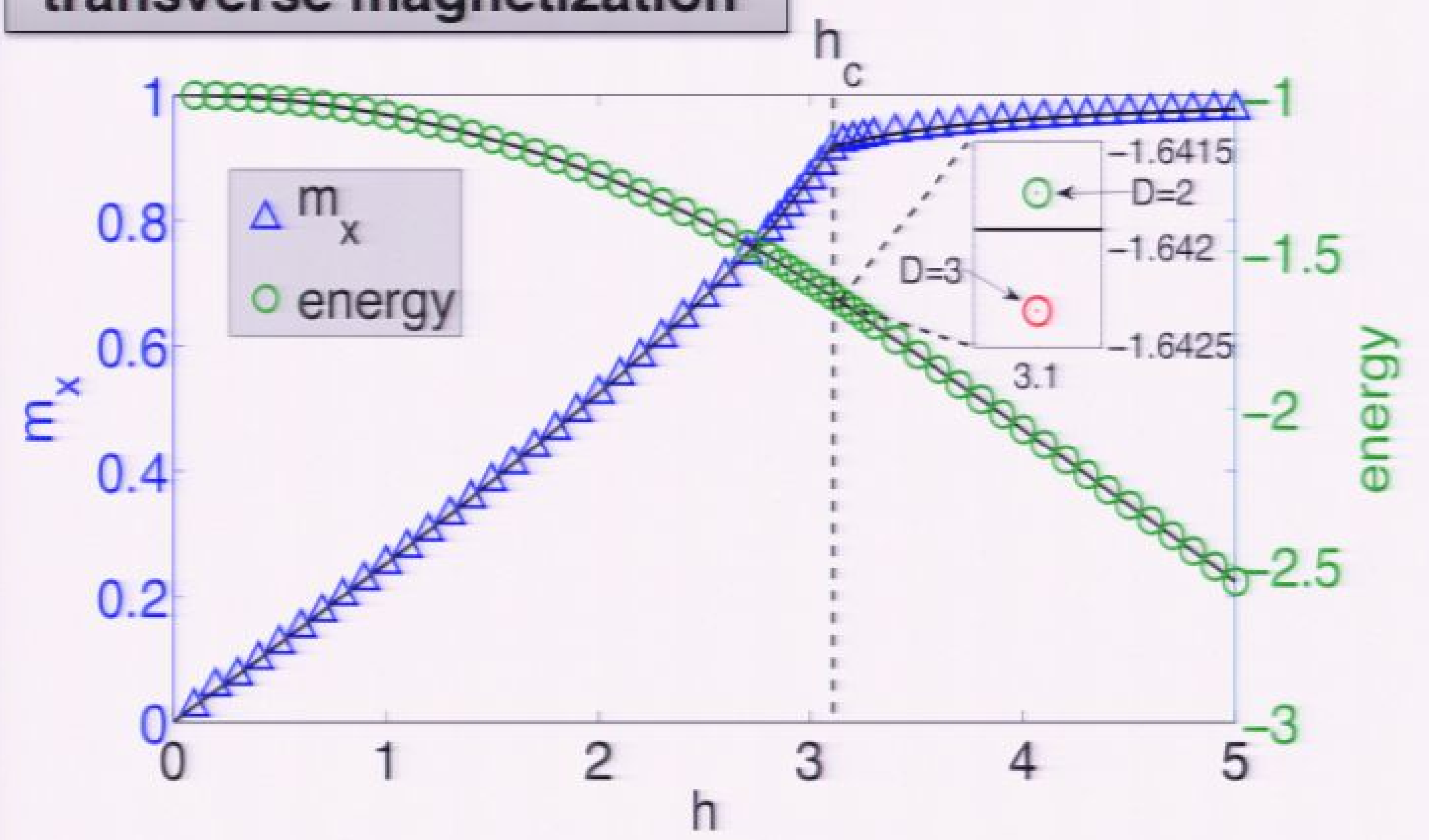
$$H = -\sum_{\langle \vec{r},\vec{r}' \rangle} \sigma_z^{\vec{r}} \sigma_z^{\vec{r}'} + h \sum_{\vec{r}} \sigma_x^{\vec{r}}$$



The critical point and the critical exponents match those from the best Quantum Monte Carlo simulation with less than 1% of relative error

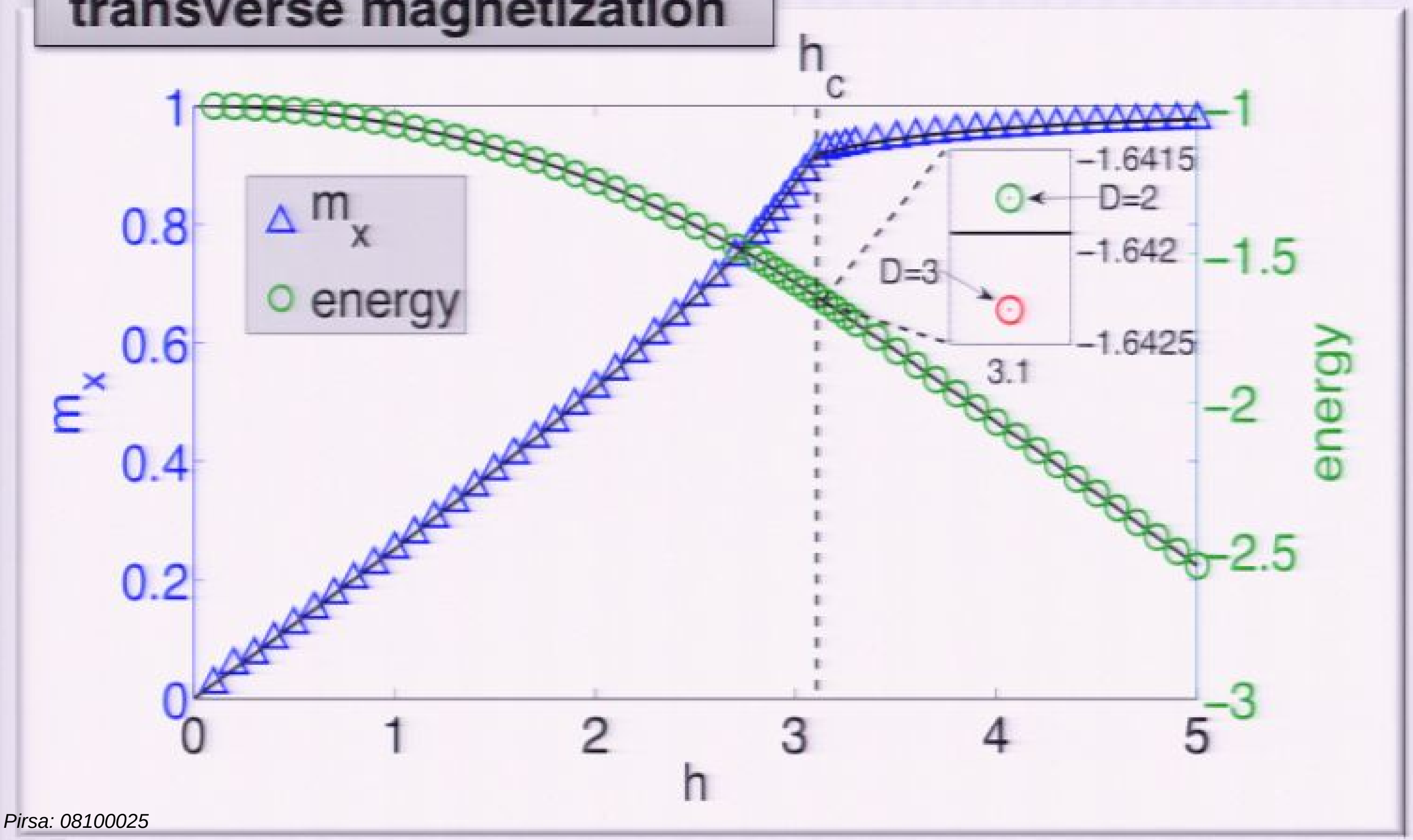
iPEPS: results

Ground state energy and transverse magnetization



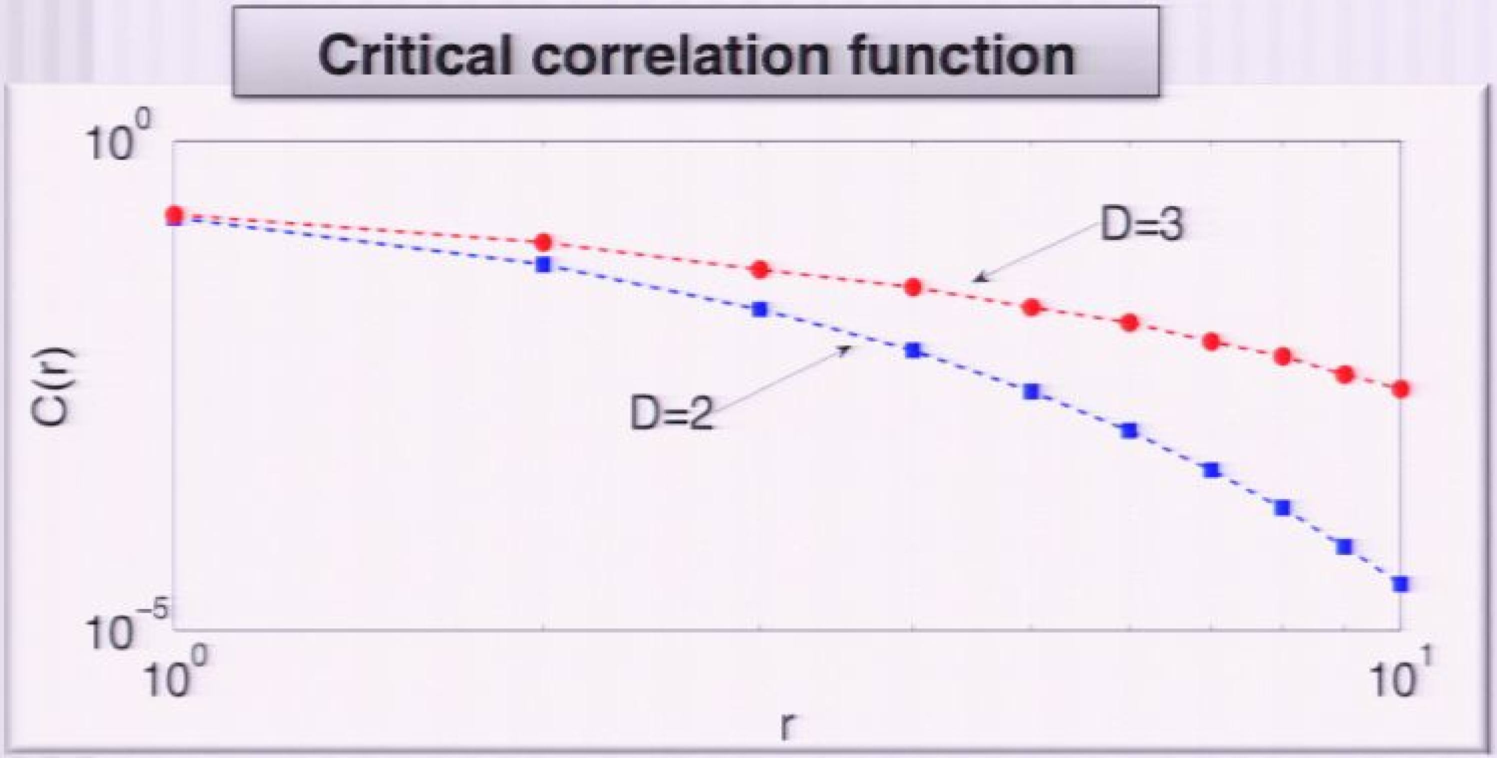
iPEPS: results

Ground state energy and transverse magnetization

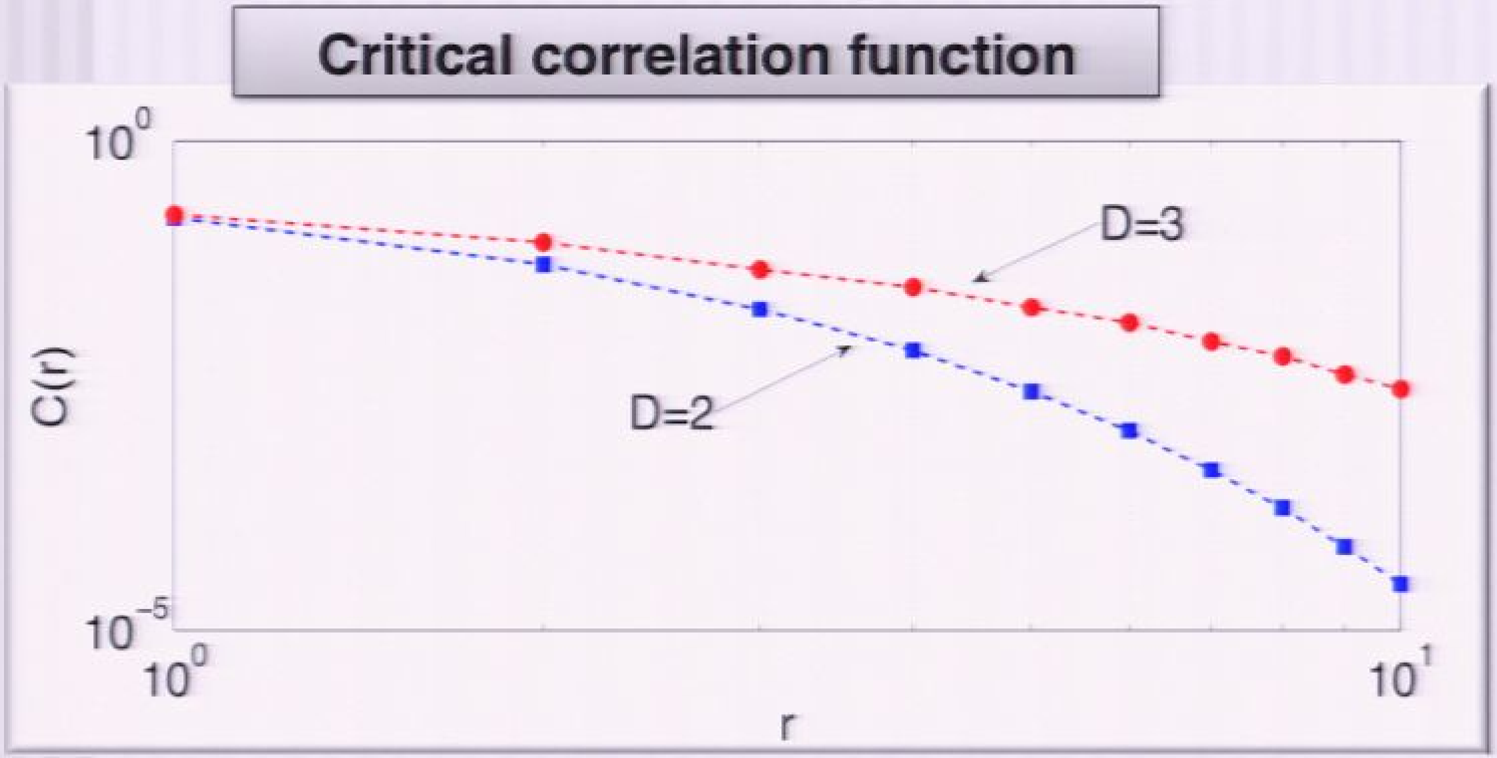


Good agreement with simple perturbative series expansions calculation (black line)

iPEPS: results



iPEPS: results

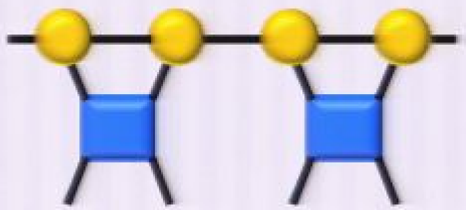


Easy to compute, and non-trivial even for small values of D

Infinite 2-dim classical systems (iTEBD+ algorithm)

The iTEBD algorithm is a method that allows to compute the evolution of infinite 1-dim quantum systems as driven by local (e.g. two-body) *unitary* operations

*G. Vidal, PRL **98**, 070201 (2007)*

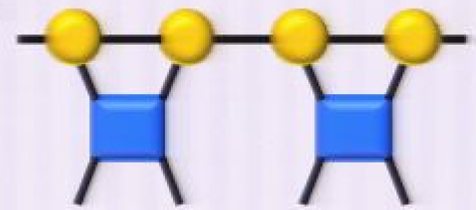


2-body gates

Infinite 2-dim classical systems (iTEBD+ algorithm)

The iTEBD algorithm is a method that allows to compute the evolution of infinite 1-dim quantum systems as driven by local (e.g. two-body) *unitary* operations

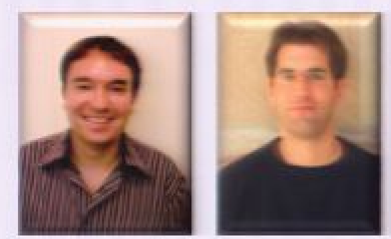
*G. Vidal, PRL **98**, 070201 (2007)*

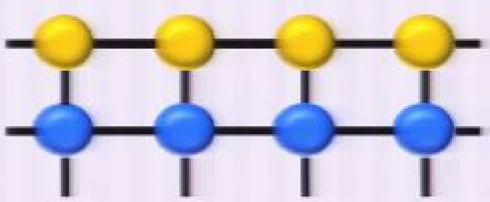


2-body gates

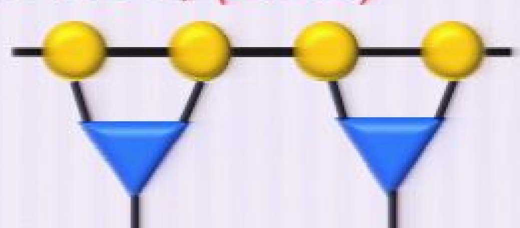
We have generalized iTEBD so that it can be used to ***simulate infinite-size classical lattice systems in 2-dim***

R. Orús, G. Vidal, PRB ?, (2008)

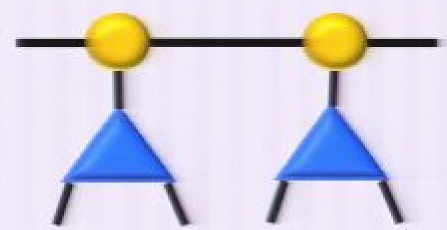




Matrix Product Operators



2-to-1 gates



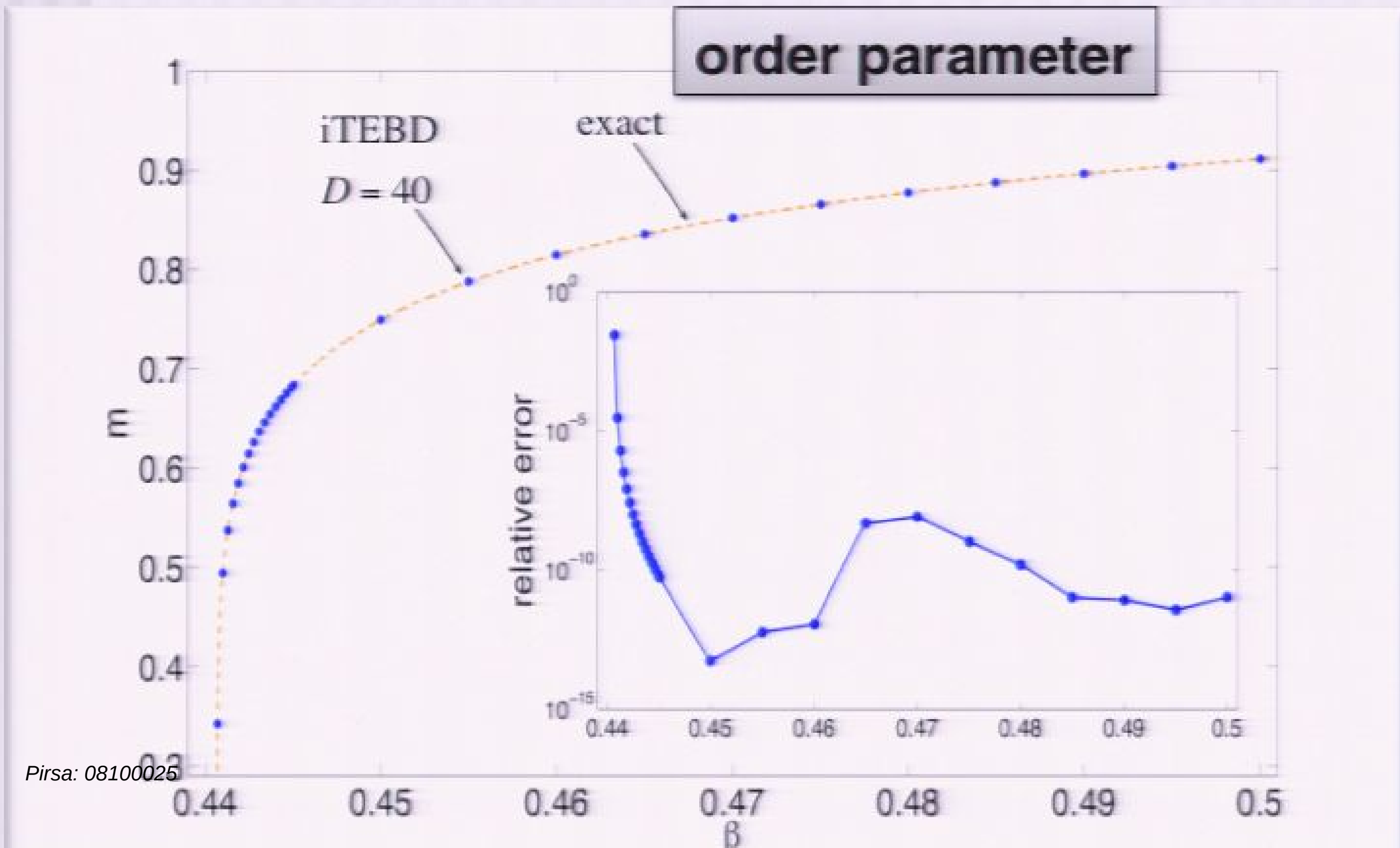
1-to-2 gates

iTEBD+: results

Benchmark: 2D classical Ising model on the infinite square lattice

$$K(s,s') = -s\, s' \qquad s,s' = \pm 1$$

$$Z(\beta) = \sum_{\{s\}} \exp\left(-\beta K\left(\{s\}\right)\right)$$



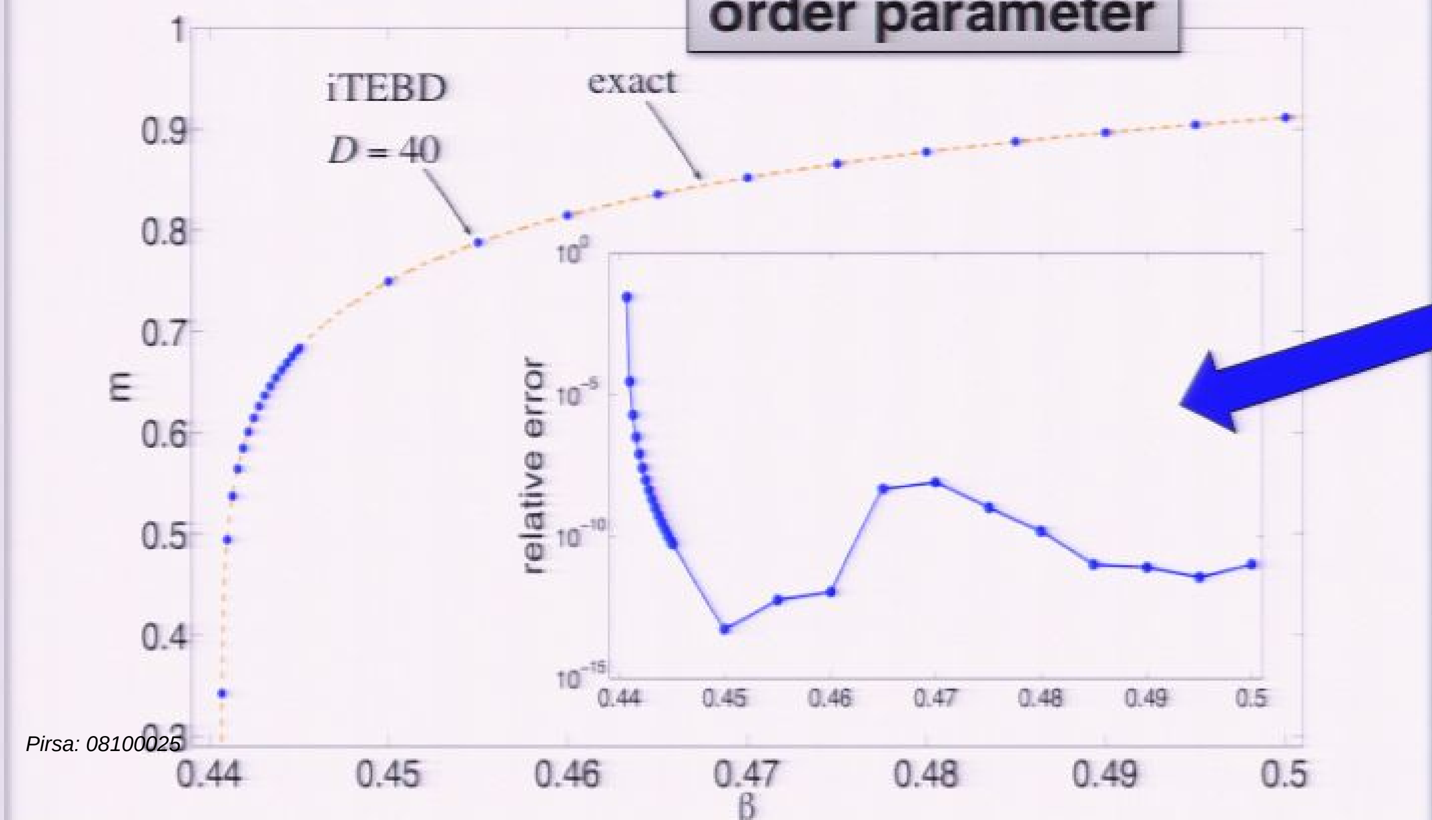
iTEBD+: results

Benchmark: 2D classical Ising model on the infinite square lattice

$$K(s,s') = -s\, s' \qquad s,s' = \pm 1$$

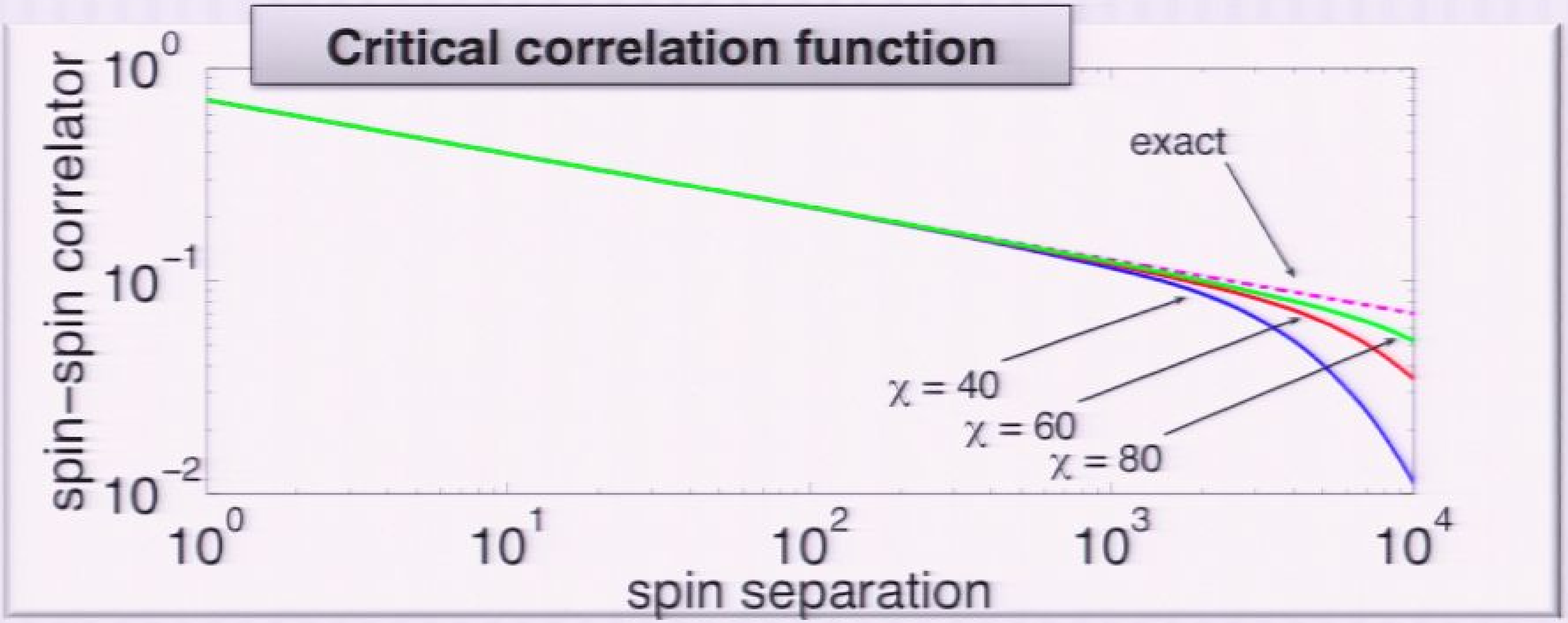
$$Z(\beta) = \sum_{\{s\}} \exp\left(-\beta K\left(\{s\}\right)\right)$$

order parameter



Small relative error close to criticality, and the critical exponent matches the exact one with 0.2% of relative error

iTEBD+: results



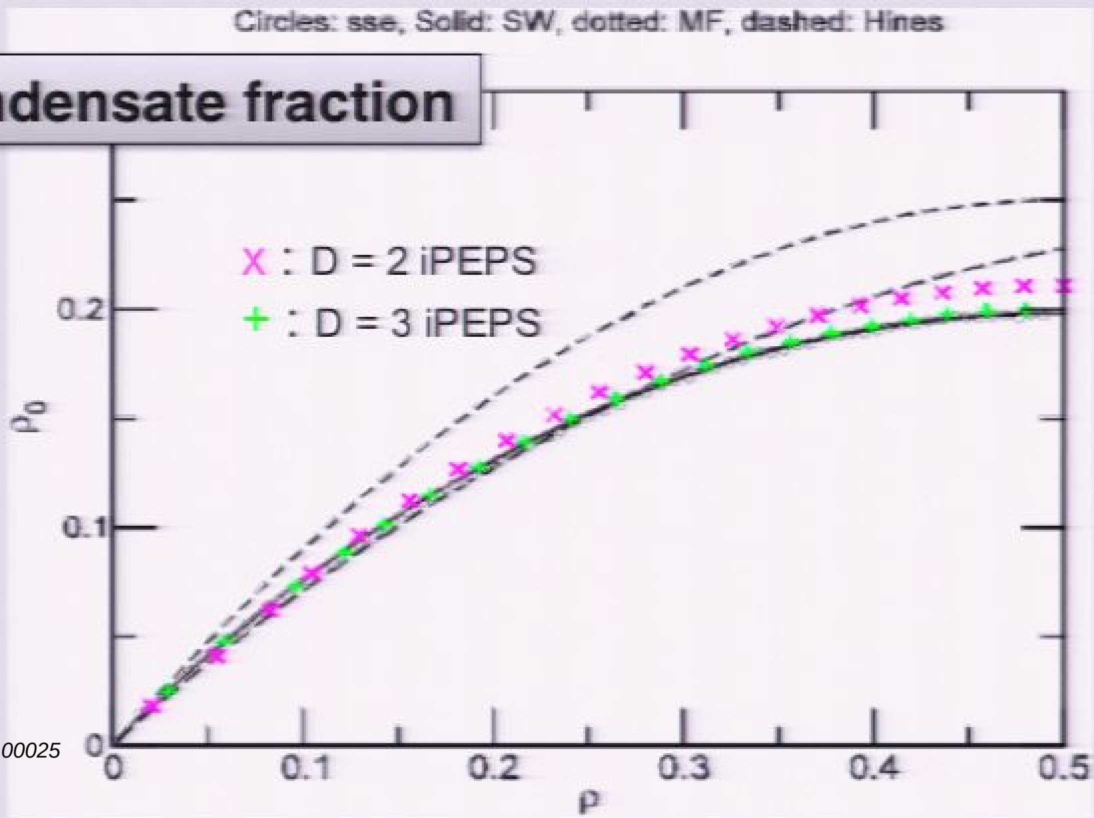
Correct power-law decay up to several thousands of sites

Hard-core Bose-Hubbard model with iPEPS

J. Jordan, R. Orús, G. Vidal, in preparation

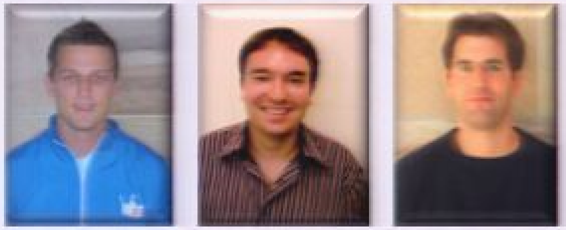
$$H_{HC} = -J \sum_{<i,j>} \left(a_i^\dagger a_j + a_j^\dagger a_i \right) - \mu \sum_i n_i$$

Condensate fraction



Hard-core Bose-Hubbard model with iPEPS

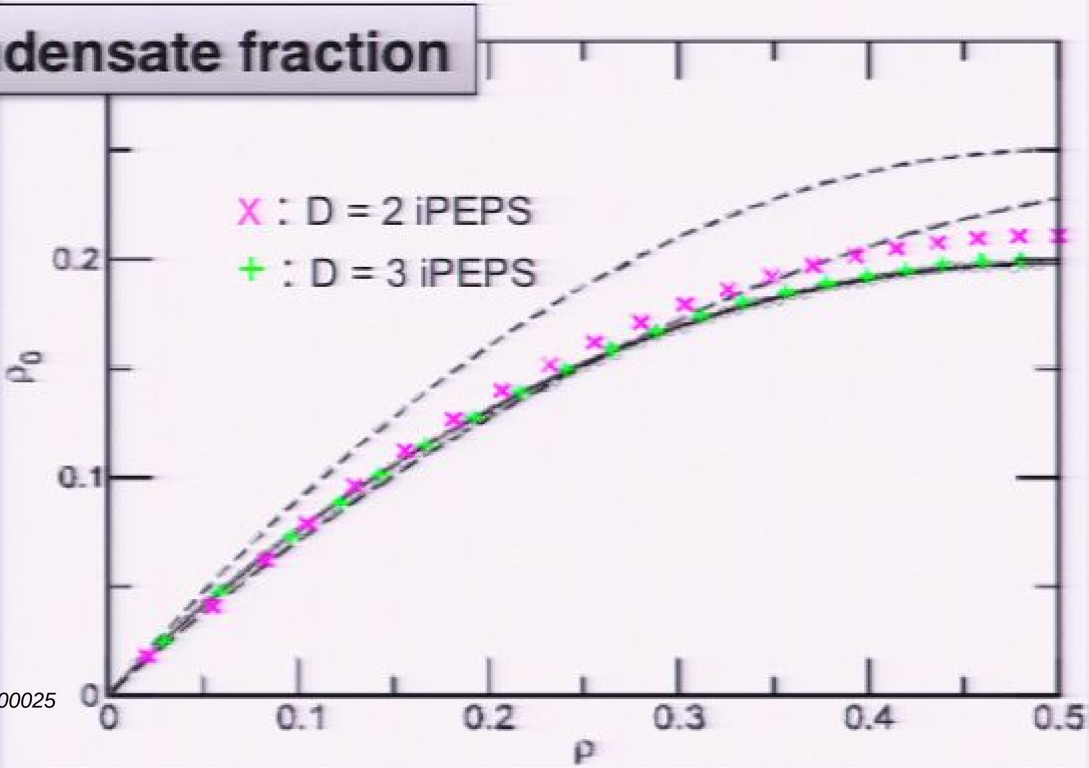
J. Jordan, R. Orús, G. Vidal, in preparation



$$H_{HC} = -J\sum_{<i,j>}\left(a_i^\dagger a_j + a_j^\dagger a_i\right) - \mu\sum_i n_i$$

Circles: sse, Solid: SW, dotted: MF, dashed: Hines

Condensate fraction



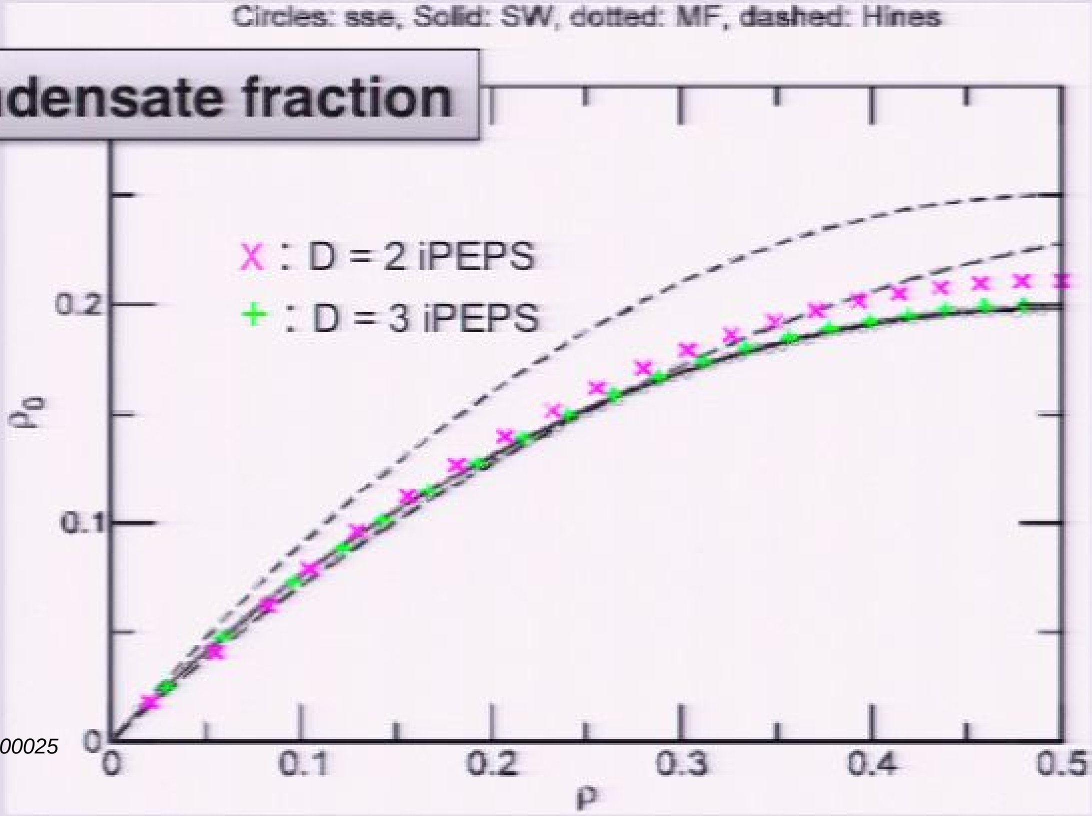
Excellent agreement with other techniques, and possibility to compute new quantities (e.g. time evolution…)

Hard-core Bose-Hubbard model with iPEPS

J. Jordan, R. Orús, G. Vidal, in preparation

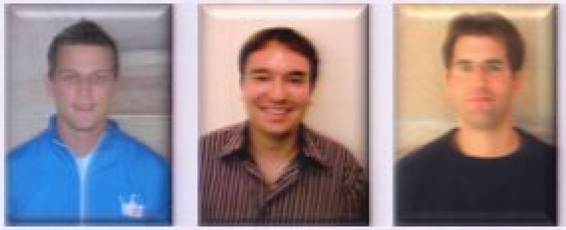
$$H_{HC} = -J \sum_{<i,j>} \left(a_i^\dagger a_j + a_j^\dagger a_i \right) - \mu \sum_i n_i$$

Condensate fraction



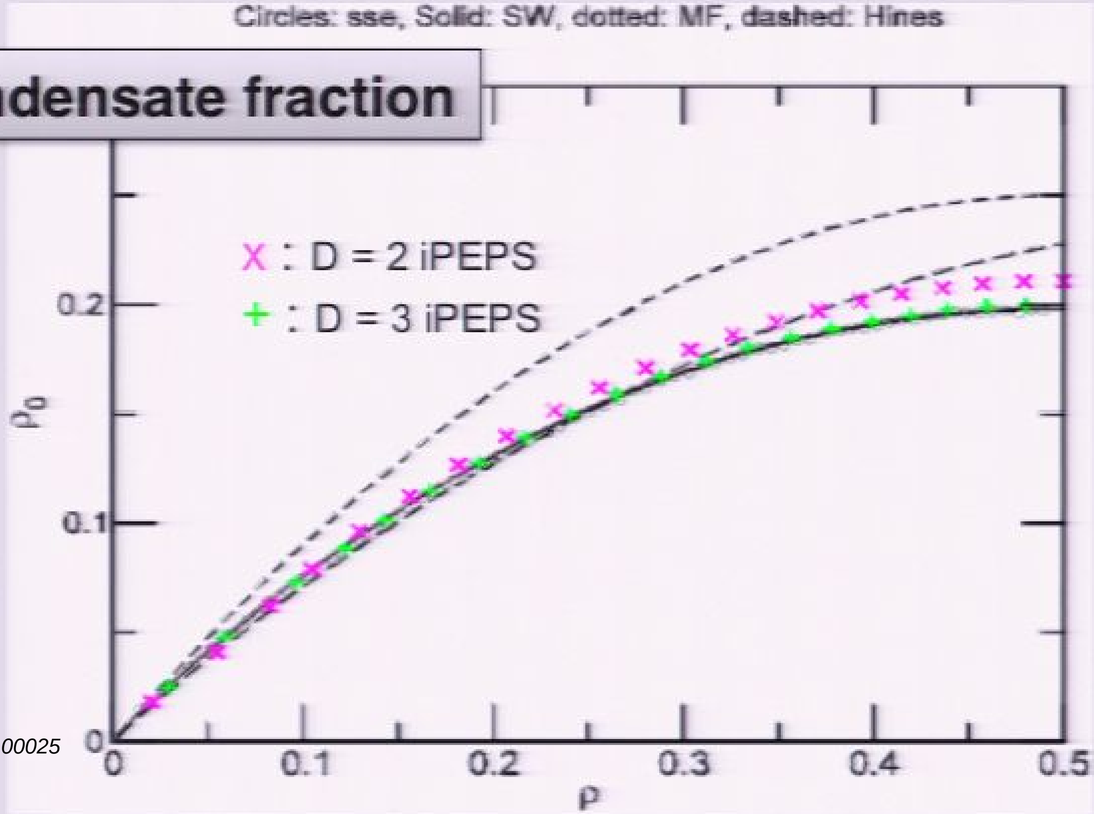
Hard-core Bose-Hubbard model with iPEPS

J. Jordan, R. Orús, G. Vidal, in preparation



$$H_{HC} = -J \sum_{<i,j>} \left(a_i^\dagger a_j + a_j^\dagger a_i \right) - \mu \sum_i n_i$$

Condensate fraction

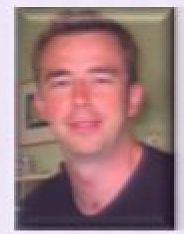


Excellent agreement with other techniques, and possibility to compute new quantities (e.g. time evolution…)

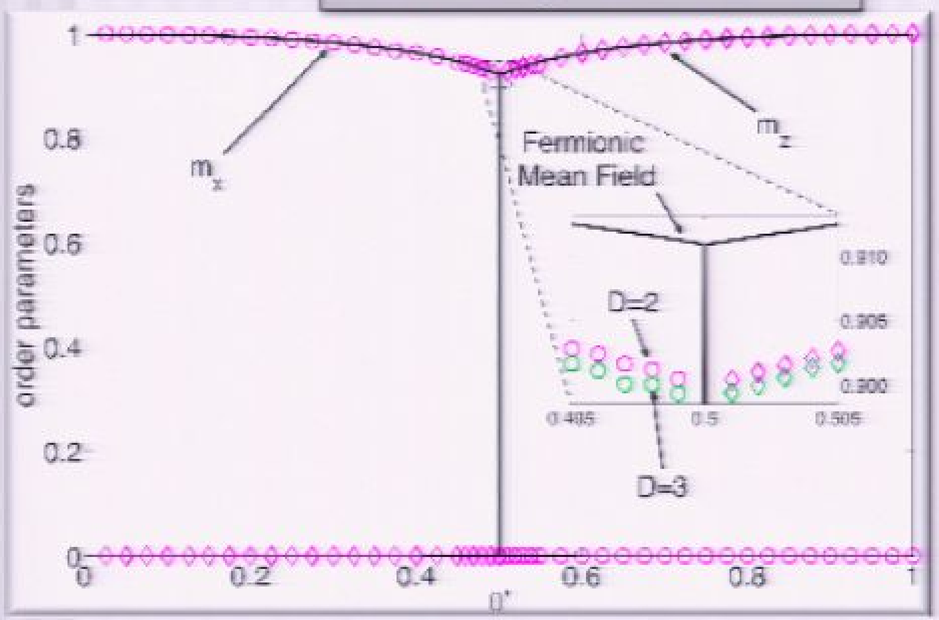
Anisotropic Quantum Orbital Compass Model with iPEPS

A. Doherty, R. Orús, G. Vidal, arXiv:0809.4068

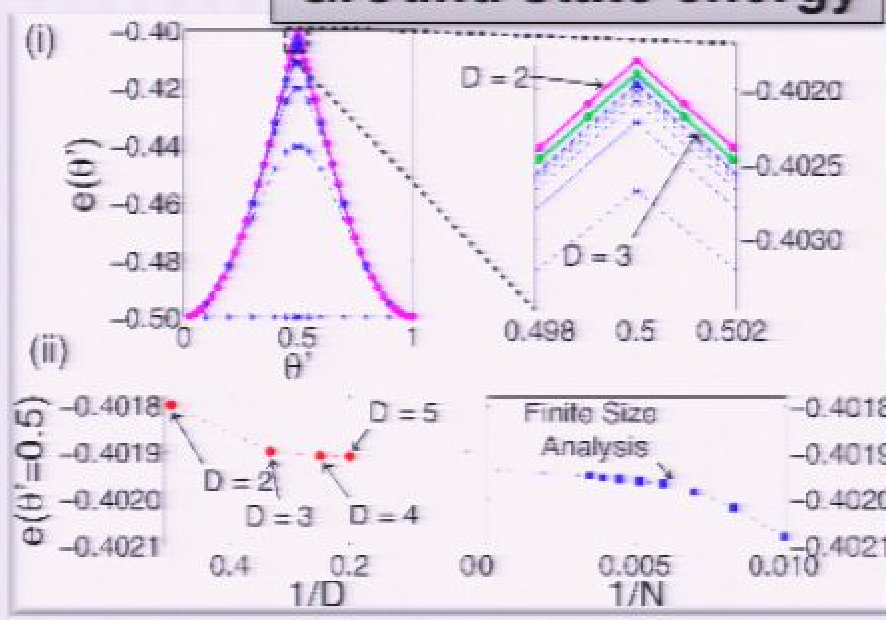
$$H = -\sin(\theta)\sum_{\vec{r}} \sigma_z^{\vec{r}}\sigma_z^{\vec{r}+\hat{i}} - \cos(\theta)\sum_{\vec{r}} \sigma_x^{\vec{r}}\sigma_x^{\vec{r}+\hat{j}} \qquad \hat{j}\ \hat{i}$$



Order parmeters



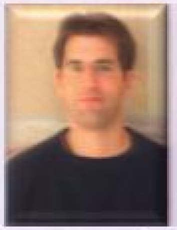
Ground state energy



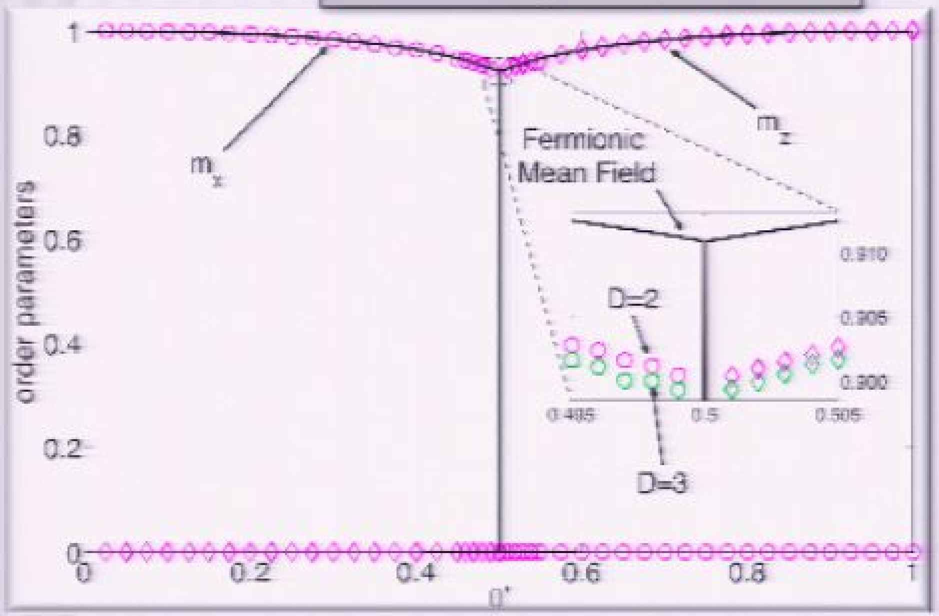
Anisotropic Quantum Orbital Compass Model with iPEPS

A. Doherty, R. Orús, G. Vidal, arXiv:0809.4068

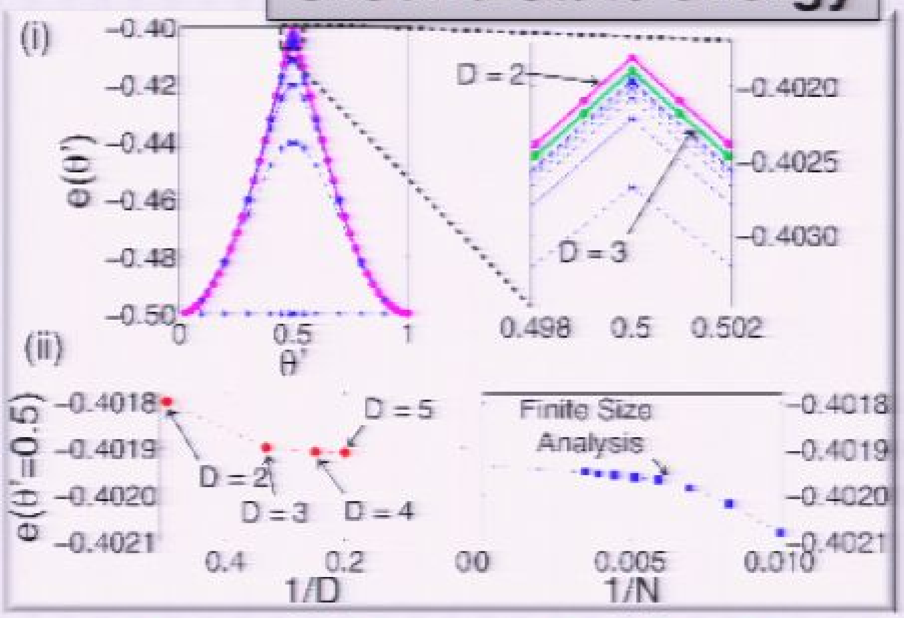
$$H = -\sin(\theta)\sum_{\vec{r}} \sigma_z^{\vec{r}} \sigma_z^{\vec{r}+\hat{i}} - \cos(\theta)\sum_{\vec{r}} \sigma_x^{\vec{r}} \sigma_x^{\vec{r}+\hat{j}} \quad \hat{j}\,\hat{i}$$



Order parmeters



Ground state energy

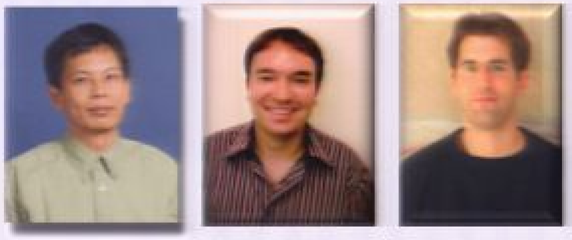


Improves upon previous results, and conclusively determines a 1st order phase transition

iPEPS and the Fidelity Approach to quantum phase transitions

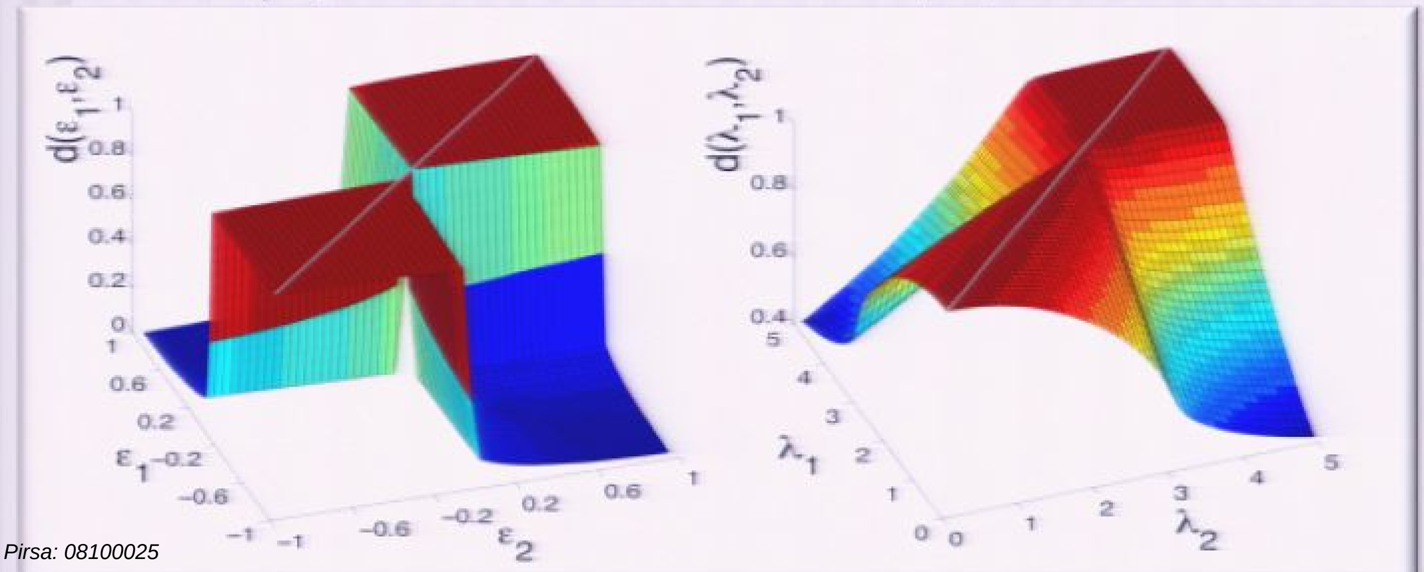
Fidelity between different ground states

H-Q. Zhou, R. Orús, G. Vidal, PRL 100, 080602 (2008)



$$H(g) \Longleftrightarrow d(g_1,g_2) = \lim_{N\to\infty} -\frac{1}{N}\log\left|\langle\Psi_0(g_1)|\Psi_0(g_2)\rangle\right|$$

$$H = -\sum_{\langle\vec{r},\vec{r}'\rangle}\sigma_z^{\vec{r}}\sigma_z^{\vec{r}'} + \varepsilon\sum_{\vec{r}}\sigma_z^{\vec{r}} \qquad H = -\sum_{\langle\vec{r},\vec{r}'\rangle}\sigma_z^{\vec{r}}\sigma_z^{\vec{r}'} + \lambda\sum_{\vec{r}}\sigma_x^{\vec{r}}$$



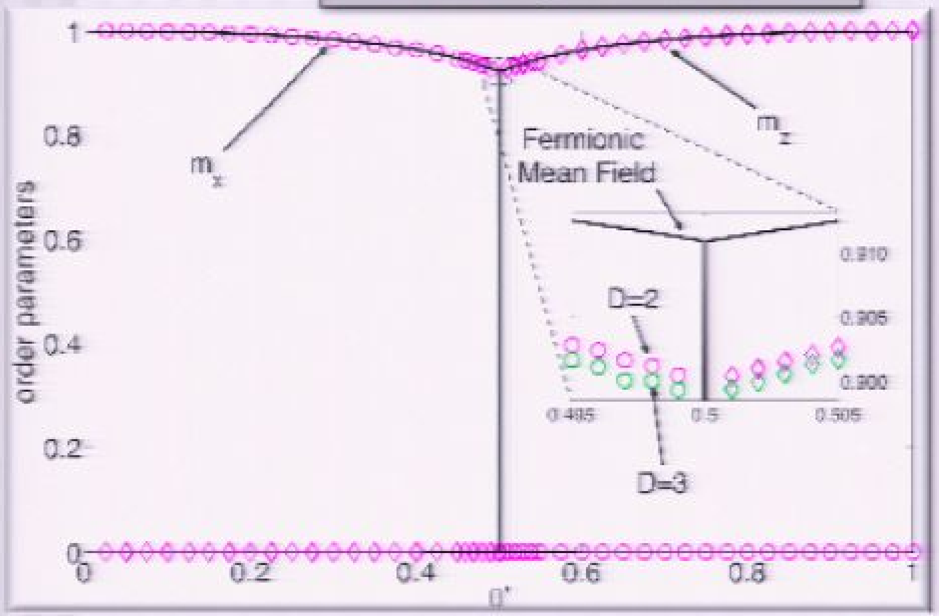
1st order QPT   2nd order QPT

Anisotropic Quantum Orbital Compass Model with iPEPS

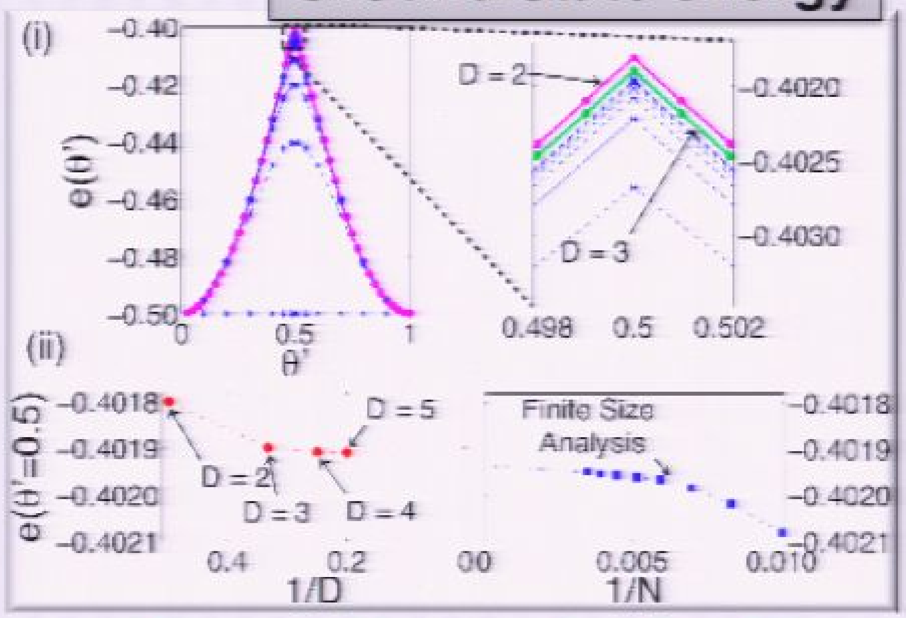
A. Doherty, R. Orús, G. Vidal, arXiv:0809.4068

$$H = -\sin(\theta)\sum_{\vec{r}} \sigma_z^{\vec{r}}\sigma_z^{\vec{r}+\hat{i}} - \cos(\theta)\sum_{\vec{r}} \sigma_x^{\vec{r}}\sigma_x^{\vec{r}+\hat{j}} \quad \hat{j}\,\hat{i}$$

Order parmeters

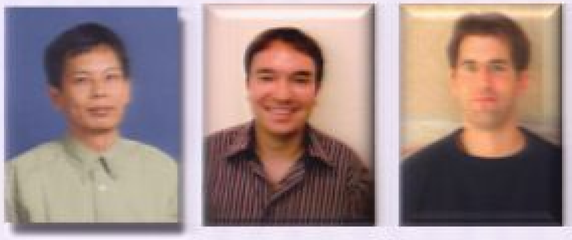


Ground state energy



Improves upon previous results, and conclusively determines a 1st order phase transition

iPEPS and the Fidelity Approach to quantum phase transitions

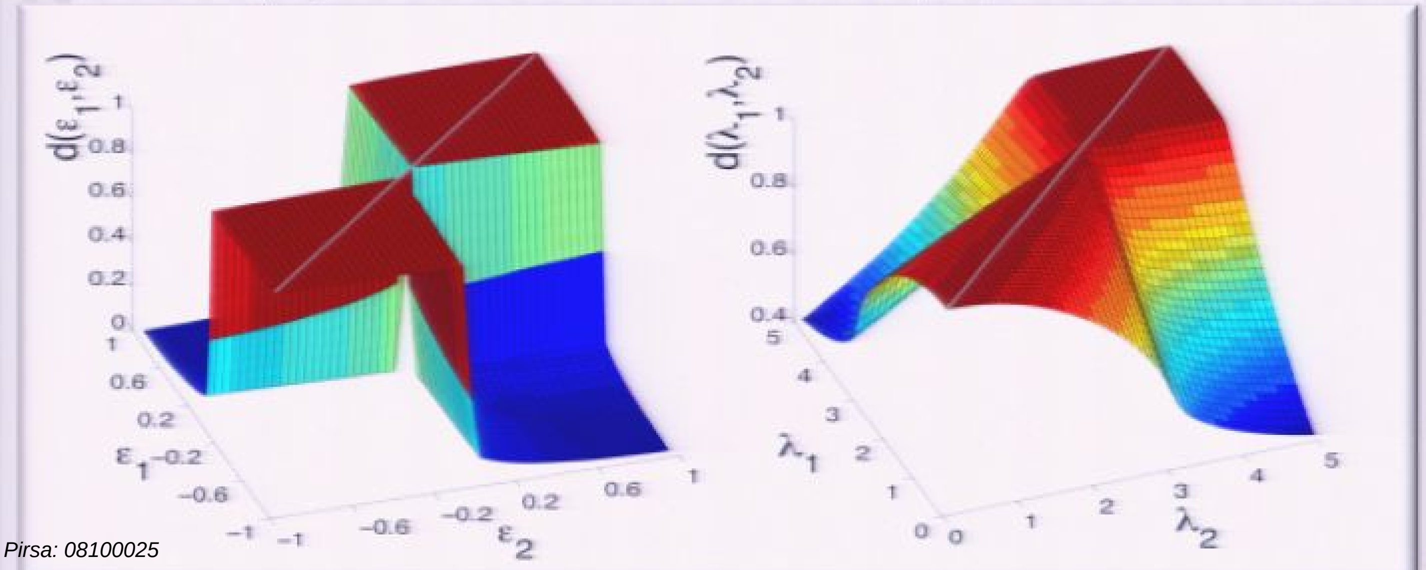


Fidelity between different ground states

H-Q. Zhou, R. Orús, G. Vidal, PRL 100, 080602 (2008)

$$H(g) \Longleftrightarrow d(g_1,g_2) = \lim_{N\to\infty} -\frac{1}{N}\log\left|\langle\Psi_0(g_1)|\Psi_0(g_2)\rangle\right|$$

$$H = -\sum_{\langle \vec r,\vec r'\rangle}\sigma_z^{\vec r}\sigma_z^{\vec r'} + \varepsilon\sum_{\vec r}\sigma_z^{\vec r} \qquad H = -\sum_{\langle \vec r,\vec r'\rangle}\sigma_z^{\vec r}\sigma_z^{\vec r'} + \lambda\sum_{\vec r}\sigma_x^{\vec r}$$



1st order QPT　　　2nd order QPT

iPEPS and the Fidelity Approach to quantum phase transitions

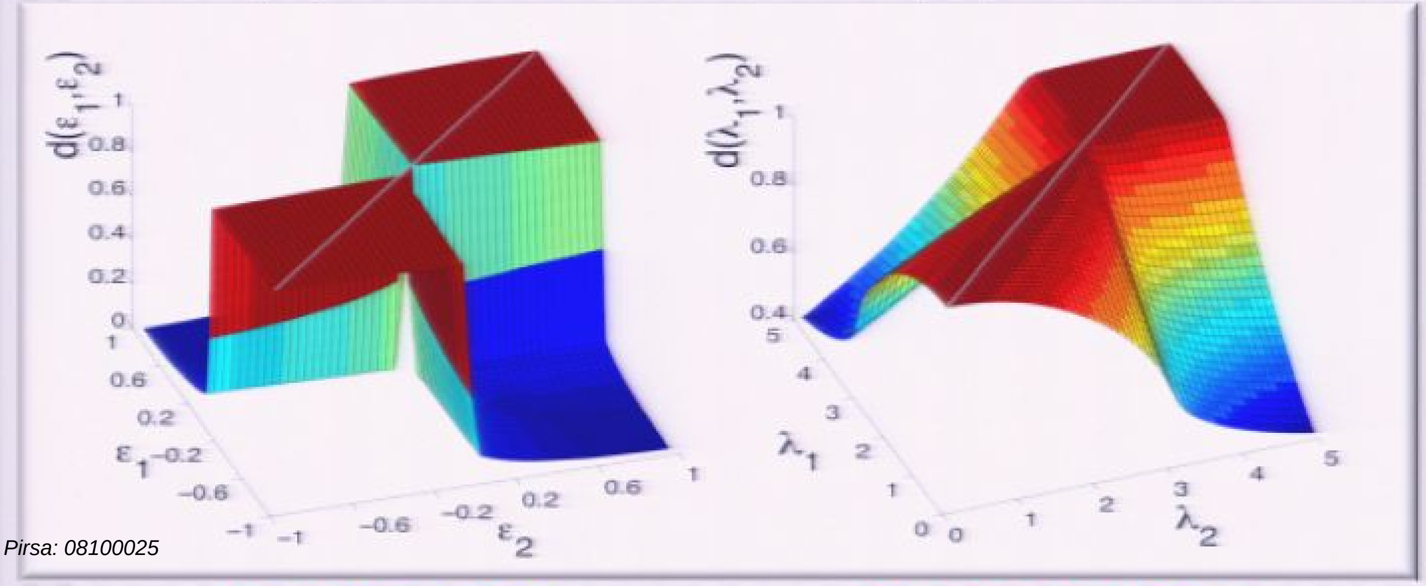
Fidelity between different ground states

H-Q. Zhou, R. Orús, G. Vidal, PRL 100, 080602 (2008)



$$H(g) \Longleftrightarrow d(g_1,g_2) = \lim_{N\to\infty} -\frac{1}{N}\log\left|\langle\Psi_0(g_1)|\Psi_0(g_2)\rangle\right|$$

$$H = -\sum_{\langle\vec{r},\vec{r}'\rangle}\sigma_z^{\vec{r}}\sigma_z^{\vec{r}'} + \varepsilon\sum_{\vec{r}}\sigma_z^{\vec{r}} \qquad H = -\sum_{\langle\vec{r},\vec{r}'\rangle}\sigma_z^{\vec{r}}\sigma_z^{\vec{r}'} + \lambda\sum_{\vec{r}}\sigma_x^{\vec{r}}$$



Easy to compute with TN, and shows the phase diagram without relying on any local order parameter

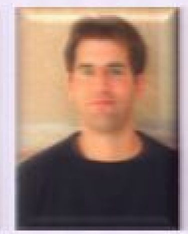
1st order QPT

2nd order QPT

2-dim quantum lattice systems with symmetries

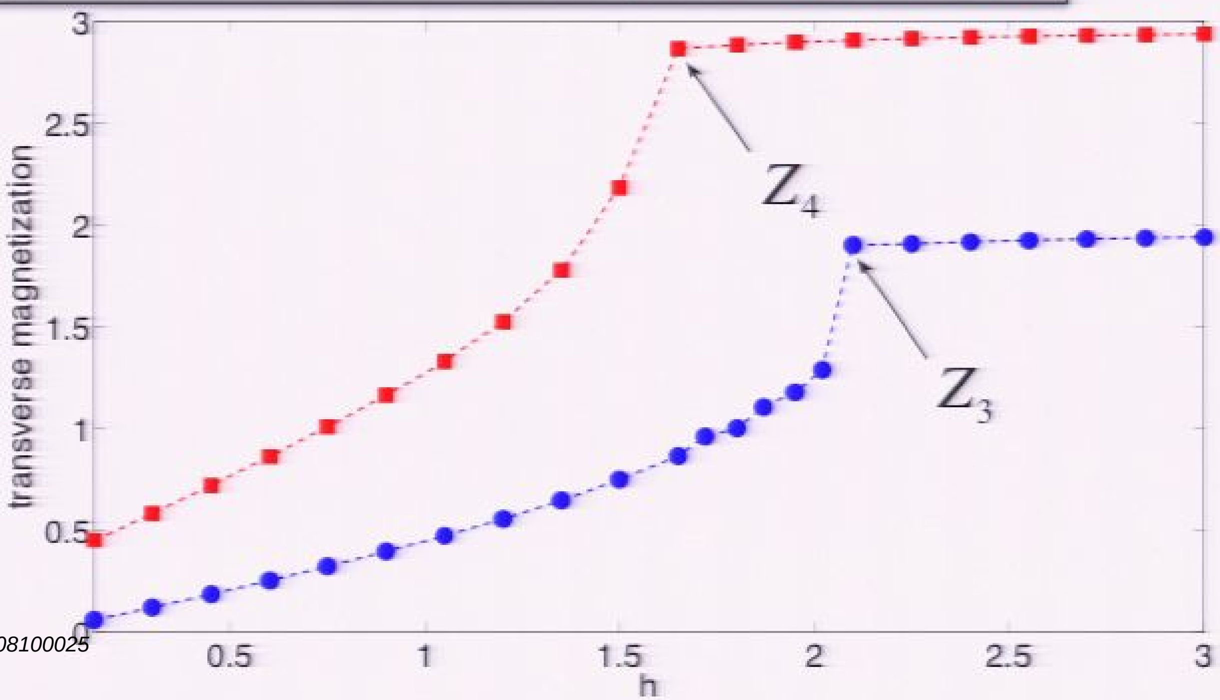
R. Orús, G. Vidal, …, work in progress

Implementation of the symmetries directly at the level of the PEPS/iPEPS



…

Zq Quantum Potts Model, 10x10 lattice



Geometric Entanglement in 1-dim Valence Bond Solids

R. Orús, arXiv:0808.0938

Spin-1 AKLT chain

$$H = \sum_{r=1}^{N-1}\left(\vec{S}^{[r]}\vec{S}^{[r+1]} + \frac{1}{3}(\vec{S}^{[r]}\vec{S}^{[r+1]})^2\right) + \pi^{[0,1]} + \pi^{[N,N+1]}$$

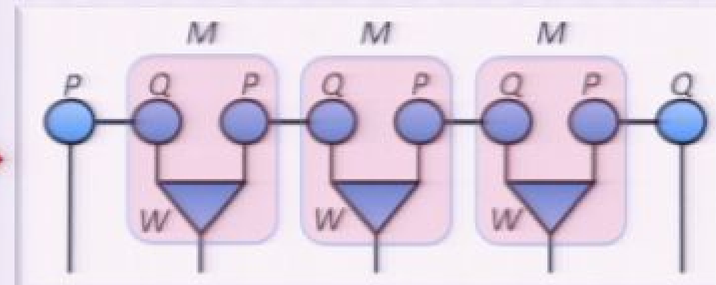
Gapped 1-dim Heisenberg-like system (Haldane conjecture)

Ground state is a Valence Bond Solid

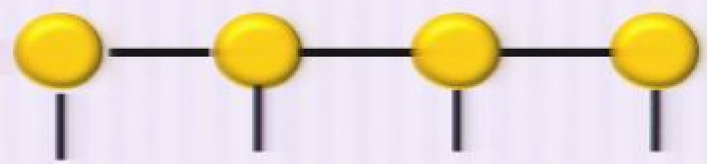
$$|\Psi\rangle = \left(\otimes_{r=1}^{N} W^{[r,r']}\right)|\Psi^-\rangle^{[0',1]}|\Psi^-\rangle^{[1',2]}\ldots|\Psi^-\rangle^{[N',N+1]}$$

Isometries — Spin ½ - singlets

TN



MPS



$$\varepsilon(L) = \log 2 - \log\left(1 + \left(-\frac{1}{3}\right)^L\right)$$

Exponentially fast saturation in L

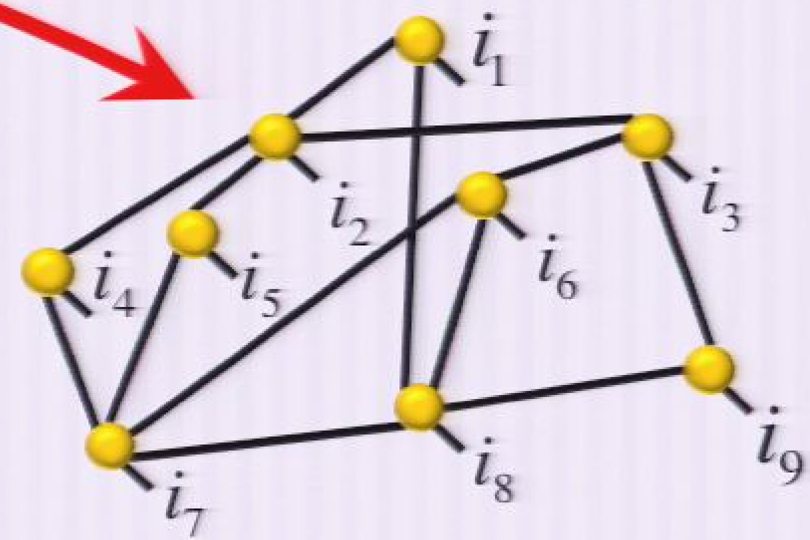
Closing the loop…

Quantum Information + Condensed Matter Physics

Entanglement in
many-body systems

Ruth Bloch: **Entanglement**
(bronze, 71 cm, 1995)

Tensor Networks



Outline

1.- Brief historical review

2.- What are Tensor Networks and why are they useful?

3.- Simulation results for 2-dim quantum systems

4.- Applications in the study of many-body entanglement

Outline

- 1.- Brief historical review
- 2.- What are Tensor Networks and why are they useful?
- 3.- Simulation results for 2-dim quantum systems
- **4.- Applications in the study of many-body entanglement**

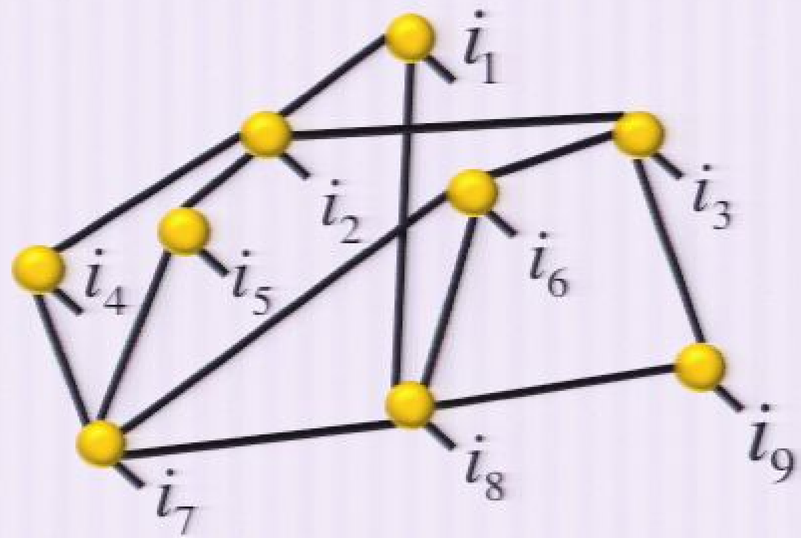
Closing the loop…

Quantum Information + Condensed Matter Physics

Entanglement in many-body systems

Ruth Bloch: **Entanglement**
(bronze, 71 cm, 1995)

Tensor Networks



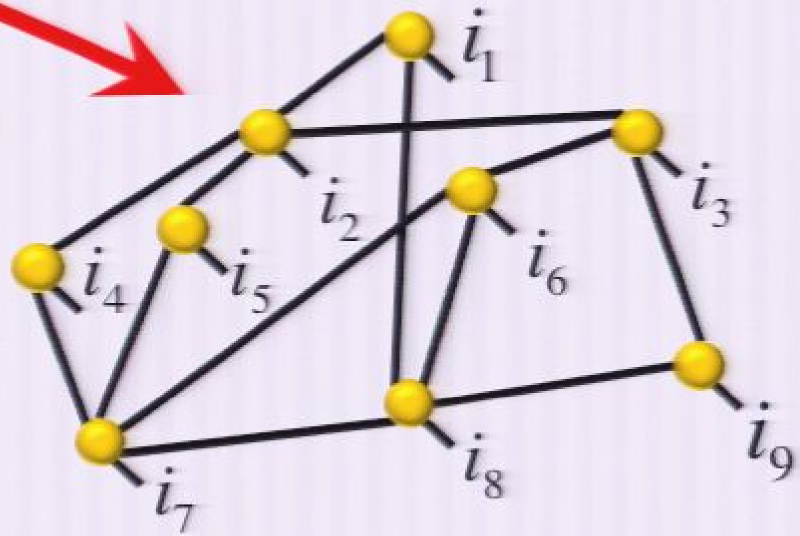
Closing the loop…

Quantum Information + Condensed Matter Physics

Entanglement in many-body systems

Ruth Bloch: ***Entanglement***
(bronze, 71 cm, 1995)

Tensor Networks



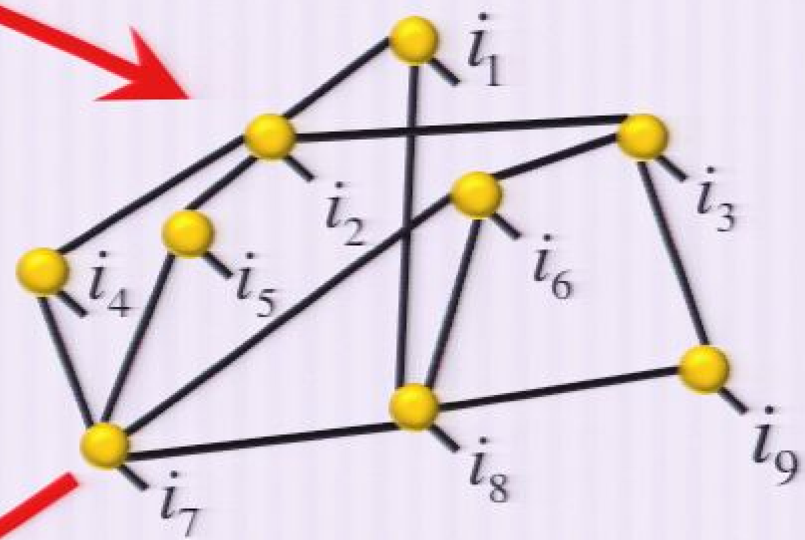
Closing the loop…

Quantum Information + Condensed Matter Physics

Entanglement in many-body systems

Ruth Bloch: ***Entanglement***
(bronze, 71 cm, 1995)

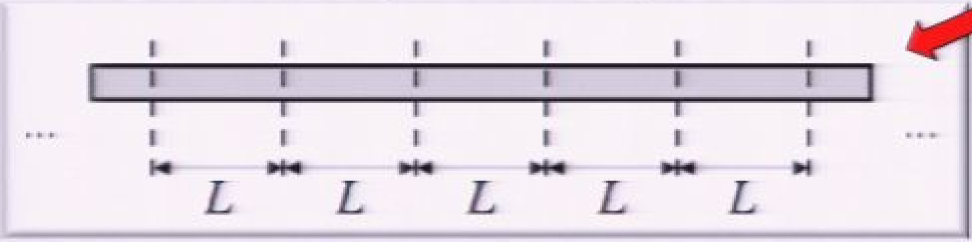
Tensor Networks



Geometric Entanglement and Conformal Field Theory

R. Orús, PRL 100, 130502 (2008)

1-dim quantum system



Divide into N blocks of length L

Global Geometric Entanglement per block:

$$\varepsilon = \lim_{N \to \infty} -\frac{1}{N} \log \left| \langle \Phi \max | \Psi \rangle \right|^2$$

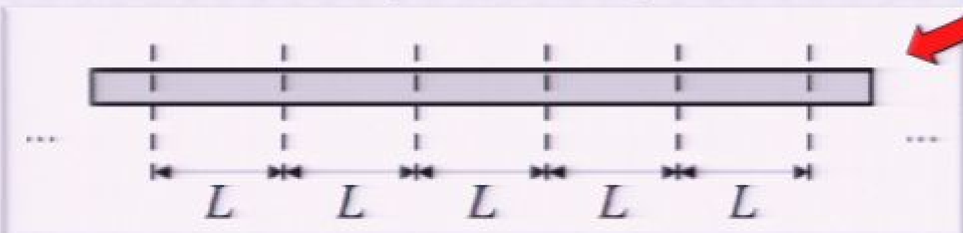
Separable state of the blocks

(Wei, Goldbart, Botero, Reznik)

Geometric Entanglement and Conformal Field Theory

R. Orús, PRL 100, 130502 (2008)

1-dim quantum system



Divide into N blocks of length L

Global Geometric Entanglement per block:

$$\varepsilon = \lim_{N\to\infty} -\frac{1}{N}\log\left|\langle \Phi \max | \Psi \rangle\right|^2$$

Separable state of the blocks

(Wei, Goldbart, Botero, Reznik)

central charge of the underlying 1+1-dim Conformal Field Theory

$$\varepsilon = \frac{c}{12}\log\left(\frac{\xi}{\eta}\right)$$ Close to criticality

$$\varepsilon < \frac{c}{6}\log\left(\frac{L}{\eta}\right)$$ At criticality

Trick: use MPS

Geometric Entanglement in 1-dim Valence Bond Solids

R. Orús, arXiv:0808.0938

Spin-1 AKLT chain

$$H = \sum_{r=1}^{N-1}\left(\vec{S}^{[r]}\vec{S}^{[r+1]} + \frac{1}{3}(\vec{S}^{[r]}\vec{S}^{[r+1]})^2\right) + \pi^{[0,1]} + \pi^{[N,N+1]}$$

Gapped 1-dim Heisenberg-like system (Haldane conjecture)

Ground state is a Valence Bond Solid

$$|\Psi\rangle = \left(\otimes_{r=1}^{N} W^{[r,r']}\right)|\Psi^-\rangle^{[0',1]}|\Psi^-\rangle^{[1',2]}\dots|\Psi^-\rangle^{[N',N+1]}$$

Isometries   Spin ½ - singlets

Geometric Entanglement in 1-dim Valence Bond Solids

R. Orús, arXiv:0808.0938

Spin-1 AKLT chain

$$H = \sum_{r=1}^{N-1}\left(\vec{S}^{[r]}\vec{S}^{[r+1]} + \frac{1}{3}(\vec{S}^{[r]}\vec{S}^{[r+1]})^2\right) + \pi^{[0,1]} + \pi^{[N,N+1]}$$

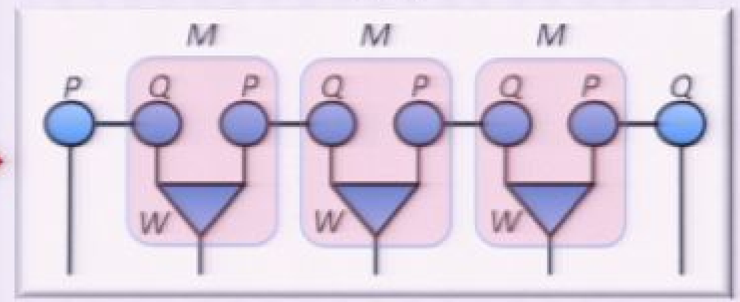
Gapped 1-dim Heisenberg-like system (Haldane conjecture)

Ground state is a Valence Bond Solid

$$|\Psi\rangle = \left(\otimes_{r=1}^{N} W^{[r,r']}\right)|\Psi^-\rangle^{[0',1]}|\Psi^-\rangle^{[1',2]}\ldots|\Psi^-\rangle^{[N',N+1]}$$

Isometries   Spin ½ - singlets

TN



Geometric Entanglement in 1-dim Valence Bond Solids

R. Orús, arXiv:0808.0938

Spin-1 AKLT chain

$$H = \sum_{r=1}^{N-1}\left(\vec{S}^{[r]}\vec{S}^{[r+1]} + \frac{1}{3}(\vec{S}^{[r]}\vec{S}^{[r+1]})^2\right) + \pi^{[0,1]} + \pi^{[N,N+1]}$$

Gapped 1-dim Heisenberg-like system (Haldane conjecture)

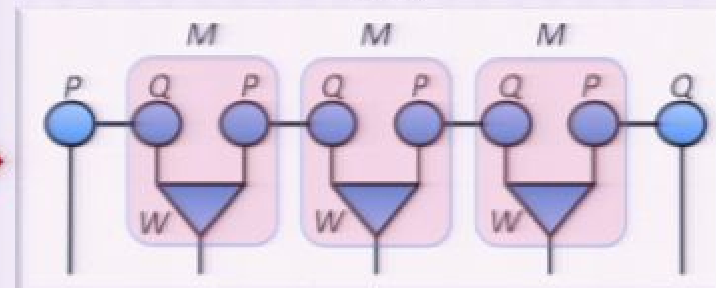
Ground state is a Valence Bond Solid

$$|\Psi\rangle = \left(\otimes_{r=1}^{N} W^{[r,r']}\right)|\Psi^-\rangle^{[0',1]}|\Psi^-\rangle^{[1',2]}\dots|\Psi^-\rangle^{[N',N+1]}$$

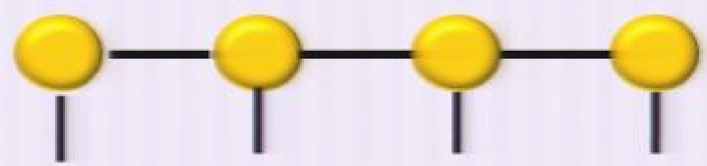
Isometries

Spin ½ - singlets

TN



MPS



Geometric Entanglement in 1-dim Valence Bond Solids

R. Orús, arXiv:0808.0938

Spin-1 AKLT chain

$$H = \sum_{r=1}^{N-1}\left(\vec{S}^{[r]}\vec{S}^{[r+1]} + \frac{1}{3}(\vec{S}^{[r]}\vec{S}^{[r+1]})^2\right) + \pi^{[0,1]} + \pi^{[N,N+1]}$$

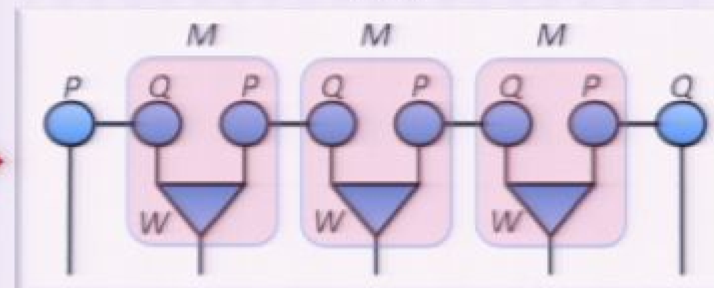
Gapped 1-dim Heisenberg-like system (Haldane conjecture)

Ground state is a Valence Bond Solid

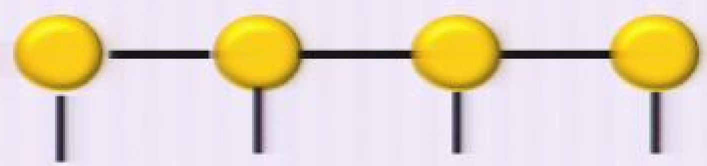
$$|\Psi\rangle = \left(\otimes_{r=1}^{N} W^{[r,r']}\right)|\Psi^-\rangle^{[0',1]}|\Psi^-\rangle^{[1',2]}\dots|\Psi^-\rangle^{[N',N+1]}$$

Isometries   Spin ½ - singlets

TN



MPS



$$\varepsilon(L) = \log 2 - \log\left(1 + \left(-\frac{1}{3}\right)^{L}\right)$$

Exponentially fast saturation in L

More work in progress…

SU(N) AKLT chain

Finite-size effects

Topological order

Mean-field theory

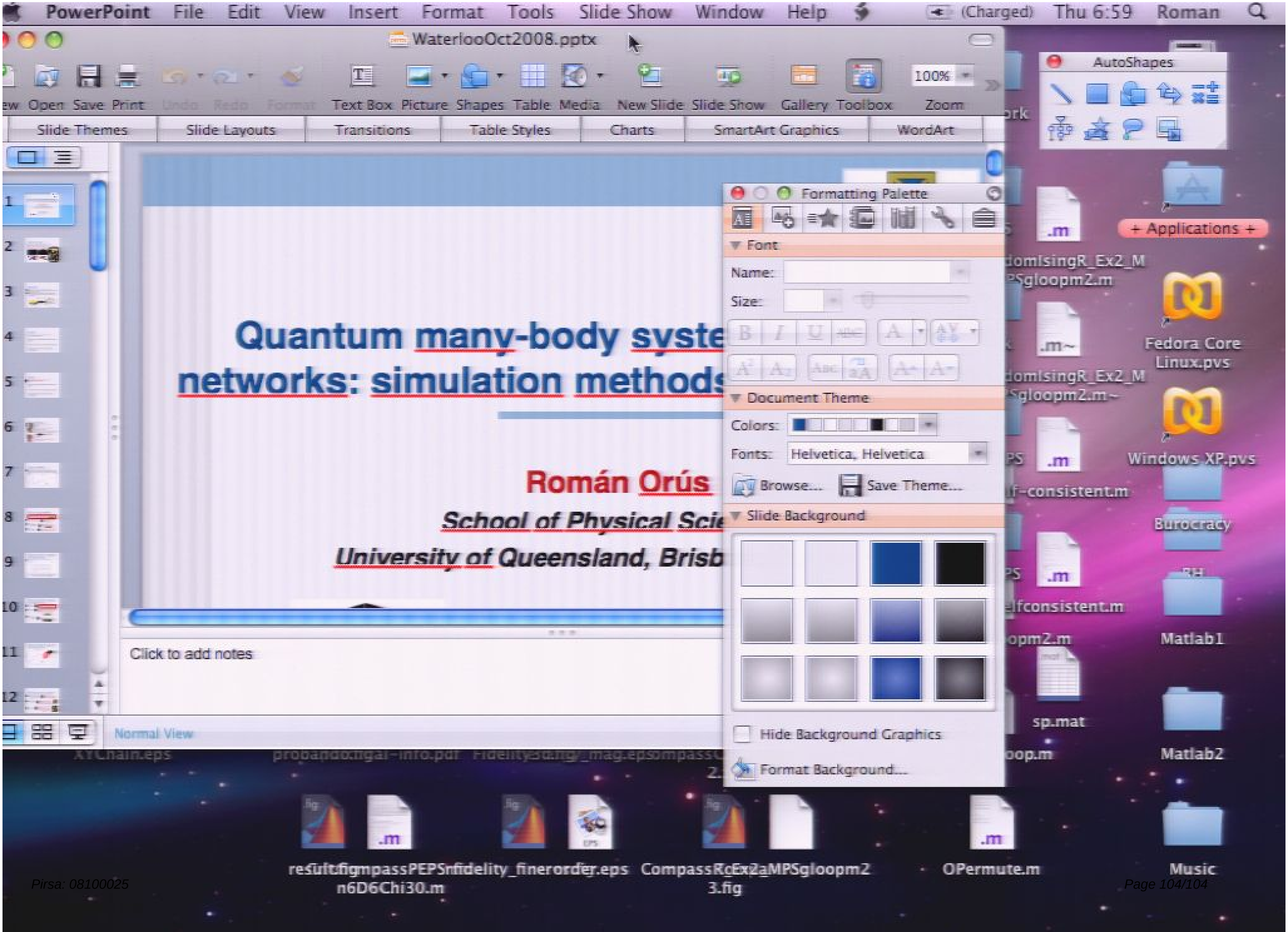
Symmetries

2-dim Bose-Hubbard model

2-dim fermionic lattice systems

…etc

Thank you!


PowerPoint File Edit View Insert Format Tools Slide Show Window Help
(Charged) Thu 6:59 Roman
WaterlooOct2008.pptx
Open Save Print Undo Redo Format Text Box Picture Shapes Table Media New Slide Slide Show Gallery Toolbox Zoom
100%
Slide Themes Slide Layouts Transitions Table Styles Charts SmartArt Graphics WordArt
Quantum many-body syste
networks: simulation methods
Román Orús
School of Physical Scie
University of Queensland, Brisb
Click to add notes
Normal View
Formatting Palette
Font
Name:
Size:
Document Theme
Colors:
Fonts: Helvetica, Helvetica
Browse... Save Theme...
Slide Background
Hide Background Graphics
Format Background...
AutoShapes
+ Applications +
Fedora Core Linux.pvs
Windows XP.pvs
Burocracy
Matlab1
Matlab2
Music
sp.mat
OPermute.m