

Title: Quantum Field Theory 1 - Lecture 4A

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

Quantization of KG field in Schrödinger picture

$$\varphi(\vec{x}) = \lim_{t \rightarrow 0} \varphi(t, \vec{x}), \quad J_1(\vec{x}) = J_1(t, \vec{x})|_{t=0}$$

Quantization of KG field in Schrödinger picture

$$\varphi(\vec{x}) = \lim_{t \rightarrow 0} \varphi(t, \vec{x}), \quad J^i(\vec{x}) = J^i(t, \vec{x})|_{t=0} = \partial_t \varphi(t, \vec{x})|_{t=0}$$
$$[\varphi(\vec{x}), \pi(\vec{y})] = [S^3(\vec{x}-\vec{y}), [\varphi(\vec{x}), \varphi(\vec{y})]] = 0, \quad [\pi(\vec{x}), \pi(\vec{y})] = 0$$

Quantization of KG field in Schrödinger picture

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$$[\varphi(\vec{x}), \pi(\vec{y})] = [S^i(\vec{x} - \vec{y}), [\bar{\varphi}(\vec{x}), \varphi(\vec{y})]] = 0, \quad [\pi(\vec{x}), \pi(\vec{y})] = 0$$

Quantization of KG field in Schrödinger picture

$$\varphi(\vec{x}) = \lim_{t \rightarrow 0} \varphi(t, \vec{x}) / t, \quad \dot{\varphi}(\vec{x}) = \dot{\varphi}(t, \vec{x}) / t \Big|_{t=0} = \partial_t \varphi(t, \vec{x}) \Big|_{t=0}$$

$$[\varphi(\vec{x}), \pi(\vec{y})] = [S(\vec{x} - \vec{y}), [\varphi(\vec{x}), \varphi(\vec{y})]] = 0, \quad [\pi(\vec{x}), \pi(\vec{y})] = 0$$

$$\varphi(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^* e^{i\vec{p}\cdot\vec{x}}) \quad |p = E_p, k_p = \sqrt{p^2 + m^2}| \quad (1)$$

$$2 \quad |p = E_p, k_p = \sqrt{p^2 + m^2} \quad (2)$$

$$\varphi(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_p e^{-ip \cdot x} + a_p^\dagger e^{ip \cdot x}) \Big|_{p=\vec{p}}, \quad \vec{p} = \sqrt{p^2 + m^2} \quad (1)$$

$$\partial_t \varphi(t, \vec{x}) \equiv J(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{e^{-iE_p t}}{\sqrt{2E_p}} \left(E_p a_p e^{-ip \cdot x} - [p] a_p^\dagger e^{ip \cdot x} \right) \Big|_{p=\vec{p}} \quad (2)$$

From here,

$$\varphi(\vec{x}') = \int \frac{d^3 p}{(2\pi)^3} \frac{e^{-iE_p t}}{\sqrt{2E_p}} \left(E_p a_p e^{-ip \cdot x'} - [p] a_p^\dagger e^{ip \cdot x'} \right) \Big|_{p=\vec{p}} \quad (3)$$

$$J(t, \chi) = \int \frac{d^3 p}{(2\pi)^3} \sqrt{2E_p} \left(q_p e^{ipx} + q_p^* e^{-ipx} \right) \int_{p^2 - E_p^2}^{\infty} \frac{d^3 k}{k^2} \sqrt{k^2 - m^2} \quad (2)$$

$$J_t(t, \chi) \equiv J(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \sqrt{2E_p} \left(q_p e^{ipx} - q_p^* e^{-ipx} \right) \quad (3)$$

From here,

$$\varphi(\vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(q_p e^{ipx} + q_p^* e^{-ipx} \right) = \int \frac{d^3 p}{(2\pi)^3} \varphi(p) e^{ipx} \quad (4)$$



$$\partial_t \mathcal{P}(t, \vec{x}) \equiv J(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(E_p q_p e^{i p \cdot \vec{x}} - [q_p^+ e^{i p_x}] \right) \quad (2)$$

From here,

$$J(\vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(q_p e^{i p_x} + q_p^+ e^{-i p_x} \right) \quad (3)$$

$$J(\vec{x}) = \int \frac{d^3 p}{(2\pi)^3} (-i) \sqrt{\frac{E_p}{2}} \left(q_p e^{i p_x} - q_p^+ e^{-i p_x} \right) = \int \frac{d^3 p}{(2\pi)^3} J_I(p) e^{i p_x} \quad (4)$$

$$J_I(p) = \int \frac{d^3 p}{(2\pi)^3} \left(q_p e^{i p_x} - q_p^+ e^{-i p_x} \right) = \int \frac{d^3 p}{(2\pi)^3} J_I(p) e^{i p_x} \quad (5)$$

Here $\varphi(\vec{p}) = \frac{1}{\sqrt{2E_{\vec{p}}}}(a_{\vec{p}} + a_{\vec{p}}^{\dagger})$, $\pi(\vec{p}) = (1/\sqrt{2})(a_{\vec{p}} - a_{\vec{p}}^{\dagger})$ (6)

Note that $\varphi(\vec{p}) = \int d\vec{x} \tilde{\rho}^{(\vec{p}, \vec{x})} \varphi(\vec{x})$, $\pi(\vec{p}) = \int d\vec{x} \tilde{\rho}^{(\vec{p}, \vec{x})} \pi(\vec{x})$ (7)

Here $\varphi(\vec{p}) = \frac{1}{\sqrt{2E_p}}(a_{\vec{p}} + a_{-\vec{p}}^*)$, $J(\vec{p}) = (-i/\sqrt{2E_p})(a_{\vec{p}} - a_{-\vec{p}}^*)$ (6)

Note that $\varphi(\vec{p}) = \int d\vec{x} \hat{c}^{\dagger}(\vec{p}, \vec{x}) \varphi(\vec{x})$, $J(\vec{p}) = \int d\vec{x} \hat{c}(\vec{p}, \vec{x}) J(\vec{x})$ (7)
Calculate commutators for $\varphi(\vec{p}_1), J_1(\vec{p}_1)$ by using (1).



Here $\varphi(\vec{p}) = \frac{1}{\sqrt{2E_{\vec{p}}}}(a_{\vec{p}} + a_{\vec{p}}^*)$, $\pi(\vec{p}) = (-i/\sqrt{2E_{\vec{p}}})(a_{\vec{p}} - a_{\vec{p}}^*)$ (6)

Note that $\varphi(\vec{p}) = \int d^3x \hat{\rho}^{(\vec{p}\vec{x})} \varphi(x)$, $\pi(\vec{p}) = \int d^3x \hat{\rho}^{(\vec{p}\vec{x})} \pi(x)$ (7)

Calculate commutators for $\varphi(\vec{p})$, $\pi(\vec{p})$ by using (1):

$$[\varphi(\vec{p}), \pi(\vec{p}')] \stackrel{(7)}{=} \int d^3x \hat{\rho}^{(\vec{p}\vec{x})} \int d^3y \hat{\rho}^{(\vec{p}'\vec{y})} [\varphi(x), \pi(y)] =$$

Here $\varphi(\vec{P}) = \frac{1}{\sqrt{2E_{\vec{P}}}}(a_{\vec{P}} + a_{\vec{P}}^*)$, $\pi(\vec{p}) = (-i)\sqrt{\epsilon_{\vec{P}}}(\dot{a}_{\vec{P}} - \dot{a}_{\vec{P}}^*)$ (6)

Note that $\varphi(\vec{P}) = \int d^3x \hat{e}^{i\vec{P}\vec{x}} \varphi(x)$, $\pi(\vec{P}) = \int d^3x \hat{e}^{i\vec{P}\vec{x}} \pi(x)$ (7)

Calculate commutators for $\varphi(\vec{P})$, $\pi(\vec{P})$ by using (1):

$$[\varphi(\vec{P}), \pi(\vec{P}')] \text{ use } \int d^3x \hat{e}^{i\vec{P}\vec{x}} \int d^3y \hat{e}^{i\vec{P}'\vec{y}} [\varphi(x), \pi(y)] = \\ = i \int d^3x \hat{e}^{i\vec{P}\vec{x}} \int d^3y \hat{e}^{i\vec{P}'\vec{y}} S^3(x-y) = i \int d^3x \hat{e}^{i(\vec{P}+\vec{P}')x} =$$

Here $\varphi(\vec{p}) = \frac{1}{\sqrt{2E_{\vec{p}}}}(a_{\vec{p}} + a_{-\vec{p}}^*)$, $\pi(\vec{p}) = (-i/\sqrt{2E_{\vec{p}}})(a_{\vec{p}} - a_{-\vec{p}}^*)$ (6)

Note that $\varphi(\vec{p}) = \int d^3x \hat{\rho}^{(\vec{p}, \vec{x})} \varphi(\vec{x})$, $\pi(\vec{p}) = \int d^3x \hat{\rho}^{(\vec{p}, \vec{x})} \pi(\vec{x})$ (7)

Calculate commutators for $\varphi(\vec{p})$, $\pi(\vec{p})$ by using (1):
 $[\varphi(\vec{p}), \pi(\vec{p}')] \stackrel{\text{use}}{=} \int d^3x \hat{\rho}^{(\vec{p}, \vec{x})} \int d^3y \hat{\rho}^{(\vec{p}', \vec{y})} [\varphi(\vec{x}), \pi(\vec{y})] =$
 $= i \int d^3x \hat{\rho}^{(\vec{p}, \vec{x})} \int d^3y \hat{\rho}^{(\vec{p}', \vec{y})} S^3(\vec{x} - \vec{y}) = i \int d^3x \hat{\rho}^{((\vec{p} + \vec{p}'), \vec{x})} =$

$(2\pi)^3 S^3(\vec{p} + \vec{p}')$, similarly, one gets $[\varphi(\vec{p}), \varphi(\vec{p}')] = 0$ and

Here $\varphi(\vec{P}) = \frac{1}{\sqrt{2E_{\vec{P}}}}(a_{\vec{P}} + a_{-\vec{P}}^*)$, $\bar{\pi}(\vec{P}) = (-i/\sqrt{2E_{\vec{P}}})(a_{\vec{P}} - a_{-\vec{P}}^*)$ (6)

Note that $\varphi(\vec{P}) = \int d^3x \hat{e}^{i\vec{P}\vec{x}} \varphi(x)$, $\bar{\pi}(\vec{P}) = \int d^3x \hat{e}^{i\vec{P}\vec{x}} \bar{\pi}(x)$ (7)

Calculate commutators for $\varphi(\vec{P})$, $\bar{\pi}(\vec{P})$ by using (1):
 $[\varphi(\vec{P}), \bar{\pi}(\vec{P}')] \stackrel{\text{use}}{=} \int d^3x \hat{e}^{i\vec{P}\vec{x}} \int d^3y \hat{e}^{i\vec{P}'\vec{y}} [\varphi(x), \bar{\pi}(y)] =$
 $= i \int d^3x \hat{e}^{i\vec{P}\vec{x}} \int d^3y \hat{e}^{i\vec{P}'\vec{y}} S^3(x-y) = i \int d^3x \hat{e}^{i((\vec{P}+\vec{P}')\vec{x})} =$
 $= (2\pi)^3 S^3(\vec{P}+\vec{P}')$, similarly, one gets $[\bar{\pi}(\vec{P}), \bar{\pi}(\vec{P}')] = 0$ and
 $[\pi(\vec{P}), \bar{\pi}(\vec{P}')] = 0$. (7)

By using (6), we get $a_{\vec{P}} = \frac{\bar{\varphi}(\vec{P}) + \bar{\pi}(\vec{P})}{2}$, $a_{\vec{P}}^t = \frac{\bar{\varphi}(\vec{P}) - \bar{\pi}(\vec{P})}{2}$,
where $\bar{\varphi}(\vec{P}) \equiv \sqrt{2E_{\vec{P}}} \varphi(\vec{P})$, $\bar{\pi}(\vec{P}) \equiv \sqrt{\frac{2}{E_{\vec{P}}}} \pi(\vec{P})$. From

By using (6), we get $a_{\vec{P}} = \frac{\bar{\varphi}(\vec{P}) + \bar{\pi}(\vec{P})}{2}$, $a_{\vec{P}'} = \frac{\bar{\varphi}(\vec{P}') - \bar{\pi}(\vec{P}')}{2}$,
 where $\bar{\varphi}(\vec{P}) \equiv \sqrt{2E_{\vec{P}}} \varphi(\vec{P})$, $\bar{\pi}(\vec{P}) \equiv \sqrt{\frac{2}{E_{\vec{P}}}} \pi(\vec{P})$. From (8)
 $[\bar{\varphi}(\vec{P}), \bar{\pi}(\vec{P}')] = -2(2\pi)^3 S^3(\vec{P}, \vec{P}')$, $[\bar{\varphi}(\vec{P}), \bar{\varphi}(\vec{P}')] = [\bar{\pi}(\vec{P}), \bar{\pi}(\vec{P}')] = 0$

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 where $\bar{\varphi}(\vec{P}) \equiv \sqrt{2E_{\vec{P}}} \varphi(\vec{P})$, $\bar{\pi}(\vec{P}) \equiv \sqrt{2E_{\vec{P}}} \pi(\vec{P})$. From (8)
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 From (9), we get: $[a_{\vec{P}}, a_{\vec{P}'}^+] = [\bar{\varphi}(\vec{P}), \bar{\varphi}(\vec{P}')] = [\bar{\pi}(\vec{P}), \bar{\pi}(\vec{P}')] = 0$
 $[a_{\vec{P}}, a_{\vec{P}'}] = [a_{\vec{P}}^+, a_{\vec{P}'}] = (2\pi)^3 S^3(\vec{P} - \vec{P}')$ and (9)

By using (6), we get $a_{\vec{p}} = \frac{\bar{\varphi}(\vec{p}) + \bar{\pi}(\vec{p})}{2}$, $a_{\vec{p}}^* = \frac{\bar{\varphi}(-\vec{p}) - \bar{\pi}(-\vec{p})}{2}$,
 where $\bar{\varphi}(\vec{p}) \equiv \sqrt{2E_{\vec{p}}} \varphi(\vec{p})$, $\bar{\pi}(\vec{p}) \equiv \sqrt{\frac{2}{E_{\vec{p}}}} \pi(\vec{p})$. From (8)
 $[\bar{\varphi}(\vec{p}), \bar{\pi}(\vec{p}')] = -2(2\pi)^3 S^3(\vec{p}, \vec{p}')$, $[\bar{\varphi}(\vec{p}), \bar{\varphi}(\vec{p}')] = [\bar{\pi}(\vec{p}), \bar{\pi}(\vec{p}')] = 0$.
 From (9), we get $[a_{\vec{p}}, a_{\vec{p}'}^*] = [a_{\vec{p}}, a_{\vec{p}'}] = (2\pi)^3 S^3(\vec{p} - \vec{p}')$ and (9)
 $[a_{\vec{p}}, a_{\vec{p}'}] = [a_{\vec{p}}^*, a_{\vec{p}'}] = 0$. (10)

Express the Hamiltonian $H = \int d^3x \left[\frac{1}{2} \bar{\pi}^2 + \frac{1}{2} (\vec{\nabla} \bar{\varphi})^2 + \frac{1}{2m} \bar{p}^2 \right]$

By using (6), we get $a_{\vec{p}} = \frac{\bar{\varphi}(\vec{p}) + \bar{\pi}(\vec{p})}{2}$, $a_{\vec{p}}^* = \frac{\bar{\varphi}(\vec{p}) - \bar{\pi}(\vec{p})}{2i}$
 where $\bar{\varphi}(\vec{p}) \equiv \sqrt{2E_{\vec{p}}} \varphi(\vec{p})$, $\bar{\pi}(\vec{p}) \equiv \frac{i}{\sqrt{2E_{\vec{p}}}} \pi(\vec{p})$. From (8)
 $[\bar{\varphi}(\vec{p}), \bar{\pi}(\vec{p}')] = -2(2\pi)^3 S^3(\vec{p}, \vec{p}')$. From (8)
 From (9), we get $[a_{\vec{p}}, a_{\vec{p}}^*] = [\bar{\varphi}(\vec{p}), \bar{\varphi}(\vec{p}')] = [\bar{\pi}(\vec{p}), \bar{\pi}(\vec{p}')] = 0$
 $[a_{\vec{p}}, a_{\vec{p}'}] = [a_{\vec{p}}^*, a_{\vec{p}'}] = (2\pi)^3 S^3(\vec{p} - \vec{p}')$ and (9)
 Express the Hamiltonian $H = \int d^3x \left[\sum \bar{\pi}^2 + \frac{1}{2} (\vec{\nabla} \vec{p})^2 + \frac{1}{2m} \vec{p}^2 \right]$
 through $a_{\vec{p}}, a_{\vec{p}}^*$.

Let us consider λT^2 term: Substitute

Let us consider $\frac{1}{2} T^2$ term:

$$\frac{1}{2} \int d^3x \bar{\pi}^2(x) = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' \int d^3(\vec{p} + \vec{p}') \bar{\pi}(\vec{p}) \bar{\pi}(\vec{p}') =$$

int.
over \vec{p}

$$\frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' S^3(\vec{p} + \vec{p}') \bar{\pi}(\vec{p}) \bar{\pi}(-\vec{p}') =$$

Let us consider $\frac{1}{2} T^2$ term:

$$\frac{1}{2} \int d^3x \bar{\pi}^2(x) = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' \int d^3(\vec{p} + \vec{p}') \bar{\pi}(\vec{p}) \bar{\pi}(\vec{p}') =$$

int.

over \vec{p}

$$= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' S^3(\vec{p} + \vec{p}') \bar{\pi}(\vec{p}) \bar{\pi}(-\vec{p}') = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \bar{\pi}(\vec{p}) \bar{\pi}(\vec{p}).$$

Let us consider $\frac{1}{2} T^2$ term:

$$\frac{1}{2} \int d^3x \bar{\pi}^2(x) = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' \int d^3p'' \int d^3p''' \bar{\pi}(p) \bar{\pi}(p') =$$

int.

over \vec{p}

$$= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' S^3(\vec{p} + \vec{p}') \bar{\pi}(p) \bar{\pi}(-p') = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \bar{\pi}(p) \bar{\pi}(-p).$$

Let us consider $\frac{1}{2} T^2$ term:

$$\frac{1}{2} \int d^3x \bar{\psi}^2(x) = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' \int d^3p'' \bar{\psi}(p) \bar{\psi}(p') \bar{\psi}(p'') \psi(p) \psi(p') \psi(p'') =$$

int.

$$\frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \int d^3p' S^3(\vec{p} + \vec{p}') \bar{\psi}(p) \bar{\psi}(-p') = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \bar{\psi}(p) \bar{\psi}(-p)$$

Similar treatment yields

Let us consider $\frac{1}{2}T^2$ term:

$$\frac{1}{2} \int d^3x \pi^2(x) = \frac{1}{2} \int \frac{1}{(2\pi)^3} d^3p_x d^3p \int d^3p' \int \frac{(\vec{r}(\vec{p}+\vec{p}'))}{(2\pi)^3} J(\vec{p}) \bar{J}(\vec{p}') =$$

int.

$$\frac{1}{2} \int \frac{1}{(2\pi)^3} d^3p d^3p' S^3(\vec{p}+\vec{p}') \bar{J}(\vec{p}) \bar{J}(\vec{p}') - \frac{1}{2} \int \frac{1}{(2\pi)^3} d^3p H(\vec{p}) \bar{J}(\vec{p})$$

(1)

Similar treatment yields $\frac{1}{2} \int d^3x \varphi^2(x) = \frac{m^2}{2} \int \frac{1}{(2\pi)^3} d^3p \varphi(\vec{p}) \bar{\varphi}(\vec{p})$

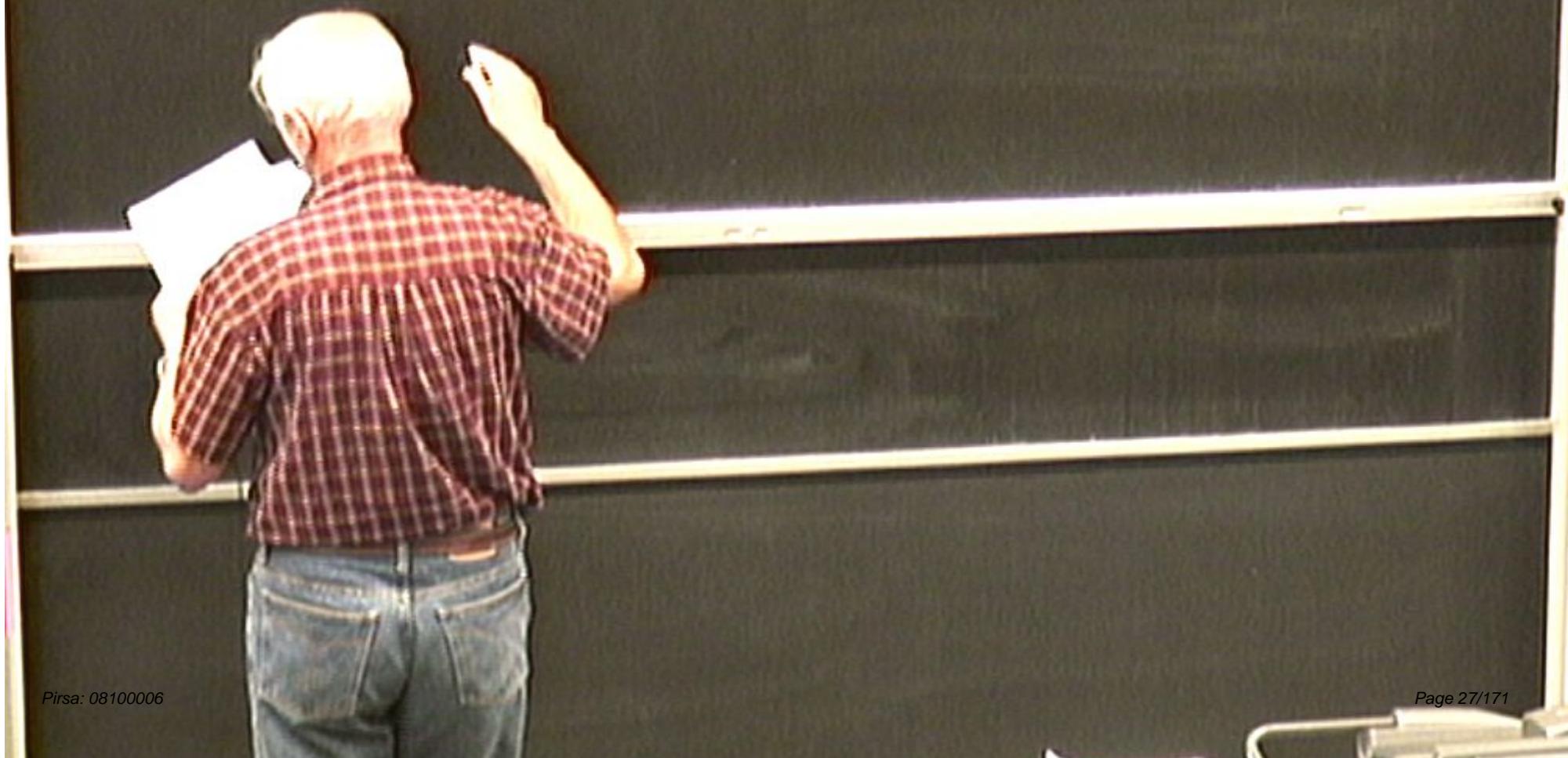
(2)

$$\text{over } \vec{P} \int \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p}') = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \bar{\psi}(\vec{p}) \bar{\psi}(\vec{p}) \quad (2)$$

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi^2(\vec{p}) = \frac{m^2}{2} \int \frac{1}{(2\pi)^3} \int d^3 p \psi(\vec{p}) \psi(-\vec{p}) \quad (3)$

The last term:

$$\frac{1}{2} \int d^3 p_x (\vec{\nabla} \varphi)^2 =$$

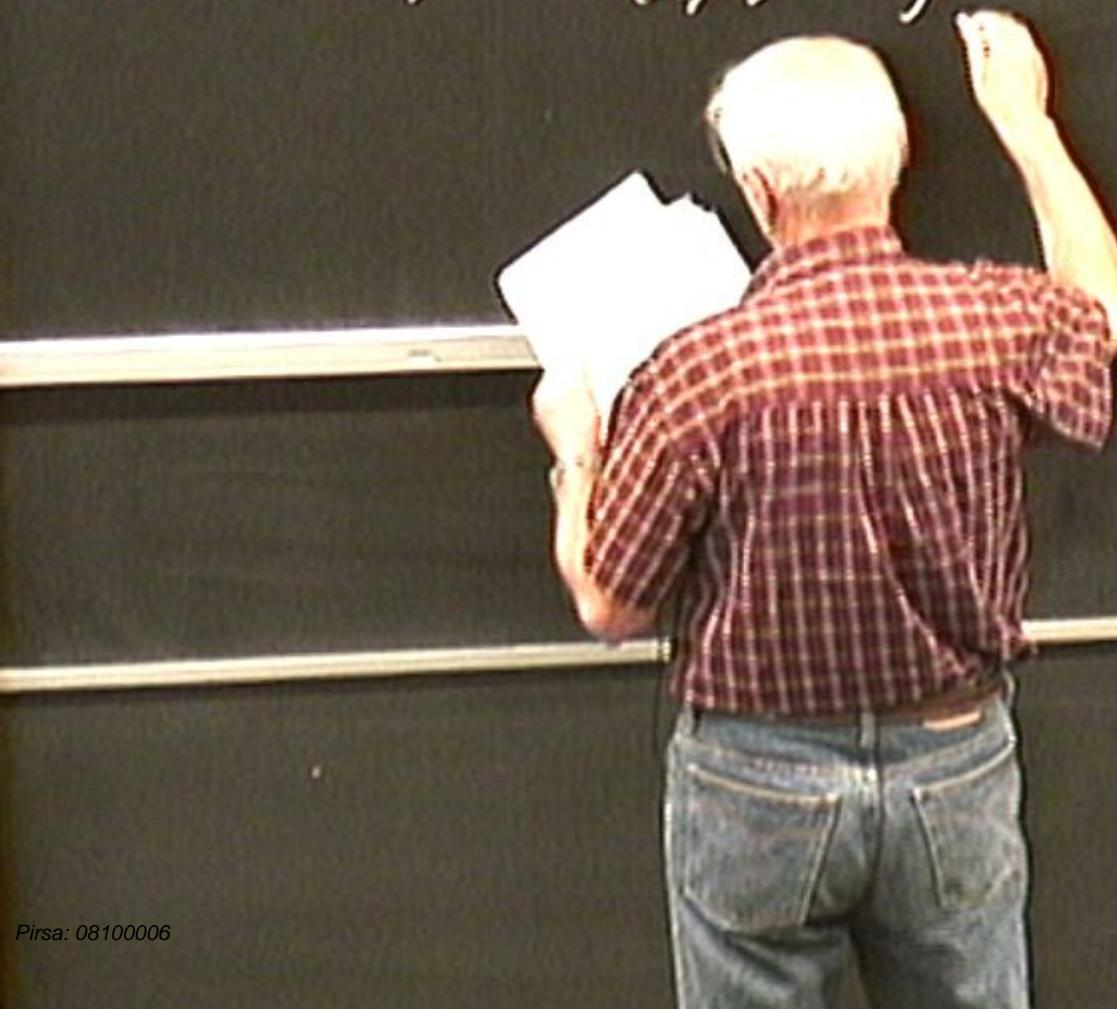


$$\overbrace{\int \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') J(\vec{p}) J(-\vec{p}')}^{\text{int. over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p J(\vec{p}) \bar{J}(\vec{p}) \quad (2)$$

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi^2(\vec{p}) = \frac{m^2}{2} \int \frac{1}{(2\pi)^3} d^3 p J(\vec{p}) \varphi(\vec{p}) \varphi(\vec{p}) \quad (3)$

The last term:

$$\frac{1}{2} \int d^3 p_x (\vec{D}\varphi)^2 = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p_x \int$$

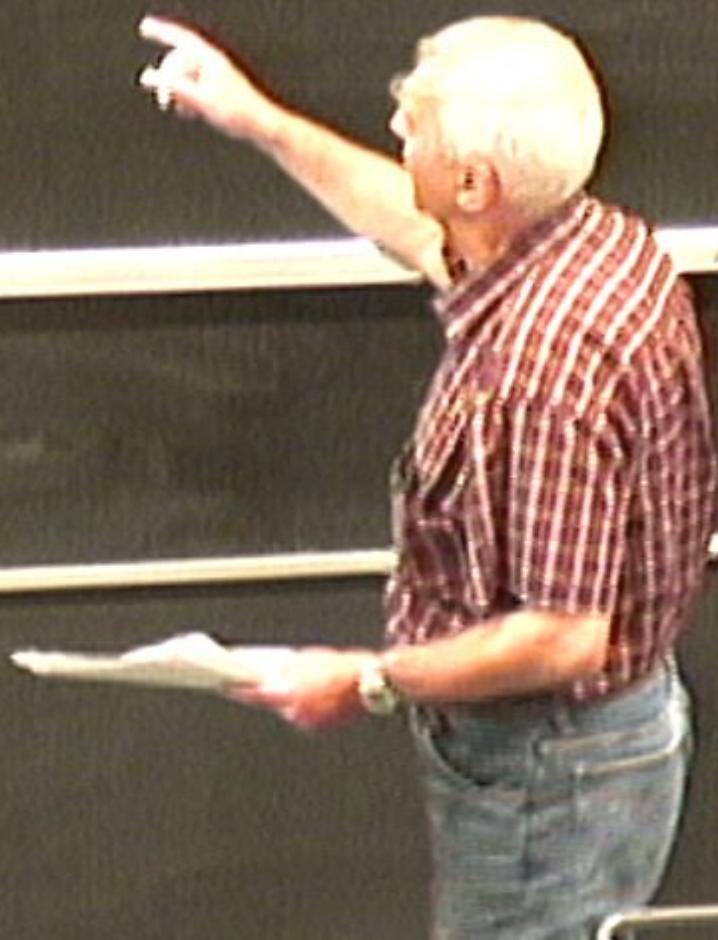


$$\overbrace{\int \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(\vec{p}') = \frac{1}{2} \int \frac{1}{(2\pi)^3} \int d^3 p \bar{\psi}(\vec{p}) \bar{\psi}(\vec{p})}^{(2)}$$

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi(\vec{p}) = \frac{m^2}{2} \int \frac{1}{(2\pi)^3} \int d^3 p \varphi(\vec{p}) \varphi(\vec{p})$

The last term:

$$\frac{1}{2} \int d^3 p_x (\vec{p} \varphi)^2 = \frac{1}{2} \int \frac{1}{(2\pi)^3} \int d^3 p_x \int d^3 p' \int d^3 p' /$$
(3)



$$\overbrace{\text{over } \vec{P}}^{\text{int.}} \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p}') = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \bar{\psi}(\vec{p}) \bar{\psi}(\vec{p})$$

(2)

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi^2(\vec{p}) = \frac{m^2}{2} \frac{1}{(2\pi)^3} \int d^3 p \psi(\vec{p}) \psi(\vec{p})$

The last term:

$$\frac{1}{2} \int d^3 p_x (\vec{\nabla} \varphi)^2 = \frac{1}{2} \frac{1}{(2\pi)^6} \int d^3 p_x \int d^3 p \int d^3 p' (\vec{p} \cdot \vec{p}')$$

(3)

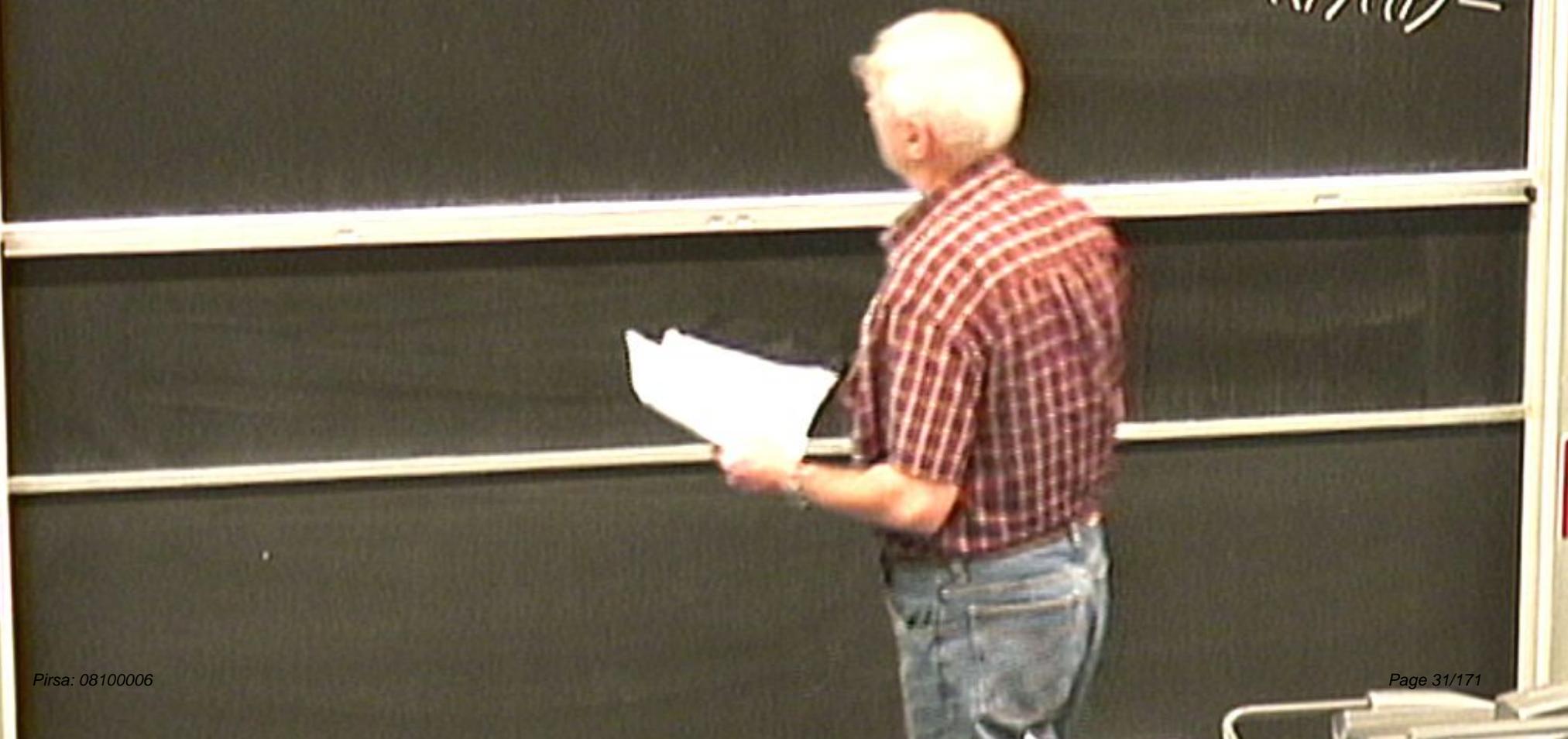
$$\overbrace{\int d^3p \int d^3p' S^3(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p}')}^{\text{over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p})$$

(2)

Similar treatment yields $\frac{m^2}{2} \int d^3x \vec{\nabla}^2 \varphi(x) = \frac{m^2}{2} \int d^3p \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p})$.

The last term:

$$\frac{1}{2} \int d^3p_x (\vec{\nabla} \varphi)^2 = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p_x \int d^3p \int d^3p' (\vec{p} \cdot \vec{p}') \rho^{(2)}(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p}') =$$



$$\overbrace{\text{over } \vec{P}}^{\text{int.}} \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') \bar{\chi}(\vec{p}) \bar{\chi}(-\vec{p}') = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \bar{\chi}(\vec{p}) \bar{\chi}(\vec{p})$$

Similar treatment yields $\frac{m^2}{2} \int d^3 x \varphi^2(x) = \frac{m^2}{2} \frac{1}{(2\pi)^3} \int d^3 p \varphi(p) \varphi(-p)$.

The last term:

$$\frac{1}{2} \int d^3 x (\vec{D}\varphi)^2 = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 x \int d^3 p \int d^3 p' ((\vec{p} \cdot \vec{p}') \rho^{(2)}(\vec{p}^2 + \vec{p}'^2) \varphi(\vec{p}) \varphi(\vec{p}')) =$$

=

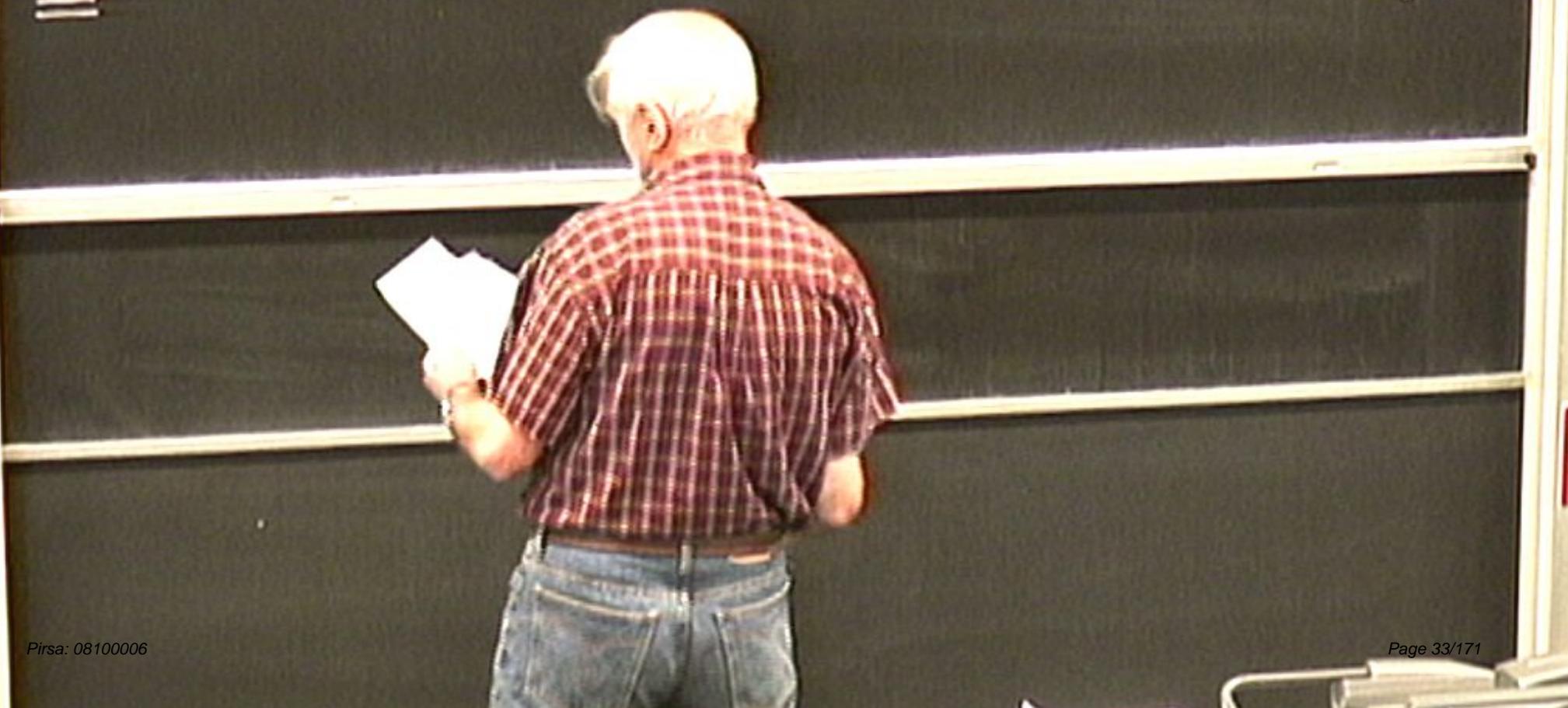
$$\overbrace{\int \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') J(\vec{p}) J(-\vec{p}')}^{int. over \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p J(\vec{p}) \bar{J}(\vec{p})$$

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi(\vec{p}) = \frac{m^2}{2} \int \frac{1}{(2\pi)^3} d^3 p J(\vec{p}) \varphi(\vec{p})$.

The last term:

$$\frac{1}{2} \int d^3 p x (\vec{p} \varphi)^2 = \frac{1}{2} \frac{1}{(2\pi)^6} \int d^3 x \int d^3 p \int d^3 p' (\vec{p} \cdot \vec{p}') \rho(x^2 \delta^3(\vec{p} + \vec{p}')) (A\vec{p}) \varphi(\vec{p}) \varphi(\vec{p}') =$$

=



$$\overbrace{\int d^3p \int d^3p' S^3(\vec{p} + \vec{p}') J(\vec{p}) J(-\vec{p}')}^{\text{over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p J(\vec{p}) \bar{J}(\vec{p})$$

Similar treatment yields $\frac{m^2}{2} \int d^3x \varphi(x)^2 = \frac{m^2}{2} \int d^3p J(\vec{p}) \bar{J}(\vec{p})$.

The last term:

$$\frac{1}{2} \int d^3x (\vec{\nabla} \varphi)^2 = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3x \int d^3p \int d^3p' ((\vec{p} \cdot \vec{p}') \rho^{(2)}(\vec{p} + \vec{p}'))$$

$$= \frac{1}{2} \frac{1}{(2\pi)^3}$$

$$\overbrace{\int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') J(\vec{p}) J(-\vec{p}')}^{\text{int. over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p J(\vec{p}) \bar{J}(\vec{p})$$

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi(\vec{p}) = \frac{m^2}{2} \frac{1}{(2\pi)^3} \int d^3 p J(\vec{p}) \varphi(\vec{p}) \varphi(-\vec{p})$.

The last term:

$$\begin{aligned} \frac{1}{2} \int d^3 p \partial_x (\vec{p} \varphi)^2 &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \partial_x \int d^3 p' \int d^3 p'' (\vec{p} \cdot \vec{p}') \partial_x^2 (\vec{p} + \vec{p}'') \\ &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \vec{p}^2 \varphi(\vec{p}) \varphi(-\vec{p}) \end{aligned}$$



$$\overbrace{\int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') J(\vec{p}) J(-\vec{p}')}^{\text{over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p J(\vec{p}) \bar{J}(\vec{p})$$

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi(\vec{p}) = \frac{m^2}{2} \frac{1}{(2\pi)^3} \int d^3 p J(\vec{p}) \varphi(\vec{p}) \bar{\varphi}(\vec{p})$.

The last term:

$$\begin{aligned} \frac{1}{2} \int d^3 p \partial_\mu (\vec{p} \varphi)^2 &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \partial_\mu \int d^3 p' \left((\vec{p} \cdot \vec{p}') \partial_\mu^2 (\vec{p}^2 + \vec{p}'^2) \right) \\ &= -\frac{1}{(2\pi)^3} \int d^3 p \vec{p}^2 \varphi(\vec{p}) \varphi(-\vec{p}) \end{aligned}$$



$$\overbrace{\int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p}')}^{\text{int. over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p})$$

(2)

Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi(\vec{p}) = \frac{m^2}{2} \frac{1}{(2\pi)^3} \int d^3 p \varphi(\vec{p}) \varphi(-\vec{p})$.

The last term:

$$\begin{aligned} \frac{1}{2} \int d^3 p \partial_x (\vec{p} \varphi)^2 &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 x \int d^3 p \int d^3 p' (\vec{p} \cdot \vec{p}') \partial_x^2 (\vec{p}^2 + \vec{p}'^2) \\ &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \vec{p}^2 \delta^{(3)}(\vec{p}) \varphi(-\vec{p}) \end{aligned}$$

$$\overbrace{\int d^3 p \int d^3 p' S^3(\vec{p} + \vec{p}') \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p}')}^{\text{int. over } \vec{p}} = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \bar{\psi}(\vec{p}) \bar{\psi}(-\vec{p})$$

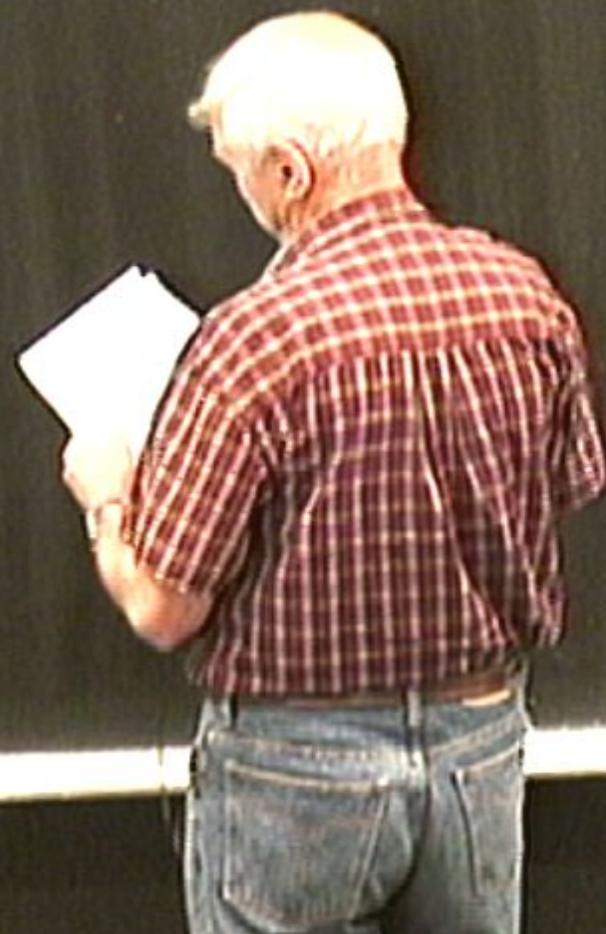
Similar treatment yields $\frac{m^2}{2} \int d^3 p \varphi(\vec{p}) = \frac{m^2}{2} \frac{1}{(2\pi)^3} \int d^3 p \varphi(\vec{p}) \varphi(-\vec{p})$.

The last term:

$$\begin{aligned} \frac{1}{2} \int d^3 p \partial_x (\vec{p} \varphi)^2 &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \partial_x \int d^3 p' \int d^3 p'' (\vec{p} \cdot \vec{p}') \partial_x (\vec{p} \cdot \vec{p}'') \varphi(\vec{p}) \varphi(\vec{p}'') \\ &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \vec{p}'^2 \varphi(\vec{p}) \varphi(-\vec{p}) \quad (14) \end{aligned}$$

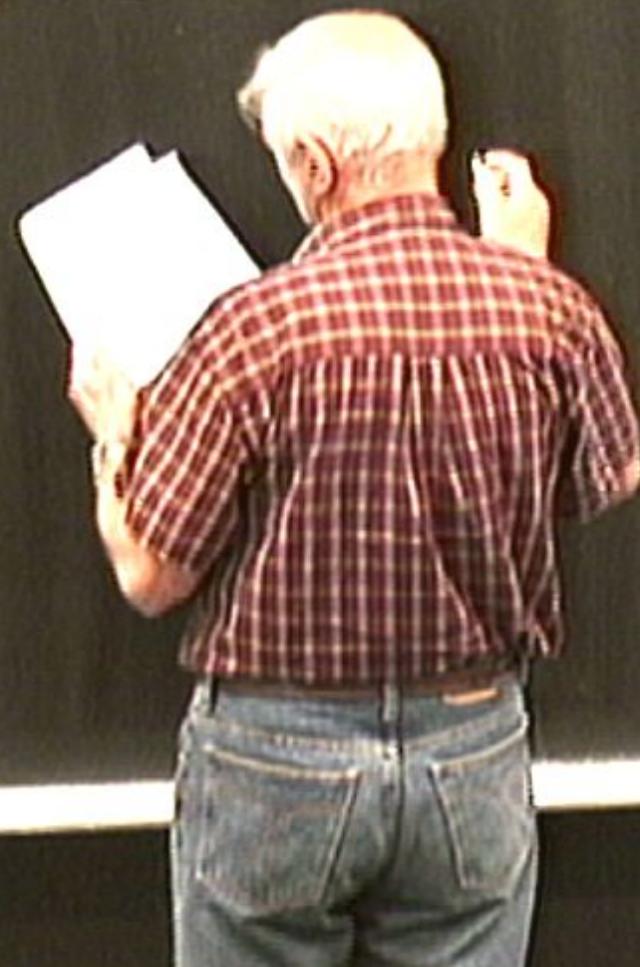


Adding these terms, we get:



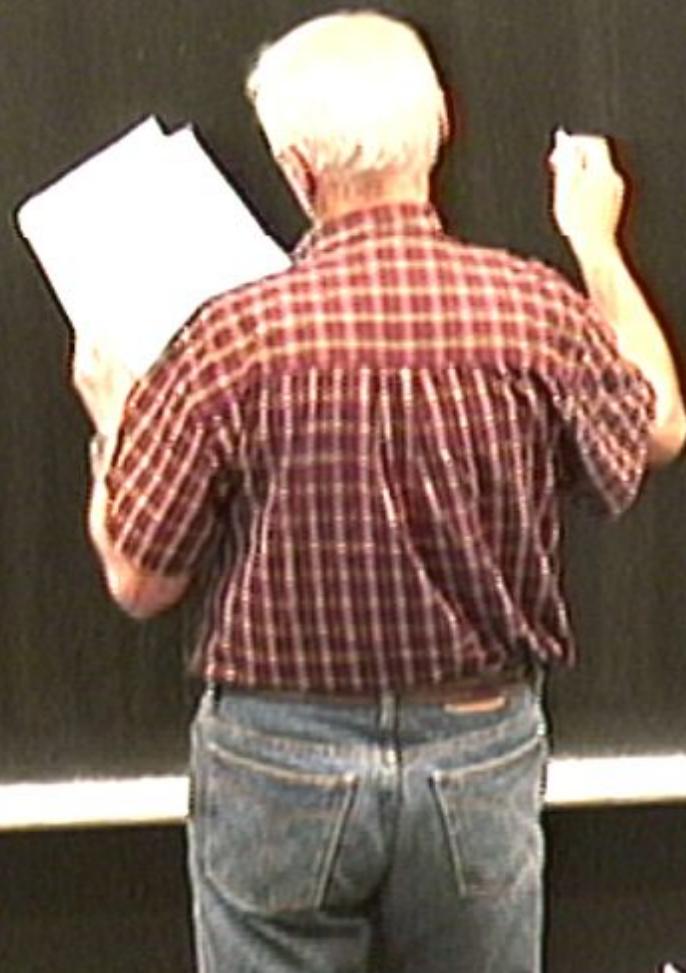
Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) \right]$$



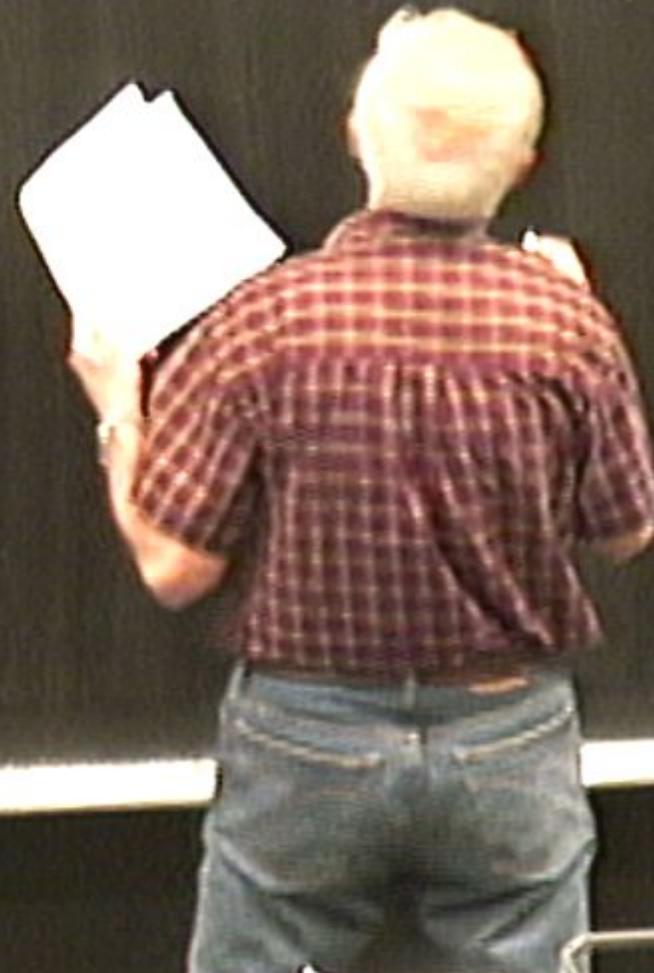
Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \right]$$



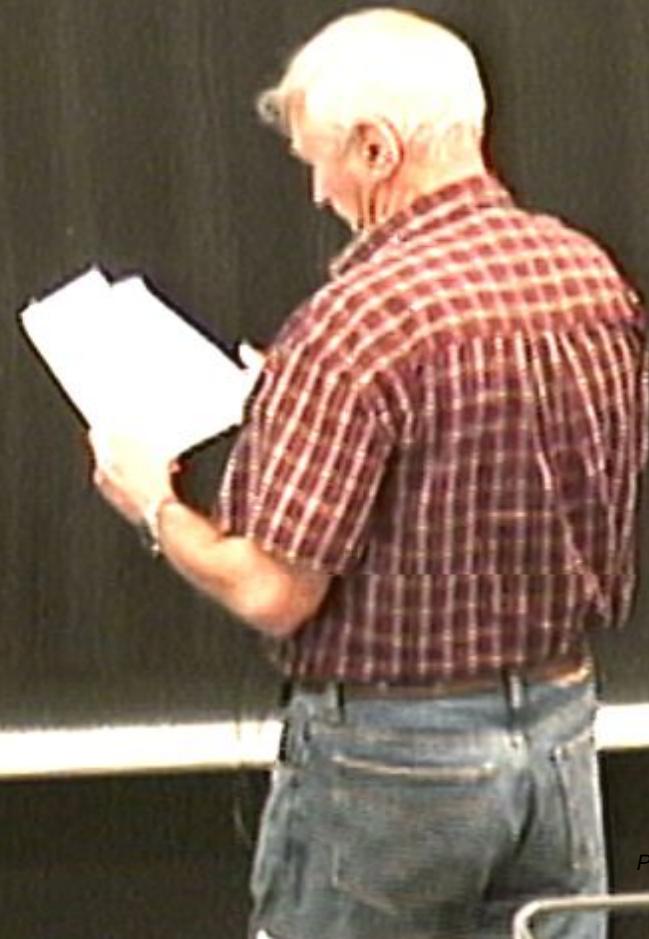
Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (p^2 + m^2) \varphi(p) \varphi(-p) \right]$$



Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (p^2 + m^2) \varphi(p) \varphi(-p) \right] \quad (15)$$



Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (p^2 + m^2) \varphi(p) \varphi(-p) \right] \quad (15)$$

By using Eq. (6), we get:

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By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(p) \varphi(-p) \right] \quad (15)$$

By using Eq. (6), we get

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[a_p a_{-p}^\dagger - a_{-p} a_p^\dagger \right]$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[a_{\vec{p}} a_{\vec{p}}^\dagger - a_{\vec{p}} a_{-\vec{p}} - a_{-\vec{p}}^\dagger a_{\vec{p}} + a_{-\vec{p}}^\dagger a_{-\vec{p}} \right]$$

Adding these terms, we get:

$$H = \frac{1}{2m\beta} Sd^3P \left[\frac{1}{2} \pi(\vec{p}) \pi(-\vec{p}) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{m\beta} Sd^3P \sum_{\vec{p}} [a_{\vec{p}} a_{-\vec{p}} - a_{-\vec{p}}^+ a_{\vec{p}}^+ + a_{\vec{p}}^+ a_{-\vec{p}} + a_{\vec{p}} a_{-\vec{p}}^+]$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(p) \varphi(-p) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[a_p a_{-p}^\dagger - a_{-p} a_{-p}^\dagger - a_p^\dagger a_{-p}^\dagger + a_{-p}^\dagger a_{-p} + a_p a_p^\dagger + a_p^\dagger a_p \right]$$

Adding these terms, we get:

$$H = \frac{1}{2\pi\beta} S d P P \left[\frac{1}{2} \pi(\beta) \pi(-P) + \frac{1}{2} (P^2 + m^2) \varphi(P) \varphi(-P) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{\pi\beta} S d P P \left[\underbrace{\alpha_P \alpha_P^\dagger - \alpha_{-P}^\dagger \alpha_{-P}}_{\text{annihilation}} - \alpha_{-P}^\dagger \alpha_P^\dagger + \alpha_P^\dagger \alpha_{-P} + \alpha_P \alpha_{-P}^\dagger + \alpha_{-P} \alpha_P + \alpha_{-P}^\dagger \alpha_{-P}^\dagger \right]$$

Adding these terms, we get:

$$H = \frac{1}{2m\beta} \nabla \beta P \left[\frac{1}{2} \nabla(\beta) \nabla(-P) + \frac{1}{2} (\vec{P}^2 + m^2) \varphi(P) \psi(-P) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{m\beta} \nabla \beta P \left[\underbrace{\alpha_{\vec{P}} \alpha_{\vec{P}}^*}_{\alpha_{\vec{P}} \alpha_{-\vec{P}}} - \alpha_{\vec{P}} \alpha_{-\vec{P}} - \alpha_{-\vec{P}}^* \alpha_{\vec{P}} + \alpha_{\vec{P}}^* \alpha_{-\vec{P}} + \underbrace{\alpha_{\vec{P}} \alpha_{-\vec{P}}^*}_{\alpha_{\vec{P}} \alpha_{\vec{P}}^*} + \alpha_{-\vec{P}}^* \alpha_{\vec{P}} \right]$$

Adding these terms, we get:

$$H = \frac{1}{2\pi\beta} \int dP \left[\frac{1}{2} \pi(P) \pi(-P) + \frac{1}{2} (\bar{P}^2 + m^2) \varphi(P) \varphi(-P) \right] \quad (15)$$

By using Eq. (6), we get:

$$\begin{aligned} H = & \frac{1}{2} \frac{1}{2\pi\beta} \int dP \left[\underbrace{\alpha_{\vec{P}}}_{\alpha_{\vec{P}}} \alpha_{-\vec{P}} - \underbrace{\alpha_{\vec{P}}^*}_{\alpha_{\vec{P}}^*} \alpha_{-\vec{P}}^* + \alpha_{\vec{P}}^* \alpha_{-\vec{P}} + \alpha_{\vec{P}} \alpha_{-\vec{P}}^* \right] \\ & + \alpha_{\vec{P}} \alpha_{-\vec{P}} + \alpha_{-\vec{P}}^* \alpha_{\vec{P}}^* \end{aligned}$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$\begin{aligned} &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\underbrace{\alpha_p^\dagger \alpha_p^\dagger}_{\alpha_{-p}\alpha_p} - \underbrace{\alpha_p^\dagger \alpha_{-p}}_{\alpha_{-p}\alpha_p^\dagger} - \alpha_{-p}^\dagger \alpha_{-p}^\dagger + \alpha_{-p}^\dagger \alpha_{-p} + \underbrace{\alpha_p^\dagger \alpha_{-p}^\dagger}_{\alpha_{-p}\alpha_{-p}^\dagger} + \right. \\ &\quad \left. \alpha_{-p}^\dagger \alpha_p^\dagger + \alpha_{-p}^\dagger \alpha_{-p}^\dagger + \alpha_{-p}^\dagger \alpha_{-p} \right] \end{aligned}$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \rho(\vec{p}) \delta(-p) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\cancel{a_p^\dagger} - \cancel{a_p^\dagger} \cancel{a_{-p}} - \cancel{a_{-p}^\dagger} \cancel{a_p^\dagger} + \cancel{a_p^\dagger} \cancel{a_{-p}} + \cancel{a_p^\dagger} \cancel{a_p^\dagger} + \right. \\ \left. + \cancel{a_p^\dagger} \cancel{a_{-p}} + \cancel{a_{-p}} \right]$$

Adding these terms, we get:

$$H = \frac{1}{2m\beta} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(p) \varphi(-p) \right] \quad (15)$$

By using Eq. (6), we get

$$H = \frac{1}{2} \frac{1}{m\beta} \int d^3p \left[\underbrace{\alpha_p^\dagger \alpha_{-\vec{p}}^\dagger}_{\text{creation}} - \underbrace{\alpha_{\vec{p}}^\dagger \alpha_{-\vec{p}}}_{\text{annihilation}} + \alpha_{\vec{p}}^\dagger \alpha_{-\vec{p}} + \alpha_{\vec{p}}^\dagger \alpha_{-\vec{p}}^\dagger \right]$$
$$+ \underbrace{\alpha_{\vec{p}}^\dagger \alpha_{-\vec{p}}}_{\text{creation}} + \cancel{\alpha_{-\vec{p}}^\dagger \alpha_{\vec{p}}} + \alpha_{\vec{p}}^\dagger \alpha_{-\vec{p}}$$

Adding these terms, we get:

$$H = \frac{1}{2\pi\beta} S d P P \left[\frac{1}{2} \pi(\beta) \pi(-P) + \frac{1}{2} (\bar{P}^2 + m^2) \varphi(P) \varphi(-P) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{\pi\beta} S d P P \left[\underbrace{\alpha_P \alpha_{-P}^*}_{\text{cancel}} - \alpha_{-P} \alpha_P^* + \cancel{\alpha_P^* \alpha_{-P}} + \cancel{\alpha_P^* \alpha_P} + \cancel{\alpha_P \alpha_P^*} + \cancel{\alpha_P \alpha_{-P}^*} + \cancel{\alpha_{-P} \alpha_P^*} + \cancel{\alpha_{-P} \alpha_{-P}} \right]$$

Adding these terms, we get:

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(p) \pi(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(p) \varphi(-p) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\underbrace{\alpha_p \alpha_{-p}^*}_{\text{crossed}} - \underbrace{\alpha_{-p} \alpha_p^*}_{\text{crossed}} + \underbrace{\alpha_p^* \alpha_{-p}}_{\text{crossed}} + \underbrace{\alpha_p \alpha_{-p}^*}_{\text{crossed}} + \right. \\ \left. + \underbrace{\alpha_p \alpha_{-p}}_{\text{crossed}} + \underbrace{\alpha_{-p} \alpha_p^*}_{\text{crossed}} + \alpha_p^* \alpha_{-p} \right]$$

Adding these terms, we get:

$$H = \frac{1}{2\pi\beta^3} \int dP \left[\frac{1}{2} \pi(\vec{P}) \pi(-\vec{P}) + \frac{1}{2} (\vec{P}^2 + m^2) \varphi(\vec{P}) \varphi(-\vec{P}) \right] \quad (15)$$

By using Eq. (6), we get:

$$\begin{aligned} H = & z \int dP \left[\underbrace{\alpha_{\vec{P}} \alpha_{\vec{P}}^\dagger}_{\cancel{\alpha_{\vec{P}} \alpha_{-\vec{P}}}} - \cancel{\alpha_{\vec{P}} \alpha_{-\vec{P}}} - \cancel{\alpha_{\vec{P}}^\dagger \alpha_{\vec{P}}} + \cancel{\alpha_{\vec{P}}^\dagger \alpha_{-\vec{P}}} + \cancel{\alpha_{-\vec{P}} \alpha_{\vec{P}}} + \cancel{\alpha_{\vec{P}} \alpha_{\vec{P}}^\dagger} \right. \\ & \left. + \alpha_{\vec{P}}^\dagger \alpha_{-\vec{P}} + \cancel{\alpha_{\vec{P}}^\dagger \alpha_{\vec{P}}} + \cancel{\alpha_{\vec{P}} \alpha_{-\vec{P}}} \right] \end{aligned}$$

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(\vec{p}) \pi(-\vec{p}) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\underbrace{\alpha_{\vec{p}} \alpha_{\vec{p}}^*}_{\cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}}} - \cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}} - \cancel{\alpha_{\vec{p}}^* \alpha_{\vec{p}}^*} + \cancel{\alpha_{\vec{p}}^* \alpha_{-\vec{p}}} + \cancel{\alpha_{-\vec{p}} \alpha_{\vec{p}}^*} + \cancel{\alpha_{-\vec{p}} \alpha_{-\vec{p}}^*} \right]$$

$$+ \cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}} + \cancel{\alpha_{\vec{p}}^* \alpha_{\vec{p}}^*} + \cancel{\alpha_{-\vec{p}}^* \alpha_{-\vec{p}}}$$



$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \bar{\psi}(p) \bar{\psi}(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \bar{\psi}(p) \psi(p) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\underbrace{\bar{a}_p \bar{a}_p^\dagger - a_p^\dagger a_{-p}}_{\text{creation and annihilation operators}} - \cancel{\bar{a}_p^\dagger \bar{a}_{-p}^\dagger} + \cancel{\bar{a}_p^\dagger a_{-p}} + \cancel{\bar{a}_p a_{-p}^\dagger} + a_p^\dagger a_p \right]$$
$$+ \cancel{\bar{a}_p^\dagger a_{-p}} + \cancel{\bar{a}_{-p}^\dagger a_p} + \cancel{\bar{a}_p^\dagger a_p}$$

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(\vec{p}) \pi(-\vec{p}) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\cancel{a_{\vec{p}} a_{\vec{p}}^*} - \cancel{a_{\vec{p}} \cancel{a_{-\vec{p}}}} - \cancel{\cancel{a_{\vec{p}}^* a_{\vec{p}}}} + \cancel{a_{\vec{p}}^* a_{-\vec{p}}} + a_{\vec{p}} a_{\vec{p}}^* \right. \\ \left. + \cancel{a_{\vec{p}} \cancel{a_{-\vec{p}}}} + \cancel{a_{-\vec{p}} a_{\vec{p}}^*} + \cancel{a_{-\vec{p}}^* a_{-\vec{p}}} \right] =$$



$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \bar{\psi}(p) \bar{\psi}(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \bar{\psi}(p) \psi(p) \right] \quad (15)$$

By using Eq. (6), we get:

$$H = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\underbrace{\bar{a}_p \bar{a}_{-p}^* - \cancel{a_p^* \bar{a}_{-p}} - \cancel{\bar{a}_p^* \bar{a}_p^*} + \cancel{\bar{a}_{-p}^* \bar{a}_{-p}^*} + a_p^* a_{-p}^*}_{+ \cancel{a_p^* \bar{a}_{-p}} + \cancel{\bar{a}_{-p}^* a_p^*} + \cancel{\bar{a}_p^* a_{-p}}} \right] =$$

else

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(\vec{p}) \pi(-\vec{p}) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$\begin{aligned} H &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\cancel{a_{\vec{p}} a_{\vec{p}}^*} - \cancel{a_{\vec{p}} a_{-\vec{p}}} - \cancel{a_{\vec{p}}^* a_{-\vec{p}}} + \cancel{a_{-\vec{p}}^* a_{-\vec{p}}} + a_{\vec{p}} a_{\vec{p}}^* \right. \\ &\quad \left. + \cancel{a_{\vec{p}} a_{-\vec{p}}} + \cancel{a_{-\vec{p}} a_{\vec{p}}^*} + \cancel{a_{-\vec{p}}^* a_{-\vec{p}}} \right] = \\ &\quad \text{use } a_{\vec{p}} a_{\vec{p}}^* = a_{\vec{p}}^* a_{\vec{p}} + \end{aligned}$$

$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \bar{\psi}(p) \bar{\psi}(-p) + \frac{1}{2} (\vec{p}^2 + m^2) \bar{\psi}(p) \psi(p) \right] \quad (15)$$

By using Eq. (6), we get:

$$\begin{aligned} H &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3p \left[\underbrace{\bar{a}_p \bar{a}_p^\dagger}_{\alpha_p} - \underbrace{\bar{a}_p^\dagger a_{-p}}_{\alpha_{-p}} - \underbrace{\bar{a}_p^\dagger a_p^\dagger}_{\alpha_{-p}^\dagger} + \underbrace{\bar{a}_p^\dagger a_{-p}}_{\alpha_{-p}} + \underbrace{a_p a_p^\dagger}_{\alpha_p} \right. \\ &\quad \left. + \underbrace{\bar{a}_p^\dagger a_{-p}}_{\alpha_{-p}} + \underbrace{\bar{a}_{-p}^\dagger a_p}_{\alpha_{-p}^\dagger} + \underbrace{a_{-p}^\dagger a_{-p}}_{\alpha_{-p}^\dagger} \right] = \\ &\text{use } \bar{a}_p \bar{a}_p^\dagger = \bar{a}_p^\dagger a_p + 2(\pi)^3 \delta^3(0) \end{aligned}$$

By using eq. (6)) we get

$$H = \frac{1}{2} \frac{1}{\alpha_B} \frac{1}{\beta_B} S d^3 p_F [\cancel{a_p a_{-p}} - \cancel{a_p a_{-p}} - \cancel{a_p^* a_{-p}^*} + \cancel{a_p^* a_{-p}^*} + \cancel{a_p^* a_{-p}^*} + \cancel{a_p^* a_{-p}^*}]$$
$$+ \cancel{a_p a_{-p}} + \cancel{a_p^* a_{-p}^*} + \cancel{a_p^* a_{-p}^*} =$$

use $a_p a_{-p}^* = a_p^* a_p + 2(1/3)S^3(0)$

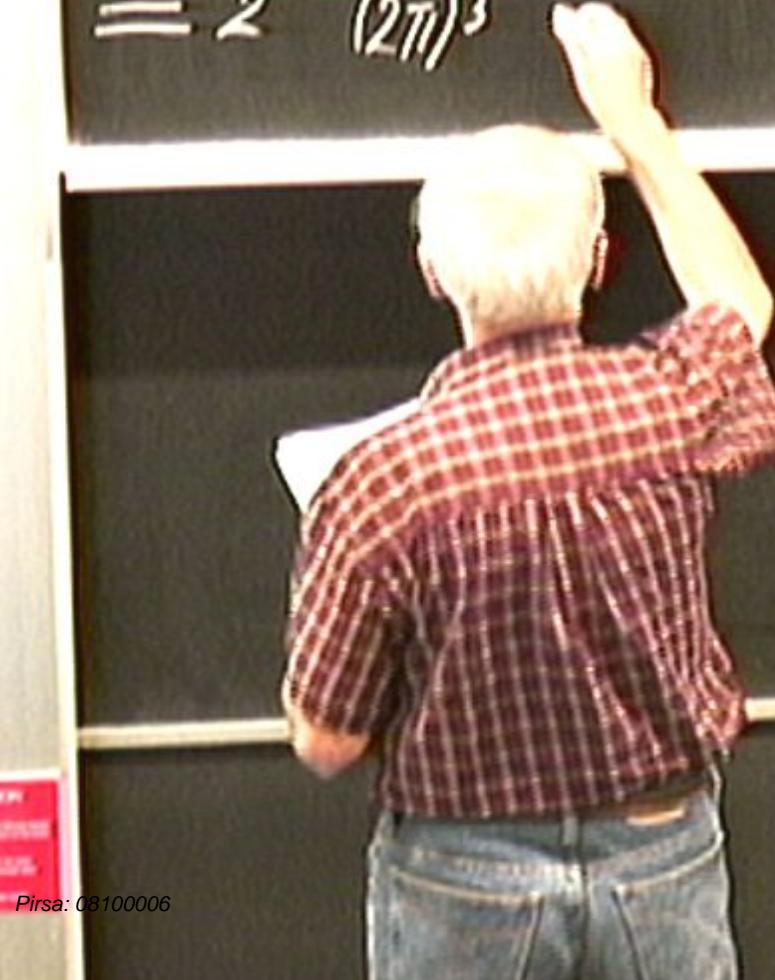
$$+ \underbrace{\alpha_p \alpha_{-\vec{p}}}_{\text{cancel}} + \underbrace{\alpha_{-\vec{p}}^* \alpha_p}_{\text{cancel}} + \boxed{\alpha_{-\vec{p}}^* \alpha_{-\vec{p}}} =$$

use $\alpha_p \alpha_{\vec{p}}^* = \alpha_p^* \alpha_p + 2(1/\beta) S^3(0)$

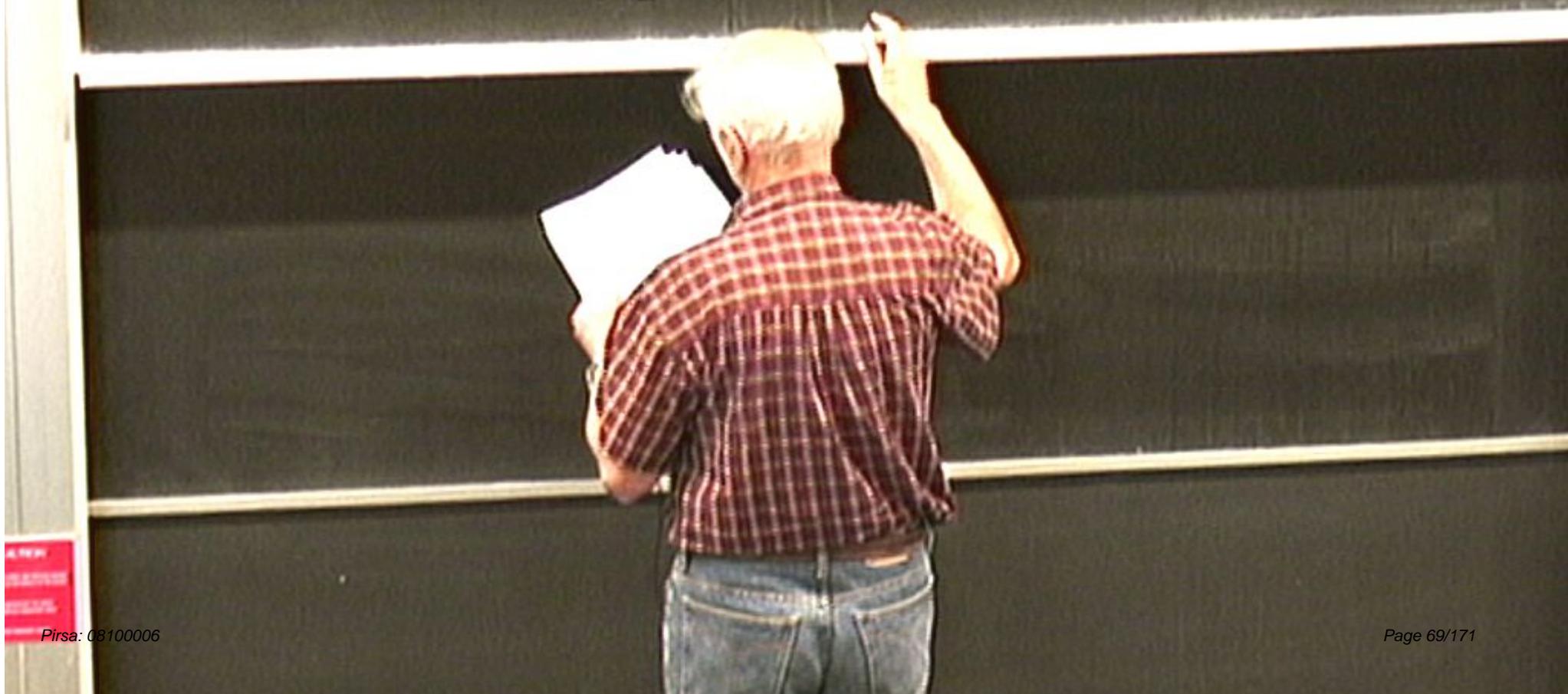
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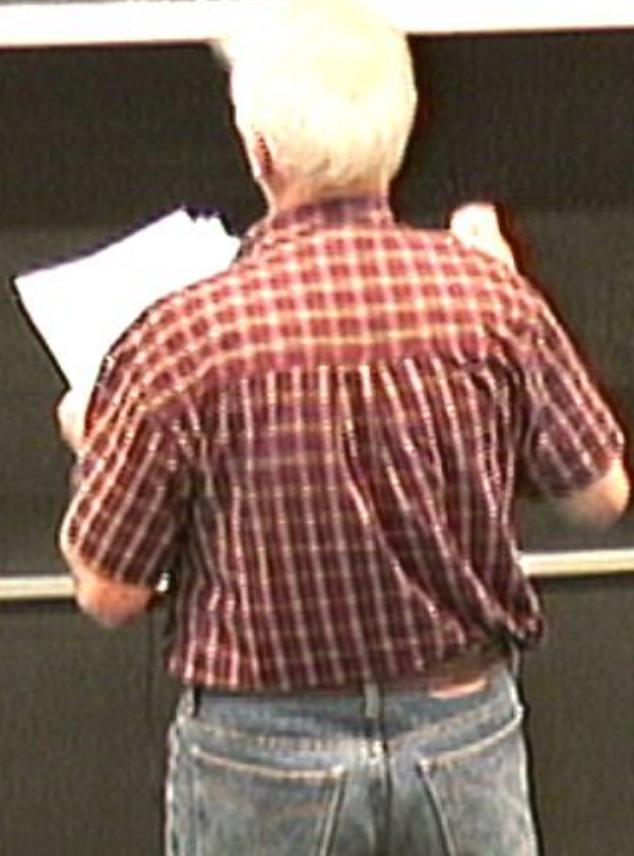
$$\begin{aligned}
 H &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' [\underbrace{a_p a_{p'}^* - a_{p'} a_{-p}^*}_{\cancel{a_p a_{-p}^*}} - \cancel{a_p^* a_{-p}} + \cancel{a_{-p}^* a_{-p}} + \cancel{a_p a_{p'}^*}] \\
 &+ \underbrace{a_p a_{-p}^*}_{\cancel{a_p a_{-p}^*}} + \underbrace{a_{-p} a_p^*}_{\cancel{a_{-p} a_p^*}} + \underbrace{a_{-p}^* a_{-p}}_{\cancel{a_{-p}^* a_{-p}}} = \\
 &\text{use } a_p a_{p'}^* = a_p^* a_p + 2(\pi)^3 \delta^3(0) \\
 &= \frac{1}{2} \frac{1}{(2\pi)^3}
 \end{aligned}$$



$$\begin{aligned}
 H = & \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' \left[\underbrace{a_p a_{-p}}_{\cancel{a_p a_{-p}}} - \underbrace{a_p^* a_{-p}}_{\cancel{a_p^* a_{-p}}} - \underbrace{\cancel{a_p^*} \cancel{a_{-p}}}_{\cancel{a_p^* a_{-p}}} + \underbrace{a_{-p}^* a_{-p}}_{\cancel{a_{-p}^* a_{-p}}} + \underbrace{a_p a_p^*}_{\cancel{a_p a_p^*}} \right] \\
 & + \underbrace{a_p a_{-p}}_{\cancel{a_p a_{-p}}} + \underbrace{a_{-p}^* a_p}_{\cancel{a_{-p}^* a_p}} + \underbrace{a_{-p}^* a_{-p}}_{\cancel{a_{-p}^* a_{-p}}} = \\
 & \text{use } a_p a_{-p}^* = a_p^* a_{-p} + 2(\pi)^3 \delta^3(0) \\
 = & \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' [a_p^* a_{-p}]
 \end{aligned}$$



$$\begin{aligned}
 H = & \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \left[\underbrace{a_{\vec{p}} a_{\vec{p}}^\dagger}_{\cancel{a_{\vec{p}} a_{\vec{p}}^\dagger}} - \cancel{a_{\vec{p}} a_{-\vec{p}}} - \cancel{a_{\vec{p}}^\dagger a_{\vec{p}}} + \cancel{a_{\vec{p}}^\dagger a_{-\vec{p}}} + a_{\vec{p}} a_{\vec{p}}^\dagger \right. \\
 & \left. + \cancel{a_{\vec{p}} a_{-\vec{p}}} + \cancel{a_{-\vec{p}} a_{\vec{p}}^\dagger} + \cancel{a_{-\vec{p}}^\dagger a_{-\vec{p}}} \right] = \\
 & \text{use } a_{\vec{p}} a_{\vec{p}}^\dagger = a_{\vec{p}}^\dagger a_{\vec{p}} + 2(\pi)^3 \delta^3(0) \\
 = & \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p E_{\vec{p}} [a_{\vec{p}}^\dagger a_{\vec{p}} + a_{\vec{p}} a_{\vec{p}}^\dagger]
 \end{aligned}$$

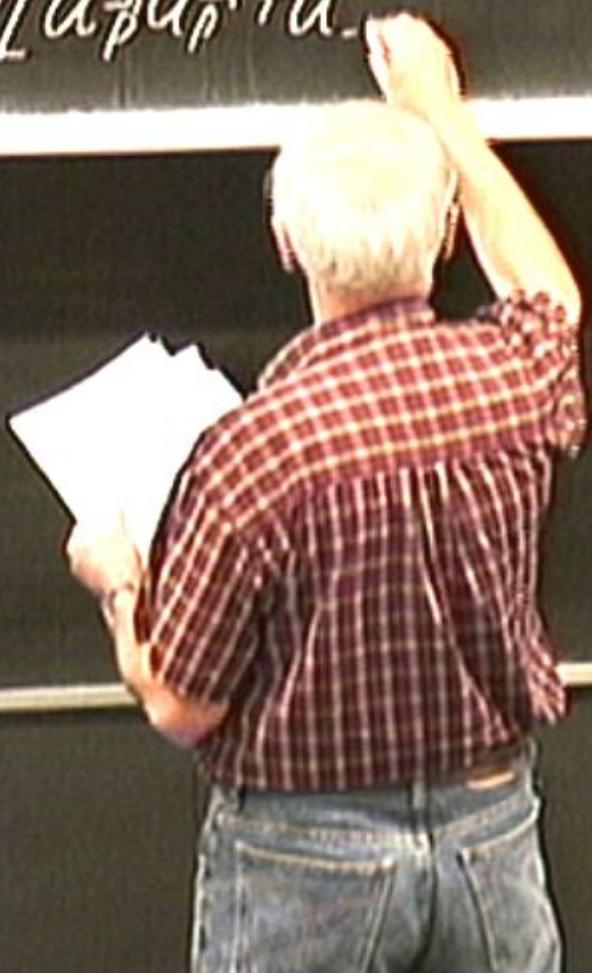


$$H = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{2} \pi(\vec{p}) \pi(-\vec{p}) + \frac{1}{2} (\vec{p}^2 + m^2) \varphi(\vec{p}) \varphi(-\vec{p}) \right] \quad (15)$$

By using Eq. (6), we get:

$$\begin{aligned} H &= \frac{1}{4} \frac{1}{(2\pi)^3} \int d^3p E_{\vec{p}} \left[\cancel{a_{\vec{p}} a_{\vec{p}}^*} - \cancel{a_{\vec{p}} a_{-\vec{p}}} - \cancel{\cancel{a_{\vec{p}} a_{\vec{p}}^*}} + \cancel{a_{\vec{p}}^* a_{-\vec{p}}} + a_{\vec{p}} a_{\vec{p}}^* \right. \\ &\quad \left. + \cancel{a_{\vec{p}} a_{-\vec{p}}} + \cancel{a_{-\vec{p}} a_{\vec{p}}^*} + \cancel{a_{-\vec{p}} a_{-\vec{p}}} \right] = \\ &\text{use } \cancel{a_{\vec{p}} a_{\vec{p}}^*} = \cancel{a_{\vec{p}} a_{-\vec{p}}} = \cancel{a_{-\vec{p}} a_{\vec{p}}^*} = \cancel{a_{-\vec{p}} a_{-\vec{p}}} = a_{\vec{p}}^* a_{\vec{p}} + 2(1/3) \delta^3(0) \\ &= \pm \frac{1}{2} \int d^3p E_{\vec{p}} [a_{\vec{p}}^* a_{\vec{p}} + a_{-\vec{p}}^* a_{-\vec{p}}] \end{aligned}$$

$$\begin{aligned}
 H &= \frac{e}{(2\pi)^3} \int d^3 p \left[\underbrace{\alpha_{\vec{p}} \alpha_{\vec{p}}^*}_{\cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}}} - \underbrace{\alpha_{\vec{p}} \alpha_{-\vec{p}}^*}_{\cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}}} - \cancel{\alpha_{\vec{p}} \alpha_{\vec{p}}^*} + \cancel{\alpha_{\vec{p}} \alpha_{\vec{p}}^*} + \alpha_{\vec{p}}^* \alpha_{\vec{p}} \right] \\
 &+ \cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}} + \cancel{\alpha_{-\vec{p}} \alpha_{\vec{p}}} + \cancel{\alpha_{-\vec{p}} \alpha_{-\vec{p}}} = \\
 &\text{use } \alpha_{\vec{p}} \alpha_{\vec{p}}^* = \alpha_{\vec{p}}^* \alpha_{\vec{p}} + 2(\pi)^3 \delta^3(0) \\
 &= \frac{1}{(2\pi)^3} \int d^3 p E_{\vec{p}} [\alpha_{\vec{p}}^* \alpha_{\vec{p}} + \alpha_{\vec{p}}^*]
 \end{aligned}$$

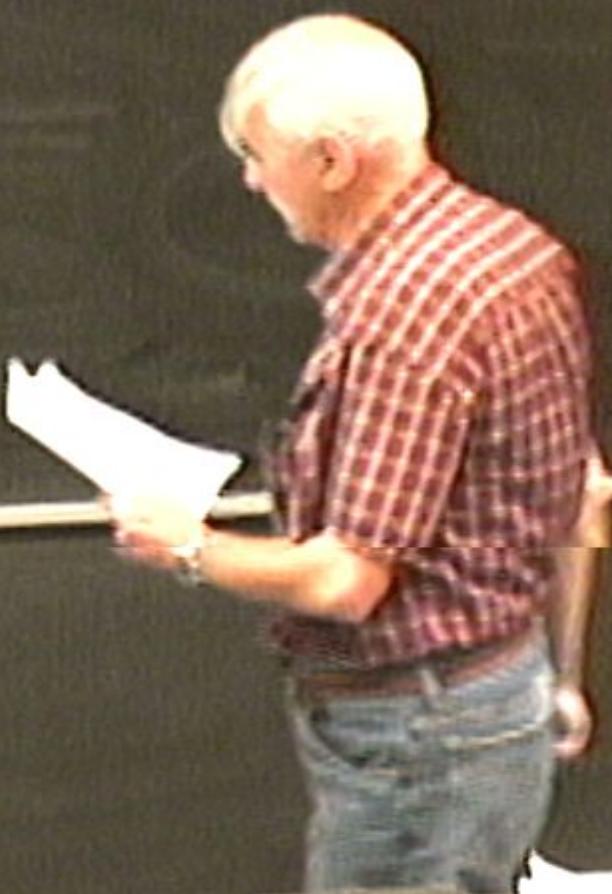


$$\begin{aligned}
 H &= \frac{1}{4} (2\pi)^3 \int d^3 p \left[\underbrace{\alpha_{\vec{p}} \alpha_{\vec{p}}^*}_{\cancel{\alpha_{\vec{p}} \alpha_{\vec{p}}^*}} - \cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}} - \cancel{\alpha_{\vec{p}}^* \alpha_{\vec{p}}} + \cancel{\alpha_{\vec{p}}^* \alpha_{-\vec{p}}} + \cancel{\alpha_{\vec{p}} \alpha_{\vec{p}}^*} \right] \\
 &+ \cancel{\alpha_{\vec{p}} \alpha_{-\vec{p}}} + \cancel{\alpha_{-\vec{p}} \alpha_{\vec{p}}} + \cancel{\alpha_{\vec{p}}^* \alpha_{-\vec{p}}} = \\
 &\text{use } \alpha_{\vec{p}} \alpha_{\vec{p}}^* = \cancel{\alpha_{\vec{p}} \alpha_{\vec{p}}^*} + 2(1/\beta) S^3(0) \\
 &= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \left[\alpha_{\vec{p}}^* \alpha_{\vec{p}} + \alpha_{-\vec{p}}^* \alpha_{-\vec{p}} \right] + \frac{1}{(2\pi)^3}
 \end{aligned}$$



$$\begin{aligned}
 & \pi = 4 \langle \alpha \beta | S^3 P | \underbrace{\alpha_{\vec{P}} \alpha_{\vec{P}} - \alpha_{\vec{P}} \alpha_{\vec{P}}}_{\cancel{\alpha_{\vec{P}} \alpha_{\vec{P}}}} - \cancel{\alpha_{\vec{P}} \alpha_{\vec{P}}} + \cancel{\alpha_{\vec{P}} \alpha_{\vec{P}}} + \cancel{\alpha_{\vec{P}} \alpha_{\vec{P}}} \\
 & + \cancel{\alpha_{\vec{P}} \alpha_{-\vec{P}}} + \cancel{\alpha_{-\vec{P}} \alpha_{\vec{P}}} + \cancel{\alpha_{-\vec{P}} \alpha_{-\vec{P}}} \rangle = \\
 & \text{use } \alpha_{\vec{P}} \alpha_{\vec{P}}^* = \alpha_{\vec{P}}^* \alpha_{\vec{P}} + 2(1/3)S^3(0) \\
 & = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p E_{\vec{P}} [\alpha_{\vec{P}}^* \alpha_{\vec{P}} + \alpha_{-\vec{P}}^* \alpha_{-\vec{P}}] + \frac{1}{(2\pi)^3} \int d^3 p E_{\vec{P}} (2\pi)^3 S^3(0)
 \end{aligned}$$

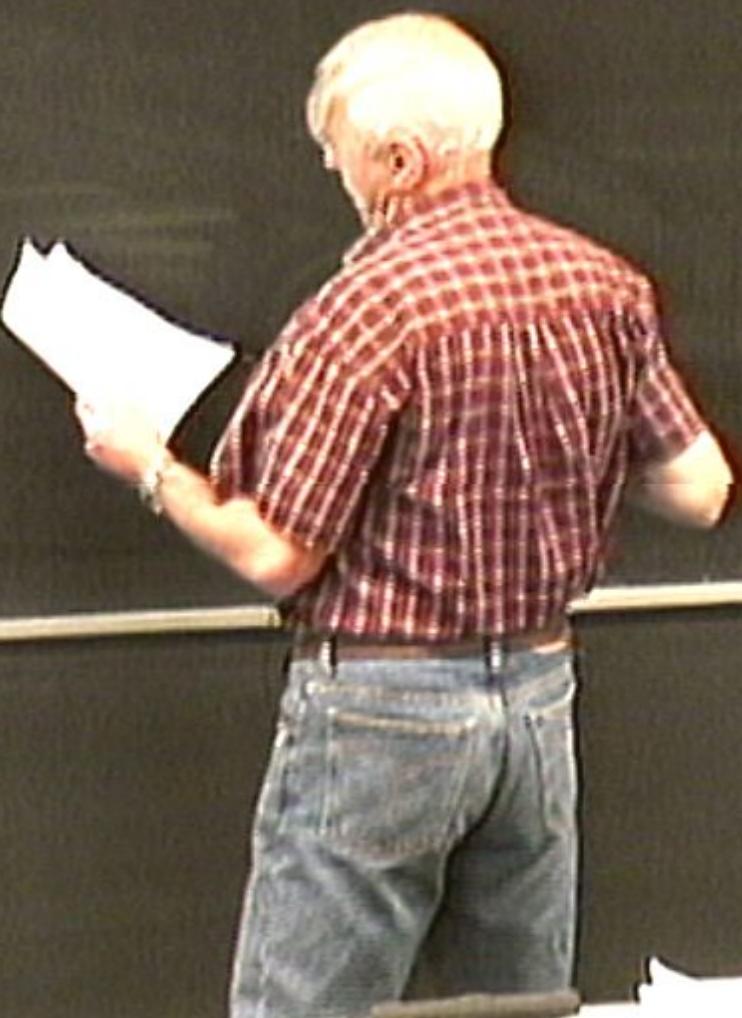
$$\text{use } \hat{a}_p^* \hat{a}_p = \hat{a}_p^* a_p + 2(\pi\beta) S^3(0)$$
$$= \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3 p \langle \hat{p} \left[\hat{a}_p^* a_p + a_p^* a_{-p} \right] \rangle + \frac{1}{(2\pi)^3} \int d^3 p \langle \hat{p} \rangle (2\pi)^3 S^3(0)$$



What is meaning



What is meaning of $s^3(0)$?



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p})$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p, \vec{x})}$$

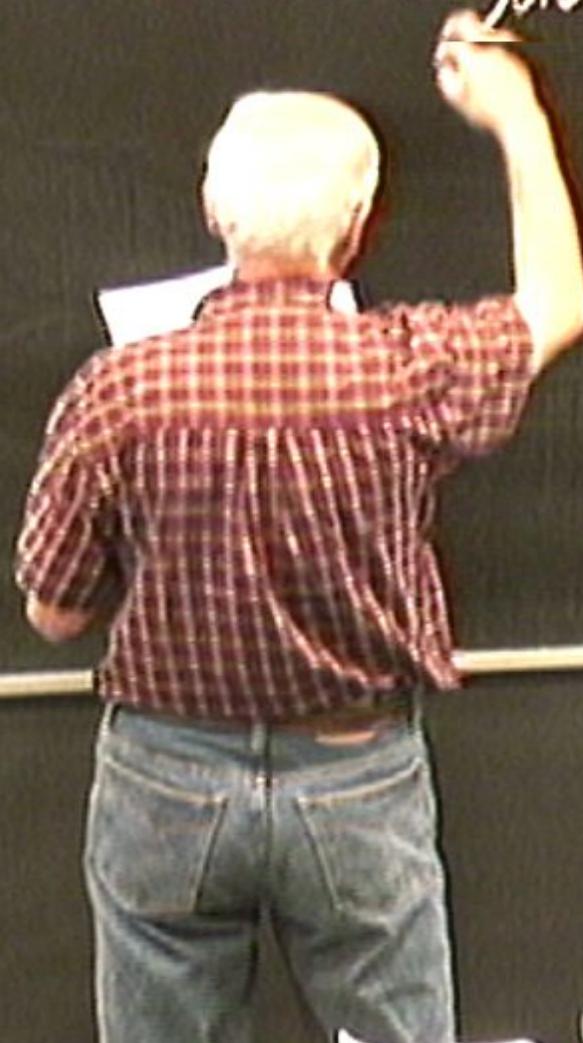
What is meaning of $\xi^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, \rho^{(1)}(\vec{x}) \Rightarrow$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p}\cdot\vec{x}} \Rightarrow (2\pi)^3 \delta^3(x) \int d^3x =$$



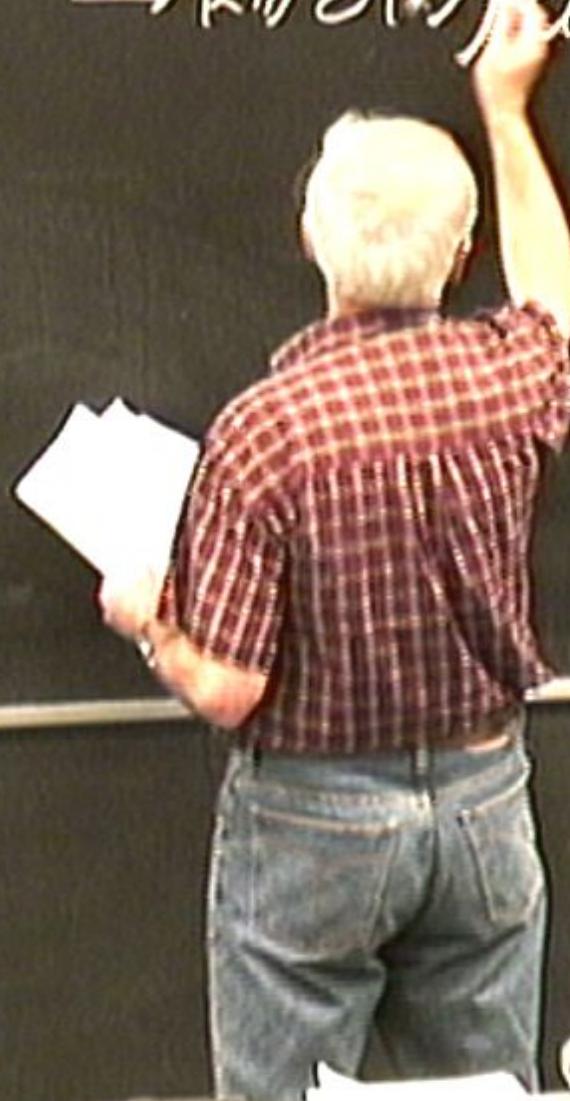
What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p} \cdot \vec{x}} \Rightarrow (2\pi)^3 \delta^3(x) \int d^3x =$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p}\cdot\vec{x}} \Rightarrow (2\pi)^3 \delta^3(0) \int d^3x =$$



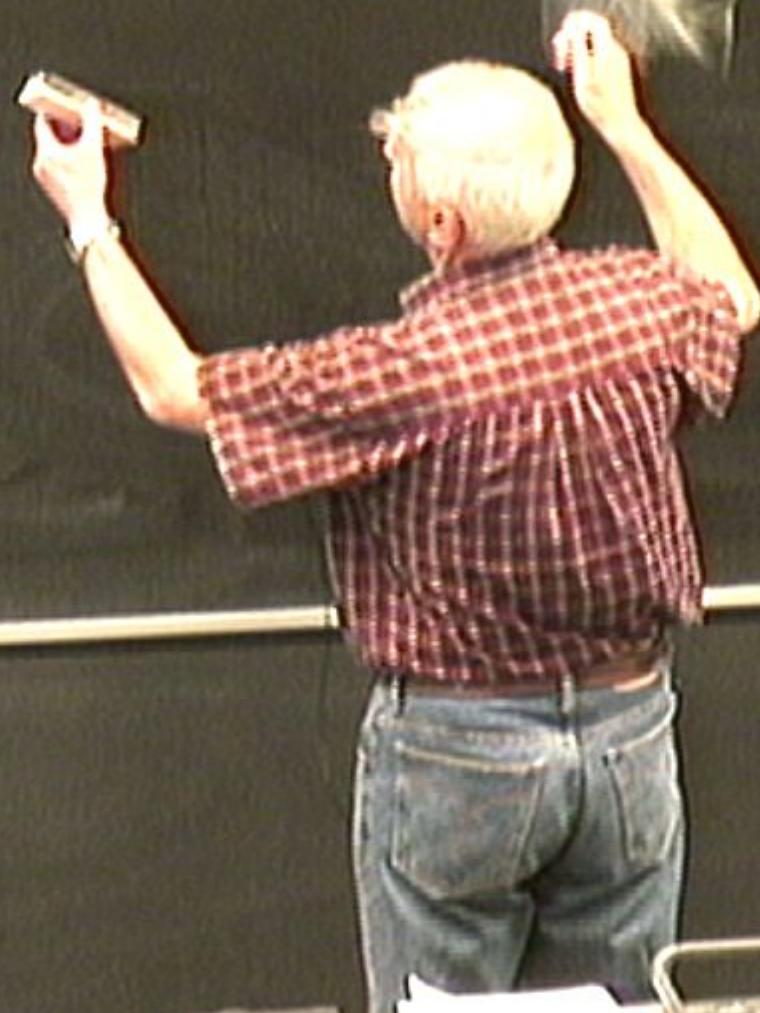
What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p}\cdot\vec{x}} \Rightarrow (2\pi)^3 \delta^3(0)$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p,x)} \Rightarrow (2\pi)^3 \delta^3(0) =$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p}\cdot\vec{x}} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p} \cdot \vec{x}} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p,x)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

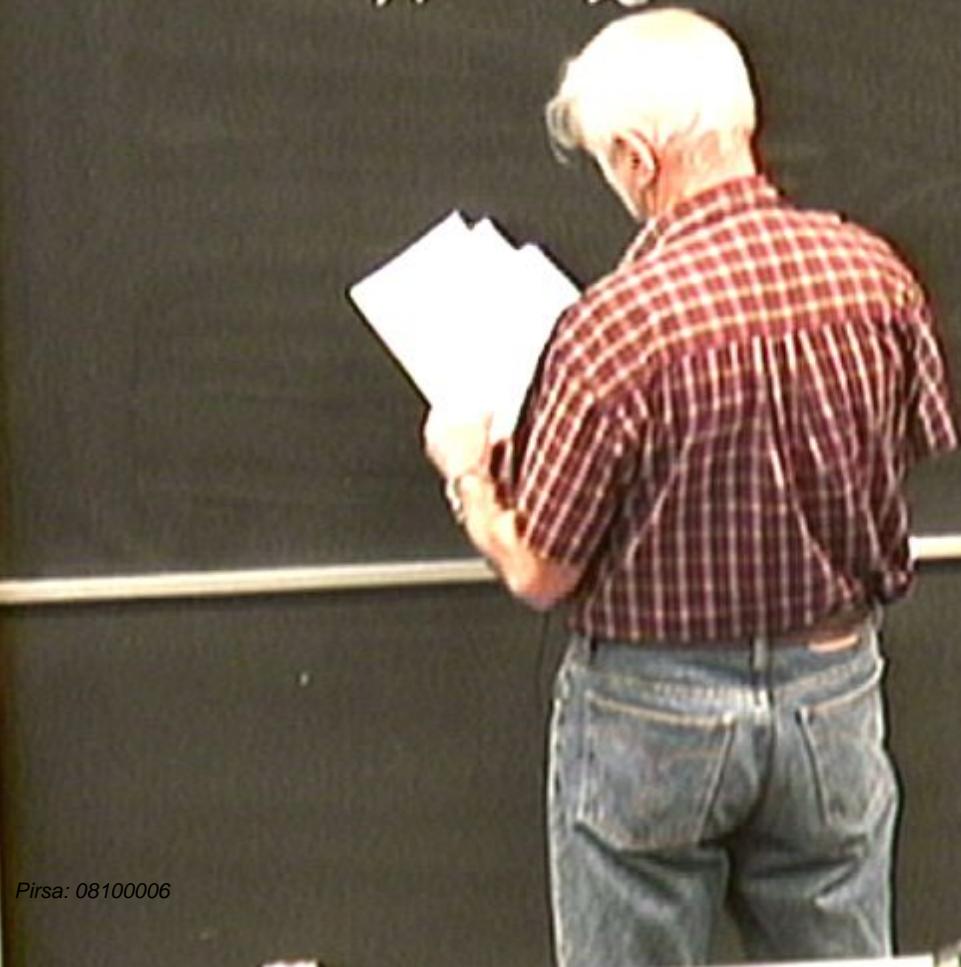
Then,



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p} \cdot \vec{x}} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{1}{2}$



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{1}{2} (2\pi)^3 \int d^3p$

What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \delta^{(3)}(\vec{p} - \vec{p}_x) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (k\pi)^3 \int d^3p E_p a_p^\dagger a_p + \frac{V}{(2\pi)^3}$

What is meaning of $\delta^3(0)$?

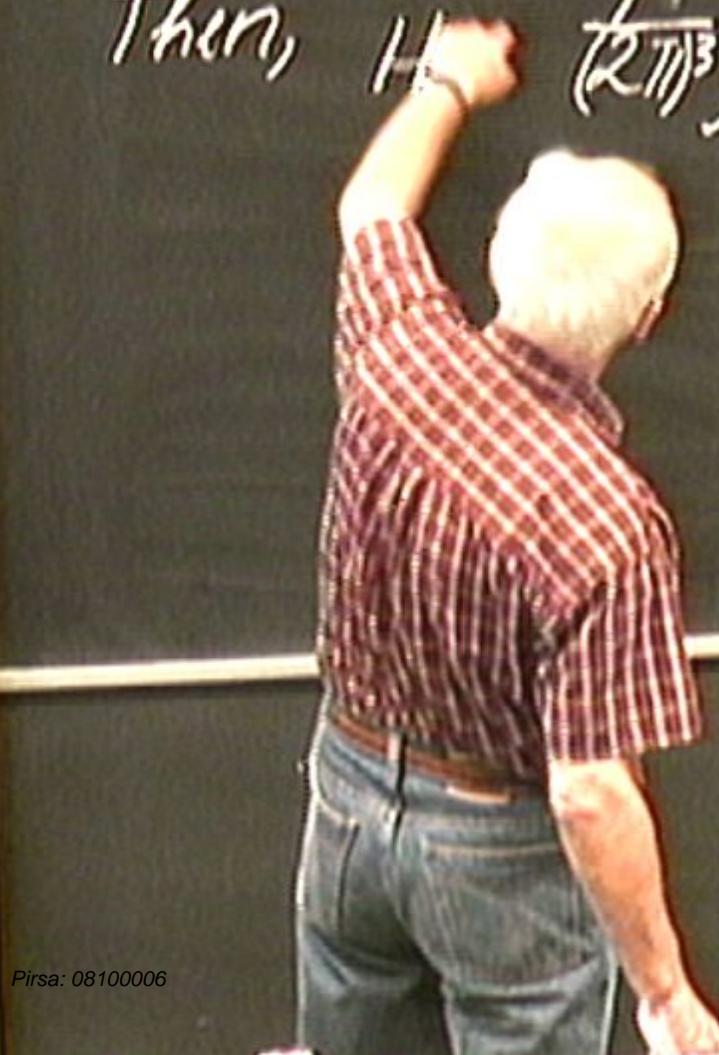
$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \delta^{(3)}(\vec{p} - \vec{x}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{1}{(2\pi)^3} \int d^3p E_{\vec{p}} \hat{a}_{\vec{p}}^{\dagger} \hat{a}_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2}$.

What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p,x)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $\langle \hat{a}_\vec{p}^\dagger \hat{a}_\vec{p} \rangle = \frac{V}{(2\pi)^3} \int d^3p E_\vec{p} a_\vec{p}^\dagger a_\vec{p} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_\vec{p}}{\epsilon}$.



What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p,x)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p E_{\vec{p}} \hat{a}_{\vec{p}}^{\dagger} \hat{a}_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2}$.

Oscillator

What is meaning of $\delta^3(\vec{0})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \delta^{(3)}(\vec{p} - \vec{k}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (k\pi)^3 \int d^3p E_{\vec{p}} a_{\vec{p}}^* a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2}$.

Oscillate



What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{1}{2m^3} \left(\int d^3p E_p a_p^\dagger a_p + \frac{V}{(2\pi)^3} \int d^3p \frac{E_p}{2} \right)$.

Oscillator $H_{osc} =$

What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p E_{\vec{p}} a_{\vec{p}}^+ a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2}$.

Oscillator $H_{osc} = \alpha$



What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p} \cdot \vec{x}} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{1}{(2\pi)^3} \int d^3p \, E_{\vec{p}} a_{\vec{p}}^{\dagger} a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \, \frac{E_{\vec{p}}}{V}$.

Oscillator H_{os}

What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \, e^{i\vec{p} \cdot \vec{x}} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p \, E_p a_p^\dagger a_p + \frac{V}{(2\pi)^3} \int d^3p \, \frac{E_p}{2}$.

Oscillator $H_{osc} = \omega a^\dagger a$

What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \delta^{(3)}(\vec{p}, \vec{x}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{1}{(2\pi)^3} \int d^3p E_{\vec{p}} a_{\vec{p}}^* a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{V}$.

Oscillator $H_{osc} = \omega a^* a + \frac{\omega}{2}$

What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3p \delta^3(\vec{p}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3p = V$$

Then, $H = (2\pi)^3 \int d^3p E_{\vec{p}} a_{\vec{p}}^{\dagger} a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p E_{\vec{p}}$.

Oscillator $H_{osc} = \omega a^{\dagger} a + \frac{\omega}{2}$ ← zero point energy

What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p)} \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p E_p a_p^\dagger a_p + \frac{V}{(2\pi)^3} \int d^3p \frac{E_p}{2}$

Oscillator $H_{osc} = \omega a^\dagger a + \frac{\omega}{2} \leftarrow$ zero point energy



What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p)} \rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = \frac{(2\pi)^3}{V} \int d^3p E_{\vec{p}} a_{\vec{p}}^{\dagger} a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{V}$

Oscillator $H_{osc} = \omega$ $\downarrow \leftarrow$ zero point energy \uparrow zero point energy

What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3p \delta^3(\vec{p}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3p = V$$

Then, $H = (2\pi)^3 \int d^3p E_{\vec{p}} a_{\vec{p}}^{\dagger} a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2}$.

Oscillator $H_{osc} = \omega a^{\dagger} a + \frac{\omega}{2}$ ← zero point energy (zero point energy infinite)

What is meaning of $\xi^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \rho^{(p)} \Rightarrow k\pi^3 \delta(0) = \int d^3x = V$$

Then, $H = \frac{1}{2} k\pi^3 \int d^3p E_p \hat{a}_p^\dagger \hat{a}_p + \frac{V}{(2\pi)^3} \int d^3p \frac{E_p}{2}$.

Oscillator $H_{osc} = \omega \sum \leftarrow$ zero point energy \leftarrow zero point energy (infinite),

$$\frac{V}{(2\pi)^3} \int d^3p \frac{E_p}{2} \gamma_1$$

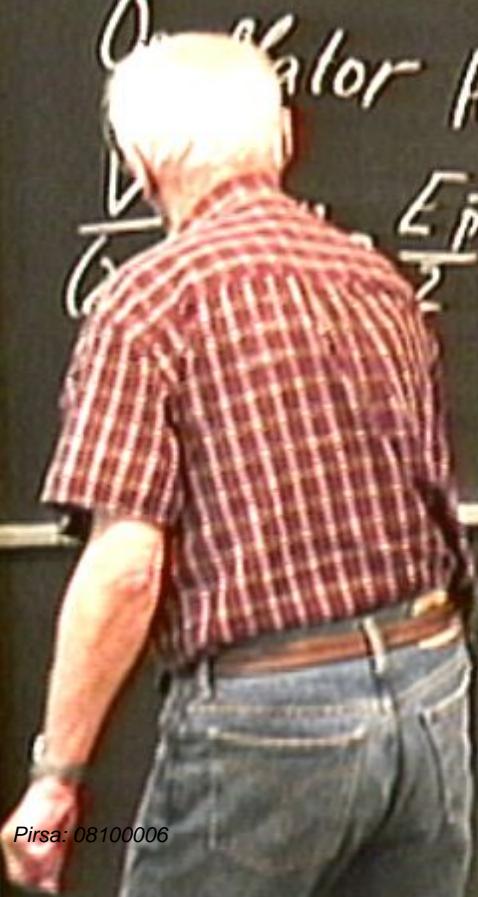
What is meaning of $\delta^3(0)$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \delta^{(3)}(\vec{p}, \vec{x}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p E_p a_p^\dagger a_p + \frac{V}{(2\pi)^3} \int d^3p \frac{E_p}{2}$.

Scalar $H_{osc} = \omega a^\dagger a + \frac{\omega}{2}$ ← zero point energy (zero point energy (infinite)).

$\frac{E_p}{2}$ yields infinite energy density -



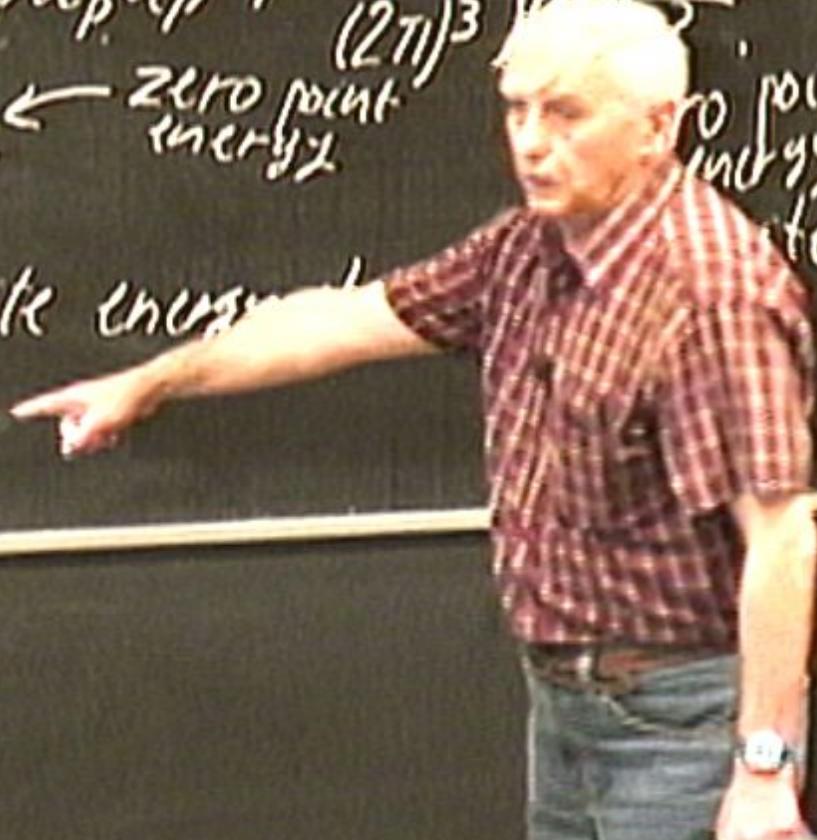
What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3x \delta^{(3)}(\vec{p}, \vec{x}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p E_p \hat{a}_p^\dagger \hat{a}_p + \frac{V}{(2\pi)^3} \int d^3p \langle \vec{p} \rangle$

Oscillator $H_{osc} = \omega a^\dagger a + \sum \leftarrow$ zero point energy

$$\frac{V}{(2\pi)^3} \int d^3p \frac{E_p}{2} \text{ yields infinite energy}$$
$$= \frac{1}{2} \frac{V}{(2\pi)^3} \int d^3p \sqrt{\vec{p}^2 + m^2}.$$



What is meaning of $\delta^3(\vec{p})$?

$$(2\pi)^3 \delta^3(\vec{p}) = \int d^3p \delta^3(\vec{p}) \Rightarrow (2\pi)^3 \delta^3(0) = \int d^3x = V$$

Then, $H = (2\pi)^3 \int d^3p E_{\vec{p}} a_{\vec{p}}^{\dagger} a_{\vec{p}} + \frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2}$.

Oscillator $H_{osc} = \omega a^{\dagger} a + \frac{\omega}{2}$ zero point energy zero point energy (infinite).

$$\frac{V}{(2\pi)^3} \int d^3p \frac{E_{\vec{p}}}{2} \text{ yields infinite energy density} =$$
$$= \frac{1}{(2\pi)^3} \int d^3p \sqrt{\vec{p}^2 + m^2}.$$

Without gravity, this infinite term cannot be observed.

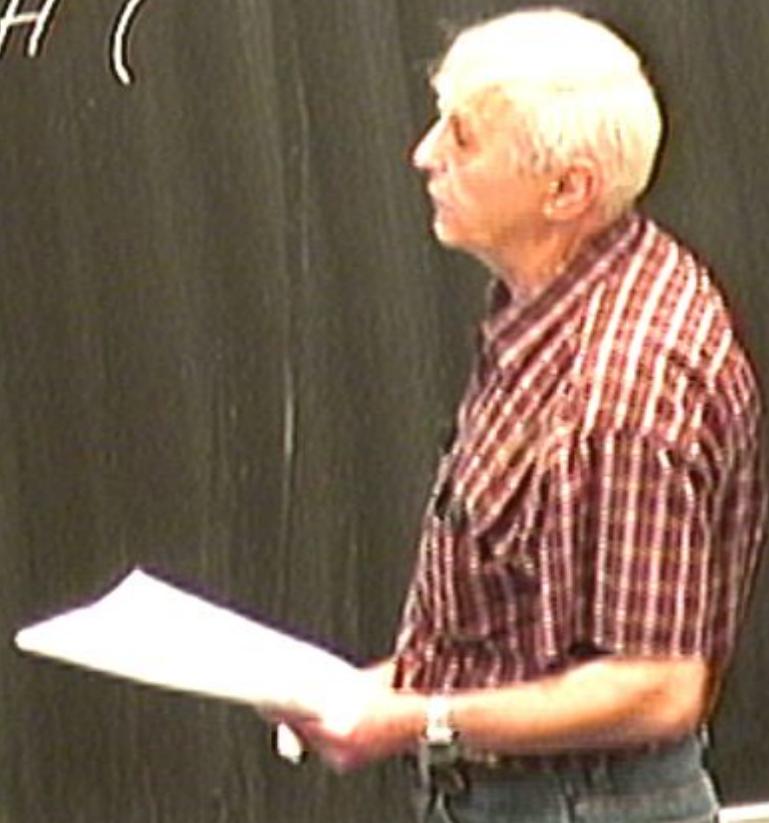
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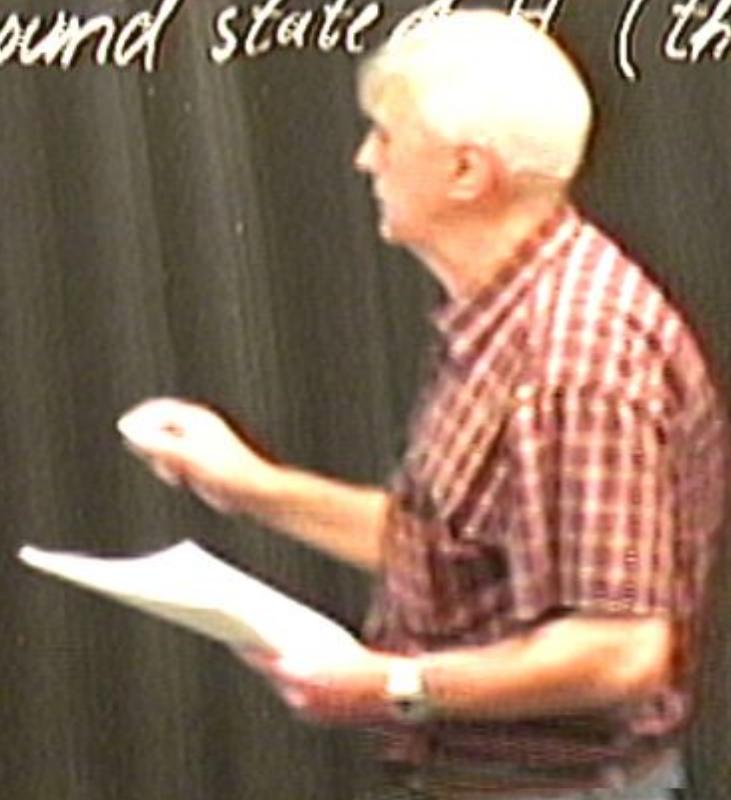
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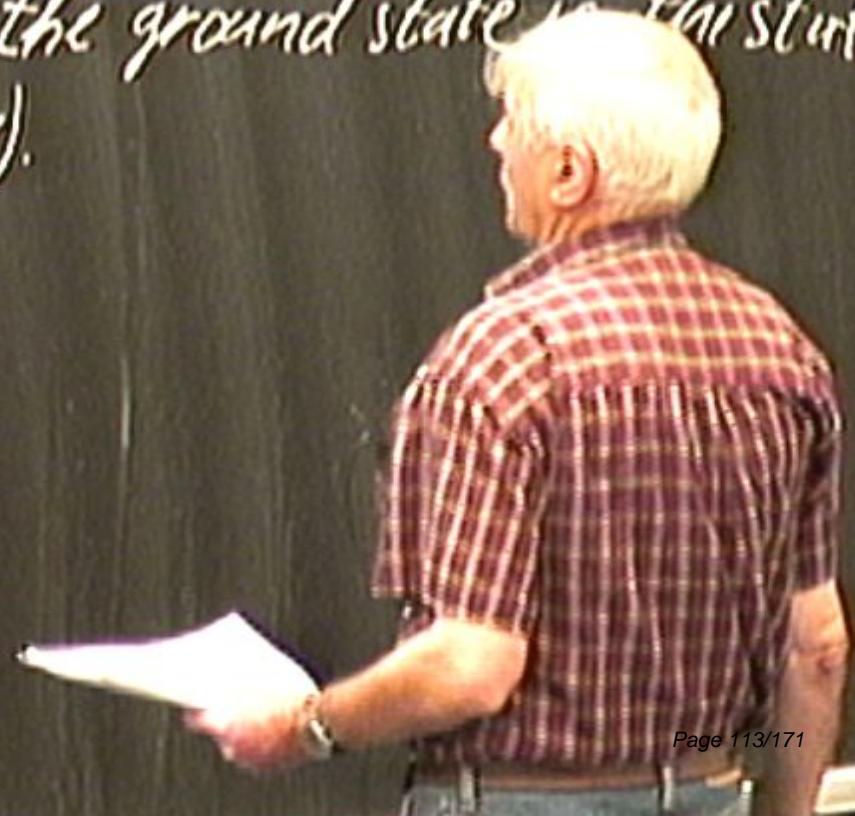
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$$\rightarrow H = \langle \alpha | \int d^3 p E_p a_\beta^\dagger a_\beta | \beta \rangle \quad \text{Using this } H, \text{ we get}$$
$$[H, a_\beta^\dagger] = E_p a_\beta^\dagger, \quad [H, a_\beta] = -E_p a_\beta$$

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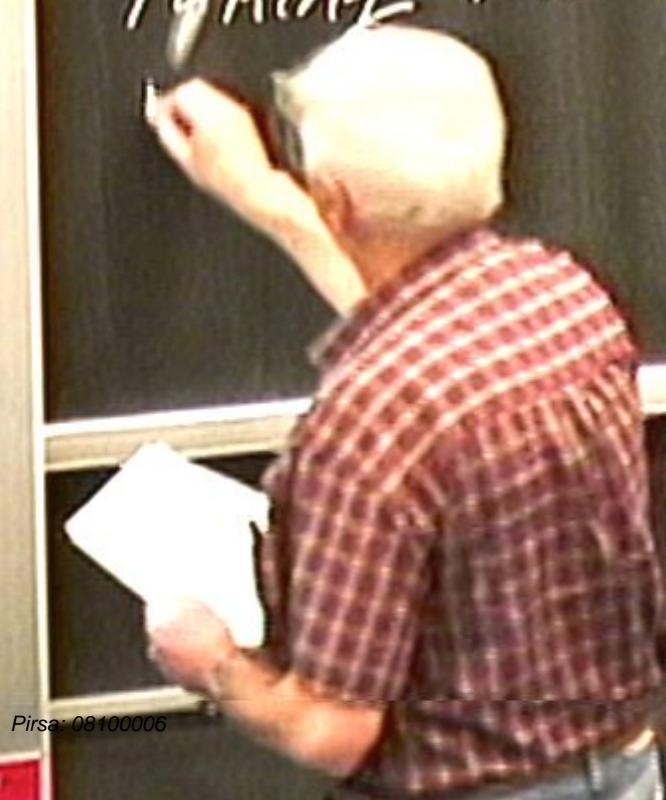
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Taking momentum operator $\vec{P} = -\int d^3 x J(x) \vec{\nabla} \psi(x)$,



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$$[H, a_p^\dagger] = E_p a_p^\dagger, [H, a_p] = -E_p a_p.$$

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$$[H, a_p^\dagger] = E_p a_p^\dagger, \quad [H, a_p] = -E_p a_p. \quad (17)$$

Taking momentum operator $\vec{P} = -i \int (B - \nabla \phi) \vec{\nabla} \psi(x)$, we find $\vec{P} = \int \frac{d^3 p}{(2\pi)^3} \vec{p} a_p^\dagger a_p$

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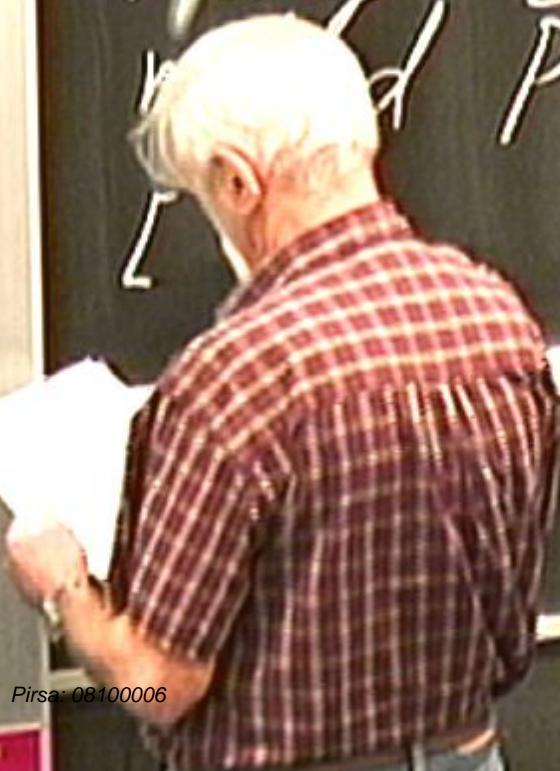
$$\rightarrow H = \frac{1}{(2\pi)} \int d^3 p E_p a_p^\dagger a_p \quad (16) \quad \text{Using this H, we get}$$
$$[H, a_p^\dagger] = E_p a_p^\dagger, \quad [H, a_p] = -E_p a_p. \quad (17)$$

Taking momentum operator $\vec{P} = -\int d^3 p \vec{p} a_p^\dagger a_p$, we find $\vec{P} = \int \frac{d^3 p}{(2\pi)^3} \vec{p} a_p^\dagger a_p$ (18)

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$$\rightarrow H = \frac{1}{(2\pi)^3} \int d^3 p E_p a_p^\dagger a_p \quad (6) \quad \text{Using this H, we get}$$
$$[H, a_p^\dagger] = E_p a_p^\dagger, \quad [H, a_p] = -E_p a_p. \quad (17)$$

Taking momentum operator $\vec{P} = -\int d^3 x J(x) \vec{\nabla} \psi(x)$,
and $\vec{P} = \int \frac{d^3 p}{(2\pi)^3} \vec{p} a_p^\dagger a_p \quad (18)$, and $[\vec{P}, a_p^\dagger] = \vec{p} a_p^\dagger$,



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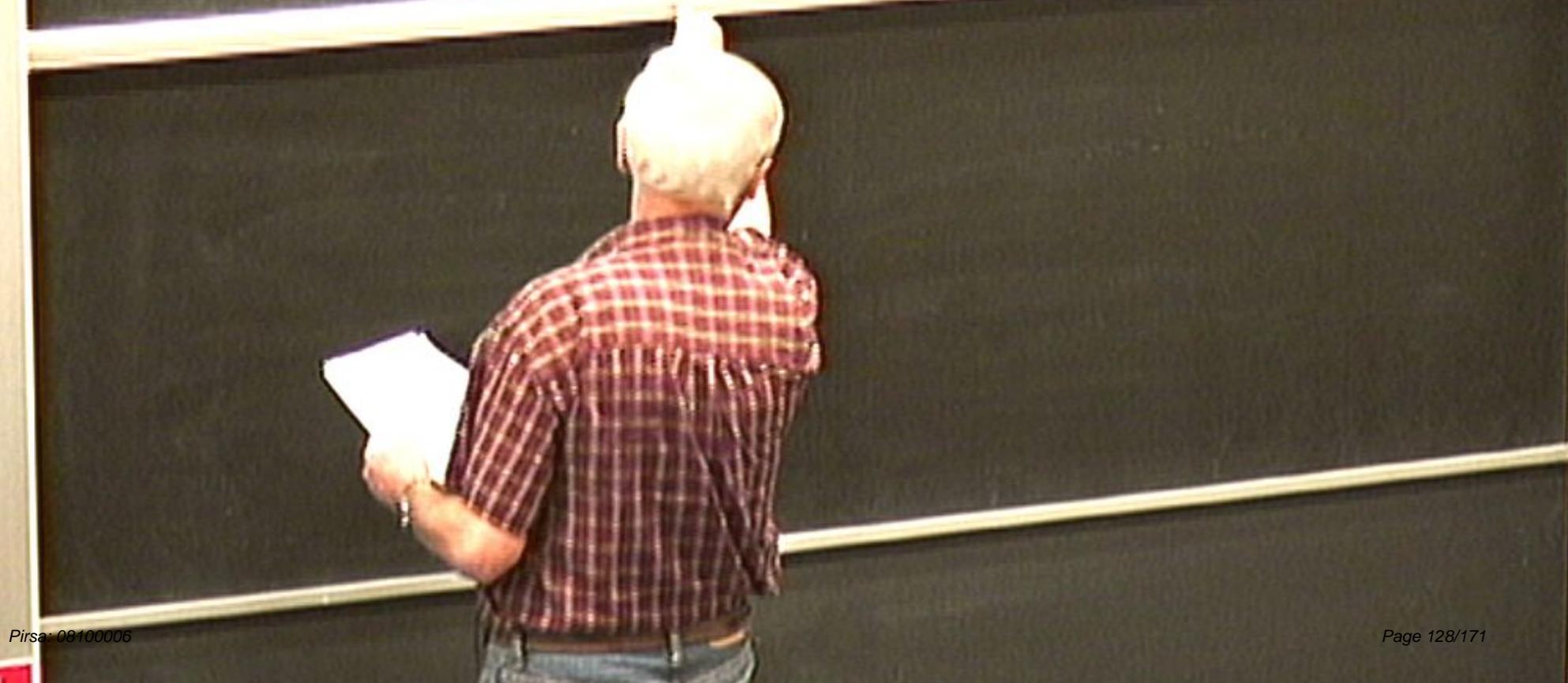
$$\rightarrow H = \frac{1}{m} \int d^3 p E_p a_p^\dagger a_p \quad (16) \quad \text{Using this H, we get}$$
$$[H, a_p^\dagger] = E_p a_p^\dagger, \quad [H, a_p] = -E_p a_p. \quad (17)$$

Taking momentum operator $\vec{P} = -\int d^3 x J(x) \vec{\nabla} \psi(x)$, we find $\vec{P} = \int d^3 p \vec{p} a_p^\dagger a_p$ (18), and $[\vec{P}, a_p^\dagger] = \vec{p} a_p^\dagger$,

$$[\vec{P}, a_p] = -\vec{p} a_p$$

$$[H, a_{\vec{p}}^{\dagger}] = E_{\vec{p}} a_{\vec{p}}^{\dagger}, \quad [H, a_{\vec{p}}] = -E_{\vec{p}} a_{\vec{p}}. \quad (17)$$

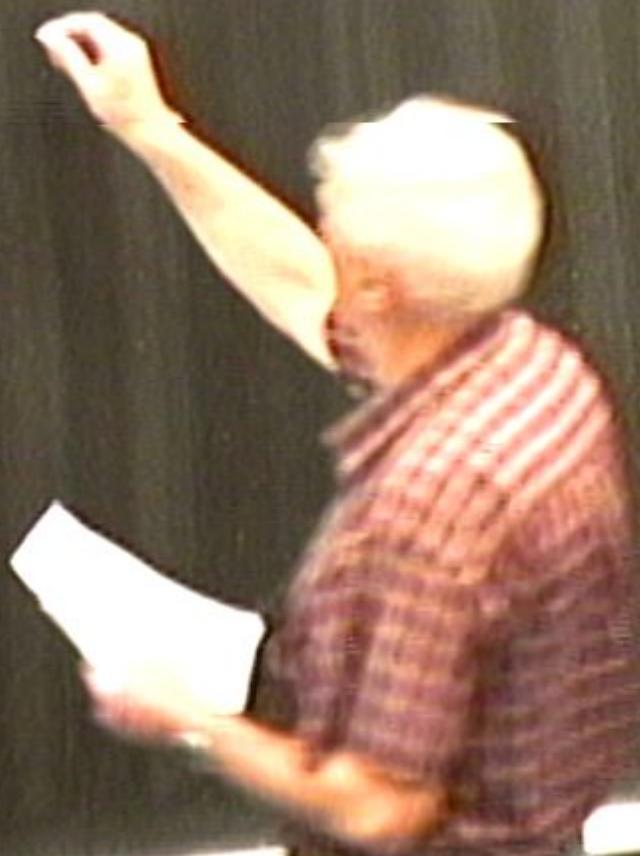
Taking momentum operator $\vec{P} = -i\hbar \vec{x} \cdot \vec{\nabla} \psi(x)$, we find $\vec{P} = \int \frac{d^3 p}{(2\pi)^3} \vec{p} a_{\vec{p}}^{\dagger} a_{\vec{p}}$ (18), and $[\vec{P}, a_{\vec{p}}^{\dagger}] = i\hbar \vec{p} a_{\vec{p}}^{\dagger}$, $[\vec{P}, a_{\vec{p}}] = -i\hbar \vec{p} a_{\vec{p}}$. (19).



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Other states are constructed as:

$$a_p^+ |0\rangle$$

(one-particle state)

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Other states are constructed from $|0\rangle$:

$a_p^+|0\rangle$, $a_p^+a_q^+|0\rangle$
(one-particle state) (two-particle states)

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(one-particle state) (two-particle states)

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(one-particle) (two-particle
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(one-particle) (two-particle
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$$H|a_p^+|0\rangle = E_p|a_p^+|0\rangle = \sqrt{\vec{P}_p m^2}|a_p^+|0\rangle, |\vec{P}a_p^+|0\rangle$$

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Other states are constructed as:

$a_{\vec{p}}^+|0\rangle$, $a_{\vec{p}}^+|0\rangle$, ... Use (17) and (19).
(one-particle state) (two-particle state)

$$H a_{\vec{p}}^+|0\rangle = -\sqrt{\vec{p}_i m^2} a_{\vec{p}}^+|0\rangle; \quad \vec{P} a_{\vec{p}}^+|0\rangle = \vec{p} a_{\vec{p}}^+|0\rangle$$

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$$H a_{\vec{p}}^+ |0\rangle = E_p a_{\vec{p}}^+ |0\rangle = \sqrt{\hbar m^2} a_{\vec{p}}^+ |0\rangle; \quad \vec{P} a_{\vec{p}}^+ |0\rangle = \vec{p} a_{\vec{p}}^+ |0\rangle$$

$$H a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle =$$

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$$H a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle = (E_{\vec{p}} + E_{\vec{q}})$$



Other states are constructed as:
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(one-particle state) (two-particle states)

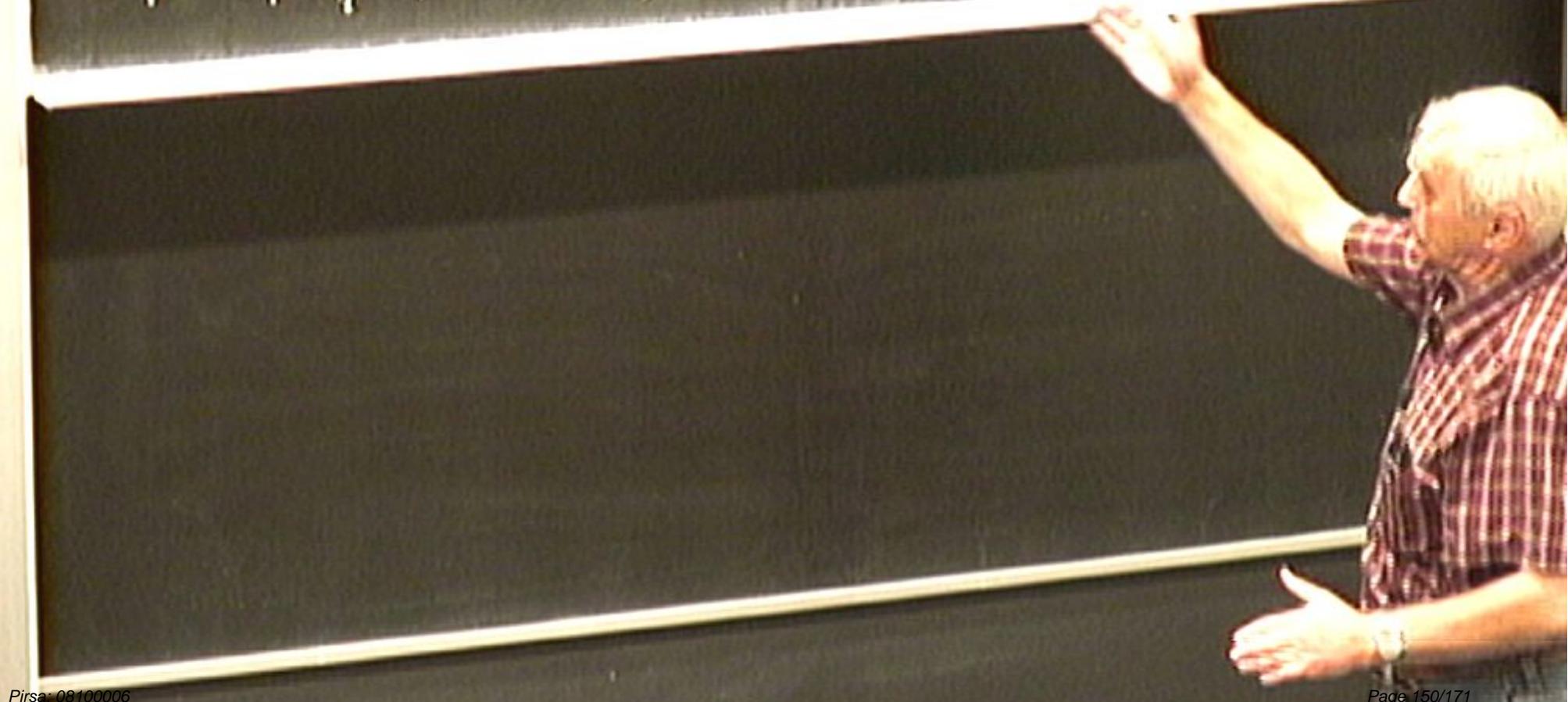
$$H a_{\vec{p}}^+ |0\rangle = E_{\vec{p}} a_{\vec{p}}^+ |0\rangle = \sqrt{\vec{p}_i m^2} a_{\vec{p}}^+ |0\rangle; \quad \vec{P} a_{\vec{p}}^+ |0\rangle = T a_{\vec{p}}^+ |0\rangle$$
$$H a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle = (E_{\vec{p}} + E_{\vec{q}}) a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle,$$



(one-particle state) (two-particle states)

$$H|a_p^t(0)\rangle = E_p a_p^t(0) = \sqrt{P_m^2} |a_p^t(0)\rangle; \quad \vec{P} a_p^t(0) = P a_p^t(0)$$

$$H|a_p^t a_q^t(0)\rangle = (E_p + E_q) |a_p^t a_q^t(0)\rangle, \quad \vec{P} a_p^t a_q^t(0) = (\vec{p} + \vec{q}) |a_p^t a_q^t(0)\rangle$$



$$\hat{a}_P^+ = \hat{a}_P^+ a_P^- + 2(\pi\beta) S^3(0)$$

What is the meaning of $S^3(0)$? Vladimir Fok

$$S^3(P) = \int d^3x \ell^{(P)} \rightarrow (2\pi)^3 S^3(0) = \int d^3x = V$$

$$H = (2\pi)^3 \int d^3P E_P \hat{a}_P^+ \hat{a}_P^- + \frac{V}{(2\pi)^3} \int d^3P$$

$$\text{or } H_{\text{esc}} = \omega a \dot{a} + \frac{\omega}{2} \leftarrow \begin{array}{l} \text{zero point} \\ \text{energy} \end{array}$$

$$\delta \hat{a}_P^{\dagger} = \hat{a}_P^{\dagger} a_P + 2(\pi\beta) S^3(0)$$

$$\frac{1}{\beta} \int d^3 P \vec{E}_P [\hat{a}_P^{\dagger} a_P + a_{-P}^{\dagger} a_{-P}] + \frac{1}{(2\pi\beta)} \int d^3 P \vec{E}_P (2\pi)^3 S^3(0)$$

What is meaning of $S^3(0)$? Vladimir Fock

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or $H_{esc} = \omega a^{\dagger} a + \frac{V}{2}$ ← zero point energy → point energy

Let us describe physical states:

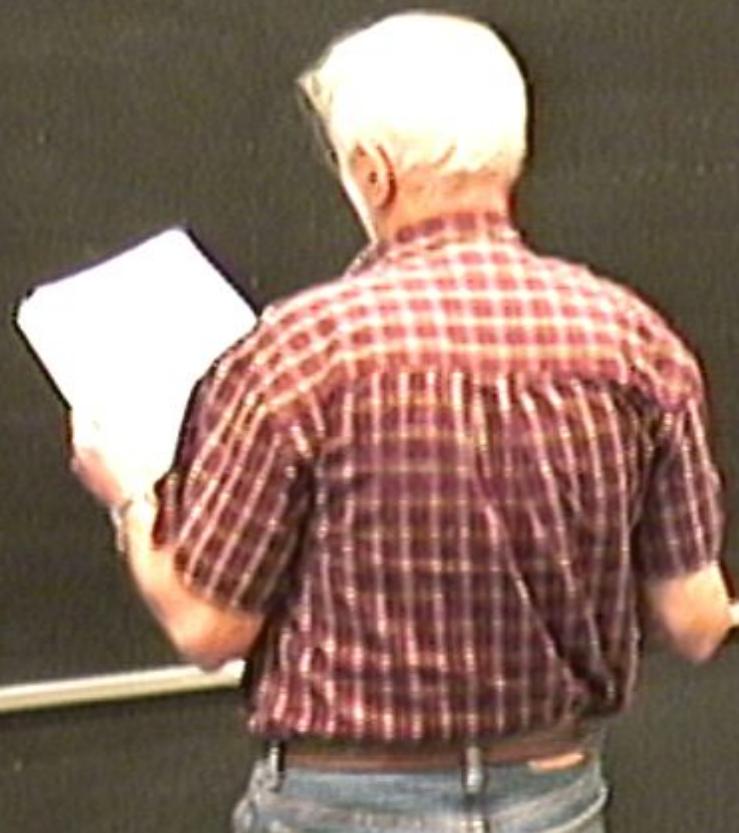
Define the ground state $|0\rangle$ as that for which $a_{\vec{p}}|0\rangle = 0$. Then $H|0\rangle = 0$, $\vec{P}|0\rangle = 0$.

Other states are constructed as:

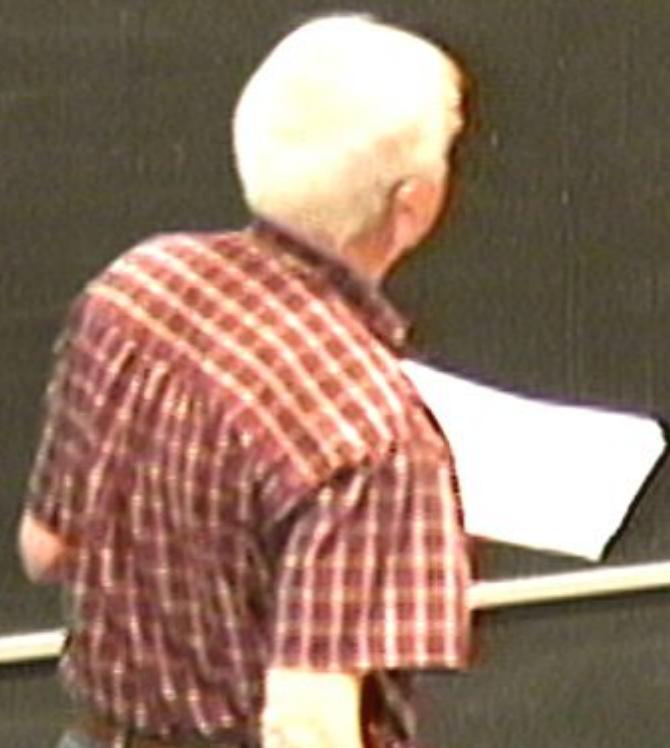
$a_{\vec{p}}^+|0\rangle$, $a_{\vec{p}}^+a_{\vec{q}}^+|0\rangle, \dots$ Use (17) and (19).
(one-particle) (two-particle
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$$|0\rangle = E_p a_{\vec{p}}^+ |0\rangle = \sqrt{\vec{p} + m^2} a_{\vec{p}}^+ |0\rangle; \quad \vec{P} a_{\vec{p}}^+ |0\rangle = \vec{p} a_{\vec{p}}^+ |0\rangle$$
$$= (\vec{E}_{\vec{p}} + \vec{E}_{\vec{q}}) a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle, \quad \vec{P} a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle = (\vec{p} + \vec{q}) a_{\vec{p}}^+ a_{\vec{q}}^+ |0\rangle.$$

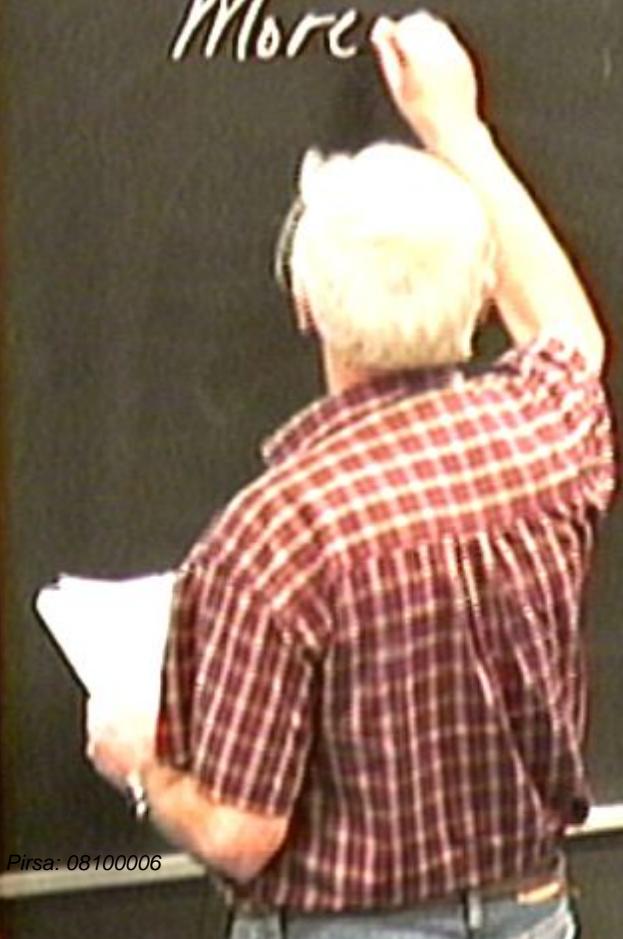
The statistics:



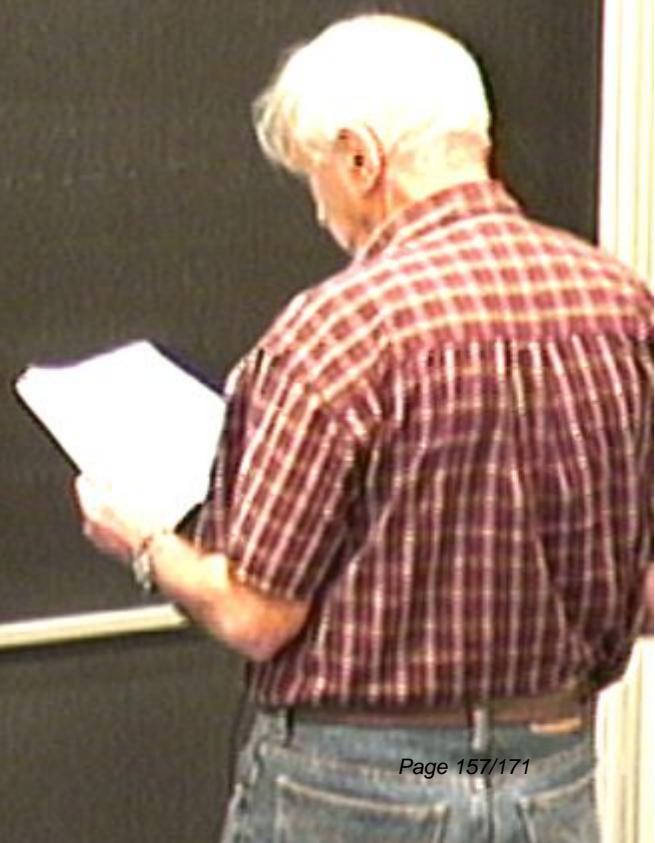
The statistics: $a_T a_T^\dagger |0\rangle = a_Q^\dagger a_Q^\dagger |0\rangle$



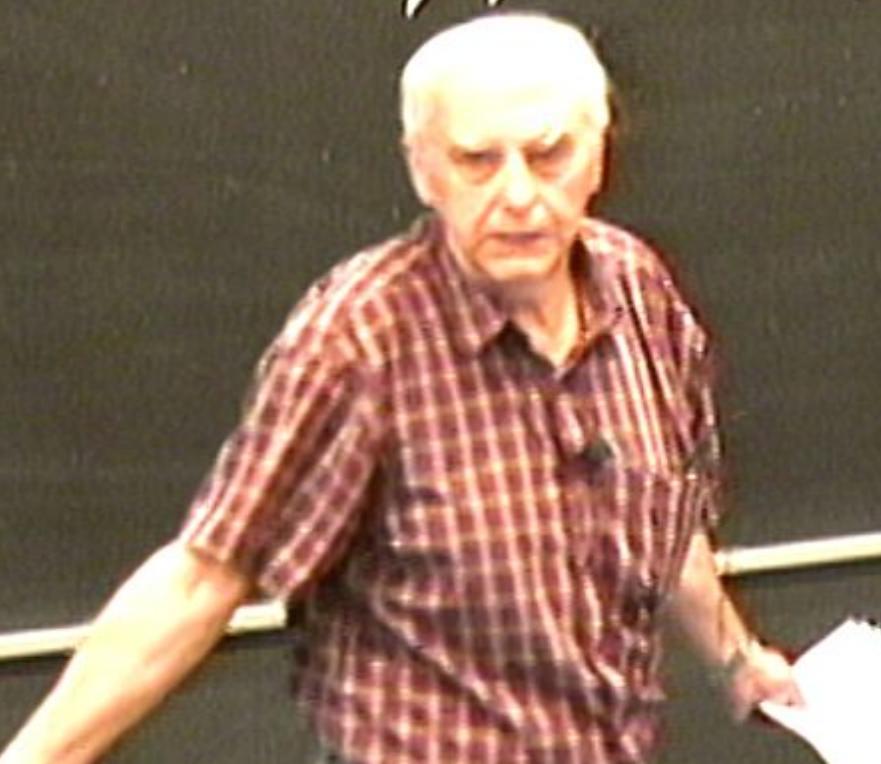
The statistics: $a^\dagger_r a^\dagger_q |0\rangle = a^\dagger_q a^\dagger_r |0\rangle$: states with interchange particles are identical.
More



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The statistics: $a_{\vec{p}}^{\dagger}a_{\vec{q}}|10\rangle = a_{\vec{q}}^{\dagger}a_{\vec{p}}|10\rangle$: states with interchange particles are identical.
Moreover, the mode with fixed \vec{p} can contain arbitrary number of particles: $(a_{\vec{p}}^{\dagger})^n|10\rangle$. We show that KG particles satisfy Bose-Einstein statistics.

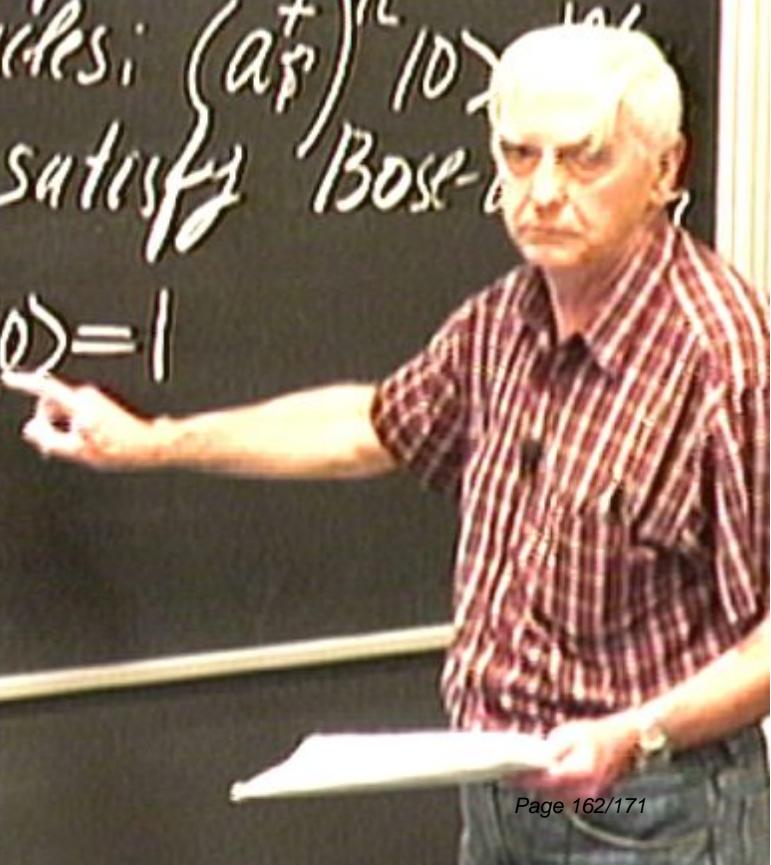
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Normalization of states:

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say that KG particles satisfy Bose-Einstein statistics.

Normalization of states: $\langle \Phi | \Phi \rangle = 1$



The statistics: $a_p a_q^\dagger |0\rangle = a_q^\dagger a_p^\dagger |0\rangle$: states with interchange particles are identical.
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Normalization of states. $\langle \Phi | 0 \rangle = 1$, then
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The statistics: $a_{\vec{p}}^{\dagger} a_{\vec{q}} |0\rangle = a_{\vec{q}}^{\dagger} a_{\vec{p}} |0\rangle$: states with interchange particles are identical.
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Normalization of states. $\langle 0 | 0 \rangle = 1$, then
 $| \vec{P} \rangle = a_{\vec{P}}^{\dagger} |0\rangle$, and $\langle \vec{P} | \vec{q}' \rangle = \langle 0 | a_{\vec{P}}^{\dagger} a_{\vec{q}}^{\dagger} |0\rangle = (2\pi)^3 \delta^3(\vec{P} - \vec{q}')$.

The statistics: $a_{\vec{p}} a_{\vec{q}}^{\dagger} |0\rangle = a_{\vec{q}}^{\dagger} a_{\vec{p}}^{\dagger} |0\rangle$: states with interchange particles are identical.
Moreover, the mode with fixed \vec{P} can contain arbitrary number of particles: $(a_{\vec{P}}^{\dagger})^n |0\rangle$. We say that KG particles satisfy Bose-Einstein statistics.

Normalization of states. $\langle \Phi | 0 \rangle = 1$, then $\langle \vec{p} | 0 \rangle$, and $\langle \vec{p} | \vec{q} \rangle = \langle 0 | a_{\vec{p}} a_{\vec{q}}^{\dagger} | 0 \rangle = (2\pi)^3 \delta^3(\vec{p} - \vec{q})$.
From (10) \uparrow

The statistics: $a_{\vec{p}} a_{\vec{q}}^{\dagger} |0\rangle = a_{\vec{q}}^{\dagger} a_{\vec{p}}^{\dagger} |0\rangle$: states with interchange particles are identical.
Moreover, the mode with fixed \vec{P} can contain arbitrary number of particles: $(a_{\vec{p}}^{\dagger})^n |0\rangle$
say that KG particles satisfy Bose-Einstein statistics.

Normalization of states. If $|0\rangle = 1$, then
 $| \vec{P} \rangle = a_{\vec{p}}^{\dagger} |0\rangle$, and $\langle \vec{P} | \vec{q} \rangle = \langle 0 | a_{\vec{p}} a_{\vec{q}}^{\dagger} |0\rangle = h \delta^{(3)}$

from (10)

The statistics: $a_p a_q^\dagger |0\rangle = a_q^\dagger a_p^\dagger |0\rangle$: states with interchange particles are identical.
Moreover, the mode with fixed \vec{p} can contain arbitrary number of particles: $(a_{\vec{p}}^\dagger)^n |0\rangle$. We say that KG particles satisfy Bose-Einstein statistics.

realization of states: $\langle \vec{p} | 0 \rangle = 1$, then
 $= a_{\vec{p}}^\dagger |0\rangle$, and $\langle \vec{p} | \vec{q} \rangle = \langle 0 | a_{\vec{p}} a_{\vec{q}}^\dagger |0\rangle = \frac{1}{(2\pi)^3} \delta^3(\vec{p} - \vec{q})$.
from (10) \uparrow



The statistics: $a_{\vec{p}} a_{\vec{q}}^\dagger |0\rangle = a_{\vec{q}}^\dagger a_{\vec{p}}^\dagger |0\rangle$: states with interchange particles are identical.
 Moreover, the mode with fixed \vec{P} can contain arbitrary number of particles: $(a_{\vec{P}}^\dagger)^n |0\rangle$. We say that KG particles satisfy Bose-Einstein statistics.

Normalization of states: $\langle \Phi | 0 \rangle = 1$, then
 $| \vec{P} \rangle = a_{\vec{P}}^\dagger |0\rangle$, and $\langle \vec{P} | \vec{q}' \rangle = \langle 0 | a_{\vec{P}} a_{\vec{q}}^\dagger |0\rangle = \frac{1}{(2\pi)^3} S^3(\vec{P} - \vec{q})$.
 From (10) \uparrow

$$H = \frac{1}{2\pi\hbar} \left[\hat{P}_x^2 [2\lambda(1)N(\lambda) + 2(\lambda^2 - 1)N(\lambda)] \right] \quad (6)$$

By using Eq. (6), we get

$$\begin{aligned} H &= \frac{1}{2\pi\hbar} \left[\hat{P}_x^2 \left[a_1 a_2^* - a_1^* a_2 - \cancel{a_1^* a_2^*} \cancel{a_1 a_2} + a_2 a_1^* \right] + \right. \\ &\quad \left. + a_2^* a_1^* + \cancel{a_1^* a_2} + \cancel{a_2 a_1} \right] = \\ &\quad \text{But } a_1 a_2^* = a_2 a_1^* + 2\delta_{12} S^z(0) \\ &= \frac{1}{2\pi\hbar} \left[\hat{P}_x^2 \left(a_1 a_2^* + a_2 a_1^* \right) + \frac{1}{2\pi\hbar} \left(\hat{P}_x^2 \right)^2 S^z(0) \right] \end{aligned}$$

What is meaning of $S^z(0)$? Without loss

$$(2\pi\hbar)^2 S^z(0) = \int d\mathbf{r} \psi^* \psi \rightarrow K_B S^z(0) = S^z(0) = V$$

$$\text{Then, } H = \frac{1}{2\pi\hbar} \left[\hat{P}_x^2 \left(a_1 a_2^* + a_2 a_1^* \right) + V \right]$$

$$\text{Orillator } H_o = \hbar \omega_o + \frac{V}{2}$$

$$\begin{aligned} \frac{V}{2\pi\hbar} \int d\mathbf{r} \psi^* \psi &\text{ will update to} \\ &= \frac{1}{2\pi\hbar} \int d\mathbf{r} \sqrt{\psi^* \psi} \end{aligned}$$

Define the ground state $\left| \psi_0 \right\rangle$ such that
 $a_1^* | \psi_0 \rangle = 0$. Then $H | \psi_0 \rangle = 0$.

Other states are considered as:
 $a_1^* | \psi_1 \rangle, a_2^* | \psi_2 \rangle, \dots$ like (17) and (19).
 $S^z(0) = 0$ (inherent state)

$$\begin{aligned} H | \psi_1 \rangle &= E_1 | \psi_1 \rangle = \sqrt{\hbar^2 k_B T_1} | \psi_1 \rangle, \quad \text{Right} \cdot \text{Right} \\ H | \psi_2 \rangle &= (E_2 - E_1) | \psi_2 \rangle, \quad \text{Left} \cdot \text{Right} \cdot (L \cdot R)^2 \cdot \text{Right} \end{aligned}$$

The statistics: $a_1^* a_2^* | \psi_0 \rangle$ and $a_2^* a_1^* | \psi_0 \rangle$ states with interchange particles are identical.
 Moreover, the mode with fixed N contains arbitrary number of particles, $| a_1^* \rangle^{N_1} | \psi_0 \rangle$. We say that K_B particles satisfy Boltzmann statistics.
 Normalization of states, $\Psi | \psi \rangle = 1$, then
 $| \Psi \rangle = | \psi_0 \rangle + | \psi_1 \rangle + | \psi_2 \rangle + \dots$

By using (6), we get $a_P = \frac{\bar{Q}(P)\bar{H}(P)}{2}$, $a'_P = \frac{\bar{Q}(P) - \bar{A}(P)}{2}$
 where $\bar{Q}(P) = \sqrt{E_F} Q(P)$, $\bar{H}(P) = \sqrt{E_F} H(P)$. From (8)
 $[Q(P), H(P)] = -2(2\pi)^3 S^3(P, P)$, $[Q(P), \bar{H}(P)] = \sqrt{E_F} [H(P), \bar{H}(P)] = 0$
 From (9), we get $[a_P, a_{P'}] = [a'_P, a'_{P'}] = 0$, $[a_P, \bar{H}(P')] = -(2\pi)^3 S^3(P - P', P)$ and $\langle \bar{Q}(P), \bar{H}(P') \rangle = 0$
 Express the hamiltonian (13) into $H =$
 $\lambda [2\pi + \frac{1}{2}(\vec{V}P)^2; \frac{1}{2}\lambda \vec{q}^2]$
 $+ (13)$

$$H = \frac{1}{2\pi\hbar} \int d^3p [(\vec{p}^2/2m) + V(\vec{r}) + \frac{1}{2}(\vec{p}^2m)^2/(2\hbar^2)] \delta(\vec{r}, \vec{p}) \quad (6)$$

By using Eq (6), we get:

$$\begin{aligned} H &= \frac{1}{2\pi\hbar} \int d^3p \left[\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p - \alpha_1 \alpha_2 \cdots \alpha_{p-1} \alpha_p \alpha_{p+1} \alpha_{p+2} \cdots \right. \\ &\quad \left. + \alpha_2 \alpha_3 \cdots \alpha_{p-1} \alpha_p + \alpha_{p+1} \alpha_{p+2} \cdots \alpha_n \alpha_1 \right] = \\ &= \frac{1}{2\pi\hbar} \int d^3p \sum_{\sigma} \alpha_1 \alpha_2 \cdots \alpha_p \alpha_{p+1} \cdots \alpha_n \delta(\vec{r}, \vec{p}) \end{aligned}$$

What is meaning of $\delta^3(\vec{r})$? Without loss

$$(2\pi\hbar)^3 \delta^3(\vec{r}) = \int d^3p \delta^3(\vec{r}, \vec{p}) \Rightarrow k_B T \delta^3(\vec{r}, \vec{p}) = \sqrt{V}$$

$$\text{Then, } H = \frac{1}{2\pi\hbar} \int d^3p \left(E_F \delta^3(\vec{r}, \vec{p}) + \frac{\sqrt{V}}{(2\pi\hbar)^3} \delta^3(\vec{r}, \vec{p}) \right)$$

$$\text{Or rather } H = \text{constant} + \frac{1}{2} \int d^3p \frac{\sqrt{V}}{(2\pi\hbar)^3} \delta^3(\vec{r}, \vec{p}) \quad (\text{constant})$$

$\frac{1}{2\pi\hbar} \int d^3p \frac{\sqrt{V}}{(2\pi\hbar)^3}$ gives infinite mass density = $= \frac{1}{2\pi\hbar} \int d^3p \sqrt{V}/\hbar^2$.

Define the ground state $\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p = 1$

$$\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p = 0 \quad \text{Then } H = 0$$

Other states are considered as:

$\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \alpha_{p+1} \cdots \alpha_n$ (Eq (17)) and (18).
 $\alpha_1 \alpha_2 \cdots \alpha_n \alpha_{p+1} \cdots \alpha_n \alpha_1$ (Eq (19))

$$H \alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \alpha_{p+1} \cdots \alpha_n = E_F \alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \alpha_{p+1} \cdots \alpha_n$$

$$H \alpha_1 \alpha_2 \cdots \alpha_n \alpha_{p+1} \cdots \alpha_n \alpha_1 = (E_F + \epsilon) \alpha_1 \alpha_2 \cdots \alpha_n \alpha_{p+1} \cdots \alpha_n \alpha_1$$

The statistics: $\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p$ states with interchange particles are identical.
 Moreover, the mode with fixed P contains arbitrary number of particles, $\langle \alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \rangle_{P, T}$. We say that $\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p$ satisfies Boltzmann statistics. If $\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p = 1$, then $\langle \alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \rangle_{P, T} = 1$. $\langle \alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \rangle_{P, T} = \langle \alpha_1 \alpha_2 \cdots \alpha_n \alpha_p \rangle_{P, T} \delta(\alpha_1 \alpha_2 \cdots \alpha_n \alpha_p - 1)$