

Title: Thinking Inside the Box: Weakly Measuring Postselected Ensembles

Date: Sep 29, 2008 11:45 AM

URL: <http://pirsa.org/08090071>

Abstract: The presumed irreversibility of quantum measurements (whatever they are) leads, in conventional approaches to quantum theory, to an asymmetry between state preparation and post-selection. Is it possible that a trajectory can be predicted from the former, yet not inferred from the latter? Especially in light of the exciting applications of non-unitary operations (i.e., postselection) in quantum information, it becomes timely to reconsider how much one can say about a post-selected subensemble. I will review the weak-measurement formalism of Aharonov, Vaidman et al., and discuss some applications and extensions. These will include a proposed experiment to study the duration of the tunneling process (a question controversial since the 1930s) and a recently completed experiment aiming to '\resolve\' Hardy\'s retrodiction paradox.

Thinking Inside the Box: Weakly Measuring Postselected Ensembles

Aephraim M. Steinberg
Centre for Q. Info. & Q. Control
Institute for Optical Sciences
Dept. of Physics, U. of Toronto



Pirsa: 08090071

The Clock and The Quantum, Perimeter Institute
29 September 2008



Page 2/96

DRAMATIS PERSONÆ

Toronto quantum optics & cold atoms group:

Postdocs: Luciano Cruz Morgan Mitchell (→ ICFO)
Matt Partlow(→Energetiq)Marcelo Martinelli (→ USP)

Optics: Rob Adamson Kevin Resch(→Wien →UQ→IQC)
Lynden(Krister) Shalm Jeff Lundeen (→Oxford)
Xingxing Xing Jason Ng Sacha Kocsis

Atoms: Jalani Kanem (→Imperial)Stefan Myrskog (→BEC→ ECE)
Mirco Siercke (→ ...?) Ana Jofre(→NIST →UNC)
Samansa Maneshi Chris Ellenor Chris Paul
Rockson Chang Chao Zhuang Xiaoxian Liu

UG's: Ardavan Darabi, Amanda O'Halloran, Nick Chisholm, Eva Markowski

Some helpful theorists:

Daniel James, Pete Turner, Michael Spanner, Howard Wiseman, János Bergou, John Sipe, Paul Brumer, ...



Pirsa: 08090071



Canadian Institute for



DRAMATIS PERSONÆ

Toronto quantum optics & cold atoms group:

Postdocs: Luciano Cruz Morgan Mitchell (→ ICFO)
Matt Partlow (→ Energetiq) Marcelo Martinelli (→ USP)

Optics: Rob Adamson Kevin Resch (→ Wien → UQ → IQC)
Lynden (Kristen) Shalm Jeff Lundeen (→ Oxford)
Xingrong Xing Jason Ng Sacha Kocsis

Atoms: Jolani Haner (→ Imperial) Stefan Myrskog (→ BEC → ECE)
Mirko Siercke (→ ...) Ana Jofre (→ NIST → UNCO)
Samansa Maneshi Chris Ellenor Chris Paul
Rockson Chang Chao Zhuang Xiaoxian Liu

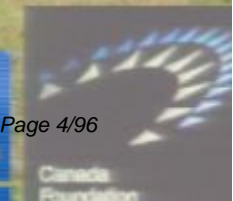
UG's: Ardavan Darabi, Amanda O'Halloran, Nick Chisholm, Eva Markowski

Some helpful theorists:

Daniel James, Pete Turner, Michael Spanner, Howard Wiseman, Janos Bergou, John Sipe, Paul Bruner,



Pirsa: 08090071



Measurement: this is not your father's observable

SUMMARY



- Weak measurement on postselected quantum systems (conditional quantum measurements)
- An alternate & unjustified derivation
- Tunneling times as an ongoing motivation
- A simple example & implementation (3-box problem)
- “Interaction-free” measurement & Hardy’s Paradox
- Which-path measurement, complementarity, & uncertainty



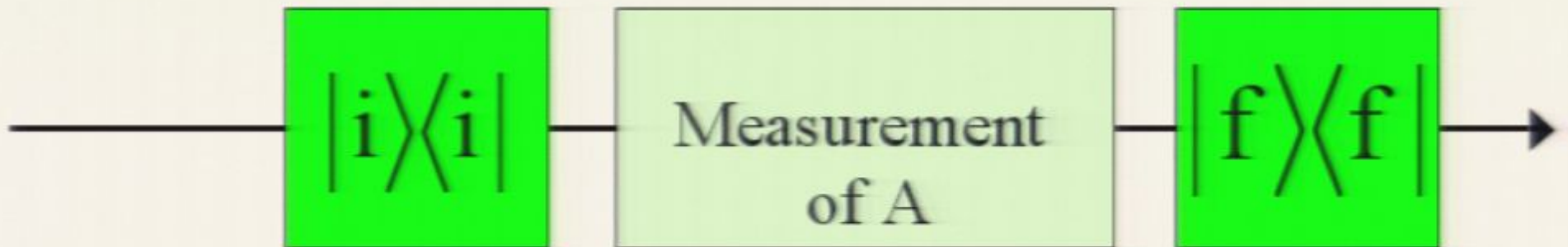
Can we talk about what goes on behind closed doors?

**(“Postselection” is the big new buzzword in QIP...
but how should one describe post-selected states?)**

Conditional measurements (Aharonov, Albert, and Vaidman)

AAV, PRL 60, 1351 ('88)

Prepare a particle in $|i\rangle$... try to "measure" some observable A ...
postselect the particle to be in $|f\rangle$



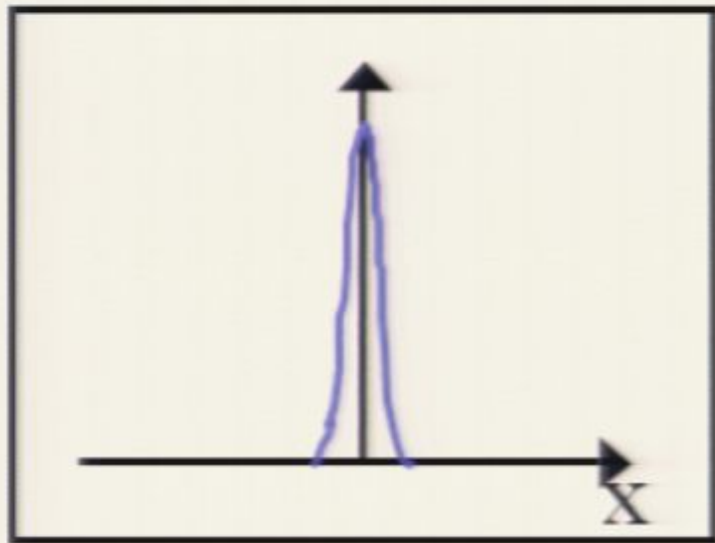
Does $\langle A \rangle$ depend more on i or f , or equally on both?
Clever answer: both, as Schrödinger time-reversible.
Conventional answer: i , because of collapse.

Reconciliation: measure A "weakly."
Poor resolution, but little disturbance.

$$\Rightarrow A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}$$

A (von Neumann) Quantum Measurement of A

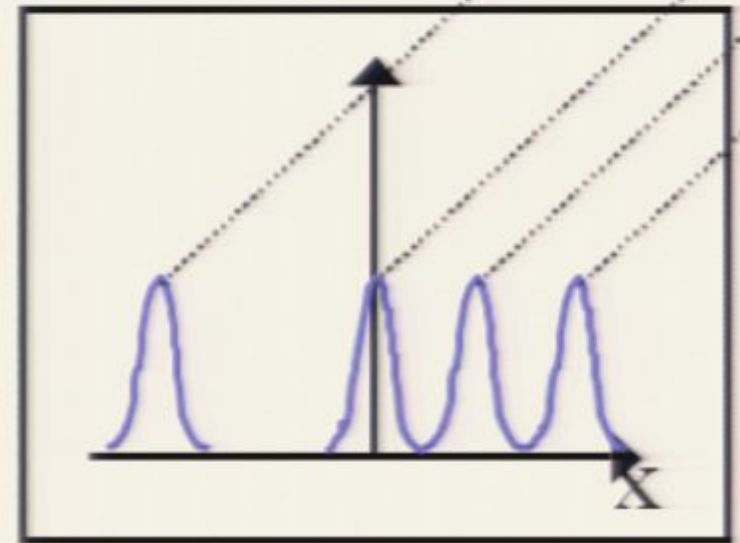
Initial State of Pointer



$$H_{\text{int}} = gAp_x$$

System-pointer
coupling

Final Pointer Readout



Well-resolved states

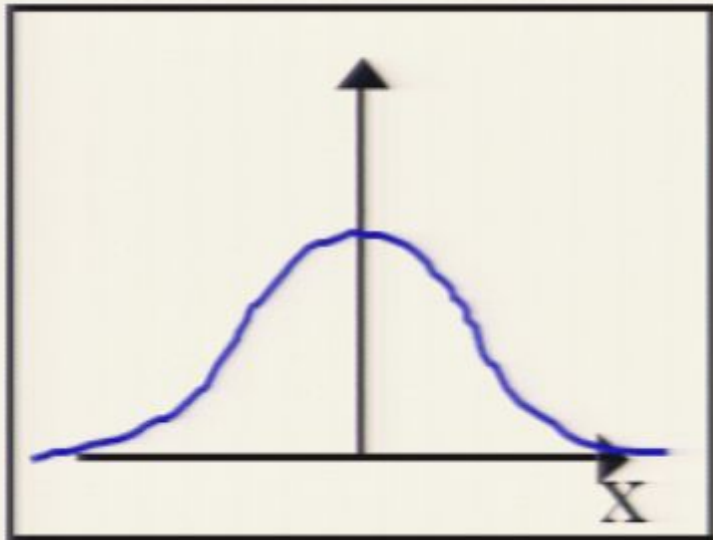
System and pointer become entangled



Decoherence / "collapse"
Large back-action

A Weak Measurement of A

Initial State of Pointer

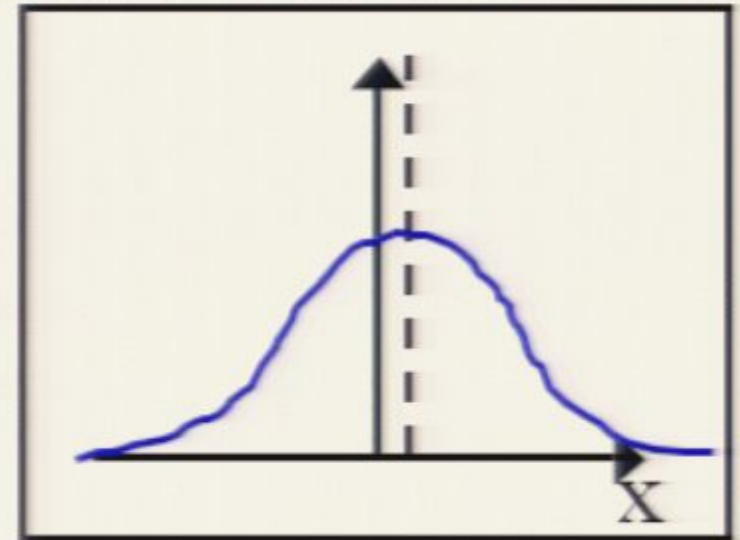


$$H_{\text{int}} = gAp_x$$



System-pointer
coupling

Final Pointer Readout



Poor resolution on each shot.

Negligible back-action (system & pointer separable)

Strong: $|\Psi\rangle_s \phi_p(x) \rightarrow \sum_i c_i |\psi_i\rangle_s \phi_p(x - ga_i)$

Weak: $|\Psi\rangle_s \phi_p(x) \rightarrow |\Psi\rangle_s \phi_p(x - g\langle A_s \rangle)$

Bayesian Approach to Weak Values

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

$$P(A \& B) = \langle \text{Proj}(B) \text{Proj}(A) \rangle = \langle \psi | B \rangle \langle B | A \rangle \langle A | \psi \rangle$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A_i|f) = \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle}.$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

$$P(A \& B) = \langle \text{Proj}(B) \text{Proj}(A) \rangle = \langle \psi | B \rangle \langle B | A \rangle \langle A | \psi \rangle$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$P(A_i|f) = \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

$$P(A \& B) = \langle \text{Proj}(B) \text{Proj}(A) \rangle = \langle \psi | B \rangle \langle B | A \rangle \langle A | \psi \rangle$$

$$\langle A \rangle_f = \sum_i a_i \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle} = \frac{\langle |f\rangle \langle f| A \rangle}{\langle |f\rangle \langle f| \rangle}.$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$P(A_i|f) = \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

$$P(A \& B) = \langle \text{Proj}(B) \text{Proj}(A) \rangle = \langle \psi | B \rangle \langle B | A \rangle \langle A | \psi \rangle$$

$$\langle A \rangle_f = \sum_i a_i \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle} = \frac{\langle |f\rangle \langle f| A \rangle}{\langle |f\rangle \langle f| \rangle}.$$

$$\langle A \rangle_{fi} = \frac{\langle i | |f\rangle \langle f| A | i \rangle}{\langle i | |f\rangle \langle f| | i \rangle} = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}.$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A_i|f) = \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle}.$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

$$P(A \& B) = \langle \text{Proj}(B) \text{Proj}(A) \rangle = \langle \psi | B \rangle \langle B | A \rangle \langle A | \psi \rangle$$

$$\langle A \rangle_f = \sum_i a_i \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle} = \frac{\langle |f\rangle \langle f| A \rangle}{\langle |f\rangle \langle f| \rangle}.$$

$$\langle A \rangle_{fi} = \frac{\langle i | |f\rangle \langle f| A | i \rangle}{\langle i | |f\rangle \langle f| | i \rangle} = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}.$$

$$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}$$

Bayesian Approach to Weak Values

$$\langle A \rangle_{\text{wk}} = \sum_j a_j P(j|i, f),$$

$$P(A|B) \equiv \frac{P(A \& B)}{P(B)}.$$

$$P(A_i|f) = \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle}.$$

$$\begin{aligned} P(A) &= \langle \text{Proj}(A) \rangle = \langle |A\rangle \langle A| \rangle \\ &= \langle \psi | A \rangle \langle A | \psi \rangle = |\langle A | \psi \rangle|^2. \end{aligned}$$

$$P(A \& B) = \langle \text{Proj}(B) \text{Proj}(A) \rangle = \langle \psi | B \rangle \langle B | A \rangle \langle A | \psi \rangle$$

$$\langle A \rangle_f = \sum_i a_i \frac{\langle \text{Proj}(f) \text{Proj}(A_i) \rangle}{\langle \text{Proj}(f) \rangle} = \frac{\langle |f\rangle \langle f| A \rangle}{\langle |f\rangle \langle f| \rangle}.$$

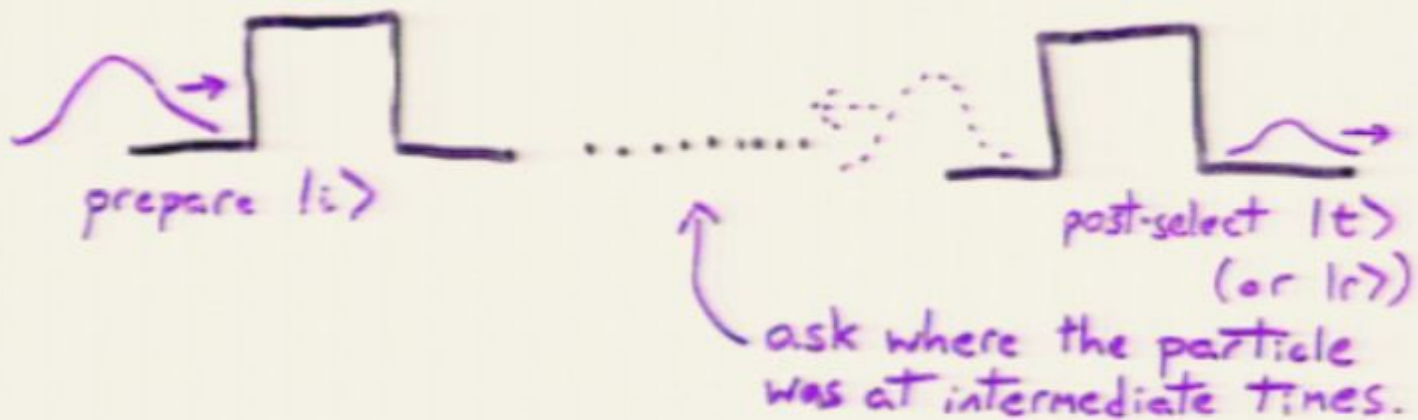
$$\langle A \rangle_{fi} = \frac{\langle i | |f\rangle \langle f| A | i \rangle}{\langle i | |f\rangle \langle f| | i \rangle} = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}.$$

$$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}$$

Note: this is the same result you get from actually performing the QM calculation (see A&V).

Weak measurement & tunneling times

How does this apply to tunneling?



$$P(x) = |\psi(x)|^2 = \underbrace{\langle \psi | x \rangle \langle x | \psi \rangle}_{\text{Proj}(x)}$$

The prob. of being at x is just the expectation value of the projector onto x .

Conditional probability distributions

$$\begin{aligned}\text{Bayes's thm.} \Rightarrow P(x | \text{trans}) &= \frac{P(x \& \text{trans})}{P(\text{trans})} \\ &= \frac{\langle 1|t \rangle \langle t|1 \rangle \langle x| \rangle}{\langle 1|t \rangle \langle t| \rangle} \\ &= \frac{\langle i|t \rangle \langle t|x \rangle \langle x|i \rangle}{\langle i|t \rangle \langle t|i \rangle} = \frac{\langle t|x \rangle \langle x|i \rangle}{\langle t|i \rangle}\end{aligned}$$

Precisely A&V's result. $P(x|\text{trans}) = \frac{1}{T} \psi_T^*(x) \psi_I(x).$

- We can write the prob. distrib. of either trans. or refl. particles, as a function of time.
- We can integrate over time & over the barrier to obtain a total "conditional dwell time."
- BUT: these results are complex.

A problem...

These expressions can be complex.

Much like early tunneling-time expressions derived via Feynman path integrals, et cetera.

THE MEANING OF 'WEAK MEASUREMENTS' WITH COMPLEX VALUES

$$\tau_T \rightarrow \tau_d - i \tau_{BL}$$

"Has anyone ever seen a stopwatch with complex numbers on the dial?"



A solution...

But consider a quantum-mechanical stopwatch.

$$\psi(x) \sim e^{-(x-t)^2/4\sigma^2}$$

← some inevitable uncertainty

$$t \text{ complex} \Rightarrow \psi \sim e^{-(x - \text{Re } t)^2/4\sigma^2} e^{i x \text{Im } t/2\sigma^2} \dots$$

↑ hand shifts by $\text{Re } t$
↑ picks up momentum of $\hbar \text{Im } t/2\sigma^2$
↑ normalization

This is precisely the meaning of weak (or conditional) measurements.

$\text{Re } t$ describes clock hand's position shift
(e.g., Larmor precession).

$\text{Im } t$ describes back-action
(e.g., spin aligning with \underline{B} .)

For large σ , back-action vanishes, but position shift of hand remains constant.

A solution...

But consider a quantum-mechanical stopwatch.

$$\psi(x) \sim e^{-(x-t)^2/4\sigma^2}$$

← some inevitable uncertainty

t complex $\Rightarrow \psi \sim e^{-(x - \text{Re } t)^2/4\sigma^2} e^{i x \text{Im } t/2\sigma^2} \dots$

↑
hand shifts
by $\text{Re } t$
↑
picks up
momentum
of $\hbar \text{Im } t/2\sigma^2$
↑
normalization

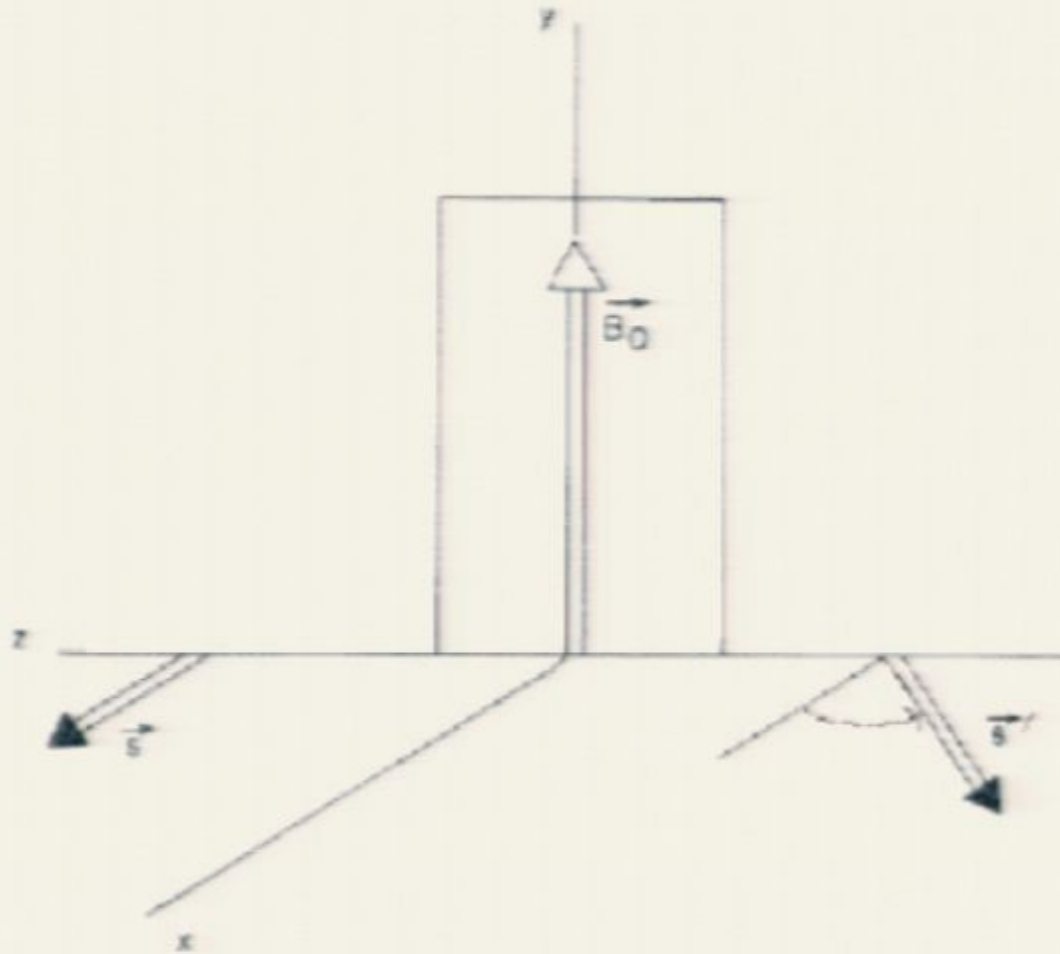
This is precisely the meaning of weak (or conditional) measurements.

$\text{Re } t$ describes clock hand's position shift
(e.g., Larmor precession).

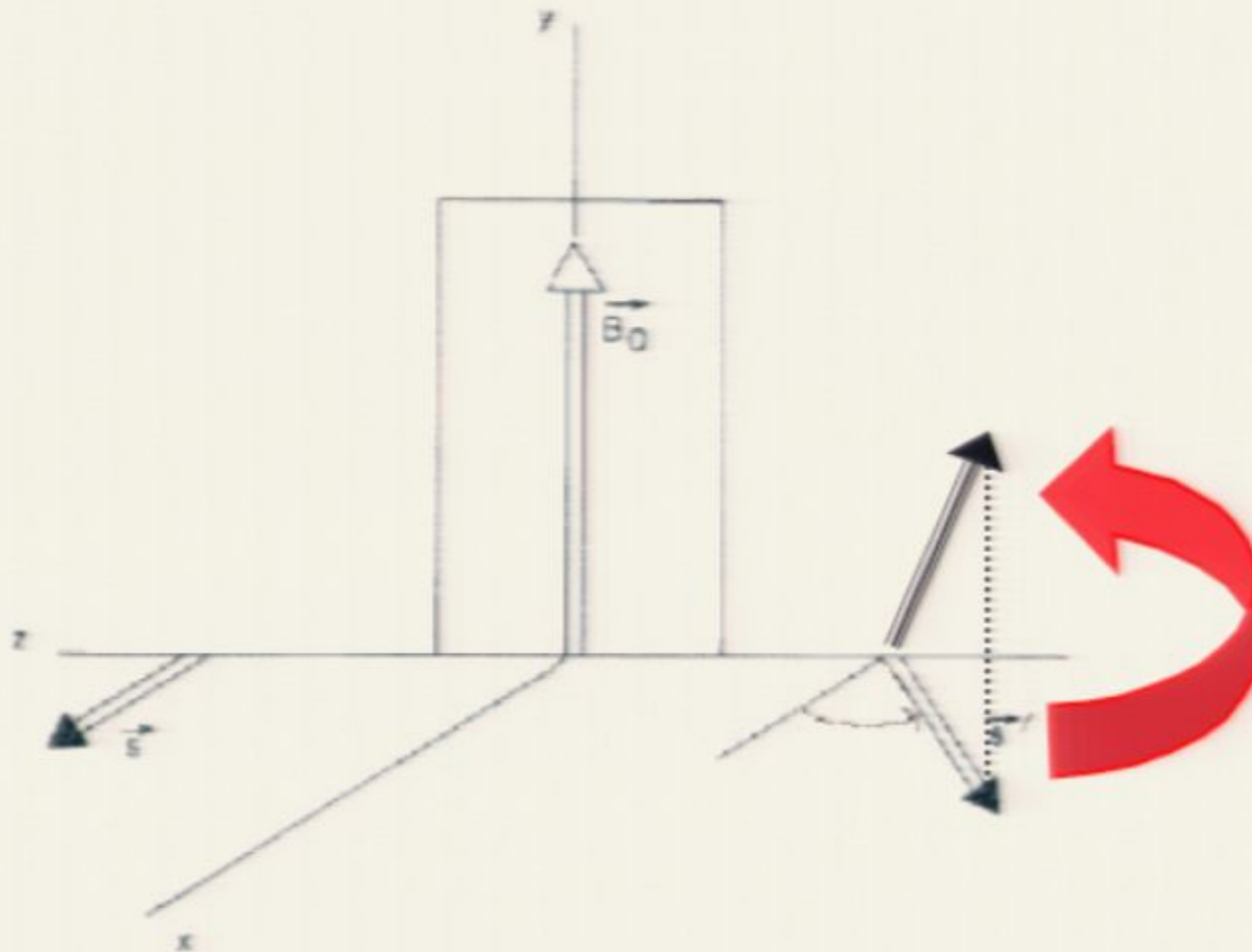
$\text{Im } t$ describes back-action
(e.g., spin aligning with \underline{B} .)

For large σ , back-action vanishes, but position shift of hand remains constant.

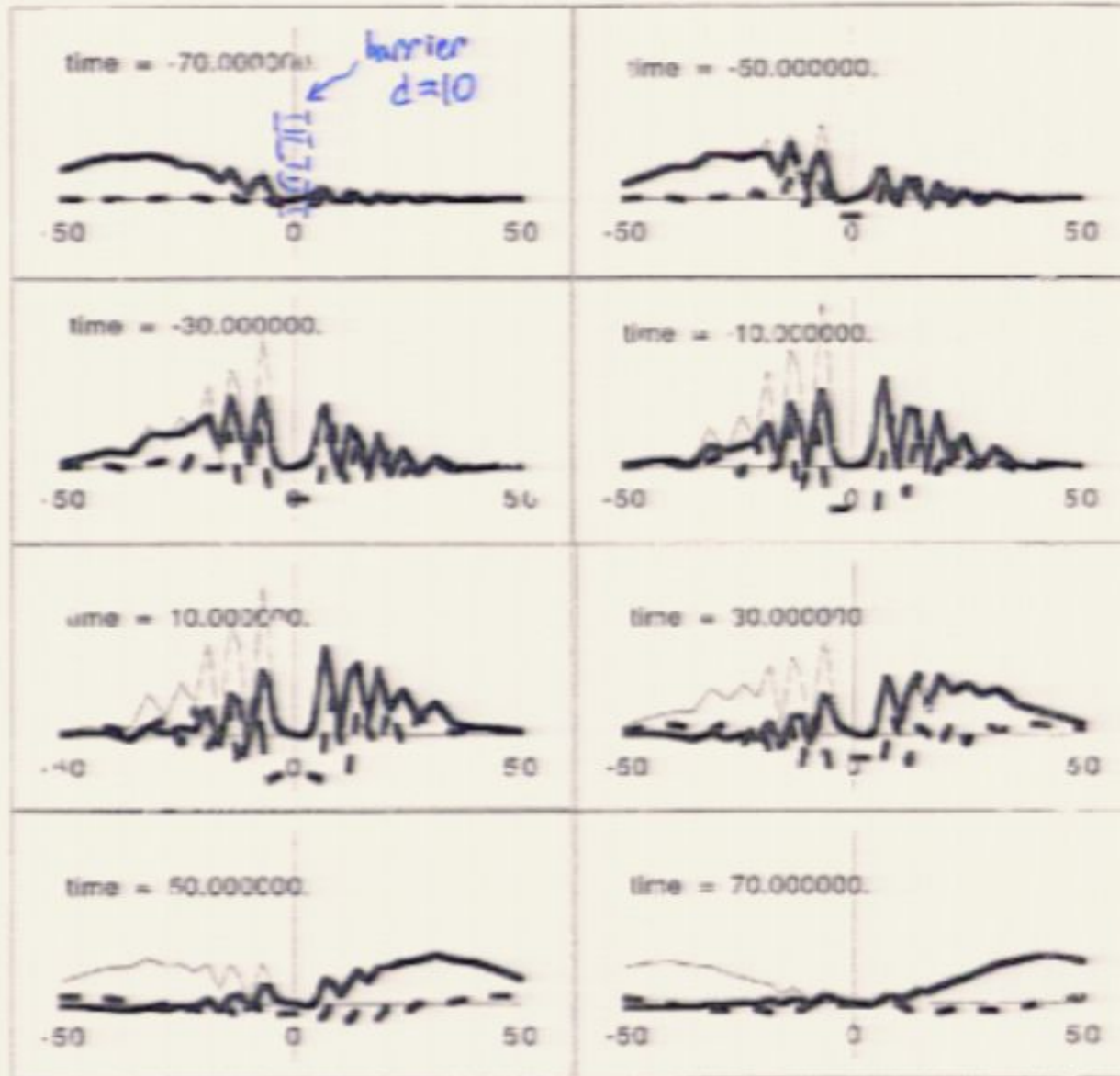
Cf. Büttiker's observation about the Baz'-Rybachenko Larmor clock



Cf. Büttiker's observation about the Baz'-Rybachenko Larmor clock



Conditional $P(x)$ for tunneling

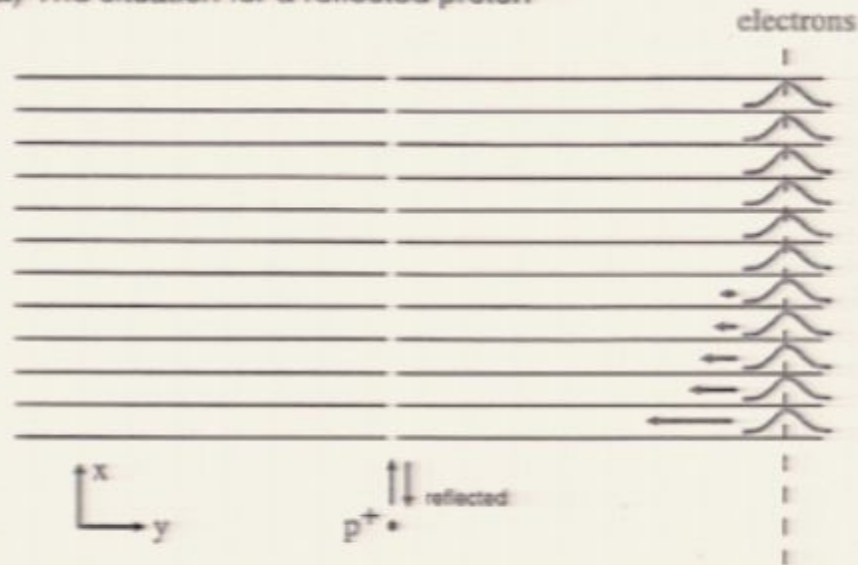


— Prob. distrib. of tunneling particle
 - - - " " " reflected "
 "Back-action" on quantum measurement device

What does this *mean* practically?

A la recherche du temps perdu

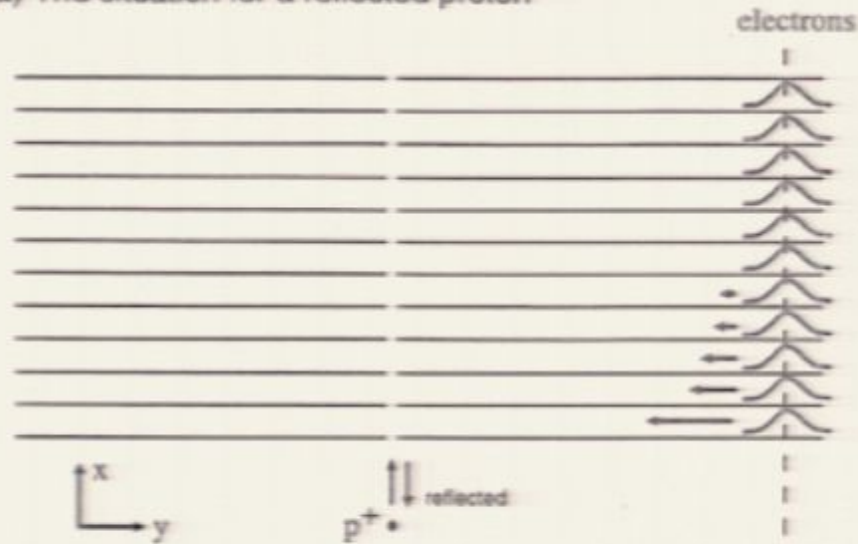
(a) The situation for a reflected proton



What does this *mean* practically?

À la recherche du temps perdu

(a) The situation for a reflected proton



(b) The situation for a transmitted proton

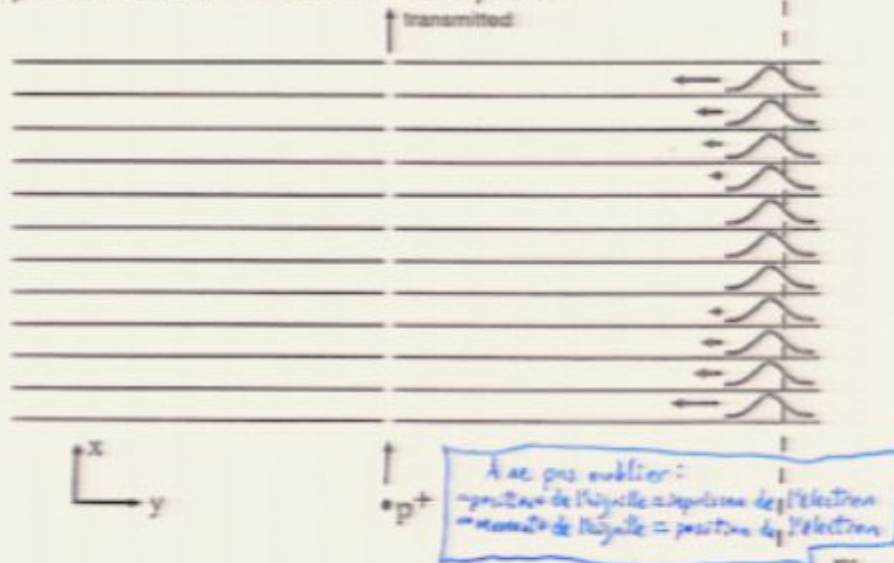
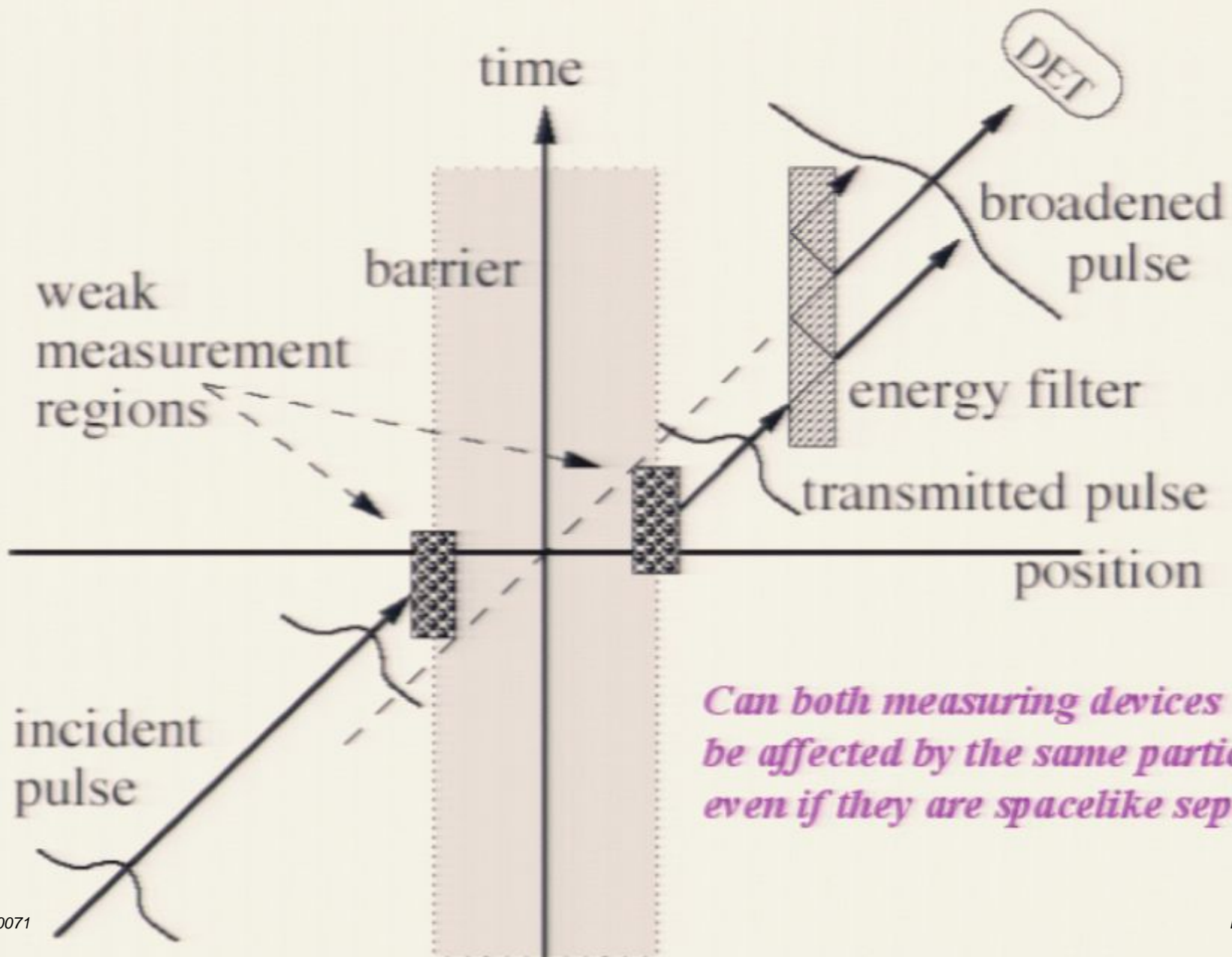


Figure 1

Just *how* nonlocal are particles?



How many ways are there to be in two places at one time?

We all know even a quantum particle may not affect particles at spacelike separations.

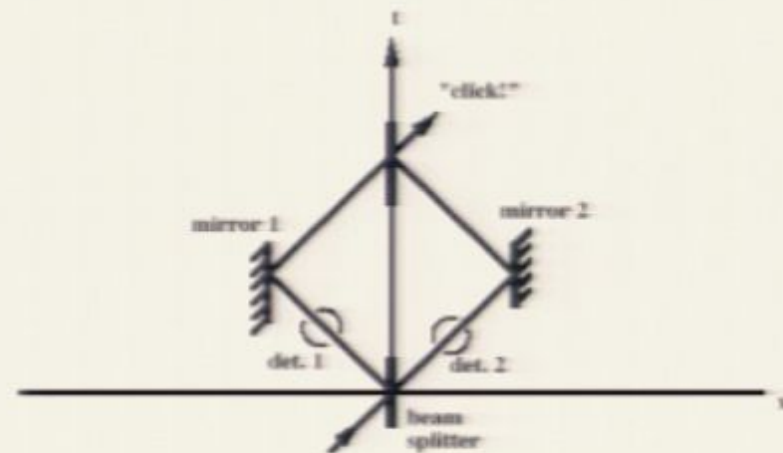
But even a classical cause may have two effects which are spacelike *from each other*.

On the other hand, a classical *particle* may not have such effects.

Neither would a single photon split into two paths of an interferometer.

If, from an ensemble of particles, each affects only one region of spacetime, then the *difference* between the two will grow noisier.

Perhaps the nonlocality of a tunneling particle is something deeper?



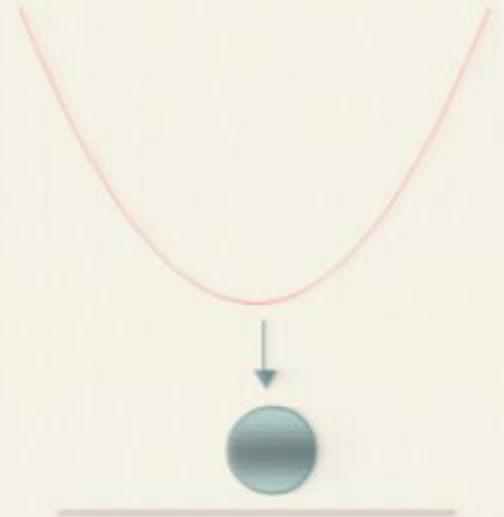
● ● ● Planned experimental sequence

- BEC in magnetic trap



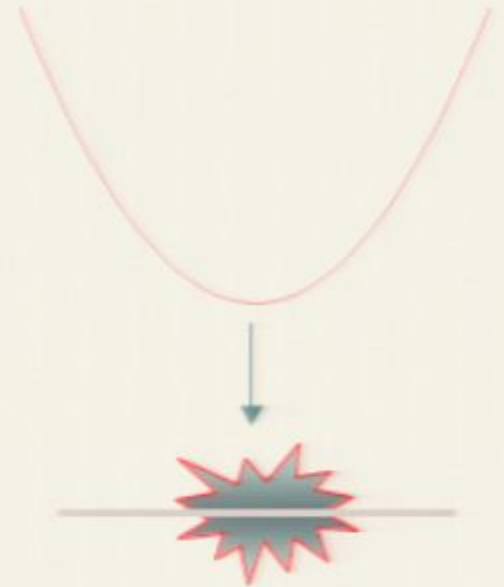
Planned experimental sequence

- BEC in magnetic trap
- Turn off trap, free expansion of condensate for 5 ms

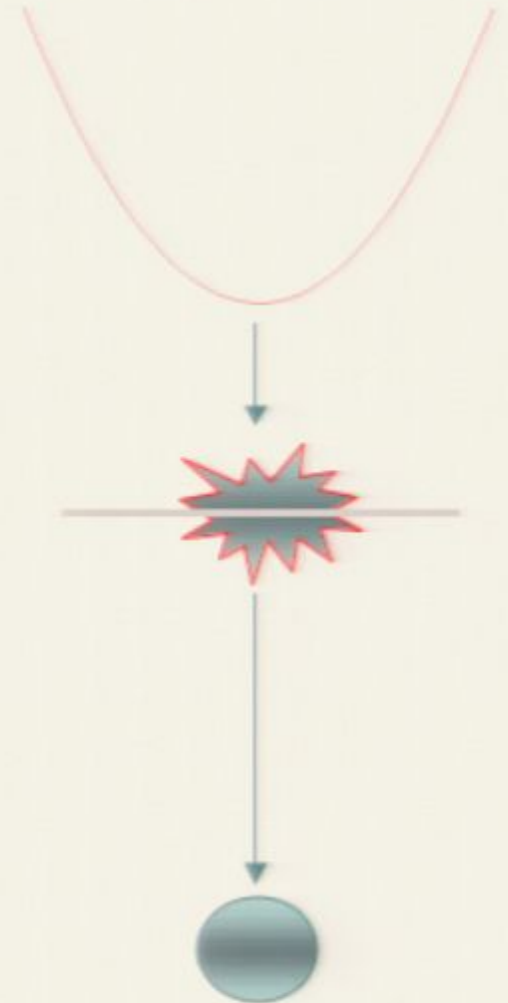


Planned experimental sequence

- BEC in magnetic trap
- Turn off trap, free expansion of condensate for 5 ms
- Interaction with barrier

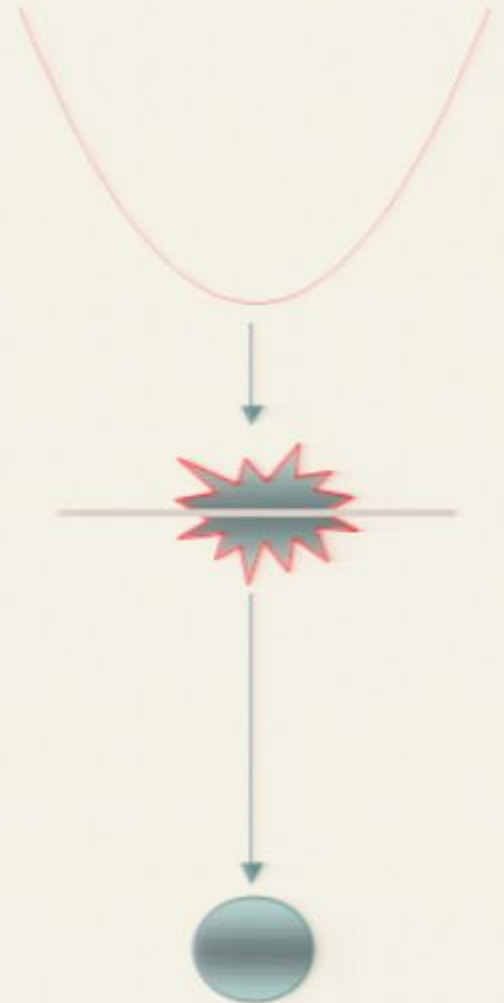
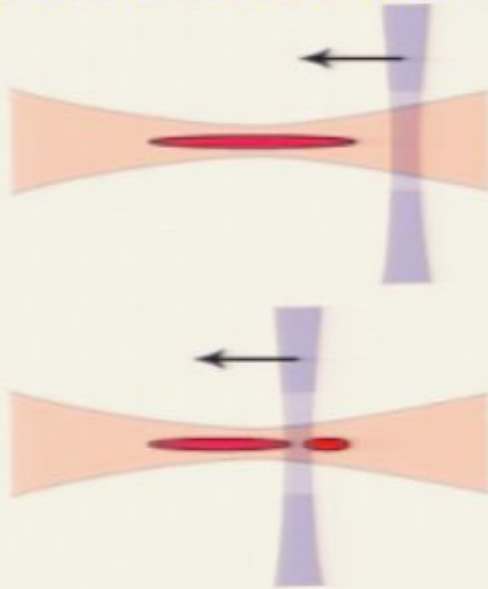


Planned experimental sequence

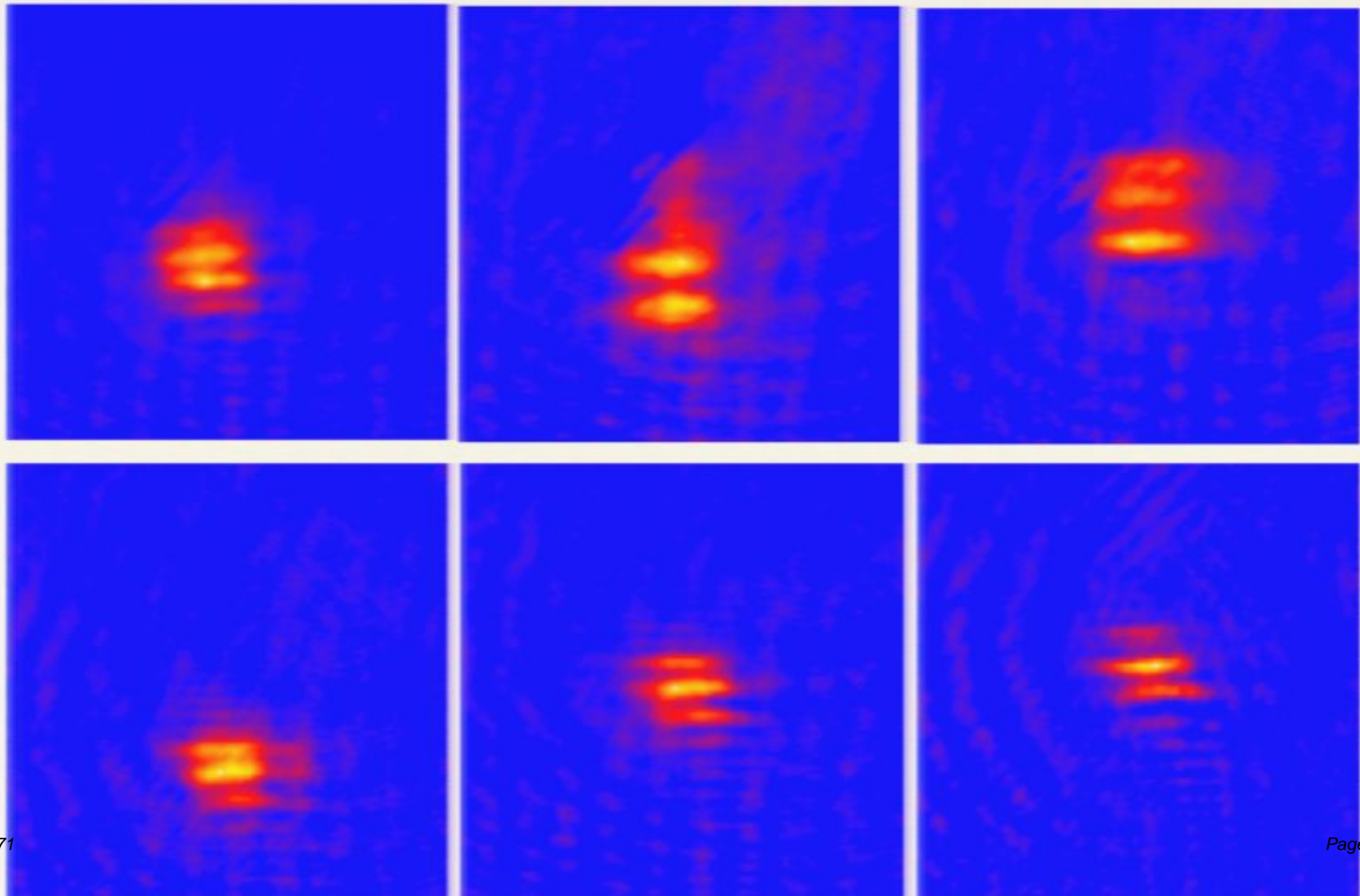


Planned experimental sequence

Second-generation plan:
slide the barrier across a one-
dimensional, horizontal cloud.



● ● ● | Not tunneling yet, but atom interference after collision with a thin optical barrier...





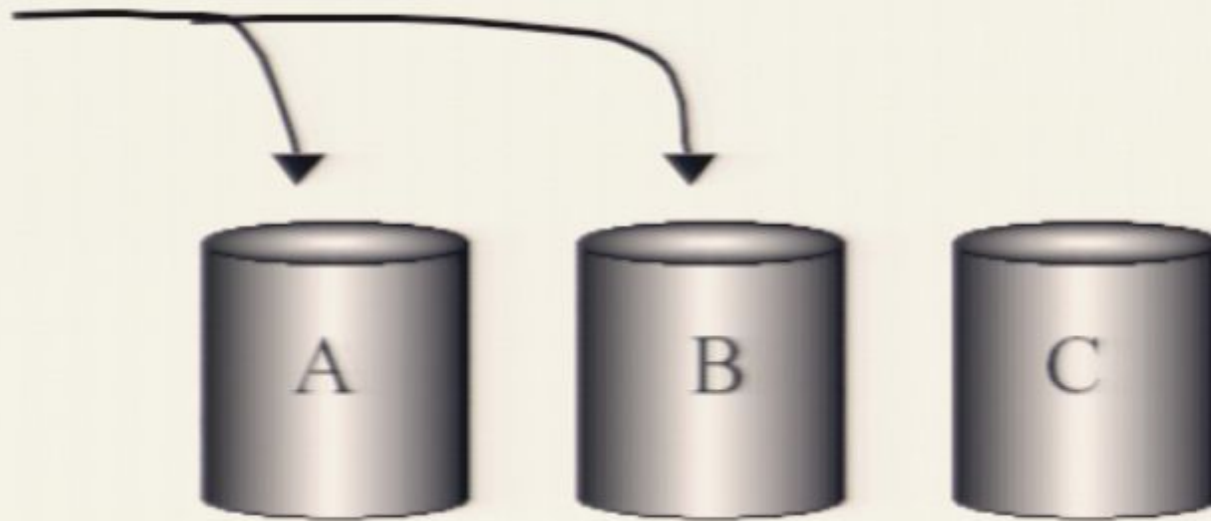
Quantum Let's Make a Deal

Predicting the past...

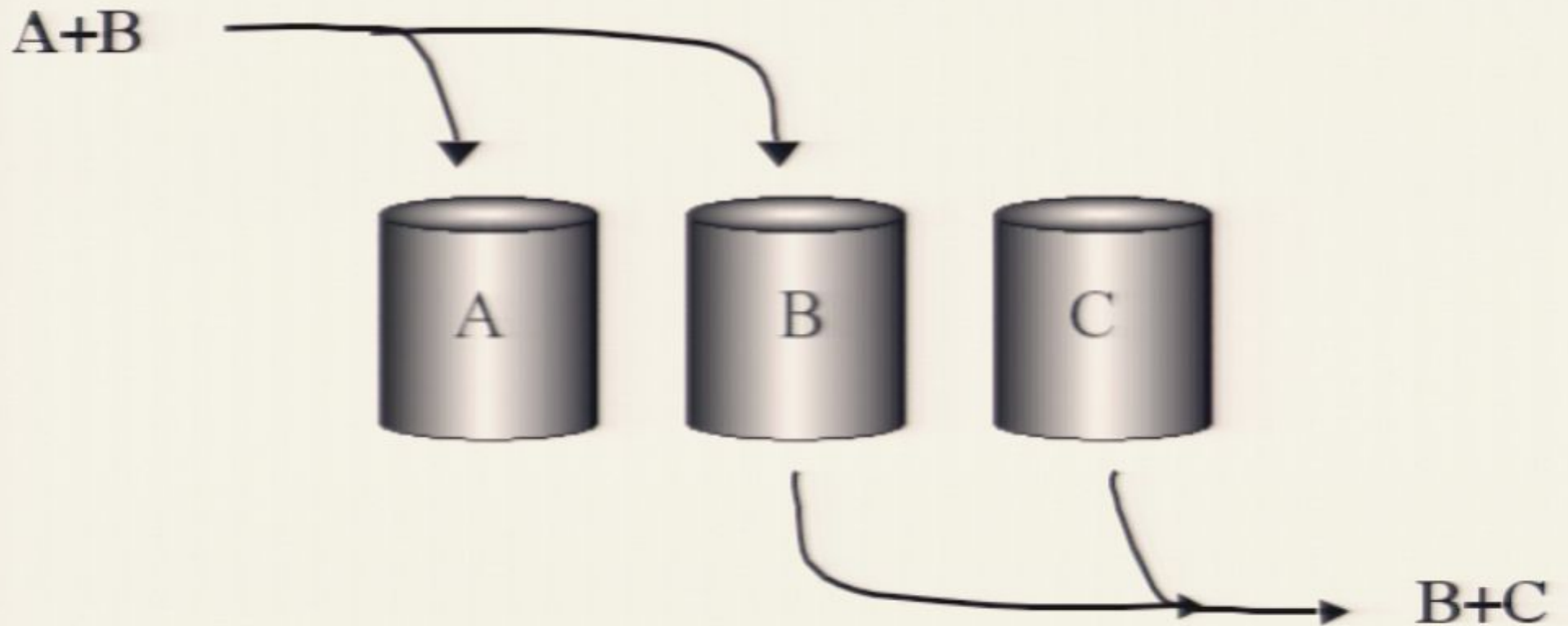


Predicting the past...

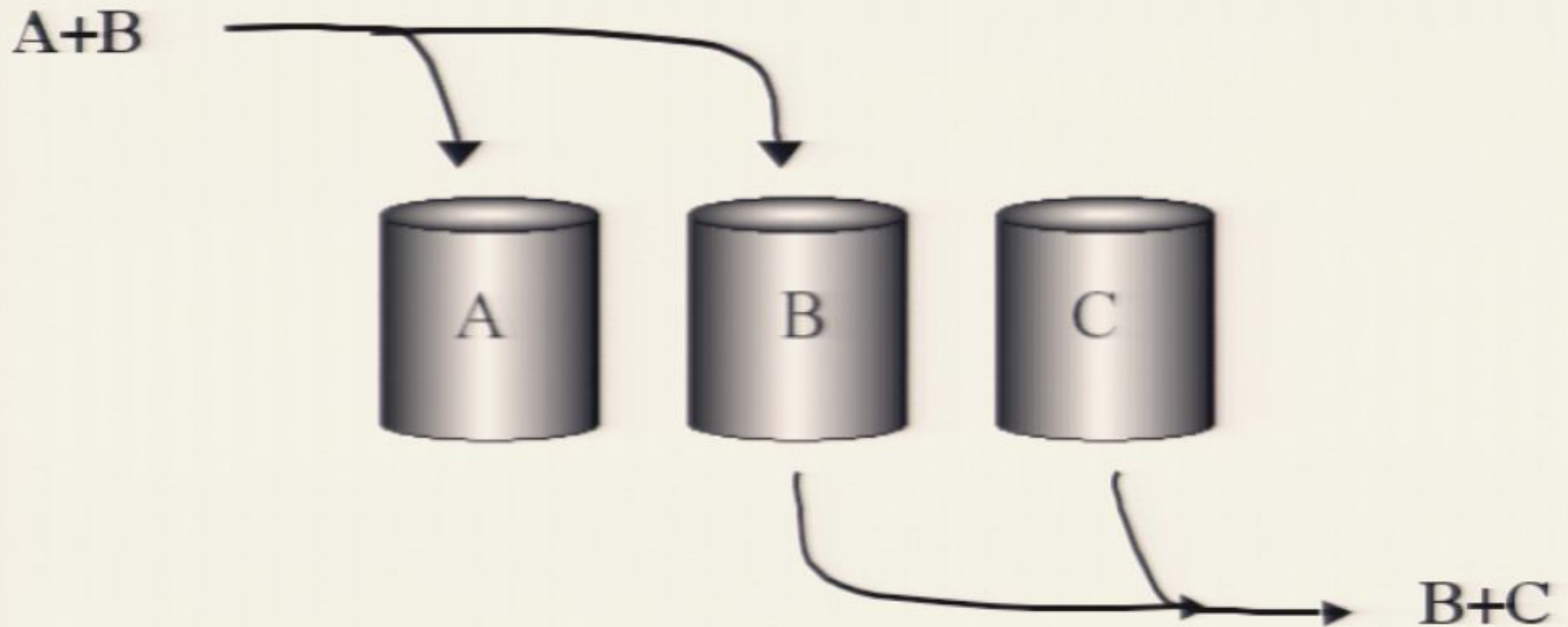
A+B



Predicting the past...

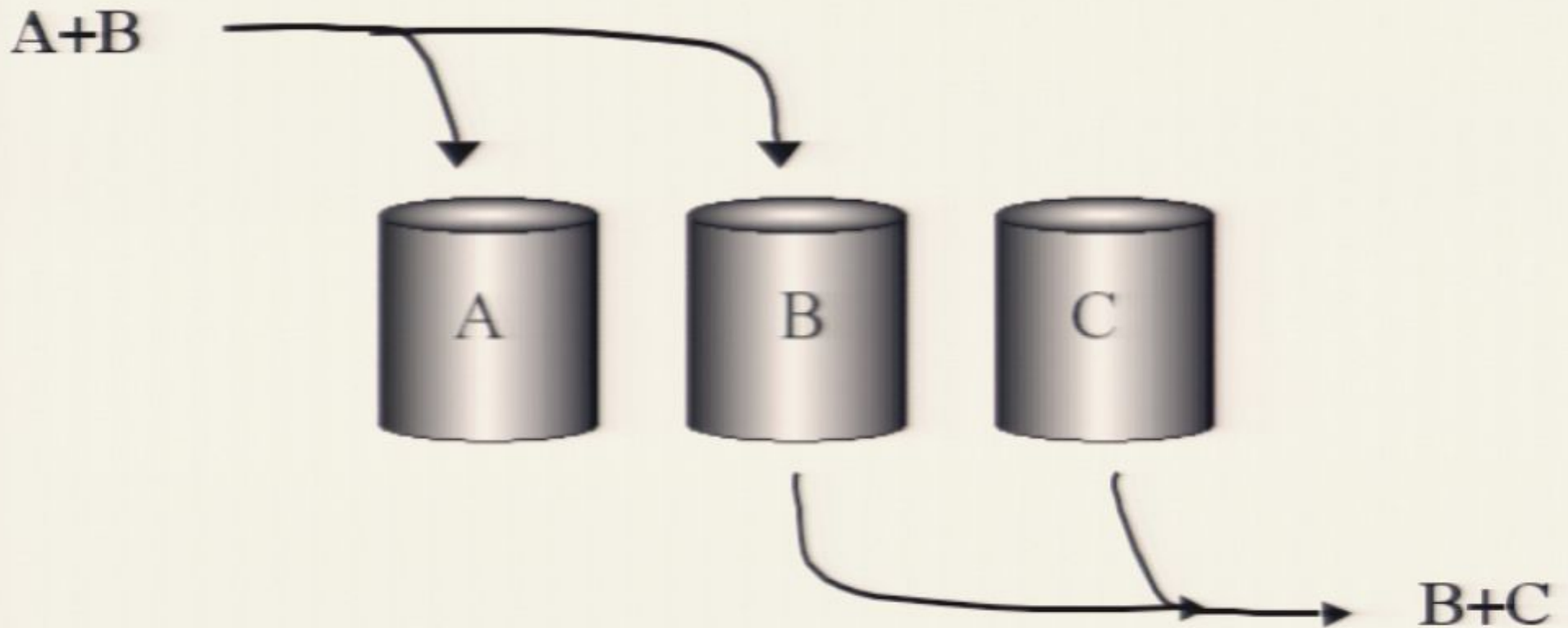


Predicting the past...



What are the odds that the particle was in a given box (e.g., box B)?

Predicting the past...



What are the odds that the particle
was in a given box (e.g., box B)?

It had to be in B, with 100% certainty.

Consider some redefinitions...

In QM, there's no difference between a box and any other state (e.g., a superposition of boxes).

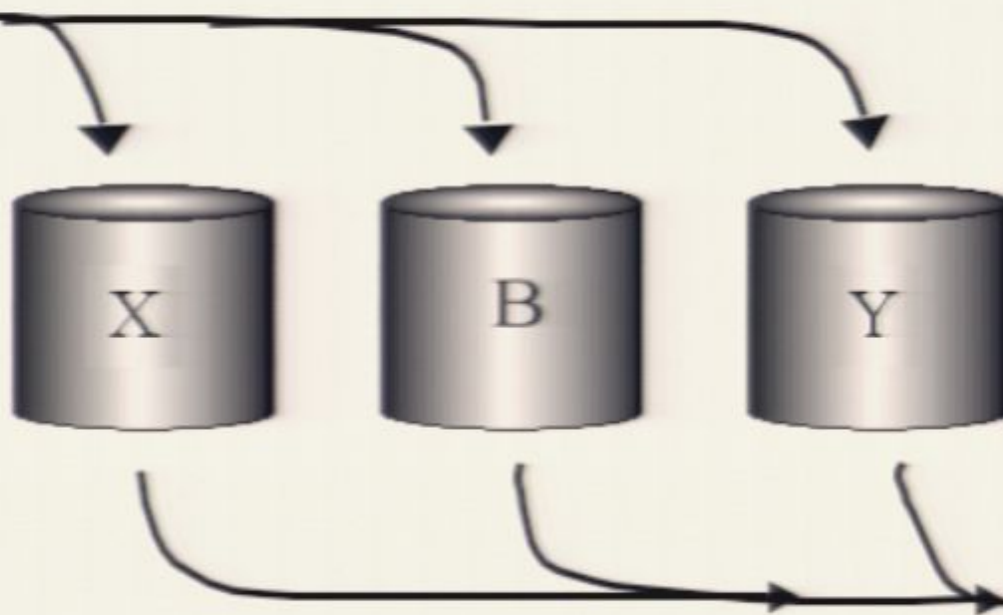
What if A is really $X + Y$ and C is really $X - Y$?

Consider some redefinitions...

In QM, there's no difference between a box and any other state (e.g., a superposition of boxes).

What if A is really $X + Y$ and C is really $X - Y$?

$$A + B \\ = X + B + Y$$



$$B + C = \\ X + B - Y$$

A redefinition of the redefinition...

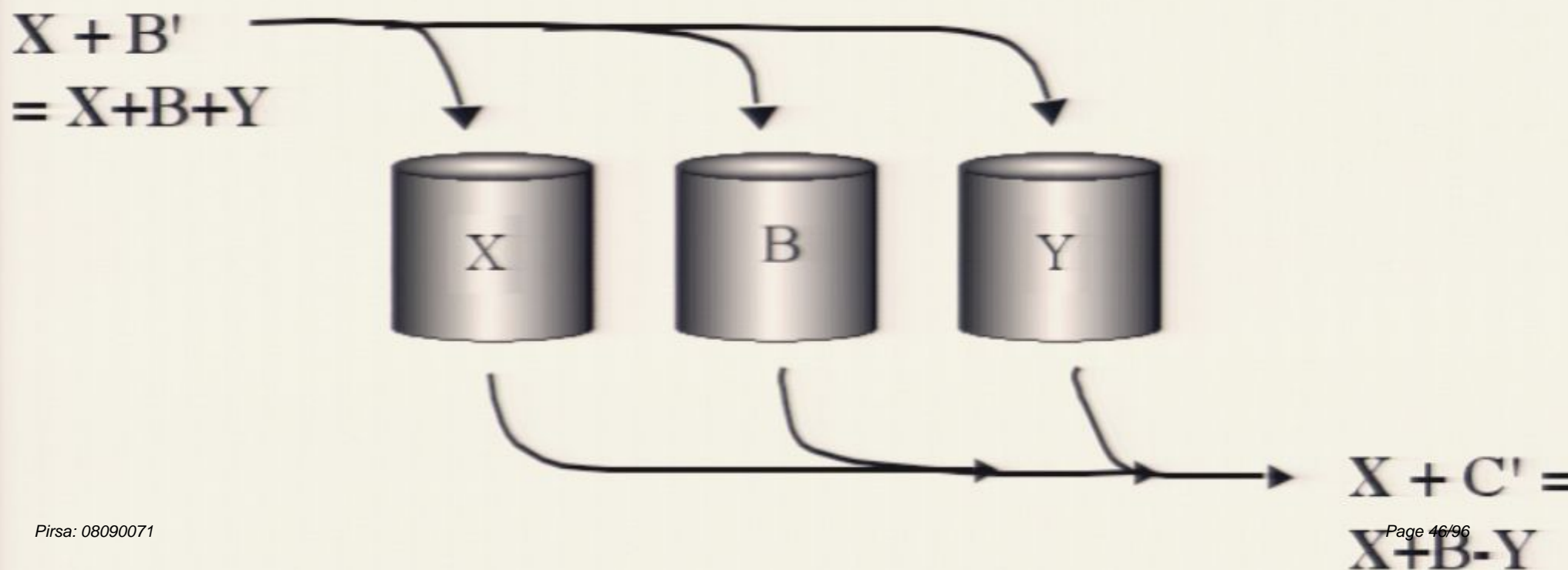
$$X + B' \\ = X+B+Y$$



$$X + C' = \\ X+B-Y$$

A redefinition of the redefinition...

So: the very same logic leads us to conclude the particle was definitely in box X.

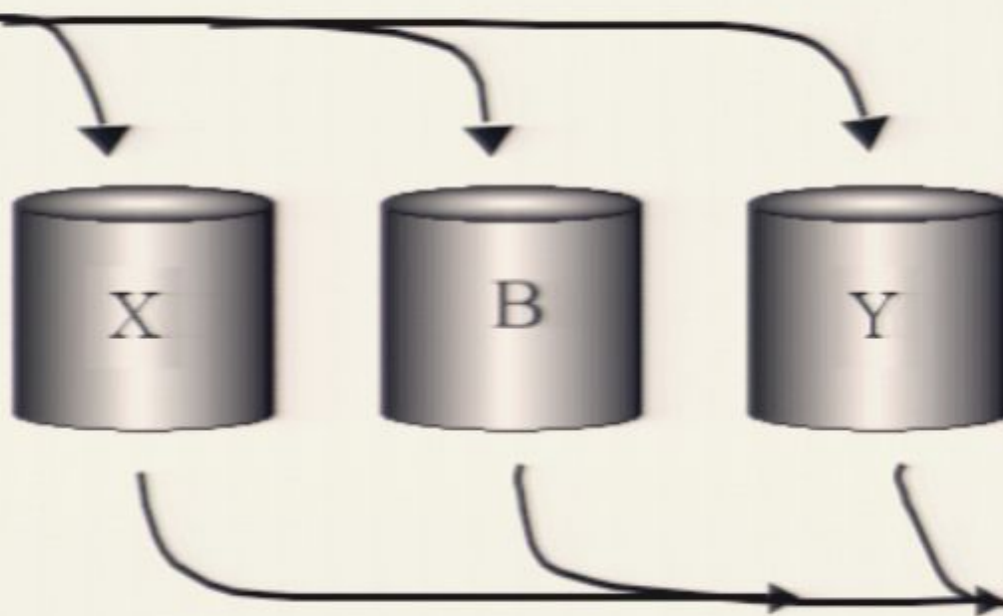


Consider some redefinitions...

In QM, there's no difference between a box and any other state (e.g., a superposition of boxes).

What if A is really $X + Y$ and C is really $X - Y$?

$$A + B \\ = X + B + Y$$



A redefinition of the redefinition...

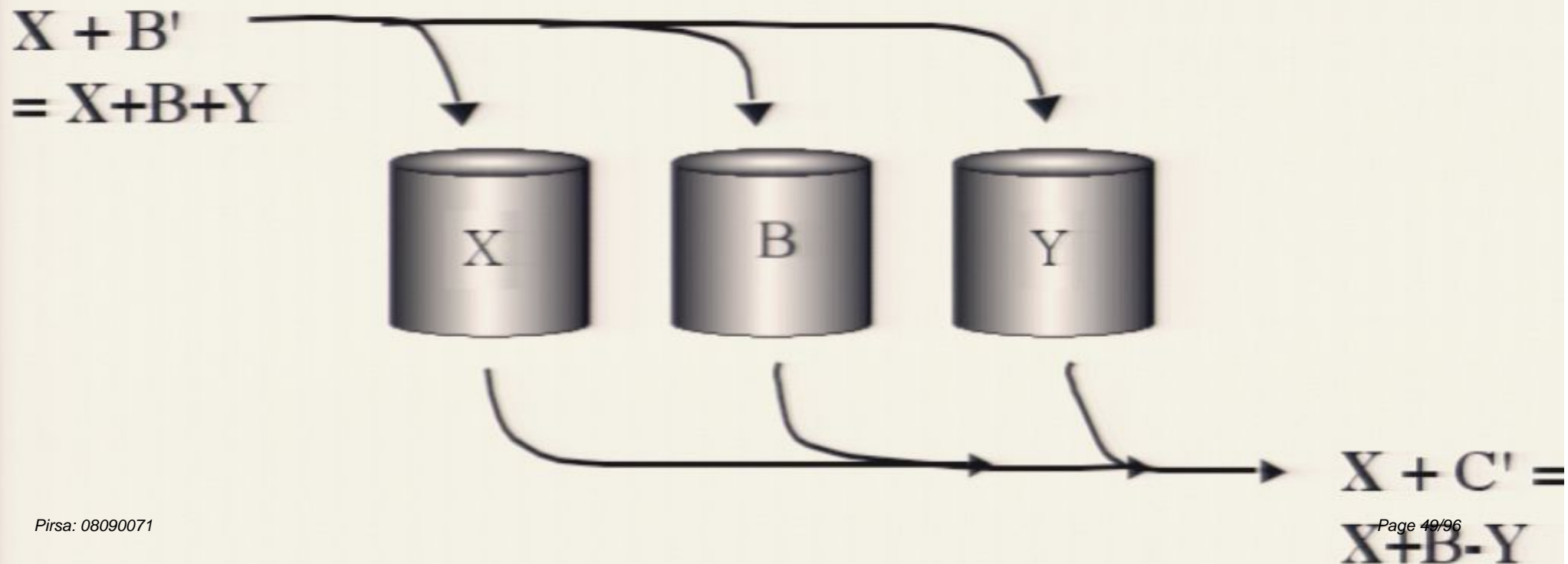
$$X + B' \\ = X+B+Y$$



$$X + C' = \\ X+B-Y$$

A redefinition of the redefinition...

So: the very same logic leads us to conclude the particle was definitely in box X.



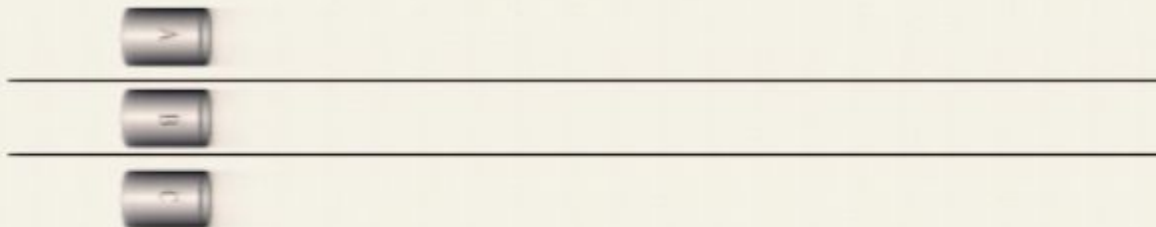
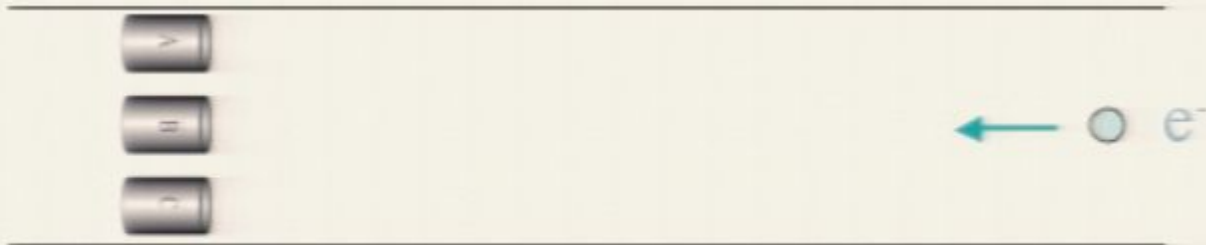
A Gedankenexperiment...



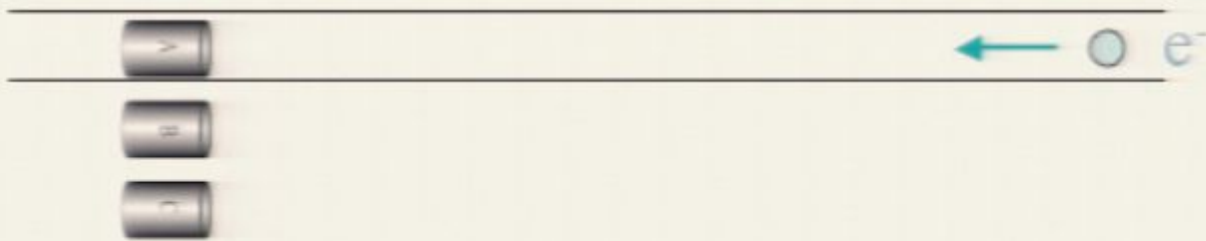
A Gedankenexperiment...



A Gedankenexperiment...



A Gedankenexperiment...



The 3-box problem: weak msmts

Prepare a particle in a symmetric superposition of three boxes: $A+B+C$.

Look to find it in this other superposition: $A+B-C$.

Ask: between preparation and detection, what was the probability that it was in A? B? C?

The 3-box problem: weak msmts

Prepare a particle in a symmetric superposition of three boxes: $A+B+C$.

Look to find it in this other superposition: $A+B-C$.

Ask: between preparation and detection, what was the probability that it was in A? B? C?

$$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle} \quad \longrightarrow \quad \begin{aligned} P_A &= \langle |A\rangle \langle A| \rangle_{wk} = (1/3) / (1/3) = 1 \\ P_B &= \langle |B\rangle \langle B| \rangle_{wk} = (1/3) / (1/3) = 1 \\ P_C &= \langle |C\rangle \langle C| \rangle_{wk} = (-1/3) / (1/3) = -1. \end{aligned}$$

The 3-box problem: weak msmts

Prepare a particle in a symmetric superposition of three boxes: $A+B+C$.

Look to find it in this other superposition: $A+B-C$.

Ask: between preparation and detection, what was the probability that it was in A? B? C?

$$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle} \quad \longrightarrow \quad \begin{aligned} P_A &= \langle |A\rangle \langle A| \rangle_{wk} = (1/3) / (1/3) = 1 \\ P_B &= \langle |B\rangle \langle B| \rangle_{wk} = (1/3) / (1/3) = 1 \\ P_C &= \langle |C\rangle \langle C| \rangle_{wk} = (-1/3) / (1/3) = -1. \end{aligned}$$

Questions:

were these postselected particles really all in A *and* all in B?
can this negative "weak probability" be observed?

An "application": N shutters

Aharonov *et al.*, PRA 67, 42107 ('03)

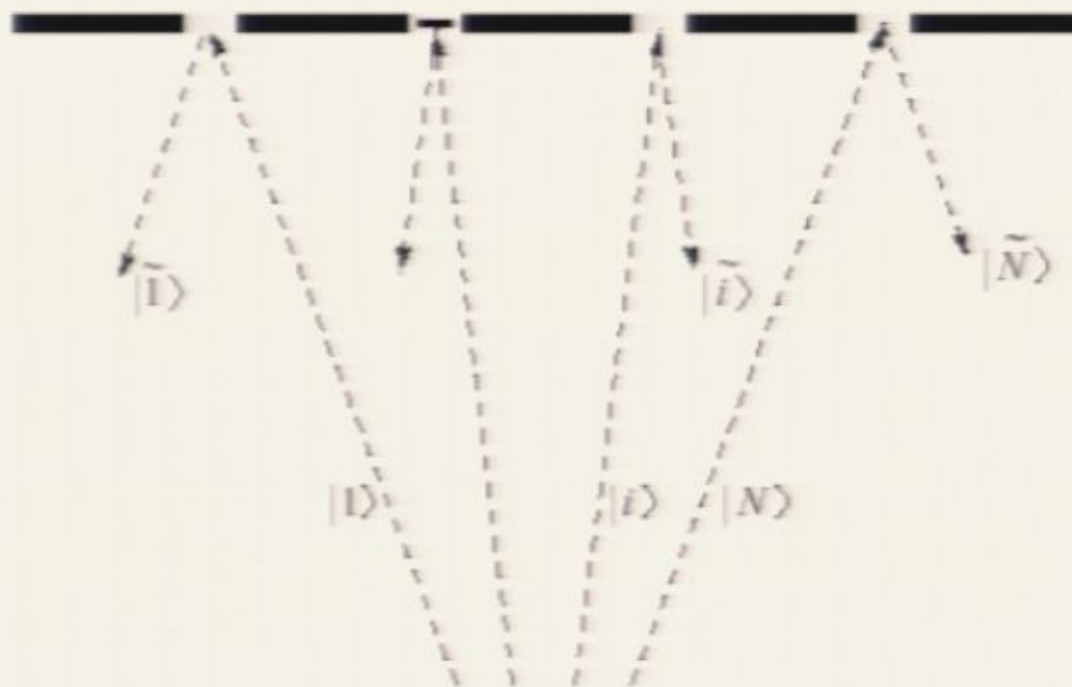
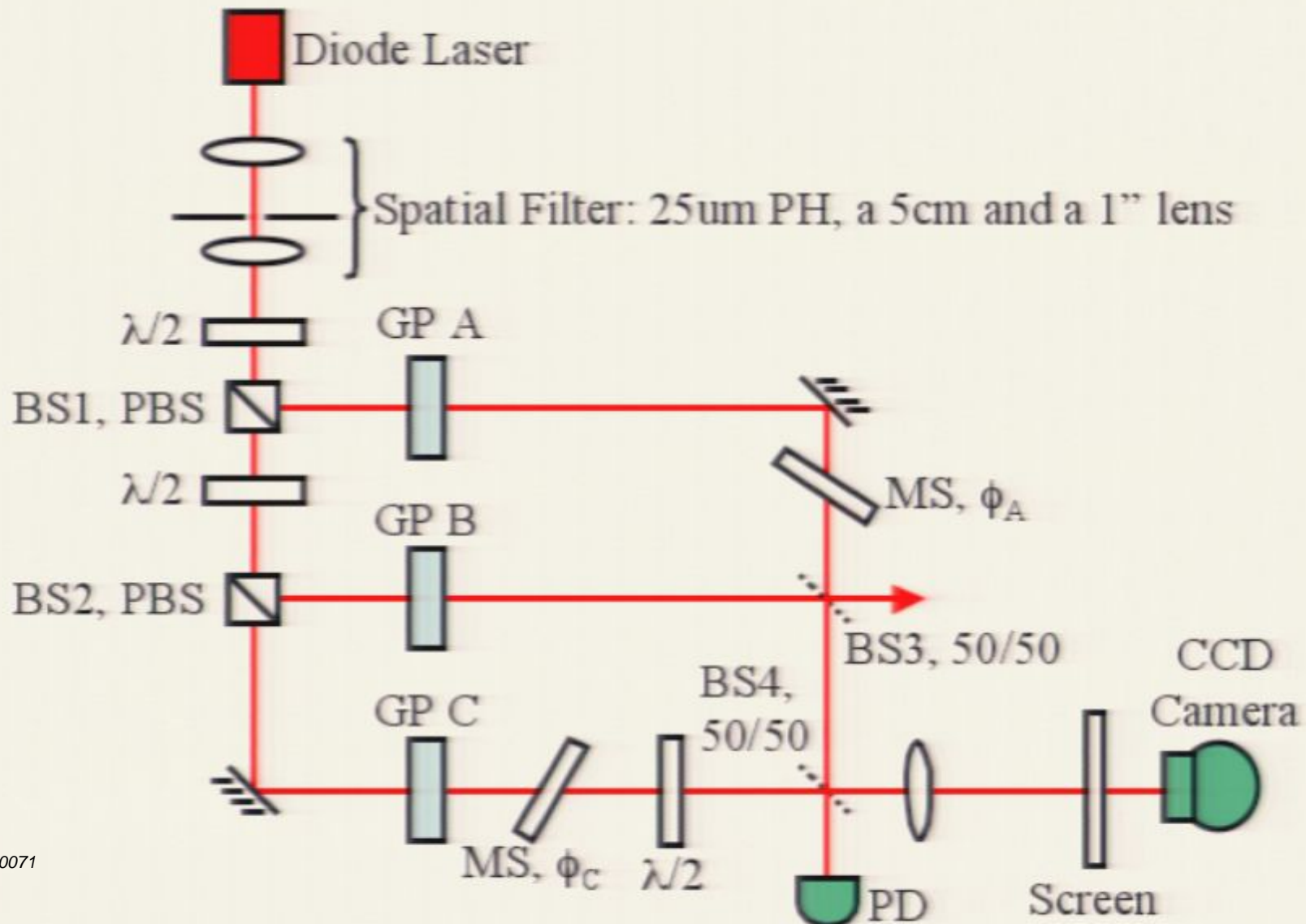


Fig. 1. A single photon arrives at N slits, but a single shutter reflects the photon as if there were shutters in every slit.

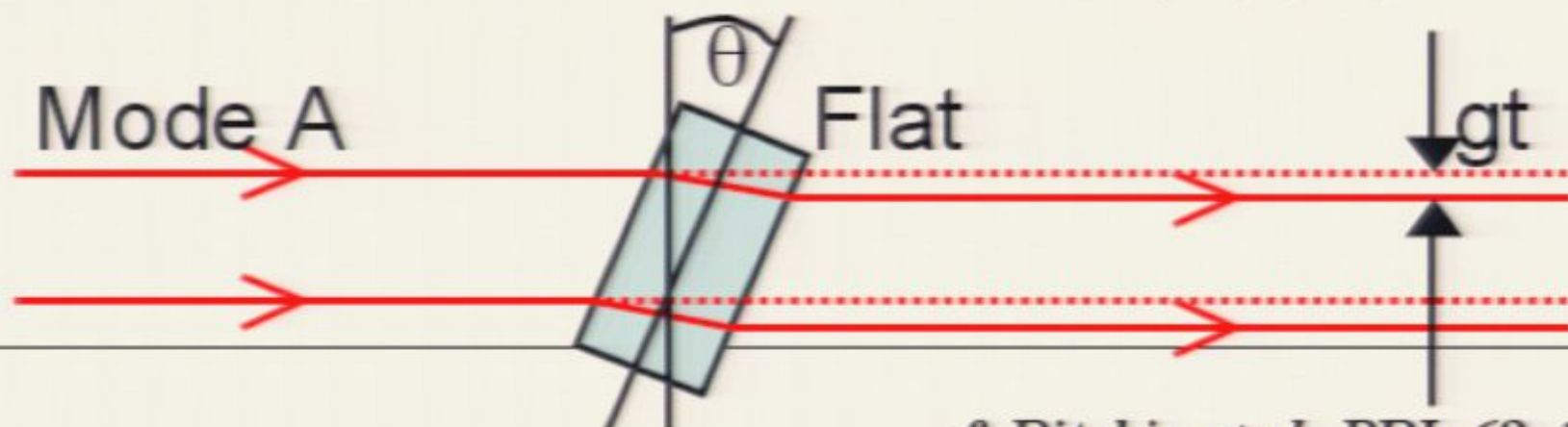
The implementation – A 3-path interferometer

(Resch *et al.*, Phys Lett A 324, 125('04))



The pointer...

- Use transverse position of each photon as pointer
- Weak measurements can be performed by tilting a glass optical flat, where effective
$$H_{\text{int}} = g|A\rangle\langle A|p_x$$



cf. Ritchie *et al.*, PRL 68, 1107 ('91)

The position of each photon is uncertain to within the beam waist...

a small shift does not provide any photon with distinguishing info.

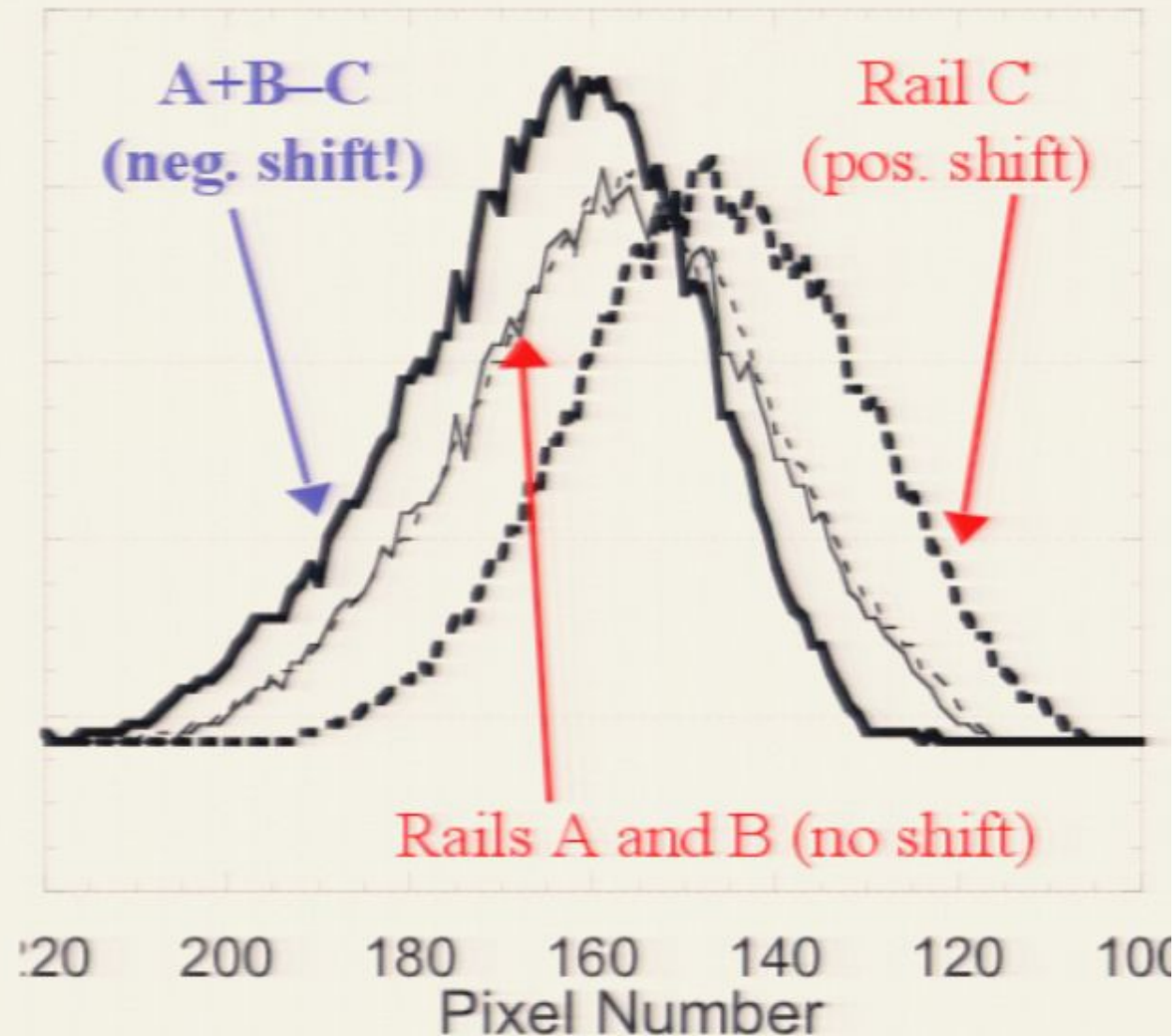
But after many photons arrive, the shift of the beam may be measured

A negative weak value for $\text{Prob}(C)$

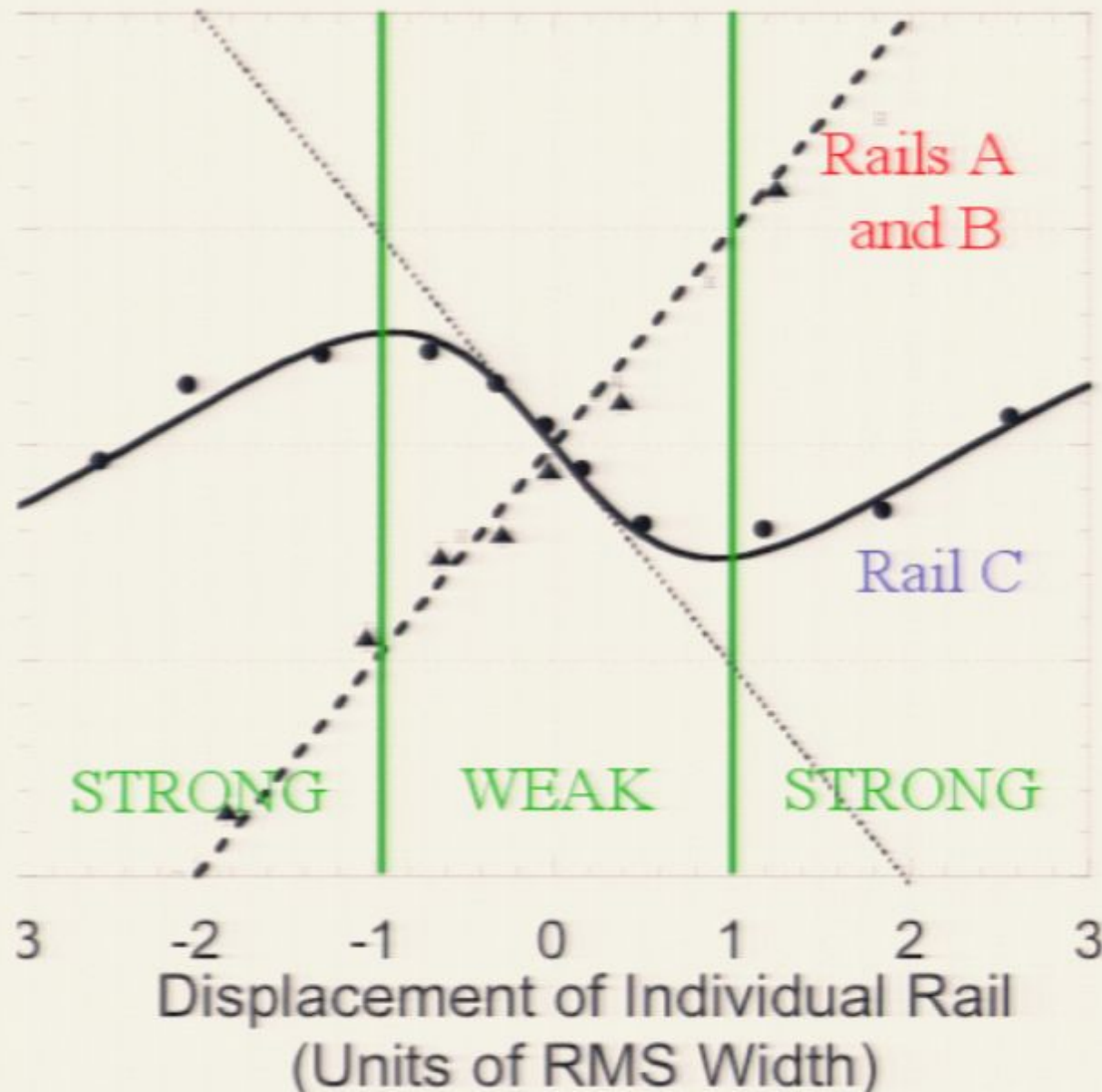
Perform weak msmt
on rail C.

Post-select either A,
B, C, or $A+B-C$.

Compare "pointer
states" (vertical
profiles).



Data for P_A , P_B , and P_C ...



Is the particle "really" in 2 places at once?

- If P_A and P_B are both 1, what is P_{AB} ?
- For AAV's approach, one would need an interaction of the form

$$H_{\text{int}} = g|A\rangle\langle A||B\rangle\langle B|p_x$$

OR: STUDY CORRELATIONS OF P_A & P_B ...

- if P_A and P_B always move together, then the uncertainty in their difference never changes.
- if P_A and P_B both move, but *never together*, then $\Delta(P_A - P_B)$ must increase.

Practical Measurement of P_{AB}

Resch & Steinberg, PRL 92, 130402 ('04)

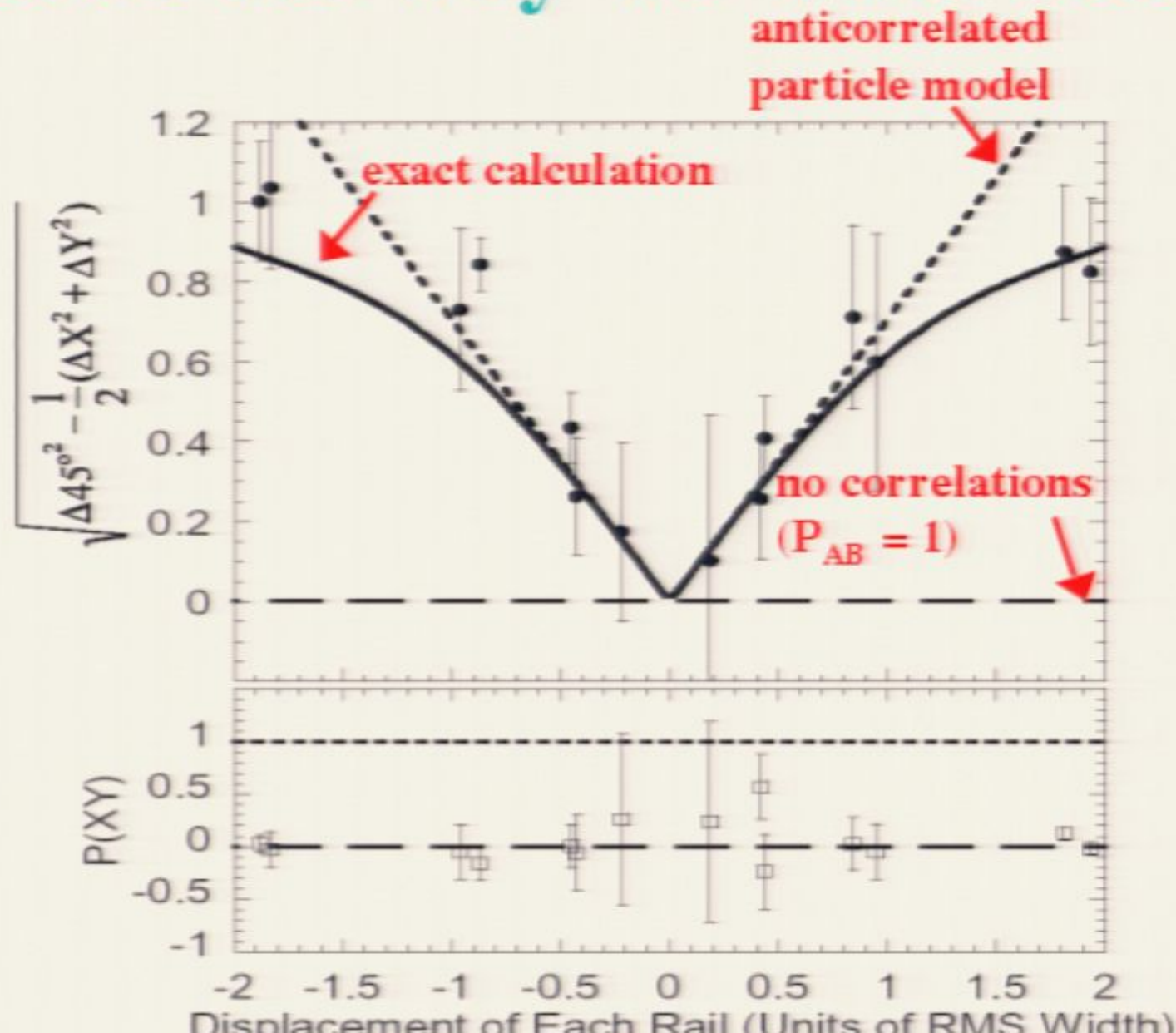
Use two pointers (the two transverse directions) and couple to both A and B; then use their correlations to draw conclusions about P_{AB} .

$$H_{\text{int}} = g_A |A\rangle\langle A| p_x + g_B |B\rangle\langle B| p_y$$

We have shown that the real part of P_{ABW} can be extracted from such correlation measurements:

$$\text{Re}(P_{ABW}) = \frac{2\langle xy \rangle}{g_A g_B t^2} - \text{Re}(P_{AW}^* P_{BW})$$

Non-repeatable data which happen to look the way we want them to...



Practical Measurement of P_{AB}

Resch & Steinberg, PRL 92, 130402 ('04)

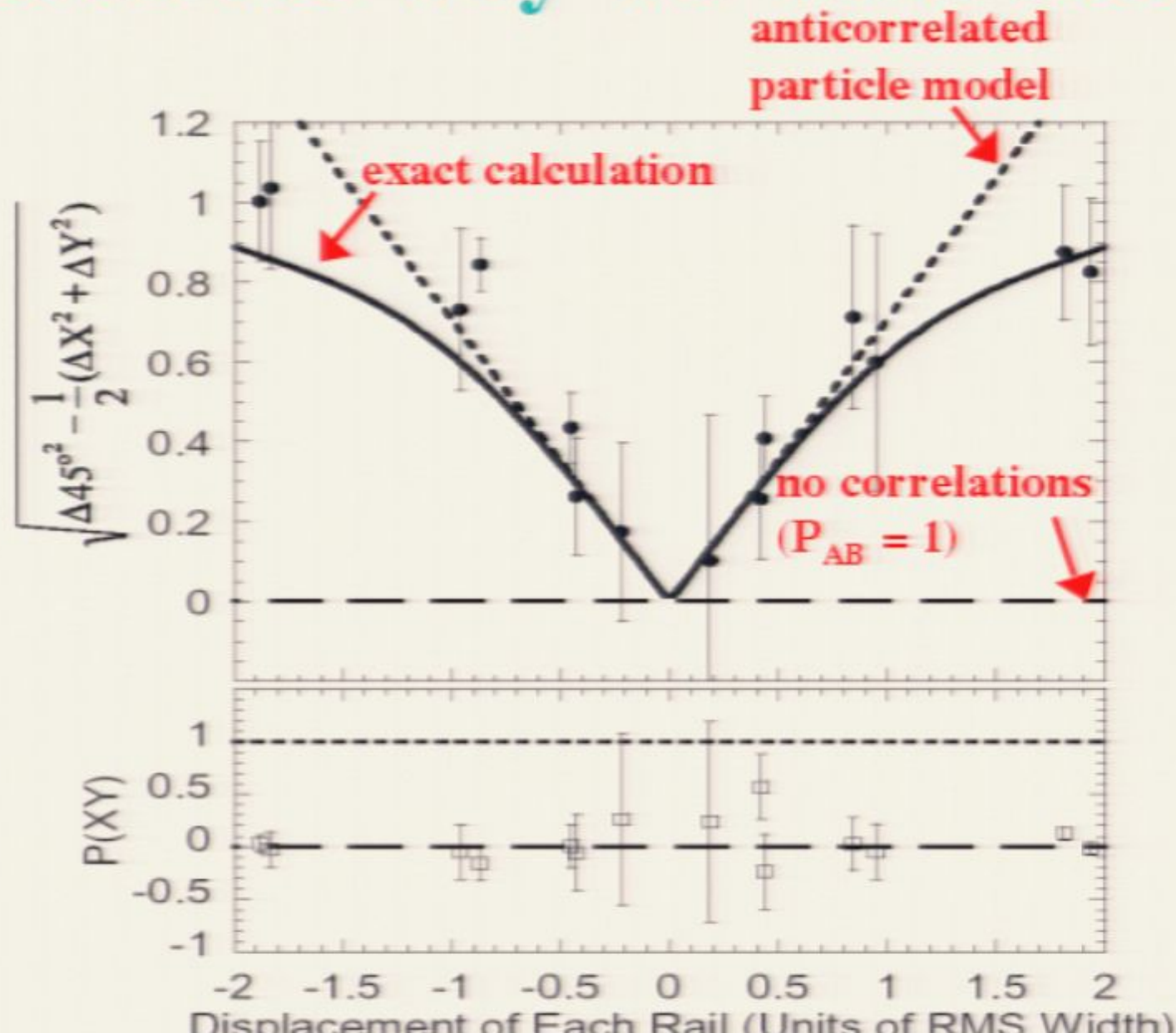
Use two pointers (the two transverse directions) and couple to both A and B; then use their correlations to draw conclusions about P_{AB} .

$$H_{\text{int}} = g_A |A\rangle\langle A| p_x + g_B |B\rangle\langle B| p_y$$

We have shown that the real part of P_{ABW} can be extracted from such correlation measurements:

$$\text{Re}(P_{ABW}) = \frac{2\langle xy \rangle}{g_A g_B t^2} - \text{Re}(P_{AW}^* P_{BW})$$

Non-repeatable data which happen to look the way we want them to...



The joint probabilities

Probabilities	A	not A	A or not A
B	0	1	1
not B	1	-1	0
B or not B	1	0	

And a final note...

The result should have been obvious...

$$\begin{aligned} |A\rangle\langle A| |B\rangle\langle B| \\ = |A\rangle\langle A|B\rangle\langle B| \end{aligned}$$

is identically zero because
A and B are orthogonal.

Even in a weak-measurement sense, a particle
can never be found in two *orthogonal* states at
the same time.

(So much for “serious” nonlocality of a tunneling
particle as well...)

The joint probabilities

Probabilities	A	not A	A or not A
B	0	1	1
not B	1	-1	0
B or not B	1	0	



“Quantum Seeing in the Dark”

" Quantum seeing in the dark "

" Quantum seeing in the dark "

(AKA: "Interaction-free" measurement,
aka "Vaidman's bomb")

A. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)

P.G. Kwiat, H. Weinfurter, and A. Zeilinger, Sci. Am. (Nov., 1996)

" Quantum seeing in the dark "

(AKA: "Interaction-free" measurement,
aka "Vaidman's bomb")

A. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)

P.G. Kwiat, H. Weinfurter, and A. Zeilinger, Sci. Am. (Nov., 1996)

Problem:

Consider a collection of bombs so sensitive that a collision with any single particle (photon, electron, etc.) is guaranteed to trigger it.

Suppose that certain of the bombs are defective, but differ in their behaviour in *no way* other than that they will not blow up when triggered.

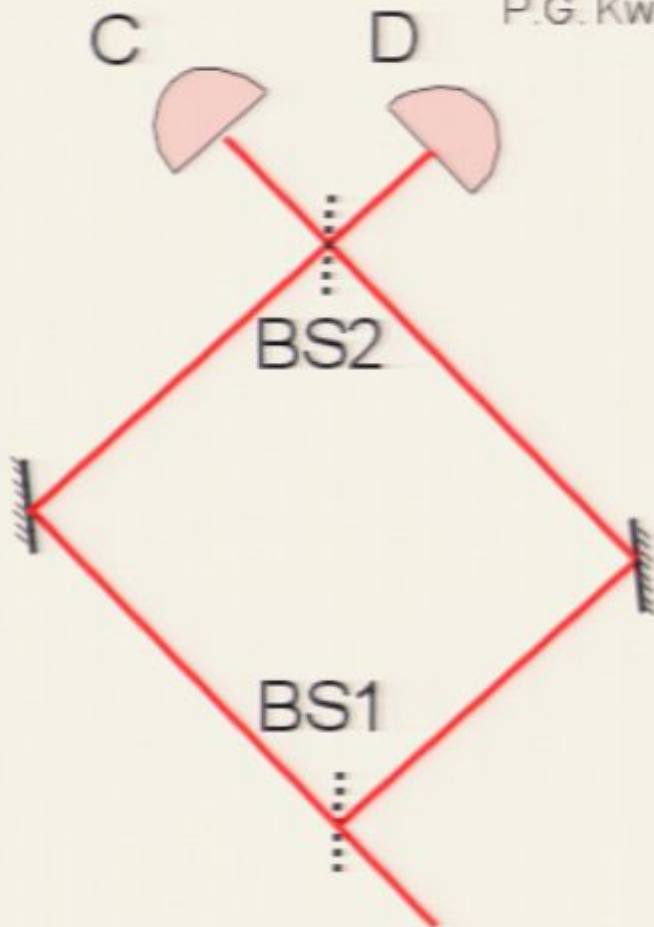
Is there any way to identify the working bombs (or some of them) without blowing them up?

" Quantum seeing in the dark "

(AKA: "Interaction-free" measurement,
aka "Vaidman's bomb")

A. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)

P.G. Kwiat, H. Weinfurter, and A. Zeilinger, Sci. Am. (Nov., 1996)



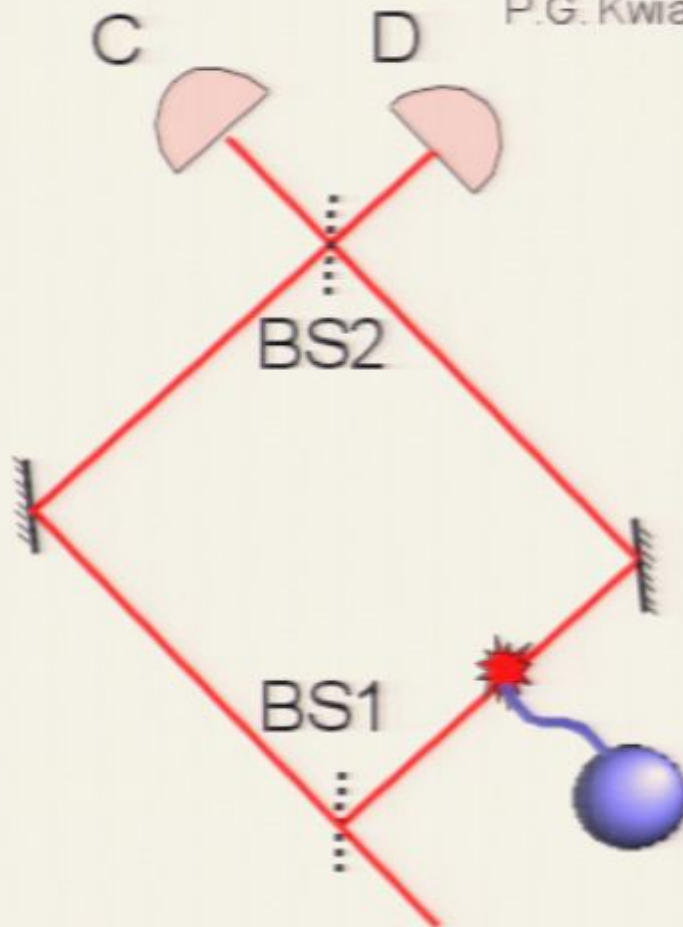
Bomb absent:
Only detector C fires

" Quantum seeing in the dark "

(AKA: "Interaction-free" measurement,
aka "Vaidman's bomb")

A. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)

P.G. Kwiat, H. Weinfurter, and A. Zeilinger, Sci. Am. (Nov., 1996)



Bomb absent:
Only detector C fires

Bomb present:

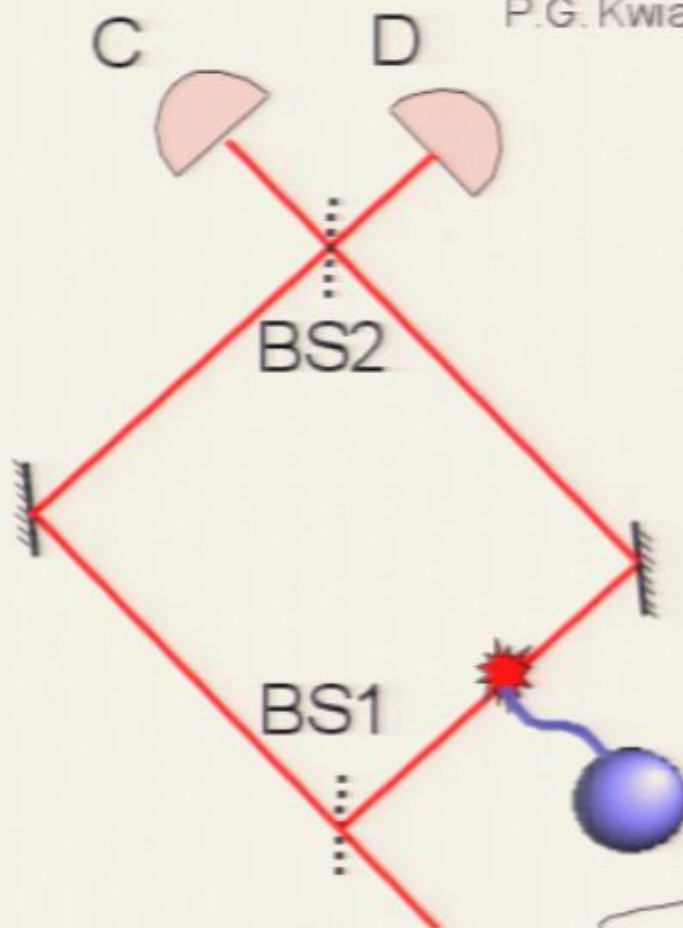
"boom!"	1/2
C	1/4
D	1/4

" Quantum seeing in the dark "

(AKA: "Interaction-free" measurement,
aka "Vaidman's bomb")

A. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)

P.G. Kwiat, H. Weinfurter, and A. Zeilinger, Sci. Am. (Nov., 1996)



Bomb absent:
Only detector C fires

Bomb present:

"boom!" $\frac{1}{2}$

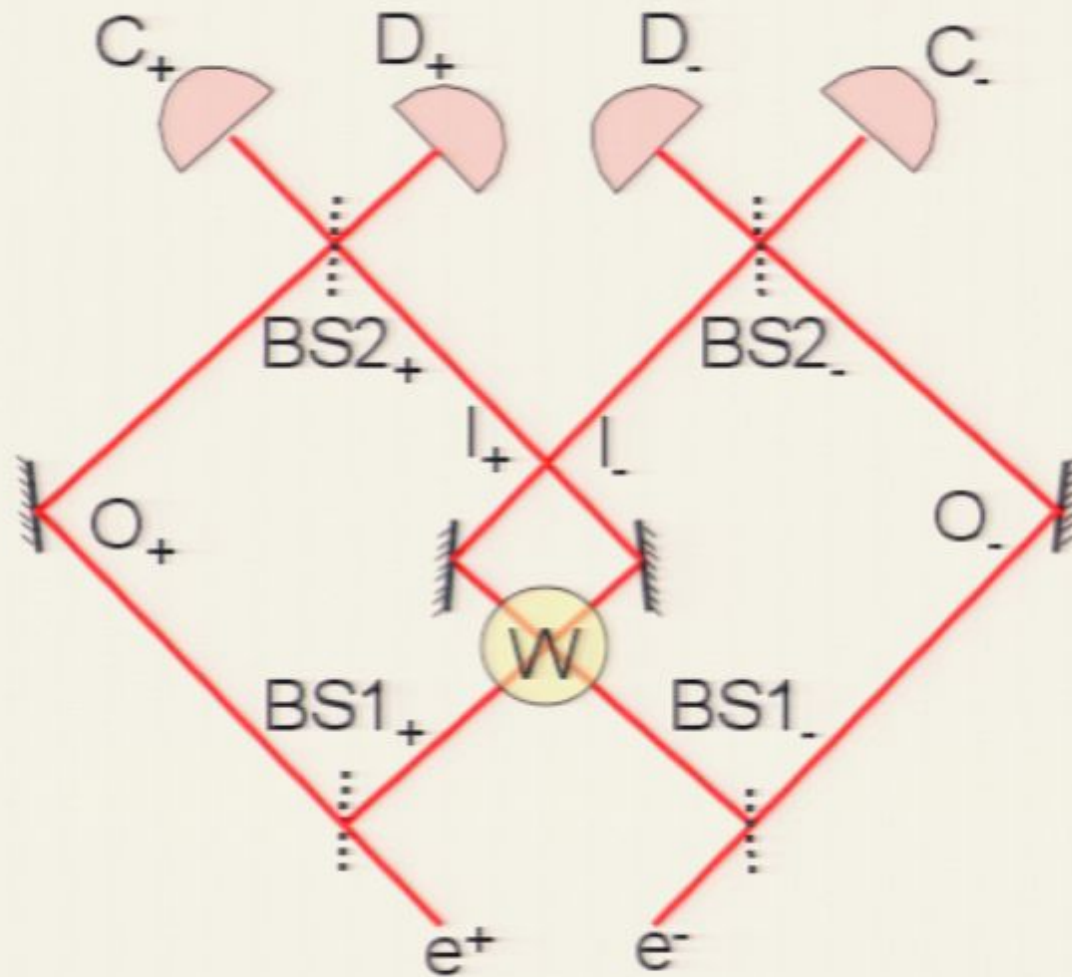
C $\frac{1}{4}$

D $\frac{1}{4}$

*The bomb must be there... yet
my photon never interacted with it.*

Hardy's Paradox

(for Elitzur-Vaidman "interaction-free measurements")



$D_+ \rightarrow e^-$ was in

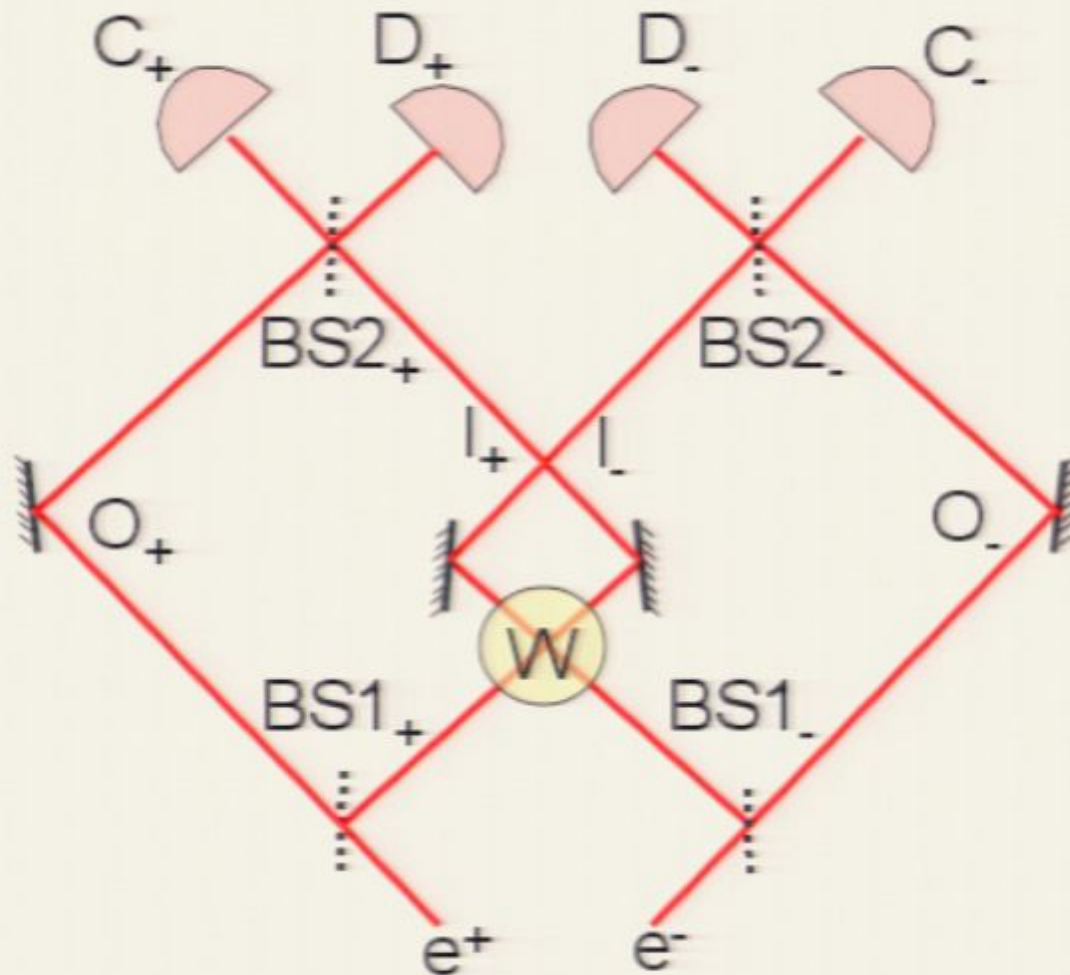
$D_- \rightarrow e^+$ was in

$D_+ D_- \rightarrow ?$

But ... if they
were
both in, they
should
have
annihilated!

Hardy's Paradox

(for Elitzur-Vaidman "interaction-free measurements")



Outcome	Prob
D_+ and C_-	1/16
D_- and C_+	1/16
C_+ and C_-	9/16
D_+ and D_-	1/16
Explosion	4/16

The two-photon switch...

OR: Is SPDC really the time-reverse of SHG?

The two-photon switch...

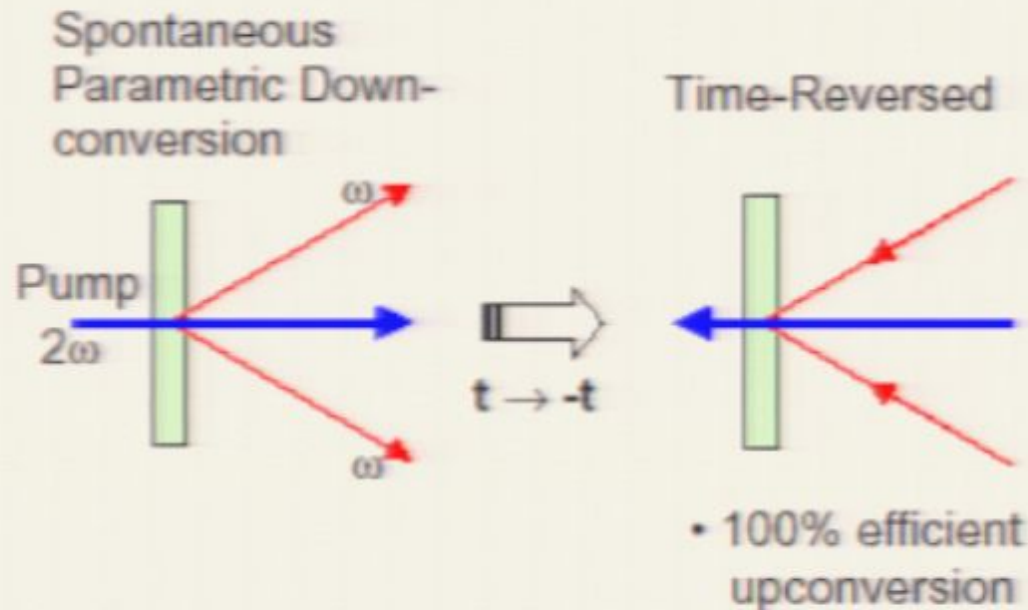
OR: Is SPDC really the time-reverse of SHG?

(And if so, then why doesn't it exist in classical e&m?)

The two-photon switch...

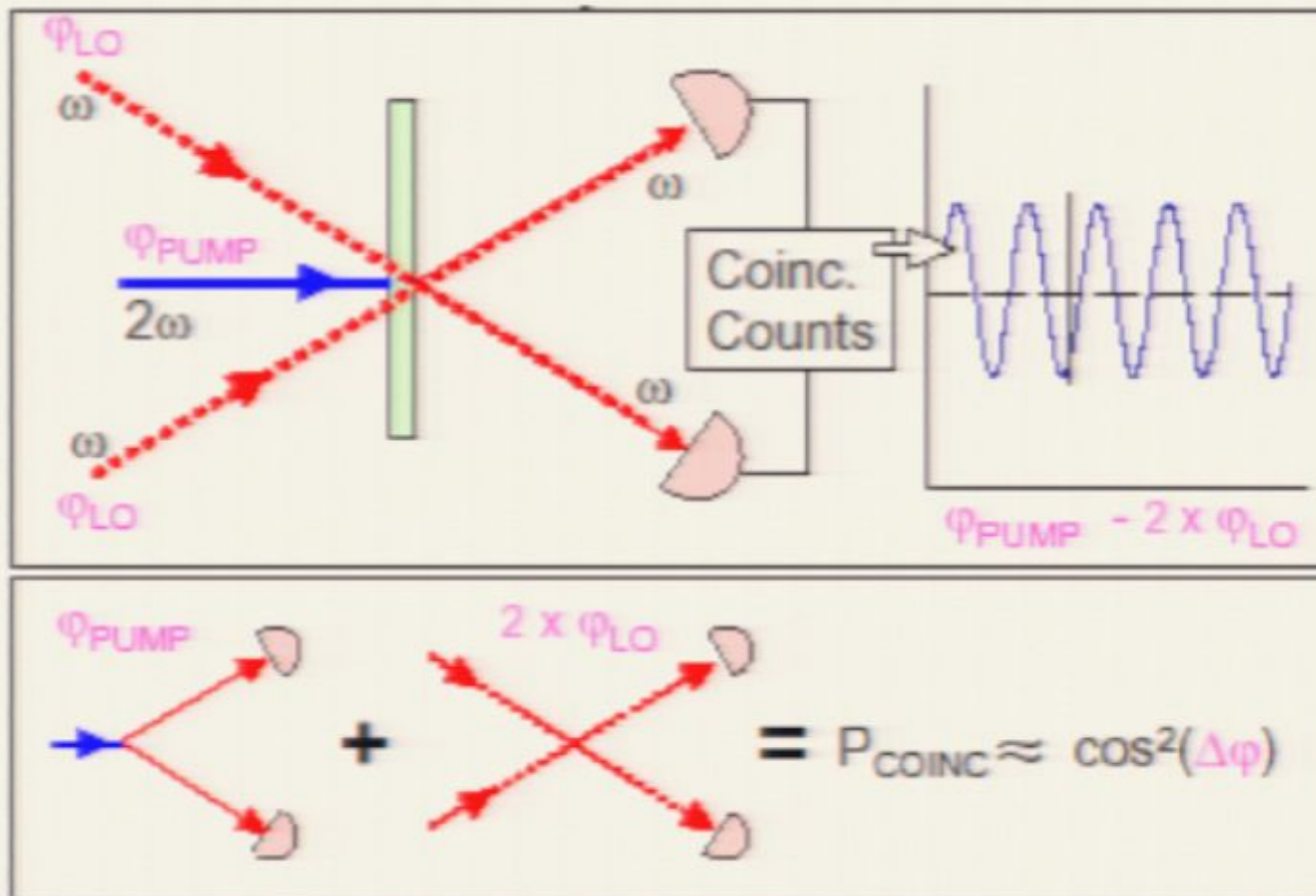
OR: Is SPDC really the time-reverse of SHG?

(And if so, then why doesn't it exist in classical e&m?)

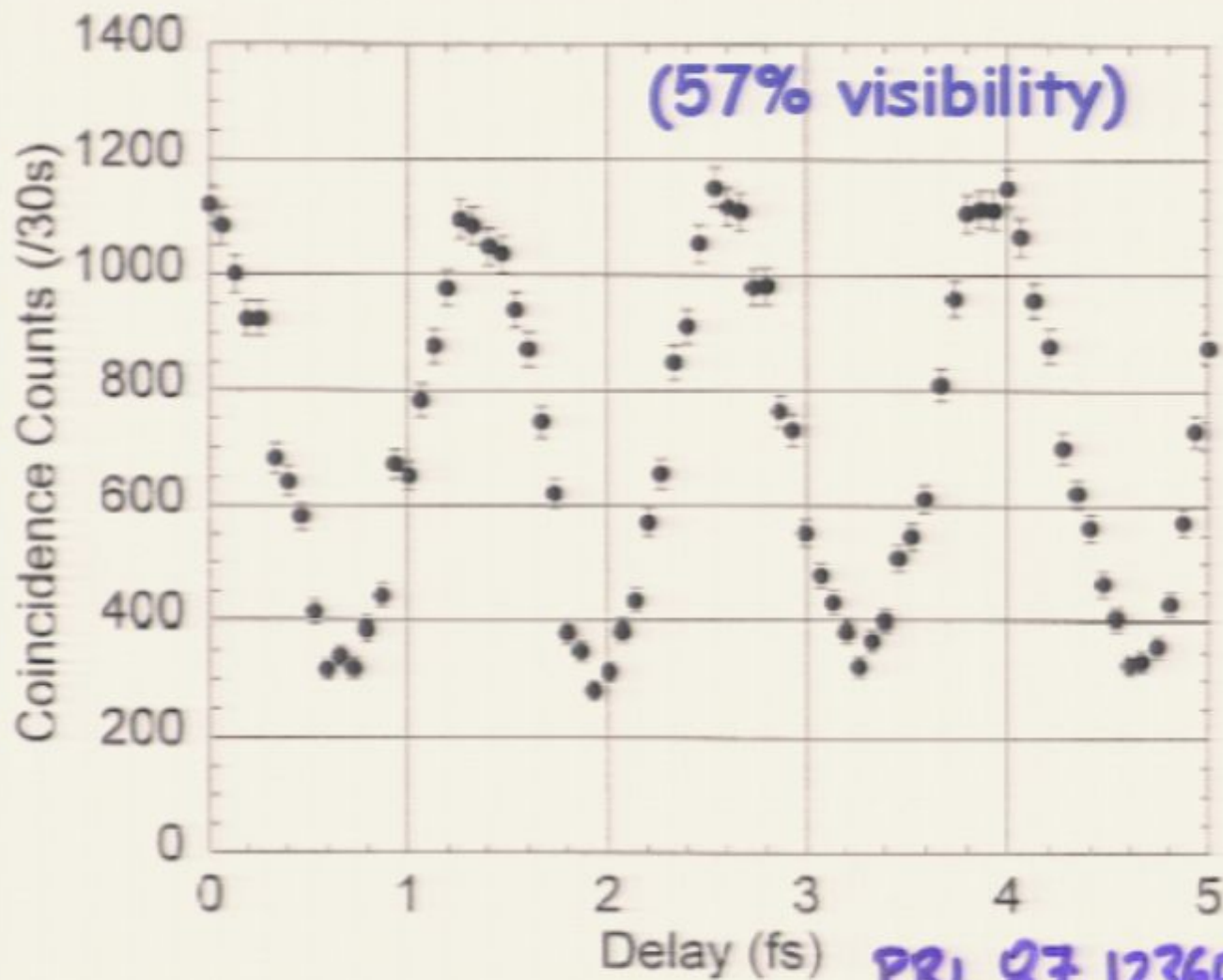


The probability of 2 photons upconverting in a typical nonlinear crystal is roughly 10^{-10} (as is the probability of 1 photon spontaneously down-converting).

Quantum Interference

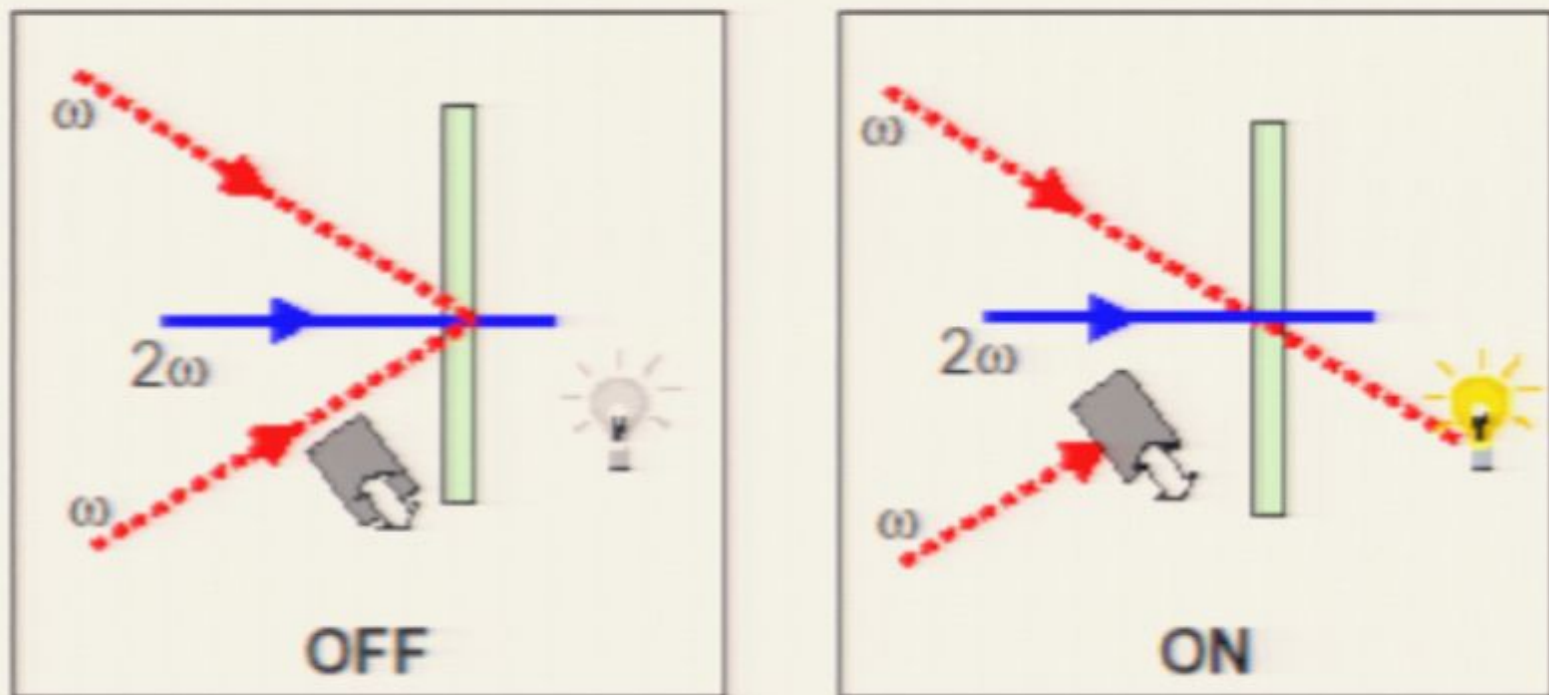


Suppression/Enhancement of Spontaneous Down-Conversion



Photon-photon transmission switch

- Phase chosen so that coincidences are eliminated



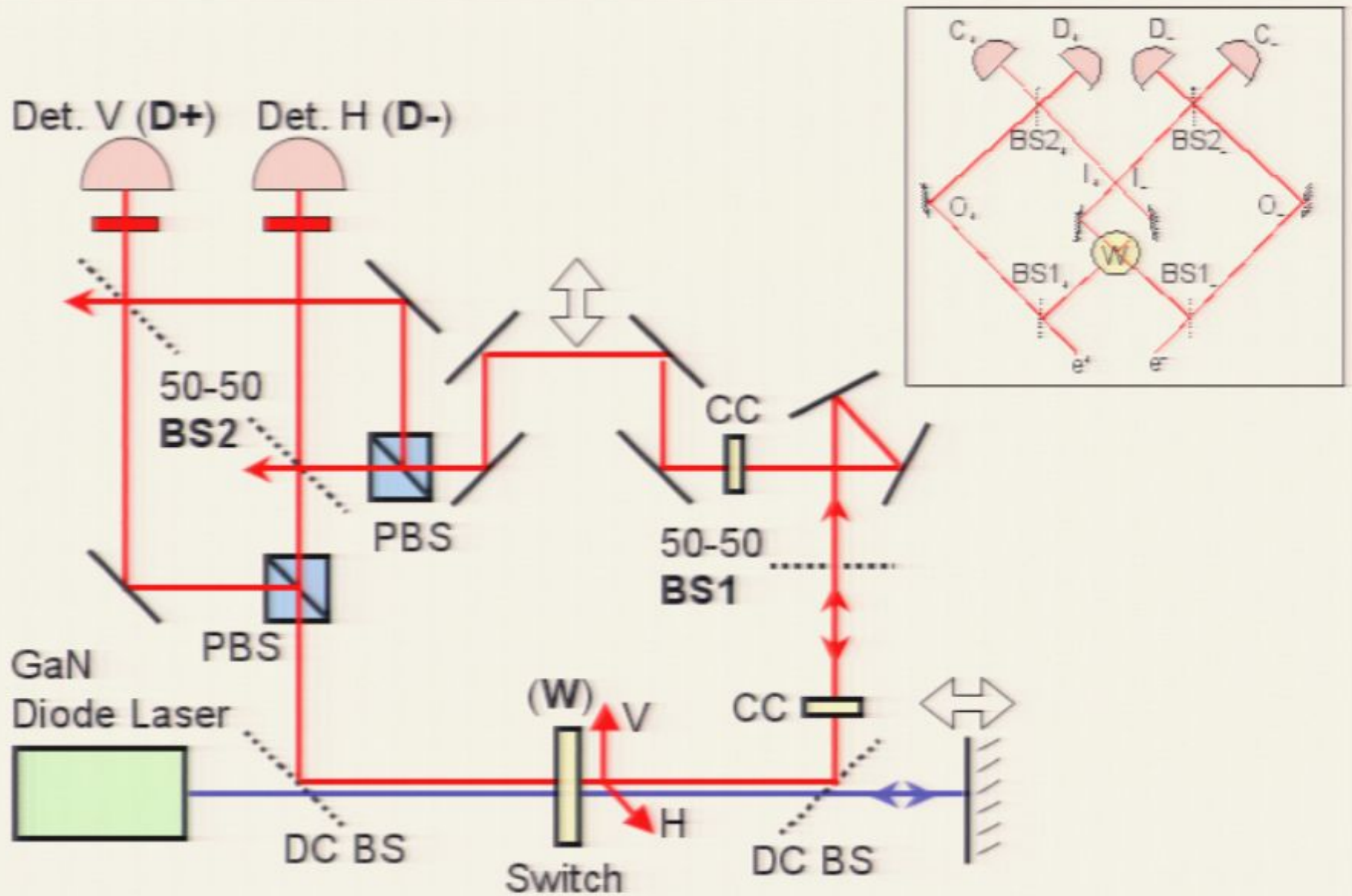
On average, less than one photon per pulse.

One photon present in a given pulse is sufficient to switch off transmission.

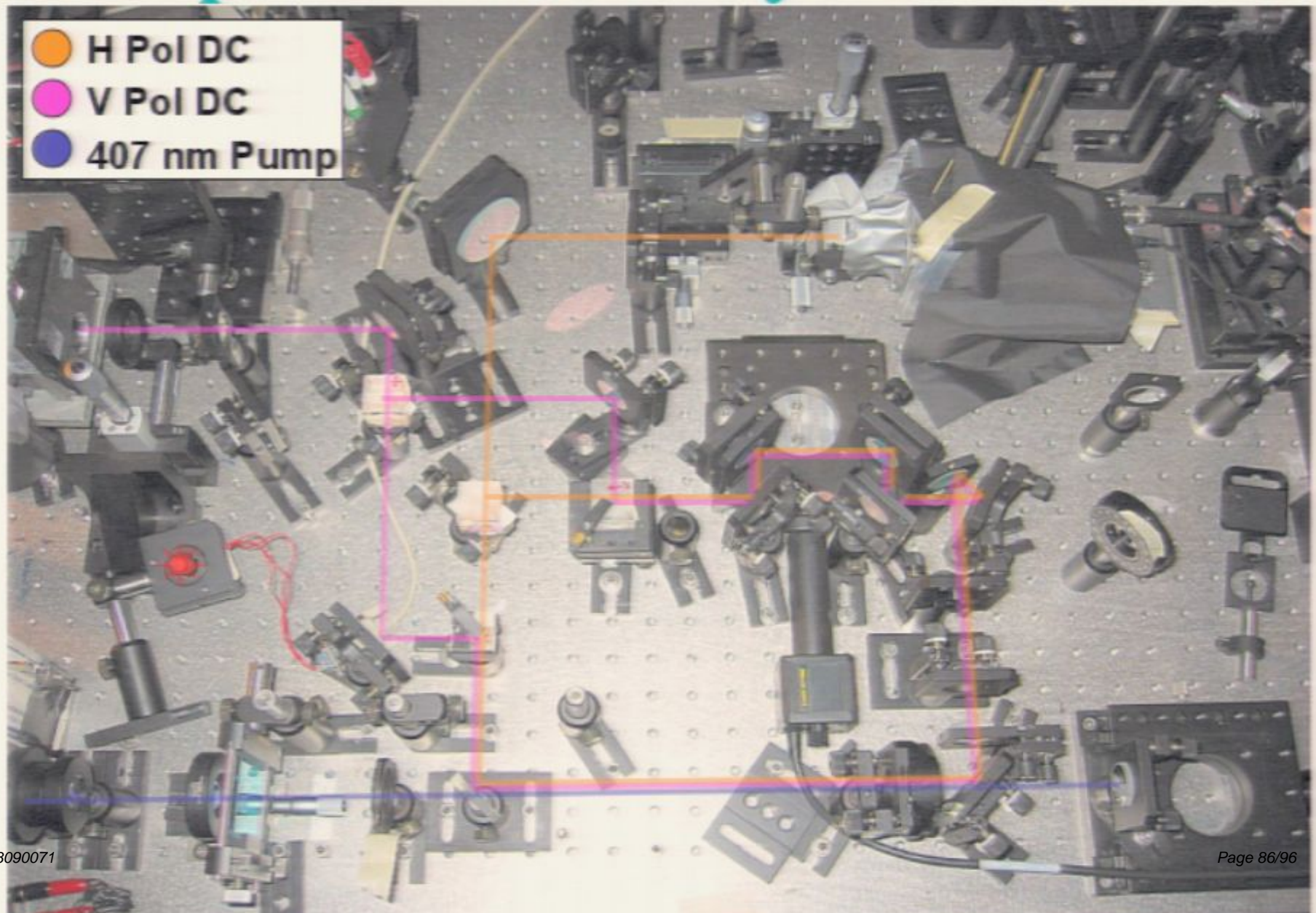
The photons upconvert with near-unit eff. (Peak power approx. mW/cm

The blue pump serves as a catalyst, enhancing the interaction by 10^{10} .

Experimental Setup



Using a “photon switch” to implement Hardy’s Paradox



But what can we say about where the particles were or weren't, once D^+ & D^- fire?

Probabilities	e- in	e- out	
e+ in			1
e+ out			0
	1	0	

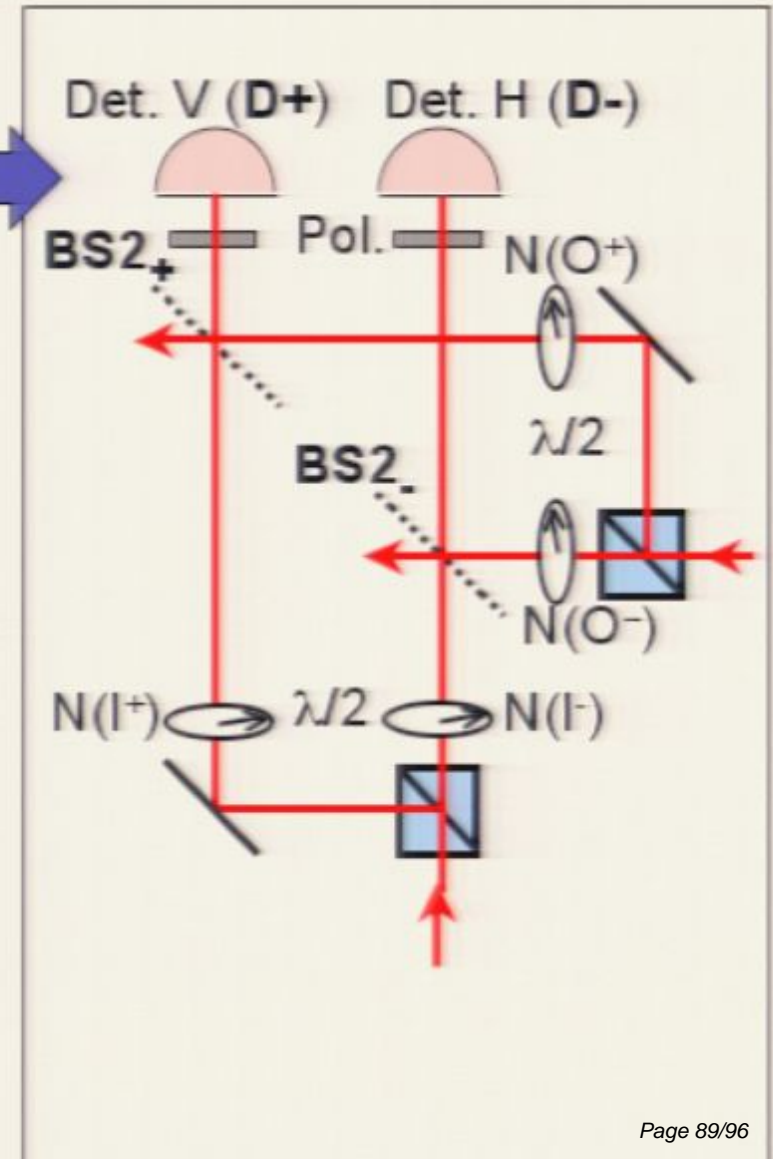
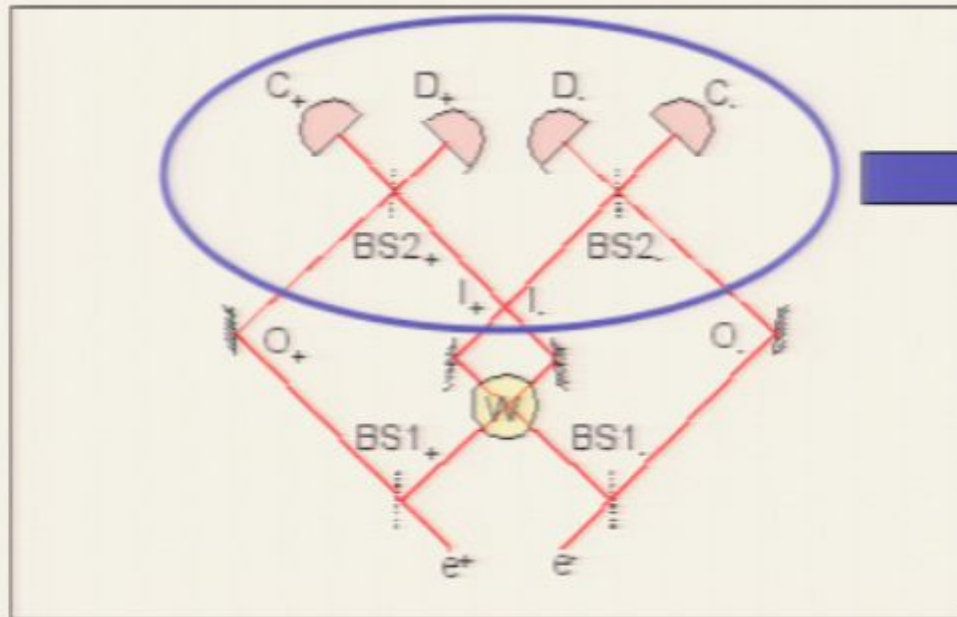
But what can we say about where the particles were or weren't, once D^+ & D^- fire?

Probabilities	e- in	e- out	
e+ in	0	1	1
e+ out	1	-1	0
	1	0	

In fact, this is precisely what Aharonov *et al.*'s weak measurement formalism predicts for any sufficiently gentle attempt to “observe” these probabilities...

Weak Measurements in Hardy's Paradox

Y. Aharonov, A. Botero, S. Popescu, B. Reznik, J. Tollaksen, PLA 301, 130 (2002); quant-ph/0104062



# In Arm	N(I ⁺)	N(O ⁺)	
N(I ⁺)	0	1	1
N(O ⁺)	1	-1	0
	1	0	

Weak Measurements in Hardy's Paradox

Ideal Weak Values

	$N(I^-)$	$N(O^-)$	
$N(I^+)$	0	1	1
$N(O^+)$	1	-1	0
	1	0	

Experimental Weak Values ("Probabilities"?)

	$N(I^-)$	$N(O^-)$	
$N(I^+)$	0.243 ± 0.068	0.663 ± 0.083	0.882 ± 0.015
$N(O^+)$	0.721 ± 0.074	-0.758 ± 0.083	0.087 ± 0.021
	0.925 ± 0.024	-0.039 ± 0.023	



The Bohr-Einstein (and Scully-Walls) Debates...

Which-path controversy (Scully, Englert, Walther vs the world?)

[Reza Mir *et al.*, New. J. Phys. 9, 287 (2007)]

Which-path measurements destroy interference.

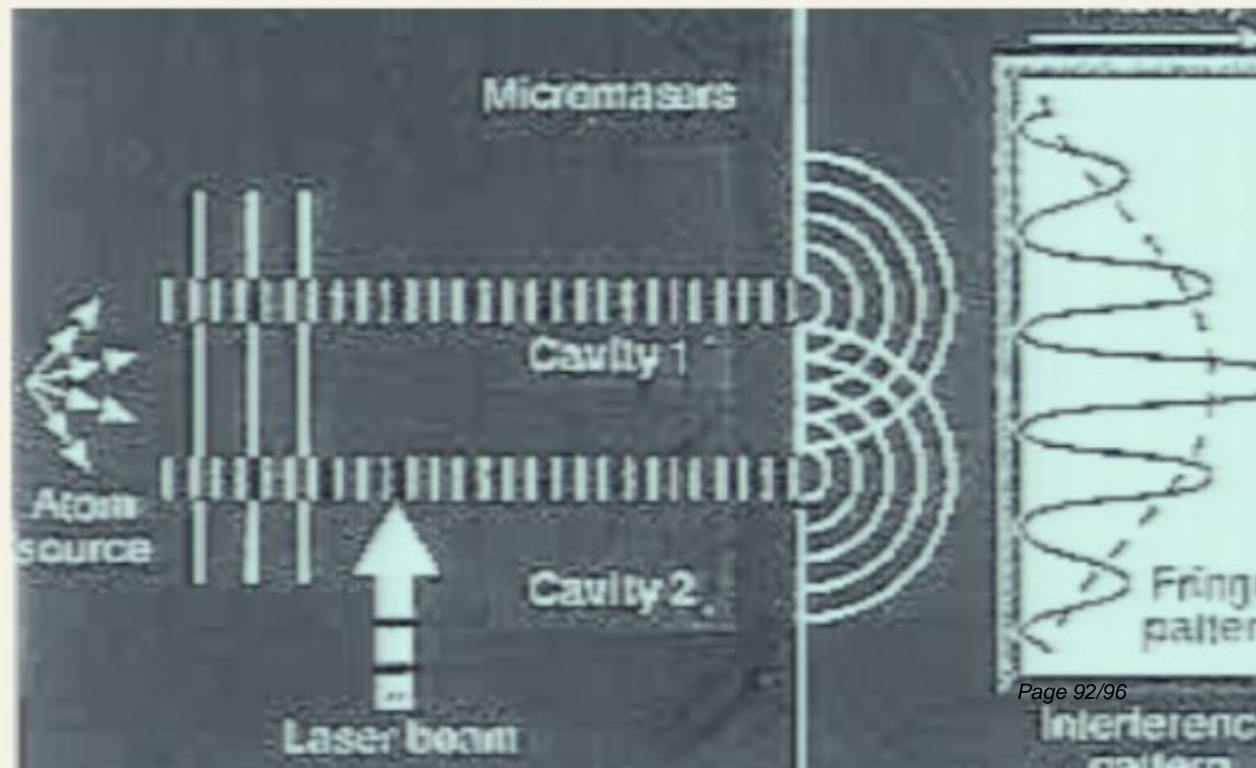
This is usually explained via measurement backaction & HUP.

Suppose we use a *microscopic* pointer.

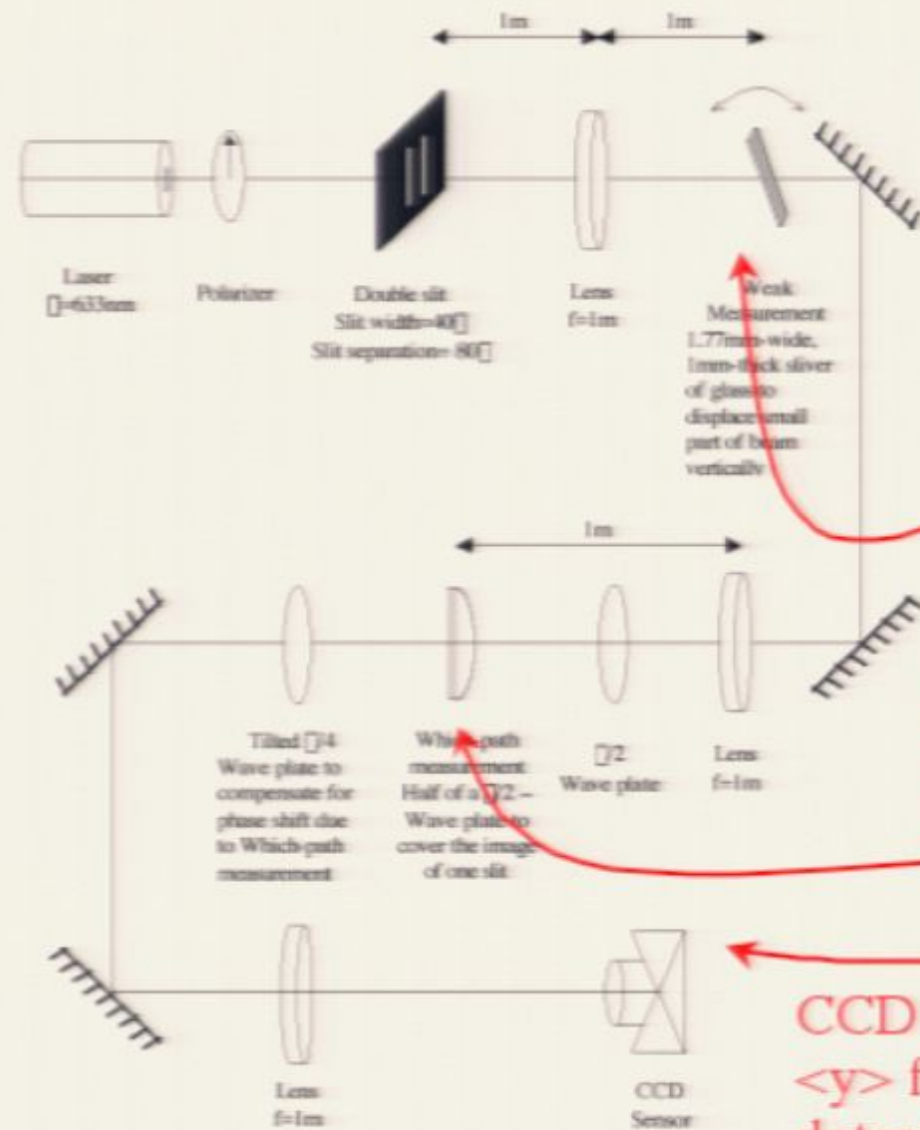
Is this really irreversible, as Bohr would have all measurements?

Need it disturb momentum?

Which is «more fundamental» – uncertainty or complementarity?



Convoluted implementation...

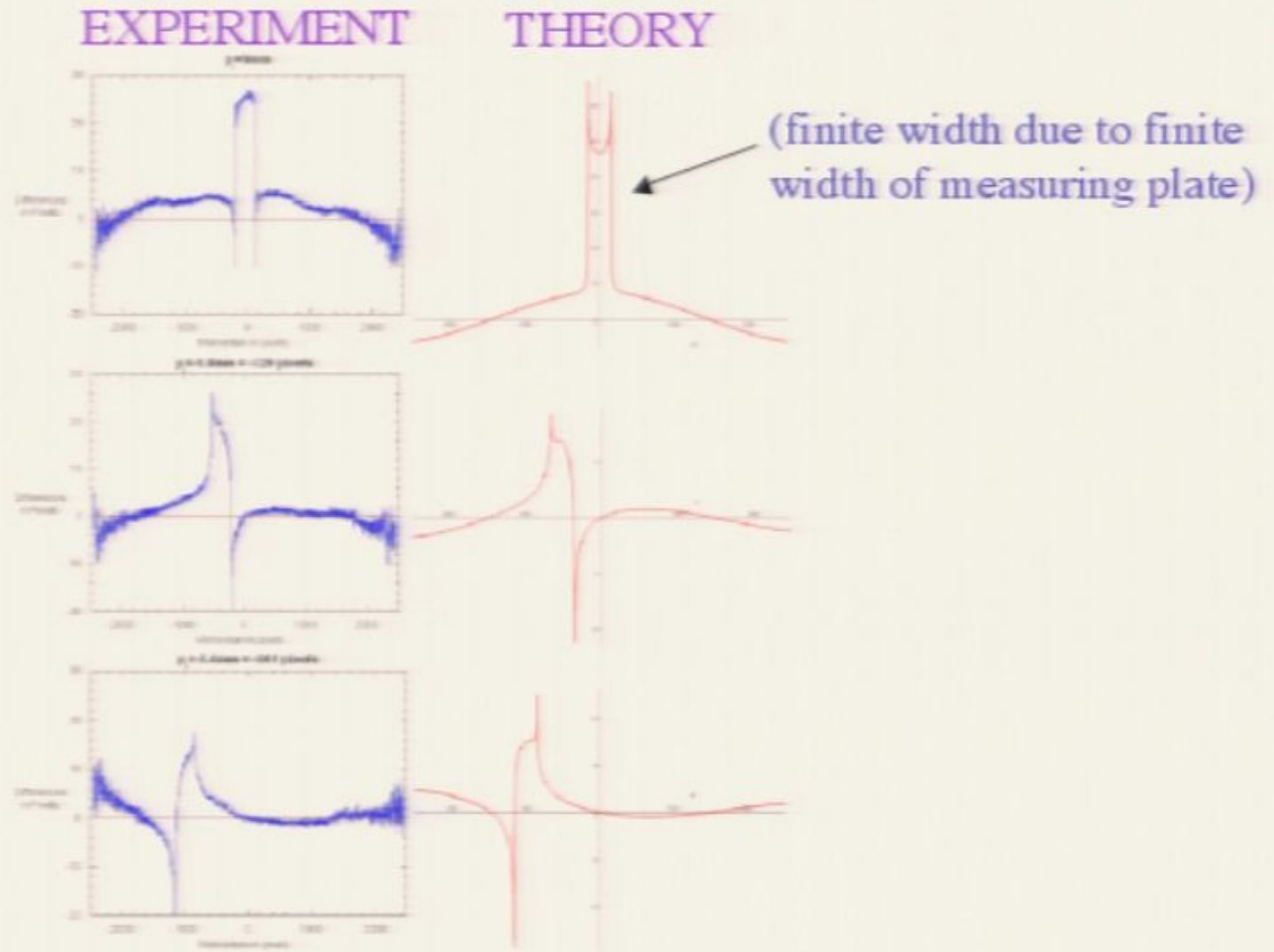


Glass plate in focal plane measures $P(p_i)$ weakly (shifting photons along y)

Half-half-waveplate in image plane measure path strongly

CCD in Fourier plane measures $\langle y \rangle$ for each position x ; this determines $\langle P(p_i) \rangle_{wk}$ for each final momentum p_{fi}

A few distributions $P(p_i | p_f)$

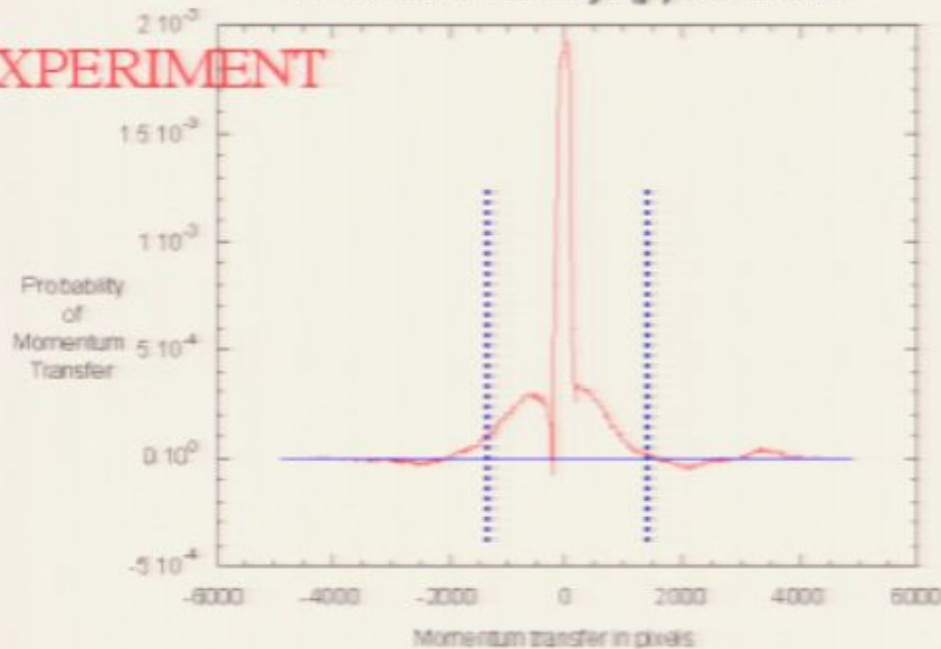


Note: not delta-functions; i.e., momentum may have changed.
Of course, these "probabilities" aren't always positive, etc etc.

The distribution of the integrated momentum-transfer

Normalized Probability $P(p)$ Distribution

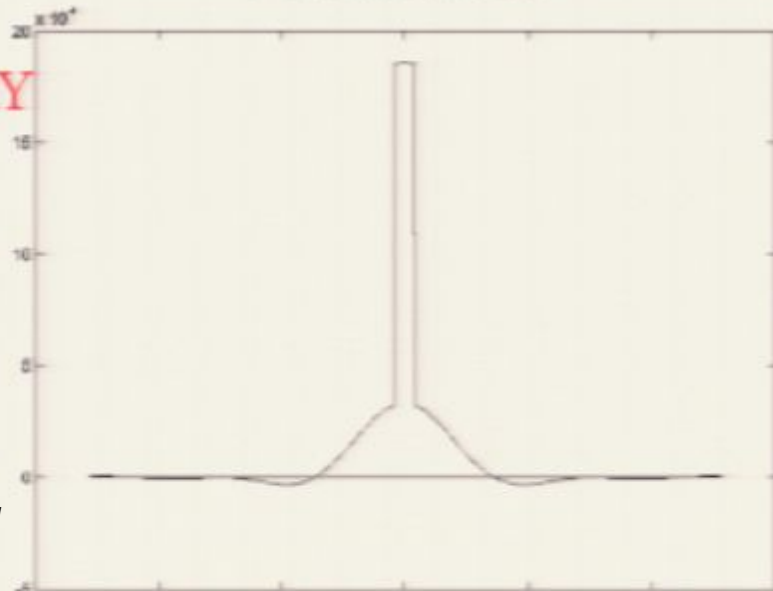
EXPERIMENT



Note: the distribution extends well beyond h/d .

On the other hand, all its moments are (at least in theory, so far) 0.

THEORY



The former fact agrees with Wall *et al*; the latter with Scully *et al*.

For weak distributions, they may be reconciled *because the distributions may take negative values in weak measurement*.

The moral of the story

1. Post-selected systems often exhibit surprising behaviour which can be probed using weak measurements.
2. These weak measurements may “resolve” various paradoxes... admittedly while creating new ones (negative probability)!
3. The two camps in the *welcher Weg* controversy are both supported by weak measurement, with no contradiction.
4. All of the claims in Hardy’s “paradox” are borne out by weak measurement, again with no contradiction: retrodiction (and “intradiction,” to mangle some jargon) is alive and well in quantum mechanics.
5. A postselected particle can be certain to have been in each of two places at the same time, yet can never be in both at the same time.
6. A series of tunneling-time experiments is still under preparation at U of T. So is an experiment to weakly measure the Bohm trajectories in a two-slit interferometer (based again on a proposal by Howard Wiseman).