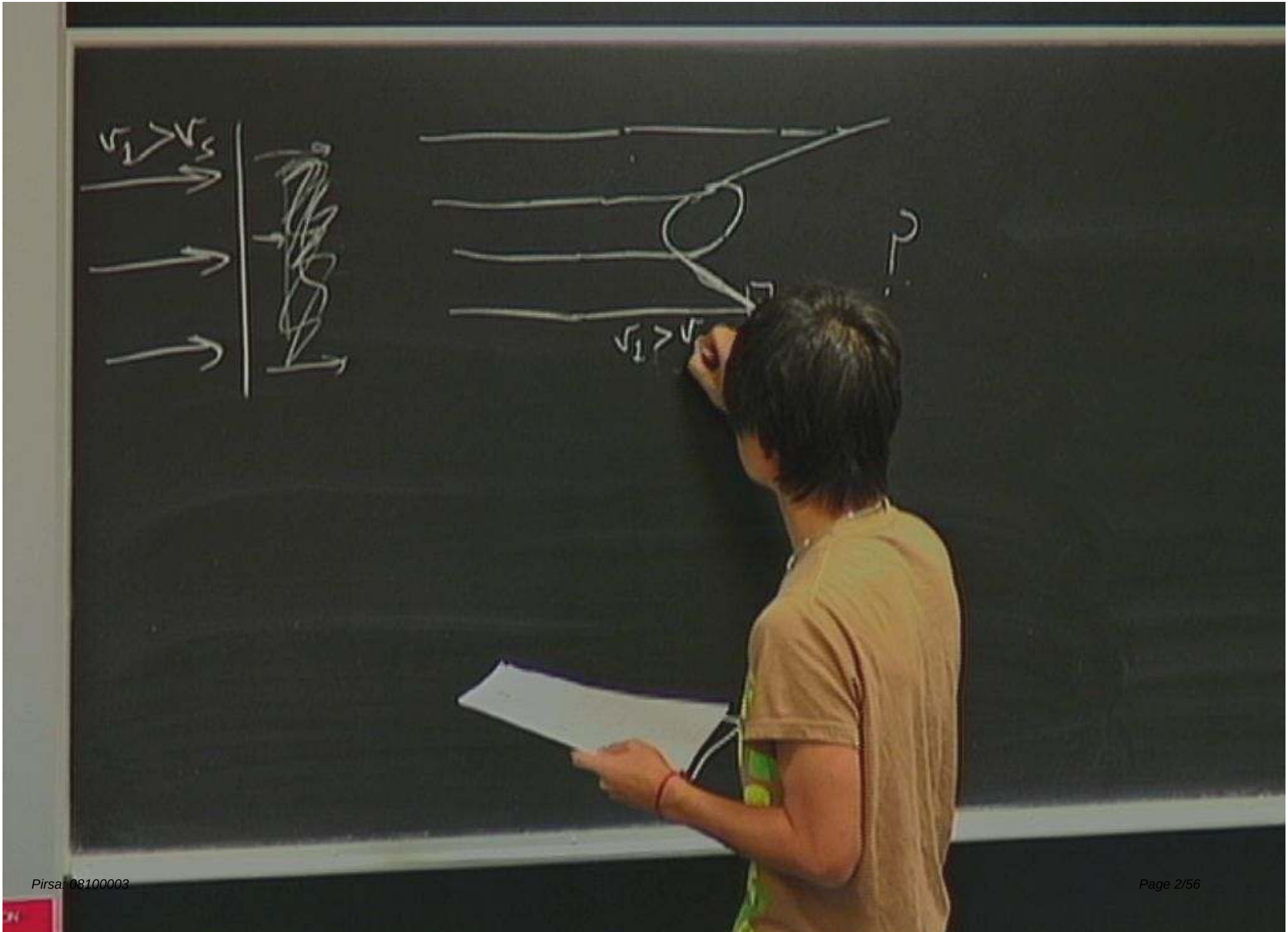
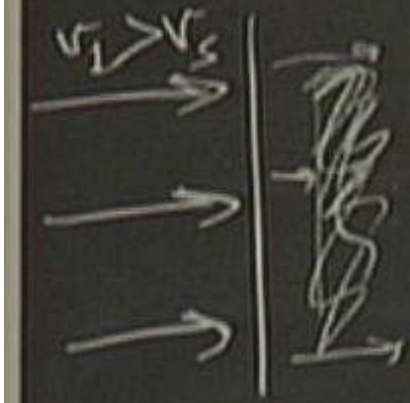
Title: Astrophysics and Cosmology through Problems - 7A

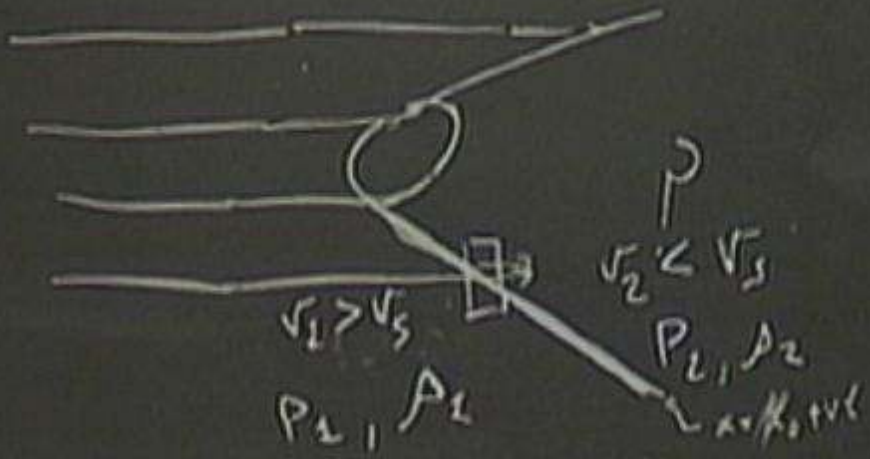
Date: Oct 16, 2008 10:00 AM

URL: http://pirsa.org/08100003

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.


$v_1 > v_2$
?
$v_1 > v$

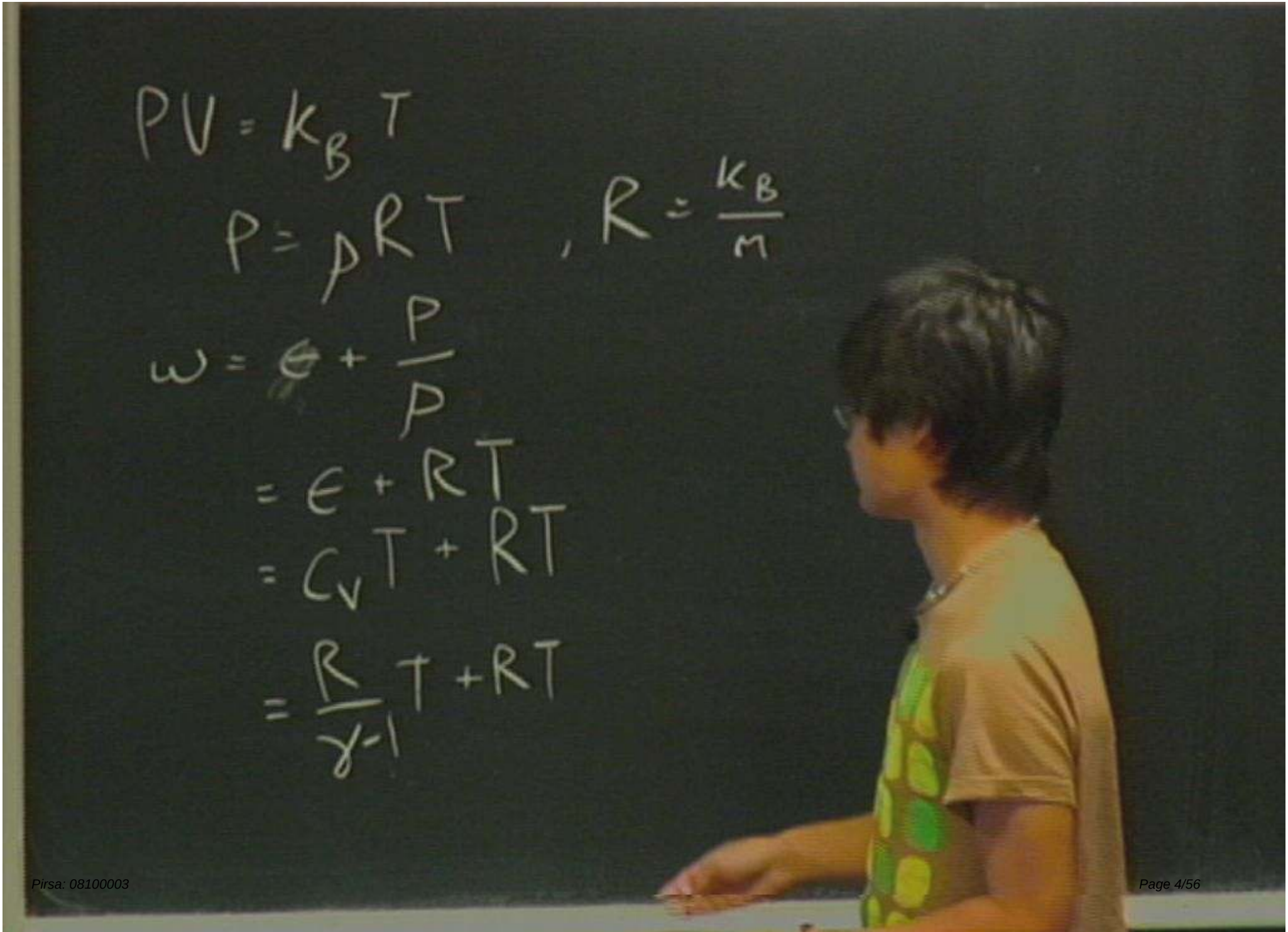





$$dM_1 = dM_2$$

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

$$\rho_1 v_1 = \rho_2 v_2$$


$PV = k_B T$
$P = \rho R T$ , $R = \frac{k_B}{m}$
$\omega = \epsilon + \frac{P}{\rho}$
$= \epsilon + RT$
$= C_V T + RT$
$= \frac{R}{\gamma - 1} T + RT$

$$PV = k_B T$$

$$P = \rho R T \quad , \quad R = \frac{k_B}{m}$$

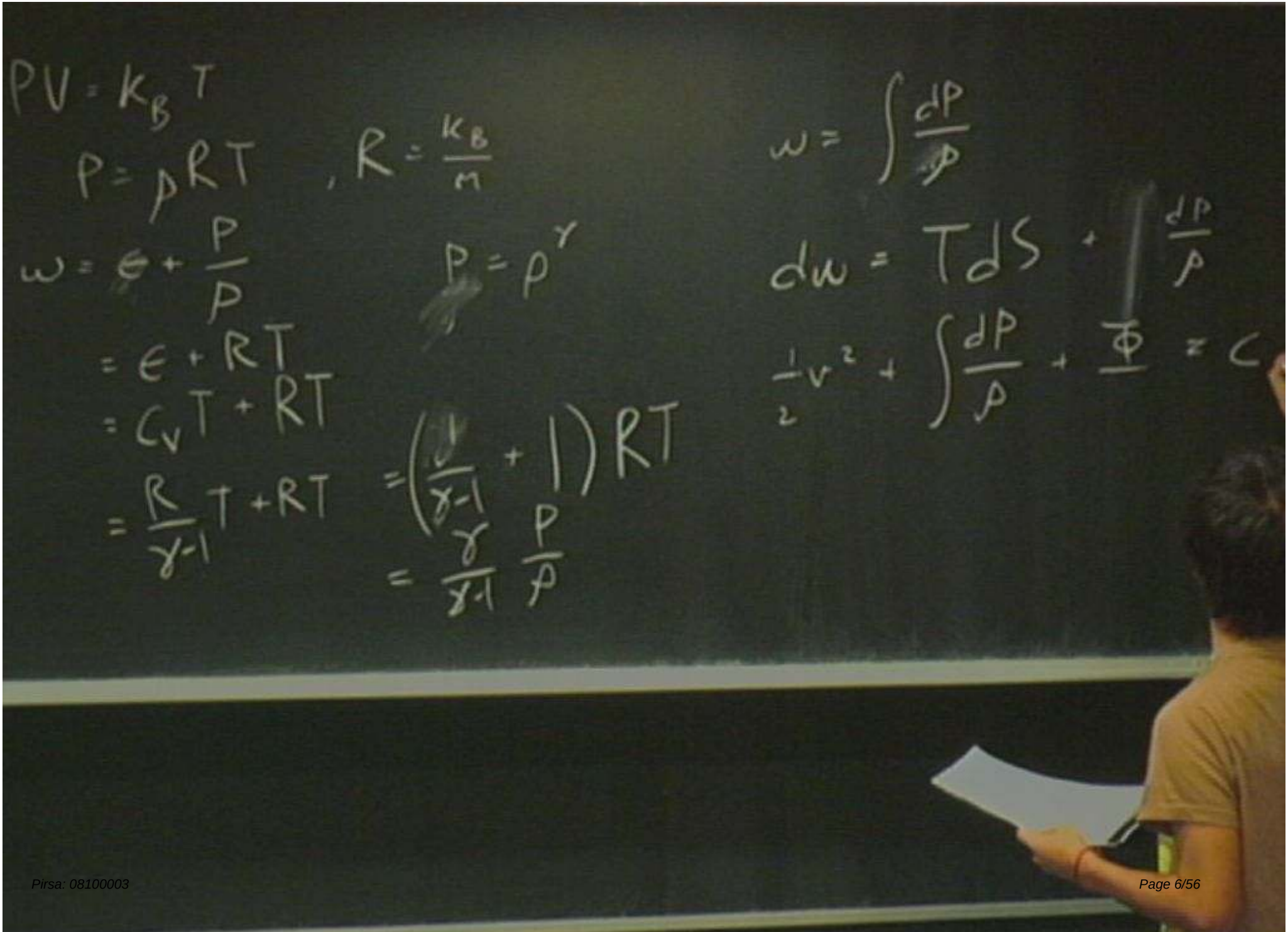
$$\omega = \epsilon + \frac{P}{\rho} \qquad P = \rho^{\gamma}$$

$$= \epsilon + RT$$

$$= C_V T + RT$$

$$= \frac{R}{\gamma - 1} T + RT \qquad = \left( \frac{1}{\gamma - 1} + 1 \right) RT$$

$$= \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$$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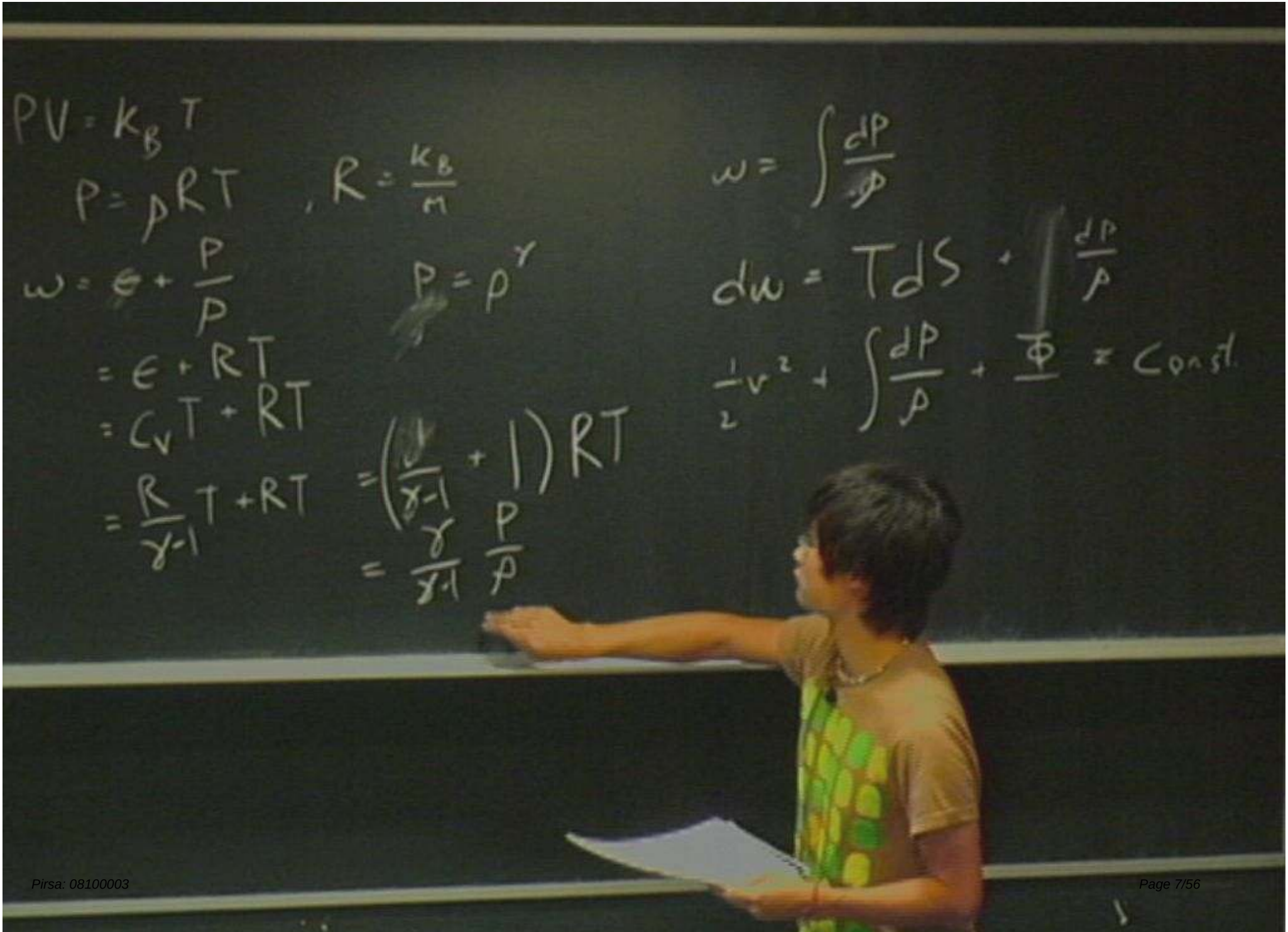
$P = \rho^\gamma$
$= \left(\frac{1}{\gamma - 1} + 1\right) RT$
$= \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$
$\omega = \int \frac{dP}{\rho}$
$dw = TdS + \frac{dP}{\rho}$
$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = C$


$PV = k_B T$
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$dw = TdS + \frac{dP}{\rho}$
$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = \text{Const.}$

$$PV = k_B T$$

$$P = \rho R T \quad , R = \frac{k_B}{m}$$

$$P = \rho^\gamma$$

$$w = \int \frac{dP}{\rho}$$

$$w = \epsilon + \frac{P}{\rho}$$

$$= \epsilon + RT$$

$$= C_V T + RT$$

$$= \frac{R}{\gamma - 1} T + RT \quad = \left( \frac{1}{\gamma - 1} + 1 \right) RT$$

$$= \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$$

$$dw = T dS + \frac{dP}{\rho}$$

$$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = \text{Const}$$

$$\frac{1}{2} v_1^2 + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} = \frac{1}{2} v_2^2 + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2}$$

$PV = k_B T$

$P = \rho R T \quad , R = \frac{k_B}{m}$

$P = \rho^\gamma$

$\omega = \int \frac{dP}{\rho}$

$\omega = \epsilon + \frac{P}{\rho}$

$= \epsilon + RT$

$= C_V T + RT$

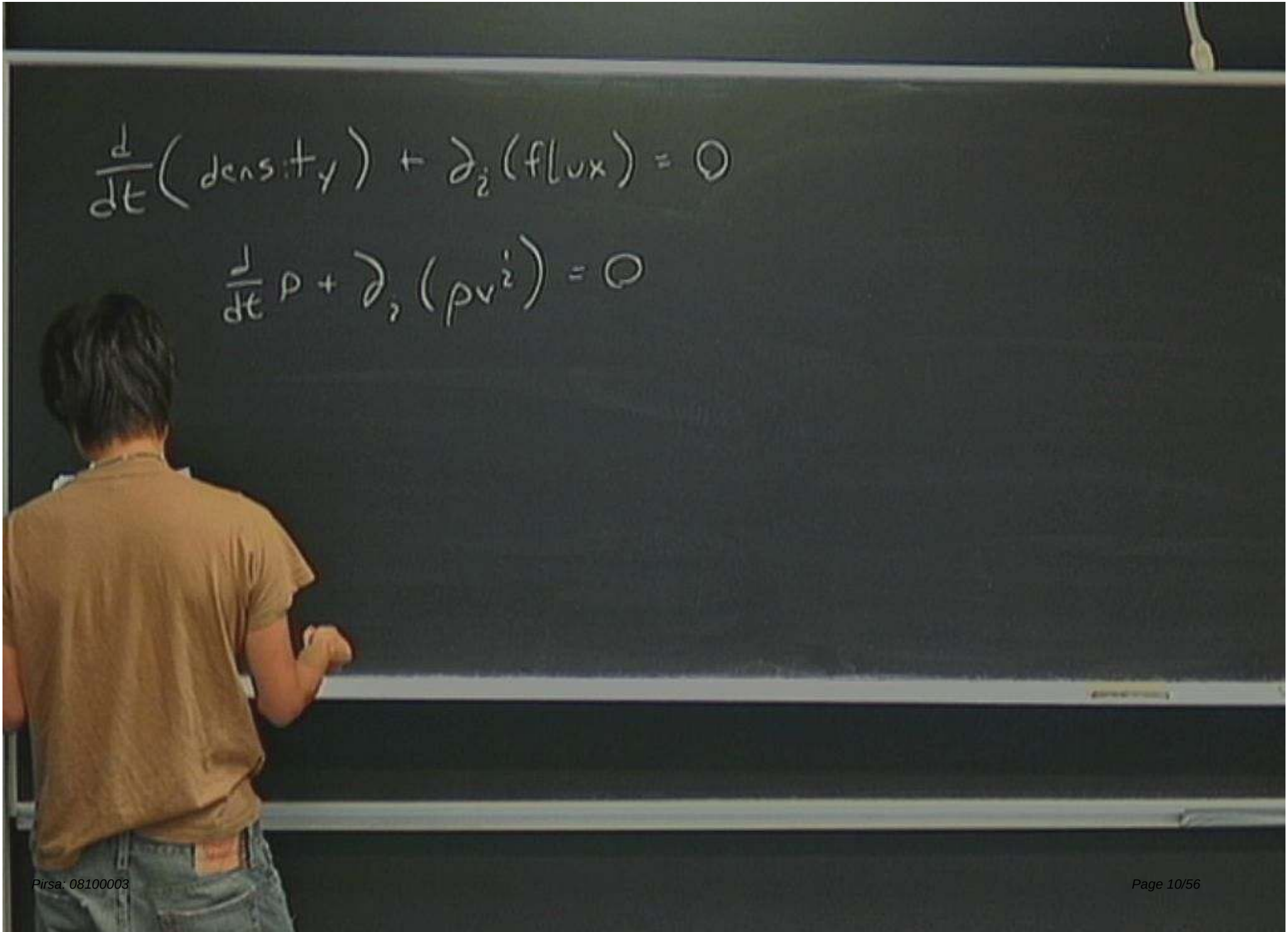
$= \frac{R}{\gamma - 1} T + RT \quad = \left( \frac{1}{\gamma - 1} + 1 \right) RT$

$= \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$

$d\omega = T dS + \frac{dP}{\rho}$

$\frac{1}{2} v^2 + \int \frac{dP}{\rho} + \Phi = \text{Const}$

$\frac{1}{2} v_2^2 + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2} = \frac{1}{2} v_2^2 + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2}$


$\frac{d}{dt}(\text{density}) + \partial_i(\text{flux}) = 0$
$\frac{d}{dt} p + \partial_i(\rho v^i) = 0$

$$\frac{d}{dt}(\text{density}) + \partial_i(\text{flux}) = 0$$

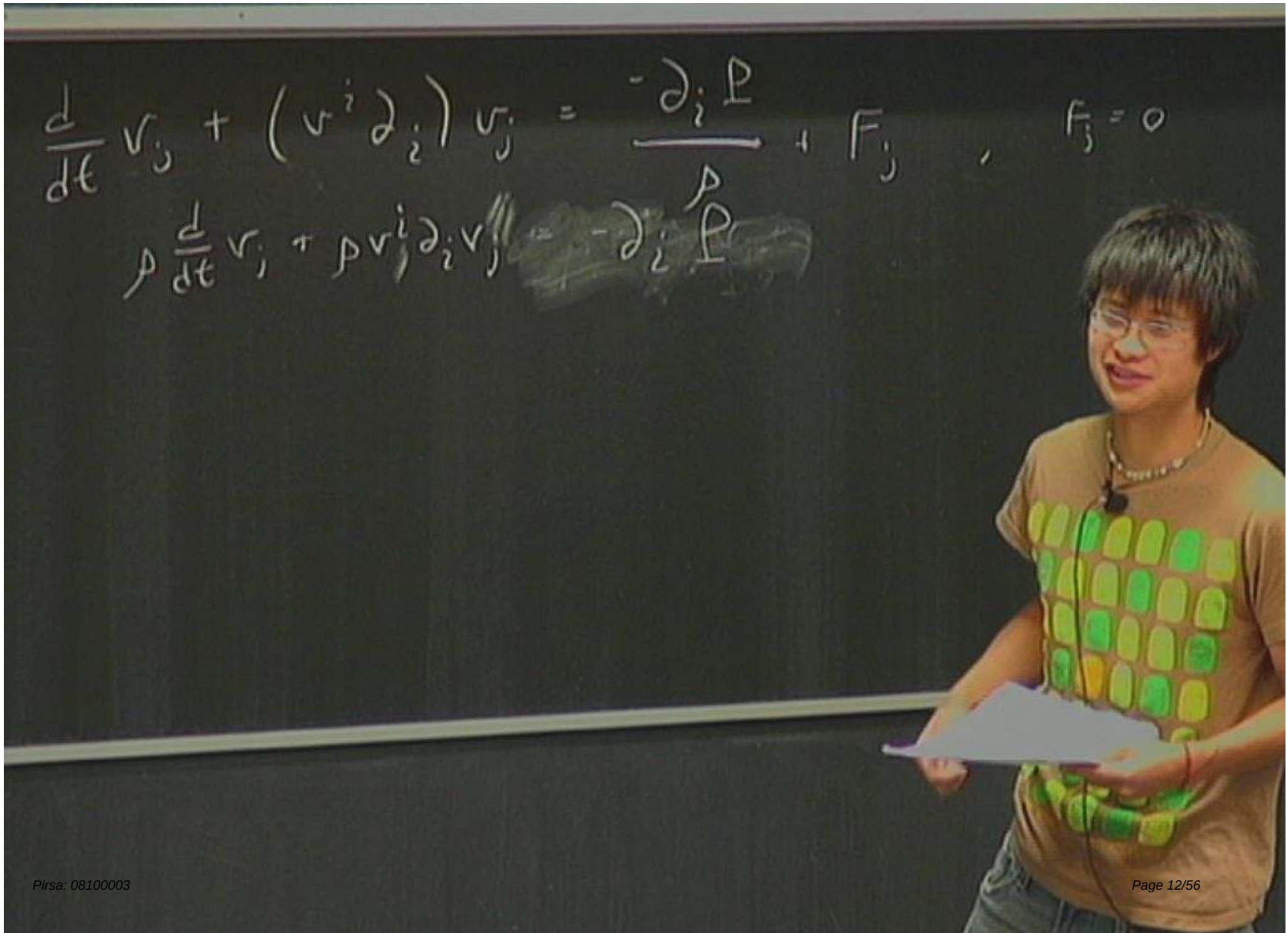
$$\frac{d}{dt}\rho + \partial_i(\rho v^i) = 0$$

$$v_j \frac{d}{dt}\rho + v_j \partial_i(\rho v^i) = 0$$

$$v_j \frac{d}{dt}\rho + v_j v^i \partial_i \rho + \rho v_j \partial_i v^i = 0$$

$$\frac{d}{dt} v_j + (v^i \partial_i) v_j = \frac{-\partial_j P}{\rho} + F_j \quad , \quad F_j = 0$$

$$\rho \frac{d}{dt} v_j + \rho v^i_j \partial_i v^j = -\partial_i P$$



$$\frac{d}{dt} v_j + (v^i \partial_i) v_j = \frac{-\partial_j P}{\rho} + F_j \quad , \quad F_j = 0$$

$$\rho \frac{d}{dt} v_j + \rho v^i_j \partial_i v_j = -\partial_j P$$

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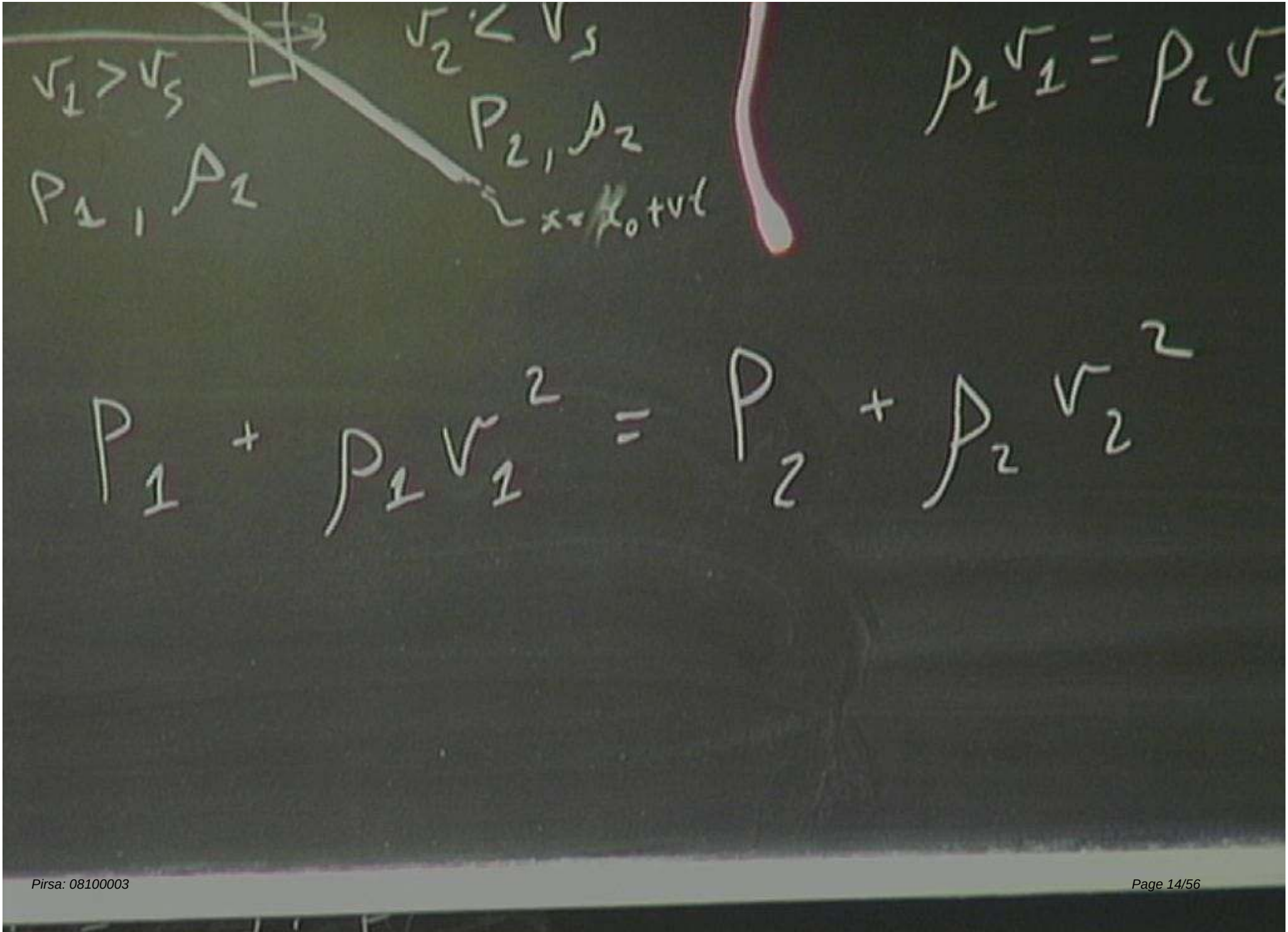
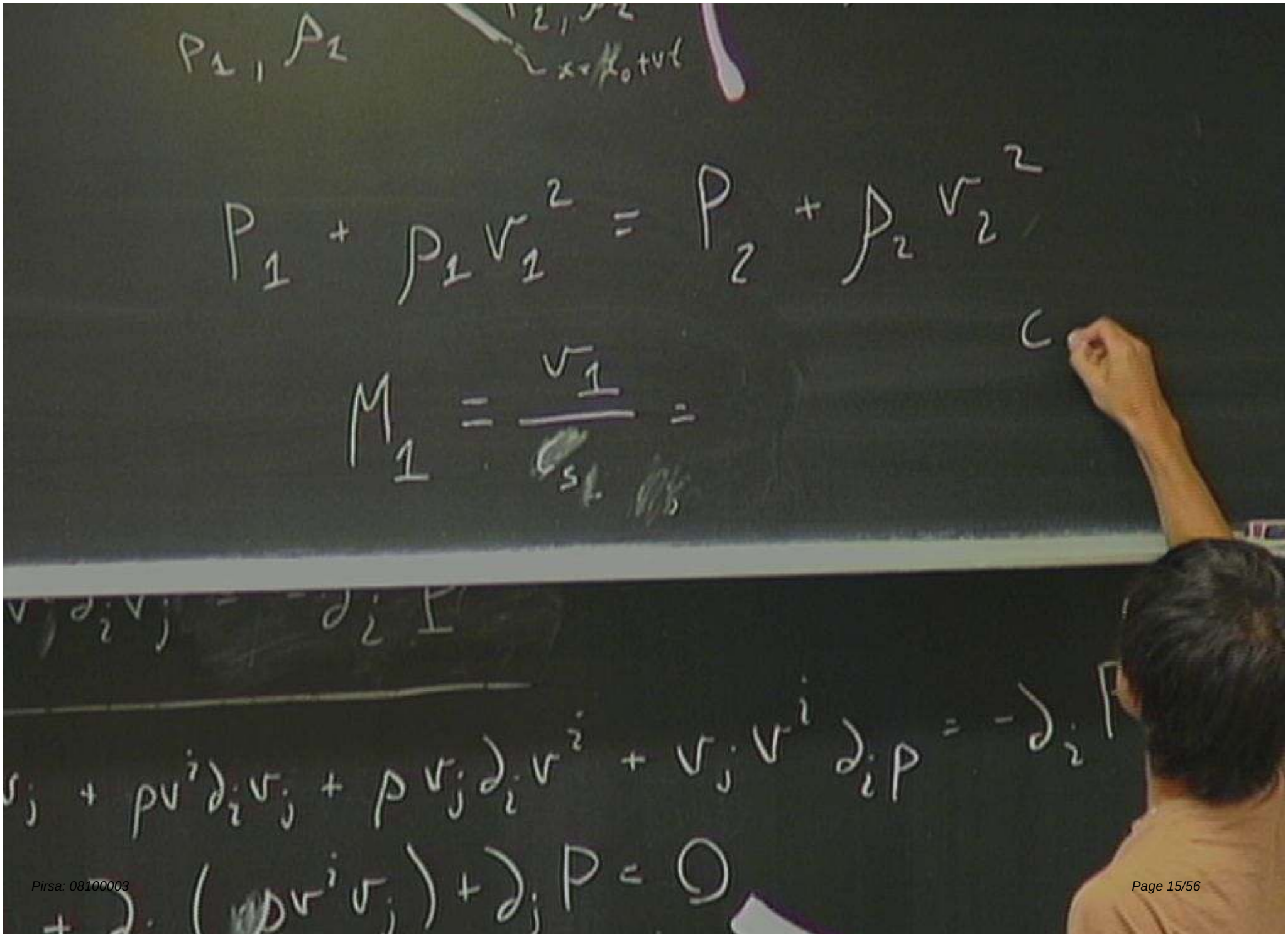
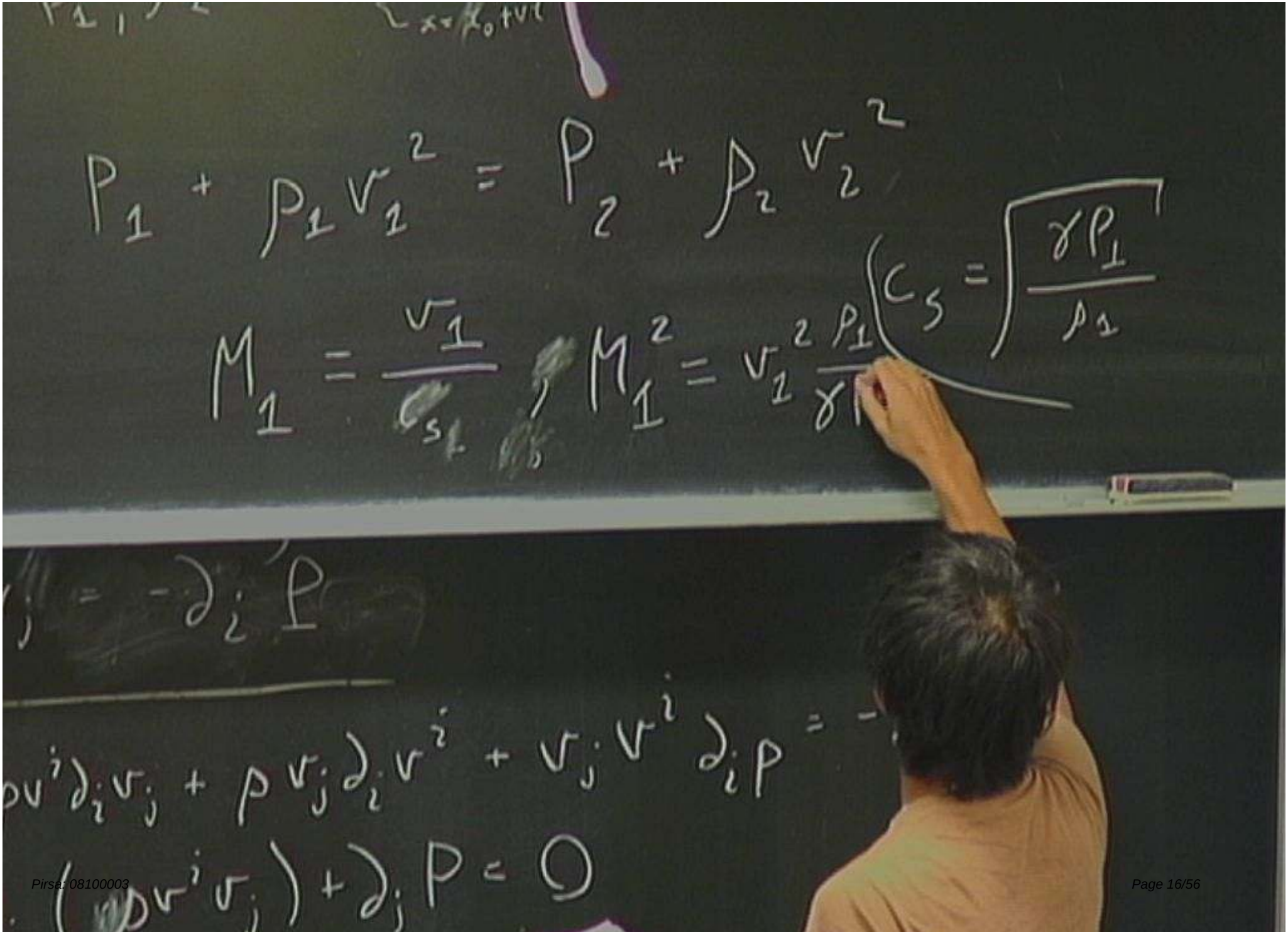
$$v_j \frac{d}{dt} \rho + \rho \frac{d}{dt} v_j + \rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho = -\partial_j P$$

$$\frac{d}{dt}(\rho v_j) + \partial_i (\rho v^i v_j) + \partial_j P = 0$$

$$T^i_j = P \delta^i_j + \rho v^i v_j$$


$v_1 > v_s$
$v_2 < v_s$
$P_2, \rho_2$
$P_1, \rho_1$
$x = x_0 + vt$
$\rho_1 v_1 = \rho_2 v_2$
$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$


$P_1, \rho_1$
$P_2$
$x = x_0 + vt$
$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$
$M_1 = \frac{v_1}{c_{s1}} =$
$v_j \partial_i v_j = -\partial_i P$
$v_j + \rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho = -\partial_i P$
$+ \partial_i (\rho v^i v_j) + \partial_j P = 0$


$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$
$M_1 = \frac{v_1}{c_{s_1}}$
$M_1^2 = v_1^2 \frac{\rho_1}{\gamma P_1}$
$c_s = \sqrt{\frac{\gamma P_1}{\rho_1}}$
$\partial_i P$
$\rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho$
$\partial_i (\rho v^i v_j) + \partial_j P = 0$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

$$M_1 = \frac{v_1}{c_{s_1}} \; ; \; M_1^2 = v_1^2 \frac{\rho_1}{\gamma P_1} \qquad \left( c_s = \sqrt{\frac{\gamma P_1}{\rho_1}} \right.$$

$$\partial_j P = -\partial_i P$$

$$\rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i \rho = -$$

$$\partial_i (\rho v^i v_j) + \partial_j P = 0$$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

$$M_1 = \frac{v_1}{c_{s_1}} \quad ; \quad M_1^2 = v_1^2 \frac{\rho_1}{\gamma P_1} \qquad \left( c_s = \sqrt{\frac{\gamma P_1}{\rho_1}} \right)$$

$$\partial_j P = -\partial_i P$$

$$\rho v^i \partial_i v_j + \rho v_j \partial_i v^i + v_j v^i \partial_i P = -\partial_i P$$

$$\partial_i (\rho v^i v_j) + \partial_j P = 0$$

$$P_2 = \frac{\gamma - 1}{2\gamma} \rho_2 v_1^2 + \frac{P_1}{\rho_1} \rho_2 - \frac{\gamma - 1}{2\gamma} \rho_2 v_2^2$$

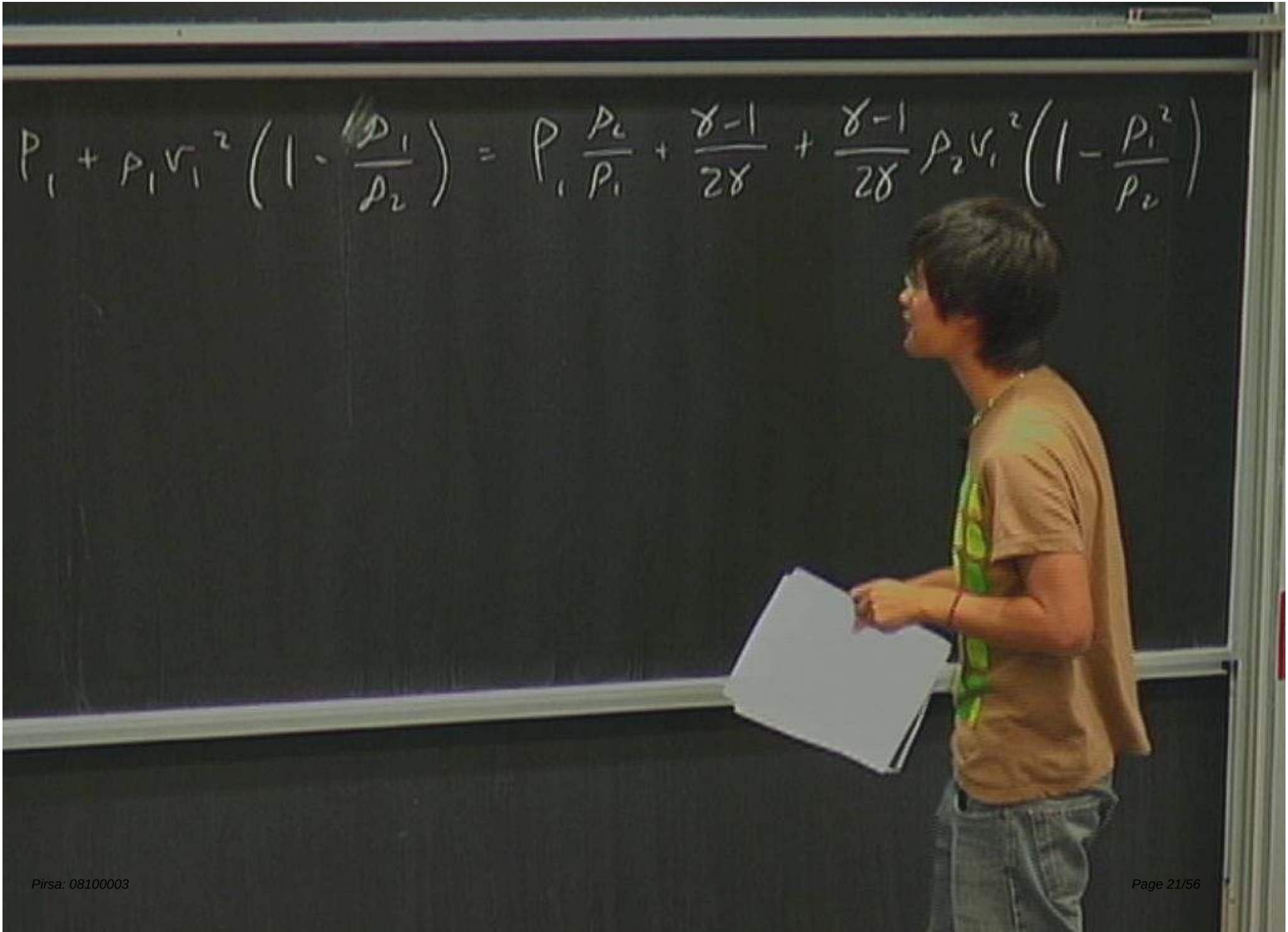
$$= P_1 \frac{\rho_2}{\rho_1} + \frac{\gamma - 1}{2\gamma} \rho_2 \left( v_1^2 - v_2^2 \right)$$

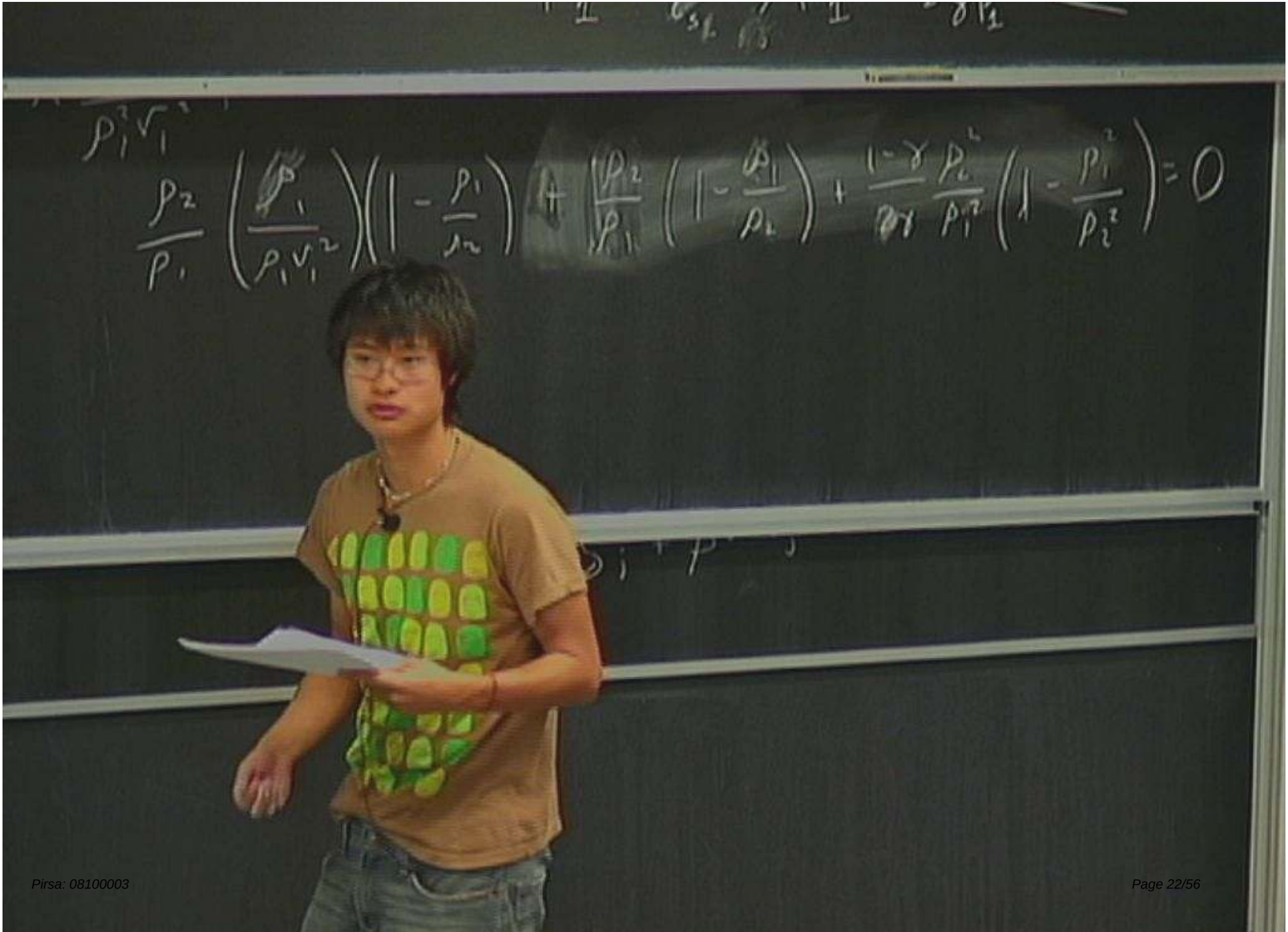
$$P_2 = \frac{\gamma-1}{2\gamma}\rho_2 v_1^2 + \frac{P_1}{\rho_1}\rho_2 - \frac{\gamma-1}{2\gamma}\rho_2 v_2^2$$

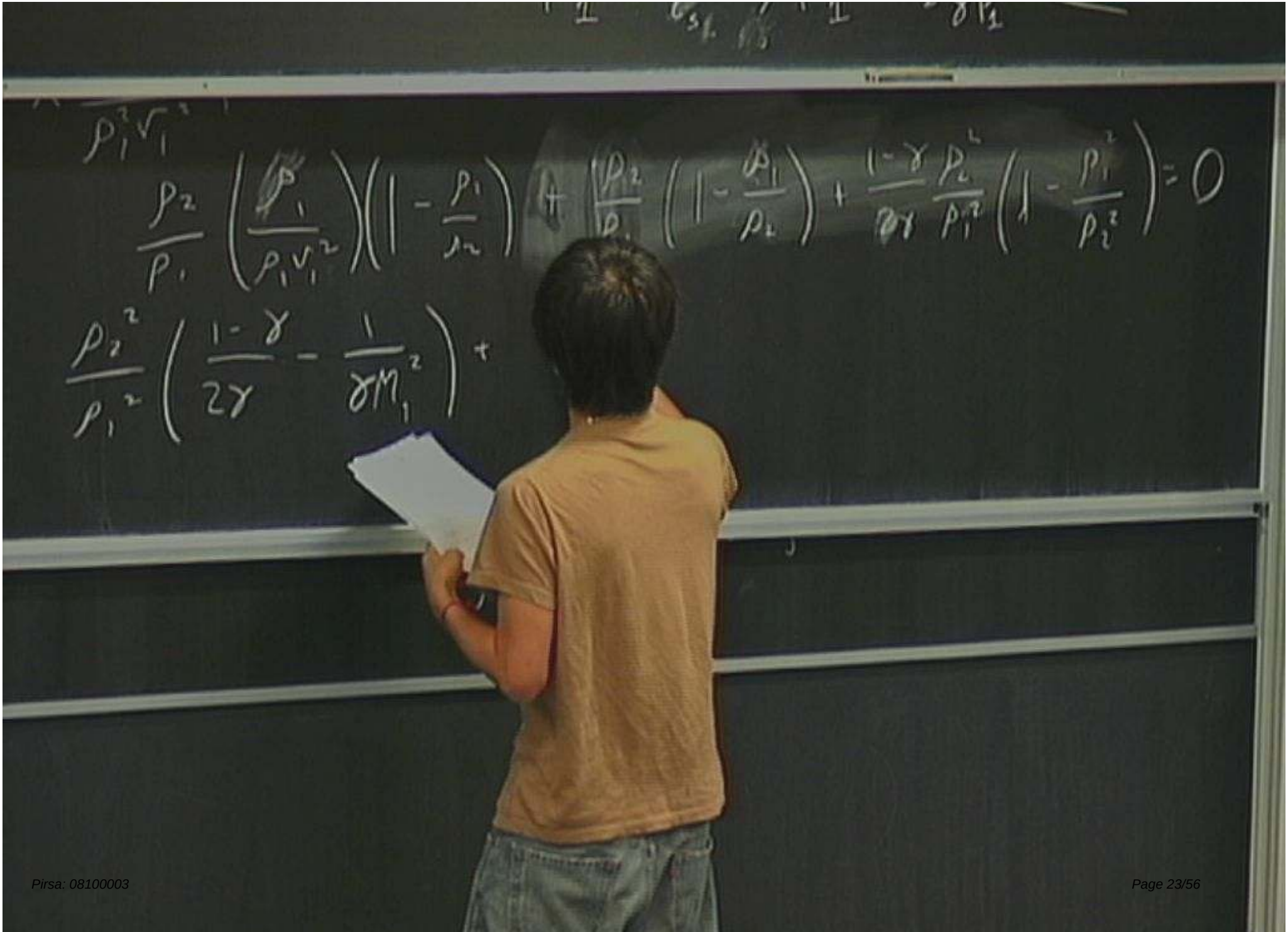
$$= P_1\frac{\rho_2}{\rho_1} + \frac{\gamma-1}{2\gamma}\rho_2\left(v_1^2 - v_2^2\right)$$

$$P_1 + \rho_1 v_1^2 - \rho_2 v_2^2 = P_1\frac{\rho_2}{\rho_1} + \frac{\gamma-1}{2\gamma}\rho_2\left(v_1^2 - v_2^2\right)$$

$$v_2^2 = \frac{\rho_1^2 v_1^2}{\rho_2^2}$$


$P_1 + \rho_1 v_1^2 \left(1 - \frac{\rho_1}{\rho_2}\right) = P_1 \frac{P_2}{P_1} + \frac{\gamma-1}{2\gamma} + \frac{\gamma-1}{2\gamma} \rho_2 v_1^2 \left(1 - \frac{P_1^2}{P_2}\right)$


$\frac{p_2}{p_1}\left(\frac{p_1}{\rho_1 v_1^2}\right)\left(1-\frac{p_1}{p_2}\right)+\frac{p_2}{p_1}\left(1-\frac{p_1}{p_2}\right)+\frac{1-\gamma}{\gamma}\frac{p_2}{p_1}\left(1-\frac{p_1}{p_2}\right)=0$

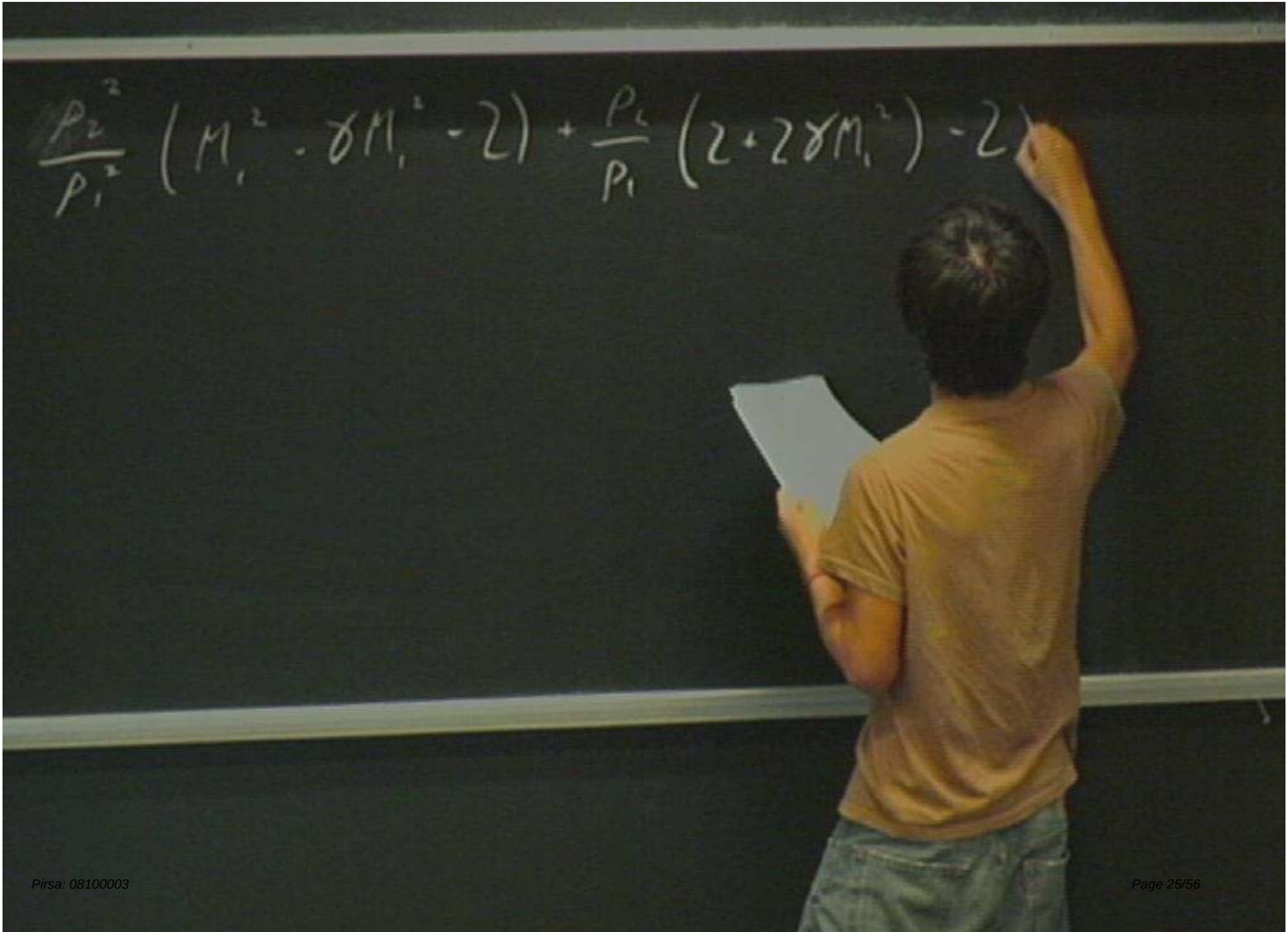

$\frac{p_2}{p_1}\left(\frac{p_1}{\rho_1 v_1^2}\right)\left(1-\frac{p_1}{p_2}\right)+\left(\frac{p_2}{p_1}\left(1-\frac{p_1}{p_2}\right)+\frac{1-\gamma}{2\gamma}\frac{p_2^2}{p_1^2}\left(1-\frac{p_1^2}{p_2^2}\right)\right)=0$
$\frac{p_2^2}{p_1^2}\left(\frac{1-\gamma}{2\gamma}-\frac{1}{\gamma M_1^2}\right)+$

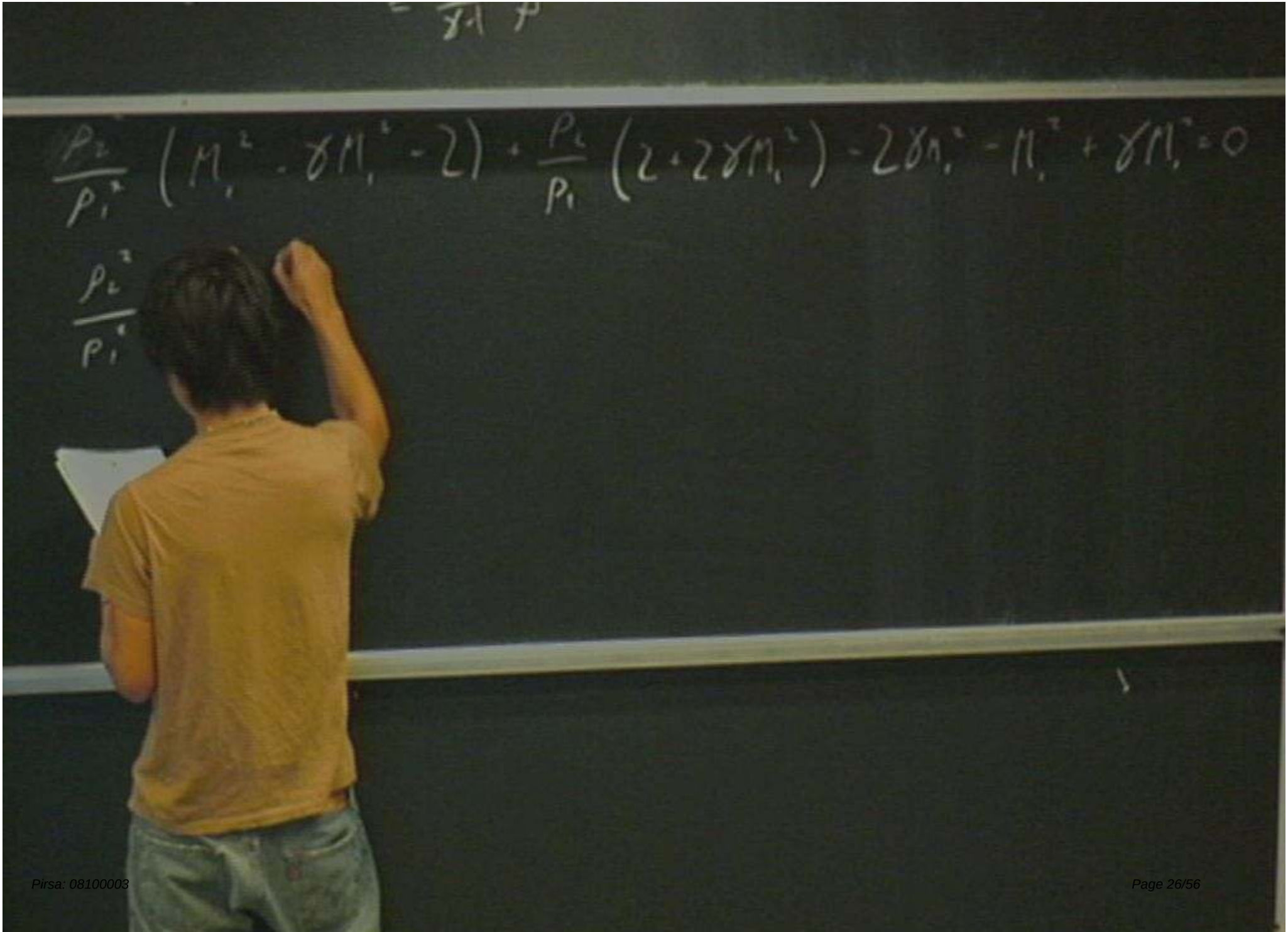
$$\rho_1 v_1^2$$

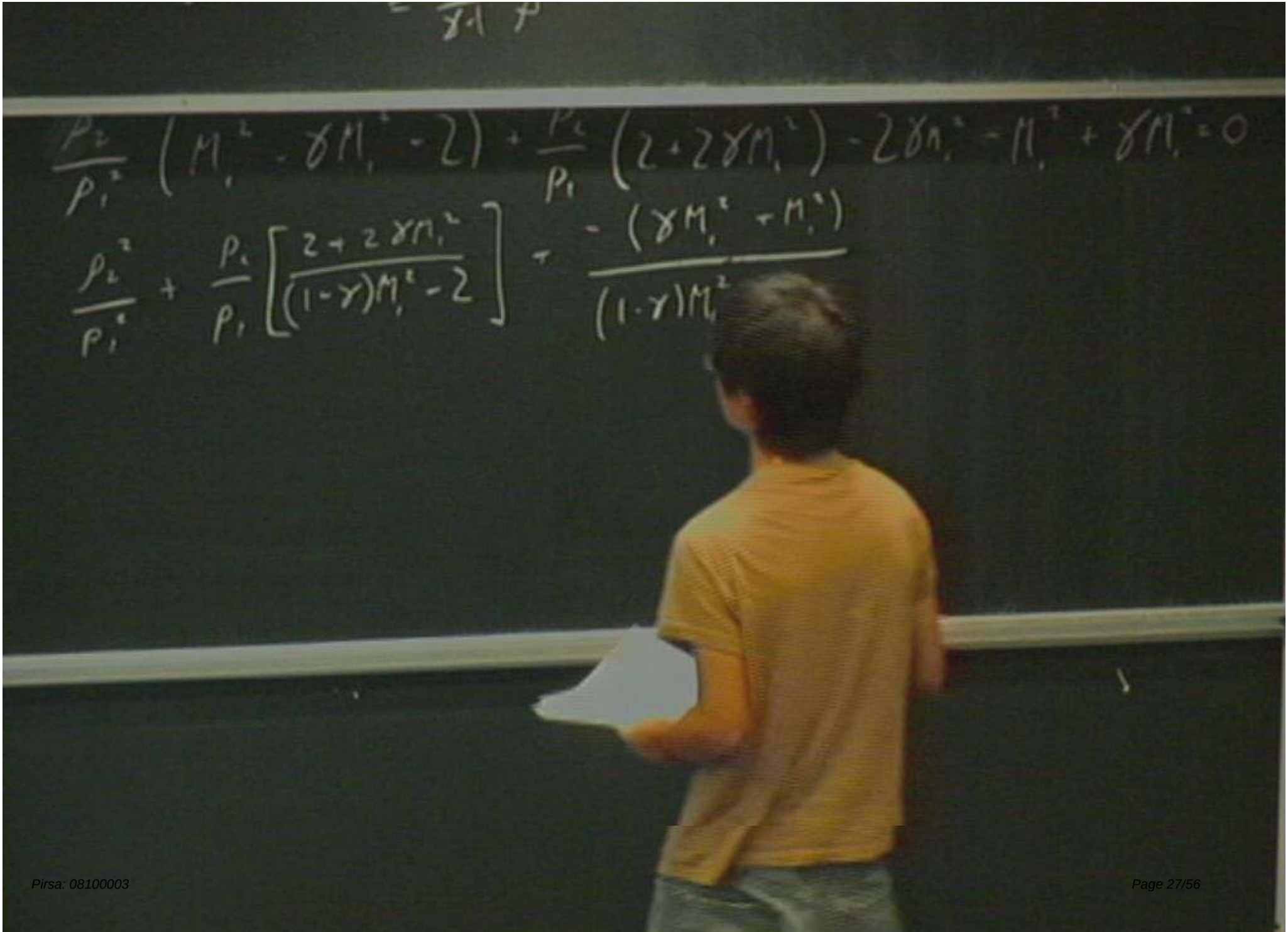
$$\frac{p_2}{p_1}\left(\frac{p_1}{\rho_1 v_1^2}\right)\left(1-\frac{p_1}{p_2}\right)+\left(\frac{p_2}{p_1}\left(1-\frac{p_1}{p_2}\right)+\frac{1-\gamma}{2\gamma}\frac{p_2^2}{p_1^2}\left(1-\frac{p_1^2}{p_2^2}\right)\right)=0$$

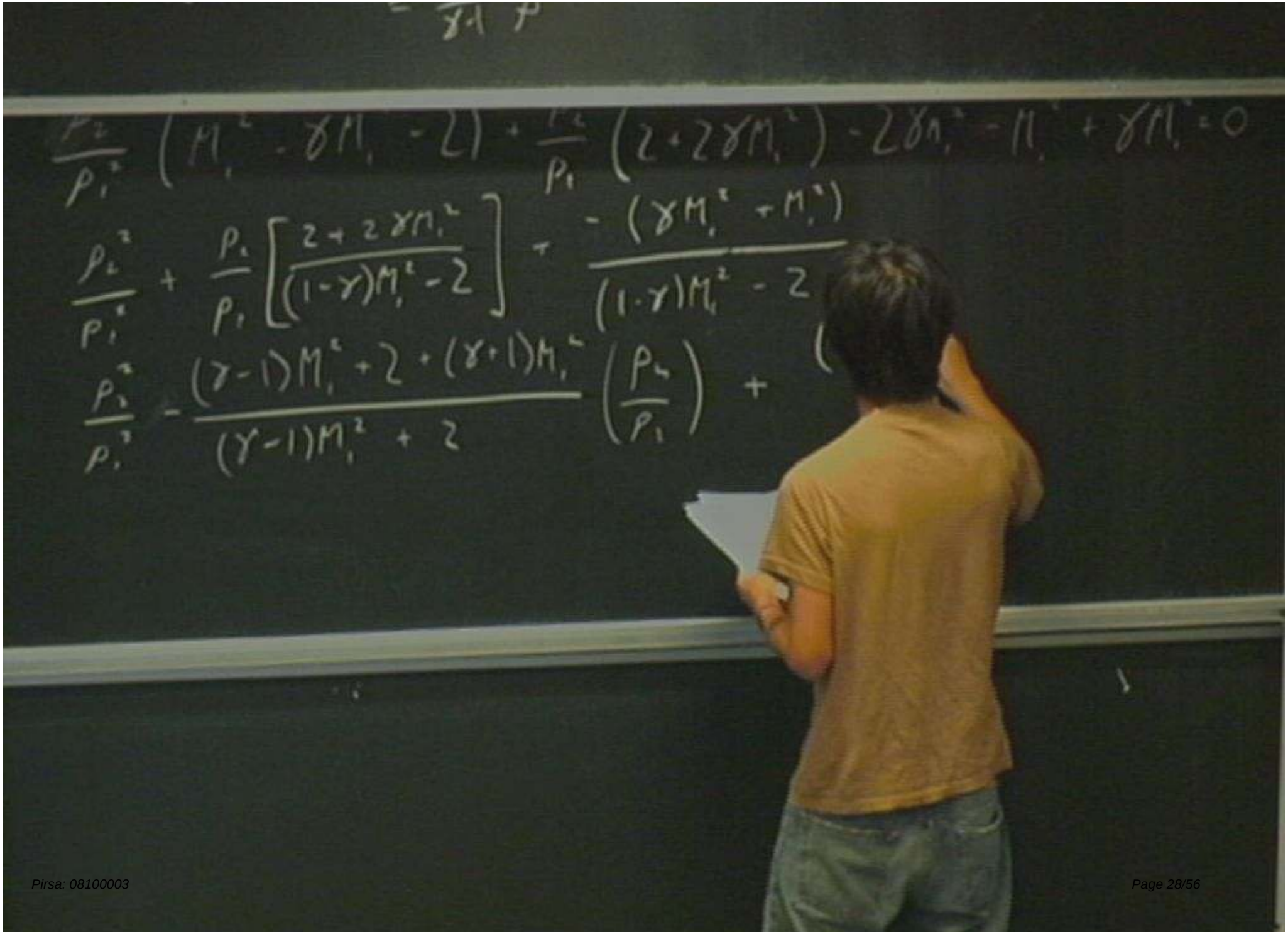
$$\frac{p_2^2}{p_1^2}\left(\frac{1-\gamma}{2\gamma}-\frac{1}{\gamma M_1^2}\right)+\frac{p_2}{p_1}\left(\frac{1}{\gamma M_1^2}+1\right)-1-\frac{1-\gamma}{2\gamma}=0$$

$$T_{ij}=\rho S_{ij}+\rho$$


$\frac{P_2^2}{P_1^2}\left(M_1^2 \cdot \gamma M_1^2 - 2\right) + \frac{P_2}{P_1}\left(2 + 2\gamma M_1^2\right) - 2$


$\frac{p_2}{p_1^2}\left(M_1^2 \cdot \gamma M_1^2 - 2\right) + \frac{p_2}{p_1}\left(2 + 2\gamma M_1^2\right) - 2\gamma M_1^2 - M_1^2 + \gamma M_1^2 = 0$
$\frac{p_2^2}{p_1^2}$

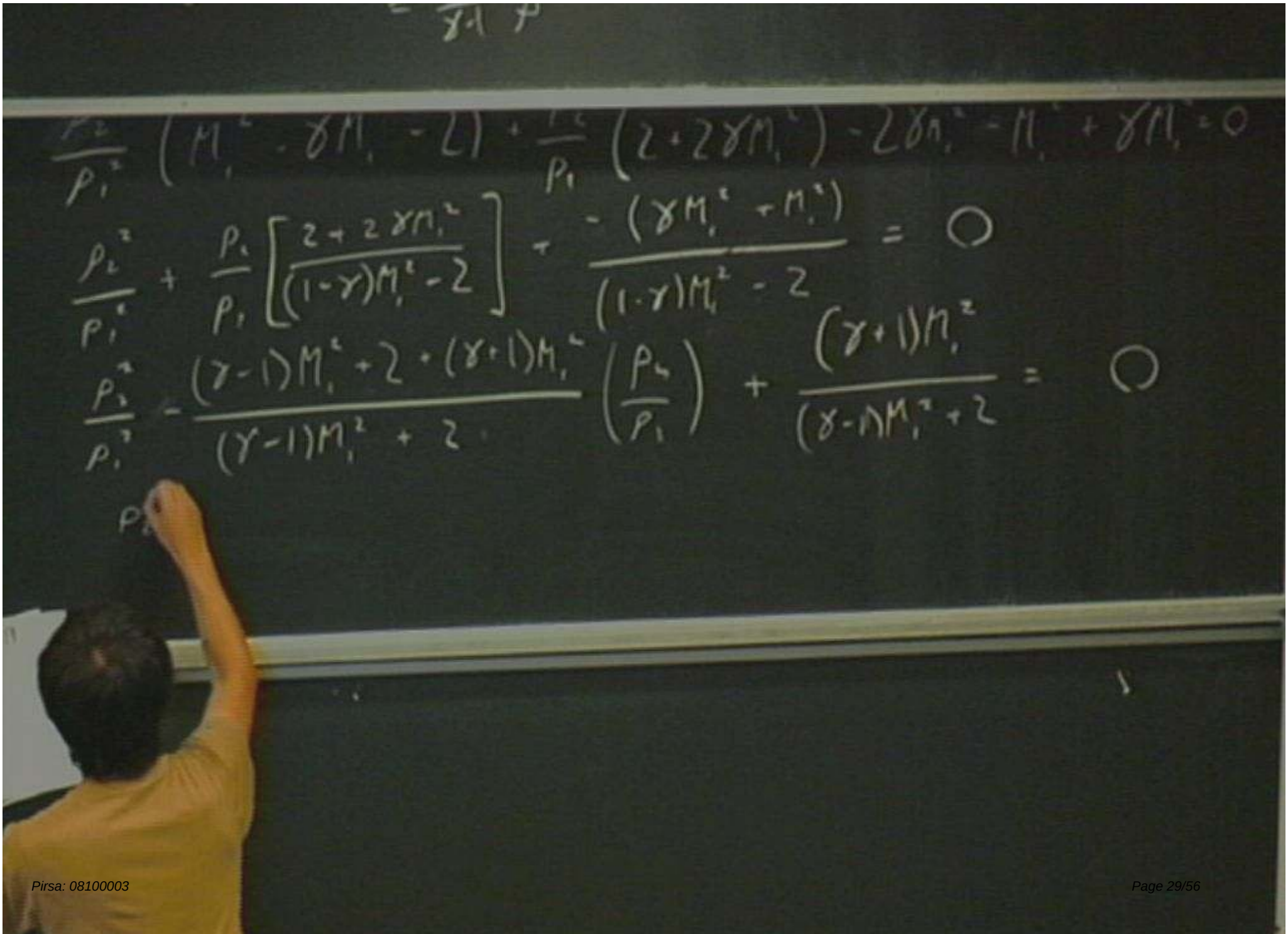

$\frac{P_2}{P_1^2}\left(M_1^2 \cdot \gamma M_1^2 - 2\right) + \frac{P_2}{P_1}\left(2 + 2\gamma M_1^2\right) - 2\gamma M_1^2 - M_1^2 + \gamma M_1^2 = 0$
$\frac{P_2^2}{P_1^2} + \frac{P_2}{P_1}\left[\frac{2 + 2\gamma M_1^2}{(1-\gamma)M_1^2 - 2}\right] + \frac{-(\gamma M_1^2 + M_1^2)}{(1-\gamma)M_1^2}$



$$\frac{P_2}{P_1}\left(M_1^2 - \gamma M_1 - 2\right) + \frac{P_2}{P_1}\left(2 + 2\gamma M_1^2\right) - 2M_1^2 - M_1^2 + \gamma M_1^2 = 0$$

$$\frac{P_2^2}{P_1^2} + \frac{P_2}{P_1}\left[\frac{2 + 2\gamma M_1^2}{(1-\gamma)M_1^2 - 2}\right] + \frac{-(\gamma M_1^2 + M_1^2)}{(1-\gamma)M_1^2 - 2}$$

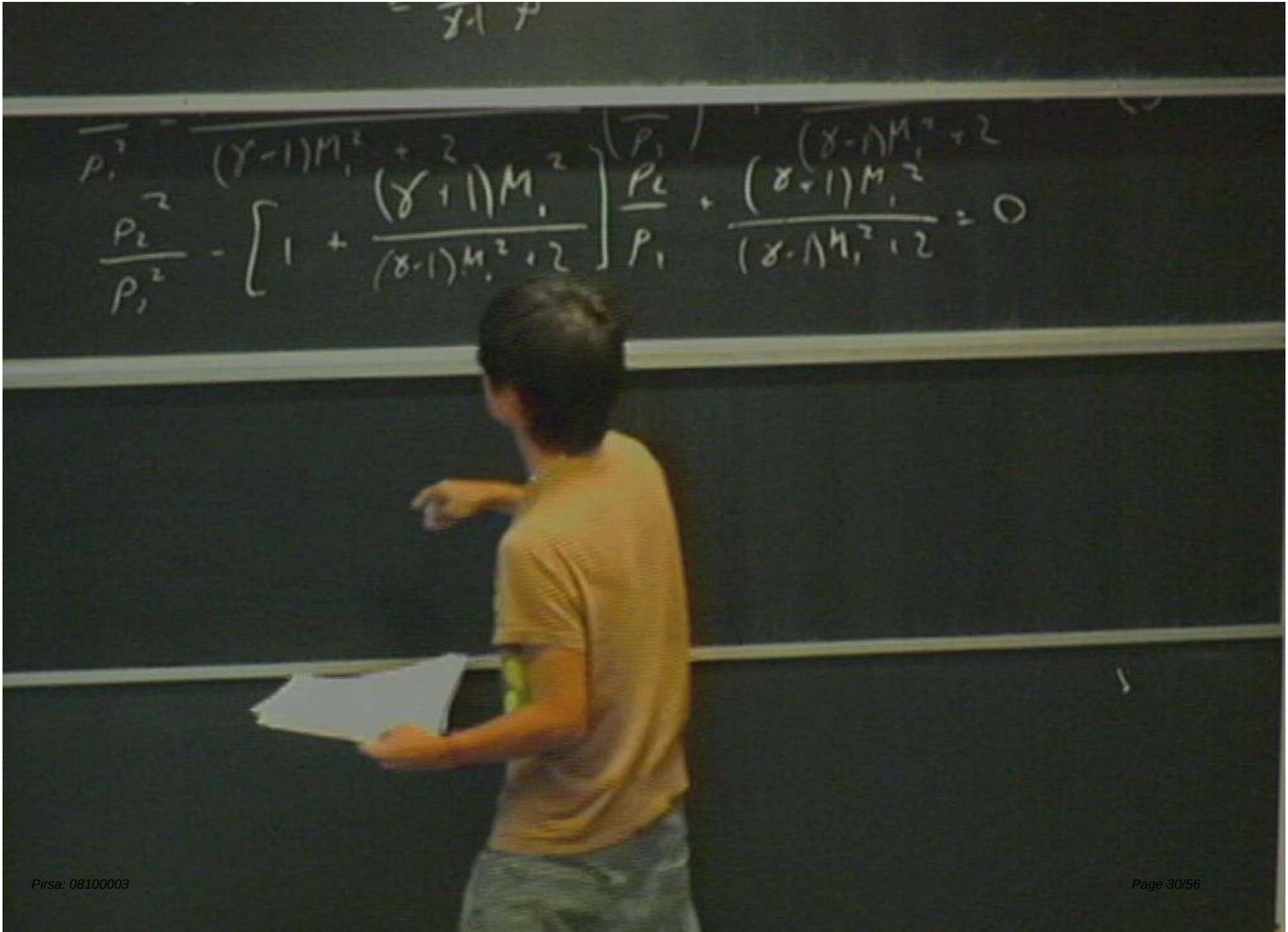
$$\frac{P_2^2}{P_1^2} = \frac{(\gamma-1)M_1^2 + 2 + (\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}\left(\frac{P_2}{P_1}\right) + \left($$

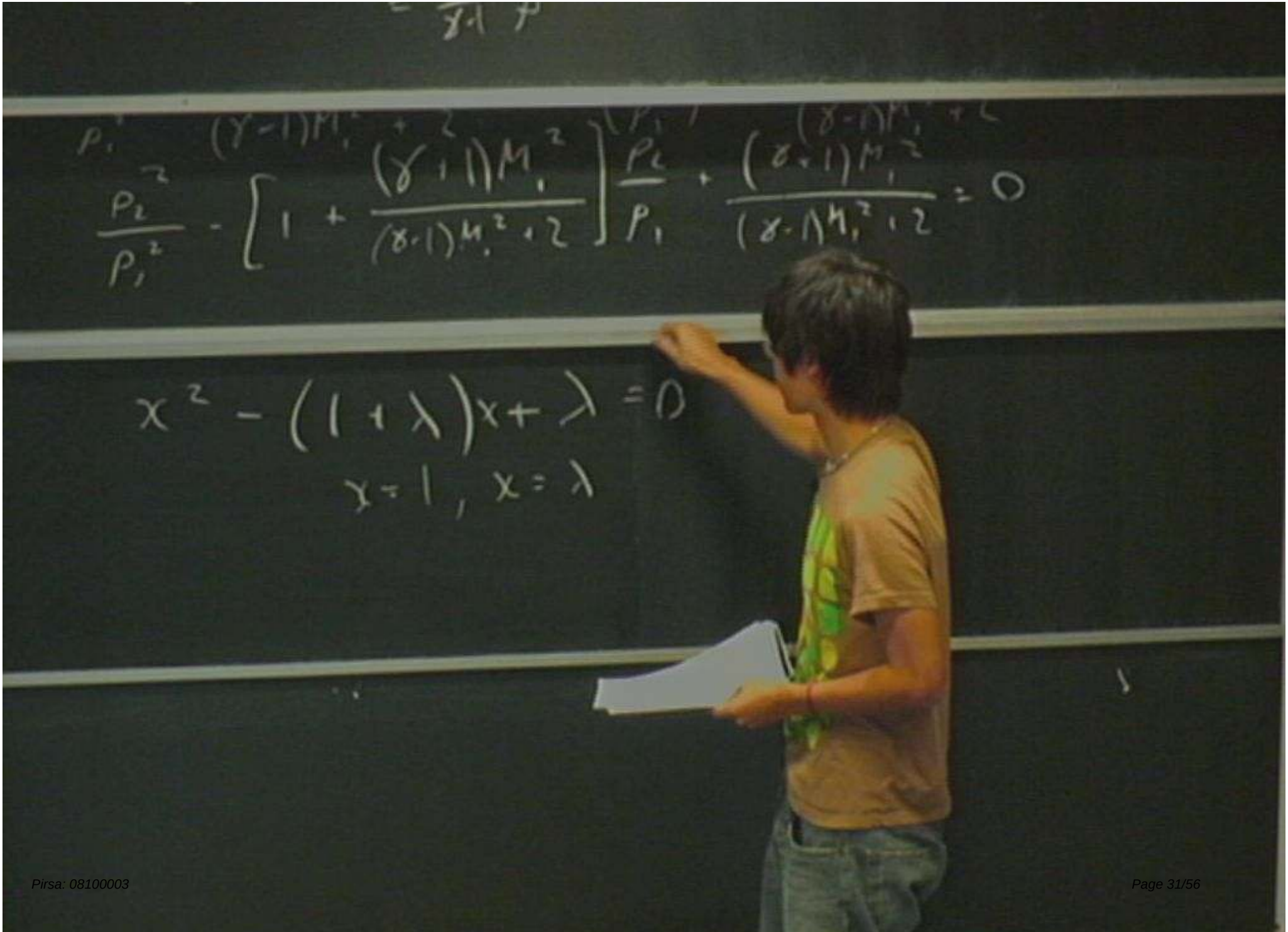


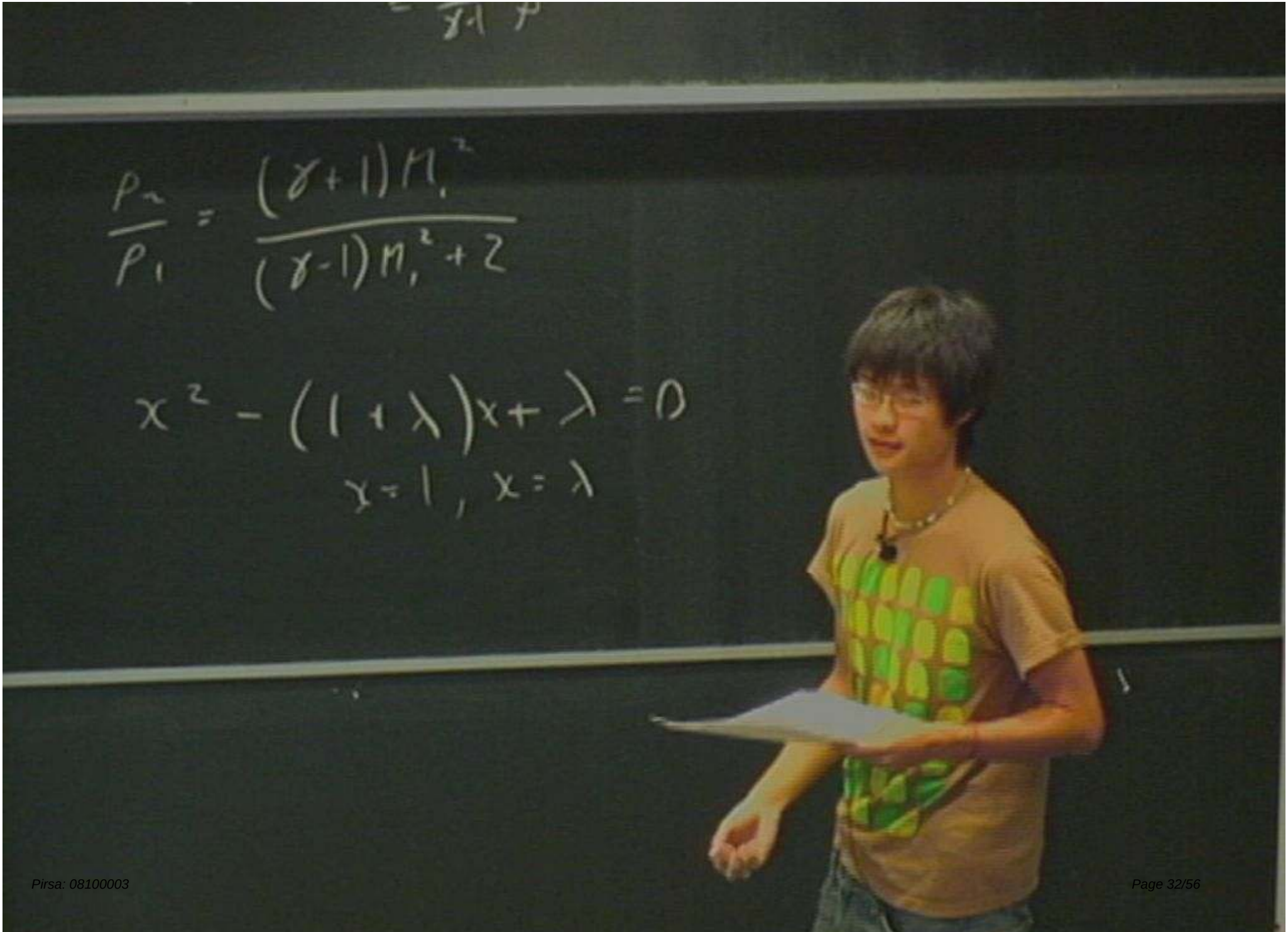
$$\frac{p_2}{p_1^2}\left(M_1^2\cdot\gamma M_1^2-2\right)+\frac{1}{p_1}\left(2+2\gamma M_1^2\right)-2M_1^2-M_1^2+\gamma M_1^2=0$$

$$\frac{p_2^2}{p_1^2}+\frac{p_2}{p_1}\left[\frac{2+2\gamma M_1^2}{(1-\gamma)M_1^2-2}\right]+\frac{-(\gamma M_1^2+M_1^2)}{(1-\gamma)M_1^2-2}=0$$

$$\frac{p_2^2}{p_1^2}-\frac{(\gamma-1)M_1^2+2+(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\left(\frac{p_2}{p_1}\right)+\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}=0$$


$\frac{P_2}{P_1^2} - \left[1 + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]\frac{P_2}{P_1} + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} = 0$


$\frac{P_2}{P_1^2} - \left[1 + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]\frac{P_2}{P_1} + \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} = 0$
$x^2 - (1+\lambda)x + \lambda = 0$
$x = 1, \quad x = \lambda$


$\frac{P_2}{P_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$
$x^2 - (1+\lambda)x + \lambda = 0$
$x = 1, \quad x = \lambda$

$$\frac{p_2}{p_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \qquad , \frac{v_2}{v_1} = \frac{p_1}{p_2}$$

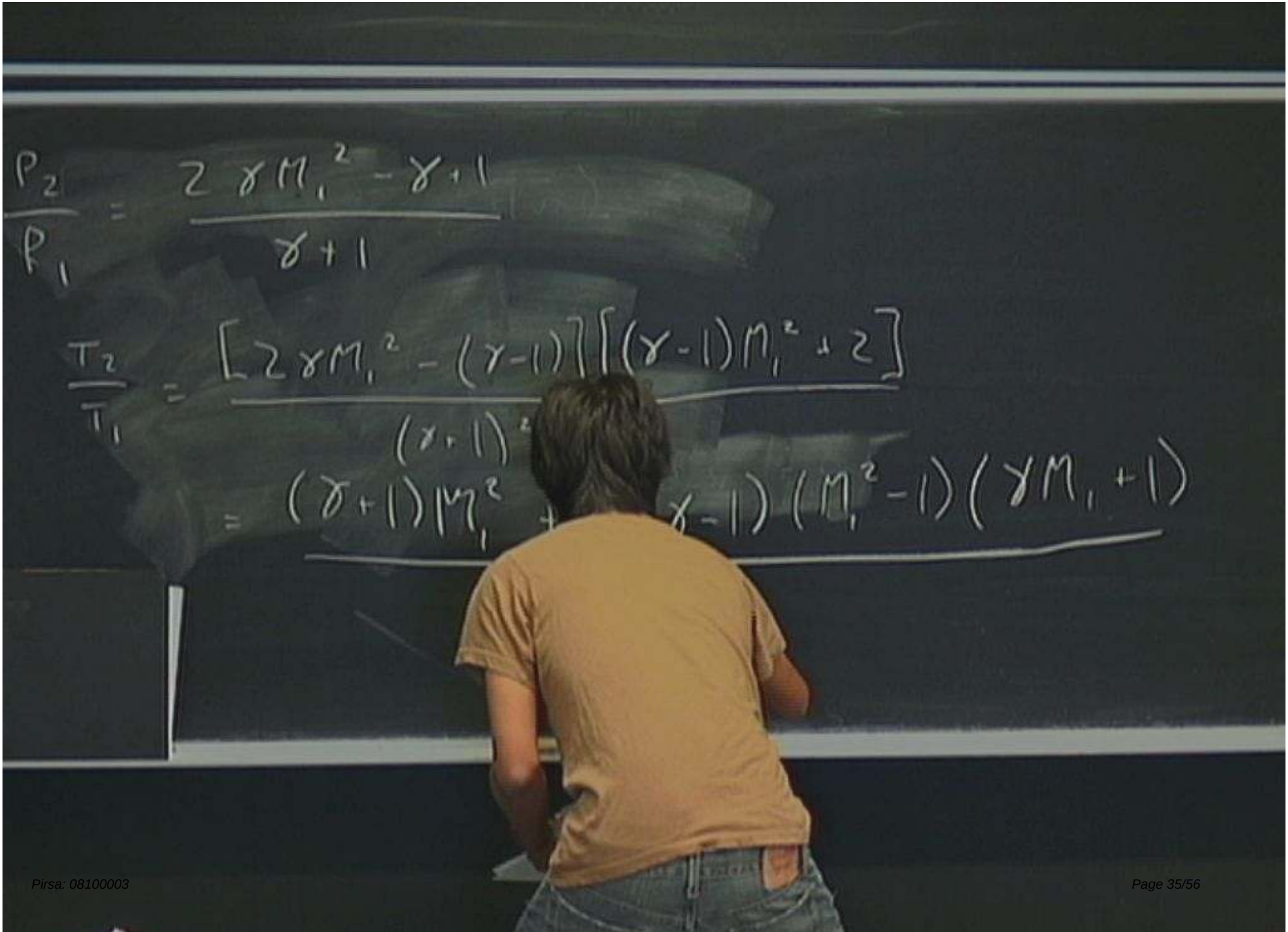
$$x^2 - (1+\lambda)x + \lambda = 0$$

$$x = 1 \quad , \quad x = \lambda$$

$$\frac{p_2}{p_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \qquad , \frac{v_2}{v_1} = \frac{p_1}{p_2}$$

$$x^2 - (1+\lambda)x + \lambda = 0 \qquad \frac{RT_2}{RT_1} = \frac{p_2}{p_1}\left(\frac{p_1}{p_2}\right)$$

$$x = 1 \quad , \quad x = \lambda$$


$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$
$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma-1)][(\gamma-1)M_1^2 + 2]}{(\gamma+1)^2}$
$= \frac{(\gamma+1)M_1^2 + (\gamma-1)(M_1^2-1)(\gamma M_1^2+1)}{}$

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma - 1)][(\gamma - 1)M_1^2 + 2]}{(\gamma + 1)^2 M_1^2}$$

$$= \frac{(\gamma + 1)M_1^2 + 2(\gamma - 1)(M_1^2 - 1)(\gamma M_1 + 1)}{(\gamma + 1)M_1^2}$$

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$$

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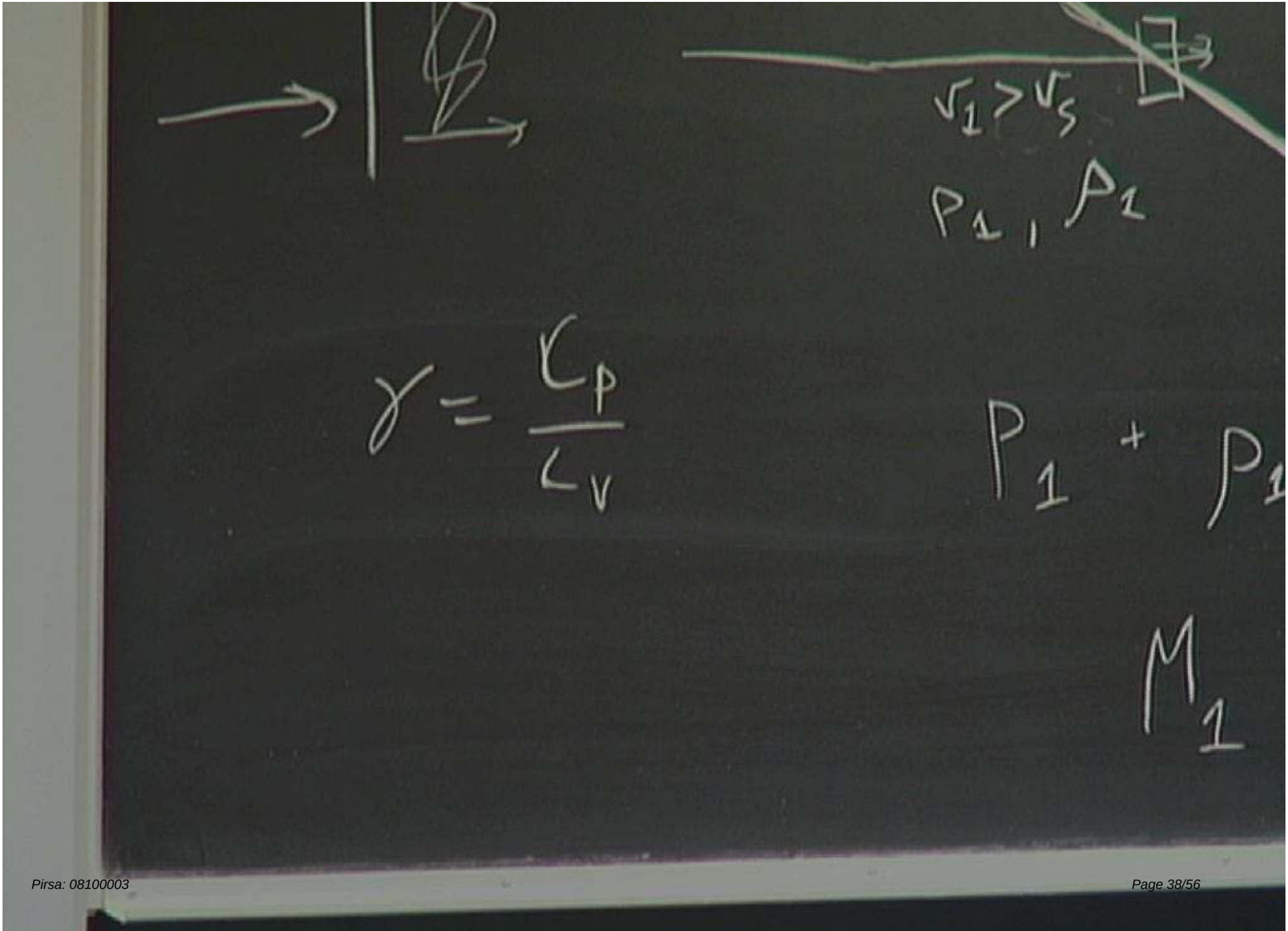
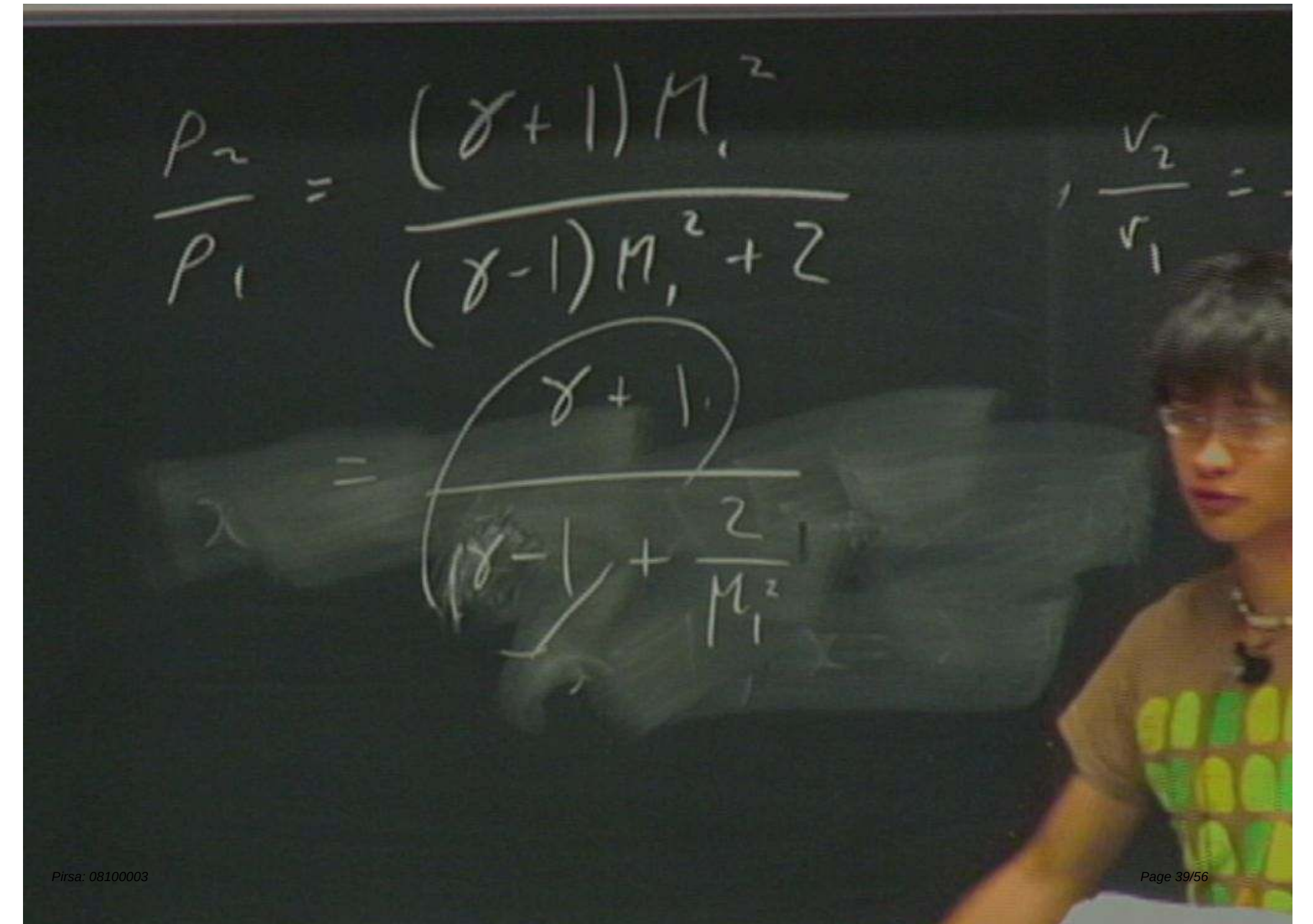

$v_1 > v_s$
$\rho_1, p_1$
$\gamma = \frac{c_p}{c_v}$
$P_1 + \rho_1$
$M_1$

$$\frac{p_2}{p_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

$$\frac{v_2}{v_1} =$$

$$= \frac{\gamma+1}{(\gamma-1)+\frac{2}{M_1^2}}$$



$$\frac{p_2}{p_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}, \quad \frac{v_2}{v_1} =$$

$$= \frac{\gamma+1}{(\gamma-1)+\frac{2}{M_1^2}}$$

$$\frac{p_2}{p_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

$$\frac{v_2}{v_1} =$$

$$= \frac{\gamma+1}{(\gamma-1)+\frac{2}{M_1^2}} \simeq \frac{\gamma+1}{\gamma-1}$$

$$(\gamma - 1) M_1^2 + 2$$

$$= \frac{\gamma + 1}{(\gamma - 1) + \frac{2}{M_1^2}} \simeq \frac{\gamma + 1}{\gamma - 1}$$

$$\frac{RT}{RT}$$

$$\frac{\frac{7}{3}}{\frac{1}{3}} = 7$$

$$\rho v^2 + P - \mathbb{P} = \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma_{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta\left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \zeta\left(\frac{\partial v}{\partial x}\right)^2$$

$$\rho v^2 + P - \mathbb{P} = \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma_{xx} \frac{\partial v}{\partial x} = \frac{1}{2}\eta\left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \zeta\left(\frac{\partial v}{\partial x}\right)^2 \qquad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$= \frac{1}{2}\eta\left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \zeta \frac{\partial v}{\partial x}$$

$$\simeq \left(\eta + \zeta\right)\frac{\partial v}{\partial x}$$

$$\rho v^2 + P - \|P\| = \text{const.}$$

$$\|P\| = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma^{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta\left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \zeta\left(\frac{\partial v}{\partial x}\right)^2 \qquad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$\sigma^{xx} = \frac{1}{2}\eta\left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \zeta\frac{\partial v}{\partial x}$$

$$\simeq \left(\eta + \zeta\right)\frac{\partial v}{\partial x}$$

$$\rho v^2 + P - \mathbb{P} = \text{const.} \Rightarrow \rho v^2 + P - \eta \frac{\partial v}{\partial x} \sim \text{const.}$$

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

$$\sigma_{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta \left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \xi \left(\frac{\partial v}{\partial x}\right)^2 \quad , \quad V_{xx} \simeq \frac{\partial v}{\partial x}$$

$$\sigma^{xx} = \frac{1}{2}\eta \left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \xi \frac{\partial v}{\partial x}$$

$$\simeq \left(\eta + \xi\right) \frac{\partial v}{\partial x} \quad , \quad \xi \simeq \eta \text{ or } \xi \ll \eta$$

$$\rho v^2 + P - \mathbb{P} = \text{const.} \Rightarrow \rho v^2 + P - \eta \frac{\partial v}{\partial x} \sim \text{const.}$$

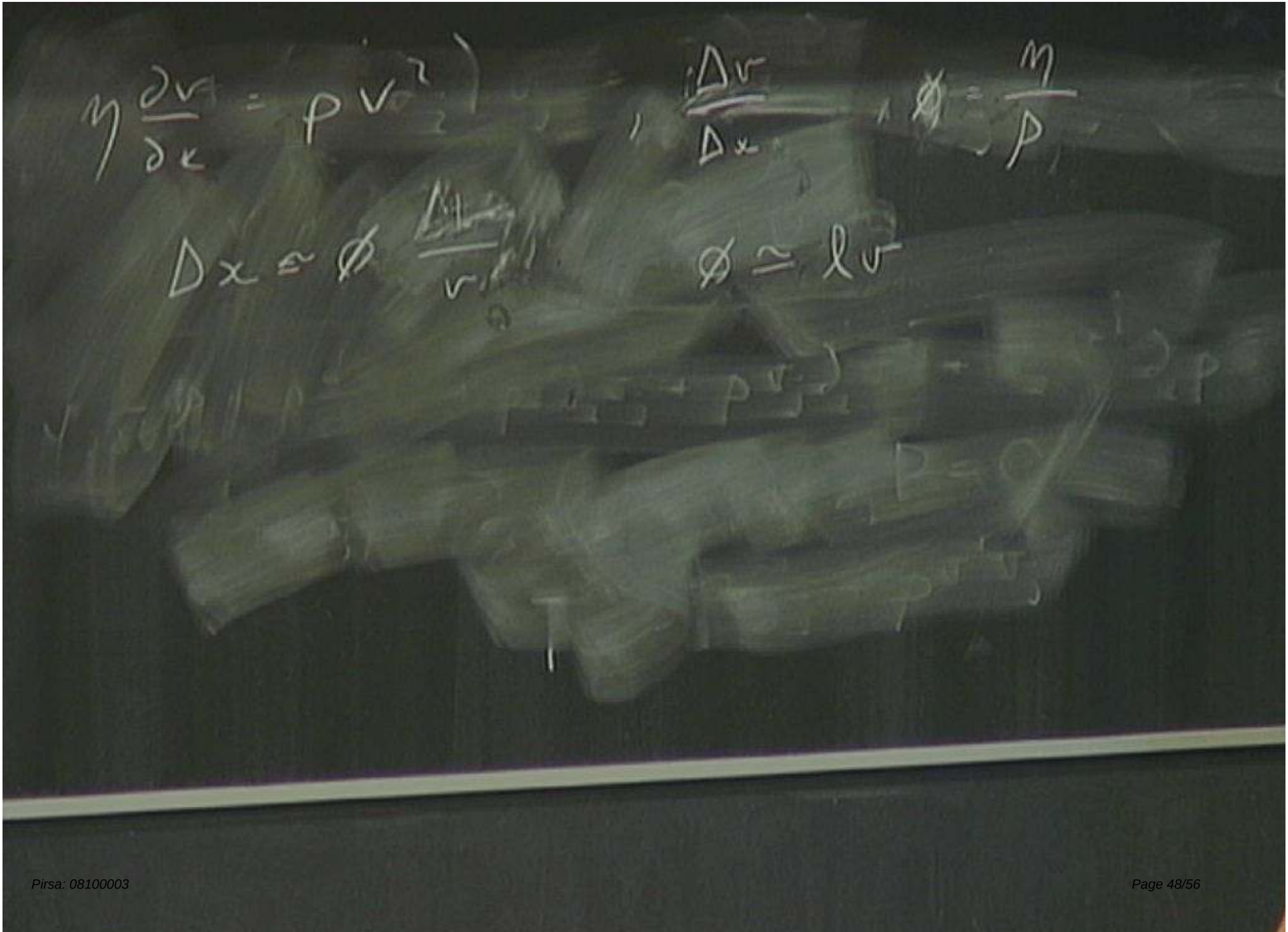
$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & - & \cdot \\ \tau_{yx} & \sigma_{yy} & \cdot \\ \tau_{zx} & - & \sigma_{zz} \end{pmatrix}$$

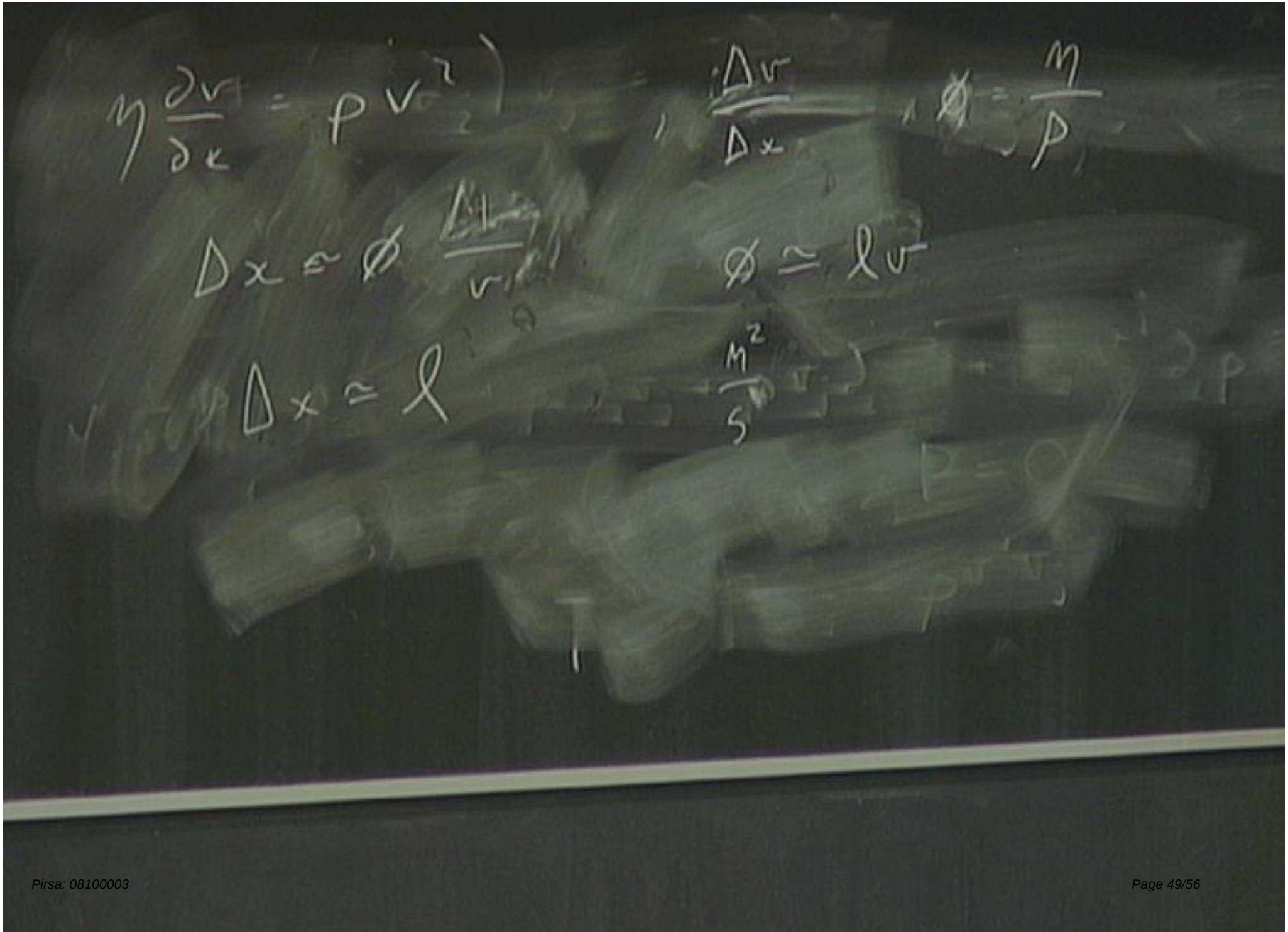
$$\rho v^2 + P - \sigma_{xx} = \text{const.}$$

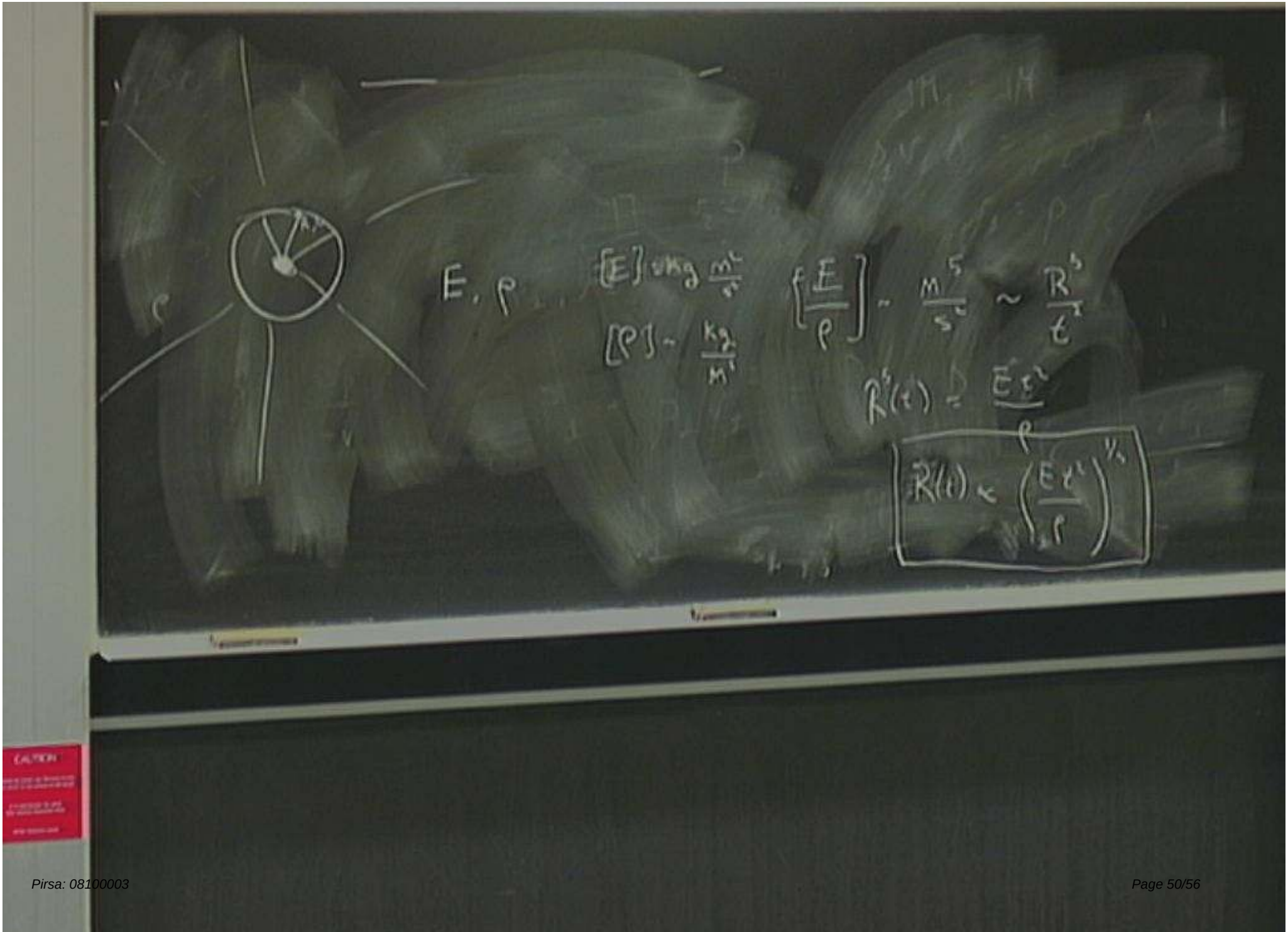
$$\sigma_{xx} \frac{\partial \vec{v}}{\partial x} = \frac{1}{2}\eta \left(2V_{xx} - \frac{2}{3}\frac{\partial v}{\partial x}\right)^2 + \xi \left(\frac{\partial v}{\partial x}\right)^2, \quad V_{xx} \simeq \frac{\partial v}{\partial x}$$

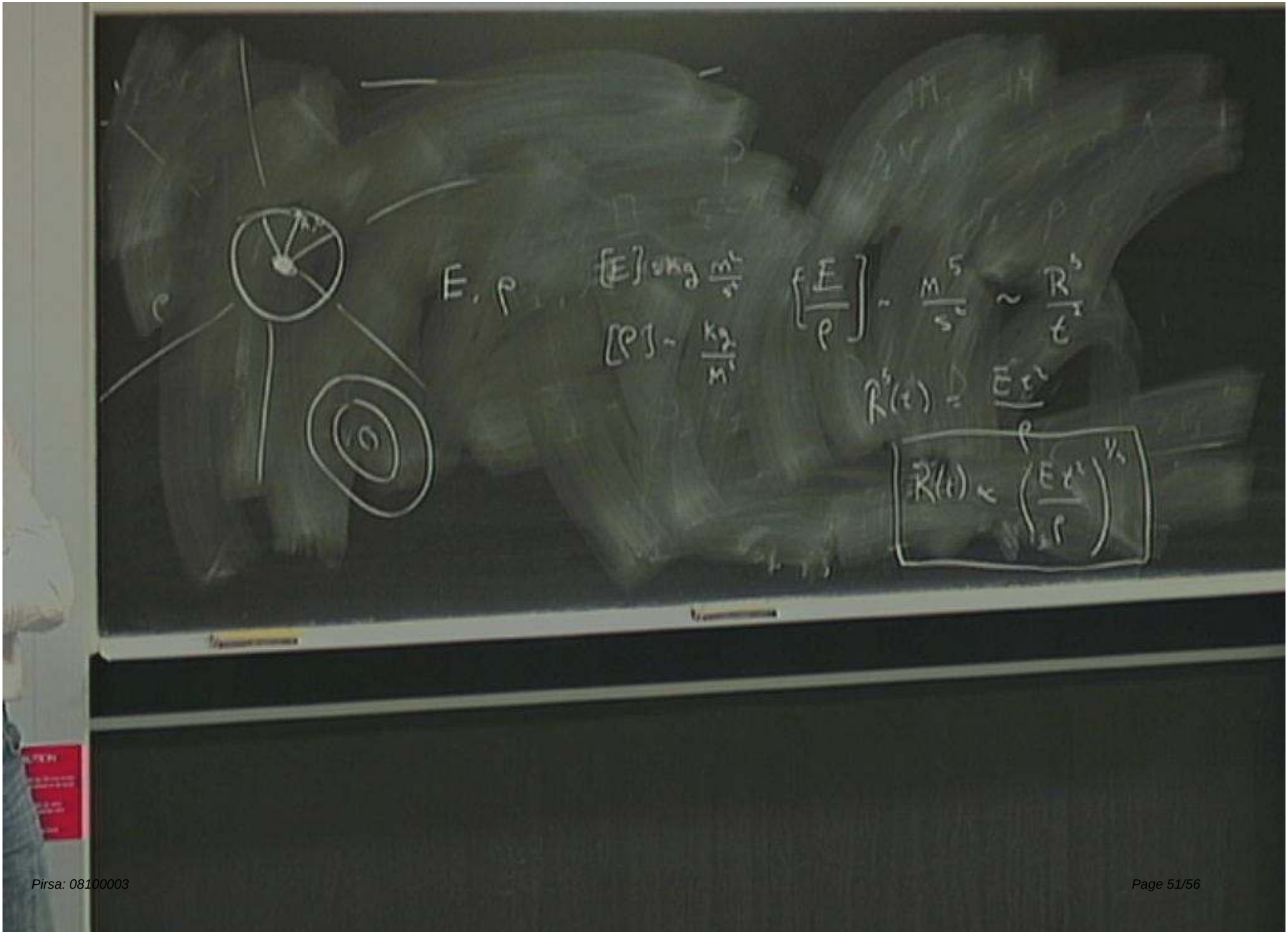
$$\sigma^{xx} = \frac{1}{2}\eta \left(\frac{4}{3}\right)^2 \frac{\partial v}{\partial x} + \xi \frac{\partial v}{\partial x}$$

$$\simeq \left(\eta + \xi\right) \frac{\partial v}{\partial x}, \quad \xi \simeq \eta \text{ or } \xi \ll \eta$$




$\eta \frac{\partial v}{\partial x} = \rho v^2$
$\frac{\Delta v}{\Delta x}$
$\nu = \frac{\eta}{\rho}$
$\Delta x \simeq \nu$
$\nu \simeq \ell v$
$\Delta x \simeq \ell$
$\frac{M^2}{S}$


E, ρ
[E] ~ kg m²/s²
[ρ] ~ kg/m³
[E/ρ] ~ m⁵/s² ~ R⁵/t²
R⁵(t) ≈ Et²/ρ
R(t) ≈ (Et²/ρ)^1/5


E, ρ
[E] ~ kg m²/s²
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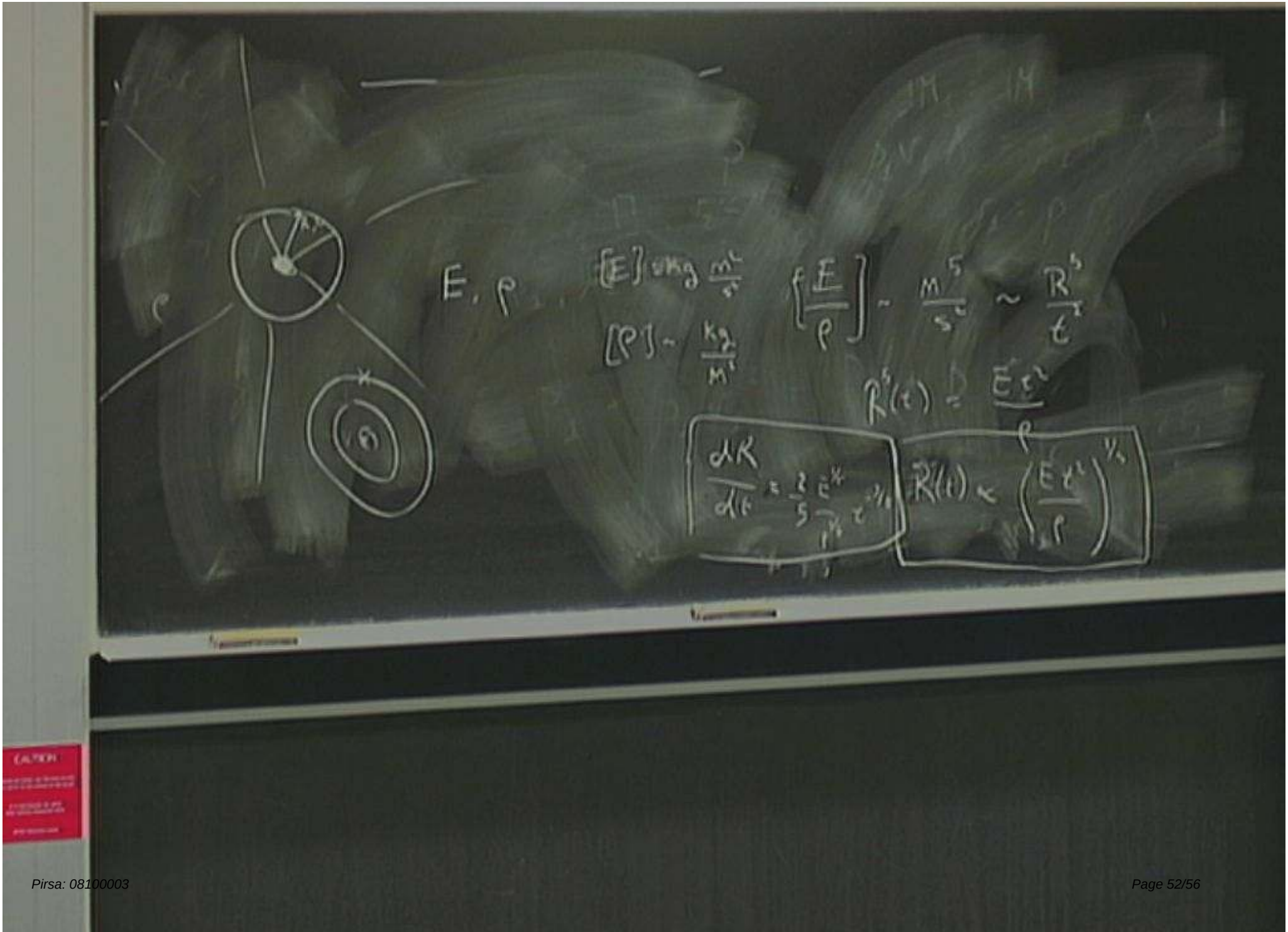
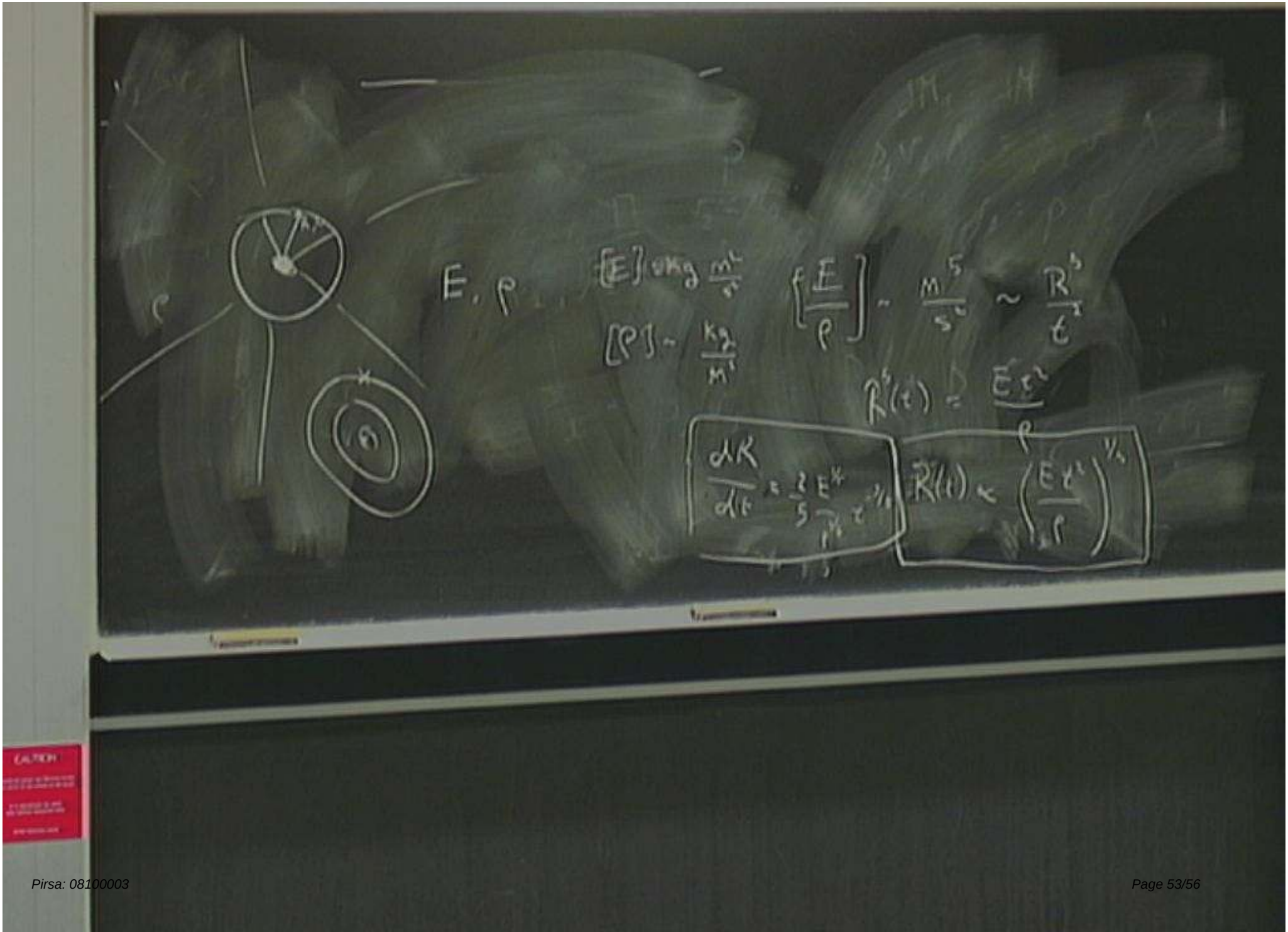
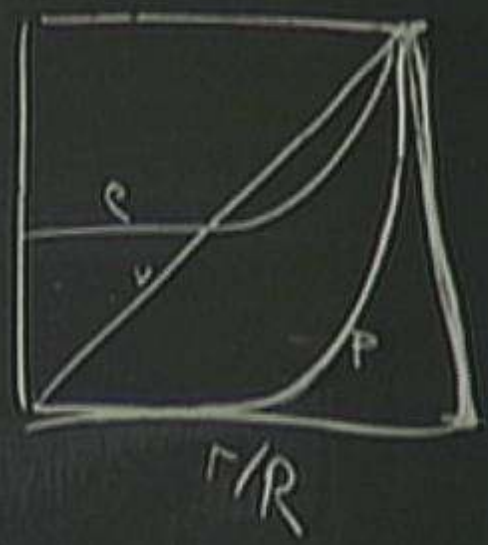

E, ρ
[E] ~ M L²/T²
[ρ] ~ M/L³
[E/ρ] ~ M⁵/T² ~ R⁵/t²
R⁵(t) ≈ E t²/ρ
dR/dt ≈ (2/5) E^½ ρ^-½ t^-½
R(t) ≈ (E t²/ρ)^⅕

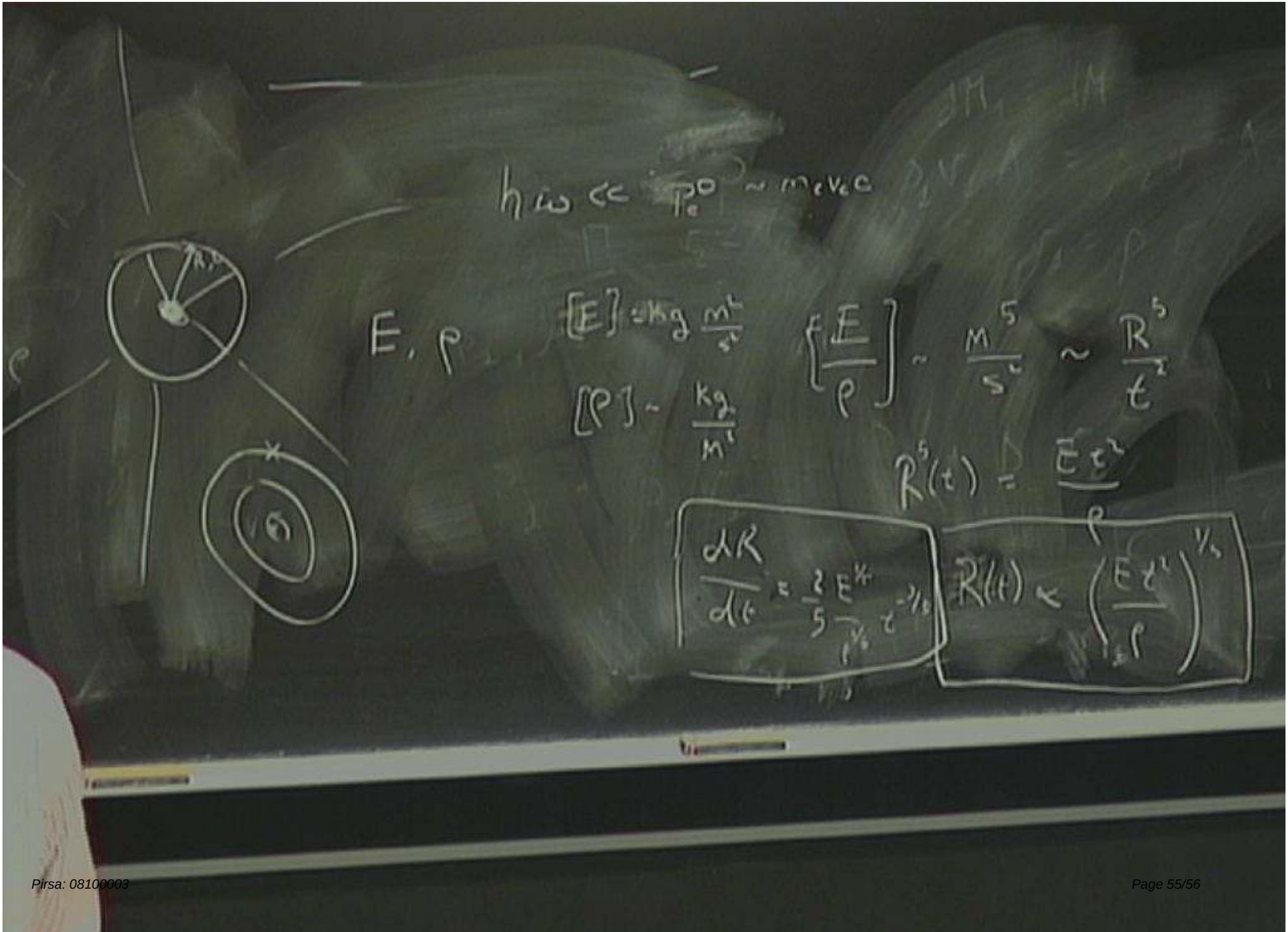

E, ρ
[E] ~ kg m²/s²
[ρ] ~ kg/m³
[E/ρ] ~ m⁵/s² ~ R³/t²
R³(t) ≈ E t²/ρ
dK/dt = (3/5) E^(3/2) ρ^(-1/2) t^(-1/2)
R(t) ≈ (E t²/ρ)^(1/5)

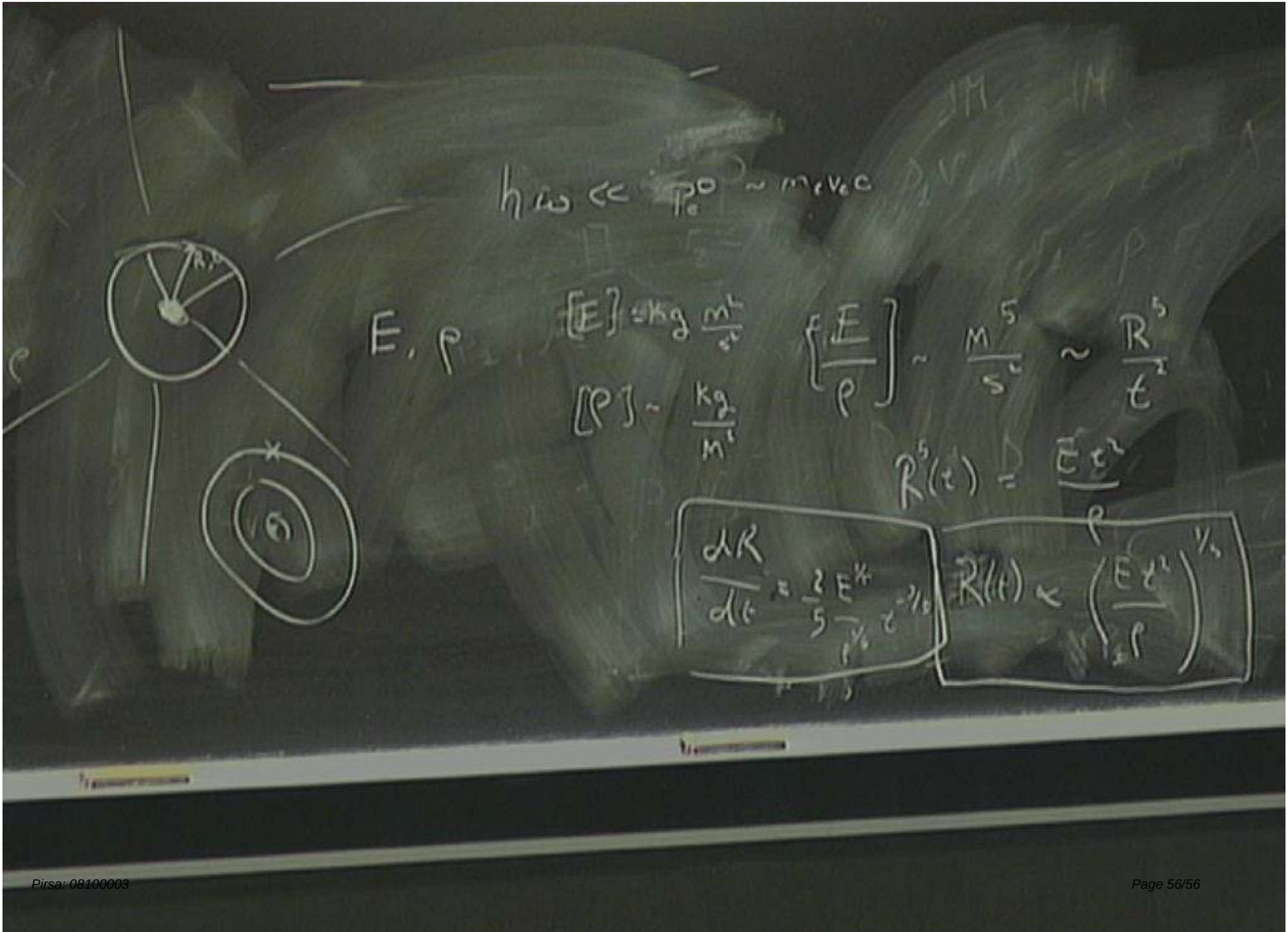
$$\frac{1}{\gamma} \rho_2 \left( v_1^2 - v_2^2 \right)$$

$$= p_1 \frac{\rho_2}{\rho_1} + \frac{\gamma - 1}{2\gamma} \rho_2 \left( v_1^2 - v_2^2 \right)$$

$$\frac{\rho_1^2 v_1^2}{\rho_2^2}$$




$h\omega \ll p_e c \sim m_e v_e c$
$E, \rho$
$[E] = kg \frac{m^2}{s^2}$
$[\rho] \sim \frac{kg}{m^3}$
$\left[\frac{E}{\rho}\right] \sim \frac{m^5}{s^2} \sim \frac{R^5}{t^2}$
$R^5(t) = \frac{E t^2}{\rho}$
$\frac{dR}{dt} = \frac{2}{5} E^{1/5} \rho^{-1/5} t^{-3/5}$
$R(t) \propto \left(\frac{E t^2}{\rho}\right)^{1/5}$


$\hbar\omega \ll p_e c \sim m_e v_e c$
E, ρ
$[E] = kg\,\frac{m^2}{s^2}$
$[\rho] \sim \frac{kg}{m^3}$
$\left[\frac{E}{\rho}\right] \sim \frac{m^5}{s^2} \sim \frac{R^5}{t^2}$
$R^5(t) = \frac{E}{\rho} t^2$
$\frac{dR}{dt} = \frac{2}{5} E^{1/5} \rho^{-1/5} t^{-3/5}$
$R(t) \propto \left(\frac{E}{\rho} t^2\right)^{1/5}$