Title: A Candidate of a Psi-Epistemic Theory

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Abstract: In deBroglie-Bohm theory the quantum state plays the role of a guiding agent. In this seminar we will explore whether this is a universal feature shared by all hidden variable theories, or merely a peculiarity of the deBroglie-Bohm theory. We present the bare bones of a theory in which the quantum state represents a probability distribution and does not act as a guiding agent. The theory is also psi-epistemic according to Spekken\'s and Harrigan\'s definition. For simplicity we develop the model for a 1D discrete lattice but the generalization to higher dimensions is straightforward. The ontic state consists of a definite particle position and in addition possible non-local links between spatially separated lattice points. These non-local links comes in two types: directed links and non-directed links. Quantum superposition manifests itself through these links. Interestingly, this ontology seems to be the simplest possible and immediately suggested by the structure of quantum theory itself. For N lattice points there are N*3^(N(N-1)) ontic states growing exponentially with the Hilbert space dimension N as expected. We further require that the evolution of the probability distribution on the ontic state space is dictated by a master equation with non-negative transition rates. It is then easy to show that one can reproduce the Schroedinger equation if an only if there are positive solutions to a gigantic system of linear equations. This is a highly non-trivial problem and whether there exists such positive solutions or not is not clear at the moment. We end by speculating how one might incorporate gravity into this theory by requiring permutation invariance of the dynamical evolution law.

A Candidate of a ψ-Epistemic Theory

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Is the WaveFunction Just a Probability Distribution?

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Introduction & Heuristic Ideas

Bell Quotes

"The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explaination, the account of all evidence must be expressed in classical terms'."

Bell Quotes

"The classical world of course is described quite directly - 'as it is'. We could specify for example the actual positions $\Lambda_1, \Lambda_2, \ldots$ of material bodies, such as the switches defining experimental conditions and the pointers, or print, defining experimental results. Thus in contemporary [quantum] theory the most complete description of the state of the world as a whole, or any part of it extending to the classical domain, is of the form

$$(\Lambda_1, \Lambda_2, \ldots, \psi)$$

with both classical variables and one or more quantum-mechanical wave functions."

Bell Quotes

"It is then implicit that the apparatus of 'observation', or, better, of experimentation, and the experimental results, are real and localized. We will have to do as best we can with these rather ill-defined. . . beables, while hoping always for a more serious reformulation of quantum mechanics where the. . . beables are explicit and mathematical rather than implicit and vague."

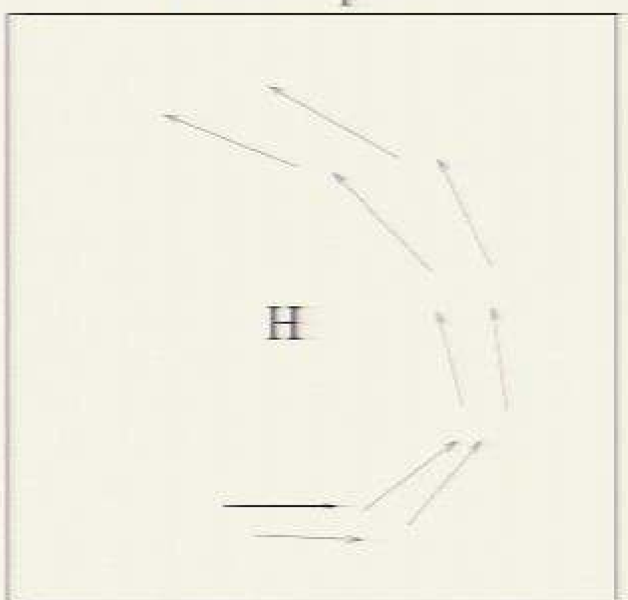
The "Hidden Variable" Program

- To develop empirically adequate theories in which the beables (which represnts things like tables and chairs) are mathematical and explicit is the basic idea behind the "hidden variable" program.
- "Hidden variables" is a fantastically stupid name for variables that are meant to represent tables, chairs, outcomes...
- Better word for 'hidden variable' is 'ontic variable', or 'beables'.

Classical Mechanics vs deBroglie-Bohm Theory

Classical Mechanics

Phase Space



$$\dot{q} = \frac{\partial H}{\partial p} \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

$$\rho_{eq.}(q,p) = const.$$

deBroglie-Bohm Theory

Configuration Space

$$\dot{q} = \frac{\hbar}{m} Im \frac{\nabla \psi}{\psi} \qquad i\frac{\partial \psi}{\partial t} = \hat{H}\psi$$

$$\rho_{eq.}(q,t) = |\psi(q,t)|^2$$

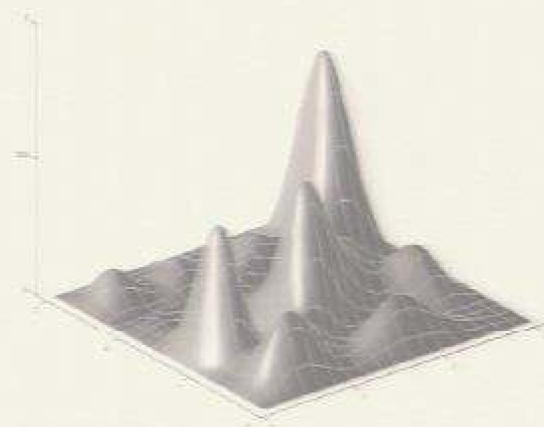
deBroglie-Bohm Theory in a 2D Box

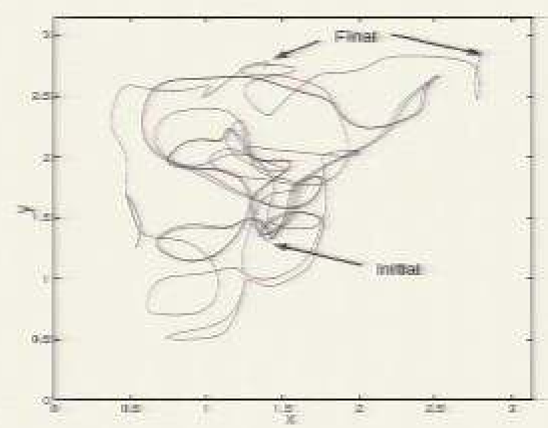
- The wavefunction $\psi(x, y)$ evolves according the the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + V\psi$$

- The Bohmian corpuscle $\mathbf{x} = (x, y)$ evolves according to deBroglie's guiding equation

$$\dot{\mathbf{x}} = Im\frac{\nabla\psi}{\psi} = \nabla S \qquad \psi = Re^{iS}$$





t at 0

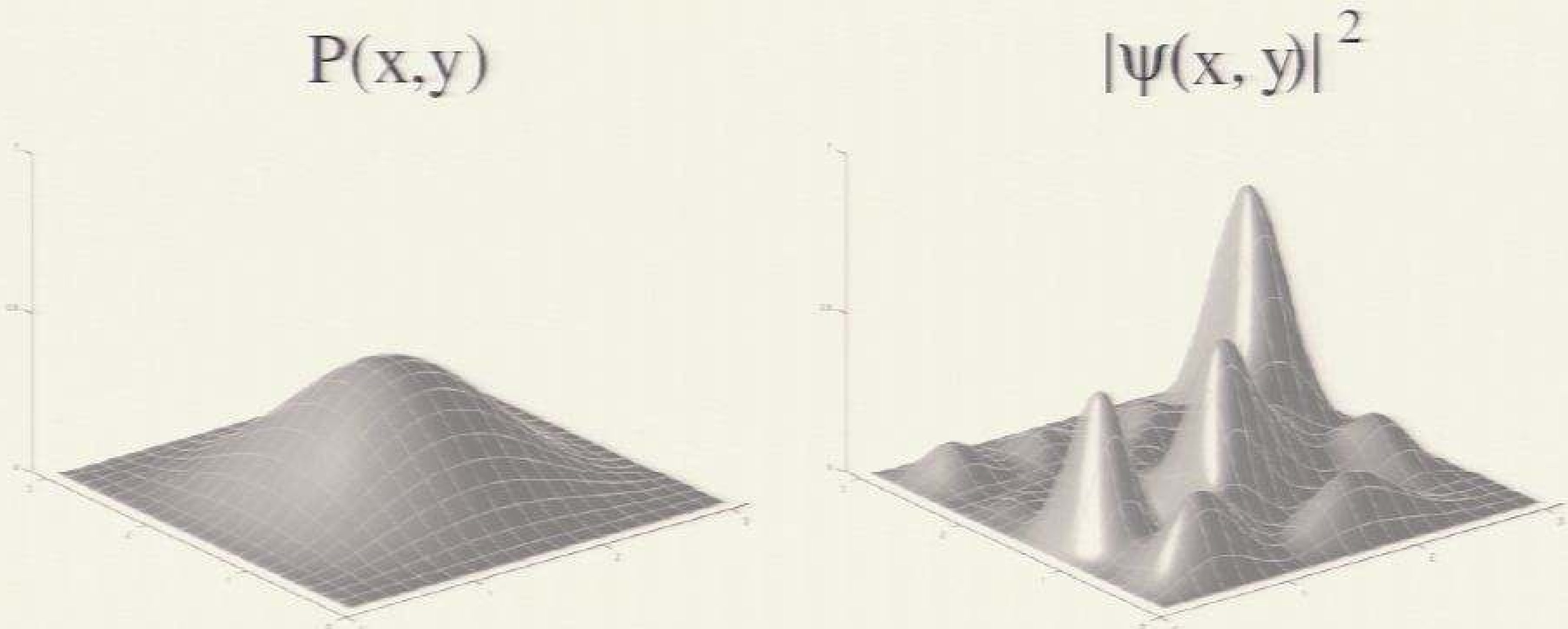


Figure: At $t = 0$ we start with a non-equilibrium distribution $p(x, y)$ which is different from the equilibrium distribution $|\psi(x, y)|^2$

t at 2π

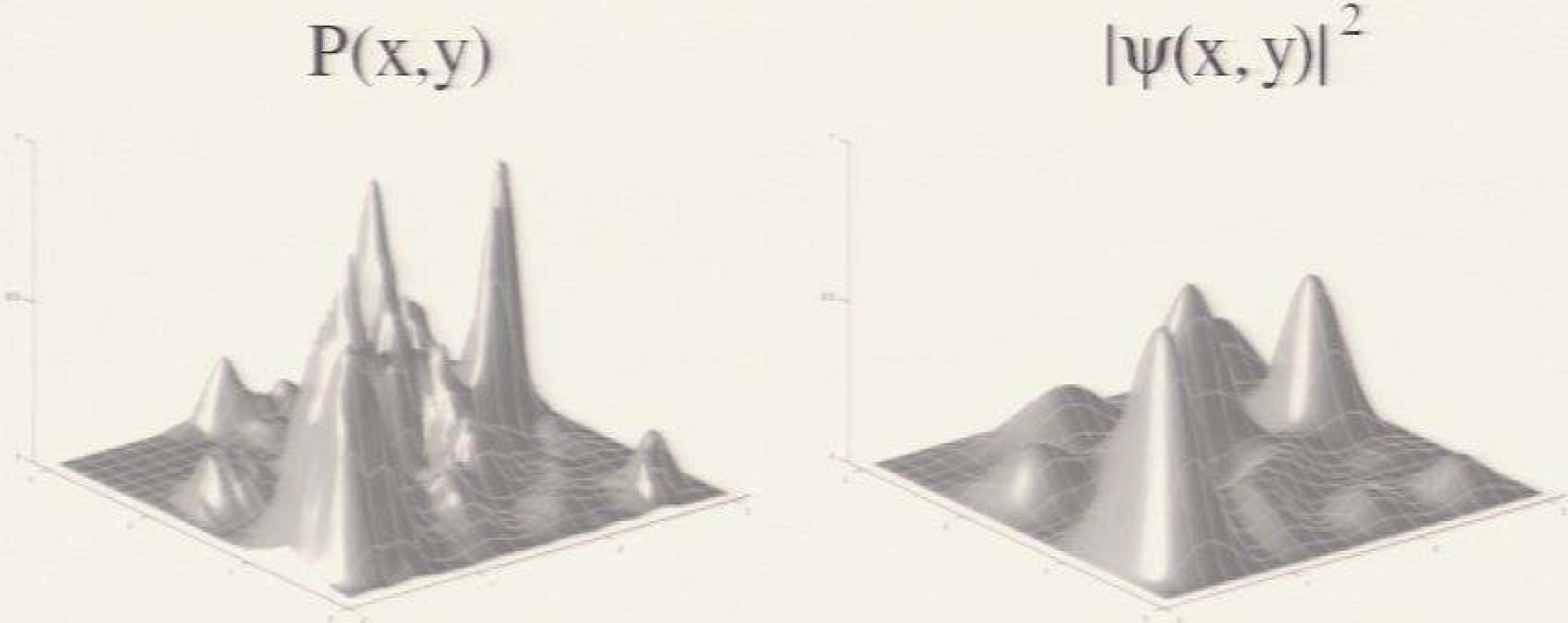


Figure: At $t = 2\pi$ in our units it is clear that the course grained distribution $P(x, y)$ is relaxing towards the equilibrium distribution: $P(x, y) \rightarrow |\psi(x, y)|^2$

t at 4π

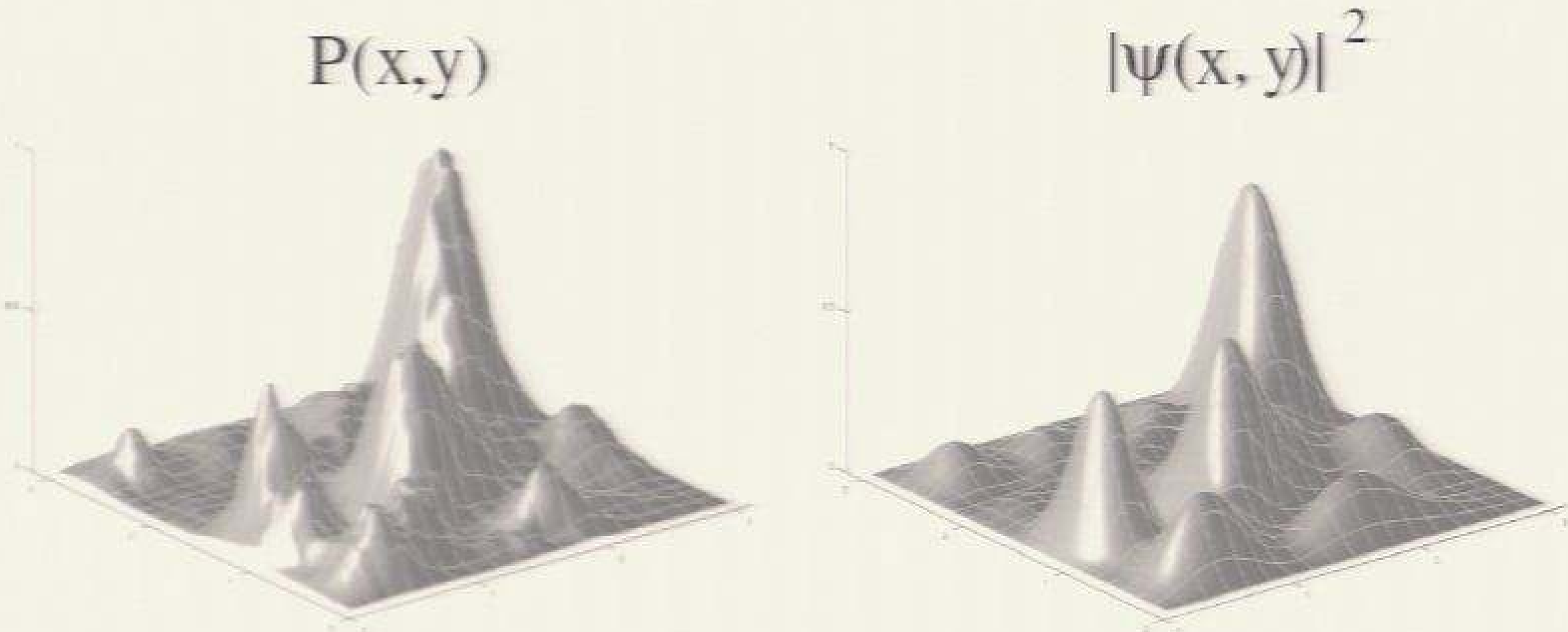


Figure: At $t = 4\pi$ in our units the course grained distribution $P(x, y)$ is almost identical to the equilibrium distribution $|\psi(x, y)|^2$.

Classical vs Quantum Branching

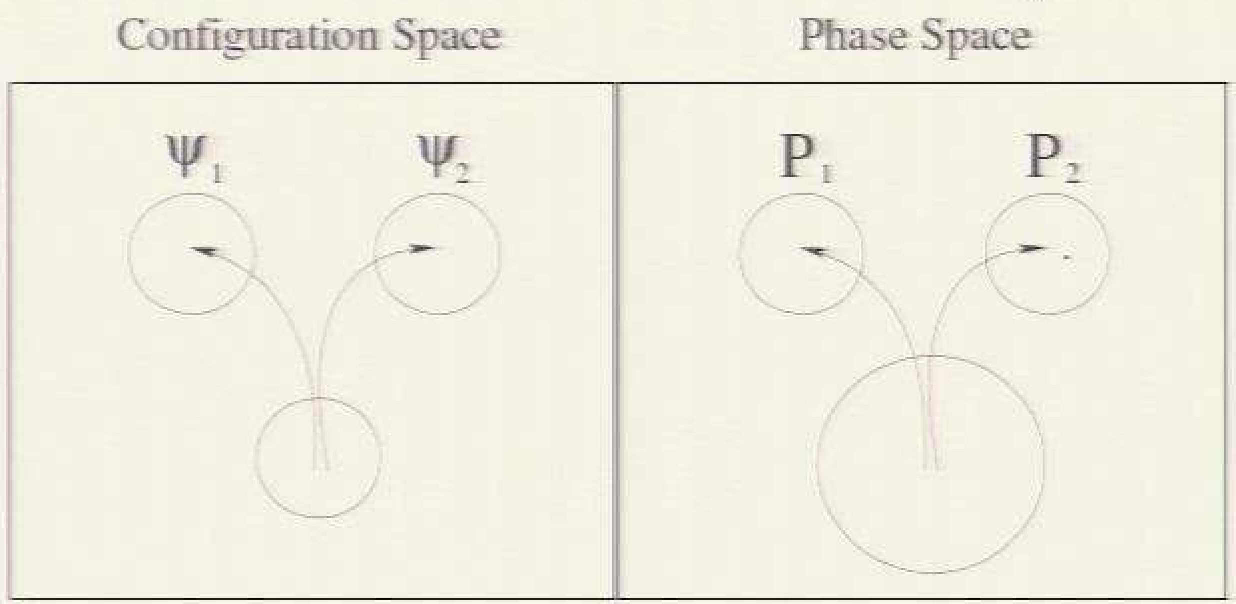


Figure: The quantum state branches into macroscopically distinct states just as the phase space distribution in classical mechanics. In deBroglie-Bohm theory the wavefunction ψ is a guiding field and not a probability distribution.

The ψ-Epistemic Program

- The ψ-epistemic program is about finding an ontological model in which the quantum state represents only a probability distribution.
- It is also about developing exact mathematical criteria for deciding when a ontological model could be regarded as ψ-epistemic.
- For example, the deBroglie-Bohm theory should not be ψ-epistemic according to any reasonable definition. The quantum state is primarily a guiding agent choreographing the motion of point-particles (or field configurations in QFT).
- Many of the features of deBroglie-Bohm theory are universal so it could be that there are no ψ-epistmic theories compatible with the quantum statistics.

Exploring the Dynamical Dimension

- Spekkens, Barett, Hardy, Harrigan, et. al. have introduced a reasonable mathematical constraint that ψ-epistemic theories should satisfy.
- We would now like to introduce a new logically independent constraint by considering dynamics.
- The intuition is that we would like to exclude models like deBroglie-Bohm theory in which the quantum state guides an individual system, thus not acting as a probability distribution.
- This will not be a kinematical constraint but rather a mathematically sharp constraint on the dynamics of an ontological model.

Dynamical Evolution of the Ontic State

Figure: Evolution of an ontic state

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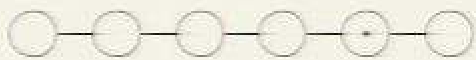


Figure: Evolution of an ontic state

Remarks

- Notice that we are assuming that the world can be decomposed into a sequence of 'instants'.
- This assumption can of course be false. But notice that standard quantum theory has the same structure $|\psi(t)\rangle$.
- Let us now discuss the evolution of ensembles, or probability distributions $\rho(\lambda)$ on the ontic state space Λ.

The Master Equation

Master Equation: $$\dot{\rho}_i = \sum_j T_{ij}\rho_j - T_{ji}\rho_i$$

Conditions on the transition rates T_{ij}:

- Off-diagonal components are non-negative: $T_{ij} \geq 0, \; i \neq j$
- Diagonal components T_{ii} do not enter the master equation and are therefore arbitrary and completely irrelevant.
- The transition rates are allowed to be infinite (Georgi&Tumulka '03, math/0312294).

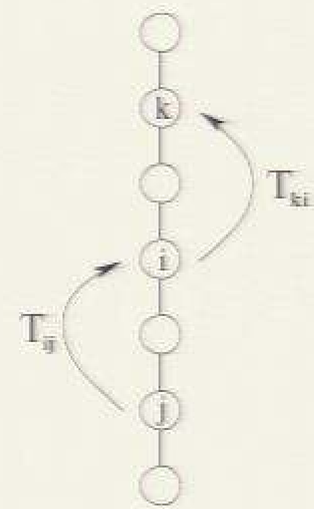
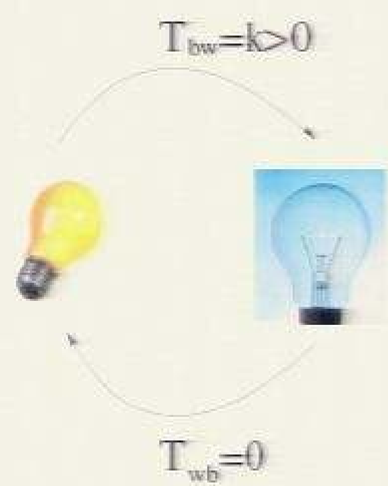


Figure: The transition rates T_{ij} represents the probility per unit time for a transition $j \to i$.

Example: Light Bulb

$$\begin{aligned}\dot{\rho}_w &= T_{wb}\rho_b + T_{ww}\rho_w - T_{bw}\rho_w - T_{ww}\rho_w = -k\rho_w \\ &\Rightarrow \rho_w(t) = Ce^{-kt}\end{aligned}$$

- Note that k can be as large as we want but not negative.
- Also note that no matter how large k is the light bulb will never be definitely broken.
- In order to reach the definitely broken state k needs to become infinite. For example, if $T_{bw}(t) = \frac{1}{\sqrt{|t|}}$ then the light bulb will definitely be in the broken state for $t > 0$.



Dynamical Constraint on ψ-epistemic Theories

- In deBroglie-Bohm theory the transition rates can be shown to depend on the quantum state.
- This means that the quantum state in that theory does not play the role of a probability distribution.
- Thus, we should impose the following the constraint:

Dynamical Constraint on ψ-epistemic models:

The transition rates T_{ij} in a ψ-epistemic ontological model have to be independent of the quantum state.

Dynamical Constraint on ψ-epistemic Theories

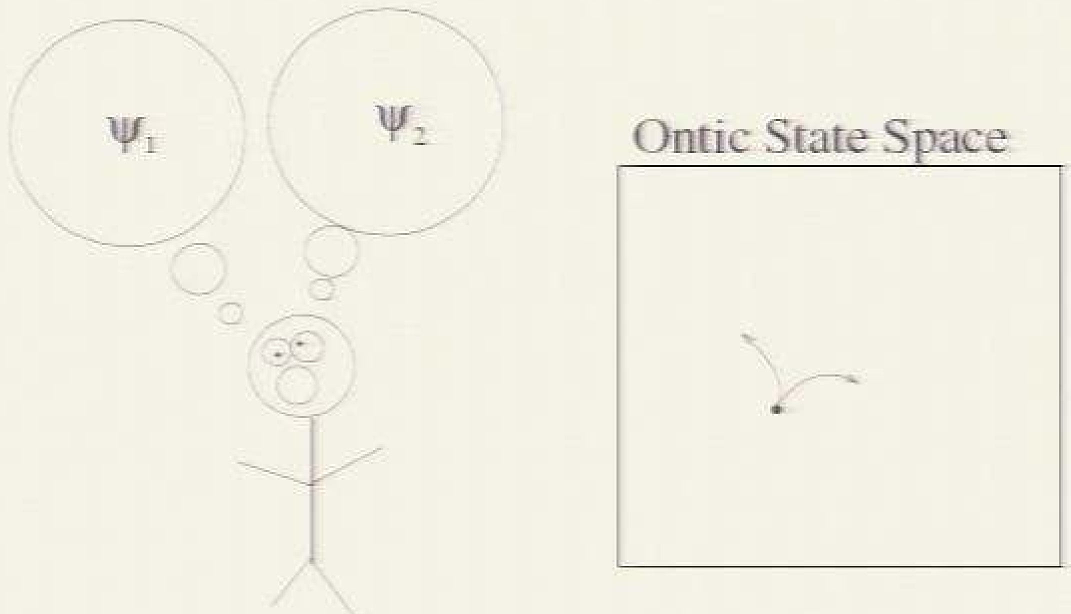


Figure: If we assume that ψ only represents a probability distribution it would be absurd if the transition rates depend on ψ!

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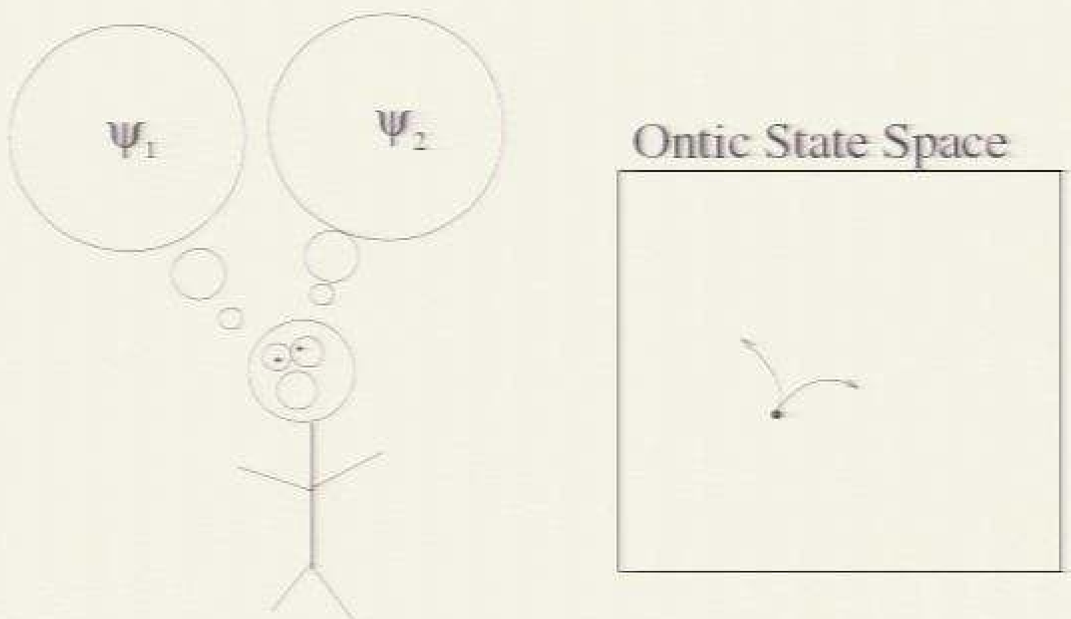

Figure: If we assume that ψ only represents a probability distribution it would be absurd if the transition rates depend on ψ!

Preliminaries

- Consider an N-dimensional quantum system with the quantum state $\hat{\rho}$ and Hamiltonian $\hat{H}$.
- In a specific basis $|i\rangle$ the quantum state and Hamiltonian can be represented as a Hermitian matrices $\hat{\rho} \sim \rho_{ij}$ and $\hat{H} \sim H_{ij}$.
- In this basis the Schrödinger equation reads:

$$\dot{\rho}_{ij} = i \sum_k \rho_{ik} H_{kj} - H_{ik} \rho_{kj}.$$

Introducing ontic properties

- Think of $i = 1, \ldots, N$ as an ontic property and $\rho_i = \rho_{ii}$ as the probability of the system of having that property.
- Thus $\lambda = i$ and $\rho(\lambda) = \rho_i$.
- The evolution of the probability distribution ρ_i is given by

$$\dot{\rho}_i = \dot{\rho}_{ii} = i \sum_j \rho_{ij} H_{ji} - H_{ij} \rho_{ji}.$$

Introducing transition rates

- In order for the model to be an ontological model the probability distribution ρ_i has to obey a master equation:

$$\dot{\rho}_i = \sum_j T_{ij}\rho_j - T_{ji}\rho_i$$

where $T_{ij} \geq 0$ for $i \neq j$.

- Let $J_{ij} = i(\rho_{ij}H_{ji} - H_{ij}\rho_{ji})$ and notice the anti-symmetry: $J_{ij} = -J_{ji}$.
- Our task is now to find transition rates T_{ij} so that

$$\sum_j T_{ij}\rho_j - T_{ji}\rho_i = \sum_j J_{ij}$$

Finding transition rates

- One solution is to take (Bell '87)

$$T_{ij} = \begin{cases} \frac{J_{ij}}{\rho_j} & \text{if } J_{ij} \geq 0 \\ 0 & \text{if } J_{ij} < 0 \end{cases}$$

- Note that with this particular choice the transition rates depend on the quantum state. Therefore, the theory does not count as a genuine ψ-epistemic theory.
- Is it possible, within the framework of Bell-type ontological models, to find transition rates that does not depend on the quantum state?
- NO!

Proof by Contradiction

- We proceed by proof by contradiction. Therefore assume that T_{ij} does not depend on the quantum state.
- This implies in particular that $\sum_j T_{ij}\rho_j - T_{ji}\rho_i$ cannot depend on off-diagonal component $\rho_{i\neq j}$ of the quantum state.
- But since $\sum_j T_{ij}\rho_j - T_{ji}\rho_i = \sum_j J_{ij}$ we have that $\sum_j J_{ij}$ cannot depend on the off-diagonal components either.
- But $\sum_j J_{ij} = 0$ depends only on the off-diagonal components $\rho_{i\neq j}$!
- Thus we have reached a contradiction and we have to give up the assumption that the transition rates T_{ij} do not depend on the quantum state.

A Candidate for a ψ-epistemic model

Key Idea

- It is important to note that we could derive a contradiction precisely because the probability distribution ρ_i did not uniquely determine the quantum state.

$$\sum_j i(\rho_{ij} H_{ji} - H_{ij}\rho_{ji}) = \sum_j T_{ij}\rho_j - T_{ji}\rho_i$$

- It is therefore seems necessary to develop a new class of ontological models so that the epistemic probability distribution $\rho(\lambda)$ uniquly determines the quantum state.
- To do this it is necessary to introduce more ontological properties.

Today's Misleading Statement

Not all 'observables' can be given beable status, for they do not all have simulateneous eigenvalues, i.e. do not all commute. J. S. Bell '84.

Introducing a Candidate for a ψ-epistemic model

- The density matrix and the Hamitonian are Hermitian operators, therefore, we can express any quantum state and any Hamilton as

$$\hat{\rho} = \sum_A c_A T_A \qquad \hat{H} = \sum_A b_A T_A.$$

where the operators T_A are N^2 generators of $U(N)$, i.e. a basis for the space of Hermitian operators.

Choosing a Operator Basis

- One simple choice of basis for the space of Hermitian operators is

$$M^{ab} = \frac{1}{2}(|a\rangle\langle b| + |b\rangle\langle a|) \qquad N^{ab} = \frac{i}{2}(|a\rangle\langle b| - |b\rangle\langle a|)$$

- Notice first the symmetry properties: $\hat{M}^{ab} = \hat{M}^{ba}$ and $\hat{N}^{ab} = -\hat{N}^{ba}$.
- This means that there are

$$\frac{N(N-1)}{2} + \frac{N(N+1)}{2} = N^2$$

operators as required.

Operator Basis Illustrated for $N = 3$ Hilbert Space

$$C\hat{M}^{22} \quad --- \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A\hat{M}^{13} \quad --- \quad \begin{pmatrix} 0 & 0 & A \\ 0 & 0 & 0 \\ A & 0 & 0 \end{pmatrix}$$

$$B\hat{N}^{13} \quad --- \quad \begin{pmatrix} 0 & 0 & iB \\ 0 & 0 & 0 \\ -iB & 0 & 0 \end{pmatrix}$$

Properties of the Operators $\hat{M}^{ab}$ and $\hat{N}^{ab}$.

- The diagonal operators $\hat{M}^{aa} = |a\rangle\langle a|$ constitute a set of N commuting projection operators and have eigenvalues $0, 1$.
- The off-diagonal ones $\hat{M}^{a \neq b}$ are elementary permutation operators (or 'swap' operators) swapping the points $a \leftrightarrow b$. The eigenvalue spectra is $0, \pm\frac{1}{2}$.
- The antisymmetric operators $\hat{N}^{ab}$ are also a type of elementary permutation operators with eigenvalue spectra is $0, \pm\frac{1}{2}$. They are there because we are dealing with a Hilbert space over the complex numbers.

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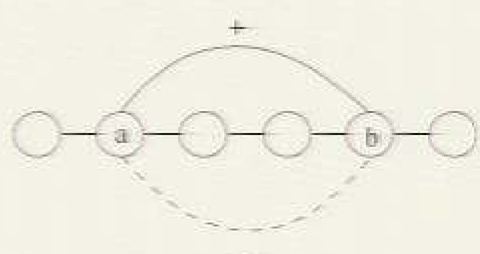
Natural Ontology Associated to the Choice of Operator Basis

- The N commuting projection operators $\hat{M}^{aa} = |a\rangle\langle a|$ could represent the position PVM on a discrete lattice with N points.
- Just like in deBroglie-Bohm theory we will assume that the only one of the projectors $\hat{M}^{aa}$ "has value 1" and the rest "have value 0".
- Thus, corresponding to the PVM $\{\hat{M}^{aa}\}$ we associate an ontic property X which we interpret as a particle occupying a definite position X on the lattice. The position property can take on the values $X = 1, \ldots, N$.

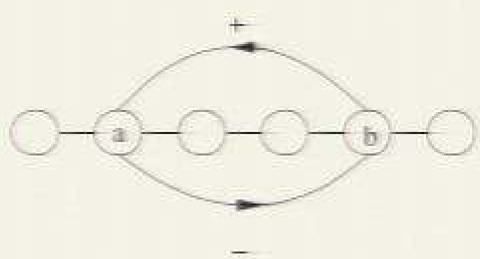
Natural Ontology Associated to the Choice of Operator Basis

- The off-diagonal "symmetric" operator $\hat{M}^{ab}$ (for some $a \neq b$) have eigenvalues 0 and $\pm\frac{1}{2}$.
- Since we only have three distinct eigenvalues the ontic property μ_{ab} corresponding to this operator only needs three distinct attainable values: $\mu_{ab} = 0, \pm\frac{1}{2}$.
- We interpret $\mu_{ab} = 0$ to mean that there is no link between the points a and b. Eigenvalue $\mu_{ab} = +\frac{1}{2}$ correspond to a non-directed "bosonic" link and $\mu_{ab} = -\frac{1}{2}$ to a non-directed "fermionic" link.



Natural Ontology Associated to the Choice of Operator Basis

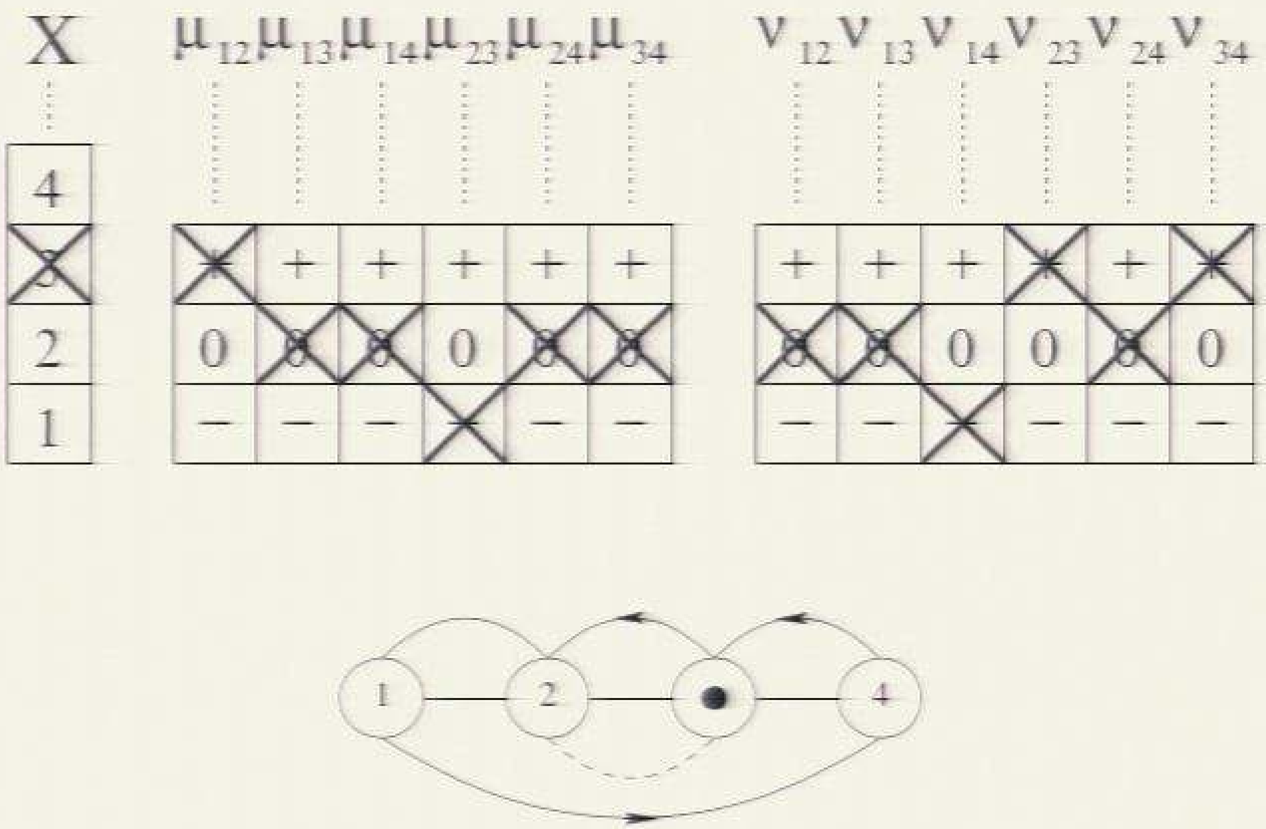
- The antisymmetry of the operators $\hat{N}^{ab}$ immediately suggest that these represent directed links. Eigenvalue $\nu_{ab} = 0$ means no link, eigenvalue $\nu_{ab} = +\frac{1}{2}$ means that there is a link starting from b and ending up at a, and $\nu_{ab} = -\frac{1}{2}$ that there is a link starting from a and ending up at b.



Picking a Specific Ontic State

X	μ_{12}	μ_{13}	μ_{14}	μ_{23}	μ_{24}	μ_{34}	ν_{12}	ν_{13}	ν_{14}	ν_{23}	ν_{24}	ν_{34}
4												
3	+	+	+	+	+	+	+	+	+	+	+	+
2	0	0	0	0	0	0	0	0	0	0	0	0
1	−	−	−	−	−	−	−	−	−	−	−	−

Ontology Illustrated



Expectation Values

- As stated before the quantum state can be expressed in the basis

$$\hat{\rho} = \sum_{a,b} c_{ab}\hat{M}^{ab} + d_{ab}\hat{N}^{ab}$$

where $c_{ab} = c_{ba}$ and $d_{ab} = -d_{ba}$.

- We also have

$$\langle \hat{M}^{ab} \rangle = Tr(\hat{\rho}\hat{M}^{ab}) = c_{ab} \qquad \langle \hat{N}^{ab} \rangle = Tr(\hat{\rho}\hat{N}^{ab}) = d_{ab}.$$

Introducing an Probability Distribution

- We now introduce a probability distribution $\rho(X;\mu_{12},\mu_{13},\ldots,\nu_{12},\nu_{13},\ldots)$ such that we reproduce the expectation values

$$
\begin{aligned}
c_{aa} &= \sum_{X,\mu,\nu} \mu_{aa}\rho(X;\mu,\nu) = \sum_{X,\mu,\nu} \delta_{Xa}\rho(X;\mu,\nu) = \sum_{\mu,\nu} \rho(a;\mu,\nu \\
c_{a\neq b} &= \sum_{X,\mu,\nu} \mu_{ab}\rho(X,\mu,\nu) \\
d_{a\neq b} &= \sum_{X,\mu,\nu} \nu_{ab}\rho(X,\mu,\nu)
\end{aligned}
$$

where μ and ν are multi-indices.

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c_{a\neq b} &= \sum_{X,\mu,\nu} \mu_{ab}\rho(X,\mu,\nu) \\
d_{a\neq b} &= \sum_{X,\mu,\nu} \nu_{ab}\rho(X,\mu,\nu)
\end{aligned}
$$

where μ and ν are multi-indices.

Introducing an Probability Distribution

- Recall that $\mu_{aa} = \delta_{a\bar{a}}$ for some $\bar{a}$. This implies that $\sum_a c_{aa} = \sum_{\mu,\nu} \rho(X;\mu,\nu) = 1$ since ρ is assumed to be a normalized probability distribution. Thus $Tr(\hat{\rho}) = 1$ just means that the probability distribution $\rho(X;\mu,\nu)$ is normalized.

Quantum Non-Equilibrium Distributions and Relaxation

- It is clear that the distribution $\rho(X;\mu,\nu)$ uniquely determines the quantum state $\hat{\rho}$. From the distribution $\rho(X;\mu,\nu)$ we can compute all the expectation values c_{ab} and d_{ab}, which in turn uniquely determines the quantum state through $\hat{\rho} = \sum_{ab} c_{ab}\hat{M}^{ab} + d_{ab}\hat{N}^{ab}$.
- Note that there will be distributions $\rho(X;\mu,\nu)$ such that the correspoding matrix $\hat{\rho}$ is not a positive operator, i.e. a proper density matrix. Such distributions are examples of quantum non-equilibrium states.
- Distributions $\rho(X;\mu,\nu)$ for which the corresponding $\hat{\rho}$ is a positive operator we denote quantum equilibrium distributions.

Quantum Non-Equilibrium Distributions and Relaxation

- It is easy to prove that in all ψ-epistemic ontological models quantum non-equilibrium distributions are logically possible. Thus, one must provide an explanation why the quantum equilibrium distributions are found in nature.
- Spekkens, Bartlett, Rudolph, et. al.: impose epistemic constraints on probability distribution.
- Gibbs' stirring: different distributions ρ_{eq} and ρ evolving according to the same law tends to converge $\rho \rightarrow \rho_{eq}$ given that the system is "suffienctly mixing".
- Boltzmann type explanation.

Constraints on the Transistion Rates Imposed by the Schrödinger Equation

The Schrödinger Equation

By making use of the symmetry properties of c_{ab} and d_{ab} and the commutator algebra:

$$[\hat{M}^{ab}, \hat{M}^{cd}] = \frac{1}{2i}(\delta^{ac}\hat{N}^{bd} + \delta^{ad}\hat{N}^{bc} + \delta^{bc}\hat{N}^{ad} + \delta^{bd}\hat{N}^{ac})$$

$$[\hat{M}^{ab}, \hat{N}^{cd}] = \frac{1}{2i}(-\delta^{ac}\hat{M}^{bd} + \delta^{ad}\hat{M}^{bc} - \delta^{bc}\hat{M}^{ad} + \delta^{bd}\hat{M}^{ac})$$

$$[\hat{N}^{ab}, \hat{N}^{cd}] = \frac{1}{2i}(\delta^{ac}\hat{N}^{bd} - \delta^{ad}\hat{N}^{bc} - \delta^{bc}\hat{N}^{ad} + \delta^{bd}\hat{N}^{ac})$$

The Schrödinger Equation

... the Schrödinger equation can be shown to take the form

$$\dot{c}_{ef} = \sum_{d} c_{ed} b_{fd} - d_{ed} a_{fd} + c_{fd} b_{ed} - d_{fd} a_{ed}$$

$$\dot{d}_{ef} = \sum_{d} c_{ed} a_{fd} + d_{ed} b_{fd} - c_{fd} a_{ed} - d_{fd} b_{ed}$$

Condition Imposed by the Schrödinger Equation

- As before we require the probability distribution $\rho(\lambda) = \rho(X;\mu,\nu)$ evolves according to a master equation.

$$\dot{\rho}(X;\mu,\nu) = \sum_{\bar{X},\bar{\mu},\bar{\nu}} T(X;\mu,\nu|\bar{X};\bar{\mu},\bar{\nu})\rho(\bar{X};\bar{\mu},\bar{\nu}) - T(\bar{X};\bar{\mu},\bar{\nu}|X;\mu,\nu)\rho(X;\mu,\nu)$$

Condition Imposed by the Schrödinger Equation

- Introduce the symbols

$$u_{ab}(\bar{\mu},\bar{\nu}) = u_{ba}(\bar{\mu},\bar{\nu}) =$$

$$\sum_{X,\mu,\nu}(\mu_{ab}-\bar{\mu}_{ab})T(X,\mu,\nu|\bar{X},\bar{\nu},\bar{\nu}) - \sum_{d}\bar{\mu}_{ed}b_{fd} - \bar{\nu}_{ed}a_{fd} + \bar{\mu}_{fd}b_{ed} - \bar{\nu}_{fd}a$$

$$v_{ab}(\bar{\mu},\bar{\nu}) = -v_{ba}(\bar{\mu},\bar{\nu}) =$$

$$\sum_{X,\mu,\nu}(\nu_{ab}-\bar{\nu}_{ab})T(X,\mu,\nu|\bar{X},\bar{\nu},\bar{\nu}) - \sum_{d}\bar{\mu}_{ed}a_{fd} + \bar{\nu}_{ed}b_{fd} - \bar{\mu}_{fd}a_{ed} - \bar{\nu}_{fd}b$$

Condition Imposed by the Schrödinger Equation

- The Schrödinger equation then gives rise to the following N^2 conditions:

$$\sum_{\bar{X},\bar{\mu},\bar{\nu}} u_{ab}(\bar{X};\bar{\mu},\bar{\nu})\rho(\bar{X};\bar{\mu},\bar{\nu}) = 0$$

$$\sum_{\bar{X},\bar{\mu},\bar{\nu}} v_{ab}(\bar{X};\bar{\mu},\bar{\nu})\rho(\bar{X};\bar{\mu},\bar{\nu}) = 0$$

- Thus the inner product of $\rho(\bar{X};\bar{\mu},\bar{\nu})$ with $u_{ab}(\bar{X};\bar{\mu},\bar{\nu})$ and $v_{ab}(\bar{X};\bar{\mu},\bar{\nu})$ should be zero.

Condition Imposed by the Schrödinger Equation

- The most immediate way to make this happen is to simply put $u_{ab}(\bar{X};\bar{\mu},\bar{\nu})=0$ and $v_{ab}(\bar{X};\bar{\mu},\bar{\nu})=0$.
- The transition rates as given by these equations will not depend on the quantum state $\hat{\rho}$ so this is potentially a genuine ψ-epistemic theory.
- However, one can show that this is way too restrictive and leads to absurd dynamics and that the equations do not even have a solution for a 2D Hilbert space.
- A less restrictive requirement is that the Schrödinger equation should hold only for equilibrium distributions.

$$\sum_{\bar{X},\bar{\mu},\bar{\nu}} u_{ab}(\bar{X};\bar{\mu},\bar{\nu})\rho_{eq.}(\bar{X};\bar{\mu},\bar{\nu})=0$$

$$\sum_{\bar{X},\bar{\mu},\bar{\nu}} v_{ab}(\bar{X};\bar{\mu},\bar{\nu})\rho_{eq.}(\bar{X};\bar{\mu},\bar{\nu})=0$$