Title: Quantum Field Theory 1 - Lecture 6A

Date: Oct 15, 2008 10:30 AM

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

Causality in Relativistic QFT

$$(x-y) = \langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}e^{-ip\cdot(x-y)}$$

Causality in Relativistic QFT

$$D(x-y) = \langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec p}}e^{-ip\cdot(x-y)}$$

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\left[a_{\vec p}e^{-ip\cdot x} + a^{\dagger}_{\vec p}e^{ip\cdot x}\right]$$

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Contribution due to $\langle 0|a_{\vec{p}}a^\dagger_{\vec{q}}|0\rangle = (2\pi)^3\delta^3(\vec{p}-\vec{q})$.

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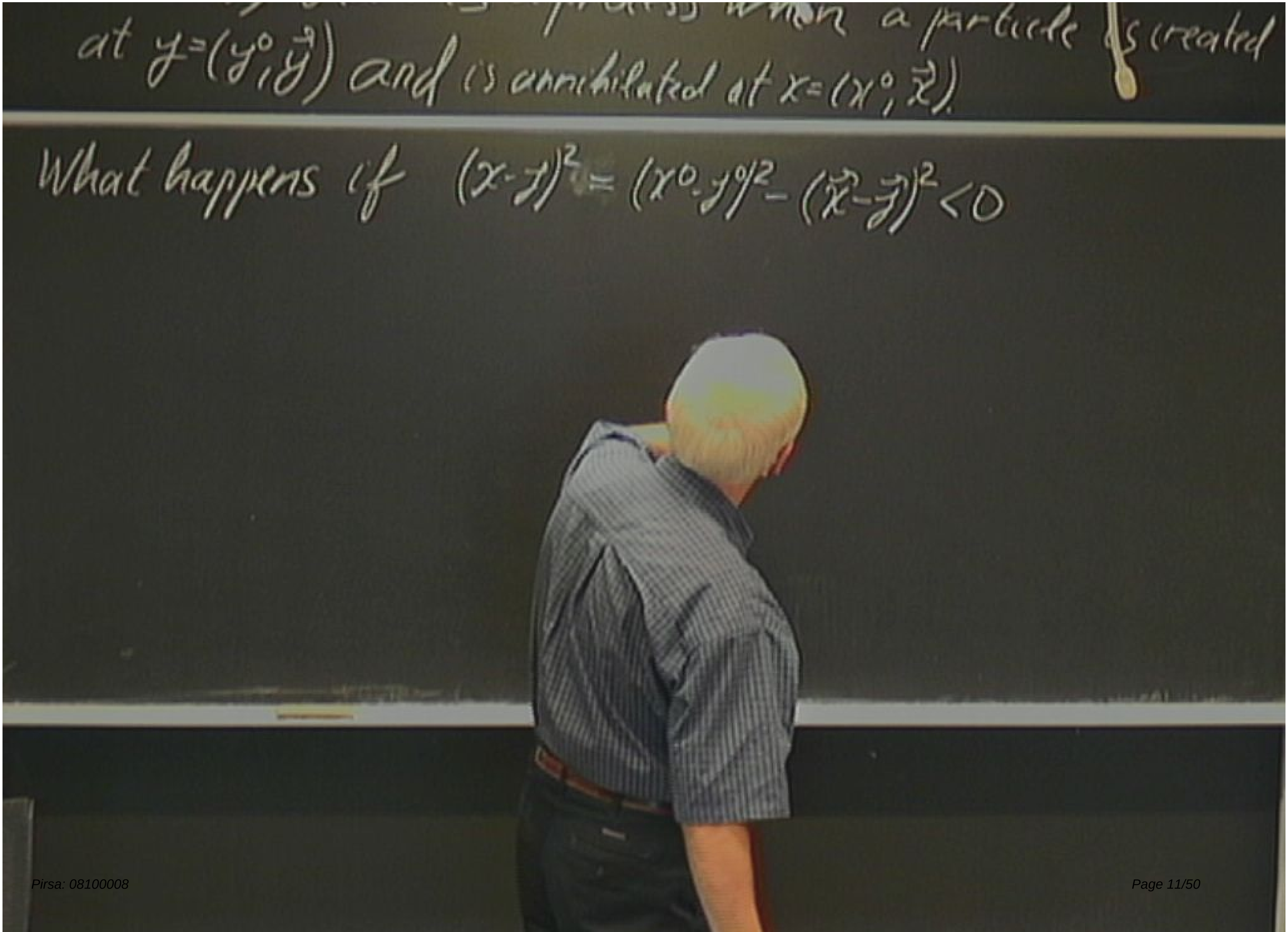
$D(x-y)$ describes a process when a particle is created at $y = (y^0, \vec y)$ and is annihilated at $x = (x^0, \vec x)$.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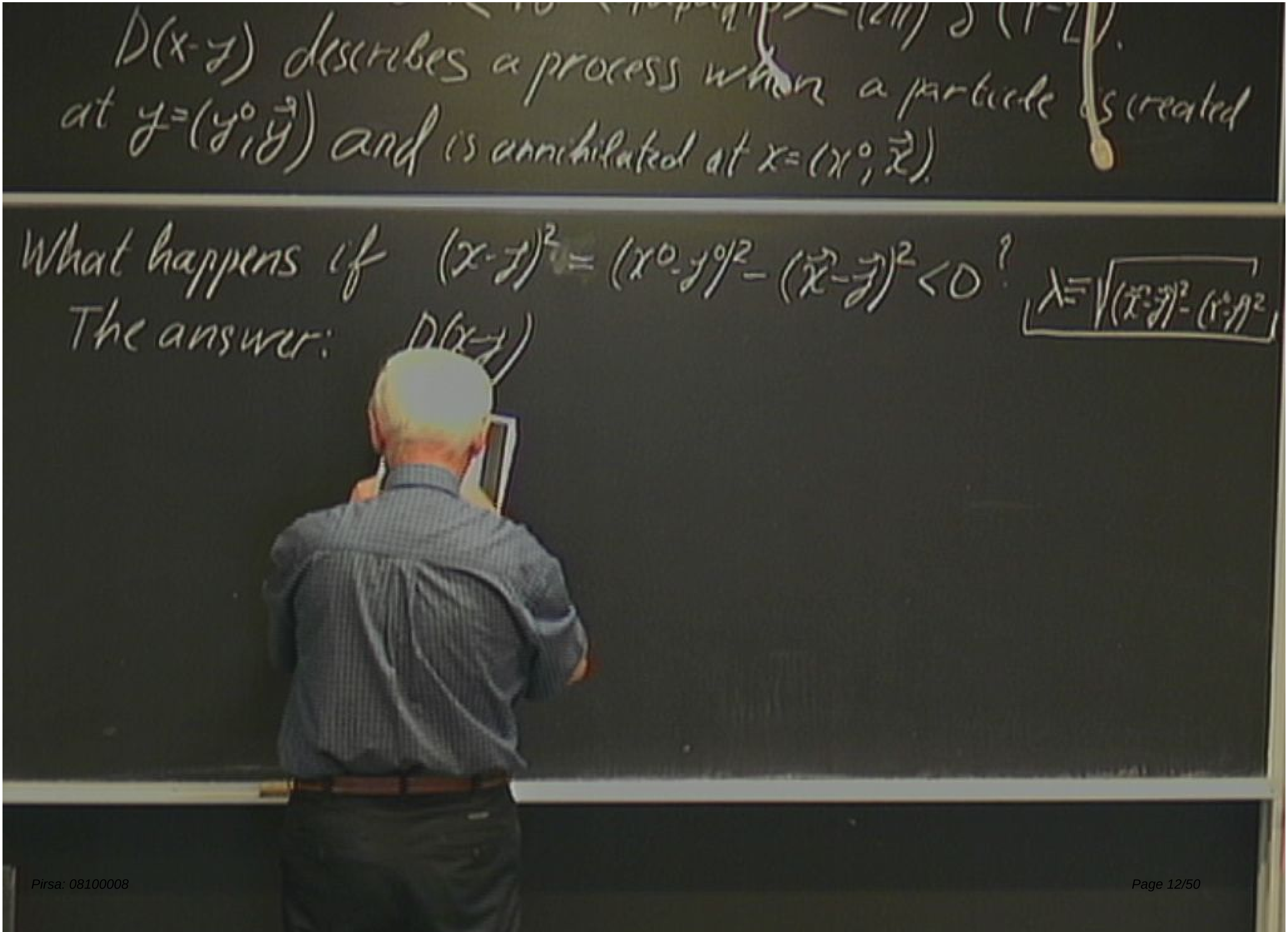
at $y=(y^0,\vec{y})$ and is annihilated at $x=(x^0,\vec{x})$.
What happens if $(x-y)^2=(x^0-y^0)^2-(\vec{x}-\vec{y})^2<0$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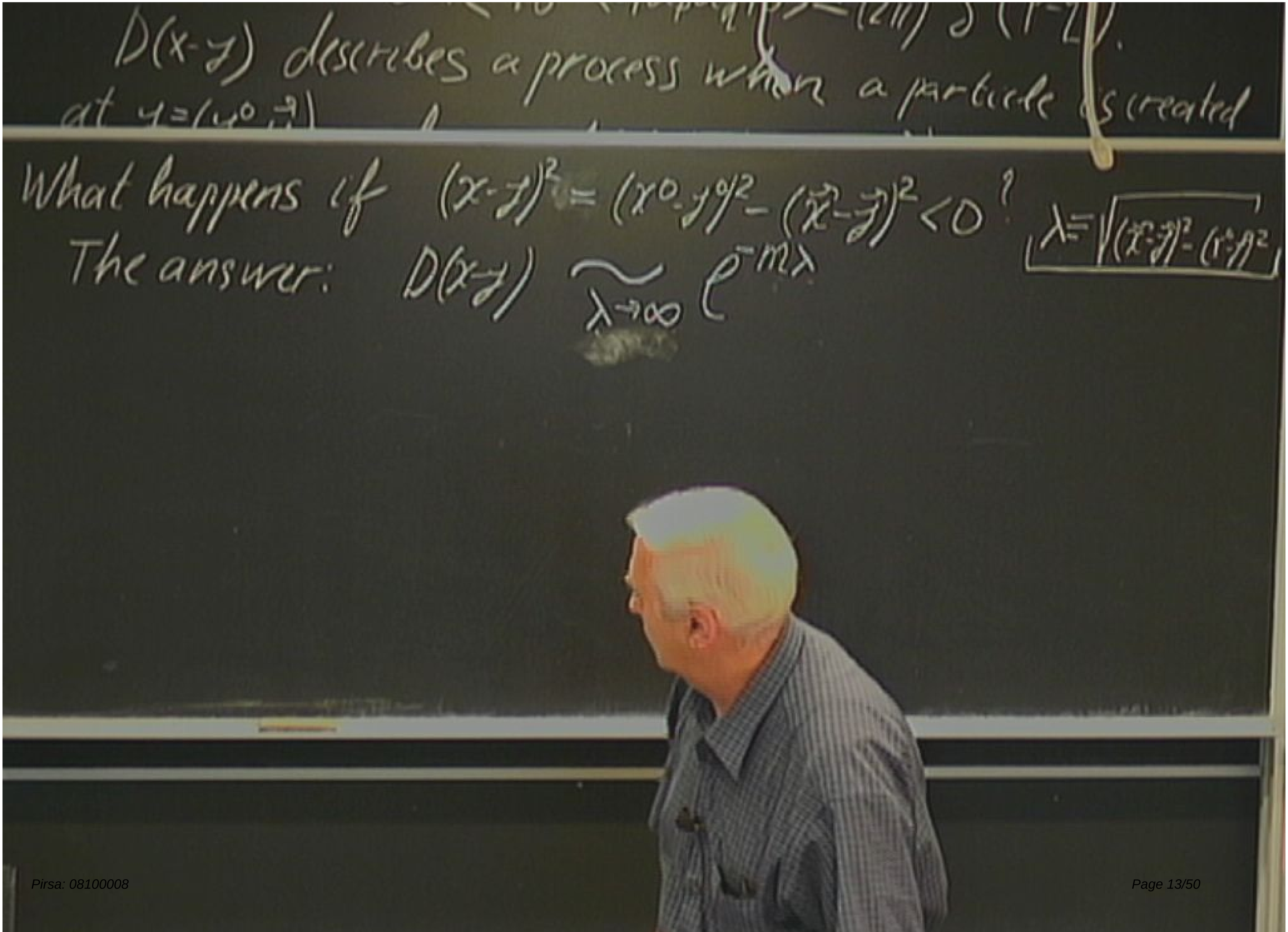
What happens if $(x-y)^2=(x^0-y^0)^2-(\vec{x}-\vec{y})^2<0$?
$\boxed{\lambda=\sqrt{(\vec{x}-\vec{y})^2-(x^0-y^0)^2}}$
The answer: $D(x-y)$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The answer: $D(x-y) \underset{\lambda\to\infty}{\sim} e^{-m\lambda}$

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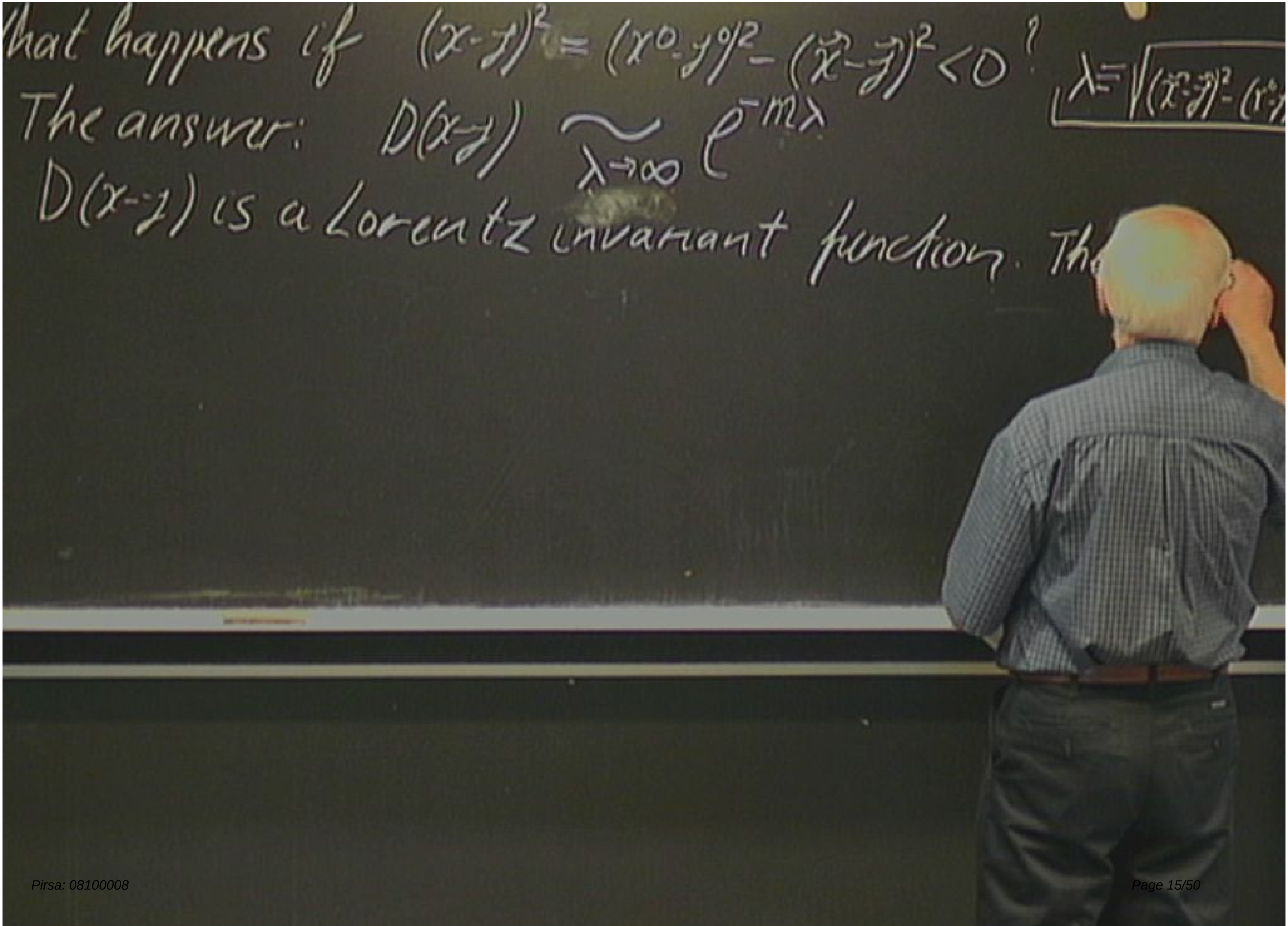
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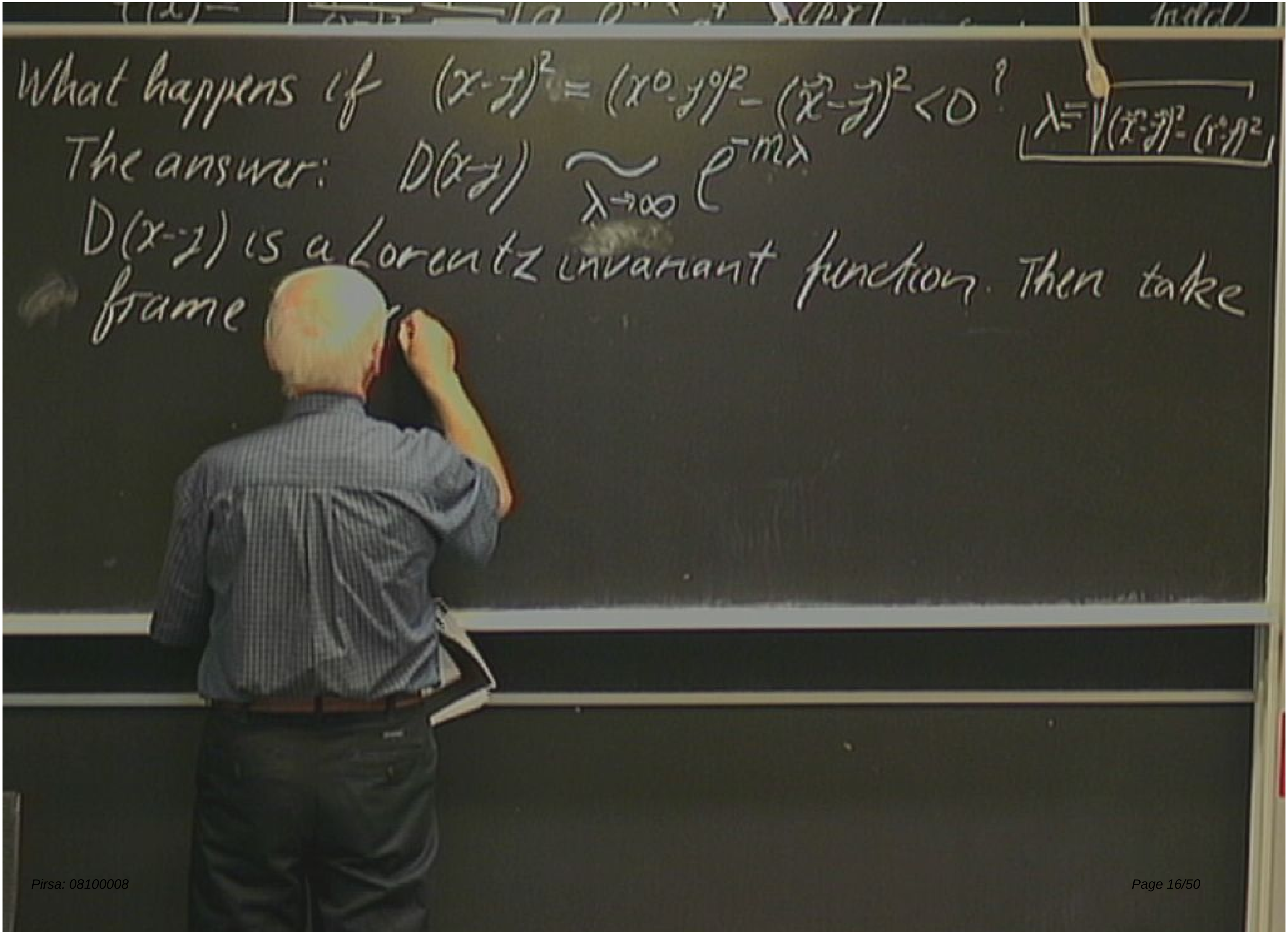
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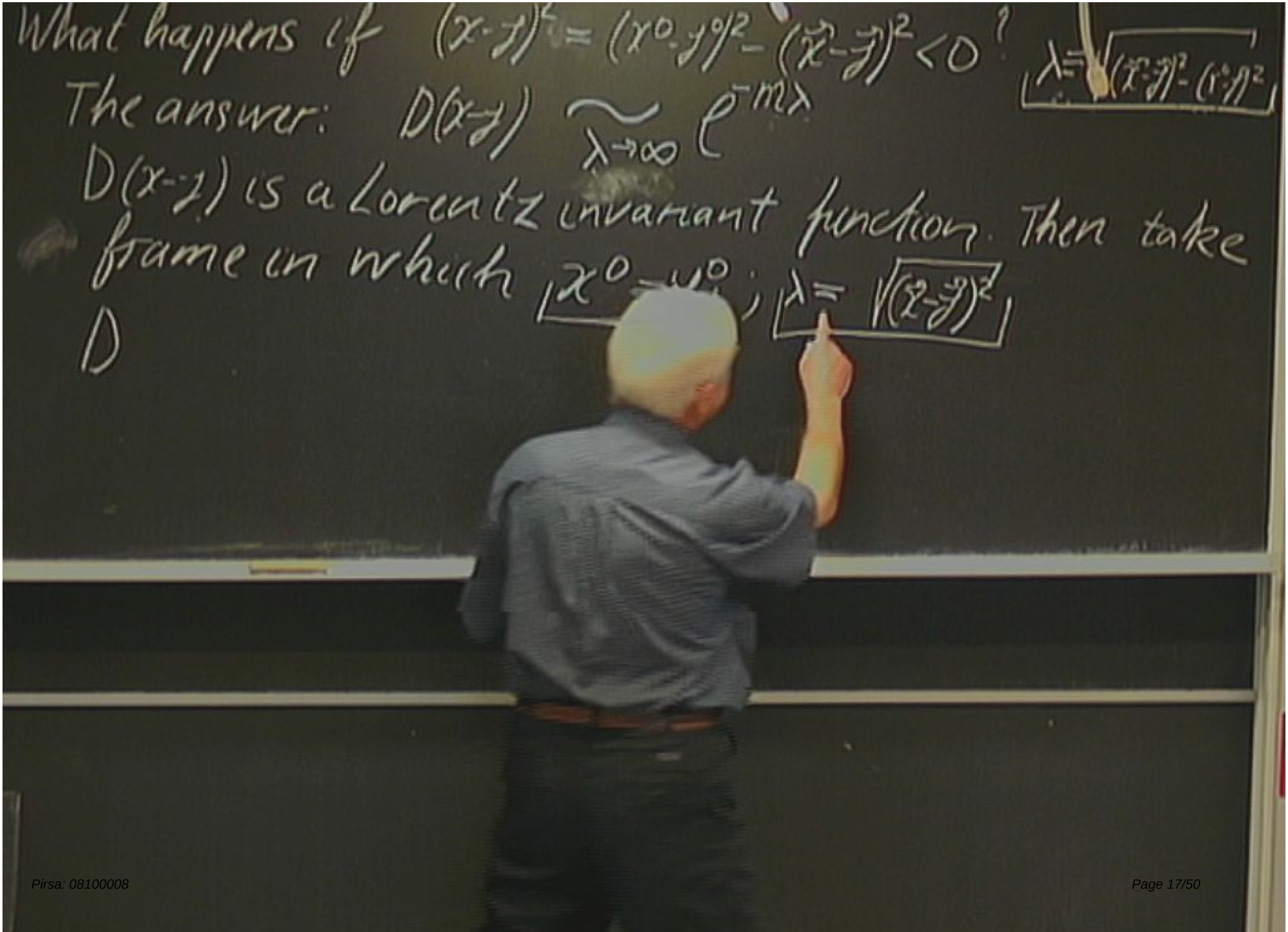

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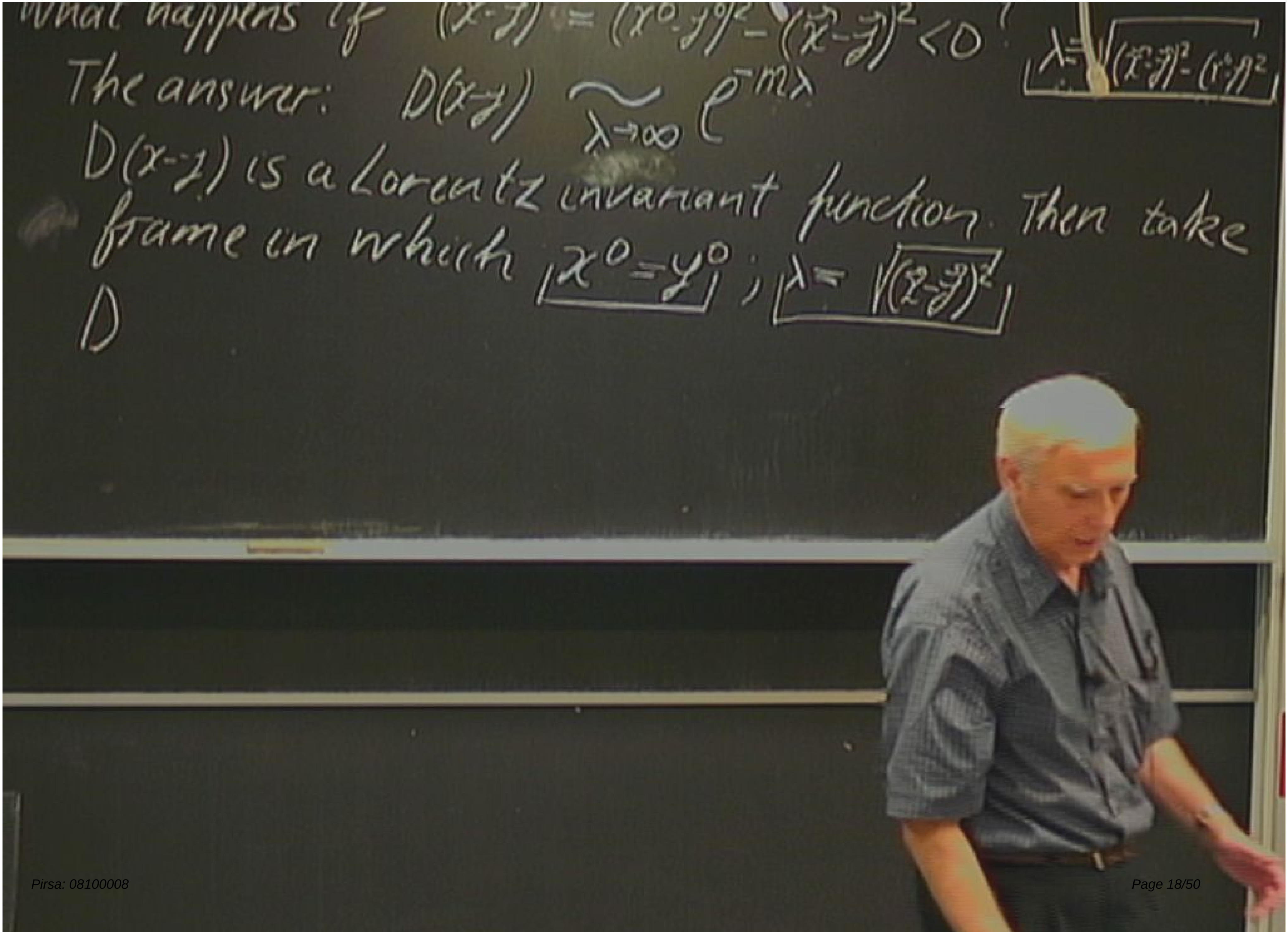
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$= p^2 \sin\theta \, d\theta \, d\varphi \, dp$

$\angle \varphi$

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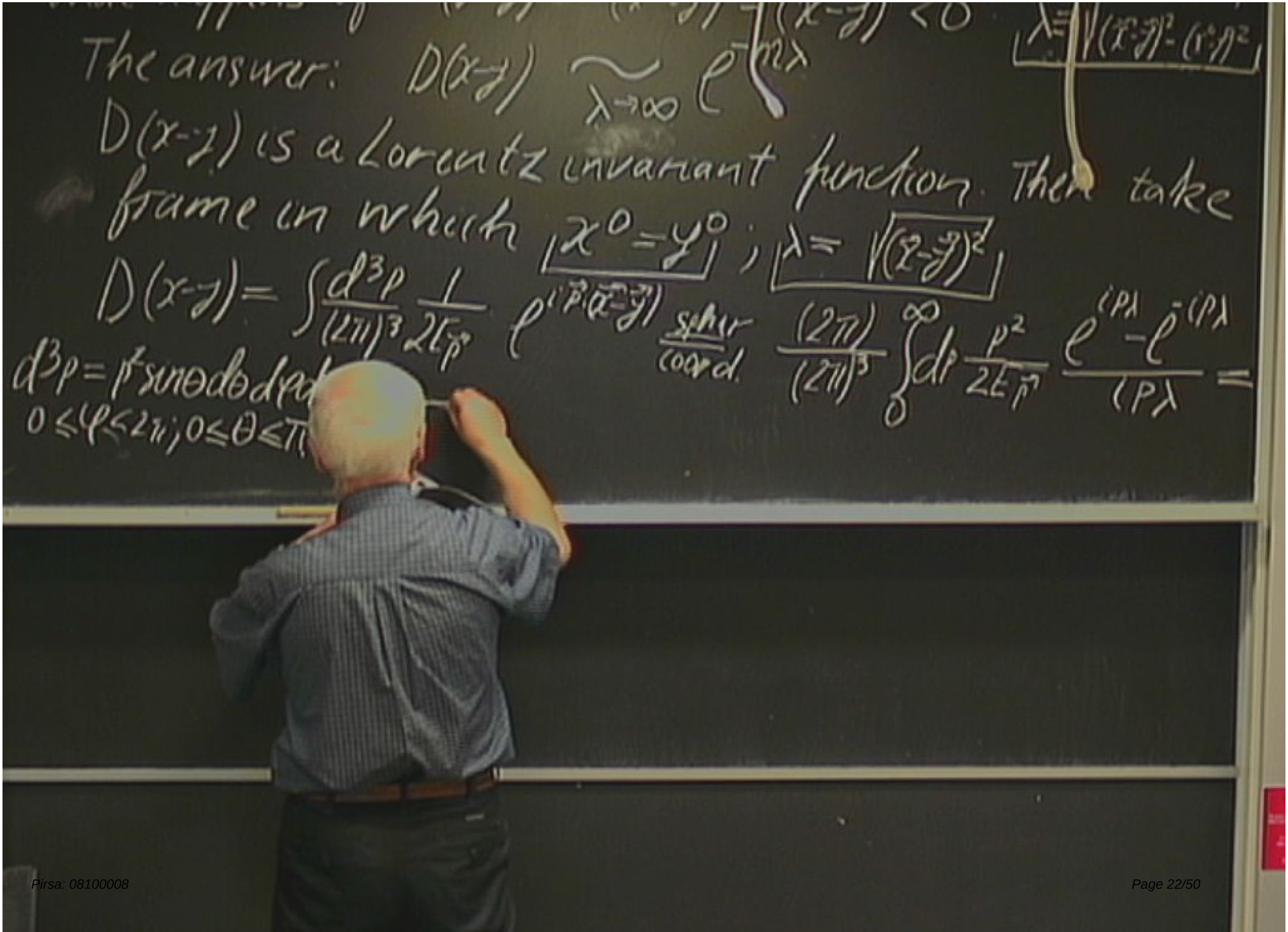
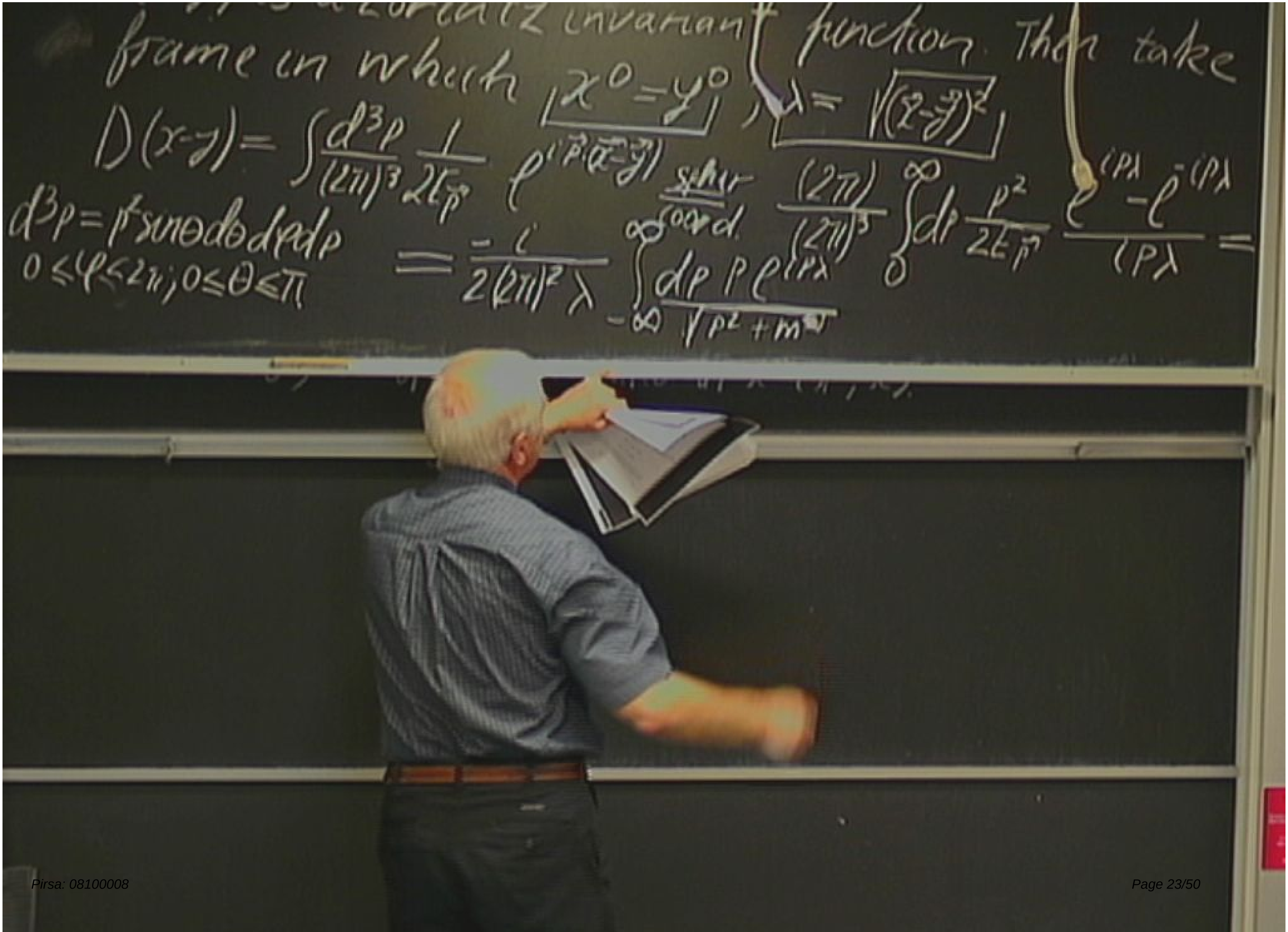
$d^3p = p^2 \sin\theta\, d\theta\, d\varphi\, dp$

$0 \le \varphi \le 2\pi;\ 0 \le \theta \le \pi$



The answer: D(x-y) ~ e^{-mλ} (λ→∞)
λ = √((x⃗-y⃗)² - (x⁰-y⁰)²)
D(x-y) is a Lorentz invariant function. Then take frame in which x⁰=y⁰ ; λ = √((x⃗-y⃗)²)
D(x-y) = ∫ d³p/(2π)³ 1/(2E_p) e^{ip⃗(x⃗-y⃗)}
spher coord.
(2π)/(2π)³ ∫₀^∞ dp p²/(2E_p) (e^{ipλ} - e^{-ipλ})/(ipλ) =
d³p = p² sinθ dθ dφ dp
0 ≤ φ ≤ 2π ; 0 ≤ θ ≤ π


invariant function. Then take
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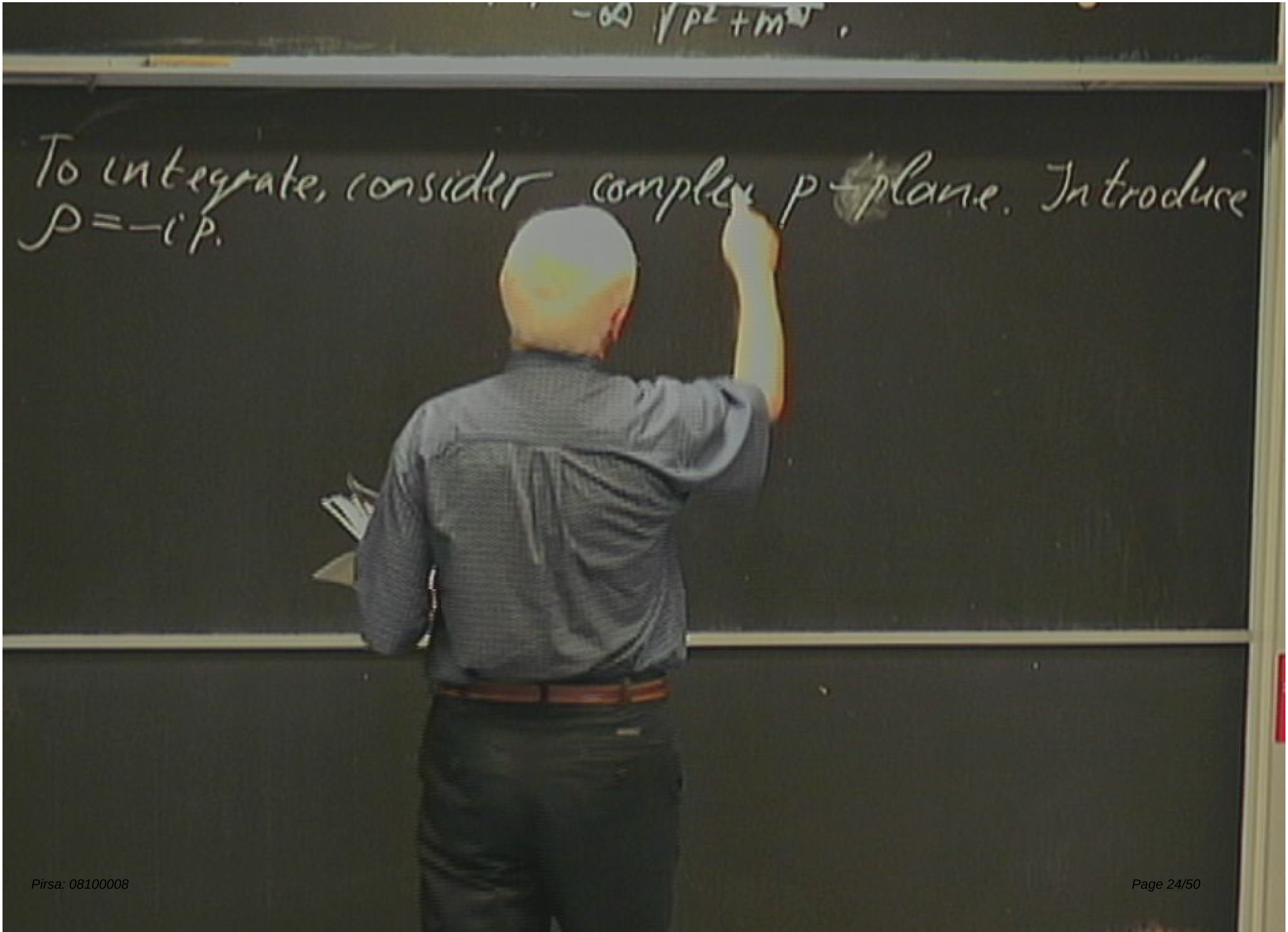
d^3p = p^2 sinθ dθ dφ dp
0 ≤ φ ≤ 2π; 0 ≤ θ ≤ π
spher coord
(2π)/(2π)^3 ∫_0^∞ dp p^2/(2E_p) (e^{ipλ} - e^{-ipλ})/(ipλ) =
= -i/(2(2π)^2 λ) ∫_{-∞}^{∞} dp p e^{ipλ}/√(p^2+m^2)


$-\infty$ $\sqrt{p^2+m^2}$.
To integrate, consider complex p-plane. Introduce
$\rho = -ip$.

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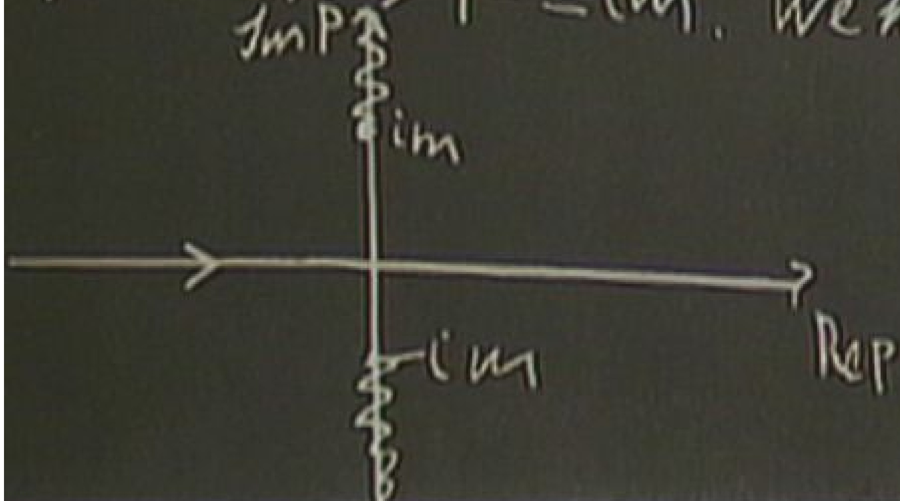
To integrate, consider complex p-plane. Introduce $\rho = -ip$. Look at $\frac{1}{\sqrt{p^2+m^2}}$: it is equal to ∞ at $p^2 = -m^2 \Rightarrow p = \pm im$. We have two branch cuts:



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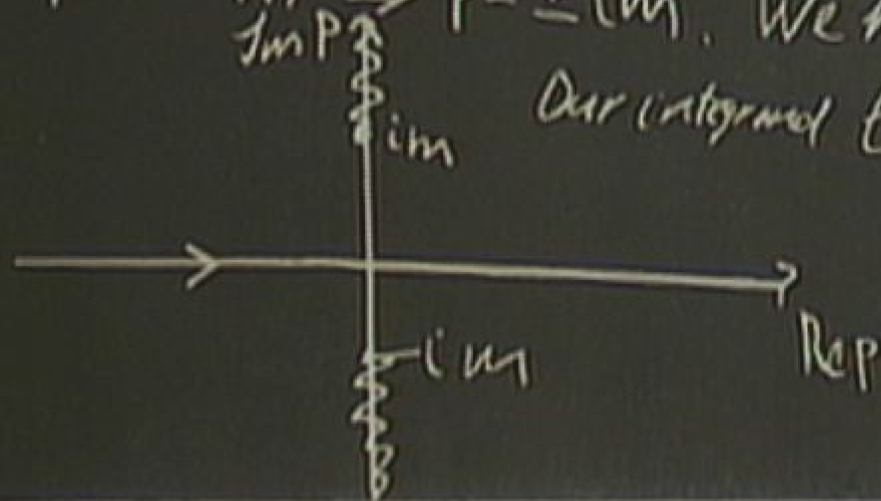

Our integrand $e^{-ipx} \Rightarrow e^{iax} e^{-bx}$

deformation of the contour.

$-\infty \quad \sqrt{p^2+m^2}\,.$

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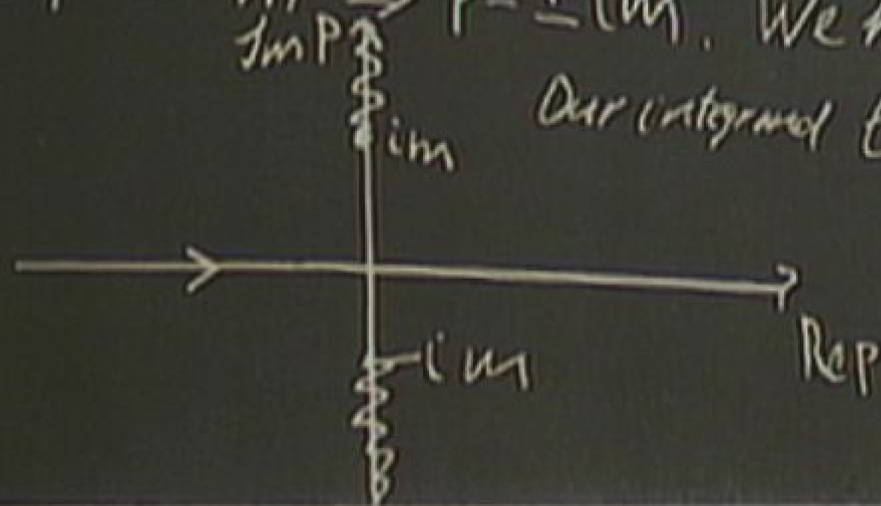

Our integrand e^{ipx} ... $e^{iax}\,e^{-bx}$:

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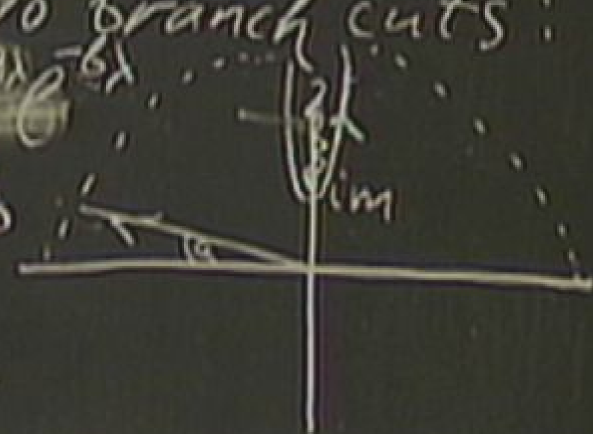

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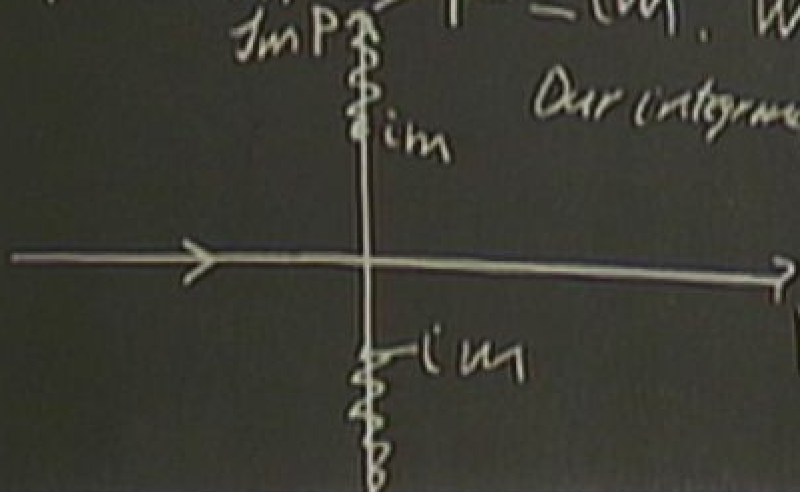

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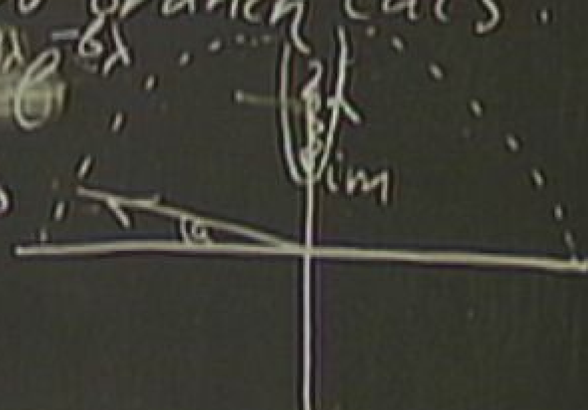

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Then, our expression (1) becomes $\frac{1}{4\pi^2\lambda}\int_m^\infty d\rho \frac{\rho e^{-\rho\lambda}}{\sqrt{\rho^2-m^2}} \Rightarrow$

$\xRightarrow{\lambda\to\infty}$

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$\xrightarrow{\lambda\to\infty} \sim e^{-m\lambda}$

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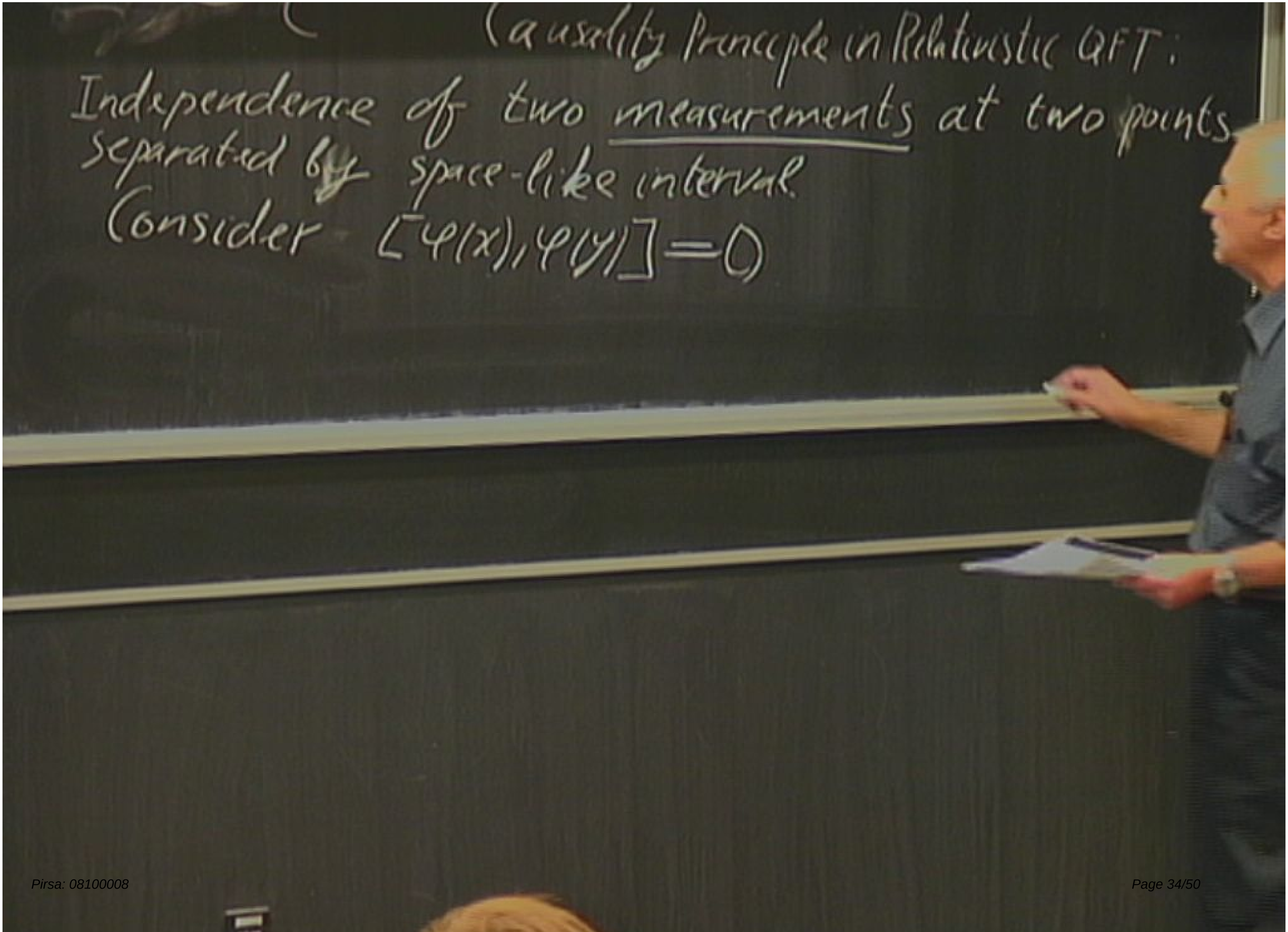
Causality Principle in Relativistic QFT:


Causality Principle in Relativistic QFT:
Independence of two measurements at two points separated by space-like interval.
Consider $[\varphi(x), \varphi(y)] = 0$

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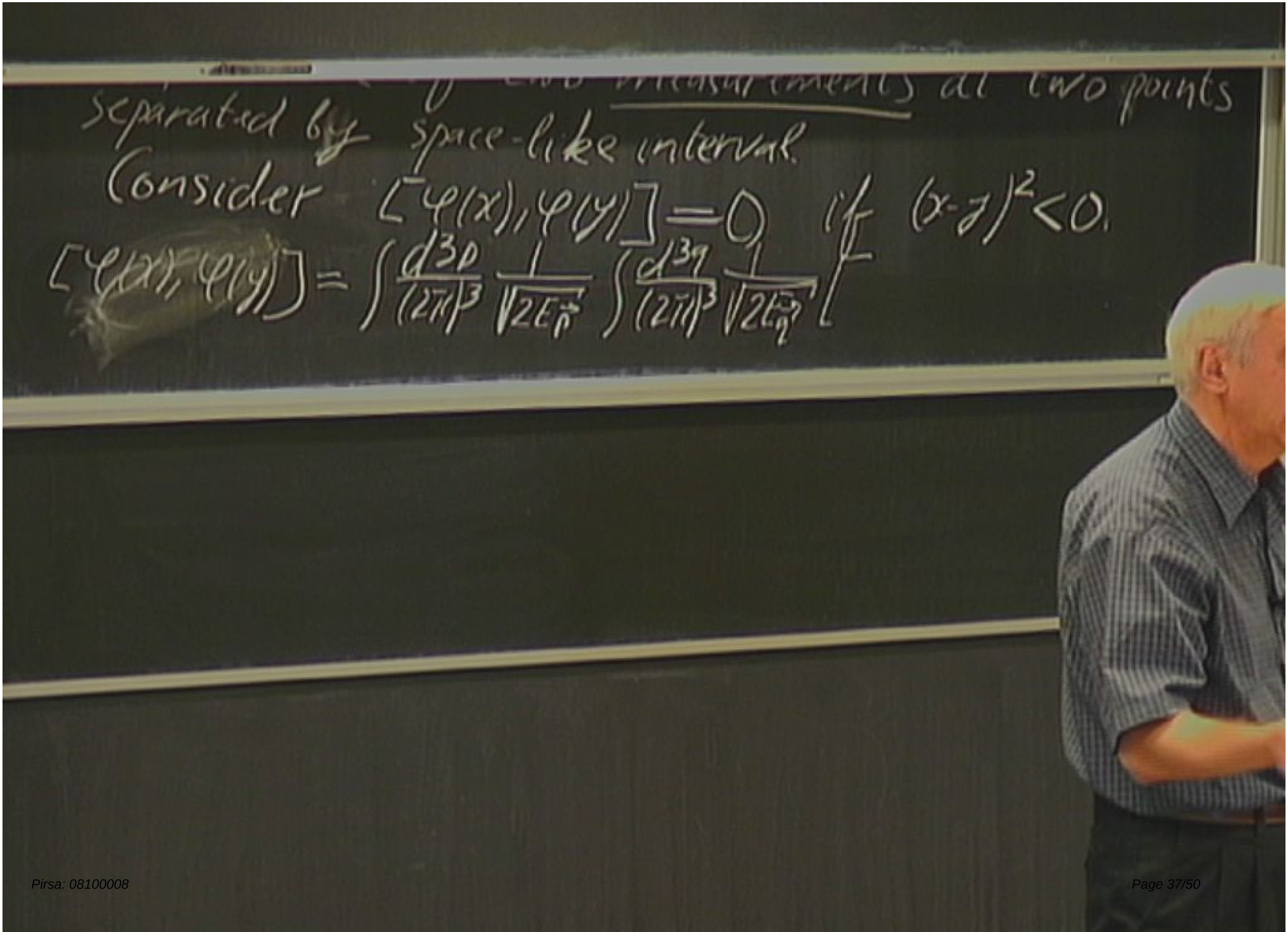
measurements at two points
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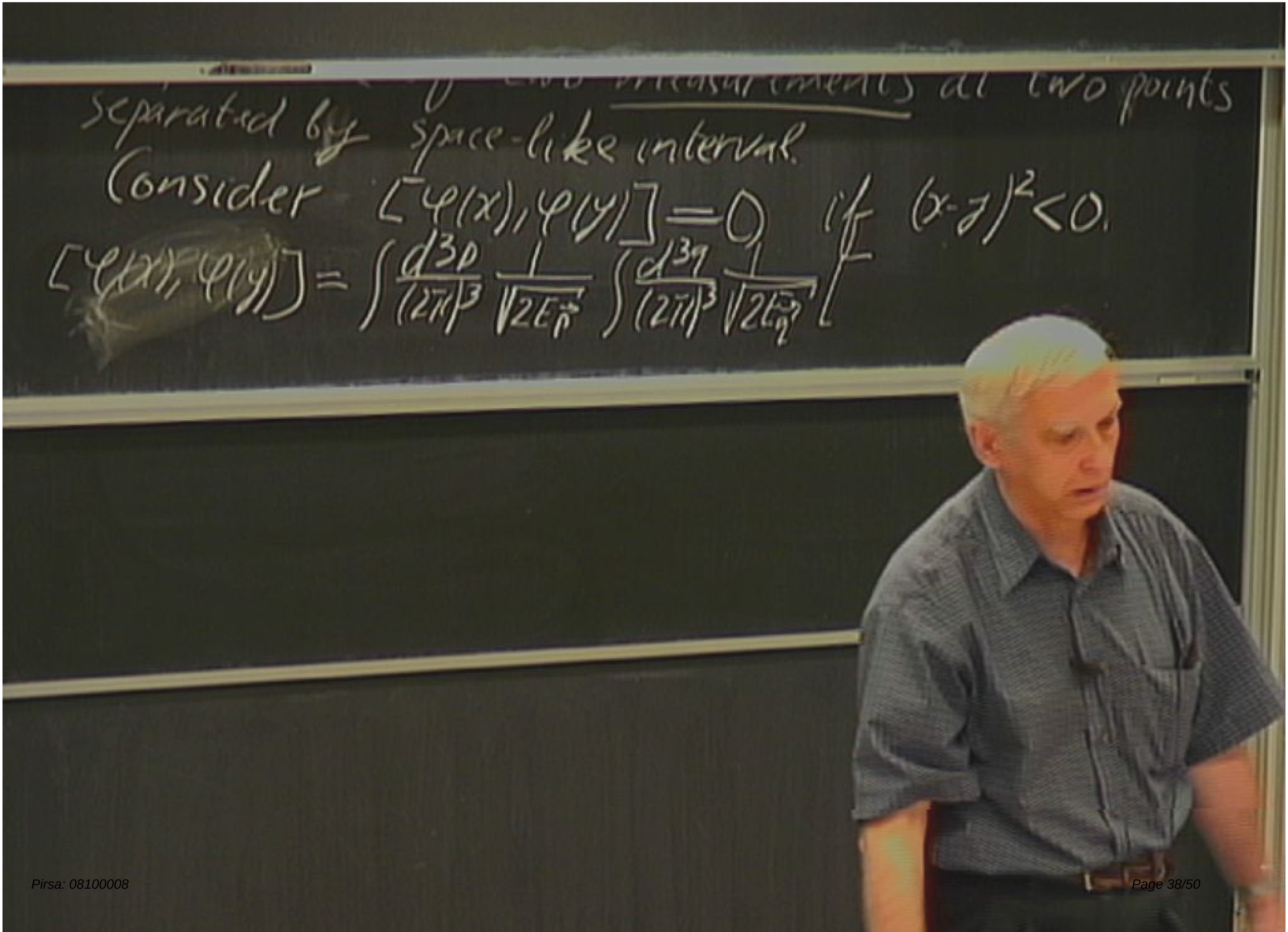
Consider $[\varphi(x), \varphi(y)] = 0$ if $(x-y)^2 < 0$.
$[\varphi(x), \varphi(y)] = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \int \frac{d^3q}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{q}}}} [$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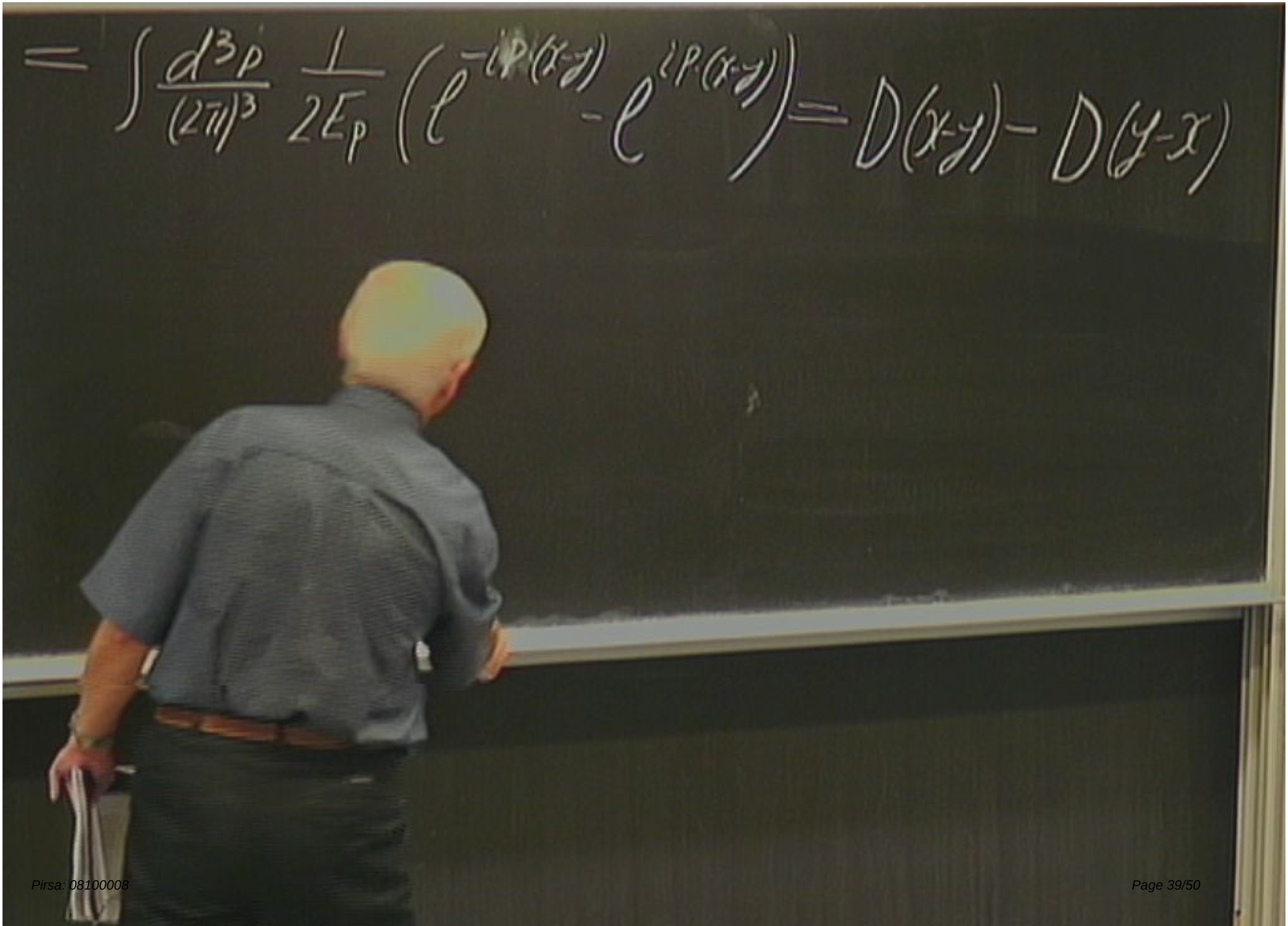
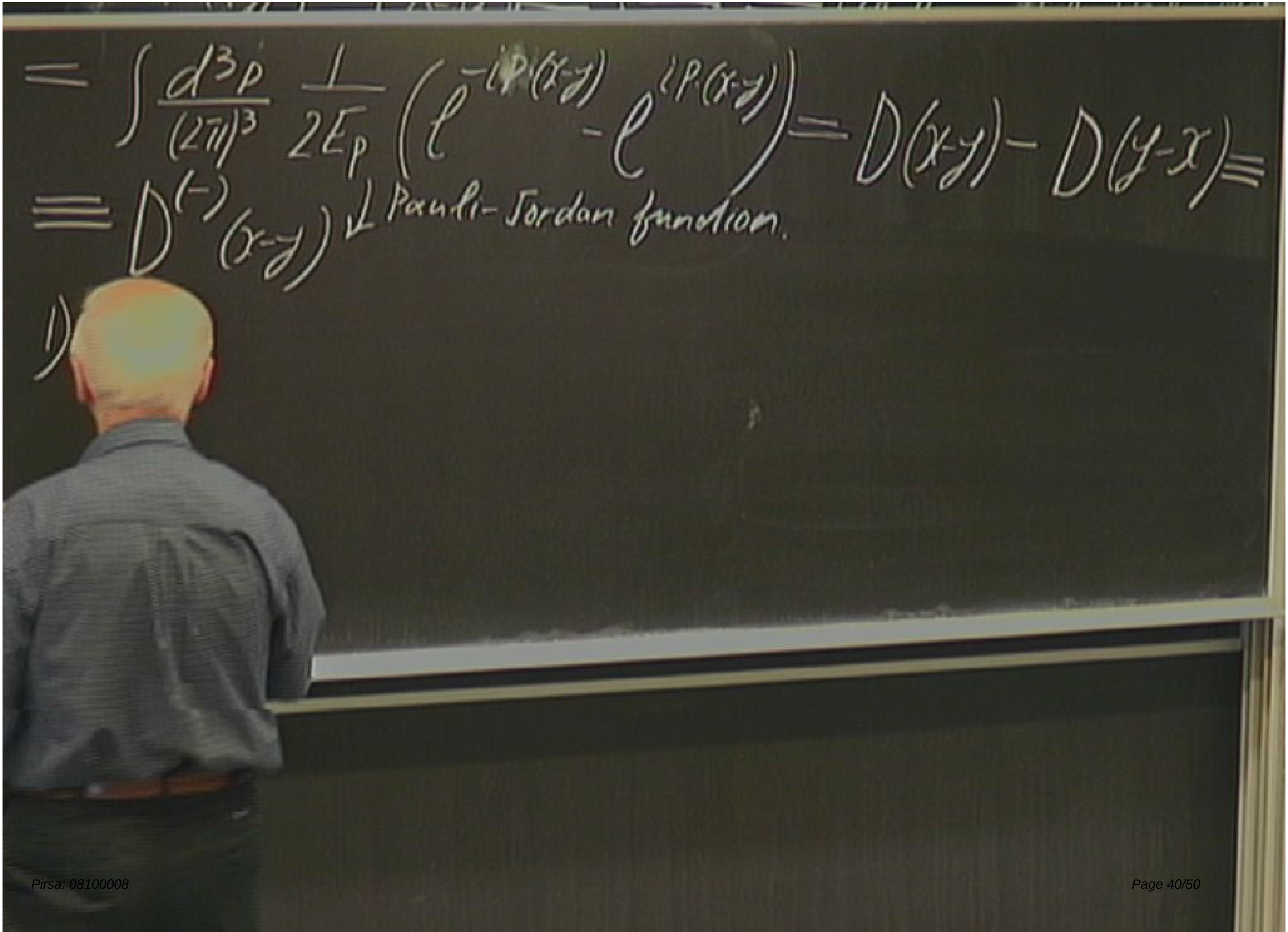

$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \left(e^{-ip(x-y)} - e^{ip(x-y)} \right) = D(x-y) - D(y-x)$



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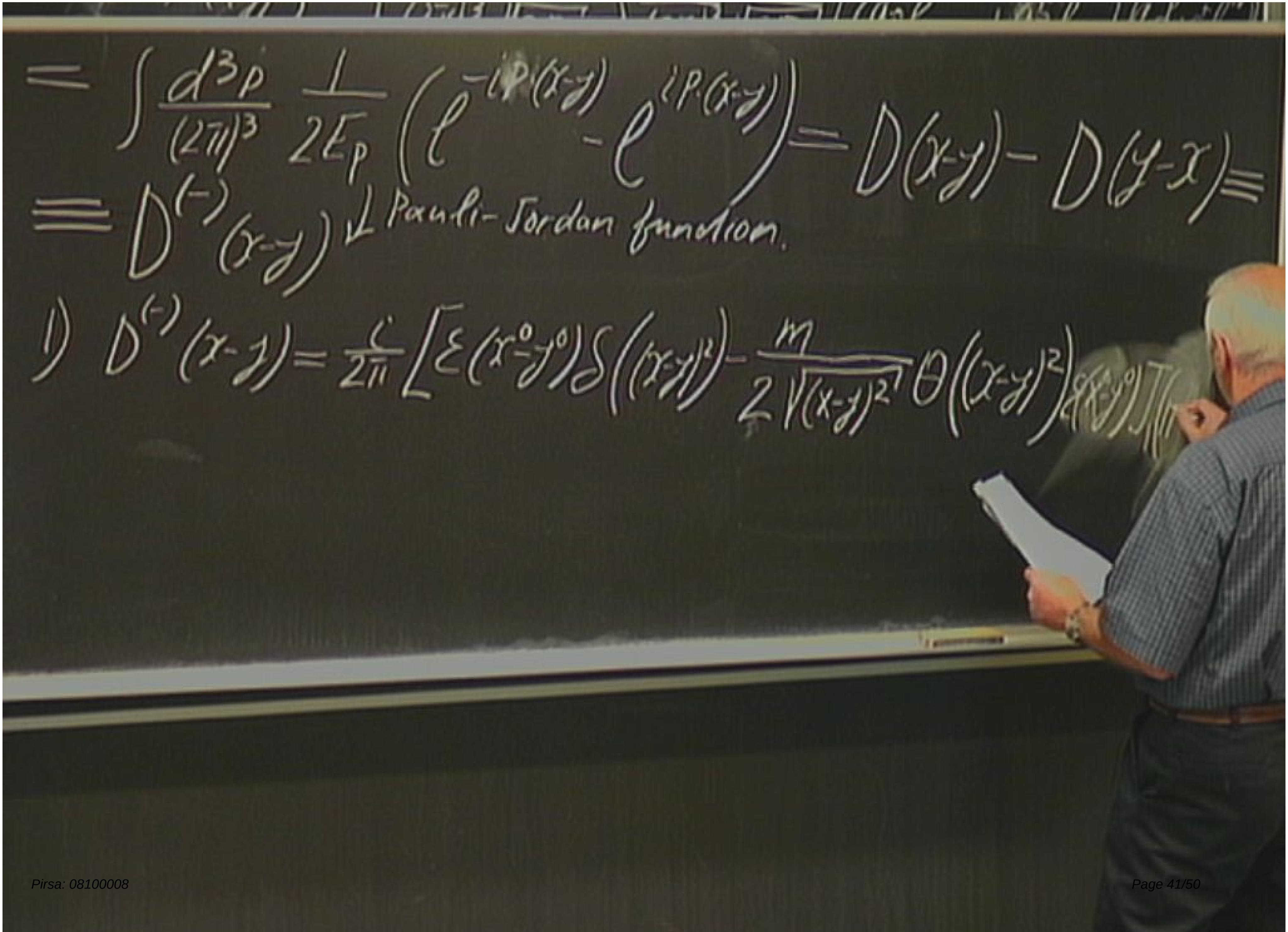
$\equiv D^{(-)}(x-y)$ Pauli-Jordan function.


$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right) = D(x-y) - D(y-x) \equiv$
$\equiv D^{(-)}(x-y)$ ↓ Pauli-Jordan function.
1) $D^{(-)}(x-y) = \frac{i}{2\pi} \Big[\varepsilon(x^0-y^0)\, \delta\big((x-y)^2\big) - \frac{m}{2\sqrt{(x-y)^2}}\, \theta\big((x-y)^2\big)\, \varepsilon(x^0-y^0)\, J_1($

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right) = D(x-y) - D(y-x) \equiv$$

$$\equiv D^{(-)}(x-y)$$ Pauli–Jordan function

$$\theta(x) = \begin{cases} 1 & \text{if } x>0 \\ 0 & \text{if } x<0 \end{cases} \quad \varepsilon(x) = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases}$$

1) $$D^{(-)}(x-y) = \frac{i}{2\pi}\left[\varepsilon(x^0-y^0)\,\delta\left((x-y)^2\right) - \frac{m}{2\sqrt{(x-y)^2}}\,\theta\left((x-y)^2\right)\varepsilon(x^0-y^0)\,J_1\left(m\sqrt{(x-y)^2}\right) \right]$$

Take $(x-y)^2<0$ (spatial like interval) ; Then $D^{(-)}(x-y)=0$.

Remarks and corrections:

1. $\cancel{J^{\mu\nu} = x^{\mu}\int d^3x\, T^{0\nu} - x^{\nu}\int d^3x\, T^{0\mu}}$ $\xrightarrow{\text{correct}}$

$$J^{\mu\nu} = \int d^3x \left[x^{\mu} T^{0\nu} - x^{\nu} T^{0\mu} \right].$$

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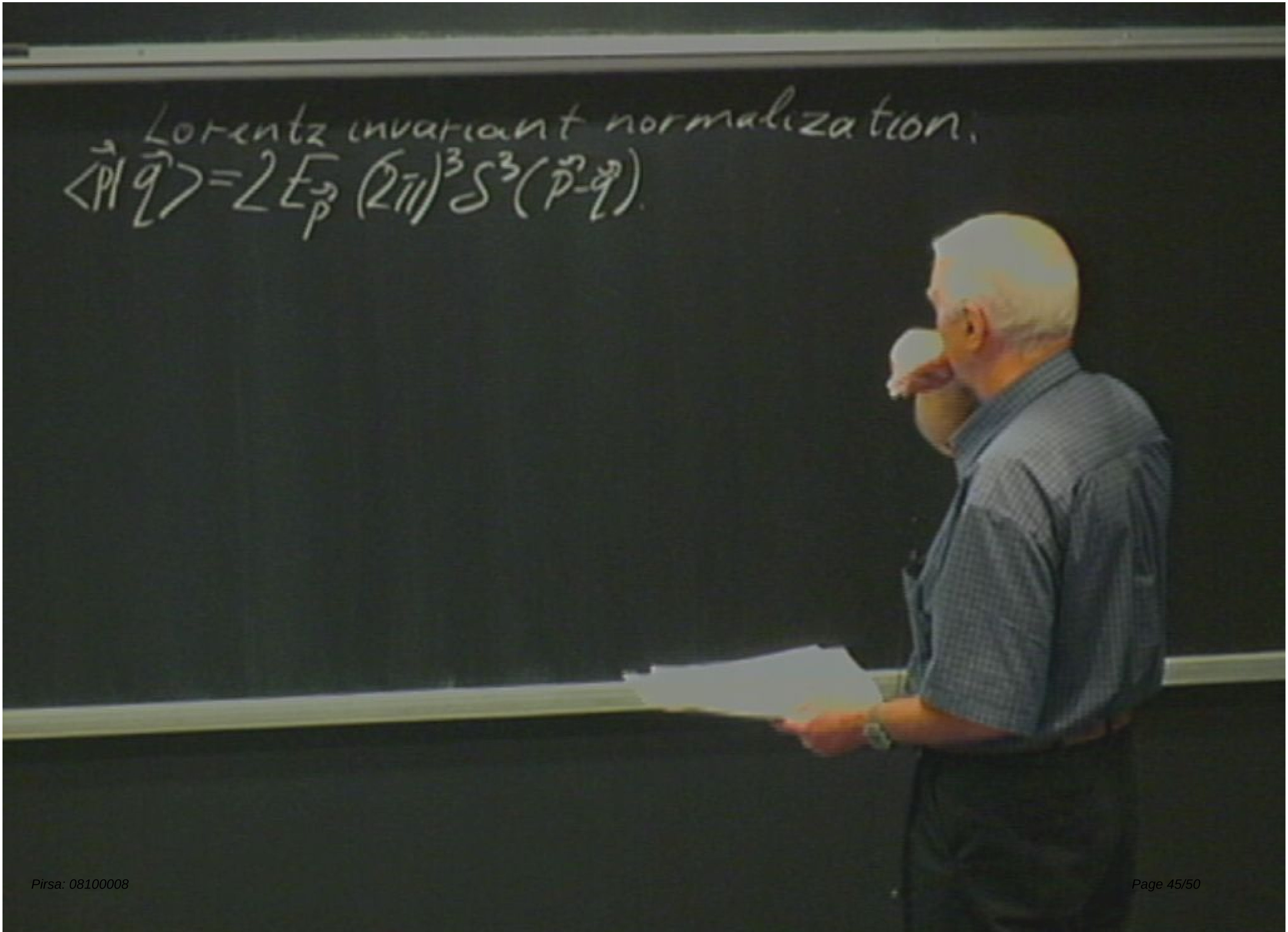
$$J^{\mu\nu} = \int d^3x \left[x^\mu T^{0\nu} - x^\nu T^{0\mu} \right].$$

2. $U(\Lambda)|0\rangle = |0\rangle$; $\underline{U(a)|0\rangle = |0\rangle}$. $\quad U(a) = e^{i P^\mu a_\mu}$

$P^0|0\rangle \equiv H|0\rangle = 0, \quad \vec{P}|0\rangle = 0$

$$P^0 = \int \frac{d^3p}{(2\pi)^3} E_{\vec{p}}\, a^\dagger_{\vec{p}} a_{\vec{p}} \qquad \vec{P} = \int \frac{d^3p}{(2\pi)^3} \vec{p}\, a^\dagger_{\vec{p}} a_{\vec{p}}.$$


Lorentz invariant normalization.
$\langle \vec{p} | \vec{q} \rangle = 2 E_{\vec{p}} (2\pi)^3 \delta^3(\vec{p} - \vec{q})$

Lorentz transf $\langle \vec{p} | \vec{q} \rangle = \sqrt{2E_{\vec{p}}}\sqrt{2E_{\vec{q}}}\langle 0 | a_{\vec{p}}\, a^{\dagger}_{\vec{q}} | 0 \rangle \overset{\Lambda}{\Longrightarrow}$

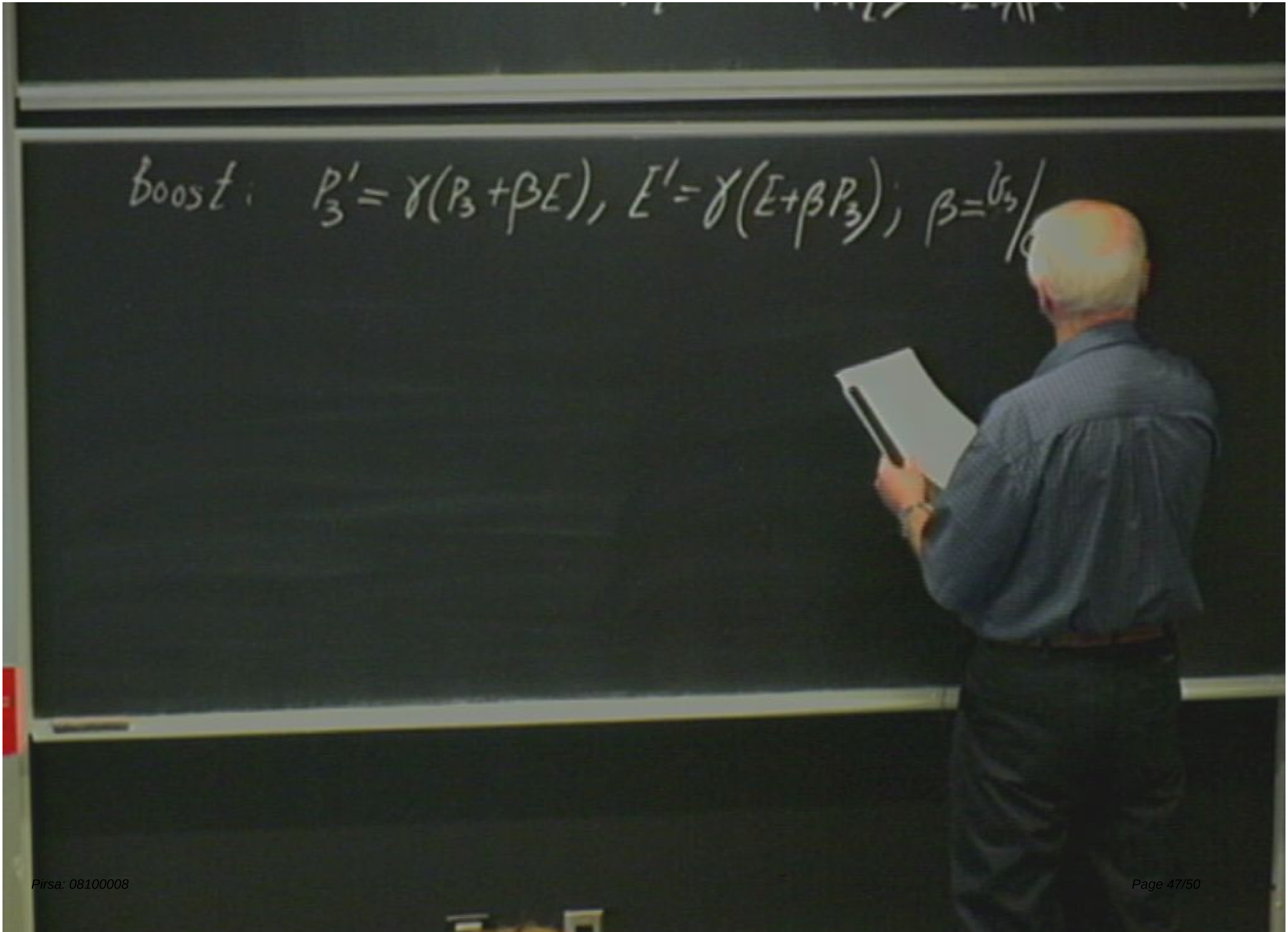
$\Rightarrow \sqrt{2E_{\vec{p}}}\sqrt{2E_{\vec{q}}}\langle \vec{p} | U^{\dagger}(\Lambda) U(\Lambda) | \vec{q} \rangle = \sqrt{2E_{\vec{p}}}\sqrt{2E_{\vec{q}}}\langle 0 | a_{\vec{p}} U^{\dagger}(\Lambda) U(\Lambda) a^{\dagger}_{\vec{q}} | 0 \rangle =$

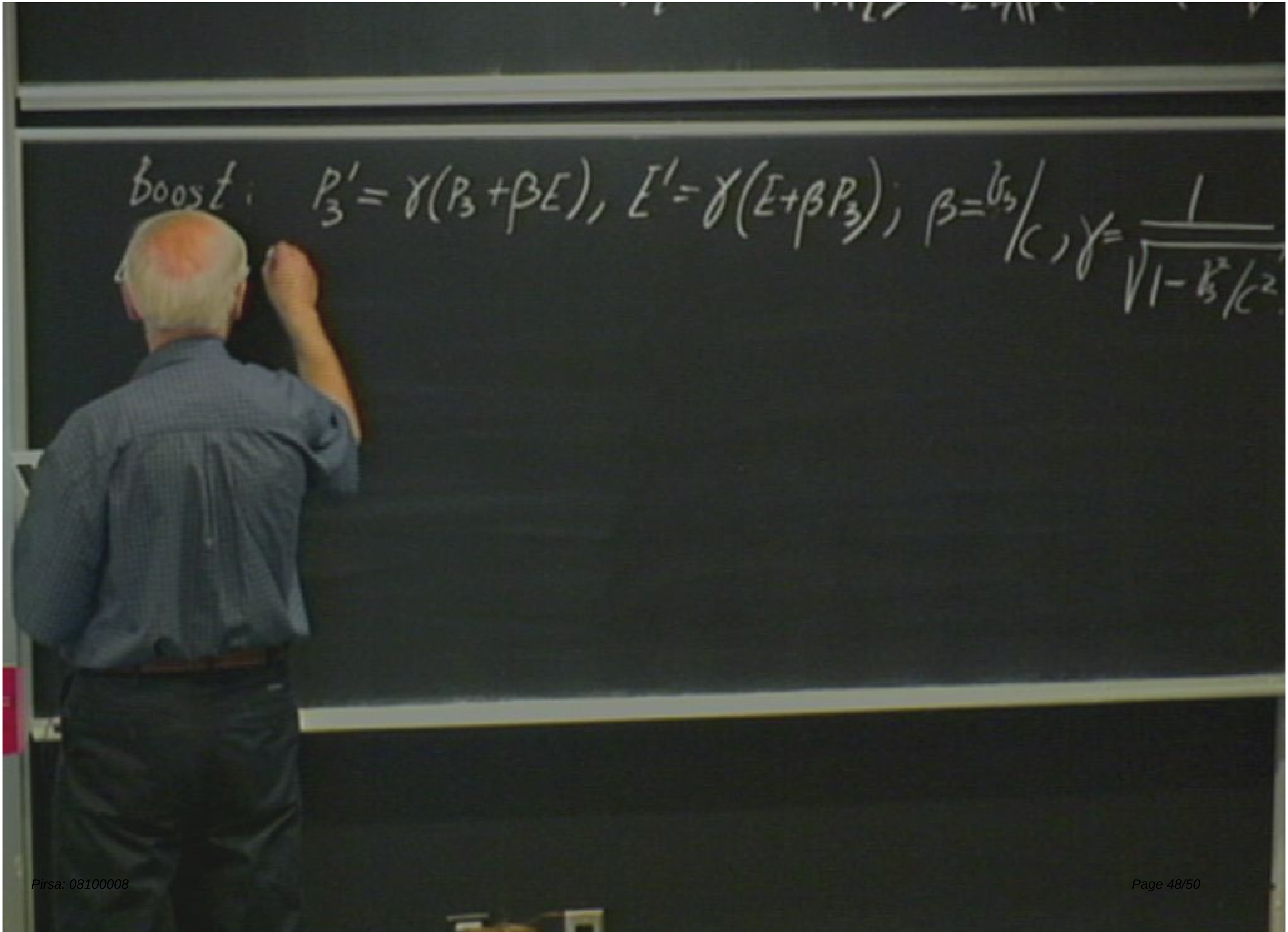
$U^{\dagger}(\Lambda) = U^{-1}(\Lambda)$

$= 2E_{p}(2\pi)^3 \delta^3(\vec{p} - \vec{q}).$

On the other hand, $\langle \vec{p} | U^{\dagger}(\Lambda) U(\Lambda) | \vec{q} \rangle = \langle \Lambda \vec{p} | \Lambda \vec{q} \rangle = 2E_{\Lambda p}(2\pi)^3 \delta^3(\Lambda\vec{p} - \Lambda\vec{q})$

$(\ldots)\frac{\ldots}{(2\pi)^3}\, a^{\dagger}_{\vec{p}} a_{\vec{p}}.$


boost: $P_3' = \gamma(P_3 + \beta E)$, $E' = \gamma(E + \beta P_3)$; $\beta = v_3/$


boost: $P_3' = \gamma(P_3 + \beta E)$, $E' = \gamma(E + \beta P_3)$; $\beta = v_3/c$, $\gamma = \frac{1}{\sqrt{1 - v_3^2/c^2}}$

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Evaluate $\delta^{(3)}(\vec{p} - \vec{q}) \underset{\delta(f(x) - f(x_0)) = \frac{1}{|f'(x_0)|}\delta(x - x_0)}{\overset{use}{=\!=}} \delta^{(3)}(\vec{p}' - \vec{q}')\dfrac{dp_3'}{dp_3} =$

$$= \delta^{(3)}(\vec{p}' - \vec{q}')\gamma\left(1 + \beta\frac{dE}{dp_3}\right) = \delta^3(\vec{p}' - \vec{q}')\frac{\gamma}{E}(E + \beta p_3) = \delta^3(\vec{p}' - \vec{q}')\frac{E'}{E} \Rightarrow$$

$$\Rightarrow E\,\delta^{(3)}(\vec{p} - \vec{q}) = E'\,\delta^{(3)}(\vec{p}' - \vec{q}').$$

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Evaluate $\delta^{(3)}(\vec{p} - \vec{q})$ $\underline{\underline{\text{use}}}$ $\delta^{(3)}(\vec{p}' - \vec{q}')\dfrac{dp_3'}{dp_3} =$

$\delta(f(x) - f(x_0)) = \dfrac{1}{|f'(x_0)|}\delta(x - x_0)$

$$= \delta^{(3)}(\vec{p}' - \vec{q}')\,\gamma\left(1 + \beta\frac{dE}{dp_3}\right) = \delta^3(\vec{p}' - \vec{q}')\,\frac{\gamma}{E}(E + \beta p_3) = \delta^3(\vec{p}' - \vec{q}')\,\frac{E'}{E} \Rightarrow$$

$$\Rightarrow E\,\delta^{(3)}(\vec{p} - \vec{q}) = E'\,\delta^{(3)}(\vec{p}' - \vec{q}').$$