

Title: Does a Computer have an Arrow of Time?

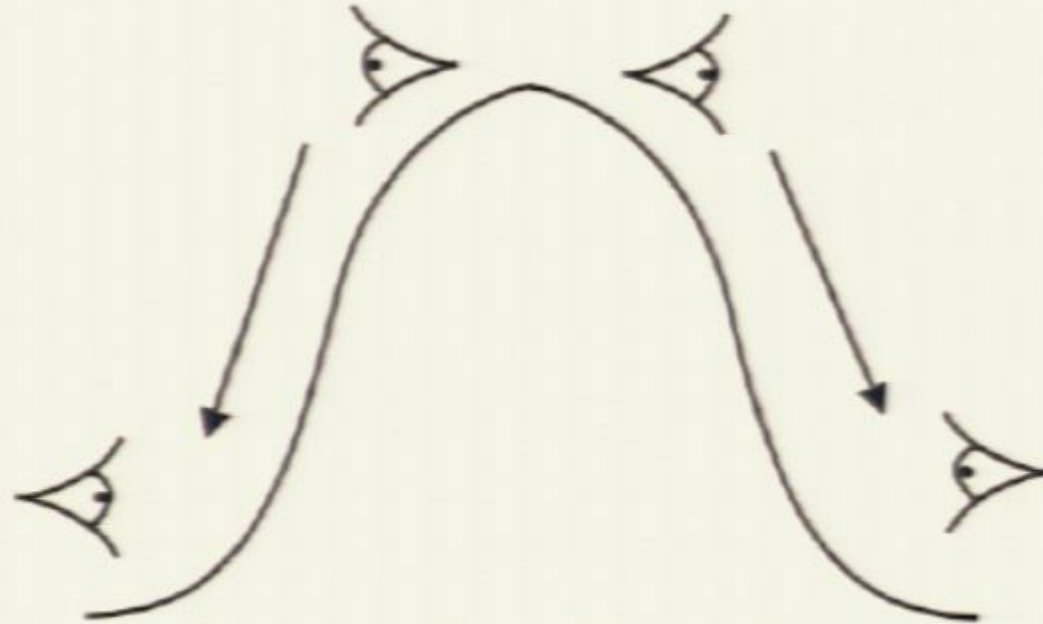
Date: Sep 30, 2008 02:00 PM

URL: <http://pirsa.org/08090081>

Abstract: It has sometimes - though usually informally - been suggested that the psychological arrow can be reduced to the thermodynamic arrow through information processing properties of the brain. In this talk we demonstrate that this particular suggestion cannot succeed, as, insofar as information processing (at least in the sense of a classical computer) has an arrow of time, it is not governed by the thermodynamic arrow.



# Does a Computer have an Arrow of Time?



"For the universe, the two directions of time are indistinguishable, just as in space there is no up and down. However, just as at a particular place on the earth's surface we call 'down' the direction toward the center of the earth, so will a living being in a particular time interval of such a single world distinguish the direction of time toward the less probable state from the opposite direction (the former toward the past, the latter toward the future)."

*Boltzmann, 1895*



# Minds, Memories and Computers





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".. when a computer records an item in memory, the total amount of disorder in the Universe increases. The direction of time in which a computer remembers the past is the same as that in which disorder increases"

*Hawking, 1987*



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"Computations are accompanied by dissipation, so much so that one of the principal issues for Intel's Itanium chip is its power consumption ... More fundamentally, Landauer ... has shown that computation requires irreversible processes and heat generation"

*Schulman, 2005*



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  - The mind is a computer
  - Certain computational processes are only possible in an entropy increasing direction





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    - We show that this is not the case



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    - All computational processes can take place in entropy decreasing, as well as entropy increasing, universes.



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  - What kind of computational process?



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  - What of Landauer's Principle?





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- Questions (and hopefully answers!)
  - What do I mean by an entropy increasing/decreasing universe?
  - What kind of computational process? What is the physical process?
  - What of Landauer's Principle?
  - What else might be responsible for the alignment?



Entropy Increasing  
Entropy Decreasing





# Entropy Increasing Entropy Decreasing



- An entropy increasing universe:



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- An entropy increasing universe:
  - A marked asymmetry in physical processes



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    - Thermal systems are initially well described by the canonical distribution on the accessible state space.



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    - No equivalent conditions on final statistical states.



Entropy Increasing  
Entropy Decreasing





# Entropy Increasing Entropy Decreasing



- An entropy *decreasing* universe:



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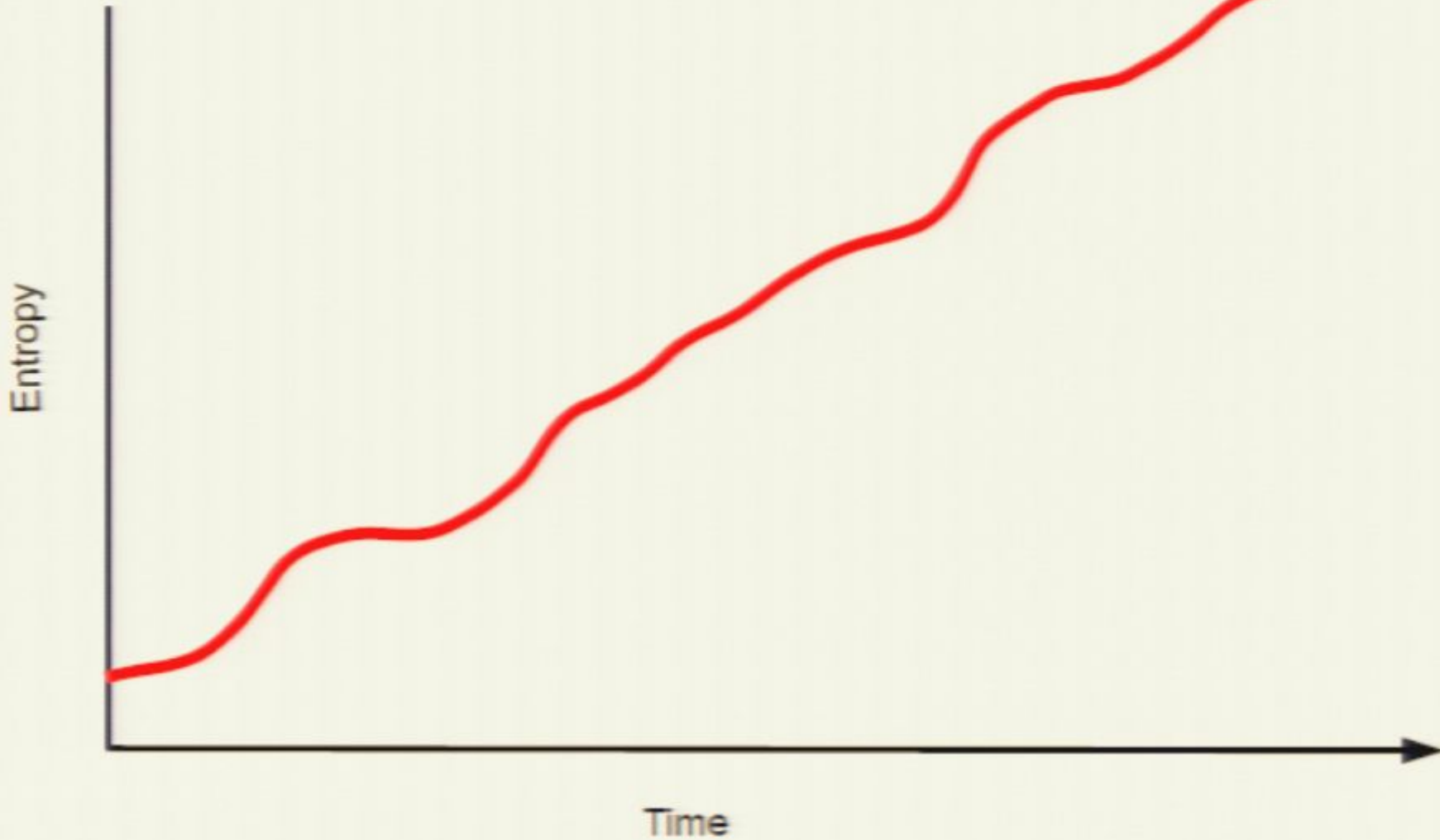


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    - No equivalent conditions on *initial* statistical states.
- But *NOT* necessarily just a time reverse of our *particular* universe...



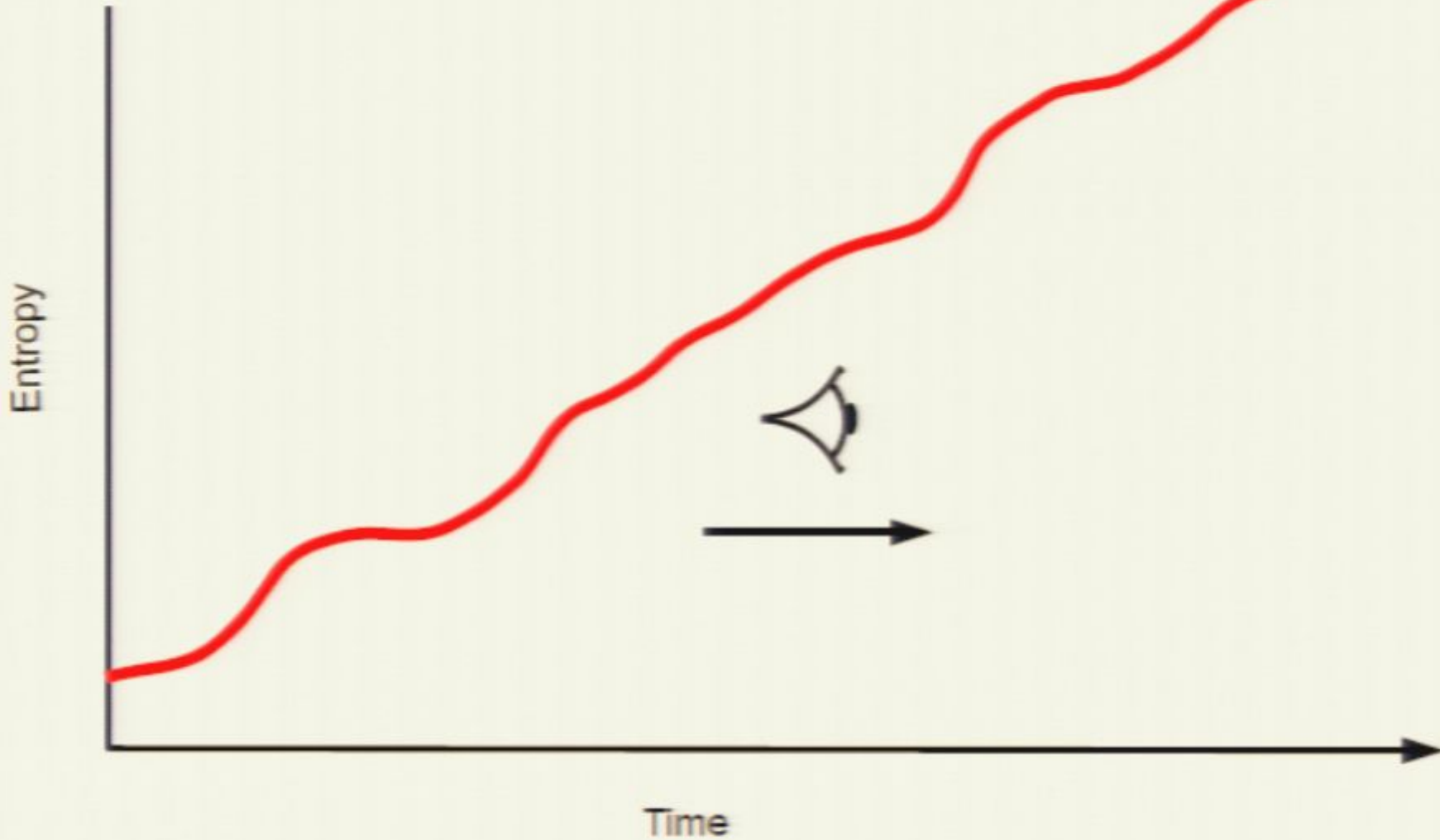


# Different Arrows?



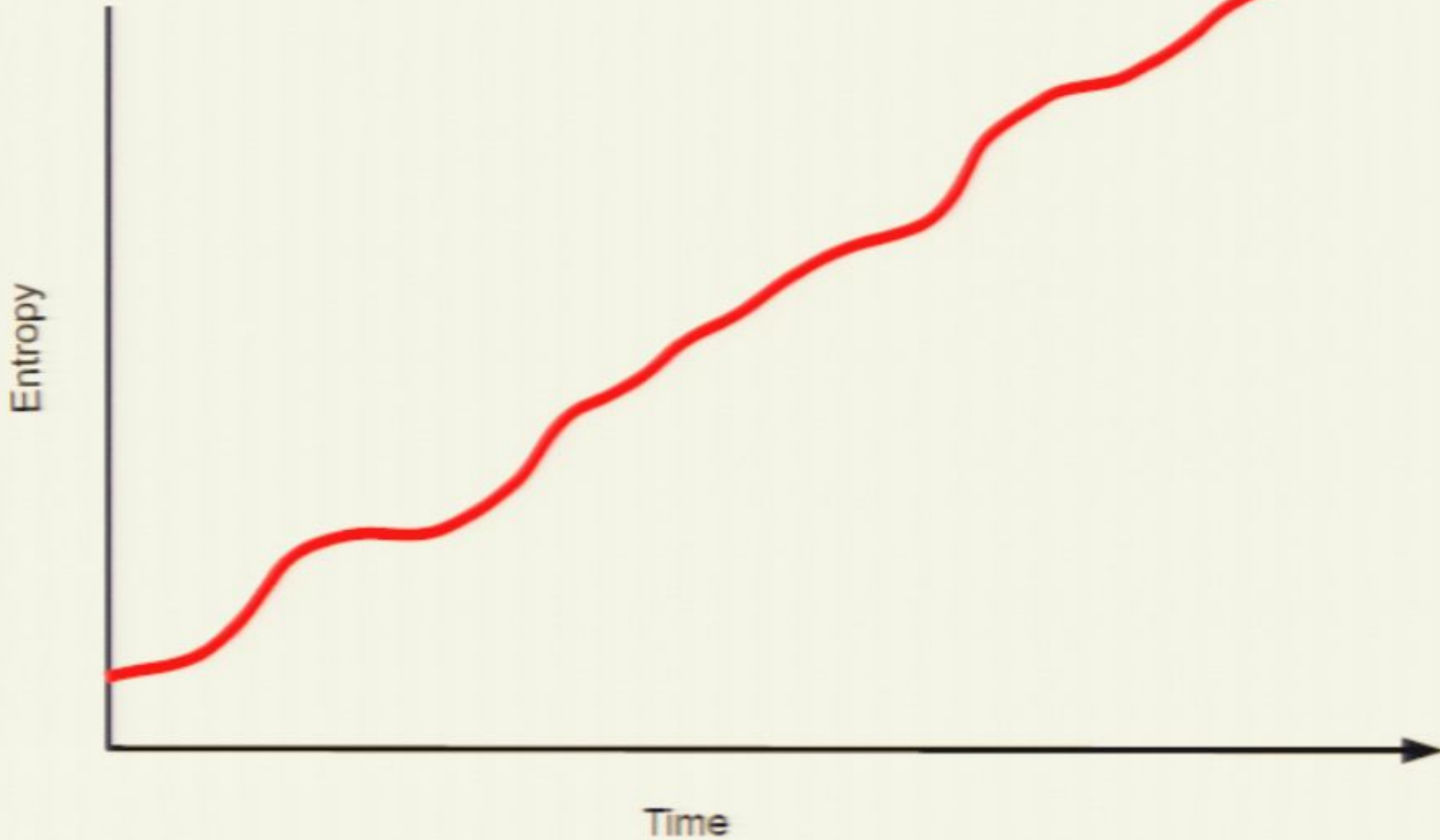


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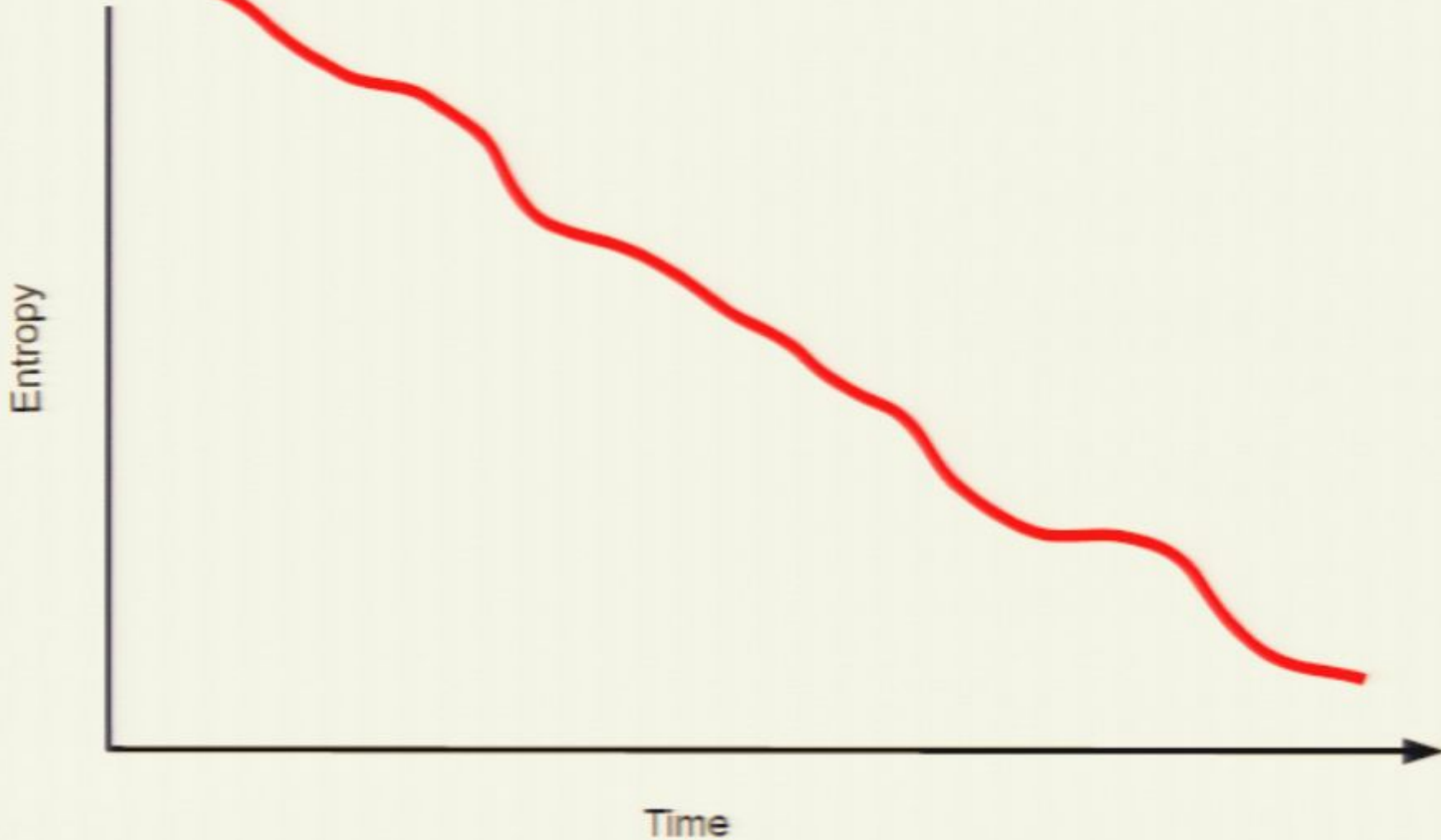


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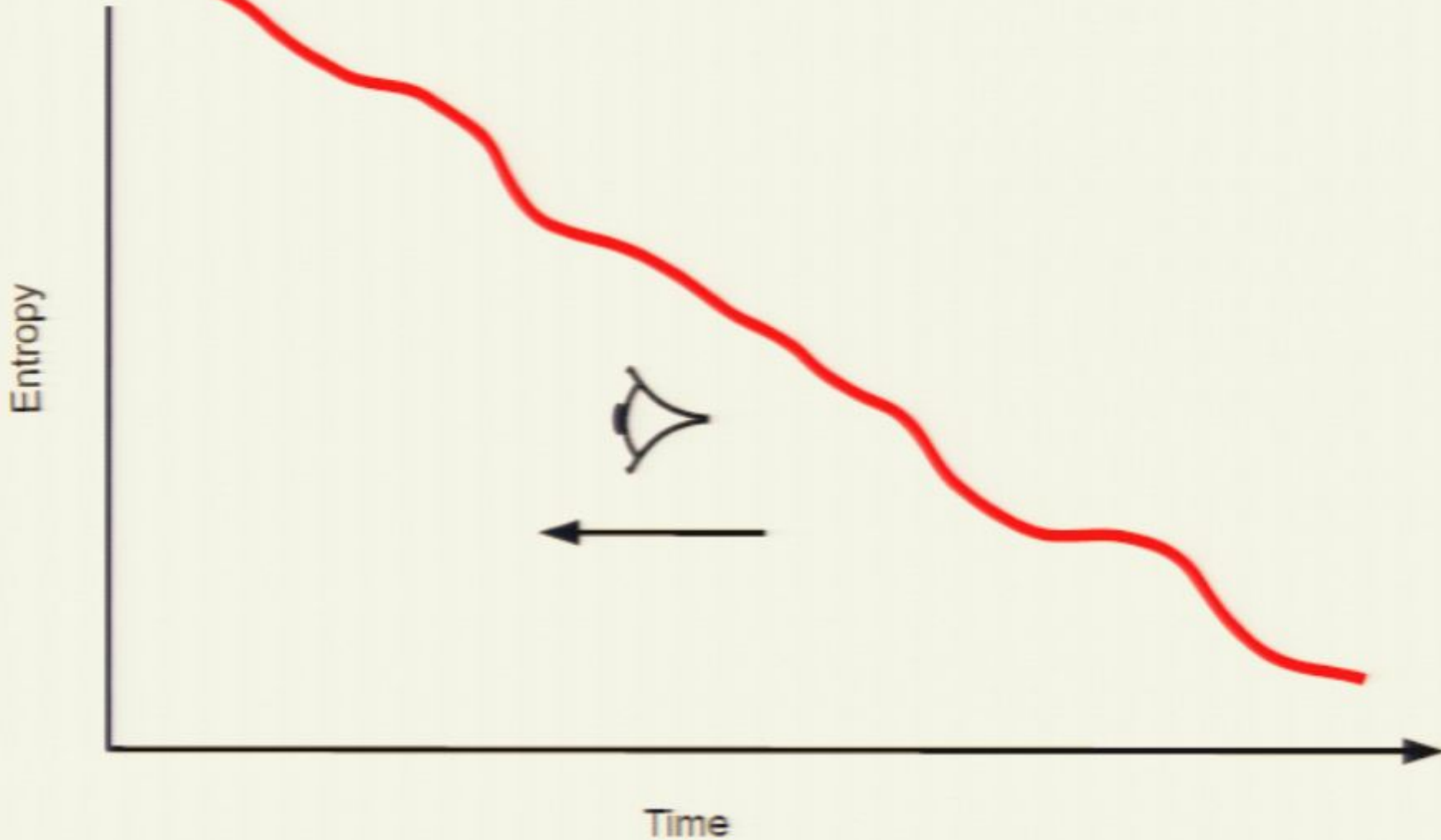


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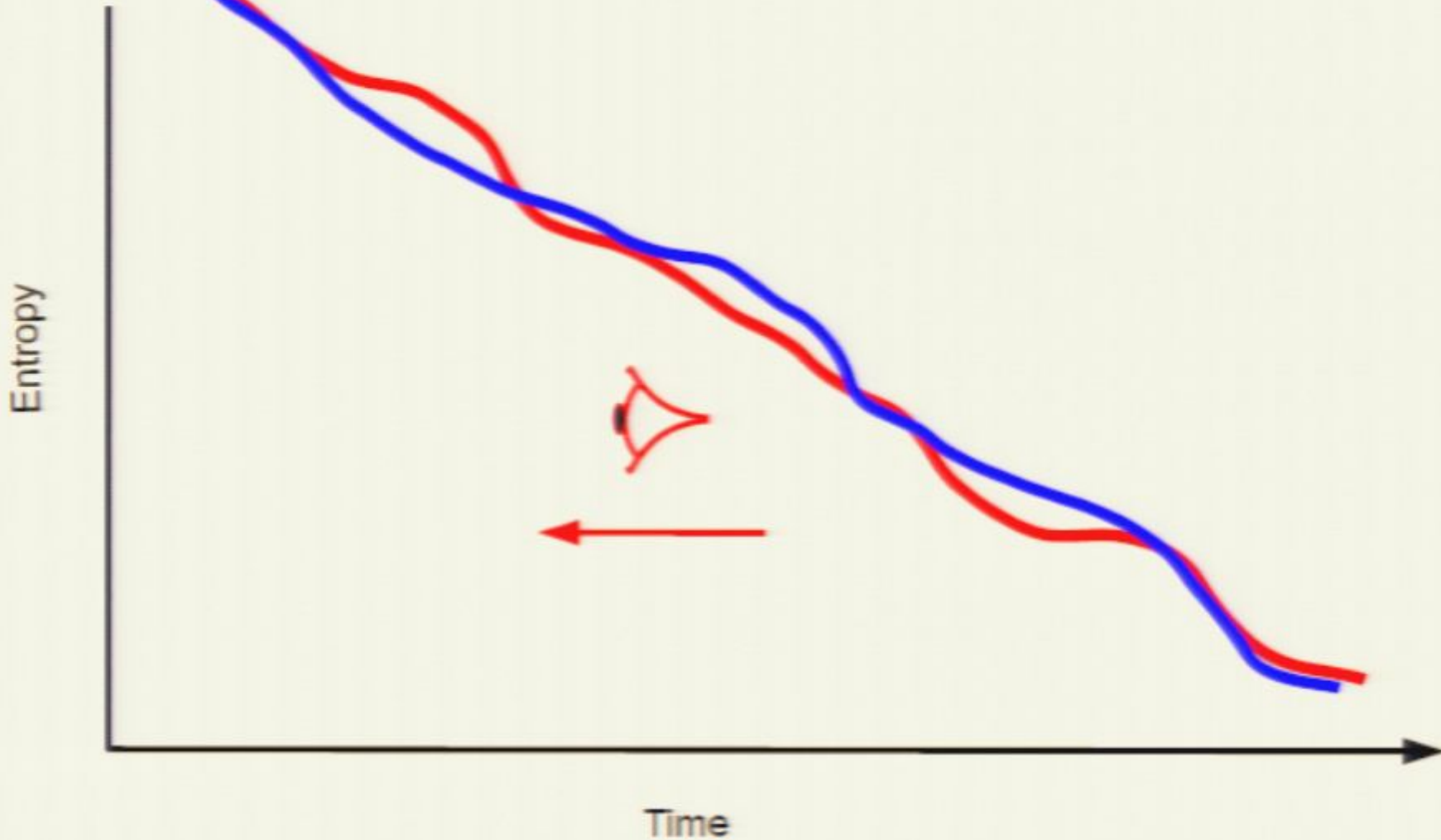


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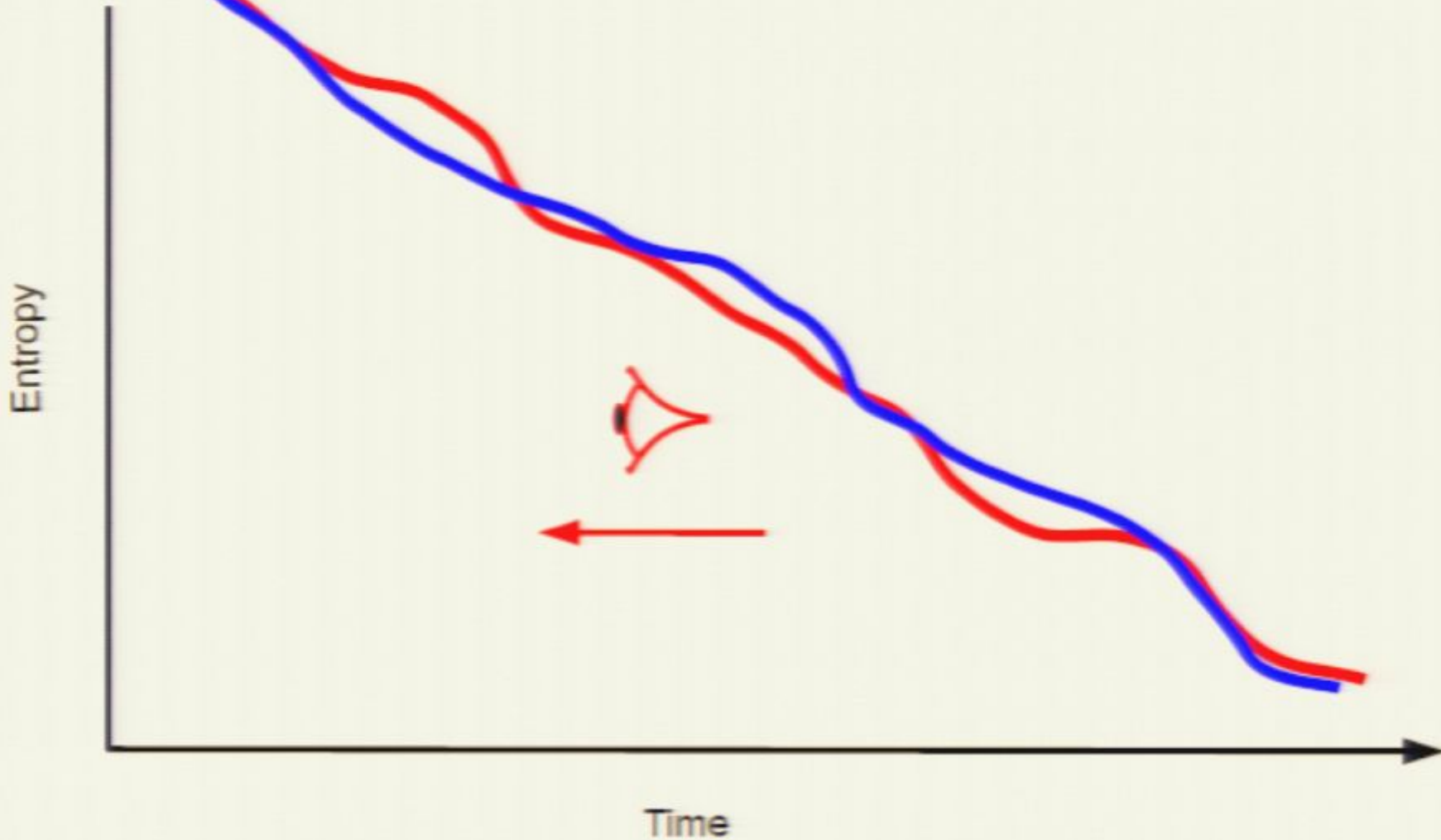


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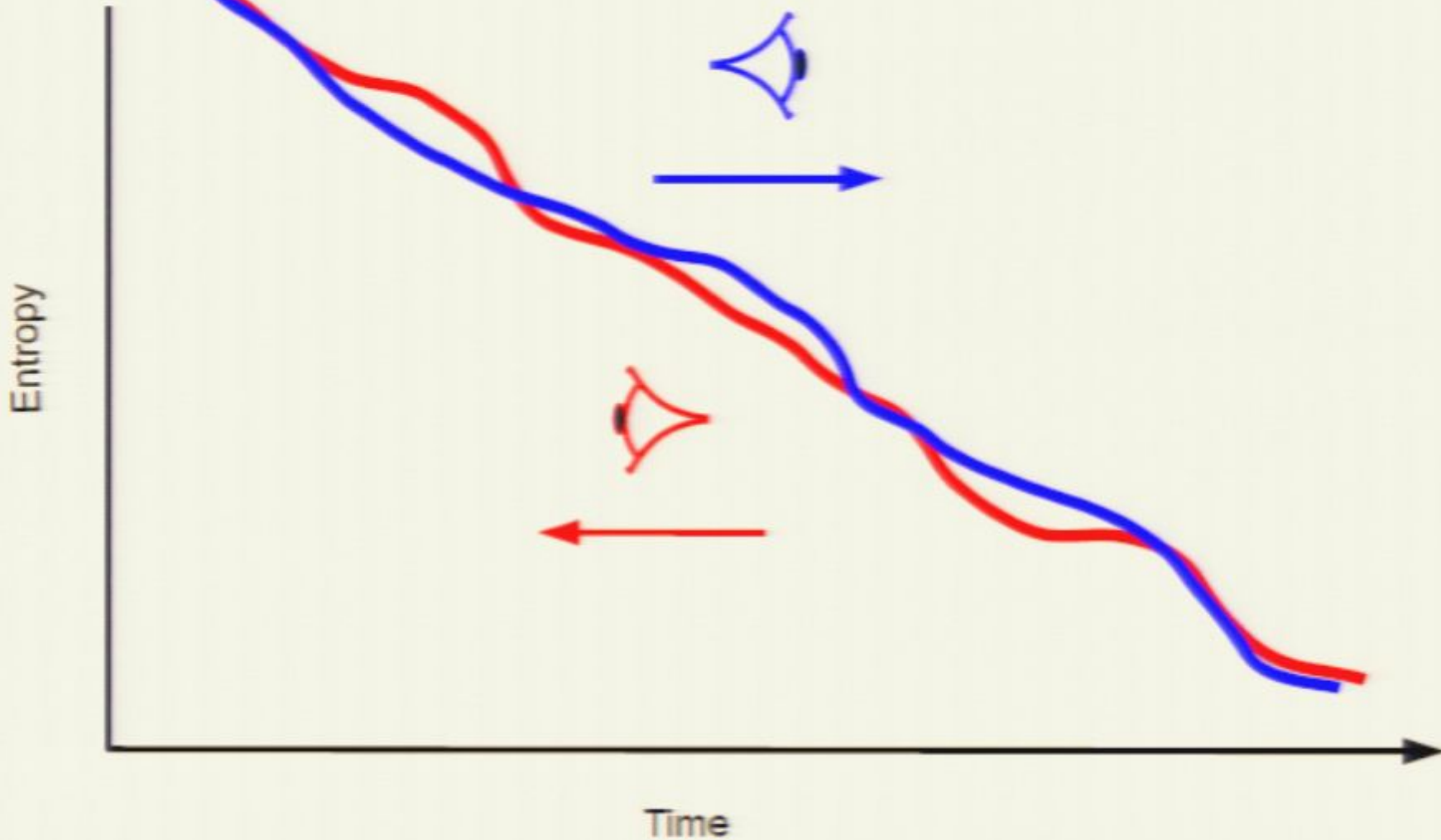


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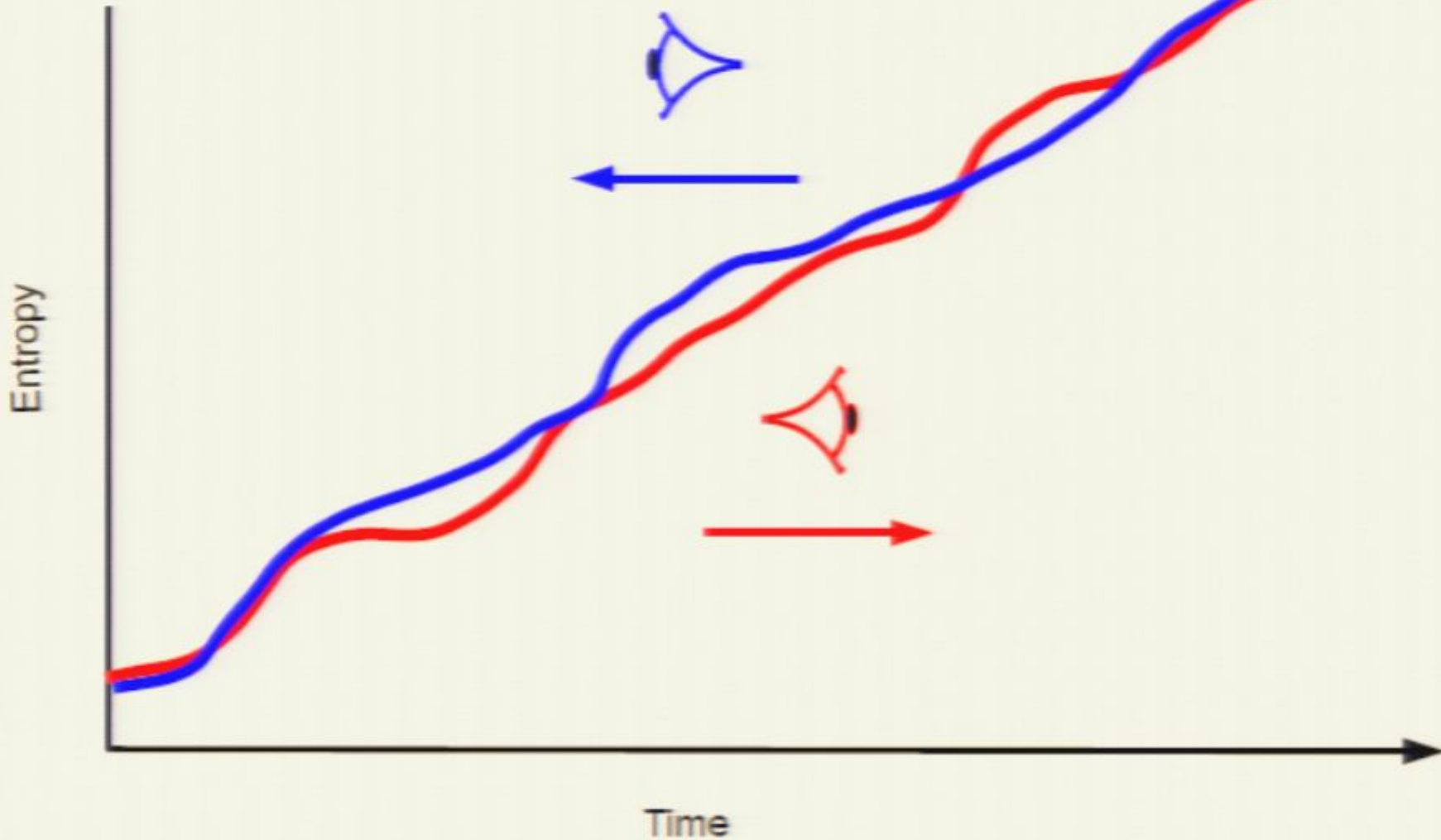
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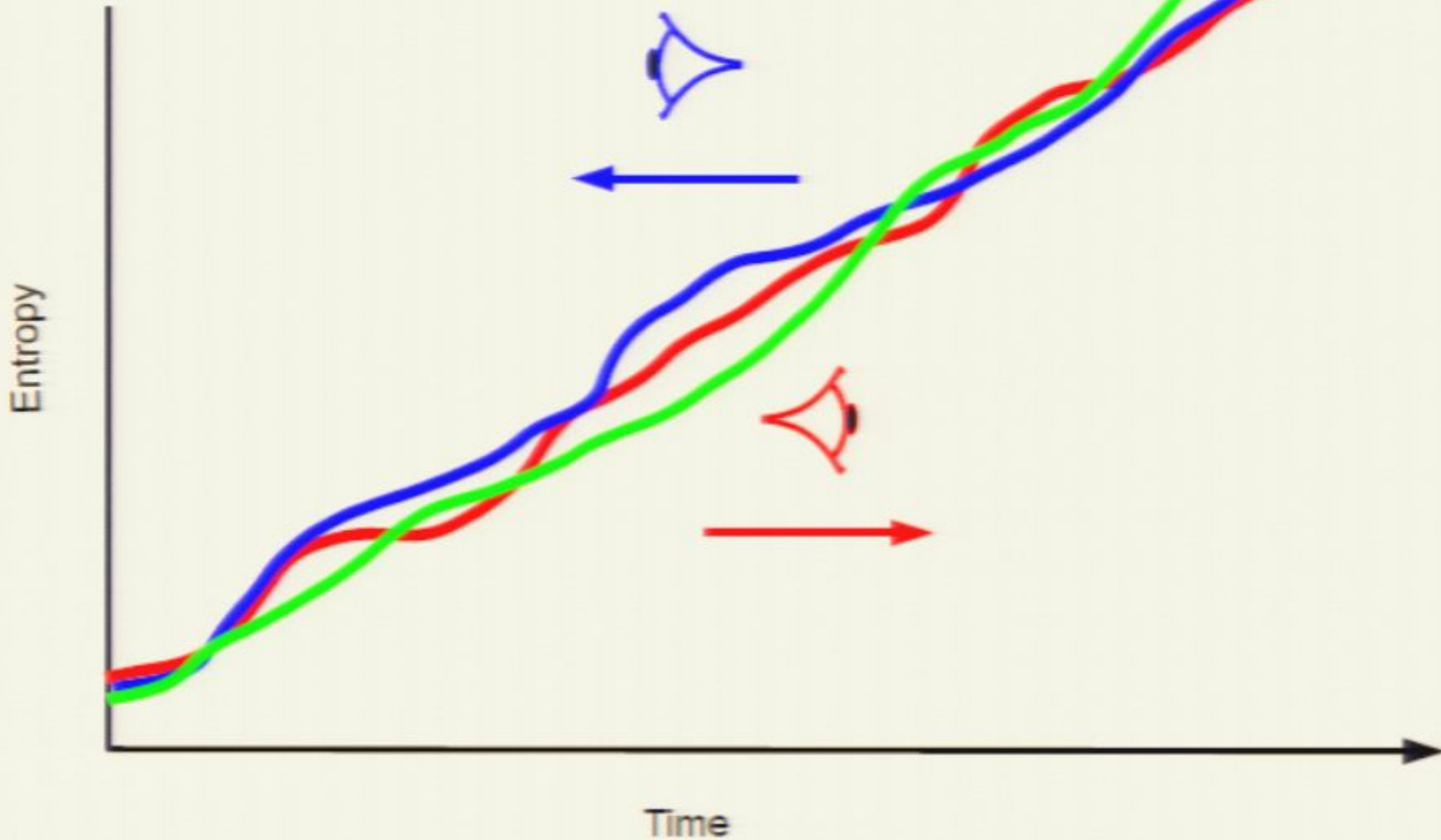


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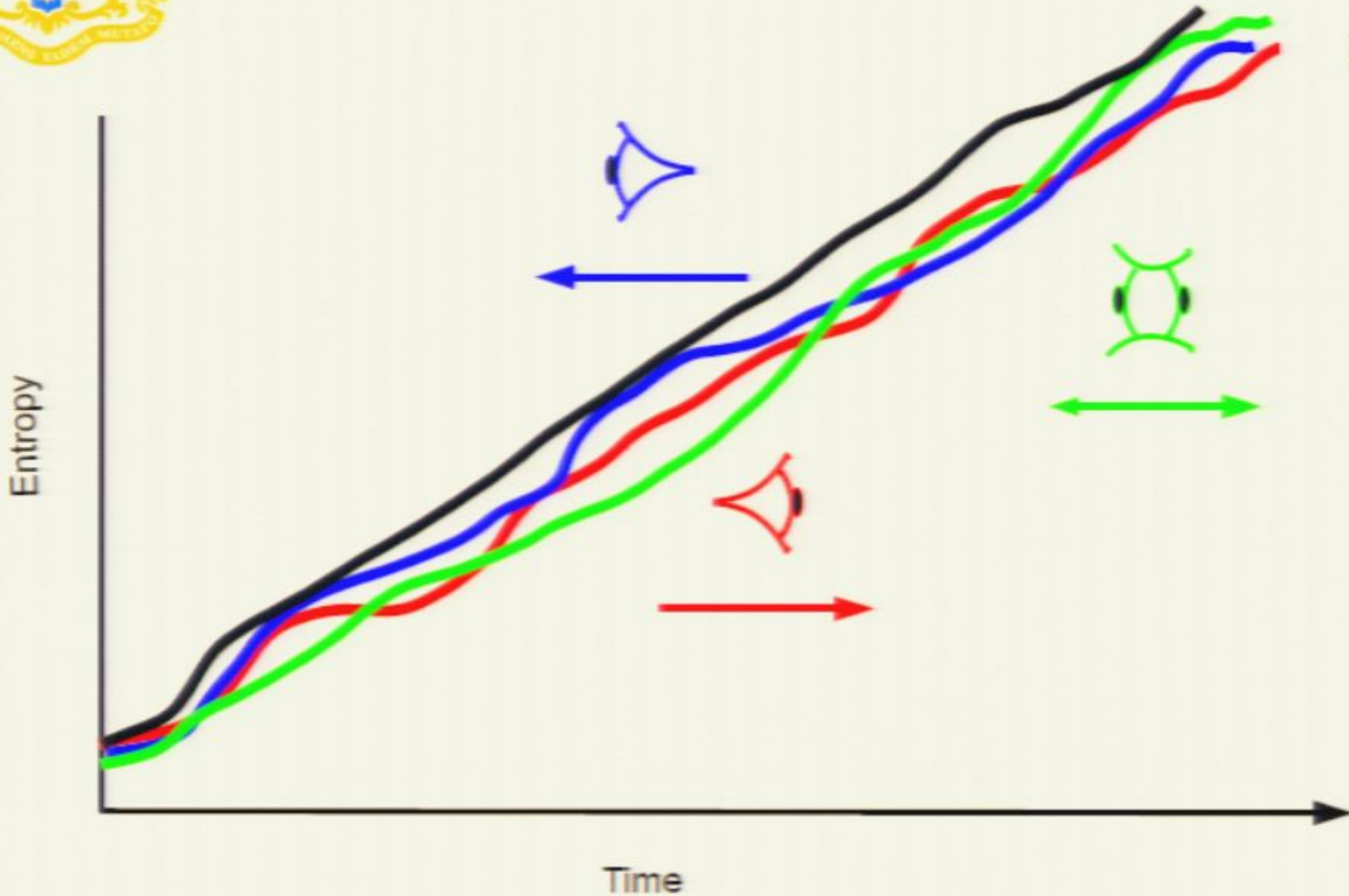


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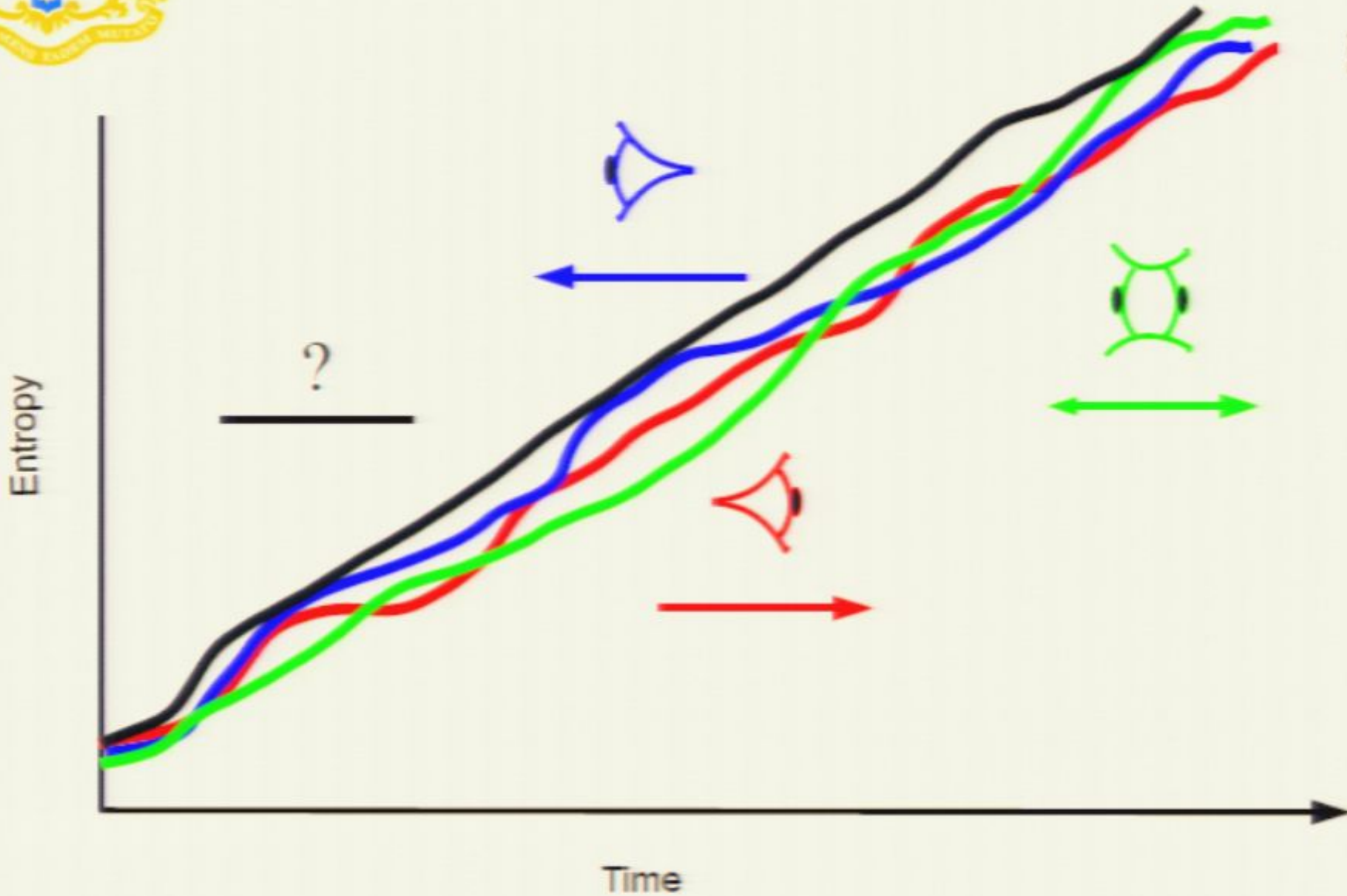


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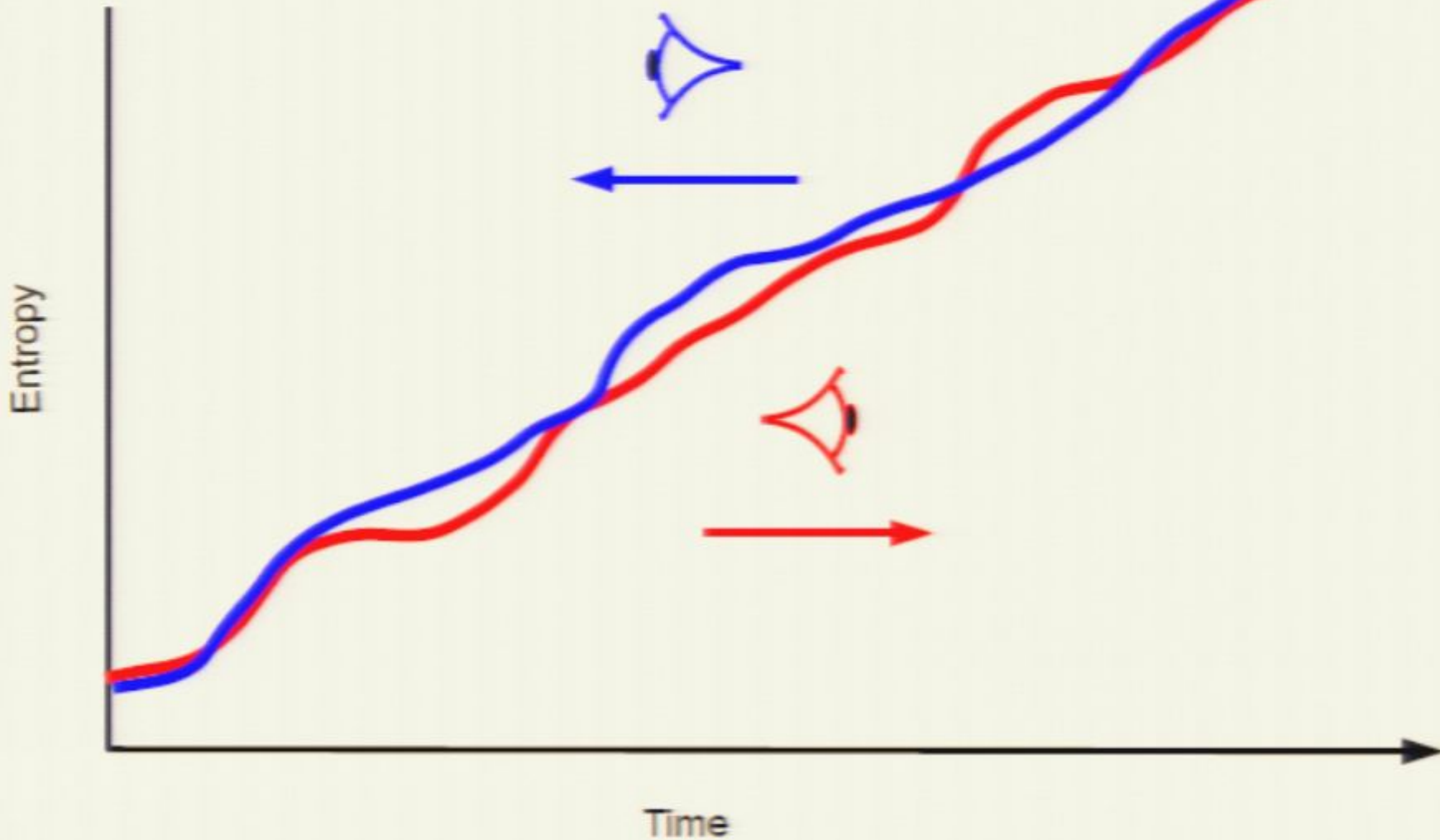


# Different Arrows?





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# Computation and Logic





# Computation and Logic



## Logical Operations

AND

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1



# Computation and Logic



## Logical Operations

### AND

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

### NOT

A	$\bar{A}$
0	1
1	0





# Computation and Logic



## Logical Operations

AND

A	B		A.B
0	0		
0	1		0
1	0		
1	1		1

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AND			A.B
A	B		
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0	1		
1	0		
1	1		1

NOT			$\bar{A}$
A			
0			1
1			0





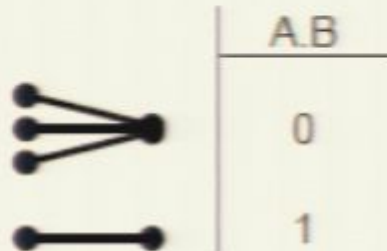
# Computation and Logic



## Logical Operations

A	B
0	0
0	1
1	0
1	1

AND



NOT





# Computation and Logic



## Reversible Logical Operations

### CCNOT

A	B	C	D	E	F
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
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input



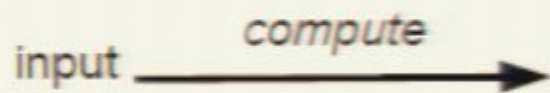
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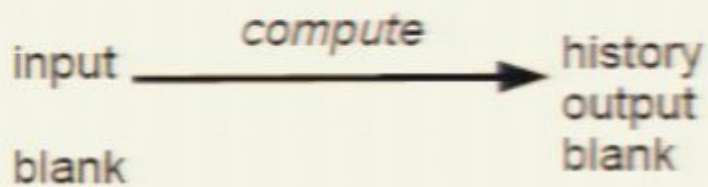
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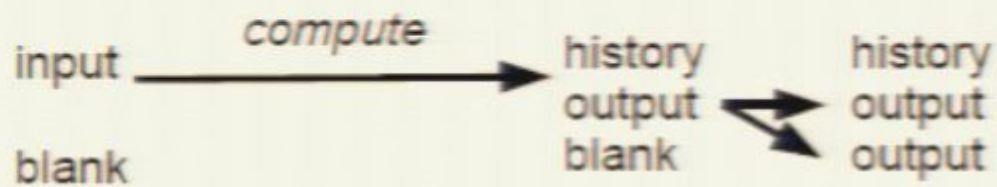
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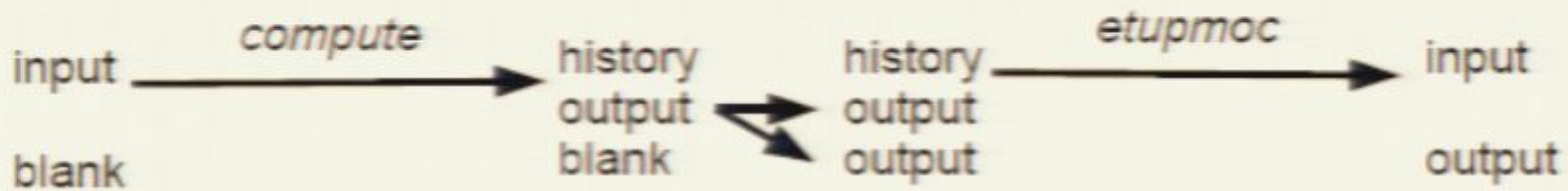
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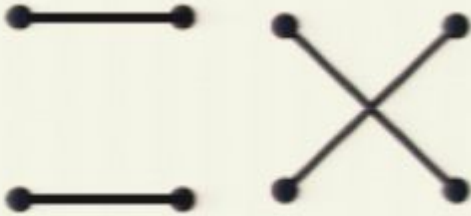
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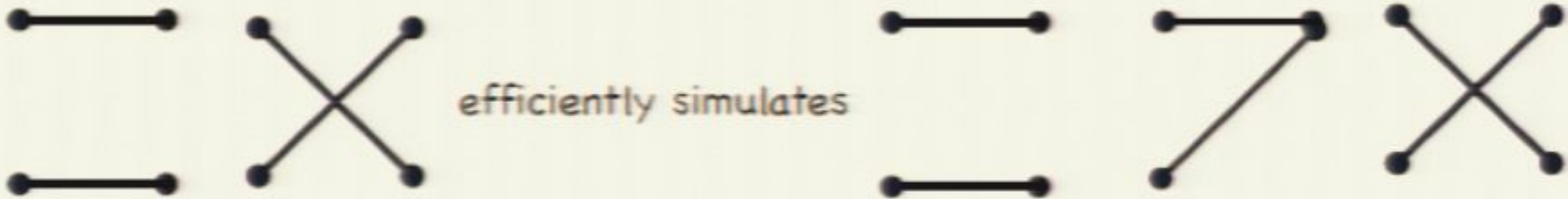
*but they cannot efficiently simulate:*







# Computation and Logic



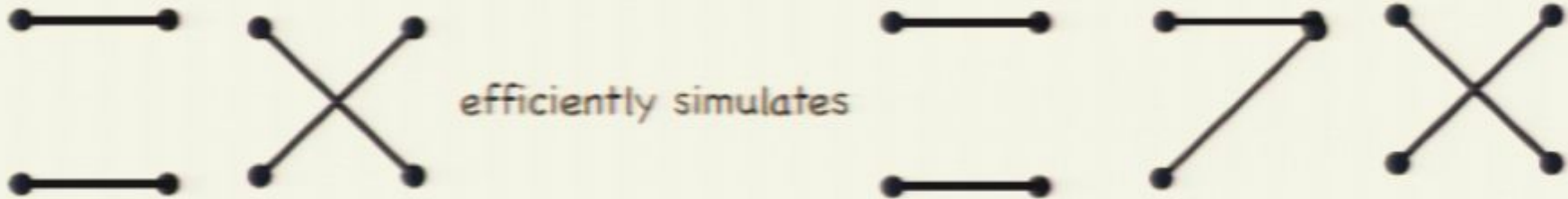
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BPP Complexity Class  
Probabilistic Turing Machines



# Computation and Logic



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but they cannot efficiently simulate:



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Defined by the conditional probabilities of any an output,  $b$ , given an input,  $a$   $P(b|a)$



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The reverse transformation of *information* “ $L^*$ ” is:

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)} = \frac{P(b|a)P(a)}{\sum_a P(b|a)P(a)}$$



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It is not hard to see:

- if the input,  $a$ , to  $L$ , occurs with probability  $P(a)$ , then following

$L$  with  $L^*$  restores the original probability distribution,  $P(a)$  :  $(L^*)L = I$



# Logically reversing computations



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Defined by the conditional probabilities of any an output,  $b$ , given an input,  $a$   $P(b|a)$

The reverse transformation of *information* " $L^*$ " is:

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)} = \frac{P(b|a)P(a)}{\sum_a P(b|a)P(a)}$$

It is not hard to see:

- if the input,  $a$ , to  $L$ , occurs with probability  $P(a)$ , then following

$L$  with  $L^*$  restores the original probability distribution,  $P(a)$  :  $(L^*)L = I$

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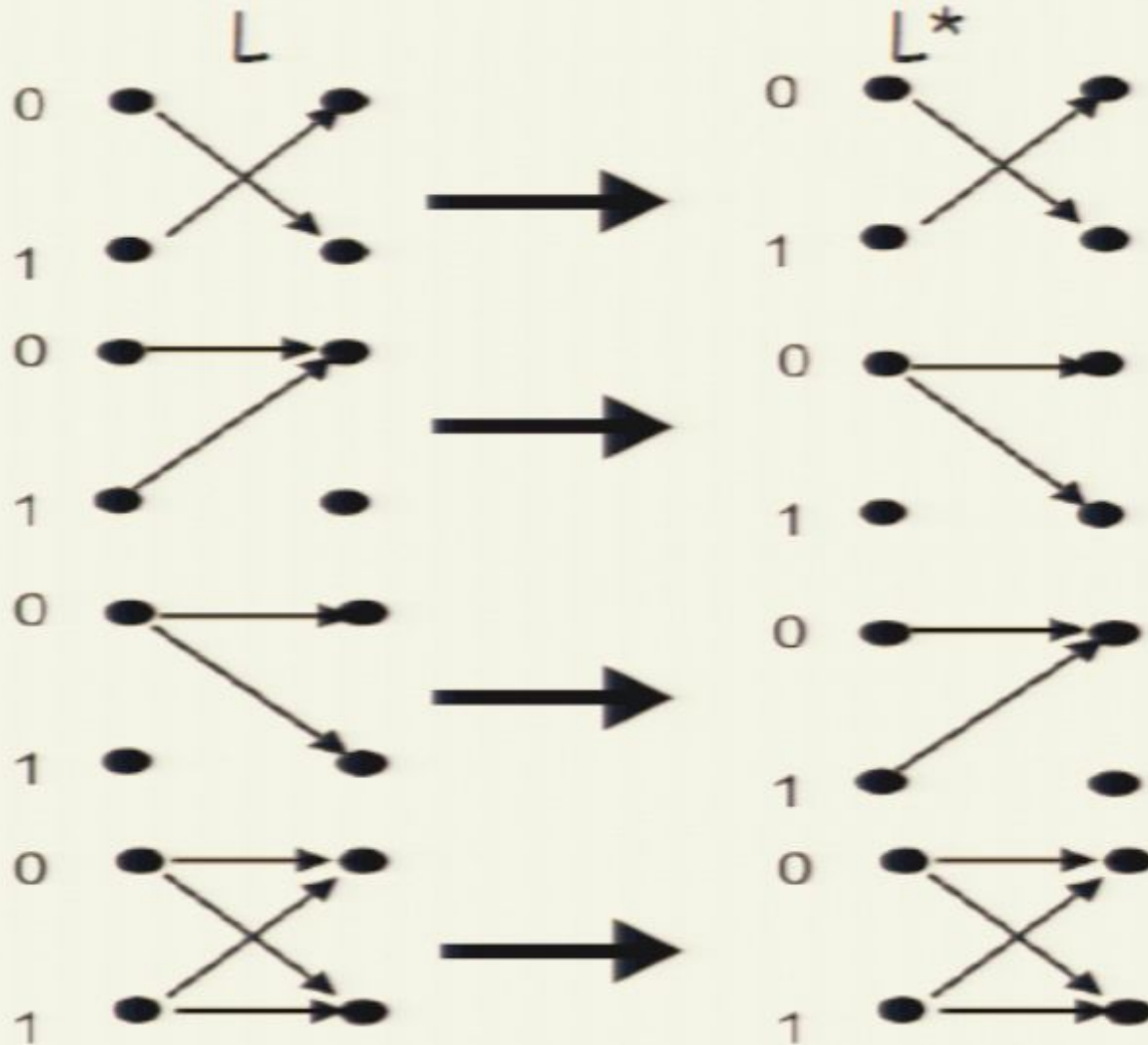
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How does this relate to the conclusion of the previous slides?



# Landauer in an entropy increasing universe





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$$\left( \sum_i \mu(A_i) \mu(B_i) \right) \mu(E_0') = \mu(A_0) \left( \sum_i \mu(B_i) \mu(E_i') \right) \leq \mu(A_0) \left( \sum_i \mu(B_i) \right) \mu(E_f')$$

Still a coarse grained entropy increase.



# (De-)Correlations with the world





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How does the entropy increase appear?



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$$.\cup_i A_i \otimes B_i \otimes E_i \subseteq .\cup_i A_i \otimes B_i \otimes E_f$$

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$$\begin{aligned}
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 \sum_i \mu(A_i) \mu(B_i) \mu(E_i) \leq \left( \sum_i \mu(A_i) \mu(B_i) \right) \left( \sum_j \mu(E_j) \right) & \qquad E_f = \cdot \cup_i E_i
 \end{aligned}$$





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Inaccessibility of microscopic correlations with the environment.



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$$\cdot \cup_i A_i \otimes B_i$$



# Other Possibilities







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## Causal handles (*Albert, Kutach, Loewer*)

- The imposition of an initial condition hypothesis, but no final condition hypothesis, constrains counterfactual reasoning. Minor perturbations in the microstate now can have unconstrained future consequences but cannot have unconstrained past consequences. Criticisms: it is not clear if an initial condition in the remote past really does constrain past consequences in the near past: a remote future condition would not appear to constrain near future choices. (*Frisch, Price & Weslake*)



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- In an entropy increasing universe there may be more *evolutionary advantage* to the development of self-replicating systems which utilise information that *has been* gathered, than in an entropy decreasing universe. Entropy increasing universes have a macroscopic predictability, so gathered information is a good predictor of the future. Entropy decreasing universes have an unpredictability, so gathered information is not necessarily of use.



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## An additional condition

- Perhaps the psychological arrow is simply independent of the thermodynamic arrow? Causal agents must agree on the direction of the causal arrow (for self consistency) but the fact that it is entropy increasing, not decreasing, may be just a contingent fact about this universe, and it could have been otherwise.



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  - Just that if it is a consequence, it cannot be via computational properties of the brain.



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