Title: The Theory of Duration and Clocks

Date: Sep 28, 2008 10:00 AM

URL: http://pirsa.org/08090064

Abstract: In 1898, PoincarÃ© identified two fundamental issues in the theory of time: 1)What is the basis for saying that a second today is the same as a second tomorrow? 2) How can one define simultaneity at spatially separated points? PoincarÃ© outlined the solution to the first problem { which amounts to a theory of duration { in his 1898 paper, and in 1905 he and Einstein simultaneously solved the second problem. Einstein\'s daring and elegant approach so gripped the imagination of theoreticians, especially after Minkowski\'s introduction of spacetime,that the definition of duration, and with it the theory of clocks, has received virtually no attention for over a century. This is a remarkable state of affairs and is a major cause of the conceptual confusion surrounding the problem of time in the canonical approach to the creation of a quantum theory of gravity. In my talk I shall develop PoincarÃ©\'s outline into a potentially definitive theory of duration and clocks.

THE THEORY OF DURATION AND CLOCKS

Julian Barbour

The Clock and the Quantum, Perimeter Institute, September 28–October 2, 2008

Classical nonrelativistic theory for a dynamically closed universe.

Classical relativistic theory on Wednesday at 2 pm

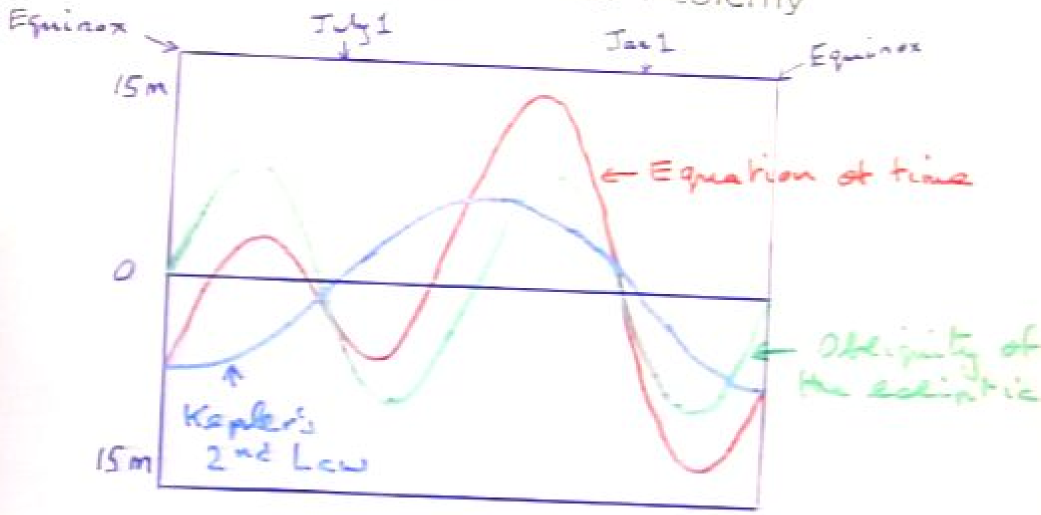
For quantum conjectures, see my book The End of Time or papers cited on my website www.platonia.com

OVERVIEW

1. Historical Antecedents
2. The Moon's Anomalous Acceleration
3. The Theory of Duration and Clocks
4. The Idea of Internal Time
5. Candidates for Internal Time
6. Tait's Inertial System
7. Conclusions

1. HISTORICAL ANTECEDENTS

1. The Equation of Time: Ptolemy



Newton: "It may be that there is no such thing as an equable motion whereby time may be accurately measured ... duration ought to be distinguished from what are only sensible measures thereof and from which we deduce it by means of the astronomical equation."

Feynman's quip

Carl Neumann's inertial clock (1870).

2. THE MOON'S ANOMALOUS ACCELERATION

Two possible causes of lunar anomaly:

Absorption of gravity or slowing of Earth's spin.

If latter, how is duration to be defined?

Define time so that $E =$ const for solar system.

Dynamical theory used to convert

labelling of time into **metric of time**.

1898: Poincaré's **two** problems with time:

Definition of **duration** and **simultaneity**.

Einstein and Minkowski's daring treatment of simultaneity eclipsed interest in duration.

Poincaré's misplaced conventionalism.

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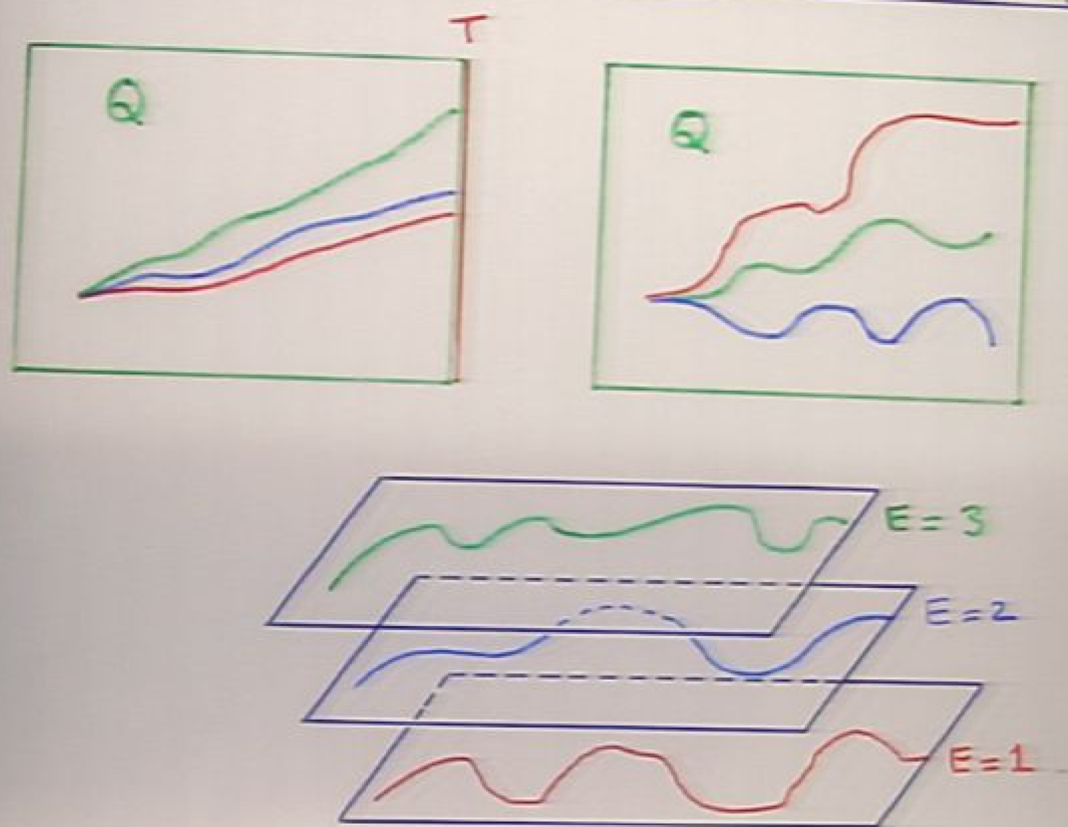
REPRESENTATIONS OF UNIVERSE'S HISTORY



History of the universe is not a curve traversed at some speed but simply a curve (in Q).

Each point on the curve is an entire

THE THEORY OF TIME AND CLOCKS

“It is utterly impossible to measure change by time.”

$$A_J = 2\int_A^B d\lambda \sqrt{(E-V)\sum_i \frac{m_i}{2}\frac{d\mathbf{x}_i}{d\lambda}\cdot\frac{d\mathbf{x}_i}{d\lambda}} \approx$$

$$\boxed{\sqrt{2}\int_A^B \sqrt{(E-V)\sum_i m_i\,\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}\ \text{(Jacobi 1843)}}$$

Lipschitz, Darboux

The **canonical momenta** $\mathbf{p}_i = \sqrt{\frac{E-V}{T}}\, m_i \frac{d\mathbf{x}_i}{d\lambda}$

$\approx \boxed{\sqrt{\frac{E-V}{2}}\,\frac{m_i\,\delta\mathbf{x}_i}{\sqrt{m_i\,\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}}}$ satisfy the **constraint**

‘Hypotenuse’

$\sum_i \frac{\mathbf{p}_i\cdot\mathbf{p}_i}{2m_i} = E-V$, and equations of motion are

$$\frac{d}{d\lambda}\left(\sqrt{\frac{E-V}{T}}\, m_i \frac{d\mathbf{x}_i}{d\lambda}\right) = -\sqrt{\frac{T}{E-V}}\frac{\partial V}{\partial \mathbf{x}_i}.$$

Choose λ to make $T = E - V$ and recover

‘Hypotenuse’

Newton's $m_i \frac{d^2\mathbf{x}_i}{dt^2} = -\frac{\partial V}{\partial \mathbf{x}_i}$ with

ephemeris time $\boxed{\delta t \approx \sqrt{\frac{\frac{1}{2}\sum m_i\,\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}{E-V}}}$

“Time is deduced from the

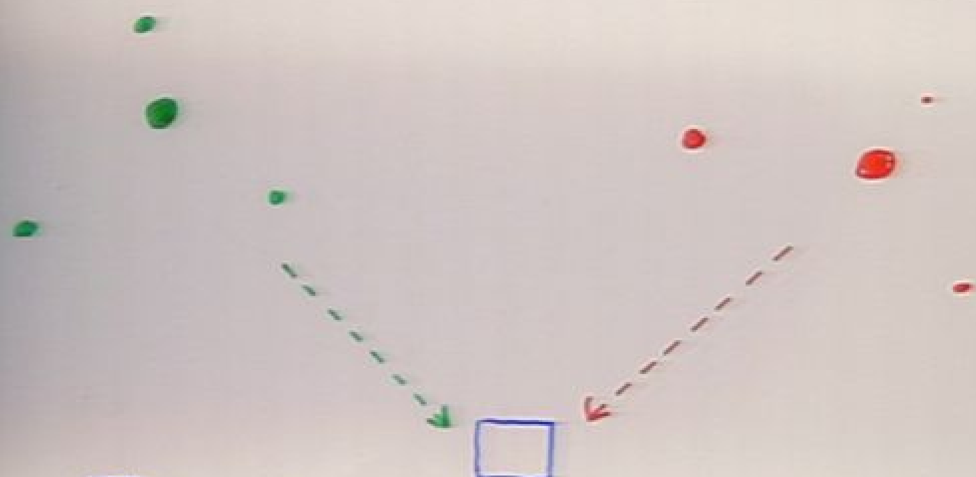
UNIQUE CRITERION OF GOOD CLOCKS

Bryce DeWitt's answer not satisfactory

First define duration (theoretician's task).

Then a clock is any dynamical system that marches in step with duration.

Any two clocks must march in step.



Time signals must march in step

Action $\delta A = 2\sqrt{(E-V)\sum_i \frac{m_i}{2}\delta \underline{x}_i \cdot \delta \underline{x}_i}$ is useless.

Ephemeris time $\delta t = \sqrt{\frac{\frac{1}{2}\sum_i m_i \delta \underline{x}_i \cdot \delta \underline{x}_i}{E-V}}$

A DYNAMICALLY CLOSED UNIVERSE IS ITS OWN UNIQUE CLOCK

$$\delta t = \sqrt{\frac{\frac{1}{2}\sum_i m_i \, \delta \underline{x}_i \cdot \delta \underline{x}_i}{E - V}}$$

Every mass in the universe contributes.

No truly isolated subsystems exist

Quantum processes must surely unfold with respect to this δt.

The theory of time presupposes space.

Galileo: "He that attempts natural philosophy without geometry is lost."

THE IDEA OF INTERNAL TIME

$$A_{\text{Newton}} = \int \mathrm{d}t\,(T - V),\ T = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i$$

Parametrized particle dynamics:

$$A_{\text{PPD}} = \int \mathrm{d}\lambda\,(\bar{T}/t' - t'V),\ \bar{T} = \frac{1}{2}\sum_{i=1}^{N} m_i \mathbf{x}_i{}' \cdot \mathbf{x}_i{}'$$

Invariant under reparametrization

$$\boxed{\lambda \to \lambda^*(\lambda).}$$

Canonical momenta are

$$p_t \equiv \frac{\partial L}{\partial t'} = \frac{-\bar{T}}{t'^2} - V,\ \ \mathbf{p}_i \equiv \frac{\partial L}{\partial \mathbf{x}_i{}'} = \frac{m_i \mathbf{x}_i{}'}{t'}.$$

Therefore

$$\boxed{\sum_{i=1}^{N} \frac{\mathbf{p}_i \cdot \mathbf{p}_i}{2m_i} + p_t + V \equiv 0,}$$

Arnowitt, Deser, and Misner: "In possessing this covariance, general relativity is precisely analogous to the parametrized form of dynamics in which the Hamiltonian and time are introduced as a conjugate pair of variables

Moment of inertia about centre of mass:

$$I = \sum_i m_i \mathbf{x}_i^{\mathrm{cm}} \cdot \mathbf{x}_i^{\mathrm{cm}} \equiv \sum_{i<j} \frac{m_i m_j}{M} r_{ij}^2, \quad M = \sum_i m_i$$

If V is homogeneous of degree k in the r_{ij}, then

$$\ddot{I} = 2\sum_i m_i \dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i + 2\sum_i m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i.$$

By Newton's second law $m_i \ddot{\mathbf{x}}_i = -\partial V/\partial \mathbf{x}_i$ and the definition of T,

$$\ddot{I} = 4T - 2\sum_i \mathbf{x}_i \cdot \frac{\partial V}{\partial \mathbf{x}_i}.$$

By Euler's theorem for homogeneous functions $\sum_i \mathbf{x}_i \cdot \partial V/\partial \mathbf{x}_i = kV$, so that, using $T = E - V$, we obtain the *Lagrange–Jacobi relation*:

$$\ddot{I} = 4(E - V) - 2kV.$$

For **celestial mechanics** $k = -1$ and $V < 0$, so that $\ddot{I} = 4E - 2V$ and $\ddot{I} > 0$ if $E \geq 0$. Thus I is concave upwards and must tend to infinity as $t \to +\infty$ and $t \to -\infty$. Any system with $E \geq 0$ is unstable.

Virial theorem: if a system has virialized, so that $\dot{I} \approx 0$, then $4E = (2k+4)V$, which establishes a relationship between the kinetic and potential energies in such a system.

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CANDIDATE INTERNAL TIMES

Inertial Motion: $V = 0$ and $E > 0$ since $T > 0$. Then $\ddot{I} = 4E$ and

$$\dot{I} = 4Et + D, \; I = 2Et^2 + Dt + F.$$

Thus $\dot{I} \propto t$ and is a good clock. I is not $\propto t$ (nonlinear and nonmonotonic).

Newtonian Gravity: If $E \geq 0$, then $\ddot{I} = 4E - 2V$, so $\dot{I}(t)$ is monotonic but not linear: it is a good *time label* but not a good clock (very like York time).

Scale Invariance: $k = -2$. Interactions are present, but the inertial relation holds $\ddot{I} = 4E$:

$$\dot{I} = 4Et + D, \; I = 2Et^2 + Dt + F.$$

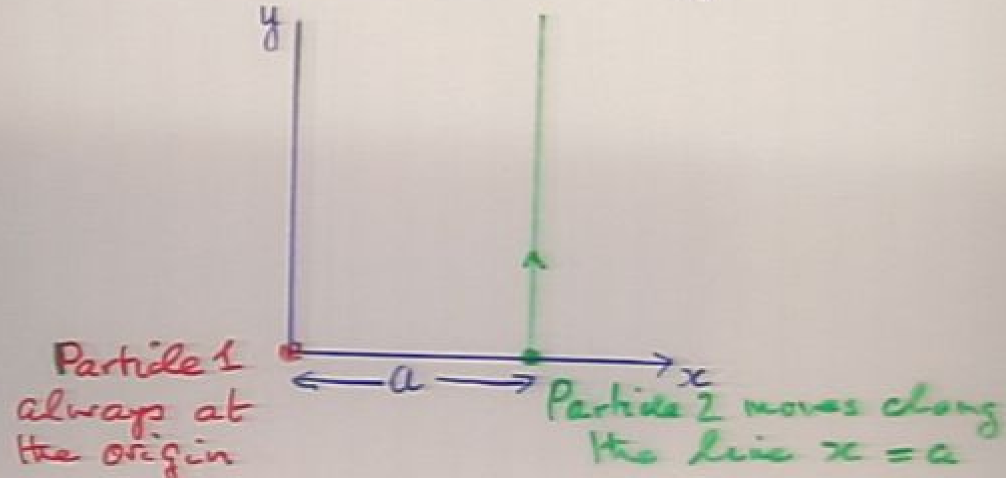
For any E, $\dot{I}$ is a good internal clock (non-trivial since interactions present).

If $E = 0, D \neq 0$, then $I = Dt + F$ becomes a good clock, but $\dot{I}$ stops ($\dot{I} = D$)!

If $E = 0$ and $D = 0$, then I stops too.

TAIT'S INERTIAL SYSTEM (1883)

Given 'snapshots' of 3 particles showing their separations r_{ij}, how many 'snapshots' are needed to confirm that they are moving inertially?



$$x_1 = y_1 = z_1 = 0$$

$$x_2 = a,\ y_2 = t,\ z_2 = 0$$

$$x_3 = \alpha + ut,\ y_3 = \beta + vt,\ z_3 = \gamma + wt$$

The problem contains 7 unknowns

Each snapshot contains 3 data, but the time at which it is taken is unknown. Thus, 2 data effectively obtained; 4 snapshots give 8 data. Inertial system will be determined with one confirmation of Newton's 1st Law.

For large N, there are
$6(N-2)+1 = 6N-11$ unknowns

Each snapshot gives $2+3(N-3) = 3N-7$ data

Two snapshots gives $3N-14$ data, which is 3 less than the number needed.

This '2 and a bit' problem is the evidence for absolute space

To be continued on Wednesday at 2pm

CONCLUSIONS

1. Instants of time are complete configurations of the universe.
2. Duration emerges from a timeless geodesic principle on the configuration space of the universe (if closed).
3. The idea of internal time is untenable.
4. The universe is its own unique clock.