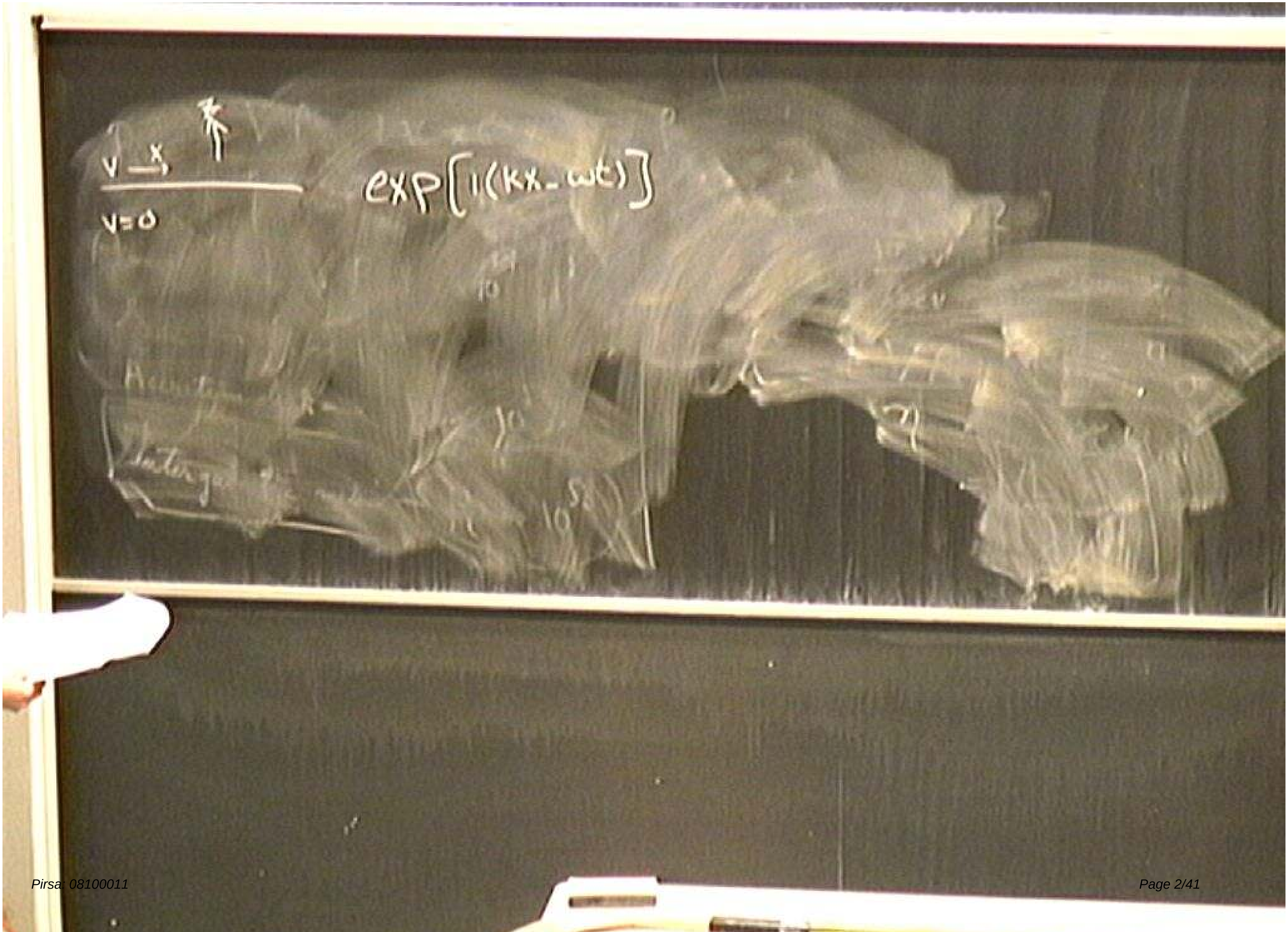
Title: Astrophysics and Cosmology through Problems - 5B

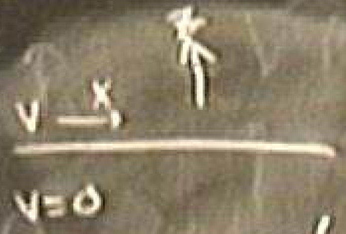
Date: Oct 02, 2008 12:30 PM

URL: http://pirsa.org/08100011

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

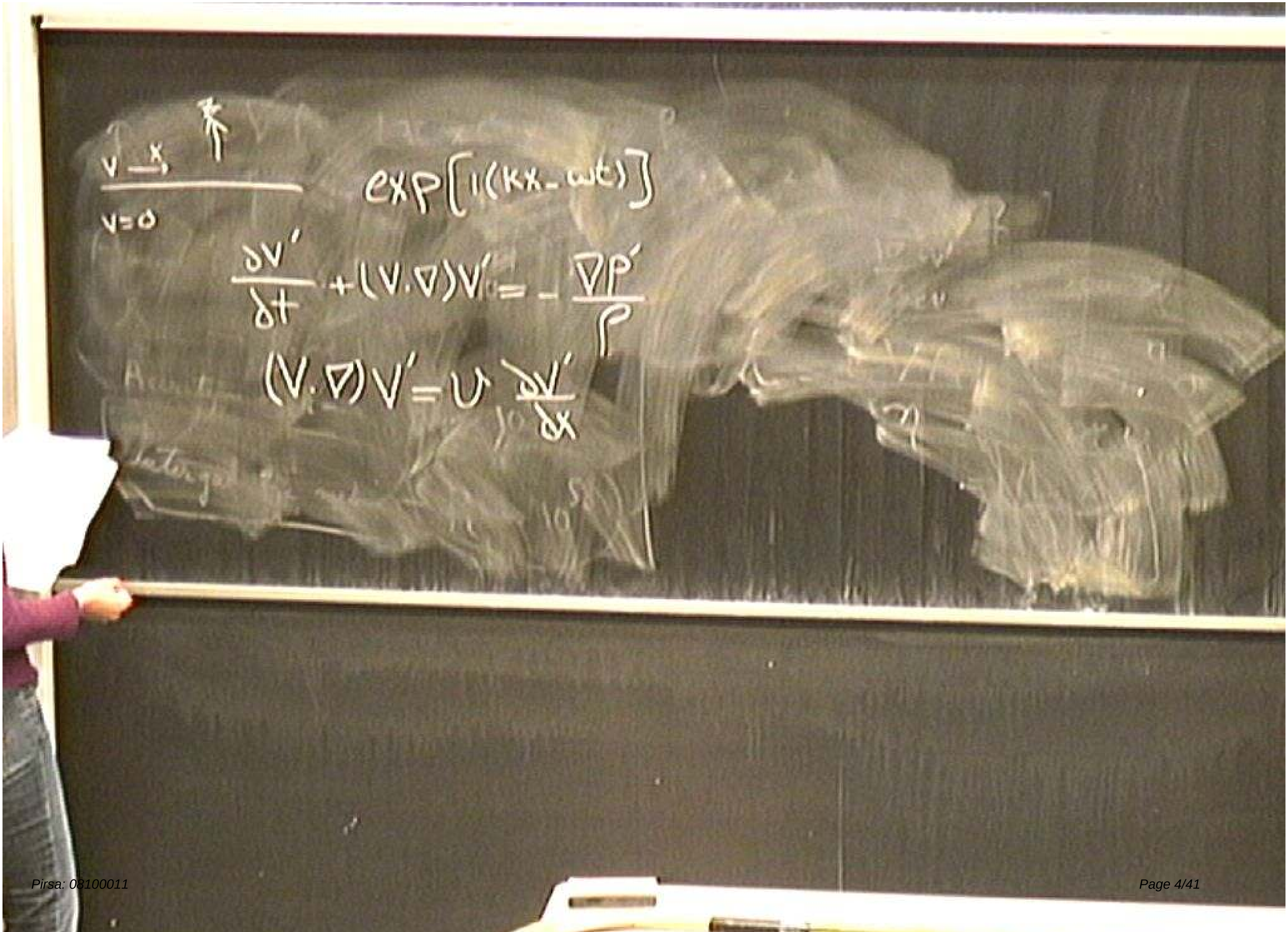


$$\exp[i(kx - \omega t)]$$



$\exp[i(kx - \omega t)]$

$$\frac{\partial V'}{\partial t} + (V \cdot \nabla)V' = -\frac{\nabla p'}{\rho}$$



$\overset{z}{\uparrow}$

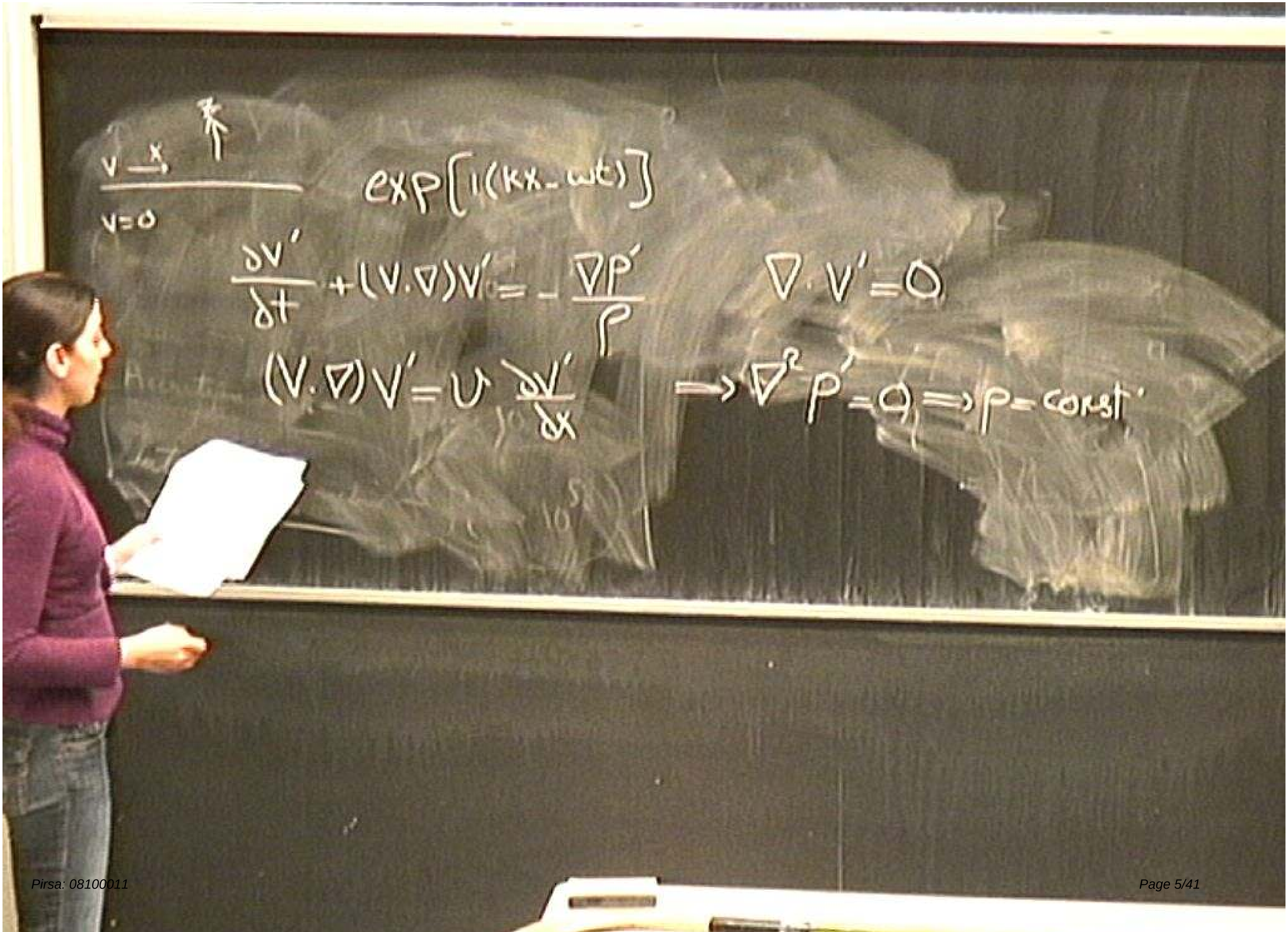
$v \longrightarrow x,$

$v = 0$

$$\exp[i(kx - \omega t)]$$

$$\frac{\partial v'}{\partial t} + (V.\nabla)V' = -\frac{\nabla p'}{\rho}$$

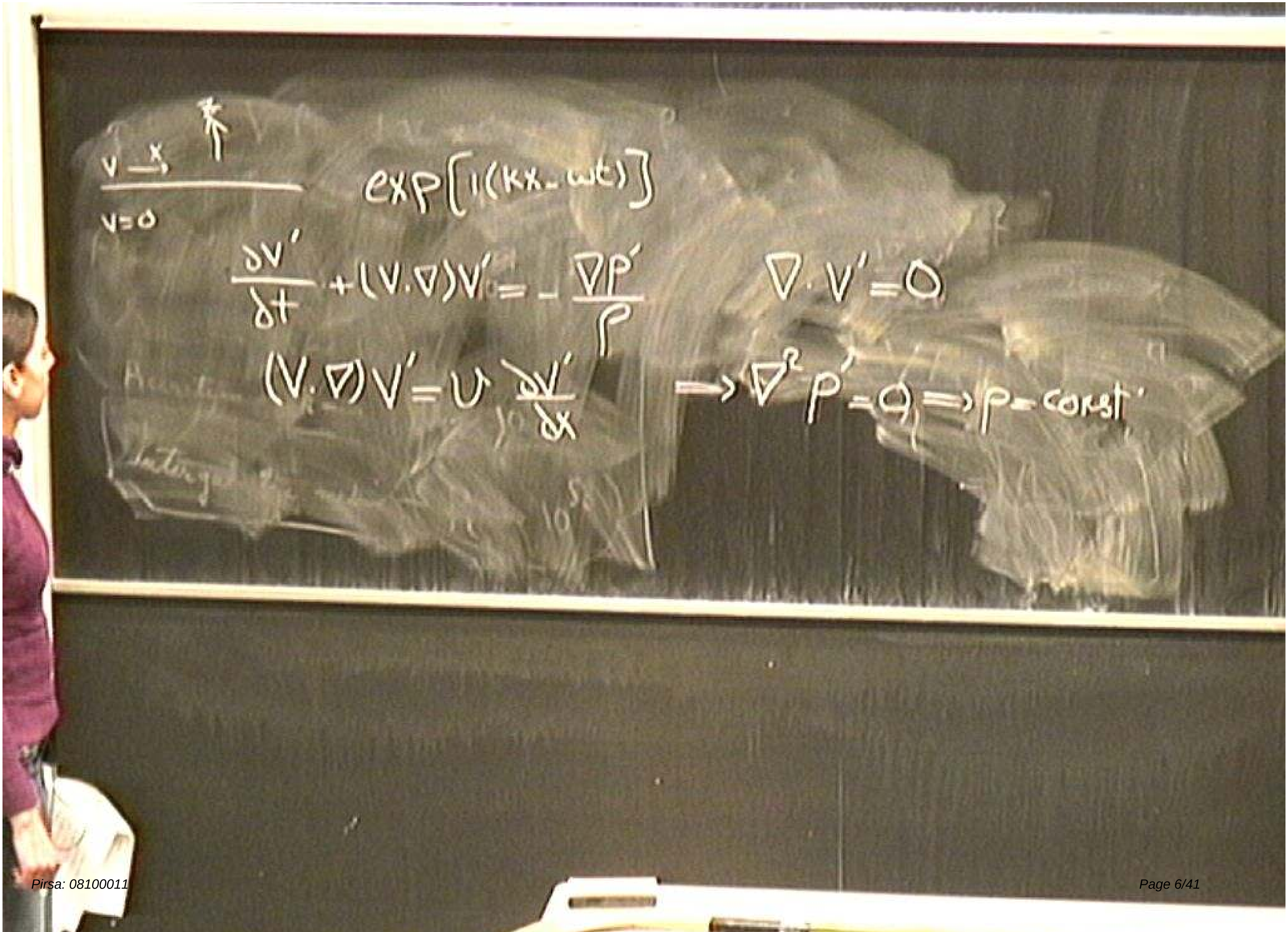
$$(V.\nabla)V' = U\frac{\partial V'}{\partial x}$$



$$\exp[i(kx-\omega t)]$$

$$\frac{\partial V'}{\partial t}+(V\cdot\nabla)V'=-\frac{\nabla P'}{\rho} \qquad \nabla\cdot V'=0$$

$$(V\cdot\nabla)V'=U\frac{\partial V'}{\partial x} \Longrightarrow \nabla^2 P'=0 \Longrightarrow P=\text{const}'$$



$$\exp[i(kx - \omega t)]$$

$$\frac{\partial V'}{\partial t} + (V.\nabla)V' = -\frac{\nabla p'}{\rho} \qquad \nabla . V' = 0$$

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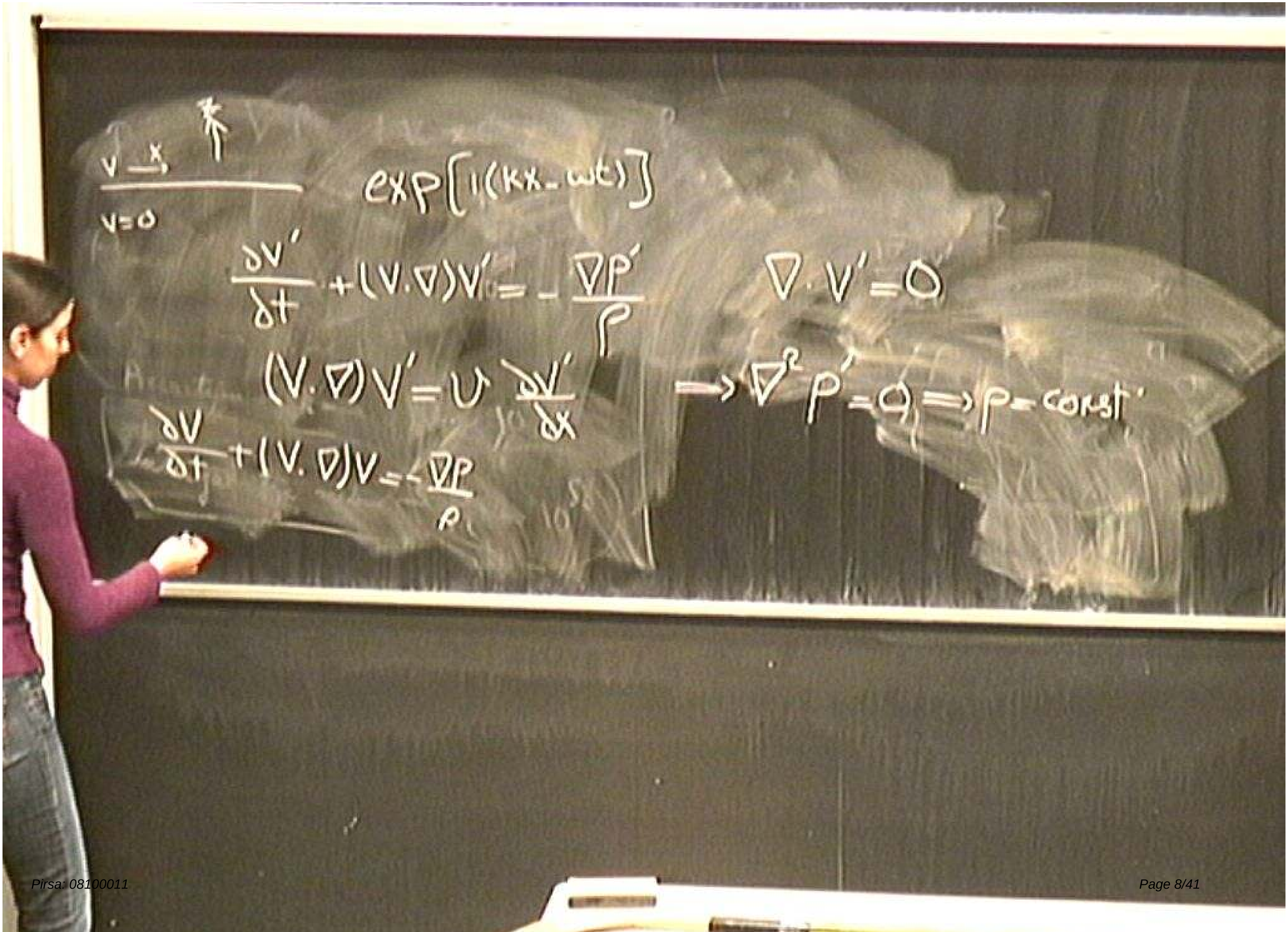
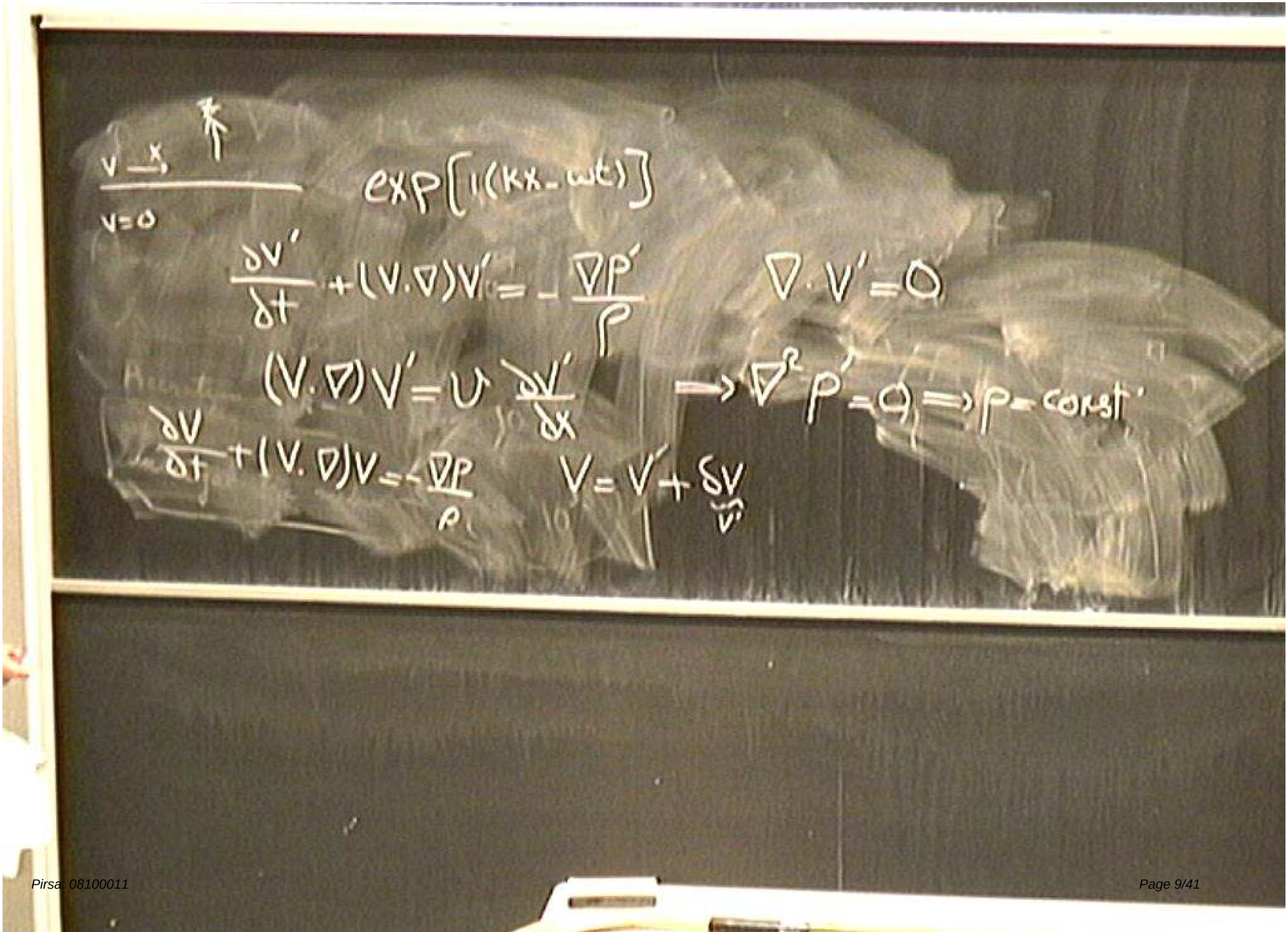


$$\exp[i(kx - \omega t)]$$

$$\frac{\partial v'}{\partial t} + (V.\nabla)v' = -\frac{\nabla p'}{\rho} \qquad \nabla \cdot v' = 0$$

$$(V.\nabla)v' = U\frac{\partial v'}{\partial x} \longrightarrow \nabla^2 p' = 0 \Longrightarrow p = \text{const}'$$

$$\frac{\partial V}{\partial t} + (V.\nabla)V = -\frac{\nabla P}{\rho}$$



$v \xrightarrow{x}$ $\overset{k}{\uparrow}$

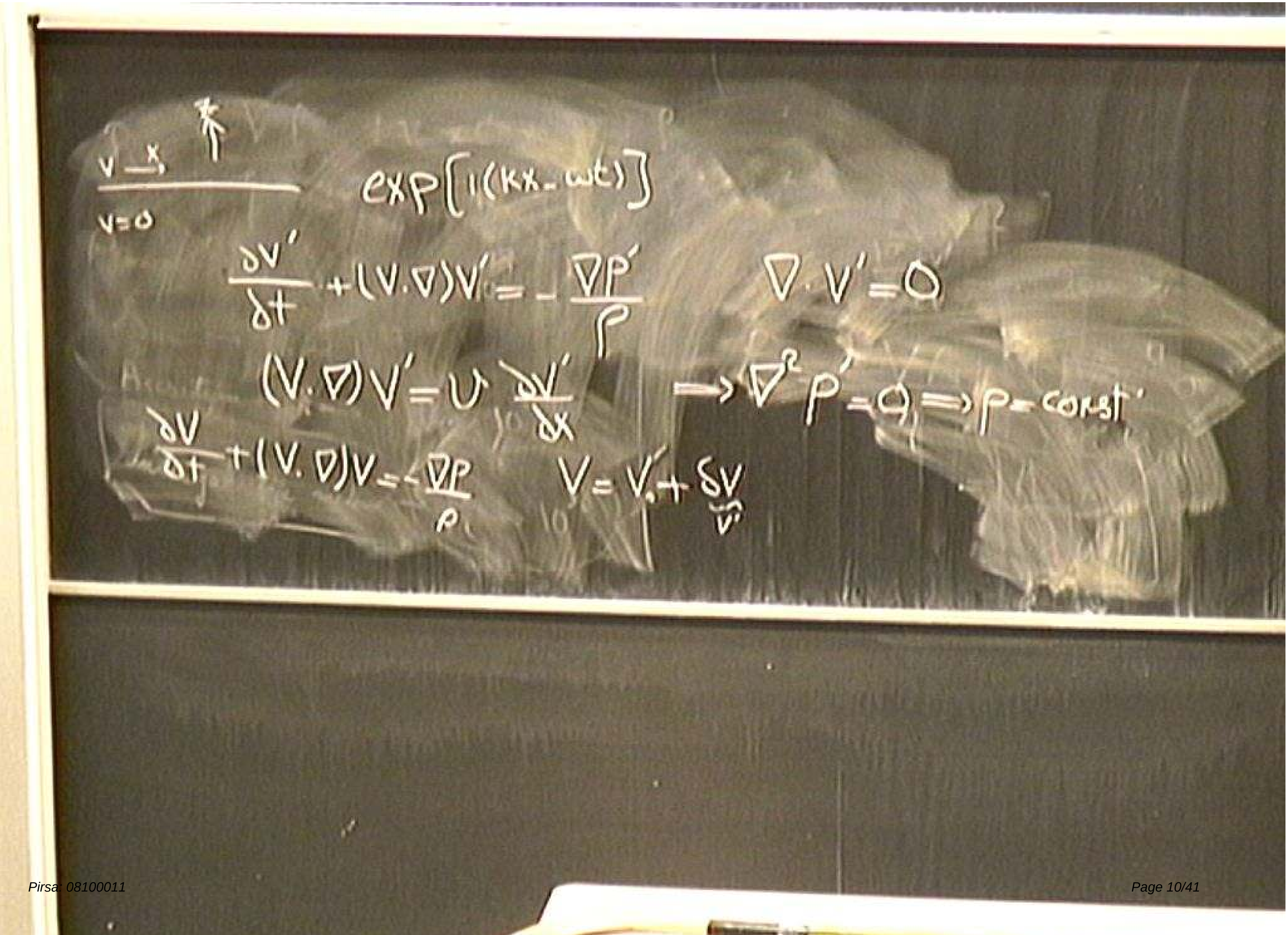
$v=0$

$\exp\left[i(kx-\omega t)\right]$

$$\frac{\partial v'}{\partial t} + (V.\nabla)V' = -\frac{\nabla p'}{\rho} \qquad \nabla \cdot V' = 0$$

$$(V.\nabla)V' = U\frac{\partial V'}{\partial x} \Longrightarrow \nabla^2 p' = 0 \Longrightarrow p = \text{const}'$$

$$\frac{\partial V}{\partial t} + (V.\nabla)V = -\frac{\nabla P}{\rho} \qquad V = V' + \underset{\tilde{v}}{\delta V}$$



$v \xrightarrow{x}$

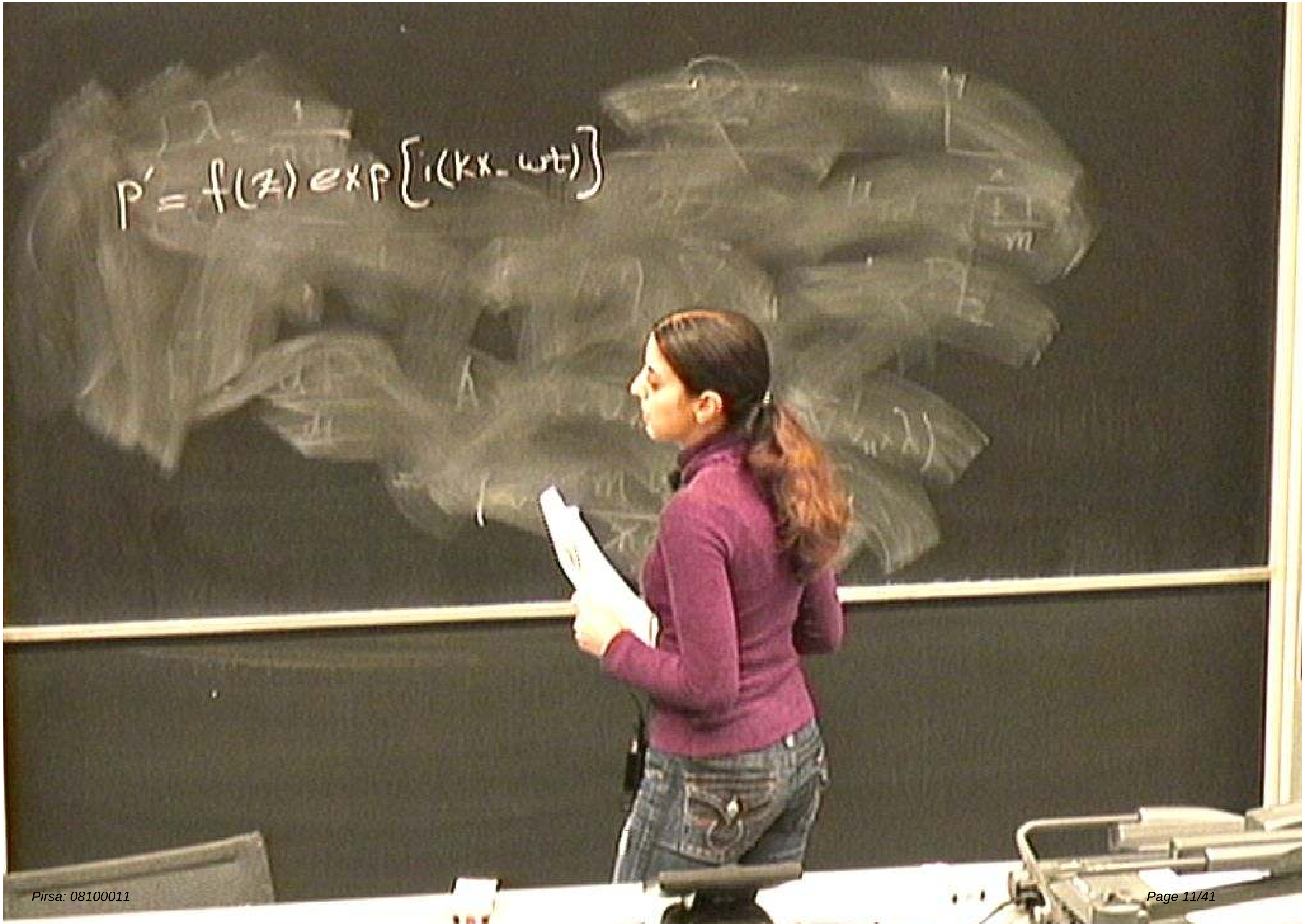
$v=0$

$\exp[i(kx - \omega t)]$

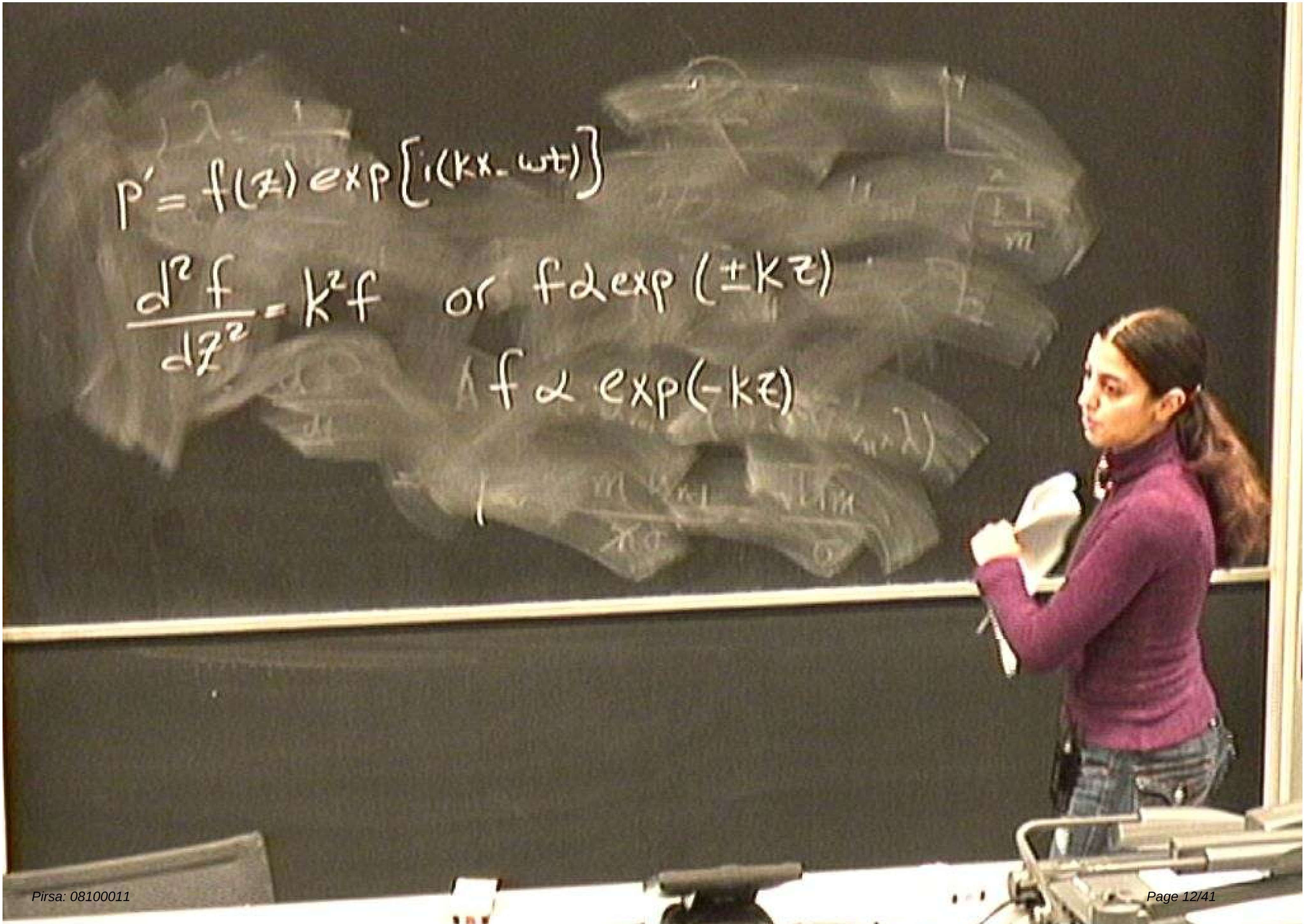
$$\frac{\partial v'}{\partial t} + (V.\nabla)V' = -\frac{\nabla p'}{\rho} \qquad \nabla . V' = 0$$

$$(V.\nabla)V' = U\frac{\partial V'}{\partial x} \implies \nabla^2 p' = 0 \implies p = \text{const}'$$

$$\frac{\partial V}{\partial t} + (V.\nabla)V = -\frac{\nabla P}{\rho} \qquad V = V'_0 + \underset{\tilde{v}}{\delta V}$$



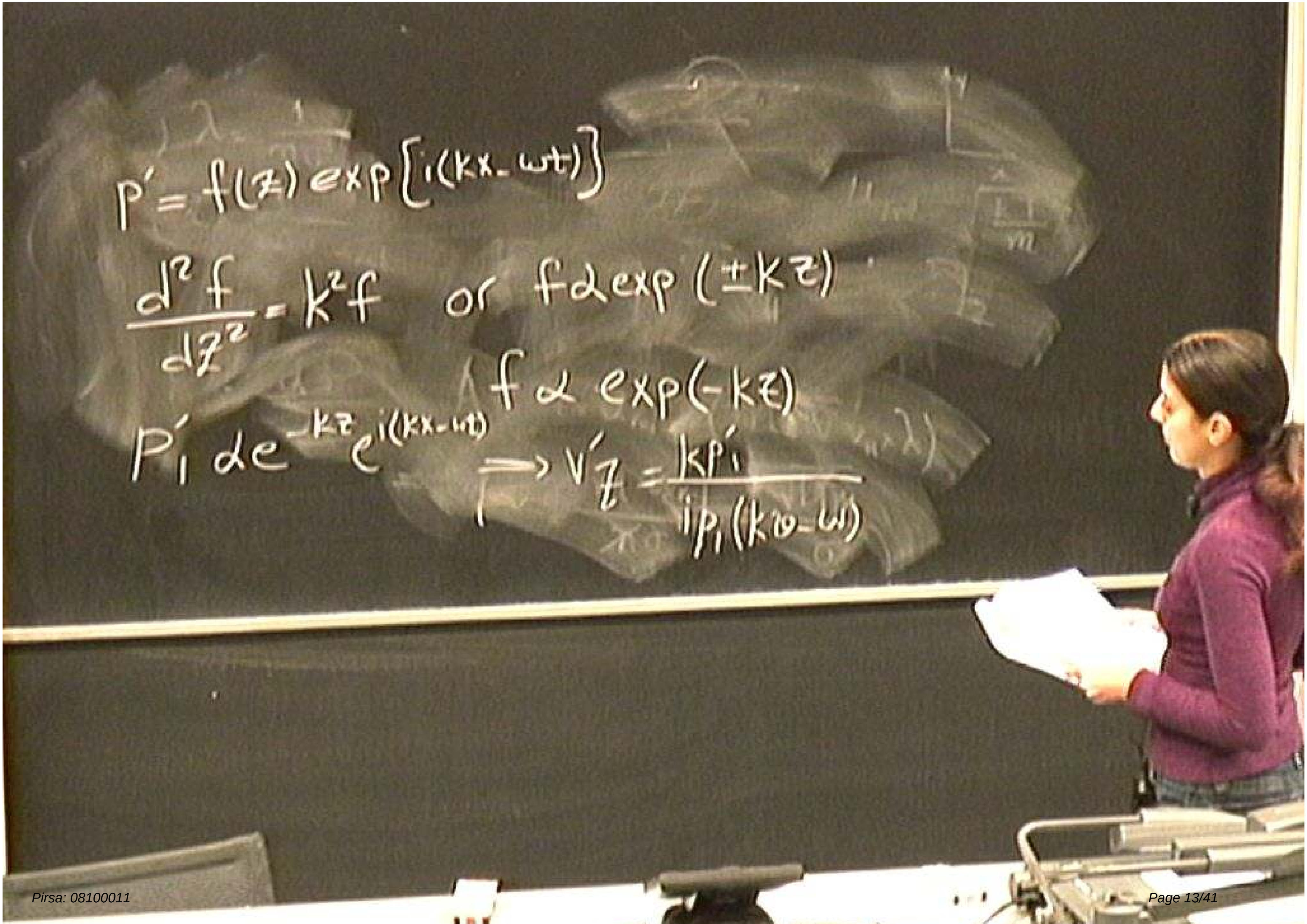
$$p' = f(z) \exp\left[i(kx - \omega t)\right]$$



$$p' = f(z)\exp\left[i(kx - \omega t)\right]$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

$$f \propto \exp(-kz)$$



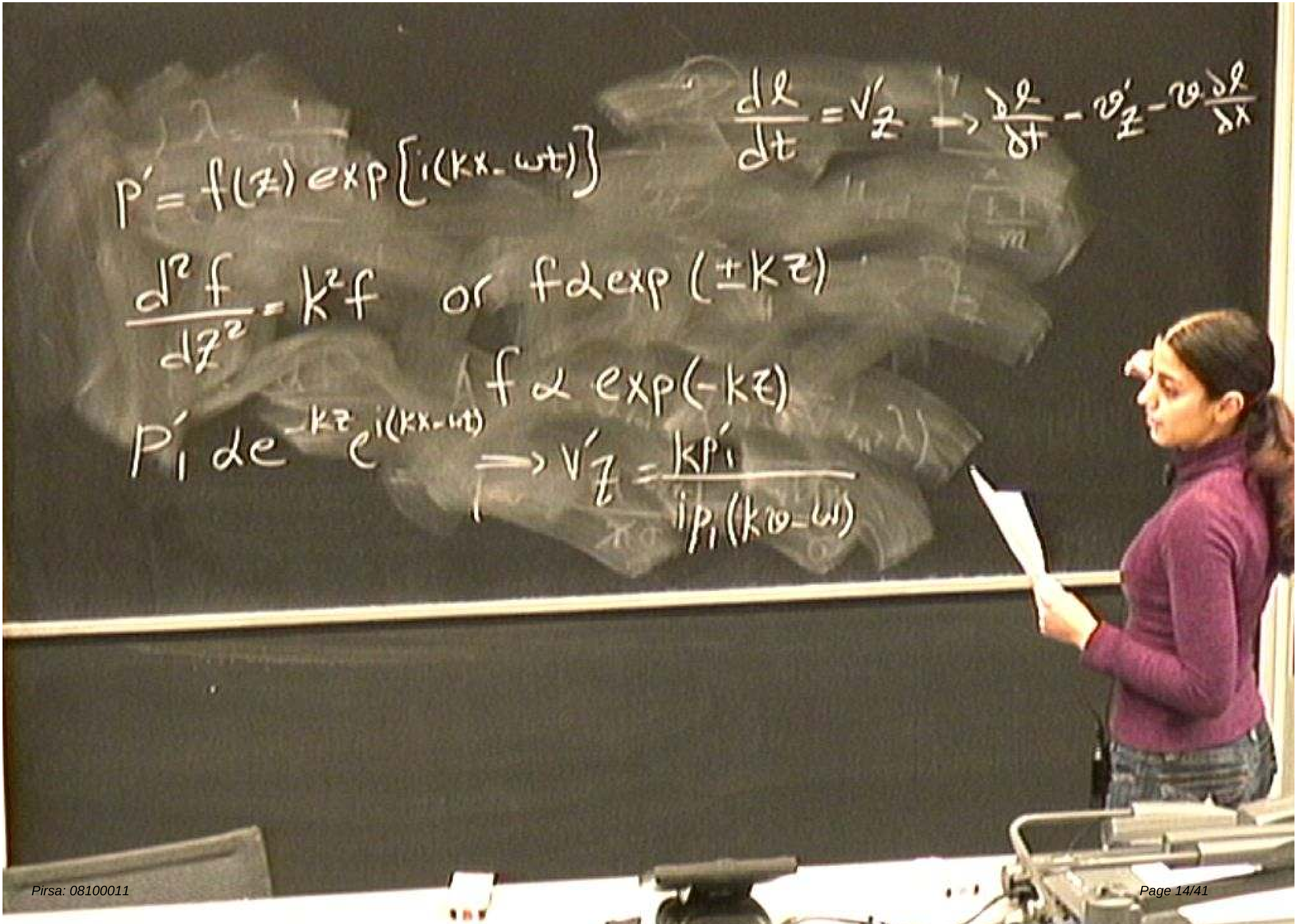
$$p' = f(z)\exp\left[i(kx - \omega t)\right]$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

$$f \propto \exp(-kz)$$

$$P'_1 \propto e^{-kz} e^{i(kx-\omega t)}$$

$$\Longrightarrow v'_z = \frac{kP'_1}{i\rho_1(k\upsilon - \omega)}$$



$$\frac{d\ell}{dt} = v'_z \Longrightarrow \frac{\partial \ell}{\partial t} - v'_z - v\frac{\partial \ell}{\partial x}$$

$$p' = f(z)\exp\left[i(kx - \omega t)\right]$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

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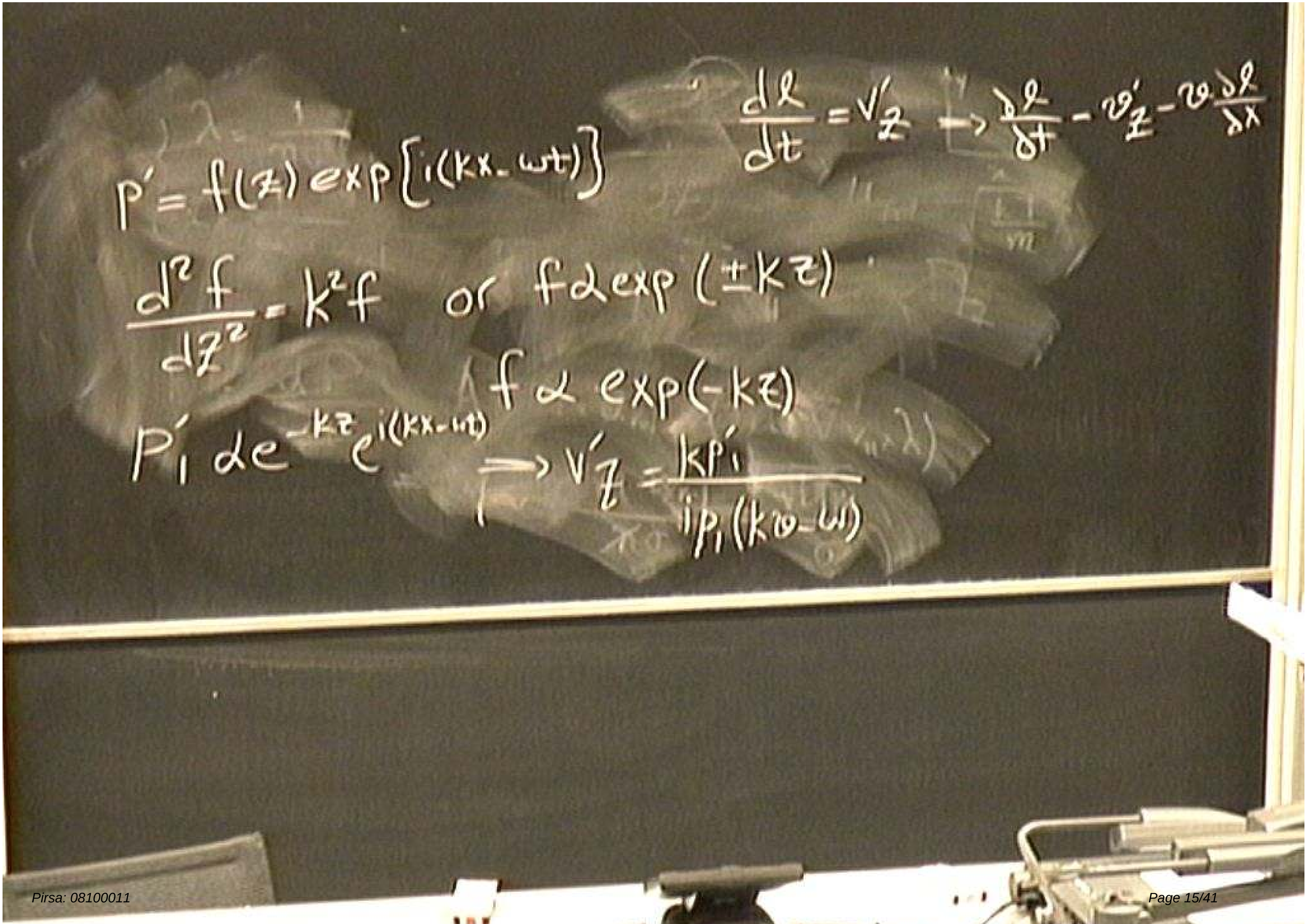
$$p'_1 \propto e^{-kz} e^{i(kx-\omega t)}$$

$$\Longrightarrow v'_z = \frac{kp'_1}{i\rho_1(kv - \omega)}$$



$$\frac{d\ell}{dt} = v'_z \longrightarrow \frac{\partial \ell}{\partial t} - v'_z - v\frac{\partial \ell}{\partial x}$$

$$p' = f(z)\exp\left[i(kx - \omega t)\right]$$

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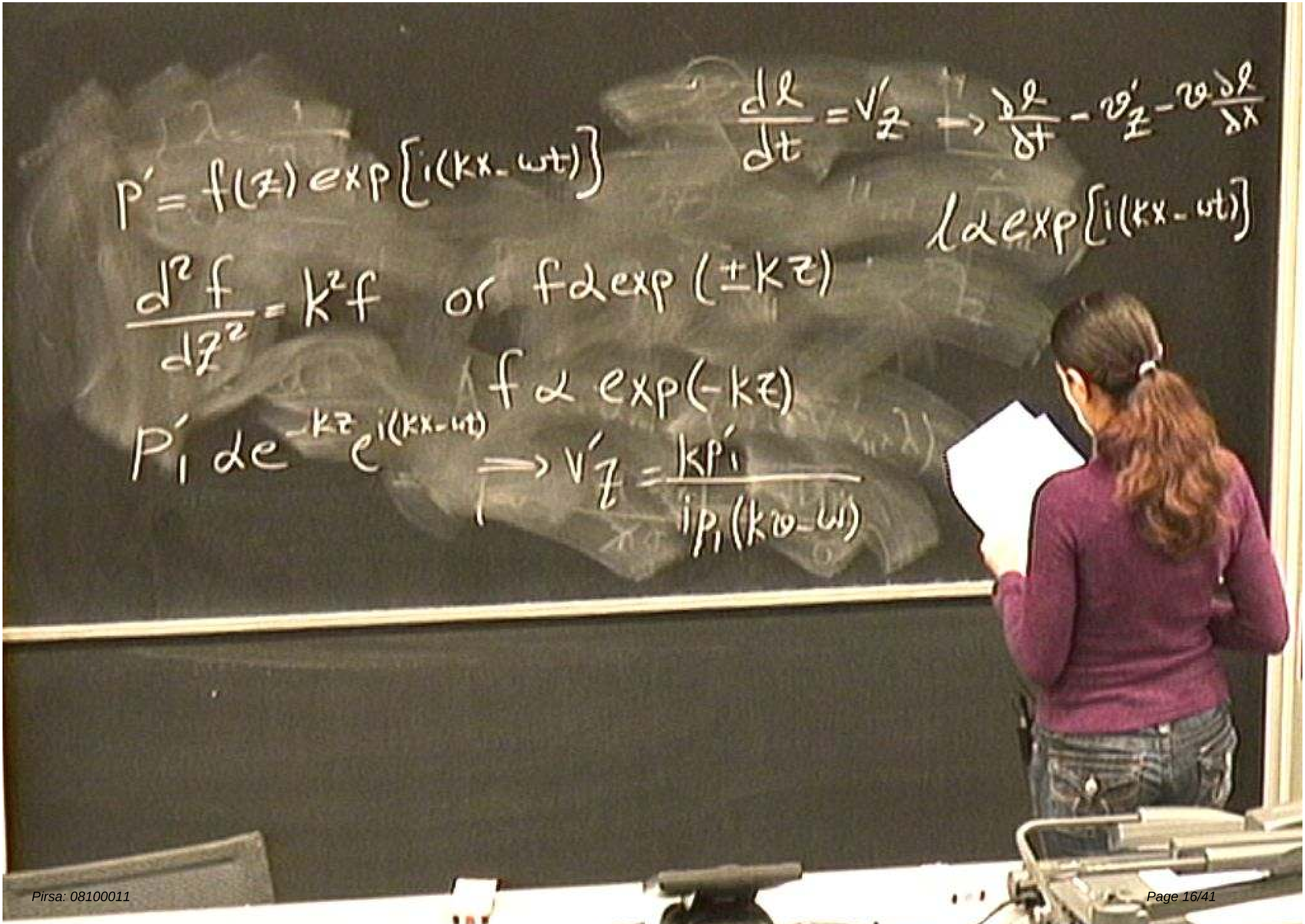

$$p' = f(z) \exp\left[i(kx - \omega t)\right]$$

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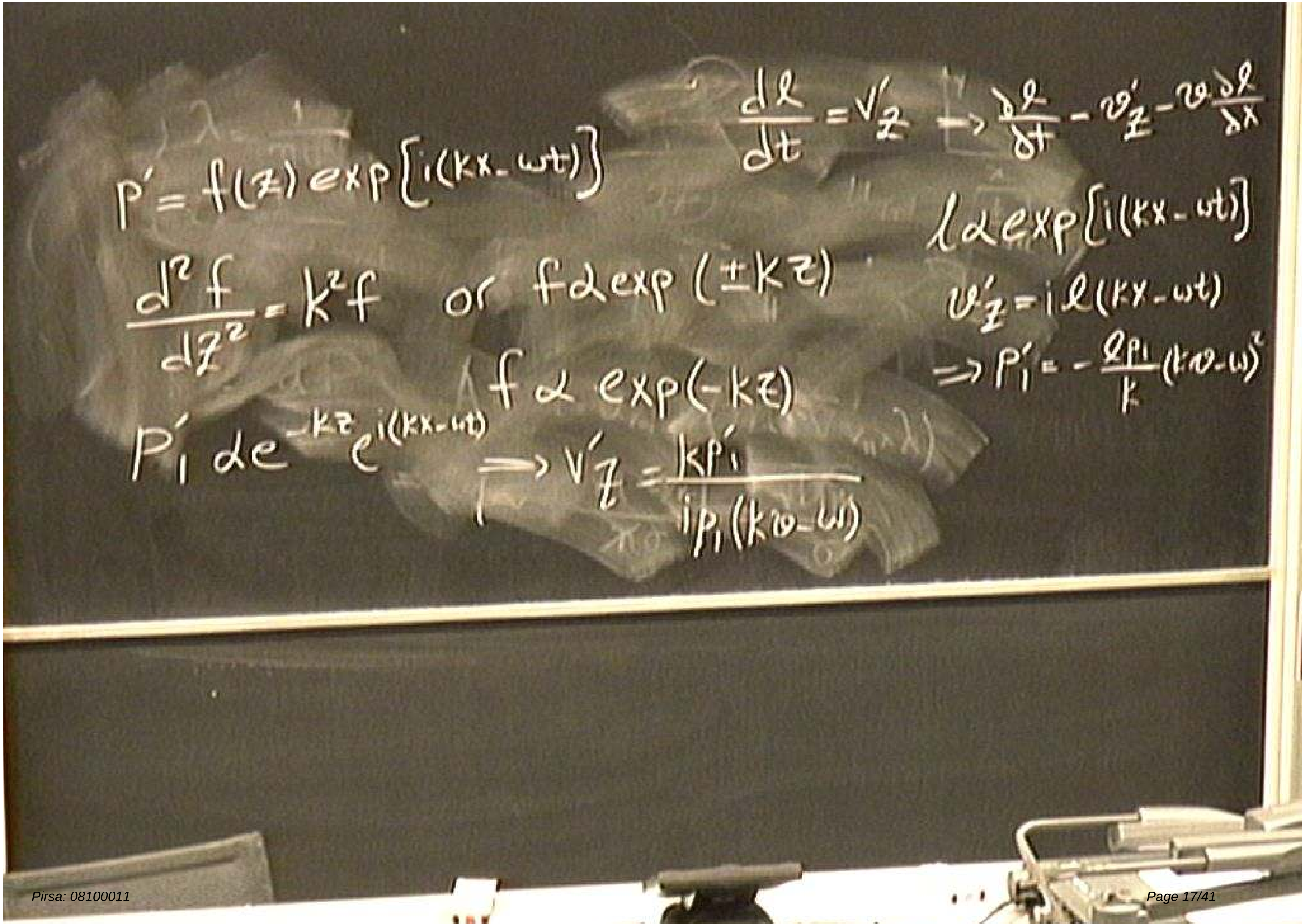
$$p_1' \propto e^{-kz} e^{i(kx-\omega t)}$$

$$\Longrightarrow v_z' = \frac{k p_1'}{i \rho_1 (kv - \omega)}$$

$$\frac{d\ell}{dt} = v_z' \longrightarrow \frac{\partial \ell}{\partial t} - v_z' - v \frac{\partial \ell}{\partial x}$$

$$\ell \propto \exp\left[i(kx - \omega t)\right]$$



$$p' = f(z)\exp\{i(kx - \omega t)\}$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

$$f \propto \exp(-kz)$$

$$p_1' \propto e^{-kz} e^{i(kx-\omega t)}$$

$$\Rightarrow v_z' = \frac{k p_1'}{i \rho_1 (kv - \omega)}$$

$$\frac{d\ell}{dt} = v_z' \Rightarrow \frac{\partial \ell}{\partial t} = v_z' - v\frac{\partial \ell}{\partial x}$$

$$\ell \propto \exp[i(kx - \omega t)]$$

$$v_z' = i\ell(kx - \omega t)$$

$$\Rightarrow p_1' = -\frac{\ell \rho_1}{k}(kv - \omega)^2$$

$$\frac{d\ell}{dt} = v'_z \Longrightarrow \frac{\partial \ell}{\partial t} - v'_z - v\frac{\partial \ell}{\partial x}$$

$$p' = f(z)\exp\left[i(kx - \omega t)\right]$$

$$\ell \propto \exp\left[i(kx - \omega t)\right]$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

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$$\Longrightarrow P'_1 = -\frac{\ell \rho_1}{k}(kv - \omega)^2$$

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$$P'_1 \propto e^{-kz} e^{i(kx - \omega t)}$$

$$\Longrightarrow v'_z = \frac{kP'_1}{i\rho_1(kv - \omega)}$$

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$$\ell \propto \exp\left[i(kx - \omega t)\right]$$

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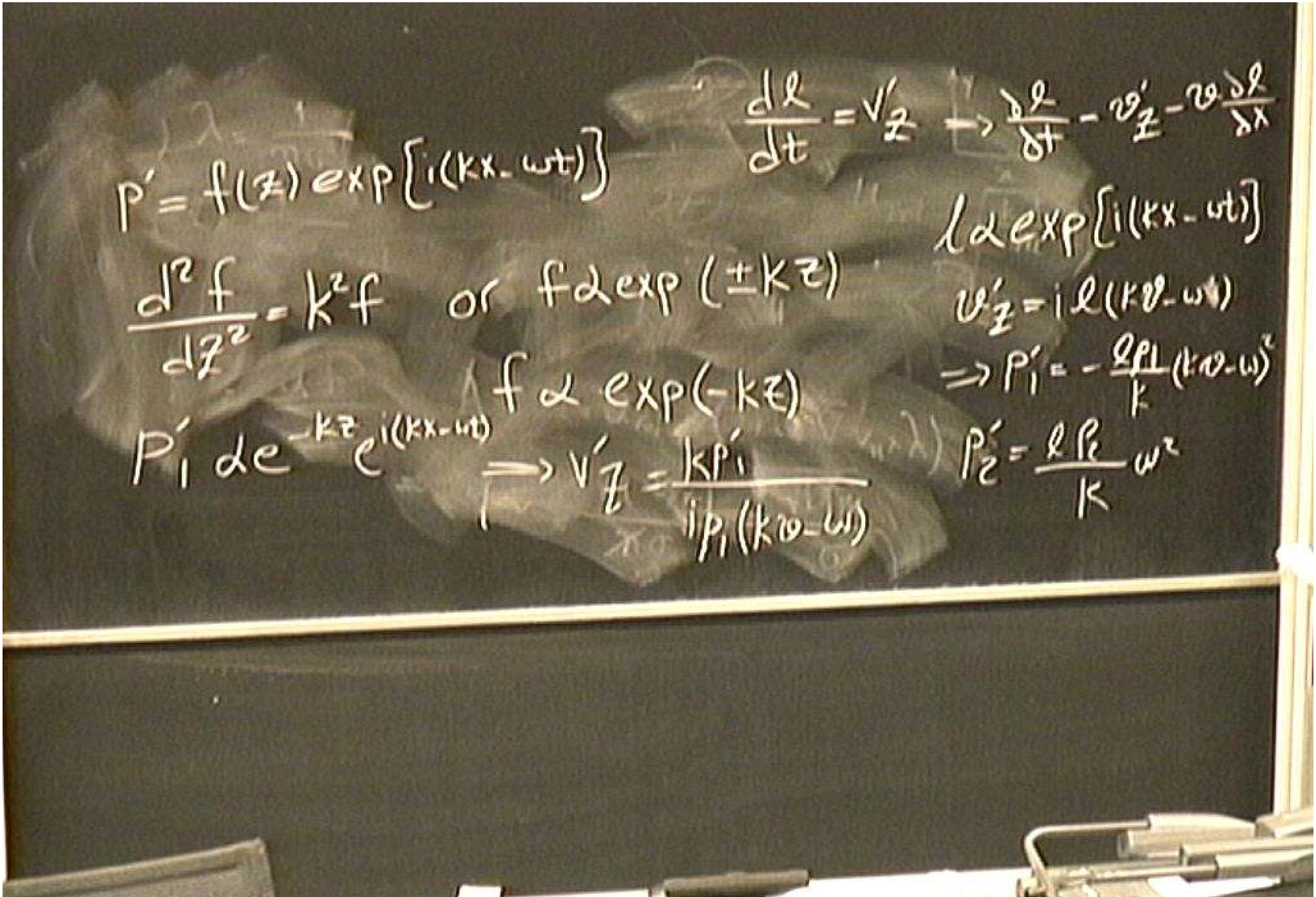

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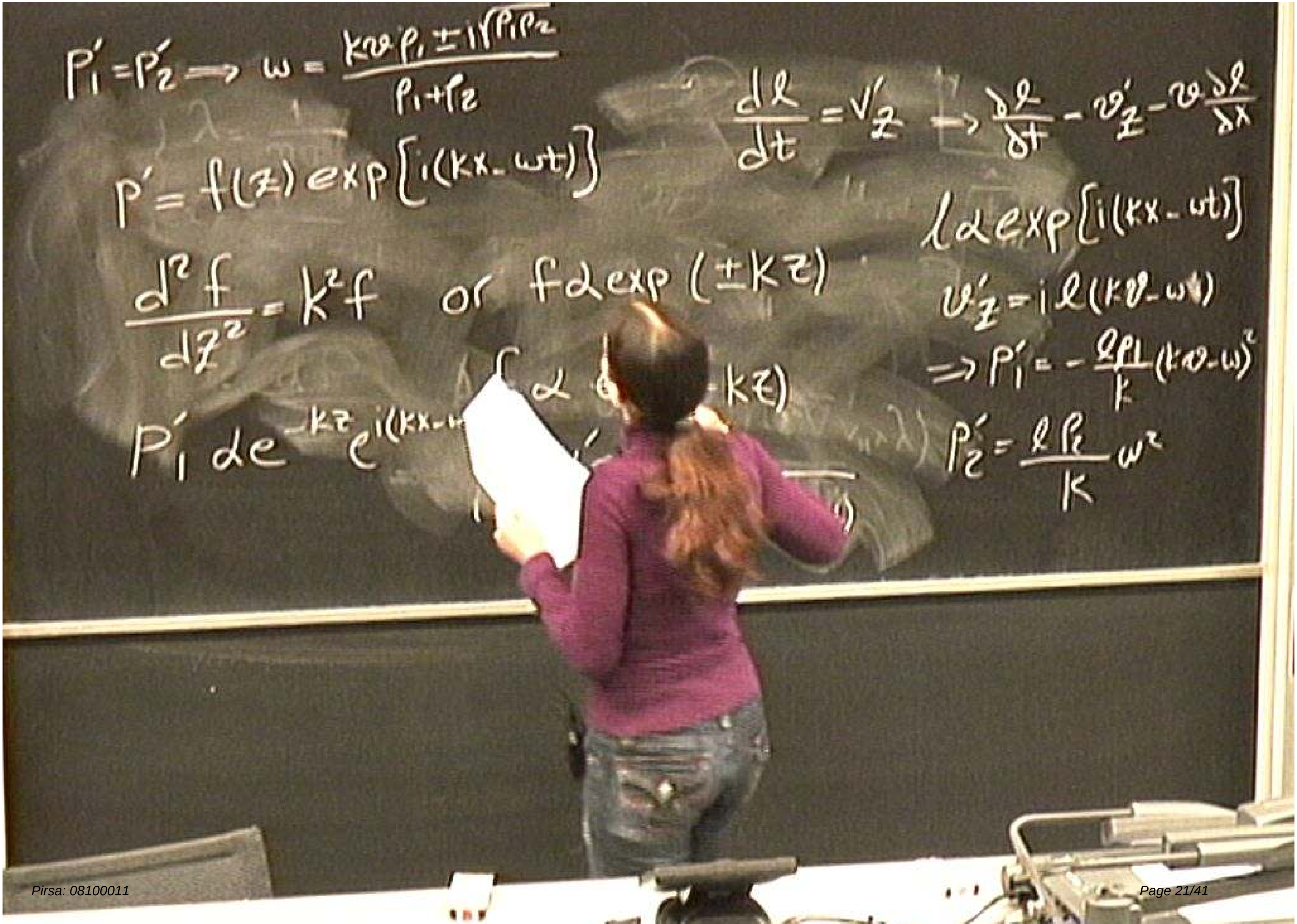
$$\frac{d\ell}{dt} = v'_z \Rightarrow \frac{\partial \ell}{\partial t} = v'_z - v\frac{\partial \ell}{\partial x}$$

$$\ell \propto \exp\left[i(kx - \omega t)\right]$$

$$v'_z = i\ell(kv - \omega)$$

$$\Rightarrow p'_1 = -\frac{\ell \rho_1}{k}(kv-\omega)^2$$

$$p'_2 = \frac{\ell \rho_2}{k}\omega^2$$



$$P_1' = P_2' \Longrightarrow \omega = \frac{kv\rho_1 \pm i\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2}$$

$$p' = f(z)\exp\left[i(kx-\omega t)\right]$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

$$P_1' \propto e^{-kz} e^{i(kx-}$$

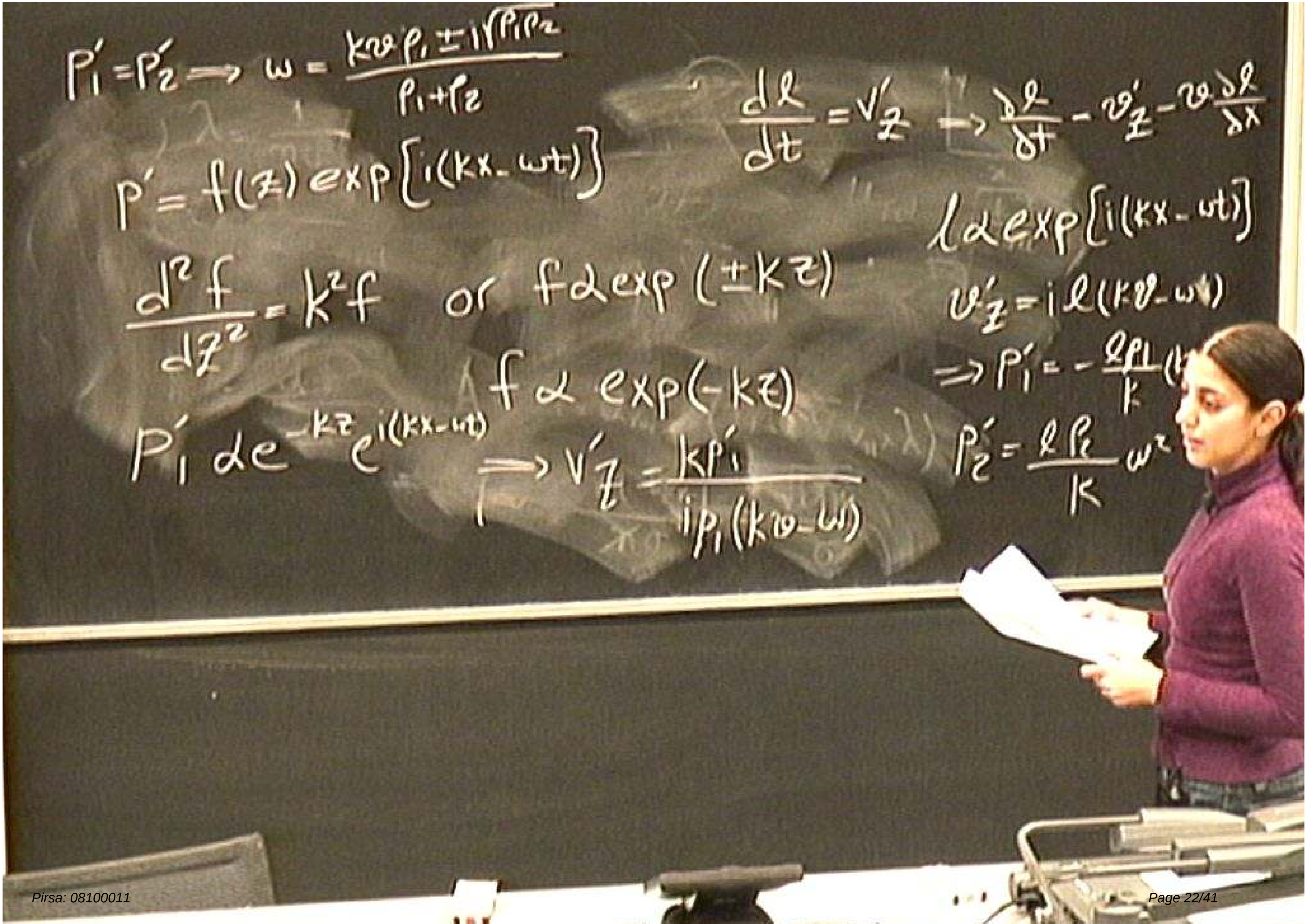
$$\frac{dl}{dt} = v_z' \longrightarrow \frac{\partial l}{\partial t} - v_z' - v\frac{\partial l}{\partial x}$$

$$l \propto \exp\left[i(kx-\omega t)\right]$$

$$v_z' = il(kv-\omega)$$

$$\Longrightarrow P_1' = -\frac{l\rho_1}{k}(kv-\omega)^2$$

$$P_2' = \frac{l\rho_2}{K}\omega^2$$



$$P_1' = P_2' \Longrightarrow \omega = \frac{kv\rho_1 \pm i\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2}$$

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$$P_1' \propto e^{-kz} e^{i(kx-\omega t)}$$

$$\Longrightarrow v_z' = \frac{kP_1'}{i\rho_1(kv-\omega t)}$$

$$\frac{d\ell}{dt} = v_z' \longrightarrow \frac{\partial \ell}{\partial t} - v_z' - v\frac{\partial \ell}{\partial x}$$

$$\ell \propto \exp\left[i(kx-\omega t)\right]$$

$$v_z' = i\ell(kv-\omega)$$

$$\Longrightarrow P_1' = -\frac{\ell\rho_1}{k}($$

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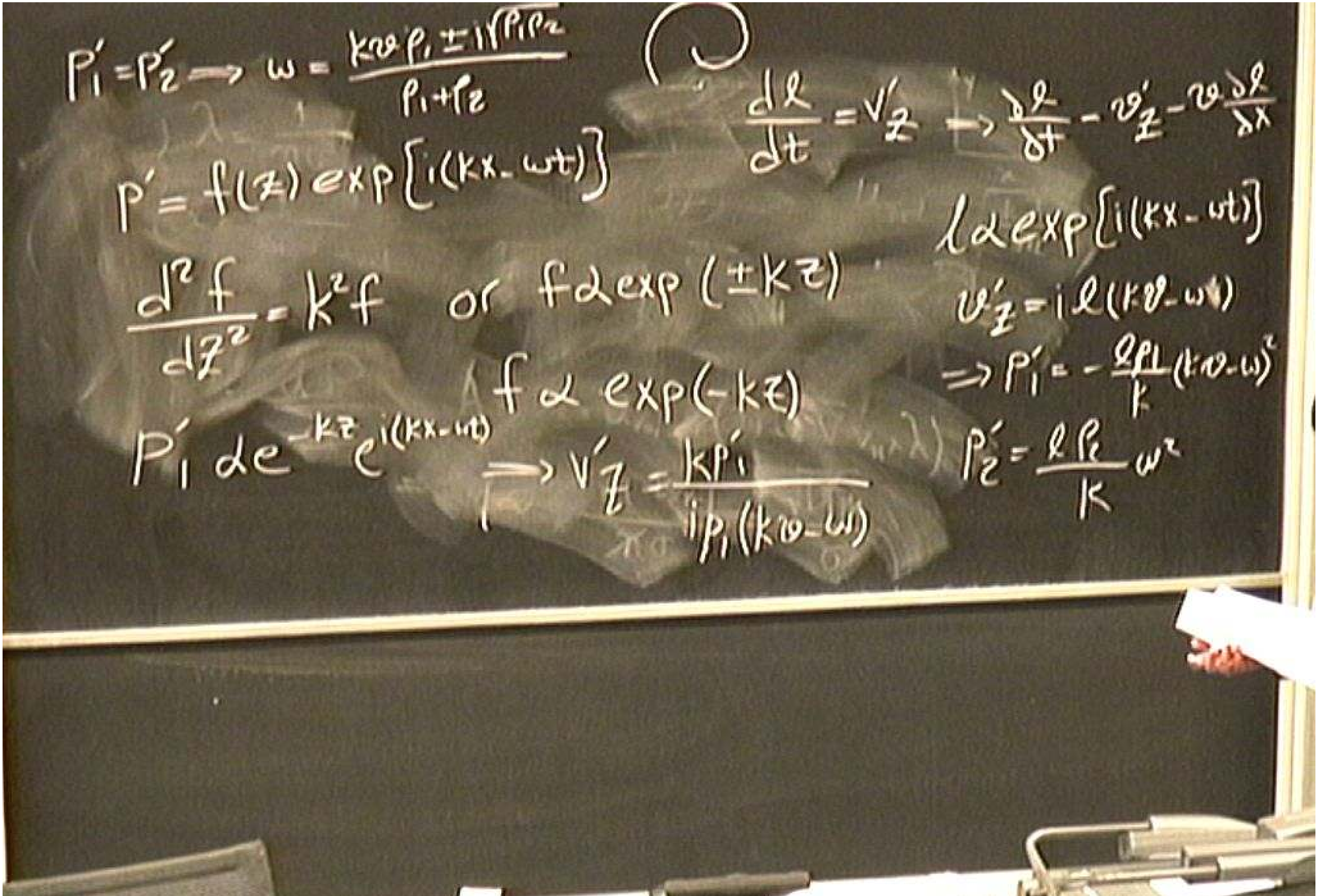

$$P_1' = P_2' \Longrightarrow \omega = \frac{kv\rho_1 \pm i\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2}$$

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$$\frac{d\ell}{dt} = v_z' \Longrightarrow \frac{\partial \ell}{\partial t} = v_z' - v\frac{\partial \ell}{\partial x}$$

$$\ell \propto \exp\left[i(kx - \omega t)\right]$$

$$v_z' = i\ell(kv - \omega)$$

$$\Longrightarrow P_1' = -\frac{\ell\rho_1}{k}(kv-\omega)^2$$

$$P_2' = \frac{\ell\rho_2}{K}\omega^2$$

$$P_1' = P_2' \Longrightarrow \omega = kv\left(\frac{\rho_1 \pm i\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2}\right)$$

$$P' = f(z)\exp\left[i(kx-\omega t)\right]$$

$$\frac{d^2 f}{dz^2} = k^2 f \quad \text{or} \quad f \propto \exp(\pm kz)$$

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$$\frac{d\ell}{dt} = v_z' \Longrightarrow \frac{\partial \ell}{\partial t} - v_z' - v\frac{\partial \ell}{\partial x}$$

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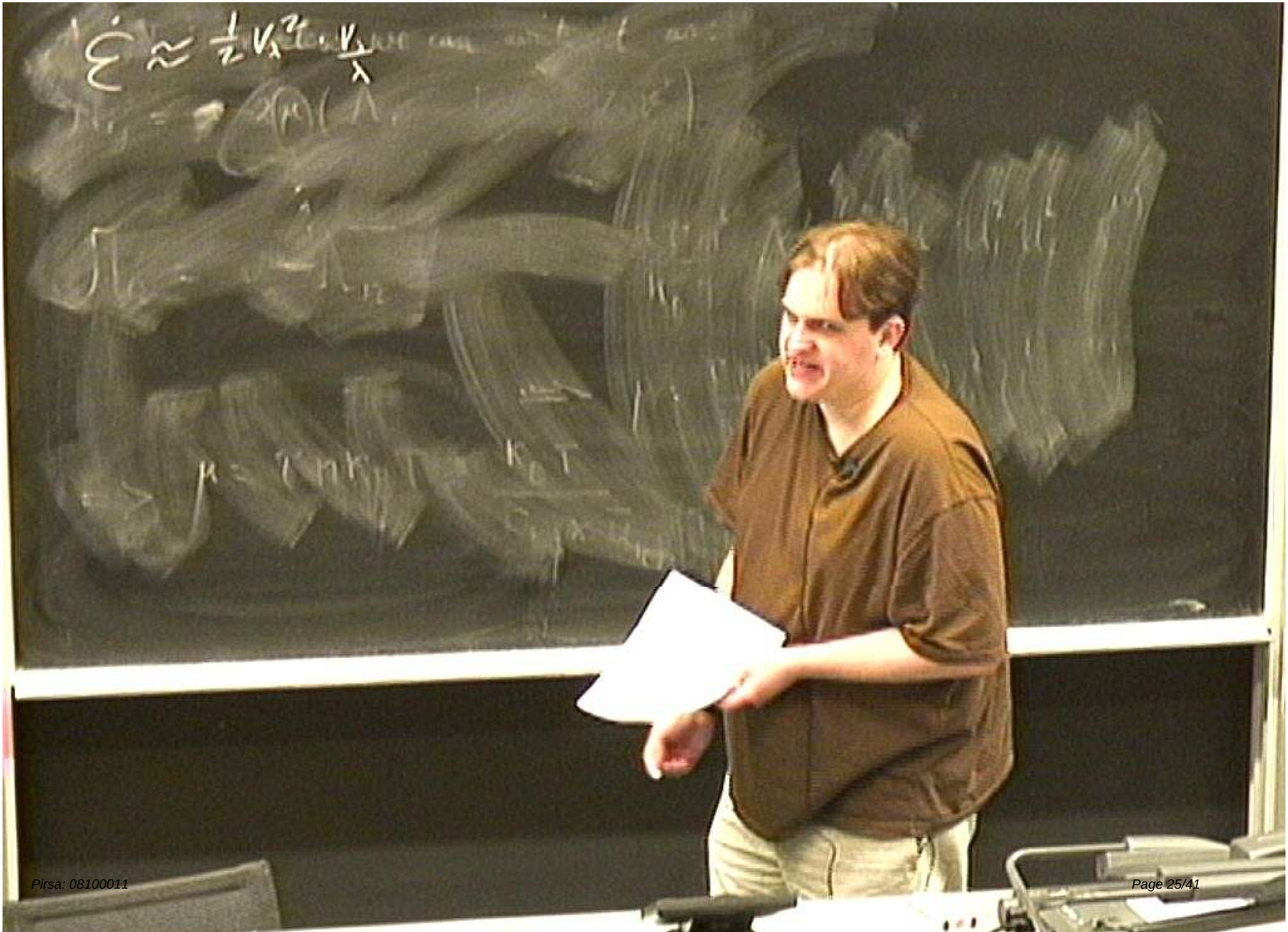
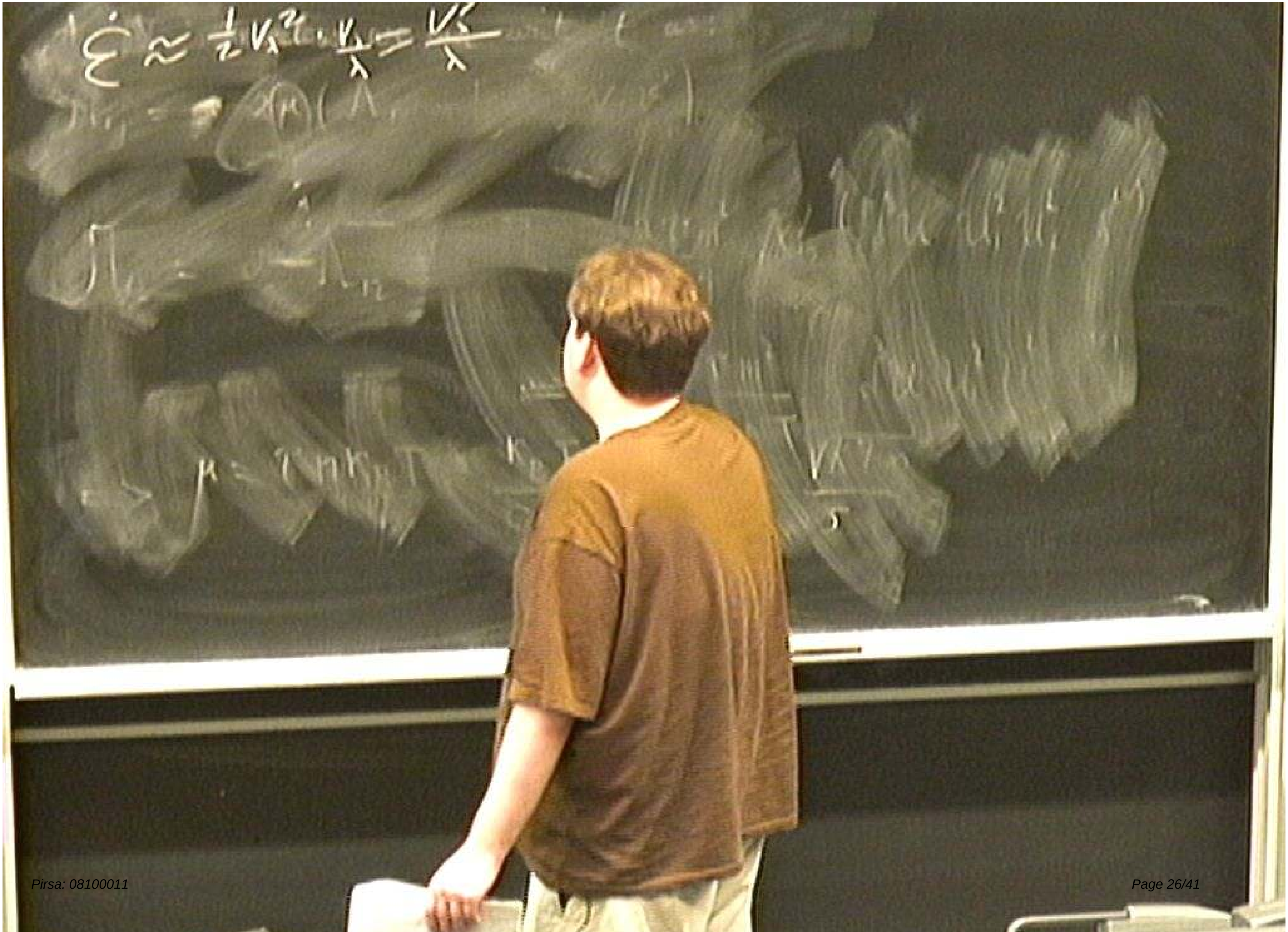
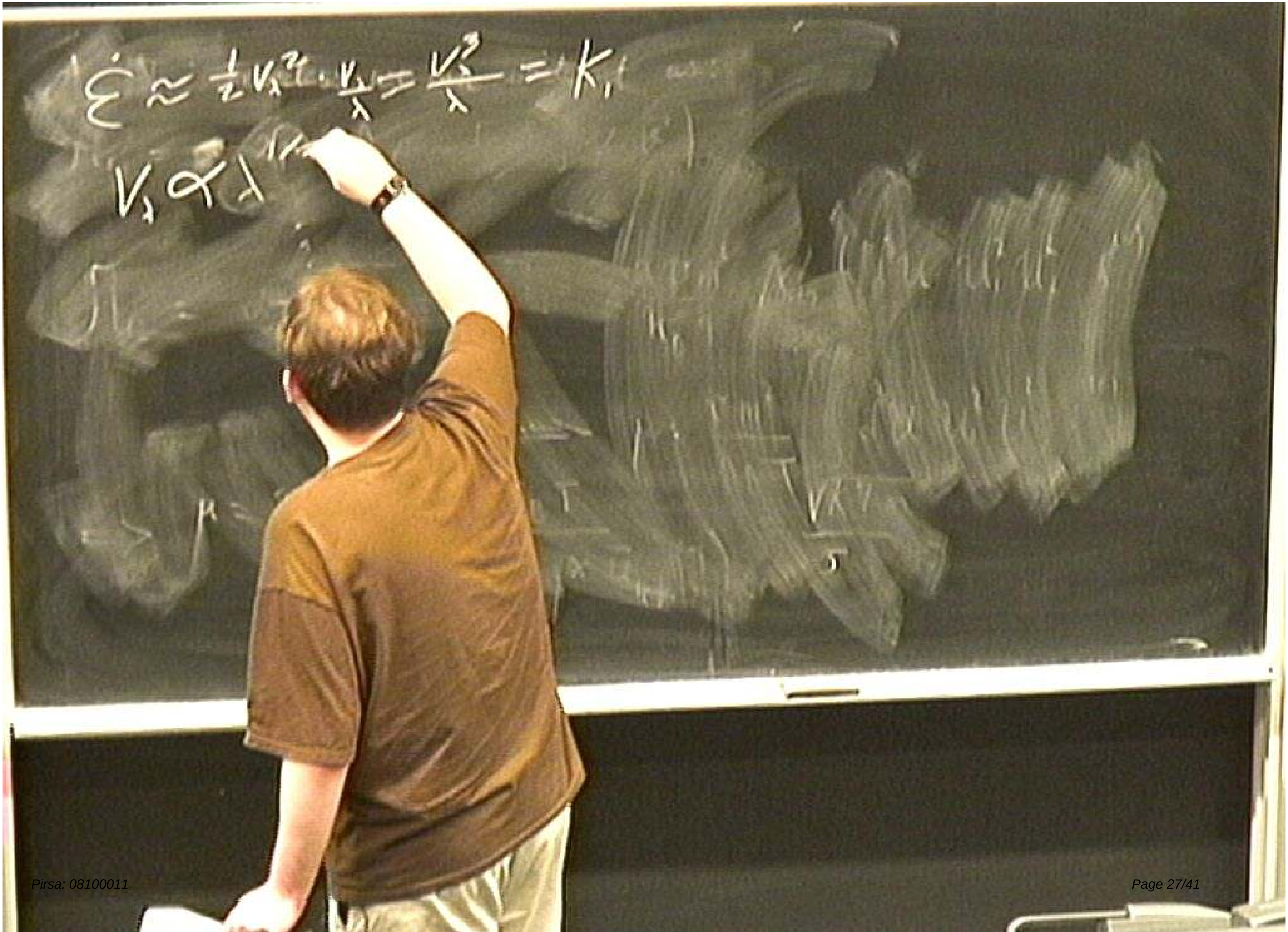
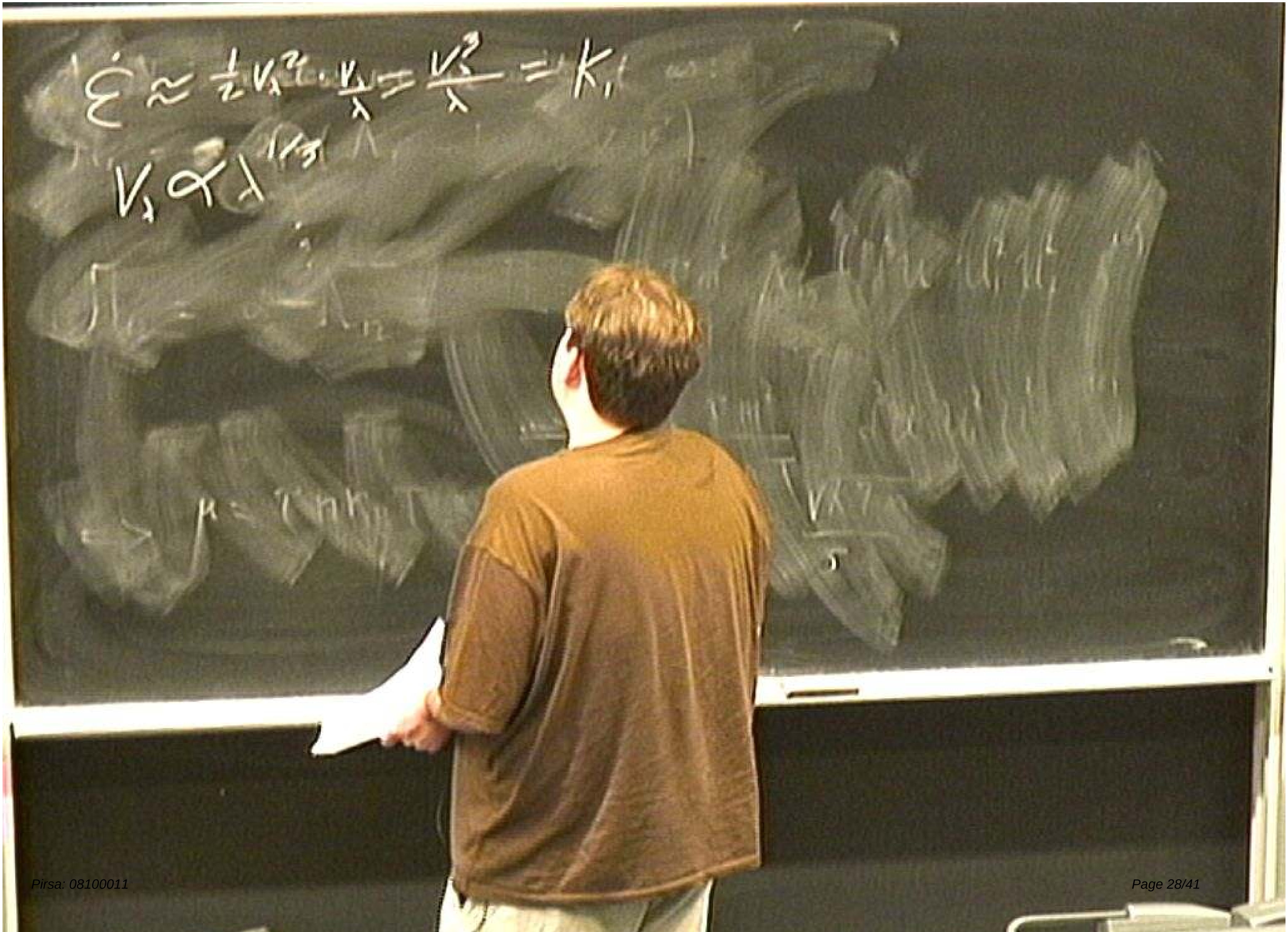

$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \frac{v_\lambda}{\lambda}$

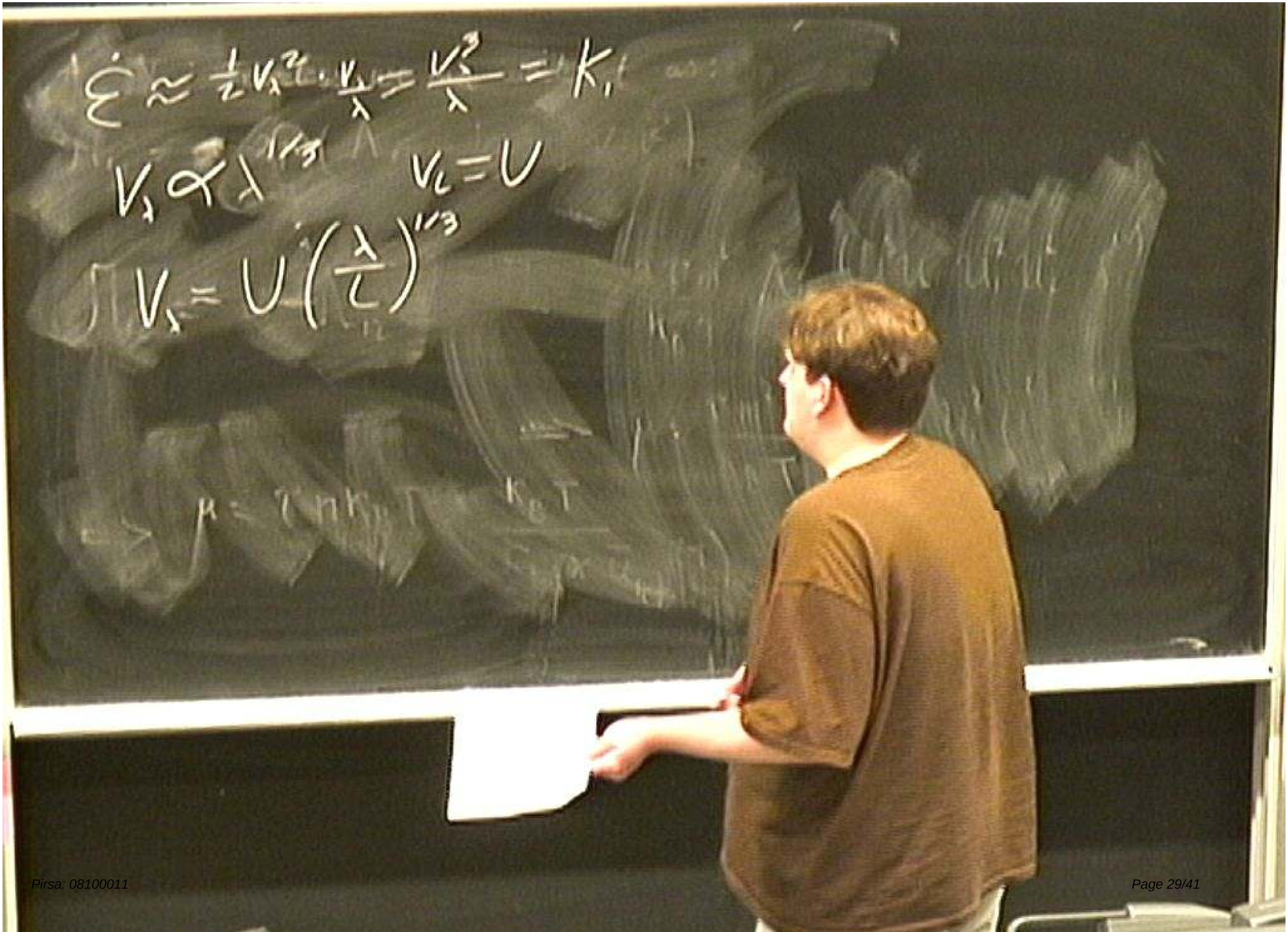

$\dot{\epsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda}$


$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1$
$v_\lambda \propto \lambda^{1/}$



$$\dot{\varepsilon} \approx \frac{1}{2} V_\lambda^2 \cdot \frac{V_\lambda}{\lambda} = \frac{V_\lambda^3}{\lambda} = K_1$$

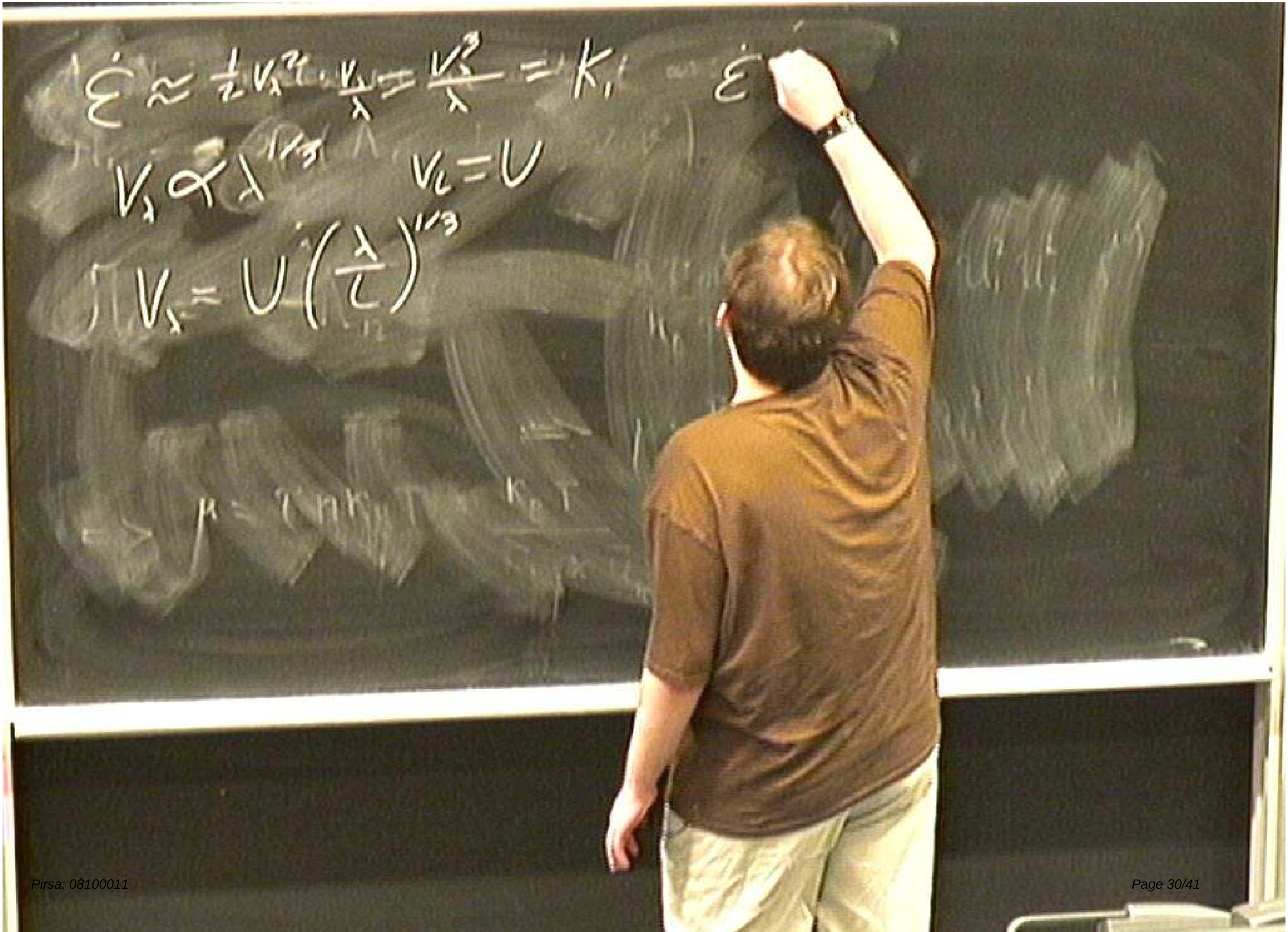
$$V_\lambda \propto \lambda^{1/3}$$



$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1$$

$$v_\lambda \propto \lambda^{1/3} \qquad v_L = U$$

$$v_\lambda = U\left(\frac{\lambda}{L}\right)^{1/3}$$



$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon}$$

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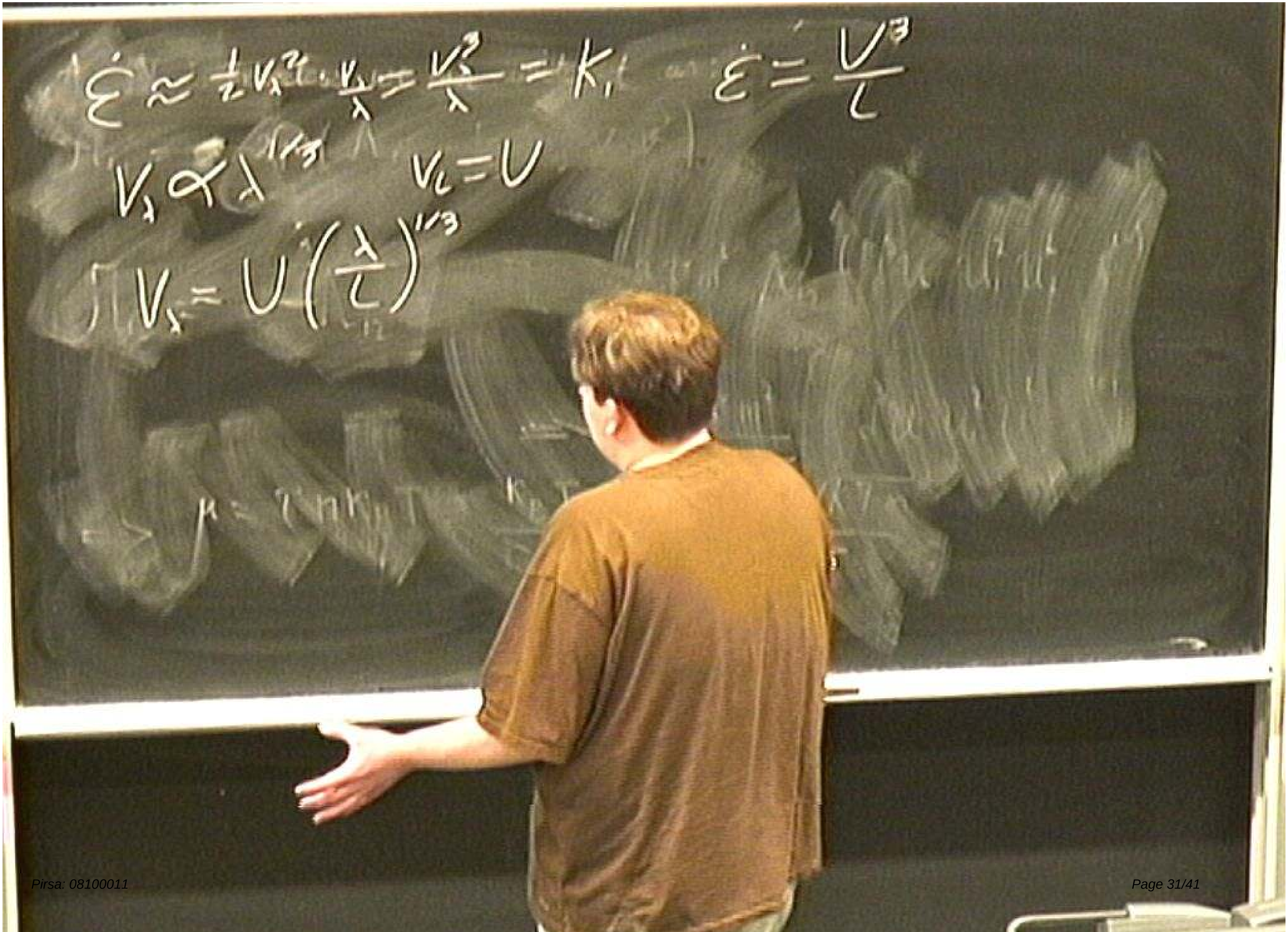

$$\dot{\varepsilon} \approx \frac{1}{2} V_\lambda^2 \cdot \frac{V_\lambda}{\lambda} = \frac{V_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{V^3}{L}$$

$$V_\lambda \propto \lambda^{1/3} \qquad V_L = V$$

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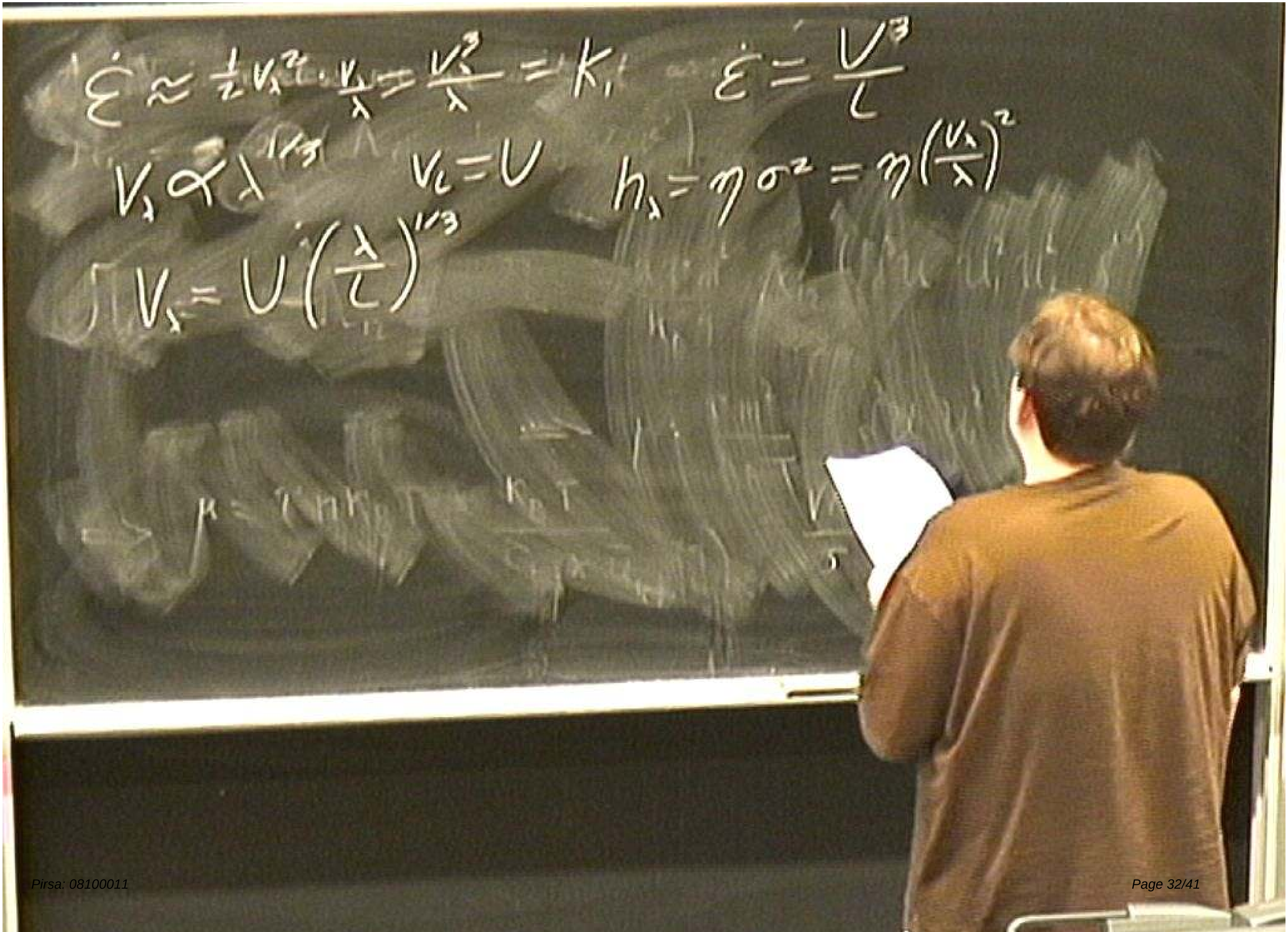
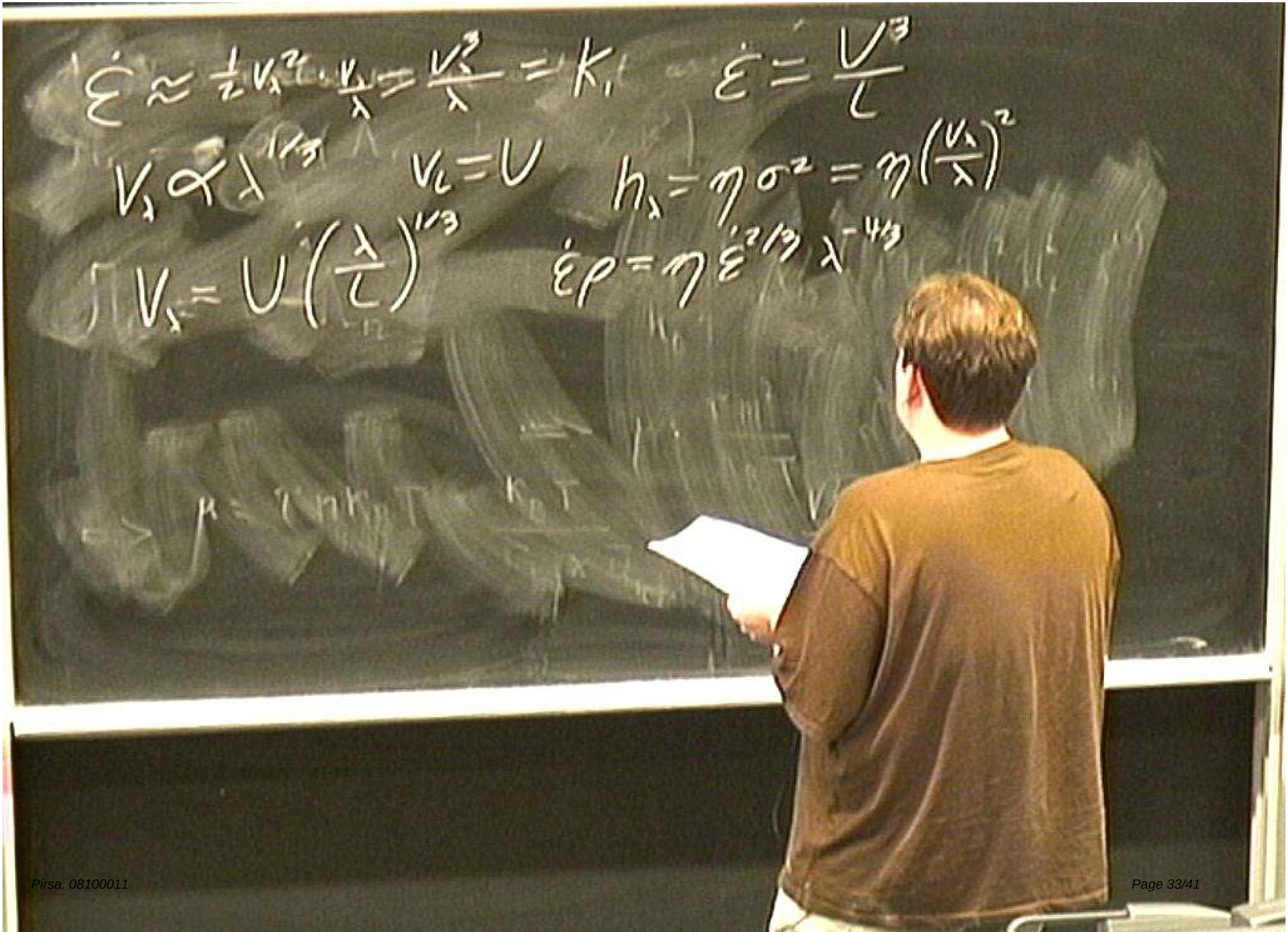


$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{V^3}{L}$$

$$v_\lambda \propto \lambda^{1/3} \qquad v_L = V \qquad h_\lambda = \eta \sigma^2 = \eta \left(\frac{v_\lambda}{\lambda}\right)^2$$

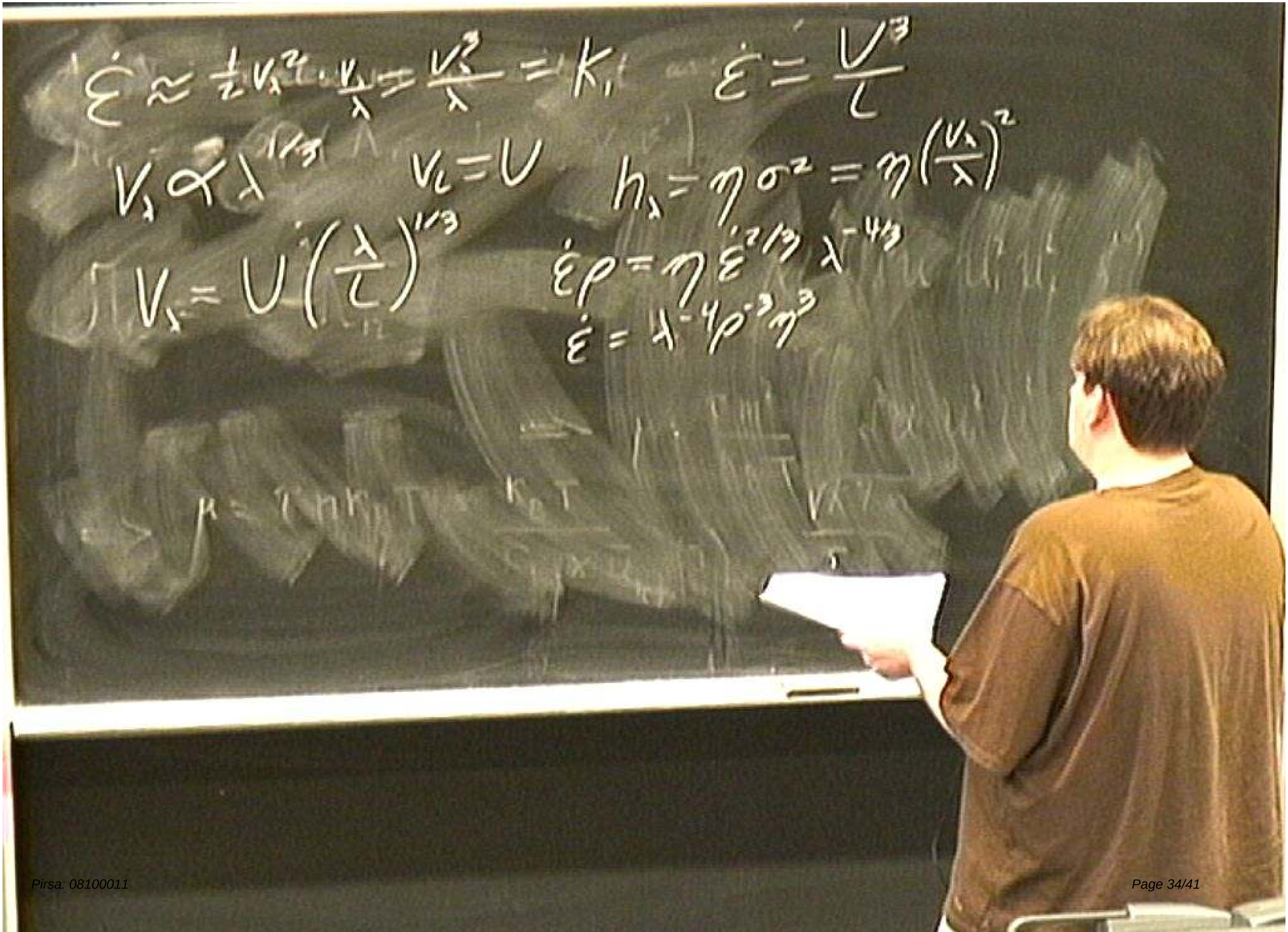
$$v_\lambda = V \left(\frac{\lambda}{L}\right)^{1/3}$$



$$\dot{\varepsilon} \approx \frac{1}{2} V_\lambda^2 \cdot \frac{V_\lambda}{\lambda} = \frac{V_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{V^3}{L}$$

$$V_\lambda \propto \lambda^{1/3} \qquad V_L = U \qquad h_\lambda = \eta\sigma^2 = \eta\left(\frac{V_\lambda}{\lambda}\right)^2$$

$$V_\lambda = U\left(\frac{\lambda}{L}\right)^{1/3} \qquad \dot{\varepsilon}\rho = \eta\,\dot{\varepsilon}^{2/3}\lambda^{-4/3}$$

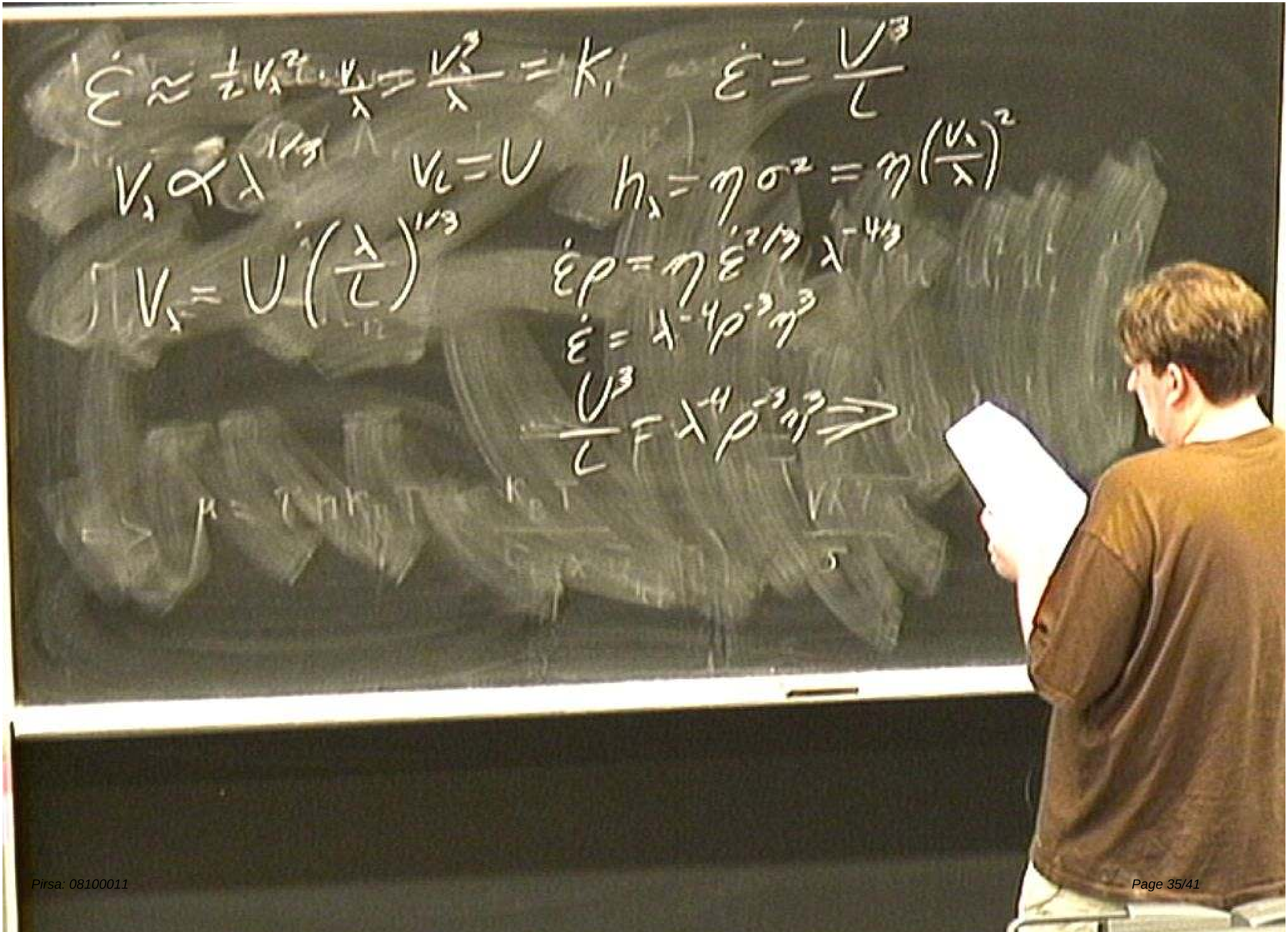


$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{V^3}{L}$$

$$v_\lambda \propto \lambda^{1/3} \qquad v_L = V \qquad h_\lambda = \eta \sigma^2 = \eta \left(\frac{v_\lambda}{\lambda}\right)^2$$

$$v_\lambda = V\left(\frac{\lambda}{L}\right)^{1/3} \qquad \dot{\varepsilon}\rho = \eta\, \dot{\varepsilon}^{2/3} \lambda^{-4/3}$$

$$\dot{\varepsilon} = \lambda^{-4} \rho^{-3} \eta^3$$



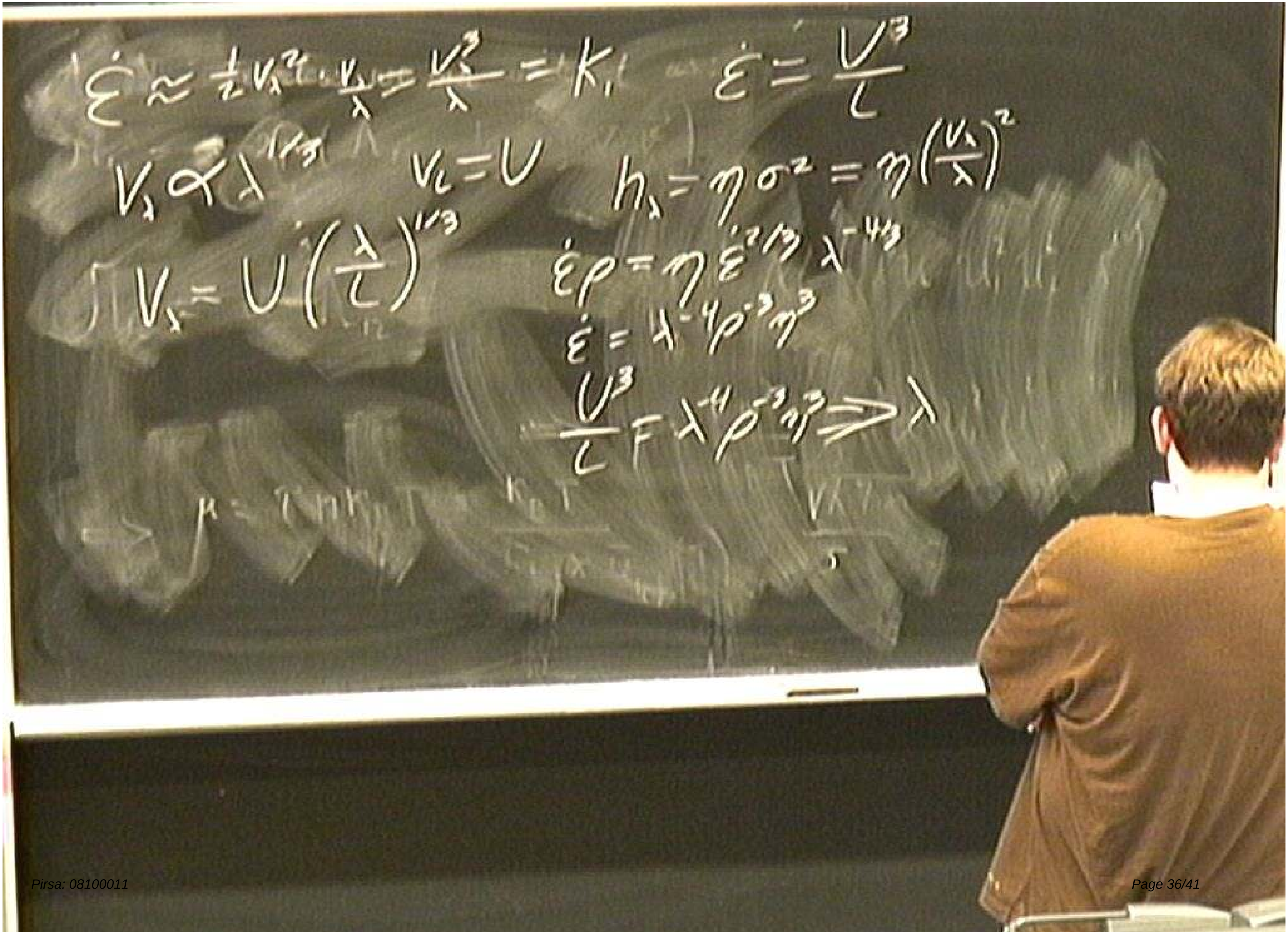
$$\dot{\varepsilon} \approx \tfrac{1}{2}V_\lambda^2 \cdot \frac{V_\lambda}{\lambda} = \frac{V_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{V^3}{L}$$

$$V_\lambda \propto \lambda^{1/3} \qquad V_L = U \qquad h_\lambda = \eta\sigma^2 = \eta\left(\frac{V_\lambda}{\lambda}\right)^2$$

$$V_\lambda = U\left(\frac{\lambda}{L}\right)^{1/3} \qquad \dot{\varepsilon}\rho = \eta\,\dot{\varepsilon}^{2/3}\lambda^{-4/3}$$

$$\dot{\varepsilon} = \lambda^{-4}\rho^{-3}\eta^3$$

$$\frac{U^3}{L} = \lambda^{-4}\rho^{-3}\eta^3 \Rightarrow$$



$$\dot{\varepsilon} \approx \frac{1}{2} V_\lambda^2 \cdot \frac{V_\lambda}{\lambda} = \frac{V_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{U^3}{L}$$

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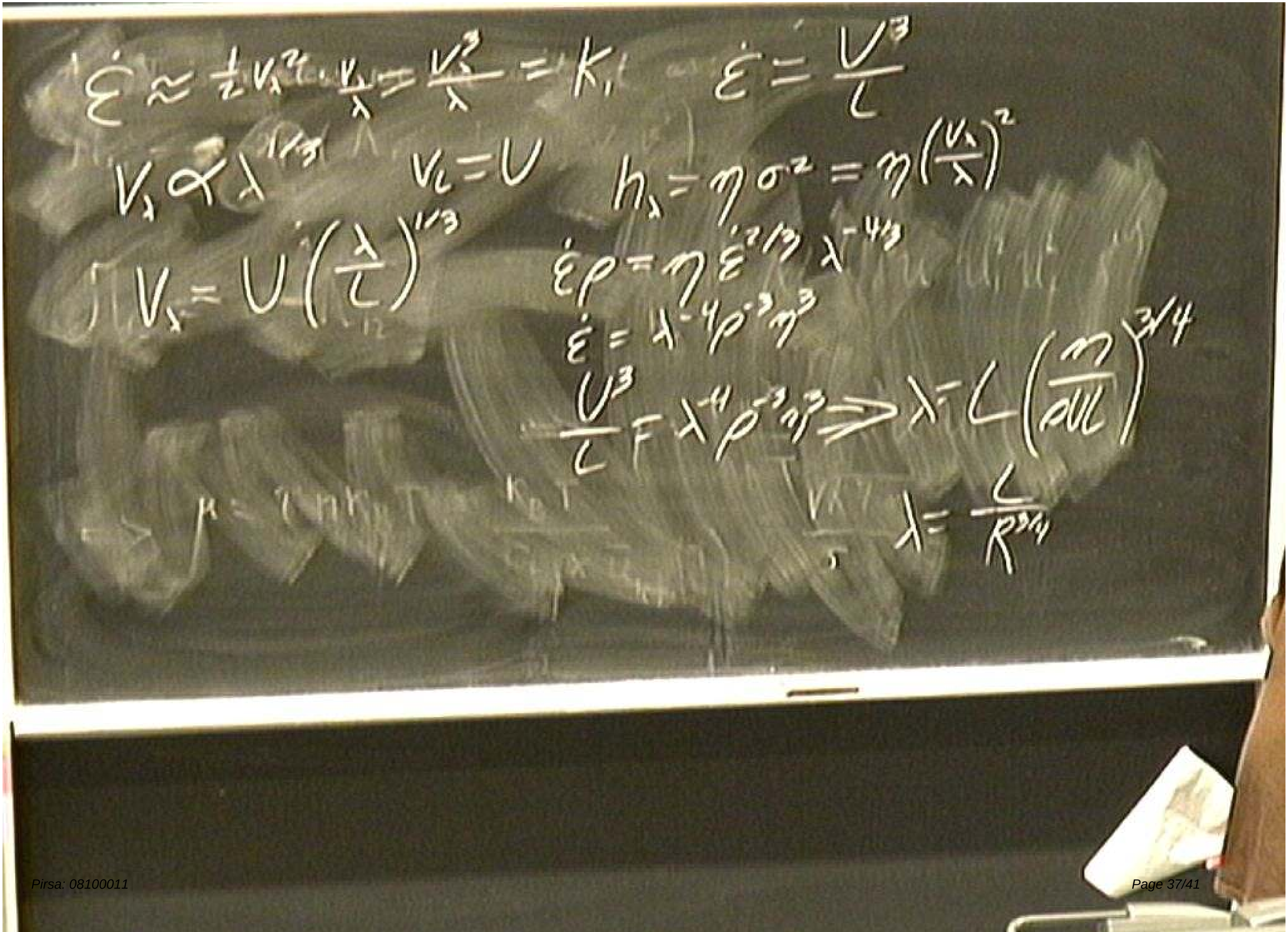
$$V_\lambda = U \left(\frac{\lambda}{L}\right)^{1/3} \qquad \dot{\varepsilon} \rho = \eta \dot{\varepsilon}^{2/3} \lambda^{-4/3}$$

$$\dot{\varepsilon} = \lambda^{-4} \rho^{-3} \eta^3$$

$$\frac{U^3}{L} F \lambda^{-4} \rho^{-3} \eta^3 \Rightarrow \lambda$$



$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{U^3}{L}$$

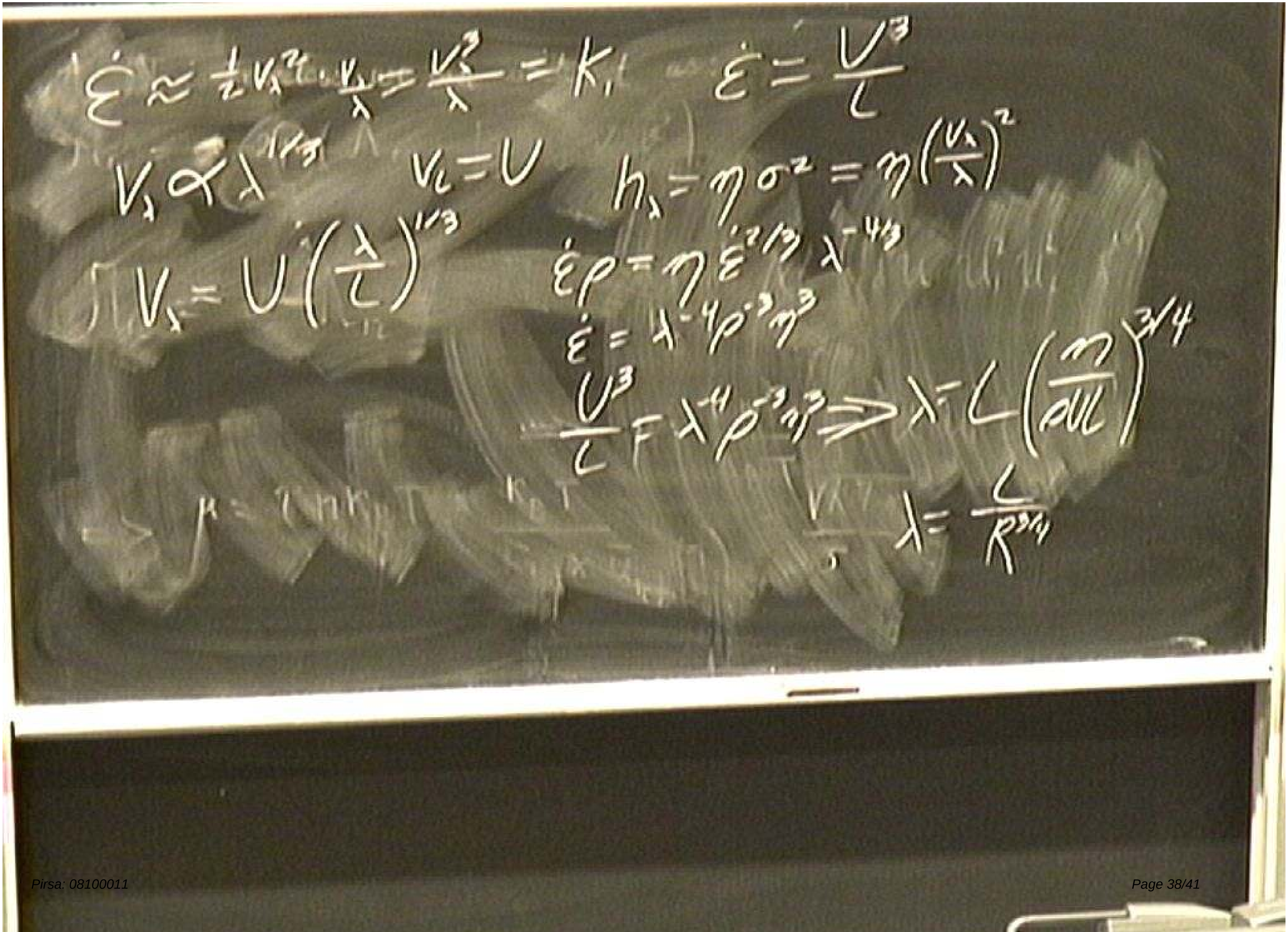
$$v_\lambda \propto \lambda^{1/3} \qquad v_L = U \qquad h_\lambda = \eta\sigma^2 = \eta\left(\frac{v_\lambda}{\lambda}\right)^2$$

$$v_\lambda = U\left(\frac{\lambda}{L}\right)^{1/3} \qquad \dot{\varepsilon}\rho = \eta\,\dot{\varepsilon}^{2/3}\lambda^{-4/3}$$

$$\dot{\varepsilon} = \lambda^{-4}\rho^{-3}\eta^3$$

$$\frac{U^3}{L} = \lambda^{-4}\rho^{-3}\eta^3 \Rightarrow \lambda = L\left(\frac{\eta}{\rho U L}\right)^{3/4}$$

$$\lambda = \frac{L}{R^{3/4}}$$



$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{U^3}{L}$$

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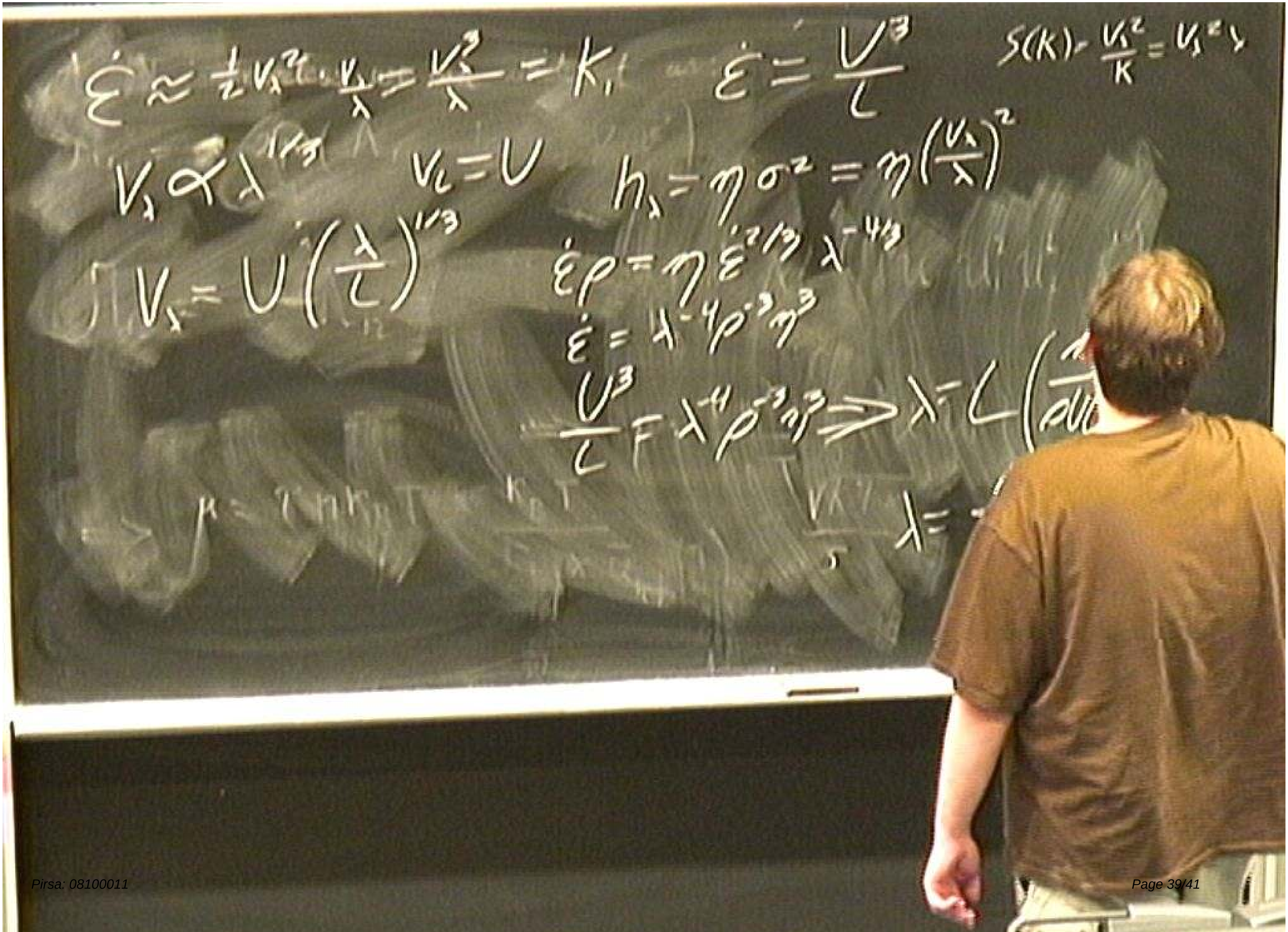


$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1$

$\dot{\varepsilon} = \frac{U^3}{L}$

$S(k) \sim \frac{v_\lambda^2}{k} = v_\lambda^2 \lambda$

$v_\lambda \propto \lambda^{1/3}$

$v_L = U$

$h_\lambda = \eta \sigma^2 = \eta \left(\frac{v_\lambda}{\lambda}\right)^2$

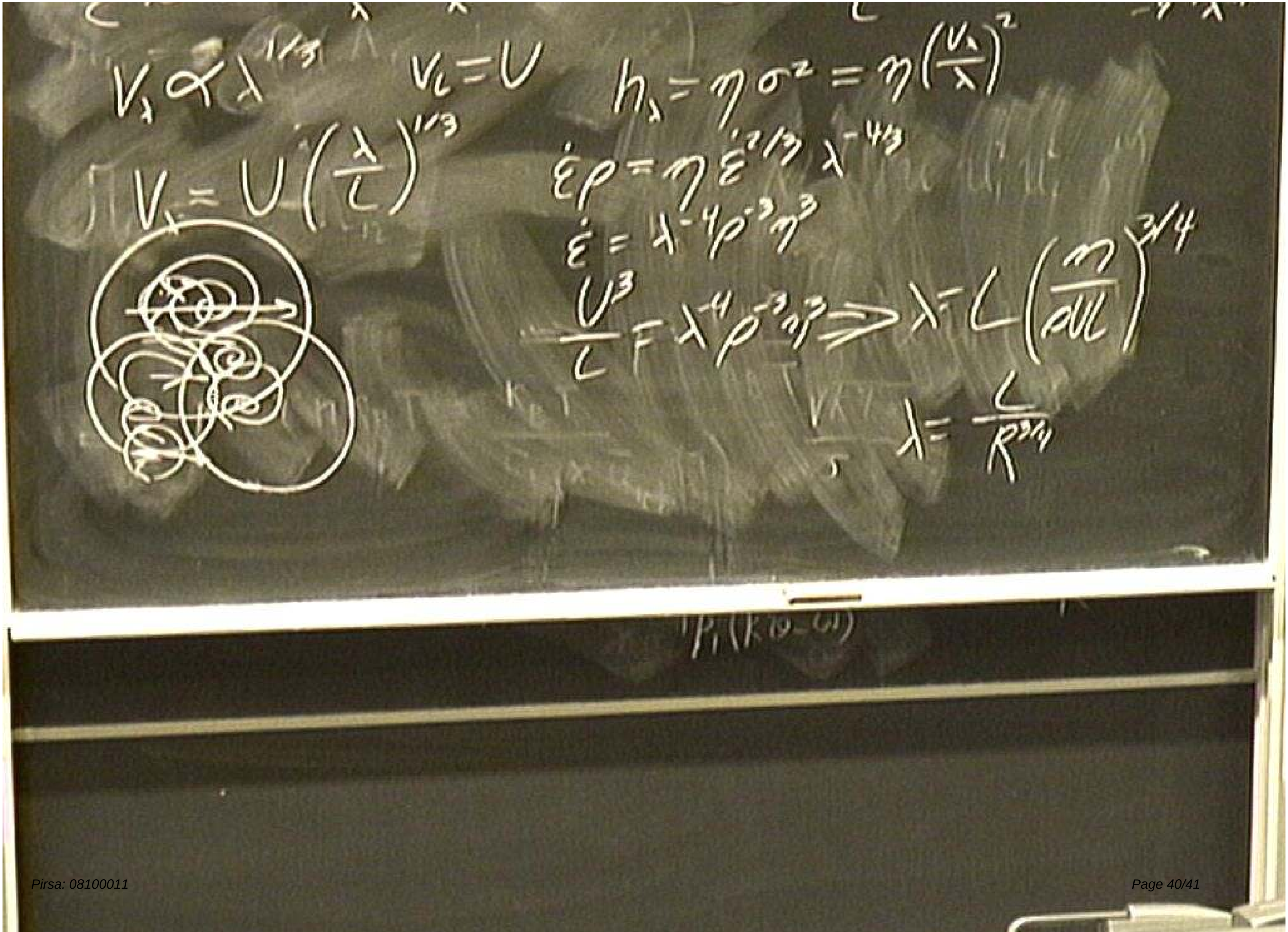
$v_\lambda = U\left(\frac{\lambda}{L}\right)^{1/3}$

$\dot{\varepsilon}\rho = \eta \dot{\varepsilon}^{2/3} \lambda^{-4/3}$

$\dot{\varepsilon} = \lambda^{-4}\rho^{-3}\eta^3$

$\frac{U^3}{L} F \lambda^{-4}\rho^{-3}\eta^3 \Rightarrow \lambda^{-} L \left(\frac{}{\rho U}\right.$

$\lambda =$



$V_\lambda \propto \lambda^{1/3}$ $\quad V_L = U$

$h_\lambda = \eta \sigma^2 = \eta\left(\frac{V_\lambda}{\lambda}\right)^2$

$V_\lambda = U\left(\frac{\lambda}{L}\right)^{1/3}$

$\dot{\varepsilon}\rho = \eta\,\dot{\varepsilon}^{2/3}\lambda^{-4/3}$

$\dot{\varepsilon} = \lambda^{-4}\rho^{-3}\eta^{3}$

$\frac{U^3}{L} = \lambda^{-4}\rho^{-3}\eta^{3} \Rightarrow \lambda = L\left(\frac{\eta}{\rho U L}\right)^{3/4}$

$\lambda = \frac{L}{R^{3/4}}$

$$\dot{\varepsilon} \approx \frac{1}{2} v_\lambda^2 \cdot \frac{v_\lambda}{\lambda} = \frac{v_\lambda^3}{\lambda} = K_1 \qquad \dot{\varepsilon} = \frac{U^3}{L}$$

$$S(k) = \frac{v_\lambda^2}{k} = v_\lambda^2 \lambda = \dot{\varepsilon}^{2/3} \lambda^{5/3}$$

$$v_\lambda \propto \lambda^{1/3} \qquad v_L = U \qquad h_\lambda = \eta \sigma^2 = \eta \left(\frac{v_\lambda}{\lambda}\right)^2$$

$$v_\lambda = U \left(\frac{\lambda}{L}\right)^{1/3}$$

$$\dot{\varepsilon} \rho = \eta \dot{\varepsilon}^{2/3} \lambda^{-4/3}$$

$$\dot{\varepsilon} = \lambda^{-4} \rho^{-3} \eta^3$$

$$\frac{U^3}{L} F \lambda^{-4} \rho^{-3} \eta^3 \Rightarrow \lambda = L \left(\frac{\eta}{\rho U L}\right)^{3/4}$$

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