Title: Hawking Boxes and Invariant Sets - A New Look at the Foundations of Quantum Theory and the Associated Role of Gravity

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Abstract: We start by studying the non-computational geometry of fractionally-dimensioned measure-zero dynamically-invariant subsets of phase space, associated with certain deterministic nonlinear dissipative dynamical systems. Then, by studying the asymptotic states of the Hawking Box, the existence of such invariant subsets is conjectured for gravitationally-bound systems. The argument hinges around the phase-space properties of black holes. Like Penrose, it is assumed that phase-space volumes shrink when the contents of the Hawking Box contain black holes. However, unlike Penrose, we do not argue for any corresponding phase-space divergence when the Box does not contain black holes. We now make the hypothesis that these invariant phase-space subsets play a primitive role in fundamental physics; specifically that the state of the universe (â€œrealityâ€•) lies on such an invariant subset (now and hence forever). Attention is focussed on the implications of this hypothesis for the foundations of quantum theory. For example, what are referred to as â€œmeasurementsâ€• of the quantum state, are defined in terms of symbolic dynamics on the invariant set, relative to some partition of the invariant set. This immediately leads to the notion that any theory which treats these invariant sets as primitive, must be contextual (since counterfactual perturbations almost certainly take states off the measure-zero invariant set and hence to â€œunrealâ€• regions of phase space where the symbolic partition is undefined). This in turn leads to a new perspective, both on the foundations of quantum theory and on the role of gravity in formulating these foundations. In particular, a measurement-free Neo-Copenhagen Interpretation of quantum theory, based on the Invariant Set Hypothesis will be presented.

Hawking Boxes and Invariant Sets:

A New Look at the Foundations of Quantum Physics

Tim Palmer

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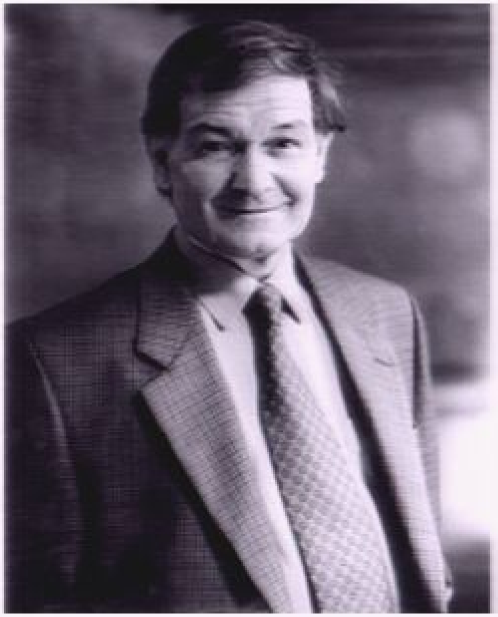

General Relativity has taught us that space-time geometry provides the basis for understanding the role of gravity in classical physics.

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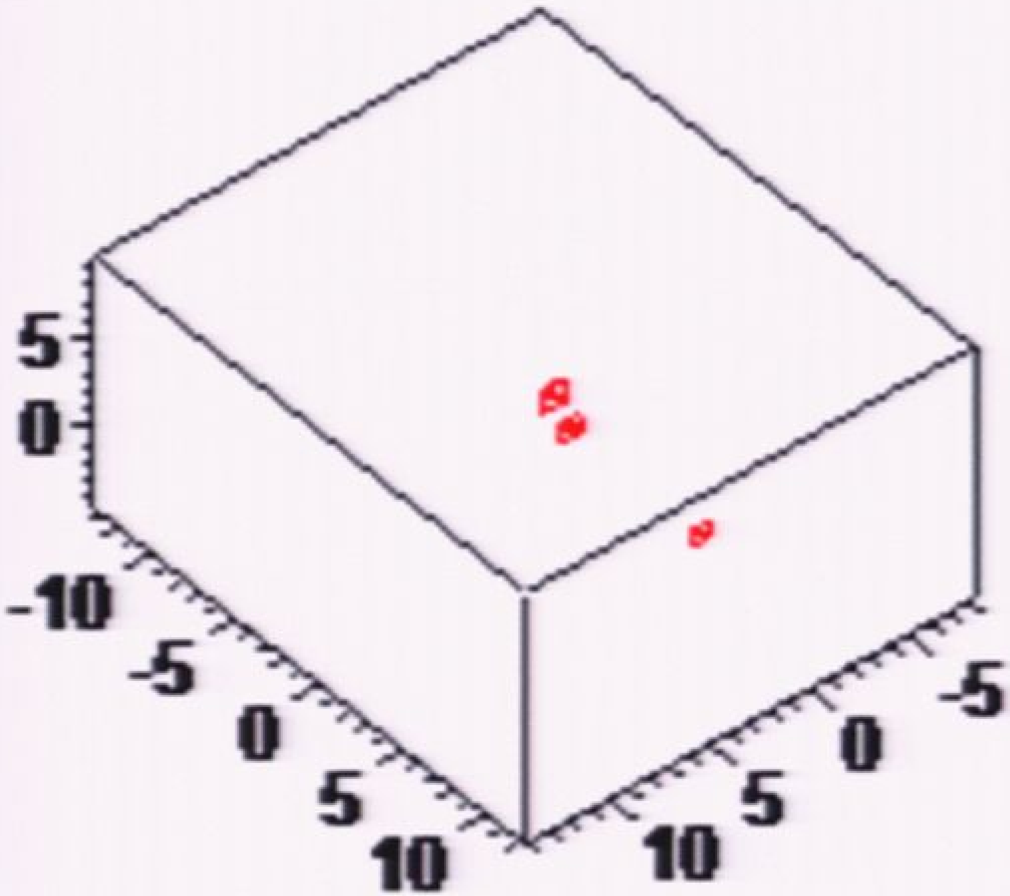
Here I want to propose that certain types of (non-computable) phase-space geometry may provide the basis for understanding the key role of gravity in quantum physics.

…leading in turn to a new perspective on the foundations of quantum theory which is atemporal on the one hand, but causal and realistic on the other.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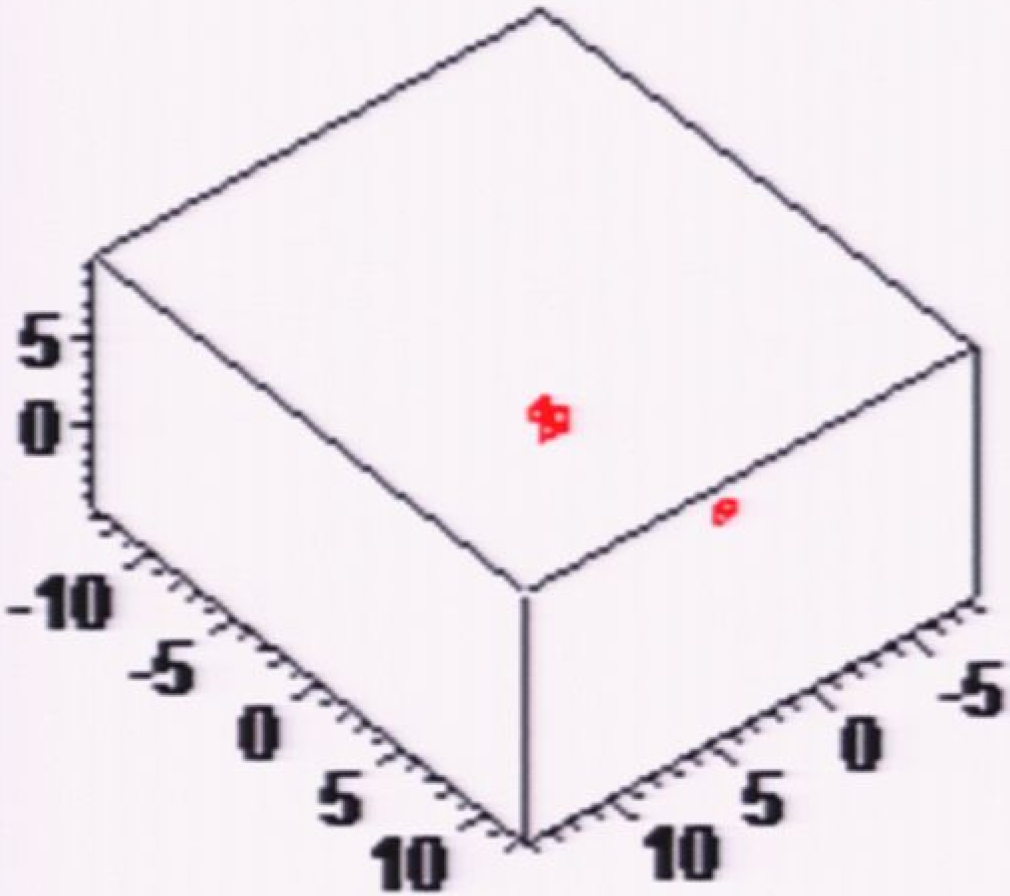
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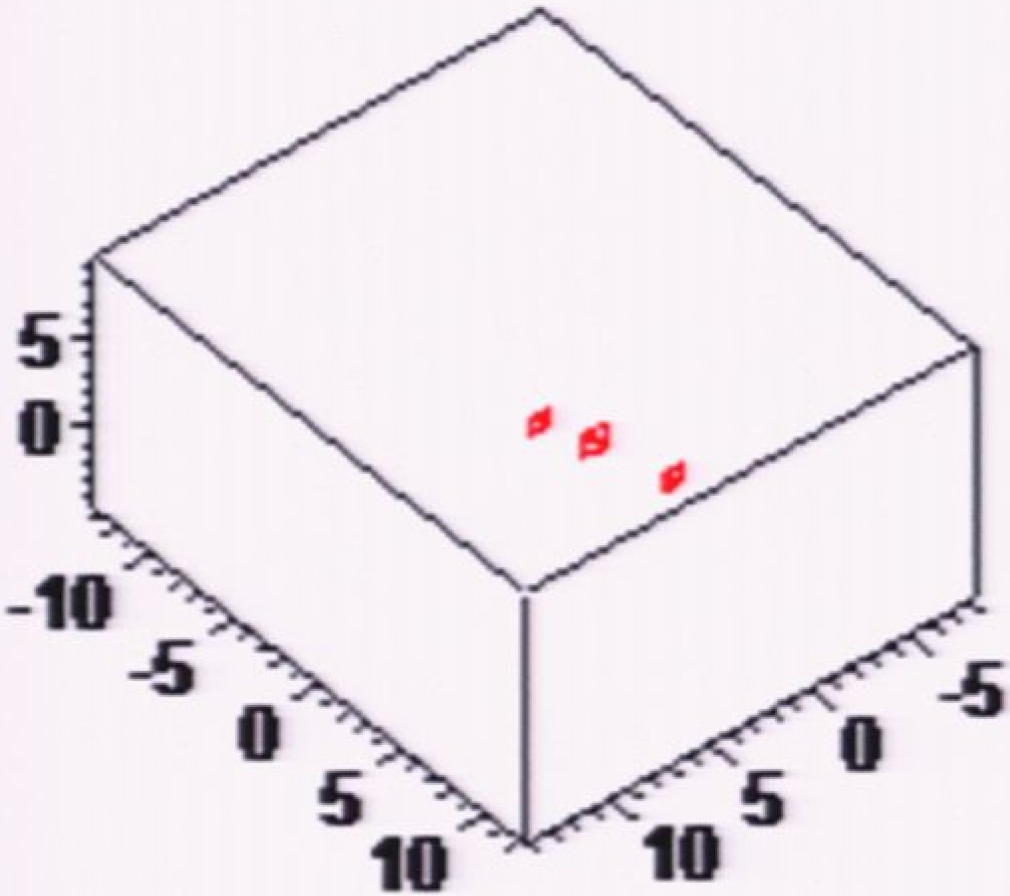
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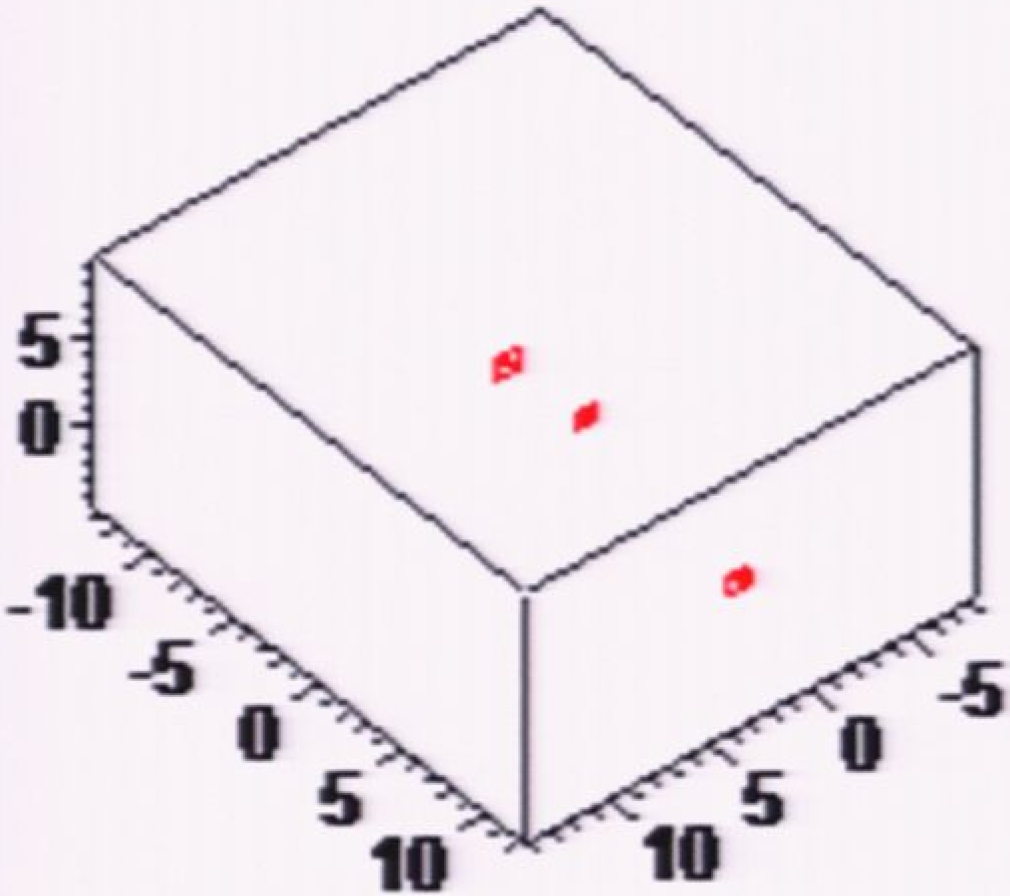
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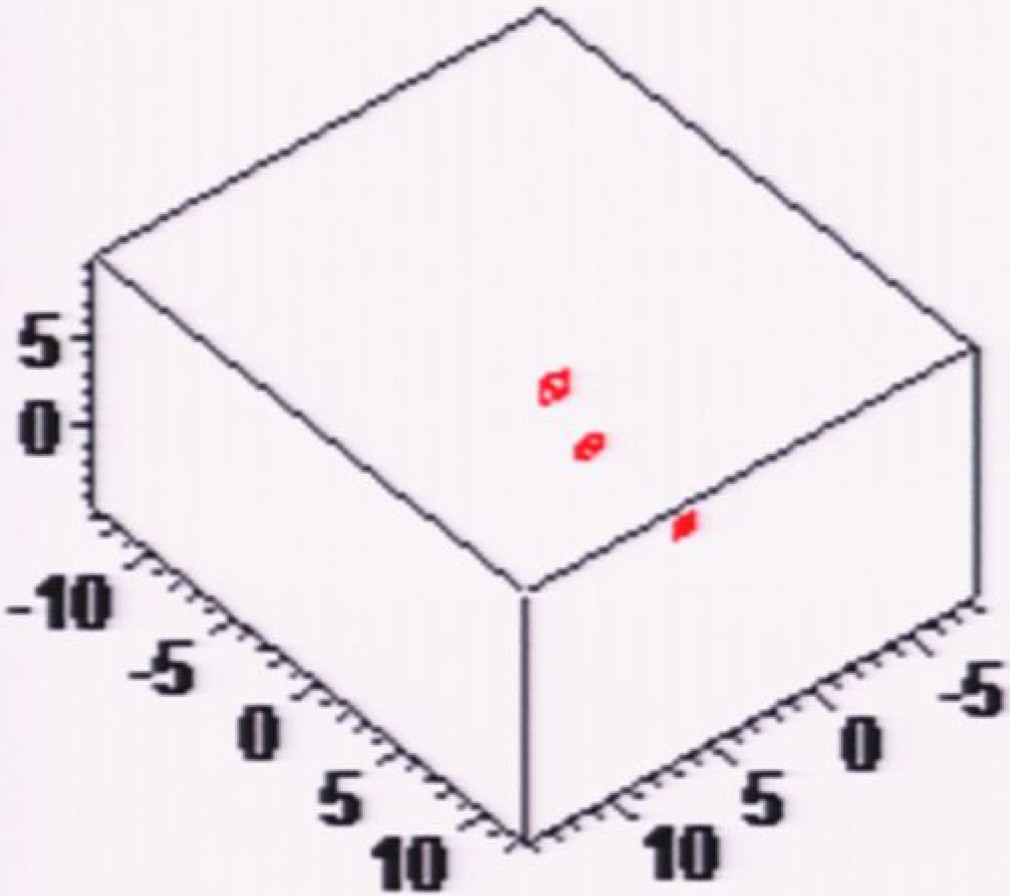

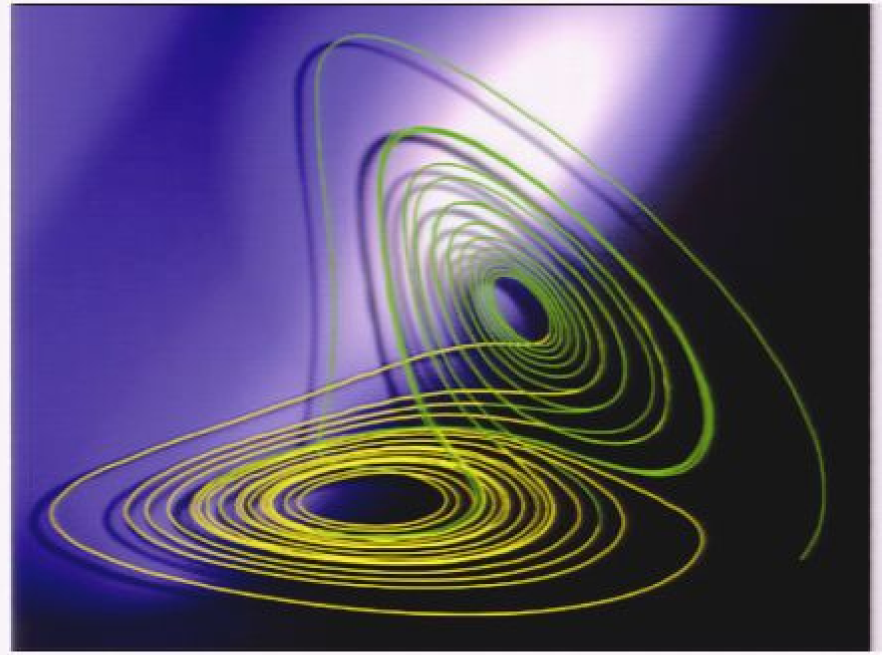

$$\dot{X} = -\sigma X + \sigma Y$$

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Fractal Geometry

Set of affine transformations with rational coefficients

Composition of affine transformations

Fractal Invariant Set

Non-trivial properties of the Invariant Set

Eg does the Invariant Set intersect a given line segment?

Theory of Computatio

Alphabet of symbols

Concatenation of symbols

Complement of the language accepted by a Turing Machine

Computationally undecidable propositions.

Post Correspondence Problem

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Post Correspondence Problem

Given a collection of dominos, eg

$$\left\{\begin{bmatrix} \dfrac{b}{ca} \end{bmatrix}, \begin{bmatrix} \dfrac{a}{ab} \end{bmatrix}, \begin{bmatrix} \dfrac{ca}{a} \end{bmatrix}, \begin{bmatrix} \dfrac{abc}{c} \end{bmatrix}\right\}$$

Can we make a list of dominos (repetitions allowed) so that the string on the top matches the string on the bottom? In this case yes, ie

$$\begin{bmatrix} \dfrac{a}{ab} \end{bmatrix}\begin{bmatrix} \dfrac{b}{ca} \end{bmatrix}\begin{bmatrix} \dfrac{ca}{a} \end{bmatrix}\begin{bmatrix} \dfrac{a}{ab} \end{bmatrix}\begin{bmatrix} \dfrac{abc}{c} \end{bmatrix}$$

The PCP is to determine whether a collection of dominos has a match. The problem is unsolvable by algorithms.

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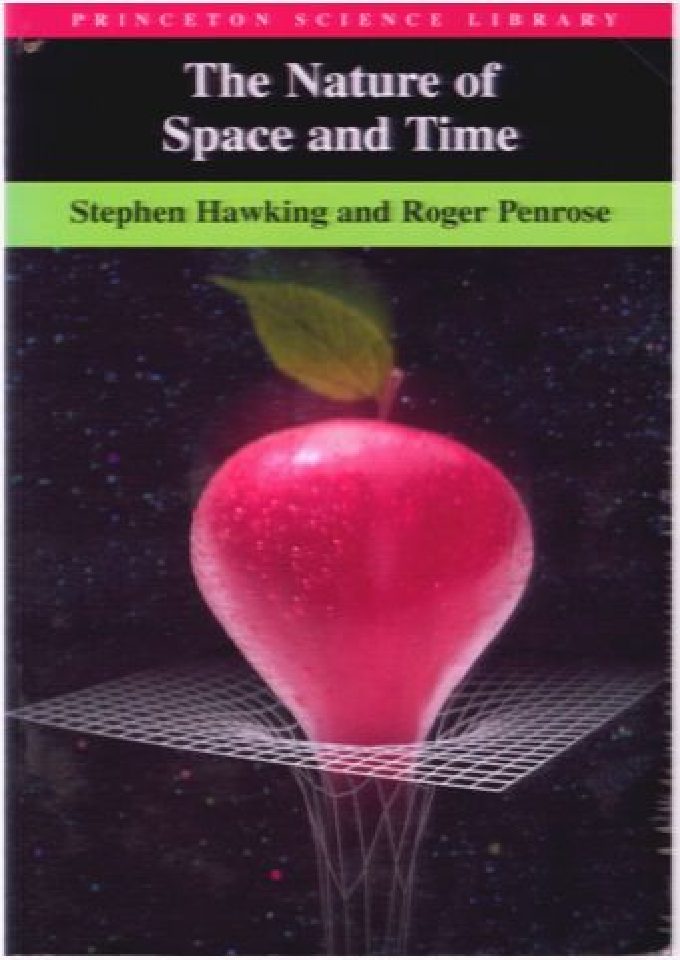
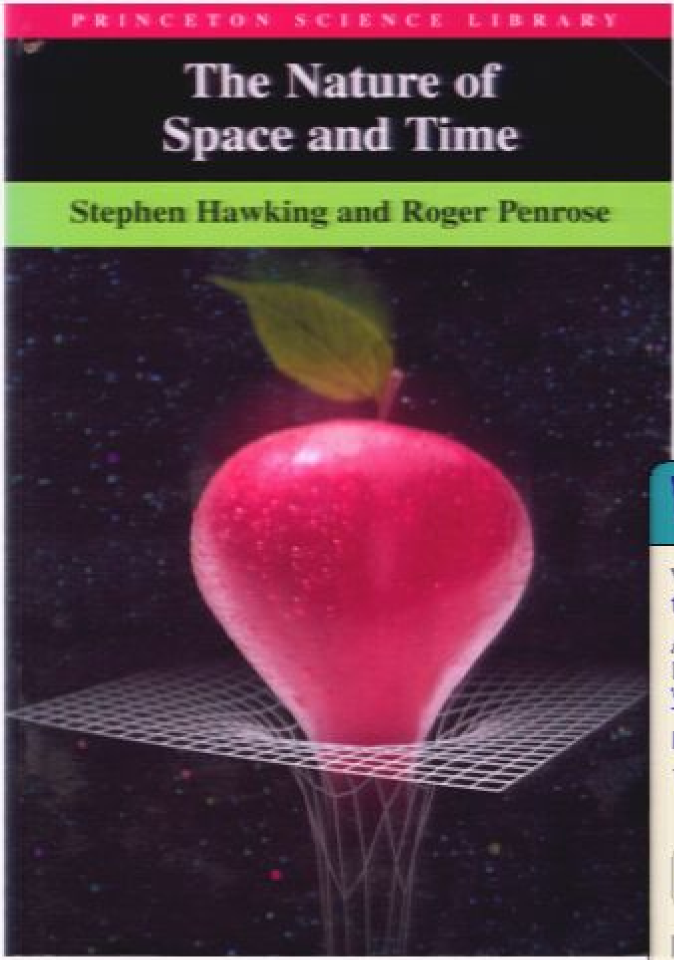
Hawking Box


PRINCETON SCIENCE LIBRARY
The Nature of
Space and Time
Stephen Hawking and Roger Penrose

Hawking Box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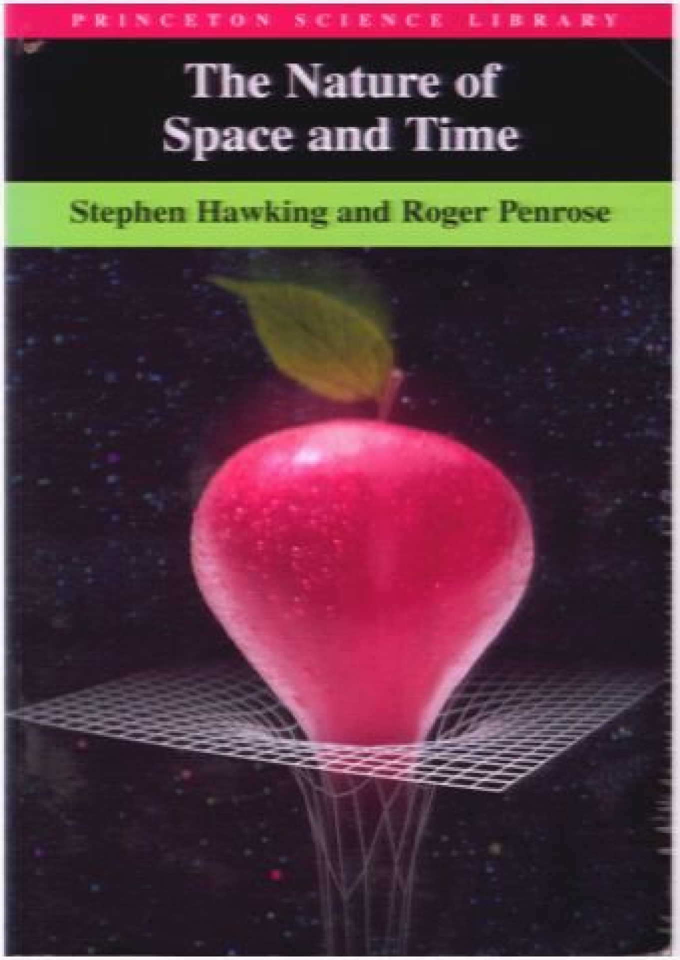
vulScan Application is trying to access the trusted zone.
Application: vulscan.exe
Destination IP: 127.0.0.1:Port 9592
View Properties
More Information Available:
This program has previously asked for Internet access.
SmartDefense Advisor
More Info...
Remember this setting.
Allow
Deny

Hawking Box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Hawking Box

- Penrose. Loss of phase space volume due to black hole formation/evaporation. Need "by Liouville's Theorem" for corresponding gain in phase space volume in other parts of phase space. Motivation for gravitationally-induced state-vector reduction.

- Hawking. To be consistent with quantum unitarity, no loss of phase space volume due to black hole formation/evaporation.

From Penrose, 2000

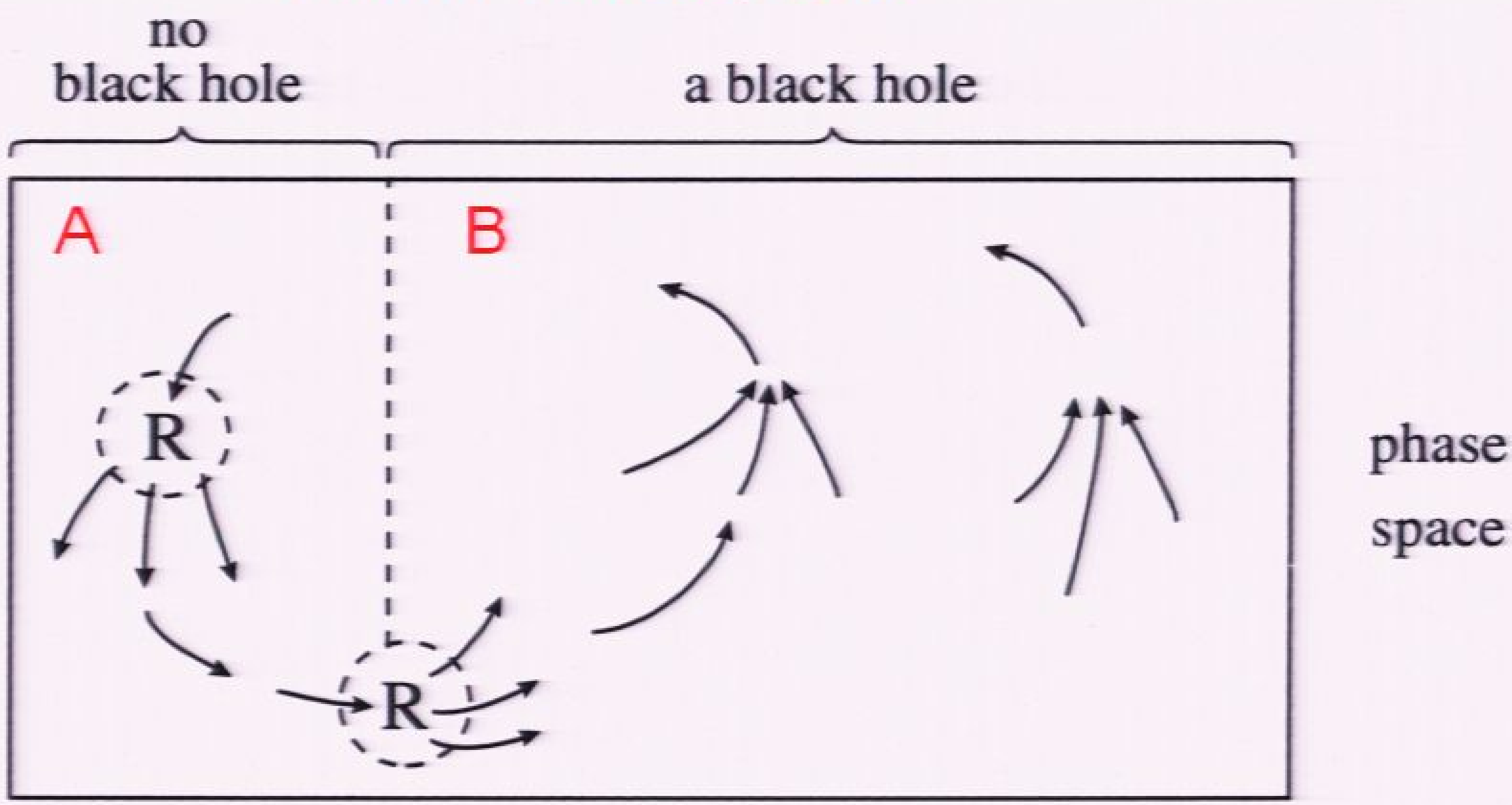

Figure 4.4 Loss of phase-space volume occurs when a black hole is present. This may be balanced against regain of phase-space volume due to wave function collapse **R**.

Liouville's Equation

$$\frac{\partial \rho}{\partial t}+\sum_{k=1}^{N}\frac{\partial}{\partial X_k}\left(\rho \dot{X}_k\right)=0$$

For irreversible system

$$\sum_{k=1}^{N}\frac{\partial \dot{X}_k}{\partial X_k}\neq 0.$$

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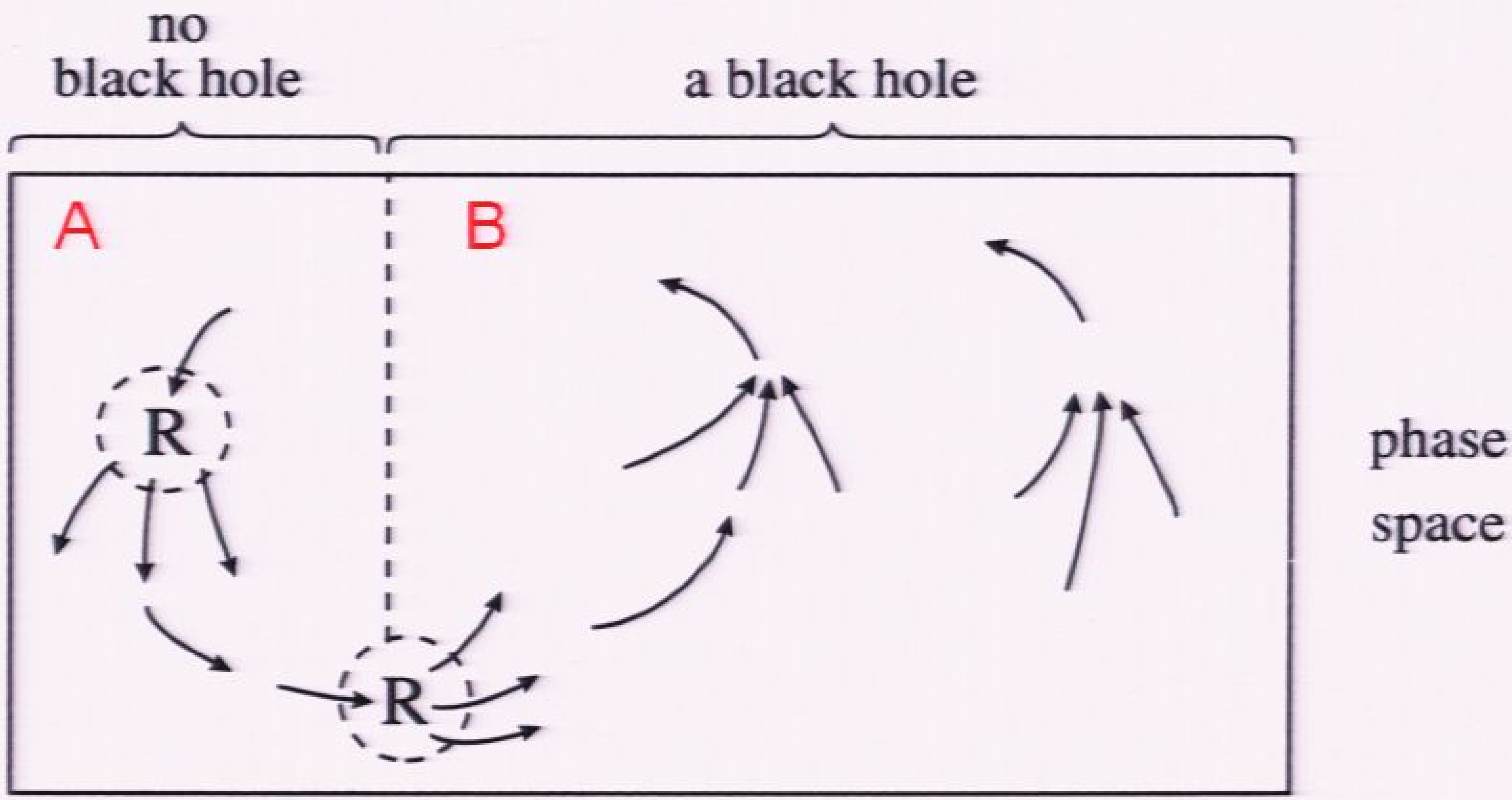

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Hawking Box

Palmer.

Loss of phase space volume due to black hole formation/evaporation.

But no corresponding gain in phase-space volume elsewhere.

Consider asymptotic invariant sets of states - fixed point/limit cycle/fractals are all possible.

On the invariant set, no loss of phase-space volume cf Hawking.

Fractal structure in A is inherited from irreversible black-hole processes in B.

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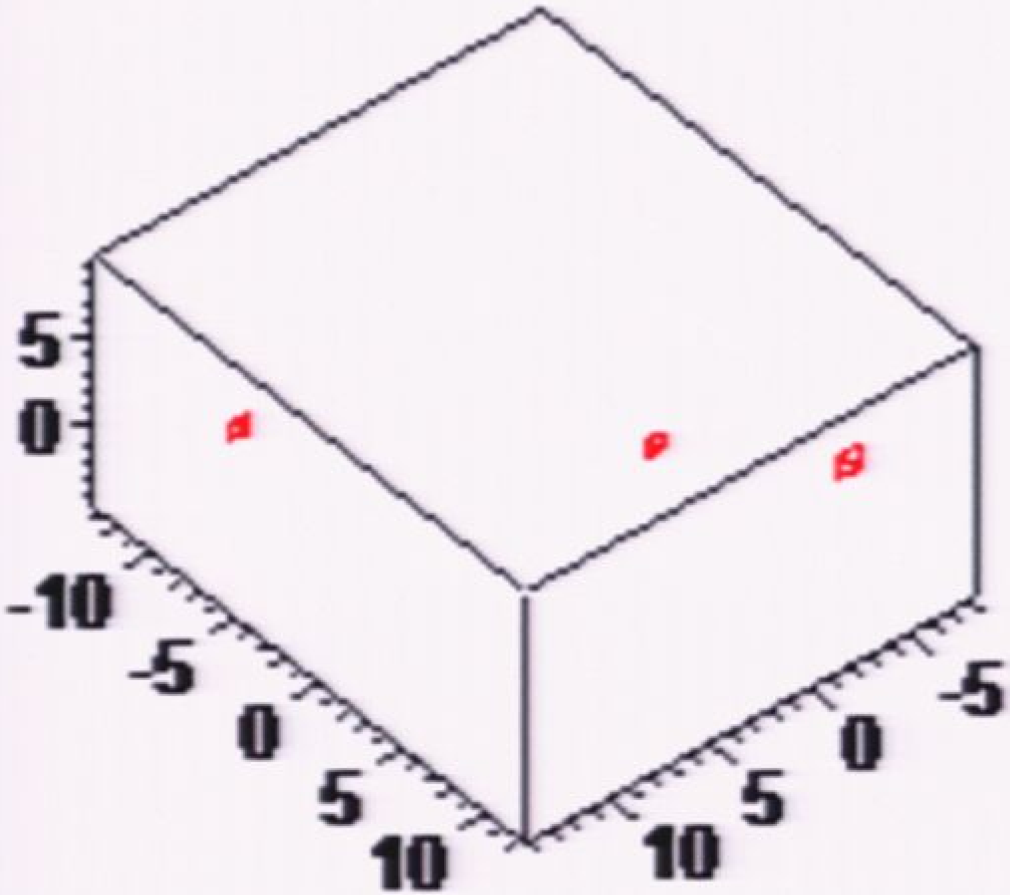

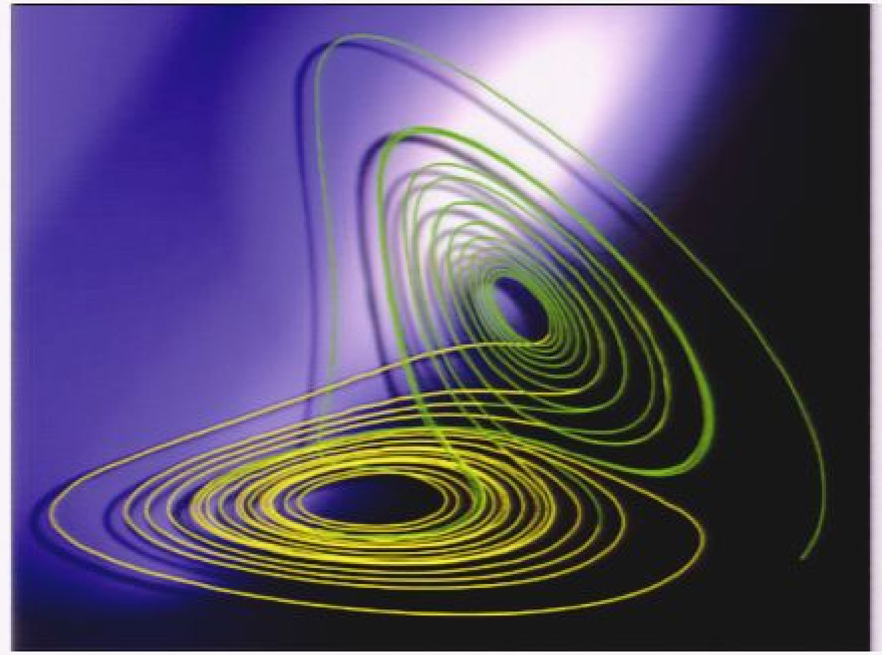

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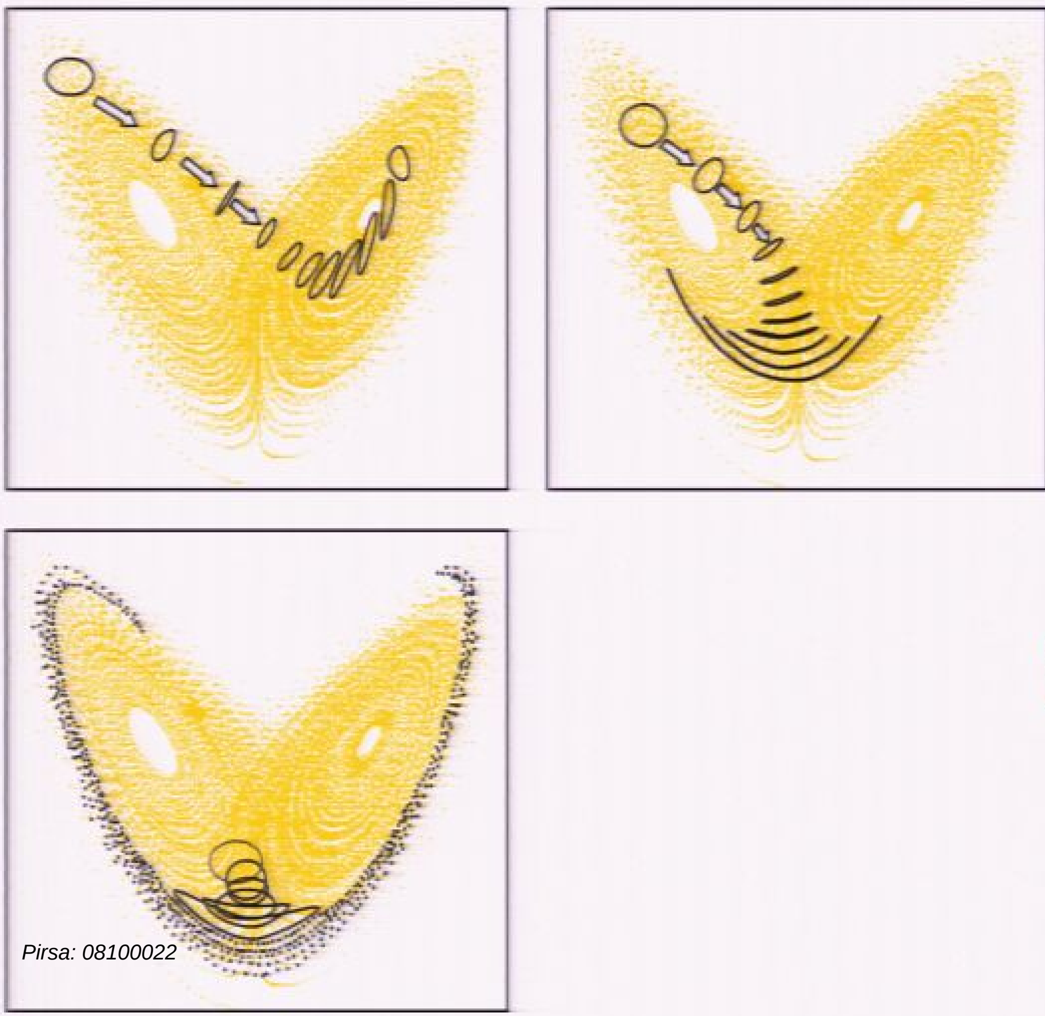

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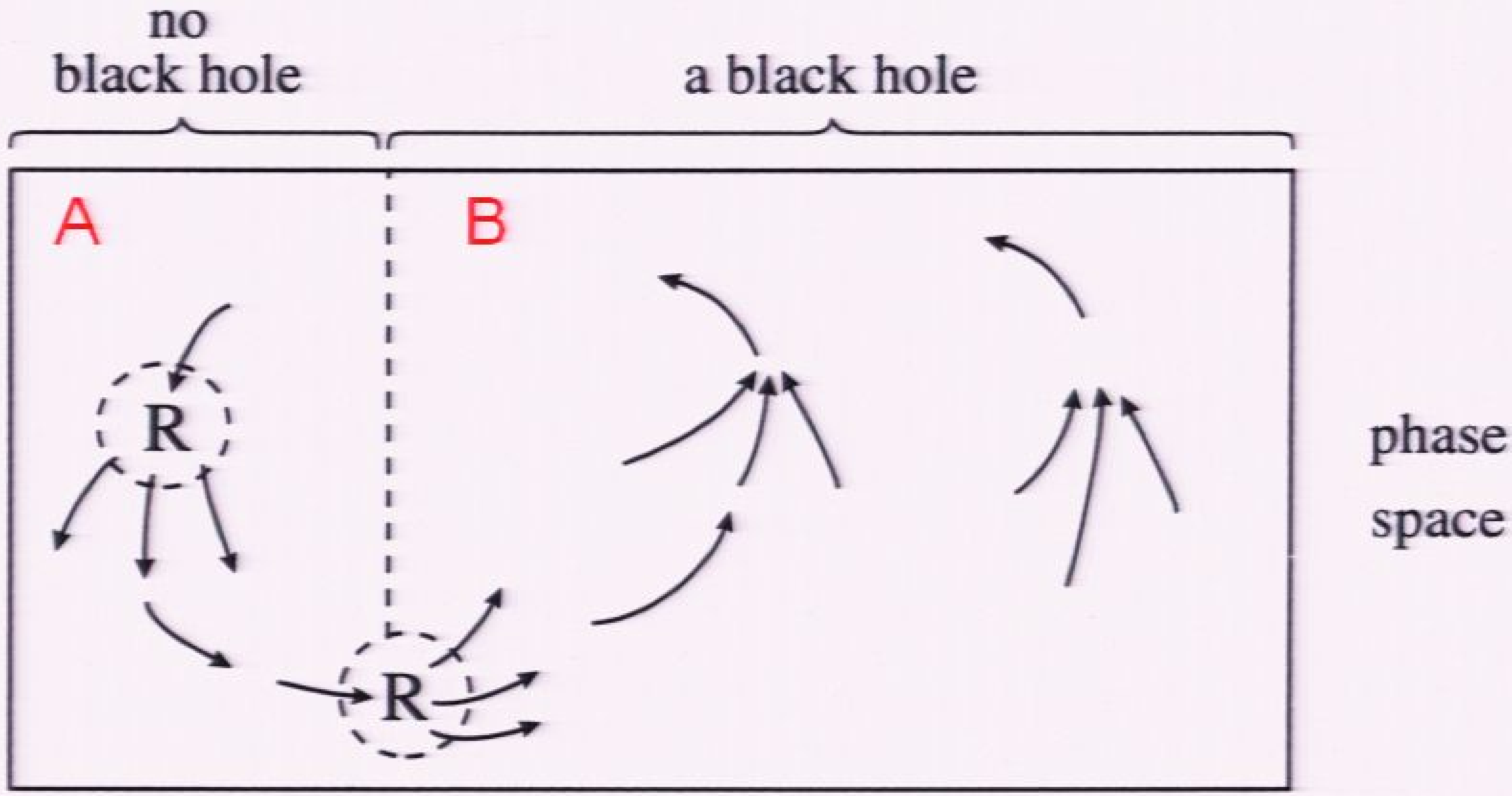


Figure 4.4 Loss of phase-space volume occurs when a black hole is present. This may be balanced against regain of phase-space volume due to wave function collapse **R**.

Suppose that the evolution of the universe car
be described as a locally-causal deterministic
dynamical system D with state ψ, and fractal
invariant subset I_D.

In addition suppose:

Invariant Set Hypothesis

$$\psi \in I_D$$

Nearby points on (a Poincaré cross section of) the invariant se
could represent states of the universe in different aeons.

By hypothesis, the notion of "physical reality" is only attributable to those points on the invariant set.

Nb The notion of invariance is a bedrock of physics.

Invariant Sets as General Relativistic Descriptors of Chaos in General Relativity

by

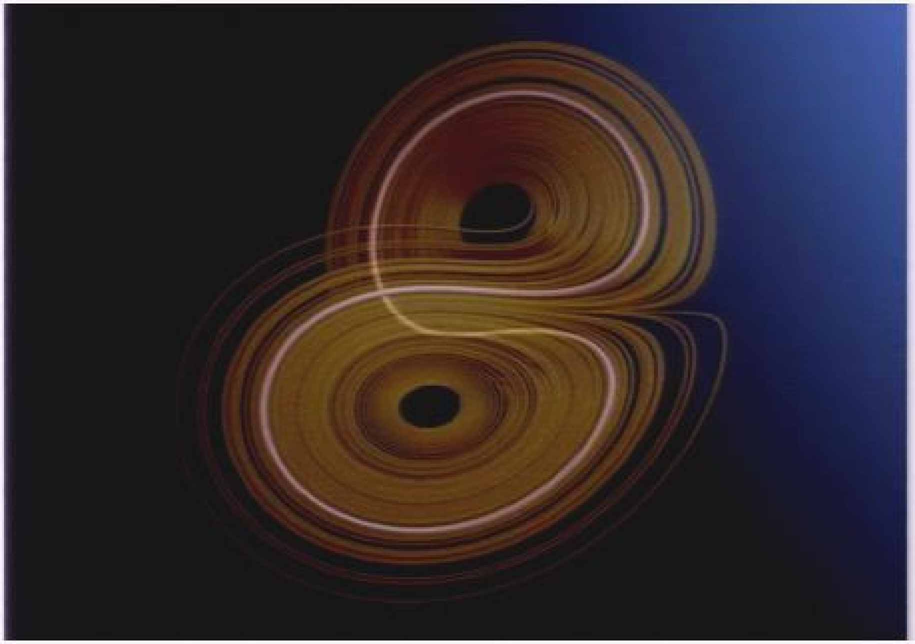
N.J.Cornish

gr-qc/9709036

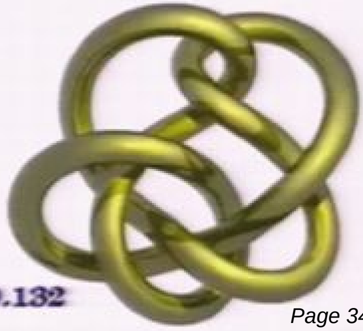
"..It is clear that the standard..measures of chaos [such as Lyapunov exponents] have to be abandoned in general relativity…The problem is solved [by introducing] fractal dimensions and symbolic codings as [diffeomorphism] invariant descriptors of chaos in general relativity. Central to both of these methods is the concept of a chaotic **invariant set** of orbits."

$\psi \in I_D$ How?

- God initialised the universe on I_D
- The global geometry of I_D is a more primitive concept than the differential equations D whose asymptotic behaviou generates I_D



Lorenz Knots are Prime, Fibred with Non-Negative Signature



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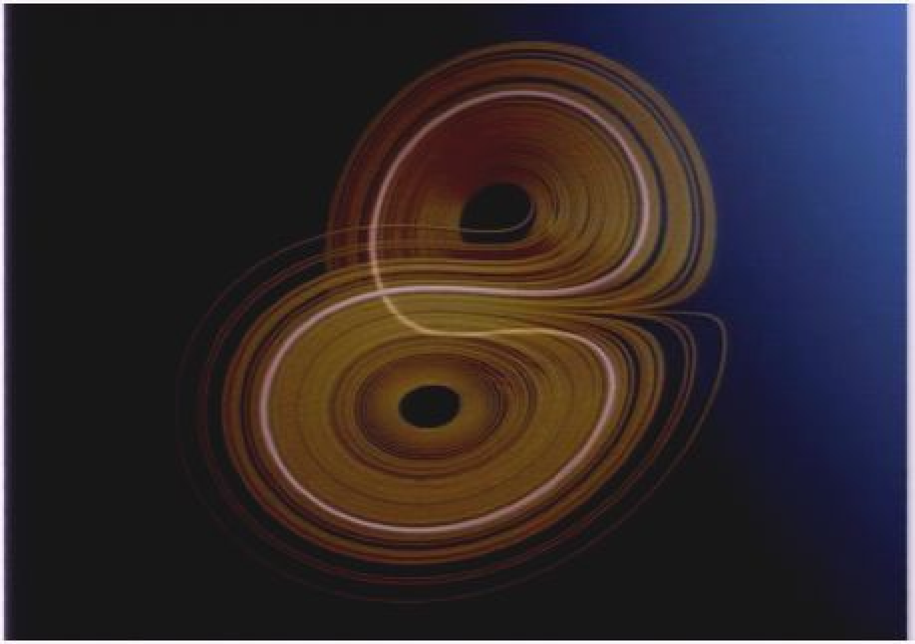

Lorenz Knots are Prime, Fibred with Non-Negative Signature

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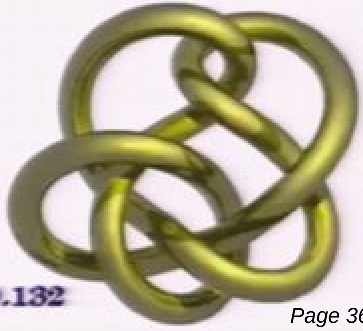

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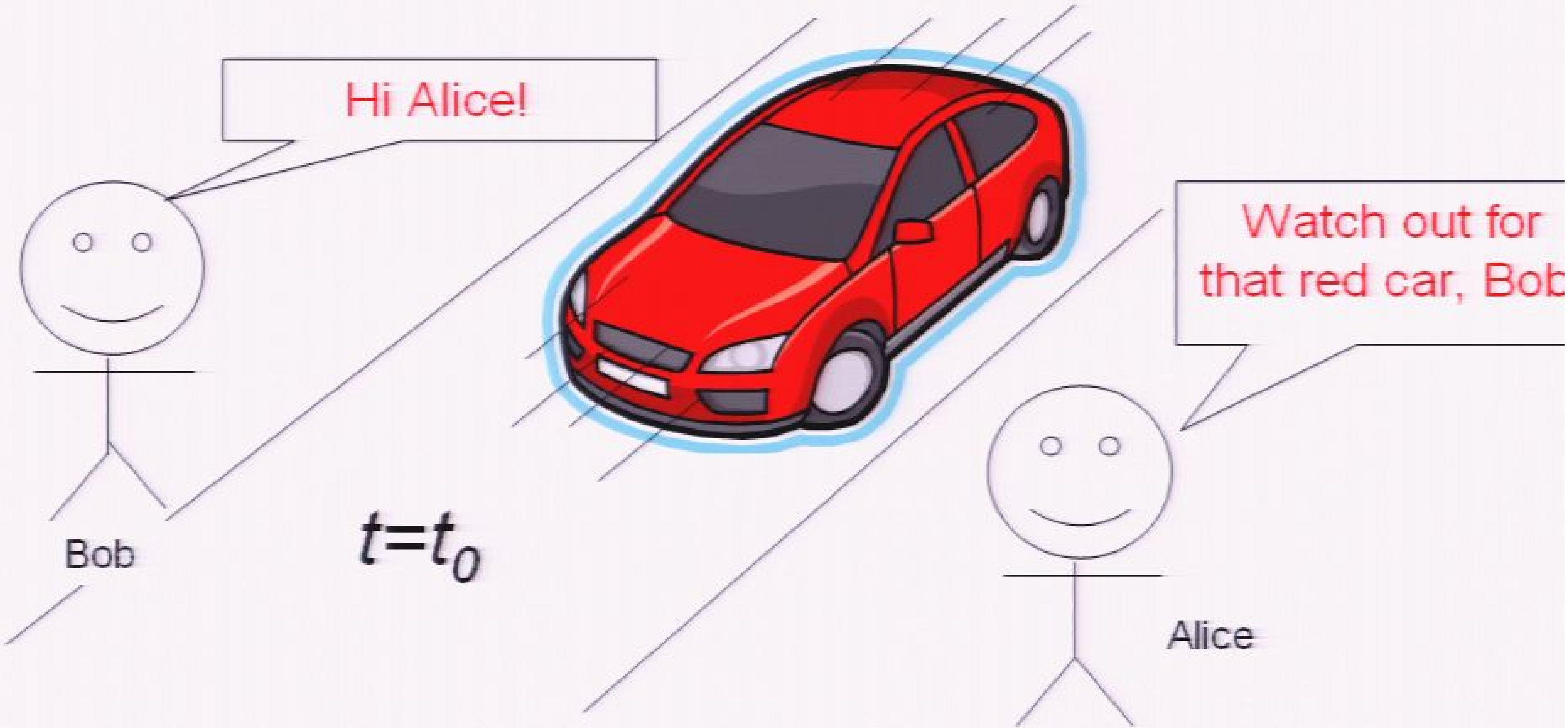
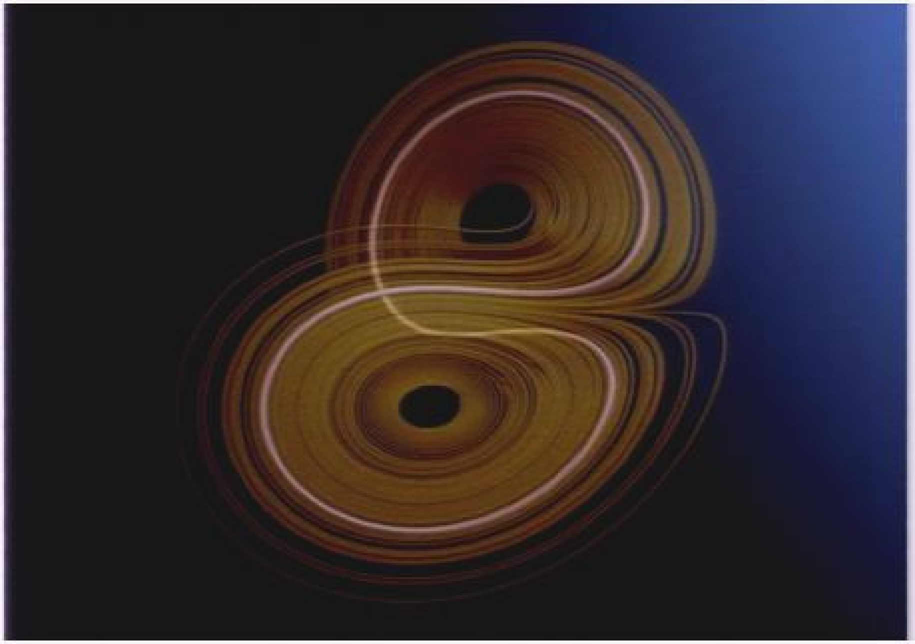
A Counterfactual



$t = t_0$

Just as well Bob didn't cross the road at that precise moment; if he had, he would have been hit by the speeding red car.



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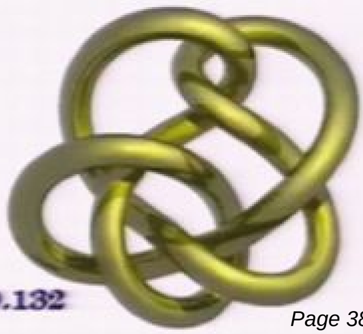

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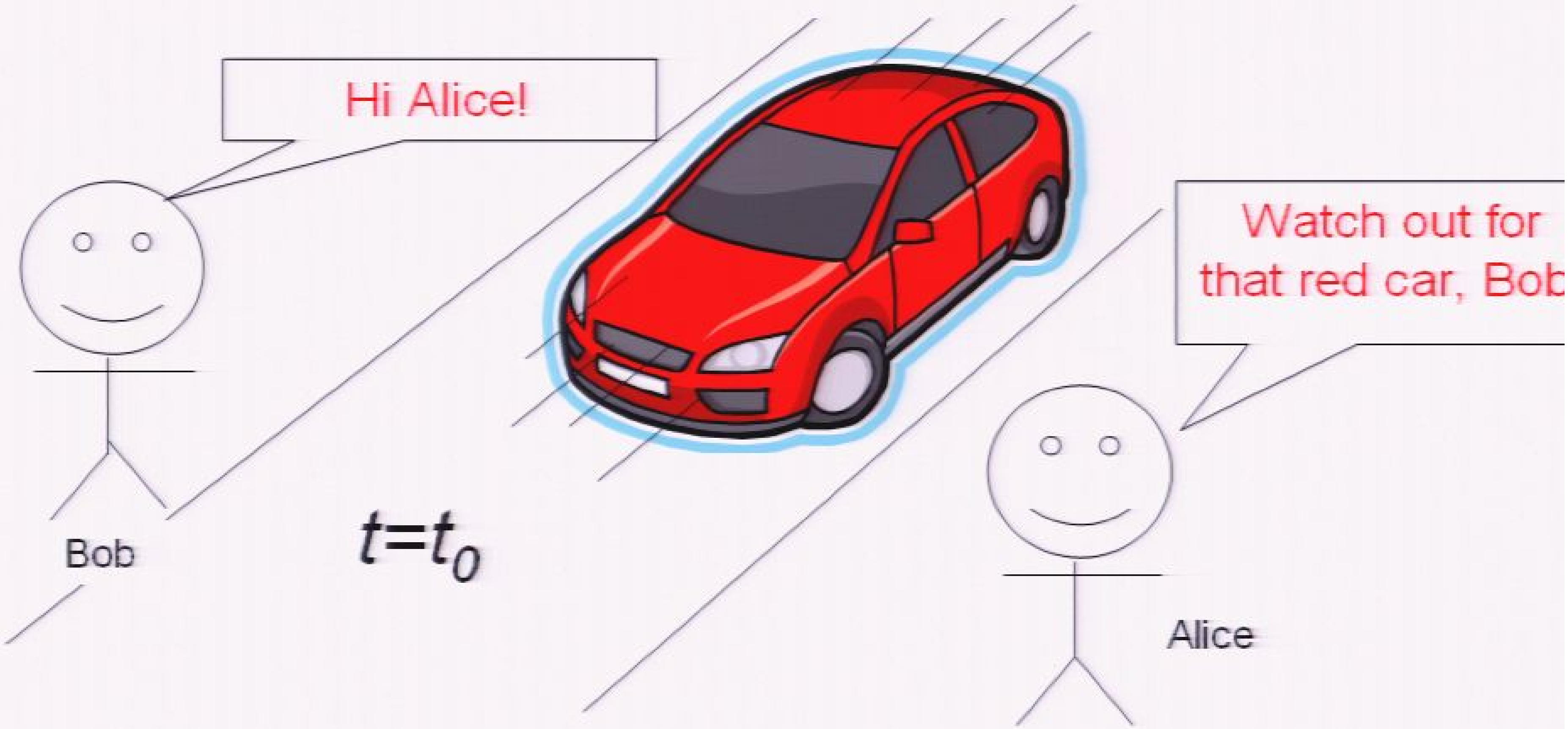

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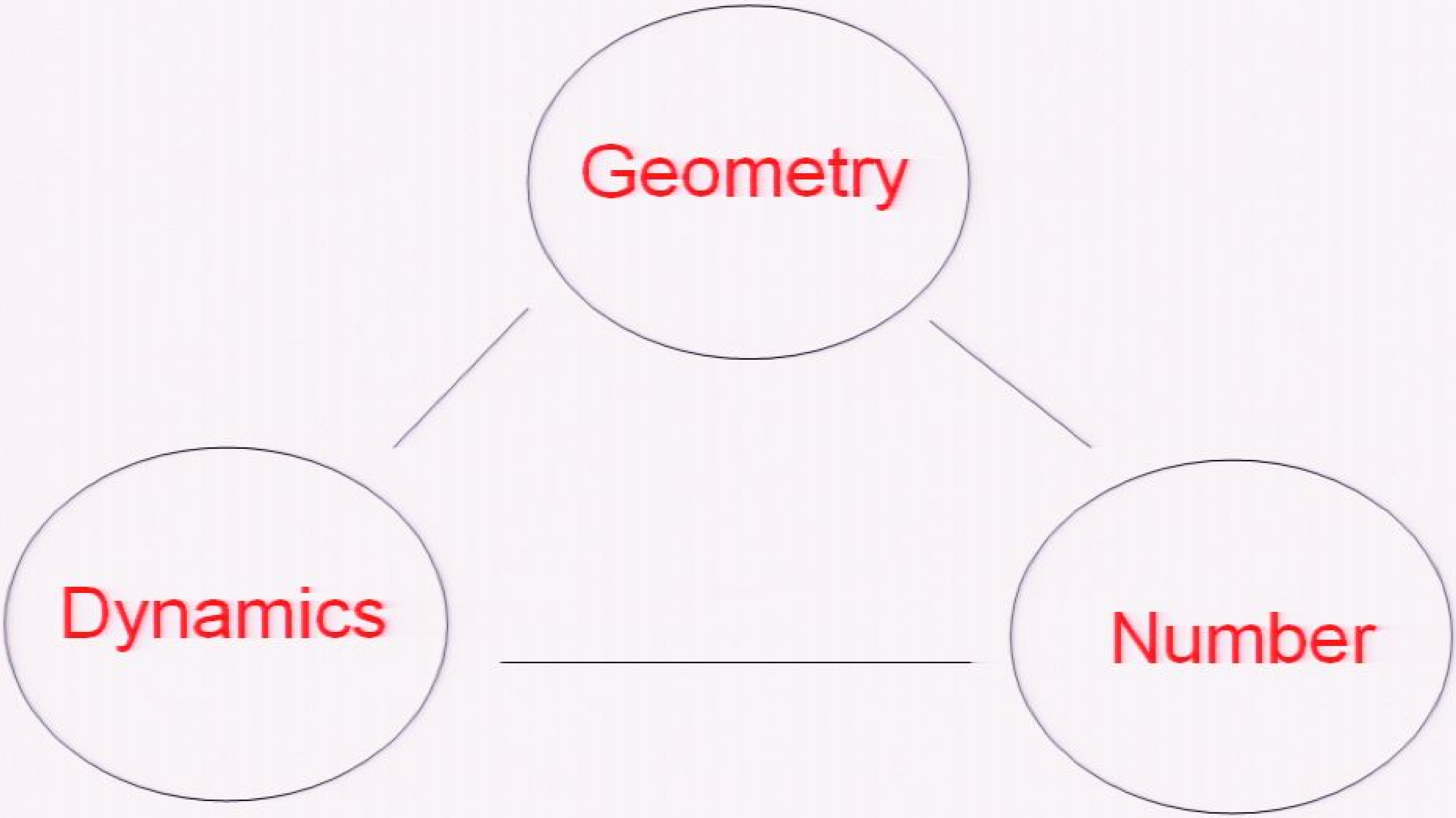
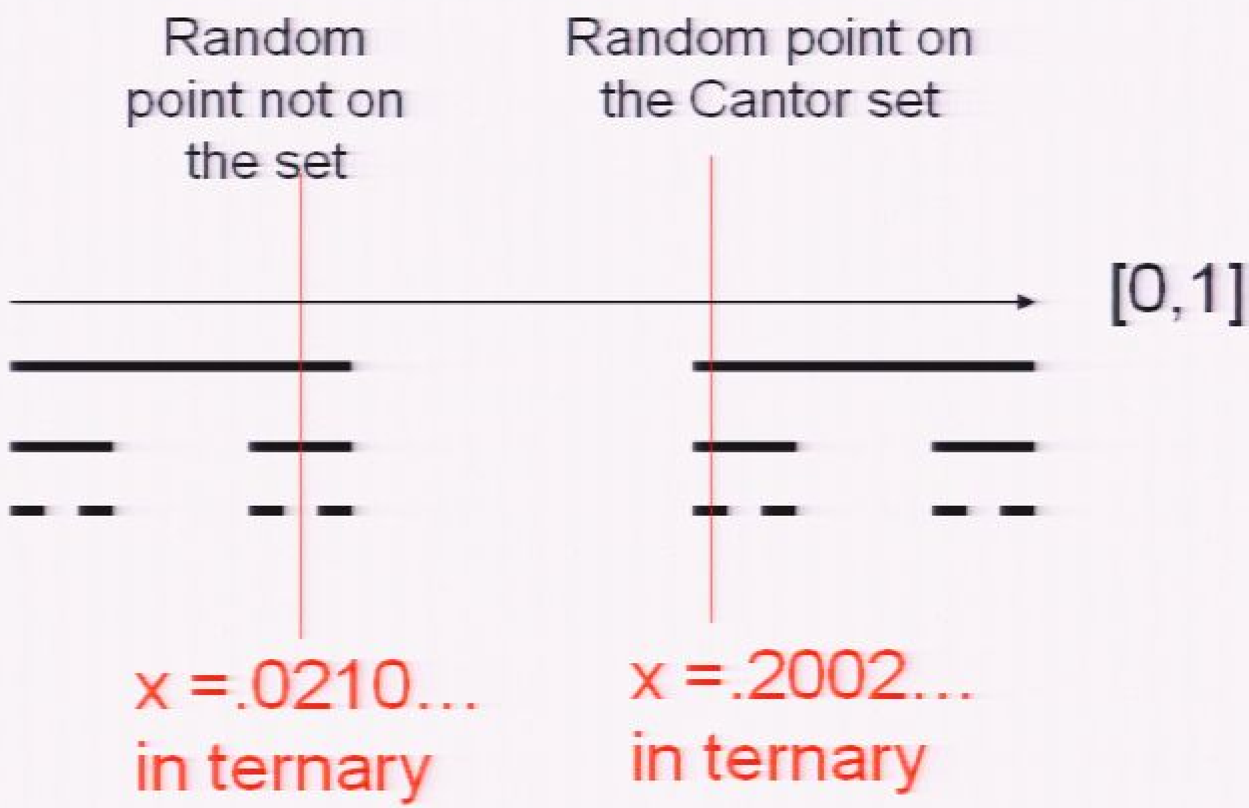
True, false or neither?


Geometry
Dynamics
Number

A simple fractal invariant set



$$\boxed{.2002..+.0210\ldots=.2212}$$

More generally, represent a point *p* on I_D by a vector of non-normal base-*b* numbers with missing digits, and a random perturbation ε by a vector of base-*b* normal numbers.

$$p+\varepsilon \not\in I_D$$

The random, dynamically-unconstrained, perturbation to Bob's across road momentum takes the state of the universe off the invariant set – by the Invariant Set Hypothesis, the counterfactual state is not a state of physical reality.

Hence neither true nor false that Bob would have been hit by the red car if he had crossed the road.

Similarly neither true nor false that Bob would have measured the spin of a given particle as "up" if his Stern-Gerlach device had been oriented differently

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states on the invariant set.

- Hence by the Invariant Set Hypothesis, it is not meaningful to regard a sub-system as having any intrinsic properties independent of the experiments by which us humans gain empirical information about the sub-system.
- This is reminiscent of the Copenhagen Interpretation, but in a deterministic realistic setting
- Reconciliation of Einstein and Bohr et al?

Invariant Set Hypothesis and the Copenhagen Interpretation

- By the Invariant Set Hypothesis, it is not meaningful to regard an individual sub-system as having any intrinsic properties independent of the invariant set on which the state of the universe as a whole evolves.
- Experiments reveal information about a sub-system to us humans.
- These same experiments are necessarily components of states on the invariant set.
- Hence by the Invariant Set Hypothesis, it is not meaningful to regard a sub-system as having any intrinsic properties independent of the experiments by which us humans gain empirical information about the sub-system.
- This is reminiscent of the Copenhagen Interpretation, but in a deterministic realistic setting
- Reconciliation of Einstein and Bohr et al?

I am presuming that the dynamics *D* are locally causal – does this conflict with quantum theory?

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Bell: Free Variables and Local Causality.

"Quantum mechanics is not locally causal and cannot be embedded in a locally causal theory. That conclusion depends on treating certain experimental parameters, typically the orientation of polarisation filters, as free variables."

Gerard ‘t Hooft, 2007: The Free-Will Postulate in Quantum Mechanics. quant-ph/0701097

“The so-called “free-will axiom” is an essential ingredient in many discussion concerning hidden variables in quantum mechanics….Our axiom, to be referred to as the “unconstrained initial state” condition has consequences similar to free will, but does not clash with determinism….We must demand that our model [of nature] gives credible scenarios for a universe for any choice of the initial conditions.”

The Invariant Set Conjecture

Variables can be freely varied only over the invariant set I_D of the universe. Because this rules out the possibility of counterfactual measurements (eg "what if I had measured the particle with this orientation instead of that"), the invariant set hypothesis prevents D being constrained by Bell inequalities

Classical Theory

- Includes all theory for which deterministic differential or difference equations are primary.
- In such theories states are not constrained to lie on invariant sets, even if such sets exist.
- ‘tHooft’s Unrestricted Initial State condition and hence Bell inequalities hold for such theories.

Objections to a Restriction of Free Will

- We can choose by whim
  - Takens Embedding Theorem
- Makes us seem no better than automata
  - I am conscious
  - I acknowledge as real, the world around me
  - By the Invariant Set Hypothesis, I therefore acknowledge as real (a state on the invariant set) something that I cannot prove algorithmically to be real
  - I am not an automaton!

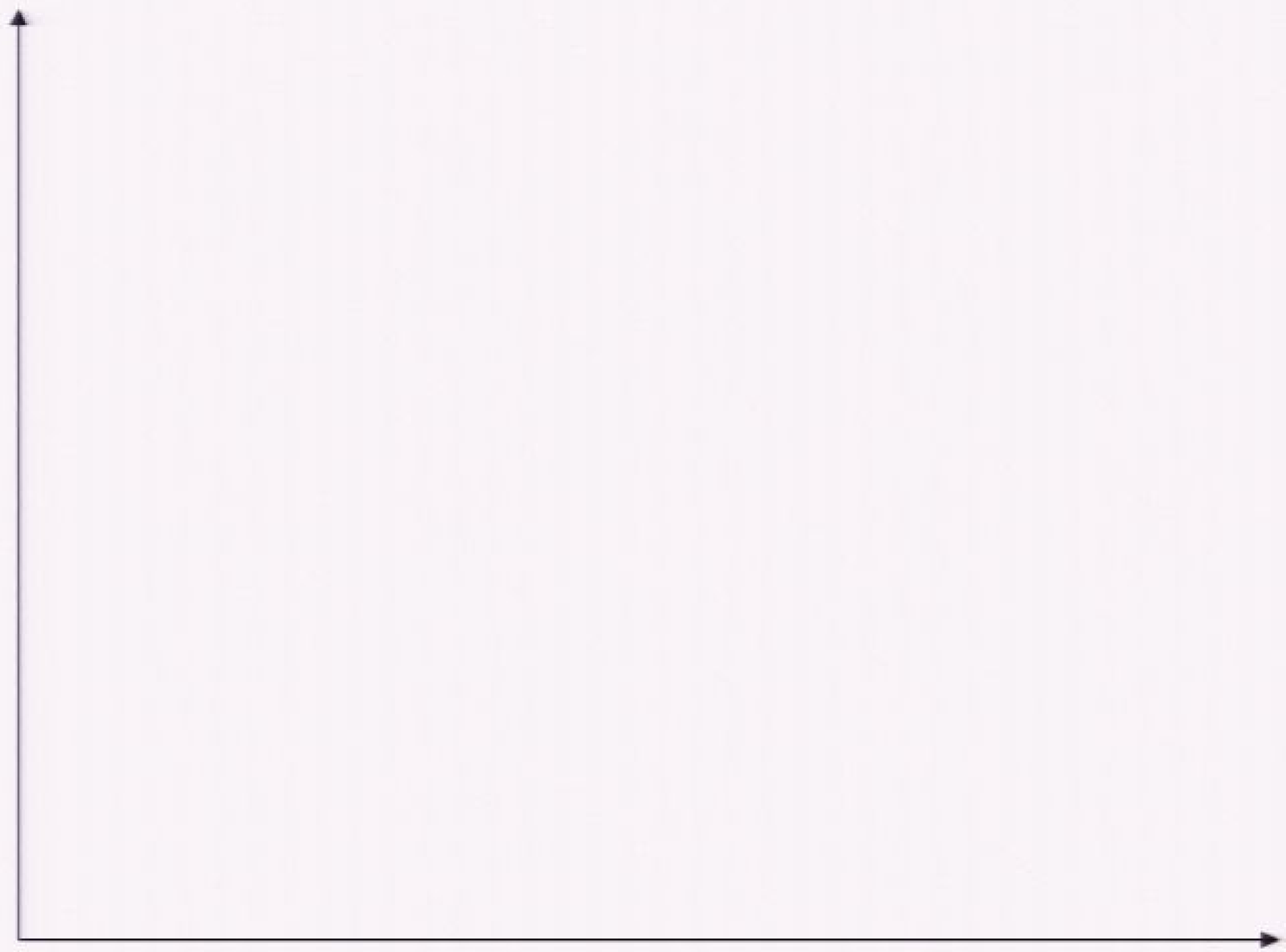
Towards the Complex Hilbert Space

The Invariant Set Hypothesis is consistent with the Kochen-Specker theorem, that there are no non-contextual hidden variable theories.

The Kochen-Specker theorem is derived using the language of Hilbert Spaces.

Could we work backwards using the Invariant Set Hypothesis as axiom, to derive the Hilbert Space in quantum theory?

What does the Invariant Set Look Like?



Measure zero, nowhere dense, so can’t see it!

Non-computable so can’t compute it!

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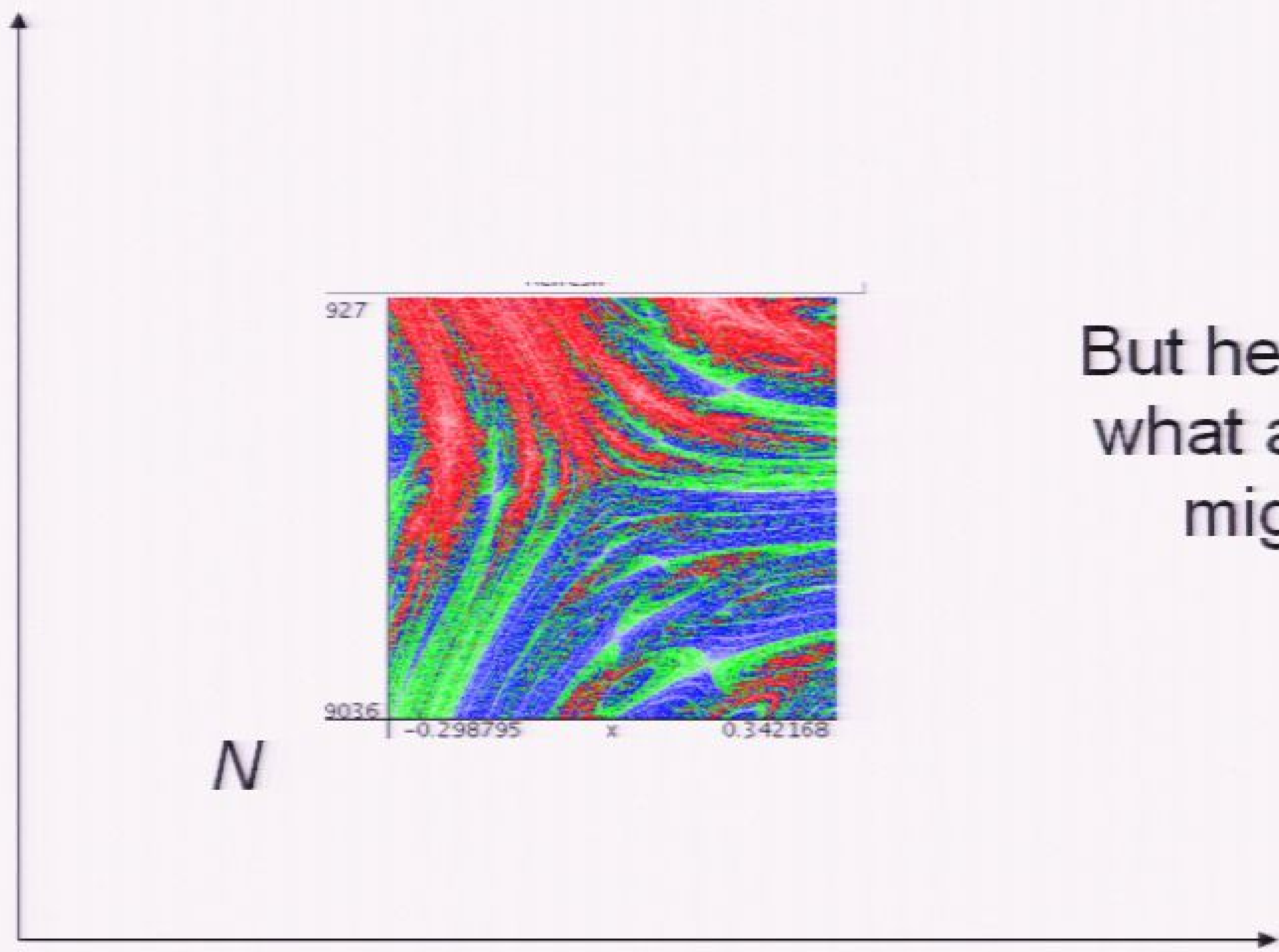
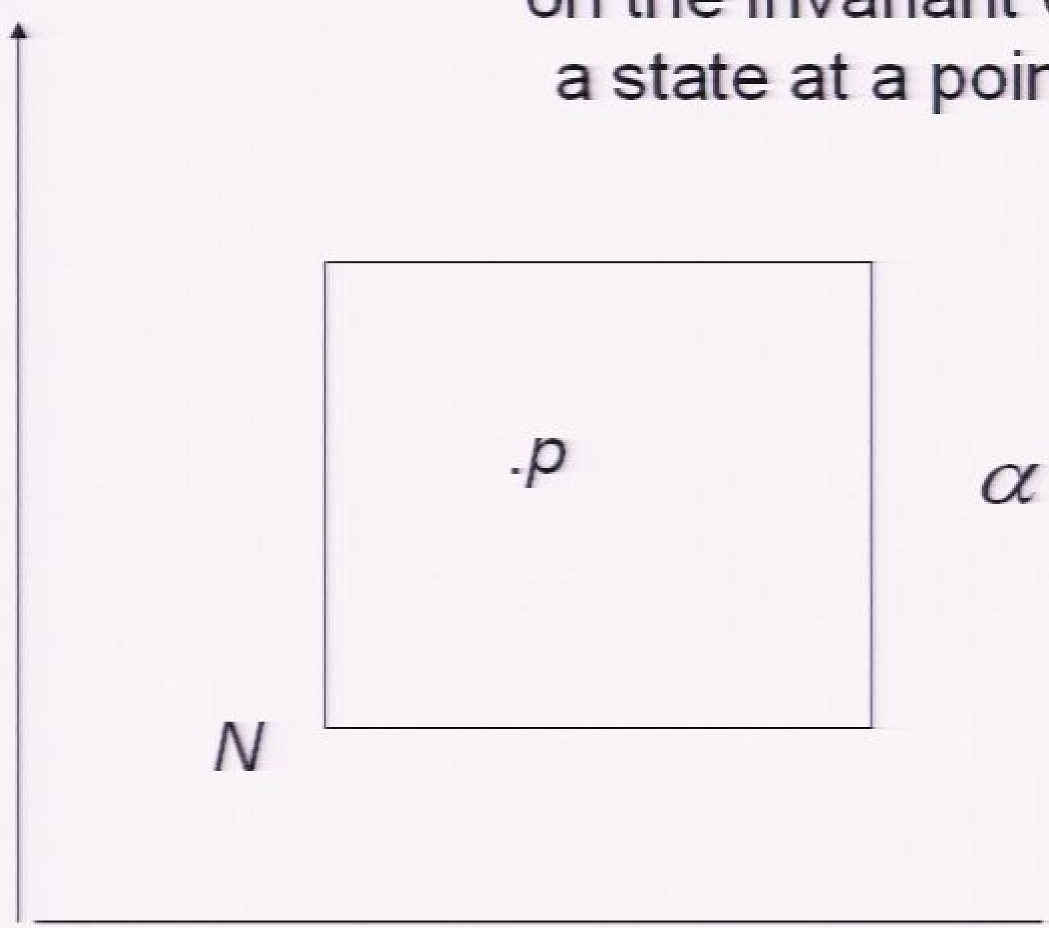


But here's part of what an "oracle" might see!

Measure zero, nowhere dense, so can't see it!

Non-computable so can't compute it!

In a theory blind to the fine-scale structure on the Invariant Set, then we can represent a state at a point *p* on the invariant set by



$$\alpha|red\rangle + \beta|blue\rangle$$

$$\alpha + \beta = 1 \quad \alpha, \beta \geq 0 \in \mathbb{R}$$

α is the relative density of red points in *N*, β is the relative density of blue points in *N*

A simple modification could be

$$\alpha|red\rangle + \beta|blue\rangle + \gamma|green\rangle$$

$$\alpha + \beta + \gamma = 1 \quad \alpha,\beta,\gamma \in \mathbb{R}$$

and γ denotes the density of points in N not on the invariant set.

But we must be able to ingest information acquired empirically (ie by experiment) into our theory.

However, by the Invariant Set Hypothesis, it is impossible to perform an experiment on a putative state which is not on the invariant set.

As such cannot ingest empirical information about γ.

Consider instead

$$\alpha|red\rangle + \beta|blue\rangle$$

$$\alpha^2 + \beta^2 = 1 \quad \alpha, \beta \in \mathbb{R}$$

α^2, β^2 denote the density of red/blue points on the invariant set in N.

Not a simple probability mixture, but now another problem- not enough degrees of freedom

- Consider a type of sub-system which, over time, is found to be oriented in all possible directions in physical 3-space relative to the mass distribution of the rest of the universe.
- A theory which is blind to the fine-scale structure of the invariant set must be prepared for the possibility that, at any time, the sub-system is oriented in any conceivable direction.

$$\alpha|red\rangle + \beta|blue\rangle$$

$$\alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

Three degrees of freedom and not in the form of a probability mixture.

Based on the SO(3) → SU(2) homomorphism, the coefficients α, β vary according to unitary transformations as the orientation in 3-space is varied.

The superposed state is a mathematical device which allows us to describe states of subsystems in a theory which is blind to the fine scale structure of the underlying invariant set.

No superposed states in an “oracle theory” based on the Invariant Set Hypothesis.

Oracle Theory

- Use symbolic dynamics to represent states on the invariant set, eg

$$S = \{blue, blue, yellow, red, blue, white, \ldots\}$$

relative to some N-element partition of the invariant set.

- Can equivalently represent S by $\log_2 N$ correlated bit strings.
- Treat individual bit strings as primitive sub-systems

Consider the bit string

$$S = \{a_1, a_2, a_3, a_4, \ldots\}$$

$$a_i \in \{red, blue\}$$

and define

$$\phi(red) = blue \quad \phi(blue) = red$$

$$-S = \{\phi(a_1), \phi(a_2), \phi(a_3), \phi(a_4), \ldots\}$$

$$\Rightarrow \quad -(-S) = S$$

and

$$\mathbf{e}_1(S) = \{\, \phi(a_2), a_1, a_4, \phi(a_3), \ldots \}$$
$$\mathbf{e}_2(S) = \{\, \phi(a_3), \phi(a_4), a_1, a_2, \ldots \}$$
$$\mathbf{e}_3(S) = \{\, a_4, \phi(a_3), a_2, \phi(a_1), \ldots \}$$

related by quaternionic multiplication….

$$\Rightarrow \;\; \mathbf{e}_j \circ \mathbf{e}_j (S) \; = \mathbf{e}_1 \circ \mathbf{e}_2 \circ \mathbf{e}_3 (S) = -S$$

Oracle Theory

- Use symbolic dynamics to represent states on the invariant set, eg

$$S = \{blue, blue, yellow, red, blue, white, \ldots\}$$

relative to some N-element partition of the invariant set.

- Can equivalently represent S by $\log_2 N$ correlated bit strings.
- Treat individual bit strings as primitive sub-systems

The superposed state is a mathematical device which allows us to describe states of subsystems in a theory which is blind to the fine scale structure of the underlying invariant set.

No superposed states in an “oracle theory” based on the Invariant Set Hypothesis.

The Measurement Problem

- Measurement reveals to us humans, information about sub-systems
- In quantum theory, updating the state vector with this information leads to a “collapse of the wavefunction”
- No superposed states, updating or collapse needed in the Oracle Theory.
- Rather, the measuring apparatus plays a key role in defining the structure of the invariant set on which the sub-system evolves.
- Cf Freeman Dyson’s Interpretation of quantum theory
  - Statements about the past cannot in general be made in quantum mechanical language
  - The role of the observer in quantum mechanics is to make the distinction between the past and the future

Gravity

- Gravity plays a key role in determining the structure o the invariant set.
- Eg, as discussed, black-hole dynamics play a key rol in determining the reduced fractal dimension of the invariant set and hence aspects of its fine-scale structure in remote parts of state space.
- This is additional to the more local effects of gravity e associated with interaction between a sub-system ar its measuring apparatus as discussed by Penrose ar others.

$$\rightarrow \quad D_{\text{Classical}}$$

Differential equations
representing the dynamics
of classical physics:
Newton, Maxwell,
Einstein, Lorenz….

$$\rightarrow \quad D_{\text{Classical}} \quad \rightarrow \quad D_{\text{Quantum}}$$

Differential equations representing the dynamics of classical physics: Newton, Maxwell, Einstein, Lorenz….

Differential/difference equations representing the dynamics of quantum physics: Schrödinger, Dirac, string theory, LQG…

$$\rightarrow \quad D_{\text{Classical}} \quad \rightarrow \quad D_{\text{Quantum}}$$

Differential equations representing the dynamics of classical physics: Newton, Maxwell, Einstein, Lorenz….

Differential/difference equations representing the dynamics of quantum physics: Schrödinger, Dirac, string theory, LQG…

Is the direct route from $D_{\text{Classical}}$ to D_{Quantum}, the right route?

$$D_{\text{Classical}} \rightarrow D_{\text{Quantum}}$$

$$I_D$$

$$\nearrow \qquad\qquad \searrow$$

$$D_{\text{Classical}} \qquad\qquad D_{\text{Quantum}}$$

$$I_D$$

The global atemporal state-space geometry of fractal invariant sets

$$\nearrow \qquad \searrow$$

$$D_{\text{Classical}} \qquad\qquad D_{\text{Quantum}}$$