Title: Exotic Phases and Phase Transitions from Geometrical Frustration

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Abstract: In frustrated systems, competing interactions lead to complex phase diagrams and sometimes entirely new states of matter. Frustration often arises from the lattice geometry, leaving the system delicately balanced between a variety of possible orders. A number of normally weak effects can lead to a lifting of this degeneracy. For example, I will discuss how quantum fluctuations can stabilize a supersolid phase, where the system is at once both a crystal and a superfluid. Frustrated magnets are promising candidates for realizing spin liquid phases with exotic \'topological order\', and new kinds of quantum phase transitions that have no classical analog. Bio: Ashvin Vishwanath received his MSc from IIT Kanpur in 1996, and PhD from Princeton in 2001. After holding the Papparlardo fellowship at MIT, he joined UC Berkeley in 2004, where he is currently associate professor of physics. He specializes in theoretical condensed matter physics, especially magnetism, superconductivity and other correlated quantum phenomena in solids and cold atomic gases. Ashvin is the recipient of several awards, most recently a Sloan fellowship (2004), a Hellman foundation fellowship (2006), and an NSF CAREER Award (2007).

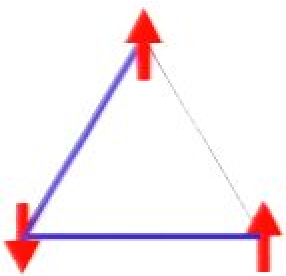
Novel Phases and Phase Transitions from Geometric Frustration

Ashvin Vishwanath

UC Berkeley

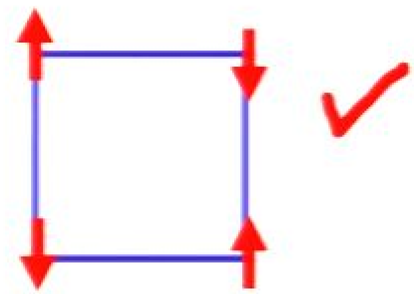
Geometric Frustration

- Frustration ~ degeneracy of classical ground states.



$$E = J \sum_{i,j\ \mathbf{neighbors}} S_i S_j$$

$$J > 0, S_i = \pm 1$$ (Classical Spin)



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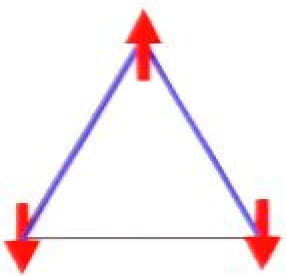

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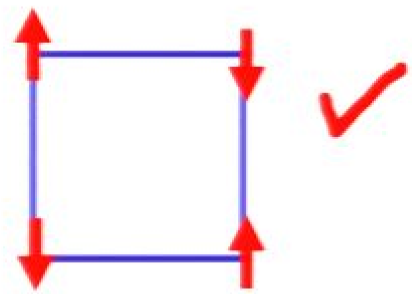
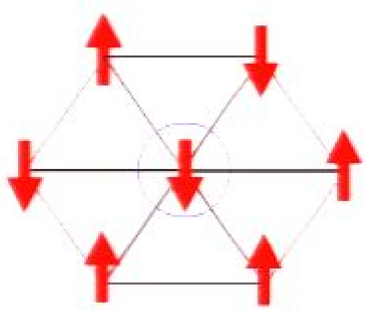
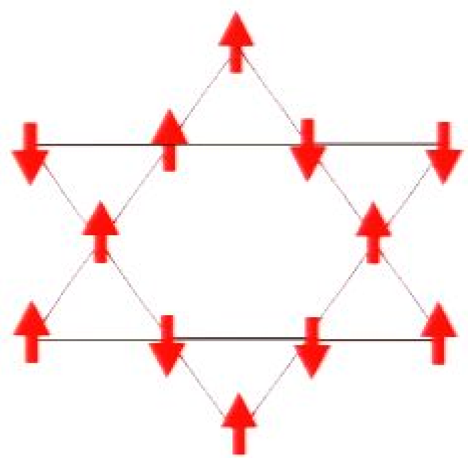
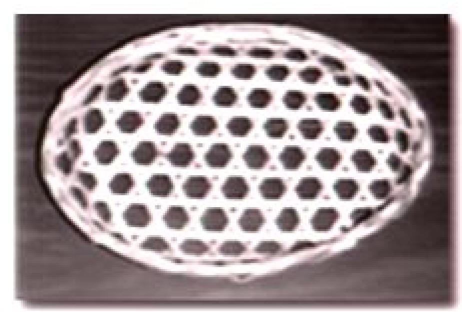


Triangular Lattice:
#of ground states= $e^{0.323\mathrm{N}}$
(N sites)



Kagome Lattice: $e^{0.502\mathrm{N}}$





Geometrical Frustration in Ice

[CONTRIBUTION FROM THE CHEMICAL LABORATORY OF THE UNIVERSITY OF CALIFORNIA]

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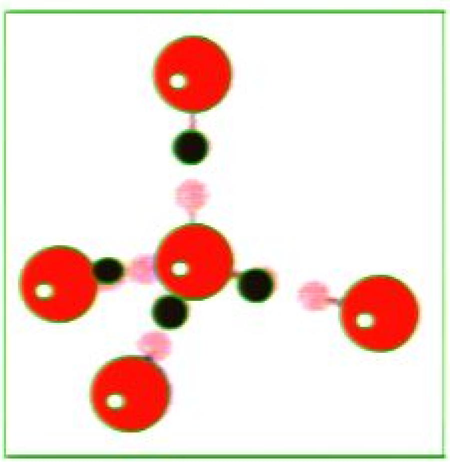
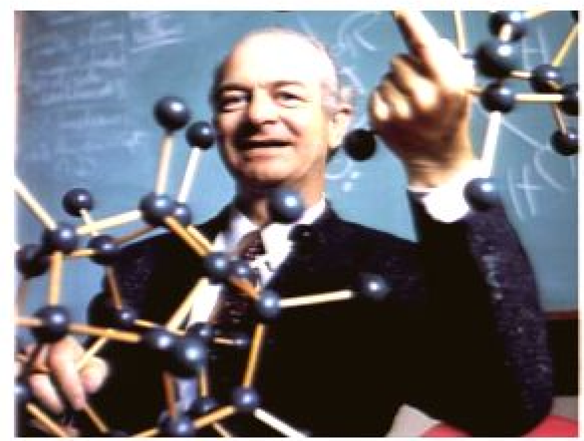


Idea: **Hydrogens** remain disordered giving entropy



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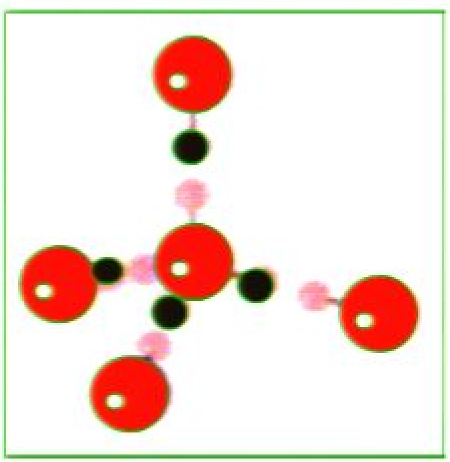

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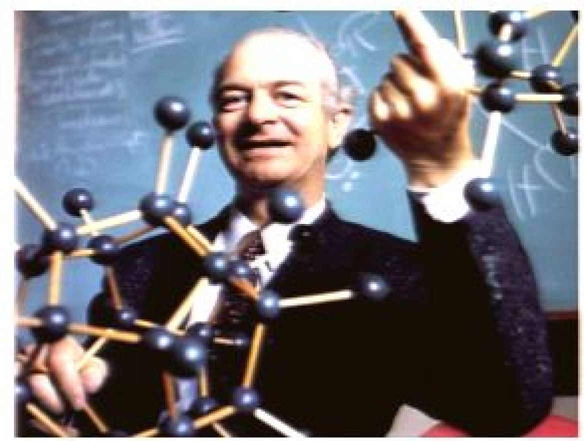


$$\Omega = 2^{2N} \times \left(\frac{6}{16}\right)^{N} \quad S = \left(\frac{R}{N}\right) \log \Omega$$

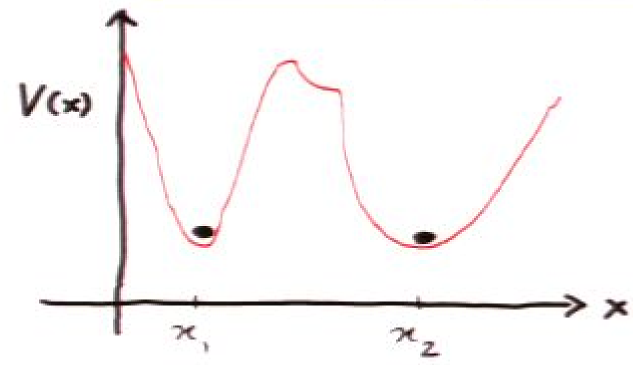
$$S_{Pauling} = 0.806\ \mathbf{cal/mol\ ^oK}$$

Geometric Frustration & Relief

Outline

1. Frustration relief
   - by residual interactions: *eg.* complex orders from lattice couplings.
   - Quantum or thermal fluctuations (order by disorder). Eg. supersolid order on triangular lattice.

Classically degenerate



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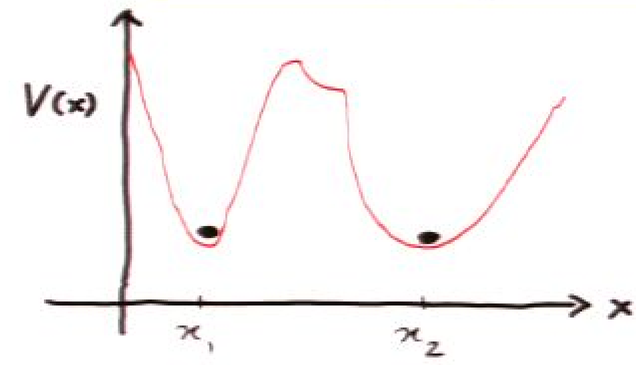

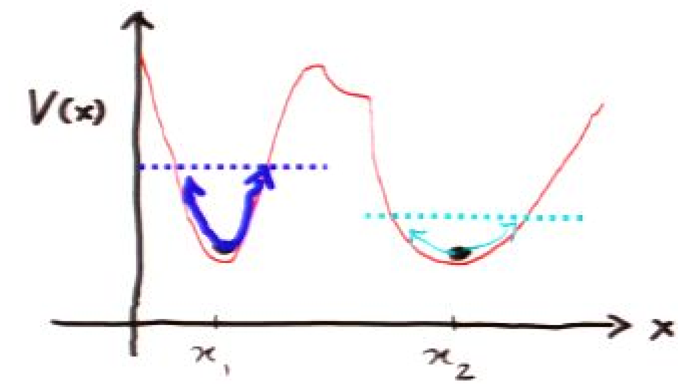

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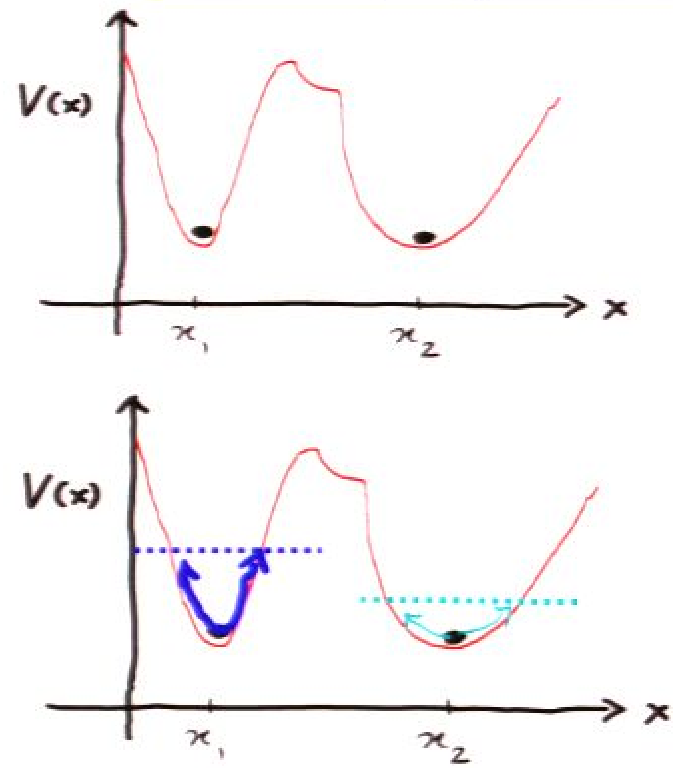

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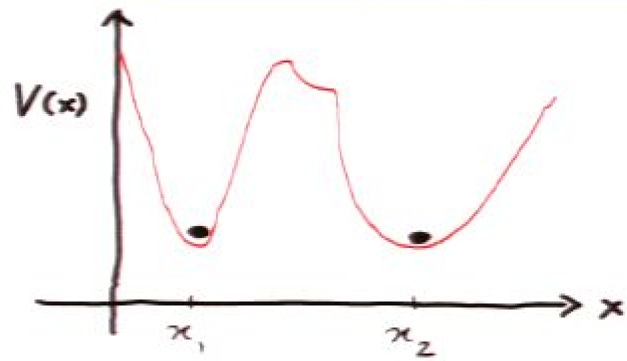
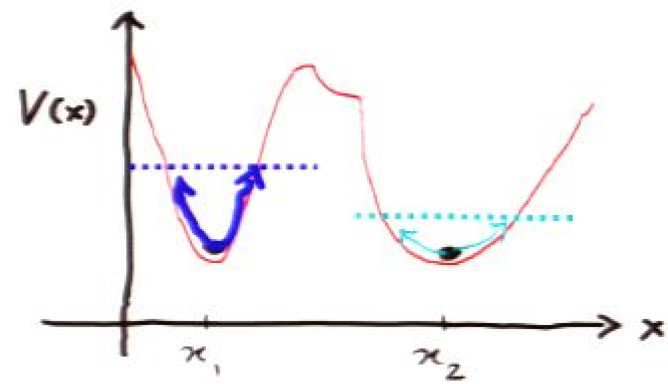
2. Quantum spin liquids.

- Quantum phases and transitions beyond Landau's paradigm.

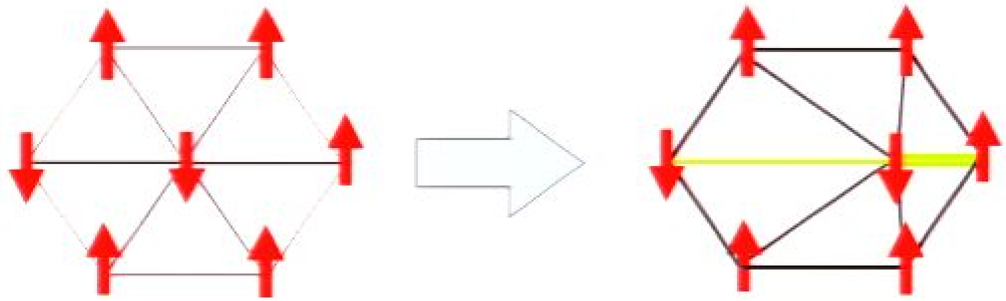
Classically degenerate





Strong tunneling

Frustration and Relief - Lattice coupling



$$E = \sum_{i,j\,\mathbf{nbrs}} (J + \partial_r J \cdot [\delta r_i - \delta r_j]) S_i S_j + K \sum_i \delta r_i^2$$

Restoring force — Change in *J*

More realistic spin model: $S_i S_j \rightarrow \vec{\mathbf{S}}_\mathbf{i} \cdot \vec{\mathbf{S}}_\mathbf{j}$

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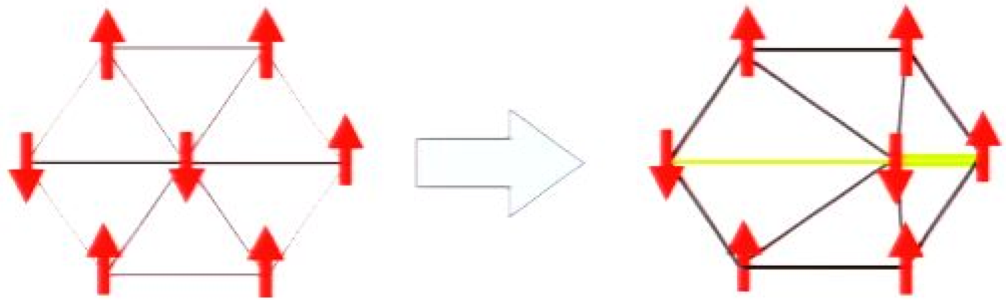

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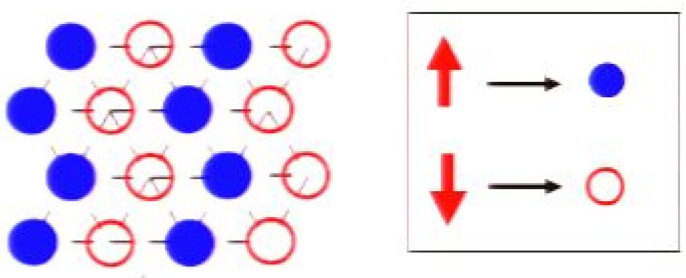


Z state

Zigzag stripes

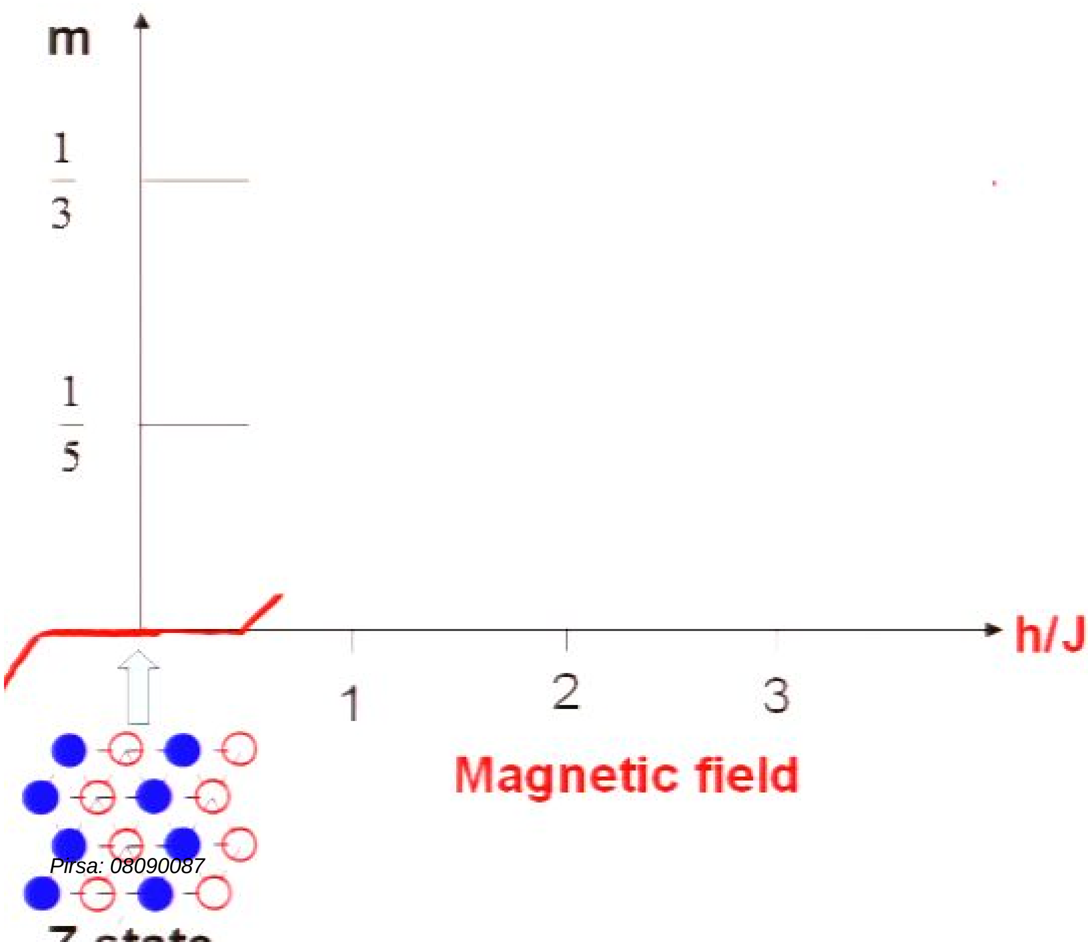
Observed in Triangular lattice magnet $CuFeO_2$

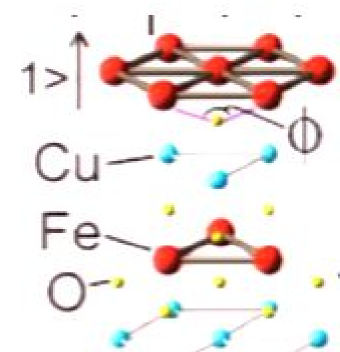
Magnetization Plateaus and $CuFeO_2$

Magnetization plateaus

(magnetization constant with changing field)

c=0.15, D_z=0.01J





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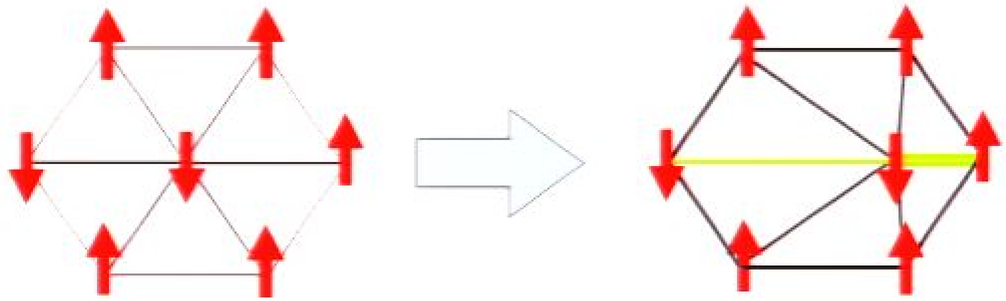

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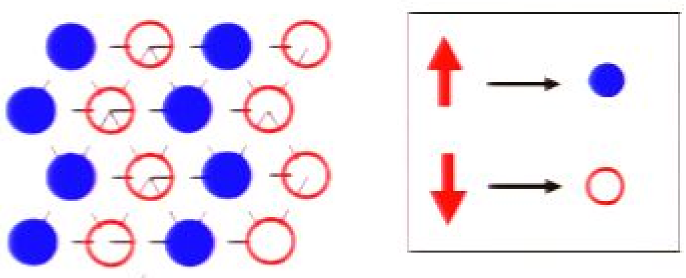

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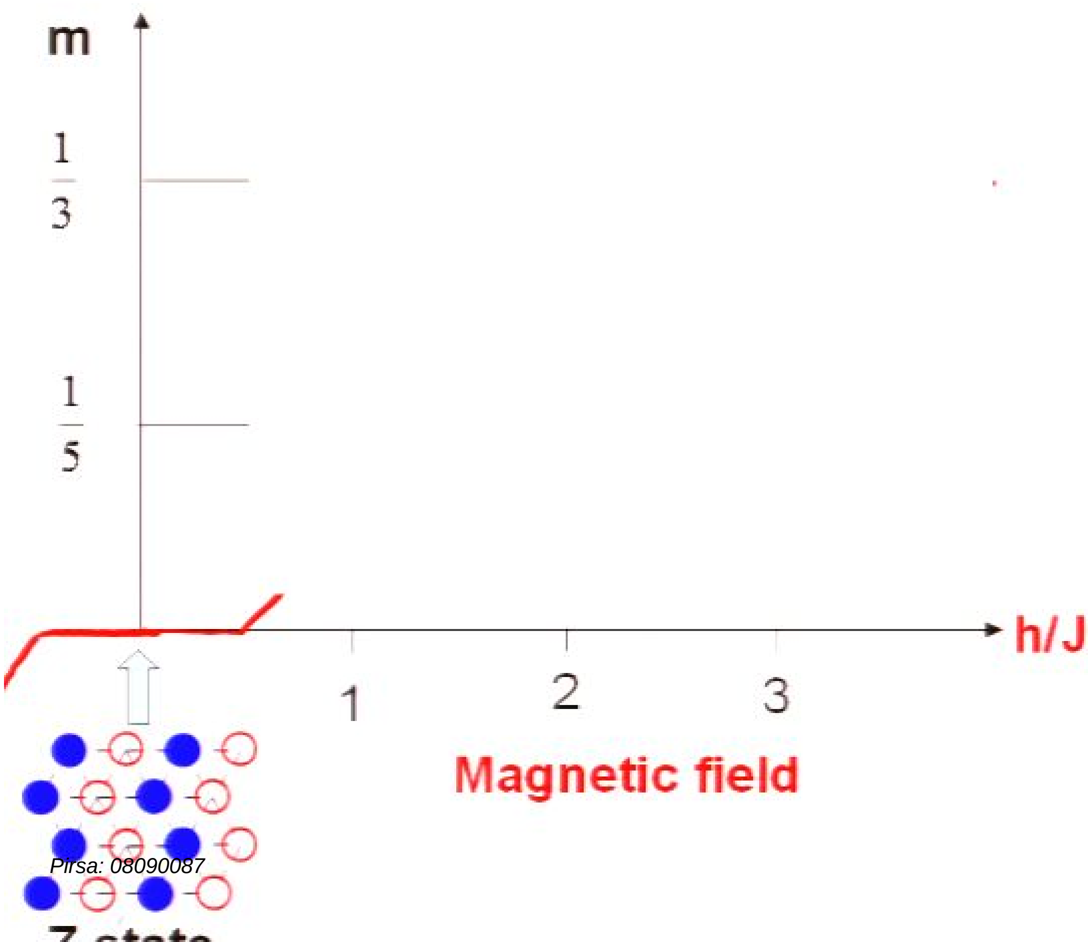

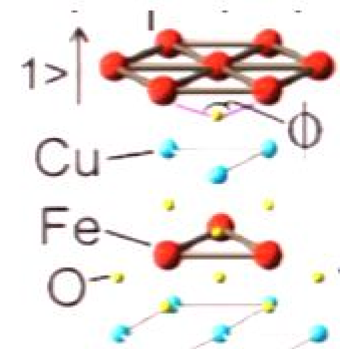

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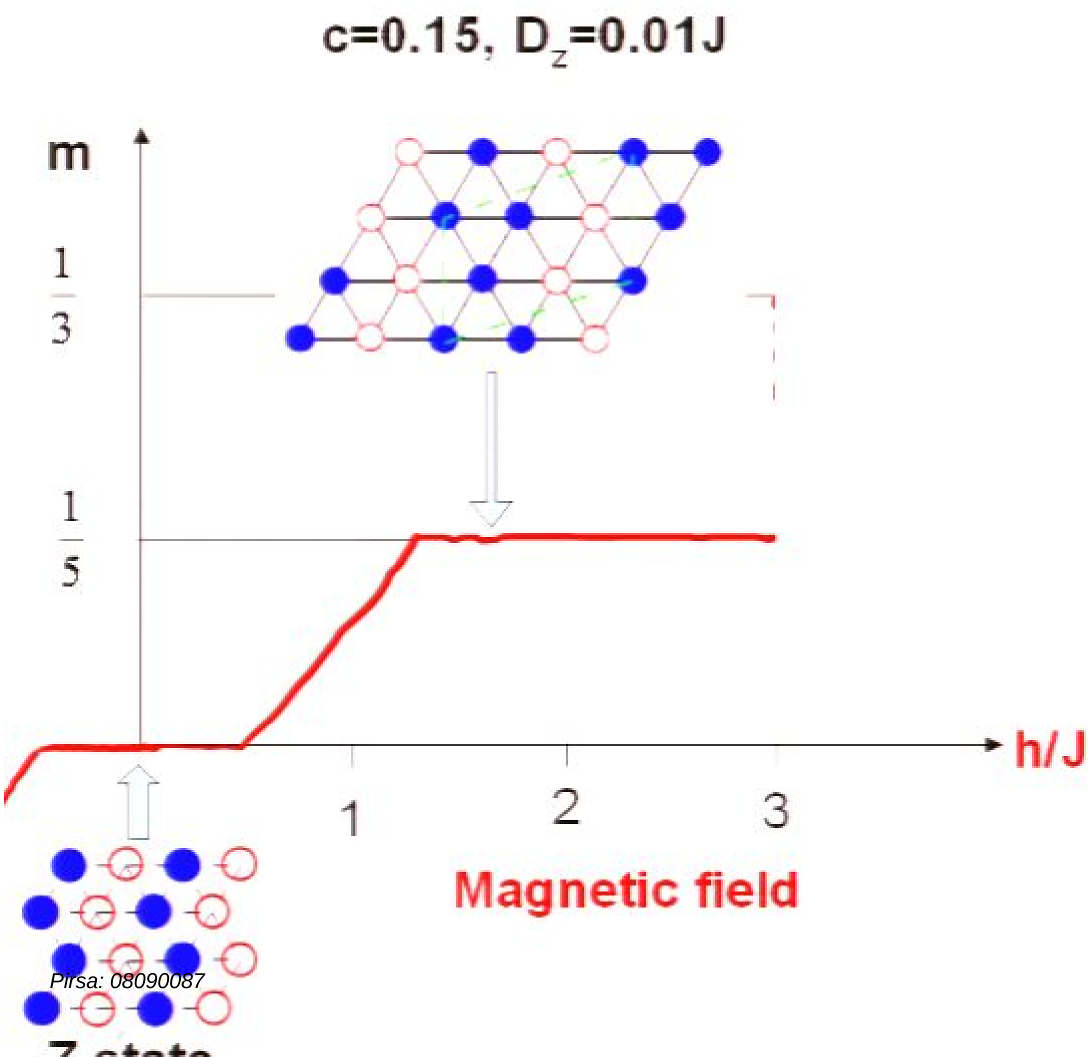

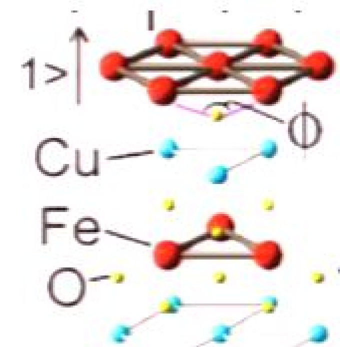

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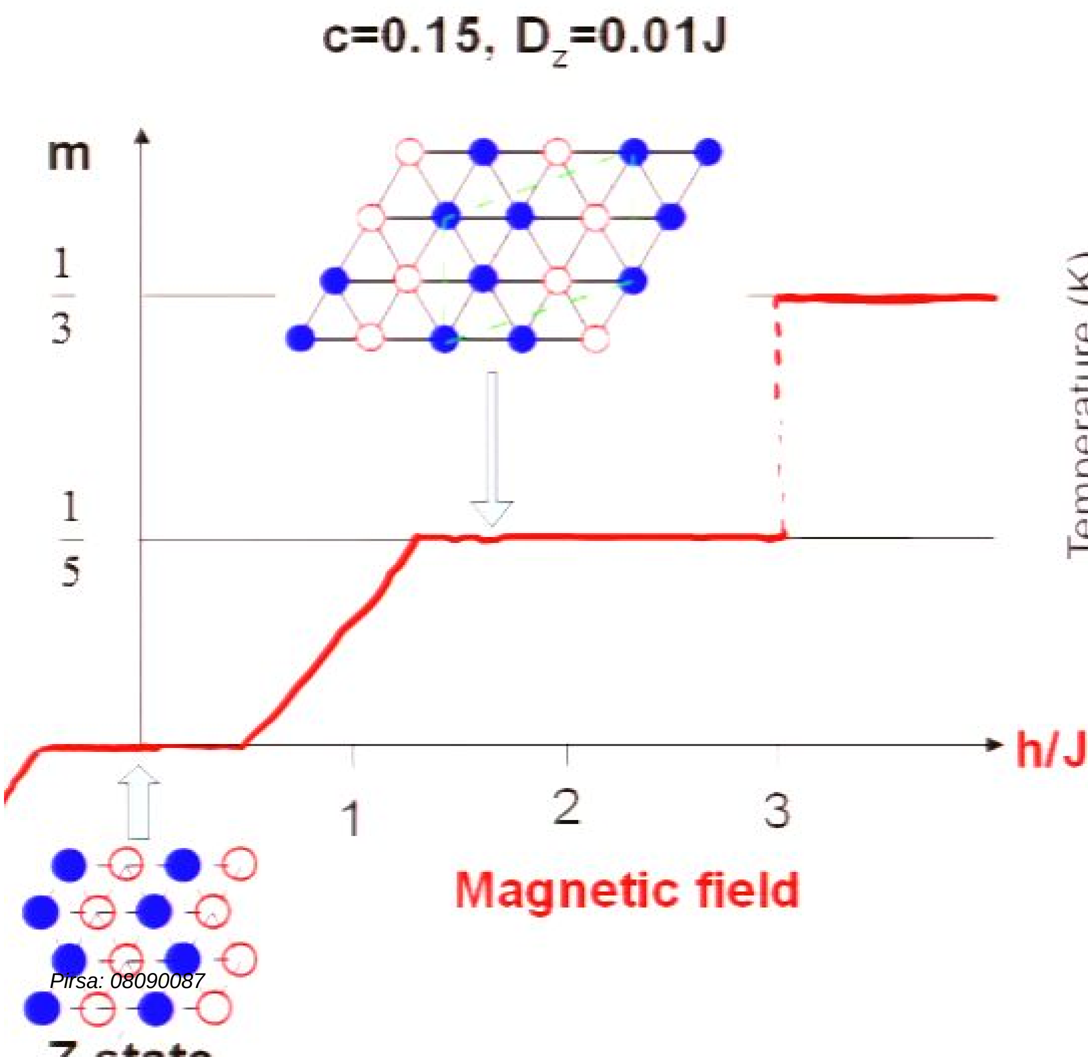
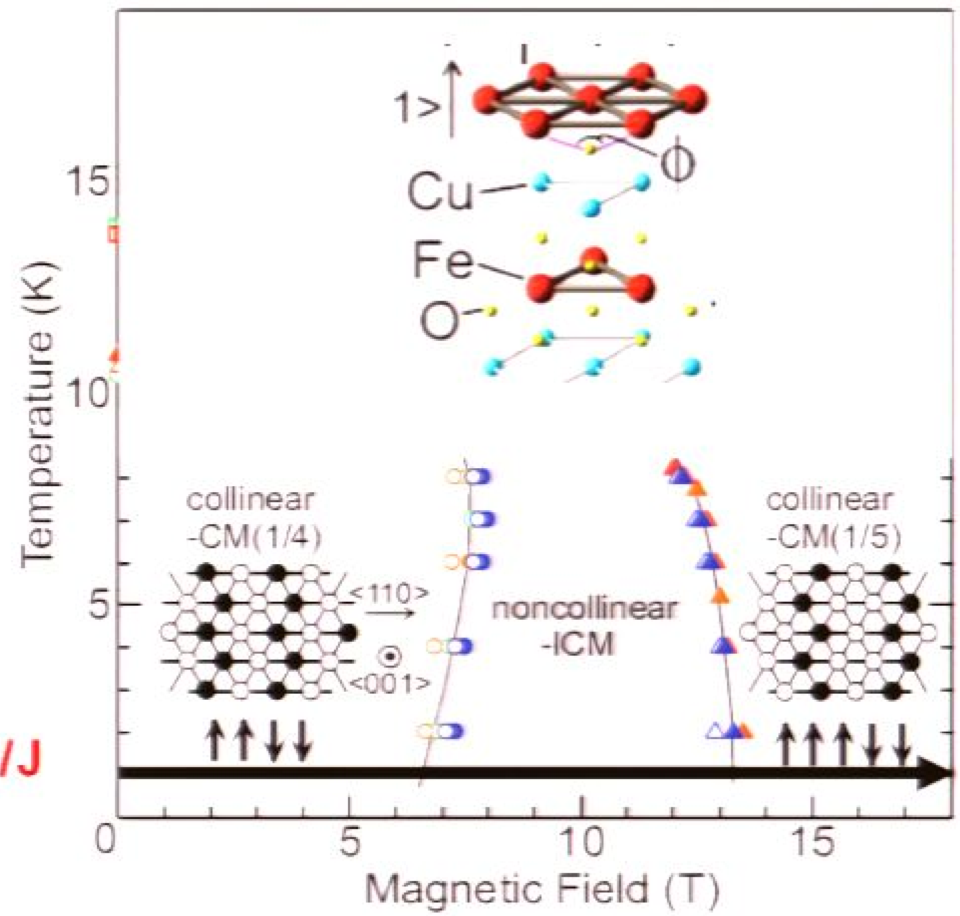


Phases of Triangular magnet $CuFeO_2$



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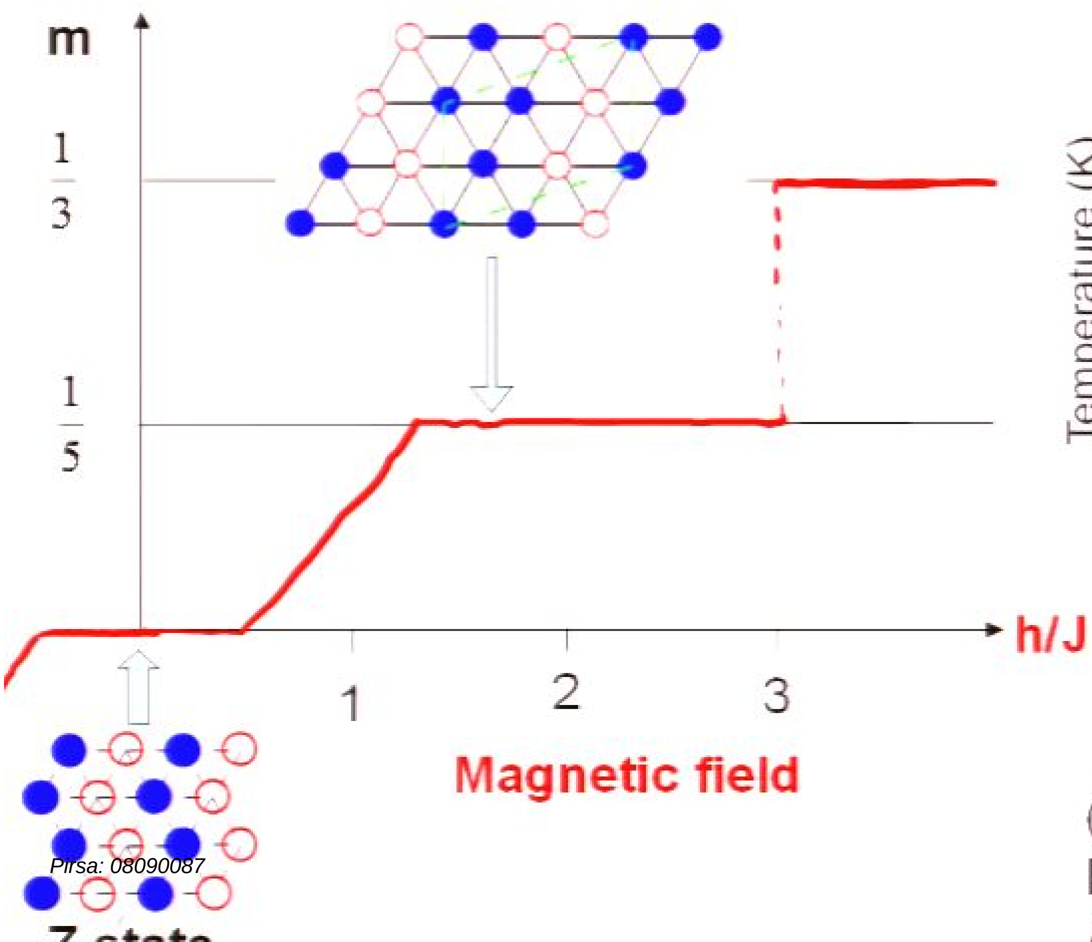

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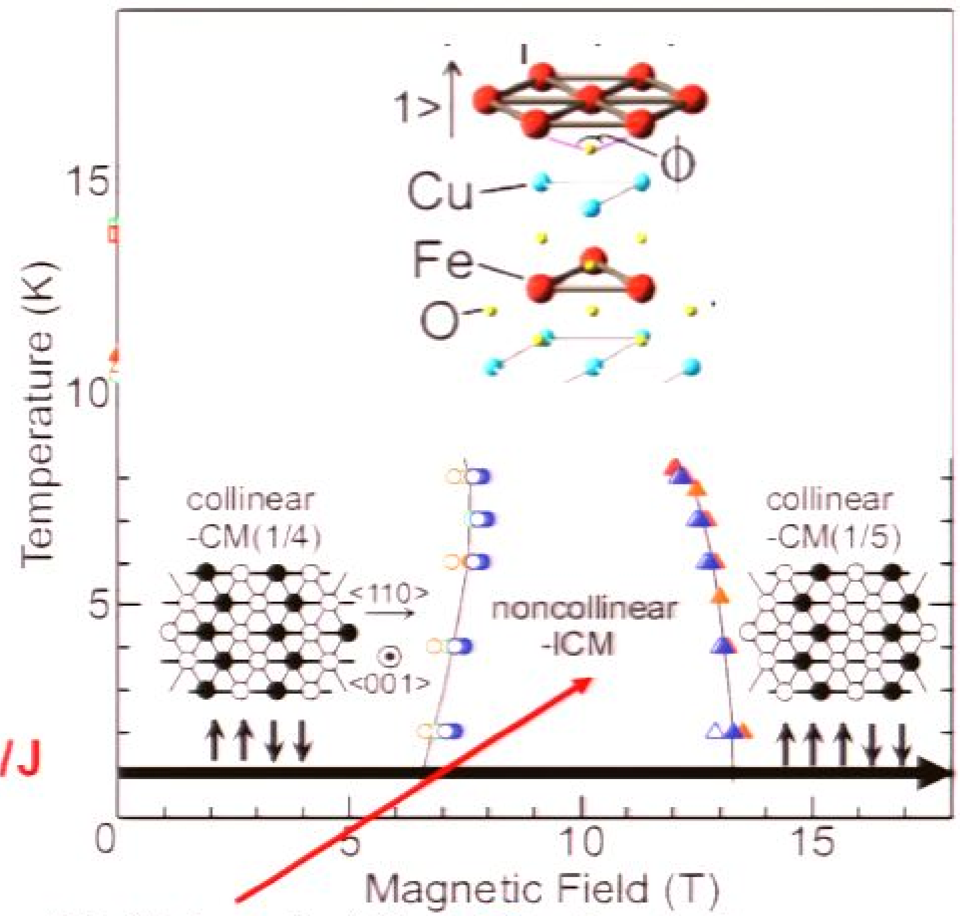


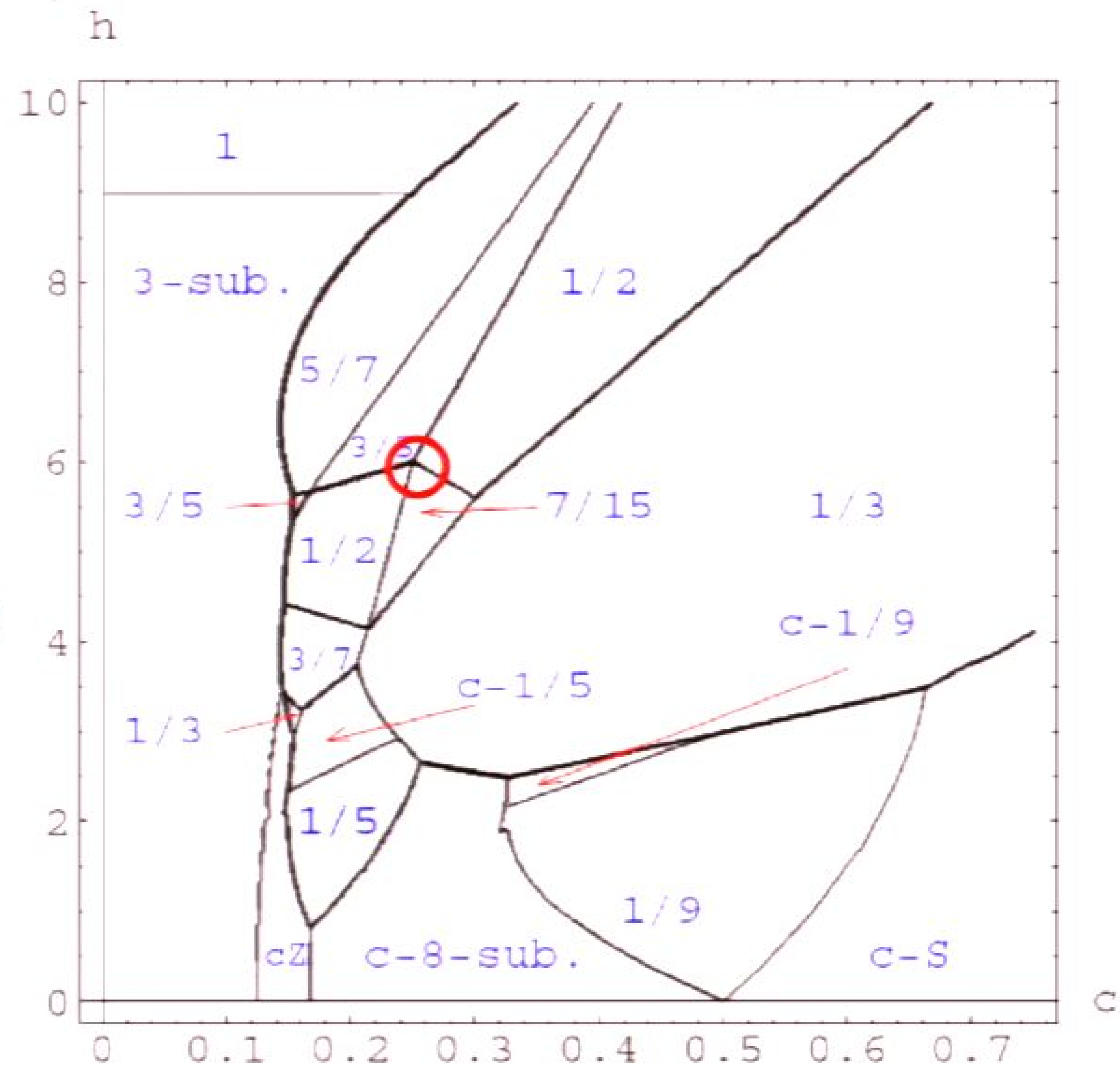
(Multi-ferroic (Also **P**): Control Magnetism with Electric fields. Applications to memories?)

Complex Phase Structure of Frustrated Systems

Spin-phonon Phase Diagram in a field

Complex Phase Structure of Frustrated Systems

- Violations of "Gibbs Phase Rule"
  - 4 phases meet at a point due to frustration (accidental degeneracy).



Spin-phonon Phase Diagram in a field

Frustration can lead to complex orders – which may even be useful

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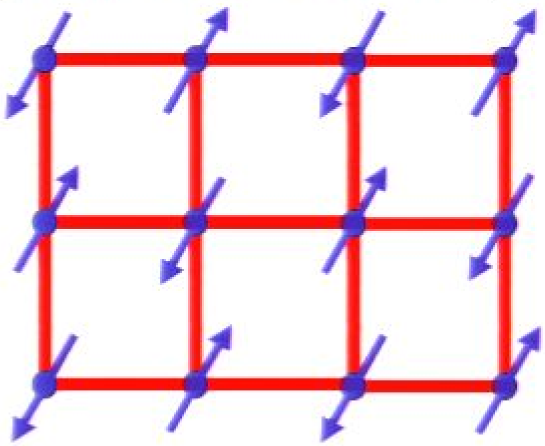
Quantum Spin Liquids

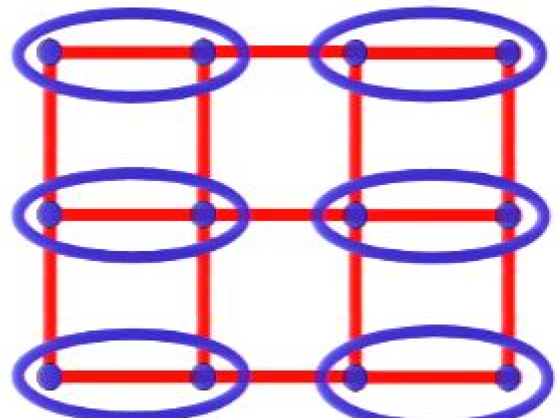
- Conventional phases – distinguished by Landau order parameters ('Standard Model'). Captures *broken* symmetry.

magnetic order: break spin symmetry

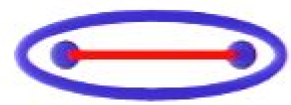


$$\hat{n}_r = (-1)^{x+y} \vec{S}_r$$

spin solid: break lattice symmetries



$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



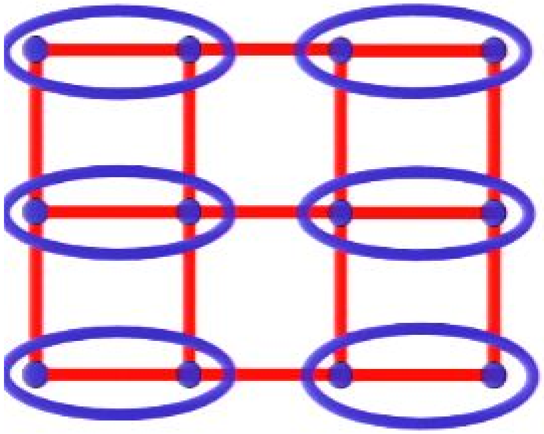
$$\psi = (-1)^x \vec{S}_r \cdot \vec{S}_{r+\hat{x}} + i(-1)^y \vec{S}_r \cdot \vec{S}_{r+\hat{y}}$$

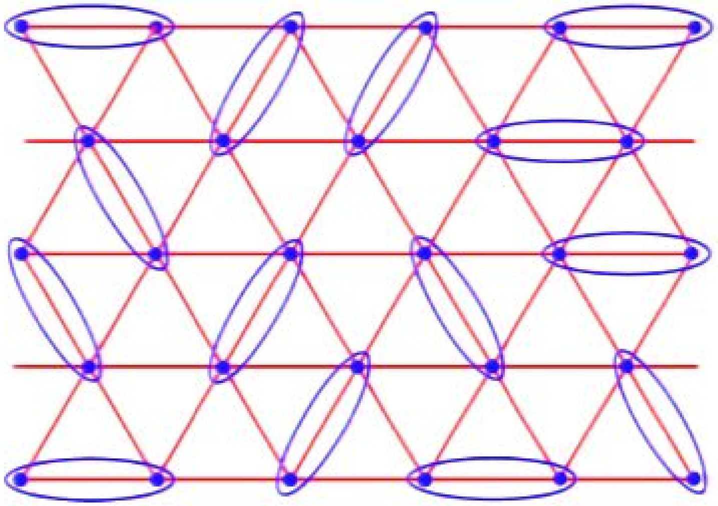
•Quantum spin liquid – interacting spins, *but not ordered*.

•Known in D=1 (spin chains).

What about D=2,3?? Frustrated lattices ideal candidates.

Quantum Spin Liquids-
Resonating Valence Bonds (RVB)





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RVB spin liquid: liquid of valence bonds (Anderson '73)

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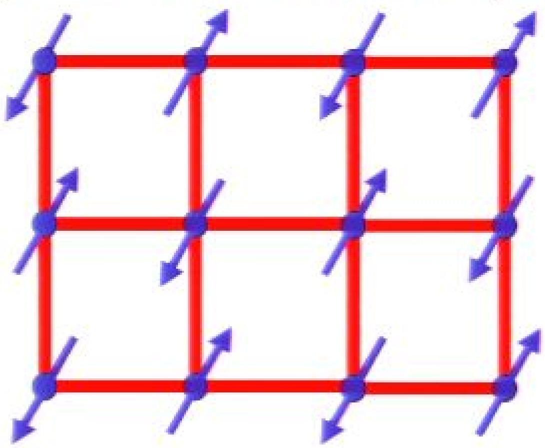

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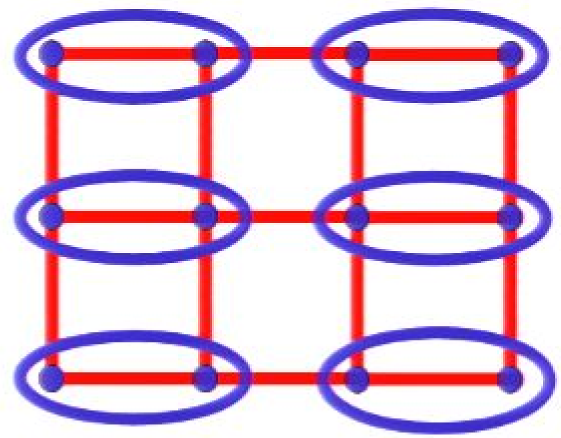

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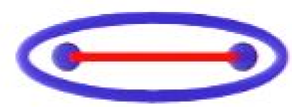

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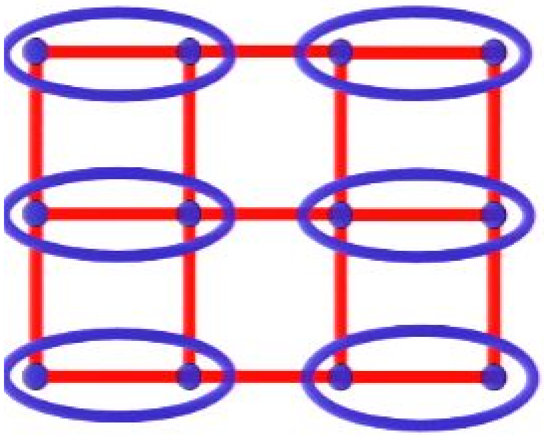

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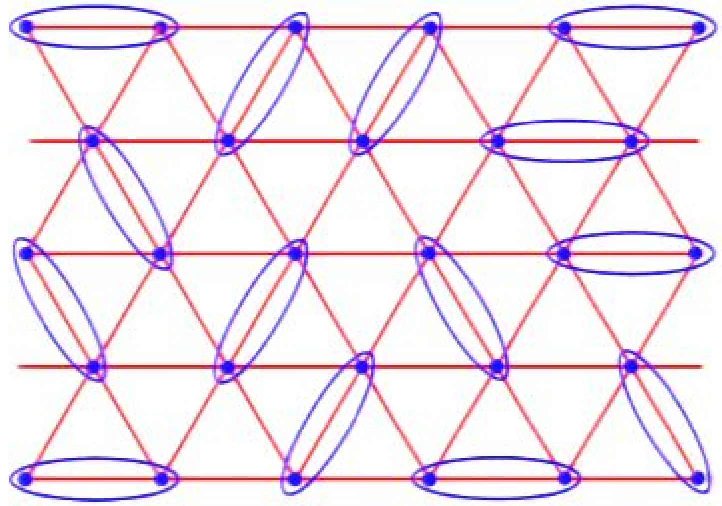

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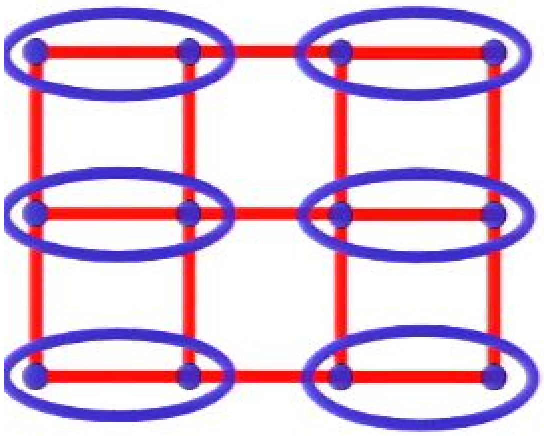

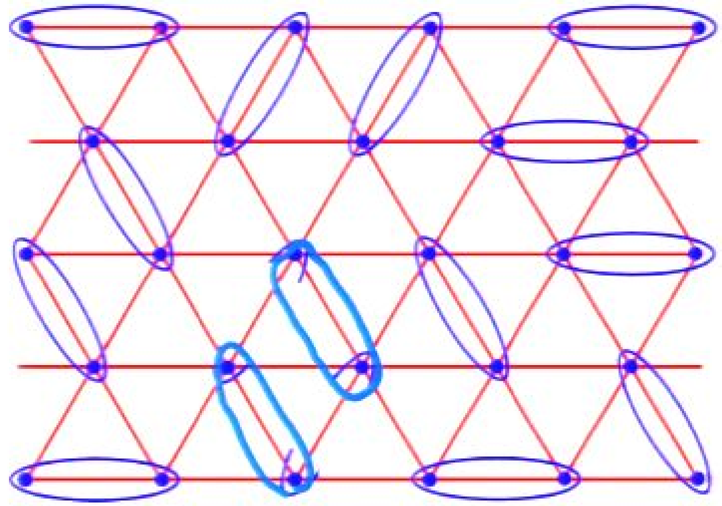

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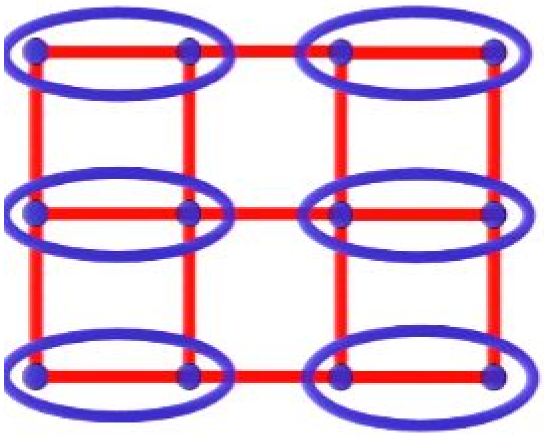

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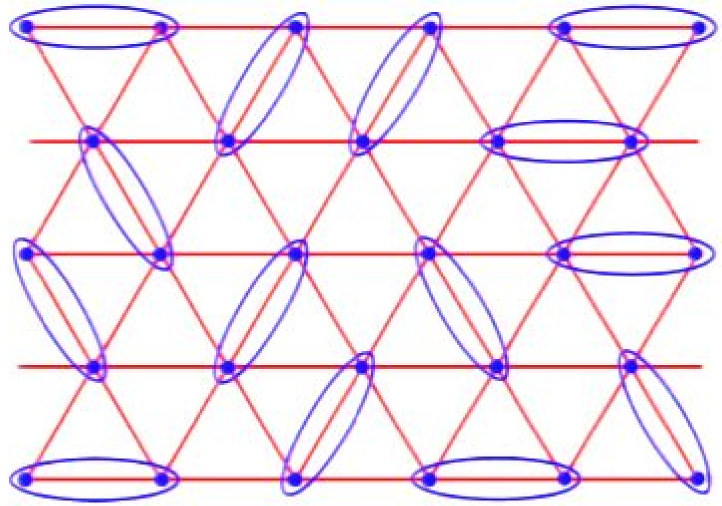

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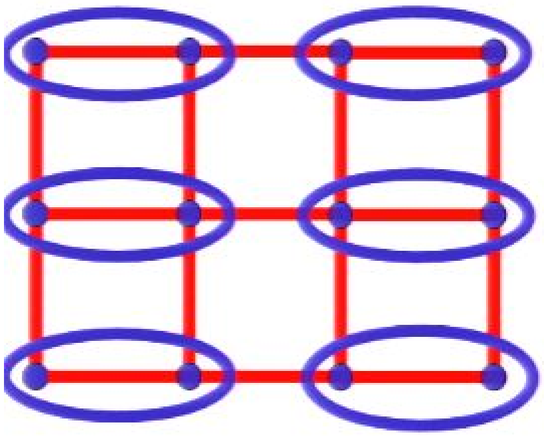

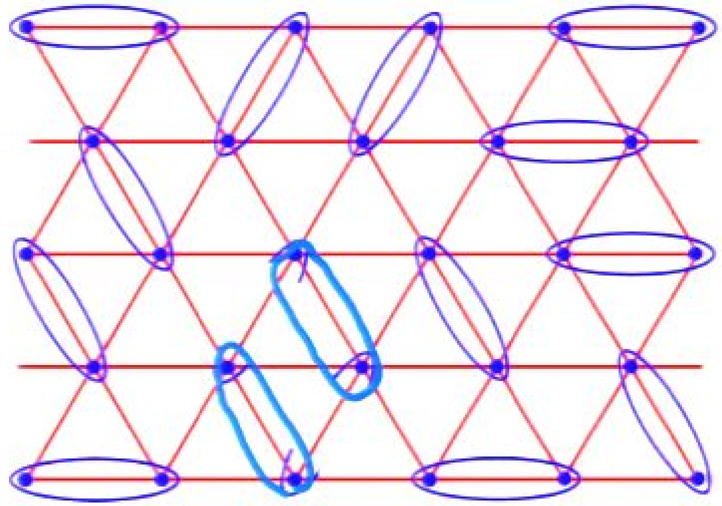

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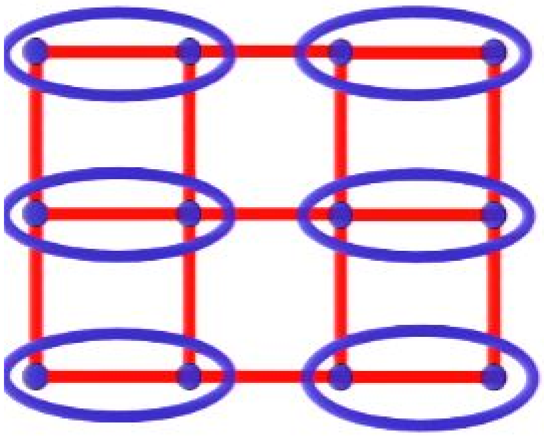

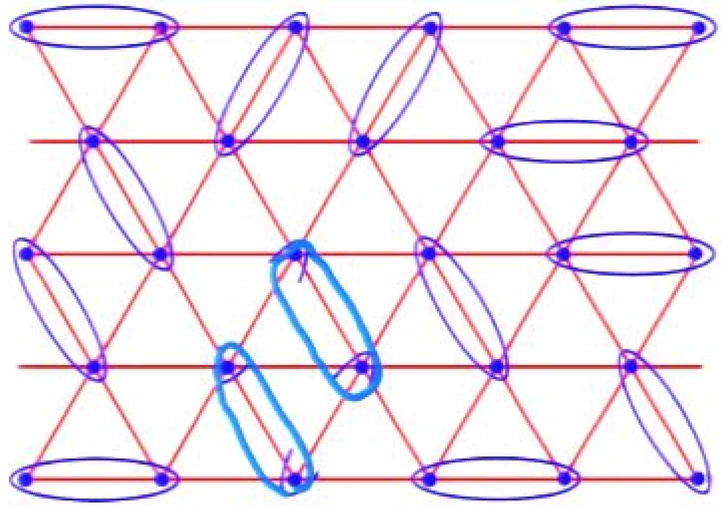

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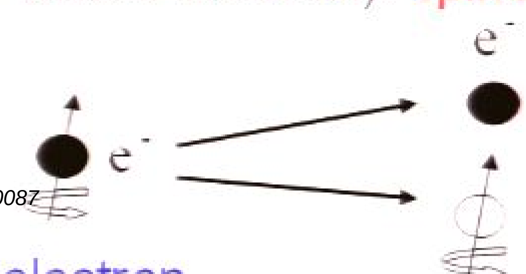
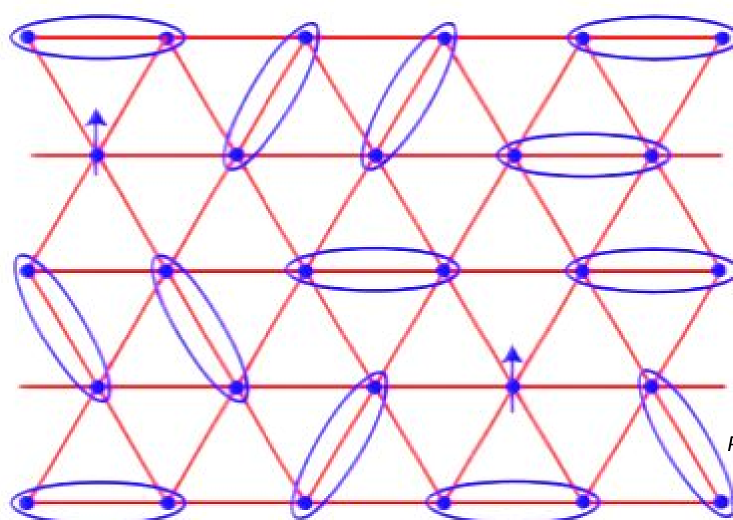
Properties:

1. Fractionalized excitations (S=1/2)
   - excitation with S=1/2 (uncharged – unlike electron). spinons

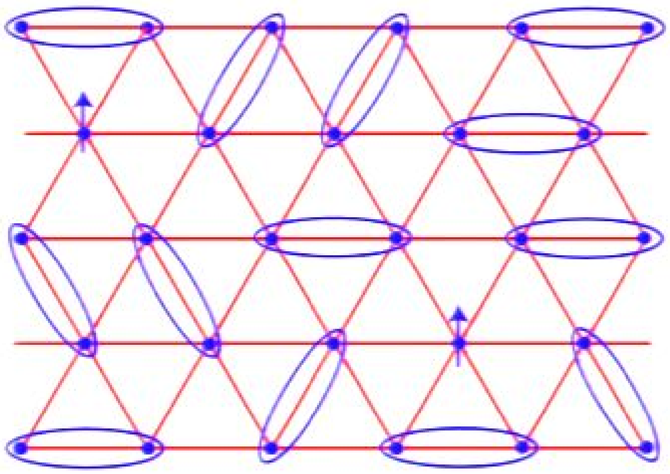




RVB Quantum Spin Liquids: Properties

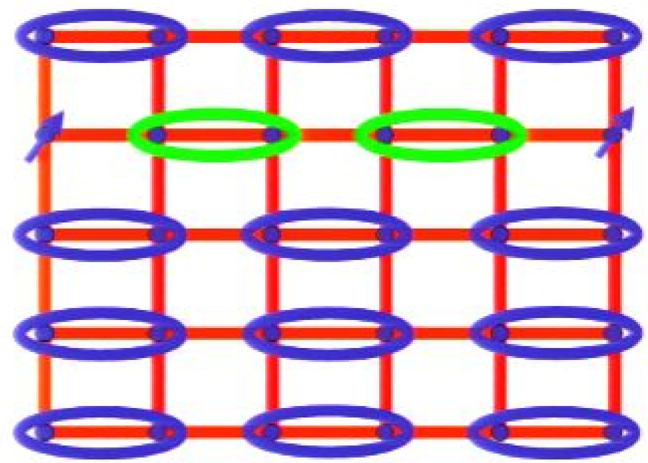
2. Described by a *deconfined gauge theory* (emergent property).

Spin liquid – finite energy cost for pair.



deconfined

Spin solid –energy cost linear in separation



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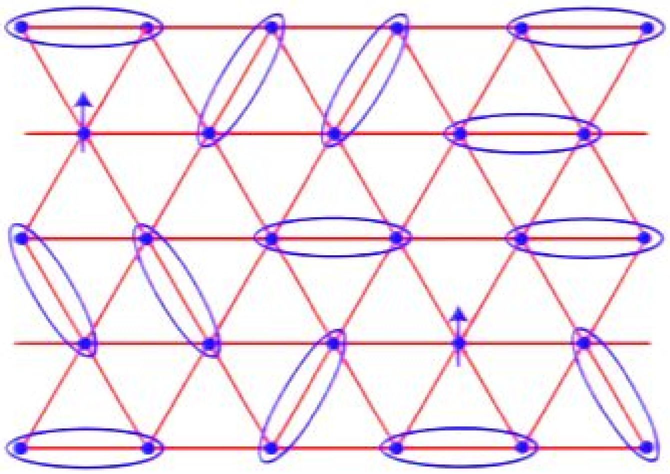

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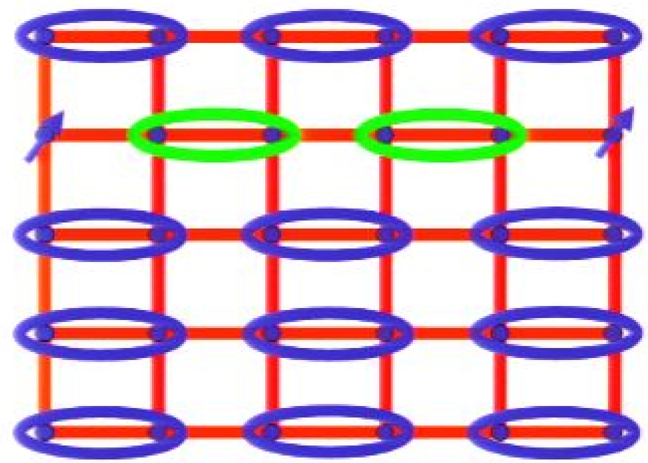


confined

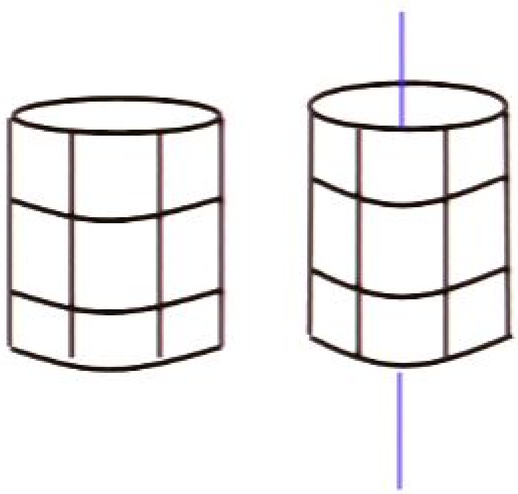
Here, Ising electrodynamics (Gauge group Z_2):

$$E = 0,1$$

$$B = 0,\pi$$

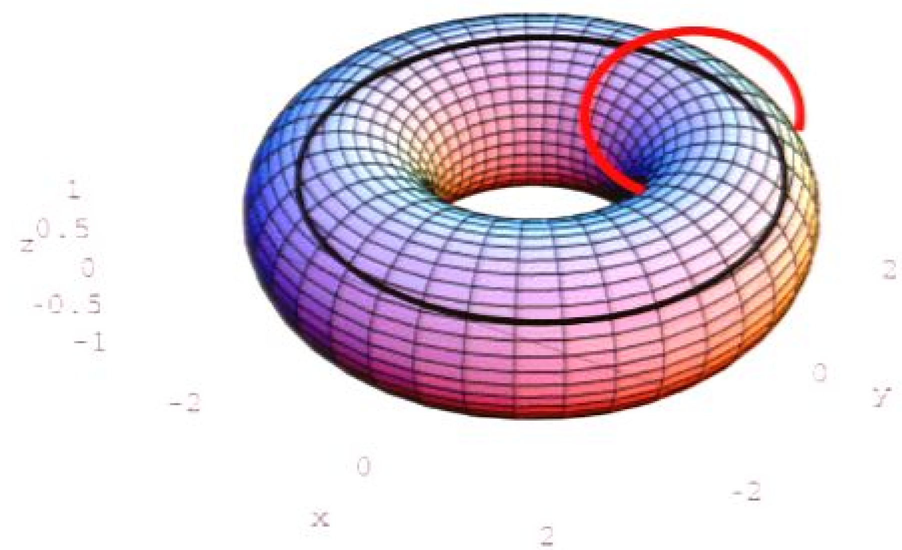
RVB Quantum Spin Liquids: Properties

3. Topological order (Wen) - ground state degeneracy depends on *topology*. No local operator can distinguish ground states.



B=0 B=π

2 fold degeneracy on cylinder



4 fold degeneracy on torus

- "Flux" detected *only* via Aharanov-Bohm effect.
- Intrinsic protection of quantum information – topological quantum computing.

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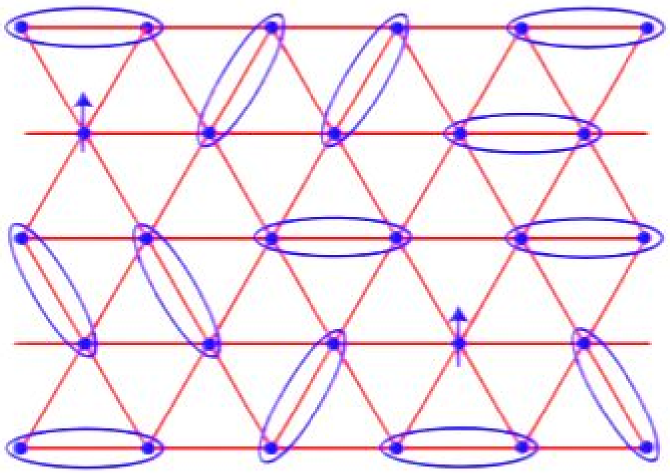

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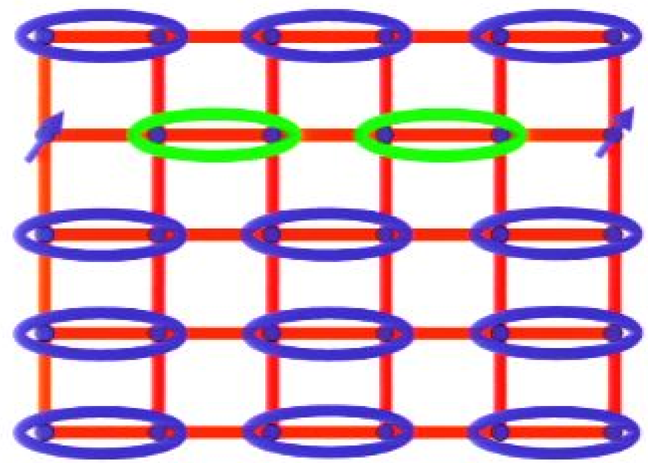

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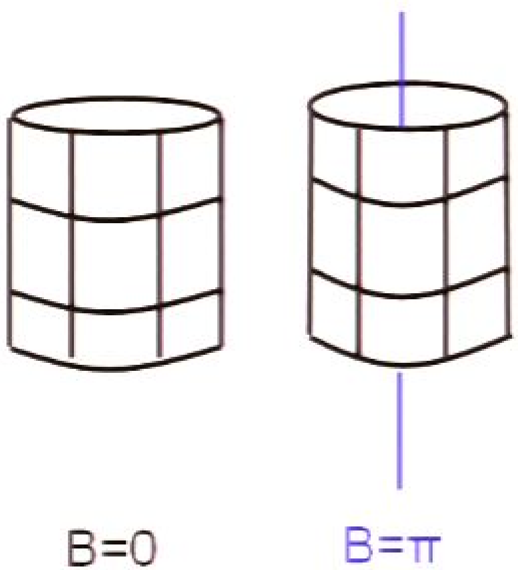

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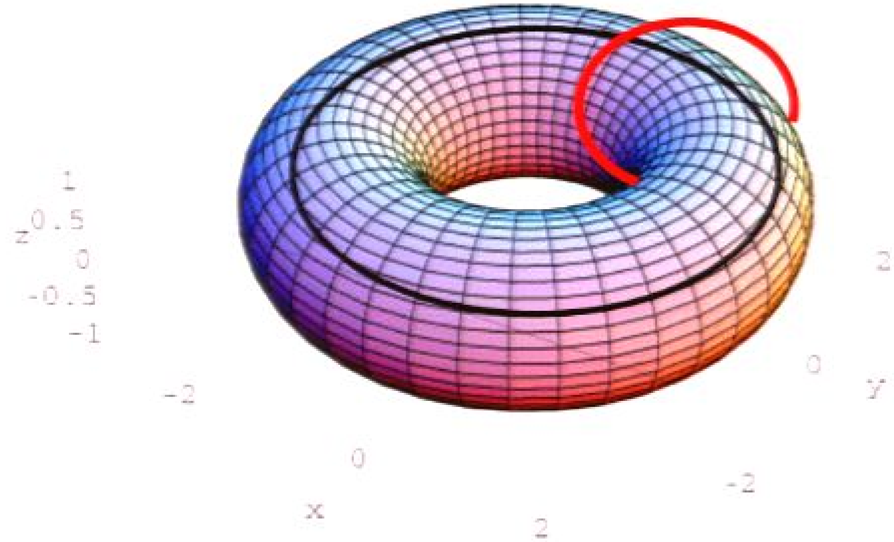

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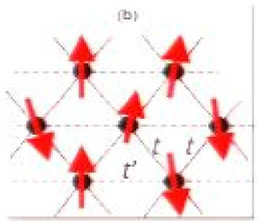
Experimental Sightings?

- Expected Signatures
  - no order, no spin freezing, no finite temperature transition.
  - decay of spin-1 excitation into spinons
  - Depending on nature of spin liquid state – other characteristic signatures (eg. metallic thermal conductivity in an insulator)

1. κ-$(ET)_2Cu_2(CN)_6$ a spin ½ triangular lattice quantum magnet, *with* J=250Kelvin
   But no ordering down to T=0.032Kelvin.



Shimizu et al. 2004

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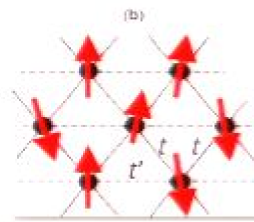

Shimizu et al. 2004

2. $ZnCu_3(OH)_6Cl_2$ (herbertsmithite) a spin ½ Kagome magnet,
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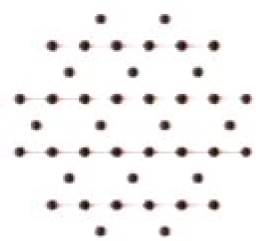

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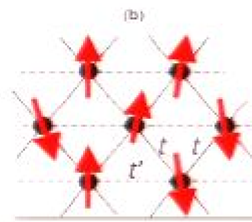

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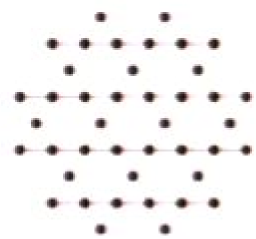
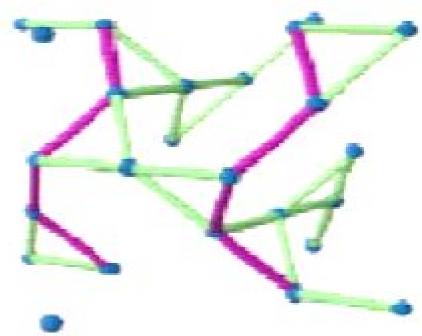


Helton et al. 2006

3. $Na_4Ir_3O_8$ a spin ½ magnet on a 3D *hyperkagome* lattice,
   J=600K
   But no ordering down to T=2K



Okamoto et al. 2007

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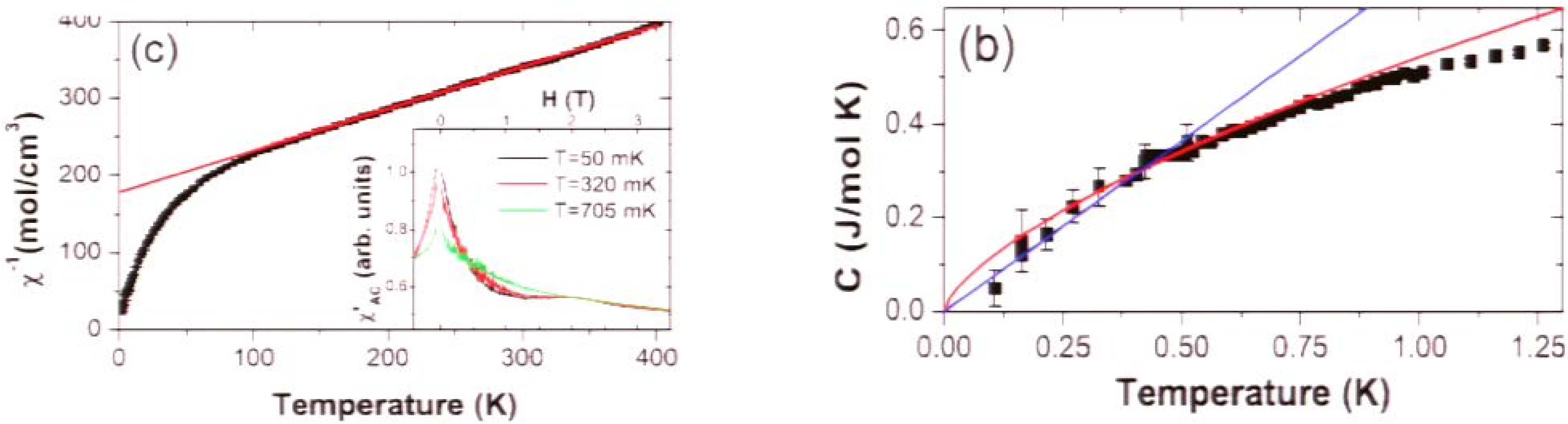


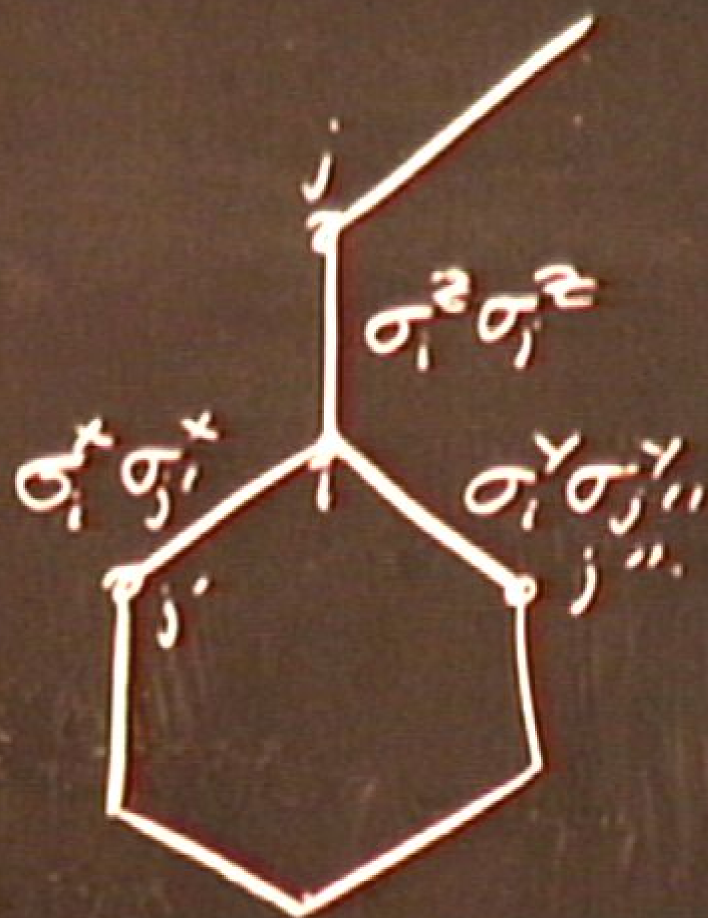
For all three materials, as T=>0, finite susceptibility and specific heat C ~T or T^2

No gap – critical spin liquid? (not simple RVB spin liquid)

Potential Problems:

- Impurity effects
- Spin rotation symmetry breaking couplings – Dzyaloshinskii-Moriya terms.

Spin liquids are a novel state of matter.
Experimental candidates now exist.


j
σ_i^z σ_j^z
σ_i^x σ_j^x
i
σ_i^y σ_j''^y
j'
j''

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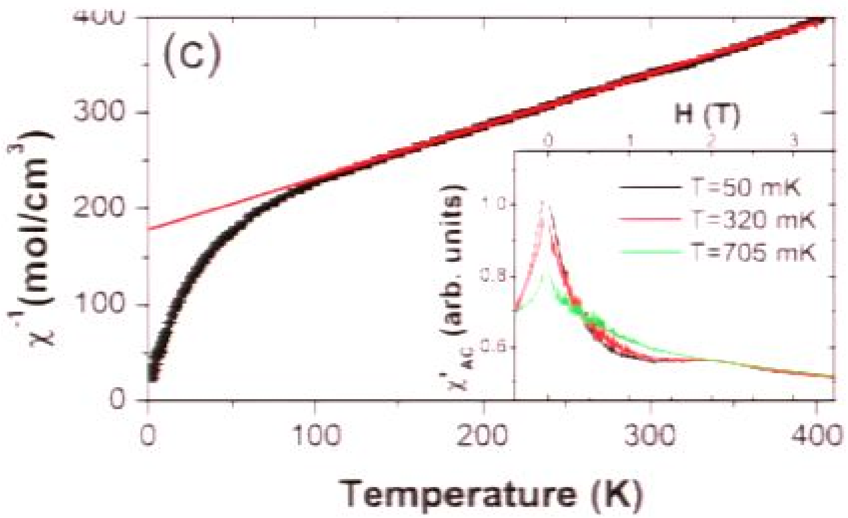

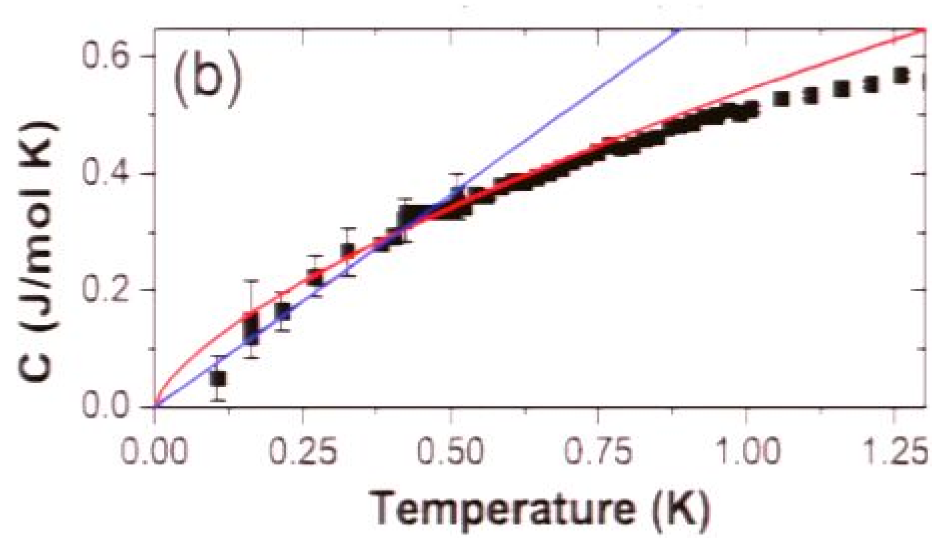

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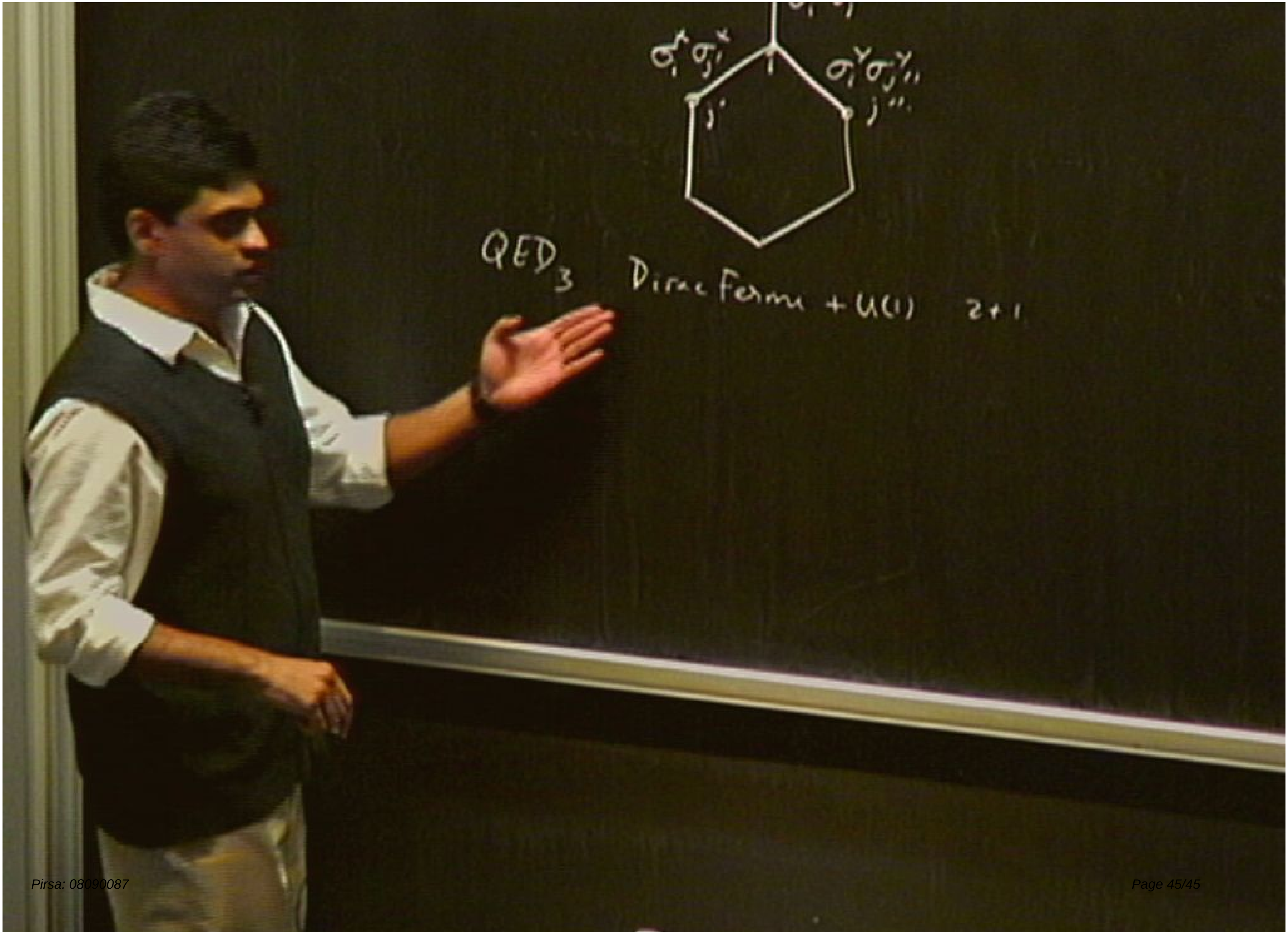
QED3
Dirac Fermi + U(1)
2+1