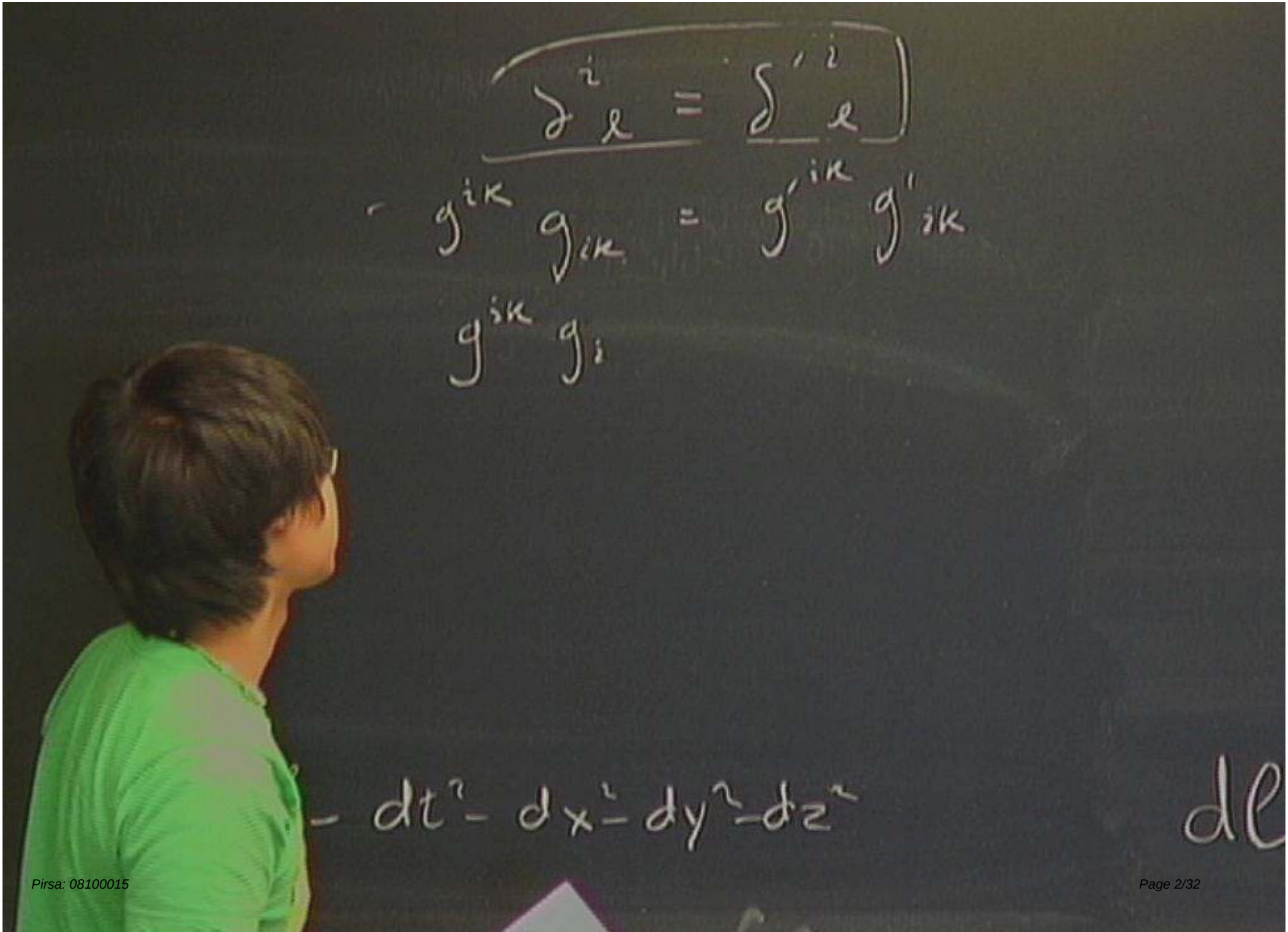
Title: Astrophysics and Cosmology through Problems - 9B

Date: Oct 30, 2008 12:30 PM

URL: http://pirsa.org/08100015

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

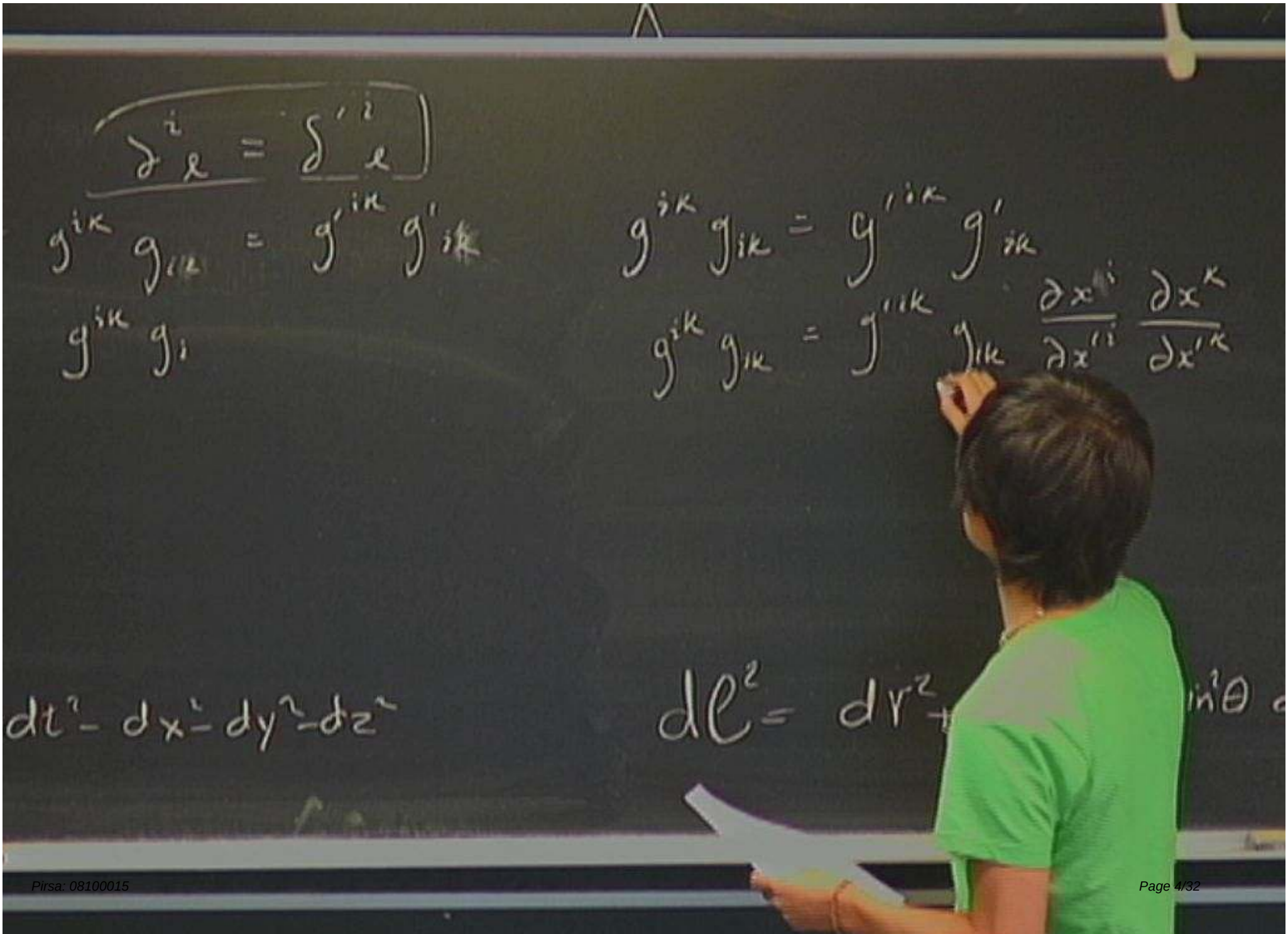

$\delta^i_k = \delta'^i_k$
$g^{ik} g_{ik} = g'^{ik} g'_{ik}$
$g^{ik} g_i$
$dt^2 - dx^2 - dy^2 - dz^2$
$d\ell$

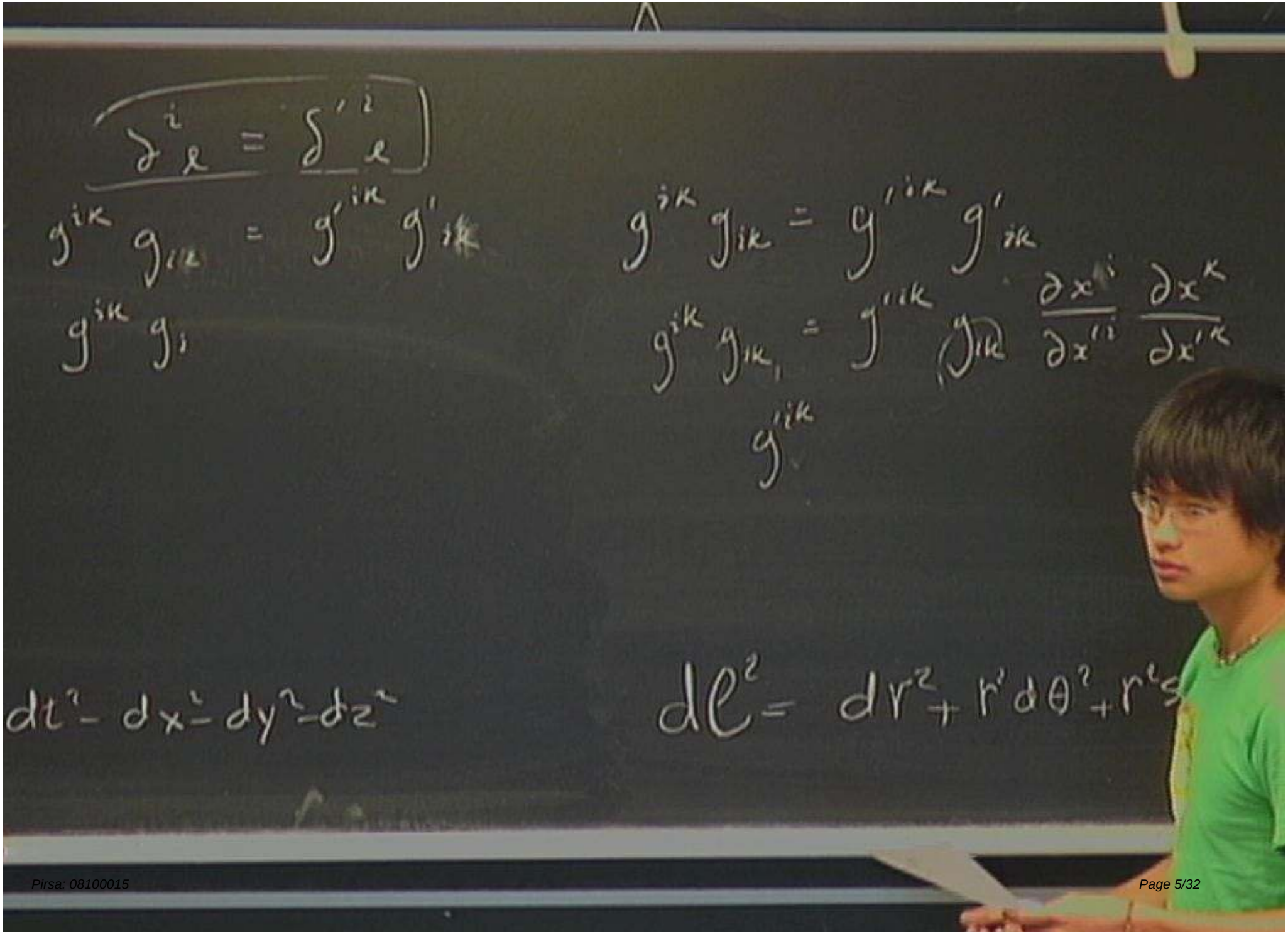
$$\delta^i_k = \delta'^i_k$$

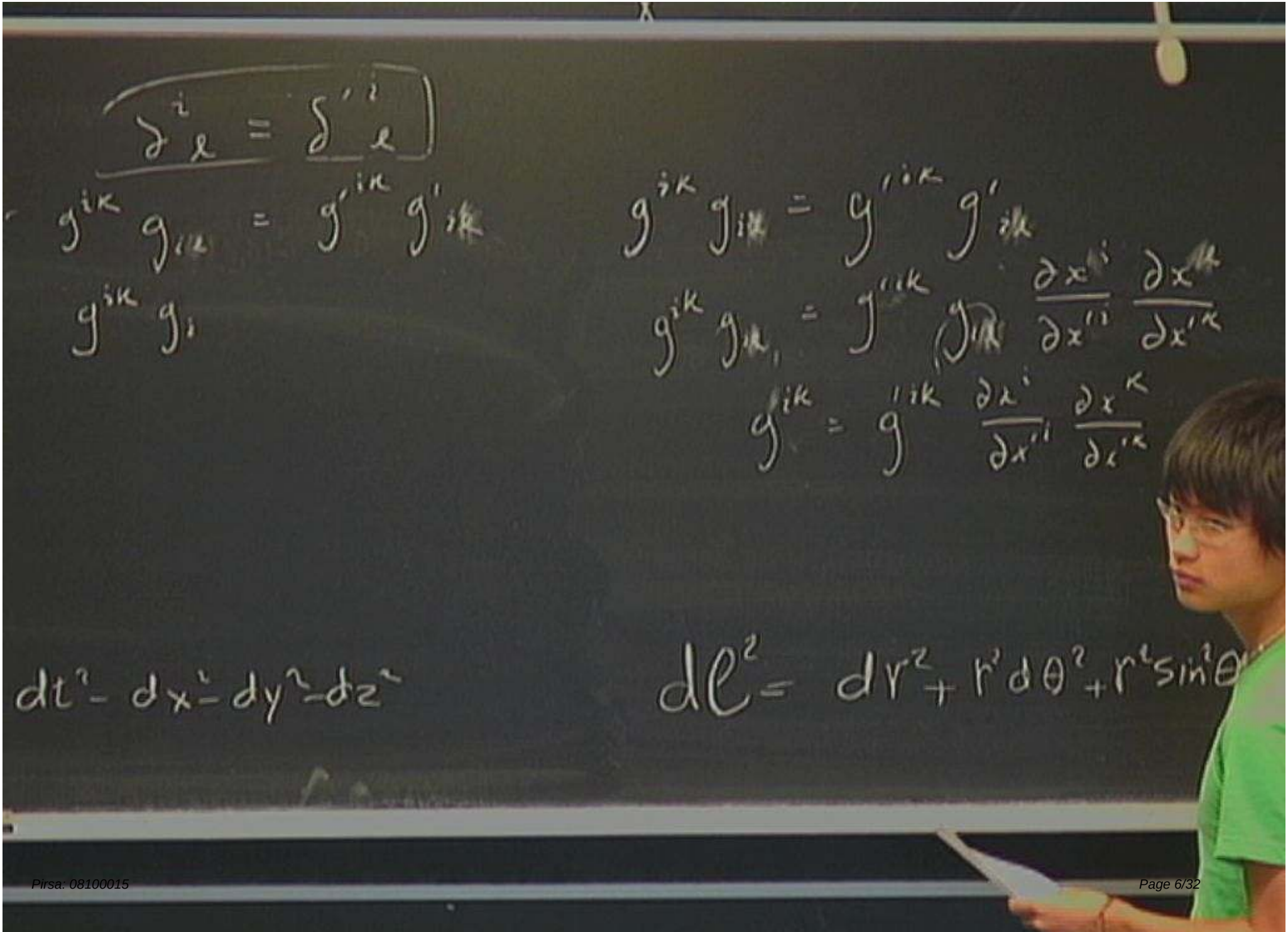
$$g^{ik} g_{ik} = g'^{ik} g'_{ik} \qquad g^{ik} g_{ik} = g'^{ik} g'_{ik}$$

$$g^{ik} g_i$$

$$^2 - dt^2 - dx^2 - dy^2 - dz^2 \qquad dl^2 = dr^2 + r^2 d\theta$$


$\delta^i_k = \delta'^i_k$
$g^{ik} g_{ik} = g'^{ik} g'_{ik}$
$g^{ik} g_i$
$g^{ik} g_{ik} = g'^{ik} g'_{ik}$
$g^{ik} g_{ik} = g'^{ik} g_{ik} \frac{\partial x'^i}{\partial x^i} \frac{\partial x^k}{\partial x'^k}$
$dt^2 - dx^2 - dy^2 - dz^2$
$dl^2 = dr^2 +$
$\sin^2\theta$


$\delta^i_k = \delta'^i_k$
$g^{ik} g_{ik} = g'^{ik} g'_{ik}$
$g^{ik} g_i$
$g^{ik} g_{ik} = g'^{ik} g'_{ik}$
$g^{ik} g_{ik} = g'^{ik} g_{ik} \frac{\partial x^i}{\partial x'^i} \frac{\partial x^k}{\partial x'^k}$
g'^{ik}
$dt^2 - dx^2 - dy^2 - dz^2$
$dl^2 = dr^2 + r^2 d\theta^2 + r^2$


$\delta^i_k = \delta'^i_k$
$g^{ik} g_{ik} = g'^{ik} g'_{ik}$
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$g^{ik} g_{ik} = g'^{ik} g'_{ik} \frac{\partial x'^i}{\partial x'^i} \frac{\partial x'^k}{\partial x'^k}$
$g^{ik} = g'^{ik} \frac{\partial x^i}{\partial x'^i} \frac{\partial x^k}{\partial x'^k}$
$dt^2 - dx^2 - dy^2 - dz^2$
$dl^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta$

$$\delta^i_k = \delta'^i_k$$

$$g^{ik} g_{ik} = g'^{ik} g'_{ik}$$

$$g^{ik} g_{ik} = g'^{ik} g'_{ik}$$

$$g^{ik} g_{ik} = g'^{\alpha\beta} \frac{\partial x^i}{\partial x'^\alpha} \frac{\partial x^k}{\partial x'^\beta}$$

$$g^{ik} = \left| g'^{\alpha\beta} \frac{\partial x^i}{\partial x'^\alpha} \frac{\partial x^k}{\partial x'^\beta} \right|$$

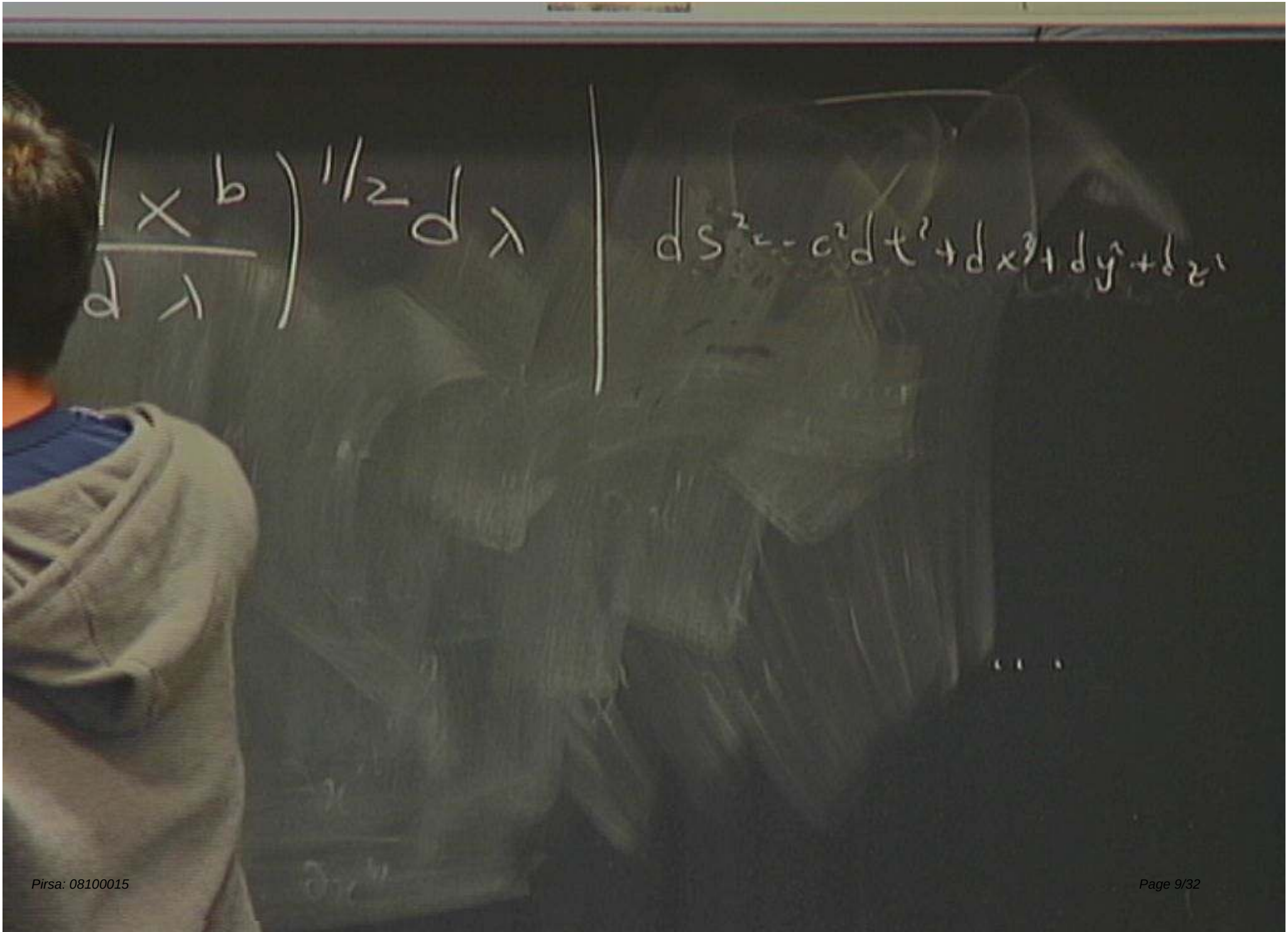
$$g'^{\alpha\beta} = g^{ik} \frac{\partial x'^\alpha}{\partial x^i} \frac{\partial x'^\beta}{\partial x^k}$$

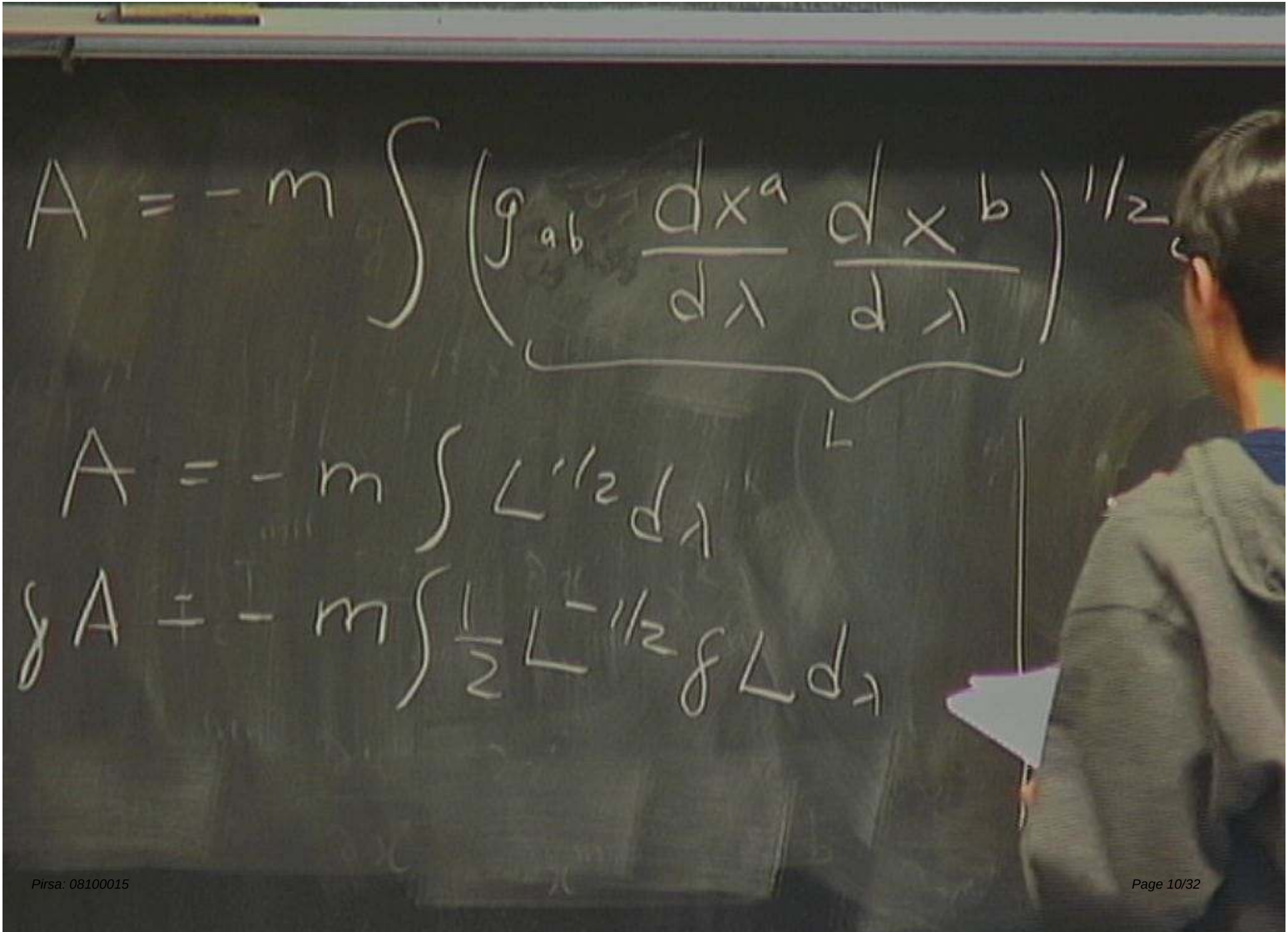
$$dl^2 = dx^2 - dy^2 - dz^2$$

$$dl^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

$$\frac{\partial x^l}{\partial x'^m} \qquad \frac{\partial x'^k}{\partial x^m}$$

$$A = -m \int \left(g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} \right)^{1/2} d\lambda$$


$\left(\frac{dx^b}{d\lambda}\right)^{1/2} d\lambda$
$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$


$A = -m \int \left(g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} \right)^{1/2} d\lambda$
L
$A = -m \int L^{1/2} d\lambda$
$\delta A = -m \int \frac{1}{2} L^{-1/2} \delta L \, d\lambda$

$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$

$d\ell^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d$

$$A = -m \int \left(g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} \right)^{1/2} d\lambda$$

$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$

$$A = -m \int L^{1/2} d\lambda \qquad L$$

$$\delta A = -m \int \frac{1}{2} \delta L \, d\lambda$$

$$L = g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} \qquad \lambda = s$$

$$= -c^2 \left(\frac{dt}{ds}\right)^2 + \left(\frac{dx}{ds}\right)^2 + \cdots + \left(\frac{dz}{ds}\right)^2$$

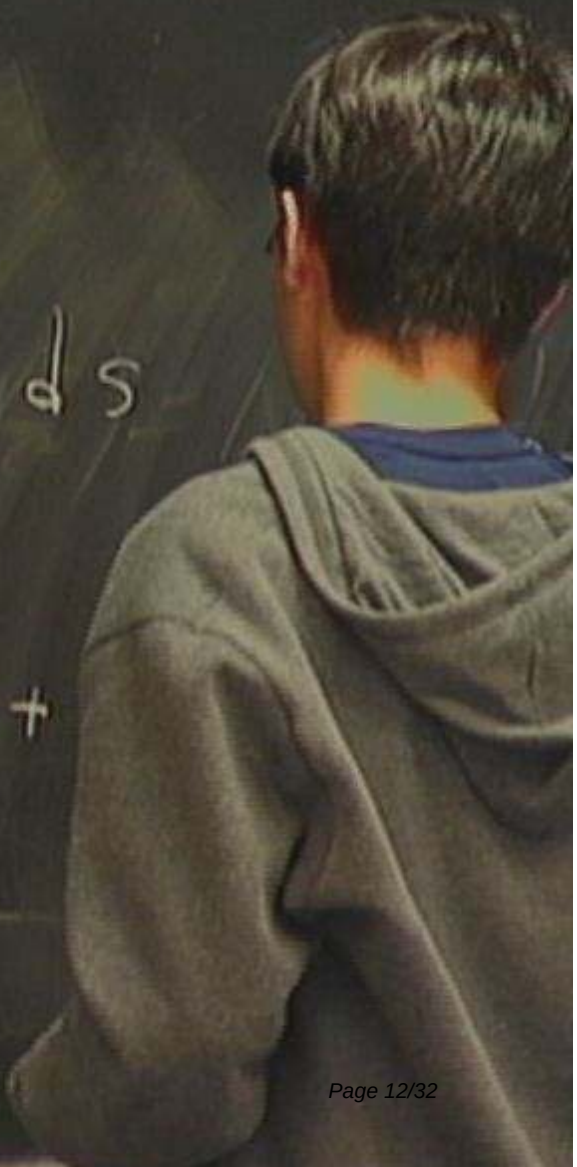
$$= 1$$

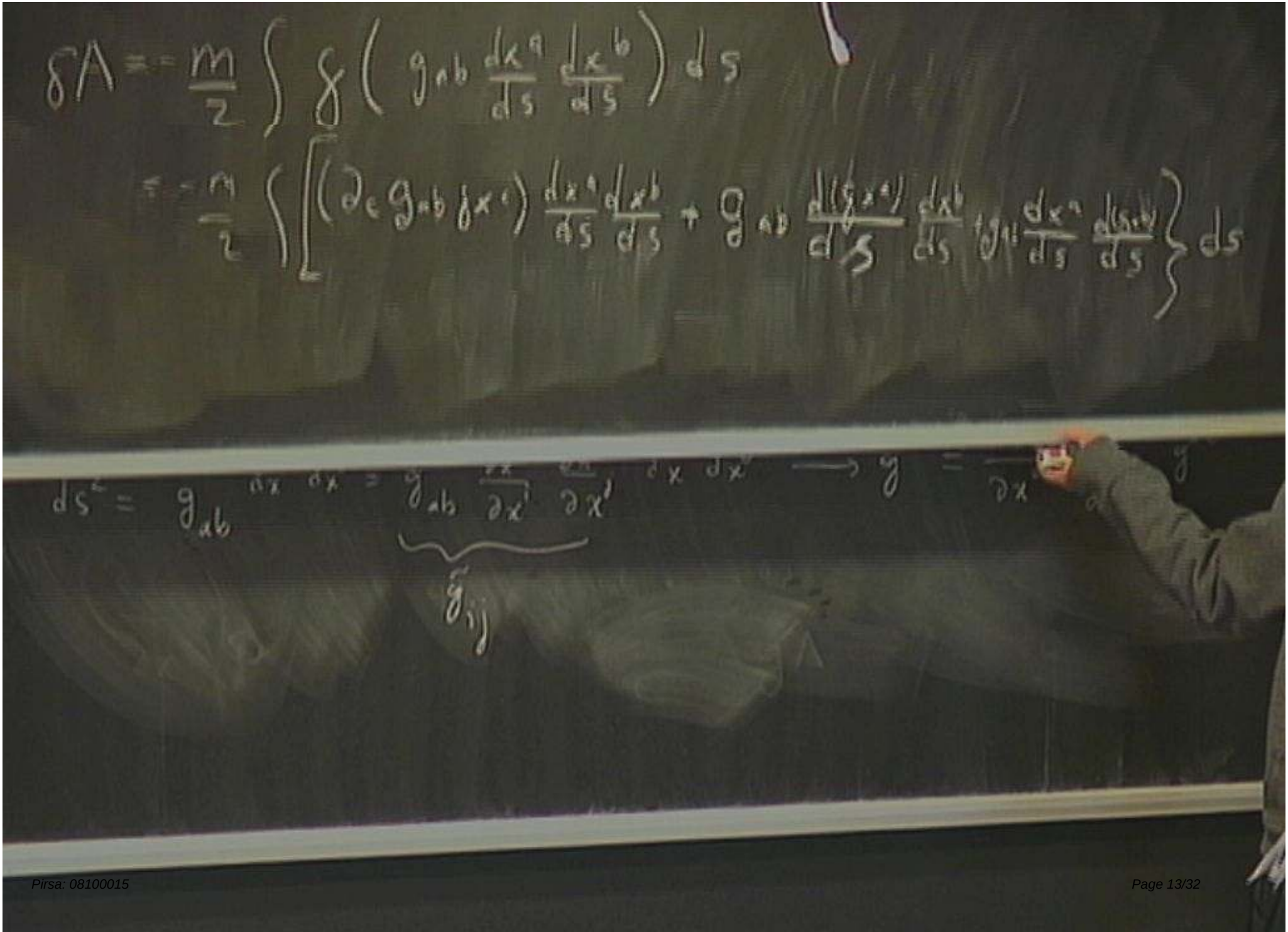
$$x^a \to x^a + \delta x^a$$

$$g^{ab} \to g^{ab} + \partial_c g^{ab}\, \delta x^c$$

$$\delta A = -\frac{m}{2} \int \delta \left(g_{ab} \frac{dx^a}{ds} \frac{dx^b}{ds} \right) ds$$

$$= -\frac{m}{2} \int \left[(\partial_c g_{ab}\, \delta x^c) \frac{dx^a}{ds} \frac{dx^b}{ds} + \right.$$




$\delta A = -\frac{m}{2}\int \delta\left(g_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds}\right) ds$
$= -\frac{m}{2}\int\left\{\left[\left(\partial_c g_{ab}\,\delta x^c\right)\frac{dx^a}{ds}\frac{dx^b}{ds} + g_{ab}\frac{d(\delta x^a)}{ds}\frac{dx^b}{ds} + g_{ab}\frac{dx^a}{ds}\frac{d(\delta x^b)}{ds}\right\} ds$
$ds^2 = g_{ab}\,dx\,dx = g_{ab}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\,dx'\,dx' \longrightarrow g =$
$\bar{g}_{ij}$

$$\delta A = -\frac{m}{2} \int \left(g_{ab} \frac{dx^a}{ds} \frac{dx^b}{ds} \right) d$$

$$= -\frac{m}{2} \int \left[(\partial_c g_{ab}\, \delta x^c) \frac{dx^a}{ds} \frac{dx^b}{ds} + g_{ab} \frac{d(\delta x^a)}{ds} \frac{dx^b}{ds} + g_{ab} \frac{dx^a}{ds} \frac{d(\delta x^b)}{ds} \right\} ds$$

Calculate

$$\int g_{ab}$$

Calculate B

$$\int g_{ab} \frac{d\delta x^a}{ds} \frac{dx^b}{ds} d\lambda$$

$$= g_{ab} \frac{dx^b}{ds} \delta x^a \Big| - \int \frac{d}{ds}\left(g_{ab} \frac{dx^b}{ds}\right) \delta x^a \, ds$$

$$= -\int \left\{ \frac{\partial g_{ab}}{ds} \frac{dx^b}{ds} + g_{ab} \frac{d^2 x^b}{ds^2} \right\} \delta x^a \, ds$$

$$\int g_{ab} \frac{dx^a}{ds} \frac{d\delta x^b}{ds} ds$$

$$= -\int \left\{ \frac{dg_{ab}}{ds} \frac{dx^a}{ds} + g_{ab} \frac{d^2x^a}{ds^2} \right\} \delta x^b \, ds$$

$\frac{dx^b}{ds}) \delta x^a ds$

$\int \delta x^a ds$

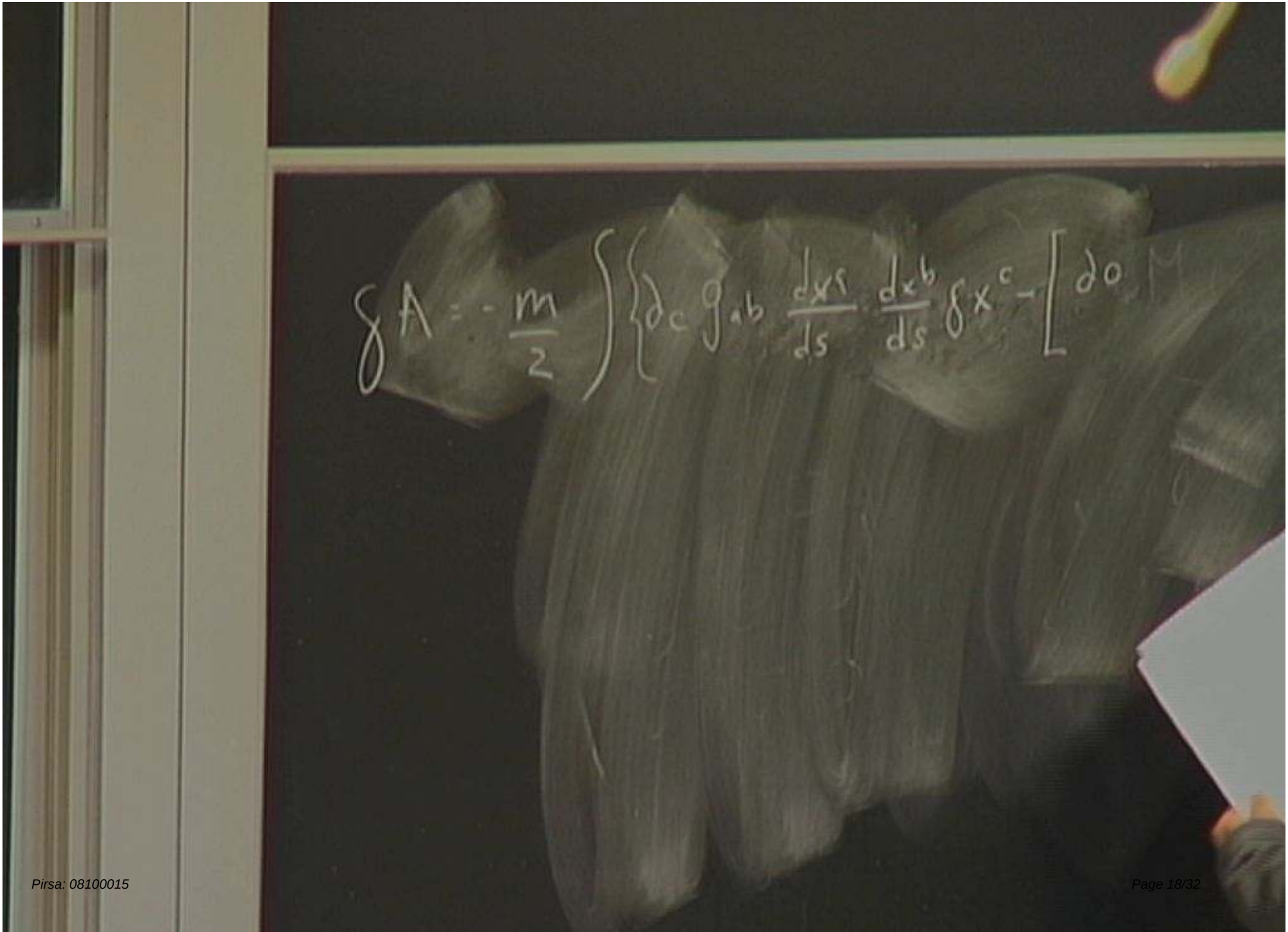
Term C

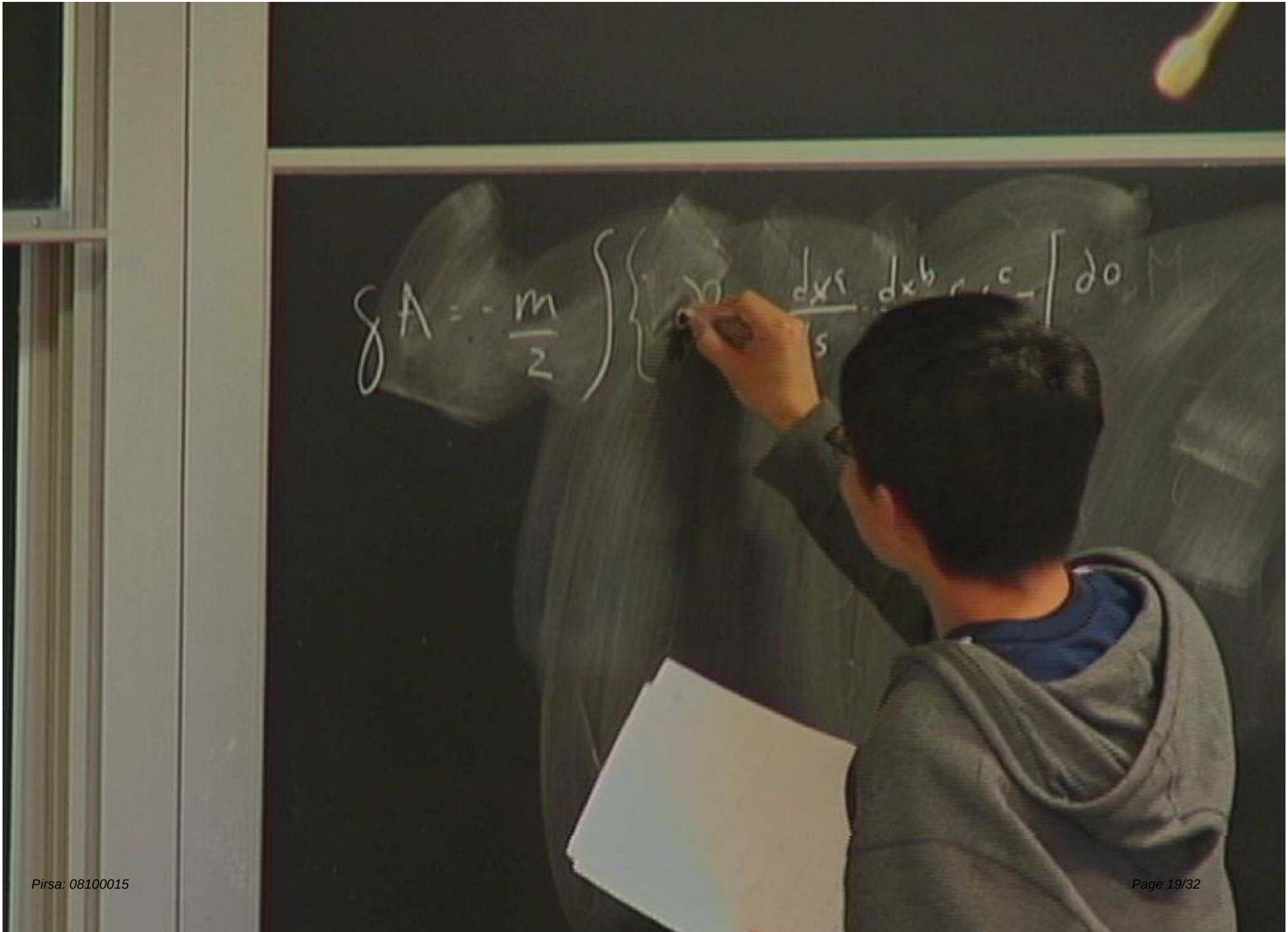
$$\int g_{ab}\,\frac{dx^a}{ds}\,\frac{d\,\delta x^b}{ds}\,ds$$

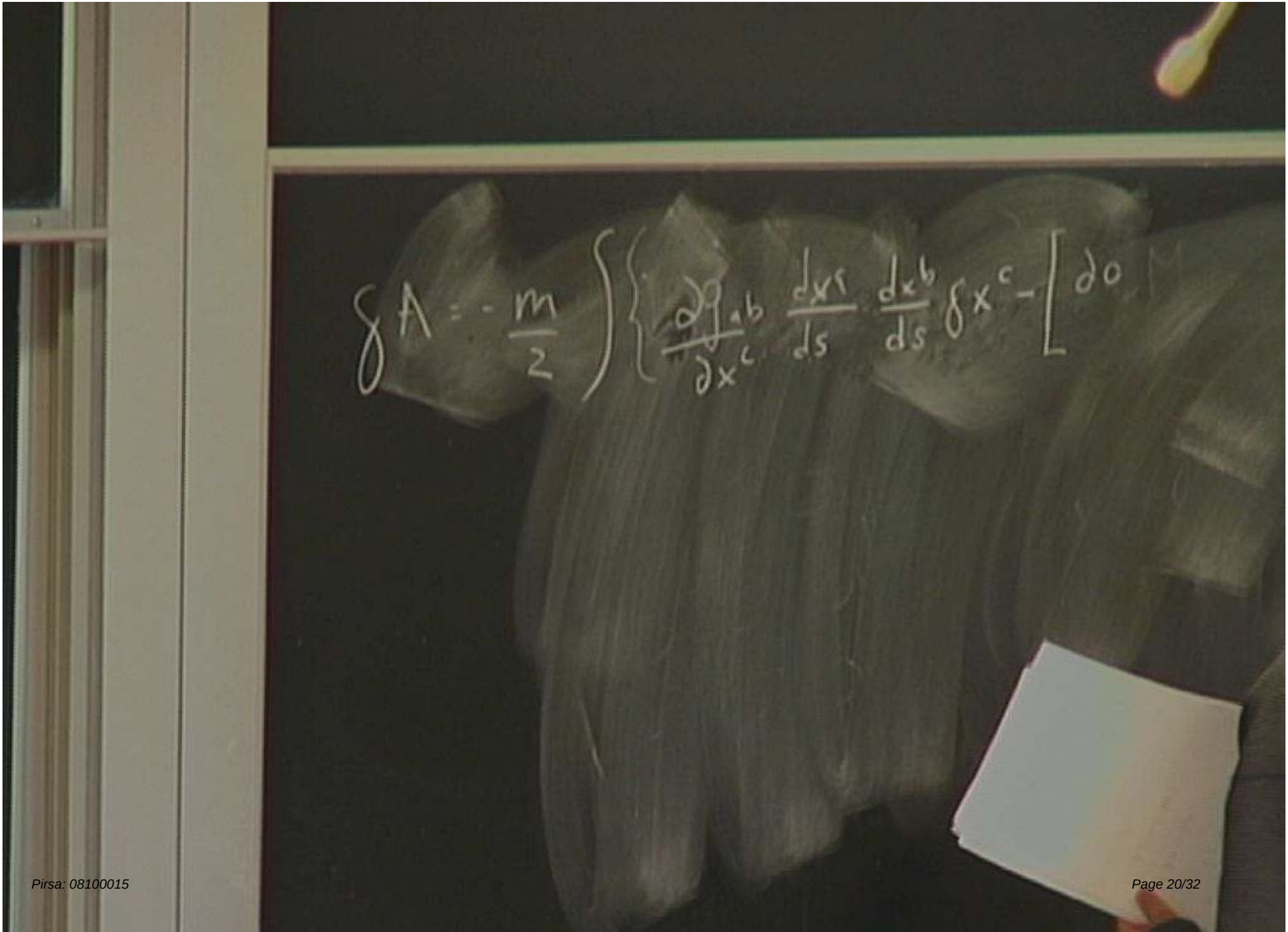
$$= -\int \left\{ \frac{dg_{ab}}{ds}\,\frac{dx^a}{ds} + g_{ab}\,\frac{d^2x^a}{ds^2} \right\} \delta x^b\,ds$$

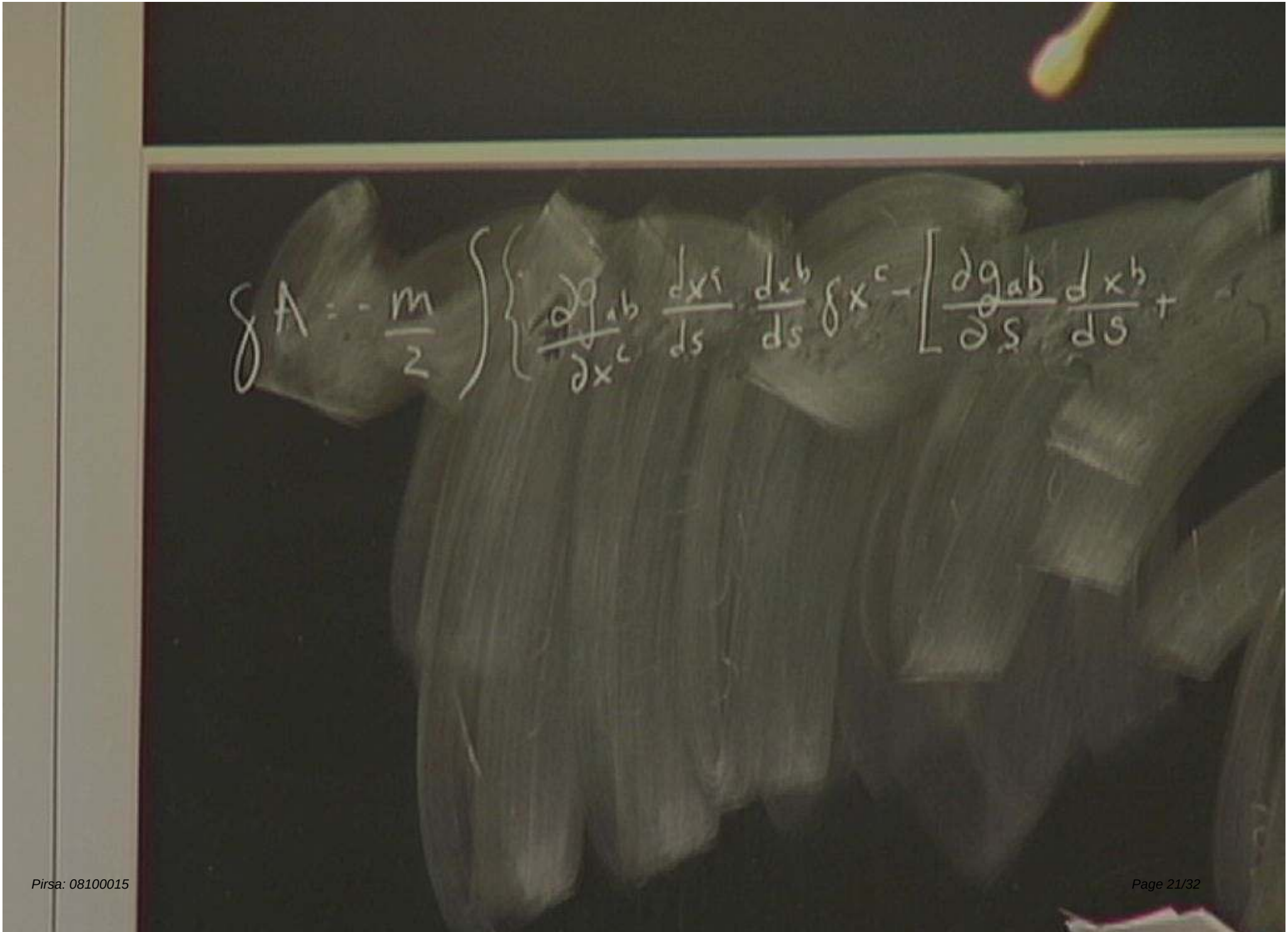
$\left(\frac{dx^b}{ds}\right)\delta x^a\,ds$

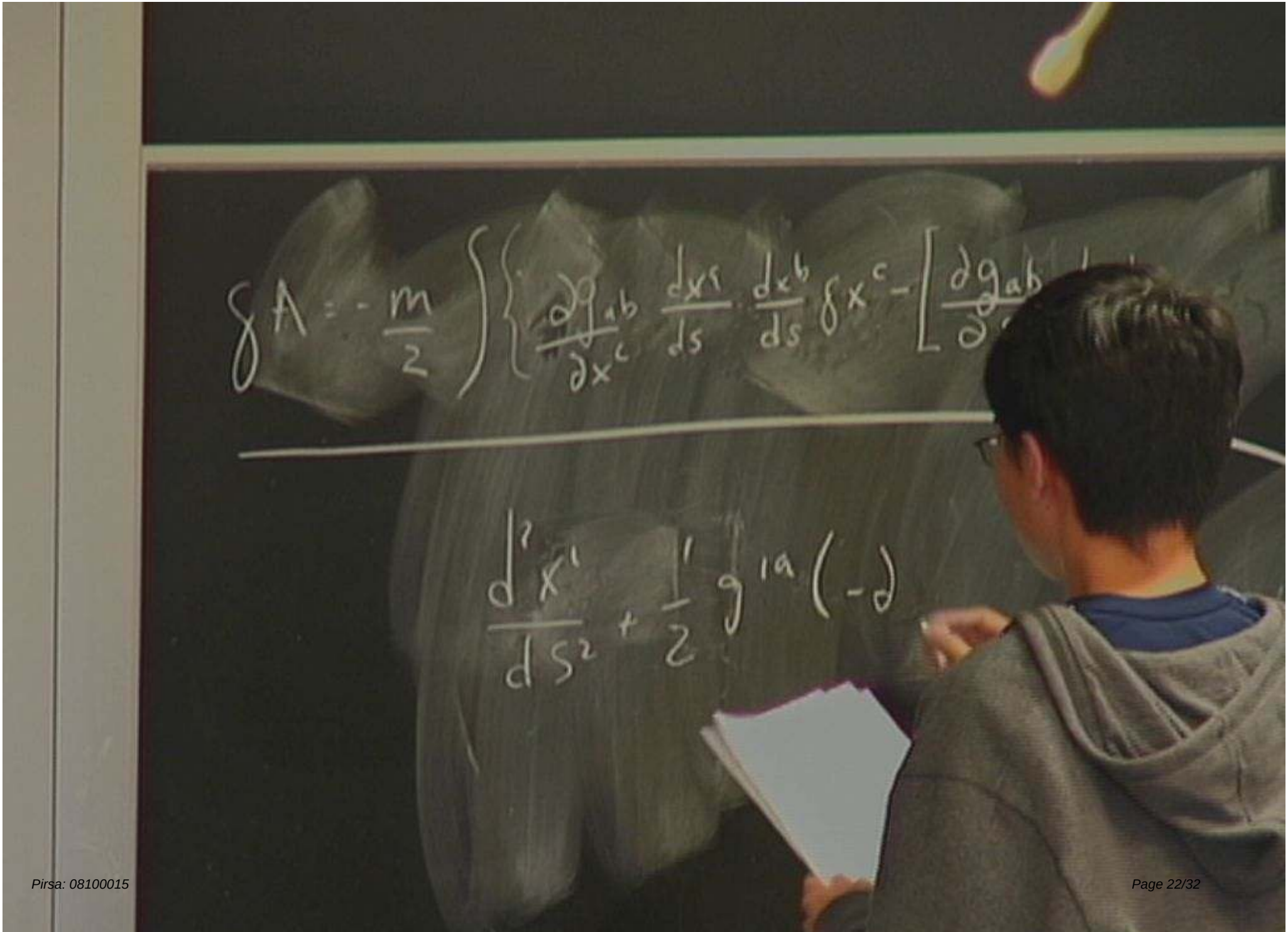
$\int \delta x^a\,ds$


$\delta A = -\frac{m}{2} \int \{ \partial_c g_{ab} \frac{dx^a}{ds} \frac{dx^b}{ds} \delta x^c - [\partial$


δA = - m/2

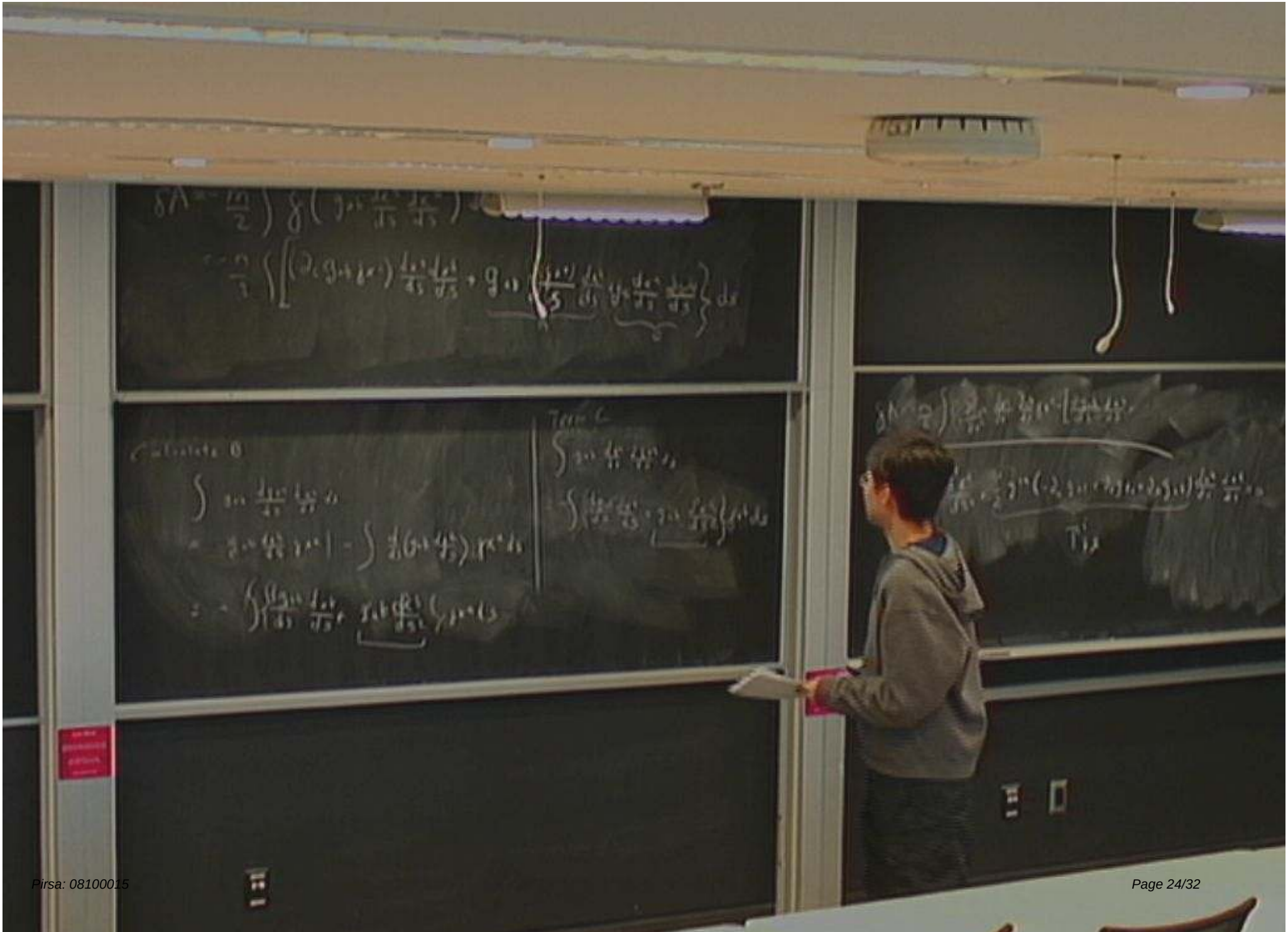

$\delta A = -\frac{m}{2} \int \left\{ \frac{\partial g_{ab}}{\partial x^c} \frac{dx^a}{ds} \frac{dx^b}{ds} \delta x^c - \left[d\sigma \right.$


$\delta A = -\frac{m}{2} \int \left\{ -\frac{\partial g_{ab}}{\partial x^c} \frac{dx^a}{ds} \frac{dx^b}{ds} \delta x^c - \left[\frac{\partial g_{ab}}{\partial s} \frac{dx^b}{ds} + \right. \right.$


$\delta A = -\frac{m}{2}$
$\frac{d^2x^1}{ds^2} + \frac{1}{2}$

$$\frac{m}{2}\int\left\{\frac{\partial g_{ab}}{\partial x^c}\frac{dx^a}{ds}\frac{dx^b}{ds}\delta x^c - \left[\frac{\partial g_{ab}}{\partial s}\frac{dx^b}{ds} + \right.\right.$$

$$\frac{d^2x^i}{ds^2} + \underbrace{\frac{1}{2}g^{ia}\left(-\partial_a g_{kl} + \partial_k g_{la} + \partial_l g_{ak}\right)}_{\Gamma^i_{kl}}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0$$



Calculate B

$$\int g_{ab} \frac{d\delta x^a}{ds} \frac{dx^b}{ds} d\lambda$$

$$= g_{ab} \frac{dx^b}{ds} \delta x^a \Big| - \int \frac{d}{ds}\left(g_{ab} \frac{dx^b}{ds}\right) \delta x^a \, ds$$

$$= -\int \left\{ \frac{dg_{ab}}{ds} \frac{dx^b}{ds} + \underbrace{\Gamma_{ab} \frac{d^2x^b}{ds^2}} \right\} \delta x^a \, ds$$

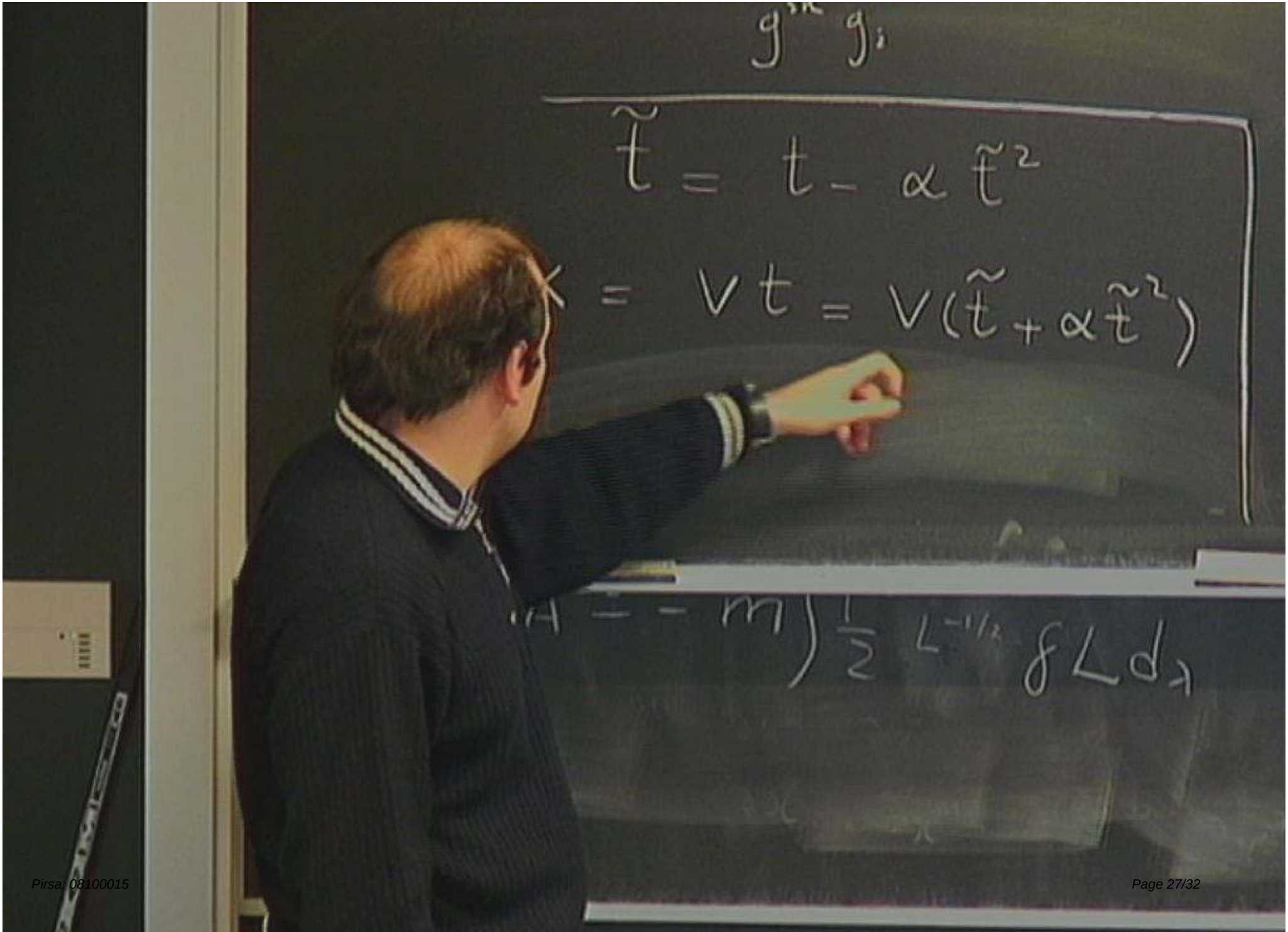
Ter

$$\int$$

$$= -\int$$

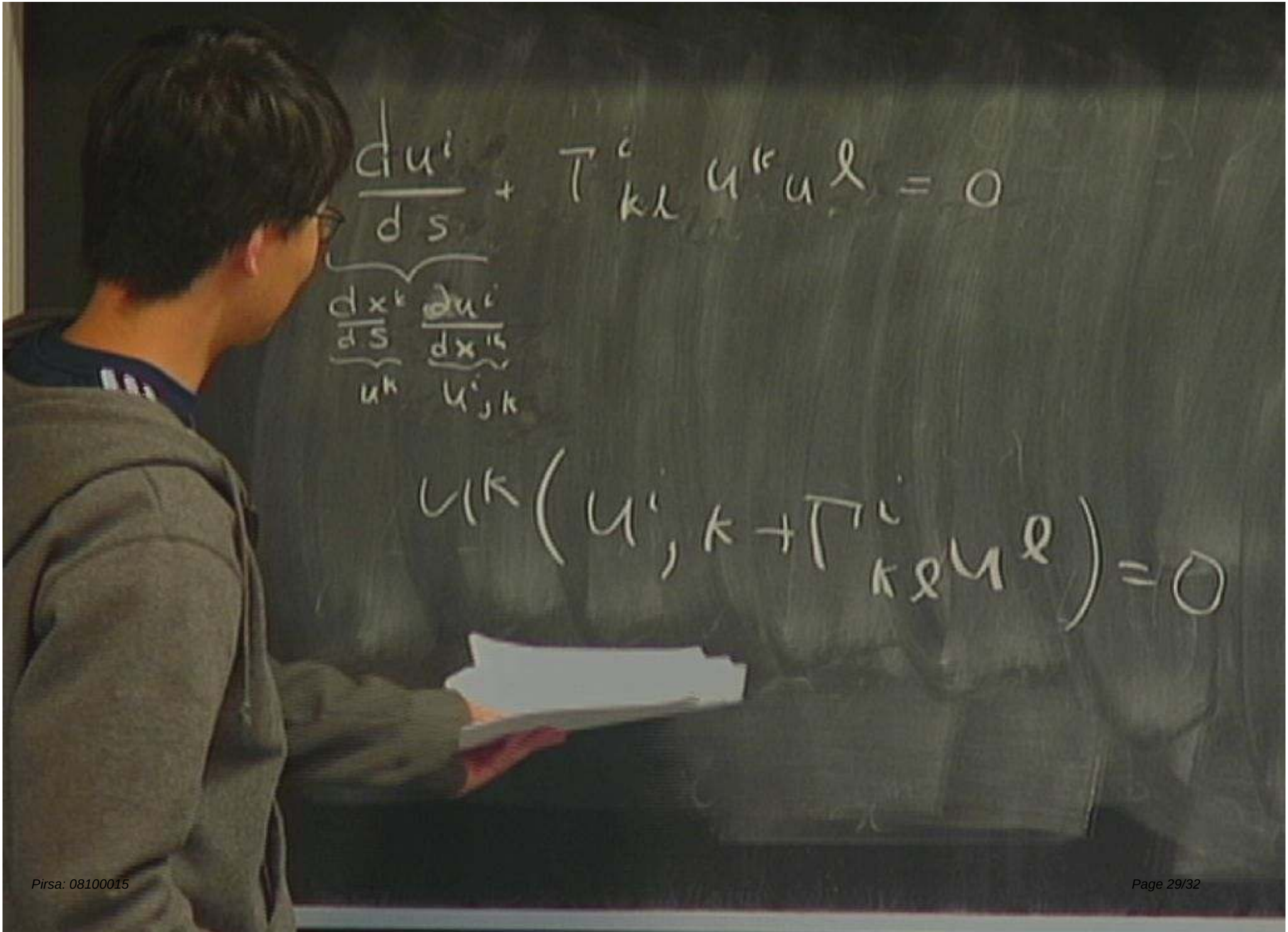
$$\delta A = -\frac{m}{2}\int\left\{\frac{\partial g_{ab}}{\partial x^c}\frac{dx^a}{ds}\frac{dx^b}{ds}\delta x^c - \left[\frac{\partial g_{ab}}{\partial s}\frac{dx^b}{ds}\right.\right. +$$

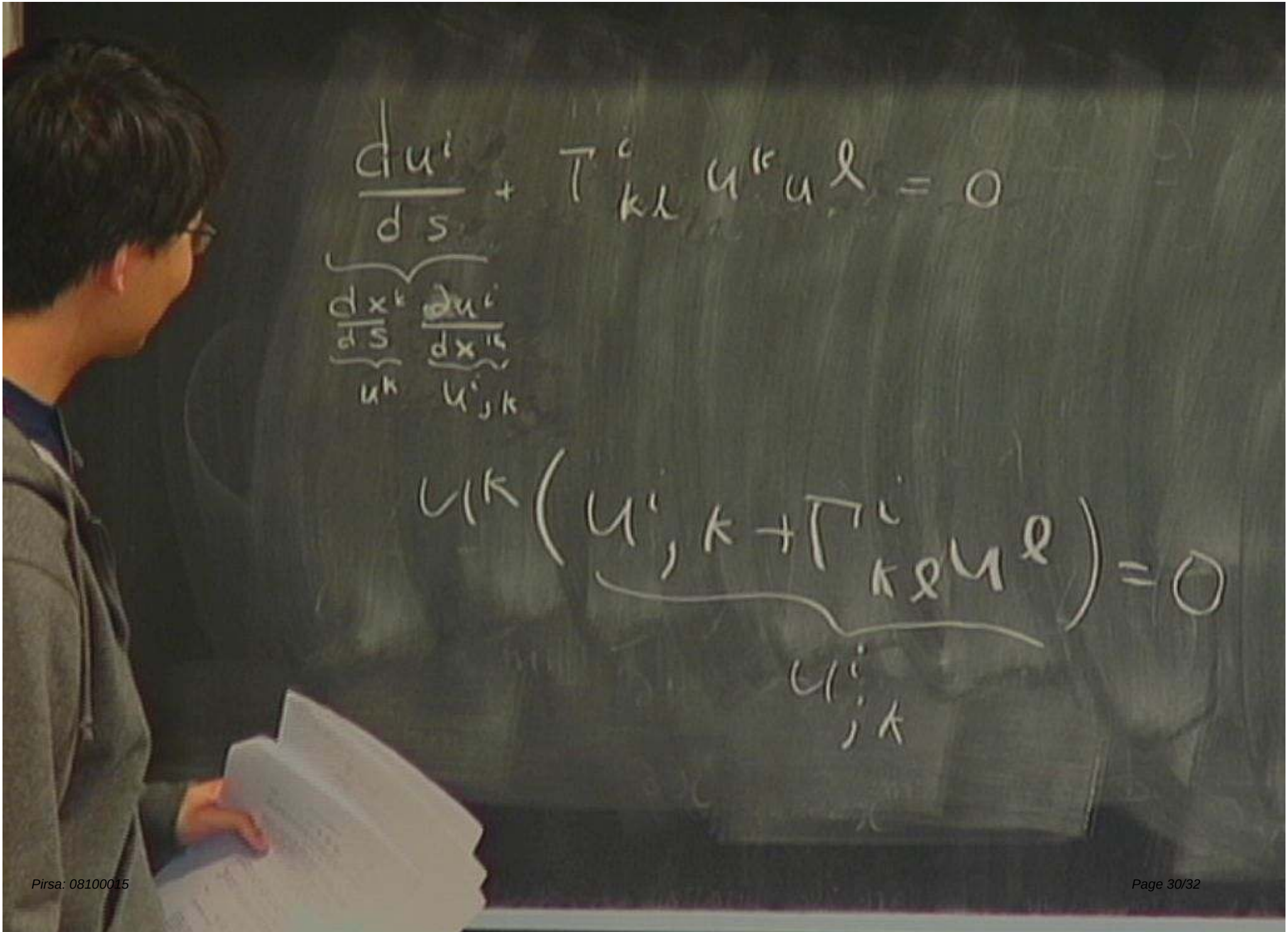
$$\frac{d^2x^i}{ds^2} + \underbrace{\frac{1}{2}g^{ia}\left(-\partial_a g_{kl} + \partial_k g_{la} + \partial_l g_{ak}\right)}_{\Gamma^i_{kl}}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0$$


$\tilde{t} = t - \alpha \tilde{t}^2$
$= Vt = V(\tilde{t} + \alpha \tilde{t}^2)$

$$\delta A = -\frac{m}{2}\int\left\{\frac{\partial g_{ab}}{\partial x^c}\frac{dx^a}{ds}\frac{dx^b}{ds}\delta x^c - \left[\frac{\partial g_{ab}}{\partial s}\frac{dx^b}{ds}\right.\right. +$$

$$L^{-1/2}\left[\frac{d^2x^l}{ds^2} + \underbrace{\frac{1}{2}g^{la}\left(-\partial_a g_{kl} + \partial_k g_{la} + \partial_l g_{ak}\right)}_{\Gamma^l_{k\ell}}\frac{dx^k}{ds}\frac{dx^\ell}{ds}\right] = 0$$


$\frac{du^i}{ds} + \Gamma^i_{kl} u^k u^l = 0$
$\frac{dx^k}{ds} \frac{\partial u^i}{\partial x^k}$
$u^k \quad u^i{}_{,k}$
$u^k \left(u^i{}_{,k} + \Gamma^i_{kl} u^l \right) = 0$


$\frac{du^i}{ds} + \Gamma^i_{kl} u^k u^l = 0$
$\frac{dx^k}{ds} \frac{du^i}{dx^k}$
$u^k \quad u^i_{;k}$
$u^k \left(u^i_{;k} + \Gamma^i_{kl} u^l \right) = 0$
$u^i_{;k}$

$$\frac{du^i}{ds} + \Gamma^i_{kl} u^k u^l = 0$$

$$\frac{dx^k}{ds} \frac{\partial u^i}{\partial x^k}$$

$$u^k \quad u^i_{;k}$$

$$u^k \left(u^i_{;k} + \Gamma^i_{kl} u^l \right) = 0$$

$$u^i_{;k}$$

$$\frac{du^i}{ds} + \Gamma^i_{kl} u^k u^l = 0$$

$$\frac{dx^k}{ds} \frac{\partial u^i}{\partial x^k}$$

$u^k \quad u^i_{;k}$

$$u^k \left(u^i_{;k} + \Gamma^i_{kl} u^l \right) = 0$$

$$u^i_{;k}$$