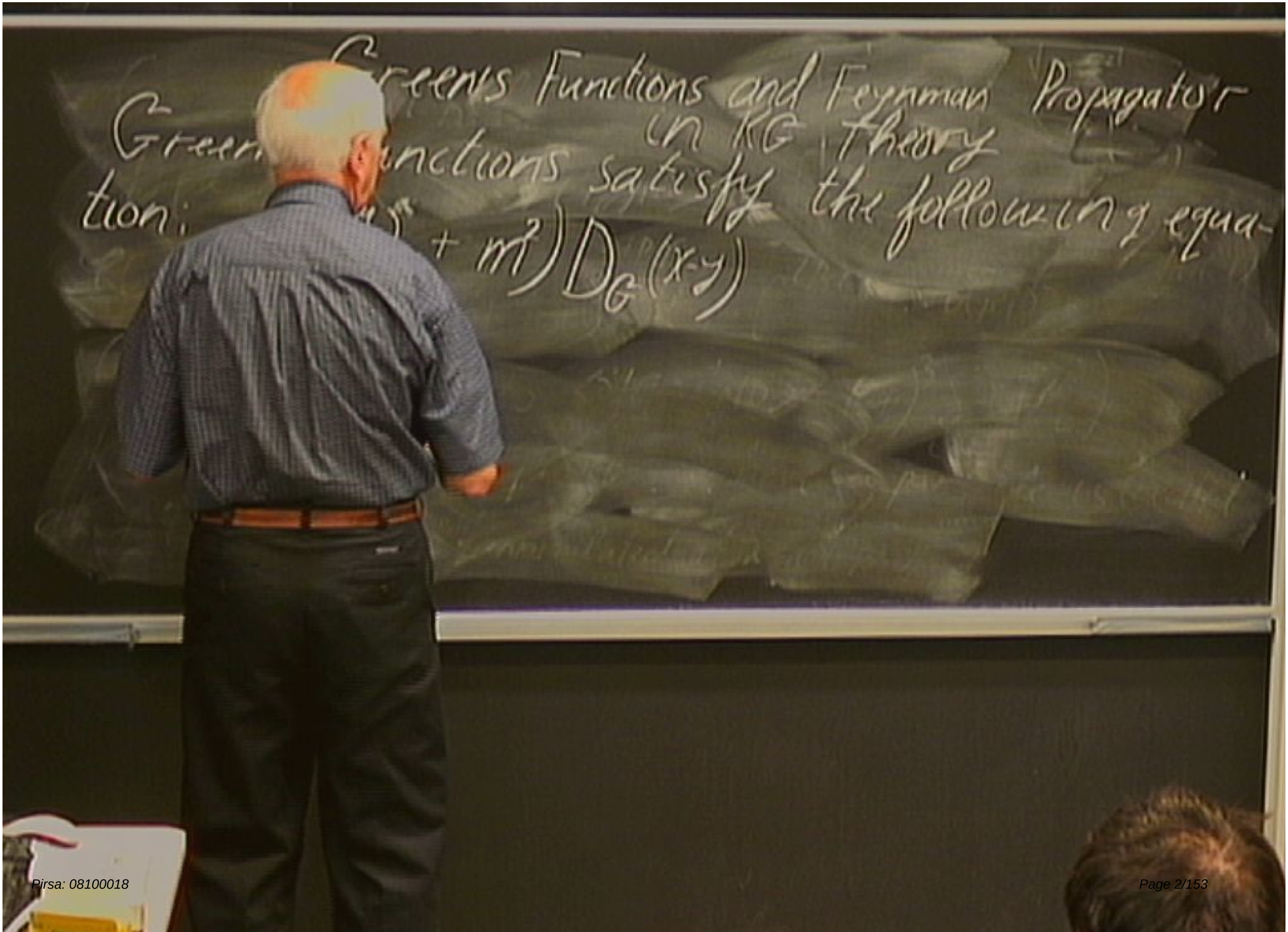
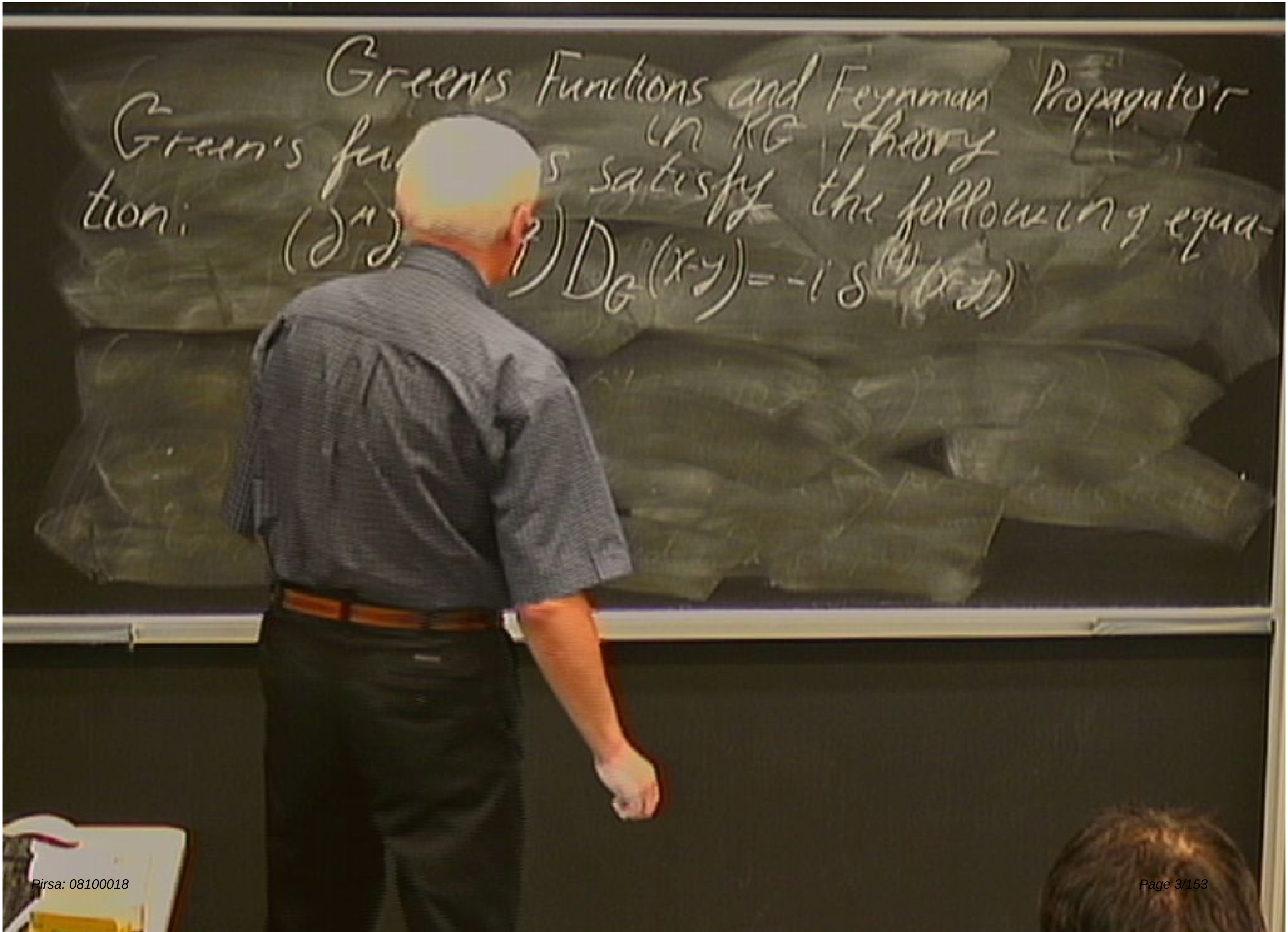
Title: Quantum Field Theory 1 - Lecture 6B

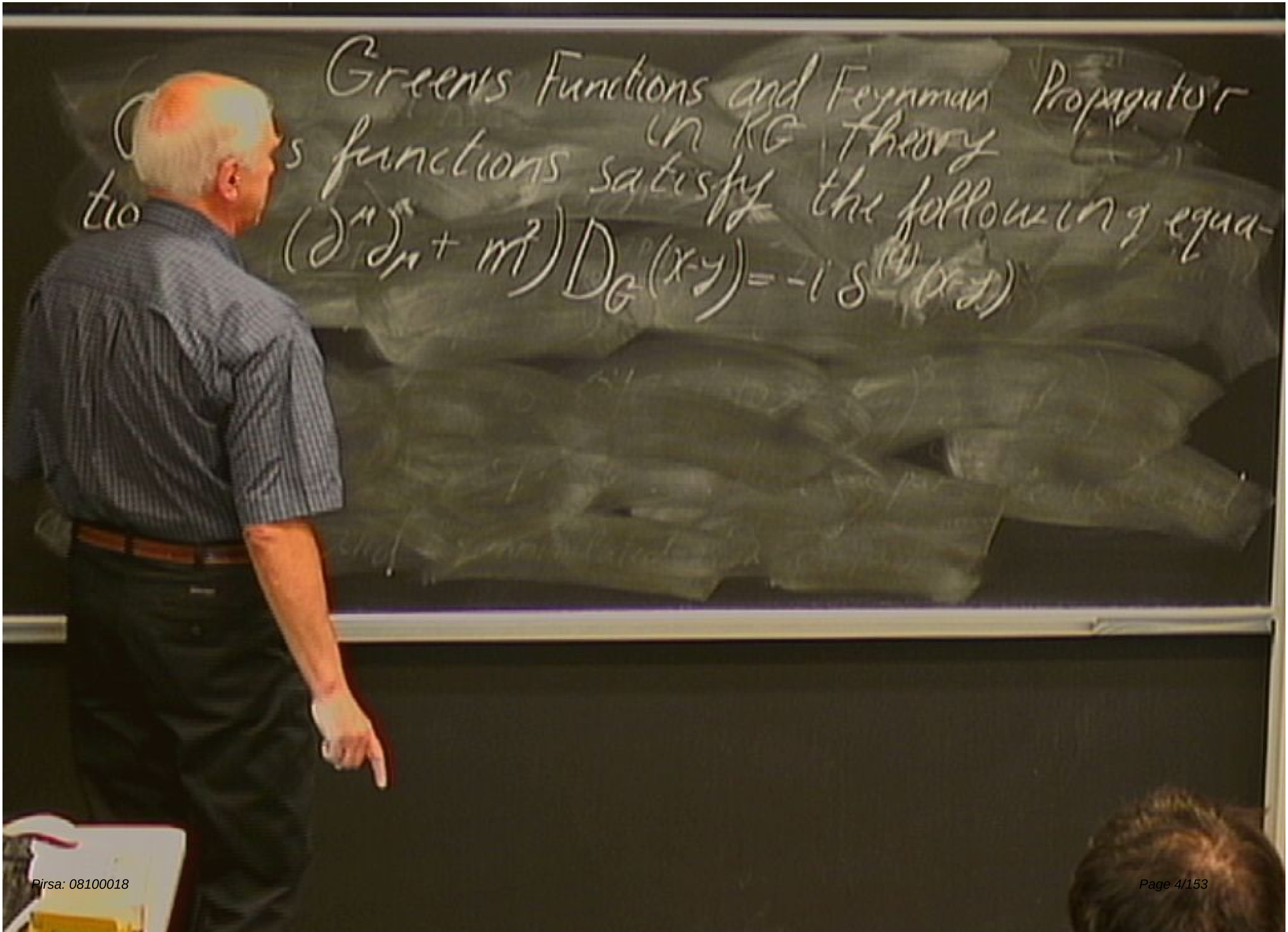
Date: Oct 15, 2008 03:30 PM

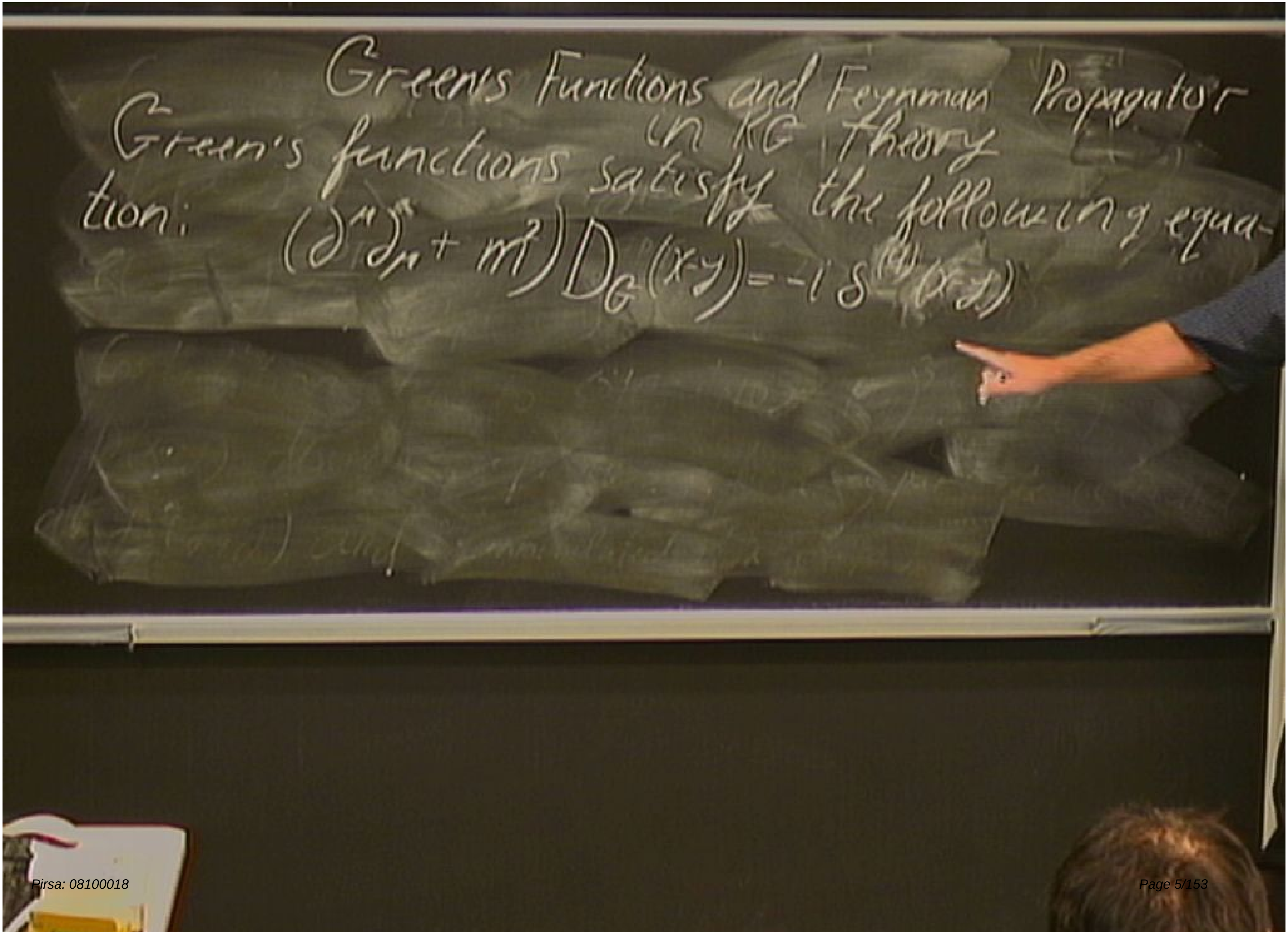
URL: http://pirsa.org/08100018

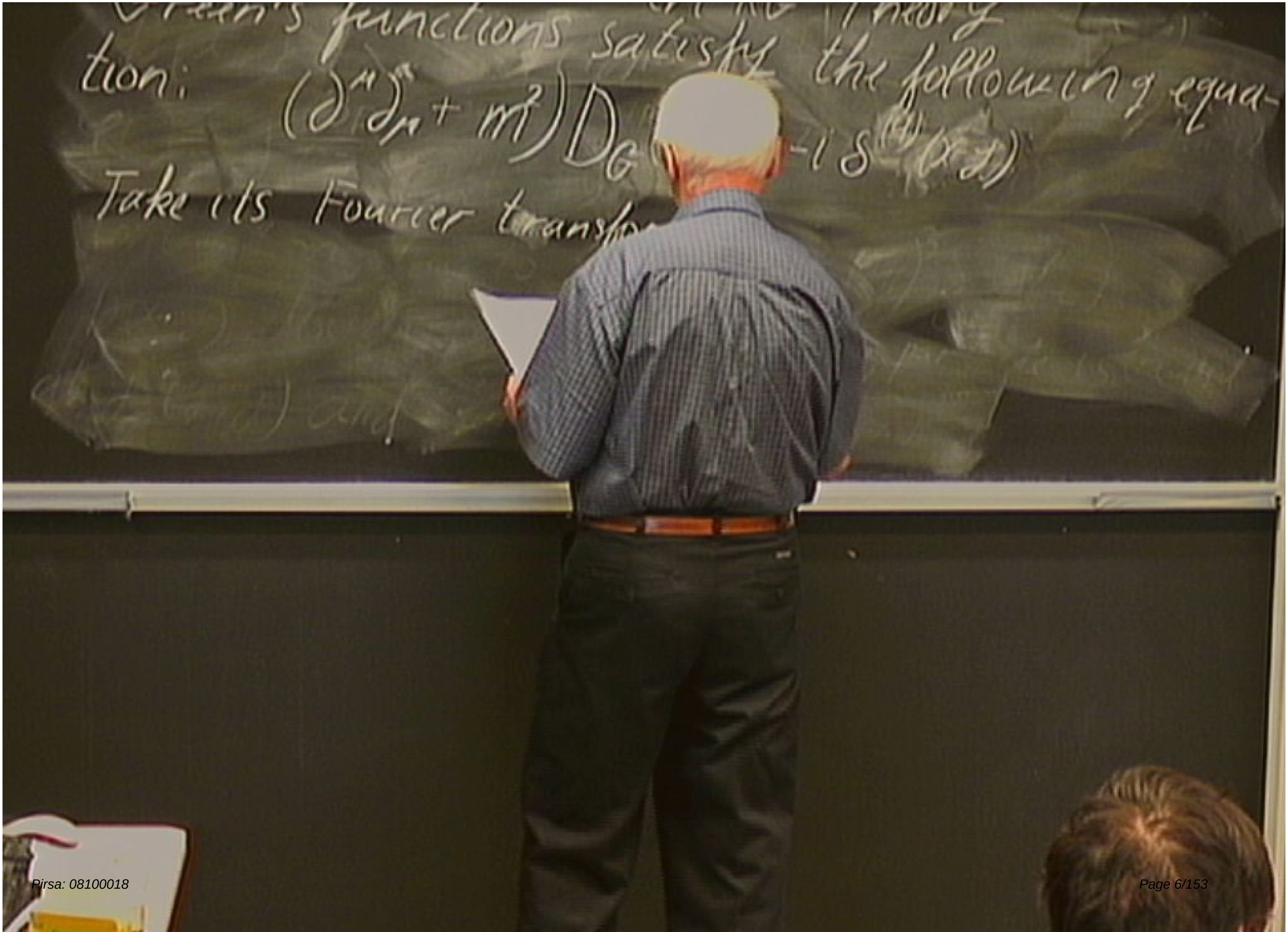
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

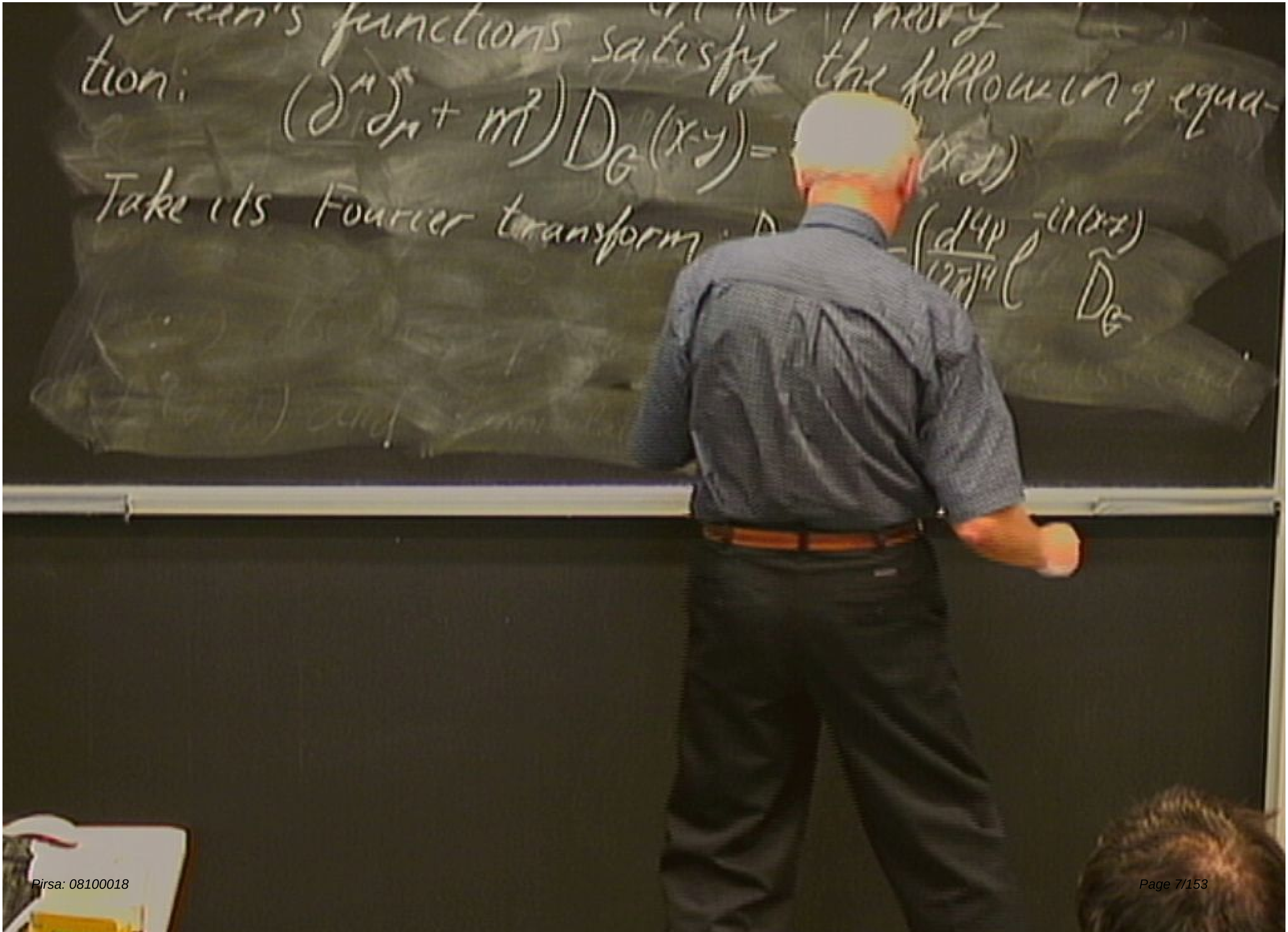

Greens Functions and Feynman Propagator
in KG theory
satisfy the following equa-
tion:
+ m²) D_G(x-y)

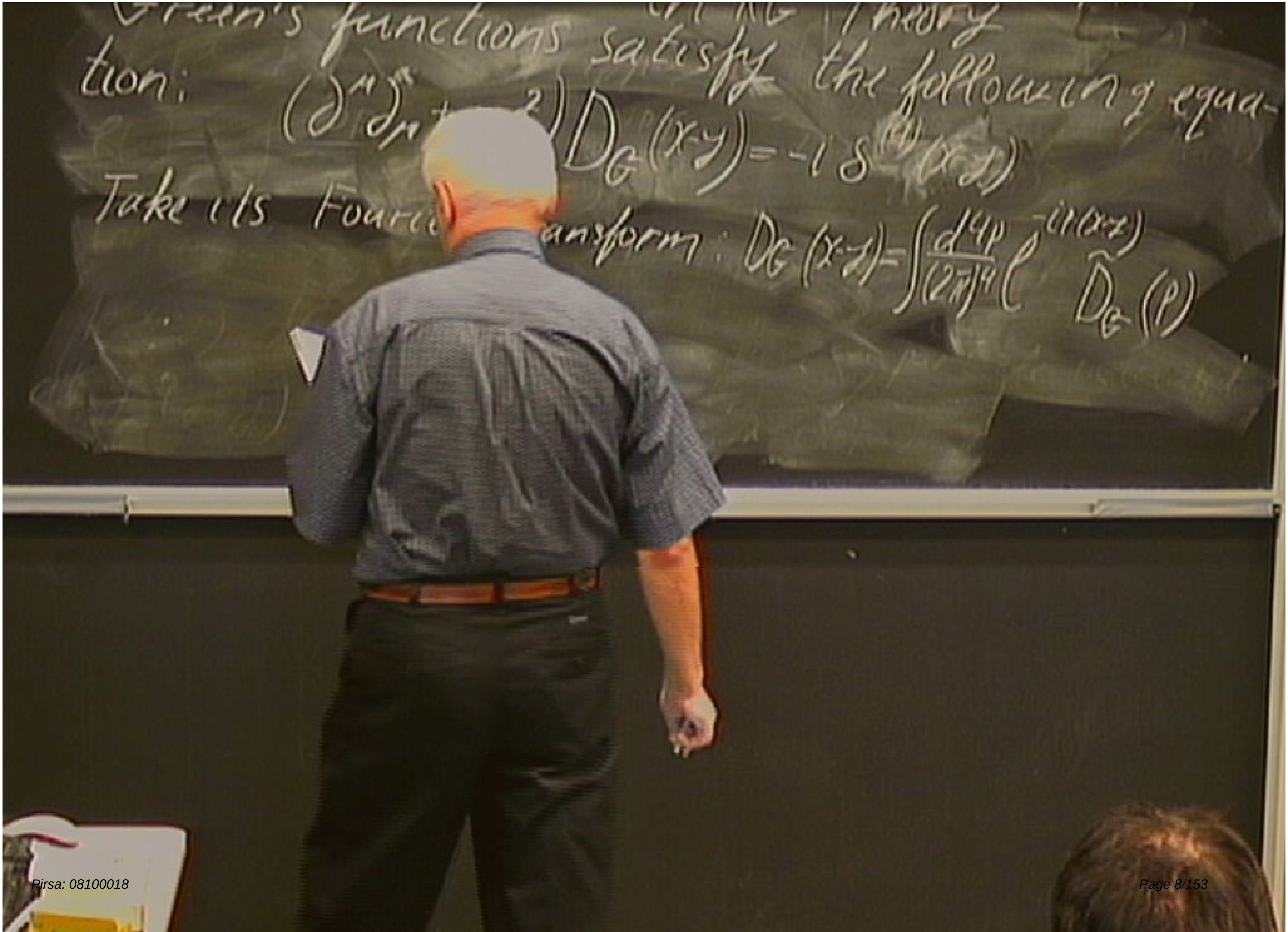

Greens Functions and Feynman Propagator
in KG theory
Green's fu s satisfy the following equa-
tion: D_G(x-y) = -i δ

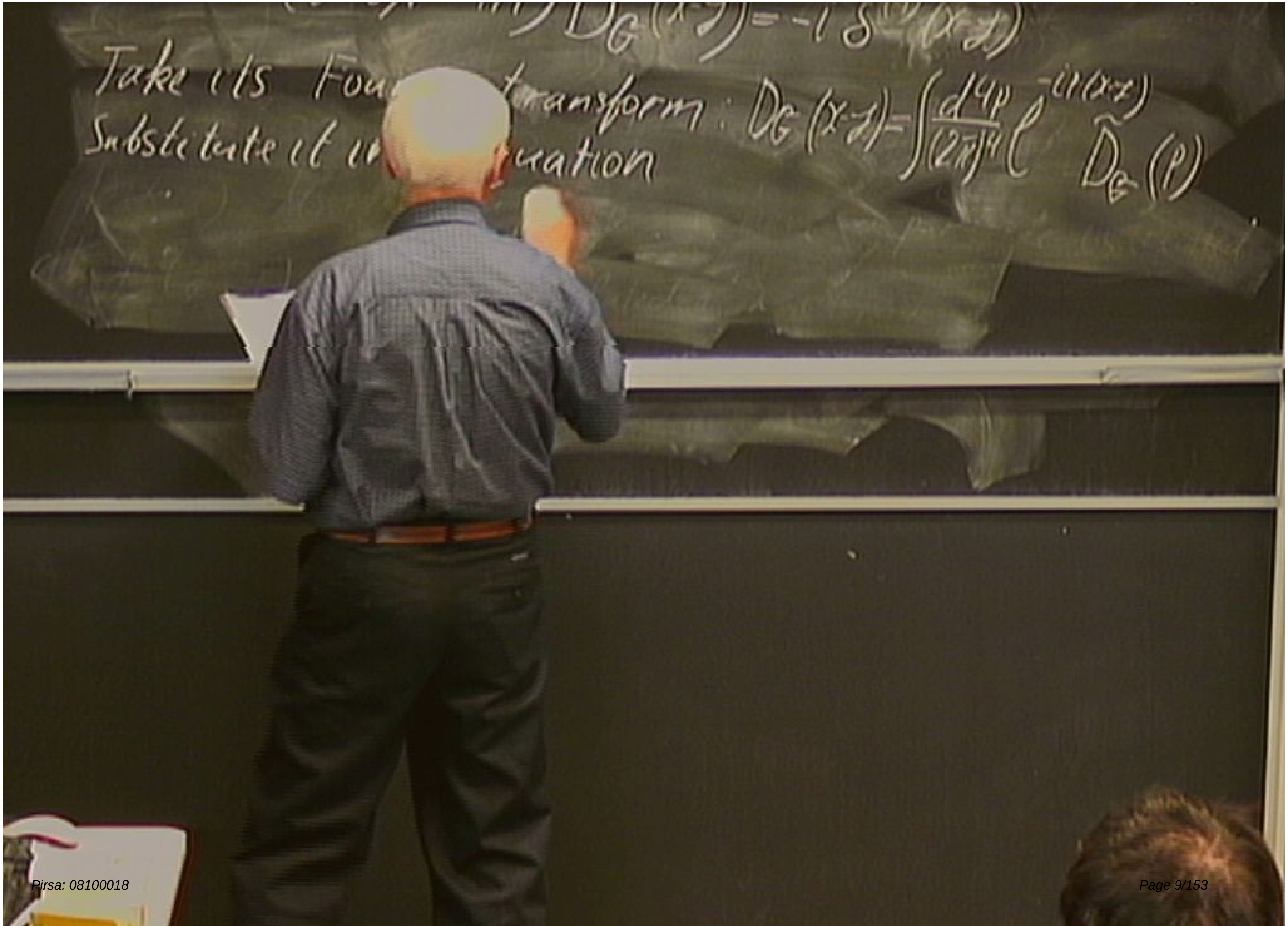

Greens Functions and Feynman Propagator
in KG theory
s functions satisfy the following equa-
tio
$(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\,\delta^{(4)}(x-y)$

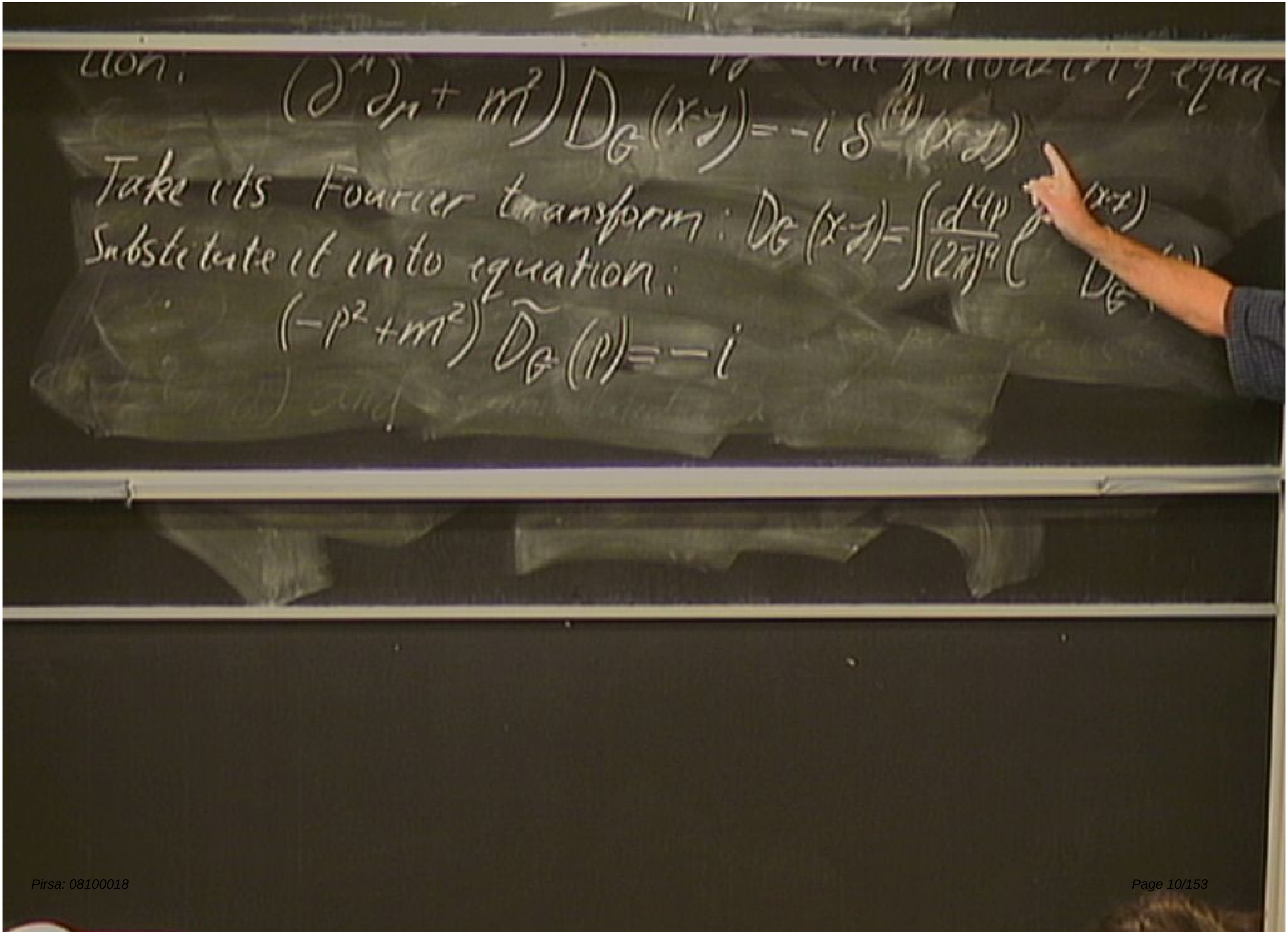

Greens Functions and Feynman Propagator
in KG theory
Green's functions satisfy the following equa-
tion: $(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\,\delta^{(4)}(x-y)$

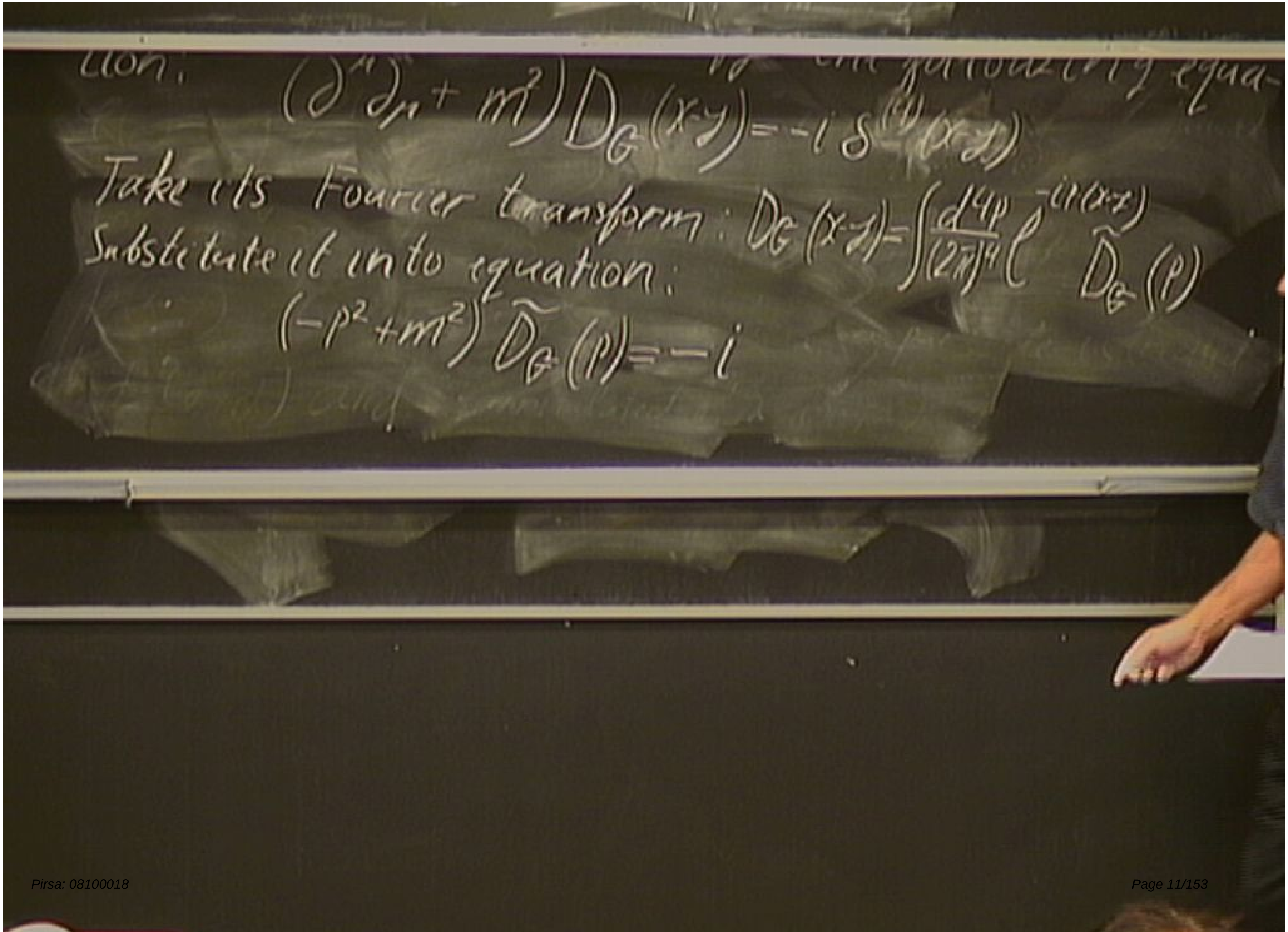

functions satisfy the following equa-
tion: $(\partial^\mu \partial_\mu + m^2) D_G$
Take its Fourier

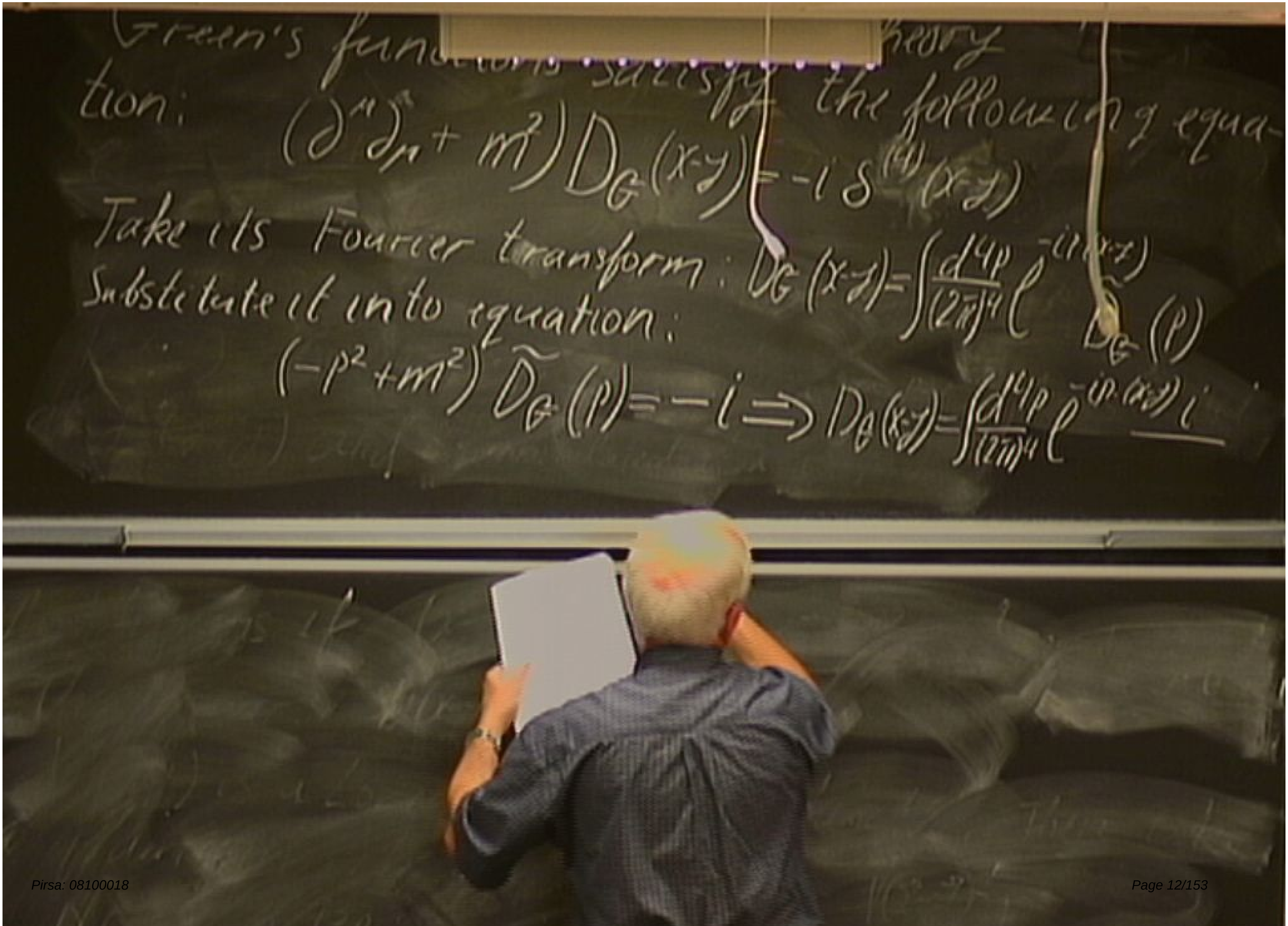

functions satisfy the following equa-
tion:
Take its Fourier transform


functions satisfy the following equa-
tion:
Take its

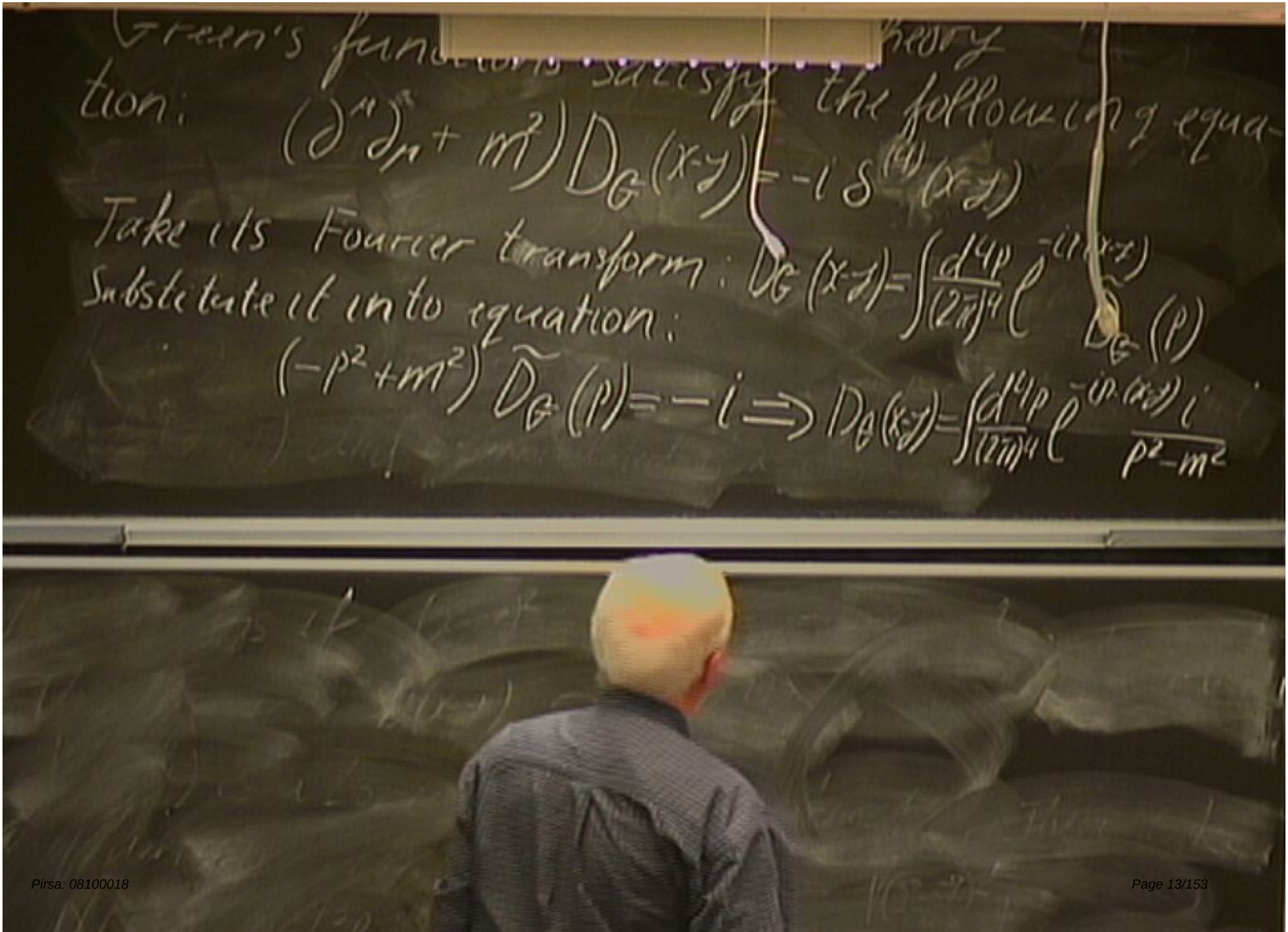

Take its Fou transform :
Substitute it in uation

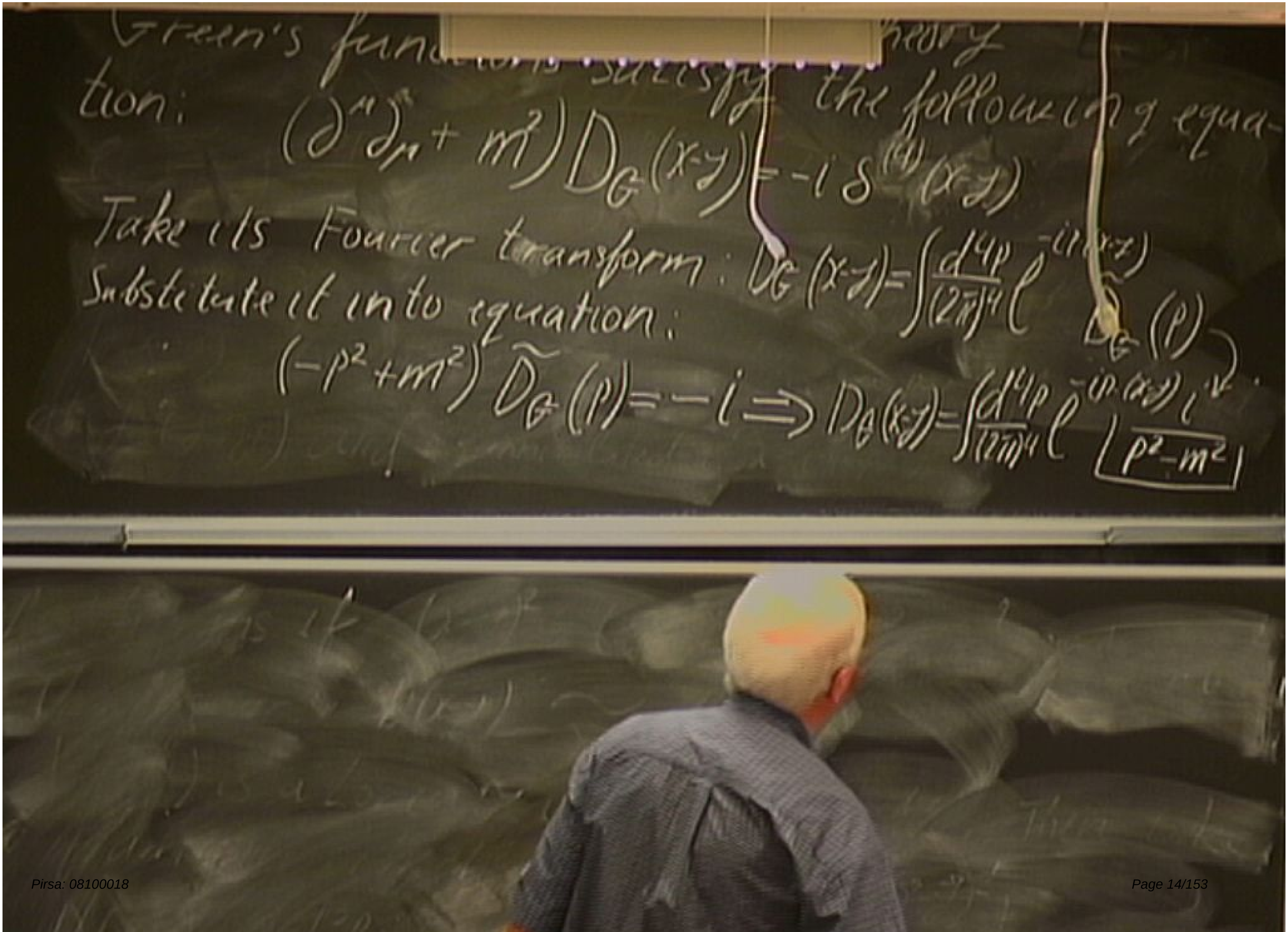

tion: $(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\,\delta^{(4)}(x-y)$
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e$
Substitute it into equation:
$(-p^2+m^2)\,\tilde{D}_G(p) = -i$

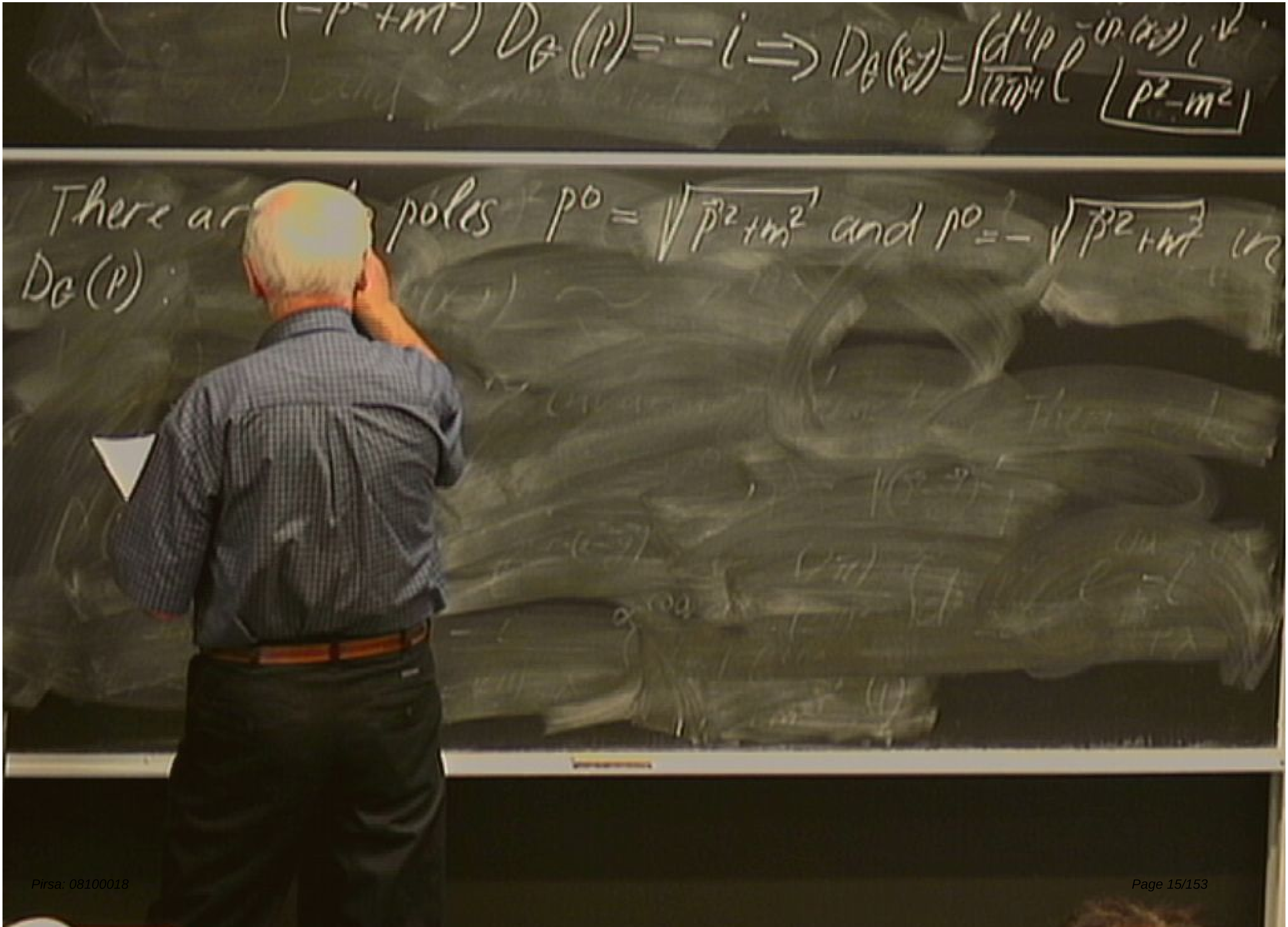

tion: $(\partial^\mu\partial_\mu + m^2) D_G(x-y) = -i\delta^{(4)}(x-y)$
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \tilde{D}_G(p)$
Substitute it into equation:
$(-p^2+m^2)\tilde{D}_G(p) = -i$

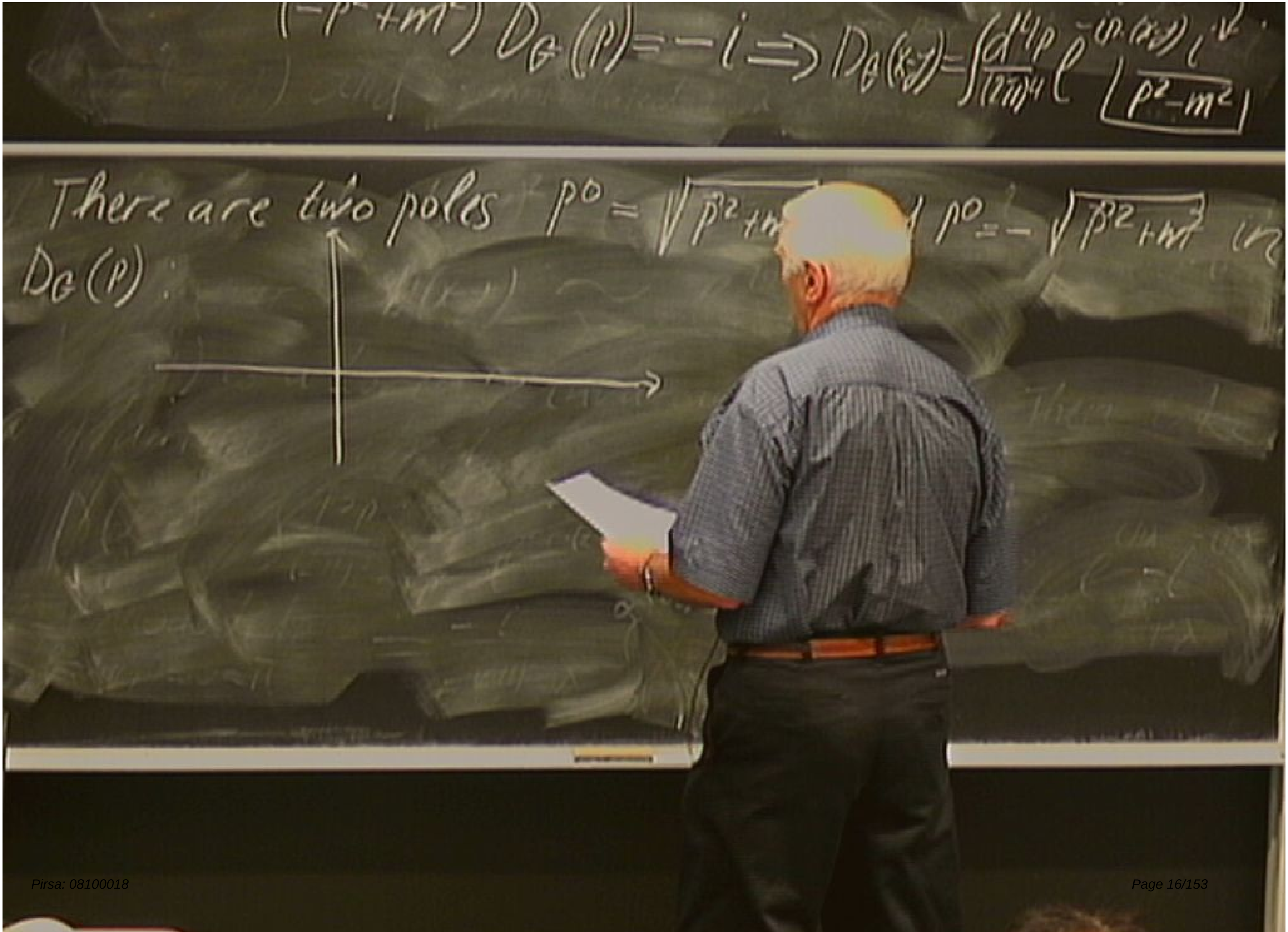


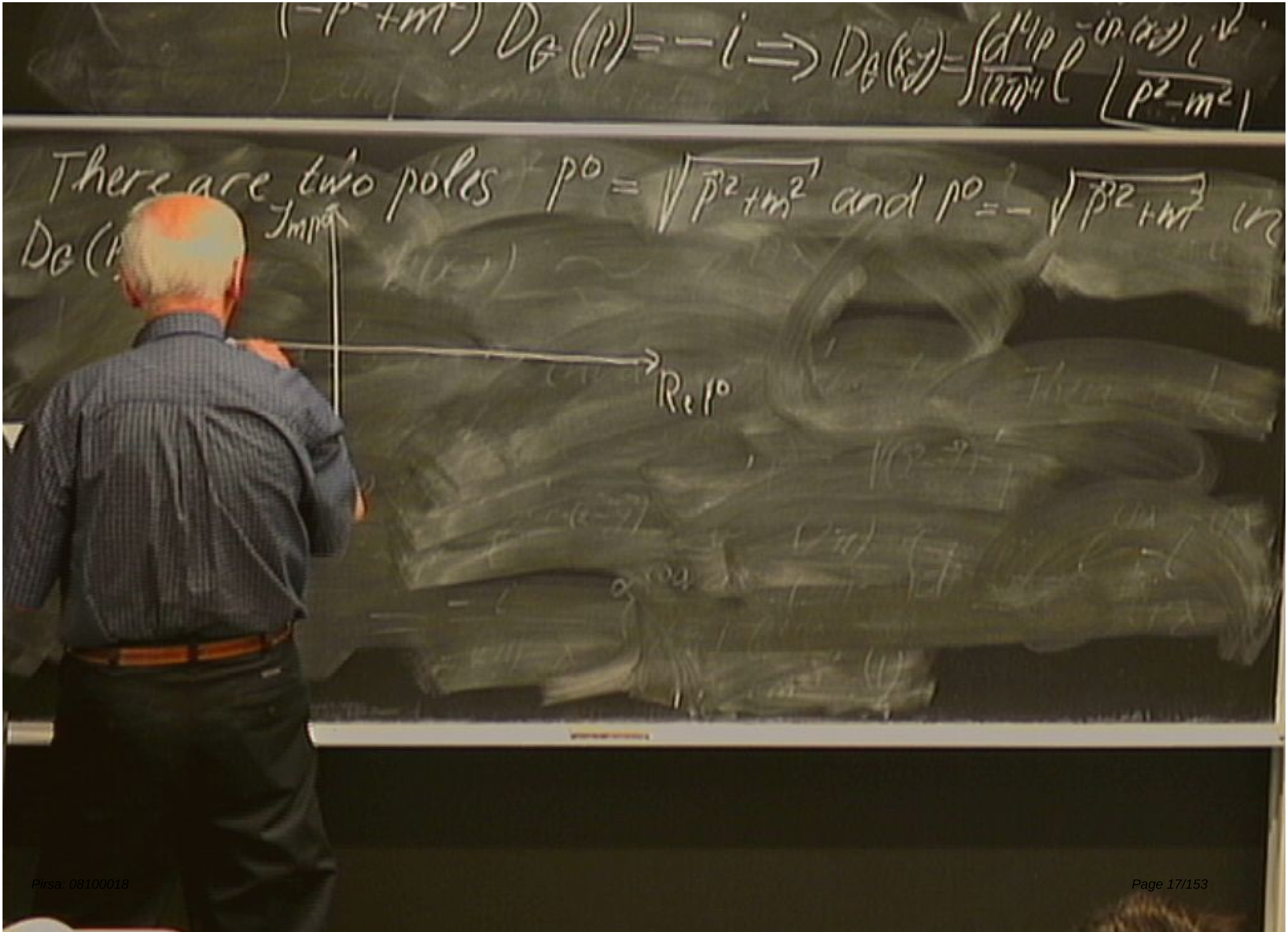
satisfy the following equa-
tion: $(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\delta^{(4)}(x-y)$
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \tilde{D}_G(p)$
Substitute it into equation:
$(-p^2+m^2)\tilde{D}_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} i$


Green's fun... theory
... satisfy the following equa-
tion: $(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\delta^{(4)}(x-y)$
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \tilde{D}_G(p)$
Substitute it into equation:
$(-p^2+m^2)\tilde{D}_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{p^2-m^2}$


the following equa-
tion: $(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\delta^{(4)}(x-y)$
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \tilde{D}_G(p)$
Substitute it into equation:
$(-p^2+m^2)\tilde{D}_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2-m^2}$

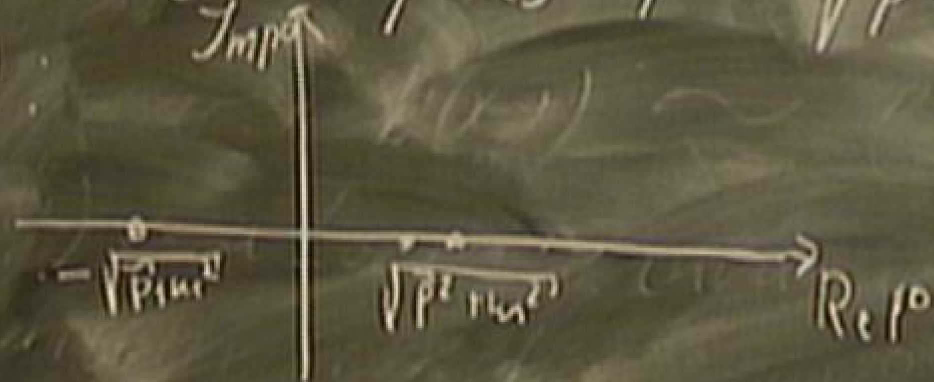

(-p²+m²) D_G(p) = -i ⇒ D_G(x-y) = ∫ d⁴p/(2π)⁴ e^{-ip(x-y)} i/(p²-m²)
poles p⁰ = √(p⃗²+m²) and p⁰ = -√(p⃗²+m²) in
D_G(p) :

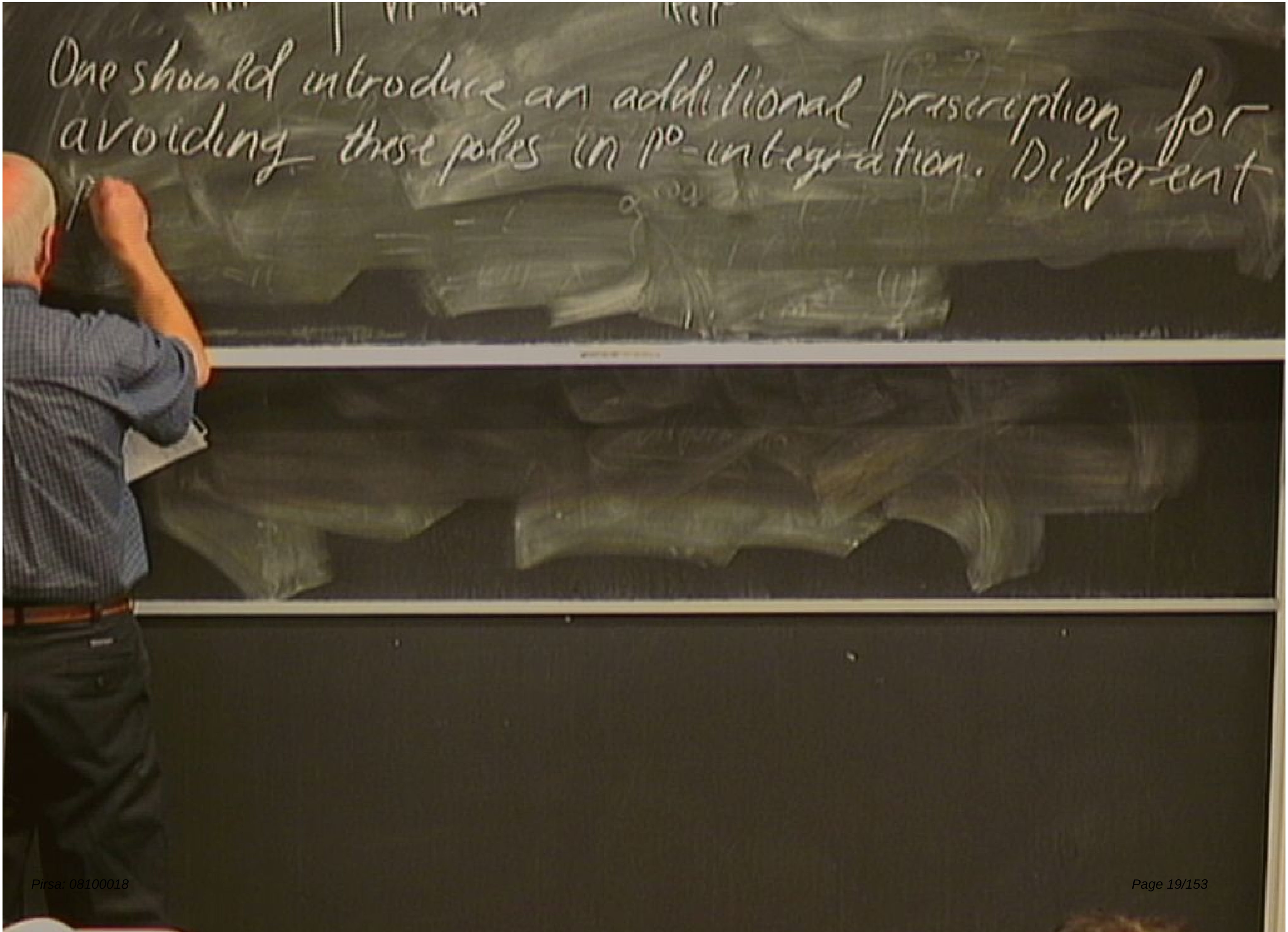

$D_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{p^2-m^2}$
There are two poles
$D_G(p)$:

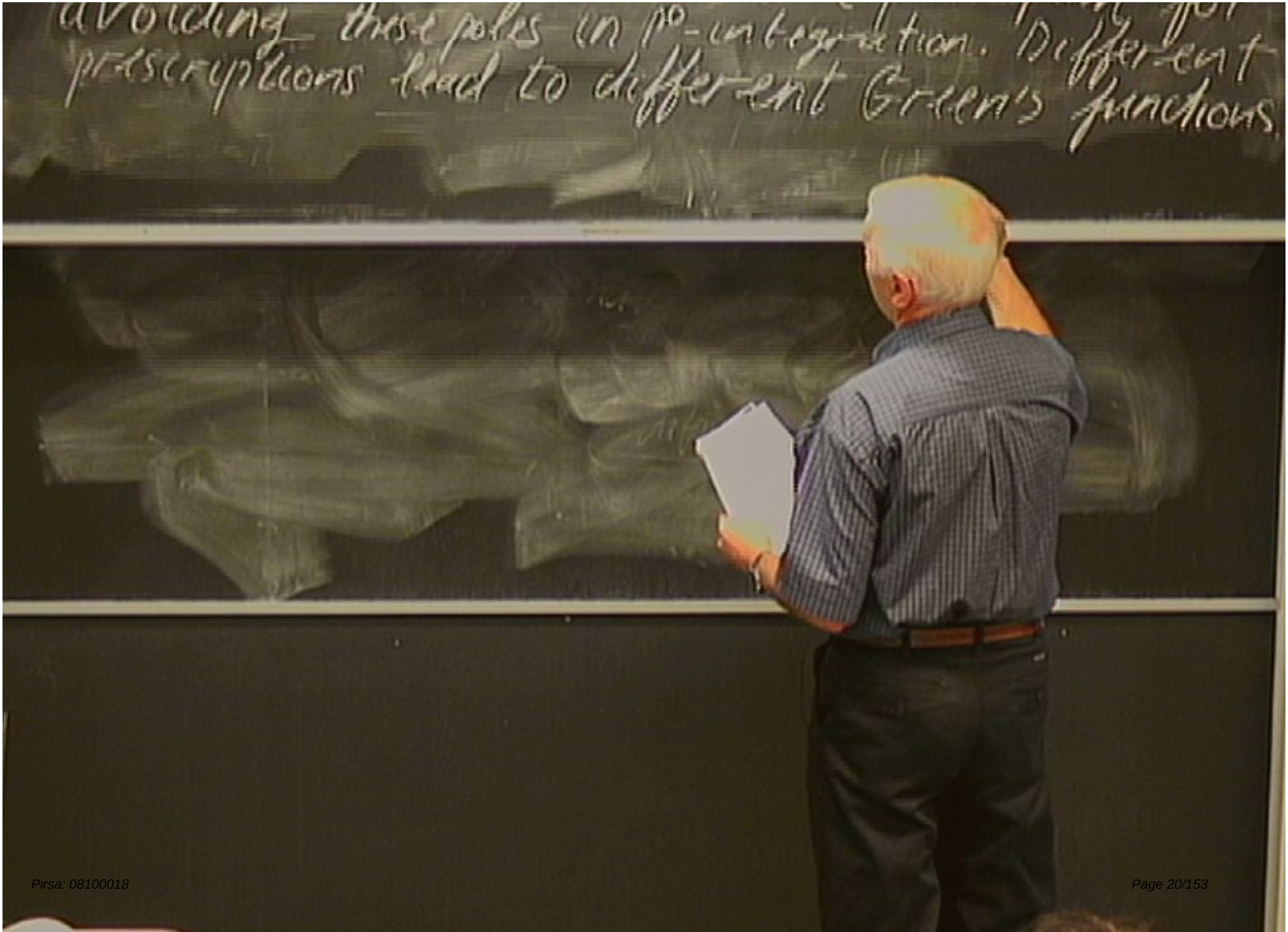

There are two poles p⁰ = √(p⃗²+m²) and p⁰ = −√(p⃗²+m²) in
Im p⁰
Re p⁰

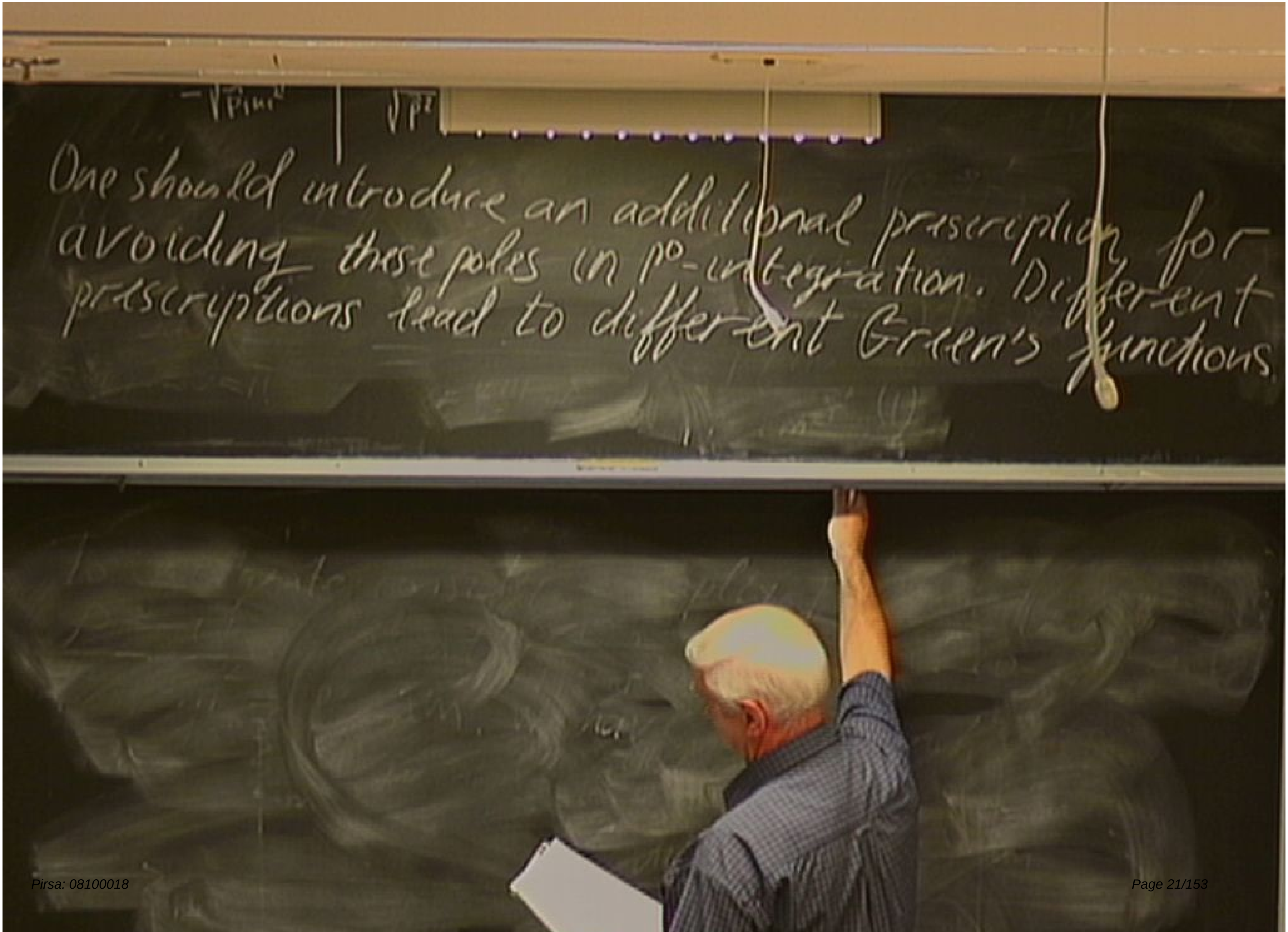
$$(-p^2+m^2)\, D_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{p^2-m^2}$$

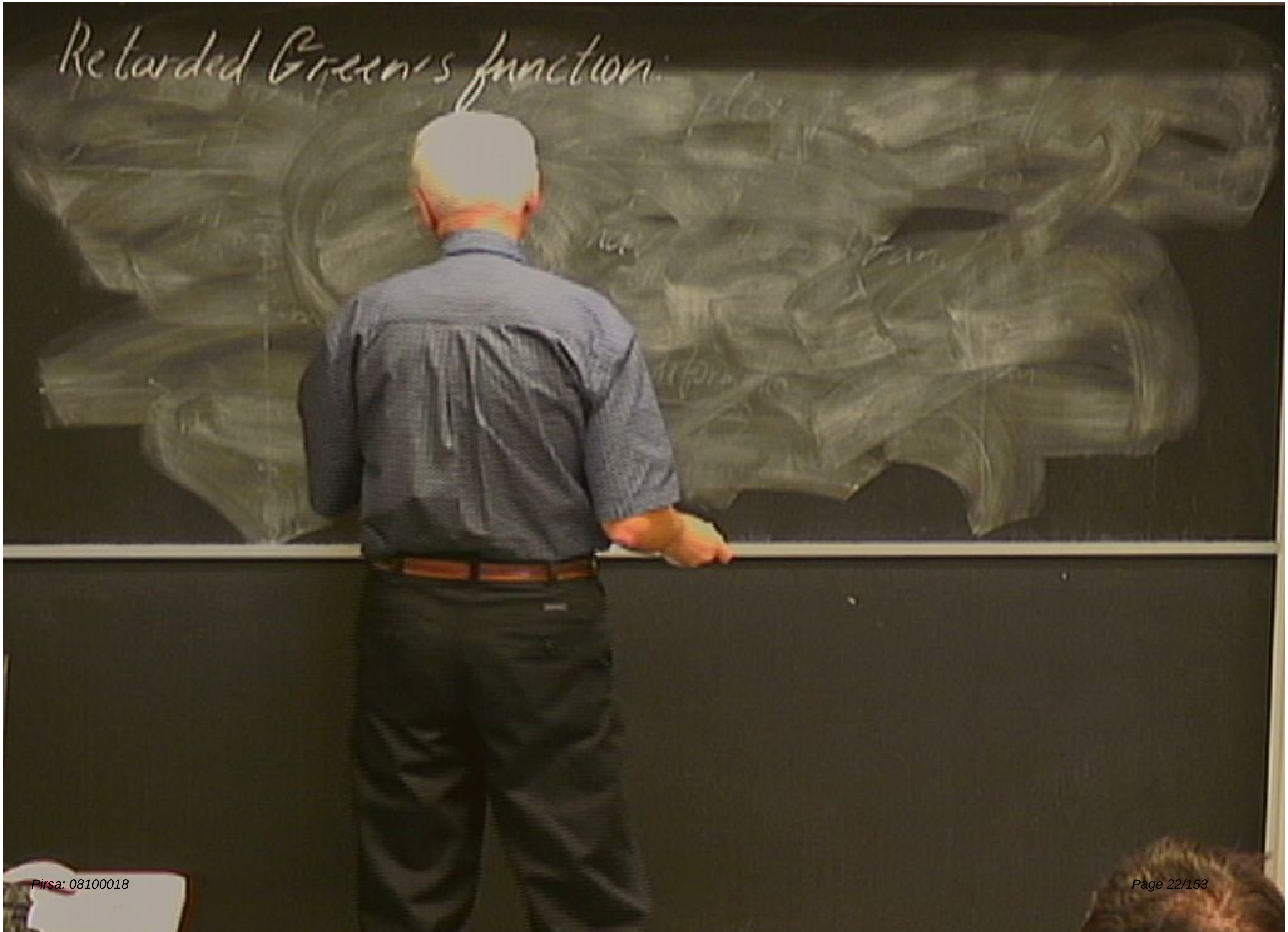
There are two poles $p^0 = \sqrt{\vec{p}^2+m^2}$ and $p^0 = -\sqrt{\vec{p}^2+m^2}$ in $D_G(p)$:

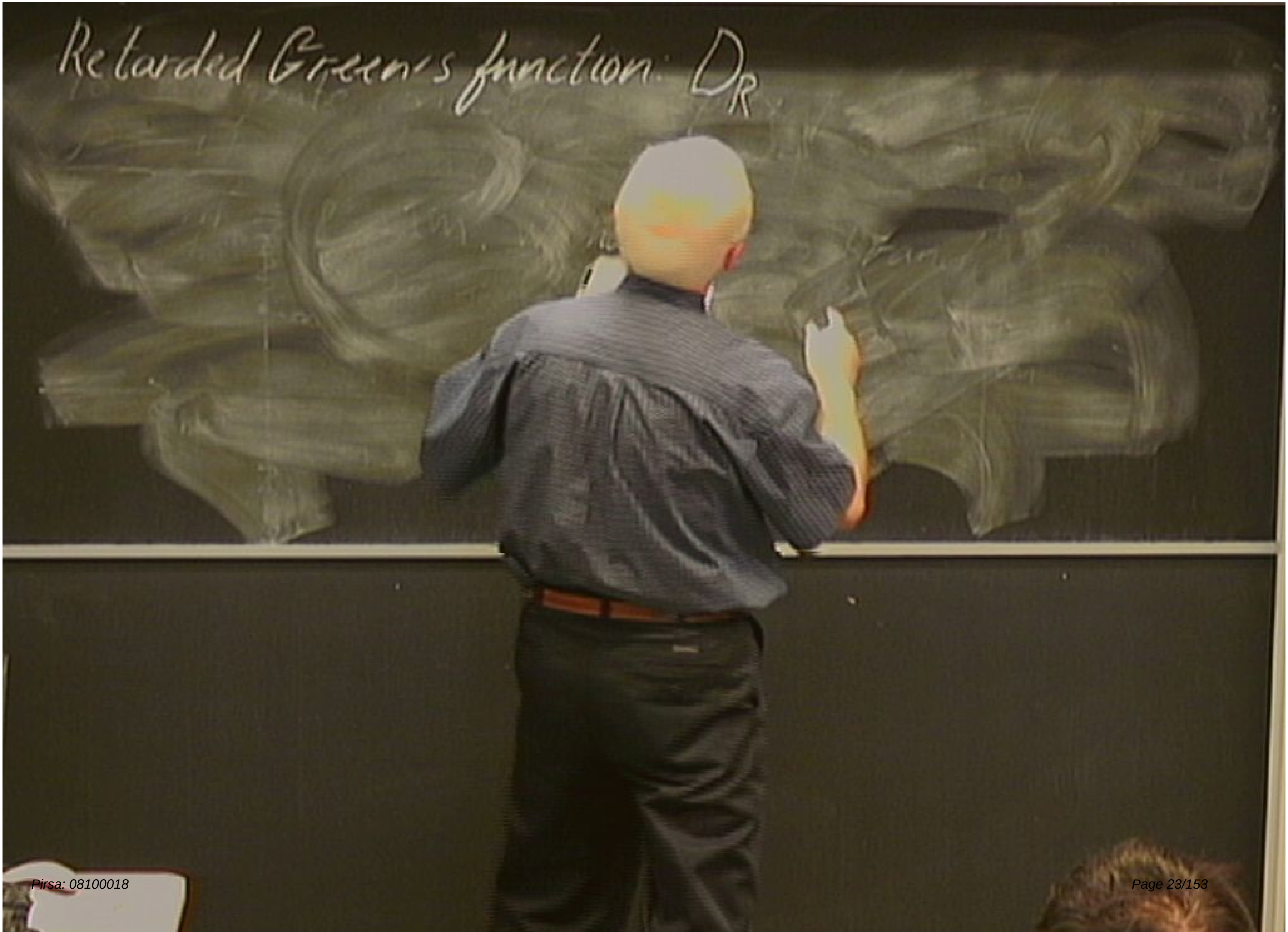


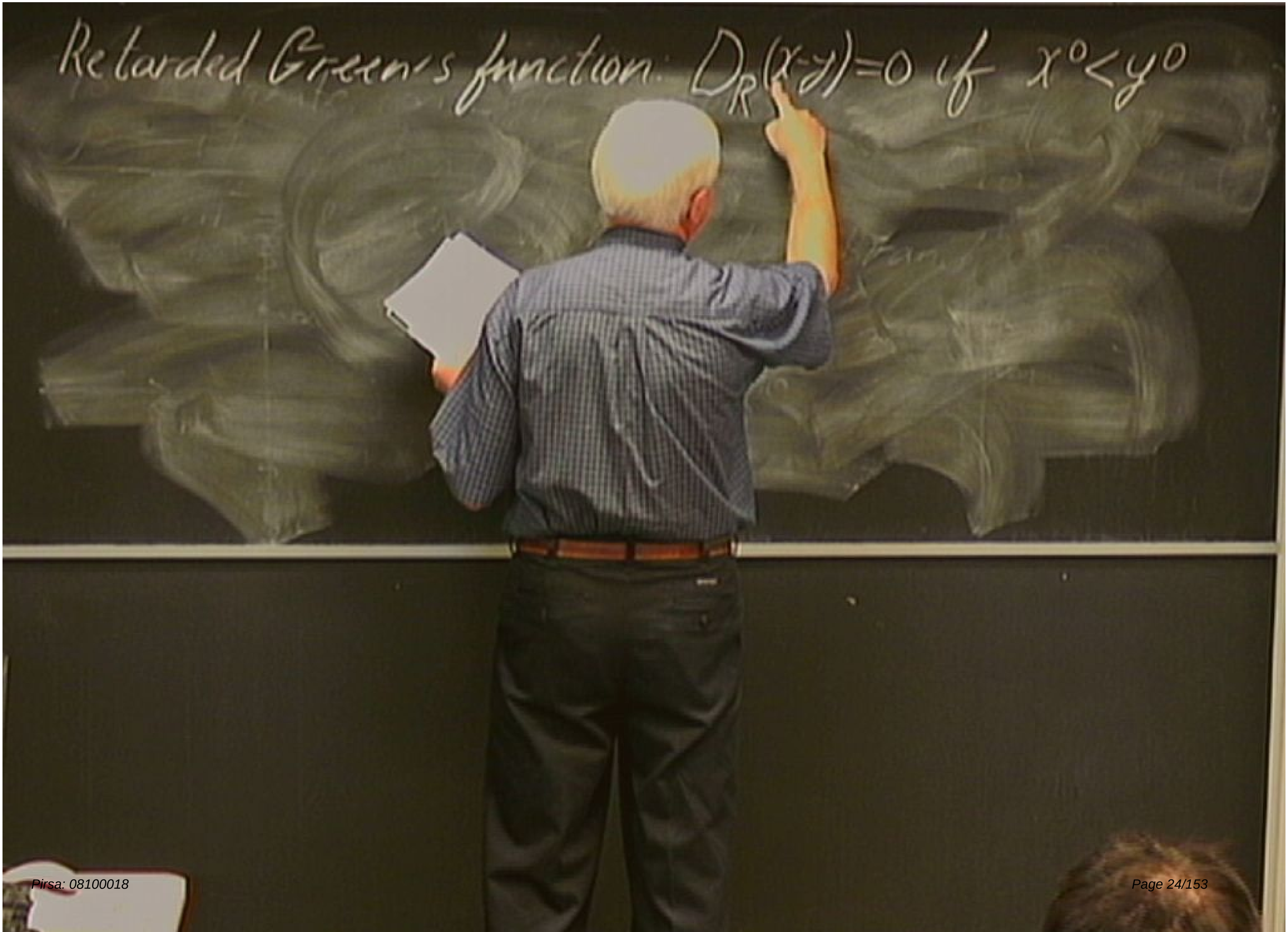

One should introduce an additional prescription for
avoiding these poles in p⁰-integration. Different

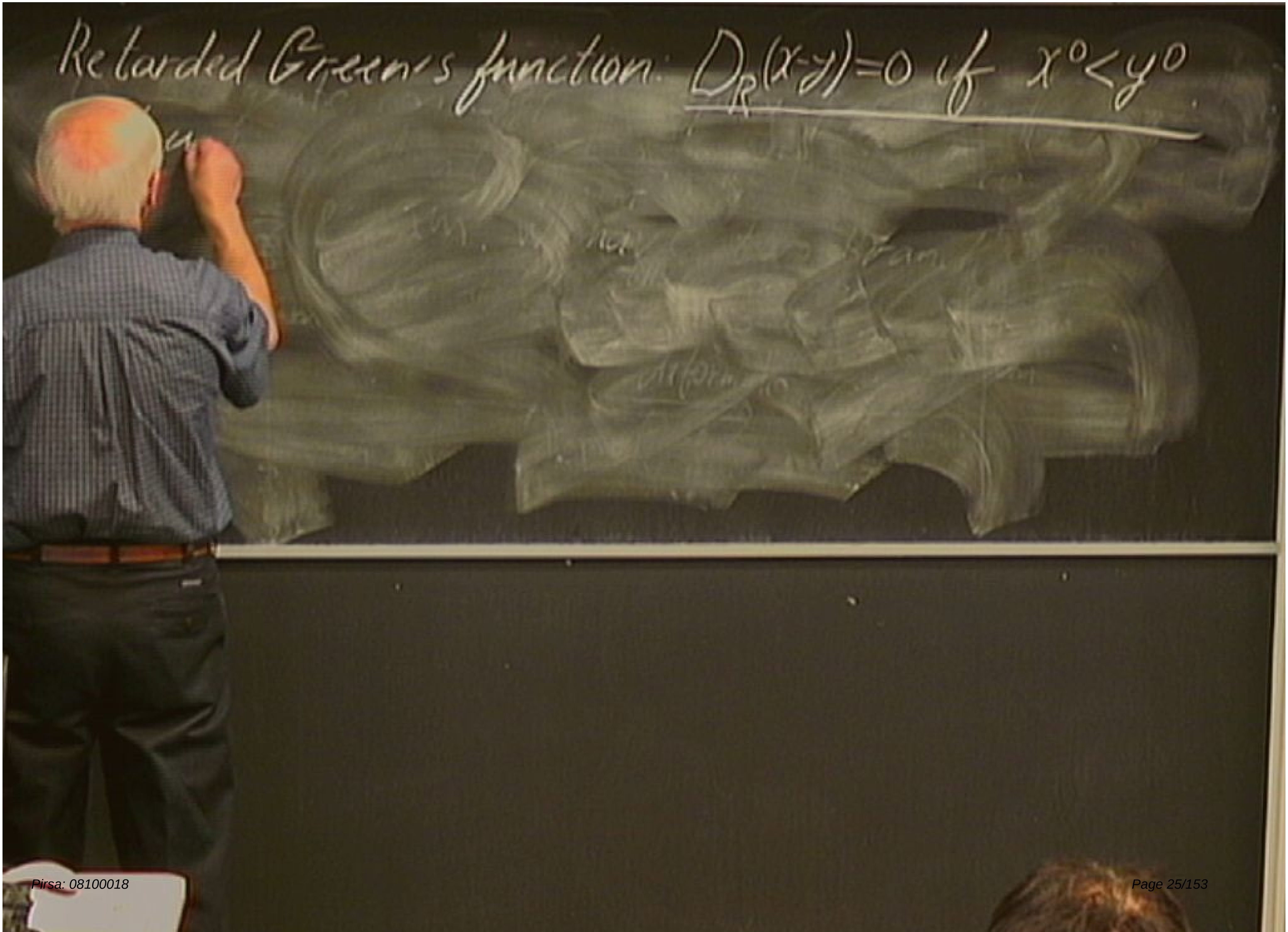

avoiding these poles in p°-integration. Different
prescriptions lead to different Green's functions.

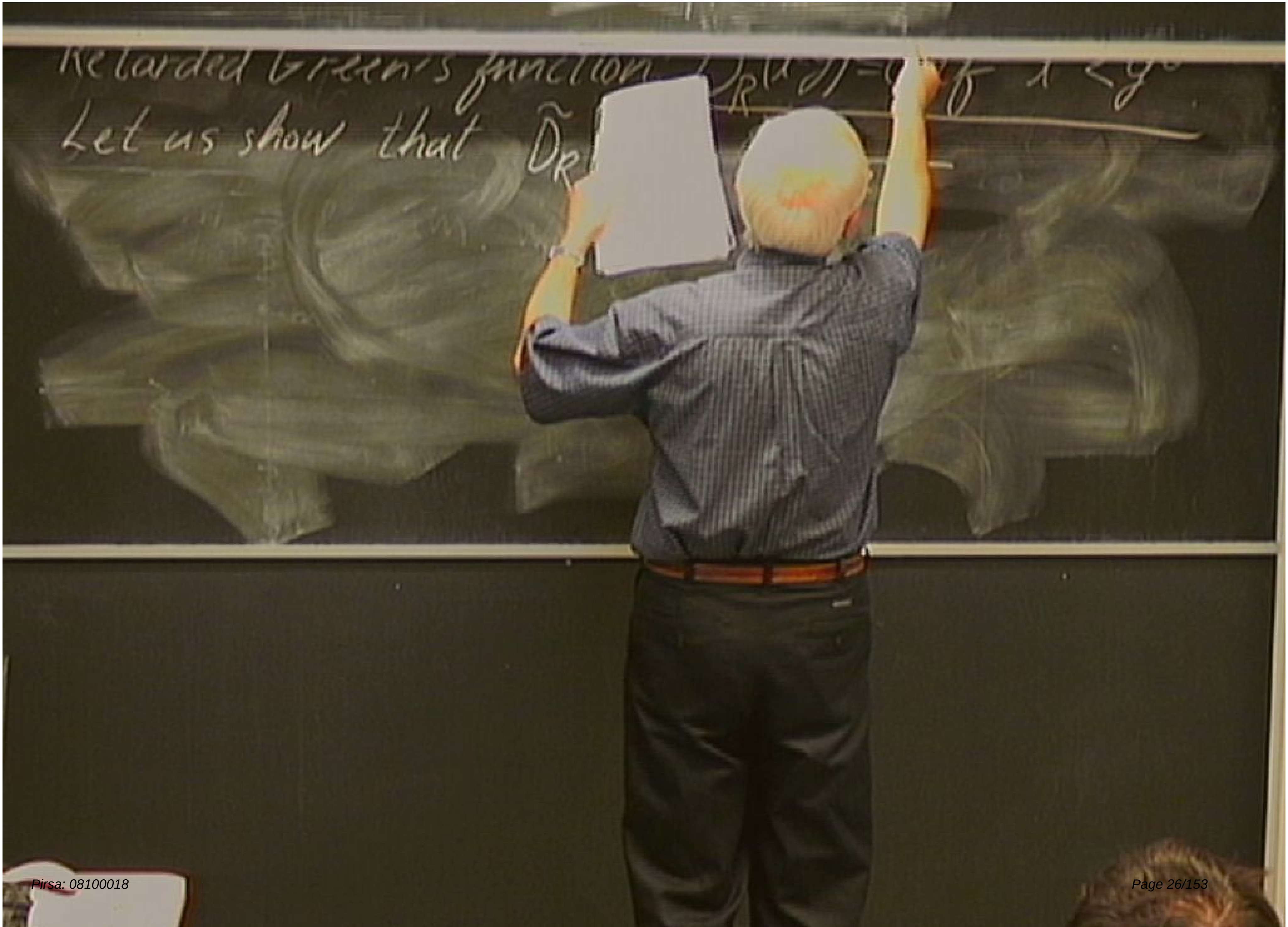

One should introduce an additional prescription for
avoiding these poles in p0-integration. Different
prescriptions lead to different Green's functions

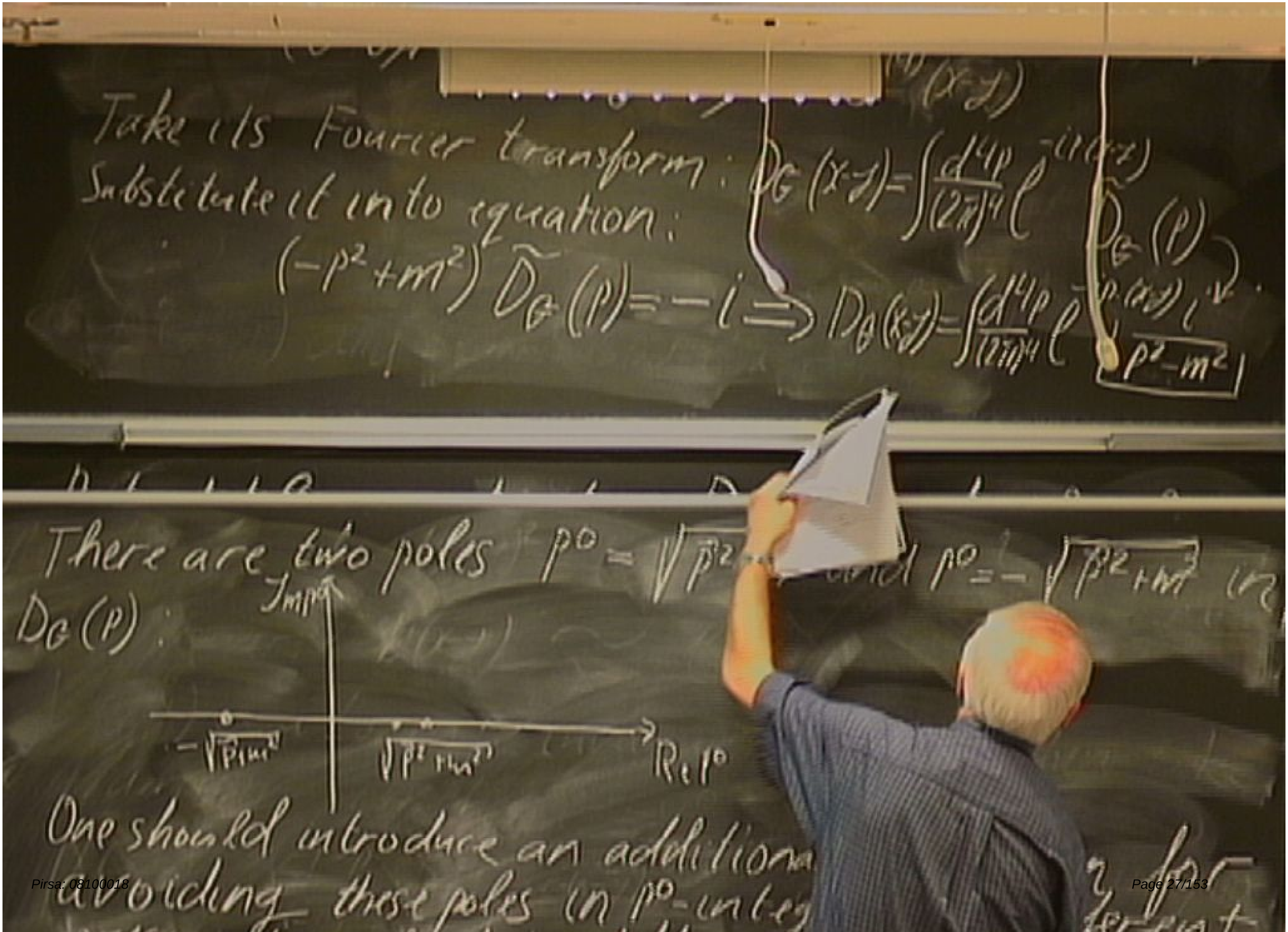

Retarded Green's function:

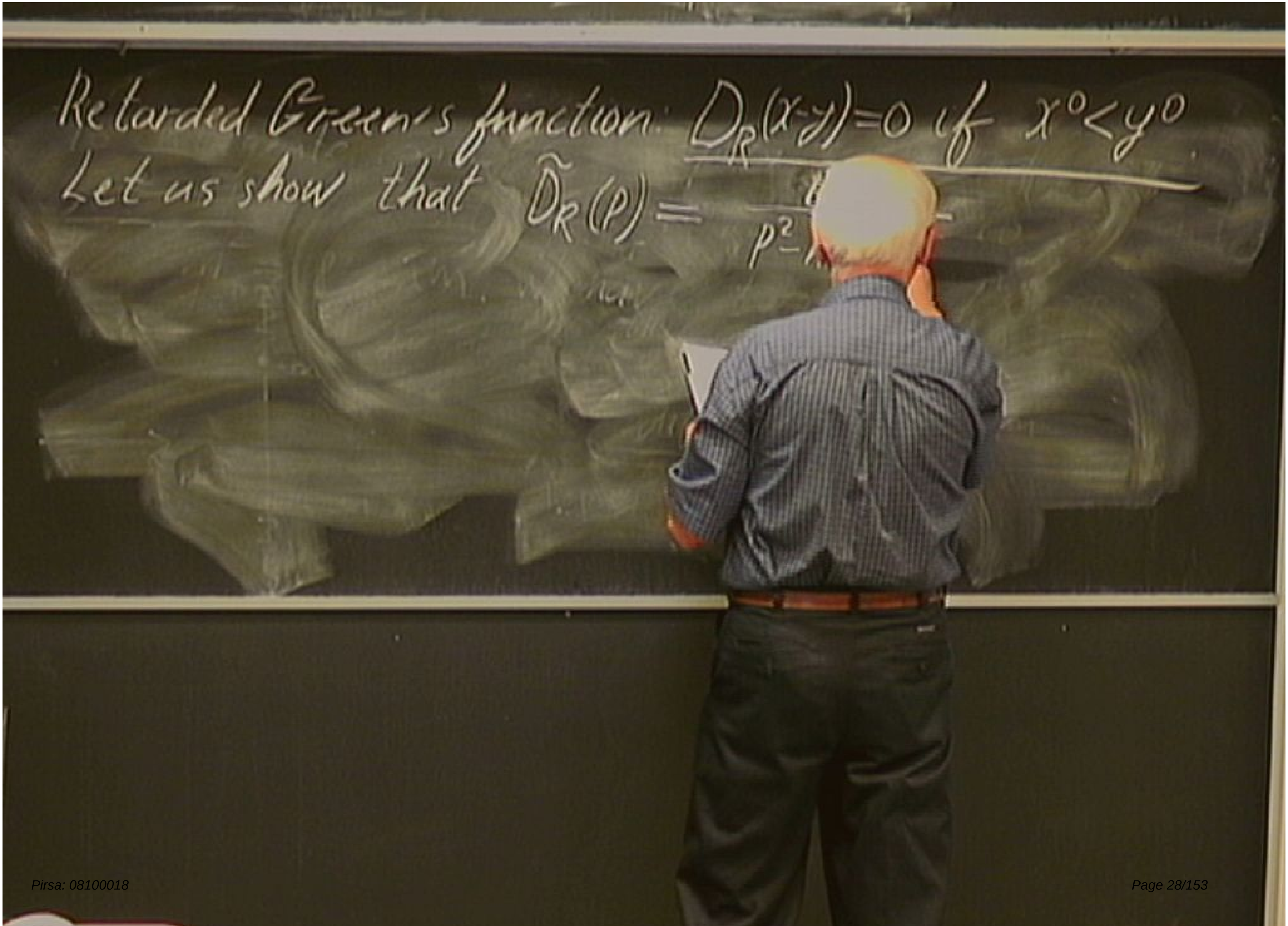

Retarded Green's function: D_R

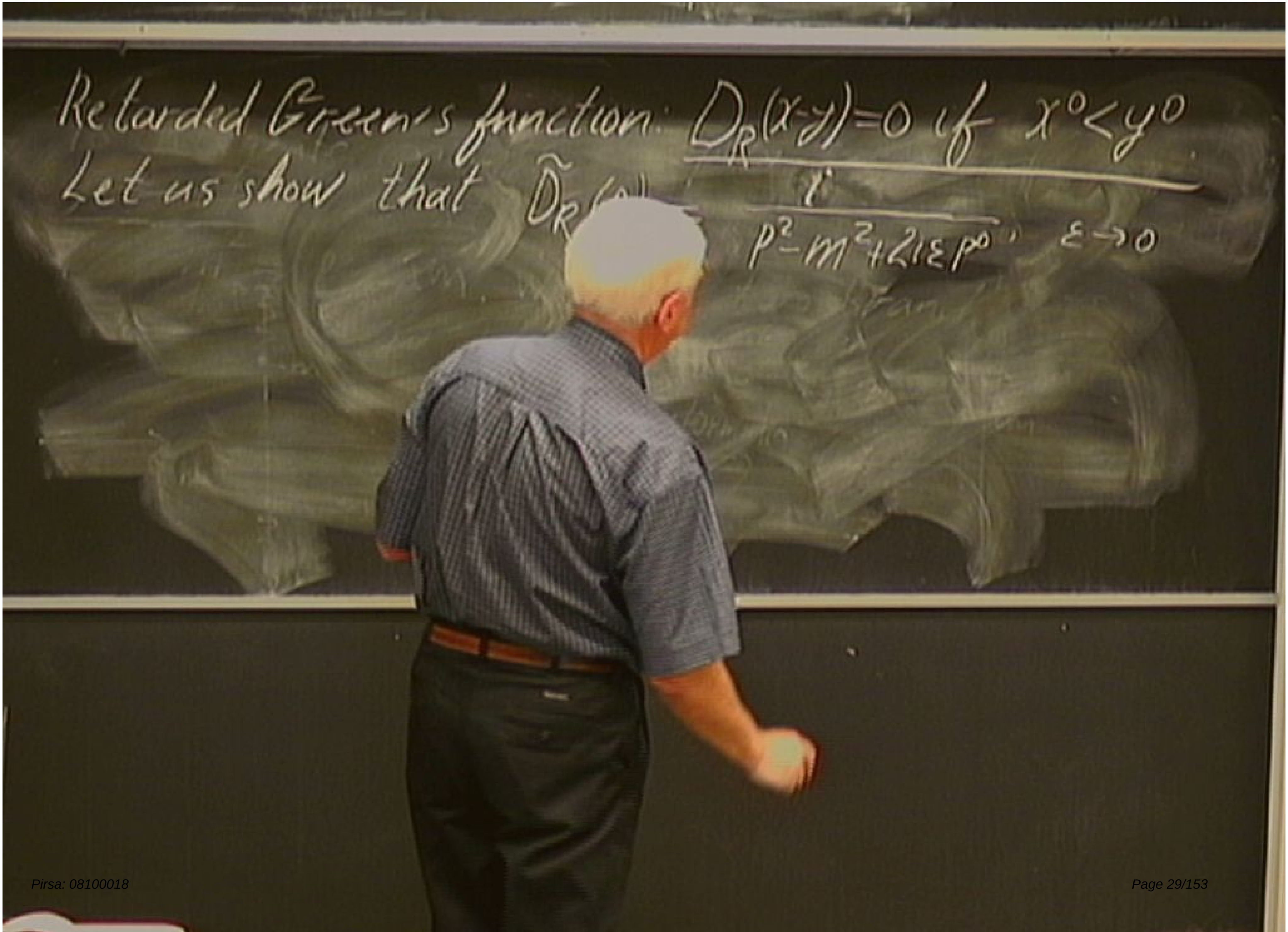

Retarded Green's function: D_R(x-y)=0 if x^0<y^0

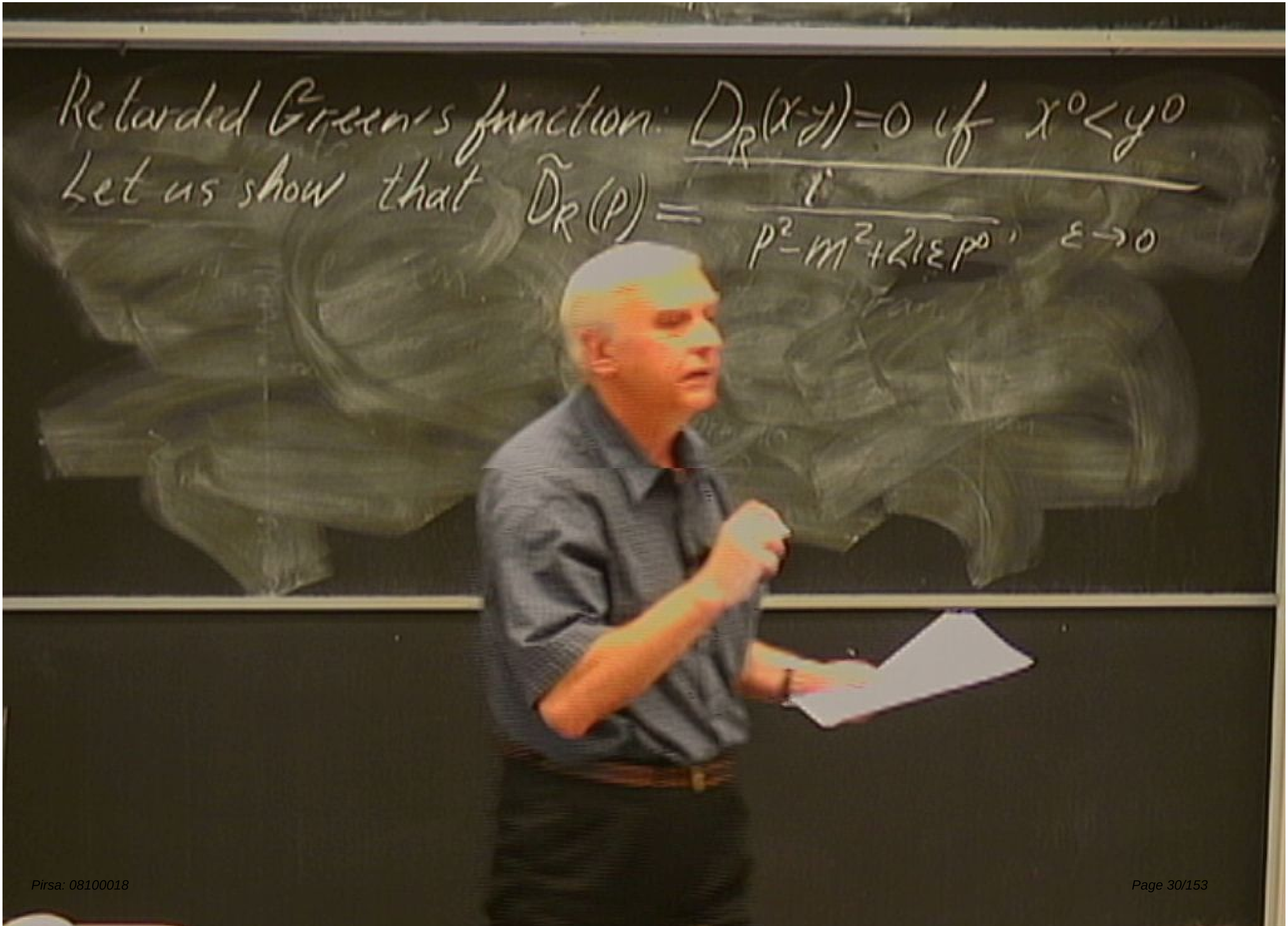

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$

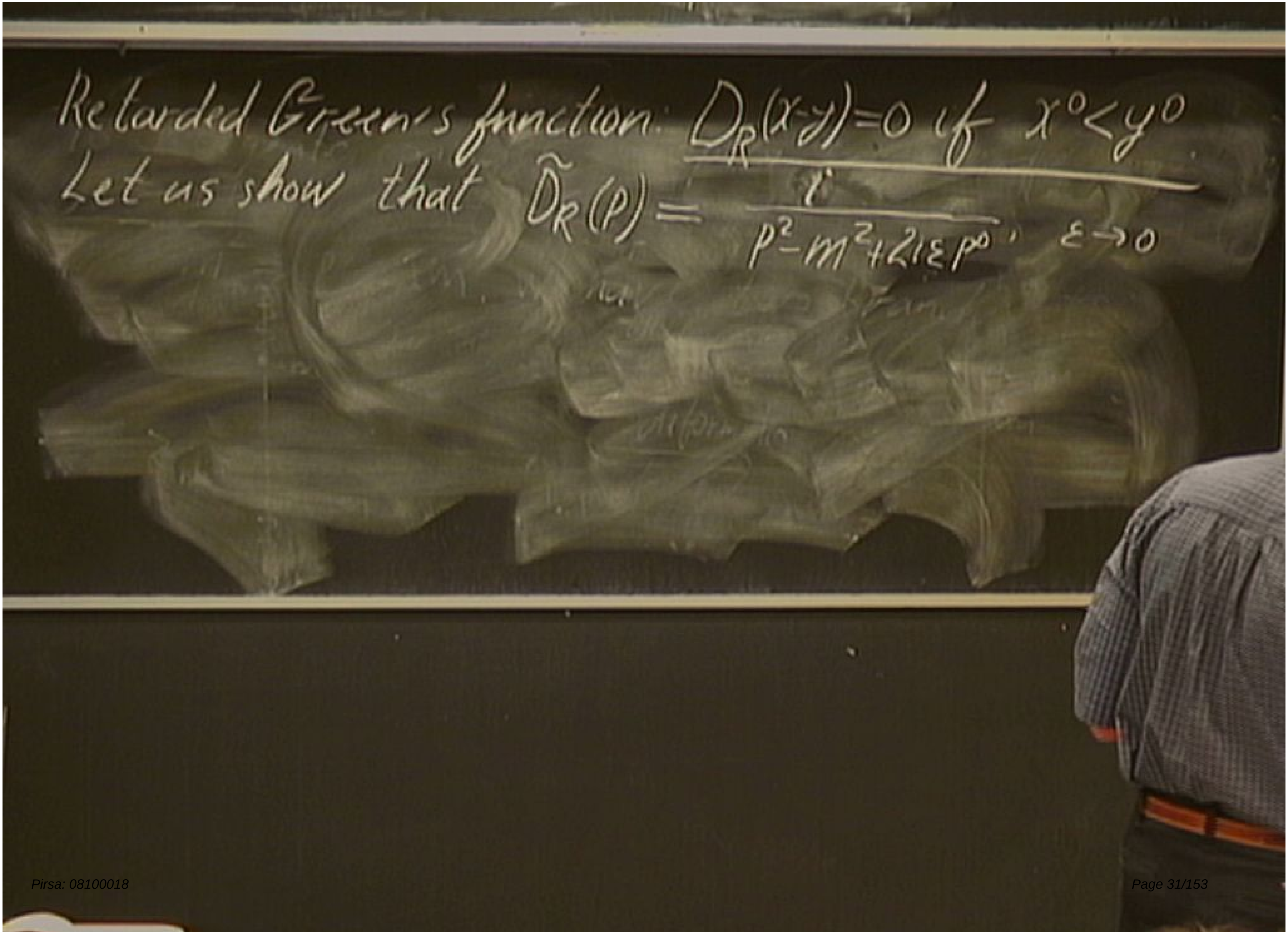

Retarded Green's function
Let us show that

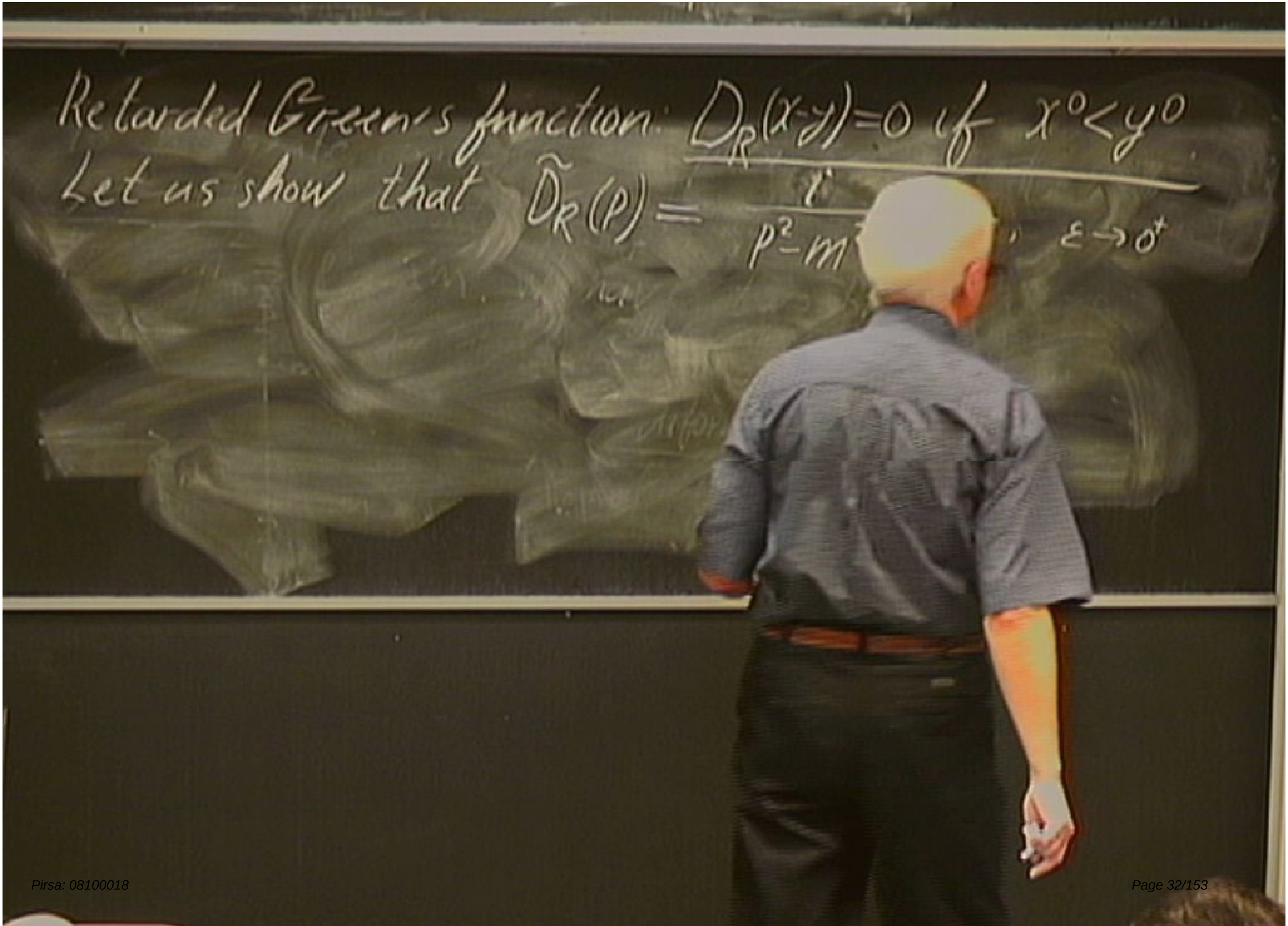

Take its Fourier transform: D_G(x-y) = ∫ d^4p/(2π)^4 e^{-ip(x-y)} D̃_G(p)
Substitute it into equation:
(-p^2+m^2) D̃_G(p) = -i ⇒ D_G(x-y) = ∫ d^4p/(2π)^4 e^{-ip(x-y)} i/(p^2-m^2)
There are two poles p^0 = √(p⃗^2 and p^0 = -√(p⃗^2+m^2)
D_G(p):
Im p^0
Re p^0
One should introduce an additiona for
avoiding these poles in p^0-integ

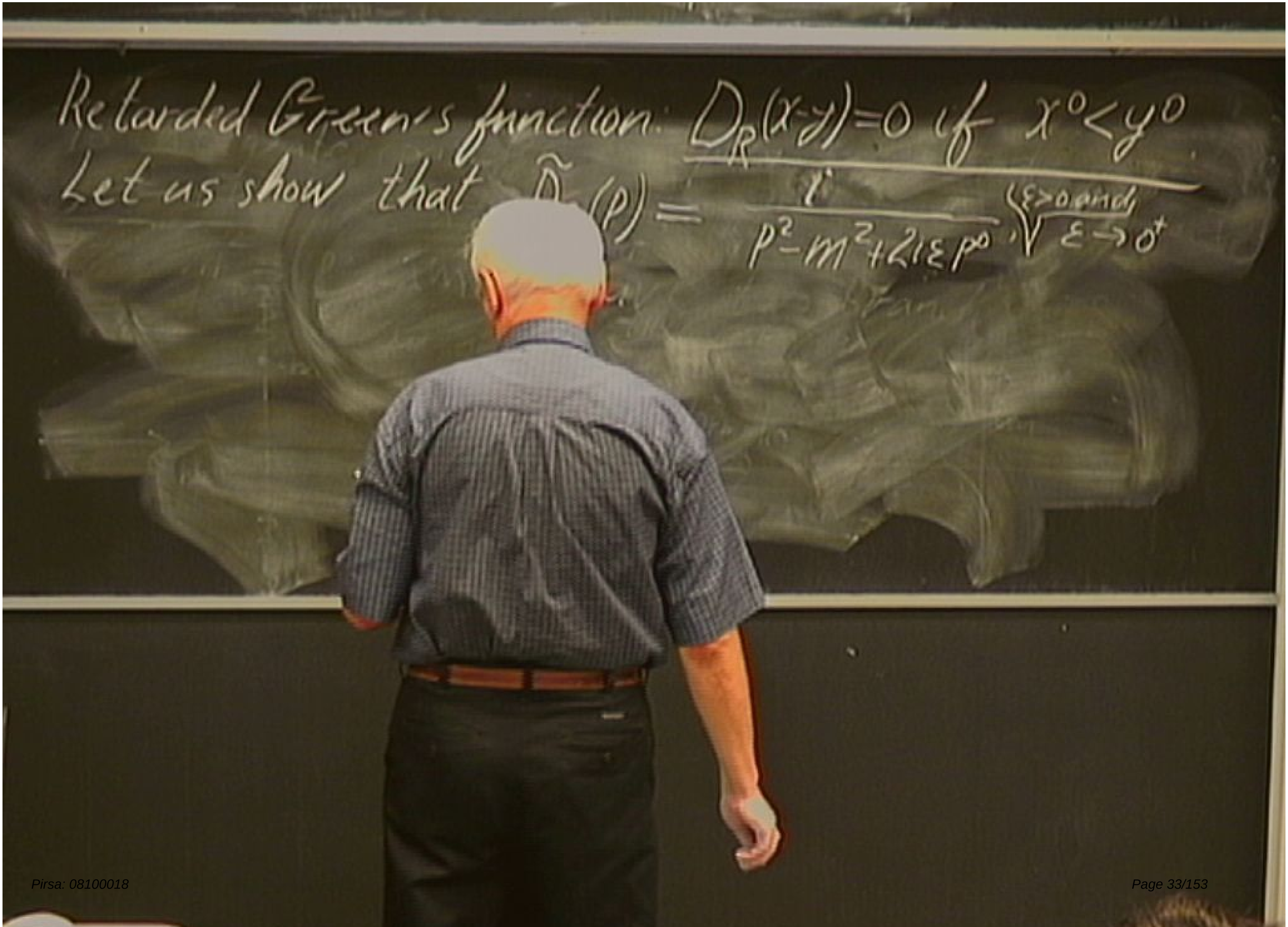

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R(p) =$
p^2-

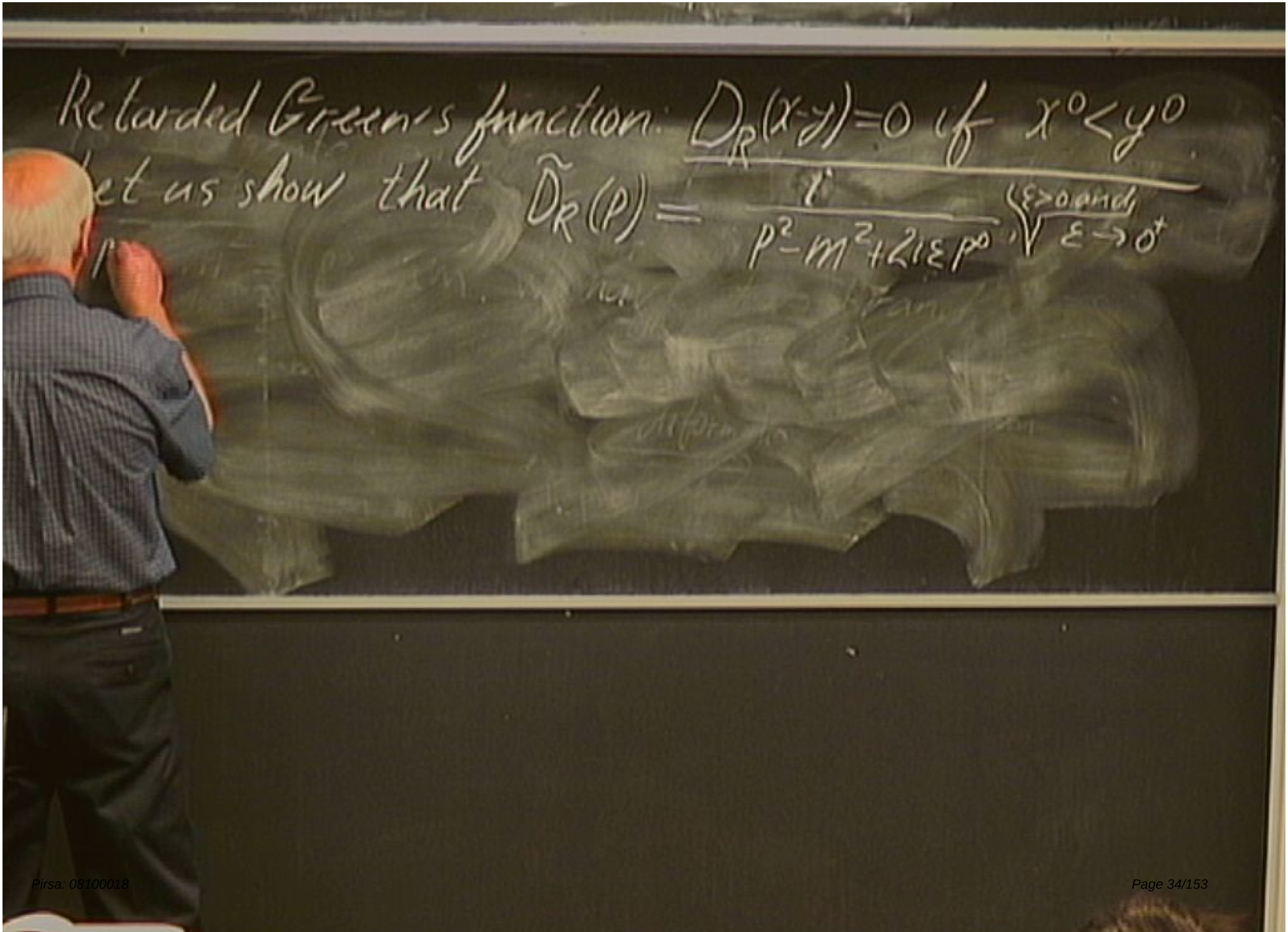

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R$
$\frac{i}{p^2-m^2+2i\varepsilon p^0}$, $\varepsilon\to 0$

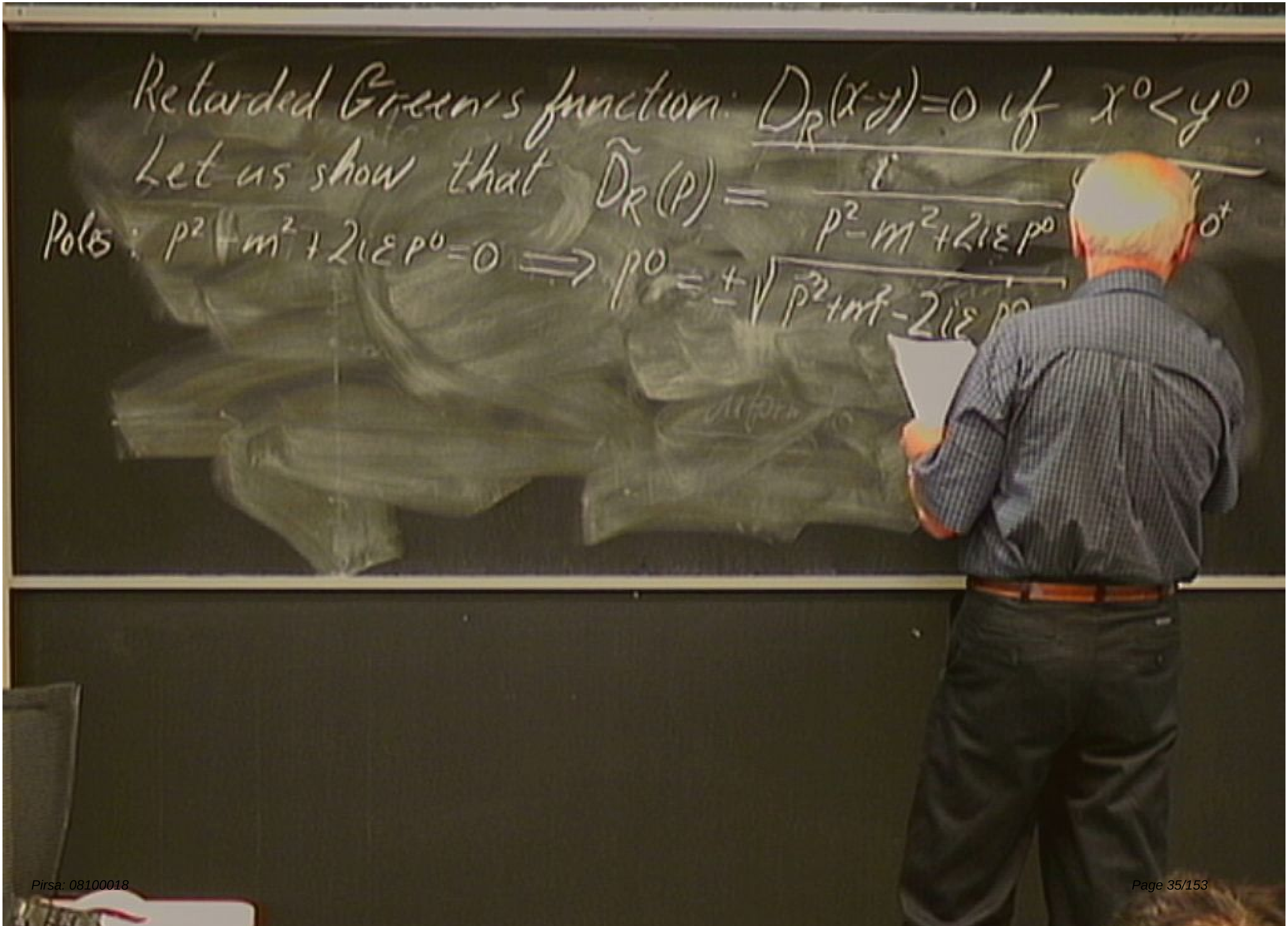

Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / (p^2-m^2+2iεp^0), ε→0

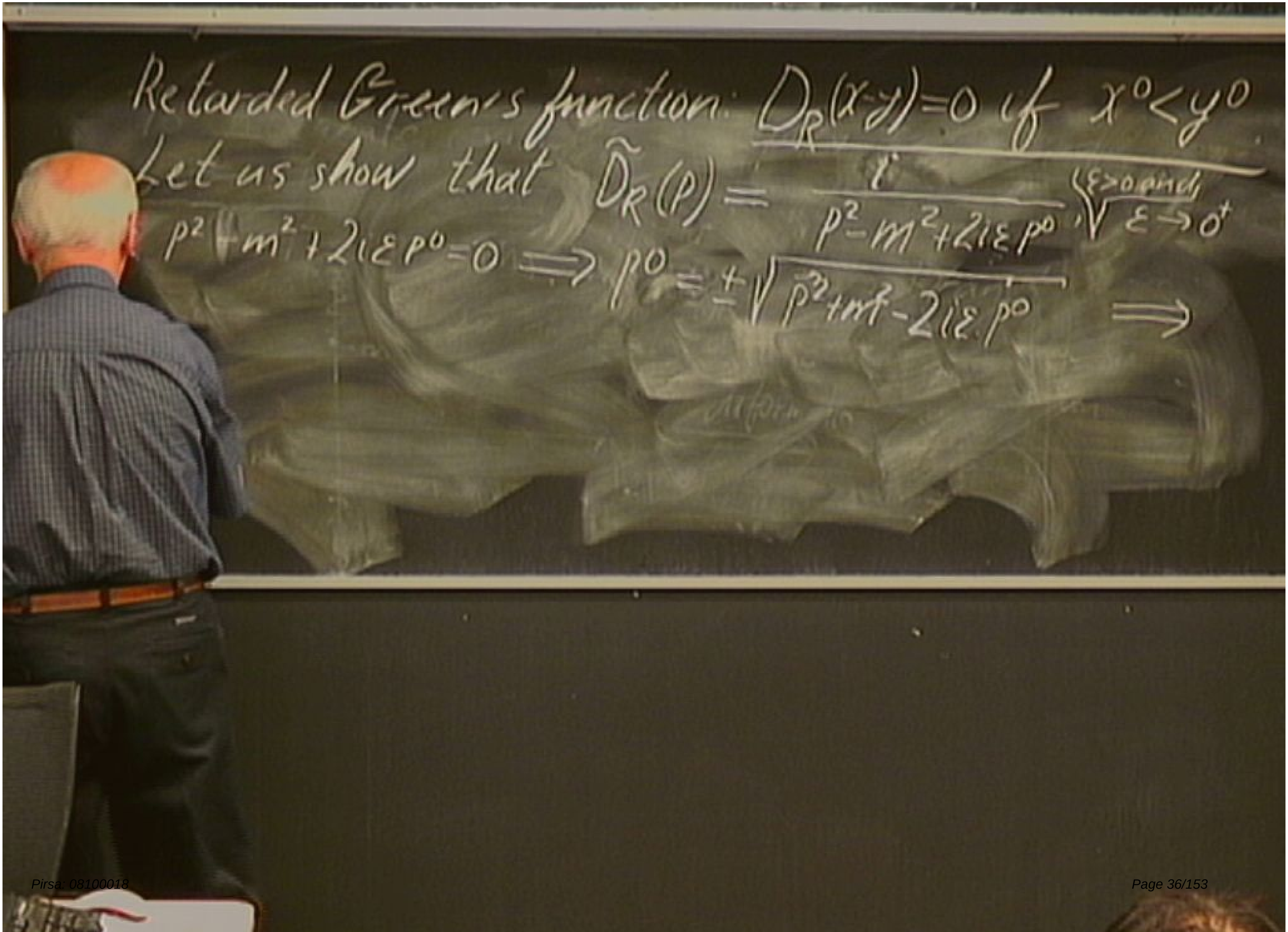

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R(p) = \frac{i}{p^2-m^2+2i\varepsilon p^0}$, $\varepsilon \to 0$

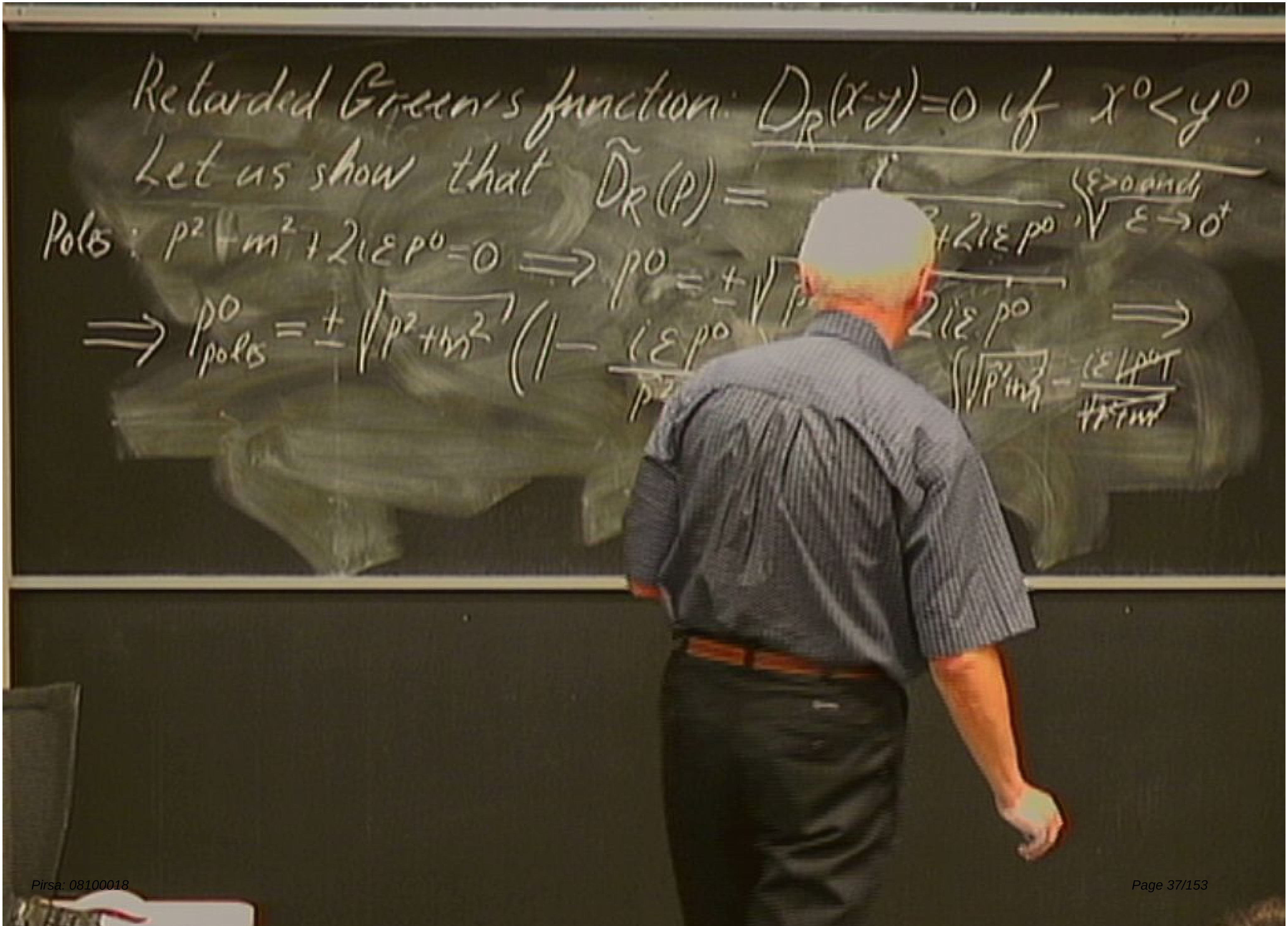

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R(p) = \frac{i}{p^2-m^2 \ldots}$, $\varepsilon \to 0^+$

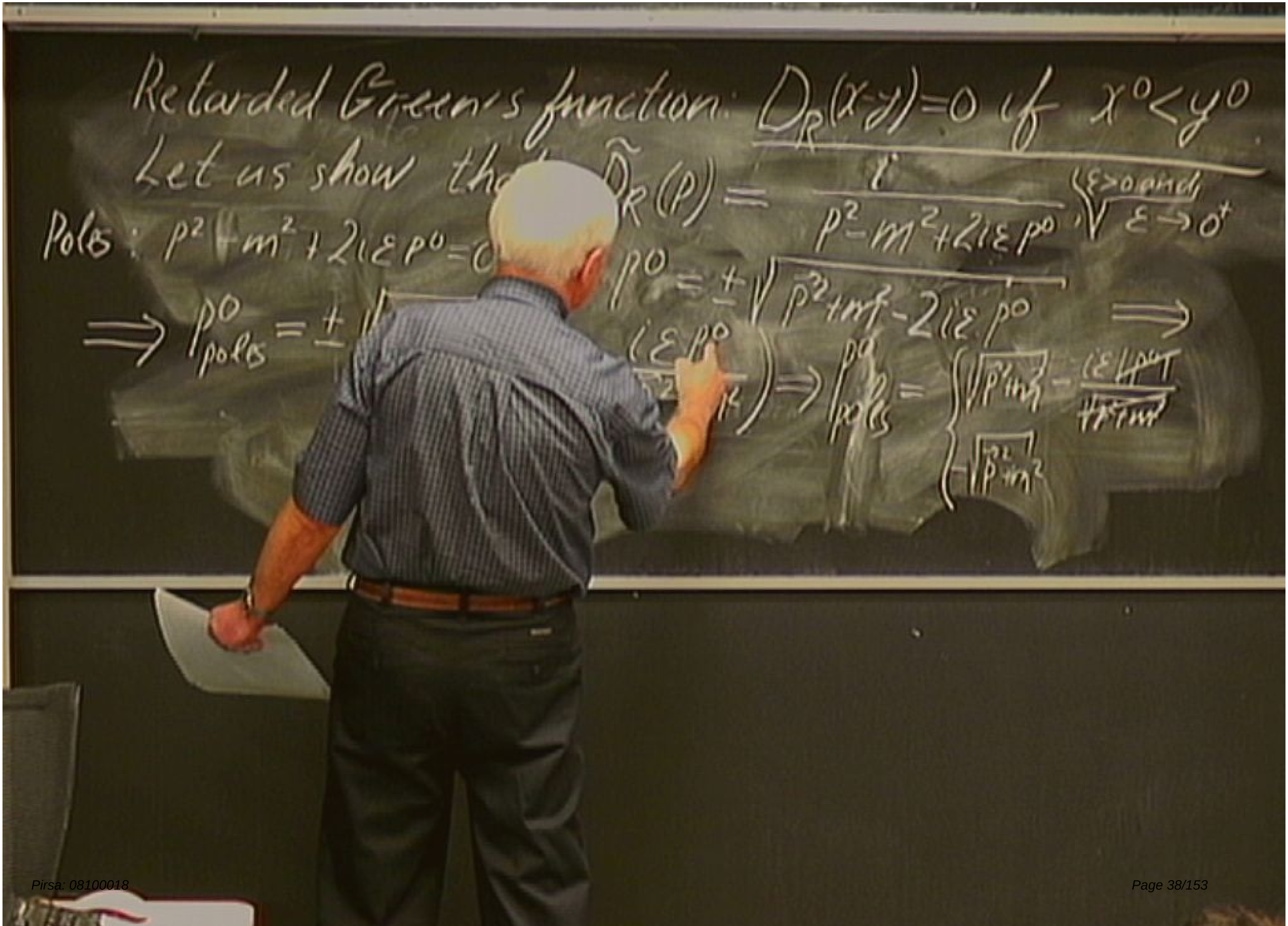

Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / (p^2-m^2+2iεp^0), ε>0 and ε→0^+

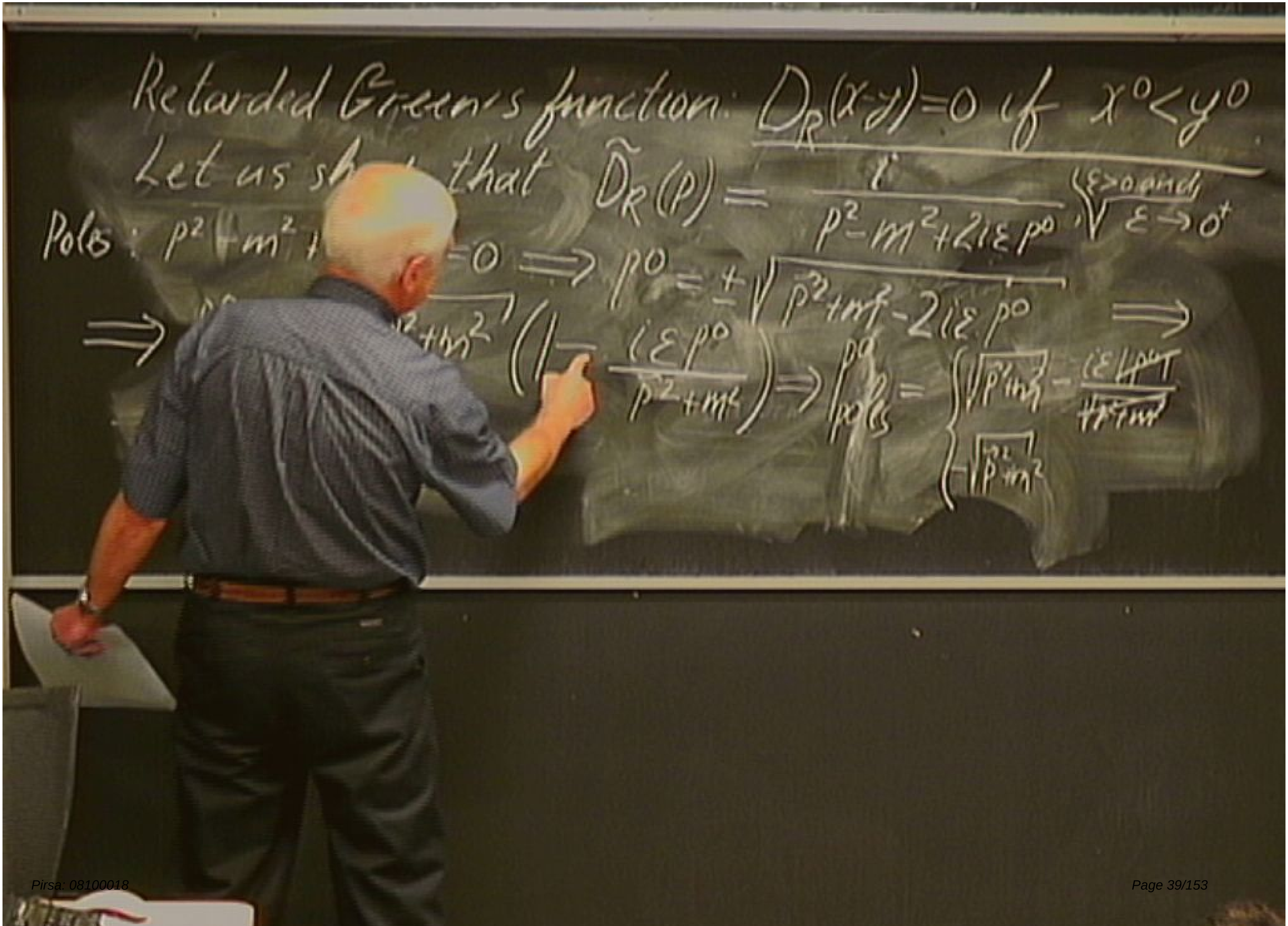

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R(p) = \frac{i}{p^2-m^2+2i\varepsilon p^0}$ ($\varepsilon>0$ and $\varepsilon\to 0^+$)

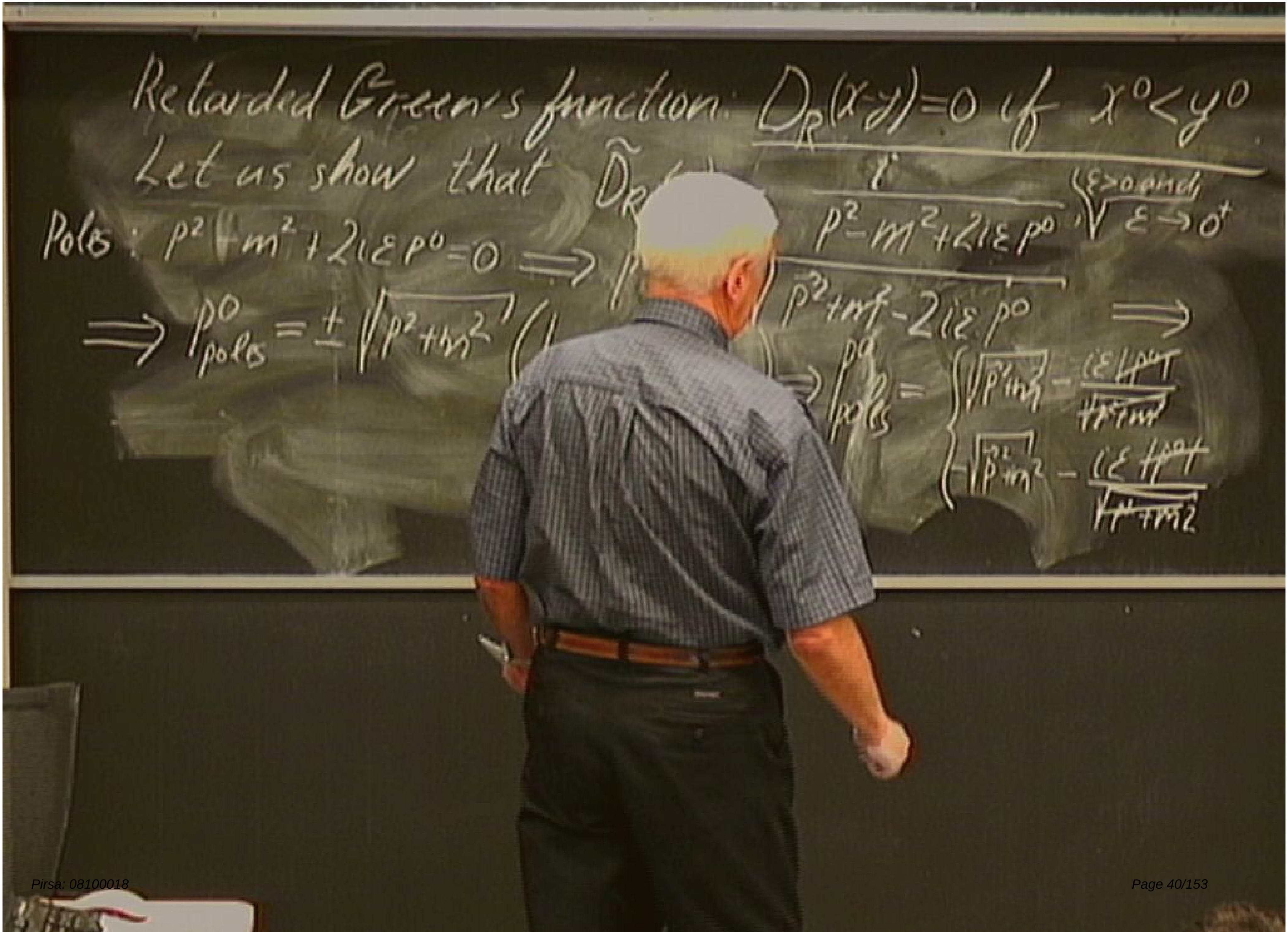

Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / (p^2-m^2+2iεp^0)
Poles: p^2-m^2+2iεp^0=0 ⟹ p^0=±√(p⃗^2+m^2-2iεp^0

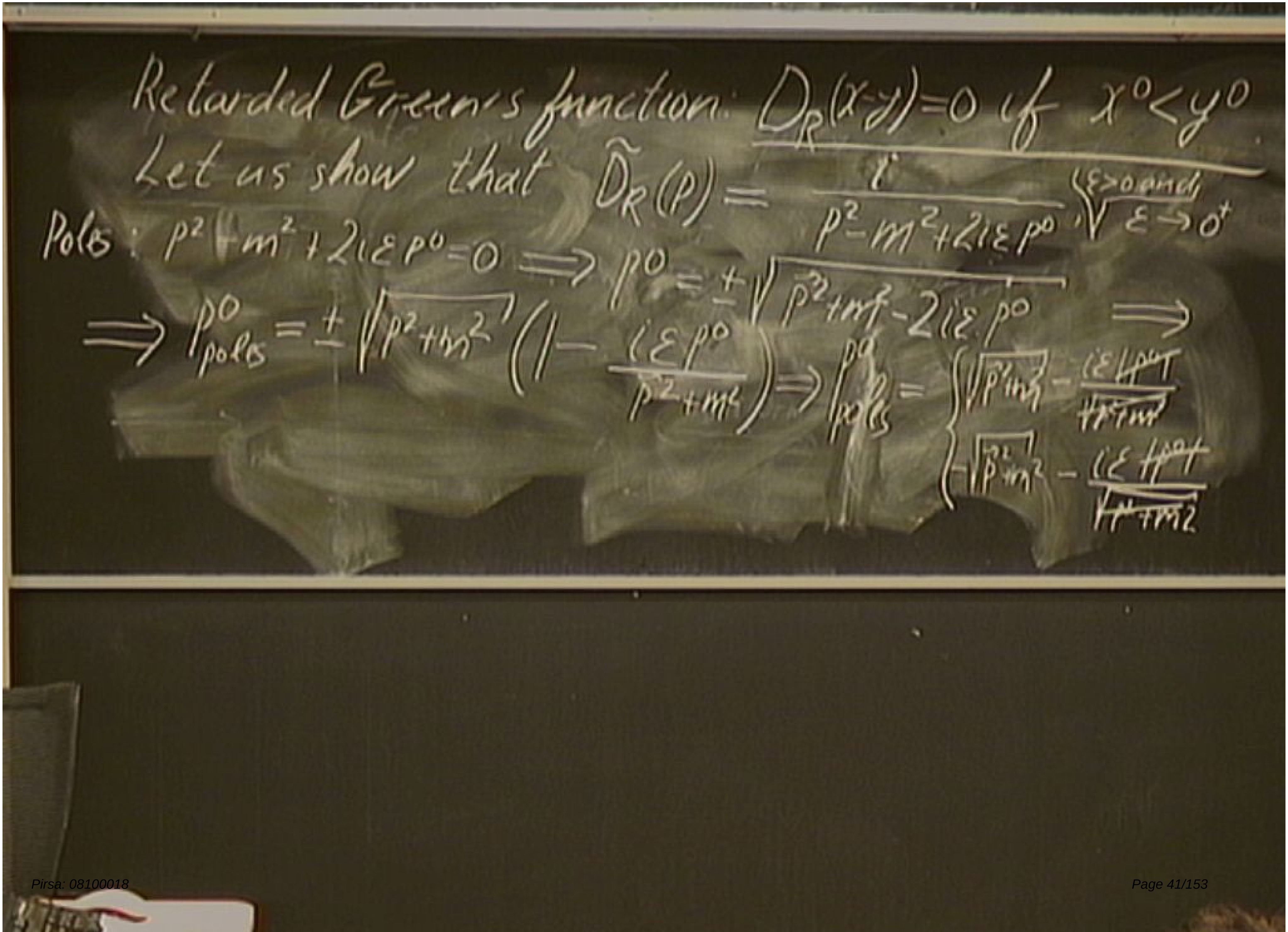

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R(p) = \frac{i}{p^2-m^2+2i\varepsilon p^0}$, ($\varepsilon>0$ and $\varepsilon \to 0^+$)
$p^2-m^2+2i\varepsilon p^0=0 \Longrightarrow p^0 = \pm\sqrt{\vec{p}^2+m^2-2i\varepsilon p^0} \Longrightarrow$

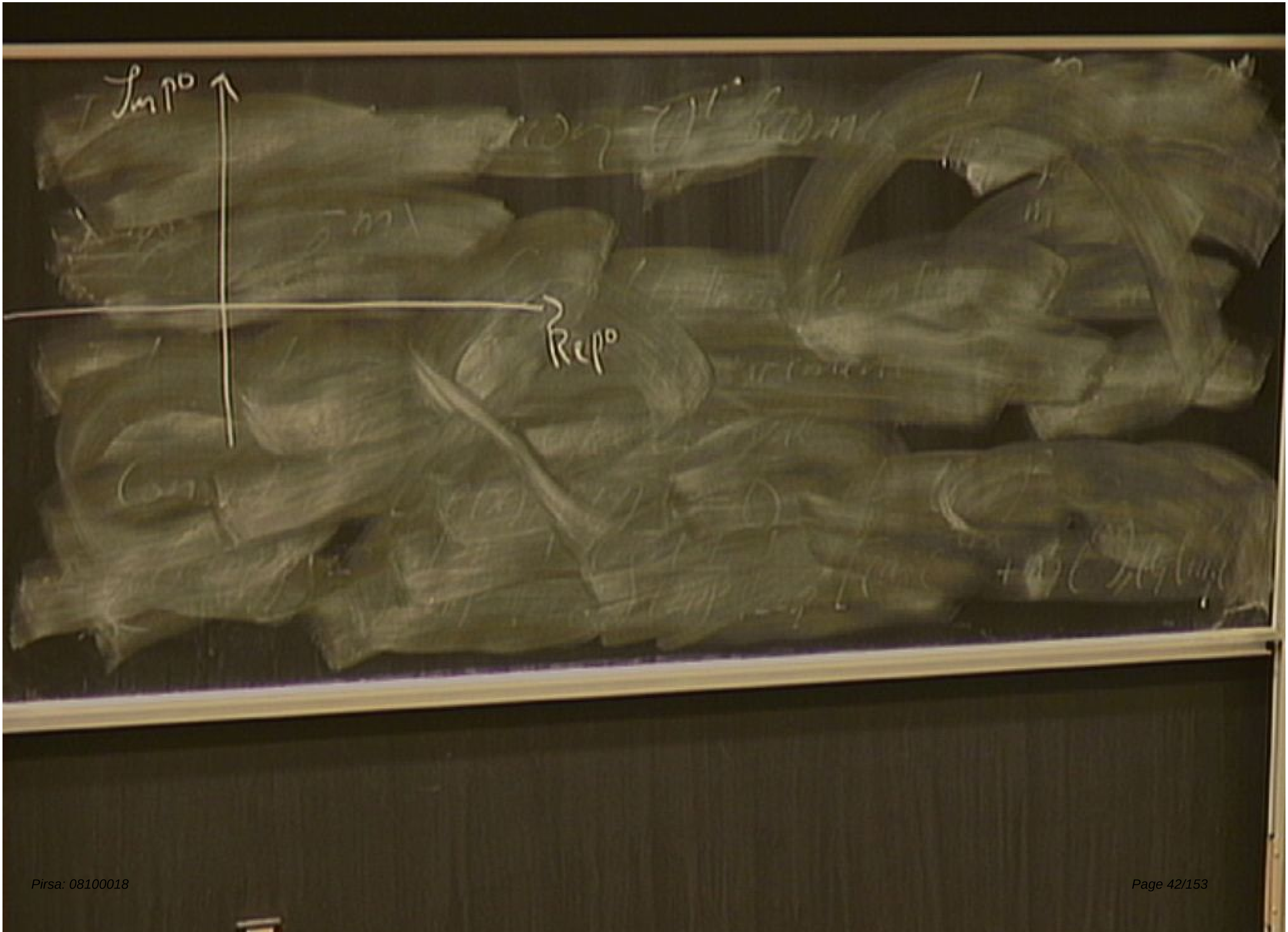

Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) =
(ε>0 and ε→0+)
Poles: p^2+m^2+2iεp^0=0 ⟹ p^0=±√
⟹ p^0_poles = ±√(p^2+m^2)(1−

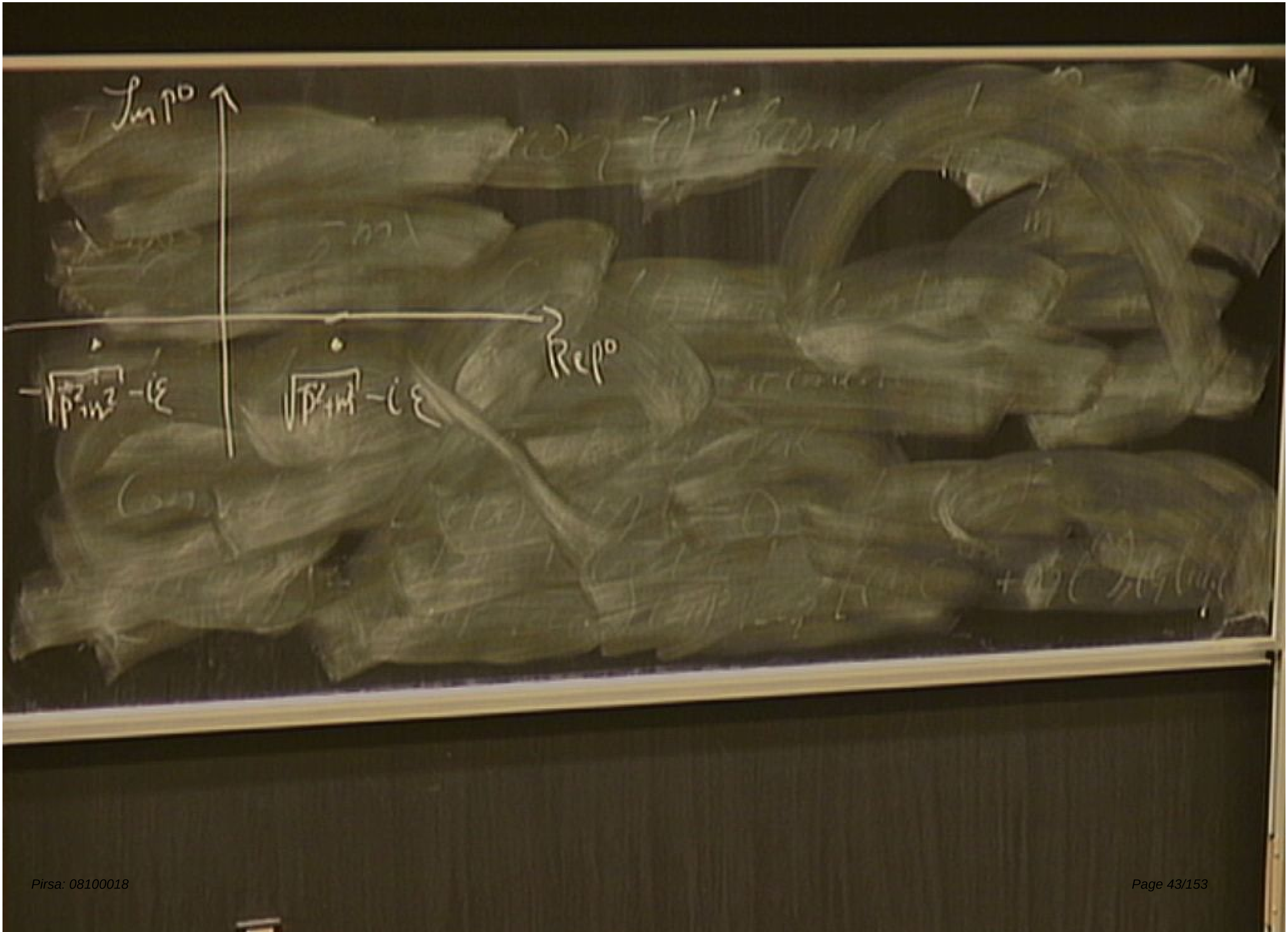

Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / (p^2-m^2+2iεp^0)
ε>0 and ε→0^+
Poles: p^2-m^2+2iεp^0=0
p^0 = ± √(p⃗^2+m^2-2iεp^0) ⟹
⟹ p^0_poles = ±
⟹ p^0_poles =
√(p⃗^2+m^2) - iεp^0 / √(p⃗^2+m^2)
-√(p⃗^2+m^2)

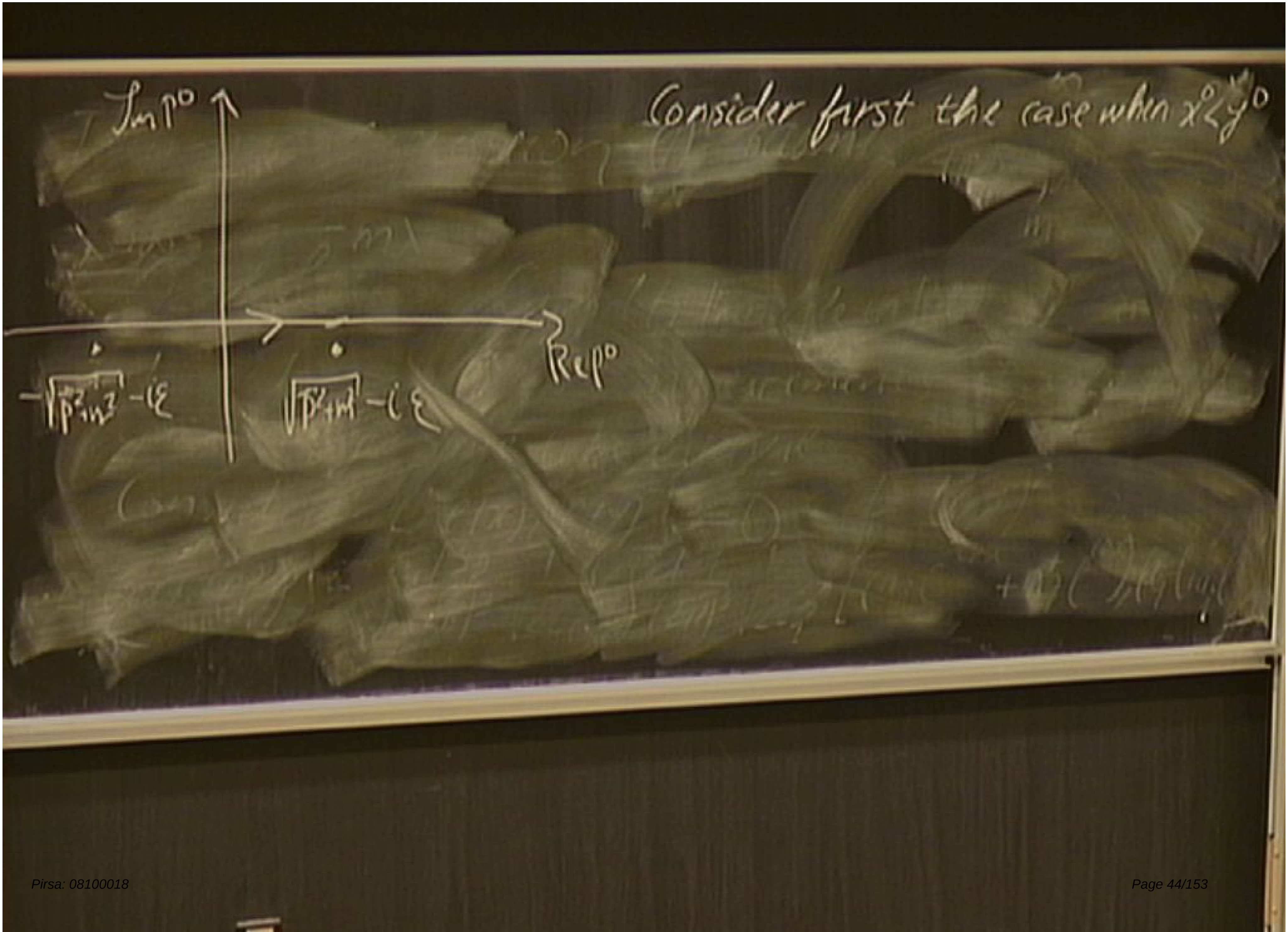

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
that $\tilde{D}_R(p) = \frac{i}{p^2-m^2+2i\varepsilon p^0}$ ($\varepsilon>0$ and, $\varepsilon \to 0^+$)
Poles:
$\Longrightarrow p^0 = \pm\sqrt{\vec{p}^2+m^2-2i\varepsilon p^0} \Longrightarrow$
$p^0_{poles} = \sqrt{\vec{p}^2+m^2} - \frac{i\varepsilon |p^0|}{\sqrt{\vec{p}^2+m^2}}$
$-\sqrt{\vec{p}^2+m^2}$

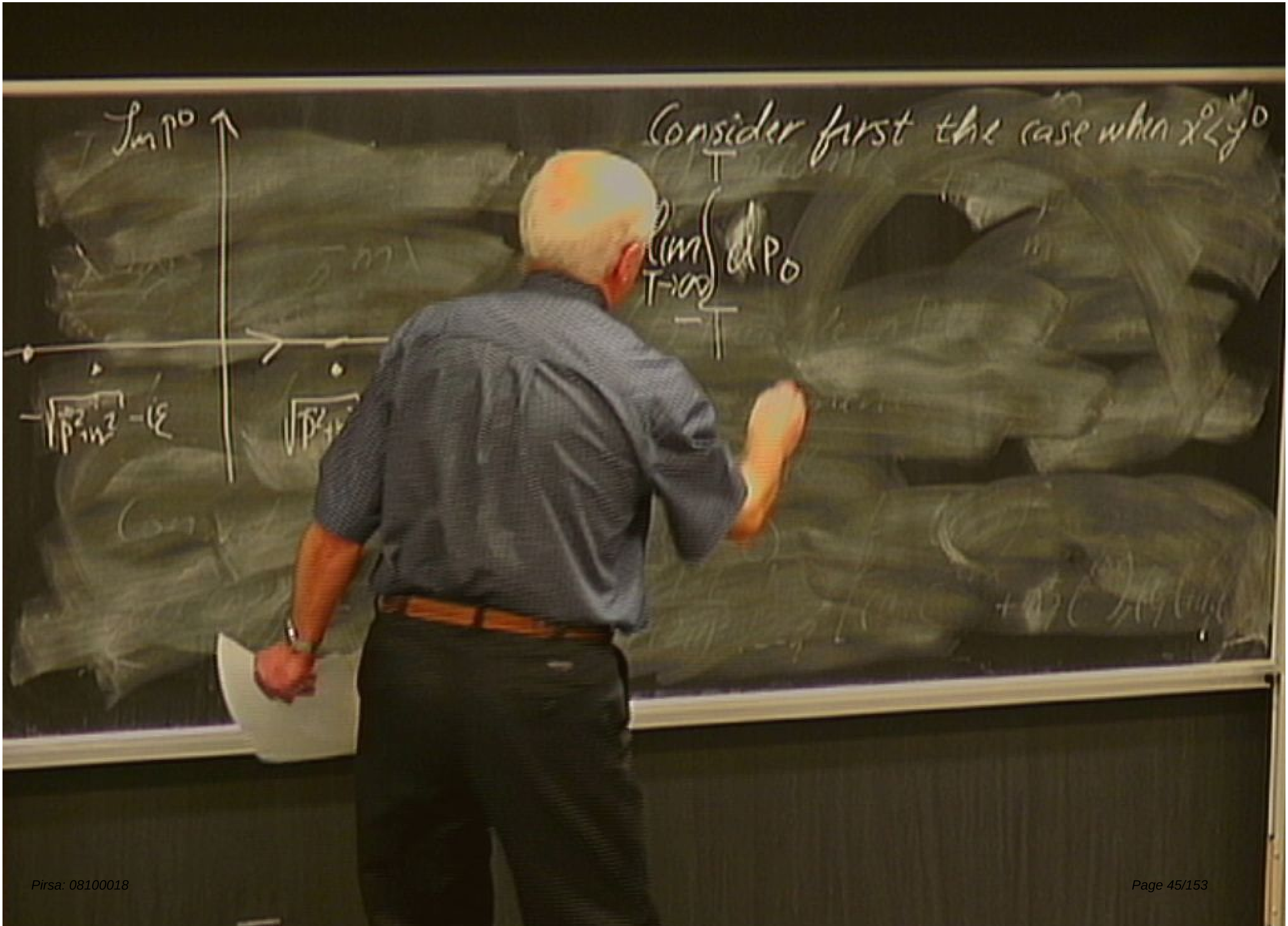

Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R
i / p^2-m^2+2iεp^0
(ε>0 and ε→0^+
Poles: p^2 - m^2 + 2iε p^0 = 0 ⟹
p^2+m^2-2iε p^0 ⟹
⟹ p^0_poles = ± √(p^2+m^2)

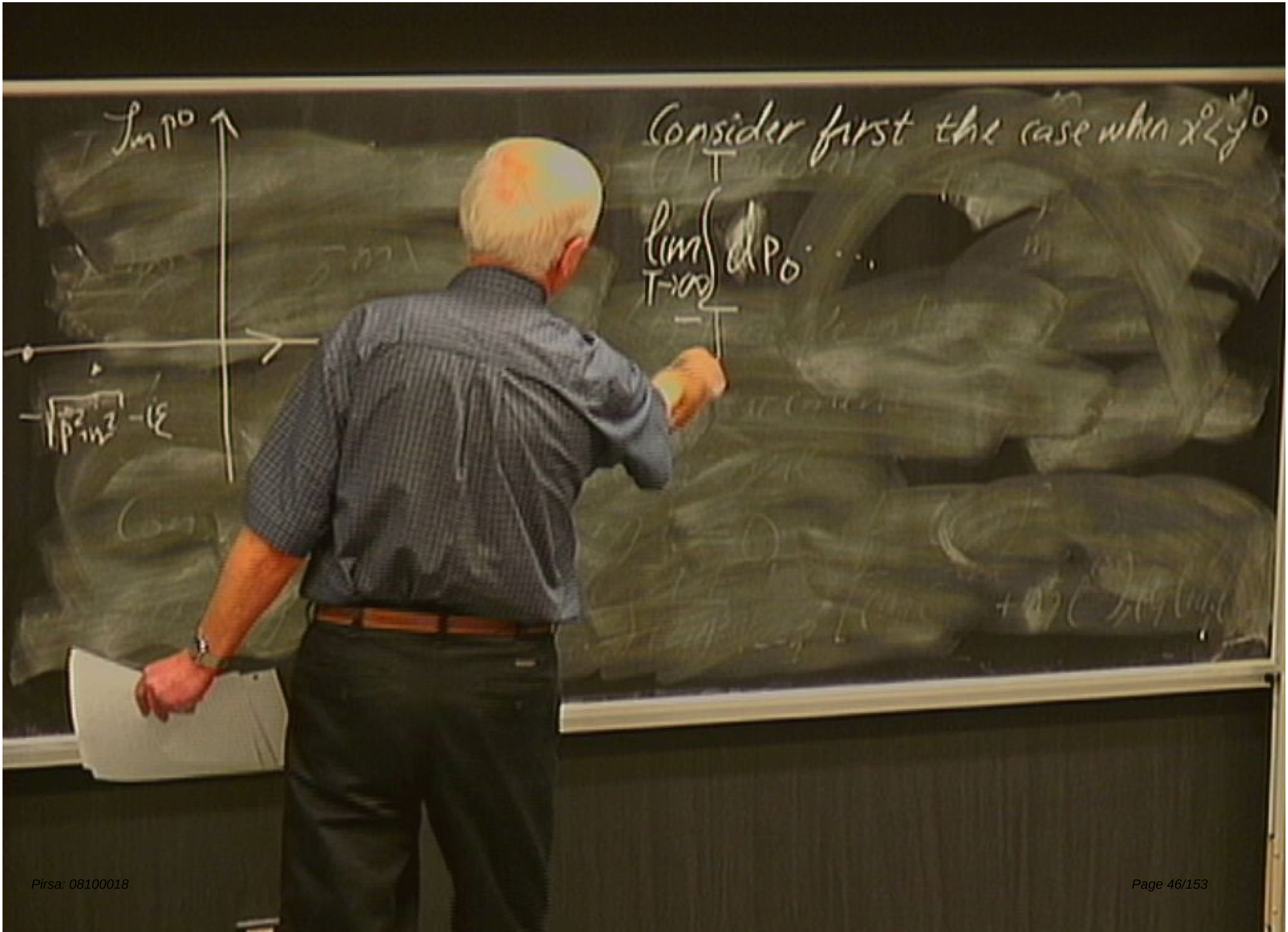

Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$
Let us show that $\tilde{D}_R(p) = \frac{i}{p^2-m^2+2i\varepsilon p^0}$
($\varepsilon>0$ and $\varepsilon \to 0^+$)
Poles: $p^2-m^2+2i\varepsilon p^0=0 \Rightarrow p^0=\pm\sqrt{\vec{p}^2+m^2-2i\varepsilon p^0} \Rightarrow$
$\Rightarrow p^0_{poles}=\pm\sqrt{\vec{p}^2+m^2}\left(1-\frac{i\varepsilon p^0}{\vec{p}^2+m^2}\right) \Rightarrow p^0_{poles}=$
$\sqrt{\vec{p}^2+m^2}-\frac{i\varepsilon}{\sqrt{\vec{p}^2+m^2}}$
$-\sqrt{\vec{p}^2+m^2}-\frac{i\varepsilon}{\sqrt{\vec{p}^2+m^2}}$

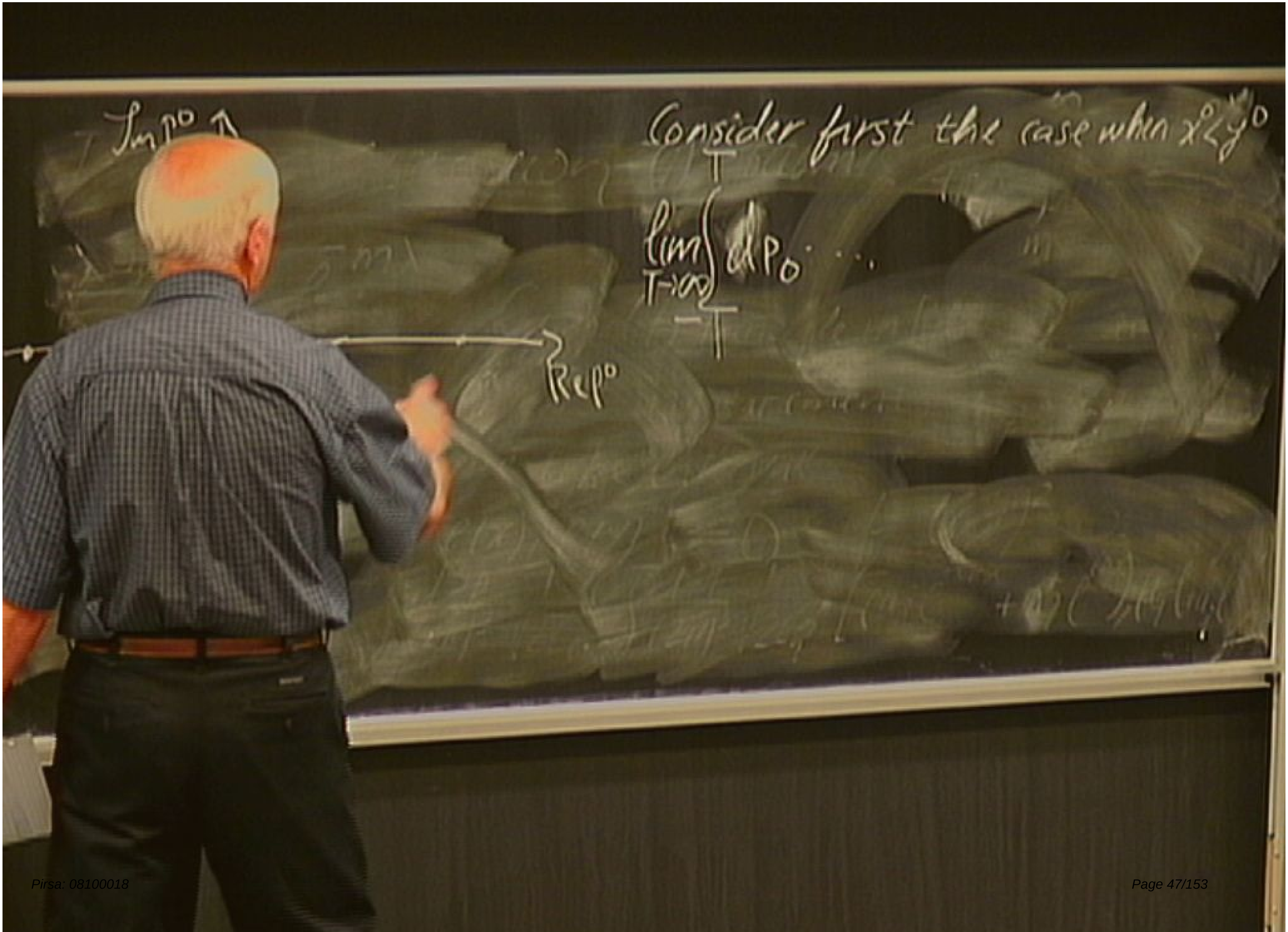

Im p⁰
Re p⁰

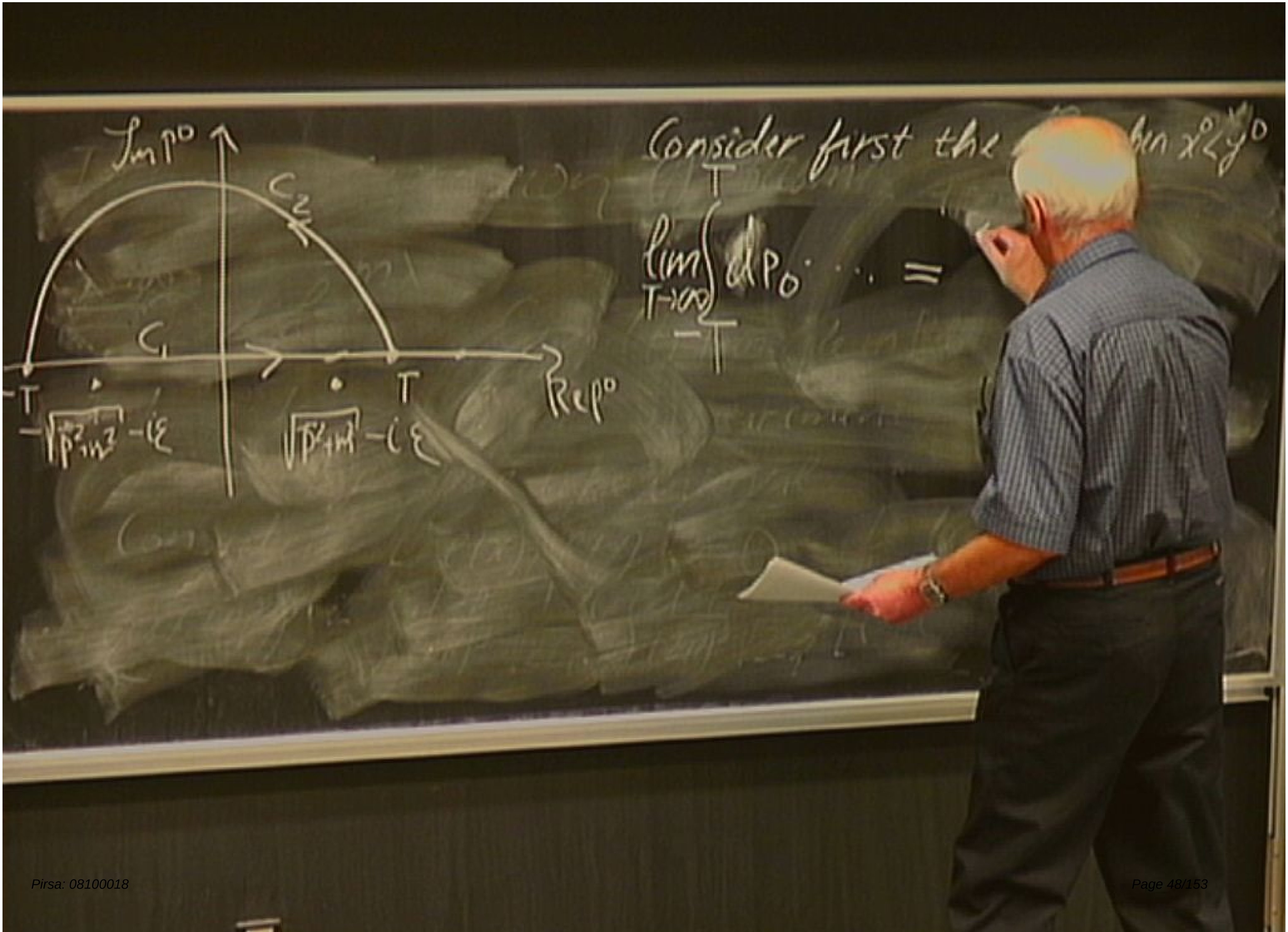

Im p^0
Re p^0
-√(p²+m²) - iε
√(p²+m²) - iε

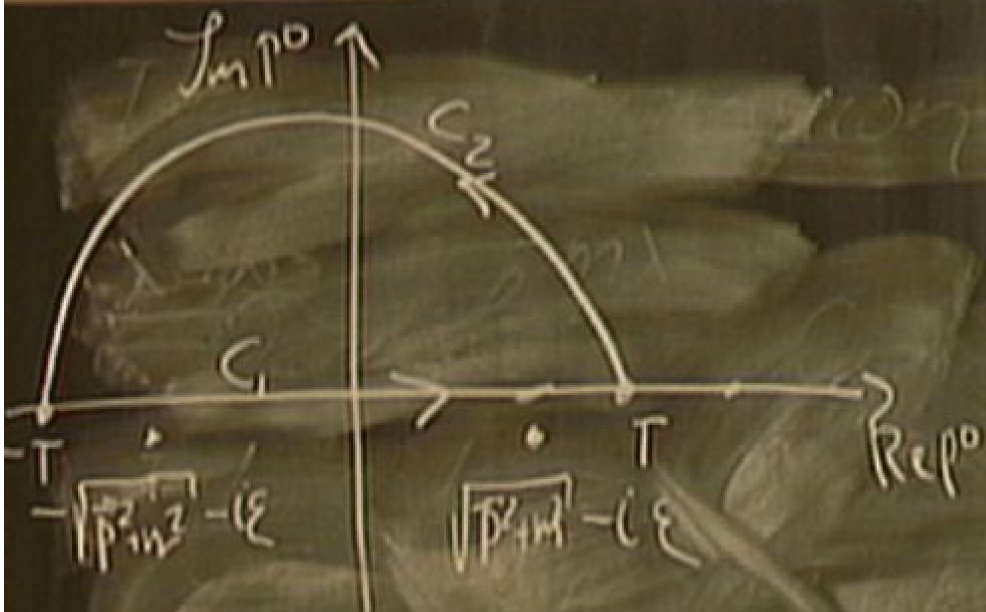

Consider first the case when x⁰ < y⁰
Im p⁰
Re p⁰
−√(p²+m²) − iε
√(p²+m²) − iε


Consider first the case when x⁰<y⁰
Im p⁰
lim ∫ dp₀
T→∞


Im p0
Consider first the case when x0 < y0
lim ∫ dp0 · · ·
-√(p²+m²) -iε

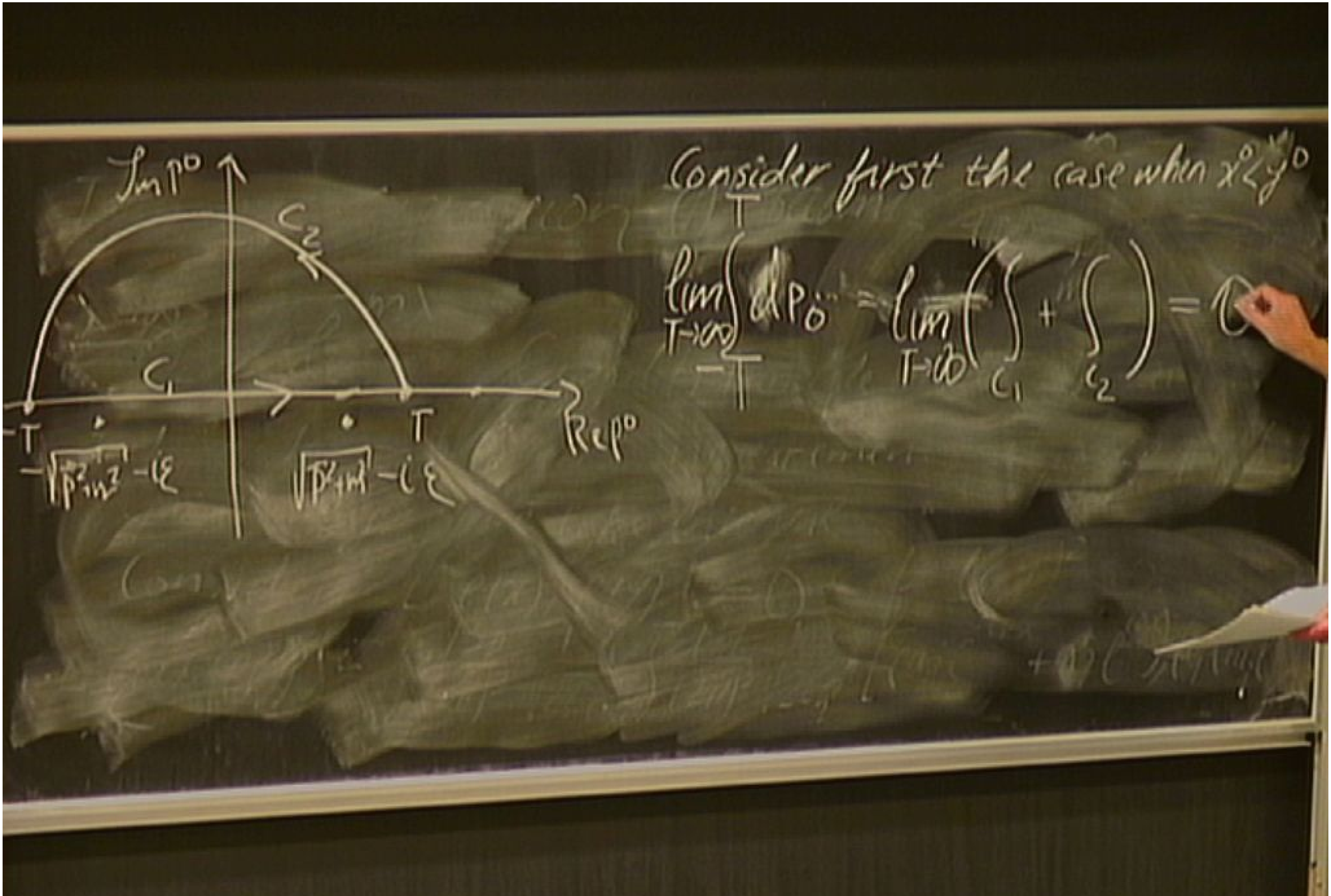

Im p0
Re p0
Consider first the case when x0 < y0
lim T→∞ ∫ dp0 . . .

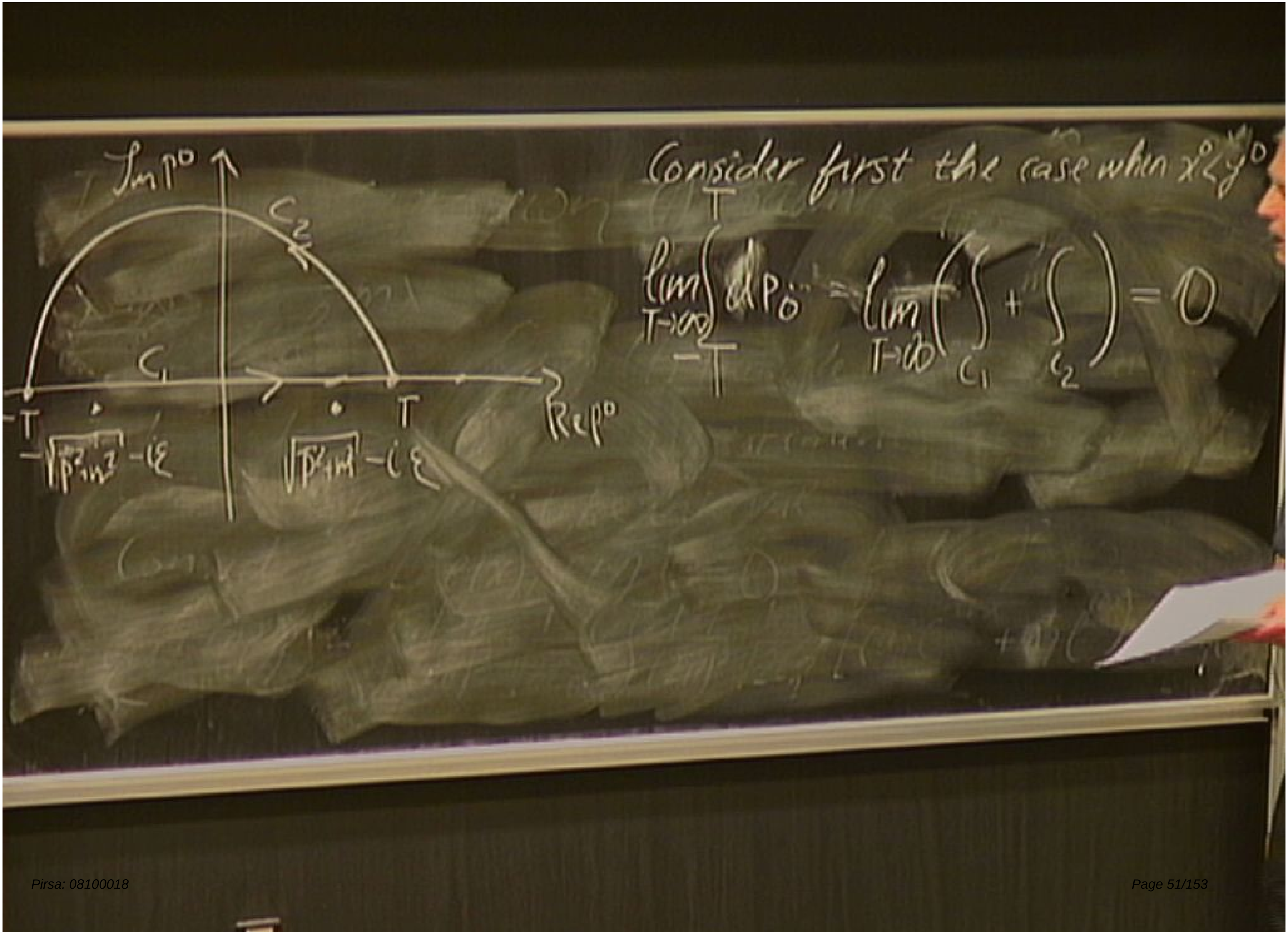

Im p^0
C_2
C_1
-T
T
Re p^0
Consider first the
lim
T→∞

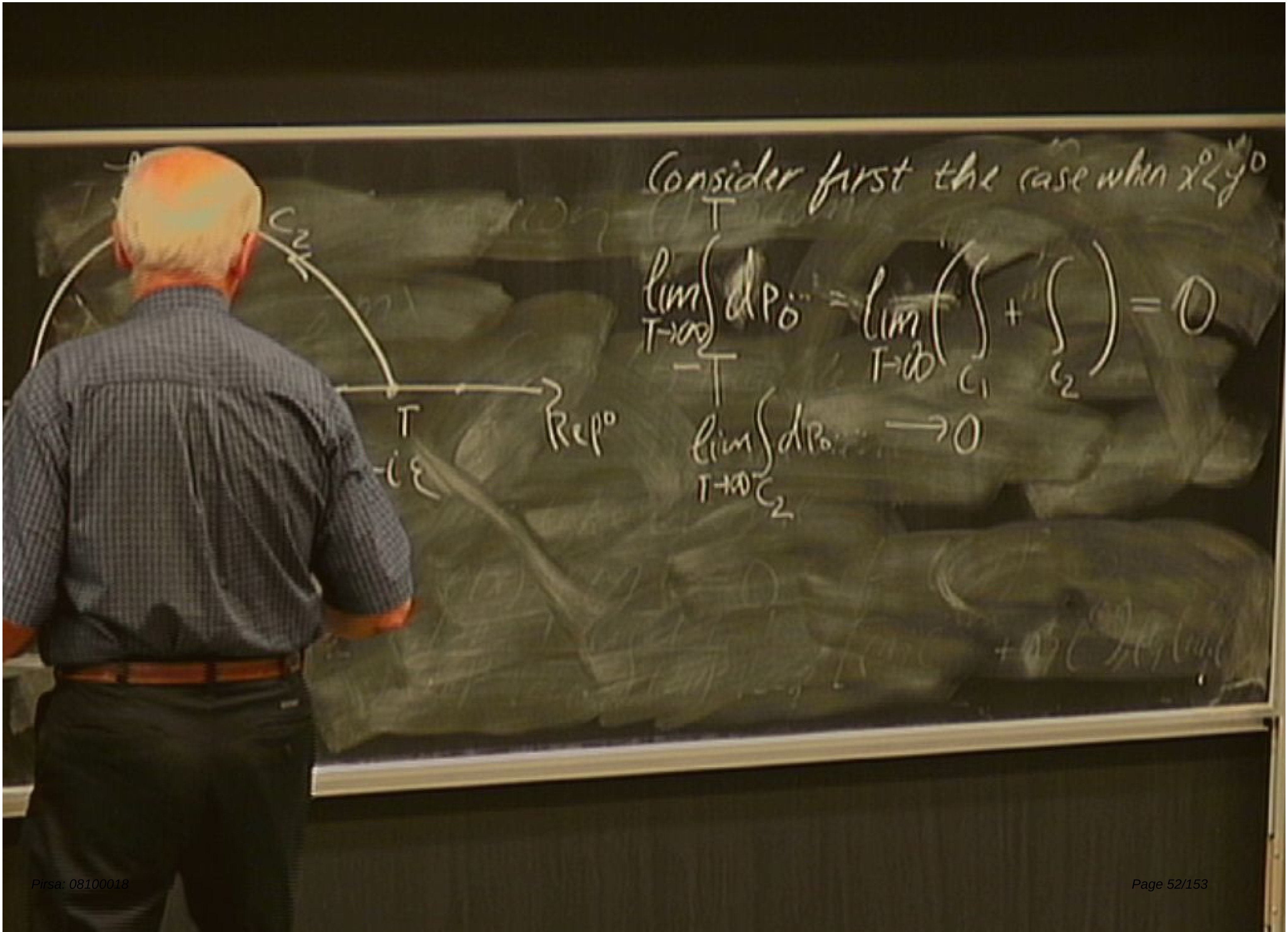


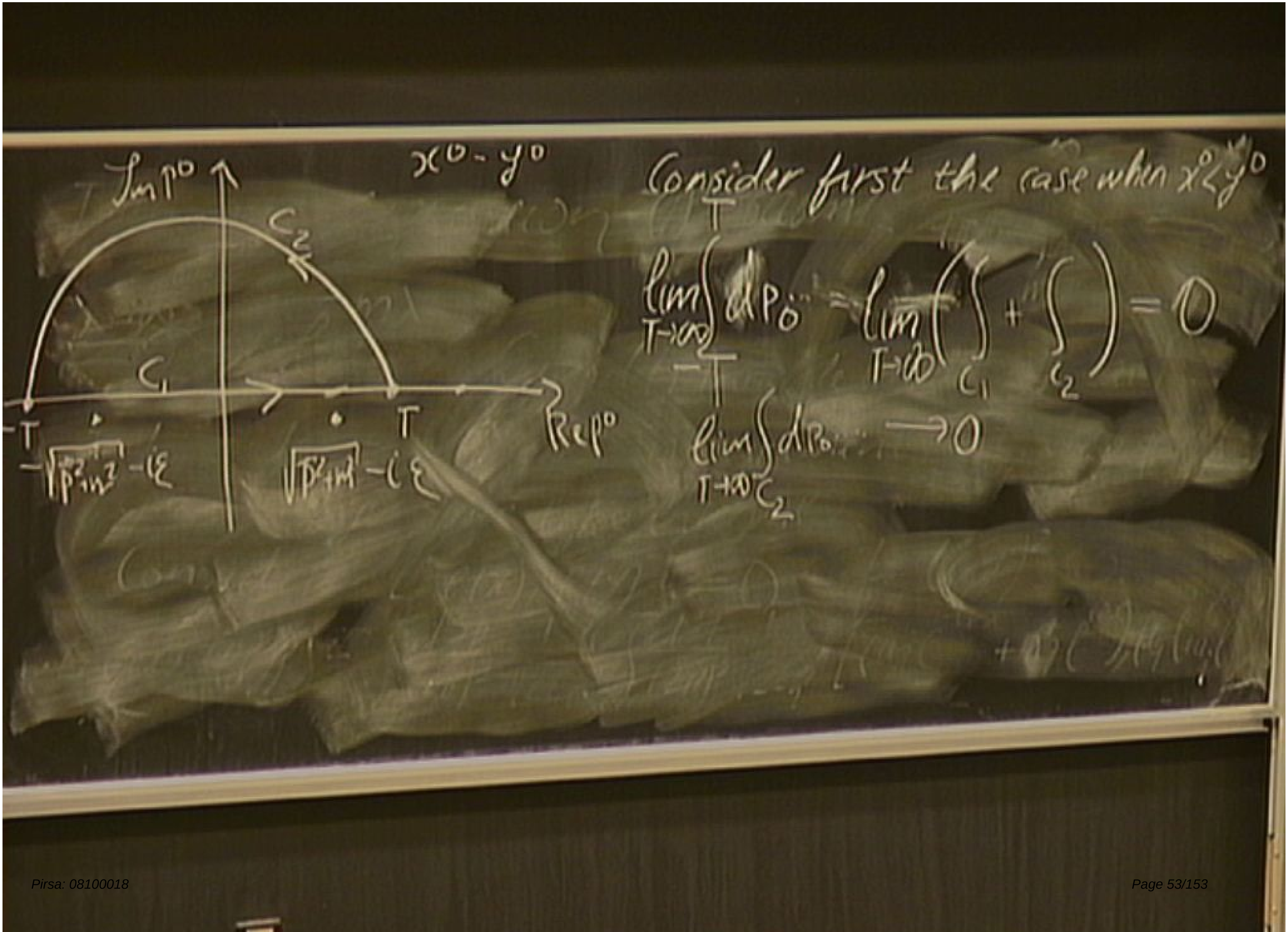
Consider first the case when $x^0 < y^0$

$$\lim_{T\to\infty}\int_{-T}^{T} dp_0 \cdots = \lim_{T\to\infty}\left(\int_{C_1} + \int_{C_2}\right)$$


Consider first the case when x⁰ < y⁰
Im p⁰
Re p⁰
C2
C1
-T
T
-√(p²+m²) - iε
√(p²+m²) - iε
lim T→∞ ∫ from -T to T dp0 ... = lim T→∞ (∫C1 + ∫C2) = 0


Consider first the case when x⁰ < y⁰
Im p⁰
Re p⁰
C2
C1
-T
T
lim T→∞ ∫ from -T to T dp0 ... = lim T→∞ (∫ C1 + ∫ C2) = 0


Consider first the case when x0 < y0
C2
T
Re p0
-iε
lim ∫ dp0 … = lim (∫ + ∫) = 0
T→∞ -T T→∞ C1 C2
lim ∫ dp0 … → 0
T→∞ C2



Im p⁰
x⁰ - y⁰
Consider first the case when x⁰ < y⁰
C₂
C₁
-T
T
Re p⁰
-√(p²+m²) - iε
√(p²+m²) - iε
lim_{T→∞} ∫_{-T}^{T} dp₀ ... = lim_{T→∞} (∫_{C₁} + ∫_{C₂}) = 0
lim_{T→∞} ∫_{C₂} dp₀ ... → 0

prescriptions lead to different Green's functions

Green's functions in KG Theory

Green's functions satisfy the following equation:

$$(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\delta^{(4)}(x-y)$$

Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \tilde{D}_G(p)$

Substitute it into equation:

$$(-p^2+m^2)\tilde{D}_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2-m^2}$$

prescriptions lead to different Green's functions

Green's functions in KG Theory

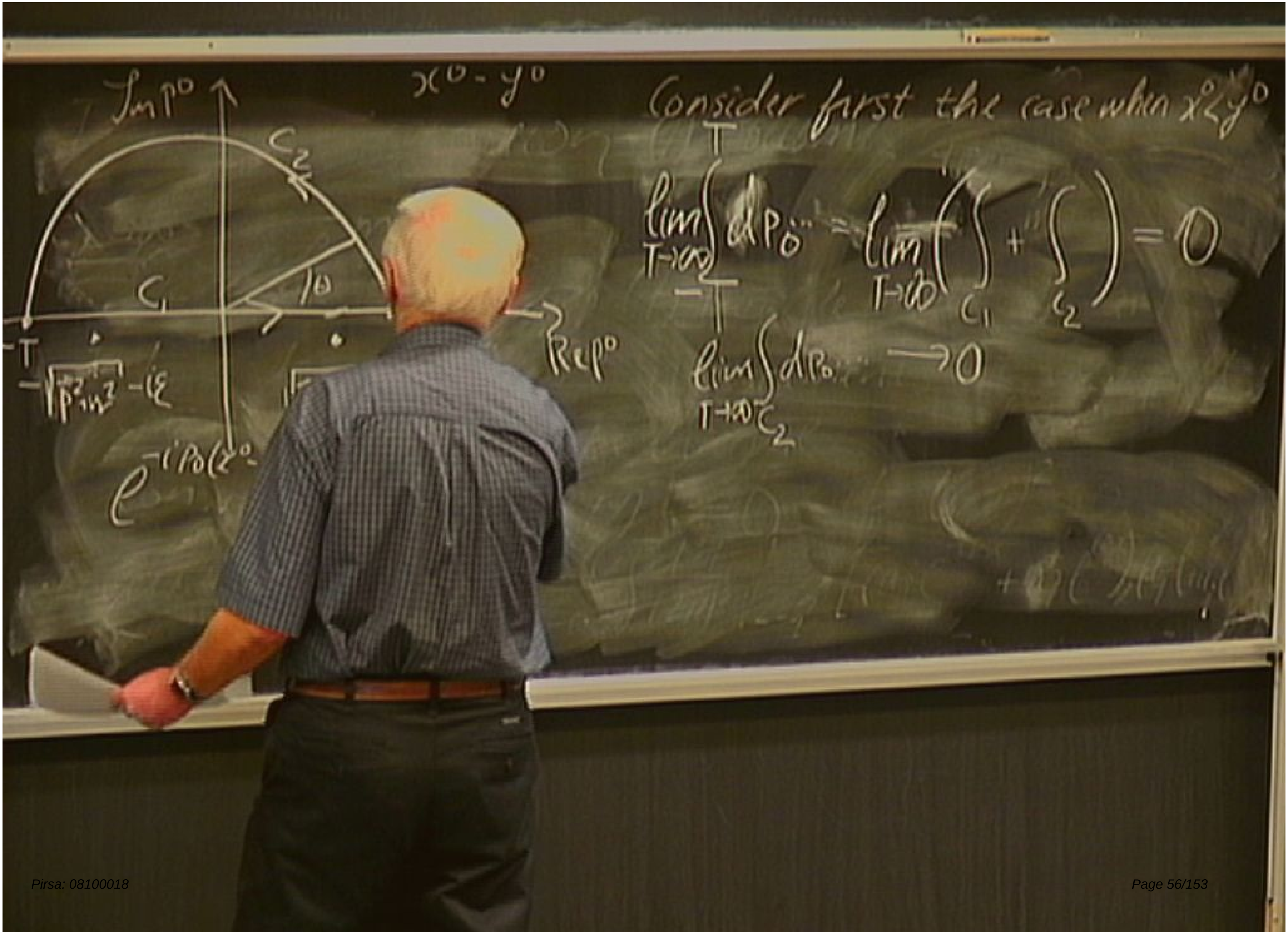
Green's functions satisfy the following equation:

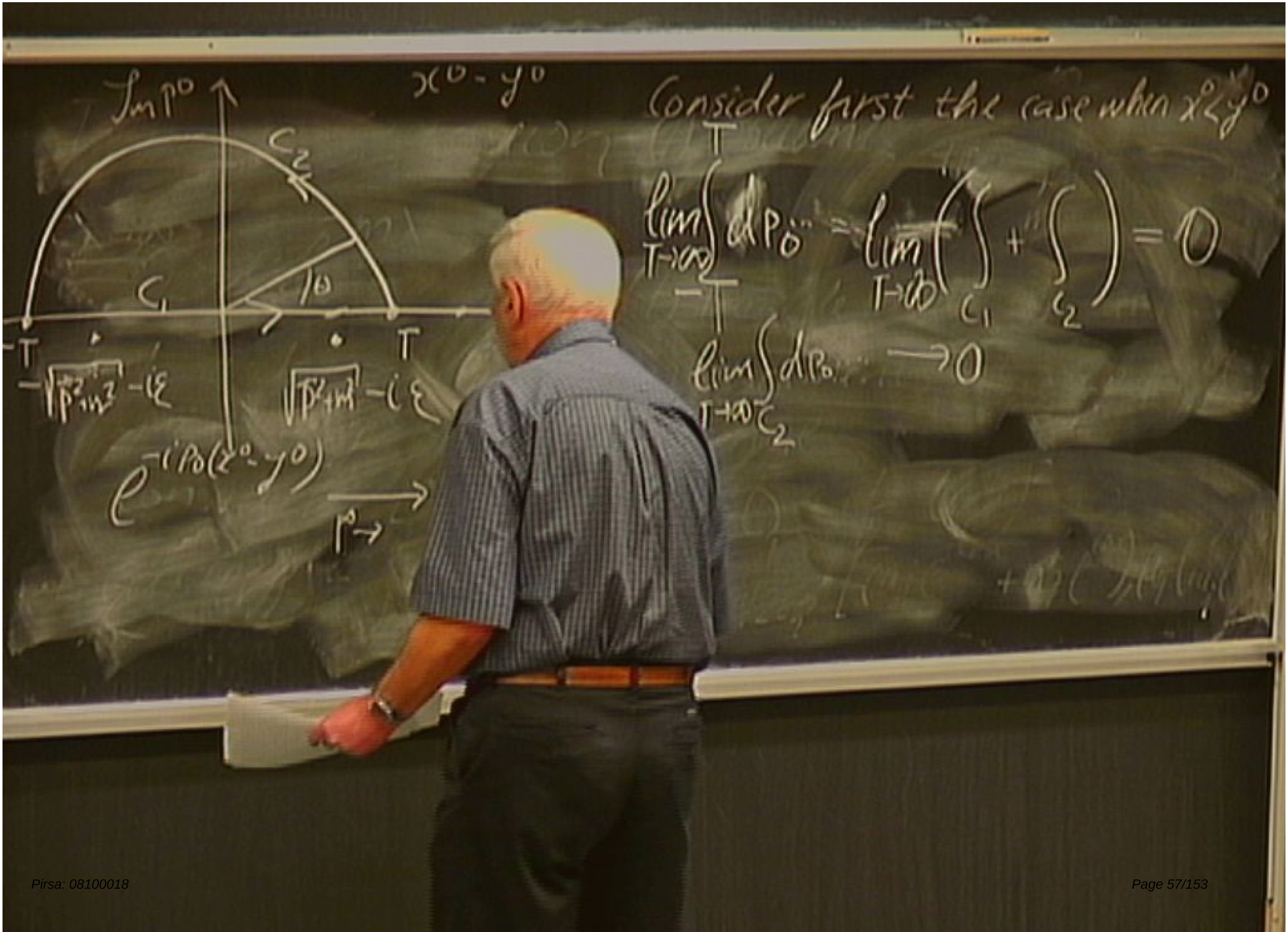
$$(\partial^\mu \partial_\mu + m^2) D_G(x-y) = -i\,\delta^{(4)}(x-y)$$

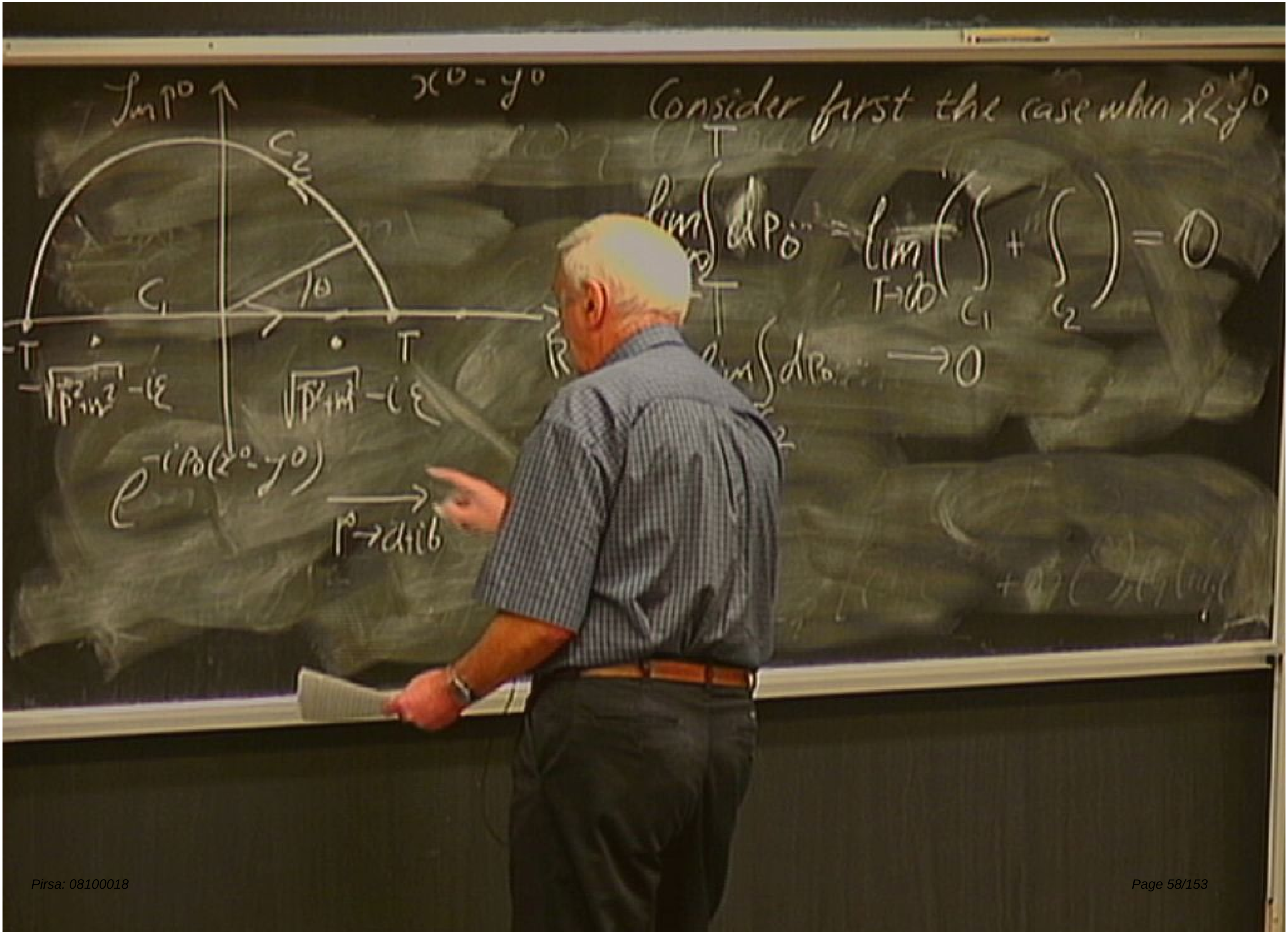
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \tilde{D}_G(p)$

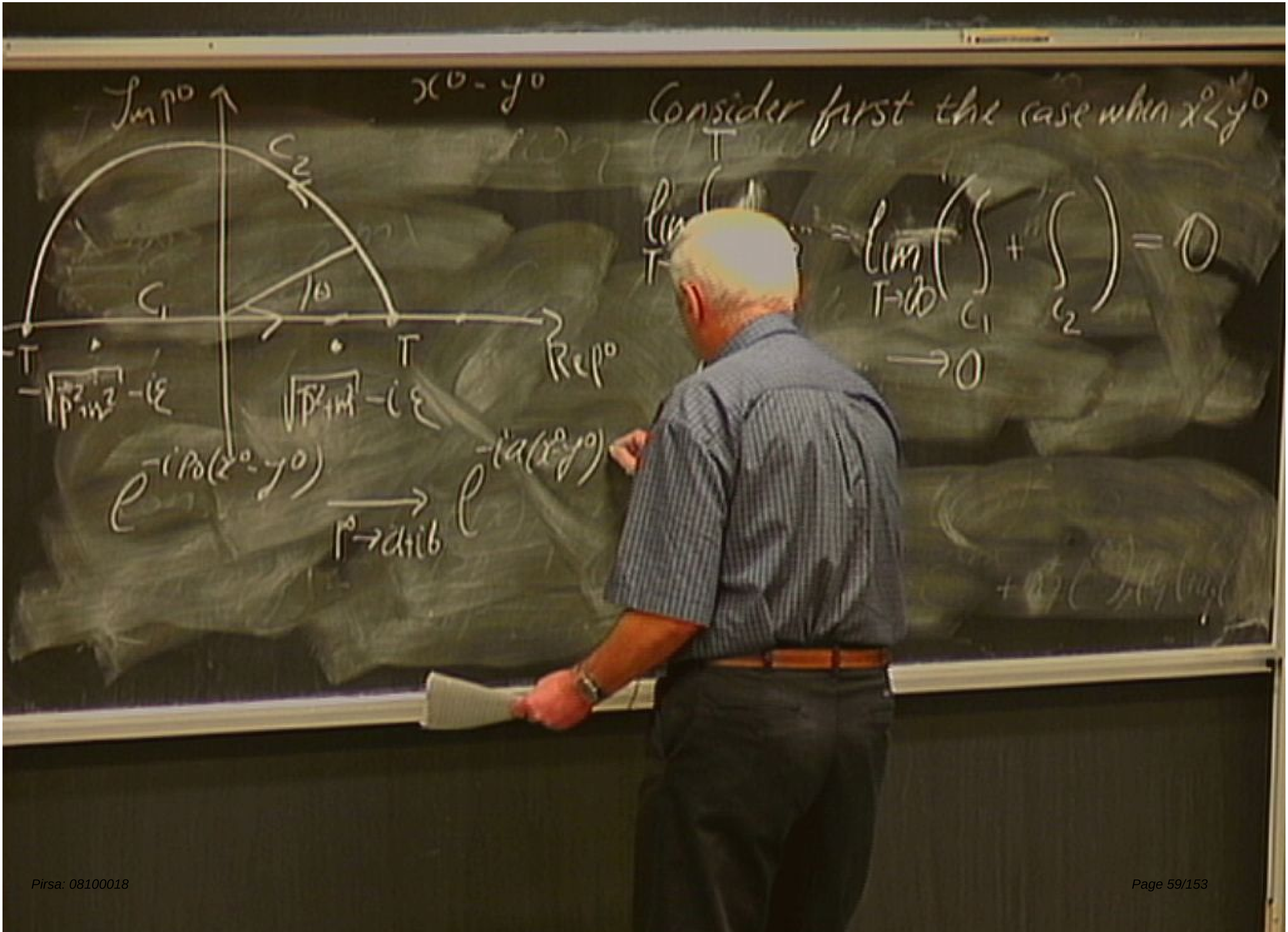
Substitute it into equation:

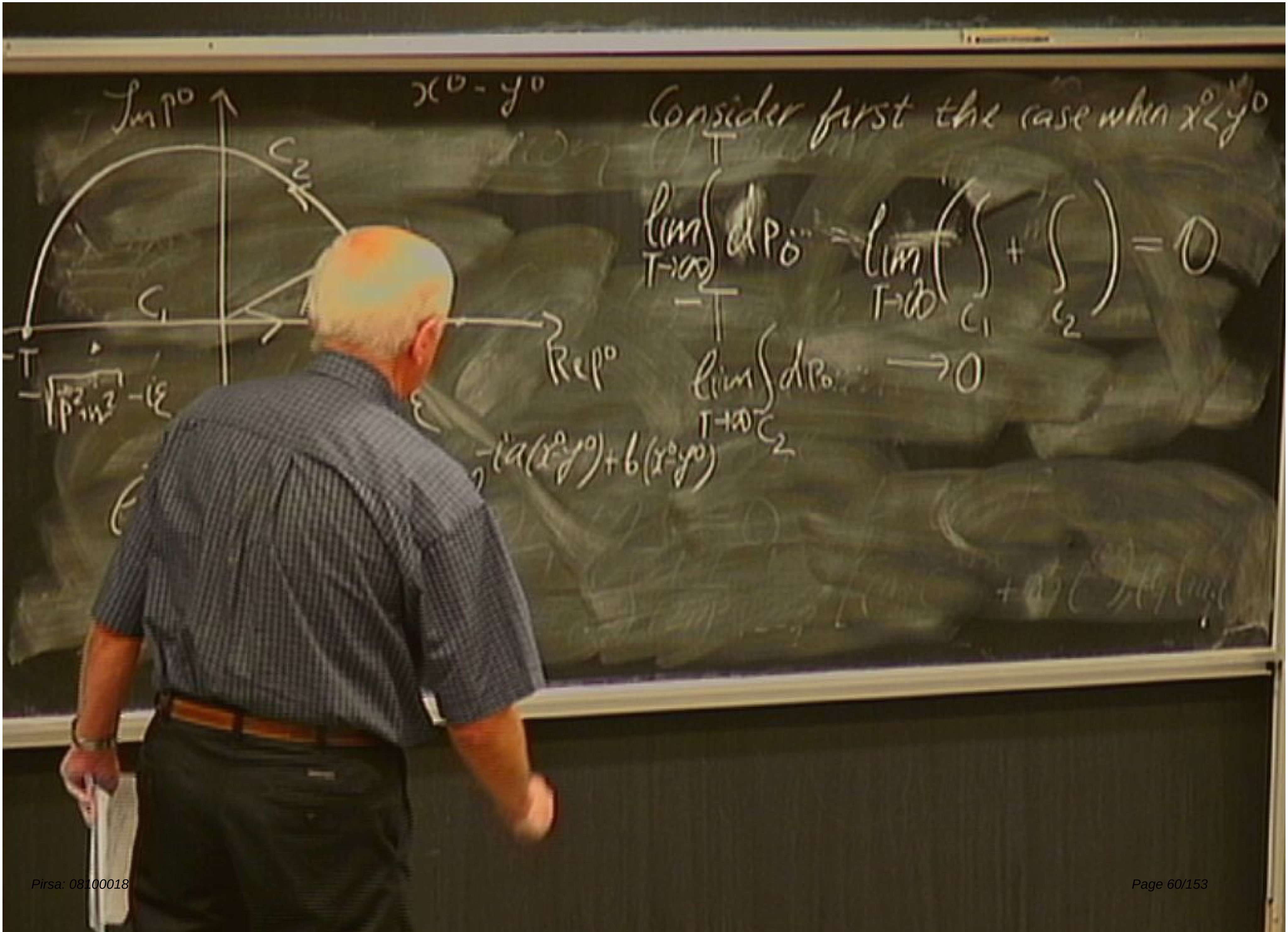
$$(-p^2+m^2)\tilde{D}_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2-m^2}$$

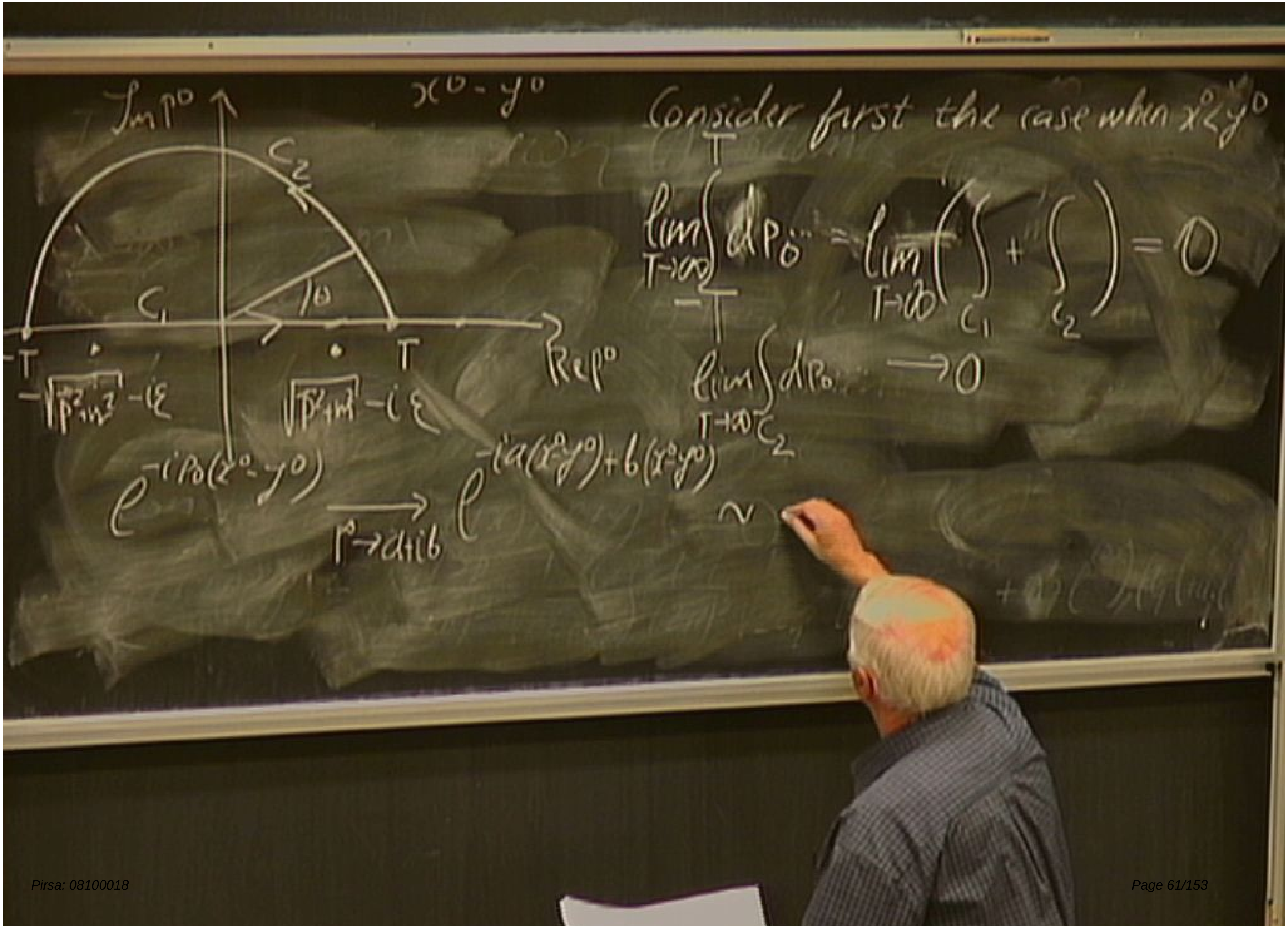

Consider first the case when $x^0 < y^0$
$x^0 - y^0$
$\mathrm{Im}\, p^0$
$\mathrm{Re}\, p^0$
C_1
C_2
θ
$-T$
$-\sqrt{\vec{p}^2+m^2} - i\epsilon$
$e^{-ip_0(x^0-}$
$\lim_{T\to\infty} \int_{-T}^{T} dp_0 \cdots = \lim_{T\to\infty} \left(\int_{C_1} + \int_{C_2} \right) = 0$
$\lim_{T\to\infty} \int_{C_2} dp_0 \cdots \to 0$

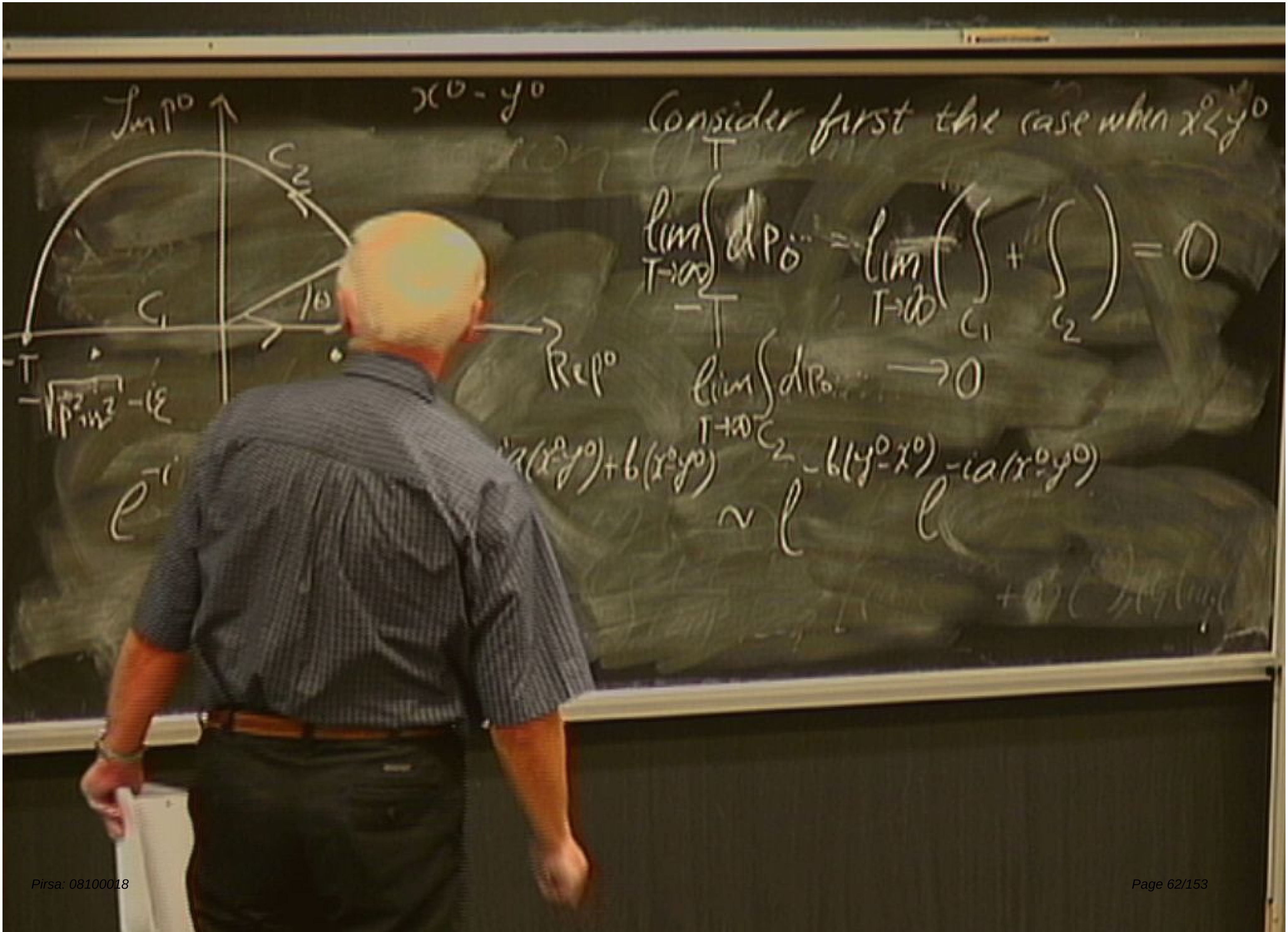

Im p0
x0 - y0
Consider first the case when x0 < y0
C2
C1
θ
-T
T
lim T→∞ ∫ dp0 = lim T→∞ (∫C1 + ∫C2) = 0
lim T→∞ ∫C2 dp0 → 0

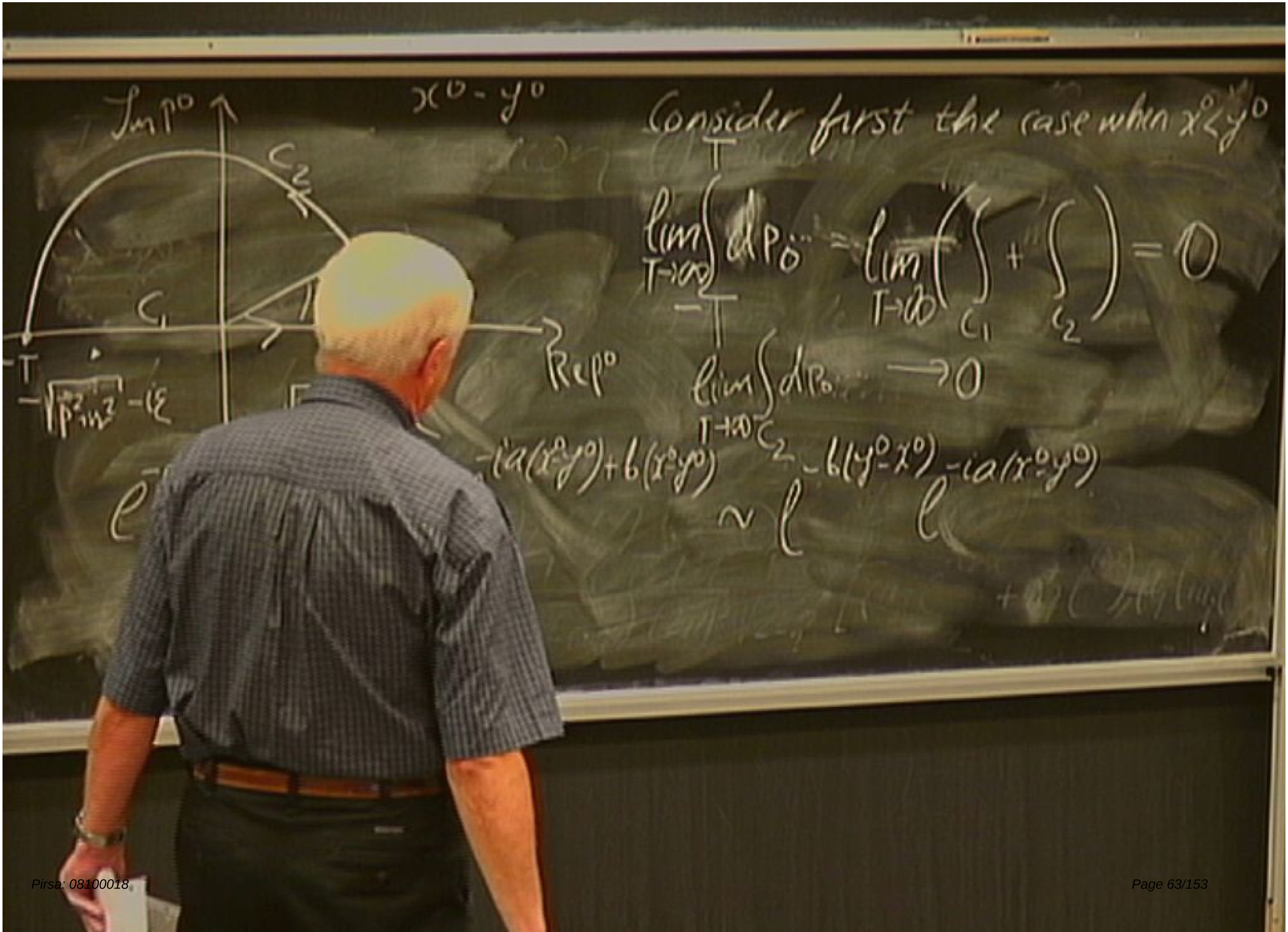

Consider first the case when x⁰ < y⁰
Im p⁰
x⁰ - y⁰
C₂
C₁
T
R
-T
lim ∫ dp₀ ... = lim (∫ + ∫) = 0
T→∞ C₁ C₂
lim ∫ dp₀ ... → 0
C₂
p→a+ib

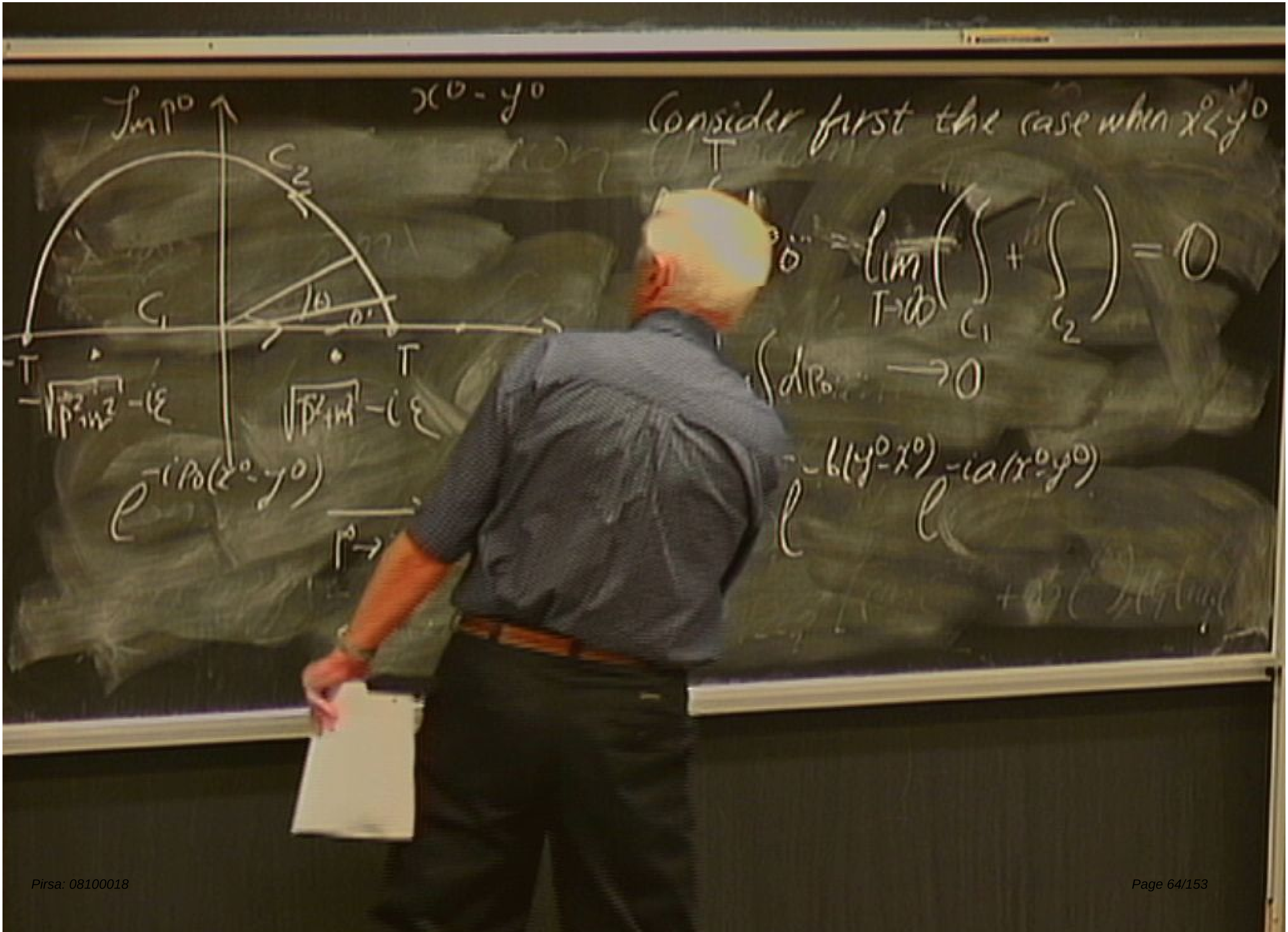

Im p⁰
x⁰ - y⁰
Consider first the case when x⁰ < y⁰
C₂
C₁
θ
-T
T
Re p⁰
→ 0
= 0
p⁰ → a+ib

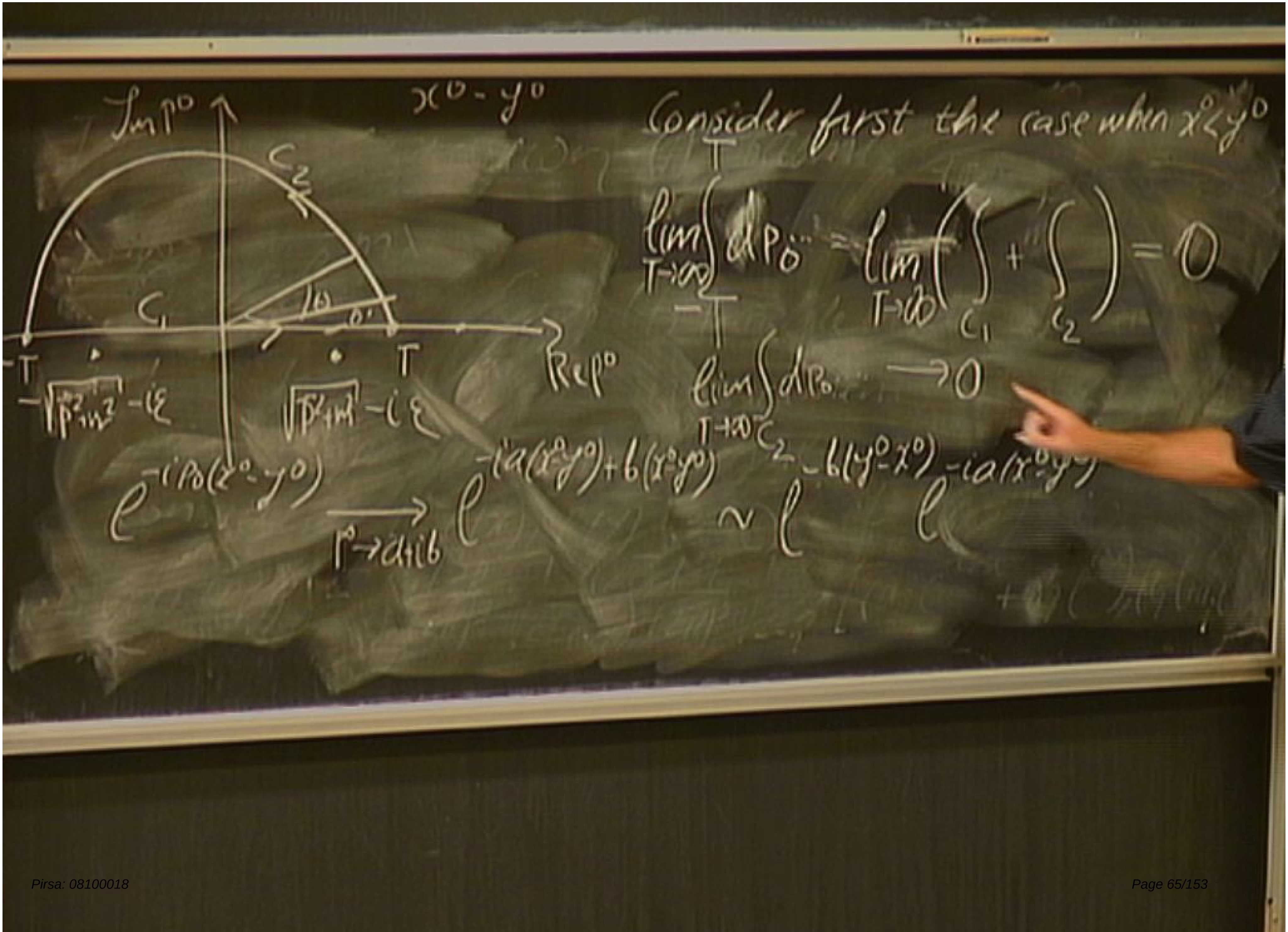

Im p⁰
x⁰ - y⁰
Consider first the case when x⁰ < y⁰
C₂
C₁
-T
Re p⁰
→ 0
= 0

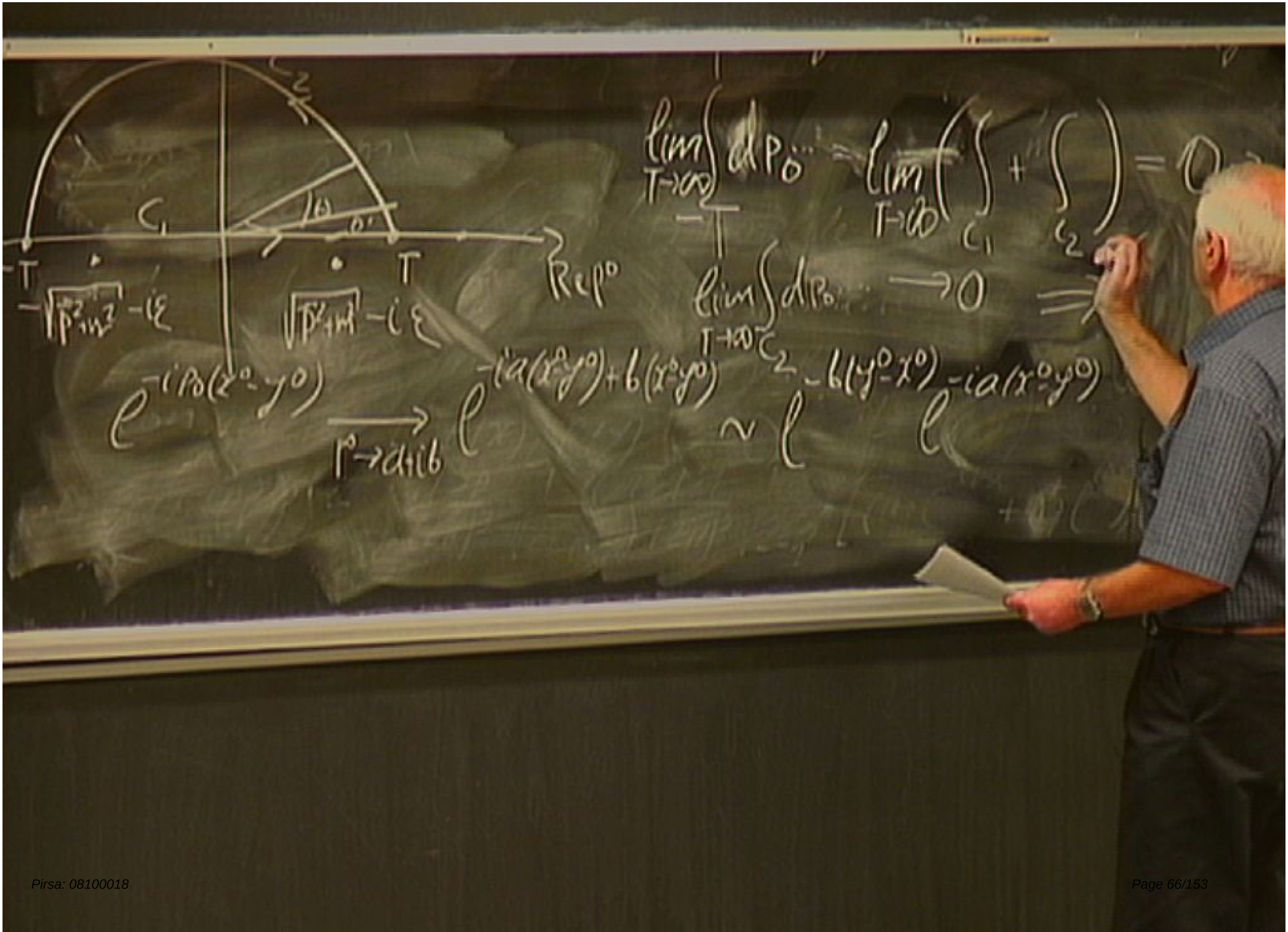

Im p0
x0 - y0
Consider first the case when x0 < y0
C2
C1
θ
-T
T
Re p0
lim T→∞ ∫ dp0 ... = lim T→∞ (∫C1 + ∫C2) = 0
lim T→∞ ∫C2 dp0 ... → 0
e^{-ip0(x0-y0)} → e^{-ia(x0-y0)+b(x0-y0)}
p0 → a+ib
~

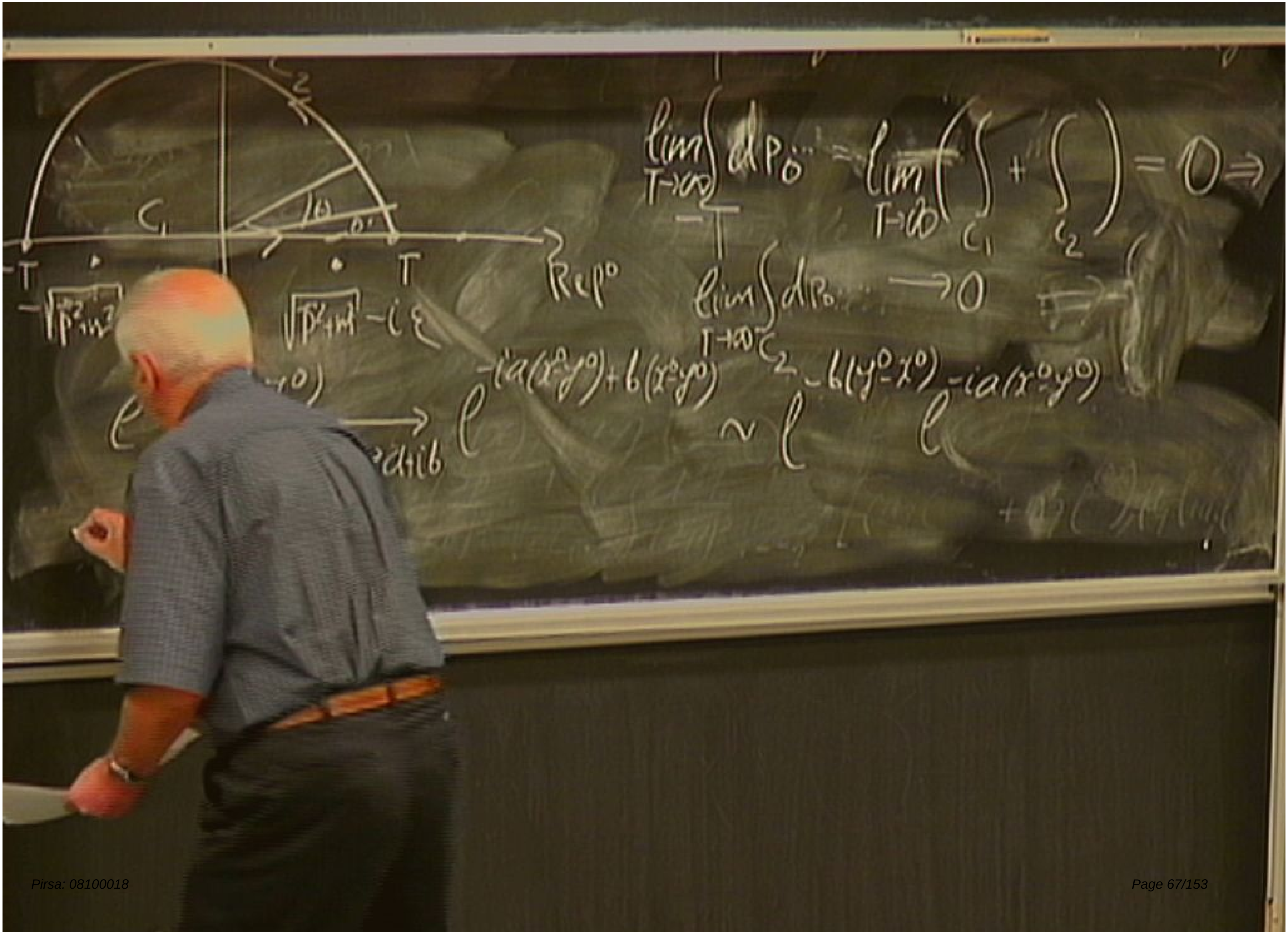

Consider first the case when x⁰<y⁰
Im p⁰
x⁰ - y⁰
Re p⁰
C1
C2
-T
= 0
→ 0

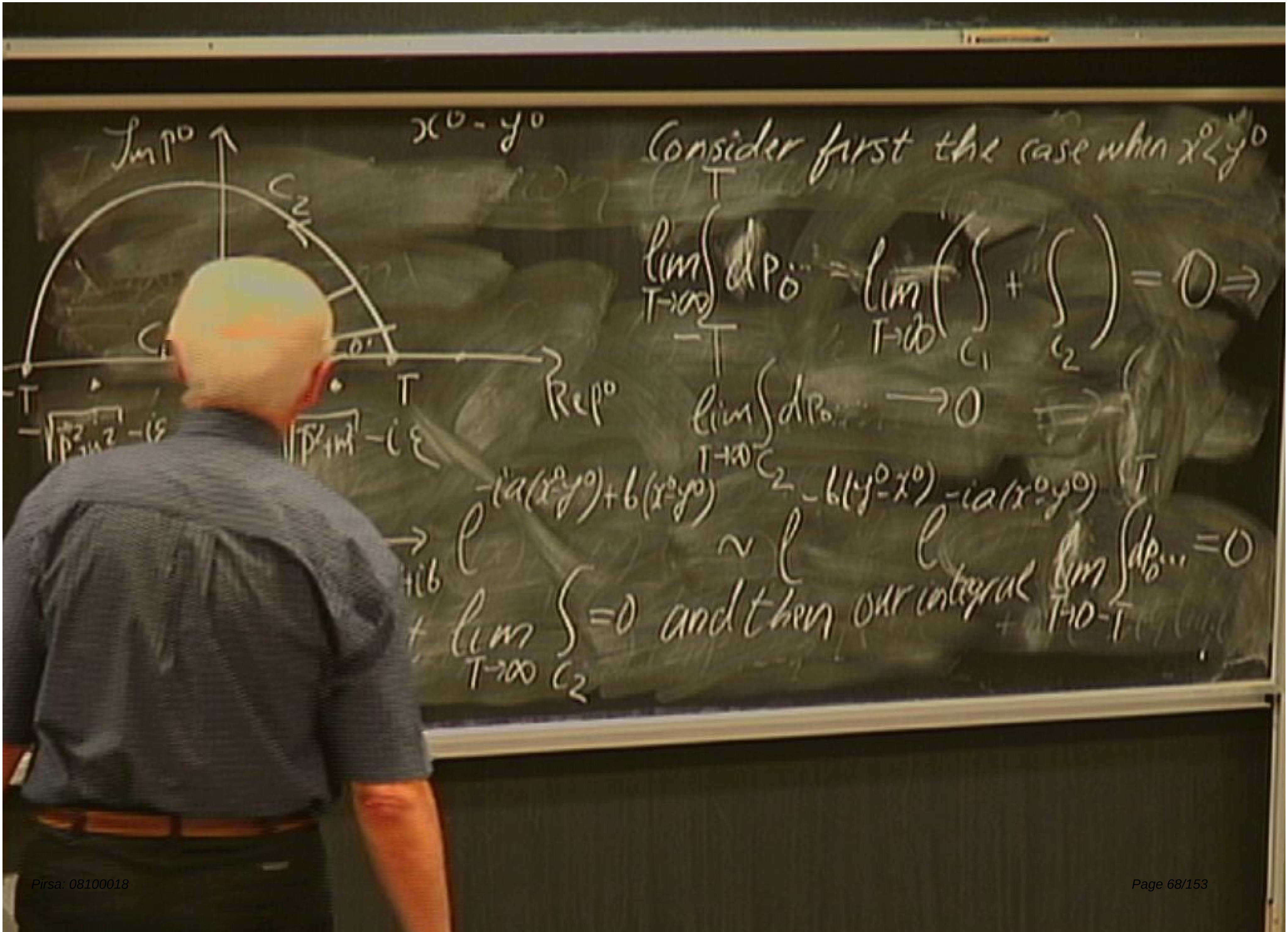

Im p^0
x^0 - y^0
Consider first the case when x^0 < y^0
C_2
C_1
Re p^0
-T
lim T→∞ ∫_{-T}^{T} dp_0 ... = lim T→∞ (∫_{C_1} + ∫_{C_2}) = 0
lim T→∞ ∫_{C_2} dp_0 ... → 0
~ e^{-b(y^0 - x^0)} e^{-ia(x^0 - y^0)}

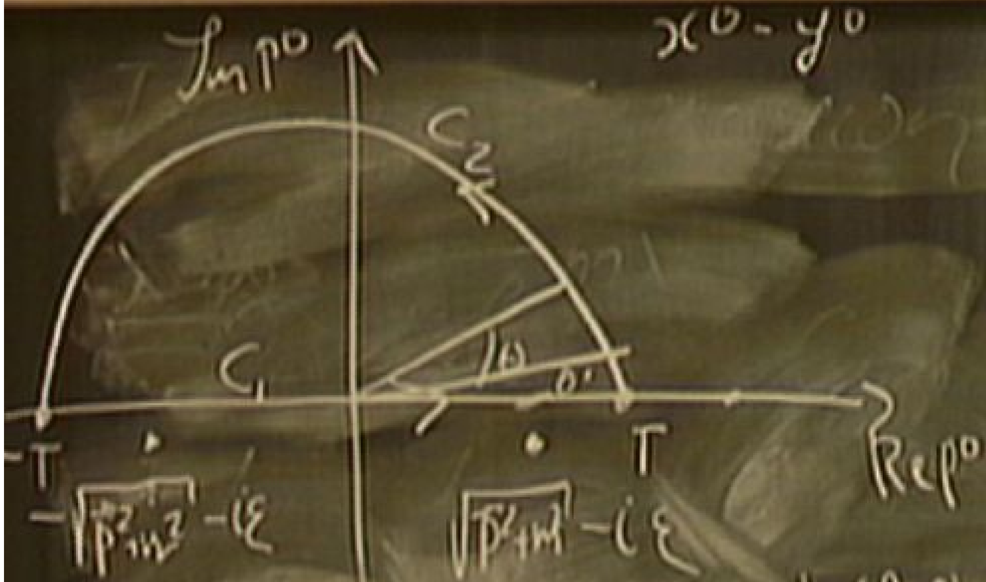

Consider first the case when x⁰<y⁰
Im p⁰
x⁰ - y⁰
C₂
C₁
-T
T
= lim (∫ + ∫) = 0
T→∞
∫dp₀ → 0


Consider first the case when x⁰ < y⁰
Im p⁰
x⁰ - y⁰
C₂
C₁
Re p⁰
-T
T
-√(p²+m²) - iε
√(p²+m²) - iε
lim T→∞ ∫ dp₀ ... = lim T→∞ (∫C₁ + ∫C₂) = 0
lim T→∞ ∫C₂ dp₀ ... → 0
e^(-ip₀(x⁰-y⁰)) → e^(-ia(x⁰-y⁰)+b(x⁰-y⁰))
p⁰ → a+ib
~ e^(-b(y⁰-x⁰)) e^(-ia(x⁰-y⁰))


C2
C1
-T
T
Re p0
=0
→0


C2
C1
-T
T
Re p0
= 0 ⇒
→ 0


Im p0
x0 - y0
Consider first the case when x0 < y0
C2
C1
-T
T
Re p0
lim T→∞ ∫ dp0 ... = lim T→∞ (∫C1 + ∫C2) = 0 ⇒
lim T→∞ ∫C2 dp0 ... → 0
e^(-ia(x0-y0)+b(x0-y0)) ~ e^(-b(y0-x0)) e^(-ia(x0-y0))
lim T→∞ ∫C2 = 0 and then our integral lim T→∞ ∫ dp0 ... = 0



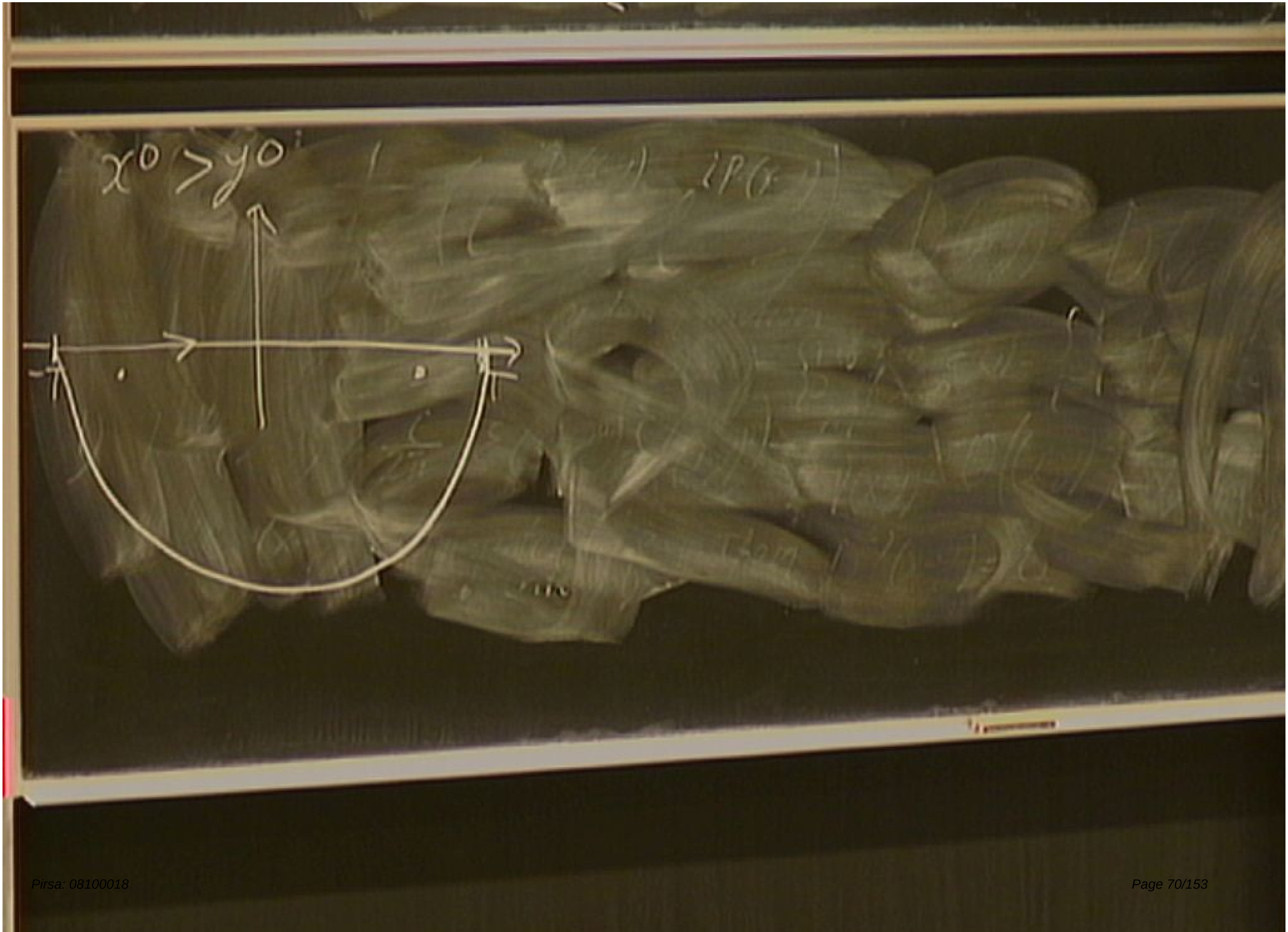
Consider first the case when $x^0 < y^0$

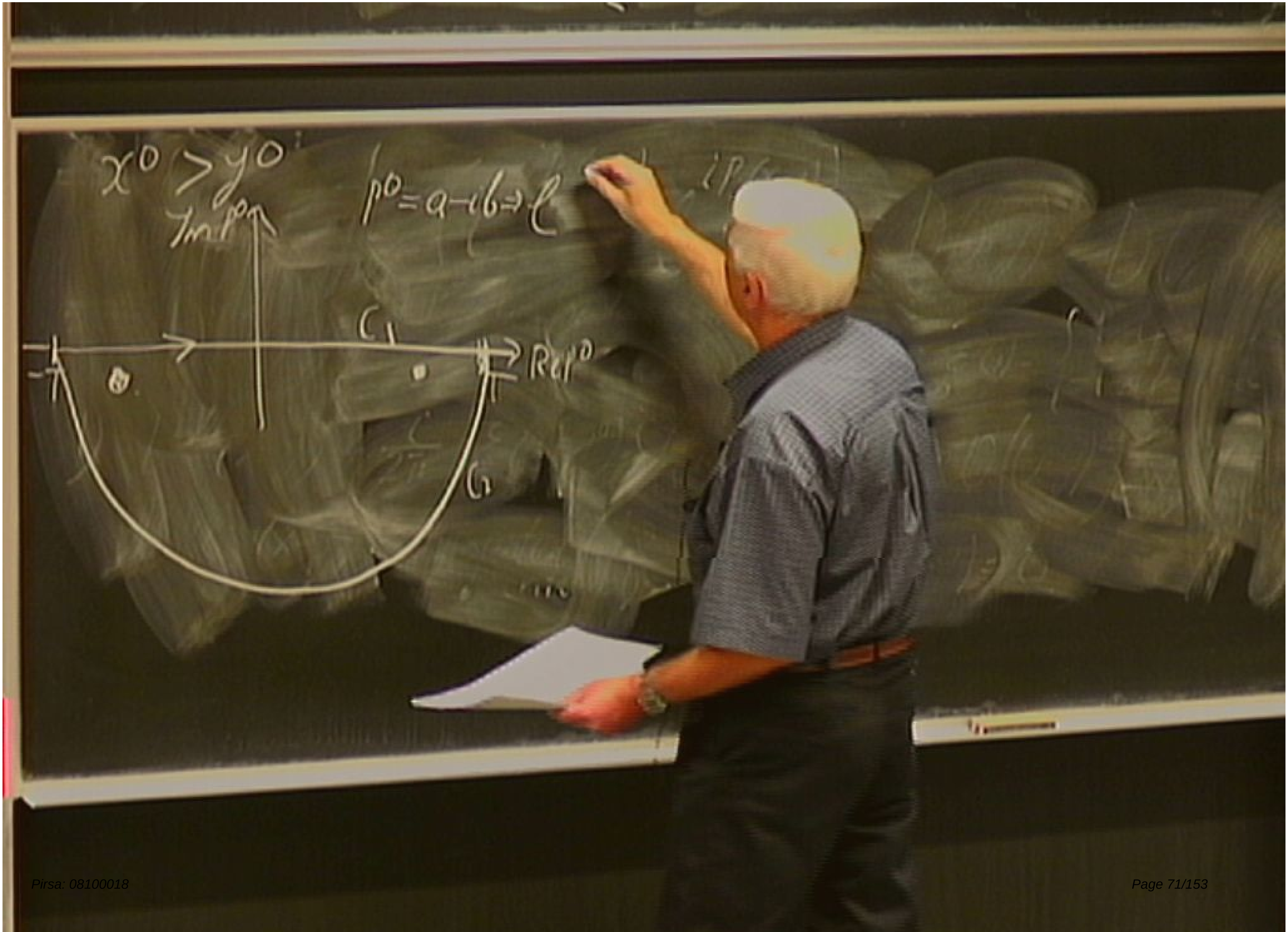
$$\lim_{T\to\infty}\int_{-T}^{T} dp_0 \cdots = \lim_{T\to\infty}\left(\int_{C_1} + \int_{C_2}\right) = 0 \Rightarrow$$

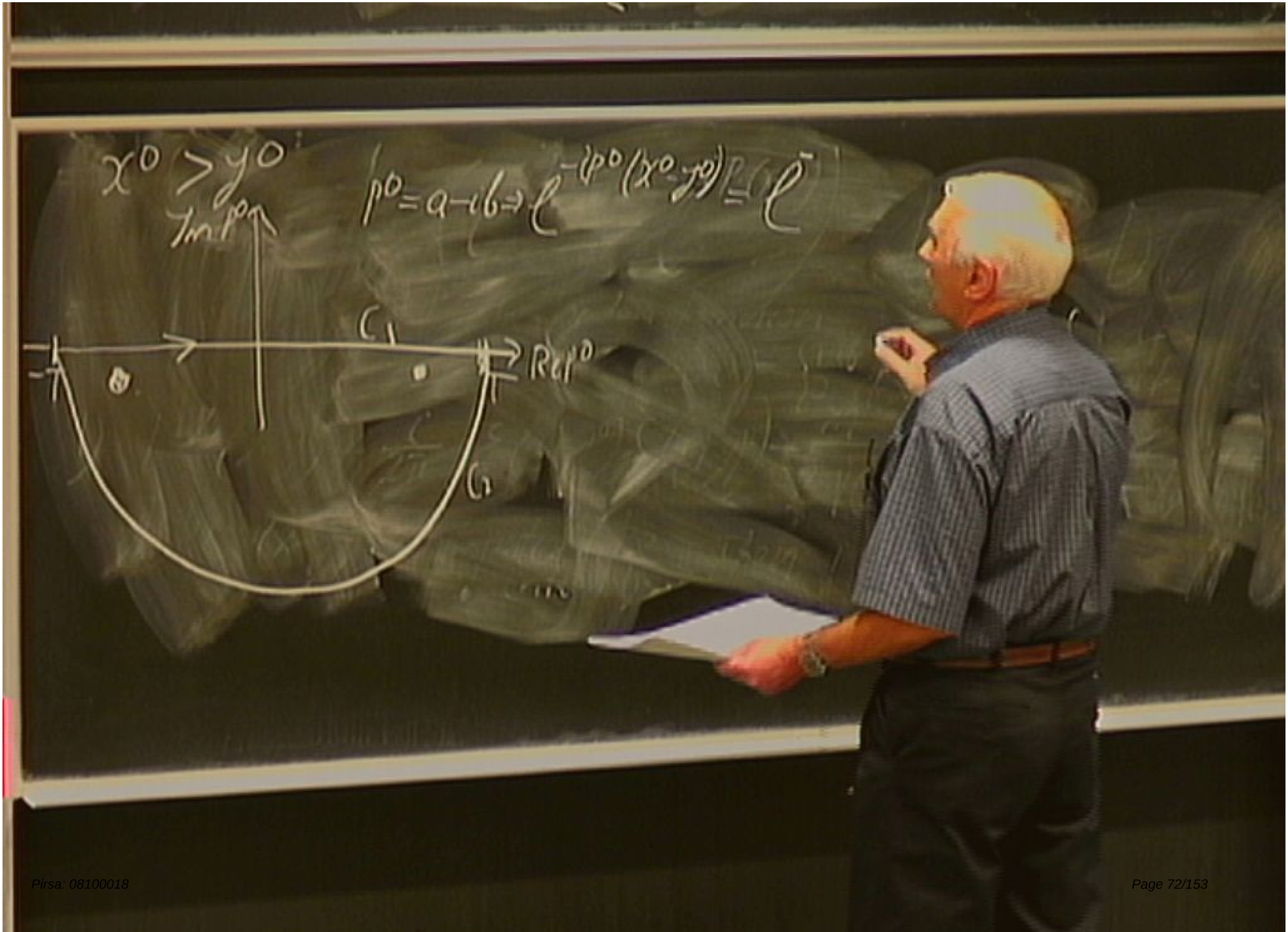
$$\lim_{T\to\infty}\int_{C_2} dp_0 \cdots \longrightarrow 0$$

$$e^{-ip_0(x^0-y^0)} \xrightarrow[p^0\to a+ib]{} e^{-ia(x^0-y^0)+b(x^0-y^0)} \sim e^{-b(y^0-x^0)}\, e^{-ia(x^0-y^0)}$$

We conclude that $\lim_{T\to\infty}\int_{C_2} = 0$ and then our integral $\lim_{T\to\infty}\int_{-T}^{T} dp_0 \cdots = 0$.

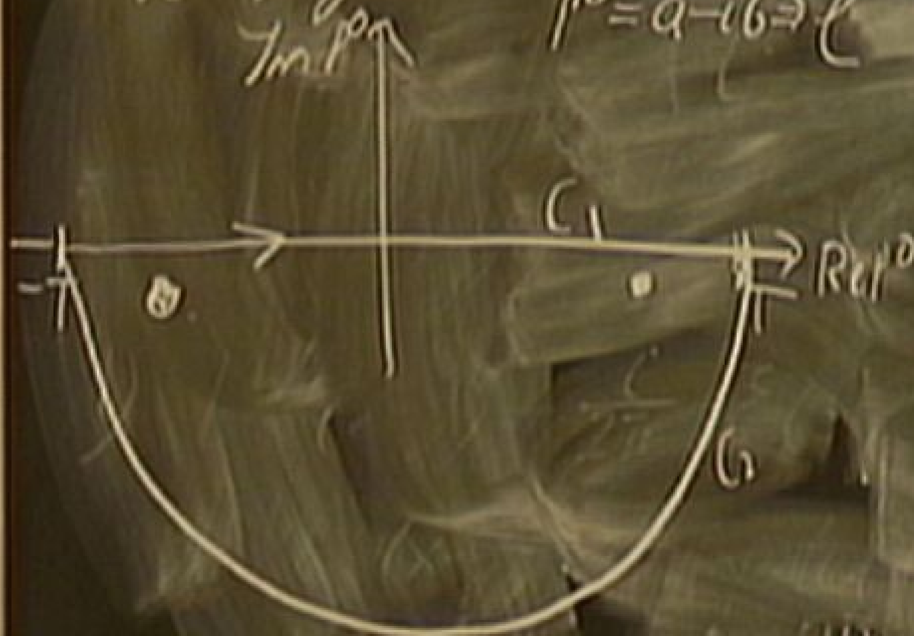

$x^0 > y^0$

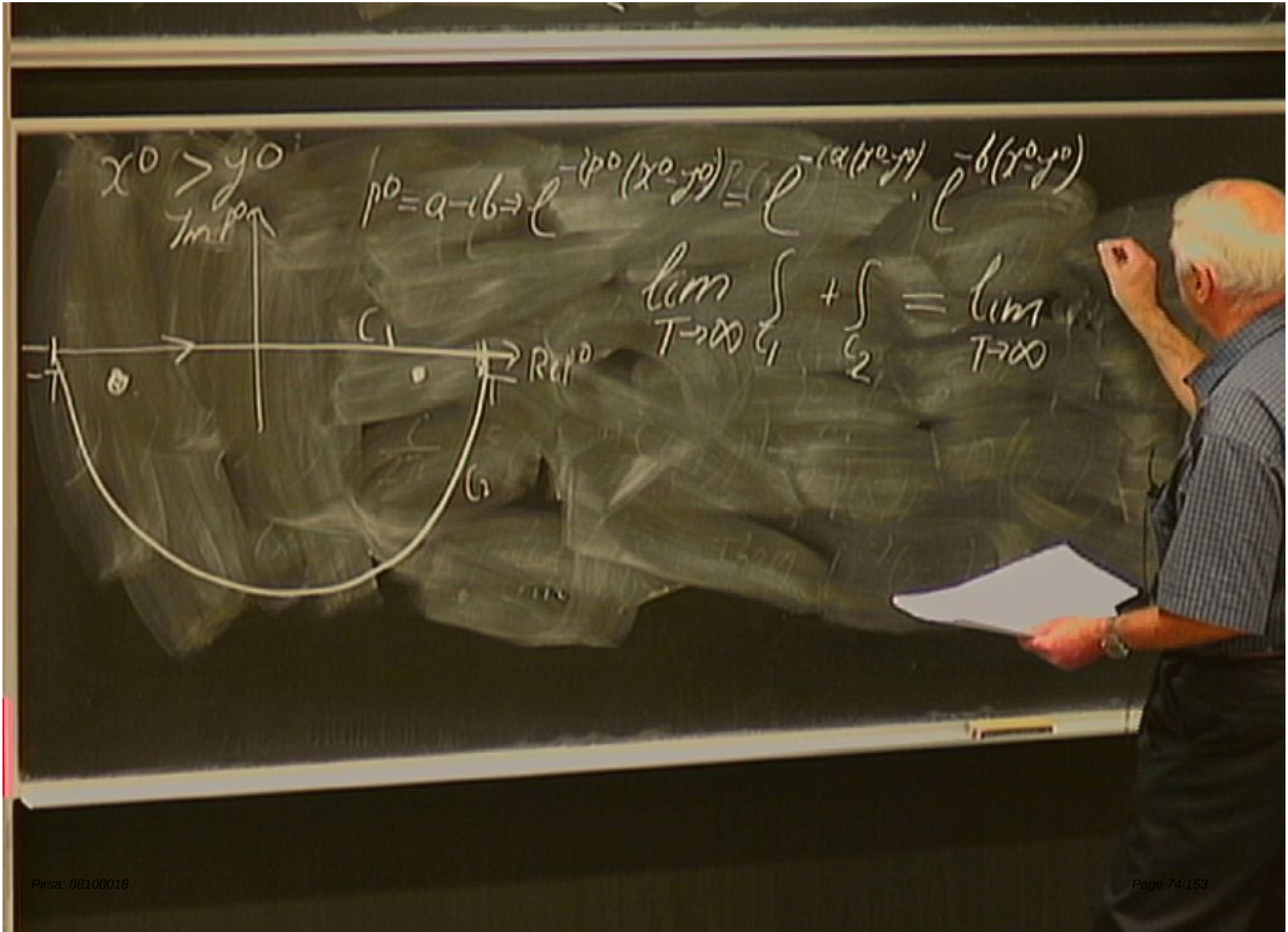

x^0 > y^0
p^0 = a - ib → ℓ
Im p^0
Re p^0
C_1
C_2

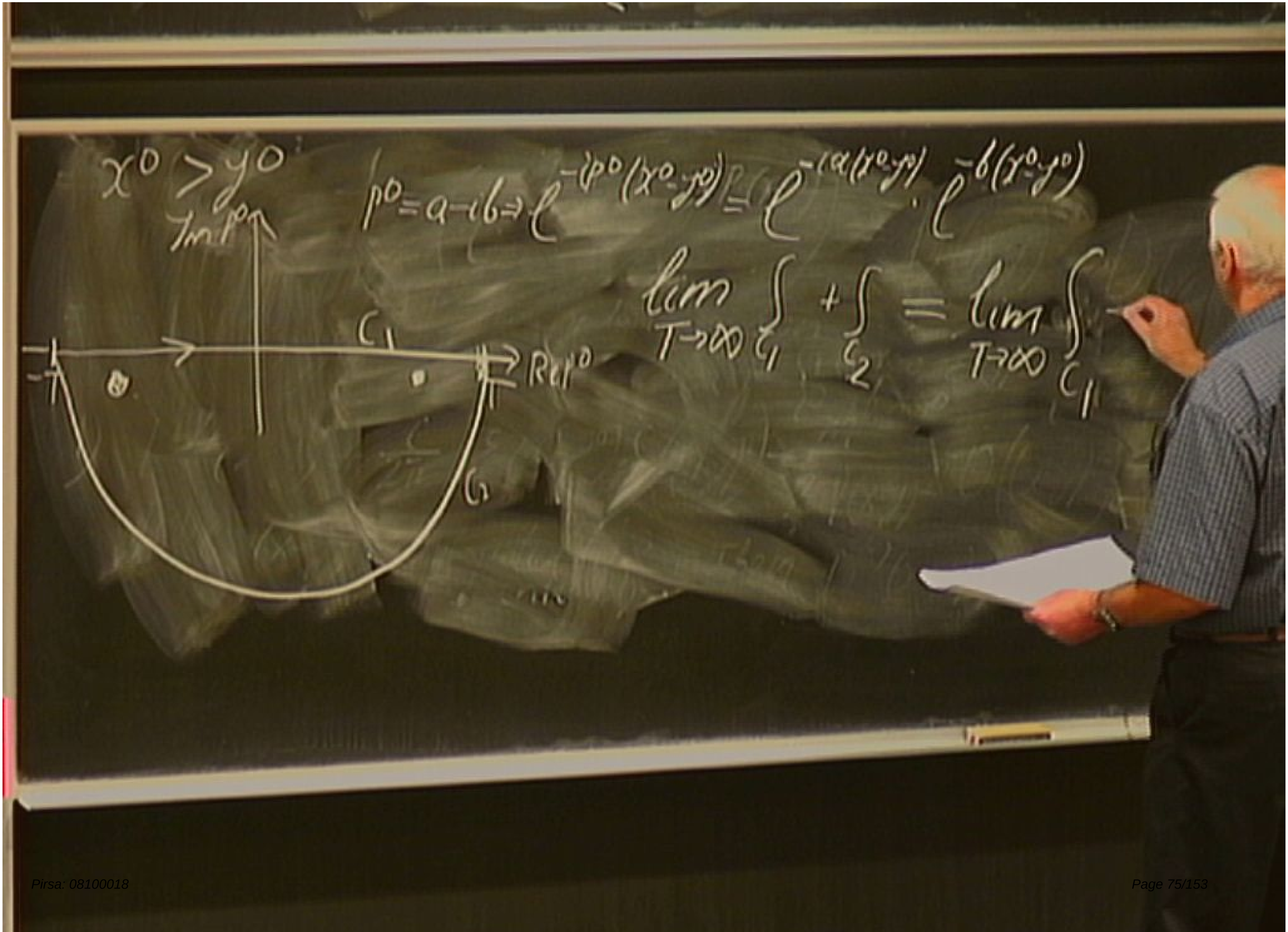

x⁰ > y⁰
Im p⁰
p⁰ = a − ib ⇒ e^{−ip⁰(x⁰−y⁰)} =
Re p⁰
C₁
C₂

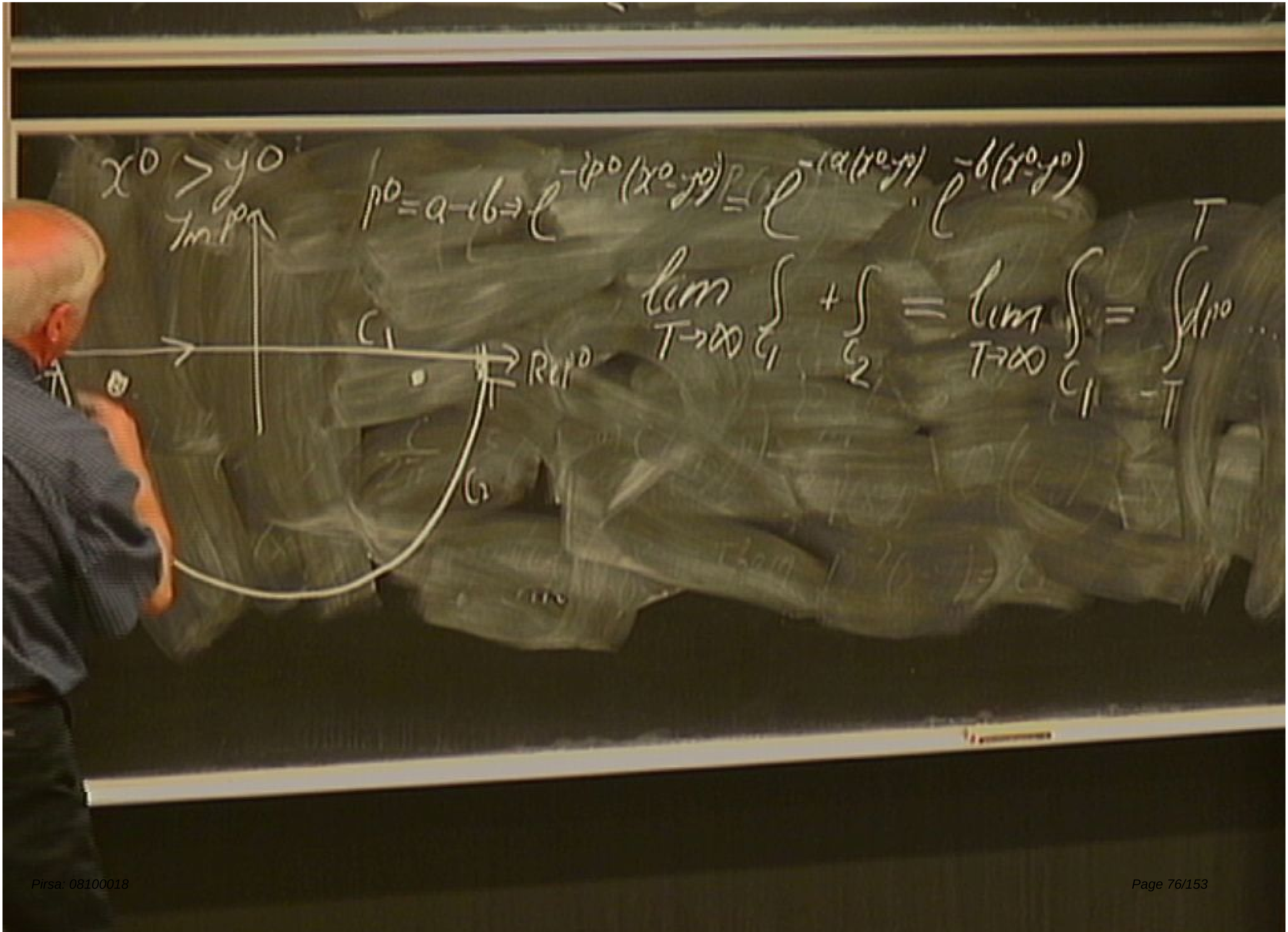
$x^0 > y^0$

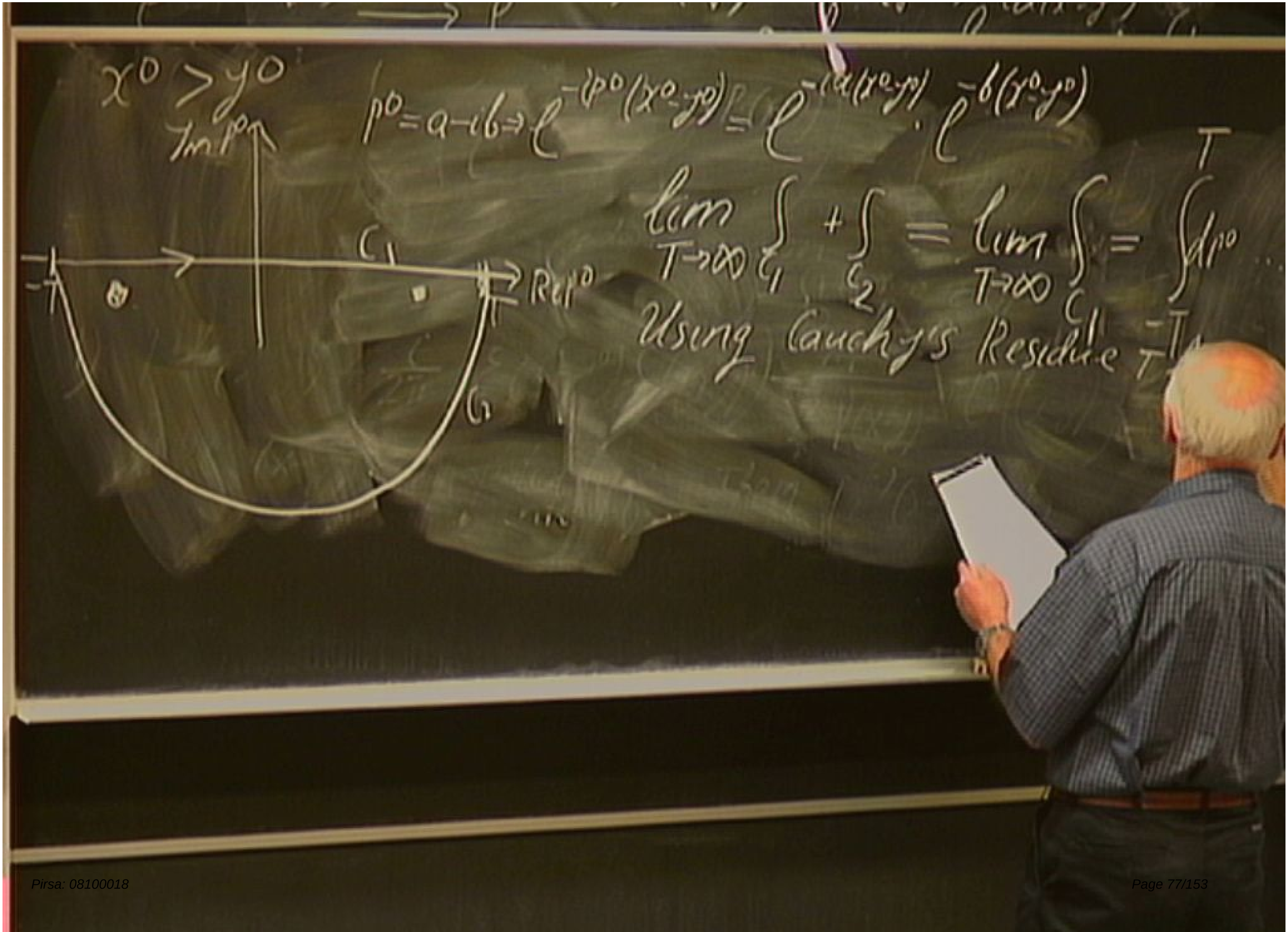
$p^0 = a - ib \Rightarrow e^{-ip^0(x^0-y^0)} = e^{-ia(x^0-y^0)} \cdot e^{-b(x^0-y^0)}$

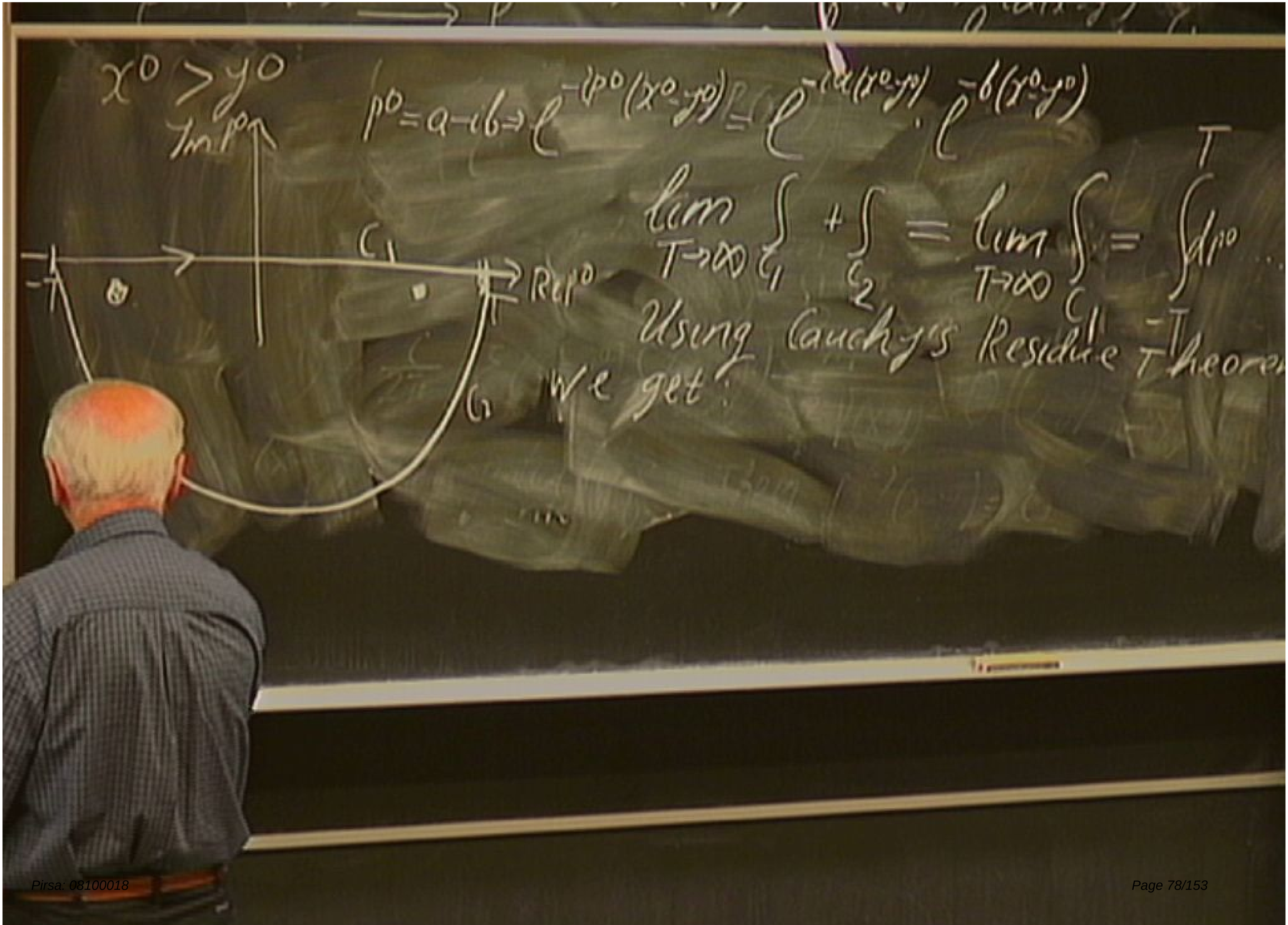


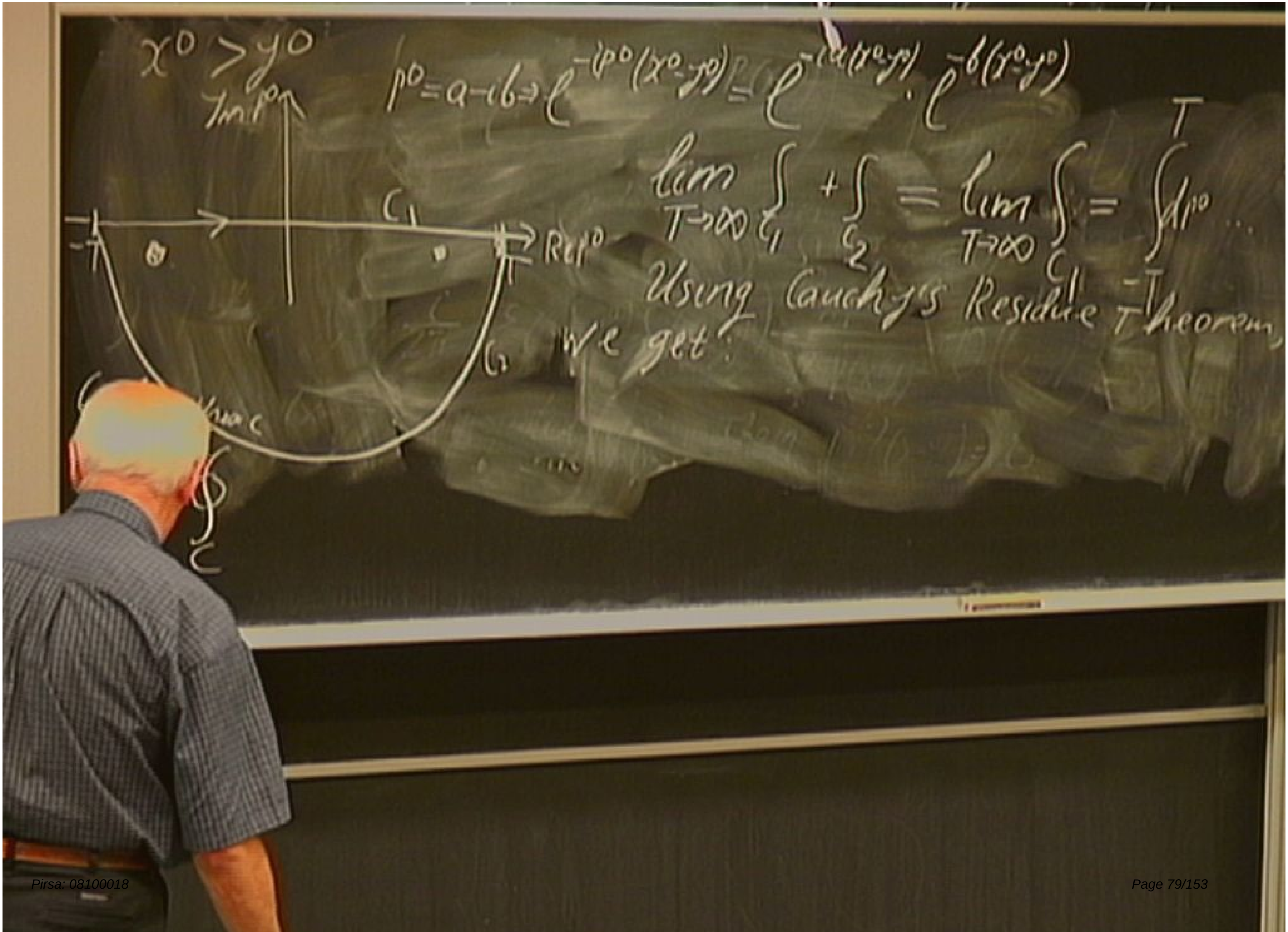

x0 > y0
Im p0
Re p0
-T
T
C1
C2
p0 = a - ib ⇒ e^-ip0(x0-y0) = e^-ia(x0-y0) e^-b(x0-y0)
lim T→∞ ∫C1 + ∫C2 = lim T→∞

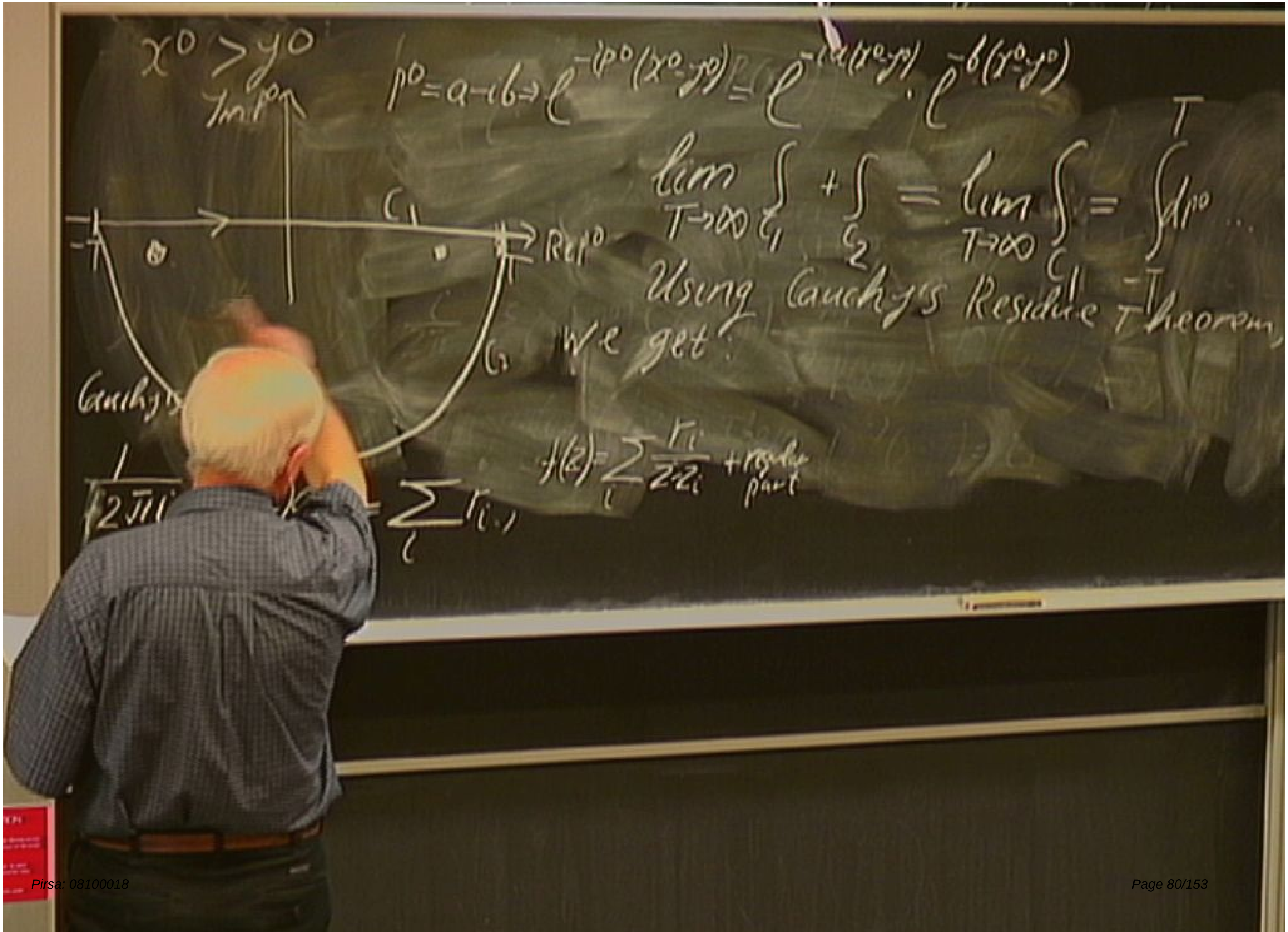

x^0 > y^0
p^0 = a - ib ⇒ e^{-ip^0(x^0-y^0)} = e^{-ia(x^0-y^0)} · e^{-b(x^0-y^0)}
lim_{T→∞} ∫_{C_1} + ∫_{C_2} = lim_{T→∞} ∫_{C_1}
Im p^0
Re p^0
-T
C_1
C_2

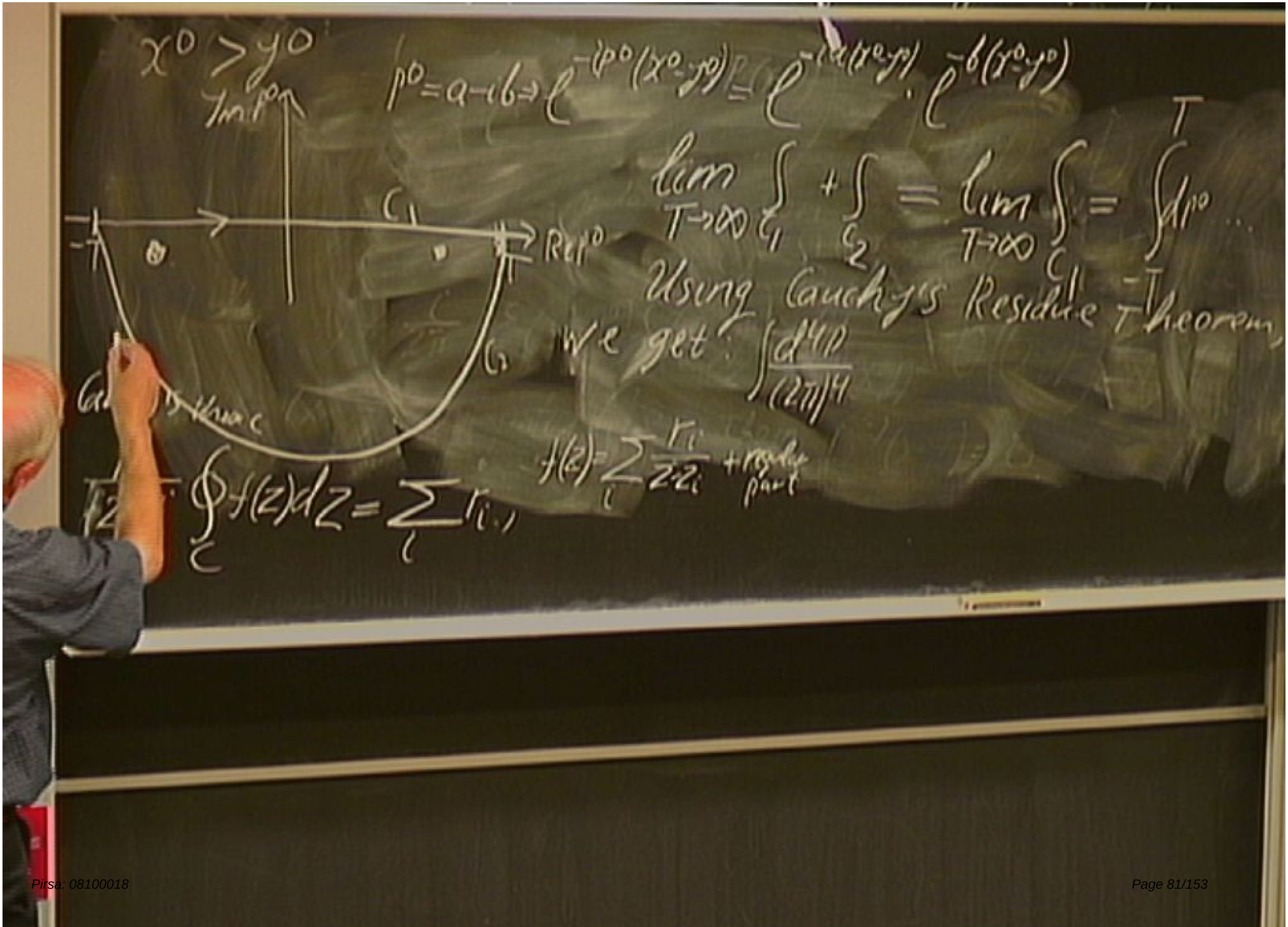

x0 > y0
p0 = a - ib ⇒ e-ip0(x0-y0) = e-ia(x0-y0) e-b(x0-y0)
Im p0
Re p0
C1
C2
lim T→∞ ∫C1 + ∫C2 = lim T→∞ ∫C1 = ∫-TT dp0 ...

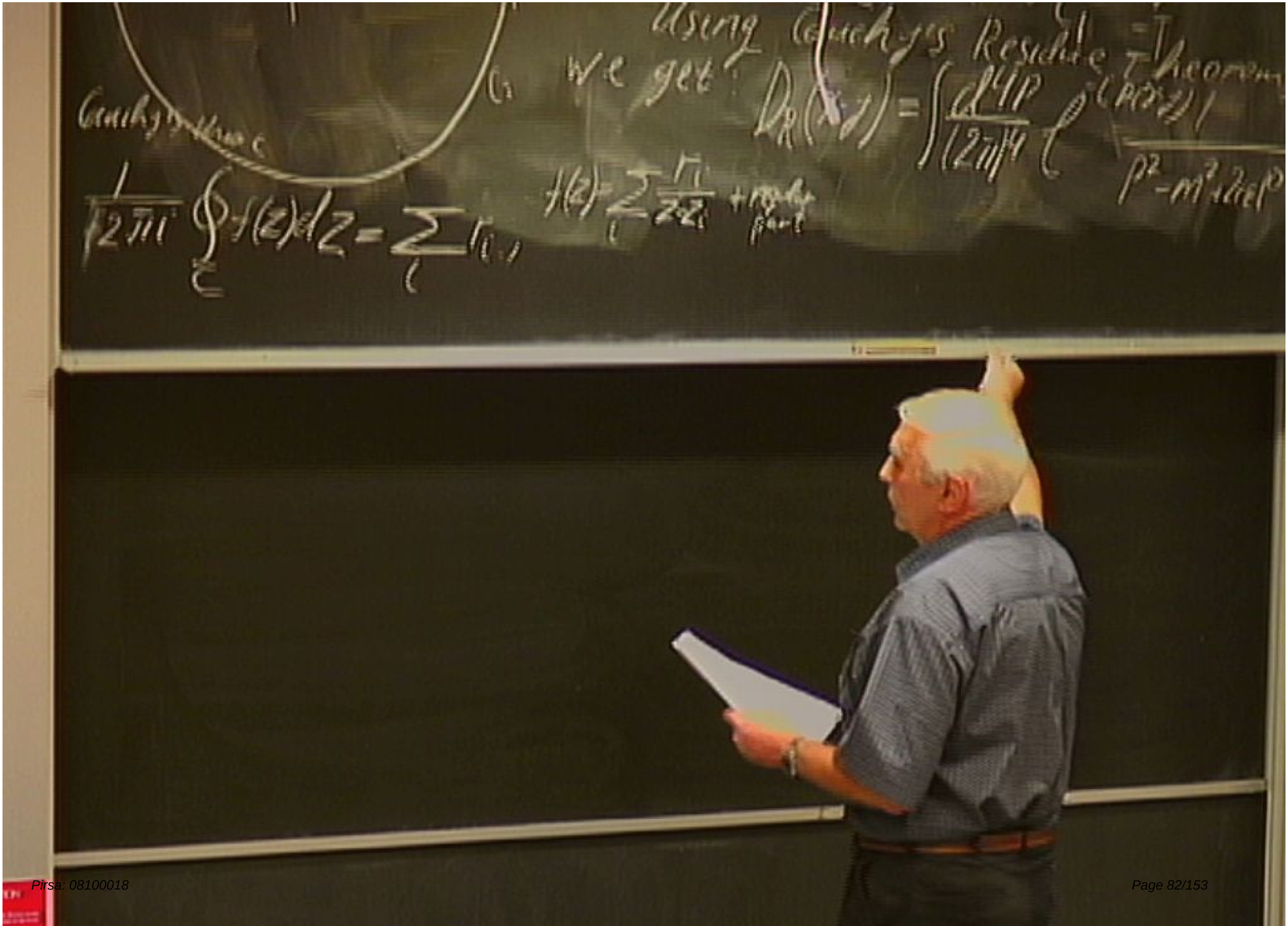

x0 > y0
Im p0
Re p0
-T
T
C1
C2
p0 = a - ib ⇒ e^(-ip0(x0-y0)) = e^(-ia(x0-y0)) · e^(-b(x0-y0))
lim T→∞ ∫C1 + ∫C2 = lim T→∞ ∫C = ∫-T T dp0 ...
Using Cauchy's Residue

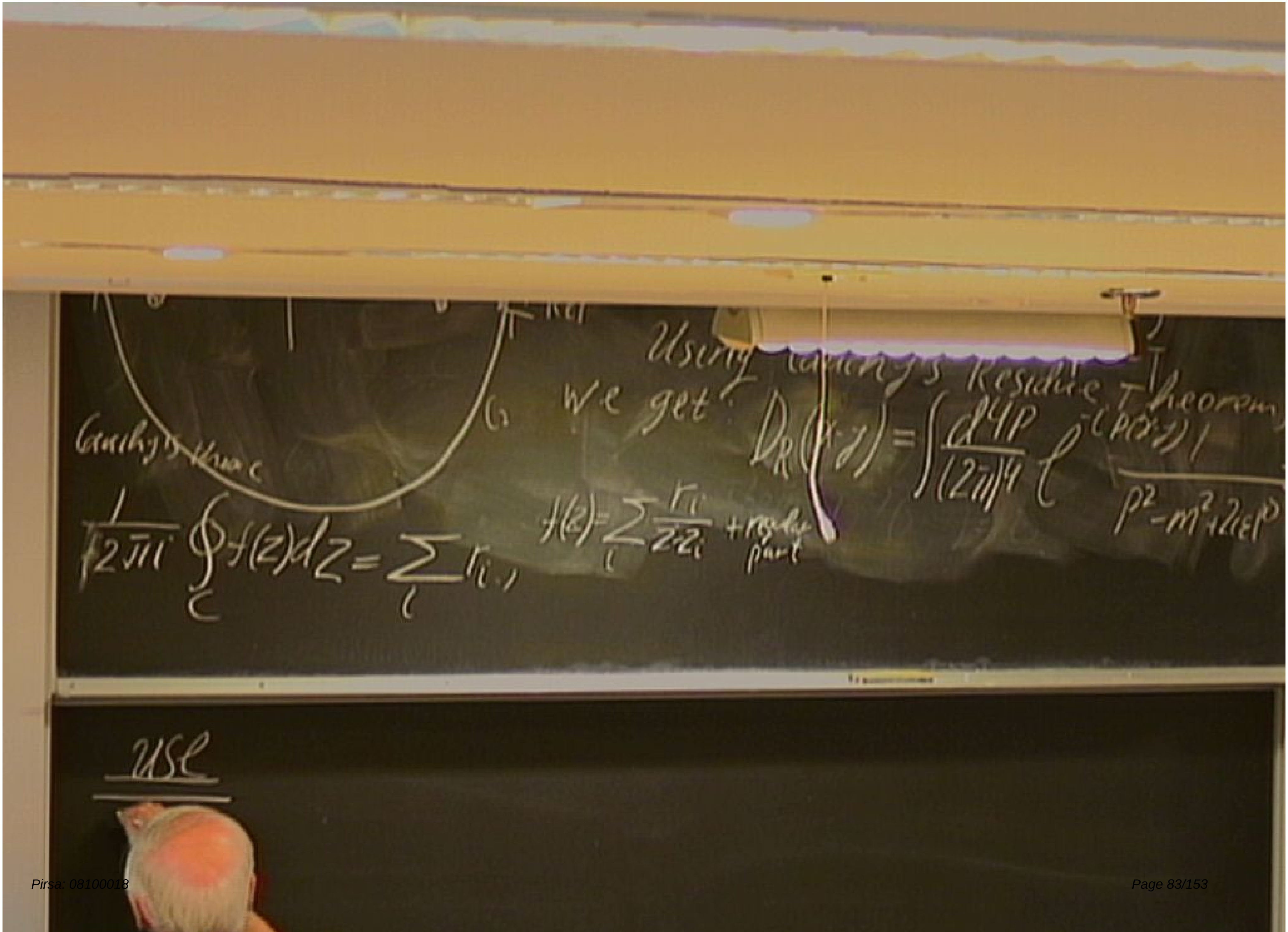

x⁰ > y⁰
Im p⁰
Re p⁰
-T
T
C1
C2
p⁰ = a - ib ⇒ e^{-ip⁰(x⁰-y⁰)} = e^{-ia(x⁰-y⁰)} · e^{-b(x⁰-y⁰)}
lim T→∞ ∫C1 + ∫C2 = lim T→∞ ∫C1 = ∫ from -T to T dp⁰ ...
Using Cauchy's Residue Theorem
we get:

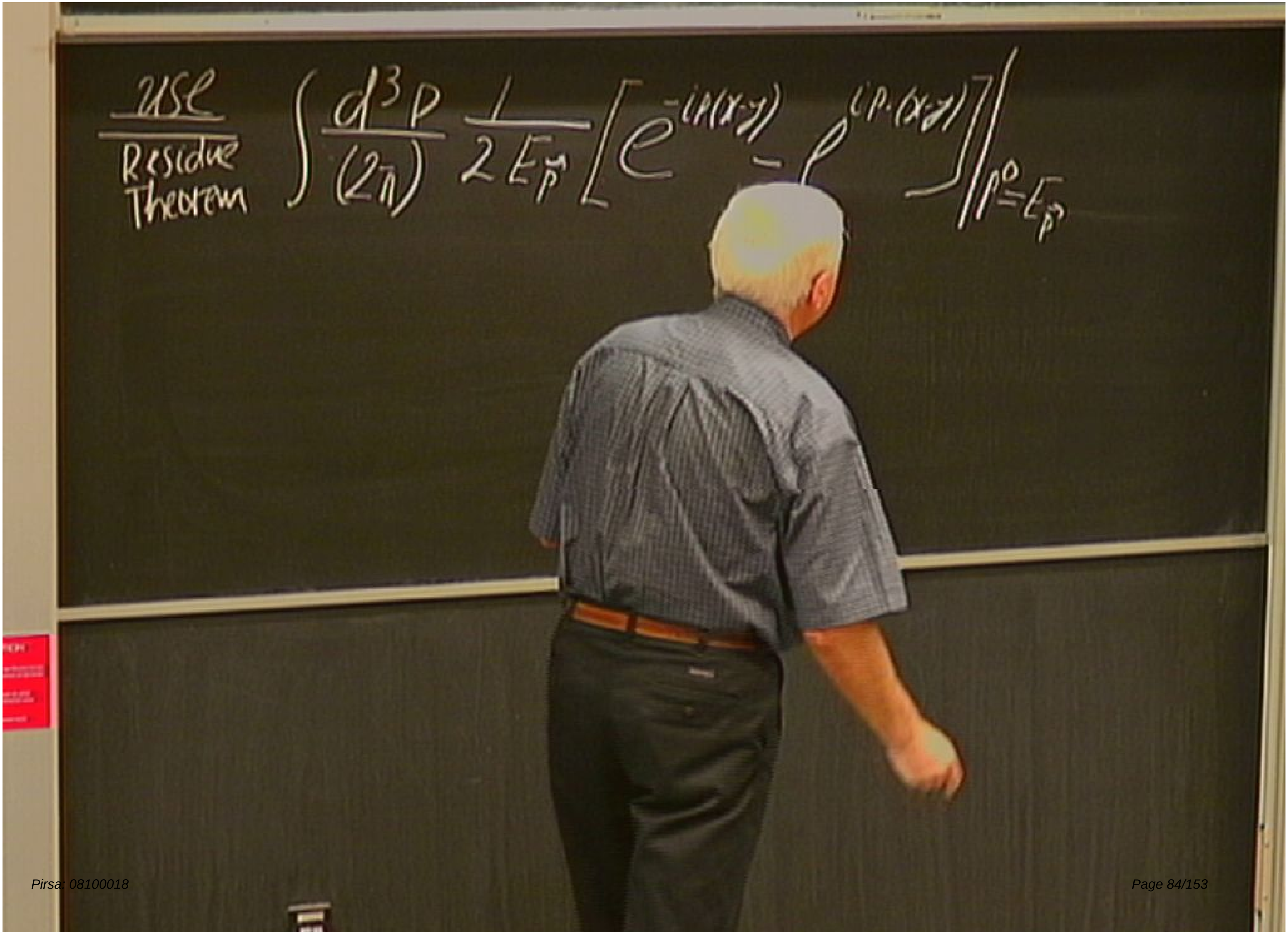

x0 > y0
Im p0
p0 = a - ib ⇒ e-ip0(x0-y0) = e-ia(x0-y0) · e-b(x0-y0)
lim T→∞ ∫C1 + ∫C2 = lim T→∞ ∫C = ∫dp0 ...
C1
C2
Re p0
Using Cauchy's Residue Theorem,
we get:

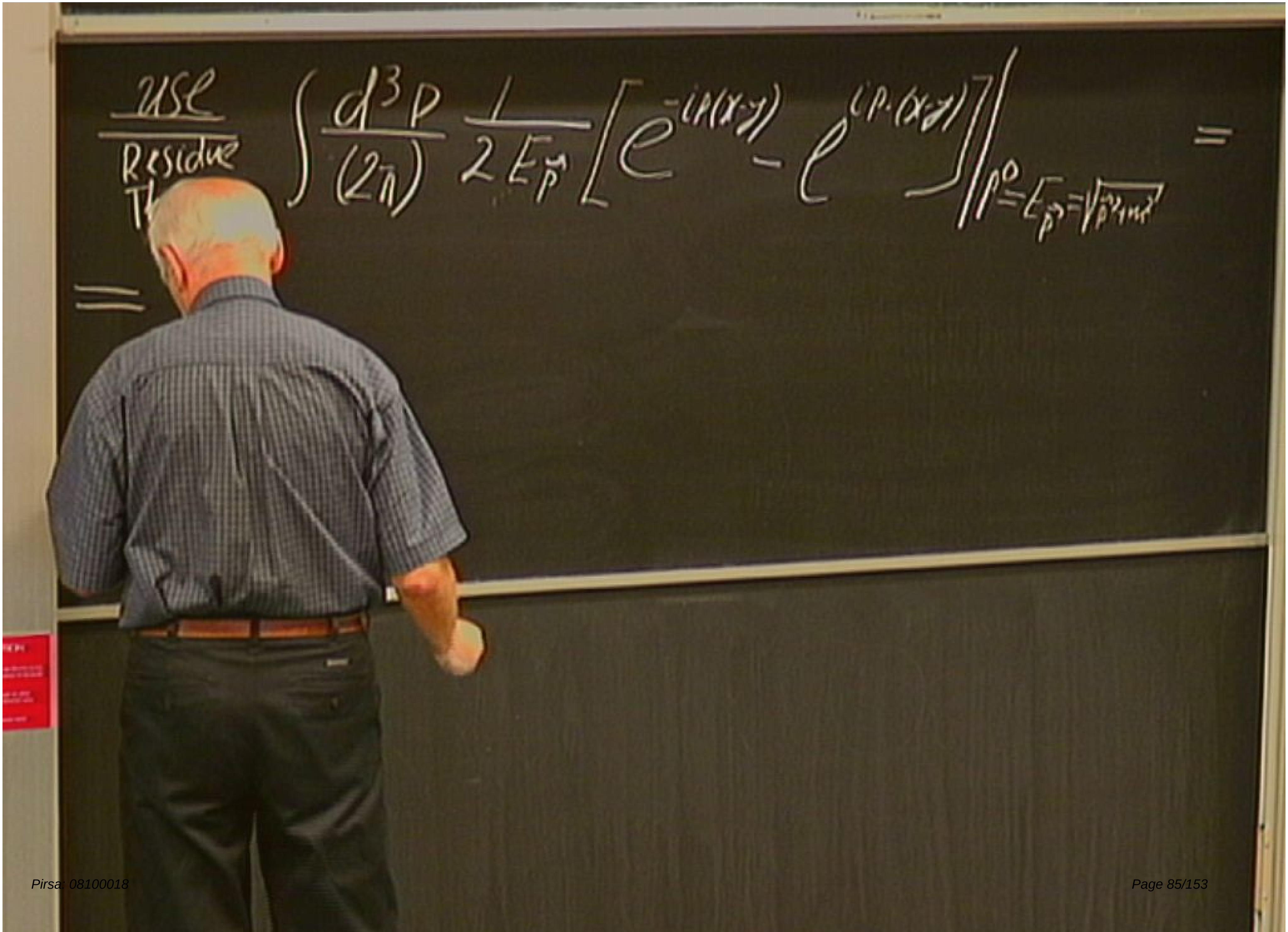

x^0 > y^0
p^0 = a - ib
Im p^0
Re p^0
C_1
C_2
lim T→∞
Using Cauchy's Residue Theorem,
we get:
f(z) = Σ r_i / (z - z_i) + regular part

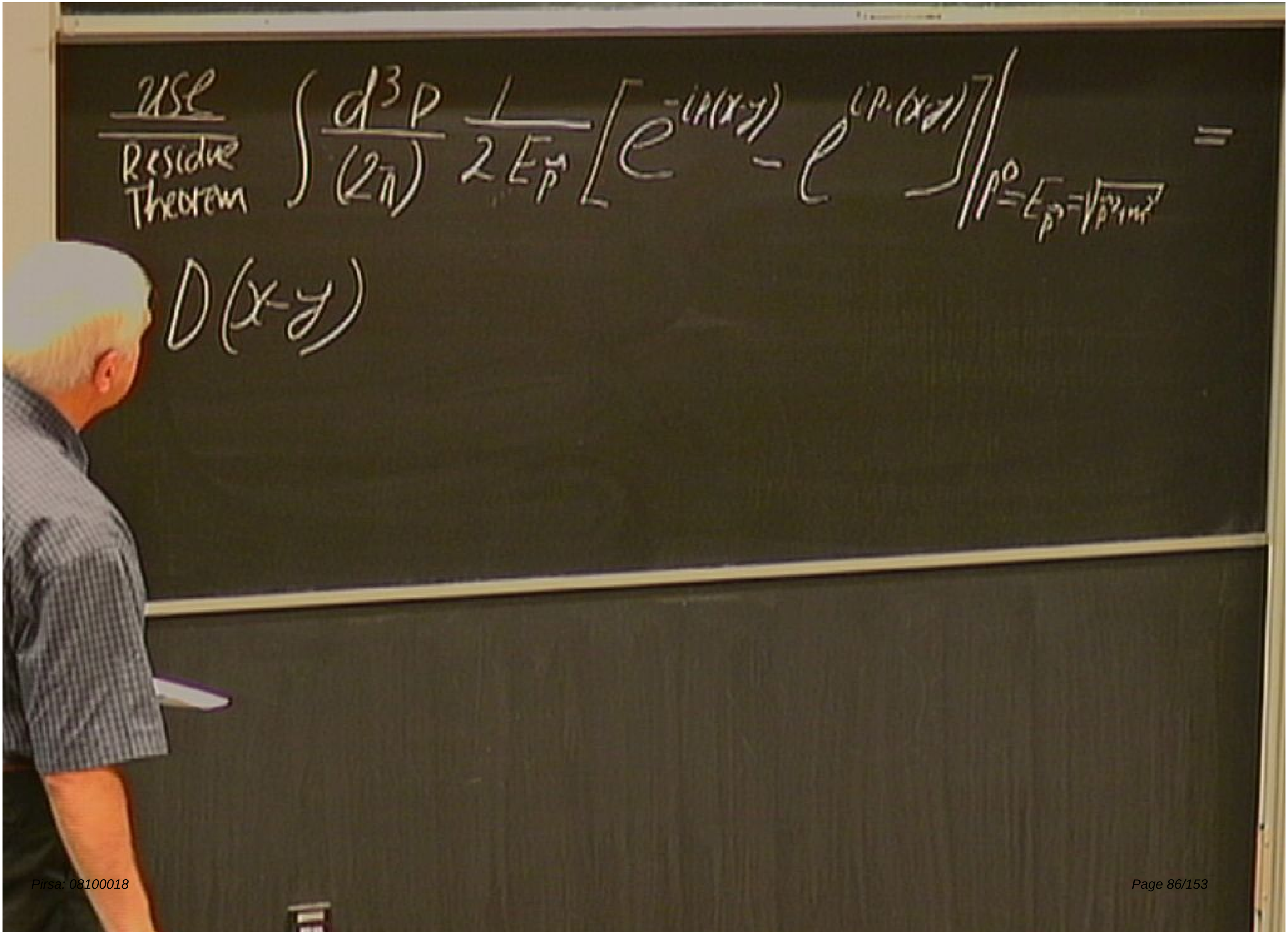

$x^0 > y^0$
Im p^0
$p^0 = a - ib \Rightarrow e^{-ip^0(x^0-y^0)} = e^{-ia(x^0-y^0)} \cdot e^{-b(x^0-y^0)}$
$\lim_{T\to\infty} \int_{C_1} + \int_{C_2} = \lim_{T\to\infty} \oint_C = \int_{-T}^{T} dp^0 \ldots$
Re p^0
C_1
C_2
Using Cauchy's Residue Theorem
we get: $\int \frac{d^4p}{(2\pi)^4}$
$f(z) = \sum_i \frac{r_i}{z - z_i}$ + regular part
$\oint_C f(z)dz = \sum_i r_i$

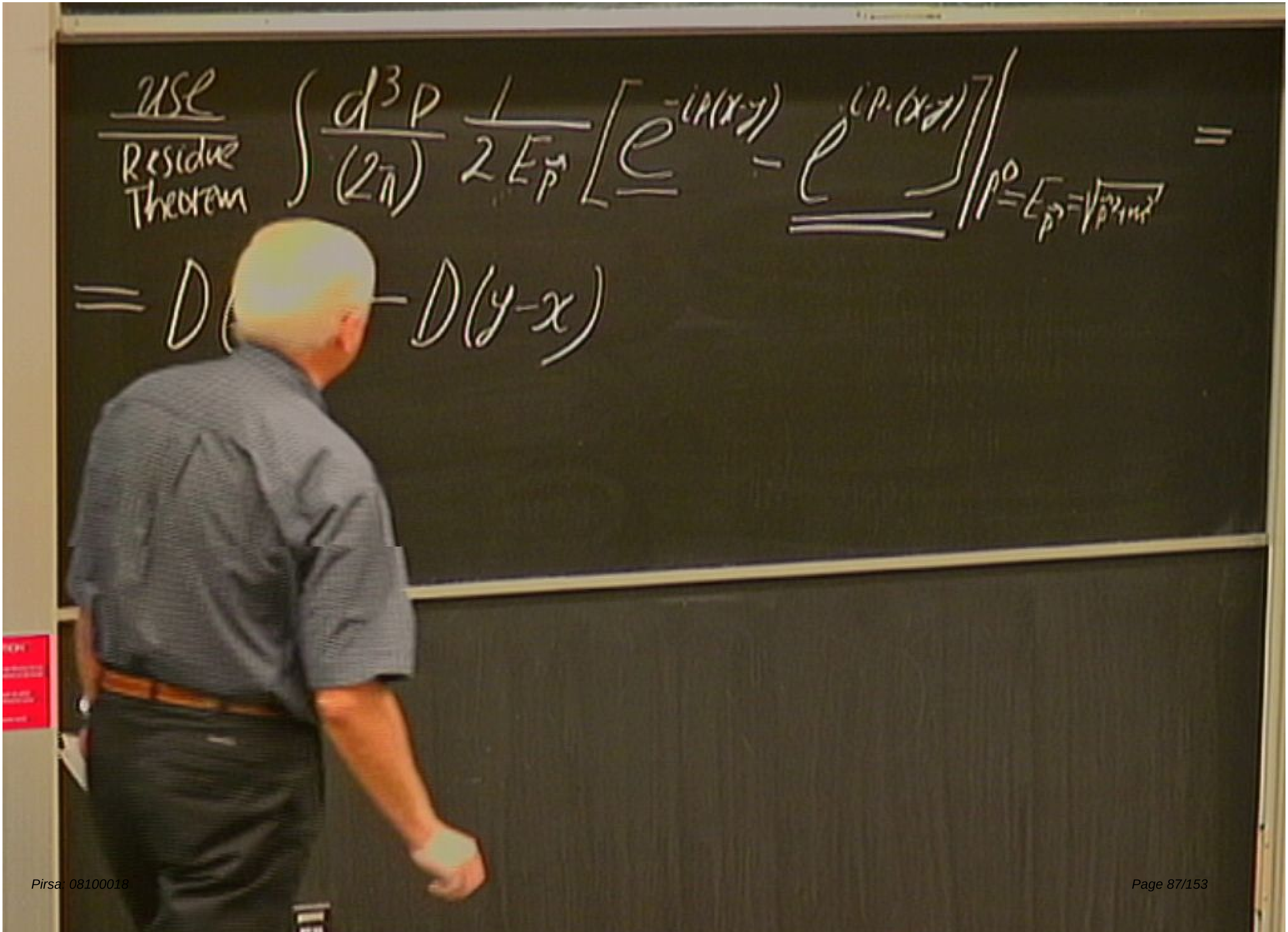

Using Cauchy's Residue Theorem
we get:

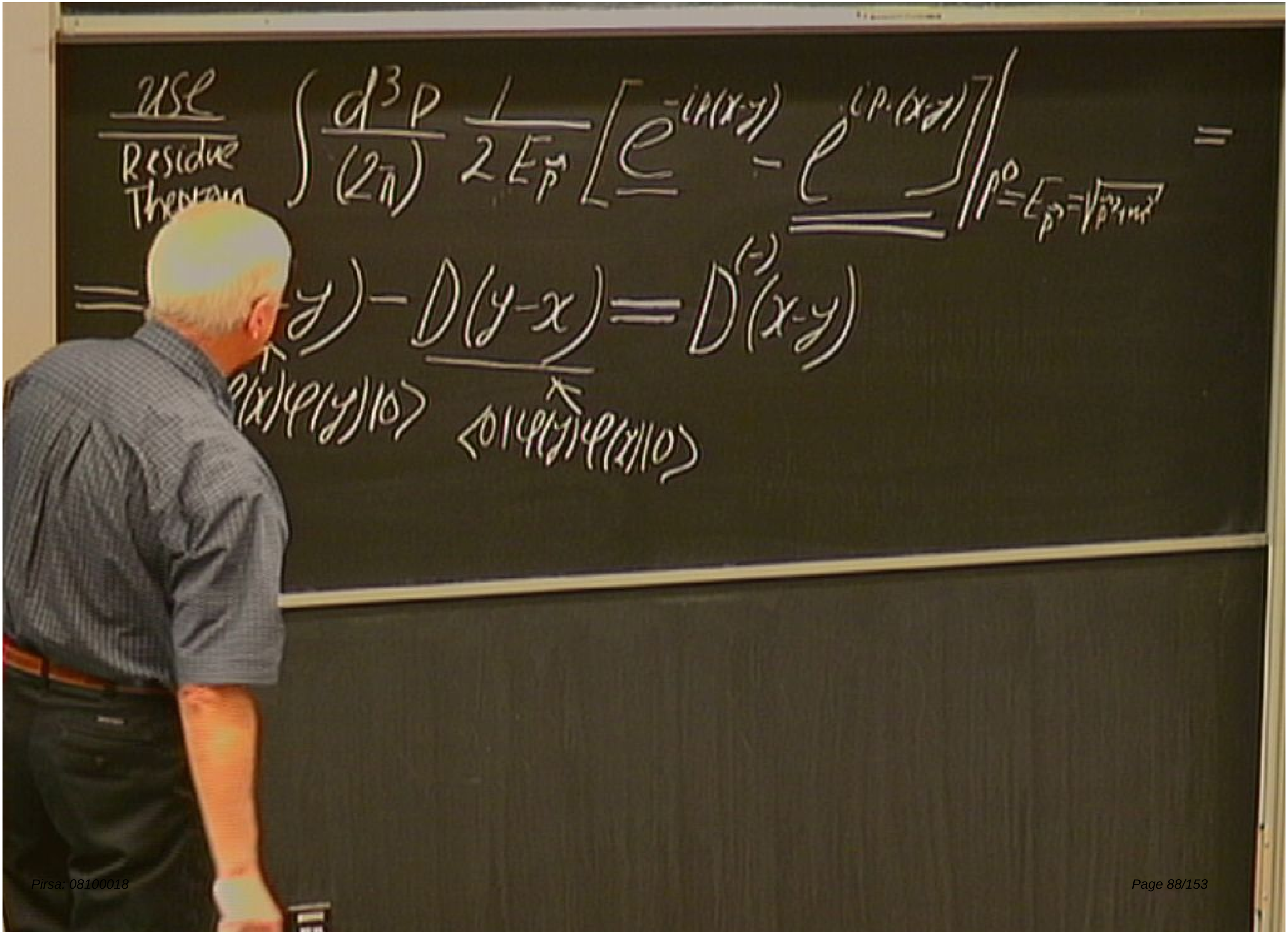

Using
Residue Theorem
we get:
use

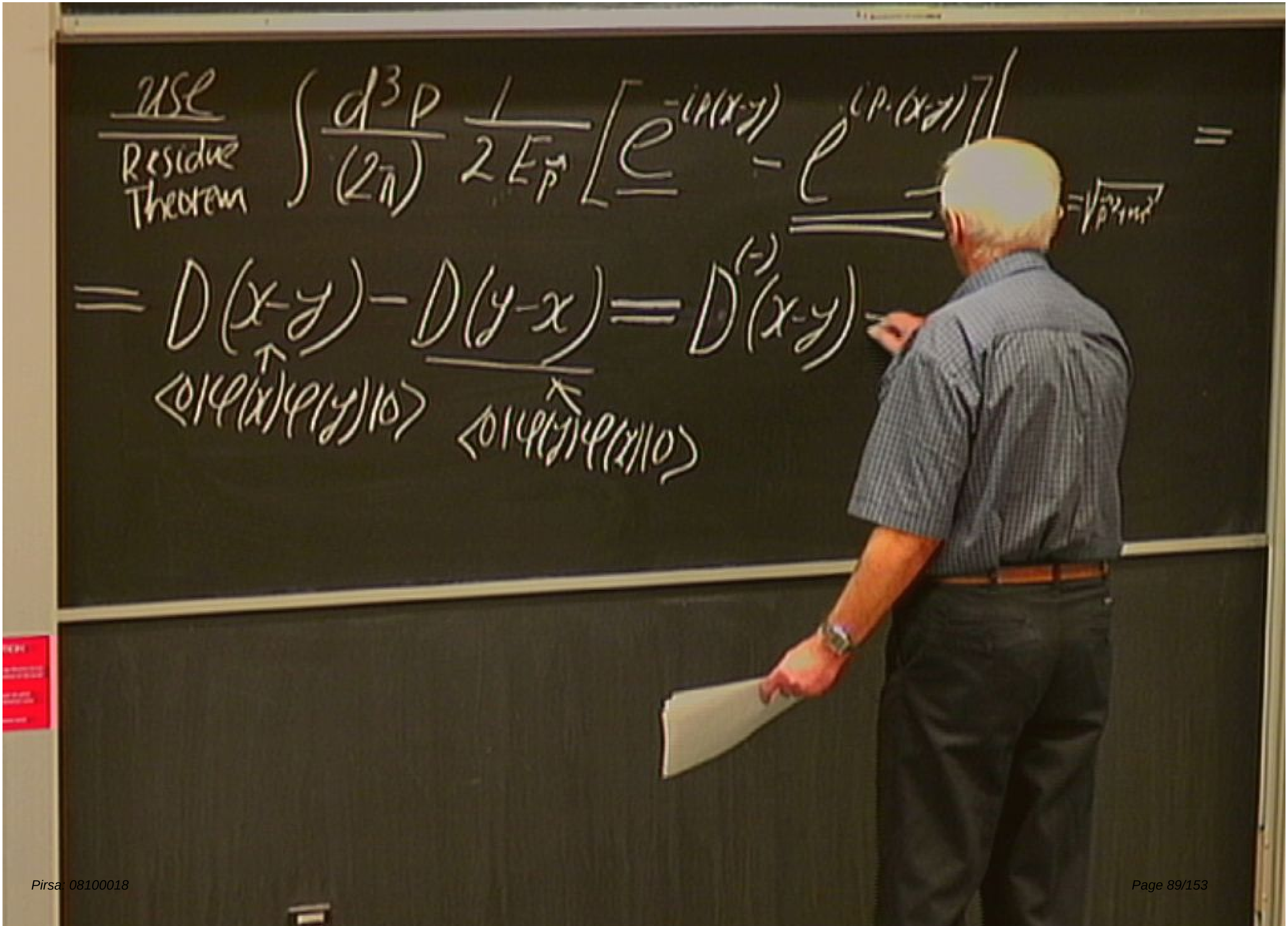

use
Residue Theorem
∫ d³p/(2π) 1/(2E_p) [e^{-ip(x-y)} - e^{ip·(x-y)}] |_{p⁰=E_p}

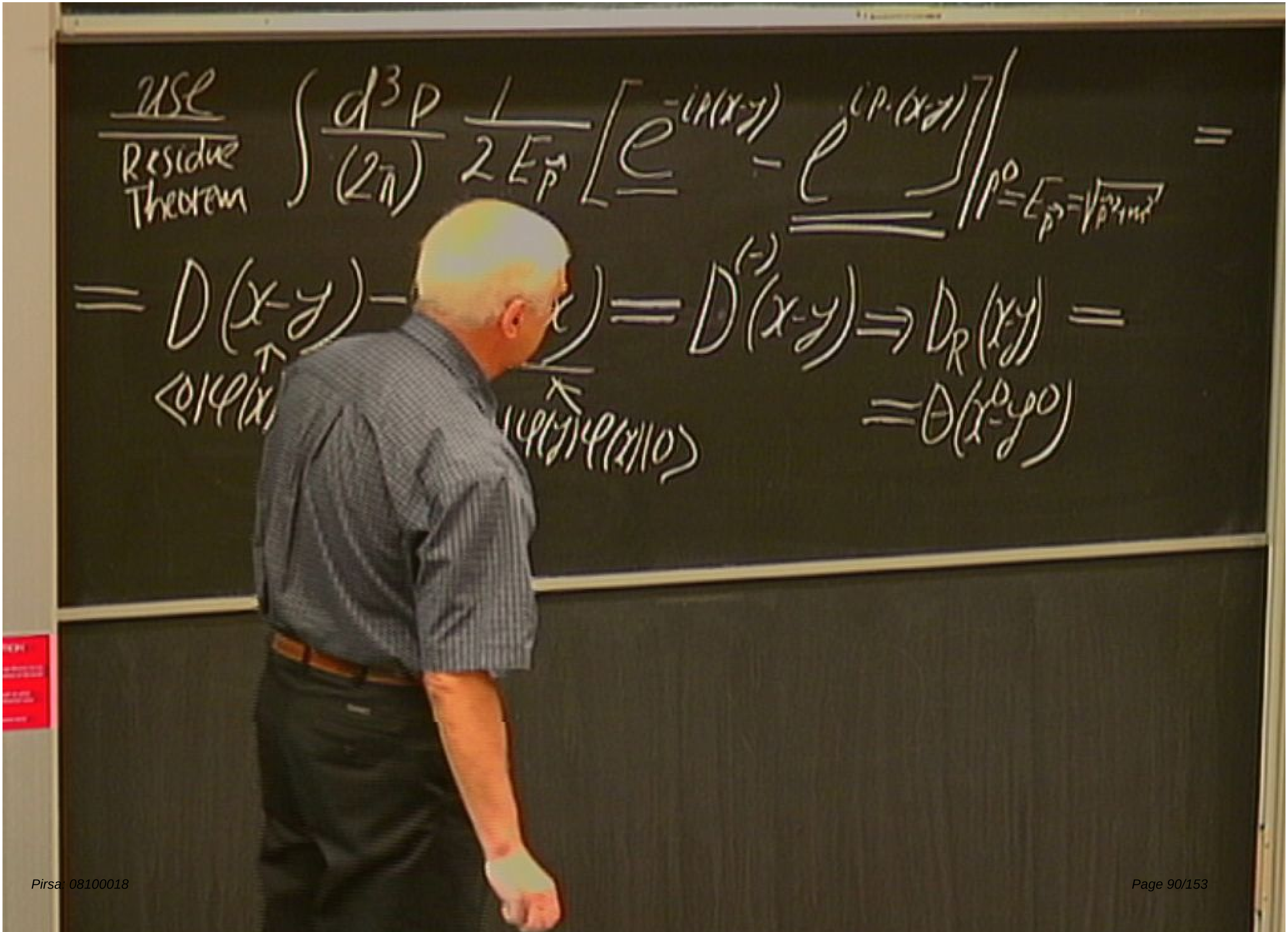

use
Residue
Th
$\int \frac{d^3p}{(2\pi)} \frac{1}{2E_{\vec p}} \left[e^{-ip(x-y)} - e^{ip\cdot(x-y)}\right]\Big|_{p^0=E_{\vec p}=\sqrt{\vec p^2+m^2}} =$
$=$

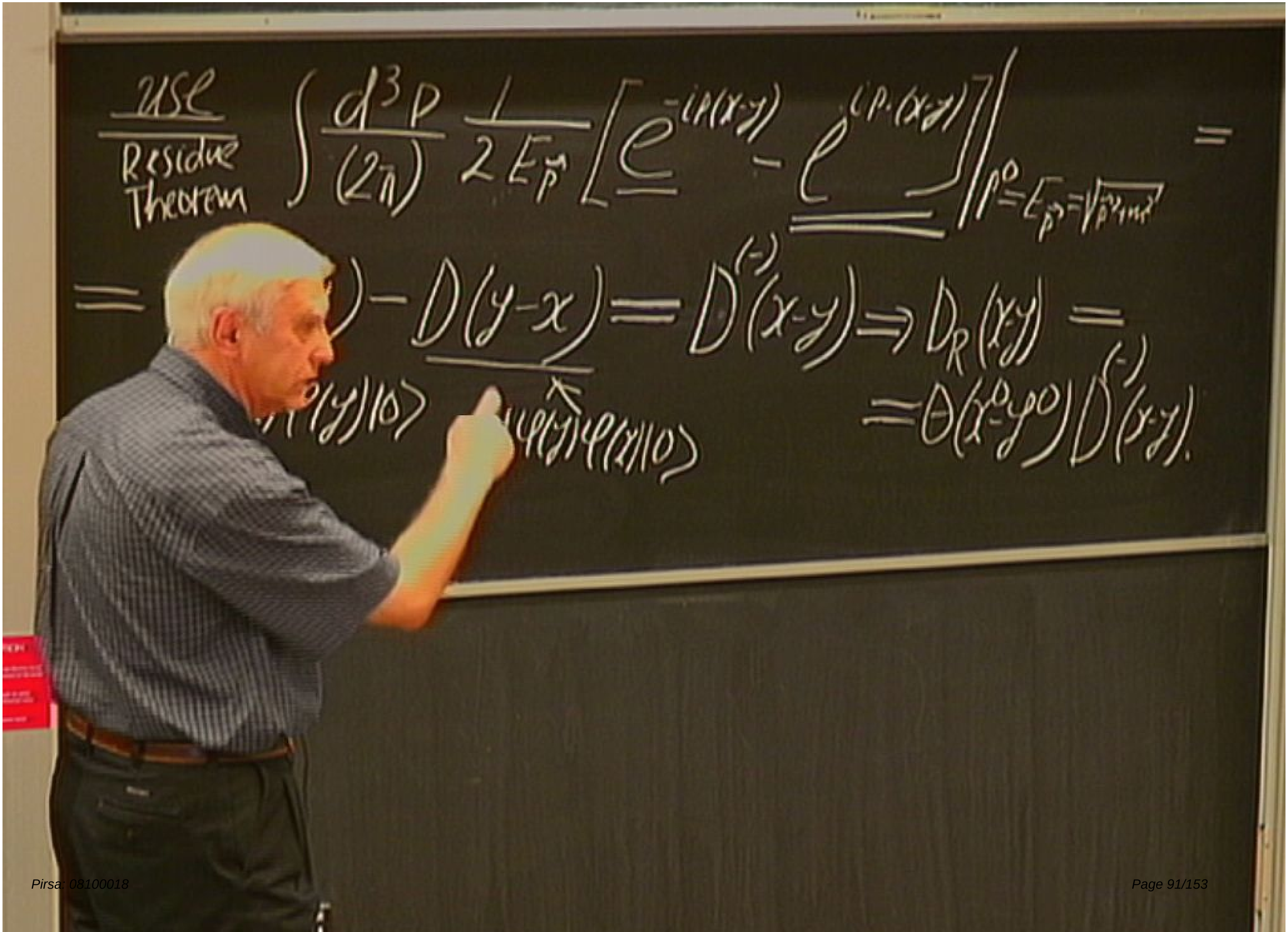

use
Residue Theorem
$\int \frac{d^3p}{(2\pi)} \frac{1}{2E_p} \left[e^{-ip(x-y)} - e^{ip\cdot(x-y)} \right] \Big|_{p^0=E_p=\sqrt{p^2+m^2}} =$
$D(x-y)$

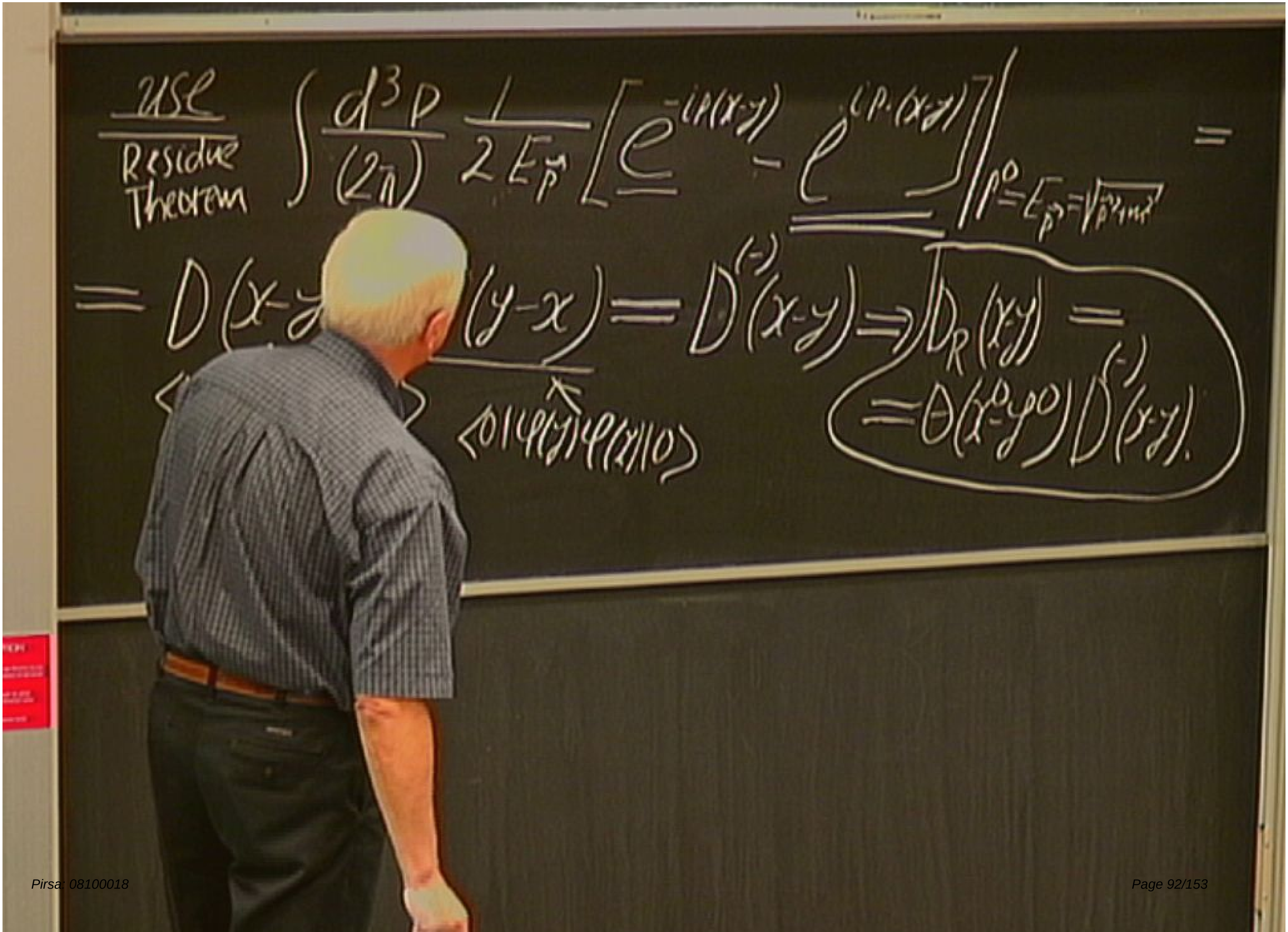

use
Residue Theorem
$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec p}} \left[e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right] \Big|_{p^0 = E_{\vec p} = \sqrt{\vec p^2 + m^2}} =$
$= D(\ \ ) - D(y-x)$

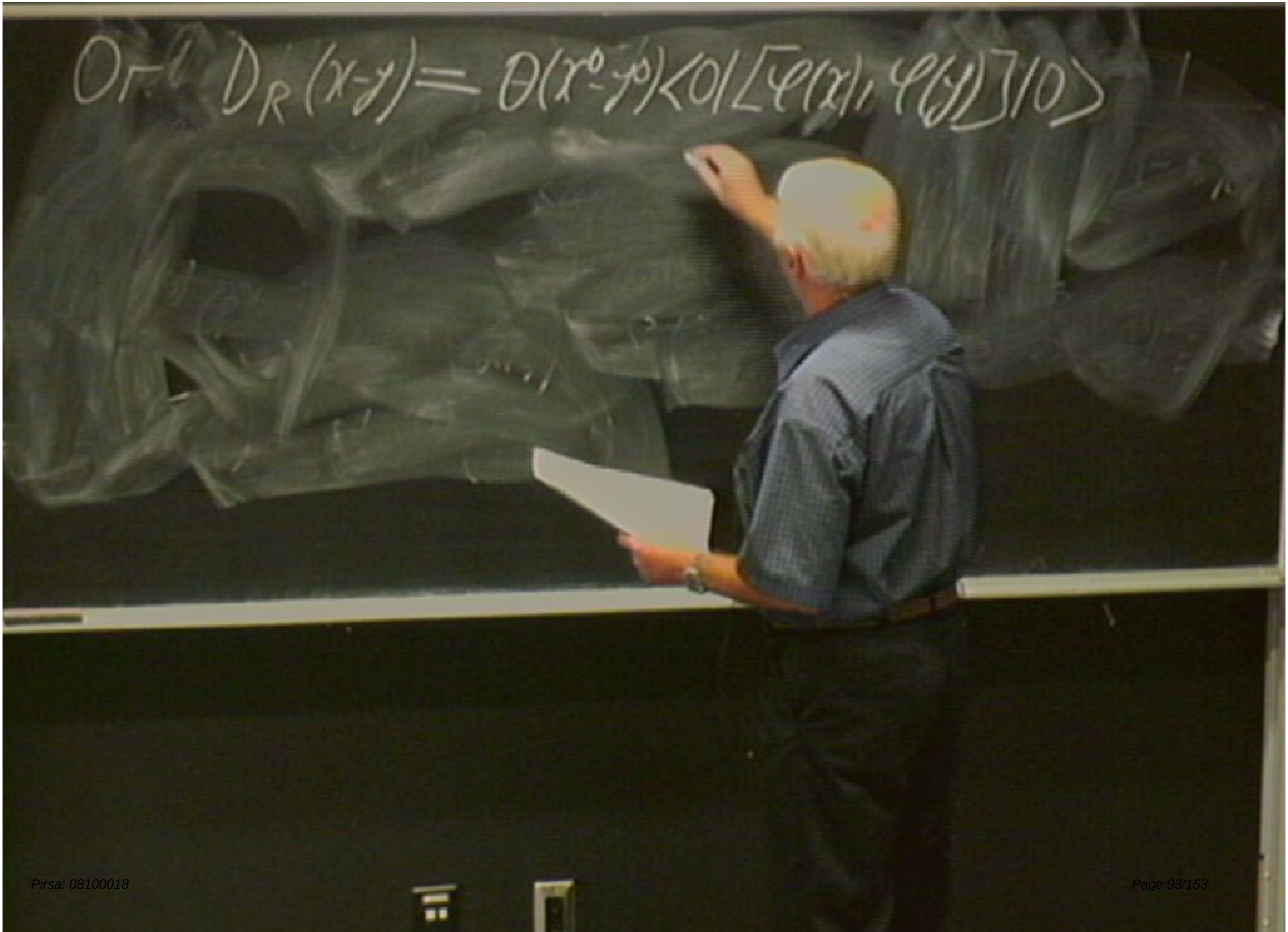

use Residue Theorem
∫ d³p/(2π) 1/2E_p [e^{-ip(x-y)} − e^{ip·(x-y)}] | p⁰=E_p=√(p²+m²) =
= y) − D(y−x) = D^(−)(x−y)
(x)φ(y)|0⟩
⟨0|φ(y)φ(x)|0⟩

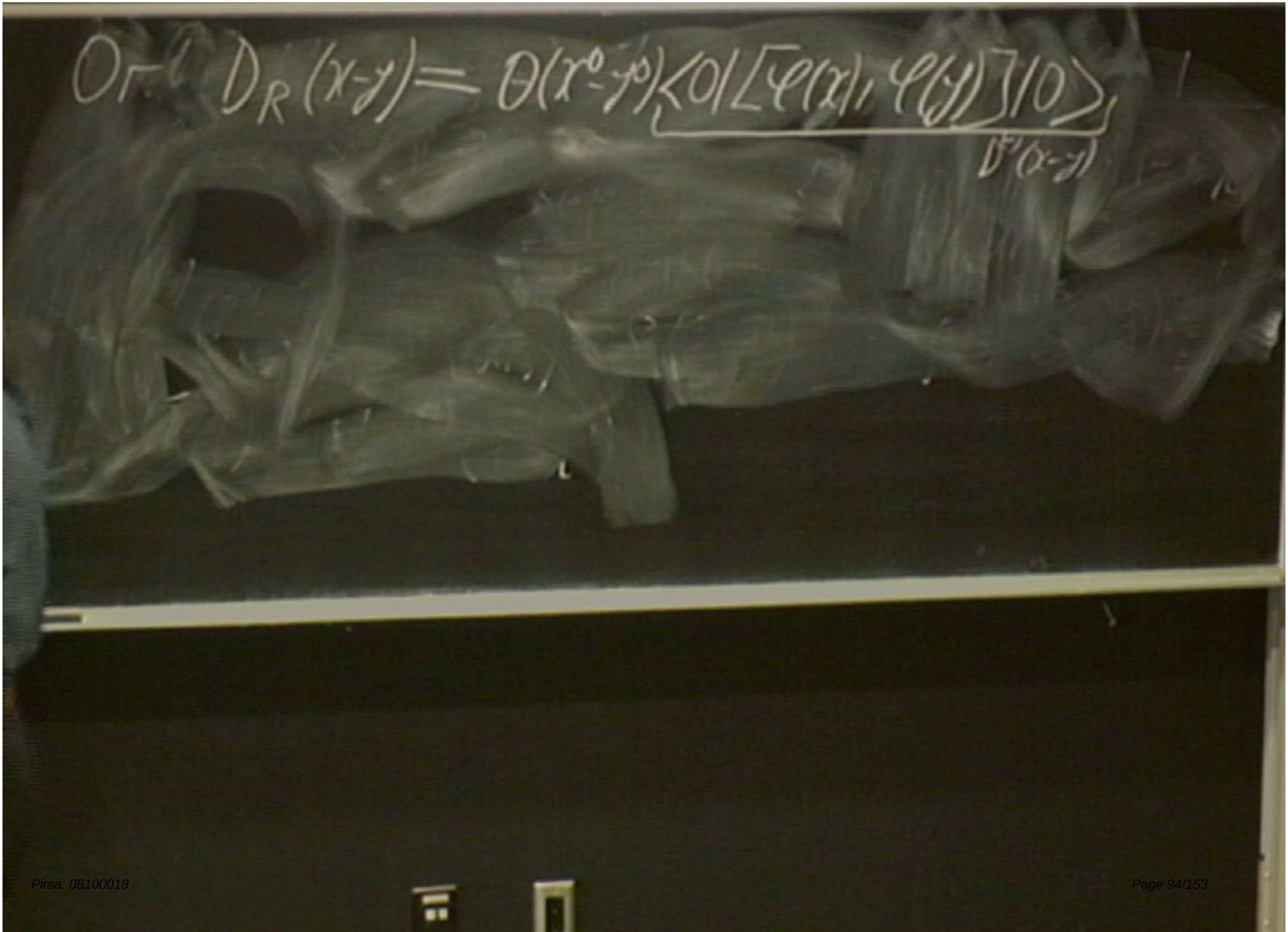

use
Residue Theorem
∫ d³p/(2π) 1/(2E_p) [e^{-ip(x-y)} - e^{ip·(x-y)}] =
E_p = √(p²+m²)
= D(x-y) - D(y-x) = D^{(-)}(x-y)
<0|φ(x)φ(y)|0>
<0|φ(y)φ(x)|0>


use
Residue Theorem
∫ d³p/(2π) 1/2E_p [e^{-ip(x-y)} - e^{ip·(x-y)}] | p⁰=E_p=√(p²+m²) =
= D(x-y) -
= D^{(-)}(x-y) ⇒ D_R(x-y) =
<0|φ(x)
φ(y)φ(x)|0>
= Θ(x⁰-y⁰)

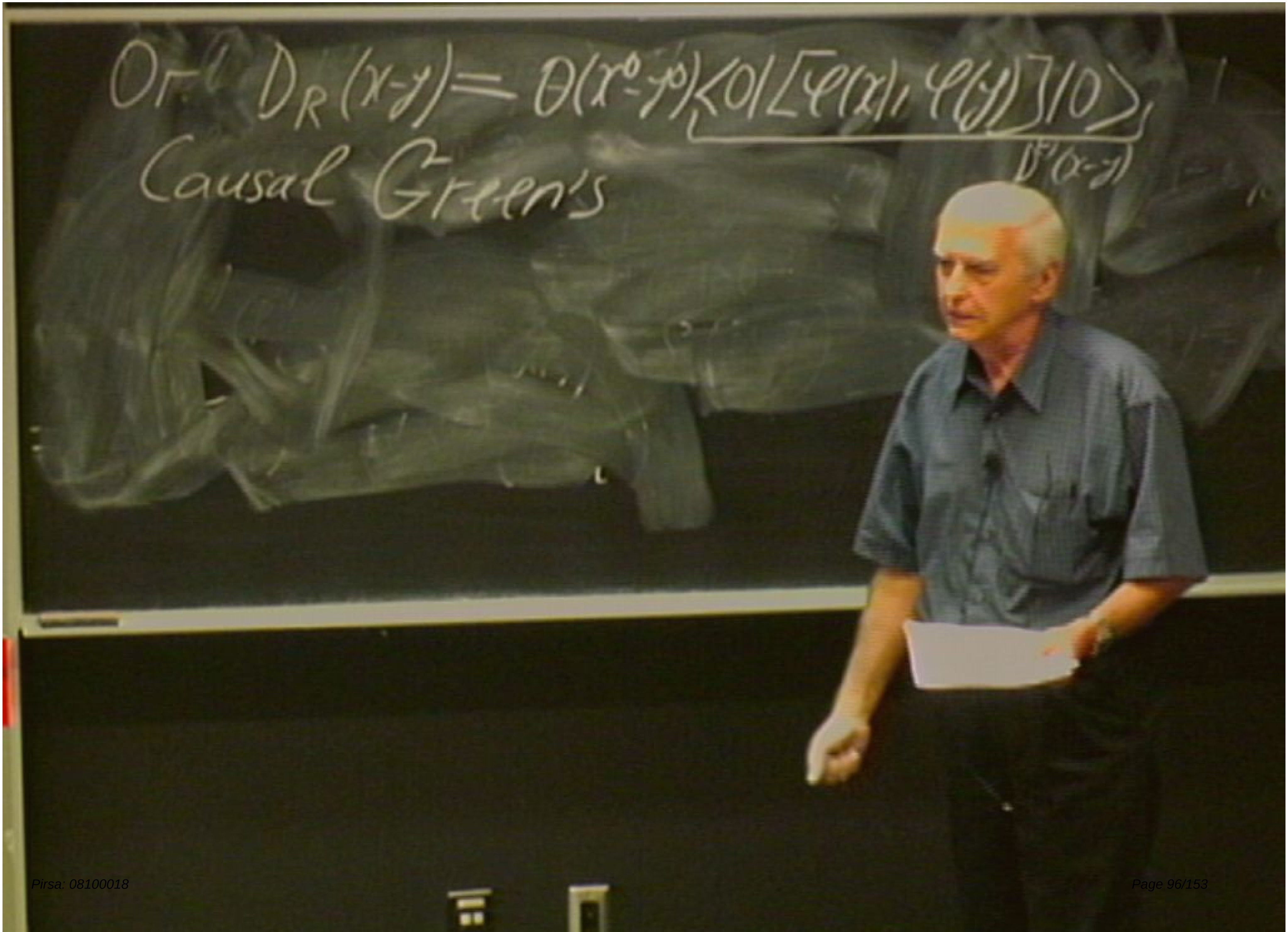

use
Residue Theorem
− D(y−x) = D(−)(x−y) ⇒ DR(x−y) =
= θ(x0−y0) D(−)(x−y).


use
Residue
Theorem


Or: D_R(x-y) = θ(x^0-y^0)<0|[φ(x), φ(y)]|0>


Or: D_R(x-y) = θ(x⁰-y⁰)<0|[φ(x), φ(y)]|0>
D'(x-y)

Or: $$D_R(x-y) = \theta(x^0-y^0)\underbrace{\langle 0|[\varphi(x),\varphi(y)]|0\rangle}_{D'(x-y)}$$


Or: D_R(x-y) = θ(x⁰-y⁰)<0|[φ(x), φ(y)]|0>
Causal Green's

Or: $D_R(x-y) = \theta(x^0-y^0)\underbrace{\langle 0|[\varphi(x),\varphi(y)]|0\rangle}_{D'(x-y)}$

Causal Green's function (or Feynman Propagator)

$D_F(x-y)$

prescriptions lead to different Green's functions

Green's functions in KG Theory

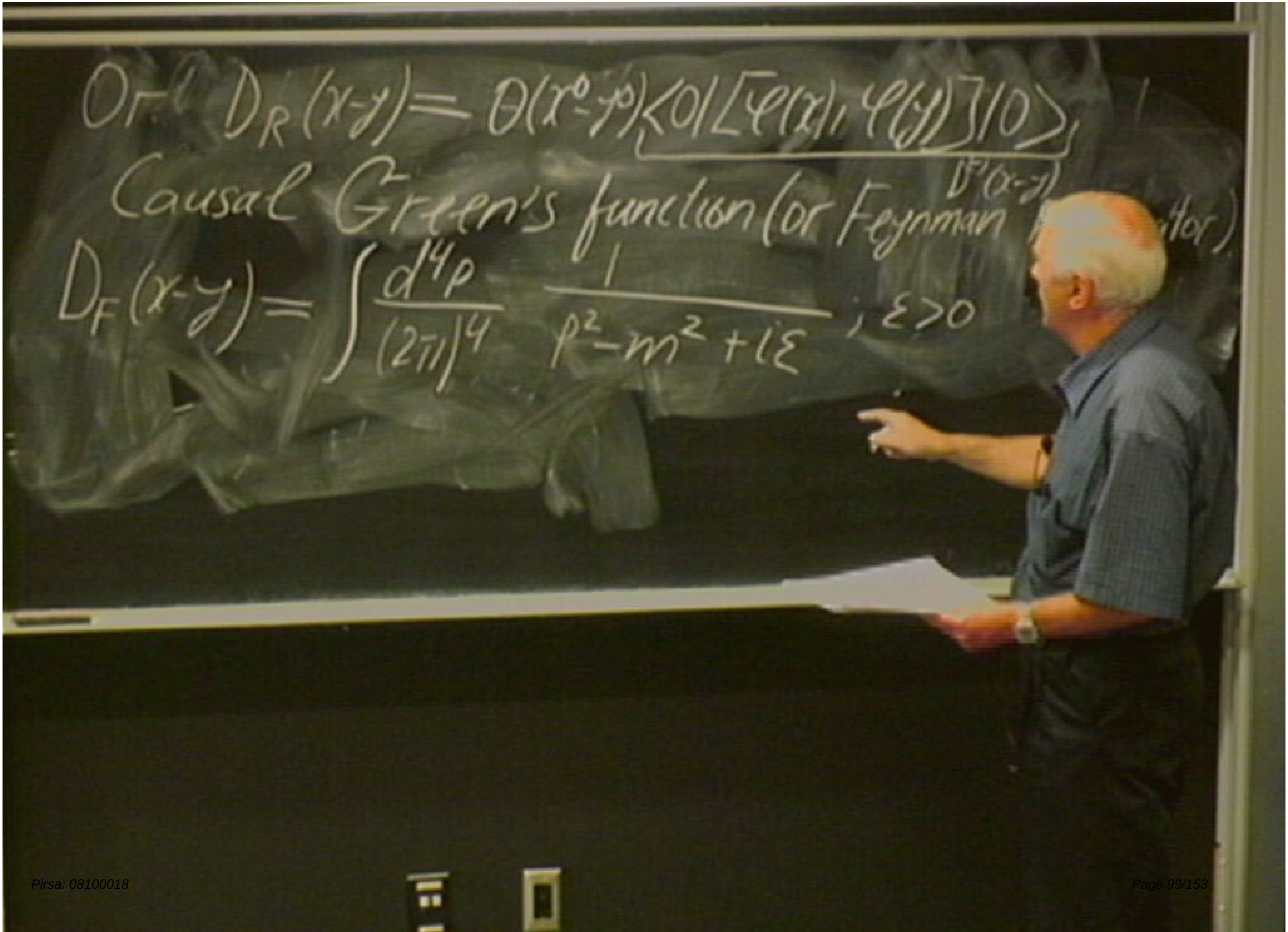
Green's functions satisfy the following equation:

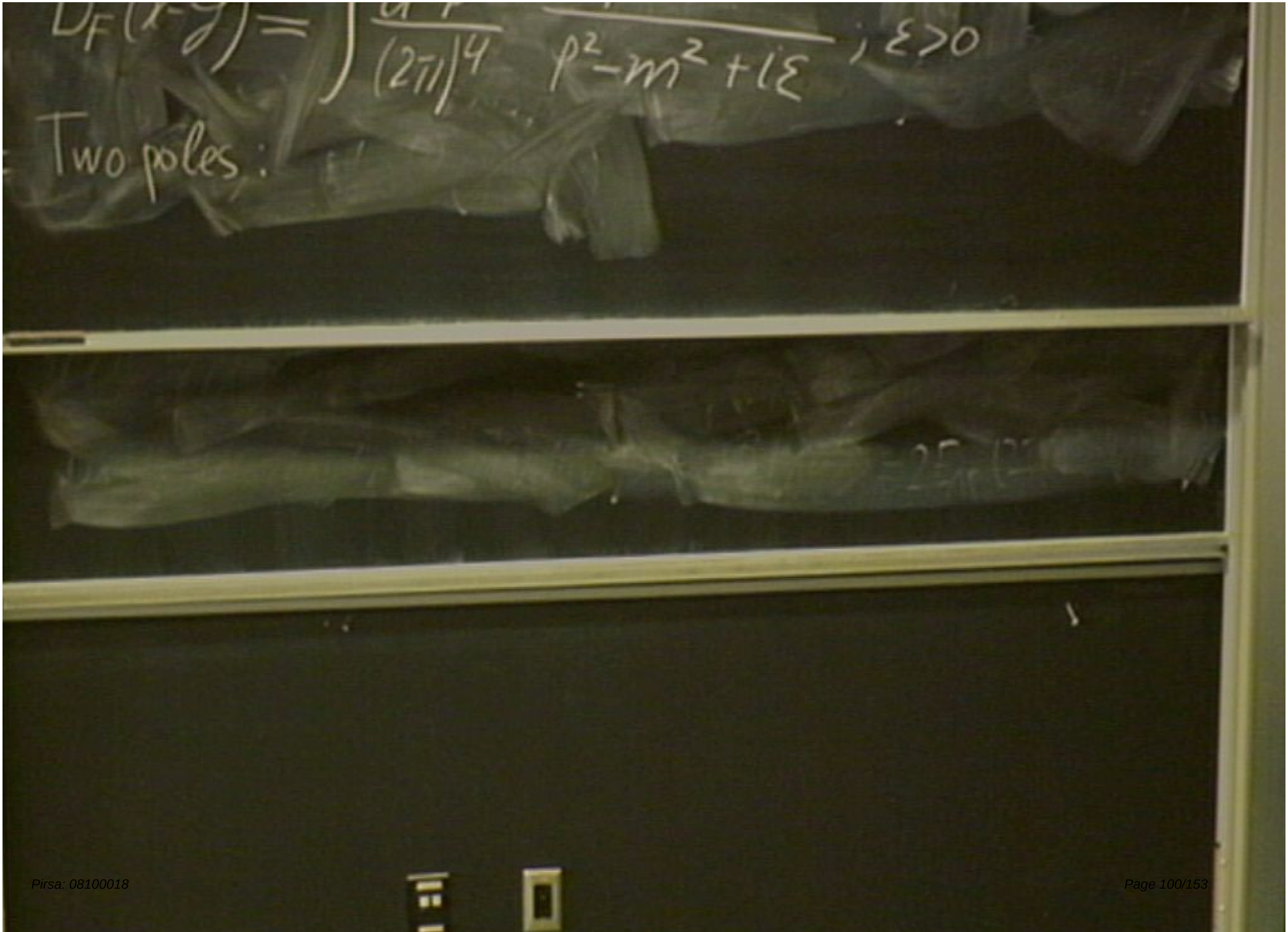
$$(\partial^\mu\partial_\mu + m^2)\, D_G(x-y) = -i\,\delta^{(4)}(x-y)$$

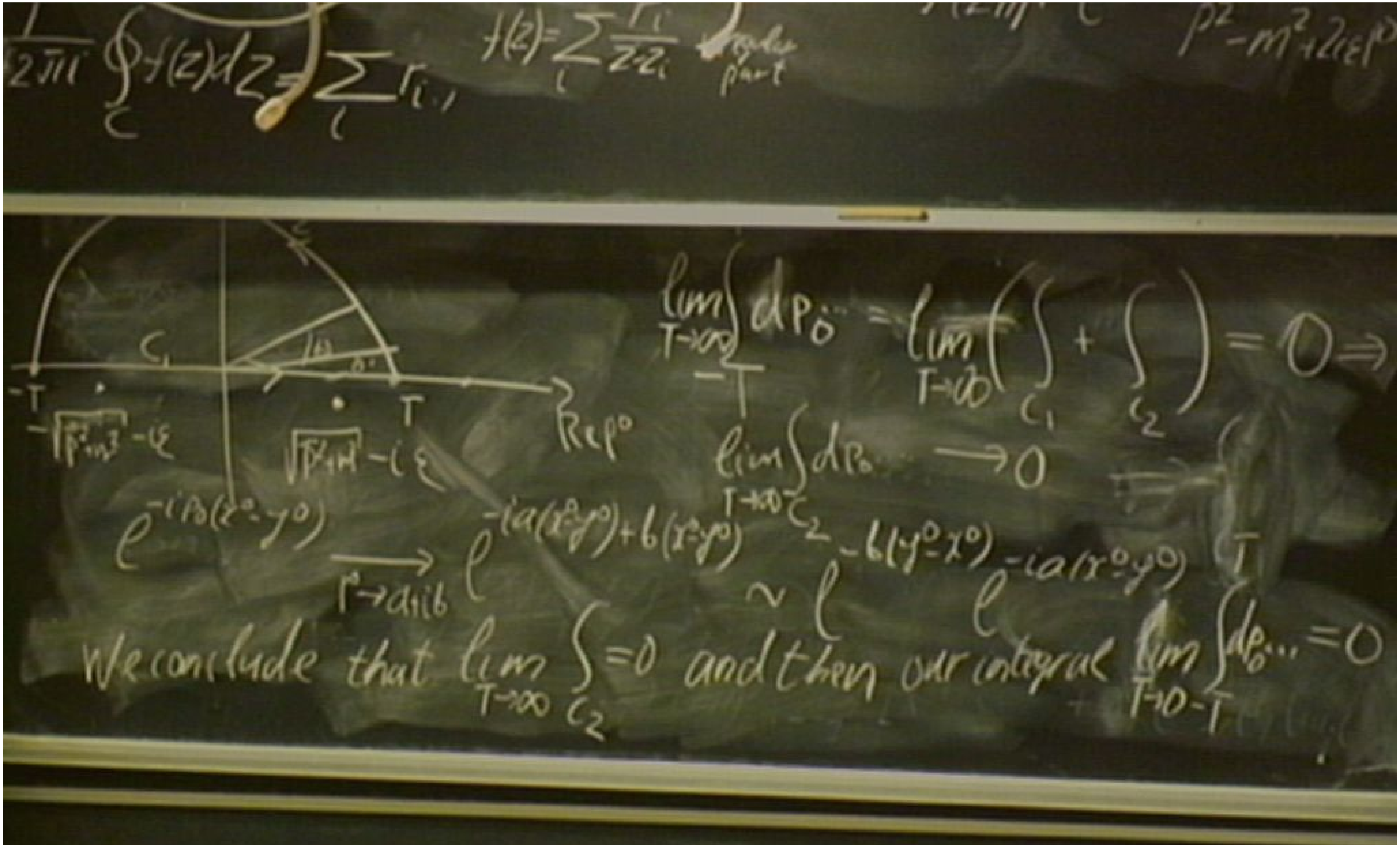
Take its Fourier transform: $D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}\, \tilde{D}_G(p)$

Substitute it into equation:

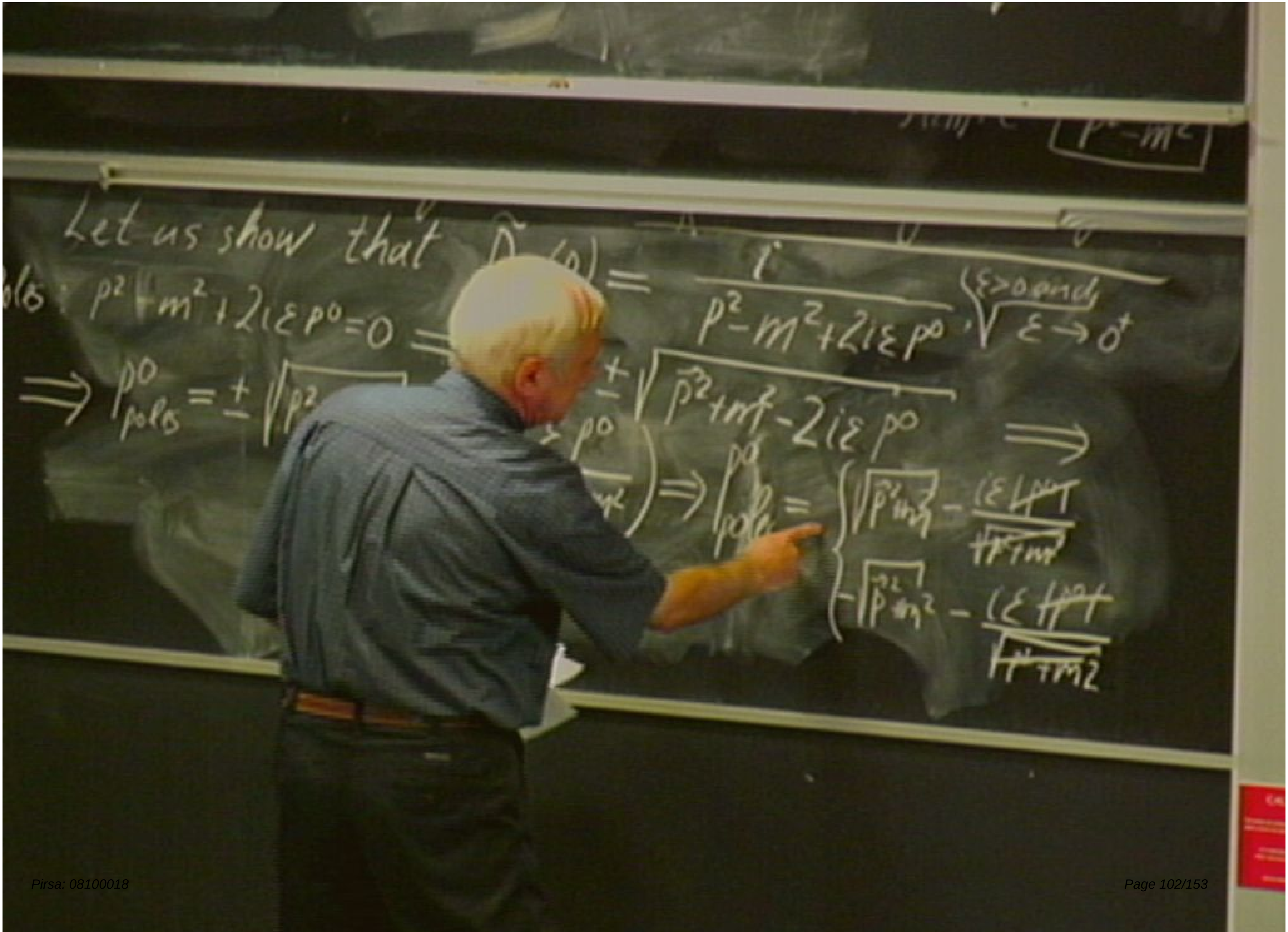
$$(-p^2+m^2)\,\tilde{D}_G(p) = -i \Rightarrow D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2-m^2}$$


Or: $D_R(x-y) = \theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$
$D'(x-y)$
Causal Green's function (or Feynman
$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2-m^2+i\varepsilon}$; $\varepsilon > 0$


; ε>0
Two poles :



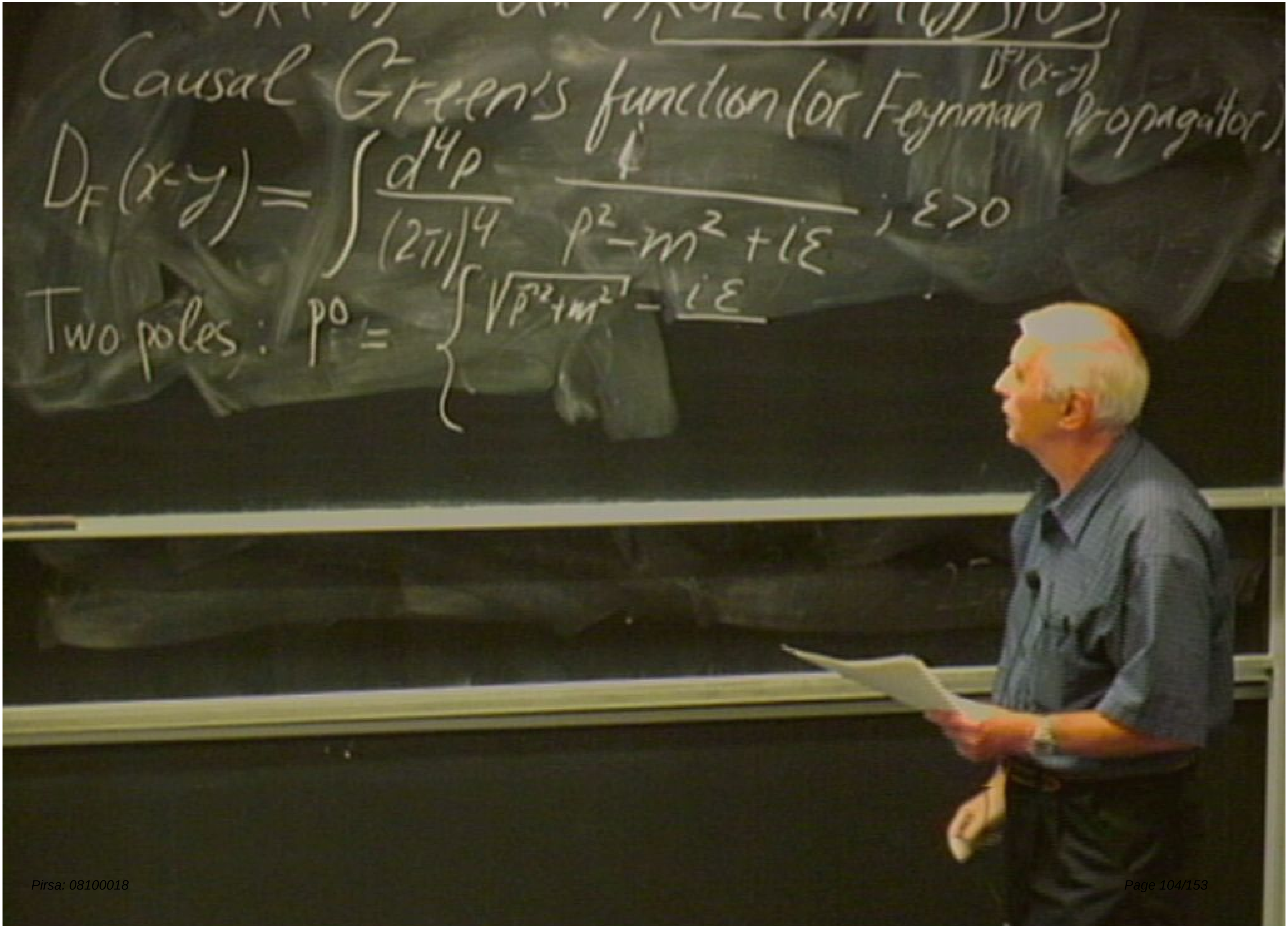
$\frac{1}{2\pi i}\oint_C f(z)\,dz = \sum_i r_i$
$f(z) = \sum_i \frac{r_i}{z - z_i}$ + regular part
$p^2 - m^2 + 2i\epsilon p^0$
C_1
$-T$
T
$-\sqrt{\vec{p}^2+m^2} - i\epsilon$
$\sqrt{\vec{p}^2+m^2} - i\epsilon$
$\mathrm{Re}\, p^0$
$\lim_{T\to\infty}\int_{-T}^{T} dp_0 \cdots = \lim_{T\to\infty}\left(\int_{C_1} + \int_{C_2}\right) = 0 \Rightarrow$
$\lim_{T\to\infty}\int_{C_2} dp_0 \cdots \longrightarrow 0$
$e^{-ip_0(x^0-y^0)} \xrightarrow{p^0\to a+ib} e^{-ia(x^0-y^0)+b(x^0-y^0)} \sim e^{-b(y^0-x^0)}\, e^{-ia(x^0-y^0)}$
We conclude that $\lim_{T\to\infty}\int_{C_2} = 0$ and then our integral $\lim_{T\to 0}\int_{-T}^{T} dp_0 \cdots = 0$


Let us show that
ε>0 and
ε→0+

Let us show that $\tilde{D}_R(p) = \dfrac{i}{p^2 - m^2 + 2i\varepsilon p^0}$, $\left\{ \varepsilon > 0 \text{ and } \varepsilon \to 0^+ \right.$

Poles: $p^2 - m^2 + 2i\varepsilon p^0 = 0 \Longrightarrow p^0 = \pm\sqrt{\vec{p}^2 + m^2 - 2i\varepsilon p^0} \Longrightarrow$

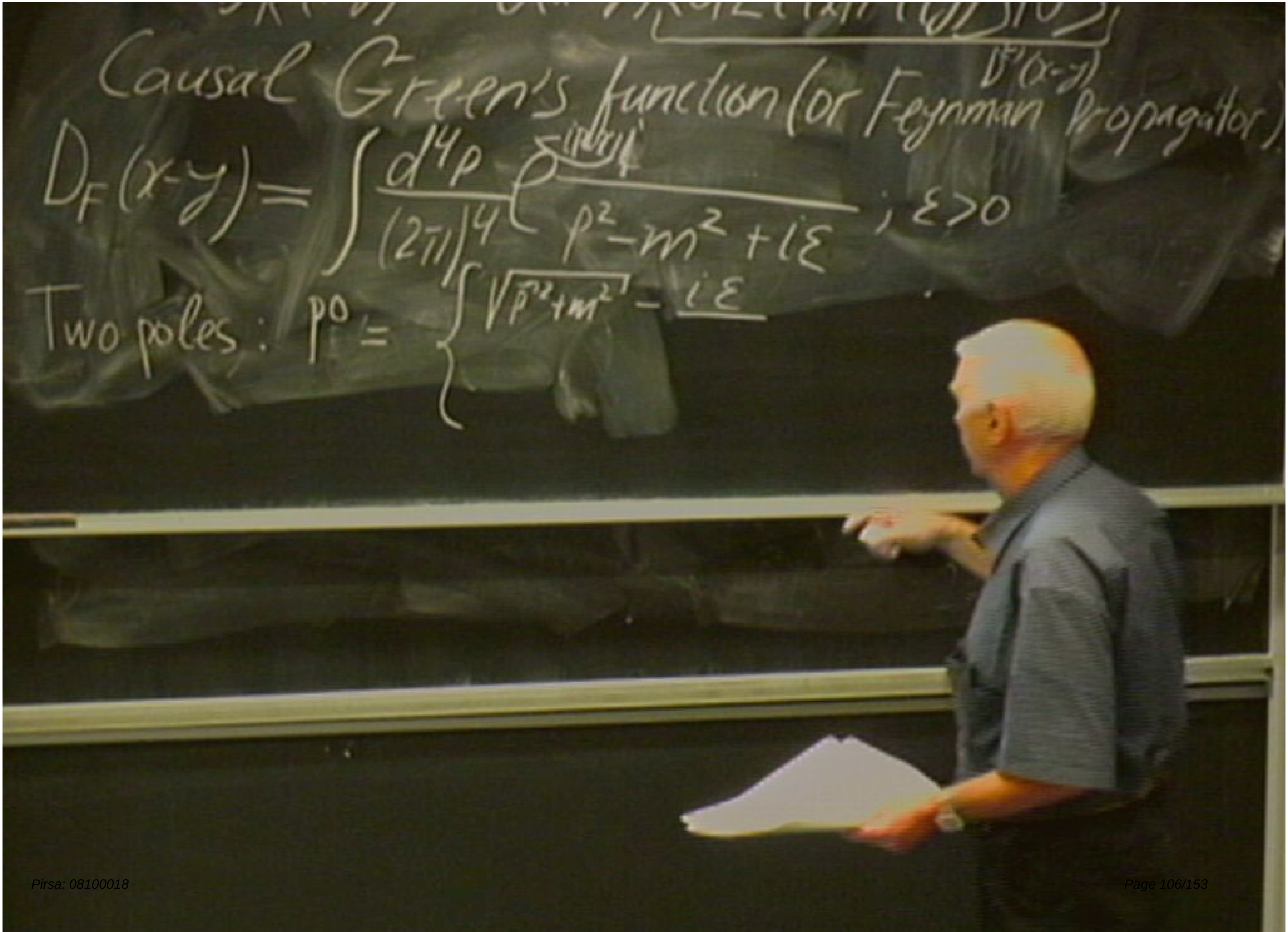
$$\Longrightarrow p^0_{poles} = \pm\sqrt{\vec{p}^2 + m^2}\left(1 - \frac{i\varepsilon p^0}{\vec{p}^2 + m^2}\right) \Longrightarrow p^0_{poles} = \begin{cases} \sqrt{\vec{p}^2 + m^2} - \dfrac{i\varepsilon |p^0|}{\sqrt{\vec{p}^2 + m^2}} \\ -\sqrt{\vec{p}^2 + m^2} - \dfrac{i\varepsilon |p^0|}{\sqrt{\vec{p}^2 + m^2}} \end{cases}$$


Causal Green's function (or Feynman Propagator)
$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon}$; $\varepsilon>0$
Two poles: $p^0 = \sqrt{\vec{p}^2+m^2} - i\varepsilon$

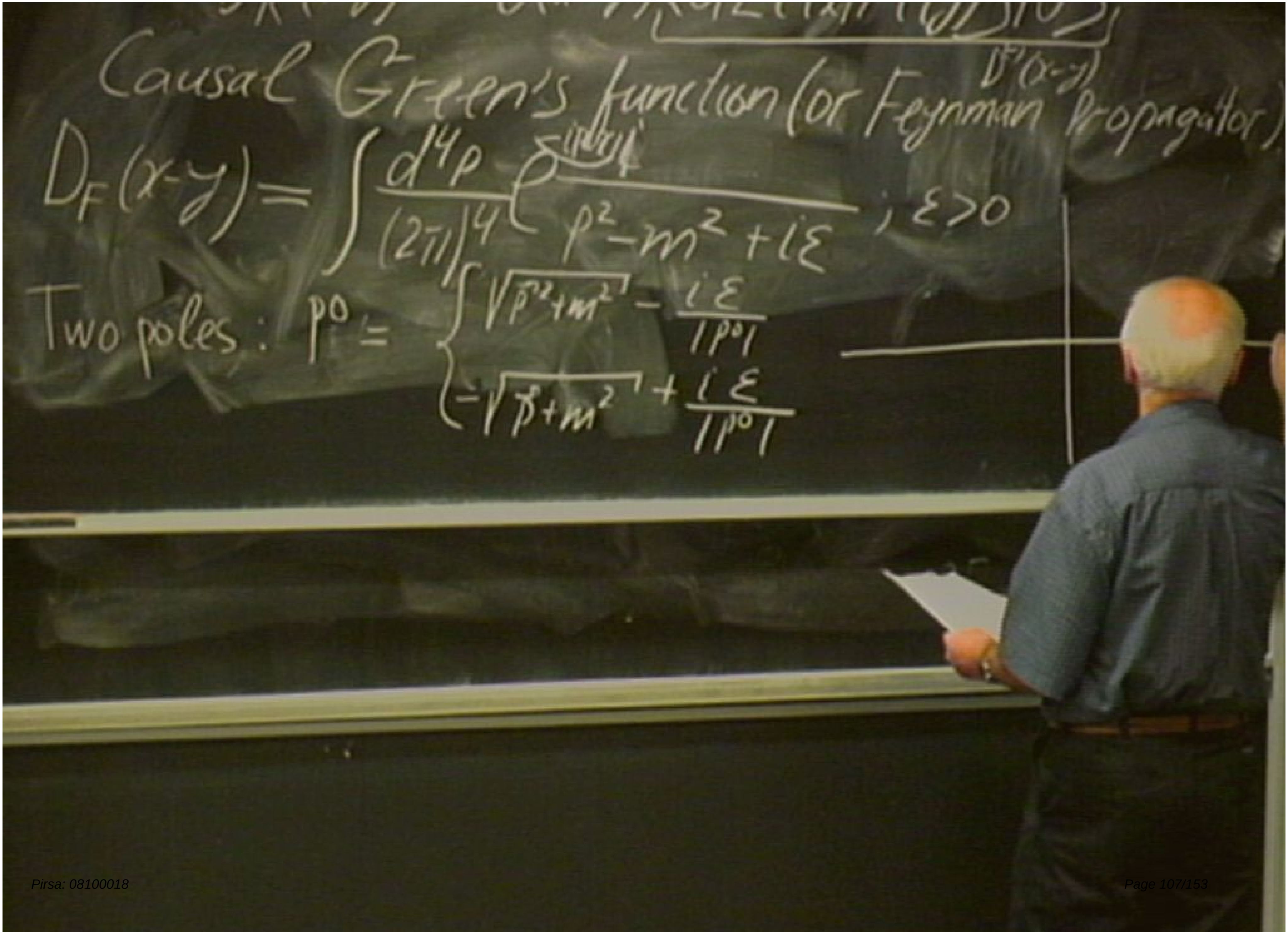
Causal Green's function (or Feynman Propagator)

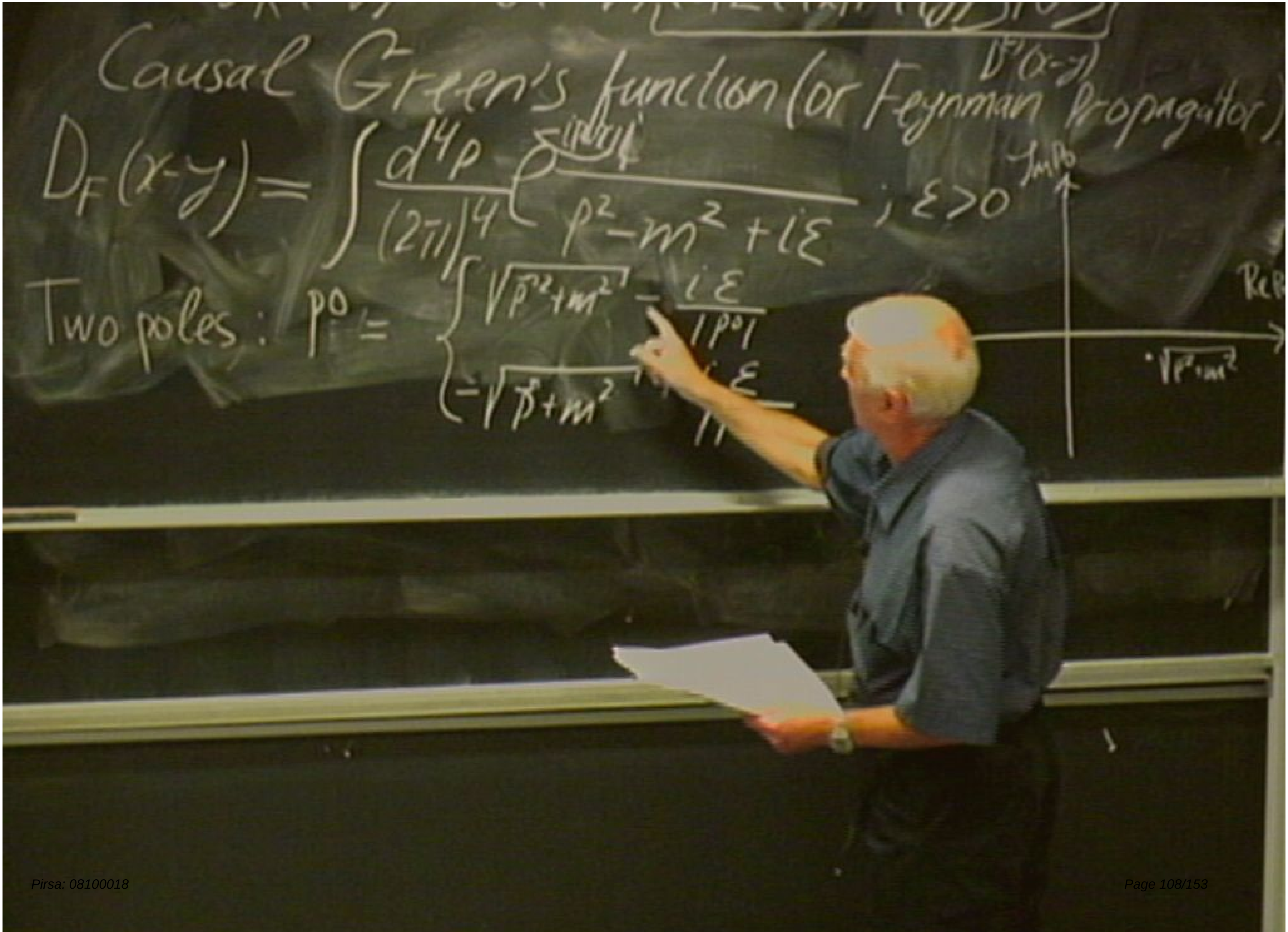
$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} e \frac{i}{p^2 - m^2 + i\varepsilon} \; ; \; \varepsilon > 0$$

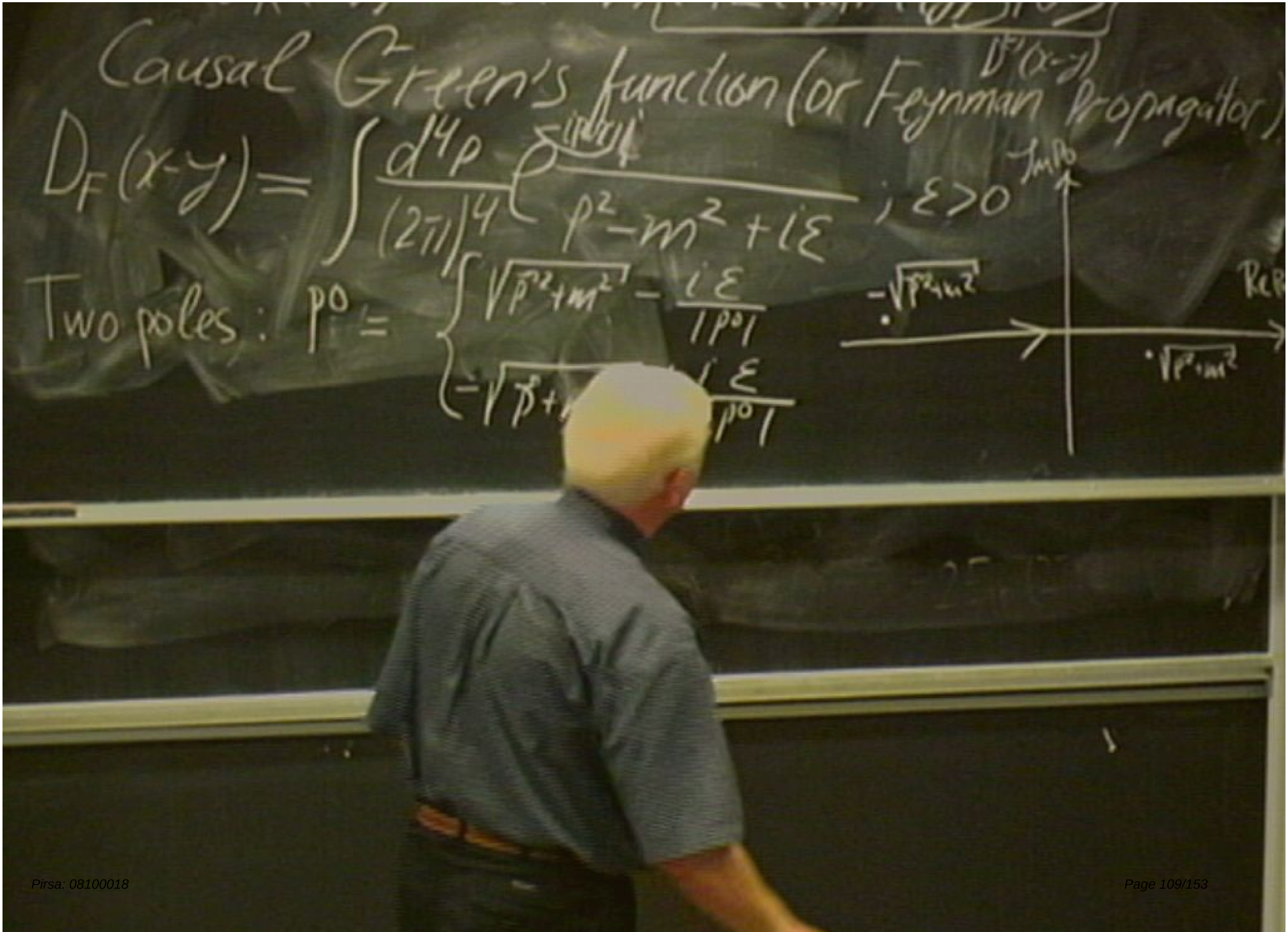
Two poles: $p^0 = \begin{cases} \sqrt{\vec{p}^2 + m^2} - i\varepsilon \\ \end{cases}$



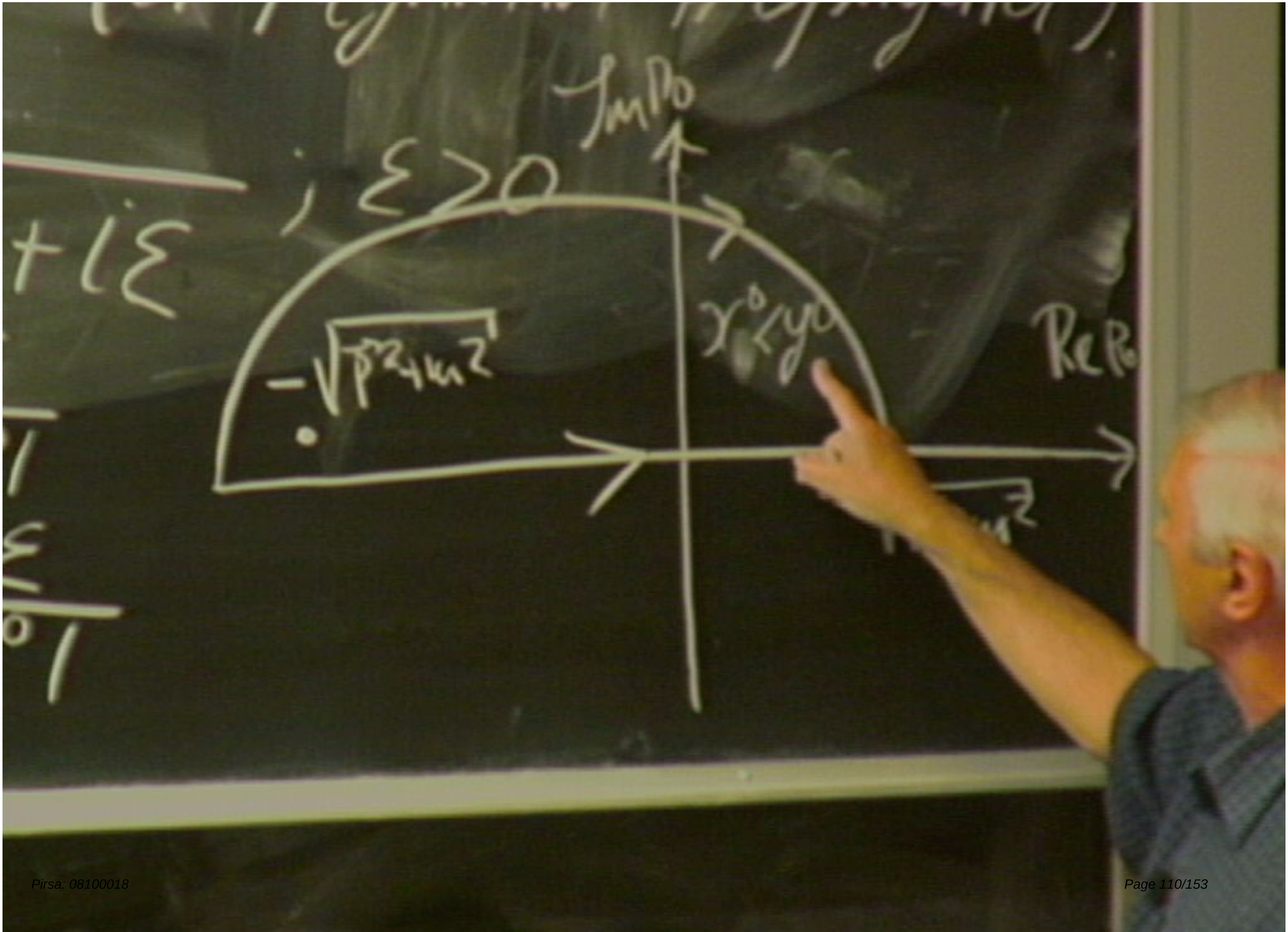
Causal Green's function (or Feynman Propagator)
$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{\dots}}{p^2-m^2+i\varepsilon}$; $\varepsilon > 0$
Two poles: $p^0 = \sqrt{\vec{p}^2+m^2} - i\varepsilon$

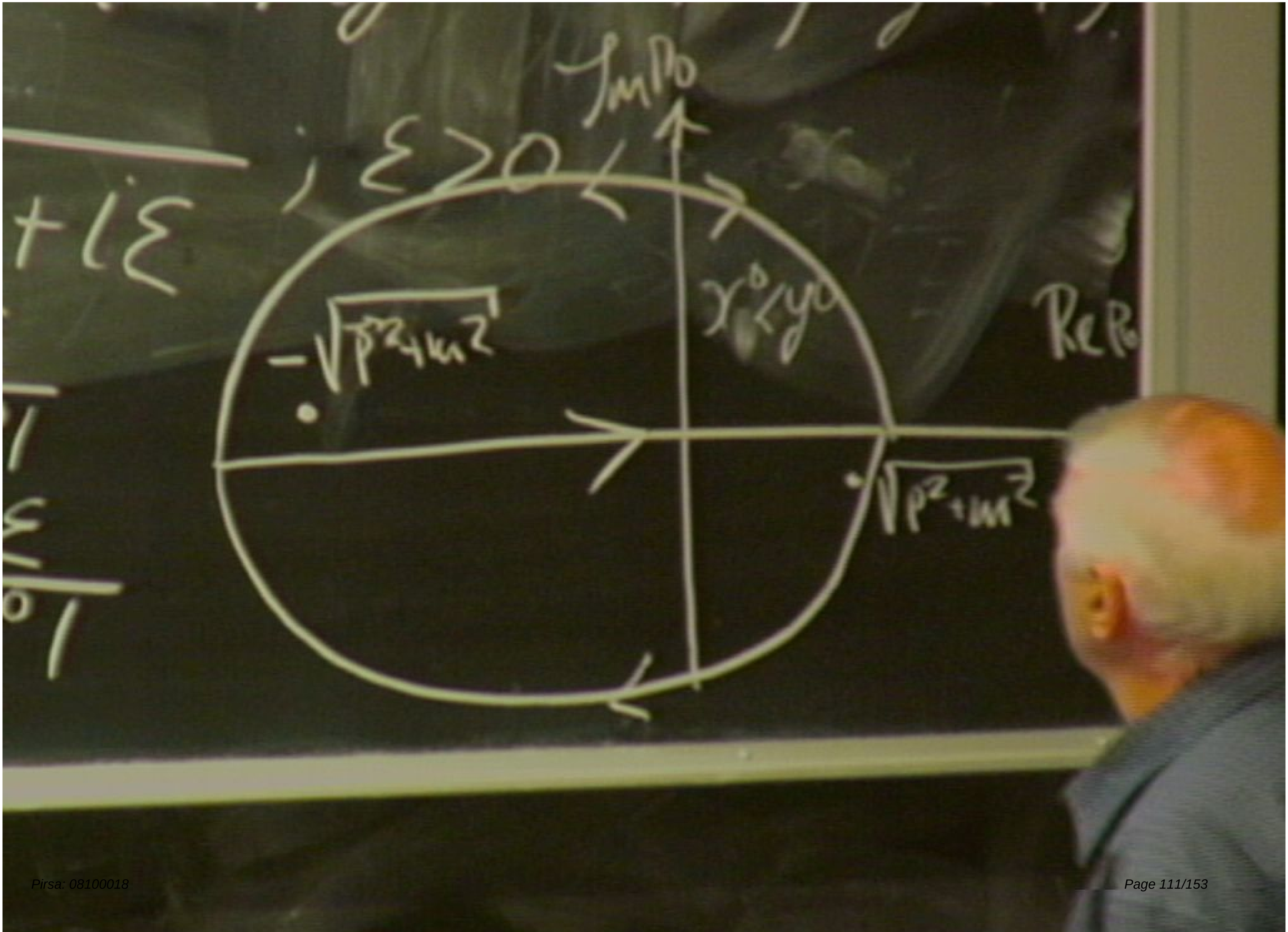

Causal Green's function (or Feynman Propagator)
$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2 - m^2 + i\epsilon}$; $\epsilon > 0$
Two poles: $p^0 = \begin{cases} \sqrt{\vec{p}^2+m^2} - \frac{i\epsilon}{|p^0|} \\ -\sqrt{\vec{p}^2+m^2} + \frac{i\epsilon}{|p^0|} \end{cases}$

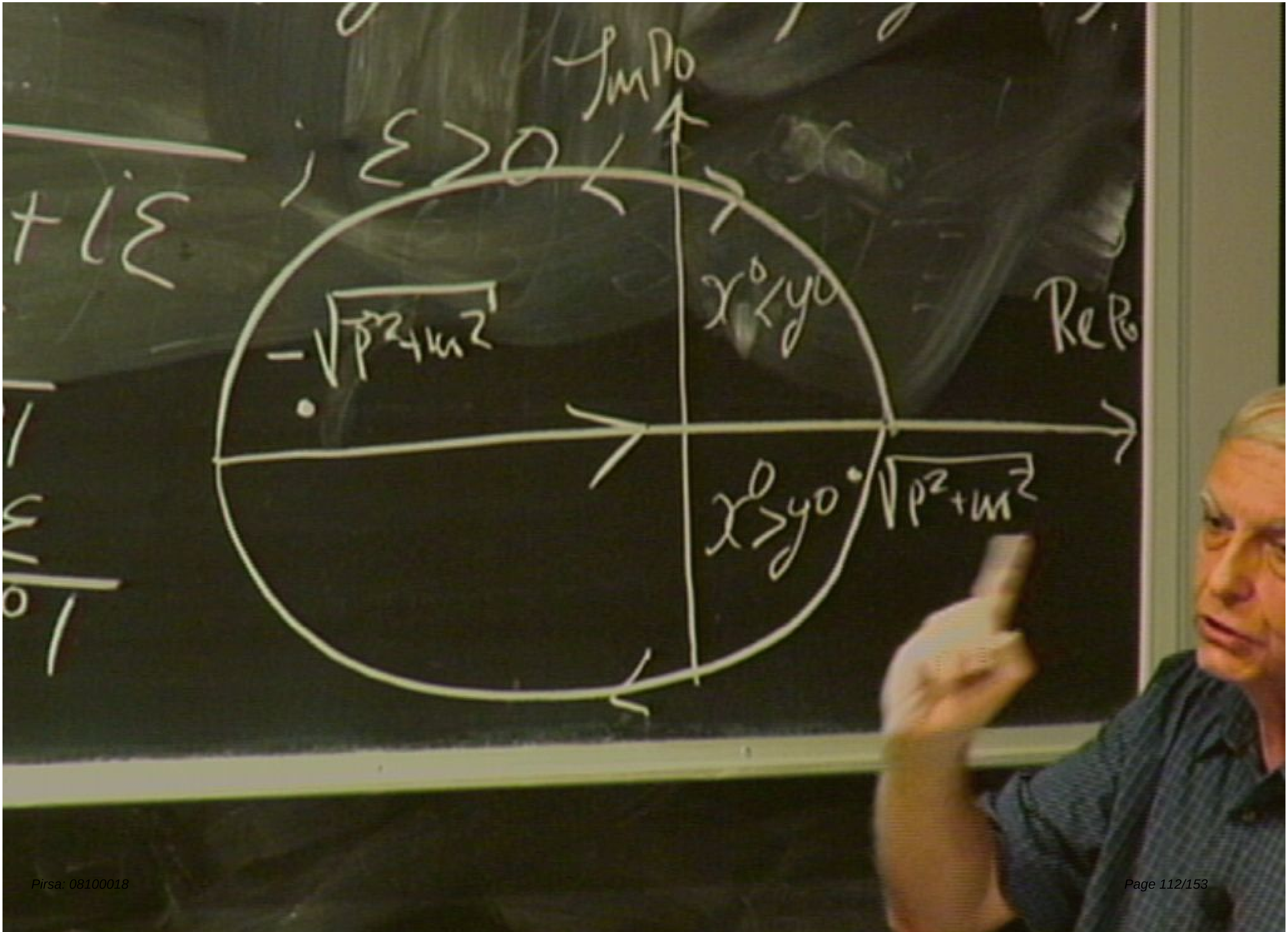

Causal Green's function (or Feynman Propagator)
Two poles:
ε>0
Re p

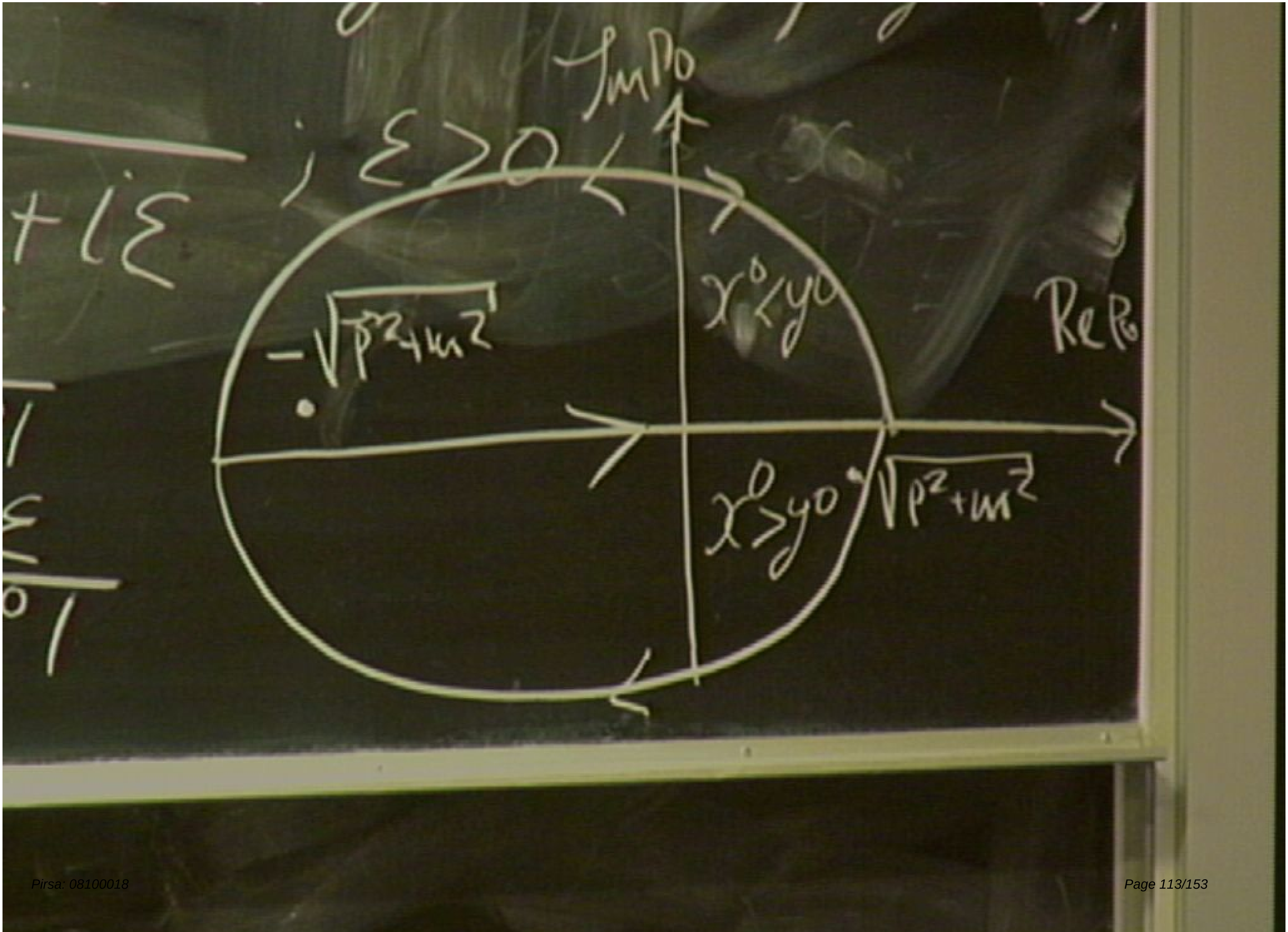


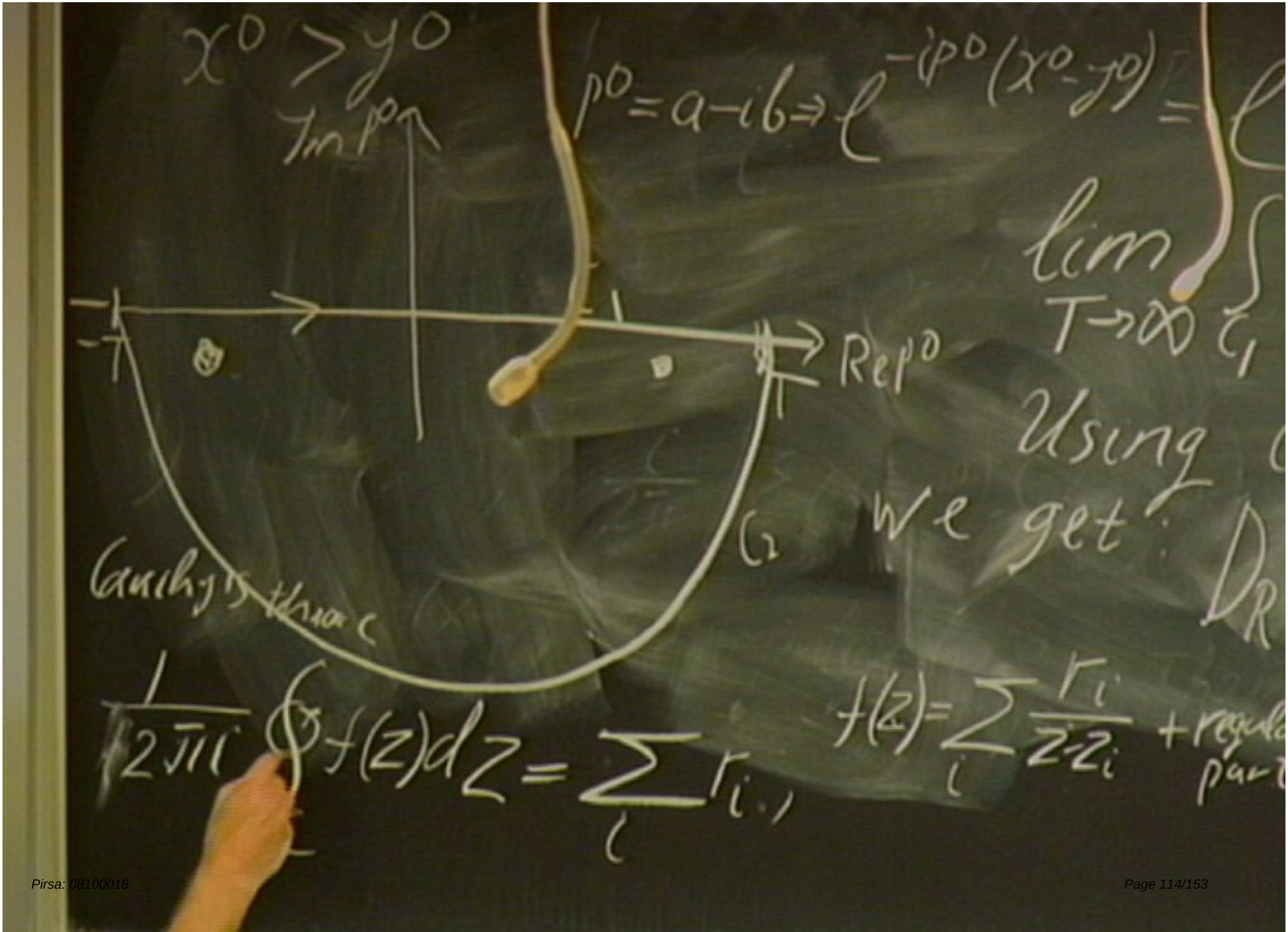
Causal Green's function (or Feynman Propagator)
$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2-m^2+i\epsilon}$; $\epsilon>0$
Two poles: $p^0 = \begin{cases} \sqrt{\vec{p}^2+m^2} - \frac{i\epsilon}{|p^0|} \\ -\sqrt{\vec{p}^2+\ldots} + \frac{i\epsilon}{|p^0|} \end{cases}$
Im p_0
Re p
$-\sqrt{\vec{p}^2+m^2}$
$\sqrt{\vec{p}^2+m^2}$

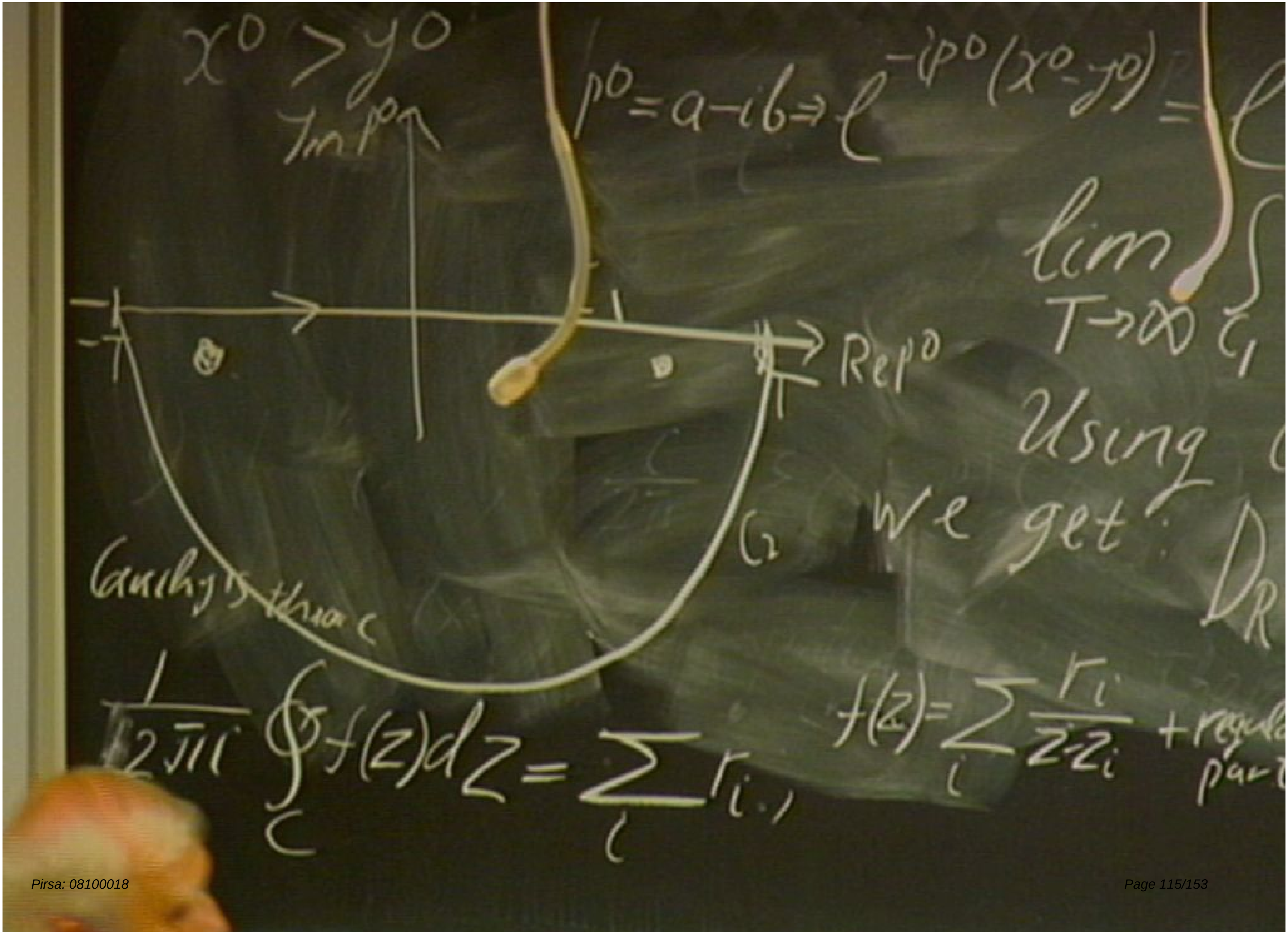

Im p0
; ε > 0
+iε
−√(p²+m²)
x⁰ < y⁰
Re p0

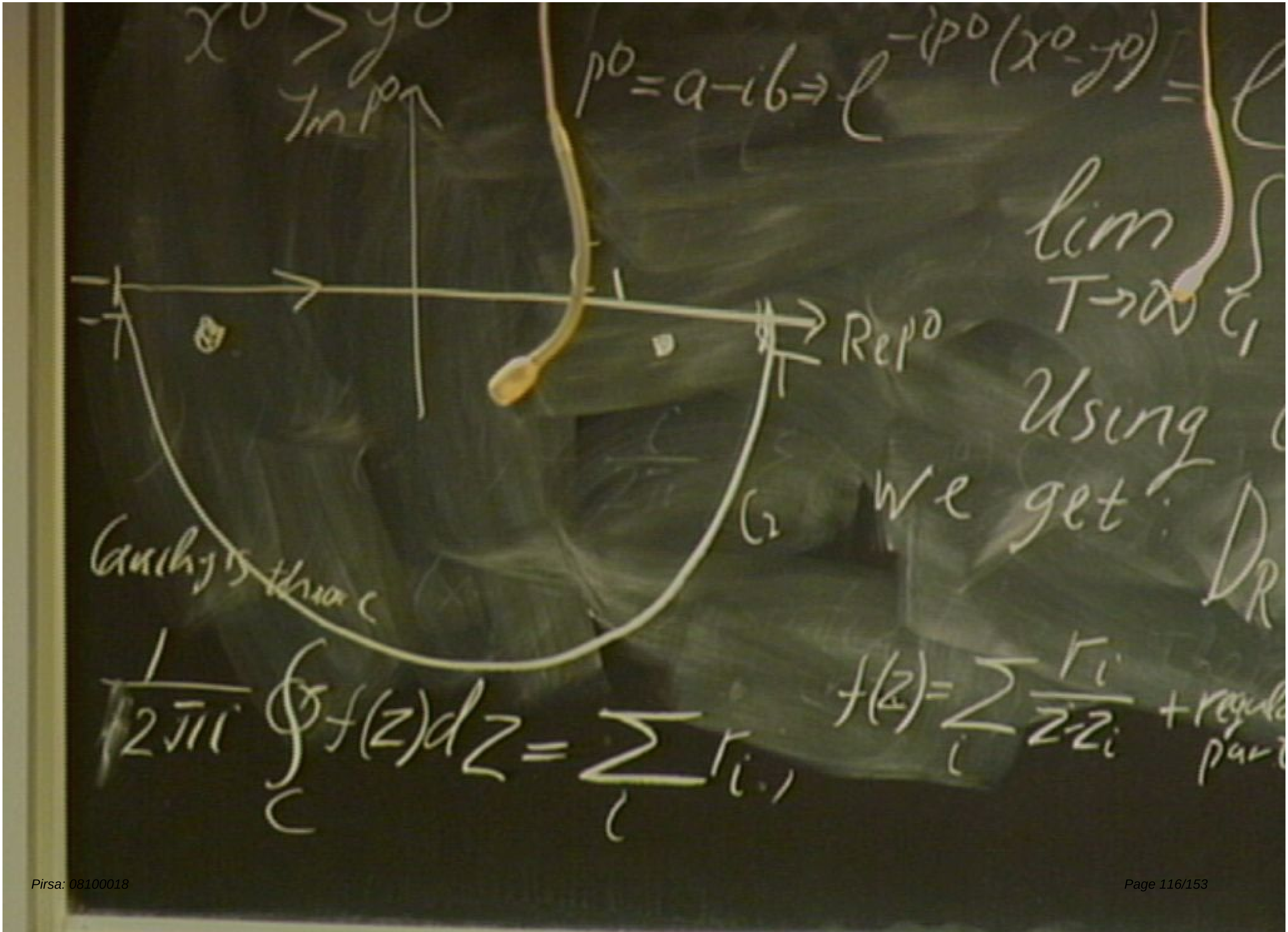

+iε
ε>0
Im p0
Re p0
x⁰<y⁰
−√(p²+m²)
√(p²+m²)

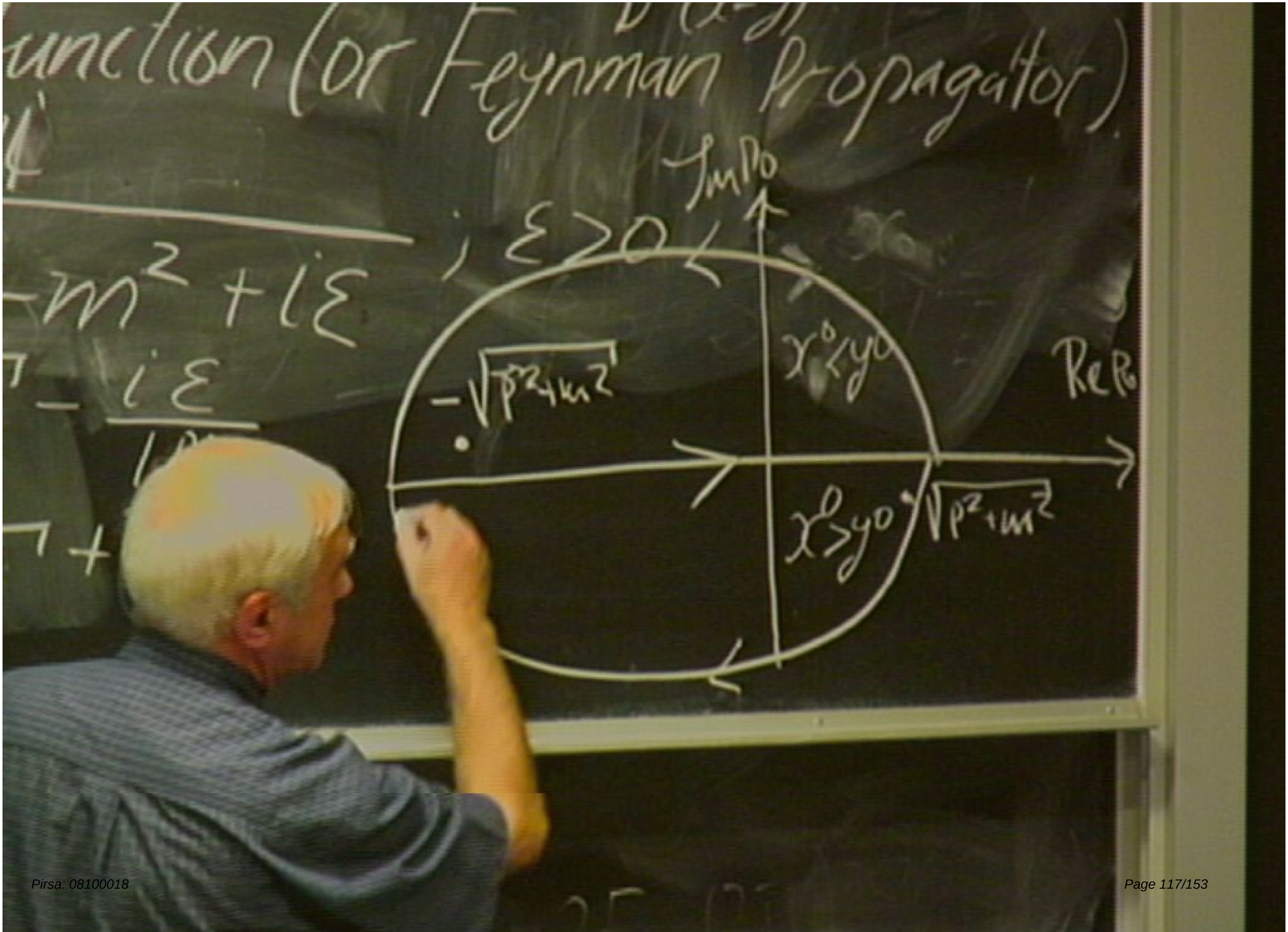

Im p0
; ε > 0
+iε
−√(p²+m²)
x0<y0
Re p0
x0>y0
√(p²+m²)

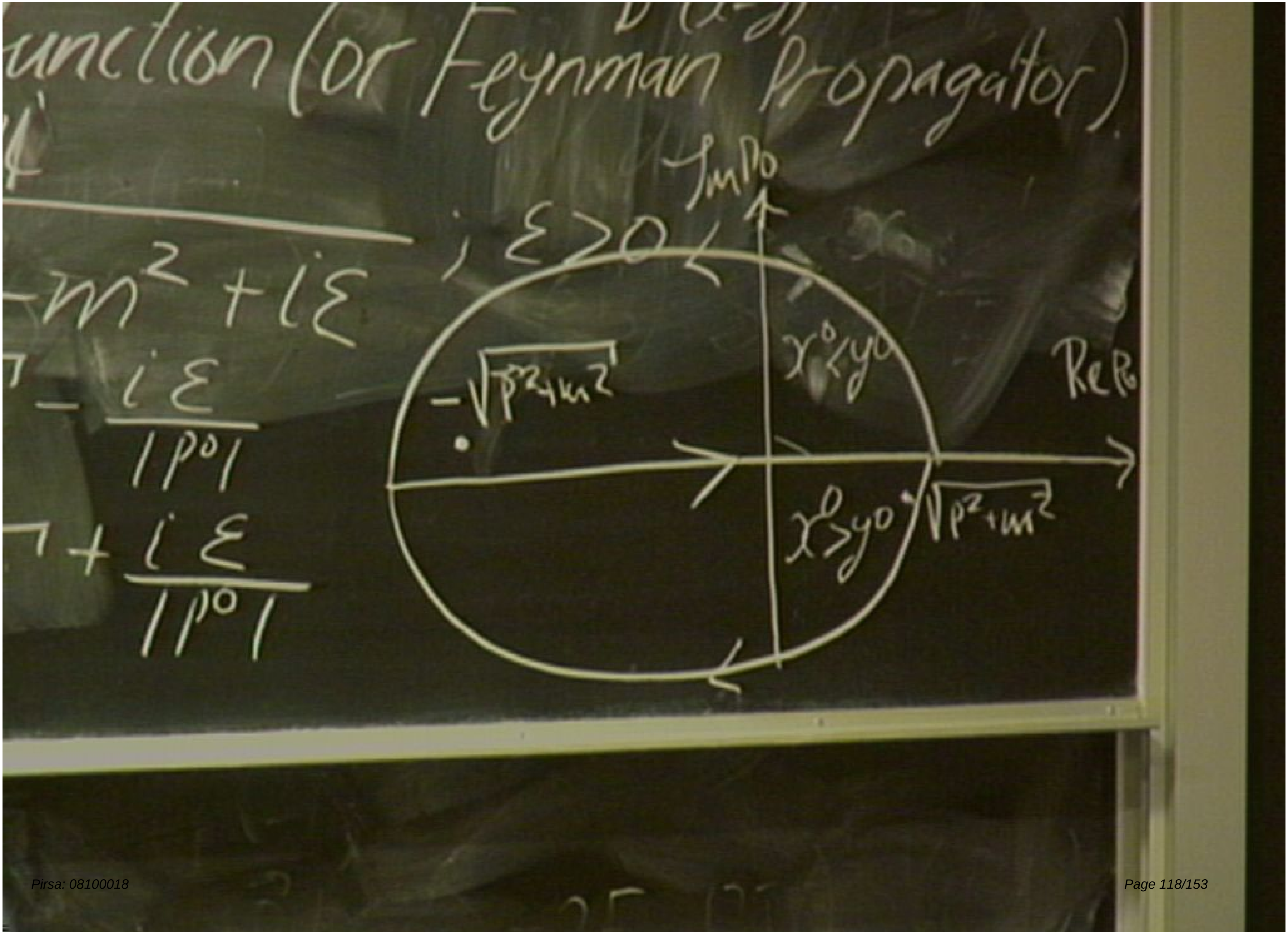

Im p0
ε > 0
+iε
−√(p²+m²)
x⁰ < y⁰
Re p0
x⁰ > y⁰
√(p²+m²)

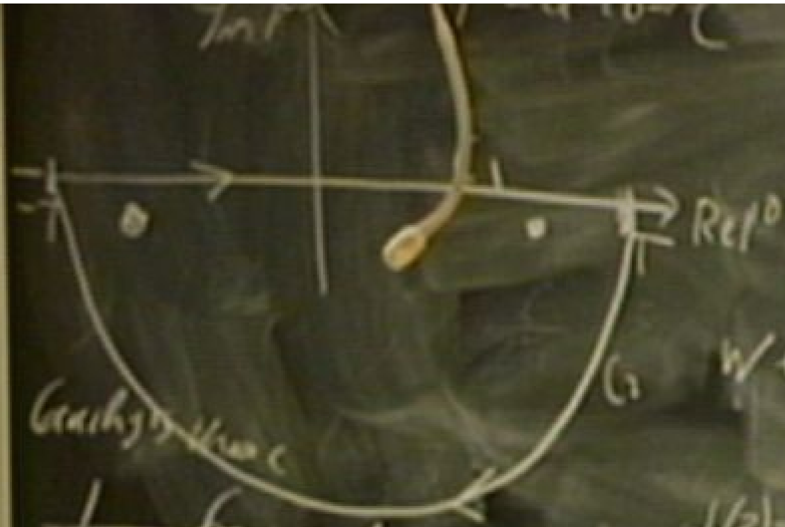

x0 > y0
Im p0
p0 = a - ib ⇒ e^{-ip0(x0-y0)} =
lim
T→∞ C1
Re p0
Using
we get: DR
C2
Cauchy's theorem
1/(2πi) ∮ f(z)dz = Σ_i r_i
f(z) = Σ_i r_i/(z-z_i) + regular part


$x^0 > y^0$
$p^0 = a - ib \Rightarrow e^{-ip^0(x^0-y^0)} =$
$\mathrm{Im}\, p^0$
$\mathrm{Re}\, p^0$
$\lim_{T\to\infty} \int_{C_1}$
Using
we get: D_R
C_2
$\frac{1}{2\pi i} \oint_C f(z)\,dz = \sum_i r_i$
$f(z) = \sum_i \frac{r_i}{z - z_i}$ + regul
part


$x^0 > y^0$
$p^0 = a - ib \Rightarrow e^{-ip^0(x^0-y^0)} =$
Im p^0
Re p^0
C_2
lim
$T \to \infty$ C_1
Using
we get: D_R
Cauchy's theorem
$\frac{1}{2\pi i}\oint_C f(z)dz = \sum_i r_i$
$f(z) = \sum_i \frac{r_i}{z - z_i}$ + regular part


unction (or Feynman Propagator)
, ε>0 ↓
m² + iε
iε
Im p0
Re p0
-√(p²+m²)
√(p²+m²)
x⁰<y⁰
x⁰>y⁰


unction (or Feynman Propagator).
Im p0
, ε > 0
m² + iε
iε
|p0|
+ iε
|p0|
−√(p²+m²)
x0<y0
x0>y0
√(p²+m²)
Re p0



$$\lim_{T\to\infty}\int_{C_1} + \int_{C_2} = \lim_{T\to\infty}\int_{C_1} = \int_{-T}^{T} dp^0 \ldots$$

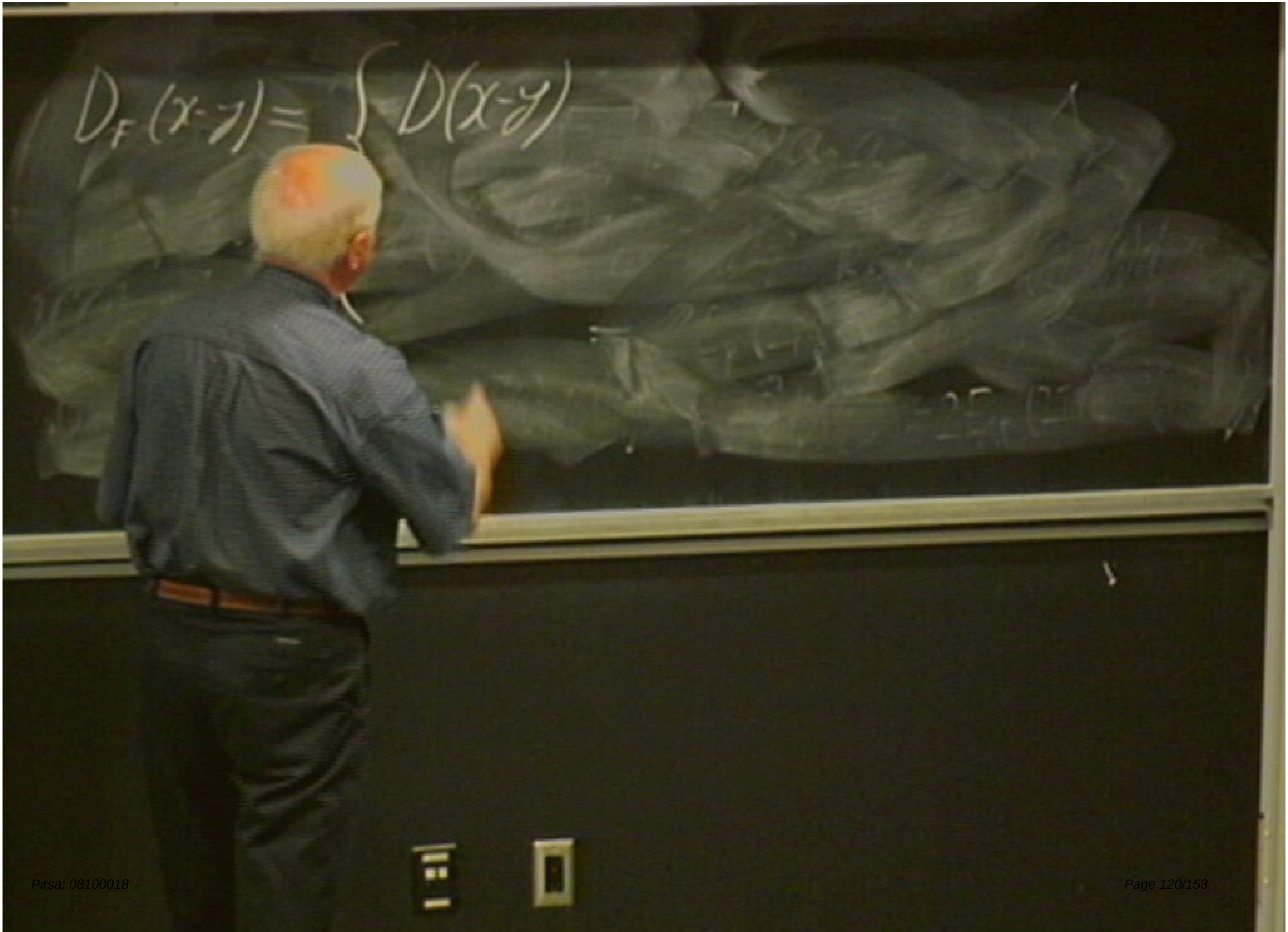
Using Cauchy's Residue Theorem, we get

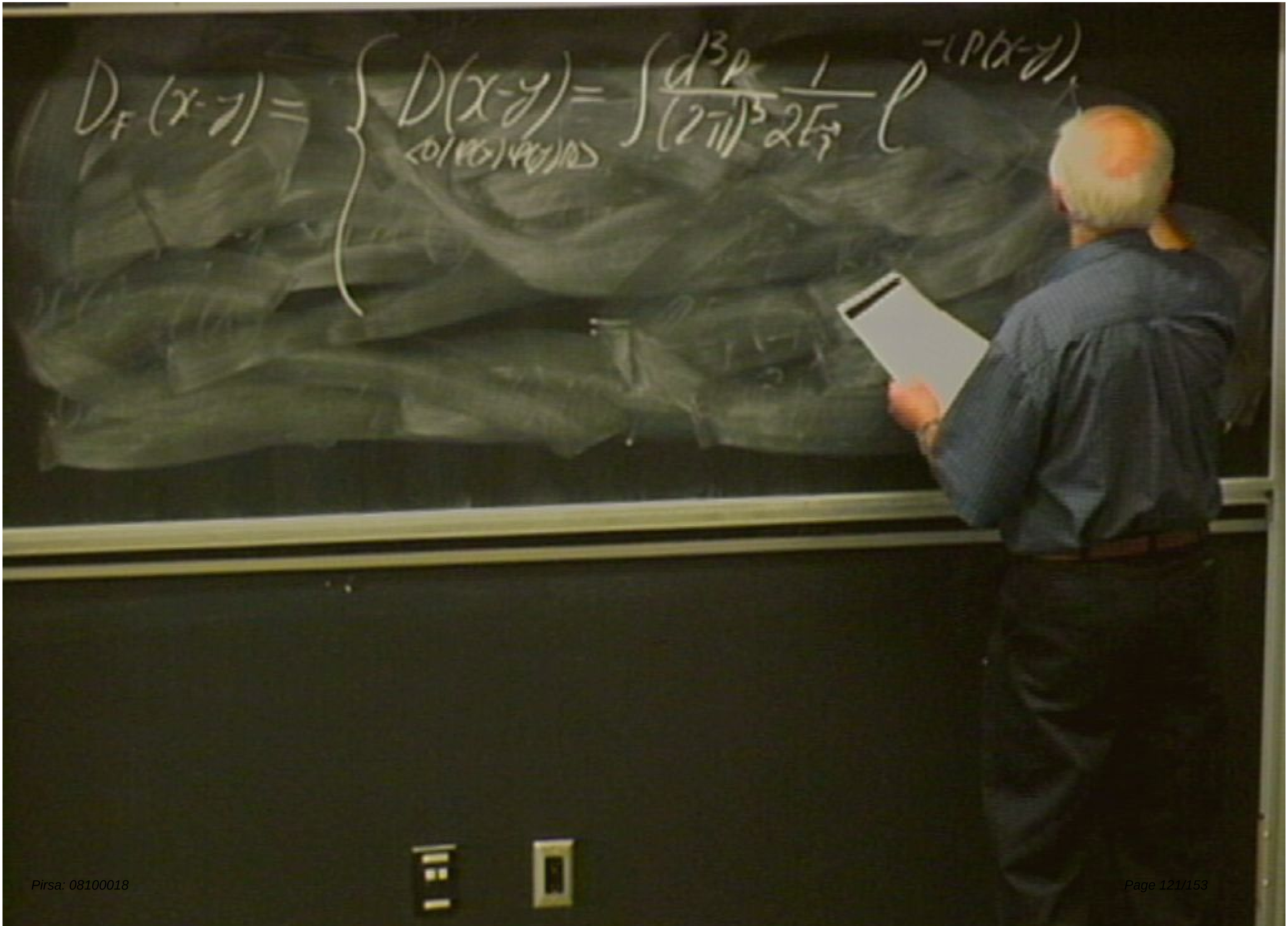
$$D_R(x-y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip(x-y)} \frac{i}{p^2-m^2+2i\epsilon p^0}$$

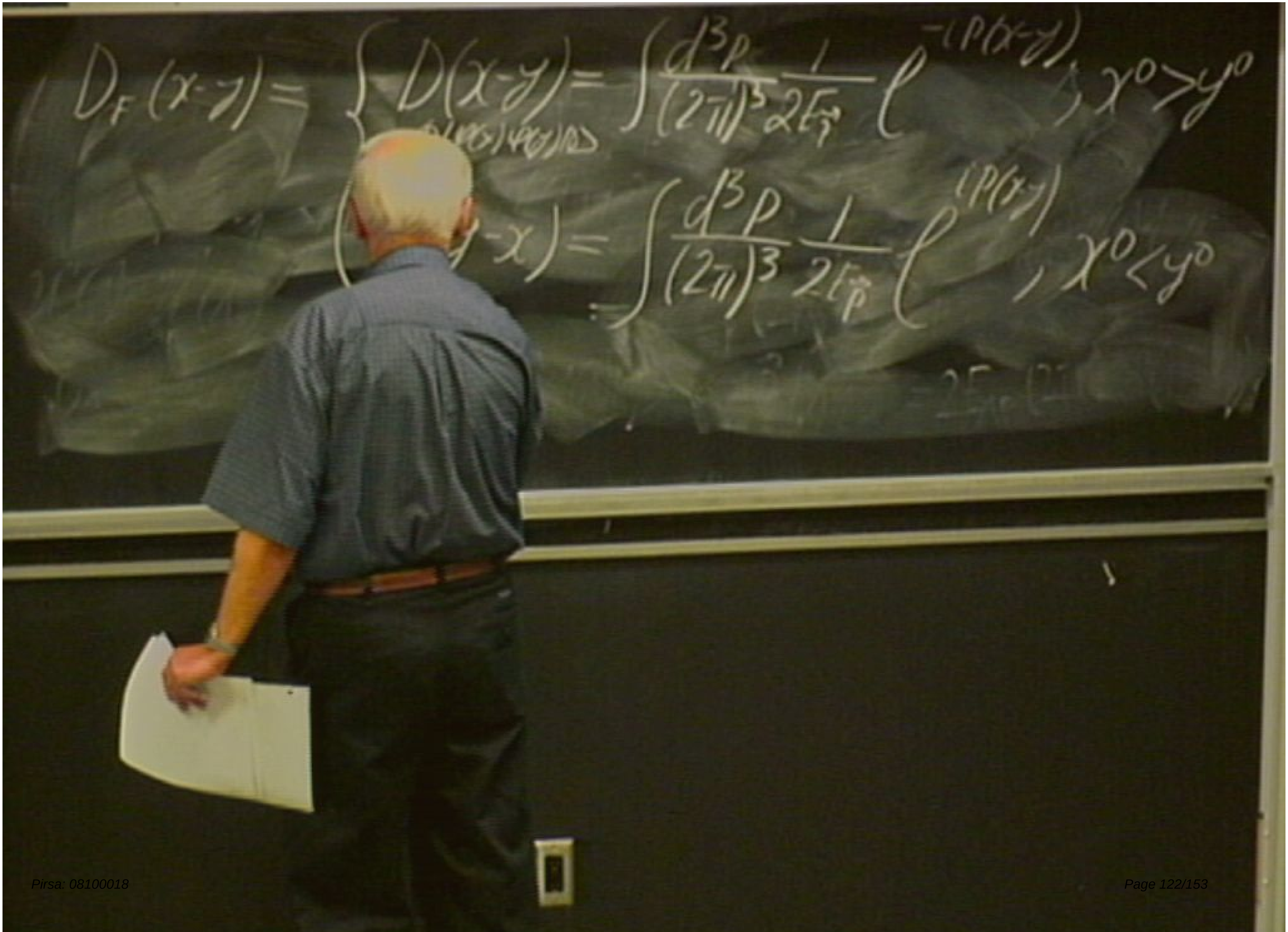
$$\frac{1}{2\pi i}\oint_C f(z)\,dz = \sum_i r_i, \qquad f(z) = \sum_i \frac{r_i}{z-z_i} + \text{regular part}$$

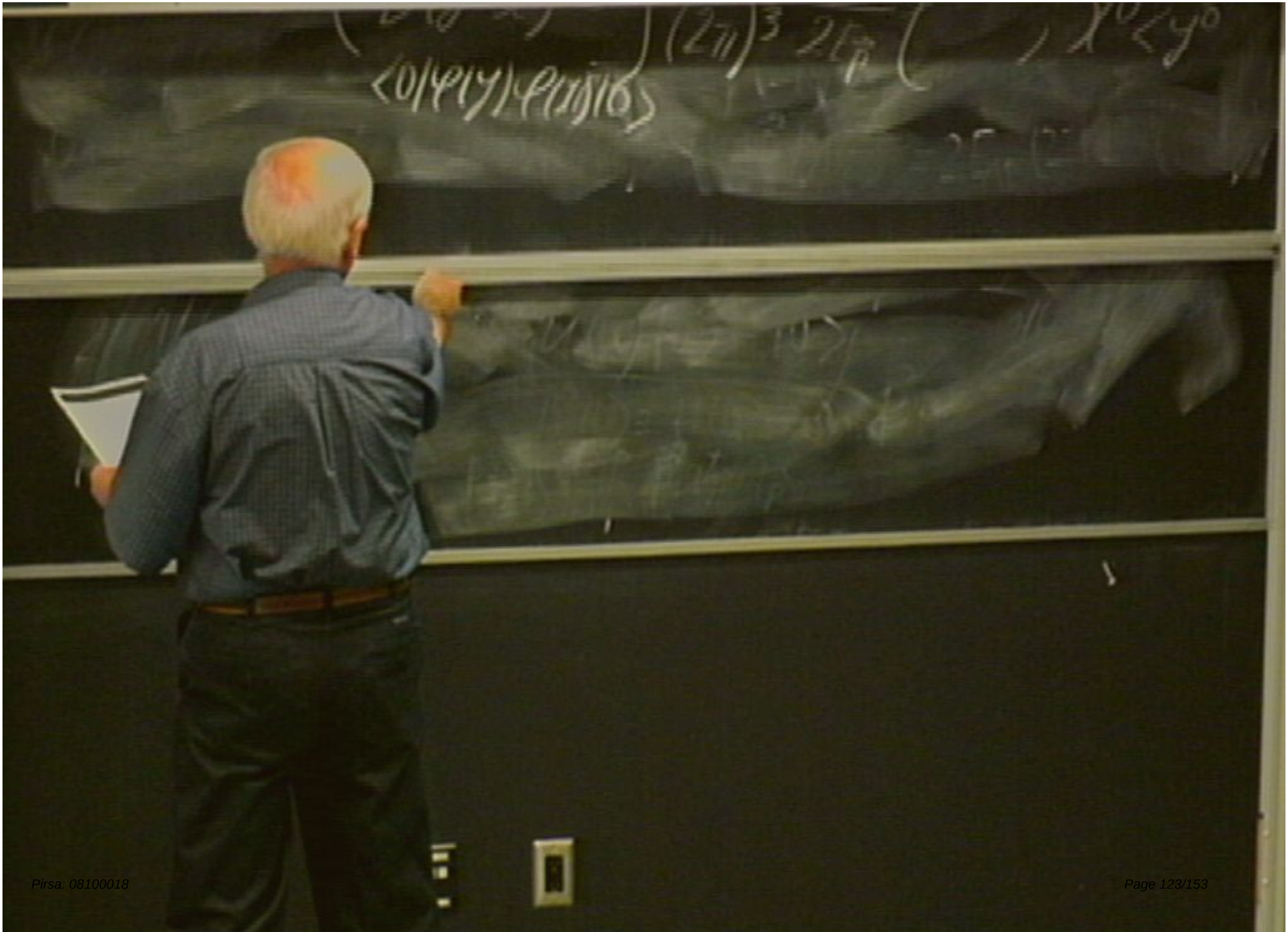
$$e^{-ip_0(x^0-y^0)} \xrightarrow[p^0\to a+ib]{} e^{-ia(x^0-y^0)+b(x^0-y^0)} \sim e^{-b(y^0-x^0)}\, e^{-ia(x^0-y^0)}$$

We conclude that $\lim_{T\to\infty}\int_{C_2} = 0$ and then our integral $\lim_{T\to\infty}\int_{-T}^{T} dp_0 \ldots = 0$

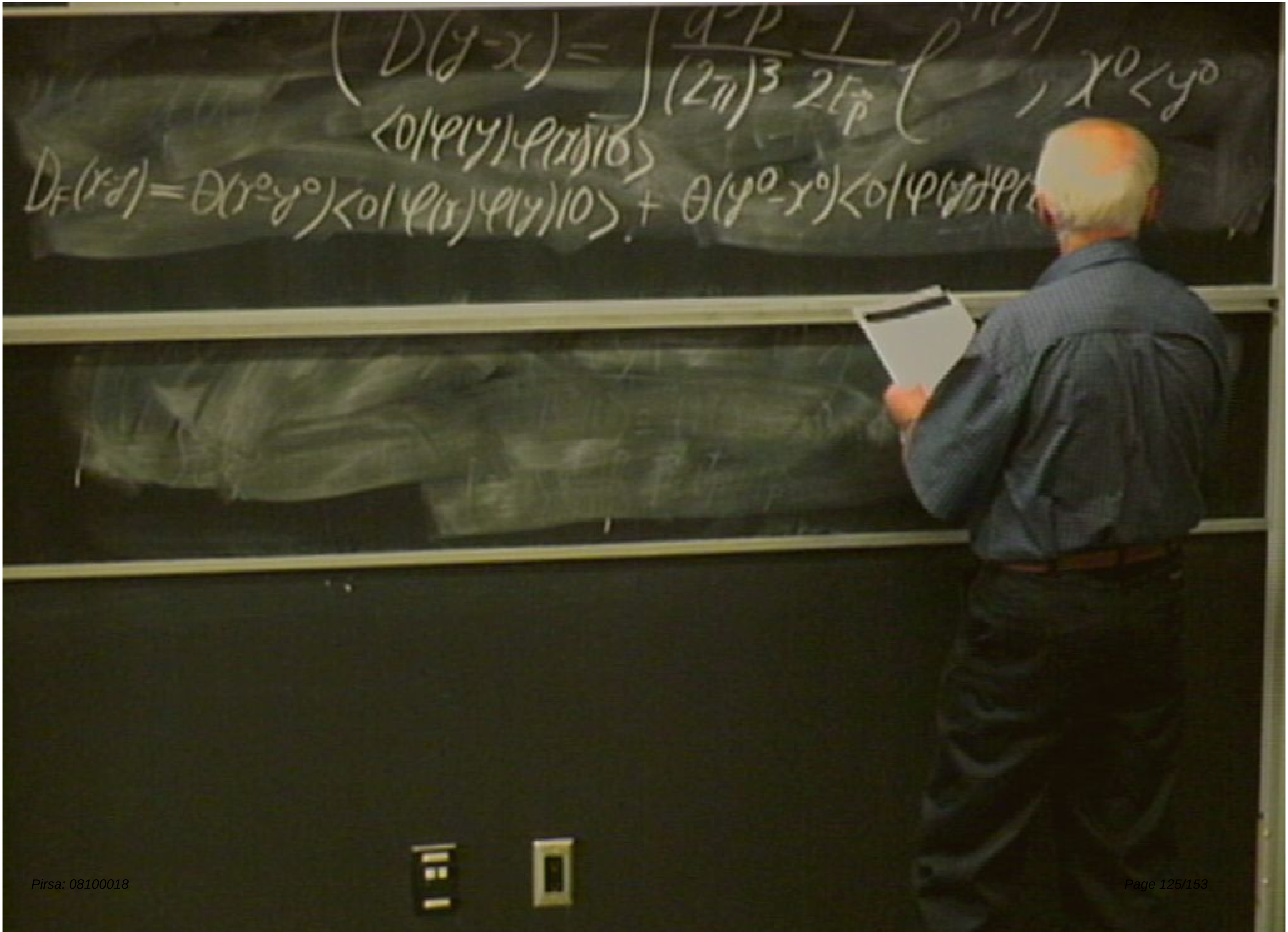

$D_F(x-y) = \int D(x-y)$

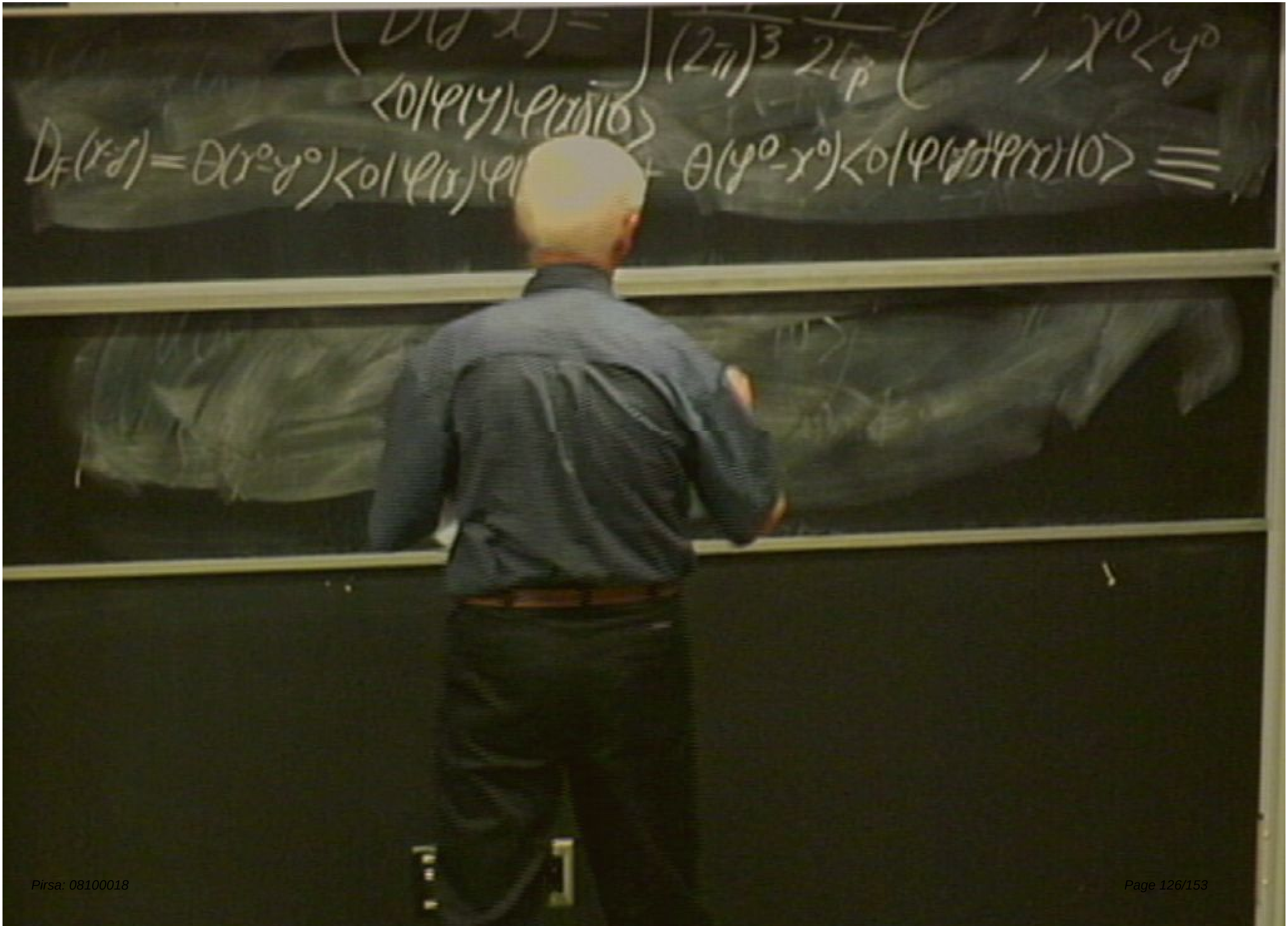

D_F(x-y) = { D(x-y) = ∫ d³p/(2π)³ 1/2E_p e^{-ip(x-y)}
<0|φ(x)φ(y)|0>


$D_F(x-y) = \begin{cases} D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip(x-y)}, & x^0 > y^0 \\ D(y-x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{ip(x-y)}, & x^0 < y^0 \end{cases}$

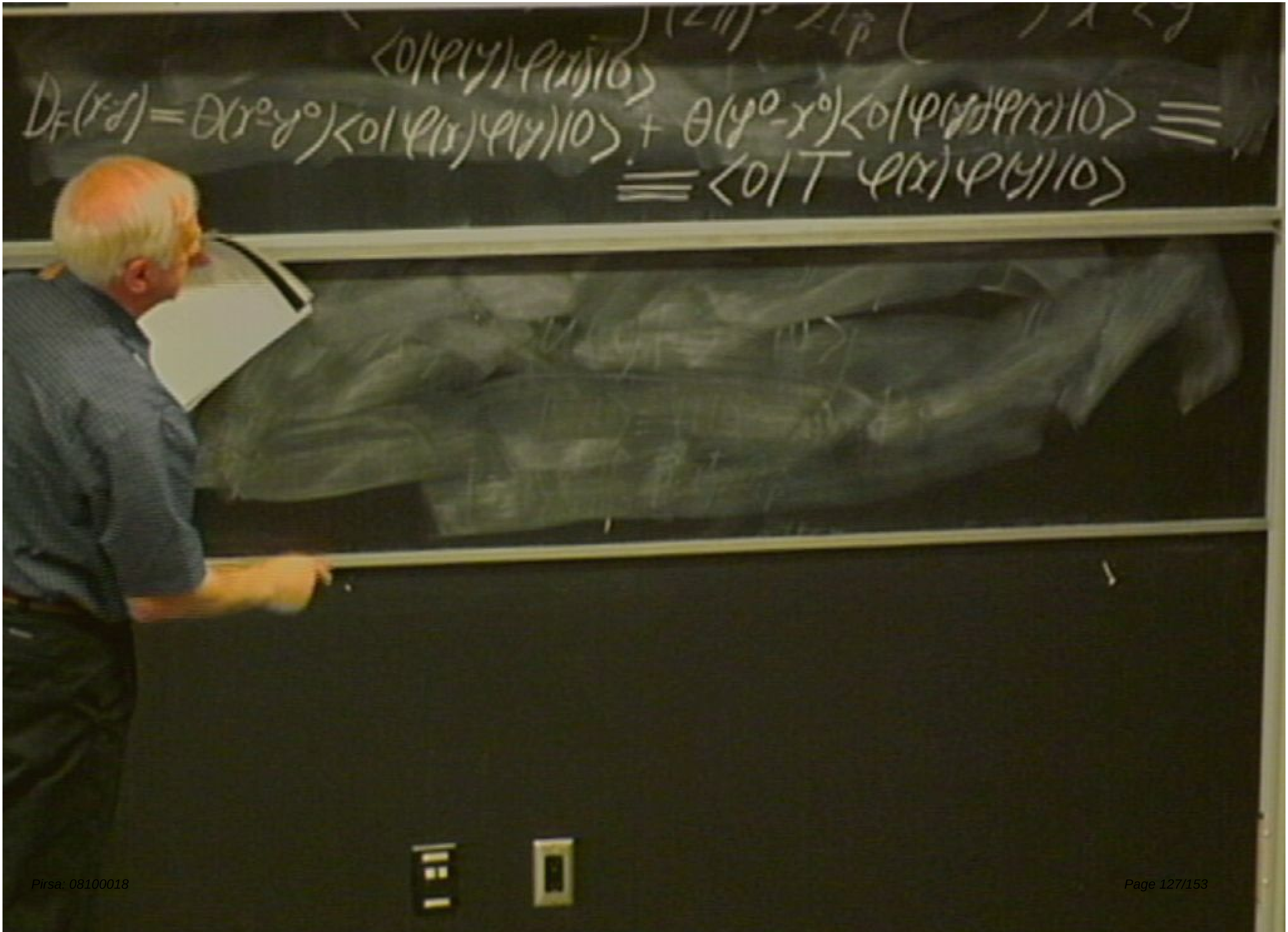


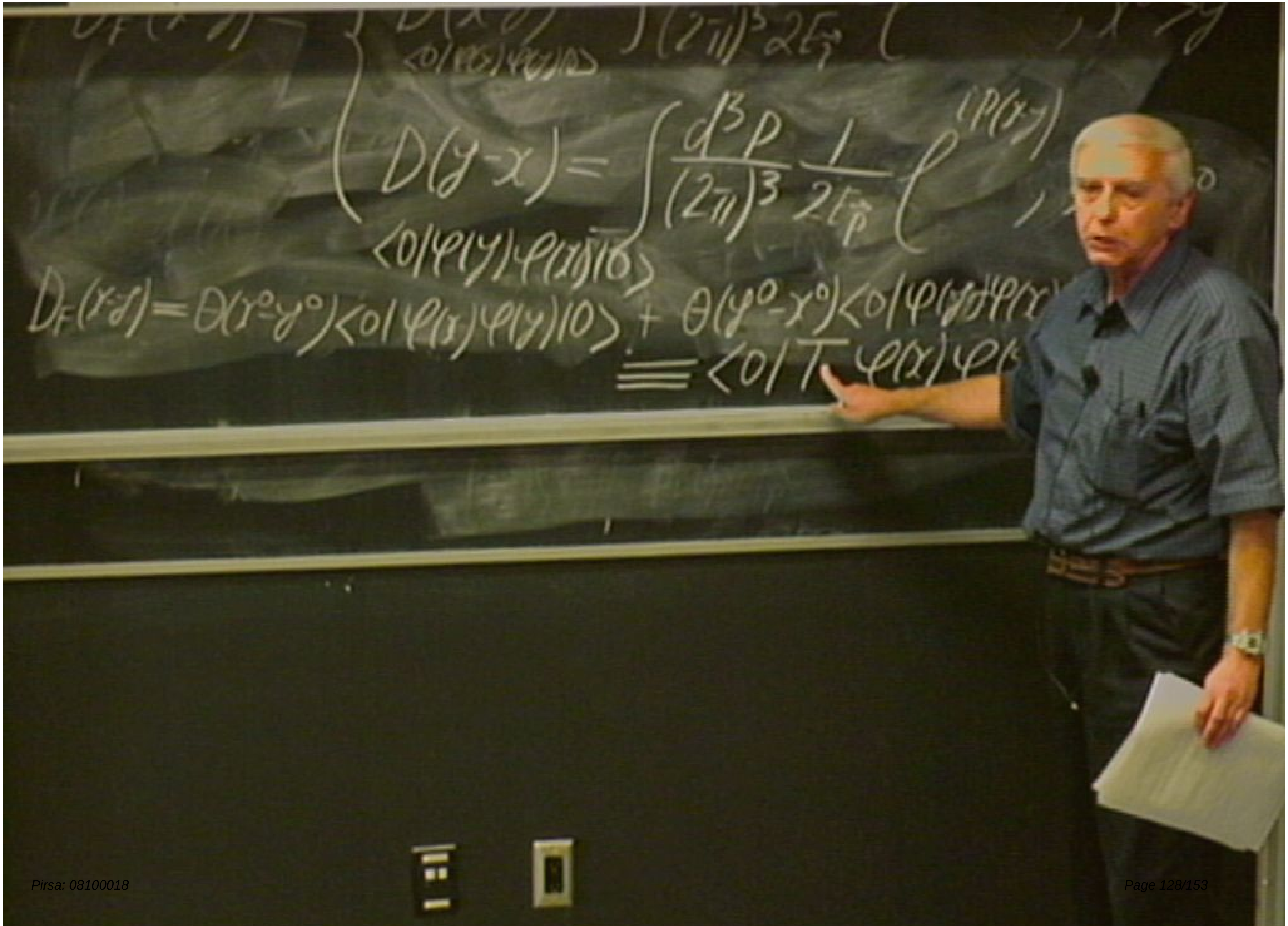
$$D_F(x-y) = \begin{cases} \underset{\langle 0|\varphi(x)\varphi(y)|0\rangle}{D(x-y)} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} e^{-ip(x-y)}, & x^0 > y^0 \\ \underset{\langle 0|\varphi(y)\varphi(x)|0\rangle}{D(y-x)} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} e^{ip(x-y)}, & x^0 < y^0 \end{cases}$$

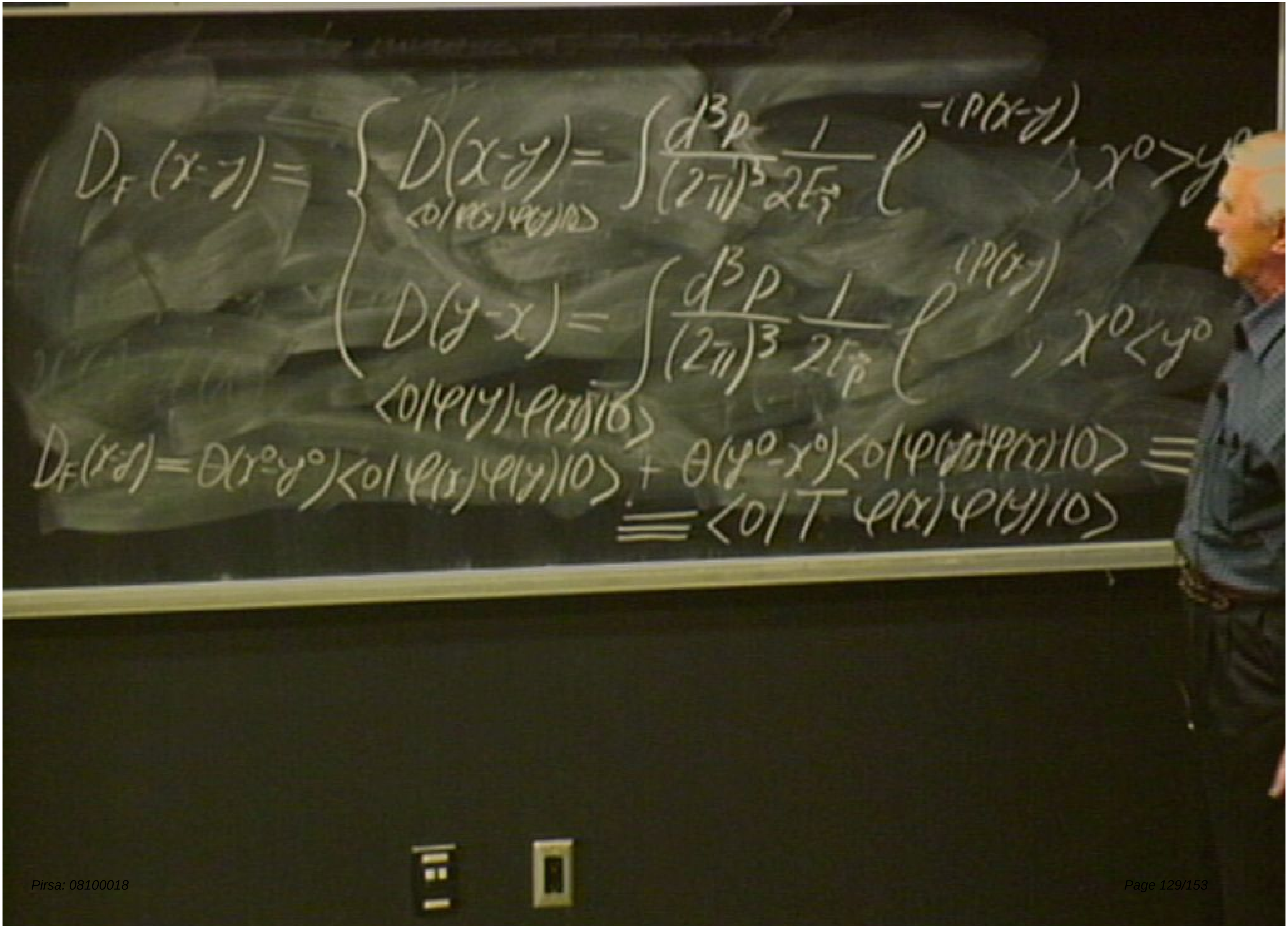

D(y-x) = ∫ d³p/(2π)³ 1/2E_p e , x⁰ < y⁰
<0|φ(y)φ(x)|0>
D_F(x-y) = θ(x⁰-y⁰)<0|φ(x)φ(y)|0> + θ(y⁰-x⁰)<0|φ(y)φ(x

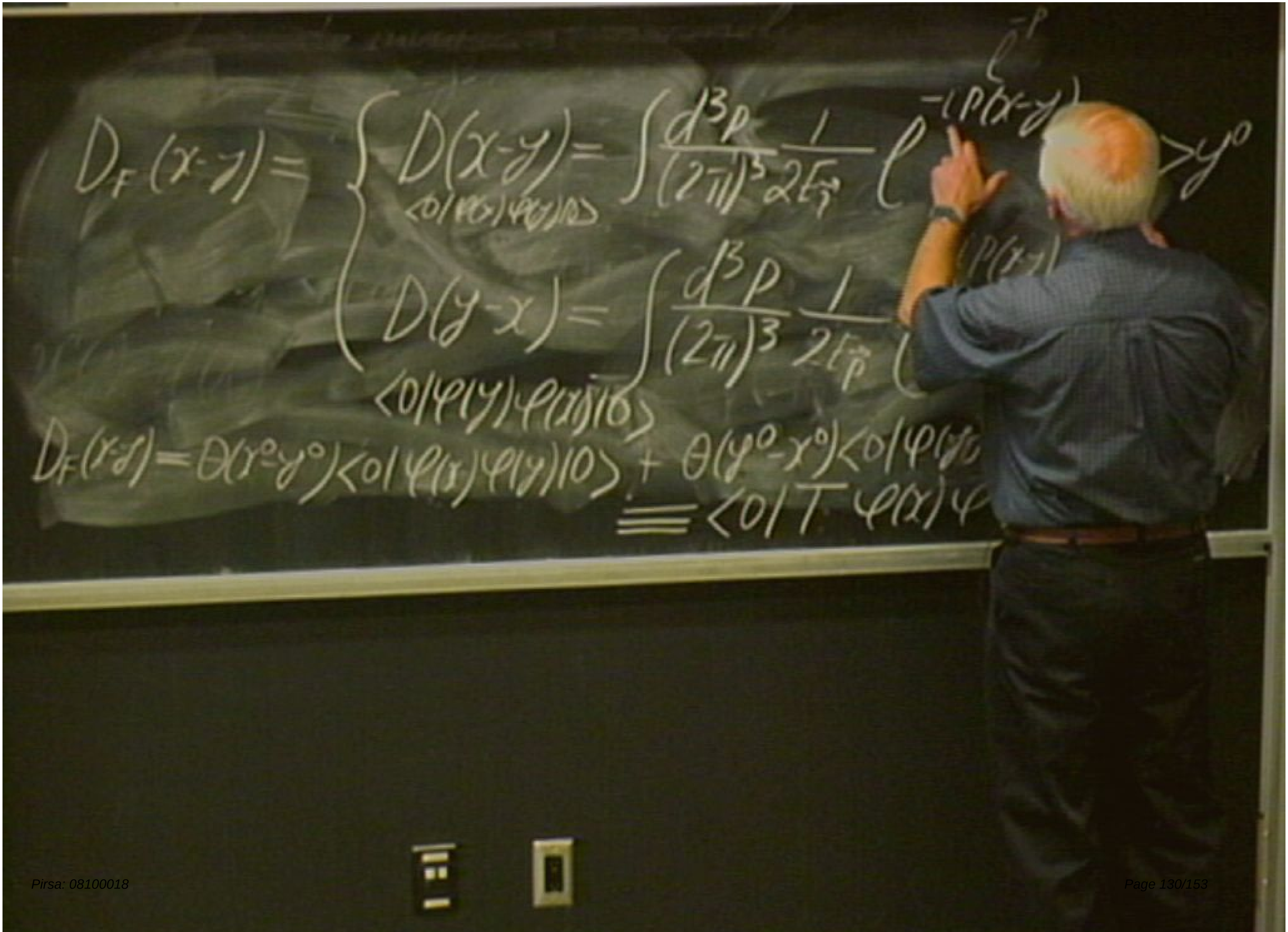


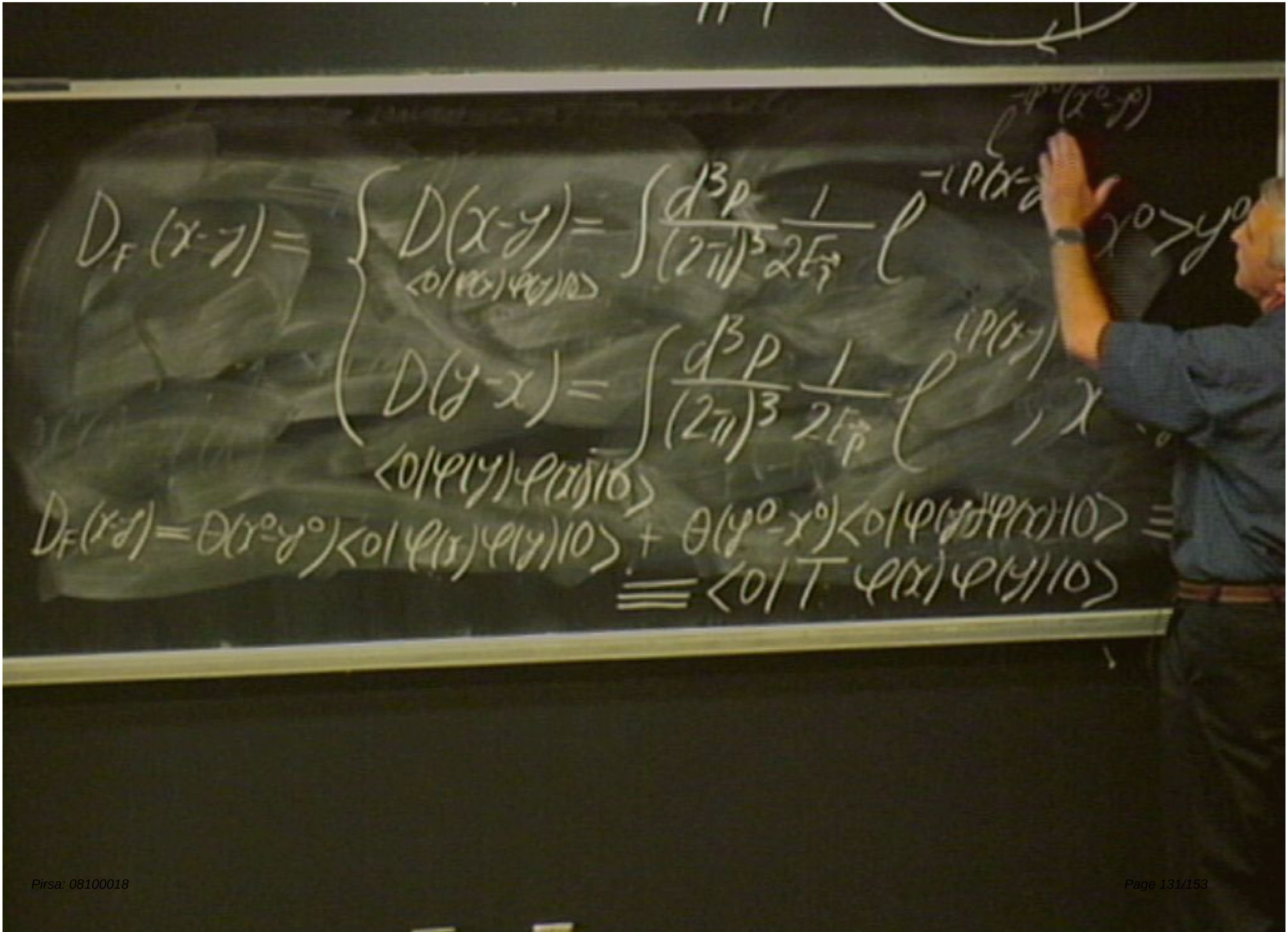
⟨0|φ(y)φ(x)|0⟩
D_F(x-y) = θ(x⁰-y⁰)⟨0|φ(x)φ(
+ θ(y⁰-x⁰)⟨0|φ(y)φ(x)|0⟩ ≡

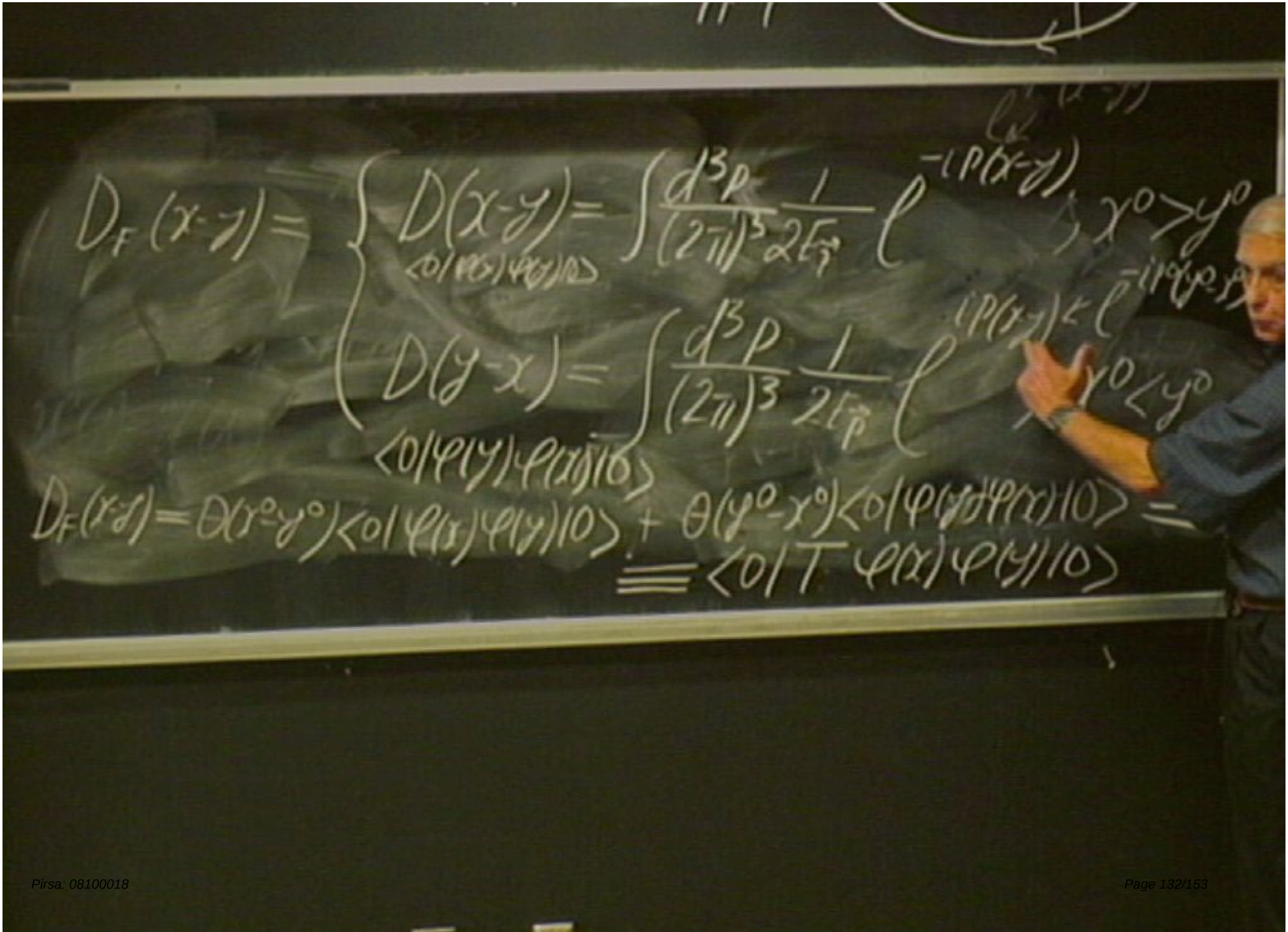

⟨0|φ(y)φ(x)|0⟩
D_F(x-y) = θ(x⁰-y⁰)⟨0|φ(x)φ(y)|0⟩ + θ(y⁰-x⁰)⟨0|φ(y)φ(x)|0⟩ ≡
≡ ⟨0|T φ(x)φ(y)|0⟩


D(y-x) = ∫ d³p/(2π)³ 1/2E_p e^{ip(y-x)}
⟨0|φ(y)φ(x)|0⟩
D_F(x-y) = θ(x⁰-y⁰)⟨0|φ(x)φ(y)|0⟩ + θ(y⁰-x⁰)⟨0|φ(y)φ(x)
≡ ⟨0|T φ(x)φ(


$D_F(x-y) = \begin{cases} D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} e^{-ip(x-y)}, & x^0 > y^0 \\ D(y-x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} e^{ip(x-y)}, & x^0 < y^0 \end{cases}$
$\langle 0|\varphi(x)\varphi(y)|0\rangle$
$\langle 0|\varphi(y)\varphi(x)|0\rangle$
$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle \equiv$
$\equiv \langle 0|T\,\varphi(x)\varphi(y)|0\rangle$


D_F(x-y) =
D(x-y) = ∫ d³p/(2π)³ 1/2E_p e^{-ip(x-y)}   > y⁰
<0|φ(x)φ(y)|0>
D(y-x) = ∫ d³p/(2π)³ 1/2E_p
<0|φ(y)φ(x)|0>
D_F(x-y) = θ(x⁰-y⁰)<0|φ(x)φ(y)|0> + θ(y⁰-x⁰)<0|φ(y
≡ <0|T φ(x)φ


D_F(x-y) =
D(x-y) = ∫ d³p/(2π)³ 1/(2E_p) e^{-ip(x-y)}   x⁰ > y⁰
<0|φ(x)φ(y)|0>
D(y-x) = ∫ d³p/(2π)³ 1/(2E_p) e^{ip(x-y)}   x
<0|φ(y)φ(x)|0>
D_F(x-y) = θ(x⁰-y⁰)<0|φ(x)φ(y)|0> + θ(y⁰-x⁰)<0|φ(y)φ(x)|0> ≡
≡ <0|T φ(x)φ(y)|0>


$D_F(x-y) = \begin{cases} D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec p}} e^{-ip(x-y)}, & x^0 > y^0 \\ D(y-x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec p}} e^{ip(x-y)}, & x^0 < y^0 \end{cases}$
$\langle 0|\varphi(x)\varphi(y)|0\rangle$
$\langle 0|\varphi(y)\varphi(x)|0\rangle$
$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle =$
$\equiv \langle 0|T\,\varphi(x)\varphi(y)|0\rangle$

$$D_F(x-y) = \begin{cases} D(x-y) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\, e^{-ip(x-y)}, & x^0 > y^0 \\ \quad \langle 0|\varphi(x)\varphi(y)|0\rangle & \\ D(y-x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\, e^{ip(x-y)}, & x^0 < y^0 \\ \quad \langle 0|\varphi(y)\varphi(x)|0\rangle & \end{cases}$$

$$e^{ip(x-y)} \leftarrow e^{-ip(y-x)}$$

$$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle \equiv \langle 0|T\,\varphi(x)\varphi(y)|0\rangle$$

$$D_F(x-y) = \begin{cases} \underbrace{D(x-y)}_{\langle 0|\varphi(x)\varphi(y)|0\rangle} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec p}} e^{-ip(x-y)}, & x^0 > y^0 \\ \underbrace{D(y-x)}_{\langle 0|\varphi(y)\varphi(x)|0\rangle} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec p}} e^{ip(x-y)}, & x^0 < y^0 \end{cases}$$

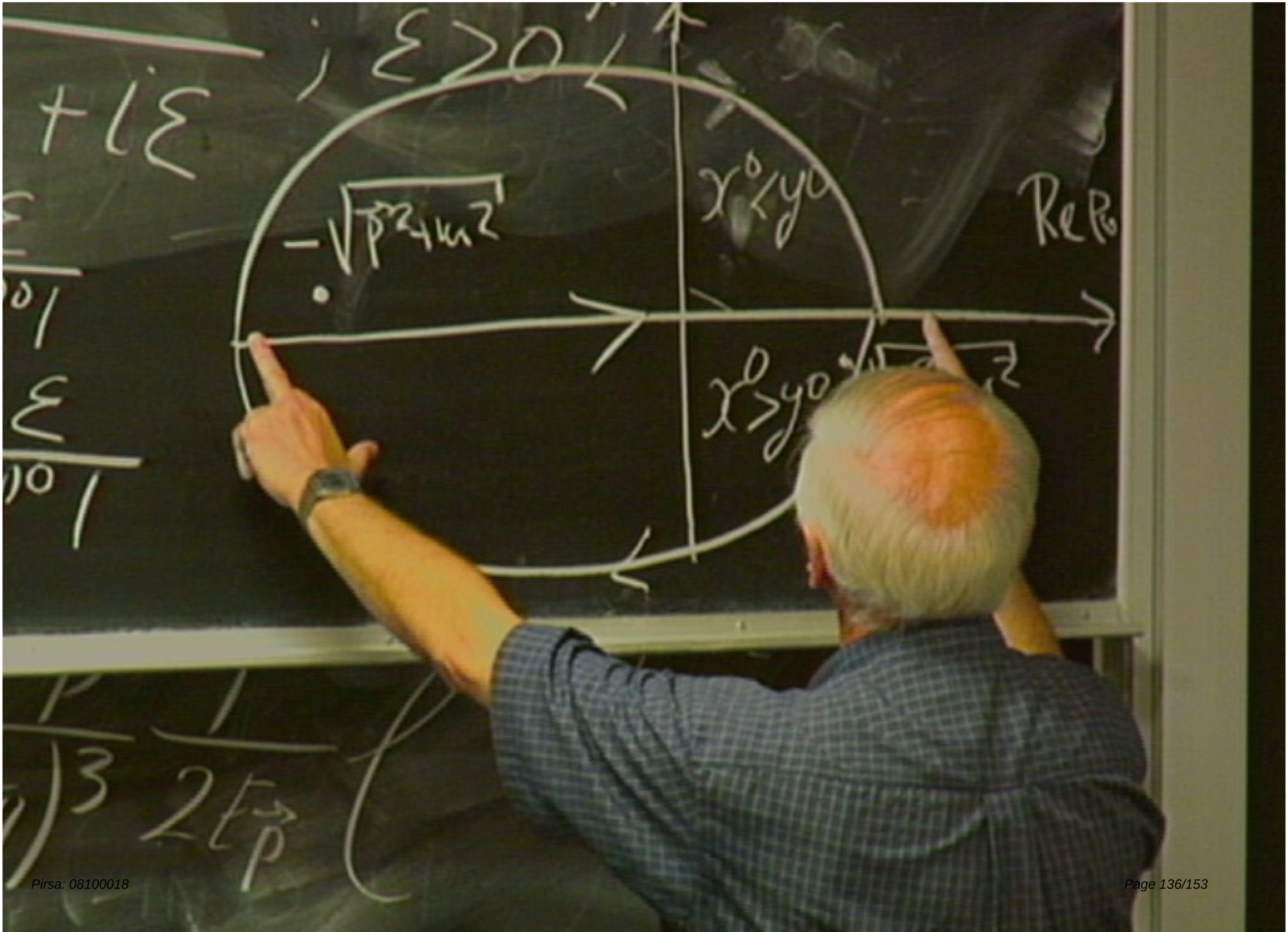
$$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle \equiv$$

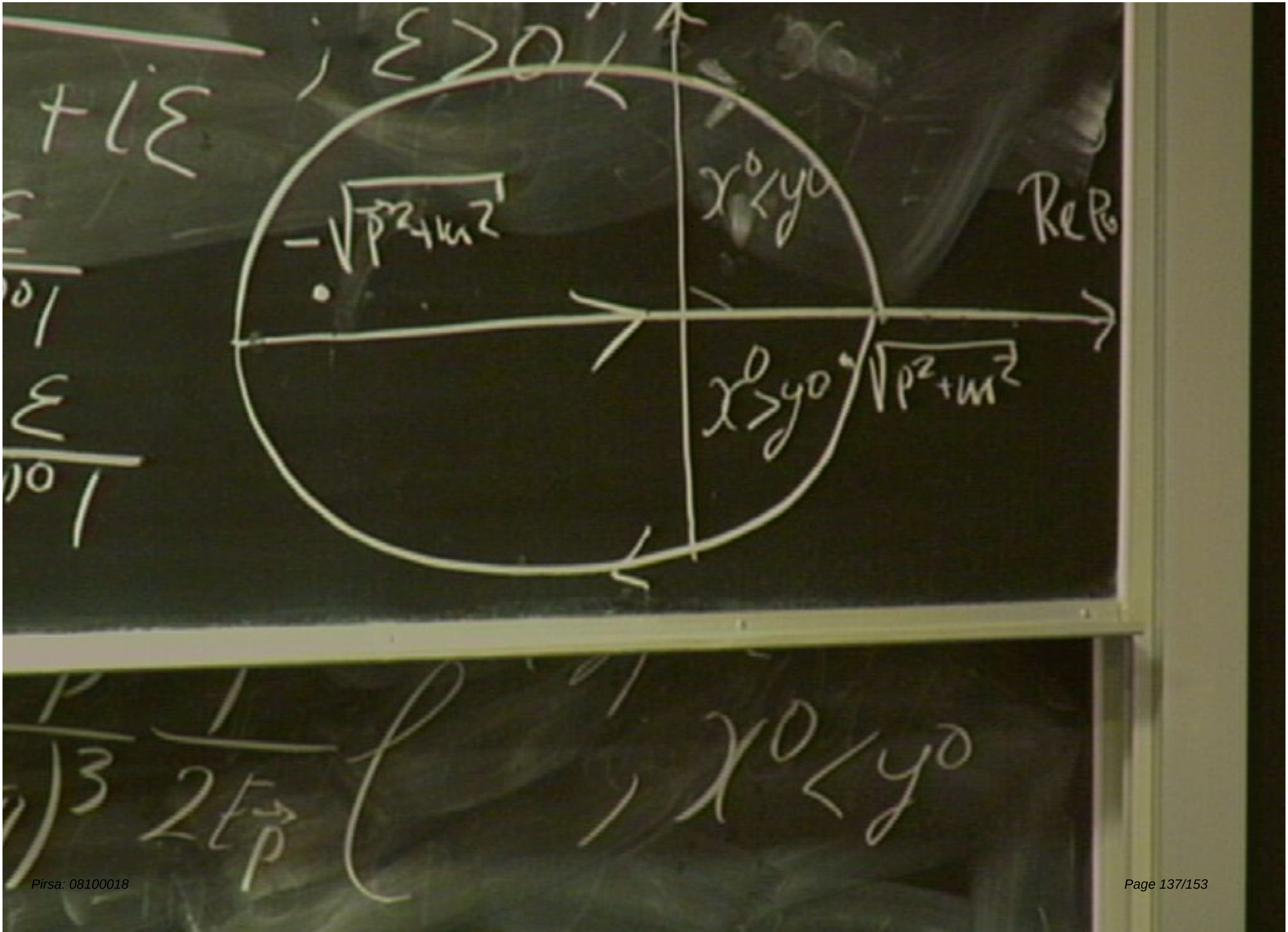
$$\equiv \langle 0|T\,\varphi(x)\varphi(y)|0\rangle$$

$$D_F(x-y) = \begin{cases} D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec p}} e^{-ip(x-y)}, & x^0 > y^0 \\ \quad \langle 0|\varphi(x)\varphi(y)|0\rangle & \\ D(y-x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec p}} e^{ip(x-y)} \leftarrow e^{-ip(y-x)}, & x^0 < y^0 \\ \quad \langle 0|\varphi(y)\varphi(x)|0\rangle & \end{cases}$$

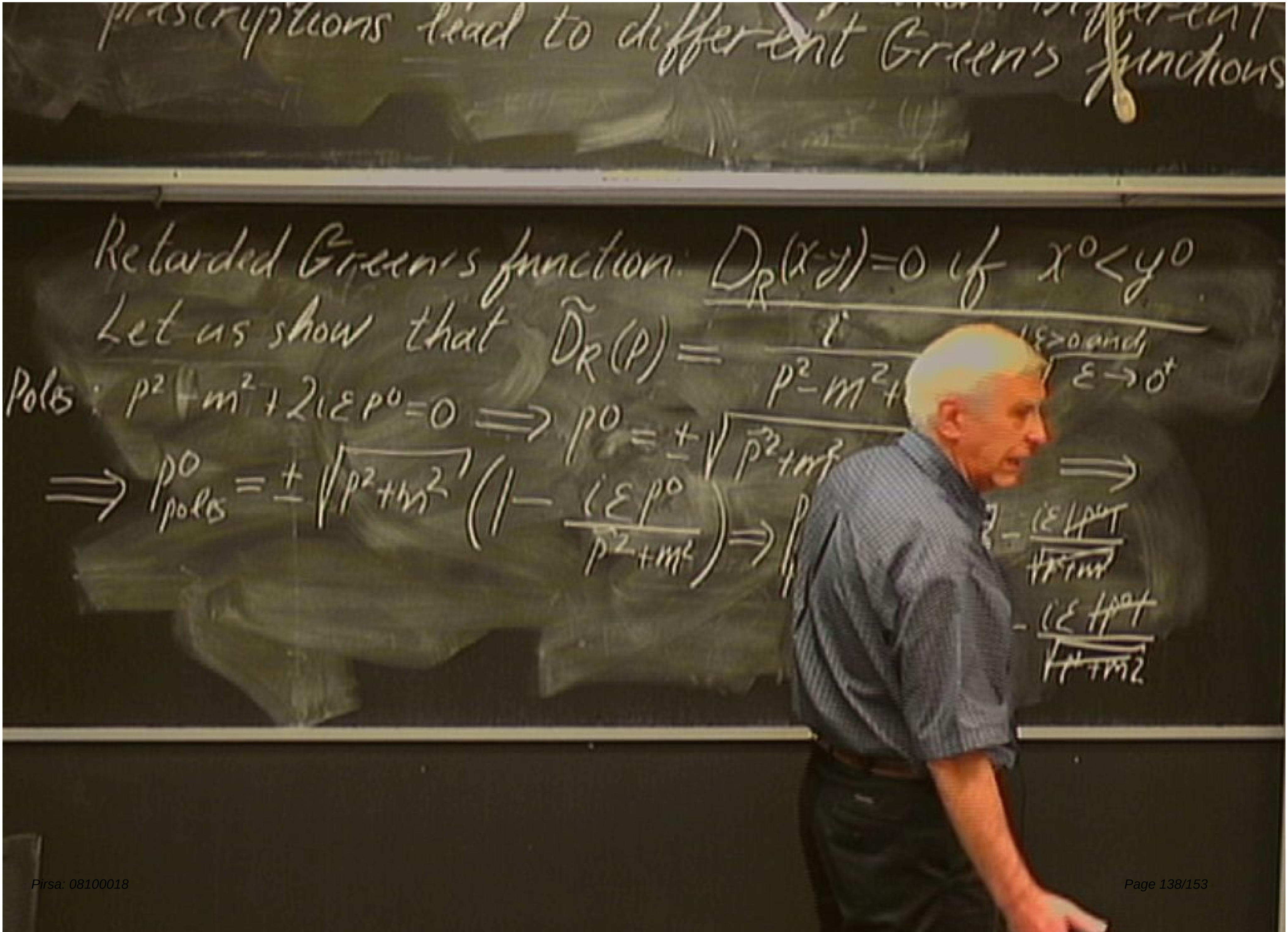
$$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle \equiv$$

$$\equiv \langle 0|T\, \varphi(x)\varphi(y)|0\rangle$$





+iε
ε>0
$-\sqrt{p^2+m^2}$
$x^0<y^0$
Re p0
$x^0>y^0$
$\sqrt{p^2+m^2}$
$2E_{\vec{p}}$
$x^0<y^0$


prescriptions lead to different Green's functions
Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / (p^2-m^2+ ... ε>0 and ε→0^+
Poles: p^2-m^2+2iεp^0=0 ⟹ p^0 = ± √(p⃗^2+m^2 ...
⟹ p^0_poles = ± √(p⃗^2+m^2) (1 - iεp^0 / (p⃗^2+m^2)) ⟹

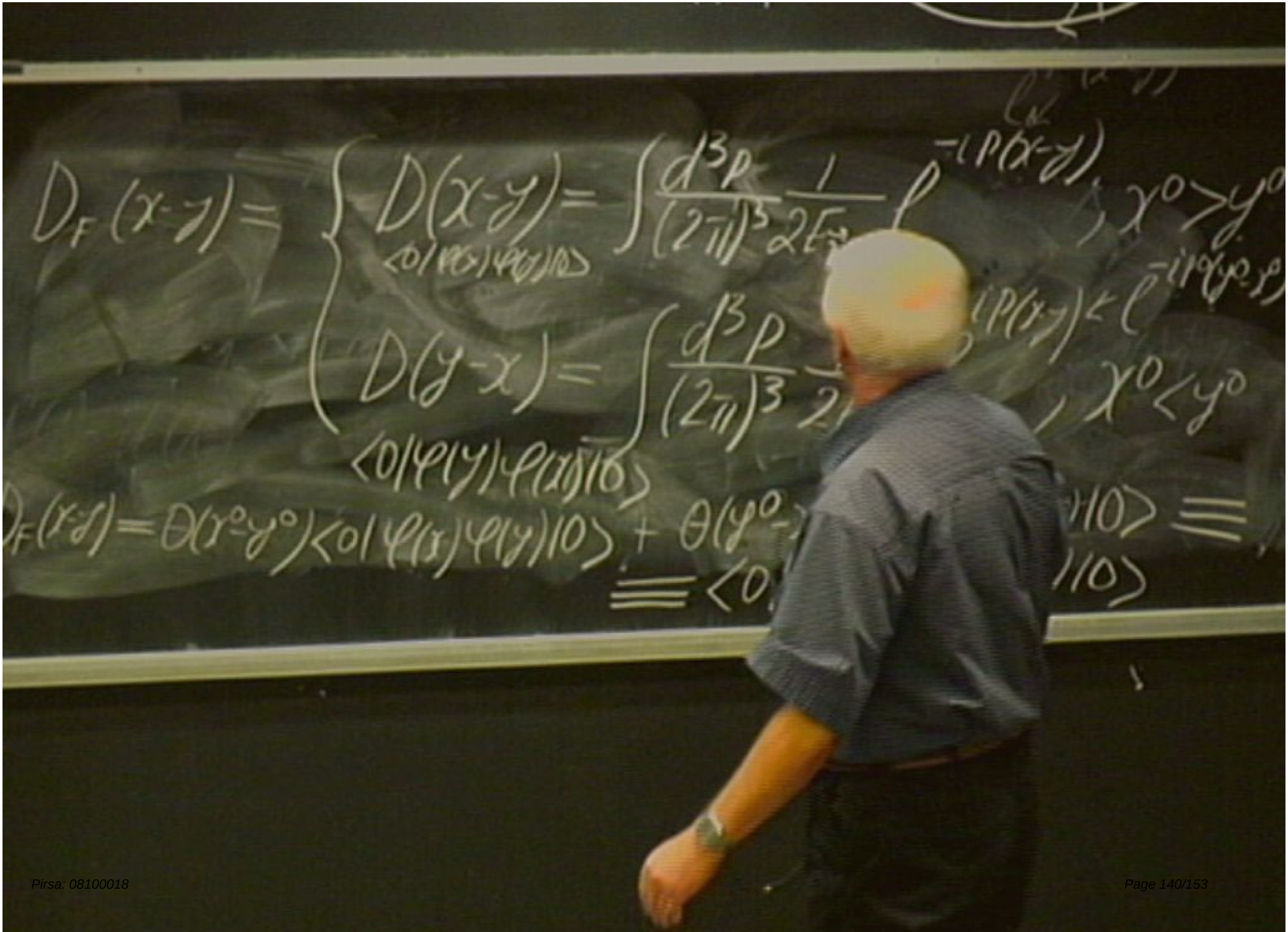
prescriptions lead to different Green's functions

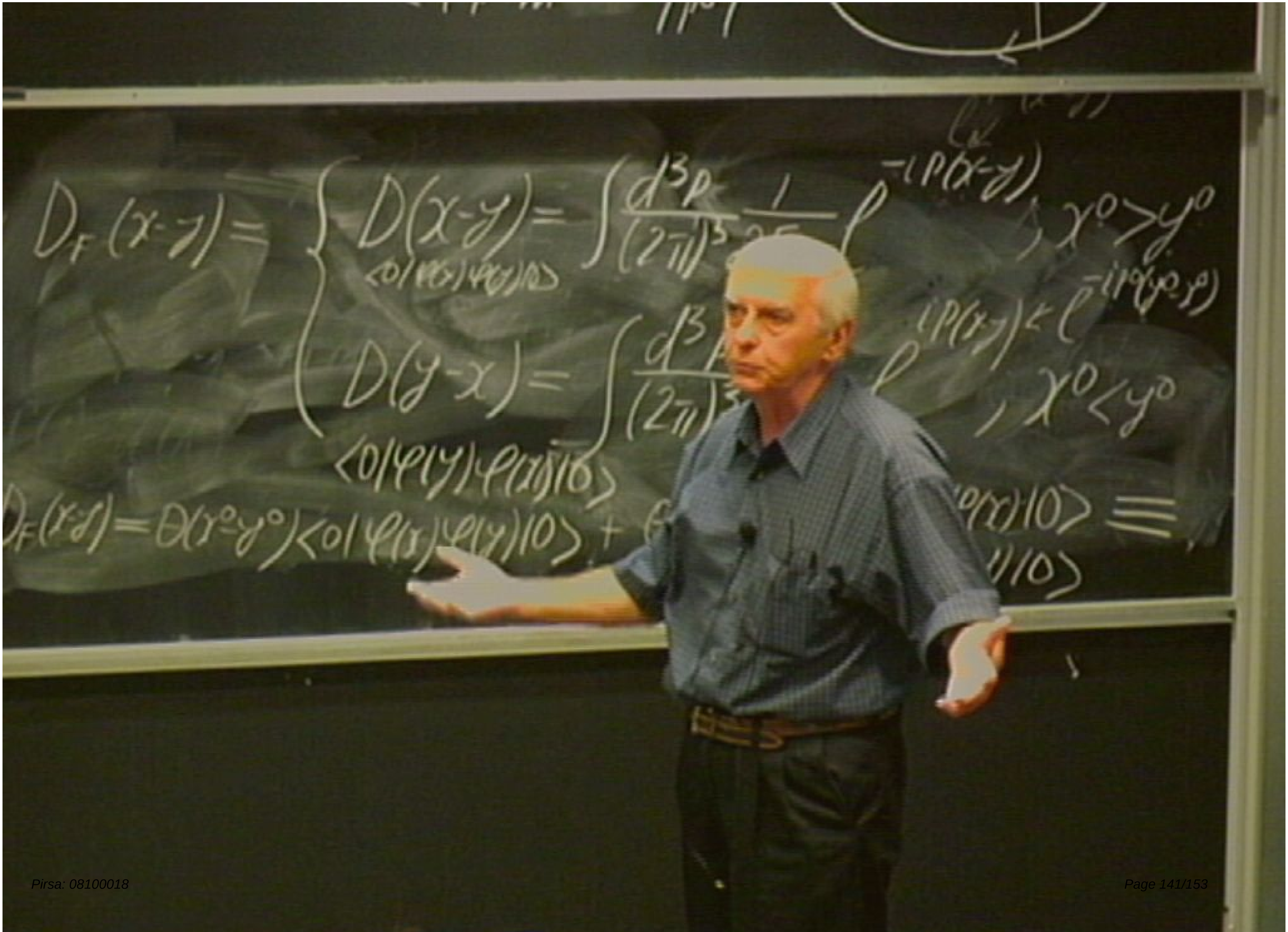
Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$

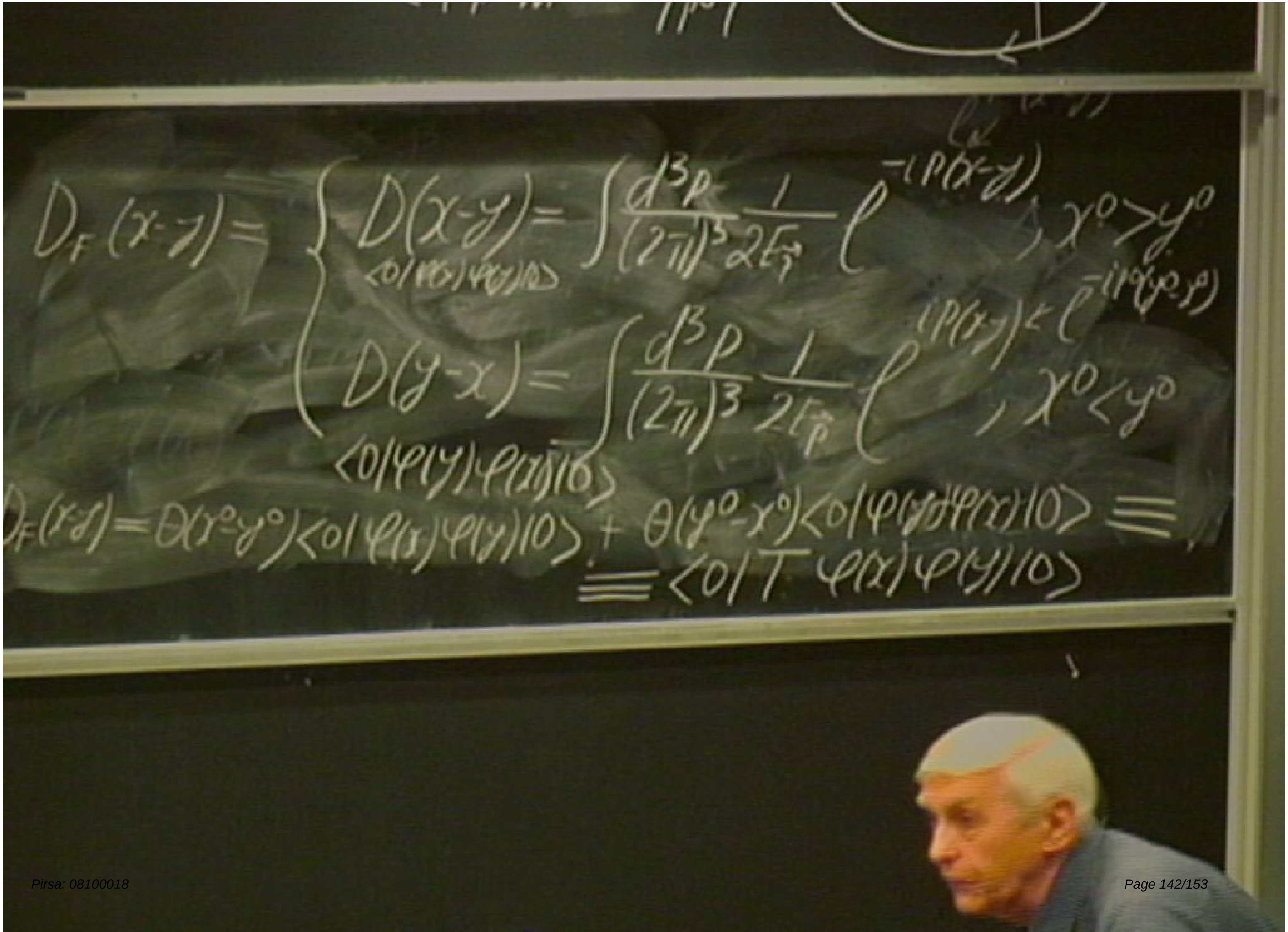
Let us show that $\tilde{D}_R(p) = \dfrac{i}{p^2-m^2+2i\varepsilon p^0}$ ($\varepsilon>0$ and $\varepsilon \to 0^+$)

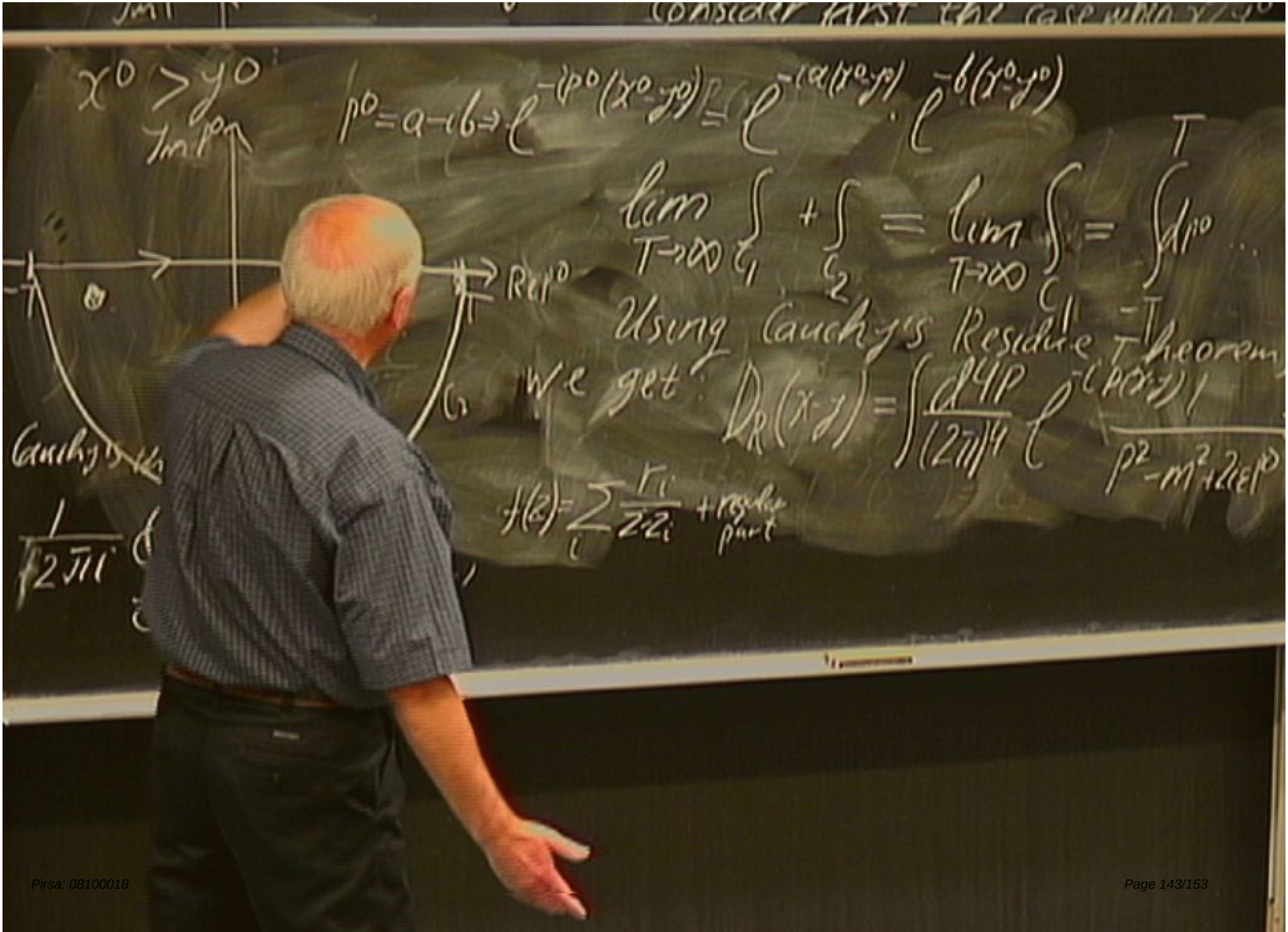
Poles: $p^2-m^2+2i\varepsilon p^0=0 \Longrightarrow p^0=\pm\sqrt{\vec{p}^2+m^2-2i\varepsilon p^0} \Longrightarrow$

$$\Longrightarrow p^0_{poles}=\pm\sqrt{\vec{p}^2+m^2}\left(1-\frac{i\varepsilon p^0}{\vec{p}^2+m^2}\right) \Longrightarrow p^0_{poles}=\begin{cases}\sqrt{\vec{p}^2+m^2}-\dfrac{i\varepsilon|p^0|}{\sqrt{\vec{p}^2+m^2}}\\ -\sqrt{\vec{p}^2+m^2}-\dfrac{i\varepsilon|p^0|}{\sqrt{\vec{p}^2+m^2}}\end{cases}$$


$D_F(x-y) =$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E}$
$\langle 0|\varphi(x)\varphi(y)|0\rangle$
$x^0 > y^0$
$D(y-x) = \int \frac{d^3p}{(2\pi)^3}$
$x^0 < y^0$
$\langle 0|\varphi(y)\varphi(x)|0\rangle$
$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-$

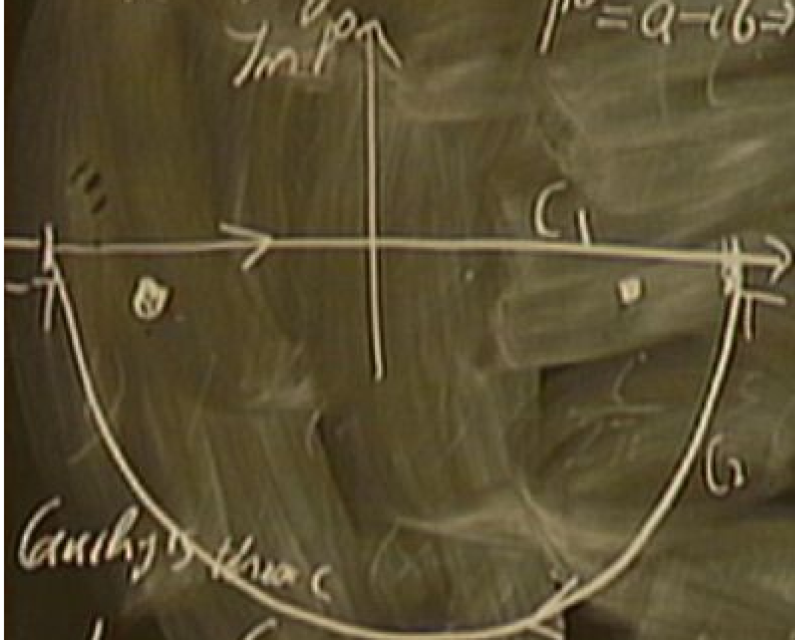

D_F(x-y) =
D(x-y) =
<0|φ(x)φ(y)|0>
x^0 > y^0
D(y-x) =
x^0 < y^0
<0|φ(y)φ(x)|0>
D_F(x-y) = θ(x^0-y^0)<0|φ(x)φ(y)|0> +


D_F(x-y) = D(x-y) = ∫ d^3p/(2π)^3 1/2E_p e^{-ip(x-y)}, x^0 > y^0
<0|φ(x)φ(y)|0>
D(y-x) = ∫ d^3p/(2π)^3 1/2E_p e^{ip(x-y)}, x^0 < y^0
<0|φ(y)φ(x)|0>
D_F(x-y) = θ(x^0-y^0)<0|φ(x)φ(y)|0> + θ(y^0-x^0)<0|φ(y)φ(x)|0> ≡
≡ <0|T φ(x)φ(y)|0>


$x^0 > y^0$
$p^0 = a - ib \Rightarrow e^{-ip^0(x^0-y^0)} = e^{-ia(x^0-y^0)} \cdot e^{-b(x^0-y^0)}$
Im p^0
Re p^0
$\lim_{T\to\infty} \int_{C_1} + \int_{C_2} = \lim_{T\to\infty} \int_{C} = \int_{-T}^{T} dp^0 \ldots$
Using Cauchy's Residue Theorem
we get: $D_R(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}$
$f(z) = \sum_i \frac{r_i}{z - z_i}$ + regular part

Consider first the case when $x^0 > y^0$

$x^0 > y^0$



$$p^0 = a - ib \Rightarrow e^{-ip^0(x^0-y^0)} = e^{-ia(x^0-y^0)} \cdot e^{-b(x^0-y^0)}$$

$$\lim_{T\to\infty} \int_{C_1} + \int_{C_2} = \lim_{T\to\infty} \int_{C} = \int_{-T}^{T} dp^0 \ldots$$

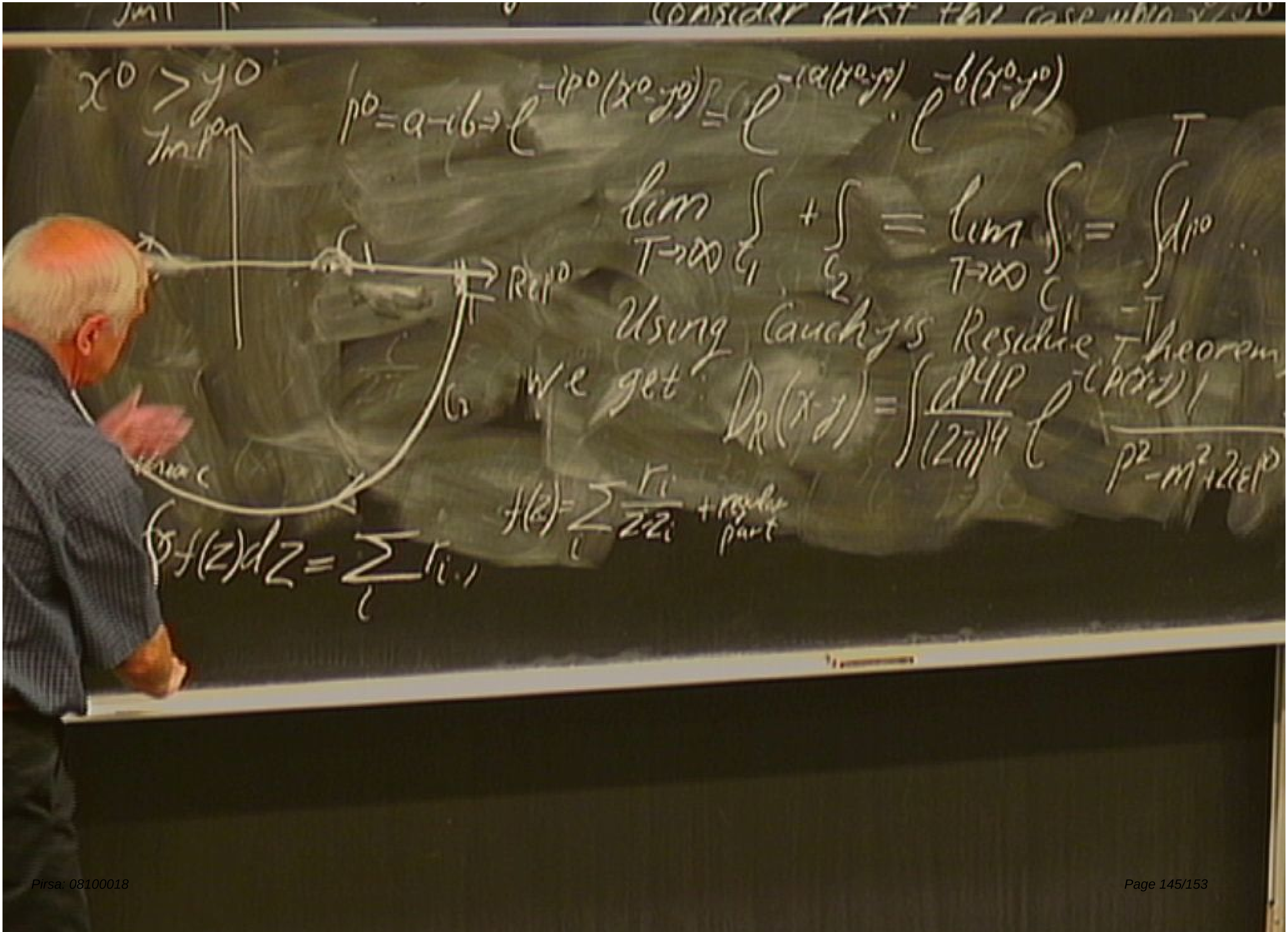
Using Cauchy's Residue Theorem we get

$$D_R(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2 - m^2 + 2i\epsilon p^0}$$

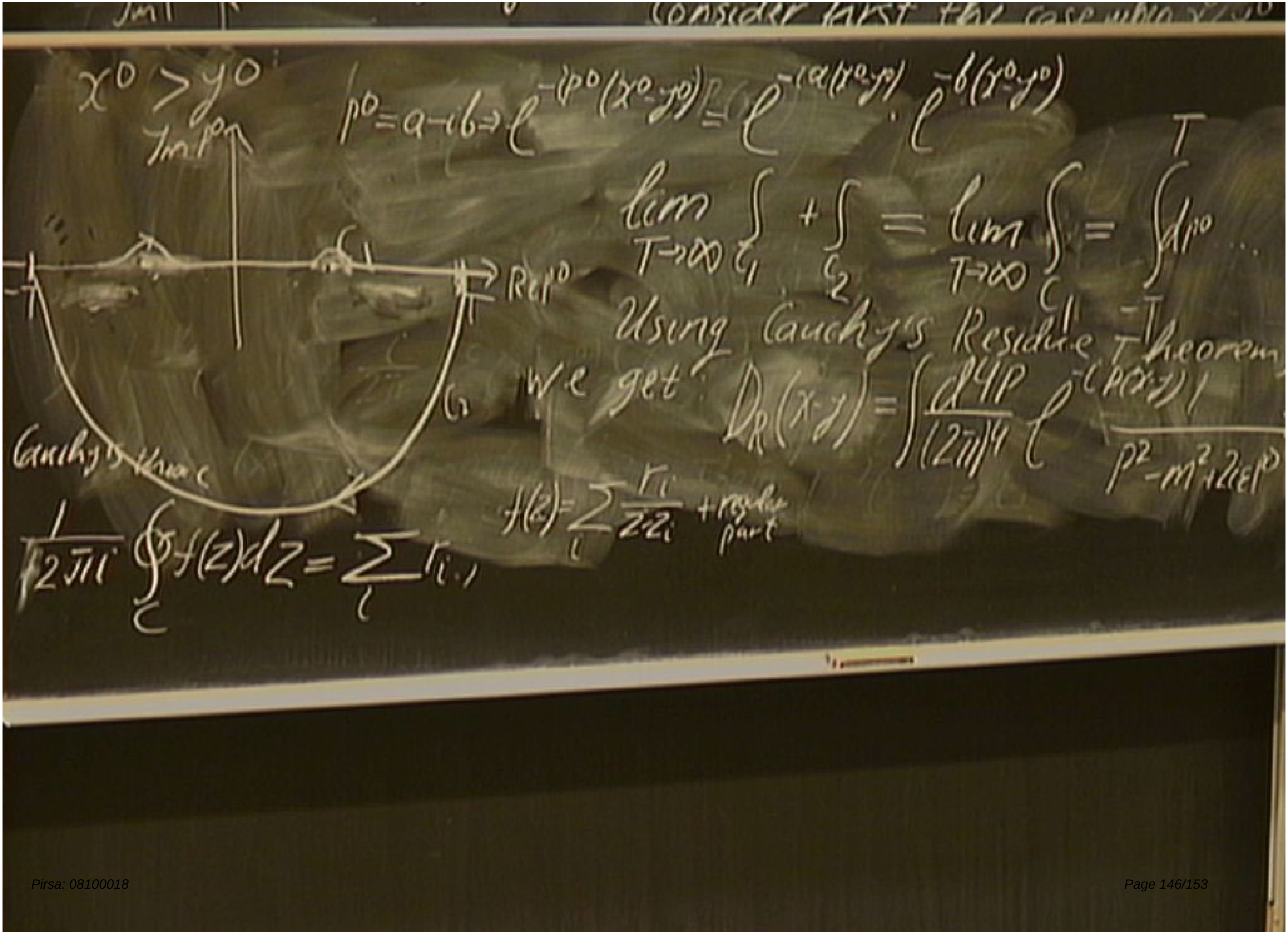
Cauchy's theorem

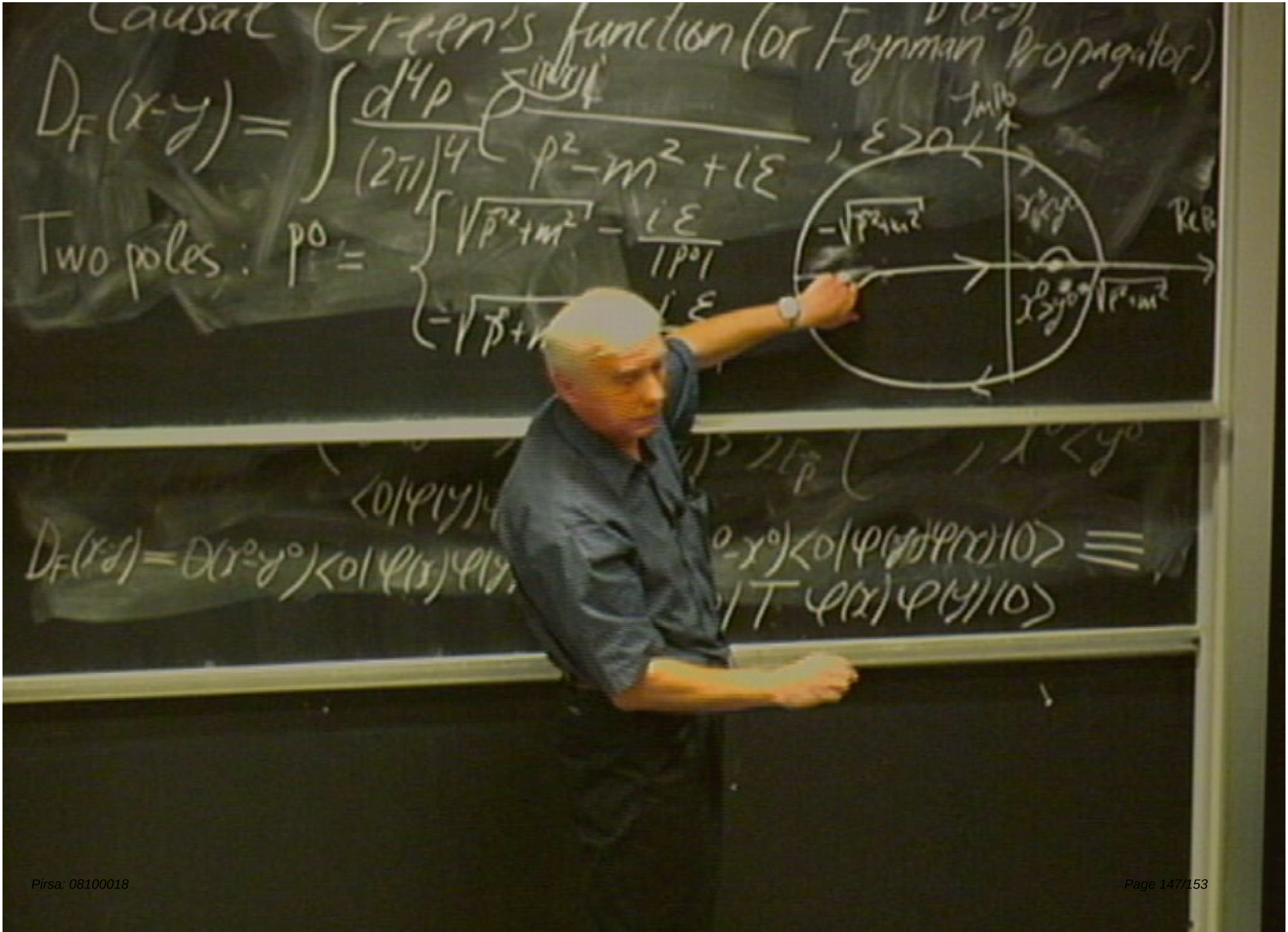
$$\frac{1}{2\pi i} \oint_C f(z)\, dz = \sum_i r_i \,.$$

$$f(z) = \sum_i \frac{r_i}{z - z_i} + \text{regular part}$$



Consider first the case when
x⁰ > y⁰
Using Cauchy's Residue Theorem
we get

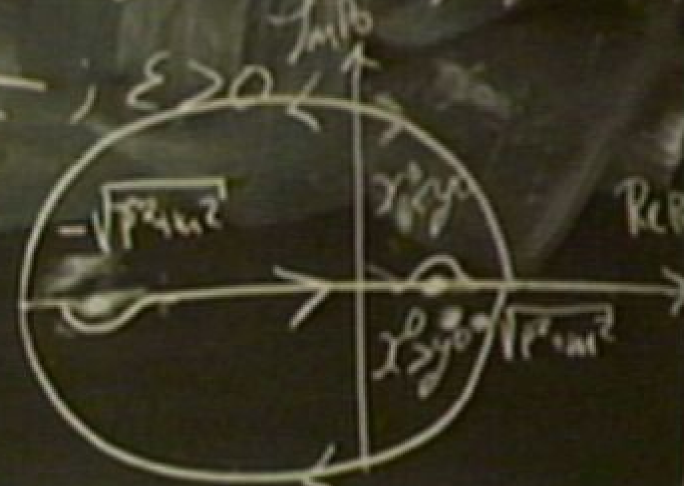

x⁰ > y⁰
Im p⁰
p⁰ = a − ib ⇒ e^{−ip⁰(x⁰−y⁰)} = e^{−ia(x⁰−y⁰)} · e^{−b(x⁰−y⁰)}
lim_{T→∞} ∫_{C1} + ∫_{C2} = lim_{T→∞} ∫_C = ∫_{−T}^{T} dp⁰ ...
Re p⁰
C1
C2
Using Cauchy's Residue Theorem
we get D_R(x−y) = ∫ d⁴p/(2π)⁴ e^{−ip(x−y)} i/(p² − m² + 2iεp⁰)
Cauchy's theorem
f(z) = Σ_i r_i/(z−z_i) + regular part
1/(2πi) ∮_C f(z) dz = Σ_i r_i


Causal Green's function (or Feynman Propagator)
Two poles:

Causal Green's function (or Feynman Propagator)

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2-m^2+i\varepsilon}, \quad \varepsilon>0$$

Two poles: $p^0 = \begin{cases} \sqrt{\vec{p}^2+m^2} - \frac{i\varepsilon}{|p^0|} \\ -\sqrt{\vec{p}^2+m^2} + \frac{i\varepsilon}{|p^0|} \end{cases}$

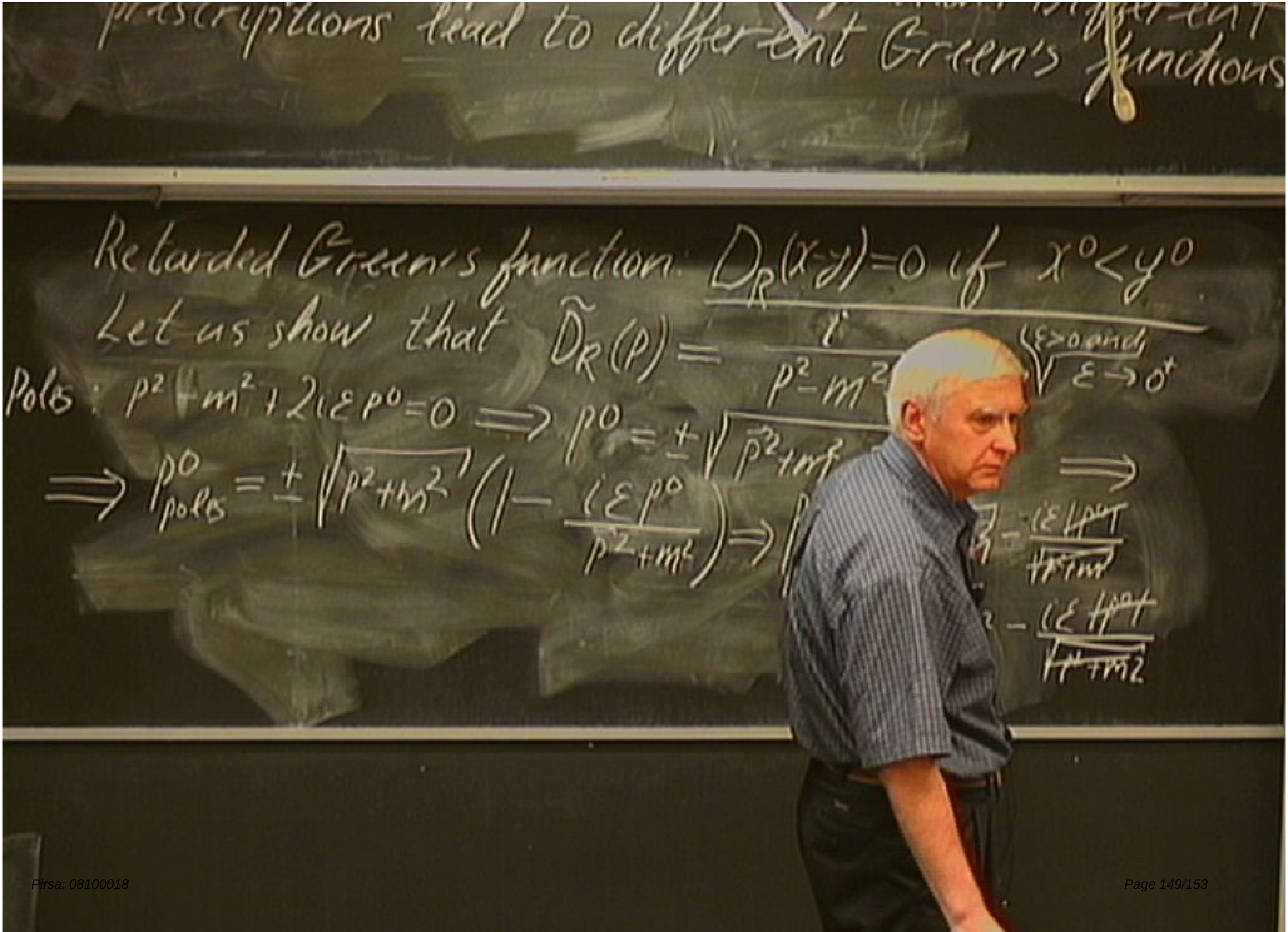


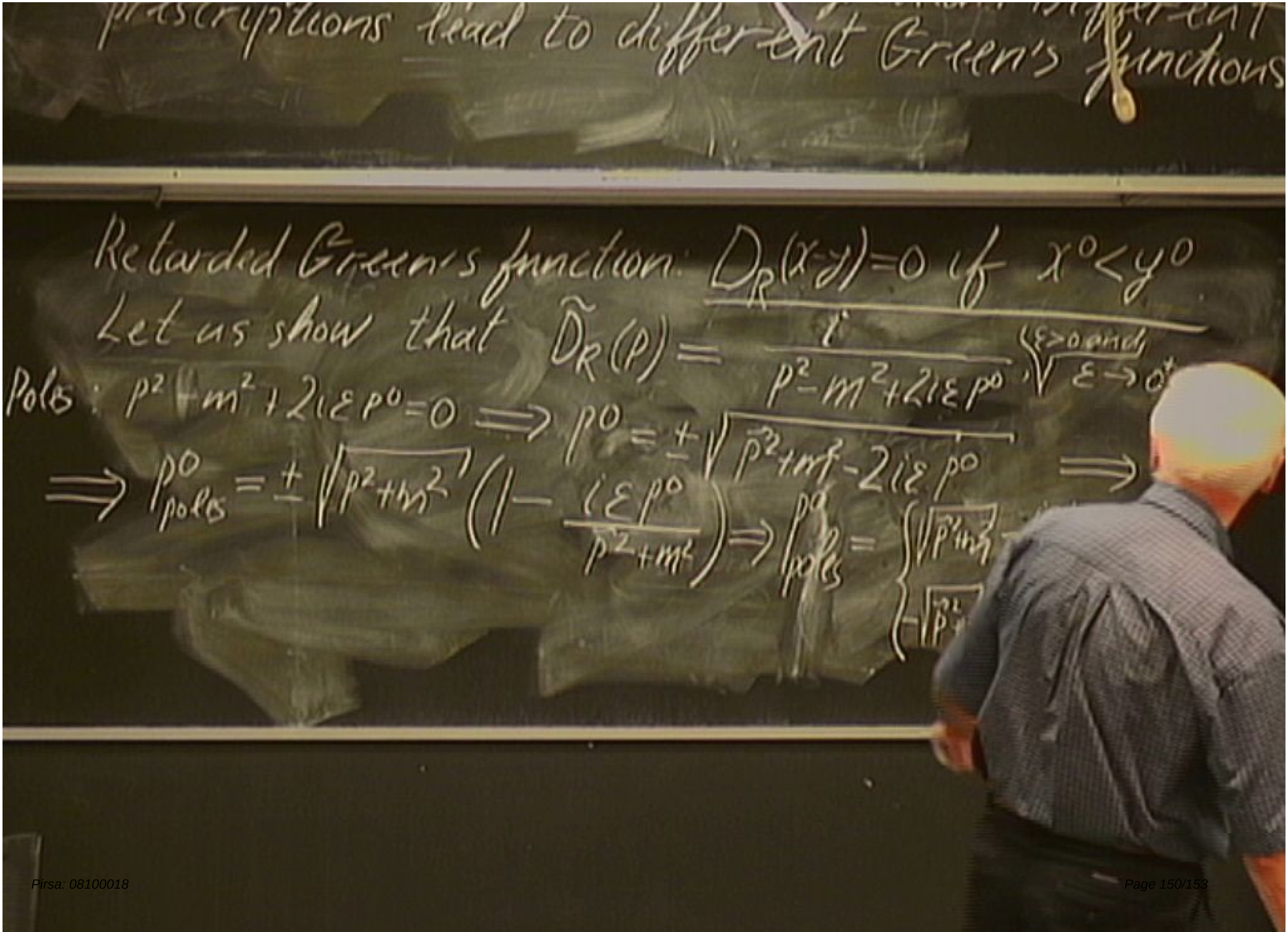
$x^0<y^0$

$\langle 0|\varphi(y)\varphi(x)|0\rangle$

$$D_F(x-y) = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle \equiv$$

$$\equiv \langle 0|T\,\varphi(x)\varphi(y)|0\rangle$$


prescriptions lead to different Green's functions
Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / p^2-m^2
ε>0 and ε→0+
Poles: p^2-m^2+2iεp^0=0 ⇒ p^0=±√(p⃗^2+m^2)


prescriptions lead to different Green's functions
Retarded Green's function: D_R(x-y)=0 if x^0<y^0
Let us show that D̃_R(p) = i / (p²-m²+2iεp^0)
(ε>0 and ε→0+)
Poles: p²-m²+2iεp^0=0 ⟹ p^0 = ± √(p⃗²+m²-2iεp^0) ⟹
⟹ p^0_poles = ± √(p⃗²+m²) (1 - iεp^0/(p⃗²+m²)) ⟹ p^0_poles =

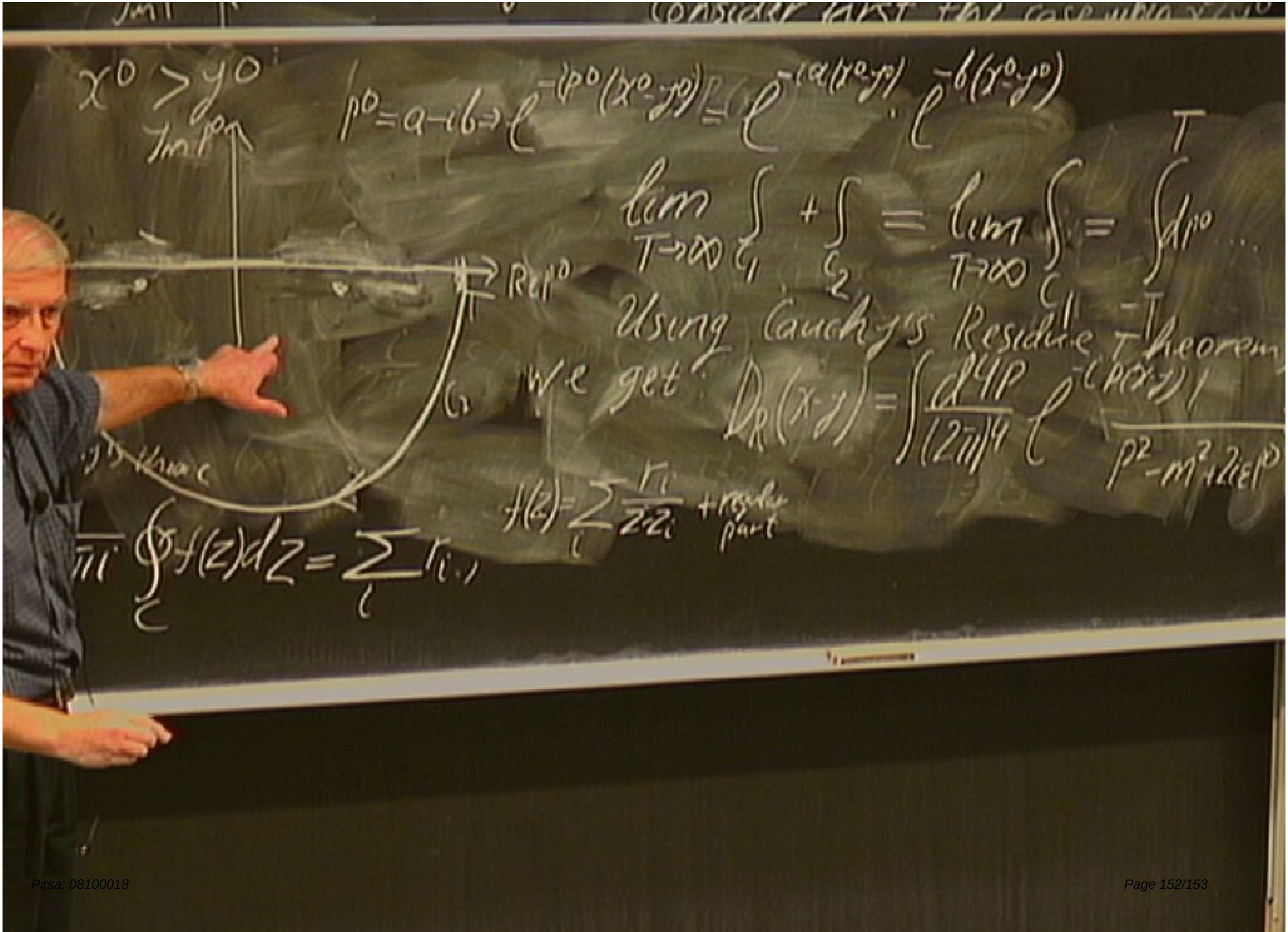
prescriptions lead to different Green's functions

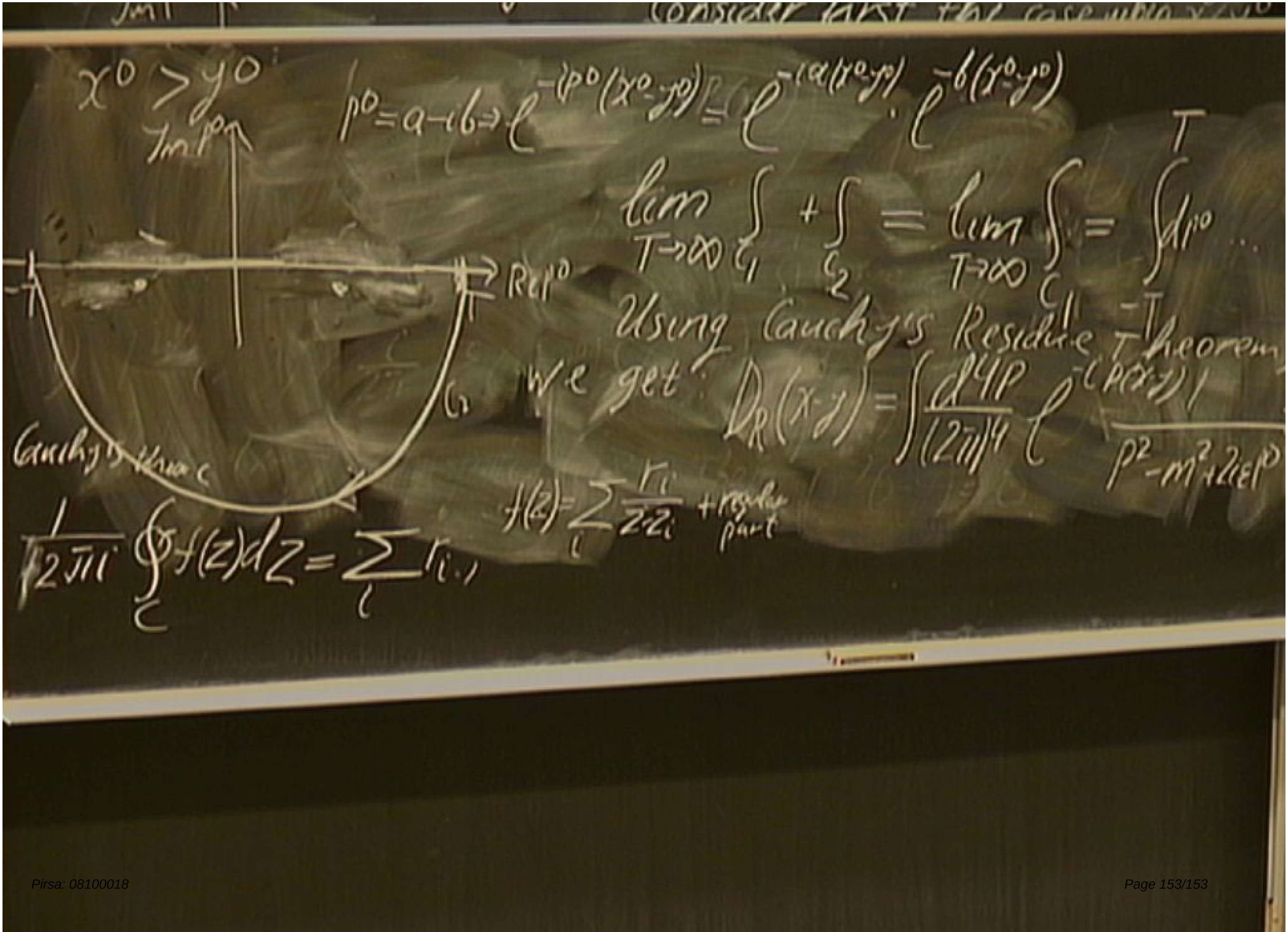
Retarded Green's function: $D_R(x-y)=0$ if $x^0<y^0$

Let us show that $\tilde{D}_R(p) = \frac{i}{p^2-m^2+2i\varepsilon p^0}$ ($\varepsilon>0$ and $\varepsilon\to 0^+$)

Poles: $p^2-m^2+2i\varepsilon p^0=0 \Longrightarrow p^0=\pm\sqrt{\vec{p}^2+m^2-2i\varepsilon p^0} \Longrightarrow$

$$\Longrightarrow p^0_{poles}=\pm\sqrt{\vec{p}^2+m^2}\left(1-\frac{i\varepsilon p^0}{\vec{p}^2+m^2}\right) \Longrightarrow p^0_{poles} = \begin{cases} \sqrt{\vec{p}^2+m^2}-i\varepsilon \\ -\sqrt{\vec{p}^2+m^2}-i\varepsilon \end{cases}$$


x0 > y0
Using Cauchy's Residue Theorem
we get
+ residue part


Consider first the case where x0 > y0
x0 > y0
Im p0
Re p0
p0 = a - ib → e^(-ip0(x0-y0)) = e^(-ia(x0-y0)) · e^(-b(x0-y0))
lim T→∞ ∫C1 + ∫C2 = lim T→∞ ∫C = ∫ from -T to T dp0 ...
Using Cauchy's Residue Theorem
we get: DR(x-y) = ∫ d4p/(2π)4 e^(-ip(x-y)) / (p2 - m2 + 2iεp0)
Cauchy's theorem
1/(2πi) ∮C f(z) dz = Σi ri
f(z) = Σi ri/(z-zi) + regular part