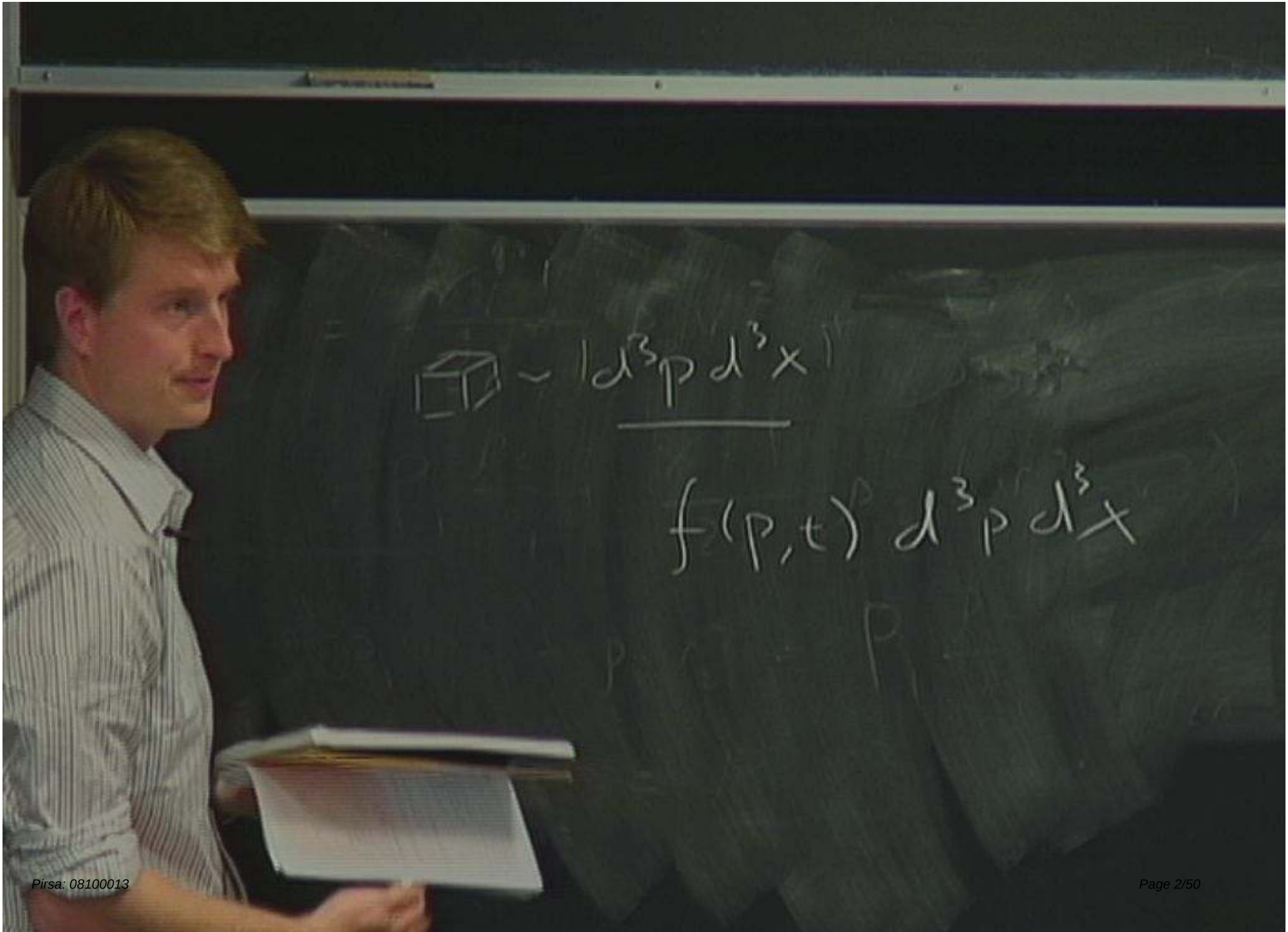
Title: Astrophysics and Cosmology through Problems - 7B

Date: Oct 16, 2008 12:30 PM

URL: http://pirsa.org/08100013

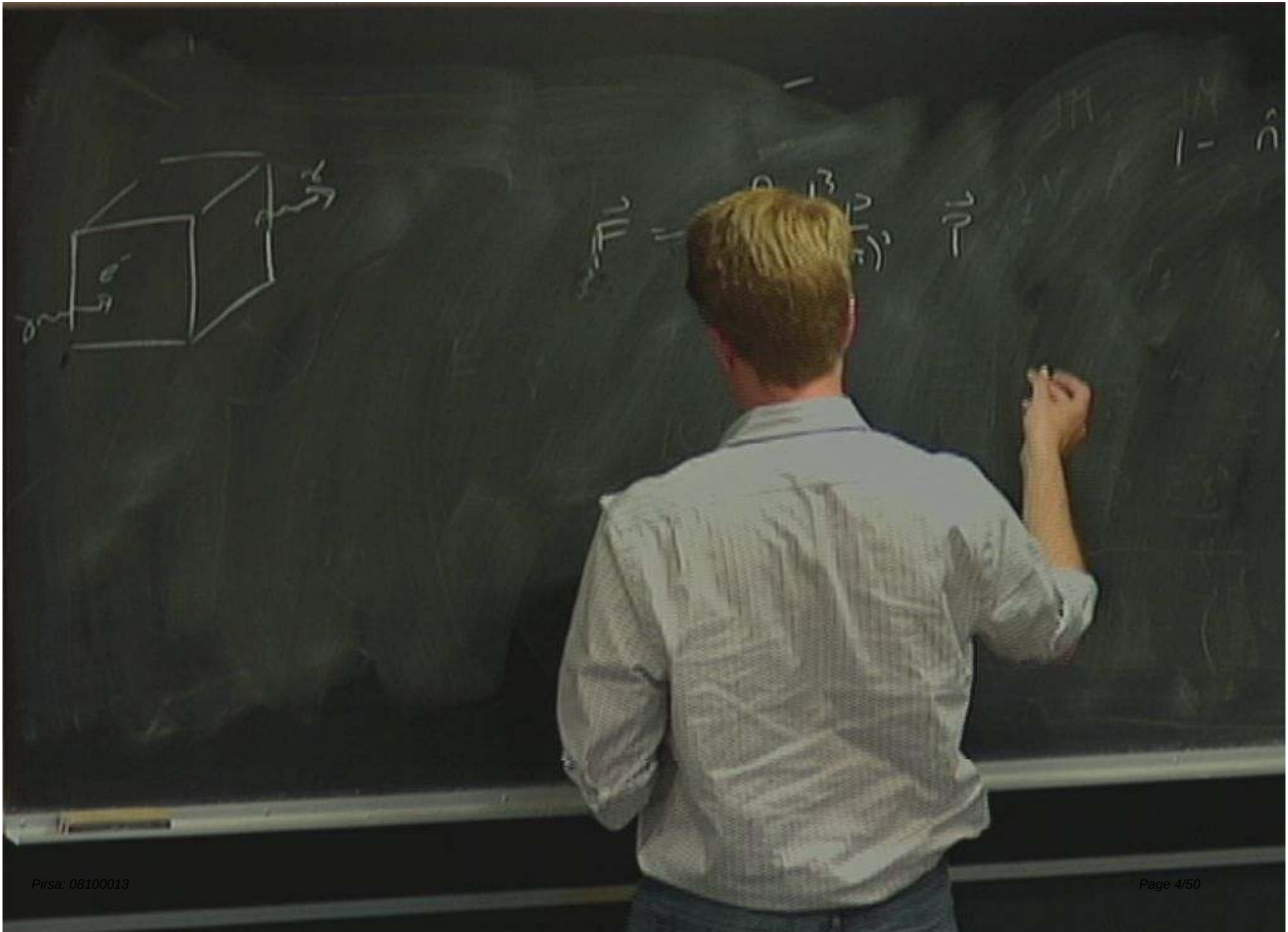
Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

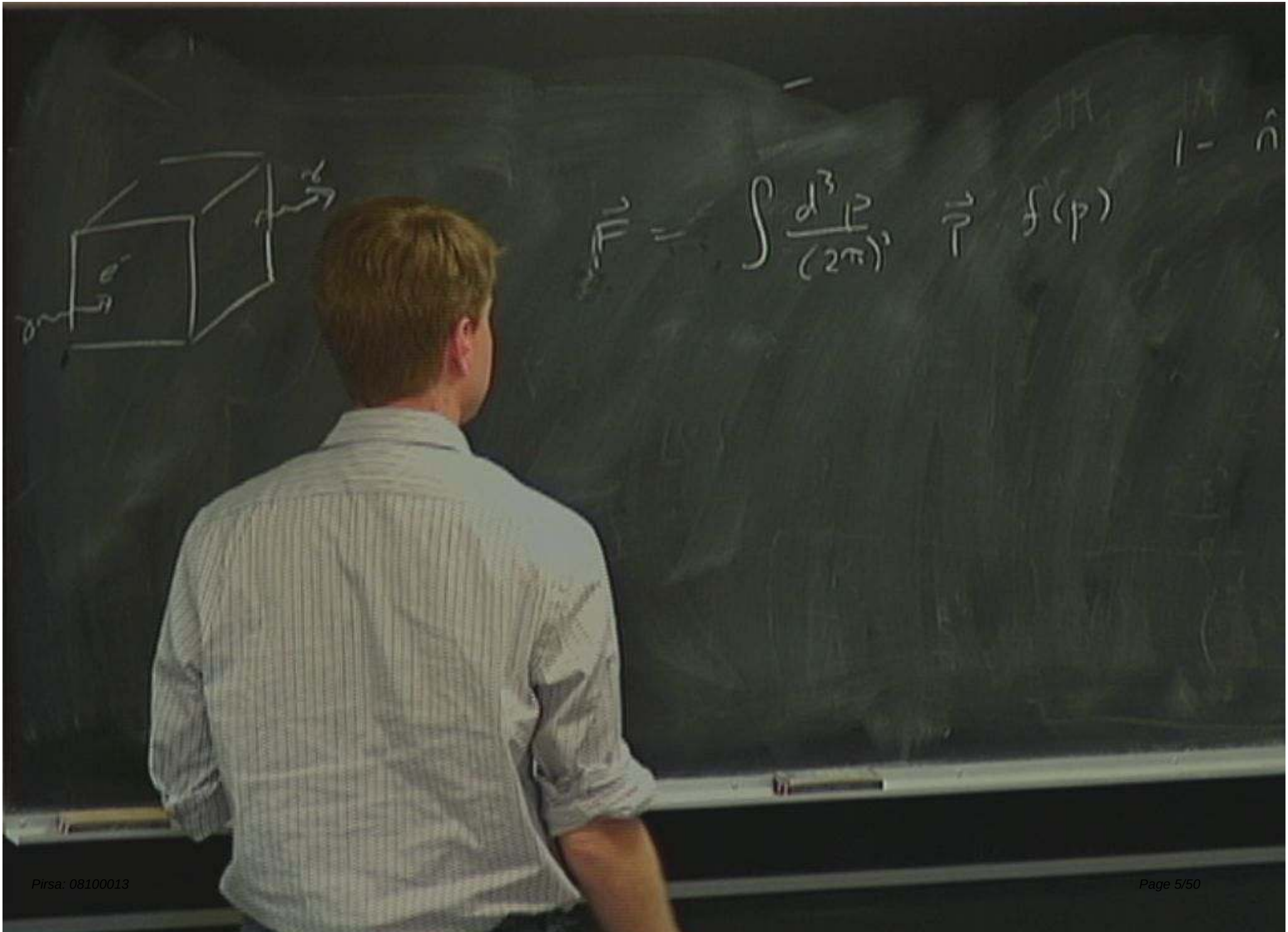

$\sim \int d^3p\, d^3x$
$f(p,t)\, d^3p\, d^3x$

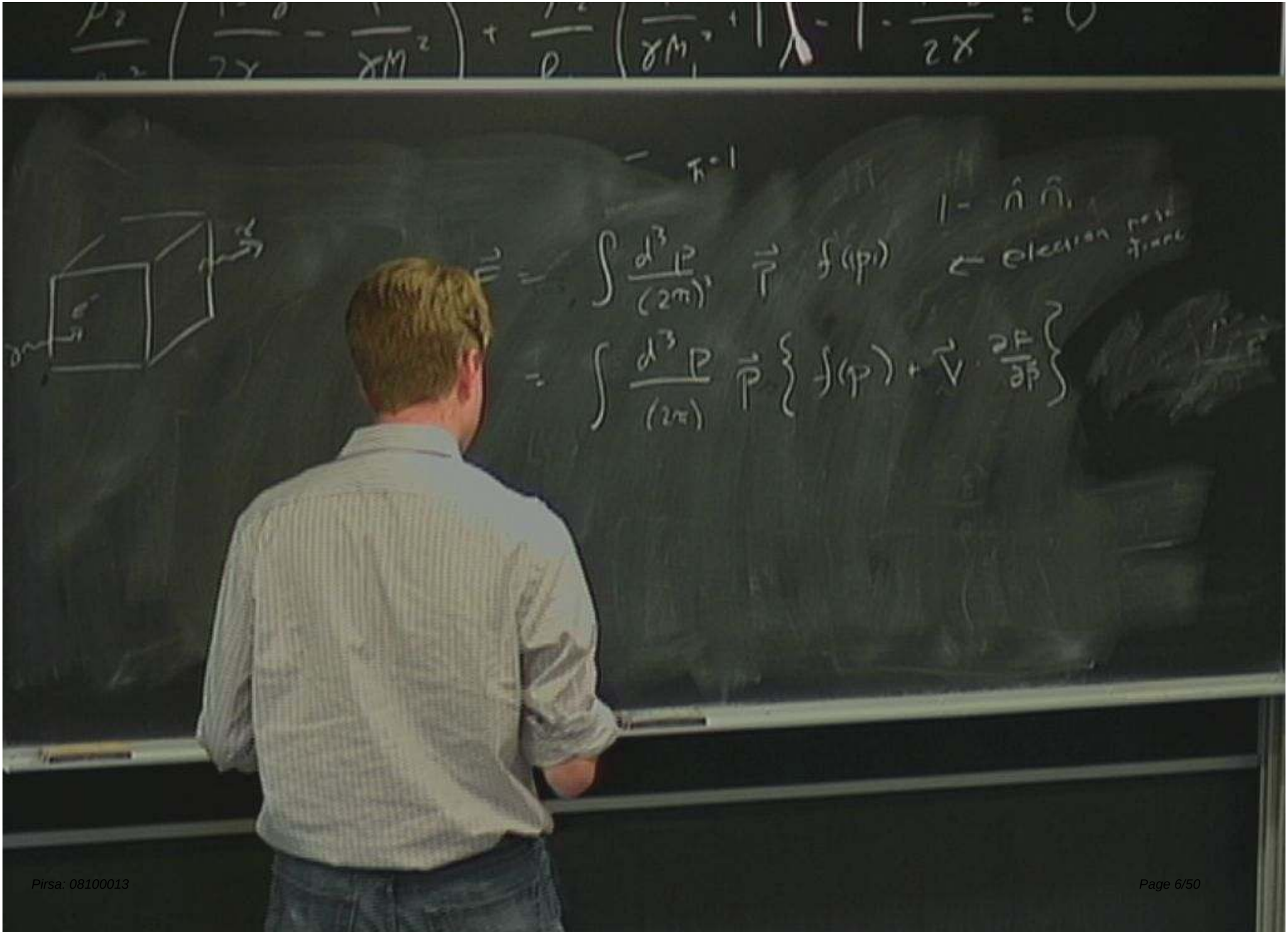
$\sim |d^3p\, d^3x|$

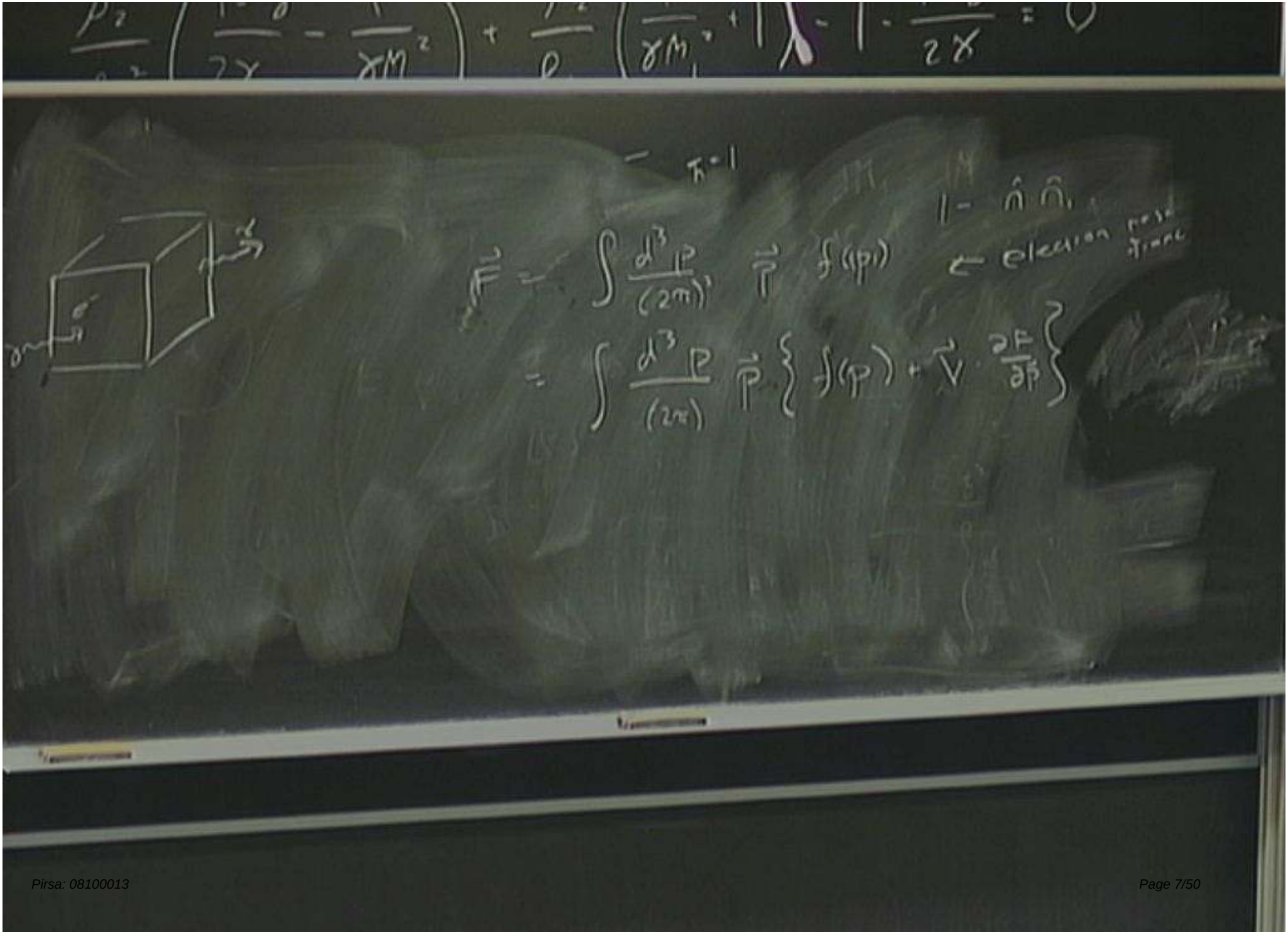
$f(p,t)\, d^3p\, d^3x$

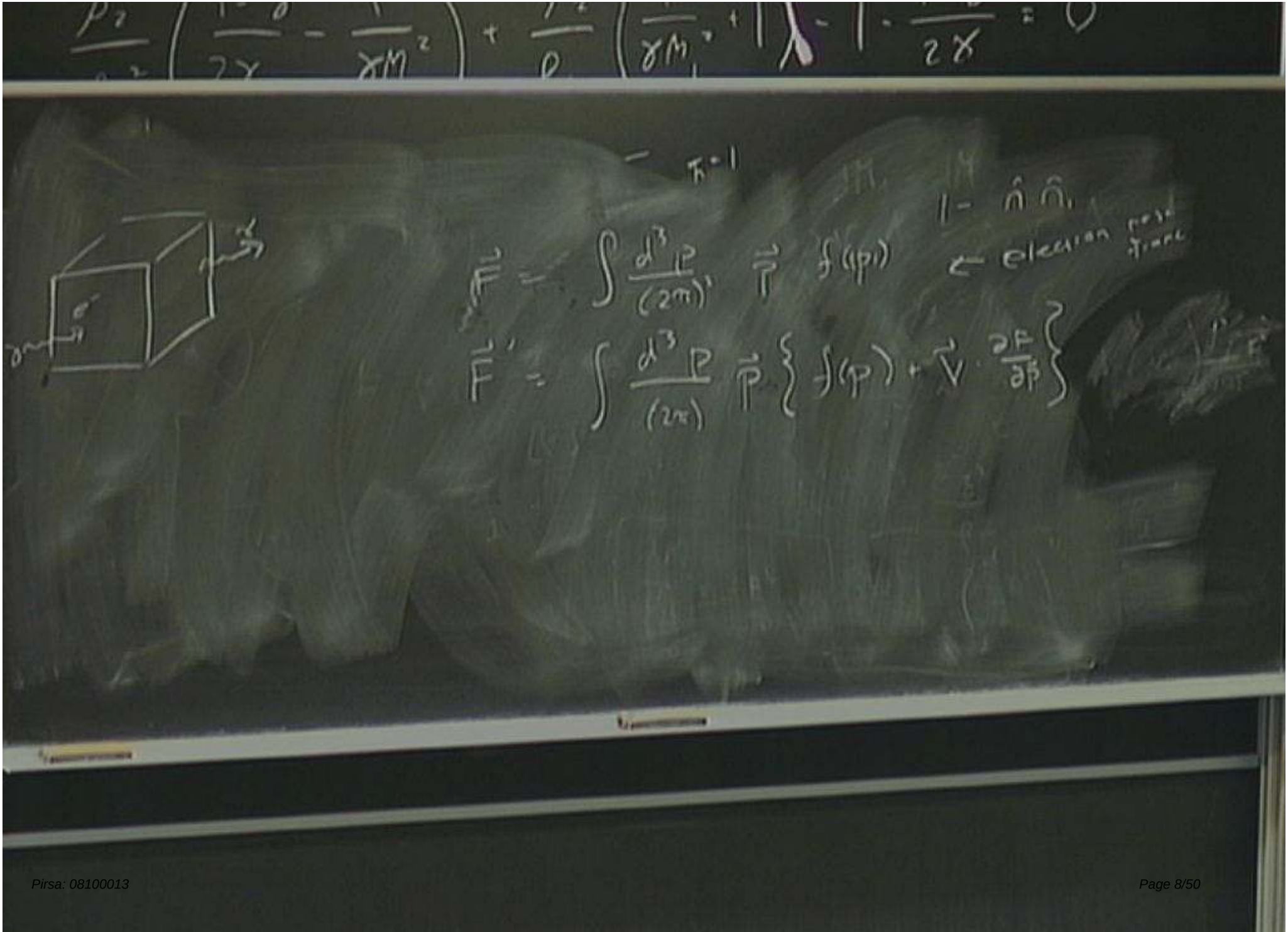
$f(\vec{p}+p'\vec{v},\, t) \simeq f(p,t) + \vec{v} \cdot \frac{\partial f}{\partial \vec{p}}(p,t)$

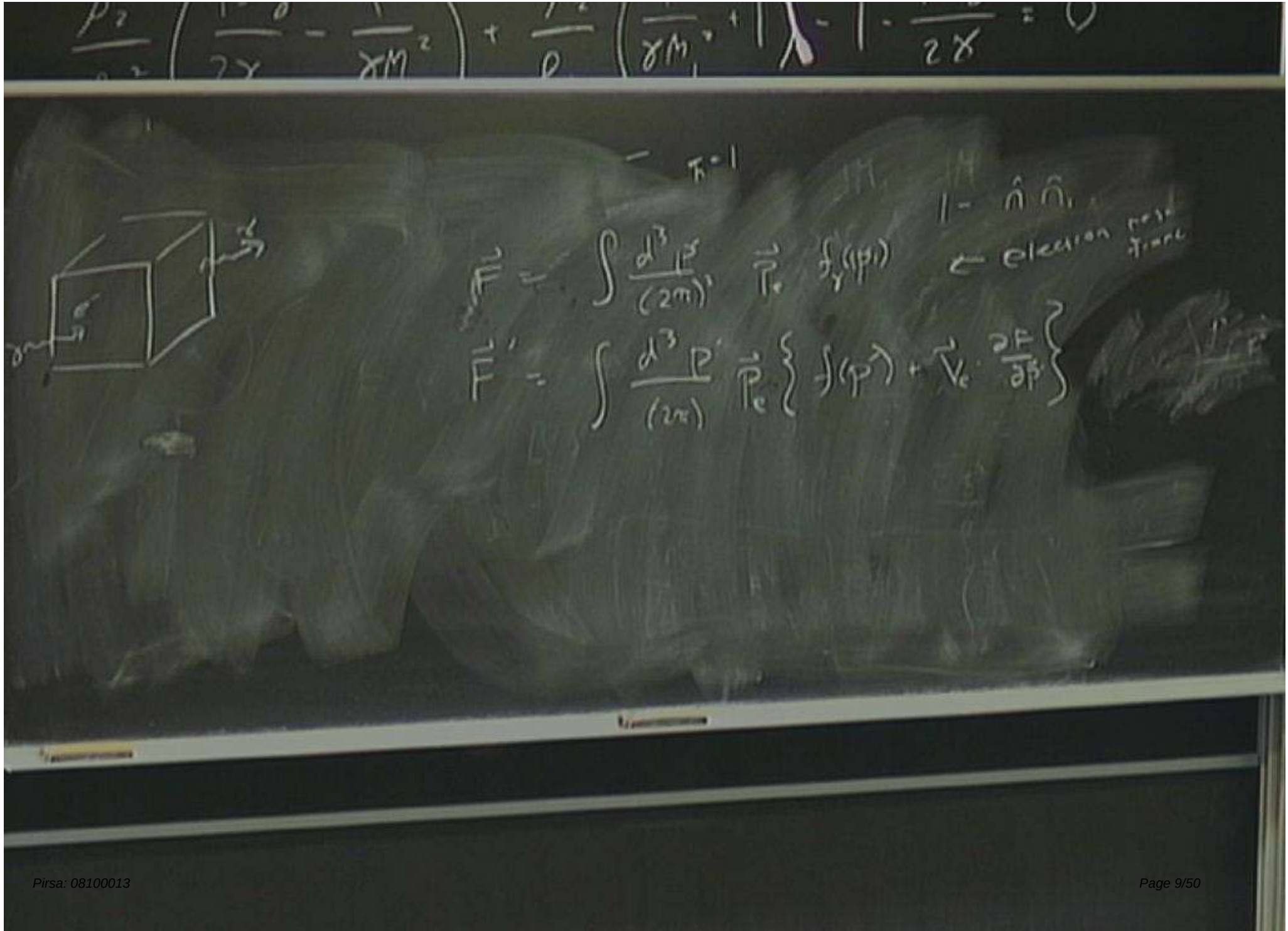



$\vec{F} = \int \frac{d^3p}{(2\pi)^3} \vec{p} f(p)$


$\int \frac{d^3p}{(2\pi)^3} \vec{r} f(p)$
$\int \frac{d^3p}{(2\pi)^3} \vec{p} \left\{ f(p) + \vec{v} \cdot \frac{\partial f}{\partial p} \right\}$


$\vec{F} = \int \frac{d^3p}{(2\pi)^3} \vec{p} f(p)$
← electron rest frame
$= \int \frac{d^3p}{(2\pi)} \vec{p} \left\{ f(p) + \vec{v} \cdot \frac{\partial F}{\partial \vec{p}} \right\}$


$\vec{F} = \int \frac{d^3p}{(2\pi)^3} \vec{p} f(p)$
← electron rest frame
$\vec{F}' = \int \frac{d^3p}{(2\pi)} \vec{p} \left\{ f(p) + \vec{v} \cdot \frac{\partial F}{\partial \vec{p}} \right\}$

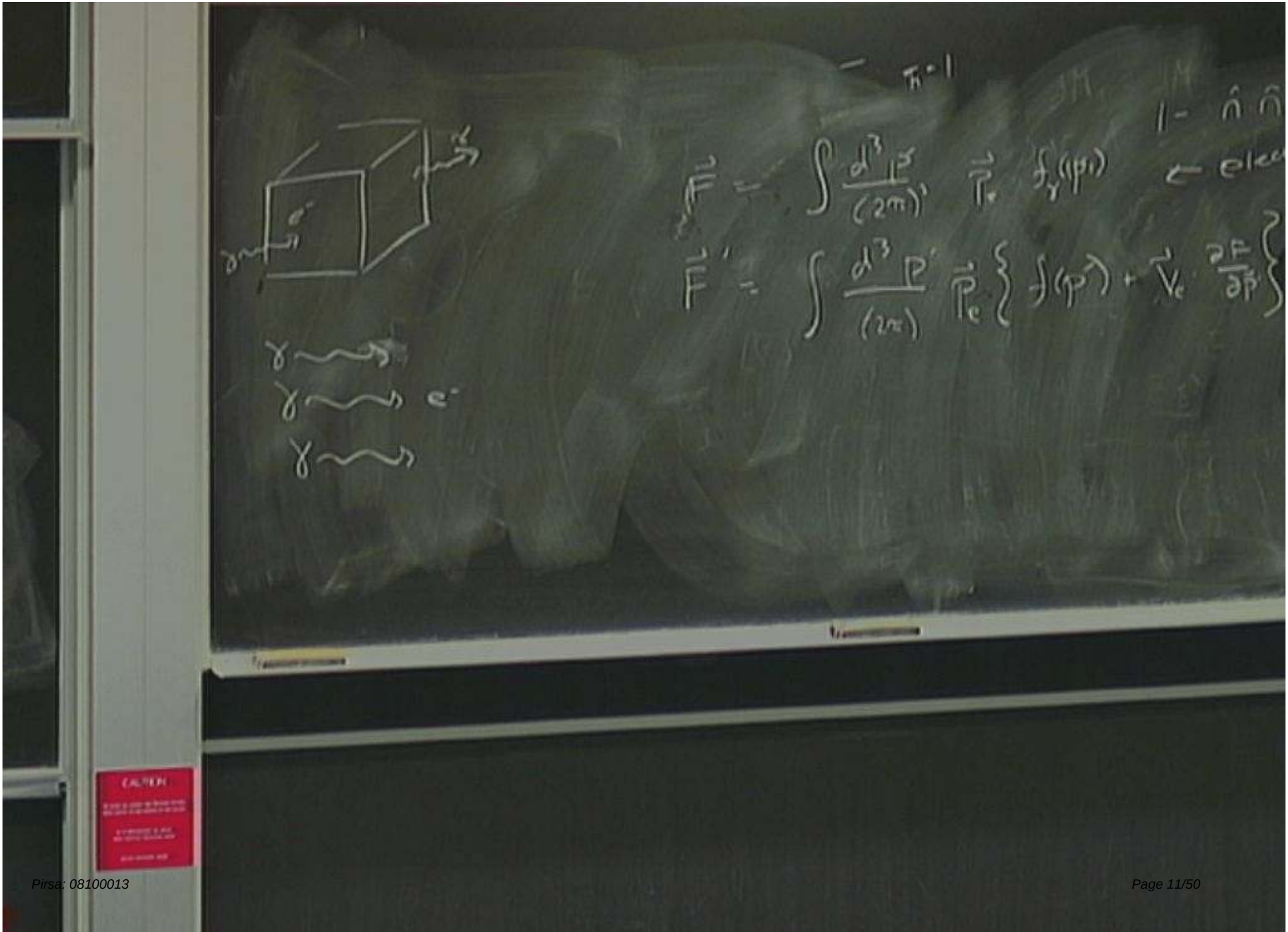

$\vec{F} = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e f_\gamma(p')$
← electron rest frame
$\vec{F}' = \int \frac{d^3p'}{(2\pi)} \vec{p}_e \left\{ f(p') + \vec{v}_e \cdot \frac{\partial F}{\partial p'} \right\}$

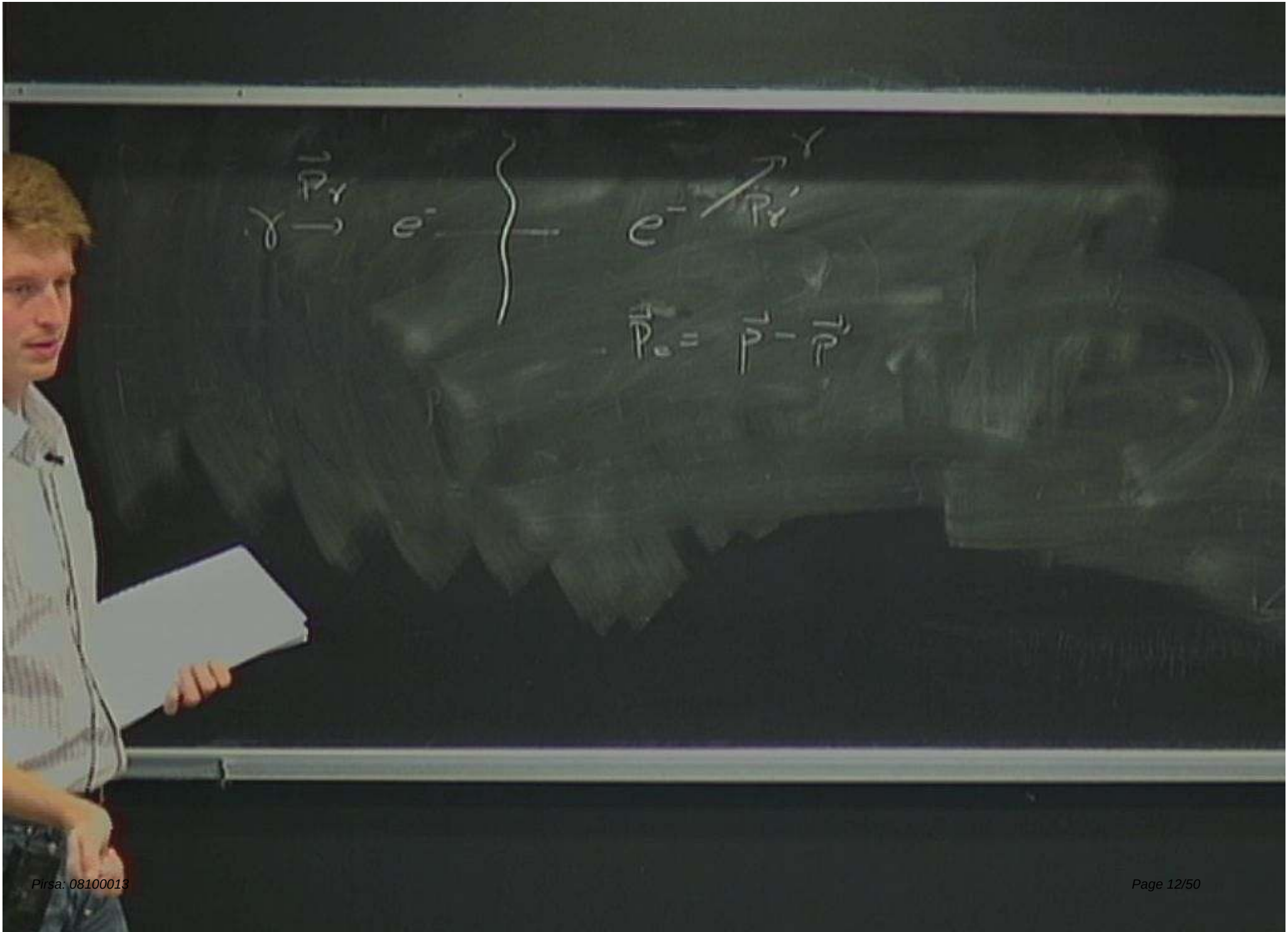
$\square \sim |d^3p\, d^3x|$

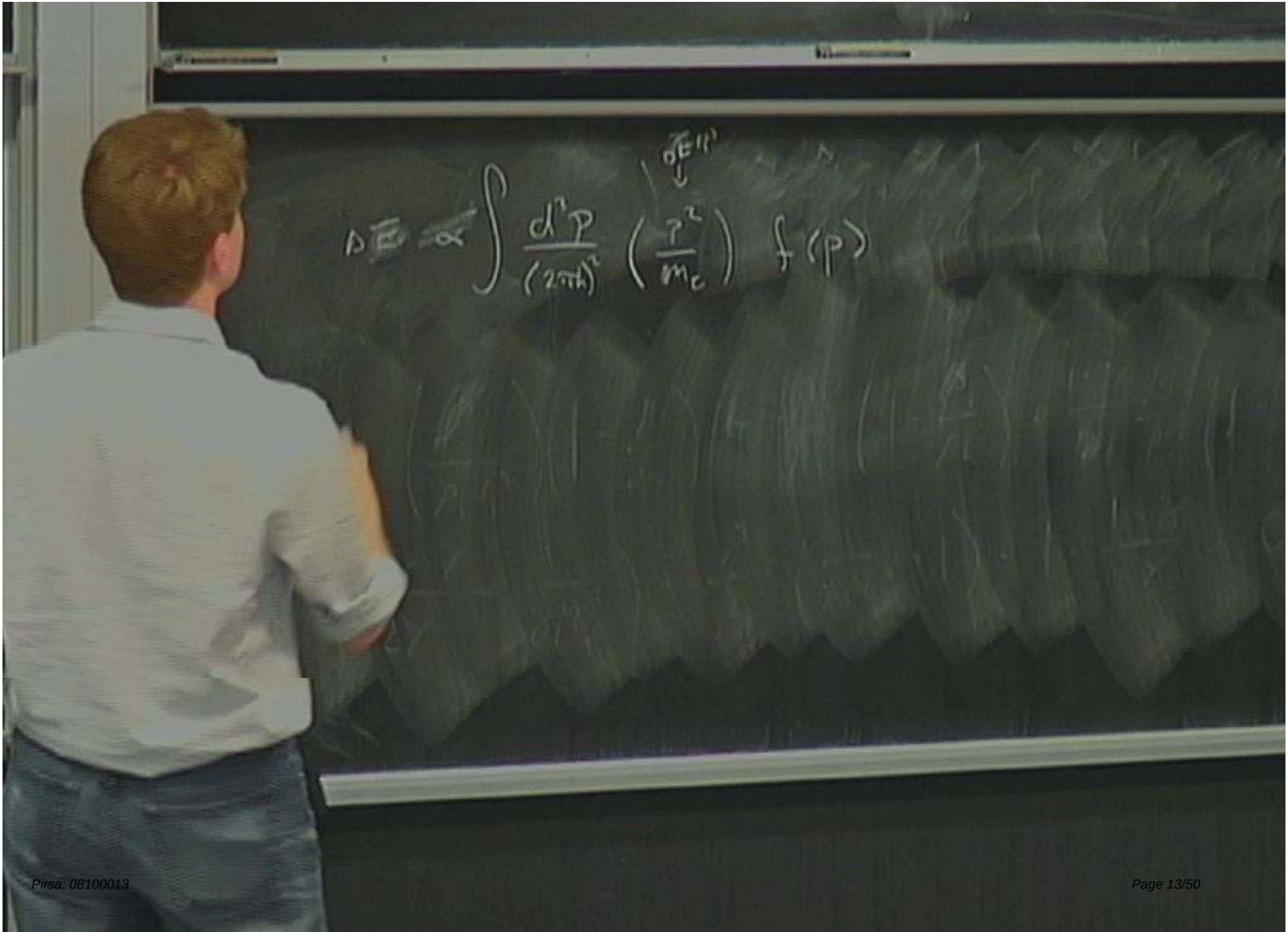
$f(p,t)\, d^3p\, d^3x$ $\vec{v}_e$

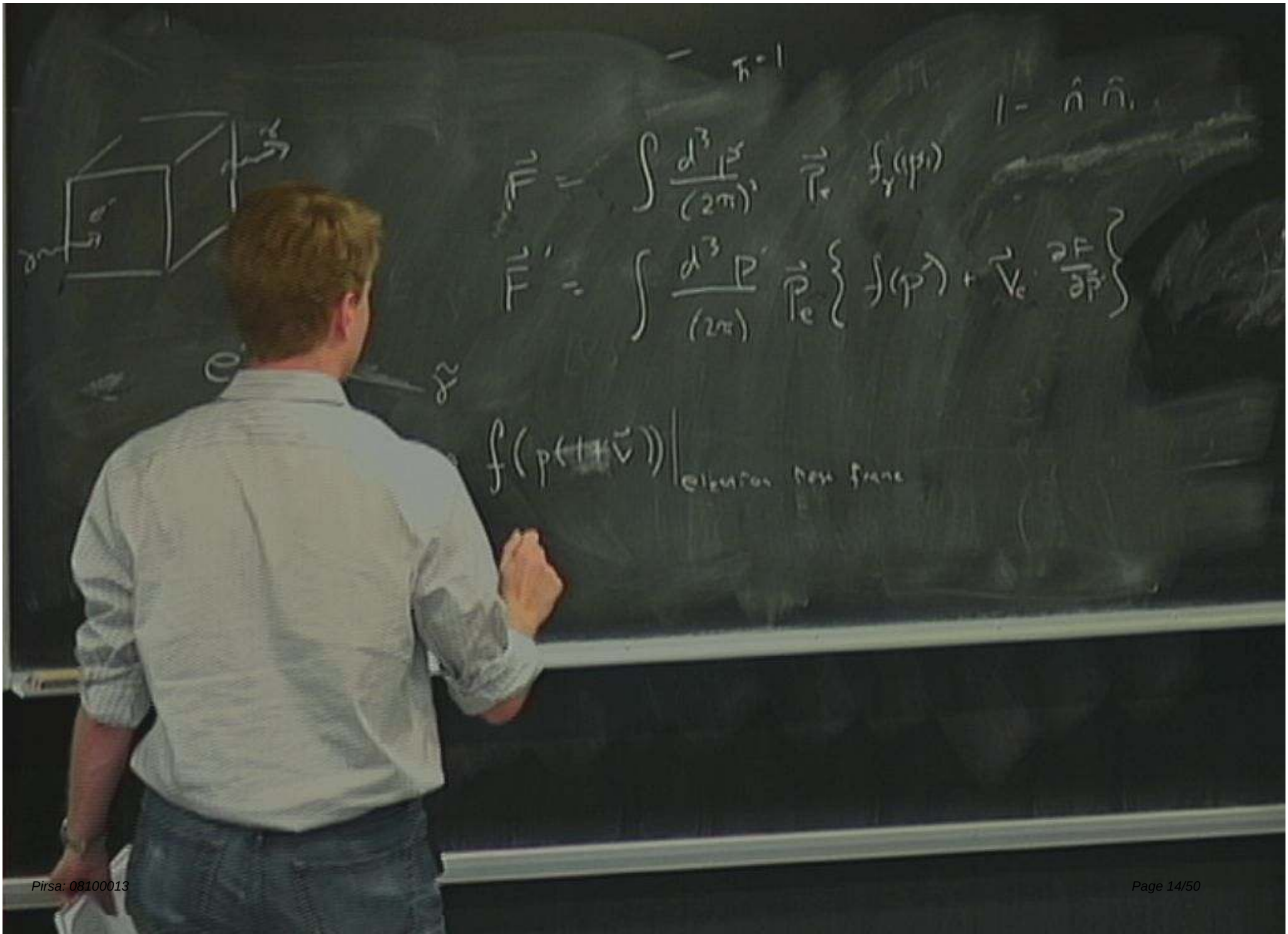
phase space dist.

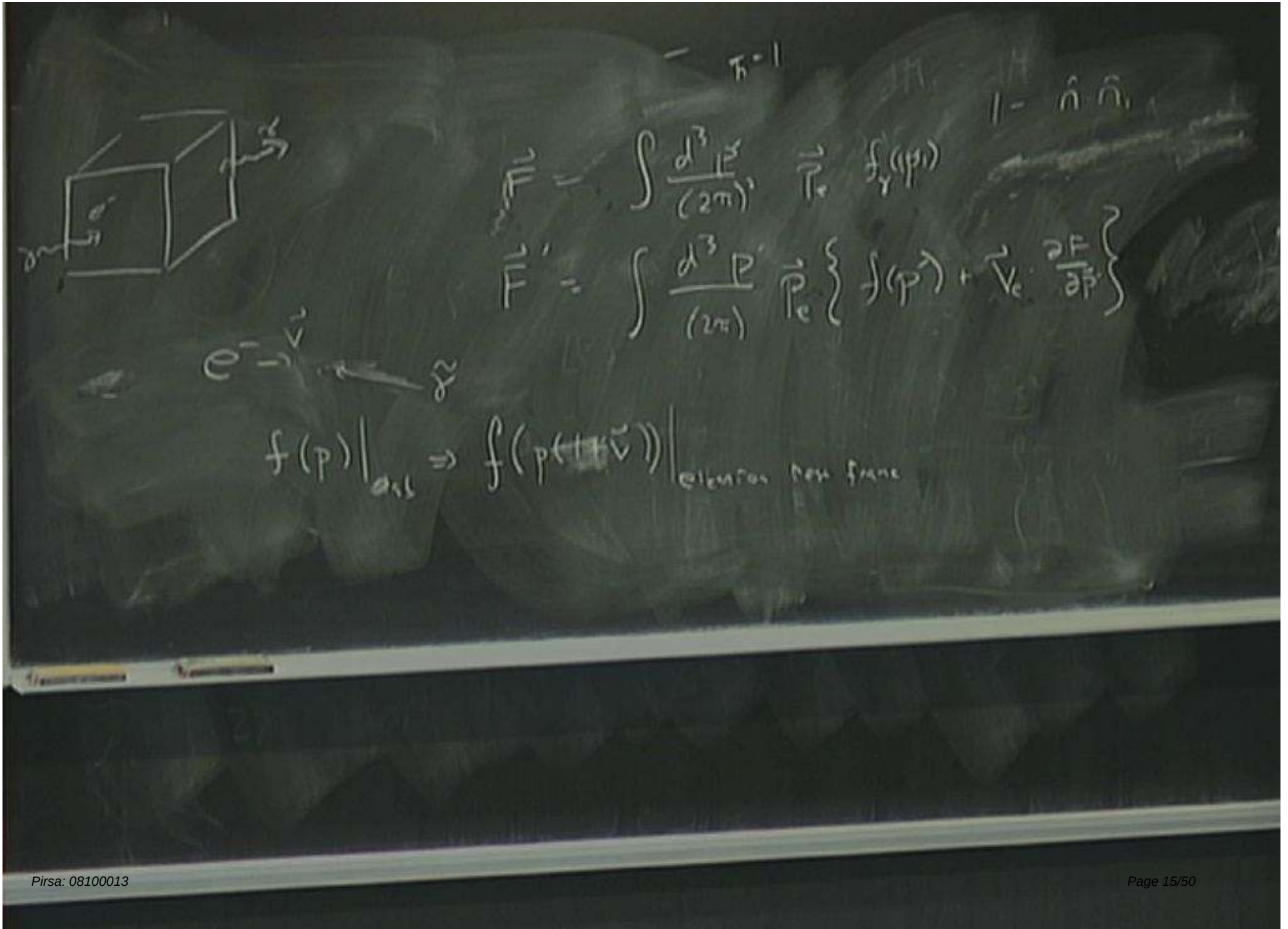
$f(\vec{p}+p\vec{v}, t) \simeq f(p,t) + \vec{v} \cdot \frac{\partial f}{\partial \vec{p}}(p,t)$

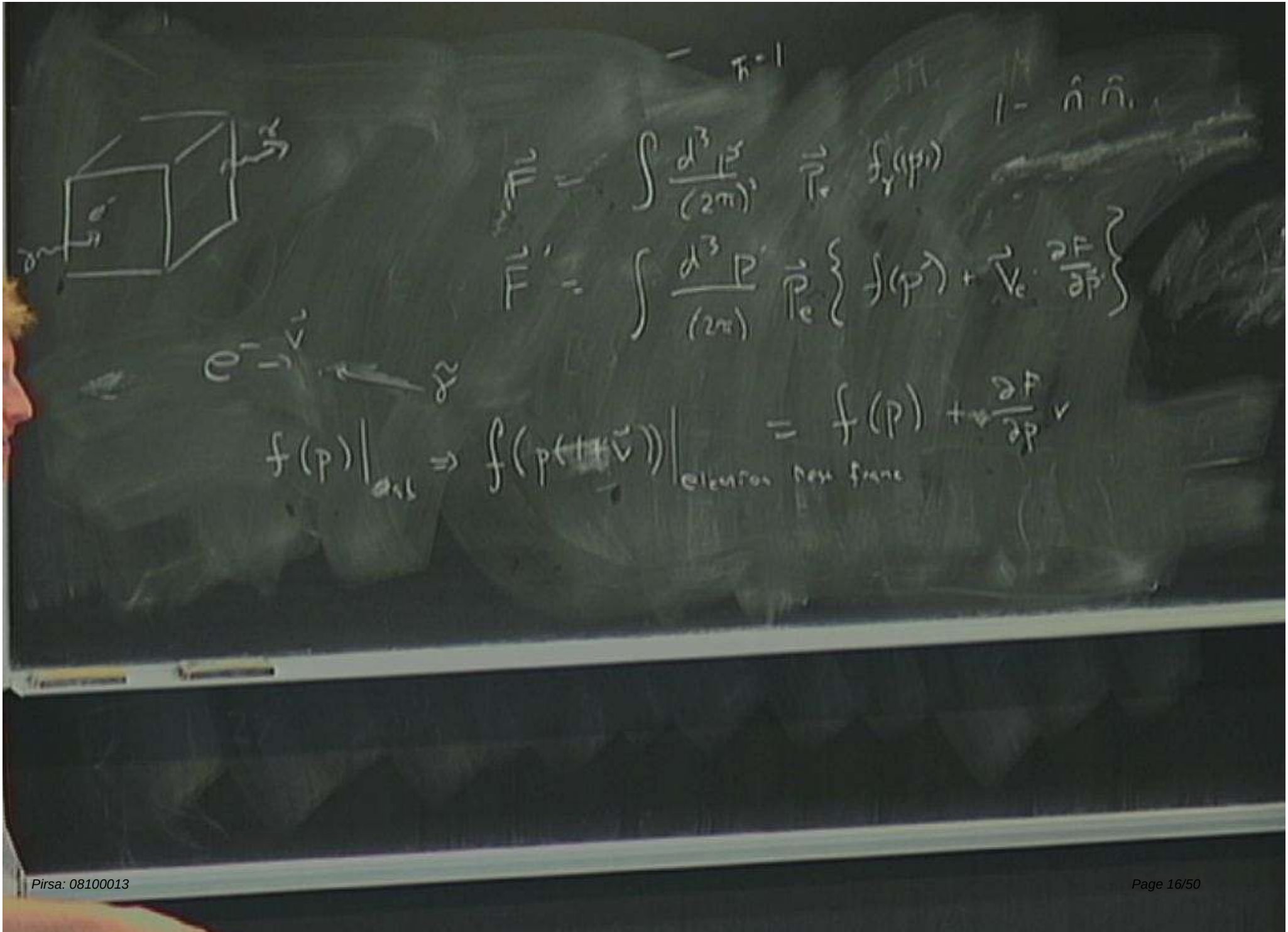

$\vec{F} = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e f_\gamma(p')$
$\vec{F}' = \int \frac{d^3p'}{(2\pi)} \vec{p}_e \left\{ f(p') + \vec{v}_e \frac{\partial F}{\partial \vec{p}'} \right\}$
γ
e^-


γ
$\vec{p}_\gamma$
e^-
e^-
$\vec{p}_{\gamma'}$
$\vec{p}_e = \vec{p} - \vec{p}'$

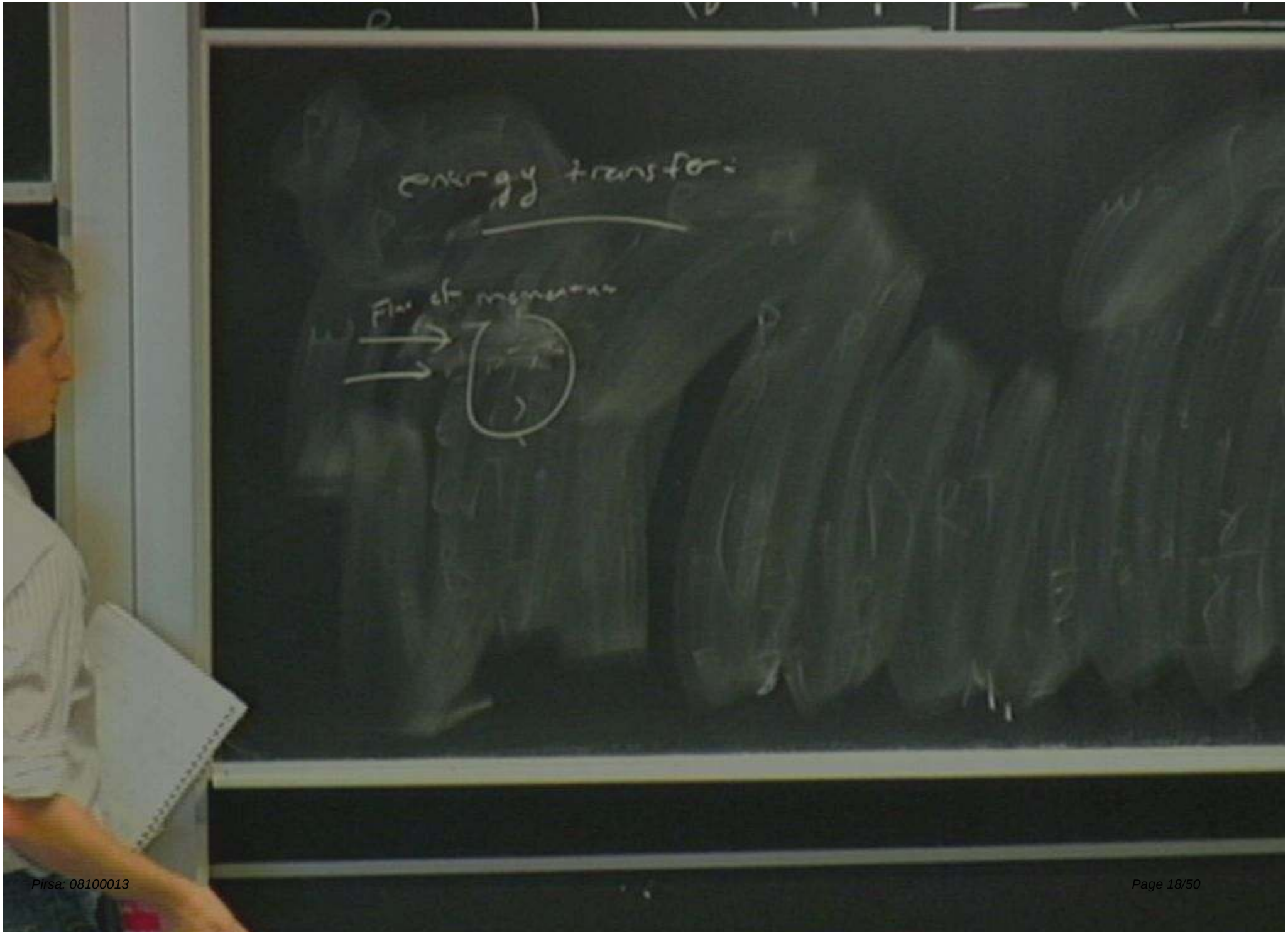

$\Delta E \propto \int \frac{d^3p}{(2\pi\hbar)^3} \left(\frac{\partial E}{\partial p}\right)^2 f(p)$

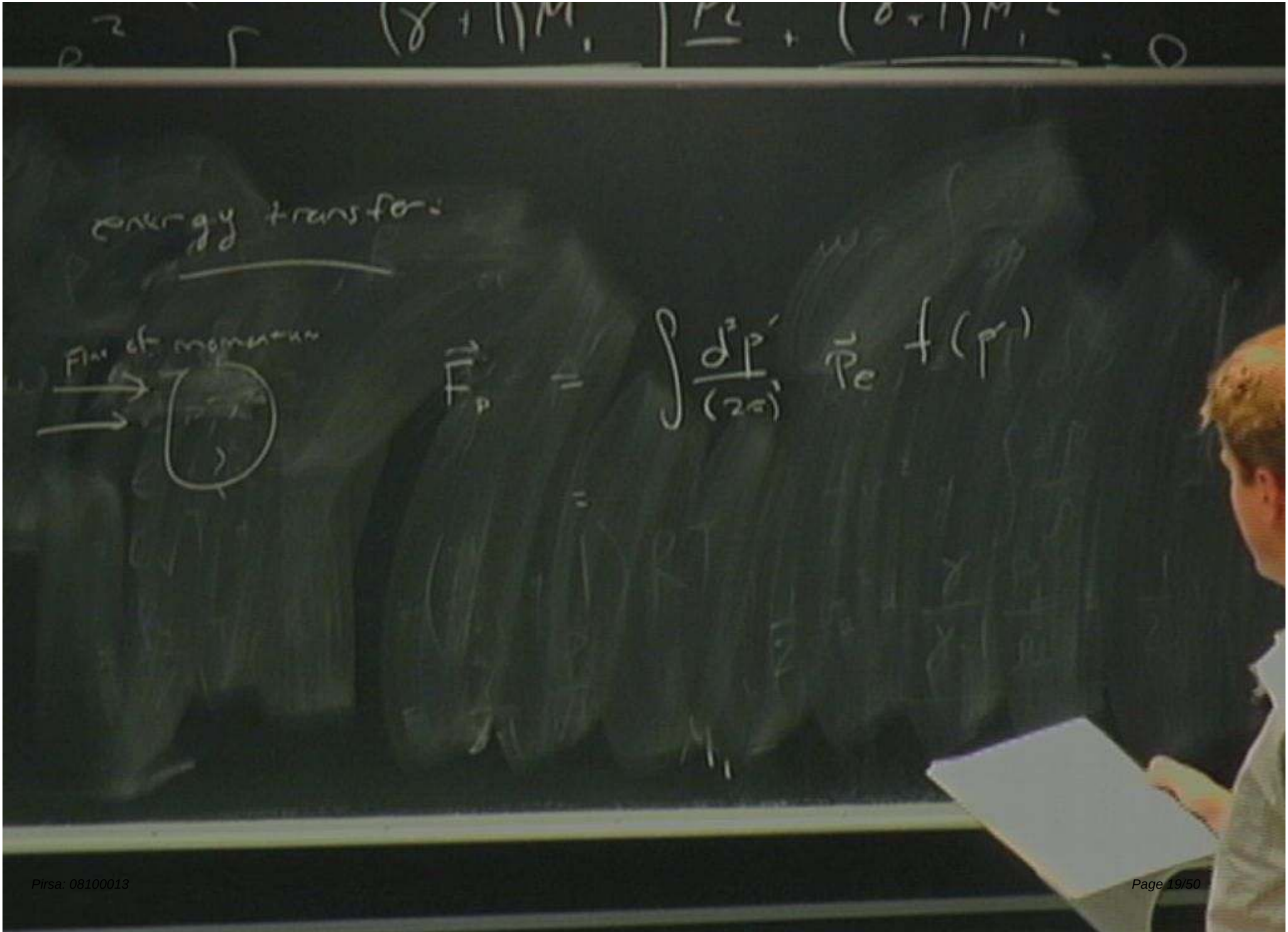

$\vec{F} = \int \frac{d^3p}{(2\pi)^3} \vec{p}_e f_\gamma(p)$
$\vec{F}' = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e \left\{ f(p') + \vec{v}_e \frac{\partial f}{\partial \vec{p}} \right\}$
$f(p(1+\vec{v}))\Big|_{\text{electron rest frame}}$

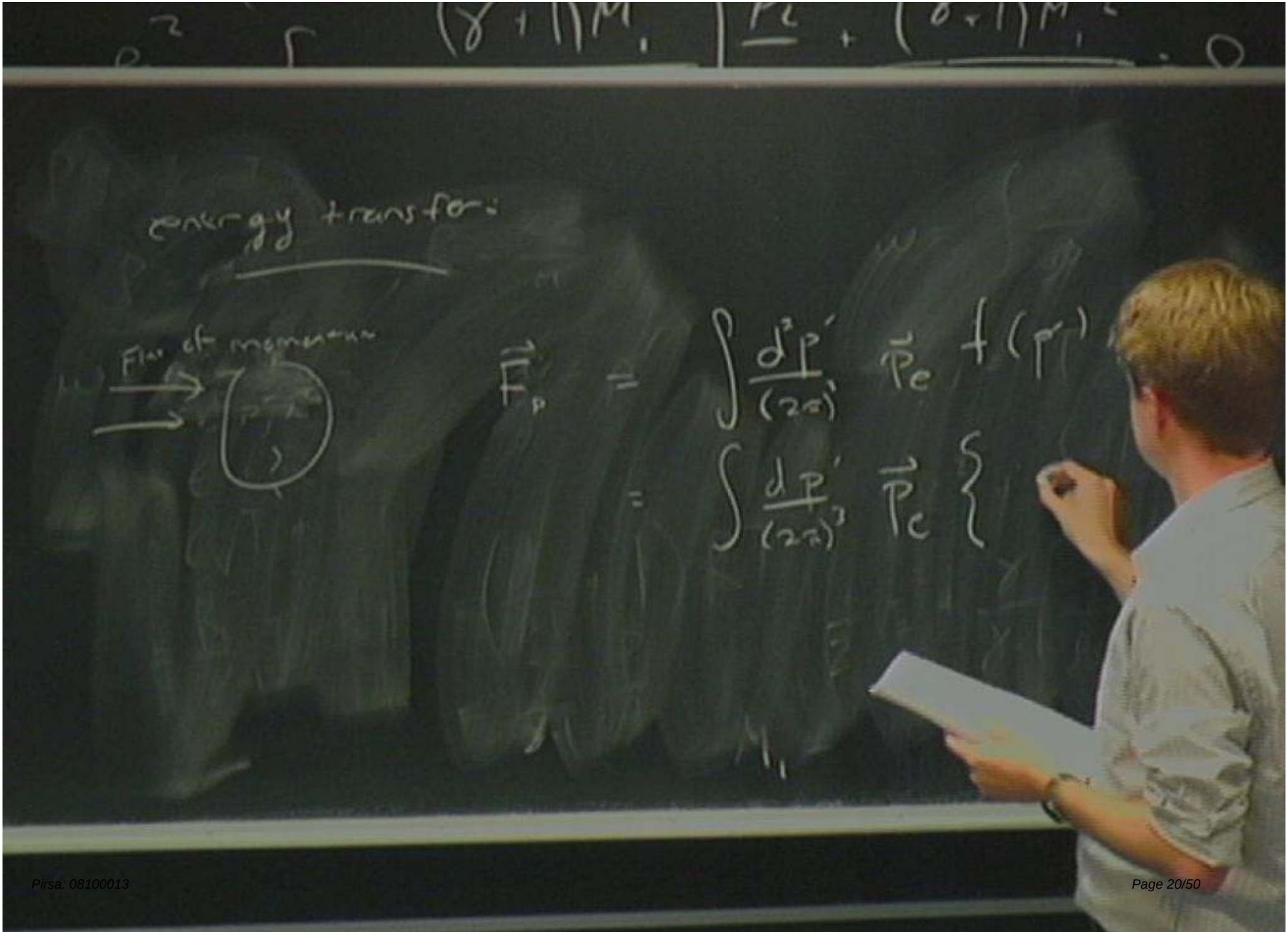

$\hbar = 1$
$1 - \hat{n}\hat{n}_1$
$\vec{F} = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e f_\gamma(p')$
$\vec{F}' = \int \frac{d^3p'}{(2\pi)} \vec{p}_e \left\{ f(p') + \vec{v}_e \frac{\partial F}{\partial \vec{p}'} \right\}$
$e^- \to \vec{v}$
$\tilde{\gamma}$
$f(p)\big|_{lab} \Rightarrow f(p(1+\vec{v}))\big|_{\text{electron rest frame}}$


$\hbar = 1$
$1 - \hat{n}\,\hat{n}$
$\vec{F} = \int \frac{d^3p}{(2\pi)^3}\, \vec{p}_e\, f_\gamma(p)$
$\vec{F}' = \int \frac{d^3p}{(2\pi)}\, \vec{p}_e \left\{ f(p) + \vec{v}_e \cdot \frac{\partial F}{\partial p} \right\}$
$e^- \to \vec{v}$
$\tilde{\gamma}$
$f(p)\big|_{lab} \Rightarrow f(p(1+\vec{v}))\big|_{\text{electron rest frame}} = f(p) + \frac{\partial F}{\partial p} v$

energy transfer


energy transfer:

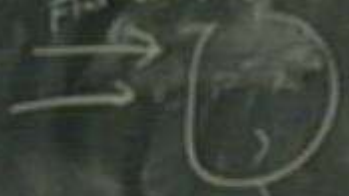

energy transfer:
Flux of momentum
$\vec{F}_p = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e f(p')$


energy transfer:
Flux of momentum
$\vec{F}_p = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_c f(r')$
$= \int \frac{dp'}{(2\pi)^3} \vec{p}_c \{$

$$\frac{P_1'}{P_1} = \frac{(\gamma-1)M_1^2 + 2}{(\gamma+1)M_1^2}$$

$$\frac{P_2}{P_1} + \frac{(\gamma-1)M_1^2 + 2}{(\gamma+1)M_1^2} = 0$$

energy transfer:

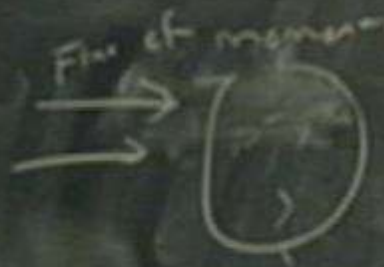


$$\vec{F}_p = \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e f(p')$$

$$= \int \frac{d^3p'}{(2\pi)^3} \vec{p}_e \left\{ f(p) + p_\nu \frac{\partial f}{\partial p} \right\}$$

$(\gamma+1)M_1^2$ $\quad \rho_L + (\gamma+1)M_1^2$

energy transfer:



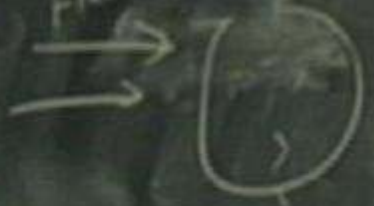
$$\vec{F} = \int \frac{d^3p'}{(2\pi)^3}\, \vec{p}\, f(p')$$

$$= \int \frac{dp'}{(2\pi)^3}\, \vec{p} \left\{ f(p) + p\, v \cdot \frac{\partial F}{\partial p} \right\}$$

$(\gamma+1)M_1^2$ $\quad \rho_2 + (\gamma+1)M_1^2$

energy transfer:

Flux of momentum

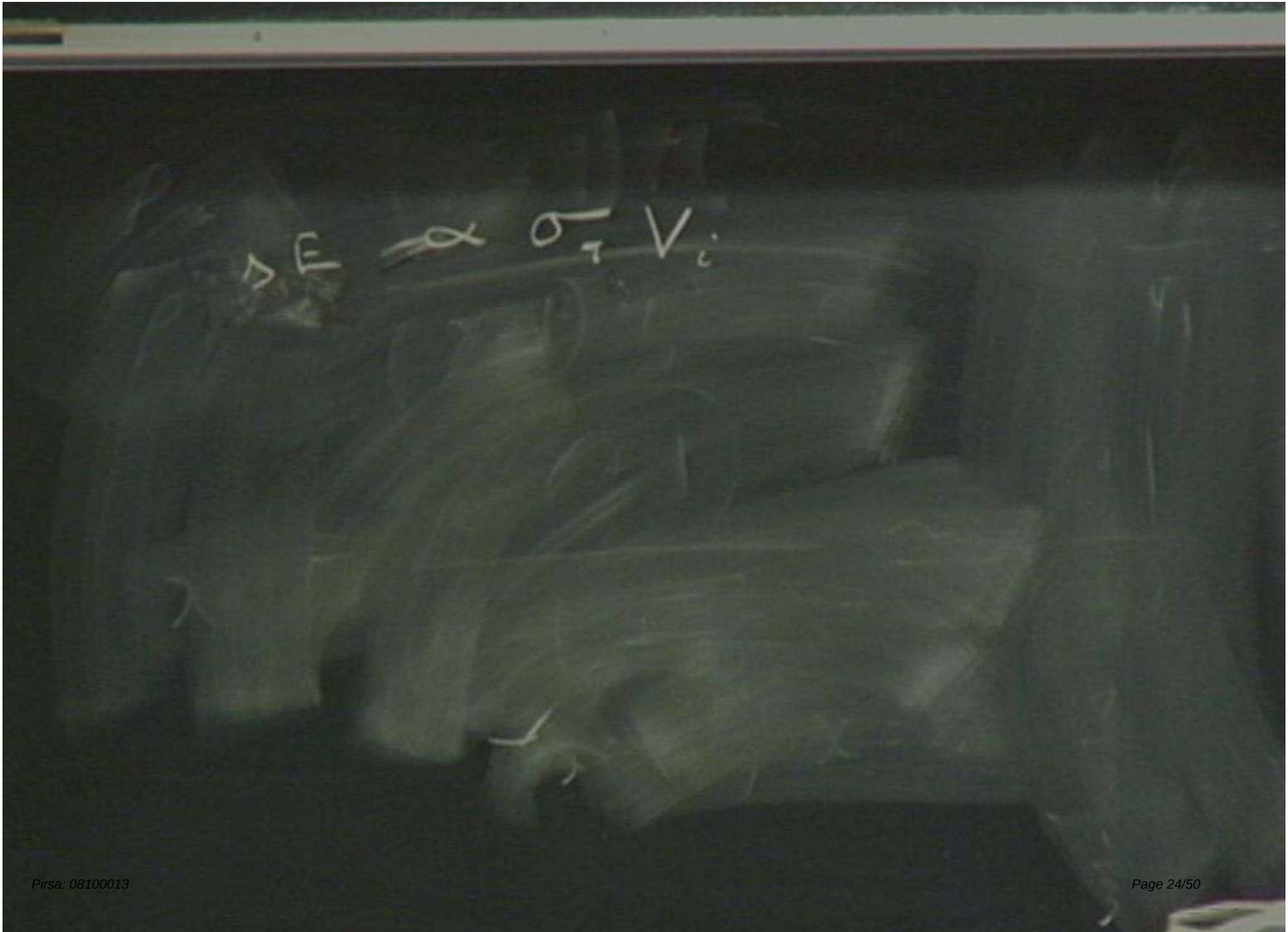


$$\vec{F} = \int \frac{d^3p'}{(2\pi)^3}\, \vec{p}\, f(p')$$

$$= \int \frac{dp'}{(2\pi)^3}\, \vec{p} \left\{ f(p) + p\, v \cdot \frac{\partial F}{\partial p} \right\}$$

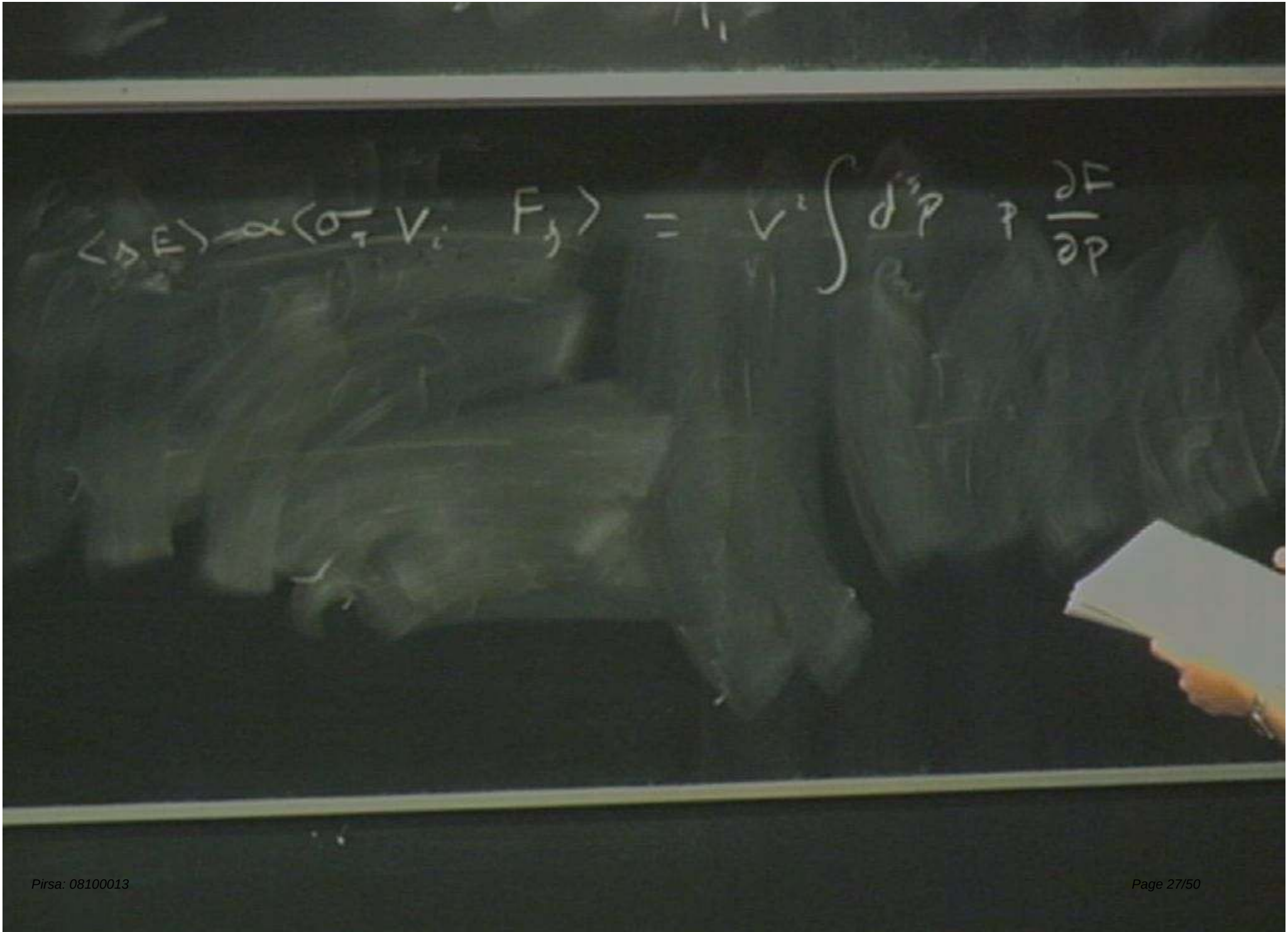
$$= \vec{F}_{old} + v \cdot \int \frac{d^3p}{(2\pi)^3}\, p\, \frac{\partial F}{\partial p}$$

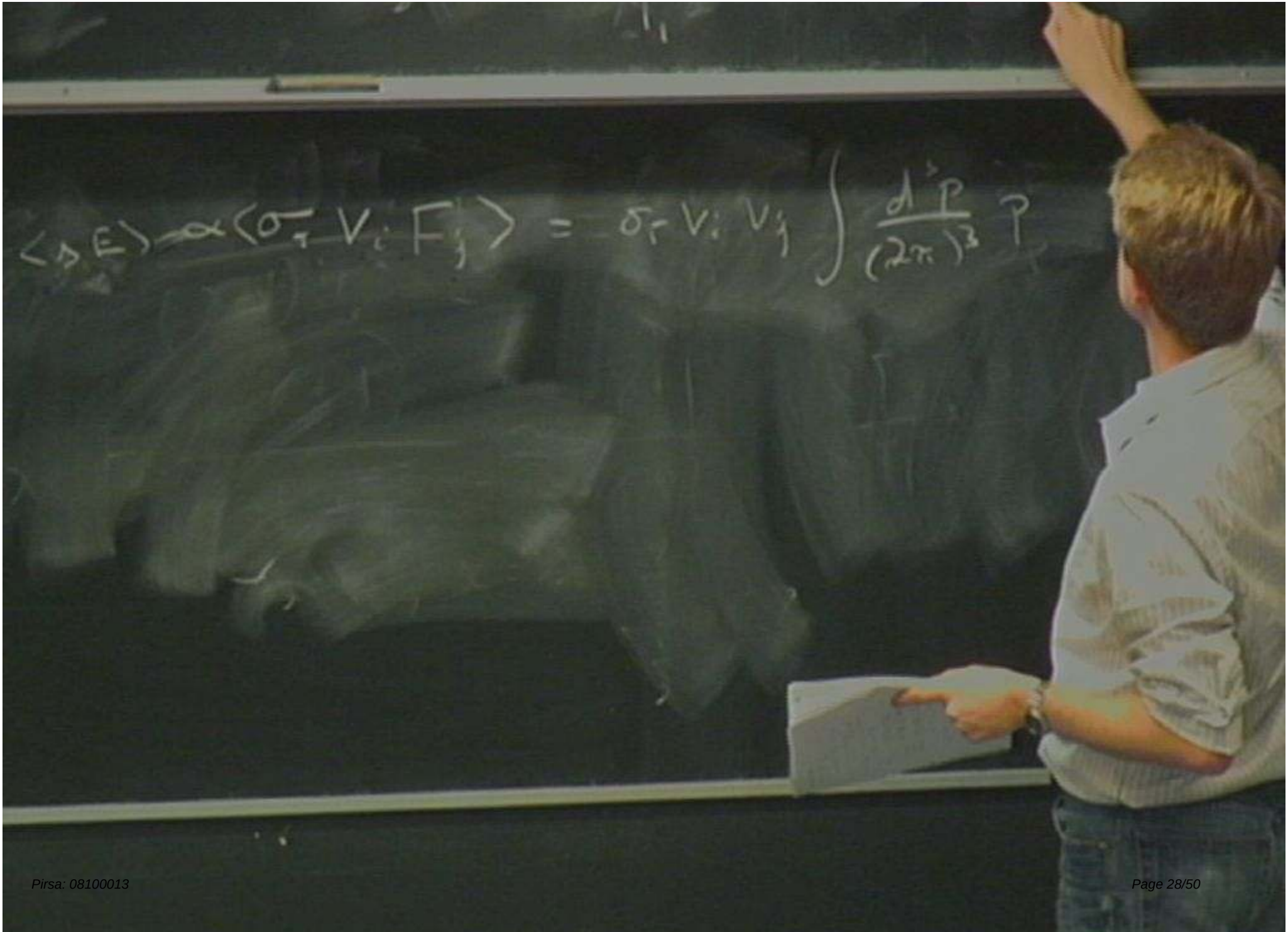
M_1


ΔE ∝ σ_T V_i

$\langle \Delta E \rangle \propto \langle \sigma_T, V_i \quad F_j \rangle$

$\langle \Delta E \rangle \propto \langle \sigma_\tau V_i \quad F_j \rangle =$


$\langle \Delta E \rangle \propto \langle \sigma_T V_i \quad F_j \rangle = V^2 \int d^3p \; f \frac{\partial F}{\partial p}$


$\langle \Delta E \rangle \propto \langle \sigma_\tau V_i F_j \rangle = \sigma_\tau V_i V_j \int \frac{d^3 p}{(2\pi)^3} p$

$$\langle \Delta E \rangle \propto \langle \sigma_{ij} V_i F_j \rangle = \sigma_{ij} V_i V_j \int \frac{d^3p}{(2\pi)^3} p_i \frac{\partial F}{\partial p_j} \quad (p_j)$$

$$= F_{old} + v_j \cdot \int \frac{d^3p}{(2\pi)^3} \vec{p}\, p \frac{\partial f}{\partial p}$$

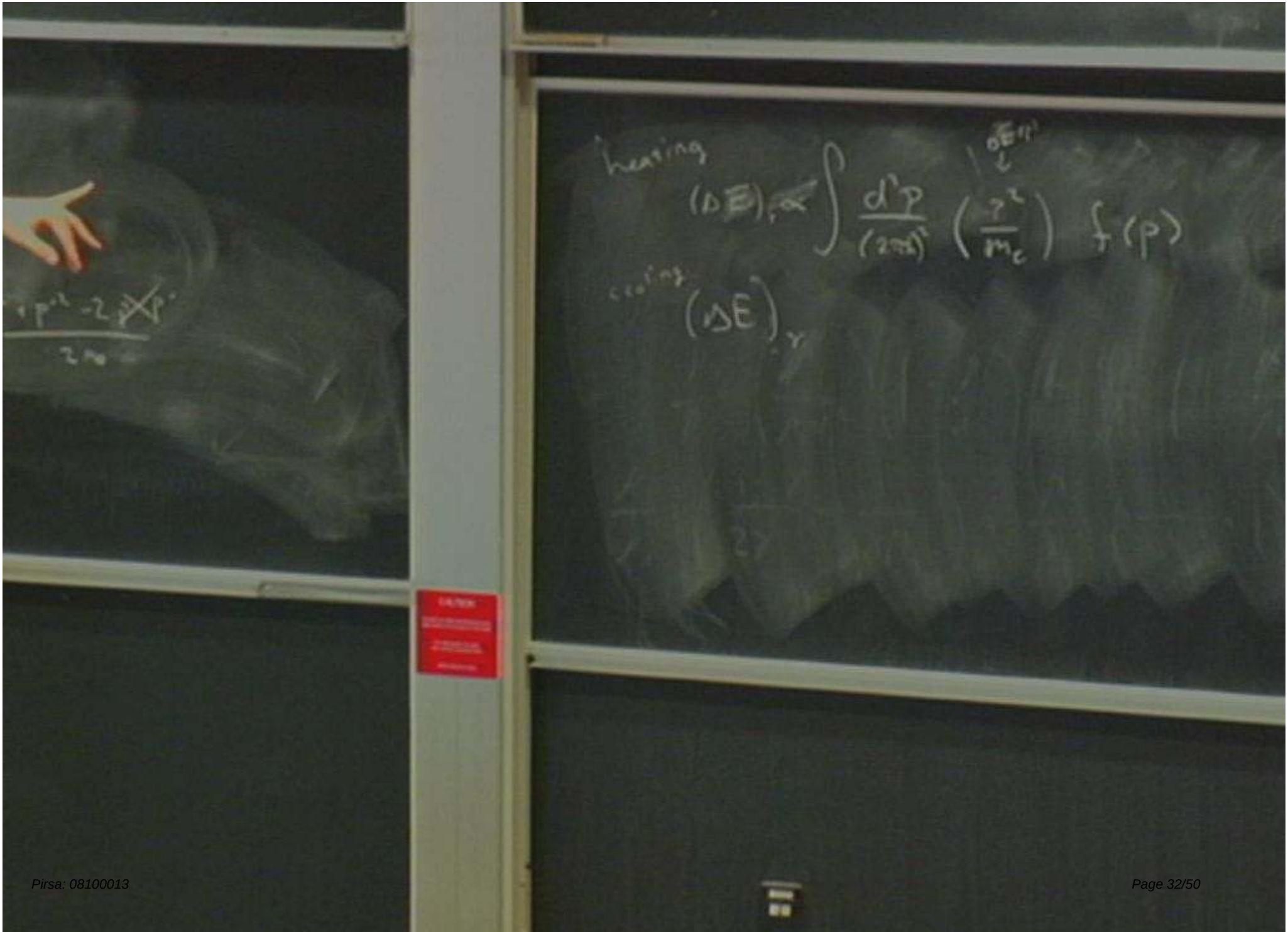
$$\langle \Delta E \rangle \propto \langle \sigma_T v_i F_j \rangle = \sigma_T \langle v_i v_j \rangle \int \frac{d^3p}{(2\pi)^3} p_i \frac{\partial F}{\partial p_j} \langle p_j | \rangle$$

$$= \sigma_T \frac{I}{m} \int \frac{d^3p}{(2\pi)^3} p^2 \left(\frac{\partial f}{\partial p}\right)$$

$$= F_{old} + V_j \cdot \int \frac{d^3p}{(2\pi)^3} \vec{p}\, p \frac{\partial f}{\partial p}$$

$$\langle \Delta E \rangle \propto \langle \sigma_T V_i F_j \rangle = \sigma_T \langle V_i V_j \rangle \int \frac{d^3p}{(2\pi)^3} p_i \frac{\partial F}{\partial p_j} \langle p_j | \rangle$$

$$= \sigma_T \left(\frac{I}{m}\right) \int \frac{d^3p}{(2\pi)^3} p^2 \left(\frac{\partial F}{\partial p}\right)$$


heating
cooling

heating

$$(\Delta E)_\gamma \propto \int \frac{d^3p}{(2\pi\hbar)^3} \left(\frac{p^2}{m_e}\right) f(p)\left(1 + f(p)\right) \qquad f \sim e^{-p/kT}$$

($\delta E''$)

cooling

$$(\Delta E)_\gamma = (\text{other board})$$

heating

$$(\Delta E)_\gamma \propto \int \frac{d^3p}{(2\pi\hbar)^2} \overset{\Delta E(p)}{\left(\frac{p^2}{m_e}\right)} f(p)\left(1 + f(p)\right)$$

cooling

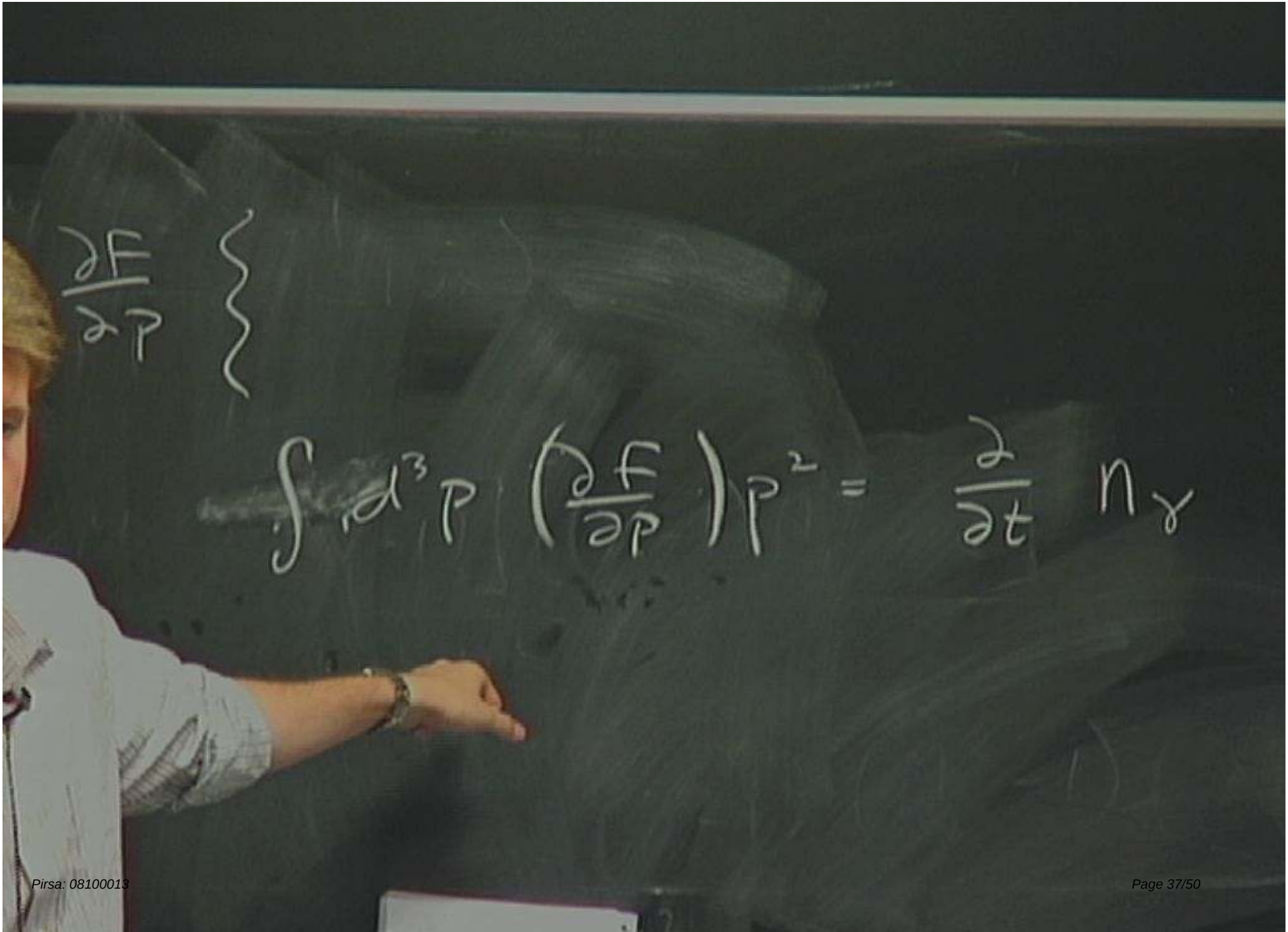
$$+ \; (\Delta E)_\gamma = (\text{other board})$$

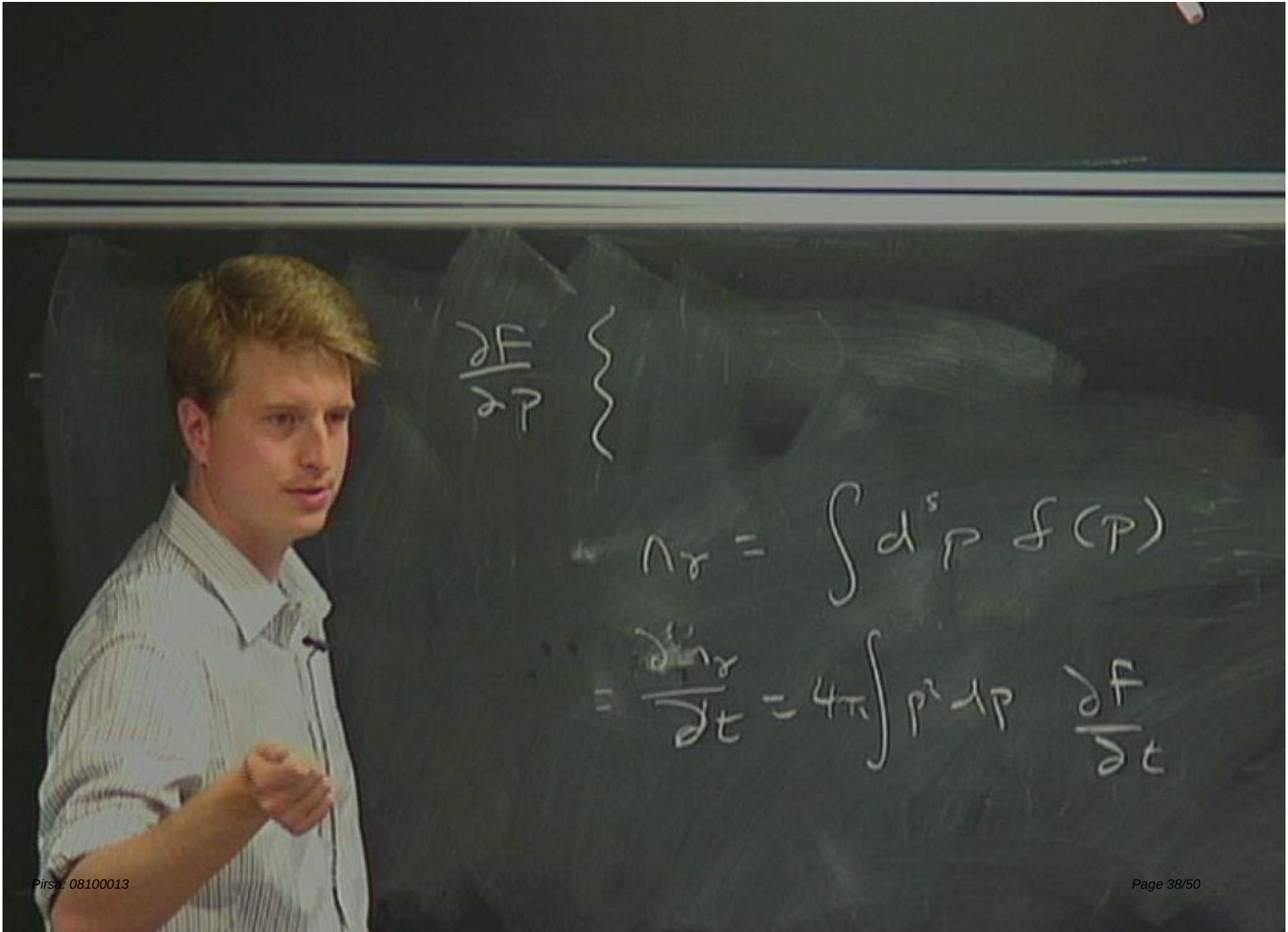
$$\text{Net } \Delta E_e = \frac{\sigma_T}{m_e} \int \frac{d^3p}{(2\pi)^3} p^2 \left\{ T_e \frac{\partial F}{\partial p} + f(1+f) \right\}$$

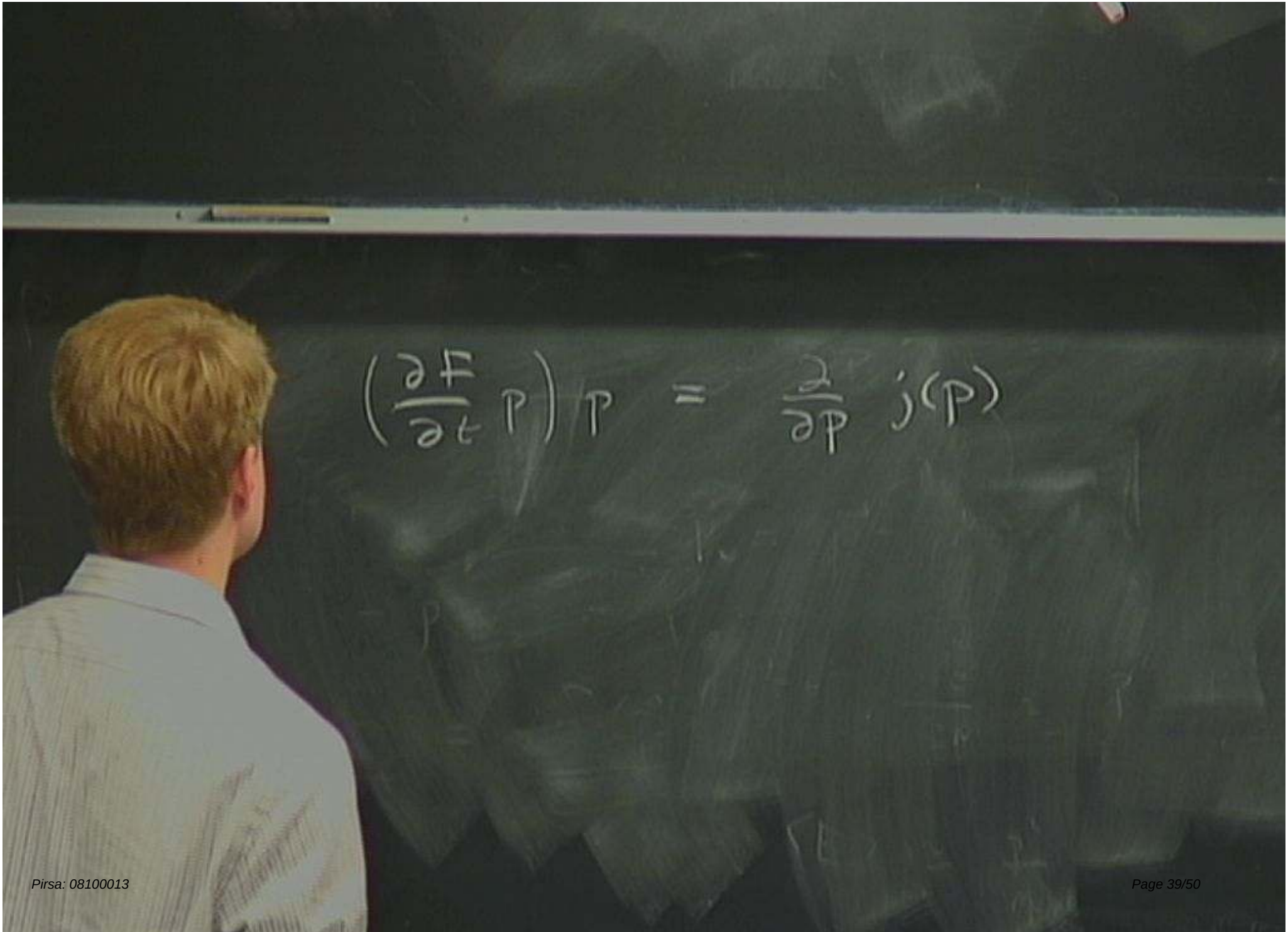
$$f(p) = \frac{1}{e^{\frac{p^2+M}{T}} - 1}$$

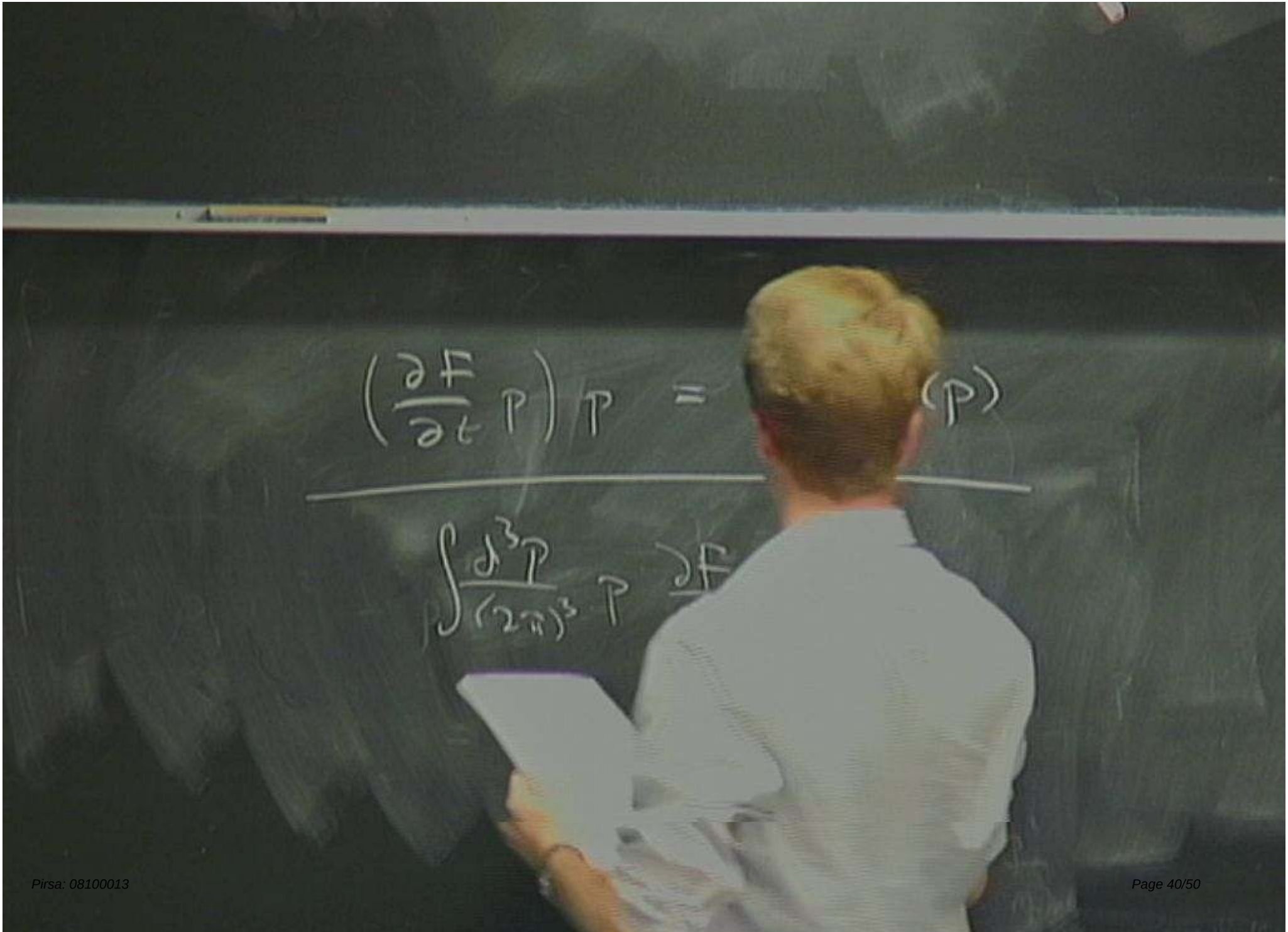
$$f(p) = \frac{1}{e^{\frac{p+\mu}{T}} - 1}$$

$$\Delta E_\gamma = -\Delta E_e$$


$\frac{\partial F}{\partial p}$
$\int d^3p \left(\frac{\partial f}{\partial p}\right) p^2 = \frac{\partial}{\partial t} n_\gamma$

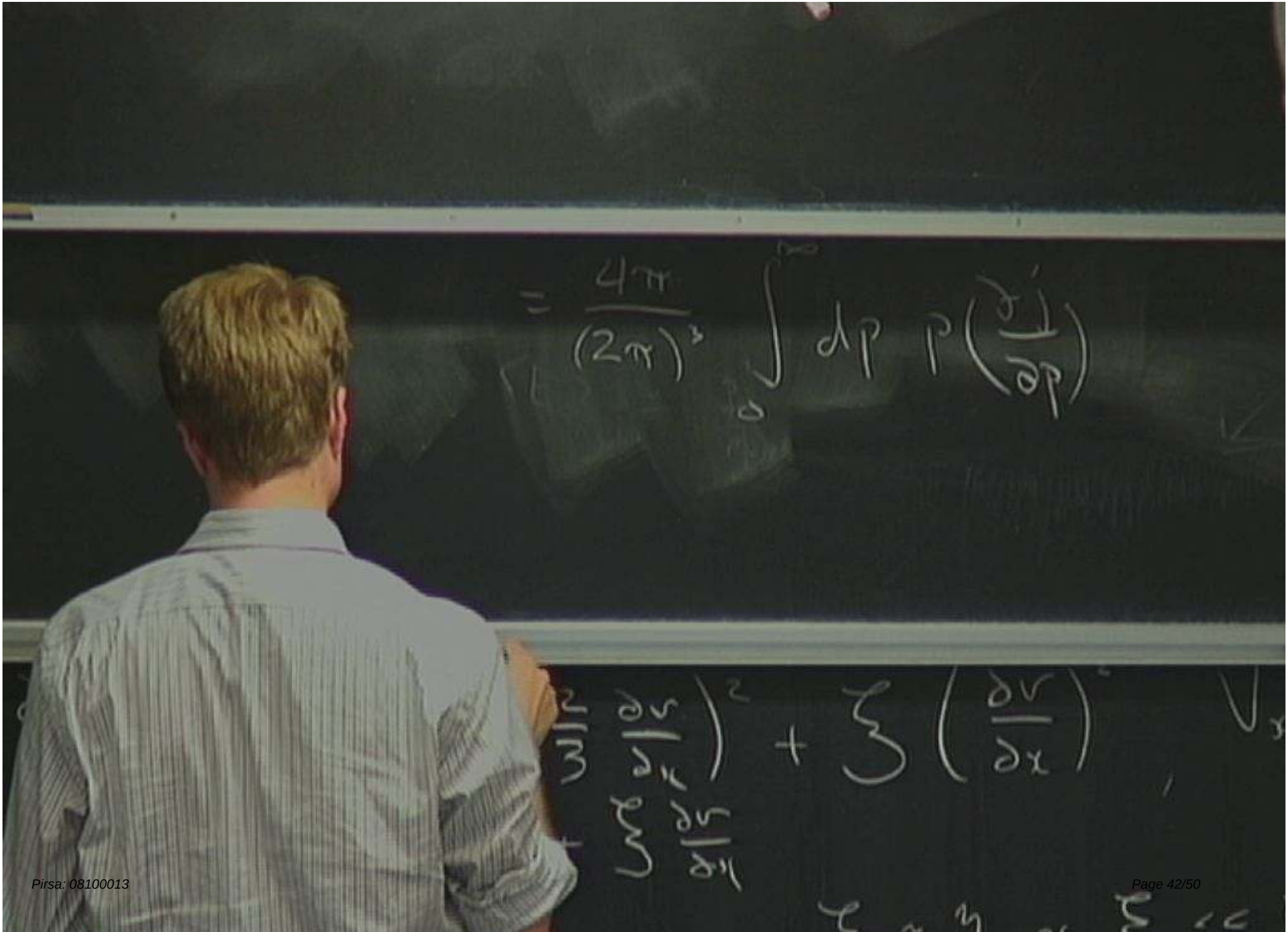

$\frac{\partial F}{\partial p}$
$n_\gamma = \int d^3p\, f(p)$
$= \frac{\partial n_\gamma}{\partial t} = 4\pi \int p^2 dp \frac{\partial f}{\partial t}$


$\left(\frac{\partial F}{\partial t} p\right) p = \frac{\partial}{\partial p} j(p)$


$\left(\frac{\partial f}{\partial t} p\right) p =$
(p)
$\int \frac{d^3 p}{(2\pi)^3} p \frac{\partial f}{}$

$$\left(\frac{\partial f}{\partial t} p\right) p = \frac{\partial}{\partial p} j(p)$$

$$\int \frac{d^3p}{(2\pi)^3} p \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \left[\frac{4\pi}{(2\pi)^3} \int_0^\infty p^3 dp\, f(p)\right]$$


$= \frac{4\pi}{(2\pi)^3} \int_0^\infty dp\, p \left(\frac{\partial}{\partial p}\right)$
$+ \xi \left(\frac{\partial v}{\partial x}\right)$

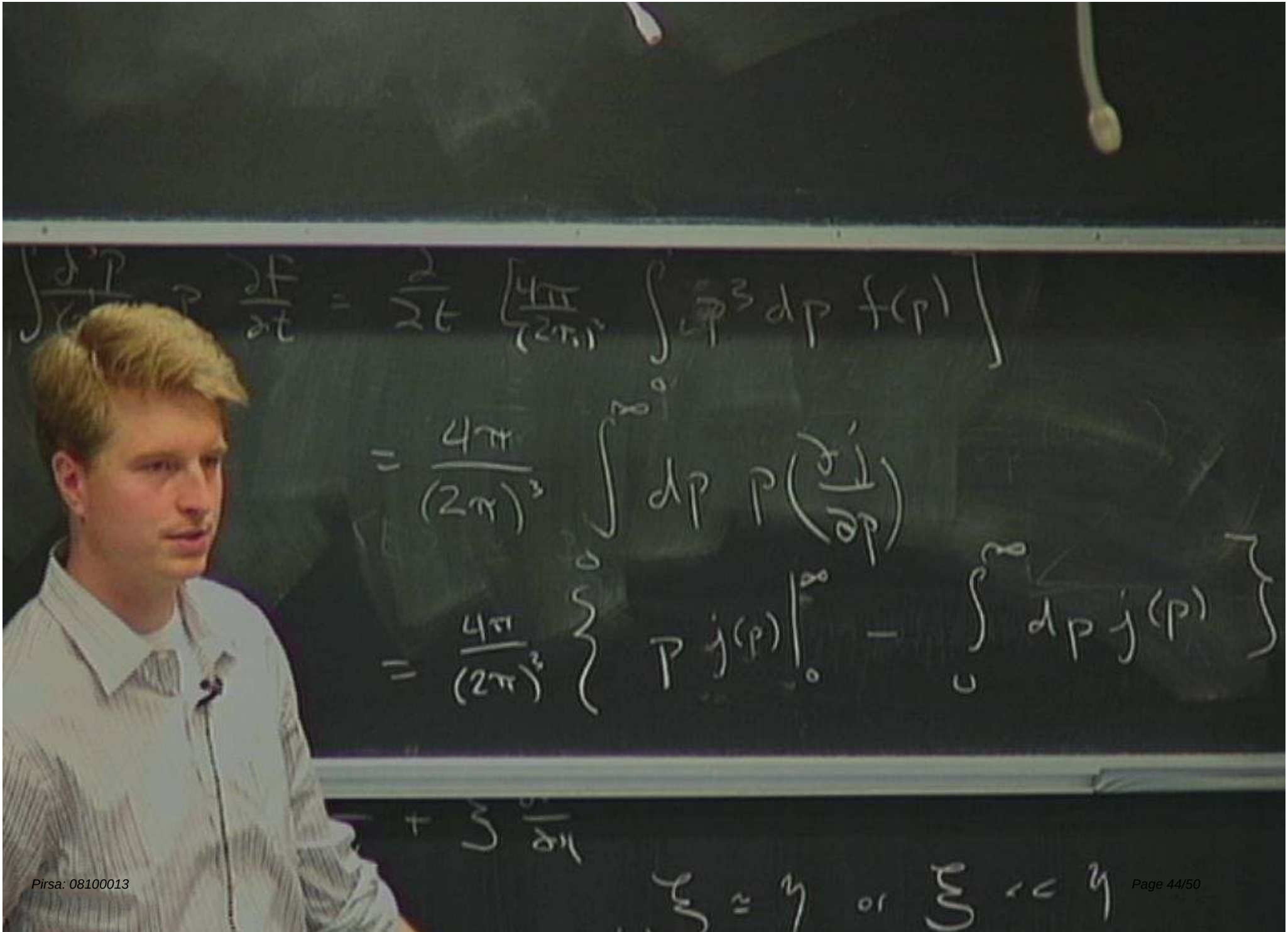
$$= \frac{4\pi}{(2\pi)^3} \int_0^\infty dp\, p \left(\frac{\partial j}{\partial p}\right)$$

$$= \frac{4\pi}{(2\pi)^3} \left\{ p\, j(p) \Big|_0^\infty - \int_0^\infty dp\, j(p) \right\}$$

$$\frac{1}{2}\eta \left(2V_{xx} - \frac{2}{3}\nabla\cdot\vec{v}\right)^2 + \zeta \left(\nabla\cdot\vec{v}\right)^2 \qquad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$\frac{1}{2}\eta \left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \zeta \frac{\partial v}{\partial x}$$

$$\left(\eta + \zeta\right) \frac{\partial v}{\partial x} \qquad \zeta \simeq \eta \text{ or } \zeta < \eta$$


$\int \frac{d^3p}{(2\pi)^3} p \frac{\partial f}{\partial t} = \frac{2}{2t} \left[\frac{4\pi}{(2\pi)^3} \int_0^\infty p^3 dp\, f(p) \right]$
$= \frac{4\pi}{(2\pi)^3} \int_0^\infty dp\, p \left(\frac{\partial j}{\partial p} \right)$
$= \frac{4\pi}{(2\pi)^3} \left\{ p\, j(p) \Big|_0^\infty - \int_0^\infty dp\, j(p) \right\}$
$+ 3 \frac{\partial}{\partial t}$
$\xi \approx \eta$ or $\xi < \eta$

$$\left(\frac{\partial f}{\partial t}\, p\right) p = \frac{\partial}{\partial p}\, j(p)$$

$$\text{Scattering rate} = \int \frac{d^3p}{(2\pi)^3}\, p\, \frac{\partial f}{\partial t} = \frac{2}{2t}\left[\frac{4\pi}{(2\pi)^3}\int_0^\infty p^3\, dp\, f(p)\right]$$

$$= \frac{4\pi}{(2\pi)^3}\int_0^\infty dp\; p\left(\frac{\partial j}{\partial p}\right)$$

$$= \frac{4\pi}{(2\pi)^3}\left\{ p\, j(p)\Big|_0^\infty - \int_0^\infty dp\, j(p) \right\}$$

$$\left(\frac{\partial f}{\partial t} p\right) p = \frac{\partial}{\partial p} j(p)$$

$$\text{Heating rate} = \int \frac{d^3p}{(2\pi)^3} p \frac{\partial f}{\partial t} = \frac{2}{2t} \left[\frac{4\pi}{(2\pi)^3} \int_0^\infty p^3 dp \, f(p) \right]$$

$$= \frac{4\pi}{(2\pi)^3} \int_0^\infty dp \, p \left(\frac{\partial j}{\partial p} \right)$$

$$= \frac{4\pi}{(2\pi)^3} \left\{ p \, j(p) \Big|_0^\infty - \int_0^\infty dp \, j(p) \right\}$$

$$-\frac{4\pi}{(2\pi)^3}\int_0^\infty d\mathbf{p}\; j(\mathbf{p}) = -\frac{4\pi}{(2\pi)^3}\frac{n_e\sigma_T}{m_e}\int_0^\infty p^4\left\{\frac{1}{T_e}\frac{\partial F}{\partial p} + f(1+f)\right\}$$

$$j(\mathbf{p})$$

$$-\frac{4\pi}{(2\pi)^3}\int_0^\infty dp\; j(p) = -\frac{4\pi}{(2\pi)^3}\frac{n_e\sigma_T}{m_e}\int_0^\infty p^4\left\{\frac{T_e}{}\frac{\partial F}{\partial p} + f(1+f)\right\}dp$$

$$j(p) = \frac{n_e\sigma_T}{m_e}\,p^4\left\{T_e\frac{\partial F}{\partial p} + f(1+f)\right\}$$

$$-\frac{4\pi}{(2\pi)^3}\int_0^\infty dp\; j(p) = -\frac{4\pi}{(2\pi)^3}\frac{n_e\sigma_T}{m_e}\int_0^\infty p^4\left\{T_e\frac{\partial f}{\partial p} + f(1+f)\right\}dp$$

$$j(p) = \frac{n_e\sigma_T}{m_e}p^4\left\{T_e\frac{\partial f}{\partial p} + f(1+f)\right\}$$

$$p^2 \frac{\partial f}{\partial t} - \frac{\partial}{\partial p} j(p) = \frac{n_e \sigma_T}{m_e} \frac{\partial}{\partial p} p^4 \left\{ T_e \frac{\partial f}{\partial p} + f(1+f) \right\}$$

$$\frac{\partial f}{\partial t} = \left[\frac{n_e \sigma_T}{m_e} T \right] \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left[\frac{\partial f}{\partial x} + f(1+f) \right] \right]$$

heating

$$(\Delta E)_\gamma \propto \int \frac{d^3p}{(2\pi\hbar)^2} \overset{\Delta E(p)}{\left(\frac{p^2}{m_e} \right)} f(p) \underline{\left(1 + f(p) \right)}$$

$$\boxed{x \equiv \frac{p}{T}}$$

$$(\Delta E)_\gamma = (\text{other board})$$

$$\text{Net } \Delta E_e = \frac{\sigma_T}{m_e} \int \frac{d^3p}{(2\pi)^3} p^2 \left\{ T_e \frac{\partial f}{\partial p} + f(1+f) \right\}$$