Title: Quantum Field Theory 1 - Lecture 8B

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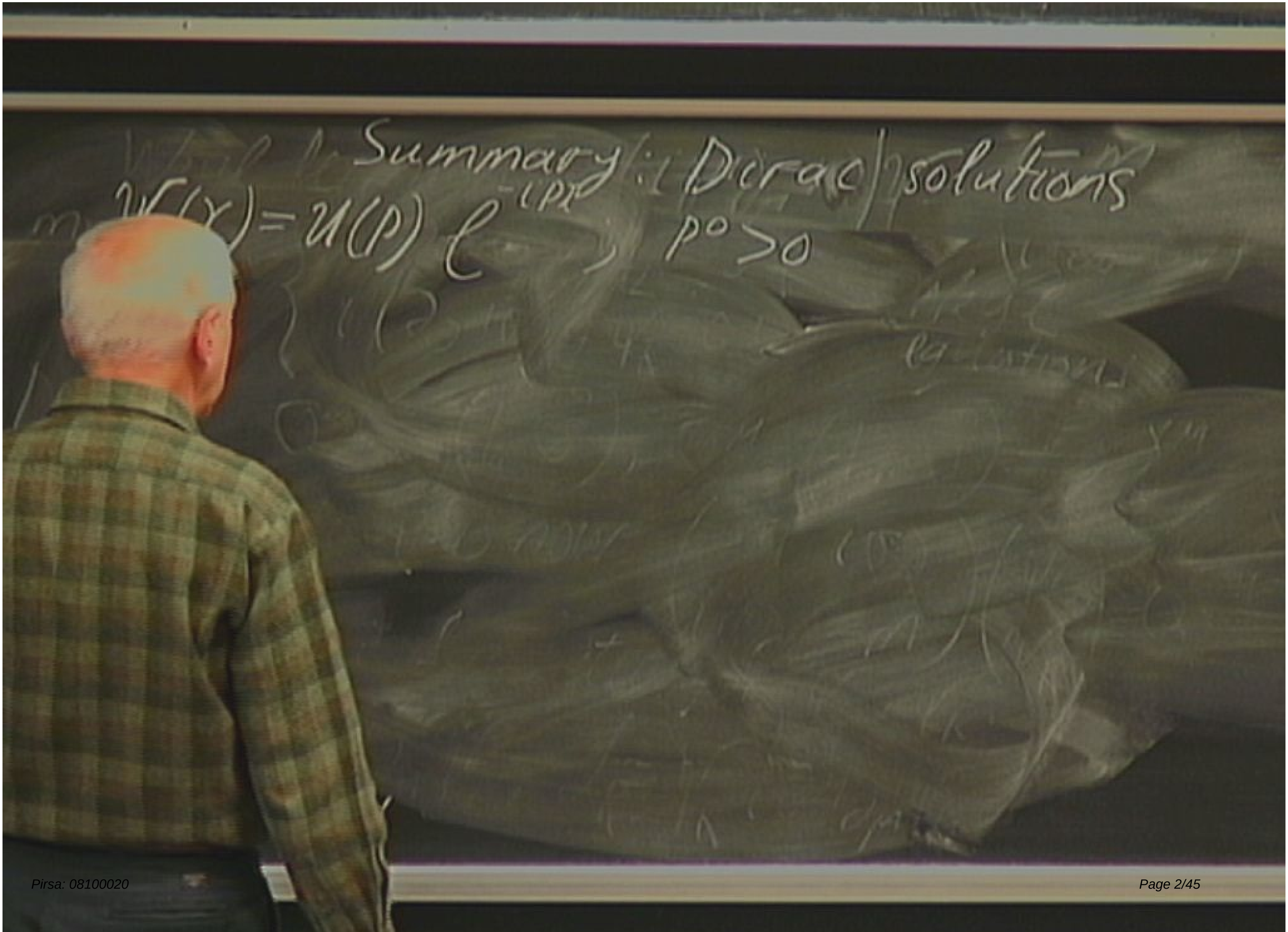
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario


Summary: Dirac solutions
ψ(x) = u(p) e^{-ipx}, p^0 > 0

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$\Psi(x) = u(p)\, e^{-ipx}$; $p^0 > 0$; $p^0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$

Two linearly independent solutions: $u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}$, $s = 1, 2$

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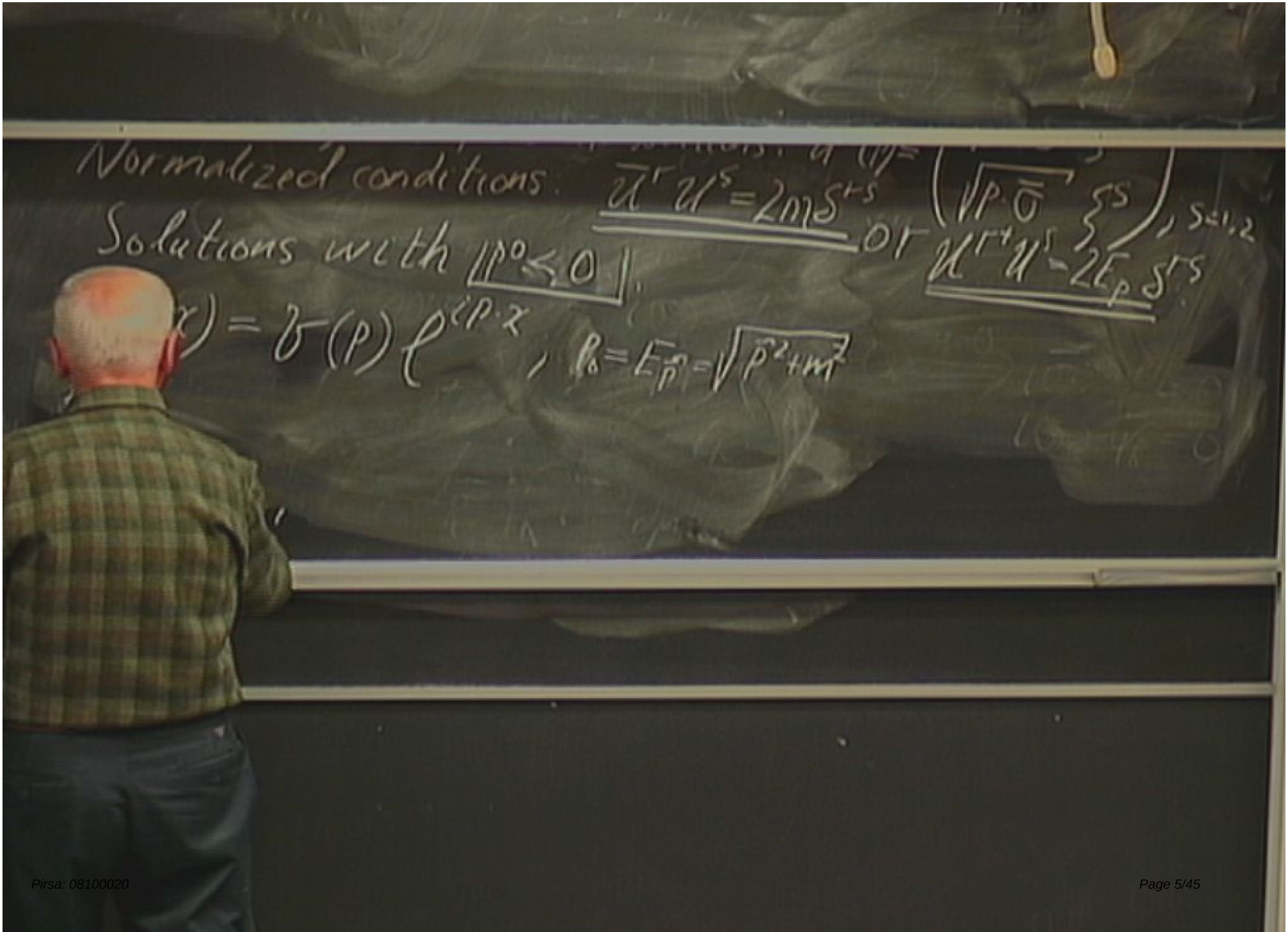
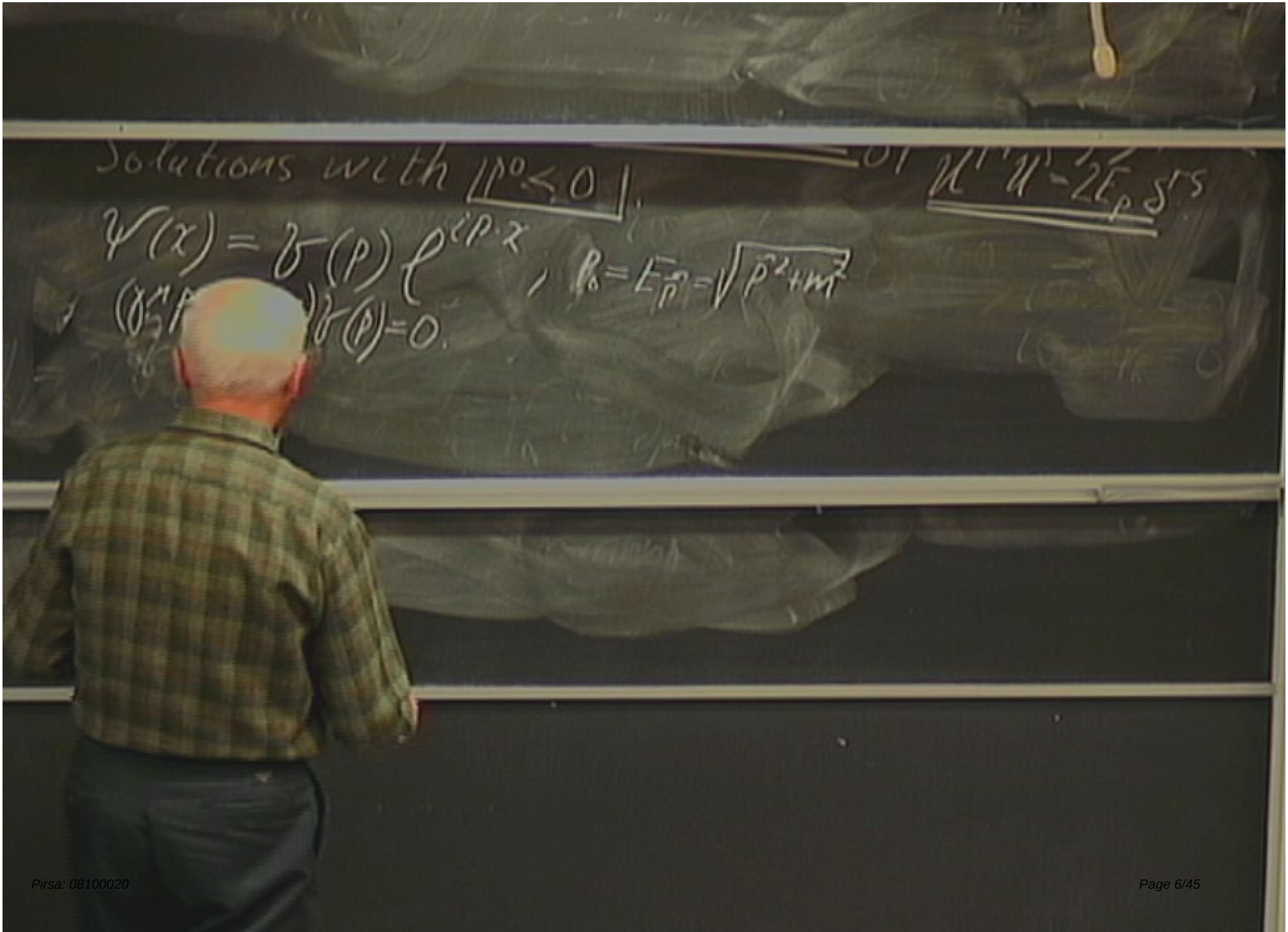
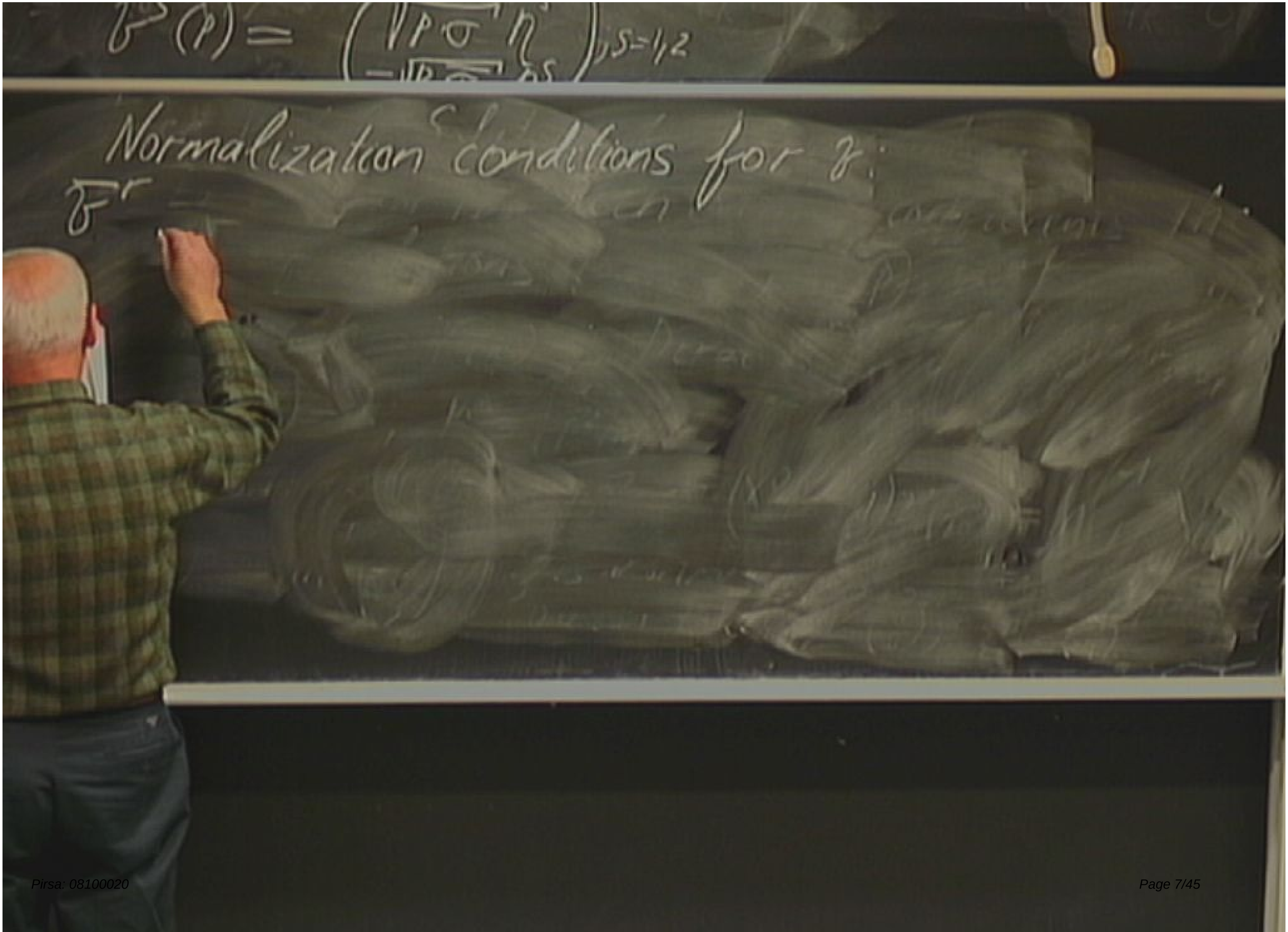
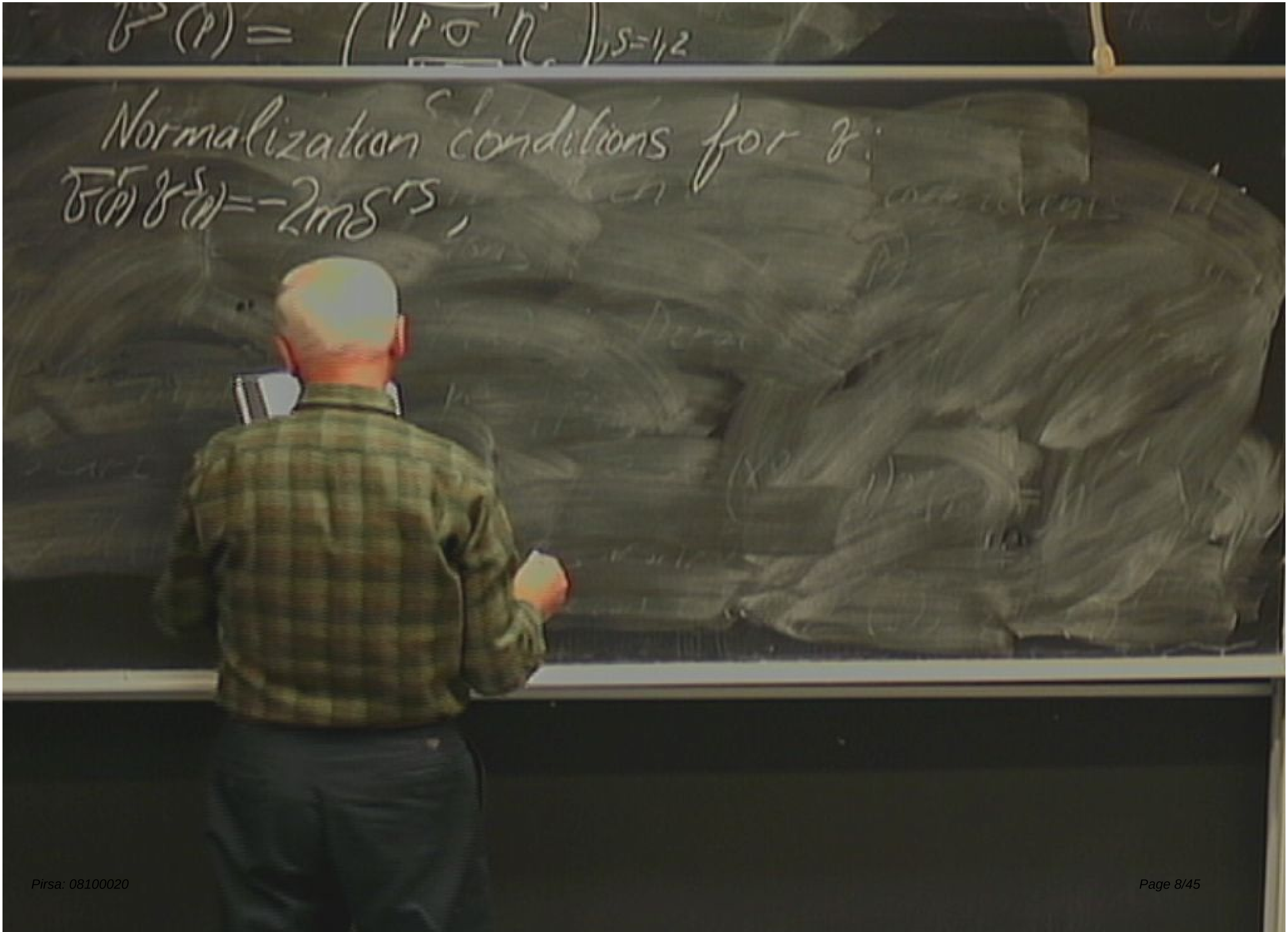
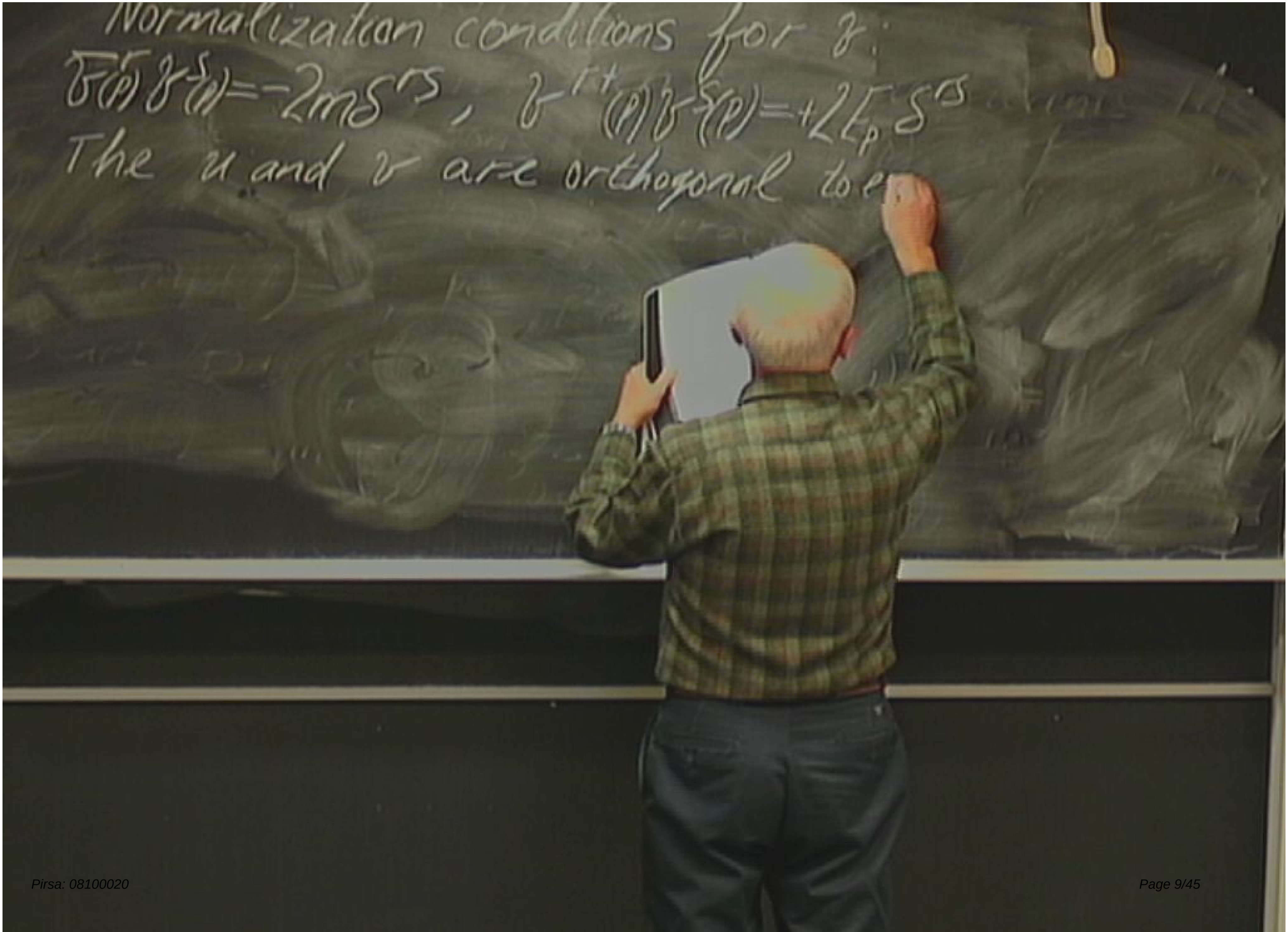
Normalized conditions: $\bar{u}^r u^s = 2m$

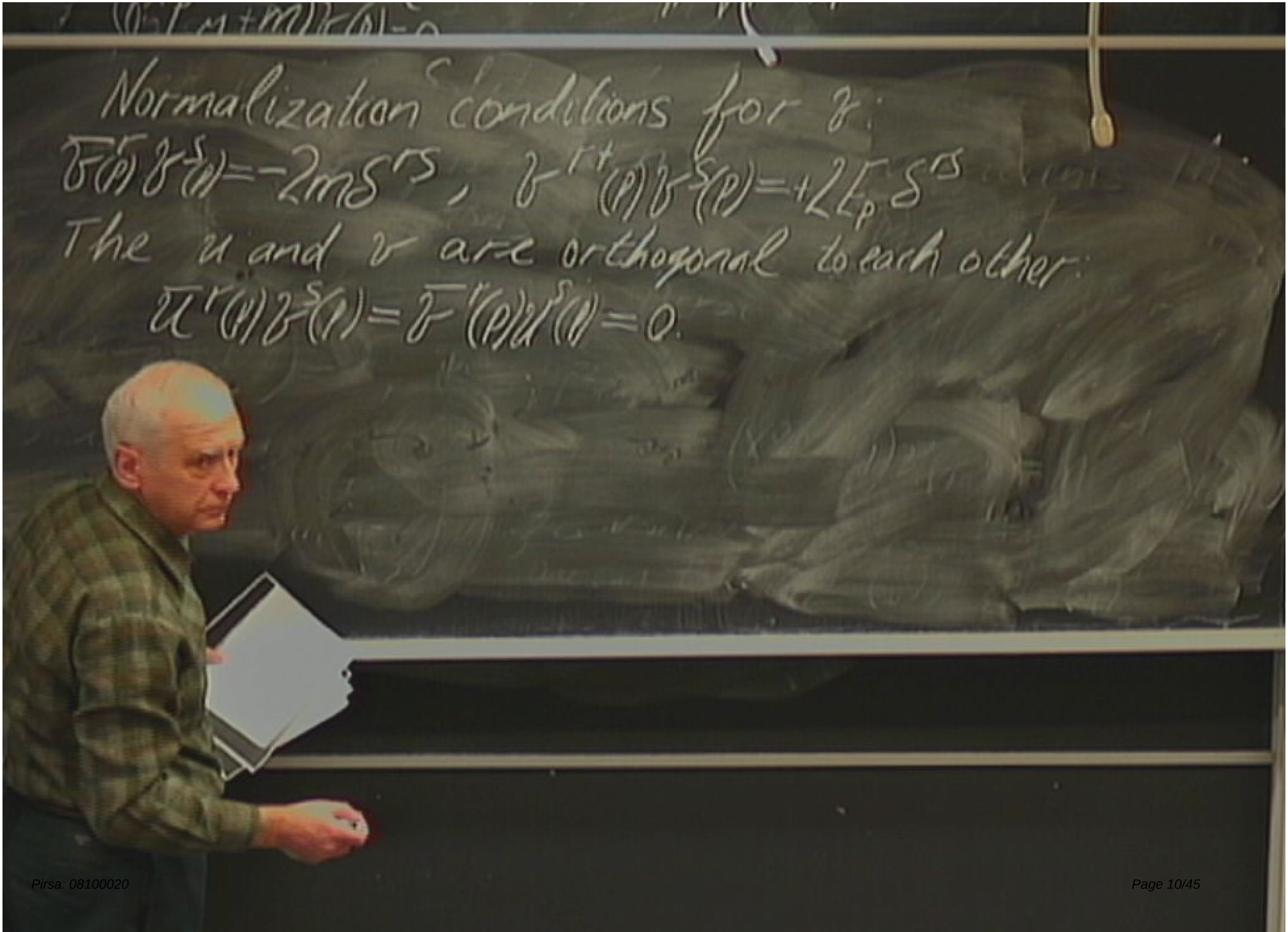

Normalized conditions: $\bar{u}^r u^s = 2m\delta^{rs}$ or $u^{r\dagger} u^s = 2E_p \delta^{rs}$
$\sqrt{p\cdot\bar{\sigma}}\,\xi^s$, $s=1,2$
Solutions with $p^0 < 0$
$x) = v(p)\, e^{ip\cdot x}$, $p_0 = E_p = \sqrt{\vec{p}^2 + m^2}$

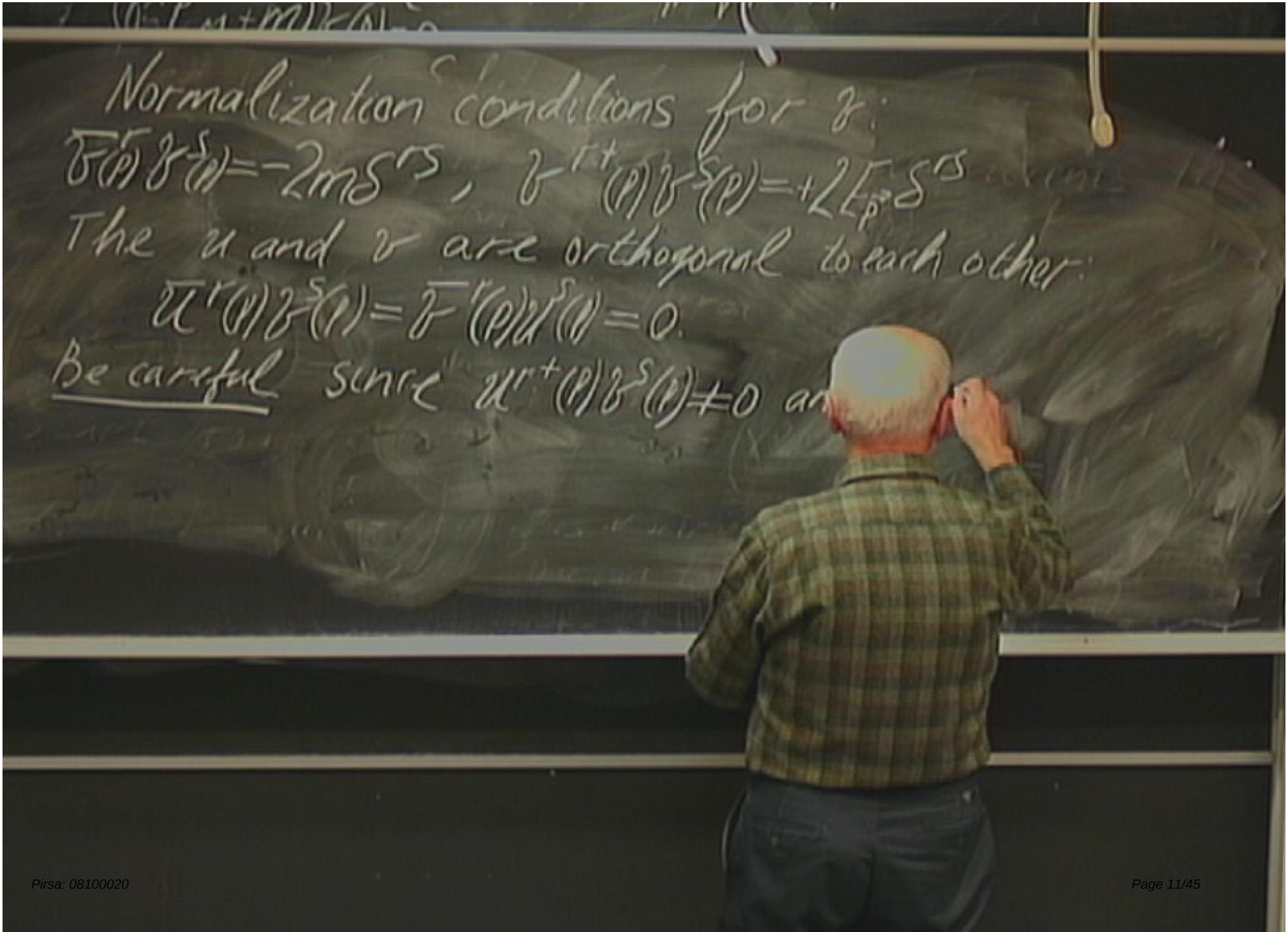

Solutions with $p^0 < 0$.
$\psi(x) = v(p)\, e^{ip\cdot x}$, $p_0 = E_p = \sqrt{\vec{p}^2 + m^2}$
$v(p) = 0$.

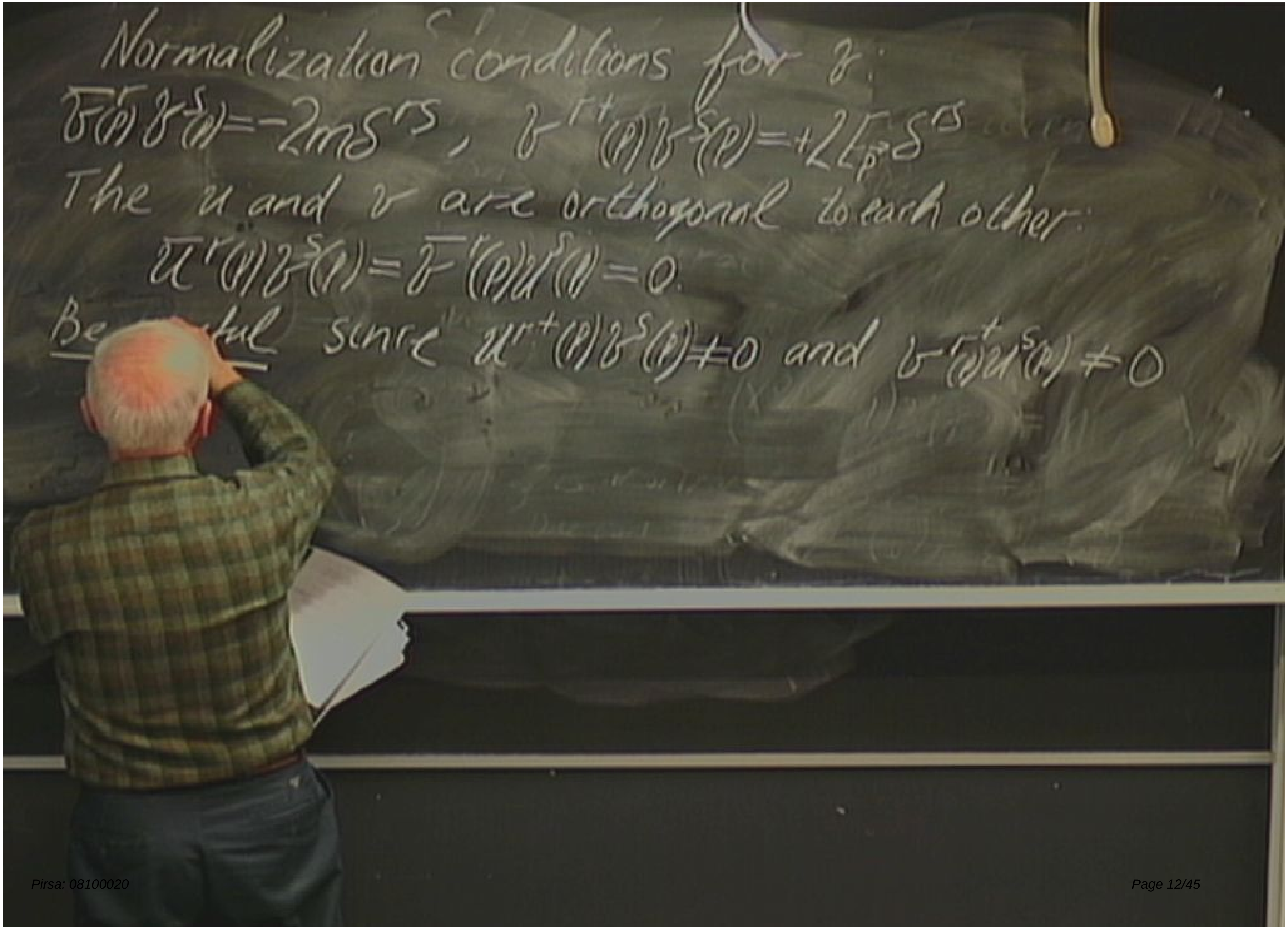

Normalization conditions for γ:
s=1,2

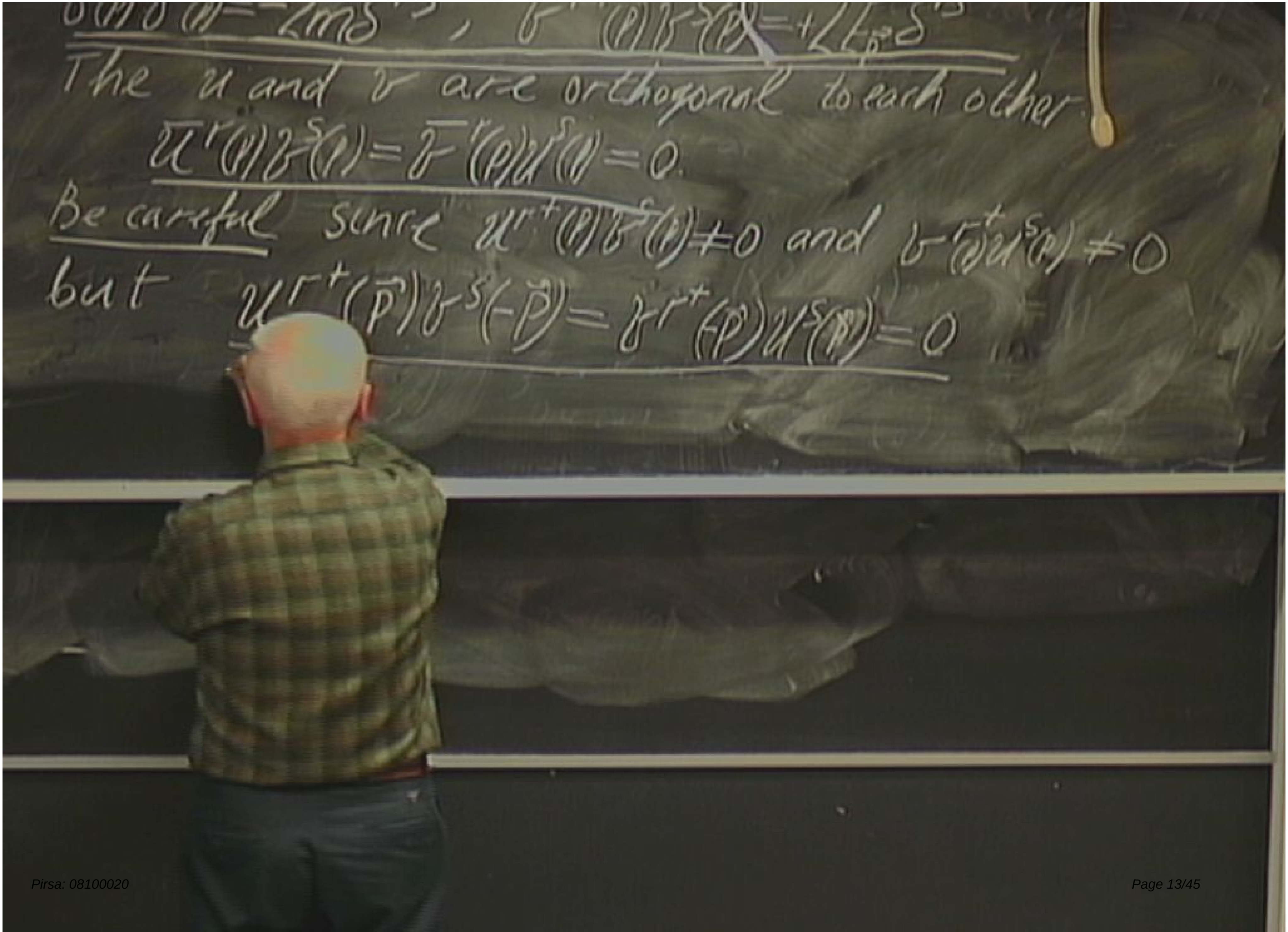

Normalization conditions for v:
v̄ˢ(p) vʳ(p) = −2m δʳˢ ,


Normalization conditions for γ:
$\bar{\gamma}^r(p)\gamma^s(p) = -2m\delta^{rs}$, $\gamma^{r\dagger}(p)\gamma^s(p) = +2E_p\delta^{rs}$
The u and v are orthogonal to


Normalization conditions for v:
$\bar{v}^r(p)v^s(p) = -2m\delta^{rs}$, $v^{r\dagger}(p)v^s(p) = +2E_p\delta^{rs}$
The u and v are orthogonal to each other:
$\bar{u}^r(p)v^s(p) = \bar{v}^r(p)u^s(p) = 0$.


Normalization conditions for v:
$\bar{v}^r(p) v^s(p) = -2m\delta^{rs}$, $v^{r\dagger}(p) v^s(p) = +2E_p \delta^{rs}$
The u and v are orthogonal to each other:
$\bar{u}^r(p) v^s(p) = \bar{v}^r(p) u^s(p) = 0.$
Be careful since $u^{r\dagger}(p) v^s(p) \neq 0$ an


Normalization conditions for v:
$\bar{v}^r(p)\, v^s(p) = -2m\,\delta^{rs}$, $v^{r\dagger}(p)\, v^s(p) = +2E_p\,\delta^{rs}$
The u and v are orthogonal to each other:
$\bar{u}^r(p)\, v^s(p) = \bar{v}^r(p)\, u^s(p) = 0$.
Be ful since $u^{r\dagger}(p)\, v^s(p) \neq 0$ and $v^{r\dagger}(p)\, u^s(p) \neq 0$



The u and v are orthogonal to each other
$\bar{u}^r(p) v^s(p) = \bar{v}^r(p) u^s(p) = 0$.
Be careful since $u^{r\dagger}(p) v^s(p) \neq 0$ and $v^{r\dagger}(p) u^s(p) \neq 0$
but $u^{r\dagger}(\vec{p}) v^s(-\vec{p}) = v^{r\dagger}(-\vec{p}) u^s(\vec{p}) = 0$

$(\gamma^\mu p_\mu + m)v(p) = 0.$

$$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar\sigma}\,\eta^s \end{pmatrix}, \; s=1,2$$

Normalization conditions for v:

$\bar{v}^r(p)\,v^s(p) = -2m\delta^{rs}$, $v^{r\dagger}(p)\,v^s(p) = +2E_p\delta^{rs}$

The u and v are orthogonal to each other:

$\bar{u}^r(p)\,v^s(p) = \bar{v}^r(p)\,u^s(p) = 0.$

Be careful since $u^{r\dagger}(p)\,v^s(p) \neq 0$ and $v^{r\dagger}(p)\,u^s(p) \neq 0$

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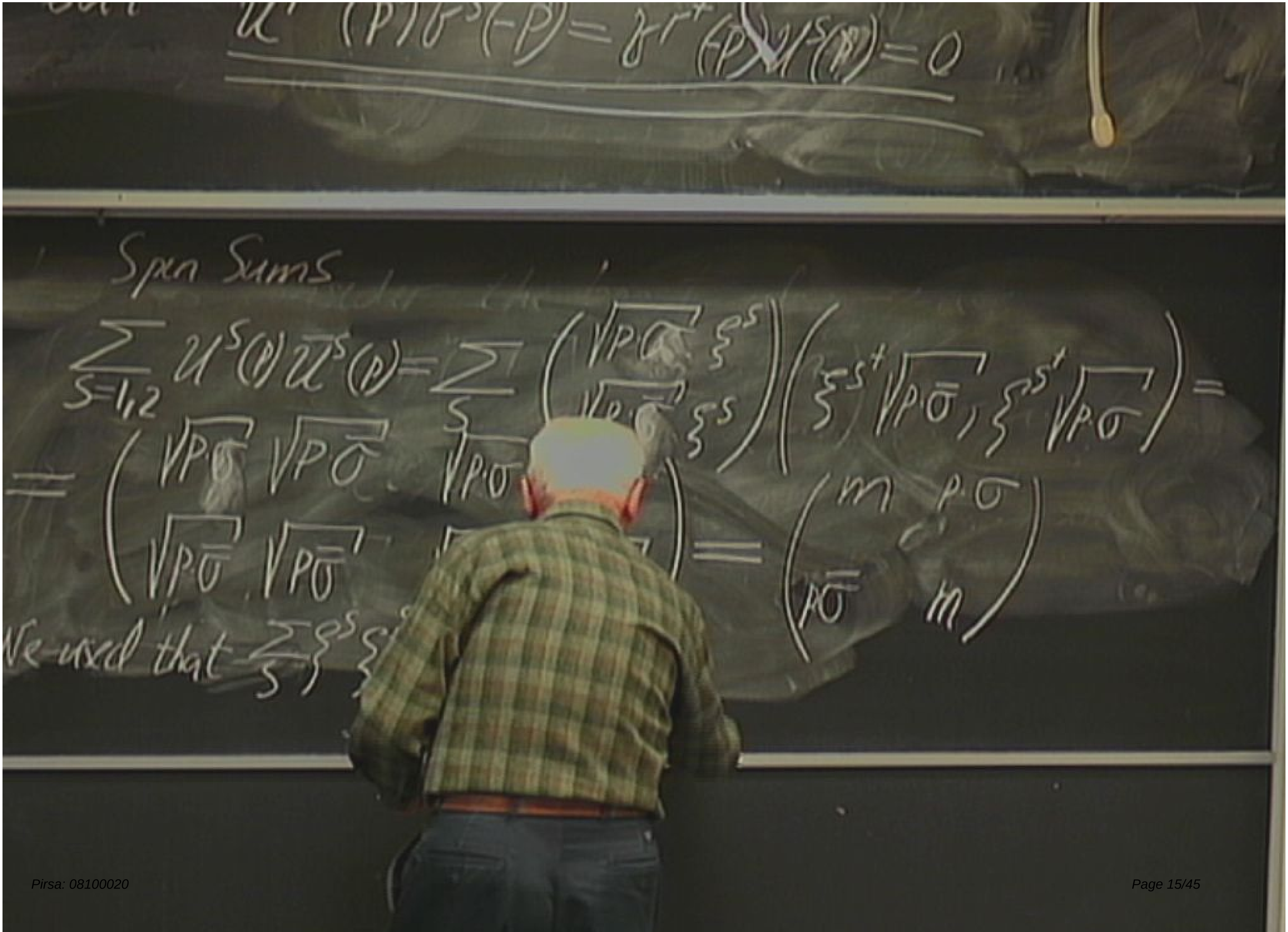
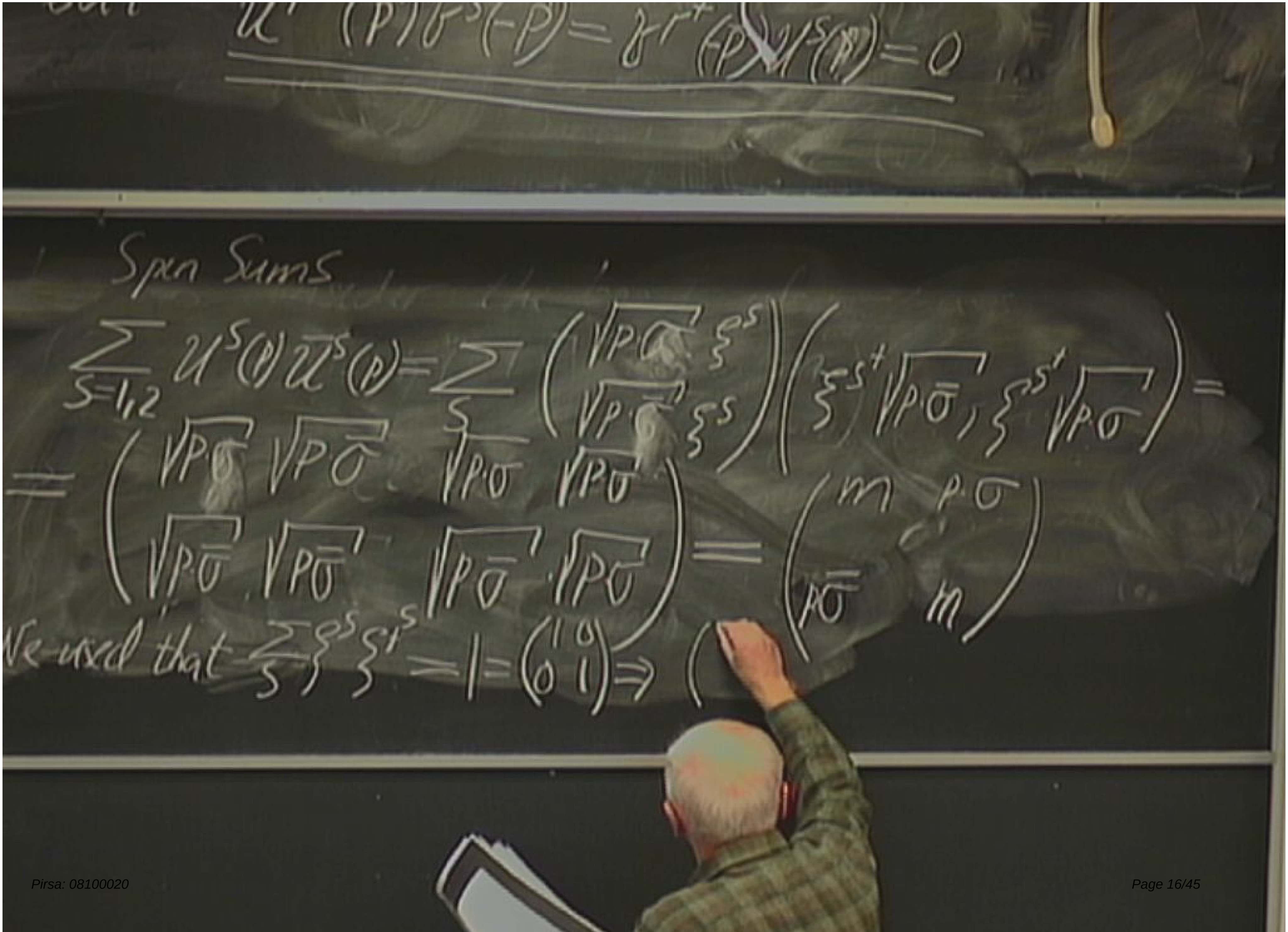

Spin Sums
We used that


$\bar{u}^{r}(p)\, v^{s}(-p) = \bar{v}^{r}(-p)\, u^{s}(p) = 0$
Spin Sums
$\sum_{s=1,2} u^s(p)\,\bar{u}^s(p) = \sum_s \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s \end{pmatrix} \left(\xi^{s\dagger}\sqrt{p\cdot\bar{\sigma}},\ \xi^{s\dagger}\sqrt{p\cdot\sigma} \right) =$
$= \begin{pmatrix} \sqrt{p\cdot\sigma}\sqrt{p\cdot\bar{\sigma}} & \sqrt{p\cdot\sigma}\sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\bar{\sigma}} & \sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma} \end{pmatrix} = \begin{pmatrix} m & p\cdot\sigma \\ p\cdot\bar{\sigma} & m \end{pmatrix}$
We used that $\sum_s \xi^s \xi^{s\dagger} = 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow ($

$$u^{r\dagger}(p)\,v^s(-p) = v^{r\dagger}(-p)\,u^s(p) = 0$$

Spin Sums

$$\sum_{s=1,2} u^s(p)\bar{u}^s(p) = \sum_s \begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar\sigma}\,\xi^s\end{pmatrix}\left(\xi^{s\dagger}\sqrt{p\cdot\bar\sigma},\ \xi^{s\dagger}\sqrt{p\cdot\sigma}\right) =$$

$$= \begin{pmatrix}\sqrt{p\cdot\sigma}\sqrt{p\cdot\bar\sigma} & \sqrt{p\cdot\sigma}\sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar\sigma}\sqrt{p\cdot\bar\sigma} & \sqrt{p\cdot\bar\sigma}\cdot\sqrt{p\cdot\sigma}\end{pmatrix} = \begin{pmatrix} m & p\cdot\sigma \\ p\cdot\bar\sigma & m\end{pmatrix}$$

$$\sum_s \xi^s\xi^{s\dagger} = 1 = \begin{pmatrix}1&0\\0&1\end{pmatrix} \Rightarrow \begin{pmatrix}1\\0\end{pmatrix}(1\ \ 0) + \begin{pmatrix}0\\1\end{pmatrix}\cdot(0\ \ 1) = \begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}0&0\\0&1\end{pmatrix}$$

$$u^{r\dagger}(p)\, v^s(-p) = v^{r\dagger}(-p)\, u^s(p) = 0$$

Spin Sums

$$\sum_{s=1,2} u^s(p)\bar{u}^s(p) = \sum_s \begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar\sigma}\,\xi^s\end{pmatrix}\begin{pmatrix}\xi^{s\dagger}\sqrt{p\cdot\bar\sigma}, & \xi^{s\dagger}\sqrt{p\cdot\sigma}\end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma}\sqrt{p\cdot\bar\sigma} & \sqrt{p\cdot\sigma}\sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar\sigma}\sqrt{p\cdot\bar\sigma} & \sqrt{p\cdot\bar\sigma}\sqrt{p\cdot\sigma}\end{pmatrix} = \begin{pmatrix} m & p\cdot\sigma \\ p\cdot\bar\sigma & m\end{pmatrix}$$

We used that $\sum_s \xi^s \xi^{s\dagger} = 1 = \begin{pmatrix}1&0\\0&1\end{pmatrix} \Rightarrow \begin{pmatrix}1\\0\end{pmatrix}\cdot(1\ 0) + \begin{pmatrix}0\\1\end{pmatrix}\cdot(0\ 1) = \begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}0&0\\0&1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

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Spin Sums

$$\sum_{s=1,2} u^s(p)\,\bar{u}^s(p) = \sum_s \begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s\end{pmatrix}\begin{pmatrix}\xi^{s\dagger}\sqrt{p\cdot\bar{\sigma}}, & \xi^{s\dagger}\sqrt{p\cdot\sigma}\end{pmatrix} =$$

$$= \begin{pmatrix}\sqrt{p\sigma}\sqrt{p\bar{\sigma}} & \sqrt{p\cdot\sigma}\sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar{\sigma}}\sqrt{p\bar{\sigma}} & \sqrt{p\bar{\sigma}}\cdot\sqrt{p\sigma}\end{pmatrix} = \begin{pmatrix} m & p\cdot\sigma \\ p\bar{\sigma} & m\end{pmatrix}$$

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$u^{r\dagger}(p)\, v^s(-p) = v^{r\dagger}(-p)\, u^s(p) = 0$

Spin Sums

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$$= \begin{pmatrix} \sqrt{p\sigma}\sqrt{p\bar\sigma} & \sqrt{p\sigma}\sqrt{p\sigma} \\ \sqrt{p\bar\sigma}\sqrt{p\bar\sigma} & \sqrt{p\bar\sigma}\cdot\sqrt{p\sigma} \end{pmatrix} = \begin{pmatrix} m & p\cdot\sigma \\ p\bar\sigma & m \end{pmatrix} = \gamma p + m$$

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Similarly, for $v^{(s)}$, $\sum_{s=1}^{2} v^s(p)\bar{v}^s(p) = \gamma p - m$

$$\sum_{s=1}^{2} u^s(p)\bar{u}^s(p) = \gamma p + m$$

Similarly, for $v^{(s)}$, $\sum_{s=1}^{2} v^s(p)\bar{v}^s(p) = \gamma p - m$

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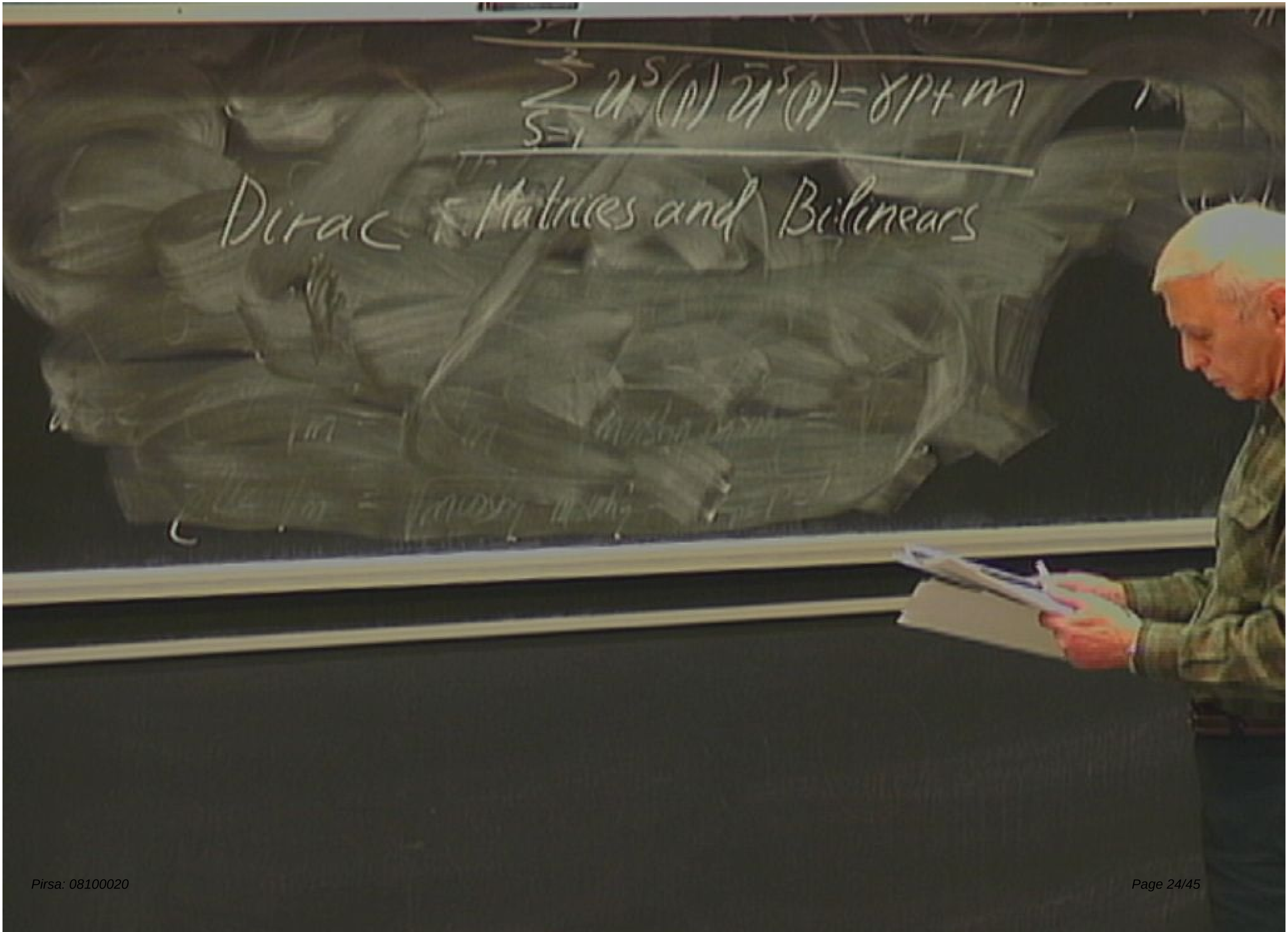
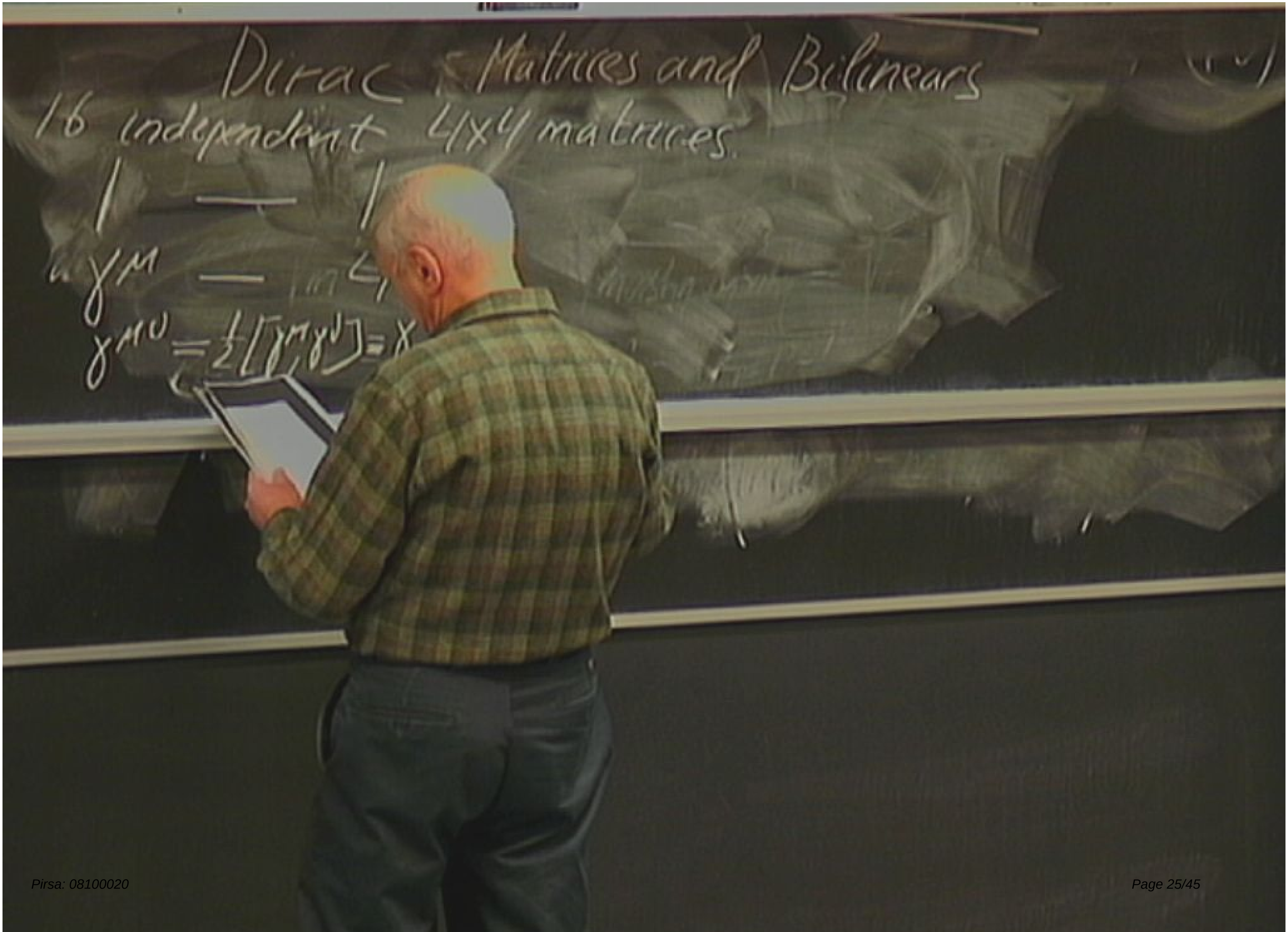
Similarly, for $v^{(s)}$, $\sum_{s=1}^{2} v^s(p)\bar{v}^s(p) = \gamma p - m$ $\quad \gamma \cdot p = \gamma^\mu p_\mu$

$\sum_{s=1}^{2} u^s(p)\bar{u}^s(p) = \gamma p + m$


$\sum_{s=1}^{2} u^s(p)\,\bar{u}^s(p) = \gamma p + m$
Dirac Matrices and Bilinears


Dirac Matrices and Bilinears
16 independent 4x4 matrices
1 — 1
γ^μ — 4
γ^μν = ½[γ^μ, γ^ν] =

Dirac Matrices and Bilinears

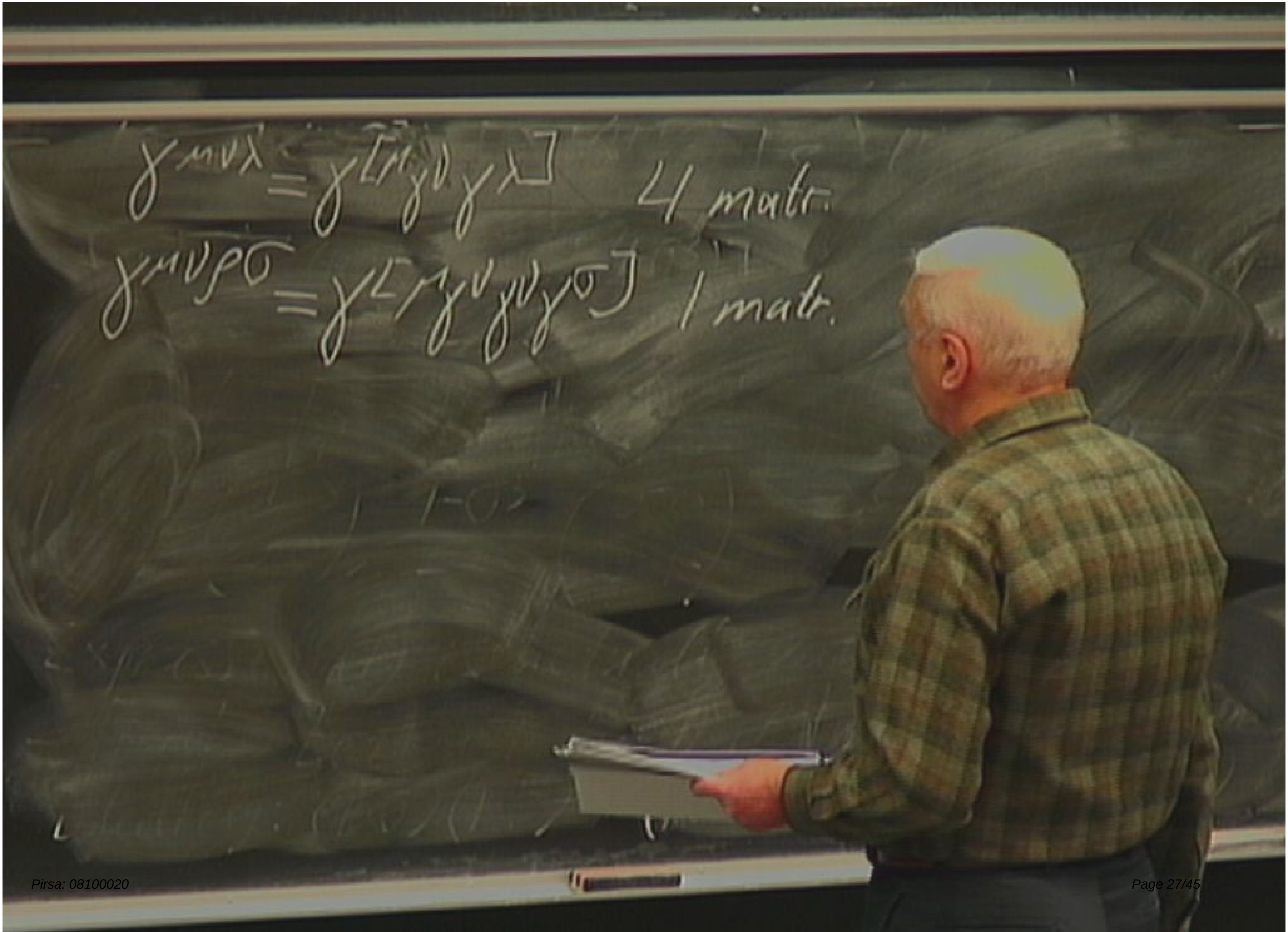
16 independent 4x4 matrices.

1 — 1 matr.

γ^μ — 4 matr.

$\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu] = \gamma^{[\mu}\gamma^{\nu]} \equiv -i\sigma^{\mu\nu}$ — 6 matr

$\gamma^{\mu\nu\lambda}$


$\gamma^{\mu\nu\lambda} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\lambda]}$ 4 matr.
$\gamma^{\mu\nu\rho\sigma} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]}$ 1 matr.

$\gamma^{\mu\nu\lambda} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\lambda]}$ 4 matr.

$\gamma^{\mu\nu\rho\sigma} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]}$ 1 matr.

$$\bar\psi \gamma^{\mu\nu} \psi \xrightarrow{\Lambda} \bar\psi \Lambda_{1/2}^{-1} \left(\tfrac{1}{2}[\gamma^{\mu}, \gamma^{\nu}]\right) \Lambda_{1/2} \psi$$

$\gamma^{\mu\nu\lambda} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\lambda]}$ 4 matr.

$\gamma^{\mu\nu\rho\sigma} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]}$ 1 matr.

$\bar\psi \gamma^{\mu\nu} \psi \xrightarrow{\Lambda} \bar\psi \Lambda_{1/2}^{-1} \left(\frac{1}{2}[\gamma^{\mu},\gamma^{\nu}]\right) \Lambda_{1/2} \psi = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} \bar\psi \gamma^{\alpha\beta} \psi.$

$\frac{1}{2} \Lambda_{1/2}^{-1} (\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}) \Lambda_{1/2} = \frac{1}{2} \Lambda_{1/2}^{-1} \gamma^{\mu} \Lambda_{1/2} \Lambda_{1/2}^{-1} \gamma^{\nu} - \frac{1}{2} \Lambda_{1/2}^{-1} \gamma^{\nu} \Lambda_{1/2} \Lambda_{1/2}^{-1} \gamma^{\mu} \Lambda_{1/2} -$

$$\gamma^{\mu\nu\lambda} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\lambda]} \quad 4 \text{ matr.}$$

$$\gamma^{\mu\nu\rho\sigma} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]} \quad 1 \text{ matr.}$$

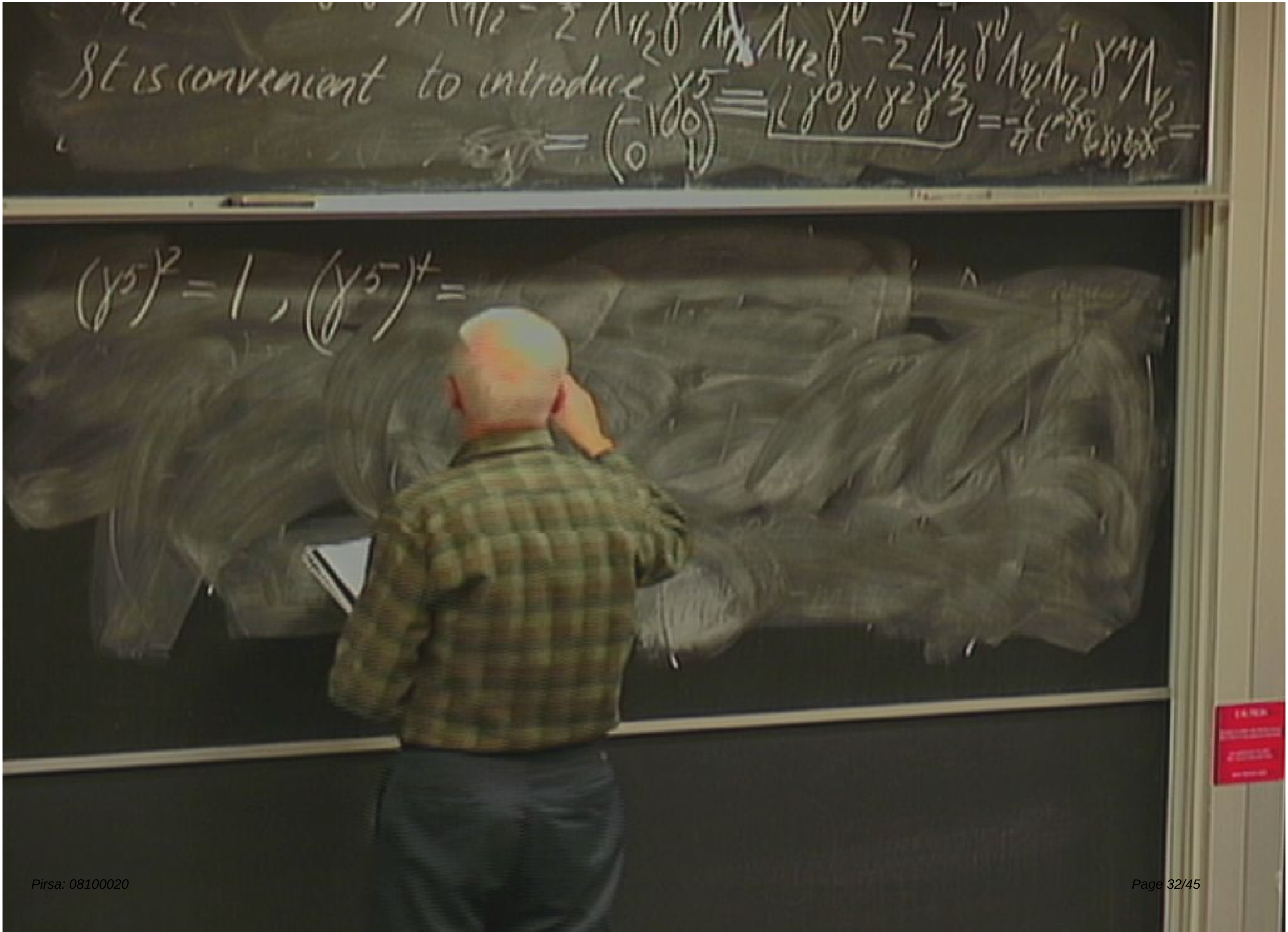
$$\bar{\psi}\gamma^{\mu\nu}\psi \xrightarrow{\Lambda} \bar{\psi}\Lambda_{1/2}^{-1}\left(\tfrac{1}{2}[\gamma^{\mu},\gamma^{\nu}]\right)\Lambda_{1/2}\psi = \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}\bar{\psi}\gamma^{\alpha\beta}\psi.$$

$$\tfrac{1}{2}\Lambda_{1/2}^{-1}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})\Lambda_{1/2} = \tfrac{1}{2}\Lambda_{1/2}^{-1}\gamma^{\mu}\Lambda_{1/2}\Lambda_{1/2}^{-1}\gamma^{\nu} - \tfrac{1}{2}\Lambda_{1/2}^{-1}\gamma^{\nu}\Lambda_{1/2}\Lambda_{1/2}^{-1}\gamma^{\mu}\Lambda_{1/2}$$

$$\bar\psi \gamma^{\mu\nu} \psi \longrightarrow \bar\psi \Lambda_{1/2}^{-1}\left(\tfrac{i}{2}[\gamma^{\mu},\gamma^{\nu}]\right)\Lambda_{1/2}\, \psi = \Lambda_{\alpha}{}^{\mu}\Lambda_{\beta}{}^{\nu}\, \bar\psi \gamma^{\alpha\beta} \psi.$$

$$\frac{1}{2}\Lambda_{1/2}^{-1}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)\Lambda_{1/2} = \frac{1}{2}\Lambda_{1/2}^{-1}\gamma^\mu\Lambda_{1/2}\Lambda_{1/2}^{-1}\gamma^\nu - \frac{1}{2}\Lambda_{1/2}^{-1}\gamma^\nu\Lambda_{1/2}\Lambda_{1/2}^{-1}\gamma^\mu\Lambda_{1/2}$$

It is convenient to introduce $\gamma^5 \equiv \boxed{i\gamma^0\gamma^1\gamma^2\gamma^3}$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$(\gamma^5)^2 = 1$, $(\gamma^5)^\dagger =$

It is convenient to introduce $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma =$

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$(\gamma^5)^2 = 1$, $(\gamma^5)^\dagger = \gamma^5$, $\{\gamma^5, \gamma^\mu\} = \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = -\gamma^\mu \Rightarrow$

$\Rightarrow [\gamma^5, S^{\mu\nu}] = 0$

$\ldots \gamma^\mu \Lambda_{1/2} - \frac{i}{2}\Lambda_{1/2}\gamma^\mu\Lambda_{1/2} \ldots \Lambda_{1/2}\gamma^0 - \frac{1}{2}\Lambda_{1/2}^{\dagger}\gamma^0\Lambda_{1/2}\Lambda_{1/2}^{-1}\gamma^\mu\Lambda_{1/2}$

It is convenient to introduce $\gamma^5 \equiv \boxed{i\gamma^0\gamma^1\gamma^2\gamma^3} = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma =$

$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$(\gamma^5)^2 = 1, \quad (\gamma^5)^\dagger = \gamma^5, \quad \{\gamma^5, \gamma^\mu\} = \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = -\gamma^\mu \Rightarrow$

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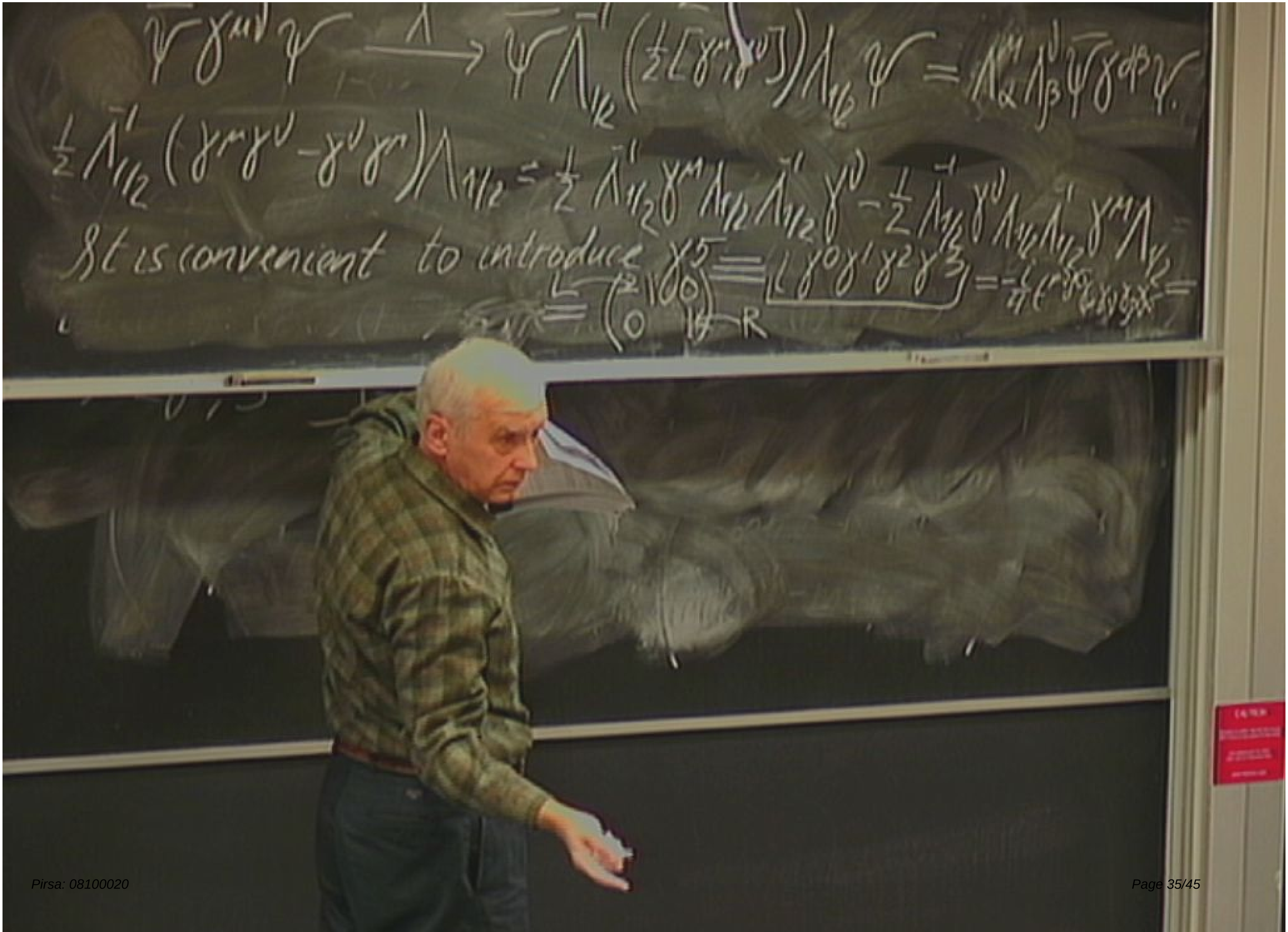
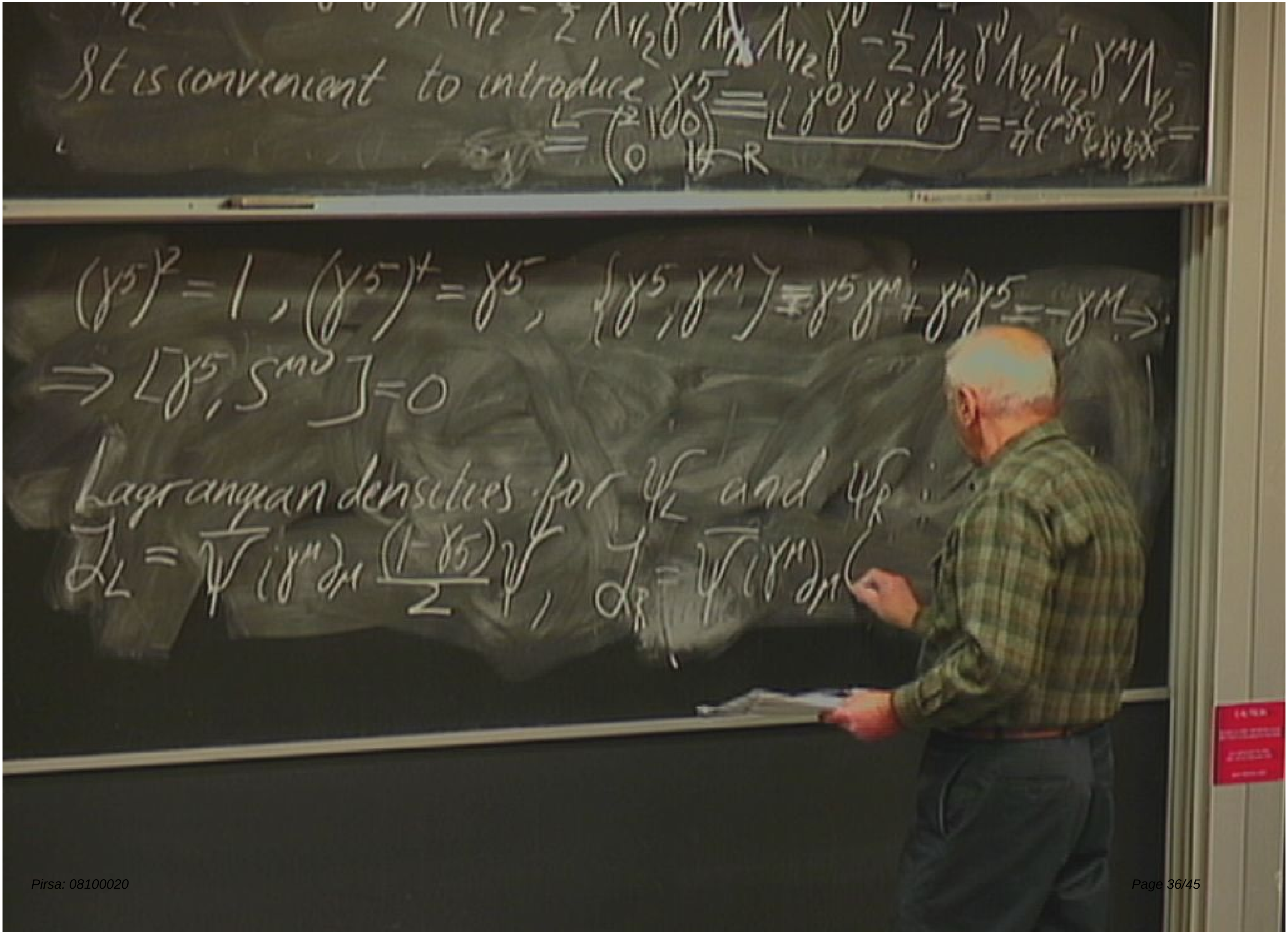

$\bar\psi\gamma^{\mu\nu}\psi \xrightarrow{\Lambda} \bar\psi\Lambda^{-1}_{1/2}\left(\frac{1}{2}[\gamma^\mu,\gamma^\nu]\right)\Lambda_{1/2}\psi = \Lambda^\mu_\alpha\Lambda^\nu_\beta\bar\psi\gamma^{\alpha\beta}\psi.$
$\frac{1}{2}\Lambda^{-1}_{1/2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\Lambda_{1/2} = \frac{1}{2}\Lambda^{-1}_{1/2}\gamma^\mu\Lambda_{1/2}\Lambda^{-1}_{1/2}\gamma^\nu - \frac{1}{2}\Lambda^{-1}_{1/2}\gamma^\nu\Lambda_{1/2}\Lambda^{-1}_{1/2}\gamma^\mu\Lambda_{1/2}$
It is convenient to introduce $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon^{\alpha\beta\gamma\delta}\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta =$
$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ L R


It is convenient to introduce $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
$(\gamma^5)^2 = 1$, $(\gamma^5)^\dagger = \gamma^5$, $\{\gamma^5, \gamma^\mu\} = \gamma^5\gamma^\mu + \gamma^\mu\gamma^5$
$\Rightarrow [\gamma^5, S^{\mu\nu}] = 0$
Lagrangian densities for ψ_L and ψ_R:
$\mathcal{L}_L = \bar{\psi} i\gamma^\mu\partial_\mu \frac{(1-\gamma_5)}{2}\psi$, $\mathcal{L}_R = \bar{\psi} i\gamma^\mu\partial_\mu ($

$$\ldots \Lambda_{1/2} - \frac{1}{2}\Lambda_{1/2}\gamma^{\mu}\Lambda_{1/2}\ldots\Lambda_{1/2}\gamma^0 - \frac{1}{2}\Lambda_{1/2}\gamma^0\Lambda_{1/2}\Lambda_{1/2}^{-1}\gamma^{\mu}\Lambda_{1/2}$$

It is convenient to introduce $\gamma^5 \equiv \boxed{i\gamma^0\gamma^1\gamma^2\gamma^3} = -\frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma =$

$$\equiv \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{matrix} L \\ R \end{matrix}$$

$$(\gamma^5)^2 = 1,\ (\gamma^5)^\dagger = \gamma^5,\ \{\gamma^5, \gamma^\mu\} \equiv \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0 \Rightarrow$$

$$\Rightarrow [\gamma^5, S^{\mu\nu}] = 0$$

Lagrangian densities for ψ_L and ψ_R:

$$\mathcal{L}_L = \bar{\psi}\,(i\gamma^\mu\partial_\mu)\frac{(1-\gamma_5)}{2}\psi,\quad \mathcal{L}_R = \bar{\psi}\,(i\gamma^\mu\partial_\mu)\frac{(1+\gamma_5)}{2}\psi.$$

1 scalar 1

γ^μ vector 4

$\sigma^{\mu\nu}$

2)

1	scalar	1
γ^μ	vector	4
$\sigma^{\mu\nu}$	tensor	6
$\gamma^\mu\gamma^5$	pseudo-vector	4
γ^5	pseudo-scalar	1
		16.

1 scalar 1

γ^μ vector 4

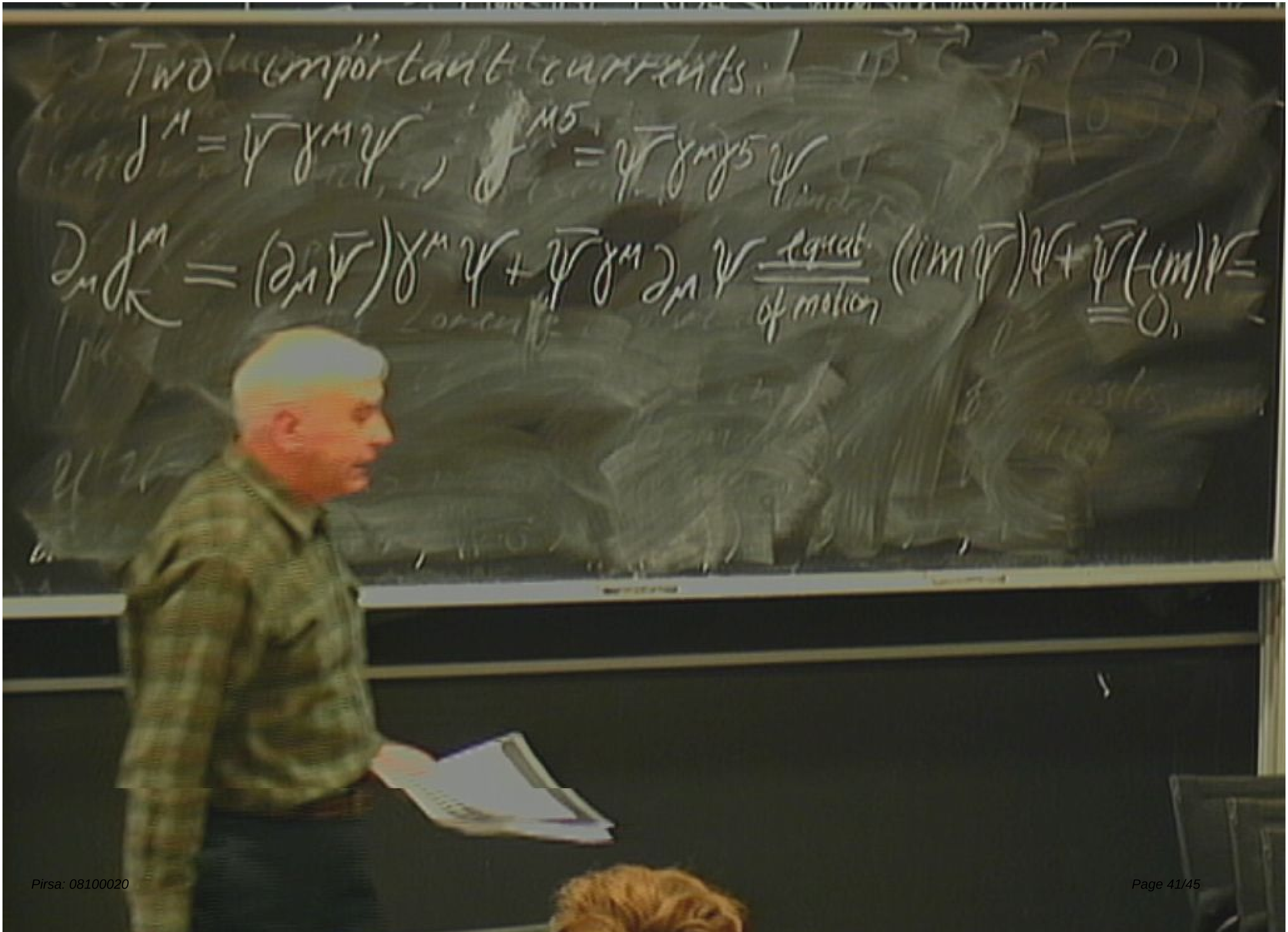
$\sigma^{\mu\nu}$ tensor 6

$\gamma^\mu\gamma^5$ pseudo-vector 4

γ^5 pseudo-scalar 1

$16.$

$\bar{\psi}\gamma^5\psi \xrightarrow{P} -\bar{\psi}\gamma^5\psi.$


Two important currents:
$J^\mu = \bar\psi \gamma^\mu \psi$, $J^{\mu 5} = \bar\psi \gamma^\mu \gamma^5 \psi$
$\partial_\mu J^\mu = (\partial_\mu \bar\psi)\gamma^\mu \psi + \bar\psi \gamma^\mu \partial_\mu \psi$
equat. of motion
$= 0$

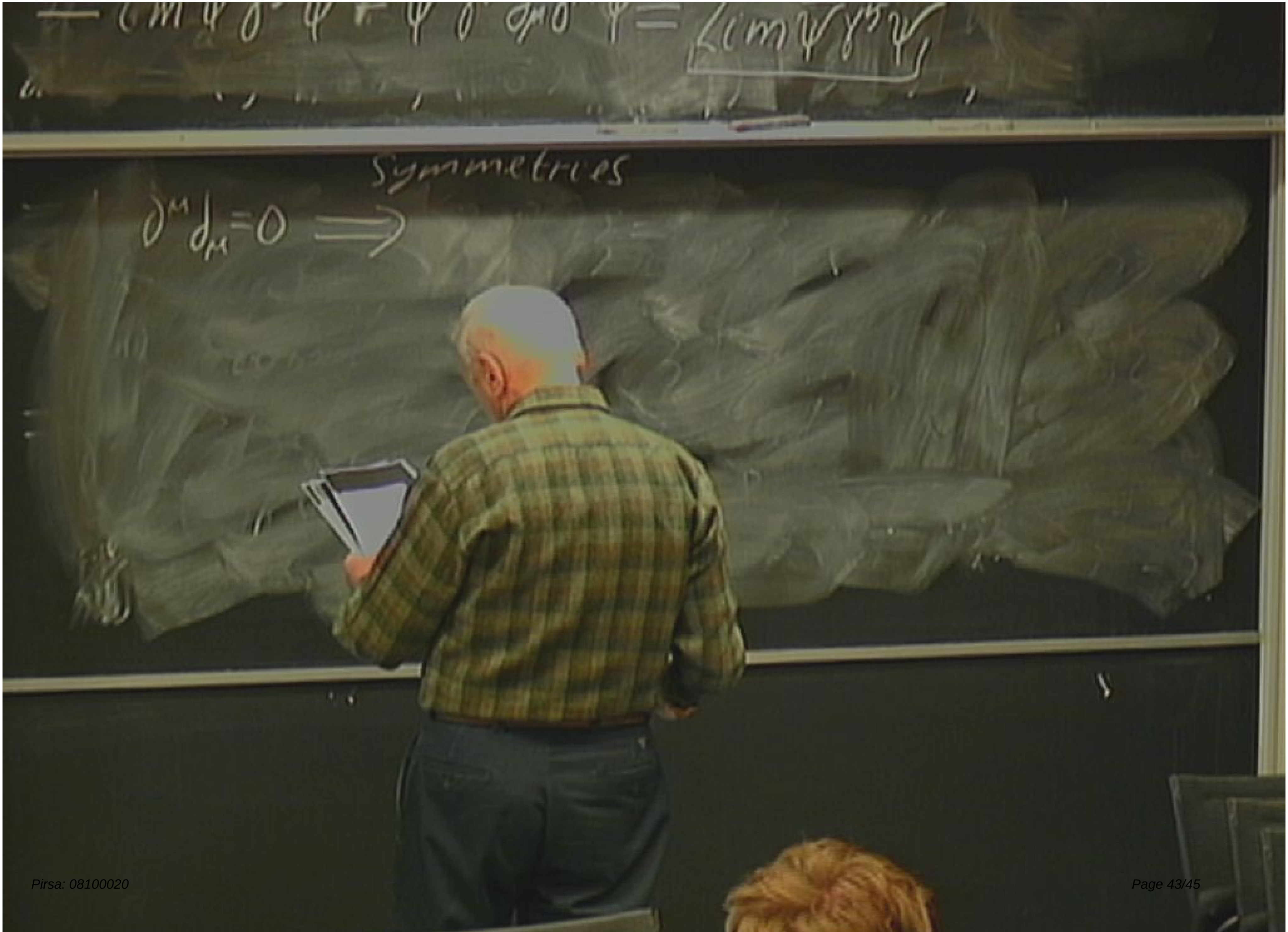
Two important currents:

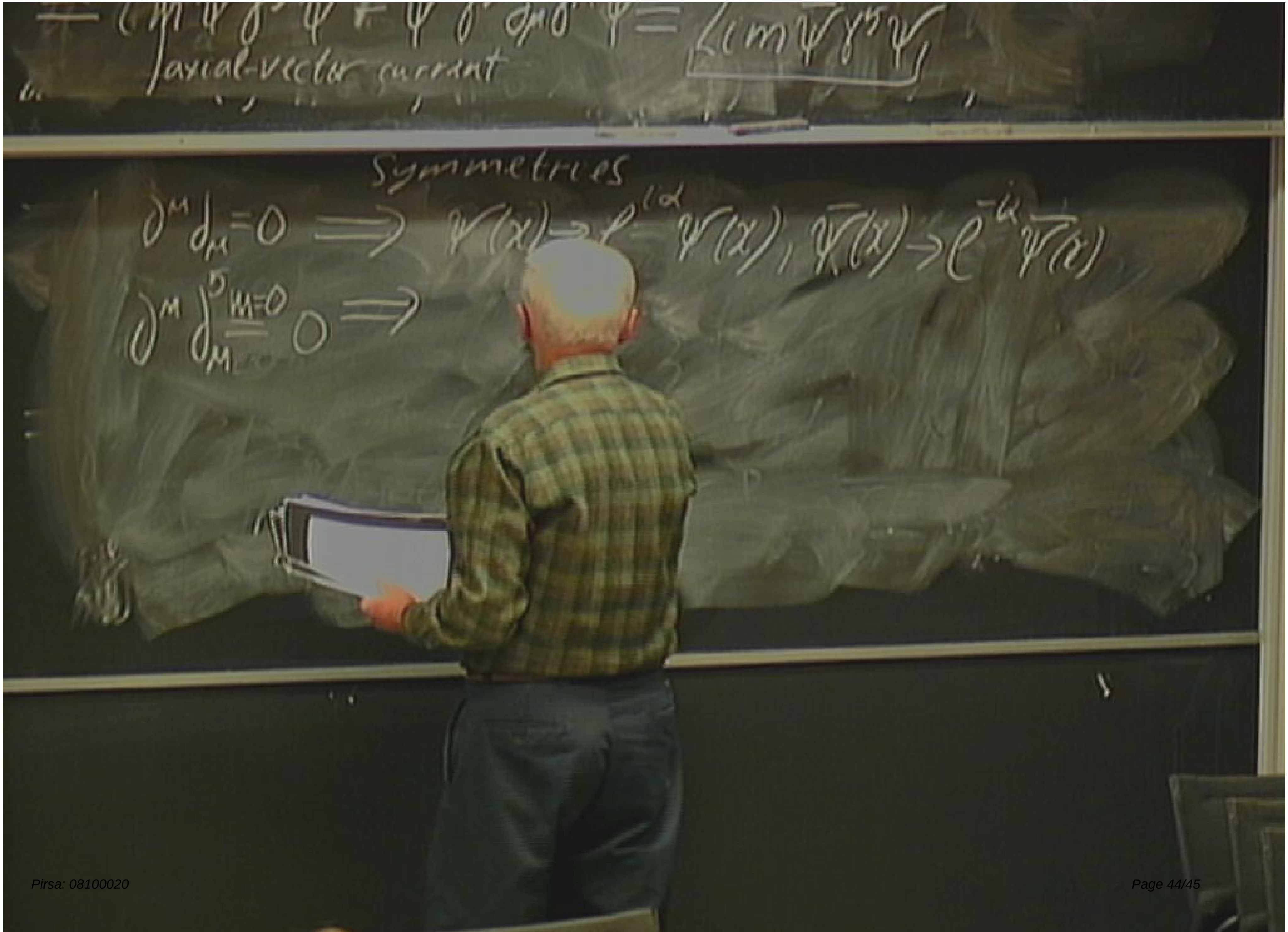
$$j^\mu = \bar\psi\gamma^\mu\psi, \quad j^{\mu 5} = \bar\psi\gamma^\mu\gamma^5\psi.$$

$$\partial_\mu j^\mu = (\partial_\mu\bar\psi)\gamma^\mu\psi + \bar\psi\gamma^\mu\partial_\mu\psi \underset{\text{of motion}}{\overset{\text{equat.}}{=}} (im\bar\psi)\psi + \bar\psi(-im)\psi = 0.$$

$$\partial_\mu j^{\mu 5} = 2im\bar\psi\gamma^5\psi \quad : \quad \partial_\mu j^{\mu 5} = (\partial_\mu\bar\psi)\gamma^\mu\gamma^5\psi + \bar\psi\partial_\mu\gamma^\mu\gamma^5\psi =$$

$$= im\bar\psi\gamma^5\psi + \bar\psi\gamma^5\partial_\mu\gamma^\mu\psi = \boxed{2im\bar\psi\gamma^5\psi},$$


Symmetries
$\partial^\mu j_\mu = 0 \Longrightarrow$


axial-vector current
Symmetries

$$= i m \bar\psi \gamma^5 \psi + \bar\psi \gamma^5 \partial_\mu \gamma^\mu \psi = \boxed{2 i m \bar\psi \gamma^5 \psi}$$

axial-vector current

symmetries

$$\partial^\mu J_\mu = 0 \implies \psi(x) \to e^{i\alpha}\psi(x),\ \bar\psi(x) \to e^{-i\alpha}\bar\psi(x)$$

$$\partial^\mu J^5_\mu \overset{m=0}{=} 0 \implies \psi(x) \to e^{i\alpha\gamma_5}\psi(x),\ \bar\psi(x) \to \bar\psi(x) e^{i\alpha\gamma_5}$$

chiral transformations

useful ~~two chiral~~ currents.

$$J^\mu_L = \bar\psi \gamma^\mu \frac{1-\gamma_5}{2}\psi,\quad J^\mu_R = \bar\psi \gamma^\mu \frac{1+\gamma_5}{2}\psi,\quad \bar\psi = \psi^+ \gamma^0 \implies \psi^+ e^{-i\alpha\gamma_5}\gamma_0 = \psi^+ \gamma_5 e^{i\alpha\gamma_5} = \bar\psi e^{i\alpha\gamma_5}$$

chiral currents