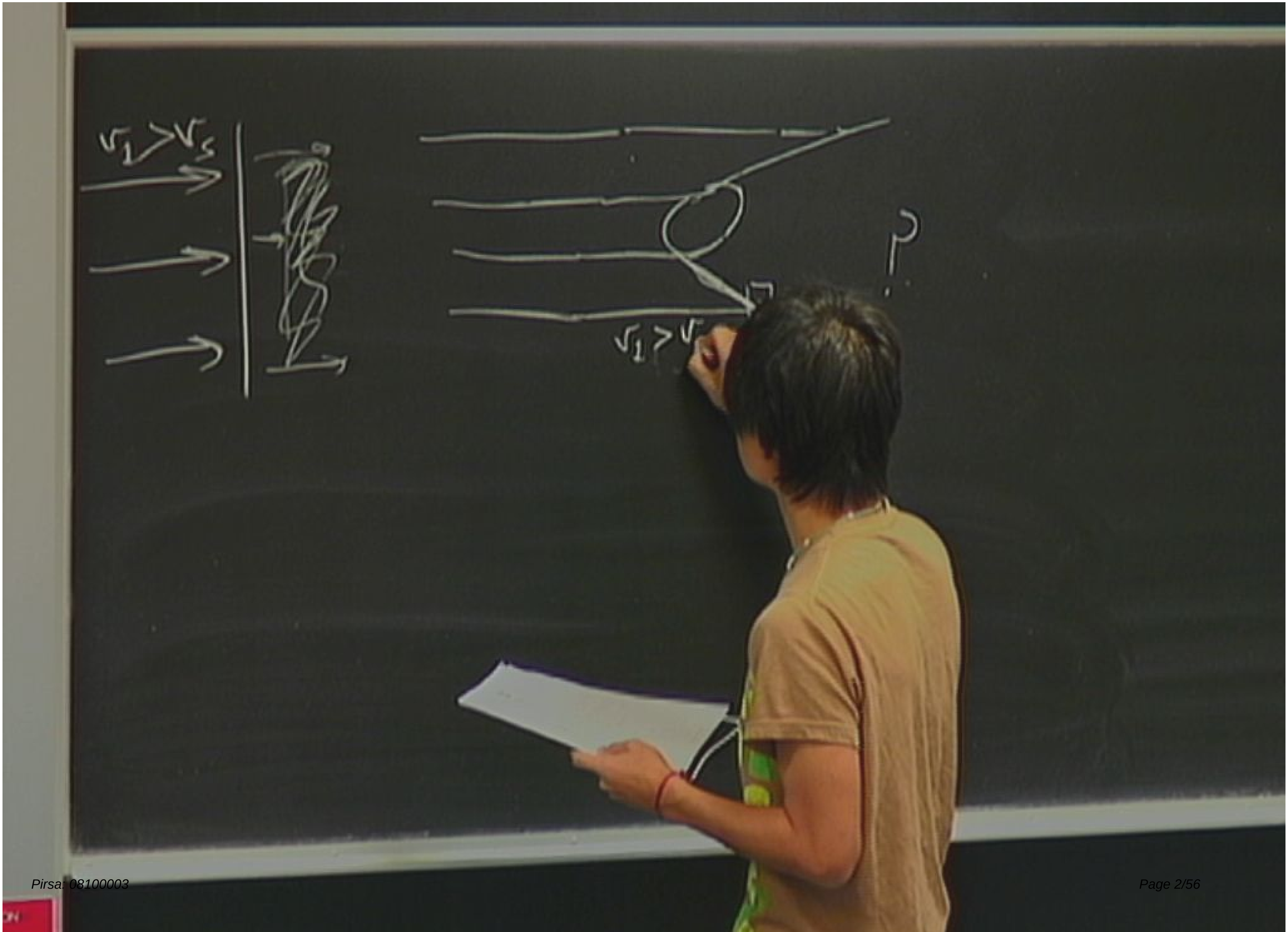
Title: Astrophysics and Cosmology through Problems - 7A

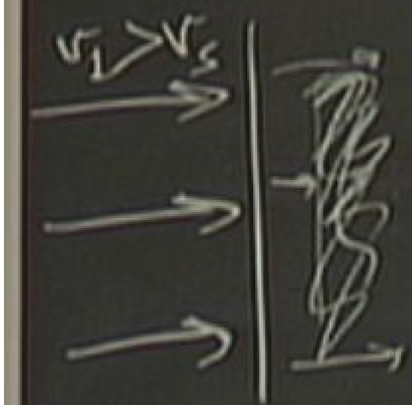
Date: Oct 16, 2008 10:00 AM

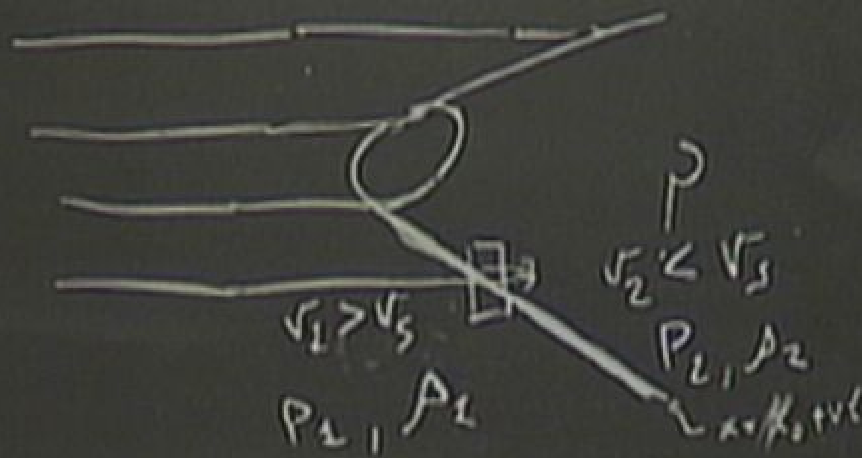
URL: http://pirsa.org/08100003

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.



v1 > vs
?
v1 > v





$$dM_1 = dM_2$$

$$\rho_1 v_1 \cancel{A_1} = \rho_2 v_2 \cancel{A_2}$$

$$\rho_1 v_1 = \rho_2 v_2$$

$$PV = k_B T$$

$$P = \rho R T \quad , \quad R \doteq \frac{k_B}{m}$$

$$\omega = \epsilon + \frac{P}{\rho}$$

$$= \epsilon + RT$$

$$= C_v T + RT$$

$$= \frac{R}{\gamma - 1} T + RT$$

$$PV = k_B T$$

$$P = \rho R T \quad , \quad R = \frac{k_B}{m}$$

$$\omega = \epsilon + \frac{P}{\rho} \qquad P = \rho^{\gamma}$$

$$= \epsilon + RT$$

$$= C_v T + RT$$

$$= \frac{R}{\gamma-1} T + RT = \left(\frac{1}{\gamma-1} + 1\right) RT$$

$$= \frac{\gamma}{\gamma-1} \frac{P}{\rho}$$

$$PV = k_B T$$

$$P = \rho R T \quad , \quad R = \frac{k_B}{m}$$

$$w = \epsilon + \frac{P}{\rho}$$

$$= \epsilon + RT$$

$$= C_v T + RT$$

$$= \frac{R}{\gamma - 1} T + RT$$

$$= \left(\frac{1}{\gamma - 1} + 1 \right) RT$$

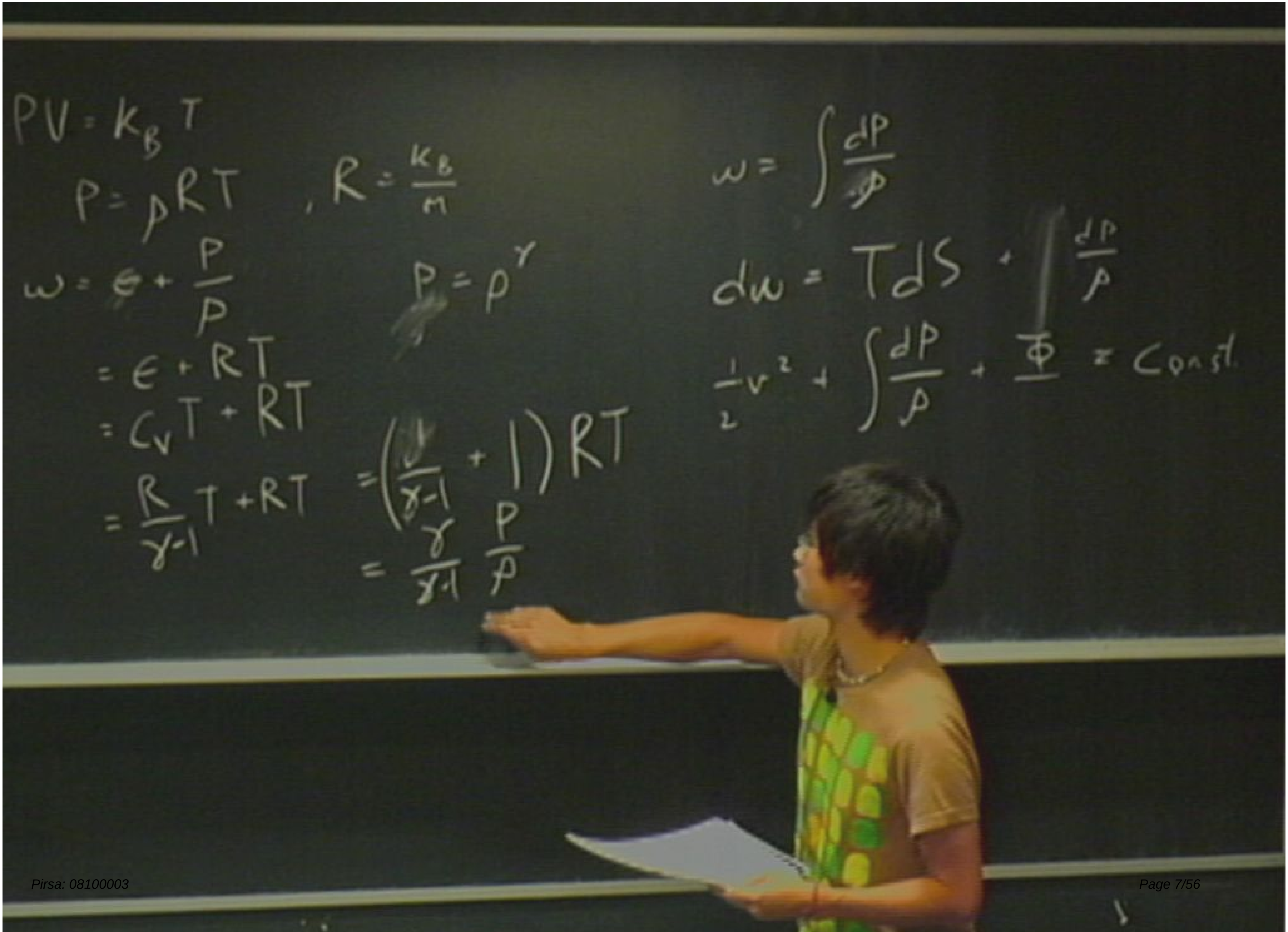
$$= \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$$

$$P = \rho^{\gamma}$$

$$w = \int \frac{dP}{\rho}$$

$$dw = T dS + \frac{dP}{\rho}$$

$$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = C$$


$PV = k_B T$
$P = \rho R T$, $R = \frac{k_B}{m}$
$\omega = \epsilon + \frac{P}{\rho}$
$= \epsilon + RT$
$= C_v T + RT$
$= \frac{R}{\gamma - 1} T + RT$
$= \left(\frac{1}{\gamma-1} + 1\right) RT$
$= \frac{\gamma}{\gamma-1} \frac{P}{\rho}$
$P = \rho^\gamma$
$\omega = \int \frac{dP}{\rho}$
$dw = TdS + \frac{dP}{\rho}$
$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = Const$

$$PV = k_B T$$

$$P = \rho R T \quad , \quad R = \frac{k_B}{m}$$

$$w = \epsilon + \frac{P}{\rho}$$

$$= \epsilon + RT$$

$$= C_V T + RT$$

$$= \frac{R}{\gamma-1} T + RT \quad = \left(\frac{1}{\gamma-1} + 1\right) RT$$

$$= \frac{\gamma}{\gamma-1} \frac{P}{\rho}$$

$$P = \rho^{\gamma}$$

$$w = \int \frac{dP}{\rho}$$

$$dw = T\,dS + \frac{dP}{\rho}$$

$$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = \text{Const}$$

$$\frac{1}{2} v_1^2 + \frac{\gamma}{\gamma-1} \frac{P_1}{\rho_1} = \frac{1}{2} v_2^2 + \frac{\gamma}{\gamma-1} \frac{P_2}{\rho_2}$$

$PV = k_B T$

$P = \rho R T \quad , \quad R = \frac{k_B}{m}$

$$\omega = \epsilon + \frac{P}{\rho}$$
$$= \epsilon + RT$$
$$= C_v T + RT$$
$$= \frac{R}{\gamma - 1} T + RT = \left(\frac{1}{\gamma-1} + 1\right) RT$$
$$= \frac{\gamma}{\gamma-1} \frac{P}{\rho}$$

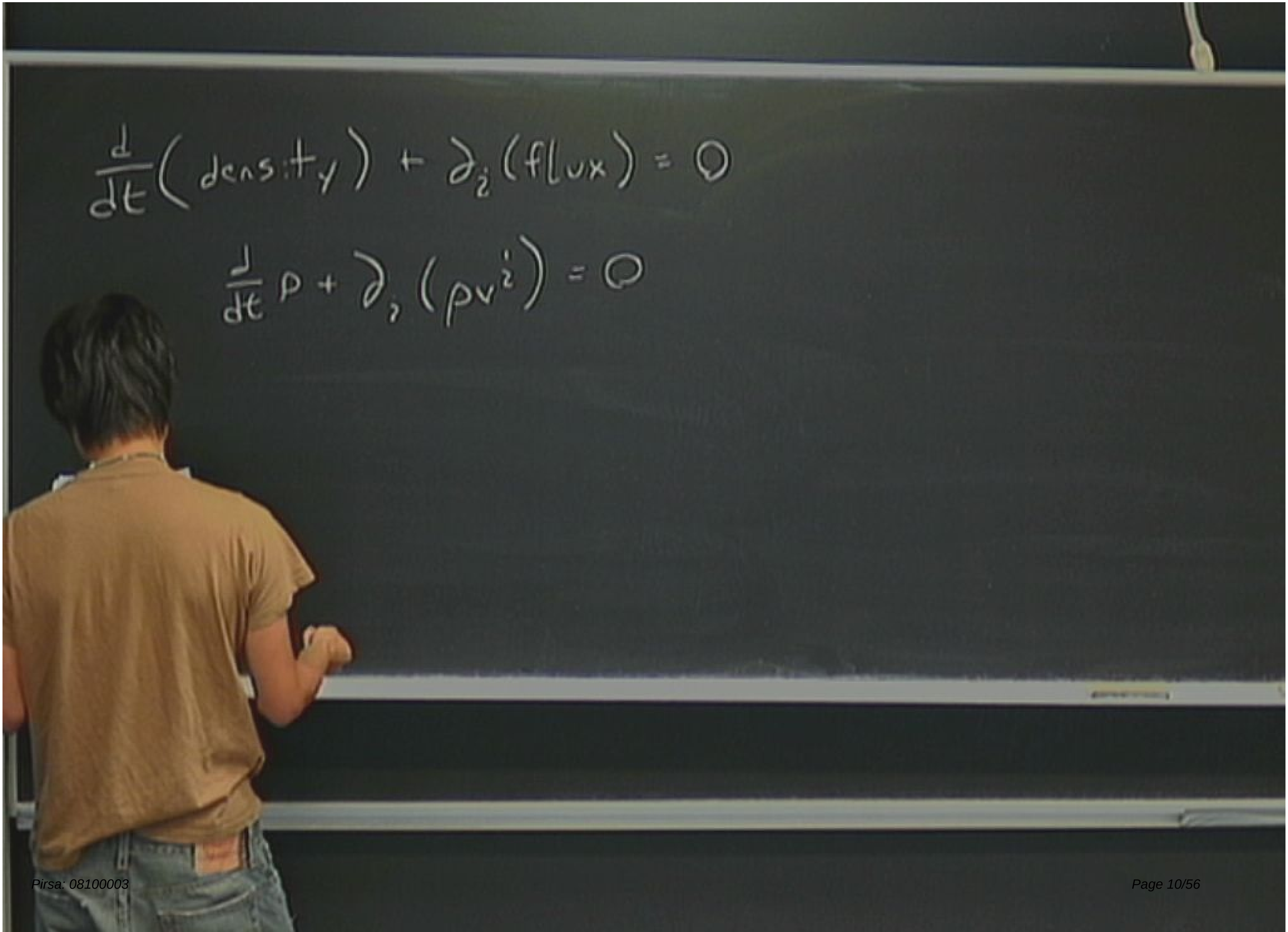
$P = \rho^\gamma$

$$\omega = \int \frac{dP}{\rho}$$

$$d\omega = T\,dS + \int \frac{dP}{\rho}$$

$$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = \text{Const}$$

$$\frac{1}{2} v_1^2 + \frac{\gamma}{\gamma-1} \frac{P_1}{\rho_1} = \frac{1}{2} v_2^2 + \frac{\gamma}{\gamma-1} \frac{P_2}{\rho_2}$$

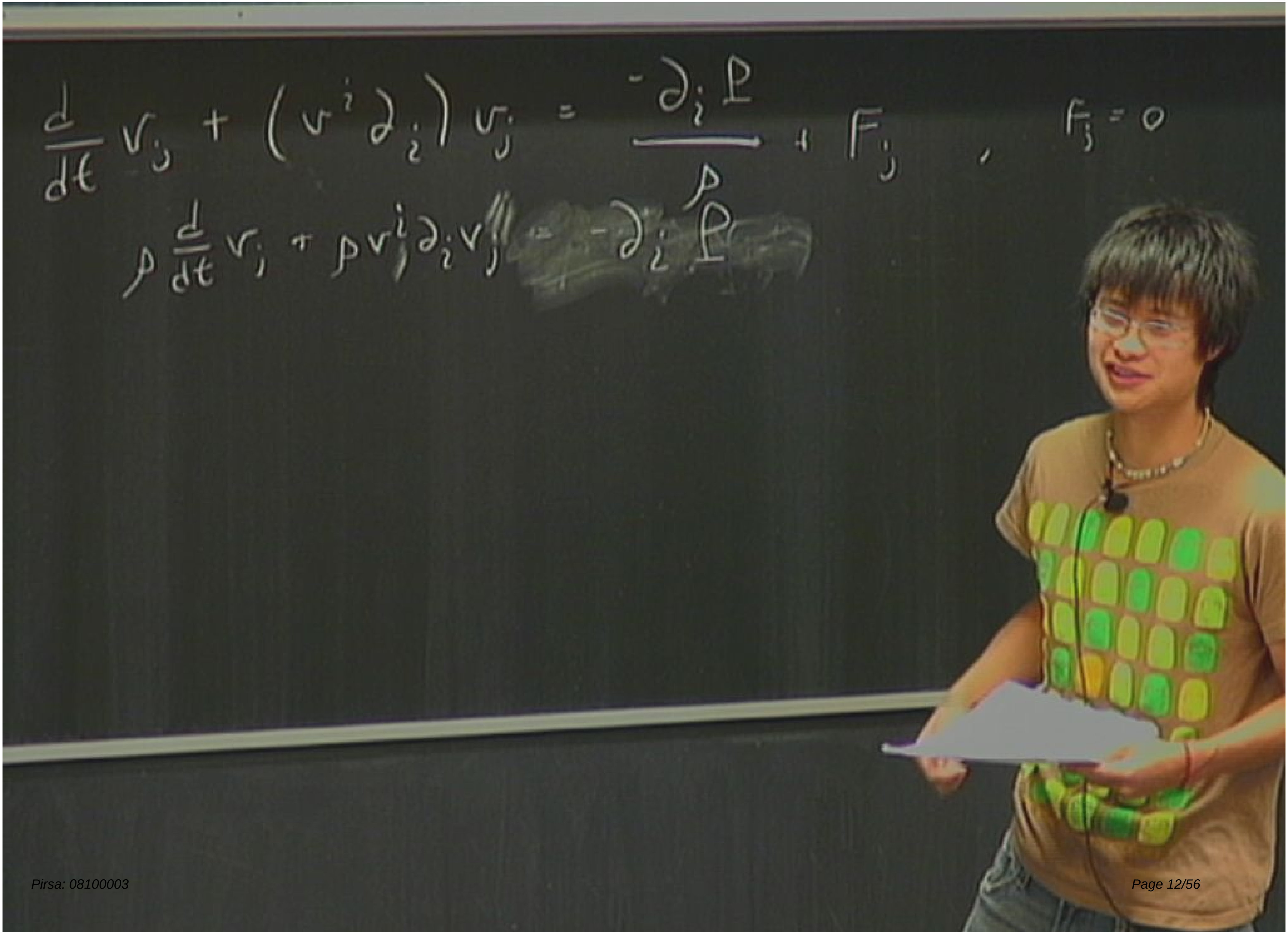

$\frac{d}{dt}(\text{density}) + \partial_i(\text{flux}) = 0$
$\frac{d}{dt}\rho + \partial_i(\rho v^i) = 0$

$$\frac{d}{dt}(\text{density}) + \partial_i(\text{flux}) = 0$$

$$\frac{d}{dt}\rho + \partial_i(\rho v^i) = 0$$

$$v_j \frac{d}{dt}\rho + v_j \partial_i(\rho v^i) = 0$$

$$v_j \frac{d}{dt}\rho + v_j v^i \partial_i \rho + \rho v_j \partial_i v^i = 0$$


$\frac{d}{dt} v_j + (v^i \partial_i) v_j = \frac{-\partial_i P}{\rho} + F_j \quad , \quad F_j = 0$
$\rho \frac{d}{dt} v_j + \rho v^i_j \partial_i v_j = -\partial_i P$

$$\frac{d}{dt} v_j + (v^i \partial_i) v_j = \frac{-\partial_i P}{\rho} + F_j \,, \quad F_j = 0$$

$$\rho \frac{d}{dt} v_j + \rho v^i \partial_i v_j = -\partial_i P$$

$$v_j \frac{d}{dt} \rho + \rho \frac{d}{dt} v_j + \rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho = -\partial_i P$$

$$\frac{d}{dt}(\rho v_j) + \partial_i (\rho v^i v_j) + \partial_j P = 0$$

$$T^i{}_j = P \delta^i_j + \rho v^i v_j$$

$v_2 < v_s$

$v_1 > v_s$

$\rho_1 v_1 = \rho_2 v_2$

P_2, ρ_2

P_1, ρ_1

$x = x_0 + vt$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

P_1, ρ_1

$x = x_0 + vt$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

c

$$M_1 = \frac{v_1}{c_{s1}} =$$

$$v_j \partial_i v_j = -\partial_i P$$

$$v_j + \rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho = -\partial_i P$$

$$+ \partial_i (\rho v^i v_j) + \partial_j P = 0$$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

$$M_1 = \frac{v_1}{c_{s1}} \qquad M_1^2 = v_1^2 \frac{\rho_1}{\gamma P_1} \qquad c_s = \sqrt{\frac{\gamma P_1}{\rho_1}}$$

$$\ldots = -\partial_i P$$

$$\rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho = - \ldots$$

$$\partial_i (\rho v^i v_j) + \partial_j P = 0$$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

$$M_1 = \frac{v_1}{c_{s1}} \qquad M_1^2 = v_1^2 \frac{\rho_1}{\gamma P_1} \qquad \left(c_s = \sqrt{\frac{\gamma P_1}{\rho_1}} \right)$$

$$\ldots v_j = -\partial_i P$$

$$\ldots \rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i P = - \ldots$$

$$\ldots \left(\rho v^i v_j \right) + \partial_j P = 0$$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

$$M_1 = \frac{v_1}{c_{s1}} \qquad M_1^2 = v_1^2 \frac{\rho_1}{\gamma P_1} \qquad \left(c_s = \sqrt{\frac{\gamma P_1}{\rho_1}} \right)$$

$$= -\partial_i P$$

$$\rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i P = -\partial_i P$$

$$(\rho v^i v_j) + \partial_j P = 0$$

$$P_2 = \frac{\gamma - 1}{2\gamma} \rho_2 v_1^2 + \frac{P_1}{\rho_1} \rho_2 - \frac{\gamma - 1}{2\gamma} \rho_2 v_2^2$$

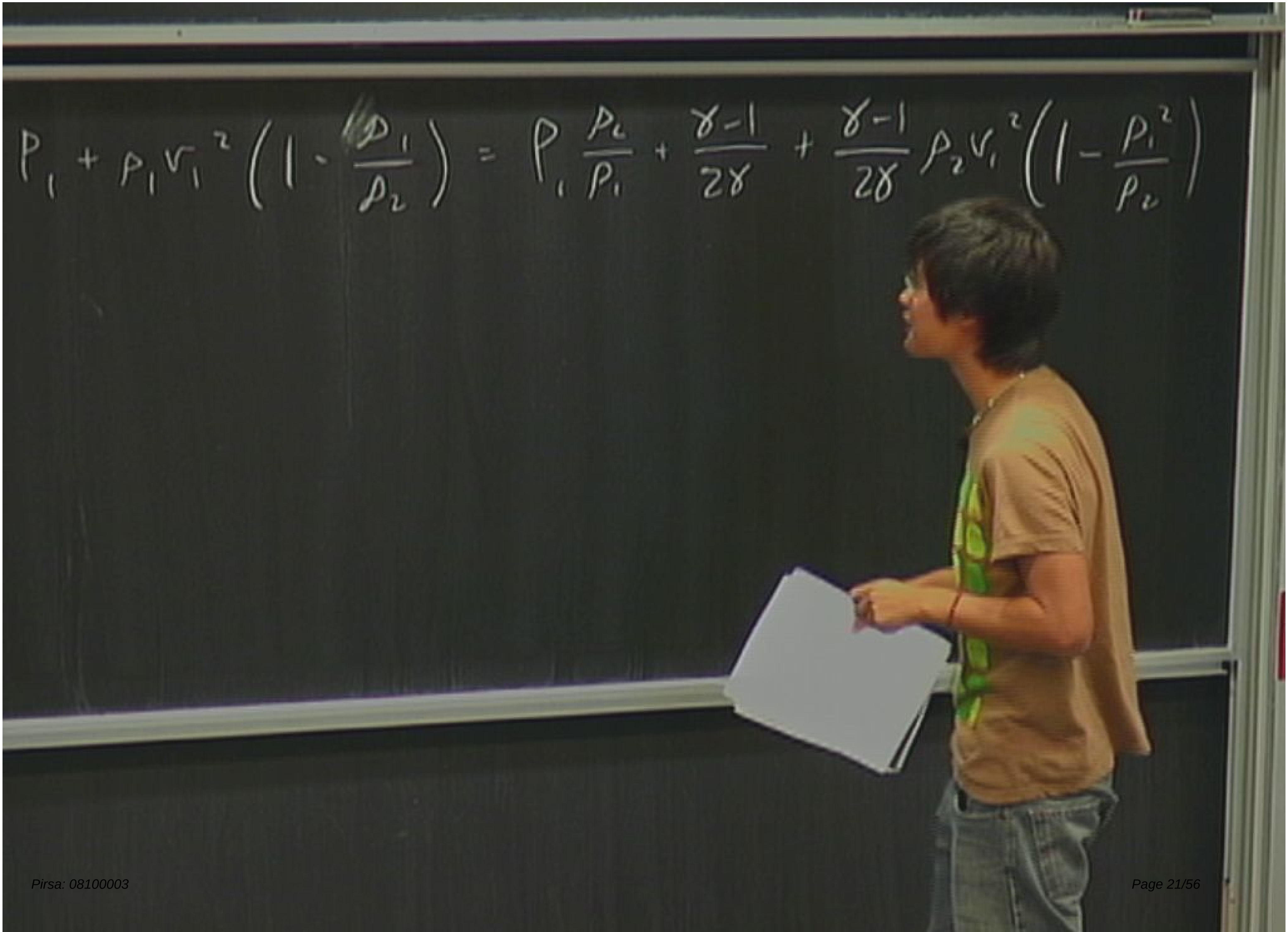
$$= P_1 \frac{\rho_2}{\rho_1} + \frac{\gamma - 1}{2\gamma} \rho_2 \left(v_1^2 - v_2^2 \right)$$

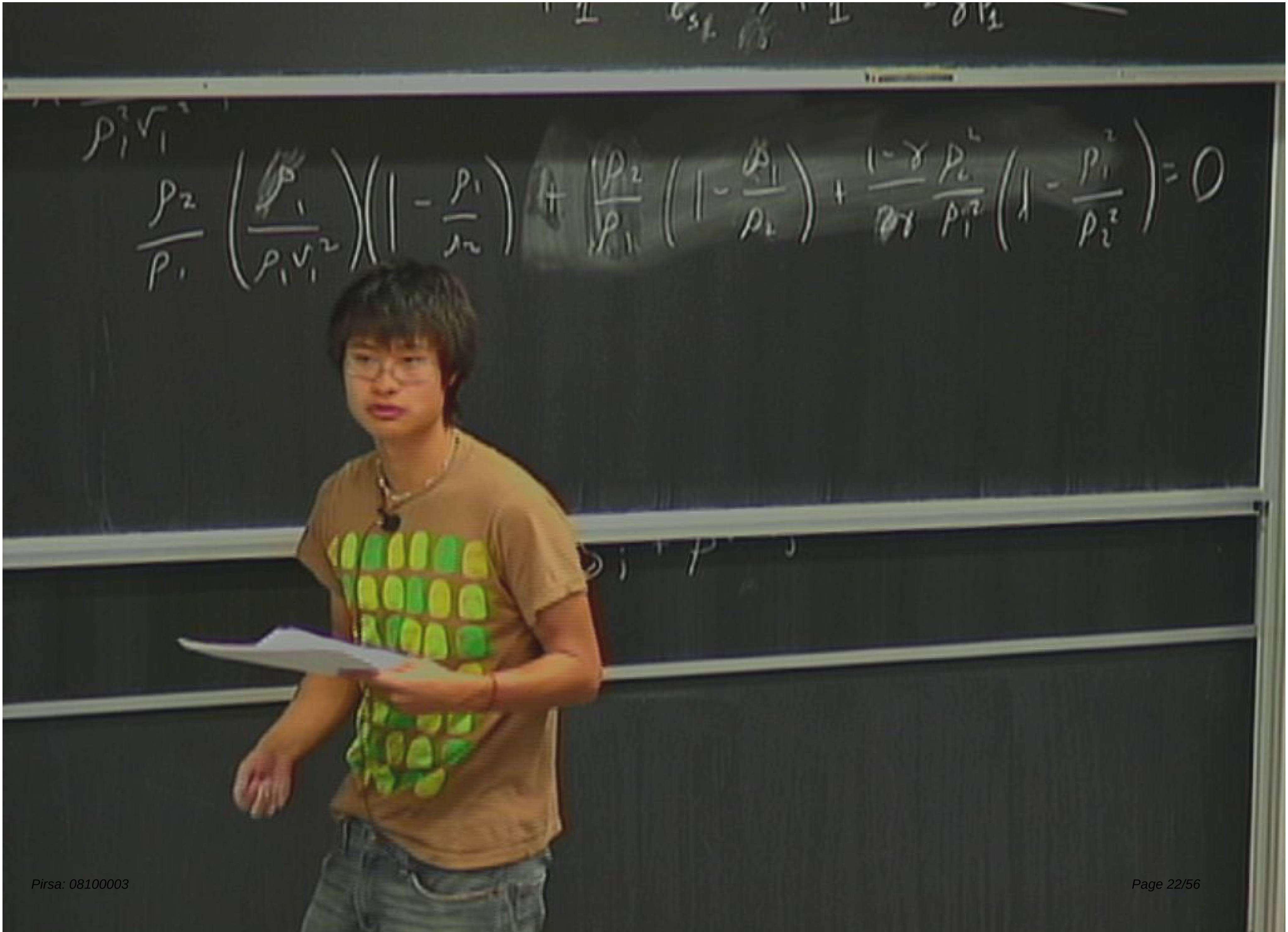
$$P_2 = \frac{\gamma - 1}{2\gamma} \rho_2 v_1^2 + \frac{P_1}{\rho_1} \rho_2 - \frac{\gamma - 1}{2\gamma} \rho_2 v_2^2$$

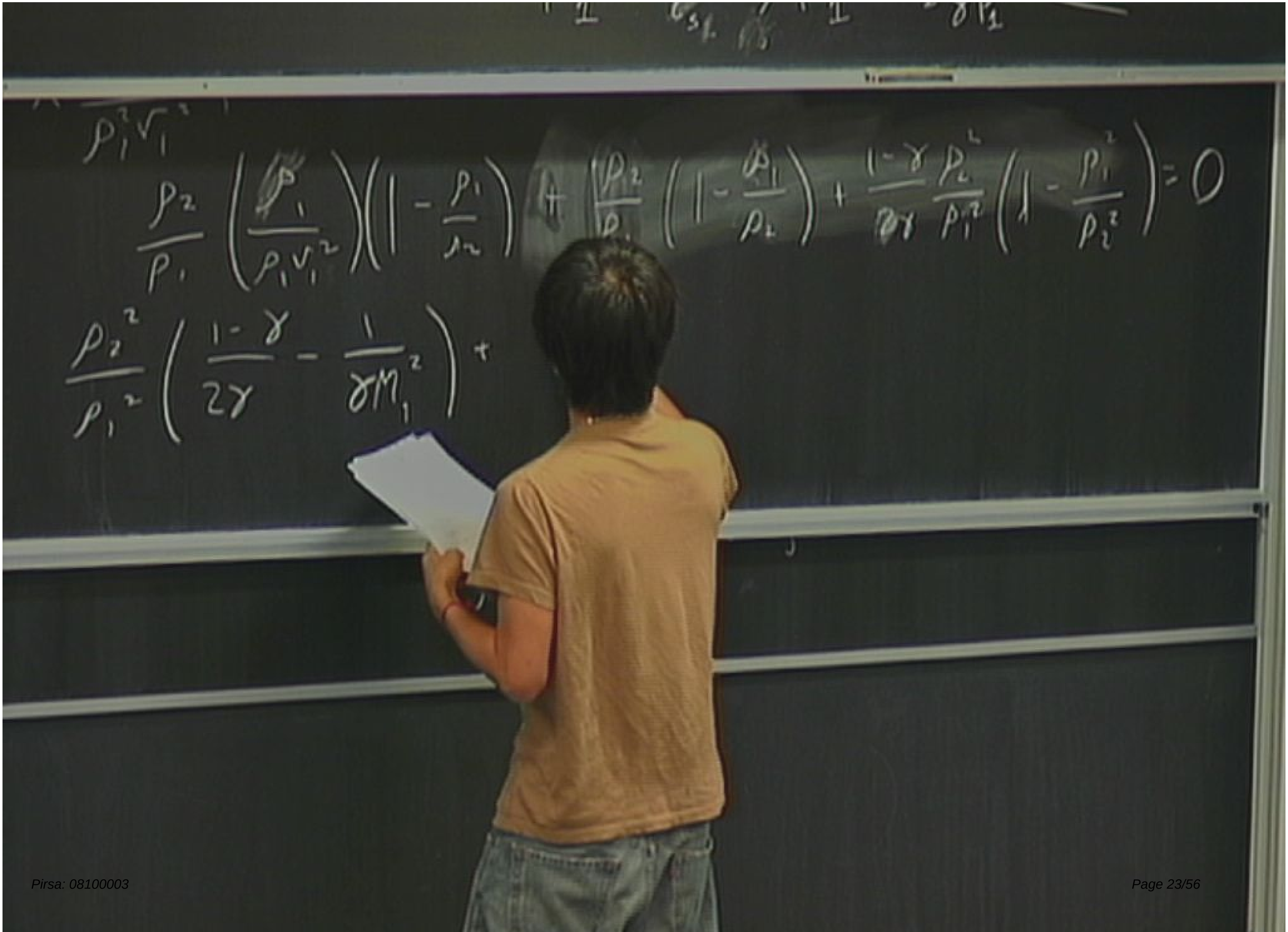
$$= P_1 \frac{\rho_2}{\rho_1} + \frac{\gamma - 1}{2\gamma} \rho_2 \left(v_1^2 - v_2^2 \right)$$

$$P_1 + \rho_1 v_1^2 - \rho_2 v_2^2 = P_1 \frac{\rho_2}{\rho_1} + \frac{\gamma - 1}{2\gamma} \rho_2 \left(v_1^2 - v_2^2 \right)$$

$$v_2^2 = \frac{\rho_1^2 v_1^2}{\rho_2^2}$$


$P_1 + \rho_1 v_1^2 \left(1 - \frac{\rho_1}{\rho_2}\right) = P_1 \frac{\rho_2}{\rho_1} + \frac{\gamma-1}{2\gamma} + \frac{\gamma-1}{2\gamma} \rho_2 v_1^2 \left(1 - \frac{\rho_1^2}{\rho_2}\right)$

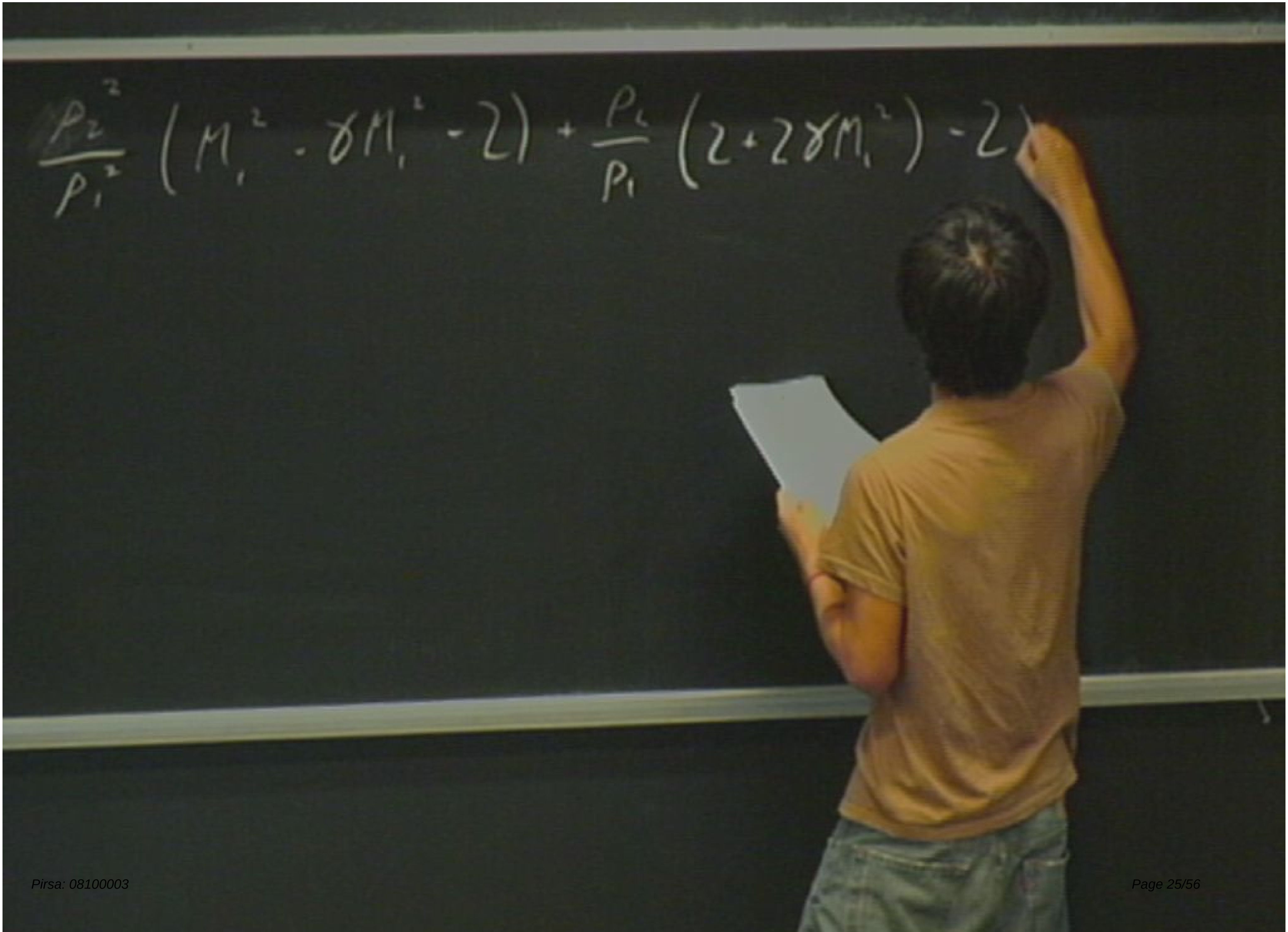


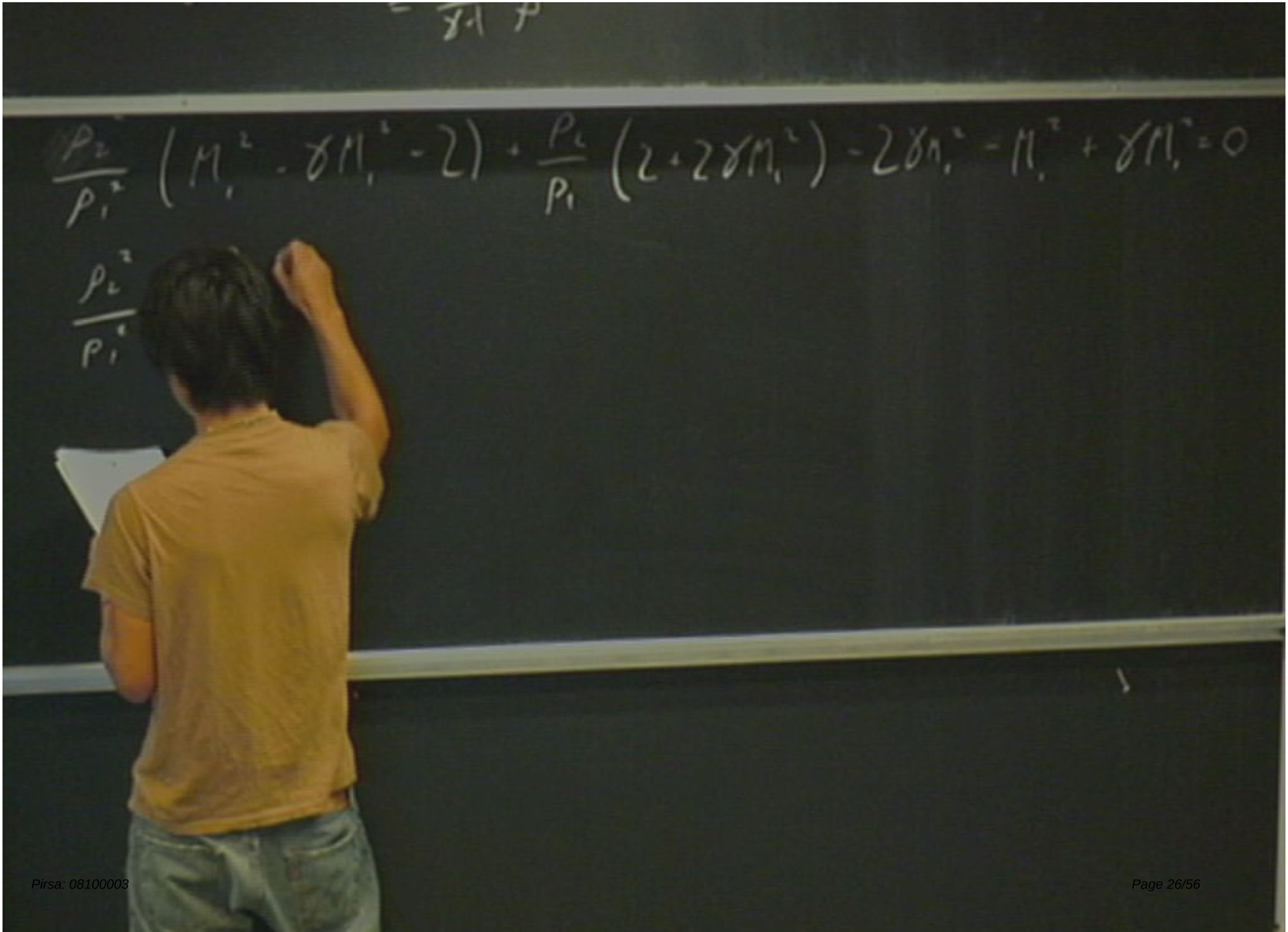

$\frac{\rho_2}{\rho_1}\left(\frac{1}{\rho_1 v_1^2}\right)\left(1-\frac{\rho_1}{\rho_2}\right)+\frac{\rho_2}{\rho_1}\left(1-\frac{\rho_1}{\rho_2}\right)+\frac{1-\gamma}{2\gamma}\frac{\rho_2^2}{\rho_1^2}\left(1-\frac{\rho_1^2}{\rho_2^2}\right)=0$
$\frac{\rho_2^2}{\rho_1^2}\left(\frac{1-\gamma}{2\gamma}-\frac{1}{\gamma M_1^2}\right)+$

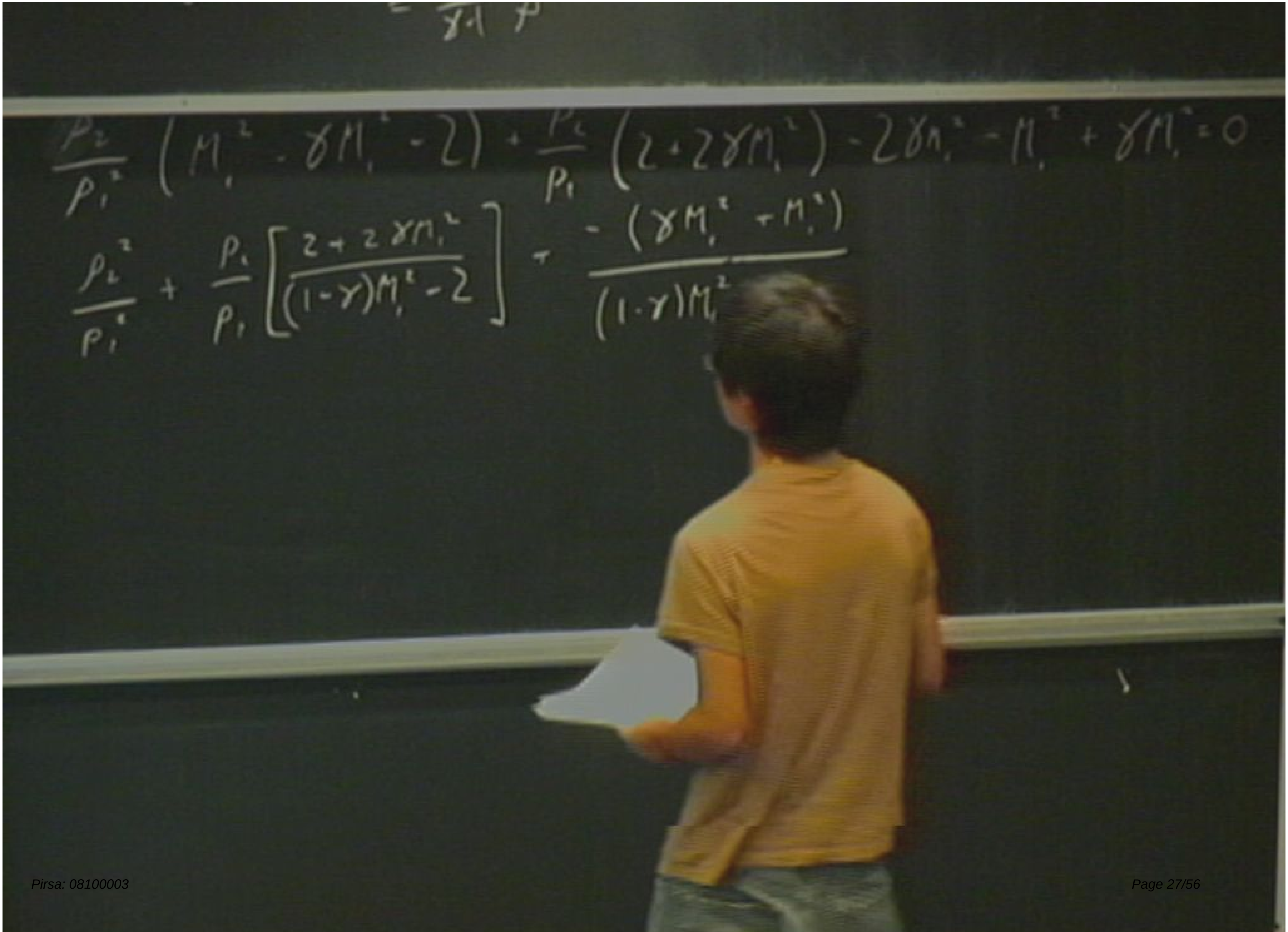
$$\frac{\rho_2}{\rho_1}\left(\frac{p_1}{\rho_1 v_1^2}\right)\left(1-\frac{\rho_1}{\rho_2}\right)+\frac{p_2}{p_1}\left(1-\frac{p_1}{p_2}\right)+\frac{1-\gamma}{2\gamma}\frac{\rho_2^2}{\rho_1^2}\left(1-\frac{\rho_1^2}{\rho_2^2}\right)=0$$

$$\frac{\rho_2^2}{\rho_1^2}\left(\frac{1-\gamma}{2\gamma}-\frac{1}{\gamma M_1^2}\right)+\frac{\rho_2}{\rho_1}\left(\frac{1}{\gamma M_1^2}+1\right)-1-\frac{1-\gamma}{2\gamma}=0$$

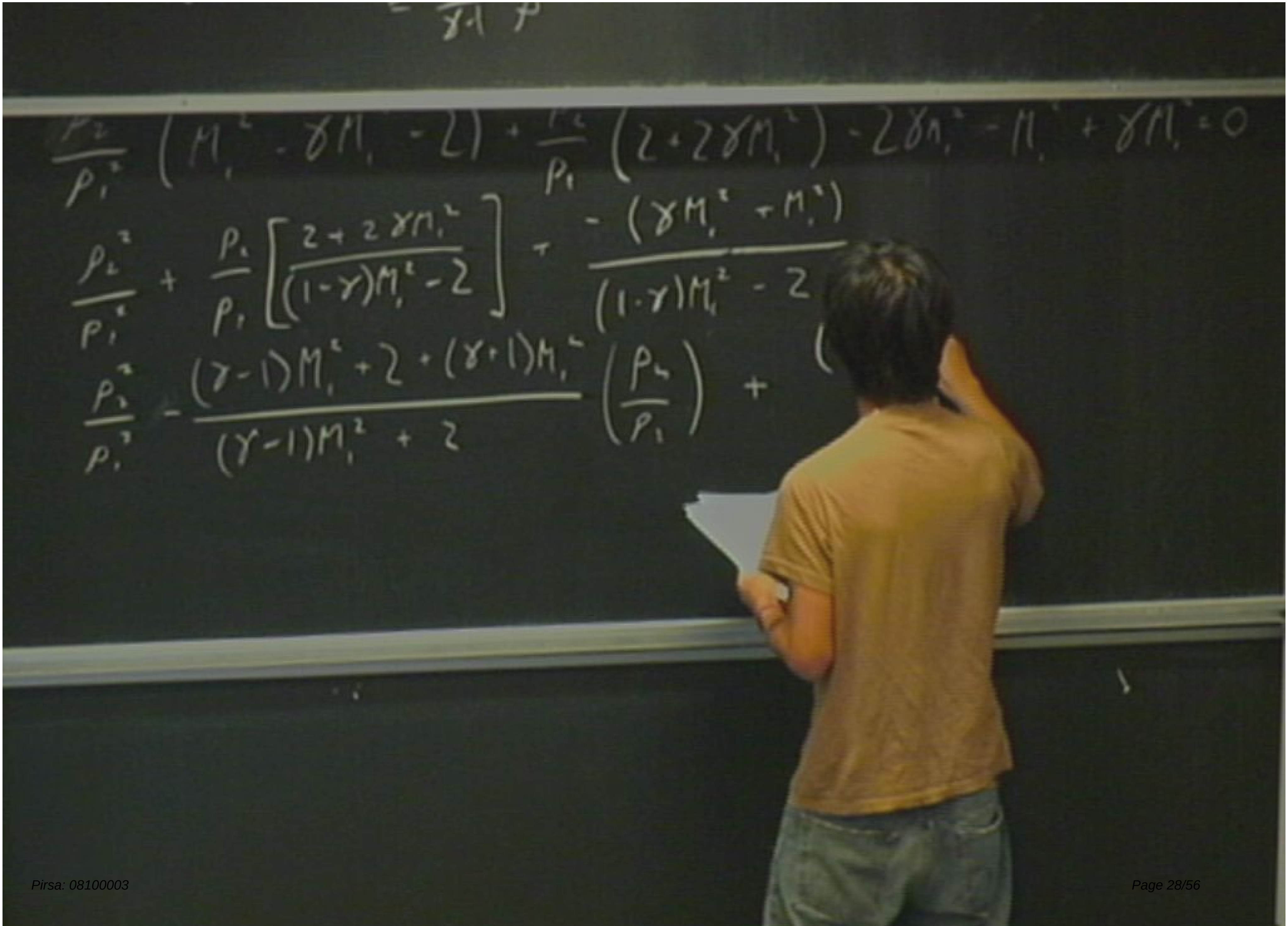
$$T^i_j = P\delta_i + \dots$$


$\frac{\rho_2^2}{\rho_1^2}(M_1^2 - \gamma M_1^2 - 2) + \frac{\rho_2}{\rho_1}(2 + 2\gamma M_1^2) - 2$


$\frac{\rho_2^2}{\rho_1^2}(M_1^2 - \gamma M_1^2 - 2) + \frac{\rho_2}{\rho_1}(2 + 2\gamma M_1^2) - 2\gamma M_1^2 - M_1^2 + \gamma M_1^2 = 0$
$\frac{\rho_2^2}{\rho_1^4}$



$\frac{\rho_2}{\rho_1^2}\left(M_1^2 - \gamma M_1^2 - 2\right) + \frac{\rho_2}{\rho_1}\left(2 + 2\gamma M_1^2\right) - 2\gamma M_1^2 - M_1^2 + \gamma M_1^2 = 0$
$\frac{\rho_2^2}{\rho_1^4} + \frac{\rho_2}{\rho_1}\left[\frac{2 + 2\gamma M_1^2}{(1-\gamma)M_1^2 - 2}\right] + \frac{-(\gamma M_1^2 + M_1^2)}{(1-\gamma)M_1^2}$

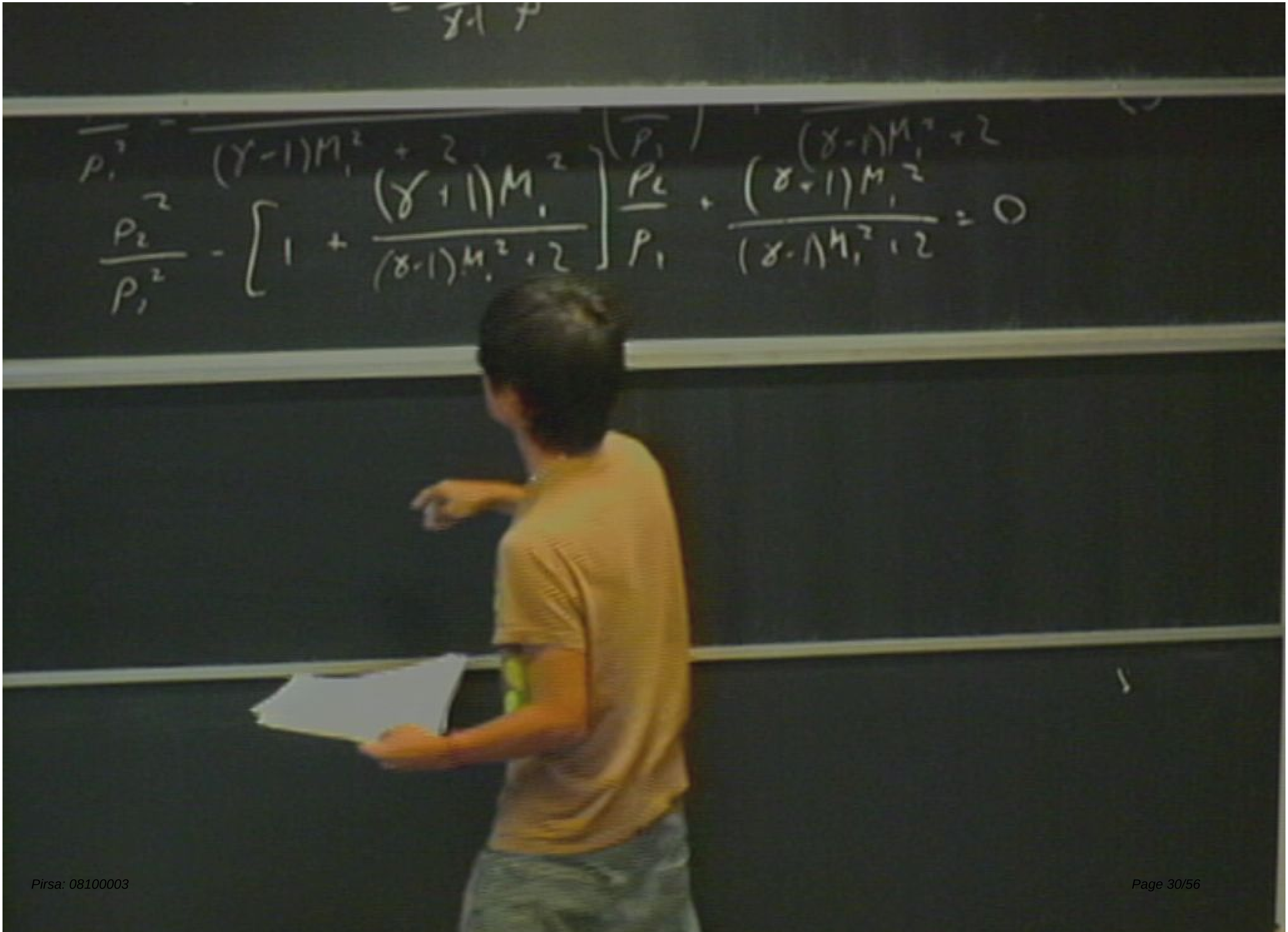

$\frac{p_2}{p_1^2}(M_1^2 - \gamma M_1 - 2) + \frac{p_2}{p_1}(2+2\gamma M_1^2) - 2\gamma M_1^2 - M_1^2 + \gamma M_1^2 = 0$
$\frac{p_2^2}{p_1^2} + \frac{p_2}{p_1}\left[\frac{2+2\gamma M_1^2}{(1-\gamma)M_1^2-2}\right] + \frac{-(\gamma M_1^2 + M_1^2)}{(1-\gamma)M_1^2-2}$
$\frac{p_2^2}{p_1^2} - \frac{(\gamma-1)M_1^2 + 2 + (\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}\left(\frac{p_2}{p_1}\right) + ($

$$\frac{p_2^2}{p_1^2}\left(M_1^2 - \gamma M_1^2 - 2\right) + \frac{p_2}{p_1}\left(2 + 2\gamma M_1^2\right) - 2\gamma M_1^2 - M_1^2 + \gamma M_1^2 = 0$$

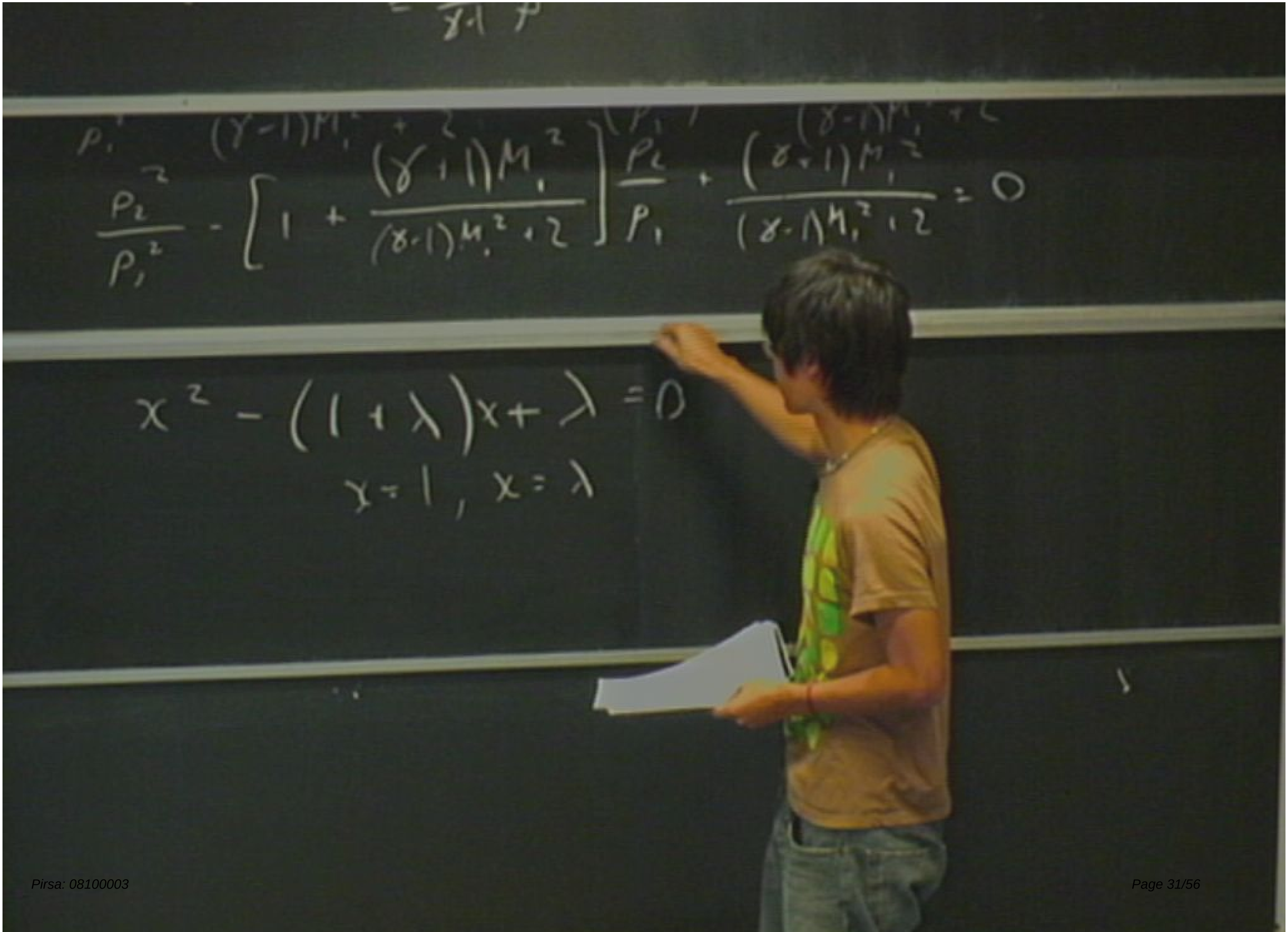
$$\frac{p_2^2}{p_1^2} + \frac{p_2}{p_1}\left[\frac{2 + 2\gamma M_1^2}{(1-\gamma)M_1^2 - 2}\right] + \frac{-(\gamma M_1^2 + M_1^2)}{(1-\gamma)M_1^2 - 2} = 0$$

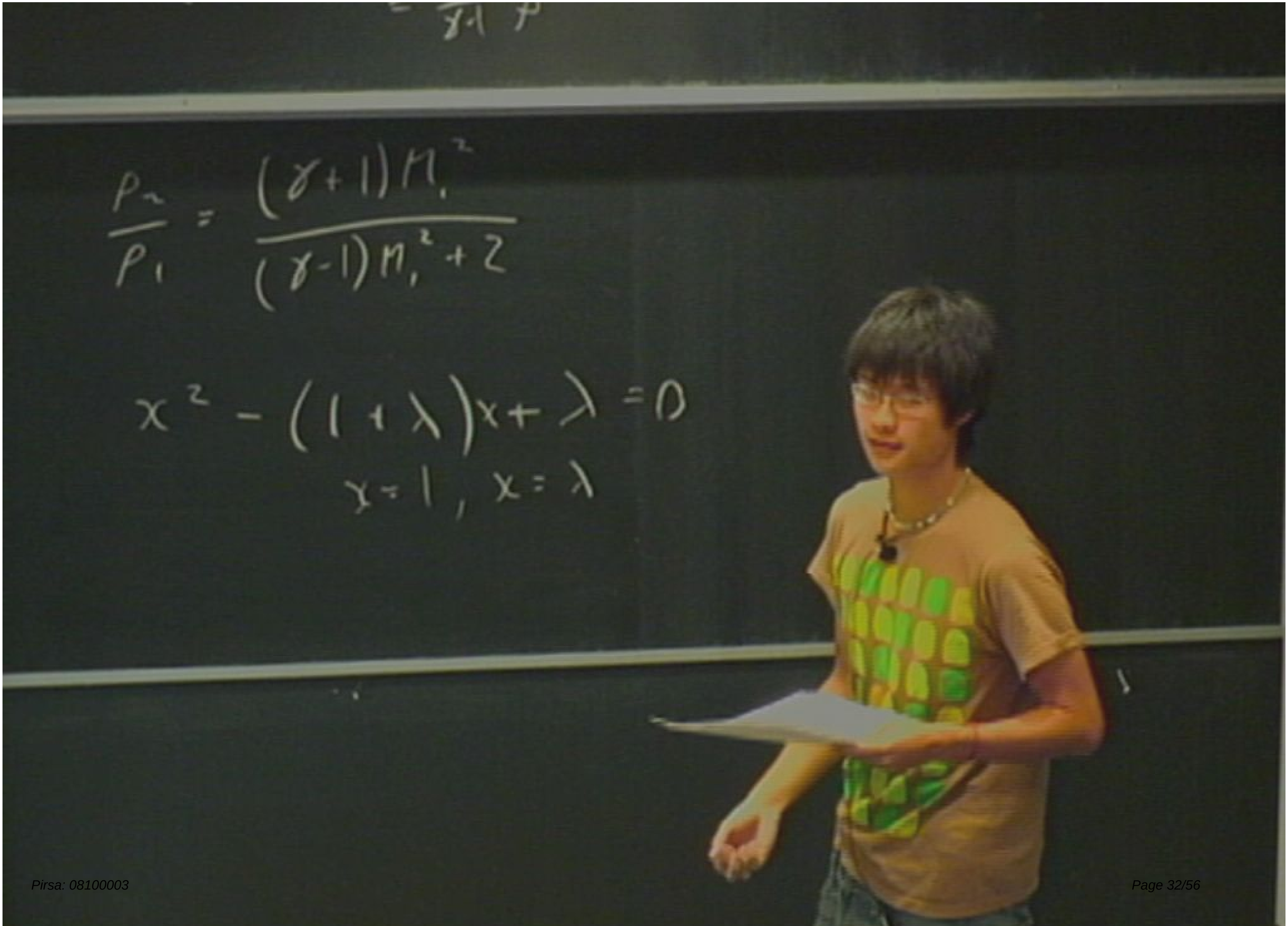
$$\frac{p_2^2}{p_1^2} - \frac{(\gamma-1)M_1^2 + 2 + (\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}\left(\frac{p_2}{p_1}\right) + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2} = 0$$

p_2



$\frac{p_2^2}{p_1^2} - \left[1 + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]\frac{p_2}{p_1} + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} = 0$


$\frac{\rho_2^2}{\rho_1^2} - \left[1 + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]\frac{\rho_2}{\rho_1} + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} = 0$
$x^2 - (1+\lambda)x + \lambda = 0$
$x=1,\ x=\lambda$


$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$
$x^2 - (1+\lambda)x + \lambda = 0$
$x = 1, \; x = \lambda$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}, \quad \frac{v_2}{v_1} = \frac{\rho_1}{\rho_2}$$

$$x^2 - (1+\lambda)x + \lambda = 0$$

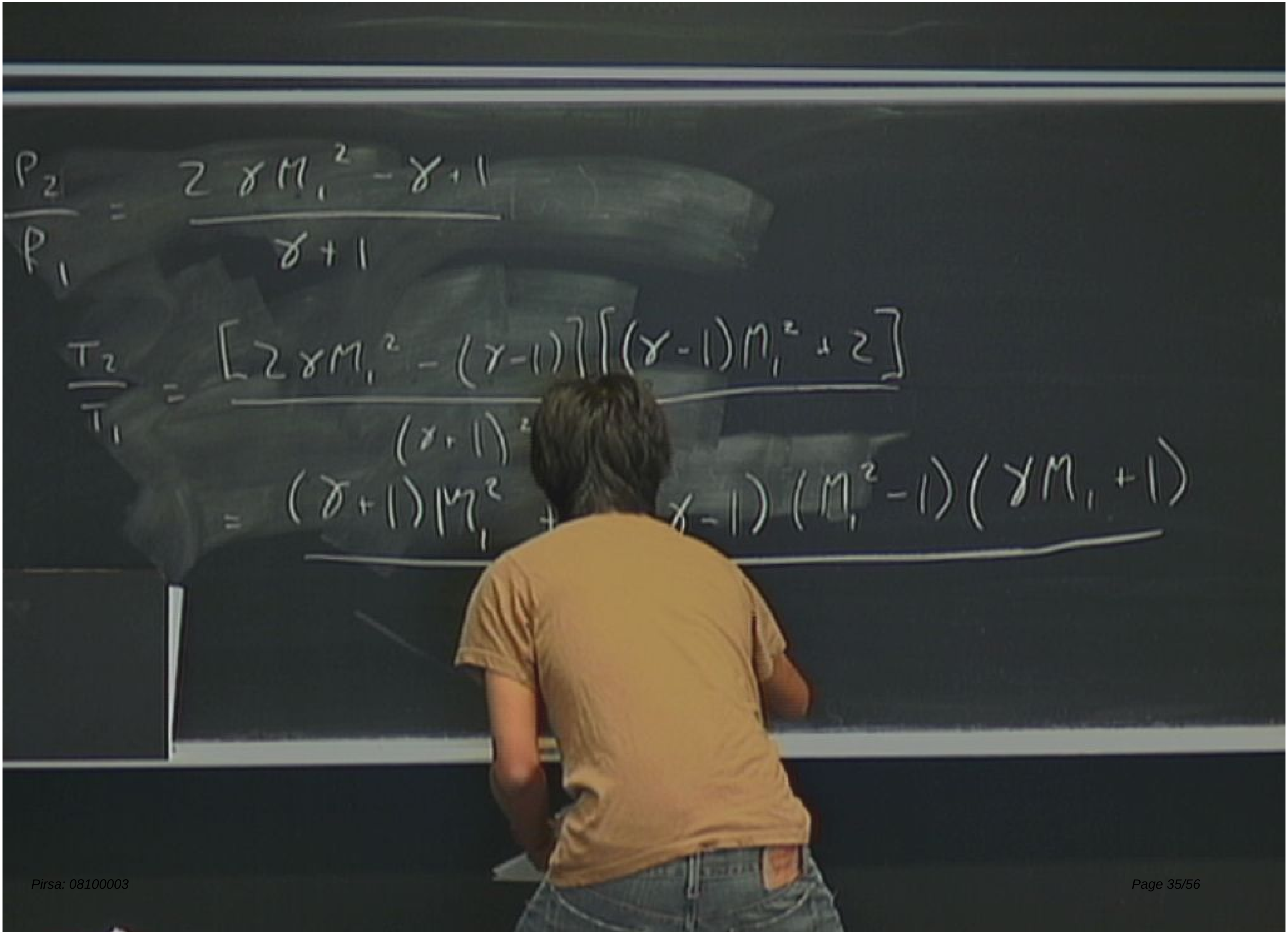
$$x = 1, \quad x = \lambda$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}, \quad \frac{v_2}{v_1} = \frac{\rho_1}{\rho_2}$$

$$\frac{RT_2}{RT_1} = \frac{P_2}{\rho_2}\left(\frac{\rho_1}{P_1}\right)$$

$$x^2 - (1+\lambda)x + \lambda = 0$$

$$x = 1, \quad x = \lambda$$



$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$
$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma - 1)][(\gamma - 1)M_1^2 + 2]}{(\gamma + 1)^2}$
$= (\gamma + 1)M_1^2$
$\gamma - 1)(M_1^2 - 1)(\gamma M_1 + 1)$

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$$

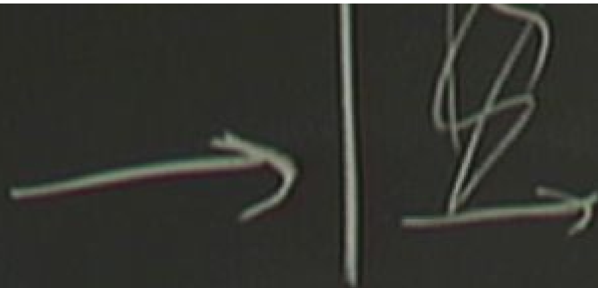
$$\frac{T_2}{T_1} = \frac{\left[2\gamma M_1^2 - (\gamma - 1)\right]\left[(\gamma - 1) M_1^2 + 2\right]}{(\gamma + 1)^2 M_1^2}$$

$$= \frac{(\gamma + 1) M_1^2 + 2(\gamma - 1)(M_1^2 - 1)(\gamma M_1 + 1)}{(\gamma + 1) M_1^2}$$

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{\left[2\gamma M_1^2 - (\gamma-1)\right]\left[(\gamma-1)M_1^2 + 2\right]}{(\gamma+1)^2 M_1^2}$$

$$= \frac{(\gamma+1)M_1^2 + 2(\gamma-1)(M_1^2-1)(\gamma M_1 + 1)}{(\gamma+1)M_1^2}$$



$v_1 > v_s$

ρ_1 , ρ_2

$$\gamma = \frac{C_p}{C_v}$$

$P_1 + \rho_1$

M_1

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \qquad , \quad \frac{v_2}{v_1} = \ldots$$

$$= \frac{\gamma+1}{(\gamma-1) + \frac{2}{M_1^2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \quad , \quad \frac{v_2}{v_1} =$$

$$= \frac{\gamma+1}{\left(\gamma-1+\frac{2}{M_1^2}\right)}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\,, \quad \frac{v_2}{v_1} = $$

$$= \frac{\gamma+1}{(\gamma-1)+\frac{2}{M_1^2}} \simeq \frac{\gamma+1}{\gamma-1}$$

$$(\gamma-1)M_1^2 + 2 \qquad \qquad p_2$$

$$= \frac{\gamma+1}{\gamma-1+\frac{2}{M_1^2}} \simeq \frac{\gamma+1}{\gamma-1} \qquad \frac{R\bar{T}}{RT}$$

$$\frac{\frac{7}{3}}{\frac{1}{3}} = 7$$

$$\rho v^2 + P - \mathbb{P} = \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma \cancel{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2} \eta \left(2 V_{xx} - \frac{2}{3} \frac{\partial v}{\partial x} \right)^2 + \zeta \left(\frac{\partial v}{\partial x} \right)^2$$

$$\rho v^2 + P - \mathbb{P} = \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta\left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \zeta\left(\frac{\partial v}{\partial x}\right)^2, \quad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$= \frac{1}{2}\eta\left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \zeta \frac{\partial v}{\partial x}$$

$$\simeq (\eta + \zeta)\frac{\partial v}{\partial x}$$

$$\rho v^2 + P - \mathbb{P} = \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma \cancel{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta\left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \zeta\left(\frac{\partial v}{\partial x}\right)^2, \quad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$\sigma^{xx} = \frac{1}{2}\eta\left(\frac{4}{3}\right)^2\frac{\partial v}{\partial x} + \zeta\frac{\partial v}{\partial x}$$

$$\simeq (\eta + \zeta)\frac{\partial v}{\partial x}$$

$$\rho v^2 + P - \mathbb{P} = \text{const.} \Rightarrow \rho v^2 + P - \eta \frac{\partial v}{\partial x} \sim \text{const}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta\left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \xi\left(\frac{\partial v}{\partial x}\right)^2, \quad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$\sigma^{xx} = \frac{1}{2}\eta\left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \xi \frac{\partial v}{\partial x}$$

$$\simeq (\eta + \xi)\frac{\partial v}{\partial x}, \quad \xi \simeq \eta \text{ or } \xi \ll \eta$$

$$\rho v^2 + P - \mathbb{P} = \text{const.} \Rightarrow \rho v^2 + P - \eta \frac{\partial v}{\partial z} \sim \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma \cancel{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2} \eta \left(2V_{xx} - \frac{2}{3} \frac{\partial v}{\partial x} \right)^2 + \xi \left(\frac{\partial v}{\partial x} \right)^2, \quad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$\sigma^{xx} = \frac{1}{2} \eta \left(\frac{4}{3} \right)^2 \frac{\partial v}{\partial x} + \xi \frac{\partial v}{\partial x}$$

$$\simeq (\eta + \xi) \frac{\partial v}{\partial x}, \quad \xi \approx \eta \text{ or } \xi \ll \eta$$

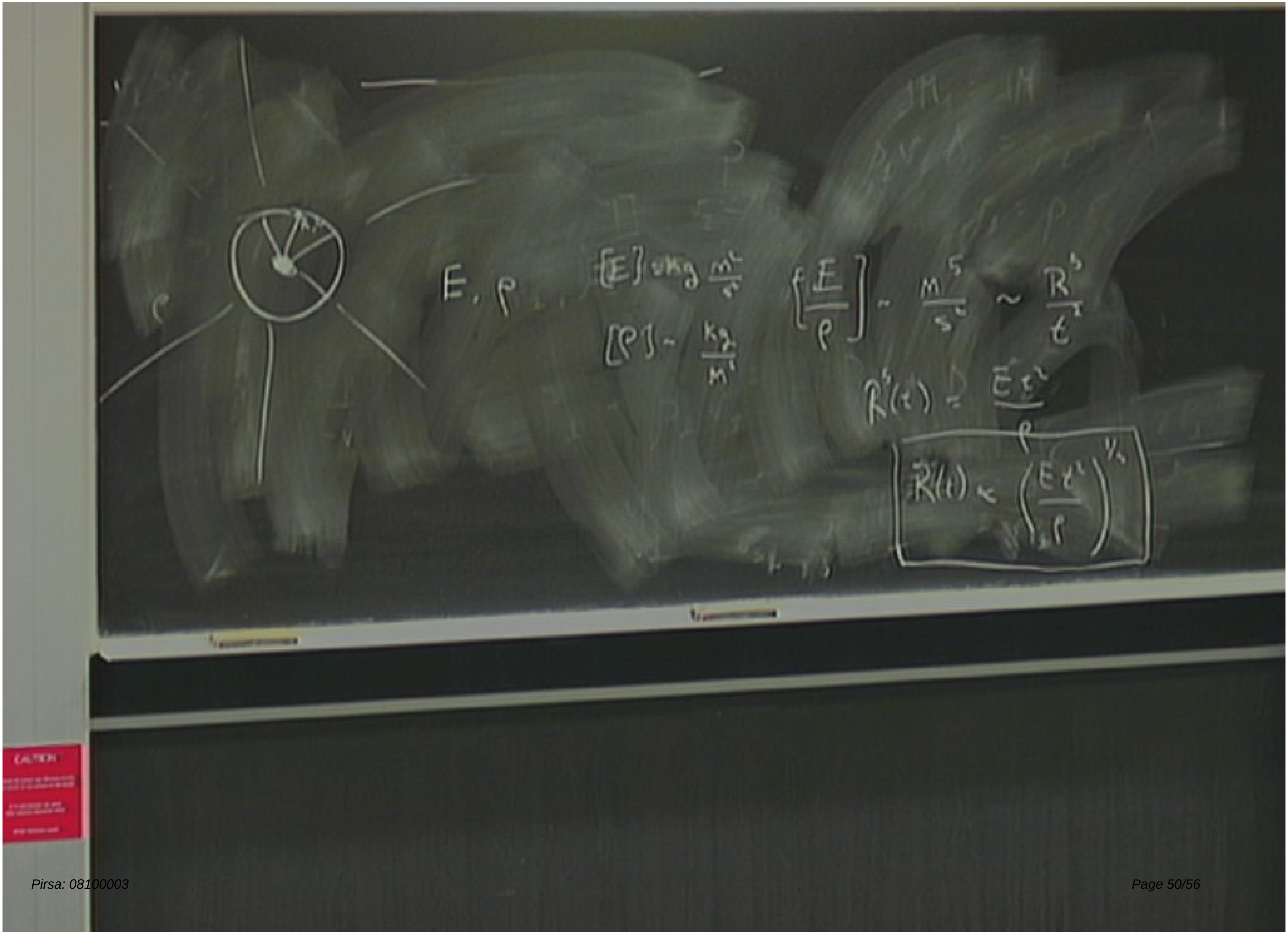
$$\eta \frac{\partial v}{\partial x} = \rho v^2, \quad \frac{\Delta v}{\Delta x}, \quad \phi = \frac{\eta}{\rho}$$

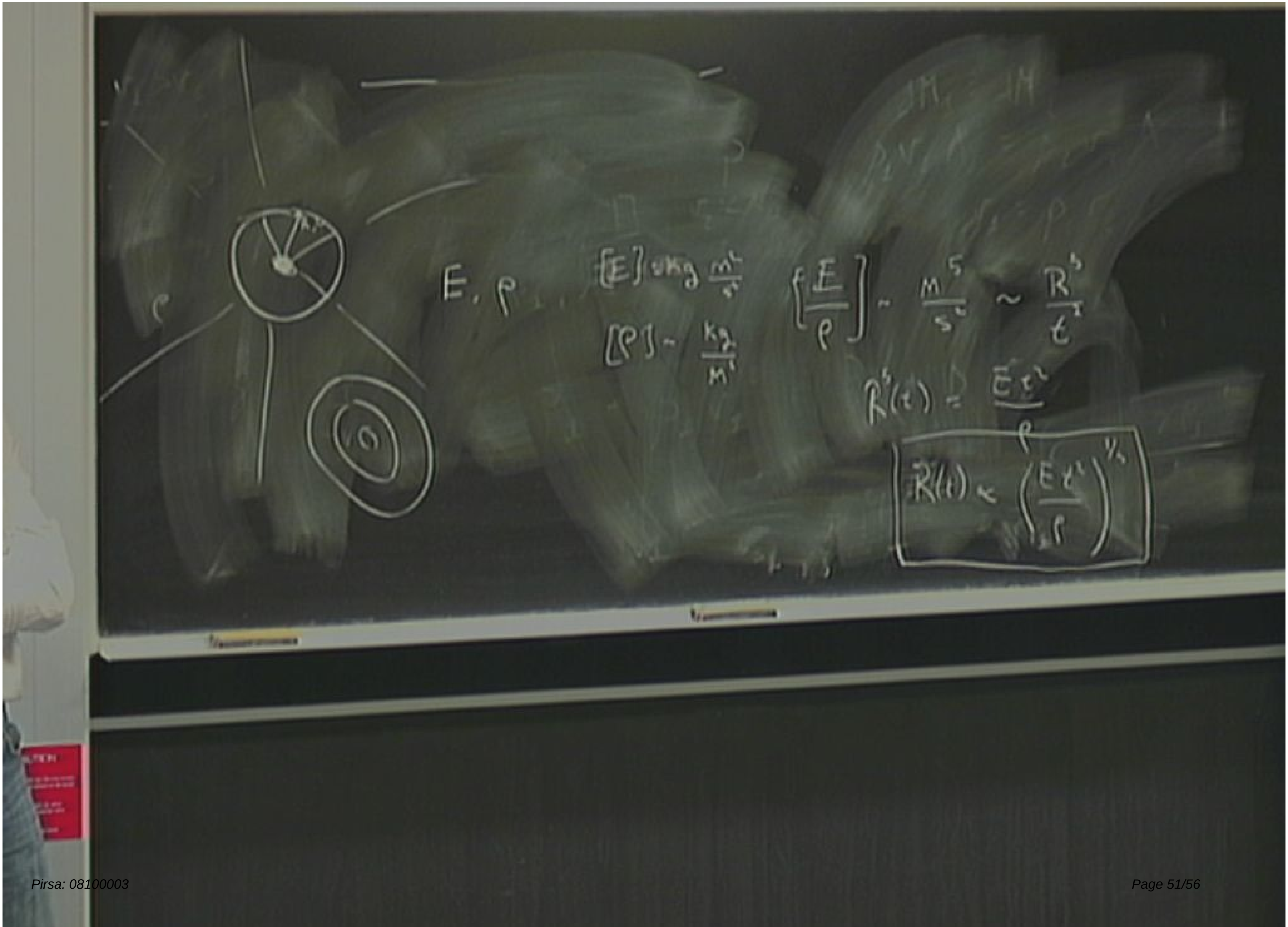
$$\Delta x \simeq \phi \frac{\Delta v}{v^2} \qquad \phi \simeq l v$$

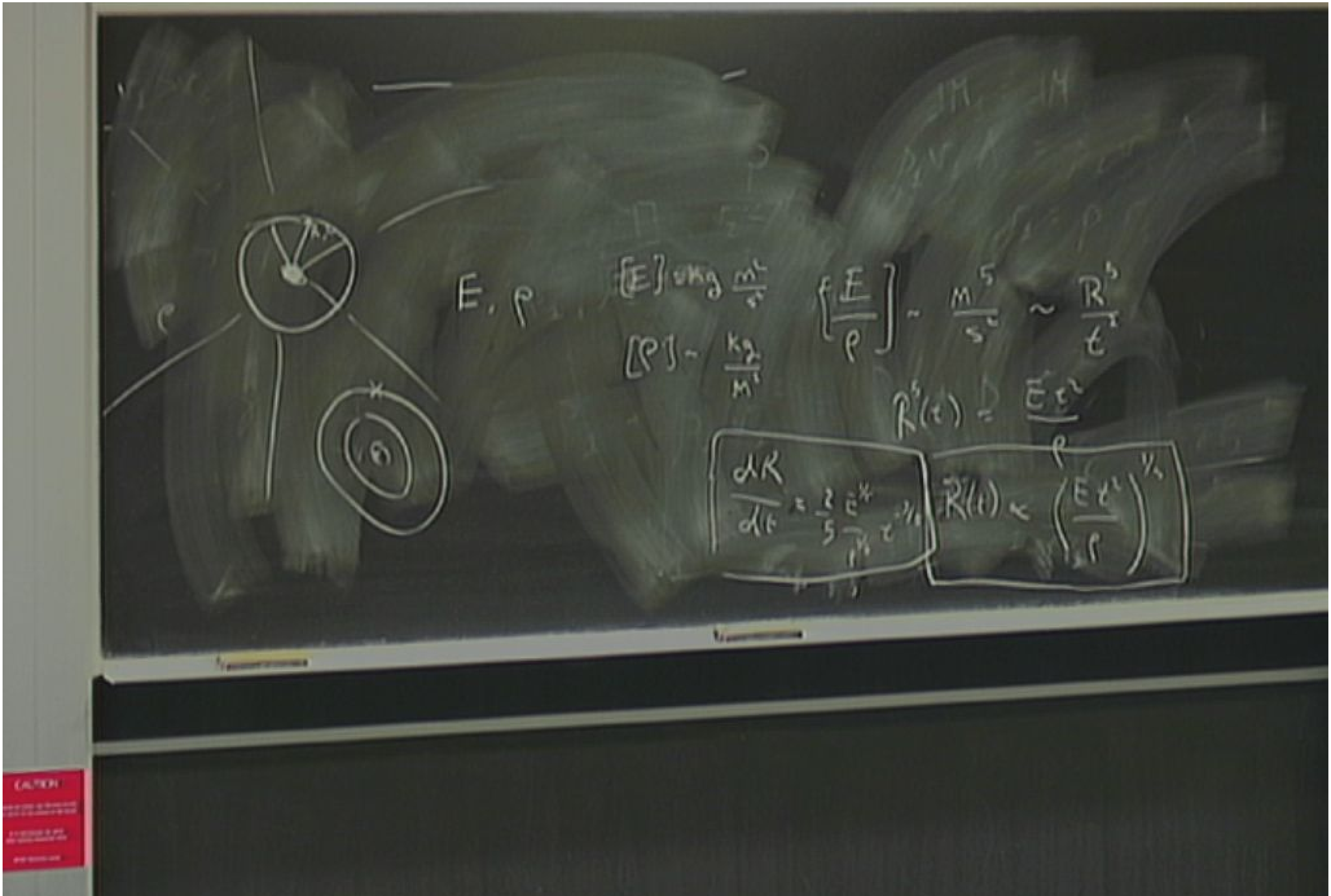
$$\eta \frac{\partial v}{\partial x} = \rho v^2 \qquad \frac{\Delta v}{\Delta x} \qquad \phi = \frac{\eta}{\rho}$$

$$\Delta x \simeq \phi \frac{\Delta v}{v} \qquad \phi \simeq l v$$

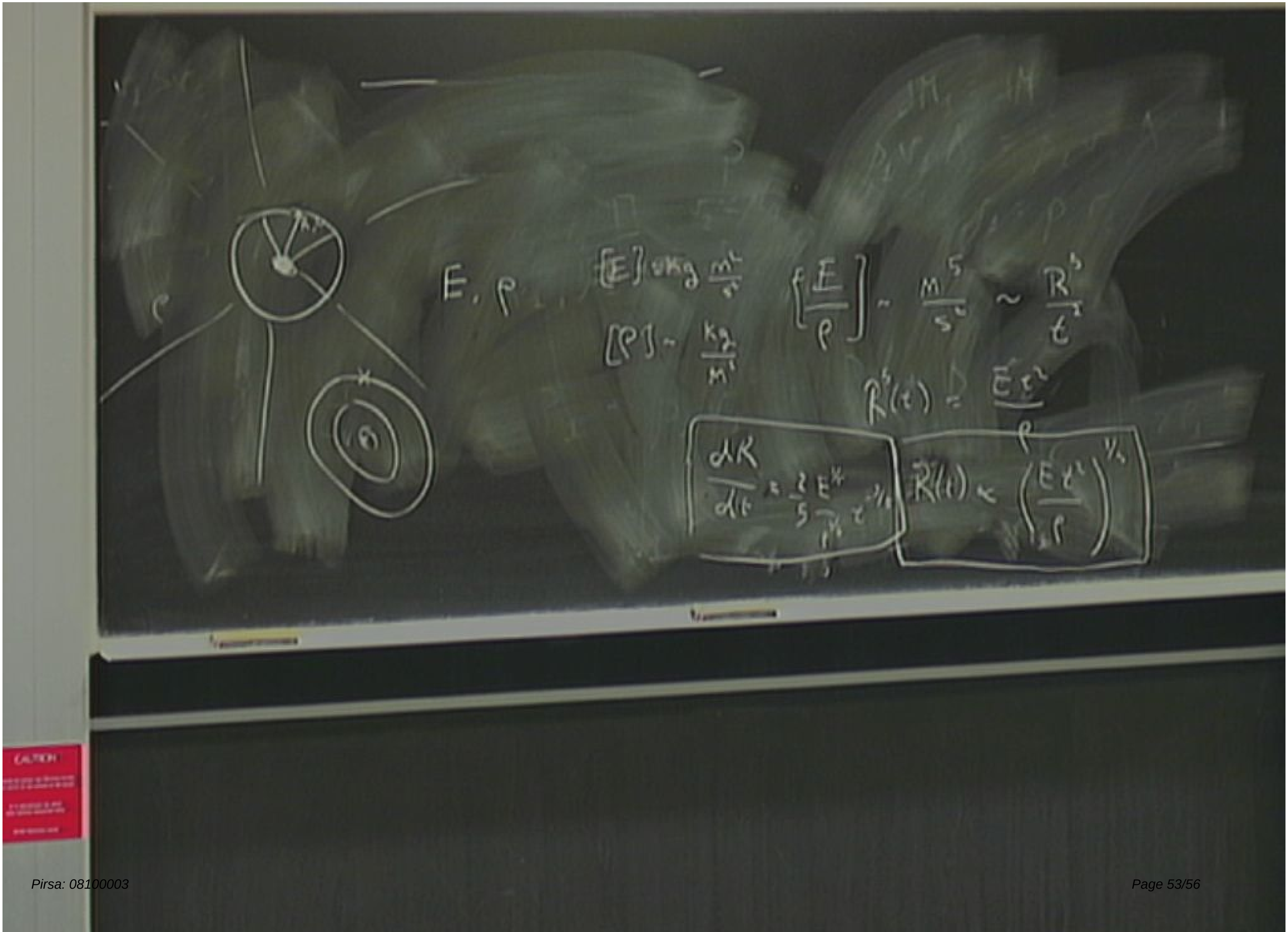
$$\Delta x \simeq l \qquad \frac{M^2}{S}$$


E, ρ
$[E] \sim kg \frac{m^2}{s^2}$
$[\rho] \sim \frac{kg}{m^3}$
$\left[\frac{E}{\rho}\right] \sim \frac{m^5}{s^2} \sim \frac{R^5}{t^2}$
$R^5(t) \doteq \frac{E t^2}{\rho}$
$\boxed{R(t) \sim \left(\frac{E t^2}{\rho}\right)^{1/5}}$


E, ρ
$[E] \sim kg\,\frac{m^2}{s^2}$
$[\rho] \sim \frac{kg}{m^3}$
$\left[\frac{E}{\rho}\right] \sim \frac{m^5}{s^2} \sim \frac{R^5}{t^2}$
$R^5(t) \stackrel{?}{=} \frac{E t^2}{\rho}$
$\boxed{R(t) \sim \left(\frac{E t^2}{\rho}\right)^{1/5}}$



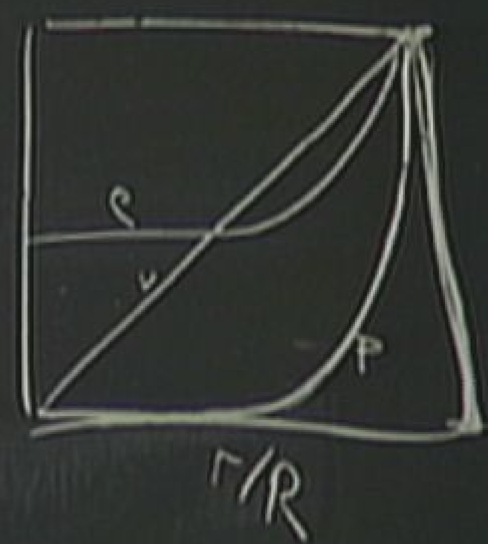
E, ρ
$[E] \sim kg \frac{m^2}{s^2}$
$[\rho] \sim \frac{kg}{m^3}$
$\left[\frac{E}{\rho}\right] \sim \frac{m^5}{s^2} \sim \frac{R^5}{t^2}$
$R^5(t) \overset{?}{=} \frac{E t^2}{\rho}$
$\frac{dR}{dt} = \frac{2}{5} \frac{E^{1/5}}{\rho^{1/5}} t^{-3/5}$
$R(t) \propto \left(\frac{E t^2}{\rho}\right)^{1/5}$


E, ρ
[E] ~ kg m²/s²
[ρ] ~ kg/m³
[E/ρ] ~ m⁵/s² ~ R⁵/t²
R⁵(t) = E t²/ρ
dR/dt = 2/5 E^(1/5)/ρ^(1/5) t^(-3/5)
R(t) ∝ (E t²/ρ)^(1/5)

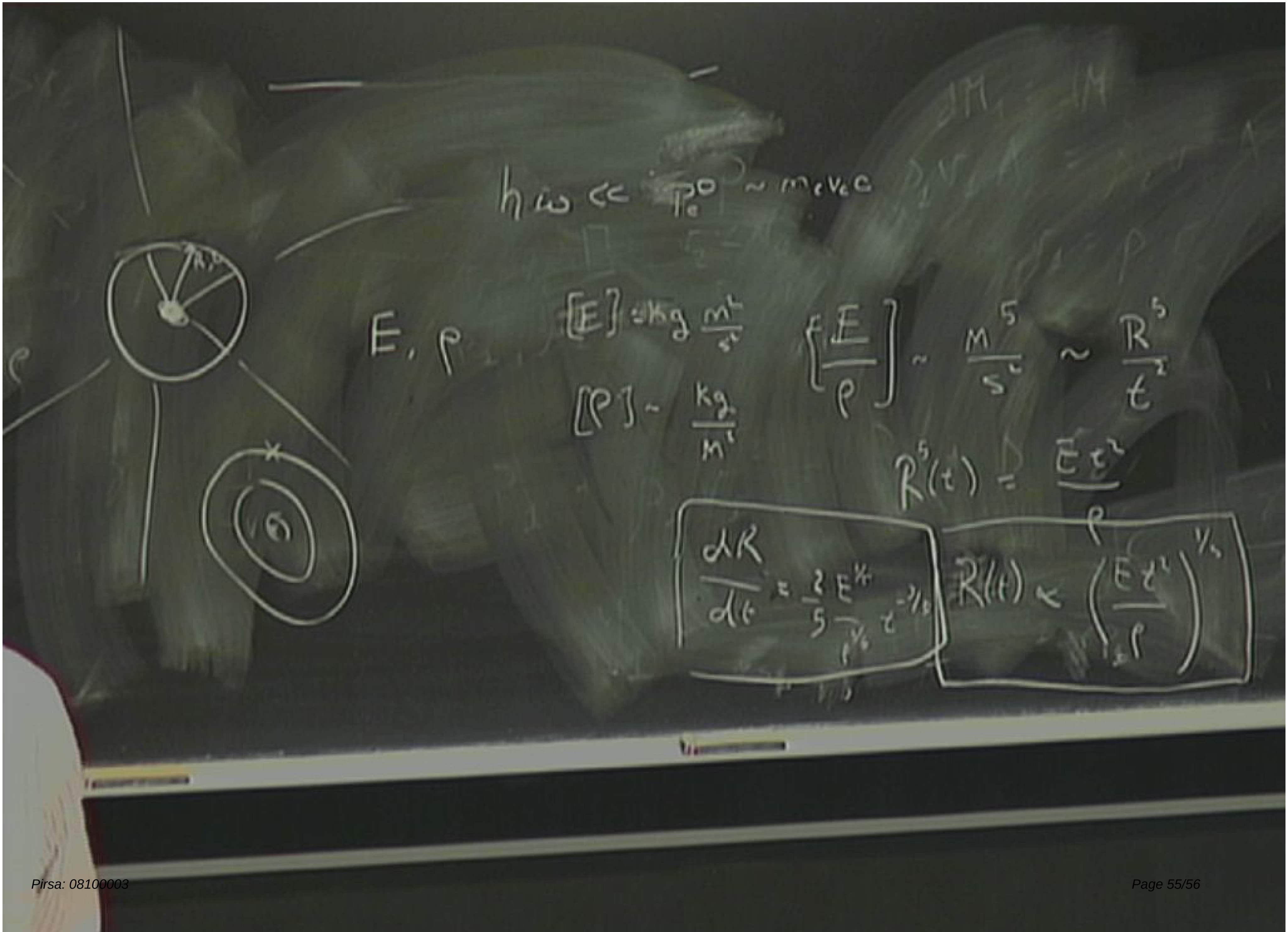
$$\rho_2 \left(v_1^2 - v_2^2\right)$$

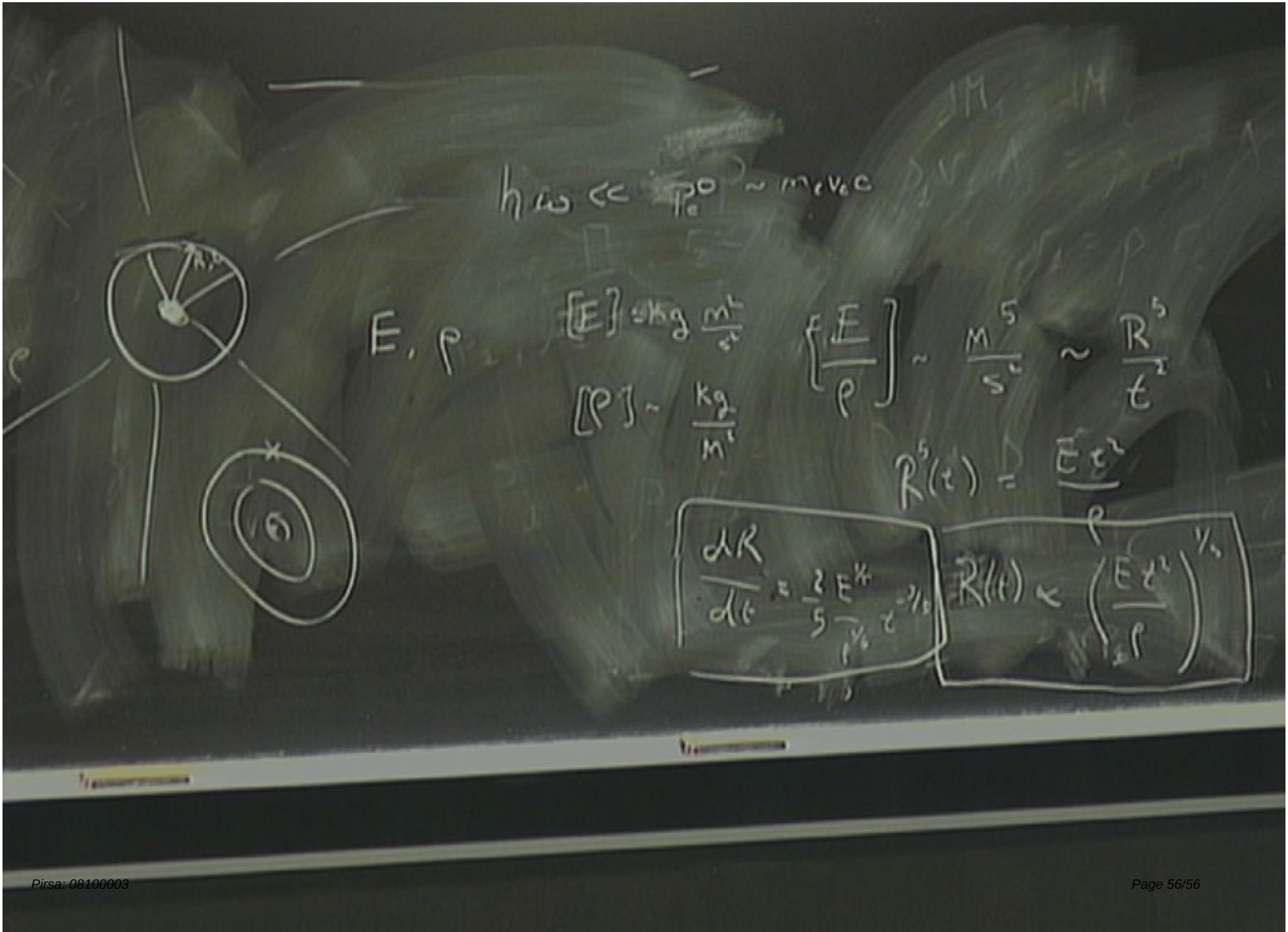
$$= P_1 \frac{\rho_2}{\rho_1} + \frac{\gamma - 1}{2\gamma} \rho_2 \left(v_1^2 - v_2^2\right)$$

$$\frac{\rho_1^2 v_1^2}{\rho_2^2}$$



$$\big) \, S) \, \partial_t$$


ħω << p_e^0 ~ m_e v_e c
E, ρ
[E] ~ kg m²/s²
[ρ] ~ kg/m³
[E/ρ] ~ m⁵/s² ~ R⁵/t²
R⁵(t) ≈ Et²/ρ
dR/dt = 2/5 E^{1/5}/ρ^{1/5} t^{-3/5}
R(t) ∝ (Et²/ρ)^{1/5}



$\hbar\omega \ll p_e^0 \sim m_e v_e c$
E, ρ
$[E] \sim kg\, \frac{m^2}{s^2}$
$[\rho] \sim \frac{kg}{m^3}$
$\left[\frac{E}{\rho}\right] \sim \frac{m^5}{s^2} \sim \frac{R^5}{t^2}$
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