

Title: Astrophysics and Cosmology through Problems - 6B

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Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

$$N = g \int_0^{E_F} \frac{V d^3 p}{(2\pi\hbar)^3} = \frac{V p_F^3}{3\pi^2 \hbar^3}$$

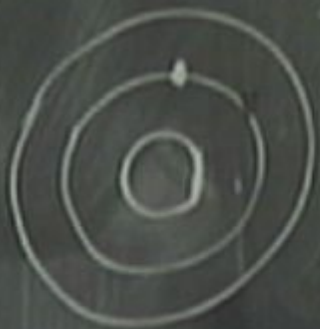
$$\Delta p \Delta x \sim \hbar$$

$$\Sigma = \frac{p^2}{2m}, \quad E_{\text{avg}} = \int \Sigma N = \frac{3}{10} \left( \frac{6\pi^2}{g} \right)^{2/3} \frac{\hbar^2}{m} \left( \frac{N}{V} \right)^{2/3} N$$

$$\Sigma = pc, \quad E_{\text{avg}} = \int \Sigma N = \frac{3}{4} \left( \frac{6\pi^2}{g} \right)^{1/3} \frac{\hbar c}{m} \left( \frac{N}{V} \right)^{1/3} N$$

$$E_c = \frac{Zc^2}{a}, \quad a = \left( \frac{Zv}{N} \right)^{1/3}$$

$$E_C = \frac{Z e^2}{a}, \quad a = \left( \frac{Z \cdot v}{N} \right)^{1/3}$$



$$E_c = \frac{Z e^2}{a}, \quad a = \left( \frac{Z \cdot v}{N} \right)^{1/3} = \left( \frac{Z \cdot m}{\rho} \right)^{1/3} = \left( \frac{Z}{n} \right)^{1/3}$$



$$\frac{Z}{v} = \frac{\rho}{m}$$

$$E_c = \frac{Z e^2}{a}, \quad a = \left( \frac{Z v}{N} \right)^{1/3} = \left( \frac{Z m}{\rho} \right)^{1/3} = \left( \frac{Z}{n} \right)^{1/3}$$



$$\frac{Z}{v} = \frac{\rho}{m}$$

$$E_K = \frac{E}{N} \ll E_c \Rightarrow \rho \gg \left( \frac{m e^2}{\hbar} \right)^3 \left( \frac{\rho A}{Z} \right)$$

$$P_{nr} = \frac{2}{3} \frac{E_{nr}}{V}$$

$$= \frac{(3\bar{u}^2)^{2/3}}{5} \frac{\hbar^2}{m} \left(\frac{Z}{A}\right)^{5/2} \left(\frac{\rho}{m_p}\right)^{5/2} \equiv \lambda_{nr} \rho^{5/2}$$

$$P_r = \frac{1}{3} \frac{E}{V} = \frac{1}{4} (3\bar{u}^2)^{2/3} \frac{1}{\hbar c} \left(\frac{Z}{A}\right)^{4/3} \left(\frac{\rho}{m_p}\right)^{4/3} \equiv \lambda_r \rho^{4/3}$$

$$PV = nKT \quad \left\{ \begin{array}{l} P = \frac{2}{3} \frac{E}{V} \\ E = \frac{3}{2} nKT \end{array} \right.$$

$$P_{nr} = \frac{2}{3} \frac{E_{nr}}{V}$$

$$= \frac{(3\bar{n}^2)^{2/3}}{5} \frac{\hbar^2}{m} \left(\frac{Z}{A}\right)^{5/4} \left(\frac{\rho}{m_p}\right)^{5/4} \equiv \lambda_{nr} \rho^{5/4}$$

$$P_r = \frac{1}{3} \frac{E}{V} = \frac{1}{4} (3\bar{n}^2)^{2/3} \frac{\hbar c}{A} \left(\frac{Z}{A}\right)^{4/3} \left(\frac{\rho}{m_p}\right)^{4/3} \equiv \lambda_r \rho^{4/3}$$

Polytrope

$$PV = nKT \quad \left\{ P = \frac{2}{3} \frac{E}{V} \right.$$

$$\Sigma = \frac{3}{2} KT$$



$$\frac{Z}{V} = \frac{\rho}{m} = \rho \cdot \frac{Z}{m_p A}$$

$$= \rho \cdot dA - (\rho + d\rho) dA = -d\rho dA$$

$$v_{\lambda} = u \left( \frac{\lambda}{L} \right)^3 = \frac{uL}{\alpha} \lambda^3$$



$$\frac{Z}{v} = \frac{\rho}{m} = \rho \cdot \frac{Z}{m_p A}$$

$$F_p = p \cdot dA - (p + dp) dA = -dp dA$$

$$F_G = \frac{G m(r) dm}{r^2}, \quad dm = \rho dr dA \quad \frac{dM(r)}{dr} = \rho 4\pi r^2$$

$$F_G = F_p \Rightarrow -dp dA = \frac{G M(r) \rho dr dA}{r^2} \Rightarrow \frac{dp}{dr} = \frac{G M(r)}{r^2} \rho$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = 4\pi G \rho$$

$$\Rightarrow \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 dP}{\rho dr} \right) = -4\pi G \rho$$

$$\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\phi}{dr} \right) = 4\pi G \rho$$

$$\Rightarrow \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\rho}{\rho dr} \right) = -4\pi G \rho$$

$$\rho = \lambda r r^{\frac{5}{3}}$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = 4\pi G \rho$$

$$\Rightarrow \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 dP}{\rho dr} \right) = -4\pi G \rho$$

$$\rho_{nr} = \lambda \rho^\gamma, \quad \frac{dP}{dr} = \frac{dP}{d\rho} \frac{d\rho}{dr} = \lambda \rho^{\gamma-1} \frac{d\rho}{dr}$$

$$\rho(r) \propto R^{-6} f(r/R)$$

$$R - L$$

$$r - L$$

$$\rho(r) \propto R^{-6} f(r/R)$$

$$R - L$$

$$r - L$$

$$\lambda_{nr} - L^4 M^{-2/3} T^{-2}$$

$$G - L^3 M^{-1} T^{-2}$$

$$\rho - L^{-3} M$$

$$\rho(r) \propto R^{-6} f(r/R)$$

$$R - L$$

$$r - L$$

$$\lambda_{nr} = L^4 \cdot M^{-2/3} \cdot T^{-2}$$

$$G = L^3 \cdot M^{-1} \cdot T^{-2}$$

$$\rho = L^{-3} \cdot M$$

$$\frac{r}{R}$$

$$\frac{\lambda_{nr}^3}{G^3 \rho_0 R^6}$$



$$P(r) \propto R^{-6} f(r/R)$$

$$R - L$$

$$r - L$$

$$\lambda_{nr} - L^4 M^{-2/3} T^{-2}$$

$$G - L^3 M^{-1} T^{-2}$$

$$\rho - L^{-3} M$$

$$\frac{r}{R}, \quad d_1$$

$$\frac{\lambda_{nr}^3}{G^3 \rho_0 R^6}, \quad d_2$$

$$d_2 = f\left(\frac{r}{R}\right)$$

$$f(d_1, d_2) = k$$

$$\frac{\lambda_{nr}^3}{G^3 \rho_0 R^6} = f$$

$$f_1(d_1) = f_2(d_2)$$

$$\frac{G^3 \rho_0 R^6}{\lambda_{nr}^3} = f$$

$$P_0 = R^{-6} f\left(\frac{r}{R}\right)$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dp}{dr} \right) = - \frac{12 \pi \epsilon_0 \epsilon_1}{5 \lambda_{HH}^2} p$$

$$p(r) = \frac{M}{r^2} \Rightarrow \left[ p(r) = p_0 \left( \frac{r_0}{r} \right)^2 \right]$$

$$p = \frac{R}{x}$$

$r^2 \quad dr \quad r^2 \quad |$

$$\frac{1}{r^2} \frac{d}{dr} \left( \rho \frac{dr}{dr} \right) = - \frac{12\pi G}{5\lambda_{Na}} \rho$$

$$[\rho] \sim \frac{M}{L^3} \Rightarrow \left[ \rho(r) = \rho_0 \left( \frac{r}{R} \right)^2 \right]$$

$r = R \cdot x$

$$\frac{1}{(R \cdot x)^2}$$

$$\Rightarrow -dp dA = \frac{G M(r) \rho dr dA}{r^2} \Rightarrow \frac{dp}{dr} = \frac{G M(r)}{r^2} \rho$$

$$\frac{2}{3} \frac{d}{dr} \left( \frac{\rho^2}{\rho_0^2} \frac{d\rho}{dr} \right) = - \frac{12\pi \zeta}{5 \lambda_{\text{min}}} \rho$$

$$[\rho] \sim \frac{1}{L^3} \Rightarrow \left[ \rho(r) = \rho_0 \left( \frac{r}{R} \right)^2 \right]$$

$$r = R x$$

$$\left( \frac{(R x)^2}{\rho_0^2 g^{1/3}} \frac{d(\rho_0 g)}{d(R x)} \right) = \frac{1}{R^2} \frac{1}{R} \frac{R^2}{\rho_0^2} \frac{\rho_0}{R} \left[ \frac{1}{x^2} \frac{d}{dx} \left( \frac{x^2}{g^{1/3}} \frac{dg}{dx} \right) \right]$$

$$\frac{1}{(Rx)'} \frac{d}{d(Rx)} \left( \frac{(Rx)^2}{\rho_0^{1/3} g^{1/3}} \frac{d(\rho_0 g)}{d(Rx)} \right) = \frac{1}{R^2} \frac{1}{R} \frac{R^2}{\rho_0^{1/3}} \frac{\rho_0}{R} \left[ \frac{1}{R^2} \frac{d}{dx} \left( \frac{x^2}{g^{1/3}} \frac{dg}{dx} \right) \right]$$

$$\frac{1}{v} = \frac{f}{v} \cdot \frac{2}{m_p A}$$

$$F = (dp) dA = -dp dA$$

$$F_G = dm = \rho dr dA \quad \frac{dM(r)}{dr} = \rho 4\pi r^2$$

$$F_G = \frac{M(r) \rho dr dA}{r^2} \Rightarrow \frac{dp}{dr} = \frac{G M(r)}{r^2} \rho$$

$\mathcal{L}(Kx) / R \cong P_0 R / R \cong R / P_0 \cong R / \mathfrak{m}$

$$\begin{aligned}
 \frac{P_0^2}{R^2} \left[ \begin{array}{c} \text{ord} \\ \text{dim} \end{array} \right] &= \left( \begin{array}{c} -12\pi G \\ 5\lambda \end{array} \right) P_0 \mathfrak{g} \\
 \left[ \begin{array}{c} \phantom{\text{ord}} \\ \phantom{\text{dim}} \end{array} \right] &= \left( \begin{array}{c} \phantom{-12\pi G} \\ \phantom{5\lambda} \end{array} \right) R^2 P_0^{\frac{11}{3}} \mathfrak{g}
 \end{aligned}$$



$$\frac{P_0^{2/3}}{R^2}$$

$$\left[ \begin{array}{c} \text{dim} \\ \text{dim} \end{array} \right]$$

$$= \left( \begin{array}{c} -12\pi G \\ 5\lambda \end{array} \right)$$

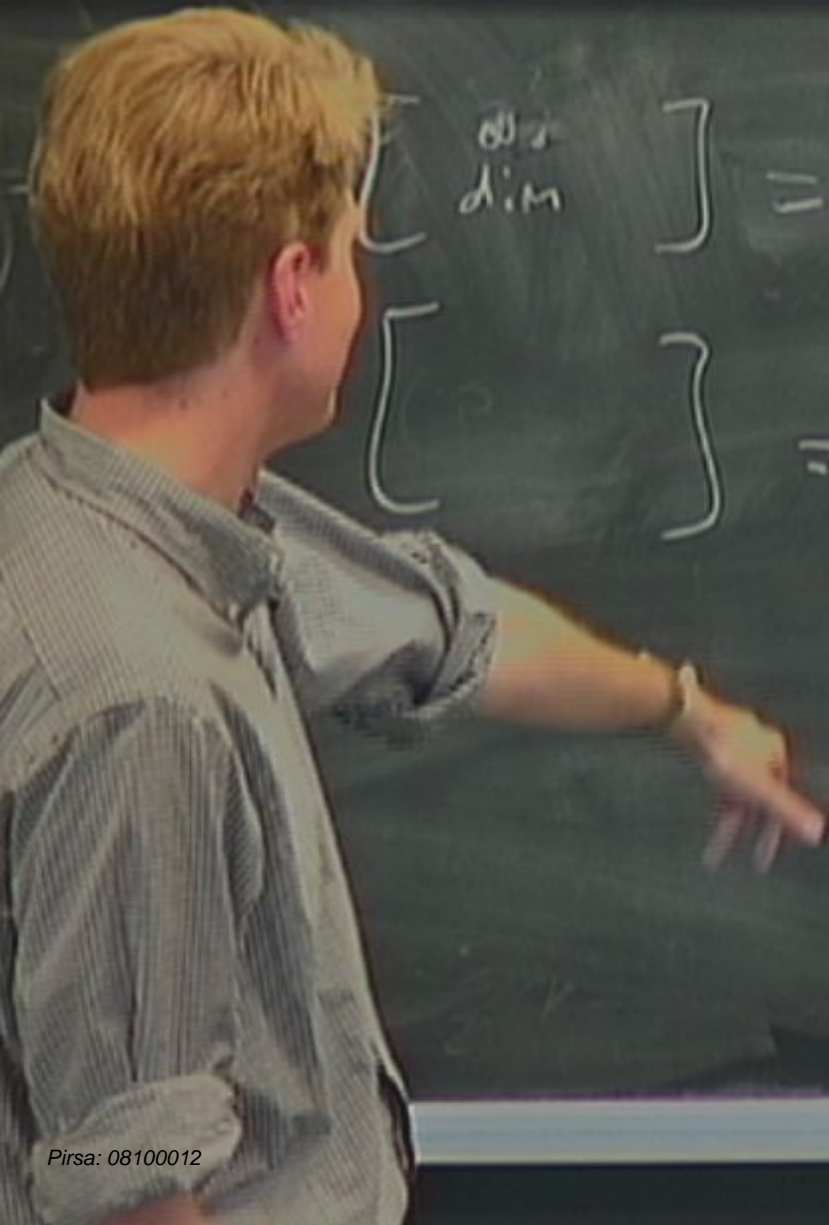
$$P_0 \quad \varphi$$

$$\left[ \begin{array}{c} \phantom{\text{dim}} \\ \phantom{\text{dim}} \end{array} \right]$$

$$= \left( \begin{array}{c} \phantom{-12\pi G} \\ \phantom{5\lambda} \end{array} \right)$$

$$R^2 \quad P_0^{1/3}$$

$$\varphi$$



$$\left[ \begin{array}{c} \text{or} \\ \text{dim} \end{array} \right] = \left( \begin{array}{c} -12\pi G \\ 5\lambda \end{array} \right) \rho_0 \quad \varphi$$
$$\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} = \left( \begin{array}{c} 5 \\ 5 \\ \times \end{array} \right) \rho_0^{1/3} \quad \varphi$$

$$\begin{array}{c} \rho_0 \\ \rho_0 \end{array} \quad \begin{array}{c} \rho_0 \\ \rho_0 \end{array}$$



$$\frac{p_0^2}{R^2}$$

$$\left[ \begin{array}{c} \text{dim} \\ \text{dim} \end{array} \right]$$

$$= \left( \begin{array}{c} -12\pi G \\ 5\lambda \end{array} \right) p_0 \varphi$$

$$\left[ \begin{array}{c} \\ \\ \\ \end{array} \right]$$

$$= \left( \begin{array}{c} 5 \\ 5 \\ \lambda \end{array} \right) p_0^2 \varphi$$

$$\begin{array}{c} R^r \\ R^0 \end{array} \vdots \begin{array}{c} p_0^2 \\ p_0 \end{array} \vdots$$

$$\frac{p_0^{2/3}}{R^2}$$

$$\left[ \begin{array}{c} \text{dim} \\ \text{dim} \end{array} \right]$$

$$= \left( \begin{array}{c} -12\pi G \\ 5\lambda \end{array} \right) p_0 \varphi$$

$$\left[ \right]$$

$$= \left( \begin{array}{c} 5 \\ \lambda \end{array} \right) \frac{p_0^{2/3}}{R^2} \varphi$$

$$\begin{array}{c} R^r \\ R^0 \end{array} \begin{array}{c} \vdots \\ \vdots \end{array} \begin{array}{c} p_0^r \\ p_0^0 \end{array} \begin{array}{c} \vdots \\ \vdots \end{array}$$