Title: Astrophysics and Cosmology through Problems - 6B

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Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

$$N = g\int_0^{E_F} \frac{V\,d^3p}{(2\pi\hbar)^3} = \frac{V p_F^3}{3\pi^2\hbar^3}$$

$$\Delta p\,\Delta x \sim \hbar$$

$$\varepsilon = \frac{p^2}{2m}, \quad E_{nr} = \int \varepsilon N = \frac{3}{10}\left(\frac{6\pi^2}{g}\right)^{2/3} \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3} N$$

$$\varepsilon = pc, \quad E_r = \int \varepsilon N = \frac{3}{4}\left(\frac{6\pi^2}{g}\right)^{1/3} \hbar c \left(\frac{N}{V}\right)^{1/3} N$$

$$E_c = \frac{Ze^2}{a}, \quad a = \left(\frac{Z \cdot V}{N}\right)^{1/3}$$

$$E_C = \frac{Ze^2}{a}\ , \quad a = \left(\frac{ZV}{N}\right)^{1/3}$$

$$E_C = \frac{Ze^2}{a}, \quad a = \left(\frac{Z\,V}{N}\right)^{1/3} = \left(\frac{Z\,m}{\rho}\right)^{1/3} = \left(\frac{Z}{n}\right)^{1/3}$$

$$\frac{N}{V} = \frac{\rho}{m}$$

$$E_c = \frac{Ze^2}{a} \quad ; \quad a = \left(\frac{ZV}{N}\right)^{1/3} = \left(\frac{Z m}{\rho}\right)^{1/3} = \left(\frac{Z}{n}\right)^{1/3}$$

$$\frac{N}{V} = \frac{\rho}{m}$$

$$E_K = \frac{E}{N} \ll E_c \Rightarrow \rho \gg \left(\frac{me^2}{\hbar}\right)^3 \left(\frac{m_p A}{Z}\right)$$

Z'

$$P_{nr} = \frac{2}{3}\frac{E_{nr}}{V}$$

$$PV = NkT \quad E = \frac{3}{2}kT \quad \Big\} \; P = \frac{2}{3}\frac{E}{V}$$

$$= \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{\rho}{m_p}\right)^{5/3} \equiv \lambda_{nr}\rho^{5/3}$$

$$P_r = \frac{1}{3}\frac{E}{V} = \frac{1}{4}(3\pi^2)^{2/3}\hbar c\left(\frac{Z}{A}\right)^{4/3}\left(\frac{\rho}{m_p}\right)^{4/3} \equiv \lambda_r \rho^{4/3}$$

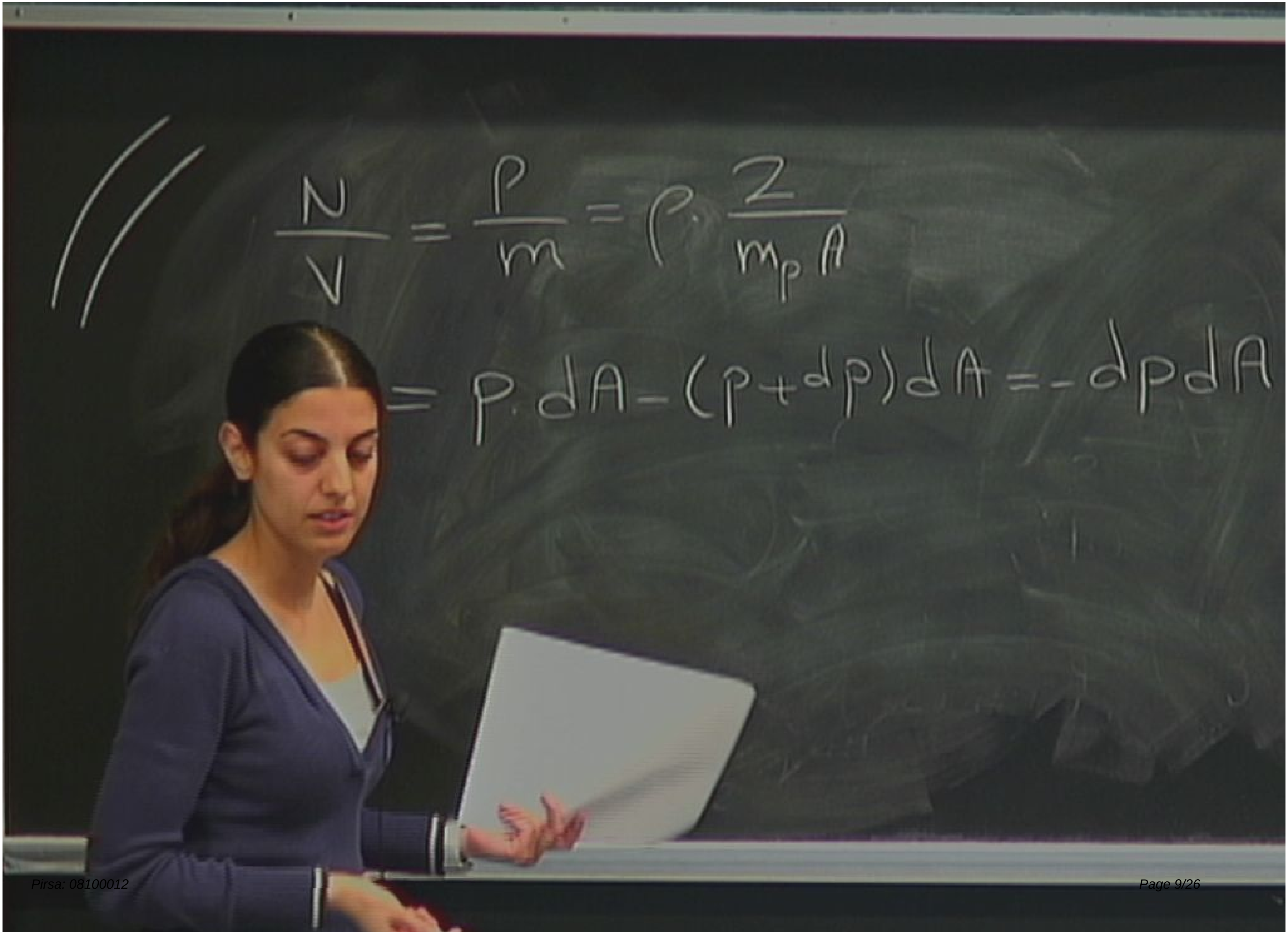
$$P_{nr} = \frac{2}{3}\frac{E_{nr}}{V}$$

$$\left.\begin{aligned} pV &= nkT \\ E &= \frac{3}{2}kT \end{aligned}\right\} p = \frac{2}{3}\frac{E}{V}$$

$$= \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{\rho}{m_p}\right)^{5/3} \equiv \lambda_{nr}\,\rho^{5/3}$$

$$P_r = \frac{1}{3}\frac{E}{V} = \frac{1}{4}(3\pi^2)^{2/3}\hbar c\left(\frac{Z}{A}\right)^{4/3}\left(\frac{\rho}{m_p}\right)^{4/3} \equiv \lambda_r\,\rho^{4/3}$$

Polytrope


$\frac{N}{V} = \frac{\rho}{m} = \rho \cdot \frac{Z}{m_p A}$
$= P \cdot dA - (P + dP)\, dA = -dP\, dA$

$$U_\lambda = u\left(\frac{\lambda}{L}\right)^3 = \underbrace{uL^{-3}}_{\alpha}\,\lambda^3$$

$$\frac{N}{V} = \frac{\rho}{m} = \rho \cdot \frac{Z}{m_p A}$$

$$F_p = p\,dA - (p + dp)\,dA = -dp\,dA$$

$$F_G = \frac{G\,M(r)\,dm}{r^2}\,, \quad dm = \rho\,dr\,dA \qquad \frac{dM(r)}{dr} = \rho\,4\pi r^2$$

$$F_G = F_p \Longrightarrow -dp\,dA = \frac{G\,M(r)\,\rho\,dr\,dA}{r^2} \Longrightarrow \frac{dp}{dr} = \frac{G\,M(r)}{r^2}\,\rho$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 d\varphi}{dr}\right) = 4\pi G\rho$$

$$\Longrightarrow \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 dP}{\rho \cdot dr}\right) = -4\pi G\rho$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 d\varphi}{dr}\right) = 4\pi G\rho$$

$$\Longrightarrow \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 dP}{\rho\, dr}\right) = -4\pi G\rho$$

$$P_{nr} \equiv \lambda_{nr}\,\rho^{5/3}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 d\varphi}{dr}\right) = 4\pi G\rho$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 dP}{\rho\, dr}\right) = -4\pi G\rho$$

$$P_{nr} \equiv \lambda \rho^{\gamma}, \quad \frac{dP}{dr} = \frac{dP}{d\rho}\frac{d\rho}{dr} = \lambda\gamma\rho^{\gamma-1}\frac{d\rho}{dr}$$

$$\rho(r) \propto R^{-6} f(r/R)$$

$$R \sim L$$

$$r \sim L$$

$$\rho(r) \propto R^{-6} f(r/R)$$

$$R - L$$

$$r - L$$

$$\lambda_{nr} - L^4 M^{-7/3} T^{-2}$$

$$G - L^3 M^{-1} T^{-2}$$

$$\rho - L^{-3} M$$

$$\rho(r) \propto R^{-6} f(r/R)$$

$$R - L$$

$$r - L$$

$$\lambda_{nr} - L^4 \cdot M^{-2/3} \cdot T^{-2}$$

$$G - L^3 \cdot M^{-1} T^{-2}$$

$$\rho - L^{-3} \cdot M$$

$$\frac{r}{R}, \quad \frac{\lambda_{nr}^{3}}{G^3 \rho_0 R^6}$$

$\rho(r) \propto R^{-6} f(r/R)$

$R - L$

$r - L$

$\lambda_{nr} - L^4 \cdot M^{-2/3} \cdot T^{-2}$

$G - L^3 \cdot M^{-1} \cdot T^{-2}$

$\rho - L^{-3} \cdot M$

$\frac{r}{R}$, $\frac{\lambda_{nr}^3}{G^3 \rho_0 R^6}$

d_1 $\quad d_2$

$f(d_1, d_2) = K$

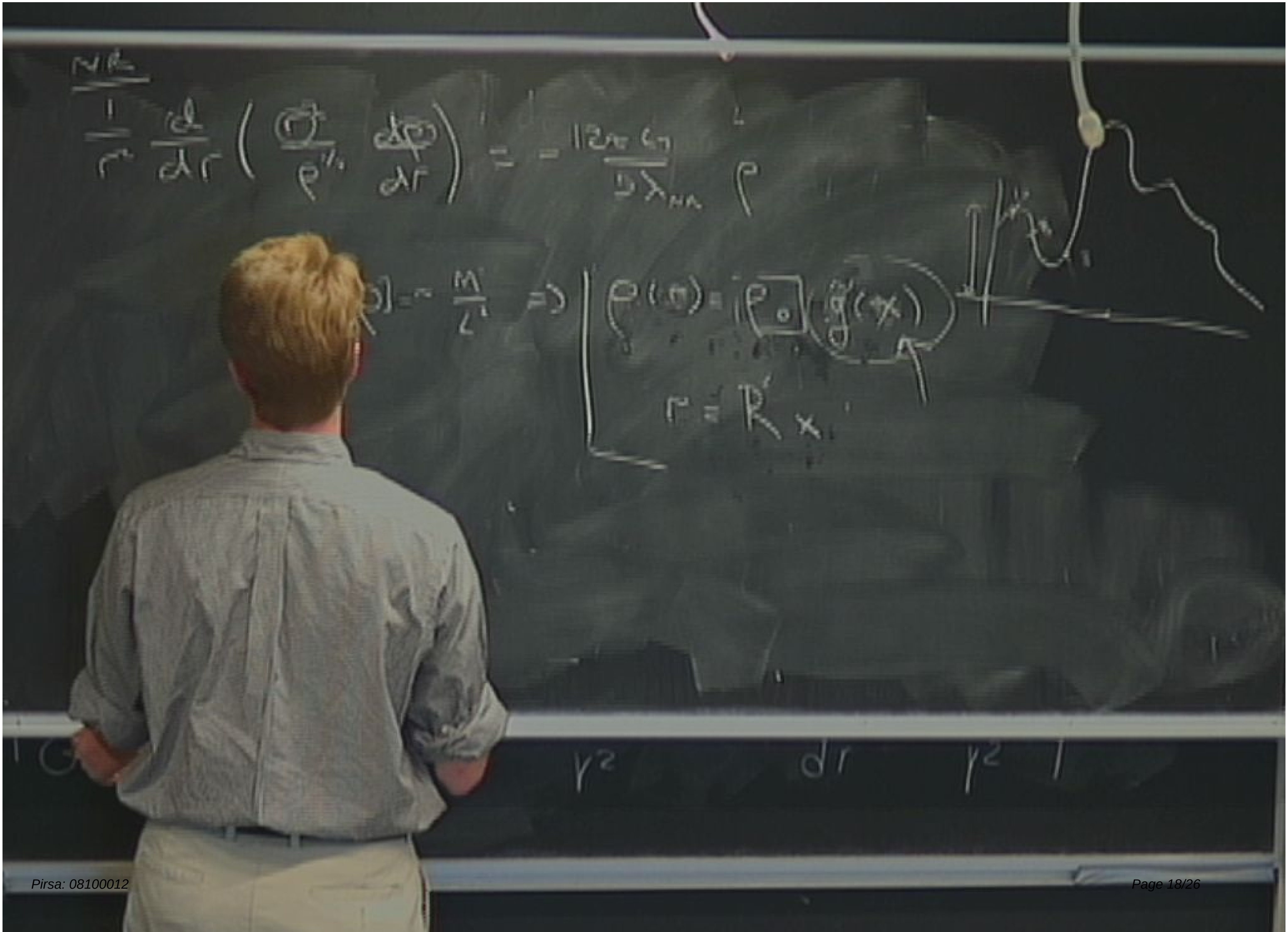
$f_1(d_1) = f_2(d_2)$

$d_2 = f\left(\frac{r}{R}\right)$

$\frac{\lambda_{nr}^3}{G^3 \rho_0 R^6} = f$

$\frac{G^3 \rho_0 R^6}{\lambda_{nr}^3} = f$

$\rho_0 = R^{-6} f\left(\frac{r}{R}\right)$


ρ(r) = ρ0 g(x)
r = R x

NR

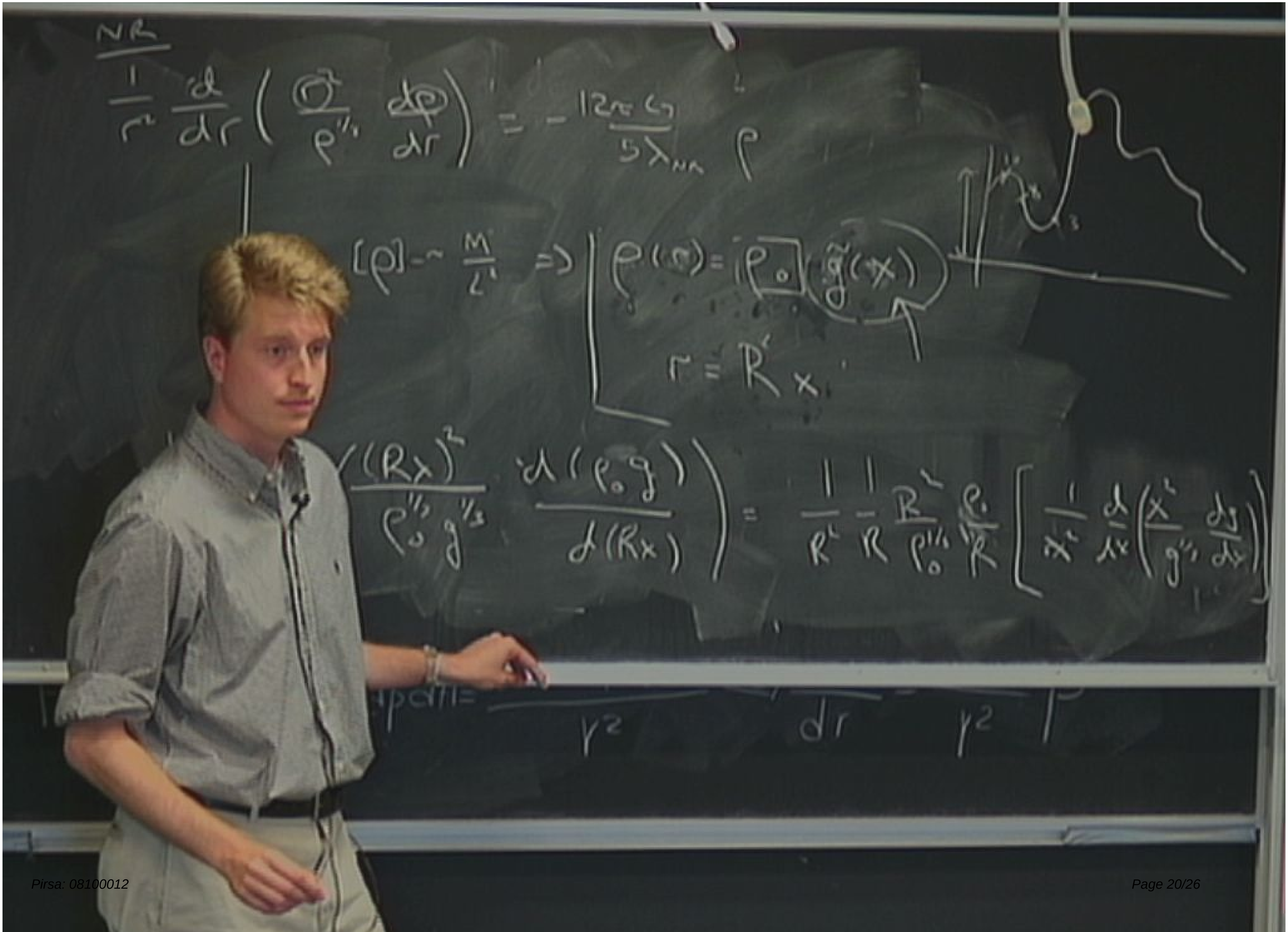
$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho^{1/3}}\frac{d\rho}{dr}\right) = -\frac{12\pi G}{5\lambda_{NR}}\rho$$

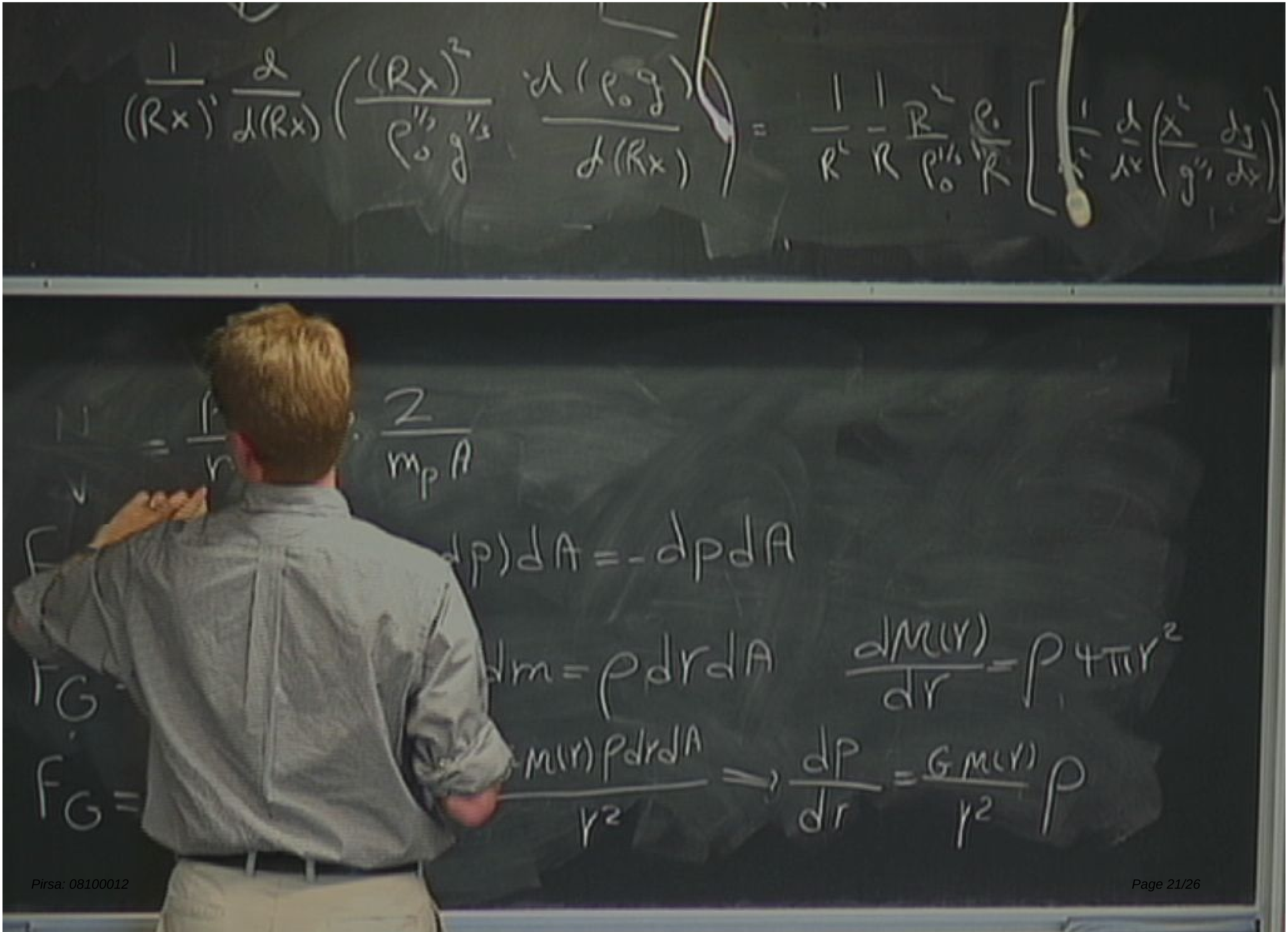
$$[\rho] \sim \frac{M}{L^3} \Rightarrow \rho(r) = \boxed{\rho_0}\, g(x)$$

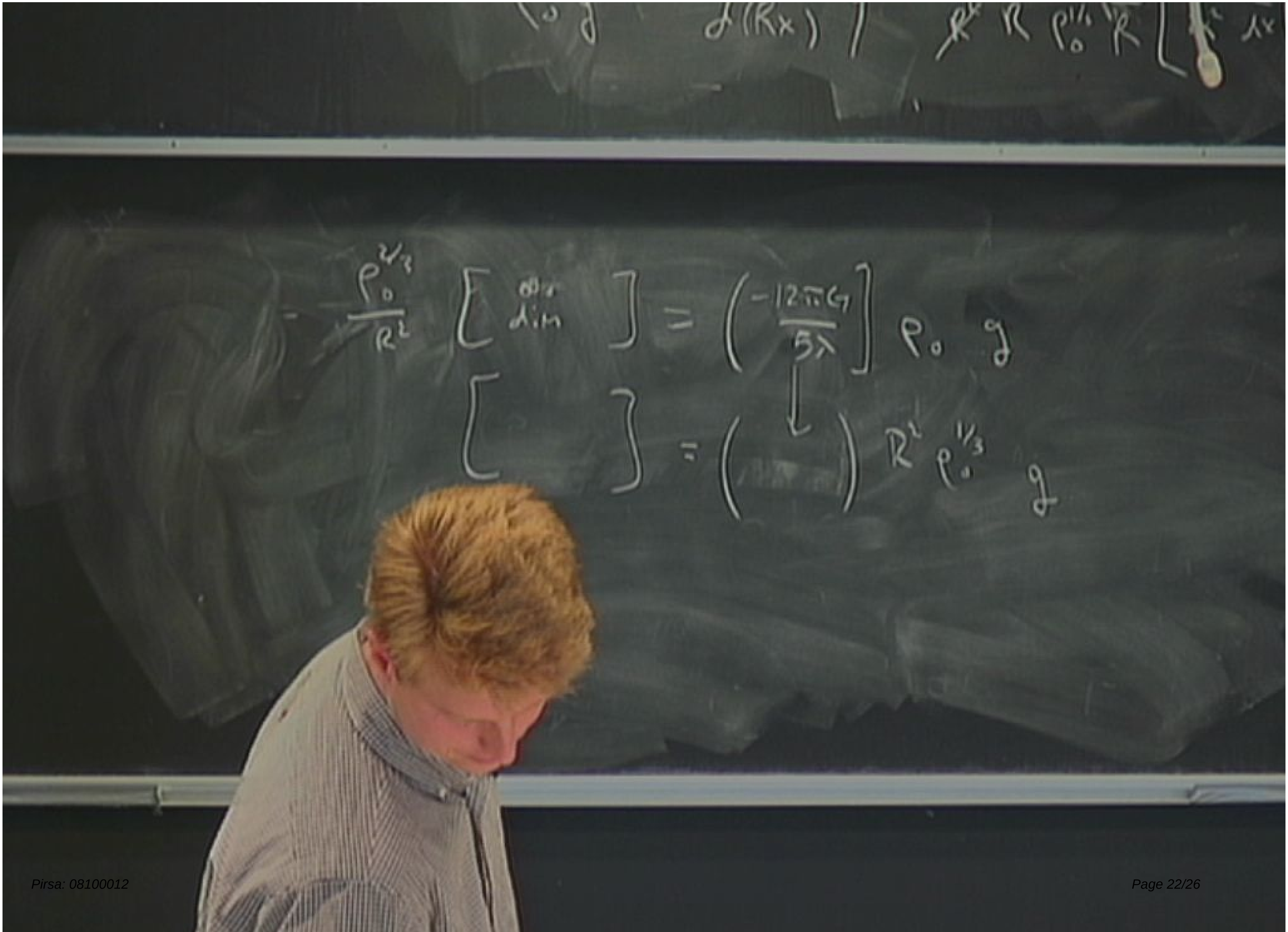
$$r = R x$$

$$\frac{1}{(Rx)^2}$$

$$\Rightarrow -dp\,dA = \frac{GM(r)\,\rho\,dr\,dA}{r^2} \Rightarrow \frac{dp}{dr} = \frac{GM(r)}{r^2}\rho$$

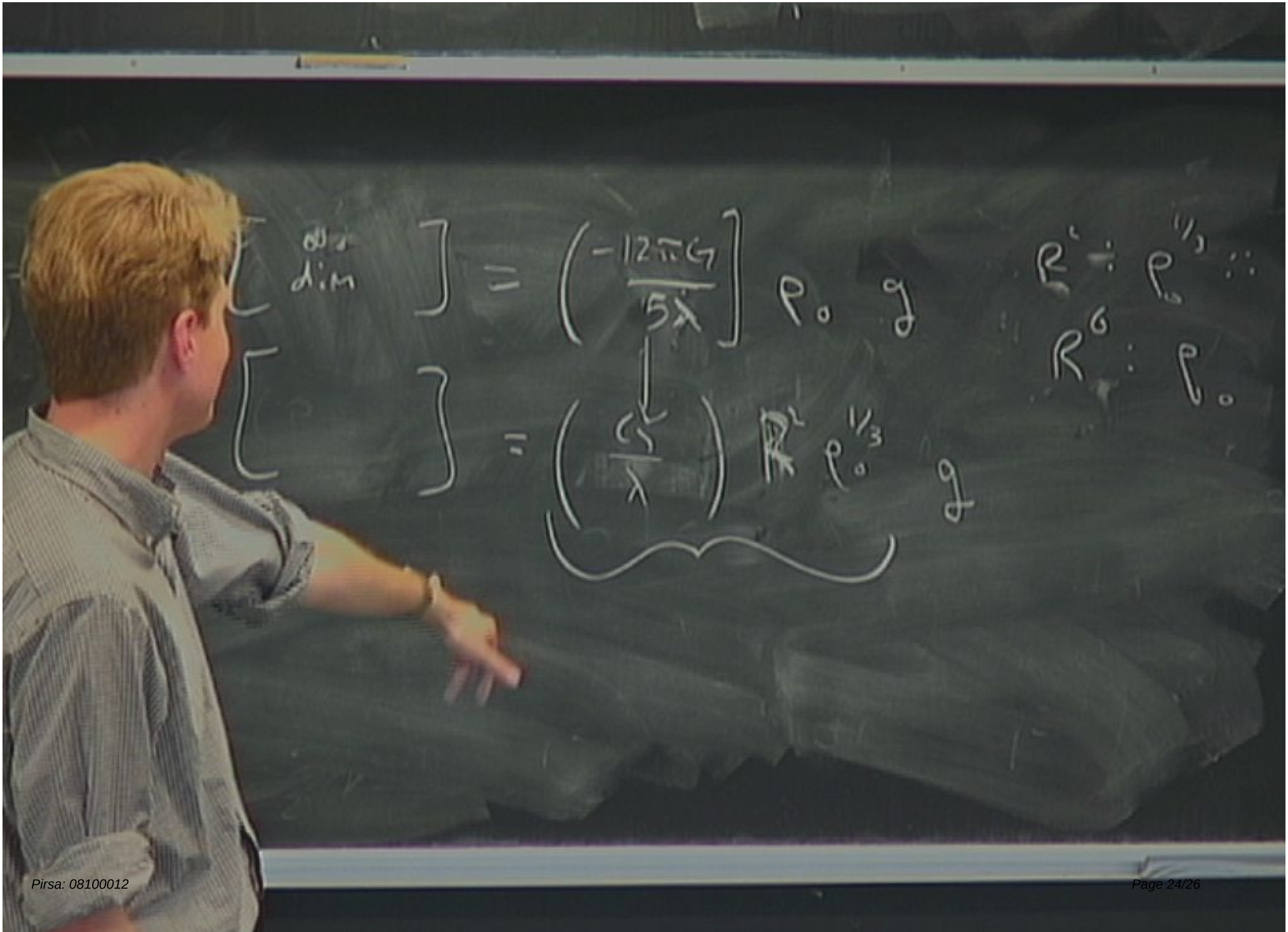



dp)dA = - dpdA
dm = ρdrdA
dM(r)/dr = ρ4πr²
dP/dr = GM(r)/r² ρ



$$\frac{\rho_0^{2/3}}{R^2}\left[\ \text{dim}\ \right] = \left(\frac{-12\pi G}{5\lambda}\right] \rho_0 \quad g$$

$$\left[\qquad\right] = \underbrace{\left(\qquad\right) R^2 \rho_0^{1/3}}\quad g$$


dim

$$\frac{\rho_0^{2/3}}{R^2}\left[\begin{matrix}\text{dim} \\ \text{d:M}\end{matrix}\right] = \left(\frac{-12\pi G}{5\lambda}\right]\rho_0 \quad g \qquad R^2 : \rho_0^{1/3} ::$$

$$R^6 : \rho_0$$

$$\Big[\qquad\Big] = \underbrace{\left(\frac{G}{\lambda}\right) R^2 \rho_0^{1/3}}\quad g$$

$$\frac{\rho_0^{2/3}}{R^2}\left[\begin{matrix}\mathrm{dim} \\ \mathrm{dim}\end{matrix}\right] = \left(\frac{-12\pi G}{5\lambda}\right] \rho_0 \; g \qquad R^2 : \rho_0^{1/3} ::$$

$$R^6 : \rho_0$$

$$\left[\quad\right] = \underbrace{\left(\frac{G}{\lambda}\right) R^2 \rho_0^{1/3}}\; g$$