Title: Quantum Field Theory 1 - Lecture 7B

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

Then, $\bar{\psi}\psi = \sum_{\alpha=1}^{4} \bar{\psi}_\alpha \psi_\alpha$ is a scalar.

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \qquad \psi \sim \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \ \bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3, \bar{\psi}_4)$$

Euler-Lagrange (EL) equation for $\bar{\psi} \Rightarrow (i\gamma^\mu \partial_\mu - m)\psi$; EL equation for $\psi \Rightarrow -i\partial^\mu \bar{\psi}\gamma^\mu - m\bar{\psi} = 0.$

(Λx); In our case M

$$\bar{\Lambda}_{\frac{1}{2}}^{-1} \gamma^{\mu} \Lambda_{\frac{1}{2}} =$$

$\left.\begin{matrix} & i \\ 0 & \end{matrix}\right)$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; $\{\sigma^i, \sigma^j\} = 2\delta^{ij}$;

$(\sigma^i)^{+} = \sigma^i$

are unitary equivalent. The dimen-

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on for $\psi \Rightarrow -i\partial_\mu \bar{\psi}\gamma^\mu - m\bar{\psi} = 0.$

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$$\overline{\psi}\gamma^\mu\psi \xrightarrow{\Lambda} \overline{\psi}\,\underbrace{\Lambda_{\frac{1}{2}}^{-1}\gamma^\mu \Lambda_{\frac{1}{2}}}\psi$$

$$(S^{0i})^\dagger = -S^{0i};\ S^{0i}\gamma^0 = \frac{i}{2}[\gamma^0,\gamma^i]\gamma^0 = -\gamma^0 S^{0i} \Rightarrow (S^{\mu\nu})^\dagger\gamma^0 = \gamma^0 (S^{\mu\nu}).$$

Then, $\bar{\psi}\psi = \sum_{\alpha=1}^{4} \bar{\psi}_\alpha \psi_\alpha$ is a scalar.

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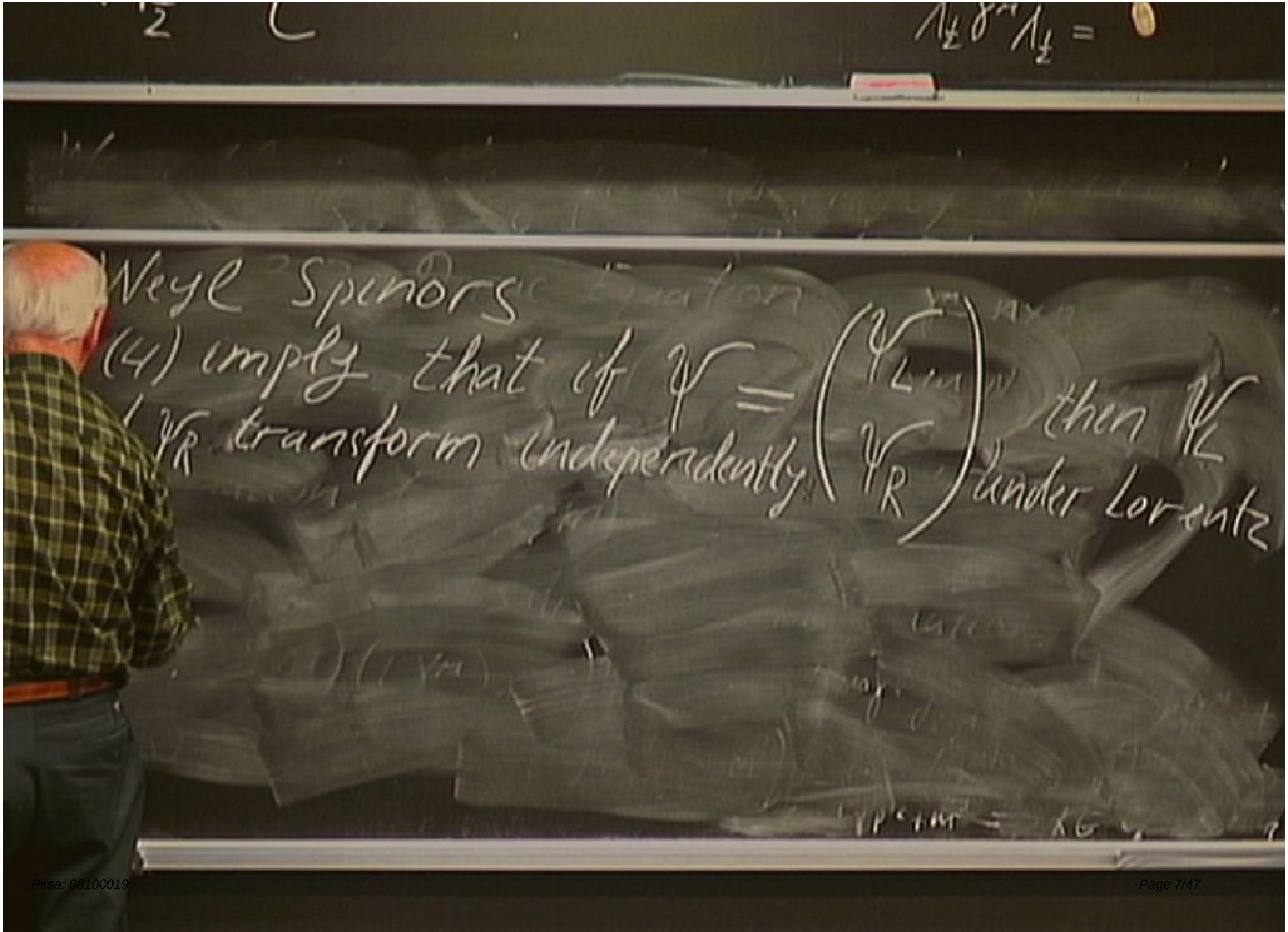
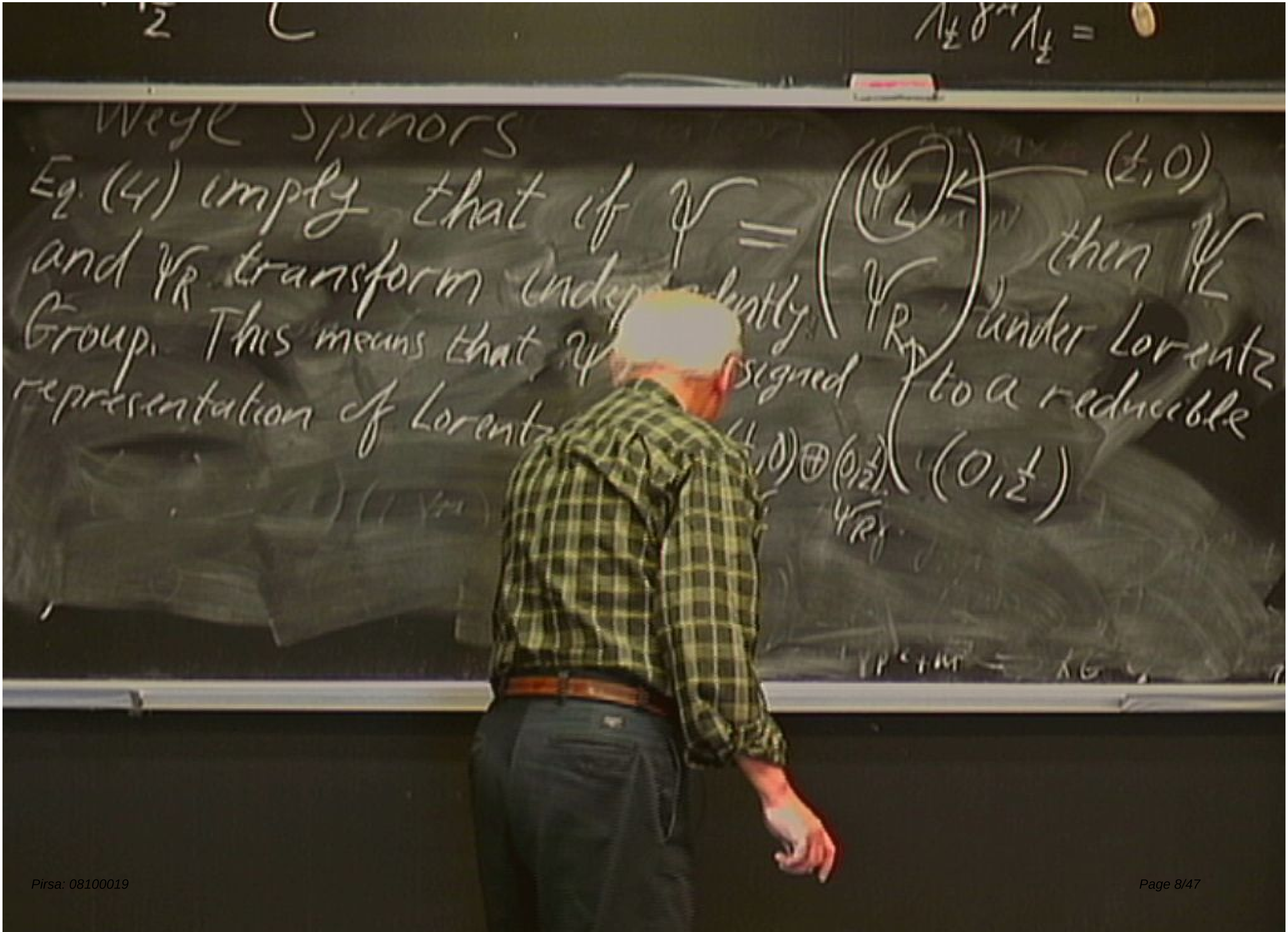
Euler-Lagrange (EL) equation for $\bar{\psi} \Rightarrow (i\gamma^\mu\partial_\mu - m)\psi$; EL equation for $\psi \Rightarrow -i\partial_\mu\bar{\psi}\gamma^\mu - m\bar{\psi} = 0$.

$$\bar{\psi}\gamma^\mu\psi \xrightarrow{\Lambda} \bar{\psi}\,\boxed{\Lambda_{\frac{1}{2}}^{-1}\gamma^\mu\Lambda_{\frac{1}{2}}}\,\psi = \Lambda^\mu_{\ \nu}\,\bar{\psi}\gamma^\nu\psi;\ \bar{\psi}\gamma^\mu\psi \text{ is a vector}$$


Weyl Spinors
(4) imply that if ψ = (ψ_L ψ_R) then ψ_L
ψ_R transform independently under Lorentz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(½,0)
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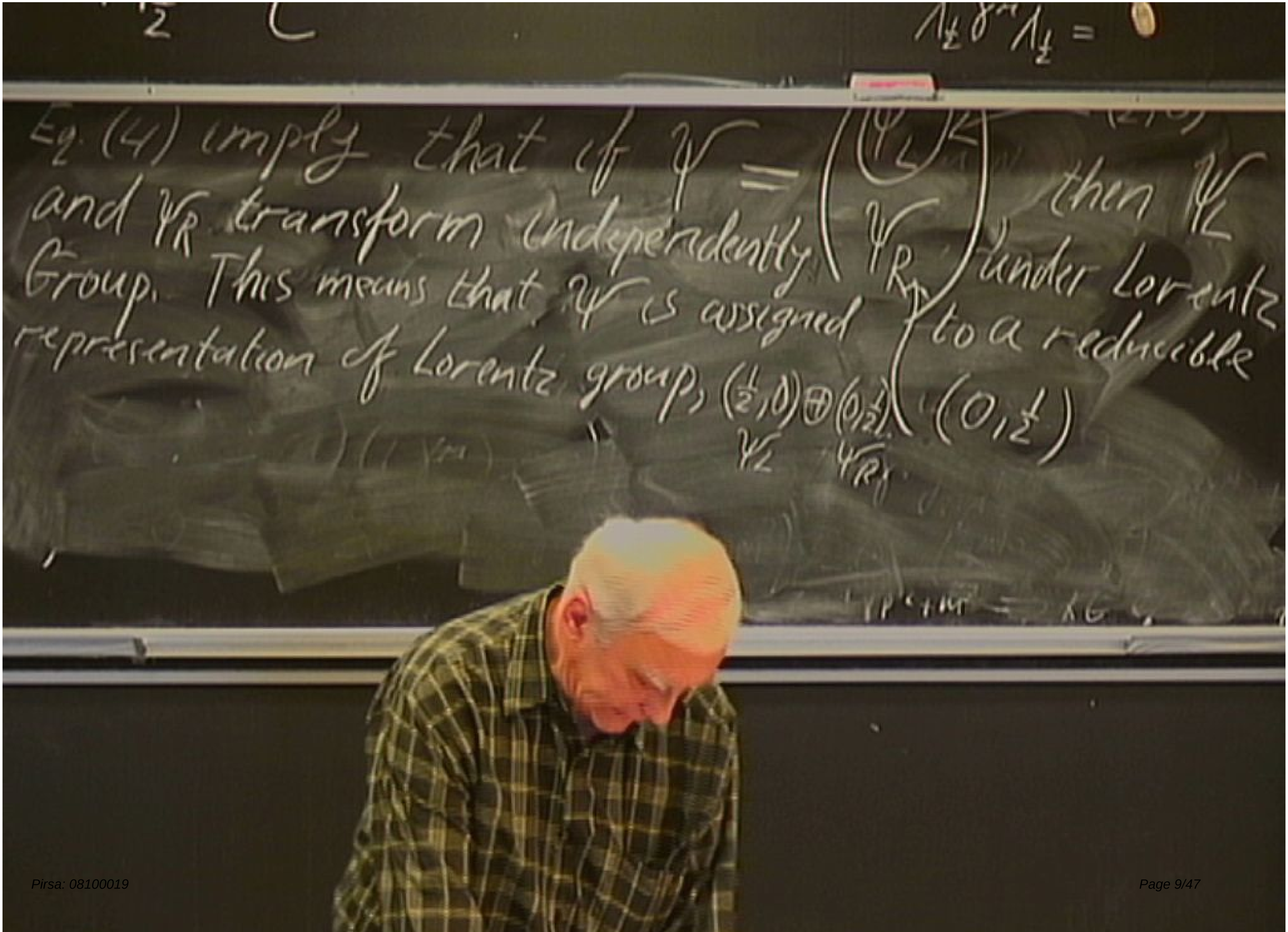
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representation of Lorentz group, (1/2,0)⊕(0,1/2)
ψL ψR
(0,1/2)

Eq. (4) imply that if $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ then ψ_L and ψ_R transform independently under Lorentz Group. This means that ψ is assigned to a reducible representation of Lorentz group, $\underset{\psi_L}{(\tfrac{1}{2},0)} \oplus \underset{\psi_R}{(0,\tfrac{1}{2})}$ $(0,\tfrac{1}{2})$

$\Lambda_{\frac{1}{2}} = e^{-\frac{i}{2}\omega_{\mu\nu} S^{\mu\nu}}$.

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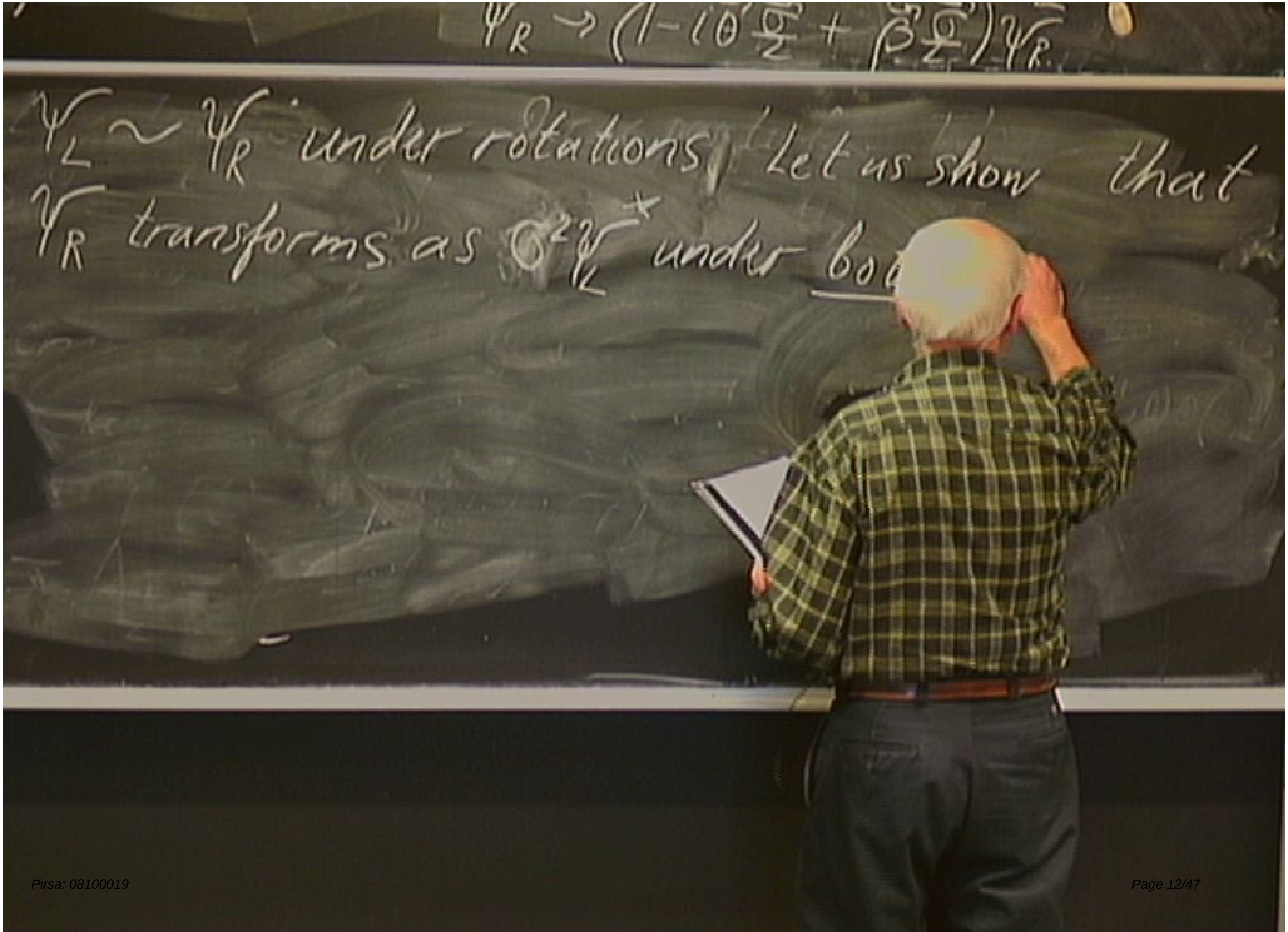
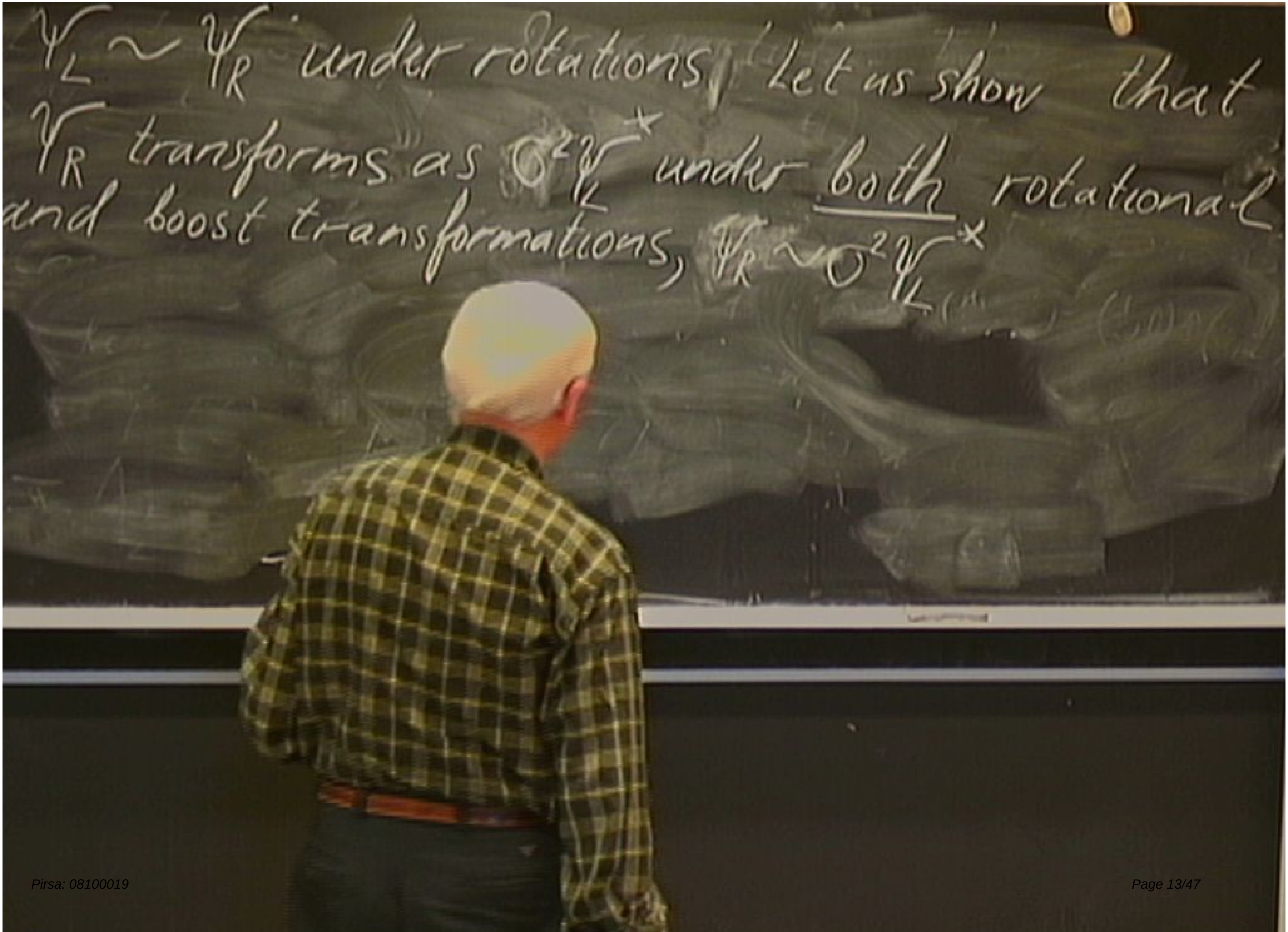
$$\Lambda_{\frac{1}{2}} = e^{-\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}} \xrightarrow[\text{trans}]{\text{infin.}} \psi_L \rightarrow \left(1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}}{2}\right)\psi_L$$

ψ_L ψ_R


$\psi_R \to (1 - i\theta\cdot\frac{\sigma}{2} + \beta\cdot\frac{\sigma}{2})\psi_R$
$\psi_L \sim \psi_R$ under rotations. Let us show that
ψ_R transforms as $\sigma^2\psi_L^*$ under bo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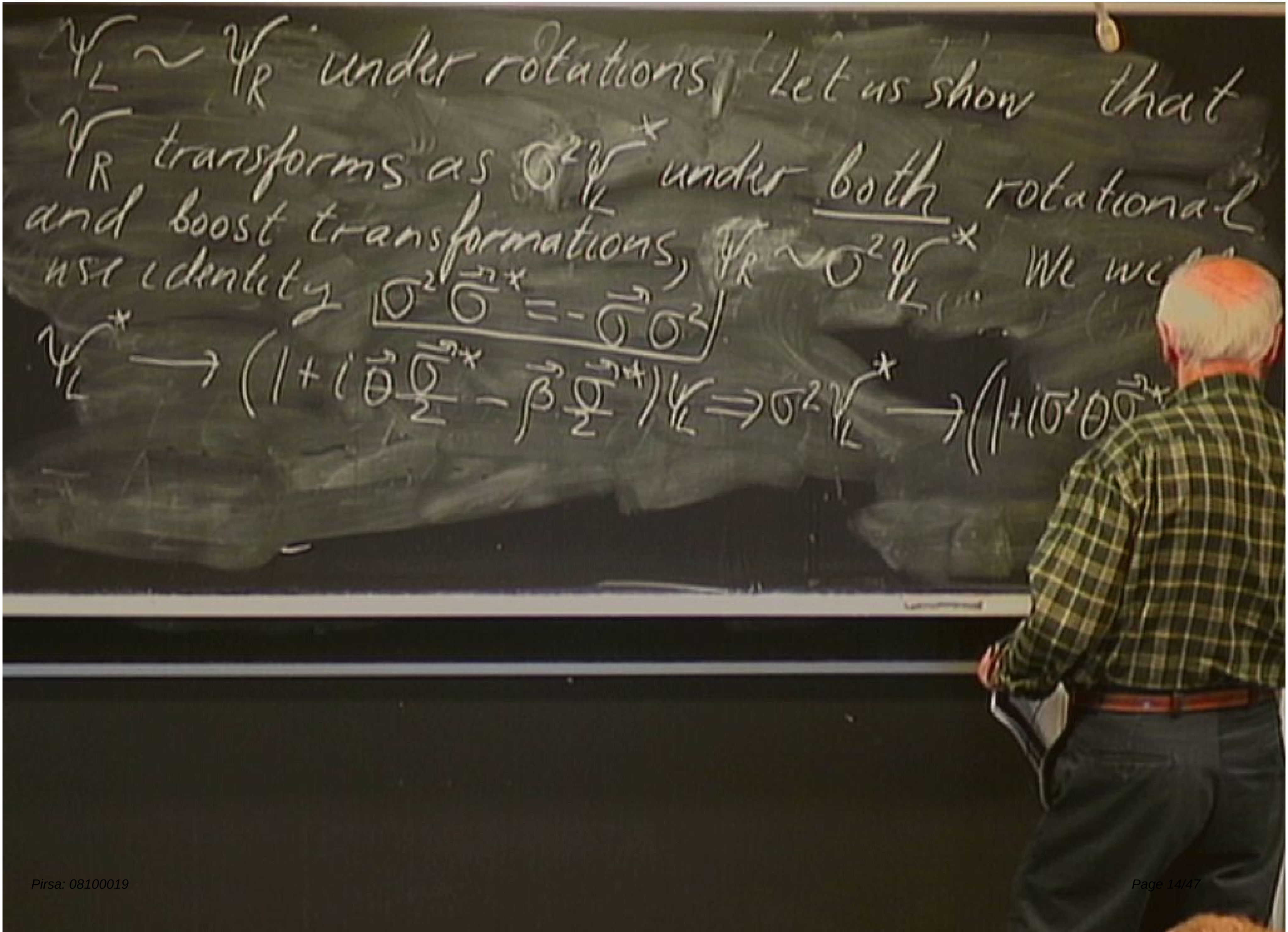
and boost transformations, $\psi_R \sim \sigma^2\psi_L^*$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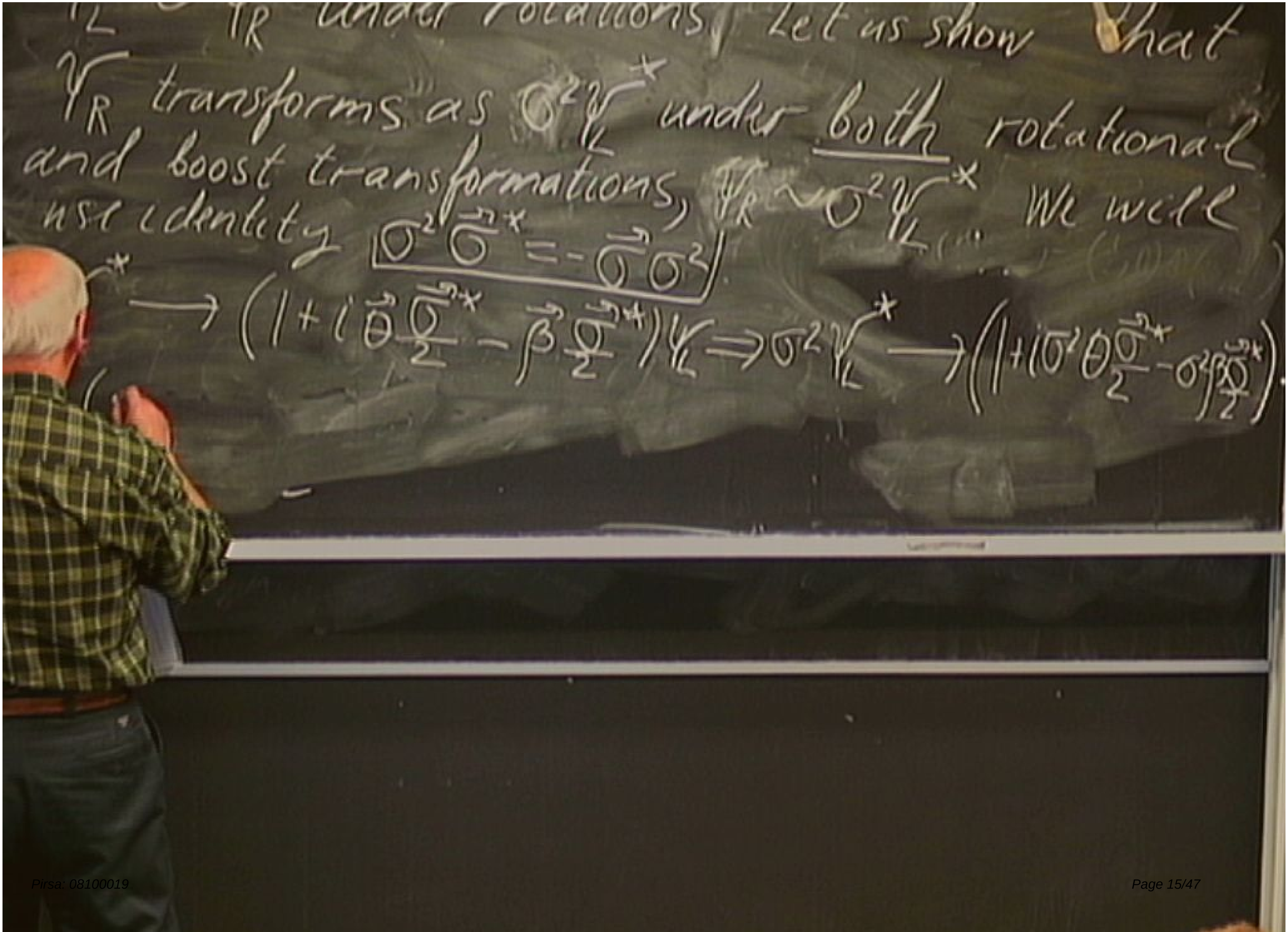
use identity $\sigma^2\vec{\sigma}^* = -\vec{\sigma}\sigma^2$
$\psi_L^* \longrightarrow (1 + i\vec{\theta}\frac{\vec{\sigma}^*}{2} - \vec{\beta}\frac{\vec{\sigma}^*}{2})\psi_L^* \Rightarrow \sigma^2\psi_L^* \longrightarrow (1 + i\sigma^2\theta\vec{\sigma}^*


ψ_L … ψ_R under rotations. Let us show that
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and boost transformations, $\psi_R \sim \sigma^2\psi_L^*$. We will
use identity $\boxed{\sigma^2\vec{\sigma}^* = -\vec{\sigma}\sigma^2}$
$\psi_L^* \longrightarrow \left(1 + i\vec{\theta}\frac{\vec{\sigma}^*}{2} - \vec{\beta}\frac{\vec{\sigma}^*}{2}\right)\psi_L \Rightarrow \sigma^2\psi_L^* \longrightarrow \left(1 + i\sigma^2\theta\frac{\vec{\sigma}^*}{2} - \sigma^2\beta\frac{\vec{\sigma}^*}{2}\right).$

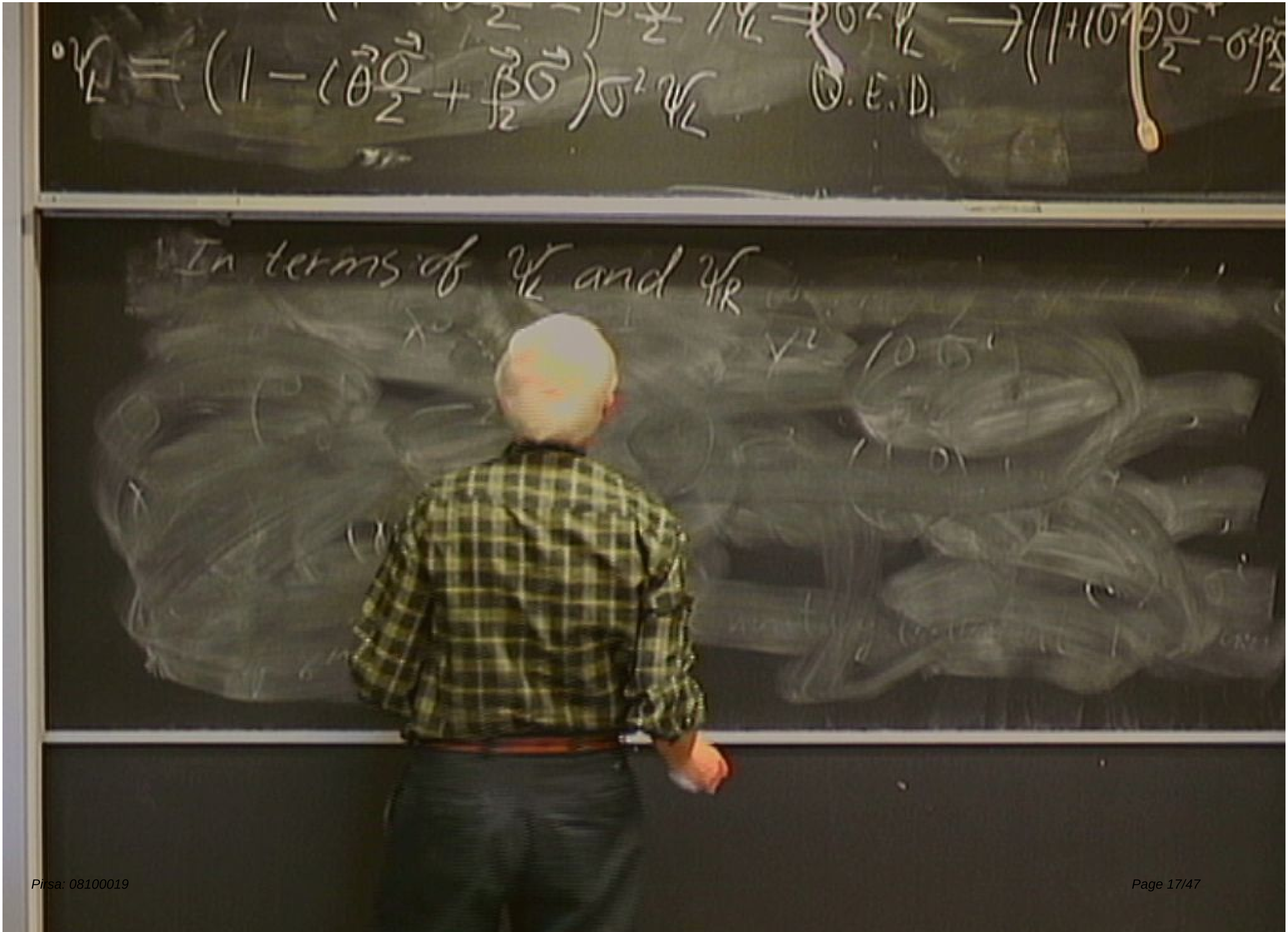
trans $\psi_L \to \left(1 - i\vec\theta\cdot\frac{\vec\sigma}{2} - \vec\beta\cdot\frac{\vec\sigma}{2}\right)\psi_L$

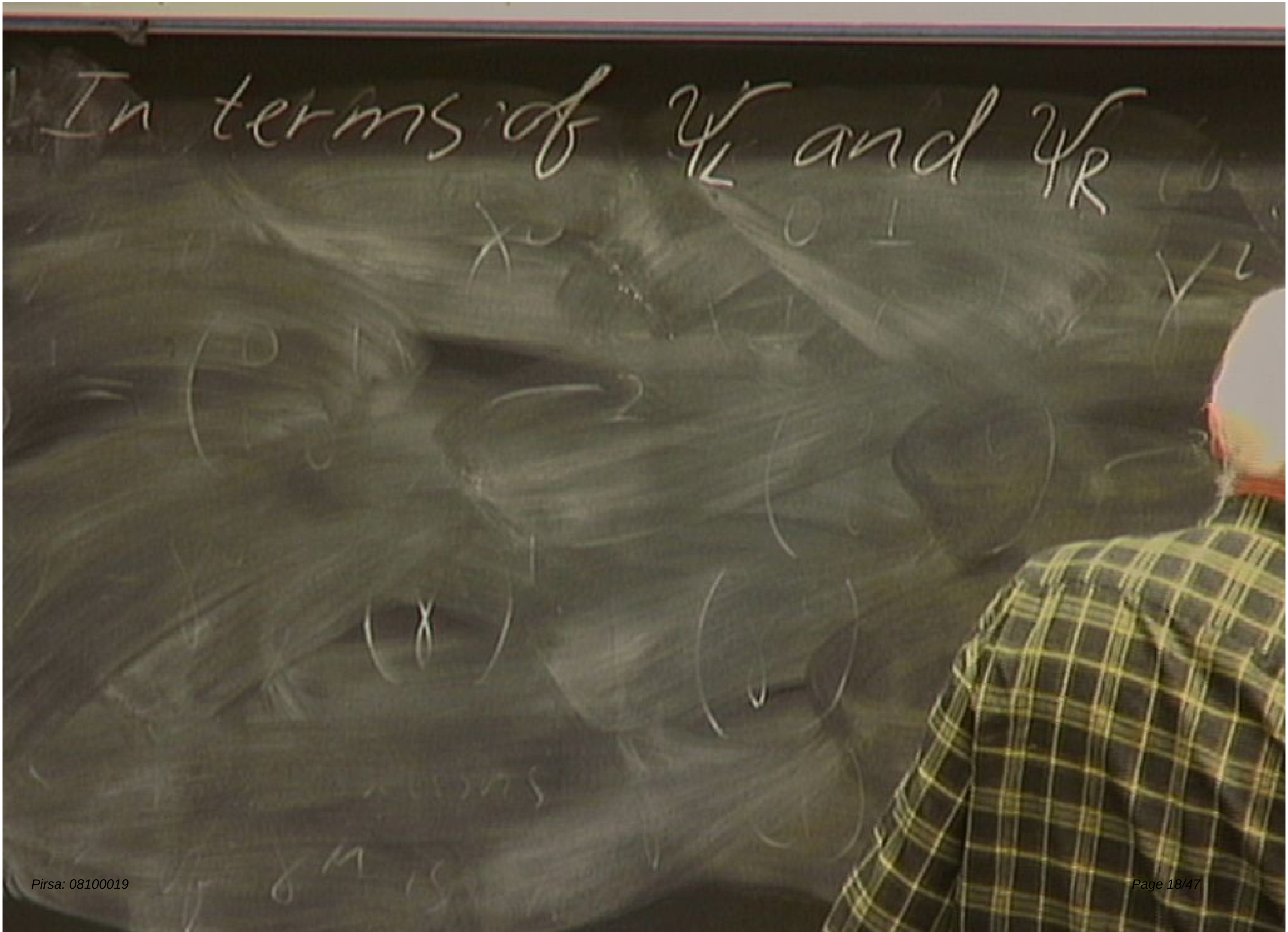
$\psi_R \to \left(1 - i\vec\theta\cdot\frac{\vec\sigma}{2} + \vec\beta\cdot\frac{\vec\sigma}{2}\right)\psi_R$

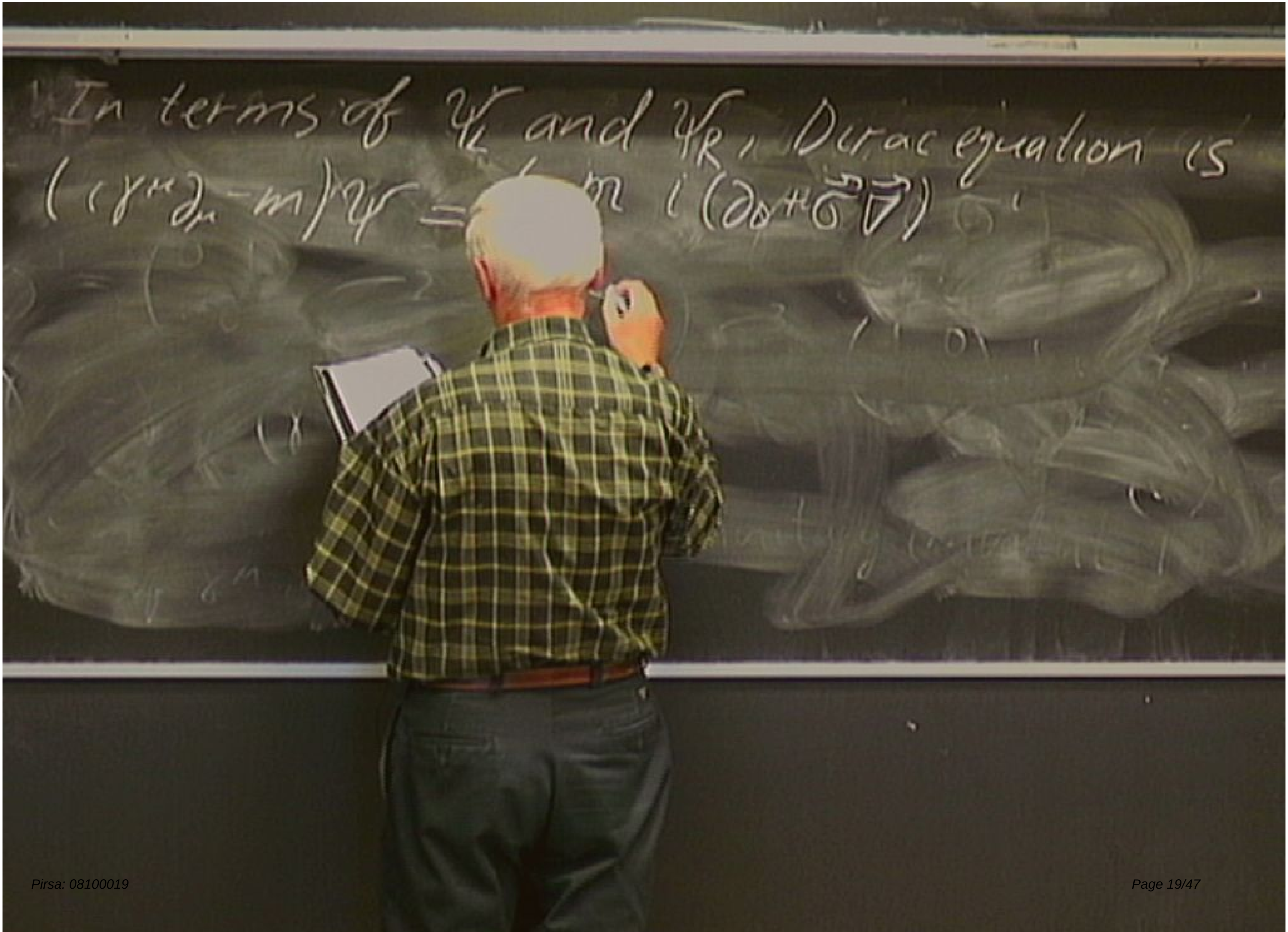
$\psi_L \sim \psi_R$ under rotations. Let us show that ψ_R transforms as $\sigma^2\psi_L^*$ under both rotational and boost transformations, $\psi_R \sim \sigma^2\psi_L^*$. We will use identity $\boxed{\sigma^2\vec\sigma^* = -\vec\sigma\sigma^2}$

$$\psi_L^* \longrightarrow \left(1 + i\vec\theta\frac{\vec\sigma^*}{2} - \vec\beta\frac{\vec\sigma^*}{2}\right)\psi_L \Rightarrow \sigma^2\psi_L^* \longrightarrow \left(1 + i\sigma^2\theta\frac{\vec\sigma^*}{2} - \sigma^2\vec\beta\frac{\vec\sigma^*}{2}\right)\cdot\psi_L$$

$$= \left(1 - i\vec\theta\frac{\vec\sigma}{2} + \frac{\vec\beta}{2}\vec\sigma\right)\sigma^2\psi_L$$


ψL = (1 − (θ⃗ σ⃗/2 + β⃗/2 σ⃗) σ² ψL   Q.E.D.
In terms of ψL and ψR


In terms of ψ_L and ψ_R


In terms of ψ_L and ψ_R, Dirac equation is
$(i\gamma^\mu\partial_\mu - m)\psi =$
$m \quad i(\partial_0+\vec{\sigma}\cdot\vec{\nabla})$

In terms of ψ_L and ψ_R, Dirac equation is

$$(i\gamma^\mu \partial_\mu - m)\psi = \begin{pmatrix} -m & i(\partial_0 + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_0 - \vec{\sigma}\cdot\vec{\nabla}) & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0 .$$

$m=0$, ψ_L and ψ_R are not mixed.

In terms of ψ_L and ψ_R, Dirac equation is

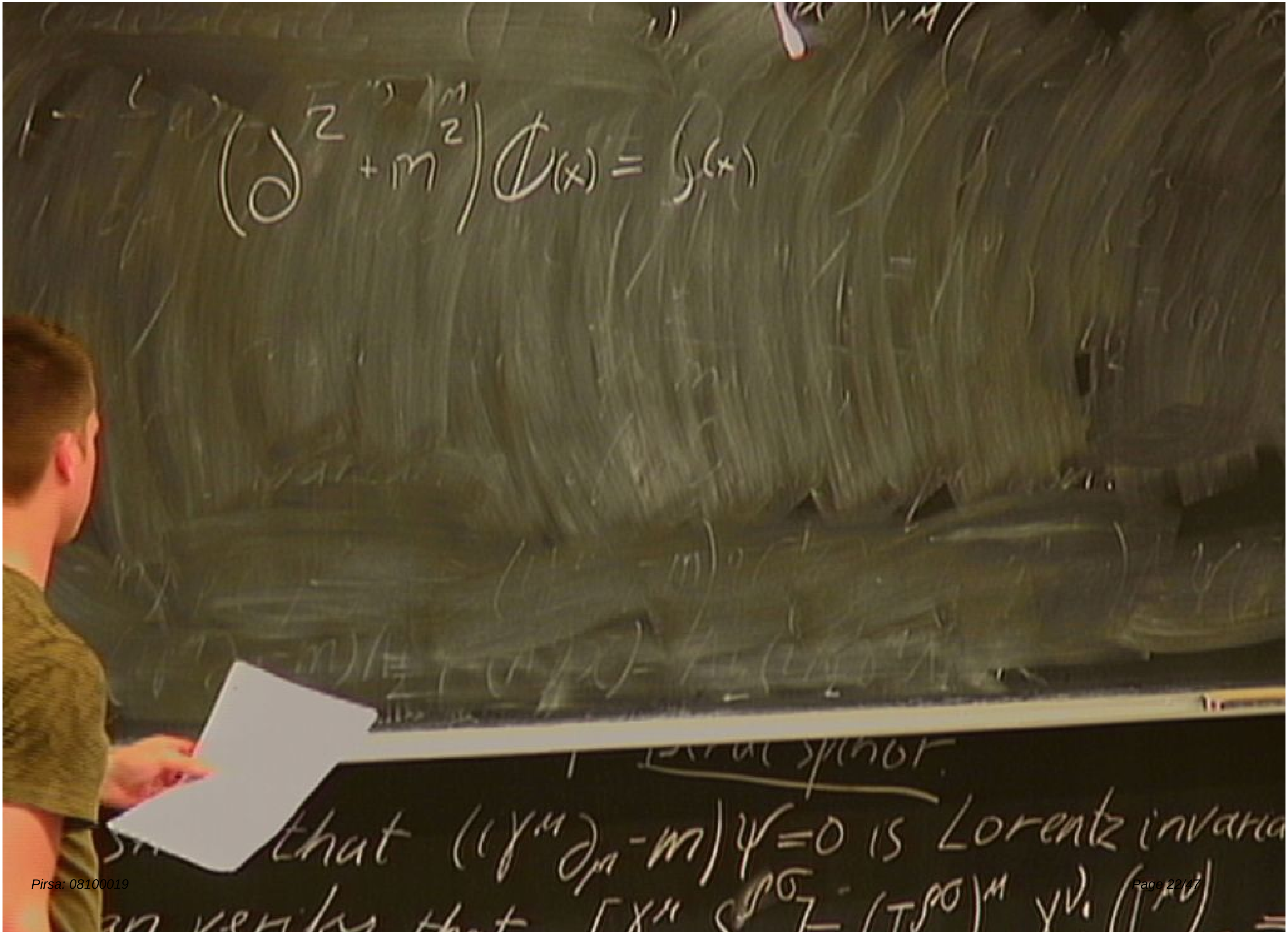
$$(i\gamma^\mu\partial_\mu - m)\psi = \begin{pmatrix} -m & i(\partial_0 + \vec{\sigma}\vec{\nabla}) \\ i(\partial_0 - \vec{\sigma}\cdot\vec{\nabla}) & -m \end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

For $m=0$, ψ_L and ψ_R are not mixed

$$i(\partial_0 - \vec{\sigma}\vec{\nabla})\psi_L = 0$$

$$i(\partial_0 + \vec{\sigma}\vec{\nabla})\psi_R = 0 \quad \Longleftarrow \text{Weyl equations}$$

ψ_L, ψ_R are Weyl fields.


(∂² + m²)Φ(x) = j(x)
Dirac spinor
that (iγ^μ∂_μ - m)ψ = 0 is Lorentz invaria

$$(\partial^2 + m^2)\,\phi(x) = J(x)$$

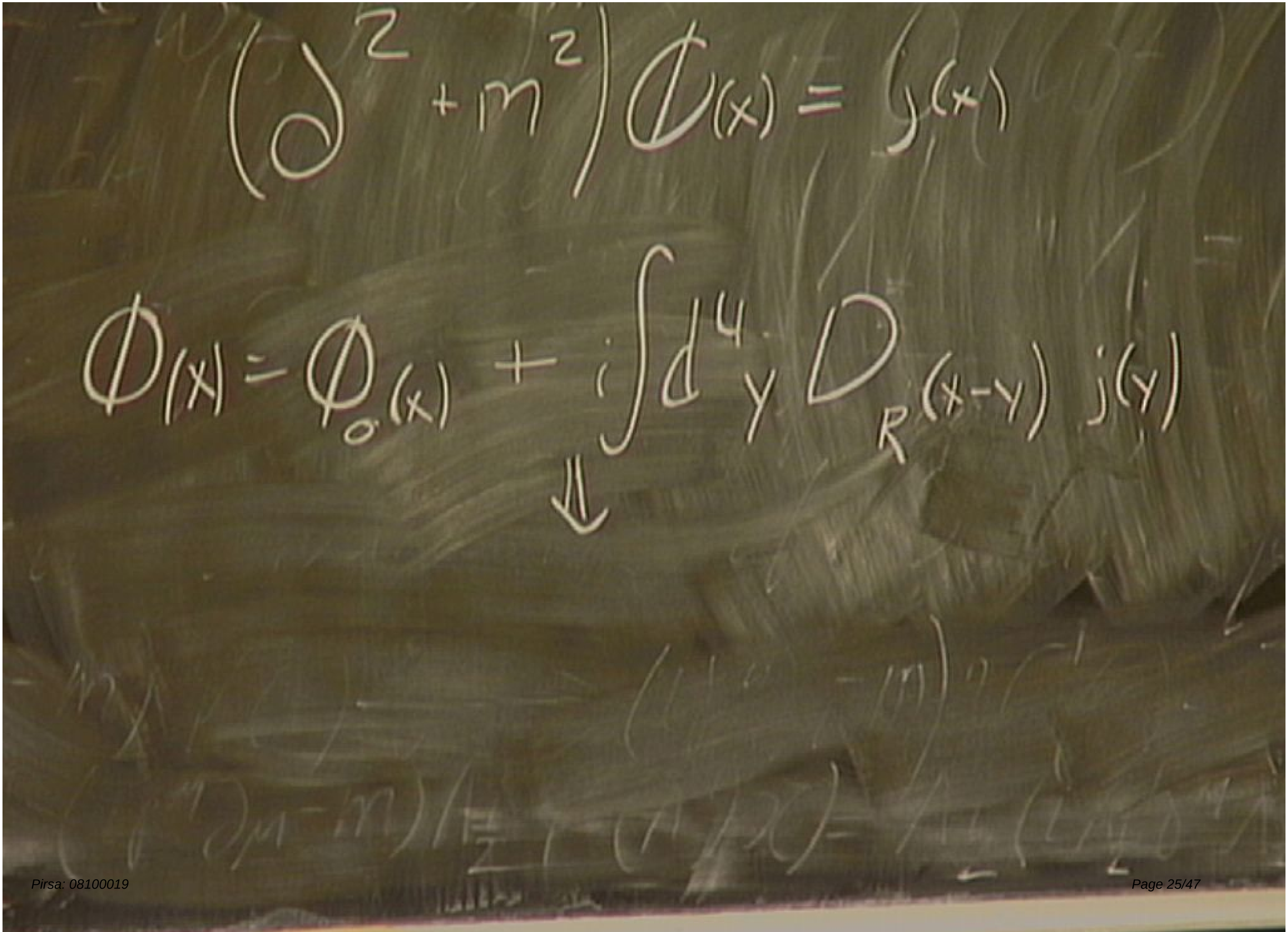
$$(\partial^2 + m^2)\,G(x) = \delta^{(4)}(x)$$

Dirac spinor

Let us show that $((\gamma^\mu \partial_\mu) - m)\psi = 0$ is

$$(\partial^2 + m^2)\,\phi(x) = J(x)$$

$$(\partial^2 + m^2)\,G(x) = \delta^{(4)}(x)$$



$(\partial^2 + m^2)\phi(x) = j(x)$
$\phi(x) = \phi_0(x) + i\int d^4y\, D_R(x-y)\, j(y)$
$\Downarrow$

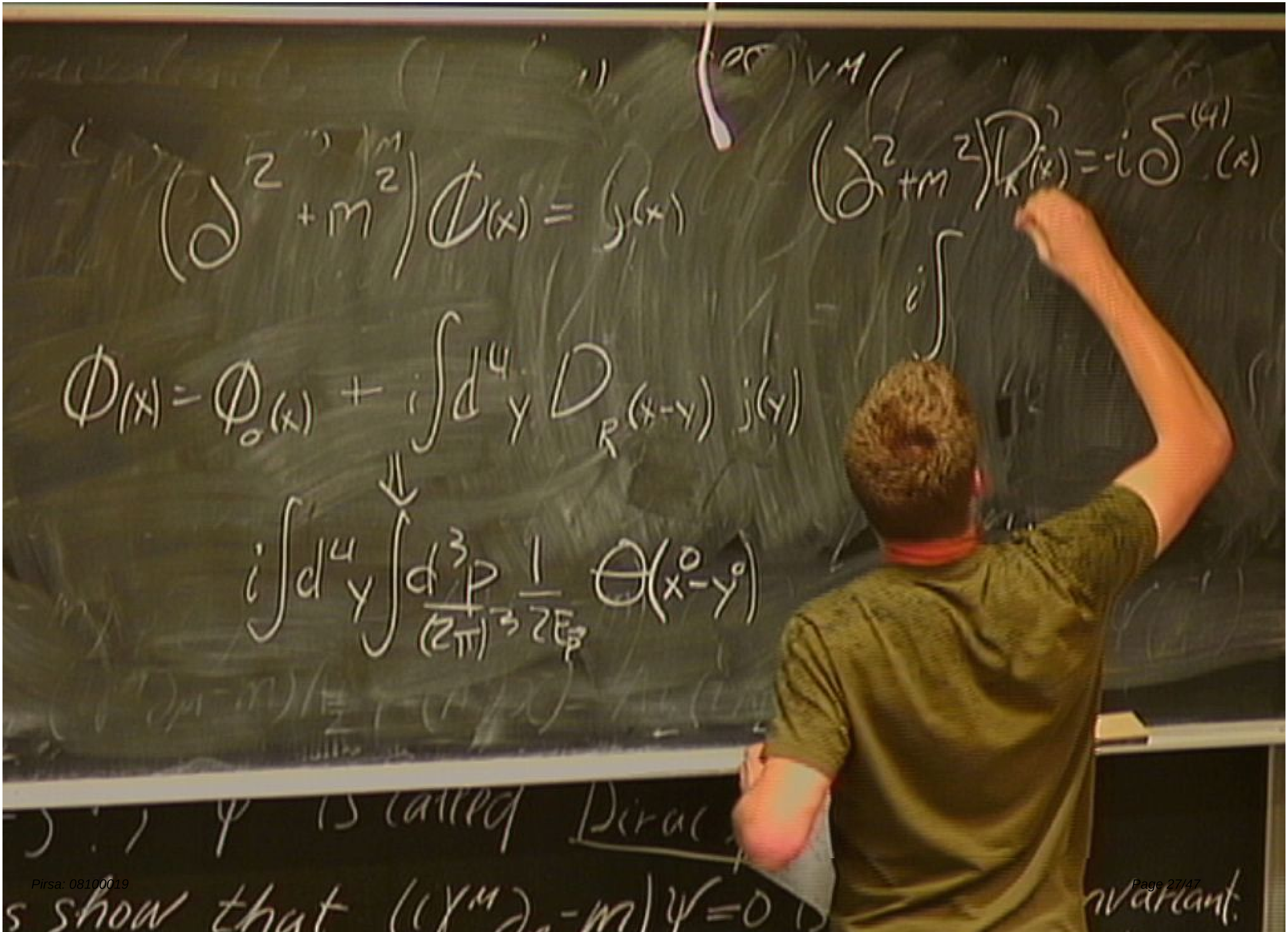
$$(\partial^2 + m^2)\,\phi(x) = j(x)$$

$$\phi(x) = \phi_0(x) + i\int d^4y\, D_R(x-y)\, j(y)$$

$$\Downarrow$$

$$i\int d^4y \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\,\theta(x^0-y^0)$$

is called Dirac spinor



$(\partial^2 + m^2)\phi(x) = j(x)$
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$\Downarrow$
$i\int d^4y \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \Theta(x^0 - y^0)$
is called Dirac
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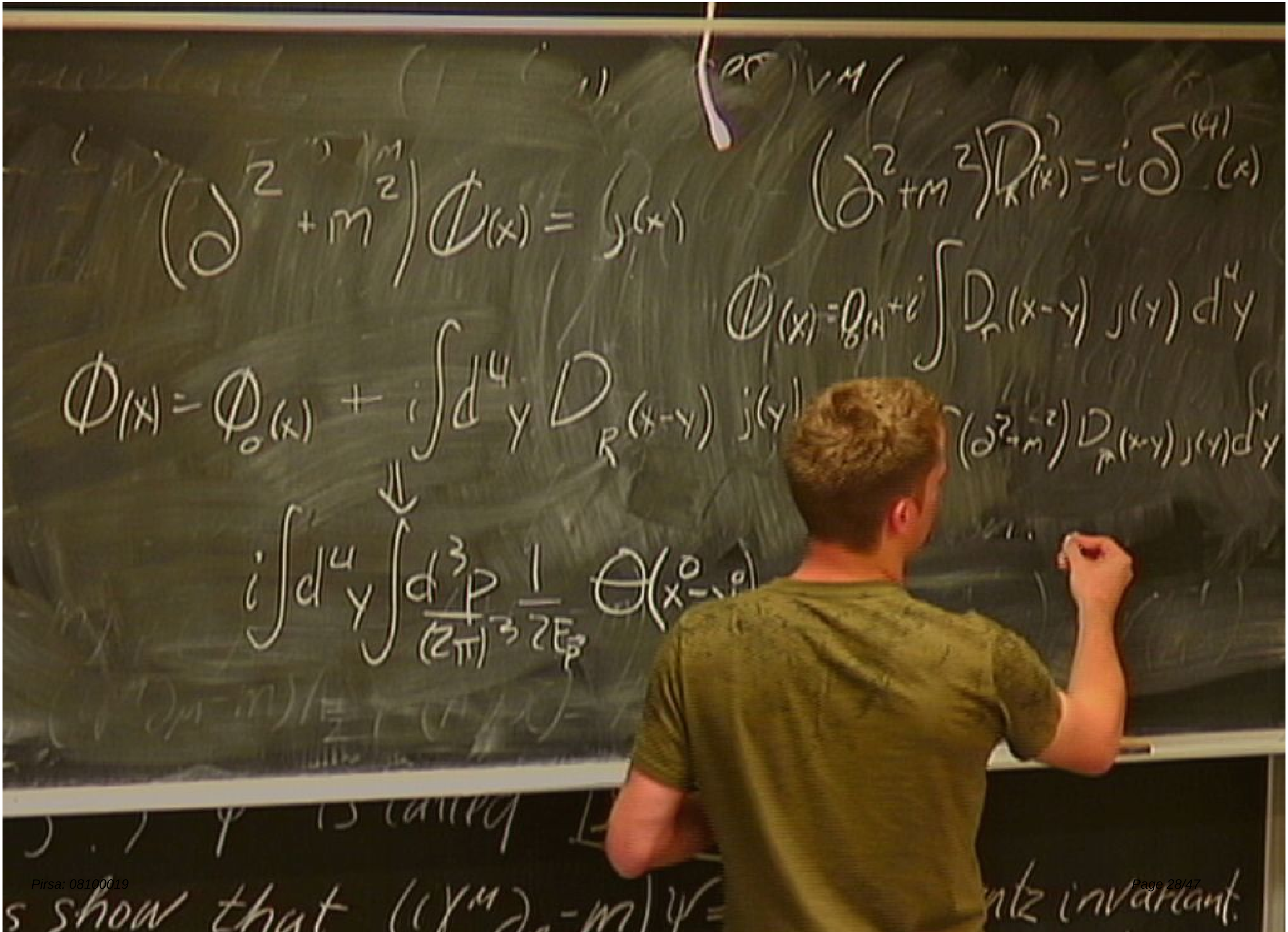
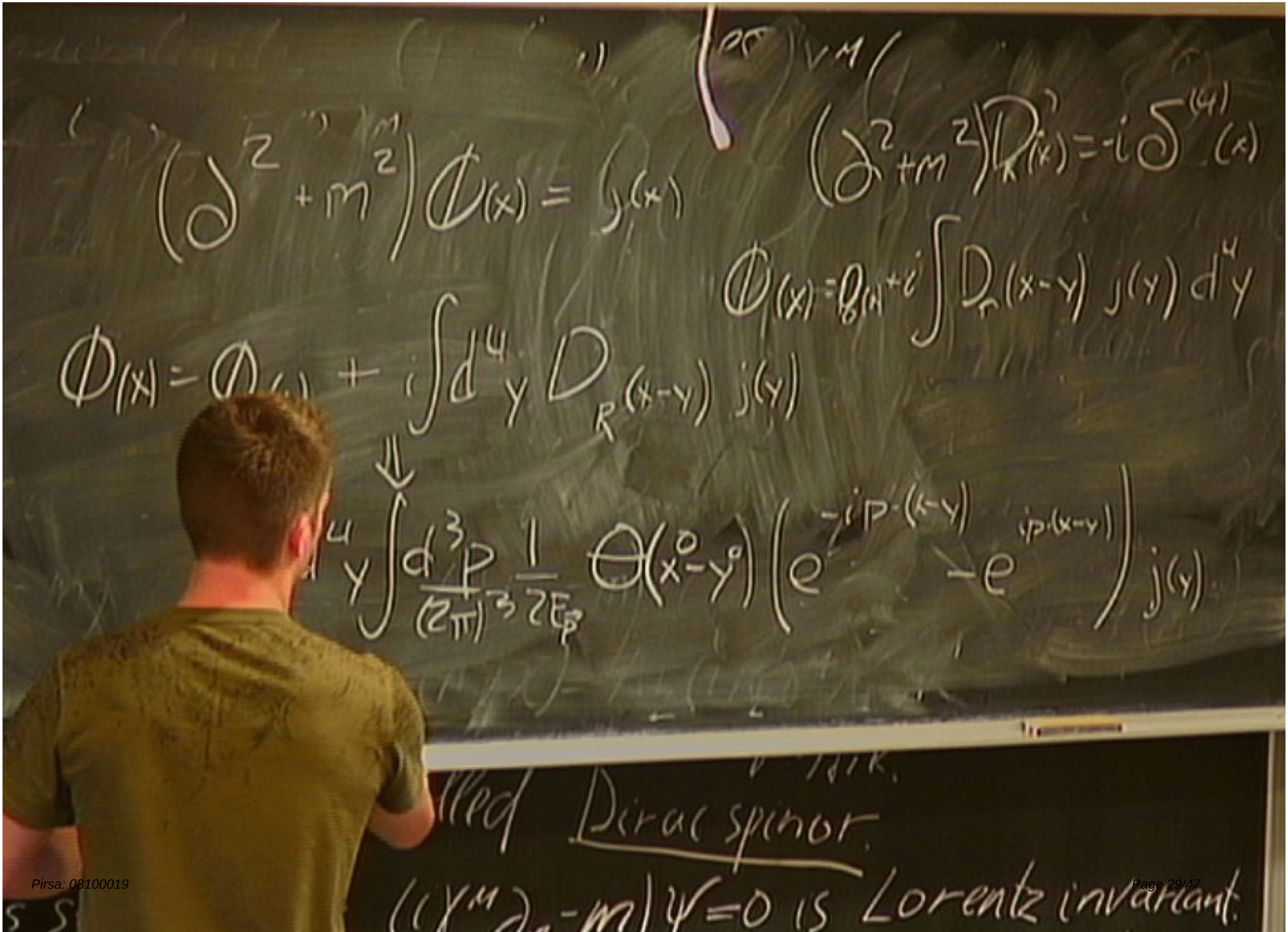
nvariant.


(∂² + m²) φ(x) = j(x)
(∂² + m²) D_R(x) = -i δ⁽⁴⁾(x)
φ(x) = φ₀(x) + i ∫ D_R(x-y) j(y) d⁴y
φ(x) = φ₀(x) + i ∫ d⁴y D_R(x-y) j(y)
i ∫ d⁴y ∫ d³p/(2π)³ 1/(2E_p) Θ(x⁰-y⁰)


$(\partial^2 + m^2)\phi(x) = j(x)$
$(\partial^2 + m^2)D_R(x) = -i\delta^{(4)}(x)$
$\phi(x) = \phi_0(x) + i\int D_R(x-y)\, j(y)\, d^4y$
$\phi(x) = \phi_0(x) + i\int d^4y\, D_R(x-y)\, j(y)$
$\Downarrow$
$d^4y \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\,\theta(x^0-y^0)\left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right) j(y)$
lled Dirac spinor.
$(i\gamma^\mu\partial_\mu - m)\psi = 0$ is Lorentz invariant.

$$(\partial^2 + m^2) D_R(x) = -i\delta^{(4)}(x)$$

$$(\partial^2 + m^2)\phi(x) = j(x)$$

$$\phi(x) = \phi_0(x) + i\int D_R(x-y)\, j(y)\, d^4y$$

$$\phi(x) = \phi_0(x) + i\int d^4y\, D_R(x-y)\, j(y)$$

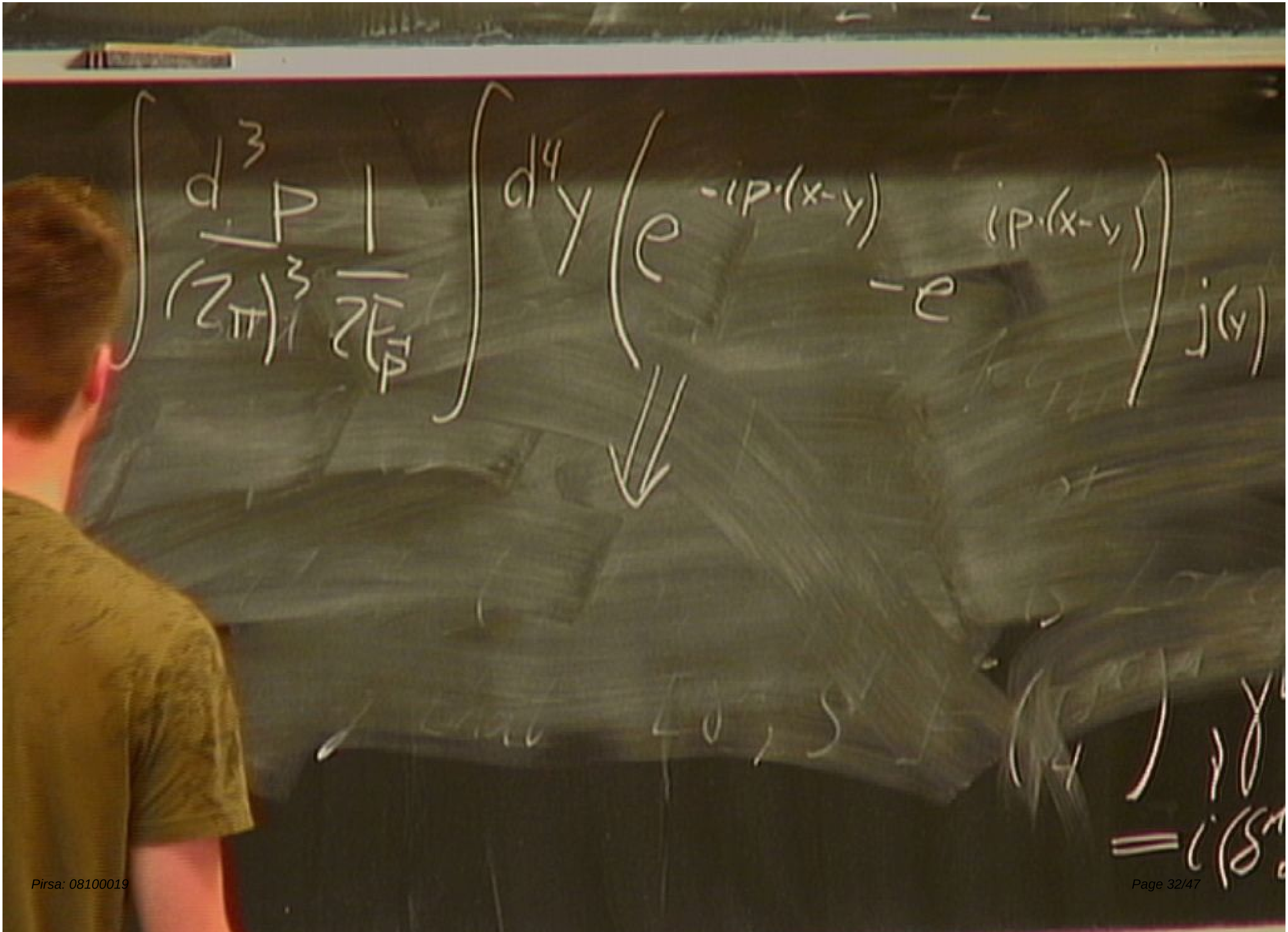
$$\Downarrow$$

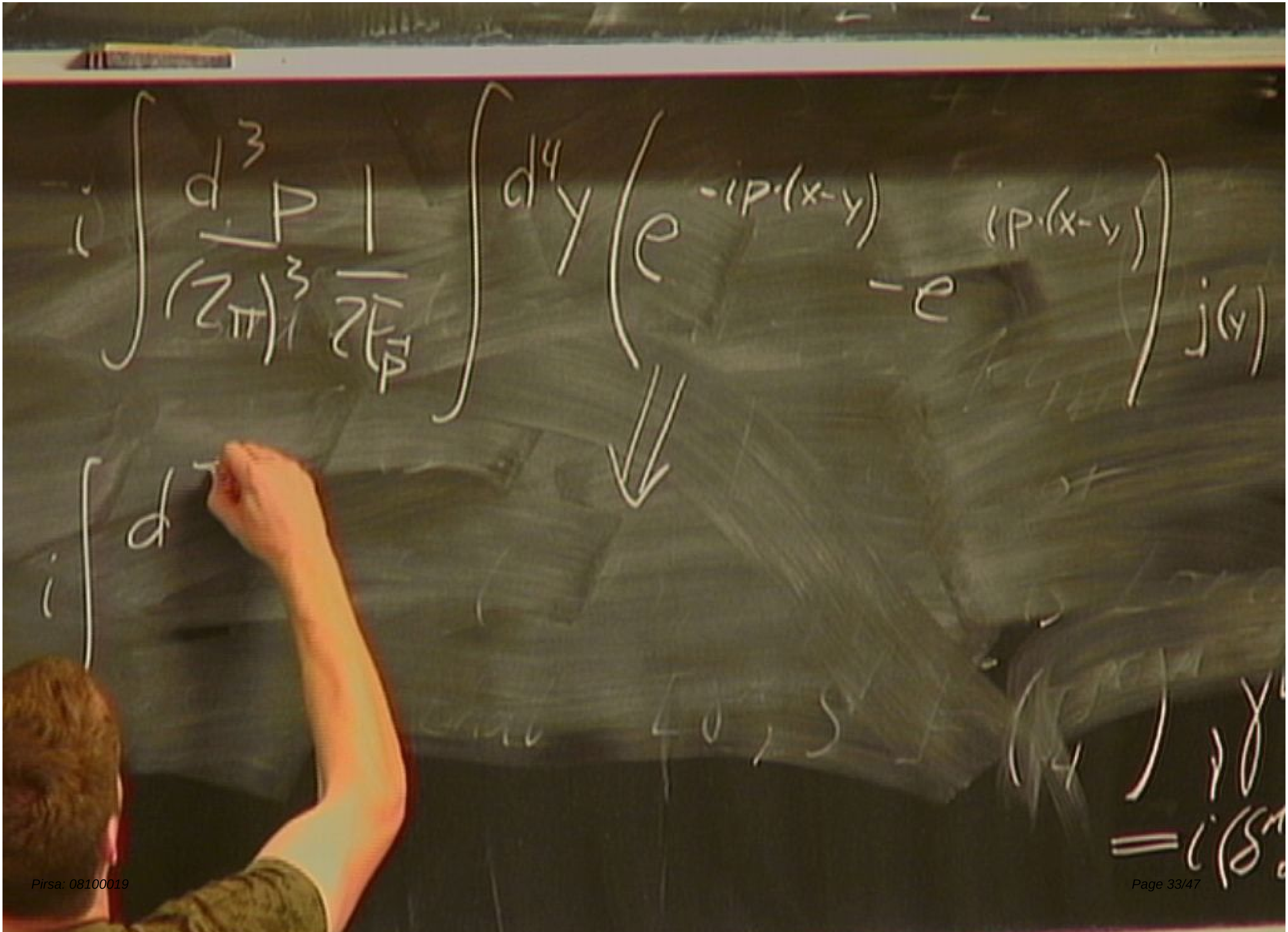
$$\underline{i\int d^4y \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\,\theta(x^0-y^0)\left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right) j(y)}$$

is called Dirac spinor.

w that $(i\gamma^\mu\partial_\mu - m)\psi = 0$ is Lorentz invariant.

$$i\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}$$


$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \int d^4y \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right) j(y)$


$i\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\int d^4y\left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right) j(y)$
$i\int d$

$$i\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\int d^4y\left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right) j(y)$$

$$\Downarrow$$

$$i\int \frac{d^3p}{(2\pi)^3}\frac{e^{-ip\cdot x}}{2E_{\vec{p}}}\int d^4y\, e^{ip\cdot y} j(y)$$

$$= i(\delta$$

$$i\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec p}}\int d^4y\left(e^{-ip\cdot(x-y)}-e^{ip\cdot(x-y)}\right)j(y)$$

$$\Downarrow$$

$$i\int \frac{d^3p}{(2\pi)^3}\frac{e^{-ip\cdot x}}{2E_{\vec p}}\left[\int d^4y\, e^{ip\cdot y}\, j(y)\right]$$

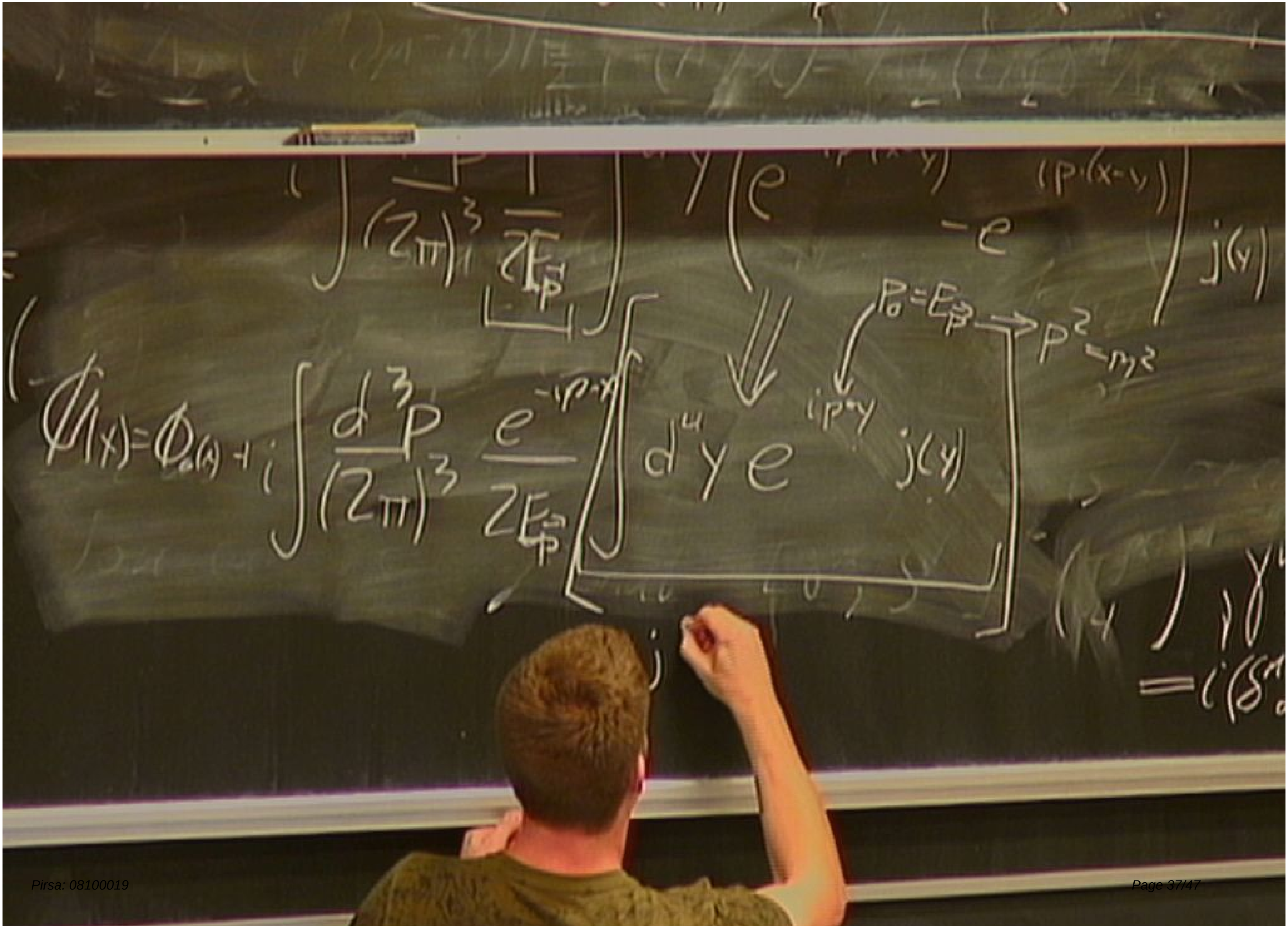
$$= i\left(\delta \ldots\right.$$

$$i\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec p}}\int d^4y\left(e^{-ip\cdot(x-y)}-e^{ip\cdot(x-y)}\right)j(y)$$

$$\Downarrow \qquad p_0=E_{\vec p}\Rightarrow p^2=m^2$$

$$+i\int\frac{d^3p}{(2\pi)^3}\frac{e^{-ip\cdot x}}{2E_{\vec p}}\left[\int d^4y\, e^{ip\cdot y}j(y)\right]$$

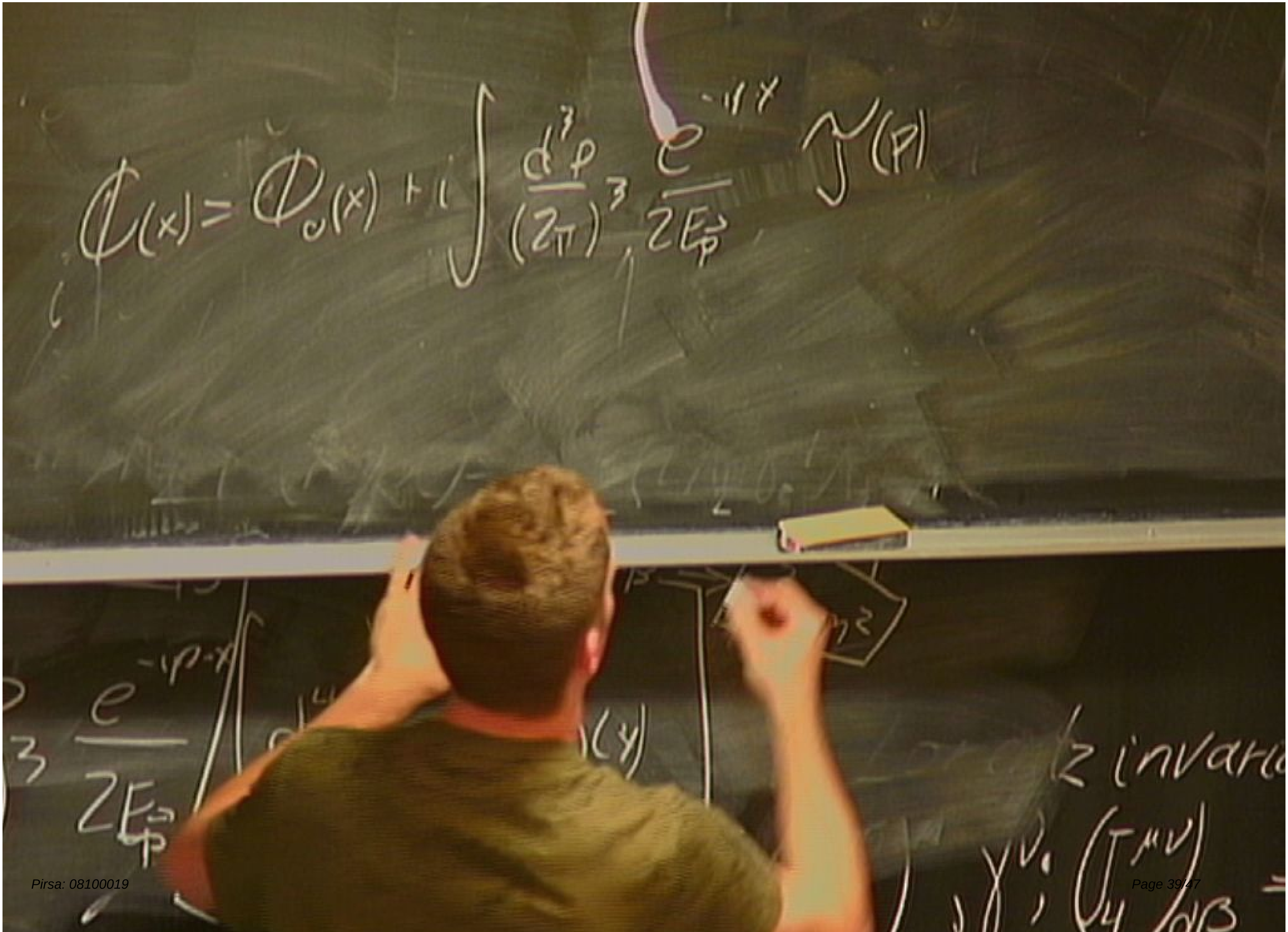
$$= i(\delta^{\mu}$$

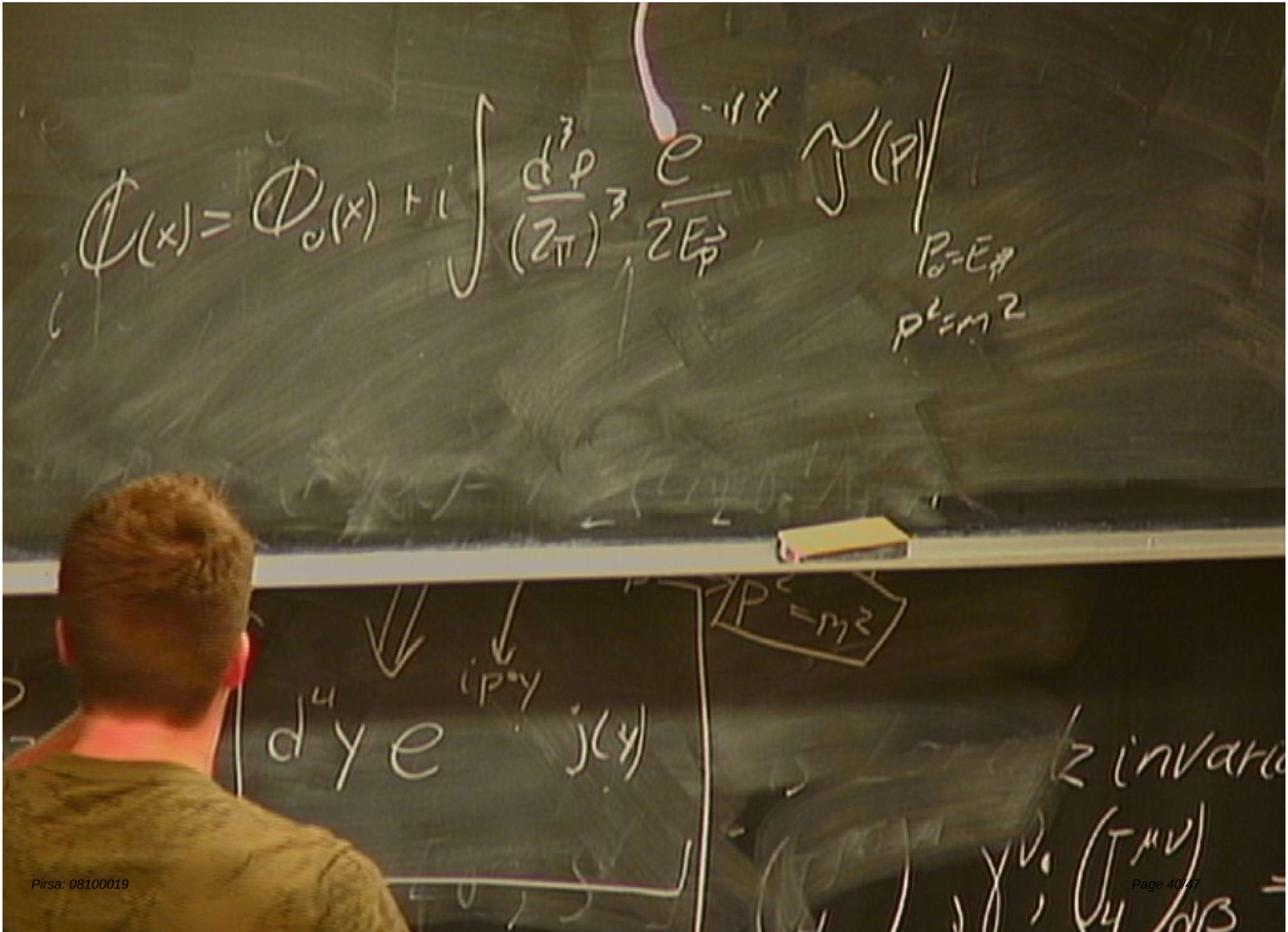


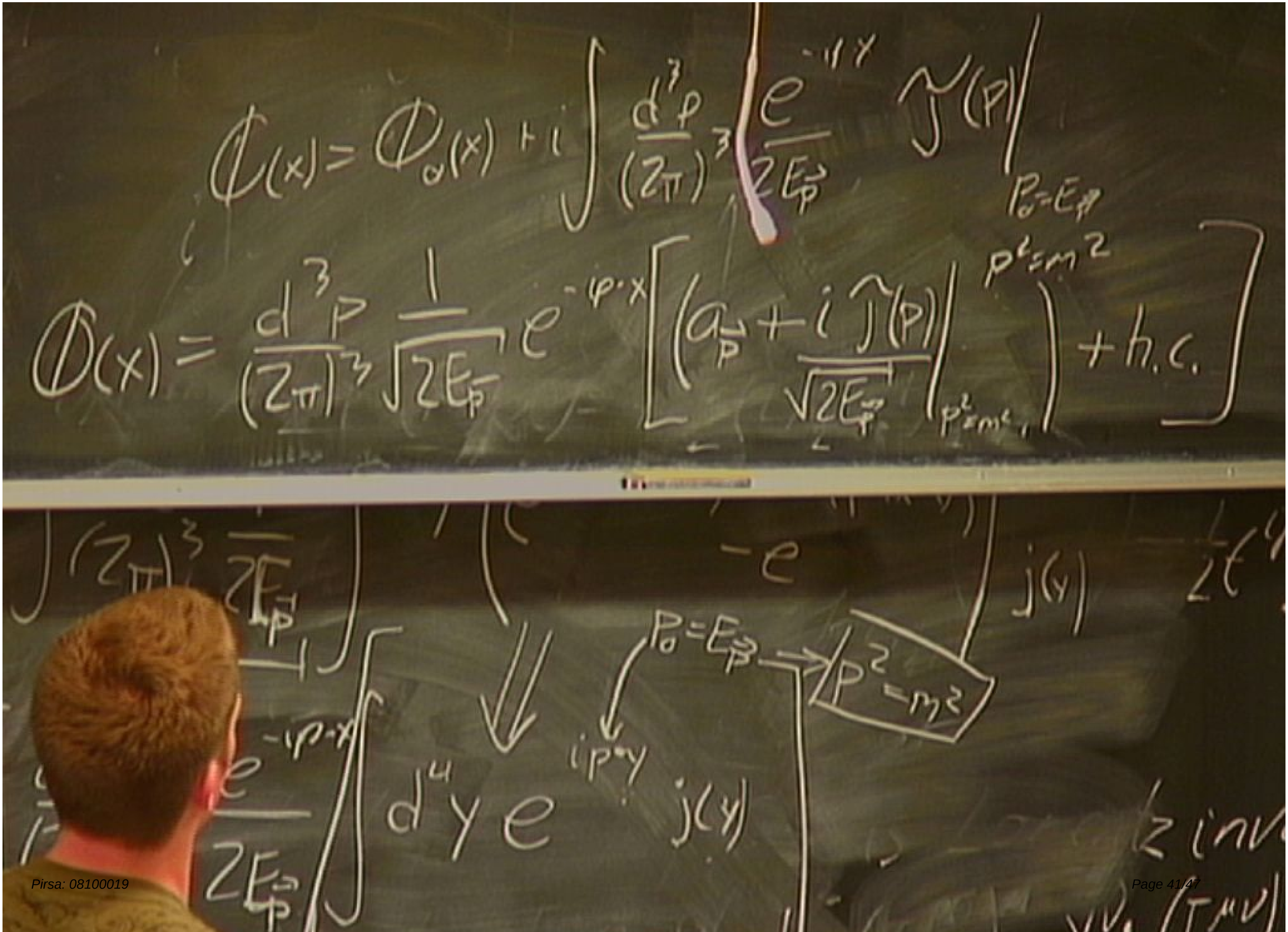
φ(x) = φ0(x) + i ∫ d3p/(2π)3 e−ip·x/2Ep ∫ d4y eip·y j(y)
p0 = Ep → p2 = m2
j(y)

$$\phi(x) = \phi_0(x) + i\int \frac{d^3p}{(2\pi)^3}\,\frac{e^{-ip\cdot x}}{2E_{\vec{p}}}\,\underbrace{\int d^4y\, e^{ip\cdot y}\, j(y)}_{\tilde{j}(p)}$$

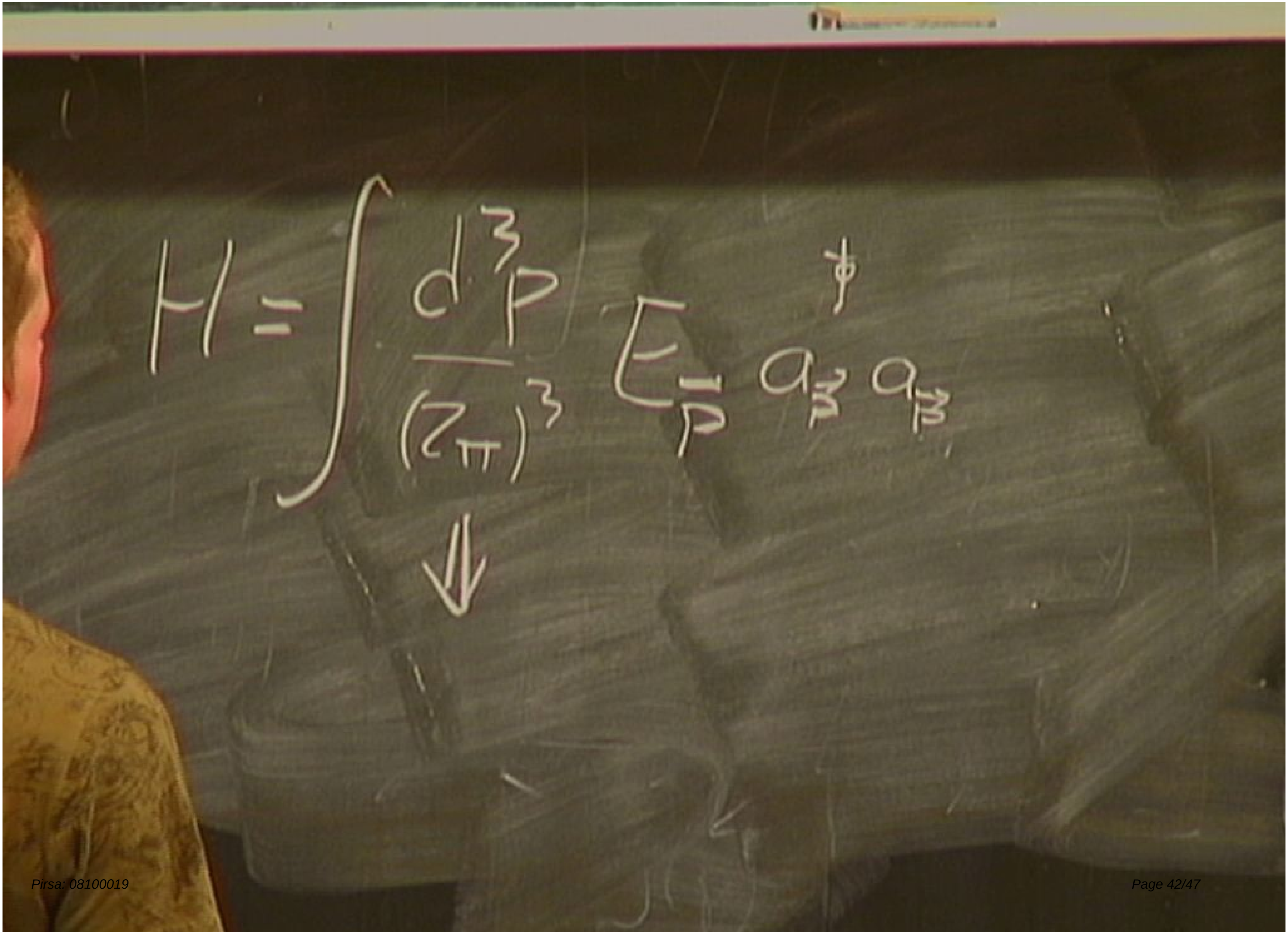
$p_0 = E_{\vec{p}} \rightarrow p^2 = m^2$


$\Phi(x) = \Phi_0(x) + i\int \frac{d^3p}{(2\pi)^3} \frac{e^{-ip\cdot x}}{2E_{\vec p}} \tilde{J}(p)$


$\Phi(x) = \Phi_0(x) + i\int \frac{d^3p}{(2\pi)^3} \frac{e^{-ip\cdot x}}{2E_{\vec{p}}} \tilde{J}(p)\Big|_{p_0=E_{\vec{p}}}$
$p^2=m^2$
$p^2=m^2$
$d^4y\, e^{ip\cdot y}\, j(y)$



Φ(x) = Φ_0(x) + i ∫ d³p/(2π)³ e^{-ip·x}/2E_p j̃(p) |_{p_0=E_p}
Φ(x) = d³p/(2π)³ 1/√(2E_p) e^{-ip·x} [(a_p + i j̃(p)/√(2E_p) |_{p²=m²}) + h.c.]
p² = m²
j(y)
p_0 = E_p → p² = m²
d⁴y e^{ip·y} j(y)


$H = \int \frac{d^3p}{(2\pi)^3} E_{\vec{p}}\, a^\dagger_{\vec{p}} a_{\vec{p}}$

$$H = \int \frac{d^3p}{(2\pi)^3} E_{\vec{p}}\, a^\dagger_{\vec{p}}\, a_{\vec{p}}$$

$$\Downarrow$$

$$\langle 0 | H | 0 \rangle$$

$$H = \int \frac{d^3p}{(2\pi)^3} E_{\vec{p}} \underline{a^\dagger_{\vec{p}} a_{\vec{p}}}$$

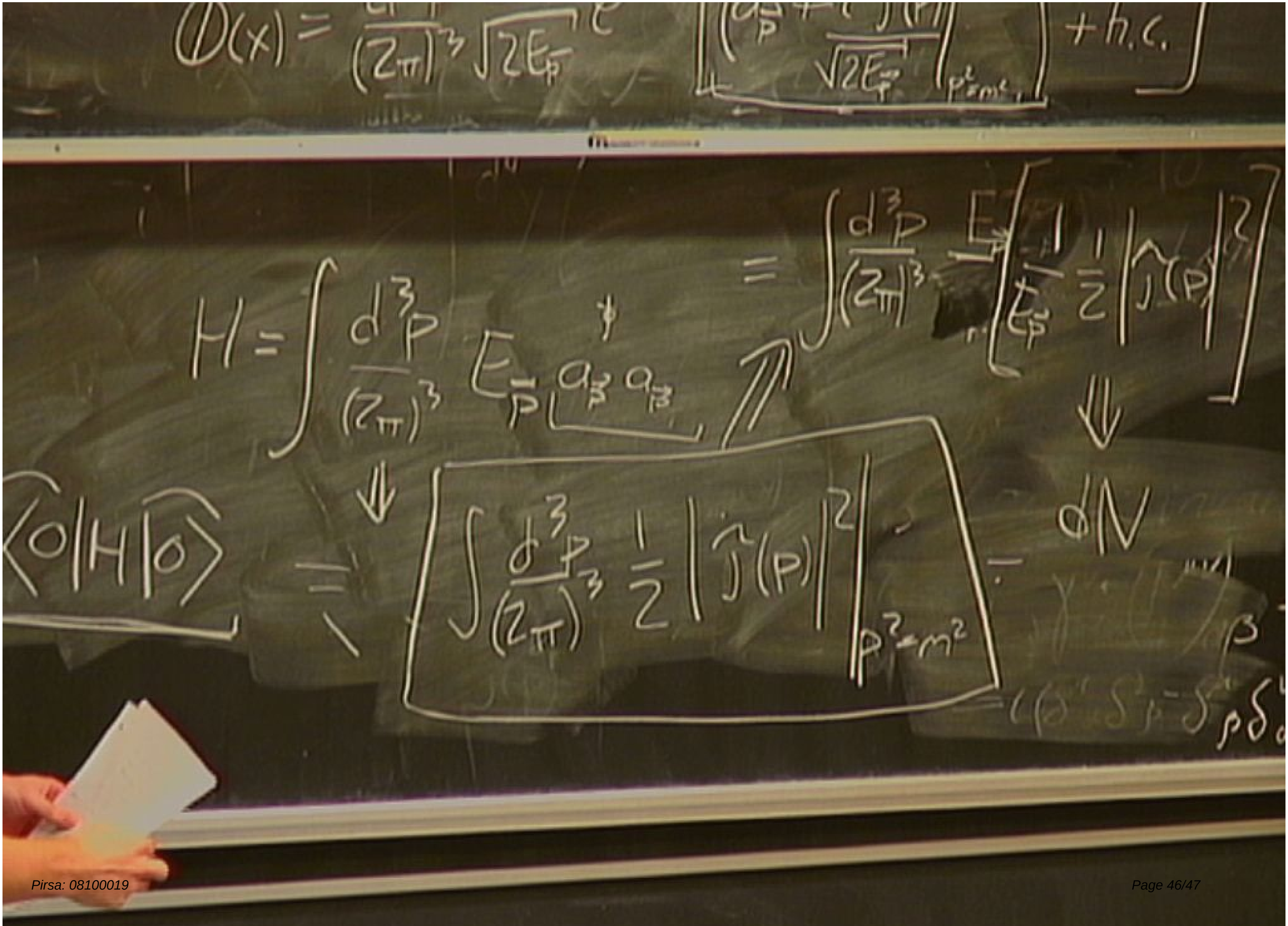
$$\Downarrow$$

$$\underline{\langle 0 | H | 0 \rangle} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \left| \tilde{j}(p) \right|^2 \Big|_{p^2=m^2}$$

$$H = \int \frac{d^3p}{(2\pi)^3} E_{\vec{p}} \, \underline{a^\dagger_{\vec{p}} a_{\vec{p}}}$$

$$\Downarrow$$

$$\underline{\langle 0 | H | 0 \rangle} = \boxed{\int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \left| \tilde{j}(p) \right|^2 \Big|_{p^2=m^2}}$$



$+ h.c.$
$H = \int \frac{d^3p}{(2\pi)^3} E_p a_p^\dagger a_p$
$= \int \frac{d^3p}{(2\pi)^3} E_p \left[\frac{1}{E_p} \frac{1}{2} |\tilde{j}(p)|^2 \right]$
$\langle 0|H|0 \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} |\tilde{j}(p)|^2 \Big|_{p^2=m^2}$
dN

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\, e^{-ipx}\left[\left(a_{\vec p} + i\,\frac{\tilde{j}(p)}{\sqrt{2E_{\vec p}}}\Big|_{p^2=m^2}\right) + h.c.\right]$$

$$N = \int dN = \int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{E_{\vec p}}\,\frac{1}{2}\left|\tilde{j}(p)\right|^2\right]$$

$$H = \int \frac{d^3p}{(2\pi)^3}\, E_{\vec p}\, \underbrace{a^\dagger_{\vec p}\, a_{\vec p}}$$

$$= \int \frac{d^3p}{(2\pi)^3}\, E_{\vec p}\left[\frac{1}{E_{\vec p}}\,\frac{1}{2}\left|\tilde{j}(p)\right|^2\right] \Downarrow dN$$

$$\Downarrow$$

$$\underline{\langle 0|H|0\rangle} = \boxed{\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{2}\left|\tilde{j}(p)\right|^2\Big|_{p^2=m^2}}$$