Title: Topologically Massive AdS Gravity

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Abstract: In an asymptotically anti-de Sitter space, three-dimensional topologically massive gravity has some remarkable properties, which suggest interesting applications to quantum gravity. Unfortunately, though, the theory appears to be unstable, even at the special \'chiral\' value of the coupling. I will discuss recent work, and recent controversies, in this field.

Topologically Massive AdS Gravity

Steve Carlip
U.C. Davis

(work with Stanley Deser, Andrew Waldron, Derek Wise)
(additional work with Russell Cosgrove in progress)

Perimeter Institute
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or. . .

A Eulogy for

Topologically Massive AdS Gravity

(probably)

Why Three-Dimensional Gravity?

- Classically exactly soluble
- Finite number of degrees of freedom
- Good model for conceptual issues (observables, "problem of time," etc.)
- Interesting black holes
- Interesting playground for AdS/CFT correspondence

Why *Not* Three-Dimensional Gravity?

- Classically exactly soluble
- Finite number of degrees of freedom
- Problems with sum over topologies and AdS/CFT correspondence

Cosmological constant $\Lambda = -1/\ell^2$
Sum over all asymptotically AdS manifolds with fixed boundary data
Witten, Maloney, Manschot: Result doesn't look like a CFT partition function
(unless you sum over two chiral sectors separately—but why?)

Topologically Massive AdS Gravity

$$I_{\text{E-H}} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R+\frac{2}{\ell^2}\right)$$

$$I_{CS} = \frac{1}{32\pi G}\int d^3x\epsilon^{\mu\nu\rho}\left(\Gamma^{\sigma}_{\mu\tau}\partial_\nu\Gamma^{\tau}_{\rho\sigma}+\frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\lambda}\Gamma^{\lambda}_{\rho\sigma}\right)$$

$$I_{TMG} = I_{\text{E-H}}+\frac{1}{\mu}I_{CS}$$

Field equations:

$$G_{\mu\nu}-\frac{1}{\ell^2}g_{\mu\nu}+\frac{1}{\mu}C_{\mu\nu}=0$$

where $C_{\mu\nu}=\epsilon_{(\mu}{}^{\rho\sigma}\nabla_{|\rho|}G_{\nu)\sigma}$ is the Cotton tensor.

– Third-order equations of motion
– Solutions include all “pure gravity” solutions, including BTZ black hole
– But also local propagating modes

Danger sign:

$G_{\textit{Newton}} > 0 \;\Rightarrow$ propagating modes have negative energy

– With no cosmological constant, just take $G < 0$:
  no propagating modes in Einstein gravity, so this is OK (probably. . .)
– With $\Lambda < 0$, taking $G < 0 \;\Rightarrow$ BTZ black holes have negative mass

Li, Song, and Strominger: problem may go away at "chiral" coupling $\mu\ell = \pm 1$
At this coupling, asymptotic Virasoro algebra is chiral, cancellations appear,. . .
Does this work?

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Why chiral coupling might be nice

1. Asymptotic symmetries:

   Asymptotic symmetries in AdS_3 consist of two Virasoro algebras

   Kraus and Larsen, Solodhukin: central charges are

   $$c_{\pm} = \frac{3\ell}{2G}\left(1 \pm \frac{1}{\mu\ell}\right)$$

$\Rightarrow$ at $\mu\ell = \pm 1$, a larger group of diffeos extends to infinity

2. Perturbation theory:

   Let $\mathcal{D}^{\pm}{}_{\alpha}{}^{\beta} = \delta_{\alpha}^{\beta} \pm \ell\epsilon_{\alpha}{}^{\beta\gamma}\bar{\nabla}_{\gamma}, \quad \mathcal{D}^{M}{}_{\alpha}{}^{\beta} = \delta_{\alpha}^{\beta} + \frac{1}{\mu}\epsilon_{\alpha}{}^{\beta\gamma}\bar{\nabla}_{\gamma}$

   Then the weak field equations (in harmonic gauge) are

   $$(\mathcal{D}^{+}\mathcal{D}^{-}\mathcal{D}^{M}h)_{\mu\nu} = 0$$

$\Rightarrow$at $\mu\ell = \pm 1$, $\mathcal{D}^{M} = \mathcal{D}^{\pm}$; "massive graviton" disappears

Grumiller and Johannson: new mode with $(\mathcal{D}^{+})^{2}h = 0$ but $\mathcal{D}^{+}h \neq 0$

But asymptotically $\sim \ln z$: AdS but not Fefferman-Graham

Giribet, Kleban, Porrati: descendants of new mode can be Fefferman-Graham

Constraint Analysis

Constraints are complicated—third derivatives, second class constraints, etc.
—but can reveal physical degrees of freedom

First-order form: $e^a = e^a{}_\mu dx^\mu$, $\omega_a = \frac{1}{2}\epsilon_{abc}\omega^{bc}{}_\mu dx^\mu$

$$I_{\text{E-H}} = \frac{1}{8\pi G}\int\left[e^a \wedge \left(d\omega_a + \frac{1}{2}\epsilon_{abc}\omega^b \wedge \omega^c\right) + \frac{1}{6}\frac{1}{\ell^2}\epsilon_{abc}e^a \wedge e^b \wedge e^c\right]$$

$$I_{\text{CS}} = \frac{1}{16\pi G}\int\left[\omega^a \wedge \left(d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^b \wedge \omega^c\right)\right]$$

$$I_{\text{TMG}} = I_{\text{E-H}} + \frac{1}{\mu}I_{\text{CS}} + \int \beta^a \wedge \left[de_a + \epsilon_{abc}\omega^b \wedge e^c\right]$$

New connection $A^a = \omega^a + \mu e^a$

$$I_{TMG} = \frac{1}{\mu} I_{CS}[A] + \int \left[\beta^a \left(D_A e_a - \mu \epsilon_{abc} e^b \wedge e^c \right) - \alpha \epsilon_{abc} e^a e^b e^c \right]$$

with $\alpha = \frac{1}{3}\left(\mu^2 - \frac{1}{\ell^2}\right)$

Diagonalizes Poisson brackets:

$$\left\{ A^a{}_i, A^b{}_j \right\} = \frac{\mu}{2} \eta^{ab} \epsilon_{ij}, \qquad \left\{ e^a{}_i, \beta^b{}_j \right\} = \eta^{ab} \epsilon_{ij}$$

Primary constraints:

$$J_a = -\frac{2}{\mu} \epsilon^{ij} \left(F_{aij} + \frac{\mu}{2} \epsilon_{abc} \beta^b{}_i e^c{}_j \right)$$

$$B_a = -\epsilon^{ij} \left(D_i \beta_{aj} - 2\mu \epsilon_{abc} \beta^b{}_i e^c{}_j - 3\alpha \epsilon_{abc} e^b{}_i e^c{}_j \right)$$

$$T_a = -\epsilon^{ij} \left(D_i e_{aj} - \mu \epsilon_{abc} e^b{}_i e^c{}_j \right)$$

Secondary constraint:

$$\Delta = \epsilon^{ij} \beta^c{}_i e_{cj}$$

Degrees of freedom

Six first class constraints: J^a, $\hat{B}^a$
Four second class constraints: T^a, Δ
$\Rightarrow 18 - 2 \times 6 - 4 = 2$ phase space degrees of freedom
(*independent of couplings*)

J^a, $\hat{B}^a$ generate local Lorentz transformations, diffeomorphisms

Asymptotic symmetries

Near AdS boundary

$$ds^2 \sim \ell^2 d\rho^2 + e^{2\rho}(\ell^2 d\varphi^2 - dt^2)$$

Asymptotic diffeos parametrized by functions $f(\varphi + t/\ell)$, $\bar{f}(\varphi - t/\ell)$

Diagonalize commutators of constraints:

$$L_\pm[\xi] = \hat{B}[\xi] + \left(\mu \pm \frac{1}{\ell}\right) J[\xi]$$

Find pair of Virasoro algebras with central charges

$$c_\pm = \frac{3\ell}{2G}\left(1 \pm \frac{1}{\mu\ell}\right)$$

and classical conformal weights

$$Q_\pm[\xi] = \left(1 \pm \frac{1}{\mu\ell}\right) Q_\pm^{E\text{-}H}[\xi]$$

Chiral coupling $\mu\ell = \pm 1$ is *interesting*, but no jump in degrees of freedom

Perturbative Analysis

Background AdS, units $\ell = 1$; Poincaré coordinate patch

$$d\bar{s}^2 = \frac{1}{z^2}\left(2dx^+dx^- + dz^2\right)$$
$$ds^2 = d\bar{s}^2 + h_{\mu\nu}dx^\mu dx^\nu$$

Various gauge choices:

light front: $h_{\mu -} = 0$

Fefferman-Graham: $h_{\mu z} = 0$

Light front: one independent degree of freedom $\varphi = z^2 h_{++}$ with action

$$I = -\frac{1}{2}\int d^3x\sqrt{-\bar{g}}\left[\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi\right] \qquad \text{with } m^2 = (\mu+2)^2 - 1$$

Single scalar field!

Gauge-Invariant description:

Let

$$\mathcal{H}_{\mu\nu} = \left[G_{\mu\nu} - g_{\mu\nu}\right]_{linear}$$

Find

$$\left[2\partial_+\partial_- + \partial_z^2 - \frac{1}{z^2}\left({m_{\mu\nu}}^2 + \frac{3}{4}\right)\right]\left(\frac{1}{\sqrt{z}}\mathcal{H}_{\mu\nu}\right) = 0$$

"scalar" fields with a mass depending on chirality:

component	mass^2
$\mathcal{H}_{++}$	$(\mu-2)^2-1$
$\mathcal{H}_{+z}$	$(\mu-1)^2-1$
$\mathcal{H}_{+-}, \mathcal{H}_{zz}$	μ^2-1
$\mathcal{H}_{-z}$	$(\mu+1)^2-1$
$\mathcal{H}_{--}$	$(\mu+2)^2-1$

$\mu = \pm 1 \Rightarrow$ one of these masses is $m^2 = -1$:
Breitenlohner-Freedman bound, allowed for a scalar.

What about *curvature* asymptotics?

component	asymptotics near $z = 0$	$\mu = 1$ case
$\mathcal{H}_{++}$	$z^{\mu-1}$	z^2
$\mathcal{H}_{+z}$	z^{μ}	z
$\mathcal{H}_{+-}, \mathcal{H}_{zz}$	$z^{\mu+1}$	z^2
$\mathcal{H}_{-z}$	$z^{\mu+2}$	z^3
$\mathcal{H}_{--}$	$z^{\mu+3}$	z^4

But additional boundary at $z \to \infty$

Wave packets

Denote $h_{++} = h$.

Inner product

$$(h_1, h_2) = -i \int_\Sigma dx dz\, z^3 (h_1 \overleftrightarrow{\partial}_t h_2^*)$$

Orthonormal modes $h_{\omega k}$, $h^*_{\omega k}$:

$$h_{\omega k} = \sqrt{\frac{\omega}{4\pi E}} \frac{1}{z} J_{\mu+2}(\omega z) e^{ikx - iEt} \qquad \text{with } E = \sqrt{\omega^2 + k^2}$$

$$h = \int d\omega dk \left[a(\omega, k) h_{\omega k} + a^*(\omega, k) h^*_{\omega k} \right]$$

$$(h, h) = \int d\omega dk \left[a(\omega, k)^2 - a^*(\omega, k)^2 \right]$$

Energy:

$$H = -\frac{1}{16\pi G}\int d\omega dk\sqrt{k^2+\omega^2}|a(\omega,k)|^2$$

So...
Start with arbitrary local initial data for h, with finite energy and compact support.
Data propagates like scalar field, reaches AdS boundary in finite time.
$\Rightarrow$ instability.

Next possible loophole: light front gauge is nonlocal;
maybe other components of h are bad.

To see, look at Fefferman-Graham gauge.

Let

$$\left[2z^2\partial_+\partial_- + (z\partial_z + 2)^2\right] X = 0$$

$$h_{++} = -2\partial_+^2(z\partial_z + 2)X$$
$$h_{+-} = 2\partial_+\partial_-(z\partial_z + 2)X$$
$$h_{--} = -2\partial_-^2(z\partial_z - 2)X$$
$$h_{z\pm} = h_{zz} = 0$$

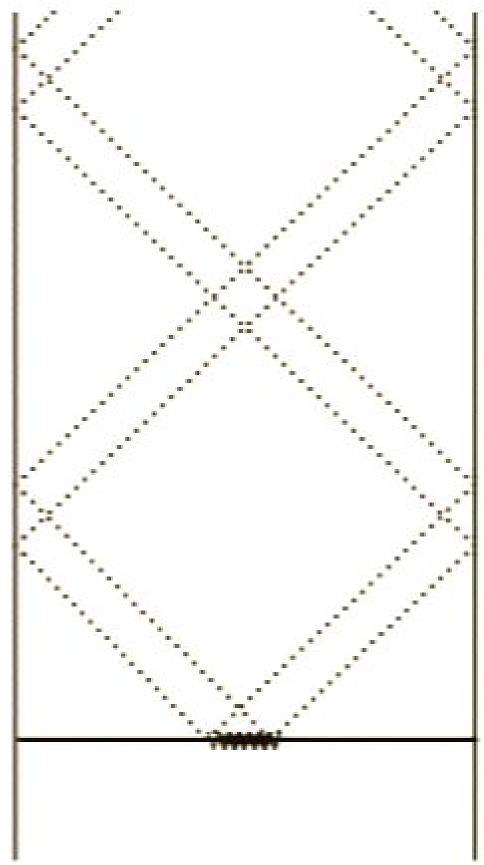
Then these $h_{\mu\nu}$ satisfy the linearized field equations for topologically massive AdS gravity with $\mu = 1$.

$$\left\{Y(x,z), \dot{Y}(x',z')\right\} = -4\pi\hbar G z \delta(x - x')\delta(z - z')$$
$$\text{with } Y = z\partial_z(z\partial_z + 2)^2 X$$

Note: $X = \frac{1}{z^3}\varphi$, where φ is an AdS scalar with $m^2 = -1$

Start with initial data for X with compact support

Finite (negative) energy – nothing to diverge



Greens function calculation (preliminary): at boundary, generically $h_{--} \sim \frac{1}{z^2}$

Open questions/loopholes

– How does chiral boundary show up in bulk?

At boundary, $T_{--} = 0$; bulk calculation is hard. . .

Deser and Tekin: background Killing vector $\xi^\mu \rightarrow \xi^\mu + \frac{1}{2\mu}\epsilon^{\mu\rho\sigma}\bar{\nabla}_\rho\xi_\sigma$

gives chiral structure for $\mu = \pm 1$

– "Outside in": Fefferman-Graham expansion near boundary

Solodhukin: lowest order equation for $\partial_z h_{--}$ disappears at chiral coupling

Higher order equations of motion?

– Last chance for theory:

Linearization instability? (May need to look at next order: hard!)

Lower bound on energy?

– What about $\Lambda = 0$? Topological instability?