Title: The Two State Vector Formalism.

Date: Sep 28, 2008 03:00 PM

URL: http://pirsa.org/08090067

Abstract: A brief review of the Two State Vector Formalism (TSVF) will be presented. It will be argued that we need to consider also backwards evolving quantum state because information given by forwards evolving quantum states is not complete. Both past and future measurements are required for providing complete information about quantum systems. Peculiar properties of pre- and post-selected quantum systems which can be efficiently analyzed in the framework of the TSVF and which can be observed using weak measurements will be described. An example is a particle reaching a certain location without being on the path that leads to and from this location. An extension of the TSVF to multiple space-time points will be discussed.

The two-state vector formalism of quantum mechanics

- The backwards evolving quantum state
- The ABL rule and quantum puzzles
- Weak measurement and weak values
- Counterfactual computation controversy

  or

  Where is the pre- and post-selected particle?
- When the worlds split in the MWI?

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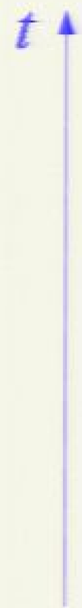
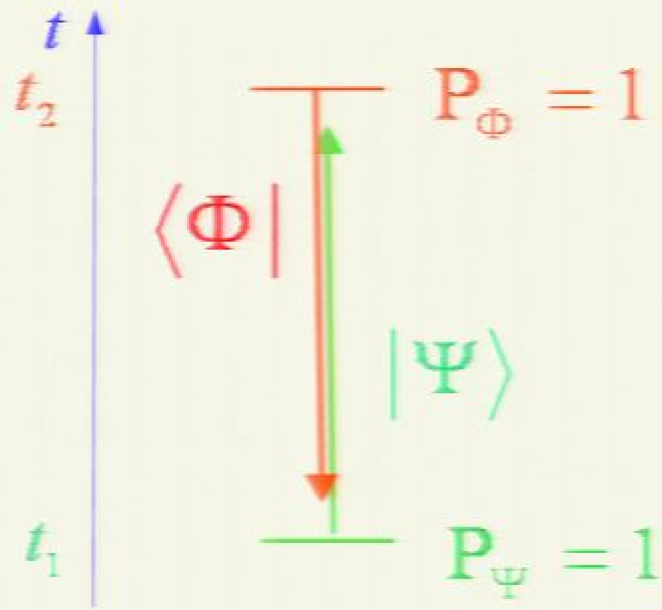
The two-state vector



The two-state vector



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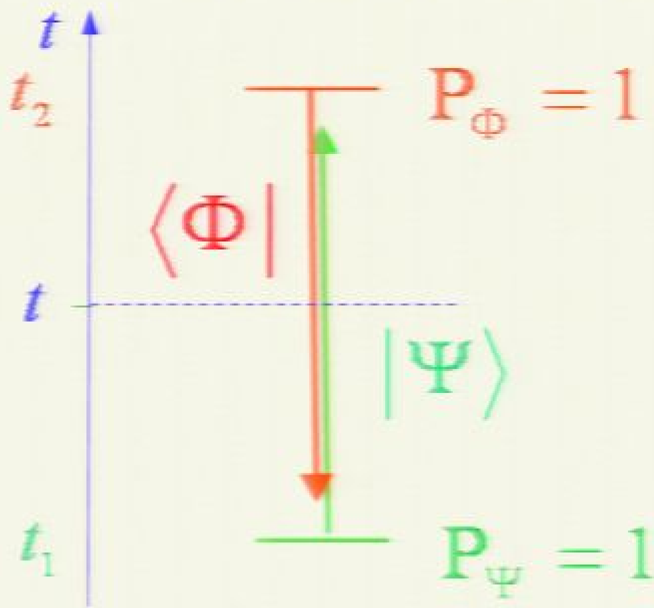
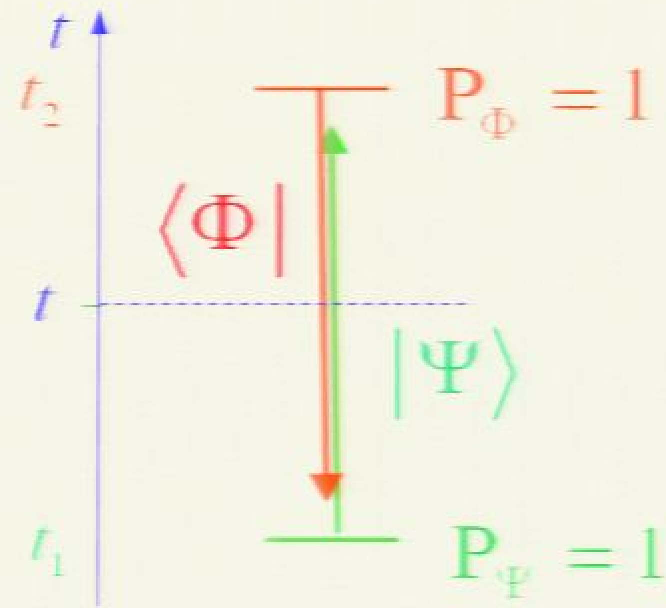


$$\langle \Phi | \quad | \Psi \rangle$$

The two-state vector



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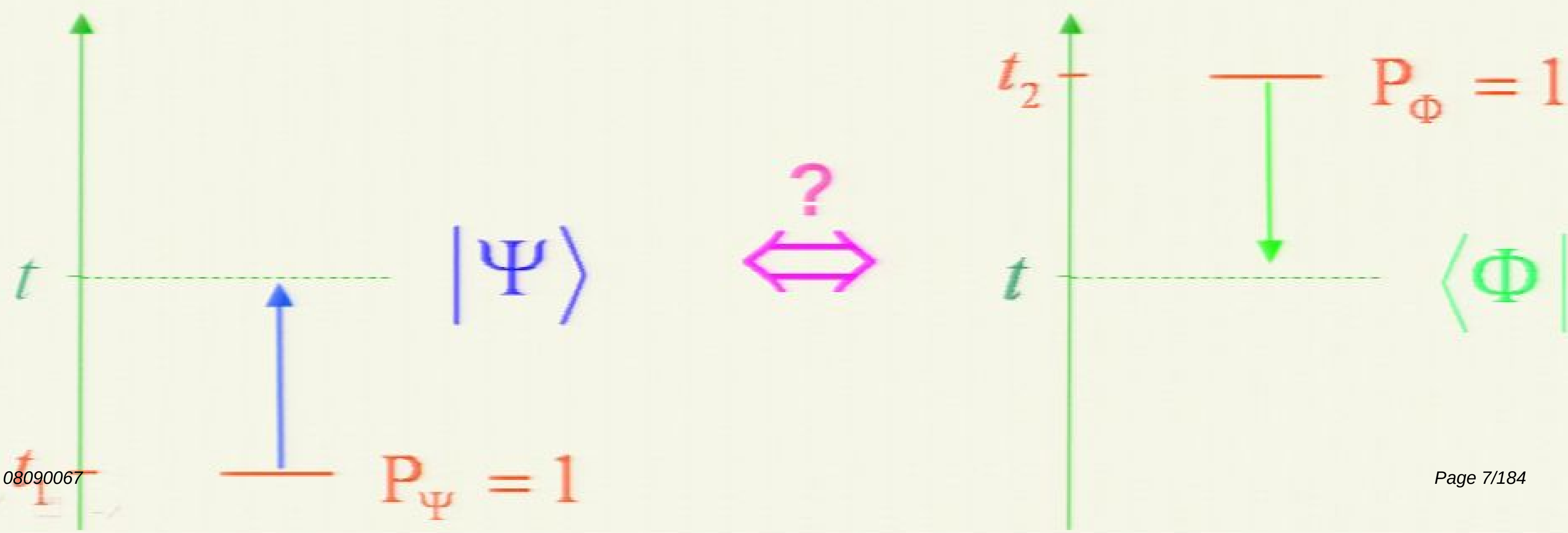

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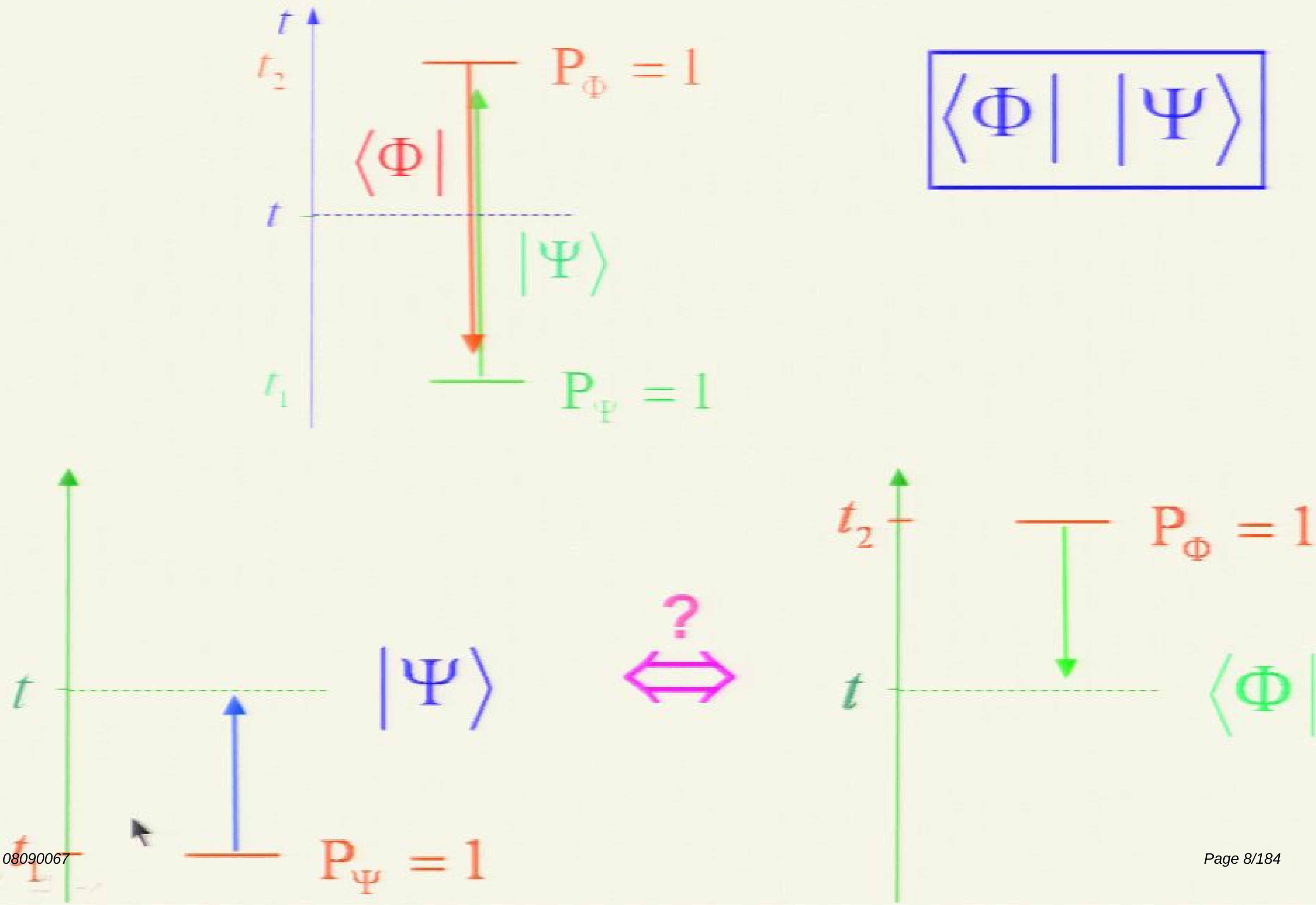

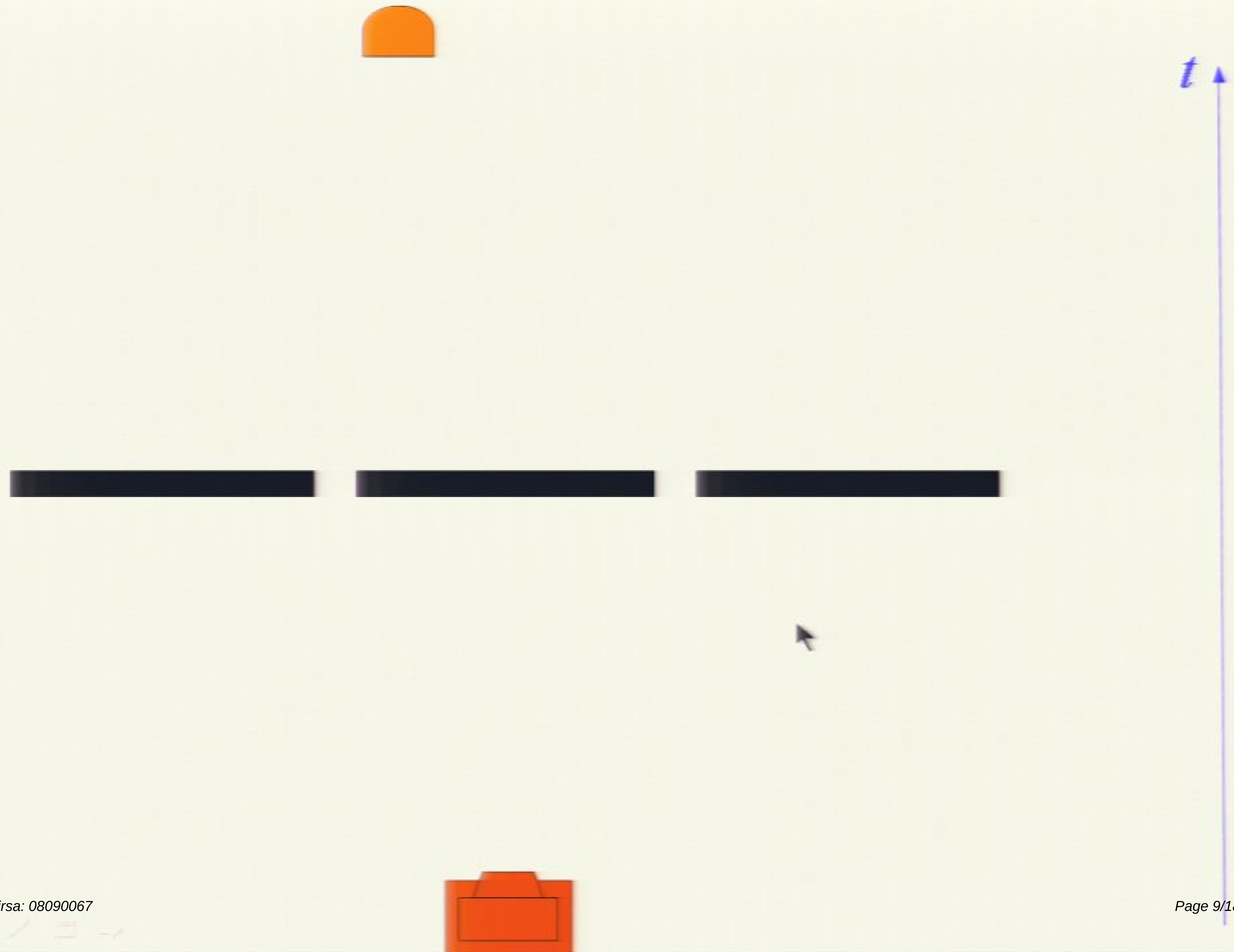
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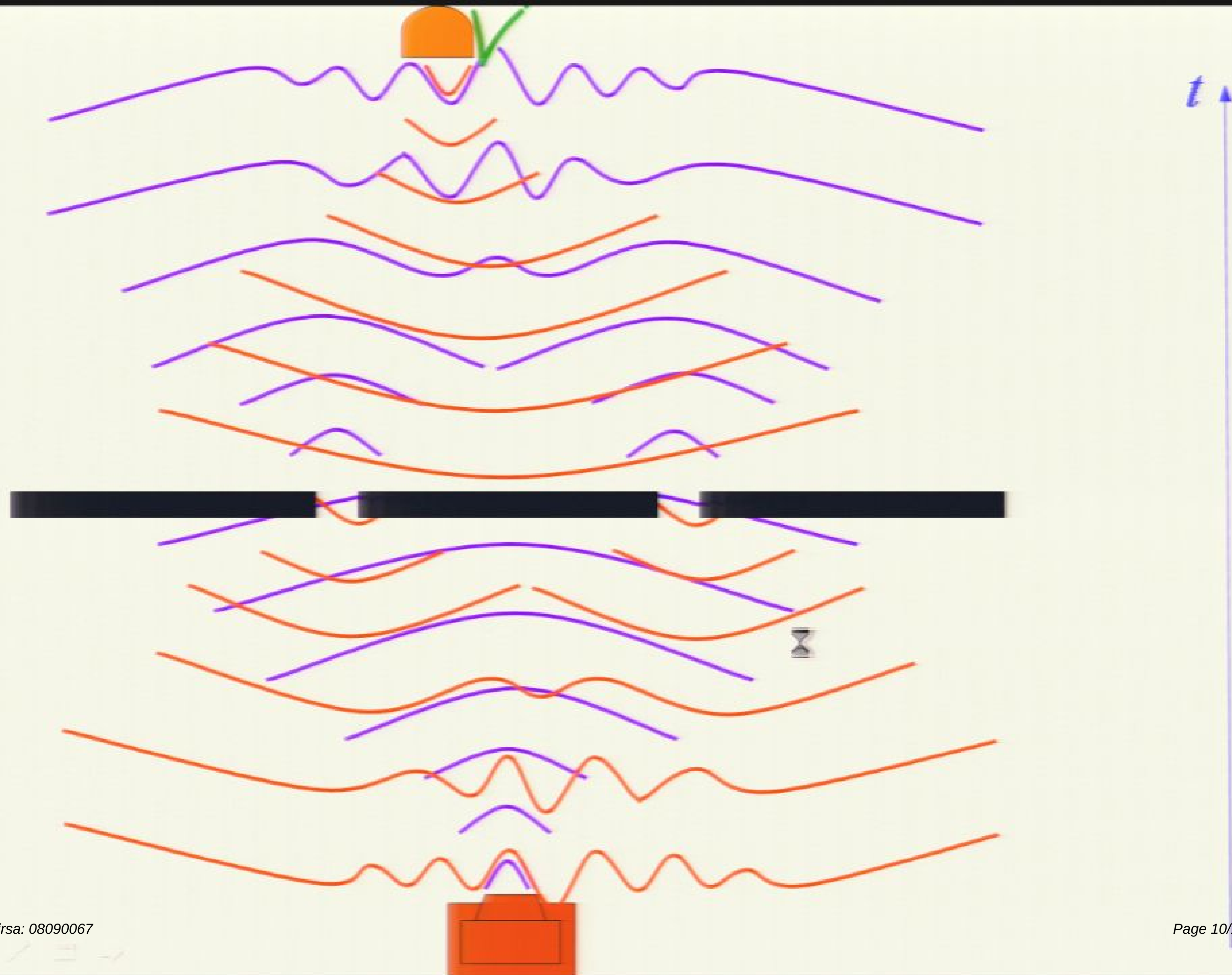
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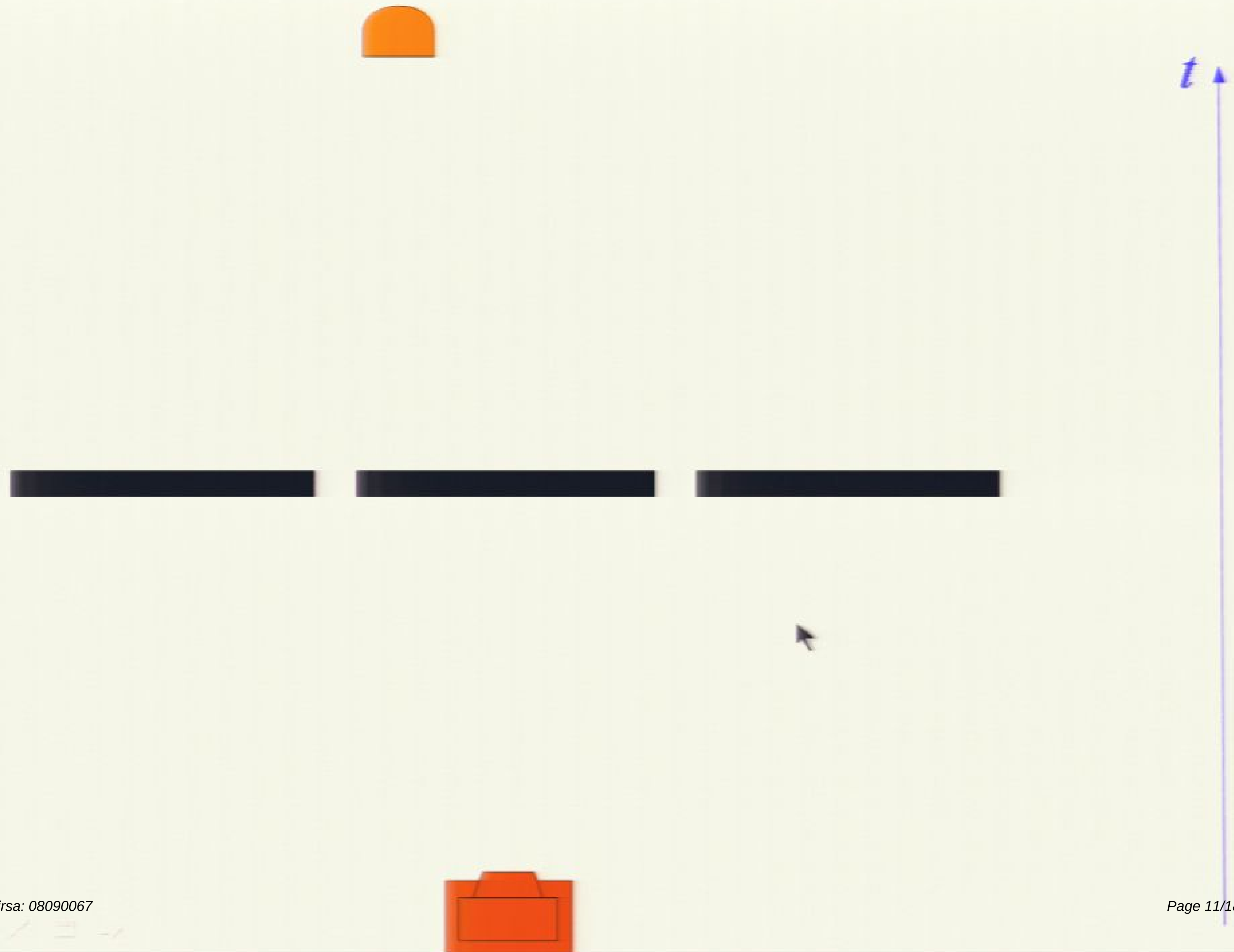
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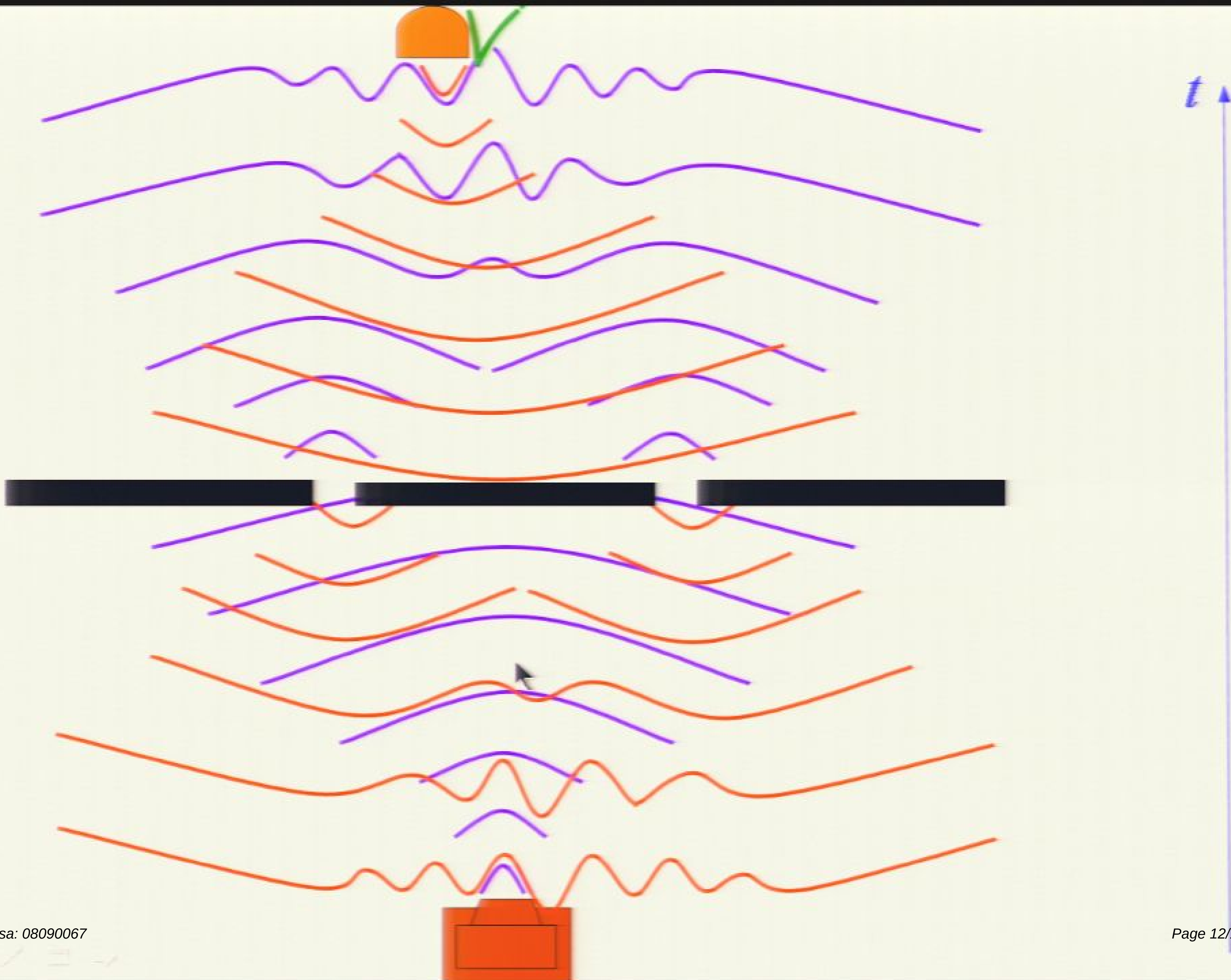
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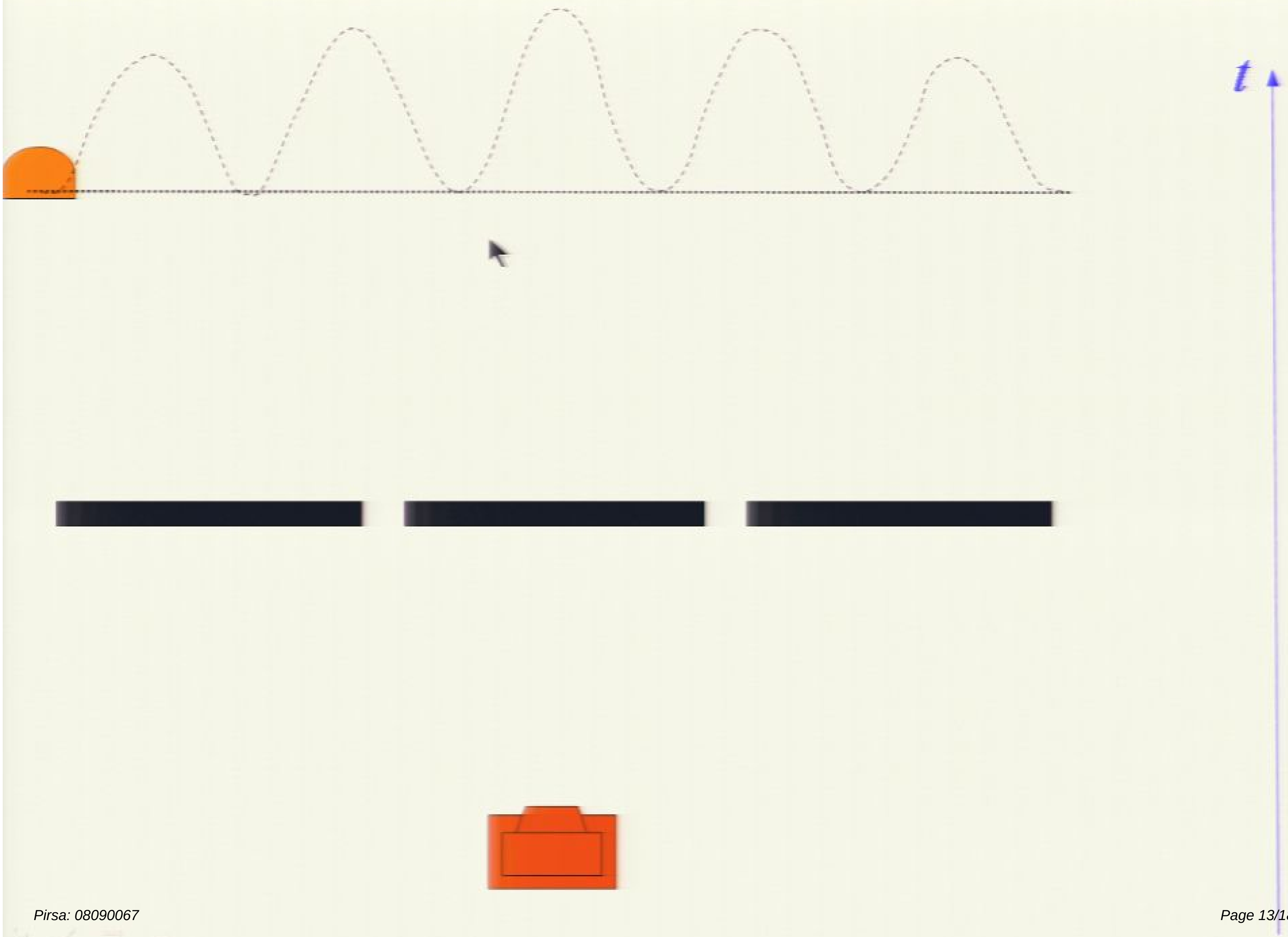
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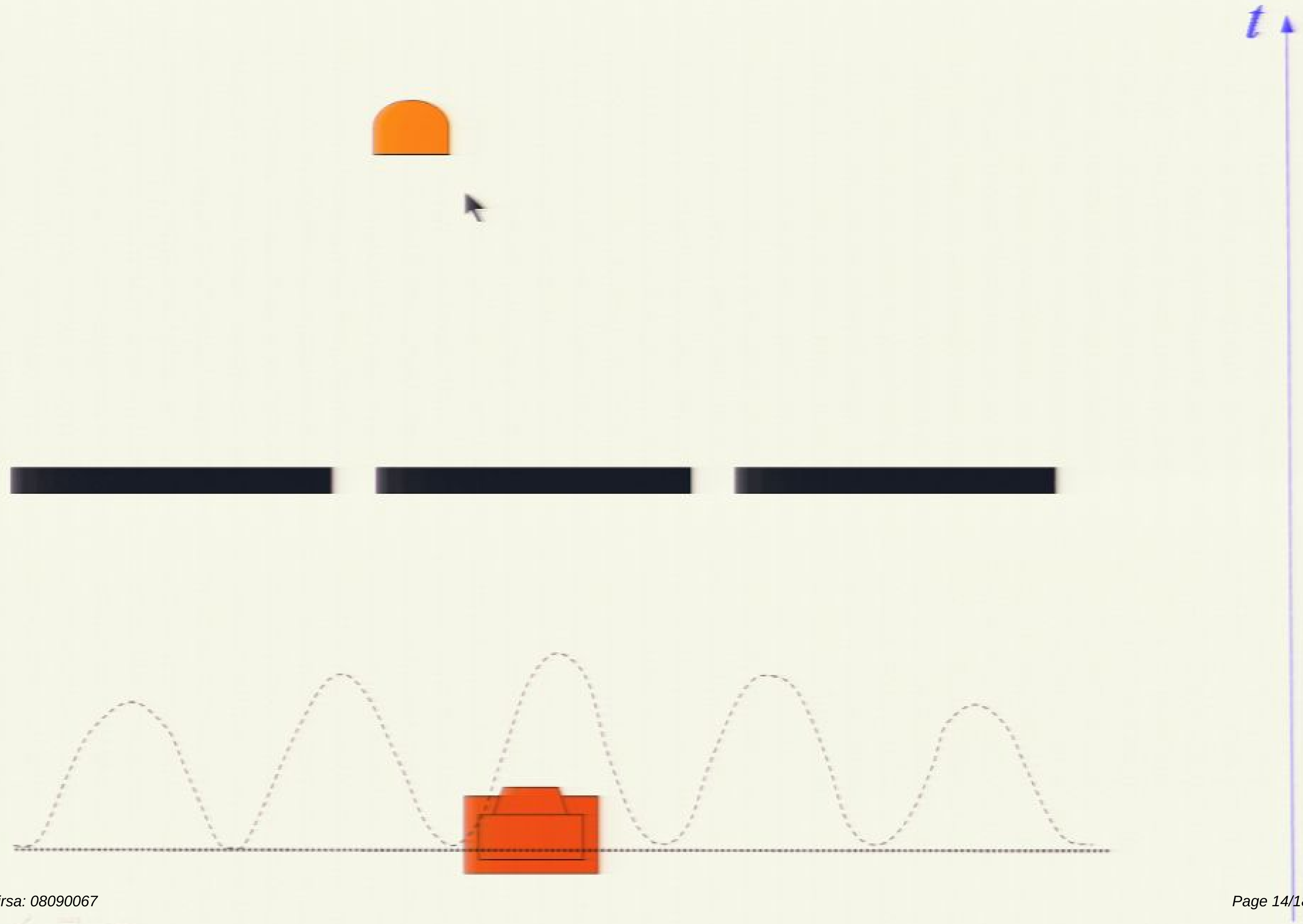
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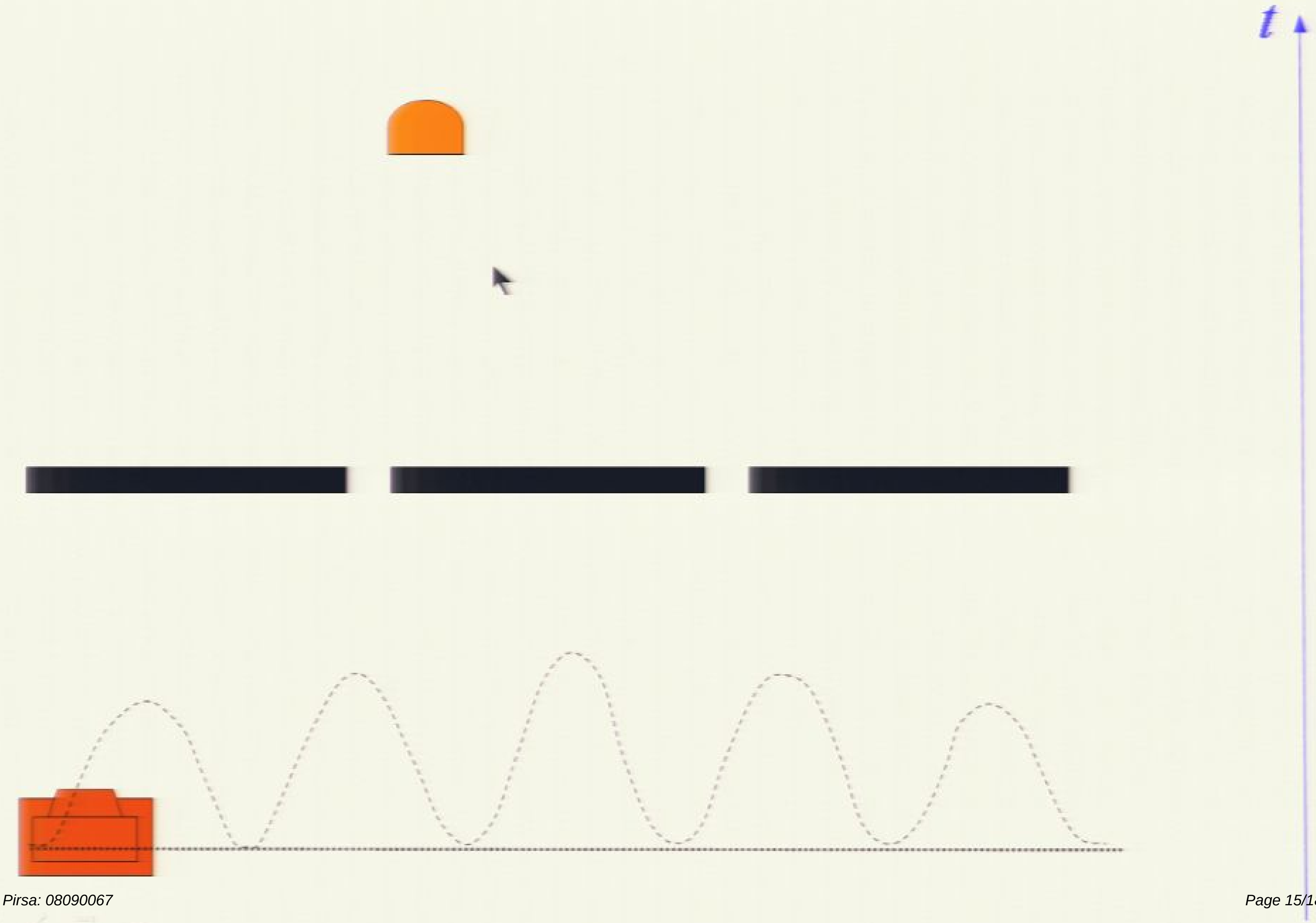
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The standard (one-state vector) description of a quantum system at time

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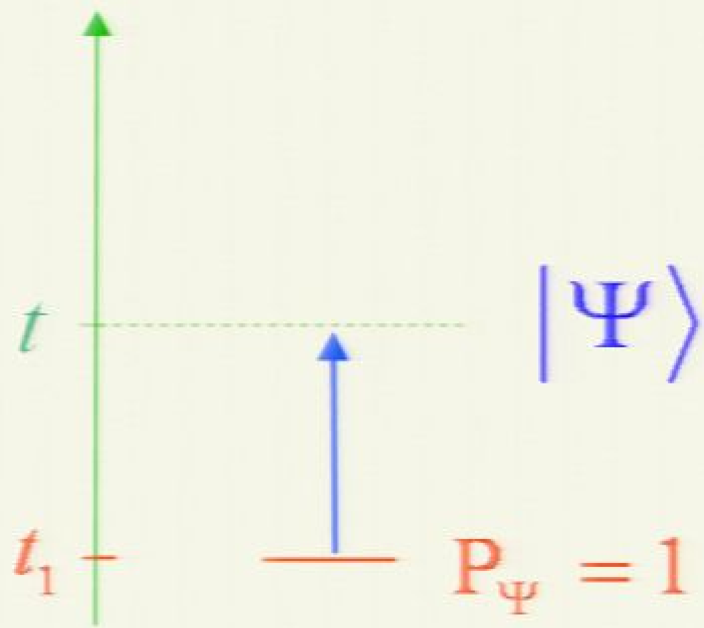
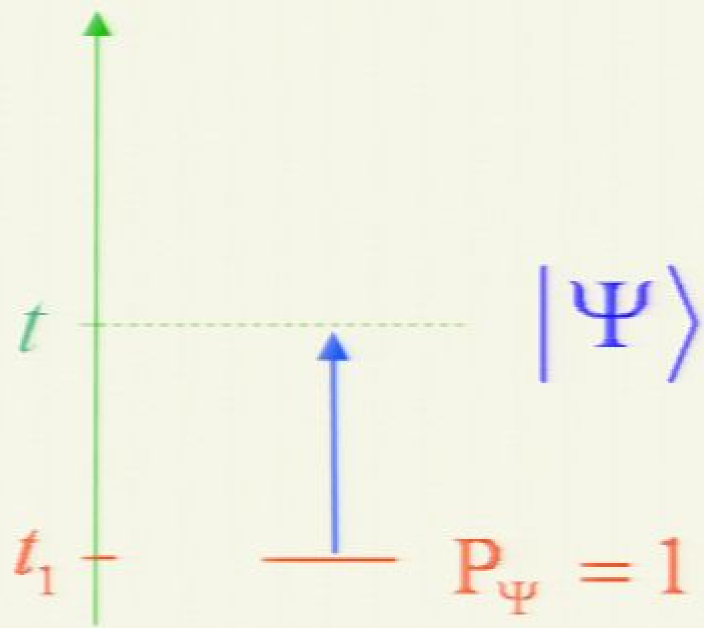


The standard (one-state vector) description of a quantum system at time

$$H_{FREE} = 0$$



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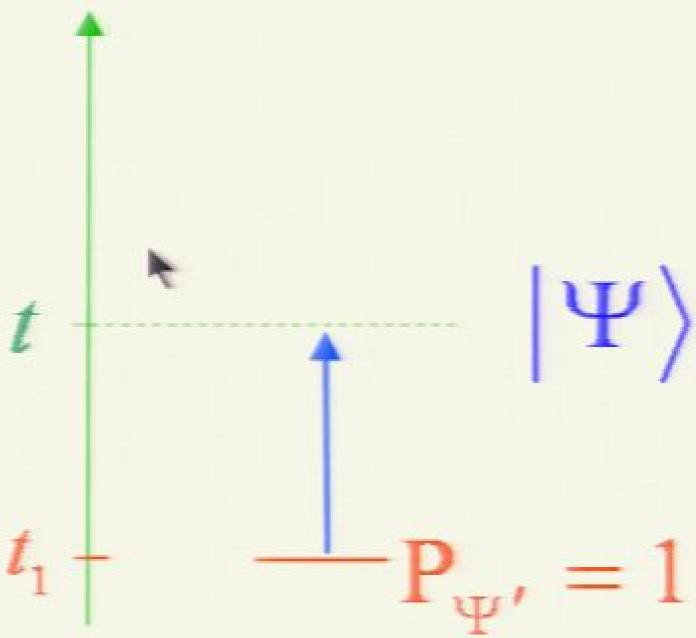
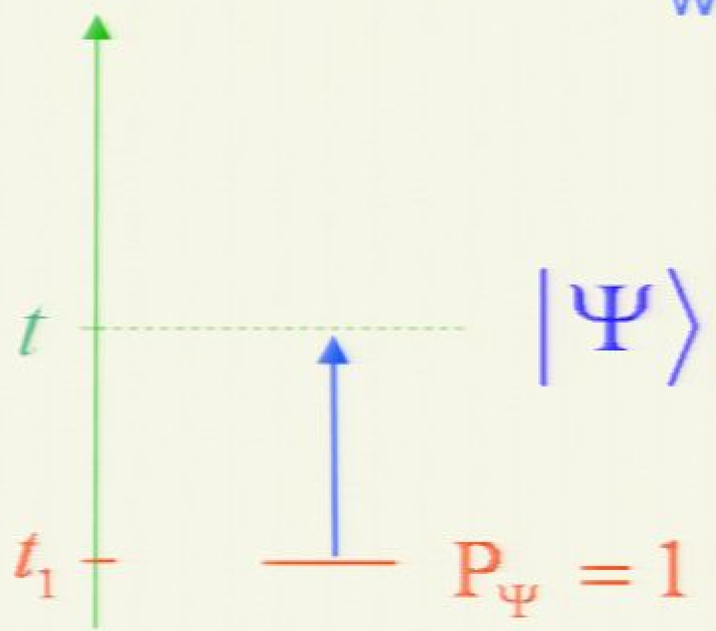


$$H_{FREE} \neq 0$$

$$|\Psi\rangle = e^{-i\int_{t_1}^{t} H_{FREE}\,dt} |\Psi'\rangle$$

The standard (one-state vector) description of a quantum system at time

We assume: $\boxed{H_{FREE} = 0}$



The standard (one-state vector) description of a quantum system

$$\left|\Psi(t)\right\rangle$$

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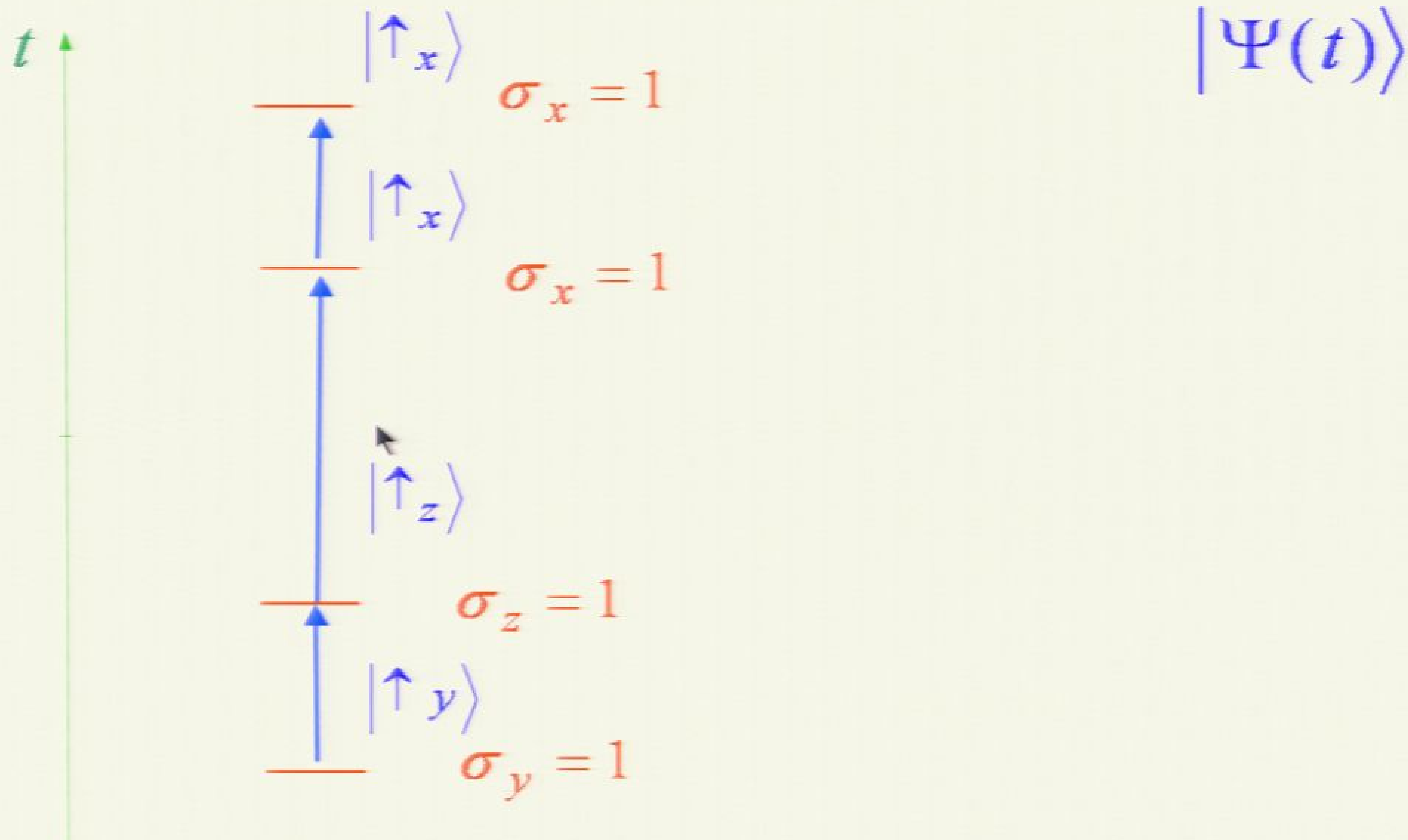

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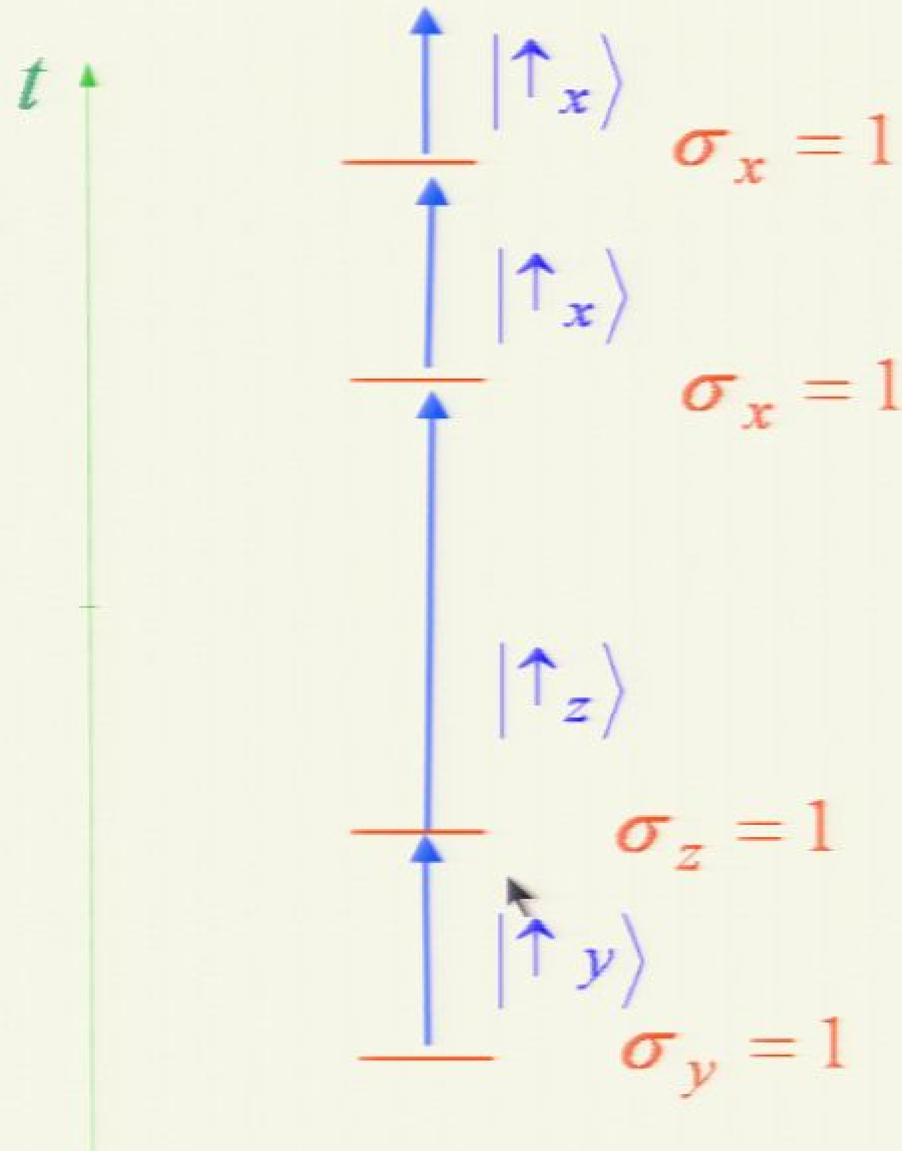
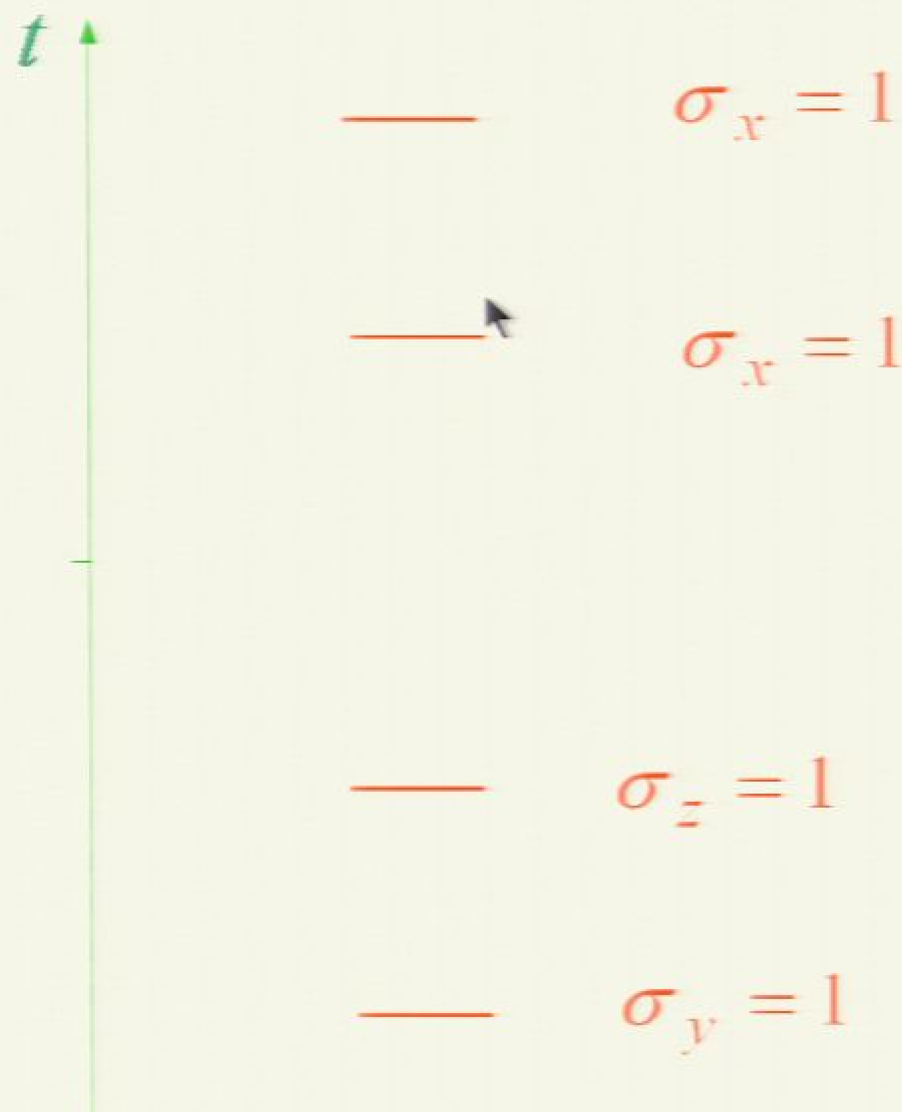


The time reversal of $|\Psi(t)\rangle$



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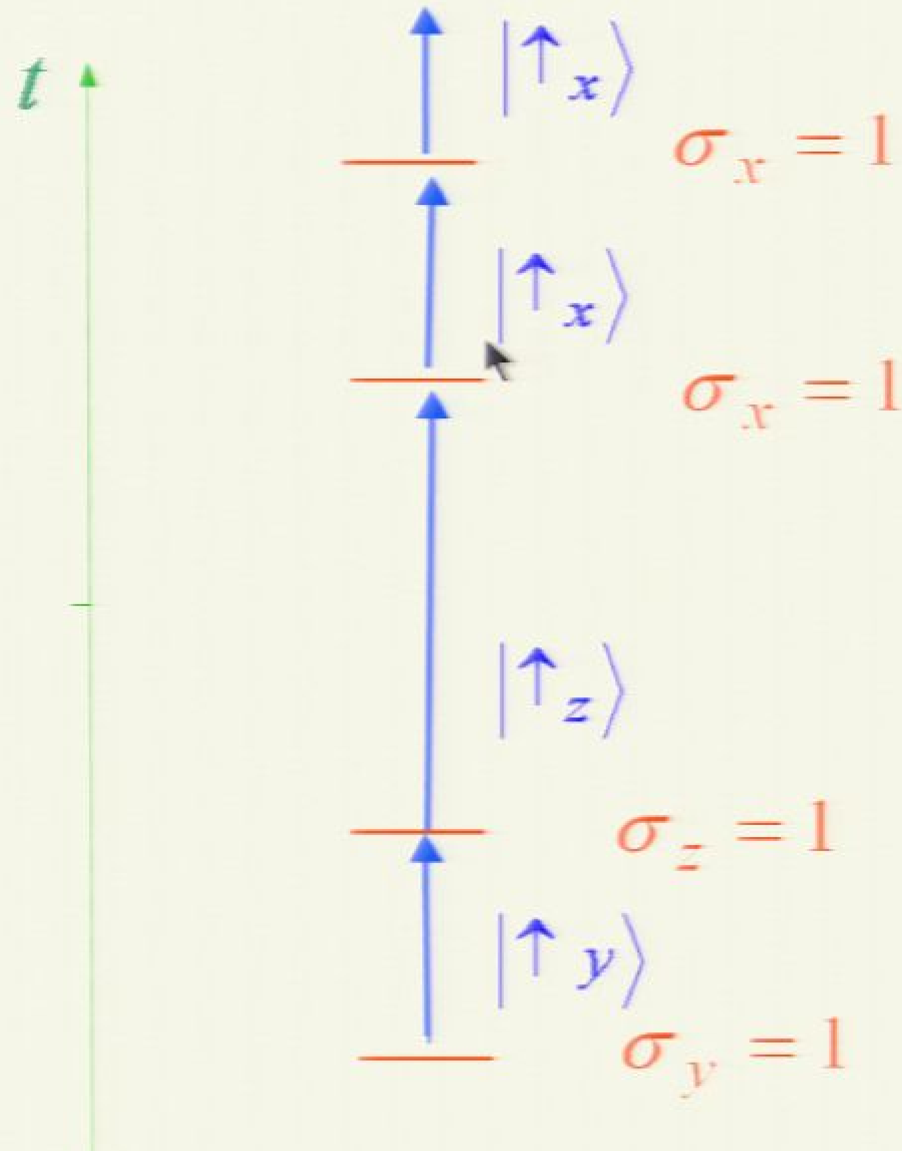


The backwards evolving quantum state $\langle \Phi(t) |$

t

$$\sigma_x = 1$$

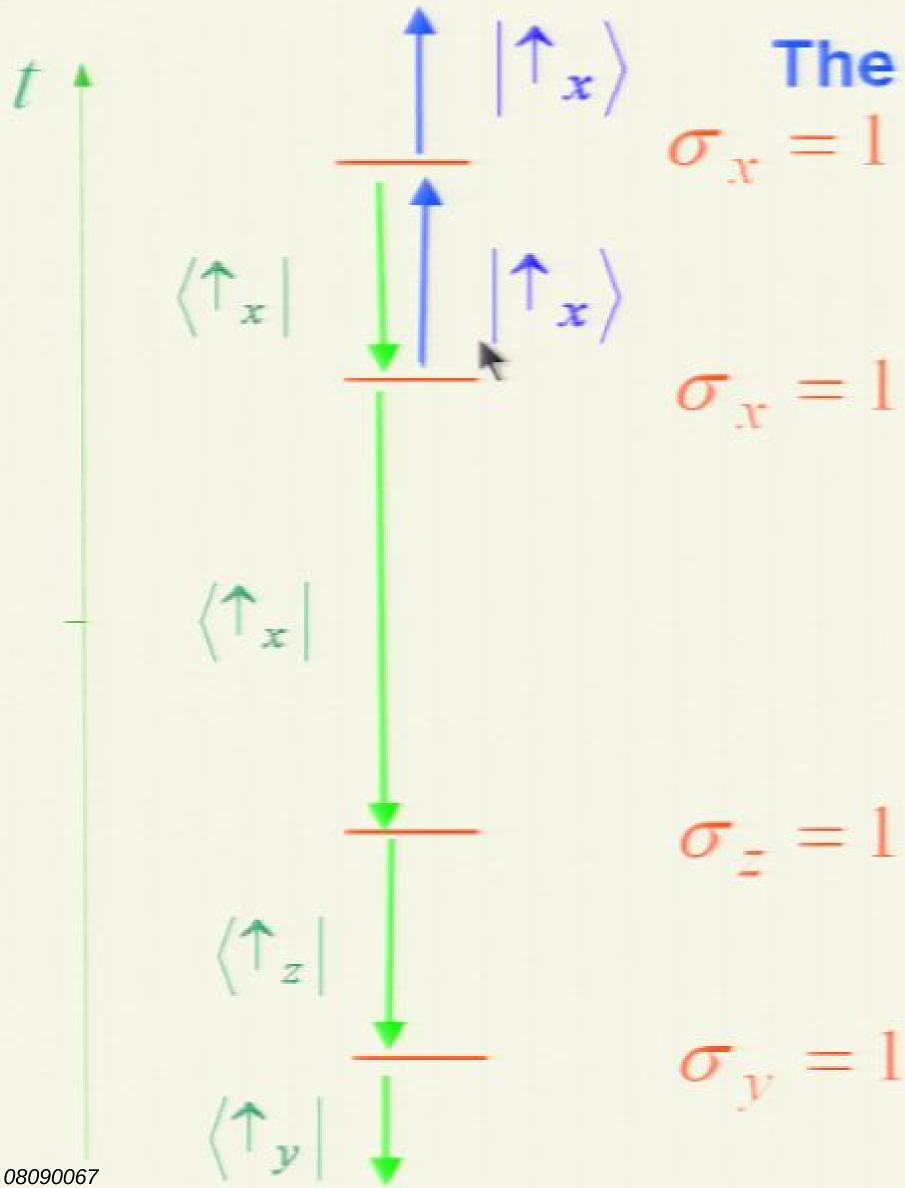
$$\sigma_x = 1$$

$$\sigma_z = 1$$

$$\sigma_y = 1$$

The backwards evolving quantum state $\langle \Phi(t) |$

The time reversal of $| \Psi(t) \rangle$



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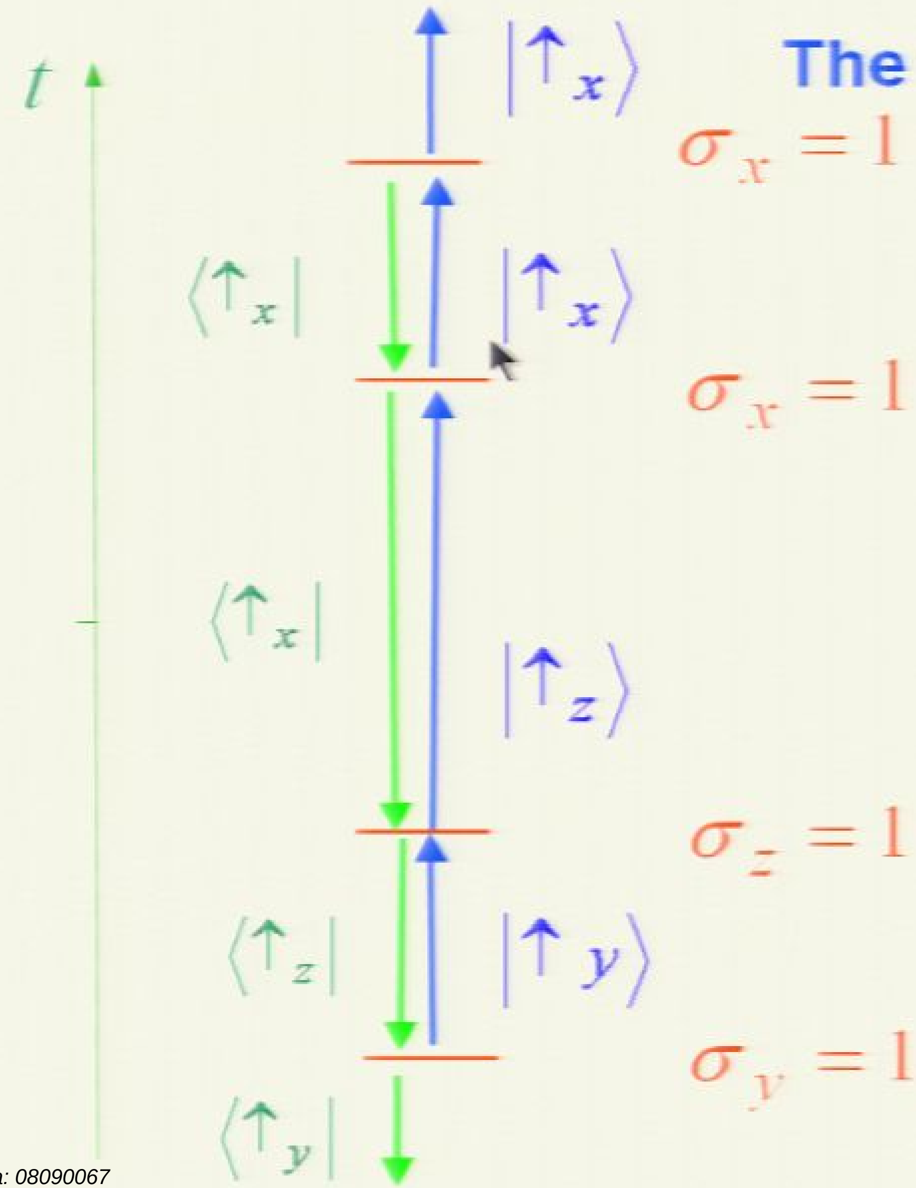

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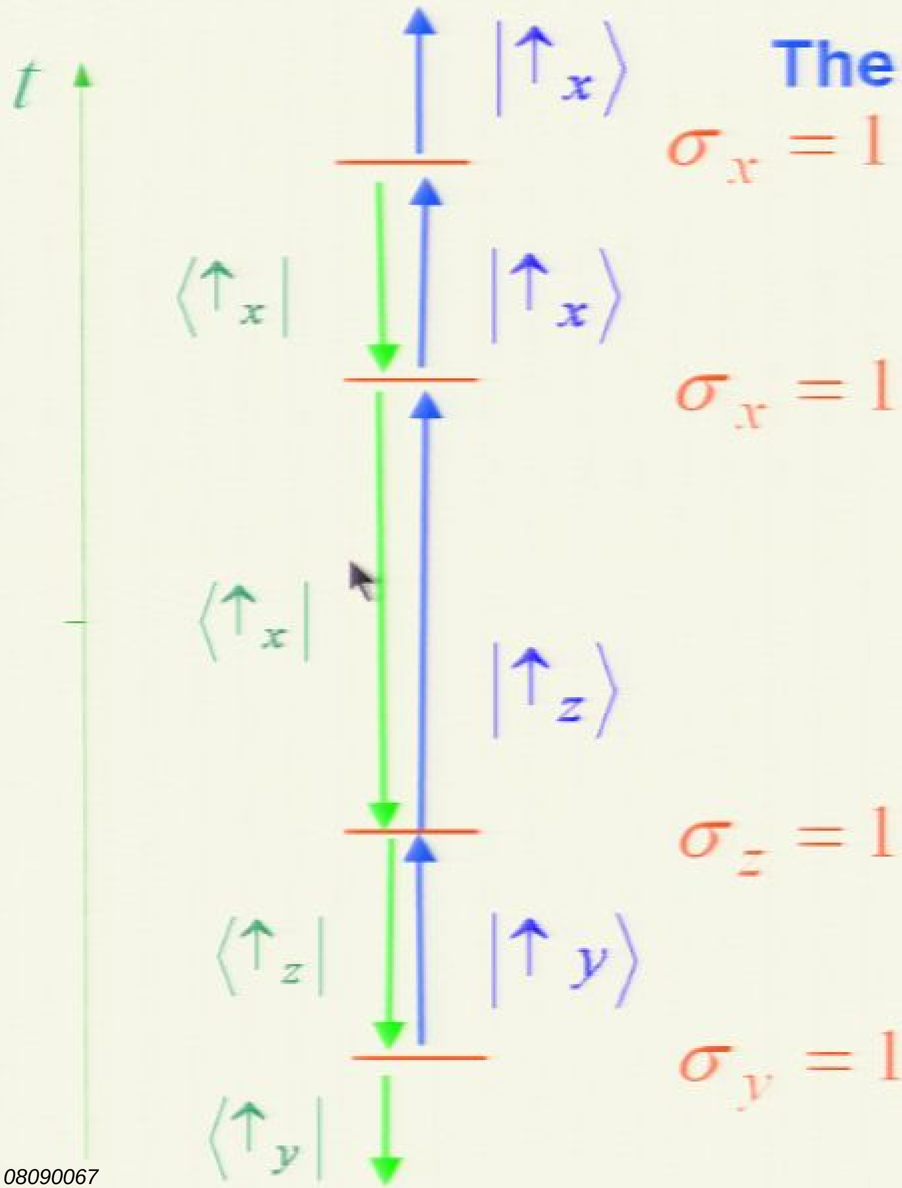
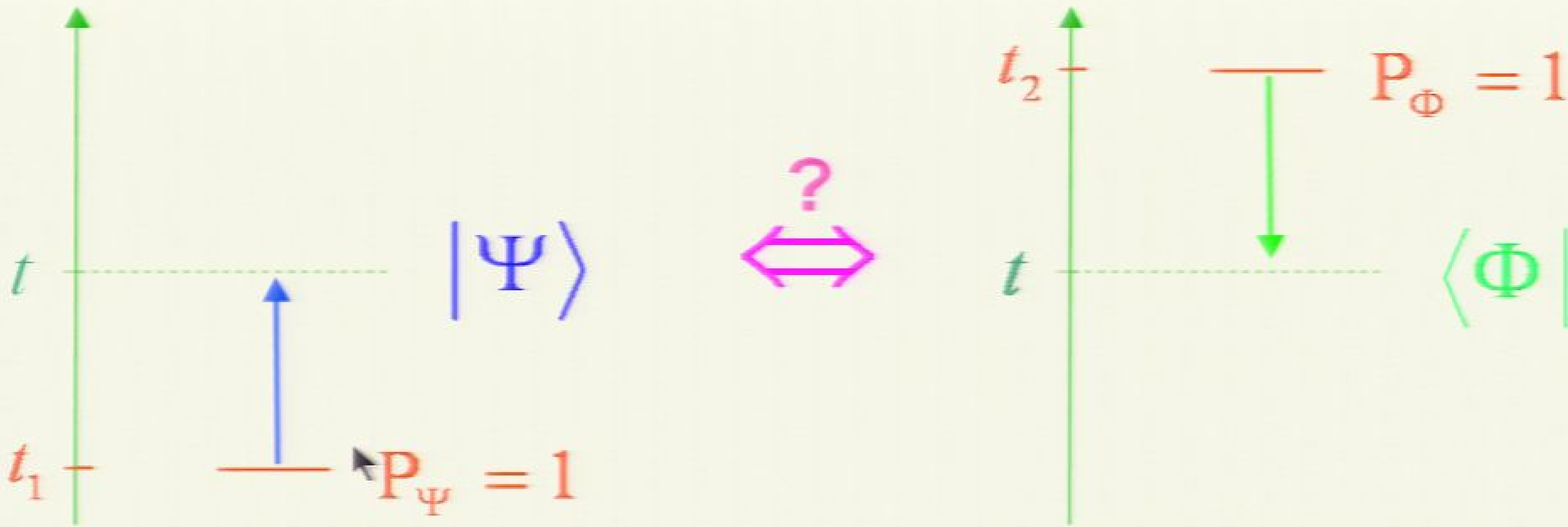
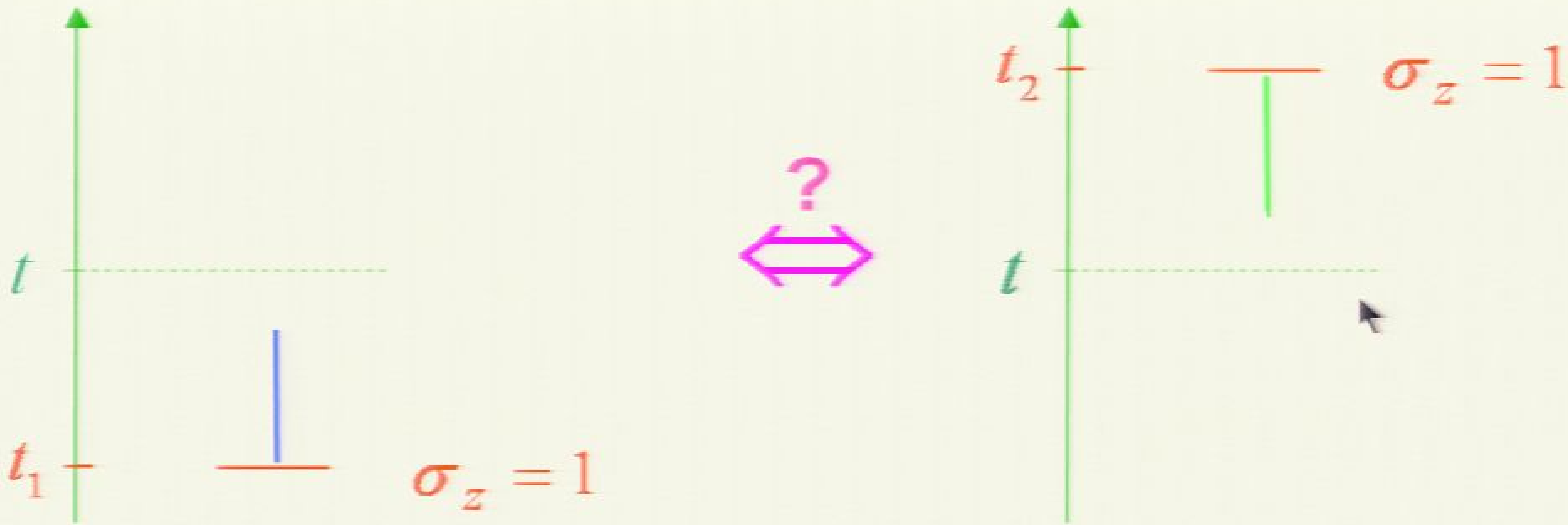


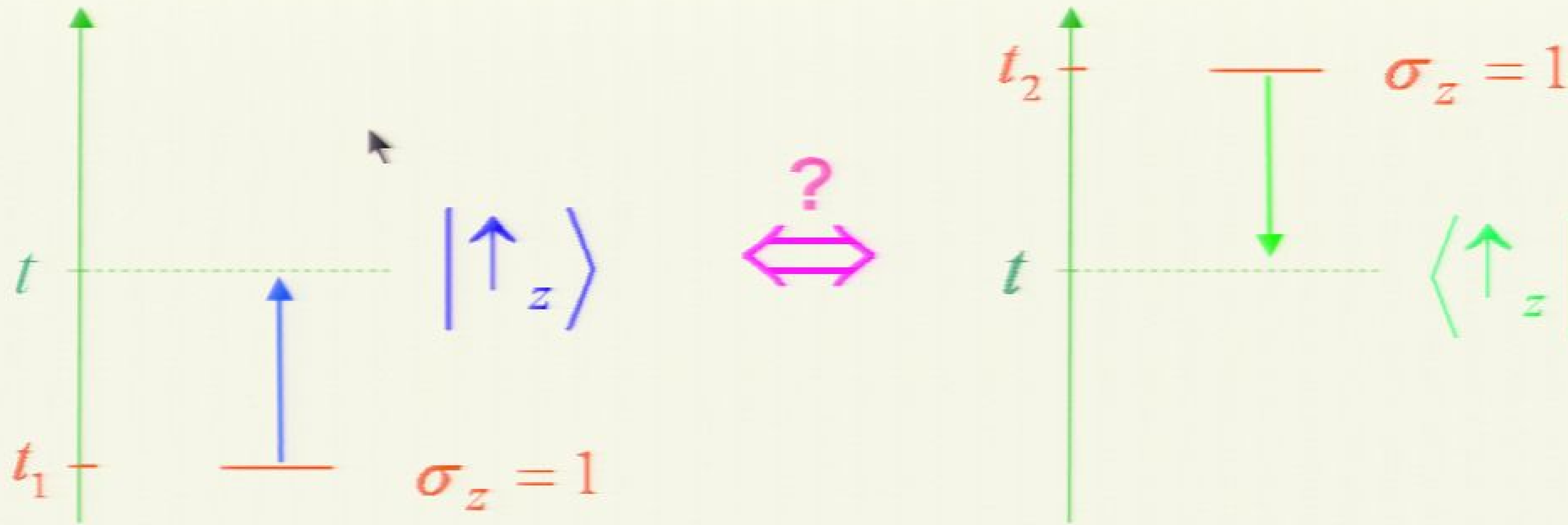
The two-state vector

$$\boxed{\langle \Phi | \quad | \Psi \rangle}$$

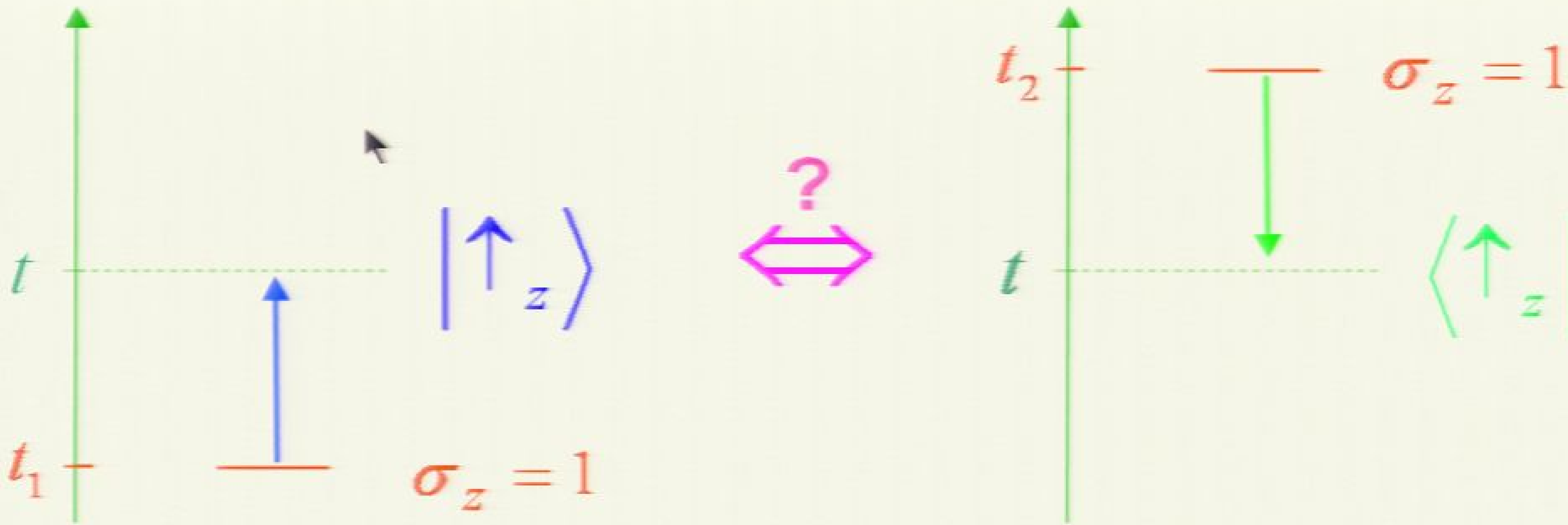

t
$|\Psi\rangle$
t_1
$P_\Psi = 1$
?
$\Leftrightarrow$
t_2
$P_\Phi = 1$
t
$\langle\Phi|$



t_2
$\sigma_z = 1$
?
t
t
t_1
$\sigma_z = 1$



At time *t*:



At time *t*:

$\text{Prob}(\uparrow_z) = 1$

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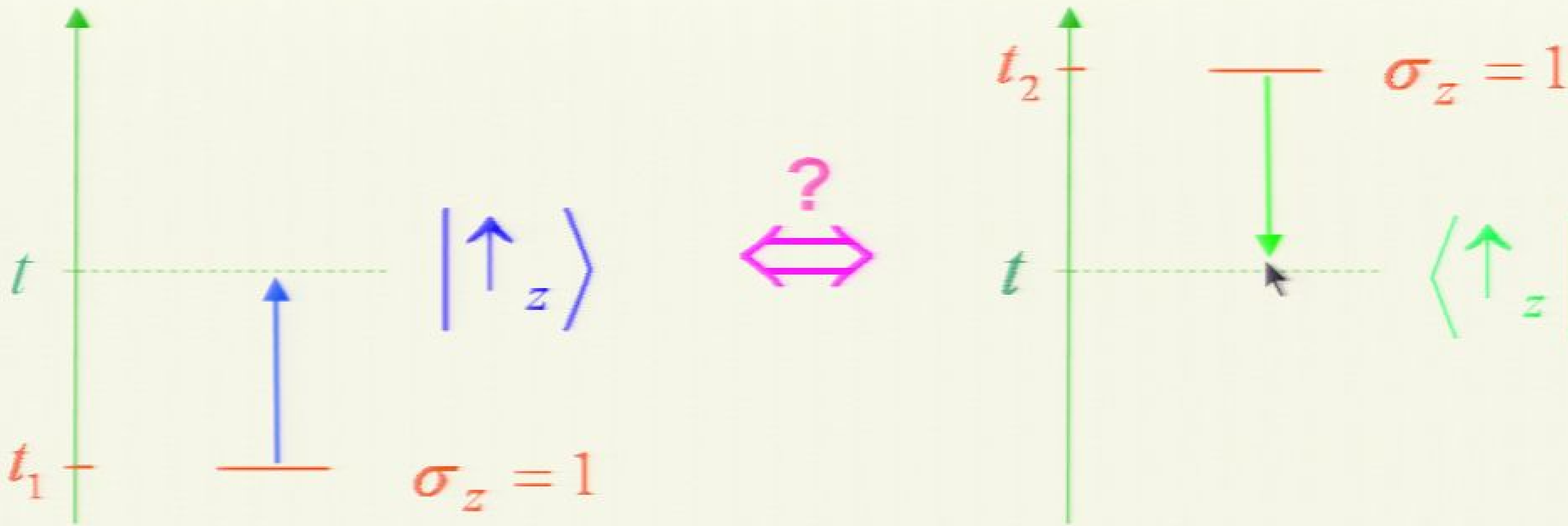
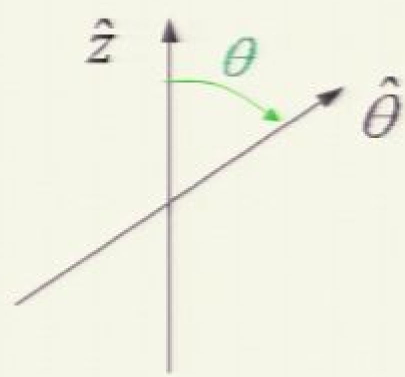


At time *t*:

$$\mathrm{Prob}(\uparrow_z) = 1$$

$$\mathrm{Prob}(\uparrow_\theta) = \cos\frac{\theta^2}{2}$$



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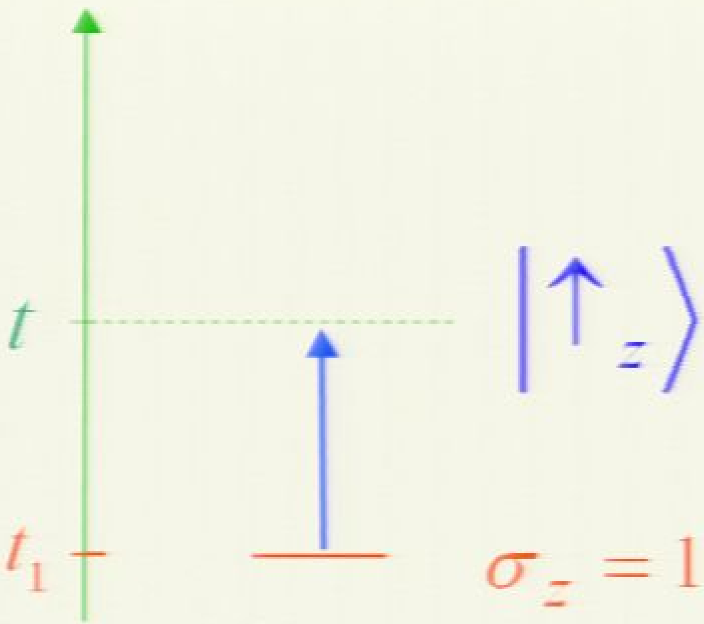
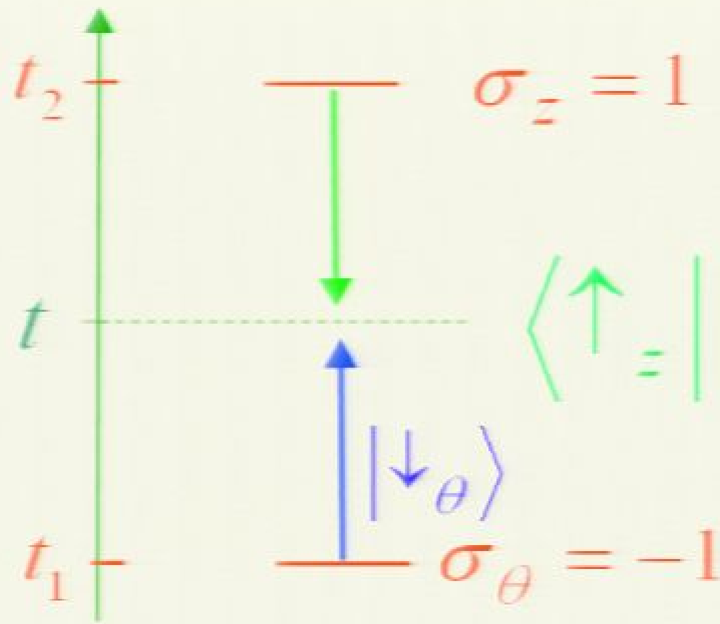


?
$\Leftrightarrow$



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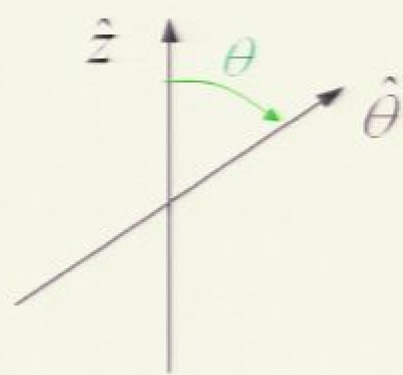
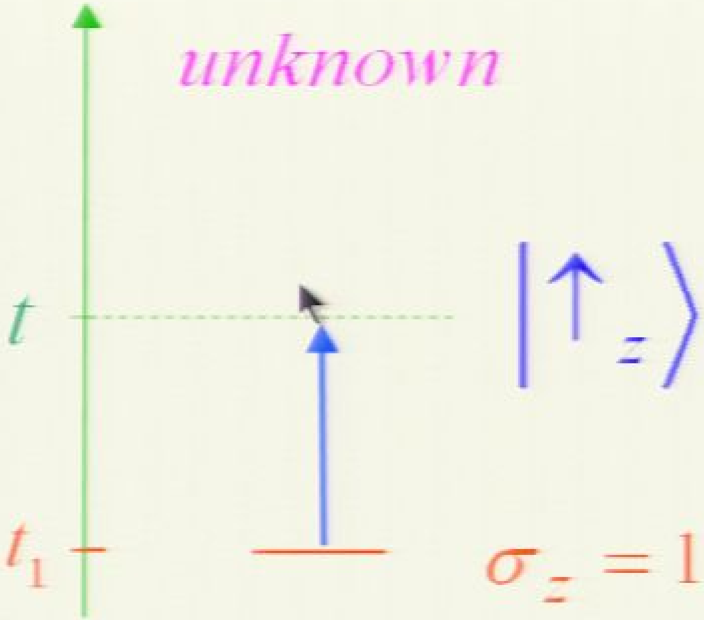
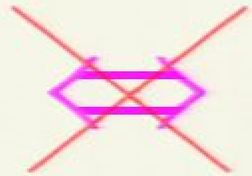
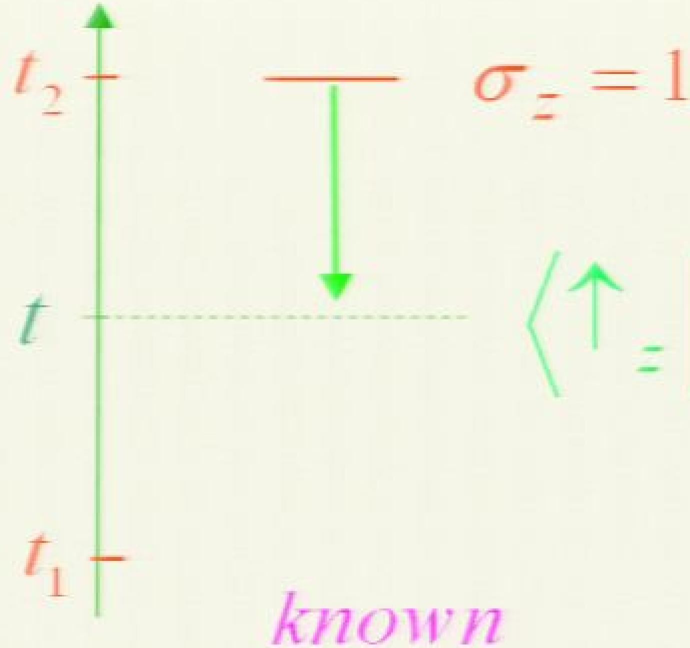


$\text{Prob}(\uparrow_z) = 1$

$\text{Prob}(\uparrow_\theta) = 0$







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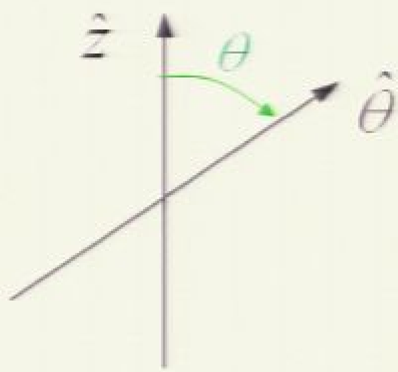
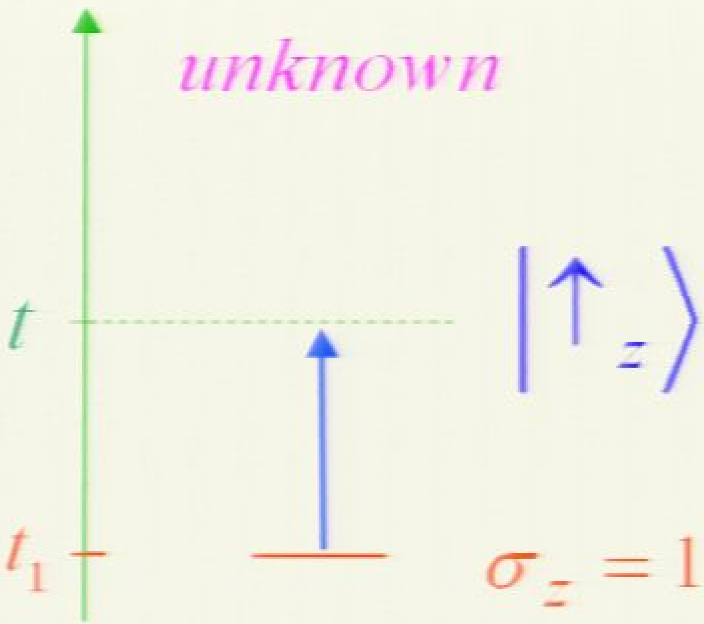
$\text{Prob}(\uparrow_\theta) = \cos\frac{\theta^2}{2}$

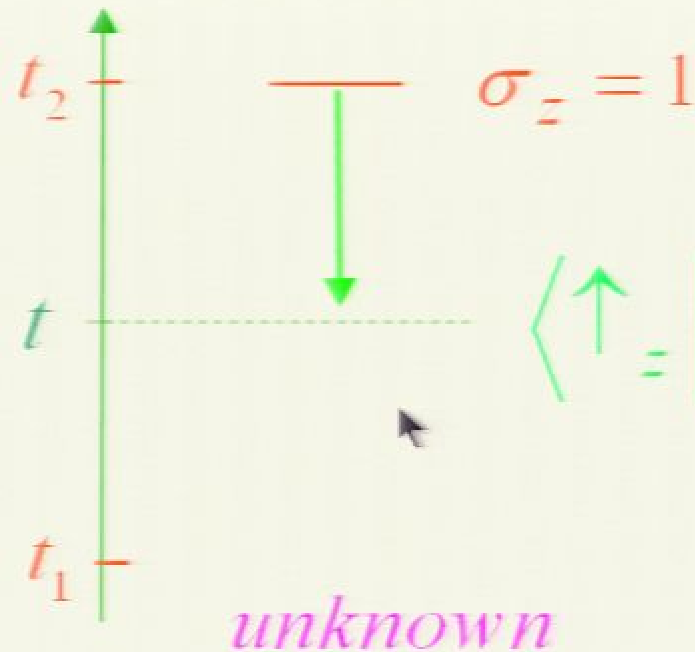


$\text{Prob}(\uparrow_z) = 1$

$\text{Prob}(\uparrow_\theta) = ?$



$\Leftrightarrow$



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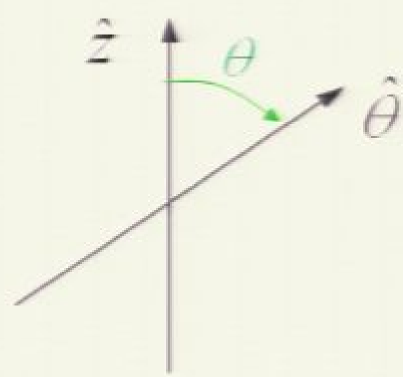

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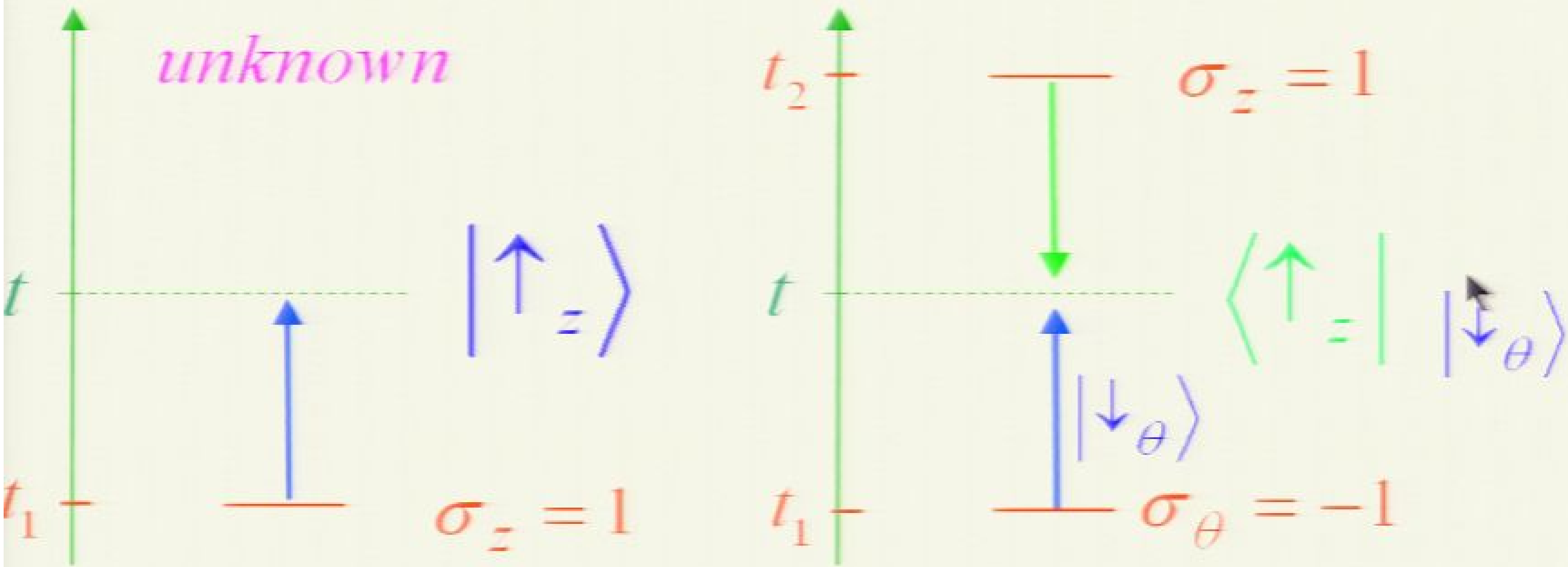
Erasing the past



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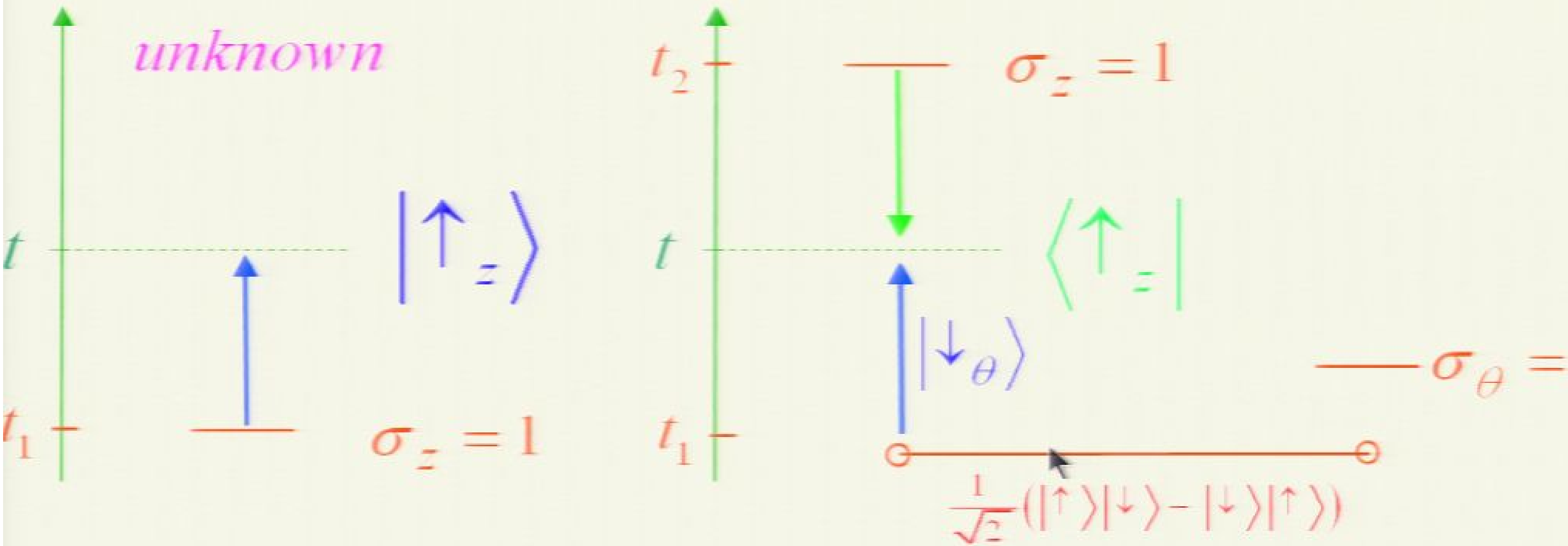
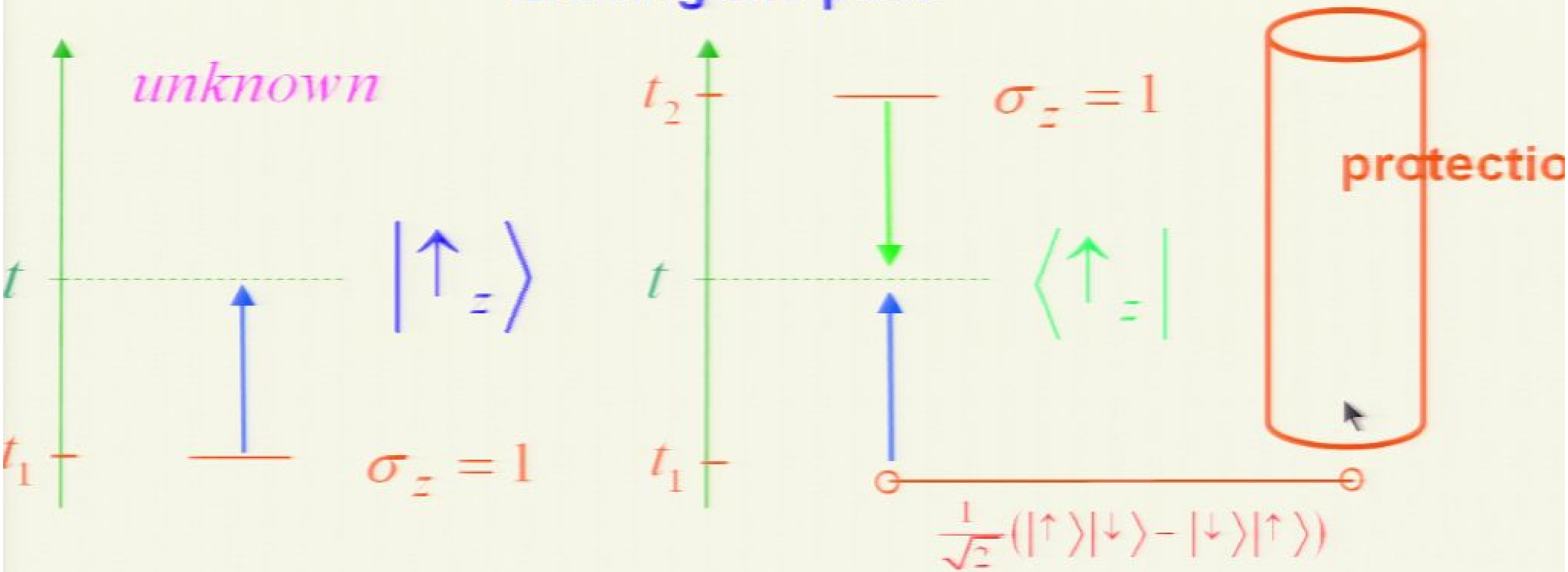


Hint: Alternative past for the same present

Erasing the past



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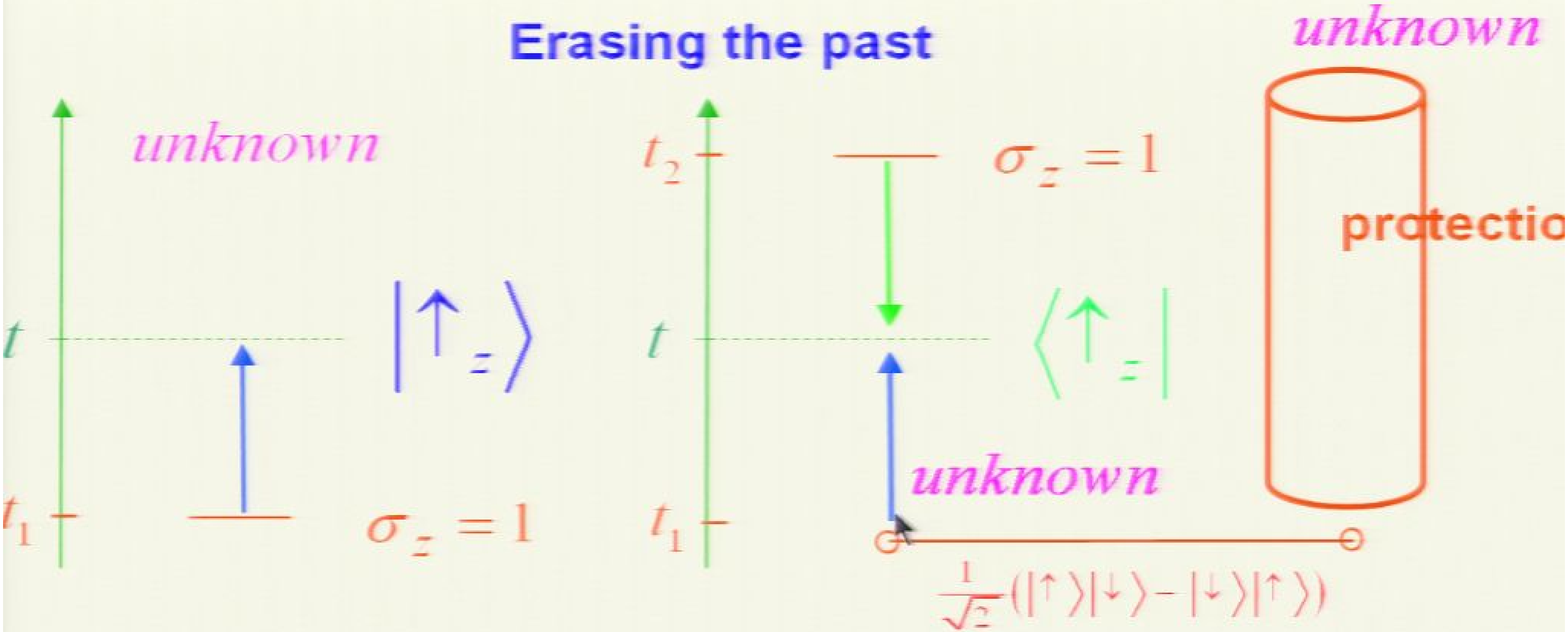

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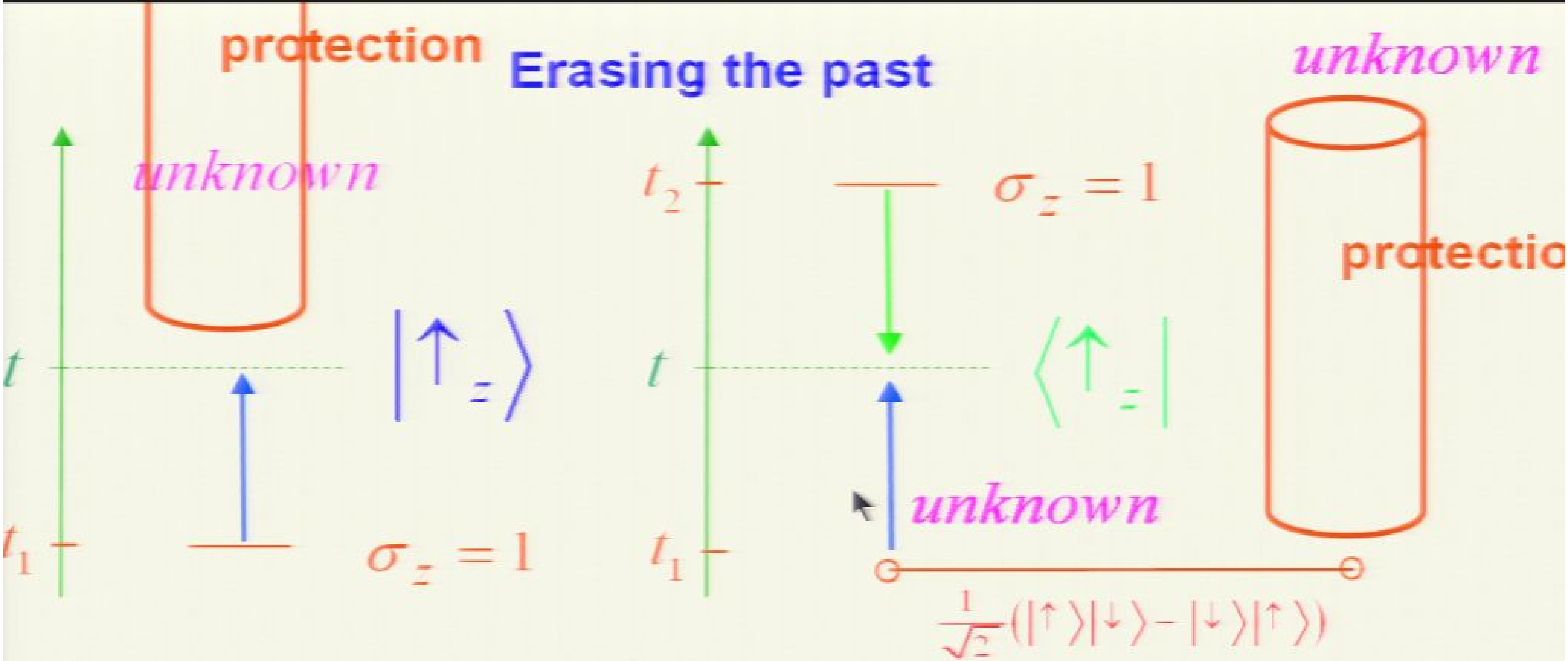
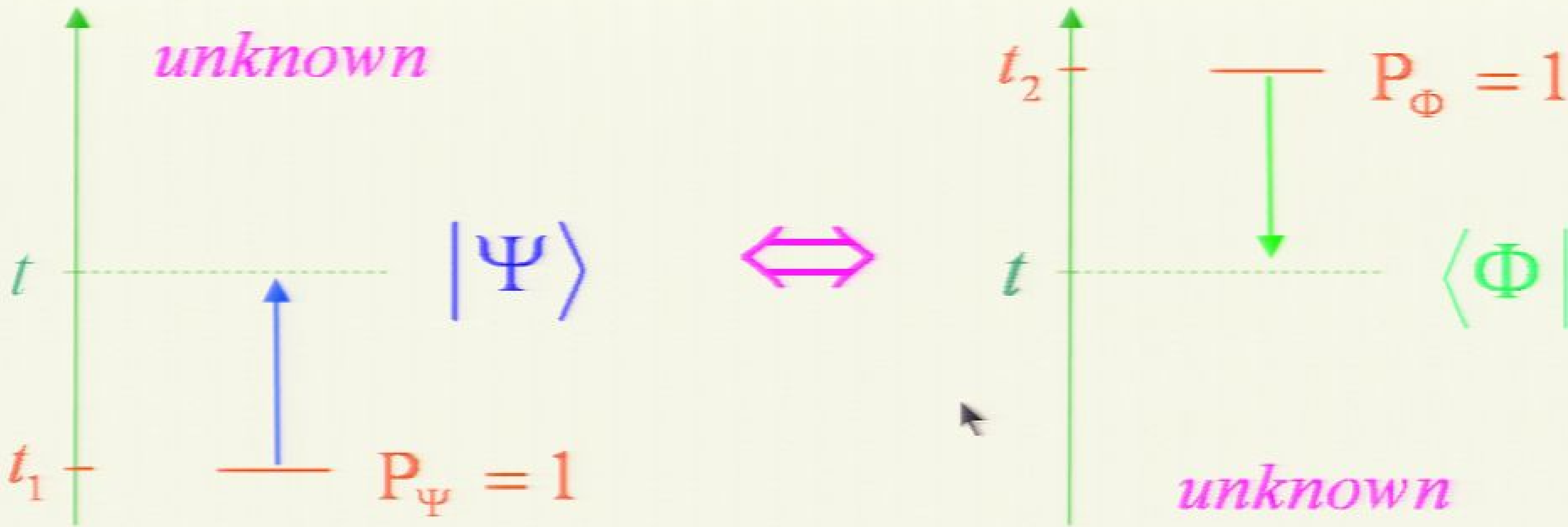
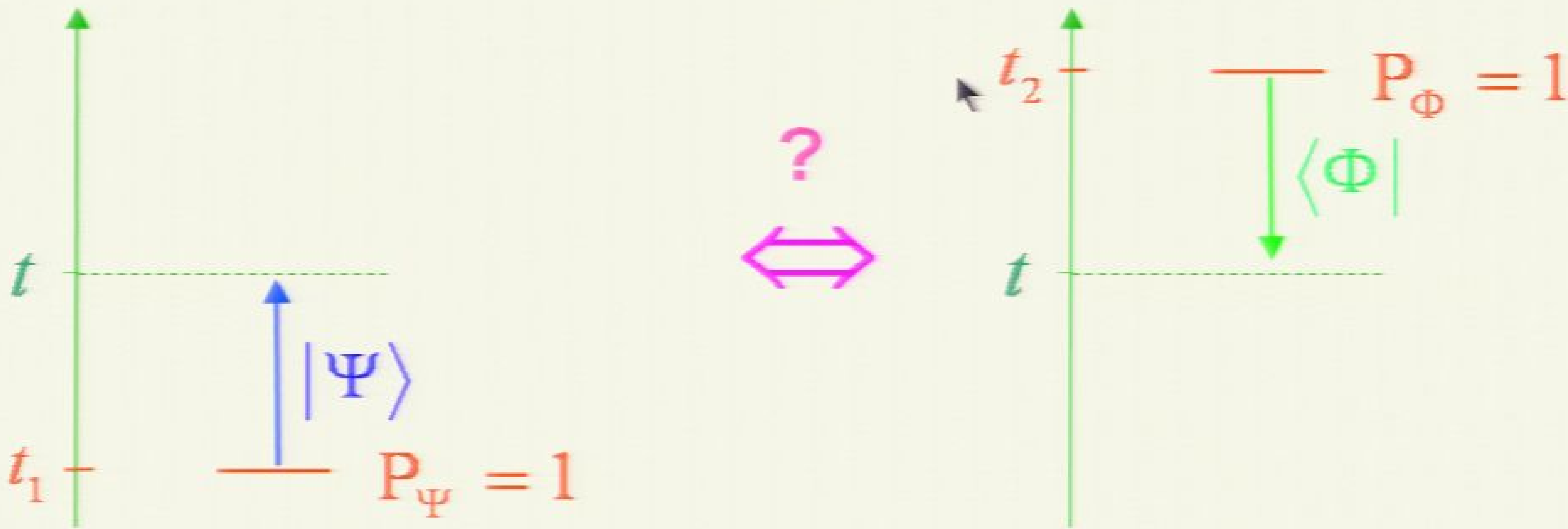



unknown
t
$|\Psi\rangle$
t_1
$P_\Psi = 1$
$\Leftrightarrow$
t_2
$P_\Phi = 1$
t
$\langle\Phi|$
unknown

Are there any differences between what can be done to $\langle\Phi|$ and $|\Psi\rangle$



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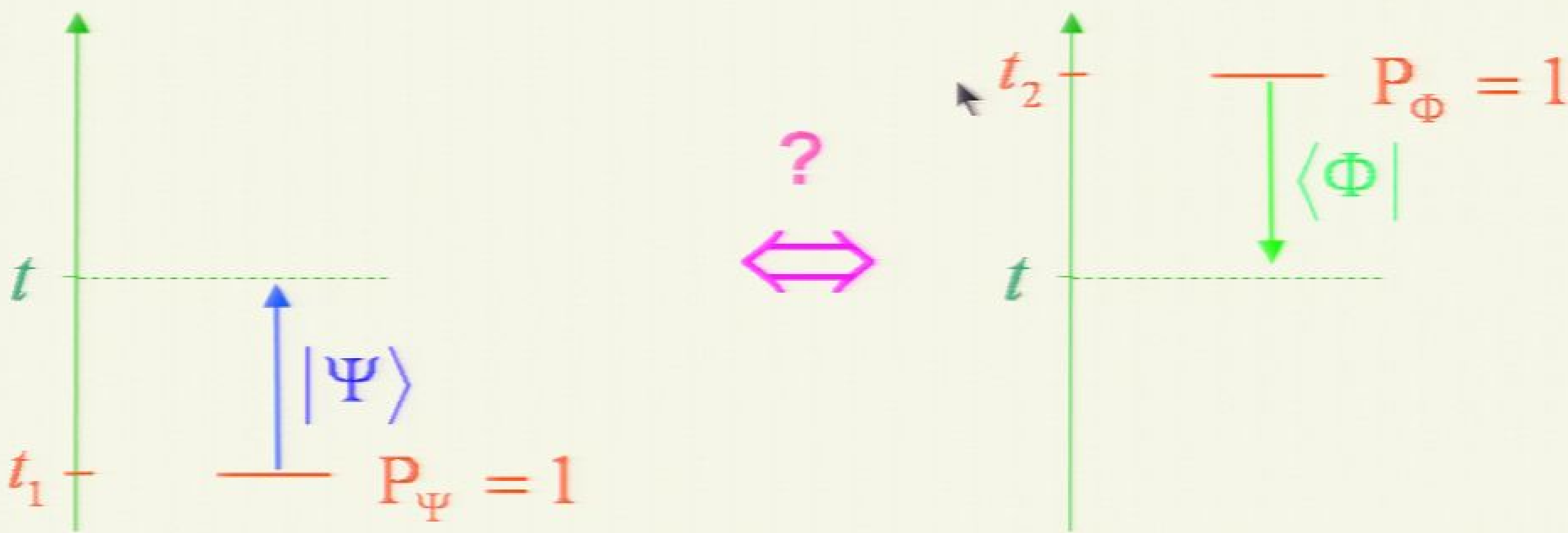
Nondemolition (von Neumann) measurements



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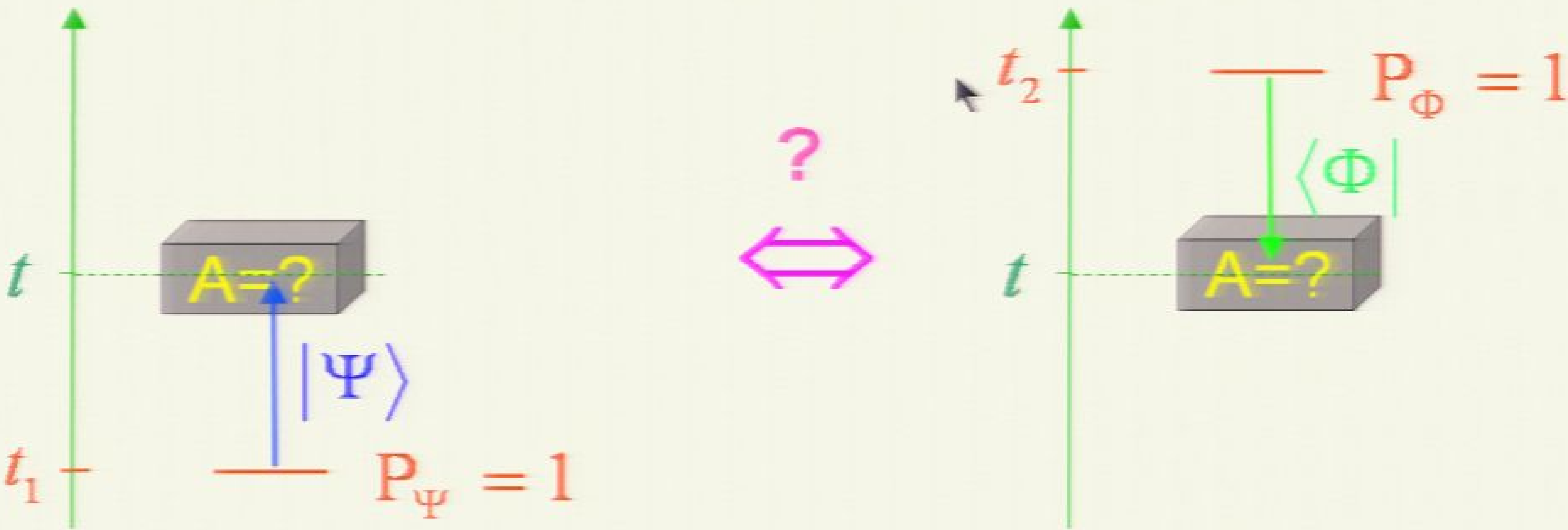

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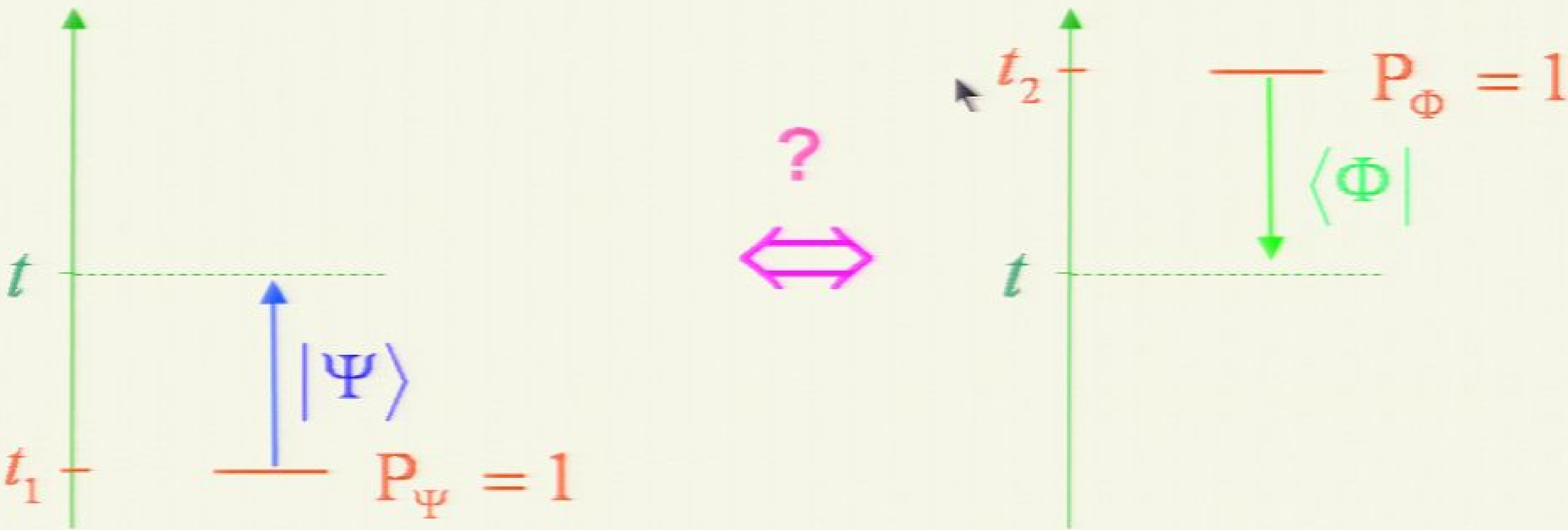
Unitary transformation



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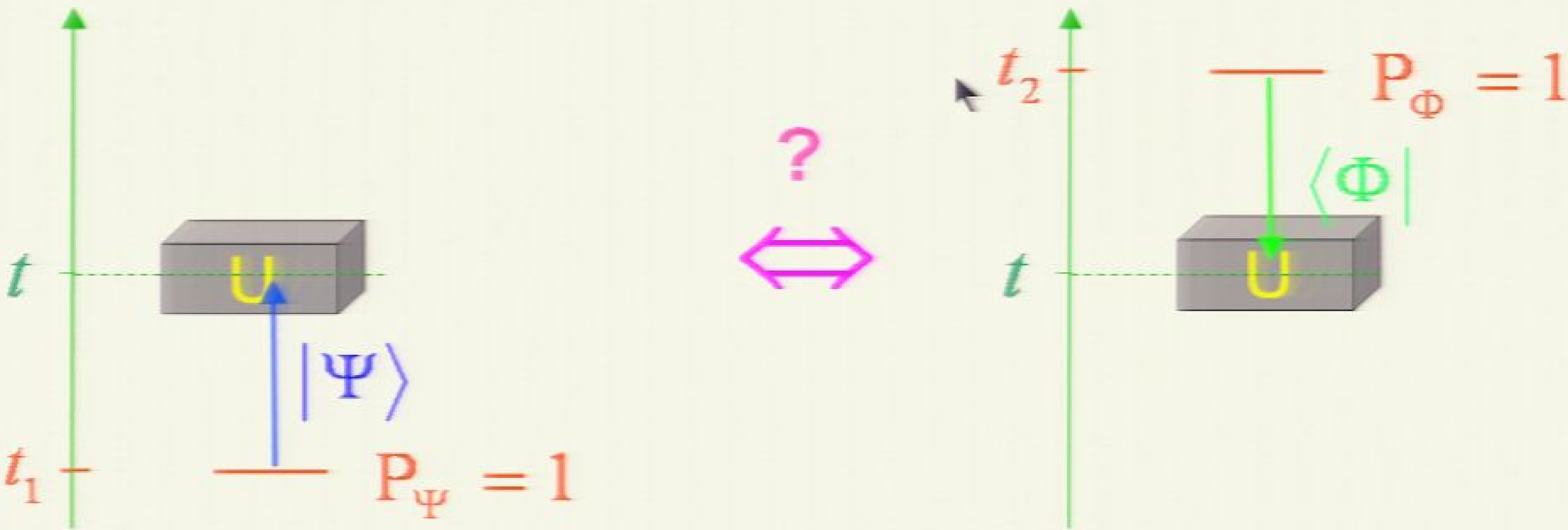

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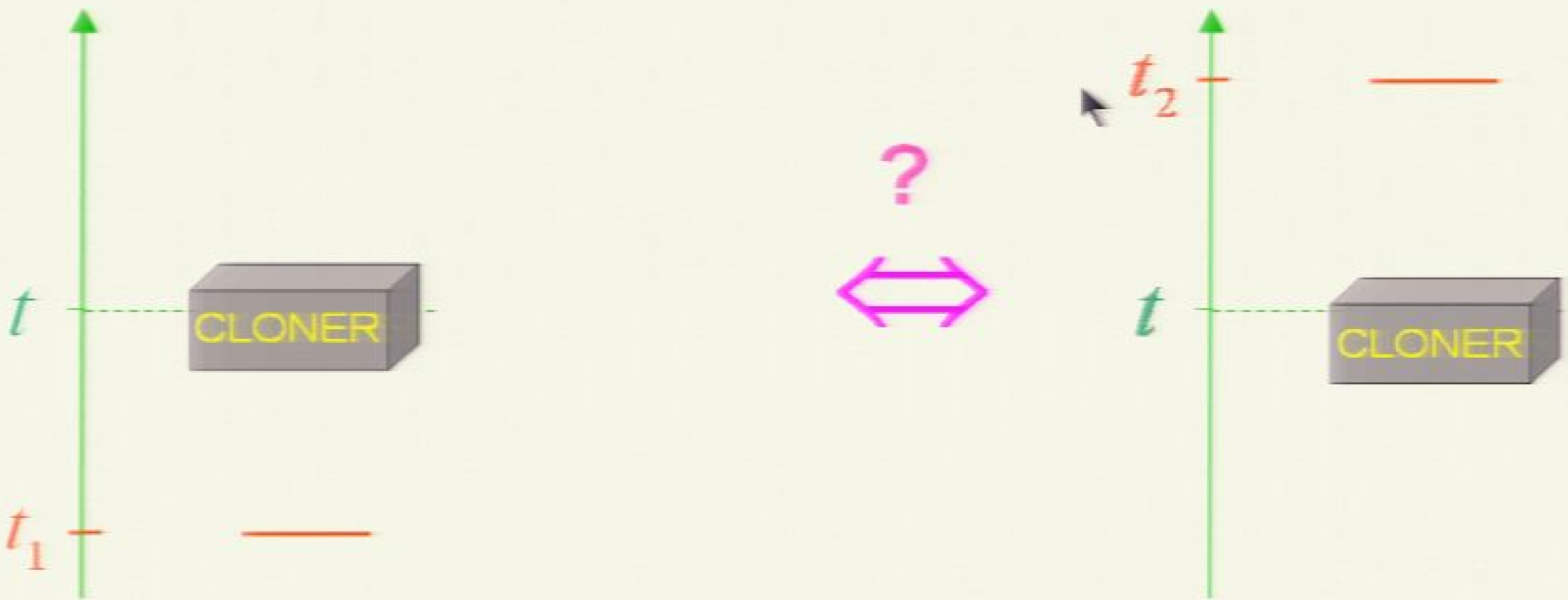
L. Vaidman, J. Phys. A 40, 3275 (2007)

No cloning theorem?



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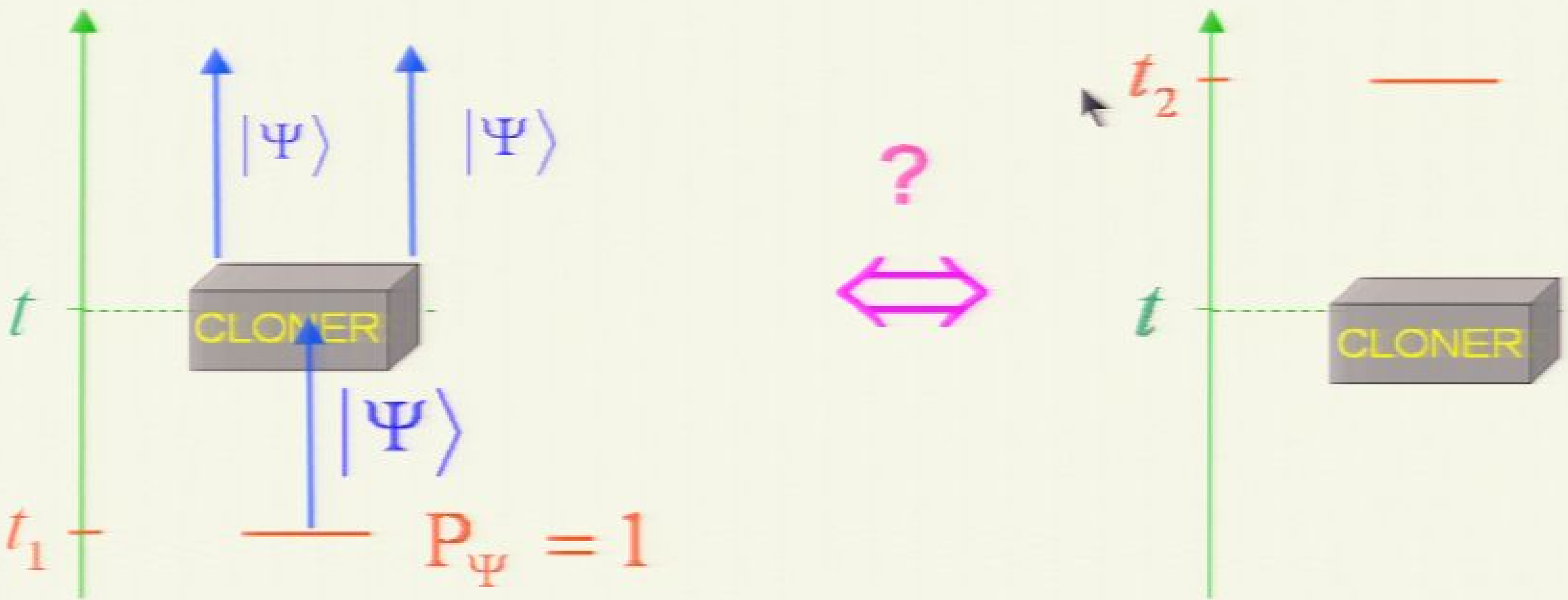

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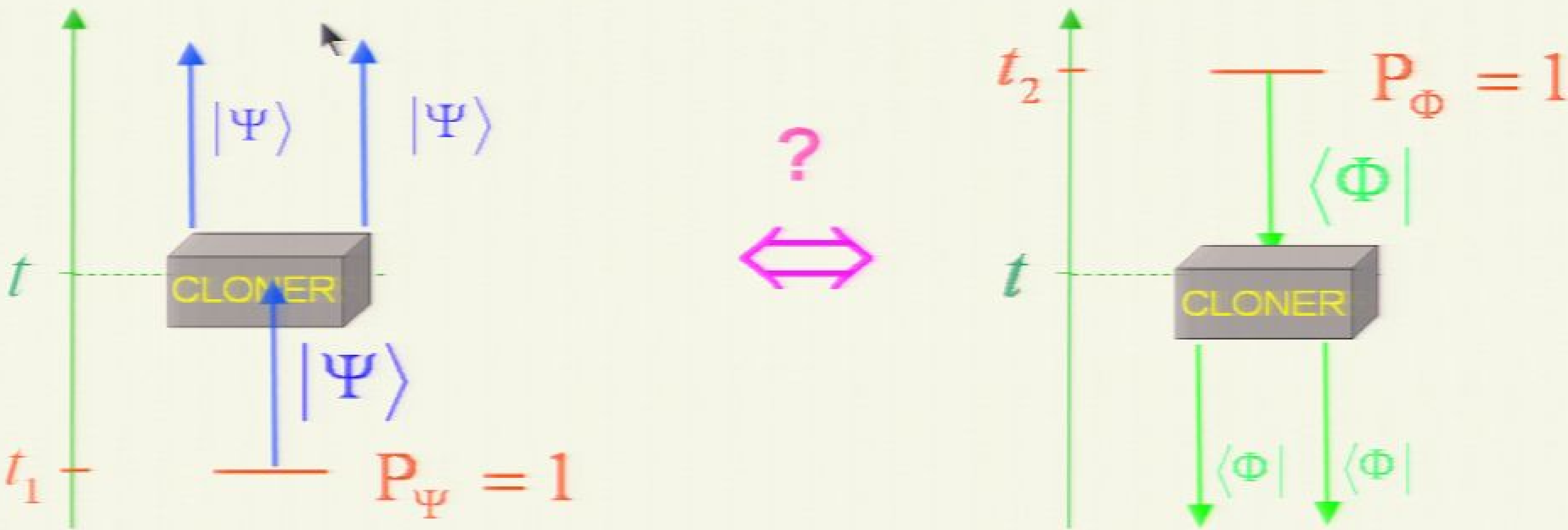

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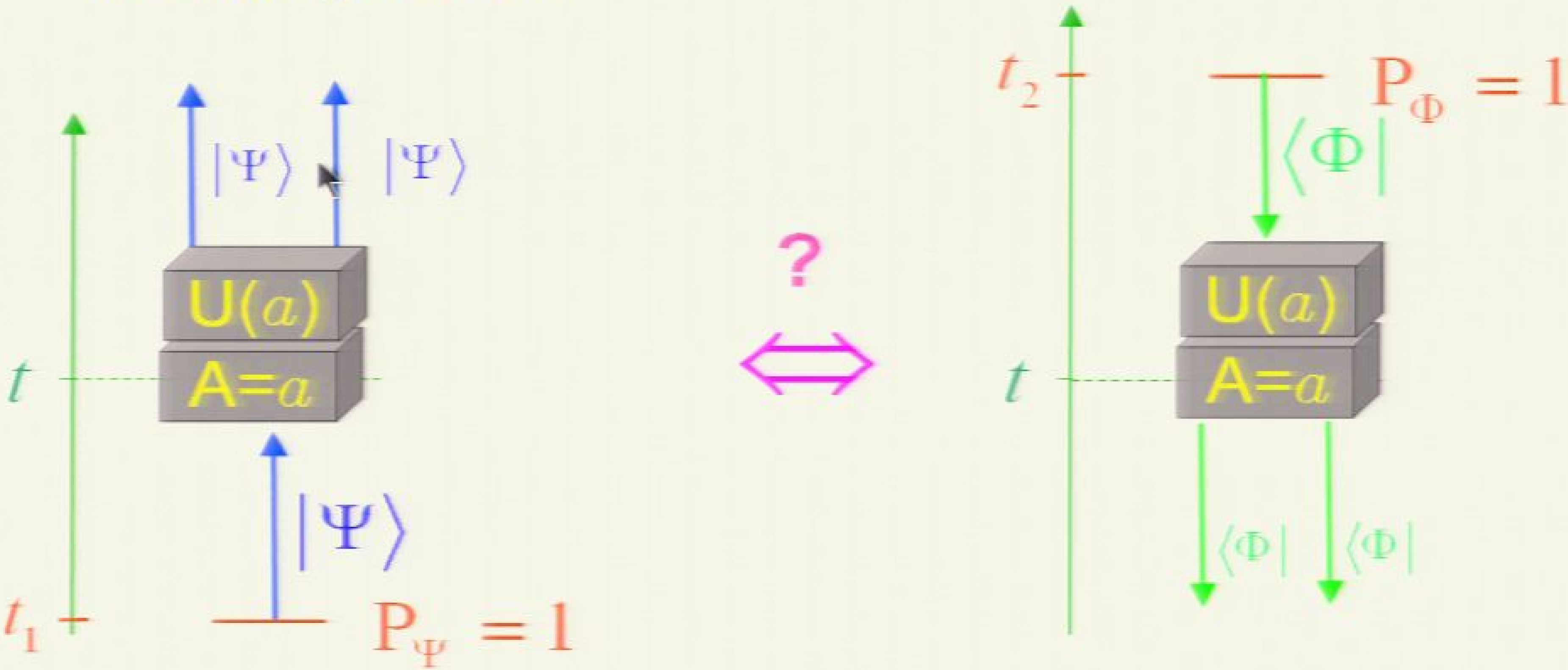
No cloning theorem



Proof of no cloning theorem for backward evolving quantum state

Proof of no cloning theorem for backward evolving quantum state

Cloner exists $\Longrightarrow$ We can send signals to the past

Proof of no cloning theorem for backward evolving quantum state

Cloner exists $\Rightarrow$ We can send signals to the past $\Rightarrow$ Cloner does not exist

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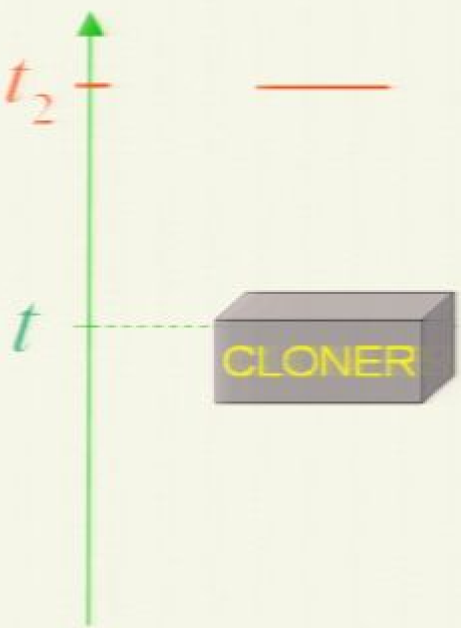

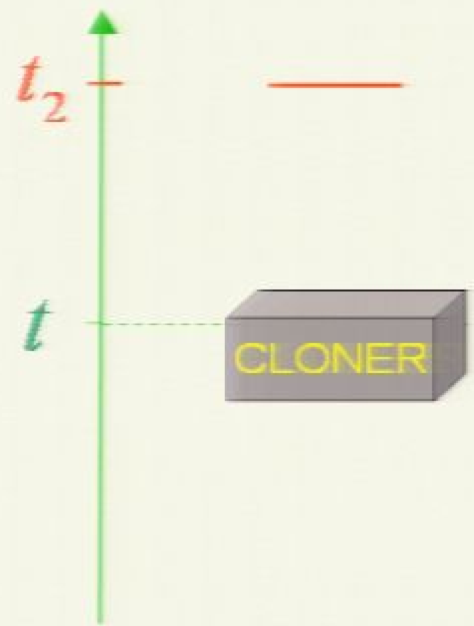

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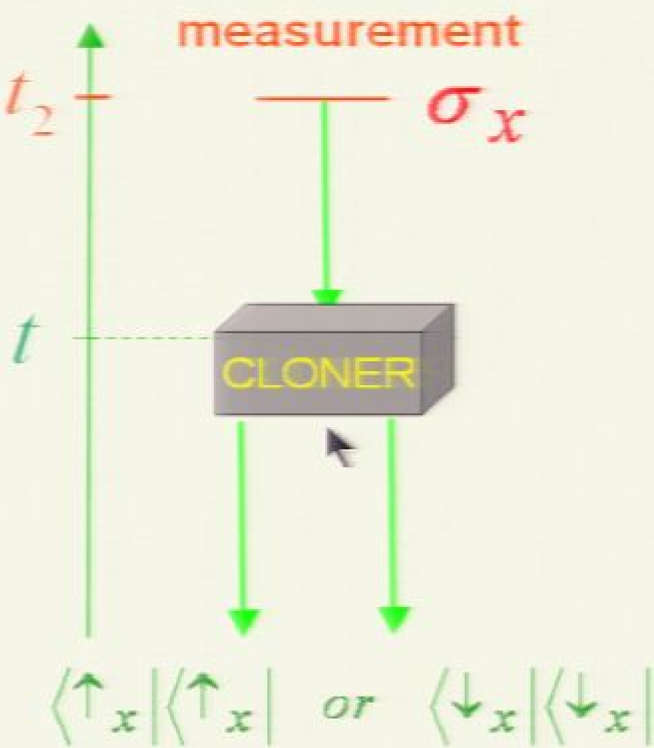

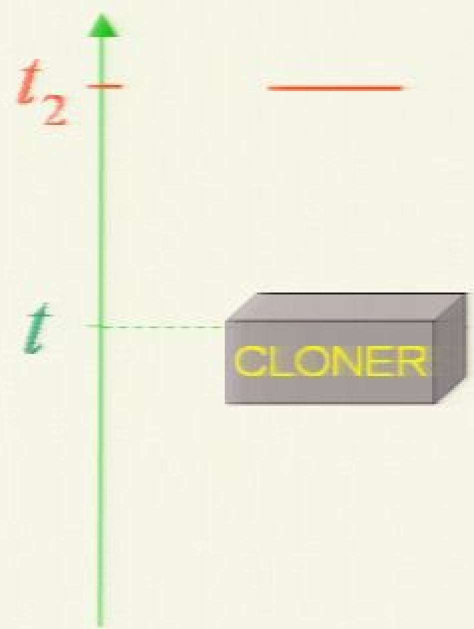

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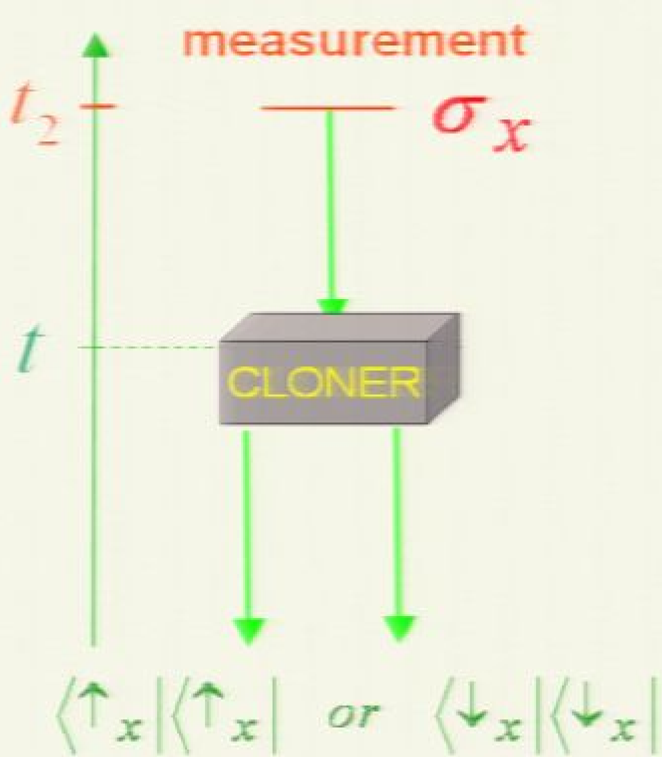
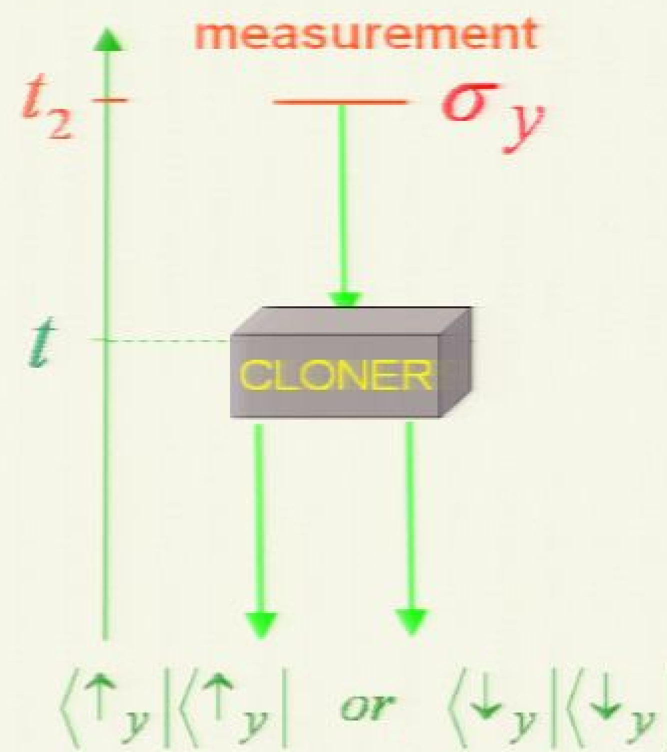
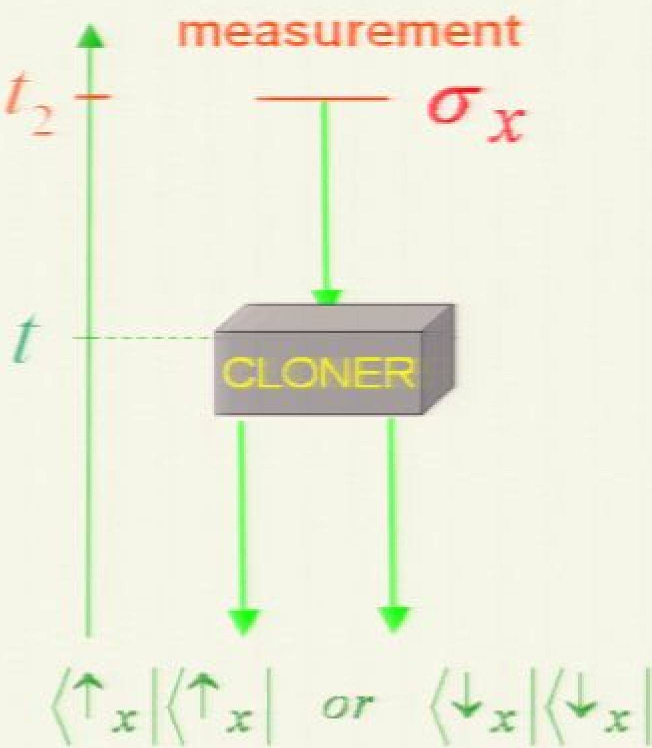


or



Proof of no cloning theorem for backward evolving quantum state

Cloner exists $\Longrightarrow$ We can send signals to the past $\Longrightarrow$ Cloner does not exist



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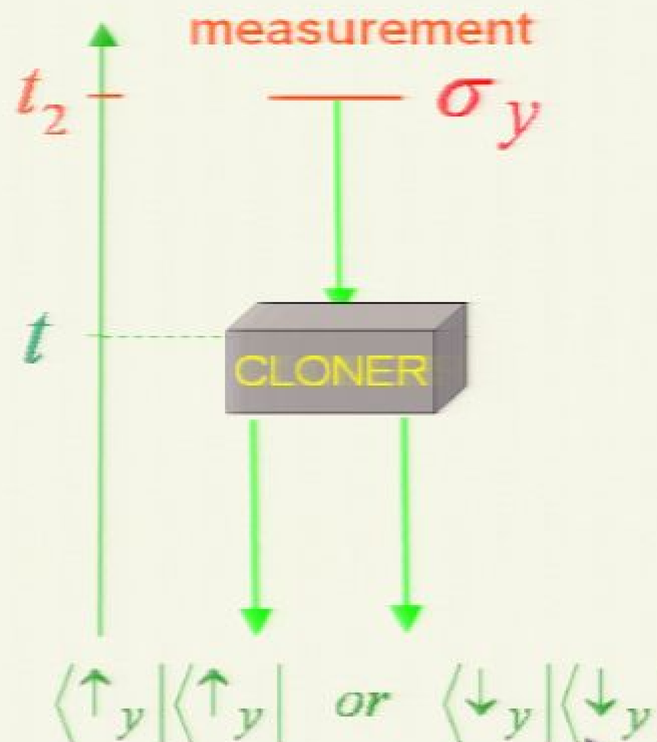

mixture of $\langle\uparrow_x|\langle\uparrow_x|$ *and* $\langle\downarrow_x|\langle\downarrow_x|$ $\neq$ mixture of $\langle\uparrow_y|\langle\uparrow_y|$ *and* $\langle\downarrow_y|\langle\downarrow_y|$

Are there any differences between what can be done to $\langle\Phi|$ and $|\Psi\rangle$

Nondemolition (von Neumann) measurements	No
Unitary transformation	No
No cloning theorem	No
Teleportation	
Nonlocal nondemolition measurements Aharonov, Albert, and Vaidman, *PRD* 34, 1805 (1986)	
Nonlocal demolition measurements Vaidman, *PRL* 90, 010402 (2003)	

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Vaidman and Nevo, *IJMP* B 20, 1528 (2006)

Nonlocal demolition measurements

Easier for backwards evolving states

$\langle \Phi | \longrightarrow | \Phi^* \rangle$ Possible

$| \Psi \rangle \longrightarrow \langle \Psi^* |$ Impossible

Are there any differences between what can be done to $\langle\Phi|$ and $|\Psi\rangle$

Vaidman and Nevo, *IJMP* B 20, 1528 (2006)

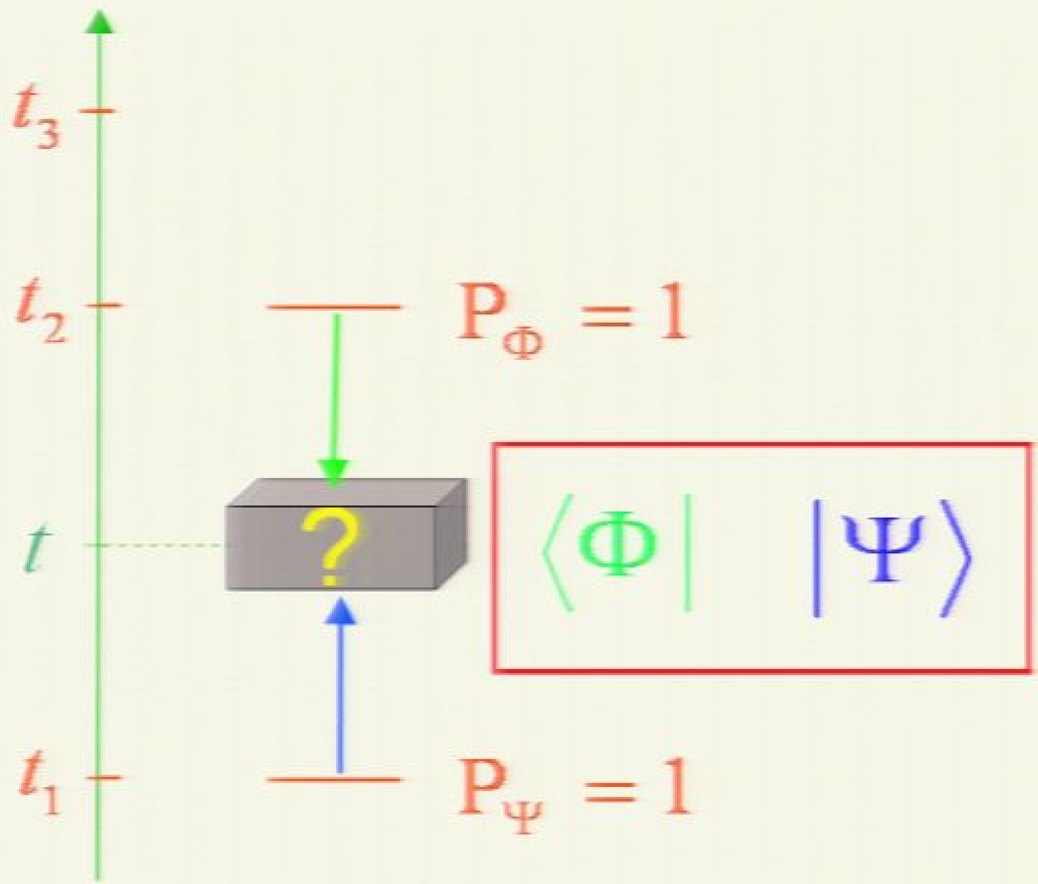
Nonlocal demolition measurements

Easier for backwards evolving states

$\langle\Phi| \longrightarrow |\Phi^*\rangle$ **Possible**

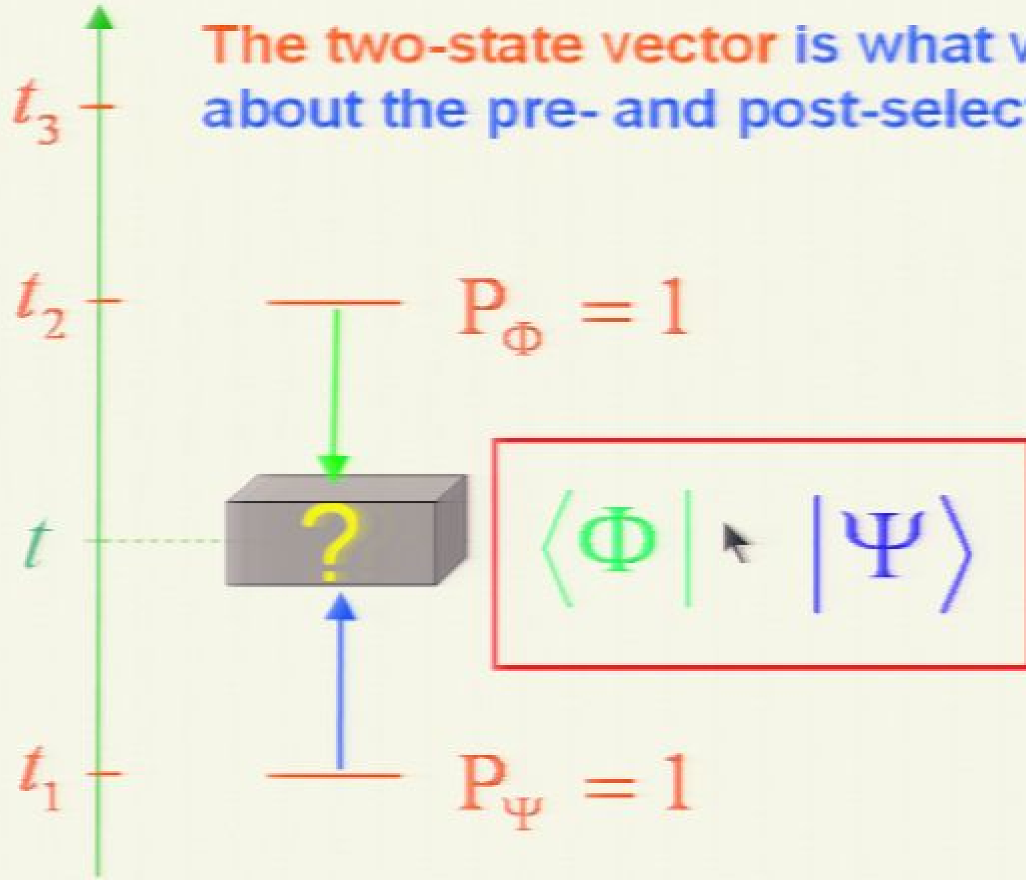
$|\Psi\rangle \longrightarrow \langle\Psi^*|$ **Impossible**

The two-state vector is a complete description of a system at time t



The two-state vector is a complete description of a system at time *t*

The two-state vector is what we can say now (t_3) about the pre- and post-selected system at time *t*



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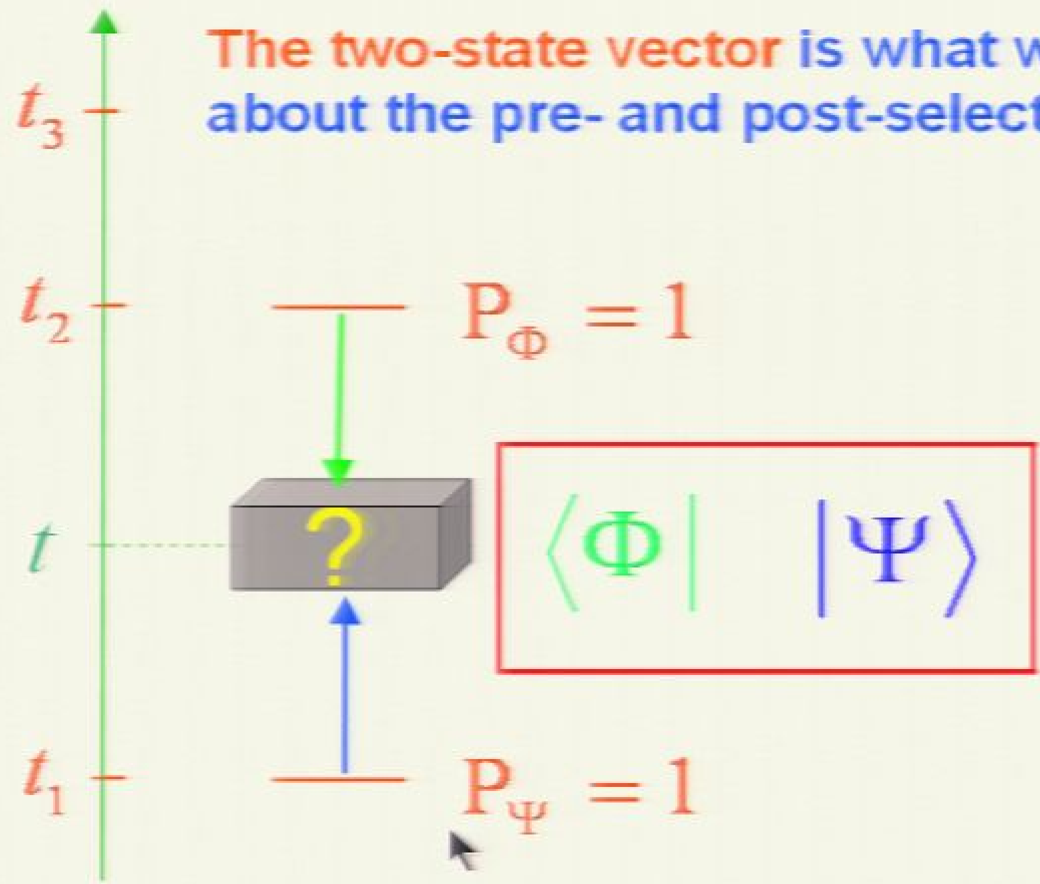
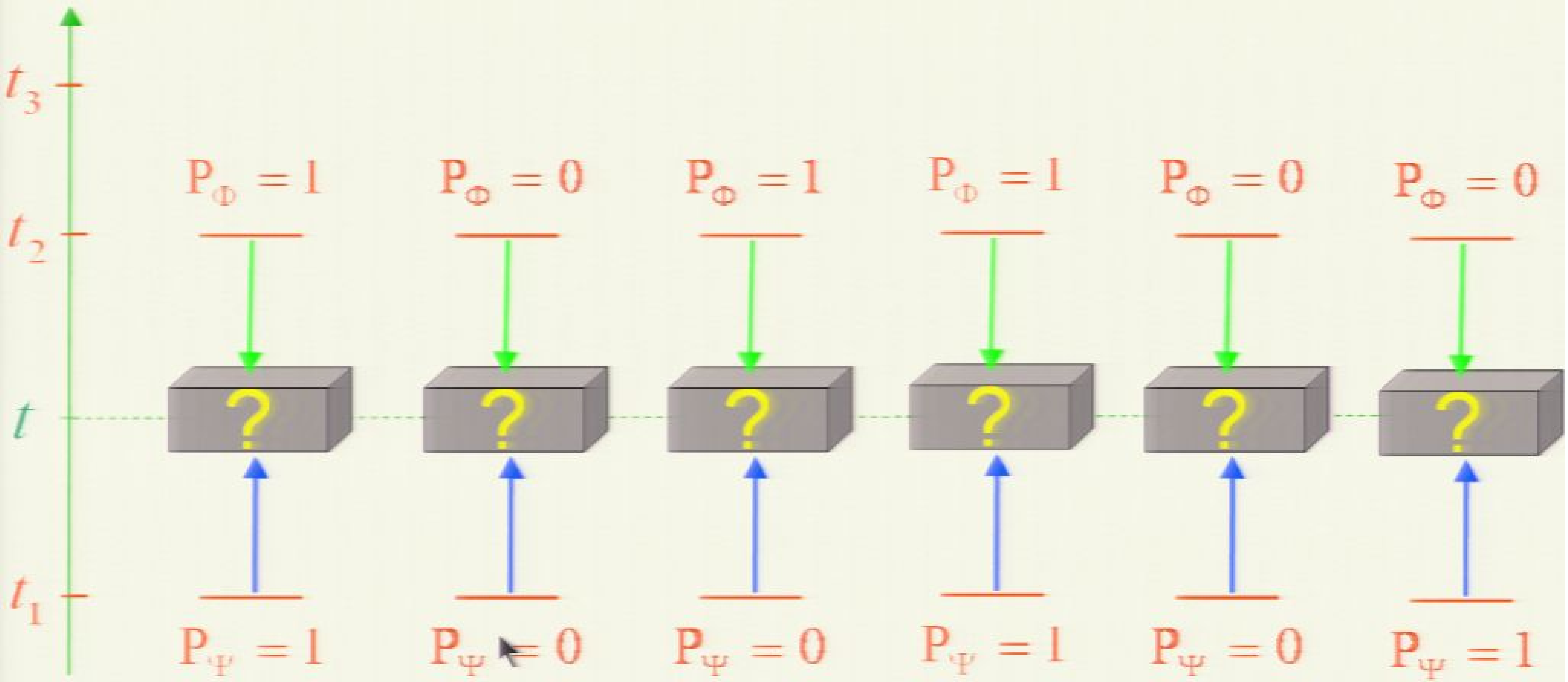
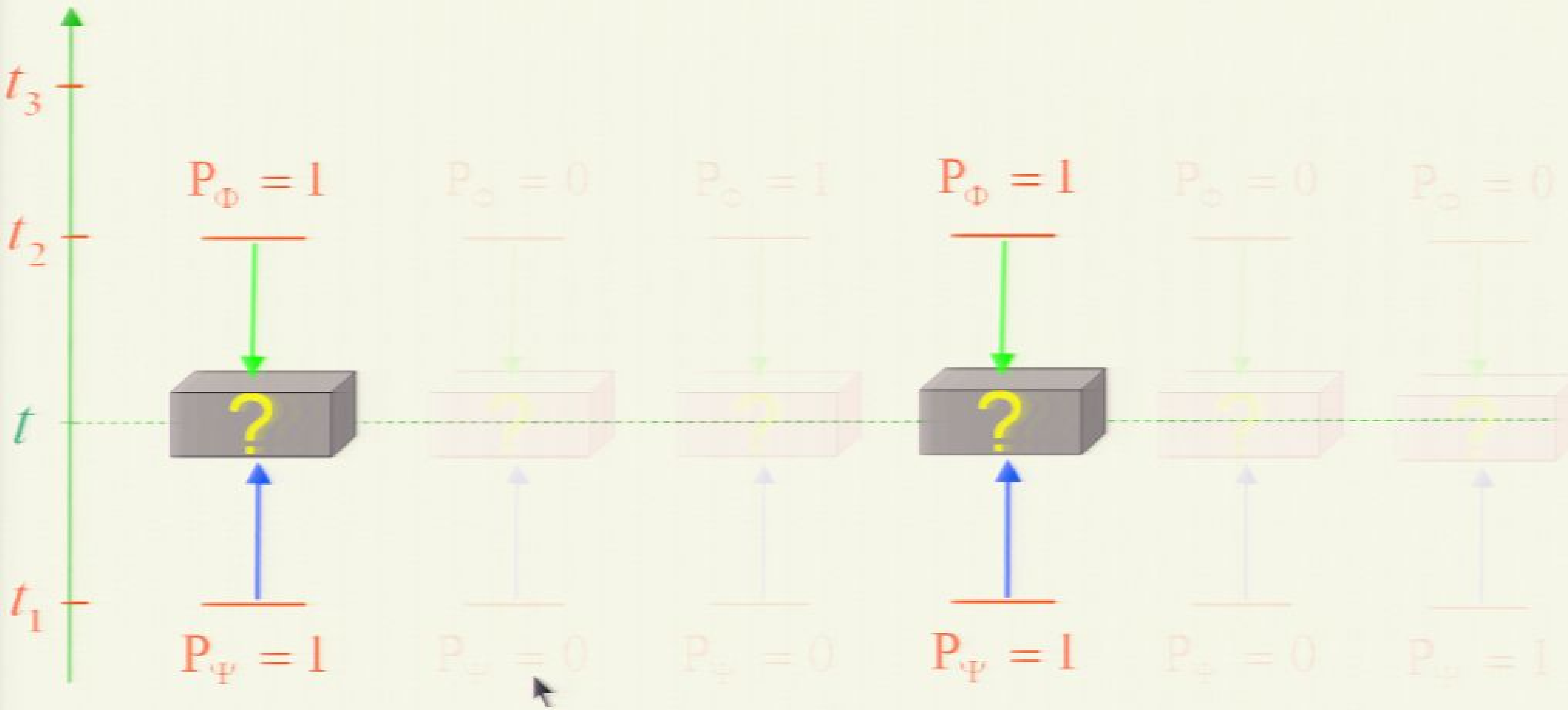


The two-state vector describes a single pre- and post-selected system, but to test predictions of the two-state vector we need a pre- and post-selected ensemble

The pre- and post-selected ensemble



The pre- and post-selected ensemble



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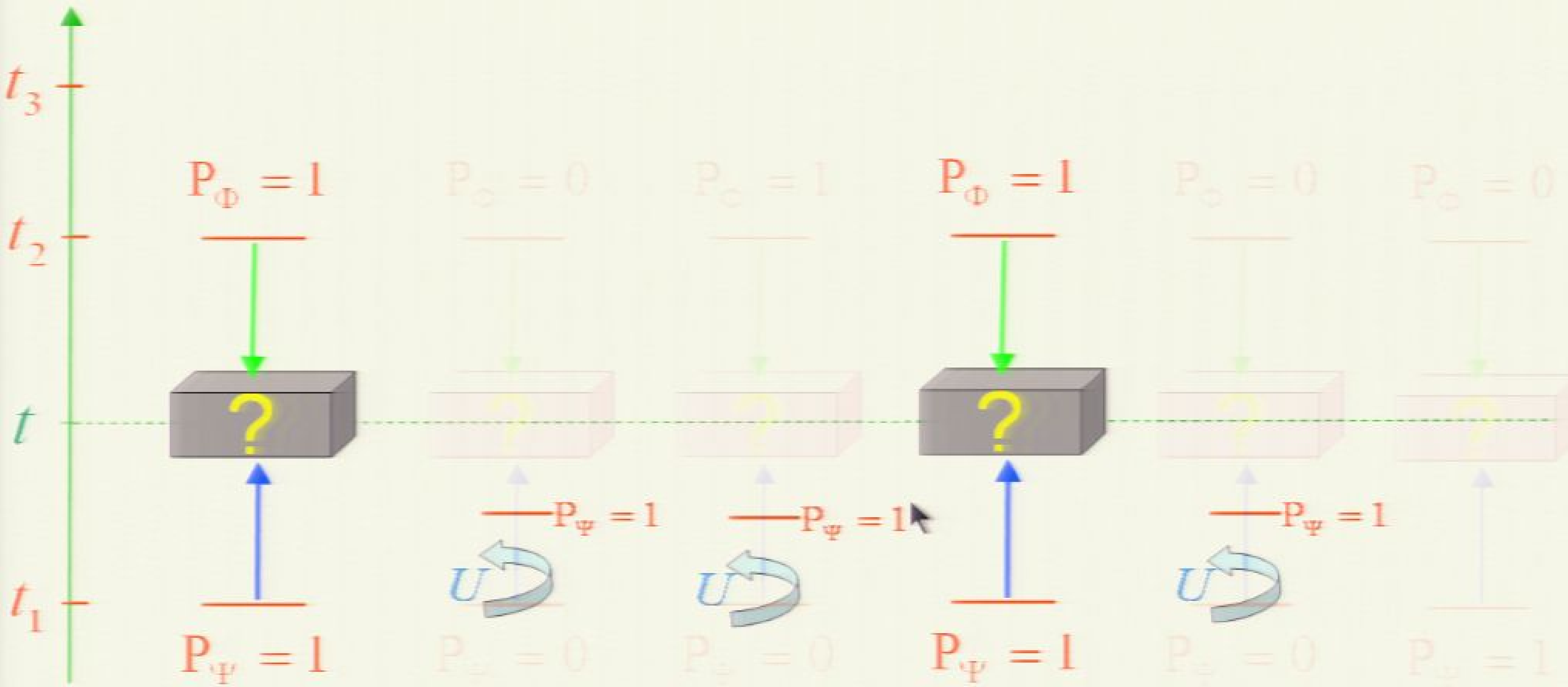
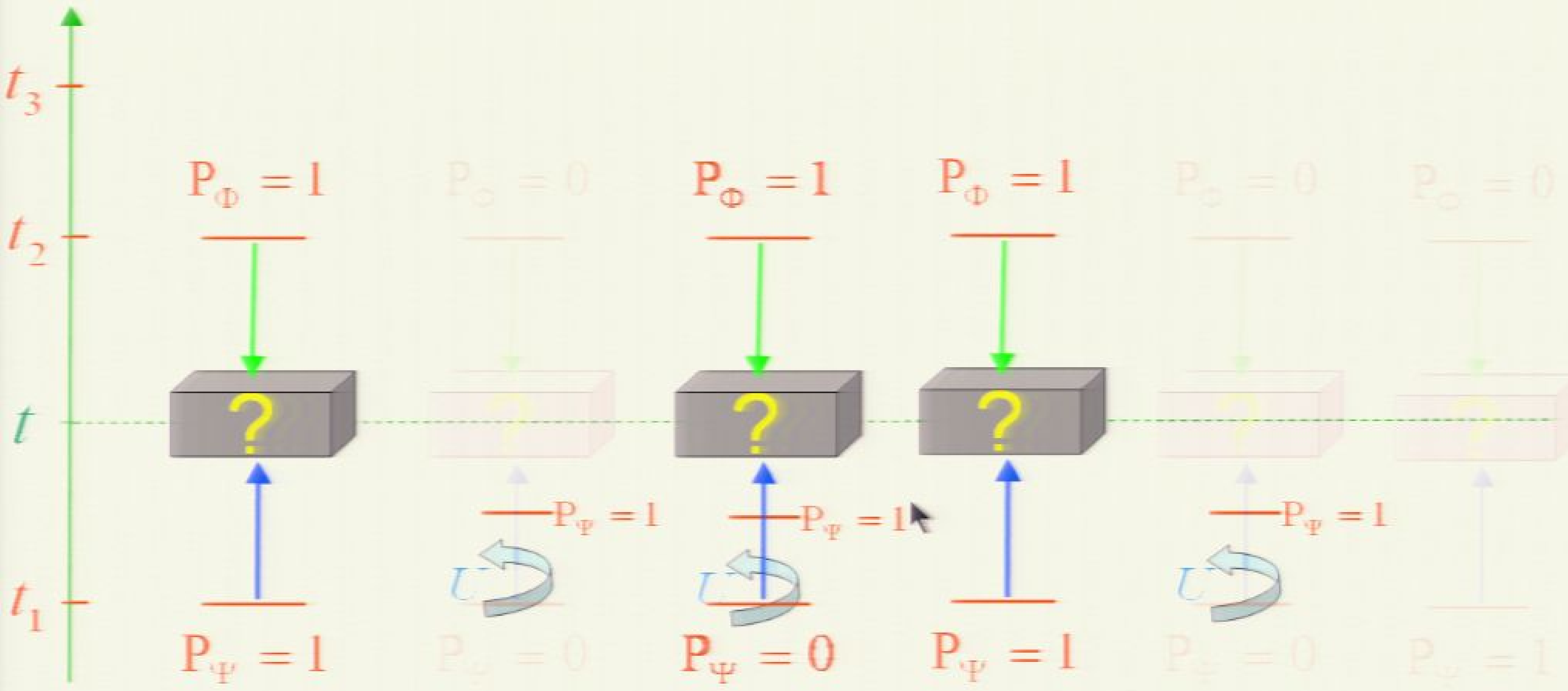
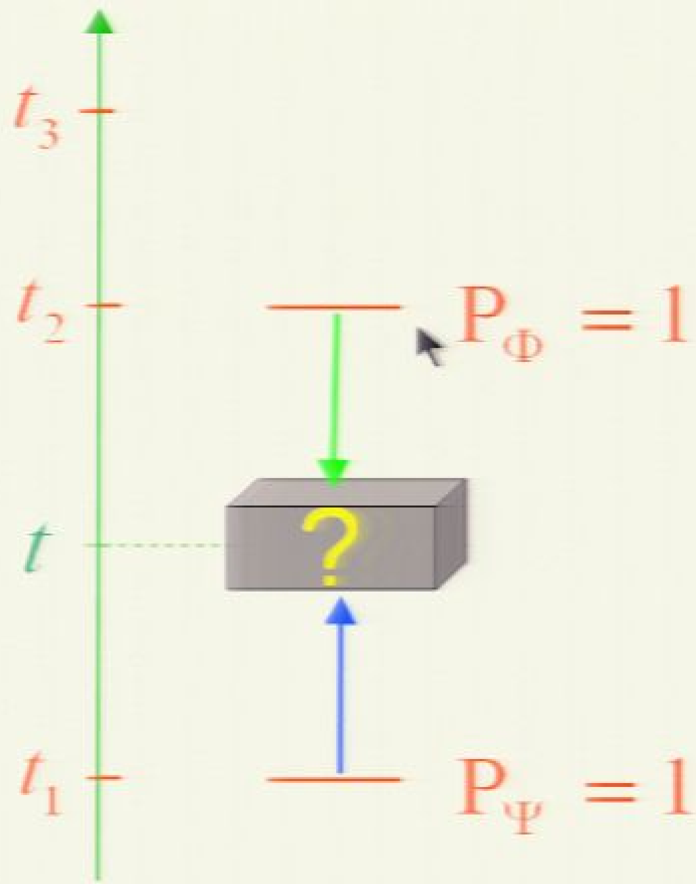


The pre- and post-selected ensemble

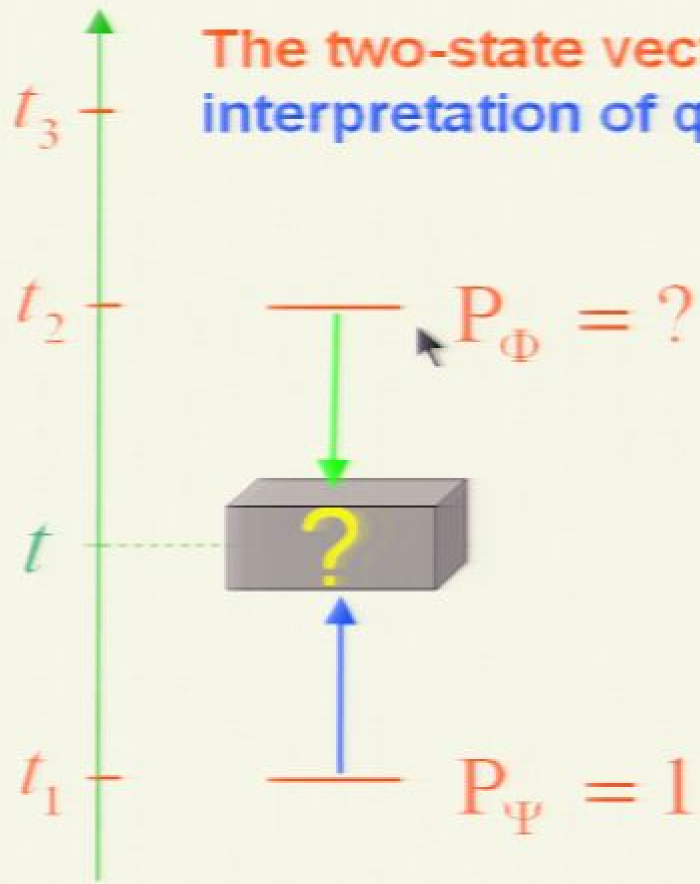


A single pre- and post-selected system



A single pre- and post-selected system

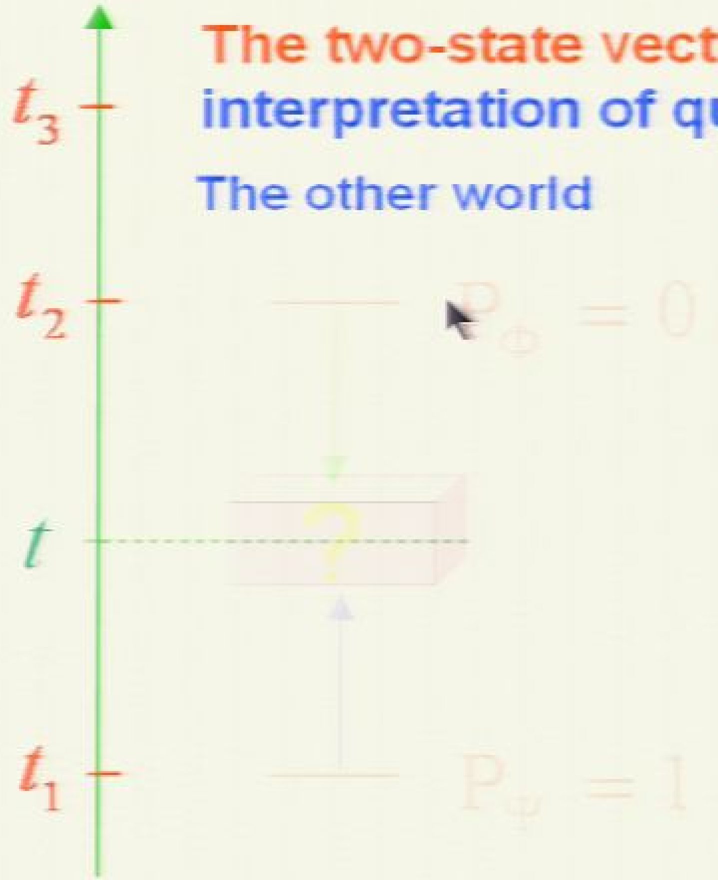
The two-state vector in the framework of the many-worlds interpretation of quantum mechanics



A single pre- and post-selected system

The two-state vector in the framework of the many-worlds interpretation of quantum mechanics

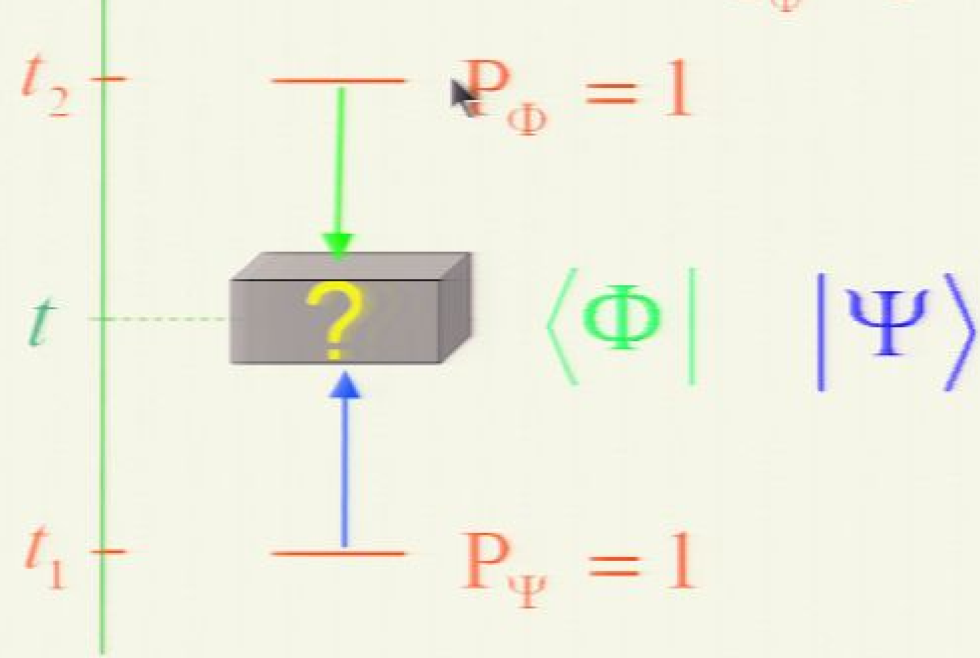
The other world $\quad P_{\Phi} = 0$



A single pre- and post-selected system

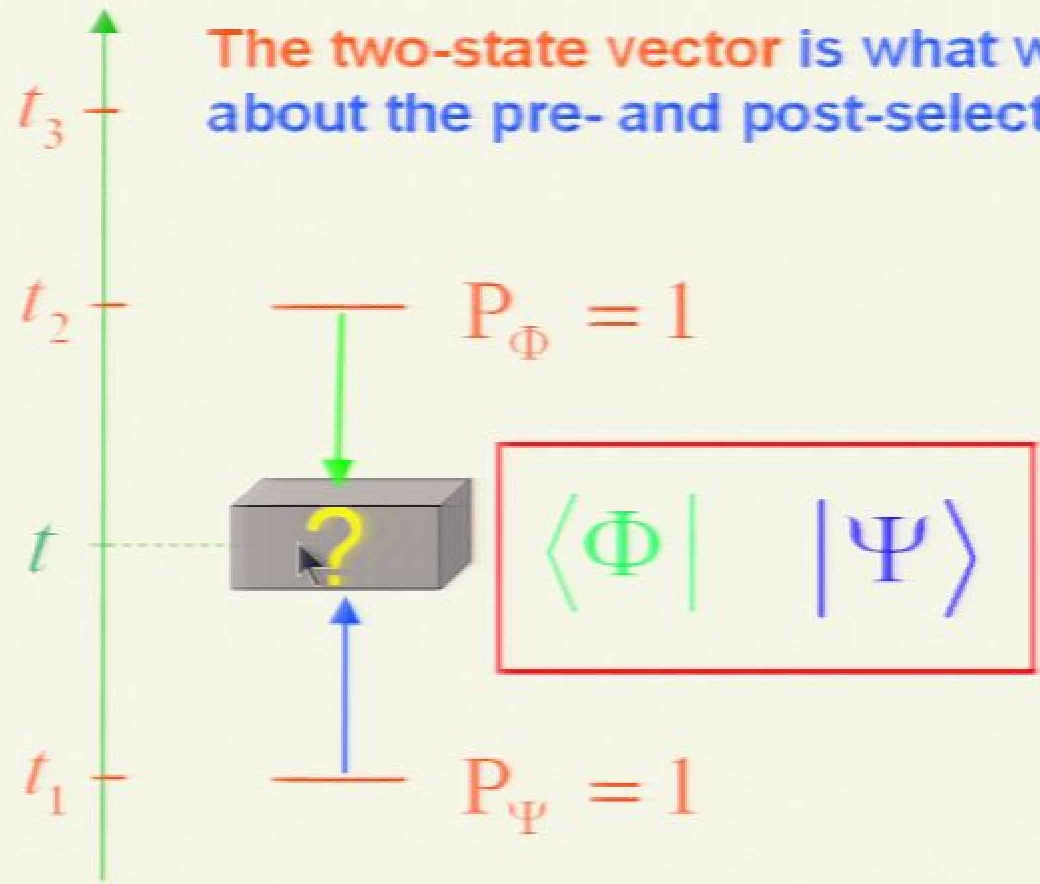
The two-state vector in the framework of the many-worlds interpretation of quantum mechanics

This world $\quad P_\Phi = 1$



The two-state vector is a complete description of a system at time t

The two-state vector is what we can say now (t_3) about the pre- and post-selected system at time t



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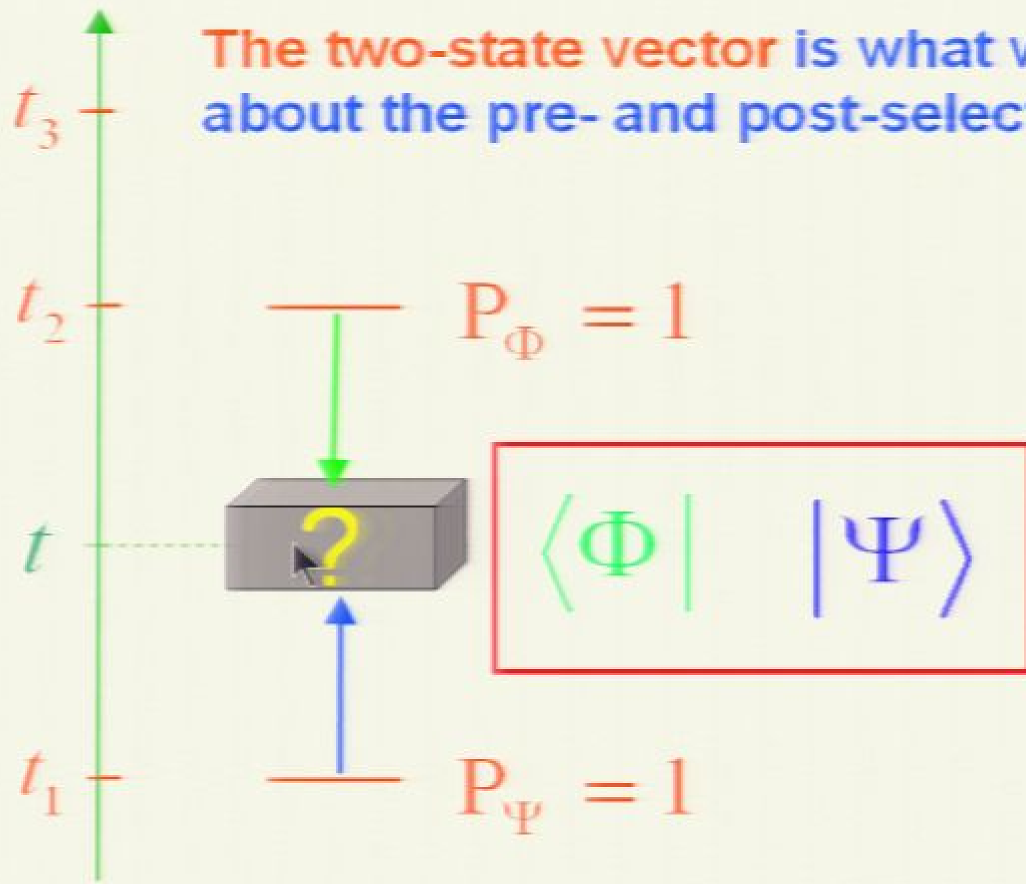
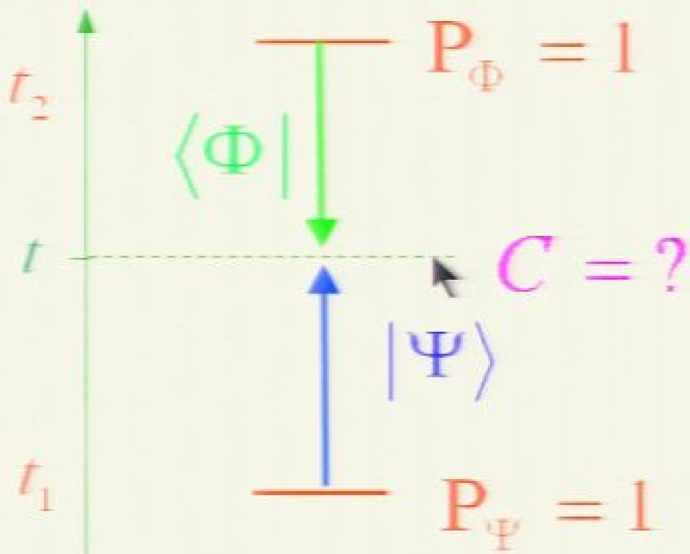


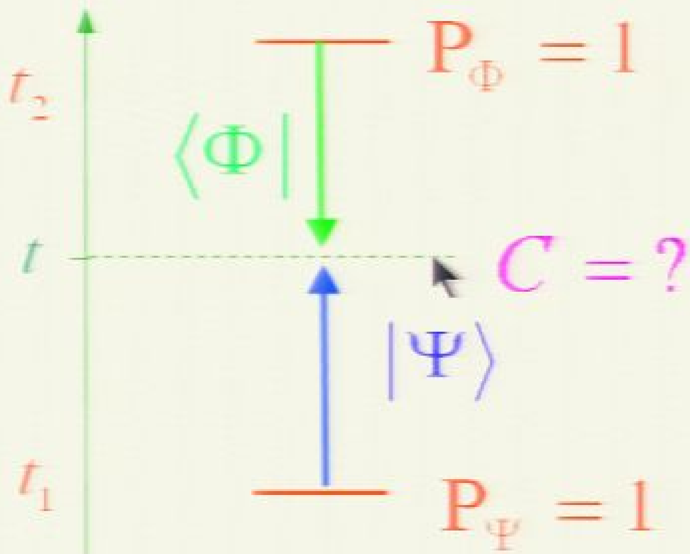
So, what can we say?

Measurements performed on a pre- and post-selected system described by the two-state vector: $\langle\Phi| \quad |\Psi\rangle$



Measurements performed on a pre- and post-selected system described by the two-state vector: $\langle \Phi | \quad | \Psi \rangle$

The Aharonov-Bergmann-Lebowitz (ABL) formula:



$$\text{Prob}(C = c) = \frac{\left| \langle \Phi | \text{P}_{C=c} | \Psi \rangle \right|^2}{\sum_i \left| \langle \Phi | \text{P}_{C=c_i} | \Psi \rangle \right|^2}$$

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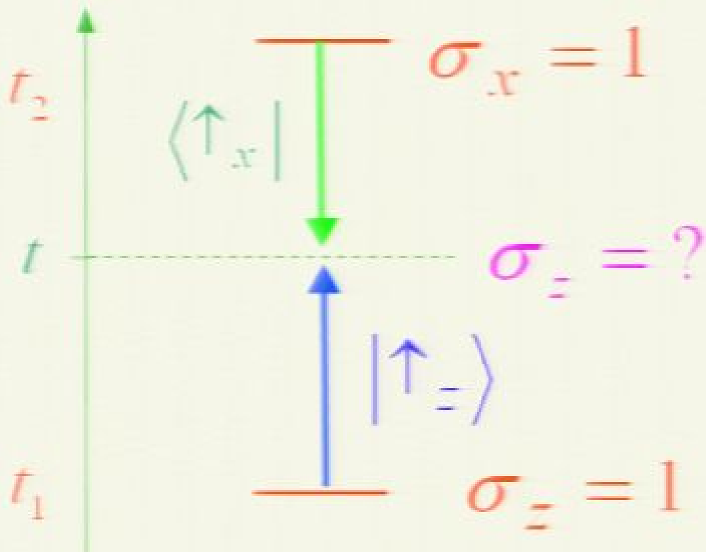

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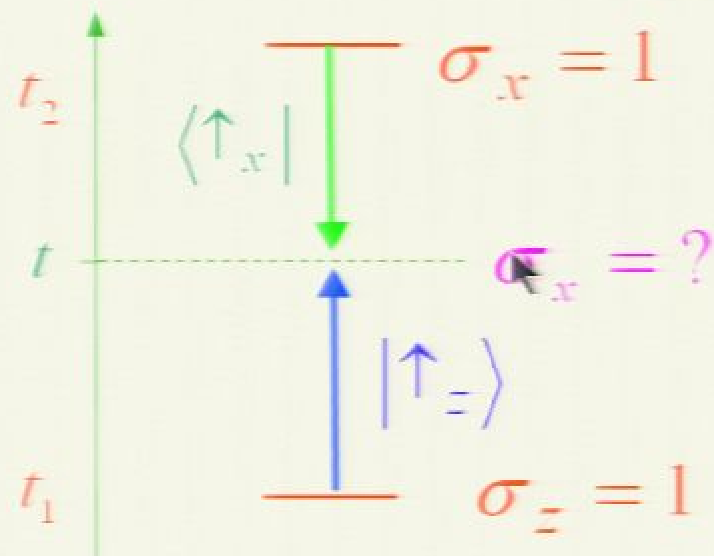
$$\text{Prob}(\uparrow_z) = \frac{\left| \langle \uparrow_x | \mathrm{P}_{\uparrow_z} | \uparrow_z \rangle \right|^2}{\left| \langle \uparrow_x | \mathrm{P}_{\uparrow_z} | \uparrow_z \rangle \right|^2 + \left| \langle \uparrow_x | \mathrm{P}_{\downarrow_z} | \uparrow_z \rangle \right|^2} = 1$$

Measurements performed on a pre- and post-selected system described by the two-state vector: $\langle \Phi | \quad | \Psi \rangle$

The Aharonov-Bergmann-Lebowitz (ABL) formula:



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Measurements performed on a pre- and post-selected system described by the two-state vector: $\langle\Phi| \quad |\Psi\rangle$

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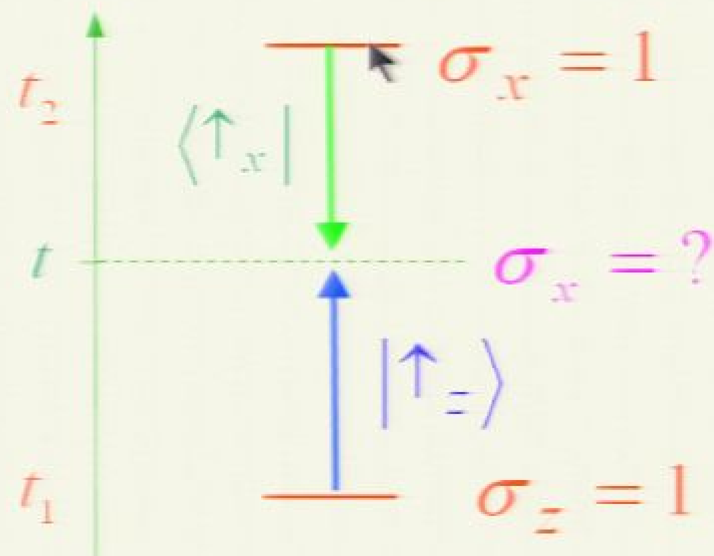

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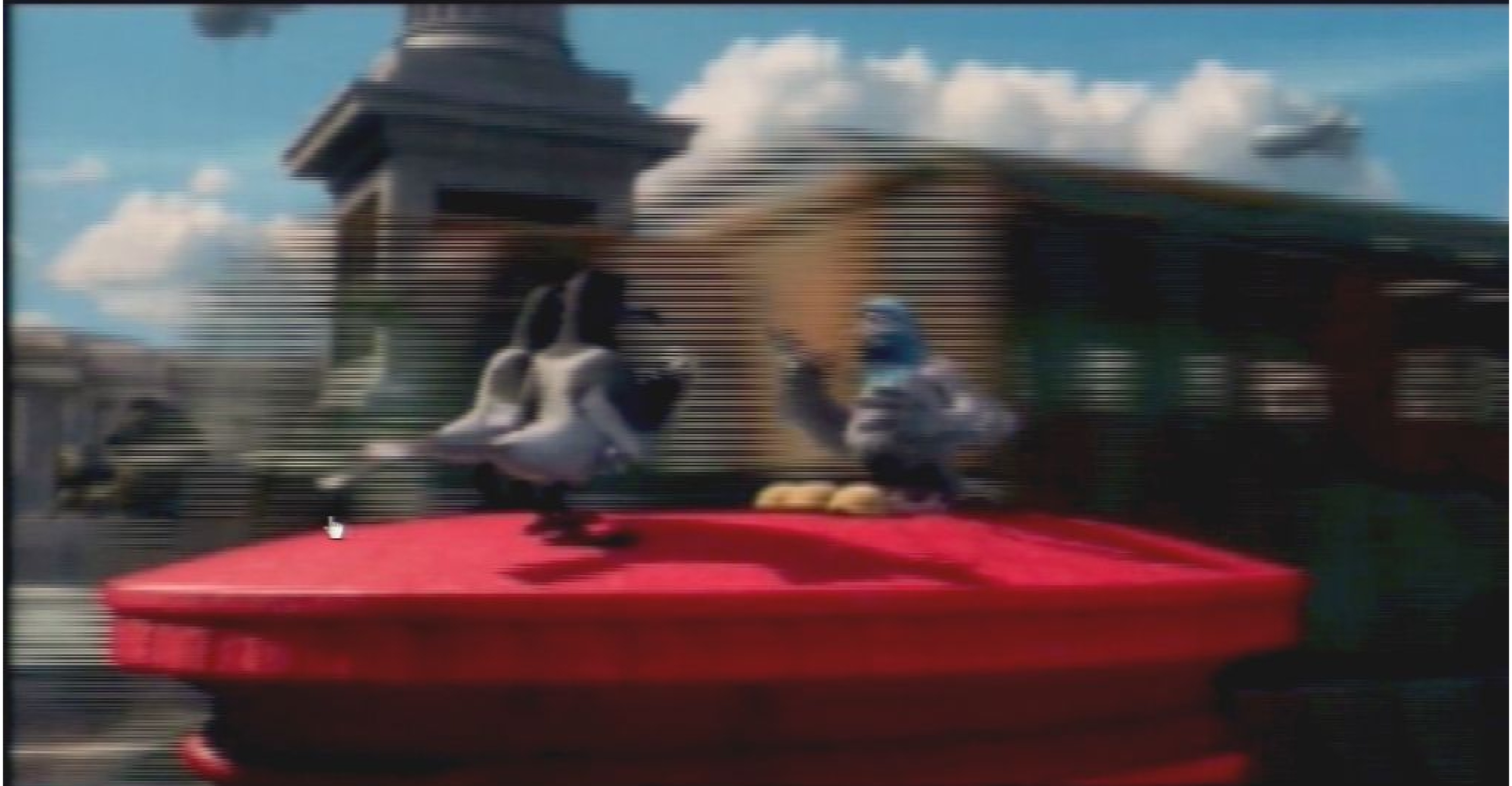
Can we arrange at time *t*:

$$\begin{cases}\sigma_x = 1 \\ \sigma_z = 1 \\ \sigma_y = 1\end{cases} \quad ?$$

$$\mathrm{Prob}(\uparrow_x) = \frac{\left|\langle\uparrow_x|\mathrm{P}_{\uparrow_x}|\uparrow_z\rangle\right|^2}{\left|\langle\uparrow_x|\mathrm{P}_{\uparrow_x}|\uparrow_z\rangle\right|^2 + \left|\langle\uparrow_x|\mathrm{P}_{\downarrow_x}|\uparrow_z\rangle\right|^2} = 1$$

PRL 58, 1385 (1987)


3 CUPS GAME



NEW RULES

You win when you do **not** find the ball

NEW RULES

You win when you do **not** find the ball

You can look only under one of the **two** cups

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You can look only under one of the **two** cups

The dealer does not see your action, but he can look at the ball later and **cancel** a particular run of the game

NEW RULES

You win when you do **not** find the ball

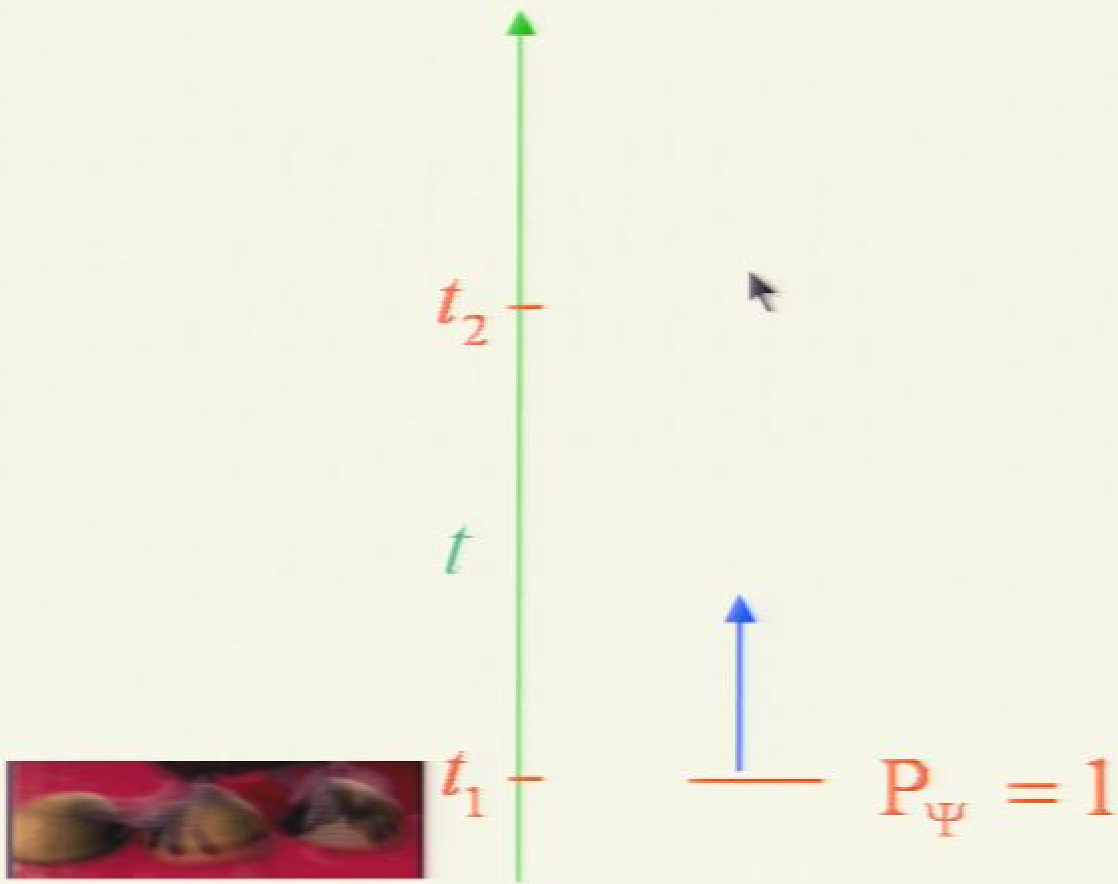
You can look only under one of the **two** cups

The dealer does not see your action, but he can look at the ball later and **cancel** a particular run of the game

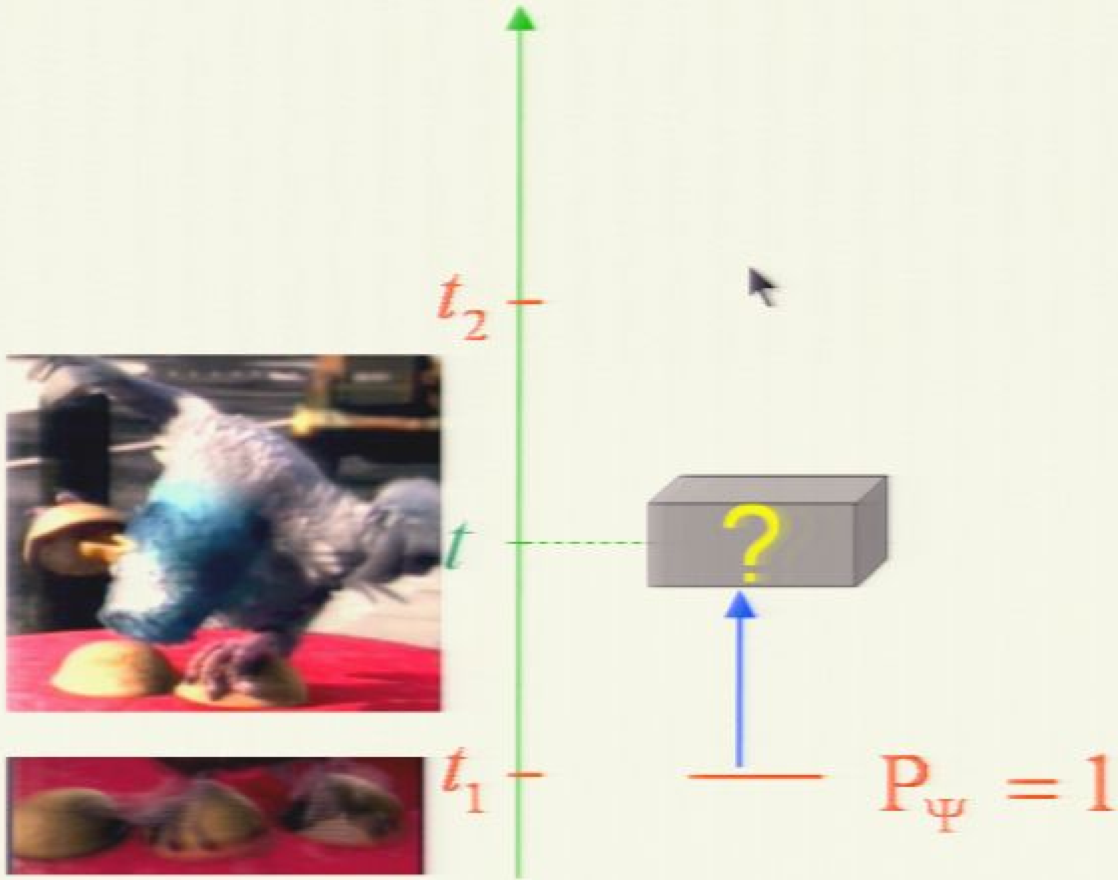
Quantum dealer can win without cheating!

The ball is a pre- and post-selected system

The ball is a pre- and post-selected system



The ball is a pre- and post-selected system



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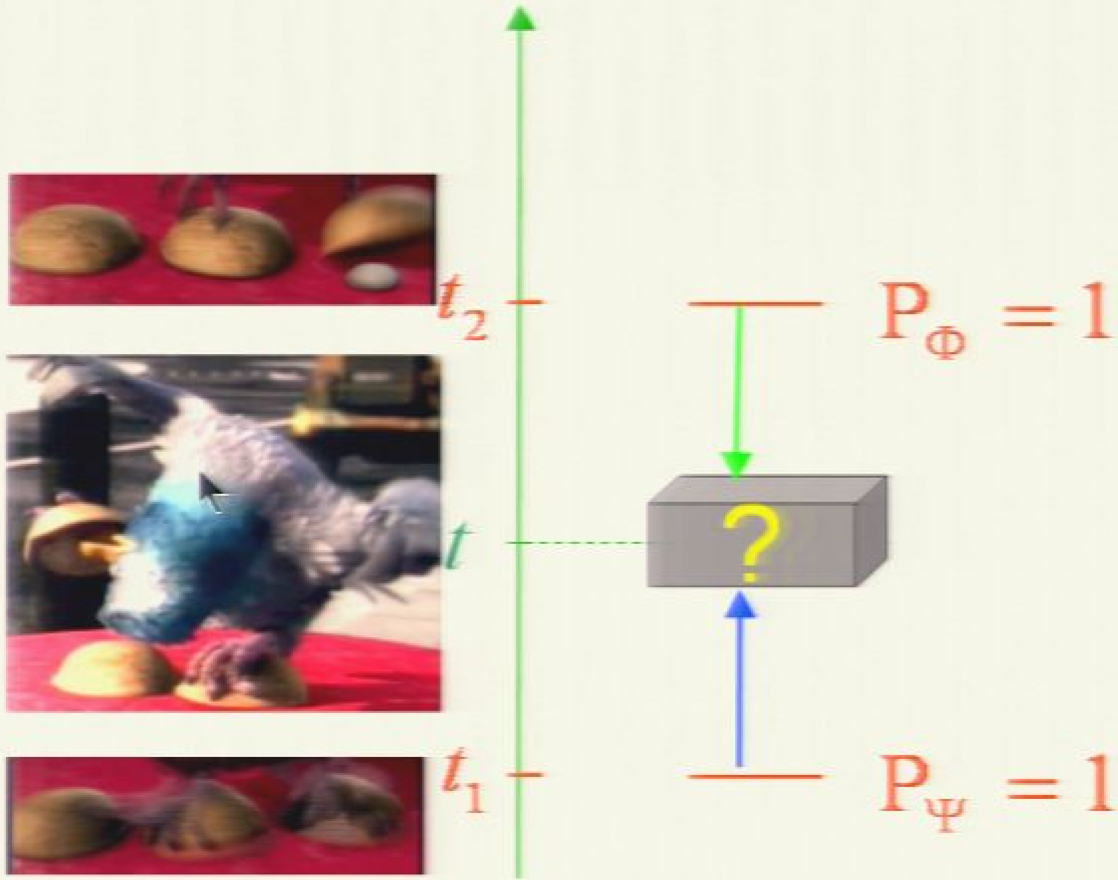

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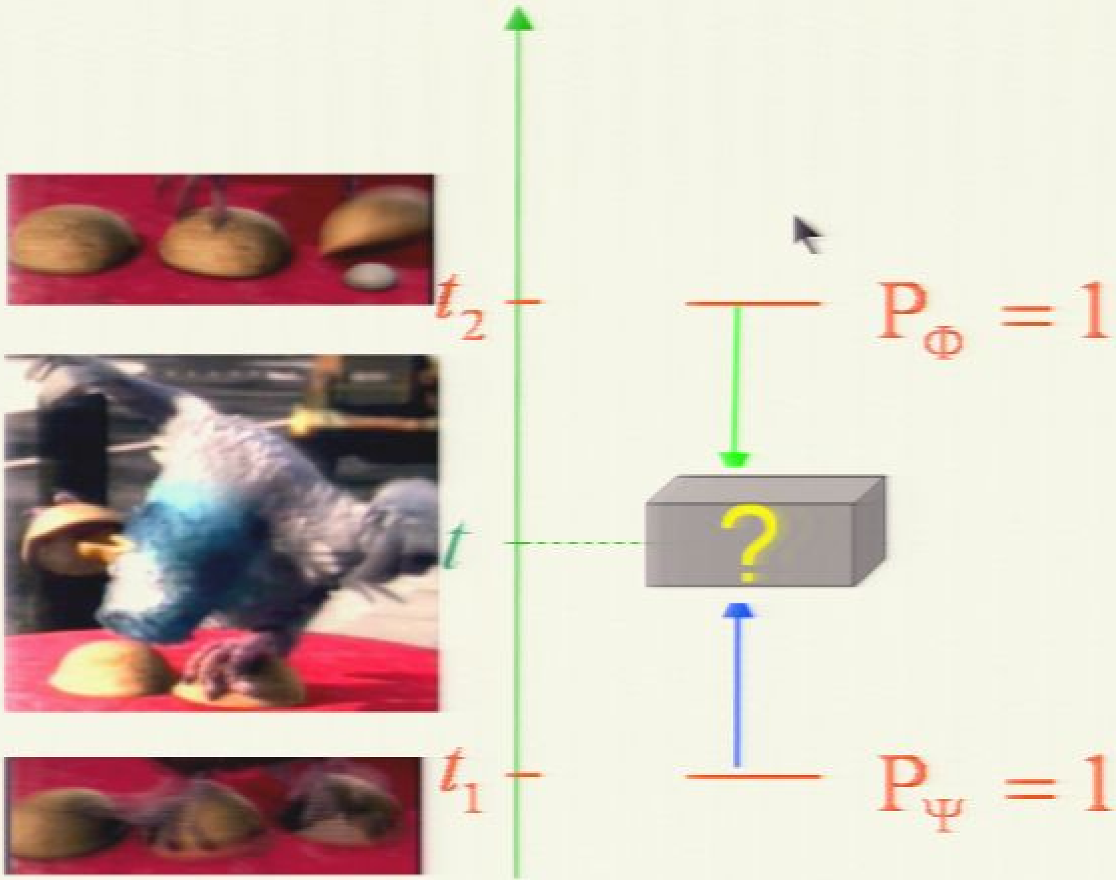


The two-state vector

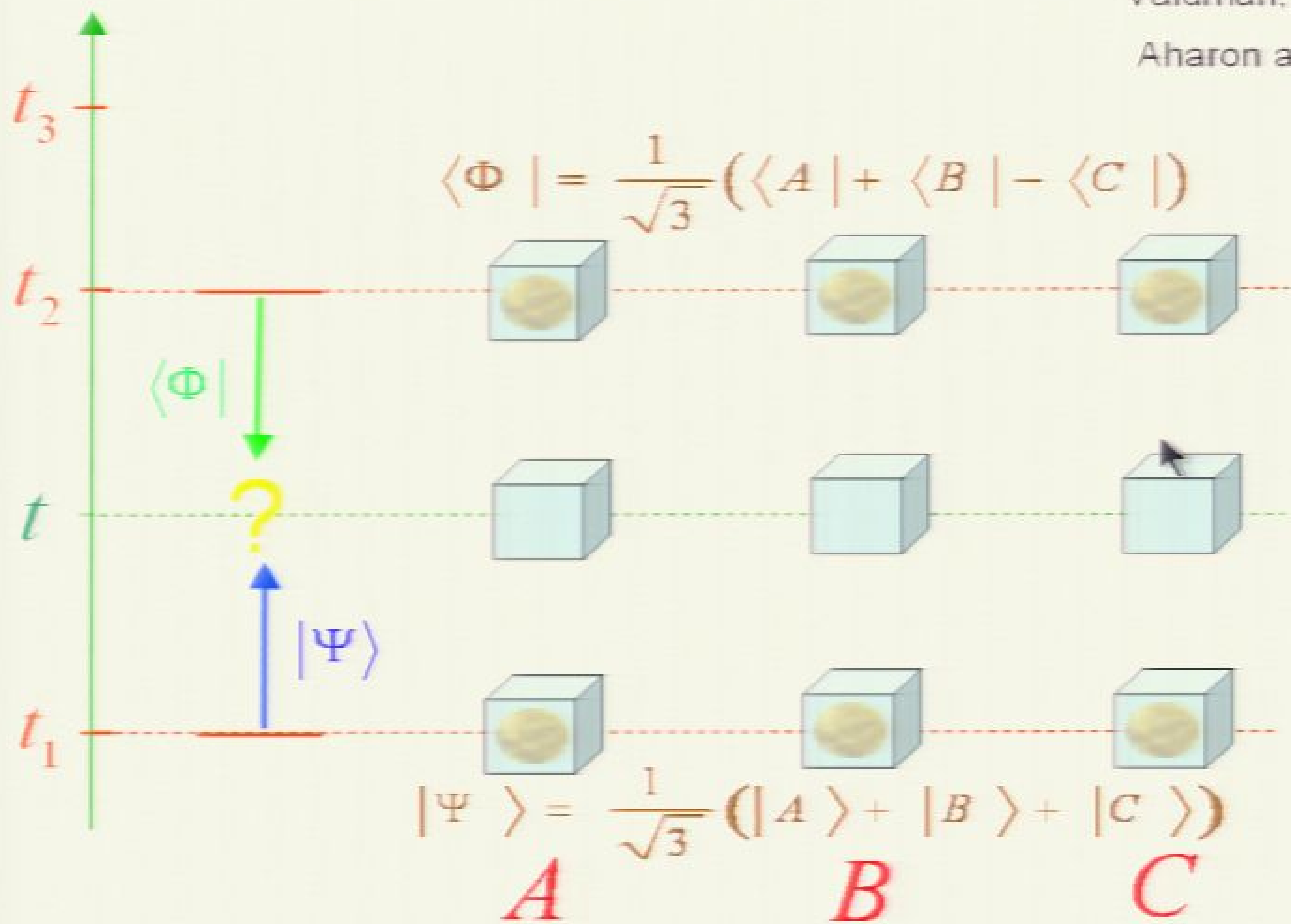
$$\langle \Phi | \quad | \Psi \rangle$$

The 3-boxes paradox

Aharonov and Vaidman, *JPA* **24**, 2315 (1991

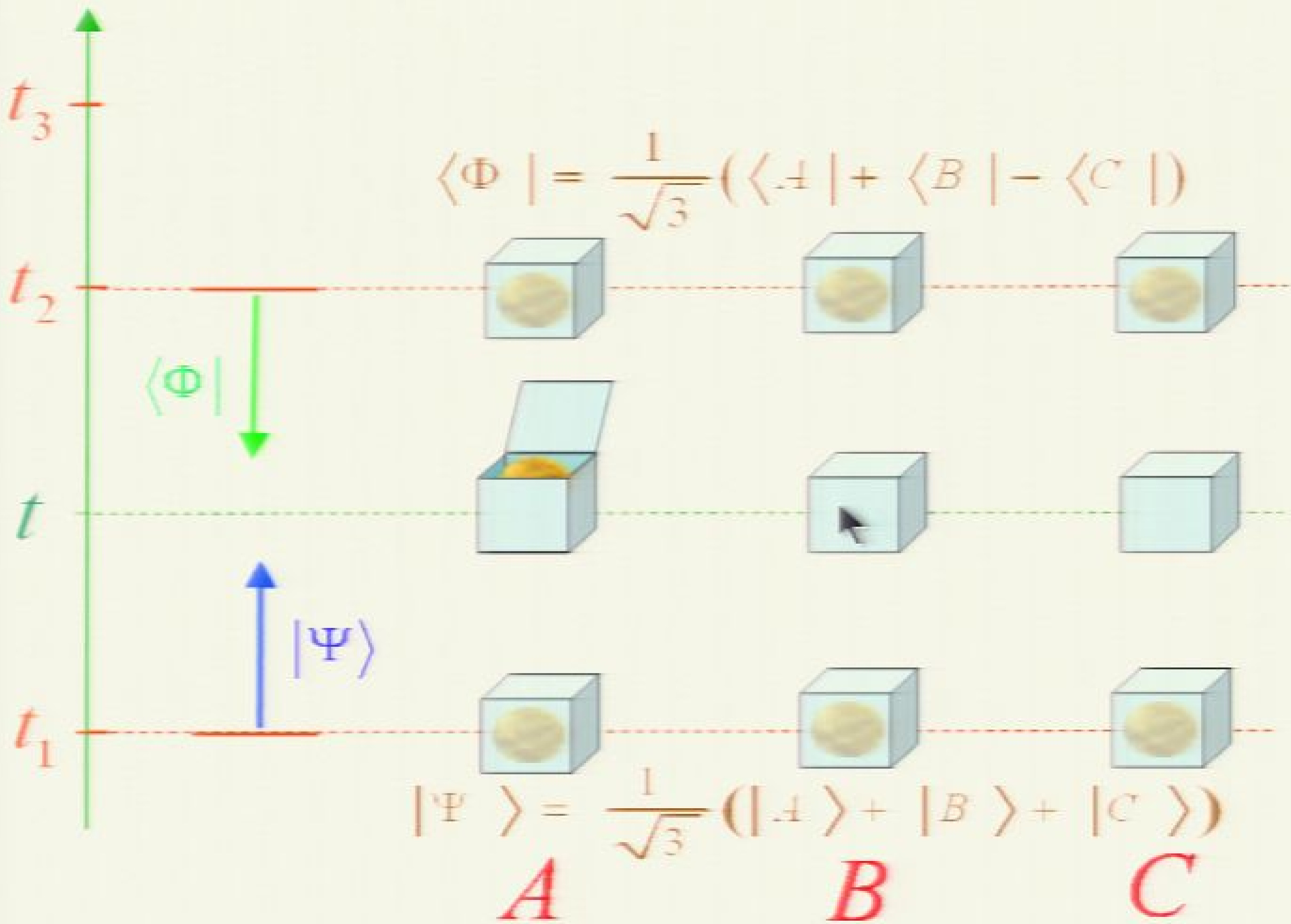
Vaidman, *Found. Phys.* **29**, 865 (1999)

Aharon and Vaidman, *PRA* **77**, 052310 (2008



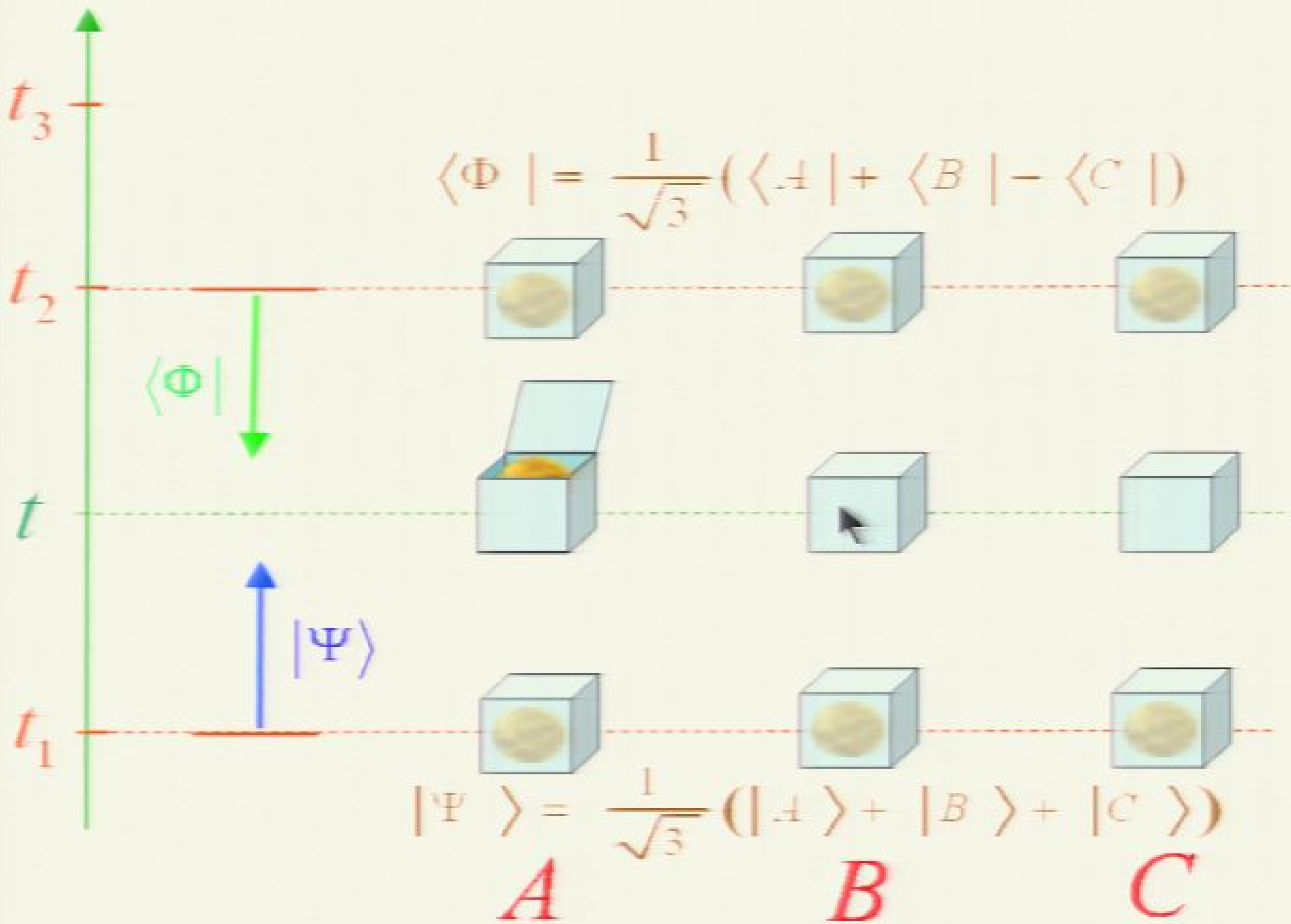
Where is the ball?

The three box paradox



It is always in A

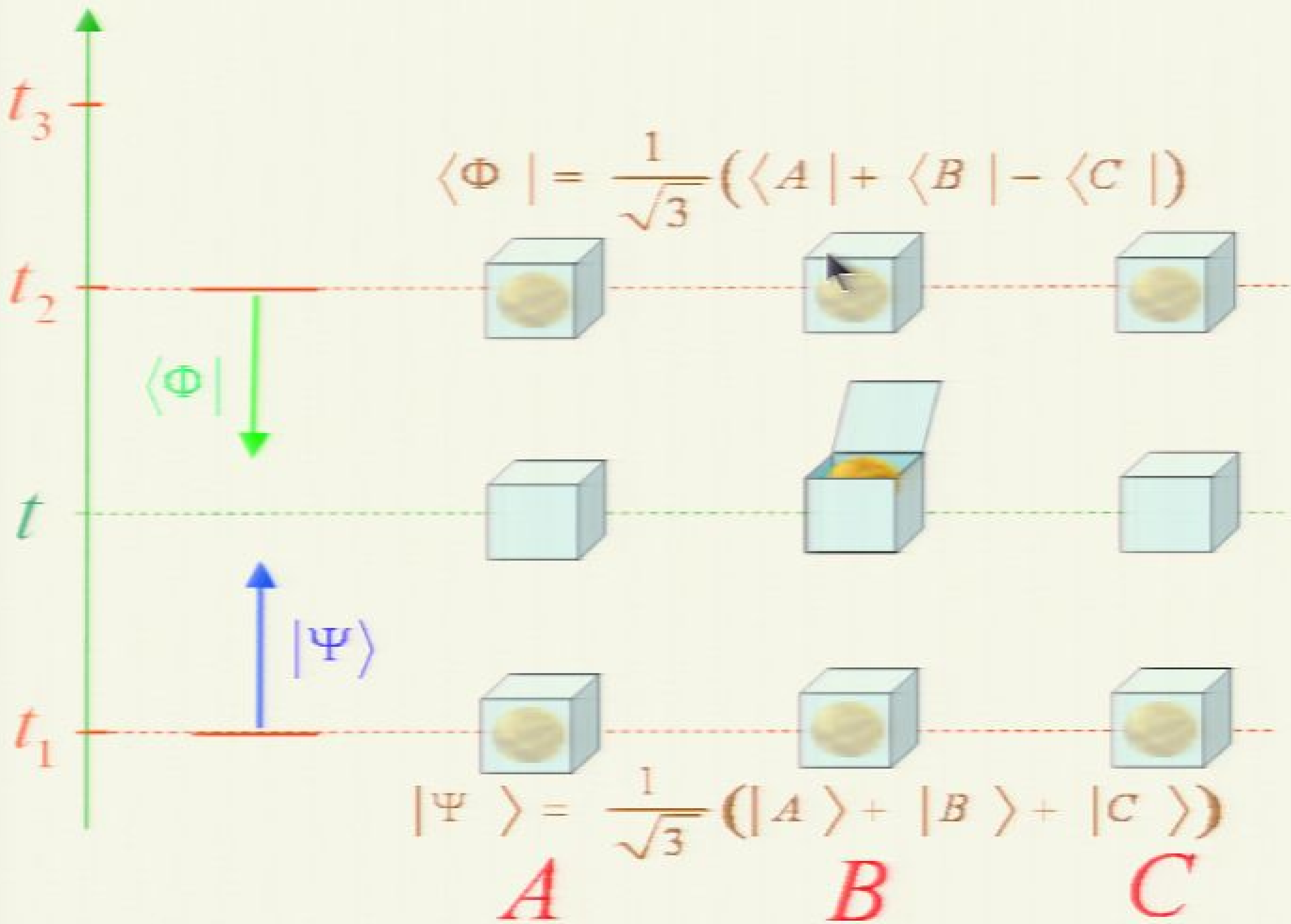
The three box paradox



It is always in A

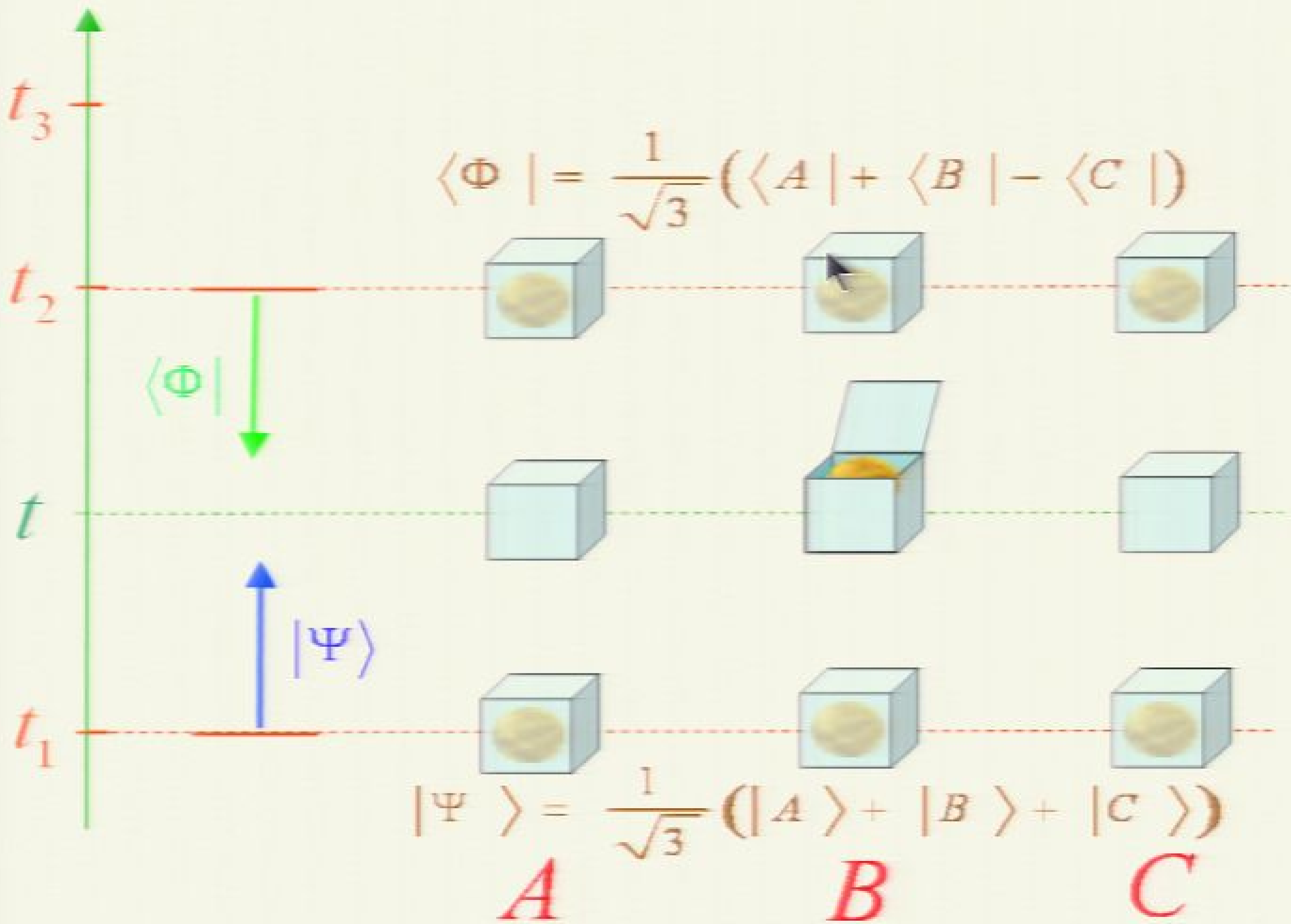
$$\mathrm{Prob}(\mathbf{P}_A = 1) = \frac{\left|\left(\langle A| + \langle B| - \langle C|\right)\mathbf{P}_A\left(|A\rangle + |B\rangle + |C\rangle\right)\right|^2}{\left|\left(\langle A| + \langle B| - \langle C|\right)\mathbf{P}_A\left(|A\rangle + |B\rangle + |C\rangle\right)\right|^2 + \left|\left(\langle A| + \langle B| - \langle C|\right)\mathbf{P}_{B\cup C}\left(|A\rangle + |B\rangle + |C\rangle\right)\right|^2} = 1$$

The three box paradox



It is always in B

The three box paradox

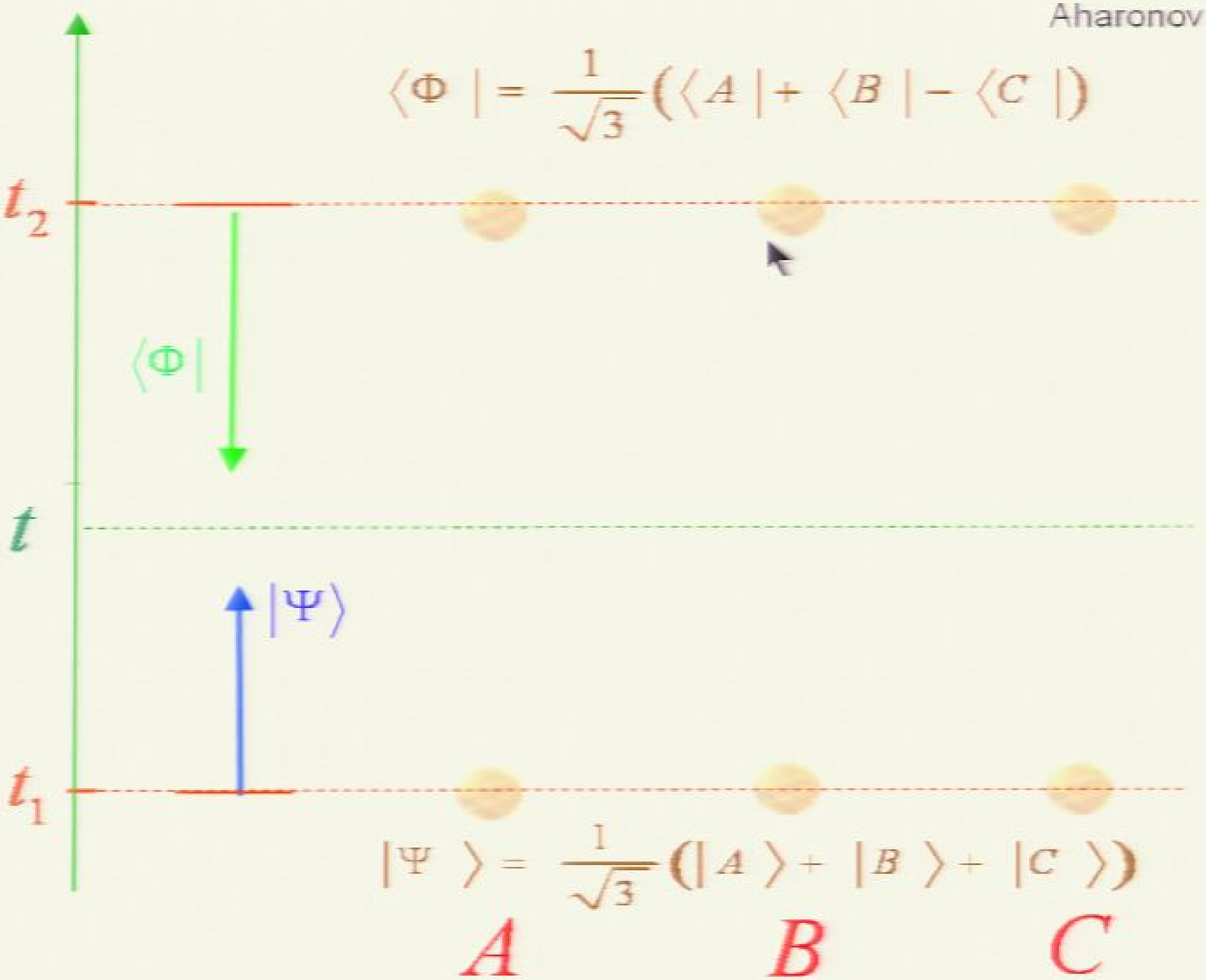


It is always in B

$$\mathrm{Prob}(\mathbf{P}_B = 1) = \frac{\left|\left(\langle A| + \langle B| - \langle C|\right)\mathbf{P}_B\left(|A\rangle + |B\rangle + |C\rangle\right)\right|^2}{\left|\left(\langle A| + \langle B| - \langle C|\right)\mathbf{P}_B\left(|A\rangle + |B\rangle + |C\rangle\right)\right|^2 + \left|\left(\langle A| + \langle B| - \langle C|\right)\mathbf{P}_{A\cup C}\left(|A\rangle + |B\rangle + |C\rangle\right)\right|^2} = 1$$

A single photon “sees” two balls

Aharonov and Vaidman *PR* A **67**, 042107 (20



A single photon “sees” two balls

Aharonov and Vaidman *PR* A **67**, 042107 (20

$$\langle \Phi | = \frac{1}{\sqrt{3}}\left(\langle A | + \langle B | - \langle C |\right)$$

t_2

$\langle \Phi |$

t

$|\Psi\rangle$

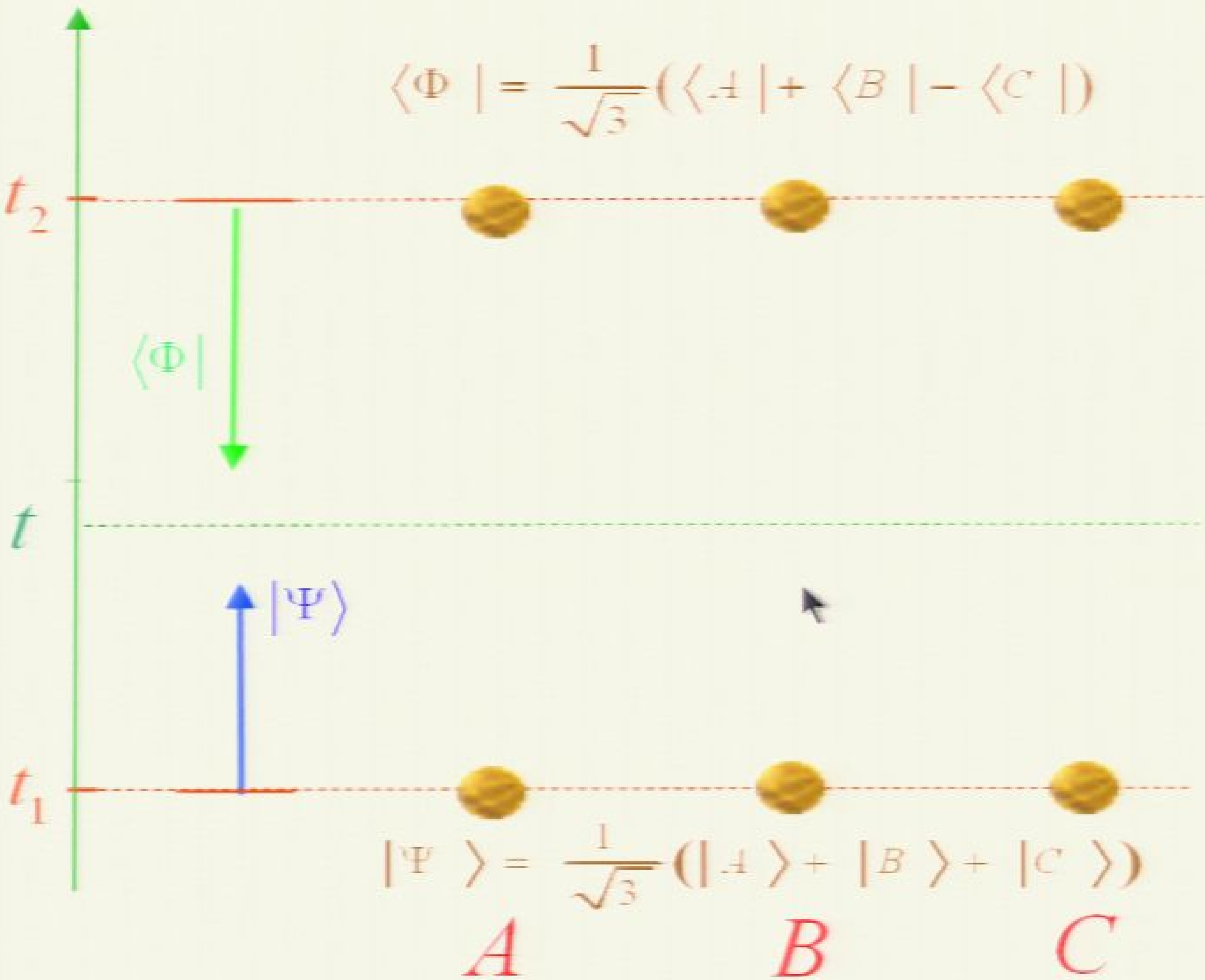
t_1

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle + |C\rangle\right)$$

A *B* *C*

It scatters exactly as if there were two balls

Any weak coupling "feels" two balls



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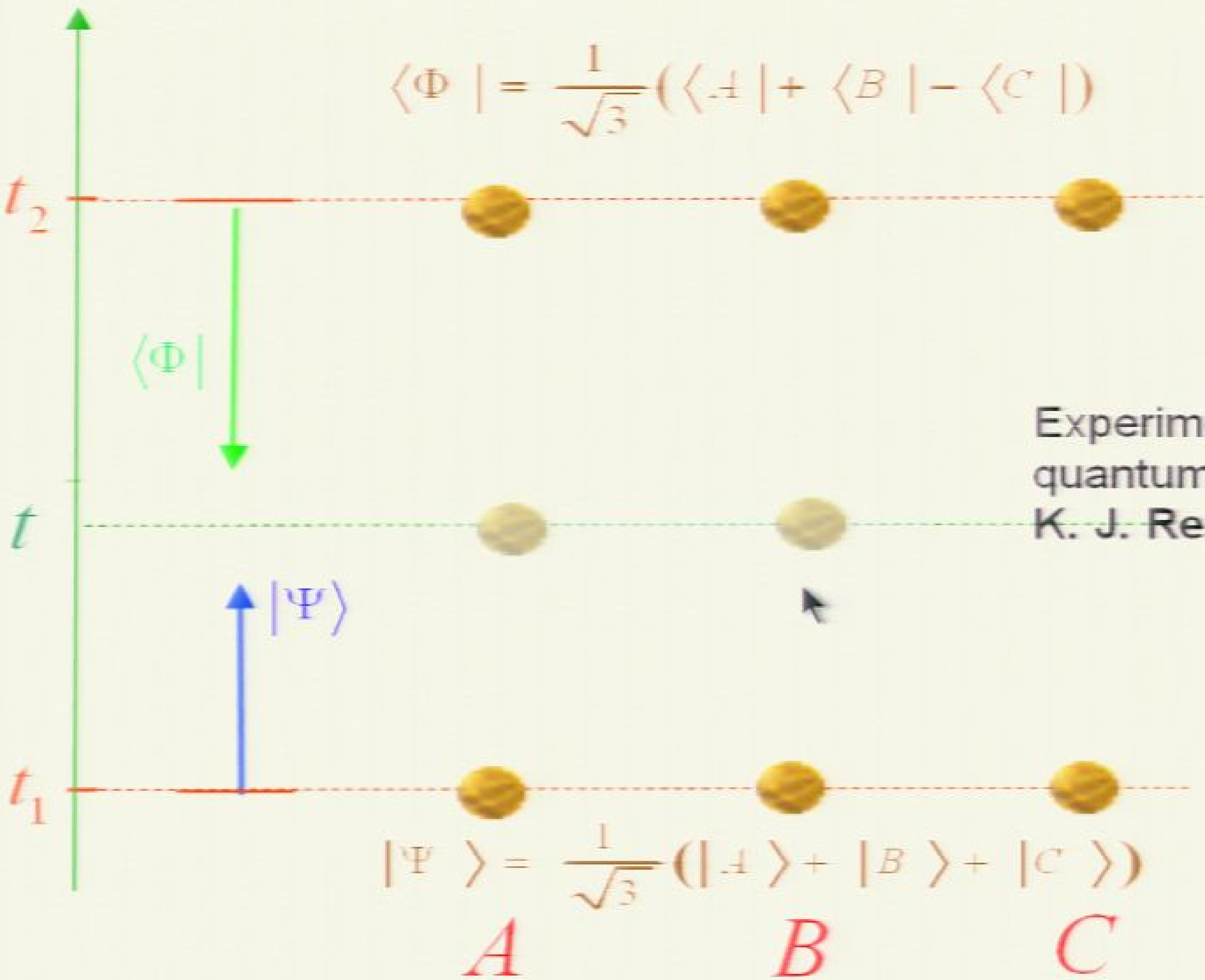
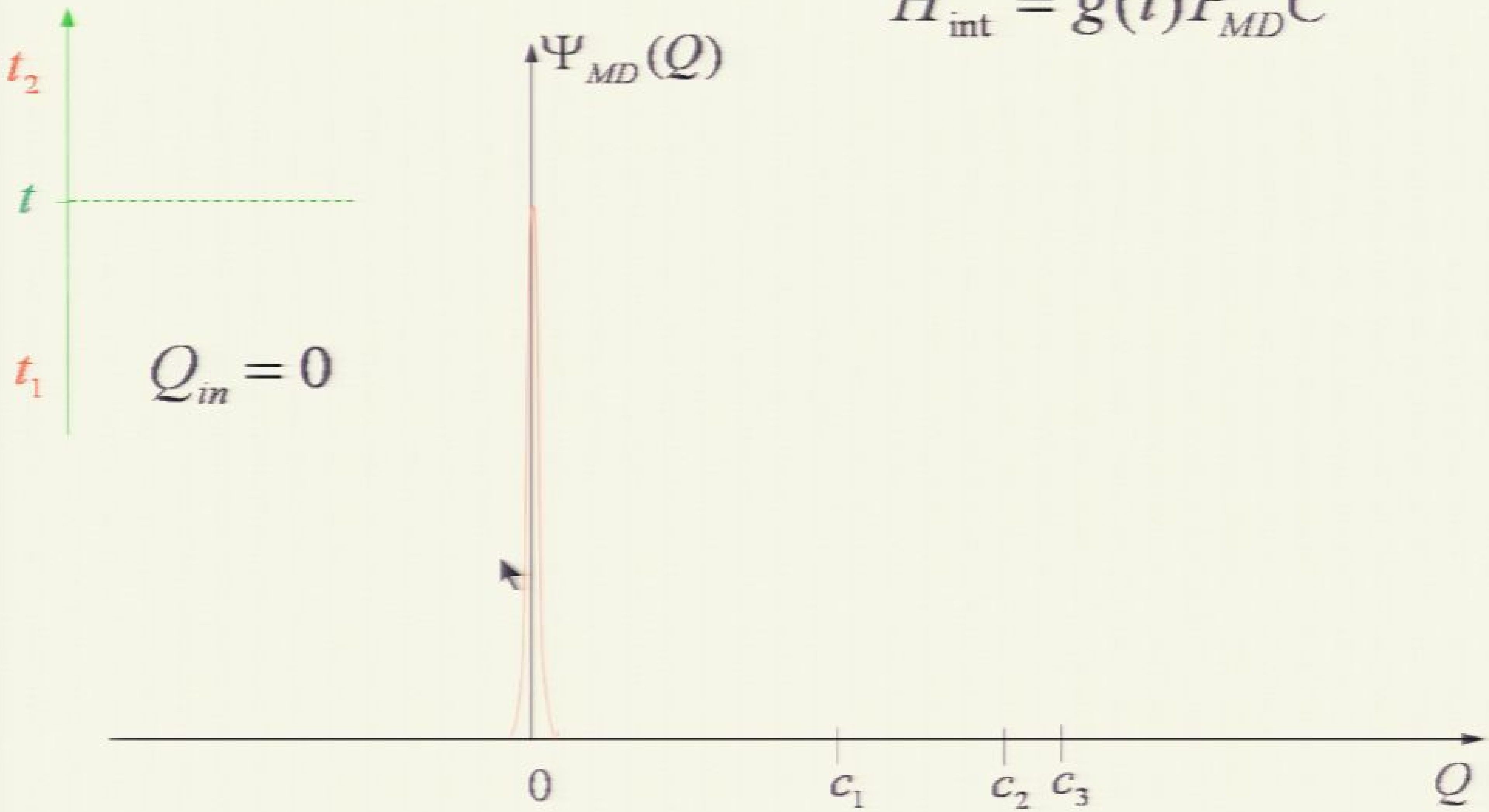


Experimental realization of the quantum box problem
K. J. Resch et al. PLA 324 125 (2004)

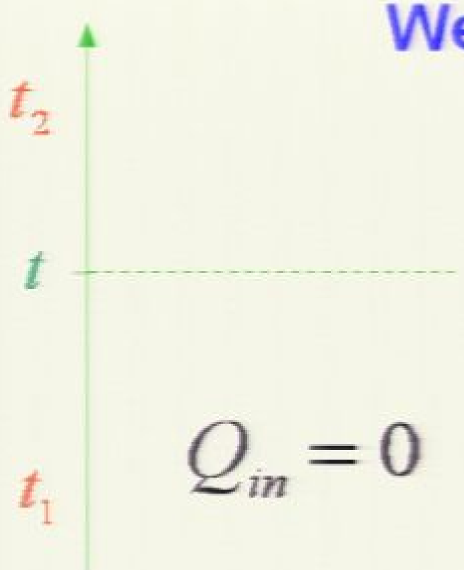
Weak Measurements

Quantum measurement of C

$$H_{\text{int}} = g(t) P_{MD} C$$



Weak quantum measurement of C

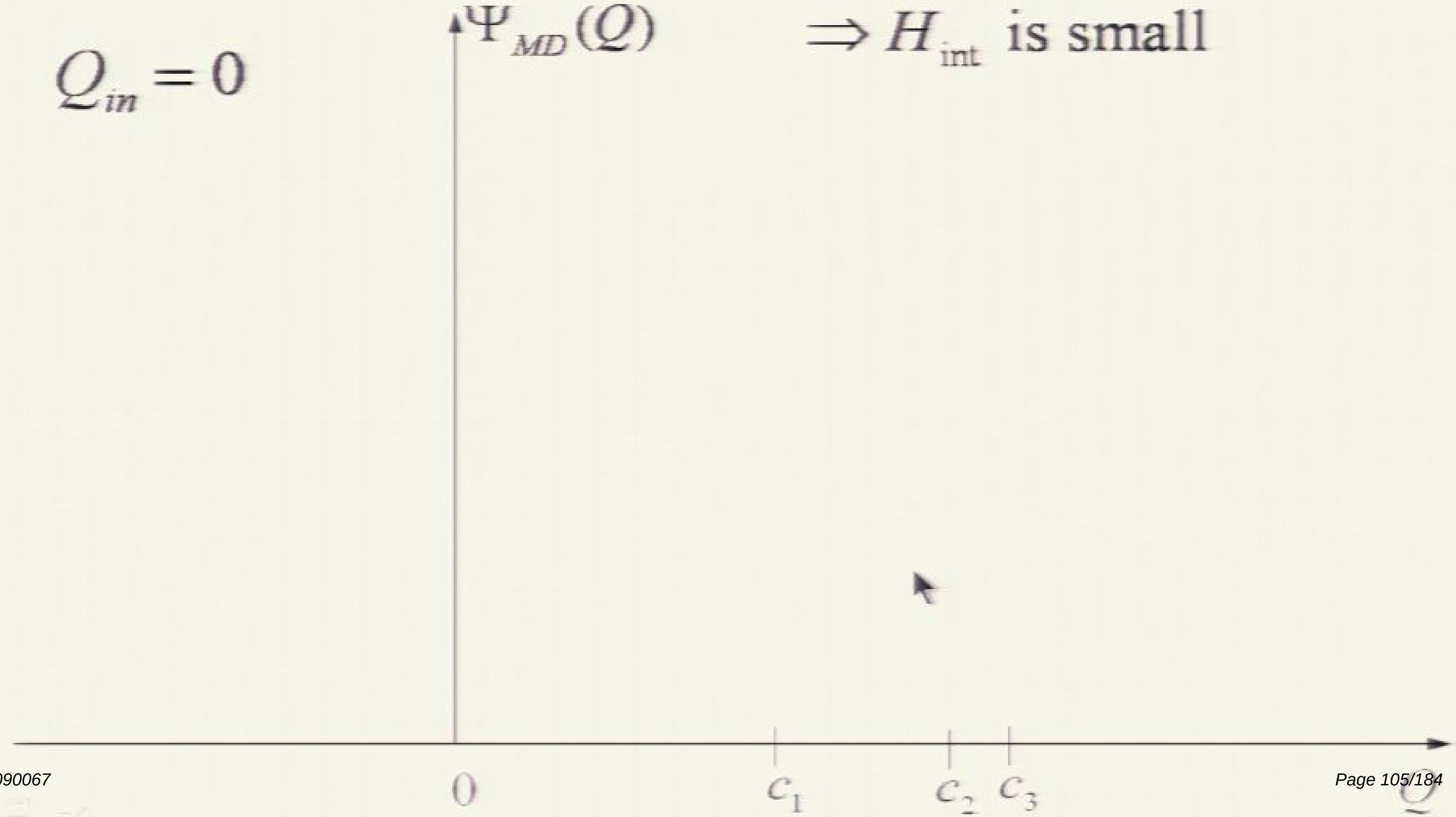


$$H_{\text{int}} = g(t) P_{MD} C$$

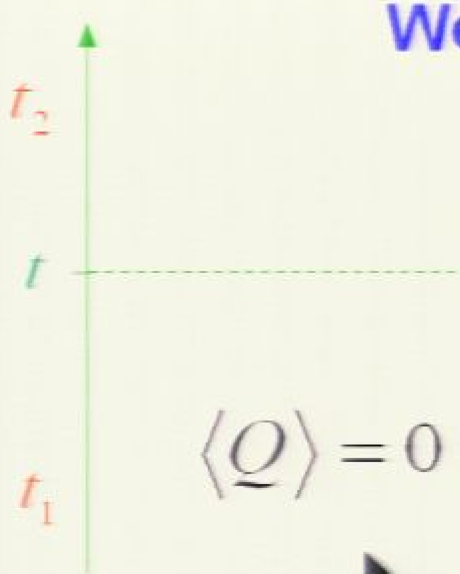
$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$

$$Q_{in} = 0$$



Weak quantum measurement of C

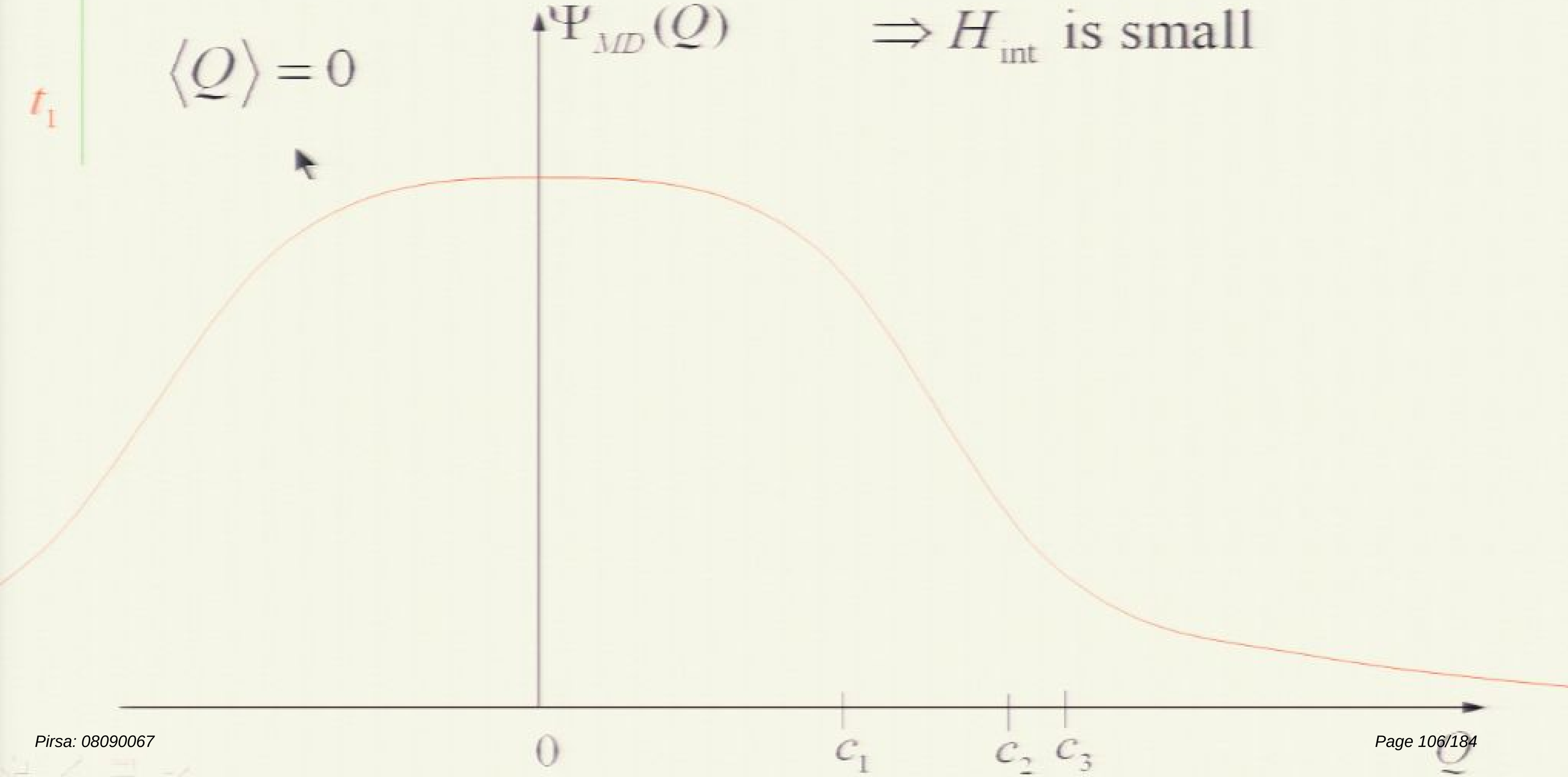


$$H_{\text{int}} = g(t) P_{MD} C$$

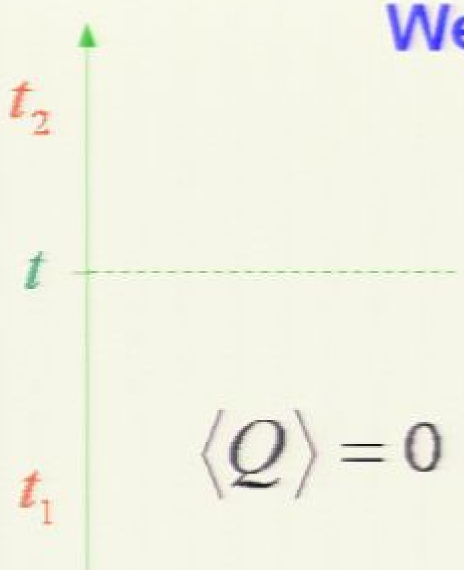
$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$

$$\langle Q \rangle = 0$$



Weak quantum measurement of C



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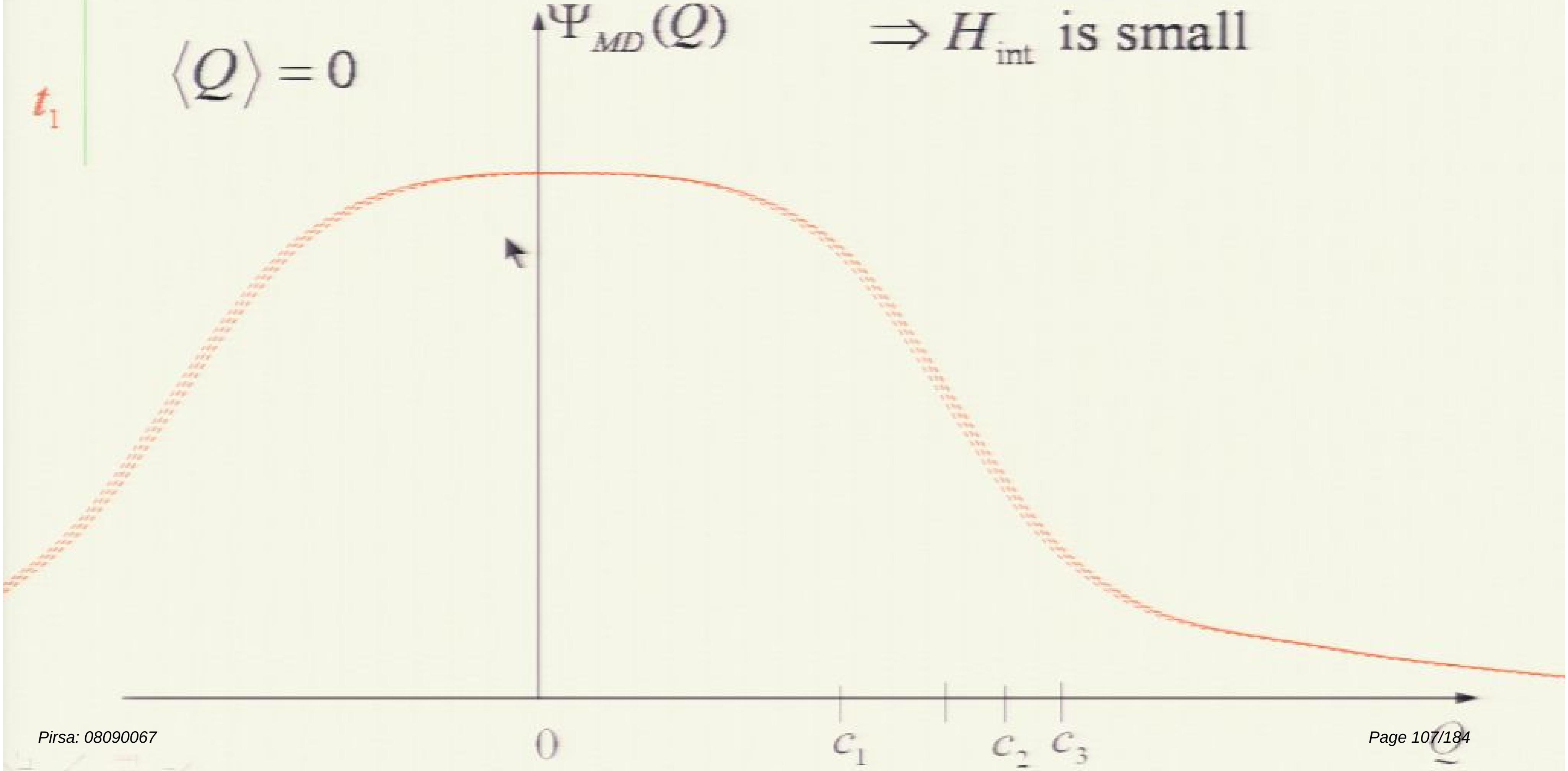
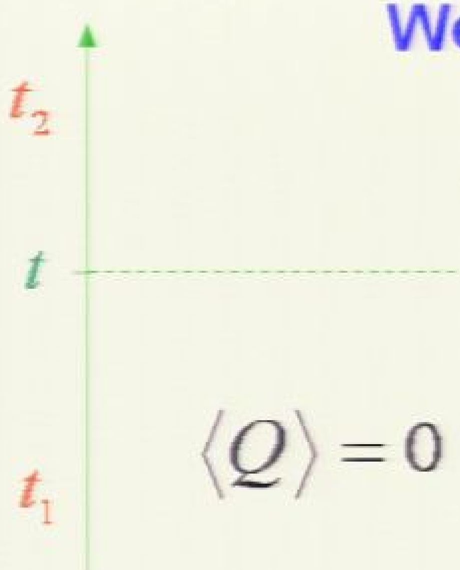
$$\langle Q \rangle = 0$$



Weak quantum measurement of C

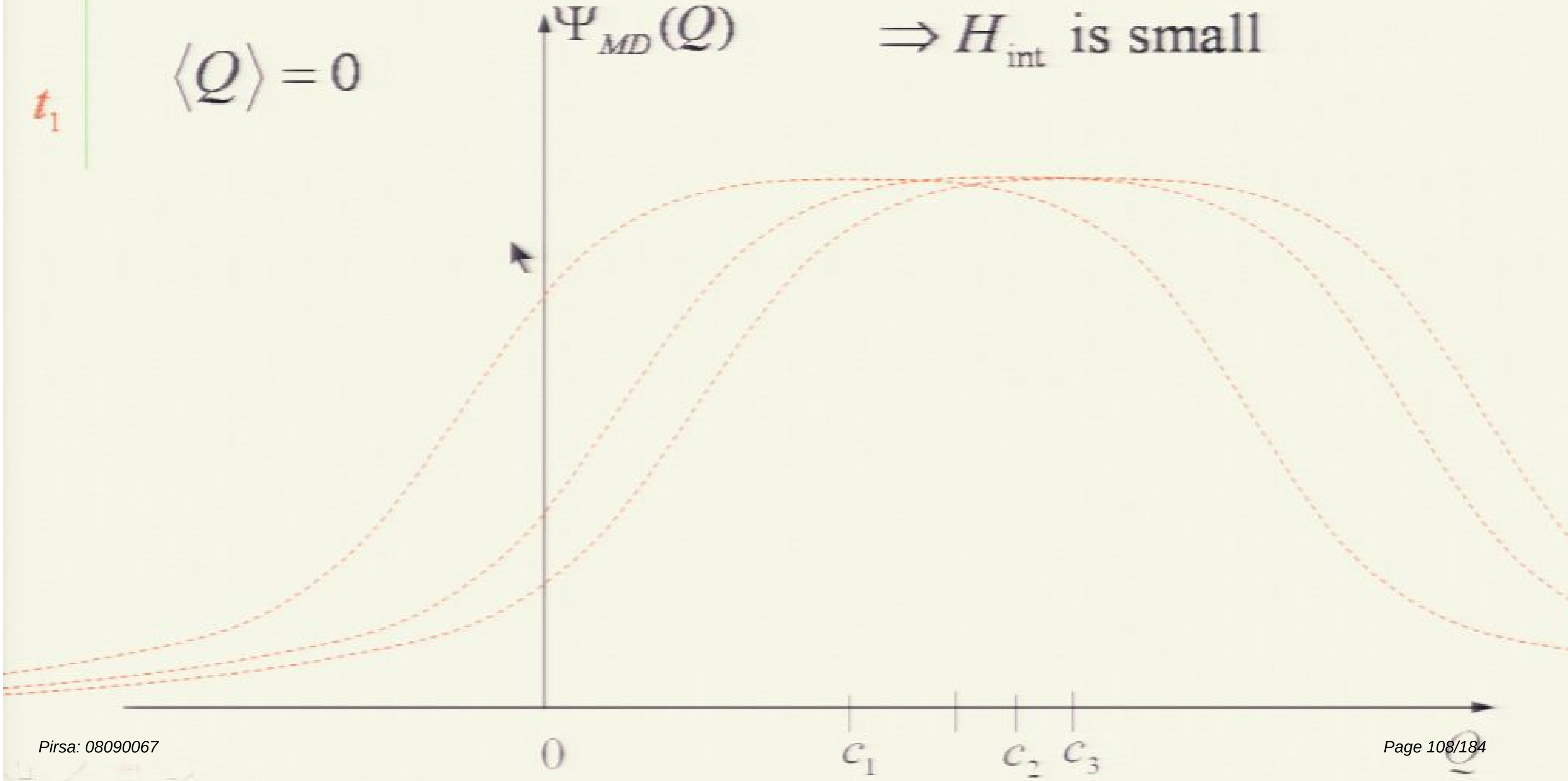


$$H_{\text{int}} = g(t) P_{MD} C$$

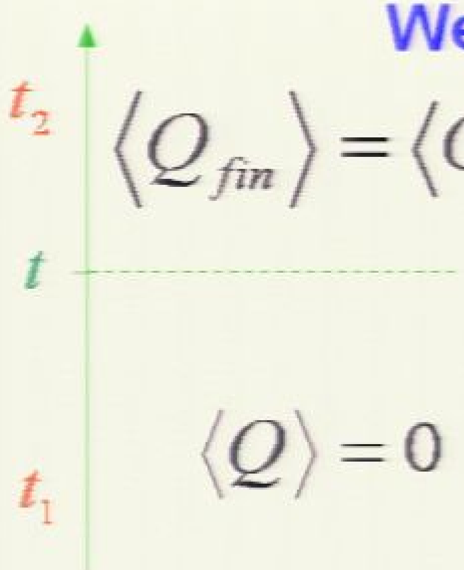
$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$

$$\langle Q \rangle = 0$$



Weak quantum measurement of C



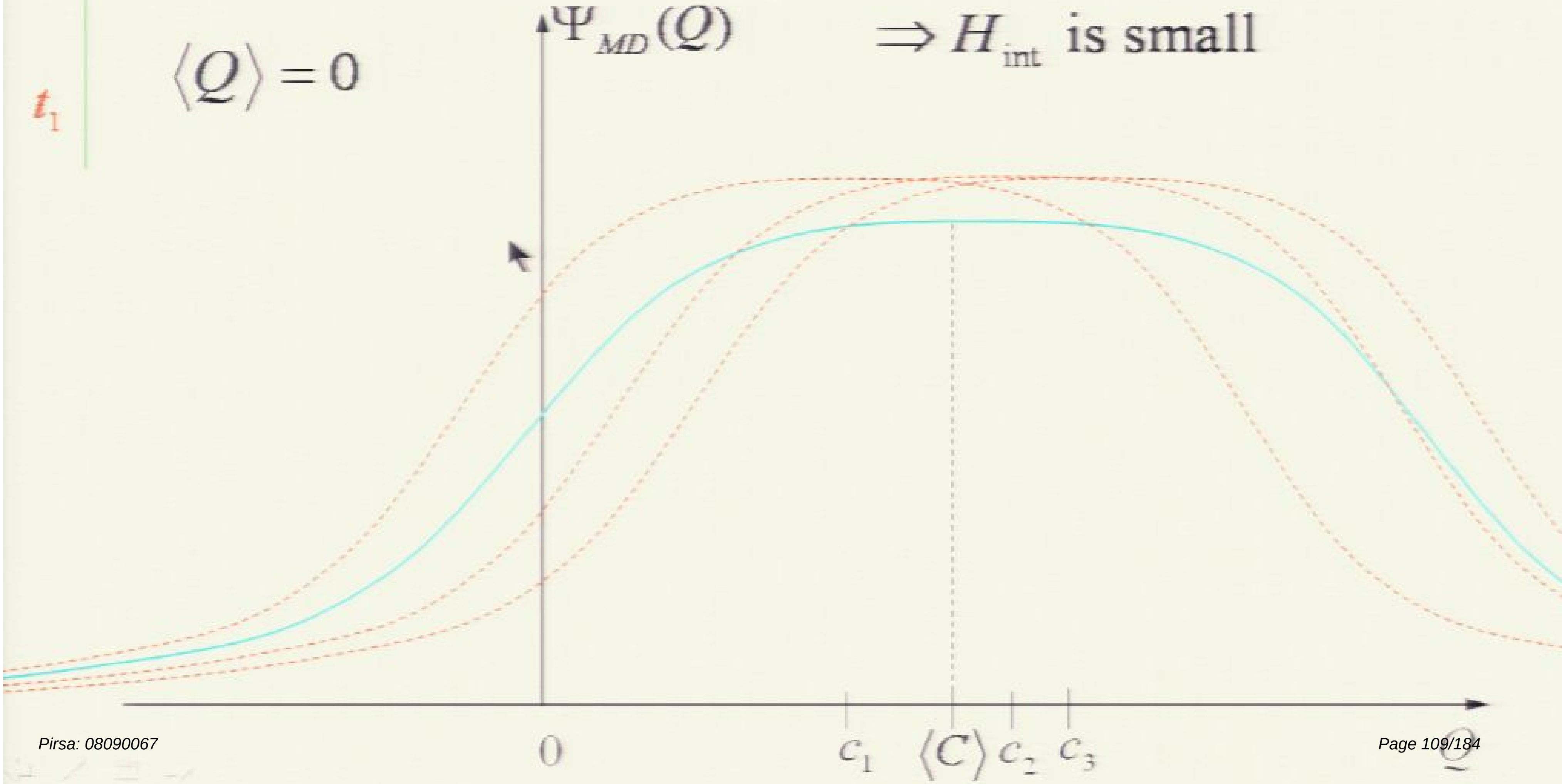
$$\langle Q_{fin} \rangle = \langle C \rangle$$

$$\langle Q \rangle = 0$$

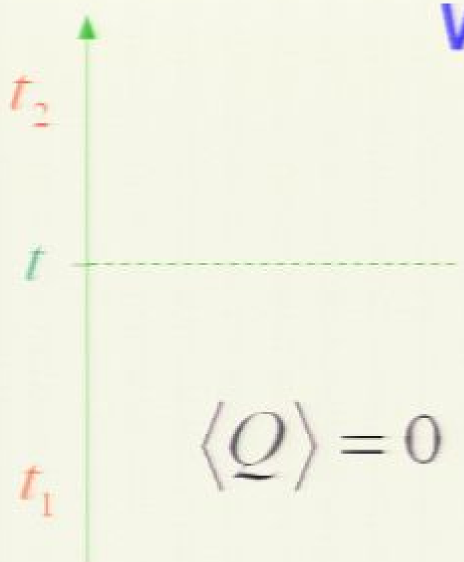
$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$



Weak measurement of C with post-selection

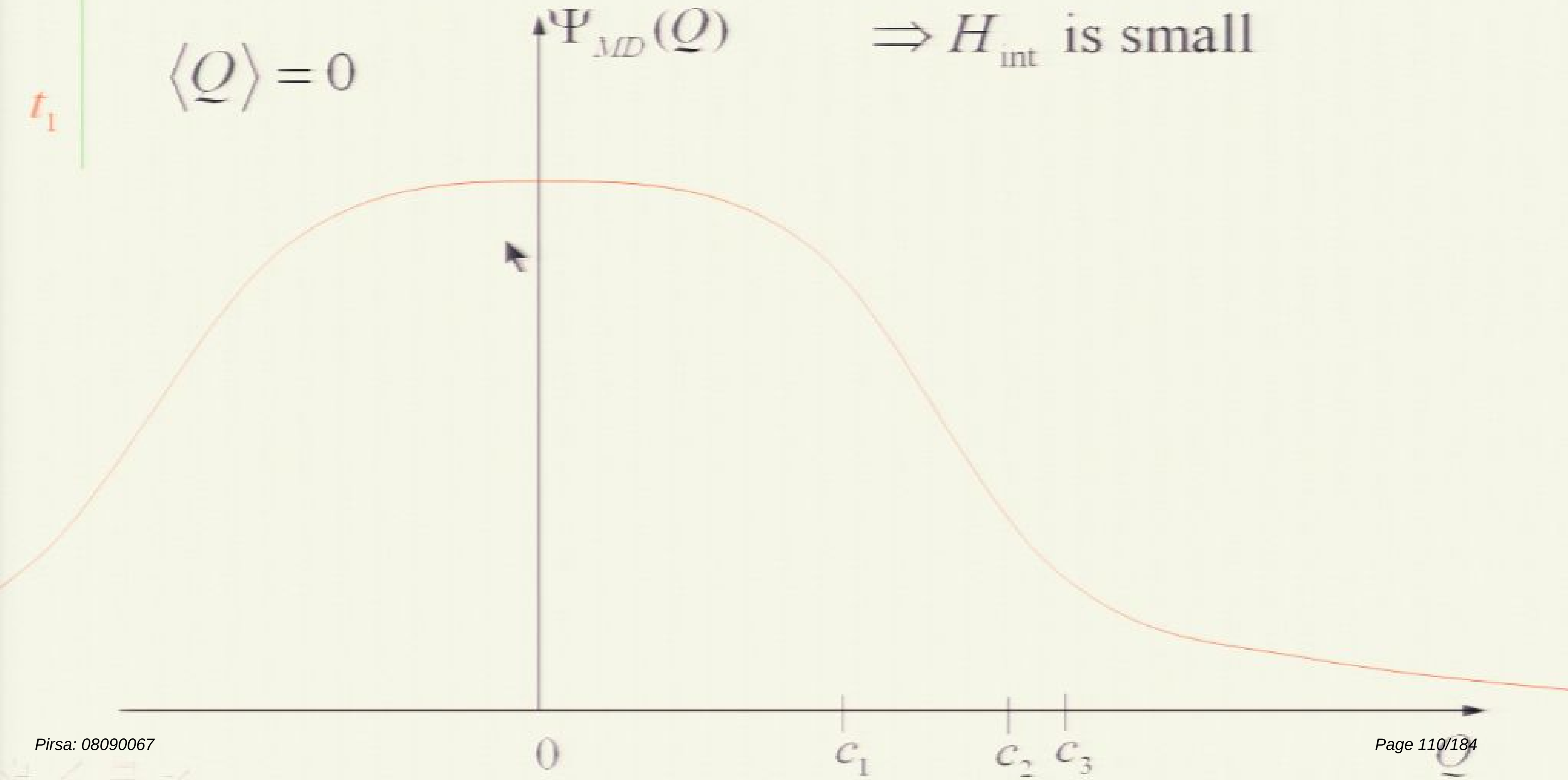


$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$

$$\langle Q \rangle = 0$$



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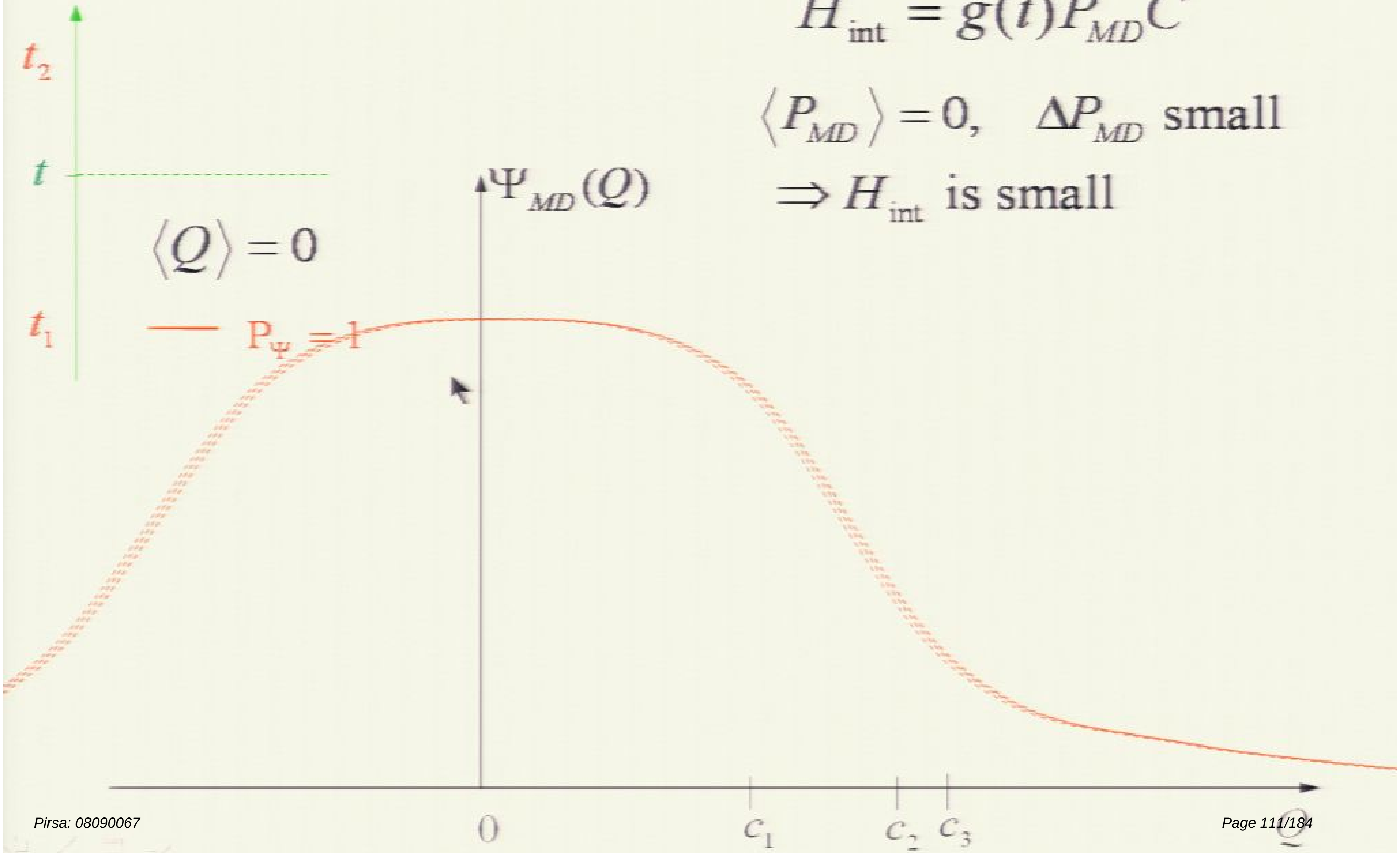

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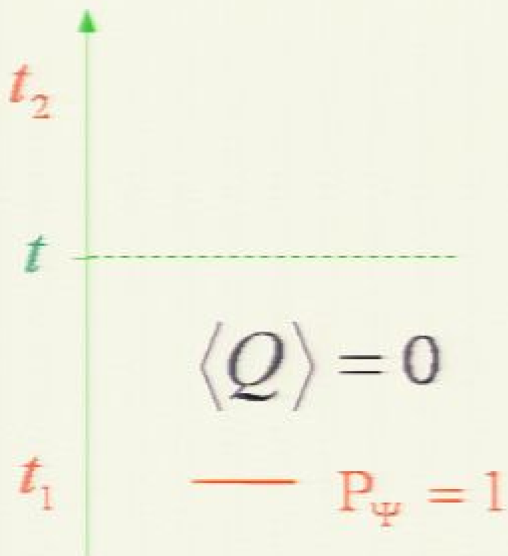

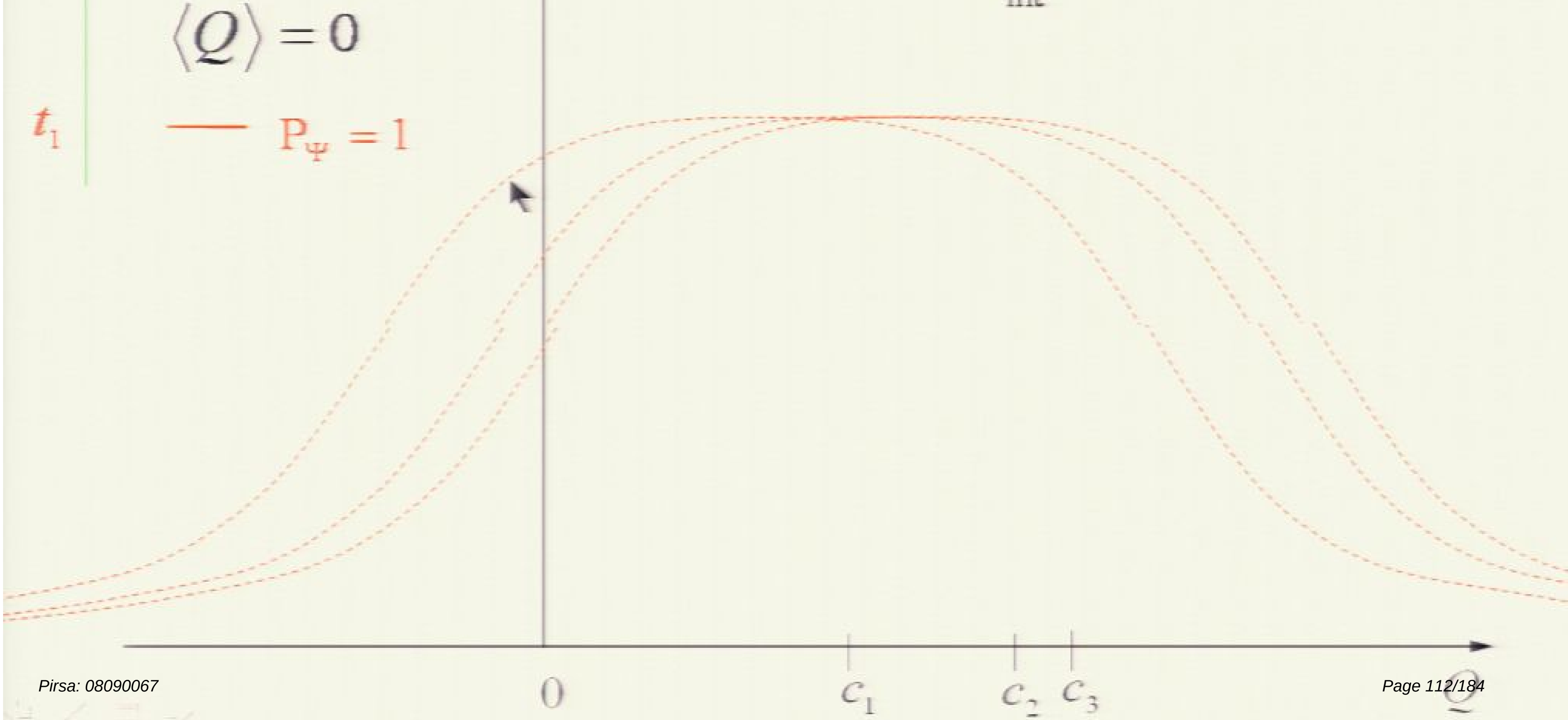

Weak measurement of C with post-selection

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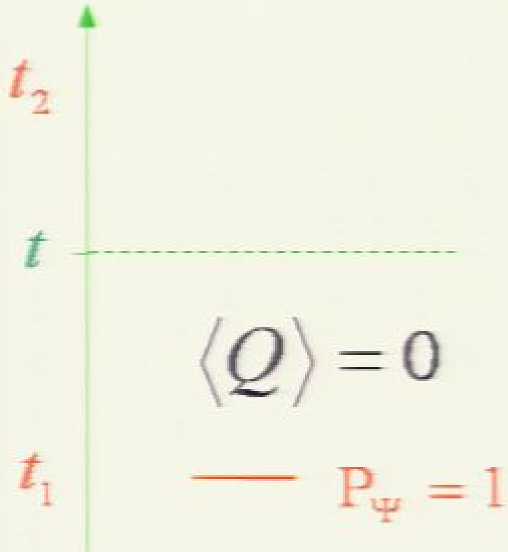
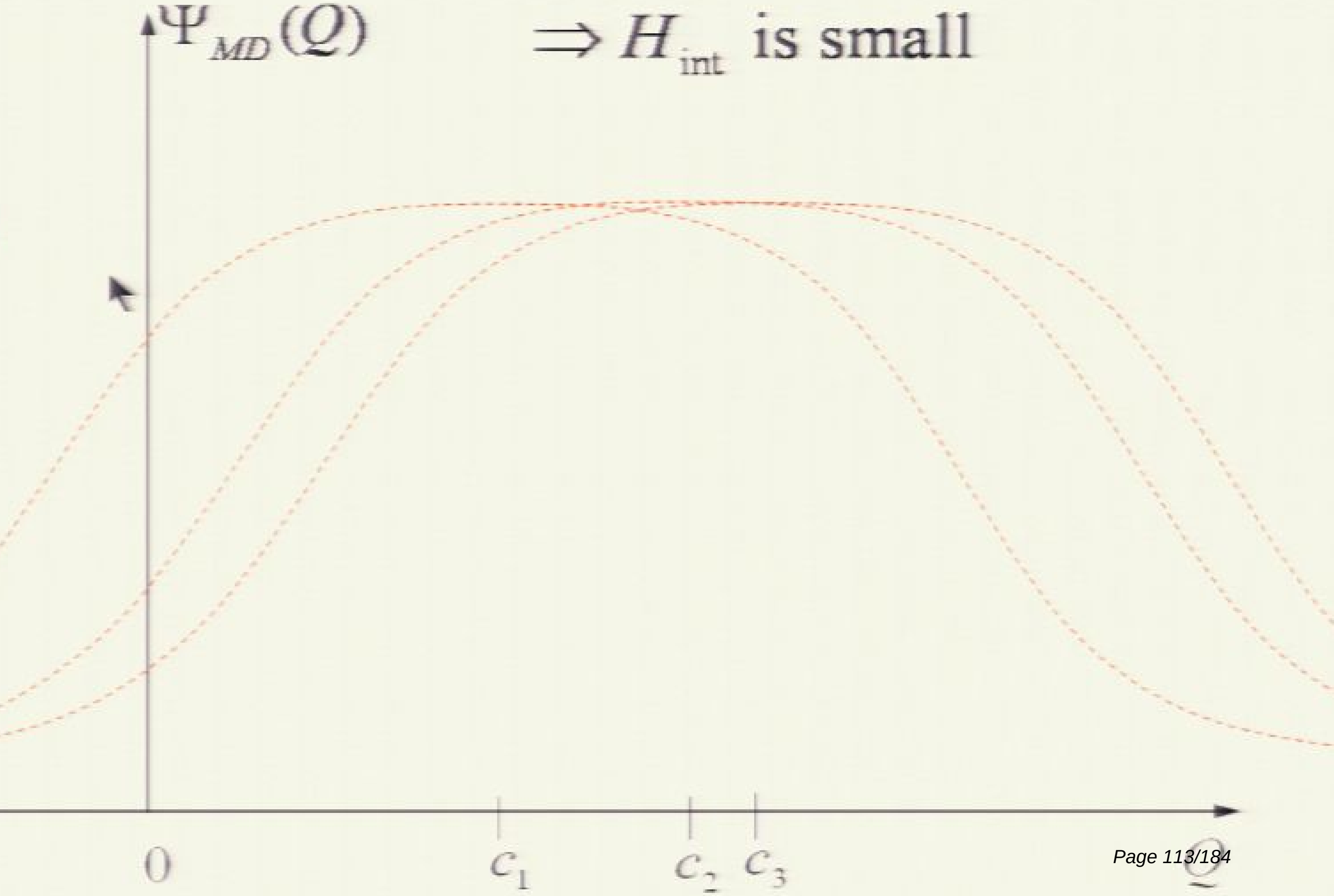


$$\langle Q \rangle = 0$$



Weak measurement of C with post-selection

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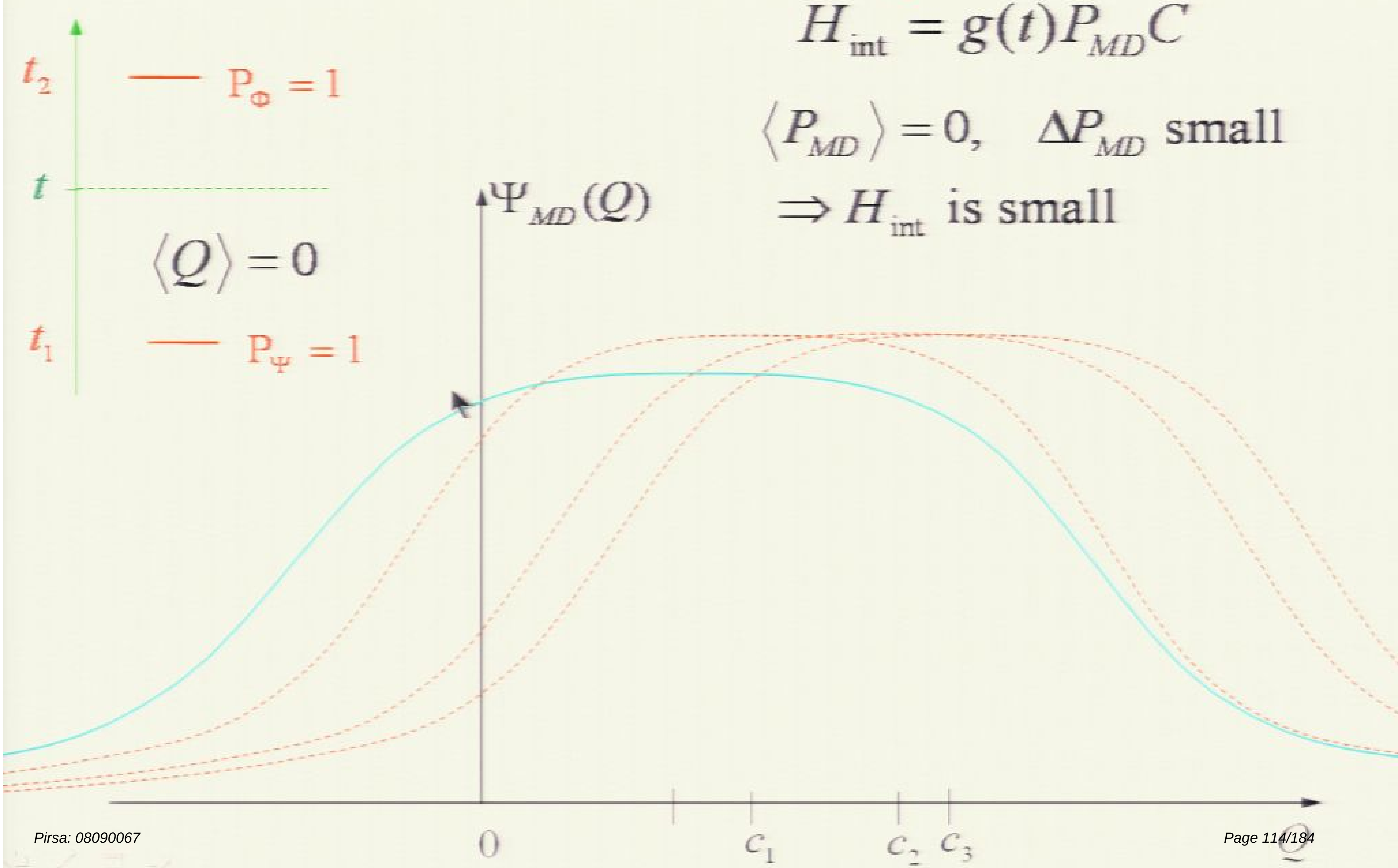
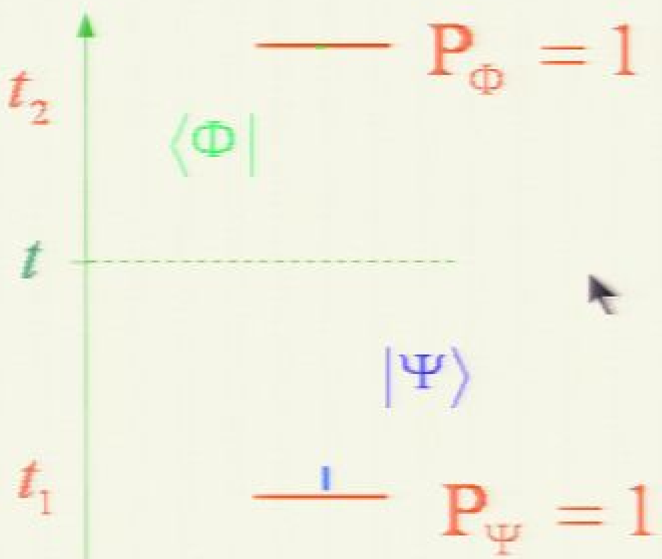
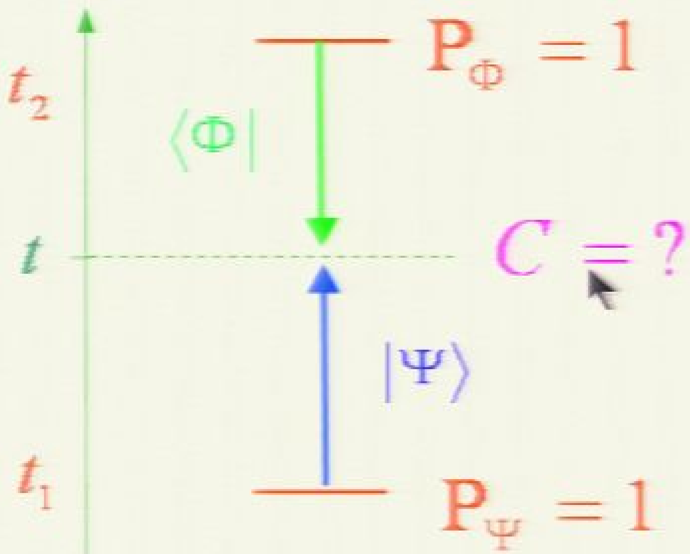


The outcomes of weak measurements are weak values



The outcomes of weak measurements are weak values

Weak value of a variable C of a pre- and post-selected system described at time t by the two-state vector $\langle\Phi| \; |\Psi\rangle$



$$\boxed{C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}}$$

The outcomes of weak measurements are weak values

Weak measurement of C with post-selection

$$\langle Q_{fin} \rangle = C_w$$

t_2 — $P_\Phi = 1$

t

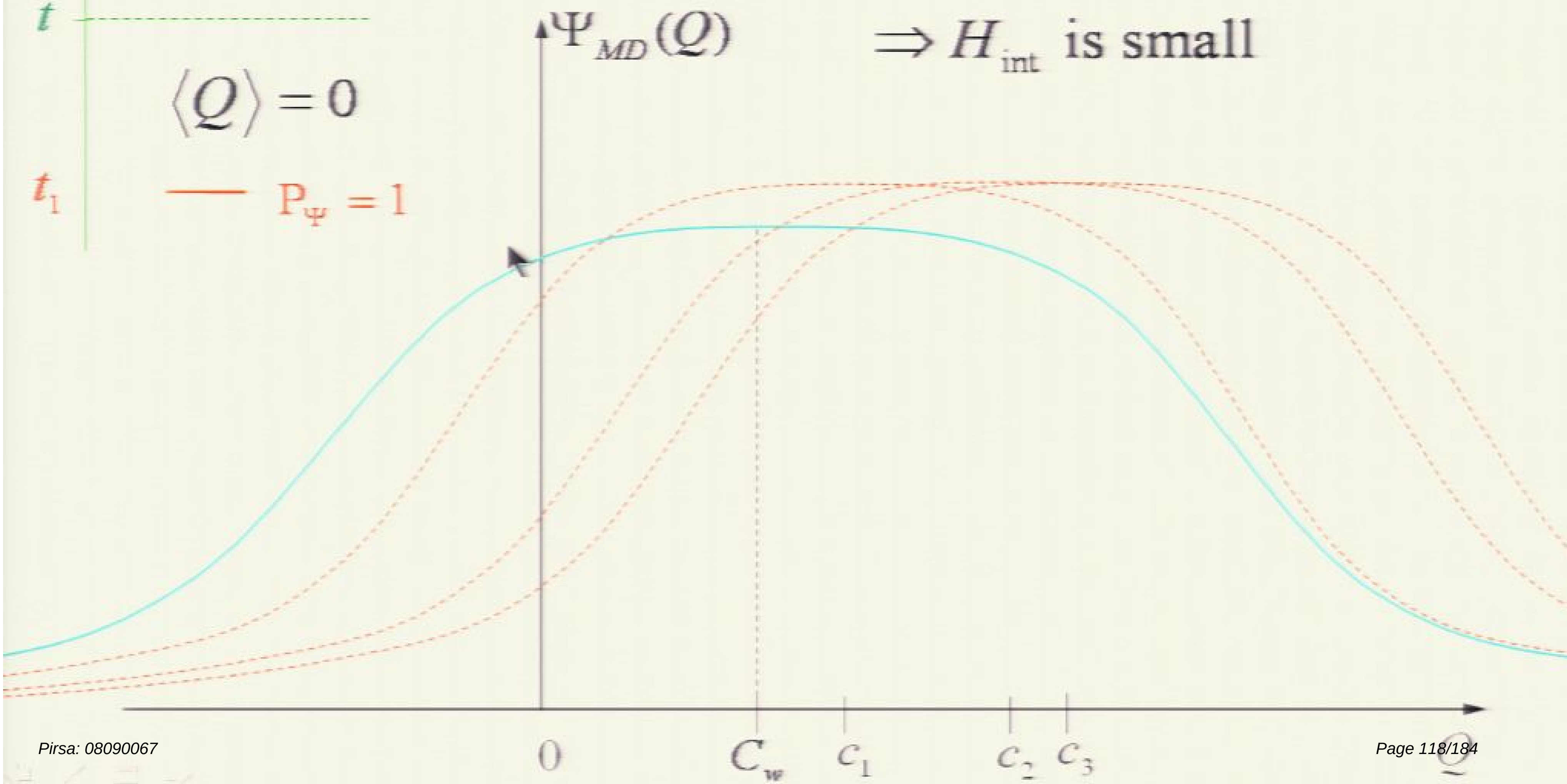
$$\langle Q \rangle = 0$$

t_1 — $P_\Psi = 1$

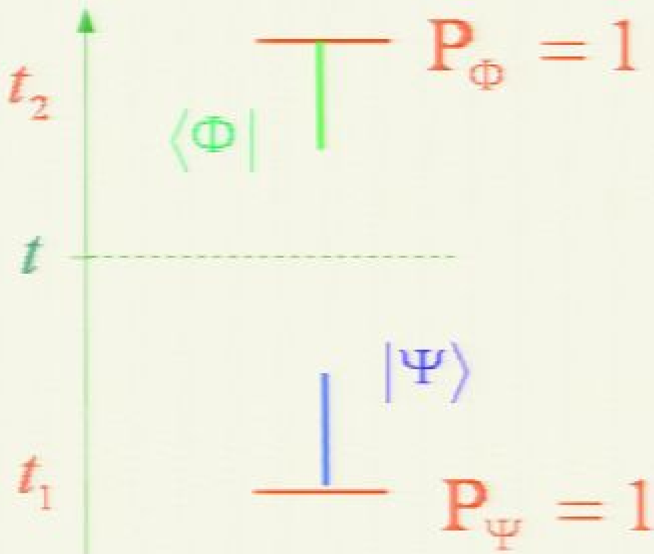
$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$

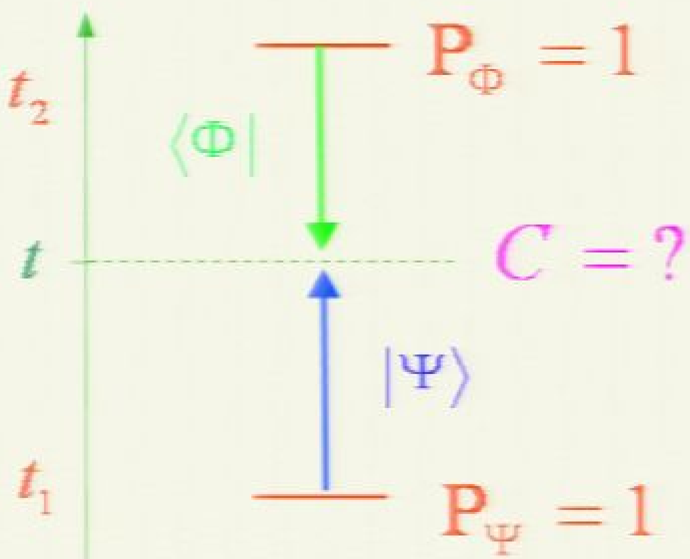


The outcomes of weak measurements are weak values



The outcomes of weak measurements are weak values

Weak value of a variable *C* of a pre- and post-selected system described at time *t* by the two-state vector $\langle\Phi| \; |\Psi\rangle$



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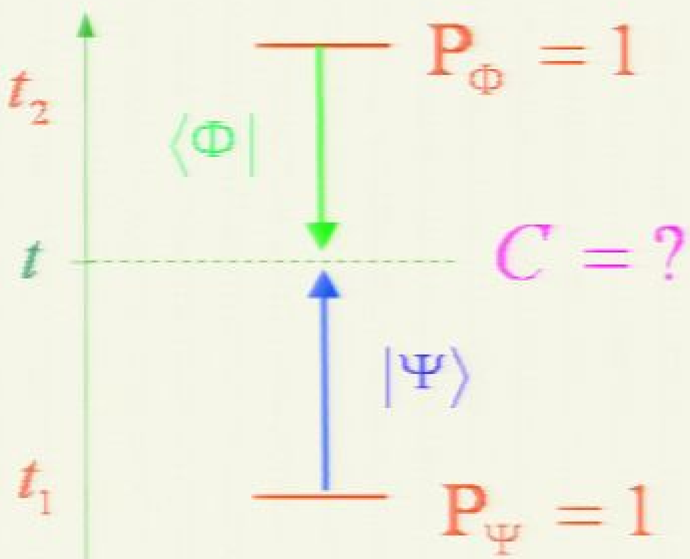
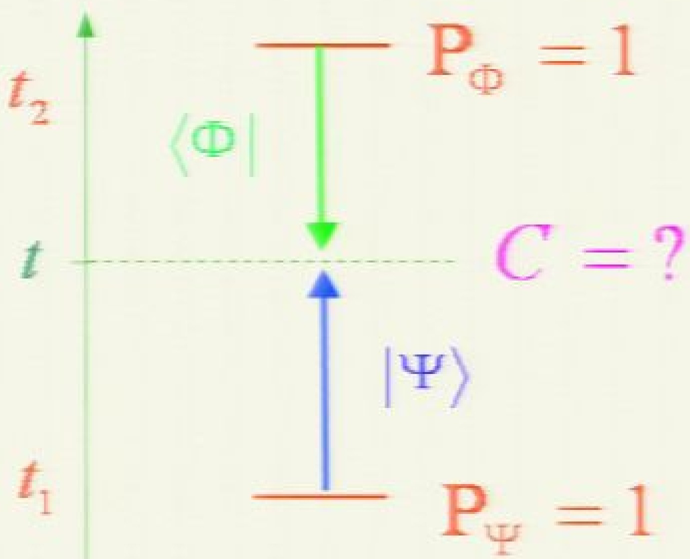


$$C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}$$

$$(A+B)_w = A_w + B_w$$

The outcomes of weak measurements are weak values

Weak value of a variable C of a pre- and post-selected system described at time t by the two-state vector $\langle\Phi|\ |\Psi\rangle$



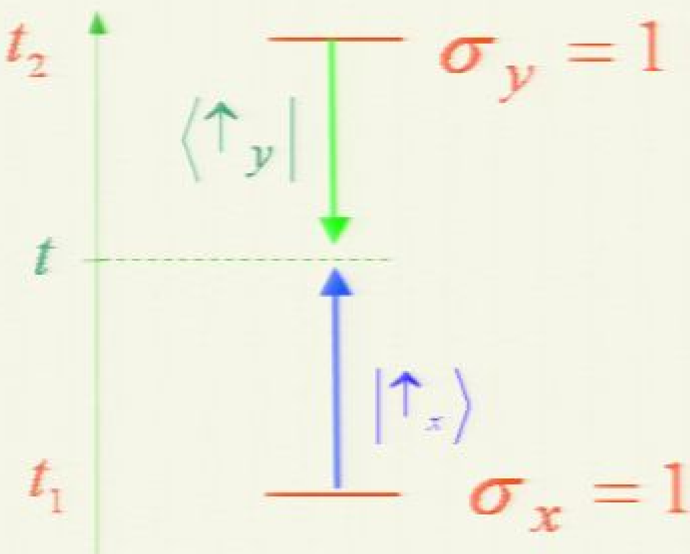
$$C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}$$

$$\left(A+B\right)_w = A_w + B_w$$

$$\left(AB\right)_w \neq A_w B_w$$

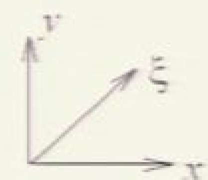
The outcomes of weak measurements are weak values

Weak value of a variable *C* of a pre- and post-selected system described at time *t* by the two-state vector $\langle \Phi | \; | \Psi \rangle$



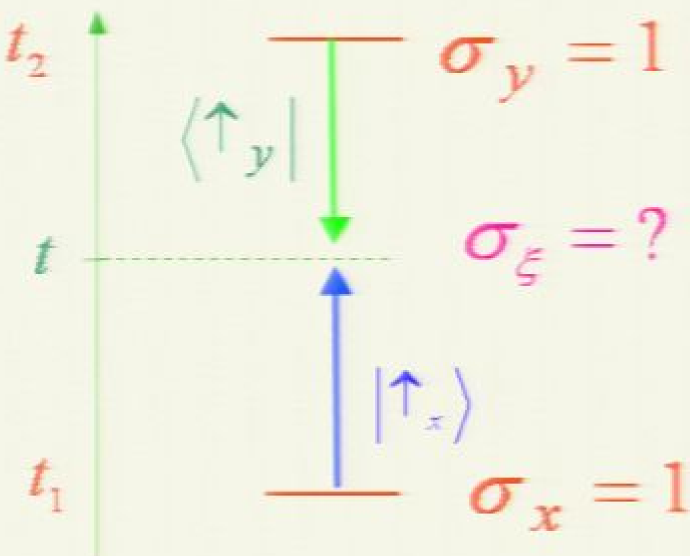
$$C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$$

$$\sigma_\xi \equiv \frac{\sigma_x + \sigma_y}{\sqrt{2}}$$



The outcomes of weak measurements are weak values

Weak value of a variable C of a pre- and post-selected system described at time t by the two-state vector $\langle\Phi|\;|\Psi\rangle$

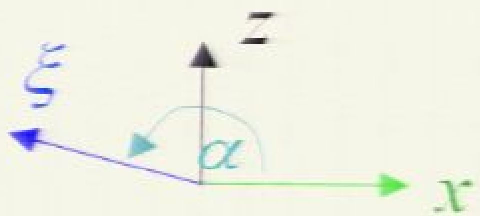


$$C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}$$

$$\sigma_\xi \equiv \frac{\sigma_x + \sigma_y}{\sqrt{2}}$$

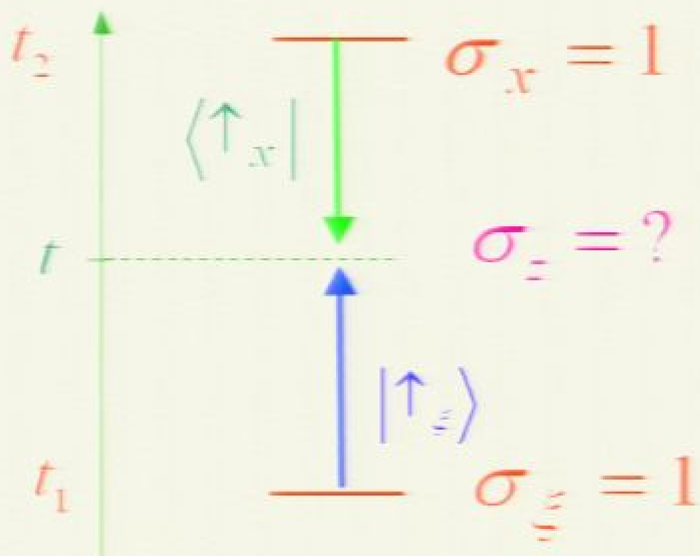
$$(\sigma_\xi)_w = \frac{\langle\uparrow_y|\sigma_\xi|\uparrow_x\rangle}{\langle\uparrow_y|\uparrow_x\rangle} = \frac{\langle\uparrow_y|\frac{\sigma_x+\sigma_y}{\sqrt{2}}|\uparrow_x\rangle}{\langle\uparrow_y|\uparrow_x\rangle} = \sqrt{2}$$

The outcomes of weak measurements are weak values

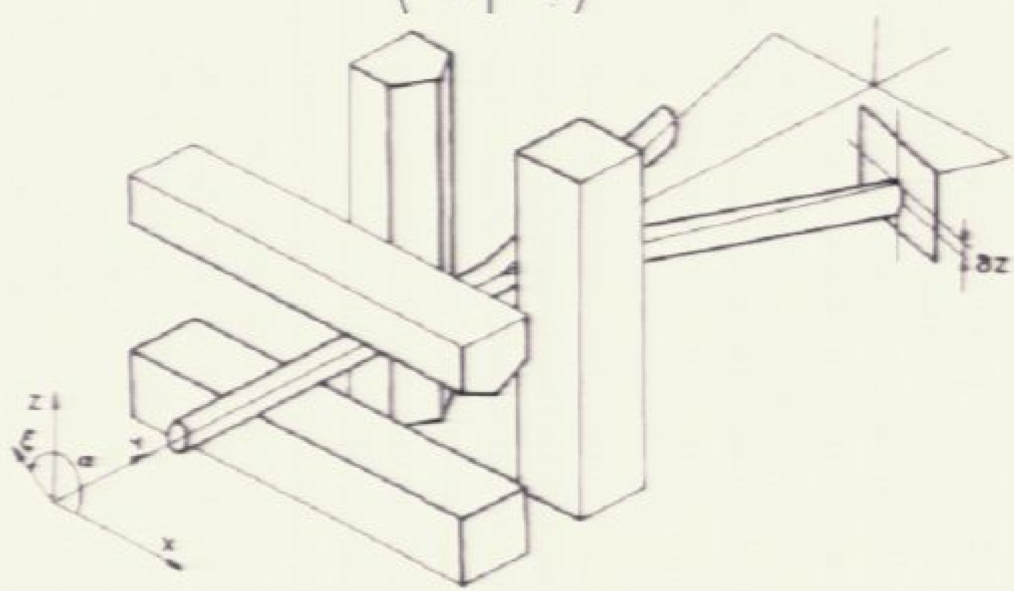


How the result of a measurement of a component of the spin of a spin-*1/2* particle can turn out to be 100

Y. Aharonov, D. Albert, and L. Vaidman **(AAV) PRL 60, 1351 (1988)**



$$(\sigma_z)_w = \frac{\langle \uparrow_x | \sigma_z | \uparrow_\xi \rangle}{\langle \uparrow_x | \uparrow_\xi \rangle} = \tan\frac{\alpha}{2}$$



Realization of a measurement of a ``weak value''

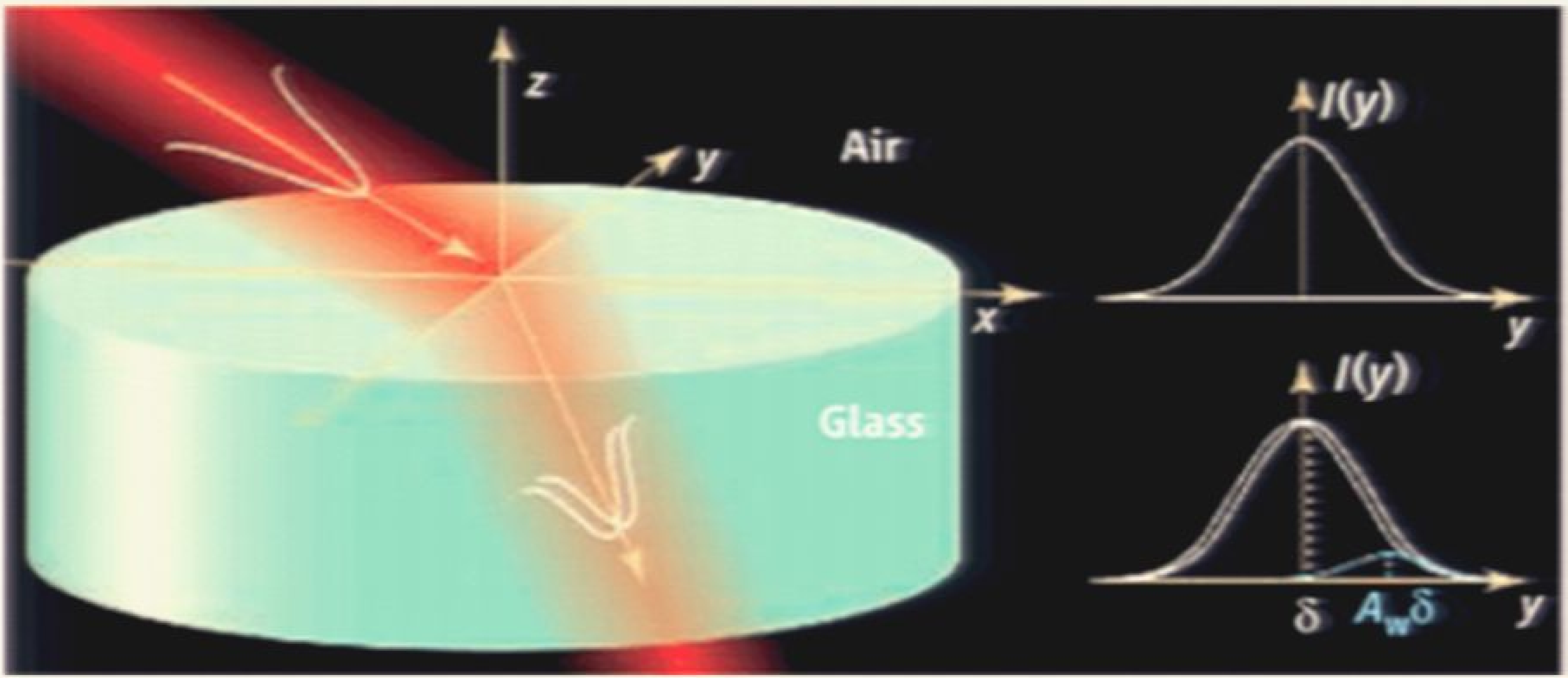
N. W. M. Ritchie, J. G. Story, and R. G. Hulet

Phys. Rev. Lett. **66**, 1107-1110 (1991)

Science 8 February 2008: **Amplifying a Tiny Optical Effect**

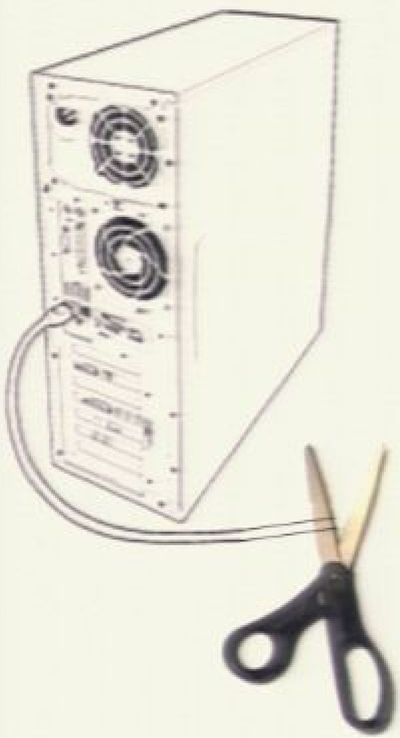
K. J. Resch

"In the first work on weak measurement (AAV), it was speculated that the technique could be useful in amplifying and measuring small effects. Now, 20 years later, this potential has finally been realized."



Observation of the Spin Hall Effect of Light via Weak Measurements

O. Hosten and P. Kwiat



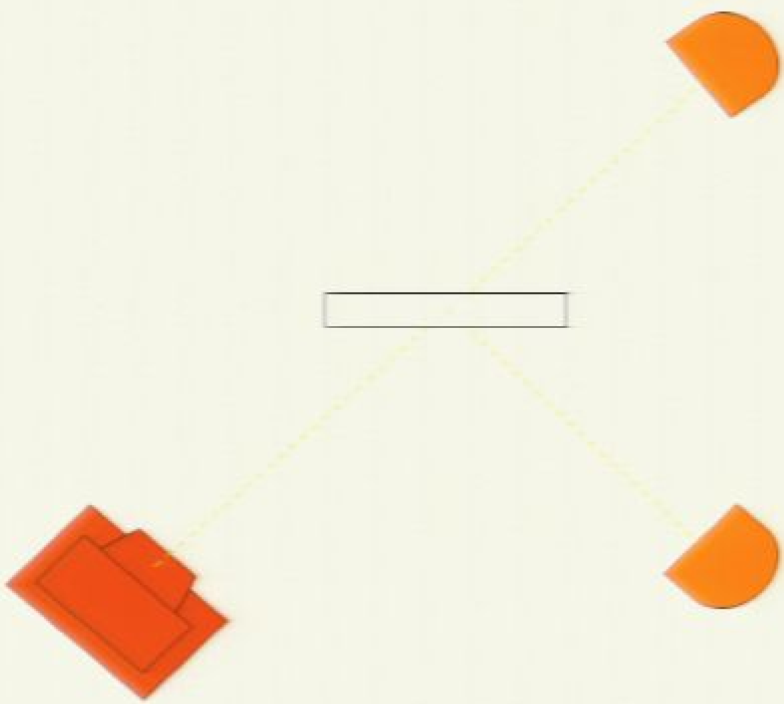
Counterfactual Computation:

FINDING THE RESULT OF A COMPUTATION WITHOUT RUNNING THE COMPUTER

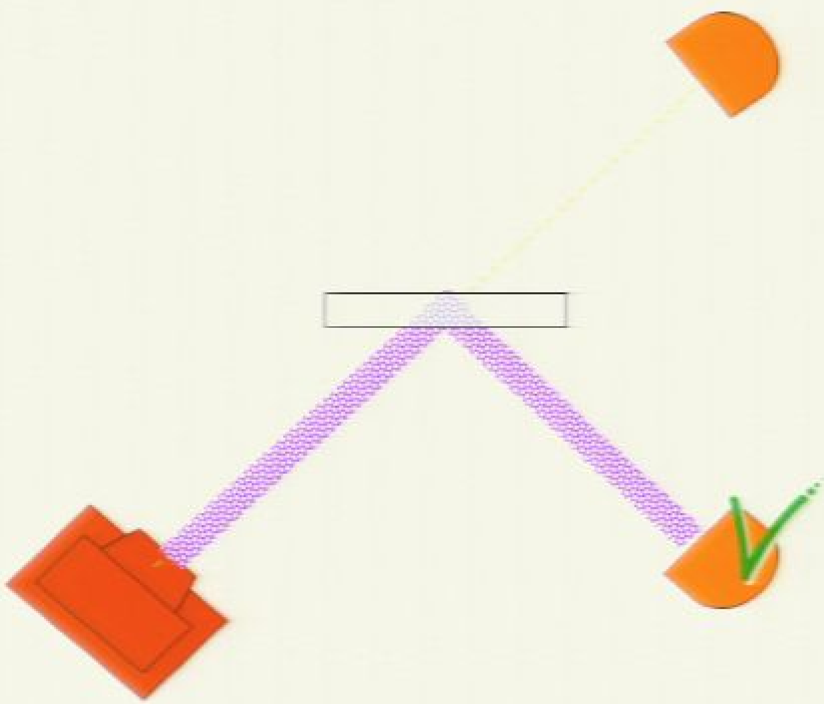
or

Where is the pre- and post-selected particle?

Where Is the Quantum Particle between Two Measurements?



Where Is the Quantum Particle between Two Measurements?



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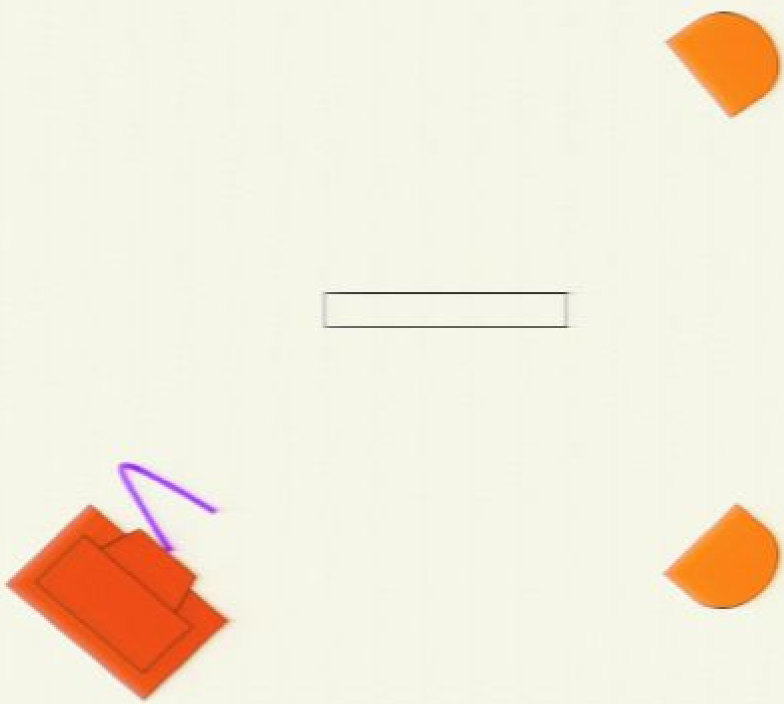
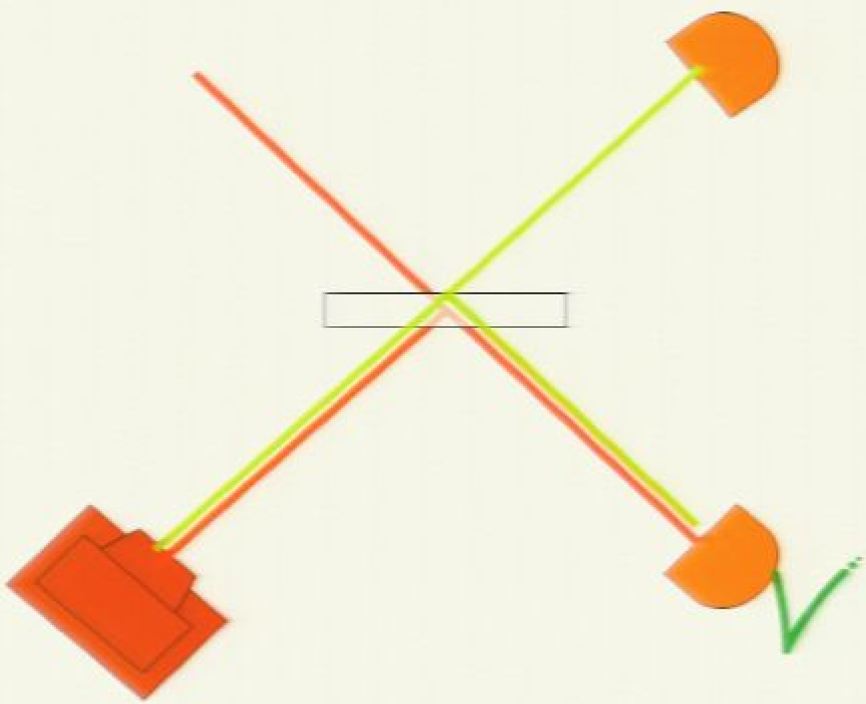
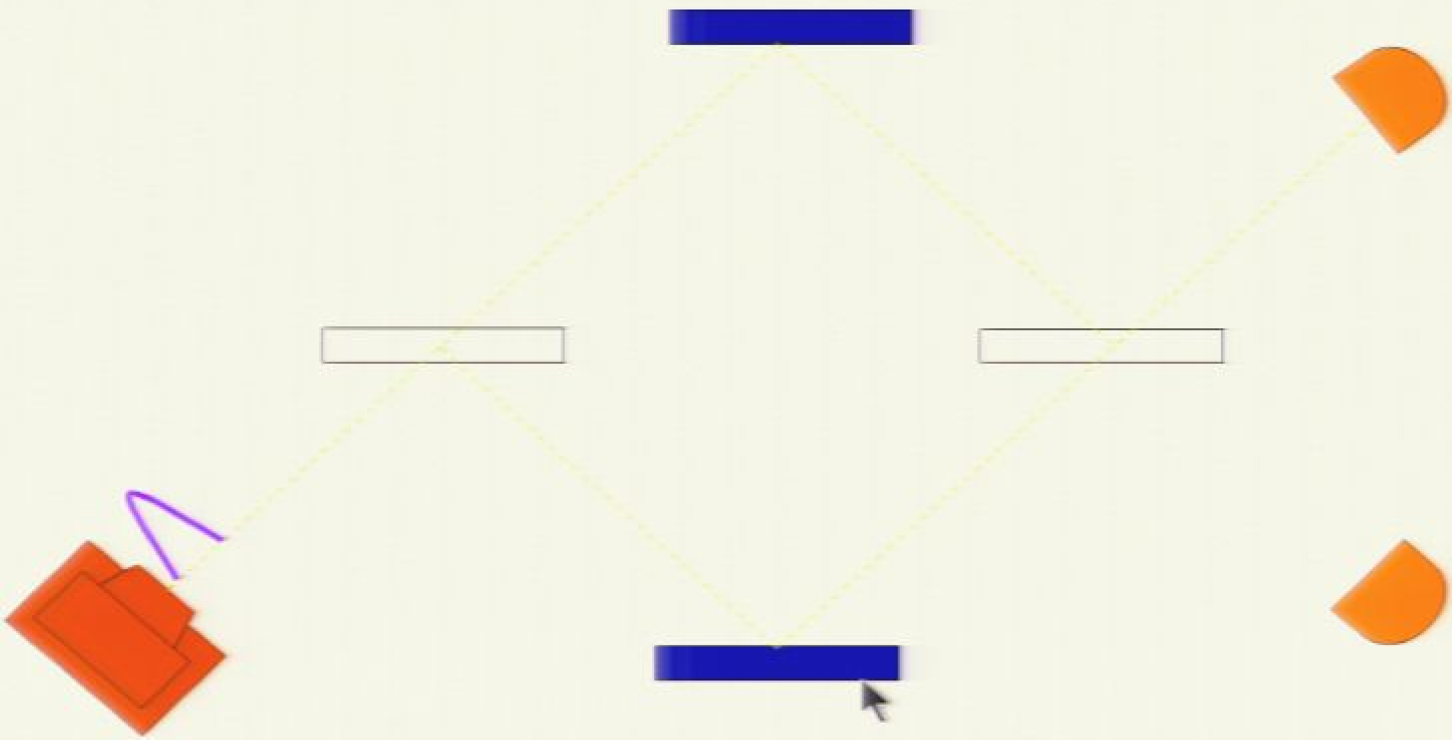


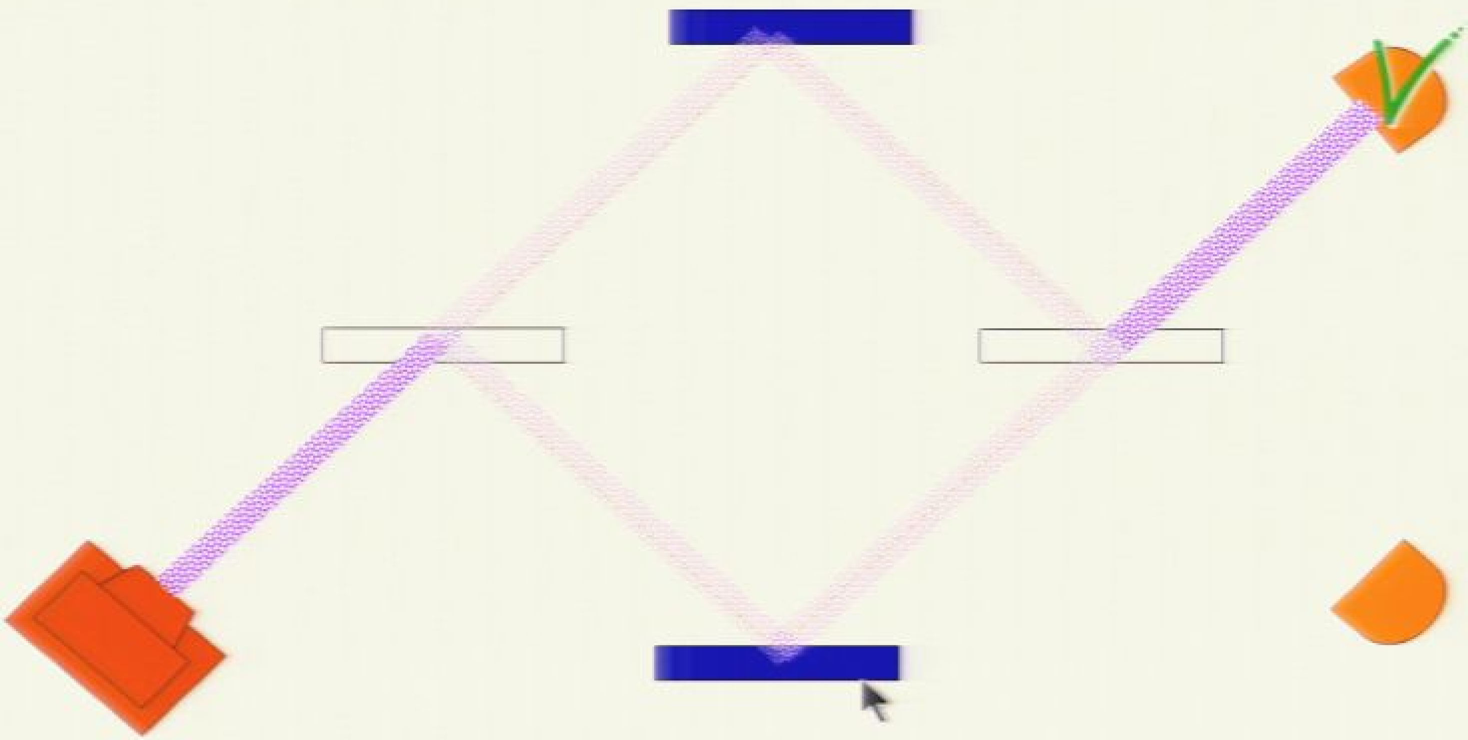
Where Is the Quantum Particle between Two Measurements?



Mach-Zehnder Interferometer



Mach-Zehnder Interferometer



Interaction-free measurement

A. Elitzur and L. Vaidman
Found. Phys. **23**, 987 (1993) .

SUPER MINE : explodes when any particle "touches" it

interacts only through explosion

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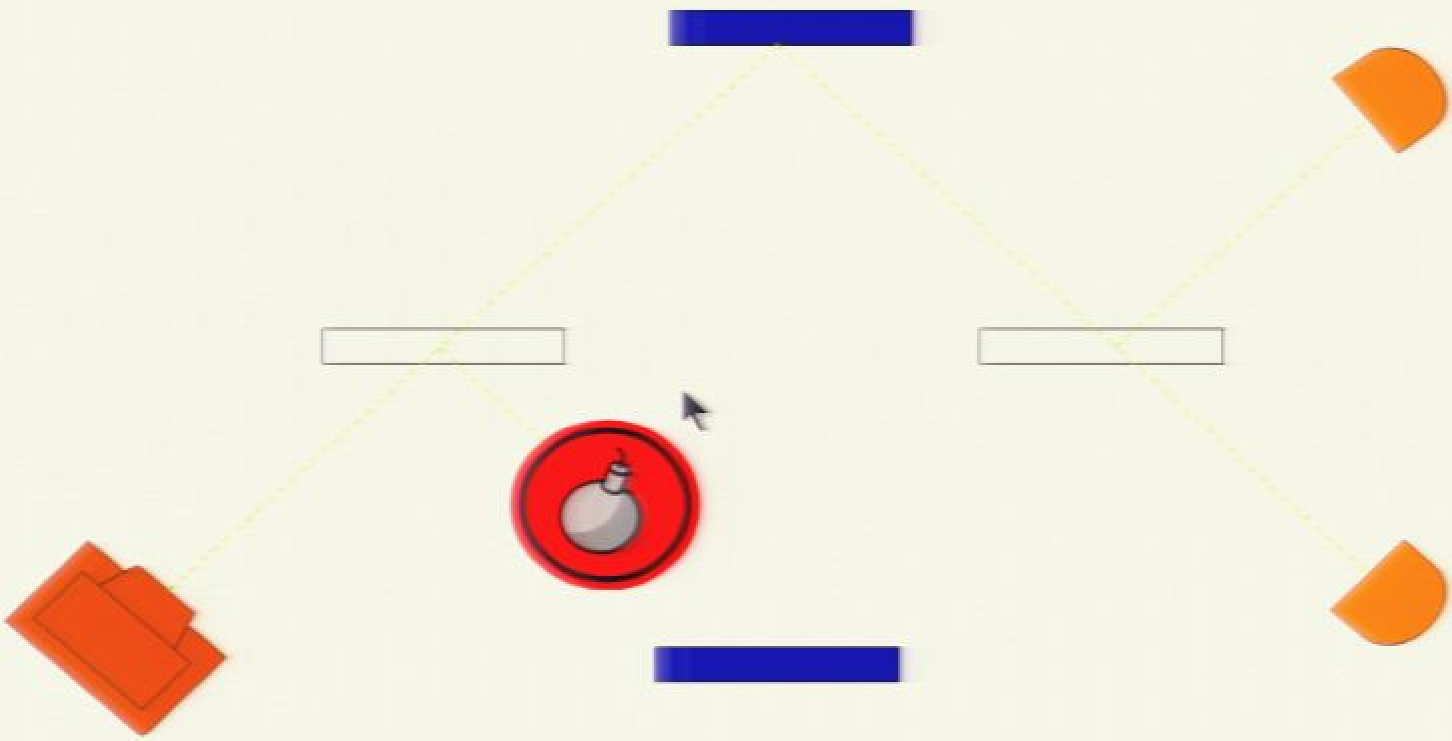
explodes when any particle "touches" it

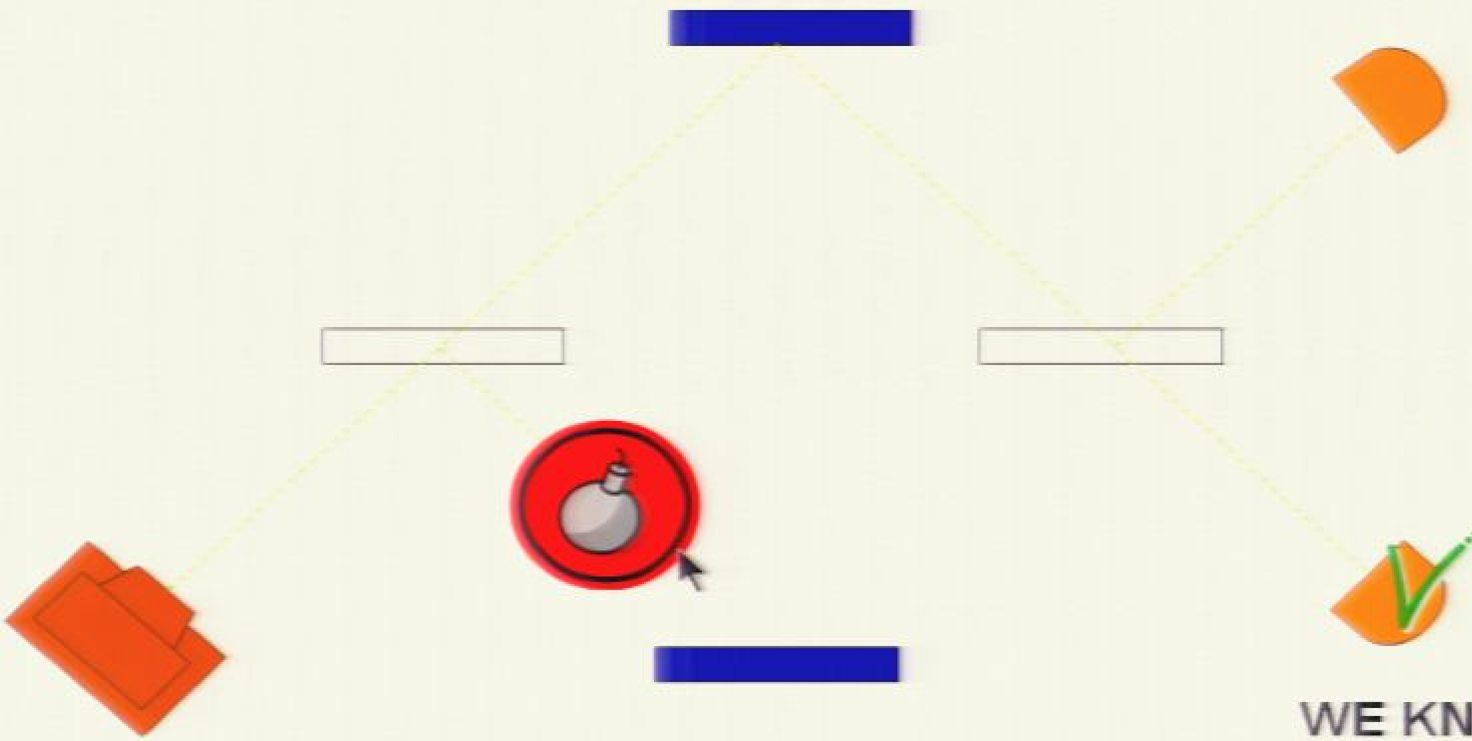
interacts only through explosion

HOW TO FIND THE SUPER MINE WITHOUT EXPLODING IT?

Interaction-free measurement

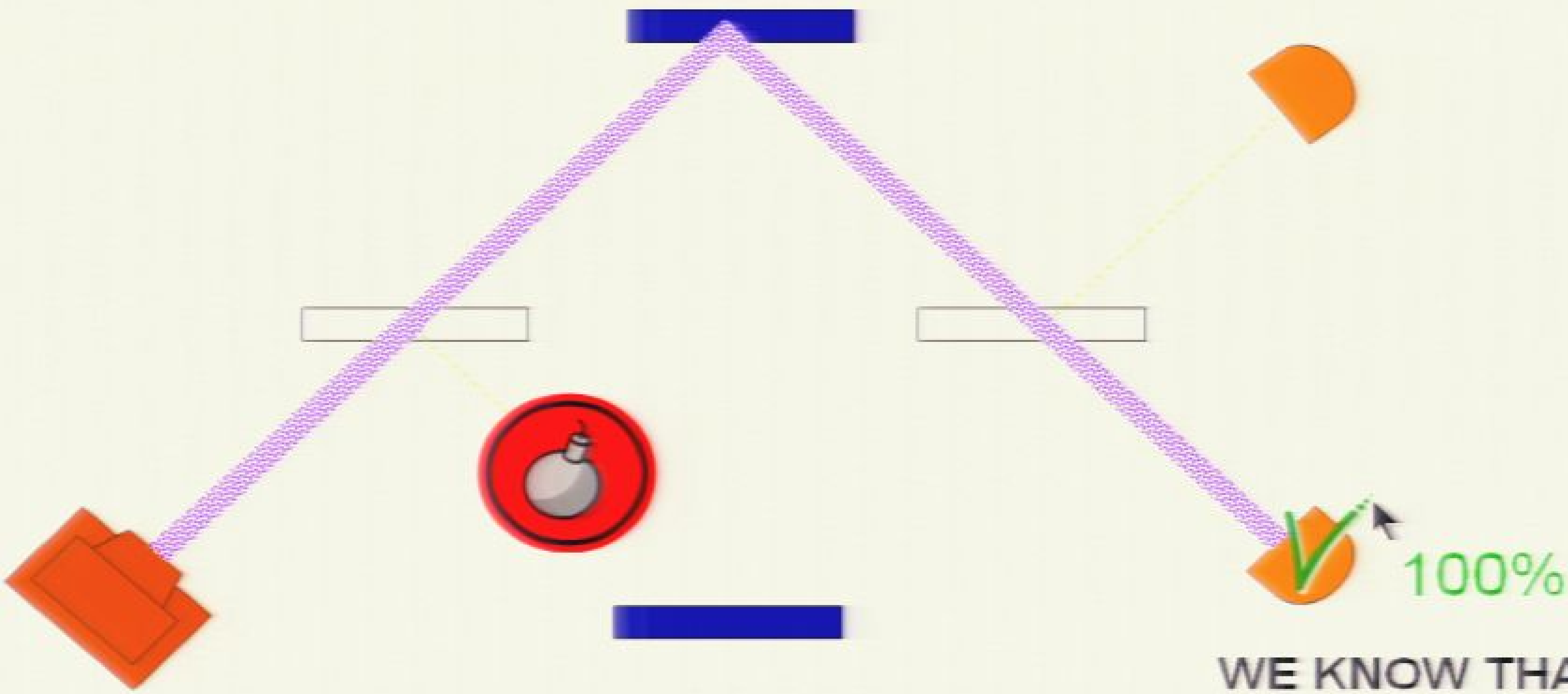


Interaction-free measurement



WE KNOW THAT
THE SUPER MINE
IS THERE AND IT
IS STILL INTACT

Interaction-free measurement



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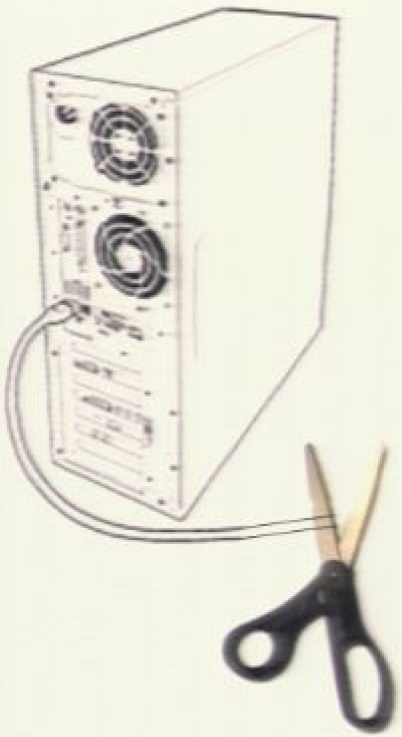
R. Jozsa, LNCS 1509, 103(199

Counterfactual Computation:

FINDING THE RESULT OF A COMPUTATION WITHOUT RUNNING THE COMPUTER



R. Jozsa, LNCS 1509, 103(199

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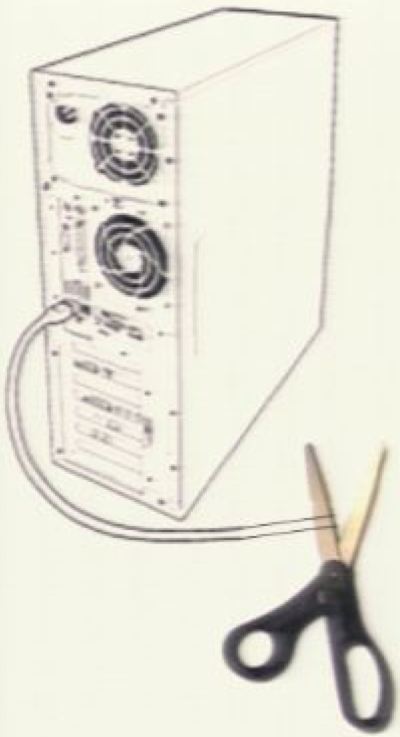
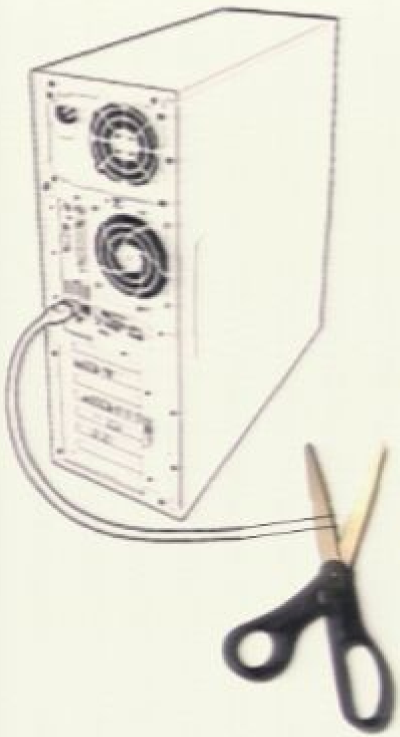


Computer is "running" = a photon passes through



R. Jozsa, LNCS 1509, 103(199

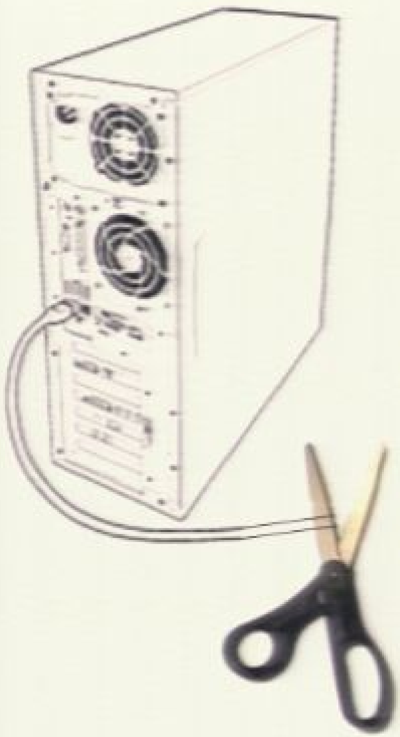
Counterfactual Computation:

FINDING THE RESULT OF A COMPUTATION WITHOUT RUNNING THE COMPUTER

Computer is "running" = a photon passes through

The computer calculates f which might be 1 or 0

R. Jozsa, LNCS 1509, 103(199



Counterfactual Computation:

FINDING THE RESULT OF A COMPUTATION WITHOUT RUNNING THE COMPUTER

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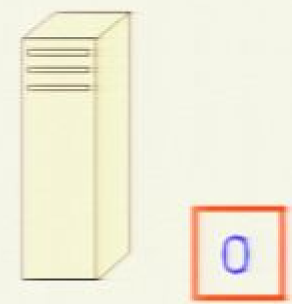
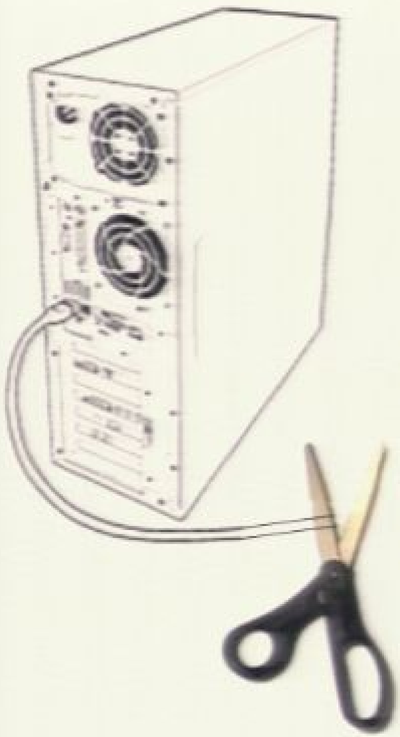
The computer calculates f which might be 1 or 0

Outcome "0" the photon is not disturbed





R. Jozsa, LNCS 1509, 103(199

Counterfactual Computation:

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The computer calculates f which might be 1 or 0

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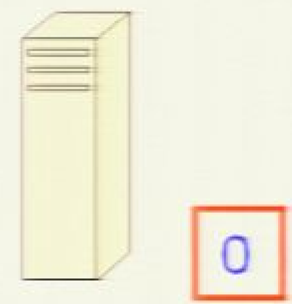
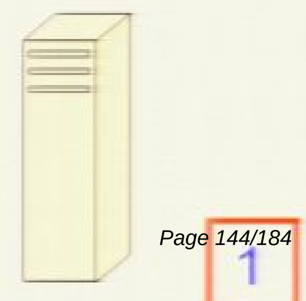
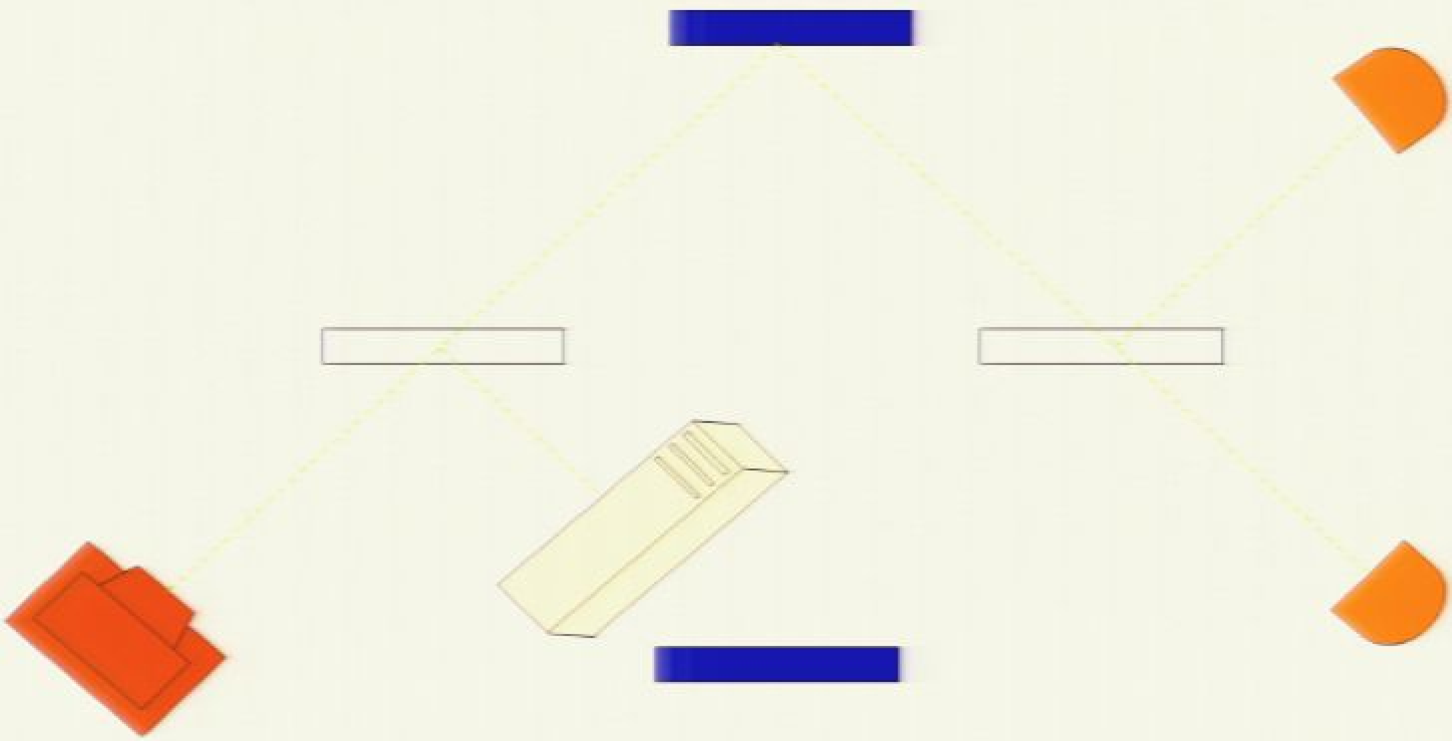


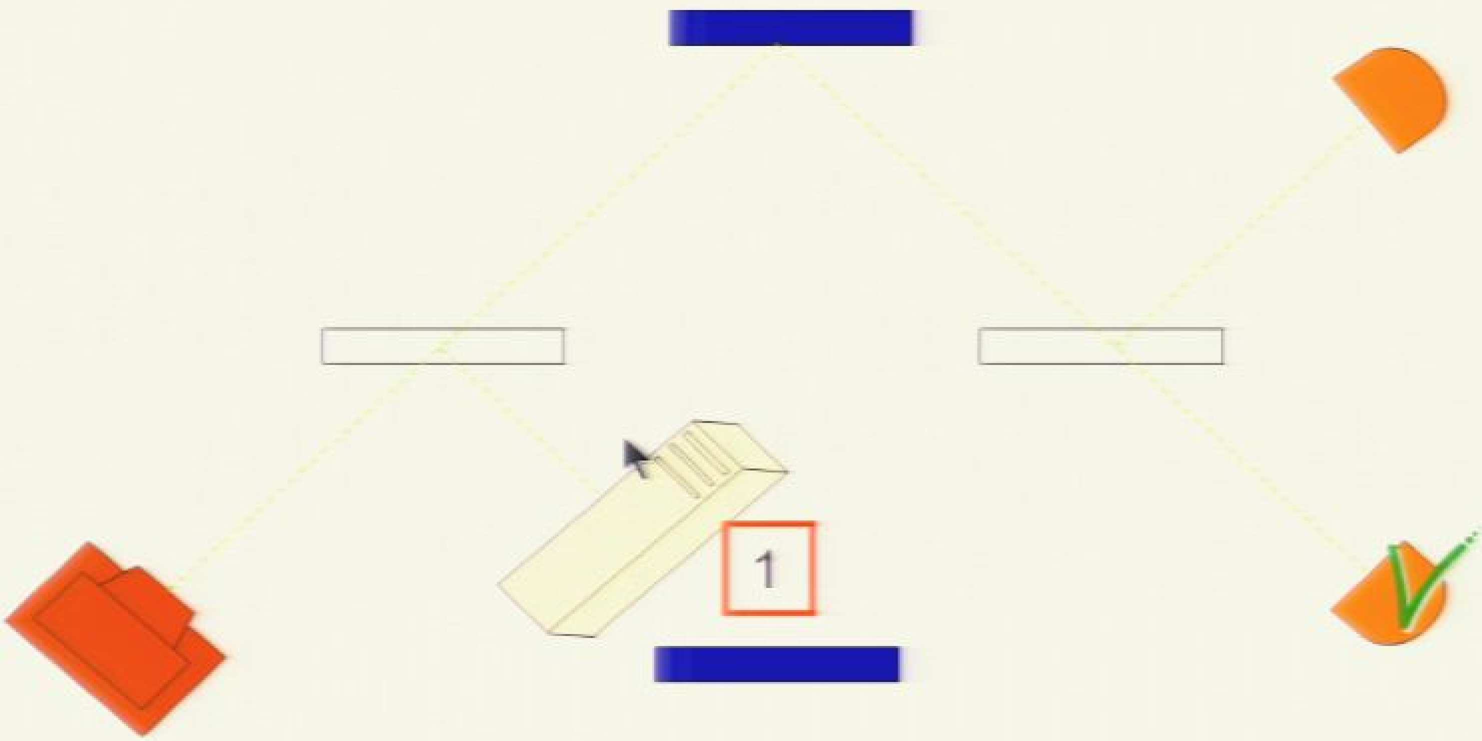
Outcome "1" the photon is absorbed



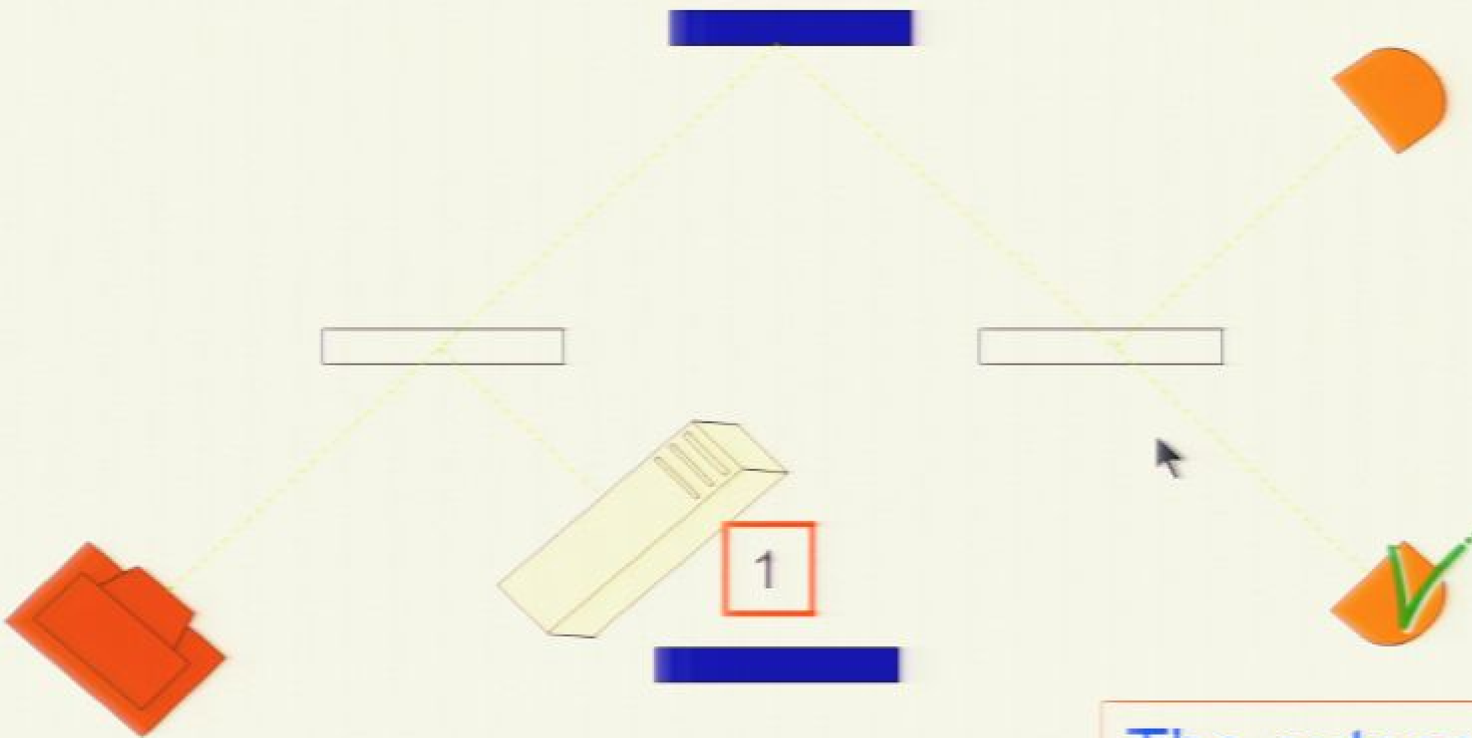
Counterfactual computation



Counterfactual computation

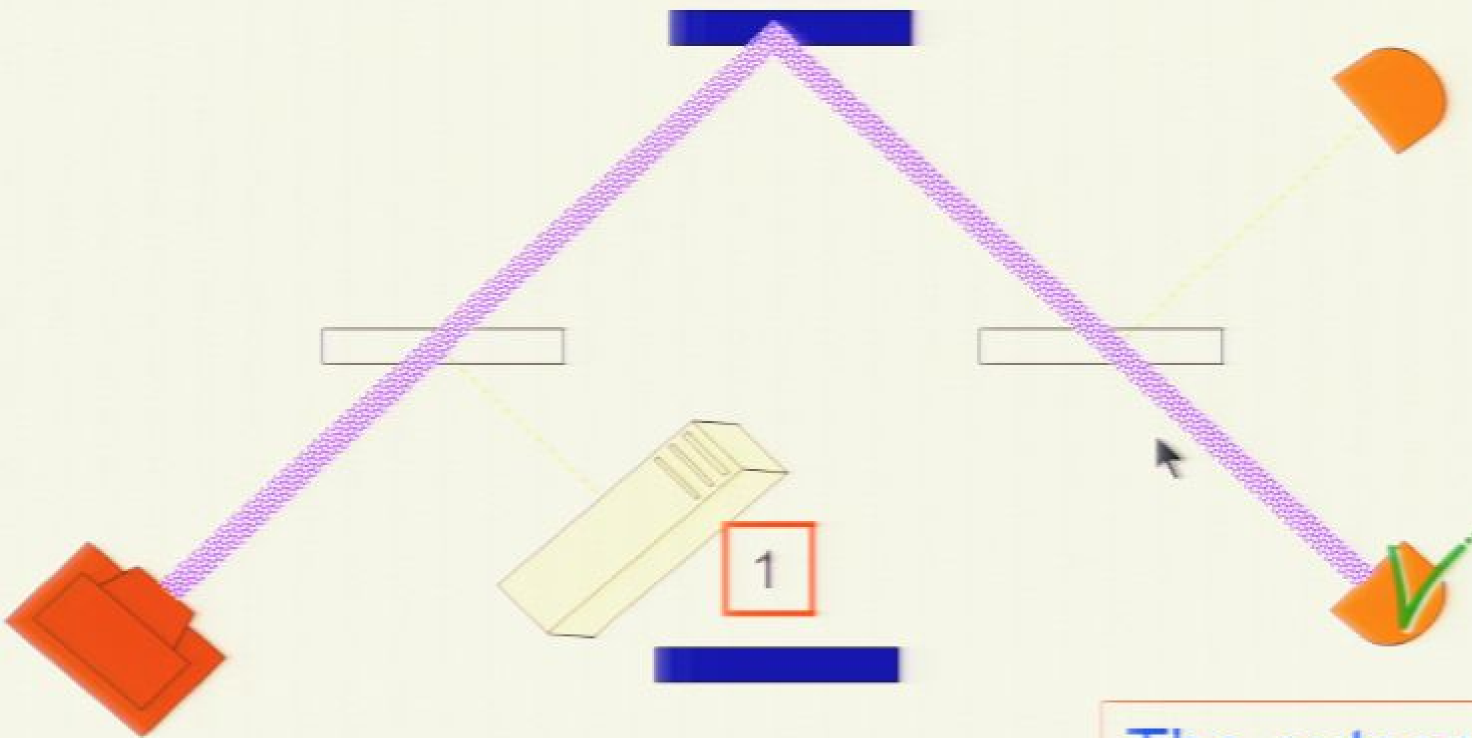


Counterfactual computation



The outcome is 1. The computer was not running

Counterfactual computation



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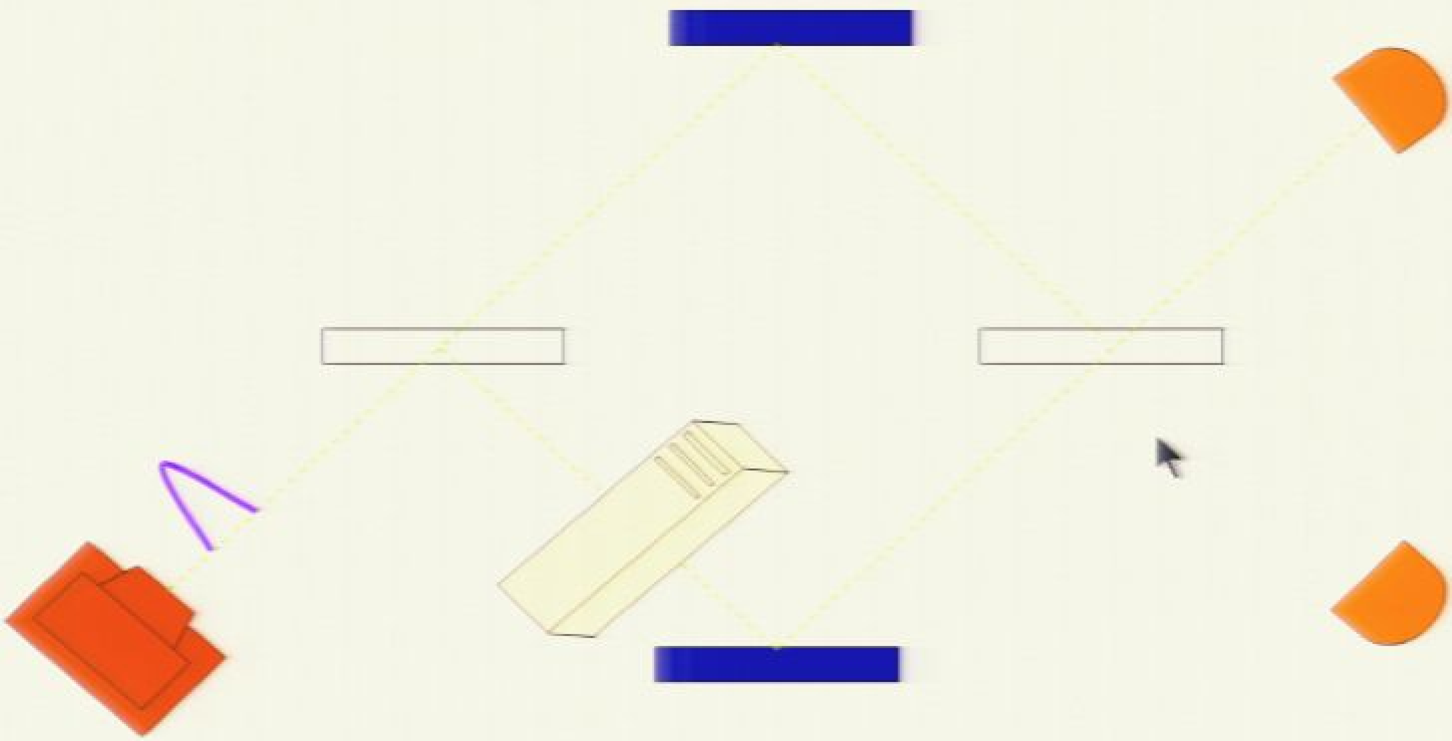
Counterfactual computation



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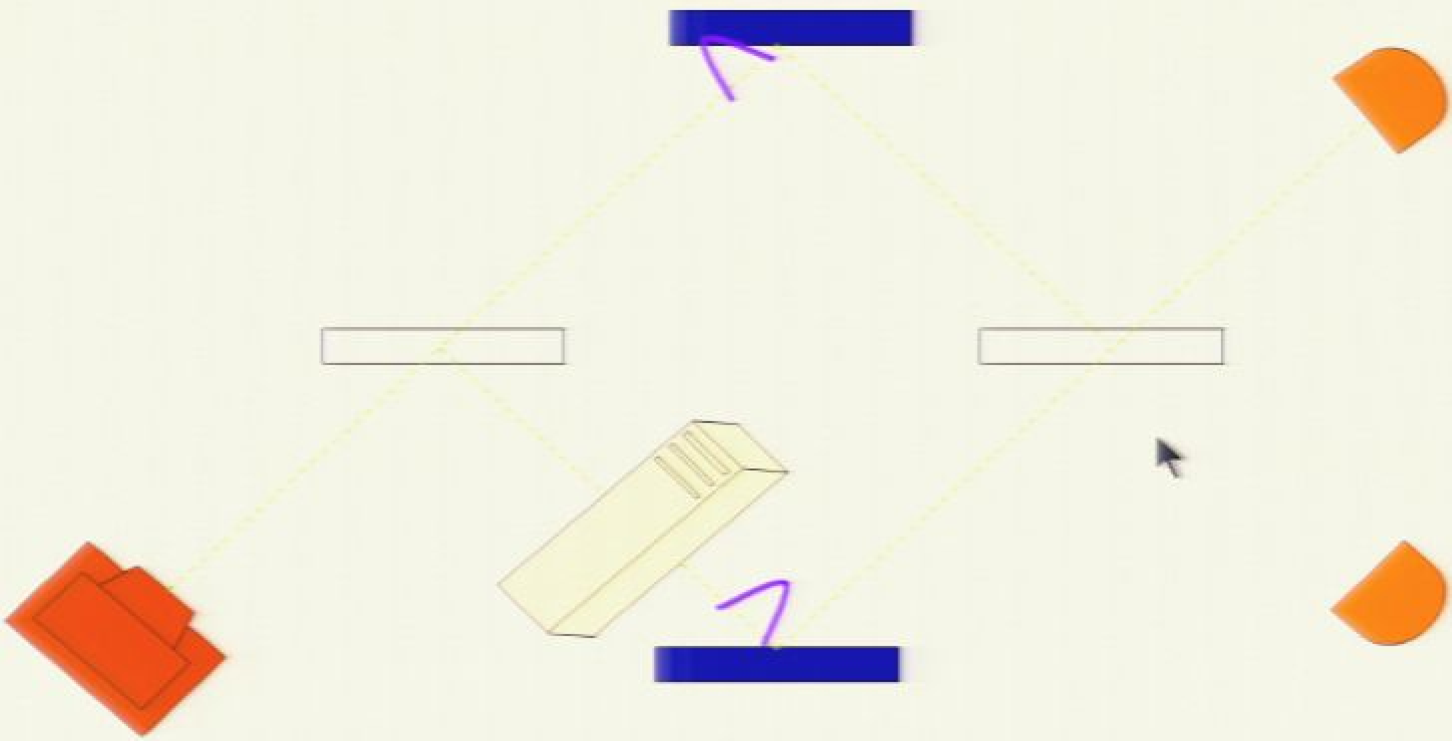

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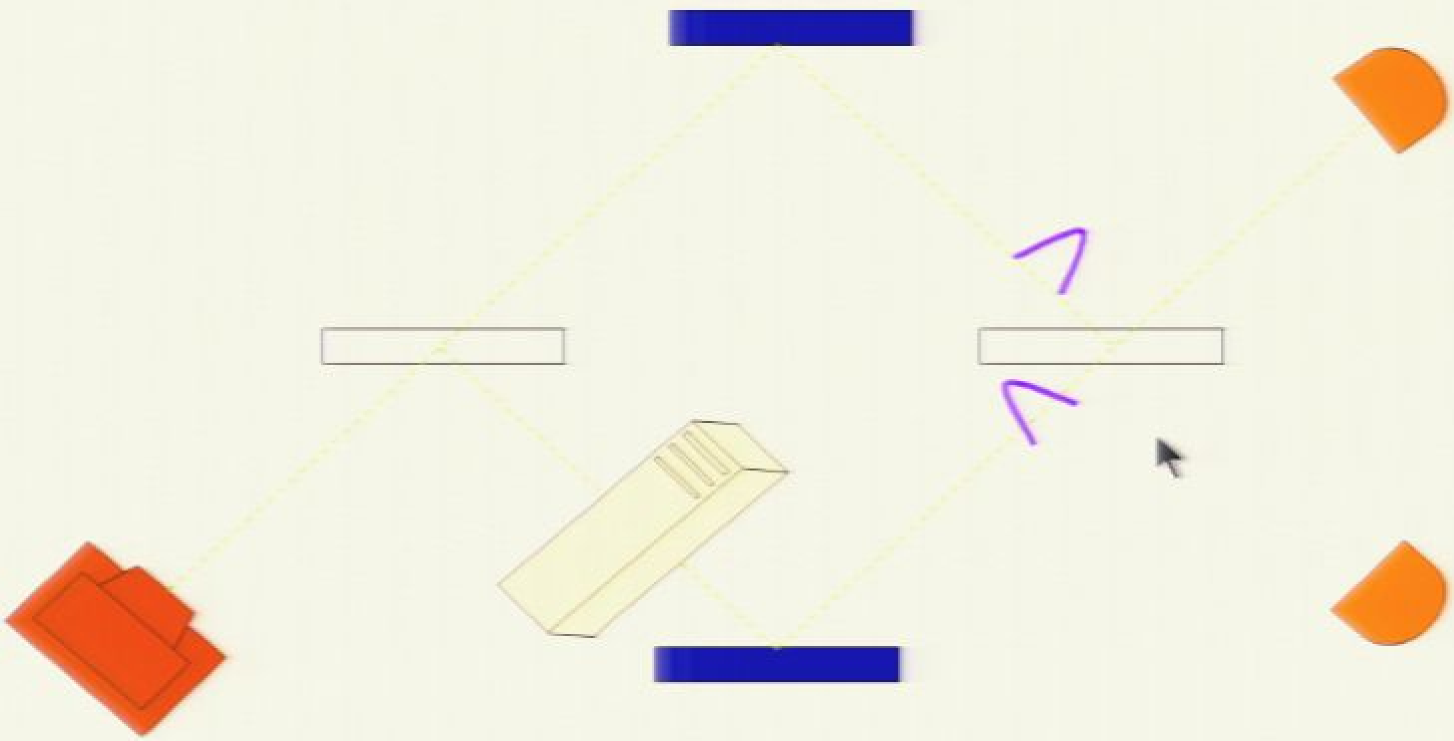
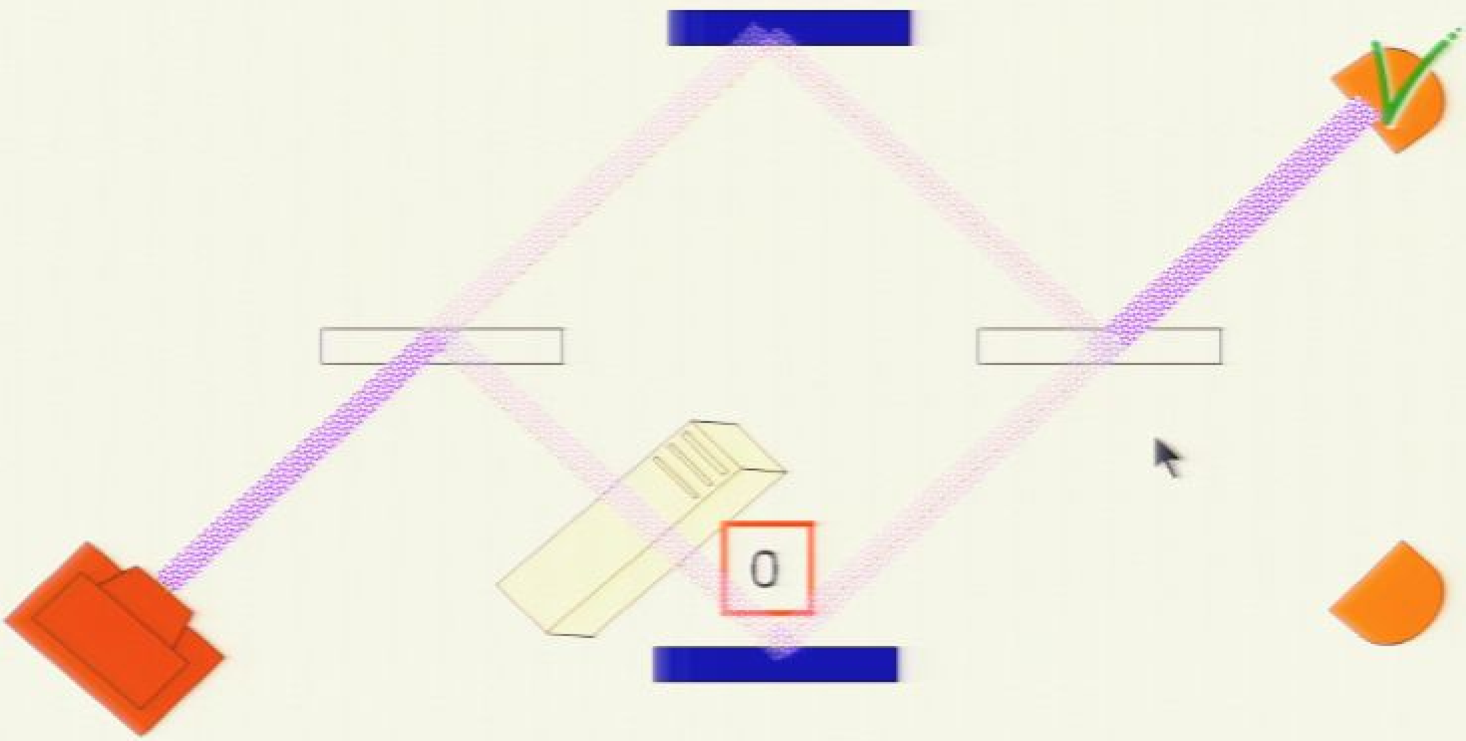
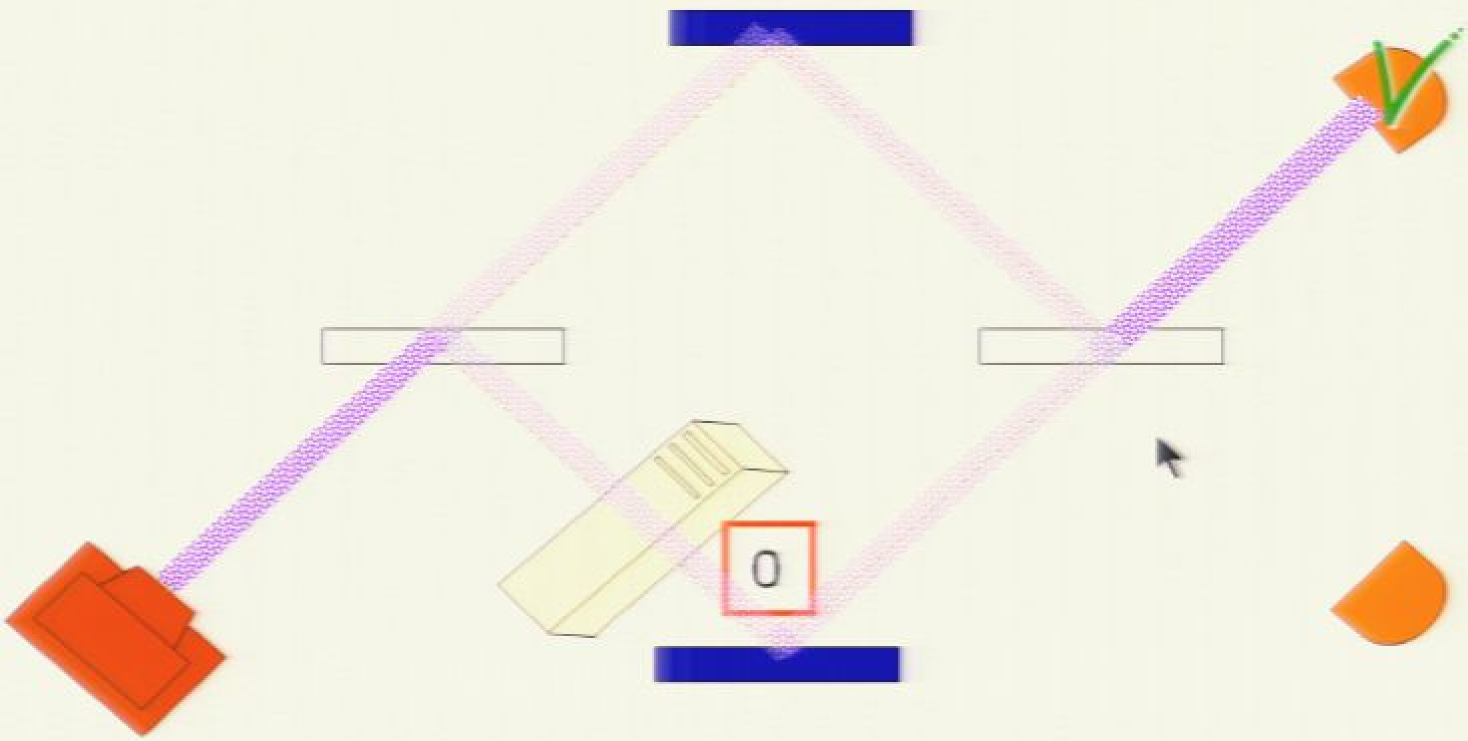


Counterfactual computation



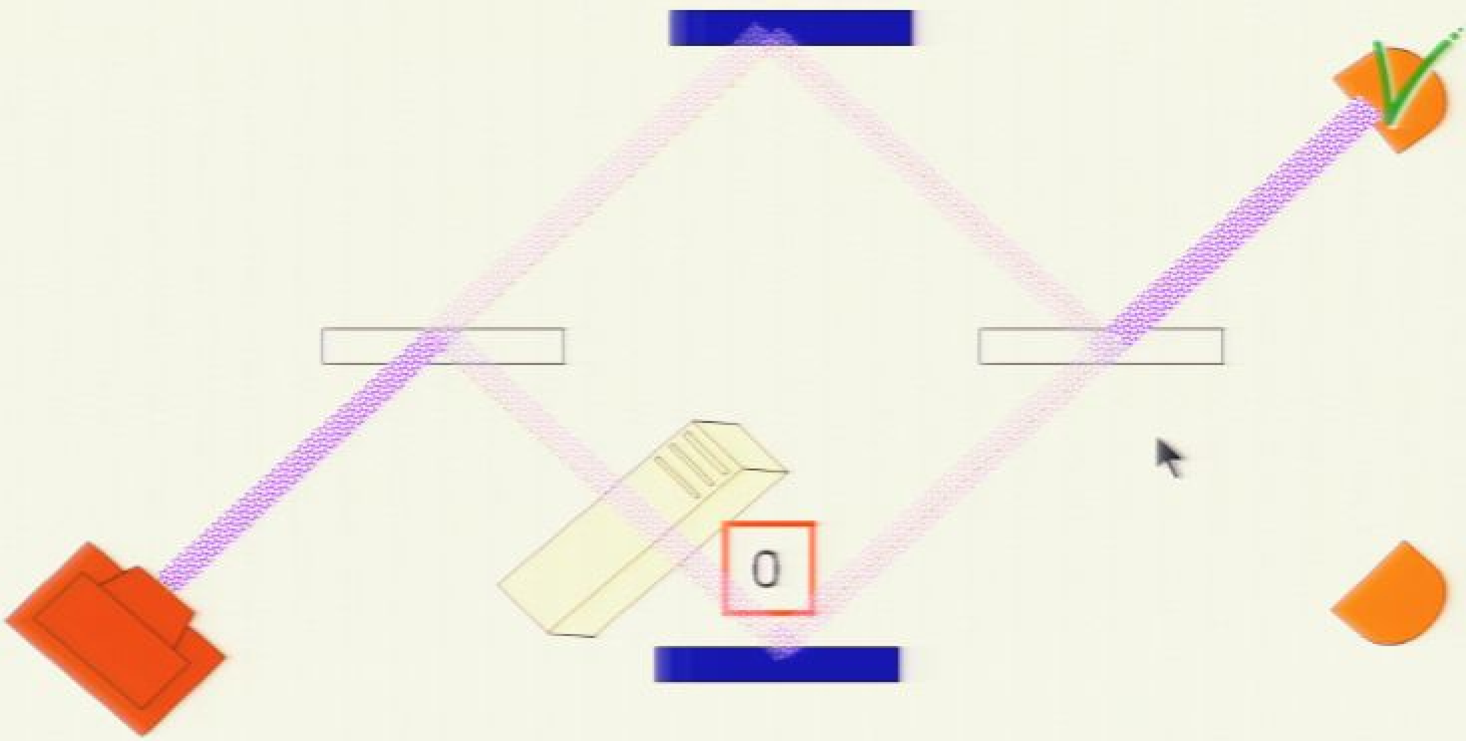
The outcome is 0. The computer was running

Counterfactual computation



The outcome is 0. The computer was running

Counterfactual computation only for one outcome



The outcome is 0. The computer was running

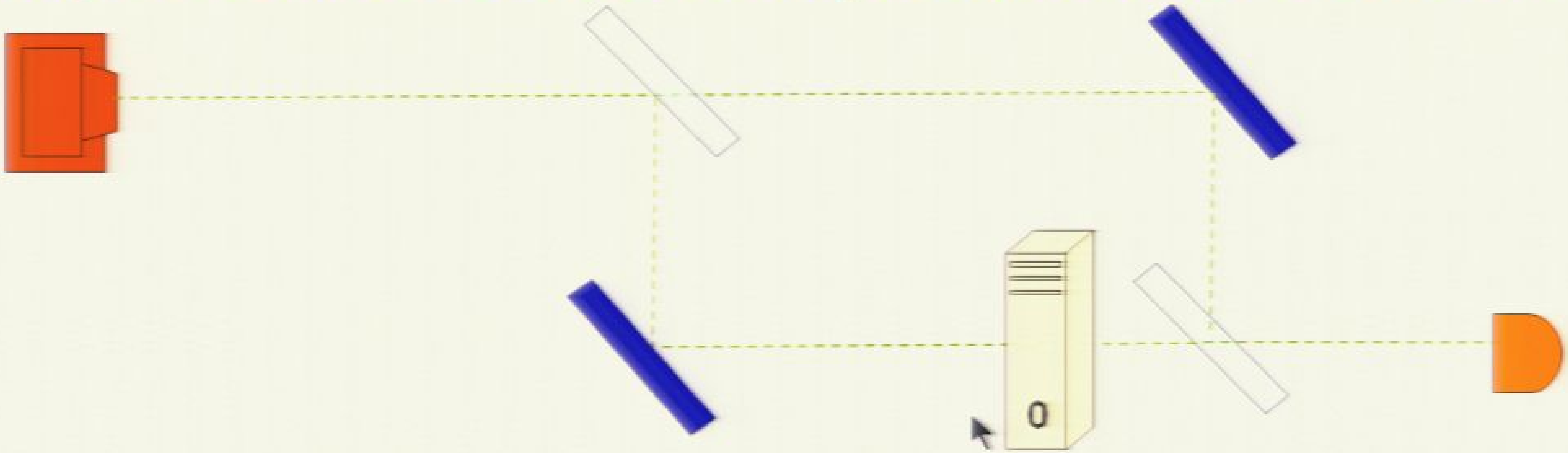
Kwiat: Counterfactual computation for all outcomes is possible

Hosten,…Kwiat, *Nature* **439**, 949 (2006)

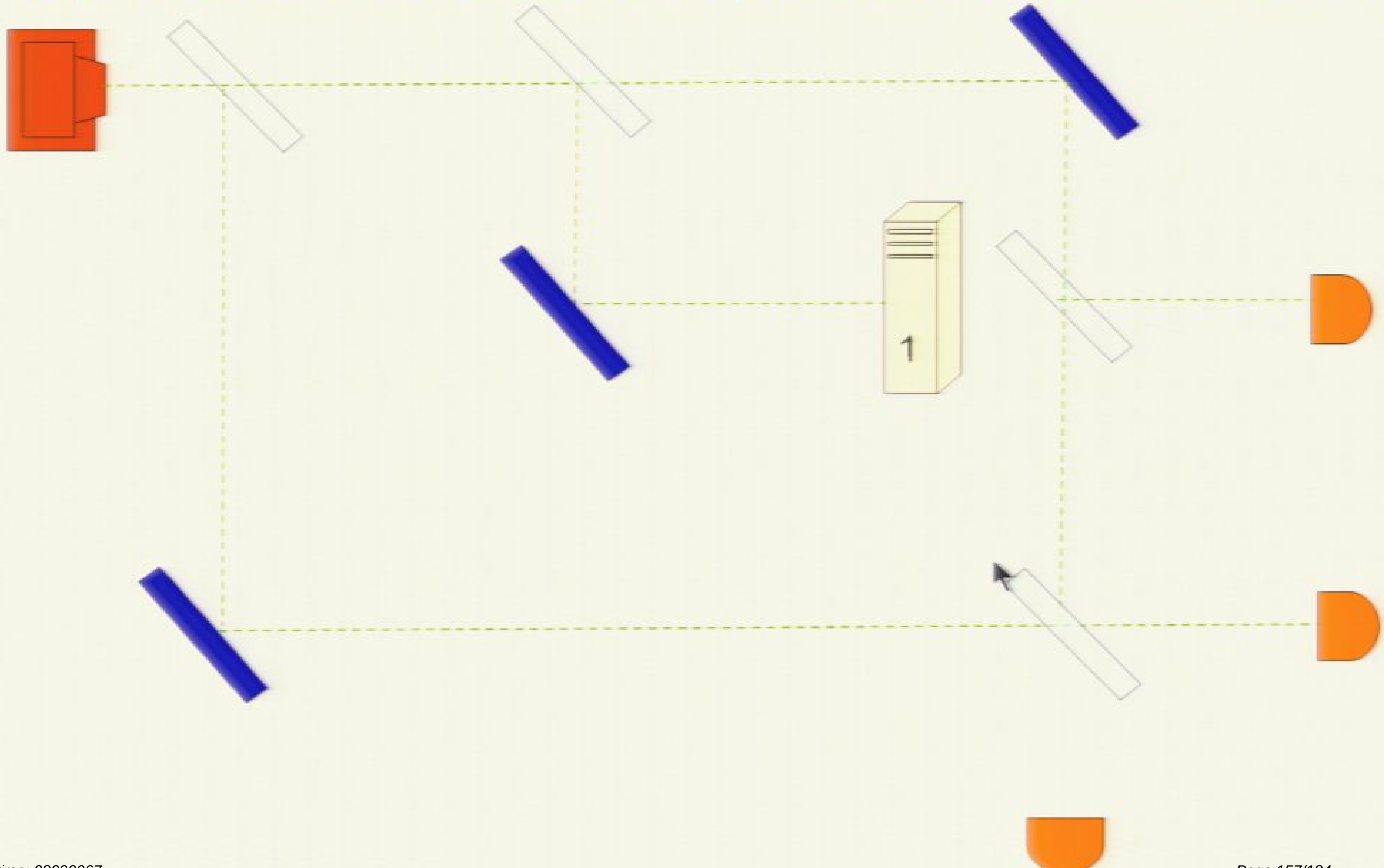
$\Downarrow$

Counterfactual computation for outcome "0" is possible

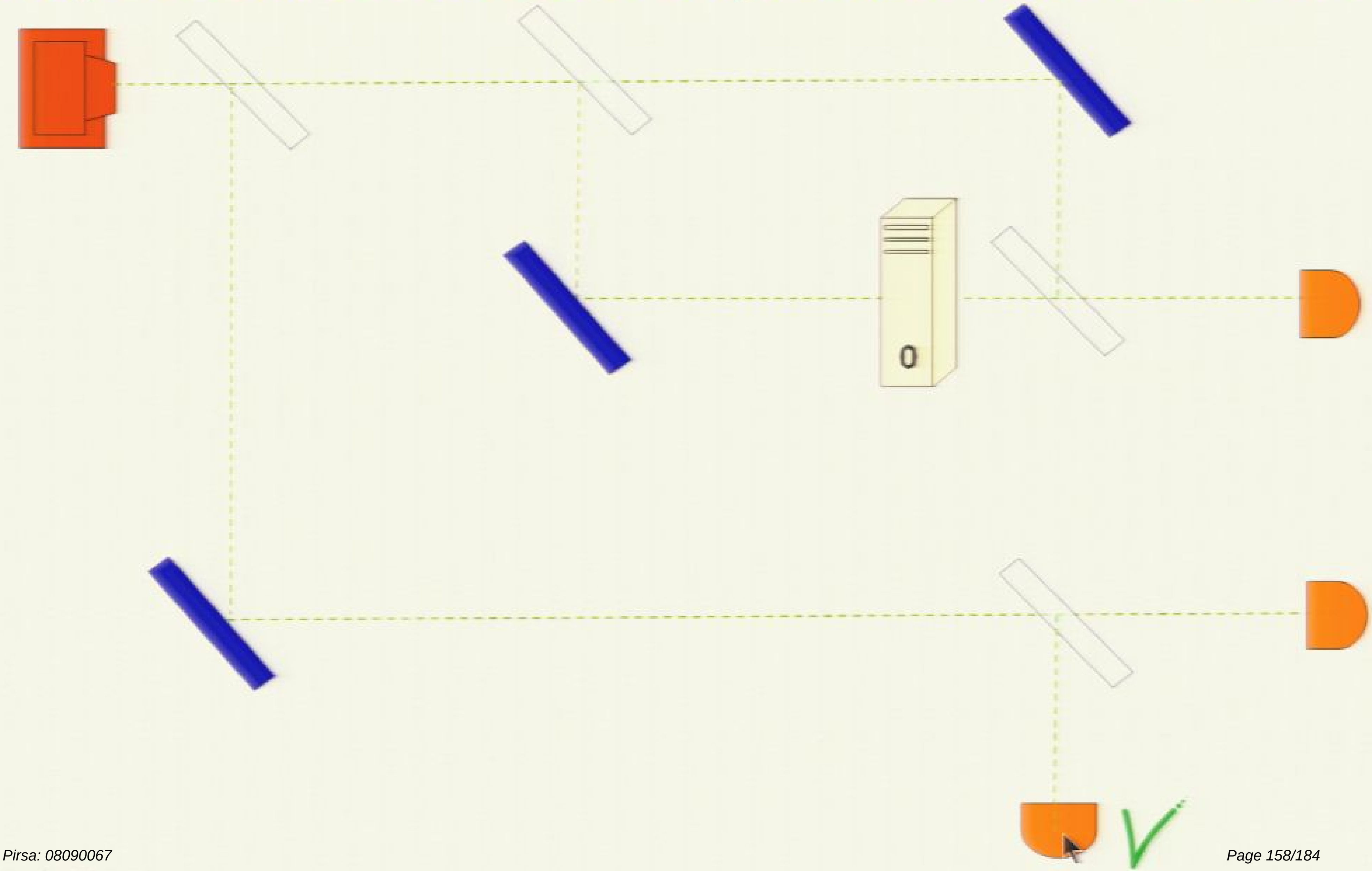
Simple Counterfactual Computation with Outcome 0



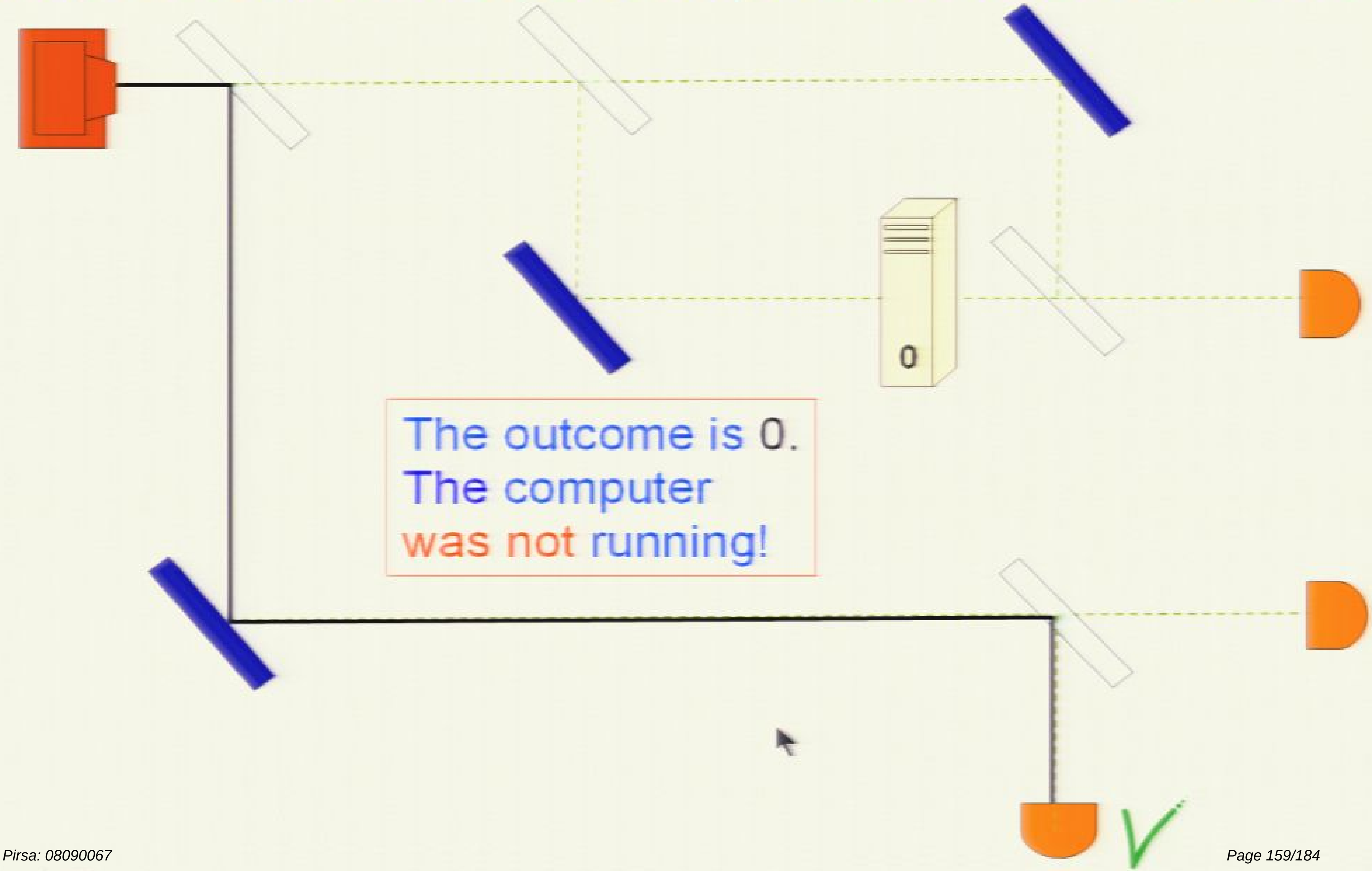
Simple Counterfactual Computation with Outcome 0



Simple Counterfactual Computation with Outcome 0



Simple Counterfactual Computation with Outcome 0



The Impossibility of the Counterfactual Computation for all Possible Outcomes

L.Vaidman, PRL 98, 160403 (2007)

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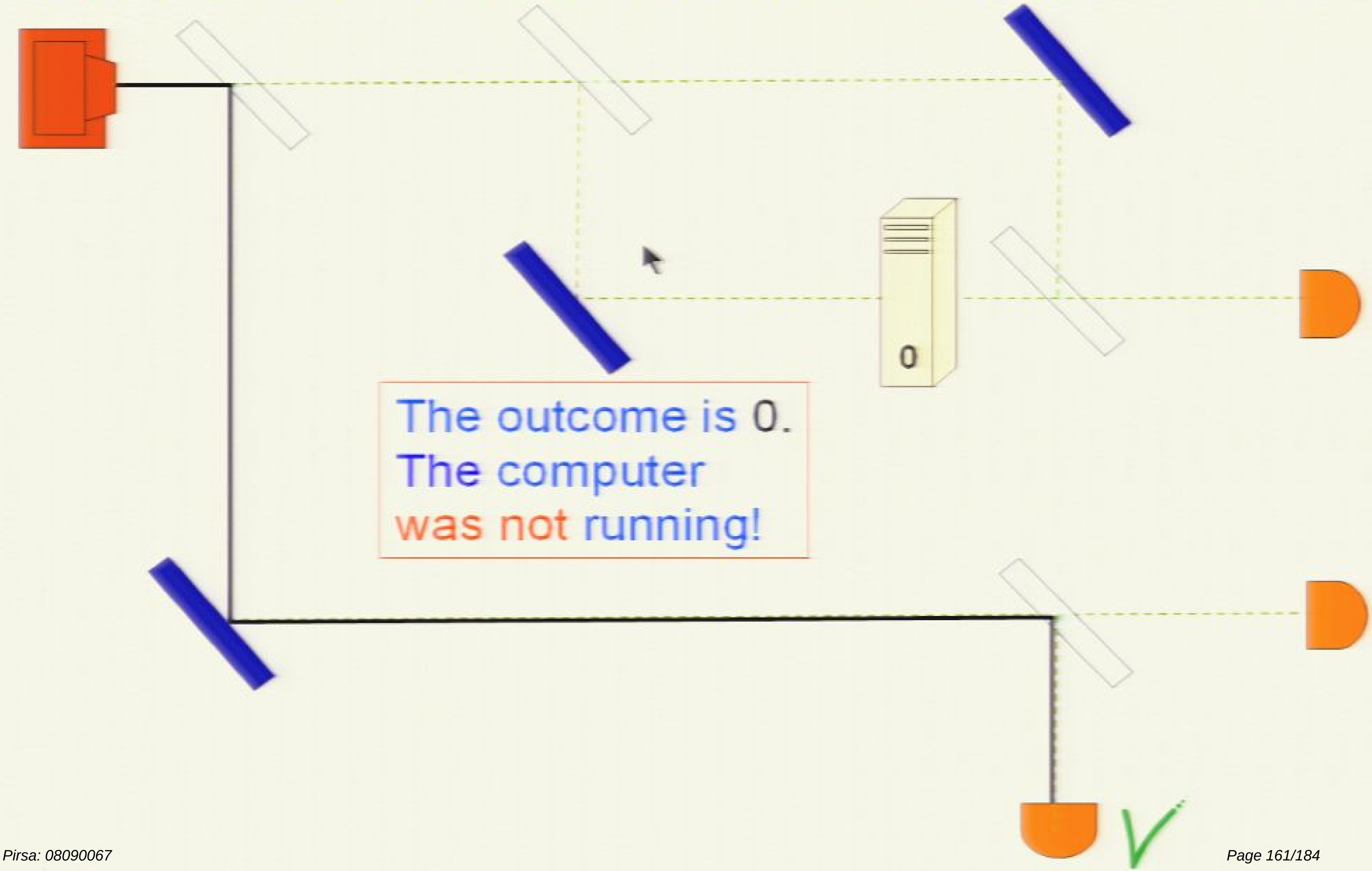

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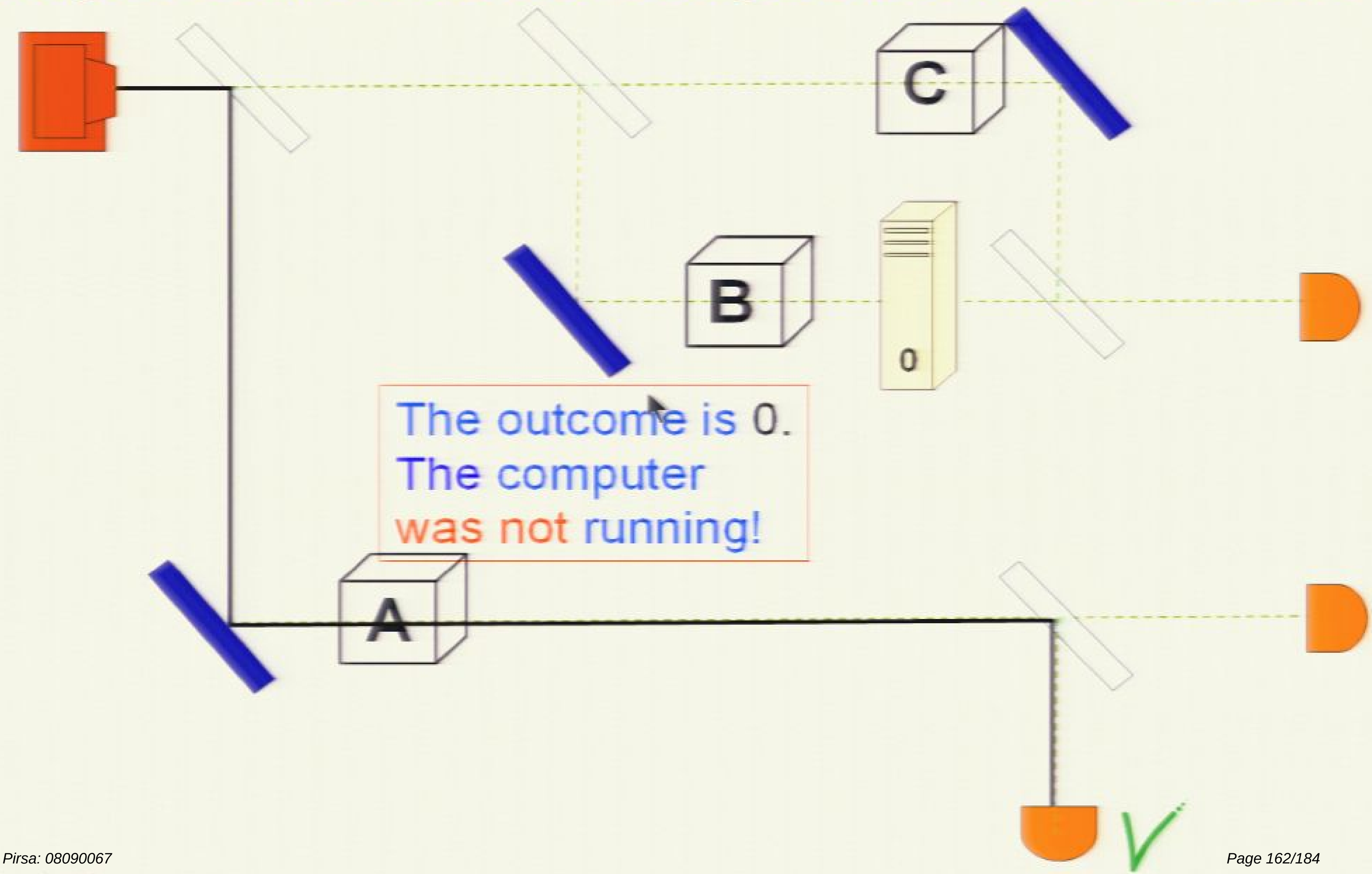
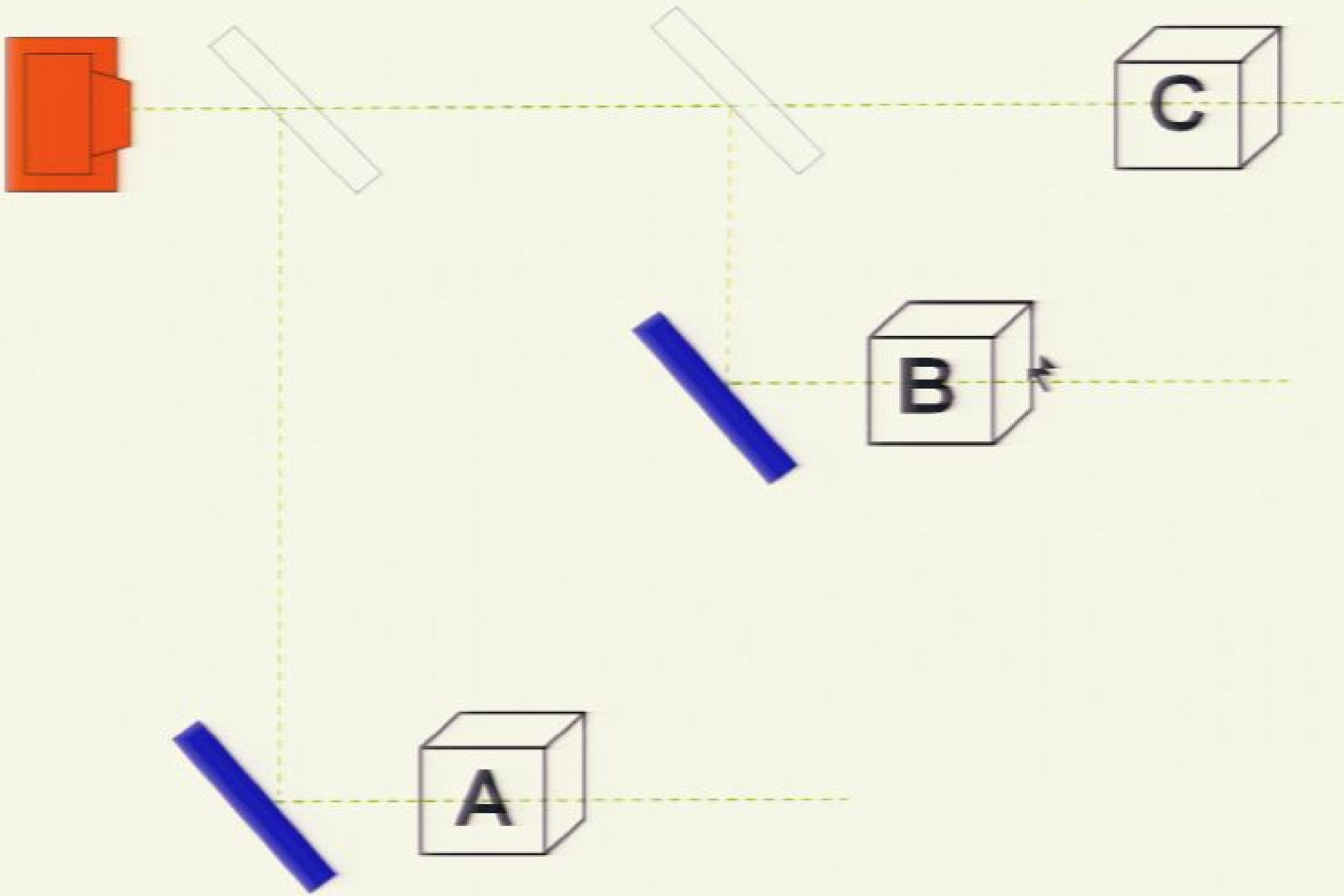


Kwiat's scheme = 3-boxes paradox



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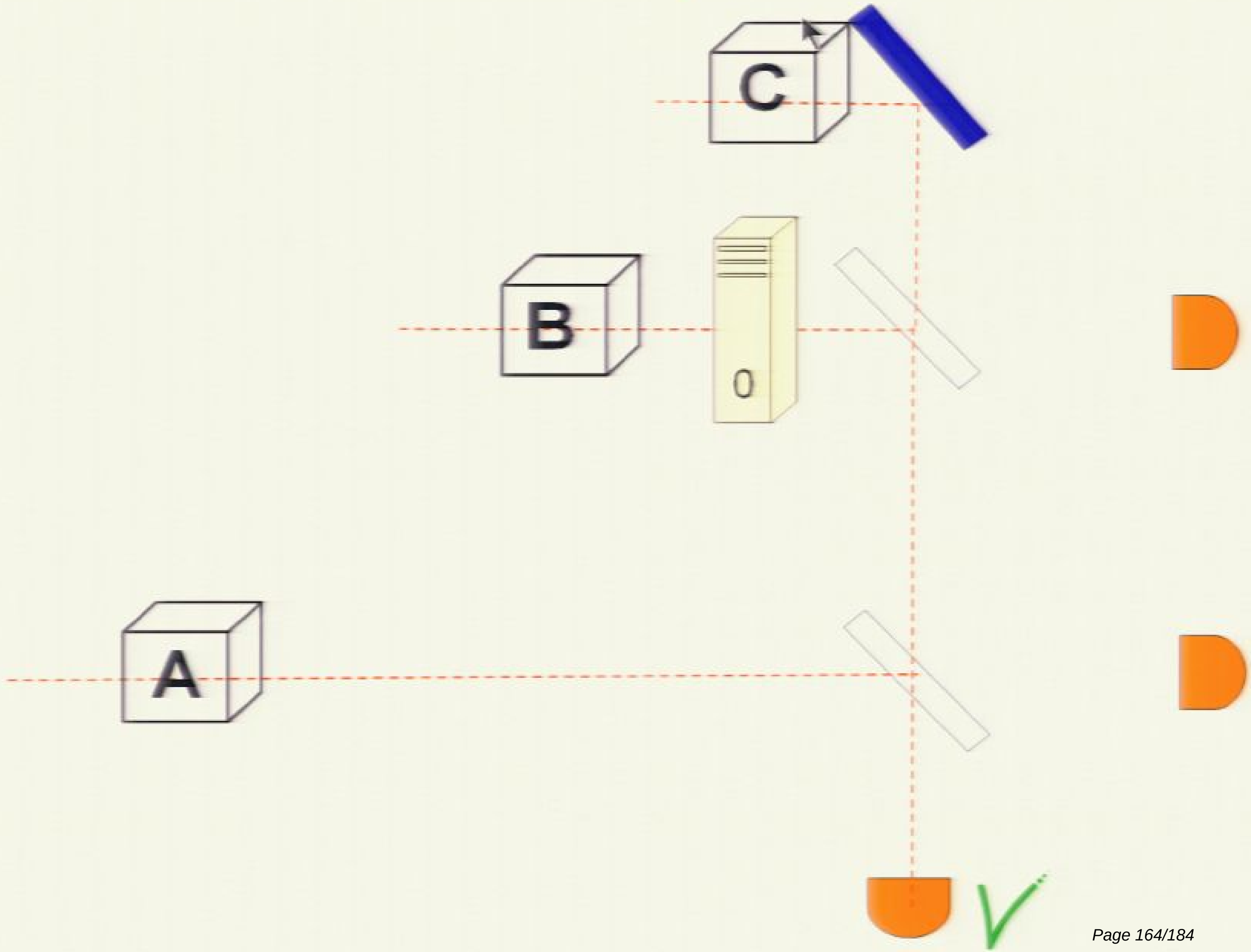

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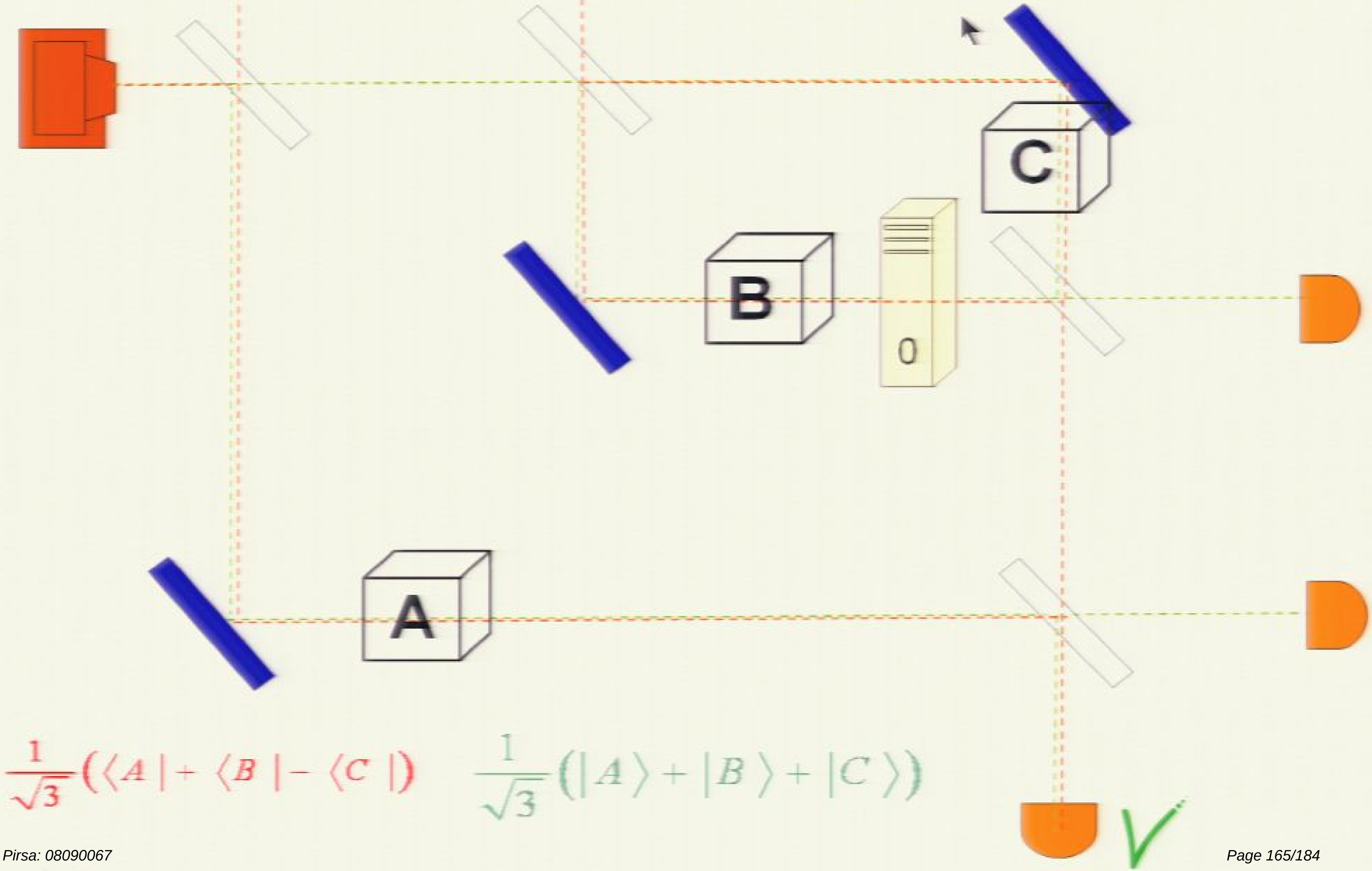

$$\frac{1}{\sqrt{3}}\left(\langle A| + \langle B| - \langle C|\right) \qquad \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle + |C\rangle\right)$$

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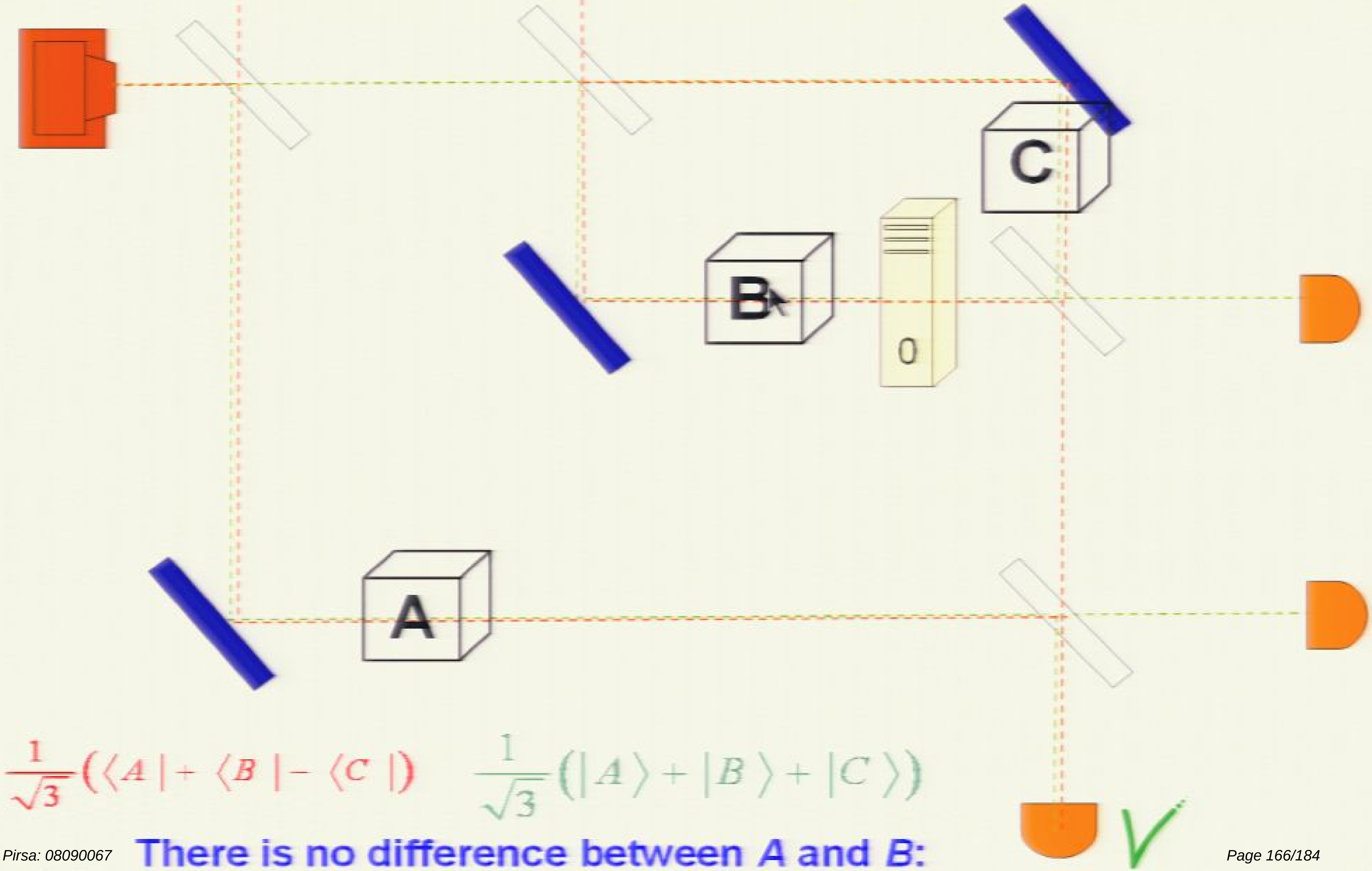

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There is no difference between *A* and *B*:

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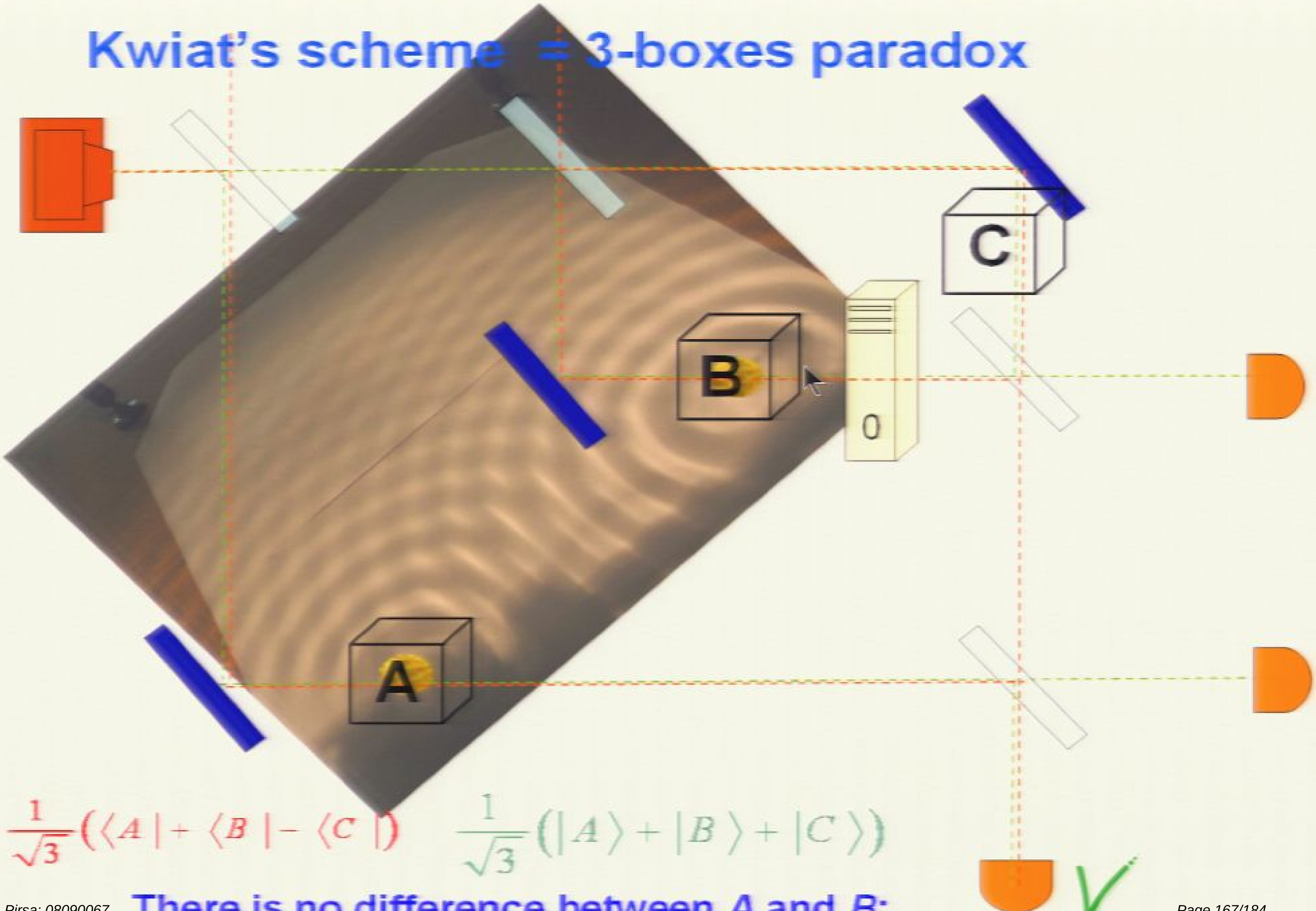

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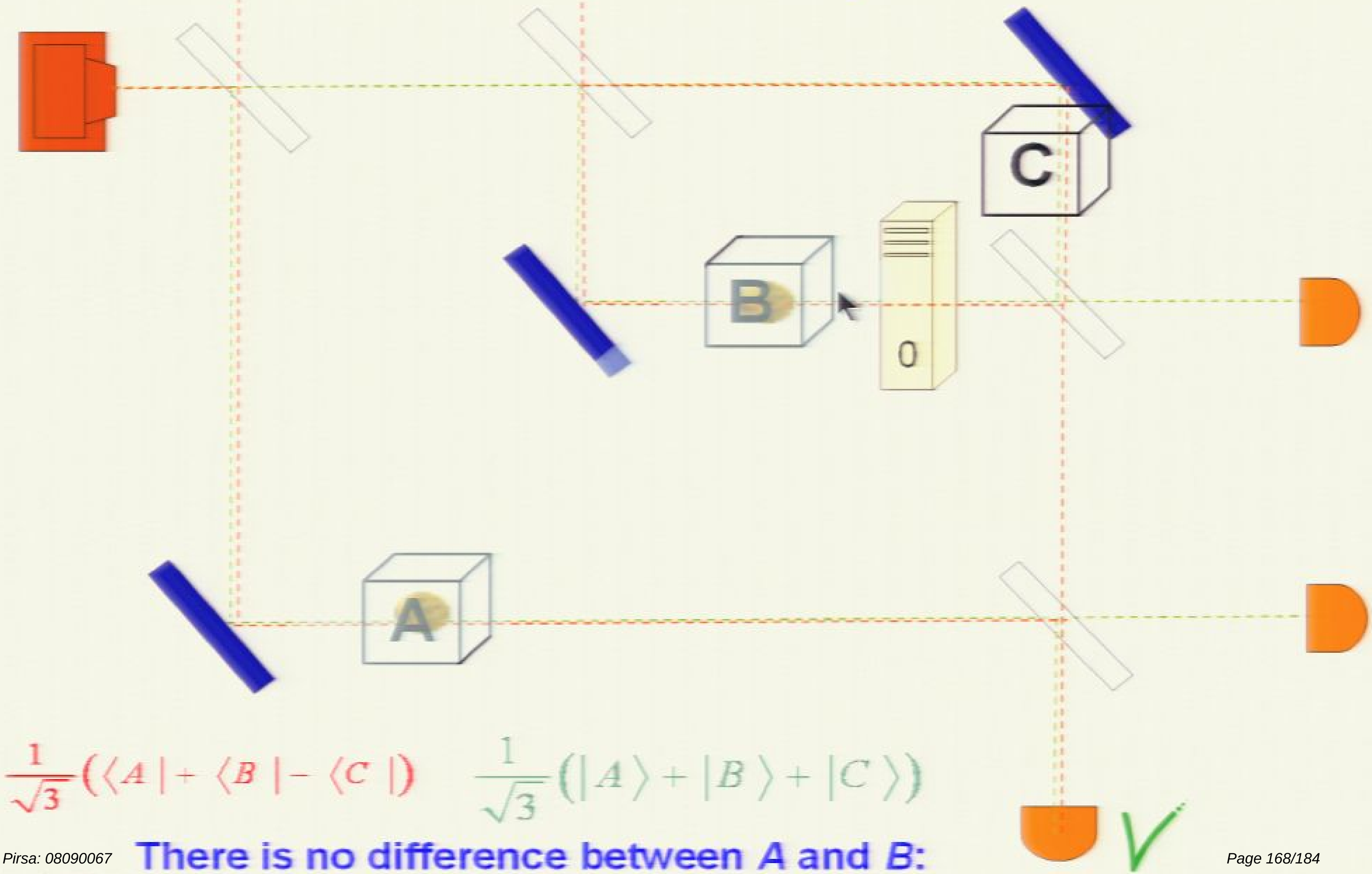

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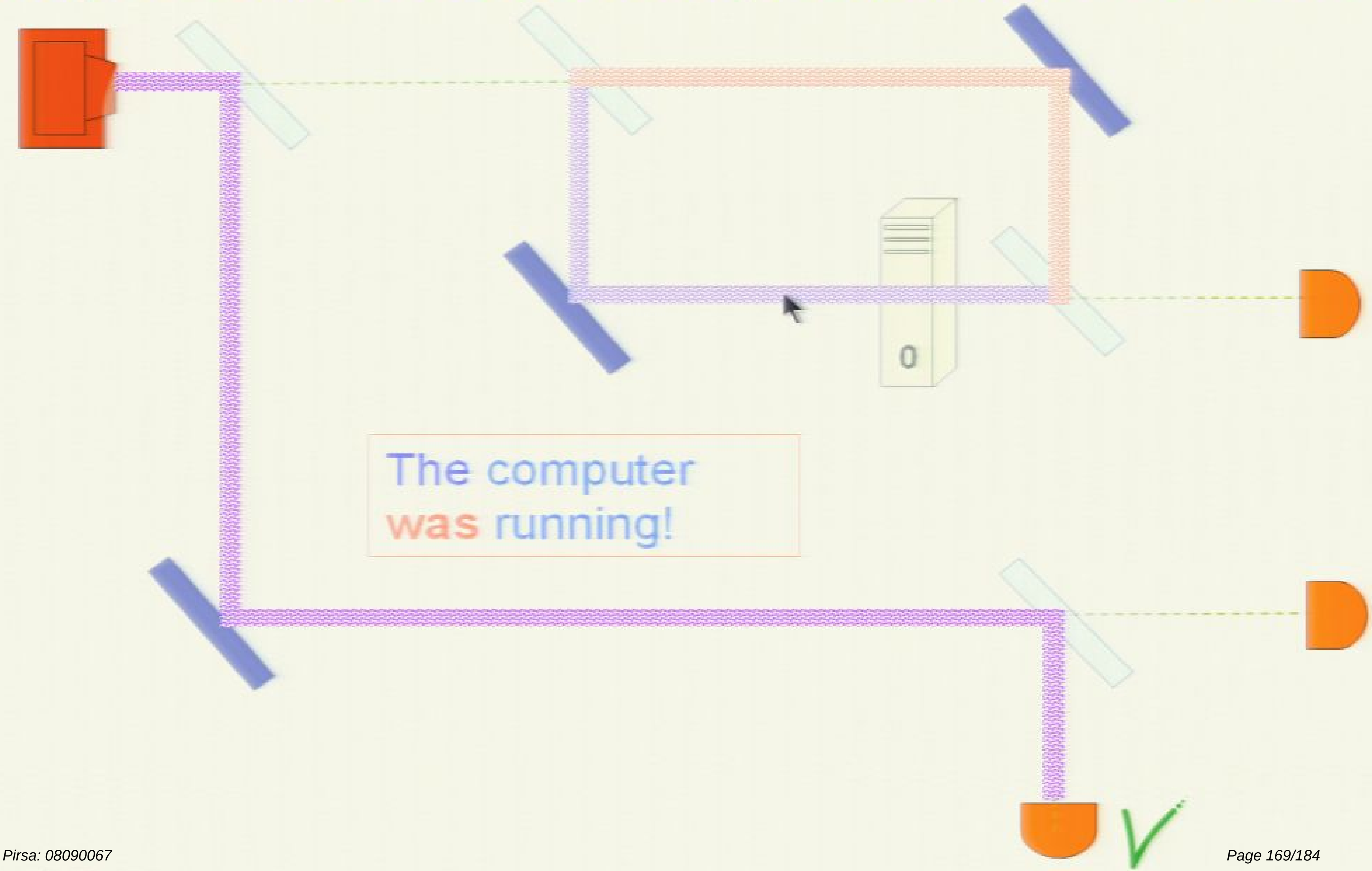
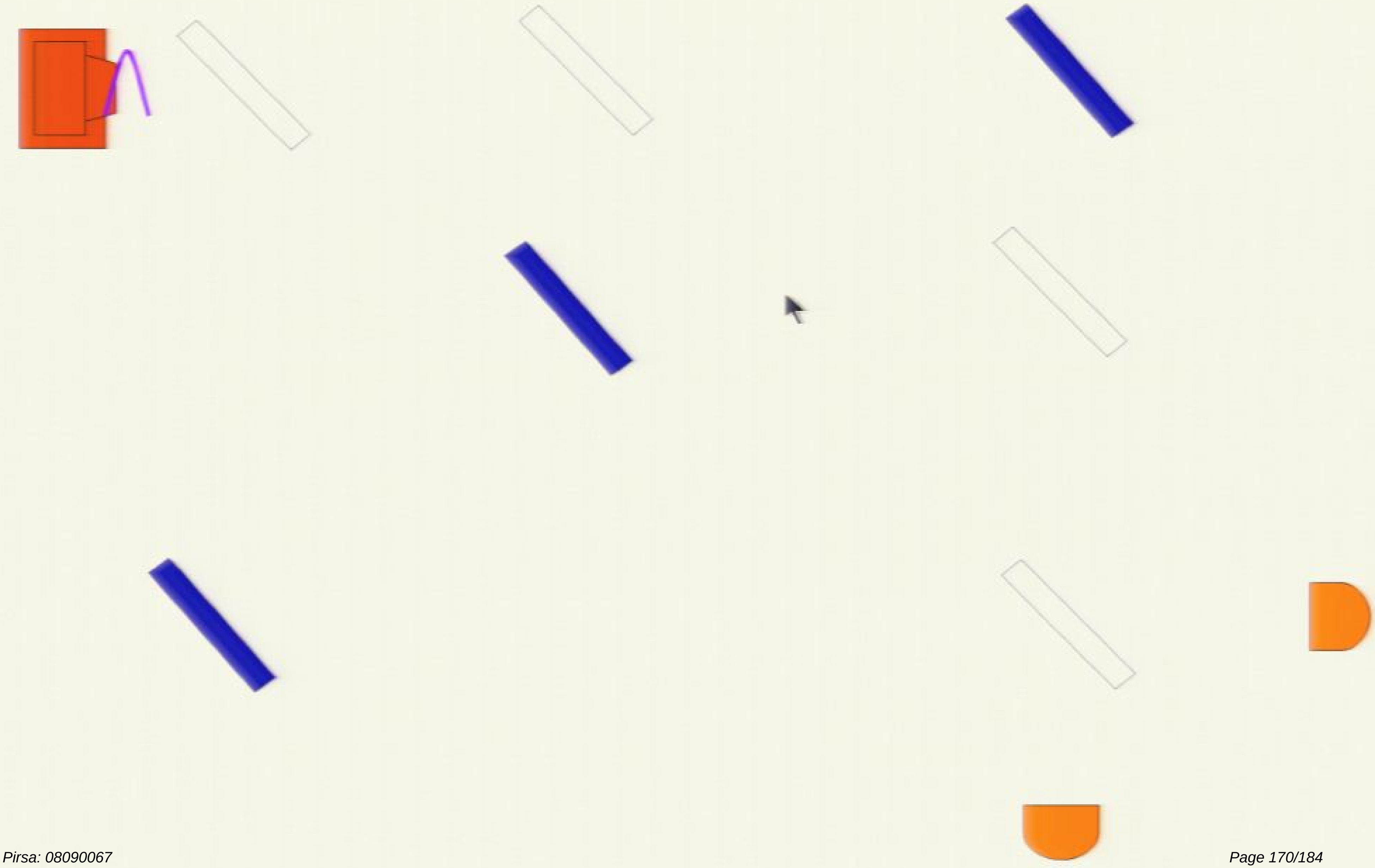


Where is the pre- and post-selected particle?



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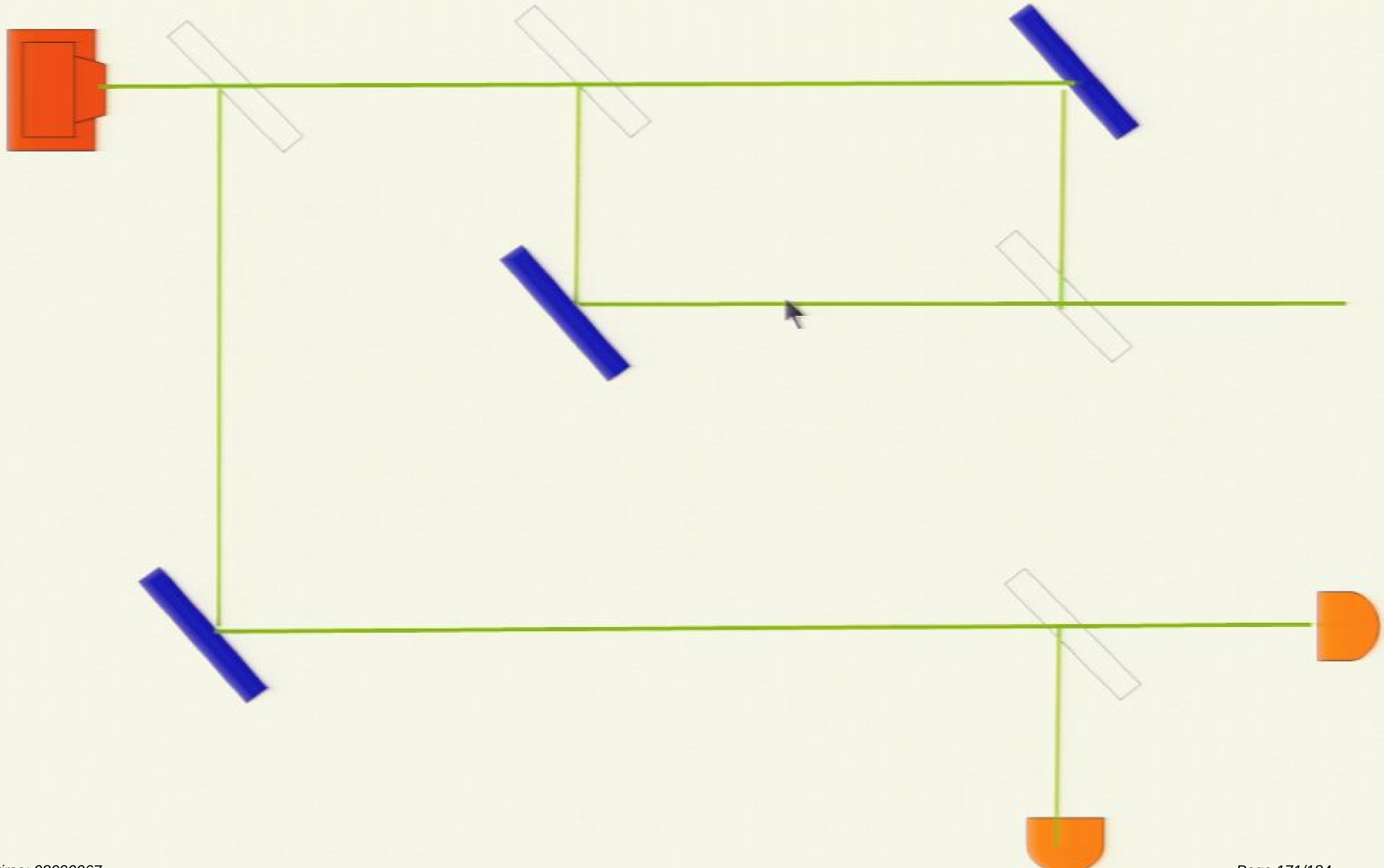

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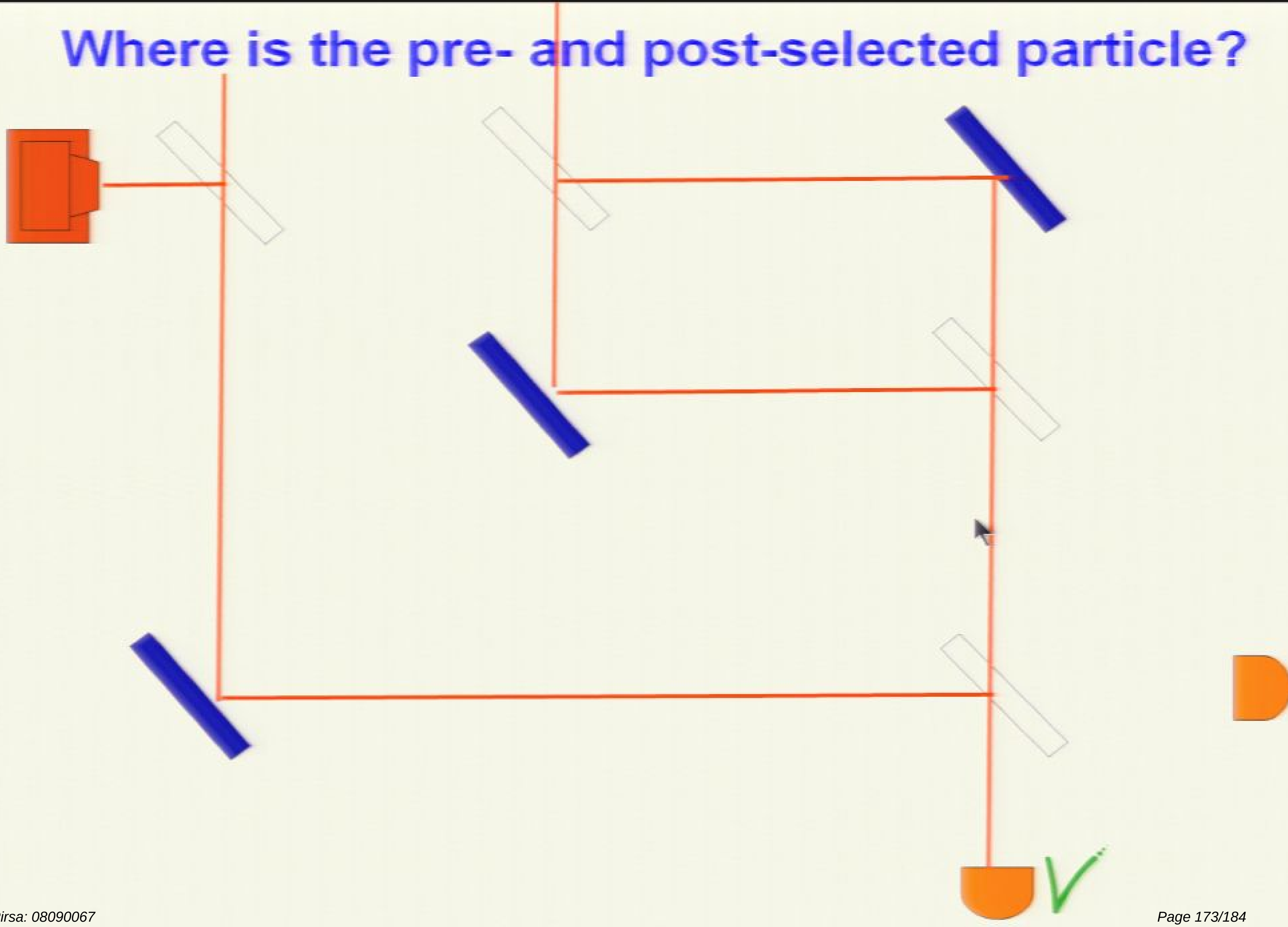

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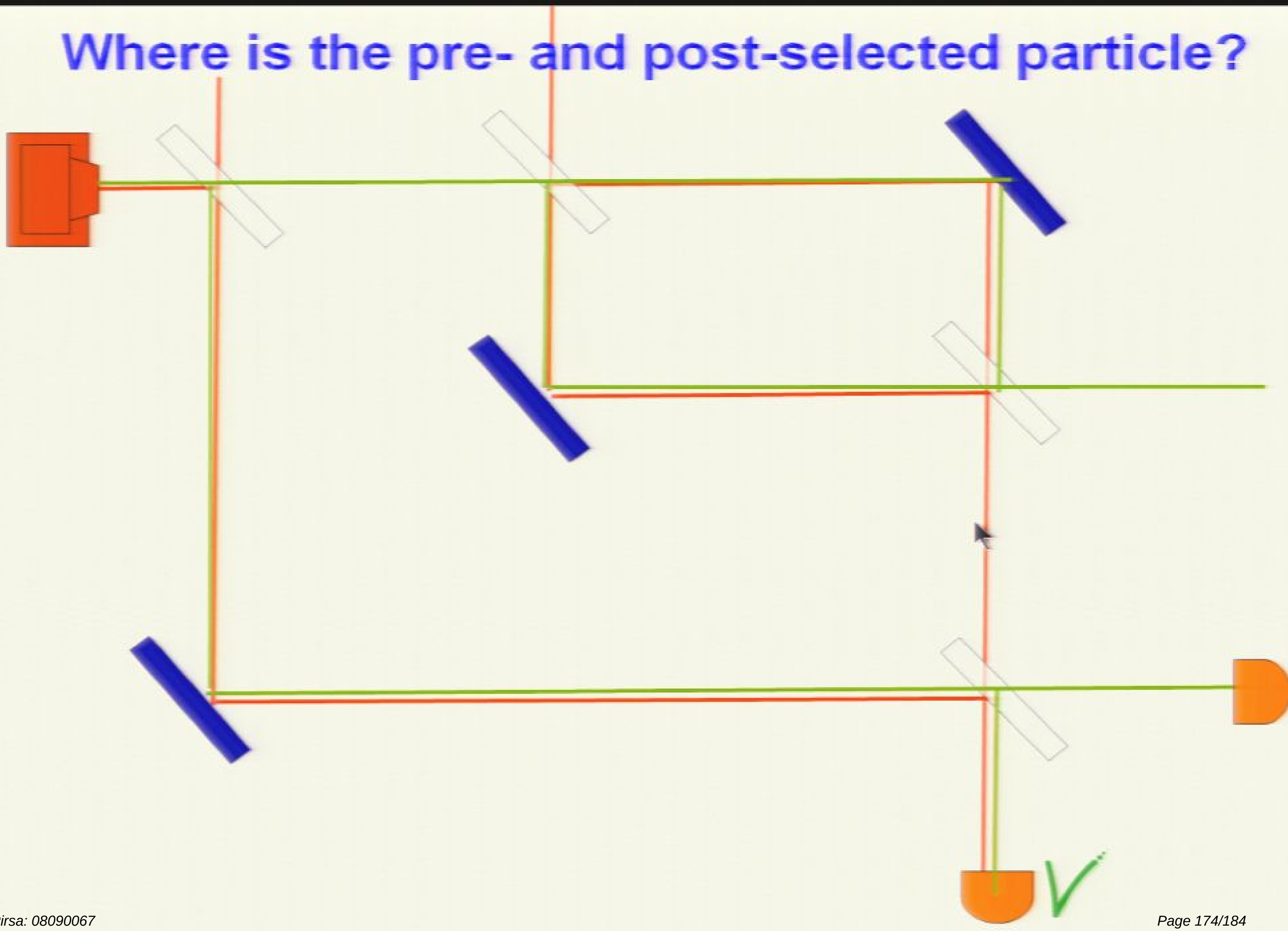

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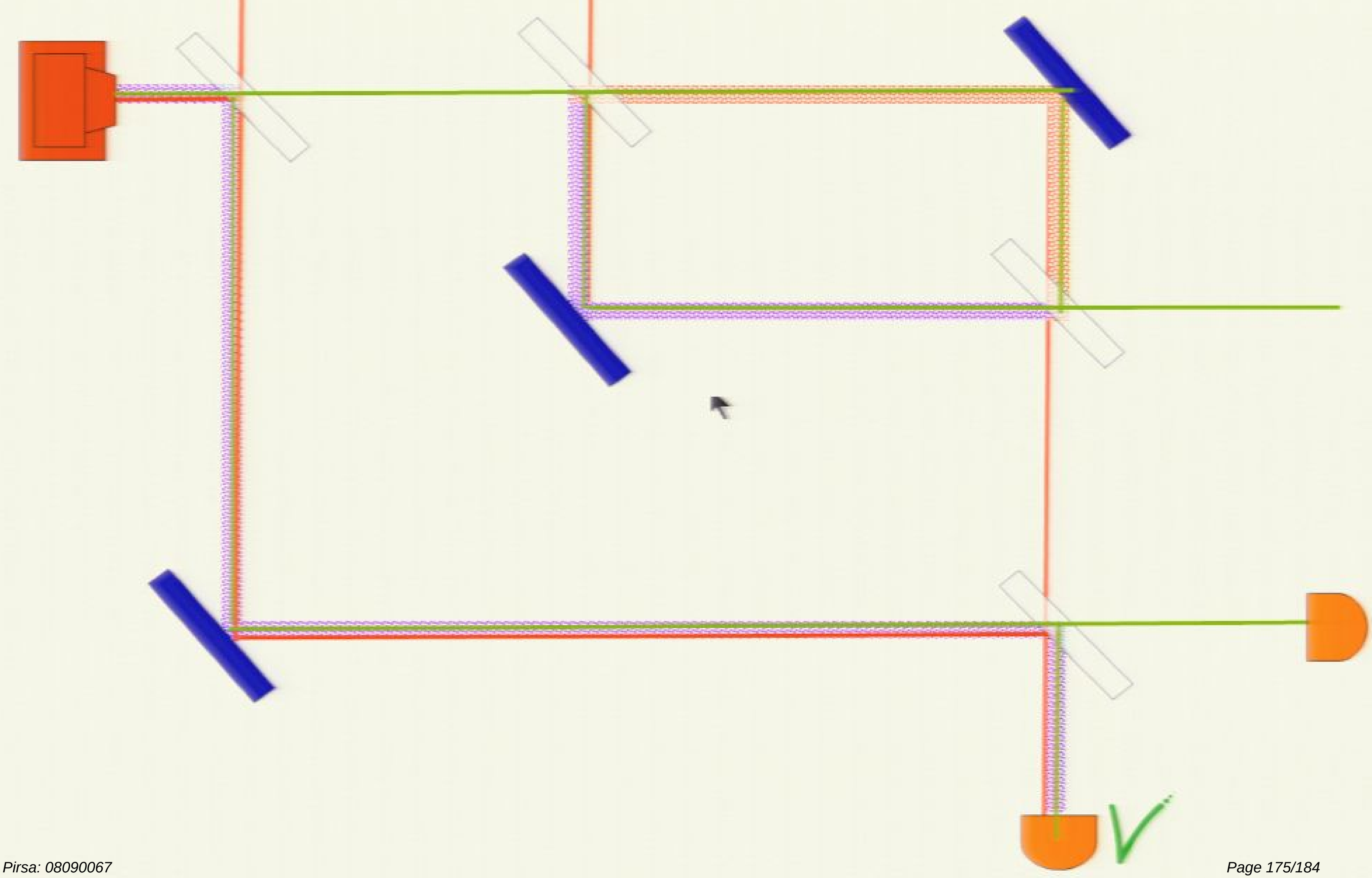

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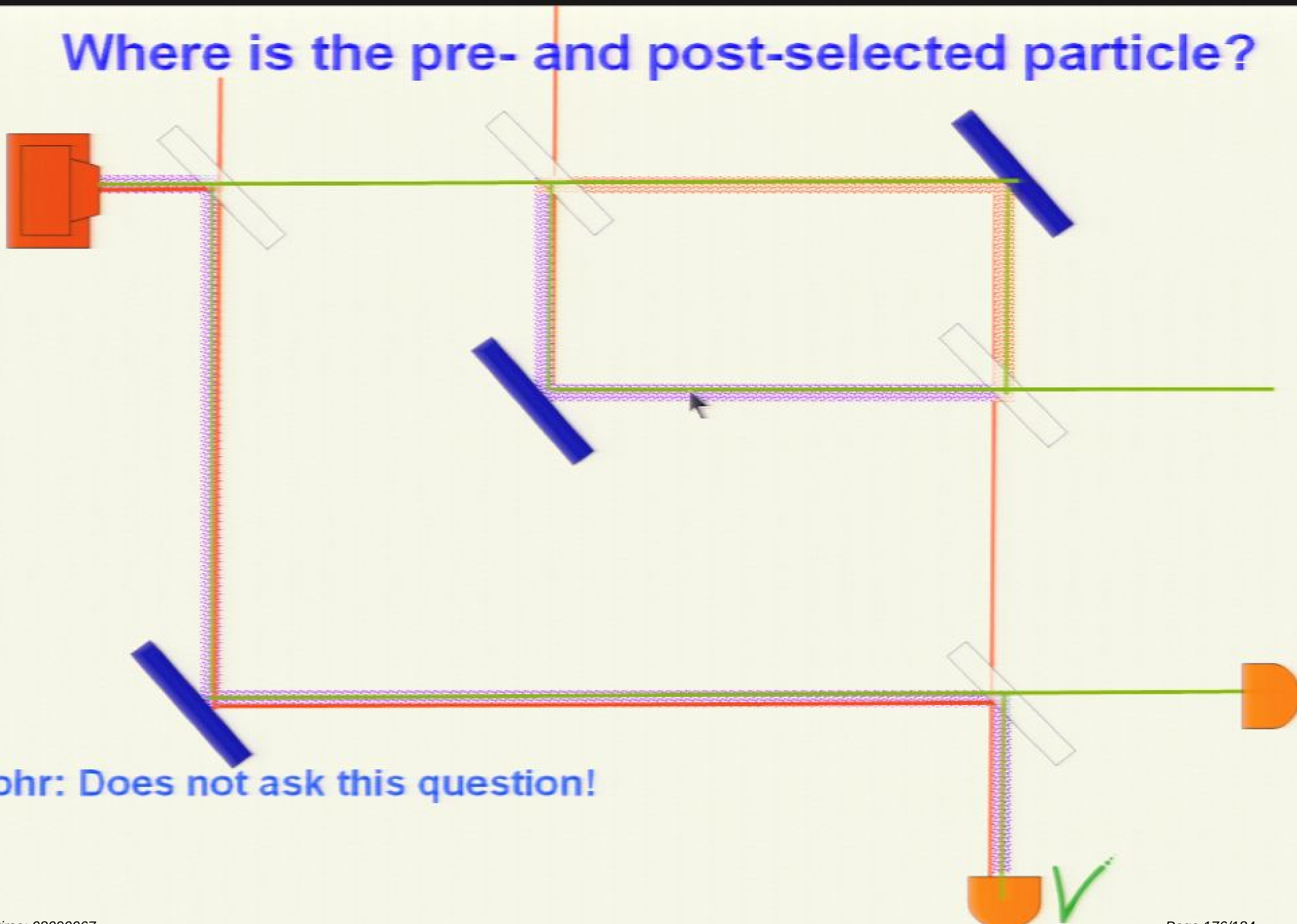

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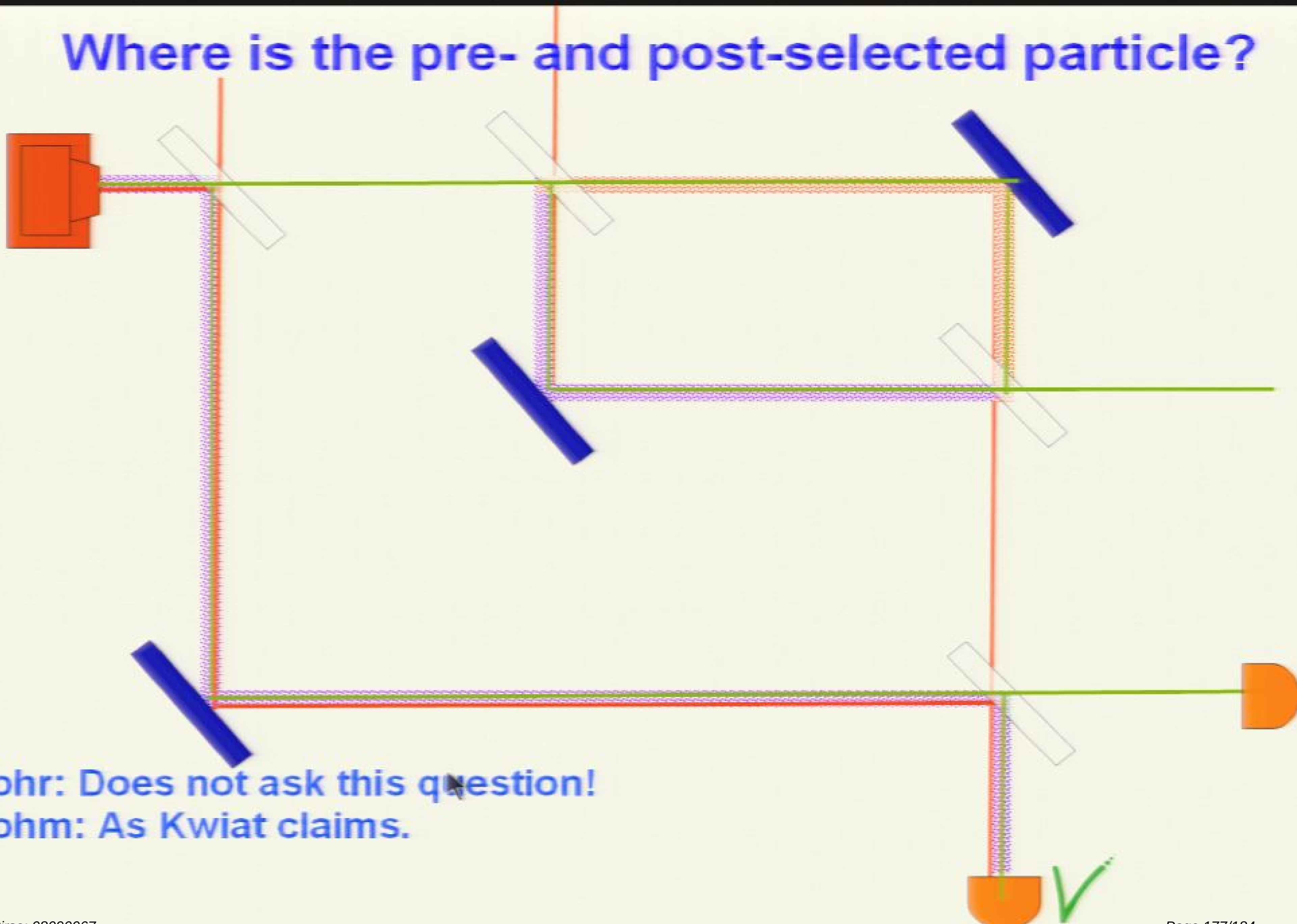

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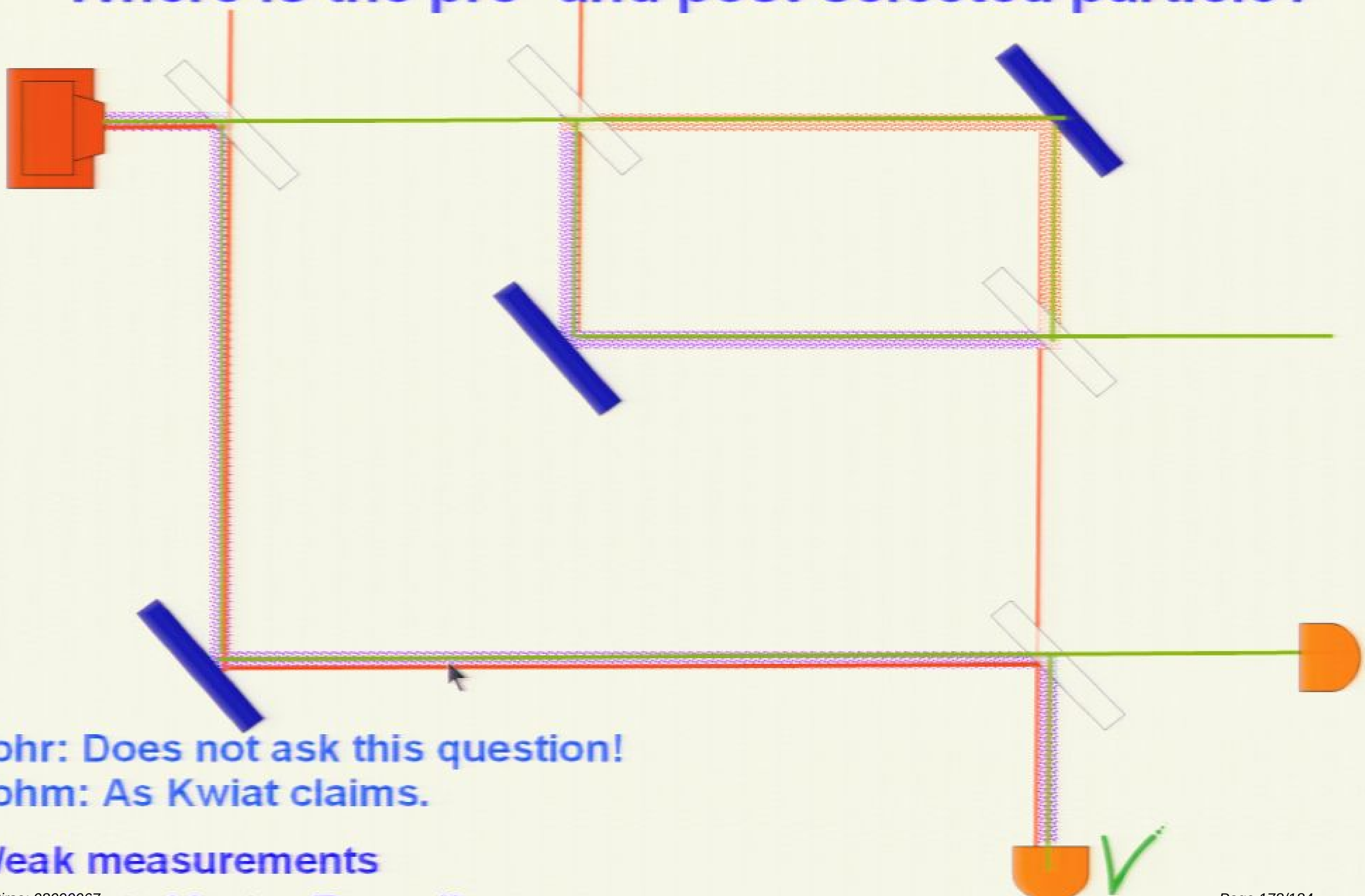

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Weak measurements
Two-State Vector Formalism

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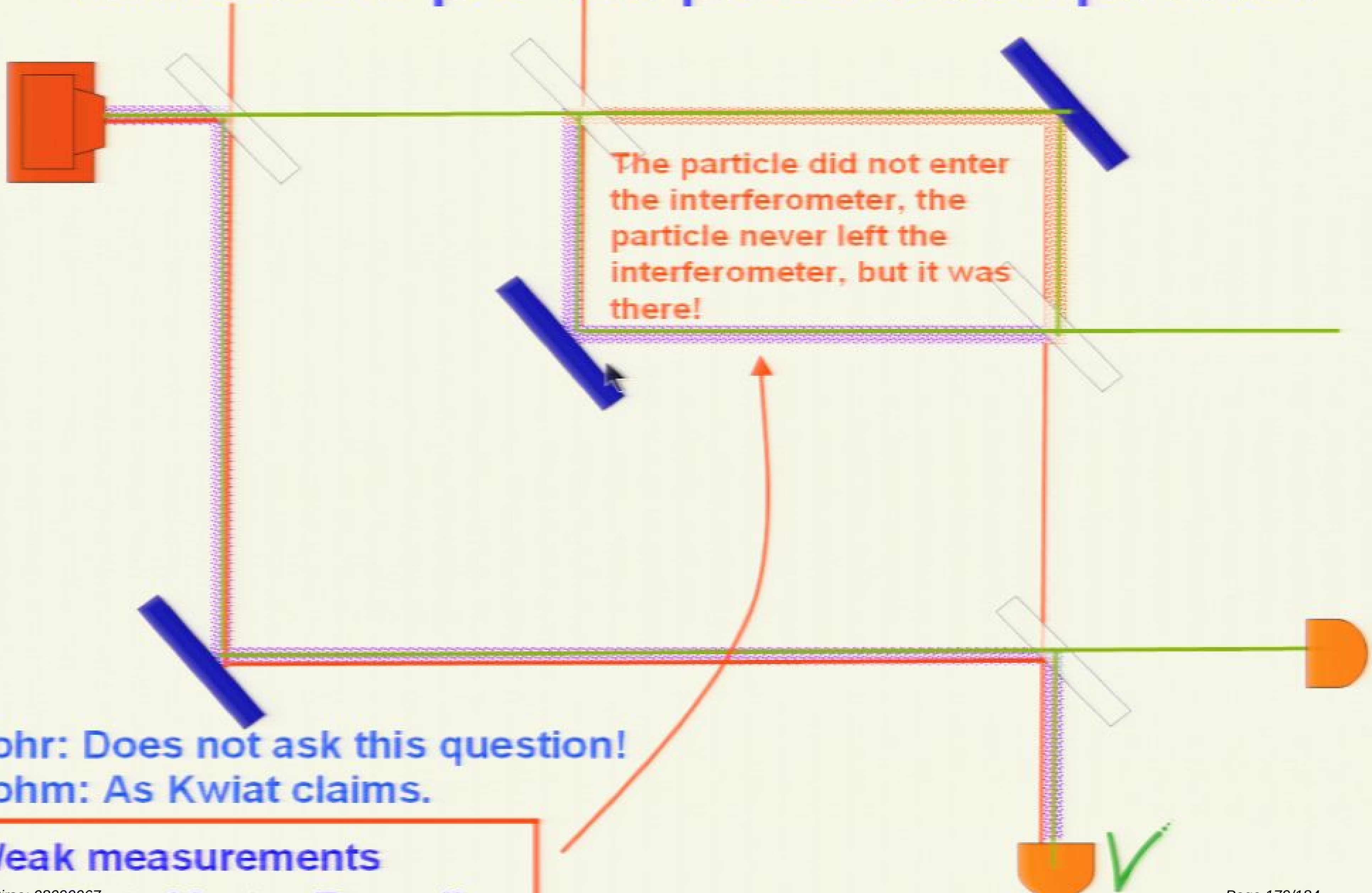

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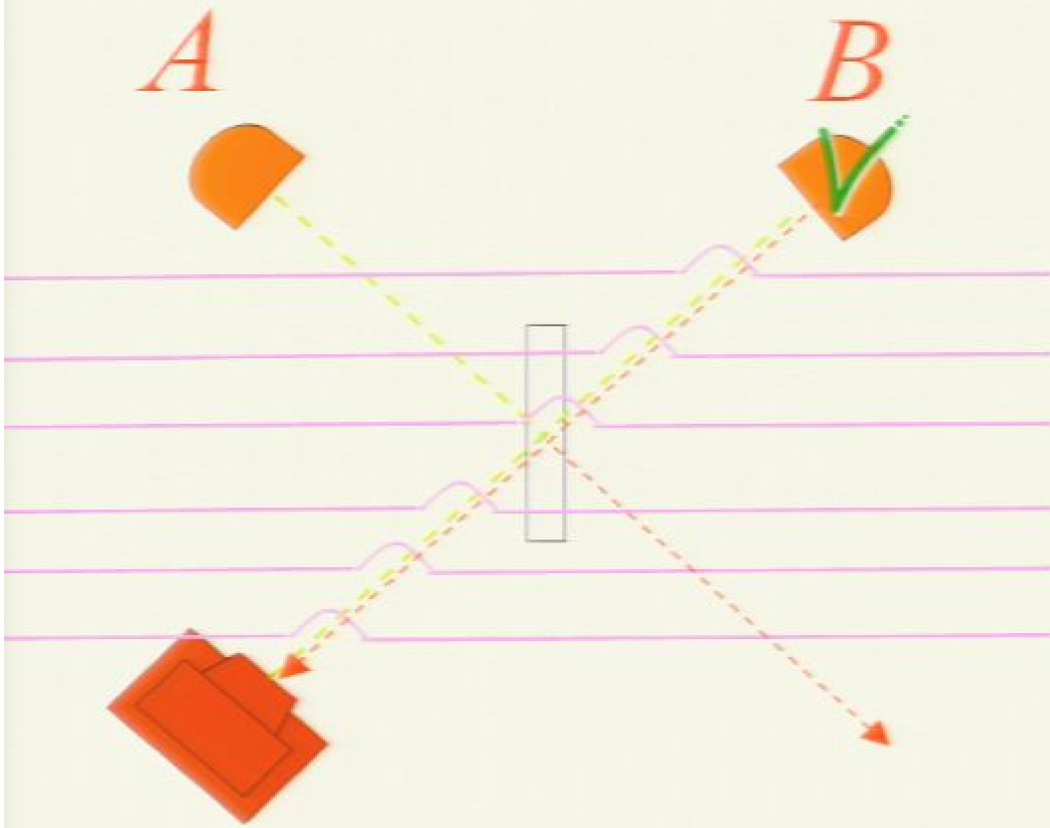
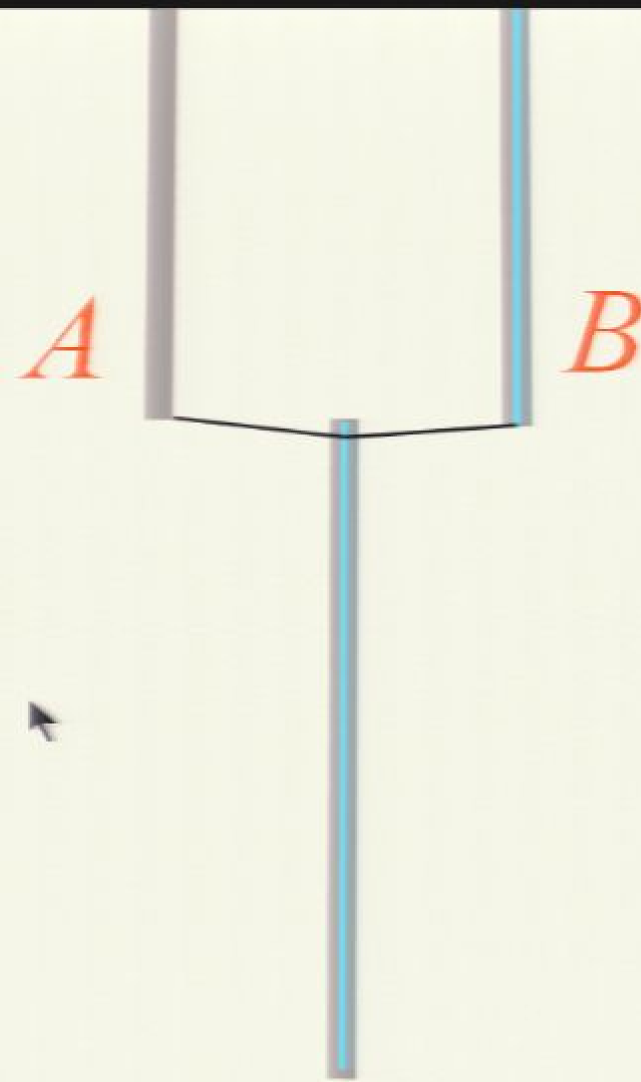
Weak measurements
Two-State Vector Formalism

When the worlds split?



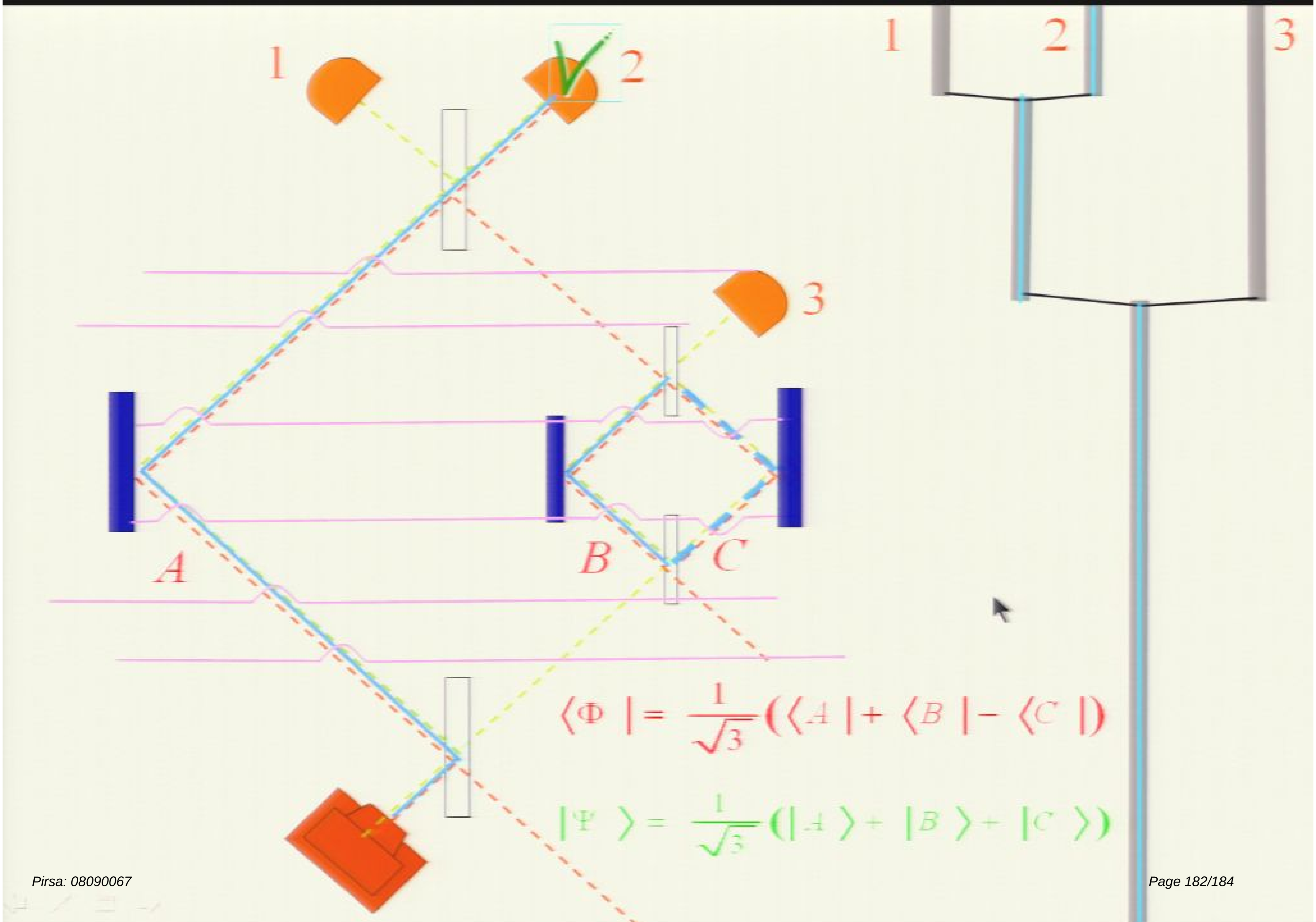


A world consist of:

•"classical" macroscopic objects rapidly measured by the environment,

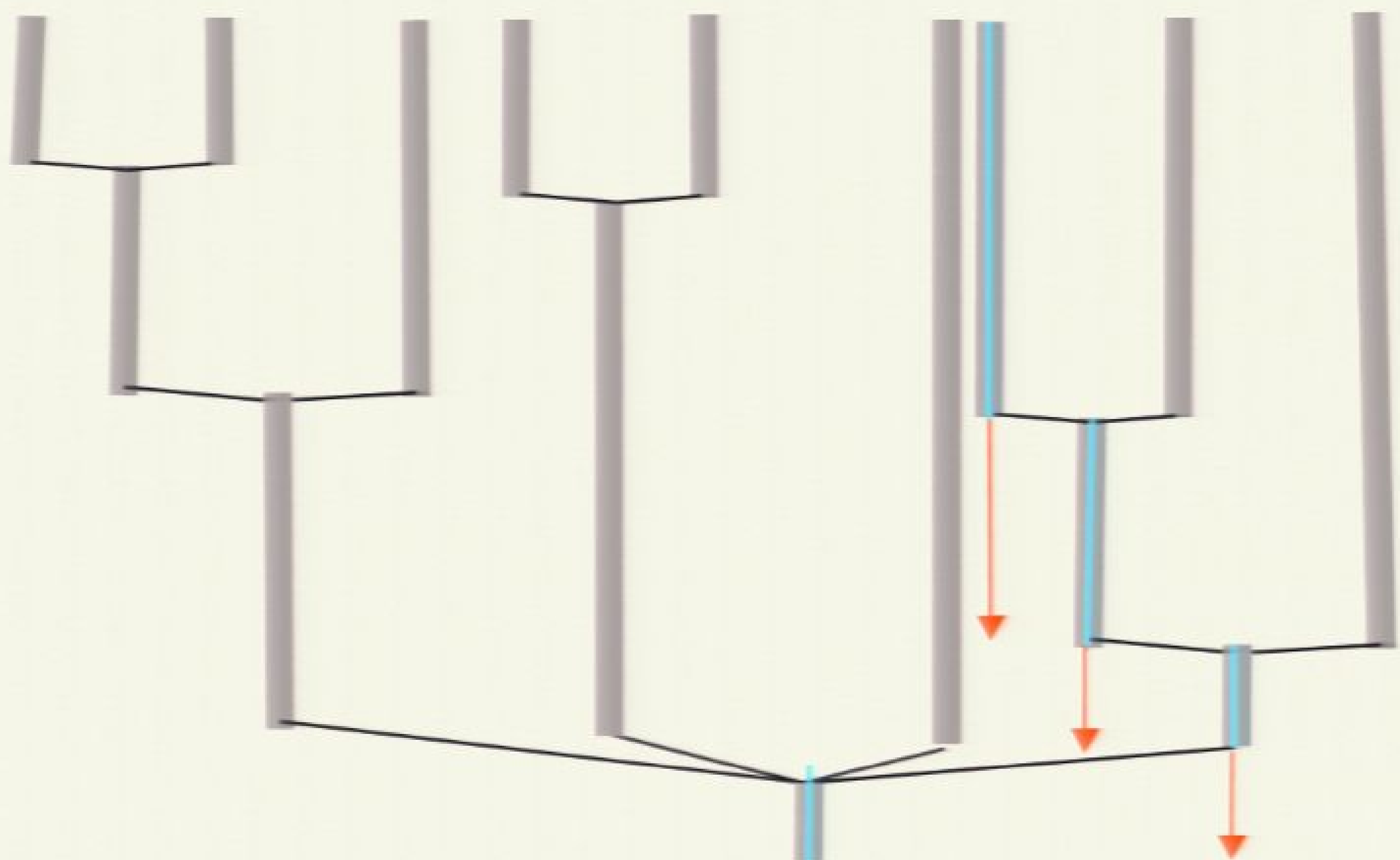
• quantum objects measured only occasionally (at world splitting events) which described by the two-state vectors,

• weakly coupled quantum objects


1
2
3
1
2
3
A
B
C
⟨Φ| = 1/√3 (⟨A| + ⟨B| − ⟨C|)
|Ψ⟩ = 1/√3 (|A⟩ + |B⟩ + |C⟩)

Forward evolving branch of the universal wave function does not describe all we should know about a world.

The (different) backward evolving state has to be added. to this world. It is created by the future measurement, so splitting of worlds happens in the future.



Conclusions

The TSV is a complete description of pre- and post-selected quantum systems in which forwards and backwards evolving states enter on equal footing.

Any system coupled weakly enough to pre- and post-selected quantum system "feels" weak values of quantum observables

Weak measurement procedure is an amplification scheme for observation of tiny effects

"Weak reality" leads to a modification of the branching picture of the MWI

The TSVF is another way to look at standard quantum mechanics, but it provides a convenient framework for its modification.